Abstract

In this research, static and dynamic voltage stability of distribution systems are studied, with a focus on systems with distributed generation. A voltage dependent model is used for distribution loads, which converts the static voltage stability analysis into a linear problem and allows for real-time solutions.

For static voltage stability studies, an index is defined that contains critical data on load and system characteristics. The static voltage behavior of distribution systems, either with or without DG, is studied by analyzing the P-V curves and the proposed voltage stability index at each node of the system. Besides identifying the weakest buses of the system in terms of voltage profile and voltage stability, this approach also allows the extension of the classical voltage stability solution to the increasingly important case of distributed generation placement - where the system is more likely to face voltage transients. The method can also be used in resilience studies.

For dynamic voltage stability studies, the Shifted Frequency Analysis (SFA) method is used to evaluate the system transients and its dynamic voltage behavior during and right after being subjected to a change or disturbance in the system. Various scenarios are discussed, including scenarios in which the voltage transients due to DG cause voltage instabilities in the system. SFA and EMTP solutions are compared with each other, and with the static analysis results. SFA and EMTP results also verify the validity of the proposed voltage stability index.

The proposed voltage assessment method facilitates real-time decision making, topological reconfiguration to strengthen voltage stability robustness, and DG placement. Different scenarios and distribution system topologies such as looped or meshed distribution...
systems, as well as microgrids and autonomous/islanded energy grids can be included in the solution.
Preface

All of the work done in this research and the corresponding publications have been designed, conducted and analyzed by myself under the supervision of Professor José R. Martí.

Different Sections of this dissertation have led to a number of currently under review publications in well-known technical journals. The corresponding chapters adapted for inclusion in the peer review publications are as following:

Chapters 2 and 4

Chapters 3 and 5
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Glossary

DG                  Distributed Generation
dc                  Direct current
EMTP                ElectroMagnetic Transients Program
ms                  Millisecond
LPF                 Linear Power Flow
P                   Active Power
PoC                 Point of Collapse
P-Q                 Active Power - Reactive Power
pu                  Per unit
P-V                 Active Power-Voltage
Q                   Reactive Power
SFA                 Shifted Frequency Analysis
S/SE                South/ South East
ZI                  Constant Impedance (Z) – Current (I)
ZIP                 Constant Impedance (Z) – Current (I) – Power (P)
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Dedication

This thesis is dedicated to my beloved parents, who have dedicated their lives to their children.
Based on IEEE/CIGRE definition, voltage stability refers to the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance from a given initial operating condition. Instability that may result occurs in the form of a progressive fall or rise of voltages of some buses. A possible outcome of voltage instability is loss of load in an area, or tripping of transmission lines and other elements by their protections leading to cascading outages that in turn may lead to loss of synchronism of some generators. Some voltage collapses may not be within the unstable region, however they can lead to a slower-occurring voltage instability in the system.

Voltage stability in distribution systems has been understood for decades, and has also been referred to as load stability [1], [2]. Voltage stability is determined by parameters such as system and load characteristics, the generating units' behavior, and the interactions of both continuous and discrete controls and protections.

Voltage/Load stability is trending towards a more challenging issue in modern distribution systems in both static and dynamic stages, given the ongoing increase in load demands, as well as restructuring trends in modern distribution systems. Therefore, in many cases, the maximum loading of some distribution system buses have to be constrained by voltage stability limits, rather than by thermal limits (as shown in Section 4.2 of this dissertation); however the former has not received enough attention. As an example, a voltage instability problem in a distribution network in the S/SE Brazilian system caused a major blackout in 1997,
which was widespread to the corresponding transmission system, leading to the failure and trip-off of a major DC link [3].

On the other hand, due to environmental impacts and transmission costs of traditional electricity generation, there is a push for alternative technologies that can assure not only an electricity supply of high quality and low price, but also the efficient use of natural resources. One of these alternative technologies is Distributed Generation (DG) [1].

A major advantage of DG is the power generation in closer proximity to consumers; hence reducing losses in transmission systems. In case of using clean energy (e.g. solar and wind) the environmental impacts of power generation is also reduced. In addition, DG provides the opportunity for autonomous islanded operation.

In recent years, the integration of distributed generation with distribution systems has been increasing. DG can impact distribution system features such as voltage profile, voltage stability, power flow, power quality, reliability, and control and protection, given its intermittent nature (in case of renewable energies) and its smaller inertia during transients and system faults, compared to large traditional generating units. Depending on the level of penetration, the impacts on the voltage stability can be significant.

If the penetration level of DG is low compared to main power generating units (1%–5%), the impacts are minor. However, if the penetration level of DG increases to 20%–30%, or in the case of microgrids or autonomous energy systems, where DG is the main source of power generation, the impact of DG on the distribution system and its voltage is of significant importance.

Voltage stability analysis can be conducted using dynamic or static approaches, as discussed in the following Sections.
1.1 Static Voltage Stability

Static voltage stability analysis is intended to evaluate the voltage stability and loadability margins, after the system has transitioned to a new operating state because of a disturbance or change in the system - without studying the transients during this transition and in-between the two operating states.

Static voltage stability analysis is traditionally done using power flow methods. Conventional static voltage stability analysis techniques usually detect saddle-node bifurcations, or voltage collapse points, in order to determine the loadability limits of the power system. These methods traditionally apply continuation power flow methods, or Point of Collapse (PoC) methods [4], [5].

Standard power flow formulations are based on the solution of a system of nonlinear equations which are computationally expensive due to the iterations needed. In the case of weakly-meshed distribution systems or distribution systems with DG, the direction of power flow can be reversed when the DG exceeds the local load, resulting in potential complications with the conventional power flow methods. As a result, different power flow situations should be considered, which will consume more time for the voltage stability analysis. The opposite load flow can also interfere with conventional voltage control and protection systems [6], [7]. Using the method proposed in this research, power flow calculations are avoided, thus preventing complications or excess processing times resulted from changes in the direction of the flow of power.

As we approach distribution load levels, the voltage dependency of power consumed by the load becomes more critical, where the response of loads to voltage changes occurring over a period of time can affect voltage stability. For transient voltage stability, and for the final stages
of slower-occurring static voltage instabilities, the normal constant power (voltage in-dependent) load model is not valid; therefore, the voltage dependent characteristics of loads become very important [6].

In this research, we introduce a novel method for static and dynamic voltage stability analysis of distribution systems, using a voltage-dependent load model, which can be easily implemented on any load type and distribution system configuration, either with or without DG.

Using a voltage dependent model for the system loads, it is possible to linearize the system solution and represent the distribution system behind a given load as a 2-node Thévenin equivalent.

We also propose an easy-to-implement voltage stability index, based on load characteristics and distribution system specifications as well as DG characteristics, which contains critical data on distribution system node voltages such as voltage stability status, and sensitivity of the voltage to power changes. This index can rank the system buses based on their voltage profile and voltage stability status, and can be used as a practical tool to identify best candidates for installation of new DG, in order to improve voltage stability and elevate undervoltages without causing overvoltage in any of the system buses.

The proposed linear method does not include any iterative calculations or conventional power flow solutions, thus it can be used in real-time, which is a significant advantage, particularly for large distribution systems with thousands of nodes.
1.2 Dynamic Voltage Stability

Dynamic voltage stability analysis is intended to evaluate the system voltage behavior during and shortly after a change or disturbance in the distribution system, such as system faults, loss of generation, or circuit contingencies, as well as sudden or incremental changes in system loads. In other words, it evaluates the transient voltage behavior of the system in between two steady state operating modes.

Dynamic voltage stability studies can be quite complicated, and not many studies in this regard exist in the literature.

We use the Shifted Frequency Analysis (SFA) method to study the transient system behavior. Shifted Frequency Analysis is based on finding an EMTP solution [9] in the shifted frequency domain, and then transitioning the solution back into the real time domain. By shifting the original signal by the value of the fundamental frequency, one can transform the signal to DC domain (0 Hz).

This allows for the use of larger simulation time steps than is possible in the original time domain. The dynamic behavior of the system is reflected on the changes in magnitude and phase of the original signal. Because only the envelope around the fundamental frequency is simulated, larger time steps can be used than in EMTP methods, thus resulting in faster analysis times [10]. The method offers a number of possibilities for its widespread application in power systems dynamic simulations [11], and for real-time operation and control applications.

The SFA method transfers the signal including its fundamental frequency, from real time domain to SFA domain by eliminating the fundamental frequency terms. Then using a discretization procedure similar to EMTP, the system transients are studied in SFA domain, using a considerably larger time step than the time step needed in real time domain. Finally, if
required, the solution is transferred back to the real time domain, including the fundamental frequencies.

Static voltage stability analysis studies are not capable of analyzing the behavior of the system during transients, therefore cannot detect any possible instability in the system caused by transient frequencies.

As shown in the coming chapters, there are scenarios where the system loses its stability during transients, although the static voltage stability studies suggest the system will stay functional.

1.3 Research Objectives

Given the previously discussed subjects, the main objectives of this PhD research are as the following:

1.3.1 Objective 1: Obtaining a Linear Model for Distribution Systems including Distributed Generation – Valid for both Unbalanced and Balanced Distribution Systems

Given that distribution systems are directly involved with load levels, generally there is a need for major amount of studies to be conducted on distribution systems e.g. power flow, short circuit, and voltage stability studies. Most of these studies include sets of non-linear equations, which are computationally expensive. On the other hand, many modern distribution systems are huge in size and are continuously growing due to ongoing increases in load demands; this makes the studies even more complex and time-consuming. As a result, it is significant if we can find a
trustable linear model to use on both balanced and non-balanced distribution systems, which can accurately simulate the behavior of the system, and therefore converts the traditional non-linear equations of the system to a set of linear equations that can represent results in real-time.

Considering distributed generation and its impacts on the distribution system behavior, the mentioned approach becomes of even greater significance, if it can also include the distributed generation.

1.3.2 Objective 2: Real Time Steady State Voltage Stability Analysis of Distribution Systems

For real time operations and online decision making procedures by operators, it is very important to have access to real-time data on the system voltage characteristics. This will help operators, before making any changes in the system, to make sure the system’s voltage behavior will stay within the acceptable ranges - as well as to improve the system voltage stability.

Conventional voltage stability analysis methods are not real time and mostly have to solve iterative computations. Obtaining a linear model for the distribution system (as per objective 1) could be the first step towards the real time voltage analysis. After that we will need to define a procedure and/or parameters to analyze the distribution system voltage stability in real time.

1.3.3 Objective 3: Identifying Nodes with Weak Voltage Profile as well as Most Resilient Nodes

In order to have a system with sound and stable voltage behavior, we need to improve the voltage profile of the parts of the system which are facing deficiencies; such as voltage drops, or
operating close to the loadability limits. If these deficiencies are not detected and resolved, more severe problems in the system can happen either in short term or long term operation modes. Therefore, fast and accurate identification of the weak buses in a distribution system, using the distribution system and loads data, is very important.

