PREDICTION OF CUTTING FORCES AT THE TOOL TIP USING DRIVE CURRENT FOR FIVE-AXIS MACHINES

by

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Abstract

The current trend in industry is to achieve intelligent, Computer Numerical Controlled (CNC) machine tools which can monitor its performance and take corrective actions automatically during machining operations. Cutting forces are the most accepted indicators of the tool condition, load on the machine and part, and abnormalities in the machining operations. The objective of this thesis is to predict the cutting forces from the current drawn by each drive during five axis machining operations.

The cutting forces generated at the tool–workpiece contact zone are transmitted to the three translational and two rotary drive motors through ball screws and gear boxes. The torque received by individual motors is transformed as disturbance current by the motor amplifiers. The cutting force transmitted to each feed drive acts as a disturbance to the closed loop servo controller, which reacts by supplying torque command in addition to the torque required to overcome the friction and inertial motions. The accurate prediction of cutting forces from the motor current measurements requires the separation of the effects of cutting and inertial motion forces from the total motor current values.

The transfer function between the applied force at the tool tip and motor current is identified at each drive. The effects of structural modes are canceled through extended Kalman Filter designed for each drive. Both Coulomb and Viscous Friction forces have been identified, and their effects are also removed from the state measurements of all drives. The cutting forces at the tool tip are predicted by applying extended Kalman Filter on motor current signals, and transmitting them to the tool tip through forward kinematic model of the machine, the contributions are proven using machining tests conducted on a five axis machining center.
Preface

This thesis is entitled as “Prediction of Cutting Forces at the Tool Tip Using Drive Current for Five-Axis Machines”, which presents the research conducted by Tugce Tuysuz, based on the initial research question by the supervisor, Dr. Yusuf Altintas. This work has been completed in the Manufacturing Automation Laboratory (MAL) at the University of British Columbia. The proposed methodology in this manuscript is original, unpublished, independent work by the author.

I was leading the investigation of the thesis, responsible for experimental design, data collection and the major development of identification algorithm. Dr. Yusuf Altintas was the supervisory author on this project and was involved in problem statement formulation, supervision throughout the project.

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To my mother,
Chapter 1: Introduction

Five-axis machining centers are widely used to machine complex parts in aerospace, automotive, die/mold industries on a single setup with a better tool accessibility. The objective of the virtual machining research is to identify the problems in the process ahead of currently practiced costly physical trials. Therefore, prediction of the cutting forces felt by the cutter and the workpiece in three Cartesian directions \( (F_x, F_y, F_z) \) and the cutting torque \( (\tau_z) \) at the tool tip (Figure 1.1) is crucial for process monitoring and optimization, which have been widely studied through modeling the physics of the cutting process in literature. However, these cutting process models lack the strength to accurately predict the tool tip cutting forces due to approximations and assumptions made in the models.

![Figure 1.1: Cutting forces and torque in machining](image)

Figure 1.1: Cutting forces and torque in machining
Five-axis machine tools have three translational and two rotational axes. Typical configuration of a five-axis machining center having Y and Z translational drives on spindle, and X translational along with A and C rotary drives on machine table is shown in Figure 1.2. In addition to the frictional, gravitational, and inertial forces under the operating conditions, machine drives handle the extra load due to cutting forces since they are transmitted to the drive motor through the kinematic chain, mechanical links and servo drive circuits.

**Figure 1.2:** Kinematic configuration of the investigated five-axis machining center [1]

The amplitude and phase of the transmitted forces are felt as the distorted torque (current) on the motor as seen in Figure 1.3 where the estimated torque is obtained by multiplying the measured current with a constant gain and compared against the monitored torque and it is illustrated in Figure 1.4.

The cutting force acts as a disturbance force on each drive. Monitoring the current and the torque drawn from the drive motors can lead to monitoring and control of the machining
process to avoid tool failure and excessive tool deflections, while detecting abnormalities of the machining operations. The ultimate goal is to predict the cutting forces and torque using the drive current along the toolpath during machining without adding extra sensors to the cutting region on the machine.

**Figure 1.3:** The measured and the estimated torque results for single-axis cutting without considering the drive dynamics

The objective of this thesis is to develop a model that can predict the tool tip forces \( F_{\text{tool tip}} \) in three and five-axis virtual machining applications using the current drawn (torque generated, \( \tau_{\text{motor}} \)) by each feed drive motor during cutting. The mathematical model for transforming the drive forces \( F_{\text{drive}} \) to the tool tip by considering machine kinematics, friction \( \tau_{\text{friction}} \) and transfer functions of the machine elements \( H_{\text{disturbance}} \) is developed.
Figure 1.4: Illustrated test set-up for the thesis scope

The true forces at the tool tip from the measured current are obtained through the designed Kalman Filter, which have been verified with three and five-axis cutting tests. The scope of the thesis is illustrated with the combined partially shown control block diagram of a machine tool drive in Figure 1.5.

Figure 1.5: Partial control block-diagram of a machine tool drive combined with the tool tip force prediction from the measured motor torque
This thesis is organized as follows: the related literature is reviewed in Chapter 2. The experimental modeling of feed drive friction and identification of feed drive disturbance dynamics are presented in Chapter 3. A new tooltip cutting force prediction model from the measured drive current using the machine kinematics of the investigated five-axis machine tool is developed in Chapter 4. The experimental verifications for three and five-axis machining applications are given in Chapter 5. Conclusions and future research directions are presented in Chapter 6.
Chapter 2: Literature Review

2.1 Introduction

Cutting force measurement is one of the most significant indicators of machining condition and quality. Therefore, the cutting forces need to be monitored for the detection of tool wear and failure, and avoidance of part tolerance violations for increased productivity [2]. Recently, sensor-less machining became an important objective in manufacturing research to achieve robust tool condition monitoring and adaptive control of the cutting processes.

The objective of this thesis is to predict cutting forces and torque from the current drawn from each drive motor that are already available in the CNC systems. Although there is no need to install additional sensors on the machine [3], the transmission of the force to motors’ electrical circuits as disturbances is distorted by the friction and dynamics of both structural and electrical components of the drive chain.

The relevant literature in predicting the force from the drives is reviewed in this chapter.

2.2 Cutting Force Estimation Using Motor Current (Indirect Monitoring)

The cutting forces at the tool tip are transmitted to the servo motor shaft as a disturbance torque which is proportional to the armature current. Kim and Chu [4] studied cutting force prediction from the feed drive current for single-axis cutting considering the drive friction. The drive friction is empirically obtained as a function of the federate and the workpiece weight. The authors obtained an empirical curve between the feed force measured at the tool tip and the associated drive current after a series of cutting tests.

Stein et al. [5, 6] experimentally investigated the relation between the feed force and the feed motor current in turning process while neglecting the dynamic compliance of the feed drive.
The authors measured the feed drive current and subtracted the air cutting current to account for friction on the drive. A sensitivity analysis for the current drawn from the feed drive motor was performed, and they showed the sensitivity of the measured current signals to both viscous and Coulomb friction on the drive system. Altintas [7] and Lee et al. [8] estimated the cutting forces from the feed motor current to detect the tool breakage. Both studies used low spindle speed to avoid amplitude and phase distortion caused by the transfer function between the cutting force and motor current. They assumed constant Coulomb friction on the drive train.

Mannan and Broms [9] investigated the temperature dependency of the feed motor current measurements since the ratio between the cutting forces and the drive current should not change in experimental observations. They concluded that the input current increases with motor temperature.

Jeong and Cho [10] proposed an empirical approach to model the relationship between measured the cutting forces and the feed drive current. They recorded the drive current, angular velocity and acceleration when the machine started to move. They curve fitted the variation of current with angular velocity and acceleration to eliminate the frictional and inertial effects on the drive. The ratio between the cutting force and the drive current during cutting was obtained by scanning different tooth passing frequencies without violating the bandwidth of the current sensor.

Li et al. [11] proposed a tool wear monitoring system for turning based on estimation of the feed force from the measured feed drive current using an adaptive neuro-fuzzy approach, which requires several sets of experiments to approximate the cutting force. Variation of the air cutting current with the feed speed is empirically obtained and used for isolating the cutting
current from the measured total current. Change in the feed force with time was used as the metric to evaluate the tool wear.

