Energy-Efficient User Association and Power Allocation In a Two-tier Heterogeneous Network

by

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Abstract

The number of mobile devices is exponentially increasing these years. Driven by new generation wireless devices, the exponential increasing of data traffic triggers great challenge of wireless network to meet the communications requirements. Heterogeneous networks provide flexible deployments for operators to improve spectrum efficiency and increase coverage.

Global warming and climate change have been a growing worldwide concern. The mobile industry is contributing to carbon dioxide emission through network operations and mobile equipments. Therefore, energy-efficient design has emerged as a promising technique in heterogeneous networks. We study the energy efficiency problem for downlink transmissions by jointly considering user association and power allocation in a two-tier heterogeneous network. The energy efficiency is maximized under certain prescribed quality-of-service requirement and maximum power limit constraint. Convex relaxation and decomposition method are employed to solve this problem. We use a convex optimization method to obtain a user association solution. A gradient-based algorithm is used to solve the power allocation problem. Then, an iterative joint user association and power allocation algorithm is proposed to maximize the downlink energy efficiency of the system. Simulation results show that the proposed algorithm has improved energy efficiency when compared with the existing schemes.

Preface

This thesis is based on [C1]. My supervisor, Prof. Julian Cheng, co-authored the publication and supervised all my research work.

Refereed Conference Publication

C1. **G. Ye**, H. Zhang, H. Liu, J. Cheng, and V. C. M. Leung, "Energy Efficient Joint User Association and Power Allocation in a Two-Tier Heterogeneous Network," accepted by the IEEE GLOBECOM 2016

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List of Acronyms

Acronyms	Definitions
1G	First Generation
2G	Second Generation
3G	Third Generation
4G	Fourth Generation
5G	Fifth Generation
AMPS	Advanced Mobile Phone System
AWGN	Additive White Gaussian Noise
FDE	Frequency Domain Equalization
GPRS	General Packet Radio Service
GPS	Global Positioning System
GSM	Global System for Mobile Communications
IMT	International Mobile Telecommunications
IP	Internet Protocol
ITU	International Telecommunication Union
ITU-R	International Telecommunications Union-Radio communications sector

LTE-Advanced	Long Term Evolution-Advanced
NGMN	Next Generation Mobile Networks
NMT	Nordic Mobile Telephone
OFDMA	Orthogonal Frequency Division Multiple Access
QoS	Quality-of-Service
Q1	First Quarter
SINR	Signal-to-Interference-plus-Noise Ratio
ST	Secondary Transmitter

List of Symbols

Symbols	Definitions
$\arg\max\{\cdot\}$	Points of the domain of the function at which the function values are maximized
lim	The limit of the function
$\log_2(\cdot)$	The log function with base 2
$\max\{\cdot\}$	The maximum value of the function
$\min\{\cdot\}$	The minimum value of the function
s.t.	Subject to

Acknowledgements

I would like to take this opportunity to give my warm and grateful thanks to my supervisor, Prof. Julian Cheng, for his guidance, advice, encouragement and support. I will continue to be influenced by his rigorous scholarship, clarity in thinking, and professional integrity.

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My thanks to all my dear colleagues and friends for their generosity and support during my study at The University of British Columbia. I would like to thank my dear colleagues for always being available for discussions and sharing their academic experiences.

Finally, I would like to express my deepest gratitude to my parents for their patience, understanding, support, and unconditional love over all these years.

To My Loving Parents

Chapter 1

Introduction

1.1 Background and Motivation

Communication systems have significant roles in modern society. The evolution of communication systems began with the use of drums, smoke signal and semaphore in early history [1]. The development of electrical technology made it possible to develop electrical communication systems. The early experiment in electrical telegraphy employed multiple wires to visually represent Latin letters and numerals. The first working telegraph was built by Francis Ronalds in 1816. The telephone was invented in the 1870s. Therefore, the revolution of communication systems allowed for instant communication across long distance. After the discovery of radio waves, communication systems using radio signals was demonstrated. In 1864, James Clerk Maxwell postulated wireless propagation, which was verified and demonstrated by Heinrich Hertz in 1880 and 1887. After a few decades from the time when the telephone was invented, radio communications was born. In 1895, Marconi demonstrated the first radio transmission from the Isle of Wight to a tugboat 18 miles away and the time of wireless communication had begun. Radio technology improved rapidly to enable transmissions over larger distance with better quality. Mobile telephones became available in the 1940s. However, early devices were bulky and the network supported only a few simultaneous conversations. During the 1950s and 1960s, AT&T offered Mobile Telephone Service. The researchers at AT&T Bell Laboratories developed the cellular concept to solve the capacity problem [2]. Cellular technology allowed reuse of frequencies in small adjacent areas covered by relatively low powered transmitters to reduce the interference, as the power of a transmitted signal falls off with distance. During the 1980s, the first generation (1G) mobile telecommunication systems were built for commercial use. The first automatic analog cellular system, NTT's system, was deployed in Tokyo in 1979. In 1981, the Nordic mobile telephone (NMT) was used in Nordic countries, Switzerland, the Netherlands, Eastern Europe and Russia. The first analogue cellular system widely deployed in North America was the Advanced Mobile Phone System (AMPS). Second generation (2G) mobile telecommunication networks were commercially launched in Finland by Radiolinja in 1991. The networks used the global system for mobile communications (GSM) standard [1]. One of the main differences between 1G and 2G telecommunication networks was that the 2G communication systems was based on digital communications. This major change of mobile telecommunication systems was driven by the needs of higher capacity and speed. 2G systems were also more efficient on the spectrum and introduced data services for mobiles. The first major step in the evolution of GSM networks towards 3G occurred with the introduction of General Packet Radio Service (GPRS). This packet switched approach routed individual packets of data from the transmitter to the receiver, while using the same circuit that to be used by different users. This evolution allowed circuits to be used more efficiently [3]. During the early 1980s, the International Telecommunication Union (ITU) developed a new telecommunication technology which is the third generation telecommunication technology (3G). It cost fifteen years to develop the specifications and standards of 3G technology. The first commercial launch of 3G was in Japan on October 1st, 2001. 3G networks offered higher data rate and greater security than their 2G predecessors. The bandwidth and location information available to 3G devices give rise to applications in Global Positioning System (GPS), location-based services, mobile Internet access, video calls and mobile TV [4]. A new generation of cellular standards has appeared approximately every tenth year since 1G systems were introduced. In March 2008, the

International Telecommunications Union-Radio communications sector (ITU-R) specified a set of requirements for the fourth generation mobile telecommunication (4G) technology standards, named the International Mobile Telecommunications Advanced (IMT-Advanced) specification [5]. A major difference between 4G system and the earlier generations is that a 4G system supports all-Internet Protocol (IP) based communication, such as IP telephony, instead of traditional circuit-switched telephony service. 4G candidate systems abandoned the spread spectrum radio technology used in 3G systems, used the orthogonal frequency division multiple access (OFDMA) multi-carrier transmission and other frequency domain equalization (FDE) schemes.

Since Marconi first demonstrated the radio transmission, wireless communication system has passed through a great evolution. Over past few decades, wireless communication system has experienced dramatic growth and become an essential part of modern life. The next generation mobile networks (NGMN) alliance defines the requirements for fifth generation (5G) networks as the network can provide data rates of tens of megabits per second for tens of thousands of users, several hundreds of thousands of simultaneous connections for massive wireless sensor network and coverage improved [6]. As we can observe that new mobile generations are typically assigned new frequency bands and wider spectral bandwidth per frequency channel. However, there is little room for larger channel bandwidths and new frequency bands. Another challenge for next generation networks is power consumption, as the explosive growth in the number of mobile users. One of the techniques for next generation mobile networks is heterogeneous networks. By shorten the distance between the basestation and the users, heterogeneous networks can offload data traffic and reduce power consumption of the network [7]. By deploying small cells within a macrocell, operators can use heterogeneous networks to help expand the coverage and improve the system capacity.

Although heterogeneous network is a promising technique, there are some technical obstacles such as power allocation, subchannel allocation, user association and energy efficiency. In this thesis, we will focus on the energy efficiency aspect, and use optimization techniques to study the energy-efficient user association and power allocation problem in a two-tier heterogeneous network.

1.2 Literature Review

Nowadays, energy crisis and global warming problems are two major problems affecting our modern society. As the explosive growth in the number of mobile users, energy consumption in wireless communication has increased dramatically in recent years [8]. The rapidly increasing number of new generation mobile devices requires higher data rate. This is a challenge for wireless communication systems to meet the data rate demand with limited energy consumption for green communication purpose. This requirement has led to a need for exploring wireless communication systems that reduce power consumption [9]. Effective network planning is essential to deal with the increasing number of mobile users. Operators improved the network planning by implementing efficient modulation and coding schemes, increasing capacity with new radio spectrum and using multi-antenna techniques. However, these methods alone are insufficient in the crowded environments and at cell edges. One effective method is to shorten the distance between the basestation and the users. Heterogeneous network has been widely studied to offload data traffic and reduce power consumption of the network. In [10], the authors developed a testbed to affirm the functions of a heterogeneous radio network, which has abilities to select one system from multiple communication systems and to aggregate multiple systems. Simulation results showed the performance improvement of the heterogeneous network compared to a single network. The authors in [11] examined the downlink transmissions of a heterogeneous cellular network that contained multiple tiers of transmitters. They provided analysis of the distribution of the signal-to-interference-plus-noise ratio (SINR) at an arbitrarily-located user. In a heterogeneous network, a cell edge user can potentially be associated with either the macrocell basestation or a small cell basestation, resulting in different user experiences. Therefore, user association is an important problem in heterogeneous networks.

User association, which is an indispensable research topic in heterogeneous network, has great impact on the system performance. Most of the existing works in user association for heterogeneous networks focused on received SINR and sum rate maximization. In [12], a low-complexity user association algorithm was designed to maximize the logarithmic utility. The numerical results demonstrated that a load-aware association significantly improved resource utilization and mitigated the congestion of macro basestations. The authors in [13] proposed joint cell association and bandwidth allocation schemes for heterogeneous networks to maximize the network sum log-rate in order to achieve proportional fairness. They also showed that the cell range expansion strategy failed to provide good performance when the wireless backhauling constraints for the small cells were considered for cell association. To minimize the potential delay related to the sum of the inverse of the per-user SINRs, a joint optimization of user association, channel selection and power control in heterogeneous networks was considered in [14]. However, when deploying small cells within macrocells in heterogeneous networks, energy consumption can potentially result in a large operational expenditure and it becomes challenging for operators to achieve data rate demand while limiting their electric bill. Therefore, energy efficiency is becoming increasingly important for future green communication in heterogeneous networks. The authors in [15] studied the user association problem in cognitive heterogeneous networks where several small cells backhaul their traffic to the neighboring cells. They showed the high energy efficiency potential of their proposed algorithm when compared with existing user association algorithms based on reference signal received power, range expansion and minimum pathloss. A joint user association and energy-efficient subcarrier allocation algorithm was proposed in [16]. The impact of user's maximum transmission power and minimum rate requirements on energy efficiency and throughput were investigated through illustrative results. In [17], a dynamic network selection mechanism in cooperative heterogeneous networks using evolutionary game theory was designed to optimize the user perceived quality-of-service (QoS).