Also, to improve the system resilience, it is very important to determine the most stable and robust nodes of the system, in order to install the most critical loads; therefore the chances that the system recovers the fastest after a disturbance will highly increase.

1.3.4 Objective 4: Distributed Generation Placement

Given the upward trend for using DG in power systems, distributed generation placement in distribution systems is of great importance. DG units in general tend to increase the voltage at their location of installation; therefore it is important to find appropriate locations within a distribution system to install the DG in order to not cause overvoltages in the system. Also DG naturally adds extra transient frequencies to the system, which results in the correct placement of DG to be even more delicate.

In addition, one of the solutions to improve the voltage at some nodes with weak voltage profile (as in objective 3) is to install DG at or nearby these nodes. Finding an appropriate methodology for DG placement, which can cover all the aforementioned aspects, is very practical in modern distribution systems.
1.3.5 Objective 5: Explicit Transient Voltage Stability Analysis of Distribution Systems with Distributed Generation

In general, transient voltage stability studies in distribution systems have not received enough attention in the existing literature. However, transient waveform values in voltage can go further than the distribution system permissible operating limits, and initiate the protection and control devices.

Furthermore, transients can result in voltage instabilities in the system; specifically transients due to a change in the system, while the system is already recovering from another stress. Given the small inertia of DG in general, and the intermittent nature of renewable DG (e.g. solar, wind), the chances for such incidents in the presence of DG increase.

These scenarios usually are not detectable by static voltage stability analysis methods, because they are not capable of studying the system behavior during transient modes. Therefore, the act of transient voltage stability analysis in distribution systems particularly in the systems with DG, as well as autonomous energy systems, is very important - in order to prevent any voltage instability in the distribution system which can correspondingly lead to the transmission system as well.

1.3.6 Objective 6: Reduced Time Transient State Voltage Stability Analysis

Transient study methods usually take longer times to complete the system analysis, since they typically simulate and analyze the system state in small time step intervals. If we can find a method for transient voltage stability analysis that can use larger time steps, and still maintain the accuracy of the results, it would save big portion of the analysis time.
The existing tools which can be used for transient voltage stability analysis, study the system including its fundamental frequency as well as the system deviated transient frequencies. If we can by any means limit the studied frequency range, we would be able to use larger time steps and therefore reduce the analysis time; this would be of practical importance in control and real-time operations of huge distribution systems with a large number of nodes.
Chapter 2:
Modelling of Distribution System

The conventional load representation as constant P-Q becomes less accurate as we get closer to the actual loads level. The load’s power dependency on voltage has an important impact on the actual power consumption. For example as shown in [12] in the BC Hydro distribution system, by decreasing the substation voltage by 1%, the active and reactive demands decrease by 1.5% and 3.4%, respectively.

For the slower forms of voltage instability, a key question is whether the normal constant power (voltage independent) load model is valid [8]. In this research, we use a load model which represents the voltage dependency of loads using a simple curve-fitting technique. This voltage dependent load model represents the load as a combination of admittance and a current source, and can be easily implemented on any system topology, including DG.

2.1 Voltage-Dependent Load Models

A common formula for conveying the dependency of active and reactive power consumption on voltage magnitude is the exponential model [5], [13]:

\[
\frac{P(V)}{P_0} = \left(\frac{V}{V_0}\right)^\alpha
\]  

(1)

\[
\frac{Q(V)}{Q_0} = \left(\frac{V}{V_0}\right)^\beta
\]  

(2)
Where \( P(V) \) and \( Q(V) \) are the load voltage dependent active and reactive power. \( V \) is the actual feeding voltage magnitude, and \( P_0, Q_0 \) and \( V_0 \) represent the nominal values for the load active and reactive power and the feeding voltage, respectively. Also, \( \alpha \) and \( \beta \) are the active and reactive power exponents, respectively, which can be extracted from measurements. Some typical values for these exponents are given in [14].

In addition to the exponential load model, the polynomial load model has also been widely used in power system studies. This model, also known as ZIP model, consists of three major components modelling the voltage dependent behavior of the load; including a constant impedance (\( Z \)), a constant current (\( I \)), and a constant power (\( P \)) terms:

\[
\frac{P(V)}{P_0} = F_Z \left( \frac{V}{V_0} \right)^2 + F_I \left( \frac{V}{V_0} \right) + F_P
\]  
(3)

\[
\frac{Q(V)}{Q_0} = F'_Z \left( \frac{V}{V_0} \right)^2 + F'_I \left( \frac{V}{V_0} \right) + F'_P
\]  
(4)

Where subscripts \( Z, I \) and \( P \) in \( F \) and \( F' \) constants, stand for constant-impedance, constant-current and constant-power contributions, respectively. Note that there are only two independent parameters in (3) and (4), subjected to the following constraints:

\[
\begin{align*}
F_Z + F_I + F_P &= 1 \\
F'_Z + F'_I + F'_P &= 1
\end{align*}
\]  
(5)
2.1.1 ZI Voltage Dependent Load Model

The model we use in this study is a fitted ZI model, as follows:

\[
P(V) = C_Z \left( \frac{V}{V_0} \right)^2 + C_I \left( \frac{V}{V_0} \right) \tag{6}
\]

\[
Q(V) = C'_Z \left( \frac{V}{V_0} \right)^2 + C'_I \left( \frac{V}{V_0} \right) \tag{7}
\]

In this formulation, constants \( C \) and \( C' \) are calculated by a curve fitting procedure. Note that there is only one independent parameter in (6) and (7), subjected to the following constraints:

\[
\begin{align*}
C_Z + C_I &= 1 \\
C'_Z + C'_I &= 1 
\end{align*} \tag{8}
\]

These constraints are actually being included in the equations already. Because when the feeding voltage is equal to the nominal voltage \( (V = V_0) \), the power consumed by the load is correspondingly equal to the nominal power \( (P = P_0 \text{ and } Q = Q_0) \). So we will have the following which are the same as the constraints given in (8):

\[
P_0 \left\{ \begin{array}{c} 
C_Z \left( \frac{V_0}{V_0} \right)^2 + C_I \left( \frac{V_0}{V_0} \right) 
\end{array} \right. \tag{9}
\]

\[
Q_0 \left\{ \begin{array}{c} 
C'_Z \left( \frac{V_0}{V_0} \right)^2 + C'_I \left( \frac{V_0}{V_0} \right) 
\end{array} \right. \tag{10}
\]

In the exponential model, the exponents \( \alpha \) and \( \beta \) given in (1) and (2) shall be calculated with a fitting procedure. While in the polynomial model, the polynomial coefficients \( F \) and \( F' \) given in (3) and (4) shall be determined. In the proposed fitted polynomial model, the
coefficients $C$ and $C'$ given in (6) and (7) are of the same type as in (3) and (4), except that the zero-order term in the voltage has been eliminated; this removes the constant P-Q term in the analysis.

The proposed model approximates the load voltage dependency using an impedance $Z$, which represents the quadratic voltage dependent term, and a current source $I$, which represents the linear voltage dependent term. Coefficients $C_Z, C_I, C'_Z, C'_I$ can be found through a fitting procedure, which can be formulated in terms of a simple convex quadratic optimization problem. The fitting objective would be to minimize the difference between the fitted approximation and the measured data for a finite number of points within a range of operating voltages. The optimization (minimization) problem can be formulated as following [13]:

$$f (C_Z, C_I) = \sum_{i=1}^{n} \left( C_Z V_i^2 + C_I V_i - P_i \right)^2$$  \hspace{1cm} (11)

$$f' (C'_Z, C'_I) = \sum_{i=1}^{n} \left( C'_Z V_i^2 + C'_I V_i - Q_i \right)^2$$  \hspace{1cm} (12)

With the following constraints, correspondingly:

$$C_Z + C_I = 1$$  \hspace{1cm} (13)

$$C'_Z + C'_I = 1$$  \hspace{1cm} (14)

Where $V_i$ is a voltage value sample within the operating voltage range, and $P_i$ and $Q_i$ are the corresponding measured active and reactive power consumed by the load, at the $V_i$ voltage. Given the constraints in (13) and (14), the number of variables in (11) and (12) reduces
to only one variable each; therefore, the optimization problem becomes a simple linear equation, by taking the derivative of the $f$ functions.

By replacing $C_i$ with $C_z$ based on (13) constraint, and using Kuhn–Tucker conditions for optimality, $C_z$ can be obtained from the following:

$$
C_z = \sum_{i=1}^{n} \frac{(V_i^2 - V_i)(P_i - V_i)}{\sum_{i=1}^{n} (V_i^2 - V_i)^2}
$$

(15)

$V_i$, $P_i$ and $Q_i$ values can be obtained directly from load measurements or from other load modelling formula given in (1) - (4).

Negative values may be obtained for the coefficients in ZIP and ZI load models, depending on the load type. These negative values do not affect the mathematical simulations; however they may not have any physical equivalents [13].

Figure 1 shows the exponential load model, ZIP load model, and ZI load model (also known as the Linear Power Flow (LPF) model), as well as the experimental measurements for a three-phase induction motor [15]. The coefficient values for the three models are also shown. Figure 1 depicts the accuracy of these models compared with the actual measured values, as well as the negligible differences between the individual models. ZIP model is slightly more accurate than the other two models.
Figure 1  The LPF, exponential, and ZIP load model, as well as experimental measured values for a three-phase induction motor.
2.1.2 Load Equivalents

In order to obtain the load equivalent model in a distribution system, we can write the power formula at each node of the system as below:

\[ \bar{I}^* = \frac{\bar{S}}{\bar{V}} \]  \hspace{1cm} (16)

Where \( \bar{I} \), \( \bar{S} \) and \( \bar{V} \) are complex values. Considering the voltage dependency of P and Q as discussed in Section 2.1.1, we can write:

\[ \bar{I} = \frac{P(V) - jQ(V)}{V_{re} - jV_{im}} \]  \hspace{1cm} (17)

Where \( V_{re} \) and \( V_{im} \) are the real and imaginary parts of the voltage at the node, respectively.

By using the equations (6) and (7) for P and Q, the above formula can be written as the following:

\[ I_{re} = \frac{P_0 C_Z V_{re} + Q_0 C'_Z V_{im}}{V_0^2} + \frac{P_0 C_I V_{re} + Q_0 C'_I V_{im}}{V_0 V} \]  \hspace{1cm} (18)

\[ I_{im} = \frac{P_0 C_Z V_{im} - Q_0 C'_Z V_{re}}{V_0^2} + \frac{P_0 C_I V_{im} - Q_0 C'_I V_{re}}{V_0 V} \]  \hspace{1cm} (19)

Where \( I_{re} \) and \( I_{im} \) are the real and imaginary parts of the current at the node, respectively.

