Abstract

The current use of glass as a building material is mostly restricted to glazing applications. However, contemporary architecture is seeing an increasing demand in providing building transparency, leading to unconventional load bearing applications for glass. The inherent beneficial glass properties, such as a high compressive strength and efficient building physics characteristics, aid in justifying the use of glass. However, the brittle nature of glass and the high sensitivity to microscopic surface flaws hinder engineers from fully exploiting the material. Coupled with the fact that current manufacturing practices limit glass products in standardized sizes, as well as the lack in general design provisions, engineers face a big challenge in providing reliable design solutions.

This thesis firstly gives an overview of relevant glass properties that motivate the current treatment from a theoretical and practical standpoint. Secondly, it presents a parametric investigation of mechanically bolted and adhesively bonded glass elements to understand the in-plane loaded behaviour. It is shown that adhesively bonded joints outperform the bolted configurations. It is further shown that for a given bolted glass panel assembly, an optimum bolt hole diameter exists where the resulting stresses are minimized. As for the adhesively bonded joints, increasing the bond area reduces the stresses up to a certain threshold.

The third part of the thesis presents two unique case studies where glass is used in structural applications. The first case study describes the development of an all-glass sandwich panel assembly in the context of ribbed mirrors for astronomical telescopes. The second case study analyzes a pedestrian bridge comprised of timber and glass components acting in a composite configuration. Code-based and common material limits for glass are found to be satisfied. Both case studies highlight the potential for glass to be used in such applications and serve as precursors for further research.
Preface

This thesis is an original and unpublished work by the author, Dan Irwin J. Dela Peña, under the direct supervision of Dr. Siegfried Stiemer and Dr. Thomas Tannert. Specific portions of the work conducted by the author in this thesis include:

- Literature review on relevant works performed in the specific research field.
- Computer modelling of connection configurations for parametric studies.
- Modifying and extending past analyses done on ribbed telescope mirrors.
- Creating and analyzing the glass-timber bridge model for given a set of dimensions and materials.

Identification of the research topic was done by the student’s graduate supervisors, Dr. Siegfried Stiemer and Dr. Thomas Tannert. Original ANSYS models for analyzing ribbed glass telescope mirrors that were subsequently modified for Chapter 4 were obtained from a UBC civil engineering graduate student, Saman Hashemi. Conceptual designs of the timber-glass bridge used as a second case study were provided by the Fraunhofer Institute for Manufacturing Technology and Advanced Materials, Germany. The preliminary analysis work for the glass-timber bridge has been included in an abstract submission for the 2016 World Conference on Timber Engineering.
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Mom and dad, thanks for the chicken adobo and rice.
To my parents, Teodoro & Vicky, and my sister, Iris Mae.
CHAPTER 1: Introduction

1.1 Glass: A General Overview

Glass use dates back to around 3500 BC in Egypt and Eastern Mesopotamia primarily in the form of common household appliances. Since then, glass can be found in many other applications, from laboratory equipment, to electronic devices, and glazing in the automotive and building industries. With the advent of advanced processing techniques such as heat-treating, tempering, and laminating, the boundary within which glass can be implemented is continually being pushed further. In the context of buildings and structures, the shift towards non-conventional applications of glass is further motivated by the fact that contemporary architecture is experiencing an increasing demand for transparency (Louter C. P., 2007).

Several precedents exist which feature the shift from the traditional use of glass as an infill material to load bearing applications. Prime examples are evidenced by many of the Apple Inc. stores found all over the world, e.g. the vestibule of an Apple store located on 5th Avenue in New York is comprised of a glass cube that forms over a square opening that descends down to the store level. The cube measures 33 ft. (10 m) on edge and is comprised of vertical glass fins at 5 ft. 5 in. (1.65 m) centres on all four sides. The fins support a lattice of an intersecting grid of glass beams and are heat-treated and laminated to strengthen the overall structure against imposed loads (O'Callaghan & Coult, 2008). Additionally, the bolted connections have been specially detailed to provide restraint while shear is being transferred within the plane of the façade (O'Callaghan & Coult, 2008).

Aside from its aesthetic appeal, glass is known to be a fairly versatile material; capable of meeting various requirements depending on the application at hand. The material on its own is durable and resistant to most chemical and environmental exposure. The gap between glass panes in insulated units can be filled with inert gas to provide excellent thermal properties and adequate acoustic attenuation (Weller et al., 2009). Laminating glass sheets together with an adhesive interlayer can increase the safety features of the final product. The strength and properties of glass can also be manipulated depending on the adapted manufacturing and processing technique (Weller et al., 2009).
The greatest drawback of the material is the total lack of ductility when compared to other common materials used in structures. Due to the microstructure of glass, the material is inherently sensitive to defects stemming from sharp edges, holes, and random microscopic surface flaws. However, research done by Louter (2007), Ber et al. (2014), and Slivansky (2012) have shown that when strategically coupled with other materials, significant post-yielding behaviour can be attained.

### 1.2 Research Need and Objectives

Despite the growth in demand for glass in load-bearing applications, the aforementioned benefits, as well as the proven viability of the material in composite configurations, certain qualities still hinder the material from being fully exploited. For one, current manufacturing techniques limit the ability for designers to realize long-span structural members. Typical widths are limited to 3210 mm in Europe, while this restriction is slightly relieved to 4000 mm in the Asian market (Wilson, 1999). Coupled with the fact that glass suffers from microscopic surface flaws leading to brittle failure modes, engineers are challenged with providing reliable design solutions that meet both architectural and structural requirements.

Significant research has been done on various bolted systems, different types of adhesives, and clamped connection configurations. However, most previous projects involved extensive prototype testing aimed at predicting the strength of structural glass connections (Overend, 2005). The few numerical or analytical-based investigations have not yielded a general consensus to deal with connected glass components. Most numerical approaches are also computationally expensive and not practical for engineering purposes. As a result, current glass design provisions, such as ASTM E1300 (US Standard), prEN 13474 (European draft Standard), and CAN/CGSB 12.20 (Canadian Standard), solely deal with the strength of out-of-plane loaded glass with assumed – and often limiting – boundary conditions. ASTM E1300 for example is only applicable to vertical or sloped glazing in buildings for which the specified design loads consist of wind, snow, and self-weight with a total combined magnitude less than or equal to 15 kPa.

This research project aims to contribute in the understanding of the behaviour of glass connections by investigating the following issues:
1) The influence of various parameters (bolt hole diameter, edge and end distances, friction, and different bushing materials) on the stress distribution around bolted glass connections.

2) The influence of various parameters (bonding area, edge and end distances, and different adhesive materials) on the stress distribution around adhesively-bonded connections.

3) The applicability of structural glass in two case studies, specifically in a purely glass sandwich panel assembly and in conjunction with timber as a composite pedestrian bridge.

1.3 Research Methodology

Fundamental aspects of glass as a material are summarized in Chapter 2 to provide background on current design methods. Analytical and empirical results pertaining to connected glass are subsequently summarized and used in validating finite element (FE) models in Chapter 3. The numerical work is performed on ANSYS by generating parametric input scripts that may be modified to suit specific analyses and post-processing needs; selected scripts are included as appendices. Chapter 4 presents FE analyses on two case studies that highlight the potential and applicability of glass in load bearing applications. The first describes an all-glass sandwich panel assembly in the context of an on-going research project with an industrial partner, Empire Dynamic Structures Limited, for the development of novel telescope mirrors for use in astronomical observatories. The second case study utilizes some work from a project for the Fraunhofer Institute for Manufacturing Technology and Advanced Materials (IFAM). This involves the design and analysis of a glass-timber pedestrian bridge with adhesively-bonded connections.
CHAPTER 2: State-of-the-Art Review

Background information into glass as a building material and the underlying concepts of the current treatment of glass in structural applications are provided in this chapter. Closed form analytical solutions and empirical methods from past experimental works and their limitations are subsequently presented to quantitatively describe the stress states of bolted and adhesively-bonded glass connections.

2.1 Glass as a Building Material

2.1.1 Material Properties

Glass is an amorphous solid material with intrinsic properties that lends its usefulness in many practical applications, from glazing in structures, to flat panel displays on electronic devices, and as biomedical tools (Kongsuwan et al., 2012). Different types of glass (i.e. soda lime silicate and borosilicate) can be produced by adjusting the chemical composition. For structural applications, soda lime silicate glass is dominantly used due to the popular “float” manufacturing process. Borosilicate glass is sometimes used for special applications (fire protection glazing, heat resistant glazing) since the material has a relatively low coefficient of thermal expansion to accommodate large variations in temperature without undergoing thermal shock (Bos et al., 2008). The typical chemical composition and physical properties for both glass types as per EN 572-1:2004 and EN 1748-1-1:2004 are summarized in Table 1 and Table 2.
### Table 1: Chemical Composition of Soda Lime Silica and Borosilicate Glass (EN 572-1:2004 & EN 1748-1-1:2004).

<table>
<thead>
<tr>
<th>Chemical Composition</th>
<th>Chemical Formula</th>
<th>Soda Lime Silica Glass [%]</th>
<th>Borosilicate Glass [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silica Sand</td>
<td>SiO$_2$</td>
<td>69-74</td>
<td>70-87</td>
</tr>
<tr>
<td>Calcium Oxide</td>
<td>CaO</td>
<td>5-14</td>
<td>-</td>
</tr>
<tr>
<td>Soda</td>
<td>Na$_2$O</td>
<td>10-16</td>
<td>0-8</td>
</tr>
<tr>
<td>Magnesia</td>
<td>MgO</td>
<td>0-6</td>
<td>-</td>
</tr>
<tr>
<td>Alumina</td>
<td>Al$_2$O$_3$</td>
<td>0-3</td>
<td>0-8</td>
</tr>
<tr>
<td>Boron Oxide</td>
<td>B$_2$O$_3$</td>
<td>-</td>
<td>7-15</td>
</tr>
<tr>
<td>Potassium Oxide</td>
<td>K$_2$O</td>
<td>-</td>
<td>0-8</td>
</tr>
<tr>
<td>Other</td>
<td>-</td>
<td>0-5</td>
<td>0-8</td>
</tr>
</tbody>
</table>

### Table 2: Physical Properties of Soda Lime Silica and Borosilicate Glass (EN 572-1:2204 & EN 1748-1-1:2004).

<table>
<thead>
<tr>
<th>Mechanical Properties</th>
<th>Symbol</th>
<th>Units</th>
<th>Soda Lime Silica Glass</th>
<th>Borosilicate Glass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>$\rho$</td>
<td>[kg/m$^3$]</td>
<td>2500</td>
<td>2200-2500</td>
</tr>
<tr>
<td>Elastic Modulus</td>
<td>$E$</td>
<td>[GPa]</td>
<td>70</td>
<td>60-70</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>$\nu$</td>
<td>[-]</td>
<td>0.23</td>
<td>0.2</td>
</tr>
<tr>
<td>Coefficient of Thermal Expansion</td>
<td>$\alpha_T$</td>
<td>[K$^{-1}$]</td>
<td>$9 \times 10^{-6}$</td>
<td>$(3.2-6.0) \times 10^{-6}$</td>
</tr>
<tr>
<td>Knoop Hardness</td>
<td>$HK_{0.1/20}$</td>
<td>[GPa]</td>
<td>6</td>
<td>4.5-6</td>
</tr>
<tr>
<td>Transition Temperature</td>
<td>$T_g$</td>
<td>[$^\circ$C]</td>
<td>530</td>
<td>560</td>
</tr>
</tbody>
</table>
Glass exclusively behaves in a linear elastic manner; exhibiting brittle fracture without any plastic deformations. The lack of slip planes prevents local stress concentrations to be effectively reduced through the redistribution of stresses. As a consequence, the strength of glass is highly dependent on the condition of the surface, the size of the glass element, and the loading intensity and duration. Theoretically, the tensile strength of glass calculated from the atomic and ionic bonds is in the range of 6.5 GPa to 8.0 GPa (Weller et al., 2009). However, the presence of mechanical surface flaws invisible to the naked eye dramatically reduces the tensile capacity. Practical values in the range of 30 MPa to 80 MPa are used for engineering purposes (Weller et al., 2009). Manufacturer-rated products such as those provided by Saint Gobin specify values of 40 MPa for annealed glass and 120 MPa to 200 MPa for toughened glass (Saint Gobain, n.d.).

The compressive strength of glass is much larger than the tensile strength since surface flaws do not grow or fail when glass is loaded in compression. Irrespective of any surface flaws, the theoretical compressive strength lies in between 400 MPa and 900 MPa for normally used silicate glasses (Weller et al., 2009). However, given that flaw directions are highly variable, and coupled with Poisson’s ratio effect, tensile stresses may still be caused in unexpected regions of the glass. This random propagation of mechanical flaws can also lead to instability and buckling which in turn causes an element’s tensile strength to be exceeded long before the critical compressive stresses are reached (Haldimann, 2006).

Stress concentrations within materials occur at irregularities due to many forms of defects. Griffith (1920) quantitatively reconciles the discrepancy between the theoretical and practical material strengths of brittle materials by introducing a so-called critical crack depth as a function of the elastic modulus, the surface energy, and the stress level. The implications behind the pioneering principles developed in Griffith’s work effectively formed the fundamental understanding of fracture mechanics theory used today. The work was later extended by Irwin (1957) to not only account for general material plasticity, but to establish a concept that can be used for engineering applications: A glass element fails under tension if the stress intensity factor of one crack reaches and exceeds some critical stress intensity threshold. The corresponding critical crack depth is expressed in Equation 1 and shown in Figure 1.

\[
c_{cr} = \frac{1}{\pi} \left( \frac{K_{IC}}{\gamma_{a}} \right)^{2}
\]

(1)
Where:

- \( c_{cr} \) : Critical crack depth.
- \( K_{lc} \) : Critical stress intensity factor.
- \( Y \) : Crack geometry coefficient.
- \( \sigma \) : Applied stress perpendicular to the crack direction.

**Figure 1**: Half-penny Crack Width (Lawn, 1993).

The critical stress intensity factor represents the elastic stress intensity near the crack tip. Different factors are used for different loading conditions. Mode 1 corresponds to normal separation of the crack under tensile stresses. The crack geometry coefficient accounts for different crack length to specimen width ratios. Despite this fact, a phenomenon known as subcritical crack growth occurs in glass, where the crack size progresses even when the size is below the critical crack depth (Badalassi et al., 2014). Subcritical growth is an extensive material behaviour and is an extensive topic on its own, therefore, it is not described in full detail in this
thesis. The main implication of subcritical growth is the premature failure of glass specimens than otherwise predicted by conventional fracture mechanics theory.

### 2.1.2 Production Methods

Depicted in Figure 2 is an overview of the most common production methods for glass. Different end products are realized depending on the adapted processing technique.

![Glass Production Processes and Products Overview](Haldimann, 2006).
Out of all of the production methods, the float process is by far the most popular and accounts for approximately 90% of flat glass production worldwide (Haldimann, 2006). The main advantage of the process lies in the fact that panes are formed by melting constituent materials in a furnace and poured continuously into a shallow pool of molten tin. Tin remains as a liquid in temperatures between 232°C and 2270°C and has a high specific weight compared to glass. The material is also immiscible; therefore, as the molten glass forms over the tin layer, mixing does not occur. The glass eventually cools, producing a panel that is superior in optical quality compared to glass produced by any other means. The annealing lehr (a temperature-controlled kiln) is used to gradually cool the glass sheets under a controlled environment to ensure that residual stresses do not form in the final product. A simplified schematic of the float process is shown in Figure 3.

Since one side of the glass forms facing the tin bath and the other side faces air, some diffusion of tin atoms occur between the glass-tin interface and the mechanical behaviour of the tin side is marginally lower than the air side (Badalassi et al., 2014). However, this is not due to the diffusion of tin into the glass, but to the contact of the tin side with the transport rollers in the cooling area which causes surface defects that reduce the strength.

**2.1.3 Processing Methods**

Glass products with different shapes, performance, and appearance to meet particular needs can be achieved by processing the basic glass product by various means. Common secondary processing techniques include (Haldimann, 2006):
• Mechanical working (cutting) with diamond-tipped saws or water jets to standardized ribbon sizes (3.21 m by 6.0 m).
• Edge working (arising, grinding, polishing) to remove irregularities caused by cutting and drilling.
• Application of coatings.
• Curving by hot or cold bending techniques.
• Tempering or thermal toughening.
• Heat-soaking to reduce the potential for nickel-sulfide-induced breakage in use.
• Laminating for enhanced post-breakage performance, safety on impact, bullet resistance, fire resistance, and acoustic insulation.
• Surface modification processes.
• Insulating glass unit assemblies to reduce heat loss, reduce solar gain, and enhance acoustic performance.

A series of commonly assembled types of glass is shown in Figure 4. These glass units represent the most widely used glass configurations in the building industry for an assortment of applications and requirements.

![Figure 4: Basic Types of Glass Units (Haldimann, 2006).](image)
Glass can be tempered to modify and enhance the material strength for more stringent demands. This secondary process is typically used to replace annealed monolithic glass sheets, or coupled with other secondary processing techniques such as laminating and insulating, to produce stronger products. The main idea of tempering glass is to create a distribution of stresses along the glass pane that includes tensile stresses in the core and compressive stresses near the surface. During the thermal tempering process, float glass is placed in highly elevated temperatures between 620 °C to 675 °C (~100 °C above the transition temperature) in a furnace and subsequently quenched by jets of cold air. The surface cools rapidly compared to the core region, resulting in immediate residual tensile stresses near the surfaces and compressive stresses in the core. The viscous nature of glass within this temperature range enables the glass to relax the tensile stresses developed near the surface. As the surface of the glass is cooled below the lower limit of the glass transition temperature range, solidification occurs and relaxation stops immediately. The core region eventually cools, but is resisted by the hardened surface, resulting in a characteristic stress distribution of tension in the core and compression near the surface. Therefore, the likelihood of glass failure, which is mainly governed by the growth of surface flaws, is reduced since any applied tensile surface stresses must first overcome the residual compressive stresses imparted to the glass element. An example is illustrated in Figure 5 showing the qualitative comparison of the resultant tensile stresses between annealed and tempered glass samples under an applied tensile load.
The overall strength of the final product is further controlled by the extent of tempering the material is subjected to. Two basic levels are used to characterize the tempering process: heat-strengthened and fully tempered. Both heat-strengthened and fully tempered glass products are fundamentally similar in the sense that basic annealed glass specimens are heated to high temperatures and subsequently cooled to create the residual stress distribution as previously described. The main difference between both types lies in the fact that the cooling of fully tempered glass products are accelerated compared to heat-strengthened glass (PPG Glass Education Center). Full tempering has the overall effect of creating higher surface compression (~4-5 times stronger than annealed glass) compared to heat-strengthened glass (~2 times stronger than annealed glass). The post-breakage fracture pattern of glass is also significantly altered depending on the type of tempering technique used. Fracture is a function of the stored energy in glass from residual stresses and any applied stresses (Haldimann, 2006). Since fully tempered glass can store the most energy compared to heat-strengthened and annealed glass, the post-
breakage behaviour is characterized by the sudden disintegration into many small shards. While annealed glass fractures into large shards, heat-strengthened glass provides a median between fairly good structural performance and sufficiently sized fracture pattern for safety applications. Typical fracture patterns for annealed, heat-strengthened, and fully tempered glass are shown in Figure 6.

Figure 6: Fracture Pattern of Annealed Glass (Left), Heat-strengthened Glass (Middle), and Fully Tempered Glass (Right) (Weller et al., 2009).
2.2 Structural Glass Applications

2.2.1 Past Experimental Research

All-glass load-bearing structural members capable of meeting strength and serviceability requirements have not fully been realized primarily due to the fact that glass suffers from random microscopic surface flaws. Much research has been conducted to obtain a sufficient data for statistical studies. However, inputs such as loading and equipment setup are specific to the study at hand, making it difficult to generalize quantitative conclusions between each study.

Sable and Kalnins (2013) performed numerical and experimental studies to investigate the performance of glass as load-bearing staircase landing platforms. Both annealed and tempered glass specimens were loaded out-of-plane to examine the effects of different treatments. They found that the general load deformation behaviour depends on the quality of the final product; all annealed samples with polished edges were able to withstand higher loads and deflections consistently between all polished samples. The authors also confirmed the notion that tempered glass is able to withstand higher loads and deflections compared to annealed glass, while the stiffness between both glass types are essentially equivalent. For design purposes, the authors recommend a deflection limit ranging from $L/300$ to $L/250$ for annealed glass, while these margins may be increased by 70% for tempered glass.