Resource allocation, such as power allocation and bandwidth allocation, has been widely studied to maximize energy efficiency [18]. In [19], subchannel power allocation was considered to maximize the system energy efficiency under certain QoS constraints for uplink and downlink transmissions in a single cell network. Simulation results showed that the energy-efficient design improved energy efficiency compared with the conventional spectral-efficient design. The authors in [20] proposed a link adaptation and resource allocation technique to maximize energy efficiency in an OFDMA system by fixing circuit power and transmit power. In [21], power allocation and sensing time that to determine the occupation status of the subchannels were considered to maximize energy efficiency in small cells. The authors proposed an iterative power control algorithm and a near optimal sensing time scheme with the consideration of the imperfect hybrid spectrum sensing. Power allocation and bandwidth allocation were jointly considered in [22] to maximize energy efficiency in a single cell system while guaranteeing the QoS requirements. They investigated the energy efficiency tradeoff between downlink and uplink, as well as among users. Resource management for energy efficiency in heterogeneous networks has also been widely studied. Energy-efficient power control schemes in multichannel macro-femto networks were investigated in [23]. The authors proposed two energy-efficient power control schemes for downlink transmissions in multichannel macro-femto networks, which were gradient-based distributed power control scheme and energy-efficient game-based power control scheme. The authors of [24] investigated spectrum sharing and resource allocation to improve energy efficiency for heterogeneous cognitive radio networks. They formulated the resource allocation problem as a three-stage Stackelberg game and applied the backward induction method to solve the problem. In [25], the authors focused on the uplink energy-efficient resource allocation in OFDMA cognitive radio networks consisting of multiple secondary transmitters. They investigated a joint subchannel allocation and power control strategy with employing a linear pricing technique to maximize each individual secondary transmitter's (ST) energy efficiency. To maximize the energy efficiency for each individual user, the authors in [26] investigated energy-efficient bandwidth and power allocation in a heterogeneous network.

Different from the existing work, in this thesis, our objective is to maximize the system energy efficiency of a two-tier heterogeneous network, while jointly considering user association and power allocation. An iterative user association and power allocation algorithm is developed.

1.3 Thesis Organization and Contributions

This thesis consists of six chapters. Chapter 1 presents the background of the evolution of communication systems. The motivation of the development of wireless communication systems and cellular networks are the increasing number of users and the demand of higher capacity. However, in modern mobile communications, power and bandwidth are scarce resource and are usually limited in wireless communication system. In order to satisfy the QoS demand of communications and the need of environmental friendly networks, the development of green communication network is the inevitable. Therefore, we study the energy-efficient power allocation and user association problem in a two-tier OFDMA based heterogeneous network.

Chapter 2 provides detailed background for the thesis. We first introduce the development of the heterogeneous networks and followed by the introduction of two most popular research objectives: user association and resource management. Convex optimization method is an important mathematic tool to solve these problems and it is briefly introduced in this section. As the energy-efficient design has an important role in modern communications, a detailed introduction of energy efficiency in communication systems is provided.

Chapter 3 provides the system model of this thesis. An energy-efficient two-tier heterogeneous network optimization framework is designed. We propose the system model to maximize the total downlink energy efficiency with the consideration of user association and power allocation in a two-tier heterogeneous network. The maximum transmit power constraints of each small cell basestation and the minimum downlink data rate requirement of each user are considered.

Chapter 4 provides the optimization conditions and solutions to the energy-efficient user association and power allocation problem. We prove that the formulated problem in Chapter 3 is a non-convex optimization problem and we use the decomposition method to solve the original problem. By decomposing the original non-convex problem into two subproblem, we solve both the energy-efficient user association and energy-efficient power allocation problems.

Chapter 5 provides the algorithms that we propose to solve the energy-efficient problem we formulated and the numerical results of the proposed algorithms. We develop a gradient based algorithm to solve the energy-efficient power allocation subproblem and an iterative algorithm to solve the energy-efficient user association and power allocation problem. We use simulation results to demonstrate the effectiveness of the proposed iterative algorithm and compare it with two schemes.

Chapter 6 summarizes the thesis and introduces our contributions in this work. In addition, some future works related to our current research are suggested.

Chapter 2

Heterogeneous Wireless Communication Networks and Energy Efficiency

In this chapter, we present detailed background knowledge about user association and resource management in heterogeneous networks. We first introduce the motivation for the development of heterogeneous networks and the characteristic of heterogeneous networks. The basic concept of energy-efficient user association and resource management are also addressed. Finally, the basic convex optimization knowledge related to user association and resource allocation is presented.

2.1 Overview of Heterogeneous Networks

As mobile devices have becoming essential tools in modern life, traditional macrocell networks face several challenges. According to [27], the number of mobile broadband subscriptions grew at a rate of 35 percent year-on-year in the first quarter (Q1) of 2014 and was reaching 2.3 billions. The amount of data usage per subscription also continues to grow steadily. Together, these factors contributed to a 65 percent growth in mobile data traffic between Q1 2013 and Q1 2014 [27]. Figure 2.1 shows a stable trend of data traffic growth. The number of mobile data subscriptions is increasing rapidly, and driving growth in data traffic along with a continuous increase in the average data

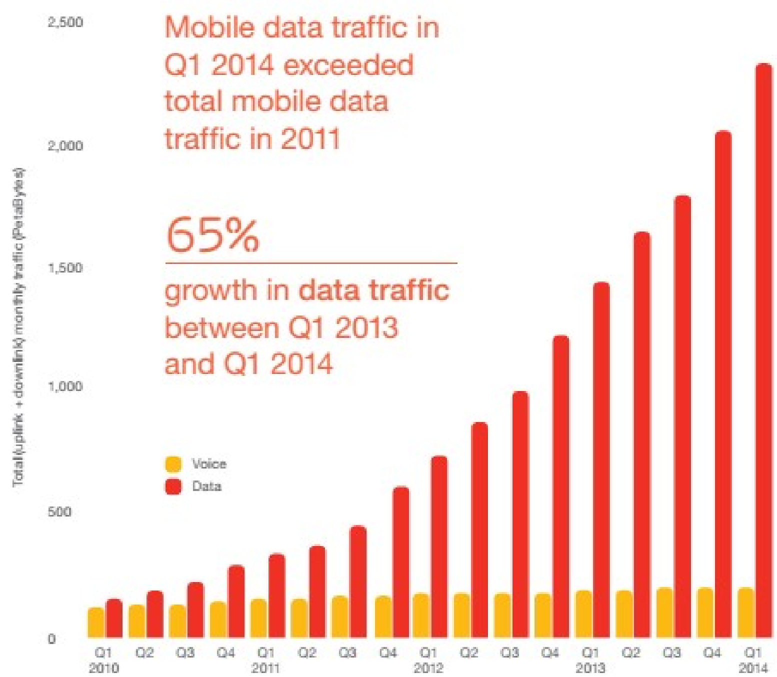

Figure 2.1: Global traffic in mobile networks [27].

volume per subscription.

The increasing indoor communication demand has also become one of the obstacles for macrocell networks as the macrocell networks provide limited coverage to indoor users. According to a survey, more than 50 percent of voice and 70 percent of data traffic take place in the indoor environment [28]. Although the existing macrocell network services provide coverage to some of the indoor users, the severe building wall penetration losses can cause user dissatisfaction when data transmissions cannot meet their demands. To address the explosive growth in data demands driven by the increasing number of smart phones, tablets, and other mobile devices, network operators will have to significantly increase the capacity of their networks [29]. The heterogeneous network, where low-power low-complexity basestations are overlaid on conventional macro basestations, is being considered as a promising paradigm for increasing system capacity and coverage in a cost effective way [30]. By deploying low-power nodes such as pico and femto basestations in addition to the macro basestations, the conventional cellular system is split into multi-tier topology, and users can be off-loaded to the small cells. For example, the long term evolution-advanced (LTE-Advanced) standard proposed improvement to network-wide spectral efficiency by employing a mix of macro, pico and femto basestations [31]. The heterogeneous networks are also expected to provide better coverage and higher throughput [32]. Although the heterogeneous network is a promising technique, the hierarchical layering of cells could introduce technical obstacles. Resource management is one fundamental limiting factor to the heterogeneous network performance.

2.1.1 Resource Management

Wireless channels undergo a wide range of impacts such as fading, shadowing and path loss. The state of wireless channels varies with time, frequency and space. As a result, it will cause variations as the users in different geographical locations, frequency or times have different received signal power. These variations create time diversity, frequency diversity, spatial diversity and multiuser diversity in the received signal power. Resources such as transmit power, frequency bands, transmit antennas, etc., can then be allocated dynamically to different users. In fact, it is well established that dynamic resource allocation schemes can result in much better performance compared to the static resource allocation schemes [33].

For a heterogeneous network, basestations from different layers usually have different resource constraints, which create more diversities than the single cell networks. Therefore, resource allocation has become an important aspect when studying heterogeneous networks. When allocating resources across multiple cells, determining which basestation transmit certain resource to which user should also be taken into consideration. Therefore, resource allocation problem in heterogeneous networks is highly coupled with the user association decision.

2.1.2 User association in Heterogeneous Networks

User Association defines a set of rules for assigning users to the different basestations available in the network. A decision to associate a user with one basestation will affect not only the performance of the user but the performance of the network. In conventional homogeneous cellular networks, user association is usually based on downlink received signal strength. In a heterogeneous network, this association rule may not be suitable for the case where the macrocells and the small cells are resource constrained. To balance the load, a user can potentially be associated with a small cell even though the received power from a macro basestation is higher. However, this may cause severe interference if the radio resources are not carefully partitioned among cells. Therefore, the resource allocation and user association should be optimized jointly [34]. Many association rules have been proposed based on different resource allocation schemes and different objectives. One of the objectives is energy-efficient heterogeneous networks design.

2.2 Energy Efficiency in Communication System

During the past decades, much effort has been made to enhance the throughput of network. However, the severe energy crisis and global warming problems are affecting our modern society and the need for developing green communication network becomes an inevitable trend. Therefore, how to transmit more data with limited power consumption in such networks and devices is an urgent task.

Energy efficiency is commonly defined as the information bits per unit transmit energy. Bits/Hz per Joule is commonly used as the energy efficiency metric in wireless networks [23], [24], [35], [36]. For energy-efficient communication, it is desirable to send the maximum amount of data with a given amount of energy. The unit achievable data rate, which is also known as spectrum efficiency, is $r = \log_2(1 + \frac{pg}{\sigma_0^2})$, where p is transmit power, σ_0^2 is additive white Gaussian noise (AWGN) power and g is channel power gain between transmitter and receiver. Given any amount of energy ΔE that consumed in a duration ΔT, we have $\Delta E = p\Delta T$. Therefore, the energy efficiency is defined as

$$\eta_{EE} = \frac{r\Delta T}{\Delta E} = \frac{r}{p} \tag{2.1}$$

bits per Hertz per Joule.