In distribution systems, considering the voltage angle of the feeding substation as reference (zero value), the imaginary component of the voltage is often smaller than its real component by several orders of magnitude [13]. Therefore we can assume:
\[ \frac{V_{re}}{V} \approx 1 \quad (20) \]
\[ \frac{V_{im}}{V} \approx 0 \quad (21) \]

Which we can use to simplify \( I_{re} \) and \( I_{im} \) given in (18) and (19) as follows:

\[ I_{re} = \frac{P_0 C \cdot V}{V_0^2} V_{re} + \frac{Q_0 C' \cdot V}{V_0^2} V_{im} + \frac{P_0 C \cdot I}{V_0} \quad (22) \]
\[ I_{im} = \frac{P_0 C \cdot V}{V_0^2} V_{im} - \frac{Q_0 C' \cdot V}{V_0^2} V_{re} - \frac{Q_0 C' \cdot I}{V_0} \quad (23) \]

The current drawn by a typical load with the given admittance of \( \bar{Y} \), including the conductance of \( G \) and susceptance of \( B \), can be calculated through the following formula:

\[ \bar{I} = \bar{Y} \bar{V} = (G + jB)(V_{re} + j V_{im}) = (GV_{re} - BV_{im}) + j(BV_{re} + GV_{im}) \quad (24) \]

Therefore, the real and imaginary components of the load current will be:

\[ I_{re} = GV_{re} - BV_{im} \quad (25) \]
\[ I_{im} = BV_{re} + GV_{im} \quad (26) \]

Comparing (22) and (23), with (25) and (26), we can obtain the equivalent parameters of ZI voltage dependent load model as follows:
\[ G = \frac{P_0 C_z}{V_0^2} \]  

\[ B = -\frac{Q_0 C_z'}{V_0^2} \]

Where \( G \) is the equivalent load conductance, and \( B \) is the equivalent load susceptance.

The constant terms in (22) and (23) can also be modelled as real and imaginary constant current sources; \( I_p \) and \( I_q \), correspondingly:

\[ I_p = \frac{P_0 C_I}{V_0} \]  

\[ I_q = -\frac{Q_0 C_I'}{V_0} \]

Figure 2 represents the equivalent components for voltage dependent loads at two nodes of a distribution system.

![Figure 2](image)

Figure 2  Part of a distribution system using voltage dependent load model.
Using the conventional classification of loads into residential, commercial and industrial, as recommended by IEEE [16], we can obtain accurate estimates of the coefficients for voltage-dependent load models, without the need for direct measurements. However, in smart distribution systems, load measurements can be taken at the load location and even at different time spans, resulting in highly accurate models for the loads [17]. Furthermore, for systems without smart devices, the main contributors to the load composition along a feeder, and their voltage dependent behavior, are normally available.

Specific types of loads which are constant power can also be included in the model. Given the fact that the amount of constant power loads in distribution systems is very small compared to voltage-dependent loads, such loads can be modelled by being merged with other voltage dependent loads and calculating the load constant parameters for the combinational load.

Typical values for ZIP and ZI load model coefficients for different load compositions are given in Table 1 [13].

| Table 1  Voltage-Dependent Load Model Coefficients for Different Load Compositions |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                | $F_Z$ | $F_I$ | $F_P$ | $F'_Z$ | $F'_I$ | $F'_P$ | $C_Z$ | $C_I$ | $C'_Z$ | $C'_I$ |
| **Industrial** | -0.07 | 0.24  | 0.83  | 1.00   | 0.00   | 0.00   | -0.90 | 1.90  | 1.00   | 0.00   |
| **Commercial** | 0.16  | 0.80  | 0.04  | 3.26   | -3.10  | 0.84   | 0.12  | 0.88  | 2.42   | -1.42  |
| **Residential**| 0.24  | 0.62  | 0.13  | 2.44   | -1.94  | 0.50   | -0.10 | 1.1   | 1.93   | -0.93  |
2.1.3 DG Model

There are 3 major types of DG control; namely P-V control, current control, and P-Q control. In order to model DG in distribution systems, buses connected to DG with either a current controller or a P-Q controller, can be considered as P-Q buses, and modelled with the proposed voltage dependent model. Buses connected to DG through a P-V controller, can be modelled as P-V bus. However, the IEEE P1547 Standard [18] specifies that DG units should not actively regulate distribution system voltages. An attempt by a DG to regulate the distribution system voltage can conflict with existing voltage regulation schemes applied by the utility to regulate the same, or nearby, voltages [19]. Thus, DG units with P-V controllers are not recommended, and their use is extremely limited.

In the most common contractual obligations of DGs in North America [20], as well as most of the existing literature for distribution systems with distributed generation, there are no reactive power requirements for DG.

Modelling buses with DG as P-Q buses with controlled reactive power injection is also valid for electronically-coupled DG units where reactive power contribution is independent from the interface bus voltage. Most of the dominant types of DG in the market are electronically coupled [21], [22]. The reduced-order model for such DG can be used along with the proposed voltage dependent model. Also depending on the type of the DG interface with the distribution system, the interface model impedance can be easily implemented in series with the DG.
2.2 Distribution System Model

In Sections 2.1.1 and 2.1.2 the distribution load model used in this study was discussed. In this Section we introduce the modelling of the rest of the distribution system as seen by a particular load bus using the Compensation Technique [23] to find the system reduced equivalents.

We can obtain multi-node and multi-branch Thévenin equivalents of a linear network by turning off all of the network’s internal sources and applying an external unit current source at the terminal, where we want to find the equivalent. Under these conditions, the corresponding terminal voltage will be equal to the equivalent Thévenin impedance as seen from that terminal. This is explained next.

Suppose we connect two independent systems \((N1)\) and \((N2)\) through a switch \((S1)\) as shown in figure 3, a current \(I_a\) will flow from system \(N1\) to system \(N2\). As an example, \(N1\) can be a distribution system and \(N2\) can be an electrical load or a DG.

![Figure 3](image_url)

**Figure 3 Independent Electrical Systems Connection Model.**
As seen by the system $N_1$, connection of $N_2$ is the same as connecting a current source $I_a$ to $N_1$, as shown in figure 4. Where, $I_a$ is determined by $N_2$.

Figure 4  Equivalent Current Source instead of System N2, as seen from N1.

Solving the $N_1$ system, there would be two independent terms in the voltage and current equations of each branch or node; one resulting from the internal sources of $N_1$, and the other resulting from the external current source $I_a$.

By source superposition, this is equivalent to solving the system for two different scenarios, as shown in figure 5. In the system on the left, the internal sources are ON and the external current source is OFF. In the system on the right, the internal sources are OFF and the external current source is ON.
Figure 5 Source Superposition for System N1. (A current source is considered OFF if Open-circuited, and a voltage source is considered OFF if Short-circuited).

Meanwhile, from the N2 point of view, system N1 can be represented just by its equivalent Thévenin, without making any difference from outside the system. So, if a current source with unity magnitude is connected to system N1, \( I_a = 1 \) in figure 5, the voltage measured at the N1 terminals will be equal to the system equivalent multi-node Thévenin impedance as seen from those terminals. In a Multi-node system as shown in figure 6 - where 1 to n represent the system nodes - similarly if we turn the internal sources off and connect the current source \( i_n = 1 \) to node n, the voltage at the node will be equal to the equivalent Thévenin impedance seen at the node \( V_n = Z_{th} \).
To prove this, we can write the power equations for the multi-node system shown in figure 6 as follows; in which $[Z_{th}]$ is the system Thévenin equivalent impedance matrix:

$$[V] = [Z_{th}] . [I]$$

(31)

By injecting a unit external current source to node $k$, while leaving the other node terminals open-circuited and turning off all the system internal sources, (31) can be expanded as follows; where $Z_{kk}$ corresponds to the system total equivalent impedance as seen at node $k$:

$$\begin{cases} 
I_j = 0 & j \neq k \\
I_j = 1 & j = k 
\end{cases}$$

(32)

$$\begin{bmatrix} 
V_1 \\
\vdots \\
V_n 
\end{bmatrix} = 
\begin{bmatrix} 
Z_{11} & \cdots & Z_{1n} \\
\vdots & \ddots & \vdots \\
Z_{n1} & \cdots & Z_{nn} 
\end{bmatrix} 
\begin{bmatrix} 
0 \\
\vdots \\
0 
\end{bmatrix} . 
\begin{bmatrix} 
I_k = 1 \\
\vdots \\
0 
\end{bmatrix}$$

(33)

$$V_k = Z_{kk} \cdot I_k = Z_{kk}$$

(34)
Using the proposed linear voltage dependent load model in Section 2.1.2 and figure 2, we can calculate the nodal admittance matrix \( \mathbf{Y} \) for any distribution system, as a multi-node system. Correspondingly, we can calculate the \( V_k \) value solving the following matrix equations, in which \( Y_{kj} \) is the mutual admittance at node \( k \) because of node \( j \):

\[
\mathbf{[Y]} \cdot \mathbf{[V]} = \mathbf{[I]} \quad (35)
\]

\[
\begin{bmatrix}
    Y_{11} & \cdots & Y_{1n} \\
    \vdots & \ddots & \vdots \\
    Y_{n1} & \cdots & Y_{nn}
\end{bmatrix}
\begin{bmatrix}
    V_1 \\
    \vdots \\
    V_n
\end{bmatrix}
= \begin{bmatrix}
    I_1 \\
    \vdots \\
    I_n
\end{bmatrix} \quad (36)
\]

\[
\begin{bmatrix}
    Y_{11} & \cdots & Y_{1n} \\
    \vdots & \ddots & \vdots \\
    Y_{n1} & \cdots & Y_{nn}
\end{bmatrix}
\begin{bmatrix}
    V_1 \\
    \vdots \\
    V_n
\end{bmatrix}
= \begin{bmatrix}
    0 \\
    \vdots \\
    0
\end{bmatrix} \quad I_k = 1 \quad (37)
\]

\[
\begin{bmatrix}
    \vdots \\
    V_k \\
    \vdots \\
\end{bmatrix}
= \begin{bmatrix}
    Y_{11} & \cdots & Y_{1n} \\
    \vdots & \ddots & \vdots \\
    Y_{n1} & \cdots & Y_{nn}
\end{bmatrix}^{-1}
\begin{bmatrix}
    0 \\
    \vdots \\
    0
\end{bmatrix} \quad I_k = 1 \quad (38)
\]

The calculated \( V_k \) is equal to the equivalent Thévenin impedance seen at node \( k \), as per (34), which we represent with \( Z_{th,k} \) from now on:

\[
Z_{th,k} = V_k \quad (39)
\]

The calculated equivalent Thévenin impedance (\( Z_{th,k} \)) includes the equivalent impedance of the load connected to node \( k \) (\( Z_{load,k} \)), as well as the equivalent impedance of the distribution
system connected to node \( k \) (\( Z_{\text{sys},k} \)). \( Z_{\text{sys},k} \) also includes the equivalent impedances of all other loads connected to the rest of the system nodes, as well.

\[
Z_{\text{th},k} = Z_{\text{load},k} \parallel Z_{\text{sys},k}
\]  

(40)

In order to find the actual equivalent impedance of the whole system seen by the load at node \( k \) (\( Z_{\text{sys},k} \)), we have to exclude the equivalent load impedance (\( Z_{\text{load},k} \)) from the calculated \( Z_{\text{th},k} \). Accordingly, \( Z_{\text{sys},k} \) is obtained from following:

\[
Z_{\text{sys},k} = \frac{Z_{\text{load},k} \cdot Z_{\text{th},k}}{Z_{\text{load},k} - Z_{\text{th},k}}
\]  

(41)

In which \( Z_{\text{th},k} \) is given in (39), and \( Z_{\text{load},k} \) is calculated by the voltage dependent load model given in (42) as per figure 2, and \( G_k \) and \( B_k \) are calculated from (27) and (28), respectively:

\[
Z_{\text{load},k} = \frac{1}{G_k + j B_k}
\]  

(42)

By calculating \( Z_{\text{sys},k} \) at each node separately, we can thus form the Thévenin equivalent impedance matrix of the distribution system “seen by the load” \( [Z_{\text{sys},k}] \). Any type of system topology, including looped distribution systems, can be modelled using this method.
In order to perform the equivalent Thévenin impedance matrix calculations, the voltage-dependent current sources given in (29) and (30) as shown in figure 2, should be open-circuited. The internal voltage sources in the distribution system should be short circuited as well, including the main feeding primary voltage source at the feeding substation - usually numbered as node 1. For this purpose, we assume a very small impedance connected to the feeding terminal, since we cannot lose this strategic node in our matrix calculations. For the analysis of the system introduced in Section 4.1 we have assumed a conductance equal to \( G = 10^{10} \) connected to the feeding substation node.