Veer et al. (2005) attempted to tackle the uncertainty in quantifying an allowable design strength for annealed and tempered float glass. Several beams 1000 mm long, 125 mm and 150 mm deep, and 10 mm thick were cut, polished, and subjected to four point bending. Out-of-plane buckling was restricted to minimize second order effects. The authors obtained mean failure stresses of 43.7 MPa (std. dev. = 11.8%) and 140 MPa (standard deviation = 14.8%) for the 125 mm deep annealed and tempered glass samples, respectively. Meanwhile, the 150 mm tempered samples failed at 88.4 MPa on average (standard deviation = 22.3%). The authors noticed that the cracks generally exhibited a fan shape pattern that emanate from an initial point in the tensile zone. Interestingly, this fan shape was also seen for tempered glass, contrary to the expected sudden disintegration upon overloading. This behaviour implies that the cracks run at a higher speed than the disintegration of glass due to the release of the pre-stress energy.
One generalized conclusion that can be drawn upon from related research projects is the viability of glass in composite configurations. Louter and Veer (2007) experimented on scale 1:4 models of an 18 m reinforced glass beam. Various stainless steel reinforcement configurations were placed along the tensile zone of monotonically loaded glass in four point bending. Slivansky (2012) conducted similar experiments aimed at comparing the behaviour of reinforced glass to unreinforced glass specimens. Ber et al. (2014) focused on integrating glass and timber in composite shear wall panels. An overarching observation of significant post-breakage performance was noted from all of the conducted studies.

2.2.2 Deterministic Approach

The random behaviour of glass naturally leads to deterministic approaches in predicting the failure load, usually done by standardized testing (Haldimann, 2006). In Europe for example, data is either gathered through coaxial double ring tests or four point bending tests. Statistical analyses of the results have been found to closely match a Weibull distribution by fitting two parameters to the experimental results. Weibull (1939) postulated that the failure probability of brittle materials is governed by the following expression:

\[ P_f = 1 - \exp(-B^*) \]  \hspace{1cm} (2)

Where:

\( P_f \) : Probability of failure for brittle materials.

\( B^* \) : Risk of rupture.

The risk of rupture must take into account the surface conditions and the stress distribution since these particular factors dictate the strength of glass. A widely accepted study conducted by Beason and Morgan (1984) has led to the formulation of the Glass Failure Prediction Model (GFPM) that forms the backbone for ASTM E1300 and CAN/CGSB 12.20. They define the risk of rupture as:

\[ B^* = \bar{B} \int_{\text{Area}} [\bar{\sigma}(x,y)\bar{\sigma}_{max}(q,x,y)] \bar{m}dA \]  \hspace{1cm} (3)
Where:
\[ \tilde{k} : \text{Weibull surface flaw parameter 1.} \]
\[ \tilde{m} \text{ : Weibull surface flaw parameter 2.} \]
\[ \tilde{c}(x, y) : \text{Biaxial stress correction factor as a function of the location on the plate surface.} \]
\[ \tilde{\sigma}_{\text{max}}(q, x, y) \text{ : Maximum equivalent principal stress as a function of the lateral load, } q \text{, and the location on the plate surface.} \]

The biaxial stress concentration factor is a function of the minimum to maximum principal stress ratio and the surface flaw parameter, \( \tilde{m} \), as shown in Beason and Morgan (1984). It accounts for the risk experienced by a flaw oriented with respect to the stress field when a plate is subjected to a uniform stress with unequal principal stresses. For the purposes of this thesis, the biaxial stress concentration factor is simplified by assuming that the minimum principal stress is equal to the maximum principal stress. In addition, it is assumed that the entire surface area is subjected to a constant uniform pressure, leading to the following expression for the risk of rupture:

\[ B^* = \tilde{k} A \tilde{\sigma}_{\text{max}}^\tilde{m} \]

\[ (4) \]

### 2.2.3 Design Methods

The GFPM forms the basis for the standards ASTM E1300 and CAN/CGSB 12.20. Comprehensive tables are provided in ASTM E1300 to determine the required thickness of glass plates using a target probability of failure (termed probability of breakage in the standard) of 0.8%. This number represents the likelihood that 8 lites (panels) per 1000 monolithic annealed glass panels fail. It should be noted that the American variant is limited to vertical and sloped glazing exposed to a uniform lateral load and comprised of monolithic, laminated, or insulated rectangular glass elements. The specified design loads may consist of wind, snow, and self-weight with a total combined magnitude less than or equal to 15 kPa. The practice applies to elements with continuous lateral simple supports along one, two, three, or four edges, while insulated glass units are assumed to be supported on all four sides. Finally, the practice is not meant to be applied to wired, patterned, etched, sandblasted, drilled, notched, or grooved glass,
or any elements meant to be used for balustrades, floor panels, aquariums, structural members, and shelves. In general, the design is verified through the following equation:

\[ q \leq LR = NFL \times GTF \times LS \]  \hspace{1cm} (5)

Where:

\[ q \] : Uniform lateral load.

\[ LR \] : Factored load resistance based on 3 s load duration.

\[ NFL \] : Non-factored load.

\[ GTF \] : Glass type factor.

\[ LS \] : Load share factor (applicable for double-glazed insulating glass units).

Deflection requirements are either provided by prescribed tables or alternatively calculated as follows:

\[ w = t \times \exp[(r_0 + r_1)(x + r_2)x^2] \]  \hspace{1cm} (6)

\[ r_0 = 0.553 - 3.83\left(\frac{a}{b}\right) + 1.1\left(\frac{a}{b}\right)^2 - 0.0969\left(\frac{a}{b}\right)^3 \]  \hspace{1cm} (7)

\[ r_1 = -2.29 + 5.83\left(\frac{a}{b}\right) - 2.17\left(\frac{a}{b}\right)^2 + 0.2067\left(\frac{a}{b}\right)^3 \]  \hspace{1cm} (8)

\[ r_2 = 1.485 - 1.908\left(\frac{a}{b}\right) + 0.815\left(\frac{a}{b}\right)^2 - 0.0822\left(\frac{a}{b}\right)^3 \]  \hspace{1cm} (9)

\[ x = \ln\left[\ln\left(\frac{q(ab)^2}{4EI^4}\right)\right] \]  \hspace{1cm} (10)

Where:

\[ w \] : Center of glass deflection.

\[ t \] : Plate thickness.

\[ a \] : Long dimension.
The Canadian standard (CAN/CGSB 12.20) deals with soda lime silicate glass in a similar manner to ASTM E1300. However, unlike the 3-second reference load duration that is used in the ASTM E1300, CAN/CGSB 12.20 is based on 60-second load duration. This is mainly due to the fact that the Canadian Standard was published in 1989 when the 3-second load duration was not yet introduced. In general, partial factors and load combination factors are applied to the action loads and factors that account for glass type, treatment, load duration, and load sharing are applied to the pane resistance. The European draft standard (prEN 13474) differs from its North American counterpart solely due to the fact that it is based on an allowable stress design philosophy. Specifics of the Canadian and European codes are not included since CAN/CGSB 12.20 is similar to ASTM E1300. The prEN 13474 assumes a different design philosophy and is therefore not directly comparable to the North American versions.

2.2.4 Case Studies

Despite the strict limitations in design practices for glass, engineers and architects alike are increasingly using glass as a focal component in load-bearing applications. Apple Inc. already extensively incorporates glass in their stores, as seen from the many iconic landmarks in New York (O'Callagahan, 2003).

Sandwich panel assemblies may also be another viable design application for glass. Typical sandwich panels are comprised of two outer stiff, strong skins and a thick, lightweight core to adequately transfer loads (Petras, 1998). Such systems are typically found in aircraft structures, building panels, and refrigerated transportation containers, to name a few. The separation of the stiff outer skins by a lightweight core increases the moment of inertia of the overall assembly and subsequently increases the bending moment and buckling resistance of the panel. Karlsson and Åström (1996) provides a very comprehensive review of the recent developments and trends of both sandwich materials and processing routes. While the core is usually made up of expandable polymer foam (i.e. polyurethane, polystyrene, polyvinyl chloride, polymethacryl imide, polyether imide, polyphenolics), fibre-reinforced polymers (glass, carbon, thermoset and
thermoplastic resins) make up the outer skins. To date, no current research is focused at looking into the macro-scale use of glass materials in sandwich configurations. The inherent clarity, thermal insulative property, and acoustic efficiency are secondary characteristics that are commonly associated with sandwich assemblies. However, the primary load-bearing characteristics have not yet been proven.

Steel is a popular material choice to be used in conjunction with glass components. In 2008, a U-shaped skywalk was publicly opened in the Hualapai Nation, Arizona. The skywalk cantilevers 21 m past the Grand Canyon cliff walls, providing an eye-opening view of the horizon and the canyon floor 350 m away (Lochsa Engineering, n.d.). Steel bridge box beams support five-layered tempered glass sidings and decks to withstand code-required seismic loads and high wind pressures. More recently in the Hunan province of China, a glass bridge was constructed to span a distance of 430 m between two cliffs in the Zhangjiajie Grand Canyon, hovering over a 300 m vertical clearance (Turner Broadcasting System, 2015). Originally a wooden bridge, it has been replaced by a steel frame supporting two 12 mm thick laminated and heat-strengthened glass. While the two projects showcase a more important role for glass used in engineered structures, one may argue that the overall structural integrity is provided by the steel supports. Thus, opportunities still exist for innovation in terms of using different material combinations or even using glass as a standalone material component in bridge structures.
2.3 Bolted Connections

The majority of glass panels used for building envelopes is commonly connected by some form of bolted assembly. The popular use of mechanical fasteners is attributed to its robustness in being easily assembled or replaced for maintenance purposes. Despite this fact, the necessitated drilling of glass does not detract from the fact that irregularities are introduced into a sensitive and brittle material, which is detrimental to its capability to resist imposed loads. In general, the behaviour of bolted glass, or any material in a broader sense, highly depends on the loading and boundary conditions. Thus, the scope of the ensuing discussion and subsequent analyses performed in this thesis is limited to in-plane loaded glass panels with infinite and finite dimensions with a concentrically placed hole.

2.3.1 Infinite Panel Width with a Concentric Hole

In-plane loaded structural components irrespective of the material are analyzed by applying a stress concentration factor to amplify nominally calculated stresses (Timoshenko & Woinowsky-Krieger, 1959):

\[ \sigma_{max} = K_t \sigma_n \]  \hspace{1cm} (11)

\[ \sigma_n = \frac{p}{t(B-d_h)} \]  \hspace{1cm} (12)

Where:

\[ \sigma_{max} : \] Maximum tensile stress.

\[ \sigma_n : \] Nominal tensile stress.

\[ K_t : \] Stress concentration factor.

Equations describing the state of stresses of a body with a hole have been widely studied and can be formulated from fundamental fracture mechanics theory. Timoshenko and Woinowsky-Krieger (1959) have shown that for an infinite plate with a hole under uniaxial tension (Figure 7), the radial, tangential, and shear stress distributions immediately near the hole vicinity are
expressed as a function of the radial distance from the centre of the plate and the direction of the applied load as follows:

\[
\sigma_r = \frac{\sigma}{2} \left(1 - \frac{a}{r^2}\right) + \frac{\sigma}{2} \left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right) \cos 2\phi \tag{13}
\]

\[
\sigma_\phi = \frac{\sigma}{2} \left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma}{2} \left(1 + \frac{3a^4}{r^4}\right) \cos 2\phi \tag{14}
\]

\[
\tau_{r\phi} = -\frac{\sigma}{2} \left(1 + \frac{3a}{r^2} + \frac{2a^2}{r^2}\right) \sin 2\phi \tag{15}
\]

Where:

\(\sigma_r\) : Radial stress.

\(\sigma_\phi\) : Tangential stress.

\(\tau_{r\phi}\) : Shear stress.

\(a\) : Radius being considered (Figure 7).

\(r\) : Panel hole radius (Figure 7).

\(\phi\) : Angle relative to the applied tensile stress (Figure 7).
It should be noted from the above equations that at the edge of the hole, only tangential stresses exist and a maximum value of $3\sigma$ is obtained for $\phi = \pm 90^\circ$ (perpendicular to the line of action of the applied stress. The constant value of 3 is commonly referred to as the stress concentration factor. The tangential stresses equals the applied stress when $\phi = 0^\circ$. As for the radial stress, at $\phi = 0^\circ$, they are initially zero at the edge of the hole, but eventually decrease and subsequently increase, reaching the value of applied stress, $\sigma$.

The brunt of research efforts are solely focused on resulting tensile stresses and not so much on the compressive stresses imparted on glass panels. This comes as no surprise since the actual tensile strength of glass as previously discussed, is significantly lower than the compressive strength. Additionally, solutions for the resulting compressive stresses are highly non-linear and require heavy computational work that is impractical for engineering purposes. Nonetheless, the importance of investigating the compressive behaviour of glass stems from the stochastic distribution of flaws that can induce tension loading in regions that are compressed. The first accepted study quantifying the behaviour of contacting bodies was developed by Heinrich Hertz.
Hertz postulated that two contacting bodies behave as linearly elastic cylindrical half-spaces. Friction effects are further neglected to simplify the derivation of his equations. However, the validity of the presented solutions is solely for cylinder lengths that are significantly bigger than the diameter of the cylindrical bodies. Thus, the approach is not valid for thin glass panels with a bolt approximately similar in length to the glass thickness.

Persson extended Hertz’ theory by discarding the initial assumption of elastic half spaces when the contact arc subtends a large angle (Ciavarella & Decuzzi, 2001). For bolted panels with infinite dimensions comprised of similar frictionless materials subjected to in-plane loads, the contact compressive stress around the edge of the hole is expressed by the following dimensionless equation:

\[ q_{rad}(y) = \frac{R p_r(\phi)}{p^*} = \frac{2}{\pi \sqrt{b^2+y^2}} \left( \frac{\sqrt{b^2-y^2}}{1+y^2} \right) + \frac{1}{2 \pi b^2 (1+b^2)} \ln \left( \frac{\sqrt{b^2+1} + \sqrt{b^2-y^2}}{\sqrt{b^2+1} - \sqrt{b^2-y^2}} \right) \] \hfill (16)

\[ p^* = \frac{P}{t} \] \hfill (17)

\[ b = \tan \left( \frac{\alpha}{2} \right) \] \hfill (18)

\[ y = \tan \left( \frac{\phi}{2} \right) \] \hfill (19)

Where:

- \( q_{rad}(y) \): Dimensionless contact compressive stress.
- \( R \): Hole radius = bolt body radius.
- \( p_r(\phi) \): Radial pressure distribution around the edge of the hole.
- \( p^* \): Load per unit panel thickness.
- \( \alpha \): Semi-angle of the contact segment.

By setting \( \phi \) and consequently, \( y \), to zero, a simplified expression for the dimensionless radial contact compressive stress is expressed in the following equation:
Ciavarella and Decuzzi (2001) further improves upon Persson’s work by considering dissimilar bolt and panel frictionless material combinations. They found that the dimensionless radial stress at the edge of the hole is distributed by the following equations:

\[ q_{rad,\text{max}} = \frac{ap_r(\phi=0)}{p^*} = \frac{2 |b|}{\pi \sqrt{b^2+1}} + \frac{\ln(\sqrt{b^2+1}+|b|)}{\pi b^2(1+b^2)} \]  

(20)

Further manipulating the governing equations, Ciavarella and Decuzzi (2001) found that the dimensionless tangential stress distribution can be expressed in terms of the radial stress distribution as follows:

\[ q_{rad}(y) = \frac{ap_r(\phi)}{p^*} = \frac{2}{\pi \sqrt{b^2+1}} \frac{\sqrt{b^2-y^2}}{1+y^2} + \frac{1}{\pi} \left(1 - B^*\right) \ln \left(\frac{\sqrt{b^2+1}+\sqrt{b^2-y^2}}{\sqrt{b^2+1}-\sqrt{b^2-y^2}}\right) \]  

(21)

\[ B^* = \frac{2b^4+2b^2-1}{b^2(b^2+1)} \]  

(22)

2.3.2 Finite Panel Width with a Concentric Bolt Hole

The equations proposed by Timoshenko and Woinowsky-Krieger (1959), Persson (Ciavarella & Decuzzi, 2001), and Ciavarella and Decuzzi (2001) are limited to infinite panel dimensions with frictionless contacting bolt and panel bodies. However, much research has been done to account for finite panel widths and non-uniform loads stemming from the bolt bearing against the hole. Howland (1930) conducted the most basic and fundamental study that provides a simple solution to quantify the stresses for a semi-finite plate with a concentric hole. He found that for a semi-finite panel subjected to uniform stress, the maximum tangential stress at the hole circumference can be approximated by amplifying the nominal stresses with a stress concentration factor as a function of the hole diameter and perpendicular panel width as follows:

\[ K_t = 2 + 0.284 \left(1 - \frac{d_h}{B}\right) - 0.6 \left(1 - \frac{d_h}{B}\right)^2 + 1.32 \left(1 - \frac{d_h}{B}\right)^3 \]  

(24)

Where:
\[ K_t : \quad \text{Stress concentration factor.} \]
\[ d_h : \quad \text{Panel hole diameter.} \]
\[ B : \quad \text{Panel width (perpendicular to the direction of the applied load).} \]

The presented solution is widely used in the engineering community to approximate the peak stresses for in-plane loaded thin plates with a concentric hole, especially since at small \( d_h/B \) ratios, the factor converges to a value of 3, which is equal to that given by an infinitely wide panel. However, the solution given by Howland (1930) disregards the interaction between the bolt and the panel. Comprehensive work was done by Frocht and Hill (1940) to account for the foregoing discrepancy by experimentally investigating high-strength aluminum alloy plates loaded by steel and aluminum bolts for \( d_h/B \) ratios ranging from 0.086 to 0.76. Frocht and Hill (1940) conclusively quantified a stress concentration factor as a function of the hole diameter and panel width with the following expression:

\[
K_t = 12.882 - 52.714 \left( \frac{d_h}{B} \right) + 89.762 \left( \frac{d_h}{B} \right)^2 - 51.667 \left( \frac{d_h}{B} \right)^3 \quad \text{.................................} \quad (25)
\]

Theocaris (1956) extended Howland’s solution for infinitely wide panels and performed a similar study to Frocht and Hill for \( d_h/B \) ratios of 0.2 to 0.5. Rather than considering the conventional definition of net tension when defining the nominal tensile stress, he refers to the nominal stress based on the loaded bearing area as follows:

\[
\sigma_n = \frac{P}{d_ht} \quad \text{.................................................................} \quad (26)
\]

Where:

- \( P \): Applied tensile load.
- \( t \): Panel thickness.

From his work, an expression for the stress concentration factor was found to be in the following format:
\[ K_t = 0.288 + 8.82 \left( \frac{d_h}{B} \right) - 23.196 \left( \frac{d_h}{B} \right)^2 + 29.168 \left( \frac{d_h}{B} \right)^3 \]  \hspace{1cm} (27)

Equation 27 is further manipulated so that the nominal stress is defined based on the net area in tension by applying a factor to the solution presented by Theocaris (1956). This is done so that the different stress concentration factors can be directly compared against each other. The stress concentration factor is therefore expressed by the following equation (Pilkey & Pilkey, 2008):

\[ K_t = \left( \frac{B}{d_h} - 1 \right) \left[ 0.288 + 8.82 \left( \frac{d_h}{B} \right) - 23.196 \left( \frac{d_h}{B} \right)^2 + 29.168 \left( \frac{d_h}{B} \right)^3 \right] \]  \hspace{1cm} (28)

In a study done by Duerr (2006), a series of theoretical and experimental studies on semi-finite pinned connections over the past 65 years is summarized. Data points taken directly taken from the studies he considered are accumulated and a best-fit line is used to define the stress concentration factor as a function of the distance between the hole edge and the edge of the panel and the panel hole diameter:

\[ K_t = 1.5 + 2.5 \left( \frac{b_e}{d_h} \right) - \left( \frac{b_e}{d_h} \right)^2 \]  \hspace{1cm} (29)

Where:

\( b_e \): Distance between the edge of the hole and the edge of the panel (Figure 8).