Shinno et al. [12] proposed a process monitoring and adaptive control system using disturbance observer for single axis ultra-precision machining. The estimated cutting forces by the disturbance observer agreed well with the ones directly measured by the dynamometer on a linear motor driven aerostatic table system.

As a migration from single axis cutting to multi-axis cutting, Kim et al. [13] proposed a Kalman Filter disturbance observe and an artificial neural network methods to predict the cutting forces on drives for two-axis milling of a circular part using the feed drive system model. Friction induced external torque on the drives were estimated from air cutting tests, which was subtracted from the total torque induced by cutting. The estimated cutting forces using both methods were compared against dynamometer results, and it was concluded that, the disturbance observer cannot keep track the cutting forces if the feed direction is reversed during cutting.

Auchet et al. [14] experimentally identified the transfer function between the input voltages of magnetic bearings of a high speed milling spindle and the cutting forces by conducting an impact hammer test on the stationary spindle in x-y plane of the machine. The researchers ran single-axis milling tests and compared experimentally measured dynamometer
forces with the ones obtained from the magnetic bearing voltage with the identified transfer function. The mismatch in the comparison was attributed to inaccuracy in the identified transfer function at high spindle speeds.

2.3 Cutting Force Measurement (Direct Monitoring)

The most common method of cutting force and torque monitoring is direct measurement using quartz piezoelectric stationary and rotary force dynamometers. Table (stationary) dynamometers are mounted on the work table and the workpiece is fixed to the dynamometer while the rotary dynamometers are attached to the spindle head as a tool holder for cutting force and torque measurement to validate the developed cutting process mechanics and force based process optimization models for single and multi-axis machining operations [15-21].

However, costly dynamometers have a limited use in industrial operations since they interfere with the cutting due to deployment into the work volume. Also, the limitation on the workpiece size and the bandwidth, and mounting either on the work table or the spindle head introduce further constraints [22].

Alternatively, Byrne and O'Donnell [23] used a spindle integrated force sensor (SIFS) and the motor power on a direct driven spindle motor for online monitoring of drilling operations. Jun et al. [24] investigated the feasibility of adding a Kistler force ring into machine tool’s spindle for intelligent process monitoring and diagnosis. It was experimentally shown that change in spindle dynamics due to addition of the force ring, drift and cross-talks in the measurement signal are inevitable. Also, air-cutting is required to compensate for the extra forces in the measured forces due to spindle rotation.

Park and Altintas [25] developed the force and displacement sensors integrated to the spindle housing to monitor milling operations. They compensated the dynamic disturbance of
the structural modes of the sensing system with an extended Kalman filter, hence widened the frequency bandwidth of the sensors.

2.4 Static Force Transformation Analysis on Machine Tools

All three components of the cutting forces and the cutting torque at the tool tip \((F_x, F_y, F_z, \tau_z)\) are in line with linear drives of three-axis machine tools. However, these forces are transmitted to the linear and rotary drives of five-axis machines through the kinematic chain of the machine tool. The kinematic model, which defines the relationship between the position, velocity, acceleration, and forces at the tool tip and each drive, is dependent on the geometric configuration of five-axis machines.

The Denavit–Hartenberg notation method (D–H notation) is adopted in this research to describe the geometry relationship between two coordinate systems and to model the transmission of the force to five drives of the machine tool [26, 27].

Lamikiz et al. [28] presented a kinematics module of five-axis machines for different configurations based on Denavit-Hartenberg Theory. Elemental errors have been studied for three well-known multi-axis machine configurations and the use of proposed methodology can be used to define machine positions to decrease tool tip errors. Local coordinate frames are required on each drive module for Denavit-Hartenberg (D-H) approach, which has been widely used to solve robot kinematics [29].

Dynamics analyses i.e. force/torque transformation by (D-H) Theory has taken place with concepts developed for the parallel or serial mechanism. Chang et al. [30] used the (D-H) Theory in the analysis of both kinematics and dynamics of parallel manipulators. They investigated the driving force variation of the hybrid five-degree-of-freedom machine tool and studied that force reactions on the joints and actuator driving forces are obtained on the basis of various cutter
locations and a given external cutting force. Subudhi and Morris [31] presented a dynamic modelling technique for a manipulator with flexible links and joints and the coordinate systems of the each link assigned using D-H notation rules. Chang et al. [32] worked on the dynamic analysis and modelling for the parallel kinematic machine are derived on the basis of the Denavit–Hartenberg (D–H) notation method. The actuator driving force variations for different cutting paths are obtained using the developed dynamic modelling.

There is a lack of study reported in the literature to predict five-axis cutting forces from drive current/torque using the force transformation by considering the kinematics of five axis machine tools.

2.5 Summary

In this chapter, previous studies for cutting force estimation by direct and indirect monitoring, the challenge especially in sensor-less machining have been reviewed. The past attempts in monitoring current for cutting force prediction have been reviewed. The previous studies for modeling the force transformation on machine tools and serial-parallel manipulators, the challenges in torque prediction especially on rotary drives have been reviewed.
Chapter 3: Modeling and Identification of Feed Drive Dynamics

3.1 Introduction

In this chapter, the feed drive dynamics of the five-axis machining center are identified to be used in cutting force prediction model (Chapter 5). In Section 3.2, Coulomb and viscous friction dynamics of drives are experimentally identified using the least squares parameter estimation method. The friction characteristics with Stribeck effect are obtained by moving the axes both in positive and negative directions at various speeds. In Section 3.3 the disturbance transfer functions of the feed drives are identified by conducting impact modal tests on each drive. The identified modal characteristics of the drives are combined with the friction dynamics model in Section 3.3. Kalman Filter is designed to estimate the cutting force while compensating the dominant modes of all five drives in Section 3.4. The results and contributions of the chapter are summarized Section 3.5.

The five-axis machining center with a Heidenhain CNC controller is used for the experimental validations (Figure 3.1). The machine has Y and Z translational drives on spindle side, and X translational along with A and C rotary drives on machine table side. The machine drives are equipped with current sensors that can be used to measure the motor current (torque) commands.
The motor constant \((K_t)\), axis pitch \((h)\), and transmission ratio \((r_g)\) of all five-axis drives are presented in Table 3.1 which are used to estimate the motor torque from the motor current.

### Table 3.1: Motor Constant Parameters of Five Axes read by Heidenhain Controller [1]

<table>
<thead>
<tr>
<th>Axis Name</th>
<th>Motor constant, (K_t) (Nm/A)</th>
<th>Pitch, (h)</th>
<th>Transmission ratio, (r_g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>1.964</td>
<td>12 (mm/rev)</td>
<td>(h / 2\pi) (mm/rad)</td>
</tr>
<tr>
<td>Y</td>
<td>1.964</td>
<td>12 (mm/rev)</td>
<td>(h / 2\pi) (mm/rad)</td>
</tr>
<tr>
<td>Z</td>
<td>1.964</td>
<td>12 (mm/rev)</td>
<td>(h / 2\pi) (mm/rad)</td>
</tr>
<tr>
<td>A</td>
<td>1.522</td>
<td>3 (deg/rev)</td>
<td>3 / 360 (deg/deg)</td>
</tr>
<tr>
<td>C</td>
<td>2.106</td>
<td>4 (deg/rev)</td>
<td>4 / 360 (deg/deg)</td>
</tr>
</tbody>
</table>

### 3.2 Experimental Modeling of Feed Drive Friction

In order to predict the cutting forces from the measured drive current, the current consumed by the friction must be identified. In the literature, this is commonly addressed [4] by conducting air cutting experiments to measure the current (torque) caused by drive friction.
The axis friction torque \( \tau_f(\omega, \tau_m) \) can be embedded with linear axis dynamics as shown in Figure 3.2 using the axis velocity \( \omega \) and motor torque \( \tau_m \).

