Improving Critical Infrastructure Resilience with Application to Power Distribution Networks

by

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Our modern societies are dependent on the functioning of infrastructure systems that support economic prosperity and quality of life. These infrastructure systems face an increasingly set of threats, natural or man-made disasters, that can cause significant physical, economic, and social disruptions. Recent extreme events have shown that total protection can not be accomplished. Therefore, Critical Infrastructure Protection strategies should focus not only on the prevention of these events but also on the response and recovery following them. This shift is realized by the concept of infrastructure resilience. In this thesis, we address the problem of assessing and improving infrastructure resilience. The contributions of this thesis focus on modelling, simulation, and optimization of infrastructure systems with respect to their resilience to extreme events. We first develop a resilience assessment framework for interdependent infrastructure systems. The developed framework provides a quantitative means to assess infrastructure resilience by introducing a generalized resilience index. To account for the inherent complexity due to infrastructure interdependencies, we use the Infrastructure Interdependency Simulator (i2Sim) framework for modelling and simulating the studied infrastructure. The resilience improvement problem is formulated using the proposed resilience index as a resources allocation optimization problem. The problem aims at finding the best allocation of available resources such as power and water to mitigate the consequences of a disaster. Two solutions algorithm are proposed to solve the problem: the first one uses a
simulation-optimization approach based on the Ordinal Optimization theory, and the second one uses a Linear Programming formulation. Results of both algorithms show that infrastructure resilience can be greatly improved by efficient allocations of available resources. In addition, a prioritization methodology is developed to assess decision makers to direct resilience investment to the most important components in the infrastructure. Finally, an optimal power distribution network reconfiguration algorithm is developed to complement the two resources allocation algorithms by solving the technical feasibility problem of the power distribution network. A heuristic computationally inexpensive optimization algorithm is developed based on Graph theory for solving this problem. The proposed algorithms are tested using different test cases and promising results are achieved.
Preface

The contributions pointed in this dissertation have led to a number of already published, or currently under preparation for publications in journals and conferences. My research work and all my publications have been done by me under the supervision of Prof. José R. Martí. The co-authors of the publications have provided us with constructive feedback.

The outcomes of each chapter in terms of publications are as follows. A major part of chapter 3 was first presented in the 10th International Conference On Critical Information Infrastructures Security 2015 and is scheduled to appear as a book chapter in the proceedings book:


Work presented in chapter 4 section 4.3 was published in the 2014 IEEE Canada International Humanitarian Technology Conference (IHTC):


Work presented in chapter 4 section 4.5 was published in the EIC Climate Change
Technology Conference 2015:

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<td>DAS</td>
<td>Distribution Automation System</td>
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<td>DR-NEP</td>
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Finally, I would like to thank my family, my father, my mother, my wife, and my children for their unconditional love and support. Special thanks go to my beloved wife who has played a crucial role in supporting me through the stressful times of my PhD studies.
Dedication

This dissertation is dedicated to my beloved family
Chapter 1

Introduction

1.1 Motivation

Our modern societies are dependent on the functioning of infrastructure systems that support economic prosperity and quality of life. These infrastructure systems face an increasingly set of threats, natural or man-made disasters, that can cause significant physical, economic, and social disruptions. The impact of these disasters could be limited to local communities, such as earthquakes, or could be global, such as the 2010 Iceland volcanic ash cloud which affected global air travel. Infrastructure systems operators have been continuously working on improving systems safety and security through traditional risk and reliability frameworks and guidelines. The traditional risk and reliability methods assume different failure scenarios and try to understand the reasons for their occurrence. Then, measures or practices are recommended to minimize the impact of these failures.

Over the last decades, disasters have caused significant loss of lives and essential services. These losses caused major setback for economic and social development in the affected countries. According to the annual disaster statistical review published by the Centre for Research on The Epidemiology of Disasters (CRED) in Belgium,
330 natural-related disasters were reported around the world in 2013 only. In 2013, more than 21,000 lives were lost and the economic losses were estimated to be US$ 118.6 billion [1]. Taking a wider look, it appears that the trend of economic losses is increasing over the last decades as shown in Fig. 1.1.

In recent years, there is an increasing effort being devoted to protecting Critical Infrastructures (CI) systems from natural and man-made disasters. Every country defines its CI systems according to its national priorities. In Canada, Public Safety Canada refers to CI as the “processes, systems, facilities, technologies, networks, assets and services essential to the health, safety, security or economic well-being of Canadians and the effective functioning of government. Critical infrastructure can
be stand-alone or interconnected and interdependent within and across provinces, territories and national borders. Disruptions of critical infrastructure could result in catastrophic loss of life, adverse economic effects, and significant harm to public confidence."

Infrastructure systems are large and complex which makes it impractical to consider all possible failure scenarios. It is then acknowledged that if total security cannot be achieved more effort should be devoted to planning effective response and recovery. As a result, there has been a paradigm shift in recent years from risk and reliability concepts toward resilience concepts [6]. This shift was realized by several governmental initiatives such as The Critical Infrastructure Resilience Study conducted by the US National Infrastructure Advisory Council (NIAC) [7] and The Critical Infrastructure Preparedness and Resilience Research Network (CIPRNet) program established by the European Union (EU) [8].

Infrastructure systems are not unique static entities. They share mutual interdependencies which are complex to analyze and manage [9]. They interact and adapt to their changing and dynamic operations. As systems grow in size and complexity, more interdependencies are introduced. These systems are typically integrated and controlled in distributed and loosely manner. Haimes et al [10] describe them as ‘emergent systems’ and show that while the cost of protecting emergent systems is high, more attention should be paid to improving their resilience. Also, Marti [3] highlights the role of emergent behavior of interdependent infrastructure systems in disaster response.

The ultimate goal of this research project is to improve the resilience of infrastructure systems by mitigating the impact of extreme events. Although domain-specific methodologies and techniques have been developed for disasters mitigation and response for different infrastructure systems, there is a need for a “global” view in dealing with complex and adaptive interdependent systems. In an attempt
to achieve a “global resiliency”, this research project considers infrastructure interdependencies in the process of improving the overall resilience of infrastructure systems.

Of particular interest in this project are the power distribution networks which are directly connected to other CI systems. Since electrical energy is a vital resource for the operation of CI systems such as hospitals and water networks, disruption of power supply often causes a high degree of disturbance to their operations. Recent incidents have raised major concerns on the resilience of power distribution networks against extreme events as it has been reported that they suffer more damages than generation and transmission networks [11]. Emphasis has been placed on improving the resilience of power distribution networks by governmental agencies and utilities [7][2]. One of the research and development needs in the area of power distribution resilience is the “coupling of electric restoration models to other infrastructure models” as shown in Figure 1.2. This research project attempts to contribute to this research area by combining a power distribution reconfiguration model with an interdependent infrastructure restoration model.

<table>
<thead>
<tr>
<th>Area</th>
<th>R&amp;D Needs</th>
<th>R&amp;D Projects</th>
</tr>
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| Design, preparedness, and planning | Design of segmented and agile distributed system | • Emergency controls, segmentation, and communications  
• Microgrid to feeder integration  
• Multiscale modeling: distribution and transmission  
• Real-time database with speed and accuracy |
| Big data and analytics | Stochastic and uncertainty | • Robust control to uncertain data  
• Predictive models |
| Operational response and system recovery | Proactive assessment of damage (automated calls to customers, smart meters) | • Damage assessment  
• Unmanned aerial vehicles to support real-time Google Maps  
• New devices to support degradation identification  
• Hardening of communications |
| Situational awareness | Decision support to determine restoration priorities | • Development of architecture  
• State estimation with new data and new devices  
• Cyberphysical degradation and the necessary understanding to respond to it when it occurs  
• Three-phase state estimation |

Figure 1.2: DOE Resilient Electrical Distribution Grid R&D Needs [2].
1.2 Problem Statement and Research Objectives

A disruption to an infrastructure system causes potential cascading interruptions to other infrastructure systems. Therefore, it is essential to make infrastructure systems more resilient to such disruptions. The problem of improving infrastructure resilience can be summarized in the following questions:

1. How can we model interdependent infrastructure systems for resilience analysis?
2. How can we measure the resilience of interdependent infrastructure systems?
3. What can be done to improve their resilience?

In order to answer the above questions, the following objectives are set for this research project:

1. To develop a resilience assessment framework for interdependent infrastructure systems. This framework should be able to capture the required characteristics of a resilient infrastructure.

2. To develop a resilience measure that can be used to assess the resilience level of a given infrastructure. This measure should be flexible enough to capture the defined characteristics and connected to the operational details of the infrastructure to show the improvement or degradation of resilience after each operational change.

3. To formulate the resilience improvement problem as an optimization problem and provide solution algorithms for this problem.

4. To model the power distribution reconfiguration problem in the context of infrastructure interdependency.
1.3 Thesis Contributions and Structure

Resilience-informed decisions will make a major shift in the field of Critical Infrastructure Protection (CIP). In the wake of recent extreme events, infrastructure operators have realized the need to extend the traditional reliability and risk strategies to resilience-based strategies. The contributions in this thesis provide effective tools for developing these strategies. The thesis presents original contributions focusing on modelling, simulation, and optimization of infrastructure systems with respect to their resilience to extreme events. The main contributions of this thesis are summarized in the following list:

1. Development of a resilience assessment framework for interdependent infrastructure systems.

2. Introduction of a comprehensive resilience index that is applicable across different domains and applications.

3. Formulation of the resilience improvement problem as a resources allocation optimization problem.


5. Development of several solution algorithms for solving the resources allocation problem.


The structure of this thesis is as follows: Chapter 1 introduces the main focus of the thesis and discusses the motivation for the research project and its objectives. Chapter 2 discusses the modelling and simulation framework for the interdependent CI systems. Based on this modelling and simulation framework, a resilience
assessment framework is proposed in chapter 3. The proposed resilience index is also discussed in chapter 3. Using this resilience index, two resources allocation algorithms are formulated and solved in chapter 4. Also, a prioritization methodology for ranking CI systems components is proposed in this chapter. In chapter 5, the resilience of power distribution networks is discussed and an optimal network reconfiguration problem is formulated and solved for restoring power supply to CI systems. A summary of contributions and future research directions are provided in chapter 6.
Chapter 2

Critical Infrastructure Modelling and Simulation

2.1 Introduction

Critical Infrastructure (CI) systems are physical or virtual systems that are made of a number of components that work collectively to provide key services to society. Examples of these systems include: electrical power system, water distribution system, telecommunication system, and healthcare system. Every nation has defined a set of infrastructure systems that are considered to be “critical”. Table 2.1 lists the defined critical infrastructure systems in Canada, the United States, and the European Union. One of the main concerns in studying critical infrastructure systems is their growing complexity. This complexity causes infrastructure systems to exhibit unpredictable behavior during extreme events. Modelling and simulation approaches have proven to be very effective in predicting this behavior.

2.1.1 Applications of Critical Infrastructure Modelling

There are many applications that require modelling and simulation of critical infrastructure systems. Some of these applications are predictive analysis, such as...
Table 2.1: List of Critical Infrastructure Systems in Different Nations.

<table>
<thead>
<tr>
<th>Country</th>
<th>Critical Infrastructure Systems</th>
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<tr>
<td>Canada</td>
<td>Health&lt;br&gt;Food&lt;br&gt;Finance&lt;br&gt;Water&lt;br&gt;Information and Communication Technology&lt;br&gt;Safety&lt;br&gt;Energy and Utilities&lt;br&gt;Manufacturing&lt;br&gt;Government&lt;br&gt;Transportation</td>
</tr>
<tr>
<td>USA</td>
<td>Emergency Services&lt;br&gt;Energy&lt;br&gt;Food and Agriculture&lt;br&gt;Healthcare and Public Health&lt;br&gt;Water and Wastewater&lt;br&gt;Transportation&lt;br&gt;Nuclear Reactors, Materials, and Waste&lt;br&gt;Information Technology&lt;br&gt;Government Facilities&lt;br&gt;Financial Services&lt;br&gt;Defence Industrial Base&lt;br&gt;Dams&lt;br&gt;Commercial Facilities&lt;br&gt;Chemical&lt;br&gt;Communications&lt;br&gt;Critical manufacturing</td>
</tr>
<tr>
<td>EU</td>
<td>Energy&lt;br&gt;Nuclear Industry&lt;br&gt;Information and Communication Technology&lt;br&gt;Water&lt;br&gt;Food&lt;br&gt;Health&lt;br&gt;Financial&lt;br&gt;Transport&lt;br&gt;Chemical Industry&lt;br&gt;Space&lt;br&gt;Research Facilities</td>
</tr>
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“What-if scenarios”; others are exploratory analysis, such as system optimization studies. A list of some common applications follows:

**Risk and Reliability Analysis** The objective of this type of applications is to identify undesirable events, estimate their probability of occurrence, and then study their consequences. The last step is where CI modelling and simulation is needed.

**Resilience Analysis** The objective of this type of applications is to study the system behavior after the disruptive event and the effectiveness of its reaction.

**Optimization** Optimization can be used in a very wide range of applications, such as cost minimization, output maximization, allocation of limited resources, and placement of certain components. Typically, a CI model is required to evaluate the objective function of the optimization problem.

**Investment Planning** The objective of this type of applications is to find the best investment strategy for the infrastructure. It can be a long-term planning; e.g. where and when to build power generation plants, or a short-term planning; e.g. which type of generation to use to meet a current demand. In many cases an optimization technique is used in this application. The CI model is used to evaluate the planning strategy.

**Operational Support** In this type of applications, CI models are used to support decision-making process during operations or to provide support to control systems. For example, a power system model is used to recommend the optimum topology for the power grid given the current operational status. Also, an interdependent infrastructure model is used to provide decision support during disasters in emergency control centers.

The above list is not distinct but rather provides examples of where CI models can be used. These applications may overlap in some cases. For example, an infras-
structure model can be used for optimization and reliability analysis to find the best structure or topology that provides the highest reliability level.

2.1.2 Challenges of Critical Infrastructure Modelling

Modelling interdependent critical infrastructure poses several challenges. One of the main challenges is the increased complexity due to interdependencies. A first step in the modelling process is to identify types of interdependencies. Rinaldi et al. [9] identified four main types of interdependencies: physical, cyber, geographical, and logical. Difficulty arises when modelling these interdependencies within a CI model. Another challenge is the huge number of components to be included in the model. Consider, for example, building a model comprising of a power distribution system and a water distribution system. Each one of the system comprises of many individual components; e.g. transformers and cables in the power system and pumps and pipes in the water system. This complexity typically introduces a tradeoff problem in the modelling: the more components to be modeled (low-level details), the more accurate is the model, but the more expensive its computation becomes.

Other modelling challenges include incompatibility of simulation requirements, such as time-step sizes and computational algorithms, unavailability of appropriate data due security and confidentiality constraints, and scarcity of simulation case studies. In addition to these challenges, modelling different infrastructure systems requires certain knowledge and expertise of the modeled systems which can be a challenge for the modeler.

2.2 Literature Review on CI Modelling and Simulation

Infrastructure modelling and simulation has gained an increasing attention within the scientific community over the past decade. Many efforts in this area are related to CIP programs. Modelling and simulating the critical infrastructure systems are key elements in many CIP programs. Since infrastructure systems are highly in-
terconnected and interdependent, the concept of infrastructure interdependencies is present in most of the modelling and simulation works. There are different ways in which interdependent infrastructure modelling and simulation can be classified and compared. Table 2.2 presents the different classifications and comparison criteria used by some scholars.

**Table 2.2:** Different Classifications and Comparisons Criteria in Infrastructure Modelling and Simulation Literature.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Year</th>
<th>Classification/Comparison Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pederson et al. [14]</td>
<td>2006</td>
<td>Infrastructure Modelling and Simulation Technique Integrated vs. Coupled Models</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hardware/Software Requirements Intended User Maturity Level</td>
</tr>
<tr>
<td>Eusgeld et al. [15]</td>
<td>2008</td>
<td>Modelling Focus Methodical Design Strategies Types of Interdependencies Types of Events</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Course of Triggered Events Data Needs Monitoring Area Modelling and Simulation Paradigms Maturity</td>
</tr>
<tr>
<td>Satumtira et al. [16]</td>
<td>2010</td>
<td>Mathematical Method Utilized Modelling Objective Scale of Analysis Quality and Quantity of Input Data</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Targeted Discipline End User Type</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Network-based Approaches Others</td>
</tr>
</tbody>
</table>

Although most of the scholarly work on CI modelling and simulation is recent, there are some early conceptual work that motivates the recent development in the
field. One of the earliest works is Leontief’s input-output model of the economy [18] which describes the interconnectedness among different sectors of the economy for the study of economic equilibrium. Agent-based modelling is also one of the earliest approaches used by researchers for studying the behavior of infrastructure systems [19]. Recently, several modelling and simulation approaches based on network analysis, complex networks and graph theory, and system dynamics have been proposed.

An Input-output Inoperability Model (IIM) was proposed by Haims and Jian [20] to study risk in CI. The model extends the Leontief’s input-output model using the concept of inoperability, which is defined as the inability of an infrastructure system to perform its intended function. In this model, $x = Ax + c$ where $x$ represents the risk on operability of infrastructure and $A$ is an interconnectedness matrix. Based on this model, one can measure the effects on infrastructure inoperability given some perturbations. Crowther and Haimes. [21] extended the IIM model to the demand-reduction IIM model. In this extended model, the inoperability is defined as the normalized loss with respect to the nominal production output of the infrastructure. Another extension is also proposed by DAgostino et al. [22] in which a Markov Chain evolution law is used to replace the Leontief equilibrium conditions. The IIM model and its extensions are useful for analyzing failures propagation among interdependent infrastructure systems. However, they provide a macroscopic view and cannot analyze the detailed components level.

Another common approach for modelling and simulating CI is the Agent-based approach. Several tools were developed using this approach, including: Aspen [14] by Sandia national lab, SMART [14] by Argonne national lab, and CIMS [14] by Idaho national lab. In agent based modelling, every infrastructure system (or component) is represented by an agent. The agent has an objective and a behavior. The agents are built in a distributed manner and communicate with each other. Agent
based modelling can be useful for conducting scenario based analysis. One of its advantages is the modularity in building the models. However, accuracy and details of agent based models are dependent on the assumptions made during constructing the agent’s behavior which may affect the simulation results.

Several CI models were built using network based approaches. These models exploit the network structure (or topology) to capture infrastructure interdependencies. Zhang and Peeta [23] used a multilayer infrastructure network to capture supply-demand mechanisms within interdependent infrastructure systems. Graph-theoretic structural networks are very common in modelling CI, e.g. [24], [25], [26], and [27]. Specific network models have also been used to model CI such as Petri-Nets [28], and network flow models [29]. Network based approaches are capable of capturing the network (topological) characteristics of infrastructure systems which makes them useful for studying failure and disruption scenarios or identifying critical parts of the system under study. However, these approaches are broad and require more modelling effort to be implemented for any specific domain. A discussion on the usefulness and limitations of complex network approaches for power system studies is presented in [30]. System Dynamics (SD) models have been used by several US national labs to build a tool for modelling interdependent CI systems. The tool was named Critical Infrastructure Protection/Decision Support System (CIP/DSS) and it was coupled with different optimization models [17].

Existing CI modelling and simulation approaches are broad and serve different objectives. They overlap in many aspects. Selecting a proper modelling approach is highly dependent on the objective of the study. In this thesis, the Infrastructure Interdependency Simulator (i2Sim) modelling approach is used for modelling infrastructure systems. The i2Sim modelling approach is described in the next section.
2.3 Modelling Framework

This thesis uses the Infrastructure Interdependency Simulator (i2Sim) for modelling critical infrastructure systems. The i2Sim is a functional modelling approach that allows for capturing the interdependent interactions between different infrastructure systems. The i2Sim modelling approach is described in more details in the following subsections.

2.3.1 i2Sim Modelling and Simulation Framework

Today’s infrastructure systems comprise a huge number of components that are essential for their operations. A single infrastructure system can comprise thousands of components between the source of its service or material and the end user. Consider, for example, the power delivery system. There are hundreds if not thousands of components between the power generation facility and the end user of electricity e.g. transformers, transmission lines, breakers, substations etc. Modelling all of these components together is impractical due to computational limitations. The problem becomes more complex when considering multiple infrastructure systems together. To model and simulate these large systems at the level of their interactions requires levels of abstractions.