The system Thévenin equivalent voltage \( (E_{th,k}) \) at each node can also be easily calculated by open-circuiting the load at the node and calculating the node open-circuit voltage.

We can now model the distribution system at each node as a two-bus Thévenin equivalent circuit as shown in figure 7, where the distribution system equivalent impedance \( (Z_{sys_{th,k}}) \) and the load equivalent impedance \( (Z_{load_{th,k}}) \) are calculated from (41) and (42), and \( I_{p,k} \) and \( I_{q,k} \) are obtained from (29) and (30), respectively.

Note that the main generating unit in the system is always modelled as a voltage source. In order to avoid a model consisted only of current sources - where there is no damping in the system - in the case of microgrids or autonomous energy systems, the major DG in the system shall be modelled as a voltage source.
Figure 7  Reduced Thévenin Equivalent of the Distribution System at Node $k$. 
Chapter 3:
Voltage Stability Analysis Tools

Voltage stability can be analyzed using static or dynamic approaches. The methods and the tools we have used in this research to conduct static and dynamic analyses are discussed in the following.

3.1 Static Voltage Stability Analysis

Having a simple equivalent system as the distribution system model as shown in figure 7 in Section 2.2, we can graph the P-V curve at each node of the distribution system.

At any given node of the system, it is ideal to operate as far as possible from the P-V collapse point, which specifies the “Theoretical Maximum Loadability” at that node. Therefore, in order to have stronger voltage/load stability and a superior voltage profile, a higher load impedance is required compared to the equivalent distribution system impedance seen at the node.

In practice, the “Practical Maximum Loadability” at each node in a distribution system is smaller than its theoretical value. Aside from the required safety margins from the P-V curve collapse point, contingency studies under the N-1 contingency criterion are one of the restrictive factors significantly limiting the loadability at each node of the distribution system. Based on the N-1 contingency criterion, distribution systems are required to maintain steady voltages in all buses of the system, under any - and including the worst case - contingency scenario. Contingencies can shift the original P-V curve collapse point substantially, and therefore
decrease the practical maximum loadability at the node. The proposed method in this research can widely be used in contingency studies as well, by simply implementing the changes in system configuration, based on the contingency scenario, and calculating a new system equivalent model.

Reduced system voltage at the feeding bus of the distribution system is another factor that can significantly reduce the practical maximum loadability, by shifting the P-V curve and its collapse point. The operating voltage magnitude of the system feeding bus can be lower than 1 \( pu \); at times it can even be intentionally decreased, in order to let the utility companies to meet peak demand requirements, without load shedding.

Based on ANSI C84.1 standard, in distribution systems, the optimal utilization voltage range (Range A) is within the [90%-105%] of the nominal voltage, and the acceptable utilization voltage range (Range B) is within the [86.7%-105.8%] of the nominal voltage. Since we are limited to operating within the permissible ranges, the actual voltage operating points for many load types, do not usually fall close to the P-V curve nose point. However, in the case of using extra load compensation techniques, which can lead to significant changes in the power factor, the P-V voltage collapse point can be shifted inside (or very close to) the distribution voltage limit range. In such case, voltage stability limits also play a major role in determining the maximum loadability. The above is exemplified in Section 4.2.

### 3.1.1 Voltage Stability Index

In this research, we introduce a Voltage Stability Index \( (L) \) at each node, as below:

\[
L_k = \frac{\left| Z_{\text{load},k} - Z_{\text{sys} \_ {th},k} \right|}{\left| Z_{\text{sys} \_ {th},k} \right|}
\]  

(43)
This index contains critical data regarding the load characteristics, distribution system configuration and values, loads connected to other buses of the system, as well as all DG units connected to the distribution system. $L$ indicates how far the operating point is from the maximum theoretical loadability limit.

As shown in Sections 4.1 and 4.2, this index can be used as a practical and simple tool to rank system buses and find the strongest and weakest nodes of the system from a voltage stability and voltage profile point of view. It can also identify the best bus candidates to install new DG in the system in order to improve voltage stability of the buses with lowest voltage stability index value, and elevate undervoltages, without causing overvoltage in any of the system buses as discussed in Section 4.3.

The minimum value for $L$ at each node, in order to maintain voltage stability, shall be determined by contingency studies, while considering the compensation technique impacts on the load characteristics, as well as the distribution system voltage limits. Thermal limits and distribution system permissible operating voltage range should also be considered in finding the maximum loadability at any node in the system.

### 3.2 Dynamic Voltage Stability Analysis

The system dynamic voltage behavior is analyzed using the ZI voltage dependent load model introduced in Section 2.1.1 - given the importance of the voltage dependency of load's power consumption in dynamic voltage stability analysis. Using the proposed load models, one distribution system can be modelled by its equivalent Thévenin components, using the Compensation Technique described in Section 2.2. Thus, Shifted Frequency Analysis can be used as a tool to study the transient voltage behavior of the distribution system model with a
focus on the systems with distributed generation; where the system is more likely to go through transient instabilities because of the intermittent nature of DG and its small inertia compared to main power generation stations.

Fundamental conceptual derivation of the Shifted Frequency Analysis is discussed in the Section 3.2.1, as a practical tool for study of system transient frequencies within an envelope around the system fundamental frequency. By filtering the fundamental steady state term, the size of time step in SFA can be a lot larger than the size needed for EMTP studies, which is a benefit for distribution systems with big number of nodes.

### 3.2.1 Shifted Frequency Analysis

AC power systems in steady state operate at a fundamental frequency - which is 60 Hz in North America. During dynamic swings, however, the frequency of the system can temporarily shift to higher or lower values within a band around the 60 Hz fundamental. The fundamental frequency can be considered as a carrier frequency in communication systems, which carries on the sidebands of information on the dynamic behavior (even though in the power system case, the fundamental frequency itself can change in magnitude and phase) [10].

The frequency spectrum of a real signal has to be always symmetrical with respect to the vertical zero-frequency axis. This means that a real signal exhibits Hermitian symmetry between the negative and positive frequency components [8]. Meanwhile, the negative frequency components do not carry any additional information, being absolutely symmetrical to the positive frequency term. In other words, the system positive frequencies are enough to thoroughly define the system data; thus in regular computational analysis the negative frequencies are usually not considered.
As indicated in [24] a signal is analytic with a real dc component, if and only if its imaginary part is the Hilbert transform of its real part. Having a real signal \( x(t) \), its associated analytic signal is defined as:

\[
\begin{align*}
  x_{\text{analytic}}(t) &= x(t) + jH[x(t)] \\
  H[x(t)] &= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau
\end{align*}
\]  

(44)  

(45)

A typical sinusoidal signal in power systems can be represented as:

\[
  x(t) = A \cos( w_s t + \theta)
\]  

(46)

By applying The Fourier Transform \( \mathcal{F} \) to (46), the frequency spectrum of the real signal is obtained as follows:

\[
  \mathcal{F}[x(t)] = \mathcal{F}[A \cos( w_s t + \theta)] = \frac{1}{2} \left( A e^{j\theta} \delta(w-w_s) + A e^{-j\theta} \delta(w+w_s) \right)
\]  

(47)

Equation (47) also shows that a real signal consists of symmetrical frequencies \( + w_s \) and \( - w_s \) for the given \( x(t) \).

In order to obtain the analytic signal as per (44), for the \( x(t) \) given in (46), we can write:

\[
  \begin{align*}
    x_{\text{analytic}}(t) &= x(t) + jH[x(t)] = A \cos( w_s t + \theta) + j \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{A \cos( w_s \tau + \theta)}{t-\tau} d\tau \\
    x_{\text{analytic}}(t) &= A \cos( w_s t + \theta) + j A \sin( w_s t + \theta) = A e^{j\theta} e^{jw_s t}
  \end{align*}
\]  

(48)  

(49)

By applying The Fourier Transform to (49), the frequency spectrum of the analytic signal
is obtained as follows:

\[
F[x_{\text{analytic}}(t)] = F[A \cos(w_s t + \theta) + j A \sin(w_s t + \theta)] = \\
\frac{1}{2} [A e^{j\theta} \delta(w - w_s) + A e^{-j\theta} \delta(w + w_s)] + j \left[\frac{1}{2} j (A e^{-j\theta} \delta(w + w_s) - A e^{j\theta} \delta(w - w_s)) \right] \\
= A e^{j\theta} \delta(w - w_s)
\]

Equation (50) shows that the analytic signal can be expressed in terms of the positive frequencies (+ w_s for the given x(t)).

In order to transfer the analytic signal in (49) from the real time domain to the SFA domain, we define a Rotation Transform R[.]:

\[
R[x(t)] = x(t) e^{-jw_s t}
\]

\[R[.]\] shifts the system frequency spectrum by - w_s as indicated below:

\[x_{\text{SFA}}(t) = R[x_{\text{analytic}}(t)] = x_{\text{analytic}}(t) e^{-jw_s t} = A e^{j\theta} e^{jw_s t} e^{-jw_s t} = A e^{j\theta}
\]

By applying the Fourier Transform to (52), the frequency spectrum of the signal in the SFA domain (SFA signal) is obtained as:

\[
F[x_{\text{SFA}}(t)] = F[A e^{j\theta}] = A e^{j\theta} \delta(0) = A e^{j\theta}
\]

Equation (53) shows that the SFA signal does not include the fundamental frequency and its base frequency is 0. In case of any dynamic transients in the system, this signal will be the base carrier for extra frequencies in the system, deviated from the fundamental frequency.

