

Energy-Efficient Power Allocation and Wireless Backhaul Design in Heterogeneous Small Cell Networks

by

Hao Liu

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The undersigned certify that they have read, and recommend to the College of Graduate Studies for acceptance, a thesis entitled: ENERGY-EFFICIENT POWER ALLOCATION AND WIRELESS BACKHAUL DESIGN IN HETEROGENEOUS SMALL CELL NETWORKS submitted by HAO LIU in partial fulfilment of the requirements of the degree of Master of Applied Science

Prof. Julian Cheng, Faculty of Applied Science/School of Engineering

Supervisor, Professor (please print name and faculty/school above the line)

Dr. Md. Jahangir Hossain, Faculty of Applied Science/School of Engineering

Supervisory Committee Member, Professor (please print name and faculty/school above the line)

Dr. Yang Cao, Faculty of Applied Science/School of Engineering

Supervisory Committee Member, Professor (please print name and faculty/school above the line)

Dr. Zheng Liu, Faculty of Applied Science/School of Engineering

University Examiner, Professor (please print name and faculty/school above the line)

External Examiner, Professor (please print name and faculty/school above the line)

July 4, 2016

(Date Submitted to Grad Studies)

Additional Committee Members include:

(please print name and faculty/school above the line)

(please print name and faculty/school above the line)

Abstract

The widespread applications of wireless services and dense devices access have triggered huge energy consumption. Due to the environmental and financial considerations, energy-efficient design in wireless networks has become an inevitable trend. Since the macrocell cannot satisfy the increasing data requirements of users, heterogeneous small cell network is one of the promising techniques to provide wireless service. However, backhaul is the bottle neck in the deployment of heterogeneous small cell networks. To address the challenges of backhaul design and energy efficiency, we study the energy-efficient power allocation and wireless backhaul bandwidth allocation in orthogonal frequency division multiple access heterogeneous small cell networks. Different from the existing resource allocation schemes that maximize the throughput, the studied scheme maximizes energy efficiency by allocating both transmit power of each small cell base station to each user and unified bandwidth for backhauling, according to the channel state information and the circuit power consumption. The problem is formulated as a non-convex nonlinear programming problem and then it is decomposed into two convex subproblems. A near optimal iterative resource allocation algorithm is designed to solve the resource allocation problem. A suboptimal low-complexity approach is also developed by exploring the inherent structure and property of the energy-efficient design. Simulation results demonstrate the effectiveness of the proposed algorithms when compared with the existing schemes.

Preface

This thesis is based on [C1, SJ1]. My supervisor, Prof. Julian Cheng, co-authored the publication and supervised all my research work.

Refereed Conference Publications

C1. H. Liu, H. Zhang, J. Cheng, and V. C. M. Leung, “Energy efficient power allocation and backhaul design in heterogeneous small cell networks,” *Proceedings of the IEEE International Conference on Communications (ICC)*, Kuala Lumpur, Malaysia, May 23–27, 2016.

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SJ1. H. Liu, H. Zhang, J. Cheng, and V. C. M. Leung, “Downlink energy efficiency of power allocation and wireless backhaul bandwidth allocation in heterogeneous small cell networks,” submitted to *IEEE Trans. Wireless Commun.* on February 01, 2016.

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List of Acronyms

Acronyms	Definitions
1G	First Generation
2G	Second Generation
3G	Third Generation
3GPP	3rd Generation Partnership Project
4G	Fourth Generation
AMPS	Advanced Mobile Phone System
AWGN	Additive White Gaussian Noise
BS	Base Station
CDMA	Code Division Multiple Access
FDMA	Frequency Division Multiple Access
GABS	Gradient Assisted Binary Search
GSM	Global System for Mobile Communications
LTE-Advanced	Long Term Evolution-Advanced
MIMO	Multiple-Input Multiple-Output
NMT	Nordic Mobile Telephone

List of Acronyms

OFDM	Orthogonal Frequency Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access
QoS	Quality-of-Service
SINR	Signal-to-Interference-plus-Noise Ratio
SIR	Signal-to-Interference Ratio
SMS	Short Message Service
SNR	Signal-to-Noise Ratio
TD-SCDMA	Time Division-Synchronous Code Division Multiple Access
WCDMA	Wideband Code Division Multiple Access
WiMAX	Worldwide Interoperability for Microwave Access

List of Symbols

Symbols	Definitions
$\arg \max \{ \cdot \}$	Points of the domain of the function at which the function values are maximized
\lim	The limit of the function
$\log_2(\cdot)$	The log function with base 2
$\max \{ \cdot \}$	The maximum value of the function
$\min \{ \cdot \}$	The minimum value of the function
$O(\cdot)$	The time complexity of an algorithm
s.t.	Subject to

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To My Loving Parents

Chapter 1

Introduction

1.1 Background and Motivation

The demand for information access promotes the development of communication technology, and mobile communication technology enables users to get rid of the constraints of wire communications. Therefore, wireless communication is playing an important role in people's everyday lives. Since Guglielmo Marconi's research team successively demonstrated the first radio transmission from the Isle of Wight to a tugboat 18 miles away in 1895 [2], the era of wireless communication had begun. The earliest mobile communication system can be traced back to 1920s. In 1928, students from Purdue University invented the super heterodyne radio receiver working at 2 MHz, and Detroit police quickly installed that mobile receiver in police patrol cars for managing traffic [3], which was the beginning of modern cellular mobile communication network. A cellular network or mobile network is a kind of communication network where the last link is wireless. A cellular network consists of cells and each cell is served by at least one fixed location transceiver which is known as a base station (BS). The BS serves the users for transmission of voice, data and others. The first generation (1G) mobile telecommunication technology for commercial use was invented in 1980s and three main analog communication standards were introduced in those years. One standard is Nordic mobile telephone (NMT), which was used in Nordic countries, Switzerland, the Netherlands, Eastern Europe and Russia. Another two standards are the advanced mobile phone system (AMPS) used in North America and Australia [4], and the total access commu-

nications system (TACS) used in the United Kingdom. In 1991, the second generation (2G) cellular telecommunication networks were commercially launched on the global system for mobile communications (GSM) standard in Finland by Radiolinja [5]. While radio signals on 1G networks are analog, radio signals on 2G networks are digital. Three primary benefits of 2G networks over their predecessors are that phone conversations were digitally encrypted; 2G systems significantly improved spectrum efficiency, which means more mobile users were allowed in 2G systems; and 2G introduced data services for mobile, such as short message service (SMS). In order to satisfy the growing demand for data service, several telecommunications companies introduced wireless mobile Internet services in the third generation (3G) based on code division multiple access (CDMA) technique [6]. There were three main standards proposed for 3G including wideband code division multiple access (WCDMA), CDMA2000 and time division-synchronous code division multiple access (TD-SCDMA) to improve system capacity and data rate [7]. Due to the development of digital signal processing, integrated circuit technology and other new technologies, mobile communication technology has made a rapid progress. In 2011, the fourth generation (4G) of mobile telecommunications technology was proposed. As opposed to earlier generations, all 4G candidate systems replaced spread spectrum radio technology used in 3G systems by orthogonal frequency division multiple access (OFDMA) multi-carrier transmission, making it possible to transmit high bit rates despite extensive multi-path radio propagation. The peak data rate can be further improved by smart antenna arrays for multiple-input multiple-output (MIMO) communications. For 4G systems, the 3rd Generation Partnership Project (3GPP) proposed Long Term Evolution-Advanced (LTE-Advanced) [8] system and Worldwide Interoperability for Microwave Access (WiMAX) Forum proposed WiMAX-Advanced technology [9].

Wireless communication networks have experienced tremendous growth in the past few decades, wireless service have migrated from the conventional voice-centric services

to data-centric services. It is obvious that one main target for modern wireless communication is to provide higher capacity wireless links for end-users. Since it is well known that the achievable data rate is limited by transmit power and transmission bandwidth, one straightforward way to meet the quality-of-service (QoS) demands of end-users is to increase the transmit power and bandwidth. However, transmission bandwidth and power are valuable and scarce resources in wireless mobile communication systems, and one could never use those resources as much as he desires. To overcome such contradiction, researchers have proposed techniques such as power allocation, subchannel allocation and the heterogeneous network. Heterogeneous small cell network is a typical multi-tier transmission scheme which can increase the mobile system capacity. OFDMA can use the bandwidth more efficiently than those previous transmission schemes such as CDMA and frequency division multiple access (FDMA) [10]. Therefore, heterogeneous small cell network with OFDMA has been adopted by modern mobile communication networks.

There are several key challenging problems associated with heterogeneous small cell network systems: energy efficiency, power allocation, user association, bandwidth allocation and backhauling. In this thesis, we will focus on the energy-efficient power allocation and backhaul bandwidth allocation problem in heterogeneous small cell networks and study the optimization techniques for resource allocation in heterogeneous small cell network with OFDMA.

1.2 Literature Review

With the explosive growth of wireless communications, it is shown that higher capacity wireless links are expected to meet the increasing QoS demands of multimedia applications. These high data rate links also result in increasing device power consumption. The next generation communication systems need to provide higher data rate with limited power and bandwidth due to the rapidly increasing demands for multime-

dia services and resource scarcity. Designing energy-efficient wireless communication system becomes an emerging trend because of rapidly increasing system energy costs and rising requirements of communication capacity [11–13]. According to [14] and [15], the radio access part is a major energy consumer in conventional wireless cellular networks, which accounts for up to more than 70 percent of the total energy consumption. It is reported that the total energy consumed by the infrastructure of cellular wireless networks, wired communication networks, and Internet takes up more than 3 percent of the worldwide electric energy consumption and the portion is expected to increase rapidly in the future [16]. Therefore, increasing the energy efficiency of typical wireless networks is important to overcome the challenges raised by the rising demands of energy consumption. In recent years, energy-efficient system design has been received much attention in academia. In [17], the impact of cell sizes on energy efficiency in cellular networks was studied. Several cross-layer approaches were also developed to obtain more gain over the independent layer design for energy efficiency [18].

With the exponential growth of mobile service requirements, macrocell network cannot satisfy all users' requirement for data service. In order to solve this problem, heterogeneous small cell network has been proposed to provide higher system capacity and data rates, and improve the system coverage with low infrastructure cost [19–21]. Many important problems related to heterogeneous small cell networks such as interference mitigation, resource allocation, and QoS provisioning were addressed to reap the potential gains [22–24].

Resource allocation, such as power allocation and bandwidth allocation, has been widely used to maximize the energy efficiency under power limit and QoS requirements in heterogeneous small cell networks. Power allocation for energy efficiency has been widely studied in the literature. The distributed power control game was studied in [25] to maximize the energy efficiency of transmission for secondary users in cognitive radio networks and an optimal power control problem was formulated as a repeated

game. The authors in [26] studied energy-efficient power control and receiver design in cognitive radio networks and a non-cooperative power control game for maximizing energy efficiency of secondary users was considered with a fairness constraint and interference threshold. The authors of [27] formulated the energy-efficient spectrum sharing and power allocation in heterogeneous cognitive radio networks with femtocells as a Stackelberg game and they proposed a gradient based iteration algorithm to obtain the Stackelberg equilibrium solution to the energy-efficient resource allocation problem. Many works have been done to consider bandwidth allocation for energy efficiency. In [28], the authors studied the joint service pricing and bandwidth allocation for energy and cost efficiency at the operator level in a multi-tier network where an operator deploys heterogeneous small cell networks, and they formulated the problem as a Stackelberg game. The problem of joint link selection, power and bandwidth allocation for energy efficiency maximization for multi-homing networks was investigated in [29]. A new energy-efficient scheme was presented in [30] to statistically meet the demands for QoS during the bandwidth allocation for wireless networks.

Convex optimization is one of the most effective mathematical modeling tools to explore resource allocation problem in wireless communication networks. The authors in [24] proposed a resource allocation scheme for cochannel femtocells to maximize system capacity under QoS and interference constraints. They formulated the power and sub-channel allocation problem as a mixed-integer programming problem and transformed it into a convex optimization problem, and the proposed problem was solved by the dual decomposition method. In [31], the authors formulated the network resource allocation problem as a convex optimization problem to maximize system throughput and minimize delay under a variety of realistic QoS and fairness constraints in wireless cellular and ad hoc networks. The globally optimal solutions were computed efficiently through polynomial time interior point methods. In [32], the authors analyzed power control problem in wireless cellular networks in high signal-to-interference ratio (SIR) and medi-

um to low SIR regimes. In high SIR regime, the formulated non-convex problems were transformed into convex optimization problems in the form of geometric programming and were effectively solved for global optimality. In the medium to low SIR regime, the problem could only be solved through the approach of successive convex approximation.

In this thesis, we define backhaul as the connection between macro BS and small cell BSs, and it is necessary to jointly consider the backhaul and radio access network. Several related works considered the backhaul to improve energy efficiency in wireless networks. The authors of [33] studied energy efficiency of resource allocation in multi-cell OFDMA downlink networks where the limited backhaul capacity, the circuit power consumption and the minimum required data rate were considered. The resource allocation problem for energy-efficient communication with limited backhaul capacity was formulated. In [34], an energy-efficient model of small cell backhaul networks with Gauss-Markov mobile models was proposed. In [35], the authors maximized system energy efficiency in OFDMA small cell networks by optimizing backhaul data rate and emission power, and they proposed a joint forward and backhaul link optimization scheme by taking both the power consumption of forward links and the backhaul links into consideration.

In this thesis, we study the energy-efficient power allocation and backhaul bandwidth allocation in heterogeneous small cell networks. A near optimal iterative resource allocation algorithm and a suboptimal low-complexity approach are proposed. Unlike the existing works in the literature, we take power allocation for small cell BSs and bandwidth allocation for backhauling together into consideration in heterogeneous small cell networks to maximize energy efficiency of all small cell users.

