Mass and Geometric Measurements of Binary Radio Pulsars

by

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Abstract

One of the primary, long-term goals in high-energy astrophysics is the measurement of macroscopic parameters that constrain the equation of state for compact stellar objects. For neutron stars, known to be composed of the densest matter in the Universe, measurements of their masses and sizes are of considerable importance due to the poorly understood processes that govern their interiors. Measurements of relativistic "post-Keplerian" effects in binary systems can be used to significantly constrain viable equations of state, test modern theories of gravitation, verify binary-evolution models that predict correlations between certain binary parameters, and determine the Galactic neutron-star mass distribution that is expected to reflect different supernovae mechanisms and evolutionary paths.

In this thesis, we use established pulsar-timing techniques to analyze signals from radio pulsars in 25 binary systems, as well as from one pulsar in a hierarchical triple system, in order to detect perturbations from Keplerian motion of the bodies. We characterize observed relativistic Shapiro timing delays to derive estimates of the component masses and inclination angles in 14 pulsar-binary systems, and measure a large number of secular variations due to kinematic, relativistic and/or third-body effects in the majority of binary systems studied here. We find a wide range of pulsar masses (m_p) , with values as low as $m_p = 1.18^{+0.10}_{-0.09} \text{ M}_{\odot}$ for PSR J1918–0642 and as high as $m_p = 1.928^{+0.017}_{-0.017} \text{ M}_{\odot}$ for PSR J1614–2230 (both 68.3% credibility), and make new detections of the Shapiro-delay signal in four binary systems for

the first time. In the relativistic PSR B1534+12 binary system, we derive an accurate and precise rate of geodetic precession of the pulsar-spin axis – due to secular variations of electromagnetic pulse structure – that is consistent with the prediction from general relativity. In the PSR B1620-26 triple system, we discuss ongoing efforts to simultaneously model both "inner" and "outer" orbits and tentatively measure secular variations of all "innerorbital" elements; we show that these variations are likely due to third-body interactions between the smaller orbit and outer companion, which can eventually be used to constrain orientation angles and possibly the pulsar mass in the near future.

Preface

All text, figures, and results presented in this dissertation are original products of the author, E. Fonseca, under the supervision of I. H. Stairs. Nonetheless, much of the pulsar data presented in this thesis were obtained in collaboration with many observational astronomers:

- In Chapter 2, the successful proposal for the ongoing NANOGrav "P2945" observing program at the Arecibo Observatory was written by E. Fonseca, using simulated results provided by J. A. Ellis, X. Siemens, J. Cordes, and D. R. Madison. The P2945 data presented in Figure 2.2 were collected by over 15 pulsar astronomers, including E. Fonseca. A list of the observers for the Arecibo and GBT timing data is given in the "Author Contributions" section of the study published by Arzoumanian et al. (2015b).
- In Chapter 3, all text, results and figures were generated by E. Fonseca
 the principal investigator of the project and accepted for publication in the Astrophysical Journal in September 2016 (see Fonseca et al.,
 2016). The interpretation of observed secular variations and analysis
 of Shapiro timing delays were performed by E. Fonseca, with guidance
 and insight provided by I. H. Stairs, D. J. Nice, T. T. Pennucci, J. A.
 Ellis, S. M. Ransom, and P. B. Demorest. The PAL2 Bayesian software discussed and used in this chapter was developed by J. A. Ellis.
 A portion of the data set was obtained for a targeted observing program devised and described by Pennucci (2015). The data presented

in this chapter were collected by over 15 pulsar astronomers, including E. Fonseca, as described in the "Author Contributions" section of the submitted study conducted by Fonseca et al. (2016). We are grateful for useful comments on the manuscript sent by C. Bassa, and for useful discussion with C. Ng and P. C. C. Freire.

- In Chapter 4, all GUPPI data collected after the start of the 2012 year were obtained through successful proposals led by E. Fonseca, and collected by E. Fonseca, I. H. Stairs and Z. Arzoumanian. Earlier observations with the Arecibo and NRAO facilities were performed by S. E. Thorsett and F. Camilo. Observations collected with the Effelsberg and Lovell telescopes were conducted by a large number of people over many years: the observations from the Effelsberg telescope were maintained, processed and provided by N. Caballero and M. Kramer; and data collected at Jodrell Bank were maintained, processed and provided by A. Lyne and B. Stappers.
- In Chapter 5, the majority of results and text in Sections 5.2 and 5.3 were first presented by Fonseca et al. (2014), which was based on a M. Sc. thesis written by E. Fonseca. However, the PUPPI observations presented in Section 5.4 successfully proposed for by E. Fonseca, I.H. Stairs and S. E. Thorsett are processed and analyzed for the first time in this dissertation.

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Chapter 1

Introduction

This dissertation is set to be completed nearly fifty years after the discovery of radio pulsars made by Jocelyn Bell-Burnell (and first summarized by Hewish et al., 1968). The discovery itself was a decisive moment in the history of physics and astronomy as it confirmed the existence of *neutron* stars – tiny, compact stars comprised mostly of neutrons that are supported against gravitational collapse by quantum-mechanical degeneracy pressure and nuclear interactions – and demonstrated that rotating neutron stars with beamed radiation along their magnetic poles could be observed at radio frequencies as pulsars. Over 2,500 pulsars are currently known to reside in the Galaxy, according to the catalog maintained by the Australia Telescope National Facility (ATNF; Manchester et al., 2005)¹; the number continues to grow as large-scale surveys search for them with increasing sensitivity (e.g. Manchester et al., 2001; Coenen et al., 2014). In the five decades since their discovery, pulsars have repeatedly been used to directly address open problems in physics and astronomy; they continue to yield game-changing results with exquisitely high precision. Indeed, two Nobel Prizes in physics have so far been awarded for results obtained by studying pulsars: one Prize² for the

¹http://www.atnf.csiro.au/people/pulsar/psrcat/

²http://www.nobelprize.org/nobel_prizes/physics/laureates/1974/

discovery of pulsars; and a second Prize³ for the confirmation that gravitational waves exist in Nature using pulsars in relativistic binary systems.

As shown in all subsequent chapters of this thesis, radio pulsars in gravitationally-bound orbital systems serve as probes of gravitation and binaryformation mechanisms. Many aspects regarding the evolution of their progenitor orbits can be inferred from precise measurements of the five basic Keplerian orbital parameters and the observed spin properties (see Lorimer, 2008, for a review). Pulsars within relativistic binary systems further exhibit a variety of "post-Keplerian" (PK) effects that can be used to measure additional parameters of each system, such as the binary-component masses and system orientation (Damour & Deruelle, 1985, 1986). PK measurements offer uniquely powerful constraints on the internal structure of ultra-compact objects (e.g. Lattimer & Prakash, 2004) and the inferred mass distribution of the neutron-star population (Thorsett & Chakrabarty, 1999; Özel et al., 2012; Kiziltan et al., 2013). Binary radio pulsars provide a desirable environment to test gravitational theory and understand late-stage stellar evolution with high precision.

The purpose of this doctoral thesis is to present results obtained from detailed analyses of radio pulsars in binary systems. In this introductory chapter, we present an overview of the current physical paradigm of pulsars, the phenomena that affect radio-frequency observations of pulsars, and general analysis techniques that are employed in all projects discussed in subsequent chapters. This context is necessary for the interpretation of results presented throughout the dissertation. In Section 1.1, we provide a brief summary of the current understanding of pulsars as neutron stars, general pulsar properties, and a brief overview of the evolution of pulsars in binary systems. In Section 1.3, we outline the procedure for modeling pulsar data that is used for all radio pulsars studied below. In Section 1.4, we explicitly discuss the models used to describe binary motion for eccentric

³http://www.nobelprize.org/nobel_prizes/physics/laureates/1993/

and nearly-circular pulsar-binary systems. In Section 1.2, we briefly discuss the observing systems used over the years to collect the pulsar data we use in the subsequent chapters of this dissertation. In Section 1.5, we discuss the relativistic and geometric phenomena that can be measured using binary pulsars and the information that can be learned from modeling these effects. In Section 1.6, we summarize the information presented in this chapter and briefly outline the content of the following chapters presented in this thesis.

1.1 The Physics and Phenomenology of Pulsars

All known pulsars in our Galaxy collectively span a wide range of pulse periods, with the smallest spin periods on the order of 1 ms. Observations of individual radio pulsars yield unique parameters that describe each neutron star's spin properties, the tenuous interstellar medium along the observer's line of sight to the pulsar, and any kinematic terms associated with secular or orbital motion. However, an evolving census of these parameters for the Galactic pulsar population has allowed for a qualitative picture of a pulsar to be formed with wide acceptance in the astrophysical community. In this section, we describe the general picture of a pulsar, expected spin properties of radio pulsars that will be exploited throughout this thesis, and an overview of pulsars in binary systems. We reserve discussion of the quantitative models that describe the various physical effects observable using pulsars for Section 1.3.

1.1.1 Pulsars are Neutron Stars

The theoretical discovery of neutron stars was first made by Baade & Zwicky (1934) shortly after the discovery of the neutron in 1932 by James Chadwick. Walter Baade and Fritz Zwitcky argued that supernova explosions generally

represent the transition from a fusion-powered star to a degenerate object that is mostly composed of neutrons. Pulsars therefore represent the remnants of old, massive stars that have aged through the branches of stellar evolution and ultimately suffered extensive (but not complete) gravitational collapse. The association of pulsars with neutron stars was first made in the discovery study undertaken by Hewish et al. (1968), though radiallypulsating white dwarfs could have also explained their observations at the time. Shortly after this discovery, the neutron-star picture was solidified with subsequent discoveries of radio pulsars at the heart of the Crab and Vela nebulae (Staelin & Reifenstein, 1968; Large et al., 1968), which both have pulse periods far smaller than the physically allowed pulsation rates for compact objects. The mechanisms proposed by Pacini (1967) and Gold (1968), which accounted for observed high-energy radiation at the center of supernova remnants as well as the range and regularity of pulse periods known at the time, quickly established pulsars are rotating neutron stars.

Initial calculations of the internal structure for neutron stars were first performed by Oppenheimer & Volkoff (1939), who considered a cold Fermi-Dirac gas while neglecting thermal and nuclear sources of pressure; they predicted that such objects must have masses around 0.7 M_{\odot} and be very small in size, with radii $R \sim 10$ km. Modern calculations that use numerical supercomputing and account for repulsive nuclear forces predict that neutron stars can have a maximum mass somewhat less than 3 M_{\odot}, depending on the non-nucleonic composition under consideration, before undergoing complete gravitational collapse into a black hole (e.g. Kiziltan et al., 2013). The maximum mass for neutron stars therefore depends on the equation of state (EOS) that governs their internal structure (e.g. Lattimer & Prakash, 2015). Indeed, one of the driving motivations for studying pulsars in binary systems is to determine the masses of the observed neutron stars, which can be used to place constraints on viable EOSs (e.g. Özel & Freire, 2016).

1.1.2 A "Lighthouse" in the (Cosmic) Darkness

The conventional model of a pulsar illustrates a highly-magnetized neutron star that emits cone-shaped beams of radiation at radio wavelengths along its magnetic poles. A small region centered on each polar cap is threaded by open magnetic-field lines which accelerates charged plasma away from the surface of the pulsar; the charged plasma produces photons during this acceleration into a conic area. The extreme electromagnetic fields around the vicinity of a neutron star produce a nebulous sphere of charged particles that co-rotates with the compact object, generally referred to as a "magnetosphere" (Goldreich & Julian, 1969; Sieber & Wielebinski, 1973). Despite nearly five decades of pulsar research, there is still no theoretical framework that fully describes the broad-band, frequency-dependent, luminous radio emission, though the strong external magnetic fields are expected to play an integral role in their production (e.g. Rankin, 2015, and references therein). Modern research in magnetosphere structure seeks to obtain solutions for non-idealized electromagnetic fields and radiation mechanisms (e.g. Kalapotharakos et al., 2012), while accounting for the implied changes of magnetospheric properties observed in "intermittent" pulsars (e.g. Kramer et al., 2006a; Lorimer et al., 2012).

Radio pulsars are observed to be highly-polarized sources, with large degrees of linearly-polarized intensity (L) and comparatively weaker circularlypolarized emission (V). Along with the Stokes Q and U polarization parameters of the observed radio emission, pulsar signals can be generally represented by the Stokes vector $\vec{S} = (I, Q, U, V)$, where $I \ge \sqrt{Q^2 + U^2 + V^2}$ is the total intensity of the pulsar signal and $L = \sqrt{Q^2 + U^2}$. If the entire polarization state is well measured, the polarization position angle (Ψ) can be computed to be

$$\Psi = \frac{1}{2}\arctan\left(\frac{U}{Q}\right).$$
(1.1)

and is a function of the phase of pulsar rotation (ϕ). Radhakrishnan & Cooke (1969) developed a model that explains smooth, swing-like variations in Ψ as a swing of the plane of linear polarization that is tied to the magnetic field lines, assuming that the magnetic field is dipolar. This model of polarization orientation is generally referred to as a *rotating vector model* (RVM), and the assumption of dipolar geometry relates Ψ to intrinsic geometric parameters that describe the relative orientation of the spin and magnetic axes of the pulsar:

$$\tan(\Psi - \Psi_0) = \frac{\sin\alpha\sin(\phi - \phi_0)}{\sin(\alpha + \beta)\cos\alpha - \cos(\alpha + \beta)\sin\alpha\cos(\phi - \phi_0)},$$
 (1.2)

where α is the misalignment angle between the spin and magnetic axes, β is the minimum angle between the magnetic axis and the line of sight, and (ϕ_0, Ψ_0) are fiducial values of ϕ and Ψ , respectively. In principle, polarization data can be used to constrain the geometry of pulsars; in practice, however, few radio pulsars exhibit values of Ψ as a function of ϕ that are well-modeled by Equation 1.2.

It is important to note that Equations 1.1 and 1.2 are derived under the assumption that Ψ is measured in the clockwise direction on the plane of the sky. This is inconsistent with the general convention, maintained by the International Astronomical Union (IAU), that position angles defined within the plane of the sky are measured in the counter-clockwise direction from celestial North (Everett & Weisberg, 2001). We nonetheless use these equations in this dissertation, while noting that any results obtained by using Equations 1.1 and 1.2 can be converted to the convention-standard values by applying the appropriate change in angular basis:

$$\alpha_{\rm IAU} = \pi - \alpha \tag{1.3}$$

$$\beta_{\rm IAU} = -\beta \tag{1.4}$$

In summary, the "cosmic lighthouse" model describes the observed polarized radiation from pulsars as a periodic "slice" of the radio-emission cone that occurs once per rotation. Naturally, a distant observer will see a radio pulse so long as the emission cone contains the observer's line of sight. The lighthouse model makes no assumptions about conic structure of the radio beam and, indeed, many different pulsars have complex (i.e. non-Gaussian, multiple-component) pulse profiles. In order to classify a spinning neutron star as a pulsar, the model does assume that the vector of spin angular momentum is misaligned with the axis of magnetic poles, i.e. $\alpha > 0$, which is necessary in order for the beamed radiation to appear as pulses to the distant observer. However, values of α can be quite small (e.g. Stairs et al., 1999).

1.1.3 Observed Characteristics

There are several important observational properties of radio pulsars that are considered to be general among the population and are well understood, despite the underlying uncertainty in neutron-star structure and the nature of the radio emission mechanism. For instance, the rotation of pulsars is observed to be remarkably stable; the regularity in pulsar rotation is maintained by the neutron star's large moment of inertia $I_{\rm rot} \sim 10^{45}$ g cm². Such rotational stability is a defining characteristic of radio pulsars and was immediately seen in PSR B1919+21⁴, the first pulsar that was discovered by

⁴Radio pulsars, designated with "PSR" that stands for <u>pulsating source of radio emission</u>, are formally given names based on their equatorial coordinates as measured at some reference epoch. Thus, PSR B1534+12 has a right ascension of $15^{h}34^{s}$ and a declination of 12 degrees North of the celestial equator, relative to the B1950 reference epoch. The J2000 name for the same pulsar is PSR J1537+1155.

Jocelyn Bell-Burnell, as a steady period of elapsed time between radio pulses $(P_{\rm s})$ that corresponds to the period of pulsar rotation. Current instrumentation at radio telescopes allows for precise measurements of each pulse's time of arrival (TOA), such that TOA uncertainties for comparatively bright pulsars are at the microsecond level. A classic example of such precision can be seen in PSR J0437-4715, the closest and brightest radio pulsar known, where a recent analysis by Reardon et al. (2016) used 5065 TOAs collected of 15 years and determined the spin frequency $\nu_{\rm s} = 1/P_{\rm s}$ to be

$\nu_{\rm s} = 173.6879458121843 \pm 0.0000000000005 \ {\rm s}^{-1}.$

The precision in current measurements of pulsar rotation is comparable to the atomic-transition clocks used for maintaining terrestrial timescales (e.g. Matsakis et al., 1997). In fact, there has been recent work undertaken to exploit the rotational stability of the most stable pulsars to develop and maintain an independent timescale based on pulsar-TOA measurements (e.g. Hobbs et al., 2012).

Radio pulsars are also observed to spin with intrinsically increasing spin periods over time. This physically corresponds to a loss of rotational kinetic energy $(E_{\rm rot})$ in pulsar rotation; if the pulsar has an angular rotation frequency $\Omega_{\rm s} = 2\pi/P_{\rm s}$, then

$$\dot{E}_{\rm rot} = \frac{dE_{\rm rot}}{dt} = \frac{d}{dt} \left(\frac{1}{2}I_{\rm rot}\Omega_{\rm s}^2\right) = I_{\rm rot}\Omega_{\rm s}\dot{\Omega}_{\rm s} = 4\pi^2 I_{\rm rot}\frac{\dot{P}_{\rm s}}{P_{\rm s}^3}.$$
 (1.5)

The loss of rotational energy in spinning neutron stars, generally referred to as "spin-down" in the literature, supplies power to their immediate environments and produces high-energy radiation. This was first proposed by Pacini (1967) as a mechanism for the observed high-energy radiation in the Crab nebula, within a year before the observational discovery of neutron stars made by Jocelyn Bell-Burnell. If one assumes that the spin-down is purely due to magnetic dipole radiation, then the strength of the magnetic field at the neutron-star surface can be determined up to a factor of $\sin \alpha$. For a radio pulsar with radius R = 10 km and $\alpha = \pi/2$, the surface field strength is

$$B(r = R) = \sqrt{\frac{3c^3}{8\pi^2} \frac{I_{\rm rot}}{R^6 \sin^2 \alpha} P_{\rm s} \dot{P}_{\rm s}}$$
(1.6)

$$= 3.29 \times 10^{19} \sqrt{P_{\rm s} \dot{P}_{\rm s}}$$
 Gauss. (1.7)

Figure 1.1 shows a plot of all known pulsars with observed $P_{\rm s}$ and $\dot{P}_{\rm s}$ values in the ATNF catalog. This figure is comparable to the Hertzsprung-Russell diagram for cluster stars, in that observed groupings of pulsars reflect underlying physical processes that contributed to their current states. "Normal" pulsars typically have rotation periods $P_{\rm s} \sim 1$ s and spin-down rates $\dot{P}_{\rm s} \sim 10^{-15}$, and are referred to as "normal" because they are generally observed to be isolated objects with no binary companions. A small number of exceptions exist, such as PSR J1740-3052 (e.g. Madsen et al., 2012), where slow-spinning pulsars are observed to orbit companions whose stellar evolution has had little or no impact on the spin evolution of the pulsar. As such, the rotation of normal pulsars is understood to have only been affected by spin-down through magnetic dipole radiation after the formative supernova event. Recent studies have shown that the magnetic field of normal pulsars can decay with timescales as short as 10^5 years (Igoshev & Popov, 2015), though this remains a controversial subject of study.

1.1.4 Binary Millisecond Pulsars

In stark contrast to the normal pulsars, millisecond pulsars (MSPs) – with exceptionally stable rotation periods $P_{\rm s} < 20$ ms and spin-down rates $\dot{P}_{\rm s} < 10^{-17}$ – are understood to be the end products of prolonged, stable mass transfer onto a neutron star from an evolving (sub)giant progenitor com-



Figure 1.1: $\dot{P}_{\rm s}$ versus $P_{\rm s}$ for all known radio pulsars with available spin and spin-down measurements. The blue lines correspond to values of both parameters that yield the denoted surface magnetic field (*B*) for a neutron star with $I_{\rm rot} = 10^{45}$ g cm². Red points denote 24 binary pulsars that are studied in Chapter 3 of this thesis, while the cyan and green points represent PSRs B1620-26 and B1534+12 that are studied in Chapters 4 and 5, respectively. As we discuss in Section 4.4, there are several different physical mechanisms (other than pure spin-down) that bias the observed value of $\dot{P}_{\rm s}$ for B1620-26.

panion. This long-term "recycling" process due to Roche-lobe overflow of the companion's outermost layer increases the neutron star's spin frequency while circularizing its orbit and reducing the magnetic-field strength over the course of accretion (e.g. Alpar et al., 1982). The post-accretion spin parameters are therefore changed from what they were when the neutron star was born; in Figure 1.1, this is generally seen as a migration from the larger population of normal pulsars to the smaller MSP population. The resultant companion object will likely be a low-mass white dwarf (WD), but, in principle, it is possible for the companion to be fully evaporated by the high-energy radiation from the spun-up neutron star (Ruderman et al., 1989). Furthermore, recent discoveries have implied that current eccentric MSP binaries may have once been a part of progenitor triple systems that have since undergone disruption to form their current states (e.g. Freire et al., 2011). Even with their binary origin, there are a number of pulsars within the MSP population that are observed to be isolated and believed to have undergone gravitational disruption of their progenitor orbits (Lorimer, 2008).

If there are no external perturbations from nearby stars, the dissipative tidal interactions due to stable mass transfer between components will govern the dynamical evolution of the orbit up to the termination of transfer (e.g. Phinney, 1992; Tauris & Savonije, 1999). Therefore, the post-accretion orbital elements will likely depend on several accretion-related factors. A notable prediction is a correlation between the resultant mass of the WD companion (m_c) and post-accretion orbital period (P_b) for "wide" binary systems (with $P_b > 1$ day; e.g. Tauris & Savonije, 1999), where numerical simulations of mass transfer showed that

$$m_{\rm c} = \left(\frac{P_{\rm b}}{b}\right)^{1/a} + c \tag{1.8}$$

where the values of (a, b, c) weakly depend on the metallicity of the progenitor companion. One can therefore compute an expected value of m_c if the companion is known or expected to currently be a low-mass WD. Evolutionary models can therefore be used in conjunction with pulsar-timing measurements to constrain additional parameters of interest, such as the pulsar mass (m_p) and the inclination of the orbit relative to the plane of the sky (i).

1.2 Data Acquisition and Instrumentation

In its raw form, radio-frequency radiation from pulsars is collected with a directional antenna and processed through a series of filters, amplifiers and mixers to produce a usable signal stream (e.g. Chapter 5 of Lorimer & Kramer, 2005). Hewish et al. (1968) used an array of dipole antennas and a pen-chart recorder to make the first (low-frequency, narrow-band) observations of PSR B1919+21. Modern observations of radio pulsars use large single-dish telescopes (or an array of small single-dish telescopes), receivers with large bandwidths and sophisticated computer hardware to make measurements with comparatively greater sensitivity and localization of the radio sources. The most common radio-pulsar observations use receivers centered on frequencies that collectively span the range 0.1-10 GHz.

Upon reception and initial processing of the radiation into voltage, the proceeding steps in real-time signal processing depend on the desired type of observation. For blind-search observations, where parts of the sky with no known pulsars are searched to potentially discover new sources, the mixed and amplified stream is processed through spectrometers and decomposed into "channelized" data contained within finite, contiguous frequency channels. The sub-banded signal is detected and recorded for offline search of pulsations using software such as the PRESTO suite.⁵ Pulsar-searching observations typically occur at low frequencies since pulsars are typically brighter and telescope beams are wider at low receiver frequencies. For observations of known pulsars, it is standard practice to further process the data stream in real time to remove the dispersive effect of the interstellar medium from the broadband signal and average successive, low-S/N pulses together, given a sufficiently accurate measure of the pulsar's apparent spin period at the time of observation. The resultant data are higher-S/N pulses within each frequency channel across the receiver band; each of these folded profiles,

⁵http://www.cv.nrao.edu/~sransom/presto/

comprised of a large number of individual radio pulses detected within each channel and within some chosen sub-integration timescale, is then recorded for subsequent offline analysis. In what follows below, we refer to the process of dispersion removal prior to the recording and integration of data as *de-dispersion*.

Any given TOA analyzed in this dissertation was obtained through one of two broadband de-dispersion techniques, depending on the type of pulsar signal processor (or *backend*) used at the time of observation. For recent generations of pulsar backends, the determination the full Stokes polarization vector is done in software using two input channels with orthogonal sense of polarization, regardless of the de-dispersion technique.⁶ In this section, we briefly describe the two de-dispersion methods used throughout this work.

1.2.1 Dispersion from the Interstellar Medium

The Galaxy is filled with a cold, generally tenuous collection of dust, ions, neutral gas and free electrons that make up the interstellar medium (ISM) between stars, with a higher concentration of material within the Galactic disk. A broadband electromagnetic signal that propagates through such a medium with free-electron number density n_e will experience a dispersion in the wave packet that culminates in a nonzero lag between signal components at different frequencies (e.g. Jackson, 1962).

Radio telescopes typically use a variety of receivers with different bandwidths that are tuned to different central frequencies; therefore, TOAs for the same pulse observed at different channel frequencies will typically have a lag between them due to the ISM along the line of sight between the observer and the pulsar. The time delay for ISM dispersion between signals recorded at two different receiver-channel frequencies f_1 and f_2 is given as $\Delta_{\rm DM} =$

⁶The earliest generations of pulsar backends required the use of additional hardware, such as adding and multiplying polarimeters (e.g. von Hoensbroech & Xilouris, 1996), in order to preserve signal-phase information and derive the components of the Stokes polarization vector.

 $C(f_1^{-2} - f_2^{-2}) \times \text{DM}$, where $C = (4.148808 \pm 0.000003) \times 10^3 \text{ MHz}^2 \text{ pc}^{-1} \text{ cm}^3$ s is a collection of physical constants⁷ and $\text{DM} = \int_0^d n_e(l) dl$ is the pulsar's *dispersion measure*, an electron column-density parameter. In practice, DM is usually measured relative to an infinite frequency, $f_2 = \infty$, so that the DM of a TOA measured at frequency $f = f_2$ can be rewritten to yield

$$\Delta_{\rm DM} = \frac{(C \times \rm DM)}{f^2}.$$
 (1.9)

If left uncorrected, pulse profiles from a pulsar with nonzero DM that are obtained at different observing frequencies will produce a smeared profile when they are averaged together. The DM is therefore another defining characteristic of radio pulsars that directly affects the precision to which TOAs can be measured.

1.2.2 Incoherent De-Dispersion

The oldest data that are presented in this dissertation, most of which were collected in the 1990s, were obtained using pulsar backends that employed the *incoherent de-dispersion* technique for correcting ISM-related delays between receiver channels (e.g. Lorimer & Kramer, 2005). This "brute-force" method of de-dispersion was typically employed using analogue filter bank spectrometers which first decomposed the mixed voltage signal into a number of spectral channels across the observed bandwidth. The incoherent de-dispersion technique uses the general form of Equation 1.9, $\Delta t = C \times \text{DM} \times (f_{\text{ref}}^{-2} - f_{\text{chan}}^{-2})$, where f_{ref} is taken to be the central frequency of the observed receiver band and f_{chan} is the frequency of the channel under consideration, to compute and directly apply the predicted timing delay. While this technique is effective and simple to implement, one of the major disadvantages of incoherent de-dispersion is the imperfect removal of DM timing delays within each indi-

⁷The uncertainty in C reflects the experimental uncertainties in the electron's charge and mass that are used in the computation of the dispersion constant.

vidual channel of finite width; a residual smearing of the pulse will occur in each channel and will therefore limit TOA precision depending on the channel width. The Mark III pulsar backend (Stinebring et al., 1992) is a classic example of a signal processor that used the incoherent de-dispersion method for DM-delay removal with channels widths of 10 MHz.

1.2.3 Coherent De-Dispersion

Modern pulsar instrumentation uses improved floating-point precision and bit-sampling technology to fully de-disperse the incoming radio signal in software, and in real time. This technique is generally referred to as *coherent dedispersion*. Hankins & Rickett (1975) showed that, in the frequency domain, the raw complex voltage detected by the radio telescope (V_f) is proportional to the intrinsic complex voltage produced at the point of emission from the pulsar $(V_{f,int})$; they are related to one another through the *transfer function* (H), which acts as a type of filter and fully characterizes the ISM-dispersion effect on the received broadband signal:

$$V_f(f + f_{cen}) = H(f + f_{cen})V_{f,int}(f + f_{cen}),$$
 (1.10)

where

$$H(f + f_{\text{cen}}) = \exp\left[i\frac{2\pi Cf^2}{(f + f_{\text{cen}})f_{\text{cen}}^2}\text{DM}\right].$$
 (1.11)

The inverse transfer function, H^{-1} , can therefore be computed and applied to V_f given some nominal value of DM to recover the original form of the pulsar signal prior to dispersion. Software filter banks are then applied to decompose the signal into spectral channels with the smearing of each pulse due to ISM dispersion completely removed.

The majority of data for all pulsars studied in this dissertation was collected using pulsar backends that employed (and still currently employ) the coherent de-dispersion method. The specific coherent-de-dispersion signal processors used are discussed in the following chapters.

1.3 Pulsar Timing, in a Nutshell

Radio pulsars are observed as faint, periodic flashes of electromagnetic radiation at radio frequencies. So long as the radio beam sweeps across their line of sight, an observer will see pulses from the rotating neutron star. After many subsequent observations of this (isolated) pulsar are made over a several-month timescale, the observer will eventually note that the pulsar's spin frequency (ν_s) is decreasing over time due to spin-down. Since spindown rates are typically very small, such an observer can construct a general "timing model" that describes the spin frequency of the isolated pulsar as a Taylor-expanded function of time (e.g. Lorimer & Kramer, 2005),

$$\nu_{\rm s}(t) = \nu_{\rm s}(t_0) + \dot{\nu}_{\rm s,0}(t-t_0) + \frac{1}{2}\ddot{\nu}_{\rm s,0}(t-t_0)^2 + \dots$$
(1.12)

where the time derivatives are evaluated at the reference epoch t_0 . In this idealized case of an isolated neutron star, the observer can gradually extend their data set and make refined measurements on the spin parameters, including higher-order time derivatives in their model when needed.

In practice, however, this process of modeling the observed spin behavior is nontrivial and ultimately more rewarding. For instance, single-dish radio telescopes on Earth do not reside in an inertial reference frame with respect to a given pulsar; the Earth is in orbit about the Solar System Barycentre (SSB) and also spins about its own precessing rotation axis at the decreasing sidereal rate. Also, a given pulsar is likely to have non-zero secular motion relative to the SSB due to asymmetries in the supernova explosion that formed the neutron star. Furthermore, pulsars in binary/triple systems will undergo orbital motion that induces regular Doppler shifts in the observed spin frequency. Finally, observations of pulsars made at different telescopereceiver frequencies will not record identical TOAs for the same pulses, but rather a lag that changes with the receiver frequency due to ISM dispersion.

The robust construction of a *timing model*⁸ for a radio pulsar therefore requires an explicit account of all relevant physical processes that affect the pulsar's observed spin behavior. This procedure, referred to as *pulsar timing* throughout pulsar literature and in this thesis, is the foundational method for studying radio pulsars. As discussed in this section and Section 1.4, pulsar timing yields direct measurements of parameters that describe spin properties, astrometry (i.e., position, proper motion, and parallax), frequencydependent effects associated with the tenuous interstellar medium (ISM), and orbital motion. An accurate pulsar-timing model can also be used to *fold* (i.e. shift to a common phase and average) observed pulses obtained over many days or years to form an integrated, high signal-to-noise pulse profile, which is useful for obtaining high-precision TOAs and resolving polarization properties. In this section, we briefly discuss the procedure for measuring TOAs as well as the theoretical models that describe many of the effects observable with pulsar timing. We reserve a full and separate discussion of binary timing models for Section 1.4, since the analysis of binary/triple motion of pulsars is the central theme of this dissertation.

1.3.1 TOA Estimation

Radio telescopes use hydrogen-maser atomic clocks at each site location in order to assign a time stamp at the beginning of each recorded data stream. For pulsars with pre-existing timing solutions, obtained from iterative analyses of the first TOAs measured after their discovery, a series of consecutive pulses are folded together modulo the computed pulse period in order to form a high-S/N pulse; its time stamp is the average of all time stamps of the middle of each individual sub-integration that formed the folded profile.

TOAs are then determined from a cross correlation between the observed

⁸Timing models are also referred to as "timing solutions" and "pulsar ephemerides", and we use these terms interchangeably throughout the text.

pulse profile (P) and a template profile $(S_{\rm P})$. A template profile can be determined from the folding of many previously-recorded pulses, or can be created as a multi-component model made with an appropriate number of Gaussian distributions (e.g. Kramer, 1994). In general, P and $S_{\rm P}$ are linearly related to each other,

$$P(t) = a + bS_{\rm P}(t - \tau) + n(t)$$
(1.13)

where $0 < t < P_{\rm s}$, *a* is a baseline offset, *b* is a scale factor, τ is the phase shift between *P* and *S*_P, in units of the local value of *P_s*, and *n(t)* is a random, time-dependent term that quantifies observed noise in the recorded data stream. The *n(t)* term is typically negligible for high S/N profiles, and so the cross correlation can be done in a straightforward manner to determine τ . The TOA is then defined to be the sum of the mid-point time of the observation with τ multiplied by the local value of *P_s*. In practice, the cross correlation is performed in the frequency domain using Fourier transforms for high-precision determination of TOAs (Taylor, 1992).

1.3.2 The Timing Model for Isolated Pulsars

TOAs are locally measured quantities, collectively referred to as topocentric TOAs (t_{top}) , that can be recorded with several telescopes located across the Earth. In order for Equation 1.12 to be applied successfully, the t_{top} measurements must be transformed to the equivalent times measured relative to the SSB. For isolated pulsars, this amounts to: applying appropriate clock corrections; accounting for orbital motion within the Solar System; modeling for transverse motion of the pulsar; and correcting delays between TOAs collected at different receiver frequencies. We discuss each of these time delays and their respective models in this subsection.

The following principles, corrections, physical mechanisms and fitting procedures described above are embodied in two standard pulsar-timing software packages: TEMPO⁹ and TEMPO2¹⁰ (Hobbs et al., 2006). We use both tools as indicated throughout this dissertation.

Local Clock Transformations

Since hydrogen-maser clocks at different telescopes maintain local observatory time, biases in parameter estimate will occur should TOAs collected from different observatories be combined without the appropriate clock corrections. TOAs are first corrected to the Coordinated Universal Time (UTC) maintained by the Global Positioning System (GPS), and then corrected further to Terrestrial Time, or $TT(BIPM)^{11}$, that is based on a number of maintained atomic clocks across the globe. Clock corrections (generally referred to as $\Delta_{\rm C}$ below) also account for the non-uniform rotation of the Earth and include leap seconds when needed.

Dynamical Effects in the Solar System

Time delays associated with orbital motion within the Solar System can be separated into three distinct components. The most prominent of these orbital effects is the Römer timing delay ($\Delta_{R_{\odot}}$), which describes the classical (i.e. non-relativistic) propagation delay experienced by an observer at different points in space over the course of the orbit. The Shapiro timing delay in the Solar System ($\Delta_{S_{\odot}}$), one of the four classic tests of GR first proposed by Shapiro (1964), accounts for general-relativistic propagation delays from varying spacetime curvature due to the presence of Solar System bodies near the observer's line of sight. The Einstein timing delay ($\Delta_{E_{\odot}}$) is a cumulative effect of general-relativistic gravitational redshift of the pulsar signal due to the Solar System bodies, as well as the special-relativistic time dilation due to Earth's motion. The functional form for these effects are given as (e.g.

⁹http://tempo.sourceforge.net/

¹⁰http://sourceforge.net/projects/tempo2/

¹¹http://www.bipm.org/en/bipm-services/timescales/
Lorimer & Kramer, 2005),

$$\Delta_{\rm R\odot} = -\frac{1}{c} (\vec{r}_{\rm SSB} + \vec{r}_{\rm obs}) \cdot \hat{s} \tag{1.14}$$

$$\Delta_{\rm S\odot} = -2\sum_{i} \frac{Gm_i}{c^3} \ln\left[\frac{\hat{s}\cdot\vec{r}_{i,\oplus}+r_{i,\oplus}}{\hat{s}\cdot\vec{r}_{i,\rm PSR}+r_{i,\rm PSR}}\right]$$
(1.15)

$$\Delta_{\rm E\odot} = \frac{1}{c^2} \sum_{i} \int \left(\frac{Gm_i}{r_{i,\bigoplus}} + \frac{1}{2} v_{\bigoplus}^2 \right) dt \tag{1.16}$$

and the various terms are summarized as follows. The \hat{s} unit points from the SSB in the direction of the pulsar. The $\vec{r}_{\rm SSB}$ vector points from the SSB to the center of mass of Earth, while the $\vec{r}_{\rm obs}$ vector points from the Earth's center of mass to the location of the observatory where TOAs were recorded. The \vec{r}_i terms denote relative vectors between the observatory on Earth and the *i*-th Solar System body $(\vec{r}_{i,\bigoplus})$, or between the *i*-th body and the pulsar $(\vec{r}_{i,\rm PSR})$. The m_i terms are the gravitational masses for the *i*-th body, \vec{v}_{\bigoplus} is the orbital velocity of the Earth, *G* is Newton's gravitational constant and *c* is the speed of light.

The body-specific terms in Equations 1.14-1.16 are independently computed using a Solar-system planetary ephemeris, maintained by the NASA Jet Propulsion Laboratory¹², that specifies the masses and past/future threedimensional locations for the Sun, Moon, and all eight planets. We used the DE421 or DE430 ephemerides as specified in each chapter. The masses and components of the various \vec{r} vectors are held fixed during the construction of a timing model, so that the only quantities left to determine are the components of the \hat{s} vector, which yield to the two-dimensional coordinates of the pulsar on the sky. Any significant transverse motion of the pulsar relative to the SSB will lead to a secular change in \hat{s} , and so the proper-motion terms can be directly measured should \hat{s} change over time.

¹²ssd.jpl.nasa.gov/eph_info.html

For sufficiently nearby pulsars, with distances d < 1 kpc, a radio-timing signature of the *astrometric parallax* ($\varpi = d^{-1}$) can be measured in TOA data. The periodic timing delay introduced by such a signature has the following functional form (Backer & Hellings, 1986),

$$\Delta_{\varpi} = -\frac{\varpi}{2c} [(\vec{r}_{\rm SSB} + \vec{r}_{\rm obs}) \times \hat{s}]^2 \tag{1.17}$$

where the components of $\vec{r}_{\rm SSB}$ and $\vec{r}_{\rm obs}$ are determined from the same Solar System ephemeris described above, and \hat{s} is gradually resolved from an application of $\Delta_{\rm R\odot}$ (Equation 1.14).

Evolution in the ISM

The ISM is a naturally dynamic and turbulent environment. This evolution was first seen as measurable variations in DM for the Crab and Vela pulsars (Rankin & Roberts, 1971; Isaacman & Rankin, 1977), and is readily seen in TOAs collected for isolated and binary pulsars with modern broadbandoriented instrumentation (e.g. Keith et al., 2013; Lam et al., 2015). In the presence of such variations, a single value of DM will not sufficiently characterize the frequency dependence of TOA data and will introduce biases in other timing-model parameters. The TEMPO software package contains a built-in feature, called "DMX", for measuring DM at some prescribed time interval, which can be as low as one day for a per-epoch determination of DM. While this type of modeling introduces more degrees of freedom within the timing model, it nonetheless allows for more robust determination of the time dependence in DM over the span of the data set.

Intrinsic Evolution of the Pulse Profile

For many pulsars with different DMs and timing-model parameters, the integrated, high-S/N pulse profile has been shown to yield slightly different shapes when observing with different receivers, or when using a single receiver with a large bandwidth. This frequency-dependent evolution is believed to be an intrinsic property of the poorly-understood mechanism for radio emission, and can be interpreted as the emission at different frequencies occurring at different heights from the polar cap (Komesaroff, 1970).

As with the general radio-emission mechanism, there is no sufficient framework that can fully account for the observed variations in profile shape as a function of frequency for all pulsars. Moreover, recent work has shown that the degree of profile evolution can vary substantially between radio pulsars, with several pulsars shown no significant shape variations over a wide range of receiver frequencies (e.g. Arzoumanian et al., 2015b). For pulsars with significant profile evolution, the cross-correlation of broadband data with a single profile template with produce TOA lags as a function of frequency that generally differs from the f^{-2} dependence of timing delays due to DM.

When needed, we use the heuristic model developed by Arzoumanian et al. (2015b) that modeled the frequency-dependent (FD) evolution of the profile shape in the following manner:

$$\Delta t_{\rm FD} = \sum_{i=1}^{n} c_i \log \left(\frac{f}{1 \text{ GHz}}\right)^i \tag{1.18}$$

where f is the channel frequency of the observed TOA, n is the user-defined number of log-polynomial terms to be used, and c_i is the *i*-th coefficient, such that i = 1, 2, ... n. The c_i terms are the allowed free parameters in TEMPO and TEMPO2. In practice, pulsars that show no significant change in pulse structure across observing frequency do not require an application of $\Delta t_{\rm FD}$, whereas pulsars with comparatively large changes require up to five log-polynomial terms when applying Equation 1.18.

Determination of the Model and Noise Processes

A timing solution of an isolated pulsar can be obtained so long as it explicitly models the aforementioned physical effects and applies the appropriate clock corrections. This amounts to transforming the observed topocentric TOAs to an inertial reference frame, taken to be the SSB^{13} ,

$$t = t_{\rm top} + \Delta_{\rm C} + \Delta_{\rm R\odot} + \Delta_{\rm S\odot} + \Delta_{\rm E\odot} + \Delta_{\varpi} + \Delta_{\rm DM} + \Delta_{\rm FD}$$

and using these corrected/transformed TOAs to model the spin behavior through Equation 1.12. The timing model therefore consists of free parameters that describe the spin frequency and its derivatives, astrometry, the pulsar's DM, and any significant variations in DM. The parameters that best describe the observed TOAs are determined using a least-squares fitting procedure, where the best-fit parameters minimize the χ^2 goodness-of-fit statistic,

$$\chi^{2} = \sum_{i}^{N_{\text{TOA}}} \left(\frac{(\nu_{\text{s,mod}}(t) - \nu_{\text{s}})^{2}}{\sigma_{i}^{2}} \right).$$
(1.19)

In Equation 1.19, N_{TOA} is the number of TOAs being modeled and σ_i is the uncertainty in the *i*-th TOA.

The use of Equation 1.19 and the corresponding covariance matrix of timing-model parameters assumes that the *timing residuals* – the difference between measured TOAs and those predicted from a given timing model – are uncorrelated between data subsets collected with different receivers and backends. In other words, the best-fit residuals are assumed to form a normal distribution with a mean of zero that reflects a white-noise random process. However, older-generation TOA data have been shown to harbor systematic errors associated with limits in instrumentation from incoherent de-dispersion and low-bit resolution, despite the appearance of "flat" best-fit residuals that indicate a good fit of the timing model.

¹³The acceleration due to gravity from a solar-mass star at a distance d = 1 pc is $Gm/d^2 \sim 10^{-13} \text{ m s}^{-2}$, while the acceleration due to gravity from the Galactic center at the SSB is $\sim 10^{-11} \text{ m s}^{-2}$. Both accelerations are considerably smaller in order of magnitude when compared to the acceleration due to gravity of the Sun at Earth, which is $\sim 10^{-8} \text{ m s}^{-2}$. Therefore, the SSB is a comparatively more "inertial" frame of reference.

A historic, *ad hoc* solution to this problem is the slight alteration of TOA measurement uncertainties for data subsets to values that ultimately produce the desired best-fit statistic, which is usually taken to be $\chi^2_{\rm red} = \chi^2/N_{\rm DOF} \approx 1$, where $N_{\rm DOF}$ is the number of degrees of freedom. Additional uncertainty can be added in quadrature (by an amount $\sigma_{\rm q}$) and/or as a multiplicative factor ($\sigma_{\rm f}$), such that

$$\sigma = \sigma_{\rm f} \sqrt{\sigma_{\rm o}^2 + \sigma_{\rm q}^2} \tag{1.20}$$

and $\sigma_{\rm o}$ is the original TOA uncertainty. For TOAs collected with modern, coherent-de-dispersion backends, $\sigma_{\rm f} \approx 1$ and $\sigma_{\rm q} \sim 1 \ \mu {\rm s}$. The corresponding values of $\sigma_{\rm f}$ and $\sigma_{\rm q}$ for historic processors can be larger than the modern-backend values by an order of magnitude.

Several bright, nearby pulsars show "red" noise (e.g. for PSR B1937+21; Kaspi et al., 1994; Verbiest et al., 2009) that appear as non-random structure in TOA residuals and can produce biases in timing-model parameters when left unaccounted, even after modeling all seemingly relevant physical effects. Red noise – also referred to as *timing noise* in the literature – is believed to reflect real instabilities in pulsar rotation due to torque fluctuations associated with the superfluid interior and/or the co-rotating magnetosphere (e.g. Shannon & Cordes, 2010). Historically, a practical solution towards modeling red-noise signatures in timing residuals is the use of higher-order time derivatives in ν_s (e.g. Arzoumanian et al., 1994).

A more sophisticated method for addressing red noise in TOA residuals was recently proposed by Coles et al. (2011), where a "generalized" least-squares (GLS) method accounts for correlated noise in TOA residuals through a linear transformation of the covariance matrix that whitens TOA residuals; the transformed data can then be used with Equation 1.19 to yield accurate, unbiased timing parameters.

1.4 Timing Delays from Binary Motion

In principle, the timing delays for radial displacement from binary motion of a pulsar have a similar, additive form to those that describe the Earth's motion in the solar system (Equations 1.14-1.16). The general timing formula for TOA correction and transformation is finally given as (e.g. Lorimer & Kramer, 2005)

$$t = t_{\rm top} + \Delta_{\rm C} + \Delta_{\rm R\odot} + \Delta_{\rm S\odot} + \Delta_{\rm E\odot} + \Delta_{\varpi} + \Delta_{\rm DM} + \Delta_{\rm R} + \Delta_{\rm S} + \Delta_{\rm E} \quad (1.21)$$

where $\Delta_{\rm R}$, $\Delta_{\rm S}$, and $\Delta_{\rm E}$ are the Römer, Shapiro and Einstein timing delays of the pulsar-binary system. However, the parameters that describe a pulsarbinary system are not known beforehand. Indeed, a pulsar must be reobserved several times upon discovery in order to measure significant changes in the observed spin period due to radial motion in an inclined orbit; the observed shifts can be used to constrain and make initial estimates of the binary parameters, though an explicit timing model is eventually required to precisely determine the orbital elements. Once a binary timing model is successfully applied to TOA data, future observations can be made to refine $\Delta_{\rm R,S,E}$ and the binary parameters that describe them. If relevant to the system, future TOAs can eventually yield estimates of variations in the orbital elements over time due to one or more effects described below and in subsequent chapters.