To cope the above modelling challenge, we use the modelling framework proposed by Martí [3], namely the Infrastructure Interdependencies Simulator (i2Sim) framework. The i2Sim Simulator is an event-driven time-domain simulator for modelling infrastructure interdependencies. It uses a cell-channel approach to build a simulation environment that provides a multi-system representation of multiple Critical Infrastructures CI at multiple hierarchical levels (local, municipal, provincial, etc.). The simulator assesses in real time the effects of resources allocation decisions in a given scenario.

In the i2Sim modelling framework, every infrastructure system is modeled inde-
pendently with its own detailed model. The detailed model is then represented by an equivalent. This equivalent represents an abstraction of the system and provides the required solution at the interaction level with other infrastructure systems. The equivalents of all the systems form the i2Sim model. This model is then solved to find the interaction parameters between the different systems; e.g. flow of resources. The solution of the equivalents model (i2Sim model) can be then fed to the underlying detailed models to update their solutions. The i2Sim framework integrates the dissimilar equivalents by providing a common ontological framework built upon a cell-channel approach. This ontology is described in the following section.

2.3.2 i2Sim Ontology

The i2Sim ontology defines a common framework to combine multiple dissimilar infrastructure systems. This framework captures the interactions among the systems and has sufficient information to simulate their behavior. The i2Sim ontology defines three main types. Each type has several components as depicted in Fig. 2.1. The i2Sim components are defined as follow:

**Tokens** These are the resources needed or produced in the infrastructure system, e.g. electricity, water, and medicines.
Cells These are the production units in i2Sim models. They take input and produce output tokens. A hospital cell is an example. It takes input tokens, such as electricity, water, and medicines, and produces output token, patents treated.

Channels These are the connecting elements in i2Sim models. They receive output tokens from different cells and transport them as inputs to other cells.

Distributors These are the allocation units in i2Sim models. They map the detailed topology of the infrastructure into the i2Sim model. Also, distributors are the decision elements if resources allocation is desirable.

Aggregators These are the additive elements in i2Sim models. They combine two outputs of the same token into one channel.

External Tokens These are the tokens (resources) that are brought in from outside the i2Sim model.

Sources These are the producers of the external tokens. Sources represent infrastructure systems that are not included in the i2Sim model.

Sinks Sinks are the components that send internal tokens to outside the i2Sim model.

Reservoirs These are the storage elements in the i2Sim model.

Physical Mode Modifiers Physical mode modifiers represent the physical damage which results in decrease in the output of the infrastructure system; e.g. failure of one transformer in an electrical substation.

Human Mode Modifiers Human mode modifiers represent the human factors that affect the output of the infrastructure system; e.g. tiredness of rescue teams.
**Information Modifiers** Information modifiers represent the knowledge effect on the operation of the infrastructure system; e.g. inability of controlling a circuit breaker due loss of monitoring signals.

The i2Sim model is interfaced with external infrastructures that are not included in the model, using source and sink elements. Any cell in the model can have many input tokens with different types but produces only one output token. A distributor has one input and multiple outputs of the same token type. An aggregator has multiple inputs and one output of the same token type. Critical infrastructure systems, such as power substations, water stations, and hospitals, are modeled using the i2Sim components: cells, channels, tokens, distributors, aggregators, sources, sinks, and modifiers. Figure 2.2 shows an example of an i2Sim model. In this example, a power substation supplying two hospitals is modeled as an i2Sim cell whose output is connected to a distributor through a channel and the distributor has two (output) channels connected to the two hospitals (cells). The source in the model represents the electrical power system supplying the power substation; i.e. high voltage transmission network. The sinks represent the output, treated patients in this example.

### 2.3.3 i2Sim Models

An infrastructure i2Sim model consists of a combination of i2Sim components: tokens, sources, sinks, cells, channels, distributors, aggregators, reservoirs, and modifiers. The cell and the channel are the two key components in every i2Sim model. The detailed models for cells and channels are described below.

**i2Sim Cell Model**

An i2Sim cell represents a production unit. It takes input resources (tokens) and produces an output resource (token). A conceptual i2Sim cell model is shown in
Fig. 2.2: An example of an i2Sim model.

Fig. 2.3. The output of the cell is a function of the input tokens and the modifiers:

\[ y(t) = f(x_1(t), ..., x_i(t), m_1(t), ..., m_j(t)) \]  

(2.1)

where \( y(t) \) is the output of the cell at time \( t \), \( x_1(t) \) to \( x_i(t) \) are the input tokens to the cell at time \( t \), \( m_1(t) \) to \( m_j(t) \) are the modifiers values at time \( t \), \( i \) is the number of tokens in the model, \( j \) is the number of modifiers in the model.

The input-output relationships can be represented by a function (or a table) describing the operation of the cell. The output level of the cell is determined by the availability of input resources, level of physical damage to the cell or channel, and the effect of possible modifiers. The availability of the input resources is determined by the solution of the i2Sim solver at every time step. The level of physical damage is modeled in i2Sim using the Physical Mode (PM) parameter which can be an
external or internal input to the model. The modifiers are inputs to the cell that are not directly used to produce the output but they can impact its level. For example, a control signal from SCADA can be a modifier to a power substation cell.

In the i2Sim framework, the possible output of the cell is discretized into five levels: 100%, 75%, 50%, 25%, and 0%. A colour code, shown in Fig. 2.3, is used to show the output level of the production cell during simulation. Each colour corresponds to an output level measured by its operability state in percentages of its rated output. The concept of output discretization into a finite number of levels facilitates the description of the cell function in cases such as a hospital where the cell production is known from experience.

**i2Sim Channel Model**

The i2Sim channel model describes a transportation function with two parameters: a time delay and losses:

\[ y(t) = \alpha x(t - \tau) \]  \hspace{1cm} (2.2)
where $y(t)$ is the output of the channel at time $t$, $x(t - \tau)$ is the input to the channel, $\tau$ is the time delay, and $\alpha$ is the degradation factor. Note that $y$ and $x$ represents the same token type, e.g. electricity or water. The concept of output discretization described in the cell’s model is also applied in the channel’s model.

### 2.3.4 Integrating i2Sim with Domain Simulators

As described above, the i2Sim framework allows the interaction between the i2Sim models and the detailed systems models. This interaction enables i2Sim solutions to respect the technical constraints imposed by the physical behaviour of the modelled infrastructure systems, e.g. voltage limits in power system. There are different ways in which the interaction is done. For example, a power system model can be used to represent the function inside an i2Sim cell model, as shown in Figure 2.4. In this example, the output of the power station cell and its distributor is determined by the power flow results calculated by the power system simulator. Another example of the interactions is using i2Sim sources. A source in an i2Sim model can represent an external water system. The value of the source can be calculated by running a water system model using a water system simulator. The mapping in this integration is a challenging task due to the complexity of the modelled infrastructure systems. It is a modelling criteria and can be case specific. For example, an i2Sim cell can be combined with a distributor to depict a specific topology, as shown in Figure 2.4.

The i2Sim framework was used in the Disaster Response Network Enabled Platform (DR-NEP) project [31] to integrate different simulators. DR-NEP is a web services based software platform that integrates different simulators by communicating their results to each other. It uses a common Enterprise Service Bus (ESB) and a database for establishing this communication. This design creates a distributed computing architecture for the purpose of assisting decision making. Every simulator is connected to DR-NEP using a software adapter that listens to the ESB for instructions.
to run simulations, gather inputs from simulators and the database, and push results from the simulators back into the database. Once the simulators and the adapters are configured, a controller in the ESB pushes input into the simulators at regular intervals, which can be predetermined before the simulation. In addition, DR-NEP offers web pages and mapping services for researchers and disaster responders to coordinate with each other and visualize resources flow and infrastructures operability. The interactions between DR-NEP components are shown in Fig. 2.5.

2.4 Mathematical Formulation of The i2Sim Framework

The i2Sim simulator is implemented in MATLAB/Simulink environment. An i2Sim Simulink toolbox has been developed to construct i2Sim models[3]. There are several
advantages for this implementation, such as user friendly, easy to construct models, and compatibility with wide range of other MATLAB tools. However, there are no analytical formulations for the constructed models. This limits i2Sim optimization applications to simulation-based optimization techniques, e.g., Genetic Algorithm and Reinforcement Learning. These techniques use the model as a black-box for objective function evaluation during the optimization process. One application of an i2Sim simulation optimization is presented in this thesis in Chapter 4. For mathematical optimization, a mathematical formulation for i2Sim model needs to be constructed. The following sections describe the mathematical formulation of i2Sim models.

2.4.1 i2Sim Cell Model Approximation

The output of an i2Sim cell, as in Eq 2.1, is a function of the input resources (or tokens), the modifiers, and the physical damage of the infrastructure. Marti [3] proposed the idea of the Human Readable Table (HRT) to model the input-output relationship in the i2Sim cells. An example of an HRT is shown in Fig. 2.6. The
HRT for a Hospital

<table>
<thead>
<tr>
<th>Operability</th>
<th>Patients per hour</th>
<th>Electricity (kW)</th>
<th>Water (L/h)</th>
<th>Doctors</th>
<th>Nurses</th>
<th>Physical Integrity</th>
</tr>
</thead>
<tbody>
<tr>
<td>100%</td>
<td>20</td>
<td>100</td>
<td>1,000</td>
<td>4</td>
<td>8</td>
<td>100%</td>
</tr>
<tr>
<td>75%</td>
<td>15</td>
<td>50</td>
<td>500</td>
<td>3</td>
<td>6</td>
<td>80%</td>
</tr>
<tr>
<td>50%</td>
<td>10</td>
<td>30</td>
<td>300</td>
<td>2</td>
<td>4</td>
<td>50%</td>
</tr>
<tr>
<td>25%</td>
<td>7</td>
<td>20</td>
<td>200</td>
<td>2</td>
<td>3</td>
<td>20%</td>
</tr>
<tr>
<td>0%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0%</td>
</tr>
</tbody>
</table>

Figure 2.6: An example a Human Readable Table [3].

Output of an i2Sim cell is determined by the minimum available input to the cell, i.e. the output variable is limited by the minimum of the input variables. For the example HRT shown in Figure 2.6 let us assume the inputs $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$, and $m_1(t)$ have the values as indicated by the green circles, then the output $y(t)$ is 10 patients per hour which corresponds to a 50% operability (indicated by the red circle). Mathematically, this function can be written as:

$$y(t) = \min \{f_1(t), f_2(t), ..., f_n(t)\}$$  \hspace{1cm} (2.3)
example, $x$ and $y$ represent the cell’s inputs and $z$ represents the cell’s output. A linear function is used for $x$ and $y$ as follows:

\[
\begin{align*}
  f_1 &= x \\
  f_2 &= y \\
  z &= \min \{ f_1, f_2 \}
\end{align*}
\]

For building the i2Sim system of equations, curve fitting techniques can be used to derive a closed-form function for the HRT without the operator $\min$. One possible approach is to use a quadratic function approximation in the form

\[
z = f(x, y) = ax + by + cxy + dx^2 + ey^2 + f
\]

where $a$, $b$, $c$, $d$, $e$, and $f$, are the function coefficients which can be found by minimizing the least square error between the fitted data and the HRT data. Figure 2.7 and Figure 2.8 show the function approximation. For the mathematical optimization formulation, one can use the approximated function which makes the problem a non-linear optimization problem. Alternatively, Equation (2.4) can be used to form a linear optimization problem, as will be explained in Chapter 4.

### 2.4.2 i2Sim System of Equations

The i2Sim system of equations describes the interrelations between interdependent infrastructure systems. This system of equations is composed of cells’ equations, aggregators’ equations, and distributors’ equations. Let us consider an i2Sim model with $n$ cells, $m$ aggregators, and $k$ distributors. We will denote each token as $x_n$, where the subscript $n$ indicates the token produced by cell $n$, e.g., electricity from the power station. Let us assume that each cell is approximated by a quadratic
Coefficients: a=0.38, b=0.38, c=1.04, d=-0.4, e=-0.4, f=-0.04

\[ z = 0.38x + 0.38y + 1.04xy - 0.4x^2 - 0.4y^2 - 0.04 \]

**Figure 2.7:** Function coefficients and function evaluation at some points

**Figure 2.8:** A plot of HRT fitted function

function as described in (2.5). For \( n \) cells, we can write:

\[
\begin{align*}
    f_1 &= a_1x_{11} + b_1x_{12} + c_1x_{11}x_{12} + d_1x_{11}^2 + e_1x_{12}^2 + f_1 \\
    f_2 &= a_1x_{21} + b_2x_{22} + c_2x_{21}x_{22} + d_2x_{21}^2 + e_2x_{22}^2 + f_2 \\
    \vdots \\
    f_n &= a_nx_{n1} + b_nx_{n2} + c_nx_{n1}x_{n2} + d_nx_{n1}^2 + e_nx_{n2}^2 + f_n
\end{align*}
\]

(2.6)
where $x_{n1}$ and $x_{n2}$ are the two inputs to cell $n$. The aggregators combine different inputs into one output of the same token. For $m$ aggregators, we can write

$$xm_1 = x_{11} + x_{12} + \ldots + x_{1j}$$
$$xm_2 = x_{21} + x_{22} + \ldots + x_{2j}$$

... 

... 

$$xm_n = x_{m1} + x_{m2} + \ldots + x_{mj}$$

where $xm$ is the aggregator’s output, $j$ is the number of inputs, and $x_{m1}$ to $x_{mj}$ are the inputs to aggregator $m$. The distributors’ functions represent the structure of the infrastructure which distributes the resources (tokens). For $k$ distributors, we can write

$$xk_1 = ak_{11}x_{11} + ak_{12}x_{12} + \ldots + ak_{1i}x_{1i}$$
$$xk_2 = ak_{21}x_{21} + ak_{22}x_{22} + \ldots + ak_{2i}x_{2i}$$

... 

... 

$$xk_k = ak_{k1}x_{k1} + ak_{k2}x_{k2} + \ldots + ak_{ki}x_{ki}$$

where $xk$ is the distributor’s output, $i$ is the number of outputs, and $x_{k1}$ to $x_{ki}$ are the outputs of distributor $k$. The parameters $ak_{k1}$ to $ak_{ki}$ are called the distribution ratios for distributor $k$. For example, if a power substation supplies 4 MW to two hospitals, 1 MW to hospital 1 and 3 MW to hospital 2, then the distribution ratios for the distributor will be 0.25 and 0.75 respectively. The summation of the parameters $ak_{k1}$ to $ak_{ki}$ for every distributor is always 1. Note that the variables $x_{n1}$, $x_{n2}$, $x_{jm}$, and $x_{ki}$ are intermediate variables which can be expressed in terms of tokens $x_1$ to
The cells’ equations (2.6), aggregators’ equations (2.7), and distributors’ equations (2.8) constitute the i2Sim system of non-linear equations. The non-linearity of the system comes from the cells’ equations (2.6). The remaining equations (aggregators and distributors) are called network equations since they represent the structure of the infrastructure. At every time step, this system is solved to find the flow value of each token. The solution of the system at every time step gives the evolution of the infrastructure dynamics along the time line of the scenario under study.

2.5 Model Development for the Case Study

The foregoing sections describe the i2Sim formulation for modelling interdependent CI. This section presents how this formulation is used to model the case study in this thesis. The model of this case study is used to demonstrate the effectiveness of the proposed approaches in this thesis for improving CI resilience. The model can show the impact of a disruption scenario on the modelled infrastructure systems. Also, it can show the effectiveness of the response activities during the studied scenarios.

The case study represents the metropolitan area of a major Canadian city. The data set used in constructing the model was collected during a project for supporting the Emergency Management team in the city. Data sources include public reports, private reports, and interviews with infrastructure operators. The modelled infrastructure includes hospitals, power system, and water system. The i2Sim model for the case study is shown in Figure 2.9. The model consists of 10 production cells, 7 distributors, and 5 aggregators.
Figure 2.9: Main i2Sim model for the case study
2.5.1 Infrastructure Models

Power

The case study model includes five electrical substations: P1, P2, P3, P4, and P5. These substations are distribution system substations which receive power from the high voltage transmission system. The inputs to the five substations are modelled as sources. Each substation is represented by an i2Sim production cell. The output of the i2Sim goes into a distributor which distributes the output power to other i2Sim cells. Since only CI systems are considered in the model, the power supply to non-critical loads such as residents is represented by a distributor connection to a sink. The number of outputs for each distributor and the ratio values are all abstracted from the available data to map the power system configuration. A general model for an electrical substation is shown in Figure 2.10.

Since the focus of this model is to study operational behavior of CI, only values of real power (in MW) are considered in the electrical substation model. In Chapter 6, the detailed power distribution system model is integrated with the i2Sim model to solve a combined restoration problem. The maximum output capacity of each substation, shown in Table 2.3, is used to form the first row of the HRT table for each substation’s model. Since the output capacity of substation P5 could not be obtained from the available data, it was assumed to be half of P1 based on the comparison between the supplied hospitals. A linear approximation is used to construct the HRT for each substation. Table 2.4 shows the HRT for substation P1.

Hospitals

There are three hospitals in the case study: Hospital 1, Hospital 2, and Hospital 3. Hospital 1 and Hospital 2 are the main hospitals in the modeled area. Hospital 3 is the women and children hospital but it is used for treating injured patients in extreme disaster scenarios. The i2Sim hospital model describes the functionality
Figure 2.10: i2Sim general model for electrical substation

Table 2.3: Output capacity of the modeled electrical substations.

<table>
<thead>
<tr>
<th>Electrical Substation</th>
<th>Output Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>300 MW</td>
</tr>
<tr>
<td>P2</td>
<td>336 MW</td>
</tr>
<tr>
<td>P3</td>
<td>336 MW</td>
</tr>
<tr>
<td>P4</td>
<td>586 MW</td>
</tr>
<tr>
<td>P5</td>
<td>150 MW</td>
</tr>
</tbody>
</table>

of the hospital during disaster scenarios. This functionality is limited to receiving trauma victims, treating them, and then discharging them. Other functions such as surgeries and outpatient care are not modeled in this case study since they are outside the context of disaster response.

Each hospital is represented by an i2Sim production cell. The output of the hospital cell is the rate at which patients can be treated, measured in patients per

Table 2.4: The HRT table for substation P1.

<table>
<thead>
<tr>
<th>PM</th>
<th>Output (MW)</th>
<th>Input (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>2</td>
<td>225</td>
<td>225</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>4</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 2.5: Treatment rate for the modeled hospitals.

<table>
<thead>
<tr>
<th>Hospital</th>
<th>Rate (Patient/hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>10</td>
</tr>
<tr>
<td>H2</td>
<td>10</td>
</tr>
<tr>
<td>H3</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 2.6: Requirements of the modeled hospitals at full capacity.

<table>
<thead>
<tr>
<th>Hospital</th>
<th>Electricity</th>
<th>Water</th>
<th>Steam/Natural Gas</th>
<th>Medical Gasses</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>20MW</td>
<td>51KL/h</td>
<td>3333 ft³/h</td>
<td>100%</td>
</tr>
<tr>
<td>H2</td>
<td>10MW</td>
<td>51KL/h</td>
<td>11458.3lbs/h</td>
<td>100%</td>
</tr>
<tr>
<td>H3</td>
<td>10MW</td>
<td>51KL/h</td>
<td>5729.2lbs/h</td>
<td>100%</td>
</tr>
</tbody>
</table>

The treatment rates for the modeled hospitals at full capacity are shown in Table 2.5. The inputs to each hospital cell are electricity (in MW), water (in KL/h), steam (in lbs/h), and medical gases (in %). The medical gases are measured in % since there were not reliable estimates for the required amount during emergency operations. Also, Hospital 1 uses natural gas for heating instead of steam. The required resources for treating patients at full capacity are shown in Table 2.6. A linear approximation is used to construct the HRT for every hospital to represent the different Physical Modes in the i2Sim model.

Water

There are two water pumping stations in the case study: W1 and W2. W1 supplies water to hospitals H1 and H2 while W2 supplies water to H3. Due to lack of data, the output of the two water stations is assumed to be only for providing water supply to the hospitals. Supplies to other consumers such as residential and commercial buildings are not included. The inputs and output data for the water station models are shown in Table 2.7.
Table 2.7: Input and output data for the i2Sim water station models.