Besides transmit power, the energy consumption also includes circuit energy consumption which represents the additional device power consumption incurred by signal processing and active circuit blocks such as analog-to-digital converter, digital-to-analog converter, synthesizer, and mixer during the transmission. Denote the circuit power as P_C, thus the overall power assumption is $P_C + p$. Energy efficiency needs to be redefined as information bits per unit energy, where an additional circuit power factor, P_C, needs to be taken into consideration. Therefore, the energy efficiency is defined as

$$\eta_{EE} = \frac{r\Delta T}{\Delta E} = \frac{r}{p + P_C}. \tag{2.2}$$

2.2.1 Convex Optimization Application in Wireless Communication Network

Wireless communication networks are essential means of communications in modern life and the number of mobile user has been through an explosive growth during the past decades. To increase operation efficiency and network capacity, many research efforts have been made in investigating effective methods for the development of wireless communication systems. One of the most common and effective mathematical tools to solve the resource allocation problem in wireless communication networks is convex optimization method.

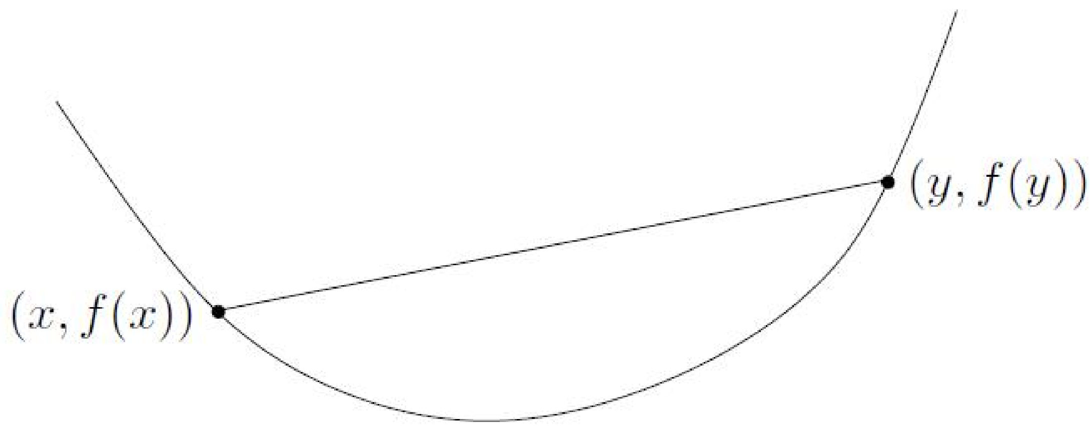

Figure 2.2: Graph of a convex function [37].

According to the definition in [37], a set $\mathcal{C}$ is convex if the line segment between any two points in $\mathcal{C}$ lies in $\mathcal{C}$, i.e., if for any $x_1, x_2 \in \mathcal{C}$, and any θ with $0 \leq \theta \leq 1$, we have

$$\theta x_1 + (1-\theta)x_2 \in \mathcal{C}. \tag{2.3}$$

A function f: $\mathbf{R}^n \rightarrow \mathbf{R}$ is convex if the domain of f, denoted by $\mathbf{dom}\ f$, is a convex set and if for all $x, y \in \mathbf{dom}\ f$, and θ with $0 \leq \theta \leq 1$, we have

$$f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta)f(y). \tag{2.4}$$

Geometrically, this inequality means that the line segment between $(x, f(x))$ and $(y, f(y))$ lies above the graph of f, which is shown in Fig. 2.2. We say f is concave if $-f$ is convex. Some operations can preserve the convexity and concavity, such as nonnegative weighted summation, and pointwise maximum operation [37].

We use the notation

$$\begin{aligned} \min \quad & f_0(x) \\ \text{s.t.} \quad & f_i(x) \le 0, i = 1, 2, ..., m \\ & h_i(x) = 0, i = 1, 2, ..., p \\ & x \in \mathcal{C} \end{aligned} \tag{2.5}$$

to describe the problem of finding an $\mathbf{x}$ that minimizes $f_0(x)$ among all x values that satisfy the conditions $f_i(x) \le 0$, $i = 1, 2, ..., m$, $h_i(x) = 0$, $i = 1, 2, ..., p$ and $x \in \mathcal{C}$. We call $x \in \mathcal{C}$ the optimization variable and f_0 the objective function or cost function. $f_i(x)$ and $h_i(x)$ are the inequality and equality constraint functions, respectively, and $\mathcal{C}$ is the constraint set. The domain of the objective and constraint functions are defined as

$$\mathcal{D} = \bigcap_{i=0}^{m} \mathbf{dom}\ f_i \cap \bigcap_{i=0}^{m} \mathbf{dom}\ h_i \cap \mathcal{C}. \tag{2.6}$$

According to the definition, the problem in (2.5) is a convex optimization problem if it satisfies the following requirements:

- the objective function must be convex,
- the the inequality constraint functions f_i $(i = 1, 2, ..., m)$ must be convex,
- the equality constraint functions h_i $(i = 1, 2, ..., p)$ must be affine[1].

Violating any one of those conditions will result in a non-convex problem. A feasible $\mathbf{x}^* \in D$ is said to be global optimal if $f_0(\mathbf{x}^*) \le f_0(\mathbf{x})$ for all $\mathbf{x}$. With a slight abuse of

[1]The affine function can be represented by matrix equation $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A}$ is a matrix and $\mathbf{b}$ is a vector of appropriate sizes.

notation, we will also refer to

$$\begin{aligned} \max \quad & f_0(x) \\ \text{s.t.} \quad & f_i(x) \leq 0, i = 1, 2, ..., m \\ & h_i(x) = 0, i = 1, 2, ..., p \\ & x \in \mathcal{C}. \end{aligned} \tag{2.7}$$

as a convex optimization problem if the objective function is concave and other conditions are satisfied.

2.3 Summary

In this chapter, we presented the essential and detailed technical background knowledge for the entire thesis. A brief description of heterogeneous networks and the motivation of investigating the resource management and user association were provided. A brief description of the energy efficiency was introduced. The basic knowledge and concepts of convex optimization were also provided.

Chapter 3

Energy Efficiency Network Modeling

In this chapter, we propose a system model for an energy-efficient two-tier heterogeneous network. We formulate the energy-efficient power allocation and user association problem to maximize the downlink energy efficiency for the two-tier heterogeneous network. We formulate the problem under QoS and total transmit power limits constraints. The formulated problem is a non-convex integer programming.

3.1 System Model

We focus on the user association and transmit power allocation in a two-tier OFDMA heterogeneous network as shown in Fig. 3.1 where J small cells are overlaid on one macrocell. The small cells share the same spectrum with macrocell. There are K users that are randomly deployed within the range of J small cells. In this work, we only consider the user association and power allocation of these K users. We denote the set of small cells by $\mathcal{S}$ and the set of all cells by $\mathcal{C}$ where $\mathcal{C} = \mathcal{S} \cup \{0\}$, and where the index 0 is introduced to denote the macrocell. Denote the set of users by $\mathcal{U}$. For simplicity, we assume that each user is assigned a different subchannel with unit bandwidth in this two-tier network.

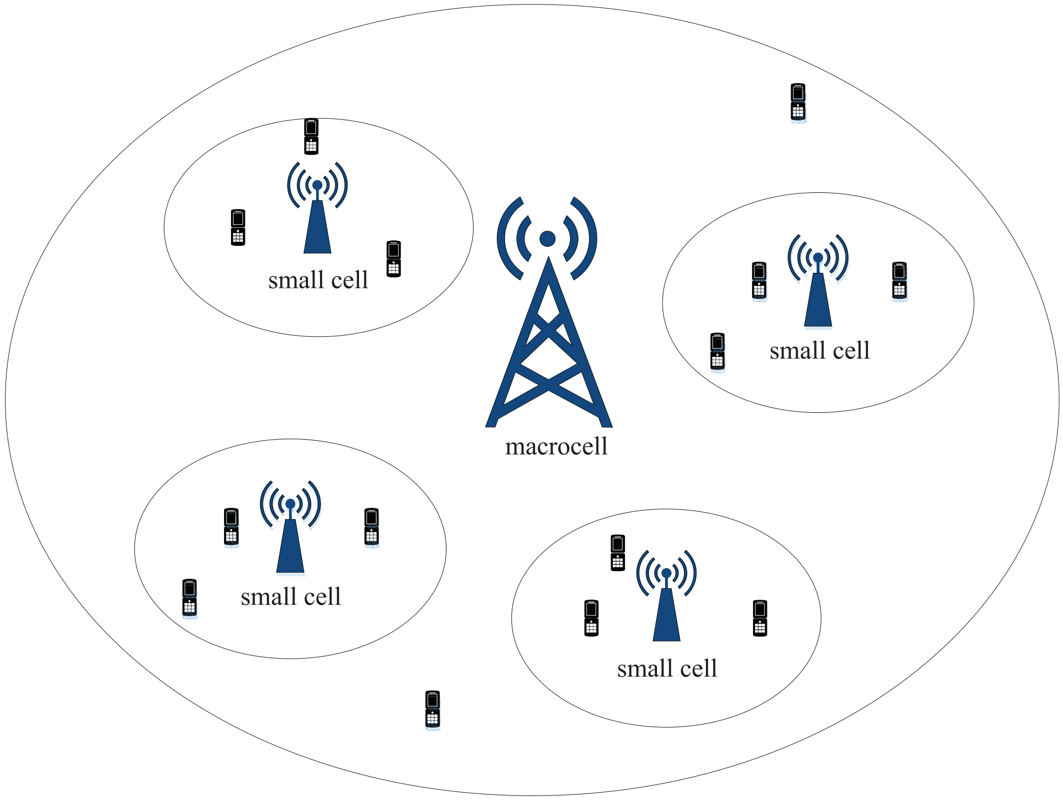

Figure 3.1: A two-tier heterogeneous network with small cells overlaid on one macrocell.

3.2 Problem Formulation

Denote H_k as the average channel power gain between the macrocell basestation and the kth user. Denote $L_{j,k}$ as the downlink channel gain between the jth small cell basestation and the kth user. $p_{j,k}$ denotes the transmit power from the jth cell basestation to the kth user, where $j = 0$ indicates the transmit power from macrocell basestation. Since each user is assigned to a different subchannel, the users associated with small cells only suffer the interference from macrocell basestation. For macrocell transmissions, we assume the interference of the kth user deployed within the jth small cell range is introduced by the jth small cell basestation and we ignore the co-channel interference[2] from other small cells. Denote N as the AWGN power. The SINR of the kth user deployed within the range of the jth small cell associated with the macrocell is

$$\gamma_{0,k} = \frac{p_{0,k} H_k}{N + p_{j,k} L_{j,k}} \tag{3.1}$$

where $p_{j,k} L_{j,k}$ is the interference power from the jth small cell basestation to the kth user in downlink transmissions. The SINR of the kth user associated with the jth small cell is

$$\gamma_{j,k} = \frac{p_{j,k} L_{j,k}}{N + p_{0,k} H_k} \tag{3.2}$$

where $p_{0,k} H_k$ is the interference power from the macrocell basestation to the kth user in downlink transmissions. For convenience, we let $G_{0,k} = H_k$, $G_{j,k} = L_{j,k}$, $I_{0,k} = N + p_{j,k} L_{j,k}$ and $I_{j,k} = N + p_{0,k} H_k$, then we rewrite

$$\gamma_{0,k} = \frac{p_{0,k} G_{0,k}}{I_{0,k}} \tag{3.3}$$

and

$$\gamma_{j,k} = \frac{p_{j,k} G_{j,k}}{I_{j,k}}. \tag{3.4}$$

The achievable data rate of the kth user associated with the jth cell is

$$r_{j,k} = \log_2 \left(1 + \gamma_{j,k}\right). \tag{3.5}$$

We introduce a binary indicator variable $x_{j,k}$, i.e., $x_{j,k} \in \{0, 1\}$ and $\sum\limits_{j \in \mathcal{C}} x_{j,k} = 1$, where $x_{j,k} = 1$ indicates the kth user is associated with the jth cell.