1.3 Thesis Organization and Contributions

This thesis consists of six chapters. Chapter 1 presents background knowledge of development and technologies for wireless communications and cellular networks. In

modern mobile communications, increasing QoS demand is the main target of system design, and therefore high transmit power and wider transmission bandwidth are desired. However, power and bandwidth are scarce resource and are usually limited in wireless communication system. Therefore, we focus on the energy-efficient resource allocation in OFDMA based heterogeneous small cell network.

Chapter 2 provides detailed technical and knowledge background for the entire thesis. First, a heterogeneous small cell network is introduced and used to provide more effective service than macrocell network. Second, resource management techniques, such as energy efficiency and backhauling, are provided. Convex optimization was introduced for resource management since it is an effective tool to solve the resource allocation problem.

In Chapter 3, an energy-efficient OFDMA heterogeneous small cell optimization framework is designed. The system model for power allocation and backhaul bandwidth allocation in heterogeneous small cell network is proposed to maximize the downlink energy efficiency for all small cell users. The corresponding problem is formulated as a nonlinear programming problem, where maximum transmit power constraints of each small cell BS to each small cell user, the downlink data rate constraint of small cell BSs and the minimum data rate between each small cell BS and each of its corresponding users are considered to provide reliable and low energy consumed downlink transmission for small cell users.

In Chapter 4, the conditions for optimization are provided and energy-efficient resource allocation problems are solved. First, we show that the formulated problem in Chapter 3 is a non-convex optimization problem and we can decompose it into two convex subproblems: one for power allocation and one for unified wireless backhaul bandwidth allocation. Second, we solve the subproblems of energy-efficient power allocation and energy-efficient backhaul bandwidth allocation.

In Chapter 5, optimization algorithms are proposed and numerical results are pro-

vided to demonstrate the effectiveness of the proposed algorithms. We first propose a near optimal iterative resource allocation algorithm and a suboptimal low-complexity approach to solve the resource allocation problem. Then we analyze the complexity for those two proposed algorithms. Finally, we use simulation results to demonstrate the effectiveness of the proposed algorithms when compared with the existing schemes.

Chapter 6 summarizes the entire thesis and lists our contributions in this thesis. In addition, some future works related to our current research are suggested.

Chapter 2

Heterogeneous Small Cell Networks and Resource Management

In this chapter, we present background knowledge about energy-efficient resource allocation in heterogeneous small cell networks. We first address the characteristic of heterogeneous small cell network and the motivation that small cell network exists, and then we introduce the basic concept of energy efficiency and backhaul. Finally, the basic convex optimization knowledge related to resource allocation is presented.

2.1 Overview of Heterogeneous Small Cell Networks

With the development of mobile Internet and the explosive growth of wireless traffic, macrocell networks face a series of challenges.

- The users' demand of mobile service has a trend of exponential growth. With the development of mobile Internet and cloud computing technology, users' demand of data service has risen rapidly. According to [36], the amount of global mobile data traffic nearly tripled over three consecutive years from 2010 to 2012 and exceeded the traffic on the entire global Internet in 2000. The development of the smart phone technology promoted the mobile network services. Those services, such as Internet video, mobile data and mobile voice, lead to an exponential growth of the

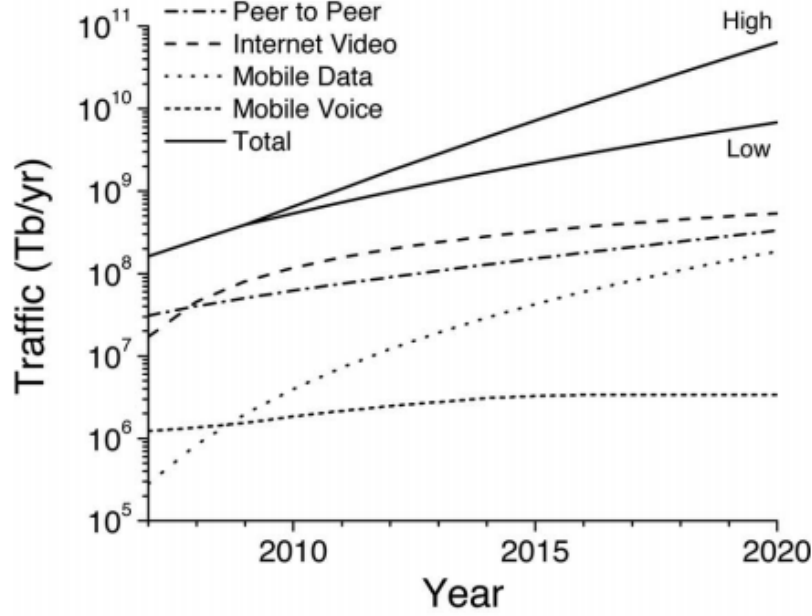


Figure 2.1: Traffic demand in terabits for North America [1].

demand for mobile data, which are shown in Fig. 2.1 [1]. The traditional cellular network cannot keep pace with the data explosion through the previous expensive and incremental methods such as increasing the amount of spectrum or deploying more macro base stations [37].

- The demand for indoor communication service has increased dramatically, but macrocell network has a limited coverage for indoor environment. According to [38], over 50 percent of the voice traffic and 70 percent of the data traffic occur in the indoor environment, and those figures seem to grow continually. Furthermore, 3G and 4G mobile communication systems are typically deployed at high frequencies and the penetration loss is huge when signals transmit between walls. Therefore, the data rate requirement of indoor users is a challenge for the coverage of macrocell.

To offload the overloaded traffics in macrocells and to enhance the coverage and capacity of the wireless networks, one method is to shorten the distance between the macro BS and user equipments. Small cells (e.g., picocells, femtocells and relay nodes) have been used to improve system capacity in hotspots for relieving the burden on overloaded macrocells, which is considered as a promising technique to provide an effective solution for the challenges in current macrocells [19, 20]. Therefore, there is no doubt that small cell has been paid much attention in recent years from academia and industry because it can help the system spatially reuse spectrum with low power consumption and improve the system coverage with low infrastructure cost deployment [21]. Heterogeneous small cell networks, where small cells are overlaid within a macrocell to improve coverage and increase system capacity beyond the initial deployment of macrocells, have been regarded as a promising approach to meet the increasing data traffic demand and coverage requirements, and to reduce energy consumption. A heterogeneous small cell network is shown in Fig. 2.2.

2.2 Resource Management

Since the demand for mobile service has an exponential growth and the scarcity of resource, resource management has drawn much attention these days. In recent years, energy-efficient system and backhauling have been proposed to help saving energy and guarantee the QoS for multi-users. Convex optimization algorithms are used in resource allocation problem for more efficient resource usage.

2.2.1 Energy Efficiency

During the past decades, much effort has been made to enhance network throughput. However, high network throughput usually implies large energy consumption, which is sometimes unaffordable for energy-aware networks or energy-limited devices. How to reduce energy consumption while meeting throughput requirements in such networks and

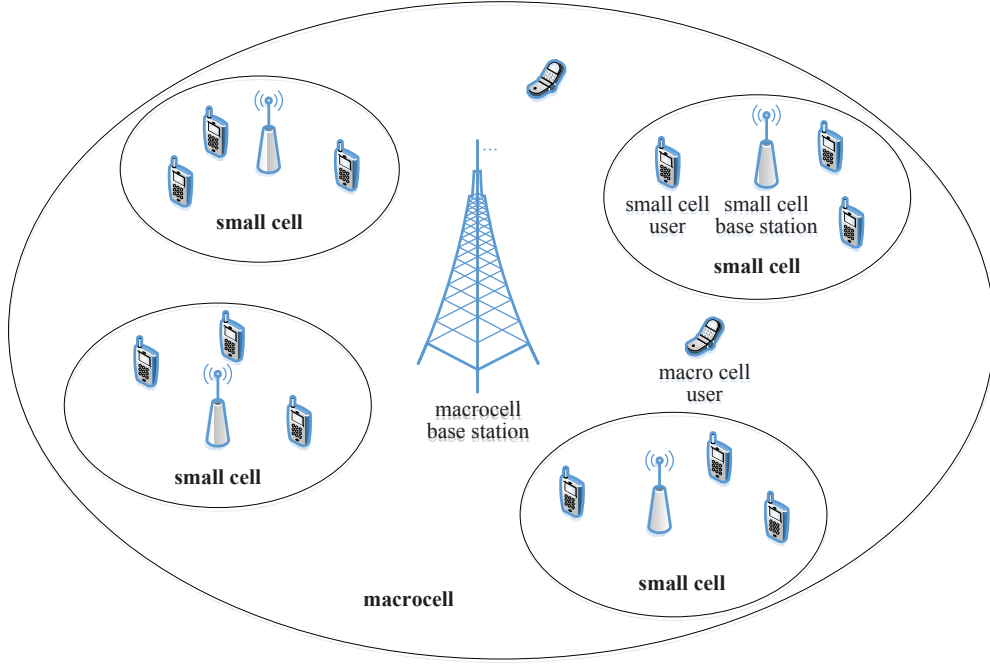


Figure 2.2: Heterogeneous small cell network.

devices is an urgent task. Therefore, energy-efficient communication system becomes an inevitable trend.

Over the past few decades, energy efficiency is commonly defined as information bits per unit transmit energy, which has been studied from the information-theoretic perspective for various scenarios [39]. For an additive white Gaussian noise (AWGN) channel, it is well known that for a given transmit power, p , and system bandwidth, W , the achievable transmission data rate is $r = W \log_2(1 + \frac{pg}{\sigma_0^2})$ bits per second, where σ_0^2 is AWGN power and g is channel power gain between transmitter and receiver. We can further write the transmission rate as $r' = \log_2(1 + \frac{pg}{\sigma_0^2})$ bits per second per Hertz. For energy-efficient communication, it is desirable to send the maximum amount of data with a given amount of energy. Given the energy ΔE consumed in duration ΔT , the energy can be rewritten as

$$\Delta E = p\Delta T. \quad (2.1)$$

Therefore, we define the energy efficiency as the ratio of the amount of data $r'\Delta T$ transmitted in duration ΔT to the amount of the given energy ΔE , which is shown as

$$\eta_{EE} = \frac{r'\Delta T}{\Delta E} = \frac{r'}{p} \quad (2.2)$$

bits per Hertz per Joule.

Except for transmit power, circuit power is also incurred by device electronics [40, 41]. Circuit power represents the additional device power consumption of devices during transmissions [42], such as digital-to-analog converters, mixers and filters, and this portion of energy consumption is independent of the transmission state. Denote the circuit power as P_C (typical value 0.1 W), thus the overall power assumption is $P_C + p$. Taking circuit energy consumption into consideration, energy efficiency needs to be redefined as information bits per unit energy (not only transmit energy) [43], where an additional circuit power factor, P_C , needs to be added in the denominator of (2.2) written as

$$\eta_{EE} = \frac{r'\Delta T}{\Delta E} = \frac{r'}{p + P_C}. \quad (2.3)$$

2.2.2 Backhaul

In a heterogeneous network, the backhaul portion of the network comprises the intermediate links between the core network or backbone network and the small cell networks. Backhaul has responsibility to carry packets to and from the core network and it acts as a bandwidth provider which guarantees QoS to the subnetwork users. Generally, backhaul solutions can be roughly categorized into wired (leased lines, copper or fiber) and wireless (point-to-point or point-to-multipoint over high-capacity radio links). Wired solution is usually expensive and often impossible to be deployed in remote areas, which makes wireless solution a more suitable and viable option. Heterogeneous wireless architecture can overcome the hurdles of wired solutions to create efficient

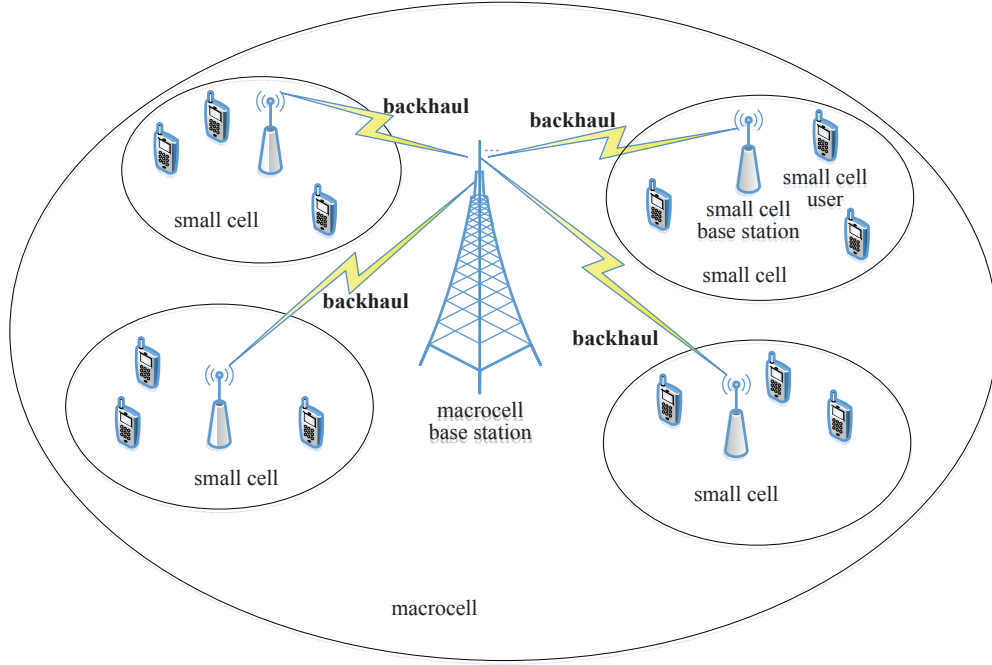


Figure 2.3: Backhaul.

large coverage areas and high capacity with relatively lower deployment cost. There is a growing demand in emerging markets where cost is usually a major factor in deciding technologies, a wireless backhaul solution is able to offer ‘carrier-grade’ services, whereas this is not easily feasible with wired backhaul connectivity [44]. In this thesis, we define that backhaul as the connection between macro BS and small cell BSs as shown in Fig. 2.3, and it is necessary to consider the joint design of backhaul and radio access network.