Figure 8 shows the frequency spectrum of a real signal as per (47). It includes the real and imaginary parts of the signal - in other words both symmetrical fundamental frequencies of the system.
Figure 8  Frequency Spectrum of a Real Signal in Phase Domain.

Figure 9 shows the frequency spectrum of the analytic signal as per (50). The analytic signal is obtained by adding the Hilbert Transferred form of the real signal to its imaginary part. This signal only includes the positive (real) fundamental frequencies; while still containing all the system data.

Figure 9  Frequency Spectrum of the Analytic Signal in Phase Domain.

Figure 10 shows the frequency spectrum of the SFA signal as per (53). The corresponding SFA signal is obtained by a rotational transformation of the analytic signal by $-\omega_s$ in modal domain, which can also be called Dynamic Phasor. This signal is transferred in
the frequency domain so that its fundamental frequency (e.g. 60 Hz) is shifted to 0 Hz. Therefore the frequency band around the fundamental frequency of the system \([60 - f_\Delta, 60 + f_\Delta]\) is exactly transferred to around zero \([- f_\Delta, + f_\Delta]\). Working within a lot smaller range of frequencies will eliminate the need for very small time steps.

![Frequency Spectrum of the SFA Signal in Modal Domain](image)

**Figure 10** Frequency Spectrum of the SFA Signal in Modal Domain.

### 3.2.2 SFA Electrical Component Models

To model the network in the SFA domain, the system components, such as resistance, inductance and capacitance, have to be mapped from the unshifted domain to the shifted (SFA) domain. This can be performed by applying the trapezoidal discretization rule. Figures 11 to 13 depict the resistance, inductance and capacitance models, respectively, in both physical phase domain and SFA modal domain, using trapezoidal discretization mapping [10].

![Resistance Model](image1)

**Figure 11** Resistance model in the phase domain and SFA modal domain.
For a discrete time solution, based on the Nyquist sampling theorem in signal processing, in order to avoid aliasing, the sampling frequency \(f_s\) should be at least twice the highest frequency contained in the signal \(f_{Nyq}\). Using trapezoidal discretization or backward Euler, in order to limit the error to less than 3% in magnitude and 0% in phase angle, the maximum frequency to be used in the calculations should be less than one fifth the Nyquist frequency, to avoid distortion [10].

\[
f_s \geq 2f_{Nyq}
\]
\[
\frac{1}{\Delta t} \geq 2 \times 5 \Delta f
\]  \hspace{1cm} (55)

For example if \(\Delta f = 2 \text{Hz}\), the bandwidth required in the SFA domain will be limited to \([-2 \text{Hz} , +2 \text{Hz}]\), resulting in a time step \((\Delta t)\) as big as 50\(\text{ms}\):

\[
\Delta t \leq \frac{1}{2 \times 5 \times 2} = 50 \text{ ms}
\]  \hspace{1cm} (56)

This larger time step in the SFA domain is possible because we are now solving for the envelope of the 60 \(\text{Hz}\) signal, not for the detailed 60 \(\text{Hz}\) waveform in the time domain. The solution in the unshifted original domain would require the solution for waveforms in the range of \([58 \text{Hz} , 62 \text{Hz}]\), which means the time step cannot go beyond 1.6 \(\text{ms}\) as per following:

\[
\Delta t \leq \frac{1}{2 \times 5 \times 62} = 1.6 \text{ ms}
\]  \hspace{1cm} (57)

The SFA solution can be more than 30 times faster than regular transient solutions, without sacrificing accuracy. As shown in [10], SFA can actually result in higher accuracy than the EMTP results.

Another aspect of the SFA method is that a finite \(\Delta t\) provides a built-in low-pass Filter that prevents frequencies higher than the Nyquist frequency from being generated by the solution process. Therefore, there is a natural filter for frequencies larger than those in the range of interest, for example beyond 62 \(\text{Hz}\) in the above example, which allows the dynamics to be concentrated around this limit for controller actions [10].
Chapter 4:

Static Voltage Stability Analysis Results

4.1 The Test System

A 90-bus distribution system, as shown in Figure 14, is studied in this chapter as the test system. Corresponding system data is provided in Appendix A. The base power is 100 MW and the error tolerance is \(1 \times 10^{-4}\). Bus 1 is the feeding bus and has a voltage magnitude of \(1.0 \pm 0.0^\circ pu\)

Figure 14 The 90 node distribution system as the study case.
The system loads are modelled using the voltage dependent load model introduced in Section 2.1.1, assuming a typical residential load composition. The distribution system is modelled separately at each node, using the Compensation technique described in Section 2.2. Then the P-V curves at each node of the distribution system are graphed and the value of voltage stability index $L$ defined in Section 3.1.1 is determined at each node of the system.

Figure 15 illustrates the Voltage Stability Index ($L$) value of different nodes of the distribution system. Table 2 also shows the sorted system nodes based on their $L$ values, where smaller $L$ values indicate weaker buses from a voltage stability point of view. At the buses with no $L$ value, there is no load connected to the bus.

![Figure 15 Voltage Stability Index ($L$) values of different nodes of the distribution system.](image)
Table 2  Distribution System Nodes Sorted based on the Voltage Stability Index ($L$) Value

<table>
<thead>
<tr>
<th>Bus #</th>
<th>Voltage Stability Index ($L$) Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>42</td>
<td>12</td>
</tr>
<tr>
<td>18</td>
<td>41</td>
</tr>
<tr>
<td>28</td>
<td>42</td>
</tr>
<tr>
<td>15</td>
<td>58</td>
</tr>
<tr>
<td>23</td>
<td>68</td>
</tr>
<tr>
<td>46</td>
<td>86</td>
</tr>
<tr>
<td>33</td>
<td>92</td>
</tr>
<tr>
<td>41</td>
<td>93</td>
</tr>
<tr>
<td>71</td>
<td>95</td>
</tr>
<tr>
<td>12</td>
<td>96</td>
</tr>
<tr>
<td>45</td>
<td>105</td>
</tr>
<tr>
<td>52</td>
<td>128</td>
</tr>
<tr>
<td>21</td>
<td>134</td>
</tr>
<tr>
<td>48</td>
<td>136</td>
</tr>
<tr>
<td>39</td>
<td>137</td>
</tr>
<tr>
<td>58</td>
<td>168</td>
</tr>
<tr>
<td>5</td>
<td>176</td>
</tr>
<tr>
<td>34</td>
<td>182</td>
</tr>
<tr>
<td>50</td>
<td>191</td>
</tr>
<tr>
<td>87</td>
<td>196</td>
</tr>
<tr>
<td>54</td>
<td>199</td>
</tr>
<tr>
<td>20</td>
<td>202</td>
</tr>
<tr>
<td>37</td>
<td>207</td>
</tr>
<tr>
<td>25</td>
<td>209</td>
</tr>
<tr>
<td>55</td>
<td>214</td>
</tr>
<tr>
<td>89</td>
<td>219</td>
</tr>
<tr>
<td>90</td>
<td>226</td>
</tr>
<tr>
<td>77</td>
<td>242</td>
</tr>
<tr>
<td>85</td>
<td>247</td>
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<tr>
<td>62</td>
<td>268</td>
</tr>
<tr>
<td>13</td>
<td>334</td>
</tr>
<tr>
<td>63</td>
<td>422</td>
</tr>
<tr>
<td>9</td>
<td>440</td>
</tr>
<tr>
<td>27</td>
<td>445</td>
</tr>
<tr>
<td>65</td>
<td>509</td>
</tr>
<tr>
<td>67</td>
<td>555</td>
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<tr>
<td>7</td>
<td>711</td>
</tr>
<tr>
<td>76</td>
<td>800</td>
</tr>
<tr>
<td>81</td>
<td>997</td>
</tr>
<tr>
<td>72</td>
<td>1004</td>
</tr>
<tr>
<td>84</td>
<td>1066</td>
</tr>
<tr>
<td>31</td>
<td>1101</td>
</tr>
<tr>
<td>80</td>
<td>1231</td>
</tr>
<tr>
<td>74</td>
<td>1422</td>
</tr>
</tbody>
</table>
The above results correspond to the reported distribution system characteristics and load values, and are comparable with results from other voltage stability studies done on the same system, where iterative power flow techniques have mainly been used [6].

4.2 Static P-V Curve Analysis

Figures 16 to 19 show the operational region of the P-V curves for the 4 weakest nodes of the test system; nodes #42, 18, 28 and 15. The operating point at each node is also shown on the curves. The system loses equilibrium and becomes unstable past the curve voltage collapse nose; therefore, the lower half of the P-V curve where the voltage collapses into the unstable region is not shown in the figures, since we are modelling only the practical behavior of the system.

![The P-V curve at Node # 42 (L42=11.655)](image)

**Figure 16** The P-V curve for Node # 42 (L42=12). Operating Point: P=1.7, V=0.9496.
Figure 17  The P-V curve for Node # 18  (L18=41).
Operating Point: $P=1.215$, $V=0.9871$.

Figure 18  The P-V curve for Node # 28  (L28=42).
Operating Point: $P=0.615$, $V=0.9877$. 
As seen in the figures above, as the $L$ index value increases, the operating voltage values increase from 0.9496 $pu$ at Node #42 and come closer to 1 $pu$. Also the maximum loadability of the nodes increases significantly from 6.74 $pu$ to 32.22 $pu$, as we move from figure 16 to figure 19. Furthermore, the slope of the P-V curve (the derivative of voltage with respect to power) around the operating point decreases, similarly; in other words the voltage of the node shows less sensitivity to changes in power, as the $L$ value increases.

Figures 20 and 21 show the P-V curves for the two nodes in the system with the highest $L$ index values; nodes #74 and 80, which are the most stable nodes of the system in terms of voltage stability.
Figure 20  The P-V curve for Node # 80  (L80= 1230.899).
Operating Point: P=0.175, V=0.9996.

Figure 21  The P-V curve for Node # 74 (L74=1422.409).
Operating Point: P=0.065, V=0.9996.
Comparing the first set of the smallest $L$ value curves with the second set of the largest $L$ values, we can see significant differences in the voltage characteristics. The maximum loadability of the buses with the highest $L$ index values (nodes #74, 80) is $75.64 \text{ pu}$ and $72.08 \text{ pu}$ respectively, which is more than 10 times larger than that of the bus with the smallest $L$ value (node #42).

Furthermore, the maximum possible operating voltage at the nodes with the highest $L$ index value is unity, whereas for the smallest $L$ index values; the bus voltage never reaches unity, even at no load. The practical operating voltages at the buses with the lowest $L$ values are also significantly lower than the practical operating voltages at the buses with the highest $L$ values, even nearly reaching undervoltage limits in some cases.

Similarly, the slope of the P-V curve around the operating point and in the acceptable distribution voltage range is steeper for the nodes with smaller $L$ values than the nodes with larger $L$ values. This indicates that the sensitivity of the bus voltages to the load variations is higher in buses with smaller $L$, resulting in lower voltage stability.