Consider the following constraints:

[2]This approximation is reasonable because small cell basestations have much smaller transmit power than the macrocell basestation.

– *Total power constraint:*

$$\sum_{k\in\mathcal{U}} x_{j,k}p_{j,k} \leq P_{\max j}, \forall j \in \mathcal{C} \tag{3.6}$$

where $P_{\max j}$ denotes the maximum transmit power of cell j.

– *QoS constraint:*

$$\sum_{j\in\mathcal{C}} x_{j,k}r_{j,k} \geq R_t, \forall k \in \mathcal{U} \tag{3.7}$$

where R_t is the minimum transmit data rate with unit bandwidth for each user.

Besides transmit power, the energy consumption also includes circuit energy consumption. We denote p_c as the average circuit power consumption of each basestation when communicating with each user. For downlink transmission, the overall power consumption of the jth basestation when communicating with the kth user is

$$P_{totj,k} = \zeta p_{j,k} + p_c \tag{3.8}$$

where $\zeta \in [0, 1]$ is the power amplifier efficiency and depends on the design and implementation of the transmitter [38]. For simplicity, we consider $\zeta = 1$. The goal of energy-efficient communications is to maximize the amount of data sent with a given amount of energy. Hence, given any amount of energy Δe consumed in a duration Δt [39], the energy efficiency corresponding to the kth user associated with the jth cell basestation is

$$\eta_{j,k} = x_{j,k}\frac{r_{j,k}}{\Delta e/\Delta t} = x_{j,k}\frac{r_{j,k}}{p_c + p_{j,k}}. \tag{3.9}$$

The energy efficiency is defined as $\eta = \sum\limits_{j}\sum\limits_{k} x_{j,k}\frac{r_{j,k}}{p_c+p_{j,k}}$, which can be interpreted as the sum of the energy efficiency of every user [24], [39].

Denote the power allocation matrix[3] as $\boldsymbol{P} = [p_{j,k}]_{(J+1)\times K}$ and the indicator matrix

[3] Power allocation for a subchannel of each user.

as $\boldsymbol{X} = [x_{j,k}]_{(J+1)\times K}$. The energy efficiency optimization problem can be formulated as

$$P1 : \max_{\boldsymbol{X},\boldsymbol{P}} \eta\left(\boldsymbol{X},\boldsymbol{P}\right) = \max_{\boldsymbol{X},\boldsymbol{P}} \sum_j \sum_k x_{j,k} \frac{r_{j,k}}{p_c + p_{j,k}} \tag{3.10}$$

$$\begin{aligned} \text{s.t.}\quad & C1 : x_{j,k} \in \{0,1\}, \forall\,(j,k) \in \mathcal{C} \times \mathcal{U} \\ & C2 : \sum_{j\in\mathcal{C}} x_{j,k} = 1, \forall k \in \mathcal{U} \\ & C3 : p_{j,k} \geq 0, \forall\,(j,k) \in \mathcal{C} \times \mathcal{U} \\ & C4 : \sum_{k\in\mathcal{U}} x_{j,k} p_{j,k} \leq P_{\max j}, \forall j \in \mathcal{C} \\ & C5 : \sum_{j\in\mathcal{C}} x_{j,k} r_{j,k} \geq R_t, \forall k \in \mathcal{U}. \end{aligned} \tag{3.11}$$

3.3 Summary

In this chapter, we proposed the framework to optimize the energy efficiency of a two-tier heterogeneous network. We designed a system model for jointly considering the user association and power allocation to maximize the downlink energy efficiency for the two-tier network. We formulated the problem with the consideration of maximum transmit power constraints of each basestation and the minimum data rate requirements for each user.

Chapter 4

Principles of Joint User Association and Power Allocation Energy Efficiency Network

In this Chapter, we notice that the problem we formulated in Chapter 3 is a non-convex integer programming. We use convex relaxation method and decomposition method to solve the problem. We find solutions to the subproblems of energy-efficient user association and energy-efficient power allocation.

4.1 Conditions of Optimality

It is obvious that the formulated objective function in (3.10) is neither convex nor concave. Moreover, since the user association indicator $x_{j,k}$ is a binary variable, the constraints in (3.11) are non-convex mixed integer constraints. Therefore, the optimization problem formulated in (3.10) and (3.11) is not a convex optimization problem. We can relax the binary variable into continuous and consider a decomposition approach to solve the energy-efficient user association and power allocation problem. We decompose the non-convex optimization problem into two convex subproblems: energy-efficient user association subproblem and energy-efficient power allocation subproblem.

4.2 Convex Relaxation and Decomposition

Since the user association indicator $x_{j,k}$ is a binary variable, the problem we formulated in (3.10) and (3.11) is non-convex mixed integer programming. To make the problem tractable, we can relax $x_{j,k}$ to be continuous. Let $0 \leq x_{j,k} \leq 1, \forall\,(j,k) \in \mathcal{C} \times \mathcal{U}$, where a fractional user association indicator can be interpreted as partial association with different cells in a user association period. Therefore, the optimization problem formulated in (3.10) and (3.11) can be modified to

$$P2: \max_{\boldsymbol{X},\boldsymbol{P}} \tilde{\eta}(\boldsymbol{X},\boldsymbol{P}) = \max_{\boldsymbol{X},\boldsymbol{P}} \sum_j \sum_k x_{j,k} \frac{r_{j,k}}{p_c + p_{j,k}} \tag{4.1}$$

$$\begin{aligned} \text{s.t.}\quad & C1: 0 \leq x_{j,k} \leq 1, \forall\,(j,k) \in \mathcal{C} \times \mathcal{U} \\ & C2: \sum_{j \in \mathcal{C}} x_{j,k} = 1, \forall k \in \mathcal{U} \\ & C3: p_{j,k} \geq 0, \forall\,(j,k) \in \mathcal{C} \times \mathcal{U} \\ & C4: \sum_{k \in \mathcal{U}} x_{j,k} p_{j,k} \leq P_{\max j}, \forall j \in \mathcal{C} \\ & C5: \sum_{j \in \mathcal{C}} x_{j,k} r_{j,k} \geq R_t, \forall k \in \mathcal{U}. \end{aligned} \tag{4.2}$$

It can be shown that the continuous variable $p_{j,k}$ and $x_{j,k}$ are separable in (4.1). Therefore, we consider a decomposition approach to solve the energy-efficient joint user association and power allocation problem.

4.3 Energy-Efficient User Association

By decomposing the problem we formulated in (4.1) and (4.2), and given $\boldsymbol{P}$, we obtain the following problem

$$P2.1: \max_{\boldsymbol{X}} \hat{\tilde{\eta}}(\boldsymbol{X}) = \max_{\boldsymbol{X}} \sum_j \Phi_j(\boldsymbol{X}) \tag{4.3}$$

$$\begin{aligned} \text{s.t.} \quad & C1: 0 \le x_{j,k} \le 1, \forall (j,k) \in \mathcal{C} \times \mathcal{U} \\ & C2: \sum_{j \in \mathcal{C}} x_{j,k} = 1, \forall k \in \mathcal{U} \\ & C3: \sum_{k \in \mathcal{U}} x_{j,k} p_{j,k} \le P_{\max j}, \forall j \in \mathcal{C} \\ & C4: \sum_{j \in \mathcal{C}} x_{j,k} r_{j,k} \ge R_t, \forall k \in \mathcal{U} \end{aligned} \tag{4.4}$$

where $\Phi_j(\boldsymbol{X})$, defined as $\Phi_j(\boldsymbol{X}) = \sum\limits_{k} x_{j,k} \frac{r_{j,k}}{p_c + p_{j,k}}$, is a concave function of $x_{j,k}$. Since all of the constraints in (4.4) are convex, the problem we formulated in (4.3) and (4.4) is a convex optimization problem.

4.3.1 Introduction of Lagrangian Method in Convex Optimization

When maximize or minimize a function subject to fixed outside conditions or constraints, it is often difficult to find a closed form for the function. The method of Lagrange multipliers is a powerful tool for solving this class of problems.

We consider an optimization problem in the standard form

$$\begin{aligned} \min \quad & f_0(x) \\ \text{s.t.} \quad & f_i(x) \le 0, i = 1, 2, ..., m \\ & h_i(x) = 0, i = 1, 2, ..., p \end{aligned} \tag{4.5}$$

with variable $x \in \mathbf{R}^n$. The basic idea in Lagrangian duality is to take the constraints in (5.1) into account by augmenting the objective function with a weighted sum of the constraint functions [37]. According to the definition in [37], we define the Lagrangian $L : \mathbf{R}^n \times \mathbf{R}^m \times \mathbf{R}^p \to \mathbf{R}$ associated with the problem (4.5) as

$$L(x, \lambda, \nu) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} \nu_i h_i(x) \tag{4.6}$$

where λ_i and ν_i are the Lagrange multipliers associated with the ith inequality constraint and the ith equality constraint. The vectors λ and ν are called the dual variables or Lagrange multiplier vectors associated with the problem (4.5).

4.3.2 Energy-Efficient User Association Solution

Since the problem we formulated in (4.3) and (4.4) is a convex optimization problem with several constraints, we can apply the Lagrangian method to solve the problem. By jointly considering (4.3) and the constraints in (4.4), we obtain the Lagrangian function associated with the problem we formulated in (4.3) and (4.4) as

$$\begin{aligned} L\left(\boldsymbol{X}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}\right) = & \sum_{j \in \mathcal{C}} \sum_{k \in \mathcal{U}} x_{j,k} \frac{r_{j,k}}{p_c + p_{j,k}} \\ & + \sum_{j \in \mathcal{C}} \lambda_j \left(P_{\max j} - \sum_{k \in \mathcal{U}} x_{j,k} p_{j,k} \right) \\ & + \sum_{k \in \mathcal{U}} \nu_k \left(\sum_{j \in \mathcal{C}} x_{j,k} r_{j,k} - R_t \right) \\ & + \sum_{k \in \mathcal{U}} \mu_k \left(1 - \sum_{j \in \mathcal{C}} x_{j,k} \right) \end{aligned} \tag{4.7}$$

where $\boldsymbol{\lambda}$, $\boldsymbol{\nu}$, $\boldsymbol{\mu}$ are the vectors of the Lagrange multipliers (also called dual variables), and they are defined as $\boldsymbol{\lambda} = [\lambda_0, \lambda_1, \ldots, \lambda_J]^T$, $\boldsymbol{\nu} = [\nu_1, \nu_2, \ldots, \nu_K]^T$ and $\boldsymbol{\mu} = [\mu_1, \mu_2, \ldots, \mu_K]^T$.