In order to compare the different stress concentration factors, the expression given by Duerr (2006) is manipulated by realizing that the panel width, \( B \), and the hole diameter, \( d_h \), is related to \( b_e \) through a geometric relationship as follows (Figure 8):

\[ \frac{B}{d_h} = \frac{d_h + 2b_e}{d_h} = 1 + \frac{2b_e}{d_h} \]  \hspace{1cm} (30)

\[ \therefore \frac{b_e}{d_h} = \frac{B}{2d_h} - \frac{1}{2} \]  \hspace{1cm} (31)
Substituting Equation 31 into Equation 29 yields a modified version of Duerr’s stress concentration factor in the following expression:

\[ K_t = 0.1825 + 1.385 \left( \frac{d_h}{b_e} \right) - 0.0675 \left( \frac{d_h}{b_e} \right)^2 \]  

(32)

### 2.3.3 Discussion of Stress Concentration Factors

The stress concentration factors considered for in-plane loaded glass panels with a concentric hole are summarized in Figure 9. As previously discussed, the Howland stress concentration factor curve is representative of a thin plate with a concentric hole subjected to uniform tensile stress. The remaining curves represent plates loaded in-plane by bolts bearing against the hole of the panel. Since it is impractical to have bolt diameters that are either relatively small or large compared to the panel width, the range of hole diameter to plate width ratios selected by the respective authors are limited.
The approximated stress concentration factors by Frocht and Hill (1940), Theocaris (1956), and Duerr (2006) are in general agreement with each other, with the Frocht and Hill and modified Theocaris solutions showing a very close fit. One noticeable result is the difference between Howland’s widely used solution and the remaining solutions. For small hole diameters relative to the plate width, the stress concentrations are much more pronounced, highlighting the importance of the bolt and panel interaction that is purely neglected by the Howland case. Additionally, at \( d_h/B > \sim 0.70 \), the solutions diverge from each other. This can be attributed to the fact that the studies conducted by the respective authors are based on data between certain \( d_h/B \) ranges. The curves represent best-fit lines that do not account for larger ratios.
2.4 Adhesive Bonded Joints

Adhesively-bonded joints are gaining popularity due to their ability of not only avoiding the weakening of glass segments through the introduction of irregularities, such systems are capable of ensuring efficient composite behaviour between the glass and supporting elements by providing an uniform load transfer mechanism (Nhamoinesu & Overend, 2012). Although bonded systems have been extensively studied either experimentally or theoretically, no unified design provisions are readily available for practical engineering purposes. This issue is described by exploring key deficiencies when it comes to the treatment of bonded joints.

2.4.1 Material Models

Structural silicones are by far the most widely used adhesives for glass applications in the building industry (Weller et al., 2009). Nonetheless, such bonds are known to be relatively thick and flexible, characteristics that enable the joint to accommodate differential thermal strains, but at the expense of loss in overall strength due to the lack of composite action (Overend et al., 2011). Studies conducted on stiffer thermosetting adhesives show that a new generation of efficient joints may be realized. Although, major gaps exist in this area due to: i) the lack of data from standardized mechanical tests for a variety of adhesives, ii) the lack of a general representative stress-strain characteristics of adhesives, and iii) insufficient long term performance data for adhesive joints (Overend et al., 2011). Further still, manufacturers release fundamentally similar products, but the resulting properties may vary depending on the loading rate, installation conditions, and geometry. In other words, the material properties are application-specific and the resulting material characteristics may vary accordingly.

Xu and Tan (2015) studied the effects of different adhesives and geometric variations for glued timber connections. Namely, the authors investigated the behaviour of tension loaded timber connections bonded by SikaPower-4720 (epoxy-based), SikaFast-5215 NT (acrylate-based), and SikaSil SG-500 (silicone-based) adhesives. The study also served to compare the performance of bonded joints to more typical dowel-type connected timber elements. For their experimental and numerical work, manufacturer-provided adhesive properties as per Table 3 were simply obtained and used.
Table 3: Basic Adhesive Properties (Xu & Tan, 2015).

<table>
<thead>
<tr>
<th>Adhesive</th>
<th>Elastic Modulus, (E) [MPa]</th>
<th>Poisson’s Ratio, (\nu) [-]</th>
<th>Basic Chemistry</th>
</tr>
</thead>
<tbody>
<tr>
<td>SikaSil SG-500</td>
<td>1.1</td>
<td>0.43</td>
<td>Acrylic</td>
</tr>
<tr>
<td>SikaFast-5215 NT</td>
<td>150</td>
<td>0.43</td>
<td>Silicone</td>
</tr>
<tr>
<td>SikaPower-4720</td>
<td>1900</td>
<td>0.43</td>
<td>Epoxy</td>
</tr>
</tbody>
</table>

Xu and Tan (2015) subsequently loaded specimens to failure, calibrated FE models to quantify the connection stiffness, and were able to validate the general understanding that the epoxy adhesive is stiffer than both acrylic-based and silicone-based adhesives. Xu and Tan were also able to conclude that glued connections are superior to dowel-type connections in terms of stiffness, but at the expense of ductility. It was additionally found that the connection stiffness for each adhesive depended on the overlap length of the bonded joint. The effect, however, decreases with an increasing adhesive stiffness.

Overend et al. (2011) conducted mechanical testing and numerical modelling in an attempt to characterize the constitutive models of the following selected bulk adhesives:

- Dow Corning DC993 (silicone): a two-part silicone adhesive cured through a polymerization reaction between the base and catalyst compounds.
- SikaForce 7550 L15 (polyurethane): a two-part polyurethane adhesive consisting of a thixotropic two-component assembly that cures through a chemical reaction to form a durable elastomer.
- 3M 2216B/A (epoxy): a two-part modified epoxy adhesive which cures by a chemical reaction between the modified epoxy and amine.
- Holdtite 3295 (2P-acrylic): a two-part acrylic adhesive cured through a chemical reaction between a resin and an amine curing agent.
- Bohle 682-T (UV-acrylic): a UV-cured acrylic adhesive based on methylacrylate resin.
The authors formulated a constitutive model of the type shown in Figure 10, where a time-independent elastic-plastic behaviour acts in parallel with a time-dependent viscoelastic model. Through a series of experiments on dumbbell specimens and curve fitting of results, the authors were able to quantify the bulk properties summarized in Table 4.

Nhamoinesu and Overend (2012) conducted studies on six candidate adhesives and by adapting a similar experimental regime, they were able to quantify the adhesive bulk properties shown in Table 5.

![Figure 10: Adhesive Constitutive Model (Overend, Jin, & Watson, 2011).](image.png)
Table 4: Adhesive Bulk Properties (Overend et al., 2011).

<table>
<thead>
<tr>
<th>Adhesive</th>
<th>Maxwell Viscoelastic</th>
<th>Elastic-Plastic Stress Strain Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$v$ [-]</td>
<td>$G_e$ [MPa]</td>
</tr>
<tr>
<td>Silicone</td>
<td>0.49</td>
<td>0.03</td>
</tr>
<tr>
<td>Polyurethane</td>
<td>0.39</td>
<td>1.50</td>
</tr>
<tr>
<td>Epoxy</td>
<td>0.46</td>
<td>201.88</td>
</tr>
<tr>
<td>2P-acrylic</td>
<td>0.39</td>
<td>195.89</td>
</tr>
<tr>
<td>UV-acrylic</td>
<td>0.30</td>
<td>336.23</td>
</tr>
</tbody>
</table>

It is evident that bulk properties for fundamentally similar adhesives are highly variable. Xu and Tan (2015) used values from provided by the manufacturers. However, these are based on specific testing parameters performed on the products, as corroborated by Overend et al. (2011) and Nhamoinesu and Overend (2012). The results highlight why adhesive bonded joints have not yet reached a stage where unified design provisions can be followed. Despite this disparity, however, subsequent analyses conducted by the authors have shown that epoxy and acrylic-based adhesives are stiffer and are capable of providing more load-bearing characteristics when compared with the widely used silicone-based adhesive joints.
**Table 5: Adhesive Bulk Properties (Nhamoinesu & Overend, 2012).**

<table>
<thead>
<tr>
<th>Adhesive</th>
<th>Maxwell Viscoelastic</th>
<th>Elastic-Plastic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\nu$ [-] $G_\nu$ [MPa]</td>
<td>$\sigma = -27568\varepsilon^2 + 1305.1\varepsilon + 0.0013 \text{ for } \varepsilon \leq 0.014$</td>
</tr>
<tr>
<td>3M DP490</td>
<td>0.38 239.0</td>
<td>$\sigma = -6552.7\varepsilon^2 + 684.03\varepsilon + 4.4918 \text{ for } \varepsilon &gt; 0.014$</td>
</tr>
<tr>
<td>Epoxy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Araldite 2047</td>
<td>0.43 211.0</td>
<td>$\sigma = -16881\varepsilon^2 + 766.43\varepsilon + 0.1549 \text{ for } \varepsilon \leq 0.025$</td>
</tr>
<tr>
<td>Acrylic</td>
<td></td>
<td>$\sigma = -1300.5\varepsilon^2 + 141.34\varepsilon + 6.3631 \text{ for } \varepsilon &gt; 0.025$</td>
</tr>
<tr>
<td>3M 7271</td>
<td>0.29 559.0</td>
<td>$\sigma = -35361\varepsilon^2 + 1283.5\varepsilon \text{ for } \varepsilon \leq 0.01$</td>
</tr>
<tr>
<td>Epoxy/Acrylic</td>
<td></td>
<td>$\sigma = -18645\varepsilon^2 + 945.9\varepsilon + 1.7463 \text{ for } \varepsilon &gt; 0.01$</td>
</tr>
<tr>
<td>3M 2216 B/A</td>
<td>0.47 192.4</td>
<td>$\sigma = -10755\varepsilon^2 + 199.14\varepsilon \text{ for } \varepsilon \leq 0.01$</td>
</tr>
<tr>
<td>Epoxy</td>
<td></td>
<td>$\sigma = -133.73\varepsilon^2 + 33.495\varepsilon + 0.6172 \text{ for } \varepsilon &gt; 0.014$</td>
</tr>
<tr>
<td>Holdtite 3295</td>
<td>0.41 219.5</td>
<td>$\sigma = -12430\varepsilon^2 + 525.25\varepsilon - 0.0061 \text{ for } \varepsilon \leq 0.015$</td>
</tr>
<tr>
<td>Acrylic</td>
<td></td>
<td>$\sigma = -886.79\varepsilon^2 + 147.7\varepsilon + 3.009 \text{ for } \varepsilon &gt; 0.015$</td>
</tr>
<tr>
<td>DC 993</td>
<td>0.48 3.9</td>
<td>$\sigma = -17.612\varepsilon^4 + 22.002\varepsilon^3 - 10.069\varepsilon^2 + 2.351\varepsilon + 0.0003$</td>
</tr>
<tr>
<td>Silicone</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.4.2 Analysis with Identical Adherends

Perhaps the most fundamental understanding of stresses in single-lap joints (SLJ) is derived from classical linear elastic analysis where the adhesive is considered to deform only in shear and the adherends are assumed to be rigid. The adhesive shear stress is therefore considered to be constant over the overlap length and can be calculated as follows:

\[
\tau = \frac{P}{b l}
\]

Where:

- \(\tau\) : Linear elastic SLJ shear stress.
- \(P\) : Applied shear load.
- \(b\) : Width of the joint.
- \(l\) : Length of the overlap.

Equation 33 is interpreted as the average shear stress acting across the adhesive layer. However, what it provides in computational simplicity, it fails in properly representing the actual joint behaviour by completely ignoring the elasticity of the adherends and the adhesive in the bonded joint. Volkersen (1938) was the first to propose a shear lag model that takes into account adherend elasticity by postulating that the adhesive deforms only in shear and the adherends deform axially in tension (Figure 11). He further stated that the shear stress distribution across the adhesive thickness, \(t_a\), does not change.

![Figure 11: Volkersen (1938) Single-lap Joint Model.](image)
The tensile force is naturally largest at the point of load application and progressively decreases to zero at the free surface of each adherend. The subsequent reduction in strain and continuity between the adhesive and adherend interface cause a non-uniform shear stress distribution in the adhesive layer in contrast to the constant shear stress calculated from classical mechanics theory. Volkersen’s closed form shear stress distribution is described by Equation 34 and Equation 35:

\[
\tau(x) = \frac{P \omega \cosh(\omega x)}{2b \sinh(\omega l/2)} + \left(\frac{t_1 - t_2}{t_1 + t_2}\right) \frac{\omega l}{2} \frac{\sinh(\omega x)}{\cosh(\omega l/2)}
\]  

(34)

\[
\omega = \sqrt{\frac{G_a}{E t_1 t_2}} \left(1 + \frac{t_1}{t_2}\right)
\]  

(35)

Where:

- \(t_1\) : Thickness of the upper adherend.
- \(t_2\) : Thickness of the lower adherend.
- \(t_a\) : Thickness of the adhesive layer.
- \(G_a\) : Adhesive shear modulus.
- \(E\) : Adherend elastic modulus.

However, Volkersen’s solution neglects the rotational effect caused by the eccentricity between the mid-planes of the adherends. Goland and Reissner (1944) addressed this issue by firstly determining the load contributions from the resulting shear and bending moments caused by the joint rotation by assuming that the adherends act as cylindrically bent plates (Figure 12).
They concluded that the resulting joint edge loads are given by the following equations:

\[ M = k \frac{P^* t}{2} \]  \hspace{1cm} (36)

\[ V = k' \frac{P^* t}{c} \]  \hspace{1cm} (37)

\[ k = \frac{1}{1 + 2 \sqrt{2} \tanh(u_2 t)} \]  \hspace{1cm} (38)

\[ u_2 = \sqrt{\frac{3(1 - v^2)}{2} \frac{1}{t} \sqrt{\frac{P^*}{tE}}} \]  \hspace{1cm} (39)

\[ k' = \frac{k c}{t} \sqrt{3(1 - v^2) \frac{P^*}{tE}} \]  \hspace{1cm} (40)

Where:

\[ M \] : Joint edge bending moment per unit width due to the load eccentricity.

\[ V \] : Joint edge shear force per unit width due to the load eccentricity.

\[ P^* \] : Load per unit width.
\[ k : \text{ Bending moment factor per Goland and Reissner (1944).} \]
\[ k' : \text{ Transverse force factor per Goland and Reissner (1944).} \]
\[ \nu : \text{ Adherend Poisson’s ratio.} \]
\[ c : \text{ Half of the joint overlap length.} \]

Equation 41 and Equation 42 denote the corresponding shear stress distribution in the adhesive layer as a function of the overlap length:

\[
\tau(x) = -\frac{P^*}{8c} \left[ \frac{\beta c}{t} (1 + 3k) \frac{\cosh \left( \frac{\beta c}{t} x \right)}{\sinh \left( \frac{\beta c}{t} x \right)} + 3(1 - k) \right] \tag{41}
\]

\[
\beta^2 = 8 \frac{E_a}{E} \frac{t}{t_a} \tag{42}
\]

Meanwhile, the transverse out-of-plane adhesive shear stress (peel stress) stemming from the induced joint edge bending moment and shear force is given by Equation 43 to Equation 48, as follows:

\[
\sigma(x) = \frac{1P^*}{\Delta c^2} \left[ R_2 \frac{\lambda^2}{2} + \lambda k' \cosh(\lambda) \cos(\lambda) \right] \cosh \left( \frac{Ax}{c} \right) \cos \left( \frac{Ax}{c} \right) + \left[ R_1 \frac{\lambda^2}{2} + \lambda k' \sinh(\lambda) \sin(\lambda) \right] \sinh \left( \frac{Ax}{c} \right) \sin \left( \frac{Ax}{c} \right) \tag{43}
\]

\[
\lambda = \gamma \frac{c}{t} \tag{44}
\]

\[
\gamma^4 = 6 \frac{E_a}{E} \frac{t}{t_a} \tag{45}
\]

\[
\Delta = \frac{1}{2} [\sin(2\lambda) + \sinh(2\lambda)] \tag{46}
\]

\[
R_1 = \cosh(\lambda) \sin(\lambda) + \sinh(\lambda) \cos(\lambda) \tag{47}
\]

\[
R_2 = -\cosh(\lambda) \sin(\lambda) + \sinh(\lambda) \cos(\lambda) \tag{48}
\]

2.4.3 Analysis with Non-identical Adherends

Bigwood and Crocombe (1989) noticed that most of the analyses done on bonded joints only considered one type of joint configuration (i.e. single or double lap). By extending the work by
Goland and Reissner (1944), they pioneered a general elastic analysis that permitted various geometries under complex loading to be discretized into a simple adherend-adhesive sandwich (Figure 13). Another notable improvement of their analysis from past studies is the effect of adherend material dissimilarity on the shear and peel stress distributions within the joint area.

![Figure 13: Bigwood and Crocombe General Adherend-Adhesive Sandwich (Bigwood & Crocombe, 1989).](image)

By assuming a state of plane strain, two uncoupled seventh and sixth order differential equations were given to describe the shear and transverse stress distributions in the adhesive layer. The full solutions explicitly given in their study require the calculation of several parameters and are not included in this thesis for brevity. Additionally, the derivation presented in their work does not include the determination of the resulting joint edge loads caused by the eccentricity between the adherend mid-planes. As with the other studies, the variation of the peel and shear stresses in the adhesive through the thickness were neglected in order to simplify the analysis and to facilitate further studies where the effect of adhesive thickness is included. Bigwood and Crocombe (1989) further presented two simplified two-parameter design formulae to calculate the adhesive shear and peel stress peaks at the overlap ends. The so-called Simplified Shear Analysis yielded Equation 49, Equation 50, and Equation 51 for separate joint edge tensile, shear, and bending moment forces, respectively. The derivation is based on an initial assumption that the transverse (peel) stress has limited contribution to the in-plane deformation of the joint. Shear compliance factors, as shown in Equation 52 and Equation 53, are defined and are used to describe the relative shear stiffness of the adhesives to the stiffness of the adherends. The superposition of the three loading cases provides an estimate of the maximum shear stress at the joint edge.
\[
\tau_T = \frac{-\alpha_1 T}{2(\alpha_1 + \alpha_2)^{0.5}} \tag{49}
\]
\[
\tau_V = \frac{3V}{4t_1} \tag{50}
\]
\[
\tau_M = \frac{3\alpha_1 M}{t_1(\alpha_1 + \alpha_2)^{0.5}} \tag{51}
\]
\[
\alpha_1 = \frac{G_a(1-v_1^2)}{E_1 t_1 t_a} \tag{52}
\]
\[
\alpha_2 = \frac{G_a(1-v_2^2)}{E_2 t_2 t_a} \tag{53}
\]

Where:

\(\tau_T\) : Simplified shear stress due to joint edge tensile load, \(T\).

\(\tau_V\) : Simplified shear stress due to joint edge shear force, \(V\).

\(\tau_M\) : Simplified shear stress due to joint edge moment, \(M\).

\(\alpha_1\) : Adherend 1 shear compliance factor.

\(\alpha_2\) : Adherend 2 shear compliance factor.

A Simplified Peel Analysis yielded Equation 54 and Equation 55 for the peel stresses caused by joint edge shear and bending moments. An underlying assumption is made that the shear stress in the adhesive layer does not contribute to the flexural behaviour of the adherends. Peel compliance factors, denoted in Equation 56 and Equation 57, have been defined to relate the bulk adhesive stiffness to the stiffness of the adherends.

\[
\sigma_v = \frac{-(2)^{0.5} \beta_1 V}{(\beta_1 + \beta_2)^{0.75}} \tag{54}
\]
\[
\sigma_M = \frac{-\beta_1 M}{(\beta_1 + \beta_2)^{0.5}} \tag{55}
\]
\[
\beta_1 = \frac{12E_a(1-v_1^2)}{E_1 t_1^2 t_a} \tag{56}
\]
The simplified design formulae are only valid for long joint overlap lengths, high adhesive elastic and shear moduli, or when the adherend Young’s modulus, Poisson’s ratio, and the adherend and adhesive thicknesses are small (Bigwood & Crocombe, 1989). Additionally, an arbitrarily selected acceptable error range of ±10% would imply peel and shear compliance factor ratios between 0.6 and 2.0 (Bigwood & Crocombe, 1989). It is recommended by the authors that the application of these equations be restricted within the aforementioned range.

