Bunching, Cooling, and Measuring of Radioactive Isotopes for Nuclear Physics

by

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Abstract

TRIUMF is currently undergoing a major expansion of its capabilities to produce exotic ion beams for experiments. In this thesis, I review a portion of the new infrastructure that I helped to design, the CANadian Rare-isotope facility with Electron-Beam ion source's Radio Frequency Quadrupole buncher and cooler. The portion that I was responsible for was the injection optics into this buncher and cooler. The studies found a solution to exceed the acceptance requirements of the device, 15 mm mrad by a factor of five, resulting in an acceptance of 67 ± 7 mm mrad for masses of ions as low as 15 AMU, as well as 72 ± 7 mm mrad for masses above 30 AMU. These results came about from simulating both the length of a tapered region in the injection optics, as well as the angle of the taper region with respect to the beam axis The optimal design of the tapered region was have a length of 50 mm at an angle of 4°. In addition to these simulations, energy spread investigations were conducted. These studies resulted in a maximum loss of 14% of the acceptance of the device when the delivered beam has an energy spread of 25%.

In addition to the design of the injection optics for the buncher and cooler, mass measurements of several short-lived isotopes at Triumf's Ion Trap for Atomic and Nuclear science were analysed. All of the investigated masses have reduced uncertainties when compared to previous values in the Atomic Mass Evaluation of 2012. Mass excesses for ^{31,32}Na were found to be 12246(14) keV and 18638(37) keV, respectively, with uncertainties being half of the smallest of those currently published in AME. Mass excess of ^{29,34,35}Al were shown to be -18207.77(37) keV, -3000.5(29) keV, and -223.7(73) keV. The mass excess of ³⁴Al has also confirmed the two-neutron separation energy cross over with ³³Mg to be 15(10) keV at a

N=21. These nuclear physics studies are discussed in context of the so-called island of inversion and nuclear correlation energy.

Preface

Chapter 1: Figures 1.1, 1.2, and 1.3 are used with permissions from References [39], [1] and [2].

Chapter 2: All figures were created by myself.

Chapter 3: Figure 3.4 is used with permission from Reference [13].

Chapter 4: Figures 4.1, 4.24.3, 4.4, 4.5, 4.6, 4.7, 4.9 are used with permissions from References [54], [19] [3], [24], [70], [49], [49], and [48]. Figure 4.8 was created by myself.

Chapter 5: Figures 5.1, 5.2, 5.3, and 5.4, as well as Table 5.1, and portions of the text are taken from a paper currently being written, from a paper I am first author on. The paper was co-authored by the following people who aided in analysis, preparation, and presentation: M. Alanssari ,A. A. Kwiatkowski, C. Andreoiu, U. Chowdhury, J. Even, A. Finlay, D. Frekers, A. T. Gallant, G. Gwinner, R. Klawitter, B. Kootte, D. Lascar, K. G. Leach, E. Leistenschneider, A. Lennarz, A. J. Mayer, B. E. Schultz, D. Short, D. Lunney, and J. Dilling.

Chapter 6: All text was created by myself.

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Glossary

Throughout the thesis many acronyms were used. For simplicity, and convenience, each one is defined here

- AC Alternating Current
- AME Atomic Mass Evaluation
- AMU Atomic Mass Unit
- ARIEL Advanced Rare IsotopE Laboratory
- **BNG** Bradbury Nielsen Gate
- CANREB CANadian Rare-isotope facility with Electron-Beam ion source
- **CPET** Cooler PEnning Trap
- **CW** Continuous Wave
- **EBIS** Electron Beam Ion Source
- EBIT Electron Beam Ion Trap
- **FWHM** Full Width Half Max
- HRMS High Resolution Mass Seperator
- IR Infrared
- ISAC-TRIUMF Isotope Seperator and ACcelerator at TRIUMF

- **ISAC** Isotope Separator and ACcelerator
- **ISAC1** Isotope Separator and ACcelerator facility 1
- **ISOL** Isotope Separator On-Line
- MCP Multi-Channel Plate
- ME Mass Excess
- MPET Measurement Penning Trap
- NSCL National Superconducting Cyclotron Laboratory
- PPB Parts Per Billion
- PPS Particles Per Second
- PTMS Penning Trap Mass Spectrometry
- **RF** Radio Frequency
- **RFQ** Radio-Frequency Quadrupole
- **RIB** Rare Isotope Beam
- **RILIS** Resonant Ionization Laser Ion Source
- **RMS** Root Mean Square
- SOI Species of Interest
- **SHM** Simple Harmonic Motion
- SRF Superconducting Radio Frequency
- TITAN TRIUMF's Ion Trap for Atomic and Nuclear science
- **TOF** Time-Of-Flight
- TOF-ICR Time-Of-Flight Ion Cyclotron Resonance
- TRILIS TRIUMF's Resonant Ionization Laser Ion Source

- UV UltraViolet
- vos Valley Of Stability

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Chapter 1

Introduction and Motivation

If I have seen farther it is by standing on the shoulders of Giants. — Sir Isaac Newton (1676)

1.1 The Nuclear Shell Model

Matter is a fundamental part of our universe, and its study has been on going for thousands of years in different forms. Today's investigations of matter have progressed so far that we are no longer content at simply studying the atoms that make up the world around us, but we are creating more and more exotic and short-lived species that have long since departed our world. How we have come to classify atoms is now both by the number of protons Z, their number of neutrons N, and their energy states. We have moved far from the standard periodic table that people learn about in secondary school and, in fact, have had to produce a much larger and more comprehensive view of what matter is consists of. The current option of choice to describe the atomic constituents of matter is the chart of nuclides, such as that in Figure 1.1. The chart of nuclides includes all known possibilities of combinations of different proton and neutron configurations. In general, the central region of the chart contains the more stable of the proton-neutron combinations, with the species further out from this Valley Of Stability (VOS) being less stable.

However this trend of decreasing life times further out from the VOS does occur, there are local regions of higher stability. These regions of enhanced stability arise



Figure 1.1: The chart of nuclides covers the ranges of all known possibilities of protons and neutrons. The black, central region displays the stable nuclides, where the yellow region indicates unstable nuclides. The dashed lines depict the stable magic numbers of protons and neutrons, where the coloured lines represent theoretical two-neutron and two-proton driplines. This figure was taken with permission from reference [39].

from the closure of nuclear shells, and occur at so-called magic proton and neutron numbers[45, 56, 57].

1.2 Motivation of High Accuracy and High Precision Mass Measurements

The discovery of the enhanced stability of nuclei with magic proton and neutron numbers paved the way for the postulation of the nuclear shell model [45, 56, 57]. The nuclear shell model gives robust results to describe properties of nuclei along the vos. However, when moving away from the valley of stability, the well-known magic numbers seem to disappear while new magic numbers appear [47, 71]. In 1975, it was found that the N = 20 neutron shell closure disappears in neutron rich Na isotopes, and this has since become known as one of the islands of inversion [73]. In recent years, this N = 20 island of inversion has been shown to

rise from a non-standard filling of the *sd* and *pf* energy levels [71]. This modified order of energy level occupation is caused by large gains in correlation energy due to the excitation of neutron pairs into the *pf* orbital. By studying regions where these features are prominent [72], as well as those regions with extreme neutronto-proton ratios, it's possible to gain further insight into nuclear structure [11]. Through this, we can further investigate the details of the shell model, and can identify new, possible features that have previously gone undetected. Examples of such are the recent measurements that showed that the inversion of the shells of N = 20 and N = 28, are in fact, a single island connected via neutron rich Mg isotopes [29, 35].

One of the most efficient ways to investigate features of nuclear structure, including binding energies and two-neutron separation energies, is through mass measurements. Mass measurements are a versatile and powerful tool which provide access to the combined effects of all interactions inside the atom. This makes mass measurements an integral part of nuclear-structure investigations and the exploration of new regions of the chart of nuclides (e.g. [47]).

There are multiple approaches to performing mass measurements where each of the various approaches reach different degrees of precision and accuracy. Although each technique has its advantages, the resultant precision that arises from techniques may be insufficient for scientific objectives. The Time-Of-Flight (TOF) mass excess determinations of ³²Na [38, 46, 61, 69, 76], for example, span a range of 2.5 MeV with uncertainties varying from 0.1 MeV to 1.3 MeV. The size of this spread hinders reliable conclusions on the underlying structure. More reliable results can be delivered via the more accurate and precise method of Penning Trap Mass Spectrometry (PTMS) [14]). Indeed, the uncertainties of the mass measurements via PTMS in this thesis (see Chapter 5) are a factor of 3.2 times smaller than that of the smallest, previously mentioned uncertainties [12].

1.3 An ARIEL Overview

As time progresses, more exotic beams are required in order to study new and interesting physics. In order to help meet the requirements of these ever increasing demands TRIUMF is currently building the Advanced Rare IsotopE Labora-



Figure 1.2: The exterior of the new ARIEL building at TRIUMF, currently under development. This picture was taken with permission from the TRIUMF home page [1].

tory (ARIEL) (1.2) to not only reach further into more extreme neutron-proton ratios, but also to expand the capabilities of the Isotope Seperator and ACcelerator at TRIUMF (ISAC-TRIUMF) to performing three simultaneous experiments[58]. Upon completion ARIEL will allow investigations of isotopes relevant for the fields of of Nuclear Physics, Astrophysics, Nuclear Medicine, and Material Science that have previously been inaccessible.