Thus, the Lagrangian dual function is given by

$$g\left(\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}\right) = \max_{\boldsymbol{X}} L\left(\boldsymbol{X}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}\right). \tag{4.8}$$

The dual problem can be expressed as

$$\min_{\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}} g\left(\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}\right) \tag{4.9}$$

$$\text{s.t.} \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu} \succeq \mathbf{0} \tag{4.10}$$

where symbol $\succeq$ denotes vector inequality, e.g., $\boldsymbol{\lambda} \succeq \mathbf{0}$ means each element of $\boldsymbol{\lambda}$ is nonnegative.

Based on the decomposition method [37], the Lagrangian function in (4.7) can be rewritten as

$$\begin{aligned} L(\boldsymbol{X}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}) &= \sum_{j \in \mathcal{C}} \sum_{k \in \mathcal{U}} L_{j,k}(\boldsymbol{X}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}) \\ &+ \sum_{j \in \mathcal{C}} \lambda_j P_{\max j} - \sum_{k \in \mathcal{U}} \nu_k R_t + \sum_{k \in \mathcal{U}} \mu_k \end{aligned} \tag{4.11}$$

where

$$L_{j,k}(\boldsymbol{X}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}) = x_{j,k} \left(\frac{r_{j,k}}{p_c + p_{j,k}} - \lambda_j p_{j,k} + \nu_k r_{j,k} - \mu_k \right). \tag{4.12}$$

The partial derivative of (4.12) can be expressed as

$$\frac{\partial L_{j,k}(\boldsymbol{X}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu})}{\partial x_{j,k}} = \tilde{H}_{j,k} - \mu_k \tag{4.13}$$

where

$$\tilde{H}_{j,k} = \frac{r_{j,k}}{p_c + p_{j,k}} - \lambda_j p_{j,k} + \nu_k r_{j,k}. \tag{4.14}$$

According to (4.14), given λ_j^i and ν_k^i, which respectively denote the Lagrangian parameters λ_j and ν_k for the ith inner iteration, $L_{j,k}(\boldsymbol{X}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu})$ implies that the kth user simply chooses the basestation that offers the highest $\tilde{H}_{j,k}$ [40]. This mechanism for updating $x_{j,k}$ is expressed as follows

$$x_{j,k}^{i+1} = \begin{cases} 1, j = j_k \\ 0, j \neq j_k \end{cases}, \forall k \in \mathcal{U} \tag{4.15}$$

where

$$j_k = \arg\max_{j \in C} \left[\frac{r_{j,k}}{p_c + p_{j,k}} - \lambda_j p_{j,k} + \nu_k r_{j,k} \right], \forall k \in \mathcal{U}. \tag{4.16}$$

According to (4.15), we can obtain binary values to the user association indicator variables $x_{j,k}$ without introducing any form of relaxation.

We use a subgradient approach to update the Lagrangian multipliers [40], [41]. Specifically, with carefully chosen step sizes, the Lagrangian multipliers are updated as

$$\lambda_j^{(i+1)} = \left[\lambda_j^{(i)} - \beta_1^{(i)}\left(P_{\max j} - \sum_k x_{j,k}^{(i+1)} p_{j,k}\right)\right]^+, \forall j \in \mathcal{C} \tag{4.17}$$

$$\nu_k^{(i+1)} = \left[\nu_k^{(i)} - \beta_2^{(i)}\left(\sum_j x_{j,k}^{(i+1)} r_{j,k} - R_t\right)\right]^+, \forall k \in \mathcal{U} \tag{4.18}$$

where $[\cdot]^+$ sets the negative value to be zero; $\beta_1^{(i)}$ and $\beta_2^{(i)}$ are the step sizes of the ith iteration ($i \in \{1, 2, \ldots, i_{\max}\}$); $i_{\max}$ is the maximum number of iterations. The step sizes should satisfy the condition

$$\sum_{i=1}^{\infty} \beta_t^{(i)} = \infty, \lim_{i \to \infty} \beta_t^{(i)} = 0, \forall t \in \{1, 2\}. \tag{4.19}$$

4.4 Energy-Efficient Power Allocation

Once the optimal solution $\boldsymbol{X}^* = [x_{j,k}^*]_{(J+1)\times K}$ is obtained from the convex problem $P2.1$, it can be used in the following problem for power allocation

$$P2.2 : \max_{\boldsymbol{P}} \hat{\hat{\eta}}(\boldsymbol{P}) = \max_{\boldsymbol{P}} \sum_j \sum_k x_{j,k}^* \Xi_{j,k}(p_{j,k}) \tag{4.20}$$

$$\begin{aligned}
\text{s.t.} \quad & C1 : p_{j,k} \geq 0, \forall (j,k) \in \mathcal{C} \times \mathcal{U} \\
& C2 : \sum_{k \in \mathcal{U}} x_{j,k}^* p_{j,k} \leq P_{\max j}, \forall j \in \mathcal{C} \\
& C3 : \sum_{j \in \mathcal{C}} x_{j,k}^* r_{j,k} \geq R_t, \forall k \in \mathcal{U}
\end{aligned} \tag{4.21}$$

where $\boldsymbol{X}^* = [x^*_{j,k}]_{(J+1)\times K}$ is the optimal solution obtained from the problem we formulated in (4.3) and (4.4). $\Xi_{j,k}(p_{j,k}) = \frac{r_{j,k}}{p_c+p_{j,k}}$ is the energy efficiency of the kth user associated with the jth cell basestation.

The concept of quasiconcavity will be used in the following discussion and is defined in [37].

Definition 1. A function f that maps from a convex set of real n-dimensional vectors, D, to a real number is called strictly quasiconcave if for any $x_1, x_2 \in D$ and $x_1 \neq x_2$,

$$f(\lambda x_1 + (1-\lambda)x_2) > \min\{f(x_1), f(x_2)\} \tag{4.22}$$

for any $0 < \lambda < 1$.

Theorem 1. For any fixed user association indicator matrix $\boldsymbol{X}^*$, if $r_{j,k}$ is strictly concave in $p_{j,k}$, the maximum achievable energy efficiency is quasiconcave in transmit power $\boldsymbol{P}$ and has an optimal $\boldsymbol{P}^*$. Moreover, $\Xi_{j,k}(p_{j,k})$ has the following properties:

(1) If $r_{j,k}$ is strictly concave in $p_{j,k}$, $\Xi_{j,k}(p_{j,k})$ is continuously differentiable and strictly quasiconcave.

(2) $\Xi_{j,k}(p_{j,k})$ is first strictly increasing and then strictly decreasing in any $p_{j,k}$.

(3) If $r_{j,k}$ is strictly concave in $p_{j,k}$, there exists a unique globally optimal transmission power matrix $\boldsymbol{P}^* = [{p_{j,k}}^*]_{(J+1)\times K}$ for (4.20), where ${p_{j,k}}^*$ is given by

$$\left.\frac{\partial r_{j,k}}{\partial p_{j,k}}\right|_{p_{j,k}=p^*_{j,k}} = \frac{r_{j,k}}{p_c + p^*_{j,k}} = \Xi_{j,k}(p^*_{j,k}) \tag{4.23}$$

and

$$\left.\frac{\partial \Xi_{j,k}}{\partial p_{j,k}}\right|_{p_{j,k}=p^*_{j,k}} = 0. \tag{4.24}$$

For strictly quasiconcave functions, if a local maximum exists, it is also globally optimal [37]. Based on Theorem 1, we can obtain the optimal solution $\boldsymbol{P}^*$ to $P2.2$.

In order to obtain the solution to $P2$, we solve the two sub-problems $P2.1$ and $P2.2$

iteratively.

4.5 Summary

In this chapter, we found the solutions to the subproblems of energy-efficient user association and energy-efficient power allocation. The problem we formulated in Chapter 3 is a non-convex integer programming. We relaxed the original problem and we found that the problem was separable. Therefore, we decomposed that problem into two convex subproblems and maximized energy efficiency for user association and power allocation separately. We designed mathematical approaches for energy-efficient user association and power allocation.

Chapter 5

Algorithm Design

In this chapter, we propose an energy-efficient user association and power allocation optimization algorithm and provide numerical results to show the effectiveness of the proposed algorithm. We first design an energy-efficient power allocation algorithm, and then we propose an iterative algorithm to solve the energy-efficient user association and power allocation problem. Then we analyze the complexity for the proposed algorithm. Finally, we use simulation results to demonstrate the effectiveness of the proposed algorithm when compared with reference schemes using the fixed power allocation and fixed user association.

5.1 Gradient Ascent Power allocation Algorithm

From the QoS constraint $\sum\limits_{j \in C} x_{j,k} r_{j,k} \geq R_t, \forall k \in \mathcal{U}$, we can observe that when $x_{j,k} = 1$, the minimum transmit power is

$$\breve{p}_{j,k} = \frac{I_{j,k}}{G_{j,k}}(2^{R_t} - 1). \tag{5.1}$$

If $p^*_{j,k} < \breve{p}_{j,k}$, then $p^*_{j,k} = \breve{p}_{j,k}$.

From (4.23) and (4.24), we can obtain the power allocation $p^*_{j,k} \in \boldsymbol{P}^*$ as

$$\left.\frac{\partial r_{j,k}}{\partial p_{j,k}}\right|_{p_{j,k}=p^*_{j,k}} = \frac{\frac{G_{j,k}}{I_{j,k}}}{(1 + \frac{G_{j,k}p^*_{j,k}}{I_{j,k}})\ln 2} = \frac{\log_2(1 + \frac{G_{j,k}p^*_{j,k}}{I_{j,k}})}{p_c + p^*_{j,k}}. \tag{5.2}$$

Therefore, we have

$$\frac{G_{j,k}p_c}{I_{j,k}\ln 2} = \left(1+\frac{G_{j,k}p^*_{j,k}}{I_{j,k}}\right)\log_2\left(1+\frac{G_{j,k}p^*_{j,k}}{I_{j,k}}\right) - \frac{G_{j,k}}{I_{j,k}}p^*_{j,k}. \tag{5.3}$$

However, it is computational costly to solve (5.3). Instead, as we discussed in Theorem 1 that $\Xi_{j,k}(p_{j,k})$ is strictly quasiconcave and first strictly increasing and then strictly decreasing in any $p_{j,k}$, we can use a gradient ascent method based on binary search assisted ascent to find the optimal transmit power matrix, and use the gradient assisted binary search (GABS) to find the optimal step size [42]. The intermediate power allocation procedure is shown in Algorithm 1.