The binary systems studied in this thesis already had long-term timing solutions that explicitly modeled their respective orbits prior to their study for this work; we update these timing solutions in an effort to measure or improve prior estimates of masses and geometry, and derive tests of gravitational theory whenever possible. In this section, we present two binary timing models that describe pulsar-binary systems and long-term changes in the orbital elements. These two general binary models exist due to compli-



Figure 1.2: A schematic of an eccentric binary orbit – shown as a solid-black ellipse, with the center of mass at the origin of the drawn coordinate system – and the various angles of pulsar/binary orientation defined in Section 1.4. The portion of the orbit shown in gray lies on the opposite side of the tangential plane of the sky. The pulsar is shown as a black dot and its direction of motion along the orbit is given by the black arrow.

cations in applying an elliptical-orbit model to a highly circular orbit, where certain timing parameters become ill-defined and numerically unstable as free parameters in a least-squares model fit. Both binary timing models are used extensively in this dissertation.

Throughout this dissertation, we adopt the IAU convention for characterizing the three-dimensional orientation of orbital planes for nearly all pulsarbinary systems.¹⁴ An example of an eccentric orbit and the angles that define

¹⁴The only exception to the angle convention in this dissertation is PSR B1534+12 (the subject of Chapter 5), which uses the convention defined by Damour & Taylor (1992): *i* is measured such that $i = 0^{\circ}$ corresponds to the orbital-angular-momentum vector pointing

its orientation is shown in Figure 1.2. In this figure, points labeled "A", "B" and "C", as well as the gray lines that connect them to the origin of the coordinate system, are drawn to help indicate how several angles are measured. The relevant angles are defined as follows: the longitude of the ascending node (Ω) is measured from the direction of celestial North to the line connecting the origin and A (which contains the point along the orbit where the pulsar pierces the tangential plane of the sky in the direction away from observers on Earth) in the plane of the sky and in the direction of celestial East, such that $0^{\circ} < \Omega < 360^{\circ}$; the argument of periastron (ω) is measured between the ascending node and the line between the origin and B, in the plane of the orbit, such that $0^{\circ} < \omega < 360^{\circ}$; and the true anomaly (u) is measured between periastron and the line connecting the origin and C, in the plane of the orbit, such that $0^{\circ} < u < 360^{\circ}$. The system inclination (i) is measured between the planes of the orbit and the sky, such that $0^{\circ} < i < 180^{\circ}$ and $i = 0^{\circ}$ corresponds to the vector of orbital angular momentum pointing in the direction towards Earth.

1.4.1 Orbits with Significant Eccentricity

A general binary orbit will have an elliptical shape that is characterized by its eccentricity (e), where a circular orbit corresponds to e = 0 and 0 < e < 1for gravitationally bound systems. An orbit with a statistically significant eccentricity has a well-defined ω (see Figure 1.2) that corresponds to the point of closest relative approach and maximum orbital speed. The Römer timing delay for a pulsar ($\Delta_{\rm R}$) in an eccentric binary system with orbital period $P_{\rm b}$ and semi-major axis $a_{\rm p}$ is dependent on the parameters that directly affect the radial motion of the pulsar along the line of sight (Blandford & Teukolsky, 1976):

away from the Earth; and Ω is measured with the opposite sense than the one used in the IAU convention, from celestial North in the direction of celestial West.

$$\Delta_{\rm R} = x \left[(\cos E - e) \sin \omega + \sin E \sqrt{1 - e^2} \cos \omega \right]$$
(1.22)

where $x = a_p \sin i/c$ is the semi-major axis projected along the line of sight due to an inclination (i) of the orbital plane, and where c is the speed of light. The "eccentric anomaly" (E) is computed using a set of "Kepler's equations" that are modified to include first-order perturbations in the orbital period and periastron argument (Damour & Deruelle, 1986):

$$E - e\sin E = n_{\rm b} \left[(t - T_0) + \frac{1}{2} \frac{\dot{P}_{\rm b}}{P_{\rm b}} (t - T_0)^2 \right]$$
(1.23)

$$u(E) = 2 \arctan\left[\sqrt{\frac{1+e}{1-e}} \tan\left(\frac{E}{2}\right)\right]$$
(1.24)

$$\omega = \omega_0 + \frac{\dot{\omega}}{n_{\rm b}} u(E) \tag{1.25}$$

where u(E) is the true anomaly of the pulsar (shown in Figure 1.2) at some time t in an inertial reference frame relative to the binary system (e.g. a coordinate system centered at the SSB), T_0 is the epoch of periastron passage, ω_0 is the value of ω measured at a time T_0 , and $n_b = 2\pi/P_b$ is the angular orbital frequency. The Keplerian timing parameters are five parameters that describe the basic properties of every eccentric binary system: $\{x, P_b, e, \omega, T_0\}$. With a measurement of Δ_R alone, a_p and $\sin i$ cannot be separately measured. Furthermore, the time derivatives in P_b and ω , as well as variations in x, are only measured in certain cases where there are intrinsic and/or kinematic effects that produce apparent changes in these three parameters over time. The physical effects that produce observable variations are discussed in Section 1.5 below.

In theory, the Römer timing delay can be written in a more general form to account for radial and tangential corrections of the eccentricity that are unique to the general-relativistic formulation of the eccentric two-body problem (Damour & Deruelle, 1985):

$$\Delta_{\rm R} = x \left[(\cos E - e_r) \sin \omega + \sin E \sqrt{1 - e_{\theta}^2} \cos \omega \right]$$
(1.26)

where $e_r = e(1+\delta_r)$, $e_{\theta} = e(1+\delta_{\theta})$, and $\{\delta_r, \delta_{\theta}\}$ are the radial and tangential deformation parameters of the orbit's eccentricity, respectively. However, Damour & Deruelle (1986) showed that δ_r cannot be separately measured from parameters related to pulsar rotation, and that δ_{θ} can be measured so long as ω has changed significantly over time through relativistic precessions and a large data span (on the order of decades) has been obtained. We therefore ignored these terms in all analyses presented in this dissertation (i.e. we hold their values fixed at $\delta_r = \delta_{\theta} = 0$) and use the Römer delay shown in Equation 1.22 to model the timing delay of eccentric orbits. We consider the likelihood of measuring δ_{θ} in the relativistic PSR B1534+12 system (Chapter 5) within the coming years in Section 6.1.

The Shapiro timing delay experienced by a pulsar ($\Delta_{\rm S}$) is also an important effect that can be present in a pulsar-binary system (Damour & Deruelle, 1985, 1986). As with the analogous Solar-System effect, the relativistic Shapiro timing delay is a measure of the change in spacetime curvature that is traced by the pulsar signal as it propagates from different points of the orbit towards an observer on Earth. The Shapiro delay is most prominent for pulsar-binary systems that appear "edge-on", i.e. when $i \to \pi/2$, where the signal will propagate closest to the companion star at superior conjunction. For an eccentric orbit, the Shapiro timing delay is given as (Damour & Deruelle, 1986)

$$\Delta_{\rm S} = -2r \ln \left[1 - e \cos E - s \left(\left(\cos E - e \right) \sin \omega + \sin E \sqrt{1 - e^2} \cos \omega \right) \right]$$
(1.27)

where r and s are the "range" and "shape" parameters of the Shapiro timing delay, respectively. In most theories of gravitation, $s = \sin i$, whereas Einstein's theory of general relativity (GR) requires that $r = T_{\odot}m_c$, where $T_{\odot} = GM_{\odot}/c^3 = 4.925490947 \ \mu s$. A significant measurement of Δ_s is therefore useful as it yields estimates of both m_c and $\sin i$ simultaneously.

The Einstein timing delay ($\Delta_{\rm E}$) is typically only measurable for highly relativistic binary systems with $e \sim 0.1$ or greater, where periastron advance is significant and both components are degenerate, compact objects with orbital periods on the order of hours, or even a few days for the most eccentric systems (Damour & Deruelle, 1985, 1986). $\Delta_{\rm E}$ is a measure of both relativistic time dilation and gravitational redshift, and is given as

$$\Delta_{\rm E} = \gamma \sin E \tag{1.28}$$

where γ is the amplitude of the effect with unit of time. The full pulsar-binary timing model, defined as the sum of the three delays given in Equations 1.22, 1.27 and 1.28, are referred to as the "DD" binary model in TEMPO and TEMPO2.

The forms of $\Delta_{\text{R,S,E}}$ shown in Equations 1.22-1.28, do not depend on a particular theory of strong-field gravitation. In other words, the $\{\dot{P}_{\rm b}, \dot{\omega}, r, s, \gamma, \delta_{\theta}\}$ parameters can be measured in a theory-independent manner, without the need to assume general relativity or any other valid gravitational theory. The DD model therefore allows for unambiguous tests of gravitation by comparing the measured PK parameters with their values as predicted by the theory in question, and we discuss this type of analysis in Section 1.5.

1.4.2 Orbits with No Significant Eccentricity

In practice, TOAs collected over several years and many orbits are needed in order to measure the eccentricity of a low-e pulsar-binary system. Until a sufficient time span is reached, ω and T_0 lose their meaning in describing the near-circular orbit and are numerically unstable parameters in the leastsquares fit of the timing model. A second binary timing model, first presented by Lange et al. (2001), was developed in order to re-parametrize the Römer and Shapiro timing delay for circular orbits in terms of a first-order expansion in *e*. This near-circular binary timing model, referred to in TEMPO and TEMPO2 as "ELL1", introduces a different set of orbital parameters:

$$\Delta_{\rm R} = x \left(\sin \Phi + \frac{\kappa}{2} \sin 2\Phi - \frac{\eta}{2} \cos 2\Phi \right) \tag{1.29}$$

$$\Delta_{\rm S} = -2r\ln\left(1 - s\sin\Phi\right) \tag{1.30}$$

where $\Phi = n_{\rm b}(t - T_{\rm asc})$ is the celestial argument of the pulsar at the pulse emission time t, $T_{\rm asc}$ is the mean epoch of ascending-node passage¹⁵, and both κ and η are collectively referred to as the Laplace-Lagrange eccentricity parameters. The low-e form of the Römer timing delay (Equation 1.29) does not include a constant additive term of -3xe/2 since pulsar timing only separately measures binary terms that vary periodically with time.

The ELL1 eccentricity parameters (κ , η , and T_{asc}) are related to the DD eccentricity parameters (e, ω , and T_0) using the following relations (Lange et al., 2001):

$$\eta = e\sin\omega \tag{1.31}$$

$$\kappa = e \cos \omega \tag{1.32}$$

$$T_{\rm asc} = T_0 - \omega/n_{\rm b}.\tag{1.33}$$

The full ELL1 timing-model fit produces estimates of η , κ and $T_{\rm asc}$ with little numerical correlation, even if the $\{\eta, \kappa\}$ parameters are not statistically significant. Secular variations in the orbital elements are measured as Taylor expansions in those parameters about $T_{\rm asc}$.

¹⁵As noted by Lange et al. (2001), the *true* time of ascending-node passage is $T_{\rm asc} + 2\eta/n_{\rm b}$.

The low-*e* expansion used for the development of the ELL1 model produces a slight degeneracy when attempting to measure the Shapiro timing delay for low-inclination, low-*e* systems. This degeneracy is best seen by computing the Fourier expansion of Equation 1.30 in terms of the orbital period of the binary system (Lange et al., 2001):

$$\Delta_{\rm S} = 2r(a_0 + b_1 \sin \Phi - a_2 \cos 2\Phi + \dots) \tag{1.34}$$

where the *a* and *b* are the even and odd harmonic amplitudes of the Fourier basis, respectively, that depend on the inclination of the system. In the case of low inclination, only the first one or two harmonic terms in Equation 1.34 will be significant, and higher order terms will be negligible; the sum $\Delta_{\rm R} + \Delta_{\rm S}$ therefore yields an expression that is identical in form to Equation 1.29 with the exception that two of the ELL1 timing parameters are modified by the harmonic amplitudes:

$$x_{\rm obs} = x + 2rb_1 \tag{1.35}$$

$$\eta_{\rm obs} = \eta + \frac{4ra_2}{x} \tag{1.36}$$

For low-inclination systems, one must therefore compute the "intrinsic" ELL1 parameters (x, η) using the observed parameters in order to determine the true values of the Keplerian elements using Equations 1.31-1.33 (see Lange et al., 2001; Freire & Wex, 2010).

Furthermore, one can use the presence of orbital harmonic structure in TOA residuals as a confirmation of the Shapiro timing delay. Figure 1.3 shows best-fit TOA residuals for PSR J1918-0642, a low-*e* binary system that is studied in Chapter 2 and uses the ELL1 timing model, as a function of the orbital phase. We see in the middle panel of Figure 1.3 that the Shapiro timing delay is not fully absorbed when only fitting for $\Delta_{\rm R}$, which is a visual indication that the system is highly inclined; this is because the higher-order



Figure 1.3: The Shapiro timing delay observed in PSR J1918-0642 (a highlyinclined binary MSP that is studied in Chapter 3) when using the ELL1 binary timing model. The blue and green points denote TOAs collected at 430 and 1400 MHz, respectively. The top panel shows the "full", intrinsic effect that $\Delta_{\rm S}$ (Equation 1.27) has on TOA residuals, assuming that the intrinsic binary parameters are known and holding $\Delta_{\rm R}$ fixed at these values (while not fitting for $\Delta_{\rm S}$). The middle panel shows the remaining harmonic structure of $\Delta_{\rm S}$ after absorption of its first two Fourier harmonic coefficients when fitting only for $\Delta_{\rm R}$. The bottom panels shows the best timing-model fit, obtained when fitting for both $\Delta_{\rm R}$ and $\Delta_{\rm S}$ simultaneously.

harmonic terms in Equation 1.34, jointly represented as the "..." term, are statistically significant and essentially represent the separately-measurable part of $\Delta_{\rm S}$.

1.5 Variations in the Orbital Elements

In the absence of observed secular variations or relativistic phenomena, the mass function (f_m) of the pulsar-binary system can be used as a measure of

the mass and inclination of the Keplerian system; it is computed using the observed Keplerian elements that describe the pulsar's orbit:

$$f_m = \frac{n_{\rm b}^2 x^3}{\mathrm{T}_{\odot}} \mathrm{M}_{\odot} = \frac{(m_{\rm c} \sin i)^3}{(m_{\rm p} + m_{\rm c})^2}.$$
 (1.37)

For strictly Keplerian motion, and without any independent knowledge of the binary companion (e.g. through optical spectroscopy and radial-velocity measurements), the intrinsic parameters of the binary system ($m_{\rm p}$, $m_{\rm c}$, i, Ω) cannot be uniquely determined. However, estimates of the companion mass can be made using Equation 1.37 since the pulsar mass is restricted by theory to be no larger than 3 M_{\odot}. A nominal value of $m_{\rm p} = 1.35 \, {\rm M}_{\odot}$ (Thorsett & Chakrabarty, 1999) can be assumed to yield estimates of $m_{\rm c}$ for a given value of sin *i*; and a minimum companion mass can be computed for the case where sin *i* = 1. Conversely, one can assume the $m_{\rm c} - P_{\rm b}$ relation for a MSP-binary system suspected to undergone significant mass transfer to compute a value of $m_{\rm c}$ using Equation 1.8, and then derive values of $m_{\rm p}$ for different inclination angles.

Many pulsar-binary systems examined in this dissertation exhibit one or more variations in their orbital elements and PK corrections of the observed orbital motion. The PK effects are particularly interesting since any contending gravitational theory must yield correct predictions of the observed PK motion as functions of fundamental quantities that uniquely describe the theory. However, several other effects can give rise to secular variations that, if left unaccounted for, will bias the secular PK variations and their interpretation. In this section, we outline several of the most dominant mechanisms that give rise to observed variations in the orbital elements.

1.5.1 Strong-Field Gravitation

Pulsar-binary systems in tight orbits with WDs or other neutron stars typically exhibit PK effects that are observed as secular changes in the orbital elements. We assumed GR to be valid throughout this dissertation in order to explicitly interpret these effects; we refer to the secular PK variations in this chapter as $(\dot{P}_{\rm b})_{\rm GR}$, $(\dot{\omega})_{\rm GR}$, $(\dot{x})_{\rm GR}$ and $(\dot{e})_{\rm GR}$, where the dots indicate derivatives in time. According to GR, each of the PK quantities (including the Shapiro r and s parameters) are functions of at least one of the two component masses (Damour & Taylor, 1992). The Shapiro-delay parameters and first-order PK variations are given as

$$(\dot{P}_{\rm b})_{\rm GR} = -\frac{192\pi}{5} (n_{\rm b} T_{\odot})^{5/3} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) \\ \times (1 - e^2)^{-7/2} \left[\frac{m_{\rm p} m_{\rm c}}{(m_{\rm p} + m_{\rm c})^{1/3}} \right]$$
(1.38)

$$(\dot{\omega})_{\rm GR} = 3(n_{\rm b}^5 T_{\odot}^2)^{1/3} (1 - e^2)^{-1} (m_{\rm p} + m_{\rm c})^{2/3}$$
 (1.39)

$$\gamma = \left(\frac{T_{\odot}^2}{n_{\rm b}}\right)^{1/3} \left[\frac{m_{\rm c}(m_{\rm p} + 2m_{\rm c})}{(m_{\rm p} + m_{\rm c})^{4/3}}\right]e\tag{1.40}$$

$$s = \left(\frac{T_{\odot}}{n_{\rm b}^2}\right)^{1/3} \left[\frac{(m_{\rm p} + m_{\rm c})^{2/3}}{m_{\rm c}}\right] x = \sin i \tag{1.41}$$

$$r = T_{\odot}m_{\rm c} \tag{1.42}$$

where the values of $m_{\rm p}$ and $m_{\rm c}$ are assumed to be in units of M_{\odot} in each of the above expressions. Throughout this thesis, we ignore the $(\dot{x})_{\rm GR} = (\dot{a}_{\rm p})_{\rm GR} \sin i/c$ and $(\dot{e})_{\rm GR}$ terms since these particular rates of change are immeasurable on the timescales spanned by the data sets analyzed below.¹⁶

Measurements of r (Equation 1.42) and s (Equation 1.41) from an observed $\Delta_{\rm S}$ alone therefore yields estimates of $m_{\rm p}$ and $m_{\rm c}$, as well as a measure of i since $s = \sin i$. Even without an observed Shapiro timing delay, estimates

¹⁶Kepler's third law requires that $(\dot{x})_{\rm GR}/x = 2(\dot{P}_{\rm b})_{\rm GR}/(3P_{\rm b})$. In the case of relativistic pulsar-binary systems, $P_{\rm b} \sim$ hours and $\dot{P}_{\rm b}$ due to GR is measurable after nearly a decade of observation. However, typical values of $x \sim$ seconds and so the value of \dot{x} due to GR is comparatively much smaller than the rate of change in period, as required by Kepler's law.

of $m_{\rm p}$ and $m_{\rm c}$ can be obtained so long as two of the other corrections due to GR are measured. A classic example of such a scenario is the Hulse-Taylor pulsar, PSR B1913+16, for which the $\dot{P}_{\rm b} - \dot{\omega} - \gamma$ combination were measured and used to derive high-precision estimates of both masses (Weisberg et al., 2010; Weisberg & Huang, 2016). Three or more observed PK effects lead to an overdetermined system of equations that can be solved to obtained a high-precision estimate of the component masses, as well as one or more "tests" of GR when using different combinations of three PK parameters to check for self-consistency with measurement uncertainties.

1.5.2 Kinematic Bias from Proper Motion

Besides the intrinsic changes within orbits from PK effects, apparent secular variations in the orbital elements will also be induced from significant relative motion between the pulsar-binary and SSB reference frames (Kopeikin, 1996). The secular variations in x and ω from proper motion (μ) – to which we refer in this study as $(\dot{x})_{\mu}$ and $(\dot{\omega})_{\mu}$ – arise from a long-term change in certain elements of orientation as the binary system moves across the sky. The kinematic terms for \dot{x} and $\dot{\omega}$ are described as trigonometric functions of i and Ω :

$$(\dot{x})_{\mu} = x\mu \cot i \sin(\Theta_{\mu} - \Omega), \qquad (1.43)$$

$$(\dot{\omega})_{\mu} = \mu \csc i \cos(\Theta_{\mu} - \Omega). \tag{1.44}$$

where Θ_{μ} is the position angle of proper motion, computed internally within TEMPO2.

1.5.3 Kinematic Bias from Acceleration

A separate kinematic bias that produces observed secular variations in orbital elements can arise from several forms of relative acceleration between the pulsar-binary and SSB systems (e.g. Damour & Taylor, 1992; Nice & Taylor, 1995), the most prominent of which are: differential rotation in the Galactic disk; acceleration in the Galactic gravitational potential vertical to the disk (e.g. Kuijken & Gilmore, 1989); and apparent acceleration due to significant proper motion (Shklovskii, 1970). The kinematic bias from relative acceleration produces a rate of change in the Doppler shift (D) in, for example, $P_{\rm b}$, such that

$$\left(\frac{\dot{P}_{\rm b}}{P_{\rm b}}\right)_D = \frac{\dot{D}}{D} = \frac{a_z}{c} - \cos b \left(\frac{\Theta_0^2}{cR_0}\right) \left(\cos l + \frac{\beta}{\sin^2 l + \beta^2}\right) + \frac{\mu^2 d}{c} \qquad (1.45)$$

where the terms in Equation 1.45 are summarized as follows: (l, b) are the Galactocentric coordinates of the MSP; R_0 is the distance between the Sun and the Galactic center; Θ_0 is the circular Galactic-rotation speed of the Sun; μ is the proper motion of the pulsar-binary system; d is the distance to the binary system; a_z is the component of acceleration in the Galactic potential that is vertical to the Galactic disk; and $\beta = (d/R_0) \cos b - \cos l$. Throughout this dissertation, we use the a_z model developed by Kuijken & Gilmore (1989), who used photometric and spectroscopic data of K-dwarf stars with known distances to measure a_z out to d = 3 kpc and found that

$$\frac{a_z}{c} = 1.08 \times 10^{-19} \left[\frac{1.25z}{(z^2 + 0.0324)^{1/2}} + 0.58z \right] \,\mathrm{s}^{-1}, \tag{1.46}$$

where $z = d \sin b$ is distance from the Galactic plane. The changing Doppler shifts ultimately produce an apparent variation in x and $P_{\rm b}$, though the effect is negligibly small for the former parameter. We refer to the component of the secular variation due to the acceleration bias as $(\dot{P}_{\rm b})_D = P_{\rm b}(\dot{D}/D)$. Several other pulsar-timing parameters (such as x and P_s) are similarly affected by the change in Doppler factors due to relative acceleration. In other words, $(\dot{x})_D = x(\dot{D}/D)$ and $(\dot{P}_s)_D = P_s(\dot{D}/D)$. However, the Doppler component of \dot{x} is generally considered to be negligibly small since $x \sim$ seconds, and the smallest known values of $P_b \sim$ hours. The Doppler component of \dot{P}_s is inseparable from the spin-down term if an accurate measure of distance is not known.

1.5.4 Periodic Variations and Annual Orbital Parallax

For sufficiently nearby pulsar-binary systems, the observed system orientation will change periodically as the Earth and the MSP orbit their respective barycenters and at their respective orbital periods. The "mixed" periodic variations in x and ω , collectively referred to as the "annual orbital parallax" (Kopeikin, 1995), depend on i, Ω , and the observed parallax (ϖ) of the pulsar-binary system:

$$x_{\rm obs} = x \left[1 - \varpi \cot i \left(\Delta_{\vec{I}_o} \sin \Omega - \Delta_{\vec{J}_0} \cos \Omega \right) \right]$$
(1.47)

$$\omega_{\rm obs} = \omega - \varpi \csc i \left[\Delta_{\vec{I}_o} \cos \Omega + \Delta_{\vec{J}_0} \sin \Omega) \right]$$
(1.48)

where ϖ is the annual astrometric parallax, and $\Delta_{\vec{I}_o} = \vec{r}_{\rm SSB} \cdot \vec{I}_0$ and $\Delta_{\vec{J}_0} = \vec{r}_{\rm SSB} \cdot \vec{J}_0$ are the time-varying projections of the SSB position vector of the Earth, defined in Equation 1.14, onto two of the orthogonal basis vectors (\vec{I}_0, \vec{J}_0) centered on the pulsar-binary system that define the celestial east and north directions, respectively. Annual orbital parallax has been detected for PSRs J0437-4715 (e.g. Verbiest et al., 2008) and J1713+0747 (e.g. Zhu et al., 2015) as significant improvements in timing-model fits when explicitly modeling the effect. In practice, annual orbital parallax does not significantly improve the estimates of timing-model parameters due to its small

signature on TOA residuals. However, annual orbital parallax can be used in conjunction with measurements of the Shapiro timing delay, annual astrometric parallax and \dot{x} due to kinematic bias (Equation 1.43) to uniquely solve for the three-dimensional geometry of the pulsar-binary system where all of these effects are measured.

1.6 This Thesis

The major goal of this dissertation is to measure and/or constrain the mass and geometric parameters of binary pulsars through the analysis of pulsartiming data and interpretation of observed variations in the orbital elements. The following chapters can be described and summarized as follows:

- in Chapter 2, we discuss the contributions that author made as a member of the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) during his graduate career;
- in Chapter 3, we present detailed analyses of 24 binary pulsars that are currently being observed as part of the NANOGrav program;
- in Chapter 4, we present a long-term and ongoing analysis of PSR B1620-26, a 11-ms pulsar within a gravitationally-bound triple system that resides within the Messier 4 globular cluster;
- in Chapter 5, we present updates of the long-term analysis of PSR B1534+12, a 37.9-ms pulsar in a 10-hr, relativistic orbit with another neutron star;
- in Chapter 6, we summarize the results obtained for this thesis, perform simulations to determine when other secular variations could be observable in PSR B1534+12, and describe possible avenues for further work.

Chapter 2

The North American Nanohertz Observatory for Gravitational Waves

The recent, direct detection of gravitational waves (GWs) at kHz frequencies using the Laser Interferometer Gravitational-Wave Observatory has officially heralded the era of observational GW astronomy (Abbott et al., 2016). Along with LIGO, several major, international efforts are currently underway in order to directly detect and characterize GWs across different parts of the GW spectrum. Several expected sources of GWs are merging black holes, merging neutron stars (e.g. Hulse & Taylor, 1975; Kramer et al., 2006b; Fonseca et al., 2014), primordial relics from the inflation era of the early Universe (e.g. Grishchuk, 2005), and cosmic strings (Vilenkin & Shellard, 1994).

NANOGrav¹ is one of three "pulsar timing array" (PTA) collaborations, comprised of faculty, researchers and students, that regularly monitor an increasing number of MSPs with the primary goal of directly detecting GWs at nanohertz frequencies. The NANOGrav collaboration members are affili-

¹http://nanograv.org/

ated with universities and research institutes within Canada and the United The other two PTAs are comprised of institutions and universi-States. ties across Europe, which are collectively referred to as the European Pulsar Timing Array (EPTA), and groups across Australian universities that are collectively referred to in literature as the Parkes Pulsar Timing Array (PPTA). The union of NANOGrav, the EPTA and PPTA is referred as the International PTA (IPTA; Manchester & IPTA, 2013). A PTA essentially serves as a Galactic-scale detector for perturbations of the spacetime metric generated by, most prominently, the expected merger of binary supermassive black holes (SMBHs) scattered across the Universe (Sazhin, 1978; Detweiler, 1979); the superposition of GW signals from SMBH binaries forms a stochastic background of nanohertz-frequency GWs. Recent work has shown that PTAs can eventually also become sensitive to individual, localizable sources of nanohertz-frequency GWs generated from nearby galaxy clusters (e.g. Sesana et al., 2009), as well as permanent post-merger deformations of spacetime referred to as GW "memory" (e.g. Madison et al., 2014).

PTA collaborations make high-precision timing observations of an array of the brightest and most stable MSPs on a regular basis using the same techniques and analysis methods discussed in Chapter 1. For NANOGrav, these measurements are made with the 305-m Arecibo Observatory in Puerto Rico and the 100-m Robert C. Byrd Green Bank Telescope (GBT) in Green Bank, West Virginia (USA). The collective goal of NANOGrav, along with the EPTA and PPTA, is the detection and characterization of nanohertzfrequency GWs within the next decade. The foundational method for detection of the stochastic background with PTAs was outlined by Hellings & Downs (1983), who proposed the cross-correlation of TOA residuals for all PTA MSPs at different sky locations to search for correlated structure due to passing GWs. The effectiveness of the cross-correlation method naturally depends on our ability to accurately measure and model the observed TOA variations with precision on the order of 10 ns, where GW structure is expected to be observable (e.g. Jenet et al., 2006; Sesana et al., 2008).

The author formally joined NANOGrav as a (graduate) student member in October 2012, shortly after the start of his Ph. D. program, though began assisting with timing observations around June 2012. After several years of undertaking data acquisition and constructing timing solutions for various NANOGrav MSPs, he became a full member of the collaboration in November 2014. During his Ph. D. career, he has attended weekly online meetings for the timing and observing groups within NANOGrav, and has presented various projects at in-person NANOGrav meetings across North American as well as two IPTA meetings in Krabi, Thailand and Banff, Canada. In this chapter, we briefly summarize the contributions made by E. Fonseca for the benefit of the NANOGrav collaboration.

2.1 Data Acquisition and Analysis

The nominal NANOGrav observing program, which formally began in 2004, observes all PTA MSPs every ~ 3 weeks using the Arecibo Telescope and/or the GBT.² In order to eventually produce high-quality pulsar data and timing solutions, observations are typically carried out using two widely-separated telescope receivers and span 20-30 minutes per receiver, per source. There are currently ~ 50 MSPs undergoing observations for the NANOGrav PTA, and the PTA increase in size by ~ 3 MSPs per year, and so observations can collectively take up ~ 50 hours per month of the observing year. Capable and experienced observers are therefore essential for the acquisition and model-construction of NANOGrav MSPs.

The author has regularly collected NANOGrav TOA data using both the Arecibo telescope and GBT during his Ph. D. career. Between November 2013 and January 2016, the author collected 180 hours' worth of pulsar

 $^{^2{\}rm Two}$ NANOGrav MSPs – PSRs J1713+0747 and B1937+21 – are observed using both observatories. All other MSPs are observed using only one of the two radio telescopes.

TOAs (Nice, 2016). This total amount of observing includes supplementary NANOGrav observations conducted for a proposal led by the author during the 2015 observing year, which is discussed in Section 2.2.

During his graduate career, the author also contributed to the cleaning and initial analysis of TOA data for five NANOGrav MSPs: PSRs J1643-1224; J1853+1303; J1910+1256; J1949+3106; and B1953+29, using the procedures discussed in Sections 1.3 and 1.4 for the inclusion of relevant timing parameters. The data acquisition and timing analyses conducted by the author, along with all full and active members of NANOGrav, culminated in the publication of the NANOGrav nine-year data set (Arzoumanian et al., 2015b).

2.2 Proposals for Observations: P2945

The success of the IPTA experiment depends on many observational and analytical factors that are currently being addressed by PTA collaborations. Standard "pulsar timing" techniques, discussed in Sections 1.3 and 1.4 above, provide the crucial means for understanding the environment and spin behavior of each MSP. However, recent studies have demonstrated a need to compensate for intrinsic "timing noise" of varying strength and properties within several NANOGrav MSPs (e.g. Shannon & Cordes, 2012; Perrodin et al., 2013; Arzoumanian et al., 2015b). Additional complications arise due to temporal variations in dispersive properties and the frequency dependence of pulse structure. However, the dedicated work of NANOGrav faculty, post-doctoral and student members has so far yielded analysis techniques and pipelines that address these issues and allow for current upper limits of the GW-signal strength. While the NANOGrav PTA is consistently becoming a more powerful tool for the detection and study of the stochastic GW background (e.g. Siemens et al., 2013), more work is needed to improve its sensitivity to individual, localizable sources of GWs.

Source	$P_s (\mathrm{ms})$	$DM (pc cm^{-3})$	LST (rise-set)	RMS residual (μ s)
J0030+0451	4.87	4.33	23:31-01:29	0.265
J1640 + 2224	3.16	18.43	15:18-18:03	0.189
J1713 + 0747	4.57	15.99	16:05-18:22	0.065
J2043 + 1711	2.38	20.70	19:21-22:06	0.136
J2317+1439	3.45	21.90	21:56-00:38	0.337

Table 2.1: Observing parameters for the five MSPs observed that are part of the P2945 program at the Arecibo Observatory. Note that the "rise" and set" times constitute times when the source enters in and exits out of the Arecibo field of view, respectively.

Arzoumanian et al. (2014) provided upper limits on the signal strength of individual-source GWs using the NANOGrav five-year data set (Demorest et al., 2013) and found that the detectability of individual-source GWs strongly depends on two factors. One factor is the angular separation of several MSPs relative to these bright GW sources, which are likely to be nearby galaxy clusters such as the Virgo and Fornax clusters (e.g. Simon et al., 2014). Localized GW signals will require nearby "detectors", so suitable MSPs with small angular separations to these individual GW sources will be necessary in order to make such detections with confidence. As mentioned above, this point is continually addressed by including several newly discovered, bright MSPs each year that are ideally distributed randomly across the sky. The second, more limiting factor involves the timing precision of MSPs close to these potential GW sources of interest. Arzoumanian et al. (2014) found that the best-timed NANOGrav pulsar – PSR J1713+0747 (e.g. Splaver et al., 2005) - vastly dominates in S/N contribution to individual-source GW detection when compared to the other 16 MSPs in their study. In principle, then, the optimal observing program would be high cadence observations of the single best-timed pulsar. However, this would not be a robust experiment, since an apparent GW signal in the timing of that single pulsar could potentially also be explained by pulsar rotation irregularities or ISM effects. In order to make a reliable detection, the GW signal needs to be seen in timing data of more than one pulsar. Thus the optimal experimental design involves highcadence observations, on timescales shorter than the nominal NANOGrav observing program, of a small number of precisely timed pulsars.

In August 2014, the author served as principal investigator and wrote a telescope proposal for an additional NANOGrav observing program that is supplementary to the nominal program discussed in Section 2.1. The telescope proposal, submitted to the Arecibo Observatory for the September 2014 deadline, requested weekly observations of five NANOGrav MSPs whose timing data are shown in Table 2.1. The scientific justification in the first proposal, given the designation "P2945" by the Arecibo Observatory³, was written by the author using results based on simulated data generated by several NANOGrav collaborators. The proposal was accepted in December 2014 with the full amount of requested observing time granted to NANOGrav, for a total of 260 hours collected for all five MSPs. Observations for P2945 formally began on 1 January 2015.

In August 2015, the author and several NANOGrav members re-submitted a telescope proposal for the continuation of the P2945 observing program. The scientific justification for the second submission was similar in form to the original version submitted in the first P2945 proposal, but included an initial timing analysis of P2945 data collected during January through August of the 2015 observing year that was performed by the author. These data are shown in Figure 2.2. The second P2945 proposal was successfully accepted and will continue throughout the 2016 observing year. These data will be published and be made publicly available in a forthcoming extension of the NANOGrav data set (Arzoumanian et al., 2016), and will likely be a part of several studies characterizing noise and DM variations using high-cadence data.

³http://www.naic.edu/vscience/schedule/2015Spring/FonsecatagP2945.pdf



Figure 2.1: TOA residuals for the five NANOGrav MSPs observed under P2945. Red and blue points represent 430 MHz and 1400 MHz data collected using the PUPPI pulsar processor currently used at Arecibo; green and magenta points represent 430 and 1400 MHz data collected with the predecessor (ASP) machine. All other points were collected with the Green Bank Telescope in West Virginia, USA. (Note: there are NANOGrav data that have been collected during the apparent gap in data around 2014-2015 as part of the nominal observing program, but they are excluded in this figure to highlight the amount of data collected under P2945.)

2.3 Contributions to NANOGrav Projects

At its core, NANOGrav is a collaborative effort that requires dedicated time and productivity from observers, data analysts and theorists alike. The recent publication of the NANOGrav nine-year data set has so far spurred a multitude of studies that probe a wide variety of scientific questions regarding:

• limits on the strength of the stochastic, nanohertz-frequency GW signal

(Arzoumanian et al., 2016),

- analysis of astrometric parameters observed in NANOGrav MSPs (Matthews et al., 2016),
- impact of interstellar scintillation in TOA precision for NANOGrav MSPs (Levin et al., 2016),
- assessment of PTA times to detection of the stochastic GW background (Taylor et al., 2016),
- assessment of noise budget in NANOGrav TOA data (Lam et al., 2016), and
- analysis of secular and PK variations in orbital parameters of NANOGrav binary MSPs.

The last of the studies listed above is the subject of Chapter 3.

The author contributed text and analysis to the publication of the NANO-Grav nine-year data set (see "Author Contributions" section of Arzoumanian et al., 2015b), and the astrometry study performed by Matthews et al. (2016). For the latter study, Matthews et al. (2016) used secular variations in $P_{\rm b}$ observed in two NANOGrav binary-MSP systems – PSRs J1614–2230 and J1909–3744 – to place formidable constraints on the distance to both binaries, using the methodology discussed in Section 1.5. These two systems and their orbital variations are discussed in Sections 3.4.4 and 3.4.10 below, respectively.

Chapter 3

The NANOGrav Nine-Year Data Set: Mass and Geometric Measurements of Binary Millisecond Pulsars¹

The NANOGrav nine-year data set (Arzoumanian et al., 2015b) contains TOAs collected for 37 MSPs, 25 of which reside in binary systems of different shapes, sizes an orbital periods. The data span for each binary MSP varies between ~ 2 to 9 years, depending on when the MSP was discovered and/or included into the NANOGrav PTA. The regular, monthly cadence and dualreceiver strategy of the NANOGrav observing program collectively yield an ideal data set for tracking long-term changes in orientation and/or relativistic phenomena over time. Moreover, long data sets with a large number of TOAs collected at different times (and different phases of each orbit) are ideal for resolving the Shapiro timing delay. The theory introduced in Sections 1.4 and 1.5 show that measurements of geometric and/or relativistic phenomena

¹This study was recently accepted by the Astrophysical Journal for publication, and is available online in the arXiv repository (Fonseca et al., 2016).

can be related to intrinsic properties of the components and orientation of the binary system, which are otherwise not uniquely accessible through strict Keplerian timing. In this chapter, we investigate the various effects observed in NANOGrav MSP-binary systems to determine the relevant mechanisms within each system, as well as extract mass and/or geometric information whenever possible.

In Section 3.1, we provide details regarding the general NANOGrav observing program as well as targeted observations that were obtained specifically for the detection of possible Shapiro timing delays in several NANOGrav MSP-binary systems. In Section 3.2, we describe the timing models and analytical methods used to derive the orbital elements, as well as theoretical constraints that can be placed on the component masses and system orientation from observed variations in the orbital elements. In Section 3.3, we discuss the methods used to characterize the physical parameters of interest, and in particular the component masses and system geometries. In Section 3.4, we discuss results obtained for select individual MSP-binary systems. Finally, in Section 3.5, we summarize the main findings of our study and provide a broader context for the implications these measurements have on understanding stellar-binary evolution and the overall mass distribution of binary MSPs.

3.1 Observations & Reduction

The full details regarding data collection, calibration, pulse arrival-time determination and noise modeling for the NANOGrav nine-year data set are provided in Arzoumanian et al. (2015b). Here we provide a brief summary of this information. The data are publicly available for download online.²

All 37 NANOGrav MSPs were observed on a monthly basis using either the 300-m William E. Gordon Arecibo Telescope in Puerto Rico or the 100-

²http://data.nanograv.org

m Robert C. Byrd Green Bank Telescope (GBT) in West Virginia, USA, as early as 2004 until late 2013. In the cases of PSRs J1713+0747 and B1937+21, both telescopes were used to monitor these MSPs. In addition to the monthly-cadence program, concentrated observing campaigns of 12 MSPs were made at specific orbital phases and were designed to maximize sensitivity to the Shapiro timing delay (Pennucci, 2015).

For the monthly observations at both telescopes, as well as the targeted Shapiro-delay campaigns at Arecibo, each MSP was observed using two radio receivers at widely separated frequencies in order to accurately measure the pulsar's line-of-sight dispersion properties on monthly timescales, and to account for any evolution in these frequency-dependent properties over time. The dual-receiver observations at Arecibo were performed contiguously during each observing session. The same measurements at the GBT were typically performed within several days of one another due to a need for retraction and extension of the prime-focus boom when switching between receivers. For the targeted Shapiro-delay observations at the GBT, only one receiver was used due to time constraints. The receivers used for the NANOGrav observations reported here were centered near: 327 MHz (at Arecibo only); 430 MHz (at Arecibo only); 820 MHz (at GBT only); 1400 MHz; and 2030 MHz (at Arecibo only).

In order to calibrate each MSP signal, pulsed broadband signals from a noise diode were recorded for several continuum radio sources of known flux density that were observed once every month during each observing year. The quasar J1413+1509 was used as the continuum source at Arecibo, while the quasar B1442+101 was similarly used at the GBT. For each receiver, two calibration scans of the same continuum source were obtained: one was centered on the continuum source, and another was obtained typically 1 degree offset from the central position. The difference in "on" and "off" calibration signals yields the conversion factor from units of machine counts to flux density. A similar noise-diode signal was obtained for each pulsar at its position during every observing session in order to convert raw voltages to flux densities using the conversion factors determined from the continuumcalibration observations.

Two generations of pulsar backend processors were used at each telescope for real-time coherent de-dispersion and folding of the signal using pre-determined ephemerides of each MSP based on early timing solutions. The identical ASP and GASP pulsar machines (Demorest, 2007; Ferdman, 2008) were used from the start of the NANOGrav observing program in 2004 until their decommissioning in 2011-2012. These backends decomposed the incoming signal into contiguous 4-MHz channels that spanned 20-64 MHz in usable bandwidth, depending on the receiver used and radio-frequencyinterference environment. The PUPPI and GUPPI machines (DuPlain et al., 2008; Ford et al., 2010), currently in use at both telescopes, can process up to 800 MHz in bandwidth using smaller, 1.5625-MHz channels. Both sets of machines generated folded pulse profiles resolved into 2048 bins across the pulsar's spin period.

Arzoumanian et al. (2015b) used the standard cross-correlation method for the determination of each folded profile's time of arrival (TOA), where a single, de-noised profile template is matched in the Fourier domain with all profiles obtained at some observing frequency and bandwidth (Taylor, 1992). Prior to correlation, we averaged data both over time (20-30 min or 2% of a MSP-binary orbit per TOA, whichever was shorter) and over a small fraction of the available bandwidth.

PSR	x (lt-s)	P_b (days)	e	$\omega (deg)$	T_0 (MJD)	η	κ	$T_{\rm asc}$ (MJD)
J0023+0923	0.03484105(11)	0.13879914244(4)	0.000025(5)	82.0(12.0)	56179.082(5)	0.000024(5)	0.000003(5)	56179.08248997(8)
J0613 - 0200	1.0914422(5)	1.198512556680(13)	0.00000443(17)	35.0(3.0)	54890.089(10)	0.0000026(2)	0.00000362(8)	54889.991808565(12)
J1012 + 5307	0.5818176(6)	0.60467271380(6)	0.0000013(17)	75.0(75.0)	54901.95(13)	0.0000012(16)	0.000003(16)	54901.95231605(11)
J1455 - 3330	32.3622120(3)	76.174567474(14)	0.00016965(2)	223.458(6)	55531.1454(14)			
J1600 - 3053	8.8016526(10)	14.348468(3)	0.000173741(11)	181.854(16)	55419.1115(6)			
J1614 - 2230	11.29119744(7)	8.68661942171(9)	0.000001333(8)	175.9(4)	55662.053(10)	0.00000096(9)	-0.000001330(7)	55658.145347857(6)
J1640 + 2224	55.329717(4)	175.460597(13)	0.00079725(2)	50.7313(15)	54784.4707(7)			
J1643 - 1224	25.0725904(3)	147.01739554(4)	0.000505752(18)	321.849(2)	54870.5948(8)			
J1713 + 0747	32.34242188(14)	67.82513826930(19)	0.0000749402(6)	176.1963(16)	53761.0327(3)			
J1738 + 0333	0.3434297(2)	0.35479073425(6)	0.0000004(10)	252.0(140.0)	55598.94(14)	-0.0000004(10)	-0.000001(9)	55598.93613993(12)
J1741 + 1351	11.0033168(4)	16.3353478266(6)	0.00000998(2)	204.00(17)	55812.321(8)	-0.00000406(3)	-0.00000912(2)	55819.25468493(3)
J1853 + 1303	40.76952255(13)	115.653786432(6)	0.000023700(6)	346.656(11)	56128.563(3)			
B1855 + 09	9.2307805(2)	12.32717119133(19)	0.00002163(2)	276.54(5)	54975.5129(19)			
J1903 + 0327	105.593463(3)	95.17411738(8)	0.43667843(2)	141.6536021(15)	55776.9743424(3)			
J1909 - 3744	1.89799095(4)	1.533449451246(8)	0.000000092(13)	179.0(13.0)	54514.49(6)	0.00000000(2)	-0.00000092(12)	54513.989936084(3)
J1910 + 1256	21.1291025(2)	58.466742058(5)	0.00023020(2)	106.013(6)	54956.3186(11)			
J1918 - 0642	8.3504665(2)	10.9131775801(2)	0.000020340(18)	219.38(6)	54893.7305(17)	-0.00001291(2)	-0.000015721(13)	54897.63652454(2)
J1949 + 3106	7.288647(7)	1.9495344177(8)	0.0000429(3)	208.0(6)	56365.552(3)	-0.0000201(5)	-0.0000379(2)	56365.97423581(3)
B1953 + 29	31.4126915(2)	117.349097292(19)	0.000330230(15)	29.483(2)	55265.7096(7)			
J2017 + 0603	2.1929208(9)	2.1984811364(4)	0.00000685(15)	177.0(3.0)	56201.626(15)	0.0000004(3)	-0.00000684(15)	56200.64259488(3)
J2043 + 1711	1.6239584(2)	1.48229078649(14)	0.00000489(13)	240.4(1.2)	56173.974(5)	-0.00000425(13)	-0.00000242(9)	56174.306240718(10)
J2145 - 0750	10.16410849(17)	6.83890250963(11)	0.000019295(19)	200.91(5)	54902.6174(9)			
J2214 + 3000	0.0590817(3)	0.4166329463(9)	0.000009(11)	345.0(72.0)	56221.96(8)	-0.000002(10)	0.000008(11)	56221.9632381(4)
J2302 + 4442	51.4299676(5)	125.93529697(13)	0.000503021(17)	207.8925(18)	56302.6599(6)			
$J_{2317+1439}$	2.313943(4)	2.45933146519(2)	0.0000007(5)	101.0(42.0)	54976.1(3)	-0.0000007(5)	0.00000015(6)	54976.609358785(14)

Table 3.1: Values in parentheses denote the 1σ uncertainty in the preceding digit(s), as determined from TEMPO2. For MSPs with both DD and ELL1 parameters listed in this table, we used the ELL1 model to describe the Keplerian orbit in the TEMPO2 fit, and then derived the corresponding DD values; the 1σ uncertainties for the derived DD parameters were computed by propagating 1σ uncertainties in the fitted ELL1 parameters. The values for PSR J1713+0747 were taken from Zhu et al. (2015).