Figure 22 shows the P-V curves of the weakest node of the system (node #42), in three different modes: normal operation, operating under extra load compensation, and operating under decreased feeding terminal voltage. These curves depict that in the case of using extra compensation, the node operating voltage - under its nominal load - can increase up to near $1 \text{ pu}$. The P-V curve nose is also shifted further, which means an increase in the theoretical maximum loadability. However, in this case the distribution system operating voltage limit (e.g. -10%) is so close to the P-V curve collapse point, that we cannot determine the practical maximum loadibility based on that. The slope of the P-V curve (derivative of voltage with respect to power) at this point ($V=0.9 \text{ pu}$) is too steep, that in case of any minor load changes, the voltage can easily enter
the unstable region. Considering mandatory safety margins from the PoC, as well as the N-1 contingency criterion, in such case the voltage stability limits are to determine the absolute maximum loadability at the node. Thermal limits are also being considered, and do not limit the loadability by far.

![The P-V curves at Node # 42](image)

**Figure 22** The P-V curves for Node # 42 in Normal Operation mode, Extra Load Compensation mode, and Decreased Feeding Voltage mode.

Figure 22 also shows the P-V curve at node #42 for the case of the distribution feeding bus voltage being smaller than unity. In this case, the loadability decreases because of the shifting of the P-V curve, and the node operating voltage drops to 0.8957 pu, which is lower than the distribution system voltage limits. This clearly portraits the weak voltage behavior of node #42, which has been earlier selected as the weakest node of the system in terms of voltage stability, and correspondingly has the smallest $L$ index value ($L_{42}=12$).
As shown above, by using the proposed linear index we can easily detect the buses with the weakest voltage profile which are operating close to voltage stability margins in real time, while considering any sudden changes in the system configuration or any system failures. It can help a system operator make real time decisions, and also improve the buses voltage profile and stability. One practical method to improve the voltage in distribution systems is the installation of new DG, at the right buses of the system, which we study in the next Section.

4.3 Distributed Generation (DG) Installation Analysis

The penetration of DG in a distribution system can increase or decrease the voltage stability margin, depending on the operation at unity, lead, or lag power factors. Asynchronous distributed generators produce real power, but they usually take reactive power from the system, which tends to offset the voltage rise caused by the real power injection. Alternatively, synchronous distributed generators generate real power and produce or consume reactive power. In the case of injection of reactive power by the DG, the voltage of the bus at the distribution system will increase. Currently, most of the installed DG units are commonly connected to operate at unity power factor to avoid interference with the voltage regulation devices connected to the system [19], [13].

In order to improve the voltage stability in a system, we use voltage sensitivity analysis to select the best bus candidates for the DG installation, conducted by testing the bus voltage sensitivity to the changes in the DG injected power. The best candidates for installing new DG in a distribution system are the buses with higher voltage sensitivity to power \((\Delta V/\Delta P)\), i.e. the buses with sharper P-V slope. These buses correspondingly have less voltage stability, and smaller loadability limits. Other factors such as the probabilistic nature of renewable resources
and other physical restrictions, as well as thermal and short circuit limits, should also be considered in the final stages of placing new DG units.

To find optimal nodes for the installation of new DG in order to maximize its impact on the system’s voltage stability, we need to select the nodes with highest voltage sensitivity. In Section 4.2 we showed that the proposed voltage stability Index \( L \) is a very efficient tool to detect nodes with high voltage sensitivity to power changes, i.e., the nodes with the lowest \( L \) values.

As shown in Section 4.2, the nodes with the lowest \( L \) values also have the lowest operating voltages, which makes them the best node candidates to install new DG to improve the voltage profile by increasing the operating voltage level \( \frac{V_{\text{with } DG}}{V_{\text{without } DG}} \). By selecting such nodes, we can ensure that the system voltage limits are not surpassed and the system will not experience overvoltages. In addition, the nodes with the largest loads, which give them priority for DG placement, already have a greater chance for a lower \( L \) value, based on the definition of the \( L \) index given in (43).

For the test system introduced in Section 4.1, in order to find the best bus candidates for installation of new DG, we start with node #42 which as per Table 2 has the smallest voltage stability index value, thus the highest sensitivity of voltage to power changes. We analyze the impacts of a typical DG of \( P=1.1 \ pu \) and \( Q=0.2 \ pu \) with a P-Q controller, at this node. We also consider node #18 as the second weakest node of the system after node #42.

Figures 23 and 24 illustrate the changes in the P-V curves of nodes #42 and #18, respectively, by installing a new DG at the node. Here, the voltage stability index value \( L \) for node #42 improves from 11.655 to 17.616, and the \( L \) value for node #18 improves from 40.542
to 60.149; this is an improvement of almost 50%. The operating voltage at node #42 also improves from 0.9496 \textit{pu} to 0.9635 \textit{pu}, and at node #18 it improves from 0.9871 \textit{pu} to 0.9911 \textit{pu}, as shown in the figures. In addition, by shifting up the P-V curve, the maximum loadability inside the distribution system permissible operating voltage range increases.

![The P-V curves for Node # 42 without and with DG](image)

Figure 23 The P-V curves for Node # 42 without and with DG. Operating Voltage without DG: $V=0.9496$ vs. with DG: $V=0.9635$. 

51
Figure 24  The P-V curves for Node # 18 without and with DG. Operating Voltage without DG: V=0.9871 vs. with DG: V=0.9911.

Figures 25 and 26 illustrate how the P-V curves of nodes #80 and #74 – which are the most stable nodes of the system and have the highest $L$ values – change by installation of a new DG at the node. As seen in the figures, the changes in the voltage profile and the P-V curves are not significant.

In these cases, the voltage stability index value ($L$) of node #80 worsens from 1231 to 505, and the $L$ value for node #74 lowers to 507 from 1422; this is a regression of almost 65%. In addition, the operating voltages of both nodes #80 and #74 exceed the nominal voltage value to over 1.0010 pu, which is not desirable.
Figure 25  The P-V curves for Node # 80 without and with DG. Operating Voltage without DG: \( V = 0.9995 \) vs. with DG: \( V = 1.0010 \).

Figure 26  The P-V curves for Node # 74 without and with DG. Operating Voltage without DG: \( V = 0.9996 \) vs. with DG: \( V = 1.0015 \).
Comparing figures 23 and 24 with figures 25 and 26, we can see how installation of DG at buses with low voltage stability index ($L$) values improves the P-V curve and thus the maximum loadability and voltage stability at that node. The voltage profile also improves and the bus operating voltage moves closer to unity. On the other hand, installation of a similar DG at buses with high $L$ value does not have a significant impact on the P-V curve and maximum loadability at that node. This scenario can actually reduce the voltage stability index value significantly, and also can cause overvoltage at the respective node and its nearby nodes. The figures above show that the proposed Voltage Stability Index ($L$), is a highly practical tool in detecting the best bus candidates for new DG installation. Note that the capacity of the feeder should also be considered, for example if necessary, the DG can be installed at the closest main feeder to a weak node.

Table 3 shows the best and worst candidate buses in the system to install a new DG, as well as their corresponding voltage stability index ($L$) values without and with installation of a new DG.
Table 3  Changes in the $L$ value of Nodes with Lowest and Highest Voltage Stability, with Installation of a typical P-Q DG.

<table>
<thead>
<tr>
<th>Node #</th>
<th>$L$ value without DG</th>
<th>$L$ value with DG</th>
</tr>
</thead>
<tbody>
<tr>
<td>42</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>18</td>
<td>41</td>
<td>60</td>
</tr>
<tr>
<td>Least Voltage Stable Buses</td>
<td>28</td>
<td>42</td>
</tr>
<tr>
<td>15</td>
<td>58</td>
<td>89</td>
</tr>
<tr>
<td>23</td>
<td>68</td>
<td>142</td>
</tr>
<tr>
<td>72</td>
<td>1004</td>
<td>358</td>
</tr>
<tr>
<td>84</td>
<td>1066</td>
<td>482</td>
</tr>
<tr>
<td>Most Voltage Stable Buses</td>
<td>31</td>
<td>1101</td>
</tr>
<tr>
<td>80</td>
<td>1231</td>
<td>505</td>
</tr>
<tr>
<td>74</td>
<td>1422</td>
<td>507</td>
</tr>
</tbody>
</table>

By installation of a new DG in the distribution system, like any other change in the system configuration, the equivalent impedance of the system seen from other system nodes, thus the value of the voltage stability index $L$ at other nodes, will change. If DG size is smaller than the load connected to the installation node, DG could reduce the equivalent impedance connected to that node, acting as a negative impedance. Therefore the total equivalent Thévenin impedance seen from other nodes of the system ($Z_{sys_{th,k}}$), especially the nodes closer to the
DG, could reduce similarly, meaning that the loadability at other nodes of the system would increase, along with the voltage stability index $L$ value. However, if DG is larger than the load at the installation node, this can change the equivalent impedance connected to that node to a negative value, which may result in increasing the total equivalent Thévenin impedance seen from other nodes of the system ($Z_{\text{sys}_{th,k}}$). In this case, the voltage stability index $L$ value in some of the other system nodes may decrease.

Figure 27 shows the best bus candidates in the present 90-bus test system for installing new DG, as well as the most stable buses in terms of voltage stability and voltage profile.
Figure 27 The best bus candidates to install new DG (shown with a DG sign), and The most voltage stable buses of the system (shown with a checkmark).
Chapter 5:

Dynamic Voltage Stability Results

5.1 Normal Operating Modes (Including DG)

The test system introduced in Section 4.1 is studied in this chapter, similarly. The system loads are modelled using the voltage dependent load model introduced in Sections 2.1.1 and 2.1.2, assuming a typical residential load composition. The distribution system is modelled separately at each node, using the Compensation technique described in Section 2.2. Finally the SFA technique introduced in Section 3.1 is used to study the system dynamics, and the SFA results in shifted domain are compared to EMTP results in the unshifted time domain.

Figures 28-33 illustrate the modal and time domain results for the load voltage and current at node #42 of the test system, using $\Delta t = 100 \ \mu s$ for SFA analysis and $\Delta t = 10 \ \mu s$ for EMTP analysis. For SFA analysis, $\Delta t$ can increase to more than 5 $ms$; while still providing reasonable results without losing any critical data.
Figure 28  SFA and EMTP results in Modal and Time Domain, respectively, for Voltage at Node #42.

Figure 29  Voltage Angle at Node #42 in SFA Modal Domain.
Figure 30  Voltage Magnitude at Node #42 in SFA Modal Domain.

Figure 31  SFA and EMTP results in Modal and Time Domain, respectively, for Load Current at Node #42.
Figure 32  SFA result for Load Current Angle at Node #42.

Figure 33  SFA result for Load Current Magnitude at Node #42.
By comparing the SFA and EMTP results in figures 28 and 31, one can see how SFA results (in red) can explicitly show the transients in the system, compared to EMTP results (in blue). As shown in the figures, the SFA is a very practical tool to analyze the transients of the system in a very clear and explicit manner.