Algorithm 1 Power allocation algorithm

Initialization: $\boldsymbol{P} = \boldsymbol{P}_o$
while no convergence **do**
  while $\sum\limits_{k\in K} x^*_{j,k}p_{j,k} \le P_{\max j}, \forall j \in \mathcal{C}$ **do**
    Use GABS to find the optimal step size $\delta^{(t+1)*}$,
    $\boldsymbol{P}^{(t+1)} = \left[\boldsymbol{P}^{(t)} + \delta^{(t+1)*}\nabla\breve{\eta}^{P}\left(\boldsymbol{P}^{(t)}\right)\right]^{+}.$
    if $p^*_{j,k} < \breve{p}_{j,k}$ **then**
      $p^*_{j,k} = \breve{p}_{j,k}.$
    end if
  end while
end while
Update $\boldsymbol{P}^*_{j,k}$ as $\boldsymbol{P}^{*(s+1)}_{j,k}$.

5.2 Iterative Energy-Efficient Algorithm

In this section, we propose an iterative algorithm as the original problem can be solved by separating the two variables and using iterations to approach the optimal solution.

According to the analysis of power allocation and user association discussed above, we propose an iterative optimization algorithm as shown in Algorithm 2.

In Algorithm 2, each user calculates (4.15) and (4.16) to obtain $\boldsymbol{X}^{(i+1)}$, and then updates $\boldsymbol{\nu}^{(i+1)}$. Each basestation updates $\boldsymbol{\lambda}^{(i+1)}$ for the $(i+1)$th inner iteration. Once the inner iteration achieves convergence, each basestation uses Algorithm 1 to allocate the power. The user association and power allocation results can be obtained once the outer iteration achieves convergence. The user association solution can be obtained by each user equipment and the power allocation solution can be obtained by each basestation. Therefore, the proposed Algorithm 2 is distributed.

Algorithm 2 Distributed joint user association and power allocation

Initialization: A feasible initial value of the transmit power vector
while no convergence (outer iteration s) **do**
  while no convergence (inner iteration i) **do**
    User strategy (inner iteration):
    for all $k \in \mathcal{U}$ **do**
      Calculate $\boldsymbol{X}^{(i+1)}$ according to (4.15) and (4.16),
      Update $\boldsymbol{\nu}^{(i+1)}$ according to (4.18).
    end for
    Update $\boldsymbol{\lambda}^{(i+1)}$ according to (4.17).
  end while
  Update user association $\boldsymbol{X}^{*(s+1)}$ as $\boldsymbol{X}^{(t+1)}$ obtained at convergence of inner iterations.
  basestation strategy (outer iteration):
  Power Allocation
  Use Algorithm 1 to find optimal transmit power vector,
  Update $\boldsymbol{P}^{*}_{j,k}$ as $\boldsymbol{P}^{*(s+1)}_{j,k}$.
end while
Return: $\boldsymbol{X}^{*(s+1)}$ and $\boldsymbol{P}^{*(s+1)}_{j,k}$ at convergence of total energy efficiency or $s = s_{\max}$.

5.3 Complexity Analysis

The asymptotic complexity of the proposed algorithms is analyzed in this subsection. In Algorithm 2, the calculation of (4.14) for every user needs JK operations, and a worst-case complexity of searching (4.15) needs JK operations in each inner iteration. Suppose the the subgradient method in Algorithm 2 requires Ω iterations to coverage, the updates of $\boldsymbol{\lambda}$ needs $O(J)$ operations and $\boldsymbol{\nu}$ needs $O(K)$ operations. Therefore, Ω

is a polynomial function of JK. According to [42], we assume the convergence rate of BASS is M. Suppose the proposed iterative Algorithm 2 requires Δ iterations to converge, the total complexity of Algorithm 2 is $O\left(\Delta(\Omega J^2 K^2 + M)\right)$.

5.4 Numerical Results

Simulation results are presented to demonstrate the effectiveness of the proposed algorithms. In our simulations, we assume that all users are uniformly distributed in each small cell coverage area, and the small cells are uniformly distributed in the macrocell coverage area. The radius of the macrocell is 300 m. The radius of each small cell is 10 m. Small cell has a minimum distance of 50 m from the macro basestation. The minimum distance between small cell basestations is 40 m. The pathloss model is based on [43]. We assume that the shadowing standard deviation between basestation and the users is 10 dB [43]. The channel fading is composed of shadowing fading, path loss, and Rayleigh fading. The AWGN power is set as σ^2=3.9811 $\times$ 10^{-14} W [44]. We assume that the maximum transmit power is 40 dBm at the macrocell basestation.

Fig. 5.1 shows the convergence of the proposed Algorithm 2 in terms of the total energy efficiency when the number of small cells is increased from 5 to 8 where $K = 100$. Each small cell is associated with 5 users. The maximum transmit power is 17 dBm in each small cell. R_t is 0.01 bps/Hz. It can be observed that Algorithm 2 takes about 15 iterations to converge, which ensures that the proposed Algorithm 2 is practical. We can also observe that with the increase of the number of small cells, the total energy efficiency has improved.

Fig. 5.2 shows the energy efficiency when $P_{\max j}$ of small cells is increased from 0.005 to 0.095 W. Each small cell is associated with 3, 4 and 5 users for the proposed Algorithm 2. Each small cell is associated with 5 users for the fixed power allocation scheme and fixed user association scheme. For fixed power allocation scheme, the power is equally allocated. The other parameters are $K = 60$, $R_t = 0.01$ bps/Hz and $J = 5$. We can observe that the improved energy efficiency performance is obtained when more users are associated with the small cells. When each small cell is associated with 5 users, the energy efficiency of the proposed Algorithm 2 is 6% more than the fixed power allocation scheme and 21% more than the fixed user association scheme. Moreover, when using the proposed Algorithm 2, the energy efficiency first increases with the power constraint because a larger power constraint leads to enlarged region of the optimizing variable. Then the energy efficiency increases slowly and converges because energy efficiency first increases and then decreases in transmit power. When the maximum transmit power is larger than the optimal transmit power, the transmit power will not increase with the maximum transmit power constraint. For the fixed power allocation scheme, the energy efficiency first increases and then decreases with the power constraint due to the quasiconcavity of the energy efficiency. The comparison of Algorithm 2 and the fixed power allocation scheme shows the Algorithm 2 can maintain a maximum energy efficiency with the increasing of the power constraint.

Fig. 5.3 shows the overall energy efficiency when the QoS constraint R_t is increased from 0.01 to 0.55 bps/Hz for the proposed Algorithm 2, the fixed power allocation scheme and the fixed user association scheme when $P_{\max j} = 14.7$ dBm and $P_{\max j} = 20$ dBm. In the simulation, we assume $K = 60$ and $J = 5$. Each small cell is associated with 5 users. We can observe that the proposed Algorithm 2 improves the energy efficiency 6% compared with the fixed power allocation scheme and 20% compared with the fixed user association scheme for $P_{\max j} = 20$ dBm. For $P_{\max j} = 14.7$ dBm, the proposed Algorithm 2 improves the energy efficiency 2% compared with the fixed power allocation scheme and 21% compared with the fixed user association scheme. The performance improvement of the proposed Algorithm 2 when $P_{\max j} = 14.7$ dBm is small because smaller power constraint leads to smaller region for the optimizing variable. It can also be observed that the total energy efficiency is reduced with an increase of R_t. Because the minimum transmit power is increased with R_t, which also leads to smaller region for the optimizing variable.

Fig. 5.4 shows the energy efficiency when the number of users each small cell associated with is increased from 3 to 7 when $P_{\max j} = 13$ dBm and $P_{\max j} = 20$ dBm. The other parameters are $K = 60$, $R_t = 0.01$ bps/Hz and $J = 5$. We can observe that the improved energy efficiency performance is obtained when more users are associated with the small cells. It also can be observed that the proposed Algorithm 2 has better performance than both reference schemes. Moreover, when $P_{\max j}$ is increased from 13 dBm to 20 dBm, the energy efficiency has improved.

Fig. 5.5 shows the energy efficiency when the number of small cells is increased from 3 to 8 for the proposed Algorithm 2 and the fixed power allocation scheme when $P_{\max j} = 20$ dBm, for the fixed user association scheme when $P_{\max j} = 13$ dBm and $P_{\max j} = 20$ dBm. Each small cell is associated 5 users. The other parameters are $K = 60$, $R_t = 0.01$ bps/Hz. We can observe that the improved energy efficiency performance is obtained when more small cells are deployed in the two-tier HetNet due to the multi-cell diversity. Moreover, the proposed Algorithm 2 improves energy efficiency compared with the fixed power allocation scheme and the fixed user association scheme for 7% and 11% respectively when $J = 8$, $P_{\max j} = 20$ dBm.

Fig. 5.6 shows the capacity when the number of small cells is increased from 3 to 8 for the proposed Algorithm 2, the fixed power allocation scheme and the fixed user association scheme when $P_{\max j} = 13$ dBm and $P_{\max j} = 20$ dBm. Each small cell is associated 5 users. The other parameters are $K = 60$, $R_t = 0.01$ bps/Hz. We can observe that the capacity is improved when more small cells are deployed in the two-tier HetNet due to the multi-cell diversity. Moreover, the proposed Algorithm 2 has lower capacity compared with the fixed power allocation scheme when $P_{\max j} = 20$ dBm. Because for the proposed Algorithm 2, the transmit power will not increase with the maximum transmit power constraint when the maximum transmit power is larger than the optimal transmit power. The proposed Algorithm 2 improves the total capacity

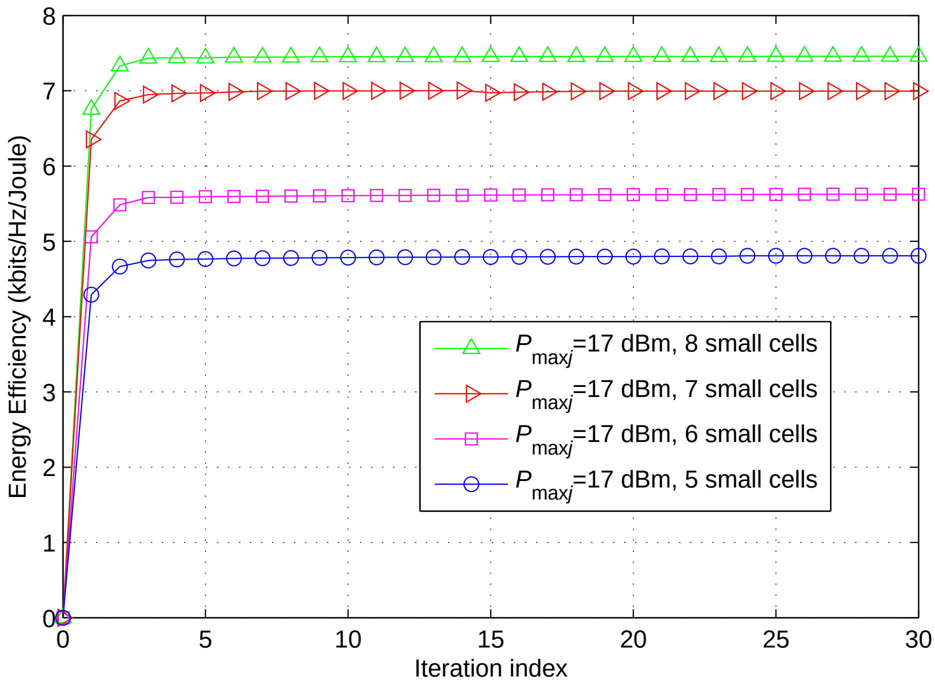

Figure 5.1: The convergence in terms of energy efficiency over the number of iterations.

compared with the fixed user association scheme when $P_{\max j} = 20$ dBm.