3.2 Binary Timing Models

We used the TEMPO2 pulsar-timing software package for the analysis of topocentric TOAs collected for all NANOGrav binary MSPs, based on solutions made publicly available by Arzoumanian et al. (2015b) that were obtained using GLS fitting. Each timing model includes parameters that describe the given pulsar's spin and spin-down rates, astrometry (i.e. eclipticcoordinate position, proper motion and annual timing parallax), DM evaluated at monthly intervals, binary motion, and evolution in pulse-profile structure as a function of observing frequency. As discussed in Chapter 2, several NANOGrav collaborators have led studies that focus on different subsets of timing parameters among the NANOGrav MSPs in order to maximize the amount of astrophysical information derivable from them. For this project, we only directly examine the measurements relevant to binary motion and any observed variations in the orbital elements.

For each binary system, five Keplerian parameters were included in the timing model. We also included timing parameters that describe secular variations in the orbital elements, and/or the Shapiro timing delay, if the least-squares fit in TEMPO2 was significantly improved, such that the F-test significance value was at least 0.0027 (i.e. each parameter is at least 3σ significant). Finally, we chose to fit for secular variations in the projected semi-major axis (x) for PSRs J1600-3053 and J1909-3744, and a secular variation in $P_{\rm b}$ of PSR J1614-2230, despite their lack of 3σ significance; the reasons for these additions are discussed in Section 3.4 below.

3.2.1 Parametrizations of the Shapiro Delay

The timing solutions developed by Arzoumanian et al. (2015b) used the "traditional" parametrization of the Shapiro timing delay discussed in Chapter 1, where $\Delta_{\rm S}$ is a function of r and s (Equation 1.27 or 1.30 for DD or ELL1 models, respectively). For this detailed study of MSP-binary systems, we also created timing solutions that used the "orthometric" parametrization of the Shapiro timing delay (Freire & Wex, 2010). The orthometric framework expresses the observed $\Delta_{\rm S}$ as a Fourier expansion across each system's orbital period and uses two different PK parameters that are derived from the harmonics of the Shapiro-delay signal to describe the relativistic effect. It is a generalized framework based on the Fourier expansion of $\Delta_{\rm S}$ introduced in Section 1.4 for the entire range of system inclination. In the orthometric framework, the PK parameters are either the third and fourth harmonic amplitudes of $\Delta_{\rm S}$ (referred to as h_3 and h_4 , respectively), or h_3 and the orthometric ratio $\varsigma = h_4/h_3$. In practice, the choice of (h_3, h_4) as PK parameters is most appropriate for low-*e* systems with $i < 60^{\circ}$, while (h_3, ς) is used for eccentric systems and low-*e* systems with $i > 60^{\circ}$.

While no new physical information is made available by its PK parameters, the orthometric parametrization reduces statistical correlation between the Shapiro-delay parameters. The orthometric model therefore provides a more numerically stable solution to the timing of binary pulsars with significant Shapiro-delay signals, particularly in low-*e* systems where $\Delta_{\rm S}$ is more difficult to measure. The available orthometric PK parameters are related to the traditional PK parameters as nonlinear functions:

$$\varsigma = \sqrt{\frac{1 - \cos i}{1 + \cos i}} \tag{3.1}$$

$$h_3 = r\varsigma^3 \tag{3.2}$$

$$h_4 = h_3\varsigma. \tag{3.3}$$

As shown by Freire & Wex (2010), the statistical significance of h_3 reflects the degree to which Δ_S is measurable and can therefore be used as a straightforward indicator for the detection of the Shapiro timing delay in a pulsar-binary system. In this study, we considered the Shapiro delay to be measurable if the estimate of h_3 was statistically significant to at least 3σ . For all systems

with significant $\Delta_{\rm S}$, as well as systems with statistically significant eccentricities that did not pass the h_3 significance test, we used the (h_3, ς) parameters to describe the Shapiro timing delay. For low-*e* systems with no significant $\Delta_{\rm S}$, we instead parameterized $\Delta_{\rm S}$ using the (h_3, h_4) combination.

Given the relations between the $(m_c, \sin i)$ and (h_3, ς) parameters in Equations 3.1 and 3.2, physical arguments require that $h_3 > 0$ and $0 < \varsigma < 1$. Equation 3.3 subsequently requires that h_4 be positive, as well. However, TEMPO2 does not impose any theoretically-motivated constraints on the Shapiro-delay parameters (traditional or orthometric) during a timing-model fit; it is therefore mathematically allowed for the Shapiro-delay terms to possess values that exceed their physical limits. Such limit discrepancies are not expected to be an issue for significant Δ_S signals, but may occur for nondetections of the Shapiro delay due to large statistical correlation between parameters when the Δ_S signal is weak.

3.3 Analyses of Mass & Geometric Parameters

We measured the Shapiro timing delay in fifteen NANOGrav binary-MSP systems, as well as many secular and PK variations in several orbital elements, based on the F-test significance criterion used by Arzoumanian et al. (2015b). In this work, we analyzed only fourteen of the fifteen MSPs with significant $\Delta_{\rm S}$ since PSR J1713+0747 was recently studied by Zhu et al. (2015) using NANOGrav and archival data sets. The other fourteen NANOGrav binary MSPs with significant $\Delta_{\rm S}$ also passed the 3σ -significance test of h_3 , as described in Section 3.2.1. The secular/PK measurements are shown in Table 3.2.
PSR	$\dot{\omega} \; (\text{deg yr}^{-1})$	$\dot{x} (10^{-12})$	$\dot{P}_{\rm b}~(10^{-12})$	$h_3 ~(\mu { m s})$	$h_4 \ (\mu s)$	ς	Detection of $\Delta_{\rm S}$?	Span (yr)
J0023+0923				0.06(5)	-0.00(6)		Ν	2.3
J0613 - 0200				0.28(3)		0.74(8)	Υ	8.6
J1012 + 5307				-0.00(9)	0.05(10)		Ν	9.2
J1455 - 3330		-0.021(5)		0.3(2)		0.7(4)	Ν	9.2
J1600 - 3053	0.007(2)	-0.0017(9)		0.39(3)		0.62(6)	Υ	6.0
J1614 - 2230			1.3(7)	2.329(11)		0.9859(2)	Υ	5.1
J1640 + 2224	-0.00028(5)	0.0145(10)		0.57(6)		0.61(8)	Υ	8.9
J1643 - 1224	•••	-0.047(3)		-0.09(13)		1.2(8)	Ν	9.0
J1713 + 0747		0.00645(11)		0.54(3)		0.73(1)	Υ	8.8
J1738 + 0333				0.02(12)	0.06(13)		Ν	4.0
J1741 + 1351		-0.0094(18)		0.46(6)		0.85(10)	Y	4.2
J1853 + 1303		0.0147(19)		0.11(11)		0.5(1.2)	Ν	5.6
B1855 + 09				1.04(4)		0.969(5)	Y	8.9
J1903 + 0327	0.0002410(13)			2.0(3)		0.70(8)	Υ	4.0
J1909 - 3744		-0.00044(16)	0.509(9)	0.868(7)		0.9381(16)	Υ	9.1
J1910 + 1256		-0.017(2)		0.3(2)		0.7(7)	Ν	8.8
J1918 - 0642				0.83(3)		0.918(8)	Υ	9.0
J1949 + 3106				2.5(5)		0.77(10)	Υ	1.2
B1953 + 29		0.011(3)		-0.1(6)		0.8(5)	Ν	7.2
J2017 + 0603				0.31(7)		0.72(8)	Υ	1.7
J2043 + 1711				0.60(3)		0.890(13)	Υ	2.3
J2145 - 0750		0.0098(19)		0.10(5)		0.94(17)	Ν	9.1
J2214 + 3000				-0.3(2)	-0.1(3)		Ν	2.1
J2302 + 4442				1.5(3)		0.55(15)	Y	1.7
J2317+1439				0.33(6)		0.49(14)	Y	8.9

Table 3.2: Values in parentheses denote the 1σ uncertainty in the preceding digit(s), as determined from TEMPO2.

3.3.1 Bayesian Analyses of Shapiro-Delay Signals

We used the procedure outlined in Appendix A to perform a statistically rigorous analysis of the fourteen MSPs in the nine-year data set with significant Shapiro-delay measurements and obtain robust estimates of $m_{\rm p}$, $m_{\rm c}$, and i. For each of the fourteen MSPs, we first created a uniform, two-dimensional $n \times n$ grid of χ^2 values for different combinations of $m_{\rm c} = r/T_{\odot}$ (Equation 1.42) and $\cos i$, where n = 200 or greater in order to minimize artifacts from interpolation. With the exception of the noise parameters, all other timingmodel parameters were allowed to vary freely when estimating the χ^2 at each grid coordinate; the noise terms were held fixed at their maximum-likelihood values as determined by Arzoumanian et al. (2015b). We used $\cos i$ instead of $\sin i$ as a grid coordinate since a collection of randomly-oriented binary systems possesses a uniform distribution in $\cos i$; this assertion is proven in Section A.2 below.

Each χ^2 map was then converted to a two-dimensional probability distribution function (PDF) by using a Bayesian likelihood density of the following form,

$$p(\text{data}|m_{\rm c}, \cos i) \propto e^{-(\chi^2 - \chi_0^2)/2}$$
 (3.4)

where χ_0^2 is the minimum value of the χ^2 distribution defined on the twodimensional grid. Bayes' theorem subsequently yields the two-dimensional posterior PDF, $p(m_c, \cos i | \text{data})$, when using the joint-uniform prior distribution of the two Shapiro-delay parameters. We then marginalized (i.e. integrated) the two-dimensional PDF over $\cos i$ to obtain the one-dimensional PDF in m_c , and marginalized over m_c to obtain the one-dimensional PDF in $\cos i$. In order to obtain a PDF in m_p , we transformed the two-dimensional $(m_c, \cos i)$ probability grid to one in the $(m_p, \cos i)$ space by applying the transformation rule for PDFs of random variables,

$$p(m_{\rm p}, \cos i | \text{data}) = p(m_{\rm c}, \cos i | \text{data}) \frac{\partial m_{\rm c}}{\partial m_{\rm p}},$$

where the partial derivative is evaluated by using the mass function (Equation 1.37, for a fixed value of $\cos i$). The expressions for map translation are provided in Section A.3.

For the two-dimensional grids, we computed χ^2 values over $0 < \cos i < 1$ and, unless otherwise noted, $0 < m_c < 1.4 M_{\odot}$. The latter upper limit approximately corresponds to the Chandrasekhar limit for a non-rotating white dwarf. (Three exceptions to this cut-off limit are PSRs J1903+0327, J1949+3106, and J2302+4442, which are discussed individually in Section 3.4 below.) The arbitrary upper limit on the companion mass does not affect the most significant $\Delta_{\rm S}$ measurements, where all non-zero probability is typically enclosed in a small, elliptical region of the $(m_c, \cos i)$ space. The cut-off value only biases estimates made for statistically weak Shapiro-delay measurements, where non-zero probability can extend to large values of m_c and low values of i; this bias is discussed in Section 3.3.2 below.

We applied the same set of χ^2 -grid and marginalization procedures described above for the fourteen timing models with significant $\Delta_{\rm S}$ that used the (h_3, ς) orthometric parametrization. However, we first created a χ^2 grid in uniform steps of the (h_3, ς) parameters, and afterwards converted the resultant likelihood density to the $(m_c, \cos i)$ probability map by using Equations 3.1 and 3.2 when applying the PDF-transformation rule.

The choice in parametrization of $\Delta_{\rm S}$ amounts to a difference in prior probabilities on the physical parameters $(m_{\rm c}, \sin i)$ when performing the MCMC or χ^2 -grid analysis described above, due to the nonlinear relation between the physical and orthometric parameters (Equations 3.1-3.3). Our first choice of prior, in $(m_{\rm c}, \cos i)$, is motivated by the expected distribution of randomly oriented binary systems – uniform in $\cos i$ – though the choice of uniform $m_{\rm c}$ is arbitrary. On the other hand, Freire & Wex (2010) argue that a statistical analysis of the orthometric parameters is preferable since h_3 and ς are related to the Fourier harmonics of $\Delta_{\rm S}$ and make no immediate assumption on the probability distributions of physical parameters. Simulations by Freire & Wex show that the one-dimensional posterior PDFs of the physical parameters will be affected in cases of low inclination, where $\Delta_{\rm S}$ is typically weaker and the posterior density is heavily influenced by the choice of prior information. For cases in which there is a highly-significant measurement of $\Delta_{\rm S}$, such that the posterior density spans a small range of parameter space, the two choices of priors give essentially the same results. We present the results obtained from both sets of priors to demonstrate the effects such choices have on our mass measurements.

MCMC Analysis of Shapiro-delay Parameters

As a check on the χ^2 -grid procedure described above, we evaluated the parameters of each binary system using a Bayesian Markov Chain Monte Carlo (MCMC; e.g. Gregory, 2005a) analysis of all timing-model parameters. In the MCMC analysis, where we used the PAL2 Bayesian inference suite³, the joint likelihood density includes all spin, astrometric, binary and noise terms as parameters to be sampled. The Bayesian analysis uses the traditional $(m_c, \cos i)$ parameterization for the Shapiro delay, along with uniform priors on these and all other timing model parameters. We analytically marginalized the joint posterior over the DM, profile-evolution, and backend-offset parameters in order to reduce computational needs.

In principle, the MCMC analysis therefore provides a more robust exploration of the parameter space and timing-model behavior than the χ^2 -grid analysis, since the MCMC method samples the noise parameters, while the χ^2 -grid holds the noise parameters fixed. Moreover, for the MCMC analysis, the computation of m_p accounts for the small uncertainty in the mass function, as it uses the posterior distributions for the Shapiro-delay and Keplerian parameters.

³https://github.com/jellis18/PAL2



Figure 3.1: Normalized posterior PDFs of $m_{\rm p}$, $m_{\rm c}$ and $\cos i$ for PSR J2043+1711. The red-solid curves were obtained from a χ^2 -grid analysis, and the blue-dashed curves were generated from an MCMC analysis of all timing-model parameters (including terms that characterize red- and white-noise processes) when drawing 10⁶ samples and using a thinning factor of 10 to reduce autocorrelation. The χ^2 -grid and MCMC methods yield nearly identical estimates of the posterior PDFs.

Figure 3.1 shows the normalized posterior PDFs of the Shapiro-delay parameters for PSR J2043+1711 (see Section 3.4.14) estimated from both the χ^2 -grid and MCMC analyses. It is clear that the χ^2 -grid and MCMC analyses yield nearly identical estimates of the posterior distributions of the component masses and $\cos i$. This consistency between methods is seen for all 14 MSPs with significant $\Delta_{\rm S}$. Thus the χ^2 -grid method is a reliable method for estimating posterior PDFs when using an adequate (fixed) noise model. All estimates reported below were obtained from the χ^2 -grid method and verified using PAL2.

Constraints from $(\dot{\omega})_{\rm GR}$ on Shapiro-delay Parameters

Both PSRs J1600-3053 and J1903+0327 exhibit statistically significant measurements of $\dot{\omega}$ and $\Delta_{\rm S}$. As discussed in Sections 3.4.3 and 3.4.9 below, the $\dot{\omega}$ measurements in these two systems are likely due to GR. We therefore generated additional χ^2 grids of the two Shapiro-delay parameters for PSRs J1600-3053 and J1903+0327 that used the statistical significance of $\dot{\omega}$ to improve our estimates of the Shapiro-delay parameters in the following manner:

- for each (m_c, cos i) coordinate on the χ² grid, we computed a value of m_p using the mass function for the given system; for the orthometric grids, we first used Equations 3.1 and 3.2 to compute m_c and cos i at each (h₃, ς) grid coordinate, and then used the mass function to compute m_p;
- we then used the values of m_p and m_c, along with the Keplerian elements of the given system, to compute (*ω*)_{GR} using Equation 1.39 at the (m_c, cos i) or (h₃, ς) grid points;
- we then held the $\dot{\omega}$ parameter fixed in the timing solution at the value given by $(\dot{\omega})_{\rm GR}$, along with the Shapiro-delay parameters, and used TEMPO2 to obtain a constrained χ^2 value.

We then used Equation 3.4 and the marginalization procedures discussed above to obtain constrained PDFs of $m_{\rm p}$, $m_{\rm c}$ and *i* from both parametrizations of $\Delta_{\rm S}$.

Constraints from geometric variations on Shapiro-delay Parameters

PSRs J1640+2224 and J1741+1351 have significant measurements of $\Delta_{\rm S}$ and secular variations in x that are likely due to proper motion, a biasing effect discussed in Section 1.5.2. However, PSR J1640+2224 also exhibits a significant $\dot{\omega}$ that is currently not well understood in terms of the various mechanisms outlined in Section 1.5 above (see Section 3.4.5 for a discussion). We therefore only analyze the observed geometric variation in x for PSR J1741+1351.

In the case of PSR J1741+1351, we generated χ^2 grids that explicitly modeled the observed \dot{x} in terms of system geometry at each grid point using Equation 1.43. The observed Shapiro delay yields a measure of sin *i*, and so an estimate of Ω can be made using the observed \dot{x} .⁴ We used the T2 binary timing model in TEMPO2, a general binary framework that uses the DD or ELL1 models when appropriate but also allows for *i* and Ω to be used as fit parameters; the T2 timing model computes both the secular and periodic variations in *x* (Equations 1.43 and 1.47) and ω (Equations 1.44 and 1.48) given the two geometric parameters.

The explicit modeling of orbital variations due to changes in geometry introduces Ω as an *a priori* unknown parameter; we therefore generated threedimensional χ^2 grids in the uniform $(m_c, \cos i, \Omega)$ and (h_3, ς, Ω) phase spaces for PSR J1741+1351, using Equation 3.4 as the likelihood density at each grid point in the three-dimensional phase space. We then appropriately translated

⁴The sign ambiguity of $\cos i$ as well as the functional form of $(\dot{x})_{\mu}$ results in four possible combinations of (i, Ω) . We discuss the possibility of breaking the degeneracy in the last paragraph of Section 3.3.1.

PSR	$i_{\rm SD}~({\rm deg})$	$i_{\dot{x}} (\text{deg})$
J0023+0923	< 56	
J1012 + 5307	< 66	
J1455 - 3330	< 85	< 77
J1643 - 1224	< 73	< 37
J1738 + 0333	< 70	
J1853 + 1303	< 74	< 63
J1910 + 1256	< 63	< 63
B1953 + 29	< 80	< 77
J2145 - 0750	< 80	< 73
J2214 + 3000	< 75	

Table 3.3: Upper limits of the system inclination for MSPs that do not have significant measurements of $\Delta_{\rm S}$; if available, we use the measured \dot{x} to compute a second, independent constraint. All upper limits are at 95% confidence.

and marginalized the three-dimensional probability maps in order to obtain one-dimensional posterior PDFs of $m_{\rm p}$, $m_{\rm c}$, i and Ω .

If the Shapiro timing delay and only \dot{x} are measured, then the $(m_c, \cos i, \Omega)$ and (h_3, ς, Ω) grid analyses will introduce a sign ambiguity in Ω due to the fact that the variation depends both $\cot i$ and $\sin(\Theta_{\mu} - \omega)$. In this case, the ambiguity in both $\cos i$ and Ω results in a four-fold degeneracy in the system orientation (i, Ω) of the orbit. However, if two or more secular and/or periodic variations are measured, then the four-fold degeneracy can be broken to determine a unique orientation of the MSP-binary orbit. We consider the relevance of annual orbital parallax for PSRs J1640+2224 and J1741+1351 below.

3.3.2 Limits on Inclination from \dot{x} and the Absence of Shapiro Delay

A constraint on the system inclination angle can still be placed using the \dot{x} measurements listed in Table 3.2 (e.g. Nice et al., 2001) for cases where the

Shapiro timing delay is not detected. This is possible since the trigonometric term for Ω in Equation 1.43 cannot exceed unity, i.e. $\sin(\Theta_{\mu} - \Omega) \leq 1$, where the equality corresponds to an alignment between the proper-motion vector and the projection of the orbital angular moment vector on the plane of the sky. The "magnitude" of the effect can therefore be written as $|\dot{x}|_{\mu,\max} = \mu x |\cot i|$, and an upper limit on the system inclination can be calculated as

$$i < \arctan\left[\frac{x\mu}{|\dot{x}|_{\text{obs}}}\right].$$
 (3.5)

We computed a 95.4%-credibility upper limit on the system inclination using Equation 3.5 and the 2σ lower limit of the \dot{x} measurements reported in Table 3.3 for systems with no detected Shapiro delay.

Another constraint on i can be placed by using a non-detection of the Shapiro timing delay. The Shapiro-delay χ^2 grids of pulsar-binary systems with no measurable $\Delta_{\rm S}$ contain zero probability in regions of the $(m_{\rm c}, \cos i)$ space that correspond to large companion masses and high inclinations. These regions can be excluded based on statistically poor timing-model fits to the NANOGrav nine-year data sets.

A complication arises from the cut-off value in m_c when generating the χ^2 grids as discussed in Section 3.3.1: the cut-off value disregards regions of the $(m_c, \cos i)$ phase space with non-zero probability density. We believe that the cut-off value in m_c is nonetheless justified since the only MSP with a suspected main-sequence-star companion is PSR J1903+0327.⁵ The inclusion of more probability density in non-detection χ^2 grids would shift the upper limit on *i* to lower values, so the upper limits on *i* we report in this study are considered to be conservative. Figures 3.2-3.10 show the χ^2 -grid and upper-limit results for all binary MSPs with no significant detection of Δ_s , and the estimates of upper limits on *i* for these systems are provided in Table 3.3.

⁵While we extended the upper limit on m_c for PSRs J1949+3106 and J2302+4442 to 5 M_{\odot} , we believe that the detections of Δ_S in their TOA residuals warrant more stringent analysis of the probability density.



Figure 3.2: Top A $(m_c, \cos i)$ probability map for PSR J0023+0923. The significance of h_3 in this system is less than 3σ , so we only compute upper limits on *i*. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability. Bottom. Posterior PDF of the derived inclination angle, obtained from the $(m_c, \cos i)$ grid shown in the top panel. The shaded blue region under the PDF contains 95% of the total probability.



Figure 3.3: Top. A $(m_c, \cos i)$ probability map for PSR J1012+5307. The significance of h_3 in this system is less than 3σ , so we only compute upper limits on *i*. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability. Bottom. Posterior PDF of the derived inclination angle, obtained from the $(m_c, \cos i)$ grid shown on the top. The shaded blue region under the PDF contains 95% of the total probability.



Figure 3.4: Top. A $(m_c, \cos i)$ probability map for PSR J1455-3330. The significance of h_3 in this system is less than 3σ , so we only compute upper limits on *i*. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability. Bottom. Posterior PDF of the derived inclination angle, obtained from the $(m_c, \cos i)$ grid shown on the top. The shaded blue region under the PDF contains 95% of the total probability.



Figure 3.5: Top. A $(m_c, \cos i)$ probability map for PSR J1643-1224. The significance of h_3 in this system is less than 3σ , so we only compute upper limits on *i*. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability. Bottom. Posterior PDF of the derived inclination angle, obtained from the $(m_c, \cos i)$ grid shown on the top. The shaded blue region under the PDF contains 95% of the total probability.



Figure 3.6: Top. A $(m_c, \cos i)$ probability map for PSR J1738+0333. The significance of h_3 in this system is less than 3σ , so we only compute upper limits on *i*. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability. Bottom. Posterior PDF of the derived inclination angle, obtained from the $(m_c, \cos i)$ grid shown on the top. The shaded blue region under the PDF contains 95% of the total probability.



Figure 3.7: Top. A $(m_c, \cos i)$ probability map for PSR J1853+1303. The significance of h_3 in this system is less than 3σ , so we only compute upper limits on *i*. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability. Bottom. Posterior PDF of the derived inclination angle, obtained from the $(m_c, \cos i)$ grid shown on the top. The shaded blue region under the PDF contains 95% of the total probability.



Figure 3.8: Top. A $(m_c, \cos i)$ probability map for PSR J1910+1256. The significance of h_3 in this system is less than 3σ , so we only compute upper limits on *i*. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability. Bottom. Posterior PDF of the derived inclination angle, obtained from the $(m_c, \cos i)$ grid shown on the top. The shaded blue region under the PDF contains 95% of the total probability.



Figure 3.9: Top. A $(m_c, \cos i)$ probability map for PSR J2145-0750. The significance of h_3 in this system is less than 3σ , so we only compute upper limits on *i*. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability. Bottom. Posterior PDF of the derived inclination angle, obtained from the $(m_c, \cos i)$ grid shown on the top. The shaded blue region under the PDF contains 95% of the total probability.



Figure 3.10: Top. A $(m_c, \cos i)$ probability map for PSR J2214+3000. The significance of h_3 in this system is less than 3σ , so we only compute upper limits on *i*. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability. *Bottom.* Posterior PDF of the derived inclination angle, obtained from the $(m_c, \cos i)$ grid shown on the top. The shaded blue region under the PDF contains 95% of the total probability.

PSR	Pulsar Mass (M_{\odot})		Companion	Mass (M_{\odot})	System Inclination (deg)	
	Trad	Ortho	Trad	Ortho	Trad	Ortho
J0613-0200	$2.3^{+2.7}_{-1.1}$	$2.1^{+2.1}_{-1.0}$	$0.21^{+0.23}_{-0.10}$	$0.19^{+0.15}_{-0.07}$	66^{+8}_{-12}	68^{+7}_{-10}
J1600-3053	$2.4^{+1.5}_{-0.9}$	$2.4^{+1.3}_{-0.8}$	$0.33\substack{+0.14 \\ -0.10}$	$0.33\substack{+0.13 \\ -0.08}$	63^{+5}_{-5}	64^{+4}_{-5}
J1614 - 2230	$1.928\substack{+0.017\\-0.017}$	$1.928\substack{+0.017\\-0.017}$	$0.493\substack{+0.003 \\ -0.003}$	$0.493\substack{+0.003 \\ -0.003}$	$89.189\substack{+0.014\\-0.014}$	$89.188\substack{+0.014\\-0.014}$
J1640+2224	$4.4^{+2.9}_{-2.0}$	$5.2^{+2.6}_{-2.0}$	$0.6^{+0.4}_{-0.2}$	$0.7^{+0.3}_{-0.2}$	60^{+6}_{-6}	58^{+6}_{-6}
J1713+0747	$1.31_{-0.11}^{+0.11}$	$1.31\substack{+0.11 \\ -0.11}$	$0.286\substack{+0.012\\-0.012}$	$0.286\substack{+0.012\\-0.012}$	$71.9_{-0.7}^{+0.7}$	$71.9\substack{+0.7 \\ -0.7}$
J1741+1351	$1.87^{+1.26}_{-0.69}$	$1.78_{-0.63}^{1.08}$	$0.32_{-0.09}^{+0.15}$	$0.31\substack{+0.13 \\ -0.08}$	66^{+5}_{-6}	66^{+5}_{-6}
B1855+09	$1.30\substack{+0.11 \\ -0.10}$	$1.31\substack{+0.12 \\ -0.10}$	$0.236\substack{+0.013\\-0.011}$	$0.238\substack{+0.013\\-0.012}$	$88.0^{+0.3}_{-0.4}$	$88.0\substack{+0.3 \\ -0.4}$
J1903+0327	$1.65_{-0.02}^{+0.02}$	$1.65\substack{+0.02 \\ -0.03}$	$1.06\substack{+0.02\\-0.02}$	$1.06\substack{+0.02\\-0.02}$	72^{+2}_{-3}	72^{+2}_{-3}
J1909-3744	$1.55_{-0.03}^{+0.03}$	$1.55\substack{+0.03 \\ -0.03}$	$0.214\substack{+0.003\\-0.003}$	$0.214\substack{+0.003\\-0.003}$	$86.33\substack{+0.09\\-0.10}$	$86.33\substack{+0.09\\-0.10}$
J1918-0642	$1.18\substack{+0.10 \\ -0.09}$	$1.19\substack{+0.10 \\ -0.09}$	$0.219\substack{+0.012\\-0.011}$	$0.219\substack{+0.012\\-0.011}$	$85.0^{+0.5}_{-0.5}$	$85.0^{+0.5}_{-0.5}$
J1949+3106	$4.0^{+3.6}_{-2.5}$	$4.0^{+3.4}_{-2.3}$	$2.1^{+1.6}_{-1.0}$	$1.9^{+1.5}_{-0.9}$	67^{+9}_{-8}	68^{+8}_{-8}
J2017+0603	$2.4^{+3.4}_{-1.4}$	$2.0^{+2.8}_{-1.1}$	$0.32_{-0.16}^{+0.44}$	$0.27\substack{+0.30 \\ -0.12}$	62^{+9}_{-12}	65^{+7}_{-11}
J2043+1711	$1.41_{-0.18}^{+0.20}$	$1.43\substack{+0.21 \\ -0.18}$	$0.175\substack{+0.016\\-0.015}$	$0.177\substack{+0.017\\-0.015}$	$83.2^{+0.8}_{-0.9}$	$83.1_{-0.9}^{+0.8}$
J2302+4442	$5.3^{+3.2}_{-3.6}$	$5.5^{+3.0}_{-3.2}$	$2.3^{+1.7}_{-1.3}$	$1.8^{+1.6}_{-1.0}$	54^{+12}_{-7}	57^{+11}_{-9}
J2317+1439	$4.7^{+3.4}_{-2.8}$	$4.1^{+3.5}_{-2.4}$	$0.7\substack{+0.5 \\ -0.4}$	$0.5\substack{+0.5 \\ -0.3}$	47^{+10}_{-7}	51_{10}^{+10}

Table 3.4: Estimate of $m_{\rm p}$, $m_{\rm c}$ and *i* for NANOGrav Binary MSPs with significant Shapiro-delay signals. All uncertainties reflect 68.3% confidence intervals. "Trad" refers to estimates made with the traditional $(m_{\rm c}, \sin i)$ Shapiro-delay model, while "Ortho" refers to those made with the orthometric (h_3, ς) model. Difference in median values and confidence intervals reflect the consequence in choosing uniform Bayesian priors on the $(m_{\rm c}, \sin i)$ or (h_3, ς) parameters for weak measurements of $\Delta_{\rm S}$. Observed secular variations used as constraints for PSRs J1600 – 3053, J1741 + 1351, and J1903 + 0327. The values for PSR J1713+0747 were taken from Zhu et al. (2015).

3.4 Results & Discussion

The traditional and orthometric parameterizations of the Shapiro timing delay yield consistent measurements of the component masses, i, and Ω (when the latter angle is measurable) in the fourteen NANOGrav MSP-binary systems with significant $\Delta_{\rm S}$ that we analyze here. We report estimates that were made using both Shapiro-delay models for each of these 14 MSPs in Table 3.4. Any differences in the estimates and confidence intervals derived from the traditional (m_c , sin i) or orthometric (h_3 , ς) probability grids reflect different Bayesian priors on those PK parameters; the most highly-inclined systems produced essentially identical estimates. These features are consistent with the expectations discussed in Section 3.3.1.

Unless otherwise specified, all numerical values with uncertainties presented below reflect 68.3% equal-tailed credible intervals; that is, we compute the credible interval by numerically integrating each (normalized) posterior PDF to values of the parameter that contain 15.9% (lower bound), 50% (median), and 84.1% (upper bound) of all probability (see Appendix A.4 for details).

3.4.1 PSR J0613-0200

PSR J0613−0200 is a 3.1-ms pulsar in a 1.2-day orbit that was discovered in a survey of the Galactic disk using the Parkes radio telescope (Lorimer et al., 1995). A previous long-term timing study of this MSP by Hotan et al. (2006) used the lack of a Shapiro-delay detection to place constraints on the companion mass and system inclination, such that $0.13 < m_c/M_{\odot} <$ 0.15 and 59° $< i < 68^{\circ}$ if $m_p = 1.3 M_{\odot}$. Two recent, independent TOA analyses of PSR J0613-0200 were performed by Reardon et al. (2016) and Desvignes et al. (2016). Reardon et al. used an 11-yr data set collected for the Parkes Pulsar Timing Array (PPTA) and did not report any secular variations or PK effects. Desvignes et al. used a 16-yr data set collected for the European Pulsar Timing Array (EPTA) to be measure a significant $\dot{P}_{\rm b} = 4.8(1.1) \times 10^{-14}$. Neither study reports a detection of the Shapiro timing delay. A recent optical-spectroscopy study did not detect the companion to PSR J0613-0200, and placed a 5σ -detection lower limit on the photometric R-band magnitude to be R > 23.8 (Bassa et al., 2015).

For the first time, we report the detection of the Shapiro timing delay in the PSR J0613-0200 system using the NANOGrav nine-year data set. It is likely that the Shapiro-delay signal in PSR J0613-0200 went undetected by Reardon et al. (2016) and Desvignes et al. (2016) because of the better sensitivity achieved with the GBT and GUPPI backend, as reflected by the factor of 2-3 improvement in TOA root-mean-square (RMS) residuals between the NANOGrav and PPTA/EPTA data sets. The χ^2 grids and marginalized PDFs for PSR J0613-0200 are shown in Figure 3.11. Our current estimates of $m_c = 0.18^{+0.15}_{-0.07} M_{\odot}$ and $i = 68^{+7}_{-10}$ degrees are consistent with the predictions made by Hotan et al. (2006), though our derived estimate of $m_p = 2.3^{+2.7}_{-1.1} M_{\odot}$ is not yet precise enough to yield a meaningful constraint on the pulsar mass.

3.4.2 PSR J1455-3330

PSR J1455-3330 is a 7.9-ms pulsar in a 76-day orbit and was discovered in a survey of the Galactic disk using the Parkes radio telescope (Lorimer et al., 1995). The long spin period of this MSP, along with its large orbit and anomalously large characteristic age, indicates potential disk instability during the transfer phase that ultimately dontated little mass to the neutron star (Li et al., 1998). A recent radio-timing analysis by Desvignes et al. (2016) reported a significant $\dot{x} = -1.7(4) \times 10^{-14}$.

We measured a significant $\dot{x} = -2.1(5) \times 10^{-14}$ in the PSR J1455-3330 system using the NANOGrav nine-year data set. Our estimate of \dot{x} is consistent with the one made by Desvignes et al. (2016) using an independent data set. We did not detect a Shapiro timing delay, as indicated by the insignificance of h_3 and unconstrained estimate of ς listed in Table 3.2.

3.4.3 PSR J1600-3053

PSR J1600-3053 is a 3.6-ms pulsar in a 14.3-day orbit that was discovered in a survey of high Galactic latitudes using the Parkes radio telescope (Jacoby et al., 2007). A recent analysis of the PSR J1600-3053 system by Reardon et al. (2016) used PPTA data to make significant measurements of \dot{x} and the Shapiro timing delay: $m_{\rm p} = 2.4(1.7) \,\mathrm{M}_{\odot}, m_{\rm c} = 0.34(15) \,\mathrm{M}_{\odot}, \sin i = 0.87(6),$ and $\dot{x} = -4.2(7) \times 10^{-15}$. Another recent and independent study by Desvignes et al. (2016) used EPTA data to measure the orthometric parameters $h_3 = 0.33(2) \,\mu$ s and $\varsigma = 0.68(5)$, consistent with the component masses and inclination measured by Reardon et al., as well as $\dot{x} = -2.8(5) \times 10^{-15}$.

We measured a significant $\dot{\omega}$ for the first time, as well as a Shapiro timing delay in the PSR J1600-3053 system. We do not yet measure a 3σ significant \dot{x} , likely because the NANOGrav data span for PSR J1600-3053 is ~6 yr, several years shorter than the EPTA and PPTA data sets. Nevertheless, we do make a tentative, ~ 2σ detection of $\dot{x} = -1.7(9) \times 10^{-15}$ and have elected to include it as a free parameter in our timing solution. Our estimates of \dot{x} and the orthometric parameters, $h_3 = 0.39(3)$ and $\varsigma = 0.62(6)$, are consistent with those made by Desvignes et al. (2016).

Our measurement of $\dot{\omega} = 7(2) \times 10^{-3} \text{ deg yr}^{-1}$ in the PSR J1600-3053 system could, in principle, be due to a combination of physical effects discussed in Section 1.5. The maximum amplitude of $(\dot{\omega})_{\mu}$ for PSR J1600-3053, computed from Equation 1.44, is $(\dot{\omega})_{\mu,\text{max}} = \mu | \csc i | \sim 10^{-6} \text{ deg yr}^{-1}$, which is two orders of magnitude smaller than the uncertainty level for the observed $\dot{\omega}$ in this MSP-binary listed in Table 3.2. Therefore, the observed $\dot{\omega}$ in the PSR J1600-3053 system cannot be due to secular variations from proper motion at the current level of precision.

The predicted GR component of $\dot{\omega}$ of PSR J1600-3053 is on the order of $10^{-3} \text{ deg yr}^{-1}$ given the Keplerian parameters of the system shown in Table

3.1, the same order of magnitude as our measured value. We therefore used the method described in Section 3.3.1 to include both $\dot{\omega}$ and the Shapirodelay parameters when generating the two-dimensional χ^2 grid. The χ^2 grids and marginalized PDFs for PSR J1600-3053 are shown in Figure 3.12; the constrained estimates of the component masses and inclination are: $m_{\rm p} =$ $2.4^{+1.5}_{-0.9} {\rm M}_{\odot}$; $m_{\rm c} = 0.33^{+0.14}_{-0.10} {\rm M}_{\odot}$; and i = 63(5) degrees. Our constrained estimates of the Shapiro delay parameters are consistent with the estimates made by Reardon et al. (2016) and Desvignes et al. (2016).

3.4.4 PSR J1614-2230

PSR J1614−2230 is a 3.2-ms pulsar in a 8.7-day orbit with a massive WD companion; this MSP was discovered in a mid-latitude radio search of unidentified EGRET gamma-ray sources using the Parkes radio telescope (Hessels et al., 2005; Crawford et al., 2006). The PSR J1614−2230 system contains one of the most massive neutron stars known, $m_{\rm p} = 1.97(4)$ M_☉, as determined by a strategic set of observations that were made and used by Demorest et al. (2010) to measure the Shapiro timing delay in this highly-inclined binary system. Demorest et al. were able to rule out nearly all models for plausible neutron-star equations of state that invoke significant amounts of exotic matter. Moreover, the PSR J1614−2230 system provided early evidence for relatively high "birth masses" of neutron stars after their formation, and before the onset of mass transfer (Tauris et al., 2011).

We made an improved measurement of the Shapiro timing delay in PSR J1614–2230 when using the NANOGrav nine-year data set, which includes a subset of the GUPPI data used by Demorest et al. (2010). The χ^2 grids and marginalized PDFs for PSR J0613–0200 are shown in Figure 3.13. The uncertainties in both $m_c = 0.493(3) M_{\odot}$ and i = 89.189(14) degrees have decreased such that the uncertainty in $m_p = 1.928(17) M_{\odot}$ is a factor of ~3 less than that made by Demorest et al. (2010).

Although there was not a formally significant measurement of orbital de-

cay, we nevertheless explored fitting for it. We measured $(\dot{P}_{\rm b})_{\rm obs} = 1.3(7) \times 10^{-12}$. This is much larger than the component expected from generalrelativistic orbital decay (Equation 1.38), $(\dot{P}_{\rm b})_{\rm GR} = -0.00042 \times 10^{-12}$. Instead, it is attributable to the change in the Doppler shift due to the pulsar motion, as discussed in Section 1.5; Equation 1.45 predicts that $(\dot{P}_{\rm b})_{\rm D} = P_{\rm b}(\dot{D}/D) = 1.36 \times 10^{-12}$ when using the pulsar distance and proper motion measured in the NANOGrav nine-year timing model, and which is consistent with the direct measurement we make here. Matthews et al. (2016) used the agreement between $(\dot{P}_{\rm b})_{\rm D}$ and the observed value as a confirmation of the parallax distance to the pulsar. The precision of $(\dot{P}_{\rm b})_{\rm obs}$ can be improved by extending the observing span backwards using pre-GUPPI archival data published by Demorest et al. (2010) and forwards (through future observations); this will eventually provide the most precise means for measuring the distance to this pulsar.

3.4.5 PSR J1640+2224

PSR J1640+2224 is a 3.1-ms pulsar in a 175-day orbit that was discovered in a Arecibo survey of high Galactic latitudes (Foster et al., 1995a,b). The companion star in this system was observed using the Palomar 5.1-m optical telescope to have an effective temperature that is consistent with an old He WD (Lundgren et al., 1995). The first dedicated radio-timing study of the PSR J1640+2224 system reported a tentative detection of the Shapiro timing delay, with $m_c = 0.15^{+0.08}_{-0.05} M_{\odot}$ and $\cos i = 0.11^{+0.09}_{-0.07}$ (Löhmer et al., 2005). However, Löhmer et al. did not derive a statistically significant constraint on m_p . A subsequent TOA analysis of the NANOGrav five-year data set (Demorest et al., 2013) used Markov chain fitting methods and noted issues with the numerical stability of the observed Shapiro timing delay (Vigeland & Vallisneri, 2014). The most recent radio-timing study by Desvignes et al. (2016) used EPTA data to measure a significant $\dot{x} = 1.07(16) \times 10^{-14}$, but did not measure a significant Shapiro delay. We measured the Shapiro timing delay, $\dot{x} = 1.45(10) \times 10^{-14}$ and $\dot{\omega} = -2.8(5) \times 10^{-4} \text{ deg yr}^{-1}$ using the NANOGrav nine-year data set for PSR J1640+2224. The χ^2 grids and marginalized PDFs of the Shapiro-delay parameters measured for this MSP are shown in Figure 3.14. Based on the Shapiro timing delay alone, we estimated that $m_c = 0.6^{+0.4}_{-0.2} \text{ M}_{\odot}$ and i = 60(6) degrees with the corresponding $m_p = 4.4^{+2.9}_{-2.0} \text{ M}_{\odot}$. The highly-significant \dot{x} , consistent with the estimate made by Desvignes et al. (2016) at the 2σ uncertainty level, is most likely due to a secular change in the inclination of the wide binary system induced by proper motion; the current data set is not sensitive to annual orbital parallax since the annual astrometric parallax was not found to be significant for PSR J1640+2224 (Matthews et al., 2016). However, we could not reconcile the 6σ -significant value of $\dot{\omega}$ with the physical mechanisms outlined in Section 1.5. In what follows below in this subsection, we explicitly discuss and reject the possibilities that were considered to explain the $\dot{\omega}$ measurement.

The general-relativistic component of $\dot{\omega}$ (Equation 1.39) cannot be the dominant term since our observed value is negative. We also rule out a significant detection of $(\dot{\omega})_{\text{GR}}$ since, given the fitted Keplerian elements listed in Table 3.1, its predicted value for large assumed component masses is on the order of 10^{-6} deg yr⁻¹. Furthermore, we reject the possibility of this measurement arising from secular orbital variations due to proper motion, since the predicted magnitude of $(\dot{\omega})_{\mu}$ (Equation 1.44) is also on the order of 10^{-6} deg yr⁻¹.

In principle, a nonzero value of $\dot{\omega}$ can arise from a spin-induced quadrupole term in the companion's gravitational potential due to classical spin-orbit coupling (Wex, 1998); this effect has been observed in pulsar-binary systems with main-sequence companions (e.g. Wex et al., 1998), and can also be observed in pulsar-WD systems in the case where a quadrupole term is induced from rapid rotation of the WD companion. This scenario was first considered in early studies of the relativistic PSR J1141-6545 system by Kaspi et al. (2000), where they noted that classical spin-orbital coupling would cause a time derivative in the system inclination angle, di/dt, that is comparable in order of magnitude to the component of $\dot{\omega}$ due to spin-orbit coupling. We used the \dot{x} measured in the PSR J1640+2224 system, the fact that $\dot{x} = d(a_p \sin i)/dt \approx (a_p \cos i) di/dt$, and the Shapiro-delay estimate of sin i to compute the time rate of change in the system inclination, and found that $di/dt \sim 10^{-6} \text{ deg yr}^{-1}$. This estimate of di/dt is two orders of magnitude smaller than the observed $\dot{\omega}$, and we therefore reject the significance of classical spin-orbit coupling in our measurement of $\dot{\omega}$ in the PSR J1640+2224 system.

While third-body effects can give rise to measurable perturbations of the pulsar-binary's Keplerian elements (e.g. Rasio, 1994), such interactions with another massive component would first be observed as large variations in $\nu_{\rm s}$ (see Chapter 4 of this thesis for an analysis of PSR B1620-26, one of two pulsar-triple systems). The NANOGrav 9-year timing solution for PSR J1640+2224 does not show such variations in spin frequency, and so there is no evidence that J1640+2224 is a triple system. Future observations of J1640+2224, along with historical data used by Löhmer et al. (2005) and the EPTA data set, will permit for even more stringent estimates of binary-parameter variations evaluated over a larger number of orbits, and ultimately yield a more robust timing solution.

3.4.6 PSR J1738+0333

PSR J1738+0333 is a 5.8-ms pulsar in a 8.5-hr orbit with a low-mass WD companion that was discovered in the Swinburne Intermediate Latitude Pulsar Survey (Jacoby, 2004). Optical spectroscopy of the WD companion yielded a significant mass ration $q = m_{\rm p}/m_{\rm c} = 8.1(2)$ and $m_{\rm c} = 0.182^{+0.007}_{-0.005} {\rm M}_{\odot}$, as well as consistent measures of the companion radius from both spectroscopy and photometry (Antoniadis et al., 2012). A radio-timing study reported the measurement of orbital decay that, after applying the correc-

tion for kinematic bias discussed in Section 1.5, is consistent with the component due to GR, yielding one of the most stringent tests on tensor-scalar theories of gravitation (Freire et al., 2012). The combination of these radio and optical analyses produced a derived estimate of $m_{\rm p} = 1.47^{+0.07}_{-0.06} \,\mathrm{M}_{\odot}$, as well as an estimate of i = 32.6(1.0) that was computed using f_m . Recent photometric observations identified optical variability of the WD companion that is consistent with pulsations of low-mass WDs (Kilic et al., 2015).

We do not measure any significant Shapiro delay or secular variations in the orbital elements. The $(m_c, \cos i) \chi^2$ grid and upper limit on *i* are shown in Figure 3.6, which is consistent with the derived estimate of *i* made by Antoniadis et al. (2012) and Freire et al. (2012).

3.4.7 PSR J1741+1351

PSR J1741+1351 is a 3.7-ms pulsar in a 16.3-day orbit that was discovered in a survey of high Galactic latitudes using the Parkes radio telescope (Jacoby et al., 2007). The Shapiro delay was initially detected in this system by Freire et al. (2006).

We detected the Shapiro timing delay in the NANOGrav nine-year data set for PSR J1741+1351, as well as a highly significant measurement of \dot{x} that we report for the first time. The annual orbital parallax is not significant for this MSP since the annual astrometric parallax was not significantly measured (Matthews et al., 2016). As discussed in Section 3.3.1 above, we nonetheless generated a three-dimensional χ^2 grid for different values of the two Shapiro-delay parameters and Ω , in order to constrain the system geometry using both measurements. Figure 3.15 shows the χ^2 -grid results for PSR J1741+1351 when first generating a three-dimensional, uniform grid in the $(m_c, \cos i, \Omega)$ parameters. The two-dimensional $(\cos i, \Omega)$ probability grid, obtained by marginalizing over m_c , illustrates a highly non-elliptical covariance between the two parameters. The constrained estimates of the Shapirodelay parameters are $m_p = 1.87^{+1.26}_{-0.69} M_{\odot}$, $m_c = 0.32^{+0.15}_{-0.09} M_{\odot}$, $i = 66^{+5}_{-6}$ degrees, and $\Omega = 317(35)$ degrees.