In this thesis, the experimental friction model of drives with Coulomb and viscous friction components are obtained by moving the axes in positive and negative directions, measuring the friction current (torque) through Heidenhain software, and curve fitting the results shown in Figure 3.3. The torque balance equation of a machine tool drive can be expressed as [11],

\[
K_c I = J_e \omega + B_c \omega + \tau_c + \tau_d
\]  

(3.1)

where \( K_c \) is the torque constant of the motor, \( I \) is the motor current, \( J_e \) is the equivalent inertia of the motor, ball screw and slide, \( B_c \) is the equivalent viscous damping of the motor, bearings, ball screw and slide, \( \tau_c \) is the Coulomb friction of the slide and other components, and \( \tau_d \) is the disturbance torque due to cutting. Once the drive reaches to constant velocity, the feed drive torque equation Eq. (3.2) in air cutting (\( \tau_d = 0 \)) can be reduced to,

\[
K_c I_m = B_c \omega + \tau_c
\]  

(3.2)

which consists of viscous damping and Coulomb friction torques.

The friction components of each drive \( (B_c, \tau_c) \) are identified for the angular speed range of \([-60, 60]\) \( \text{rad/sec} \) with \( 10 \text{ rad/sec} \) increments\( (\pm 60, \pm 50, \pm 40, \pm 30, \pm 20, \pm 10 \text{ rad/sec}) \).
The variation of the friction torque with the angular velocity of the axis is obtained by measuring the average friction current for the positive and negative directions of motion as shown in Figure 3.3.
Figure 3.3: Estimated friction for the feed drives at different velocities

Linear least squares method is employed to identify $B_c$ and $\tau_c$ from the measured motor torque (Figure 3.3). The average viscous damping coefficient and the Coulomb friction torque of each drive in positive and negative directions are obtained by curve fitting (Appendix A) and they are presented in Table 3.2.

**Table 3.2: The identified viscous and Coulomb friction for Quaser-UX600**

<table>
<thead>
<tr>
<th>Axis</th>
<th>Viscous friction [Nm/rad/s]</th>
<th>Positive Coulomb friction torque [Nm]</th>
<th>Negative Coulomb friction torque [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>0.0653</td>
<td>2.118</td>
<td>-2.176</td>
</tr>
<tr>
<td>Y</td>
<td>0.0670</td>
<td>2.049</td>
<td>-2.095</td>
</tr>
<tr>
<td>Z</td>
<td>0.0401</td>
<td>2.054</td>
<td>-2.356</td>
</tr>
<tr>
<td>C</td>
<td>0.0092</td>
<td>1.215</td>
<td>-1.574</td>
</tr>
<tr>
<td>A</td>
<td>0.0347</td>
<td>2.328</td>
<td>-2.214</td>
</tr>
</tbody>
</table>

Moreover, the cutting force is smaller than the friction force, it cannot be detected the cutting force along toolpath from the drive torque because the control system cannot detect the drive velocity of position changes.
3.3 Identification of Feed Drive Disturbance Transfer Function

Cutting forces act as disturbance on machine tool drives. The disturbance transfer function \( (H_d) \) between the disturbance force and the motor current is shown in Figure 3.4, and needs to be identified to predict the tool tip forces.

![Disturbance transfer function between disturbance torque and motor current](image)

**Figure 3.4:** Disturbance transfer function between disturbance torque and motor current

The disturbance transfer functions of Quaser-UX600 machine’s drives are obtained via impact hammer tests while the current controller of the CNC is running. The impact hammer experiments are grouped as: translational drive identification (X, Y, and Z axes) and rotary drive identification (A and C axes) cases as follows.

### 3.3.1 Disturbance Transfer Function Identification – Translational Drives

An instrumented hammer is used to excite the translational drives with an impact disturbance force applied in the direction of motion as shown in Figure 3.5, where the feed drive system of X-axis is shown as a multi-input multi-output (MIMO) system with motor torque \( (\tau_m) \) and hammer force \( (F_d) \) as inputs, and the table position \( (x) \) and the motor rotational angle \( (\theta) \) as outputs. The generic controller in Figure 3.5 includes position, velocity, and current controllers of Heidenhain system.
Impact force data in time domain is measured using the data acquisition module of CutPro® while the motor current is measured with Heidenhain software and then converted to motor torque as,

$$\tau_m = K_I I$$  \hspace{1cm} (3.3)

The measured impact force and torque signals in time domain are transformed to frequency domain through Fast Fourier Transformation (FFT) as [34]

$$F_{hammer}(j\omega) = \frac{1}{N} \sum_{n=0}^{N-1} f(nT_s) \left[ \cos \frac{2k\pi}{N} n - j \sin \frac{2k\pi}{N} n \right], \quad k = 0, 1, \ldots, \frac{N}{2}$$  \hspace{1cm} (3.4)

$$\tau_{motor}(j\omega) = \frac{1}{N} \sum_{n=0}^{N-1} \tau(nT_s) \left[ \cos \frac{2k\pi}{N} n - j \sin \frac{2k\pi}{N} n \right], \quad k = 0, 1, \ldots, \frac{N}{2}$$

where $F$ is the impact force applied to the drives, $\omega$ is the circular frequency in rad/s, $N$ is the number of samples, $T_s$ is the sampling time in seconds, and $\tau$ is the motor torque measured from the Heidenhain. Instead of using the Fourier spectra of the measured torque and the impact force, the cross spectrum of the measurements were divided by the auto spectrum of the impact force to minimize the measurement noise in the disturbance frequency response function ($H_d(j\omega)$) as [30],
where $^*$ is the complex conjugate operator. Since several impact tests are conducted on each drive, $H_d(j\omega)$ is obtained by averaging the frequency response curves obtained from the successive impact tests, and then $H_d(j\omega)$ is modal fitted in the form of Eq.(3.6) to identify the modal parameters i.e. natural frequencies $\omega_{n,i}$ and damping ratios $\zeta_i$.

$$H_d(j\omega) = \frac{\tau_m}{F} = \sum_{i=1}^{n} \frac{\alpha_i + j\beta_i\omega}{\omega_{n,i}^2 + j2\zeta_i\omega_{n,i}\omega - \omega^2}$$

(3.6)

In order to obtain the dimensionless $H_d(j\omega)$ the impact force is converted to torque ($F_h$) using the transmission ratio ($r_g$) of drives as,

$$\tau = F r_g$$

(3.7)

The introduced disturbance transfer function identification procedure is followed for X, Y, and Z translational axes, and the hammer impact is given to the drive mechanism as close as possible as exemplified for X-axis in Figure 3.6.

**Figure 3.6:** Example of translational hammer test set-up
An example of the measured motor torque and the corresponding hammer (disturbance) torque in time domain are shown in for X-axis. As seen, the transmitted disturbance force, which is converted to disturbance torque using Eq.(3.7), is not transmitted as it is and causes oscillations in drive torque due to excitation of natural modes of the feed drive structure.

Figure 3.7: Hammer Response for X-Drive in time domain
Figure 3.8: Hammer Response for Y-Drive in time domain

Figure 3.9: Hammer Response for Z-Drive in time domain
The identified modal parameters of $H_d(j\omega)$ for X, Y, and Z drives through modal fitting Eq.(3.6) are presented in Table 3.3.

**Table 3.3:** The identified modal parameters of disturbance FRF between drive torque and hammer torque for all translational axes