The new ARIEL facility will utilize a 50 MeV 10 mA Continuous Wave (CW) linear electron accelerator, as well as the CANadian Rare-isotope facility with Electron-Beam ion source (CANREB). The second beamline will take advantage of a currently inactive beamline driven by the world's largest cyclotron[4] accelerating protons to 500 MeV with a current of 100 μ A[58]. ARIEL consists of Superconducting Radio Frequency (SRF) cavities that brings the electron beam to

50 MeV, followed by a target station where photo-fission occurs. After photo-fission, radioactive isotopes are created and undergo mass separation at the CANREB High Resolution Mass Seperator (HRMS), followed by a Radio-Frequency Quadrupole (RFQ) buncher and cooler (discussed in Chapter 3). The RFQ's purpose is to package the CW beam into discrete packets, as well as to decrease the beam's energy spread. After extraction from the RFQ the beam may be sent into the Electron Beam Ion Source (EBIS) where charge breeding, increasing the charge state of ions, may occur. Finally, the radioactive beam will be filtered with a Nier Spectrometer [30] and sent to ISAC-TRIUMF where the various experiments are located. Using ARIEL in conjunction with the Isotope Separator On-Line (ISOL) technique at Isotope Separator and ACcelerator (ISAC) (see Chapter 4) will for not only allow for a farther reach from the VOS, but enable an additional two beamlines to service the ISAC experimental halls. This will be accomplished by extending a currently unused beamline from the Cyclotron, and transporting it alongside ARIEL and adding it's own target station for the ISOL technique, as shown in Figure 1.3.



Figure 1.3: Schematic of ARIEL 1) The electron gun strips electrons from the ion beam, and gives them an initial kick of energy. 2) The electrons proceed to the e-linac where they are accelerated via RF cavities to 50 MeV.
3) Magnets steer the electron beam into the target hall. 4) The electron beam strikes the target producing photons that shatter atomic nuclei in the target, creating isotopes. 5) footopes are sorted by charge and mass in the mass separator magnets. 6) Isotopes are sorted by charge and mass in the TRIUMF cyclotron into ARIEL. This Figure was taken used with permission from a private communication [2].

Chapter 2

Principles of Ion Traps

An RFQ buncher and cooler is a three-dimensional trap that confines charged particles (in our case ions) as they undergo buffer gas collisions and are grouped into bunches. In order to understand the mechanisms behind the RFQ, we must first cover the theory behind its predecessor, the linear Paul trap, which confines ions in two-dimensions.

2.1 Paul Traps

The force from an electric field or magnetic field acting on a charged particle is described by Equation 2.1 [22]. In this equation, F is the force acting on an ion with a charge of q moving with a velocity v, in an electric field E, and a magnetic field B. This force is known as the *Lorentz Force*, and this is what enables the confinement of ions.

$$\vec{F} = q(\vec{E} + (\vec{v} \times \vec{B})) \tag{2.1}$$

Using this equation, one-dimensional confinement may be accomplished simply by using three electrodes to create a potential well (Figure 2.1). This confinement is possible by having the outer two hollow cylindrical electrodes, at a matching voltage higher than that of the inner electrode. This will create a potential well that will capture positively charged ions. In order to confine negatively charged ions, the potentials would have to be reversed.



Figure 2.1: Schematic of a three-electrode, one-dimensional trap. Each electrode is a hollow cylinder, the outer electrodes have a common bias, higher than that of the center electrode. This voltage scheme will confine positively charged ions along the electrode axis. To trap negative ions, the voltages will be reversed with the center electrode at the highest potential.

In this configuration, a charged ion will repelled by the outer two electrodes, and attracted to the central electrode. Thus, the ion is subjected to one-dimensional confinement. Two-dimensional confinement requires creating a two-dimensional potential well. However, this entails having a charge distribution or an electrode at the potential minimum. Having an electrode at the potential minimum would cause any trapped ions to collide with it, and as such would defeat the purpose of the trap. In order to trap charged ions in two-dimensions, we need a potential minimum in free-space. As the Earnshaw Theorem shows, it is impossible for a charged particle to remain stable in a purely static electric field [74]. However, it is still possible to trap an ion in free-space if we take advantage of time-dependent electric fields. The simplest way to accomplish time-dependent electric field, is to apply a sinusoidally oscillating field to a quadrupole electrode configuration. In this configuration, two pairs are 180° out of phase with each other, as shown in Figure 2.2. Using this configuration correctly allows us to produce an overall restoring force.

As the two electrode pairs are exactly half an oscillation period out of phase, an



Figure 2.2: Schematic of a quadrupole that enables trapping of charged particles in two-dimensions The voltage on each pair (red or blue) is 180° out of phase with the other pair. This creates a rotating saddle-point potential, which prevents the ions from escaping in the transverse direction.

ion will experience both an attraction, and a repulsion in different directions. As the charged ion is attracted to two of the electrodes, it will move towards one of them. If the applied voltages change quickly enough, the ion will be repelled by the same electrode before the ion collides with the electrode, and move towards one of the electrodes of the opposite polarity. This field oscillation causes an overall focusing effect [31] in two-dimensions, and results in a rotating saddle-point, as shown in Figure 2.3. The time averaged potential that an ion experiences due to the switching electrodes is known as the pseudo-potential, and is shown in Equation 2.2 [16].

$$V_{pseudo}(r) = \frac{eV_{RF}^2}{4mr_0^4 \omega_{RF}^2} r^2$$
(2.2)

Thus, this pseudo-potential, V_{pseudo} , is dependent on the ion's radial distance, denoted as *r*, the distance the electrodes are spaced from the beam axis, r_0 , the ion's mass, *m*, the ions charge, *e*, the frequency of oscillation applied to the electrodes, ω_{RF} , and the applied peak to peak voltage of the electrodes, V_{RF} . Although one can apply an arbitrary pseudopotential according to Equation 2.2, not all parameter combinations will yield desirable results. Qualitatively, if ω_{RF} is not large enough,



Figure 2.3: As the quadrupole structure shown in Figure 2.2 is undergoing its time-varying potential, it creates a rotating saddle potential. When the correct parameters are used, this rotating saddle-point is responsible for confinement in two-dimensions.

it is easy to picture the ions rolling down the saddle-point and out of the trap before the electrodes are able to switch their polarity, and force the ion back towards the center. In order to determine which sets of parameters can yield stable motion, one starts with the equation of motion of a ion in the centre of a quadrupole potential [62]. The solution of Laplace's Equation for a linear quadrupole (Equation 2.3) resembles stable Simple Harmonic Motion (SHM) in one direction, and unstable SHM in the other [22].

$$\Phi = \frac{\Phi_0}{2r_0^2} (y^2 - x^2) \tag{2.3}$$

Here, Φ is the potential experienced by the ion, Φ_0 is the potential difference across the electrode structure, r_0 is the spacing of the electrodes from the central axis, and x and y represent the position of the ion in the transverse plane. The second order differential equations used to describe the transverse motion in the x and y directions are standard equations for SHM described in Equations 2.4 through 2.6 with Φ oscillating in time according to Equation 2.7.

$$\frac{d^2x}{dt^2} - \omega^2 x = 0 \tag{2.4}$$

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \tag{2.5}$$

$$\omega^2 = \frac{e\Phi_0}{mr_0^2} \tag{2.6}$$

$$\Phi_0 = V_{RF} \cos(\omega_{RF} t) \tag{2.7}$$

Due to time dependence, Equations 2.4 to 2.7 do not have closed form solutions. Thus, another technique to analyse them is needed. With substitutions and manipulation they may be made to take the form of Equation 2.8, known as the Mathieu Equation. In this equation, ξ is a generic variable that may represent either *x* or *y*. In addition to ξ , two substitutions are used and defined in Equations 2.9 and 2.10. Here, *q* is known as the Mathieu stability parameter.

$$\frac{d^2\xi}{d\tau^2} + 2q\cos(2\tau)\xi = 0 \tag{2.8}$$

$$q = \frac{2eV_{RF}}{mr_0^2\omega_{RF}^2} \tag{2.9}$$

$$\tau = \frac{\omega_{RF}t}{2} \tag{2.10}$$

The condition for stable trapping is q < 0.908 [16]. The ions will undergo two types of motion inside the trap. The first is a slower macro-motion which occurs due to the pseudo-potential, and the second is a faster micro-motion due to the driving frequency. By using Equations 2.9 and 2.10, and defining $V_{pseudo} = \frac{m\omega_{macro}^2 r^2}{2}$, we obtain Equations 2.11 and 2.12.

$$\omega_{macro}^2 = \frac{q^2 \omega_{RF}^2}{8} \tag{2.11}$$

$$\omega_{macro}^2 = \frac{2e}{mr_0^2} V_{pseudo}(r_0) \tag{2.12}$$

Extending a two-dimensional Paul trap in the third direction, it's possible to confine ions radially while travelling down the z axis. This type of trap is known as a linear Paul trap [37], and commonly referred to as an RFQ due to the fact that it operational frequency is typically in the same range as radio-waves (*MHz*). See



Figure 2.4: A schematic of a Penning trap. The use of a grounded hyperbolic ring electrode with either end capped with positive DC biased hyperbolic end-caps allows trapping along the *z* direction. By adding a homogeneous magnetic field parallel to the same axis, ion trapping is possible in three-dimensions. This configuration comes very close to providing a parabolic trapping potential along the *z* axis. It should be noted that for an ideal Penning Trap $2z_0 = r_0$.

Section 4.3.1 for more information on the uses of an RFQ.