<table>
<thead>
<tr>
<th>Water station</th>
<th>Output (Water)</th>
<th>Input (Electricity)</th>
<th>Input (Water)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>103 KL/h</td>
<td>2.4MW</td>
<td>103 KL/h</td>
</tr>
<tr>
<td>W2</td>
<td>51 KL/h</td>
<td>1.2MW</td>
<td>51 KL/h</td>
</tr>
</tbody>
</table>

2.5.2 Mathematical Formulation

The mathematical formulation for the case study follows the formulation described in section 2.4. For the power substation cells, let us denote \( y_{p1}(t), y_{p2}(t), y_{p3}(t), y_{p4}(t), \) and \( y_{p5}(t) \) as the outputs of the cells. A linear function in the form \( f = ax \) is used to describe the relationship between the input \( x \) and the output \( y_p \). Since power losses are not considered in this model, the value of the constant \( a \) is 1. Therefore, the cell model is simplified to \( y_{p}(t) = x(t) \) for each of the power substation cells. The value of \( x(t) \) is between 0 and the output capacity of the substation as in Table 2.3.

We denote \( y_{h1}(t), y_{h2}(t), \) and \( y_{h3}(t) \) as the hospital outputs. Since there are four inputs to each hospital, Equation (2.3) can be rewritten as

\[
y_{h}(t) = \min \{ f_{eh}(t), f_{wh}(t), f_{s}(t), f_{mg}(t) \} \tag{2.9}
\]

where \( f_{eh}, f_{wh}, f_{s} \) and \( f_{mg} \) are the input functions for electricity, water, steam (or natural gas), and medical gasses, respectively. Similar to the power substation cells, the hospital inputs are linearly mapped to the output of the hospital cell using the values in Table 2.6. For the water stations, we denote \( y_{w1}(t), \) and \( y_{w2}(t) \) as the water outputs. Since there are two inputs to each water station, Equation (2.3) can be rewritten as

\[
y_{w}(t) = \min \{ f_{ew}(t), f_{ww}(t) \} \tag{2.10}
\]

where \( f_{ew}, \) and \( f_{ww} \) are the input functions for electricity and water, respectively. The inputs are also linearly mapped to the output of the water station cells using
The values in Table 2.7.

There are seven distributors in the case study: five electrical distributors and two water distributors. We denote $x_k yp_n$ as the $k^{th}$ output of the $n^{th}$ power substation cell. For example, $x_2 yp_3$ is the second output of the distributor of power substation P3. For every electrical distributor, we write

$$yp_n(t) = a_1 x_1 yp_n + a_2 x_2 yp_n + \ldots + a_k x_k yp_n$$ (2.11)

Similarly, we denote $x_k yw_n$ as the $k^{th}$ output of the $n^{th}$ water station cell. Therefore, we can write

$$yw_n(t) = a_1 x_1 yw_n + a_2 x_2 yw_n + \ldots + a_k x_k yw_n$$ (2.12)

where $k$ is the number of outputs of this distributor and $a_k$ is the distribution factor such that $a_1 + a_2 + \ldots + a_k = 1$.

There are five aggregators in the case study: four electrical aggregators $AH1e$, $AH2e$, $AH3e$, and $AW1e$, and one water aggregator $AH2w$. The aggregators combine inputs of the same token type into one, as described in (2.7). For example, the electrical aggregator for hospital $H1$ can be expressed as

$$AH1e(t) = m_1 x_1 yp_3 + m_2 x_1 yp_4 + BGH1$$ (2.13)

where $x_1 yp_3$ and $x_1 yp_4$ are the power supplies from substations P3 and P4 respectively, and $BGH1$ is the backup generation source in the hospital. If the input values are expressed in per unit, then the appropriate factors $m_1$ and $m_2$ are needed to adjust the input values. Otherwise, $m_1 = m_2 = 1$.

The i2Sim system of equations for this case study consists of 10 cells’ equations, 7 distributors’ equations, and 5 aggregators’ equations. At every time step $t$, 22 equations are solved to find the flow value of each variable (electricity, wa-
ter, etc) in the modelled infrastructure systems. For a scenario of 10 hours with a time step of 5 minutes, there are \((10 \times 60/5) \times 22 = 2640\) equations. The case study was implemented using the i2Sim toolbox in MATLAB/Simulink and using the analytical mathematical formulations described above. Both implementations are fundamentally the same. However, if the model is to be used in an optimization problem, then they are different with respect to the optimization approaches. The MATLAB/Simulink implementation can be used with simulation based optimization techniques such as Genetic Algorithm, Ordinal Optimization, and Simulated Annealing. In these techniques, the model is considered as a black box and is used for evaluating the candidate solutions. In Chapter 4, an application of Ordinal Optimization is presented. The analytical formulation can be used in formulating mathematical optimization problems such as Linear Programming and Non Linear Programming. In Chapter 4, the i2Sim non linear system of equations is formulated as a Linear Programming problem.

2.5.3 Extreme Event Scenario

In this case study, we consider a damage scenario based on an earthquake event that causes major damages to several CI systems in the modeled area. Since every extreme event offers a unique set of conditions and circumstances, a general scenario was developed in consultation with the infrastructure operators during the project that provided the data set for the case study. The scenario was refined using data, such as average equipments restoration times, from public reports and literature. The scenario describes a realistic threat to the city, an earthquake, and simulates a complex situation in which simultaneous failures are encountered. The scenario aims at illustrating the importance of allocating scarce resources during extreme events. The sequence of events for the scenario is described below.

\[
T=00:00 \quad \text{Normal operations at all critical infrastructure systems.}
\]
An earthquake with a magnitude of 6 degrees hits the modelled area resulting in the following damages:

- Failures at two main transformers at substation P4 causing 75% reduction of power output
- Major leaks in water supply pipes to hospitals H2 and H3
- Failure of one of the circuits (cables) supplying power from substation P5 to water station W2

Backup generators come online.

Backup generator at water station W1 runs out of fuel.

Transformer 1 at substation P4 is back to service. All backup generators run out of fuel.

Power supply from substation P5 to water station W2 is restored.

Transformer 2 at substation P4 is restored.

Water supplies to hospitals H2 and H3 are restored.

Normal operations at all critical infrastructure systems are restored.

Typically, restoration activities continue until all services are restored to all users. In the early phases of the restoration activities, all efforts are directed toward restoring services to essential users. After that, the efforts are directed toward the remaining users. The later phases are usually slower and take longer duration, days and sometimes weeks. Since the focus of this work is on CI systems, the scenario events are limited to restoring services to the three hospitals in the model. The performance of the three hospitals in this scenario is shown in Figure 2.11. The scenario events start at Time Step=1, where each time step is 5 minutes. Theoretically, the simulation can be considered an event-driven simulation in which the execution is
done when there is a triggering event. However, the development of this work takes into consideration its applicability to real time applications in which the events are not known in advance and there are updates at every time step. The results of this scenario are considered the base case scenario and will be compared with the results from the proposed optimization algorithms in Chapter 4.

2.6 Conclusion

This chapter presents the i2Sim modelling framework. The description of i2Sim models and their mathematical formulations are presented. This framework is used throughout this thesis for modelling CI systems. The description of the case study modelling is also presented in this chapter. In Chapter 3, a resilience assessment methodology is developed based on the i2Sim framework. In Chapter 4, optimization applications for CI resilience improvement are presented using different optimization techniques.
In Chapter 5, an i2Sim model is integrated with a power distribution model to solve a power restoration problem considering infrastructure interdependencies.
Chapter 3

Resilience of Critical Infrastructure

3.1 Introduction

Resilient infrastructure systems such as electric power, water, and health care are essential for minimizing the impact of extreme events. Building a resilient infrastructure is an important goal for every nation’s Critical Infrastructure Protection (CIP) program. One of the first steps toward this goal is developing an evaluation methodology that enables decision makers to quantify the infrastructure’s resilience. The methodology is then used to evaluate the possible measures for improving infrastructure resilience.

As infrastructure systems are coupled and interdependent, failures can propagate from one infrastructure system to another causing catastrophic consequences. Scala et. al. [32] have shown that couplings between infrastructure systems have two competing impacts on cascade failures: increasing the withstand capability and increasing the total failure probability. Therefore, an effective resilience assessment approach must incorporate the interdependencies among infrastructure systems into
the analysis. However, this incorporation is not an easy task. Although modelling and simulation tools are available for studying different aspects of infrastructure systems [14], conducting a comprehensive cross-infrastructure resilience analysis poses several challenges. One of these challenges is the absence of universal measures or metrics that articulate the resilience of an infrastructure. Another challenge is finding the appropriate data for the study, especially data related to failures or incidents. Infrastructure systems operators are reluctant to share information due to regulation, security, and competition.

In this chapter, we propose an effective cross-infrastructure resilience assessment framework. It incorporates three main steps: defining resilience attributes, modelling critical infrastructure, and measuring resilience. The assessment framework can be used to evaluate and optimize preparedness, response, and mitigation plans against natural and man-made disasters. Section 3.2 of this chapter presents a literature review on existing resilience definitions and assessment methods. Section 3.3 discusses the use of the resilience concept within CIP analysis. The proposed resilience assessment framework is presented in section 3.4. An illustrative example is discussed in section 3.5. Section 3.6 presents the generalized resilience index. Section 3.7 discusses cyber-physical interactions within resilience assessment. A discussion on how to improve the infrastructure resilience is presented in section 3.8 and a conclusion is presented in section 3.9.

3.2 Literature Review on CI Resilience

3.2.1 Resilience Definition

The word resilience has been used in different disciplines such as ecology and health sciences for a long time. Resilience was first defined at the system level by Holling in 1973 [33]. Holling defined resilience as a measure of the persistence of systems and of their ability to absorb change and disturbance and still maintain the same
relationships between populations or state variables [34]. Since the time when the first definition was proposed, researchers in social, ecological, and economic systems have proposed other definitions. Some of them are general while others are domain specific. Francis et al [35] presented a survey of resilience definitions from different disciplines.

Within the Critical Infrastructure Protection community, much effort has been devoted in the past to explore and study the concepts of risk, reliability and security. However, the concept of resilience is still relatively new. In a survey conducted by the NIAC, many power companies executives indicated that “while reliability is relatively easy to define and measure, resilience is more difficult.” [36].

The definitions of resilience in the context of critical infrastructure systems have evolved from the existing definitions in other fields. Infrastructure system resilience is generally regarded as the ability of the system to withstand a disturbance and recover back to its initial state [34]. Dalziell et al [37] describes resilience as the overarching goal of a system to continue to function to the fullest possible extent in the face of stress to achieve its purpose. Based on this definition, resilience is a function of the systems vulnerability and its adaptive capacity, where adaptive capacity is the ability of the system to respond to external changes and recover from internal damages [37]. Haimes et al [10] proposes a definition that is more related to the disaster response activities: Resilience is the ability of the system to withstand a major disruption within acceptable degradation parameters and to recover within acceptable cost and time. Recently, several governmental reports defined resilience as a key component in their CIP programs. For instance, the US National Infrastructure Advisory Council (NIAC) defines infrastructure resilience as [7]:

the ability to reduce the magnitude and/or duration of disruptive events.

The effectiveness of a resilient infrastructure or enterprise depends on its
ability to anticipate, absorb, adapt to, and/or rapidly recover from a potentially disruptive event.

Looking at the different definitions, one can notice commonalities and differences. Figure 3.1 shows some of the key resilience characteristics as identified by CIP community definitions and other disciplines. Attributes such as ability to recover and to adapt were incorporated in several proposals. Some of the definitions consider the long-term resilience by including a planning component [38] [7], others think about resilience as an emerging behaviour after a disturbance [34]. Most of the proposed definitions include ‘the ability to withstand’ or ‘absorb’ a disturbance as a key attribute. However, Madni et. al. [39] argues that this attribute is the definition of survivability while resilience is the ability to bounce back. In general, it is difficult to select any of the discussed definitions as ‘the best’ or ‘the global’ definition for resilience as they were developed to serve different objectives and perspectives. MacAskill and Guthrie [40] argue that “a strict consensus on the definition of resilience is not practical or perhaps not ever possible”. They suggest that a flexibility in the definition is required to be adopted in different contexts.

### 3.2.2 Resilience Assessment

Since the introduction of a system resilience definition by Holling in 1973 [33], researchers have proposed different approaches and methodologies for assessing and evaluating system resilience. In the context of critical infrastructure systems, Biringer et al [34] classify resilience assessment approaches into three general categories: structural, performance based, and hybrid. Structural approaches use the structure or topology of the system to evaluate its resilience. Performance based approaches evaluate the system resilience by measuring its performance before and after a disruption. Hybrid approaches combine both: structural and performance based. It has been acknowledged that resilience assessment or evaluation is not an easy task.
It requires not only the static and dynamic properties of the systems but also other factors such as economic and human factors [39]. A survey of resilience assessment methods and frameworks for infrastructure systems is presented in Table 3.1. Several conclusions can be drawn from this survey. First, the civil (structural) engineering community is one of the earliest areas in the engineering discipline to adapt the resilience concept and use it in seismic-related research. Therefore, there are more contributions in the literature from the civil (structural) engineering community than other areas. Second, the resilience assessment approach is highly dependent on the study definition and scope. Moreover, more work is being done on quantifying the resilience level using some performance indices. Finally, most of the work in assessing infrastructure systems resilience is recent (last few years) which reflects the increased attention to this topic within the research community.

Figure 3.1: Key characteristics of resilience definitions from different disciplines.
### Table 3.1: Literature survey of resilience assessment in infrastructure.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Year</th>
<th>Approach</th>
<th>Context/System</th>
</tr>
</thead>
<tbody>
<tr>
<td>[41]</td>
<td>2003</td>
<td>Hybrid (Quantitative)</td>
<td>Seismic/Infrastructure</td>
</tr>
<tr>
<td>[42]</td>
<td>2004</td>
<td>Performance (Quantitative)</td>
<td>Seismic/Infrastructure</td>
</tr>
<tr>
<td>[43]</td>
<td>2006</td>
<td>Performance (Quantitative)</td>
<td>Transportation</td>
</tr>
<tr>
<td>[44]</td>
<td>2007</td>
<td>Structural (Quantitative)</td>
<td>Seismic resilience</td>
</tr>
<tr>
<td>[45]</td>
<td>2009</td>
<td>Structural (Quantitative)</td>
<td>Risk/Transportation</td>
</tr>
<tr>
<td>[46]</td>
<td>2009</td>
<td>Performance (Quantitative)</td>
<td>Infrastructure</td>
</tr>
<tr>
<td>[47]</td>
<td>2009</td>
<td>Performance (Quantitative)</td>
<td>Telecommunication</td>
</tr>
<tr>
<td>[48]</td>
<td>2009</td>
<td>Hybrid (Quantitative)</td>
<td>Hurricane/Infrastructure</td>
</tr>
<tr>
<td>[49]</td>
<td>2010</td>
<td>Performance (Quantitative)</td>
<td>Seismic/Health Care Facilities</td>
</tr>
<tr>
<td>[50]</td>
<td>2010</td>
<td>Structural (Quantitative)</td>
<td>Risk/Transportation</td>
</tr>
<tr>
<td>[51]</td>
<td>2010</td>
<td>Performance (Quantitative)</td>
<td>Transportation</td>
</tr>
<tr>
<td>[52]</td>
<td>2011</td>
<td>Structural (Quantitative)</td>
<td>Risk/Transportation</td>
</tr>
<tr>
<td>[53]</td>
<td>2011</td>
<td>Performance (Quantitative)</td>
<td>Transportation</td>
</tr>
<tr>
<td>[54]</td>
<td>2011</td>
<td>Performance (Quant. &amp; Quali.)</td>
<td>Petrochemical</td>
</tr>
<tr>
<td>[55]</td>
<td>2012</td>
<td>Performance (Quantitative)</td>
<td>Generic</td>
</tr>
<tr>
<td>[56]</td>
<td>2012</td>
<td>Performance (Quantitative)</td>
<td>Infrastructure</td>
</tr>
<tr>
<td>[57]</td>
<td>2012</td>
<td>Performance (Quantitative)</td>
<td>Transportation</td>
</tr>
<tr>
<td>[58]</td>
<td>2012</td>
<td>Performance (Quantitative)</td>
<td>Power Transmission Grid</td>
</tr>
<tr>
<td>[59]</td>
<td>2014</td>
<td>Hybrid (Quantitative)</td>
<td>Infrastructure</td>
</tr>
</tbody>
</table>

### 3.3 Resilience Concept within Critical Infrastructure Protection Analysis

There are different analyses that CIP organizations use to ensure the safety of infrastructure systems. Reliability, Risk, and Vulnerability analysis are commonly used in CIP studies. There are some overlapping aspects within the concepts of Reliability, Risk, Vulnerability, and Resilience. This section addresses the relationships between these terms.

Reliability can be defined as “the probability of a device (or system) performing its purpose adequately for the period of time intended under the operating conditions encountered” [60]. One of the key problems in reliability analysis is to determine the reliability of a complex system from knowledge of the individual components’ reliability. Reliability analysis is concerned with the internal behaviour of the infrastructure and limits the analysis to the failure time. However, resilience analysis extends reliability analysis in two ways: it considers external sources of failures in
the analysis and it considers the reaction of the system (response) after the failure.

Risk is a more general concept than reliability. Risk analysis is based on three questions [61]: what can happen?, how likely will it happen?, and if it does happen, what are the consequences?. By answering these questions, one can identify potential threats and develop strategies to reduce or avoid the risk of these threats. Resilience analysis, on the other hand, is concerned with the ability of the system to deal with these threats and return the system to its functional state. Therefore, resilience analysis can be viewed as an extended part of the traditional risk analysis. Another view is to consider risk analysis as an input to resilience analysis. In both views, resilience analysis answers the question: how will the system react after the event?, which is not answered by traditional risk analysis.

Vulnerability analysis is often viewed as part of risk analysis in which Risk = Threat x Vulnerability x Impact. Vulnerability as defined by Haines [62] is “the manifestation of the inherent states of the system that can be exploited to adversely affect that system”. He also argues that vulnerability and resilience are “two sides of the same coin”: vulnerability analysis focuses on protecting the system and resilience analysis focuses on restoring the system. Therefore, a vulnerability analysis may identify the weak points in the system but may not address how the system behaves in case it is attacked. The latter aspect is covered by resilience analysis.

In general, reliability, risk, vulnerability, and resilience are all essential analysis in critical infrastructure protection studies. They focus on different dimensions of the infrastructure under study. Resilience analysis complements the other traditional analysis by extending the scope to cover the infrastructure response to disruptive events.
3.4 Resilience Assessment Framework

Resilience assessment is needed for decision support to quantify the effectiveness of preparedness investments and activities. An effective preparedness plan improves the reaction of critical infrastructure following a disruption or a disaster. Prior to conducting resilience assessment, one needs to define what aspects of the system under study constitute a resilient system. A proper modelling is also required to study the behaviour of the system after disruptions. In addition, a metric (or metrics) needs to be formulated to measure resilience. In this chapter, we propose a resilient assessment framework consisting of three stages: defining resilience attributes, building an infrastructure model, and measuring resilience, as shown in Figure 3.2. More details on each stage are given in the following subsections.

3.4.1 Critical Infrastructure Systems Attributes

Since resilience is a multifaceted concept, it is imperative to assess resilience within the context of interest. The context of this thesis is critical infrastructure systems, such as power networks, water networks, and health facilities, with a particular interest in disaster response operations. Therefore, it is helpful to define the systems
attributes that constitute their resilience. From the perspective of this thesis, a resilience assessment framework should encompass the following attributes:

**Static** This attribute describes the physical static parameters of the infrastructure. These parameters provide information about the components and topology of each system. Examples of these parameters include electrical network topology, capacity parameters of water pumps, and number of routes leading to a specific site. The interdependencies between the different systems are also described by this attribute. We should point out here that the interdependence relationships related to systems structure are measured here. Other interdependencies are captured by the other two attributes defined in the proposed framework.

**Dynamic** This attribute describes the dynamic behaviour of the infrastructure systems. Aspects such as emergency preparedness, response management, and recovery activities can all be measured in this attribute. For example, how the available resources are allocated, how failures propagate through the infrastructure, and how long it takes the infrastructure to return to its normal performance level.

**Decision** This attribute describes the decision factors whose contributions are essential to the overall infrastructure resilience. Examples of these factors include decisions to allocate scarce resources, policies dictating command and control during disastrous events, and scheduling of available maintenance (or rescue) teams.

The above attributes are defined as linearly independent Eigen-attributes that influence the overall Critical Infrastructure (CI) resilience. The information given by these attributes provides insights on the systems capabilities to withstand, absorb, and recover from a disruption. The next section describes how these attributes can
be modelled within the proposed resilience assessment framework.