2.2.3 Convex Optimization based Resource Management

Mobile wireless networks are invented and act as essential means of communications to provide reliable data transmission among many users. With the exponential increase of users’ demand for mobile service, wireless communication system is hard to satisfy all requests due to the resource scarcity. Therefore, managing available communication resources, such as power and bandwidth, has drawn much attention. Many research

efforts have been made in investigating effective methods to increase operation efficiency and network capacity for the development of wireless communication systems. Convex optimization is one effective mathematical modeling tool to explore resource allocation problem in wireless communication networks. The convex optimization methods were used extensively in modeling, analyzing and designing of communication systems [45, 46]. Theoretically, convex optimization is appealing since a local optimum is also a global optimum for a convex problem.

According to the definition of convex function in [45], a function $f: \mathbf{R}^n \rightarrow \mathbf{R}$ is convex if the domain of f , denoted by $\mathbf{dom} f$, is a convex set¹ and if for any two points $x_1, x_2 \in \mathbf{dom} f$, and θ with $0 \leq \theta \leq 1$, we have

$$f(\theta x_1 + (1 - \theta)x_2) \leq \theta f(x_1) + (1 - \theta)f(x_2). \quad (2.4)$$

Geometrically, this inequality means that the line segment between $(x_1, f(x_1))$ and $(x_2, f(x_2))$ lies above the graph of f , which is shown in Fig. 2.4. We say f is concave if $-f$ is convex. Convexity and concavity will be preserved under nonnegative weighted summation, positive scaling, and pointwise maximum operation.

An optimization problem with arbitrary equality and inequality constraints can always be written in the following standard form [45]

$$\begin{aligned} \min \quad & f_0(\mathbf{x}) \\ \text{s.t.} \quad & f_i(\mathbf{x}) \leq 0, i = 1, 2, \dots, m \\ & h_i(\mathbf{x}) = 0, i = 1, 2, \dots, p \\ & \mathbf{x} \in S \end{aligned} \quad (2.5)$$

to find \mathbf{x} that minimizes $f_0(\mathbf{x})$ among all \mathbf{x} values that satisfy the conditions $f_i(\mathbf{x}) \leq 0$, $i = 1, 2, \dots, m$, $h_i(\mathbf{x}) = 0$, $i = 1, 2, \dots, p$ and $\mathbf{x} \in S$, where f_0 is called the objective

¹If a set C is convex, the line segment between any two points in C lies in C .

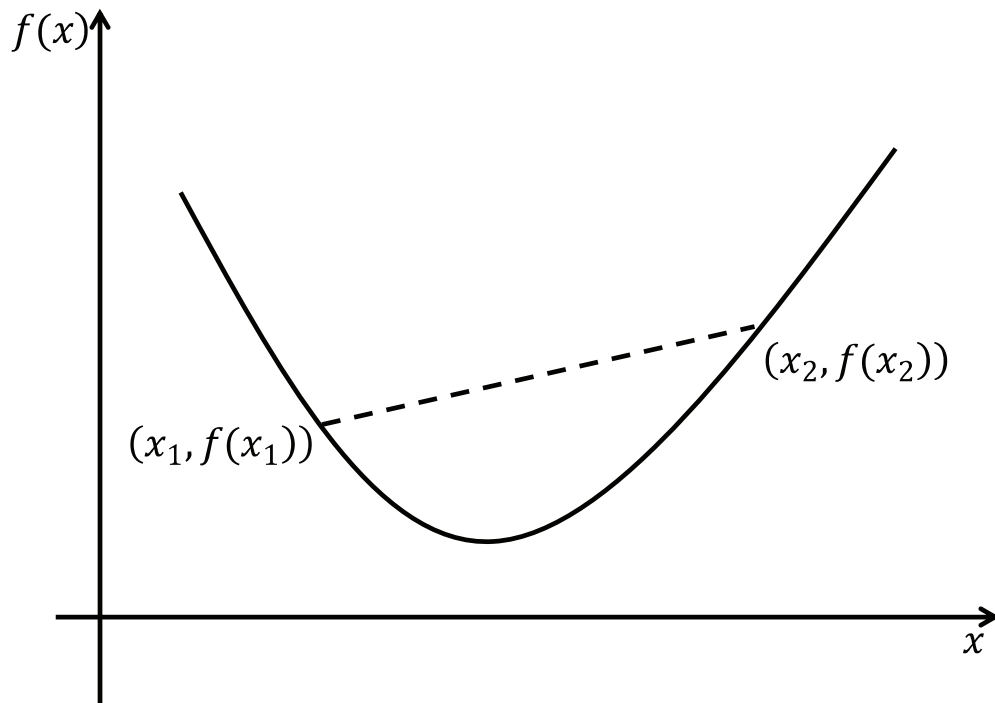


Figure 2.4: Graph of a convex function.

function or cost function; $f_i(\mathbf{x})$ and $h_i(\mathbf{x})$ are the inequality and equality constraint functions, respectively, and S is the constraint set. The domain of the objective and constraint functions is defined as

$$D = \left(\bigcap_{i=1}^m \text{dom } f_i \right) \cap \left(\bigcap_{i=1}^m \text{dom } h_i \right) \cap S. \quad (2.6)$$

The problem in (2.5) is a convex optimization problem if the objective function and the inequality constraint functions $f_i(\mathbf{x}) (i = 1, 2, \dots, m)$ are convex; equality constraints $h_i(\mathbf{x}) (i = 1, 2, \dots, p)$ are affine functions²; and the set S is convex. Violating any one of those conditions will result in a non-convex problem. A feasible $\mathbf{x}^* \in D$ is said to be global optimal if $f_0(\mathbf{x}^*) \leq f_0(\mathbf{x})$ for all \mathbf{x} . Notice that when we want to find the maximum value of the objective function, we can rewrite the formulation as

$$\begin{aligned} \max \quad & f_0(\mathbf{x}) \\ \text{s.t.} \quad & f_i(\mathbf{x}) \leq 0, i = 1, 2, \dots, m \\ & h_i(\mathbf{x}) = 0, i = 1, 2, \dots, p \\ & \mathbf{x} \in S. \end{aligned} \quad (2.7)$$

Problem (2.7) is still a convex optimization problem if the objective function is concave and the other conditions are the same as problem (2.5).

When formulating the resource allocation problems in wireless networks, it often happens that the formulated objective and constraint functions are non-convex. Thus, the problem cannot be solved by a convex optimization method. Fortunately, many optimization problems have hidden convexity and can be equivalently transformed into convex problems. In this thesis, we first formulate the energy-efficient power allocation and backhaul bandwidth allocation problem in heterogeneous small cell networks as a non-convex problem, and then decompose it into two convex subproblems.

²The affine function can be represented by matrix equation $\mathbf{Ax} = \mathbf{b}$, where \mathbf{A} is a matrix and \mathbf{b} is a vector of appropriate size.

2.3 Summary

In this chapter, we presented the essential technical background knowledge for the entire thesis. A brief description of heterogeneous small cell networks and the motivations of this kind of mobile network were provided. Then, the background knowledge and basic concepts on energy efficiency and backhaul were introduced. Finally, the basic knowledge about convex optimization were provided.

Chapter 3

Resource Allocation Modeling

In this chapter, we design an energy-efficient OFDMA heterogeneous small cell optimization framework and propose a system model for power allocation and backhaul bandwidth allocation to maximize the downlink energy efficiency for all small cell users. We formulate the problem as a nonlinear programming problem under QoS, transmit power and backhaul data rate constraints.

3.1 System Model

We consider a heterogeneous small cell network as shown in Fig. 3.1 with a single macro BS, J small cells deployed within the macrocell range and K users randomly located in each small cell.

The small cells share the same spectrum with macrocell. In this work, the unified wireless backhaul bandwidth allocation is investigated. The unified bandwidth allocation factor $\beta \in [0, 1]$, which is the fraction of bandwidth assigned for wireless backhauling at all small cell BSs within a macrocell range. For simplicity, all small cells are assumed to have the same bandwidth allocation factor. We assume that the multiple antenna technology is used in the macro BS and each small cell corresponds to a beamforming group, so the interference for wireless backhaul between different small cells can be neglected. The antenna array size at macro BS is N , which is much greater than the beamforming group size B and the number of small cells, $N \gg B$ and $N \gg J$. In this work, we also assume that $B \geq J$. Each small cell BS is equipped with single

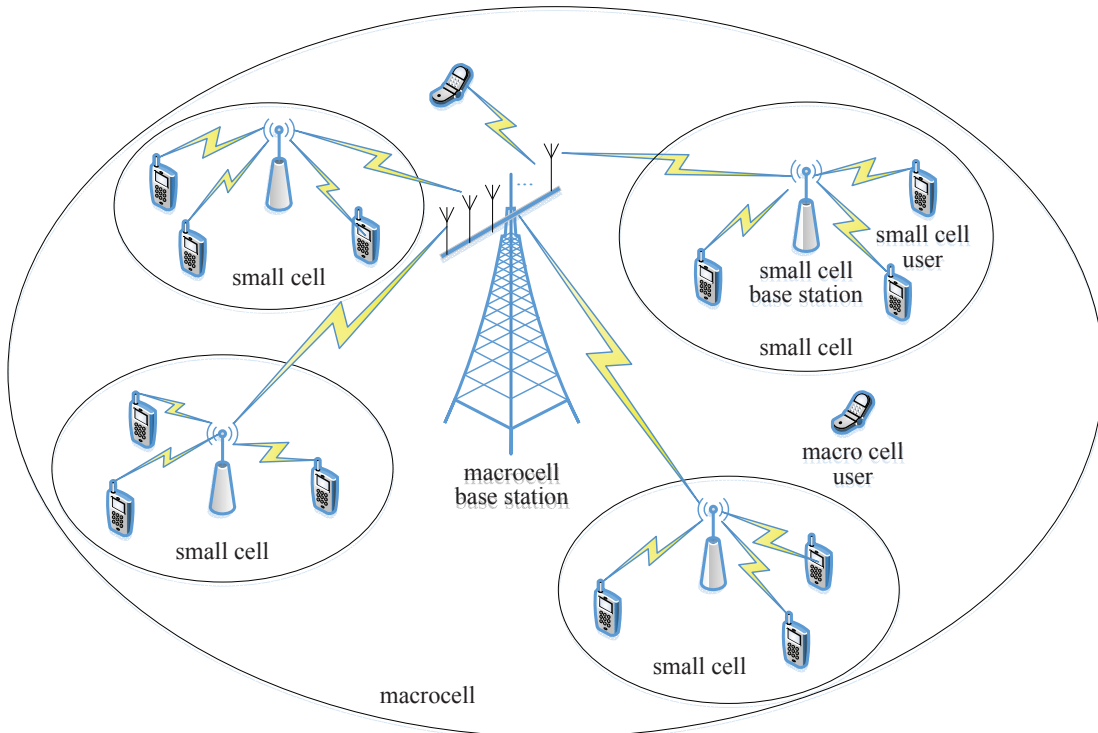


Figure 3.1: Topology of a heterogeneous small cell network.

antenna. OFDMA technology is used in each small cell to support the communication between BS and users.

3.2 Problem Formulation

Our objective is to maximize downlink energy efficiency of all small cell users through transmit power allocation and unified wireless backhaul bandwidth allocation in heterogeneous small cell networks. Let $g_{j,k}$ be the channel power gain between the j th small cell BS and its k th user, and denote G_j as the channel power gain between macro BS and the j th small cell BS, where $j \in \{1, 2, \dots, J\}$, $k \in \{1, 2, \dots, K\}$. Let $p_{j,k}$ denote the transmit power from the j th small cell BS to its k th user, and let $\mathbf{P} = [p_{j,k}]_{J \times K}$ denote the power allocation matrix. Then the received signal-to-noise ratio (SNR) in the wireless backhaul downlink of small cell j is given by

$$\gamma_j = \frac{P_0 G_j}{\sigma^2} \quad (3.1)$$

where P_0 is the transmit power of the macro BS and σ^2 is the AWGN power.

We assume that different users in each small cell use different subchannels and co-channel interference between small cells as part of the thermal noise because of the severe wall penetration loss and low power of small cell BSs [24]. The received signal-to-interference-plus-noise ratio (SINR) of small cell user k associated with small cell j is given by

$$\gamma_{j,k} = \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}} \quad (3.2)$$

where $I_{j,k}$ is the interference power introduced by macro BS $I_{j,k} = P_0 G_{j,k}$, where $G_{j,k}$ is the channel power gain between macro BS and the k th user in the j th small cell. The achievable data transmission rate between the j th small cell BS and its k th user is

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determined by

$$r_{j,k} = \left(\frac{1-\beta}{K} \right) \log_2 (1 + \gamma_{j,k}). \quad (3.3)$$

Therefore, we have the relation between $r_{j,k}$ and $p_{j,k}$ as

$$\begin{aligned} p_{j,k} &= (2^{\frac{Kr_{j,k}}{1-\beta}} - 1) \frac{\sigma^2 + I_{j,k}}{g_{j,k}} \\ r_{j,k} &= \left(\frac{1-\beta}{K} \right) \log_2 \left(1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}} \right). \end{aligned} \quad (3.4)$$

Except for transmit power, circuit power is also incurred by device electronics in small cell BSs [40, 41]. Circuit power represents the additional device power consumption of devices during transmissions [42], such as digital-to-analog converters, mixers and filters, and this portion of energy consumption is independent of the transmission state. If we denote the circuit power as P_C , the overall power assumption of the j th small cell BS to the k th user is $P_C + p_{j,k}$.