2.2 Penning Traps

In this section, the theory behind the hyperbolic Penning trap will be discussed. It was this variety of trap that was used to produce the measurements presented in Chapter 5. The hyperbolic Penning trap consists of a hyperbolic ring electrode, two hyperbolic caps with a DC bias, and a homogeneous magnetic field (Figure 2.4). Using this configuration, three-dimensional confinement is possible. The addition of the magnetic field will cause an ion with a component of its velocity perpendicular to the z-axis to move in a circular pattern in the x and y directions, while the DC bias will confine the ion along the z axis (axes defined in Figure 2.4).

An ideal hyperbolic Penning trap's electric field [9] takes on the form of Equation 2.13, where z and r are the longitudinal and radial distances between the center of the trap, V_{DC} is the end-cap bias voltage. The characteristic length, d_0 , is further defined as shown in Equation 2.14. It should also be noted that for an ideal Penning trap, the longitudinal and radial distances are related by $2z_0 = r_0$.

$$V_{(z,r)} = \frac{V_{DC}}{2d_0^2} (z^2 - \frac{r^2}{2})$$
(2.13)

$$d_0 = \sqrt{\frac{z_0^2}{2} + \frac{r_0^2}{4}} \tag{2.14}$$

Using this potential, and transferring into a cylindrical co-ordinate system, the force that an ion will undergo is simply:

$$F = m\vec{a} = -\frac{qV_{DC}}{d_0^2}(\vec{z} - \frac{\vec{r}}{2}) - qB\dot{\vec{r}} \times \hat{z}.$$
 (2.15)

Separating the radial (r) and axial (z) motions, we obtain Equations 2.16 and 2.17.

$$\ddot{z} = -\frac{qV_{DC}}{md_0^2}z\tag{2.16}$$

$$\ddot{r} = \frac{qV_{DC}}{2md_o^2}\vec{r} - \frac{qB}{m}\dot{\vec{r}} \times \hat{z}$$
(2.17)

Once again, by inspection it is clear that ions exhibit SHM in the axial direction. Rewriting this to solve for the oscillating frequency in the axial direction, we obtain ω_z as shown in Equation 2.18.

$$\omega_z = \sqrt{\frac{qV_{DC}}{md_0^2}} \tag{2.18}$$

As the cyclotron frequency [41] is given by Equation 2.19, we may combine it with Equations 2.17 and 2.18 to obtain the radial equation of motion as presented in Equation 2.20.

$$\omega_c = \frac{q}{m}B \tag{2.19}$$

$$\ddot{\vec{r}} = \frac{\omega_z^2}{2}\vec{r} + \omega_c \dot{\vec{r}} \times \hat{z}$$
(2.20)

Rewriting Equation 2.20 into Cartesian co-ordinates, Equations 2.21 and 2.22

are obtained.

$$\ddot{\vec{x}} - \omega_c \dot{y} - \frac{\omega_z^2}{2} x = 0$$
(2.21)

$$\ddot{\vec{y}} - \omega_c \dot{x} - \frac{\omega_z^2}{2} y = 0$$
(2.22)

Defining a general co-ordinate u = x + iy these two equations may be combined into a single relation as shown in Equation 2.23.

$$\ddot{u} = -i\omega_c \dot{u} + \frac{\omega_z^2}{2}u \tag{2.23}$$

Solving for the eigen-frequencies of Equation 2.23 leads to Equation 2.24.

$$\omega_{\pm} = \frac{\omega_c}{2} \pm \frac{\omega_c}{2} \sqrt{1 - \frac{2\omega_z^2}{\omega_c^2}}$$
(2.24)

Rearranging this, we obtain Equations 2.25 to 2.27.

$$\omega_c = \omega_+ + \omega_- \tag{2.25}$$

$$\omega_c^2 = \omega_+^2 + \omega_-^2 + \omega_z^2 \tag{2.26}$$

$$\omega_z^2 = 2\omega_+\omega_- \tag{2.27}$$

As ω_z is much lower than that of ω_c , it is possible to take the Taylor series approximation of Equation 2.24 to obtain Equations 2.28 and 2.29. Here ω_- is known as the magnetron motion, and ω_+ is the reduced cyclotron motion, as it is slightly lower than that of ω_c .

$$\omega_{-} \approx \frac{\omega_{z}^{2}}{2\omega_{c}}$$
(2.28)

$$\omega_{+} \approx \omega_{c} - \frac{\omega_{z}^{2}}{2\omega_{c}}$$
(2.29)



Figure 2.5: This diagram shows the calculated trajectory of the three eigenmotions of an ion present in a Penning trap. The first motion along the \hat{z} , shown in red, is due to the axial motion ω_z , the second is a faster motion at the reduced cyclotron frequency, ω_+ shown in blue, and the final, slowest motion is at a magnetron frequency ω_- , is shown in black.

Thus, the three eigen motions of an ion in a Penning trap are ω_z in combination with ω_- and ω_+ . This motion is shown in Figure 2.5. For a more thorough derivation of these motions, please consult reference [36].

Chapter 3

CANadian Rare-isotope facility with Electron-Beam ion source Radio Frequency Quadrupole Buncher and Cooler

The CANREB buncher and cooler is a device that will take the continuous beam of isotopes from ARIEL'S HRMS, lower the energy spread of the beam via buffer gas collisions, and bunch the beam into discrete pulses of ions to be sent to the next stage of CANREB. Although simulations that were done on all the various regions of the CANREB buncher and cooler, see Figure 3.4, the focus of this chapter will be on the injection optics. The extraction optics, as well as the bunching and cooling simulations are outside of the scope of this work, but are documented in the CANREB design document [13].

It should be noted that the taper angle section, studied within Sections 3.3.2 and 3.3.3, will not be used in the final RFQ design due to several reasons. The first reason is that it was decided that in order to deliver low energy, singly charged ion beams to select experiments in ISAC, the RFQ will have to be able to utilize reverse extraction This means the injected ions will leave through the same optics as the beam originally entered. This need of reflecting the beam arises from the

optics downstream of the RFQ being designed for non-singly charged ions at higher energies (10's of keV as opposed to 100's of eV). The only other alternative lowenergy beam transport without removing the taper-section from the injection optics would be to add an additional beamline parallel to the existing highly charged ion beamline. This additional beamline would be designed for the lower energies, however this was deemed infeasible due to both funding and space limitations. As the taper-section would not be able to support the reverse extraction from the RFQ, it was decided their added benefits (in the form of an enlarged acceptance) did not outweigh the additional complications that would arise in delivering the beam to Isotope Separator and ACcelerator facility 1 (ISAC1).

3.1 Introduction to Emittance and Acceptance

An important figure of merit for beam transport is the beam emittance. The beam emittance is found by taking a transverse slice, in the x - y plane of the beam, and recording each ion's position (x) and momentum ($\vec{P'}$) on a single graph. In the case of the axial momentum, $\vec{P_{axial}}$, being much greater than that of the transverse momentum, $\vec{P_{trans}}$, the small angle approximation may be used such that the angle of each ion's trajectory with respect to the beam axis is $x' \approx \frac{P_{trans}}{P_{axial}}$. An example of the emittance plot is shown in Figure 3.1 with the area of the spanned region known as the emittance of the beam. Because emittance is the area found on a mm vs mrad plot, it has units of *mm mrad*, however, due to convention a factor of π may also added into the units of emittance creating an overall unit of π *mm mrad* (note this factor of π is not used within this report). This comes from the fact most ideal emittances are elliptical.

Ideally in a transversely symmetric potential, the shape of a beam (in the x, x' phase space) is an ellipsoid, however, beams are not limited to this shape as beams are known to become filamented with spiralling arms coming off of the body of the ellipse (See Figure 3.2). This filamentation is due to higher order effects that are beyond the scope of this work. In order to avoid over estimating the acceptance of the device due to including these filimentations, they may be removed. This removal of the filaments is accomplished by rotating the acceptance ellipse, and then removing the arms. The rotation of the ellipse is used to allow for more


Figure 3.1: In order to find the emittance of the beam at a particular cross section, each ion's position, as well as its current angle that its velocity is making with the beam axis is plotted on two separate axes.

precise cutting away of the arms, without removing sections of the main body of the ellipse.

When discussing a component it is common to refer the largest possible emittance it can accept as the *acceptance* of the device. In order to describe the beam's orientation as well as its phase space area, *Twiss parameters* [28] are used. The Twiss parameters based on the half-width of the emittance in both x and x' directions, as well as the orientation in the x, x' phase space. To describe the emittance of the beam beyond simply the area, the Twiss parameters may be utilized as described in Equation 3.1. A graphical representation of the Twiss parameters are shown in Figure 3.3. In order to quantitatively find if a beam is either converging (narrowing) or diverging (widening), α , as defined in Equation 3.4 may be used. If $\alpha > 0$ the beam is converging, if $\alpha < 0$ the beam is diverging, and finally if $\alpha = 0$ the beam section may be described as either a waist (minimum) or anti-waist (maximum). Throughout the simulations these values were calculated via Equations 3.2-3.4 with ε being defined as the emittance of a Root Mean Square (RMS).