### 3.4.2 System Modelling

Prior to measuring resilience, an infrastructure model needs to be constructed. There are a number of requirements that a model should satisfy. Some of these requirements are highlighted below:

- The model should include the required parameters for describing the specified attributes for system resilience, i.e., static, dynamic, and decision.

- As time is an integral part of resilience assessment, the model should capture the temporal behaviour of the infrastructure.

- The model should be able to include both external and internal parameters.

To meet the above requirements, this thesis uses the Infrastructure Interdependencies Simulator (i2Sim) framework. The i2Sim modelling framework is described in detail in Chapter 2 of this thesis. The i2Sim framework has been used in modelling critical infrastructure systems in disaster response applications [63] [64]. Critical infrastructure systems, such as power substations, water stations, and hospitals, are modelled using the i2Sim components: cells, channels, tokens, distributors, aggregators, sources, sinks, and modifiers. The i2Sim simulation layers shown in Figure 3.3 are used to model the system attributes described in section 3.4.1. The structure attribute can be modelled within the physical layer. For example, the topology of the infrastructure is represented by their corresponding arrangement of cells, channels, distributors, and aggregators. The physical parameters, such as power substation capacities, water pumps sizes, and required manpower resources to operate a hospital, are used to build the input-output functions. The dynamic attribute can be modelled by simulating the impact of resources allocation decisions on the performance of the infrastructure. Decision attribute can be modelled as modifiers inputs.
to the cells in the physical layer. It is worth noting that in many cases, an attribute is not necessarily captured within a single layer but can be modelled across different layers. Different parameters belonging to the same attribute can be represented differently inside the i2Sim model. For example, in the decision attribute, one can model an emergency response policy within the decision layer while a maintenance team schedule is modelled in the physical layer.

![Diagram of i2Sim simulation layers](image)

**Figure 3.3:** i2Sim simulation layers [3].

### 3.4.3 Resilience Measure

A quantitative resilience assessment is needed for decision support in planning for or designing resilient infrastructure. An index (or metric) reflecting the modelled resilience attributes is a basic measure in the assessment. One approach for defining this index is to use a performance indicator (or indicators) that captures all the required attributes. In the context of critical infrastructure protection, the performance indicator is usually related to the functionality (output) level of the system.
Thus, infrastructure resilience can be defined in terms of the deviation from the normal (healthy) performance level. This is represented graphically in Figure 3.4. $PL_N$ is the normal performance level (without any disruption) while $PL_0$ is the performance level immediately after the event. $t_0$ is the initial time of the event. $t_R$ is the recovery time when the infrastructure returns to its normal performance level.

![Graphical representation of resilience.](image)

**Figure 3.4:** Graphical representation of resilience.

In this thesis, we propose the use of the i2Sim cell’s output as a basis for defining an infrastructure resilience index. The output of an i2Sim cell is a measure of the operability of the modelled system. Moreover, it incorporates all the resilience attributes, static, dynamic, and decision, through the modelling approach described in section 3.4.2. We define $R$ as a resilience index which can be defined mathematically as follows:

$$R = \frac{1}{t_D} \int_{t_0}^{t_R} y(t) \, dt$$

(3.1)

where $y(t)$ represents the i2Sim cell’s output in per unit and $t_D = t_R - t_0$ is the duration of the event. $R$ is measured in per unit, i.e., $R \in [0, 1]$. A higher $R$ value indicates a more resilient infrastructure. As $t_D$ increases (longer time to recover), the value
of $R$ decreases (less resilient infrastructure). This is in accordance with the general concept of resilience.

The integral of $y(t)$ represents the area under the curve between $t_0$ and $t_R$ in Figure 3.4 Assuming that $y(t_i)$ is constant in the interval $[t_i, t_{i+1}]$, where $t_i$ is any time step between $t_0$ and $t_R$, the integral of $y(t)$ can be calculated by adding up the areas in every time step as shown in Figure 3.5 which can be expressed as

$$\int_{t_0}^{t_R} y(t) \, dt = \sum_{i=1}^{f} A_i$$

where $f$ is the number of time steps. The area $A_i$ is $A_i = (t_{i+1} - t_i) y(t_i)$. Assuming a constant time step and $t_{i+1} - t_i = 1$, the summation \[3.2\] can be expressed as

$$\sum_{i=1}^{f} y(t_i)$$

Therefore, $R$ can be expressed as

$$R = \frac{\sum_{i=1}^{f} y(t_i)}{t_R - t_0}$$

The performance level evaluation using i2Sim ensures that the defined attributes
of the CI resilience are measured by the proposed index. The choice of a specific i2Sim output in calculating $R$ is problem specific and depends on the desired outcomes of the study. For example, a hospital’s cell output is an appropriate index for a disaster response planning concerning immediate response after a disaster. In this case, the resilience index $R$ is expressed in terms of this output. In many cases, the decision maker needs to consider multiple CI systems in the resilience assessment study. In these cases, multiple outputs are required for evaluating the overall resilience. Thus, a proper calculation of $y(t)$ is required.

Consider the graphical representation of resilience shown in Figure 3.6 for the multiple outputs case. The overall area under the curve is composed of a number of "smaller" areas. These areas represent the different CI systems contributions to the overall resilience. A direct extension of 3.3 is

$$R = \sum_{i=1}^{j} \sum_{j=1}^{y_j(t_i)} \frac{\sum_{j=1}^{y_j(t_i)}}{t_R - t_0}$$  \hspace{1cm} (3.4)

where $j$ is the number of considered i2Sim outputs and $y_j(t_i)$ is the output of system $j$.

**Figure 3.6:** Graphical representation of resilience with constant time step and multiple outputs.
at time $i$. Equation 3.4 shows that the overall output is $y(t) = y_1(t) + y_2(t) + ... + y_j(t)$ which assumes a linear relationship. In many cases, not all the outputs have the same importance to the decision maker. For example, a power supply to recreational facilities may not be as important as a water supply to houses. To account for the decision maker preferences, we can define importance weights, $w_j$, such that

$$y(t) = w_1y_1(t) + w_2y_2(t) + ... + w_jy_j(t)$$

(3.5)

where $j$ is the number of outputs to be considered in the study. We call $R$ the “Basic Resilience Index”. Note that when calculating $y(t)$ all the outputs should be normalized to their basis.

### 3.5 Illustrative Example

In this section, we demonstrate the application of the proposed resilience assessment framework by applying it to a large university campus. The CI of the campus were modelled using the i2Sim modelling approach described in section 3.4.2. In this model, there are two power substations cells, one water station cell, one steam plant cell, and one hospital cell. The disaster events are taken from a heavy snowfall that occurred during the winter of 2006. Figure 3.7 depicts the sequence of events for the simulated scenario. The infrastructure and events data were collected during previous projects.

![Figure 3.7: Time line of the simulated events.](image-url)
The objective of this example is to show that the proposed framework can model the defined resilience attributes and measure their impact using the basic resilience index. In this example, we assume the hospital’s output as the performance level variable. Four different cases are considered to study different resilience attributes. The results are shown in Figure 3.8. Case (a) represents the original sequence of events as depicted in Figure 3.7. In this case, the i2Sim model represents the structure and topology of the infrastructure. The resilience index for this case is $R=0.38$. The other three cases represent hypothetical scenarios in which we assume that some actions can be taken to alter the sequence of events. In case (b), we assume a dynamic behaviour where the available resources are redistributed to increase the functionality of the hospital. The resilience for this case is $R=0.62$ which is higher than case (a) as expected. In case (c), we assume that some of the resources allocation are done automatically (power distribution network is reconfigured through SCADA). The resilience index for this case increased slightly $R=0.70$. Finally, we assume a human factor in case (d) where a maintenance crew is required to perform switching operation instead of automatic reconfiguration as in case (c). The resilience index for this case is $R=0.66$ which is lower than case (c). This example only shows that different actions (related to different attributes) bring different resilience levels. Whether they are better actions or how to find the best action is outside the scope of this chapter and it is discussed in the next chapter.

3.6 Generalized Resilience Index

The basic resilience index defined in 3.1 provides a basic mean to assess infrastructure resilience by considering the infrastructure performance and the recovery time. However, there are other dimensions of resilience that need to be captured and measured, such as availability of resources (or resourcefulness), cost of recovery, organizational factors, etc. Therefore, there is a need for a generalized resilience
index that is flexible to enable capturing the many dimensions of resilience.

Following the HRT concept in i2Sim defined in Chapter 2, we define the Generalized Resilience Index (GRI) as follows:

\[
GRI = \min \{ R_{D_1}, R_{D_2}, \ldots, R_{D_n} \}
\]  

(3.6)

where \( D \) is the considered resilience dimension, \( R_{D_n} \) is the dimension resilience function, and \( n \) is the number of dimensions. The GRI function can be implemented using a lookup table similar to the HRT table as shown in Table 3.2. The GRI function can be explained as follows: the infrastructure resilience is a function of \( n \) dimensions. The function \( R_{D_n} \) measures the resilience along that dimension. The overall infrastructure resilience (measured by \( GRI \)) is limited by the minimum \( R_{D_n} \) value. Each \( R_{D_n} \) value can be calculated independently using a separate function that evaluates the specific resilience dimension \( D \). Consider the following example from Table 3.2: if \( R_{D_1} = 1, R_{D_2} = 1, \) and \( R_{D_n} = 0.8 \), then \( GRI = 0.8 \). This example
Table 3.2: A GRI table example.

<table>
<thead>
<tr>
<th>GRI</th>
<th>$R_{D_1}$</th>
<th>$R_{D_2}$</th>
<th>...</th>
<th>$R_{D_n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>...</td>
<td>1</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>...</td>
<td>0.9</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>...</td>
<td>0.8</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
<td></td>
<td>:</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

could represent a recovery plan that results in a full system performance recovery ($D_1$) and an acceptable recovery time ($D_2$) but at a high recovery cost ($D_n$). In this case, the cost dimension affects the overall resilience of the system (GRI).

The GRI function provides a flexible formulation that makes it practical and useful for many domains and applications. One aspect of its flexibility lies in the ability to add as many dimensions ($D$) as needed for the study. It is possible that for every resilience definition, one can formulate its corresponding GRI function. Consider, for example, the resilience definition by Bruneau et al. [41] in which four dimensions are defined: Robustness, Rapidity, Resourcefulness, and Redundancy. One can define four columns in the GRI table $R_{D_1}$, $R_{D_2}$, $R_{D_3}$, and $R_{D_4}$ that can measure the four dimensions in the definition.

Some resilience definitions stress that the resilience of infrastructure systems is highly dependent on the type of event. These definitions emphasize that a resilience assessment should include the question: resilient to which event?. Using the proposed formulation, one can capture this event-dependence by defining a GRI table for every considered threat or event. For example, one can define two GRI tables for the same modelled system: one for an earthquake scenario and one for a terrorist attack scenario.

Another important flexibility aspect is the ability to calculate each dimension by using a specific function for that dimension. This function should be able to evaluate the contribution of the dimension to the overall resilience. We define this
function $R_D$ to map every value in the $R_D$ column to its corresponding value in the GRI column. The $R_D$ function can take any form: linear, exponential, or it can be a result of a simulation.

An additional flexibility aspect is the ability to discretize the resilience index. Similar to the widely used risk matrix in risk assessment methods, in some cases it is more convenient for the decision makers to characterize infrastructure resilience using distinct resilience levels. However, the poor resolution problem in risk matrices is avoided in the GRI function by controlling the number of rows in the GRI table. Therefore, one can define a row for each resilience level in the table to achieve the desired assessment resolution. If a continuous value is desired, one can use an approximation technique similar to the one described in Chapter 2 to find a closed form for the GRI function.

There are two conditions for defining the resilience dimensions in the GRI function: **First**, all the dimensions have to be linearly independent. For example, if the recovery cost is linearly dependent on the recovery time, then only one of them can be included in the GRI function. **Second**, the dimension function $R_D$ has to be a monotonically increasing function of GRI, i.e., as the value of $R_D$ increases, the value of GRI increases. The last condition ensures a proper implementation of the limiting factor function in the table.

### 3.6.1 GRI Function for the Case Study

For the case study in this thesis, we define the following resilience dimensions.

**Dynamic Performance Recovery** This dimension evaluates the behaviour of the system during the recover process. For this dimension, we define $D_d$ such that

$$D_d = \frac{\int_{t_0}^{t_R} y(t) \, dt}{\int_{t_0}^{t_R} y_{normal}(t) \, dt} \tag{3.7}$$

where $y(t)$ is the performance level during the event, $y_{normal}(t)$ is the normal
performance level. Equation 3.7 represents the ratio of the area under the recovered performance curve to the area under the normal performance curve. It measures the efficiency of the infrastructure’s response to the disruptive event.

**Recovered Performance Level** We define this resilience dimension as the ratio of the recovered performance level to the maximum performance drop after the event. This ratio is a measure of the infrastructure robustness. For this dimension, we define $D_y$ such that

$$D_y = \frac{y_r - y_{\text{min}}}{y_d - y_{\text{min}}}$$ \hspace{1cm} (3.8)

where $y_r$ is the performance level after completing the recovery activities, $y_{\text{min}}$ is the minimum performance level during the event, and $y_d$ is a defined reference performance level. In some cases, the system pre-event status is not its normal status. Therefore, Equation 3.8 does not assume that the pre-event performance is the desired performance after recovery. Instead, the variable $y_d$ is introduced to define the desired recovery performance level by the decision maker. Consider for example a power system that encounters a disruption at some loading point. It will be restored to a different loading point which might be different than the pre-disruption loading point depending on the time of the day. The difference between this dimension and the dynamic performance recovery dimension $D_d$ is that $D_y$ measures how much drop in performance was recovered regardless of the intermediate stages in the recovery process, while $D_d$ measures the efficiency of the recovery path to the final performance level.

**Recovery Time** We define this resilience dimension as the ratio of the time required to complete the recovery activities to a pre-defined duration. This
definition stresses that the recovery activities have to be completed within a
time frame. This is driven by the fact that many infrastructure systems op-
erators have regulations mandating the recovery of services within some time
frame. For this dimension, we define $D_t$ such that

$$D_t = \frac{T_r}{T_d} \quad (3.9)$$

where $T_r$ is the total recovery duration and $T_d$ is the pre-defined maximum
recovery duration.

**Recovery Cost** We define this resilience dimension as the ratio of the total cost
incurred by the infrastructure as a result of the disruption event to a pre-
defined cost. The total recovery cost includes the recovery activities cost and
the production losses during the recovery time. Like the recovery time, the
recovery cost cannot be unlimited and a cost limit $C_d$ is introduced. We define
$D_c$ as

$$D_c = \frac{C_r + C_l}{C_d} \quad (3.10)$$

where $C_r$ is the total cost due to recovery activities, $C_l$ is the total losses in
production during the recovery time, and $C_d$ is the maximum cost that can be
incurred.

For constructing the GRI function, we require the definitions of $R_d$, $R_y$, $R_t$, and
$R_c$. Since the recovered performance level increases proportionally with the overall
resilience, then we can define $R_d$ as

$$R_d = w_d D_d \quad (3.11)$$

Similarly, we define $R_y$ as

$$R_y = w_y D_y \quad (3.12)$$
Table 3.3: GRI table for the case study.

<table>
<thead>
<tr>
<th>GRI</th>
<th>$R_d$</th>
<th>$R_y$</th>
<th>$R_t$</th>
<th>$R_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note that since $y_{min} \leq y_r \leq y_d$, the value of $R_y$ is always between 0 and 1. For the recovery time dimension, we can define

$$R_t = \begin{cases} w_t (1 - D_t) & D_t \leq 1 \\ 0 & D_t > 1 \end{cases}$$

(3.13)

and similarly for the recovery cost dimension:

$$R_c = \begin{cases} w_c (1 - D_c) & D_c \leq 1 \\ 0 & D_c > 1 \end{cases}$$

(3.14)

The parameters $w_d$, $w_y$, $w_t$, and $w_c \in [0, 1]$ are used to represent the importance of each dimension’s contribution to the overall resilience. Therefore, the GRI function for the case study can be written as follows:

$$GRI = \min \{R_d, R_y, R_t, R_c\}$$

(3.15)

Table 3.3 shows a sample implementation of the function. Note that all values are expressed in per unit.
3.7 Cyber-Physical Interactions within Resilience Assessment

Many of the Critical Infrastructure Protection tools and strategies focus on the physical infrastructures such as power networks, water networks, and transportation. On the other hand, Information Infrastructure is typically treated separately. Due to its complex and diverse nature, specific tools and strategies are developed for Critical Information Infrastructure Protection (CIIP) activities. The International CIIP Handbook 2008/2009 [65] summarizes the initiatives undertaken by different countries and organizations for CIIP related issues.

Concepts such as risk, reliability, security, and resilience are studied within CIP in terms of the physical infrastructure attributes such as flows, pressure, and voltages. Although these attributes could provide good insights on the protection level, the cyber-physical interactions need to be considered. A large body of the existing research on cyber-physical systems highlights the difficulty of cyber-physical modelling.

In this section, we outline a general approach for studying the impact of cyber-physical interactions on critical infrastructure. First, we utilize the i2Sim multiple simulation layer to construct an Information and Communication Technologies (ICT) layer. The ICT layer can be designed using the i2Sim cell-channel approach or can be implemented using a domain-specific modelling approach. After that, the interactions between the ICT layer and the physical layer can be modelled using the i2Sim parameters in the physical layer, such as modifiers and distributors. For example, SCADA control signals can be interfaced to an i2Sim distributor to map the corresponding topology changes into the i2Sim distributor parameters. Since the focus of this chapter is resilience assessment, we consider the impact of ICT failures on the physical systems. For this purpose, we define four failure types of cyber-physical interactions as shown in Table 3.4. Each failure is mapped into the
Table 3.4: Types of cyber-physical failures.

<table>
<thead>
<tr>
<th>Failure Type</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>Fail to maintain control functionality</td>
<td>Loss of a control signal (e.g. opening a circuit breaker)</td>
</tr>
<tr>
<td>Monitor</td>
<td>Fail to maintain monitor functionality</td>
<td>Loss of a monitoring signal (e.g. water level in a tank)</td>
</tr>
<tr>
<td>Time</td>
<td>Fail to send/receive information within acceptable time frame</td>
<td>Delay of a monitoring signal</td>
</tr>
<tr>
<td>Value</td>
<td>Fail to send/receive the correct value</td>
<td>Error in a monitoring signal (e.g. voltage value)</td>
</tr>
</tbody>
</table>

i2Sim model using the appropriate parameter. For example, a delay in one of the ICT signals can be implemented using an i2Sim channel with a delay.

3.8 Improving Critical Infrastructure Resilience

The ultimate objective of resilience assessment studies is to suggest ways for improving infrastructure resilience. In many cases, it is very difficult, if not impossible, to predict the time and magnitude of the disruptive event. A resilient infrastructure is an infrastructure that withstands and survives disruptive events with minimum loss. There are two ways for improving CI resilience: 1) to be proactive (increasing the withstand capability) and 2) to be reactive (increasing the survive capability). Dalizell and McManus [37] refer to these two ways as reducing vulnerability and increasing adaptive capacity. Reducing vulnerability can be achieved by increasing robustness through hardening and redundancy. Also, ensuring adequate marginal capacity is one form of reducing vulnerability. Adaptive capacity is referred to as “the ability of a system to adjust to undesirable situations by undergoing some changes” [35]. A good example is the reconfiguration of power networks after failures to restore power supply to affected users. Reconfiguration and resources allocation are two important tools for increasing adaptive capacity. Table 3.5 and Table 3.6 list examples of actions related to infrastructure attributes described in Sec. 3.4.1 for improving infrastructure resilience.
Table 3.5: Examples of resilience actions for reducing vulnerability.

<table>
<thead>
<tr>
<th>Infrastructure Attribute</th>
<th>Reducing Vulnerability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static</td>
<td>Using underground power cables instead of overhead lines</td>
</tr>
<tr>
<td>Dynamic</td>
<td>Reducing electrical loads on substation transformers</td>
</tr>
<tr>
<td>Decision</td>
<td>Adding more security measures for personnel entering critical locations</td>
</tr>
</tbody>
</table>

Table 3.6: Examples of resilience actions for increasing adaptive capacity.