Figures 34-39 illustrate the SFA and EMTP behavior of the system when a typical DG of \( P=1.1 \text{ pu} \) and \( Q=0.2 \text{ pu} \), with a P-Q controller, is connected to node #42. Based on the static voltage stability analysis results given in table 2 in Section 4.1, bus #42 has the weakest voltage characteristics in the system, as well as the highest sensitivity of the bus voltage to power changes; therefore it is one of the best candidates in the system to install new DG.
Figure 34  SFA and EMTP results in Modal and Time Domain, respectively, for Voltage at Node #42, in the presence of a DG.

Figure 35  Voltage Angle at Node #42 in SFA Modal Domain, in the presence of a DG.
Figure 36 Voltage Magnitude at Node #42 in SFA Modal Domain, in the presence of a DG.

Figure 37 SFA and EMTP results in Modal and Time Domain, respectively, for Current at Node #42 in the presence of a DG.
Figure 38  SFA result for Current Angle at Node #42, in the presence of a DG.

Figure 39  SFA result for Current Magnitude at Node #42, in the presence of a DG.
The SFA results shown in the above figures can be used to clearly compare the two sets of figures without and with DG. These results show that DG has improved the steady state operating voltage magnitude from 0.9266 pu to 0.9506 pu. The steady state operating voltage magnitude results given by P-V curves in Sections 4.2 and 4.3 are in compliance with the SFA/EMTP results; however they are slightly different, due to higher complexity and accuracy of SFA and EMTP techniques. Specifically, the ranking of the system buses in terms of steady state voltage profile and voltage stability highly correspond to the results from SFA and EMTP techniques.

In addition to the changes in the voltage magnitude, the steady state value of the current drawn from the distribution network by the load has, expectedly, decreased from 1.671 pu to 0.6854 pu; the difference being provided by the DG.

However, besides the steady state improvement of the voltage magnitude and the load current, DG adds extra transients to both the system voltage and current, clearly shown by the SFA results in the above figures.

The SFA voltage magnitude at Node #42 without DG oscillates between 0.91214 pu and 0.9299 pu, within the first 15ms of powering the system (Fig. 28); while it oscillates within a wider range of 0.9434 pu and 0.9561 pu, connecting a DG to Node #42 (Fig. 34) within the same transient time interval. As long as the system is operating under the normal conditions, these voltage transients are usually not big enough to cause instabilities in the system; however at certain nodes of a distribution system the operating voltage may fluctuate outside of the distribution system operational voltage range during the transients, which can result in tripping of control and protection devices.

In addition, transients in a system, under specific circumstances such as a system fault, a
circuit contingency, loss of other generation in the system, or even incremental changes in system load, may lead to instabilities in the system. Such cases are discussed in Section 5.2.

Figures 40-45 illustrate the modal and time domain results for the load voltage and current at node #74 of the test system, using $\Delta t = 100 \, \mu s$ for SFA analysis. Node #74 is the strongest node in the system in terms of voltage characteristics and has the highest voltage stability index ($L$) value based on the static voltage stability analysis results given in table 2 in Section 4.1.

Comparing the SFA results shown in figures 40-45 with 28-33, we can see the range of oscillations in voltage and current magnitude and angle, at node #74 is significantly smaller than the range of oscillations at node #42. The voltage magnitude at node #74 oscillates between 0.999 $pu$ and 0.9995 $pu$ (compared to 0.91214-0.9299 $pu$ at node #42). Similarly, the voltage angle at node #74 oscillates between -0.00542° and 0.02005°, while the voltage angle at node #42 oscillates within a lot wider range of 0.2430° and 0.6562°. Correspondingly, the oscillations in the current at node #74 are also smaller compared to node #42. These results also demonstrate that the steady state voltage stability index introduced in Section 3.1.1 is a highly practical linear tool to detect the weak buses in any distribution system.
Figure 40  SFA and EMTP results in Modal and Time Domain, respectively, for Voltage at Node #74.

Figure 41  Voltage Angle at Node #74 in SFA Modal Domain.
Figure 42 Voltage Magnitude at Node #74 in SFA Modal Domain.

Figure 43 SFA and EMTP results in Modal and Time Domain, respectively, for Load Current at Node #74.
Figure 44  SFA result for Load Current Angle at Node #74.

Figure 45  SFA result for Load Current Magnitude at Node #74.
Figures 46-51, illustrate the modal and time domain behavior of the system when a DG similar to the one used for node #42, is connected to node #74. The high voltage stability index \((L)\) value of node #74 as per table 2 in Section 4.1, results in the bus to be not a good candidate for installation of new DG for long term.

Comparing the SFA results shown in figures 46-51 with the figures 40-45, we can see that DG has declined the voltage characteristics at node #74. DG has increased the steady state operating voltage magnitude from 0.9993 \(pu\) to 1.002 \(pu\), which is not desirable. These results comply with the steady state operating voltage results given by the P-V curves in Sections 4.2 and 4.3. In more dense distribution systems, installing a DG at a wrong bus can result in excessive increase in the bus voltage, and cause serious overvoltages in the system.

In this case, the DG size is greater than the load installed at the bus and thus it is injecting current to the system; therefore the voltage and current transient behavior at the bus has changed dramatically. The transient frequencies in the system seem to be reduced although the SFA voltage and current still oscillates within a wider range compared to the case where there is no DG installed at the bus.

Normally, the transient voltage oscillations caused by connecting a DG to a distribution system, cannot result in instabilities in the system; however depending on the node location and the distribution system density and configuration, these transient voltages can cause transient under/(over)voltages in the system, outside of the distribution system permissible operating voltage range, which may cause problems with the control and protection devices.

The instabilities DG can cause in a distribution system, under abnormal conditions, are discussed in Section 5.2.
Figure 46  SFA and EMTP results in Modal and Time Domain, respectively, for Voltage at Node #74, in the presence of a DG.

Figure 47  Voltage Angle at Node #74 in SFA Modal Domain, in the presence of a DG.
Figure 48  Voltage Magnitude at Node #74 in SFA Modal Domain, in the presence of a DG.

Figure 49  SFA and EMTP results in Modal and Time Domain, respectively, for Current at Node #74 in the presence of a DG.
Figure 50  SFA result for Current Angle at Node #74, in the presence of a DG.

Figure 51  SFA result for Current Magnitude at Node #74, in the presence of a DG.
5.2 Instability Operating Modes

As long as a distribution system is operating under normal conditions, usually the voltage transients resulted by DG cannot cause instabilities in the system. However under specific circumstances such as system faults, circuit contingencies, loss of other generation or a part of the feeding transmission system, or even incremental changes in system load, the transients caused by connecting DG to a distribution system may lead to instabilities in the system.

Given the intermittent nature of certain types of DG, such as wind and solar, a DG can connect to the system while the system is not working under its 100% functionality. Figures 52-53 illustrate the system voltage in a scenario where a DG, similar to the one used in the studies in Section 5.1, is connected to node #42, while there is a phase to ground short circuit fault at the other end of the distribution system at node #89. Comparing the SFA results in these figures with figures 28-29 (or 34-35), we can see that the voltage oscillates with a very high frequency within the off-the-limit range of [-3, +3] pu constantly, and does not settle towards a steady state. This is also the case for the voltage angle which oscillates between -123.5° and 23.7° with a high frequency and does not converge into a steady solution. In this case the system loses stability, although the static studies cannot detect this transient instability and suggest that the system will stay stable after connecting the DG.
Figure 52 Voltage Magnitude at Node #42 connecting a DG, during a Phase to Ground Short Circuit Fault at Node #89.

Figure 53 Voltage Angle at Node #42 in SFA Modal Domain, connecting a DG during a Phase to Ground Short Circuit Fault at Node #89.
Figures 54-55, illustrate the system voltage in a scenario where a similar DG is connected to node #74 while there is a phase to ground short circuit fault at the other end of the distribution system at node #27.

Although node #74 is the most stable node of the system in terms of voltage characteristics, in the scenario of connecting a DG to this node under an abnormality, the system similarly loses its stability. This can be seen by comparing the high frequency oscillations in the SFA voltage magnitude and angle in figures 54-55, with the SFA results shown in figures 46-47 (or 40-41). The oscillations do not damp, and the voltage does not converge to a steady state.

The difference with the scenario in node #42 is that the out of range SFA voltage oscillations shown by the simulations, are within a band around unity at node #74 (0.2278 – 1.809 pu), compared to node #42 (1.513 – 3.021 pu) which is highly out of range. Also the unstable angle oscillations are within a smaller range in the case of node #74 compared to node #42. However the transient frequencies at node #74 are higher compared to node #42.

Similar to node #42 scenario, steady state studies cannot predict the transient instability in this case, either, and suggest that the system will still keep its stability after connecting the DG to node #74.
Figure 54  Voltage Magnitude at Node #74 connecting a DG, during a Phase to Ground Short Circuit Fault at Node #27.

Figure 55  Voltage Angle at Node #74 in SFA Modal Domain, connecting a DG during a Phase to Ground Short Circuit Fault at Node #27.
Chapter 6:

Conclusion

In this dissertation a novel method for linearization of the modelling of the distribution systems and loads is presented. The proposed model can include DG as well as any type of distribution system topology, and can be used in static and dynamic voltage stability analysis of distribution systems, as well as distributed generation placement studies. A voltage stability index is introduced for evaluating and ranking the static voltage stability characteristics at each node of the system. The method of Shifted Frequency Analysis is introduced as a practical tool to study the dynamic voltage stability in distribution systems.

Dynamics in the voltage and current in a distribution system have been studied. The steady state and transient impacts of DG on the distribution system voltage have been analyzed in the static and dynamic studies, and the scenarios, in which DG can result in instabilities in the distribution system during transients, are discussed. The linear static studies results comply with and are verified by dynamic studies results; obtained by more complex methods including SFA and EMTP.

6.1 Accomplished Work and Applications

6.1.1 Load Modelling

The first step in the studies conducted in this research is introducing a ZI voltage dependent load model which allows for linearity in the distribution system modelling. This model as discussed in Section 2.1.1 consists of two major components that model the voltage
dependent behavior of the load; including a constant impedance term and a constant current term. The values for the parameters used in the model components, are obtained through a fitting procedure, which is formulated in terms of a simple convex quadratic optimization problem. Specific types of loads which are constant power, can also be included in the model.

In addition to the linearity the proposed load model provides, implementing the voltage dependency of the load power consumption in the load model is another principal aspect of this approach.

Load power dependency on voltage magnitude is important in voltage stability studies, particularly in dynamic voltage stability analysis. As we get closer to the distribution load levels, the voltage dependency of the power consumed by the load becomes more tangible and critical; to the extent where, in transient voltage stability, and in the final stages of a slower occurring static voltage instability, the normal constant power load model is not accurate anymore.