For comparison, we over-plotted the posterior PDFs obtained from a standard two-dimensional χ^2 grid over the traditional $(m_c, \cos i)$ parameters, while allowing \dot{x} and all other parameters to vary freely in each timing-model fit, as the grey lines in Figure 3.15. There are clear and significant differences between the posterior PDFs, which strongly suggest correlation between \dot{x} and one or both of the Shapiro delay parameters. The three-dimensional χ^2 grid results indicate that explicit modeling of the highly-significant kinematic term reduces correlation between the Shapiro-delay parameters and \dot{x} , and produces more sensible posterior PDFs of the component masses and system inclination that are consistent with initial results presented by Freire et al. (2006).

3.4.8 PSR B1855+09

PSR B1855+09 is a 5.4-ms pulsar in a 12.3-day orbit with a WD companion, and is also one of the earliest MSP discoveries made using the Arecibo Observatory (Segelstein et al., 1986). This MSP-binary system was the first to yield a significant measurement of the Shapiro timing delay from pulsar-timing measurements (Ryba & Taylor, 1991). The most recent longterm radio timing study determined the pulsar mass to lie within the range $1.4 < m_p < 1.8 M_{\odot}$ (95% confidence; Nice et al., 2004). Optical follow-up observations of the companion yielded a WD-cooling timescale of ~10 Gyr, which is twice as long as the characteristic age of the MSP (van Kerkwijk et al., 2000).

We made a highly significant measurement of the Shapiro timing delay when using the NANOGrav nine-year data set for PSR B1855+09. The χ^2 grids and marginalized PDFs for PSR B1855+09 are shown in Figure 3.16. Our estimates of the component masses and inclination angle – $m_{\rm p} =$ $1.30^{+0.11}_{-0.10} \,\mathrm{M}_{\odot}, m_{\rm c} = 0.236^{+0.013}_{-0.011} \,\mathrm{M}_{\odot}, \text{ and } i = 88.0^{+0.3}_{-0.4} \,\mathrm{degrees}$ – are consistent with, and more precise than, those previously made by Kaspi et al. (1994), Nice et al. (2004) and Reardon et al. (2016).

3.4.9 PSR J1903+0327

PSR J1903+0327 is a 2.1-ms pulsar in an eccentric, 95-day orbit with a main-sequence companion (Champion et al., 2008). This binary system, located within the Galactic disk, posed a significant challenge to the standard view of MSP formation since tidal interactions are expected to produce loweccentricity orbits with WD companions, as is observed for all other disk MSP-binary systems. Freire et al. (2011) performed the most recent pulsartiming analysis of PSR J1903+0327 and argued that both binary components were once members of a progenitor triple system where the main-sequence companion was in an outer orbit about an inner MSP-WD binary; this system was subsequently disrupted and produced the binary currently observed, either by a chaotic third-body interaction or full dissipation of the inner WD companion. They combined their Shapiro-delay measurement for this system with a significant measurement of $\dot{\omega}$, which they argue is due to GR, to determine the component masses and inclination with high precision: $m_{\rm p} = 1.667(21) \,{\rm M}_{\odot}; \ m_{\rm c} = 1.029(8) \,{\rm M}_{\odot};$ and 77.47(15) degrees (all 99.7% confidence). Freire et al. also measured an $\dot{x} = 0.020(3) \times 10^{-12}$ that they attributed to proper-motion bias. A recent optical analysis of radial-velocity measurements estimated the mass ratio of this system to be $q = m_{\rm p}/m_{\rm c} = 1.56(15)$ (68.3% confidence; Khargharia et al., 2012), consistent with the radio-timing estimate of q = 1.62(3) made by Freire et al.

We also independently measure a significant $\dot{\omega} = 2.410(13) \times 10^{-4} \text{ deg yr}^{-1}$ in the PSR J1903+0327 system, as well as the Shapiro timing delay indicated by the significance of h_3 listed in Table 3.2. We do not measure a significant \dot{x} . The observed $\dot{\omega}$ from our data set is consistent with the measurement made by Freire et al. (2011), and so we used the methodology discussed in Section 3.3.1 to constrain the Shapiro-delay parameters assuming that GR describes the observed periastron shift. The constrained χ^2 grids for PSR J1903+0327 are shown in Figure 3.17. From these grids, we estimated the component masses and inclination to be: $m_{\rm p} = 1.65(2) \,\mathrm{M}_{\odot}$; $m_{\rm c} = 1.06(2) \,\mathrm{M}_{\odot}$; and $i = 72^{+2}_{-3} \,\mathrm{deg} \,\mathrm{yr}^{-1}$. The estimate of $m_{\rm p}$ agrees with the Freire et al. measurement at the 68.3% credibility level, while $m_{\rm c}$ and i are consistent at about the 95.4% credibility level. We do not adjust the uncertainty in our measurement of $\dot{\omega}$ for the maximum uncertainty in $(\dot{\omega})_{\mu}$, which Freire et al. do when deriving their estimates. Our derived estimate of q = 1.56(3) also agrees with the optical measurement and Freire et al. estimate mentioned above.

3.4.10 PSR J1909-3744

PSR J1909-3744 is a 2.9-ms pulsar in a 1.5-day orbit with a WD companion (Jacoby et al., 2005). The Shapiro timing delay has previously been observed in this system with high precision, leading to the first precise mass measurement for an MSP (Jacoby et al., 2005; Hotan et al., 2006; Verbiest, 2009). Two recent, independent TOA analyses of this pulsar were performed by Reardon et al. (2016) and Desvignes et al. (2016). Reardon et al. used the PPTA data set and reported significant Shapiro-delay parameters, apparent orbital decay, and geometric variations the PSR J1909-3744 system with the following measured and derived results: $m_{\rm p} = 1.47(3)$ M_☉; $m_{\rm c} = 0.2067(19)$ M_☉; $i = 93.52(9)^{\circ}$; and $\dot{P}_{\rm b} = 0.503(6) \times 10^{-12}$. Desvignes et al. analyzed the EPTA data set and also reported estimates of the Shapiro-delay parameters, apparent orbital decay, angenet orbital decay, and geometric variations: $m_{\rm p} = 1.54(3)$ M_☉; $m_{\rm c} = 0.213(2)$ M_☉; $\sin i = 0.99771(13)$; and $\dot{P}_{\rm b} = 0.503(5) \times 10^{-12}$.

We independently measure both Shapiro-delay parameters and $\dot{P}_{\rm b}$ with high significance when using the NANOGrav nine-year data set. We also make a marginal detection of $\dot{x} = -4.4(1.6) \times 10^{-16}$ when incorporating it as a free parameter, but it does not pass the F-test criterion.

The component masses that we derived from the probability maps for J1909-3744 shown in Figure 3.18, $m_{\rm p} = 1.55(3)$ M_{\odot} and $m_{\rm c} = 0.214(3)$ M_{\odot},

agree with the estimates made by Reardon et al. (2016) and Desvignes et al. (2016). Our estimate of i = 86.33(10) degrees possesses a sign ambiguity in $\cos i$, so i = 93.67(10) is an allowed solution for our analysis; the latter estimate agrees with the Reardon et al. and Desvignes et al. measurement.

Given our measurements of the Keplerian and Shapiro-delay parameters, the expected orbital decay in this system from quadrupole gravitational-wave emission is $(\dot{P}_{\rm b})_{\rm GR} = -0.00294 \times 10^{-12}$, which is significantly less than our measurement of $\dot{P}_{\rm b}$. This low estimate of $(\dot{P}_{\rm b})_{\rm GR}$ implies that $\dot{P}_{\rm b} = 0.509(9) \times$ $10^{-12} \approx (\dot{P}_{\rm b})_D$, which agrees with the measurement and assessment made by Reardon et al. (2016) and Desvignes et al. (2016). We therefore attribute the apparent orbital decay in PSR J1909–3744 system to biases from significant acceleration between the MSP-binary and SSB reference frames. Matthews et al. (2016) used our $\dot{P}_{\rm b}$ measurement to find the distance to PSR J1909-3744 to be 1.11(2) kpc, in agreement with their timing-parallax distance of $1.07^{+0.04}_{-0.03}$ kpc.

3.4.11 PSR J1918-0642

PSR J1918-0642 is a 7.6-ms pulsar in a 10.9-day orbit with a likely WD companion that was discovered by Edwards & Bailes (2001) in a multi-beam survey of intermediate Galactic latitudes using the Parkes Radio Telescope. An optical search for the companion of PSR J1918-0642 was unsuccessful (van Kerkwijk et al., 2005), requiring that the apparent R-band magnitude of the WD be R > 24. A long-term timing study of this MSP was carried out by Janssen et al. (2010) using the Westerbork, Nançay and Jodrell Bank radio observatories at 1400 MHz for a combined timespan of 7.4 years. While only Keplerian parameters were measured, Janssen et al. (2010) combined their distance estimate to PSR J1918-0642 – based on their dispersion-measure estimate for this pulsar and the Cordes & Lazio (2001) electron-density model for the Galaxy – with the R > 24 limit, and the assumption that the white-dwarf cooling and pulsar spin-down are coeval, to further constrain the com-

panion to be a He or CO white dwarf with a thin hydrogen atmosphere. They used the mass function of the system, as well as an assumed $m_{\rm p} = 1.35 \,\mathrm{M}_{\odot}$, to compute a minimum companion mass of $m_{\rm c,min} = 0.24 \,\mathrm{M}_{\odot}$. A recent radio-timing analysis by Desvignes et al. (2016) used the EPTA data to measure the Shapiro delay in this system, with $m_{\rm p} = 1.3^{+0.6}_{-0.4} \,\mathrm{M}_{\odot}$, $m_{\rm c} = 0.23(7) \,\mathrm{M}_{\odot}$, and $\cos i = 0.09^{+0.05}_{-0.04}$.

We measured a highly-significant Shapiro timing delay in the PSR J1918-0642 binary system using the NANOGrav nine-year data set. The probability maps computed from χ^2 grids for PSR J1918-0642 are shown in Figure 3.19. The significance of h_3 in the PSR J1918-0642 system exceeds 27σ , a factor of ~ 4 better than the h_3 estimate made by Desvignes et al. (2016) when using their EPTA data set. Our precise measurements of the WD mass and inclination from the Shapiro timing delay are $m_c = 0.219^{+0.012}_{-0.011} M_{\odot}$ and i = 85.0(5) degrees regardless of choice in the parameterization of $\Delta_{\rm S}$. The derived estimate of the pulsar mass is the first precise estimate for this system, and is suggestive of a low-mass neutron star: $m_{\rm p} = 1.18^{+0.10}_{-0.09} M_{\odot}$.

3.4.12 PSR J1949+3106

PSR J1949+3106 is a 13.1-ms pulsar in a 1.9-day orbit with a massive companion that was discovered by the ongoing PALFA survey of the Galactic plane using the Arecibo telescope (Deneva et al., 2012). The initial radio-timing study by Deneva et al. used TOAs collected with the Arecibo, Green Bank, Nançay and Jodrell Bank telescopes over a four-year period to make a significant detection of the Shapiro timing delay in this system. They reported significant measurements of the orthometric parameters, $h_3 = 2.4(1) \ \mu s$ and $\varsigma = 0.84(2)$, as well as derived estimates of component masses and system inclination: $m_{\rm p} = 1.47^{+0.43}_{-0.31} \ {\rm M}_{\odot}$; $m_{\rm c} = 0.85^{+0.14}_{-0.11} \ {\rm M}_{\odot}$; and $i = 79.9^{+1.6}_{-1.9}$ degrees.

We independently measured a Shapiro timing delay in the PSR J1949+3106 using the NANOGrav nine-year data set. The probability maps computed from χ^2 grids for PSR J1949+3106 are shown in Figure 3.20; we set $m_{\rm c,max} = 5 \,\rm M_{\odot}$ when computing the χ^2 grids since the peak-probability value is nearly equal to our usual upper limit of $m_{\rm c,max} = 1.4 \,\rm M_{\odot}$. Our measurements of the orthometric parameters, $h_3 = 2.5(5) \,\mu$ s and $\varsigma = 0.77(10)$, are consistent with those made by (Deneva et al., 2012) at the 68.3% credibility level. The uncertainties in our measurements are comparatively larger due to the shorter time span of our data set and, therefore, less TOA coverage across the orbit. Our derived estimates of the component masses and inclination are subsequently much less stringent than those made by Deneva et al.: $m_{\rm p} = 4.0^{+3.6}_{-2.5} \,\rm M_{\odot}$; $m_{\rm c} = 2.1^{+1.6}_{-1.0} \,\rm M_{\odot}$; and $i = 67^{+9}_{-8}$ degrees.

3.4.13 PSR J2017+0603

PSR J2017+0603 is a 2.9-ms pulsar in a 2.2-day orbit that was initially found using the *Fermi* Large Area Telescope (LAT) as a gamma-ray source with no known associations; radio pulsations were discovered and subsequently timed from this source using the Nancay Radio Telescope and Jodrell Bank Observatory for nearly two years by Cognard et al. (2011). They used the mass function of the PSR J2017+0603 system, along with an assumed $m_{\rm p} =$ 1.35 M_☉, to compute a minimum companion mass of $m_{\rm c,min} = 0.18$ M_☉.

For the first time, we detect a Shapiro timing delay in the PSR J2017+0603 system using the NANOGrav nine-year data set, with $m_c = 0.32^{+0.44}_{-0.16} M_{\odot}$ and $i = 62^{+9}_{-12}$ degrees. The probability maps computed from χ^2 grids for PSR J2017+0603 are shown in Figure 3.21. The observed Shapiro delay in this system is currently weak since the marginalized, one-dimensional PDF of $m_p = 2.4^{+3.4}_{-1.4} M_{\odot}$ extends to large values of the neutron-star mass. However, we were able to make a significant detection using a comparatively small, 1.7-yr data set that includes targeted observations at select orbital phases discussed in Section 3.1; our measurement will improve with the inclusion of future TOAs collected at different points in the orbit.

3.4.14 PSR J2043+1711

PSR J2043+1711 is a 2.4-ms pulsar in a 1.5-day orbit that was initially found using the *Fermi* LAT as a gamma-ray source with no previously known associations. The radio counterpart was discovered using the Nancay and Green Bank Telescopes; the Shapiro delay was detected in this MSP-binary system using a timing model derived from TOAs collected with the Nancay, Westerbork and Arecibo observatories over a three-year period (Guillemot et al., 2012). At the time of the initial study performed by Guillemot et al., the Shapiro timing delay was not significant enough to yield statistically meaningful estimates of the component masses and inclination angle. They placed limits on the companion mass by assuming the validity of the m_c - P_b relation, and derived a preferred range of $0.20 < m_c < 0.22 M_{\odot}$; with this constraint, Guillemot et al. found the pulsar mass and inclination to be $1.7 < m_p < 2.0 M_{\odot}$ and i = 81.3(1.0) degrees, respectively.

The NANOGrav nine-year data set on PSR J2043+1711, which includes the targeted Shapiro-delay observations discussed in Section 3.1, yields a significantly improved measurement of the component masses and system inclination as shown in Table 3.4; the impact of the targeted observations on the significance of $\Delta_{\rm S}$ in the PSR J2043+1711 system was discussed by Pennucci (2015). The probability maps computed from χ^2 grids for PSR J2043+1711 are shown in Figure 3.22. Our improved measurements of $m_{\rm c} = 0.175^{+0.016}_{-0.015} \,{\rm M}_{\odot}$ and $i = 83.2^{+0.8}_{-0.9}$ degrees are consistent with the initial estimates made by Guillemot et al. (2012), though $m_{\rm c}$ is moderately lower than the range determined from the $m_{\rm c}$ - $P_{\rm b}$ relation. Our derived $m_{\rm p} = 1.41^{+0.21}_{-0.18} \,{\rm M}_{\odot}$ is therefore slightly below the $m_{\rm p}$ range determined by Guillemot et al. when assuming the validity of the $m_{\rm c} - P_{\rm b}$ relation.

3.4.15 PSR J2145-0750

PSR J2145-0750 is a 16-ms pulsar in a 6.8-day orbit with a white-dwarf companion and was discovered in a Parkes Telescope survey (Bailes et al., 1994). Both Phinney & Kulkarni (1994) and van den Heuvel (1994) argued that the J2145-0750 system likely experienced unstable mass transfer from "common-envelope" evolution, where the pulsar gradually expelled the outer layers of the donor, in order to explain its unusually long pulsar-spin period and massive companion compared to other binary-MSP systems. Early optical observations of the WD companion noted the difficulty in obtaining accurate photometry due to the use of a dispersion-based distance estimate and the presence of a coincident field star (Lundgren et al., 1995). However, a recent study performed by Deller et al. (2016) combined improved optical imaging with a precise VLBI distance of $d = 613^{+16}_{-14}$ pc to estimate a companion mass of $m_c \approx 0.85 \text{ M}_{\odot}$. Deller et al. also detected the orbital reflex motion of J2145-0750 through their VLBI measurements, and inferred estimates of $i = 21^{+7}_{-4}$ degrees and $\Omega = 230(12)$ degrees.⁶

We measured $\dot{x} = 0.0098(19) \times 10^{-12}$, consistent with estimates made by Reardon et al. (2016). Our estimate of $h_3 = 0.10(5) \ \mu$ s does not pass the h_3 -significance test, and so we do not formally measure a significant Shapiro timing delay from the radio-timing data alone. However, we used the estimate of $m_c = 0.83^{+0.06}_{-0.06} M_{\odot}$ made by Deller et al. (2016) as a prior distribution when computing the posterior maps for PSR J2145-0750. The resulting constraints on $\cos i$ and m_p are shown in Figure 3.23, which yield $m_p = 1.3^{+0.4}_{-0.5} M_{\odot}$ and $i = 34^{+5}_{-7}$ degrees; these estimates are consistent with those made by Deller et al., and with the upper limits on i we derive in Section 3.3.2, shown in Table 3.3.

⁶Deller et al. (2016) report their estimate of Ω using a convention that measures Ω from celestial East through North. This convention is inconsistent with the North-through-East convention we use in this work. We report their estimate of Ω relative to our convention.

3.4.16 PSR J2302+4442

PSR J2302+4442 is a 5.2-ms pulsar in a 126-day orbit that, along with PSR J2017+0603 (Section 3.4.13) was initially found using the *Fermi* LAT as a gamma-ray source with no known associations and observed in the radio using the Nançay Radio Telescope and Jodrell Bank Observatory for nearly two years by Cognard et al. (2011). They used the mass function of the PSR J2302+4442 system, along with an assumed $m_{\rm p} = 1.35 \, {\rm M}_{\odot}$, to compute a minimum companion mass of $m_{\rm c,min} = 0.3 \, {\rm M}_{\odot}$.

For the first time, we tentatively detect a Shapiro timing delay in the PSR J2302+4442 system using the NANOGrav nine-year data set. The probability maps computed from χ^2 grids for PSR J2302+4442 are shown in Figure 3.24. Due to the weak detection of $\Delta_{\rm S}$ and large correlation between r and s, the timing solution published by Arzoumanian et al. (2015b) used a fixed value of $m_{\rm c} = 0.355 \,{\rm M}_{\odot}$ that was computed from the $m_{\rm c}$ - $P_{\rm b}$ relation when fitting for all other timing parameters, including the Shapiro s parameter. In this study, we developed timing solutions using both the traditional and orthometric parameterizations of $\Delta_{\rm S}$ that allowed both PK parameters to be fitted for. The value of h_3 in the PSR J2302+4442 system exceeds 5σ and therefore passes the h_3 significance test for detection of $\Delta_{\rm S}$.

Our estimates of the companion mass and inclination are $m_c = 2.3^{+1.7}_{-1.3} M_{\odot}$ and $i = 54^{+12}_{-7}$ degrees, and the corresponding pulsar mass is $m_p = 5.3^{+3.2}_{-3.6} M_{\odot}$. We computed χ^2 grids with $m_{c,max} = 5 M_{\odot}$ since the peak-probability value of m_c exceeds the usual upper limit of $m_{c,max} = 1.4 M_{\odot}$. While the posterior PDFs of the component masses span a large range of mass values, the significant estimates of s and ς indicate a measurable constraint on the system inclination. The measurement of Δ_s will improve in significance over time since the current data set for PSR J2302+4442 only spans about 1.7 years – or ~5 orbits, given the long P_b of this MSP-binary system – and so a very small fraction of the Shapiro-delay signal has been sampled. Furthermore, given the large orbit and modest inclination, we expect to see a measurable
secular variation in x within the next few years.

3.4.17 PSR J2317+1439

PSR J2317+1439 is a 3.4-ms pulsar in a 2.5-day orbit that was discovered in a survey of high Galactic latitudes using the Arecibo Obsveratory and possesses one of the smallest eccentricities known (Camilo et al., 1993, 1996; Hobbs et al., 2004). The most recent radio-timing analysis of PSR J2317+1439 performed by Desvignes et al. (2016) did not yield any secular variations in orbital parameters or a significant measurement of the Shapiro timing delay when using their 17.3-yr EPTA data set. However, a Bayesian-timing analysis performed by Vigeland & Vallisneri (2014) used the NANOGrav fiveyear data set (Demorest et al., 2013) to measure several secular variations in the binary parameters: $\dot{P}_{\rm b} = 6.4(9) \times 10^{-12}$; $\dot{\eta} = -2(4) \times 10^{-15}$; and $\dot{\kappa} = 2.0(7) \times 10^{-14}$. Vigeland and Vallisneri noted that many of the posterior distributions for binary parameters of J2317+1439 changed slightly when using different priors for the astrometric timing parallax.

The original NANOGrav nine-year timing model for PSR J2317+1439 contains parameters that describe secular variations in x and the Laplace-Lagrange eccentricity parameters, with $\dot{\eta} = 5.0(9) \times 10^{-15} \text{ s}^{-1}$, all of which pass the F-test criterion. We found that $\dot{P}_{\rm b}$ did not pass the F-test, so it was not fitted in the original NANOGrav nine-year timing solution. Moreover, both the F-test and the h_3 -significance test indicated that the Shapiro delay was not significant, and so we also did not initially incorporate the Shapirodelay parameters.

Despite the statistical significance of $\dot{\eta}$, we do not believe that the PSR J2317+1439 system is experiencing physical processes that produce a changing eccentricity. For instance, if mass transfer between components were currently taking place, we would expect to observe a spin-up phase; instead, we observe seemingly "normal" spin-down properties and stable rotation that is typical of MSPs. The presence of a third massive body in a bound, hierar-

chical orbit about the pulsar-companion binary system would induce higherorder derivatives in spin frequency as well as additional third-body effects on the shape, size and period of the inner binary (e.g. Joshi & Rasio, 1997), most of which we do not see in the NANOGrav nine-year data set. Finally, the timescale for the observed change in η is estimated to be $\eta/\dot{\eta} \approx 0.7$ years, which is implausibly short.

Because the observed $\dot{\eta}$ is physically implausible, and because covariances between it and several other parameters distort the timing solution, we chose to hold both $\dot{\eta}$ and $\dot{\kappa}$ fixed to a value of zero (i.e. no change in the eccentricity parameters of the system) while re-fitting the nine-year timing model. In this case, we found that the significance of h_3 exceeded 3σ and therefore included the Shapiro-delay parameters. We found that \dot{x} did not pass the F-test, and so did not fit for it in our modified solution. The new timing model for PSR J2317+1439 fits the data well (reduced $\chi^2 = 1.0053$ for 2531 degrees of freedom), though the original model published by Arzoumanian et al. (2015b) that fits for $\dot{\eta}$ and $\dot{\kappa}$ better fits the TOA data (reduced $\chi^2 = 0.9966$ for 2531 degrees of freedom).

We generated two-dimensional χ^2 grids for the traditional and orthometric Shapiro-delay parameters. The probability maps and the marginalized PDFs of the component masses and system inclination are shown in Figure 3.25. Given the new binary timing model of PSR J2317+1439, we have made a weak detection of the Shapiro timing delay in this system since the twodimensional probability density extends to large m_c for low inclinations, and so the system inclination angle is not as well constrained as for the other stronger detections. Our current estimates of the component masses and inclinations are $m_p = 4.7^{+3.4}_{-2.8} M_{\odot}$, $m_c = 0.7^{+0.5}_{-0.4} M_{\odot}$, and $i = 47^{+10}_{-7}$ degrees.



Figure 3.11: Probability maps and posterior PDFs of the traditional Shapiro-delay parameters measured for PSR J0613-0200. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability defined on each two-dimensional map, respectively. In all slimmer panels, the blue solid lines represent posterior PDFs obtained from marginalizing the appropriate two-dimensional map, the vertical red-dashed lines are bounds of the 68.3% confidence interval, and the red-solid line is the median value.



Figure 3.12: Probability maps and posterior PDFs of the traditional Shapiro-delay parameters measured for PSR J0613-0200. The maps and PDFs for J1600-3053 were constrained assuming that the observed $\dot{\omega}$ is due to GR (see Section 3.4.3). The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability defined on each two-dimensional map, respectively. In all slimmer panels, the blue solid lines represent posterior PDFs obtained from marginalizing the appropriate two-dimensional map, the vertical red-dashed lines are bounds of the 68.3% confidence interval, and the red-solid line is the median value.



Figure 3.13: Probability maps and posterior PDFs of the traditional Shapiro-delay parameters measured for PSR J1614-2230. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability defined on each two-dimensional map, respectively. In all slimmer panels, the blue solid lines represent posterior PDFs obtained from marginalizing the appropriate two-dimensional map, the vertical red-dashed lines are bounds of the 68.3% confidence interval, and the red-solid line is the median value.



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Figure 3.14: Probability maps and posterior PDFs of the traditional Shapiro-delay parameters measured for PSR J1640+2224. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability defined on each two-dimensional map, respectively. In all slimmer panels, the blue solid lines represent posterior PDFs obtained from marginalizing the appropriate two-dimensional map, the vertical red-dashed lines are bounds of the 68.3% confidence interval, and the red-solid line is the median value.



Figure 3.15: Probability maps and posterior PDFs of the traditional Shapiro-delay parameters measured for PSR J1741+1351, including the PDF for Ω determined from a three-dimensional χ^2 grid. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability defined on each two-dimensional map, respectively. In all slimmer panels, the blue solid lines represent posterior PDFs obtained from marginalizing the appropriate two-dimensional map, the vertical red-dashed lines are bounds of the 68.3% confidence interval, and the red-solid line is the median value. Shown for comparison, the grey curves in the slimmer panels of PSR J1741+1351 are marginalized PDFs obtained from computing a separate, two-dimensional χ^2 grid over the $(m_c, \cos i)$ parameters while letting \dot{x} be a free parameter in each TEMPO2 fit. See Section 3.4.7 for a discussion on the visible differences in PDFs.



Figure 3.16: Probability maps and posterior PDFs of the traditional Shapiro-delay parameters measured for PSR B1855+09. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability defined on each two-dimensional map, respectively. In all slimmer panels, the blue solid lines represent posterior PDFs obtained from marginalizing the appropriate two-dimensional map, the vertical red-dashed lines are bounds of the 68.3% confidence interval, and the red-solid line is the median value.



Figure 3.17: Probability maps and posterior PDFs of the traditional Shapiro-delay parameters measured for PSR J1903+0327, using statistical significance of $\dot{\omega}$ and the assumption of GR to constrain the probability density. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability defined on each two-dimensional map, respectively. In all slimmer panels, the blue solid lines represent posterior PDFs obtained from marginalizing the appropriate two-dimensional map, the vertical red-dashed lines are bounds of the 68.3% confidence interval, and the red-solid line is the median value.



Figure 3.18: Probability maps and posterior PDFs of the traditional Shapiro-delay parameters measured for PSR J1909-3744. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability defined on each two-dimensional map, respectively. In all slimmer panels, the blue solid lines represent posterior PDFs obtained from marginalizing the appropriate two-dimensional map, the vertical red-dashed lines are bounds of the 68.3% confidence interval, and the red-solid line is the median value.



Figure 3.19: Probability maps and posterior PDFs of the traditional Shapiro-delay parameters measured for PSR J1918-0642. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability defined on each two-dimensional map, respectively. In all slimmer panels, the blue solid lines represent posterior PDFs obtained from marginalizing the appropriate two-dimensional map, the vertical red-dashed lines are bounds of the 68.3% confidence interval, and the red-solid line is the median value.



Figure 3.20: Probability maps and posterior PDFs of the traditional Shapiro-delay parameters measured for PSR J1949+3106. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability defined on each two-dimensional map, respectively. In all slimmer panels, the blue solid lines represent posterior PDFs obtained from marginalizing the appropriate two-dimensional map, the vertical red-dashed lines are bounds of the 68.3% confidence interval, and the red-solid line is the median value.



Figure 3.21: Probability maps and posterior PDFs of the traditional Shapiro-delay parameters measured for PSR J2017+0603. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability defined on each two-dimensional map, respectively. In all slimmer panels, the blue solid lines represent posterior PDFs obtained from marginalizing the appropriate two-dimensional map, the vertical red-dashed lines are bounds of the 68.3% confidence interval, and the red-solid line is the median value.



Figure 3.22: Probability maps and posterior PDFs of the traditional Shapiro-delay parameters measured for PSR J2043+1711. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability defined on each two-dimensional map, respectively. In all slimmer panels, the blue solid lines represent posterior PDFs obtained from marginalizing the appropriate two-dimensional map, the vertical red-dashed lines are bounds of the 68.3% confidence interval, and the red-solid line is the median value.



Figure 3.23: Probability maps and posterior PDFs of the traditional Shapiro-delay parameters measured for PSR J2145-0750. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability defined on each two-dimensional map, respectively. In all slimmer panels, the blue solid lines represent posterior PDFs obtained from marginalizing the appropriate two-dimensional map, the vertical red-dashed lines are bounds of the 68.3% confidence interval, and the red-solid line is the median value.



Figure 3.24: Probability maps and posterior PDFs of the traditional Shapiro-delay parameters measured for PSR J2302+4442. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability defined on each two-dimensional map, respectively. In all slimmer panels, the blue solid lines represent posterior PDFs obtained from marginalizing the appropriate two-dimensional map, the vertical red-dashed lines are bounds of the 68.3% confidence interval, and the red-solid line is the median value.



Figure 3.25: Probability maps and posterior PDFs of the traditional Shapiro-delay parameters measured for PSR J2317+1439. The inner, middle and outer red contours encapsulate 68.3%, 95.4% and 99.7% of the total probability defined on each two-dimensional map, respectively. In all slimmer panels, the blue solid lines represent posterior PDFs obtained from marginalizing the appropriate two-dimensional map, the vertical red-dashed lines are bounds of the 68.3% confidence interval, and the red-solid line is the median value.

3.5 Conclusions & Summary

We have derived estimates of binary component masses and inclination angles for fourteen NANOGrav MSP-binary systems with significant measurements of the Shapiro timing delay. Four of these fifteen Shapiro-delay signals – in PSRs J0613-0200, J2017+0603, J2302+4442, and J2317+1439 – have been measured for the first time. From the Shapiro timing delay alone, we were able to measure high-precision neutron star masses as low as $m_{\rm p} =$ $1.18^{+0.10}_{-0.09}$ M_{\odot} for PSR J1918-0642 and as high as $m_{\rm p} = 1.928^{+0.017}_{-0.017}$ M_{\odot} for PSR J1614-2230. Measurements of previously observed $\Delta_{\rm S}$ signals in the J1918-0642 and J2043+1711 systems have been significantly improved upon in this work, with the pulsar mass for PSR J2043+1711 $m_{\rm p} = 1.41^{+0.21}_{-0.18}$ M_{\odot} being measured significantly for the first time. For the fourteen MSPs with significant $\Delta_{\rm S}$, we performed a rigorous analysis of the χ^2 space for the two Shapiro-delay parameters, using priors uniform in the traditional ($m_{\rm c}$, sin *i*) and orthometric (h_3 , ς) parametrizations of the Shapiro timing delay, in order to determine robust credible intervals of the physical parameters.

Most of the NANOGrav binary MSPs exhibit significant changes in one or more of their orbital elements over time. Whenever possible, we used the statistical significance of the observed orbital variations to further constrain the parameters of the observed Shapiro timing delay when performing the χ^2 -grid analysis. Assuming the validity of GR, we further constrained the component masses in the PSR J1600-3053 and PSR J1903+0327 systems, which both experience significant periastron advance due to strong-field gravitation; the precision of our $\dot{\omega}$ measurement for PSR J1903+0327 contributed to a highly constrained estimate of $m_{\rm p} = 1.65^{+0.02}_{-0.02} \,\mathrm{M}_{\odot}$ that is consistent with previous timing studies of this MSP using an independent data set. We also used the highly-significant \dot{x} measurement in the PSR J1741+1351 system in combination with the Shapiro timing delay observed in this system, which allowed for an estimation of Ω , albeit with a large uncertainty.

System Name	$m_{\rm c}~({ m M}_{\odot})$	$P_{\rm b} \ ({\rm days})$	References (Mass Measurement, Identification)
PSR J0348+0432	0.172(3)	0.102	Antoniadis et al. (2013)
PSR J0751-1807	0.16(1)	0.263	Bassa et al. $(2006b)$, Desvignes et al. (2016)
PSR J1738+0333	$0.181^{+0.007}_{-0.005}$	0.354	Antoniadis et al. (2012)
PSR J1012+5307	0.16(2)	0.604	van Kerkwijk et al. (1996), van Kerkwijk et al. (2005)
J0247 - 25B	0.186(2)	0.667	Maxted et al. (2013)
PSR J1910 - 5959A	0.180(18)	0.837	Bassa et al. (2006a), Corongiu et al. (2012)
PSR J0337+1715i	0.19751(15)	1.629	Ransom et al. (2014) , Kaplan et al. (2014)
KOI 1224	0.22(2)	2.698	Breton et al. (2012)
KOI 74	0.22(3)	5.189	van Kerkwijk et al. (2010)
PSR J0437-4715	0.224(7)	5.741	Durant et al. (2012) , Reardon et al. (2016)
RRLYR 02792	0.260(15)	15.243	Pietrzyński et al. (2012)
PSR J0337+17150	0.4101(3)	327.257	Ransom et al. (2014)

Table 3.5: Uncertainties in $P_{\rm b}$ are suppressed due to the high precision to which they are measured. Values in parentheses denote the 1σ uncertainty in the preceding digit(s).

The relativistic Shapiro timing delay provides a direct measurement of the companion mass that is independent of the given system's evolutionary history, and that therefore can be used to test the plausibility of available binary-evolution paradigms. Figure 3.26 illustrates the $P_{\rm b}$ -vs- $m_{\rm c}$ estimates for the NANOGrav MSP-binary systems that are known or suspected to have He-WD companions, as well as a blue-shaded region that corresponds to the theoretical $m_{\rm c}$ - $P_{\rm b}$ correlation (Equation 1.8) as predicted by Tauris & Savonije (1999). PSR J1903+0327 is excluded since its companion is likely a main sequence star, while PSR J1614-2230 is excluded since its companion is a carbon-oxygen WD and is believed to have evolved through a different formation channel (Tauris et al., 2011). Figure 3.26 is recreated from the one presented by Tauris & van den Heuvel (2014). Black points denote precise measurements of $m_{\rm WD}$ in WD-binary systems examined in previous works; values and references for these data are provided in Table 3.5. The width of the shaded region represents possible correlated values of $P_{\rm b}$ and $m_{\rm WD}$ for progenitor donor stars with different chemical compositions, particularly with metallicities (Z) in the range 0.001 < Z < 0.02. While our m_c estimates generally agree with the predicted correlation, additional measurements at higher companion masses are needed in order to perform a robust exploration of the correlation parameters and their credible intervals.

At its current level of precision, the low mass of PSR J1918–0642 is interesting since this MSP possesses spin parameters that are indicative of an old neutron star that experienced significant mass transfer and a substantial spin-up phase. The implication of a low "birth mass" for neutron stars is consistent with early estimates of the initial-mass function (e.g. Timmes et al., 1996), though suggests that the neutron-star progenitor to J1918–0642 may have undergone an electron-capture supernova event (e.g. Schwab et al., 2010) which produces comparatively less-massive neutron stars. Similar conclusions have been drawn for the lighter neutron stars in the J0737–3039A/B (Ferdman et al., 2013) and J1756–2251 (Ferdman et al.,



Figure 3.26: $P_{\rm b}$ versus $m_{\rm c}$ for binary systems with He-WD companions. Red points are our new measurements (see Figure 9). Black points are WD-mass measurements made for systems listed in Table 3.5. The shaded blue region is the expected correlation between $m_{\rm c}$ and $P_{\rm b}$, computed by Tauris & Savonije (1999), for post-transfer He-WD binary systems with progenitor companions that have metallicities within the range 0.001 < Z < 0.02.

2014) double-neutron-star binary systems, though the evolutionary history of these systems (with lesser degrees of mass transfer) are understood to be different than that expected for PSR J1918-0642.

Extending the data sets of these MSPs will refine observed secular variations due to PK and/or kinematic-bias effects within the next few years. Furthermore, extending TOA coverage in orbital phase for PSRs J0613-0200, J1949+3106, J2017+0603, J2302+4442, and J2317+1439 will improve the significance of the Shapiro timing delay that we report in this study. In particular, additional TOAs collected for PSRs J1640+2224 and J2317+1439 will help in the assessment of their complex orbital behavior as seen in the NANOGrav nine-year data set for these systems. The combination of NANOGrav high-precision TOAs with archival data published in previous studies will provide more accurate timing models and a complete picture of the physical processes that affect the NANOGrav MSP orbits.

Chapter 4

Long-Term Timing of the PSR B1620–26 Triple System in Messier 4

The proto-stellar environments that eventually form binary systems can also form dynamically complex systems with three or more massive components. Hydrodynamic simulations of gravitationally-driven collapse in proto-stellar gas clouds have shown that such "multi-core" systems can be readily formed through substantial fragmentation (e.g. Boss, 1991). For the purposes of long-term study, the most stable multi-core systems are those in a *hierarchical* formation, where an "inner" system is orbited by an "outer" object with a large distance between their respective centers of mass; for these hierarchical systems, the mutual tidal perturbations are expected to influence the dynamical evolution at a rate that keeps the total system gravitationally bound on long timescales.¹ Recent observational studies have suggested that hierarchical triple systems, with three components, are indeed common in

¹For hierarchical triple systems, we refer to the orbital parameters of the inner and outer companions with subscripts "i" and "o", respectively. The details regarding the outer-orbital parameters are summarized in Section 4.2 and discussed in greater detail in Appendix B.

nature and constitute $\sim 20\%$ of all small-period binaries within the Galaxy (e.g. Rappaport et al., 2013).

At this point in time, only two radio pulsars are known to reside in hierarchical triple systems. The first of these pulsars to be discovered, PSR B1620-26, was found in a search for radio pulsations in the Messier 4 (M4) globular cluster (Lyne, 1988) and was immediately noted to belong in a binary system with a WD companion with $(m_c)_i \approx 0.3 \text{ M}_{\odot}$ (McKenna & Lyne, 1988). However, subsequent timing of PSR B1620-26 showed an unusually large eccentricity of the 191-day pulsar-WD orbit, $e_i \approx 0.025$, that contradicts the standard formation model discussed in Section 1.1.4, as well as a large time derivatives in ν_s that is unlike those seen in other stable binary pulsars (Thorsett, 1991).

It was Backer et al. (1993) who first proposed the existence of a second companion of substellar mass in the PSR B1620-26 system to explain these discrepancies, since gravitational acceleration from binary (or triple) motion can produce large variations in $\nu_{\rm s}$ from orbit-induced Doppler shifts. Using a data set that spanned nearly a decade in time, Thorsett et al. (1999) provided the first timing solution for a hierarchical pulsar-triple system that described the observed spin-frequency derivatives in terms of Doppler shifts due to two non-interacting Keplerian orbits. However, their analysis indicated that the outer-orbital companion mass $(m_c)_o \sim 0.01 \,\mathrm{M}_{\odot}$ and outer-orbital period $(P_b)_o \sim 100$ years; the orbital elements that comprise the outer-orbital Römer delay, $(\Delta_R)_o$, were found to be covariant as fit parameters and not well constrained, and so Thorsett et al. provided estimates of the outer-orbital elements for different fixed values of the outer-orbital eccentricity (e_o) .

The study of orbital variations due to hierarchical three-body interactions is expected to yield intrinsic properties of the complex dynamical system and system components, such as the component masses and mutual inclination of the inner and outer orbits (e.g. Ford et al., 2000b; Kopeikin & Vlasov, 2004). These variations can also be used as constraints to infer the evolution of three-body systems to their current states. A notable example is the recent discovery and analysis of the PSR J0337+1715 system – the other hierarchical pulsar-triple system – with two low-mass white dwarfs in short-period orbits, which used a newly developed three-body integration technique that allows for direct measurement of "interaction" parameters that quantify three-body dynamical terms tied to mutual geometry and ratios of component masses (Ransom et al., 2014). Ransom et al. also showed that a double-Keplerian timing model of the whole system that models inner-orbit perturbations as Taylor expansions does not adequately describe the complex timing behavior for three-body systems, which illustrates the need for explicit, simultaneous modeling of both orbits and their mutual interactions.

In the unique case of PSR B1620-26, the sub-stellar nature of the outer companion yields interesting implications for planet formation in the early, metal-poor Universe. Thorsett et al. (1999) determined that $(m_c)_o < 0.036 M_{\odot}$ (95% confidence), significantly smaller than the minimum mass needed for hydrogen fusion (~ 0.08 M_{\odot}), and argued that the outer companion is likely a Jupiter-mass planet. The presence of planets in globular clusters was first considered by Sigurdsson (1992), shortly after the discovery of the PSR B1257+12 planetary system (Wolszczan & Frail, 1992) and prior to the triplesystem association for PSR B1620-26 by Backer et al. (1993); Sigurdsson noted that the timescale for planet formation in globular clusters, which are among the oldest collections of stars and metal-poor environments, may not be sufficient for their creation due to the need for significant dust coagulation (Weidenschilling, 1980). However, Sigurdsson et al. (2003) used optical photometric data obtained with the Hubble Space Telescope to determine an inner-binary white dwarf age of $480(140) \times 10^6$ years through isochrone fitting of their observed color-magnitude diagram of the system. The white-dwarf age is considerably smaller than the cluster age of $12.7(4) \times 10^9$ years (Hansen et al., 2002), and supports the evolutionary scenario that the triple system formed through a recent exchange interaction between an old NS-WD system and a main-sequence/planet system in the cluster core, where the original WD companion was ejected and the newly formed neutron-star/mainsequence/planet triple was jettisoned out of the core. Subsequent accretion the inner neutron-star/main-sequence binary would eventually produce a low-mass (~0.3 M_{\odot}) white dwarf, which is consistent with the measurement of (m_c)_i = 0.34(4) M_{\odot} made by Sigurdsson et al. (2003).

The presence of the planet in M4 therefore suggest that coagulation can indeed take place on timescales shorter than the average age of globular clusters, and that dynamical disruption from nearby stars cannot entirely prevent a planet population to form within such dense stellar environments. Recent photometric measurements of globular clusters indicate that multiple stellar populations are actually common and suggest that a non-negligible amount of heavy elements is available in these astrophysical relics (Nardiello et al., 2015; Piotto et al., 2015).

In this chapter, we present current results obtained from an ongoing analysis of the hierarchical PSR B1620-26 triple system. We analyze nearly 30 years of TOA data collected with several premier radio facilities, estimate the elements of the outer orbit, and resolve secular variations of several innerorbital parameters that are likely to be (in part) due to tidal interactions from third-body effects.

4.1 Observations & Reduction

We used the TOA data set constructed by Thorsett et al. (1999) as well as TOAs collected for an additional 16+ years after their publication. In this section, we briefly summarize the observatories and pulsar backends used to collect the TOAs we analyze in the subsection section of this chapter.

The observations undertaken by Thorsett et al. (1999) were performed three different radio facilities. A portion of the Thorsett et al. data set was collected using the Very Large Array (VLA) near Socorro, New Mexico, USA, over the course of one or two days per most of the months between December 1990 to October 1998. A Princeton Mark III machine (Stinebring et al., 1992) was used to incoherently de-disperse data collected across a 50-MHz bandpass centered at 1660 MHz into a single, folded profile with a 5-minute integration time, which ultimately produced 486 TOAs. Another portion of the Thorsett et al. data set was collected with the 76-m Lovell Telescope at Jodrell Bank, England, using a 64-MHz bandpass for low observing frequencies (400, 600 MHz) and a 32-MHz bandpass for higher frequencies (1400, 1600 MHz) using analogue filter banks to incoherently de-disperse the data stream in hardware. For this dissertation, we use additional Jodrell-Bank TOAs that were collected after the Thorsett et al. study was published up to late-March 2003; in total, 608 TOAs were collected with the Lovell Telescope at Jodrell Bank. The third portion of TOAs was collected using the 43-m radio telescope at the NRAO in Green Bank, West Virginia (USA). For each observing epoch, the Spectral Processor – a fast-Fourier-transform spectrometer – was used to digitally sample 512 frequency channels across a 40-MHz bandpass centered on two observing frequencies (430 MHz and 1400 MHz) using a 5-minute integration time; before February 1991, only 256 channels across a 20-MHz bandpass were recorded. Each observation session at Green Bank consisted of recording pulsar signals using two out of four available frequency receivers; these multi-frequency observations were performed contiguously.

We extended the Thorsett et al. (1999) data set to incorporate highprecision TOAs with four different pulsar backends using the GBT, and exclusively using the 1400-MHz receiver. We used the same Spectral Processor discussed above for initial GBT observations that began in November 2001 and ended in August 2004, which yielded 137 TOAs. A portion of our extended data set was collected with the Berkeley-Caltech Pulsar Machine (BCPM; Backer et al., 1997). The BCPM was used between July 2004 and May 2009, and employed a digital filter bank to incoherently de-disperse an incoming signal across 100 MHz in bandwidth with 4-bit sampling and a user-specified number of frequency channels. A total of 307 TOAs were collected with the BCPM backend. We used the GASP coherent-dedispersion backend (first discussed in Section 3.1) between August 2005 and October 2011 to collect a total of 717 TOAs. We began using the wide-band GUPPI coherent-dedispersion backend since February 2011 and continue to use it for ongoing observations, having so far collected 440 TOAs across the full 800-MHz bandwidth. For all four GBT backends, we generally integrated successive pulsars over a 3-minute timescale and across all available bandwidth to form high-S/N, averaged TOAs.

It is important to note that the early and late portions of the GASP data overlap with segments of the BCPM and GUPPI data, respectively. Since these data are collected with the same telescope, and with the same receiver, the overlapping data essentially lead to a redundancy in a small fraction of the TOA measurements and a slight overweighting of pulsar-timing analyses to these data. However, the overlap in data is needed for accurately determining instrumental offsets between the various backends. We therefore currently retain all available TOAs for the analyses presented below.