Another general elastic analysis that deals with non-identical adherends was done by Cheng et al. (1991). They noted that the analysis performed by Bigwood and Crocombe (1989) fails to provide an explicit solution for discretizing the applied tensile load into shear and bending moment components at the edges of the joint area. They define separate edge moment factors for each adherend in the joint as follows:

\[
\beta_i = \sqrt{\frac{p^*_i}{D_i}} \text{ for } i = 1, 2
\]

\[
\beta = \sqrt{\frac{p^*}{D}}
\]

Where:

- \(\sigma_y\) : Simplified peel stress due to joint edge shear force, \(V\).
- \(\sigma_M\) : Simplified peel stress due to joint edge moment, \(M\).
- \(\beta_1\) : Adherend 1 peel compliance factor.
- \(\beta_2\) : Adherend 2 peel compliance factor.

\[
\beta_2 = \frac{12E_a(1-v^2)}{E_2t_2^2t_a}
\]
Where:

\[ D : \quad \text{Flexural rigidity of the joint.} \]

\[ D_i : \quad \text{Flexural rigidity of } i = 1 \text{ (adherend 1) or } i = 2 \text{ (adherend 2).} \]

2.4.4 Discussion on the Analytical Solutions for Bonded Joints

A series of analytical formulations, their limitations, and selected improvements by other authors have been presented. The shear stress across an adhesive layer can be computed using a simple nominal stress calculation based on infinitely rigid adherends. Volkersen (1938) improves upon this by considering material elasticities. However, Volkersen assumed that:

1. Bending deformation of the adherends and the associated tearing stresses in the adhesive is negligible.
2. The stress does not vary through the thickness of the adhesives. This has been shown to be untrue from photo-elastic experiments by Mylonas and Tuzi and Shimada (Pahoja, 1972). Therefore, the assumption results in inaccurate prediction of stresses in the area where the maximum stress occurs.
3. The members are subjected to tensile loads only. Shear force and bending loads are not considered.

Goland and Reissner (1944) try to rectify these issues, but it has been found that their formulations are limited in the following ways:

1. Only valid for adherends of the same material and identical length and thickness.
2. The stresses in the adhesive are not considered to vary through its thickness which as in the case of Volkersen’s theory, results in inaccurate prediction of stresses in the area adjacent to the overlap edge.

3. The theory does not consider external shear force or bending loads.

More general elastic analyses have been proposed by Bigwood and Crocombe (1989) and Cheng et al. (1991), but like the other works presented, these solutions are computationally heavy. Additionally, most of the works done are through experimentation and sometimes coupled with FE modelling. These methods are still not practical for engineering purposes.
CHAPTER 3: Parametric Analysis – Glass Connections

A comprehensive parametric investigation is presented for both steel-bolted and adhesively bonded connections for glass subjected to in-plane loads. ANSYS, a commercially-available FE software package, is used to generate models to capture the component-level structural behaviour of a system under specified loading and boundary conditions. The parametric investigation is meant to serve multiple purposes:

1) Investigate the effects of varying the hole diameter, edge and end distances, coefficient of static friction, and different bushing materials on the stresses imparted on glass panels.

2) Investigate the effects of bonding area, edge and end distances, and different adhesives readily available on the market on the stresses imparted on glass panels.

3) Provide a comparative summary of the load-bearing characteristics of steel-bolted and adhesively bonded connections.

4) Evaluate current analytical solutions for bolted and adhesively-bonded glass connections.

3.1 Model Development

Two benchmark FE models are generated for the bolted and adhesively-bonded glass panels through the programming language incorporated in ANSYS. An example of a general script written in the ANSYS Parametric Design Language (APDL) is included in Appendix A. The bolted connection represents one steel bolt that completely penetrates through the glass panel thickness. The adhesively-bonded connection simulates a block that is attached to the glass panel with a thin layer of adhesive. Both connection types representing the FE models and geometric parameters are depicted in Figure 14 and Figure 15.
A rectangular bonding area with an equivalent shear stiffness to the bolt connection is modelled for the adhesively-bonded connections to provide a basis on which both connection types can be compared. The mathematical formulations by Bigwood and Crocombe (1989) and Cheng et al. (1991) are also based on a rectangular-shaped bonding area. Lastly, most bonded joints employ an irregular bond area and a rectangular bond shape is one of the simplest irregular shapes that can be analyzed. The formulation first considers a unit volume, $dV$, with an applied shear stress as shown in Figure 16.
The elastic deformation of this unit volume under the influence of shear action is defined by the shear modulus, \( G \). One dimension of the rectangular area, denoted as the joint overlap length \( d_{adh} \), is set equal to the bolt diameter, \( d_{bolt} \) (Figure 14 and Figure 15). Thus, by assuming that small displacements govern, an expression for the equivalent shear stiffness represented by the adhesive width, \( b_{eq} \), can be derived as follows:

\[
G_{adh} = G_{bolt} \tag{65}
\]

\[
\frac{\tau_{x adh}}{\gamma_{x adh}} = \frac{\tau_{x bolt}}{\gamma_{x bolt}} \tag{66}
\]

\[
\frac{F I}{\Delta x A_{adh}} = \frac{F I}{\Delta x A_{bolt}} \tag{67}
\]

\[
A_{adh} = A_{bolt} \tag{68}
\]

\[
d_{adh} b_{eq} = \frac{\pi}{4} d_{bolt}^2 \tag{69}
\]

\[
b_{eq} = \frac{\pi}{4} d_{bolt} \tag{70}
\]

Where:
\( G_i \): Shear modulus of \( i = \) bolted or adhesively-bonded connection.

\( \tau_{xz_i} \): \( x-z \) plane shear stress of \( i = \) bolted or adhesively-bonded connection.

\( \gamma_{xz} \): \( x-z \) plane shear strain of \( i = \) bolted or adhesively-bonded connection.

\( F \): Applied shear force on the \( x-z \) plane.

\( \Delta x \): Unit volume deformation along the \( x \)-axis.

\( A_i \): Shear cross-section of \( i = \) bolted or adhesively-bonded connection.

\( l \): Original unit volume length.

\( d_i \): Depth of \( i = \) bolted or adhesively bonded connection.

\( b_{eq} \): Equivalent shear stiffness adhesive depth.

### 3.1.1 Element Types

Different element types found in the ANSYS library were initially considered when generating the component-level FE models. The parametric analysis investigated the stress behaviour of in-plane loaded glass sections, naturally leading to the use of simple 2D elements. However, given the difficulty experienced in defining contact between components, the 3D element, SOLID186 as shown in Figure 17, is used. The element is characterized by 20 nodes, with three degrees of freedom per node: translations and rotations along the \( x \), \( y \), and \( z \) axes. SOLID186 is meant to simulate higher order effects, allowing the element to displace in a quadratic manner (ANSYS, Inc., 2013). Thus, solutions are conditioned for better accuracy compared to elements that use linear shape functions to approximate the solutions. Various mechanical behaviours, such as plasticity, hyper-elasticity, creep, stress stiffening, larger deflections, and strain capabilities are all supported by this element.
ANSYS requires defined contacting regions when modelling physically attached bodies. Corresponding contact and target elements that are specifically designed for the parent element types are overlain to simulate real physical contacts. As such, CONTA174 and TARGE170 elements corresponding to SOLID186 are defined for all analyses. Element properties that are associated with the contact and target elements including stiffness multipliers and friction coefficients can be readily modified.
3.1.2 Material Models

The material models used in this research are summarized in Table 6.

*Table 6: Summary of Material Properties used in the Numerical Model.*

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1) “Float” glass</td>
<td>70</td>
<td>-</td>
<td>-</td>
<td>26.9</td>
<td>0.3</td>
</tr>
<tr>
<td>2) Steel</td>
<td>200</td>
<td>250</td>
<td>0</td>
<td>79.4</td>
<td>0.26</td>
</tr>
<tr>
<td>3) Aluminum Bushing</td>
<td>68.9</td>
<td>115</td>
<td>0</td>
<td>25.5</td>
<td>0.33</td>
</tr>
<tr>
<td>4) PTFE Bushing</td>
<td>0.56</td>
<td>20.5</td>
<td>0</td>
<td>0.20</td>
<td>0.4</td>
</tr>
<tr>
<td>5) POM-C Bushing</td>
<td>2.10</td>
<td>55.0</td>
<td>0</td>
<td>0.75</td>
<td>0.4</td>
</tr>
<tr>
<td>6) 2P Epoxy</td>
<td>0.660</td>
<td>45.0</td>
<td>0</td>
<td>0.240</td>
<td>0.38</td>
</tr>
<tr>
<td>7) Dow Corning 993 Silicone</td>
<td>0.0115</td>
<td>5.0</td>
<td>0</td>
<td>0.004</td>
<td>0.49</td>
</tr>
<tr>
<td>8) UV-cured Acrylic</td>
<td>1.004</td>
<td>35.0</td>
<td>0</td>
<td>0.385</td>
<td>0.30</td>
</tr>
</tbody>
</table>

The glass panels are modelled as linear elastic, whereas all other materials are simply modelled as elastic-perfectly plastic. Adhesives are widely known to exhibit viscoelastic load-deformation behaviour. Past research such as Overend et al. (2011) led to the formulation of a constitutive model that considers a time-independent elastic-plastic behaviour acting in parallel with a time-dependent viscoelastic model. To simplify the analysis, it is assumed that the forces are applied slowly in order to isolate the time-independent elastic-perfectly plastic behaviour. During model validation, elastic material properties are assumed since the analytical formulations are limited in

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the elastic range. Commonly used values for soda lime silicate “float” glass are taken from EN 572-1:2004. The steel properties are representative of commonly used ASTM A36 specifications (ASTM International, 2005). Bushings in point supported glass connections are used to confine the steel bolt and aid in relieving the stresses imparted on glass panels created by irregularities. Typical bushing material properties presented in Table 6 are extracted from exhaustive online repositories, MatWeb (MatWeb, LLC, 2016) and Merrem Kunststoffen (Merrem Kunststoffen Groep, n.d.). Values for the different adhesives used to model the bond between the block and glass components are extracted from research done by Overend et al. (2011) and Nhamoinesu and Overend (2012).

3.1.3 Analysis Types

The analyses are naturally non-linear due to the adapted material properties and the underlying mechanics of the contact problem for bolted connections. Solution controls for ANSYS, including the number of minimum and maximum iterations, subsets per iterations, and small or large displacement analysis options, have been manipulated to ensure the solutions converge while minimizing computation time. Since the parametric study is only meant to investigate the load-bearing characteristics of different connection components, the analysis of the adhesives has also been restricted to an isothermal case (i.e. the change of temperature in the system is 0). The Augmented Lagragian contact algorithm is chosen for all contact analyses. This option is an iterative series of penalty updates to find the exact Lagrange multipliers, commonly leading to better conditioning and is less sensitive to the magnitude of the contact stiffness coefficient (Johnson, n.d.). The full Newton-Raphson method is used when the program detects non-linearities. Using this solution control, the method evaluates the out-of-balance load vector: the difference between the reactionary and applied loads (ANSYS, Inc., 2013). In essence, the solution is linearly approximated and checked against a convergence criterion. The iterative procedure continues until the solution is below the threshold and the problem converges. The Newton-Raphson concept is graphically described in Figure 18.
Internal loops and conditional constructs are implemented into the APDL scripts in order to repeat the analysis for multiple dimensions and to extract the required nodal or elemental results for post-processing purposes.

### 3.1.4 Loads and Boundary Conditions

A total discrete load is applied for all analyses aside from validating the FE models against current closed-form solutions. The load is based on a maximum probability of breakage of 8 lites per 1000 (0.8%) as specified in ASTM E1300. For an annealed glass panel, the maximum allowable load can be approximated by initially considering Equation 11 in Chapter 2. The failure probability as per Weibull’s theory (Weibull, 1939) that is presented in Equation 2 is coupled with Beason and Morgan’s (1984) GFPM definition of the risk of rupture. By assuming that the glass panel is subjected to an uniform stress field, where the minimum principal stress is equivalent to the maximum principal stress, the corresponding biaxial stress correction factor,
\( \tilde{c}(x, y) \), is equal to unity. The failure probability, now defined as the probability of breakage to follow ASTM E1300 notation, is written as:

\[
P_b = 1 - \exp\left( -\tilde{k}A_b\tilde{\sigma}_{max}\tilde{m} \right) \quad \text{.......................................................... (71)}
\]

\[
A_b = \pi d_h t \quad \text{.......................................................... (72)}
\]

Where:

- \( P_b \): Probability of breakage (0.8\%) as per ASTM E1300.
- \( \tilde{k} \): Surface flaw parameter \((2.86 \times 10^{-53} \text{ N}^7/\text{m}^{12})\) as per ASTM E1300.
- \( \tilde{m} \): Surface flaw parameter \((7)\) as per ASTM E1300.
- \( \tilde{\sigma}_{max} \): Maximum equivalent stress.
- \( A_b \): Loaded bearing area.

Rearranging Equation 71 yields a deterministic expression for the maximum tensile stress in the following form:

\[
\tilde{\sigma}_{max} = \left[ \frac{-\ln(1-P_b)}{\tilde{k}A_b} \right]^{1/\tilde{m}} \quad \text{.......................................................... (73)}
\]

The nominal tensile stress is expressed as a function of a discrete applied load and the net cross sectional area as follows:

\[
\sigma_n = \frac{P}{t(B-d_h)} \quad \text{.......................................................... (74)}
\]

Where:

- \( P \): Applied tensile load (Figure 14).
- \( t \): Glass panel thickness (Figure 14).
- \( B \): Glass panel width (Figure 14).
\( d_h \): Glass panel hole diameter (Figure 14).

By substituting Equation 73 and Equation 74 into Equation 71 and rearranging for the applied load, \( P \), the maximum load for a bolted connection is represented by the following equation:

\[
P_{\text{max}} = \frac{t(B-d_h)}{K_t} \left[ -\ln(1-P_b) \right]^{1/m} \]

For an annealed glass panel with a 60 mm diameter hole, 150 mm wide, and 6 mm thick, the maximum allowable load for the stress concentration factors given by Frocht and Hill (1940), Duerr (2006), and Howland (1930) are shown in Table 7. Based on the results from the three concentration factors, a total load of 7 kN is conservatively applied for the bolted and adhesively-bonded connections except for all validation analyses.

Table 7: Maximum Allowable Load for Different Stress Concentration Factors.

<table>
<thead>
<tr>
<th>Stress Concentration Factor, ( K_t )</th>
<th>Maximum Allowable Load, ( P = P_{\text{max}} ) [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frocht and Hill (1940)</td>
<td>2.85</td>
</tr>
<tr>
<td>Duerr (2006)</td>
<td>3.22</td>
</tr>
<tr>
<td>Modified Theocaris (1956)</td>
<td>2.96</td>
</tr>
</tbody>
</table>

For all bolted connections, two nodes adjacent along the \( z \)-axis and located at the extreme faces of the steel bolt are coupled to ensure that both nodes displace equally. Half of the total load is dedicated to each node in the coupled set to simulate a bolt force hitting the glass panel body as shown in Figure 19. The bottom face of the glass panel is fixed against all translational and rotational degrees of freedom, while all faces of the bolt body are restricted to displace only along the \( y \)-axis to effectively eliminate effects from eccentricity, if any.
For all adhesively-bonded connections, the total load of 7 kN is applied as a surface pressure acting on the upper face of the block attached to the glass panel. This is achieved by dividing the total load by the equivalent width, $b_{eq}$, and the thickness, $s_{thick}$, of the attached block as shown in Figure 20. Similar to the bolted connection, all translational and rotational degrees of freedom on the bottom of the glass panel face is constrained.
3.1.5 Evaluation

Due to the brittle nature of glass (the lack of yield capacity to enable the redistribution of concentrated stresses within the body), tension is typically the governing load that causes failure in the material. Additionally, the random direction of microscopic surface flaws means that the redistribution of stresses from compressed regions may cause tensile loading in unexpected areas. Soft bushings are used in point-supported glass connections to alleviate this issue. Thus, the resulting tensile and compressive principal stresses are extracted from the bolted connection models. In a similar manner, the tensile stresses transmitted by loading the adhesively-bonded block are extracted. Adhesive shear stresses and maximum deformations are examined in order to gain a general understanding of the relative behaviour between different adhesive materials.
Stresses are the primary extracted results since the extent of which the behaviour of glass has been investigated is predominantly stress-based.

3.2 Bolted Connections

The parameters specifically studied in this portion of the work are the hole diameter for the bolt in the glass, edge and end distances, assumed static coefficient of friction, and the effect of enclosing the steel bolt with softer bushing materials. An example of the FE model after one analysis run is shown in Figure 21. The bolt body has been removed for clarity.

![Figure 21: Example Results for a Bolted Connection.](image)

3.2.1 Model Validation and Calibration

The principal tensile stresses calculated from Frocht and Hill (1940), Duerr (2006), and Theocaris (1956) for a given connection geometry are used to compare results extracted from the
FE model. Figure 22 shows the variation of principal tensile stresses with increasing hole diameter for a steel bolt-glass connection. The panel geometry is kept constant at width, depth, and thickness dimensions of 100 mm, 100 mm, and 6 mm respectively, while the hole diameter is varied from 25 mm to 85 mm. A force of 7 kN is applied to the bolt body, based on the estimated maximum force annealed glass can take for the given panel dimensions (see Equation 75).

![Model Comparison of $\sigma_1$ with Empirical Methods](image)

*Figure 22: FE Model Comparison of $\sigma_1$ to Current Empirical Equations (Frocht & Hill, 1940), (Duerr, 2006), and (Howland, 1930).*

As it can be seen, the generated model generally conforms to the principal tensile stress values given by the current empirical equations. However, deviations seem to arise for $d_h/B > \sim 0.70$, especially for the solution given by Frocht and Hill (1940).

The analytical stress distribution along the hole edge circumference as per Ciavarella and Decuzzi (2001) are compared against values taken from the FE model. Since the closed form solutions are limited to infinite panels with the bolt conforming to the hole diameter, the panel width, depth, and bolt-hole gaps are kept constant at 400 mm, 600 mm, and 0 mm. The analytical solution is also most accurate for frictionless contacting bodies of the same elastic material. In
this case, a glass bolt to glass panel with no friction input is modelled. A total load of 7 kN is applied to a 25 mm diameter snug-fit bolt and the radial and tangential dimensionless stresses as per Equation 16 and Equation 23 respectively, as well as the radial and tangential stresses from the FE model are graphically depicted in Figure 23 and Figure 24. Corresponding minimum principal stress results from varying the hole diameter (25 mm to 40 mm), load values (7 kN, 20 kN, and 30 kN), and the panel thickness (6 mm and 10 mm) are also shown in Figure 25. The FE model generally agrees well with the analytical solution, with the extracted results deviating by 5-10% on average. This is achieved by adjusting the contact stiffness coefficient to 0.1 times the contact element stiffness. The 0.1 factor additionally ensured that the solutions are accurate while still meeting convergence requirements (ANSYS, Inc., 2013).

![Dimensionless Radial Stress Distribution Comparison Between the FE Model and the Analytical Solution per Ciavarella and Decuzzi (2001)](image)

*Figure 23: Dimensionless Radial Stress Distribution; FE Model vs. Analytical Solution (Ciavarella & Decuzzi, 2001).*
Figure 24: Dimensionless Tangential Stress Distribution; FE Model vs. Analytical Solution (Ciavarella & Decuzzi, 2001).
Figure 25: FE Model Comparison of Minimum $\sigma_3$ to the Analytical Solution (Ciavarella & Decuzzi, 2001).

As a final validation step, the nodal and elemental results extracted from ANSYS are compared against each other. ANSYS presents results in two specific ways: nodal solutions and elemental solutions. Elemental results are the averaged values for each node in that element. These elements are typically transformed into their natural coordinate systems and the solutions are found by applying weights at certain quadrature points, as is the norm from finite element theory. The nodal solutions defined within the program are averaged results from elements that share these aforementioned common nodes. Therefore, large differences between “un-averaged” solutions for a particular node in each element and “averaged” solutions for a particular node considering all elements would imply insufficient mesh quality and unreliable results. The minimum principal compressive stress for the second case shown in Figure 25 ($P=7$ kN, $t=6$ mm) with a hole diameter of 40 mm is found to be on node 5226 with an approximate value of $-42$ MPa. The aforementioned node is located where the bolt force is directed on the circumference of the edge of the panel hole. All elements associated with node 5226 are isolated as shown in Figure 26. The compressive stress results for each element and the corresponding result for Node 5226 at each element are summarized in Table 8.
Figure 26: Case 2 Elements Associated with Node 5226.
Table 8: Node 5226 Principal Compressive Stresses for Associated Elements.