<table>
<thead>
<tr>
<th>Mode number $i$</th>
<th>Natural frequency $\omega_{n,i} (\text{Hz})$</th>
<th>Damping $\zeta_i (%)$</th>
<th>Real Residue $\sigma_i (\text{m} / \text{N})$</th>
<th>Imaginary Residue $\nu_i (\text{m} / \text{N})$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>X-drive</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>36.102</td>
<td>4.7</td>
<td>20.139</td>
<td>0.010</td>
</tr>
<tr>
<td>2</td>
<td>40.215</td>
<td>2.144</td>
<td>22.259</td>
<td>-7.222</td>
</tr>
<tr>
<td>3</td>
<td>42.844</td>
<td>3.505</td>
<td>35.386</td>
<td>-19.608</td>
</tr>
<tr>
<td>4</td>
<td>62.5</td>
<td>5.000</td>
<td>79.245</td>
<td>-29.282</td>
</tr>
<tr>
<td>5</td>
<td>179</td>
<td>4.233</td>
<td>-59.546</td>
<td>-86.666</td>
</tr>
<tr>
<td><strong>Y-drive</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>15</td>
<td>3.8</td>
<td>-0.824</td>
<td>-2.946</td>
</tr>
<tr>
<td>2</td>
<td>16.358</td>
<td>4.963</td>
<td>2.401</td>
<td>-0.985</td>
</tr>
<tr>
<td>3</td>
<td>31</td>
<td>2.9</td>
<td>3.427</td>
<td>-6.084</td>
</tr>
<tr>
<td>4</td>
<td>36</td>
<td>3.3</td>
<td>3.073</td>
<td>-12.692</td>
</tr>
<tr>
<td>5</td>
<td>47</td>
<td>2.8</td>
<td>-5.286</td>
<td>-2.743</td>
</tr>
<tr>
<td>6</td>
<td>52</td>
<td>5</td>
<td>0.502</td>
<td>-21.390</td>
</tr>
<tr>
<td>7</td>
<td>63.5</td>
<td>6</td>
<td>-1.721</td>
<td>-8.831</td>
</tr>
<tr>
<td>8</td>
<td>71</td>
<td>2.2</td>
<td>-6.583</td>
<td>-14.975</td>
</tr>
<tr>
<td>9</td>
<td>77.5</td>
<td>3</td>
<td>49.990</td>
<td>-132.179</td>
</tr>
<tr>
<td>10</td>
<td>78</td>
<td>1.705</td>
<td>8.121</td>
<td>48.264</td>
</tr>
<tr>
<td>11</td>
<td>85</td>
<td>2.2</td>
<td>11.759</td>
<td>-10.906</td>
</tr>
<tr>
<td>12</td>
<td>92.488</td>
<td>1.261</td>
<td>0.578</td>
<td>-3.788</td>
</tr>
<tr>
<td>13</td>
<td>96.822</td>
<td>0.860</td>
<td>1.344</td>
<td>-2.425</td>
</tr>
<tr>
<td>14</td>
<td>102</td>
<td>1.2</td>
<td>-1.926</td>
<td>-4.150</td>
</tr>
<tr>
<td>15</td>
<td>130</td>
<td>10</td>
<td>69.047</td>
<td>-6.204</td>
</tr>
<tr>
<td>16</td>
<td>180</td>
<td>4</td>
<td>-76.390</td>
<td>46.058</td>
</tr>
<tr>
<td><strong>Z-drive</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>11.5</td>
<td>63</td>
<td>-6.663</td>
<td>0.409</td>
</tr>
<tr>
<td>2</td>
<td>39</td>
<td>7</td>
<td>19.343</td>
<td>3.894</td>
</tr>
<tr>
<td>3</td>
<td>43</td>
<td>3.817</td>
<td>-0.948</td>
<td>-8.728</td>
</tr>
<tr>
<td>4</td>
<td>63.7397</td>
<td>5</td>
<td>26.038</td>
<td>-38.587</td>
</tr>
<tr>
<td>5</td>
<td>71.5</td>
<td>1.2</td>
<td>3.009</td>
<td>-1.920</td>
</tr>
<tr>
<td>6</td>
<td>86</td>
<td>0.4</td>
<td>-0.738</td>
<td>-0.737</td>
</tr>
<tr>
<td>7</td>
<td>89</td>
<td>6.2</td>
<td>90.764</td>
<td>-100.621</td>
</tr>
<tr>
<td>8</td>
<td>92.5</td>
<td>3</td>
<td>11.648</td>
<td>31.899</td>
</tr>
<tr>
<td>9</td>
<td>101</td>
<td>3</td>
<td>12.656</td>
<td>7.277</td>
</tr>
<tr>
<td>10</td>
<td>106</td>
<td>2</td>
<td>-2.574</td>
<td>4.415</td>
</tr>
</tbody>
</table>
The measured and identified (synthesized) disturbances FRF of all translational axes are favorably compared and the dominant modes of the disturbance FRF are shown in Figure 3.10, Figure 3.11, and Figure 3.12.

**Figure 3.10:** Comparison of the measured and synthesized (identified) disturbance FRF for X-drive
Figure 3.11: Comparison of the measured and synthesized (identified) disturbance FRF for Y-drive
The bandwidth of the feed drive must be ideally larger than the tooth passing frequency in order to measure the force without using force sensors. The X-drive has two dominant modes (40Hz, 62.5Hz) and the measured motor torque is distorted due to drive dynamics above the disturbance bandwidth of 200 Hz. Y-drive has five dominant modes (31 Hz, 36 Hz, 46 Hz, 52 Hz, 78Hz) with a disturbance bandwidth of 115 Hz. Similarly, Z-drive, has three dominant modes (40 Hz, 63 Hz, and 88) with a disturbance bandwidth of 150 Hz. In practical machining applications, the frequency of the cutting (disturbance) force is usually higher than the bandwidth of the drives; and thus a Kalman filter is designed to compensate for the dominant modes of all five drives in the next section.

Figure 3.12: Comparison of the measured and synthesized (identified) disturbance FRF for Z-drive
In the closed loop disturbance identification, impact tests have been conducted close to the drives and the motor corresponding current is measured while the controller is on. \( \tau_h \) and \( \tau_m \) are transformed to frequency domain using Eq.(3.4) and the disturbance FRF is obtained with Eq.(3.6). The obtained FRF is modal fitted in the form of Eq.(3.8) to identify the modal parameters i.e. natural frequencies \( \omega_{n,i} \,(Hz) \) and damping ratios \( \zeta_i \,(\%) \).

\[
\frac{\tau_m}{F} = h(s) = \sum_{j=1}^{n} \frac{r^j}{2j(s-s_1)} + \frac{\tau_{r}^{*}}{2j(s-s_1^*)} = \frac{\alpha^j + \beta^j s}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]  

The identified modal parameters of the disturbance FRF of the X-axis are given in Table 3.3.

### 3.3.2 Disturbance Transfer Function Identification - Rotary Drives

In order to determine the cutting forces from the drive current in five-axis cutting, the disturbance FRF between the motor torque and the disturbance torque applied on the rotary axes (A and C) have been measured. Unlike the translational axes, the rotary drives are connected to motor with a worm gear. Therefore, the pitch of each rotary drive is defined in \( \text{deg/deg} \) as given in Table 3.1.

The same procedure is followed as in the case of translational drives. However, the disturbance is applied as torque on the rotary drives, which is practically achieved by defining a moment arm \( d \) between the line of impact force and the rotary drive axis as shown in Figure 3.13. The disturbance torque due to impact force is defined as,

\[
\tau_h = (F_h \, d) \, r_g
\]  

where \( d \) is defined from the table center as the axes of both rotary drives pass through the center.
Figure 3.13: Top view of the machine table with applied impact force ($F_{hammer}$) at a distance ($d$) for: (a) A-drive, (b) C-drive

Once the time domain measurements are conducted, the measured motor torque ($\tau_m$) and the disturbance torque ($\tau_h$) are transformed into frequency domain using FFT (Eq.(3.4)) with measurement noise cancellation (Eq.(3.5)). The dimension-less disturbance FRF is modal fitted (Eq.(3.6)) to extract the modal parameters that are given in Table 3.4.

Table 3.4: The identified modal parameters of disturbance FRF between drive torque to hammer torque for rotary axes

<table>
<thead>
<tr>
<th>Mode number $i$</th>
<th>Natural frequency $\omega_{n,i}$ (Hz)</th>
<th>Damping $\zeta_i$ (%)</th>
<th>Real Residual $\sigma_i (m / N)$</th>
<th>Imaginary Residual $\nu_i (m / N)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A-drive</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>64.844</td>
<td>29.854</td>
<td>-103.09</td>
<td>-185.94</td>
</tr>
<tr>
<td><strong>C-drive</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>40</td>
<td>10</td>
<td>-247.14</td>
<td>-18.53</td>
</tr>
<tr>
<td>2</td>
<td>49</td>
<td>15</td>
<td>883.79</td>
<td>-217.31</td>
</tr>
<tr>
<td>3</td>
<td>113</td>
<td>2</td>
<td>-28.35</td>
<td>25.96</td>
</tr>
</tbody>
</table>
The measured motor torque and the corresponding hammer (disturbance) response in time domain are shown in Figure 3.14 and Figure 3.15 for A and C-axes, respectively. The disturbance torque is not transmitted to rotary drives as it is, and it causes oscillations in drive response.