For energy-efficient communication, it is desirable to send the maximum amount of data with a given amount of energy for small cell BSs. Hence, given the amount of energy Δe consumed in a duration Δt in each small cell BS to each user, $\Delta e = \Delta t(P_C + p_{j,k})$, the small cell BSs desire to send a maximum amount of data bits by choosing the power allocation matrix and unified backhaul bandwidth allocation factor to maximize

$$\sum_{j=1}^J \sum_{k=1}^K \frac{r_{j,k}(\beta, p_{j,k}) \Delta t}{\Delta e} \quad (3.5)$$

which is equivalent to maximizing

$$U(\beta, \mathbf{P}) = \sum_{j=1}^J \sum_{k=1}^K U_{j,k}(\beta, p_{j,k}) \quad (3.6)$$

where

$$U_{j,k}(\beta, p_{j,k}) = \frac{r_{j,k}(\beta, p_{j,k})}{P_C + p_{j,k}}. \quad (3.7)$$

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$U(\beta, \mathbf{P})$ is called energy efficiency for all small cell users and $U_{j,k}(\beta, p_{j,k})$ is the energy efficiency of the k th user in the j th small cell. The unit of the energy efficiency is bits per Hertz per Joule, which has been frequently used in the literature for energy-efficient communications [39, 47–49].

When the downlink channel state information is estimated by the small cell BSs, the resource allocation is performed by each small cell BS under the following constraints.

- *Transmit power constraint of each small cell BS to each user:*

$$0 \leq p_{j,k} \leq P_{\max}, \forall j, k \quad (3.8)$$

where P_{\max} denotes the maximum transmit power of each small cell BS to each user.

- *The downlink data rate constraint of each small cell BS:* the throughput of the small cell is given by

$$R_j = \sum_{k=1}^K r_{j,k}. \quad (3.9)$$

According to [50], the capacity of the wireless backhaul downlink for small cell j is

$$C_j = \beta \log_2 \left(1 + \frac{N-B+1}{B} \gamma_j \right). \quad (3.10)$$

The downlink wireless backhaul constraint requires

$$R_j \leq C_j \quad (3.11)$$

such that the downlink traffic of the j th small cell can be accommodated by its wireless backhaul.

- *Heterogeneous QoS guarantee:* the QoS requirement R_t should be guaranteed for each user in each small cell to maintain the performance of the communication

3.3. Summary

system

$$r_{j,k} \geq R_t. \quad (3.12)$$

Our target is to maximize the energy efficiency of power allocation and unified bandwidth allocation for wireless backhauling in heterogeneous small cell networks under power constraint and data rate requirements. Thus, the corresponding problem for the downlink can be formulated as the following nonlinear programming problem

$$\max_{\beta, \mathbf{P}} U(\beta, \mathbf{P}) = \max_{\beta, p_{j,k}} \sum_{j=1}^J \sum_{k=1}^K U_{j,k}(\beta, p_{j,k}) \quad (3.13)$$

$$\text{s.t. } C1 : 0 \leq p_{j,k} \leq P_{\max}$$

$$C2 : R_j \leq C_j \quad (3.14)$$

$$C3 : r_{j,k} \geq R_t$$

$$C4 : 0 \leq \beta \leq 1.$$

3.3 Summary

In this chapter, we proposed an energy-efficient OFDMA heterogeneous small cell optimization framework. We established a system model for power allocation and backhaul bandwidth allocation to maximize the downlink energy efficiency for all small cell users. We formulated the problem as a nonlinear programming problem with the consideration of maximum transmit power constraints of each small cell BS to each small cell user, the downlink data rate constraints of small cell BSs and the minimum data rate between each small cell BS and its corresponding users.

Chapter 4

Energy-Efficient Resource Allocation and Backhauling

In this Chapter, we present the conditions for proposed optimization problem and design the mathematical approaches for resource allocation. We first prove that the formulated problem in Chapter 3 is a non-convex optimization problem. We find that the problem is separable, so we decompose it into two convex subproblems: one for power allocation and another for unified wireless backhaul bandwidth allocation. Then, we solve the subproblems of energy-efficient power allocation and energy-efficient backhaul bandwidth allocation individually.

4.1 Conditions of Optimality

We can prove that the formulated objective function in (3.13) is not concave and we notice that the continuous variable β and $p_{j,k}$ are separable in (3.13). A detailed proof is given in Appendix A. The constraint $C2$ in (3.14) is a nonlinear non-convex constraint. The detailed proof is shown in Appendix B. Therefore, the optimization problem formulated in (3.13) and (3.14) is not convex. By fixing the transmit power \mathbf{P} , the constraint $C2$ in (3.14) becomes convex with the bandwidth allocation factor β . Therefore, we consider a decomposition approach to solve the energy-efficient resource allocation problem. We decompose the non-convex optimization problem into two convex subproblems: one for energy-efficient power allocation and one for energy-efficient

wireless backhaul bandwidth allocation.

4.2 Energy-Efficient Power Allocation

Given a unique global bandwidth allocation factor β for wireless backhauling, we demonstrate that optimal energy-efficient power allocation exists. The optimization algorithm begins with the power allocation subproblem P1 which is formulated as

$$\text{P1 : } \max_{\mathbf{P}} U(\mathbf{P}) = \max_{p_{j,k}} \sum_{j=1}^J \sum_{k=1}^K U_{j,k}(p_{j,k}) \quad (4.1)$$

$$\begin{aligned} \text{s.t. } C1 : 0 &\leq p_{j,k} \leq P_{\max} \\ C2 : R_j &\leq C_j \\ C3 : r_{j,k}(p_{j,k}) &\geq R_t \end{aligned} \quad (4.2)$$

where

$$r_{j,k}(p_{j,k}) = \left(\frac{1-\beta}{K} \right) \log_2 \left(1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}} \right) \quad (4.3)$$

is strictly concave and monotonically increasing with $p_{j,k}$ when $r_{j,k}(0) = 0$ and $p_{j,k} = 0$.

According to the concept of quasiconcavity defined in [51], a function f that maps from a convex set of real n -dimensional vectors S' to a real number is called strictly quasiconcave if for any $x_1, x_2 \in S'$ and $x_1 \neq x_2$, and λ with $0 < \lambda < 1$, we have

$$f(\lambda x_1 + (1-\lambda)x_2) > \min\{f(x_1), f(x_2)\}. \quad (4.4)$$

The optimal energy-efficient power allocation achieves maximum energy efficiency, i.e.

$$\mathbf{P}^* = \arg \max_{\mathbf{P}} U(\mathbf{P}). \quad (4.5)$$

It is proved in Appendix C that $U(\mathbf{P})$ has the following properties.

Lemma 1. If $r_{j,k}(p_{j,k})$ is strictly concave in $p_{j,k}$, $U_{j,k}(p_{j,k}) \in U(\mathbf{P})$ is strictly quasiconcave. Furthermore, $U_{j,k}(p_{j,k})$ is first strictly increasing and then strictly decreasing in any $p_{j,k}$, i.e., the local maximum of $U(\mathbf{P})$ for each $p_{j,k}$ exists at a positive finite value.

For strictly quasiconcave functions, if a local maximum exists, it is also globally optimal [51]. Hence, a unique globally optimal transmit power matrix always exists and its characteristics are summarized in Theorem 1 according to the proofs in Appendix C.

Theorem 1. If $r_{j,k}(p_{j,k})$ is strictly concave, there exists a unique globally optimal transmission power matrix $\mathbf{P}^* = \{p_{j,k}^*; (j, k) \in J \times K\}$ for $\mathbf{P}^* = \arg \max_{\mathbf{P}} U(\mathbf{P})$, for each element in \mathbf{P}^* , $p_{j,k}^* = \arg \max_{p_{j,k}} U_{j,k}(p_{j,k})$ where $p_{j,k}^*$ is given by

$$\begin{aligned} \left. \frac{\partial U_{j,k}(p_{j,k})}{\partial p_{j,k}} \right|_{p_{j,k}=p_{j,k}^*} &= 0, \quad f(p_{j,k}) = 0, \\ \text{i.e., } U_{j,k}(p_{j,k}^*) &= \frac{r_{j,k}(p_{j,k}^*)}{P_C + p_{j,k}^*} = \left. \frac{\partial r_{j,k}(p_{j,k})}{\partial p_{j,k}} \right|_{p_{j,k}=p_{j,k}^*}. \end{aligned}$$

In order to solve the power allocation problem P1, we rewrite the objective function in (4.1) as

$$\max_{p_{j,k}} U_{j,k}(p_{j,k}) = \max_{p_{j,k}} \frac{r_{j,k}(p_{j,k})}{P_C + p_{j,k}}. \quad (4.6)$$

If each small cell user could reach the maximum energy efficiency, the whole small cells could reach the maximum energy efficiency. The total data rate in each small cell could not exceed the capacity of the wireless backhaul downlink for small cell j , that is, $R_j \leq C_j$. We can approximate that the data rate for each user will be less than $\frac{C_j}{K}$, $r_{j,k}(p_{j,k}) \leq \frac{C_j}{K}$.

Thus, P1 is equivalent to

$$\text{P1.1 : } \max_{p_{j,k}} U_{j,k}(p_{j,k}) \quad (4.7)$$

$$\begin{aligned}
 \text{s.t. } C1 : 0 &\leq p_{j,k} \leq P_{\max} \\
 C2 : r_{j,k}(p_{j,k}) &\leq \frac{C_j}{K} \\
 C3 : r_{j,k}(p_{j,k}) &\geq R_t.
 \end{aligned} \tag{4.8}$$

We can rewrite $C2$ in (4.8) according to (3.4) as

$$p_{j,k} \leq \left(\frac{\sigma^2 + I_{j,k}}{g_{j,k}} \right) \left(2^{\left(\frac{\beta}{1-\beta} \right) \log_2 \left(1 + \frac{N-B+1}{B} \frac{P_0 G_j}{\sigma^2} \right)} - 1 \right). \tag{4.9}$$

We can rewrite $C3$ in (4.8) according to (3.4) as

$$p_{j,k} \geq \left(\frac{\sigma^2 + I_{j,k}}{g_{j,k}} \right) \left(2^{\frac{K R_t}{1-\beta}} - 1 \right). \tag{4.10}$$

Therefore,

$$L_{j,k} \leq p_{j,k} \leq H_{j,k} \tag{4.11}$$

where

$$L_{j,k} = \left(\frac{\sigma^2 + I_{j,k}}{g_{j,k}} \right) \left(2^{\frac{K R_t}{1-\beta}} - 1 \right) \tag{4.12}$$

$$H_{j,k} = \min \left\{ \left(\frac{\sigma^2 + I_{j,k}}{g_{j,k}} \right) \left(2^{\left(\frac{\beta}{1-\beta} \right) \log_2 \left(1 + \frac{N-B+1}{B} \frac{P_0 G_j}{\sigma^2} \right)} - 1 \right), P_{\max} \right\} \tag{4.13}$$

only if the following inequality is satisfied

$$L_{j,k} \leq H_{j,k}. \tag{4.14}$$

The energy-efficient power allocation is given by

$$\hat{p}_{j,k}^* = \arg \max_{p_{j,k}} \frac{r_{j,k}(p_{j,k})}{P_C + p_{j,k}} \tag{4.15}$$

subject to

$$L_{j,k} \leq p_{j,k} \leq H_{j,k}. \tag{4.16}$$

We can solve (4.6) by using Theorem 1 to find the optimal power allocation solution. We can also use the low-complexity iterative algorithms based on the gradient assisted binary search (GABS) algorithm proposed in [52] to realize the energy-efficient power allocation for the k th user in the j th small cell BS. The GABS algorithm is shown as follows.

Algorithm Gradient Assisted Binary Search (GABS) Algorithm

```

1: Initialization: Each small cell BS allocates the same transmit power to each user,
    $p_{j,k} > 0$ .
2: Then do  $p_{j,k}^{(1)} = p_{j,k}$ ,  $h_1 \leftarrow \frac{\partial U_{j,k}(p_{j,k})}{\partial p_{j,k}} \Big|_{p_{j,k}=p_{j,k}^{(1)}}$  and  $c > 1$  (let  $c = 2$ ).
3: if  $h_1 < 0$  then
4:   repeat
5:      $p_{j,k}^{(2)} \leftarrow p_{j,k}^{(1)}$ ,  $p_{j,k}^{(1)} \leftarrow \frac{p_{j,k}^{(1)}}{c}$ , and  $h_1 \leftarrow \frac{\partial U_{j,k}(p_{j,k})}{\partial p_{j,k}} \Big|_{p_{j,k}=p_{j,k}^{(1)}}$ 
6:   until  $h_1 \geq 0$ 
7: else
8:    $p_{j,k}^{(2)} \leftarrow p_{j,k}^{(1)} \times c$  and  $h_2 \leftarrow \frac{\partial U_{j,k}(p_{j,k})}{\partial p_{j,k}} \Big|_{p_{j,k}=p_{j,k}^{(2)}}$ 
9:   repeat
10:     $p_{j,k}^{(1)} \leftarrow p_{j,k}^{(2)}$ ,  $p_{j,k}^{(2)} \leftarrow p_{j,k}^{(2)} \times c$  and  $h_2 \leftarrow \frac{\partial U_{j,k}(p_{j,k})}{\partial p_{j,k}} \Big|_{p_{j,k}=p_{j,k}^{(2)}}$ 
11:   until  $h_2 \leq 0$ 
12: end if
13: while no convergence do
14:    $\hat{p}_{j,k}^* \leftarrow \frac{p_{j,k}^{(1)} + p_{j,k}^{(2)}}{2}$ ,  $h' \leftarrow \frac{\partial U_{j,k}(p_{j,k})}{\partial p_{j,k}} \Big|_{p_{j,k}=\hat{p}_{j,k}^*}$ 
15:   if  $h' > 0$  then
16:      $p_{j,k}^{(1)} = \hat{p}_{j,k}^*$ 
17:   else
18:      $p_{j,k}^{(2)} = \hat{p}_{j,k}^*$ 
19:   end if
20: end while
21: Output  $\hat{p}_{j,k}^*$ .
    
```

If the output $\hat{p}_{j,k}^*$ satisfies the power constraint, i.e., $\hat{p}_{j,k}^* = p_{j,k}^*$. Otherwise, we can get the maximum $U_{j,k}(p_{j,k})$ by

$$p_{j,k}^* = L_{j,k} \quad (4.17)$$

if $\hat{p}_{j,k}^* < L_{j,k}$, or we can get the maximum $U_{j,k}(p_{j,k})$ by

$$p_{j,k}^* = H_{j,k} \quad (4.18)$$

if $\hat{p}_{j,k}^* > H_{j,k}$, since $U_{j,k}(p_{j,k})$ is first strictly increasing and then strictly decreasing in any positive finite $p_{j,k}$.