<table>
<thead>
<tr>
<th>Infrastructure Attribute</th>
<th>Increasing Adaptive Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static</td>
<td>Installing new alternate feeders in power networks</td>
</tr>
<tr>
<td>Dynamic</td>
<td>Reconfiguring network topology to restore supply</td>
</tr>
<tr>
<td>Decision</td>
<td>Developing disaster preparedness plans and procedures</td>
</tr>
</tbody>
</table>

This thesis proposes new approaches for improving CI resilience through resources allocation and network reconfiguration. In Chapter 4, different optimization approaches are used to allocate infrastructure resources after some disruptive events. In Chapter 5, a new approach for reconfiguring power distribution networks is proposed to account for infrastructure interdependencies during the restoration process.

3.9 Conclusion

The primary focus of this chapter is infrastructure resilience assessment. In this chapter, we propose a resilience assessment framework. The presented framework consists of three main components. First is the definition of the attributes that make an infrastructure resilient. The second component is the modelling approach which is built upon the Infrastructure Interdependencies Simulator (i2Sim) framework. The third component is the resilience index.

As highlighted in the relevant literature, the definition and quantification of re-
silence are highly dependent on the context and dimensions of the study. For the proposed framework, the context is emergency management in physical infrastructure. Its applicability is dependent on the modelling approach, which is shown to be able to capture the required resilience attributes. A generalized resilience index is proposed to measure the defined resilience dimensions. Also, we show that the proposed framework can study the impact of cyber-physical interactions on the physical infrastructure resilience.

Infrastructure resilience can be improved by either reducing its vulnerability or increasing its adaptive capacity. This thesis focuses on increasing infrastructure adaptive capacity through resources allocation and network reconfiguration. In this thesis, we propose several optimization algorithms that use the framework described in this chapter. Details of the proposed resources allocation and reconfiguration algorithms are described in Chapter 4 and Chapter 5.
Chapter 4

Optimal Resources Allocation for Resilience Improvement

4.1 Introduction

In the previous chapter, we discussed how infrastructure resilience can be improved by increasing its adaptive capacity through resources allocation. During extreme events, different infrastructures (power networks, water networks, health system, and communication networks, etc.) are affected simultaneously. Successful response to such events requires efficient allocations of available resources. This, in turn, requires effective coordination across the infrastructure systems. Considering their complexity, allocating resources within interdependent infrastructure systems is a major challenge for decision makers. Responsive resources allocation is determined based on the evolution of the event over time. Therefore, it can be formulated as a dynamic process that can be optimized for improving its effectiveness.

In this chapter, we build upon the modelling framework discussed in Chapter 2 and the resilience assessment framework discussed in Chapter 3 to formulate the resources allocation problem as an optimization problem. Two different opti-
mization algorithms are developed. The algorithms are designed to increase the infrastructure resilience through resources allocation. The first algorithm is based on Ordinal Optimization (OO) theory [4]. The second algorithm is based on a Linear Programming (LP) formulation. The OO algorithm can be classified as one of the simulation-based optimization algorithms, in which the simulation model is used to evaluate the candidate solution during the search process. The i2Sim model is used for evaluating the candidate resources allocation in every iteration in the algorithm. The LP algorithm uses the i2Sim mathematical formulation described in section 2.4. In addition to the optimization algorithms, a prioritization methodology is proposed for ranking critical components in multiple physical infrastructures, such as power networks, water networks, and health care facilities.

The rest of this chapter is organized as follows. Section 4.2 presents a brief literature review on the related work in resources allocation within interdependent infrastructure systems. The proposed OO algorithm is presented in Section 4.3. In Section 4.4, the LP algorithm is explained. The proposed prioritization methodology is presented in Section 4.5. A discussion and a conclusion are provided in Section 4.6.

4.2 Literature Review on Resources Allocation Optimization in CI

The scholarly works on resources allocation optimization in CI is a natural extension of the CI modelling and simulation literature. Since the literature on CI modelling and simulation is covered in chapter 2 of this thesis, we focus the review in this section on the most related work on optimization applications using CI models.

Zhang and Peeta [23] proposed a multilayered network framework to model interdependent infrastructure systems. Then, they use this framework to formulate an equilibrium problem using an economic-market approach in which each infras-
tructure system is treated as an economic sector. There are two dimensions of the equilibrium problem: market and network flow. For each infrastructure system, there are producing entities associated with production functions and there are consuming entities associated with consumption functions. Also, flow costs functions are defined in the problem. The problem is formulated as a non-linear programming problem that is solved to balance the demand and supply in the network. Systems disruptions are modelled by placing capacity constraints on the producing nodes. Since the framework considers the economic interactions in addition to the physical systems interactions, its application is more useful for studying long-term recovery plans. In addition, the modelling approach does not allow capturing many of the specific operational details of each network.

Building on the work of Lee et al. [66], Cavdaroglu et al. [29] considered the scheduling problem of restoration activities for interdependent infrastructure services after major disrupted events. The interdependent layer network flow model (ILN) of Lee et al. [66] is used to model the interactions between interdependent infrastructure systems. In every network, there are supply nodes and demand nodes with some operating costs associated with every node. At every time step, the layered networks model is solved to calculate the costs considering the systems’ status at that time step. Disrupted events are modelled by removing or adding arcs between the nodes. A mixed integer programming problem is formulated to solve the restoration problem for the restoration activities immediately after the event. Although the solution of the problem provides an optimum schedule for the restoration activities, the modelling approach does not capture the functional behaviour of the network (or nodes) which might affect the restoration plans.

Holden et al. [67] developed a network flow model to account for interdependencies between infrastructure systems. In this model, each node represents an infrastructure system and has certain properties that describe its processes: pro-
duction, consumption, transshipment, and storage. Each process is associated with a cost. A linear optimization problem is formulated to minimize the total cost in the model. The authors modelled the extreme events scenarios by placing appropriate constraints on the process variables. The description of the functional processes of nodes in the model makes this work different from the other network models. The production process in the node is described using a linear relationship in terms of other nodes’ outputs. This is a fundamental difference between this model and the i2Sim models.

In addition to the above work, there has been some research on optimizing interdependent infrastructure systems using network flow models: [26], [68], and [69]. All of these models use the basic structure: A network is represented by a set of nodes and a set of edges (or arcs). A resource flow is represented by a flow variable for every edge or arc. The basic difference between these models lies in the methodology of calculating the optimum value for the flow variables.

Another approach in the literature for optimizing CI resources is to use simulation-optimization approaches for optimizing CI models such as i2Sim, Aspen, and CIMS. Although the models are built using different modelling approaches such as agent-based or system dynamics, their optimization approach is similar in which the optimization agent (or engine/model) interacts iteratively with the CI models in the process of finding an optimum solution. For example, Permann [70] used Genetic Algorithm with the CIMS models for determining restoration strategies in the case of disasters.

Following the simulation-optimization approach, Khouj et al. [71] developed an intelligent decision-making system for supporting emergency responders during disaster events. The proposed system consists of two major components: a learning agent and an infrastructure simulator. The learning agent is based on Reinforcement Learning technique which aims at developing a policy for allocating the resources
during the event. The learning agent uses experiences from either a knowledge database or a Monte Carlo simulations using the i2Sim simulation. The output of the decision-making system is the efficient allocation of resources along the event time line.

Fiedrich et al. [72] considered the problem of optimizing search and rescue activities after major natural events. A network model consisting of nodes and edges is used to represent the infrastructure. Resources are considered to be machines and equipment for the rescue and search activities. A dynamic optimization problem is solved to find an optimal resources scheduling using two simulation-optimization techniques: Simulated Annealing and Tabu Search. This work does not consider resources flow between infrastructures but it is included here for its similarity with the problem formulation presented in this thesis.

CI optimization applications are crucial for CIP planning and development. The above review shows different optimization applications employing a variety of modeling approaches and focusing on different objectives. The work presented in this thesis focuses on improving the CI resilience as its primary objective. The resilience-centered optimization application considering infrastructure interdependencies presents a new advancement to the field of CIP.

4.3 Resources Allocation using Ordinal Optimization-Based Algorithm

4.3.1 Overview of Ordinal Optimization

Ordinal Optimization OO is a new optimization theory introduced by Ho in the 1990s for providing fast Good Enough solutions for complex simulation-based optimization problems [4]. It has been applied to solve many problems in different disciplines, such as power systems [73], communication networks [74], resources allocation in manufacturing systems [75], scheduling of parallel computing systems [76], and robotics
motion control systems [77]. Ordinal optimization tries to overcome difficulties in existing optimization theories in solving problems that have exponential growth in its search space and computational complexity in its simulation models.

Ordinal Optimization is based on two main concepts: 1) **Order Comparison**: it is easier to determine order than value, i.e., determining A > B is easier than determining the value of A-B=? and 2) **Goal Softening**: instead of looking for the best for sure, we look for good enough with high probability. Using these two concepts, ordinal optimization methods provide a set of Good Enough solutions in an order of their performance. In many practical applications, it is enough to find good enough solutions instead of insisting on finding the true optimum solution which may exhaust the available computational resources. This rationale motivates the application of ordinal optimization to the resources allocation problem addressed in this thesis. During a disaster, responders are under pressure to save lives and mitigate disaster impacts. In these emergency situations, they can accept a fast “good enough” solution instead of waiting for the optimum one. Also, this application can be very useful for playing response scenarios in planning and training activities.

The application procedure of Ordinal Optimization can be summarized as follows:

1. Randomly or heuristically sample $N$ solutions from the entire solutions space.

2. Use a crude and computationally fast model to estimate the performance of these $N$ solutions.

3. Estimate the Ordered Performance Curve (OPC) class of the problem and the error level of the crude model. Then, specify the size of the good enough set $g$ and the required alignment level $K$.

4. Use the Universal Alignment Probability Tables to calculate the size of the selected set $s=f(g,k/OPC\ class, \ error\ level)$.
5. Select the observed top $s$ solutions of the $N$ as estimated by the crude model as the selected set $S$. The theory of OO ensures that $S$ contains at least $s$ truly good enough solutions with probability no less than 0.95.

As can be seen from the above procedure, the key idea of ordinal optimization is to find a selected set $S$ of solutions with an acceptable probability to be a member of the good enough set $G$ as shown in Figure 4.1. The good enough set is defined as the top $n$ solutions of the entire solution space $\Theta$. Therefore, the problem is changed from finding the optimum resources distribution over the entire space to finding a set of distributions that has an overlap with the top $n$ solutions in the entire solutions space $\Theta$.

**Ordered Performance Curve**

The size of the selected set $S$ is determined by the shape of the Ordered Performance Curve (OPC). The OPC curve is a conceptual plot of the candidate solutions as a function of their ordered performance, i.e., the best, the second best, and so on. If we perform an exhaustive evaluation of all solutions in the entire space $\Theta$ and we rank them from low to high, we can get one of the curves in Figure 4.2 where $J$ represents the evaluation function. There are five classes of OPC curves [4] as shown in Figure 4.2: Flat, Steep, Bell, U-shaped, and Neutral. The OPC class

![Figure 4.1: Ordinal Optimization [4].](image-url)
can be obtained by past experience or by a one-time pre-processing over the entire solutions of a particular problem. This step is useful to understand the shape of the search space of the problem of interest.

4.3.2 Problem Formulation

The resources allocation problem in disaster response is considered as a combinatorial constrained optimization problem. The objective of the optimization problem is to maximize the resilience of the infrastructure systems after a disruption. Since saving lives is the highest priority during emergencies, we formulate the objective function to maximize the overall resilience of the modelled infrastructure using the generalized resilience index GRI as follows:

$$\text{Maximize} \quad GRI = f\{R_d, R_y, R_t, R_c\}$$

(4.1)

The GRI function is described in chapter 3 using 3.11, 3.12, 3.13, 3.14 and 3.15. Since we are using resources allocation as a tool for improving the infrastructure resilience,
we focus this formulation on the resilience dimension that is affected by the process of resources allocation: dynamic performance recovery; i.e., $GRI = f\{R_d\}$. The performance of the infrastructure systems is measured by the hospitals’ outputs. The hospitals’ outputs are normalized to a common base since they have different capacities. This formulation is based on the assumption that all response efforts are focused on providing the needed resources to the modelled hospitals. If different assumptions are considered, the output function $y(t)$ is defined accordingly.

The constraints of the problem are the availability constraints and the capacity constraints. The availability constraints are modelled using the $i2Sim$ sources in which $S_m(t)$ represents the amount of available resource $m$ at time $t$. For example, at $t = 4$, the available electrical power from the HV system to substation P1 is limited to $S_{p1}(4) = 15\text{MW}$. The capacity constraints are modelled using the distribution ratios in $i2Sim$ distributors. For example, it is possible to have enough power at the substation but only limited power supply is delivered due to cables capacity constraints (either thermal or voltage limits) in the power networks.

The control variables of the problem are the $i2Sim$ distributors’ ratios: $a_1, a_2, \ldots, a_k$, where $k$ is the total number of distributors outputs in the model. For every set $a=[a_1 \ a_2 \ \ldots \ a_k]$, there is a corresponding output in every cell in the model. A candidate solution is the evolution of the set $a$ over the simulation time. This can be written in a matrix form as follows:

$$S_i = \begin{bmatrix} a_{1t1} & a_{1t2} & a_{1t3} & \ldots & a_{1tT} \\ a_{2t1} & a_{2t2} & a_{2t3} & \ldots & a_{2tT} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{kt1} & a_{kt2} & a_{kt3} & \ldots & a_{kTt} \end{bmatrix}$$

(4.2)

where $a_{ktT}$ is the $k^{th}$ output at time $T$ and $T$ is the total simulation time. We set an upper and lower bounds for the control variables such that $a_{k\text{min}} \leq a_k \leq a_{k\text{max}}$. 

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The upper and lower bounds are problem specific and can be determined based on the physical constraints in the model.

### 4.3.3 OO-Based Algorithm

In this section, we propose an Ordinal Optimization-based approach to solve this problem in two stages. The first stage searches through the solution space to produce candidate solutions using a feasibility crude model and the second stage searches through the candidate solutions to find the optimum one. Since most of the computational cost is due to running the i2Sim model for evaluating the candidate solutions, the decomposition of the search process is important to filter out non-feasible solutions in advance. The interaction between the algorithm and the i2Sim model is shown in Figure 4.3 and the flow chart for the Resources Allocation using Ordinal Optimization (RAOO) algorithm is shown in Figure 4.4.
Figure 4.4: RAOO Algorithm.

The first step in stage one is to sample \( N \) solutions from the search space. Instead of using the standard pseudo-random generators, we use the Low Discrepancy
Sequence (LDS) technique (alternatively called Quasi-Monte Carlo (QMC)). The LDS sampling technique employs specific number-theoretic algorithms, such as Van der Corput sequence and Halton sequence, to produce a uniform and even distribution of samples compared to the standard algorithms [78]. Figure 4.5 shows a comparison between two sample sets generated using the LDS technique and the standard pseudo-random technique. In the implementation of the LDS sampling process, each control variable is considered as an independent dimension of the sample set. For every sampling iteration, a candidate solution in the matrix form 4.2 is generated. The generated candidate solutions are then evaluated using the i2Sim crude model. The crude model is used to eliminate the non-feasible solutions using the following feasibility checks:

1. Total output of every distributor at every time step is less than or equal to 1.
2. The output of every aggregator at every time step is the sum of the inputs.
3. Each control variable is within the upper and lower limits.

The feasibility checks can be expressed mathematically as follows:

\[
\sum_{j=1}^{k} a_j t \leq 1 \\
A_m = \sum_{j=1}^{n} a_j t \\
\ a_{j}^{min} \leq a_j \leq a_{j}^{max}
\]

\[
(4.3)
\]

where \(k\) is the number of outputs in the distributor, and \(n\) is the number of inputs to the aggregator \(m\).

Once a set of feasible solutions is obtained, the size of the selected set is calculated using the following function [4]:

\[
Z(k, g) = e^{Z_1}k^{Z_2}g^{Z_3} + Z_4
\]

\[
(4.4)
\]
where $Z_1$, $Z_2$, $Z_3$, and $Z_4$ are regression constants depending on the OPC class of the problem as defined in Section 4.3.1. In the second stage of the OO-Algorithm, every candidate solution in the selected set $S$ is evaluated using the $i2Sim$ model and the optimum solution is found. Therefore, the computational efficiency is improved by limiting the number of $i2Sim$ model evaluations to the feasible solutions only.

4.3.4 Case Study Results

This section presents the numerical results of experiments based on the RAOO algorithm proposed in Section 4.3. The experiments are conducted for the case study data presented in Chapter 2 and the $i2Sim$ model is shown in Figure 2.9. The primary objective is to identify the potential value of optimizing the allocation of available resources for improving the infrastructure resilience. The extreme event scenario described in Chapter 2 is modelled by varying the Physical Modes (PM) values in the $i2Sim$ cells and channels in the model. The scenario duration is assumed to be 48 hours and the simulation time step is 5 minutes (576 time steps). The $i2Sim$ model and the OO-algorithm are implemented in MATLAB/Simulink.

We assume that all three hospitals have the same importance, i.e., $w_{H1}=w_{H2}=w_{H3}=1$. 

Figure 4.5: Sample sets using LDS vs. standard Pseudo-random generators.
The defined reference performance level $y_d$ is defined as 1 p.u. The value of the $N$ in the OO-algorithm is set to 1000. The OPC curve for the case study is shown in Figure 4.6. The resilience index was adjusted with negative values to convert the problem to a minimization problem following the convention of the OPC classes as described in section 4.3.1. The OPC plot for the case study is described by the “Bell class”, i.e., many moderate solutions, few bad solutions and few good solutions. The values of the corresponding regression constants $Z_1$, $Z_2$, $Z_3$, and $Z_4$ are 8.1998, 1.9164, -2.0250, and 10, respectively. Choosing $g=50$ and $k=1$ in 4.4, the size of the selected set $S$ is 11.

A comparison of the results from the base case scenario and the optimized scenario (by the RAOO algorithm) is shown in Table 4.1. Although the OO-algorithm does not guarantee the global optimal solution, its solution results in a considerable improvement to the infrastructure resilience. The resilience index ($R_d$) in the optimized case scenario is improved by 78% compared to the base case scenario as
Table 4.1: Comparison of results.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$R_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>0.407</td>
</tr>
<tr>
<td>Optimized Case</td>
<td>0.727</td>
</tr>
</tbody>
</table>

a result of optimizing the allocation of available resources. In both cases, the final performance level is the pre-event level and the recovery time is the same. As a result both cases have the same $R_y$ and $R_t$ values. The objective of the optimization is to increase the infrastructure resilience through increasing its adaptive capacity which is affected by the dynamic performance of the infrastructure. Therefore, $R_d$ is expected to change as a result of re-distributing the resources. This can be seen in the results in Table 4.1.

The resilience index $R_d$ in the base case scenario is $R_d=0.407$ which corresponds to the output of affected infrastructure systems without any redistribution for the available resources. When the distribution of available resources is optimized, i.e., the amount of electricity and water supplied to the hospitals are changed during the event, the value of $R_d$ is improved to 0.727. Figure 4.7 shows a comparison of hospitals’ outputs in the two scenarios. It can be seen that Hospital 1 is fully restored in the optimized scenario while the other two hospitals have fluctuations in their outputs. This can be attributed to the availability of two redundant power supplies in Hospital 1 compared to a single power supply for Hospital 2 and Hospital 3. This result emphasizes the importance of network topology in improving the infrastructure resilience.

Figure 4.8 shows a comparison of the outputs from Power Substation 2, Power Substation 4, and Water Station 1. The output of Power Substation 2 is reduced in the optimized scenario compared with the base case scenario. The output of Water Station 1 is fully restored compared in the optimized scenario compared to the base case scenario. Note that both, Power Substation 2 and Water Station 1, are
supplied from the Power substation 4. Also, both of them are the primary supplies for Hospital 1. The Water station 1 is given a higher priority, by increasing its power supply, over Power substation 2 due to its importance to Hospital 1. Since Hospital 1 has two redundant supplies of electrical power, its water supply (from Water station 1) becomes its limiting factor and consequently the output of Water station 1 becomes an important factor in the process of improving the infrastructure resilience. This result shows the importance of infrastructure interdependencies in improving infrastructure resilience. The effect of the limiting factor shows that improving one infrastructure system does not necessarily improve the overall infrastructure resilience. Such result can be very useful for decision makers to direct infrastructure’s investment to improve the limiting factors within the infrastructure.

The output of Power substation 4 does not change in both scenarios. The reason is that it receives its input from the high voltage transmission system and the prob-
lem formulation does not include any control variable for its input. Although Power substation 4 is affected by the event scenarios due to failures in its transformers, its output cannot be improved by means of reconfiguration. Recall that infrastructure resilience can be improved by either increasing the survive capability or the withstand capability. In the case of Power Substation 4, the results shows that the potential improvement in its survive capability is limited and more attention should be given to its withstand capability.

