INFLUENCE OF A WEAK LAYER ON DEPTH OF ROCK FAILURE IN UNDERGROUND LIMESTONE MINES IN HIGH HORIZONTAL STRESS ENVIRONMENTS

by

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Abstract

High horizontal stress environments have plagued the aggregate industry in shallow room and pillar mining operations, particularly in the northeastern United States. With population density increasing around Southern Ontario and environmental regulations becoming more stringent, it appears that Ontario’s aggregate industry could be looking underground in the near future as the source of material to meet the ever-increasing demand. Given that the near surface horizontal stress conditions in Southern Ontario are uniquely high ($\sigma_H/\sigma_V = 4-6$ to depths of 200m; Lo, 1978), it can be expected that slabbing and buckling failures observed in similar mining operations with lower stress regimes in the U.S. will be exacerbated in Southern Ontario. In an effort to be proactive with this expected geotechnical design issue, a distinct element analysis using UDEC was carried out to understand the mechanisms driving failure in stratified rock environments under high horizontal stress conditions as well as to observe the impact of the high horizontal stress on the maximum depth of failure in the roof where the roof is composed of limestone interbedded with a weak shale layer.

Accordingly, 116 models representing a variation of rock mass conditions subjected to stress ratios ($\sigma_H/\sigma_V$) ranging from 1 to 4 were simulated. A Voronoi tessellation was used to represent the intact rock mass directly above the excavation so that the failure profile through intact rock could be explicitly modelled.

Key conclusions from the modelling were as follows:

1) A shale layer within a defined distance from the roof of the excavation could increase the depth of failure to three times what would normally be estimated for stratified rock masses of limestone only.

2) Failure driven by the influence of a weak shale interbed occurs through diagonal fracturing, supporting an experimental conclusion published by Stimpson and Ahmed (1992), as opposed to slabbing or buckling. Slabbing and buckling are the common failure mechanisms in stratified rock masses without weak interbeds.

Therefore, it is critical to understand the interbedded nature of the rock mass comprising the roof over a mine opening so that a proper ground support design can be developed.
Preface

The topic of the research presented in this thesis was developed in collaboration with Golder Associates based on industrial challenges observed by the author and work colleagues during his previous employment with the company. The research program for the thesis was designed, carried out and analyzed solely by the author. The case studies discussed in Chapter 5 were facilitated by Golder Associates as well.
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c  Cohesion
E  Young’s Modulus
G  Shear Modulus
K  Bulk Modulus
k  In-Situ Stress Ratio
k_N Joint Normal Stiffness
k_S Joint Shear Stiffness
ϕ  Internal Friction Angle
γ  Specific Weight
ν  Poisson’s Ratio
ρ  Density
σ₁  Major Principal Stress
σ₂  Intermediate Principal Stress
σ₃  Minor Principal Stress
σ_C  Unconfined Compressive Strength
σ_H  Major Horizontal Stress
σ_h  Minor Horizontal Stress
σ_T  Tensile Strength
σ_V  Vertical Stress
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CHAPTER 1 INTRODUCTION

Limestone is a highly valued resource required for cement and aggregate production. Preference is given to mining it from surface (quarrying) as this is generally the most cost effective option. Even though, worldwide, underground mining of limestone already contributes to the supply of aggregate, building stone and lime, Canada has yet to develop its first underground limestone mine because of the abundance of quality limestone and dolostone resources available from surface; particularly in Southern Ontario around the great lakes. However, with increasing population growth, the depletion of readily accessible resources, and societal concerns for preserving environmentally sensitive areas resulting in stricter regulations for the opening of new quarry operations, underground mining is now being considered as an alternative option for accessing and developing new limestone sources.

Limestone deposits are located near existing or past water bodies (nature of deposition), which is where 90% of the population of Ontario lives (Ontario Geological Survey, 1992). The Greater Toronto Area (GTA) limestone and dolostone industry has been the most impacted because it holds the largest aggregate and construction materials markets whilst population densities and environmental regulations prevent quarry expansions and/or new quarry permitting close to these markets. For a low value, bulk commodity such as limestone, transportation costs represent a major part of the product’s end cost and therefore distance to market has a significant influence on the economic feasibility of aggregate production. Consequently, the greater transportation costs for supplying these materials to the GTA market result in higher material prices, and therefore, as documented by Thompson (1982), local underground mines in this area have potential to be more economical than surface quarries which are up to three times the distance from the market.

Underground mining of stratified deposits, such as limestone and dolostone, is typically carried out using the room and pillar mining method and presents different engineering challenges than those common in quarrying. Experience from the United States of America (US) suggests that roof failures via slabbing or buckling constitute the most prominent hazard and stability concern (Esterhuizen et al., 2008). This is related to the geology of limestone and other sedimentary rock deposits, which typically respond to applied stress like beams. Much research has been completed to date on the development of a stress arch in homogenous, stratified rock masses and Vousoir beams (Diederichs, 1999; Ran et al., 1994; Beer & Meek, 1982; Evans, 1941) to assist with establishing suitable ground support for mining operations by understanding the probable depth of roof failure in these environments. However, the
inclusion of a weak layer (creating a heterogeneous, stratified rock mass) seems to govern rock mass failure mechanisms resulting in a depth of failure beyond the anticipated depth of failure predicted using the stress arch theory. As such, ground support approaches in these environments are at best moderately successful in preventing roof failures as observed in the US. This degree of uncertainty is not acceptable/desirable from the perspective of safety and resource isolation; large failure can prevent access to high quality stone. Additionally, geotechnical misunderstanding resulting in on-going remediation to deal with ground failures significantly increases the operating costs for the mining operation and decreases the feasibility of the project.