Figure 3.2: Left: A non-cut acceptance ellipse with prominent filaments. **Center:** A post-cut acceptance ellipse with the majority of filamentation removed. Notice that the cuts are not perfect, and what is left is a *dog-bone* shape. **Right:** The superimposed processed data from the eight phases, each colour representing different phase. **Bottom Row:** A more typical acceptance ellipse shape without the dog-bone appearance as it undergoes cutting and fitting. Note: The central images have a different scale due to rotation transformations used to enable better cutting of filaments.

$$\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2 \tag{3.1}$$

$$\gamma = \frac{var(x')}{\varepsilon} \tag{3.2}$$

$$\beta = \frac{var(x)}{\varepsilon} \tag{3.3}$$

$$\alpha = \frac{-covar(x, x')}{\varepsilon}$$
(3.4)

3.1.1 Acceptance Calculations

As the RFQ provides a time varying potential to trap ions in the transverse directions, \hat{x} and \hat{y} , ions entering the RFQ at different points during the Radio Frequency (RF) cycle, will encounter different fields. In order to ensure an accurate ac-



Figure 3.3: To describe the emittance of the beam beyond simply the area, the Twiss parameters may be utilized as described in Equation 3.1. Here, the graphical representation of the Twiss parameters is shown with $\sqrt{\beta\varepsilon}$ defined as the half width of the beam, $\sqrt{\gamma\varepsilon}$ as the half width of the divergence, and α describing the orientation of the ellipse. γ is found by $\gamma = \frac{1+\alpha^2}{\beta}$.

ceptance simulation of a CW beam, the ions were simulated at eight evenly spaced RF phases from 0° to 360° during the RF cycle. The acceptance of each of these phases was viewed individually, and any large filaments were removed such that 90 % of the fitted ellipse was filled with the fitting program (See Figure 3.2). The acceptance plots in this report include results of the averaged acceptance of the eight phases that the ions were simulated to enter the RFQ.

After each phase was processed and was cut individually, the area of overlapping acceptance ellipses was calculated. Thus the overlapping acceptance is the minimum acceptance that can be guaranteed. Due to the nature of the cutting and averaging process, the average acceptance of the eight different phase injection will always be higher than the overlapping acceptance. Occasionally a single phase may skew the overlapping acceptance by being small enough to dominate the fit. In order to account for this situation when it occurred, the emittance of each phase were averaged together. The reported averaged Twiss parameters arise from averaging them across all eight phases.

3.2 Requirements

The requirements of the CANREB RFQ [13] are to maximize the efficiency of injection, extraction, and cooling of a CW beam, into an Alternating Current (AC) beam. More specifically, the RFQ must be capable of handling a beam of ions with a mass range from 15 Atomic Mass Unit (AMU) to 250 AMU. Although the exact injection energy spread is unknown, it was decided that if a simulated energy spread range of ± 50 eV could be transported without significant (> 50%) efficiency drops, the RFQ should meets specifications. In addition to this, the acceptance of the device must be, at a minimum, 15 mm mrad. In order to ensure that these requirements were met, multiple investigations were carried out for the injection region including the implementation of an immersion lens (Section 3.3.1) as well as a taper region with both the taper's angle (Section 3.3.2), and its length (Section 3.3.3).

3.3 Injection Optics Design

As mentioned previously, in order to maximize the normalized acceptance of the RFQ, simulations of the injection optics were carried out for both the immersion lens (Section 3.3.1) and taper section (Sections 3.3.2 and 3.3.3). In order to optimize both the immersion lens biasing, the tapered section's angle, as well as the taper section's length, two of the three variable had to be held constant as the third was investigated. The results of these simulations are discussed in the following sections.



Figure 3.4: The CANREB RFQ system is comprised of three distinct regions. The first being the injection optics, and the latter two being the RFQ section, and the ejection optics. For details concerning the RFQ section, as well as the ejection optics, refer to the CANREB design document [13] from which this figure was taken with permission.

3.3.1 Immersion Lens Simulations

In order to understand how the immersion lens focuses the beam, one first has to understand the physics of the Einzel lens [8] from which it is derived. The use of an Einzel lens in beamlines is quite common for focusing in both transverse directions and consists of three hollow, cylindrical electrodes that the ions will sequentially travel through. (see Figure 3.5). The outer electrodes of the Einzel lens are held at the same potential while the central cylinder is biased separately. The potential differences between the cylinders produces a focussing field. However, due to the fact that electric fields are conservative, the normalized emittance must remain constant, thus increasing the angular spread of the beam (Section 3.1. The use of an Einzel lens allows the rotation of the emittance of a beam from one of one occupying a large spatial spread and little angular spread, to one of a large angular spread with a small spatial spread. This conversion from spatial spread into angular spread can be seen in Figure 3.6.

In the case of the CANREB RFQ it was decided that an entire Einzel lens would not be needed, and as such only two of the three electrodes were incorporated (shown in Figure 3.7).

This modified Einzel lens is known as an immersion lens [5] and has the advantage of bringing the beam to a different potential. Thus, upon exiting the immersion lens, not only are the ions focused in position space, but they have decreased



Figure 3.5: A schematic of a three component Einzel lens. This Einzel lens consists of three hollow, cylindrical electrodes, with the outer two (shown in blue) have the same bias, and a different bias is applied to the central electrode (shown in green).

their longitudinal speed as they are at a higher potential energy than they originally started. This allows for more effective cooling via buffer gas collisions as the ions will encounter the gas at lower speeds, and allows the ions to be extracted afterwards with the desired energy by lowering their potential energy.

The immersion lens parameter that was investigated was the effect that the lens biasing had on the acceptance of the CANREB RFQ in conjunction with different mass ranges. As the Rare Isotope Beam (RIB) is entering the RFQ with an energy of 60 keV the simulated bias ranged from a minimum of 0 V up to a maximum of 55 kV. At this point the acceptance of the device was clearly dropping, as shown



Figure 3.6: This figure depicts the electric field, as well as example ion trajectories of the Einzel lens in Figure 3.5. The red lines are equipotential lines, the blue lines are ion trajectories (moving from the top left to bottom right), and the brown grid is the cross section of the three Enzel lenses. In this simulation, the potential is caused by biasing the two outer lenses biased identically, and at a lower potential than the center electrode. This figure was produced using Simion 8.1.

in Figure 3.8. As the simulation's parameter space included not only lens biasing, but also the length and angle of the taper section, as well as ion masses, only a sampling of the simulations will be shown in this thesis.

3.3.2 Taper Electrode Angle Simulations

The following results are an expansion upon the work at National Superconducting Cyclotron Laboratory (NSCL) by Brad Barquest in his PhD thesis [6]. The theory behind using a tapered section (Figure 3.9) is similar to that of using a funnel (see Figure 3.10), with the wider the entrance region, the easier to accept the incoming ions. If the angle (as shown in Figure 3.10 is 0° , the funnel simply becomes a hollow cylinder and ions may simply miss the entire entrance. As the taper angle



Figure 3.7: Ion trajectories, shown in blue, are focused as they pass through the immersion lens, shown as a brown grid, as they travel from the top left towards the bottom right. This figure was produced using Simion 8.1.

increases such this cylinder becomes more funnel like, the acceptance is expected to increase as less ions will miss entrance.

This effect was observed in the simulations, however, once they began to be too large, the acceptance once again decreased. In this analogy, when the angle is closer to 90° , the funnel ceases to be a funnel and starts to become a wall with a hole in the center. In this case, it is expected that the majority of ions will simply splash against the wall and not be redirected and, as such, not successfully be accepted. To visualize the effect that the angle has on the acceptance of the RFQ several screen shots of ion trajectories are shown in Figure 3.11 with the ions entering from above, and being accepted towards the bottom.

3.3.3 Taper Electrode Length Simulations

While in the designing stage of the RFQ it became clear early on that space is at a premium. As such, when designing the device, effort was taken to limit its length



Figure 3.8: This figure describes a sample scan of acceptance simulations as a function of immersion lens biases from 0 V-55 kV. This particular set of simulations were carried out with a taper section of 150 mm long with an angle of 0° .

when possible. For the injection optics, this meant limiting the length of the taper section while still maximizing the acceptance of the device. The three electrode simulated were 150 mm, 100 mm, and 50 mm.

As mentioned previously, the acceptance of the device was simulated using the parameter space of length and angle of the taper section, as well as the ion's mass. Figure 3.13 shows, there is a definite effect on acceptance of lens bias, taper angle, and the ion's mass. The maximum acceptances of the three taper lengths were compared by mapping out this parameter space for each one, and it was found that comparable acceptances were possible at each length for different angles and lens biasing. This can be seen from the shorter, 50 mm taper section's acceptance plot



Figure 3.9: A rendered drawing of the section between the some injection optics (top right) and the cooling region (bottom left). This particular image is from the 150 *mm* Taper section with an angle of 2° . In this figure, the beam enters from the upper right and travels towards the bottom left.

in Figure 3.13. After it was found to be possible for each taper section length to have comparable acceptances, when the angles were changed, it was concluded that the shortest length simulated would be used. Thus, it was decided to use the 50 mm taper section with an angle of 4° at a bias of 45 kV. Using these parameters, an acceptance of 67 ± 7 mm mrad for masses as low as 15 is expected, with an acceptance of 72 ± 7 mm mrad for masses above 30 AMU. This result exceeds the technical requirements of 15 mm mrad [13] by a factor of five.

3.3.4 Energy Spread Simulations

As it is practically impossible to have a uniform beam energy, an investigation was completed to find out the affect the ion energy spread has on the acceptance. It is not well known *a priori* what the energy spread will be as the ions enter the RFQ. For this reason, the effect of energy spread on the acceptance of the RFQ was



Figure 3.10: A diagram of a funnel with a disperse substance being poured into the top, and leaving the funnel after being redirected.

investigated within a range of \pm 50 eV from the targeted beam energy of 60 keV. This range was chosen as an estimated upper bound on the energy spread based on the experience obtained from studies of the of the NSCL RFQ [6]. It should be mentioned that the RFQ in its entirety was biased to 59.8 keV thus resulting in an ion energy of 200 eV upon entry. The energy dependence was investigated using both the 100 mm and 50 mm taper length as it was already decided that the 150 mm length was unnecessarily large.