![Figure 3.14: Hammer Response for A-Drive in time domain](image)

**Figure 3.14:** Hammer Response for A-Drive in time domain
The measured and identified disturbance FRF of rotary axes is also favorably compared, and the dominant modes of the disturbance FRF are shown in Figure 3.16 and Figure 3.17.
Figure 3.16 Comparison of the measured and synthesized disturbance FRF for A-Drive
As shown in the plots, the A-drive has one dominant mode at 64 Hz while C-drive has three dominant modes (40 Hz, 49Hz, and 113 Hz). The magnitude ratio for both drives does not approach to unity at low frequencies ($\approx 0$ Hz). In order to further investigate that static stiffness issue, static load tests are performed on both rotary drives to identify the static disturbance gain as presented in next section.

### 3.3.3 Static Load Tests for Rotary Axes

The theoretical determination of rotary drive stiffness is difficult due to the complex nature of the axes and lack of information about machine elements and their material properties. Therefore, in order to show the reason behind the unexpectedly high static stiffness behavior of the identified disturbance FRF for the rotary axes as shown for A-drive in Figure 3.18, the static
load test is conducted on both rotary drives. Static loading is achieved by positioning weights at approximately the same position where the disturbance force is given in the impact hammer tests.

![Figure 3.18: High static stiffness in A-drive disturbance FRF before the static load test](image)

The static test set-up is shown in Figure 3.19 for the A-drive. The drive is loaded gradually from 70lbs up to 100lbs on the table with 10lbs increments, and the corresponding motor torque response is measured. The static torque created by the static load is obtained by replacing the hammer force in Eq.(3.10) with the weight \( F_w \) as,

\[
\tau_s = (F_w d) r_g
\]

(3.10)
The variations of the static torque ($\tau_{st}$) and the measured motor torque ($\tau_m$) with the static load are given in Figure 3.20, and the average ratio is found as $\frac{\tau_m}{\tau_{st}} = 0.81$.

**Figure 3.19:** Static load test set-up for A-drive

**Figure 3.20:** Static loading test results for A-drive
Similar to A-drive, C-drive disturbance FRF also shows a high static stiffness behavior as shown in Figure 3.21. For C-drive static loading test, the C-axis is rotated by 90° to position weights on the corner of the table as shown in Figure 3.22. As before, the static load is applied gradually and the variations of the static torque ($\tau_{st}$) and the measured motor torque ($\tau_m$) with the static load are given in Figure 3.23, and the average ratio is found as $\frac{\tau_m}{\tau_{st}} = 0.92$ which is close to unity.

The deviation of $\frac{\tau_m}{\tau_{st}}$ from unity for both rotary drives could be attributed to the approximate distance of the static loading center to the rotation axis of the drives, and also the measurement noise in motor torque due to low motor current.

![Figure 3.21: High static stiffness in C-drive disturbance FRF before the static load test](image-url)
As seen, the quasi-static behavior \( (\approx 0Hz) \) of the identified disturbance FRF of the rotary drives cannot be captured with impact hammer tests. Hence, the results of the static loading tests are used to calibrate the quasi-static region of the identified disturbance FRF as presented in Figure 3.18 and Figure 3.21.
3.4 Kalman Filter Design for Cutting Force Estimation

A Kalman filter is implemented as a disturbance observer model for indirect measurement of cutting forces in five-axis milling process. The inputs to the Kalman filter are the measured drive current and given CNC dynamic system model. Then, the Kalman Filter estimates the values of the state variables, which include the disturbance torque to the drive. The purpose of using the Kalman filter is to suppress the undesired current component due to the acceleration or deceleration from the measured drive current.

The measured disturbance (cutting) torque, which is isolated by removing the friction component (Section 3.2) from the measured total motor torque, is distorted due to the disturbance dynamics (Section 3.3). Kalman filter is used to estimate the drive force (unknown input) from the distorted measured torque (output) of the drive motor using the disturbance FRF of the drive in a computationally efficient recursive manner as follows [35], [36].

The disturbance transfer function of each drive is considered as a linear time-invariant (LTI) discrete-time (DT) system as,

\[
\begin{align*}
    x_{[k+1]} &= Ax_{[k]} + Bu_{[k]} + \omega_{[k]} \\
    y_{[k]} &= Cx_{[k]} + Du_{[k]} + \nu_{[k]} 
\end{align*}
\]  

(3.11)

where \( x_{[k]} \in \mathbb{R}^n \) is the states vector, \( u_{[k]} \in \mathbb{R}^n \) is the unknown deterministic input (force) vector, \( y_{[k]} \in \mathbb{R}^p \) is the measurement vector at time instant \( k \), and \( \omega_{[k]} \in \mathbb{R}^n \) and \( \nu_{[k]} \in \mathbb{R}^n \) are zero-mean noises with covariance \( Q \) and \( R \), respectively [37]. The overall derivation of the mathematical expressions is detailed in [35], and only the required equations are presented here. The block diagram of the direct feedthrough input and state estimator is given in Figure 3.24.
The input estimation is obtained as,

\[ \hat{u}_{[k|k]} = M_{[k]} (y_{[k]} - C \hat{x}_{[k-k-1]}) \]  (3.12)

where \( M_{[k]} \) is the minimum variance unbiased input estimator, \( y_{[k]} \) is the output, and \( \hat{x}_{[k-k-1]} \) is the state estimate at previous time step. \( M_{[k]} \) is obtained as,

\[
\begin{align*}
M_{[k]} &= (D^\top R^{-1}_{[k]} D)^{-1} D^\top R^{-1}_{[k]} \\
R_{[k]} &= CP_{[k-k-1]} C^\top + R
\end{align*}
\]  (3.13)

where \( P_{[k-k-1]} \) is the error covariance matrix at the current time step based on the previous time step and can be found as,
\[ P_{[k+1|k]} = \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} P_{[k|k]} & P_{[w,u|k]} \\ P_{[w,u|k]} & P_{[u|k]} \end{bmatrix} \begin{bmatrix} A^T \\ B^T \end{bmatrix} + Q \] (3.14)

The individual covariance matrices of input and state estimates can be found as,

\[
\begin{align*}
P_{[u|k]} &= (D^T R^{-1} D)^{-1} \\
P_{[k|k]} &= P_{[k|k-1]} - L_{[k]} (R_{[k]} - DP_{[u|k]} D^T) L_{[k]}^T \\
P_{[w,u|k]} &= P_{[w,u|k-1]} - L_{[k]} D P_{[u|k]} 
\end{align*}
\] (3.15)

where \( L_{[k]} \) is the gain matrix and defined as,

\[ L_{[k]} = P_{[k|k-1]} C^T R^{-1} \] (3.16)

Eqs. (3.11) - (3.16) are implemented in MATLAB Simulink to compensate for the identified disturbance FRF for all five drives, which are presented in next section.

3.5 Compensation of Disturbance FRF with Kalman Filter

The Kalman filter is a linear, unbiased, and minimum error variance recursive algorithm to optimally estimate the unknown state of a dynamic system from a noisy environment [13]. The disturbance force \( (u_{[k]}) \) is estimated from the measured distorted motor torque through the Kalman filter (Section 3.4) as illustrated in Figure 3.25. It is assumed that both the disturbance force and the current (torque) signals are contaminated with system Gaussian noise, \( \omega_{[k]} \) and measurement noise, \( v_{[k]} \).

**Figure 3.25:** Estimated Force using Kalman Filter
Comparison of the uncompensated \( \frac{y_k}{u_k} \) and compensated disturbance \( \frac{\hat{u}_k}{u_k} \) FRF, and the Kalman filter FRF \( \frac{\hat{u}_k}{\hat{y}_k} \) for impact hammer tests used in Section 3.3 for all five drives are presented in Figure 3.26, Figure 3.28, Figure 3.30, Figure 3.32. As seen, the ratio of the estimated torque and to the actual impact (disturbance) torque \( \frac{\hat{u}_k}{u_k} \) approaches to unity for all five drives.

Based on mathematical proofs, input which is disturbance force must be equal to output which is estimated disturbance force that can show us the proposed model can be used to predict tool tip forces. As any deviation happens into the model while using Kalman Filter, it causes to predict tool tip forces above or below of actual force values.