<table>
<thead>
<tr>
<th>Element Identification Number</th>
<th>Element Principal Compressive Stress [MPa]</th>
<th>Node 5226 Principal Compressive Stress [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>74368</td>
<td>-37.0</td>
<td>-39.0</td>
</tr>
<tr>
<td>82800</td>
<td>-37.5</td>
<td>-41.6</td>
</tr>
<tr>
<td>82801</td>
<td>-37.6</td>
<td>-40.3</td>
</tr>
<tr>
<td>86125</td>
<td>-37.6</td>
<td>-40.4</td>
</tr>
<tr>
<td>87926</td>
<td>-38.7</td>
<td>-43.7</td>
</tr>
<tr>
<td>87928</td>
<td>-36.3</td>
<td>-45.9</td>
</tr>
<tr>
<td>92470</td>
<td>-36.6</td>
<td>-45.9</td>
</tr>
<tr>
<td>95258</td>
<td>-39.1</td>
<td>-41.3</td>
</tr>
<tr>
<td>Node 5226 FE Result</td>
<td></td>
<td>-42</td>
</tr>
</tbody>
</table>

The mesh element size is chosen to be equivalent to the glass thickness with a superimposed arbitrary factor of ¼. The extracted results indicate that the difference in individual compressive stress values for Node 5226 at each element is relatively small (3rd column in Table 8). Furthermore, nodal stresses are highly dependent on element size. Therefore, individual element principal stresses are enumerated in the second column of Table 8. The individual element principal compressive stresses are sufficiently close in value to the minimum principal compressive stress for Node 5226. Thus, it can be concluded that the chosen mesh size is sufficient for further analyses.

3.2.2 Effect of Friction

Friction is a complex non-linear phenomenon that involves surface roughness, temperature, velocity and load duration (Maniatis, 2006). The complexity of friction is further increased when dealing with dissimilar contacting bodies. In order to study the influence of friction in
conjunction with material dissimilarity, a range of static coefficients are chosen for a glass panel and aluminum bolt connection and the resulting stresses are examined. Similar infinite panel dimensions, as previously described in the preceding subsection, are used in the ensuing analyses. The static coefficients of friction are arbitrarily set to 0.0, 0.2, 0.6, and 1.0, representing a gradient of lubricated to dry contact surfaces, respectively. The importance of conducting experiments is emphasized in order to properly quantify appropriate friction coefficients since the analysis conducted in this portion of the study only assumes a constant static frictional value. In reality, the frictional constant between two sliding objects changes, as described by Iyer (2001). The radial and tangential stress distributions for the aforementioned set of static coefficients of friction given by the FE model and those from a study done by Iyer (2001) are shown in Figure 27 and Figure 28.

![Dimensionless Radial Stress Distribution for Varying Static Coefficients of Friction](image)

**Figure 27: Dimensionless Radial Stress Distribution for Varying Static Coefficients of Friction.**
Results indicate that friction slightly increases tensile stresses and reduces the compressive stresses. This general behaviour has been corroborated by Iyer (2001) who investigated the effects of friction on both the tensile and compressive stresses around bolted connections using different glass panel to bolt material combinations and different FE software. Iyer (2001) found that a static coefficient of friction value of 1.0 increased the maximum tensile stress by 30% and reduced the minimum compressive stress by 30% compared to a frictionless contact surface, irrespective of the type of bolt and glass material combinations used. However, the results obtained from the FE analysis indicate a decrease in compressive stresses by as much as 25% and an increase in tensile stresses of approximately 55%. The disparity in results occurs despite closely matching the analytical results for frictionless contact bodies given by Ciavarella and Decuzzi (2001).
At present, the effects of friction in pinned glass connections have been mostly studied numerically with finite element software. The issue with using numerical software is the uncertainty with the specific treatment of input parameters and geometries. Many control options within various programs can be manipulated. The study done by Maniatis (2006) involved calibrating the contact stiffness to match the analytical solutions. Similar calibration techniques are used in this thesis where a contact stiffness factor of 0.1 is found to closely match the minimum compressive stresses given by these analytical solutions using frictionless contacting bodies. Thus, not only is it very simple to have conflicting results, the scatter of results only further justifies the need for experimentally determining proper friction values to use. Nevertheless, a general behaviour of increased tensile stresses and decreased compressive stresses due to increased friction are two conclusions that can be confirmed for pin-supported glass connections.

3.2.3 Effect of Edge and End Distances

Table 9 summarizes the loading and geometric parameters used in investigating the effects of changing the connection geometry shown in Figure 14 for steel-bolted glass panels with an applied load of 7 kN. The resulting stress plots are shown in Figure 29 and Figure 30.

<table>
<thead>
<tr>
<th>Case</th>
<th>Material Combination (Panel-Bolt)</th>
<th>Hole Diameter, $d_h$ [mm]</th>
<th>Bolt Hole Clearance, $\Delta R$ [% $d_h$]</th>
<th>Glass Width, Height, Thick, $B \times H \times thick$ [mm]</th>
<th>Edge x End Distance, $e_d \times e_n$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Glass-Steel</td>
<td>25 to 85</td>
<td>1</td>
<td>100 x 100 x 6</td>
<td>50 x 50</td>
</tr>
<tr>
<td>2</td>
<td>Glass-Steel</td>
<td>25 to 85</td>
<td>1</td>
<td>150 x 200 x 6</td>
<td>75 x 100</td>
</tr>
<tr>
<td>3</td>
<td>Glass-Steel</td>
<td>25 to 85</td>
<td>1</td>
<td>100 x 200 x 6</td>
<td>50 x 100</td>
</tr>
<tr>
<td>4</td>
<td>Glass-Steel</td>
<td>25 to 85</td>
<td>1</td>
<td>500 x 100 x 6</td>
<td>250 x 50</td>
</tr>
</tbody>
</table>
Figure 29: Effect of Connection Geometry on the Principal Tensile Stress, $\sigma_1$. 
The resulting tensile stresses indicate a dependence on the end distance rather than the edge distance. For the following example, the end distance is varied from 50 mm to 500 mm and the corresponding tensile stresses show an overall decrease with increasing end distance. It is speculated that for smaller end distances (smaller panel dimensions), the stresses are not redistributed to unstressed areas a larger panel would accommodate. Increasing the edge distance from 50 mm to 150 mm and 500 mm does show a slight decrease in tensile stresses, but the effects are relatively insignificant considering the large increase in edge dimensions. One noticeable result is the effect of increasing the bolt hole diameter. For a given edge and end distance characterized by one of the lines in Figure 29, the tensile stresses on the glass panels can be negated, but only up to a certain hole diameter to edge distance ratio. Thus, a given panel dimension has an optimum bolt hole diameter where tensile stresses are minimized under in-plane loads.
Meanwhile, the principal compressive stress distribution shows more of a slight dependence on the edge distance than to the end distance: An increase in edge distance shows a decrease in compressive stress. It should be noted that the relative decrease of compressive stresses with increasing edge distances is very small. For an increasing hole diameter, the compressive stresses show a general non-linear behaviour. The results indicate that for a given panel dimension, the compressive stresses can be decreased, but only up to a certain point where the stresses plateau.

### 3.2.4 Effect of Different Bushing Materials

As discussed in Section 3.1.5, bushings decrease the stresses imparted on glass panels by confining the primary the hard bolt body with a relatively softer material. The efficiency of including bushings in point-supported glass connections is investigated next. Three types representing hard bushings (aluminum) and softer bushings (POM-C and PTFE) are analyzed. The values in Table 10 are used in analyzing different bushing materials that surround the steel bolt while the bushing thickness, \( bs_{th} \), is arbitrarily chosen to be 2 mm thick. A simplified sketch of the representative model for the FE analysis is depicted in Figure 31 and the corresponding results are presented in Figure 32 and Figure 33.
**Figure 31: Bolted Connection with a Bushing.**

**Table 10: Geometric Parameters for Different Bushing Materials.**

<table>
<thead>
<tr>
<th>Case</th>
<th>Material Combination (Panel-Bolt)</th>
<th>Coefficient of Static friction, $\mu_s$ [-]</th>
<th>Bolt Hole Clearance, $\Delta R$ [% $d_h$]</th>
<th>Glass Width x Height x Thick, $B \times H \times thick$ [mm x mm x mm]</th>
<th>Edge x End Distance, $e_d \times e_n$ [mm x mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Glass-Steel</td>
<td>0.7</td>
<td>1</td>
<td>$100 \times 100 \times 6$</td>
<td>$50 \times 50$</td>
</tr>
<tr>
<td>2</td>
<td>Glass-Aluminum</td>
<td>0.7</td>
<td>1</td>
<td>$100 \times 100 \times 6$</td>
<td>$50 \times 50$</td>
</tr>
<tr>
<td>3</td>
<td>Glass-PTFE</td>
<td>0.065</td>
<td>1</td>
<td>$100 \times 100 \times 6$</td>
<td>$50 \times 50$</td>
</tr>
<tr>
<td>4</td>
<td>Glass-POM-C</td>
<td>0.32</td>
<td>1</td>
<td>$100 \times 100 \times 6$</td>
<td>$50 \times 50$</td>
</tr>
</tbody>
</table>
Figure 32: Effect of Different Bushing Materials on the Principal Tensile Stress, $\sigma_1$. 

Effect of Different Bushing Materials on $\sigma_1$

- Glass-Steel (Control Case)
- Glass-Aluminum Bushing
- Glass-POM-C Bushing (Thermoplastic)
- Glass-PTFE Bushing
From Figure 32 and Figure 33, one can easily see that using different bushing materials have no significant effect on the principal tensile stresses from in-plane loaded glass connections. However, the bushings have a more pronounced effect in mitigating the resulting compressive stresses. Reductions can vary between 1% to 12% for the aluminum bushing compared to the steel bolt-glass contact case and 38% to 60% when softer bushings are used. Tensile stresses are the primary concern when it comes to the behaviour of glass, but it is reiterated that the direction and location of microscopic flaws are random. Thus, the redistribution of stresses within the material may intensify tension in regions not expected to undergo high stresses. The decrease in compressive stresses means that the resulting redistributed stresses may also be decreased and the possibility of mitigating stress intensifications from edges and microscopic flaws is increased.
3.3 Adhesively-bonded Connections

The bonding area, edge and end distances, and adhesive material as outlined in Table 6 (silicone, 2-part epoxy, UV-cured Acrylic) are all varied for the ensuing analyses. The selection of adhesives represents increasing stiffness in the stated order. The bonded block is made out of steel for most of the conducted analyses, while aluminum is used in comparing the results against the analytical solutions. This is done to closely match the elastic strength of glass. It should be noted that a rectangular bond area would yield higher stresses than circular areas due to stress concentrations. However, a rectangular shape is chosen since the analytical solutions by Bigwood and Crocombe (1989) and Cheng et al. (1991) are 2D, per unit width based formulations. Furthermore, in terms of practicality, most bonded joints are irregular, rather than circular (Bigwood & Crocombe, 1989). An example of the FE model is shown in Figure 34.

Figure 34: Example Results for an Adhesively-bonded Connection.
3.3.1 Model Validation

Bigwood and Crocombe (1989) provided a numerical example of a simplified 2D shear problem where similar adherends are subjected to a uniformly distributed load of 100 N/mm. The eccentric load path stemming from the offset in adherend mid-surfaces induces a bending moment within the actual joint. Therefore, the applied tensile load is further decomposed into shear and moments, shown in Figure 35, by applying bending moment factors as per Cheng et al. (1991). These factors define the fraction of the total offset moment reacted by direct bending moments at the end of the overlap, while the remaining moment is counter-acted by resultant shear forces. Since the parameters used in validating the FE model also involves similar linear elastic adherends, the solutions provided by Volkersen (1938) and Goland and Reissner (1944) are evaluated against the FE model and against each other. Lastly, relevant properties used in the example by Bigwood and Crocombe (1989) are summarized in Table 11.

![Figure 35: Example of Single-lap Shear as per Bigwood and Crocombe (1989).](image-url)
Table 11: Material and Geometrical Properties used to Validate the Adhesively-bonded Glass FE Model (Bigwood & Crocombe, 1989).

<table>
<thead>
<tr>
<th></th>
<th>Unit</th>
<th>Adherend 1</th>
<th>Adherend 2</th>
<th>Adhesive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus</td>
<td>$E$</td>
<td>[GPa]</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>$\nu$</td>
<td>[-]</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>Width</td>
<td>$b_{eq}$</td>
<td>[mm]</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Overlap Length</td>
<td>$d_{adh}$</td>
<td>[mm]</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Thickness</td>
<td>$t$</td>
<td>[mm]</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 12 summarizes the maximum adhesive shear stress from the FE model and the equations given by simple linear elastic stress theory, Volkersen (1938), Goland and Reissner (1944), and by the simplified design formulae from Bigwood and Crocombe (1989). Figure 36 shows a comparison of the shear stress distribution with respect to the joint overlap length for the solutions given by basic linear elastic theory, Volkersen (1938), and Goland and Reissner (1944) compared to the shear stress distribution extracted from the FE model.

Table 12: Maximum Shear Stress at the Edge of the Joint Overlap; FE Model vs. Analytical Solutions Bigwood and Crocombe (1989) and Goland and Reissner (1944).

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Joint Edge Total Maximum Shear Stress, $\tau_{max}$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Elastic/Average Stress</td>
<td>4.00</td>
</tr>
<tr>
<td>Volkersen (1938)</td>
<td>21.87</td>
</tr>
<tr>
<td>Goland and Reissner (1944)</td>
<td>26.31</td>
</tr>
<tr>
<td>Simplified Shear (Bigwood &amp; Crocombe, 1989)</td>
<td>26.18</td>
</tr>
<tr>
<td>FE Model</td>
<td>26.67</td>
</tr>
</tbody>
</table>
The values summarized in Table 12 indicate that the maximum shear stress at the ends of the joint overlap given by the FE model sufficiently matches those given by Goland and Reissner (1944), as well as those given by the simplified design formulae by Bigwood and Crocombe (1989). Figure 36 further illustrates the close match between the FE model and the analytical solution given by Goland and Reissner (1944) for the joint configuration described in Table 11. Additionally, it appears that the simplest solution from linear elastic theory significantly underestimates the peak shear stresses at the joint ends. This may be due to the fact that the adherends are assumed to be infinitely rigid. The elasticity of the adherends and the adhesive are completely neglected. As a result, the applied load is idealized to be uniformly distributed over the joint shear area. The solution given by Volkersen (1938) also seems to underestimate the...
shear stresses, especially at the edges of the joint region. This may be attributed to the underlying assumption that only shear stresses occur in the adhesively-bonded joint. In actuality, the offset in adherend mid-surfaces causes an eccentric load path for the applied tensile load. This results in both shear and tensile peel stresses acting across the joint region. As discussed, the more updated solutions given by Goland and Reissner (1944) and Bigwood and Crocombe (1989) account for these tensile peel stresses in single lap joints. Since the FE model is found to adequately match the solutions given by Goland and Reissner (1944) and Bigwood and Crocombe (1989), the models are used in studying the effects of modifying the bonding area, edge and end distances, and the use of different adhesive materials.

As a final validation step, the mesh size in the FE model is adjusted to ensure the results are constant beyond a minimum mesh threshold. The analysis indicates that the FE results are sensitive to the chosen mesh size for the adhesive material. For a given adhesive thickness of 0.1 mm, the shear stress results are sufficiently accurate for a similar value of 0.1 mm in mesh size. Therefore, the same factor is applied for all adhesive elements in the subsequent model modifications, where the element size is set to equal the chosen adhesive thickness. Figure 37 shows the Graphing Solution Tracking plot for the adhesively-bonded FE model after one analysis cycle.
The graph depicts the convergence of the non-linear adhesively-bonded FE model. The x-axis represents the number of times the program iterates through the analysis using the Newton-Rhapson Method. A user-controlled minimum value of 100 iterations is selected in this case. The F CRIT curve refers to the convergence criteria for the applied force value which is equal to the product of the VALUE multiplied by the tolerance, TOLER (Awang et al., 2016). By default, the VALUE is the square root of the sum of squares (SRSS) of the applied loads, whereas the TOLER is defined as 0.5% of the loads. As the analysis progresses, the value of F CRIT increases since the analysis is divided into steps, or portions of the total load in the context of structural analysis. The F L2 curve represents the Vector Norm or SRSS of the force imbalance for all degrees of freedom (Awang et al., 2016). It is also interpreted as the SRSS of the difference between the calculated internal force and the external force at a particular direction and at a particular node. For each subdivision, ANSYS automatically iterates for the solution until the F L2 curve falls below the F CRIT threshold, indicating that the solution is within the defined tolerance. The U CRIT and U L2 curves for displacements are analogous to the F CRIT.
and F L2 curves for forces. The plot indicates that at very low iterations, the F L2 and U L2 curve falls well below the critical limits. Thus, the model is deemed sufficiently accurate for the given mesh density.

### 3.3.2 Effect of Changing Bonding Area with Different End and Edge Distances

Summarized in Table 13 are the various geometries used in the single-lap shear adhesive joint. A steel adherend is bonded with a glass panel using the epoxy adhesive. Maximum adhesive in-plane shear and principal tensile stress results for the glass panel are shown in Figure 38 and Figure 39.

Table 13: Loading and Geometric Parameters for Glass-Epoxy-Steel Bonded Joints

<table>
<thead>
<tr>
<th>Case</th>
<th>Material Combination (Panel-Patch)</th>
<th>Overlap Length, $d_{adh}$ [mm]</th>
<th>Equivalent Width, $b_{eq}$ [mm]</th>
<th>Glass Width, Height, Thick, $B \times H \times thick$ [mm x mm x mm]</th>
<th>Edge x End Distance, $e_d \times e_n$ [mm x mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Glass-Steel</td>
<td>25 to 85</td>
<td>20 to 67</td>
<td>100 x 100 x 6</td>
<td>50 x 50</td>
</tr>
<tr>
<td>2</td>
<td>Glass-Steel</td>
<td>25 to 85</td>
<td>20 to 67</td>
<td>150 x 200 x 6</td>
<td>75 x 100</td>
</tr>
<tr>
<td>3</td>
<td>Glass-Steel</td>
<td>25 to 85</td>
<td>20 to 67</td>
<td>100 x 200 x 6</td>
<td>50 x 100</td>
</tr>
<tr>
<td>4</td>
<td>Glass-Steel</td>
<td>25 to 85</td>
<td>20 to 67</td>
<td>500 x 100 x 6</td>
<td>250 x 50</td>
</tr>
</tbody>
</table>
Figure 38: Maximum Adhesive Shear Stress for Different Glass Panel Geometries.
This analysis highlights the effect of increasing the bonding area on the maximum adhesive shear stress and glass panel principal tensile stress. Unlike the positive parabolic distribution of tensile stresses found for pin-supported connections, bonded joints exhibit a non-linear reduction in stresses for an increasing bond area. The results further indicate that for a given adhesive material, the stresses within the joint plateau irrespective of the continuous increase of the overlap dimension. Varying the edge and end distances has minimal effects on the joint stresses. Increasing the edge distance from 50 mm to 500 mm merely reduced the adhesive shear stresses by a maximum of 2 MPa, while there is a reduction of approximately 5 MPa in the glass principal tensile stresses.
3.3.3 Effect of Different Adhesives

Table 14 reiterates the material adhesive properties and shows the geometric parameters used for three representative adhesive materials. The glass panel depth (100 mm), width (100 mm) and thickness (6 mm) dimensions are kept constant throughout this portion of the analysis. The overlap length is varied from 25 mm to 85 mm, corresponding to equivalent block widths of 20 mm to 67 mm using Equation 70. The thickness of the steel and glass adherends is kept constant at 6 mm. Extracted adhesive maximum shear stresses, glass principal tensile stresses, and maximum joint deformations are illustrated in Figure 40, Figure 41, and Figure 42 respectively.

Table 14: Summarized Adhesive Parameters as specified in Table 62.