Figure 3.14 displays a typical selection of simulations that were done for angles 2° to 6° for the shorter, 50 mm taper section. From this figure, it is clear that the RFQ was designed for a beam energy of 60 keV to satisfy CANREB requirements, as the largest acceptances are in that row. However, the purpose of these simulations was not to confirm the operational target, but to confirm an energy spread of \pm 50 eV will not be detrimental (loss> 50%) to the acceptance of the RFQ. The largest acceptance for both cases is the 4° case for both mass 15 AMU and 30 AMU. The results of these simulations shows that applying a \pm 25% energy spread for the mass 15 case, only a 4% drop in the acceptance occurred. For the heavier, mass 30 AMU simulations, it was found that a \pm 25% energy spread resulted in a 14% drop in the acceptance of the device. Thus is was shown that the injection optics of the CANREB RFQ may handle a 25% energy spread without detrimental loss.



Figure 3.11: These images are a sampling of simulations with the same ions to show the effect of the taper angle. The ions in these images enter from the top. **Left:** a taper section with an angle of 0°, causes many ions to miss the point of entry. **Center:** an angle of 9° manages to capture additional ions that would have otherwise have escaped. **Right:** an angle of 90° allows nearly all of the would-be ions to escape the device. **Note:** The position and angle spread of the incoming ions was chosen to be much larger than the acceptance of the device in order to fully trace out the acceptance of the device.



Figure 3.12: This figure displays a sample of the acceptance simulations investigating the angle dependence. This set of data came from simulating a taper section of 150 mm length, with an immersion lens bias of 55 kV. As can be seen, there is clearly an angle dependence that peaks near the 2° case.



Figure 3.13: This figure depicts a typical acceptance plot as a function of taper angle, immersion lens bias, as well as mass. Similar acceptances to these plots may also be reached by different angles and biases of different length taper sections. The ion's energy in these simulations was 200 eV.



Figure 3.14: This figure summarizes the ion energy spread investigations. Energies both 50 eV above and below the ideal 60 keV energy were used at different angles for both mass 15 AMU and 30 AMU.

Chapter 4

Introduction to the Isotope Separator and ACcelerator (ISAC) and TRIUMF's Ion Trap for Atomic and Nuclear science (TITAN)

4.1 Rare Isotope Beam Facilities

For many years, TRIUMF has been providing rare and exotic nuclei to various experiments in the form of RIBs. The two varieties of RIB production are 'in-flight' facilities [15, 59], such as NSCL or RIKEN, as well as ISOL facilities [65], such as TRIUMF or ISOLDE at CERN.

The in-flight technique is accomplished by impinging heavier ion beams into relatively lighter targets. The beam may then undergo fragmentation and/or fission and form unstable nuclei. Both of these mechanisms may produce a large range of isotopes of interest [59], but it may do so at yields too low for many experiments when far from the valley of stability. As the initial beam was at relativistic speeds, the daughter nuclei will have the majority of their momenta in the axial direction.

This translates into the RIB to have a narrow angular distribution as $x' \approx \frac{P_a xial}{P_t rans}$ they are transported to experiments.

The ISOL technique [65] employed at TRIUMF consists of using a high energy proton beam at 500 MeV impinging on a many-layered target, in the case of Chapter 5 UC_x was used as this target. Once the high intensity proton beam (100 μ *A*) hits with the target, it breaks the target nuclei into a range of both stable and unstable isotopes. These isotopes will then diffuse towards the ion source where they are ionized, and subsequently extracted into the beamline. Once the nuclei arrive at the ionization region, those with an ionization energy below 6 eV will be surface ionized [10]. Species whose ionization energy is above 6 eV, such as alkaline earth metals and transition metals with ionization energies up to 9 eV [10], may be laser ionized (See Section 4.1.1 for additional information). Although the ISOL method may produce higher yields than the in-flight technique, it can suffer due to chemistry effects as the nuclei produced are nearly at rest, and may chemically interact before emerging from the target.

4.1.1 TRIUMF's Resonant Ionization Laser Ion Source (TRILIS)

As not all ionization energies for Species of Interest (SOI) are below 6 eV, TRI-UMF, as well as many other RIB facilities have had to develop other methods of ionization. A popular choice of accomplishing this is using a Resonant Ionization Laser Ion Source (RILIS). TRIUMF's Resonant Ionization Laser Ion Source (TRILIS) utilizes three TiSa lasers which may operate between 680 nm to 990 nm at 10 kHz [55]. Through the use of of frequency doubling, tripling, and quadrupling TRILIS may produce a large portion of both UltraViolet (UV) and Infrared (IR) wavelengths [7, 44, 63]. By utilizing these lasers, step-wise excitation of specific elements is possible (see Figure 4.1 for what is possible at TRIUMF). In the case of aluminium two lasers of wavelengths 308 nm and 309 nm are used to excite the atom from its split ground state. Once it's excited a non-resonant laser of 532 nm is used once again excite the aluminium atom into a state that is prone to ionization [54].



Figure 4.1: Green: Elements that have been delivered at TRIUMF via laser ionization. **Blue:** Elements that are currently undergoing testing at TRI-UMF for their feasibility with laser ionization. **Grey:** Elements that are theoretically possible via laser ionization, but have yet to be tested at TRIUMF. This figure is an updated version of that in reference [55], obtained through a private communication [54], and used with permission.

4.2 The TRIUMF Facility and ISAC

TRIUMF uses the ISOL technique with a 100 μ A and 500 MeV proton beam impinging on a target. The device accelerating the proton beam is TRIUMF's cyclotron, the largest in the world [4]. After the production of the desired isotopes ionization must occur, and the unwanted species are rejected before being delivered to an experiment. This is done through a magnetic dipole mass separator, which filters out the undesired species via their charge over mass ratio (q/m). However, as a limit of the HRMS's resolution being approximately $\frac{\delta m}{m} \approx 2500$ [34], isobars that are too close in mass to the SOI will still be present, and may, depending on experiment specific sensitivities, need to be cleaned out prior to experiments. A schematic of ISAC including the target and mass separators are shown in Figure 4.2.



Figure 4.2: Schematic of ISAC beam production and separation. The proton beam from the main cyclotron impinges on the ISAC target, resulting in a variety of isotopes. After the newly created isotopes are extracted from the ion source, they undergo two phases of mass separation, with the second being the HRMS. At this point, the mass separated beam is delivered to the various experiments in the ISAC experimental hall. This schematic is taken with permission from Reference [19].

4.3 TRIUMF's Ion Trap for Atomic and Nuclear Science (TITAN)

TRIUMF's Ion Trap for Atomic and Nuclear science (TITAN) is a mass spectrometry experiment located at ISAC-TRIUMF, in Vancouver, Canada, and makes full use of the RIB produced at TRIUMF by utilizing a high-precision hyperbolic Penning trap, as discussed in Section 2.2. In addition to the Penning trap, TITAN has two additional traps that are currently in use, including TITAN's RFQ buncher and cooler, as well as the Electron Beam Ion Trap (EBIT). An overview of the TITAN set-up is shown in Figure 4.3. Please note that the Cooler PEnning Trap (CPET), where cooling of the ions will occur, has yet to be implemented in the beamline.

4.3.1 TITAN's RFQ Buncher and Cooler

After delivery of the RIB from ISAC to TITAN 4.3, the beam will first enter TITAN's RFQ buncher and cooler [24]. The purpose of the RFQ is to take the continuous RIB from ISAC and form it into discrete bunches of ions with a decreased emittance. This is accomplished via helium buffer gas collisions (see Figure 4.4). Although ISAC is designed to provide beam of ions with a kinetic energy in the range of 12 keV to 60 keV [24], the beam's energy may be changed by simply by raising the RFQ's bias and converting the desired portion of the beam's kinetic energy into potential. When the ions enter the RFQ they begin transferring their kinetic energy into the buffer gas via collisions. In order to ensure that the ions do not simply stop due to the loss of all their kinetic energy, a decreasing electric field is applied over the course of the first 23 electrodes, dropping a total of 6.6 V (see Figure 4.4). This decreasing electric field will cause the positively charged ions to move towards the extraction optics, and as such this field is commonly refereed to as a drag field. When it is time for extraction, the last electrode's (# 24) potential is lowered to -26.6 V while electrode # 22's potential is increased to +13.4 V. This allows the accumulated ions at the bottom of the potential well to be extracted as a single bunch, while the ions currently entering the RFQ are blocked and are not allowed to exit.

As the theory of trapping transversely in an RFQ was described in Section 2.1, simply a reference to the operational parameters will be mentioned here. The



Figure 4.3: Schematic of the TITAN set-up which consists several sections. The RFQ (Section 4.3.1) receives the beam from ISAC for bunching and cooling. The second, and optional stage is charge breeding in the EBIT (Section 4.3.2). Thirdly, the ions will enter MPET (Section 4.3.3) for excitation. Finally the ions will strike an MCP to enable mass determination via the TOF technique (Section 4.4). This figure was taken with permission from the publicly available TITAN home page [3].



Figure 4.4: The schematic of the TITAN RFQ which consists of 24 electrodes forming a potential well. As the continuous beam enters the RFQ it undergoes collisions with the helium buffer gas. This causes the ions to lose kinetic energy to the buffer gas as they fall deeper into the potential well. Upon extraction, the last several electrodes change their potential to allow the newly cooled bunched beam to exit. This figure was based on that in reference [24]

TITAN RFQ has a total length of 700 mm with an r_0 of 11 mm, and is operated at a Helium pressure of approximately 0.01 mbar. The RF parameters are such that the V_{PP} is 400 V and driven by a square wave at 1.2 MHz.