![Graph showing FRFs](image)

**Figure 3.26:** Comparison of the uncompensated \( \frac{y_k}{u_k} \) and compensated disturbance \( \frac{\hat{u}_k}{u_k} \) FRF, and the Kalman filter FRF \( \frac{\hat{u}_k}{\hat{y}_k} \) for X-drive impact hammer test.
Figure 3.27: The phase of comparison of the uncompensated \( \frac{\hat{y}_k}{u_k} \) and compensated disturbance \( \frac{\hat{u}_k}{u_k} \) FRF, and the Kalman filter FRF \( \frac{\hat{u}_k}{\hat{y}_k} \) for X-drive impact hammer test.

Figure 3.28: Comparison of the uncompensated \( \frac{\hat{y}_k}{u_k} \) and compensated disturbance \( \frac{\hat{u}_k}{u_k} \) FRF, and the Kalman filter FRF \( \frac{\hat{u}_k}{\hat{y}_k} \) for Y-drive impact hammer test.
Figure 3.29: The phase of comparison of the uncompensated \( \hat{y}_k / u_k \) and compensated disturbance \( \hat{u}_k / u_k \) FRF, and the Kalman filter FRF \( \hat{y}_k / \hat{y}_k \) for Y-drive impact hammer test.

Figure 3.30: Comparison of the uncompensated \( \hat{y}_k / u_k \) and compensated disturbance \( \hat{u}_k / u_k \) FRF, and the Kalman filter FRF \( \hat{u}_k / \hat{y}_k \) for Z-drive impact hammer test.
Figure 3.31: The phase of comparison of the uncompensated $\frac{\hat{y}_k}{u_k}$ and compensated disturbance $\frac{\hat{u}_k}{u_k}$ FRF, and the Kalman filter FRF $\frac{\hat{u}_k}{\hat{y}_k}$ for Z-drive impact hammer test.

Figure 3.32: Comparison of the uncompensated $\frac{\hat{y}_k}{u_k}$ and compensated disturbance $\frac{\hat{u}_k}{u_k}$ FRF, and the Kalman filter FRF $\frac{\hat{u}_k}{\hat{y}_k}$ for A-drive impact hammer test.
Figure 3.33: The phase of comparison of the uncompensated \( \frac{\hat{y}_k}{u_k} \) and compensated disturbance \( \frac{\hat{u}_k}{u_k} \) FRF, and the Kalman filter FRF \( \frac{\hat{u}_k}{\hat{y}_k} \) for A-drive impact hammer test.

Figure 3.34: Comparison of the uncompensated \( \frac{\hat{y}_k}{u_k} \) and compensated disturbance \( \frac{\hat{u}_k}{u_k} \) FRF, and the Kalman filter FRF \( \frac{\hat{u}_k}{\hat{y}_k} \) for C-drive impact hammer test.
Figure 3.35: The phase of comparison of the uncompensated $\frac{\hat{y}_k}{u_k}$ and compensated disturbance $\frac{\hat{u}_k}{u_k}$ FRF, and the Kalman filter FRF $\frac{\hat{u}_k}{y_k}$ for C-drive impact hammer test

3.6 Summary

In this chapter, identification of the feed drive dynamics is presented for translational and rotary drives. Static load test is used to correct the unexpectedly high static stiffness of the identified disturbance FRF for the rotary drives. Kalman filter has been designed to estimate the cutting forces from the distorted motor torque values.
Chapter 4: Force Transformation on Machine Tools

4.1 Introduction

The transmitted disturbance forces on machine drives are predicted in Chapter 3. The estimated forces on the drives are transformed into the tool tip coordinate frame for the machining process and tool condition monitoring. In the following sections, the forward kinematics of five-axis Quaser-UX600 machine is modeled using the Denavit-Hartenberg (D-H) method for force transformation. The D-H approach has been used to model the kinematics of machines and to evaluate the geometrical errors in five-axis milling centers [28]; however, to the best of author’s knowledge, it is the first time used for cutting force transformation for five-axis machining.

4.2 Kinematic Chains of Quaser-UX600 Machine Tool

Five-axis serial machine tools are composed of the following kinematic elements: (1) prismatic (translational) joints, (2) revolute (rotary) joints, (3) workpiece, and (4) cutting tool as illustrated in Figure 4.1 for the investigated five-axis machine. The workpiece and the cutting tool are attached to different joints, thus there are two open kinematic chains; the workpiece chain defined from the base coordinate frame to the workpiece, and the tool chain defined from the base coordinate frame to the cutting tool frame. These two open kinematic chains form the entire kinematic chain of the machine.

The generic kinematic model for five-axis machines depends on the relative position and the orientation of the tool coordinate system \( (OXYZ)_{tool} \) with respect to the workpiece coordinate system \( (OXYZ)_{wp} \). Both frames have the motion with respect to the same base coordinate system \( (OXYZ)_{b} \).
Figure 4.1: Kinematic chains of Quaser-UX600 machining center

The base coordinate frame $(O_XYZ)_b$ is attached to the machine bed and located at the machine corner when the joint movements are zero (machine home position). The workpiece $(O_XYZ)_{wp}$ and the tool coordinate $(O_XYZ)_{tool}$ frames are attached to the workpiece and the cutting tool, respectively as shown in Figure 4.1.
Since the force transformation is defined between frames, the relative positions of drive frames with respect to each other are required as shown in Figure 4.2, where $X^*$ is the movement of X-drive, $Y^*$ is the movement of Y-drive, $Z^*$ is the movement of Z-drive.

Figure 4.2: Relative position of the coordinate frame on each joint at machine home position for Quaser UX-600
The transformation of the workpiece coordinate system into base frame is obtained by the homogenous transformation matrices as,

\[ WP \times A \times WPBB \times A \times TT T \times TT T \]

where the matrix \( i^j \) is transformation from \( j^{th} \) coordinate frame to \( i^{th} \) frame.

In order to build the generalized kinematic model, tool frame must be represented with respect to the base frame similarly as,

\[ tool \times BB \times toolBB \times toolTT T \times TT T \]

The tool tip position \( t_{pt} \) and orientation \( t_{ot} \) vectors are defined in tool frame as,

\[ t_p = t_{p,x}i + t_{p,y}j + t_{p,z}k \]
\[ t_o = t_{o,x}i + t_{o,y}j + t_{o,z}k \]

which are transformed into the workpiece frame using Eq. (4.1) and (4.2) as;

\[
\begin{bmatrix}
t_{pt}^\epsilon
t_{ot}^\epsilon
\end{bmatrix} = \left( T^b \right)^{-1} T^b \begin{bmatrix}
t_{p}^w
t_{o}^w
\end{bmatrix}
\]

For the five-axis machining center illustrated in Figure 4.1, the cutting tool is aligned with \( Z_t \) and its orientation is defined by the directional cosines in the augmented vector,

\[ t_o^i = [0 \quad 0 \quad -1 \quad 0] \]

Similarly, the tool tip position \( t_p^i \) in the tool frame can be represented by the augmented vector as,

\[ t_p^i = [0 \quad 0 \quad L \quad 0] \]

where \( L \) is the tool length and transformed into workpiece frame using the Eq. (4.7) as,
Relative positions of the coordinate frames on each joint at machine home position is schematically shown in Figure 4.2 and the structural (constant) dimensions between joint frames are listed in Table 4.1.

Table 4.1: Machine home position parameters

<table>
<thead>
<tr>
<th>D-H parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>224 cm</td>
</tr>
<tr>
<td>( d_1 )</td>
<td>149.5 cm</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>56 cm</td>
</tr>
<tr>
<td>( d_2 )</td>
<td>90 cm</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>67 cm</td>
</tr>
<tr>
<td>( d_3 )</td>
<td>177 cm</td>
</tr>
</tbody>
</table>

Based on Table 4.1 and Figure 4.2, the D-H parameters table is obtained and shown in Table 4.2 by following the standard D-H rules.