The results presented in this section illustrate that the proposed framework is able to capture key aspects of infrastructure resilience. Also, the modelling approach captures the impacts of infrastructure interdependencies on the overall infrastructure resilience. An important result is the identification of the limiting factors in the process of improving infrastructure resilience. This result can be used by decision makers to direct available investments toward improving these limiting factors.
The next question for decision makers is how to improve the limiting infrastructure system: by improving surviving capability, e.g., adding more flexibility and redundancy, or by improving withstanding capability, e.g., physical or structural support. The proposed framework and optimization algorithm can give important insights on how to plan the required investment.

4.4 Resources Allocation using Linear Programming

4.4.1 Introduction

The Resources Allocation using Ordinal Optimization algorithm described in the previous section provides a way to obtain a "good enough" solution considering the computational effort required to run a simulation model in i2Sim. The basic idea of RAOO algorithm is to have an idea about the search space of the problem, filter out the non-feasible solutions using a crude model, and then search through the feasible ones using the simulation model. Although this algorithm reduces the required number of simulation runs compared with other techniques such as Genetic Algorithm, it still requires running a number of computer simulations which makes it more expensive computationally than evaluating analytical functions.

The i2Sim model can be described using a system of equations as described in chapter 2. This system of equations is composed of cells’ equations, aggregators’ equations, and distributors’ equations. Using this system of equations, a mathematical optimization problem can be formulated. In this section, the resources allocation problem is formulated as a linear programming optimization problem. Linear programming problems can be solved efficiently using available methods such as Simplex method and Interior-Point method [79].
4.4.2 Problem Formulation

The problem formulation in this section uses the i2Sim mathematical formulation described in section 2.4. Consider an i2Sim model with $N$ cells, $n=[1,2,...,N]$, $M$ aggregators, $m=[1,2,...,M]$, and $K$ distributors, $k=[1,2,...,K]$. Every cell has an output $y_c \in [y_{c1},...,y_{cn}]$ and is given by:

$$y_c(t) = \min \{f_1(t), f_2(t), ..., f_n(t)\} \quad (4.5)$$

where $f_1$ to $f_n$ are the the functions relating every input to the output $y_c(t)$ and $n$ is the number of inputs. In the optimization problem, the cell’s output function can be replaced by a set of $n$ linear inequalities:

$$y_c(t) \leq f_1(t)$$
$$y_c(t) \leq f_2(t)$$
$$\vdots$$
$$y_c(t) \leq f_n(t) \quad (4.6)$$

Note that the inequalities are expressed in terms of $f_n$ and not in terms of the cell’s input $x_n$. For a linear formulation, $f_1$ to $f_n$ need to be linear. The i2Sim distributor is modeled by a linear equality constraint such that:

$$y_d(t) = x_1(t) + x_2(t) + ... + x_n(t) \quad (4.7)$$

where $x_1(t)$ to $x_n(t)$ are the distributor’s outputs and $y_d(t)$ is the distributor’s input. Unlike the distributor’s equation in the OO-Algorithm, the summation here does not need to be 1 since the equation is expressed in terms of the i2Sim token variables and not the distributor’s ratios. Note that the equality in this constraint implies that we are using all the output of the cell $y_c(t)$. If the scenario does not need to utilize all
the output $y_c(t)$, then we can replace the equality sign by a $\geq$ sign which will place an upper limit on the distributor’s outputs. This can be useful in case there is a cost for the resources and we only need to use some but not all of the cell’s output.

The aggregators equation is modelled by a linear equality constraint such that

$$y_a(t) = x_1(t) + x_2(t) + \ldots + x_n(t)$$

(4.8)

where $x_1(t)$ to $x_n(t)$ are the aggregator’s inputs and $y_a(t)$ is the aggregator’s output.

The objective function in this problem is to maximize the infrastructure resilience:

$$\text{Maximize } GRI = \min \{R_{D_1}, R_{D_2}, \ldots, R_{D_n}\}$$

(4.9)

This objective function is not a linear function due to the presence of Max-Min operators but it can be converted to a linear function by following the same transformation in 4.6:

$$\text{Maximize } GRI$$

such that

$$GRI \leq R_{D_1}$$

$$\vdots$$

$$GRI \leq R_{D_n}$$

(4.10)

where the functions $R_{D_1}, \ldots, R_{D_n}$ are linear with respect to the decision variables of the problem. Similar to the problem formulation in the OO-Algorithm, we focus this formulation on the resilience dimension that is affected by the process of resources allocation: dynamic performance recovery; i.e., $GRI = f\{R_d\}$. The decision variables in this formulation are the cells’ outputs and the distributors’ outputs at every time step. The Resources Allocation using Linear Programming (RALP) model is summarized in table 4.2.
Table 4.2: RALP Optimization Model

<table>
<thead>
<tr>
<th>RALP Optimization Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inputs</td>
</tr>
<tr>
<td>$N$ set of cells</td>
</tr>
<tr>
<td>$M$ set of aggregators</td>
</tr>
<tr>
<td>$K$ set of distributors</td>
</tr>
<tr>
<td>$S$ set of sources</td>
</tr>
<tr>
<td>$T$ set of time steps</td>
</tr>
<tr>
<td>$L$ set of loads (sinks)</td>
</tr>
<tr>
<td>$J$ set of outputs at distributor $k$</td>
</tr>
<tr>
<td>$I$ set of outputs at distributor $m$</td>
</tr>
<tr>
<td>$LB$ Decision variables lower bound</td>
</tr>
<tr>
<td>$UB$ Decision variables upper bound</td>
</tr>
<tr>
<td>Decision Variables</td>
</tr>
<tr>
<td>$y_{cn}(t)$ $n \in N$ cell output</td>
</tr>
<tr>
<td>$x_{kj}(t)$ $k \in K, j \in J$ distributor output</td>
</tr>
<tr>
<td>Objective Function</td>
</tr>
<tr>
<td>Maximize $GRI$</td>
</tr>
<tr>
<td>Constraints</td>
</tr>
<tr>
<td>$GRI \leq R_D$</td>
</tr>
<tr>
<td>$y_c(t) \leq f_n$ $n \in N$ Cells’ output functions</td>
</tr>
<tr>
<td>$y^k_d(t) = \sum_{j}^{J} x_{kj}$ $j \in J$ Equation for distributor $k$</td>
</tr>
<tr>
<td>$y^m_a(t) = \sum_{i}^{I} x_{mi}$ $i \in I$ Equation for aggregator $m$</td>
</tr>
<tr>
<td>$y_c(t), x_{kj}(t) \leq ub(t)$ $ub(t) \in UB$ Upper bound at time $t$</td>
</tr>
<tr>
<td>$y_c(t), x_{kj}(t) \geq lb(t)$ $lb(t) \in LB$ Lower bound at time $t$</td>
</tr>
</tbody>
</table>

4.4.3 Case Study Results

This section presents the numerical results of the experiments based on the RALP algorithm. The experiments are conducted for the case study data presented in Chapter 2 and the i2Sim model is the same one shown in Figure 2.9. The primary objective is to identify the potential value of optimizing the allocation of available resources for improving the infrastructure resilience. The extreme event scenario described in Chapter 2 is modelled by placing constraints on the corresponding variables in the model. The scenario duration is assumed to be 48 hours and the simulation time step is 5 minutes (576 time steps + 1 pre-event time step). The RALP
Table 4.3: RALP Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Variables</td>
<td>17311</td>
</tr>
<tr>
<td>No. of Time Steps</td>
<td>577</td>
</tr>
<tr>
<td>No. of Inequality Constraints</td>
<td>8656</td>
</tr>
<tr>
<td>No. of Equality Constraints</td>
<td>4039</td>
</tr>
</tbody>
</table>

Table 4.4: Comparison of results.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$R_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>0.407</td>
</tr>
<tr>
<td>Optimized Case</td>
<td>0.866</td>
</tr>
</tbody>
</table>

algorithm is implemented in MATLAB. We assume that all the three hospitals have the same importance, i.e., $w_{H1}=w_{H2}=w_{H3}=1$. The defined reference performance level $y_d$ is defined as 1 p.u. Table 4.3 shows the parameters of the RALP model for this case study.

A comparison of the results from the base case scenario and the optimized scenario by the RALP algorithm is shown in Table 4.4. The optimum value for the resilience index is 0.866 which is double the base case index. This result highlights the huge potential for improving infrastructure operations by re-allocating available resources during extreme events. Figure 4.9 shows a comparison of the hospitals’ outputs in the two scenarios. It can be seen that Hospital 1 and Hospital 2 can be fully operational if the available resources are optimized. Due to its smaller capacity compared to the other two hospitals, Hospital 3 is not fully restored.

The outputs of Power substation 2, Power substation 4, and Water station 1 are shown in Figure 4.10. The results here match the results of the RAOO algorithm in the following observations. First, nothing can be done to improve the output of Power substation 4 since it gets its output from the high voltage transmission system which is not included in the control variables of the problem. Second, the water supply from Water station 1 is recognized as an important factor in improving
Figure 4.9: Comparison of the hospitals’ outputs using RALP algorithm.

the hospitals’ outputs. One additional observation in the results here is that the optimal solution by RALP does not utilize all the outputs of Power substation 2 and Power substation 4. This result can be useful for determining the required topology improvements in order to improve the overall infrastructure flexibility in facing extreme events.

4.5 Prioritization of CI Systems

One of the main actions for improving CI resilience is to develop effective disaster preparedness plans. Disaster preparedness is a huge challenge, but the consequences of being unprepared can be devastating. Disaster preparedness is a preparation phase in the disaster management process. It includes all the activities that need to be implemented before a disaster strikes. A typical preparedness activity is infrastructure reinforcement, such as installing emergency power generators or building
Prioritization methodologies can help in developing cost-effective investment plans for disaster management agencies. In this section, a prioritization methodology is proposed for ranking critical infrastructure systems in multiple physical infrastructures, such as power networks, water networks, and healthcare facilities. The proposed methodology utilizes the i2Sim modeling framework to assess different failure scenarios. A single failure in one system, e.g., a water pumping station, can cause huge degradation in the operations of multiple infrastructures due to the mutual interactions between them. The i2Sim modelling approach used in this section allows for capturing these interactions which improves the effectiveness of the prioritization methodology.
The problem of prioritizing and ranking critical infrastructure systems has not gained enough attention and is often treated within the vulnerability and risk assessment studies. For example, a screening methodology based on Multi-Attribute Utility Theory (MAUT) and graph theory is proposed in [81] for ranking vulnerable buildings in the campus of the Massachusetts Institute of Technology. Another methodology is proposed in [82] for identifying critical sets of components in large scale technical infrastructures. This methodology is based on measuring failure consequences in an electrical power network. Also, social network analysis is used in [83] to determine the priority of a railway infrastructure assets. Most of the existing works use topological models, represented as graphs, to measure the impact on the critical infrastructure systems and only a few studies considered a functional model, e.g. [82]. Those who use functional models limit their analyses to one infrastructure system without considering interdependent interactions with other systems.

4.5.1 Critical Infrastructure Ranking

A ranking list of critical infrastructures is extremely useful in planning emergency management investments. As a basic concept, the importance of a system component depends on the damage caused by its failure or absence from the system. Therefore, a ranking approach can be developed based on measuring the impact of a failure of one (or more) critical infrastructure in a modelled area. The impact (consequence) needs to be quantified and an importance measure is used to rank the critical infrastructures. Different importance measures can be developed based on the methodology and models used in the analysis. The methodology proposed in this section can be described as follows:

1. Define the critical infrastructure systems to be included in the analysis.

2. Build the i2Sim model for the infrastructure under consideration.

3. Generate a failure scenario in which one of the considered infrastructure sys-
tems is affected.

4. Evaluate the consequence of that failure and then calculate the importance measure (IM). The importance measure is defined as the relative performance drop after the failure. It can be expressed as follows:

\[ IM = \frac{y_{\text{normal}} - y_{\text{failure}}}{y_{\text{normal}}} \]  

(4.1)

where \( y \) is the chosen performance measure.

5. Repeat Step 3 for all the infrastructure systems considered for ranking.

6. Generate the ranking list according to the IM values.

The failure scenarios in Step 3 are represented by failure sets. Each set has the infrastructure components to be affected. The number of possible failure sets increases rapidly as the set size increases and is given by

\[ \frac{k!}{(k-n)!n!} \]  

(4.2)

where \( k \) is the total number of components in the model and \( n \) is the size of the failure set. For instance, a model with 100 components has 161,700 failure sets of size 3. Enumerating all possible failure sets is impractical. One possible way is to consider only failure sets with high failure probabilities. In this section, only failure sets of sizes 1, 2, and 3 are considered. Also, the use of i2Sim modelling approach allows for considering different failure modes; e.g., total failure (0% operability level) and partial failure (50% operability level). Such consideration is not possible in many of the topological models used in the literature.
4.5.2 Case Study Results

In this section, the proposed methodology is applied to the case study described in chapter 2. The i2Sim model for this case is shown in Figure 2.9. The selected infrastructure systems from the model for the study are the two main hospitals, the four power substations, one water pumping station. Two non-critical infrastructure cells are added to account for the residential and commercial buildings. The input-output function of each cell provides the necessary behavioural information for measuring the impact on the modelled infrastructure systems. The topological information is captured by the arrangements of channels, distributors and aggregators as shown in Figure 2.9. In the context of disaster management, saving lives has the highest priority. Therefore, the total output of the two hospitals is chosen to be the performance measure for this test case. The output of the hospital model in i2Sim is the rate of treating patients, i.e. number of treated patients per hour. The importance measure is calculated as the drop in the hospitals output normalized to the rated output.

Three sizes of failure sets are considered: 1, 2, 3. The total number of combinations is 9 (for size 1) + 36 (for size 2) + 84 (for size 3) = 129 failure sets. For every failure set, two failure modes are tested: total failure (100% damage) means that the infrastructure system is totally out of service, and partial failure (50% damage) means that the infrastructure system is affected by the disaster but it can provide some outputs. An example of the partial failure mode is the failure of one main transformer in a power substation with two main transformers. It is possible to generalize the test for more partial failure modes using the i2Sim discretized operability levels as described in chapter 2.

Results for the total failure modes for the top ranking failure sets are shown in Table 4.5. It is observed that the top two critical infrastructures appear in the top of size 1 sets with IM=1, meaning that total damage to any of these two
infrastructures will completely affect the two hospitals. Even though each hospital has two redundant power supplies from two different power substations as shown in Figure 4, power station 3 can cause a major interruption to both hospitals. This can be attributed to the interdependency phenomenon since it also supplies the only water pumping station in the system.

Table 4.5: Ranking of failure sets for 100% failure mode.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Size=1</th>
<th></th>
<th>Size=2</th>
<th></th>
<th>Size=3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Failure Set</td>
<td>IM</td>
<td>Failure Set</td>
<td>IM</td>
<td>Failure Set</td>
<td>IM</td>
</tr>
<tr>
<td>1</td>
<td>W1</td>
<td>1</td>
<td>W and NC2</td>
<td>1</td>
<td>W and NC1 and NC2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>P3</td>
<td>1</td>
<td>W and NC1</td>
<td>1</td>
<td>W and H2 and NC2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>H1</td>
<td>0.625</td>
<td>W and H2</td>
<td>1</td>
<td>W and H2 and NC1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>P2</td>
<td>0.450</td>
<td>W and H1</td>
<td>1</td>
<td>W and H1 and NC2</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>P4</td>
<td>0.375</td>
<td>P4 and W</td>
<td>1</td>
<td>W and H1 and NC1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>H2</td>
<td>0.375</td>
<td>P4 and H1</td>
<td>1</td>
<td>W and H1 and H2</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>P1</td>
<td>0</td>
<td>P3 and W</td>
<td>1</td>
<td>P4 and W and NC2</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>NC2</td>
<td>0</td>
<td>P3 and P4</td>
<td>1</td>
<td>P4 and W and NC1</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>N1</td>
<td>0</td>
<td>P3 and NC2</td>
<td>1</td>
<td>P4 and W and H2</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
<td>-</td>
<td>P3 and NC1</td>
<td>1</td>
<td>P4 and W and H1</td>
<td>1</td>
</tr>
</tbody>
</table>

Results for failure sets of sizes 2 and 3 show that the ranking is dominated by the presence of the two critical infrastructures: power station 3 and water station. We know that they are critical in themselves from the results of failure sets of size 1. Therefore, they cause the highest impact regardless of which other infrastructure fails with them. Failure sets containing these two critical infrastructures can be filtered out to show the ranking of other important sets. In fact, some less critical infrastructures can cause tremendous damage if they fail simultaneously. Consider, for instance, the 6th failure set in the size 2 list in which power station 4 and hospital 1 can cause a major drop in the system performance if they fail together.

The results for the 50% failure mode are shown in Table 4.6. It can be seen that the two critical infrastructures, power station 3 and water station, are still at the top of the ranking in size 1 sets. However, the partial failure mode shows a difference in the criticality, 0.575 for power station 3 and 0.525 for water pumping station.
Table 4.6: Ranking of failure sets for 50% failure mode.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Size=1 Failure Set</th>
<th>IM</th>
<th>Size=2 Failure Set</th>
<th>IM</th>
<th>Size=3 Failure Set</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P3</td>
<td>0.575</td>
<td>P4 and H1</td>
<td>0.750</td>
<td>P4 and H1 and H2</td>
<td>0.825</td>
</tr>
<tr>
<td>2</td>
<td>W</td>
<td>0.525</td>
<td>P3 and H1</td>
<td>0.750</td>
<td>P3 and H1 and H2</td>
<td>0.825</td>
</tr>
<tr>
<td>3</td>
<td>H1</td>
<td>0.500</td>
<td>H1 and H2</td>
<td>0.750</td>
<td>P2 and P3 and H1</td>
<td>0.825</td>
</tr>
<tr>
<td>4</td>
<td>P2</td>
<td>0.325</td>
<td>W and H1</td>
<td>0.700</td>
<td>P2 and P3 and H2</td>
<td>0.775</td>
</tr>
<tr>
<td>5</td>
<td>P4</td>
<td>0.250</td>
<td>P2 and P3</td>
<td>0.650</td>
<td>W and H1 and H2</td>
<td>0.750</td>
</tr>
<tr>
<td>6</td>
<td>H2</td>
<td>0.250</td>
<td>P3 and H2</td>
<td>0.575</td>
<td>P4 and W and H1</td>
<td>0.750</td>
</tr>
<tr>
<td>7</td>
<td>P1</td>
<td>0</td>
<td>W and H2</td>
<td>0.575</td>
<td>P4 and H1 and NC2</td>
<td>0.750</td>
</tr>
<tr>
<td>8</td>
<td>NC2</td>
<td>0</td>
<td>P4 and W</td>
<td>0.575</td>
<td>P4 and H1 and NC1</td>
<td>0.750</td>
</tr>
<tr>
<td>9</td>
<td>NC1</td>
<td>0</td>
<td>P3 and W</td>
<td>0.575</td>
<td>P3 and W and H1</td>
<td>0.750</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
<td>-</td>
<td>P3 and P4</td>
<td>0.575</td>
<td>P3 and P4 and H1</td>
<td>0.750</td>
</tr>
</tbody>
</table>

Figure 4.11: Comparison of the rankings in two different failure modes.

This result suggests that different failure modes can result in different criticality rankings. This also can be seen in the comparison between the rankings of the two failure modes results shown in Figure 4.11.
Table 4.7: Comparison of results.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$R_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>0.407</td>
</tr>
<tr>
<td>RAOO</td>
<td>0.727</td>
</tr>
<tr>
<td>RALP</td>
<td>0.866</td>
</tr>
</tbody>
</table>

4.6 Discussion and Conclusion

During disasters, effective disaster response is a key characteristic of resilient CI systems. It is expected to encounter shortages of physical resources, e.g., power and water, after disasters. Resilient infrastructure systems must be able to effectively use these resources. Managing physical CI systems to optimize the use of the available resources is a challenging task. This task is formulated as a resources allocation problem during disasters. An optimum allocation of available resources can greatly improve response effectiveness. The interdependencies between different infrastructure systems play a crucial role in the optimization process.

In this chapter, two algorithms for helping disaster responders are proposed: RAOO and RALP. The objective function in both algorithms is to maximize the resilience of the infrastructure. Both algorithms use the i2Sim modelling framework for modeling and simulating the behaviour of the infrastructure. The i2Sim model accounts for the interdependencies between infrastructure systems when solving the problem. Both algorithms solve the problem effectively and improve the CI resilience compared to the base case scenario as shown in table 4.7.