5.5 Summary

In this chapter, we designed an energy-efficient user association and power allocation optimization algorithm, and provided numerical results to show the effectiveness of the proposed algorithm. We first proposed a gradient ascent algorithm to solve the power allocation problem. Then we proposed an iterative algorithm to solve the energy-efficient user association and power allocation problem. We analyzed the complexity for the proposed algorithm and used simulation results to demonstrate the effectiveness of the proposed algorithms when compared with reference schemes using the fixed power allocation and fixed user association.

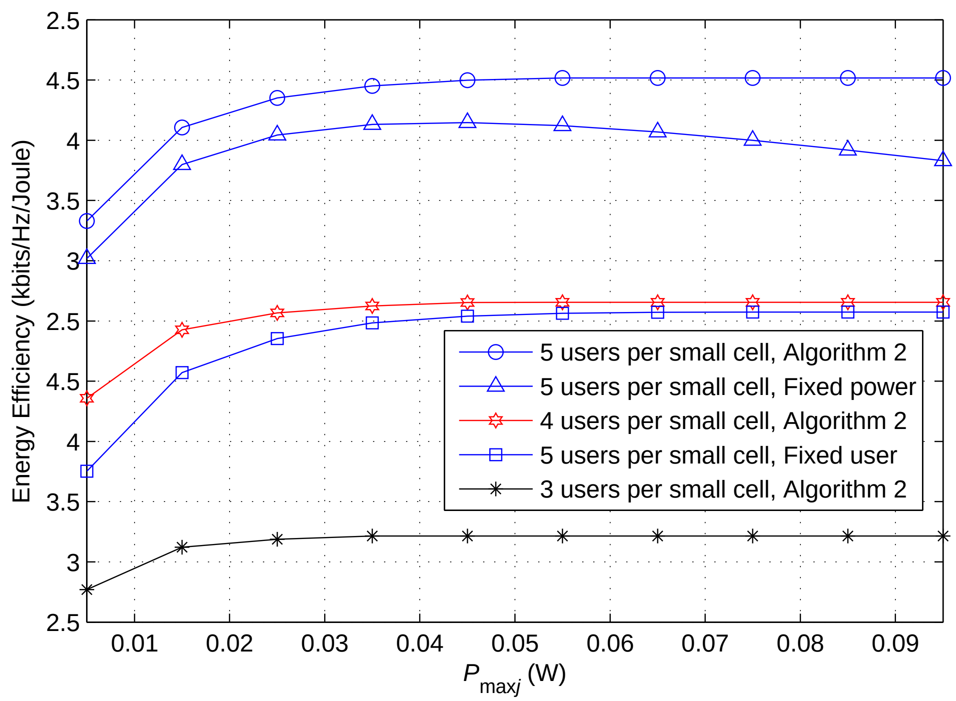

Figure 5.2: Total energy efficiency versus power constraint.

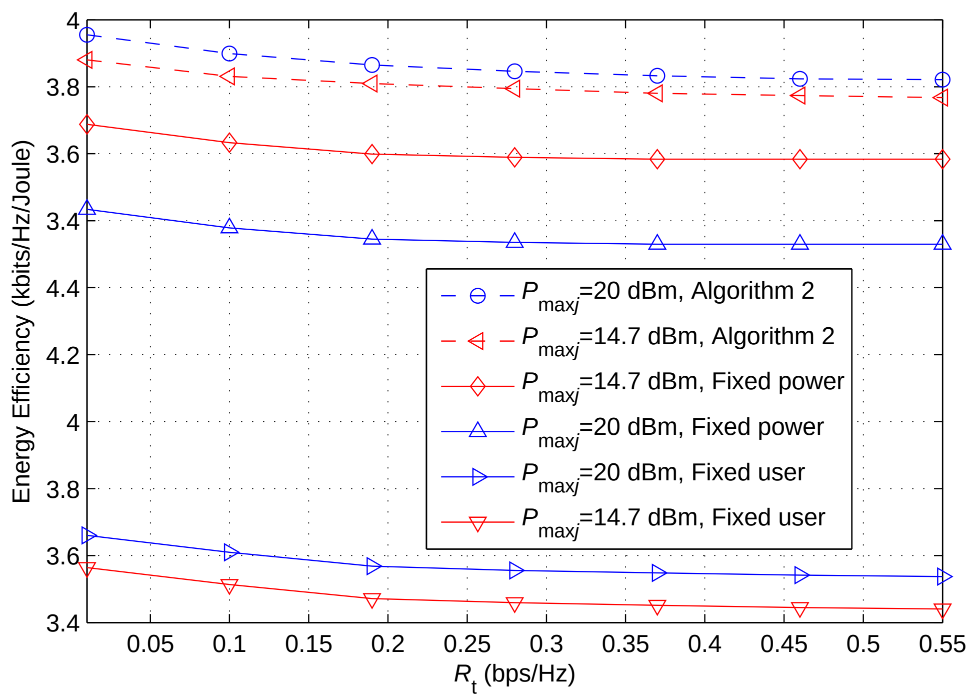

Figure 5.3: Total energy efficiency versus minimum data rate.

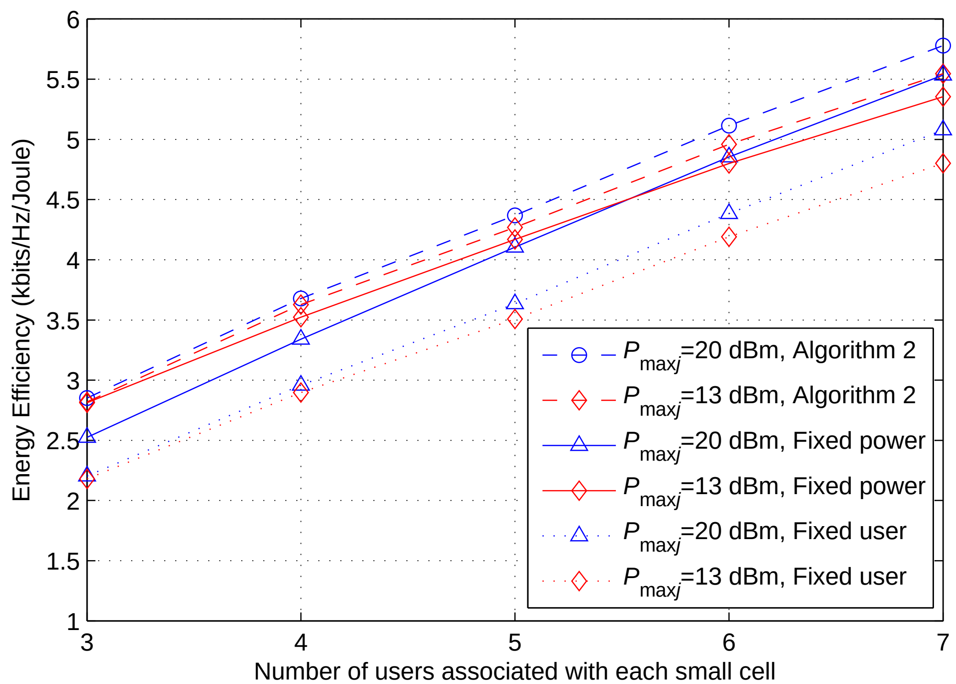

Figure 5.4: Total energy efficiency versus the number of users associated with each small cell.

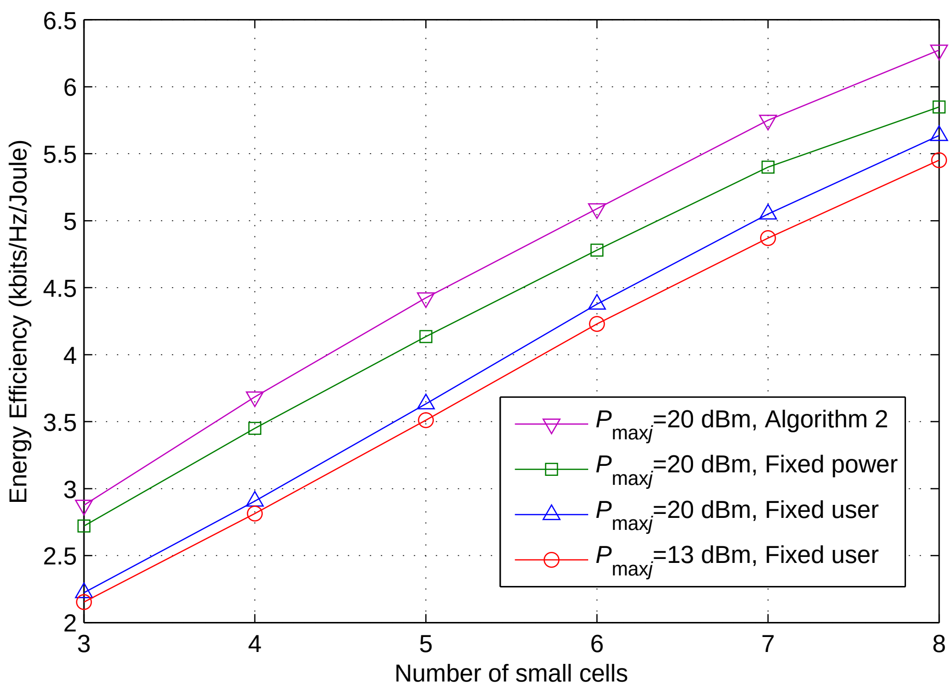

Figure 5.5: Total energy efficiency versus the number of small cells.

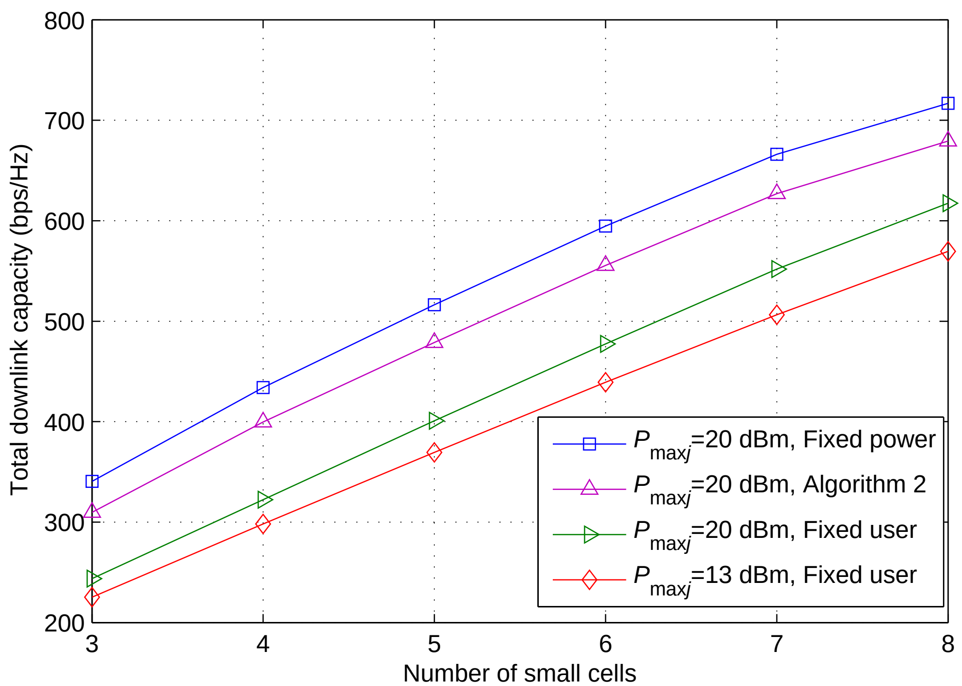

Figure 5.6: Total capacity versus the number of small cells.