For the first time, we also incorporated TOAs collected with the 100m Effelsberg Telescope in Bad Münstereifel, North Rhine-Westphalia, Germany. The Effelsberg data were processed using the coherent-dedispersion Effelsberg-Berkeley Pulsar Processor (EBPP; Backer et al., 1997). A single TOA was obtained per observing epoch using the 1410-MHz receiver, after averaging 30-minutes of successive radio pulsars across the entire ~ 64 -MHz bandwidth. In total, 47 TOAs were collected with the Effelsberg Telescope using the EBPP backend between July 1997 and March 2009.

4.2 Methods for Timing Analysis

A grand total of 3,515 TOAs collected for PSR B1620-26 are analyzed in this study. We first determined instrumental offsets between data collected with different backends and/or telescopes by using TEMPO and a fixed timing solution to fit for arbitrary time offsets between short overlapping segments of data. These offsets were then held fixed when performing the various timing analyses discussed in Section 4.3 below.

We used the TEMPO pulsar-timing package for modeling TOAs collected for PSR B1620-26. In all analyses presented in Section 4.3 below, we directly modeled the following physical effects described in Sections 1.3 and 1.4:

- the J2000 positions and proper motion terms;
- the timing parallax, fixed at a value of $\varpi = 0.59$ " determined from optical color and magnitude measurements made by Peterson et al. (1995);
- the five Keplerian elements that describe the Römer timing delay due to the inner orbit, (Δ_R)_i, along with first-order derivatives in one or more elements;
- the DM and a first-order rate of change in DM, which we hold fixed in our timing model using values first determined by Thorsett et al. (1999);
- and the spin frequency, along with one or more spin-frequency time derivatives.

It is important to note that, due to the complex nature of the triple system and its timing model, we do not yet perform a full analysis of TOA uncertainties and their slight adjustment to produce a best-fit $\chi^2_{\rm red} \approx 1.0$. While this does not affect the robustness of the timing model in fitting our

data set, it will likely affect the interpretation of measurements that possess marginal statistical significance. As discussed in Section 4.3, the lack of TOA-uncertainty analysis primarily affects the measurements of several inner-orbital secular variations. However, we show in Section 4.4 that the orders of magnitude of the observed inner-orbital secular variations are consistent with those expected by theory and likely indicate real changes in several elements.

In principle, the effects of the hierarchical outer orbit on pulsar TOAs can be modeled with an additional Römer timing delay and its outer-orbital Keplerian elements.² In this case, which we refer to as the "two-orbit" solution, the total Römer timing delay of the pulsar ephemeris is $\Delta_{\rm R} = (\Delta_{\rm R})_{\rm i} + (\Delta_{\rm R})_{\rm o}$ and two separate sets of Keplerian elements can be obtained using the appropriate eccentric or nearly-circular forms presented in Section 1.4. The simultaneous fit of both orbits, along with the other effects mentioned in the above list, allows for an unambiguous determination of the spin frequency and its first derivative due to spin-down and other kinematic effects discussed in Section 1.5.

However, if the time spanned by a given TOA data set is significantly less than the given binary's orbital period, then a fit for $\Delta_{\rm R}$ and its orbital parameters will not be well constrained and likely fail. These issues were first noted by Thorsett et al. (1999) when applying a two-orbit solution to their data set for PSR B1620-26, where the outer-orbital elements could not be uniquely obtained with sufficient numerical stability in the timing-model fit. Thorsett et al. instead fitted for $(\Delta_{\rm R})_{\rm o}$ for different fixed values of $e_{\rm o}$ and found a large variation in outer-orbital elements as a function of $e_{\rm o}$.

As pointed out by Joshi & Rasio (1997), significant spin-frequency derivatives arise from the periodic Doppler shift due to unmodeled orbital motion. For long-period orbits, one can use the observed spin-frequency derivatives

 $^{^{2}}$ Due to the wide, hierarchical orbital period of the outer companion, third-body perturbations of the inner-orbital elements can approximately be modeled as Taylor expansions of those parameters over time.

to derive the orbital elements. We used the framework first developed by Joshi & Rasio (1997) that we derived, discussed and extended in Appendix B of this dissertation, in order to relate the spin-frequency derivatives to the outer-orbital parameters measured relative to the inner binary's center of mass. The use of the Joshi & Rasio (1997) method, as well as the assumption that $m_{\rm p} + (m_{\rm c})_{\rm i} + (m_{\rm c})_{\rm o} \approx m_{\rm p} + m_{\rm WD} \approx 1.65 \, M_{\odot}$, allows one to derive an estimate of the "full" semi-major axis of the outer orbit $(a_{\rm o})$, as well as the true anomaly (u) and its time derivative (\dot{u}) . It is important to note that the Joshi & Rasio (1997) method yields direct estimates of the outer-orbital orientation angles measured in the plane of the orbit (i.e. $u_{\rm o}$ and $\omega_{\rm o}$), though does not yield any information of orbital nodes; one can compute the corresponding orientation angles of the inner binary by noting that $\omega_{\rm i} = \omega_{\rm o} + \pi$, and $u_{\rm i} = u_{\rm o} = u$.

This indirect method for fitting the outer orbit requires at least five spinfrequency derivatives in order to uniquely solve for the five outer-orbital Keplerian parameters. In the analyses presented below, we measure five or more time-derivatives with using all or subsets of our long TOA data set. However, a complication of Joshi & Rasio (1997) method arises when interpreting the sign and value of $\dot{\nu}_{\rm s}$, since it will contain a time-varying component due to the orbital Doppler shift along with the nominal components that are approximately constant across typical data spans. We elaborate on this complication in more detail in Appendix B.3 and also discuss potential components of $\dot{\nu}_{\rm s}$ and $\ddot{\nu}_{\rm s}$ due to globular-cluster dynamics in Section 4.4.

4.3 Timing Update for PSR B1620–26

During the construction of an updated timing model for PSR B1620-26, it became clear that a global, two-orbit solution that accurately models all available TOAs – where $\Delta_{\rm R} = (\Delta_{\rm R})_{\rm i} + (\Delta_{\rm R})_{\rm o}$ – could not be obtained at this time. An example of the complex behavior is shown in Figure 4.1, where TOA



Figure 4.1: TOA residuals for all data collected from PSR B1620-26, using a two-orbit solution derived from data collected between 1987 and 2010 (see Section 4.3.1 for a discussion of this model). TOAs are collected at 1400 MHz (unless otherwise noted) and are colored in the following manner: 140-ft radio telescope at Green Bank (red circles - 430 MHz, blue circles - 1400 MHz); Lovell Telescope at Jodrell Bank (green circles); VLA (magenta circles); GBT using the Spectral Processor (cyan circles), BCPM (yellow circles), GASP (black circles), and GUPPI (red crosses); and Effelsberg Telescope (blue crosses). The two-orbit solution dramatically fails to model TOAs collected after 2010.

residuals were computed using a two-orbit model with $(P_b)_o \approx 38$ years that we derive from all data taken between late 1987 and mid-2009. (We discuss this model in more detail in Section 4.3.1 below.) It is clear from Figure 4.1 that the two-orbit solution, while adequately predicting TOAs between 1987 and 2009, drastically fails to model TOA data taken after a ~ 2-year gap in 2009-2010.

We nevertheless found that all data taken before the 2-year gap could be reasonably fit with a two-orbit solution, with small non-random structure that may likely indicate significant three-body interactions. Moreover, using the Joshi & Rasio (1997) method, we successfully modeled the outer orbit with a large number of spin-frequency derivatives when examining smaller portions of the whole data set, and several measures of the outer-orbital parameters derived from these spin-frequency measurements were consistent between these subsets.

In this section, we present separate timing analyses of three TOA subsets: one subset consists of TOAs collected before the late-2009 gap in GASP data; the second subset consists of GASP TOAs collected after the 2009 gap, as well as all GUPPI TOAs; and the third analysis is performed on the entire TOA data set for PSR B1620-26. We demonstrate that both "pre-gap" and "post-gap" subsets yield a large number of spin-frequency derivatives that are consistent with an unmodeled outer orbit with a period of ~ 40 years when using the Joshi & Rasio (1997) method, described in Appendix B, for estimating orbital elements. For the data subset collected prior to the 2009 gap, we are also able to successfully fit a two-orbit model with an outer-orbital period of ≈ 38 years, which is consistent with the value derived from the spin-frequency-derivative model of the outer orbit for the same data subset. We also present our current analysis of the global data set using a single timing solution with a large number of spin-frequency derivatives as well as significant first-order changes in the inner-orbital elements, and present results obtained from the Joshi & Rasio (1997) method.

4.3.1 Analysis of "Pre-Gap" Data

We first considered the first portion of the whole data set, where this subset consists of TOAs collected between November 1987 and May 2009. This subset includes all data first published and analyzed by Thorsett et al. (1999), as well as a large portion of data collected with the Effelsberg, Green Bank and Jodrell Bank telescopes discussed in Section 4.1 above. While all available BCPM and Effelsberg data are included in this subset, only half of the GASP data (prior to the 2009 gap) are included here. In total, this subset consists of 2,876 TOAs that span nearly 22 years of observation.

We first developed a timing solution that models the outer orbit using 8 spin-frequency derivatives, while directly fitting for the inner orbit and variations in x across the data set. The best-fit residuals for this timing model are shown in the top panel of Figure 4.2 and the best-fit parameters are presented in Table 4.1. The timing model we derive from this pre-gap subset is in good agreement with the model first derived by Thorsett et al. (1999), with the largest differences in parameter values occurring in the higher-order spin-frequency derivatives. This slight discrepancy is likely due to the large degree of covariance between the higher-order derivative terms as model parameters. However, the higher-order spin-frequency derivatives we measure from the 22-yr, pre-gap subset possess the same order of magnitude and sign as those published by Thorsett et al.

When we use the first five spin-frequency derivatives and the procedure outlined in Appendix B, we estimate that the five outer-orbital Keplerian elements are

$$a_{\rm o} \approx 14 \text{ AU}$$

 $(P_{\rm b})_{\rm o} \approx 41 \text{ years}$
 $e_{\rm o} \approx 0.178$
 $\omega_{\rm o} \approx 175 \text{ degrees}$
 $(T_0)_{\rm o} \approx \text{MJD } 44420, \text{ or July } 1980$
(4.1)

The full list of outer-orbital parameters that can be derived from the Joshi & Rasio (1997) method are presented in Table 4.2. Using the above estimates and Equation B.23, we computed the outer-orbital companion mass to be $(m_{\rm c} \sin i)_{\rm o} \sim 1 \times 10^{-3} \,{\rm M}_{\odot}$. The outer-orbital period and planetary-companion

Parameter	Pre-Gap	Post-Gap				
Start date (MJD)	47104	55008				
Finish date (MJD)	54953	57231				
Reference Epoch (MJD)	51029	56290				
Astrometry						
Right Ascension, $\alpha_{\rm J2000}$	$16^{\rm h}23^{\rm m}38.21610(6)^{\rm s}$	$16^{\rm h}23^{\rm m}38.20230(2)^{\rm s}$				
Declination, $\delta_{\rm J2000}$	$-26^{\circ}31'53.880(4)$ "	$-26^{\circ}31'54.1841(15)$ "				
$\mu_{\alpha} \; (\text{arcsec yr}^{-1})$	-13.03(9)	-12.2(2)				
$\mu_{\delta} \; (\text{arcsec yr}^{-1})$	-21.7(5)	-17(1)				
ϖ (arcsec)	0.59	0.59				
	Spin					
$\nu_{\rm s} ({\rm s}^{-1})$	90.287331294248(2)	90.2873323075030(18)				
$\dot{\nu}_{\rm s} \ (10^{-15} \ {\rm s}^{-2})$	-1.74145(6)	6.02434(18)				
$\ddot{\nu}_{\rm s} \ (10^{-23} \ {\rm s}^{-3})$	1.76871(10)	1.5478(7)				
$\nu_{\rm s}^{(3)} \ (10^{-33} \ {\rm s}^{-4})$	-7.40(3)	4.3(8)				
$\nu_{\rm s}^{(4)} (10^{-41} \ {\rm s}^{-5})$	1.217(5)	2.91(17)				
$\nu_{\rm s}^{(5)} (10^{-49} \ {\rm s}^{-6})$	7.04(17)	-20(20)				
$\nu_{\rm s}^{(6)} \ (10^{-56} \ {\rm s}^{-7})$	-2.42(3)					
$\nu_{\rm s}^{(7)} \ (10^{-65} \ {\rm s}^{-8})$	-5.7(5)					
$\nu_{\rm s}^{(8)} (10^{-72} {\rm s}^{-9})$	1.83(12)	•••				
	Dispersion					
$DM (pc cm^{-3})$	62.862983	62.862983				
$\dot{\rm DM}~({\rm pc~cm^{-3}~yr^{-1}})$	-0.0006997	-0.0006997				
	Inner Binary					
$x_{\rm i} \ ({\rm lt-sec})$	64.8093323(9)	64.8091202(6)				
\dot{x}_{i}	-0.580(5)	-0.52(2)				
$\ddot{x}_{i} (10^{-22} \text{ s}^{-1})$	1.6(3)	0(3)				
e_{i}	0.025315412(9)	0.025315442(8)				
$(T_0)_i (MJD)$	51025.575809(10)	55428.760606(13)				
$(n_{\rm b})_{\rm i} \ (10^{-8} \ {\rm s}^{-1})$	6.0457079725(8)	6.0457080391(15)				
$\omega_{\rm i} \ ({\rm deg})$	117.128143(19)	117.12786(2)				
	Fit Statistics					
$\chi^2_{ m red}$	1.268	1.135				
Number of TOAs	2,876	639				
Weighted RMS residual (μs)	9.982	4.987				

Table 4.1: Best-fit parameters from TEMPO analysis of TOAs collected for PSR B1620-26 when separately analyzing two subsets collected before and after a gap in data around the year 2009. Values in parentheses denote the 1σ uncertainty in the preceding digit(s) as determined from TEMPO. Quantities with no reported uncertainties were held fixed at the listed values; see Section 4.1 for a discussion.



Figure 4.2: TOA residuals of PSR B1620-26 using data collected up to mid 2009. The top panel consists of residuals computed from a timing model where the inner orbit is directly fitted for, and the outer orbit is approximately fitted for with 8 time-derivatives in $\nu_{\rm s}$. The bottom panel shows residuals computed from a model where both Keplerian orbits are directly fitted for, along with one time-derivative in $\nu_{\rm s}$. The residual colors denote the same data sets presented in Figure 4.1.

mass we derive from the spin-frequency derivatives are consistent with the initial estimates made by Thorsett et al. (1999).

The results shown in Equation 4.1 above and Table 4.2 are significantly different from those that were derived by Ford et al. (2000a), who also used the method developed by Joshi & Rasio (1997) to derive the outer-orbital elements from spin-frequency derivatives. Ford et al. used the timing solution constructed Thorsett et al. (1999) and derived a larger eccentricity (≈ 0.45) and orbital period (~ 300 years) than we derive from the pre-gap subset. However, the Thorsett et al. solution only measured four significant spin-frequency derivatives, along with a fifth derivative that was consistent with
Parameter	FJRZ	Pre-Gap	Post-Gap	All Data	
Joshi & Rasio (1997) Parameters					
eo	0.448	0.178	0.306	0.131	
$\omega_{ m o}~({ m deg})$	135	175	227	178	
$u \; (\deg)$	18.9	165	185.3	175	
$\dot{u} (\text{rad yr}^{-1})$	0.055	0.111	0.094	0.210	
Derived Orbital Elements					
$(m_2 \sin i)_{\rm o} (10^{-3} {\rm M}_{\odot})$	6.48	0.80	1.27	0.18	
$a_{\rm o}~({\rm AU})$	56.5	14.0	13.2	9.62	
$(P_{\rm b})_{\rm o}$ (years)	330	40.9	37.8	23.2	
$(T_0)_{\rm o} ({\rm MJD})$		44420	49088	48062	

Table 4.2: Unique solutions of the Joshi & Rasio (1997) method and several derived elements for the outer orbit. In keeping with notation used in Chapter 1, u is the true anomaly and \dot{u} is its time derivative. The parameters directly measured from the Joshi & Rasio (1997) method are then used to derive the outer-orbital elements. The label "FRJZ" refers to the study conducted by Ford et al. (2000a) that used the Thorsett et al. (1999) solution and its constraint on $\nu_{\rm s}^{(5)}$.

zero at the 68.3% confidence level. We therefore consider this difference to not be problematic, as our estimate of $\nu_s^{(5)}$ is statistically significant and more robust as a timing-model parameter.

In conjunction with the TEMPO results discussed above, we used an iterative Markov Chain Monte Carlo (MCMC) procedure in order to explore the phase space spanned by the fit parameters and sample the region of best fit to obtain their posterior probability distributions for all model parameters. The MCMC method consists of a random walk in the parameter space that is generally biased towards regions of more probable parameter values where, after each step in the walk, a likelihood probability is evaluated and compared to a random number drawn from a uniform distribution between 0 and 1; the proposed step in the phase space is accepted if the likelihood is less than than the randomly drawn number, and is rejected otherwise. This process leads to the construction of a Markov chain, where the acceptance or rejection of each

element in the chain only depends on the immediately preceding element.³ While the algorithm generally forces the walk to probe the best-fit region of the parameter phase space, the probabilistic nature of the MCMC method allows for the random walk to reach regions of the parameter space that do not fit the data well. This method of model determination is therefore advantageous for exploring complex local features in the χ^2 phase space, as well as for determining more robust confidence intervals in the case of nonzero and/or nonlinear correlation.

We implemented a MCMC method using a Metropolis-Hastings algorithm that uses TEMPO to compute the χ^2 for a set of fixed parameters at each step in the Markov chain. The ratio of likelihood probability between adjacent elements in the chain is evaluated by computing the quantity $\exp(-0.5\Delta\chi^2)$, where $\Delta\chi^2$ is the change in χ^2 between the current and proposed coordinate in the parameter phase space. We used the covariance matrix (Σ) of the best-fit timing solution discussed above, computed by TEMPO, in order to account for covariance between model parameters when sampling the jointprior normal probability distribution of model parameters,

$$f(x) = \frac{1}{\sqrt{(2\pi)^k \det \Sigma}} \exp\left(-\frac{1}{2}(x-\mu)^{\mathrm{T}}\Sigma(x-\mu)\right)$$
(4.2)

where x is the vector of model parameters, μ is the vector of the mean parameter values, "T" refers to the transpose of the vector $(x - \mu)$, and k is the number of parameters (i.e. the dimension of the phase space). We performed 100,000 iterations of the MCMC method, and the step-acceptance rate for our simulation was ~ 25%. The median values and 68.3% credible intervals for all posterior distributions were consistent with the best-fit values and $1 - \sigma$ uncertainties determined by TEMPO. Figure 4.3 displays the posterior distributions and two-dimensional phase spaces of the spin-frequency derivatives used to model the pre-gap TOA data set. The MCMC method

 $^{^{3}\}mathrm{An}$ "element" of a Markov chain consists of a set of all model parameters.



Figure 4.3: MCMC results for spin-frequency derivatives when analyzing data collected prior to the 2009 gap in GASP data. The scatter plots illustrate correlation between posterior distributions, while the histograms are posterior distributions for the parameter denoted at the bottom of their respective columns.

confirms the high degree of linear correlation between the higher-order frequency derivatives used as timing-model parameters.

One of the advantages of using an MCMC method is that, in principle, the posterior distributions obtained for the spin-frequency derivatives can be converted to distributions of the outer-orbital elements through the use of the Joshi & Rasio (1997) method. We computed the posterior distributions of outer-orbital elements by applying the Joshi & Rasio (1997) method using the first five spin-frequency derivatives within each element of the Markov chain. The results of this translation are shown in Figure 4.4. All 100,000 sets of spin-frequency derivatives were successfully converted to estimates of the outer-orbital elements. The distributions are consistent with the best-fit



Figure 4.4: Posterior distributions of the outer-orbital Keplerian elements, computed from the MCMC posterior distributions of the first five spin-frequency derivatives measured from the pre-gap data set, using the Joshi & Rasio (1997) method discussed in Appendix B.

estimates shown in Equation 4.1 above.

We also derived a timing solution that explicitly models for the outer orbit, based on an extension of an earlier timing solution by Thorsett et al. (1999) that was developed over the past decade as more data was being collected. We display the best-fit residuals in the bottom panel of Figure 4.2 and present the best-fit parameters in Table 4.3. The astrometric and inner-orbital parameters agree with those that were derived from the timing solution that fitted the outer orbit with multiple spin-frequency derivatives. In the two-orbit model, the estimate of $\dot{\nu}_s$ is significantly different from the estimate obtained from the spin-frequency solution due to the fact that the outer-orbital Römer delay essentially accounts for the component of $\dot{\nu}$ due to orbital motion. The outer-orbital Keplerian parameters are generally con-

Dependent	Value (Uncentation)			
Parameter	Value (Uncertainty)			
Start date (MJD)	47104			
Finish date (MJD)	57231			
Reference Epoch (MJD)	52167			
Astrometry				
Right Ascension, α_{J2000}	$16^{h}23^{m}38.21595(6)^{s}$			
Declination, δ_{J2000}	$-26^{\circ}31'53.880(4)$ "			
$\mu_{\alpha} \; (\text{arcsec yr}^{-1})$	-12.96(9)			
$\mu_{\delta} (\operatorname{arcsec} \operatorname{yr}^{-1})$	-21.7(5)			
ϖ (arcsec)	0.59			
Spin				
$\nu_{\rm s} ({\rm s}^{-1})$	90.2873322732(10)			
$\dot{\nu}_{\rm s} \ (10^{-15} \ {\rm s}^{-2})$	-1.4415(14)			
Dispersion	n			
$DM (pc cm^{-3})$	62.862983			
\dot{DM} (pc cm ⁻³ yr ⁻¹)	-0.0006997			
Inner Binary				
$x_{\rm i} \; (\text{lt-sec})$	64.8093261(8)			
$\dot{x}_{\rm i} \ (10^{-12})$	-0.600(5)			
$\ddot{x}_i (10^{-22} \text{ s}^{-1})$	4.7(3)			
e_{i}	0.025315326(8)			
$(T_0)_i (MJD)$	51025.57574(10)			
$(n_{\rm b})_{\rm i} \ (10^{-8} \ {\rm s}^{-1})$	6.0457080216(8)			
$\omega_{\rm i} \; ({\rm deg})$	117.128020(19)			
Outer Binary				
$x_{\rm o} \ (lt-sec)$	2.422(4)			
eo	0.14836(2)			
$(T_0)_{\rm o} ({\rm MJD})$	44441(4)			
$(P_{\rm b})_{\rm o}$ (years)	38.215(19)			
$\omega_{\rm o} \ ({\rm deg})$	183.737(3)			
Fit Statistics				
$\chi^2_{\rm red}$	1.710			
Number of TOAs	2,829			
Weighted RMS residual (μ s)	11.479			

Table 4.3: Best-fit parameters from TEMPO analysis of TOAs collected prior to the 2009 gap for PSR B1620-26, using a two-Keplerian-orbit model.

sistent between the two-orbit and frequency-derivative solutions. However, several outer-orbital timing parameters possess considerably larger uncertainties than those which are normally attainable with pulsar timing. This lack of precision reflects the degeneracy obtained when directly fitting for the outer-orbital Römer timing delay using data that does not yet span a full (outer) orbit.

4.3.2 Analysis of "Post-Gap" GASP and GUPPI Data

We also separately analyzed all data collected after the gap in GASP TOAs that occurred around the year 2009. This data subset consists of post-gap GASP data and all TOAs collected with the GUPPI processor. In total, this subset consists of 639 TOAs that collectively spans over five years of observation with the Green Bank Telescope.

From the post-gap GASP and GUPPI TOAs, we derived a timing solution that used five spin-frequency derivatives to model the outer orbit; the TOA residuals for this timing solution are shown in Figure 4.5. The best-fit parameters of the post-gap timing solution are shown in Table 4.1, alongside the pre-gap solution discussed in Section 4.3.1, and indicate that only four of the five spin-frequency derivatives are significant. The frequency-derivative values are different from those quoted in the pre-gap solution because the reference epoch is different between the pre-gap and post-gap subsets. Moreover, the best-fit estimate of $\dot{\nu}_{\rm s}$ is positive, which indicates that the component of $\dot{\nu}_{\rm s}$ due to outer-orbital motion is positive and larger than the sum of the other components due to intrinsic spin-down and acceleration in the Galactic potential.

As discussed in Appendix B, we used the first four spin-frequency derivatives to find a family of solutions to the Joshi & Rasio (1997) method for different values of e_0 . The family of solutions for the directly-measurable parameters of the Joshi & Rasio (1997) method are shown in Figure 4.6. While only four significant spin-frequency derivatives are measured from the post-gap TOA subset, we can use the (insignificant) best-fit estimate of $\nu_s^{(5)}$ to obtain a unique solution of the outer orbit using the Joshi & Rasio (1997) method. We find that the five spin-frequency derivatives yield the following outer-orbital elements:

$$a_{\rm o} \approx 13 \text{ AU}$$

 $(P_{\rm b})_{\rm o} \approx 37 \text{ years}$
 $e_{\rm o} \approx 0.306$
 $\omega_{\rm o} \approx 227 \text{ degrees}$
 $(T_0)_{\rm o} \approx \text{MJD 49088, or April 1993.}$
(4.3)

which are also presented in Table 4.2 for comparison with other portions of the TOA data set for PSR B1620-26. The post-gap estimates of the outer-orbital Keplerian elements generally differ from those determined from the pre-gap data analyzed in Section 4.3.1, though the derived estimate of a_0 and $(P_b)_0$ are similar between the pre-gap and post-gap subsets. The discrepancies in e_0 , ω_0 $(T_0)_0$ between data subsets are likely due to the fact that $\nu_s^{(5)}$ is not measured with statistical significance; its constraint on the outer-orbit parameters is considerably weak and will change as its significance improves over time with the collection of additional GUPPI TOAs.

We used the same MCMC algorithm described in Section 4.3.1 above to obtain posterior distributions of all fit parameters; we then converted the spin-derivative distributions into ones for the outer-orbital Keplerian elements using the Joshi & Rasio (1997) method, which are also shown in Figure 4.7. In this case, only ~ 72% of all Markov-chain elements produced solutions of the Joshi & Rasio (1997) method. The small fraction of non-solutions to the Joshi & Rasio (1997) method is likely related to the fact that $\nu_s^{(5)}$ is consistent with zero at the 68.3% confidence level, and so a small fraction of



Figure 4.5: TOA residuals of PSR B1620-26 using data collected after the 2009 gap in GASP data, using a frequency-derivative model to account for the outer orbit. Points in red are GASP data, while points in green are GUPPI TOA residuals.

posterior samples for $\nu_{\rm s}^{(5)}$ will have orders of magnitudes and signs that are inconsistent with the component of $\nu_{\rm s}^{(5)}$ due to orbital motion. The resultant distributions of the outer-orbital Keplerian elements are wider than those derived from the pre-gap analysis in Section 4.3.1 above, which also indicate a weak constraint from the observed $\nu_{\rm s}^{(5)}$. Moreover, the distributions for $(T_0)_{\rm o}$ do not overlap, and likely indicates an inability of the Joshi & Rasio (1997) to accurately determine all outer-orbital elements based on only ~4 years of TOA data.



Figure 4.6: Numerically stable solutions of the Joshi & Rasio (1997) method using time-derivatives in ν_s measured for PSR B1620-26 for the post-gap data set. The red-solid lines are solutions found for a fixed value of e_o , while the vertical blue-dashed lines are unique solutions obtained from a full Newton-Raphson method that allows e_o to be determined.

4.3.3 Changes of Inner-Orbital Parameters over Time

Given the greatly-improved TOA precision and the uncertainties in parameter measurements⁴, the inner-orbital Keplerian elements presented in Table 4.1 are significantly different when measured relative to the pre-gap and postgap epochs of periastron, where the difference is $\Delta(T_0)_i = 4,403$ days ≈ 12 years in time. For example, the minimum difference in the e_i measurements between the pre-gap and post-gap estimates of $(T_0)_i$ is nearly 4σ when using the larger of the two uncertainties in e_i . The Δe_i measured between the two data subsets implies a rate of change $\dot{e}_i \approx \Delta e_i / \Delta(T_0)_i \approx 10^{-9} \text{ yr}^{-1}$. Similarly,

⁴The reduced χ^2 for the pre-gap and post-gap fits are slightly greater than 1.0, indicating that uncertainties for the TOAs and best-fit model parameters are underestimated.

the best-fit values of $\omega_{\rm i}$ change between the two data subsets such that the minimum difference is 12σ ; this difference implies a significant rate of change $\dot{\omega}_{\rm i} \approx \Delta(\omega)_{\rm i}/\Delta(T_0)_{\rm i} \approx -2 \times 10^{-5} \text{ deg yr}^{-1}$. The largest change occurs in the measured $(n_{\rm b})_{\rm i}$, where the minimum difference in inner-orbital frequency is nearly 46σ and the inferred $(\dot{n}_{\rm b})_{\rm i} \approx \Delta(n_{\rm b})_{\rm i}/\Delta(T_0)_{\rm i} \approx 5 \times 10^{-17} \text{ s}^{-1} \text{ yr}^{-1}$.

We consider these apparent changes in the inner-orbital elements to reflect real variations due to one or more physical mechanisms affecting the inner binary system. As we discuss in Section 4.4.3, the variation in $(P_b)_o$ is dominated by the component due to mean-field acceleration in the globularcluster potential; this measurement allows for an evaluation of the component of $\dot{\nu}_s$ do to the same mechanism, which we show to be a significant bias in the observed first-order change in spin frequency. In Section 4.4.5, we show that the estimates of \dot{e}_i and $\dot{\omega}_i$ computed above possess orders of magnitude consistent with those expected from prolonged interaction with a third companion in a wide orbit about the inner binary.

4.3.4 Global Analysis of All TOA Data

The current, complete TOA data set for PSR B1620–26 triple system spans 27+ years and is well fit by a timing solution that models the outer orbit using 15 spin-frequency derivatives. The best-fit timing solution, which directly fits for the inner orbit and first-order variations in all inner-orbital elements, is summarized by the parameters listed in Table 4.4 and shown in Figure 4.8. As with the separate pre-gap and post-gap analyses presented above, we currently do not fit for variations in DM over the time span since nearly all data collected was observed using a single receiver centered at 1400 MHz. Instead, we used the DM parameters determined by Thorsett et al. (1999) and held them fixed in our current solution. Since our primary goal is to robustly fit for $(\Delta_R)_o$, we do not consider unaccounted variations in DM to be a significant source of systematic error that prevents or heavily impacts a direct measurement of the time delay due to the outer orbit.

Parameter	Value (Uncertainty)			
Start date (MJD)	47104			
Finish date (MJD)	57231			
Reference Epoch (MJD)	52167			
Astrometry				
Right Ascension, α_{J2000}	$16^{n}23^{m}38.21276(3)^{s}$			
Declination, δ_{J2000}	$-26^{\circ}31'53.964(2)''$			
$\mu_{\alpha} (\text{arcsec yr}^{-1})$	-12.39(5)			
$\mu_{\delta} (\operatorname{arcsec} \operatorname{yr}^{-})$	-19.2(3)			
ϖ (arcsec)	0.39			
Spin				
$\nu_{\rm s} ({\rm s}^{-1})$	90.287331207875(4)			
$\nu_{\rm s} (10^{-23} {\rm s}^{-3})$	1.430(0) 1.7505(2)			
$V_{\rm S} (10^{-1} {\rm S}^{-1})$	1.7505(5)			
$\nu_{s}^{(4)}$ (10 00 s 4)	1.43(7)			
$\nu_{\rm s}^{(4)} (10^{-40} {\rm s}^{-5})$	1.46(3)			
$\nu_{\rm s}^{(3)} (10^{-48} {\rm s}^{-6})$	1.58(8)			
$\nu_{\rm s}^{(6)} \ (10^{-55} \ {\rm s}^{-7})$	-1.03(4)			
$\nu_{\rm s}^{(7)} \ (10^{-63} \ {\rm s}^{-8})$	-3.75(9)			
$\nu_{\rm s}^{(8)} (10^{-71} {\rm s}^{-9})$	7.4(4)			
$\nu_{\rm s}^{(9)}$ (10 ⁻⁷⁸ s ⁻¹⁰)	3.99(10)			
$\nu^{(10)}(10^{-86} \text{ s}^{-11})$	-46(3)			
$\nu_{\rm s}^{(11)} (10^{-93} {\rm s}^{-12})$	-3.15(9)			
$\nu_{\rm s}^{(12)}$ (10 ⁻¹⁰¹ s ⁻¹³)	2.07(18)			
$\nu_{\rm s}^{(13)}$ (10 ⁻¹⁰⁸ s ⁻¹⁴)	1.76(6)			
$\nu_{\rm s}^{(14)} \ (10^{-117} \ {\rm s}^{-15})$	-5.0(6)			
$\nu_{\rm s}^{(15)} \ (10^{-124} \ {\rm s}^{-16})$	-5.3(2)			
Dispersion	n			
$DM (pc cm^{-3})$	62.862983			
$\dot{\rm DM}~({\rm pc~cm^{-3}~yr^{-1}})$	-0.0006997			
Inner Binary				
x_i (lt-sec)	64.8092852(5)			
\dot{x}_{i}	-0.564(3)			
$\ddot{x}_{i} (10^{-22} \text{ s}^{-1})$	1.14(14)			
e_i	0.025315465(13)			
$\dot{e}_{i} (10^{-6} \text{ yr}^{-1})$	-1.01(15)			
$(T_0)_i (MJD)$	51982.790123(18)			
$(n_{\rm b})_{\rm i} (10 \ {\rm s}^{-1})$ $(n_{\rm b})_{\rm i} (10^{-24} \ {\rm s}^{-2})$	0.04370849(4) 1.60(4)			
$(n_{\rm b})_1 (10 {\rm s})$	$117\ 12850(3)$			
$\dot{\omega}_{1}$ (deg) $\dot{\omega}_{2}$ (10 ⁻⁵ deg vr ⁻¹)	-6.0(4)			
Fit Statistics				
$\frac{1}{v^2}$ 1 801				
^red Number of TOAs	3.515			
Weighted RMS residual (μs)	10.287			

Table 4.4: Best-fit parameters from TEMPO analysis of TOAs collected for PSR B1620-26 when using all data and modeling the outer orbit using spinfrequency derivatives. Values in parentheses denote the 1σ uncertainty in the preceding digit(s) as determined from TEMPO. Quantities with no reported uncertainties were held fixed at the listed values, and taken from Thorsett et al. (1999).



Figure 4.7: Posterior distributions of the outer-orbital Keplerian elements, computed from the MCMC posterior distributions of the first five spin-frequency derivatives measured from the post-gap data set, using the Joshi & Rasio (1997) method discussed in Appendix B.

The observed behavior in the pulsar's spin period across the 27-year data span can be directly computed using the measured time-derivatives in spin frequency and Equation 1.12, and is shown in Figure 4.9. The inclusion of higher-order terms in the Taylor expansion, which are due to the outer-orbital motion of the planetary companion about the inner-binary's center of mass, produces a wave-like variation in $P_{\rm s}$ over time. It is clear from Figure 4.9 that the outer orbit possesses a large orbital period, as the variation has not been fully covered and observed to repeat. This feature is consistent with initial estimate made by Thorsett et al. (1999) that $(P_{\rm b})_{\rm o}$ is much longer than their initial data set, and that the outer-orbital period is ~ 100 years. Moreover, Figure 4.9 indicates that the current value of $P_{\rm s}$ is approaching the value observed in 1987, when PSR B1620-26 was discovered by (Lyne,



Figure 4.8: TOA residuals of PSR B1620-26 using the complete data set. The timing solution used to compute these TOAs directly fits the inner orbit and using 15 spin-frequency derivatives to model the outer orbit. The residual colors denote the same data sets presented in Figure 4.1.

1988). Assuming that the observed variations are purely due a third body in a bound orbit, this feature then most likely indicates that the outer orbit will soon reach its point of inflection within the next few years. The observation of another inflection of $P_{\rm s}$ may make a numerically-stable fit for $e_{\rm o}$, and therefore $(\Delta_{\rm R})_{\rm o}$, more robust and allow for a unique determination of both sets of orbital elements from direct modeling of the orbit.

We attempted to implement the Joshi & Rasio (1997) method using the first five spin-frequency derivatives measured in the global timing model, but no convergent solution of the method could be obtained. The fitted value of $(\dot{\nu}_{\rm s})_{\rm obs} = -1.722(4) \times 10^{-17} \text{ s}^{-2}$ is two orders of magnitude smaller than the value published by Thorsett et al. (1999), which corresponds to the time in Figure 4.9 where the orbit-induced curve is turning over and the first deriva-

tive is approximately zero. Therefore, the assumption that $(\dot{\nu}_s)_{obs} \approx (\dot{\nu}_s)_{orb}$ does not hold at the quoted reference epoch of the timing solution. Moreover, as discussed in Appendix B.3 below, the various mechanisms that can contribute to an observable first-derivative in ν_s complicates a robust determination of outer-orbital elements. However, the large number of required spin-frequency derivatives are likely dominated by the outer-orbital motion, and so an analysis based of higher-order derivatives could lead to more "unbiased" estimates of the Keplerian parameters.

As described in Appendix B.3, we extended the Joshi & Rasio (1997) method by re-writing the expected derivatives due to orbital motion in terms of $\ddot{\nu}_{\rm s}$, instead of $\dot{\nu}_{\rm s}$, and thus avoiding the contributions from pulsar spin-down and various terms from kinematic, non-orbital acceleration. This extension of the Joshi & Rasio (1997) method requires a measurement of $\nu_{\rm s}^{(6)}$ in order to uniquely solve for the approximate orbital elements, which we readily made using our full, 27-yr data set. We applied the same Newton-Raphson procedure for determining the "best-fit" values of the outer-orbital elements, and derived the following orbital elements based on the higher-order derivatives up to $\nu_{\rm s}^{(6)}$:

$$a_{\rm o} \approx 10 \text{ AU}$$

 $(P_{\rm b})_{\rm o} \approx 23 \text{ years}$
 $e_{\rm o} \approx 0.131$
 $\omega_{\rm o} \approx 178 \text{ degree}$
 $(T_0)_{\rm o} \approx \text{MJD } 48062, \text{ or June } 1990.$
(4.4)

The outer-orbital eccentricity and semi-major axis are relatively consistent with the estimates made when analyzing the pre-gap TOA subset discussed in Section 4.3.1 above. However, the 23-year outer-orbital period inferred



Figure 4.9: $P_{\rm s}$ versus time for the complete TOA data set for PSR B1620-26, derived from the timing model shown in Figure 4.8.

from the higher-order frequency derivatives is more than a decade smaller than the 40-year period inferred from both the pre-gap and post-gap analyses described above. More strikingly, the outer-orbital period determined from the entire data set is slightly smaller than the time span of the set itself, meaning that a robust fit for $(\Delta_R)_o$ can theoretically be made. However, as discussed at the beginning of Section 4.3, a two-orbit solution based on the entire TOA data set is not currently attainable. Furthermore, our global estimate of $(P_b)_o$ from the extended Joshi & Rasio (1997) method is considerably smaller than the period suggested from the long-term behavior of the spin period illustrated in Figure 4.9. We discuss a variety of plausible mechanisms that could affect our estimation of the outer-orbital elements using the Joshi & Rasio (1997) method in Section 4.4 below.

For the first time, we measure first-order variations in all inner-orbital elements, as well as a significant second derivative in x_i , when deriving our timing model on the full TOA data set for PSR B1620-26. The measurement

of \dot{x}_i was first made by Arzoumanian et al. (1996), and our estimate of \dot{x}_i in Table 4.4 is consistent with the last measurement made by Thorsett et al. (1999) when computing the value at their timing-solution reference epoch. The signs and orders of magnitude of the best-fit variations are consistent with the values derived from computing changes in the elements over time between the pre-gap and post-gap estimates as discussed at the end of Section 4.3.2. As we discuss in Section 4.4.5 below, the variations in e_i and ω_i are likely due to third-body perturbations from the planetary companion.

4.4 Discussion

We demonstrated in Section 4.3 that PSR B1620-26 exhibits complex timing behavior due to a long-period outer orbit. While the analysis of data subsets yields consistent estimates of the outer-orbital elements, and a two-orbit model can be applied to the majority of our complete data set, a global analysis of the entire TOA set does not yet yield a two-orbit solution. In this section, we discuss additional sources of complication in our analyses and perform calculations to determine the likelihood of additional biases from globular-cluster dynamics, as well as to check the veracity of the observed inner-orbital secular variations.

4.4.1 Is PSR B1620–26 a Triple System?

In principle, our use of higher-order derivatives other than $\dot{\nu}_{\rm s}$ when implementing the Joshi & Rasio (1997) method generally allows for an "unbiased" estimate of the outer-orbital parameters, since it avoids any consideration of the various non-binary processes that contribute to the observed first derivative in spin frequency. However, when using the extended Joshi & Rasio (1997) method on the complete TOA data, the derived outer-orbital elements illustrate an eccentric orbit with a period of ~ 23 years. While the inferred eccentricity is generally in agreement with estimates made by analyzing subsets of all available TOAs, the derived outer-orbital period is shorter than the time span of the whole data set and implies that a direct fit of $(\Delta_{\rm R})_{\rm o}$ can be obtained. However, this is inconsistent with the observed behavior of the pulsar's spin period shown in Figure 4.9, where the measurement of higher-order frequency derivatives suggest an outer-orbital period of ~ 100 years. Moreover, we are currently not able to obtain a two-orbit solution for the entire TOA data set collected for PSR B1620-26.

At this time, it is not clear why the extended Joshi & Rasio (1997) method gives conflicting answers when applied to the entire TOA data set for PSR B1620-26. It is also unclear why the two-orbit solution discussed in Section 4.3.1, where $\Delta_{\rm R} = (\Delta_{\rm R})_{\rm i} + (\Delta_{\rm R})_{\rm o}$, currently fails to reasonably model data collected after the late-2009 TOA gap.

Given its globular-cluster association and the complications discussed above, we are confronted with the possibility that PSR B1620-26 is not actually a bound hierarchical triple system. The observed variations in TOA residuals and inner-orbital elements nonetheless indicate that a sustained gravitational interaction is occurring between the pulsar-WD binary and at least one other massive object. One possible scenario involves a hyperbolic encounter of the pulsar-WD binary with a nearby, low-mass star, likely another globular-cluster white dwarf. As with bound orbits, the hyperbolic flyby will induce a number of time-derivatives in spin frequency in the same way that we observe a large number of time-derivatives when extending the TOA data set for PSR B1620-26. Moreover, the spin-frequency analysis of the post-gap data subset for PSR B1620-26 (Section 4.3.2) yields outer-orbital elements that are consistent with the results obtained from the pre-gap subset, which could be caused by both bound and hyperbolic orbits.

While such hyperbolic encounters are possible in globular clusters, the fact that PSR B1620-26 is outside of the central regions of the cluster core (where the stellar-number density is comparatively smaller than at the center) makes this scenario unlikely. However, high-resolution, multi-wavelength

optical imaging and spectroscopy may be able to constrain the likelihood of an interacting white dwarf nearby the PSR B1620-26 system.

For the remainder of the discussion presented below, we assume that the triple-system status of PSR B1620-26 remains valid.

4.4.2 Complications from Moons and Pulsar Glitches

Assuming that the PSR B1620-26 system is indeed a hierarchical triple system, one possibility that is not considered in the above analyses is that the outer planetary object possess a moon. Lewis et al. (2008) considered the idealized scenario where a moon is in a circular orbit about a planet that is in a wider orbit with PSR B1620-26, and found that the moon could be detected through TOA perturbations if its mass is > 5% of the planet's mass and its distance from the planet's center of mass is ~ 2% of the planet-pulsar separation. Thorsett et al. (1999) estimated that the distance between the planet and the inner-binary's center of mass is ~ 35 AU, meaning that the hypothetical moon could be ~ 1 AU away from the planet. However, the TOA perturbations predicted by Lewis et al. are expected to be comparatively smaller (~1 μ s) than those seen in Figure 4.1, and so it is unlikely that a moon could be the dominant cause of the observed TOA variations.

Another possible source of complication arises if PSR B1620-26 exhibits a "glitch" – a sudden, abrupt change in its rotational period that is sometimes followed by a period of relaxation towards the pre-glitch period – sometime during or immediately after the 2009 gap. While still an active area of research, a glitch is believed to indicate a sudden transfer of angular momentum due to either a truncation of an oblate crust towards a more spherical shape (Baym et al., 1969), or the disconnection of vortices between the neutronstar superfluid interior and the remaining solid component of the star (e.g. Glampedakis & Andersson, 2009). The Crab and Vela pulsars are known to exhibit a large number of glitches during the past few decades of observation (e.g. Espinoza et al., 2011; Lyne et al., 2015). While not commonly observed in MSPs, glitches have been seen in PSR B1821-24 (Cognard & Backer, 2004) and more recently in PSR J0613-0200⁵ (McKee et al., 2016). It is currently difficult to assess whether an intrinsic glitch event has occurred around the time of the 2009 gap, since a glitch can be approximately modeled using a large number of spin-frequency derivatives and therefore serves as another biasing effect in our modeling of the outer orbit.

4.4.3 Bias in $\dot{\nu}_{\rm s}$ from Cluster Acceleration

Phinney (1992) pointed out that globular-cluster dynamics due to the collective "mean-field" potential can also contribute to a significant first-order rate of change in Doppler shifts, which affect both spin and orbital periods (as discussed in Section 1.5 above). The globular-cluster association of PSR B1620-26 therefore introduces a possible third component of $\dot{\nu}_{\rm s}$, along with the components from spin-down and kinematic acceleration. For a spherical star cluster, the component of acceleration due to mean-field cluster dynamics corresponds to a change in Doppler shift, such that

$$\left(\frac{\dot{P}_{\rm s}}{P_{\rm s}}\right)_{\rm GC} = \left(\frac{\dot{P}_{\rm b}}{P_{\rm b}}\right)_{\rm GC} = -\frac{1}{c}\frac{GM(< r)}{r^2}\frac{h}{r}$$
(4.5)

where r is the radial distance of the source to the cluster center, h is a projected distance of the source to the plane of the sky that intersects the cluster center, and M(< r) is the total mass contained within a sphere or radius r centered on the cluster core. The value of h can be positive or negative, corresponding to the pulsar being in the front half or back half of the cluster relative to a distant observer. For the first-derivative in spin frequency, we cannot uniquely separate the cluster-dynamics term from the other prominent components due to spin-down, kinematic bias and outer-orbital motion.

 $^{^5 \}rm We$ analyze TOAs collected from PSR J0613–0200 as part of the NANOGrav program in Section 3.4.1 of this dissertation.

However, we can place limits on the significance of the cluster-dynamics component (Equation 4.5) by analyzing the likely mechanism that produced the observed change in $(P_{\rm b})_{\rm i}$.