<table>
<thead>
<tr>
<th>Case</th>
<th>Modulus of Elasticity, $E_a$ [MPa]</th>
<th>Yield Stress, $f_y$ [MPa]</th>
<th>Tangent Modulus, $E_{tan}$ MPa</th>
<th>Poisson Ratio, $\nu$ [-]</th>
<th>Adhesive thickness, $adh_{thick}$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epoxy</td>
<td>660</td>
<td>45</td>
<td>0</td>
<td>0.38</td>
<td>0.5</td>
</tr>
<tr>
<td>Silicone</td>
<td>11.5</td>
<td>5.0</td>
<td>0</td>
<td>0.49</td>
<td>1.0</td>
</tr>
<tr>
<td>UV-Acrylic</td>
<td>1004</td>
<td>35</td>
<td>0</td>
<td>0.30</td>
<td>0.5</td>
</tr>
</tbody>
</table>

2 See “Table 6: Summary of Material Properties used in the Numerical Model.” for all material property sources.
Figure 40: Maximum Adhesive Shear Stress for Different Adhesive Materials.
Figure 41: Maximum Glass Principal Tensile Stress for Different Adhesive Materials.
Figure 42: Maximum Joint Deflection for Different Adhesive Materials.

Intuitively, stiffer adhesives are expected to transmit higher stresses both in the adhesive layer and glass panels. The simplest way to explain this behaviour is to observe a linear spring model. The resultant stress, or force, is directly proportional to the stiffness and the relative deflection of the joint. Under a deflection value in consideration, the stiffer material will yield higher forces and consequently, higher stresses. The higher stiffness value is also expected to produce lower joint deflections due to the better shear transfer mechanism stiff adhesives are known for. Alluding back to the previously described spring model, under a certain force value in consideration, the stiffer adhesive will theoretically yield lower relative joint deflections due to the higher stiffness value. Figure 40, which shows the variation of shear stress within the adhesive layer for an increasing bond overlap, confirms this fact. The relative stiffness of the adhesives, however, seems to play a more major role when the resulting principal tensile stresses were extracted for the glass panel. At relatively low overlap lengths, the tensile stresses are the same for all adhesives. However, the difference in tensile stresses starts to arise for increasing overlap lengths. Lastly, the maximum joint deformations are found to be the most sensitive to the
change in adhesive materials. At low overlap lengths, Figure 42 shows a large difference in joint
deflections between the flexible silicone and the stiff epoxy and acrylic-based adhesives. The
results qualitatively enforce the notion that silicone adhesives, or similar soft materials, are able
to provide enough flexibility to accommodate thermal strains. However, studies show that more
flexible adhesives have lower strengths before failure occurs either within the adhesive layer or
at the adhesive-adherend interface.
CHAPTER 4: Case Studies

4.1 All-glass Sandwich Panel

The following case study describes the development of a sandwich panel made entirely out of glass, presented in the context of ribbed telescope mirrors. While the applications of glass may be completely different, the conceptual understanding of glass in a ribbed telescope configuration can be translated over to the future design of sandwich panels for more conventional structural applications (i.e. floor and ceiling systems).

Efficient mirrors for astronomical telescopes rely on precision in order to minimize optical errors and properly observe objects in the vast cosmos millions of light years away. As such, mirror systems for astronomical purposes are typically made stiff to ensure the performance of the mirror adheres to strict deflection tolerances (Cheng J., 2009). Seeing further into the universe implies the need for larger sized telescope mirrors in order to collect more light (Starizona Adventures in Astronomy & Nature). Great strides have been taken in achieving this overall goal, as evidenced by the future Thirty Metre Telescope (TMT). The overall mirror structure is slated to comprise more than 500 hexagonal mirror segments with an estimated total weight of 1,400 tonnes. This affects not only the overall cost of constructing the observatory in which the telescope is housed in, but also the structural performance of the entire support and observatory enclosure. Under excitatory dynamic loads, such as earthquakes, a top-heavy structure can induce greater base shear at the foundations. Therefore, one of the challenges in the design of telescope mirrors is finding a balance between providing stiff elements to minimize deflections while keeping the overall weight as low as possible. While minimized deflections imply the need for larger elements, a lightweight structure is achieved by providing smaller elements. In the same way, the design and fabrication of all-glass sandwich panels will need to consider both the structural performance in terms of deflections as well as overall cost and applicability in terms of weight.

The deflection performance of lightweight mirrors is dependent on the size, weight, loading, and boundary conditions the overall structure is subjected to (Vukobratovich, 1999). For axisymmetric structures such as telescope mirrors, the most common self-weight loading, known as axial loading, is the worst case of the gravity vector acting normal to the mirror surface.
Radial deflections (the load vector parallel to the plate surface) are typically small for relatively small telescope mirrors, but can be significant for very large mirrors. However, for preliminary analysis purposes, radial deflections can be ignored. Various methods to assess optical errors from self-weight deflections include the Peak to Valley (PV) Error, Root Mean Square (RMS), and the Strehl Ratio (Nichol, n.d.). The PV Error describes the distance between the highest and lowest point on the mirror surface in terms of the wavelength of collected light, $\lambda$. It should be noted that visible light lies in the wavelength between 390 nm to 700 nm. The RMS method is meant to better characterize the entire mirror surface (wavefront) by accounting for the relative size of the defects. Through this method, the difference between the expected and measured surface deflections in various discrete locations is compared. The Strehl ratio describes the ratio of the intensity of the diffraction pattern of an aberrated (out-of-focus) image to the intensity of a perfectly focused image. For preliminary purposes, the PV Error method is a quick assessment tool to judge the deflection performance of optical telescope mirrors, as summarized in Table 15 (Schwertz, 2010):

<table>
<thead>
<tr>
<th>Surface Irregularity Tolerance Guide</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
</tr>
<tr>
<td>$\lambda$</td>
</tr>
</tbody>
</table>

**4.1.1 Past Telescopes with Ribbed Mirrors**

Empire Dynamic Structures Ltd. (DSL) is a premier provider of engineering solutions for dynamic, complex structures such as themed rides and telescopes (Empire Dynamic Structures Ltd.). They are engaged in a comprehensive research project to provide solutions for the efficient design and fabrication of telescope mirrors. From their extensive studies, one promising design option is to provide ribbed support structures that are connected to a manufactured thin mirror segment. A collaborative investigation between a UBC graduate student, Saman Hashemi, and DSL was undertaken to theoretically analyze various rib-supported mirror configurations under
gravity loads. The systematic study followed an evolutionary process where various structural configurations (uniform mirror thickness, linearly varying and parabolic mirror thicknesses, and rib-supported mirrors) were studied to determine the influence of different structural parameters on deflections. The results led to conceptually laying out four different practical mirror designs for further study in order to select a viable and efficient mirror design.

4.1.2 Analysis of Structural Configurations

In order to determine the optimum structural configuration for a telescope mirror, the efficiencies of several different configurations are examined based on the ability to resist deflections under gravitational effects. The mirror structures that are considered in this investigation are all circular with a diameter of 4 feet (1210 mm). The direct comparison for efficiency is based on a constant volume regardless of the shape and is calculated from a constant 10 mm thick mirror. All mirror configurations are pin-edge supported with the properties listed in Table 16.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus</td>
<td>E</td>
<td>[GPa]</td>
<td>50</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>ν</td>
<td>[-]</td>
<td>0.2</td>
</tr>
<tr>
<td>Density</td>
<td>ρ</td>
<td>[g/mm³]</td>
<td>2.4 x 10⁻³</td>
</tr>
<tr>
<td>Constant Volume</td>
<td>V</td>
<td>[mm³]</td>
<td>1.1675 x 10⁷</td>
</tr>
</tbody>
</table>

The main configurations considered for this portion of the analysis are as follows:

i. A dish with uniform thickness as the very basic model.
ii. A dish with linearly varying thickness as a first optimization iteration.
iii. A dish with a parabolic thickness profile as a second optimization iteration.

Glass properties are based on “Table 2: Physical Properties of Soda Lime Silica and Borosilicate Glass (EN 572-1:2204 & EN 1748-1:1:2004)” and from consultation with Dynamic Structures Ltd.
iv. A dish structure with rib supports. The top dish of the structure has a constant thickness, while the depth of the rib supports linearly vary.

The resulting edge and center dish thicknesses for the linearly varying and parabolic dish geometries are determined by optimizing for the least deflection under a constant volume of material. The final dimensions are shown in Figure 43 and Figure 44 and given in Table 17.

*Figure 43: Dish Geometry for Non-rib Supported (Left) and Supported Mirrors (Right).*
Figure 44: Resultant Optimized Dish Thicknesses and Rib Support Dimensions (Right-most).

Table 17: Optimization Results for Various Mirror Dimensions.

<table>
<thead>
<tr>
<th>Top Dish Thickness [mm]</th>
<th>Rib Supports [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Edge $e_{d_{th}}$</td>
</tr>
<tr>
<td>Constant Thickness</td>
<td>10</td>
</tr>
<tr>
<td>Linear Thickness</td>
<td>2.5</td>
</tr>
<tr>
<td>Parabolic Thickness</td>
<td>0.5</td>
</tr>
<tr>
<td>Rib-Supported</td>
<td>5.0</td>
</tr>
</tbody>
</table>
Gravitational effects are accounted for by using the density of the material and applying a static acceleration value of 9.81 m/s². For this portion of the analysis, all mirror configurations outlined in Table 17 are supported along the circumferential edge of the mirrors by pins. The pins restrict all translational movements, but allow rotational degrees of freedom. Figure 45 depicts a model with the applied gravitational acceleration located at the origin of the global coordinate system and the aforementioned edge boundary conditions.

![Figure 45: Mirror Applied Loads and Edge Boundary Conditions.](image)

The deflection results from the static analysis for the different mirror configurations are compiled in Table 18.
Table 18: FE Vertical Deflection Results for the Various Mirror Configurations.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Maximum Deflection [μm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Thickness</td>
<td>210</td>
</tr>
<tr>
<td>Linear Thickness</td>
<td>110</td>
</tr>
<tr>
<td>Parabolic Thickness</td>
<td>110</td>
</tr>
<tr>
<td>Rib-Supported (Linear variation in rib depth)</td>
<td>13</td>
</tr>
</tbody>
</table>

Varying the top dish either linearly or in a parabolic way reduces the total deflection compared to the constant thickness top dish assembly for a given volume of material. A linear variation in top dish thickness is found to perform as equally well compared to the parabolic thickness configuration. In considering construction practicality, the linearly varying top dish thickness is deemed more efficient. Lastly, it is shown that ribs can significantly reduce total gravity deflections by providing the total overall stiffness of the assembly. Figure 46 depicts the results of the rib-supported dish configuration. The centre depth of the rib supports is intentionally emphasized for illustration purposes.

![Figure 46: FE Vertical Deflection Results for the Ribbed Mirror with Linearly Varying Rib Depth.](image)

The findings emphasize the benefits of including rib supports for thin mirror structures. However, it is also shown that while ribs can play a major role in minimizing overall structural...
deflections, localized regions of non-uniform deflections can exist, as shown from Figure 46. Therefore, the study is extended in an attempt to develop rib-supported configurations that not only minimize the overall deflection of the structure, but also minimizes unwanted optical errors stemming from localized distortions along the mirror surface.

4.1.3 Analysis of Four Mirror Designs

Four practical mirror designs with varying rib configurations are considered to further optimize the usefulness of incorporating ribs to support telescope mirrors. For the purposes of the ensuing analyses, all ribs are assumed to have constant depths of 120 mm. The top dish has a diameter of 1200 mm and all glass components for each configuration is made up of 15 mm thick monolithic sections with properties as specified in Table 16. The first configuration shown in Figure 48 serves as the simplest rib-supported structure to improve upon. It is comprised of 6 ribs evenly distributed around the top dish circumference that radially extend out from the dish centre. The second practical mirror design shown in Figure 49 primarily contains 12 radially distributed ribs from the centre of the top dish. These ribs are laterally supported approximately midway between the radial rib lengths and at the edge of the top dish. The third mirror design depicted in Figure 50 adapts a star-shaped pattern to support the main dish. The fourth mirror design emulates a similar star pattern for the ribs as the third configuration, but with the addition of lateral ribs in a hexagonal pattern near the centre of the dish, as shown in Figure 51.

An example FE model with the applied gravity load and pinned boundary conditions are illustrated in Figure 47. The static gravitational acceleration is represented by the red arrow located at the origin of the global coordinate system, while three pinned boundary conditions are radially located 350 mm away from the centre of the dish structures. Table 19 summarizes the analysis results from the gravity load as well as the computed total mass of the four designs in question.
Figure 47: Rib-supported Mirror Applied Load and Boundary Conditions.

Table 19: Gravity Load Analysis Results and Computed Mass of each Practical Mirror Design.

<table>
<thead>
<tr>
<th>Mirror Design</th>
<th>Total mass [kg]</th>
<th>Maximum Deflection [μm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mirror Design 1</td>
<td>56.3</td>
<td>13.0</td>
</tr>
<tr>
<td>Mirror Design 2</td>
<td>97.0</td>
<td>3.30</td>
</tr>
<tr>
<td>Mirror Design 3</td>
<td>99.4</td>
<td>2.80</td>
</tr>
<tr>
<td>Mirror Design 4</td>
<td>103.9</td>
<td>2.70</td>
</tr>
</tbody>
</table>
Figure 48: Practical Mirror Design 1 Configuration (Left) and Deflection Plot (Right).

Figure 49: Practical Mirror Design 2 Configuration (Left) and Deflection Plot (Right).

Figure 50: Practical Mirror Design 3 Configuration (Left) and Deflection Plot (Right).
The first mirror design served as a benchmark for the remaining design configurations. Analysis results show non-uniform localized deflections at the rib support regions where the pin supports are located. The results motivated the use of edge supports which are included in the remaining three designs. From the second mirror design, including edge supports further minimizes the overall deflection; however, the deflections are still quite non-uniform across the dish surface as shown in the deflection plot above. The layout of the third rib configuration appears to alleviate this issue while still reducing self-weight deflections. Lastly, the incorporation of a hexagonal edge support near the middle of the dish from the third design insignificantly reduces deflections (0.1 microns) at the cost of 5 kg of extra material. It is therefore concluded that the third layout is the most efficient of the four designs considered.

4.1.4 Further Analyses

The studies conducted on the practical mirror designs were aimed at minimizing deflections by increasing the overall structural stiffness while providing a lightweight configuration. However, introducing irregularities such as sharp edges and corners can intensify stresses around connected regions. Therefore, the aim of this extension of the study is to investigate the stresses around connected segments and develop conceptual means of mitigating these transmitted stresses. At the same time, potential methods of reducing deflections and overall mass are presented. The third mirror design will be used for further development as a case study since it has been deemed...
as the most practical out of the four designs previously considered. The studies follow a similar evolutionary process where step-by-step modifications are implemented and studied as follows:

1) Effect of adding a 15 mm thick bottom plate on the deflections and stresses.
2) Effect of adding 80 mm diameter holes to the ribs on the deflections and stresses.
3) Combined effects of adding a 15 mm thick bottom plate and 80 mm diameter holes to the ribs.
4) Combined effects of adding a bottom plate with 120 mm holes and holes to the ribs.

The holes allow for ease in installing various instruments (strain gauges, hydraulic actuators, electrical wiring), as well as in maintaining the structure during its operational life.

Graphical results of the stress and displacement distributions for each finite element model described above are shown in Figure 52 to Figure 56. Results are further tabulated in Table 20 for both the entire sandwich structure and the mirror separately.
Figure 52: Original Design (Left), Total Structure Deflection Distribution (Right).

Figure 53: 1st Evolution (Left), Total Structure Deflection Distribution (Right).

Figure 54: 2nd Evolution (Left), Total Structure Deflection Distribution (Right).
Figure 55: 3rd Evolution (Top), Total Structure Deflection Distribution (Right).

Figure 56: 4th Evolution (Left), Total Structure Deflection Distribution (Right).
Table 20: Mirror Design 3 Further Development Analysis Results.

<table>
<thead>
<tr>
<th>Design 3 Configuration</th>
<th>Total Mass [kg]</th>
<th>von Mises Stress [MPa]</th>
<th>Maximum Deflection [μm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Total Structure</td>
<td>Mirror Component</td>
</tr>
<tr>
<td>Original</td>
<td>99.4</td>
<td>0.130</td>
<td>0.0580</td>
</tr>
<tr>
<td>1) Bottom Plate</td>
<td>140.1</td>
<td>0.110</td>
<td>0.0543</td>
</tr>
<tr>
<td>2) Holes in Ribs</td>
<td>92.9</td>
<td>0.414</td>
<td>0.0880</td>
</tr>
<tr>
<td>3) Holes in Ribs +</td>
<td>133.62</td>
<td>0.546</td>
<td>0.0936</td>
</tr>
<tr>
<td>Bottom Plate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Holes in Ribs +</td>
<td>131.18</td>
<td>0.552</td>
<td>0.0948</td>
</tr>
<tr>
<td>Holes in Bottom Plate</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.1.5 Discussion of Results

Adding a bottom plate to the original Design 3 configuration (1\textsuperscript{st} modification) intuitively helps in reducing overall deflections. Specifically, the 41\% increase in total mass from the 15 mm thick plate leads to a 48\% decrease in total deflections. The effectiveness of adding the bottom plate becomes more evident when the stresses from the self-weight load are examined. In the original configuration, the maximum von Mises stress is located around the interface region between the top mirror plate and the radial ribs. The addition of the bottom plate shifts the stresses away from the top mirror and rib interface towards the bottom plate where the supports are located.

In the separate case of including holes in the radial ribs (2\textsuperscript{nd} modification), future production and installations for glass sandwich panels will need to be approached with caution. For the specific case considered, the 6.5\% decrease in mass intensified the total deflections by as much as 27\% compared to the original configuration. Furthermore, the irregularities caused by the holes shift
the maximum stresses towards the edges of the hole. It should be noted that the stresses on the actual mirror surface are still higher compared to the original configuration. One design solution to relieve these high stressed hole regions is to reinforce the edges with the same volume of material missing for each hole.

Notable results can be seen when the effects of both including holes in the radial ribs and adding a bottom plate are combined (3rd and 4th modifications). While the overall structural deflections are reduced by as much as 9.6% compared to the original Design 3 configuration, the stresses are increased by approximately 325%. The amplified stresses are located at the supports on the bottom plate and may be attributed to the lack of in-plane stiffness provided by the radial ribs due to missing material from the holes.

Lastly, despite the analysis and modifications outlined in this case study, the total deflections are still quite too large to satisfy common rule of thumb limits for astronomical telescopes specified in Table 15. However, for the geometry and gravity loading condition considered in this section, the stress and deflection results enforce the great potential for an all-glass sandwich assembly as viable structural systems in other applications.
4.2 Glass-Timber Pedestrian Bridge

The Fraunhofer Institute for Manufacturing Technology and Advanced Materials (IFAM) is a leading independent research organization in Europe in the fields of Shaping and Functional Materials, and Adhesive Bonding Technology and Surfaces. As part of their research and development work, the conceptual design of a glass-timber composite bridge is currently being developed. While the combination of timber and glass shows great architectural expression, the design must, nonetheless, be proven to perform under imposed loading conditions to meet strength and serviceability requirements.

The unique bridge structure draws upon the inherent beneficial properties from the two main constituent materials: timber and glass. Timber is an exceptional and reliable flexural material, as evidenced by many current design codes used all over the world. Glass can be treated to withstand rigorous loading conditions. From a conceptual standpoint, the combination of both materials can provide a rigid and sturdy pedestrian bridge structure. Presented herein is a preliminary study meant to investigate the viability of the structure against specific loading conditions. While the analysis and corresponding results are by no means exhaustive, the results can aid in providing a preliminary proof of concept that can be extended for the future development of the structure.

4.2.1 Bridge Description

The bridge geometry and conceptual design was provided by IFAM. The bridge measures 10.3 m in length and 1.4 m in width. Longitudinal timber chords form the top and bottom spans of the structure and are sandwiched by double-layered tempered safety glass panels on either side of the chords by a stiff layer of 2C silane epoxy adhesive. Transverse timber beams line the length of the longitudinal chords and are anchored in place to the bottom longitudinal chords with glued-in steel rods. Tempered safety glass slabs lie on top of the transverse beams and are fixed in place to the transverse beams with a layer of flexible silicone adhesive. Neoprene bearing pads are used to simply support the bridge structure on either end. Architectural renderings provided by IFAM are included in Figure 57 and Figure 58. A simplified engineering drawing for the bridge details is shown in Figure 59.
Figure 57: Side elevation view of the IFAM bridge.