Once the beam exists the RFQ after being cooled and bunched, it may be steered to either the EBIT where it may be charge bred, or it may pass through the Bradbury Nielsen Gate (BNG) for further mass filtering [23], then enter the MPET.

4.3.2 TITAN's EBIT

In the case where additional precision is needed, the EBIT may be used to chargebreed ions into higher charged states [53]. As both electric and magnetic forces depend on the charge of an ion this is an efficient means of increasing the precision of mass measurements. The exact relation of the uncertainty in mass measurements at TITAN is shown by Equation 4.1 with q representing the charge state, Bthe magnetic field, m the ion's mass, and N being the number of ions detected.

$$\frac{m}{\delta m} \approx \frac{T_{RF} q \sqrt{N}}{m} \tag{4.1}$$



Figure 4.5: The TITAN EBIT consists of a potential well where the ions are struck by an incoming electron beam. The electron beam strips off additional electrons from the ions, thus increasing the ions charge state. Confinement during the charge breeding process is done through a parallel magnetic field, and electrostaticly biased electrodes. This image is courtesy of M. Simon.

The EBIT consists of an electron gun, a collector, and electrodes to provide a potential well along the axis, as well as a superconducting magnet to provide a magnetic field. The radial confinement of the ions is provided by the magnetic field parallel with the beam axis, as shown in Figure 4.5. In addition to radial confinement, the ions must be trapped axially during the charge breeding process. This is accomplished by using cylindrical electrodes to form a potential well. The bottom of this well is where the ion cloud forms and charge breeding occurs. The charge breeding is brought about by an electron beam, emitted from an electron-gun, typically operating at, 5 keV with a current of 400 mA [53]. When the electron beam encounters the ion cloud, it may knock off additional electrons from the ions, thus increasing the charge state of the ions. Once charge breeding has been accomplished, the wall of the potential well is lowered, and the charge breed and bunched beam will be extracted towards MPET for mass measurement.

It should be noted that charge breeding of the RIB is typically limited by two factors. The first limitation is the lifetime of the SOI. While it is true that the longer the time charge breeding occurs for abunch, a larger portion of the bunch will be ionized, if the SOI has a short enough lifetime, they may decay away before extraction. The second limitation of charge breeding ions is the ionization energy.

As the electron beam has a particular energy, the capability of charge breeding is limited to ionization energies below the e-beam energy.

4.3.3 TITAN's MPET

The MPET is the work-horse behind TITAN as it allows us to make mass measurements. The characteristic dimensions of MPET (See Section 2.2) are a $r_0 = 15$ mm, $z_0 = 11.785$ mm, and $d_0 = 11.21$ mm. As discussed in Section 2.2, axial confinement is achieved via two electrostatic endcaps, where radial confinement is accomplished via a magnetic field. In order to excite the trapped ions from the magnetron motion to the reduced cyclotron motion, an RF field is applied to the trapping region. It is by exciting the faster reduced cyclotron motion that the mass of the isotopes may be determined via Time-Of-Flight measurements as discussed Section 4.4.

4.4 Time-Of-Flight Mass Measurements in Penning Traps

At TITAN, ion masses are are determined via the Time-Of-Flight Ion Cyclotron Resonance (TOF-ICR) technique [49] in which a range of excitation frequencies are applied to the ion, centred near the expected cyclotron frequency, v_c . Whenever a higher portion of the motion is converted into the reduced cyclotron motion, it will result in a decreased TOF from MPET to the MCP (See Figure 4.7). This excitation is simply driving by quadrupole RF field within MPET.

When the axial confinement is removed and the ions are ejected out of MPET, the ion navigates a magnetic field gradient, and the radial energy is converted into axial energy. Thus an ion with a higher reduced cyclotron frequency is ejected with a higher energy, and has a decreased TOF to the MCP.

In the case of a $\omega_{RF} = \omega_c$, the amplitude of the radial motion is defined by Equation 4.2 with ρ representing the radial position, *t* is the time, k_0 is defined by Equation 4.3, r_0 is the radius of the ring electrode, and $\Delta \phi = \omega_{RF} - \omega_c$.

$$\rho^{\pm}(t) = \rho^{\pm}(0)\cos(\frac{k_o t}{2}) \pm \rho^{\mp}(0)e^{i\Delta\phi}\sin(\frac{k_0 t}{2})$$
(4.2)

$$k_0 = \frac{V_{RF}q}{2r_0^2} \frac{1}{\omega_+ - \omega_-}$$
(4.3)

In the case where the ions are injected with a pure magnetron $\rho^{-}(0) = \rho_0$ ($\rho_0 > 0$), or pure cyclotron motion, $\rho^{+}(0) = 0$, the two motions will be converted completely from one form into the other in an excitation time of $T_{RF} = \frac{\pi}{k_0}$. If an excitation time of $T_{RF} = 2\frac{\pi}{k_0}$ is applied to the ion, the ion will be again in its original state.

However, what if the driving frequency of $\omega_{RF} \neq \omega_c$? This is known as the off-resonance case and results in an incomplete conversion from one motion into the other. The relationship between an ion with an initially pure magnetron motion, and the excitation time is shown in Equation 4.4, and is graphically shown in Figure 4.6 with the Full Width Half Max (FWHM) of the central peak approximately $\frac{0.8}{T_{RF}}$ [49].

$$E_r \propto \frac{4}{k_0^2} \sin^2(\frac{k_0}{2}T_{RF})$$
 (4.4)

As the ion within MPET will have a kinetic energy of approximately E_r , as it passes through the magnetic field, it's magnetic moment of $\vec{\mu} = \frac{E_r(\omega_{RF})}{B}\hat{e}$ will cause it to experience the axial force shown in Equation 4.5.

$$\vec{F}(\boldsymbol{\omega}_{RF}, z) = -\vec{\boldsymbol{\mu}}(\boldsymbol{\omega}_{RF}) \cdot \nabla \vec{\boldsymbol{B}}(z)$$
(4.5)

The TOF of an ion between point z_0 and z_f with an initial total energy of E_0 , and with electric and magnetic fields along the axis, of strengths V(z) and B(z) may then be determined via Equation 4.6 [49].

$$T(\omega_{RF} = \int_{z_0}^{z_f} \left(\frac{m}{2[E_0 - qV(z) - \mu \,\omega_{RF} B(z)]}\right)^{\frac{1}{2}} dz$$
(4.6)

By combining Equations 4.4 and 4.6 and setting $k_0 = \frac{\pi}{T_{RF}}$ the plot on Figure 4.7.



Figure 4.6: Normalized radial energy as a function of injection time and excitation time. Here Δv_{RF} represents the phase that the ion enters the penning trap, and T_{RF} represents the excitation time. This figure uses $T_{RF} = \frac{k_{\odot}}{\pi}$. This figure was used with permission from reference [49].

4.4.1 Dipole Cleaning

As mentioned previously, the limited resolution of the pre-TITAN mass separator is $\frac{\delta m}{m} = 2500$ [34]. While this may be acceptable for some experiments it is insufficient to resolve the species of interest from isobaric contaminants. In order to expel the isobars from the trap as to prevent them from shifting the resonance frequency [18], dipole cleaning is used.

Dipole cleaning is accomplished by applying an electric RF field to the trapped ions in MPET. This is implemented by biasing an electrode pair of the four electrode slices of MPET 180° out of phase from one another (Figure 4.8). The frequency chosen is that of the eigen frequency of the unwanted isobars. When a sufficiently long excitation time is applied with a high enough amplitude, the motion of the isomer will grow large enough that it will either strike the side of trap or gain enough energy to escape the trap along the axis. Both of these options will eliminate the unwanted contaminant from our TOF spectrum [40].



Figure 4.7: The solid line represents the theoretical TOF of on and off cyclotron resonance excitation from Equation 4.6. The dashed line is the superimposed energy due to excitation. This figure was used with permission from reference [49].

4.4.2 Sources of Uncertainties

Non-Uniformity of Magnetic Field

Although extreme care was taken in the construction of TITAN'S MPET, it is impossible to to ensure a idealized magnetic field. This is due to limitations of alignment, the finite size of the solenoid, as well as imperfections in the material itself [20]. The frequency shift from these misalignments and material imperfections is given by Equation 4.7 [17].

$$\Delta v_c = \beta_2 [(z^2 - \rho_+^2) - \frac{v_-}{v_c} (\rho_+^2 + \rho_-^2)]$$
(4.7)



Figure 4.8: In order to apply dipole cleaning, two electrodes opposite from each other (in this case blue and red) undergo a time-varying potential 180° out of phase from one another. In the case of TITAN, the electrodes that apply this electric field are slices of the so-called compensation electrode in MPET.

Here, β_2 is the quadrupole coefficient of the magnetic field inhomogeneity, z is the amplitude of the axial oscillation, and ρ_- is the radius of the magnetron motion, and ρ_+ is the radius of the reduced cyclotron motion. The upper limit of the shift in cyclotron frequency at TITAN is given by Equation 4.8 with the Δ representing the difference in mass (in AMU) between the reference ion, and the SOI.

$$\frac{\Delta R}{R} < 4.3 \times 10^{-10} \frac{\Delta m}{q} \tag{4.8}$$

Relativistic Effects

As the trapped ions undergo reduced cyclotron motion, their velocity may enter the realm where relativistic effects may occur. The velocity of an ion travelling at a radius of ρ_+ at a frequency of ω_+ is simply $V_+ = \omega_+ \rho_+$. When fitting the TOF spectrum, both ρ_+ and ω_+ are determined. As an example, for the measurement of ²⁹Al (see Chapter 5) a radius of approximately 2 mm was found with a frequency of about 1 960 026 Hz. This produces a velocity of $8 \times 10^{-5}c$, with *c* being the speed of light, thus giving us a correction factor of $\gamma \approx 1.000000003$. Using this correction factor to obtain the relativistic mass, the cyclotron frequency is given by Equation 4.9, leading to an effect of 1.6 Parts Per Billion (PPB).

$$\omega_{c(rel)} = \frac{\Delta m}{q} \tag{4.9}$$

Ion-Ion Interaction

With the goal of obtaining high-precision and high-accuracy mass measurements, an emphasis must be placed on understanding what has been trapped inside of MPET. This includes not only ions of similar masses to the SOI, but also the number of ions that were successfully trapped simultaneously. Through the use of the HRMS and the BNG, the majority of contaminants may be removed before entering MPET. However if multiple SOI are trapped simultaneously, the center frequency may still be altered due to Coulomb interactions, as well as a modification of the trapping potential [66]. In order to compensate for multiple ions being trapped at once, a count-class analysis [48] may be used.