Chapter 6

Conclusions

In this chapter, we conclude the thesis by summarizing the accomplished work and suggest some potential further works.

6.1 Summary of Accomplished Work

In this thesis, we developed an optimization algorithm for energy-efficient user association and power allocation in a two-tier heterogeneous network. The obtained numerical results can show the effectiveness of the proposed design. In order to conclude the thesis, we will summarize the accomplished work as follows:

- In Chapter 2,we presented detailed background knowledge about user association and resource management in heterogeneous networks. We first introduced the motivation for the development of heterogeneous networks and the characteristic of heterogeneous networks. Then, we proposed the concept of energy-efficient user association and resource management. Finally, the basic convex optimization knowledge was presented.

- In Chapter 3, we proposed a system model for an energy-efficient two-tier heterogeneous network. We formulated the energy-efficient power allocation and user association problem to maximize the downlink energy efficiency for the two-tier heterogeneous network. We formulated the problem under minimum data requirements and total transmit power limits constraints. The formulated problem was a non-convex integer programming.

– Chapter 4 provided the conditions for optimization and mathematical approaches for user association and transmit power allocation. Firstly, we noticed that the formulated problem in Chapter 3 was a non-convex integer programming. We relaxed and decomposed it into two convex subproblems that one for user association and another for power allocation. Secondly, we solved the subproblems of energy-efficient user association and energy-efficient power allocation, respectively.

– In Chapter 5, an iterative algorithm was designed and numerical results were presented. We designed an energy-efficient power allocation algorithm, and then we proposed an iterative algorithm to solve the energy-efficient user association and power allocation problem. We analyzed the complexity for the proposed algorithm and use simulation results to demonstrate the effectiveness of the proposed algorithm when compared with reference schemes using the fixed power allocation and fixed user association.

6.2 Future Work

Besides the proposed problem in this thesis, there are still some potential directions worth further investigation. In this work, we considered the energy-efficient user association and power allocation for a two-tier heterogeneous network. Subchannel allocation is also an important resource allocation aspect and could be jointly considered along with user association for energy-efficient purpose. Moreover, spectral efficiency is also an important system performance. The relationship between energy efficiency and spectral efficiency when considering user association and power allocation is worth investigating in the future.

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Appendix

Appendix A

Proof: Denote the superlevel set of $\Xi_{j,k}(p_{j,k})$ as

$$S_\beta = \{p_{j,k} \geq 0 | \, \Xi_{j,k}(p_{j,k}) \geq \beta\}. \tag{A.1}$$

According to [37], $\Xi_{j,k}(p_{j,k}) = \frac{r_{j,k}}{p_c + p_{j,k}}$ is quasiconcave in $p_{j,k}$ if S_β is convex in $p_{j,k}$ for any real number β. For $\beta < 0$, there exists no point on the contour of $\Xi_{j,k}(p_{j,k}) = \beta$. When $\beta = 0$, only $p_{j,k} = 0$ is on the contour of $\Xi_{j,k}(p_{j,k}) = \beta$. Thus, when $\beta \leq 0$, S_β is convex. When $\beta > 0$, S_β can be rewritten as $S_\beta = \{p_{j,k} \geq 0 | \, \beta(p_c + p_{j,k}) - r_{j,k} \leq 0\}$. Since $r_{j,k}$ is strictly concave in $p_{j,k}$, $-r_{j,k}$ is strictly convex in $p_{j,k}$. Therefore S_β is strictly convex. Thus, $\Xi_{j,k}(p_{j,k})$ is strictly quasiconcave in $p_{j,k}$.

The partial derivative of $\Xi_{j,k}(p_{j,k})$ is $\frac{\partial \Xi_{j,k}(p_{j,k})}{\partial p_{j,k}} = \frac{(p_c + p_{j,k}) r'_{j,k} - r_{j,k}}{(p_c + p_{j,k})^2}$, where $r'_{j,k}$ is the first order derivative of $r_{j,k}$ in $p_{j,k}$. If $p^*_{j,k}$ exists then the solution to $\frac{\partial \Xi_{j,k}(p_{j,k})}{\partial p_{j,k}} = 0$ exists. Next, we investigate the conditions when $p^*_{j,k}$ exists.

Denote the numerator of $\frac{\partial \Xi_{j,k}(p_{j,k})}{\partial p_{j,k}}$ as

$$h(p_{j,k}) = (p_c + p_{j,k}) r'_{j,k} - r_{j,k}. \tag{A.2}$$

The derivative of $h(p_{j,k})$ in $p_{j,k}$ is

$$\begin{aligned} h'(p_{j,k}) &= r'_{j,k} + (p_c + p_{j,k}) r''_{j,k} - r'_{j,k} \\ &= (p_c + p_{j,k}) r''_{j,k} \end{aligned} \tag{A.3}$$

where $r''_{j,k}$ is the second-order partial derivative with respect to $p_{j,k}$. $r''_{j,k} < 0$ always hold-s because $r_{j,k}$ is strictly concave. Hence, $h'(p_{j,k}) < 0$ and $h(p_{j,k})$ is strictly decreasing.

Next, we investigate whether $\frac{\partial \Xi_{j,k}(p_{j,k})}{\partial p_{j,k}}$ has only one $p_{j,k}$ that satisfies $\frac{\partial \Xi_{j,k}(p_{j,k})}{\partial p_{j,k}} = 0$.

When $p_{j,k} \to \infty$, we have

$$\begin{aligned}
&\lim_{p_{j,k}\to\infty} h(p_{j,k}) \\
&= \lim_{p_{j,k}\to\infty} (p_c + p_{j,k}) r'_{j,k} - r_{j,k} \\
&= \lim_{p_{j,k}\to\infty} p_c \tfrac{G_{j,k}}{\ln 2(I_{j,k}+p_{j,k}G_{j,k})} + \tfrac{p_{j,k}G_{j,k}}{\ln 2(I_{j,k}+p_{j,k}G_{j,k})} \\
&\quad - \log_2(1 + \tfrac{p_{j,k}G_{j,k}}{I_{j,k}}).
\end{aligned} \tag{A.4}$$

For the second term of the last line in (A.4), we use the L'Hospital's rule and obtain

$$\begin{aligned}
&\lim_{p_{j,k}\to\infty} \tfrac{p_{j,k}G_{j,k}}{\ln 2(I_{j,k}+p_{j,k}G_{j,k})} \\
&= \lim_{p_{j,k}\to\infty} \tfrac{(p_{j,k}G_{j,k})'}{\left[\ln 2(I_{j,k}+p_{j,k}G_{j,k})\right]'} \\
&= \lim_{p_{j,k}\to\infty} \tfrac{G_{j,k}}{G_{j,k}\ln 2} \\
&= \tfrac{1}{\ln 2}.
\end{aligned} \tag{A.5}$$

Therefore, with $\lim\limits_{p_{j,k}\to\infty} p_c \frac{G_{j,k}}{\ln 2(I_{j,k}+p_{j,k}G_{j,k})} = 0$ and $\lim\limits_{p_{j,k}\to\infty} -r_{j,k} = -\infty$, we obtain $\lim\limits_{p_{j,k}\to\infty} h(p_{j,k}) = -\infty$.

When $p_{j,k}$ approaches 0, we have

$$\begin{aligned}
\lim_{p_{j,k}\to 0} h(p_{j,k}) &= \lim_{p_{j,k}\to 0} (p_c + p_{j,k}) r'_{j,k} - r_{j,k} \\
&= \lim_{p_{j,k}\to 0} p_c \frac{G_{j,k}}{I_{j,k}\ln 2} > 0.
\end{aligned} \tag{A.6}$$

Hence, $\frac{\partial \Xi_{j,k}(p_{j,k})}{\partial p_{j,k}}$ has a unique $p^*_{j,k}$ that satisfies $\frac{\partial \Xi_{j,k}(p_{j,k})}{\partial p_{j,k}} = 0$, and $\Xi_{j,k}(p_{j,k})$ is first strictly increasing and then strictly decreasing within its domain.

Since $\hat{\hat{\tilde{\eta}}}(\boldsymbol{P}) = \sum\limits_j \sum\limits_k x^*_{j,k} \Xi_{j,k}(p_{j,k})$ is a linear combination of $\Xi_{j,k}(p_{j,k})$, the quasiconcavity also holds for $\hat{\hat{\tilde{\eta}}}(\boldsymbol{P})$. Because for any $\boldsymbol{P}_1 \neq \boldsymbol{P}_2$ and $\lambda \in (0,1)$, $\hat{\hat{\tilde{\eta}}}(\lambda \boldsymbol{P}_1 + (1-\lambda)\boldsymbol{P}_2) \geq \min\left\{\hat{\hat{\tilde{\eta}}}(\boldsymbol{P}_1), \hat{\hat{\tilde{\eta}}}(\boldsymbol{P}_2)\right\}$ when $\Xi_{j,k}(p_{j,k})$ is first strictly increasing and then strictly decreasing in $p_{j,k}$. Moreover, constraints in (4.21) are convex, and $\hat{\hat{\tilde{\eta}}}(\boldsymbol{P})$ under these convex constraints is still quasiconcave in $\boldsymbol{P}$. Therefore, $\hat{\hat{\tilde{\eta}}}(\boldsymbol{P})$ has an optimal $\boldsymbol{P}^*$.

From the proof above, $\frac{\partial \Xi_{j,k}(p_{j,k})}{\partial p_{j,k}}$ has a unique $p^*_{j,k}$ that satisfies $\frac{\partial \Xi_{j,k}(p_{j,k})}{\partial p_{j,k}} = 0$, where $\frac{\partial \Xi_{j,k}(p_{j,k})}{\partial p_{j,k}} = \frac{(p_c+p_{j,k})r'_{j,k}-r_{j,k}}{(p_c+p_{j,k})^2}$ equals to $h(p_{j,k}) = (p_c + p_{j,k})r'_{j,k} - r_{j,k} = 0$. Then we have

$$r'_{j,k} = \frac{r_{j,k}}{p_c + p_{j,k}}. \tag{A.7}$$

■