Figure 58: Cross-sectional view of the IFAM bridge.
Figure 59: IFAM Bridge Dimensions and Details.
4.2.2 Model Development

As a preliminary proof of design concept, the pedestrian bridge was modelled on ANSYS and was subjected to different static load cases as follows:

1) Dead load from the overall self-weight.
2) Live load from a uniform 5 kPa pressure acting on the individual glass slabs.
3) Combined dead load and live load.
4) A uniformly distributed lateral load of 1.5 kN/m acting on the handrail.

The last load case is meant to provide an estimate of how much demand the glued-in steel rods should be designed for. Additional safety factors of 1.35, 1.5, and 1.5 have been applied to the dead load, live load, and uniform lateral load, respectively. A copy of the general input script is attached in Appendix C for further reference.

The bridge is symmetric about two perpendicular vertical planes oriented along the longitudinal and transverse global bridge coordinate system. Therefore, quarter symmetry boundary conditions are applied to the model so that a finer mesh density can be used and the overall computation time is reduced without compromising the accuracy of the results. Mesh element sizes of 60 mm, 40 mm, 12 mm, 20 mm, and 20 mm correspond to the timber, tempered safety glass, 2C silane epoxy, silicone, and neoprene pad respectively are used. The glued-in steel rods connecting the transverse beams to the longitudinal chords are not explicitly modelled, but the stiff connection is modelled by gluing the longitudinal and transverse bodies together. Lastly, the bottom face of the neoprene bearing pad is fixed to simulate pinned boundary conditions. A representation of the quarter-symmetric FE model is shown in Figure 60, depicting the symmetry boundary conditions, the fixed neoprene pad, and the surface area loads on the horizontal glass slabs. The gravitational acceleration is shown as a red vertical arrow located at the origin of the global coordinate system.
4.2.3 Material Properties

Orthotropic wood properties are implemented into the program while simple linear elastic materials for both the glass and the adhesives were modelled. Table 21 and Table 22 summarize the specific material properties used in analyzing the bridge structure. The wood properties are representative of Black Oak, Red, a hardwood species commonly found in the United States with an average moisture content of 12% (Kretschmann, 2010). Strictly speaking, the transverse and radial orthotropic properties of wood are different. Additionally, it is realistically impossible to determine the radial and tangential axes of the source material once the processed material arrives on-site for construction. The radial and tangential properties used are therefore averaged values between both directions.
Table 21: *Red Oak Orthotropic Mechanical Properties* (Kretschmann, 2010).

<table>
<thead>
<tr>
<th>Properties</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Elastic Modulus</td>
<td>$E_L$</td>
<td>[MPa]</td>
<td>12430</td>
</tr>
<tr>
<td>Transverse Elastic Modulus</td>
<td>$E_T$</td>
<td>[MPa]</td>
<td>1467</td>
</tr>
<tr>
<td>Radial Elastic Modulus</td>
<td>$E_R$</td>
<td>[MPa]</td>
<td>1467</td>
</tr>
<tr>
<td>Longitudinal-Radial Shear Modulus</td>
<td>$G_{LR}$</td>
<td>[MPa]</td>
<td>1106</td>
</tr>
<tr>
<td>Radial-Tangential Shear Modulus</td>
<td>$G_{RT}$</td>
<td>[MPa]</td>
<td>1007</td>
</tr>
<tr>
<td>Longitudinal-Tangential Shear Modulus</td>
<td>$G_{LT}$</td>
<td>[MPa]</td>
<td>1007</td>
</tr>
<tr>
<td>Longitudinal-Radial Poisson Ratio</td>
<td>$\nu_{LR}$</td>
<td>[-]</td>
<td>0.35</td>
</tr>
<tr>
<td>Radial-Transverse Poisson Ratio</td>
<td>$\nu_{RT}$</td>
<td>[-]</td>
<td>0.56</td>
</tr>
<tr>
<td>Longitudinal-Transverse Poisson Ratio</td>
<td>$\nu_{LT}$</td>
<td>[-]</td>
<td>0.448</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho$</td>
<td>[kg/mm$^3$]</td>
<td>650</td>
</tr>
</tbody>
</table>
Table 22: Other Bridge Material Properties.\(^4\)

<table>
<thead>
<tr>
<th>Materials</th>
<th>Elastic Modulus, (E) [MPa]</th>
<th>Poisson Ratio, (\nu) [-]</th>
<th>Density (\rho) [kg/mm(^3)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tempered Safety Glass</td>
<td>70000</td>
<td>0.23</td>
<td>2500</td>
</tr>
<tr>
<td>2C Silane Epoxy Adhesive</td>
<td>125</td>
<td>0.38</td>
<td>1000</td>
</tr>
<tr>
<td>Silicone Adhesive</td>
<td>5</td>
<td>0.4</td>
<td>1000</td>
</tr>
<tr>
<td>Neoprene Bearing Pad</td>
<td>100</td>
<td>0.3</td>
<td>1000</td>
</tr>
</tbody>
</table>

4.2.4 Results

Table 23 and Table 24 summarize some preliminary stress and displacement vector results from individual load cases, as well as combined loads acting on each bridge component. Example analysis results from the dead and live load case combinations for individual bridge components are shown in Figure 61 and Figure 62.

![Figure 61: Total Vertical Deflection (Left), von Mises Stress on Longitudinal Oak (Right).](image)

\(^4\) Material properties are derived from consultations with IFAM.
Figure 62: von Mises Stress Plots: Tempered Safety Glass (Left) and Epoxy Adhesive (Right).

Table 23: IFAM Bridge Components Maximum von Mises Stress.

<table>
<thead>
<tr>
<th>Bridge Component</th>
<th>Load Case Maximum von Mises Stress Results [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dead Load</td>
</tr>
<tr>
<td>Longitudinal Wood</td>
<td>3.58</td>
</tr>
<tr>
<td>Transverse Wood</td>
<td>1.19</td>
</tr>
<tr>
<td>Vertical Glass Panels</td>
<td>3.14</td>
</tr>
<tr>
<td>Glass Slabs</td>
<td>0.54</td>
</tr>
<tr>
<td>Epoxy Adhesive</td>
<td>1.81</td>
</tr>
<tr>
<td>Silicone Adhesive</td>
<td>0.057</td>
</tr>
</tbody>
</table>
Table 24: IFAM Bridge Components Maximum Displacement Vector Sums.

<table>
<thead>
<tr>
<th>Bridge Component</th>
<th>Load Case Displacement Vector Sums Results [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dead Load</td>
</tr>
<tr>
<td>Longitudinal Wood</td>
<td>1.88</td>
</tr>
<tr>
<td>Transverse Wood</td>
<td>1.96</td>
</tr>
<tr>
<td>Vertical Glass Panels</td>
<td>1.89</td>
</tr>
<tr>
<td>Glass Slabs</td>
<td>1.96</td>
</tr>
<tr>
<td>Epoxy Adhesive</td>
<td>1.88</td>
</tr>
<tr>
<td>Silicone Adhesive</td>
<td>1.96</td>
</tr>
</tbody>
</table>

4.2.5 Discussion of Stress and Displacement Results

The modulus of rupture of a material reflects the maximum capacity a member can sustain under bending in the elastic domain. The typical modulus of rupture for Oak at 12% moisture content is 57 MPa. The von Mises stress results indicate that both the longitudinal and transverse timber components are expected to adequately resist the effects of factored dead and live loads. As per AASHTO LRFD guidelines for the design of pedestrian bridges, deflections should be limited to 1/500 of the span for serviceability requirements (The American Association of State Highway and Transportation Officials, 2009). For a straight bridge with supports located 8.5 m away, the deflection limit would be 17 mm. While these guidelines are based on un-factored dead and live loads, the extracted deflections from the FE analysis indicate that the deflection limits for both the longitudinal and transverse wood members have been met despite applying safety factors for both the dead and live loads. It should be noted that only principal loads are considered in the analysis with no companion load contributions from snow or wind. However, given that
principal loads only utilize 12% of the maximum deflection, it is expected that contributions from factored down companion loads will still be met. 

As previously discussed, glass on its own behaves in a linear elastic manner with no significant post-yielding effects. Comparing the von Mises stress to a non-existent yield value would therefore be meaningless. Additionally, due to the sub-critical growth phenomenon, it is possible that glass can fail at regions that are not subjected to the maximum principal tensile stress. Previously described design codes treat the determination of the strength of glass in a deterministic manner. Equation 2 in Chapter 2 is firstly adjusted to account for heat-treated glass as follows:

\[
P_f = 1 - \exp\left(-\bar{k}A\sigma_{\text{max}}^m\right) \tag{76}
\]

\[
P_f = 1 - \exp\left[-\bar{k}A\left(\bar{\sigma}_{\text{max}} + \sigma_{\text{pre}}\right)^m\right] \tag{77}
\]

The crack opening stress, termed as the maximum equivalent principal stress is increased by the pre-stress strength enhancement value denoted as \(\sigma_{\text{pre}}\). With a probability of failure of 0.8% and a minimum pre-stress of 69 MPa (ASTM), the estimated maximum tensile stress for both the vertical and horizontal glass components is approximately 85 MPa. The glass is expected to be able to sustain both dead and live loads in this regard. Deflection limits as per ASTM E1300 require that for the 1350 mm deep, 750 mm wide, and 10 mm thick vertical glass panels, the deflection is limited to approximately 10 mm. The value is conservatively calculated by assuming that the full 1.5 kPa handrail load acts directly on the vertical glass panel faces. Additionally, the 1300 mm deep, 820 mm wide, and 12 mm thick horizontal glass slabs subjected to a uniform 5 kPa is limited to approximately 11 mm in center-of-glass deflection. From the extracted results, the glass components fall well below these required limits, as calculated by Equation 6.

Mechanical properties for adhesives are highly dependent on the load intensity, duration, curing time, temperature, and construction workmanship. Furthermore, values for similar proprietary adhesives also vary from manufacturer to manufacturer. Despite the common disparity of assumed values, minimum yield strengths typically used are 1-5 MPa and 45 MPa for silicones
and epoxies, respectively. The resultant extracted stresses from the FE model in Table 23 indicate that the values are below these material limits.

Overall, the preliminary analysis results show that the glass-timber bridge can be a viable option as a pedestrian structure. All bridge components are within the elastic range when compared to typical yield values, while deflection requirements based on widely accepted design codes are satisfied. The potential applicability of the structure is further justified by the fact that the results are based on imposed factors on the individual dead and live loads.
CHAPTER 5: Concluding Remarks

5.1 Parametric Analyses of Bolted and Adhesively-bonded Connections

The parametric analysis indicates that adhesively-bonded connections generally out-perform bolted connections. A maximum probable load is applied to both connection types and the resulting tensile stresses transmitted into the glass panel are generally lower in bonded than in bolted connections. This is found by running different analyses involving soft bushing materials that surround the main steel bolt body. A positive parabolic tensile stress distribution is seen for bolted connections, while the tensile stresses are inversely proportional to an increasing bond area for adhesively-bonded joints. The comparison is based on a bond area that is equal to the bolted connection in shear stiffness.

Current empirical equations by Frocht and Hill (1940), Duerr (2006), and Howland (1930) for tensile stresses, and those given by Ciavarella and Decuzzi (2001) for compressive stresses, generally agree with the results from the FE model. Beyond a hole diameter to glass width ratio of 0.6, the empirical equations start to deviate. The analytical compressive stresses can be matched numerically by setting the stiffness factor to 0.1 times the contact element stiffness and the element sizes of the glass panel and the steel bolt to $\frac{1}{4}$ times the chosen panel thickness.

The simplified design formula as per Goland and Reinsner (1944) and Bigwood and Crocombe (1989) for the adhesive connections yielded fairly similar results for the specific geometric and material parameters considered. This is achieved by specifying an element size equivalent to the adhesive layer thickness. The solution provided by Volkersen (1938) underestimates the joint edge maximum shear stresses since the additional contribution from the offset in adhered mid-surfaces are not taken into account. Simple linear elastic average shear stress theory also underestimates the maximum shear stress for bonded joints due to the fact that the adherends are assumed to be infinitely rigid and the total stress is uniformly distributed over the bond area. All in all, the maximum shear stresses within the adhesive layer and the principal tensile stresses transferred into the glass panel mostly depend on the adhesive bond area, with insignificant changes stemming from edge and end distances.
5.2 Case Studies

Two case studies are presented highlighting the potential for glass in structural applications. In the context of past studies of ribbed telescope mirrors, the development of a sandwich panel shows the effectiveness of borosilicate glass to withstand gravity loads. The connected regions can be achieved either through a bolted configuration, but the design should consider the effects of stress concentrations and component geometries on the resultant tensile stress of glass. The analysis also reveals that adhesively-bonded joints generally perform better than pinned connections. Depending on the specified application at hand, a stiffer adhesive can minimize joint deformations at the expense of added shear. The main conclusion from the analysis conducted on the IFAM Bridge is that the overall integration of timber and glass joined with adhesives can be a viable option for pedestrian bridges. Code-based limits are used to assess the preliminary performance of the structure under factored dead and live loads and all structural components are expected to be within their elastic range.

5.3 Future Research

The study presented in this thesis is approached from a purely numeric and analytical approach. Future research should focus on experimental investigations to corroborate the obtained results. For the parametric analysis of the two connection types, an adequate number of samples need to be collected for statistical analyses. Regression models can be used to assess the sensitivity of individually defined parameters for the output in question. Values used in this thesis are derived from past research on adhesive properties that are very specific to the testing regime the products are subjected to, consequently yielding different results. Further testing is required to quantify individual bulk adhesive properties.

The numerical analysis can be improved by extending the material behaviour embedded within the analysis software. Simple elastic and elastic-perfectly plastic material models are used to model all materials. Prony series (a method that extracts valuable information from a uniformly sampled signal) constants should be defined for different temperatures to effectively model the viscoelastic properties of bulk adhesives. To this end, future analyses should focus on the effects of temperature on the behaviour of in-plane loaded glass connections.
While the research field is fairly new, the goal of the work presented in this thesis is to contribute to the understanding of glass behaviour in non-conventional load bearing applications. Ultimately, it is hoped that design provisions tailored towards the treatment of glass components in major structural purposes can be established in the future and that the conducted work can be used to extend the current state of knowledge.
References


Appendix A – ANSYS Input Scripts

A.1 General Bolted Connection in Shear Example Input Script

! ** GENERAL NOTES
! UNITS IN [m], [N], [kg], [degrees Celsius]

!** TITLE
/TITLE, BOLTED CONNECTION IN SHEAR; NO BUSHING

!** PLOTCONTROLS SETTINGS

FINISH
/CLEAR,START
/SHRINK,0
/ESHAPE,1.0
/EFACET,1
/RATIO,1,1,1
/CFORMAT,32,0
/REPLOT
/PNUM,AREA,1

/NERR,0,99999999,-1,-1
/UIS,MSGPOP,4

!****************************************************************************
!
MODEL DEVELOPMENT
!****************************************************************************

!** PREPROCESSOR
/PREP7

/INPUT,Mats_Elements.txt

*CREATE,EXECUTABLEMACRO

*DO,LOAD_OPTION,1,2,1

*IF,LOAD_OPTION,EQ,1,THEN

*CFOPEN,ANALYSIS_RESULTS_S1,OUT,,

*ELSE
*CFOPEN,ANALYSIS_RESULTS_S3.OUT,\

*ENDIF

!* Modelling Submenu

*DO,dh,0.025,0.085,0.010

/PREP7
ALLSEL,ALL
VCLEAR,ALL
VDELE,ALL,,,1
EDELE,ALL
NDELE,ALL

*SET,efit,0.01
*SET,delta,efit*dh
*SET,THICK,0.006
*SET,Prob_F,0.008
*SET,m_param,7
*SET,k_param,2.86E-53
*SET,con_area,3.14*dh*THICK
*SET,x_param,1-Prob_F
*SET,m_fine,1

R,3,0,0,0.01,0.1,,
RMORE,0,0,1E20,(dh-delta)/2,0,0
RMORE,0,0,0,0

N,1,0,delta/2,0
N,2,0,delta/2,THICK

BLOCK,-H/2,H/2,-c,c,0,THICK
CYLIND,dh/2,,0,THICK,,
VSBV,1,2,,DELETE
CYL4,0,delta/2,(dh-1*delta)/2,,,THICK

ALLSEL,ALL
ASEL,S,LOC,X,H/2,H/2
VSLA,S
VATT,1,,1

ALLSEL,ALL
ASEL,S,LOC,Y,delta-dh/2,delta-dh/2
VSLA,S
VATT,2,,1
!* Meshing Submenu

ALLSEL,ALL
VSELS,MAT,,2,2
ESIZE,THICK*m_fine/2
MSHKEY,0
MSHAPE,0,3D
VSWEEP,ALL

ALLSEL,ALL
VSELS,MAT,,1,1
ESIZE,THICK*m_fine/2
MSHKEY,0
MSHAPE,0,3D
VSWEEP,ALL

ALLSEL,ALL
VSELS,MAT,,1,1
ASLV,S
ASELR,LOC,Y,dh/2,dh/2
ASELA,LOC,Y,-dh/2,-dh/2
NSLA,S,1
TYPE,2
MAT,1
REAL,3
ESYS,0
SECNUM,0
TSHAP,QUA8
ESURF,,TOP,

ALLSEL,ALL
VSELS,MAT,,2,2
ASLV,S
ASELU,LOC,Z,0
ASELU,LOC,Z,THICK
NSLA,S,1
TYPE,3
MAT,2
REAL,3
ESYS,0
SECNUM,0
TSHAP,QUA8
ESURF,,TOP,
RUNNING THE ANALYSIS

!******************************************************************************
!