The count-class analysis takes into account not only the number of trapped ions in MPET but also the efficiency of ion detection. If multiple SOI are trapped at once with the same charge state, shifts in the center frequency will be linearly related to the number of ions trapped [75]. This is the relation that count-class analysis relies on. By recording the number of ions detected on the MCP, it's possible to bin the data in such a way that the cyclotron frequency for a single ion may be extrapolated. Extrapolating the frequency and including detector efficiency is how ion-ion interaction is taken into account when reporting mass determinations. An example of this relationship for ${}^{36}\text{Ar}^+$ may be seen in Figure 4.9



Figure 4.9: The solid line represents the extrapolation of the cyclotron frequency spanning from 0 to 25 ions trapped simultaneously. This fit was based on the shown data using a least squares fit. This figure was taken from reference [48] for an analysis of ${}^{36}\text{Ar}^+$. This figure was used with permission from reference [48].

Chapter 5

Atomic Mass Measurements of Na and Al Isotopes

"The most exciting phrase to hear in science, the one that heralds the most discoveries, is not "Eureka!" but "That's funny..." — Isaac Asimov

As the benchmark for accurate and precise mass measurements, PTMS has been repeatedly used to uncover deviations from previous measurements, for Na³², one of the isotopes investigated here, Na³²'s values from the TOF technique spans a range of 2.5 MeV with uncertainties varying from 0.1 MeV to 1.3 MeV [38, 46, 61, 68, 76]. However, PTMS also has the ability to reveal smaller, novel features that may have otherwise been looked over due to a lack of resolution. An example of one of these features revealed by PTMS is the crossover of the twoneutron separation energy, S_{2n} , found in ³³Mg and ³⁴Al, as first discussed at length in [52]. When initially discovered, this crossover was considered to be due to inaccurate mass measurements [52]; however, previous mass measurements performed at TITAN [26, 52], as well as this work have further confirmed the crossover. Largescale nuclear-shell-model calculations [52] indicate that the crossover arises from large gains in correlation energy due to degenerate energy levels. A possible experimental explanation for the seeing this crossover would be the measurement of an isomer [67] that was erroneously assigned to be the ground state. Additional measurements were required to determine the excitation energy of this theoretical

isomer. This is the motivation of the mass measurements presented in this chapter.

In this chapter, I report on PTMS measurements of ^{31,32}Na and ^{29,34,35}Al performed at TITAN [33]. Particular attention was given to searching for a recently discovered, long-lived isomer of ³⁴Al [67]. Since the excitation energy is unknown, this may effect the understanding of the reported crossover of the two-neutron separation energies of ³³Mg and ³⁴Al [52]. Though an isomer was not observed in this work, we have confirmed the previous measurement of the ground-state mass [52] and, thus, the crossover in the two-neutron separation energy.

5.1 Experiment

The measurements presented in this chapter were performed at the ISAC-TRIUMF facility [32] via the ISOL technique outlined in Chapter 4. The sodium isotopes were surface-ionized, while aluminium isotopes were ionized via TRILIS [55] (see section 4.1.1). In this experiment, the determined yields ranged from 75 Particles Per Second (PPS) for ³⁴Al, to 2.0×10^6 PPS with ²⁹Al. The singly charged beam was mass separated via the dipole magnet spectrometer with $R \approx 2500$ [34] and then transported at 20 keV to the TITAN facility where it underwent further beam preparation followed by the mass measurement itself. The preparation involved the cooling and bunching of the beam via the TITAN RFQ (Section 4.3.1), and further mass separation by the use of a BNG, and finally dipole cleaning [40] (Section 4.4.1) to purify the beam. Once cleaned, the TOF-ICR technique [17] (Section 4.4) was used to determine the cyclotron frequency, v_c . As discussed, this frequency is related to the ion's mass *m*, the ion's charge *q*, and the magnetic field *B* of MPET via

$$v_c = \frac{1}{2\pi} \frac{q}{m} B. \tag{5.1}$$

With the TOF-ICR technique, the ion's TOF is measured as a function of the excitation frequency applied to the trap, after which the data are fitted with the analytic line-shape [49], whose centroid corresponds to the cyclotron frequency, as shown in Figure 5.1. The centroid of this figure corresponds to the cyclotron frequency of 34 Al⁺. In the case of 29 Al, a Ramsey excitation scheme [43, 51] was applied with two 100 ms excitation pulses separated by 300 ms.



Figure 5.1: TOF spectrum of ³⁴Al with an excitation time of 100 ms. Here $v_c \approx 1670813$ Hz. The solid curve is the analytic fit [49].

5.2 Data Analysis

Taking the ratio of frequencies between a reference ion $v_{c,ref}$, and the isotope of interest v_c , allows the magnetic field to be cancelled out as in Equation 5.2:

$$R = \frac{v_{\rm c,ref}}{v_{\rm c}} = \frac{q_{\rm ref}}{q} \cdot \frac{m}{m_{\rm ref}}.$$
(5.2)

By using this cancelling out of magnetic fields, the measurements may be calibrated by bracketing each radioactive ion measurement by reference ion measurements. This bracketing allowed for a linear interpolation of the magnetic field to the time of the radioactive ion's cyclotron frequency being determined. This ratio is, in principle, independent of the magnetic field, and thus, our primary result. The frequency ratios and their statistical uncertainties are shown in Table 5.1.

We analysed the data to determine both the systematic and statistical uncertainties. Systematic uncertainties [20] including relativistic effects (Section 4.4.2), non-linear fluctuations in the magnetic field (Section 4.4.2), anharmonicities in the trapping potential, and other mass-dependent effects were found to be negligible) as compared to statistical uncertainties (relative uncertainties of 10^{-9} vs 10^{-7}).

Species	Reference Ion	T _{ex} (ms)	R	ME _{TITAN} (keV)	ME_{Lit} (keV)	S_{2n} (keV)
³¹ Na	³⁹ K ⁺	20	1.25636549(62)	12246(14)	12261(23)	657(16)
³² Na	³⁹ K ⁺	20	1.2168586(15)	18638(37)	18810(120)	5979(38)
²⁹ Al	$^{23}Na^{+}$	100-300-100	0.793281449(10)	-18207.77(37)	-18209.0(19)	17153.51(37)
³⁴ Al	³⁹ K ⁺	50	1.14610225(44)	-2999(12)	-2990.0(72)	8042(15)
³⁴ Al	³⁹ K ⁺	71	1.14610245(24)	-3004.4(67)	-2990.0(72)	8048(11)
³⁴ Al	³⁹ K ⁺	100	1.14610227(12)	-2999.5(34)	-2990.0(72)	8042.8(93)
³⁴ Al	$^{39}K^{+}$	50,71,100	1.14610230(11)	-3000.5(29)	-2990.0(72)	8043.7(92)
³⁵ Al	$^{39}K^{+}$	50	1.11325817(25)	-223.7(73)	-220(70)	7869(10)

Table 5.1: Each of the nuclides measured in this paper is presented here, alongside the ion that was used as its reference, the excitation time in MPET T_{ex} , the ratio R, (see Equation 5.2), mass excesses *ME* from both this work and literature, as well as the two-neutron separation energy S_{2n} . For ^{31,32}Na and ³⁵Al we compare to AME [12] and for ^{29,34}Al the more recent values in reference [52]. We also used the mass measurements of ^{27,32,33}Al in [52] to calculate the S_{2n} . All species presented in this experiment had a charge state of +1. In the case of ²⁹Al, a 100-300-100 ms Ramsey excitation [43] was used. This table also presents the combined results of the 50, 71, 100 ms excitation time of ³⁴Al.

These systematic uncertainties were typically two orders of magnitude smaller than statistical (relative uncertainties of 10^{-9} versus 10^{-7}), and thus were insignificant. The uncertainty stemming from ion-ion interactions (Section 4.4.2) was determined via a count-class analysis [48] whenever yields allowed. This count class analysis takes into account the detector efficiency ($40 \pm 20\%$), as well as the number of ions detected in the same measurement. The count-class analysis extrapolates using this information to determine the cyclotron frequency of a single ion in the trap. Moreover, reference measurements of 39 K⁺ or 23 Na⁺ were alternated with the radioactive mass measurements. These calibration measurements were within one standard deviation of Atomic Mass Evaluation (AME) and had comparable uncertainties to those of our radioactive ion measurements.