4.3 Energy-Efficient Wireless Backhaul Bandwidth Allocation

Once the optimal solution $\mathbf{P}^* = \{p_{j,k}^*; (j,k) \in J \times K\}$ is obtained for the convex subproblem P1 parameterized by β , it can be used in the following subproblem P2 for the unified wireless backhaul bandwidth allocation

$$\text{P2 : } \max_{\beta} U(\beta, \mathbf{P}^*) = \max_{\beta} \sum_{j=1}^J \sum_{k=1}^K U_{j,k}(\beta, p_{j,k}^*) \quad (4.19)$$

$$\text{s.t. } C1 : 0 \leq \beta \leq 1$$

$$C2 : R_j(\beta, \mathbf{P}^*) \leq C_j(\beta, \mathbf{P}^*) \quad (4.20)$$

$$C3 : r_{j,k}(\beta, p_{j,k}^*) \geq R_t$$

In order to obtain the solution to the original problem in (3.13) and (3.14), the two subproblems P1 and P2 are solved iteratively until convergence.

Maximizing the objective function of P2 with respect to β is equivalent to maximizing $(1 - \beta)$ only, because (4.19) is a monotonically decreasing function of β . Problem P2 reduces to a feasibility problem whose solution is the smallest feasible value of β given constraints (4.20).

According to C2 in (4.20), $R_j(\beta, \mathbf{P}^*) \leq C_j(\beta, \mathbf{P}^*)$, we have

$$\beta \geq \frac{\sum_{k=1}^K \log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)}{K \log_2 \left(1 + \frac{N-B+1}{B} \frac{P_0 G_j}{\sigma^2} \right) + \sum_{k=1}^K \log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)}. \quad (4.21)$$

According to C3 in (4.20), $r_{j,k}(\beta, p_{j,k}^*) \geq R_t$, we have

$$\beta \leq 1 - \frac{K R_t}{\log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)}. \quad (4.22)$$

Therefore, we can get the optimal backhaul bandwidth allocation factor β written as

$$\beta = \max \{ \phi_j, j \in J \} \quad (4.23)$$

where

$$\phi_j = \frac{\sum_{k=1}^K \log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)}{K \log_2 \left(1 + \frac{N-B+1}{B} \frac{P_0 G_j}{\sigma^2} \right) + \sum_{k=1}^K \log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)} \quad (4.24)$$

only if R_t satisfies the following condition

$$R_t \leq \min \{ \varphi_j, j \in J \} \quad (4.25)$$

where

$$\varphi_j = \frac{\log_2 \left(1 + \frac{N-B+1}{B} \frac{P_0 G_j}{\sigma^2} \right) \log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)}{K \log_2 \left(1 + \frac{N-B+1}{B} \frac{P_0 G_j}{\sigma^2} \right) + \sum_{k=1}^K \log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)}. \quad (4.26)$$

A detailed derivation of (4.25) can be found in Appendix D.

4.4 Summary

In this chapter, we found the solutions to subproblems of energy-efficient power allocation and energy-efficient wireless backhaul bandwidth allocation, respectively. We first proved that the problem formulated in Chapter 3 is a non-convex optimization problem and we found that the problem was separable. Therefore, we decomposed that problem into two convex subproblems, which means we maximized energy efficiency for power allocation and wireless backhaul bandwidth allocation separately. Then, we designed mathematical approaches for energy-efficient power allocation and energy-efficient backhaul bandwidth allocation.

Chapter 5

Algorithm Design

In this chapter, we propose two optimization algorithms for resource management and provide numerical results for the proposed algorithms. We first design a near optimal iterative resource allocation algorithm and a suboptimal but low-complexity approach to solve the resource allocation problem, and then we analyze the complexity for those two proposed algorithms. Finally, we use simulation results to demonstrate the effectiveness of the proposed algorithms when compared with the existing schemes.

5.1 Iterative Resource Allocation Algorithm

According to the analysis of power allocation and wireless backhaul bandwidth allocation discussed in Chapter 4, we propose an iterative optimization algorithm as shown in Algorithm 1.

In Algorithm 1, each small cell BS calculates ϕ_j according to (4.24) and then sends ϕ_j to macro BS. The macro BS chooses the maximum ϕ_j to be the optimal bandwidth allocation factor β and broadcasts β to all small cell BSs.

5.2 Low-Complexity Optimization Algorithm

To reduce the complexity of Algorithm 1, we propose a low-complexity optimization algorithm where bandwidth allocation factor is calculated from the equal power allocation and we fix β to calculate the power allocation according to the scheme proposed in Chapter 4. This low-complexity optimization algorithm is shown in Algorithm 2.

Algorithm 1 Iterative Resource Allocation Algorithm

```

1: Initialization: Each small cell BS allocates the same transmit power to each user,
    $p_{j,k} > 0$  and set  $l = 1$ .
2: repeat
3:   Backhaul Bandwidth Allocation
4:   Compute optimum  $\beta$  according to (4.23).
5:   Macro BS broadcasts the updated wireless backhaul bandwidth allocation factor
   to all small cell BSs.
6:   for each small cell BS do
7:     for each small cell user do
8:       Power Allocation
9:       a) find  $\hat{p}_{j,k}^* = \arg \max U_{j,k}(p_{j,k})$  according to GABS;
10:      b) check power constraint;
11:      if  $L_{j,k} \leq \hat{p}_{j,k}^* \leq H_{j,k}$  then
12:         $p_{j,k}^* = \hat{p}_{j,k}^*$ 
13:      end if
14:      if  $\hat{p}_{j,k}^* < L_{j,k}$  then
15:         $p_{j,k}^* = L_{j,k}$ 
16:      end if
17:      if  $\hat{p}_{j,k}^* > H_{j,k}$  then
18:         $p_{j,k}^* = H_{j,k}$ 
19:      end if
20:    end for
21:  end for
22:   $l = l + 1$ .
23: until total energy efficiency convergence or  $l = L_{\max}$ .

```

5.3 Complexity Analysis

Since the problem formulated in (3.13) and (3.14) is not convex, the only way to get the optimal solution is to use the method of exhaustion. If we assume that it costs P operations to calculate $r_{j,k}$ and it costs Q operations to calculate C_j , the complexity of checking $C2$ and $C3$ in (3.14) entails $KP + K + Q$ operations and $P + 1$ operations, respectively. If we assume it costs S operations to calculate $U_{j,k}$, the complexity of obtaining the total energy efficiency of all small cell users entails $JKS + (J - 1)(K - 1)$ operations. The total complexity of getting the value of objective function in (3.13) under the constraints in (3.14) entails $KP + K + P + 1 + JKS + (J - 1)(K - 1)$

Algorithm 2 Fixed β and Optimum Power Allocation Algorithm

```

1: Initialization: Each small cell BS allocates the same transmit power to each user,
    $p_{j,k} > 0$ .
2: Backhaul Bandwidth Allocation
3: Compute optimum  $\beta$  according to (4.23).
4: Macro BS broadcasts the wireless backhaul bandwidth allocation factor to all small
   cell BSs.
5: for each small cell BS do
6:   for each small cell user do
7:     Power Allocation
8:     a) find  $\hat{p}_{j,k}^* = \arg \max U_{j,k}(p_{j,k})$  according to GABS;
9:     b) check power constraint;
10:    if  $L_{j,k} \leq \hat{p}_{j,k}^* \leq H_{j,k}$  then
11:       $p_{j,k}^* = \hat{p}_{j,k}^*$ 
12:    end if
13:    if  $\hat{p}_{j,k}^* < L_{j,k}$  then
14:       $p_{j,k}^* = L_{j,k}$ 
15:    end if
16:    if  $\hat{p}_{j,k}^* > H_{j,k}$  then
17:       $p_{j,k}^* = H_{j,k}$ 
18:    end if
19:   end for
20: end for
    
```

operations for specific $p_{j,k}$ and β values. If we assume the value of the step size for $p_{j,k}$ is a and the value of the step size for β is b , there are $\frac{1}{b}(\frac{P_{\max}}{a})^{JK}$ choices for the values of $p_{j,k}$ and β . Therefore, the complexity for the method of exhaustion is $O\left(\frac{JKS}{b}(\frac{P_{\max}}{a})^{JK}\right)$.

In Algorithm 1, the worst-case complexity of calculating bandwidth allocation factor β from (4.23) entails J operations in each iteration. If we assume that it costs Ω operations in each GABS to search the optimal power allocation without power constraint, then the worst-case complexity of finding the power allocation for every user in each small cell entails $JK(\Omega+4)$ operations in each iteration. Suppose the Algorithm 1 requires Δ iterations to converge, so the total complexity of Algorithm 1 is $O(JK\Omega\Delta)$. Since iteration is not applied in Algorithm 2, the total complexity of Algorithm 2 is $O(JK\Omega)$, which is less than that of Algorithm 1. In the simulation, the typical value

for Δ is around 16, the typical value for Ω is less than 500, and the typical values for $\frac{1}{b}$ and $\frac{P_{\max}}{b}$ are both 100. So the complexities of Algorithm 1 and Algorithm 2 are always less than that of the method of exhaustion. When the number of small cells J and the number of users in each small cell K increase, the complexity of the method of exhaustion increases exponentially, so the complexity of the method of exhaustion is much larger than the complexities of proposed two algorithms.

5.4 Numerical Results

Simulation results are presented in this section to evaluate the performance of the proposed power allocation and wireless backhaul bandwidth allocation algorithms. In the simulations, it was assumed that small cells are uniformly distributed in the macro-cell coverage area, and small cell users are uniformly distributed in the coverage area of their serving small cell. AWGN power $\sigma^2 = 3.9811 \times 10^{-14}$ W. The coverage radius of the macrocell is 500 m, and that of a small cell is 10 m. The small cell BS has a minimum distance of 50 m from the macro BS. The minimum distance between small cell BSs is 40 m. We assume that the channel fading is composed of path loss, shadowing fading, and Rayleigh fading. The pathloss model for small cell users is based on [53]. The shadowing between small cell BS and small cell users is 10 dB. At the macro BS, we assume that transmit power is 33 dBm, the antenna array size $N = 100$ and beamforming group size $B = 20$. We consider that all the small cell users have the same QoS requirement.

Figure 5.1 shows the convergence in terms of the energy efficiency of all small cell users for the proposed Algorithm 1 versus the number of iterations, where $J = 5$, $R_t = 0.01$ bps/Hz, $P_{\max} = 20$ dBm. It can be observed that the proposed method takes nearly 16 iterations before converging to the stable solution. This result, together with the previous analysis, ensures that the proposed Algorithm 1 is applicable in heterogeneous small cell networks.

Figure 5.2 shows the total energy efficiency of all small cell users when the number of users per small cell is increased from 2 to 10. The energy efficiencies of Algorithm 2 are shown when $P_{\max} = 7$ dBm, $P_{\max} = 10$ dBm and $P_{\max} = 20$ dBm, and the energy efficiency of Algorithm 1 is shown when $P_{\max} = 20$ dBm. The simulation parameters are set as $J = 5$, $R_t = 0.01$ bps/Hz. Fig. 5.2 shows that the energy efficiency performance of Algorithm 1 is 20% higher than that of Algorithm 2. It also can be seen from Fig. 5.2 that the more number of users in small cell is, the better performance is obtained because of the multi-user diversity.

Figure 5.3 shows the total energy efficiency of all small cell users when the number of small cells is increased from 3 to 15. The energy efficiencies of Algorithm 2 are shown when $P_{\max} = 7$ dBm, $P_{\max} = 10$ dBm and $P_{\max} = 20$ dBm, and the energy efficiency of Algorithm 1 is shown when $P_{\max} = 20$ dBm. The simulation parameters are set as $K = 5$, $R_t = 0.01$ bps/Hz. Fig. 5.3 indicates that more number of small cell is, the better performance is obtained. It can also be seen from Fig. 5.3 that the energy efficiency performance of Algorithm 1 is always better than that of Algorithm 2 and the gap between them becomes larger when the number of small cells increases. The energy efficiency performance of Algorithm 1 is 30% superior to that of Algorithm 2 when the number of small cells is 10.

Figure 5.4 shows the total downlink capacity of all small cell users when the number of users per small cell is increased from 2 to 10. The total downlink capacities of Algorithm 2 are shown when $P_{\max} = 7$ dBm, $P_{\max} = 10$ dBm and $P_{\max} = 20$ dBm, and the total downlink capacity of Algorithm 1 is shown when $P_{\max} = 20$ dBm. The simulation parameters are set as $J = 5$, $R_t = 0.01$ bps/Hz. Fig. 5.4 shows that the total downlink capacity of Algorithm 1 is more than 3 bps/Hz higher than that of Algorithm 2. It also can be seen from the Fig. 5.4 that the more number of users in small cell is, the better performance is obtained due to the multi-user diversity. The total downlink capacity of Algorithm 1 is 21% higher than that of Algorithm 2 when the number of

users in each small cell is over 10.

Figure 5.5 shows the total downlink capacity of all small cell users when the number of small cells is increased from 3 to 15. The total downlink capacities of Algorithm 2 are shown when $P_{\max} = 7$ dBm, $P_{\max} = 10$ dBm and $P_{\max} = 20$ dBm, the total downlink capacity of Algorithm 1 is shown when $P_{\max} = 20$ dBm. The simulation parameters are set as $K = 5$, $R_t = 0.01$ bps/Hz. Fig. 5.5 illustrates that Algorithm 1 is superior to Algorithm 2 in terms of the total downlink capacity and the gap between them becomes larger when the number of small cells increases. The total downlink capacity of Algorithm 1 is 29% larger than that of Algorithm 2 when there 14 small cells in the heterogeneous network.