### 6.1.2 Distributed Generation Modelling

DG can be modelled using the similar approach proposed for modelling the distribution system loads. Most of the DG types commonly used in the market can be modelled as P-Q type DGs, using a negative load model basically similar to the system loads model.

The reduced-order model of DG interface impedance can be used along with the proposed voltage dependent model.

### 6.1.3 Distribution System Modelling

The proposed voltage dependent model for system loads and components provides the opportunity to convert the distribution system problem to a linear solution. Thus, the system
reduced equivalents can be found in the form of linear electrical components as seen at any single node.

The model is valid for unbalanced distribution systems as well as balanced systems, while for unbalance systems, each phase shall be modelled separately.

One can obtain multi-node and/or multi-branch Thévenin equivalents of a linear network by turning off the internal sources within the network, and applying an external unit current source at the terminal where we want to find the equivalent of the system. Under these conditions, the corresponding terminal voltage will be equal to the equivalent Thévenin impedance as seen from that terminal.

In order to calculate the corresponding voltage at a specific node within a distribution system, we form the nodal admittance matrix of the system using the proposed linear load model in Section 2.1.1, and solve the corresponding matrix equations for the terminal voltage value, when there is a unit current source connected to the distribution node. This voltage value is equal to the system equivalent Thévenin impedance seen from that node, and by calculating this value for every single node of the system we can form the Thévenin equivalent matrix for the system. Having the Thévenin equivalent impedance of the system as seen at one node, we can form the two-node Thévenin equivalent circuit for the distribution system at each node.

Any type of distribution system configuration and topology, including non-radial and weakly-looped systems, can be modelled, without adding extra complexity to the analysis. DG, similarly, can be included in the model without causing complications, given the possible changes DG can make in the system for example in the direction of the flow of power. Microgrids and islanded autonomous energy systems which are majorly dependent on DG can also be modelled using this approach.
The proposed model for the distribution system including its loads and distributed generation is a linear model, and fulfills the objective number 1 of this research introduced in Section 1.3.1.

6.1.4 Static Voltage Stability Analysis

Static voltage stability analysis of distribution systems, either with or without DG, can be conducted by analyzing the P-V curve and a proposed voltage stability index at each node of the system, using the above mentioned equivalent model for the distribution system. One of the significant advantages of this linear real time method is to avoid computationally expensive iterative calculations or power flow solutions based on set of non-linear equations, which are conventionally used in static voltage stability studies.

“Theoretical Maximum Loadability” and “Practical Maximum Loadability” values at each node of the system are discussed, considering the safety margins from the P-V curve PoC, N-1 contingency criterion, reduced system feeding voltage, as well as load compensation techniques.

The proposed distribution system model and voltage analysis approach can clearly study different scenarios in the distribution system, including installation of new DG, extra load compensation, and decreased feeding terminal voltage, as shown in Section 4.2. Using this approach we can in real-time study the impact of any failure or sudden changes in the system configuration. This can, for example, help system operators make real time decisions, and also improve the system buses voltage profile and stability.

The method also allows the extension of the classical voltage stability solution to the increasingly important case of distributed generation placement, where the system is more likely
to face voltage stability problems.

The proposed methodology for the real time static voltage stability analysis of distribution system fulfills the objective number 2 of this research introduced in Section 1.3.2.

6.1.4.1 Voltage Stability Index

A voltage stability index is defined based on the load characteristics and distribution system specifications, as well as other loads and DG specifications connected to other buses of the system.

As shown in Section 4.2, this index can efficiently be used as a linear real-time tool to rank the system buses based on voltage stability and voltage profile, and find the parts of the system which (potentially) face voltage deficiencies. The buses with smaller voltage stability index values have bigger voltage drops in the system; thus operate closer to the distribution systems undervoltage limits. These buses also have smaller maximum loadability values and are operating closer to their voltage stability margins. In addition, the slope of the P-V curve around the operating point at the nodes with smaller index values is steeper, which means the derivative of voltage with respect to power is larger, therefore the voltage of the node shows more sensitivity to the changes in power. These studies show that the nodes with smallest voltage stability index values in the system are having the weakest voltage characteristics and are the most vulnerable nodes in the system in terms of voltage behavior problems. These results have been later verified by SFA and EMTP results in Section 5.1.

The voltage stability index can also identify the most stable and resilient nodes in the systems, which is very important for different types of systems resiliency studies.
Using the proposed voltage stability index for ranking the system buses based on voltage characteristics and identifying the nodes with weak voltage profile as well as most resilient nodes, fulfills the objective number 3 of this research introduced in Section 1.3.3.

The proposed voltage stability index can also identify the best candidates in the system for installation of new DG based on voltage sensitivity analysis; in order to improve voltage stability and profile, and elevate undervoltages without causing overvoltage in the system. The impacts of DG on the system voltage, at the right and wrong candidates for installation of new DG are studied in Section 4.3. These results show that the DG improves the operating voltage value as well as the loadability at the good bus candidates, while it can cause overvoltage at the buses which are not proper candidates for new DG installation.

Identifying the ideal candidate buses for new DG placement fulfills the objective number 4 of this research introduced in Section 1.3.4.

6.1.5 Dynamic Voltage Stability Analysis

Shifted Frequency Analysis is used as a tool to evaluate the transient voltage behavior of the distribution system with a focus on the systems with distributed generation; where the system is more likely to go through transient instabilities because of the intermittent nature of renewable DG, as well as DG’s small inertia compared to large power generating stations. This approach can be of significant importance in microgrids and islanded autonomous energy systems.

There are scenarios where static voltage stability studies cannot detect instabilities in the voltage, caused by transient cycles in the system. In this study, different scenarios have been discussed, including scenarios in which the voltage transients due to DG cause voltage instabilities in the system.
6.1.5.1 Shifted Frequency Analysis Technique

Conceptual mathematical derivation of the Shifted Frequency Analysis is discussed, in order to study the system transients within an envelope around the system fundamental frequency. SFA is based on finding an EMTP solution in the shifted frequency domain, and then transitioning the solution back into the real time domain.

The SFA method transfers the signal including its fundamental frequency, from real time domain to SFA domain by shifting the fundamental frequency terms to dc (0 Hz). A discretization procedure similar to EMTP is used to study the system transients in DC domain. Finally, if required, instantaneous values can be obtained directly from the SFA solution, transitioned from modal domain back to actual time domain.

Based on the Nyquist sampling theorem in signal processing to avoid aliasing in the system, and due to the filtering of fundamental steady state terms in SFA, the size of the time step in SFA can be considerably larger than the size needed for EMTP solutions, without sacrificing the accuracy. Actually as shown in other researches SFA can be more accurate than EMTP, because of eliminating the fundamental frequency terms from the calculations. This makes dynamic voltage stability assessment based on SFA, of practical value in real-time and control operations, specifically for distribution systems with a large number of nodes.

SFA provides highly clear and explicit results compared to other transient study tools (including EMTP), therefore SFA results can be used to explicitly analyze the system transient voltage behavior, and easily prevent voltage problems in the system.

Using the proposed SFA technique and therefore the possibility of using larger time steps fulfills the objective number 6 of this research introduced in Section 1.3.6.
6.1.5.2 Normal Operating Modes

System voltage and current are studied comparing SFA and EMTP solutions in the presence of DG and without DG. SFA results can clearly illustrate the dynamic behavior of the system and the deviated frequencies in the voltage and current, thus can be used as a highly practical tool to analyze the system transient behavior, as shown in Section 5.1.

SFA and EMTP are initially intended to study the transient behavior of the system; however the steady state results from these techniques are also compared to the static voltage stability results achieved in Chapter 4. In this case SFA and EMTP solutions verify the results from the linear static solutions and highly comply with them. SFA and EMTP results also verify the practicality of the voltage stability index introduced in Section 3.1.1, in terms of transient voltage stability behavior as well.

6.1.5.3 DG Operating Modes

SFA results clearly show that although the steady state behavior of the system, in the presence of DG at a right place in the system, can improve; but the transient deviations in the voltage and current usually increase. Normally, such transient voltage oscillations, caused by connecting a DG to a distribution system, cannot result in instabilities in the system; however depending on the node location and the distribution system density and configuration, these transient voltages can cause transient under/(over)voltages in the system, outside of the distribution system permissible operating voltage range, and therefore trip the protection and control devices, falsely.
The SFA results also show that depending on the distribution system specifications, installing a DG at a wrong bus can result in excessive increase in the bus voltage in steady state, and cause serious overvoltage in the system during the transients.

### 6.1.5.4 Instability Operating Modes

Transient oscillations in a system, under specific scenarios where the system is not working under its 100% functionality, may result in instabilities in the system. Such scenarios can include a fault in the system, circuit contingencies, loss of other generation or loss of a part of the feeding transmission system, as well as sudden or incremental changes in system load.

Given the intermittent nature of some renewable DG, such as wind and solar, scenarios have been studied in Section 5.2 in which a DG connects to the system while the system is already facing a short circuit fault at the other end. SFA results show that in this case the system cannot recover from the transients caused by the DG, and will lose its instability; however the static studies suggest that the system will still keep its stability, by studying only the steady state behavior of the system after connecting the DG.

These results show the importance of transient voltage stability studies in modern distribution systems.

Using the proposed methodology for analyzing the transient voltage stability in distribution systems including distributed generation, fulfills the objective number 5 of this research introduced in Section 1.3.5.
6.2 Future Research Developments

As discussed, the proposed method can be widely used for dynamic and static voltage studies in different areas of distribution systems. The future development of this research can include expanding the current research to more scenarios and distribution system types, as well as obtaining more detailed models for the studies, as discussed in the following:

6.2.1 Research Expansion

The current research can be expanded to study numerous scenarios in distribution systems under different circumstances, such as system components malfunction (including protection and control devices), loss of part of a system (including the feeding transmission system or the distribution system itself), sudden loss of a part of the load in the system, system faults, etc.

In addition, the proposed voltage stability techniques can be used for contingency studies in distribution systems.

In any of the mentioned scenarios different types of DG can also be added to the system, and the dynamic behavior of the system be studied.

Microgrids and autonomous energy systems also are one of the extremely interesting topics which can be studied in a separate research, using the proposed approaches. In addition, connection of such energy-independent systems to a traditional grid, and studying the corresponding transients in both systems can be of high interest.

Both dynamic and static characteristics of looped systems can also be studied in a separate research in real-time, considering the frequent changes in the system configuration, due to opening/closing of tie circuit breakers.
6.2.2 **Extra Detailed Models**

Work can be done to obtain more accurate measurements and therefore models for different types of load in the system. Distribution system loads can be divided into more detailed categories, and therefore one can obtain more detailed models for each type of load categories.

Similarly, work can be done to obtain more accurate models for DGs including the reduced-order model of DG interfaces.
Bibliography


Appendix

Appendix A  The 90-node Distribution System Data

This Appendix includes the system configuration data and load data of the 90-node distribution system, used in case studies in Chapters 4 and 5.
### A.1 The Distribution System Branch Configuration Data

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