We used the best-fit astrometric parameters for the global timing solution and a distance d = 1.72 kpc (Peterson et al., 1995) in order to compute the components of $(\dot{P}_{\rm b})_{\rm i}$ due to Galactic acceleration, differential rotation and significant proper motion, using Equation 1.45,

$$\left(\frac{\dot{P}_{\rm b}}{P_{\rm b}}\right)_D = \frac{\dot{D}}{D} = \frac{a_z}{c} - \cos b \left(\frac{\Theta_0^2}{cR_0}\right) \left(\cos l + \frac{\beta}{\sin^2 l + \beta^2}\right) + \frac{\mu^2 d}{c}.$$

Using the acceleration model developed by Kuijken & Gilmore (1989) for a_z , we find that these three non-globular terms produce an expected value of $(\dot{P}_b)_{i,D} = 3.78 \times 10^{-11} \text{ s}^{-1}$. This estimate of the kinematic, non-globular bias is positive and an order or magnitude smaller than the observed negative value, $(\dot{P}_b)_{i,\text{obs}} = -4.36(2) \times 10^{-10} \text{ s}^{-1}$. If the residual amount of \dot{P}_b is due to mean-field cluster acceleration, $(\dot{P}_b)_{i,\text{GC}} = (\dot{P}_b)_{i,\text{obs}} - (\dot{P}_b)_{i,D} \approx -5 \times 10^{-10}$, then it is clear from Equation 4.5 that h > 0, and that the PSR B1620-26 triple system is being accelerated away from observers on Earth.

Furthermore, assuming that $(\dot{P}_{\rm b})_{\rm i,GC} \approx -5 \times 10^{10}$ and using the left-hand side of Equation 4.5, we find that the spin-frequency derivative due to meanfield cluster acceleration is $(\dot{\nu}_{\rm s})_{\rm GC} \approx 2 \times 10^{-15} \ {\rm Hz}^{-2}$, which is comparable in order of magnitude to the value of $\dot{\nu}_{\rm s}$ published by Thorsett et al. (1999) and could therefore likely be a significant component in our measurement of the first derivative in spin frequency. While $(\dot{\nu}_{\rm s})_{\rm GC} > 0$, the corrected value of our first-derivative measurement for the two-orbit model presented in Table 4.3 is $(\dot{\nu}_{\rm s})_{\rm corr} = (\dot{\nu}_{\rm s})_{\rm obs} - (\dot{\nu}_{\rm s})_{\rm GC} < 0$, and is compatible with the expected changes in rotation from pulsar spin-down. Given the computation of $(\dot{P}_{\rm b})_{\rm i,D}$ above, the component of $\dot{\nu}_{\rm s}$ due to non-cluster kinematic acceleration is smaller in order of magnitude than the spin-down component.

4.4.4 Bias in $\ddot{\nu}_{\rm s}$ from Cluster Jerks

The discrepancy in the Joshi & Rasio (1997) estimate of $(P_{\rm b})_{\rm o}$ shown in Table 4.2 could possibly be due to further bias in several spin-frequency derivatives associated with accelerations and jerks from nearby stars in the M4 globular cluster. Blandford et al. (1987) first pointed out that globularcluster pulsars would experience observable time-varying perturbations from gravitational fields of nearby cluster members, especially those that reside in the denser environments of cluster cores. The net change in gravitational accelerations over time will produce an observable second derivative in spin frequency, $(\ddot{\nu}_{\rm s})_{\rm GC}$, that results from a time-averaged jerk. The second-order change in spin period varies with an order of magnitude of

$$\left(\frac{\ddot{P}_{\rm s}}{P_{\rm s}}\right)_{\rm GC} \sim 10^{-29} \left(\frac{\sigma}{10 \text{ km s}^{-1}}\right)^3 \left(\frac{r_{\rm c}}{1 \text{ pc}}\right)^{-2} \text{ s}^{-2}$$
(4.6)

where σ is the mean velocity dispersion of the globular cluster and r_c is characteristic radius normally taken to be the core or tidal radius. For Messier 4, $\sigma = 3.5(3)$ km s⁻¹ and $r_c \sim 40' \sim 10$ pc assuming a distance of 1.72 kpc to the cluster (Peterson et al., 1995), which suggests that $(\ddot{P}_s)_{\rm GC} \sim 10^{-33}$ s⁻¹. This is significantly smaller than the observed second-derivative in spin period, where $|(\ddot{P}_s)_{\rm obs}| \sim 10^{-27}$ s⁻¹. We therefore do not believe that biases in $\ddot{\nu}_s$ due to jerks from nearby cluster star are present in our timing model.

4.4.5 Variations of the Inner-Orbital Elements

The observed variations of x_i , e_i and ω_i are likely due to three-body interactions between the inner components and the outer planet. Arzoumanian et al. (1996) argued that the highly significant \dot{x}_i is mostly dominated by the three-body component (Equation 4.7), since the maximum component due to kinematic bias from proper motion (Equation 1.43) only constitutes ~ 10% of the observed value. To first order, the secular perturbations of hierarchical inner orbits due to outer companions are expected to exhibit the following rates of change in several elements (Rasio, 1994):

$$\dot{x}_{i} = \frac{3\pi (a_{p}\cos i)_{i}w}{2c(P_{b})_{i}}\sin 2\theta_{o}\cos(\omega_{i}+\phi_{o})$$

$$(4.7)$$

$$\dot{\omega}_{\rm i} = \frac{3\pi w}{(P_{\rm b})_{\rm i}} \left[\sin^2 \theta_{\rm o} (5\cos^2 \phi_{\rm o} - 1) - 1 \right]$$
(4.8)

$$\dot{e}_{i} = -\frac{15\pi e_{i}w}{2(P_{b})_{i}}\sin^{2}\theta_{o}\sin 2\phi_{o}$$

$$\tag{4.9}$$

where the various terms are summarized as follos: $w = (m_2/m_1)[a_i/r_o]^3$; $m_2 = (m_c)_o$; $m_1 = m_p + (m_c)_i$; $(a_p \cos i)_i$ is the product of the inner-binary cos *i* times the semi-major axis of the pulsar relative to the inner-binary center of mass; a_i is the total semi-major axis of the inner binary; and (r_o, θ_o, ϕ_o) are the spherical coordinates of the outer planet in a fixed coordinate system centered on the inner-binary center of mass, with ϕ_o measured from the inner-orbital periastron argument in the inner-orbital plane and θ_o measured relative to the inner-orbital angular momentum vector (with $\theta_o = 0$ pointing in the same direction as the angular momentum vector).

As discussed in Section 4.1 above, we have not yet fully characterized and adjusted the uncertainties in our TOA measurements for the global analysis discussed in Section 4.3.4. Moreover, Figure 4.10 shows non-random structure in TOA residuals at ~ 10 - μ s level that is likely due to an indirect fit of the outer orbit by using spin-frequency derivatives. We therefore do not yet utilize our measurements of \dot{x}_i , \dot{e}_i and $\dot{\omega}_i$ to constrain the geometry of the outer orbit. We can nevertheless perform an order-of-magnitude comparison of the observed perturbations to their expected values, given a set of assumptions motivated by our study of PSR B1620-26. For instance, we determined that $m_2 \sim 10^{-3}$ M_{\odot} in the above analyses of spin-frequency data from our TOA set, and Sigurdsson et al. (2003) determined that $(m_c)_i = 0.34(5)$ M_{\odot} from a WD cooling-sequence analysis of optical colors and magnitudes, which likely means that $m_1 \sim 2$ M_{\odot}. For the purposes of calculation, we assume an



Figure 4.10: A zoomed-in view of the TOA residuals shown in Figure 4.8. The residual colors denote the same data sets presented in Figure 4.1.

inclination of the inner binary $i = 45^{\circ}$, so that $a_{\rm p} = x_{\rm i}/\sin i \approx 0.2$ AU and so $a_{\rm i} \sim 1$ AU. If we assume that $r_{\rm o} = 30$ AU and neglect the $(\theta_{\rm o}, \phi_{\rm o})$ trigonometric terms in Equations 4.7-4.9, we find that $\dot{x}_{\rm i} \sim 10^{-13}$, $\dot{\omega}_{\rm i} \sim 10^{-5}$ yr⁻¹ and $\dot{e}_{\rm i} \sim 10^{-9}$ yr⁻¹. These order-of-magnitude estimates of the inner-orbital secular variations are consistent with the observed changes between pre-gap and post-gap analysis presented in Section 4.3.3, as well as the variations measured from the global solution derived from all data discussed in Section 4.3.4.

4.5 Further Work

As discussed above, we could not successfully apply a two-orbit model to the full, 27-year TOA data set we have collected for PSR B1620-26 at this time. We nonetheless derived comparable estimates of the outer-orbital parameters

when examining subsets of the whole TOA data set. However, we derived estimates of the outer-orbital period using the Joshi & Rasio (1997) method that are shorter than suggested from the model of the spin period as a function of time shown in Figure 4.9. We considered the possibility of significant components in $\dot{\nu}_s$ and $\ddot{\nu}_s$ due to globular-cluster dynamics, and found that there is a likely bias in the first time-derivative. Future work will assess the possibility that a pulsar glitch occurred sometime during or after the late-2009 gap, which could add further, significant bias into the spin-frequency model of the outer orbit for the global analysis if left unaccounted.

An explicit two-orbit model will likely make the measurements of innerorbital variations more robust, since the use of many spin-frequency derivatives to model the outer orbit is an incomplete parametrization of the orbit and introduces large degrees of statistical correlation between derivatives. Once a two-orbit model is obtained, we will implement an analysis of the inner-orbit perturbations similar to the method used by Joshi & Rasio (1997) and Thorsett et al. (1999) to uniquely constrain the geometry of the outer binary. The increasing significance of \ddot{x}_i over time will yield an additional constraint on the analysis of inner-orbital perturbations and will allow for direct constraints on the inner-binary mass components, which have so far not been possible.

Finally, we will eventually use the three-body integrator developed by A. Archibald for the analysis of the PSR J0337+1715 stellar triple system performed by Ransom et al. (2014), which has been shown yield additional interaction parameters and allow for unique determination of the three hierarchical-component masses.

Chapter 5

Long-Term Observations of the Relativistic PSR B1534+12 Binary System

The population of "double-neutron-star" (DNS) systems – binary orbits that consist of two neutron stars – is expected to be comparatively small due to the need for both components to undergo a supernova event; if the binary system ultimately survives and both components remain bound, then the post-formation system eccentricity is expected be $e \sim 0.1$ or greater due to the injected energy from both supernovae. Indeed, DNS systems are observationally rare as only 11 such systems are currently known of among the $\sim 2,500$ pulsars observed in the Galaxy (see Table 1 in Martinez et al., 2015). The identification of a binary system as a DNS type is less straightforward than for a pulsar-WD system, where WD companions in relatively nearby systems can be observed with sufficiently sensitive optical telescopes. However, DNS systems typically have comparable eccentricities and spin parameters, with $P_{\rm s} \sim 50$ ms, that reflect a common evolution; their evolutionary history is generally thought to consist of minimal mass transfer between components since their massive progenitor stars are expected to have evolved on short and comparable timescales (e.g. Stairs, 2003).

Despite their rarity, DNS systems currently offer the most precise means for testing the predictions of Einstein's relativity theory in the "strong-field" regime. The most relativistic pulsar-binary systems can exhibit a large number of PK secular variations of orbital elements that are measurable on decadal timescales. This is especially true for DNS systems with orbital periods on the order of hours, where relativistic secular variations are expected to be comparatively large in magnitude. A classic example of a relativistic DNS system is PSR B1913+16, famously known as the "Hulse-Taylor" system and the first pulsar-binary system to be discovered, for which the first PK measurements in a pulsar-binary system were made. The eventual measurement of orbital decay in the PSR B1913+16, consistent with the prediction from GR, provided the first evidence for the existence of gravitational radiation (Taylor & Weisberg, 1989). Long-term timing of PSR B1913+16 produced a set of pulse profiles that were observed to be secularly changing in time, consistent with the notion of geodetic precession of the pulsar's spin axis about the orbital angular momentum vector occurring in the relativistic system and ultimately leading to an evolving view of the two-dimensional radio beam (Weisberg et al., 1989). As with the PK timing parameters discussed in Section 1.5, the rate of geodetic precession (Ω_1^{spin}) can be computed under the assumption of a gravitational theory. In GR, this rate is given as (e.g. Barker & O'Connell, 1975)

$$\Omega_1^{\rm spin} = \frac{1}{2} T_{\odot}^{2/3} n_{\rm b}^{-5/3} \frac{m_{\rm c} (4m_{\rm p} + 3m_{\rm c})}{(1 - e^2)(m_{\rm p} + m_{\rm c})^{4/3}}.$$
(5.1)

However, the measurement of geodetic precession in PSR B1913+16 requires a model of the two-dimensional beam shape (Kramer, 1998), which is not immediately known. This DNS system nevertheless remains a valuable highprecision laboratory that for many years yielded only one test of GR using the $\dot{\omega}$ - $\dot{P}_{\rm b}$ - γ combination, as the Shapiro timing delay was not significant for many decades due to low inclination (Weisberg et al., 2010). However, with the aid of substantial periastron advance over the years, the Shapiro delay has recently become significantly measurable for the first time (Weisberg & Huang, 2016).

PSR B1534+12 is a 37.9-ms radio pulsar in a 10.1-hour orbit with another neutron star that currently exhibits up to six PK deviations in the system's orbital parameters and orientation due to strong-field gravity. In their first long-term analysis of PSR B1534+12, Stairs et al. (1998) provided the firstever measurements of at least five PK timing parameters from one pulsarbinary system; this also corresponded to the first time that multiple tests of GR could be performed using a single gravitationally-bound system immersed in the strong-field regime.

In 2014, we published a study that built on work done by Stairs et al. (1998, 2002) and analyzed TOAs from PSR B1534+12 that collectively span 22 years; this data was obtained exclusively with the 300-m Arecibo telescope using three generations of signal processors (Fonseca et al., 2014). We improved the quality of the best test obtained from this system using the $\gamma - s - \dot{\omega}$ combination and showed that GR is correct to within 0.17% of its predictions, nearly a factor of 8 smaller than the previous timing study performed by Stairs et al. (2002). Stairs et al. (2004) made the first quantitative (but low-precision) beam-model-independent measurement of the relativistic spin-precession rate by comparing aberration effects and precessioninduced changes in the total-intensity profile of PSR B1534+12. In our 2014 study, we confirmed this detection and achieved a precision measurement of $\Omega_1^{\rm spin} = 0.59^{+0.12}_{-0.08}$ degrees per year, which is in excellent agreement of the GR prediction of 0.51 degrees per year. We also used evolving polarization properties to uniquely determine the full orientation of the system, confirm the relativistic precession of the system, and constrain the misalignment angle between spin and orbital angular momenta to be 27 ± 3 degrees.

The majority of the timing analysis presented by Fonseca et al. (2014), as well as an initial investigation of the total-intensity pulse profiles, constituted the bulk of the M .Sc. thesis written by E. Fonseca. However, work was done after the formal granting of the M. Sc. degree that was incorporated into the Fonseca et al. (2014) publication, including the determination of the precise relativistic spin precession rate. In this chapter, we summarize the work done for the analysis of TOAs and pulse profiles from PSR B1534+12 during the Ph. D. program. In Section 5.1, we briefly discuss the logistics of data used by Fonseca et al. (2014) since most of the results presented in this chapter are derived from these data. In Section 5.2, we present an analysis of DM measurements and ISM turbulence made using the 22-yr data set. In Section 5.3, we present the tools and analyses used to quantify relativistic precession in pulse-structure data. In Section 5.4, we present initial results obtained from ongoing observations of PSR B1534+12 using the PUPPI coherent dedispersion backend. In Section 5.5, we summarize the results presented by Fonseca et al. (2014) and discussed in this chapter, as well as discuss future prospects in PUPPI observations of PSR B1534+12.

5.1 Observations & Reduction

Data were obtained exclusively with the 305-m Arecibo Observatory in Puerto Rico, using two observing frequencies and three generations of pulsar signal processors. Basic information regarding the data and backends used in this analysis are presented in Table 1 of Fonseca et al. (2014) while a more detailed account of observing information can be found in Fonseca (2012).

Part of this set of pulse profiles and times-of-arrival (TOAs) were recorded with the Mark III (Stinebring et al., 1992) and Mark IV (Stairs et al., 2000) pulsar backends. The Mark III system employed a brute-force pulse dedispersion algorithm by separating each receiver's bandpass into distinct spectral channels with a filterbank, detecting the signal within each channel, and shifting the pulse profile by the predicted amount of dispersive delay for alignment and coherent averaging. A small amount of Mark III data was obtained using the coherent-dedispersion "reticon" subsystem; these data were used only in the polarization analysis. The Mark IV machine was an instrumental upgrade which employed the now-standard coherent de-dispersion technique (Hankins & Rickett, 1975) that samples and filters the data stream prior to pulse detection. A series of digital filters applied in the frequency domain completely remove the predicted dispersion signatures while retaining even greater precision than the Mark III counterpart. See Stairs et al. (1998, 2002) for more details on these observing systems and reduction of PSR B1534+12 data obtained with these two backends.

Recent data were collected with the Arecibo Signal Processor (ASP; Demorest, 2007), a further upgrade from the Mark III/IV systems that retains the coherent de-dispersion technique, but first decomposes the signals across a bandwidth of 64 MHz into a number of 4-MHz spectral channels that depends on the observing frequency. We used data collected with the four innermost spectral channels centered on 430 MHz, and typically sixteen channels centered on 1400 MHz with some variability, due to limits in computer processing and available receiver bandpass. While the Mark IV machine used 4-bit data sampling in 5-MHz-bandpass observing mode and 2-bit sampling in 10-MHz-bandpass observing mode, ASP always used 8-bit sampling. The coherent de-dispersion filter is applied to the raw, channelized data, which are then folded modulo the topocentric pulse period within each channel and recorded to disk, preserving polarimetric information.

Observations were generally conducted at semi-regular intervals, with typical scan lengths of an hour for each frequency. Several extensive "campaign" observations were also conducted at 430 MHz, which consisted of several-hour observing sessions performed over 12 consecutive days, in order to obtain high-precision snapshots of the pulse profile at different times. Campaign sessions occurred during the summers of 1998, 1999, 2000, 2001, 2003, 2005, and 2008. We used all available data for the timing analysis, and only used most of the campaign profiles and several strong bi-monthly scans for the profile analysis. We excluded the 2008 ASP data from the profile-shape analysis due to weak, heavily scintillated signals recorded during this epoch, but used several stronger observations during this campaign for the RVM analysis (see Section 5.3 below).

We used the template cross-correlation algorithm developed by Taylor (1992) for determining pulse phases, their TOAs and uncertainties using a standard-template profile. A standard template was derived for the Mark III and Mark IV backends at each frequency by averaging several hours of consecutive pulse profiles; ASP TOAs were derived using the Mark IV templates. We added small amounts of error in quadrature or as factors to the original TOA uncertainties, in order to compensate for apparent systematic errors in TOAs. We also ignored TOAs with uncertainties greater than 10 μ s; only 10% of all available TOAs – including points affected by radio frequency interference – were excised when using this cut.

It is important to note that there is a overlap in pulse TOAs collected with the Mark IV and ASP data sets between MJD 53358 and 53601. We incorporated TOAs acquired from both machines during this era, despite the overlap, due to the substantially larger ASP bandwidth; we believe that this difference in bandwidth does not produce many redundant data points. The improvement in bit sampling between backends has a measurable effect on the pulse profile shape, as discussed in Section 5.3 below.

5.2 DM Variations in PSR B1534+12

The DM of any pulsar traces the amount of free electrons in the ISM along the observer's line of site. Three-dimensional motion of the pulsar will change the instantaneous line of sight and ultimately lead to a changing number of electrons in between the observatory and pulsar, which alters the amount of dispersion experienced by a broadband electromagnetic signal. This effect manifests itself as an inherently unpredictable change in DM over time as the Galactic electron distribution is not known.¹ If left unaccounted, the stochastic signal from DM variations will bias other timing parameters due to a suboptimal model fit. Moreover, if one seeks to test the predictions of GR or any other viable theory of gravitation, then all physical processes (whether deterministic or inherently random) must be modeled in order to obtain unbiased estimates of PK variations and the Shapiro timing delay.

In the case of PSR B1534+12, DM variations are easily observed and have dramatic, piecewise structure over the course of the 22-yr data set analyzed by Fonseca et al. (2014). These variations are shown in Figure 5.2. As first noted in the M. Sc. thesis for E. Fonseca, we chose to probe DM evolution across our data set in two different ways. For one method, shown as the black data points in Figure 5.2, we fit for $\Delta_{\rm DM}$ in 80-day bins using the Mark IV and ASP TOAs, where dual-receiver TOAs are available. For the second method, shown as solid, sloped lines in Figure 5.2, we instead fit for four distinct DM bins that account for any temporal evolution as time derivatives of DM. Both methods adequately model the DM variations in PSR B1534+12, though we ultimately chose the few-bins/gradients method as it reduced the number of free parameters in our timing model.

These long-term DM measurements are useful for a statistical analysis of turbulence within the interstellar medium (e.g. Kaspi, Taylor, & Ryba, 1994), which usually assumes that the power spectrum of spatial variations in electron density is a power law within a range of length scales (Rickett, 1990),

$$P(q) \propto q^{-\beta}, \qquad q_o < q < q_i \tag{5.2}$$

where $q = 2\pi/l$ is a spatial frequency and l is a scattering length. The frequency range in Equation 5.2 corresponds to a range between an "inner"

¹In fact, the combination of distance and DM measurements from pulsar TOA analyses have allowed for rough models of the Galactic n_e distribution to be inferred (e.g. Cordes & Lazio, 2001).



Figure 5.1: DM variations from TOAs collected from PSR B1534+12. Horizontal-dashed lines represent the timespan where the denoted pulsar backend was used. Points with uncertainties represent fits of DM to segments of data contained within 80-days bins. Black-solid lines represent fits of DM and rate of change in DM across larger segments of time. This figure was first published by Fonseca et al. (2014).

 (l_i) and "outer" (l_o) length scale where the power-law form is valid. The observed spatial fluctuations due to a relative transverse velocity v are related to a time lag τ by $l = v\tau$. The power spectrum P(q) can therefore be estimated by computing a pulse-phase structure function $D_{\phi}(\tau) = \langle [\phi(t + \tau) - \phi(t)]^2 \rangle$, where the angle brackets represent an ensemble average over observing epoch t. The pulse's electromagnetic phase ϕ is linearly related to DM, which therefore relates $D_{\phi}(\tau)$ to a DM structure function $D_{\text{DM}}(\tau) = \langle [\text{DM}(t + \tau) - \text{DM}(t)]^2 \rangle$,

$$D_{\phi}(\tau) = \left(\frac{2\pi C}{f}\right)^2 D_{\rm DM}(\tau) \tag{5.3}$$

where $C = 4.148 \times 10^3 \text{ MHz}^2 \text{ pc}^{-1} \text{ cm}^3 \text{ s}$, and f is the observing frequency in MHz. Moreover, $D_{\phi}(\tau)$ is a power law in τ within the inner length scales defined in Equation 5.2, which finally requires that

$$D_{\phi}(\tau) = \left(\frac{\tau}{\tau_0}\right)^{\beta-2} \tag{5.4}$$

where τ_0 is a normalization constant with units of time. Scintillation theory requires that $\tau_0 = \tau_d$, where τ_d is the timescale of diffractive scintillation of the signal (e.g. Cordes et al., 1985), if the inner length-scale $l_i \leq v\tau_d$.

We computed values of $D_{\phi}(\tau)$ at f = 430 MHz using the Mark IV and ASP small-bin measurements of DM shown in Figure 5.2. The Mark III DM point was measured using all Mark III TOAs collected over several years, which were generated with a different standard profile than the one used for the Mark IV and ASP data; we therefore chose to ignore this measurement in order to avoid incorporating bias in the structure function. Uncertainties in $D_{\phi}(\tau)$ were determined by propagating errors from our DM(t) measurements. Our estimate of $D_{\phi}(\tau)$ is shown in Figure 5.2, and illustrates a power-law evolution between time lags of roughly 70 and 900 days. We fitted Equation 5.4 to this segment of data, and found that

$$\beta = 3.70 \pm 0.04$$

 $\tau_0 = 3.0 \pm 0.8 \text{ minutes}$ (5.5)

which is shown as a solid black in in Figure 5.2.

The measured spectral index β is consistent with the value for a "Kolmogorov" medium, $\beta_{\text{Kol}} = 11/3$. Furthermore, β and τ_0 in Equation 5.5 are consistent with the structure-function estimates reported by Scheiner & Wolszczan (2012). Our estimate of τ_0 is also consistent with the value of τ_d measured from the autocorrelation function of a dynamic spectrum of PSR B1534+12 (Bogdanov et al., 2002).



Figure 5.2: Phase structure function D_{ϕ} as a function of time lag τ . The solid line is a best-fit model of Equation 5.4 for data with lags between 70 and 900 days. This figure was first published by Fonseca et al. (2014).

At large timescales, the structure function departs from the fitted model at a lag $\tau_o \approx 900$ days, which suggests that

$$l_o \approx 52 \left(\frac{v}{100 \text{ km/s}} \right) \text{ AU}$$
 (5.6)

Bogdanov et al. (2002) derived an interstellar scintillation (ISS) velocity of 192 km/s. They noted in their study that ISS velocities of pulsars are typically dominated by the systemic transverse component, which means that $v \approx 192$ km/s for PSR B1534+12, and $l_o \sim 100$ AU $\sim 10^{15}$ cm from Equation 5.6. This estimate is consistent with the upper limit of l_o observed for several pulsars by Phillips & Wolszczan (1991). By contrast, there is no evidence for a significant inner scale from our data set, since bins with mean values less than 70 days contain only one or two pairs of DM(t) and were therefore ignored in the analysis. We did not apply any correction for the solar-wind contribution of our DM(t) measurements, due to a covariance between the TEMPO solar-wind DM model and a fitted timing parameter that is discussed in Fonseca et al. (2014).

5.3 Geodetic Precession and Secular Evolution in Pulse Structure

An observed pulse produces a set of Stokes-vector pulse profiles, from which a TOA can be derived when cross-correlating the total-intensity profile Iwith a standard template as described in Chapter 1. As with pulsar timing, gravitational strong-field effects can give rise to observable changes in parameters that describe the electromagnetic structure of pulses, such as the total-intensity shape and polarization properties, over a variety of timescales. Such variations will occur if there is a misalignment between the spin axis of the pulsar and the orbital angular momentum vector of the binary system (de Sitter, 1916). This spin-orbit coupling, generally referred to as *geodetic precession*², ultimately leads to an evolving view of the radio beam emitted from the pulsar and different slices of the radio cone over time. The effects of relativistic spin precession on pulse structure have also been observed in PSR B1913+16 (Weisberg et al., 1989), the double-pulsar system (Breton et al., 2008), and most dramatically in PSR J1141-6545 (Manchester et al., 2010).

In order to detect such changes in PSR B1534+12, we shifted our 430-MHz profiles to a common phase using the derived DD-binary timing model described by Fonseca et al. (2014). Each set of "campaign" data, where PSR B1534+12 was observed for the entirety of its observable track in the sky for

 $^{^{2}}$ Pulsar literature also refers to geodetic precession as *relativistic spin precession*, as we do at the beginning of this chapter.

14 consecutive days, was then binned into twelve orbital-phase cumulative profiles. Several particularly strong, non-campaign scans that were taken during the observing year were integrated into single profiles recorded at their respective epochs and included in this analysis. We subsequently performed two distinct types of analyses on these total-intensity and polarization data in order to extract gravitational information from independent techniques, as described below.

5.3.1 Methodologies

For the first analysis, we employed the general model developed by Stairs et al. (2004) that establishes pulse-structure data as functions of time and location within the relativistic orbit. Estimates of the total-intensity profile shape (F) at a given epoch were derived by first applying a principal component analysis (PCA) on a set of total-intensity profiles collected over time. The first and second principal components correspond to an average (P_0) and "difference" (P_1) profile, respectively, and an observed profile within the timespan of the PCA input can be approximately represented by a linear combination of the two PCA components: $P = c_0P_0 + c_1P_1$. The coefficients c_0 , c_1 were estimated using a cross-correlation algorithm between the observed profiles and principal components in the Fourier domain, and the shape F of each profile was then estimated by calculating the ratio $F = c_1/c_0$ in order to negate epoch-dependent scintillation effects.

The shape F of a profile recorded at time t and eccentric anomaly E can also be determined using the relation

$$F_{\rm mod} = \frac{dF}{dt}t + \delta_{\rm A}F(E) + F_0 \tag{5.7}$$

where F_0 is an intercept parameter and dF/dt and $\delta_A F$ are the terms that describe secular and aberrational changes in the pulse shape, respectively. These two important terms are functions of the pulsar's precession rate Ω_1^{spin}


Figure 5.3: Difference between cumulative 2005 campaign profiles for the Mark IV and ASP backends. This figure was first published by Fonseca et al. (2014).

and the angle ϵ between the line of nodes and the projection of the spin axis on the plane of the sky:

$$\frac{dF}{dt} = F'\Omega_1^{\rm spin}\cos\epsilon\sin i \tag{5.8}$$

$$\delta_{A}F = F'\frac{\beta_{1}}{\sin i}[-\cos\epsilon S(E) + \cos i\sin\epsilon C(E)]$$
(5.9)

The parameter $F' = dF/d\zeta$ characterizes the unknown beam structure as a function of the auxiliary "viewing" angle ζ , $\beta_1 = 2\pi x/(P_b\sqrt{1-e^2})$ is the mean orbital velocity of the pulsar, and

$$C(E) = \cos[\omega + u(E)] + e \cos \omega$$
$$S(E) = \sin[\omega + u(E)] + e \sin \omega$$

are time-dependent orbital terms that depend on the true anomaly u(E). The Keplerian binary parameters in Equations 5.8 and 5.9 were determined through pulsar-timing techniques described in Chapter 1, and are presented by Fonseca et al. (2014).

We fitted Equation 5.7 to our 430-MHz data using an MCMC implementation with a Metropolis algorithm in order to obtain posterior distributions of Ω_1^{spin} , ϵ , F', and F_0 from uniform priors. We assumed that the joint likelihoo probability density of the model, $J(\Omega_1^{\text{spin}}, \epsilon, F', F_0|F, t, E)$, is a normal distribution in the χ^2 goodness-of-fit statistic for the profile-shape model,

$$J \propto \exp\left[-\frac{1}{2}\sum_{i}\left(\frac{F_i - F_{\text{mod}}(t_i, E_i)}{\sigma_i}\right)^2\right]$$
(5.10)

where σ_i is the uncertainty in F_i determined from the cross correlation between the *i*th profile and the two principal components. The results of this fitting procedure are summarized in Table 5.1 and discussed in Section 5.3.2 below.

As a second analysis, we used Equation 1.2 to fit an RVM to available polarization position-angle data on each full-sum campaign profile. As discussed in Section 1.1, an RVM fit to significant measurements of Ψ as a function of the pulse-rotation phase ϕ yields α and β simultaneously.³ While no evolution is expected in α , geodetic precession will cause β to evolve with time such that $d\beta/dt = \Omega_1^{\text{spin}} \cos \epsilon \sin i$ (Damour & Taylor, 1992). While the

³As first noted in Chapter 1, we use the angle-measurement convention adopted by Damour & Taylor (1992) when measuring and reporting values of α and β ; this convention is not consistent with the IAU standard. However, we are ultimately interested in the rates of change in the RVM orientation angles and their association with geodetic precession. The analysis of their time derivatives does not depend on choice of convention.



Figure 5.4: Full-sum profile data for PSR B1534+12 collected during the June 2003 campaign. (*Top.*) Ψ as a function of ϕ , are shown as blue points. The thick red line is the best-fit RVM for this profile. (*Bottom.*) Stokes *I*, *L* and *V* as a function of ϕ . (For clarity, uncertainties in Ψ are not shown in this figures, but were used in all RVM fits.)

sign convention used in Equation 1.2 is inconsistent with the IAU standard, we are only concerned with secular variation of β over time and its connection to Ω_1^{spin} ; the choice in sign convention will ultimately make no difference in the measured estimate of the precession rate, and we therefore choose the convention employed by Damour & Taylor (1992). The combination of MCMC and RVM analyses therefore yields a test on observed profile evolution due to relativistic spin precession from two independent measurements, since the profile-shape analysis yields an estimate of ϵ .

The differences in data quality between Mark IV and ASP profiles can be seen as slight differences in the profile shape across pulse phase, as shown in Figure 5.3. This introduced slight discrepancies in the results obtained from the PCA analysis described above when performed using all available data, which subsequently affected the derived profile shapes and MCMC results. Two separate studies between backends were not possible as the ASP era consisted of fewer profiles and a smaller timespan, with campaign data collected during the 2008 observing year being excluded from the MCMC analysis due to having many low signal-to-noise profiles. We therefore decided to perform a PCA on all Mark IV profiles only, and then use the derived principal components to estimate the shapes for all high signal-to-noise Mark IV and ASP profiles. This approach does not account for observed scintillation or profile evolution across observing frequency in the ASP data. We therefore only used ASP data collected with the two innermost frequency channels centered on 430 MHz for both analyses in order to minimize such effects.

5.3.2 Results

Results from the MCMC fit on several data sets can be found in Table 5.1, and the posterior distribution for Ω_1^{spin} derived from our Mark IV and ASP data sets is shown in Figure 5.5. We generated 3×10^5 samples for each application of the algorithm, after burning the first 5000 samples in order to remove non-convergent iterations. We provided the original results obtained

Parameter	STA04	STA04-MCMC	Mark IV	All
$\Omega_1^{\rm spin}$ (°/yr)	$0.44_{-0.16}^{+0.48}$	$0.51^{+0.10}_{-0.08}$	$0.48^{+0.09}_{-0.07}$	$0.59_{-0.08}^{+0.12}$
ϵ (°)	$\pm 103^{+10}_{-10}$	$\pm 99^{+2}_{-2}$	$\pm 118^{+10}_{-15}$	$\pm 139^{+16}_{-25}$
F'	n/a	$-5.9^{+0.9}_{-1.0}$	$-2.2^{+0.6}_{-0.7}$	$-1.3^{+0.3}_{-0.5}$
$F_0 (10^{-3})$.	$-1.5^{+0.3}_{-0.3}$	$-1.90^{+0.08}_{-0.09}$	$-1.21^{+0.08}_{-0.08}$	$6.67\substack{+0.07\\-0.07}$

Table 5.1: MCMC results for measurement of Ω_1^{spin} and the orientation angle ϵ for different data subsets. Uncertainties reflect 68.3% confidence intervals of posterior distributions. "STA04" denotes the results obtained by Stairs et al. (2004), whereas "STA04-MCMC" denotes the results obtained when implementing our MCMC analysis on the same data set used by Stairs et al.

by STA04, as well as a reproduced set of results from the STA04 data set using the MCMC algorithm, for comparison with our extended Mark IV and ASP profiles. We assumed that values of F' must be negative while using the MCMC algorithm, since the simultaneous-linear-fit technique used and described by STA04, which avoids any consideration of F', estimates that $\cos \epsilon < 0$. These results agree well with predictions from general relativity, where $\Omega_1^{\text{spin}} = 0.51^{\circ}/\text{yr}$ using the derived masses from PK timing parameters, and previous measurements made by STA04. General improvements in precision come from the new fitting procedure, which permitted direct sampling of the precession rate and other free parameters, as well as the addition of the ASP 2005 campaign and several strong bi-monthly observations.

The RVM analysis yielded values of α and β at different times using the Mark III (reticon), Mark IV and ASP campaign profiles. The values of β measured for each campaign are shown in Figure 5.6. Measurements of α , with average values of $\alpha = 103.5(3)^{\circ}$, are consistent with no evolution in time, while the values of β are found to change significantly, where $d\beta/dt = -0.23 \pm 0.02 \,^{\circ}/\text{yr}$. This is consistent with the STA04 result of $-0.21 \pm 0.03 \,^{\circ}/\text{yr}$. The assumption that general relativity is correct requires that $d\beta/dt = \Omega_1^{\text{spin}} \sin i \cos \epsilon$, and therefore yields $\epsilon = \pm 117 \pm 3^{\circ}$ (68% confidence), which agrees with the value determined from the MCMC analysis described above.



Figure 5.5: Top: MCMC posterior distribution of Ω_1^{spin} obtained from the profile-shape analysis of Mark IV and ASP data discussed in Section 5.3.2. Bottom: Markov chain for Ω_1^{spin} determined from the MCMC algorithm. This figure was first published by Fonseca et al. (2014).

With these values, the misalignment angle δ between the spin and orbital angular momentum axes can be derived through spherical trigonometry by $\cos \delta = -\sin i \sin \lambda \sin \epsilon + \cos \lambda \cos i$. The sign ambiguity in ϵ and i, as well as the requirement that $\cos i \tan \epsilon > 0$ pointed out by STA04, gives an expected value of $\delta = 27.0 \pm 3.0^{\circ}$ or $\delta = 153.0 \pm 3.0^{\circ}$. Physical arguments based on alignment of angular momenta prior to the second supernova suggest that the smaller angle is correct (Bailes, 1988), and therefore requires that $\epsilon = -117 \pm 3^{\circ}$ and $i = 77.7 \pm 0.9^{\circ}$.

The consistency between the MCMC and RVM analyses serves as an improved, independent check of precession within this relativistic binary system. These results also confirm the geometric picture of this pulsar-binary system derived in STA04.



Figure 5.6: Impact angle β between the magnetic axis and line of sight as a function of time. The black line is a best-fit slope of -0.23 ± 0.02 °/yr. This figure was first published by Fonseca et al. (2014).

5.4 Timing Observations with PUPPI

The PUPPI instrument began formal operation during the early months of the 2012 observing year at the Arecibo Observatory. We submitted telescope proposals every year since 2012 in order to collect TOAs using the PUPPI backend, which can process and record data collected with the 1400-MHz receiver that samples the full 800 MHz in bandwidth using 512 thin frequency channels. The technical specifications of PUPPI are a dramatic improvement over the ASP processors for the case of PSR B1534+12, since the ASP machine could only process up to 64 MHz in bandwidth due to limitations in hardware functionality (see Fonseca, 2012, for details). All submitted propos-

Parameters	$430 \mathrm{~MHz}$	$1400 \mathrm{~MHz}$
Full frequency range (MHz)	421 - 445	1147 - 1765
Effective Bandwidth (MHz)	20	600
Number of Channels	4	4
Integration Time (s)	180	180
Number of TOAs	984	668
RMS residual (μs)	8.47	6.32
$\chi^2_{ m red}$	1.013	1.004

Table 5.2: Parameters of TOAs from PSR B1534+12 obtained with PUPPI. Note that "effective bandwidth" means the total width of broadband data that is usable after excision of narrow-band radio frequency interference.

als for continued timing of PSR B1534+12 were accepted⁴ and all requested time was granted; this includes two additional dense campaigns occurring during the month of August in 2013 and 2015.

Our initial analysis of the PUPPI data for PSR B1534+12 is shown graphically in Figure 5.7, and statistics of the current set of PUPPI TOAs are presented in 5.4. We employed the same general methods for excision of radio frequency interference (RFI) used by Fonseca et al. (2014) when generating PUPPI TOAs and pulse profiles, in order to compare the data quality between PUPPI TOAs and those published by Fonseca et al. (2014). Moreover, we averaged sections of data collected across the bandwidth of each receiver in order to obtain 4 channelized profiles per 3-minute integration for each receiver. We used the PSRCHIVE⁵ software suite in order to reduce raw TOA data into calibrated pulse profiles, using the same methodology described in 3.1 of Chapter 3.

From the logistics shown in Table 5.4, it is immediately clear that the processing capabilities of PUPPI are superior for data collected with the 1400 MHz receiver at Arecibo; this improvement is entirely due to the greater

 $^{^4{\}rm These}$ projects were given the following designations: P2719; P2820; P2906; and P2990.

 $^{^5}$ http://psrchive.sourceforge.net/index.shtml



Figure 5.7: Current state of the timing data on PSR B1534+12. Black points are data published by Fonseca et al. (2014). Red points are TOAs measured at 430 MHz collected with PUPPI, and blue points are TOAs measured at 1400 MHz collected with PUPPI.

access of the 1400-MHz receiver bandwidth. The ASP machine was able to process up to 128 MHz in real time, though only the innermost 64 MHz was generally usable. In stark contrast, PUPPI can process and record data across the full bandwidth at 1400 MHz, though in practice only ~600 MHz of bandwidth was usable after removing channels that were affected by RFI. An example of a PUPPI observation at 1400 MHz is shown in Figure 5.8. Furthermore, the expanded bandwidth and additional pulsar signal allows us to average TOA data into a smaller number of channels across the 800-MHz bandwidth, which can help mitigate the effects of complex profile evolution across the band while folding profiles for improving S/N. For our current data set shown in Figure 5.7, we were able to derive 668 usable TOAs with the 1400-MHz receiver using PUPPI over the course of two years, which is



Figure 5.8: Heat map of TOA data for PSR B1534+12 collected with the 1400-MHz receiver, using PUPPI, on MJD 56326. Several frequency channels were excluded due to the presence of narrow-band RFI.

nearly three times as many TOAs recorded at 1400 MHz during the entire ASP era using the same receiver.

The improved consistency in dual-receiver measurements with PUPPI also allows for estimates of DM using widely separated observing frequencies on a nearly per-epoch basis. Figure 5.9 shows measurements of DM versus time for our current PUPPI data set, using 430-MHz and 1400-MHz TOA data on nearly all observing epochs, illustrating a gradual change over time similar to the pre-PUPPI DM measurements shown in Figure 5.2. The two DM points in Figure 5.9 with comparatively large uncertainties, derived from



Figure 5.9: DM versus time for PSR B1534+12, using the PUPPI backend.

PUPPI data collected at the beginning of the 2014 and 2015 observing years, were derived from the channelized 1400-MHz data set for these two epochs; no 430-MHz TOAs collected during these two observations were found to be usable after excision of RFI. It is clear that, when using the broadband PUPPI processor, the DM offset is measurable from channelized 1400-MHz receiver data alone, which was not previously possible due to a lack of signal across the bandwidth recorded by ASP. However, the lack of precision in these two PUPPI points is mostly due to the intrinsically weaker pulsar signal at 1400 MHz when compared to the profile observed at 430 MHz.

5.5 Conclusions

The timing and pulse-profile results presented by Fonseca et al. (2014) cover a wide range of astrophysical scope and application, including improved tests of GR from extended timing observations of PSR B1534+12. In Sections 5.2 and 5.3 of this chapter, we discussed additional results obtained by further analysis of TOA and profile data that was ultimately presented by Fonseca et al. (2014). We found that the DM variations seen in PSR B1534+12 were consistent with local ISM fluctuations due to a turbulent, Kolmogorov medium. We also carried out an extensive analysis of the pulse structure and observed secular variations over time, modeling the long-term changes in total-intensity pulse shape and polarization properties to determine a precise estimate of the Ω_1^{spin} .

Future long-term timing observations of PSR B1534+12 are crucial for the improvement of its formidable constraints on relativity theory using highquality Arecibo data. We have demonstrated that full use of Arecibo data, both in the timing of pulse profiles and in the electromagnetic properties of the profiles themselves, contain a wealth of information that has so far been used to obtain unique and precise measures of relativistic phenomena. We continue to use the PUPPI signal processor in an effort to improve the signal-to-noise ratio and profile quality at both proposed observing frequencies (particularly at 1400 MHz, where the source is intrinsically weaker). The factor of ~ 5 increase in bandwidth at 1400 MHz has so far provided a large boost in S/N, as shown in Figure 5.8, such that we have so far collected more useable PUPPI 1400-MHz TOAs than we have during the entire ASP era. Moreover, we have reduced and included PUPPI data collected up to March 2016 (and as early as March 2012) into our current data set and will continue to process ASP/PUPPI data collected before/after then.

The full, current timing model for PSR B1534+12 uses TOAs collected with four signal processors (including PUPPI) that collectively span 25 years of observation. We use the same timing methodology for our current model, which fits for variations in DM during the PUPPI era with a single gradient. A full evaluation of the current measurements of relativistic parameters in the PSR B1534+12 system, using the updated PK timing parameters, are



Figure 5.10: PK parameters measured in the PSR B1534+12 system, shown as colored bands in the $(m_{\rm p}, m_{\rm c})$ plane with black labels that denote the particular effect. The width of each colored band represents the 68.3% credible interval measured for the denoted parameter. The values and uncertainties of the PK timing parameters were determined from an updated timing model that used the data analyzed by Fonseca et al. (2014) and the PUPPI observations presented in Section 5.4 of this dissertation.

shown in Figure 5.10. The five PK timing parameters remain consistent with those estimated by Fonseca et al. (2014) and in the previous studies undertaken by Stairs et al. (2002, 1998); the TEMPO uncertainties in the secular PK variations ($\dot{P}_{\rm b}$ and $\dot{\omega}$) have decreased by nearly a factor of 2 since the publication of the Fonseca et al. (2014) study, which is expected since we've extended our timespan by nearly three years.

As first noted in Stairs et al. (1998), the use of the DM-based estimate of d = 0.7(2) kpc (Cordes & Lazio, 2001) in the Doppler correction for $(\dot{P}_{\rm b})_{\rm obs}$ produces an estimate of $(\dot{P}_{\rm b})_{\rm int} = (\dot{P}_{\rm b})_{\rm obs} - (\dot{P}_{\rm b})_D$ that does not agree with the predictions from the other PK parameters at the 68.3% uncertainty level. Moreover, the uncertainty in $(\dot{P}_{\rm b})_{\rm int}$ is dominated by the uncertainty in the DM-based estimate of d. This slight bias in the $\dot{P}_{\rm b}$ correction is shown in Figure 5.10 as the blue-shaded region that narrowly misses the common region of intersection for all of the other PK parameters. Fonseca et al. (2014) used this discrepancy in the observed orbital decay to derive a distance to the system when assuming the validity of GR, $d_{\rm GR} = 1.051(5)$ kpc, that produces the corresponding $(\dot{P}_{\rm b})_D$. PSR B1534+12 was recently included into the MSPSR π radio-interferometry program⁶, which will eventually produce an independent measure of d – and, ideally, a robust correction of $(\dot{P}_{\rm b})_{\rm obs}$ within the next two or three years.

⁶https://safe.nrao.edu/vlba/psrpi/home.html

Chapter 6

Concluding Remarks

As we have shown throughout this dissertation, an analysis of secular variations in orbital elements can yield constraints or significant measurements of the component masses and angles of orientation, which are otherwise not accessible when analyzing purely Keplerian motion. In a practical sense, the measurement of these variations is also important for robust determination of the full, accurate pulsar-timing model applied to TOAs from a given binary radio pulsar. Accurate timing solutions are essential for subsequent interpretation of the model parameters, and are especially important for the NANOGrav, EPTA and PPTA efforts that search for correlated structure in TOA residuals due to nanohertz-frequency gravitational waves among a large ensemble of MSPs.

In Chapter 3, we measured the relativistic Shapiro timing delay in fourteen of twenty five NANOGrav binary radio pulsars, which yielded direct measurements of m_c and $\sin i$ for each system with varying degrees of precision. Using the mass function computed from Keplerian elements, we derived values of m_p for each of these fifteen systems. The most significant measurements of m_p that we made, which typically possess relative uncertainties equal to or less than 20%, are shown in Figure 6.1. Four of the fourteen measured signals – in PSRs J0613-0200, J2017+0603, J2302+4442, and J2317+1439 – have been characterized by our analysis for the first time. It is important to note that no detections of $\Delta_{\rm S}$ were made for J2317+1439 using the EPTA data set (Desvignes et al., 2016), or for J0613-0200 using either the EPTA or PPTA data sets (Desvignes et al., 2016; Reardon et al., 2016). We attribute these differences to the superior timing precision obtained with the Arecibo Observatory and GBT using the PUPPI and GUPPI processors, respectively, as well as the targeted Shapiro-delay campaign devised by Pennucci (2015). Moreover, we improved the measurements of two previously known signals in the J1918-0642 and J2043+1711 systems (Sections 3.4.11 and 3.4.14, respectively) such that $m_{\rm p}$ has been measured with relative uncertainties of ~20% or less for the first time.