Figure 5.6 shows the total energy efficiency of all small cell users when using Algorithm 2 for power constraint P_{\max} ranging from 0 dBm to 12.79 dBm where the number of users in each small cell is 3, 4, 5. The simulation parameters are set as $J = 5$, $R_t = 0.01$ bps/Hz. Fig. 5.6 presents that the more users in each small cell, the higher total energy efficiency can be obtained, which has already been shown in Fig. 5.2. It also can be seen from the Fig. 5.6 that the larger power constraint is, the better performance is obtained. This is because the larger power constraint leads to the larger region of the optimizing variable.

Figure 5.7 shows the total energy efficiency of all small cell users when the number of users per small cell is increased from 2 to 10, for different algorithms. Algorithm 1 and Algorithm 2 are the iterative optimization algorithm and the low-complexity optimization algorithm, respectively. Algorithm 3 is an existing energy efficiency optimization algorithm with equal power allocation and Algorithm 4 is an algorithm that uses the optimal power allocation we proposed given a random β to optimize energy efficiency. All the algorithms are under the setting of $P_{\max} = 20$ dBm. Fig. 5.7 indicates that the more users in each small cell, the better performance can be obtained, which has already been shown in Fig. 5.2. It also can be seen from Fig. 5.7 that Algorithm 1 has

the best performance and then it follows by Algorithm 2, Algorithm 3 and Algorithm 4. The energy efficiency performance of Algorithm 1 is 30.5% and 56.6% higher than that of Algorithm 3 and Algorithm 4, respectively.

Figure 5.8 shows the total energy efficiency of all small cell users when the number of small cells is increased from 2 to 5, for the optimal solution and those two proposed algorithms. Since the complexity of the method of exhaustion is very high, we only consider the situation with small dimension where there are two users located in each small cell, $K = 2$. All the algorithms are under the setting of $P_{\max} = 20$ dBm and $R_t = 0.01$ bps/Hz. From Fig. 5.8, we can observe that the difference between the optimal solution and Algorithm 1 in terms of energy efficiency is very slight, which ensures the effectiveness of the proposed algorithms. The energy efficiency performance of the optimal solution is only about 7% and 24% higher than that of Algorithm 1 and Algorithm 2 when the number of small cells is 3, respectively.

5.5 Summary

In this chapter, we designed two optimization algorithms for resource management and provided numerical results for the proposed algorithms. We first proposed a near optimal iterative resource allocation algorithm and a suboptimal but low-complexity approach to solve the resource allocation problem. Then we analyzed the complexity for those two proposed algorithms. Finally, we used simulation results to demonstrate the effectiveness of the proposed algorithms by comparing them with the existing schemes.

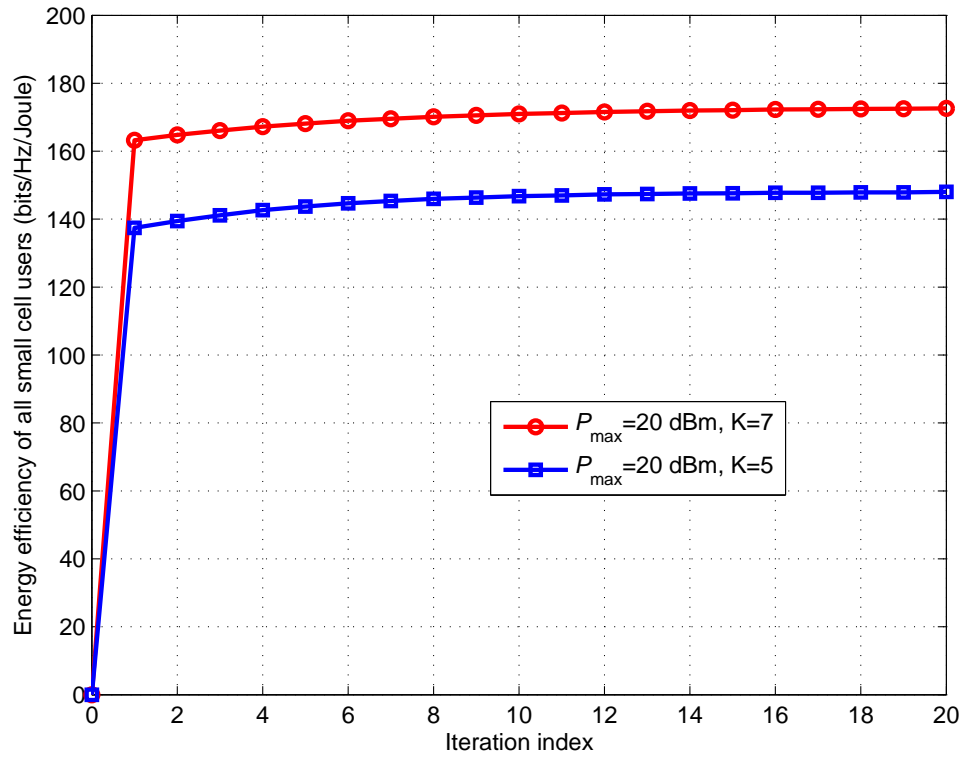


Figure 5.1: The convergence in terms of energy efficiency of all small cell users over the number of iterations.

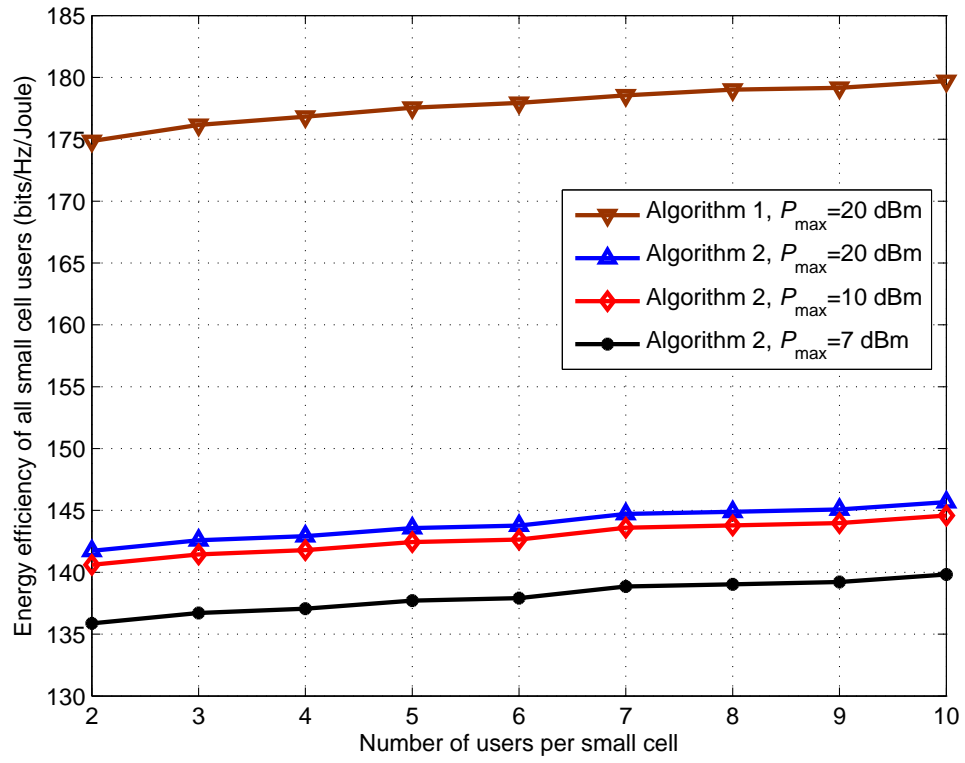


Figure 5.2: Energy efficiency versus the number of users per small cell.

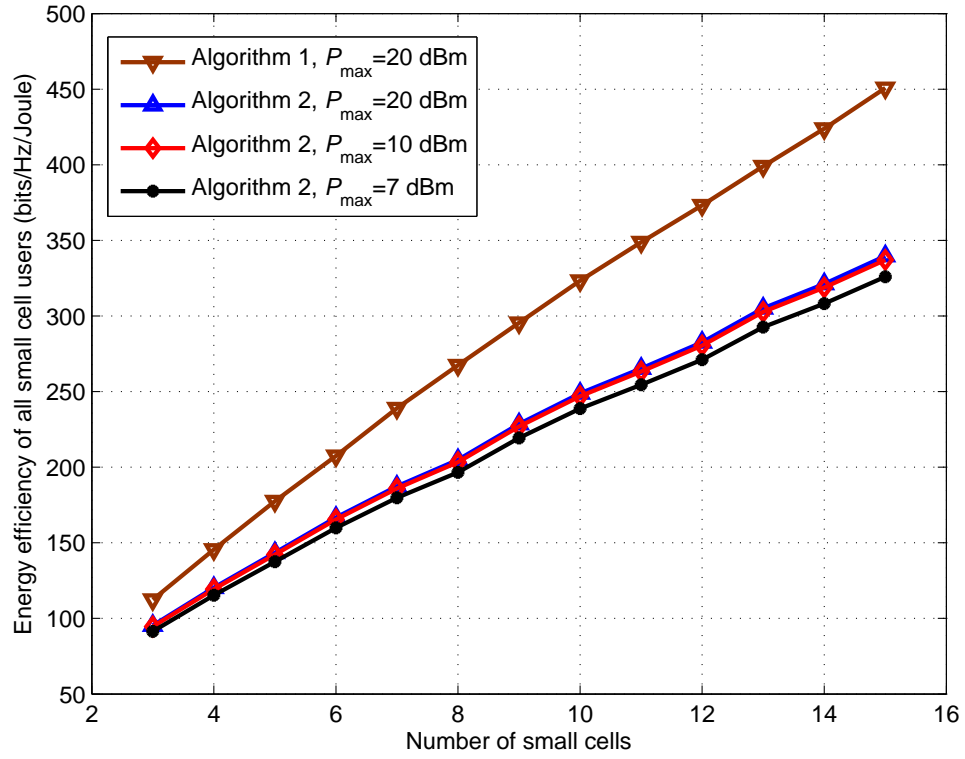


Figure 5.3: Energy efficiency versus the number of small cells.

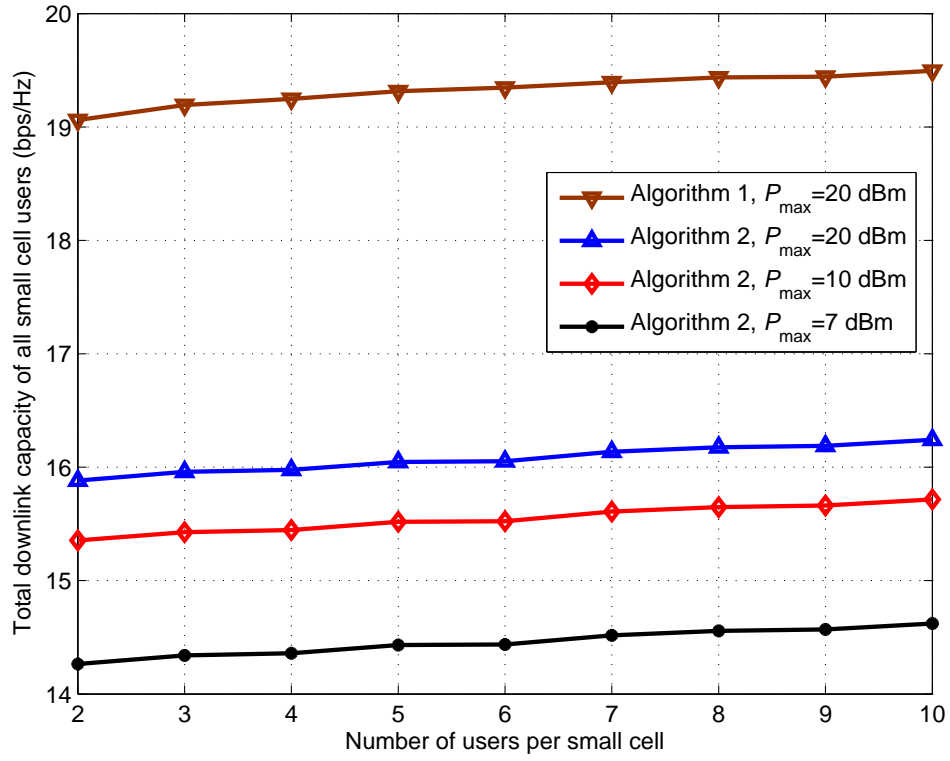


Figure 5.4: Capacity versus the number of users per small cell.

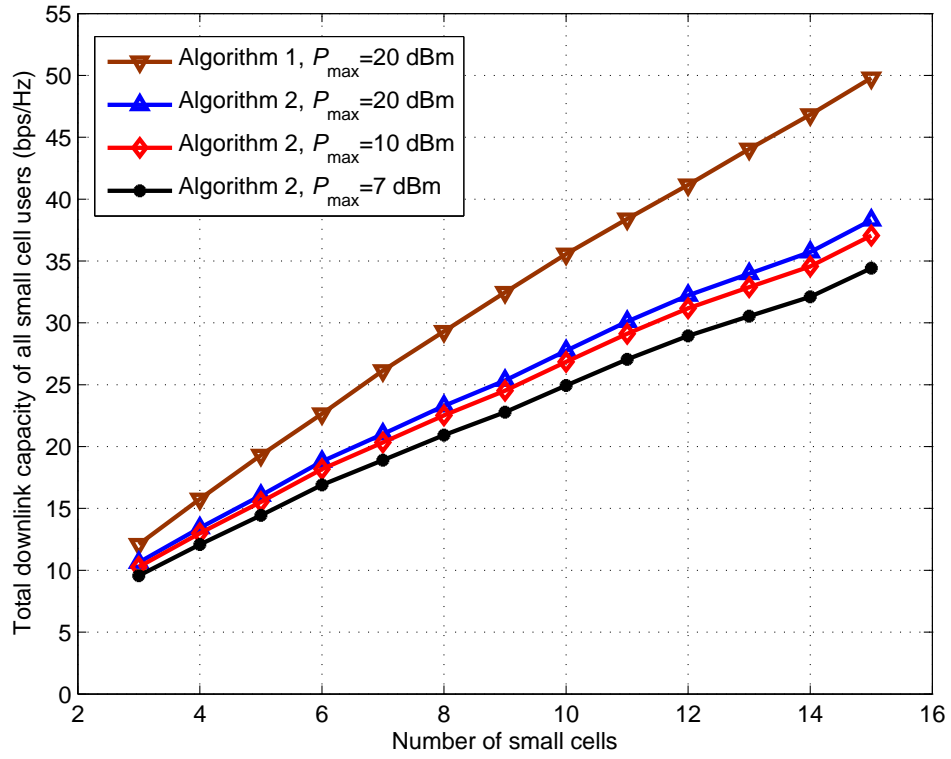


Figure 5.5: Capacity versus the number of small cells.