The RAOO and RALP differ in two aspects: the optimization technique used and the i2Sim implementation. RAOO uses a heuristic optimization technique based on the Ordinal Optimization theory. This type of optimization techniques does not guarantee an optimal solution but it provides a "good enough" solution as compared to the RALP solution as shown in table 4.7. RALP uses a linear programming formulation which is considered an exact mathematical optimization technique. The i2Sim model
in RAOO is built using the simulator implementation in the MATLAB/Simulink environment while in RALP a mathematical formulation is used to represent the i2Sim model within the linear programming formulation.

Although the RALP algorithm can provide better solutions due the use of exact mathematical optimization technique, it requires more implementation efforts especially when used by emergency responders. The RAOO algorithm can be used in a modular approach in which the optimization technique and the infrastructure model are separate modules in a disaster management system as shown in figure 4.12. The advantage of using this modular approach is that one can modify the infrastructure model with limited changes to the optimization algorithm. Also, other CI models, such as Geographical Information System (GIS), can be integrated with a minimal effort.

In addition to RAOO and RALP algorithms, a prioritization methodology for CIP decision makers is proposed. The methodology facilitates the ranking of critical components in multiple infrastructure systems. Using this methodology, emergency management agencies can direct their investments to the most critical systems (or components) in the considered locations for improving the overall resilience. The modelling approach in the proposed methodology provides several advantages. First,
it considers the interdependent relationships between different infrastructure systems. Also, it allows for simulating the functional behaviour of the modelled infrastructures in contrast to the topological models. In addition, different failure modes can be simulated which expands the scope of the analysis.

Analysis of the test case reveals some interesting results. The interdependencies among the critical infrastructures play an important role in the ranking process. For example, the importance of one of the power stations increases because it supplies power to a water station that supplies water to a hospital. Also, when different failure modes are considered, the ranking has changed. This observation stresses the importance of including infrastructure interdependencies in the analysis.
Chapter 5

Resilience of Power Distribution Networks

5.1 Introduction

Electrical power systems are among the CI systems in our modern societies. The availability of electrical power supply is essential for our daily economic, business, and social activities. In the wake of recent climate-related events, resilience of electrical systems has become a major concern for utilities and the CI governmental agencies. Power system has been operating using reliability and risk concepts which are well understood and developed concepts. However, the concept of resilience is still a new concept in power systems in general [36]. Recently, power distribution networks have received increasing attention following the movement toward Smart Grids. Since distribution networks are connected directly to end users, they have great influence on the operation of other CI systems.

In chapter 3, we presented a resilience assessment framework for generic CI systems and in chapter 4, we presented algorithms for improving resilience through optimization. In this chapter, we apply the framework in chapter 3 to assess the
resilience of electrical distribution networks. The rest of this chapter is organized as follows. In section 5.2, we present a literature review on the existing studies on power distribution systems resilience. In section 5.3, we show how the proposed framework in chapter 3 can be adapted to power distribution networks. After that, we present different strategies for improving power distribution networks resilience in section 5.4. The formulation of the optimal network reconfiguration for the CI restoration problem and the proposed solution algorithm are presented in section 5.5 and 5.6. Finally, a conclusion is presented in section 5.7.

5.2 Literature Review of Power Distribution Networks Resilience

Studies on power distribution networks resilience in the literature cover a wide spectrum of topics depending on the context and the application being investigated. In this section, we provide a literature review on the definition and assessment of resilience in power systems in general. Since optimal network reconfiguration is used in this thesis as a strategy for improving resilience, we provide a brief summary on the related work on power distribution network reconfiguration.

A large body of the research uses the terms reliability and resiliency interchangeably and only limited research has been published in the context of resilience definition and assessment for power networks. The US National Infrastructure Advisory Council (NIAC) developed a framework for establishing critical infrastructure resilience goals using the electrical distribution system as an example of CI [36]. The framework consists of the following steps: develop a common resilience construct, establish baseline resilience practices, conduct stress test of electric systems, conduct roundtables, and develop findings and recommendations. One of the important findings in the NIAC’s report is that “while reliability is relatively easy to define and measure, resilience is more difficult”. The NIAC’s report defines four features for
the electrical system resilience: Robustness, Resourcefulness, Rapid recovery, and Adaptability. Shinozuka and Chang [84] used the robustness and restoration rapidity to define disaster resilience for utility power systems. They developed a method for evaluating resilience using technical, organizational, and socio-economic dimensions. Panteli et. al. developed a power system resilience assessment methodology using sequential Monte-Carlo simulation models [85]. The methodology assesses the impact of weather events on the power system taking into consideration the repair times and human factors. The methodology does not propose a resilience measure but instead it uses two reliability indices, loss of load frequency (LOLF) and loss of load expectation (LOLE). Recent work on power system resilience focuses on different aspects such as impact and repair times estimation for weather related events [85], outage prediction models and crew allocation models [86].

Optimal network reconfiguration is one of the effective functionalities of Smart Adaptive Distribution Networks. In the literature, researchers have proposed Optimal Network Reconfiguration (ONR) to optimize different objectives, such as reliability improvement, power loss reduction, and load balancing. Different formulations and solution techniques have been proposed. Since the context of this research project is CI restoration, only work related to distribution network reconfiguration for power restoration is presented. Power distribution restoration is a multi-fold problem and has been formulated in different ways. In [87], [88], and [89] the problem was formulated as minimization of the de-energized loads (or minimization of un-served areas), subject to voltage and current constraints. Similarly, in [90] a problem formulation that minimizes load loss and utilizes available spare power resources using graph partitioning techniques was used. An objective function that minimizes switching costs is adopted in [91], and [92]. Typical constraints are voltage limits, current limits, radiality, and load constraints. In many of the above works, no knowledge about the type of loads was assumed, i.e., the problem formulation
did not distinguish between critical loads and non-critical loads. Few researchers used formulations that take importance or criticality of loads into consideration. In [87], for example, customers were classified into four priority levels and then used to define a heuristic rule in the restoration rule-based expert system. A set of restored critical loads buses was defined in [89] and a ranking based search technique was used to maximize total load current for these buses in this pre-defined set. In [93], the authors considered supplying high priority loads as constraints in a multi-objective formulation for the reconfiguration problem but it was not clear how these constraints were handled. In [94], similar to [87], four hierarchy levels of loads importance were defined and used in a heuristic search technique to solve the restoration problem. Even though these formulations take loads priority into consideration, interdependencies between them were not considered.

The network reconfiguration problem is a highly complex problem. Since the problem involves searching a finite but large search space, it is classified as a combinatorial problem. Radiality, voltage and current constraints make the problem a nonlinear mixed-integer optimization problem. Many methods have been used to solve this problem. Due to the complexity and combinatorial nature of the problem, many of the reported works in the literature use heuristic type of techniques or combine them with other optimization methods. In [95], mixed-integer programming (MIP) combined with a heuristic approach was used to minimize the number of customers without supply. A genetic algorithm was applied in [96] to find an optimal post-fault restoration strategy. The authors in [90] presented a comparative study of four heuristic algorithms: reactive Tabu search, Tabu search, Parallel Simulated Annealing, and Genetic Algorithm. The performances of these algorithms in solving the reconfiguration problem were evaluated and the reactive Tabu search was shown to outperform the remaining algorithms. Moreover, several researchers have used Graph-based algorithms for solving network reconfiguration problems such as Min-
In all of the above approaches, there are pros and cons. Mathematical programming approaches can guarantee optimal solutions but they require high computational costs especially when system size is large. Heuristic approaches are computationally more efficient but in most cases they cannot guarantee global optimality. Also, performance of some heuristic approaches is highly dependent on the effectiveness of its rules and parameters selection. This is why there is a tendency toward using hybrid solution approaches to take advantage of these different features.

5.3 Resilience Assessment Framework for Power Distribution Networks

The proposed resilience assessment framework for power distribution networks follows the general framework work described in section 3.4. It consists of the three main stages shown in Figure 5.1. Each stage is defined in terms of the power distribution network problem.

![Figure 5.1: Stages for the resilience assessment framework.](image)

As an emerging concept in power systems, resilience is not well understood. One source of the difficulty of understanding resilience is the fact that it is a multidimensional concept. Therefore, it is important to define the context of the problem when developing the framework. The general context of this thesis is critical infras-
structure’s behaviour during its disaster response operations. Table 5.1 lists some examples of the attributes that can be used to characterize the resilience of power distribution networks within our context.

**Table 5.1:** Examples of resilience attributes for the power distribution network.

<table>
<thead>
<tr>
<th>Attributes</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static</td>
<td>Network Topology</td>
</tr>
<tr>
<td></td>
<td>Capacity of transformers and feeders</td>
</tr>
<tr>
<td>Dynamic</td>
<td>Reconfiguring network topology</td>
</tr>
<tr>
<td></td>
<td>Impact of ICT technologies</td>
</tr>
<tr>
<td>Decision</td>
<td>Developing command and control plans</td>
</tr>
<tr>
<td></td>
<td>Repair and maintenance strategies</td>
</tr>
</tbody>
</table>

The defined attributes need to be modelled before they can be evaluated and measured. The model (or models) needs to describe the behaviour of the systems under study with the required level of details. The models need to capture the following:

- The operational parameters for the distribution network: voltage and angle value at every node, and line flow for every feeder.
- The topological structure of the network.
- The interdependencies between the power distribution network and other CI systems.

Two models are proposed for capturing the above characteristics: the i2Sim infrastructure model and the AC power flow model. The i2Sim model is described in details in chapter 2. It is used to account for the impact of changes in power distribution networks on the interdependent CI systems. The i2Sim model is coupled with the AC power flow model in the simulation framework. It is also possible to integrate both models into one model in a mathematical optimization problem. How the models are built and integrated is problem-specific.
After defining the required attributes and models, an index or (indices) reflecting the modelled attributes is defined. We use the Generalized Resilience Index (GRI), described in chapter 3, to formulate an index for power distribution network resilience. The GRI enables combining different resilience dimensions into one index. The definition of the required resilience dimensions depends on the problem and its specific objective. For example, we can define one dimension as the power supply to interdependent CI systems for the distribution reconfiguration problem.

5.4 Improving Power Distribution Networks Resilience

There are two general approaches for improving any CI system resilience: increasing the withstand capability and increasing the survive capability. These two approaches are also referred to as reducing vulnerability and increasing adaptive capacity. Improving power distribution networks resilience requires advances in the two directions to make power networks more resilient in the face of extreme events. Measures that can be taken to improve power distribution networks resilience are discussed next and some examples of these measures are shown in Figure 5.2.

Increasing the withstand capability makes power distribution networks less susceptible to disruptions which results in minimum damages or loss to the networks. There are different approaches for increasing the withstand capability (or reducing vulnerability) such as hardening, redundancy, assets and capacity management. Hardening is typically associated with increasing the structural integrity of the components or the system. Examples include using underground cables instead of overhead cables, and using advanced coatings materials in major equipments to mitigate water and ice damages. Hardening measures can also be extended to the information and communication technology used in power networks by using strong cyber attacks detection and protection software. Redundancy typically refers to the availability of alternative options for providing the same or similar function in case of
Figure 5.2: Approaches and investments for improving power distribution networks resilience.

losing the main component. Redundancy measures include building alternate feeders for important loads in the network and installing redundant transformers in substations. Assets and capacity management refers to enhancing the utilization of existing equipments and systems. It can be achieved through variety of practices and measures such as trees trimming for mitigating lines outages, loads balancing between feeders to avoid feeders’ overloading, and increasing automation in the network.

Increasing the survive capability ensures that the power network can overcome the disruption and resumes its functionality (fully or partially) in a timely manner. Reconfiguration and resources allocation are two effective measures for achieving that. Although these reconfiguration and resources allocation can be used any time,
they play a critical role during emergency situations when power and resources are needed in areas of greatest priority. Preparedness and response planning are also important measures for increasing the survive capability. Planning of prioritization of restoration activities and dispatching repair crew are examples of how utilities can prepare for disruptions.

After identifying the potential approaches and ways for improving resilience, utilities are faced with the challenge of choosing the appropriate approach for their power networks. Investments in improving resilience of power distribution networks are constrained by different factors including market, regulatory, and technical [36]. Resilience assessment methodologies, such as the one proposed in this thesis, are needed to evaluate different approaches and recommend the most effective one. In some cases, more than one approach can be combined to improve the overall resilience.

5.5 Optimal Network Reconfiguration for Power Distribution Networks

Power distribution networks are normally designed with mesh topologies but they are operated using radial configurations by opening switches at certain points. In a radial configuration, there is only one active path between each customer and the supplying substation. Utilities prefer radial configuration because it is cost effective, easy to plan and maintain, and also easy to coordinate effective protection schemes [99].

Distribution network reconfiguration is the process of altering the current network topology by changing the open/close status of switches. The network reconfiguration can be implemented during normal operating conditions for different reasons such as reducing power losses, balancing feeders’ loadings, or increasing networks’ reliability. In emergency situations (abnormal operating conditions), network re-
configuration is used to restore power supply to affected customers. Since there are many candidate switching combinations, finding an operational network topology is a challenging task to utility operators.

The Optimal Network Reconfiguration (ONR) problem is defined as the problem of finding an optimum network topology that maximizes or minimizes an objective function. Due to large number of switches in the network, and due to the nonlinear characteristics of the power system behaviour, the ONR problem is considered a combinatorial constrained nonlinear mixed-integer optimization problem. A typical ONR problem includes the following elements: 1) proper network models; 2) power flow calculations; 3) algorithm for making changes to the current configuration; and 4) optimization technique for guiding the algorithm to the optimum solution.

The ONR can be implemented manually by sending out operators to do the switching, or it can be done automatically using the Distribution Automation System (DAS). Automatic network reconfiguration is one of the envisioned functionalities in Smart Grids [100]. Recent advancements in Smart Grids deployments have made the automatic reconfiguration of distribution networks a practical option for utilities.

5.6 ONR for CI Restoration using A Minimum Spanning Tree Based Algorithm

Restoring power supply to CI systems after an extreme event is critical for improving the infrastructure resilience. This result is illustrated by the problems addressed in Chapter 4. However, the problem formulations in Chapter 4 do not take into consideration the technical feasibility of the power distribution networks when evaluating a candidate solution. Although some basic constraints can be tested in these formulations, such as substation capacities, other electrical constraints such as voltage limits can only be tested using specific power system models.
ONR is the complementary problem to the resources allocation problem in Chapter 4. ONR can be integrated with the resources allocation problem in i2Sim to increase the overall infrastructure resilience. The problem can be stated as follows: what is the optimum power network topology that maximizes CI systems’ outputs considering their interdependencies? We name this problem: Optimal Network Reconfiguration for CI Restoration.

The ONR for CI restoration problem brings significant computational challenges. One of the challenges is how to model CI systems’ interdependencies. In addition, formulating the ONR problem for power networks is a complex problem due to the high number of possible configurations, which increases exponentially with the number of switches in the network. Another challenge is how to account for the interdependencies within the ONR problem.

5.6.1 System Models

Three models are used for formulating the ONR for CI restoration problem. The first model is the i2Sim model which takes care of the upper level interactions between the modelled CI systems. This model calculates the outputs of every system given a specific set of supplies (including power supply). The i2Sim model is described in details in Chapter 2.

The second model is the graph model for the power distribution network. Distribution networks can be represented by an undirected graph $G(V, E)$, where $V$ is the set of $n$ vertices and $E$ is the set of $e$ edges (or lines). The root node (feeding substation) of the distribution network is denoted as node 0. All load points in the network have an active power $L_i$ [kw] $i \in 1, 2, ..., n - 1$. It is assumed that each branch (or line) of the network is represented by an edge $e \in E$ and incorporates a sectionalizing switch on it. These switches are operated to modify the network topology according to the reconfiguration process. Many distribution networks are operated typically
under radial configurations, which means that there is only one path between each load node and the root node (or feeding substation) [99]. Therefore, they can be represented as Tree Graphs [101]. A spanning tree is a sub-graph that contains all the nodes without any loops and some or all the edges such that there is only one path between any pair of nodes. Spanning trees are ideal representations of radial distribution networks [97].

The third model is the AC power flow model which is the fundamental tool for calculating the operational parameters of a general power network. The network is represented on a per phase basis and the loads on each each line are represented as constant $P$ and $Q$. The network in the model is assumed to be operating in balanced conditions. There are four parameters associated with each node (bus): voltage magnitude $|V|$, phase angle $\delta$, real power $P$, and reactive power $Q$. In a typical power flow problem, there are two unknown parameters at each node. Two sets of equations are solved: the network equations and the power equations. Given the values of the loads consumption and the values of the power supply to the network, the power flow model calculates the power flow in each line.

The basic network equation for power flow models is based on Kirchhoff’s laws and is derived from nodal analysis as follows:

$$
\begin{bmatrix}
y_{11} & y_{12} & \cdots & y_{1j} \\
y_{21} & y_{22} & \cdots & y_{2j} \\
\vdots & \vdots & \ddots & \vdots \\
y_{i1} & y_{i2} & \cdots & y_{ij}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N
\end{bmatrix}
= 
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N
\end{bmatrix}
$$

(5.1)

where the $y_{ij}$ are the elements of the admittance matrix $Y$. $V_n$ are the nodes’ voltages and $I_n$ are the current injections at each node $n$. The admittance matrix $Y$ represents the interconnection between nodes (buses) in the power network. In the resilience assessment framework, the admittance matrix can represent a static
attribute for the modelled network. The currents and voltages are complex numbers which can be represented in polar or rectangular forms. Therefore, the network equations result in complex linear simultaneous algebraic equations in terms of the currents $I$. Typically, the values of the currents $I$ are not known but the real power $P$ values are known. The power equation $S = VI^*$ is then used to express the voltage equations in terms of $P$:

$$P_n + jQ_n = V_n \sum_{k}^{K} Y_{nk}^* V_k^* \tag{5.2}$$

After separating the real and imaginary parts of (5.2), the resulting formulation is a system of algebraic nonlinear equations which can be solved using iterative techniques such as Gauss-Seidel and Newton Raphson methods [102]. For the power flow analysis of distribution networks, what is known before the analysis is the substations’ power supply and voltages, and the complex power of the loads given the load model (constant complex power, constant impedance, constant current, or a combination).

### 5.6.2 Problem Formulation

The ONR for CI restoration problem is formulated as a resilience maximization problem. The objective function is the GRI function which is expressed in terms of the output of affected CI systems after the event (evaluated by the i2Sim model). The decision variables are the i2Sim distributors’ ratios and the power distribution network
switches’ status. The problem can be stated as follows:

\[
\text{Maximize} \quad GRI = y_{ci} \\
\text{subject to} \\
\sum_{j=1}^{k} a_{jt} \leq 1 \\
A_m = \sum_{j=1}^{n} a_{jt} \\
a_{j}^{\min} \leq a_{j} \leq a_{j}^{\max} \\
V_{n}^{\min} \leq V_{n} \leq V_{n}^{\max} \\
I_{n} \leq I_{n}^{\max}
\]

Network is radial

where \( y_{ci} \) is the output of the considered infrastructure system (or systems). The first three constraints are the i2Sim constraints and the last three constraints are the power distribution constraints. Since the decision variables are discrete variables, the problem is considered as a mixed-integer nonlinear discrete optimization problem. Furthermore, the dimension of the problem is large due to the coupling of two problems: the i2Sim optimization problem and the power network topology optimization problem. For every solution for the i2Sim model, there are \( 2^n \) solutions for the power network model where \( n \) is the number of switches.

5.6.3 Solution Algorithm

To solve the above problem, we propose an iterative simulation optimization algorithm as depicted in Figure 5.3. The basic idea of the proposed algorithm is as follows: first, the i2Sim model is optimized and then an ONR problem is solved based on the i2Sim solution. There are two stages in the algorithm. The first stage solves an i2Sim optimization problem using an OO-based algorithm similar to the one pro-
posed in Chapter 4. From the solution of the i2Sim optimization problem, we extract the power supply requirements for the CI systems operations. The i2Sim solution accounts for the interdependencies with other infrastructure systems, such as water and gas. Once the power supply requirements are obtained, an ONR problem is solved in the second stage to find a feasible power distribution network topology that can meet the i2Sim solution’s requirements. The next iteration starts when a new i2Sim solution is produced along the time line of the event.