The most significant measurements of $\Delta_{\rm S}$ generally corresponded to systems with comparatively large orbital inclinations, where the Shapiro-delay parameters can be measured with low statistical correlation and with little absorption of the relativistic signal between other Keplerian timing parameters. From the Shapiro delay alone, we made high-precision estimates of pulsar masses as low as $m_{\rm p} = 1.18^{+0.10}_{-0.09}$ M_{\odot} for PSR J1918–0642 and as high as $m_{\rm p} = 1.928^{+0.017}_{-0.017}$ M_{\odot} for PSR J1614–2230 (Section 3.4.4). We used probability density maps computed from grids of χ^2 values obtained for different combinations of the Shapiro delay parameters to find accurate uncertainties of the component masses and inclination angles that reflect the statistical correlation present in our measurements.

For five pulsars studied here, we used the statistical significance of $\Delta_{\rm S}$ and one or more observed Keplerian variations to constrain our estimates of $m_{\rm p}$, $m_{\rm c}$ and *i*. For example, we confirmed that the observed $\dot{\omega}$ in the eccentric PSR J1903+0327 system (Section 3.4.9) is due to relativistic precession at its current level of precision. We assumed the validity of GR in order to use the statistical significance of $\dot{\omega}$ to further constrain the region of preferred solutions based on the χ^2 grid over the two Shapiro-delay parameters. We found that the constrained mass of PSR J1903+0327 is $m_{\rm p} = 1.65^{+0.02}_{-0.02} \, {\rm M}_{\odot}$,



Figure 6.1: Estimates of pulsar masses measured in this dissertation. Labels in red indicate that one or more secular variations were used to constrain the masses and/or geometry of the binary systems. Labels with black stars indicate the first significant measurements of the pulsar mass in the denoted binary systems.

which is consistent with the initial assessment and calculations made by Freire et al. (2011) using an independent data set obtained for the same MSP. In the case of PSR J1741+1351 (Section 3.4.7), we measured a significant $\Delta_{\rm S}$ as well as a significant change in x over time that we determined to be due to an evolving orientation of a system with significant proper motion on the sky. We similarly used the statistical significance of the observed \dot{x} to constrain the value of Ω and the Shapiro-delay parameters while acknowledging ambiguities in the sign of i and Ω .

The impact that these orbital variations can have in determining other intrinsic quantities is most dramatically seen in our analysis of PSR B1534+12 (Chapter 5), a DNS system with a compact, 10.1-hour orbit. We continue to resolve five significant PK timing parameters with improved precision, as well as a significant kinematic bias in our observed $\dot{P}_{\rm b}$, when incorporating nearly four years of data collected with the PUPPI backend after the study conducted by Fonseca et al. (2014). We assumed the validity of GR and determined the component masses to be

$$m_{\rm p} = 1.33302(17) \,\,{\rm M}_{\odot}$$
 (6.1)

$$m_{\rm c} = 1.34553(17) \,\,{\rm M}_{\odot}$$
 (6.2)

where the uncertainties reflect 68.3% confidence intervals as determined by TEMPO. At this time, the component masses for the PSR B1534+12 DNS system are the most precise neutron-star masses currently known. As a secondary study of relativistic gravity, we also conducted detailed analyses of electromagnetic pulse structure in PSR B1534+12 and quantified the effect of geodetic precession by modeling secular changes in the profile shape and polarization position angles at 430 MHz along with special-relativistic aberration of the signal due to orbital motion. We derived an estimate of $\Omega_1^{\text{spin}} = 0.59^{+0.12}_{-0.08} \text{ deg yr}^{-1}$ that is consistent with the value predicted by GR, yielding a sixth PK parameter in this system for the first time. At this point in time, only PSRs B1534+12, J0737-3039A/B (Breton et al., 2008) and B1913+16 (Weisberg & Huang, 2016) yield at least six quantifiable PK parameters within one relativistic binary system.

We also conducted a preliminary analysis of 27+ years of TOA data collected for PSR B1620-26 (Chapter 4), a radio pulsar that has long been interpreted as being embedded in a hierarchical triple system with a He WD and Jupiter-mass planet in the Messier 4 globular cluster. We measured fifteen time-derivatives in ν_s that are due to the long-period motion of the outer planet with the inner pulsar-WD binary system, though we demonstrated that $\dot{\nu}_s$ contains significant components associated with globularcluster dynamics and intrinsic pulsar spin-down. We used the method proposed by Joshi & Rasio (1997) to relate the time-derivatives in $\nu_{\rm s}$ to the outer-Keplerian elements. For the first time, we used only time-derivatives of higher order than $\dot{\nu}_{\rm s}$ to compute the outer-orbital quantities when analyzing the entire TOA data set, which allowed for a unique determination of the orbital component in $\dot{\nu}_{\rm s}$. We derived an outer-orbital period of several decades, though analyses of data subsets or the entire TOA collection yield conflicting estimates (see Section 4.3 and 4.4). The improving measurements of inner-orbital secular variations and $\ddot{x}_{\rm i}$, which we believe are due to prolonged three-body perturbations from the outer object, will eventually yield a direct measurement of the pulsar mass. Ongoing work will eventually lead to a simultaneous fit for both orbits, either using the two-orbit model with TEMPO or the novel three-body integrator first described by Ransom et al. (2014).

6.1 **Projections of Future PK Measurements**

The primary means for future work with binary pulsars is the prolonged acquisition of high-precision TOAs over time, as it allows for the eventual resolution of secular variations and other relativistic terms that contain highimpact information. For new discoveries of binary pulsars, the presence of a significant Shapiro delay can be quickly assessed using strategic observations of specific orbital phases where the signal is expected to have maximum harmonic structure. We used data collected from targeted campaigns, first described by Pennucci (2015), to make the first measurement of $\Delta_{\rm S}$ in the PSR J2302+4442 system (Section 3.4.16) and the first significant measurement of $m_{\rm p}$ in the J2043+1711 system. In both aforementioned systems, we made a measurement of $\Delta_{\rm S}$ that passed the orthometric h_3 -significance test (Freire & Wex, 2010) using data sets that spanned less than two years of observation. Continued timing of known radio pulsars will likely yield interesting relativistic effects in the coming years. For example, the PSR J1600-3053 binary system (Section 3.4.3) currently exhibits a statistically marginal change in ω over time that is consistent with the predictions of GR. The precision in the $\dot{\omega}$ measurement will improve over time such that the fractional uncertainty in its measurement decreases as $t^{-3/2}$ (Damour & Taylor, 1992). With this scaling law, a ~10% fractional uncertainty $\dot{\omega}$ will be achieved for J1600-3053 by the year 2020. Similarly, the observed $\dot{P}_{\rm b}$ in the J1909-3744 system (Section 3.4.10) is dominated by the kinematic-acceleration component, though its uncertainty possesses the same order of magnitude as the expected component of intrinsic orbital decay due to GR. Since the relative uncertainty in $\dot{P}_{\rm b}$ scales as $t^{-5/2}$, the relative uncertainty in $(\dot{P}_{\rm b})_{\rm GR}$ will reach ~20% by the year 2029. However, the measurability of $(\dot{P}_{\rm b})_{\rm GR}$ depends on the correction for the kinematic bias, and forthcoming studies will need to address the uncertainty associated with the distance-dependent correction.

A recent study of the Hulse-Taylor DNS system has yielded the first measurement of the measurable shape correction for eccentric orbits (δ_{θ} in Equation 1.26) in any binary system, as well as a marginal estimate of \dot{x} that is likely due to classical spin-orbit coupling (Weisberg & Huang, 2016). While the δ_{θ} parameter may yield another unique test of gravitation, a measurement of \dot{x} due to classical spin-orbit coupling can be directly related to the moment of inertia ($I_{\rm rot}$) for one or both neutron stars that can yield unprecedented constraints on neutron-star EOS models (e.g. Lattimer & Schutz, 2005). However, the δ_{θ} measurement contains a bias due to aberration of the signal from pulsar rotation (e.g. Damour & Taylor, 1992). Moreover, the correction for the intrinsic component of δ_{θ} due to GR, as well as the determination of $I_{\rm rot}$ from the spin-orbit-coupling component of \dot{x} , both require a measurement of several pulsar-orientation angles that have not yet been measured in the Hulse-Taylor system. It is therefore currently not possible to make a direct measurement of $I_{\rm rot}$ in the PSR B1913+16 DNS system. However, the required angles for correction of δ_{θ} and \dot{x} – namely ϵ and $\lambda = \pi - \alpha - \beta$ – have been estimated for PSR B1534+12, first by Stairs et al. (2004) and in Chapter 5 of this dissertation, by modeling the secular changes in pulse-profile shape and polarization position-angle data to ultimately derive a measure of Ω_1^{spin} . In principle, then, our geometric measurements allow for a full determination of the various components of δ_{θ} and \dot{x} , as well as a unique determination of I_{rot} . With ongoing observations of PSR B1534+12 using the PUPPI backend at the Arecibo Observatory, we may likely be able to eventually measure and correct for the component of δ_{θ} due to GR for the first time in the near future, as well as determine I_{rot} from measurements of spin-orbit coupling component of \dot{x} . However, we do not currently measure δ_{θ} or \dot{x} with statistical significance. Simulations of TOA data sets are therefore required in order to determine when in the future such measurements can be made with sufficient precision.

In order to perform robust simulations for "times of detection", we first computed the various components of δ_{θ} and \dot{x} that are possible for the PSR B1534+12 system. For δ_{θ} , we expect that only the GR and rotationalaberration components are significant:

$$(\delta_{\theta})_{\rm GR} = (T_{\odot} n_{\rm b})^{2/3} \left(\frac{\frac{7}{2} m_{\rm p}^2 + m_{\rm p} m_{\rm c} + 2m_{\rm c}^2}{(m_{\rm p} + m_{\rm c})^{4/3}} \right) \approx 5 \times 10^{-6}$$
(6.3)

$$(\delta_{\theta})_{\rm A} = \frac{P_{\rm s}}{P_{\rm b}} \frac{\csc i}{(1-e^2)^{1/2}} \frac{\sin \eta}{\sin \lambda} \approx 1 \times 10^{-6} \tag{6.4}$$

so that $(\delta_{\theta})_{\text{obs}} = (\delta_{\theta})_{\text{GR}} + (\delta_{\theta})_{\text{A}} \approx 6 \times 10^{-6}$ when using the values determined by Fonseca et al. (2014) to compute the components of δ_{θ} . For the components of \dot{x} , there are five possible terms that arise with varying degrees of significance:

$$(\dot{x})_{\rm GR} = -\frac{64}{5} \sin i (T_{\odot} n_{\rm b})^2 \frac{(1 + (73/24)e^2 + (37/96)e^4)}{(1 - e^2)^{7/2}} \frac{m_{\rm p} m_{\rm c}^2}{m_{\rm p} + m_{\rm c}}$$
$$\approx -1.32 \times 10^{-17} \tag{6.5}$$

$$(\dot{x})_{\rm D} = x \left(\frac{\dot{D}}{D}\right) \approx 5.72 \times 10^{-18} \tag{6.6}$$

$$(\dot{x})_{\rm A} = -x \frac{P_{\rm s}}{P_{\rm b}} \frac{\Omega_1^{\rm spin}}{(1-e^2)^{1/2}} \frac{(\cot\lambda\sin2\eta + \cot i\cos\eta)}{\sin\lambda} \approx -2.58 \times 10^{-17} \quad (6.7)$$

$$(\dot{x})_{\rm SO} \approx \frac{x \cot i}{c^2} \left(2 + \frac{3m_{\rm c}}{2m_{\rm p}} \right) \frac{I_{\rm rot} (2\pi/P_{\rm s}) (2\pi/P_{\rm b})^2}{(m_{\rm p} + m_{\rm c}) (1 - e^2)^{3/2}} \sin \lambda \cos \epsilon$$
$$= -1.49 \times 10^{-15} \tag{6.8}$$

 $(\dot{x})_{\mu} = (-0.943 \text{ or } -1.25) \times 10^{-15}$ (6.9)

where the above equations are taken from Damour & Taylor (1992) and the values in Equations 6.5-6.8 were computed using values estimated by Fonseca et al. (2014). The possible values for $(\dot{x})_{\mu}$ listed in Equation 6.9 were first computed by Bogdanov et al. (2002) after determining that $\Omega =$ $(70 \text{ or } 290) \pm 20$ degrees from orbital-dependent scintillation measurements. For the spin-orbit-coupling component – which we refer to in Equation 6.8 as $(\dot{x})_{SO}$ – we used the assumption made by Weisberg & Huang (2016) that the contribution from the neutron-star companion (whose rotation rate is unknown) is negligible. With these expected components determined, it is clear that $(\dot{x})_{obs} \approx (\dot{x})_{\mu} + (\dot{x})_{SO} \approx -3 \times 10^{-15}$. The component due to spinorbit coupling is therefore expected to be among the most dominant effects that produce a non-zero \dot{x} .

We can also simulate the times of detection for several other possible components of $\dot{\omega}$, due to proper-motion bias (Equation 1.44) and spin-orbit coupling. As first discussed for PSR J1640+2224 (Section 3.4.5), the spinorbit component of $\dot{\omega}$ will generally be approximately equal to $di/dt \approx$ $(\dot{x}/x)_{\rm SO} \tan i$. We therefore expect the non-GR components of $\dot{\omega}$ to be:

$$(\dot{\omega})_{\mu} = \mu \csc i \cos(\Theta_{\mu} - \Omega) = (-6.61 \text{ or } + 6.91) \times 10^{-6} \text{ deg yr}^{-1}$$
 (6.10)

$$(\dot{\omega})_{\rm SO} = (\dot{x}/x)_{\rm SO} \tan i \approx -6.67 \times 10^{-6} \, \deg \, \mathrm{yr}^{-1}$$
 (6.11)

where we used the estimates of Ω made by Bogdanov et al. (2002) to compute the expected changes from proper-motion bias. While both non-GR components are comparable in order of magnitude, the allowed values for $(\dot{\omega})_{\mu}$ possess opposite signs and produce composite values of $(\dot{\omega})_{\text{non-GR}} =$ $(\dot{\omega})_{\mu} + (\dot{\omega})_{\text{SO}} \approx (-1.33 \text{ or } 0.024) \times 10^{-5} \text{ deg yr}^{-1}$. For the purposes of simulation, we chose an intermediate value of $(\dot{\omega})_{\text{non-GR}} = -5 \times 10^{-6} \text{ deg yr}^{-1}$ to estimate an approximate time of detection.

We determined times of detection for $\delta_{\theta} = 6 \times 10^{-6}$, $\dot{x} = -3 \times 10^{-15}$ and $(\dot{\omega})_{\text{non-GR}} = -5 \times 10^{-6}$ deg yr⁻¹ for PSR B1534+12 by using the current best-fit timing solution with the DD binary model presented at the end of Chapter 5, holding all parameters fixed, and producing fake TOAs that are modeled by said timing solution. We assumed that observations began on MJD 48718 (i.e. the same start date of the real B1534+12 data set) and were conducted once every 60 days, and that each observing epoch lasted 90 minutes to produce 30 TOAs with an uncertainty of 8 μ s for each TOA. We also neglected DM variations, computed barycenter-corrected TOAs, and injected random-normal noise with a standard deviation of 1 μ s. The results from our simulations are shown in Figure 6.2, where each point with an uncertainty of model parameter determined by TEMPO when simultaneously fitting for all model parameters, including the δ_{θ} , \dot{x} and $(\dot{\omega})_{\text{non-GR}}$.

Our simulations suggest that significant, accurate estimates of \dot{x} and $(\dot{\omega})_{\text{non-GR}}$ will roughly be made starting after the year 2040. Figure 6.2 also suggests that δ_{θ} and \dot{x} are highly correlated when weakly constrained, as the best-fit estimates of both parameters track each other prior to the



Figure 6.2: Simulated times of detection for δ_{θ} , \dot{x} and $(\dot{\omega})_{\text{non-GR}}$ in the PSR B1534+12 DNS system. For ease of comparison, all values are scaled to possess an order of magnitude of unity. The solid lines indicate the expected, simulated parameter values. The dots and error bars represent the best-fit values and uncertainties determined by TEMPO when simultaneously fitting for all parameters, including δ_{θ} , \dot{x} and $(\dot{\omega})_{\text{non-GR}}$.

year 2040. Once the \dot{x} measurement becomes robust, the δ_{θ} estimates gradually becomes more accurate. However, our simulations show that δ_{θ} will not be measured on reasonable timescales. Nonetheless, the eventual measurement of \dot{x} will allow for a direction computation of $I_{\rm rot}$ using the angle and precession-rate measurements presented in Chapter 5.

6.2 The Era of Neutron-Star Mass Measurements

The first measurement of m_p with the Shapiro timing delay in a pulsarbinary was made by Ryba & Taylor (1991) in their analysis of TOAs from PSR B1855+09. Since then, a large and increasing number of pulsar-mass measurements have been made such that meaningful analyses of the neutronstar mass distribution have recently begun. The distribution of neutron-star masses can be directly inferred from available measurements of the Shapiro timing delay and its physical parameters, along with mass estimates that were derived from two or more PK variations. Recent work has shown that an increasing number of these measurements can help delineate the roles of different supernovae processes in the formation of double-neutron-star binary systems (e.g. Schwab et al., 2010) and assess the possible range of component masses for such systems (e.g. Martinez et al., 2015), as well as derive the statistics for pulsar-binary populations that have evolved along different postsupernova evolutionary paths (Özel et al., 2012; Kiziltan et al., 2013).

In this thesis, the significant estimates of m_p span a range of $1.2-1.95 \text{ M}_{\odot}$ in neutron star mass. PSRs J1614–2230 and J1918–0642 are at the high and low ends of our overall mass distribution, respectively. The low mass of PSR J1918–0642 is particularly interesting since this MSP possesses spin parameters that are typical of an old neutron star that experienced significant mass transfer and a substantial spin-up phase. Nonetheless, our measurement of $m_p = 1.18^{+0.10}_{-0.09} \text{ M}_{\odot}$ is comparatively low. The implication of a low "birth mass" for J1918–0642, believed to be an old neutron star, is inherently different from recent studies of the binary evolution of PSRs J0737–3039A/B (Ferdman et al., 2013) and J1756–2251 (Ferdman et al., 2014), which are both young pulsars believed to have been formed through electron-capture supernovae that reduce the electron-degeneracy pressure within the progenitor core and induces gravitational collapse at lower Chandrasekhar masses. Furthermore, Antoniadis et al. (2016) recently used available measurements of pulsar masses to argue that the MSP mass distribution is in fact bimodal. Given the wide range of spin and orbital periods in the MSP population and a lack of clear correlation between the period/mass parameters, the two components of the bimodal distribution derived by Antoniadis et al. likely reflect inherently different MSP birth masses, as opposed to complex processes related to the mass-transfer period.

As discussed in Section 3.5, significant measurements of $\Delta_{\rm S}$ allow for independent tests of the theoretical correlation between $P_{\rm b}$ and $m_{\rm c}$ from tidal interactions during the phase of long-term mass transfer (e.g. Tauris & Savonije, 1999). Additional measurements of $\Delta_{\rm S}$ for long-period binary systems will help constrain the $P_{\rm b} - m_{\rm c}$ relation over a wider range of $P_{\rm b}$ than is currently seen in Figure 3.26.

One of the frontier goals in high-energy astrophysics is the understanding of neutron-star structure and the microscopic processes that govern the interiors of stellar-mass objects with mass densities that exceed nuclear saturation ($\rho_{\rm sat} \approx 2.8 \times 10^{14} \text{ g cm}^{-3}$). Within the interiors of neutron stars, the relativistic pressure-density relation becomes theoretically uncertain and currently allows for a large number of proposed equations of state (EOSs) that differ in fractional composition of hadronic and pure-quark matter (e.g. Lattimer & Prakash, 2001; Lattimer & Prakash, 2004). Figure 6.3 illustrates a large number of proposed neutron-star EOSs¹, as well as the experimental constraints posed by radio-timing measurements of m_p . In the case of PSR J0348+0432 (see Antoniadis et al., 2013), optical radial-velocity estimates of the companion mass and mass ratio were combined with the radio-timing measurement of orbital decay to yield a high-precision estimate of the pulsar mass.

As shown in Figure 6.3, the low- $m_{\rm p}$ measurement made for PSR J1918-0642

¹The EOS data shown in Figure 6.3 can be downloaded at http://xtreme.as. arizona.edu/NeutronStars/.



Figure 6.3: Mass-radius relations for EOSs of neutron stars, shown as solid curves, that reflect different underlying assumptions of internal composition. The EOS data used in this figure was compiled by Özel & Freire (2016) and first shown in Figure 7 of their study. Red bands represent mass measurements made in this dissertation, while the blue band represents the mass estimate for PSR J0348+0432 made by Antoniadis et al. (2013). The lighter, gray lines represent EOSs that do not predict neutron-star masses larger than or equal to the estimate made for PSR J1614-2230 in Chapter 3 of this dissertation.

does not provide a meaningful constrain on the neutron-star EOSs since all curves predict similar mass values for a range of radii. However, the highmass estimate for PSR J1614-2230 exceeds the maximum-mass values for several EOSs, which are shown as gray lines in Figure 6.3. In principle, the PSR J1614-2230 mass measurement (first made by Demorest et al. (2010)) invalidates the gray EOSs as physically plausible models of the neutron-star interior. Estimates of larger pulsar masses in the future will therefore provide even more stringent constraints on allowed compositions and EOSs of neutron stars.

It is clear that high-precision estimates of pulsar masses offer uniquely far-reaching impact on the areas of stellar-binary evolution, supernovae and neutron-star birth masses, tests of strong-field gravitation, and the ongoing efforts to constrain the equation of state for neutron stars. Ongoing studies and future discoveries with premier radio facilities will surely aid in these efforts. The forthcoming Square Kilometre Array (SKA) is projected to detect Shapiro-delay signals from ~80% of all pulsar-binary systems with $m_c > 0.1 M_{\odot}$ and RMS TOA residuals of ~50 μ s, yielding unprecedented access into the Galactic distribution of neutron-star masses and pulsar-binary system inclinations (Watts et al., 2015). Furthermore, the SKA is expected to be sensitive to even more general-relativistic timing effects, such as the secular variation in ω due to Lense-Thirring precession (Kehl et al., 2016). We are confident that future measurements will bring forth new and exciting information to bear on these areas in the coming years.

Bibliography

- Abbott, B. P., Abbott, R., Abbott, T. D., & et al. 2016, Physical Review Letters, 116, 061102
- Alpar, M. A., Cheng, A. F., Ruderman, M. A., & Shaham, J. 1982, Nature, 300, 728
- Antoniadis, J., Freire, P. C. C., Wex, N., Tauris, T. M., Lynch, R. S., van Kerkwijk, M. H., Kramer, M., Bassa, C., Dhillon, V. S., Driebe, T., Hessels, J. W. T., Kaspi, V. M., Kondratiev, V. I., Langer, N., Marsh, T. R., McLaughlin, M. A., Pennucci, T. T., Ransom, S. M., Stairs, I. H., van Leeuwen, J., Verbiest, J. P. W., & Whelan, D. G. 2013, Science, 340
- Antoniadis, J., Tauris, T. M., Ozel, F., Barr, E., Champion, D. J., & Freire, P. C. C. 2016, ArXiv e-prints
- Antoniadis, J., van Kerkwijk, M. H., Koester, D., Freire, P. C. C., Wex, N., Tauris, T. M., Kramer, M., & Bassa, C. G. 2012, MNRAS, 423, 3316
- Arzoumanian, Z., Brazier, A., Burke-Spolaor, S., & et al. 2014, ApJ, 794, 141
- —. 2015b, ApJ, 813, 65
- —. 2016, ApJ, 821, 13

- Arzoumanian, Z., Joshi, K., Rasio, F., & Thorsett, S. E. 1996, in Pulsars: Problems and Progress, IAU Colloquium 160, ed. S. Johnston, M. A. Walker, & M. Bailes (San Francisco: Astronomical Society of the Pacific), 525
- Arzoumanian, Z., Nice, D. J., Taylor, J. H., & Thorsett, S. E. 1994, ApJ, 422, 671
- Baade, W. & Zwicky, F. 1934, Proc. Nat. Acad. Sci., 20, 254
- Backer, D. C., Dexter, M. R., Zepka, A., D., N., Wertheimer, D. J., Ray, P. S., & Foster, R. S. 1997, PASP, 109, 61
- Backer, D. C., Foster, R. F., & Sallmen, S. 1993, Nature, 365, 817
- Backer, D. C. & Hellings, R. W. 1986, Ann. Rev. Astr. Ap., 24, 537
- Bailes, M. 1988, A&A, 202, 109
- Bailes, M., Harrison, P. A., Lorimer, D. R., Johnston, S., Lyne, A. G., Manchester, R. N., D'Amico, N., Nicastro, L., Tauris, T. M., & Robinson, C. 1994, ApJ, 425, L41
- Barker, B. M. & O'Connell, R. F. 1975, ApJ, 199, L25
- Bassa, C. G., Antoniadis, J., Camilo, F., Cognard, I., Koester, D., Kramer, M., Ransom, S. R., & Stappers, B. W. 2015, ArXiv e-prints
- Bassa, C. G., van Kerkwijk, M. H., Koester, D., & Verbunt, F. 2006a, A&A, 456, 295
- Bassa, C. G., van Kerkwijk, M. H., & Kulkarni, S. R. 2006b, A&A, 450, 295
- Baym, G., Pethick, C., Pines, D., & Ruderman, M. 1969, Nature, 224, 872
- Blandford, R. & Teukolsky, S. A. 1976, ApJ, 205, 580

- Blandford, R. D., Romani, R. W., & Applegate, J. H. 1987, MNRAS, 225, 51P
- Bogdanov, S., Pruszunska, M., Lewandowski, W., & Wolszczan, A. 2002, ApJ, 581, 495
- Boss, A. P. 1991, Nature, 351, 298
- Breton, R. P., Kaspi, V. M., Kramer, M., McLaughlin, M. A., Lyutikov, M., Ransom, S. R., Stairs, I. H., Ferdman, R. D., & Camilo, F. 2008, Science
- Breton, R. P., Rappaport, S. A., van Kerkwijk, M. H., & Carter, J. A. 2012, ApJ, 748, 115
- Camilo, F., Nice, D. J., & Taylor, J. H. 1993, ApJ, 412, L37
- —. 1996, ApJ, 461, 812
- Champion, D. J., Ransom, S. M., Lazarus, P., Camilo, F., Bassa, C., Kaspi, V. M., Nice, D. J., Freire, P. C. C., Stairs, I. H., van Leeuwen, J., Stappers, B. W., Cordes, J. M., Hessels, J. W. T., Lorimer, D. R., Arzoumanian, Z., Backer, D. C., Bhat, N. D. R., Chatterjee, S., Cognard, I., Deneva, J. S., Faucher-Giguère, C.-A., Gaensler, B. M., Han, J., Jenet, F. A., Kasian, L., Kondratiev, V. I., Kramer, M., Lazio, J., McLaughlin, M. A., Venkataraman, A., & Vlemmings, W. 2008, Science, 320, 1309
- Coenen, T., van Leeuwen, J., Hessels, J. W. T., Stappers, B. W., Kondratiev, V. I., Alexov, A., Breton, R. P., Bilous, A., Cooper, S., Falcke, H., Fallows, R. A., Gajjar, V., Grießmeier, J.-M., Hassall, T. E., Karastergiou, A., Keane, E. F., Kramer, M., Kuniyoshi, M., Noutsos, A., Osłowski, S., Pilia, M., Serylak, M., Schrijvers, C., Sobey, C., ter Veen, S., Verbiest, J., Weltevrede, P., Wijnholds, S., Zagkouris, K., van Amesfoort, A. S., Anderson, J., Asgekar, A., Avruch, I. M., Bell, M. E., Bentum, M. J., Bernardi, G., Best, P., Bonafede, A., Breitling, F., Broderick, J., Brüggen,

M., Butcher, H. R., Ciardi, B., Corstanje, A., Deller, A., Duscha, S.,
Eislöffel, J., Fender, R., Ferrari, C., Frieswijk, W., Garrett, M. A., de
Gasperin, F., de Geus, E., Gunst, A. W., Hamaker, J. P., Heald, G.,
Hoeft, M., van der Horst, A., Juette, E., Kuper, G., Law, C., Mann, G.,
McFadden, R., McKay-Bukowski, D., McKean, J. P., Munk, H., Orru, E.,
Paas, H., Pandey-Pommier, M., Polatidis, A. G., Reich, W., Renting, A.,
Röttgering, H., Rowlinson, A., Scaife, A. M. M., Schwarz, D., Sluman, J.,
Smirnov, O., Swinbank, J., Tagger, M., Tang, Y., Tasse, C., Thoudam, S.,
Toribio, C., Vermeulen, R., Vocks, C., van Weeren, R. J., Wucknitz, O.,
Zarka, P., & Zensus, A. 2014, A&A, 570, A60

- Cognard, I. & Backer, D. C. 2004, ApJ, 612, L125
- Cognard, I., Guillemot, L., Johnson, T. J., Smith, D. A., Venter, C., Harding, A. K., Wolff, M. T., Cheung, C. C., Donato, D., Abdo, A. A., Ballet, J., Camilo, F., Desvignes, G., Dumora, D., Ferrara, E. C., Freire, P. C. C., Grove, J. E., Johnston, S., Keith, M., Kramer, M., Lyne, A. G., Michelson, P. F., Parent, D., Ransom, S. M., Ray, P. S., Romani, R. W., Saz Parkinson, P. M., Stappers, B. W., Theureau, G., Thompson, D. J., Weltevrede, P., & Wood, K. S. 2011, ApJ, 732, 47
- Coles, W., Hobbs, G., Champion, D. J., Manchester, R. N., & Verbiest, J. P. W. 2011, MNRAS, 418, 561
- Cordes, J. M. & Lazio, T. J. W. 2001, ApJ, 549, 997
- Cordes, J. M., Weisberg, J. M., & Boriakoff, V. 1985, ApJ, 288, 221
- Corongiu, A., Burgay, M., Possenti, A., Camilo, F., D'Amico, N., Lyne, A. G., Manchester, R. N., Sarkissian, J. M., Bailes, M., Johnston, S., Kramer, M., & van Straten, W. 2012, ApJ, 760, 100
- Crawford, F., Roberts, M. S. E., Hessels, J. W. T., Ransom, S. M., Livingstone, M., Tam, C. R., & Kaspi, V. M. 2006, ApJ, 652, 1499

- Damour, T. & Deruelle, N. 1985, Ann. Inst. H. Poincaré (Physique Théorique), 43, 107
- —. 1986, Ann. Inst. H. Poincaré (Physique Théorique), 44, 263

Damour, T. & Taylor, J. H. 1992, Phys. Rev. D, 45, 1840

- de Sitter, W. 1916, MNRAS, 77, 155
- Deller, A. T., Vigeland, S. J., Kaplan, D. L., Goss, W. M., Brisken, W. F., Chatterjee, S., Cordes, J. M., Janssen, G. H., Lazio, T. J. W., Petrov, L., Stappers, B. W., & Lyne, A. 2016, ArXiv e-prints
- Demorest, P. B. 2007, PhD thesis, University of California, Berkeley
- Demorest, P. B., Ferdman, R. D., Gonzalez, M. E., Nice, D., Ransom, S., Stairs, I. H., Arzoumanian, Z., Brazier, A., Burke-Spolaor, S., Chamberlin, S. J., Cordes, J. M., Ellis, J., Finn, L. S., Freire, P., Giampanis, S., Jenet, F., Kaspi, V. M., Lazio, J., Lommen, A. N., McLaughlin, M., Palliyaguru, N., Perrodin, D., Shannon, R. M., Siemens, X., Stinebring, D., Swiggum, J., & Zhu, W. W. 2013, ApJ, 762, 94
- Demorest, P. B., Pennucci, T., Ransom, S. M., Roberts, M. S. E., & Hessels, J. W. T. 2010, Nature, 467, 1081
- Deneva, J. S., Freire, P. C. C., Cordes, J. M., Lyne, A. G., Ransom, S. M., Cognard, I., Camilo, F., Nice, D. J., Stairs, I. H., Allen, B., Bhat, N. D. R., Bogdanov, S., Brazier, A., Champion, D. J., Chatterjee, S., Crawford, F., Desvignes, G., Hessels, J. W. T., Jenet, F. A., Kaspi, V. M., Knispel, B., Kramer, M., Lazarus, P., van Leeuwen, J., Lorimer, D. R., Lynch, R. S., McLaughlin, M. A., Scholz, P., Siemens, X., Stappers, B. W., Stovall, K., & Venkataraman, A. 2012, ApJ, 757, 89
- Desvignes, G., Caballero, R. N., Lentati, L., Verbiest, J. P. W., Champion, D. J., Stappers, B. W., Janssen, G. H., Lazarus, P., Osłowski, S., Babak,

S., Bassa, C. G., Brem, P., Burgay, M., Cognard, I., Gair, J. R., Graikou,
E., Guillemot, L., Hessels, J. W. T., Jessner, A., Jordan, C., Karuppusamy,
R., Kramer, M., Lassus, A., Lazaridis, K., Lee, K. J., Liu, K., Lyne, A. G.,
McKee, J., Mingarelli, C. M. F., Perrodin, D., Petiteau, A., Possenti, A.,
Purver, M. B., Rosado, P. A., Sanidas, S., Sesana, A., Shaifullah, G.,
Smits, R., Taylor, S. R., Theureau, G., Tiburzi, C., van Haasteren, R., &
Vecchio, A. 2016, ArXiv e-prints

- Detweiler, S. 1979, ApJ, 234, 1100
- DuPlain, R., Ransom, S., Demorest, P., Brandt, P., Ford, J., & Shelton, A. L. 2008, in Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 7019, Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, 70191D
- Durant, M., Kargaltsev, O., Pavlov, G. G., Kowalski, P. M., Posselt, B., van Kerkwijk, M. H., & Kaplan, D. L. 2012, ApJ, 746, 6
- Edwards, R. T. & Bailes, M. 2001, ApJ, 547, L37
- Espinoza, C. M., Lyne, A. G., Stappers, B. W., & Kramer, M. 2011, MNRAS, 414, 1679
- Everett, J. E. & Weisberg, J. M. 2001, ApJ, 553, 341
- Ferdman, R. D. 2008, PhD thesis, University of British Columbia
- Ferdman, R. D., Stairs, I. H., Kramer, M., Breton, R. P., McLaughlin, M. A., Freire, P. C. C., Possenti, A., Stappers, B. W., Kaspi, V. M., Manchester, R. N., & Lyne, A. G. 2013, ApJ, 767, 85
- Ferdman, R. D., Stairs, I. H., Kramer, M., Janssen, G. H., Bassa, C. G., Stappers, B. W., Demorest, P. B., Cognard, I., Desvignes, G., Theureau, G., Burgay, M., Lyne, A. G., Manchester, R. N., & Possenti, A. 2014, MNRAS, 443, 2183

- Fonseca, E. 2012, Master's thesis, The University of British Columbia, https://circle.ubc.ca/handle/2429/43498
- Fonseca, E., Pennucci, T. T., Ellis, J. A., & et al. 2016, ArXiv e-prints
- Fonseca, E., Stairs, I. H., & Thorsett, S. E. 2014, ApJ, 787, 82
- Ford, E. B., Joshi, K. J., Rasio, F. A., & Zbarsky, B. 2000a, ApJ, 528, 336
- Ford, E. B., Kozinsky, B., & Rasio, F. A. 2000b, ApJ, 535, 385
- Ford, J. M., Demorest, P., & Ransom, S. 2010, in Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 7740, Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, 77400A
- Foster, R. S., Cadwell, B. J., Wolszczan, A., & Anderson, S. B. 1995a, ApJ, 454, 826
- Foster, R. S., Wolszczan, A., & Cadwell, B. J. 1995b, in Millisecond Pulsars: A Decade of Surprise, ed. A. Fruchter, M. Tavani, & D. Backer, Vol. 72 (Astronomical Society of the Pacific Conference Series), 24
- Freire, P., Jacoby, B., Bailes, M., Stairs, I., Mott, A., Ferdman, R., Nice, D., & Backer, D. C. 2006, in American Astronomical Society Meeting Abstracts, Vol. 208, American Astronomical Society Meeting Abstracts 208, 72.06
- Freire, P. C. C., Bassa, C. G., Wex, N., Stairs, I. H., Champion, D. J., Ransom, S. M., Lazarus, P., Kaspi, V. M., Hessels, J. W. T., Kramer, M., Cordes, J. M., Verbiest, J. P. W., Podsiadlowski, P., Nice, D. J., Deneva, J. S., Lorimer, D. R., Stappers, B. W., McLaughlin, M. A., & Camilo, F. 2011, MNRAS, 412, 2763
- Freire, P. C. C. & Wex, N. 2010, MNRAS, 409, 199

- Freire, P. C. C., Wex, N., Esposito-Farèse, G., Verbiest, J. P. W., Bailes, M., Jacoby, B. A., Kramer, M., Stairs, I. H., Antoniadis, J., & Janssen, G. H. 2012, MNRAS, 423, 3328
- Glampedakis, K. & Andersson, N. 2009, Phys. Rev. Lett., 102, 141101
- Gold, T. 1968, Nature, 218, 731
- Goldreich, P. & Julian, W. H. 1969, ApJ, 157, 869
- Gregory, P. C. 2005a, Bayesian Logical Data Analysis for the Physical Sciences: A Comparative Approach with 'Mathematica' Support (Cambridge University Press)
- —. 2005b, Bayesian Logical Data Analysis for the Physical Sciences: A Comparative Approach with 'Mathematica' Support (Cambridge University Press, Cambridge, UK)
- Grishchuk, L. P. 2005, Phys. Uspekhi, 1235
- Guillemot, L., Freire, P. C. C., Cognard, I., Johnson, T. J., Takahashi, Y., Kataoka, J., Desvignes, G., Camilo, F., Ferrara, E. C., Harding, A. K., Janssen, G. H., Keith, M., Kerr, M., Kramer, M., Parent, D., Ransom, S. M., Ray, P. S., Saz Parkinson, P. M., Smith, D. A., Stappers, B. W., & Theureau, G. 2012, MNRAS, 422, 1294
- Hankins, T. H. & Rickett, B. J. 1975, in Methods in Computational Physics Volume 14 — Radio Astronomy (New York: Academic Press), 55
- Hansen, B. M. S., Brewer, J., Fahlman, G. G., Gibson, B. K., Ibata, R., Limongi, M., Rich, R. M., Richer, H. B., Shara, M. M., & Stetson, P. B. 2002, ApJ, 574, L155
- Hellings, R. W. & Downs, G. S. 1983, ApJ, 265, L39
- Hessels, J., Ransom, S., Roberts, M., Kaspi, V., Livingstone, M., Tam, C., & Crawford, F. 2005, in Astronomical Society of the Pacific Conference Series, Vol. 328, Binary Radio Pulsars, ed. F. A. Rasio & I. H. Stairs, 395
- Hewish, A., Bell, S. J., Pilkington, J. D. H., Scott, P. F., & Collins, R. A. 1968, Nature, 217, 709
- Hobbs, G., Coles, W., Manchester, R. N., Keith, M. J., Shannon, R. M., Chen, D., Bailes, M., Bhat, N. D. R., Burke-Spolaor, S., Champion, D., Chaudhary, A., Hotan, A., Khoo, J., Kocz, J., Levin, Y., Oslowski, S., Preisig, B., Ravi, V., Reynolds, J. E., Sarkissian, J., van Straten, W., Verbiest, J. P. W., Yardley, D., & You, X. P. 2012, MNRAS, 427, 2780
- Hobbs, G., Lyne, A. G., Kramer, M., Martin, C. E., & Jordan, C. 2004, MNRAS, 353, 1311
- Hobbs, G. B., Edwards, R. T., & Manchester, R. N. 2006, MNRAS, 369, 655
- Hotan, A. W., Bailes, M., & Ord, S. M. 2006, MNRAS, 369, 1502
- Hulse, R. A. & Taylor, J. H. 1975, ApJ, 195, L51
- Igoshev, A. P. & Popov, S. B. 2015, Astronomische Nachrichten, 336, 831
- Isaacman, R. & Rankin, J. M. 1977, ApJ, 214, 214
- Jackson, J. D. 1962, Classical Electrodynamics (Wiley)
- Jacoby, B. A. 2004, in IAU Symposium, Vol. 218, Young Neutron Stars and Their Environments, ed. F. Camilo & B. M. Gaensler, 129
- Jacoby, B. A., Bailes, M., Ord, S. M., Knight, H. S., & Hotan, A. W. 2007, ApJ, 656, 408
- Jacoby, B. A., Hotan, A., Bailes, M., Ord, S., & Kuklarni, S. R. 2005, ApJ, 629, L113

- Janssen, G. H., Stappers, B. W., Bassa, C. G., Cognard, I., Kramer, M., & Theureau, G. 2010, A&A, 514, A74
- Jenet, F. A., Hobbs, G. B., van Straten, W., Manchester, R. N., Bailes, M., Verbiest, J. P. W., Edwards, R. T., Hotan, A. W., Sarkissian, J. M., & Ord, S. M. 2006, ApJ, 653, 1571
- Johnston, S., Walker, M. A., & Bailes, M., eds. 1996, Pulsars: Problems and Progress, IAU Colloquium 160 (San Francisco: Astronomical Society of the Pacific)
- Joshi, K. J. & Rasio, F. A. 1997, ApJ, 479, 948
- Kalapotharakos, C., Kazanas, D., Harding, A., & Contopoulos, I. 2012, ApJ, 749, 2
- Kaplan, D. L., van Kerkwijk, M. H., Koester, D., Stairs, I. H., Ransom, S. M., Archibald, A. M., Hessels, J. W. T., & Boyles, J. 2014, ApJ, 783, L23
- Kaspi, V. M., Lackey, J. R., Mattox, J., Manchester, R. N., Bailes, M., & Pace, R. 2000, ApJ, 528, 445
- Kaspi, V. M., Taylor, J. H., & Ryba, M. 1994, ApJ, 428, 713
- Kehl, M. S., Wex, N., Kramer, M., & Liu, K. 2016, ArXiv e-prints
- Keith, M. J., Coles, W., Shannon, R. M., & et al. 2013, MNRAS, 429, 2161
- Khargharia, J., Stocke, J. T., Froning, C. S., Gopakumar, A., & Joshi, B. C. 2012, ApJ, 744, 183
- Kilic, M., Hermes, J. J., Gianninas, A., & Brown, W. R. 2015, MNRAS, 446, L26
- Kiziltan, B., Kottas, A., De Yoreo, M., & Thorsett, S. E. 2013, ApJ, 778, 66

- Komesaroff, M. M. 1970, Nature, 225, 612
- Kopeikin, S. & Vlasov, I. 2004, Phys. Rep., 400, 209
- Kopeikin, S. M. 1995, ApJ, 439, L5
- —. 1996, ApJ, 467, L93
- Kramer, M. 1994, A&AS, 107, 527
- Kramer, M. 1998, ApJ, 509, 856
- Kramer, M., Lyne, A. G., O'Brien, J. T., Jordan, C. A., & Lorimer, D. R. 2006a, Science, 312, 549
- Kramer, M., Stairs, I. H., Manchester, R. N., McLaughlin, M. A., Lyne, A. G., Ferdman, R. D., Burgay, M., Lorimer, D. R., Possenti, A., D'Amico, N., Sarkissian, J. M., Hobbs, G. B., Reynolds, J. E., Freire, P. C. C., & Camilo, F. 2006b, Science, 314, 97
- Kuijken, K. & Gilmore, G. 1989, MNRAS, 239, 571
- Löhmer, O., Lewandowski, W., Wolszczan, A., & Wielebinski, R. 2005, ApJ, 621, 388
- Lam, M. T., Cordes, J. M., Chatterjee, S., & et al. 2016, ApJ, 819, 155
- Lam, M. T., Cordes, J. M., Chatterjee, S., Jones, M. L., McLaughlin, M. A., & Armstrong, J. W. 2015, ArXiv e-prints
- Lange, C., Camilo, F., Wex, N., Kramer, M., Backer, D., Lyne, A., & Doroshenko, O. 2001, MNRAS, 326, 274
- Large, M. I., Vaughan, A. E., & Mills, B. Y. 1968, Nature, 220, 340
- Lattimer, J. H. & Prakash, M. 2004, Science, 304, 536

Lattimer, J. M. & Prakash, M. 2001, ApJ, 550, 426

- —. 2015, ArXiv e-prints
- Lattimer, J. M. & Schutz, B. F. 2005, ApJ, 629, 979
- Levin, L., McLaughlin, M. A., Jones, G., Cordes, J. M., Stinebring, D. R., Chatterjee, S., Dolch, T., Lam, M. T., Lazio, T. J. W., Palliyaguru, N., Arzoumanian, Z., Crowter, K., Demorest, P. B., Ellis, J. A., Ferdman, R. D., Fonseca, E., Gonzalez, M. E., Jones, M. L., Nice, D. J., Pennucci, T. T., Ransom, S. M., Stairs, I. H., Stovall, K., Swiggum, J. K., & Zhu, W. 2016, ApJ, 818, 166
- Lewis, K. M., Sackett, P. D., & Mardling, R. A. 2008, ApJ, 685, L153
- Li, X.-D., van den Heuvel, E. P. J., & Wang, Z.-R. 1998, ApJ, 497, 865
- Lorimer, D. R. 2008, Living Reviews in Relativity, 11, 8
- Lorimer, D. R. & Kramer, M. 2005, Handbook of Pulsar Astronomy (Cambridge University Press)
- Lorimer, D. R., Lyne, A. G., McLaughlin, M. A., Kramer, M., Pavlov, G. G., & Chang, C. 2012, ApJ, 758, 141
- Lorimer, D. R., Nicastro, L., Lyne, A. G., Bailes, M., Manchester, R. N., Johnston, S., Bell, J. F., D'Amico, N., & Harrison, P. A. 1995, ApJ, 439, 933
- Lundgren, S. C., Cordes, J. M., Foster, R. S., Wolszczan, A., & Camilo, F. 1995, BAAS, 186, 4703
- Lyne, A. G. 1988, in Gravitational Wave Data Analysis, (NATO ASI Series), ed. B. Schutz (Dordrecht: Reidel), 95