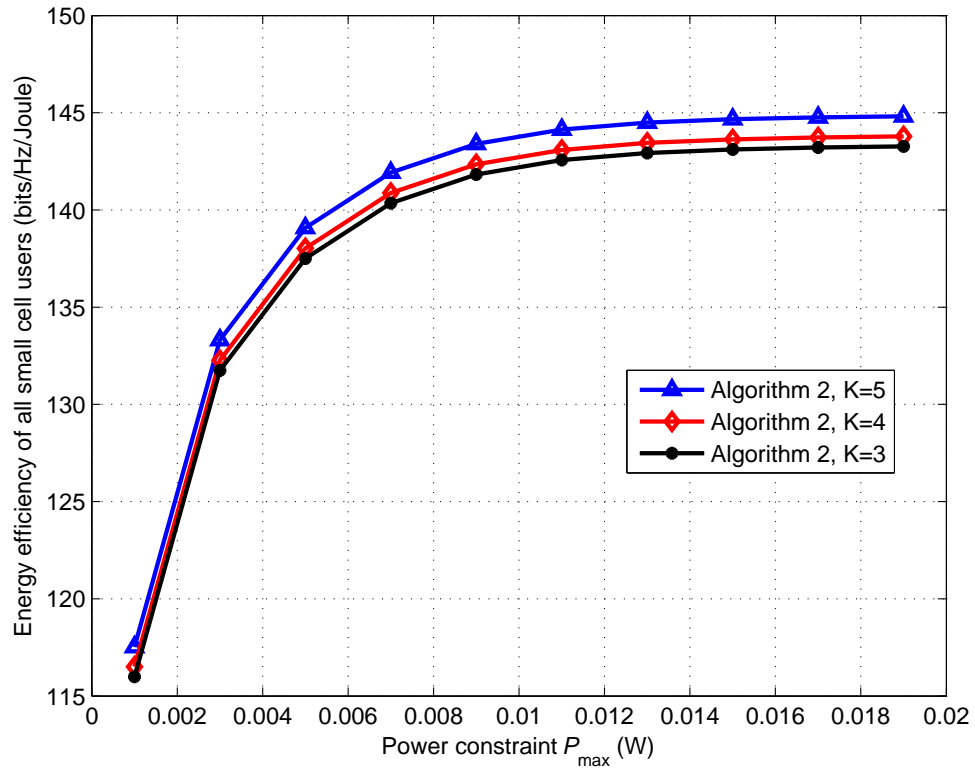


Figure 5.6: Energy efficiency versus the power constraint.

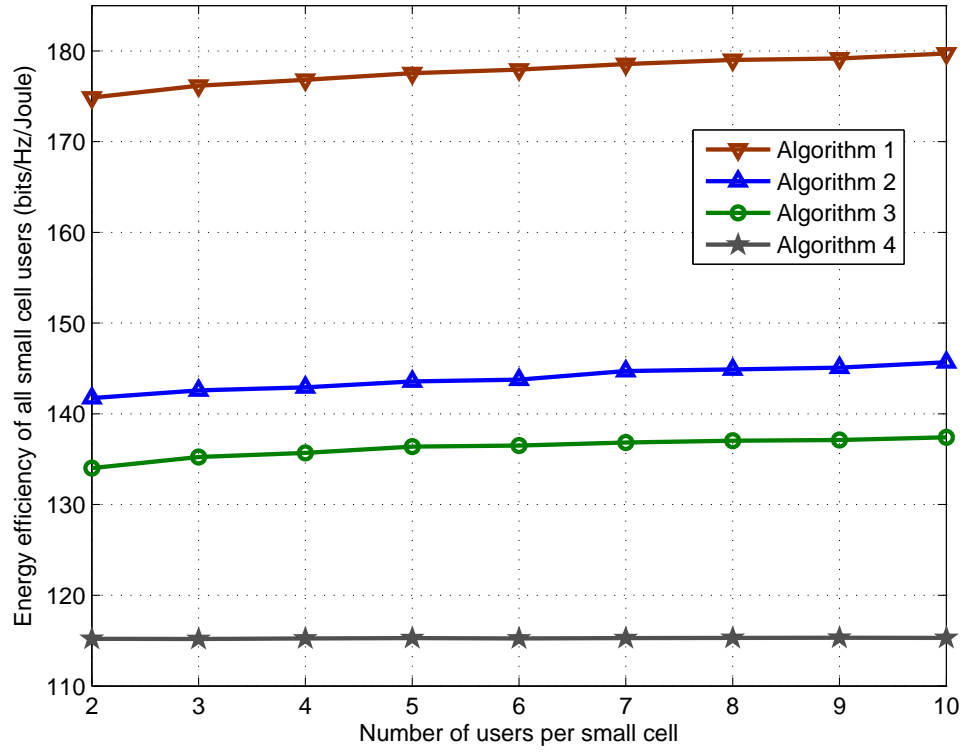


Figure 5.7: Energy efficiency comparison for different algorithms.

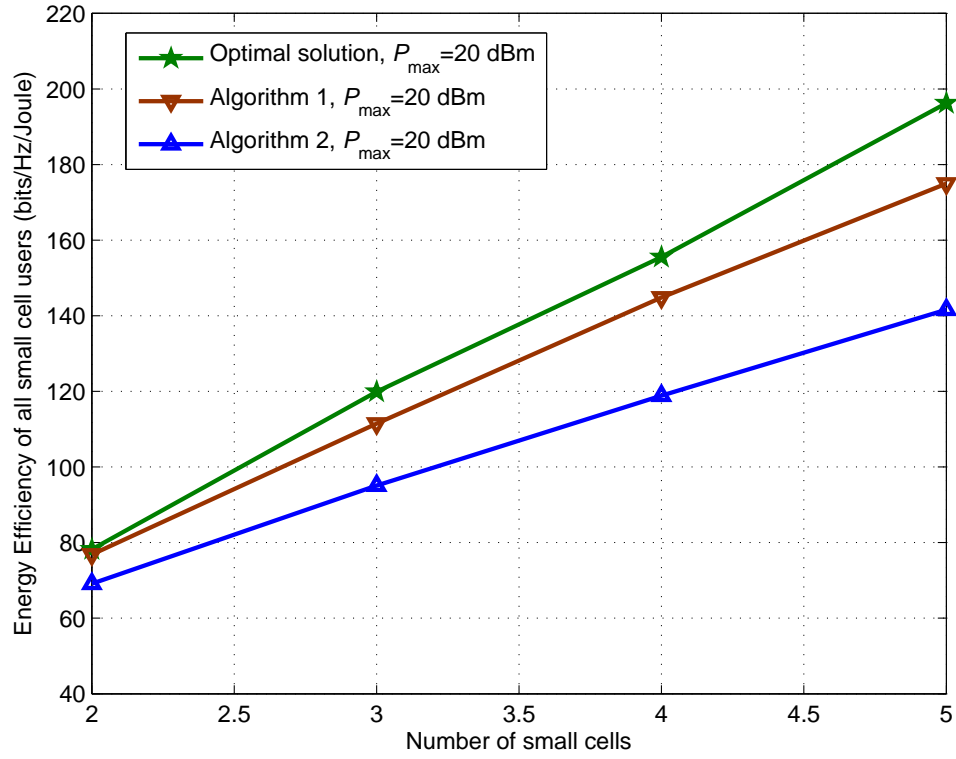


Figure 5.8: Energy efficiency comparison for the optimal solution and proposed algorithms.

Chapter 6

Conclusions

In this chapter, we conclude the thesis by summarizing the accomplished work and suggest some potential further works.

6.1 Summary of Accomplished Work

In this thesis, we developed two optimization algorithms for energy-efficient power allocation and backhaul bandwidth allocation in heterogeneous small cell networks. The numerical results establish the effectiveness of proposed design when compared with the existing schemes. To conclude the thesis, we summarize the accomplished work as follows:

- In Chapter 2, we provided detailed technical and knowledge background for the entire thesis. We first introduced heterogeneous small cell network which was used to provide more effective service than macrocell network, and then we presented some resource management techniques such as energy efficiency and backhauling. Convex optimization was introduced for resource management since it is an effective tool to solve the resource allocation problem.
- In Chapter 3, we provided energy-efficient OFDMA heterogeneous small cell optimization framework. The system model for power allocation and backhaul bandwidth allocation in heterogeneous small cell network was proposed to maximize the downlink energy efficiency for all small cell users. The corresponding problem was formulated as a nonlinear programming problem under the constraints of

QoS, transmit power and backhaul data rate.

- Chapter 4 provided the conditions for optimization and mathematical approaches for transmit power allocation and backhaul bandwidth allocation. First, we showed that the formulated problem in Chapter 3 is a non-convex optimization problem and we decomposed it into two convex subproblems: one for power allocation and another for unified wireless backhaul bandwidth allocation. Second, we solved the subproblems of energy-efficient power allocation and energy-efficient backhaul bandwidth allocation.
- In Chapter 5, suboptimal algorithms were designed and numerical results were presented. We proposed a near optimal iterative resource allocation algorithm and a suboptimal but low-complexity approach to solve the energy-efficient resource allocation problem. Then, we analyzed the complexity for those two proposed algorithms. Finally, we used simulation results to demonstrate the effectiveness of the proposed algorithms by comparing them with the existing schemes.

6.2 Future Work

Although considerable research work on resource management have already been proposed during the past few years, there are still some potential directions worth further investigation.

- In this work, we considered the power allocation and backhaul bandwidth allocation for energy efficiency. However, spectral efficiency is also an important system performance indicator to be studied. Therefore, resource allocation with backhauling for spectral efficiency is worth investigating in the future.
- For resource allocation, subchannel allocation is an important aspect. Therefore, the joint energy-efficient subchannel allocation and power allocation are worth

studying in the future.

- In this work, we considered the small cell BS with single antenna. It will be interesting to investigate small cell BS with beamforming or precoding technology using multi-antenna.

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Appendices

Appendix A

In order to prove that the objective function in (3.13), $\sum_{j=1}^J \sum_{k=1}^K U_{j,k}(\beta, p_{j,k})$, is concave, we first prove that function $U_{j,k}(\beta, p_{j,k})$ is concave.

The energy efficiency function for each small cell user is

$$U_{j,k}(\beta, p_{j,k}) = \frac{\left(\frac{1-\beta}{K}\right) \log_2 \left(1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}}\right)}{P_C + p_{j,k}}. \quad (\text{A.1})$$

The Hessian matrix for function $U_{j,k}(\beta, p_{j,k})$ can be written as

$$\mathbf{Hes}(U_{j,k}) = \begin{bmatrix} \frac{\partial^2 U_{j,k}}{\partial \beta^2} & \frac{\partial^2 U_{j,k}}{\partial \beta \partial p_{j,k}} \\ \frac{\partial^2 U_{j,k}}{\partial p_{j,k} \partial \beta} & \frac{\partial^2 U_{j,k}}{\partial p_{j,k}^2} \end{bmatrix} \quad (\text{A.2})$$

where

$$\frac{\partial^2 U_{j,k}}{\partial \beta^2} = 0 \quad (\text{A.3})$$

$$\frac{\partial^2 U_{j,k}}{\partial \beta \partial p_{j,k}} = \frac{\left(-\frac{1}{K}\right) \left(\frac{P_C + p_{j,k}}{(\ln 2) \left(1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}}\right)}\right) \left(\frac{g_{j,k}}{\sigma^2 + I_{j,k}}\right) + \left(\frac{1}{K}\right) \log_2 \left(1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}}\right)}{(P_C + p_{j,k})^2} \quad (\text{A.4})$$

$$\frac{\partial^2 U_{j,k}}{\partial p_{j,k} \partial \beta} = \frac{\left(-\frac{1}{K}\right) \left(\frac{P_C + p_{j,k}}{(\ln 2) \left(1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}}\right)}\right) \left(\frac{g_{j,k}}{\sigma^2 + I_{j,k}}\right) + \left(\frac{1}{K}\right) \log_2 \left(1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}}\right)}{(P_C + p_{j,k})^2} \quad (\text{A.5})$$

$$\begin{aligned} \frac{\partial^2 U_{j,k}}{\partial p_{j,k}^2} = & \frac{\left(\frac{1-\beta}{K}\right) \left(\frac{g_{j,k}}{\sigma^2 + I_{j,k}}\right) \left[\frac{-\left(P_C + p_{j,k}\right) \left(\frac{g_{j,k}}{\sigma^2 + I_{j,k}}\right)}{(\ln 2) \left(1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}}\right)^2} \right]}{(P_C + p_{j,k})^2} \\ & - \frac{2\left(\frac{1-\beta}{K}\right) \left[\left(\frac{1}{\ln 2}\right) \left(\frac{g_{j,k}}{\sigma^2 + I_{j,k}}\right) \left(\frac{P_C + p_{j,k}}{1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}}}\right) - \log_2 \left(1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}}\right) \right]}{(P_C + p_{j,k})^3}. \end{aligned} \quad (\text{A.6})$$