SOLUTION

/ SOLU

ANTYPE,0

NLGEOM,1
NSUBST,100,1000,10
OUTRES,ERASE
OUTRES,ALL,1
AUTOTS,1
LNSRCH,1
NEQIT,100
TIME,1

*IF, LOAD_OPTION, EQ, 1, THEN

ALLSEL, ALL
NSEL, S, LOC, Y, -c
D, ALL, UX, 0
D, ALL, UY, 0
D, ALL, UZ, 0

*ELSE

ALLSEL, ALL
NSEL, S, LOC, Y, c
D, ALL, UX, 0
D, ALL, UY, 0
D, ALL, UZ, 0

*ENDIF

ALLSEL, ALL
VSEL, S, MAT, 1, 1
ASLV, S
ASEL, R, LOC, Y, -dh/2, -dh/2
DA, ALL, UX, 0
DA, ALL, UZ, 0

ALLSEL, ALL
VSEL, S, MAT, 1, 1
ASLV, S
ASEL,R,LOC,Y,dh/2,dh/2  
DA,ALL,UX,0  
DA,ALL,UZ,0

ALLSEL,ALL  
VSEL,S,MAT,,2,2  
ASLV,S  
ASEL,R,LOC,Z,0  
DA,ALL,UX,0  
DA,ALL,UZ,0

ALLSEL,ALL  
VSEL,S,MAT,,2,2  
ASLV,S  
ASEL,R,LOC,Z,THICK  
DA,ALL,UX,0  
DA,ALL,UZ,0

ALLSEL,ALL  
NEAR1 = NNEAR(1)  
NEAR2 = NNEAR(2)  
NSEL,S,NODE,,NEAR1  
NSEL,A,NODE,,NEAR2  
CP,1,UY,ALL  
F,ALL,FY,7000/2/2

ALLSEL,ALL  
SOLVE

!*****************************************************************************
!                POST-PROCESSING ANALYSIS RESULTS                           
!*****************************************************************************

!* POSTPROCESSOR
/POST1

ALLSEL,ALL  
VSEL,S,MAT,,1,1  
ASLV,S  
ASEL,R,LOC,Y,dh/2,dh/2  
ASEL,A,LOC,Y,-dh/2,-dh/2  
NSLA,S,1

*IF,LOAD_OPTION,EQ,1,THEN

NSORT,S,1,0,1,,SELECT
*GET,N_Stress,SORT,0,MAX
*GET,Cor_node,SORT,0,IMAX
*GET,Cor_nodeX,NODE,Cor_node,LOC,X
*GET,Cor_nodeY,NODE,Cor_node,LOC,Y
*GET,Cor_nodeZ,NODE,Cor_node,LOC,Z

NSORT,U,Y,1,1.,SELECT
*GET,N_maxD,SORT,0,MAX
*GET,Cor_nodemaxD,SORT,0,IMAX
*GET,Cor_nodeDX,NODE,Cor_nodemaxD,LOC,X
*GET,Cor_nodeDY,NODE,Cor_nodemaxD,LOC,Y
*GET,Cor_nodeDZ,NODE,Cor_nodemaxD,LOC,Z

*VWRITE,dh,Cor_node,Cor_NodeX,Cor_nodeY,Cor_NodeZ,N_Stress,Cor_nodemaxD,Cor_nodeDX,Cor_NodeDY,Cor_NodeDZ,N_maxD
(E12.6,2X,F12.0,2X,E12.6,2x,E12.6,2x,E12.6,2x,E12.0,2x,F12.0,2x,E12.6,2x,E12.6,2x,E12.6,2x,E12.6,2x)

*ELSE

NSORT,S,3,0,1.,SELECT
*GET,N_Stress,SORT,0,MAX
*GET,Cor_node,SORT,0,IMAX
*GET,Cor_nodeX,NODE,Cor_node,LOC,X
*GET,Cor_nodeY,NODE,Cor_node,LOC,Y
*GET,Cor_nodeZ,NODE,Cor_node,LOC,Z

NSORT,U,Y,1,1.,SELECT
*GET,N_maxD,SORT,0,MAX
*GET,Cor_nodemaxD,SORT,0,IMAX
*GET,Cor_nodeDX,NODE,Cor_nodemaxD,LOC,X
*GET,Cor_nodeDY,NODE,Cor_nodemaxD,LOC,Y
*GET,Cor_nodeDZ,NODE,Cor_nodemaxD,LOC,Z

*VWRITE,dh,Cor_node,Cor_NodeX,Cor_nodeY,Cor_NodeZ,N_Stress,Cor_nodemaxD,Cor_nodeDX,Cor_NodeDY,Cor_NodeDZ,N_maxD
(E12.6,2X,F12.0,2X,E12.6,2x,E12.6,2x,E12.6,2x,E12.0,2x,F12.0,2x,E12.6,2x,E12.6,2x,E12.6,2x,E12.6,2x)

*ENDIF

*ENDDO

*CFCLOS

*ENDDO
*END

*USE,EXECUTABLEMACRO

!** END OF CODE.
A.2 Sandwich Panel Example Input Script

! ** GENERAL NOTES
! UNITS IN [mm], [N], [Mg], [degrees Celsius]

!** TITLE
/TITLE, ALL-GLASS SANDWICH PANEL

!** PLOTCONTROLS SETTINGS

FINISH
/CLEAR,START
/SHRINK,0
/ESHAPE,1.0
/EFACET,1
/RATIO,1,1,1
/CFORMAT,32,0
/REPLOT

/NUM,AREA,1

/NERR,0.99999999,-1.-1
/UIM,MSGPOP,4

!*****************************************************************************

!*****************************************************************************

!** MODEL DEVELOPMENT

!*****************************************************************************

!** PREPROCESSOR

/REP7
ET,1,SHELL181
MPTEMP,....
MPTEMP,1,0
MPDATA,EX,1,,50000
MPDATA,PRXY,1,,0.2
MPTEMP,....
MPTEMP,1,0
MPDATA,DENS,1,,2.4E-9

! Define the section properties

SECT,1,SHELL,.
SECDATA, 15,1,0,0,3
SECOFFSET,BOT
SECCONTROL,..., , ,
SECT,2,SHELL,,
SECDATA, 15,1,0,3
SECOFFSET,MID
SECT,3,shell,,
SECDATA, 15,1,0,3
SECOFFSET,TOP
SECCONTROL,0,0,0, 0, 1, 1, 1

! Define the geometry

CYL4,,600
WPRO,.90,
RECTNG,0,600,0,-120,
WPCSYS,-1,0
WPOFFS,173.21,0,-60
WPRO,.90,
CYL4,,40
ASBA,2.3

WPCSYS,-1,0
WPOFFS,473.21,0,-60
WPRO,.90,
CYL4,,40
ASBA,4.2

WPCSYS,-1,0
CSYS,1
AGEN,6,3,,60,
K,100,600,30,0
K,101,600,30,-120
LSTR,100,101
LSTR,7,100
LSTR,6,101
AL,6,77,78,79

CSYS,0
WPCSYS,-1,0
WPOFFS,600-((600-600*COS(30*ACOS(-1)/180))/2),150,-60
WPROTA,15
WPROTA,0,0,90
CYL4,,40
ASBA,8.9
ASEL,U,LOC,Z,0
ASEL,R,LOC,X,550,600
CSYS,1
AGEN,12,ALL,,,30,,
ALLSEL,ALL

CSYS,1
K,200,346.41,60,0
K,201,346.41,60,-120
K,202,346.41,120,0
K,203,346.41,120,-120

LSTR,200,201
LSTR,202,203
LSTR,200,202
LSTR,201,203
AL,172,173,174,175
WPCSYS,-1,0
WPOFFS,0,346.41*COS((30*ACOS(-1))/180),-60
WPRO,,90,
CYL4,,,40
ASBA,20,21
ASEL,S,AREA,,22
AGEN,6,ALL,,,60,,
ALLSEL,ALL

LSTR,100,206
LSTR,101,205
AL,220,221,206,77
WPCSYS,-1,0
WPOFFS,346.41+(0.5*346.41*COS((60*ACOS(-1))/180)),0.5*346.41*SIN((60*ACOS(-1))/180),-60
WPROTA,,90,
WPROTA,0,0,60
CYL4,,,40
ASBA,26,27
ASEL,S,AREA,,28
AGEN,6,ALL,,,60,,
ALLSEL,ALL

LSTR,100,224
LSTR,101,225
AL,266,267,227,77
WPCSYS,-1,0
WPOFFS,346.41,346.41*COS((30*ACOS(-1))/180),-60
WPROTA,,90,
CYL4,,,40
ASBA,32,33
ASEL,S,AREA,,34
! Define mesh attributes and properties

ASEL,S,LOC,Z,0
AATT,1,1,0,1
ALLSEL,ALL

ASEL,U,LOC,Z,0
ASEL,U,LOC,Z,-120
AATT,1,1,0,2

ASEL,U,SEC,,2
AATT,1,1,0,3

ALLSEL,ALL
WPOFFS,0,190,-120
CYL4,,50
ASEL,U,SECN,,1
ASEL,U,SECN,,2
ASEL,U,SECN,,3
CSYS,1
AGEN,6,ALL,,,,60,,
AATT,1,1,0,4

CSYS,0
WPCSYS,-1,0

WPOFFS,0,400,-120
CYL4,,60
ASEL,R,LOC,Y,400
CSYS,1
agen,6,all,,,,60,,
AATT,1,1,0,4
CSYS,0
WPCSYS,-1,0
ASEL,S,LOC,Z,-120

ASBA,38,39
ASBA,51,40
ASBA,38,41
ASBA,39,42
ASBA,38,43
ASBA,39,44

ASBA,38,45
ASBA,39,46
ASBA,38,47
ASBA,39,48
ASBA,38,49
ASBA,39,50

ALLSEL,ALL
ASEL,R,LOC,Z,-120
AATT,1,,1,0,3

ALLSEL,ALL
CSYS,0
WPCSYS,-1,0

AOVLAP,ALL
AGLUE,ALL

ESIZE,10,
!!  Mesh
ALLSEL,ALL
MSHKEY,0
AMESH,ALL

!***************************************************************************
!                     RUNNING THE ANALYSIS                                 |
!***************************************************************************

/SOLU
NSEL,S,LOC,Z,-120
NSEL,R,LOC,Y,0
NSEL,R,LOC,X,,-345,-350
D,ALL,UX,,,,UY,UZ

NSEL,S,LOC,Z,-120
NSEL,R,LOC,X,172.5,173.5
D,ALL,UX,,,,,UY,UZ

ALLSEL,ALL
ACEL,0,0,9810,
SOLVE

!******************************************************************************
!
ANALYSIS RESULTS AND DATA EXTRACTION
!
******************************************************************************

/POST1
ALLSEL,ALL

/TITLE,Total structure deflection due to self-weight
PLNSOL,U,Z,,
/UI,COPY,SAVE,PNG,GRAPH,COLOR,REVERSE,LANDSCAPE,NO,100

/TITLE,Total structure von Mises stress due to self-weight
PLNSOL,S,EQV,0,1.0
/UI,COPY,SAVE,PNG,GRAPH,COLOR,REVERSE,LANDSCAPE,NO,100

ALLSEL,ALL
ASEL,S,LOC,Z,0
ESLA,S
/TITLE,Mirror component deflection due to self-weight
PLNSOL,U,Z,,
/UI,COPY,SAVE,PNG,GRAPH,COLOR,REVERSE,LANDSCAPE,NO,100

/TITLE,Mirror component von Mises stress due to self-weight
PLNSOL,S,EQV,0,1
/UI,COPY,SAVE,PNG,GRAPH,COLOR,REVERSE,LANDSCAPE,NO,100

/SHRINK,0
/ESHAPE,1.0
/EFACET,1
/RATIO,1,1,1
/CFORMAT,32,0
/REPLOT

ALLSEL,ALL
ASEL,U,LOC,Z,0
ESLA,S
/UI,COPY,SAVE,PNG,GRAPH,COLOR,REVERSE,LANDSCAOPE,NO,100

!** END OF CODE.
A.3 Glass-Timber Bridge Example Input Script

! ** GENERAL NOTES
! UNITS IN [m], [N], [kg], [degrees Celsius]

! ** ANALYSIS TITLE
/TITLE, GLASS-TIMBER BRIDGE

! ** PLOTCONTROLS SETTINGS
FINISH
/CLEAR,START
/SHRINK,0
/ESHAPE,1.0
/EFACET,1
/RATIO,1,1,1
/CFORMAT,32,0
/REPLOT

!/PNUM,AREA,1

/NERR,0.99999999,-1,-1
/UIS,MSGPOP,4

*******************************************************************************

MODEL DEVELOPMENT
*******************************************************************************

/INPUT,Mat_El_Props.txt

*******************************************************************************

** SECTIONAL AND ELEMENTAL GEOMETRY, LOADING & USER-INPUTS

! Chosen Load Case (1,2,3,4,5)
! 1 = Dead Load
! 2 = Live Load on Glass
! 3 = Lateral Uniform Distributed Load on Handrail
! 4 = 1+2+3
! 5 = 1+2

!Load_case = 5

! Loads
dl_sf = 1.35
dist_sf = 1.5
hand_sf = 1.5
pres_uf = 5000
hand_uf = 1000

! General Geometry
num_sec = 6
m_fine = 2

! 2C Epoxy Adhesive
adh1_th = 0.003

! Silicone Adhesive
adh2_th = 0.005

! Vertical Tempered Safety Glass
gvnum = 2
gvt = 0.010
gvd = 1.350
gvtt = gvnum*gvt
gvl = 0.750

! Horizontal Tempered Safety Glass
ghnum = 3
ght = 0.012
ghtt = ghnum*ght
ghw = 1.300
ghl = 0.820

! Upper and Lower Longitudinal Chord Solid Oak
wd1 = 0.180
wb1 = 0.100
wl1 = 0.850
wlamt = 0.020

! Transverse Solid Oak
wd2 = 0.180
wb2 = 0.090
wl2 = 1.400
wlam2t = 0.020

! Neoprene Bearing Pad
neo_th = 0.050

!--------------------------------------------------------------------------------------------------------------------!
!** CREATING VOLUMES AND ASSIGNING ATTRIBUTES
! Longitudinal oak top and bottom chords
BLOCK,0,wl1,gvd+2*wlamt-wd1,gvd+2*wlamt, wl2/2, wl2/2+wb1
BLOCK,0,wl1,0,wd1, wl2/2,wl2/2+wb1

! Vertical Tempered Safety Glass
BLOCK,(wl1-gvl)/2,(wl1-gvl)/2+gvl,wlamt,wlamt+gvd, wl2/2-adh1_th-gvtt, wl2/2-adh1_th
BLOCK,(wl1-gvl)/2,(wl1-gvl)/2+gvl,wlamt,wlamt+gvd, wl2/2-adh1_th-gvtt, wl2/2-adh1_th

! Vertical TSG to Longitudinal Chords Adhesive
BLOCK,(wl1-gvl)/2,(wl1-gvl)/2+gvl,gvd+2*wlamt-wd1,gvd+1*wlamt, wl2/2+wb1, wl2/2+wb1+adh1_th
BLOCK,(wl1-gvl)/2,(wl1-gvl)/2+gvl,gvd+2*wlamt-wd1,gvd+1*wlamt, wl2/2-adh1_th-wl2/2
BLOCK,(wl1-gvl)/2,(wl1-gvl)/2+gvl,wlamt,wl1/2+wb1, wl2/2+wb1+adh1_th
BLOCK,(wl1-gvl)/2,(wl1-gvl)/2+gvl,wlamt,wl1/2+wb1, wl2/2-adh1_th-wl2/2

! Transverse oak beams
BLOCK,0,wb2/2,0,wd2,0,wl2/2
BLOCK,wb1-wb2/2,0,wd2,0,wl2/2

! Horizontal Adhesive
BLOCK,(wl1-ghl)/2,(wl1-ghl)/2+(wb2/2-(wl1-ghl)/2),wd2, wd2+adh2_th,0, ghw/2
BLOCK,wb1-wb2/2,0+(wb2/2-(wl1-ghl)/2),wd2, wd2+adh2_th,0, ghw/2

! Tempered Safety Glass Slab
BLOCK,(wl1-ghl)/2,(wl1-ghl)/2+ghl,wd2+adh2_th,wd2+adh2_th+ghtt,0, ghw/2

*DO,i,1,(num_sec-1),1
ALLSEL,ALL
VSEL,S,LOC,X,0,wl1
VGEN,2,ALL,,wl1*i,,,1000*i
*ENDDO

! Neoprene Bearing Pad
BLOCK,(num_sec-1)*wl1-wb2/2,0,(num_sec-1)*wl1+wb2/2,-neo_th,0, wl2/2,wl2/2+wb1

ALLSEL,ALL
VGLUE,ALL

! Longitudinal Oak Top and Bottom Chord Attributes
VSEL,S,LOC,Z,0,wd2/2, wb1
VSEL,U,LOC,Y,-neo_th,0
VATT,1,,1

! Transverse Oak Beam Attributes
! Vertical TSG Attributes
ALLSEL,ALL
VSEL,S,LOC,Z,wl2,0/wl2/2
VSEL,R,LOC,Y,0,wd2
VSEL,U,LOC,Z,wl2/2-adh1_th,wl2/2
VATT,2,,1

! Horizontal TSG Slab Attributes
ALLSEL,ALL
VSEL,S,LOC,Y,wd2+adh2_th,wd2+adh2_th+ghtt
VATT,3,,1

! Vertical TSG Adhesive Attributes
ALLSEL,ALL
VSEL,S,LOC,Z,wl2/2+wb1+adh1_th,wl2/2+wb1+adh1_th+gvtt
VATT,3,,1
ALLSEL,ALL
VSEL,S,LOC,Z,wl2/2-adh1_th-gvtt,wl2/2-adh1_th
VATT,3,,1

! Horizontal Slab Adhesive Attributes
ALLSEL,ALL
VSEL,S,LOC,Y,wd2,wd2+adh2_th
VATT,5,,1

! Neoprene Bearing Pad Attributes
ALLSEL,ALL
VSEL,S,LOC,Y,-neo_th,0
VATT,6,,1

ALLSEL,ALL
VPLOT

!--------------------------------------------------------------------------------------------------------------------

!** FE MESHING

! Vertical Adhesive
ALLSEL,ALL
VSEL,S,MAT,,4
ESIZE,adh1_th*m_fine*2
MSHKEY,1
MSHAPE,0,3D
VMESH,ALL

! Horizontal Adhesive
ALLSEL,ALL
VSEL,S,MAT,,5
ESIZE,adh2_th*m_fine*2
MSHKEY,1
MSHAPE,0,3D
VMESH,ALL

! Vertical and Horizontal TSG
ALLSEL,ALL
VSEL,S,MAT,,3
ESIZE,gvt*m_fine*2
MSHKEY,0
MSHAPE,1,3D
VMESH,ALL

! Longitudinal Oak Top and Bottom Chords
ALLSEL,ALL
VSEL,S,MAT,,1,1
ESIZE,gvt*m_fine*3
MSHKEY,0
MSHAPE,1,3D
VMESH,ALL

! Transverse Oak Beams
ALLSEL,ALL
VSEL,S,MAT,,2,2
ESIZE,gvt*m_fine*3
MSHKEY,0
MSHAPE,1,3D
VMESH,ALL

! Neoprene Bearing Pad
ALLSEL,ALL
VSEL,S,MAT,,6
ESIZE,adh2_th*m_fine*2
MSHKEY,0
MSHAPE,1,3D
VMESH,ALL

ALLSEL,ALL
/VIEW,1,0.67,0.33,-0.67
E PLOT

!***********************************************************************
! RUNNING THE ANALYSIS + POST PROCESSING
!***********************************************************************

/SOLU

!*************************************************************************

!** SOLUTION CONTROLS

!*************************************************************************

!** BOUNDARY CONDITIONS

! Applied symmetric constraints
ALLSEL,ALL
ASEL,S,LOC,X,0,0,,1
ASEL,A,LOC,Z,0,0,,1
DA,ALL,SYMM

! Supports
ALLSEL,ALL
ASEL,S,LOC,Y,-neo_th,-neo_th
DA,ALL,UY
DA,ALL,UZ
ALLSEL,ALL

!*************************************************************************

!** LOAD APPLICATION

*DO,Load_case,1,5,1

*IF,Load_case,EQ,1,THEN

! Gravitational Acceleration - Load Case 1
ALLSEL,ALL
ACEL,0,9.81*dl_s,f,0
ALLSEL,ALL
SOLVE
/TITLE,Load Case - Dead Load
/INPUT,Results.txt

*ELSEIF,Load_case,EQ,2,

! TSG Slab Surface Pressure - Load Case 2
ALLSEL,ALL
VSEL,S,MAT,,3,3
ASLV,S
ASEL,R,LOC,Y,wd2+adh2_th+ghtt,wd2+adh2_th+ghtt,,1
SFA,ALL,1,PRES,pres_uf*dist_sf
ALLSEL,ALL
SOLVE
/TITLE,Load Case - Glass Pressure (5 kPa)
/INPUT,Results.txt

*ELSEIF,Load_case,EQ,3

! Handrail Horizontal Load
ALLSEL,ALL
VSEL,S,MAT,,1
ASLV,S
ASEL,R,LOC,Y,gvd+2*wlamt-wd1,gvd+2*wlamt,,1
ASEL,R,LOC,Z, wl2/2+wb1, wl2/2+wb1,,1
SFA,ALL,1,PRES,-hand_uf*hand_sf*num_sec/wd1
ALLSEL,ALL
SOLVE
/TITLE,Load Case - Lateral Uniform Distributed Load on Handrail
/INPUT,Results.txt

*ELSEIF,Load_case,EQ,4

! Load case 1 + Load Case 2 Interaction + Load Case 3
ALLSEL,ALL
ACEL,0,9.81*dl_sf,0
ALLSEL,ALL
VSEL,S,MAT,,3,3
ASLV,S
ASEL,R,LOC,Y,wd2+adh2_th+ghtt,wd2+adh2_th+ghtt,,1
SFA,ALL,1,PRES,pres_uf*dist_sf
ALLSEL,ALL
VSEL,S,MAT,,1
ASLV,S
ASEL,R,LOC,Y,gvd+2*wlamt-wd1,gvd+2*wlamt,,1
ASEL,R,LOC,Z,wl2/2+wb1, wl2/2+wb1,,1
SFA,ALL,1,PRES,-hand_uf*hand_sf*num_sec/wd1
ALLSEL,ALL
SOLVE
/TITLE,Load Case - Superimposed DL+LL+UDL
/INPUT,Results.txt

*ELSE
! Load case 1 + Load Case 2
ALLSEL,ALL
ACEL,0.981*dl_sf,0
ALLSEL,ALL
ASEL,S,LOC,Y,wd2+adh2_th+ghtt,wd2+adh2_th+ghtt,,1
SFA,ALL,1,PRES,pres uf*dist sf
ALLSEL,ALL
SOLVE
/TITLE,Load Case - Dead Load + Live Load
/INPUT,Results.txt

*ENDIF

*ENDDO

/TITLE,Load Case - Dead Load
ALLSEL,ALL
EPlot

******************************************************************************
!Post-processing embedded within the analysis section.
******************************************************************************

! Post-processing embedded within the analysis section.

!** END OF CODE.