Based on D-H convention, the transformation between frame \( i \) and \( i-1 \) frames can be represented as [26],

\[
T_{i-1,i} = \begin{bmatrix}
\cos \theta_i & -\sin \theta_i & \cos \alpha_i & \sin \theta_i \sin \alpha_i & a_i \cos \theta_i \\
\sin \theta_i & \cos \theta_i & \cos \alpha_i & -\cos \theta_i \sin \alpha_i & a_i \sin \theta_i \\
0 & s \alpha_i & \cos \alpha_i & d_i \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (4.8)
Table 4.2: D-H parameters table of Quaser UX600 five-axis machining center

<table>
<thead>
<tr>
<th>link</th>
<th>$\theta$ (deg)</th>
<th>$d$</th>
<th>$a$</th>
<th>$\alpha$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workpiece Chain</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^xT_B$</td>
<td>+90</td>
<td>0</td>
<td>$-a_2$</td>
<td>+180</td>
</tr>
<tr>
<td>$^AT_X$</td>
<td>+90</td>
<td>$d_3 + X^*$</td>
<td>$-a_3$</td>
<td>−180</td>
</tr>
<tr>
<td>$^CT_A$</td>
<td>$90 + \theta_A^*$</td>
<td>0</td>
<td>0</td>
<td>+270</td>
</tr>
<tr>
<td>$^{WP}T_C$</td>
<td>$\theta_C^*$</td>
<td>0</td>
<td>$-a_4$</td>
<td>0</td>
</tr>
<tr>
<td>Tool Chain</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^rT_B$</td>
<td>0</td>
<td>$-d_1$</td>
<td>$a_1$</td>
<td>−90</td>
</tr>
<tr>
<td>$^ZT_Y$</td>
<td>−90</td>
<td>$Y^*$</td>
<td>0</td>
<td>+90</td>
</tr>
<tr>
<td>$^{tool}T_Z$</td>
<td>0</td>
<td>$d_2 + Z^*$</td>
<td>0</td>
<td>+180</td>
</tr>
</tbody>
</table>

The force transformation from the machine tool drives to the tool tip frame using D-H method is presented in the following section.

### 4.3 Force Transformation on Quaser-UX600 Machining Center

Force transformation for the investigated machine tool is composed of the tool chain and workpiece chain which are combined as one to represent force jacobian using the D-H convention. Firstly, frame to frame transformations are defined as,

\[
^{WP}T_B = ^xT_B ^AT_X ^CT_A ^{WP}T_C
\]

\[
^{tool}T_B = ^rT_B ^ZT_Y ^{tool}T_Z
\]

\[
^{WP}T_{tool} = \left( ^{tool}T_B \right)^{-1} ^{WP}T_B
\]

Frame to frame transformation matrices between the drives within the tool chain are obtained as follows,
\[ Base to Y - Drive: Y_T^B = \begin{bmatrix} 1 & 0 & 0 & a_t \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 -d_t \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ Y - drive to Z - drive: Z_T^Y = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & Y^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ Z - drive to Tool frame: Tool_T^Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & d_2 + Z^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

(4.10)

Frame to frame transformation matrices between the drives within the workpiece chain are,

\[ Base to X - Drive: X_T^B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -a_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ X - Drive to A - Drive: A_T^X = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & a_3 \\ 0 & 0 & -1 & d_3 + X^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ A - Drive to C - Drive: C_T^A = \begin{bmatrix} \cos(90 + \theta_A^*) & 0 & -\sin(90 + \theta_A^*) & 0 \\ \sin(90 + \theta_A^*) & 0 & \cos(90 + \theta_A^*) & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ C - Drive to WP frame: WP_T^C = \begin{bmatrix} \cos \theta_C^* & -\sin \theta_C^* & 0 & -a_4 \cos \theta_C^* \\ \sin \theta_C^* & \cos \theta_C^* & 0 & -a_4 \sin \theta_C^* \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

(4.11)
Then, the force jacobian is obtained using Eq. (4.7), Eq.(4.10) and Eq.(4.11) as

\[
[J]_{6 \times 5} = \begin{bmatrix}
\frac{\partial X}{\partial X^*} & \frac{\partial X}{\partial Y^*} & \frac{\partial X}{\partial Z^*} & \frac{\partial X}{\partial \theta_a^*} & \frac{\partial X}{\partial \theta_c^*} \\
\frac{\partial Y}{\partial X^*} & \frac{\partial Y}{\partial Y^*} & \frac{\partial Y}{\partial Z^*} & \frac{\partial Y}{\partial \theta_a^*} & \frac{\partial Y}{\partial \theta_c^*} \\
\frac{\partial Z}{\partial X^*} & \frac{\partial Z}{\partial Y^*} & \frac{\partial Z}{\partial Z^*} & \frac{\partial Z}{\partial \theta_a^*} & \frac{\partial Z}{\partial \theta_c^*} \\
\frac{\partial O_i}{\partial X^*} & \frac{\partial O_i}{\partial Y^*} & \frac{\partial O_i}{\partial Z^*} & \frac{\partial O_i}{\partial \theta_a^*} & \frac{\partial O_i}{\partial \theta_c^*} \\
\frac{\partial O_j}{\partial X^*} & \frac{\partial O_j}{\partial Y^*} & \frac{\partial O_j}{\partial Z^*} & \frac{\partial O_j}{\partial \theta_a^*} & \frac{\partial O_j}{\partial \theta_c^*} \\
\frac{\partial O_k}{\partial X^*} & \frac{\partial O_k}{\partial Y^*} & \frac{\partial O_k}{\partial Z^*} & \frac{\partial O_k}{\partial \theta_a^*} & \frac{\partial O_k}{\partial \theta_c^*} \\
\end{bmatrix}
\]

(4.12)

Force transformation from the tool tip to drive can be obtained using the jacobian as,

\[
\{\tau_{\text{drive}}\}_{5 \times 1} = [J^T]_{6 \times 5} \{F_{\text{tool}}\}_{6 \times 1}
\]

(4.13)

Since the ultimate goal of the sensor-less machining is to predict the tool tip forces using drive current, Eq. (4.13) can be re-arranged as,

\[
\{F_{\text{tool}}\}_{6 \times 1} = [J^T]_{6 \times 5}^+ \{\tau_{\text{drive}}\}_{5 \times 1}
\]

(4.14)

where $[ ]^+$ is the generalized matrix inverse. The experimental verification results for Chapters 3 and 4 with cutting tests are presented in Chapter 5.

### 4.4 Summary

In this chapter, a new force prediction model from the drive currents for the sensor-less machining is developed using D-H theory. Kinematic model of the machining center and the measured drive forces obtained from the current drawn by the drive motors are specified as the input for the force prediction model.
Chapter 5: Experimental Verification

5.1 Introduction

Several cutting experiments have been conducted to verify the proposed cutting force prediction model from the measured drive torque. The experiments have been conducted on Quaser-UX600 five-axis machining center, and they are grouped as: single-axis and two-axis flat-end milling, three-axis, and five-axis free-form surface machining with ball-end mill. The cutting conditions used in the experiments are given in Table 5.1, and each experiment was repeated twice to check for the repeatability of the results. The specifications of the cutting tools used in all experiments are given in Table 5.2.

Table 5.1: Details of model verification experiments

<table>
<thead>
<tr>
<th>Experiment Name</th>
<th>Spindle Speed [rpm]</th>
<th>Linear Feed [mm/rev/tooth]</th>
<th>Axial depth of cut [mm]</th>
<th>Workpiece material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-Axis</td>
<td>1000</td>
<td>0.20</td>
<td>4.50</td>
<td>Al7050</td>
</tr>
<tr>
<td>2-Axis Cutting</td>
<td>1000</td>
<td>0.20</td>
<td>4.50</td>
<td></td>
</tr>
<tr>
<td>3-Axis Free</td>
<td>1000</td>
<td>0.20</td>
<td>3.00 – 4.50</td>
<td></td>
</tr>
<tr>
<td>5-Axis Free</td>
<td>1000</td>
<td>0.20</td>
<td>1.50 – 8.00</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: Specifications of the tool used in verification experiments

<table>
<thead>
<tr>
<th>Tool Type</th>
<th>Nominal Tool Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-fluted Solid Flat-end Cutter</td>
<td>$\phi$19.05mm</td>
</tr>
<tr>
<td>(Data-Flute)</td>
<td></td>
</tr>
<tr>
<td>2-fluted Solid Ball-end Cutter</td>
<td>$\phi$16mm</td>
</tr>
<tr>
<td>(Sandvik)</td>
<td></td>
</tr>
</tbody>
</table>
All measurements were done on an initially flat surface in dry cutting conditions and the measurement data was collected using the data acquisition module of the machining engineering software CutPro®. Three-component Kistler 9257B table type, and four-component Kistler 9123-C1011 rotary dynamometers were used for cutting force measurements in three-axis (including single-axis and two-axis machining) and five-axis cutting experiments, respectively. In the following sections, for each test type, the finished workpiece surface is shown along with the machined surface generated in Computer Aided Manufacturing (CAM) environment using the commercial software NX 8®.