- Lyne, A. G., Jordan, C. A., Graham-Smith, F., Espinoza, C. M., Stappers, B. W., & Weltevrede, P. 2015, MNRAS, 446, 857
- Madison, D. R., Cordes, J. M., & Chatterjee, S. 2014, ApJ, 788, 141
- Madsen, E. C., Stairs, I. H., Kramer, M., Camilo, F., Hobbs, G. B., Janssen, G. H., Lyne, A. G., Manchester, R. N., Possenti, A., & Stappers, B. W. 2012, MNRAS, 425, 2378
- Manchester, R. N., Hobbs, G. B., Teoh, A., & Hobbs, M. 2005, AJ, 129, 1993
- Manchester, R. N. & IPTA. 2013, Classical and Quantum Gravity, 30, 224010
- Manchester, R. N., Kramer, M., Stairs, I. H., Burgay, M., Camilo, F., Hobbs, G. B., Lorimer, D. R., Lyne, A. G., McLaughlin, M. A., McPhee, C. A., Possenti, A., Reynolds, J. E., & van Straten, W. 2010, ApJ, 710, 1694
- Manchester, R. N., Lyne, A. G., Camilo, F., Bell, J. F., Kaspi, V. M., D'Amico, N., McKay, N. P. F., Crawford, F., Stairs, I. H., Possenti, A., Morris, D. J., & Sheppard, D. C. 2001, MNRAS, 328, 17
- Martinez, J. G., Stovall, K., Freire, P. C. C., Deneva, J. S., Jenet, F. A., McLaughlin, M. A., Bagchi, M., Bates, S. D., & Ridolfi, A. 2015, ApJ, 812, 143
- Matsakis, D. N., Taylor, J. H., & Eubanks, T. M. 1997, A&A, 326, 924
- Matthews, A. M., Nice, D. J., Fonseca, E., & et al. 2016, ApJ, 818, 92
- Maxted, P. F. L., Serenelli, A. M., Miglio, A., Marsh, T. R., Heber, U., Dhillon, V. S., Littlefair, S., Copperwheat, C., Smalley, B., Breedt, E., & Schaffenroth, V. 2013, Nature, 498, 463
- McKee, J. W., Janssen, G. H., Stappers, B. W., Lyne, A. G., Caballero, R. N., Lentati, L., Desvignes, G., Jessner, A., Jordan, C. A., Karuppusamy, R., Kramer, M., Cognard, I., Champion, D. J., Graikou, E., Lazarus, P.,

Osłowski, S., Perrodin, D., Shaifullah, G., Tiburzi, C., & Verbiest, J. P. W. 2016, ArXiv e-prints

- McKenna, J. & Lyne, A. G. 1988, Nature, 336, 226, erratum ibid., 336, 698
- Nardiello, D., Milone, A. P., Piotto, G., Marino, A. F., Bellini, A., & Cassisi, S. 2015, A&A, 573, A70
- Nice, D. 2016, private communication via e-mail sent on 8 February 2016.
- Nice, D. J., Splaver, E. M., & Stairs, I. H. 2001, ApJ, 549, 516
- Nice, D. J., Splaver, E. M., & Stairs, I. H. 2004, in Young Neutron Stars and Their Environments, IAU Symposium 218, ed. F. Camilo & B. M. Gaensler (San Francisco: Astronomical Society of the Pacific)
- Nice, D. J. & Taylor, J. H. 1995, ApJ, 441, 429
- Oppenheimer, J. R. & Volkoff, G. 1939, Phys. Rev., 55, 374
- Ozel, F. & Freire, P. 2016, Ann. Rev. Astr. Ap., 54, 401
- Ozel, F., Psaltis, D., Narayan, R., & Santos Villarreal, A. 2012, ApJ, 757, 55
- Pacini, F. 1967, Nature, 216, 567
- Pennucci, T. T. 2015, PhD thesis, The University of Virginia
- Perrodin, D., Jenet, F., Lommen, A., Finn, L., Demorest, P., Ferdman, R., Gonzalez, M., Nice, D., Ransom, S., & Stairs, I. 2013, ArXiv e-prints
- Peterson, R. C., Rees, R. F., & Cudworth, K. M. 1995, ApJ, 443, 124
- Phillips, J. A. & Wolszczan, A. 1991, ApJ, 382, L27
- Phinney, E. S. 1992, Philos. Trans. Roy. Soc. London A, 341, 39
- Phinney, E. S. & Kulkarni, S. R. 1994, Ann. Rev. Astr. Ap., 32, 591

- Pietrzyński, G., Thompson, I. B., Gieren, W., Graczyk, D., Stępień, K., Bono, G., Moroni, P. G. P., Pilecki, B., Udalski, A., Soszyński, I., Preston, G. W., Nardetto, N., McWilliam, A., Roederer, I. U., Górski, M., Konorski, P., & Storm, J. 2012, Nature, 484, 75
- Piotto, G., Milone, A. P., Bedin, L. R., Anderson, J., King, I. R., Marino, A. F., Nardiello, D., Aparicio, A., Barbuy, B., Bellini, A., Brown, T. M., Cassisi, S., Cool, A. M., Cunial, A., Dalessandro, E., D'Antona, F., Ferraro, F. R., Hidalgo, S., Lanzoni, B., Monelli, M., Ortolani, S., Renzini, A., Salaris, M., Sarajedini, A., van der Marel, R. P., Vesperini, E., & Zoccali, M. 2015, ApJ, 149, 91
- Press, W. H., Flannery, B. P., Teukolsky, S. A., & Vetterling, W. T. 1986, Numerical Recipes: The Art of Scientific Computing (Cambridge: Cambridge University Press)

Radhakrishnan, V. & Cooke, D. J. 1969, Astrophys. Lett., 3, 225

- Rankin, J. M. 2015, ApJ, 804, 112
- Rankin, J. M. & Roberts, J. A. 1971, in IAU Symposium, Vol. 46, The Crab Nebula, ed. R. D. Davies & F. Graham-Smith, 114
- Ransom, S. M., Stairs, I. H., Archibald, A. M., Hessels, J. W. T., Kaplan, D. L., van Kerkwijk, M. H., Boyles, J., Deller, A. T., Chatterjee, S., Schechtman-Rook, A., Berndsen, A., Lynch, R. S., Lorimer, D. R., Karako-Argaman, C., Kaspi, V. M., Kondratiev, V. I., McLaughlin, M. A., van Leeuwen, J., Rosen, R., Roberts, M. S. E., & Stovall, K. 2014, Nature, 520
- Rappaport, S., Deck, K., Levine, A., Borkovits, T., Carter, J., El Mellah, I., Sanchis-Ojeda, R., & Kalomeni, B. 2013, ApJ, 768, 33

Rasio, F. A. 1994, ApJ, 427, L107

- Reardon, D. J., Hobbs, G., Coles, W., Levin, Y., Keith, M. J., Bailes, M., Bhat, N. D. R., Burke-Spolaor, S., Dai, S., Kerr, M., Lasky, P. D., Manchester, R. N., Osłowski, S., Ravi, V., Shannon, R. M., van Straten, W., Toomey, L., Wang, J., Wen, L., You, X. P., & Zhu, X.-J. 2016, MNRAS, 455, 1751
- Rickett, B. J. 1990, Ann. Rev. Astr. Ap., 28, 561
- Ruderman, M., Shaham, J., & Tavani, M. 1989, ApJ, 336, 507
- Ryba, M. F. & Taylor, J. H. 1991, ApJ, 371, 739
- Sazhin, M. V. 1978, Sov. Astron., 22, 36
- Scheiner, B. & Wolszczan, A. 2012, in American Astronomical Society Meeting Abstracts, Vol. 220, American Astronomical Society Meeting Abstracts #220, 430.06
- Schwab, J., Podsiadlowski, P., & Rappaport, S. 2010, ApJ, 719, 722
- Segelstein, D. J., Rawley, L. A., Stinebring, D. R., Fruchter, A. S., & Taylor, J. H. 1986, Nature, 322, 714
- Sesana, A., Vecchio, A., & Colacino, C. N. 2008, MNRAS, 390, 192
- Sesana, A., Vecchio, A., & Volonteri, M. 2009, MNRAS, 394, 2255
- Shannon, R. M. & Cordes, J. M. 2010, ApJ, 725, 1607
- —. 2012, apj, 761, 64
- Shapiro, I. I. 1964, Phys. Rev. Lett., 13, 789
- Shklovskii, I. S. 1970, Sov. Astron., 13, 562
- Sieber, W. & Wielebinski, R. 1973, Astrophys. Lett., 13, 225

- Siemens, X., Ellis, J., Jenet, F., & Romano, J. D. 2013, Classical and Quantum Gravity, 30, 224015
- Sigurdsson, S. 1992, ApJ, 399, L95
- Sigurdsson, S., Richer, H. B., Hansen, B. M., Stairs, I. H., & Thorsett, S. E. 2003, Science, 301, 193
- Simon, J., Polin, A., Lommen, A., & et al. 2014, ApJ, 784, 60
- Splaver, E. M. 2004, PhD thesis, Princeton University, Princeton, N. J., U.S.A.
- Splaver, E. M., Nice, D. J., Arzoumanian, Z., Camilo, F., Lyne, A. G., & Stairs, I. H. 2002, ApJ, 581, 509
- Splaver, E. M., Nice, D. J., Stairs, I. H., Lommen, A. N., & Backer, D. C. 2005, ApJ, 620, 405
- Staelin, D. H. & Reifenstein, III, E. C. 1968, Science, 162, 1481
- Stairs, I. H. 2003, Living Reviews in Relativity, 5, uRL (Cited on 2008/02/16): http://relativity.livingreviews.org/Articles/lrr-2003-5
- Stairs, I. H., Arzoumanian, Z., Camilo, F., Lyne, A. G., Nice, D. J., Taylor, J. H., Thorsett, S. E., & Wolszczan, A. 1998, ApJ, 505, 352
- Stairs, I. H., Splaver, E. M., Thorsett, S. E., Nice, D. J., & Taylor, J. H. 2000, MNRAS, 314, 459, (astro-ph/9912272)
- Stairs, I. H., Thorsett, S. E., & Arzoumanian, Z. 2004, Phys. Rev. Lett., 93, 141101
- Stairs, I. H., Thorsett, S. E., & Camilo, F. 1999, ApJS, 123, 627
- Stairs, I. H., Thorsett, S. E., Taylor, J. H., & Wolszczan, A. 2002, ApJ, 581, 501

- Stinebring, D. R., Kaspi, V. M., Nice, D. J., Ryba, M. F., Taylor, J. H., Thorsett, S. E., & Hankins, T. H. 1992, Rev. Sci. Instrum., 63, 3551
- Tauris, T. M., Langer, N., & Kramer, M. 2011, MNRAS, 416, 2130
- Tauris, T. M. & Savonije, G. J. 1999, A&A, 350, 928
- Tauris, T. M. & van den Heuvel, E. P. J. 2014, ApJ, 781, L13
- Taylor, J. H. 1992, Philos. Trans. Roy. Soc. London A, 341, 117
- Taylor, J. H. & Weisberg, J. M. 1989, ApJ, 345, 434
- Taylor, S. R., Vallisneri, M., Ellis, J. A., Mingarelli, C. M. F., Lazio, T. J. W., & van Haasteren, R. 2016, ApJ, 819, L6
- Thorsett, S. E. 1991, ApJ, 377, 263
- Thorsett, S. E., Arzoumanian, Z., Camilo, F., & Lyne, A. G. 1999, ApJ, 523, 763
- Thorsett, S. E. & Chakrabarty, D. 1999, ApJ, 512, 288
- Timmes, F. X., Woosley, S. E., & Weaver, T. A. 1996, ApJ, 457, 834
- van den Heuvel, E. P. J. 1994, A&A, 291, L39
- van Kerkwijk, M., Bassa, C. G., Jacoby, B. A., & Jonker, P. G. 2005, in Binary Radio Pulsars, ed. F. Rasio & I. H. Stairs (San Francisco: Astronomical Society of the Pacific), 357
- van Kerkwijk, M. H., Bell, J. F., Kaspi, V. M., & Kulkarni, S. R. 2000, ApJ, 530, L37
- van Kerkwijk, M. H., Bergeron, P., & Kulkarni, S. R. 1996, ApJ, 467, L89
- van Kerkwijk, M. H., Rappaport, S. A., Breton, R. P., Justham, S., Podsiadlowski, P., & Han, Z. 2010, ApJ, 715, 51

Verbiest, J. P. W. 2009, ArXiv e-prints

- Verbiest, J. P. W., Bailes, M., Coles, W. A., Hobbs, G. B., van Straten, W., Champion, D. J., Jenet, F. A., Manchester, R. N., Bhat, N. D. R., Sarkissian, J. M., Yardley, D., Burke-Spolaor, S., Hotan, A. W., & You, X. P. 2009, MNRAS, 400, 951
- Verbiest, J. P. W., Bailes, M., van Straten, W., Hobbs, G. B., Edwards, R. T., Manchester, R. N., Bhat, N. D. R., Sarkissian, J. M., Jacoby, B. A., & Kulkarni, S. R. 2008, ApJ, 679, 675
- Vigeland, S. J. & Vallisneri, M. 2014, MNRAS, 440, 1446
- Vilenkin, A. & Shellard, E. P. S. 1994, Cosmic strings and other topological defects
- von Hoensbroech, A. & Xilouris, K. M. 1996, in Pulsars: Problems and Progress, IAU Colloquium 160, ed. S. Johnston, M. A. Walker, & M. Bailes (San Francisco: Astronomical Society of the Pacific), 267
- Wall, J. V. & Jenkins, C. R. 2003, Practical Statistics for Astronomers (Cambridge: Cambridge University Press)
- Watts, A., Espinoza, C. M., Xu, R., Andersson, N., Antoniadis, J., Antonopoulou, D., Buchner, S., Datta, S., Demorest, P., Freire, P., Hessels, J., Margueron, J., Oertel, M., Patruno, A., Possenti, A., Ransom, S., Stairs, I., & Stappers, B. 2015, Advancing Astrophysics with the Square Kilometre Array (AASKA14), 43
- Weidenschilling, S. J. 1980, Icarus, 44, 172
- Weisberg, J. M. & Huang, Y. 2016, ArXiv e-prints
- Weisberg, J. M., Nice, D. J., & Taylor, J. H. 2010, ApJ, 722, 1030
- Weisberg, J. M., Romani, R. W., & Taylor, J. H. 1989, ApJ, 347, 1030

- Wex, N. 1998, MNRAS, 298, 67
- Wex, N., Johnston, S., Manchester, R. N., Lyne, A. G., Stappers, B. W., & Bailes, M. 1998, MNRAS, 298, 997
- Wolszczan, A. & Frail, D. A. 1992, Nature, 355, 145
- Zhu, W. W., Stairs, I. H., Demorest, P. B., Nice, D. J., Ellis, J. A., Ransom, S. M., Arzoumanian, Z., Crowter, K., Dolch, T., Ferdman, R. D., Fonseca, E., Gonzalez, M. E., Jones, G., Jones, M. L., Lam, M. T., Levin, L., McLaughlin, M. A., Pennucci, T., Stovall, K., & Swiggum, J. 2015, ApJ, 809, 41

Appendix A

Probabilistic Analysis of Shapiro-Delay Parameters

Using an initial set of parameter values, the TEMPO pulsar-timing package creates a timing solution that best fits the supplied TOA data. The uncertainties reported along with the best-fit values are determined from the diagonal elements of the resultant covariance matrix, which is approximated by the least-squares method for model determination as based on χ^2 statistics. However, nonzero correlation between parameters can produce slightly larger degrees of uncertainty, as well as nonlinear correlation between model parameters that yield asymmetric uncertainties, that are not reflected in the reported TEMPO uncertainties.

In Chapter 3, we employed a statistically rigorous approach to determine more accurate uncertainties for the parameters of the Shapiro delay in fourteen NANOGrav binary pulsars. This approach, based on a method discussed by Splaver et al. (2002), analyzes the accuracy and behavior of the timing model for different, fixed values of m_c and $\sin i$ by generating a grid of χ^2 values. The resulting distribution of χ^2 values is then converted to a two-dimensional probability density in the $(m_c, \cos i)$ phase space using Equation 3.4,

$$p(\text{data}|m_{\rm c}, \cos i) \propto e^{-(\chi^2 - \chi_0^2)/2}$$
 (A.1)

and a series of mathematical operations are used to convert this density to a posterior probability distribution in the $(m_{\rm p}, \cos i)$ phase space.

In this Appendix, we outline the theory and procedure in more detail. We also provide the equations used to obtain the two-dimensional probability distributions and one-dimensional probability density functions (PDFs) shown in Chapter 3.

A.1 Bayesian Interpretation of χ^2 -grid Analysis

In the context of a Bayesian analysis of probabilities, the use of prior information for certain outcomes takes on an important role when determining the final, "posterior" evaluation of said outcomes. This interpretation of probability allows for a powerful set of analysis techniques, such as the Markov Chain Monte Carlo (MCMC; Wall & Jenkins, 2003; Gregory, 2005b) method used in Chapters 3, 4 and 5 to probe the parameter space, evaluate regions of larger or lower likelihood for best fits based on relative changes in χ^2 values, and bias the random walk towards regions of maximum likelihood. The fundamental concepts of Bayesian statistics are embodied in Bayes' theorem, which states the following: the posterior probability that a set of parameters describe the supplied data – p(model|data) – is given as

$$p(\text{model}|\text{data}) = \frac{p(\text{data}|\text{model})p(\text{model})}{p(\text{data})}$$
(A.2)

where p(data|model) is the likelihood function, p(model) is the prior probability of the model parameters, and p(data) is the "Bayesian evidence." The prior probability is interpreted as one's prior knowledge in the distribution of probable values for each parameter that forms the model of the data in hand. In the case where it is unknown which value a given parameter is more likely to have, a uniformly random distribution is chosen for that parameter: in the absence of prior knowledge, all values between a chosen interval are equally likely to be the parameter value.

The Splaver et al. (2002) method for determining accurate confidence intervals of the Shapiro-delay parameters from a χ^2 grid uses Equation A.2 in order to relate the likelihood density (Equation 3.4) to the joint-posterior probability density of m_c and $\cos i$. In their formulation, they compute a grid of values of m_c and $\cos i$, and assume prior probability distributions for both parameters. From this assumed form of $p(m_c, \cos i)$, the posterior distribution can be readily computed since, from Equation A.2,

$$p(m_{\rm c}, \cos i | \text{data}) \propto p(\text{data} | m_{\rm c}, \cos i)$$
 (A.3)

and the Bayesian evidence is a constant, normalizing factor. The uniform prior-probability distribution in m_c reflects our ignorance of the binary companion and its stellar type. For example, in the absence of prior information, a main-sequence companion star can have any mass, whereas a white-dwarf companion has an upper limit on physically allowed masses that corresponds to the Chandrasekhar limit. In such cases, prior information could be obtained from optical photometry and/or radial-velocity measurements from spectroscopy, which yield a measurement of the component-mass ratio. As discussed in Chapter 1, the upper limit for neutron-star companion masses is less certain but is expected from recent numerical studies to be ~ 3 M_☉. The uniform prior distribution in cos *i* is chosen to reflect random orientation of the orbit, which is discussed in more detail in Section A.2 below.

A.2 Derivation of Uniform Distribution for Randomly Oriented Orbits

Let us first consider an orbit with inclination i and longitude of ascending node Ω , such as the example orbit shown in Figure 1.2. Since the orbital angular moment vector $(\vec{S}_{\rm b})$ is perpendicular to the orbital plane, the orientation angles (i, Ω) also quantify the direction of $\vec{S}_{\rm b}$.

Let us now consider a collection of orbits with different Keplerian parameters, as well as different values of i and Ω . The set of $\vec{S}_{\rm b}$ in this sample is said to be randomly distributed if it is isotropic. It is clear that the Keplerian elements do not affect orientation, and so the magnitude of $\vec{S}_{\rm b}$ does not contribute to random orientation. We can therefore consider the probability of $\vec{S}_{\rm b}$ pointing in some direction and require isotropy when determining which functions of i and Ω should have uniformly random distributions.

The probability that $\vec{S}_{\rm b}$ points in a certain direction can be evaluated by first considering an infinitesimal solid angle $(d\phi)$ that contains the direction of $\vec{S}_{\rm b}$. If the solid angle spans a range of i and i + di, as well as a range of Ω and $\Omega + d\Omega$, then the probability that the direction of $\vec{S}_{\rm b}$ is contained within $d\phi$ is

$$p = \frac{d\phi}{\text{total solid angle}} = \frac{1}{4\pi} \sin i di d\Omega \tag{A.4}$$

We can integrate Equation A.5 to consider the probability that $\vec{S}_{\rm b}$ is contained within a finite patch of the sky, subtended by orientation angles (i_0, Ω_0) and (i, Ω) :

$$P(i,\Omega) = \frac{1}{4\pi} \int_{i_0}^{i} \sin i' di' \int_{\Omega_0}^{\Omega} d\Omega'$$

= $\frac{1}{4\pi} (\cos i_0 - \cos i) (\Omega - \Omega_0).$ (A.5)

In order to determine the probability distribution functions (PDFs) whose uniformity correspond to random orbital orientations, we must make use of two properties of probability theory:

- 1. the probability of the occurrence of two independent events A and B (e.g. a specific outcome from two rolls of dice) is given as P(A and B) = P(A)P(B), and
- 2. a random variable x is said to be uniformly distributed if its cumulative distribution function (CDF) is proportional to the variable.

Using property 1, we can view Equation A.5 as a joint probability distribution that is made up of the product of two one-dimensional probability distributions, i.e. $P(i, \Omega) = Q(i)R(\Omega)$. We also see that $Q = (\cos i_0 - \cos i)/2$ and $R = (\Omega - \Omega_0)/2\pi$ are each normalized cumulative distribution functions of *i* and Ω , respectively, since each integral in Equation A.5 yields the probability that their values lie between the bounds of integration. Using property 2, we can finally impose isotropy of $\vec{S}_{\rm b}$ by requiring that the uniformly-random variables be $\cos i$ and Ω , since $Q \propto \cos i$ and $R \propto \Omega$.

A.3 Translations of Probability Densities

As discussed in Chapter 3 and Section A.1, the computation of $p(\text{data}|m_c, \cos i)$ and use of a joint-uniform prior $-p(m_c, \cos i) \propto \text{constant} - \text{allow for a}$ straightforward determination of the posterior probability density $p(m_c, \cos i|\text{data})$ for both Shapiro-delay parameters from a map of χ^2 values. We eventually want to obtain the marginalized, one-dimensional posterior PDFs from the computed maps:

$$p(m_{\rm c}|{\rm data}) = \int_0^1 p(m_{\rm c}, \cos i |{\rm data}) d(\cos i)$$
(A.6)

$$p(\cos i | \text{data}) = \int_0^{m_{\text{c,max}}} p(m_{\text{c}}, \cos i | \text{data}) d(m_{\text{c}})$$
(A.7)

where $m_{c,max}$ is the maximum value of the companion mass defined on the χ^2 grid. It is important to note that, while $-1 < \cos i < 1$, the Shapiro $s = \sin i$ is always defined to be positive since $0 < i < \pi$. We cannot uniquely determine the true value of *i* by analyzing only the Shapiro delay, but can instead compute two values of *i* that correspond to positive and negative values of $\cos i$. We therefore restrict the χ^2 -grid values for $\cos i$ to range from 0 to 1 when only analyzing the Shapiro-delay parameters, since no new information is obtained when compute a χ^2 grid over the complete range of $\cos i$.

The binary mass function (Equation 1.37) can be solved for the value of $m_{\rm p}$ as a function of both $m_{\rm c}$ and sin *i*. In order to properly obtain the posterior PDF for $m_{\rm p}$, we must project the computed $p(m_{\rm c}, \cos i | \text{data})$ into a different phase space that is spanned by $m_{\rm p}$ along one of the two dimensions. We arbitrarily choose the $(m_{\rm p}, \cos i)$ space since, in practice, the probability density in the $(m_{\rm p}, m_{\rm c})$ space is heavily truncated to a small slice and difficult to resolve using finite bin sizes. The probability density in the $(m_{\rm p}, \cos i)$ can be determined by requiring that the marginalized posterior PDF of $\cos i$ computed in both phase spaces be equal. In other words,

$$p(\cos i | \text{data}) = \int_{0}^{\infty} p(m_{\text{p}}, \cos i | \text{data}) d(m_{\text{p}})$$
$$= \int_{0}^{\infty} p(m_{\text{c}}, \cos i | \text{data}) \left| \frac{\partial(m_{\text{c}})}{\partial(m_{\text{p}})} \right| d(m_{\text{p}})$$
(A.8)

where the second line of Equation A.8 was obtained by writing Equation A.7

as an integral over $m_{\rm p}$. The absolute value of the derivative ensures that the probability density (and marginalized PDFs) remain positive. The top and bottom forms of Equation A.8 are equal if the integrands are equal, so that

$$p(m_{\rm p}, \cos i | \text{data}) = p(m_{\rm c}, \cos i | \text{data}) \left| \frac{\partial(m_{\rm c})}{\partial(m_{\rm p})} \right|$$
(A.9)

where the notation for partial derivatives is used to emphasize that $\cos i$ is fixed in Equations A.8. The derivative must be evaluated using the binary mass function (Equation 1.37),

$$f_m = \frac{n_{\rm b}^2 x^3}{\mathrm{T}_{\odot}} = \frac{(m_{\rm c} \sin i)^3}{(m_{\rm p} + m_{\rm c})^2},$$

which is a constant quantity¹ that relates $m_{\rm p}$, $m_{\rm c}$, and $\sin i$. The derivative can be computed through implicit differentiation of Equation 1.37 to yield

$$\frac{\partial(m_{\rm c})}{\partial(m_{\rm p})} = \frac{2f_m(m_{\rm p} + m_{\rm c})}{3m_{\rm c}^2 \sin^3 i - 2f_m(m_{\rm p} + m_{\rm c})}$$
(A.10)

Equation A.9 implies that the probability distributions are continuous. In practice, however, the χ^2 grids and derived probability densities consist of finite grid bins, and so the integrals in Equation A.7 turn into discrete sums. Moreover, several grid bins in one phase space can be encompassed by a singe bin in another phase space. However, $p(m_c, \cos i | \text{data})$ can be approximately translated to the $(m_p, \cos i)$ space by averaging together the probability in finite bins across m_c that fall within a corresponding bin in m_p . The approximation becomes more accurate when smaller grid bins are used.

¹For the most relativistic binary systems (e.g. PSR B1534+12), $P_{\rm b}$ intrinsically changes over time due to the emission of gravitational radiation from the system. However, the rates of change in $P_{\rm b}$ are typically very small, and the secular change ultimately does not significantly change the value of f_m over the time span of the data set.

A.3.1 Probability in Orthometric Space

The orthometric model for the Shapiro timing delay (Freire & Wex, 2010) parametrizes the effect to using two different PK parameters. The choice of specific parameters depends on the degree of inclination and orbital eccentricity, as the orthometric method was developed by Freire & Wex to carefully reduce correlation between the PK parameters:

- if the binary model is ELL1 and i < 50 degrees, the third and fourth Fourier harmonics of $\Delta_{\rm S}$ (h_3 and h_4 , respectively) best characterize the Shapiro delay;
- if the binary model is ELLI and i > 50 degrees, or if the binary model is DD, then h_3 and the orthometric ratio $\varsigma = h_4/h_3$ are the ideal Shapirodelay parameters.

As with the traditional $(m_c, \cos i)$ parameters, a χ^2 -grid analysis can be performed using the orthometric parameters in order to evaluate correlation and compute more robust confidence intervals. The nonlinear relation between the traditional and orthometric Shapiro-delay parameters (Equations 3.1-3.3) require additional, careful translation of the posterior density derived from the χ^2 grid over the orthometric parameters to the physical parameters of interest. Furthermore, the nonlinear relation amounts to a difference in choice of prior probability densities between the traditional and orthometric parameters.

For instance, let us consider a two-dimensional posterior probability density in the (h_3, ς) phase space. This map can be converted to one in the $(m_c, \cos i)$ space by using the translation rule for multivariate probability distributions,

$$p(m_{\rm c}, \cos i | \text{data}) = p(h_3, \varsigma | \text{data}) \left| \frac{\partial(h_3, \varsigma)}{\partial(m_{\rm c}, \cos i)} \right|$$
(A.11)

where $\partial(h_3,\varsigma)/\partial(m_c,\cos i)$ is the determinant of the Jacobian matrix that maps volume elements between phase spaces:

$$\frac{\partial(h_3,\varsigma)}{\partial(m_{\rm c},\cos i)} = \det \begin{bmatrix} \frac{\partial(h_3)}{\partial(m_{\rm c})} & \frac{\partial(h_3)}{\partial(\cos i)} \\ \frac{\partial\varsigma}{\partial(m_{\rm c})} & \frac{\partial\varsigma}{\partial(\cos i)} \end{bmatrix} = \frac{\partial(h_3)}{\partial(m_{\rm c})} \frac{\partial\varsigma}{\partial(\cos i)}$$

since ς is only a function of $\cos i$. The posterior density in the orthometric space can therefore be directly translated to the $(m_c, \cos i)$ space using Equation A.11. The same procedures can then be applied to obtain the marginalized PDFs for m_p , m_c and $\cos i$.

A.4 Confidence Intervals

The primary goal of the above computations is to determine robust measures of the best-fit parameter and its degree of uncertainty. With the onedimensional posterior PDFs at hand, we take the "best-fit value" of, say, the pulsar mass as the median value of its posterior PDF. The median value of $m_{\rm p}$ (and, similarly, $m_{\rm c}$ and $\cos i$) corresponds to the point on the PDF where 50% of the probability lies above and below it. In other words, we integrate the PDF up to a value of $m_{\rm p,med}$ such that

$$\int_{0}^{m_{\rm p,med}} p(m_{\rm p}|\text{data}) = 0.5.$$
 (A.12)

The median values of m_c and $\cos i$ are computed in a similar manner using their appropriate posterior PDFs.

There are several ways to compute confidence limits from the one-dimensional posterior PDFs of the Shapiro-delay parameters. For example, Splaver (2004) define the 68% confidence interval as the range that both encompasses 68% of the total probability and spans the shortest range of the parameter values. Splaver et al. (2002) used this definition of confidence limits in their analysis of the PSR J0621+1002 binary system and its Shapiro-delay parameters. While the "shortest interval" method is a common one for uncertainty

determination, we instead choose an "equal tail" method to compute confidence limits. For the 68.3% confidence interval, we find the lower bound of the range $(m_{\rm p,lo})$ by integrating the posterior PDF up to a value that encapsulates 15.85% of all probability,

$$\int_{0}^{m_{\rm p,lo}} p(m_{\rm p}|\text{data}) = 0.1585$$
 (A.13)

and find the upper bound of the same range $(m_{p,up})$ by integrating up to a value that encapsulates 84.15% of all probability,

$$\int_{0}^{m_{\rm p,up}} p(m_{\rm p}|\text{data}) = 0.8415.$$
 (A.14)

We refer to this set of confidence intervals as "equal tail" since $m_{\rm p,lo}$ and $m_{\rm p,up}$ serve as lower and upper bounds of the posterior tails that have equal probability.

Appendix B

Spin-Derivative Model of Long-Period Orbits

The last radio-timing study of PSR B1620-26 performed by Thorsett et al. (1999) showed that a direct fit of the binary Römer timing delay for the outer orbit could fully explain the observed spin derivatives, but could not uniquely determine all outer-orbital elements. Thorsett et al. instead fitted ($\Delta_{\rm R}$)_o to their 11-year data set for different values of the outer-orbital eccentricity $e_{\rm o}$, and found that all orbital elements indeed varied with increasing $e_{\rm o}$. In particular, the smallest outer-orbital period obtained using this method, corresponding to $e_o = 0$ (a circular outer orbit), was found to be $(P_{\rm b})_{\rm o} \approx 62$ years, much longer than the 11 years spanned by their data set. The degeneracy of timing parameters for ($\Delta_{\rm R}$)_o is therefore due to significant lack of coverage across a full outer orbit.

Joshi & Rasio (1997, JR97) noted that, in the case where a significant fraction of a full pulsar-binary orbit has not been spanned by the data set, the measured Doppler-induced time derivatives of ν_s could be used to infer the orbital elements of the system. To first order in its derivation, the JR97 framework can be applied to TOAs from PSR B1620-26 since Thorsett et al. (1999) showed that both orbits could be jointly represented as the sum of two non-interacting Keplerian orbits: $\Delta_{\rm R} = (\Delta_{\rm R})_{\rm i} + (\Delta_{\rm R})_{\rm o}$. In this sense, a *hierarchical* triple system can be viewed as a "binary" system where one of the "binary" components has a mass equal to the sum of the inner-component masses, and a center of mass that is approximately equal to the center of mass of the inner-binary system.

In this section, we describe the model developed by JR97 to estimate the elements using the frequency-derivative method, and derive additional equations for higher-order time derivatives in order to analyze an updated timing solution for our current TOA data set for PSR B1620-26 that we discuss in Chapter 4.

B.1 Derivation of Orbit-Induced Spin Derivatives

We first consider a binary orbit, with eccentricity e, of two bound components with masses m_1 and m_2 . In the context of PSR B1620-26, $m_1 = m_p + (m_c)_i$ is the "primary" mass and $m_2 = (m_c)_o$ is the "companion" mass. The corresponding semi-major axes of each component are a_1 and a_2 , respectively. Due to the symmetry of the (non-relativistic) two-body problem, the eccentricities and true anomalies of both orbits about the center of mass are equal, though the periastron arguments are shifted such that $\omega_1 = \omega_2 + \pi$.

For radio pulsars, the Doppler shift in ν_s due to binary motion is generally given as $\nu_s = -\nu_{s,0}(\mathbf{v} \cdot \hat{\mathbf{n}})/c$, where \mathbf{v} is the orbital velocity of the observed component, $\hat{\mathbf{n}}$ is a unit vector pointing along the line of sight to the system, and $\nu_{s,0}$ is the spin frequency in a reference frame that is co-moving with the pulsar in its binary motion.¹ In our current timing solution, as well as the model published by Thorsett et al. (1999), the inner orbit is explicitly

¹In practice, the absolute radial motion of pulsars and pulsar-binary systems through space induces a constant Doppler shift in the true, intrinsic values of certain timing parameters (e.g. spin/binary periods, projected semi-major axis, etc.), and so the "observed" values are different than the true values.

modeled by determining the inner Römer delay; therefore, all time-derivatives in spin frequency are due to intrinsic spin-down, biases from secular motion of the system, or orbital motion of the outer binary. We discuss the effects of the non-binary components on our analysis in Section B.3 below.

Binary motion induces a number of higher-order time derivative in ν_s due to periodic Doppler shifts:

$$\nu_{\rm s}^{(l)} = \frac{d^l}{dt^l} \nu_{\rm s} \approx -\nu_{\rm s,0} \frac{(\mathbf{a}^{(l-1)} \cdot \hat{\mathbf{n}})}{c} \tag{B.1}$$

where $\mathbf{a} = \dot{\mathbf{v}}$ is the acceleration of the observed component. In the approximation of Equation B.1 we ignored terms that were nonlinear in time derivatives of $\nu_{\rm s}$, which are typically small for pulsar-binary systems. For large orbits with widely-separated components, the general-relativistic corrections of the orbital motion can be ignored for the purposes of first-order calculations; the acceleration of the pulsar is therefore given by Newtonian inverse-square law, $a = |\mathbf{a}| = kr_1^{-2}$, where

$$k = G \frac{m_2^3}{(m_1 + m_2)^2} \tag{B.2}$$

$$h = a_1(1 - e^2) \tag{B.3}$$

$$r_1^{-1} = h^{-1}(1 + e\cos u) \equiv h^{-1}A \tag{B.4}$$

(B.5)

and where $A = 1 + e \cos u$ is a function of the true anomaly u. By computing the dot product in Equation B.1 and taking derivatives, we find that the component of the first derivative in ν_s due to orbital motion is

$$\dot{\nu}_{\rm s} = -\nu_{\rm s,0} K A^2 \sin(u + \omega_1),\tag{B.6}$$

where $K = k \sin i/(h^2 c)$. While the constant K is not immediately known, the higher-order derivatives will also be proportional to K, and so we can

use Equation B.6 to write the higher-order derivatives in terms of $\dot{\nu}_{\rm s}$, which is usually measured for radio pulsars:

$$\nu_{\rm s}^{(2)} = \frac{\dot{\nu}_{\rm s}}{A^2 \sin(u+\omega_1)} B\dot{u} \tag{B.7}$$

$$\nu_{\rm s}^{(3)} = \frac{\dot{\nu}_{\rm s}}{A^2 \sin(u+\omega_1)} C \dot{u}^2 \tag{B.8}$$

$$\nu_{\rm s}^{(4)} = \frac{\dot{\nu}_{\rm s}}{A^2 \sin(u+\omega_1)} D\dot{u}^3 \tag{B.9}$$

$$\nu_{\rm s}^{(5)} = \frac{\dot{\nu}_{\rm s}}{A^2 \sin(u+\omega_1)} E \dot{u}^4 \tag{B.10}$$

$$\nu_{\rm s}^{(6)} = \frac{\dot{\nu}_{\rm s}}{A^2 \sin(u+\omega_1)} F \dot{u}^5 \tag{B.11}$$

and where the coefficients have the following form:

$$B = 2AA'\sin(u+\omega) + A^2\cos(u+\omega_1)$$
(B.12)

$$C = B' + 2\frac{BA'}{A} \tag{B.13}$$

$$D = C' + 4\frac{CA'}{A} \tag{B.14}$$

$$E = D' + 6\frac{DA'}{A} \tag{B.15}$$

$$F = F' + 8\frac{EA'}{A} \tag{B.16}$$

where the apostrophe denotes a derivative with respect to u. Equations B.7-B.11, along with the definition $A = 1 + e \cos u$, therefore show that four quantities are directly measurable using the JR97 technique: $\{e, \omega_1, u, \dot{u}\}$. At a minimum, five measured time derivatives are needed in order to uniquely solve for the four orbital parameters, since four of the orbit-induced derivatives (Equations B.7-B.10) are written in terms of $\dot{\nu}_s$.

B.2 Determination of Parameters

We used a multi-dimensional Newton-Raphson method (Press et al., 1986) in order to solve a system of nonlinear equations $\mathbf{f}^{\mathbf{k}}(\mathbf{x}) = 0$, where \mathbf{x} is a "state" vector with the unknown orbital parameters as components, and $\mathbf{f}^{\mathbf{k}}$ is a vector of functions with components $(f^k)^i = \nu_s^{(i+k)} - (\nu_s^{(i+k)})_{\text{obs}}$. The *i* index denotes the *i*-th component of $\mathbf{f}^{\mathbf{k}}$, which has the same number of elements as \mathbf{x} , while the *k* index denotes the set of spin-frequency derivatives to be used for computation of the orbital elements. For example, if $\mathbf{x} = \{\omega_o, u, \dot{u}\}$, then *i* runs from 1, 2, and 3, and the components of $\mathbf{f}^{\mathbf{k}=\mathbf{1}}$ are the three time-derivatives given by Equations B.7-B.9, minus their observed values. If instead $\mathbf{x} = \{e_o, \omega_o, u, \dot{u}\}$, the "full" state vector of orbital parameters that are directly measurable, then the components of $\mathbf{f}^{\mathbf{k}=\mathbf{1}}$ are the four timederivatives given by Equations B.7-B.10 minus their observed values. We use this notation in order to simplify the discussion below when using different sets of time-derivatives to derive the components of \mathbf{x} , which is presented in the following section of this Appendix.

Using the iterative Newton-Raphson method for finding roots of a system of equations, the best approximations of \mathbf{x} can be determined by computing

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \mathbf{J}^{-1} \mathbf{f}^{\mathbf{k}}(\mathbf{x}_n), \qquad (B.17)$$

where **J** is the Jacobian matrix of partial derivatives, with components $J^{ij} = \partial (f^k)^i / \partial x^j$. With an initial guess of **x**, Equation B.17 is computed repeatedly using the iteratively-updated state vector until a chosen criterion for the best approximation of **x** is satisfied.

For a one-dimensional Newton-Raphson problem, one typically chooses the "best fit" criterion that the function under consideration be approximately equally to a small value close to zero, say $f < 10^{-12}$. For the multidimensional case we consider here, a complication occurs from the fact that the components of $\mathbf{f}^{\mathbf{k}}$ each have different physical units and orders of magnitude. Since all components of $\mathbf{f}^{\mathbf{k}}$ are theoretically equal to zero, we scale each component of $\mathbf{f}^{\mathbf{k}}$ by factors that yield values with orders of magnitude $\sim 10^{0}$. After this arbitrary scaling, we assumed the best-fit criterion to be that the length of $\mathbf{f}^{\mathbf{k}}$ is less than 10^{-12} .

If only four time-derivatives are significantly measured, then $\mathbf{x} = \{\omega_0, u, \dot{u}\}$ and a value of e_0 must be chosen and held fixed in order to use Equation B.17 to obtain a solution of \mathbf{x} . In this way, one can obtain a "family" of solutions using the JR97 method, where a set of \mathbf{x} is determined for a range of values of e_0 . JR97 used initial estimates of their significantly-measured frequency derivatives, from $\dot{\nu}_s$ up to $\nu_s^{(4)}$, to find a family of solutions for different, fixed values of $\omega_0 = \omega_1 - \pi$, while Thorsett et al. (1999) used their updated values of the spin-frequency derivatives to find a set of solutions in terms of e_0 . For the purposes of comparison, we used the definitions adopted by Thorsett et al. (1999), where $\mathbf{x} = \{\omega_0, u, \dot{u}\}$ and k = 1, so that Equations B.7-B.9 made up the components of $\mathbf{f^{k=1}}$. We then used Equation B.17 to find the best approximations of \mathbf{x} for different, fixed values of e_0 .

If at least five time-derivatives are significant measured, then the full state vector $\mathbf{x} = \{e_0, \omega_0, u, \dot{u}\}$ can be uniquely approximated to find a single solution for $\mathbf{f^{k=1}}(\mathbf{x}) = 0$. Ford et al. (2000a) provided the first unique solution for \mathbf{x} using the time-derivatives reported by Thorsett et al. (1999), and determined the outer orbit to be highly eccentric, with $e_0 \approx 0.45$ and $(P_{\rm b})_0 \approx 308$ years. However, the value of $\nu_{\rm s}^{(5)}$ was not measured with statistical significance.

B.3 Complications from Spin-down

One of the major complications of the JR97 method is that the observed $\dot{\nu}_{\rm s}$ is not purely due to Doppler shifts from binary motion. As discussed in Section 1.1, radio pulsars generally exhibit an intrinsic spin-down in the form of a first derivative in $\nu_{\rm s}$ due to magnetic dipole radiation. Moreover, the

significant kinematic effects from proper motion, differential Galactic rotation and gravitational acceleration in the Galactic potential discussed in Section 1.5 will produce a change in Doppler shifts of $P_{\rm s}$ in the same manner observed for the orbital periods of PSRs B1534+12 (Chapter 5), J1614-2230 (Section 3.4.4) and J1909-3744 (Section 3.4.10). The observed first-derivative in spin frequency is therefore a sum of three components:

$$(\dot{\nu}_{\rm s})_{\rm obs} = (\dot{\nu}_{\rm s})_{\rm int} + (\dot{\nu}_{\rm s})_{\rm o} + (\dot{\nu}_{\rm s})_D$$
 (B.18)

where the "D" subscript denotes the component secular accelerations that produce changes in the Doppler shift that is discussed in Section 1.5. In general, the intrinsic and secular-acceleration terms in the first-derivative are not separately measurable unless $(\dot{P}_{\rm b})_D$ is significant. While $(\dot{\nu}_{\rm s})_{\rm int} < 0$ from physical arguments, the sign of $(\dot{\nu}_{\rm s})_{\rm o}$ can be positive or negative and will vary over time.

JR97 and Thorsett et al. (1999) pointed out that assuming $(\dot{\nu}_s)_o = (\dot{\nu}_s)_{obs}$ did not produce a significantly large difference in their results when compared to those obtained under the assumption that $(\dot{\nu}_s)_o = 0.01(\dot{\nu}_s)_{obs}$. This bias in (and the *ad hoc* adjustment of) the first derivative nonetheless complicates the unique determination of the outer-orbital elements.

A solution to this problem can be obtained by noting that, in the absence of timing noise and encounters with nearby stars, the second and higherorder time-derivatives in $\nu_{\rm s}$ should be entirely due to binary motion. We can therefore use Equation B.7 to put Equations B.8-B.11 in terms of $\nu_{\rm s}^{(2)}$,

$$\nu_{\rm s}^{(3)} = \frac{\nu_{\rm s}^{(2)}}{B} C \dot{u} \tag{B.19}$$

$$\nu_{\rm s}^{(4)} = \frac{\nu_{\rm s}^{(2)}}{B} D \dot{u}^2 \tag{B.20}$$

$$\nu_{\rm s}^{(5)} = \frac{\nu_{\rm s}^{(2)}}{B} E \dot{u}^3 \tag{B.21}$$

$$\nu_{\rm s}^{(6)} = \frac{\nu_{\rm s}^{(2)}}{B} F \dot{u}^4 \tag{B.22}$$

and entirely avoid the use of $\dot{\nu}_{\rm s}$ in the computations of Equation B.17 to approximate **x**. The use of these "unbiased" derivatives requires a sufficiently long data span in order to measure them and derive the elements. However, in the case of our ongoing analysis of PSR B1620-26 (see Chapter 4), we measure a large number of spin-frequency derivatives with statistical significance. We refer to the sets of derivatives listed in Equation B.19 as $\mathbf{f^{k=2}}$ in subsequent discussion, and use this approach in Section 4.3.4 in order to compare the "biased" and "unbiased" results obtained using the JR97 method.

In Section 4.4, we discuss another potential component of $\dot{\nu}_{\rm s}$ and $\ddot{\nu}_{\rm s}$ due to accelerations and jerks, respectively, from stars in the Messier 4 globular cluster in which PSR B1620–26 resides. Such components are unique to globular-cluster pulsars and can be used to constrain information on local mass densities in the cluster (Phinney, 1992; Blandford et al., 1987). However, such analyses are not currently possible with PSR B1620–26 due to the frequency-derivative model of the outer orbit, and we instead consider their impacts on measurements we present and discuss in Chapter 4.

B.4 Derived Quantities of the Orbit

The directly-measurable quantities of the JR97 method, which collectively form the components of \mathbf{x} , can be used to derive other orbital parameters of interest. JR97 derived the relations for $a_2 = m_1 a_1/m_2$ and $m_2 \sin i$, using Equations B.2 and B.3, in terms of the first time-derivative in spin frequency:

$$m_2 \sin i \approx -\frac{\dot{\nu}_{\rm s}c}{\nu_{\rm s}\sin(u+\omega)} \left(\frac{m_1^2 A^2}{G\dot{u}^4}\right)^{1/3} \tag{B.23}$$

$$a_2 = \frac{m_1 \nu_{\rm s} c A^2}{\nu_{\rm s} (m_2 \sin i) \sin(u+\omega) \dot{u}^2 (1-e^2)}$$
(B.24)

where the approximation in B.23 uses the assumption that $m_2 \ll m_1$. In the case of PSR B1620-26, $m_1 = m_p + (m_c)_i$ and $m_2 = (m_c)_o$. We assume that $m_1 \approx 1.65 \text{ M}_{\odot}$, and Thorsett et al. determined $m_2 \sim 10^{-3} \text{ M}_{\odot}$. This corresponds to $m_2/m_1 \ll 1$, which satisfies the approximation made in Equation B.23. The outer-orbital period can therefore be computed using Kepler's third law and carrying through the lowest-order approximation of low companion mass:

$$P_{\rm b} \approx 2\pi \sqrt{\frac{a_2^3}{Gm_1}} \tag{B.25}$$