5.3 Results

Using the frequency ratio in Equation 5.2, the atomic masses of the species measured were extracted using the AME mass value of the reference [12], as well as the electron binding energy (*BE*) [50], and the mass of the electron (m_e), this relation to the atomic mass is defined as

$$m = R \cdot m_{\text{ref}} - BE - m_{\text{e}}.$$
(5.3)

As the SOI in this experiment were singly charged, the amount of energy to remove a single electron *BE*, compared to the measurement's statistical uncertainty, was negligible and could be ignored. A convenient way to describe masses is by the difference between an atom's mass and the number of present nucleons. This quantity is refereed to as the Mass Excess (ME) of the atom, and is defined by Equation 5.4 where the mass is in units of AMU, and A is the number of nucleons present in atom.

$$ME = (Mass - A) \times 931.49432 \, MeV \tag{5.4}$$

The resulting ME values are presented in Table 5.1 alongside the literature values. MEes of ³¹Na and ³²Na were found to be 12246(14) keV and 18638(37) keV, respectively. The uncertainties of these new measurements are half of the smallest currently published in AME. While our measurement of ³¹Na is 0.6σ from the



Figure 5.2: A sample of mass measurements with the horizontal lines centred around the AME indicating the 1σ confidence level. As can be seen our new value, TITAN 2015, disagrees with AME by 1.4σ . Values taken from [46, 61, 69, 76]

values in AME, we find a deviation of 1.4σ for ³²Na, as shown in Figure 5.2.

The TITAN-measured mass excesses for 29,34,35 Al are listed in Table 5.1. Those of 29,34 Al agree with our prior measurements [52] within a 1 σ and 2 σ uncertainties, but also have lower uncertainties. The *ME* value of 35 Al presented in Table 5.1 agrees with AME [12] and has an improved precision by a factor of ten over the previous values determined via the TOF technique [38, 46, 61, 68, 76].

During the experiment and data analysis, we paid special attention to ³⁴Al due to the possible presence of a long-lived isomer [67]. Such an isomer could have been produced in the ISAC production target and delivered simultaneously with

the ground state. To identify the constituent species in the beam we turned the laser ionization of ³⁴Al on and off. Note that due to Doppler broadening in the ionization region, both the ground state of ³⁴Al and its isomer would be ionized with the same laser-excitation scheme. Stable ³¹PH₃⁺ accounted for at least 75% of the surface-ionized beam. Thus, more than 90% of the beam at the MPET MCP produced with laser ionization, was ³⁴Al⁺. The stable ³¹PH₃⁺ was identified via its v_c in the MPET.

If an isomer were present during the mass measurements, a second resonance feature would be visible in its TOF distribution (see for ex. [42]). In the analysis, we attempted to fit the data with a fitting function for both a single and double resonance; however, no second resonance was observed. Therefore, only one species was identified. In an effort to clarify which nuclear state of ³⁴Al was being measured, we used multiple v excitation times as the isomer and ground state have very different half-lives with the isomer's being 26(1) ms [67] and the ground state of 56.3(5) ms [60]. The various excitation times used were 50 ms, 71 ms, and 100 ms. Due to variations in the ISAC yields, as well as data being taken for ³⁴ intermittently with other elements, it was not possible to normalize the count rates between the various measurements of ³⁴Al. These variations in yield were observed to change from a fraction of surface-ionized ³⁴Al⁺ to surface-ionized contaminant ions of $\approx 12\%$ at the start of the experiment to $\approx 3\%$ at the end of the experiment, and as such a comparison of the count rates could not be used to distinguish the ratio of the ground-state to the isomer if a mixture were present. A linear extrapolation of count rates for ³⁴ was not possible due to a 48 hour period between the first round of measurements of ³⁴Al with an excitation time of 100 ms, and the second round with the excitation time of 50 ms. This gap was due to taking measurements of our other SOI. However, as the shortest excitation time was nearly twice the half-life of the short lived isomer, a maximum of 14% of those delivered from the target station would survive long enough to be observed. This drop in the number of detected isomers is even more apparent with the longer excitation time of 100 ms with a maximum of 4% of the original isomer yield possibly being measured, further lowering our ability to detect it.

In order for TITAN to observe a separation of two species at a difference of FWHM with the longest excitation, 100 ms, the mass difference would need to
be as large as 200, keV. At this point, a second TOF resonance would be considered fully resolved. An illustration of fitting with an isomer present is shown in Figure 5.3 for various isomer to ground state ratios, as well as mass differences. However, as the short excitation scheme used would reduce the quantity of the isomer to 14% of its original value, we do not expect a yield sufficient to distinguish it from the ³⁴Al ground state.

As no second resonance was observed, and since the isomer yield could be at most 14% of our total ${}^{34}Al$ yield, it was concluded that only the ground state was in fact, the species measured. The only other known isomeric state [64] was observed via a Coulomb excitation experiment, and its half-life is of the order of nanoseconds.

The measured mass excess of 34 Al agrees with that of TITAN's previous measurement of -2990.0(72) keV [52], but with a smaller uncertainty due to better statistics. Using this updated mass excess, two-neutron separation energies were found to overlap for 33 Mg and 34 Al. The two-neutron separation energy tabulated in Table 5.1 is defined as

$$S_{2n}(N,Z) = -m(N,Z) + m(N-2,Z) + 2m_n.$$
(5.5)

Thus, with our observations, the two-neutron separation energy crossover of ${}^{33}Mg$ and ${}^{34}Al$ at N = 21 is confirmed with an overlap of 15(10) keV.

The importance of the aluminium masses derives from their transitional nature as they border the island of inversion. In addition to agreeing with previous measured mass values, the ^{29,34,35}Al values presented here support large-scale nuclear shell model calculations [52], the values of Mg were obtained from [25]. In these predictions, ^{34,35,36}Al have mixed *sd* and *pf* nuclear orbitals. The relative gains in correlation energy peak at N = 21,22 for the aluminium isotopes, which can be seen in the trend of the two-neutron separation energy flattening from ³⁴Al to ³⁶Al. Figure 5.4 shows this for Al, and Mg for the N = 19 - 21 region, and shows the an overlap at N = 21. This overlap is known as the crossover at N = 21. The present TITAN measurements are in agreement with this flattening of the ^{34,35}Al two-neutron separation energy. This is significant as the current AME value of ³⁶Al [12] does not agree with this flattening. The TOF mass mea-



Figure 5.3: TOF resonance simulations for ³⁴*Al* for the ground state and isomer with various differences in their masses, and yield ratios. The leftmost vertical line represents the v_c of the isomer, and the rightmost vertical line represents the v_c of the ground state, $v_c \approx 1670813$ Hz A: a difference of 100 keV and an isomer ratio of anywhere from 50% to 0% of trapped ions. **B:** a difference of 200 keV was simulated. **C:** a difference of 300 keV.



Figure 5.4: Two-neutron separation energies comparing the experimental results with those from literature, including an expanded view of the N = 21 crossover. Solid circles represent values taken from AME [12], open circles represent values determined by TITAN measurements, triangles represent energies determined using both AME and TITAN measurements together, and lines represent theoretical values [52]. Note that theoretical calculations for Na have yet to be calculated. 56

surements of ³⁶Al currently span ≈ 0.3 MeV with uncertainties at a maximum of ≈ 0.4 MeV [46, 61, 68, 76]. A discrepancy of approximately 0.5 MeV with the literature value [12] would bring the measured mass in agreement with the calculated value and strongly support the predicted gains in correlation energy[52].

This correlation energy would be due to a neutron pair crossing the N=20 shell gap, and occupying close lying orbitals in the pf shell [21]. This is also known as a two-particle-two-hole excitation. The new state is now constructed of the two-particle-two-hole excitations may then lead to a lowering of the ground state energy, leading to the flattening of the ${}^{21-23}$ Al two-neutron separation energy.

Chapter 6

Conclusions and Outlook

In this thesis, I have presented my results of simulations and designs of the new CANREB buncher and cooler. The results of the simulations have shown that the design is adequate for the specified requirements. This includes being capable of handling a mass range from 15 to 250 AMU, as well as an energy spread of 50 eV. In addition to this, the acceptance of the device is a factor of five times larger than those outlined in the design document [13], thus creating a tool that would facilitate tuning of the beamline.

In addition to aiding ARIEL in the process of providing a larger range of isotopes to the scientific program at TRIUMF, I have presented mass measurements of species that are currently available at ISAC. These mass measurements of 31,32 Na, as well as 29,34,35 Al help aid our understanding in the region of the island of inversion [27], and have joined TITAN's previous mass measurements of the island of inversion now spanning $^{29-32}$ Na, $^{29-34}$ Mg, and $^{29-35}$ Al. The flattening of the S_{2n} from 34 Al to 36 Al is supported by our investigations, and support the predictions of large correlation gains in 34,35 Al [52]. Although extensive efforts were used in the search for an isomer of 34 Al, none was observed. This supports the claims in Reference [52] of a crossover in the two-neutron separation energy of the 33 Mg and 34 Al ground states.

Although the measurements presented in this paper aid in our understanding of the island of inversion, more measurements must be carried out to further our understanding of the inversion mechanism itself. Spin-parity measurements will play a role in investigating the inversion mechanism, and as such spectroscopic information is still needed. The structure of the ³⁴Al is still under debate [52, 67], as its observation has been elusive at radioactive-isotope-beam facilities, and thus far its production has been achieved only via decay from its parent, ³⁴Mg [67]. TITAN has recently demonstrated the capacity to recapture the daughter from a radioactive decay for subsequent mass measurements. The mass of ³⁶Al is now well within TITAN's reach, and its mass may substantiate the predicted gains in correlation energy [52] and the flattening of S_{2n} for N = 21 - 23 observed in the aluminium isotopic chain.

In the future, TITAN will continue to investigate masses in the island of inversion and the underlying physics therein, as well as the physics behind isotopes that have previously been unavailable at TRIUMF.

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