5.2 Experimental Results

The drive force transformations are carried out for two-axis, three-axis and five-axis toolpaths to obtain the tool tip forces that are employed to verify the cutting force prediction model. For verification, the current data from all axes is fed-back to the A/D board of the signal processing computer. The sampling time is 100 μs.

5.2.1 Single-Axis Cutting Experiment

The purpose for single-axis experiments was to check the model validation in pure horizontal feed conditions. In order to obtain a steady cut in single-axis cutting with a flat end mill, multiple straight cutting experiments were run to verify the static force transformation. An image of the straight cut and its 3D CAD model are demonstrated in Figure 5.1. For the single-axis cutting tests parameters given in Table 5.1, the measured and predicted cutting forces in y-axis direction of the machine are compared in Figure 5.2.
As seen from Figure 5.2, the proposed cutting force prediction model from the drive current using the Kalman filter gives a good match for single-axis cutting along y-direction.
5.2.2 Two-Axis Cutting Experiment

The purpose for two-axis cutting experiments was to check the model validation in partially in X-direction and Y-direction conditions since this is the second step to consider simultaneous movement of multiple linear drives for machining applications.

XY direction tests in flat-end milling experiments were used since they are the simplest multi-axis control cases experiencing the two linear drives effect. The simulation of the cutting process in CAM environment is also shown in Figure 5.3.

Figure 5.3: Comparison of the experimentally measured and the predicted cutting forces

The measured and simulated cutting forces in two principal directions of the machine tool (x,y) are compared and the results are presented in Figure 5.4 and Figure 5.5, accordingly.
As seen, the cutting forces evaluated from the feed drive current and transformed to the tool tip are in an in good agreement with the forces measured with the table dynamometer.
5.2.3 Three-Axis Cutting Experiment

The three-axis ball-end milling experiments have been conducted on a free-form surface which has repeated inclined sections for roughing operation. Since the surface has several upward and downward inclined segments, a three-axis toolpath was generated using NX 8® for 1000 rpm spindle speed, 0.2 mm/rev/tooth feed per tooth, and 4.5 mm maximum axial depth of cut. The actual cut surface and the associated CAM surface are demonstrated in Figure 5.6.

**Figure 5.6:** Three-axis cutting test workpiece (left) and its 3D CAD model (right)

The measured and predicted tool tip forces in three principal directions of the machine \((F_x, F_y, \text{and } F_z)\) are compared in Figure 5.7, Figure 5.8, and Figure 5.9.
Figure 5.7: Comparison of the experimentally measured and predicted cutting forces, $F_x$

Figure 5.8: Comparison of the experimentally measured and predicted cutting forces, $F_y$
Figure 5.9: Comparison of the experimentally measured and predicted cutting forces, $F_z$

Measured and simulated cutting forces in z-direction of the machine tool are compared in Figure 5.9 where the toolpath is compared separately as experimental and simulated process times matched in reasonable region.

5.2.4 Five-Axis Cutting Experiment

Last set of experiments have been conducted on a free-form surface which provides a smooth change in tool orientation for the associated five-axis toolpath. The purpose of the experiment was to transform the measured drive forces into the cutting forces at the tool tip in five-axis cutting conditions. A five-axis toolpath was generated using NX 8® for 1000 rpm spindle speed, 0.2 mm/rev/tooth feed per tooth, and 8 mm maximum axial depth of cut by defining the tool orientation relative to the surface. The actual cut surface and the related CAM surface are shown in Figure 5.10.
Figure 5.10: Five-axis cutting test workpiece (left) and its 3D CAD model (right)

The measured and predicted tool tip forces in three principal directions of the machine ($F_x, F_y, \text{and } F_z$) are compared in Figure 5.11-Figure 5.13.

Figure 5.11: Comparison of the experimentally measured and predicted cutting forces, $F_x$
Figure 5.12: Comparison of the experimentally measured and predicted cutting forces, $F_y$

Figure 5.13: Comparison of the experimentally measured and predicted cutting forces, $F_z$
The cutting forces evaluated from the measured feed drive current and transformed to the tool tip are in a good agreement with the force measurements by the rotary dynamometer attached to the cutting tool during cutting except the highlighted regions in Figure 5.11-Figure 5.13. The highlighted region for X-drive is between 6s – 7s and for Y-drive is between 4s – 5s and between 7.7s – 8.3s, and for Z-drive is between 4s – 5s and between 7.9s – 8.3s. The discrepancy in these regions can be attributed to inaccuracy in drive inertia and identified drive friction characteristics, and also the change in the drive velocity since the designed Kalman Filter only compensates for the structural dynamics of the drives. As seen in Figure 5.14, the velocity (acceleration is similar) of all drives change direction so as the friction and inertial forces.
Figure 5.14: The measured drive velocities during five-axis toolpath cutting

The measured cutting forces by the rotary dynamometer have a better tracking capability than the transformed forces obtained from the drive current for the toolpaths having frequent or large direction changes in drive motion. That limitation of the proposed estimation method can be further improved by considering the drive velocity and acceleration changes since the Kalman Filter is a model based disturbance estimator. There is a direct velocity change in Y-drive between $4.2s - 5s$ and between $7.7s - 8.3s$ which is clearly seen as an example in Figure 5.15.
Figure 5.15: The predicted cutting force using drive current in five-axis machining in y-direction, $F_y$ (zoomed for Figure 5.12)

5.3 Summary

In this chapter, verification of the proposed cutting force prediction model from the drive current is presented via single-axis and two-axis machining with flat-end cutter, three-axis and five-axis free-form surface cutting experiments with ball-end mill. Tests were designed to predict the cutting forces using the drive current from the simplest to complex toolpaths. It is observed that, the proposed force prediction model can be used in the sensor-less machining. The thesis is ended with the concluding remarks, contributions of this study and future research directions in next chapter.
Chapter 6: Conclusions and Future Research Directions

This thesis presents a new method to predict the cutting forces from the current drawn by the feed drives. The motor current reflects the inertial loads of all active drives during velocity changes along the tool path, the effects of servo control system and structural dynamics of the drive chain, friction in the guides and cutting loads during machining. The accuracy of predicting the cutting forces is dependent on the precision of the mathematical model that describes the transmission of forces from the tool tip to servo motor as the required current. The cutting forces are predicted from the measured motor current with the following methods introduced in the thesis:

The elements of the close loop transfer function for each position control servo are identified experimentally. The rigid body motion of each drive is modeled by the inertias reflected on each drive. The motor – amplifier circuit is defined as a gain, and the Coulomb and viscous friction on each feed drive are identified from experiments. The digital control parameters for each drive are obtained from the CNC directly.

The disturbance frequency response function between the high frequency force acting at the tool tip-workpiece contact and the drive motor’s response torque is measured using static and impact loading tests. The static loading yields the gain of the disturbance transfer function, while impact excitation reveal the response of the motor at the structural modes of the machine elements along the tool tip- motor chain.

The motor current command, position and velocity of each drive are sampled from the CNC system at discrete time intervals directly. The effects of Coulomb and viscous friction are removed, and the remaining is passed through an extended Kalman Filter which compensates the effects of the structural modes in the drive chain. The filtered and compensated disturbance...
torque on each drive, which represents the cutting force transmitted to the motor, is transformed to tool axis coordinates using the five-axis kinematics model of the machine. The directly measured cutting forces are favorably compared with the values predicted by the proposed novel method in single, two, three and five-axis machining experiments within the bandwidth of the drive’s disturbance transfer function zones.

The proposed model can be enhanced by considering the effects of trajectory generation algorithms which shape the velocity and acceleration values during multi-axis contour machining applications. The uncertainties in the transfer function parameters can be minimized by an on-line calibration during machining.
Bibliography