For simplicity, we denote that

$$M_1 = \frac{\partial^2 U_{j,k}}{\partial \beta \partial p_{j,k}} = \frac{\partial^2 U_{j,k}}{\partial p_{j,k} \partial \beta} \quad (\text{A.7})$$

and

$$M_2 = \frac{\partial^2 U_{j,k}}{\partial p_{j,k}^2}. \quad (\text{A.8})$$

Therefore, we can write the Hessian matrix for function $U_{j,k}(\beta, p_{j,k})$ as

$$\mathbf{Hes}(U_{j,k}) = \begin{bmatrix} 0 & M_1 \\ M_1 & M_2 \end{bmatrix}. \quad (\text{A.9})$$

It is obvious that the Hessian matrix is not negative semi-definite, so the function $U_{j,k}(\beta, p_{j,k})$ is not concave and the objective function in (3.13), $\sum_{j=1}^J \sum_{k=1}^K U_{j,k}(\beta, p_{j,k})$, is not concave. ■

Appendix B

In this Appendix, we prove the $C2$ in (3.14) is not a convex constraint. The $C2$ in (3.14) can be written as

$$\sum_{k=1}^K \left(\frac{1-\beta}{K} \right) \log_2 \left(1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}} \right) \leq \beta \log_2 \left(1 + \frac{N-B+1}{B} \frac{P_0 G_j}{\sigma^2} \right). \quad (\text{B.0.1})$$

We denote function $Z(\beta, p_{j,k})$ as

$$Z(\beta, p_{j,k}) = \sum_{k=1}^K \left(\frac{1-\beta}{K} \right) \log_2 \left(1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}} \right) - \beta \log_2 \left(1 + \frac{N-B+1}{B} \frac{P_0 G_j}{\sigma^2} \right) \leq 0. \quad (\text{B.0.2})$$

The Hessian matrix for function $Z(\beta, p_{j,k})$ can be written as

$$\mathbf{Hes}(Z) = \begin{bmatrix} \frac{\partial^2 Z}{\partial \beta^2} & \frac{\partial^2 Z}{\partial \beta \partial p_{j,k}} \\ \frac{\partial^2 Z}{\partial p_{j,k} \partial \beta} & \frac{\partial^2 Z}{\partial p_{j,k}^2} \end{bmatrix} \quad (\text{B.0.3})$$

where

$$\frac{\partial^2 Z}{\partial \beta^2} = 0 \quad (\text{B.0.4})$$

$$\frac{\partial^2 Z}{\partial \beta \partial p_{j,k}} = \frac{\left(-\frac{1}{K} \right) \left(\frac{g_{j,k}}{\sigma^2 + I_{j,k}} \right)}{(\ln 2) \left(1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}} \right)} \quad (\text{B.0.5})$$

$$\frac{\partial^2 Z}{\partial p_{j,k} \partial \beta} = \frac{\left(-\frac{1}{K} \right) \left(\frac{g_{j,k}}{\sigma^2 + I_{j,k}} \right)}{(\ln 2) \left(1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}} \right)} \quad (\text{B.0.6})$$

$$\frac{\partial^2 Z}{\partial p_{j,k}^2} = \frac{-\left(\frac{1-\beta}{K} \right) \left(\frac{g_{j,k}}{\sigma^2 + I_{j,k}} \right)^2}{(\ln 2) \left(1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}} \right)^2}. \quad (\text{B.0.7})$$

For simplicity, we denote that

$$M'_1 = \frac{\partial^2 Z}{\partial \beta \partial p_{j,k}} = \frac{\partial^2 Z}{\partial p_{j,k} \partial \beta} \quad (\text{B.0.8})$$

and

$$M'_2 = \frac{\partial^2 Z}{\partial p_{j,k}^2}. \quad (\text{B.0.9})$$

Therefore, we can write the Hessian matrix for function $Z(\beta, p_{j,k})$ as

$$\mathbf{Hes}(Z) = \begin{bmatrix} 0 & M'_1 \\ M'_1 & M'_2 \end{bmatrix}. \quad (\text{B.0.10})$$

It is obvious that Hessian matrix is not positive semi-definite, so the function $Z(\beta, p_{j,k})$ is not convex and $C2$ in (3.14) is not a convex constraint. ■

Appendix C

In order to prove the properties of $U(\mathbf{P})$ in Lamma 1, we first focus on $U_{j,k}(p_{j,k})$ and then we can get the properties of $U(\mathbf{P})$.

According to [51], we denote the α -superlevel sets of $U_{j,k}(p_{j,k})$ as

$$S_\alpha = \{p_{j,k} \geq 0 \mid U_{j,k}(p_{j,k}) \geq \alpha\} \quad (\text{C.0.1})$$

where $p_{j,k}$ is nonnegative. Based on the propositions from [51], $U_{j,k}(p_{j,k})$ is strictly quasiconcave if and only if S_α is strictly convex for any real number α . In this case, when $\alpha < 0$, no points exist on the contour $U_{j,k}(p_{j,k}) = \alpha$. When $\alpha = 0$, only $p_{j,k} = 0$ is on the contour $U_{j,k}(0) = \alpha$. Hence, S_α is strictly convex when $\alpha \leq 0$. Now, we investigate the case when $\alpha > 0$. We can rewrite the S_α as $S_\alpha = \{p_{j,k} \geq 0 \mid \alpha P_C + \alpha p_{j,k} - r_{j,k}(p_{j,k}) \leq 0\}$. Since $r_{j,k}(p_{j,k})$ is strictly concave with respect to $p_{j,k}$, $-r_{j,k}(p_{j,k})$ is strictly convex with respect to $p_{j,k}$. Therefore, S_α is strictly convex. Hence, we have the strict quasiconcavity of $U_{j,k}(p_{j,k})$.

Next, we can obtain the partial derivative of $U_{j,k}(p_{j,k})$ with $p_{j,k}$ as

$$\frac{\partial U_{j,k}(p_{j,k})}{\partial p_{j,k}} = \frac{(P_C + p_{j,k})r'_{j,k}(p_{j,k}) - r_{j,k}(p_{j,k})}{(P_C + p_{j,k})^2} = \frac{f(p_{j,k})}{(P_C + p_{j,k})^2} \quad (\text{C.0.2})$$

where $f(p_{j,k}) = (P_C + p_{j,k})r'_{j,k}(p_{j,k}) - r_{j,k}(p_{j,k})$, $r'_{j,k}(p_{j,k})$ is the first partial derivative of $r_{j,k}(p_{j,k})$ with respect to $p_{j,k}$. If $p_{j,k}^*$ exists such that $\left. \frac{\partial U_{j,k}(p_{j,k})}{\partial p_{j,k}} \right|_{p_{j,k}=p_{j,k}^*} = 0$, it is unique, i.e., if there is a $p_{j,k}^*$ such that $f(p_{j,k}^*) = 0$. In the following, we investigate the conditions when $p_{j,k}^*$ exists.

The derivative of $f(p_{j,k})$ is

$$f'(p_{j,k}) = (P_C + p_{j,k})r''_{j,k}(p_{j,k}) \quad (\text{C.0.3})$$

where $r''_{j,k}(p_{j,k})$ is the second partial derivative of $r_{j,k}(p_{j,k})$ with respect to $p_{j,k}$. Since $r_{j,k}(p_{j,k})$ is strictly concave in $p_{j,k}$, so $r''_{j,k}(p_{j,k}) < 0$, $f'(p_{j,k}) < 0$. Hence, $f(p_{j,k})$ is strictly decreasing.

$$\begin{aligned} \lim_{p_{j,k} \rightarrow \infty} f(p_{j,k}) &= \lim_{p_{j,k} \rightarrow \infty} ((P_C + p_{j,k})r'_{j,k}(p_{j,k}) - r_{j,k}(p_{j,k})) \\ &= \lim_{p_{j,k} \rightarrow \infty} (P_C r'_{j,k}(p_{j,k}) + p_{j,k} r'_{j,k}(p_{j,k}) - r_{j,k}(p_{j,k})) \end{aligned} \quad (\text{C.0.4})$$

where

$$r'_{j,k}(p_{j,k}) = \left(\frac{1-\beta}{K} \right) \left(\frac{g_{j,k}}{\sigma^2 + I_{j,k}} \right) \left(\frac{1}{\ln 2} \right) \left(\frac{1}{1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}}} \right) \quad (\text{C.0.5})$$

and

$$\lim_{p_{j,k} \rightarrow \infty} r'_{j,k}(p_{j,k}) = 0 \quad (\text{C.0.6})$$

so we have

$$\lim_{p_{j,k} \rightarrow \infty} P_C r'_{j,k}(p_{j,k}) = 0. \quad (\text{C.0.7})$$

According to the L'Hopital's rule, it is easy to show that

$$\begin{aligned} \lim_{p_{j,k} \rightarrow \infty} p_{j,k} r'_{j,k}(p_{j,k}) &= \lim_{p_{j,k} \rightarrow \infty} \left(\frac{1-\beta}{K} \right) \left(\frac{g_{j,k}}{\sigma^2 + I_{j,k}} \right) \left(\frac{1}{\ln 2} \right) \left(\frac{p_{j,k}}{1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}}} \right) \\ &= \lim_{p_{j,k} \rightarrow \infty} \left(\frac{1-\beta}{K} \right) \left(\frac{g_{j,k}}{\sigma^2 + I_{j,k}} \right) \left(\frac{1}{\ln 2} \right) \left(\frac{1}{\frac{g_{j,k}}{\sigma^2 + I_{j,k}}} \right) \\ &= \lim_{p_{j,k} \rightarrow \infty} \left(\frac{1-\beta}{K} \right) \left(\frac{1}{\ln 2} \right) \end{aligned} \quad (\text{C.0.8})$$

$$\lim_{p_{j,k} \rightarrow \infty} (-r_{j,k}(p_{j,k})) = \lim_{p_{j,k} \rightarrow \infty} \left[- \left(\frac{1-\beta}{K} \right) \log_2 \left(1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}} \right) \right] = -\infty \quad (\text{C.0.9})$$

so

$$\lim_{p_{j,k} \rightarrow \infty} f(p_{j,k}) < 0. \quad (\text{C.0.10})$$

Besides,

$$\begin{aligned} \lim_{p_{j,k} \rightarrow 0} f(p_{j,k}) &= \lim_{p_{j,k} \rightarrow 0} ((P_C + p_{j,k})r'_{j,k}(p_{j,k}) - r_{j,k}(p_{j,k})) \\ &= P_C r'_{j,k}(p_{j,k}^{(0)}) - r_{j,k}(p_{j,k}^{(0)}) \end{aligned} \quad (\text{C.0.11})$$

where $p_{j,k}^{(0)}$ denotes $p_{j,k} = 0$

$$\begin{aligned} r'_{j,k}(p_{j,k}^{(0)}) &= \left(\frac{1-\beta}{K} \right) \left(\frac{g_{j,k}}{\sigma^2 + I_{j,k}} \right) \left(\frac{1}{\ln 2} \right) \left(\frac{1}{1 + \frac{p_{j,k} g_{j,k}}{\sigma^2 + I_{j,k}}} \right) \Big|_{p_{j,k}=0} \\ &= \left(\frac{1-\beta}{K} \right) \left(\frac{g_{j,k}}{\sigma^2 + I_{j,k}} \right) \left(\frac{1}{\ln 2} \right) \end{aligned} \quad (\text{C.0.12})$$

$$r_{j,k}(p_{j,k}^{(0)}) = 0 \quad (\text{C.0.13})$$

$$\lim_{p_{j,k} \rightarrow 0} f(p_{j,k}) = \left(\frac{1-\beta}{K} \right) \left(\frac{P_C g_{j,k}}{\sigma^2 + I_{j,k}} \right) \left(\frac{1}{\ln 2} \right) > 0. \quad (\text{C.0.14})$$

Therefore, together with $\lim_{p_{j,k} \rightarrow \infty} f(p_{j,k}) < 0$, we obtain that $p_{j,k}^*$ exists and $U_{j,k}(p_{j,k})$ is first strictly increasing and then strictly decreasing in $p_{j,k}$. After achieving the maximum energy efficiency of every user in each small cell, the total energy efficiency of all small cells users can be maximized.

Lemma 1 is readily obtained. ■

Appendix D

From (4.21) and (4.22), we can get the interval for the wireless backhaul bandwidth allocation factor β . Problem P2 can be solved only if the upper bound of β larger or equal to its lower bound

$$\begin{aligned}
1 - \frac{KR_t}{\log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)} &\geq \frac{\sum_{k=1}^K \log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)}{K \log_2 \left(1 + \frac{N-B+1}{B} \frac{P_0 G_j}{\sigma^2} \right) + \sum_{k=1}^K \log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)} \\
&\geq \frac{KR_t}{\log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)} \quad (\text{D.0.1}) \\
R_t &\leq \frac{\log_2 \left(1 + \frac{N-B+1}{B} \frac{P_0 G_j}{\sigma^2} \right) \log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)}{K \log_2 \left(1 + \frac{N-B+1}{B} \frac{P_0 G_j}{\sigma^2} \right) + \sum_{k=1}^K \log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)}.
\end{aligned}$$

Therefore, we have the condition for R_t to guarantee that the problem P2 is solvable

$$R_t \leq \min \left\{ \frac{\log_2 \left(1 + \frac{N-B+1}{B} \frac{P_0 G_j}{\sigma^2} \right) \log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)}{K \log_2 \left(1 + \frac{N-B+1}{B} \frac{P_0 G_j}{\sigma^2} \right) + \sum_{k=1}^K \log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)} \right\}. \quad (\text{D.0.2})$$

In order to facilitate the representation, we denote φ_j as

$$\varphi_j = \frac{\log_2 \left(1 + \frac{N-B+1}{B} \frac{P_0 G_j}{\sigma^2} \right) \log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)}{K \log_2 \left(1 + \frac{N-B+1}{B} \frac{P_0 G_j}{\sigma^2} \right) + \sum_{k=1}^K \log_2 \left(1 + \frac{p_{j,k}^* g_{j,k}}{\sigma^2 + I_{j,k}} \right)}. \quad (\text{D.0.3})$$

So (D.0.2) can be rewritten as

$$R_t \leq \min \{ \varphi_j, j \in J \} . \tag{D.0.4}$$