**Figure 5.3:** Overall flow of the proposed iterative simulation optimization algorithm.

The ONR problem in the second stage is solved using a heuristic method based on a Minimum Spanning Tree (MST) algorithm. The problem aims at finding a radial network topology that delivers the required power to the CI systems. The MST algorithm identifies the possible paths for the power to flow and returns a spanning tree graph which ensures compliance with the radially constraint. There are several algorithms for finding a MST from a given graph such as Prim’s algorithm and Kijkstra’s algorithm. From the perspective of our problem, there is no advantage in choosing one algorithm over another. We select Prim’s algorithm for developing our
solution algorithm here. Details on MST graphs and Prim’s algorithm are provided in appendix A.

The solution strategy for the ONR problem in the second stage is composed of the following main steps:

1. **Build system models:** In this step, the AC power flow model and the corresponding graph model for the distribution network are built based on the initial system data. Then, they are updated based on the outage data. The outage data can be in the form of line failures, equipment failures, or reduction in the available power supply from the high voltage system at the supplying substations.

2. **Build graph model and check connectivity of the loads:** In this step, all switches in the network are closed to form a meshed network. After that, the graph model is formulated and the connectivity of the graph is checked. If there are any nodes that are not connected, then they are removed from the model.

3. **Run the MST algorithm:** In this step, an initial weight is assigned to each line (edge) in the network. The weights assignment is as follows:

   \[ W_i = \begin{cases} 
   1/S_{i}^{\text{rated}} & \text{for lines with normally-closed switches} \\
   M & \text{for lines with normally-open switches} 
   \end{cases} \]  

where \( W_i \) is the weight of line \( i \), \( S_{i}^{\text{rated}} \) is its rated loading capacity in MVA, and \( M \) is a large arbitrary number. Prim’s algorithm is then used to find a minimum spanning tree for the network graph in which the sum of all weights in the tree is minimum. One can influence the tree construction by assigning certain weights to certain edges, e.g., assigning high weights to un-wanted edges. The weights assignments in (5.4) is designed to serve two objectives: the
The first objective is to produce a configuration as close as possible to the pre-failure configuration and the second objective is to utilize the edges with higher capacity ratings as much as possible. The first objective is facilitated by assigning large weights to the normally open switches, i.e., driving the algorithm away from selecting them. The second objective is facilitated by assigning $1/S_i^{rated}$ to the remaining edges, i.e., lines with higher capacity have lower weights and higher probability to be included in the tree.

4. **Run the AC power flow model and check feasibility** Once a candidate radial configuration is selected by the MST algorithm, power flow calculations are performed using the standard Newton Raphson method. If the power flow converges to a solution, then the constraints are checked. If no constraints are violated, then a feasible radial configuration is obtained. The solution is output and the algorithm is ended. If there are violations, or if the power flow does not converge, then go to the next step.

5. **Perform loads management** The loads management step performs two functions: loads transfer and loads shedding. If the candidate configuration has a feasible power flow with line capacity violations, then the loads transfer is attempted first. If the power flow has no solution or there are voltage violations, then loads shedding is attempted. After that, the algorithm will go to step 4. This process is repeated until a feasible network configuration is obtained.

The flow chart for the ONR solution algorithm is shown in Figure 5.4. The proposed algorithm explores the candidate topologies to restore the power distribution network with a maximum supply to the CI systems as identified by the i2Sim solution. The MST algorithm ensures that the candidate topology meets the radiality constraint. Since the pre-failure configuration is considered as the reference...
configuration, the weights assignments in the MST algorithm are used to produce a candidate configuration that is close to the pre-failure configuration.

In case of power flow violations, loads transfer or loads shedding is performed, depending on the violation as described in step 5. The loads transfer and loads shedding procedures are described as follows:

**Loads transfer** This function aims at balancing the loads in the network by transferring some loads from the overloaded feeder to another feeder. If there are more than one overloaded feeder, then feeders with CI nodes are treated first. The function proceeds as follows. First, it searches for a line $m$ with an open switch connected to the overloaded feeder $k$. This is done by searching all paths between the receiving node at the overloaded line and the root node (supplying substation). If there is no candidate line in any path, then loads’
transfer cannot be performed and loads shedding should be attempted. If there is candidate line $m$ for switching, then the spare capacity for every line in the path to the root node is calculated as follows:

$$S_{\text{spare}} = S_{\text{rated}} - \sqrt{P^2 + Q^2}$$

(5.5)

If the load to be transferred is greater than the minimum spare capacity for the path to the root node, then load transfer is not possible. If not, a switching exchange is done to transfer the load while maintaining the radiality of the configuration (closing the candidate line’s switch and opening the switch before the receiving node). The procedure continues until all overloaded lines are treated.

**Loads shedding** The nodes in the network are divided into two sets: critical nodes $N_c$ and non-critical nodes $N_{nc}$. Loads shedding is attempted first by shedding loads from the $N_{nc}$ set. The algorithm starts by searching for leaf nodes on the feeder with power flow violations. In graph theory, leaf nodes are nodes with degree one, i.e., have only one connecting edge to the graph tree (loads at the end of the feeder). If we remove a leaf node, the graph is still a tree. This feature is very useful in our load shedding procedure since we want to maintain a radial topology all the time. Once leaf nodes on the overloaded feeder are identified, nodes that are in the non-critical set $N_{nc}$ are shed. The procedure continues shedding one node at a time until all load points in the set $N_{nc}$ are taken out of the network. If there still are power flow violations, leaf nodes that are in the set $N_c$ are considered for shedding.

### 5.6.4 Test Cases

In this section, the proposed MST-based algorithm is tested in two cases using two test systems that were constructed by combining the i2Sim model described in chapter
and two of the available power distribution test systems in the literature. The first test case uses the i2Sim model and the 33-nodes power distribution test system [103]. This power distribution test system has 33 nodes (loads) and 37 edges (representing switches). The initial configuration of the 33-node test system is shown in Figure 5.5. The abstracted cells in the i2Sim models need to be mapped into their corresponding nodes in the power network model.

Failures scenarios are mapped into the power network by opening the corresponding switches. Also, CI systems are assigned to their corresponding load points (nodes) in the power network. The following assignments are used in this test case: Hospital 1 is represented by two nodes, nodes 8 and 9, since it is bigger than Hospital 2 which is represented by node 25. Water station 1 is represented by node 21. Since Hospital 3 and water station 2 are supplied from a separate power substation, they are not modelled in this test case. Different failure scenarios are tested representing single line failure and multiple lines failures. Results for the reconfiguration problem are only reported in this section since the i2Sim optimization results are presented in Chapter 4. Table 5.2 shows the obtained results for the ONR for CI restoration for the considered scenarios.

In the first three scenarios, a single line failure due to the disaster events is considered. This causes some nodes to be out of service. The objective of the MST-based algorithm is to restore the power supply to the critical nodes as requested by the i2Sim solution. Only one of the scenarios restores a full power supply to the critical nodes without returning to i2Sim (scenario 1 in the table). The initial tree configuration by the algorithm produces line capacity violations triggering the loads management subroutine which transfers loads to another feeder without the need to shed any loads. For the other two single failure scenarios, loads transfer and loads shedding have to be performed since the initial tree configuration returns non-feasible solutions as loads management fails to maintain power supply to critical
nodes. Similar results are observed for the last three scenarios that consider two line failures at a time. The minimum nodal voltages for the final configuration in the obtained solutions are within the voltage limits as shown in Figure 5.6.

The second test case uses the i2Sim model and the 70-node power distribution test system. The power distribution system has 70 nodes (loads) and 79 edges (lines) arranged in 4-feeders configuration as shown in Figure 5.7. The mapping between the i2Sim model and the power distribution model is as follows: nodes 11 and 12 represent Hospital 1, node 58 represents Hospital 2, node 38 represents Hospital 3, node 27 represents Water Station 1, and node 62 represents Water Station 2. Results for this case are shown in Table 5.3. Six scenarios are tested. Due to the availability of several alternate lines, the power supply can be restored to the critical
Table 5.2: Obtained solutions for the 33-nodes network reconfiguration

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Failure Location</th>
<th>$P_{\text{critical}}$ (%)</th>
<th>$P_{\text{shed}}$ (%)</th>
<th>$V_{\text{min}}$ (P.U.)</th>
<th>Open Switches</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20-21</td>
<td>100</td>
<td>0</td>
<td>0.9079</td>
<td>20-21,9-15,12-22,18-33,25-29</td>
</tr>
<tr>
<td>2</td>
<td>6-7</td>
<td>89.10</td>
<td>25.98</td>
<td>0.9424</td>
<td>6-7,14-15,17-18,21-22,25-29,8-9,18-33,7-8</td>
</tr>
<tr>
<td>3</td>
<td>8-9</td>
<td>89.10</td>
<td>18.17</td>
<td>0.9303</td>
<td>8-9,8-21,12-22,18-33,25-29-9,15</td>
</tr>
<tr>
<td>4</td>
<td>91-20,30-31</td>
<td>100</td>
<td>11.31</td>
<td>0.9229</td>
<td>12-13,18-33,19-20,21-22,25-29,30-31</td>
</tr>
<tr>
<td>5</td>
<td>5-6,15-16</td>
<td>52.73</td>
<td>55.32</td>
<td>0.9807</td>
<td>5-6,8-21,25-29,12-22</td>
</tr>
<tr>
<td>6</td>
<td>23-24,26-27</td>
<td>63.64</td>
<td>45.76</td>
<td>0.9399</td>
<td>8-21,23-24,26-27,12-22,18-33</td>
</tr>
</tbody>
</table>

* Percentage of critical loads supply where 100% means all critical loads are fully supplied.
** Loads taking out by loads shedding as a percentage of total load.

nodes in four of these scenarios. One of the differences between this case and the first case is the availability of a second power substation which adds more flexibility to the reconfiguration process. However, voltage violations limit the restoration of some nodes in some scenarios. Even though this case has a more flexible topology, maintaining the final configuration voltages within the limits is more challenging than in the first case as can be seen in Figure 5.8.

In six of the tested 12 scenarios, the i2Sim solution cannot be accommodated by the power distribution network. This result shows the importance of adding the technical feasibility of the modelled infrastructure systems to the resources allocation problem during disaster management. The results in Figure 5.9 show the effectiveness of the proposed ONR algorithm compared with the no-reconfiguration cases. As a result of the MST weights assignment strategy, the final topology after the reconfiguration procedure is close to the pre-event topology which leads to a reduced number of switching operations, which in turn increases the speed of restoration.
Figure 5.6: Minimum voltage in the final configuration for every scenario in the 33-nodes test cases.

5.7 Conclusion

This chapter discusses the resilience assessment of power distribution networks in light of the generic framework proposed in Chapter 3. While there are different ways of improving the resilience of power distribution networks, as described in section 5.4, optimal network reconfiguration is used in this chapter as a means of resilience improvement. This chapter presents a problem formulation, a solution algorithm, and test cases results for the ONR for CI restoration problem. The MST-based solution algorithm is used to complement the resources allocation problem in Chapter 4. The proposed solution algorithm for the ONR problem is a heuristic algorithm which does not guarantee a global optimum solution. However, the use of a fast graph theoretic algorithm makes it computationally efficient and fast which, in turns, makes more applicable to real time applications.
Figure 5.7: Initial configuration of the 70-node test system. The dotted lines represent normally open switches.
<table>
<thead>
<tr>
<th>Scenario</th>
<th>Failure Location</th>
<th>$P_{\text{critical}}$ (%)</th>
<th>$P_{\text{shed}}$ (%)</th>
<th>$V_{\text{min}}$ (P.U.)</th>
<th>Open Switches</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3-4</td>
<td>100</td>
<td>6.49</td>
<td>0.9001</td>
<td>3-4-5-4-10,7-68,9-50,22-67,29-64,45-60,38-43,39-59,21-27,9-15</td>
</tr>
<tr>
<td>6</td>
<td>4-10,33-34</td>
<td>88.73</td>
<td>15.47</td>
<td>0.9052</td>
<td>4-10,12-13,33-34,9-50,37-38,15-46,22-67,29-64,45-60,38-43,39-59,21-27,15-67</td>
</tr>
</tbody>
</table>

* Percentage of critical loads supply where 100% means all critical loads are fully supplied.
** Loads taking out by loads shedding as a percentage of total load.
**Figure 5.8:** Minimum voltage in the final configuration for every scenario in the 70-nodes test cases.

**Figure 5.9:** Comparison of ONR for CI restoration results.
Chapter 6

Conclusion

This thesis focuses on modelling, simulation, analysis, and optimization of critical infrastructure systems with respect to their resilience to extreme events. It attempts to provide a standardized methodology for assessing infrastructure resilience. It also formulates the resilience improvement problem as an optimization problem and proposes different algorithms for its solution. The main conclusions of the contributions in this thesis are summarized in the following paragraphs.

A Resilience Assessment Framework for Interdependent Infrastructure Systems has been developed. The presented resilience assessment framework brings an original contribution to the analysis of critical infrastructure systems. It provides a quantitative means to assess infrastructure resilience using a generalized measure that is comparable across different systems’ contexts and structures. The generalized resilience index provides a flexible formulation that can be applied to different domains and applications. The i2Sim modeling approach is used to account for the inherent complexity due to CI systems’ interdependencies. The results presented in Chapter 3 and Chapter 4 show that infrastructure interdependencies play an important role in the resilience improvement process.

Resources Allocation Algorithms for CI Systems have been developed for
assessing disaster response during disruptions. Two optimization problems have been formulated for selecting the proper allocation of available resources (power, water, etc.) to enhance the overall resilience of the infrastructure. The two formulations test two different paradigms in optimization: simulation-based optimization and exact mathematical optimization. The formulations show that the problem is computationally intensive, especially for large scale infrastructure systems. The simulation-based optimization algorithm RAOO provides good solutions compared to the exact optimization algorithm RALP, which provides the optimum solution. However, RAOO can be a more practical solution due its modularity and ease of implementation. The results of both algorithms prove that infrastructure resilience can be greatly improved by efficient allocations of available resources.

A Prioritization Methodology for Interdependent CI Systems has been developed. This methodology can be used to direct investments to the most critical components of the systems. It utilizes the i2Sim modeling framework to assess different failure scenarios. The use of i2Sim allows for capturing the mutual interactions between different CI systems. Unlike other methodologies in the literature, the proposed methodology considers the functional properties of the modelled infrastructure systems along with the topological properties. Results of the case studies show that hidden interdependencies can change the ranking of components when considered. Also, the proposed methodology can be extended to consider multiple failure sets to simulate more scenarios.

An Optimal Power Distribution Network Reconfiguration for CI Restoration Algorithm has been developed. This algorithm is used to complement the two resources allocation algorithms to check for their technical feasibility. The objective of this algorithm is to find the best power distribution network topology that maximizes the power supply to the CI systems. Results of the case studies show that not every solution by the resources allocation algorithm can be implemented.
when considering the technical models of the power distribution network. Optimal network reconfiguration (ONR) is used as a technique for solving this feasibility problem. The ONR problem is solved using a graph-theoretic algorithm based on Minimum Spanning Trees that provide feasible solutions.

6.1 Future Research Directions

In this section, different future research directions are proposed. These research directions can be followed to overcome some current limitations or to extend the contributions of this thesis to the field of critical infrastructure resilience.

Improvements to the proposed Resilience Assessment Framework: There are different aspects in which the proposed framework can be improved or extended. One of these aspects is related to the uncertainty in real-life systems. The presented resilience assessment framework uses deterministic system dynamics to describe the response of the CI systems to a disturbance. Since uncertainty is a key feature in real-life systems (and events), the proposed framework can be extended to account for such uncertainty. A probabilistic approach can be used to describe the systems’ parameters using some probability density functions. The resilience index also can be represented using a stochastic process similar to the one used in [56]. The use of probabilistic approaches can be seen as a way of transitioning the traditional reliability theory to the new concept of resiliency.

Another aspect for improvement is related to the attributes’ definition in the framework. Experts’ opinions are typically used in defining the required resilience attributes for a given system. However, these opinions may have some conflicting directions or may not agree on the level of importance of a particular system attribute. A systematic approach for defining the attributes can be useful in this regard. Multi-criteria decision analysis techniques such as Analytical Hierarchy Process (AHP) and Multi-attribute Utility Theory (MAUT) can be used for this purpose.
Applications to other CI systems and Cyber-Physical Interdependencies: The focus of this thesis is on a high-level abstracted model of interdependent infrastructure systems and also on an application to power distribution networks. The results in Chapter 5 show that analysis of the detailed technical model of the infrastructure system (e.g. power networks) can improve the optimization solution obtained by the high-level model (i2Sim model). In Chapter 5, the focus is on power distribution networks when integrated with the i2Sim models. One way of extending this focus is to consider detailed models of other infrastructure systems such as water distribution networks and transportation networks. Another way is to consider cyber-physical interdependencies between the information and communication networks and other infrastructure systems. Of particular interest is the resilience assessment of Smart Grids which includes both the power network and the information and communication networks. The use of communication networks in Smart Grids allows one to isolate failures and restore power supply more quickly, which improves the resilience of the infrastructure. However, new challenges and risks emerge due to cyber-physical interdependencies. For example, mapping cascading failures among the two systems is a challenging task due to the nature of the interconnection between components (one to one, one to many, or combination). Also, the use of information technologies increases the risks of cyber intrusions which makes cyber security a major concern for infrastructure operators.

Integration with other CIP tools for Decision Support during Disasters: The resilience assessment framework combined with the i2Sim simulator, and the optimization models proposed in Chapters 4 and 5 can all be integrated to form a decision support system for disaster management. The decision support system can also integrate other CIP tools which can provide more information and capabilities. One example is the Geographic Information System GIS. The GIS can provide location information which can be used in the optimization problems to speed up
the restoration process. Other tools that can be integrated are the weather forecast models and the seismic assessment models. The modularity of the developed framework and optimization models in this thesis allows the applicability of this integration.

**Computational Efficiency Improvement**: Simulation and optimization of CI models is computationally expensive and data intense. For the integrated decision support system described above to be realized, the computational efficiency of the models need to be improved. One source of the computational difficulty in the proposed models in this thesis is the initiation of multiple i2Sim simulations within the MATLAB/Simulink environment. One approach of dealing with this difficulty is to use one of the parallel processing techniques which allows for running multiple simulations simultaneously. Another way of improving the computational efficiency is to use a data structure algorithm which allows storing and exchanging data between different models more efficiently.
Bibliography


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Appendix A

Minimum Spanning Trees

In graph theory, a tree is a connected graph that contains no cycles (loops). A spanning tree is a tree that contains all the nodes (vertices) of the graph. A graph may have many different spanning trees. If every edge (link) has a weight associated with it, then the graph is called a weighted graph. A Minimum Spanning Tree MST is a spanning tree of a weighted graph such that the weight of the tree (sum of weights of all edges) is minimum.

The problem of minimum spanning tree arises in many applications, such as communication networks, circuit designs, transportation, and data structure algorithms [105]. The desire in all applications is to find a network that minimizes or maximizes the sum of the weights. The weights in the graph of the network are designed to represent a parameter of particular interest to the problem. For example, the weights could represent the cost of transporting some material from one location to another and the objective of the MST problem is to find a network with a minimum total cost.

There are efficient algorithms for finding the minimum spanning tree of a given graph. The earliest algorithm was developed in 1926 by Otakar Boruvka [105]. Two of the most common algorithms are Kruskal [106] and Prime [107]. Prime’s algorithm
is one of the most commonly used algorithms for finding minimum spanning trees and it is used in the network reconfiguration problem presented in this thesis. Primes’ algorithm tries to build a spanning tree starting from a root node by selecting the edge with a smallest weight until it spans all nodes in the graph. This process makes it a greedy algorithm. Prime’s algorithm for a graph $G(V,E)$ is described as follows:

1. Start with a root node $r$. Set $T = r$ and $E = \phi$. $T$ is the tree’s nodes set and $V - T$ is the unselected nodes from the graph.

2. Find a minimum weight edge $e$ that connects one node $v$ from $V - T$ to $r$.
   Add the $e$ node $v$ to the tree set $T$ and update the edges set $E$.

3. Choose another node from the set $V - T$.

4. Find a minimum weight edge $e$ that connects one node $v$ from $V - T$ to $T$.
   Add the node $v$ to the tree set $T$ and update the edges set $E$.

5. If $V - T = \phi$, then end. If not, go to step 3.

Prime’s algorithm is applied to the example graph (adopted from [108]) shown in Figure A.1.
Figure A.1: An application of Prim’s algorithm to an example graph.