Exacerbating this situation in Southern Ontario is that the area is known to have abnormally high horizontal in-situ stresses. The relative alignment of these high stresses and the weak layer in the stratified rock mass increases the potential for excessive slabbing/overbreak (depth of failure) where the thickness between the roof and weak layer is insufficient to allow for proper redistribution of the induced stresses onto adjacent pillars.

The National Institute for Occupational Safety and Health (NIOSH) based in Pennsylvania has funded numerous observational studies over the past 20 years in an effort to reduce the number of roof failures in US limestone mines under high horizontal stress conditions. The outcome of these studies (Esterhuizen et al., 2008; Esterhuizen et al., 2007; Iannacchione et al., 2001) was a series of design criteria for developing a room and pillar mine so that the impact of the high horizontal stresses is minimized by allowing suitable redistribution of stresses:

1) Increase the total proportion of headings driven parallel to the direction of the maximum horizontal stress.
2) Reduce the number and size of cross-cuts driven perpendicular to the direction of maximum horizontal stress.
3) Drive cross-cuts into existing headings instead of having the heading and cross-cut faces meeting at an intersection.
4) Offset cross-cuts to create only three-way intersections which are more stable in stressed environments, and so that if failure does develop in the cross-cut then it is truncated by a rib pillar.
5) Maintain a wedge-shaped mining front parallel to the direction of the maximum horizontal stress.
This design criteria, although having proven effective at reducing the number of roof failures in these conditions, is solely based on stress redistribution and does not account for the coupled impact of stress and structure. As such, roof failures are still occurring characterized by failure depths beyond those expected which indicates that a better understanding of the mechanisms generated from this coupled relationship of stress and structure in these environments is necessary.

This thesis presents the research and numerical modelling completed to understand the impacts of a weak layer within a stratified rock mass (such as an interbedded limestone and shale sedimentary sequence) on the maximum depth of failure in high horizontal stress environments. Based on observations from the US where a smaller high horizontal stress regime exists than in Southern Ontario, the depth of failure truncates at the weak layer above the failure depth predicted using stress arch theory. However, it is expected that at some depth the weak layer will be too far from the excavation proving inconsequential to roof stability and conventional stress arch theory will control the depth of failure. By understanding the governing mechanisms leading to increased depth of failure and the distance in which a weak layer proves inconsequential, hopefully more effective ground support programs, from a safety and cost perspective, can be designed and implemented when underground limestone and dolostone mining imminently begins in Southern Ontario.

1.1 Thesis Structure

This thesis has been structured so that the thesis flows from a review of current practices, theories and specifics regarding the ground conditions in Southern Ontario in Chapters 2 through 6, and progresses to the author’s research methodology, results, conclusions and research limitations in Chapters 7 through 10.

Chapter 2 provides a general background of limestone mining discussing its purpose in the industry and how it has changed over the years, the mining methodology, and limestone’s viability as an underground resource. This background is intended to highlight the importance of limestone within the aggregate and construction industries worldwide and hopefully establish an appreciation for the probable direction of limestone mining in the future.

Chapter 3 focusses on the potential for limestone mining in Southern Ontario, outlining the geological setting, current state of the surface mining (quarrying) industry and the potential target formations for underground limestone mining in the province.
Chapter 4 details the mining conditions in Southern Ontario providing the background geotechnical information (in-situ stress and rock mass structure) in which limestone mines are most likely to be constructed within. This information sets the basis for the conditions represented in the numerical modelling portion of this research. Finally, international experience in which the aggregate industry of Southern Ontario can draw upon in preparation for exploiting underground limestone and dolostone resources.

Chapter 5 discusses failure mechanisms, transitioning from an elementary classification of failure modes (stress and structurally controlled failures) to specific modes of failure observed in stratified rock masses. The effects of high horizontal stress on failure modes and the phenomenon of arching are also visited. The chapter concludes with two case studies that summarize the failures observed by the author in limestone mines under high horizontal stresses in the northeastern United States.

Chapter 6 provides a background on numerical modelling and discusses the selection of the distinct element approach using the commercial software, UDEC, to carry out the research.

Chapter 7 highlights the importance of model calibration, particularly pertaining to rock mass properties, and steps through the process of establishing suitable micro-mechanical properties required for the Voronoi tessellation employed in the numerical modelling.

Chapter 8 details the engineering model and methodology for assessing the influence of a weak layer on the depth of failure under variable in-situ stress ratios and bedding thicknesses.

Chapter 9 summarizes the results from the numerical modelling and discusses trends observed at each bedding thickness and in-situ stress ratio. Limitations to the research are also detailed at the end of this chapter.

Chapter 10 concludes the thesis by discussing the key outcomes and contributions of the research to the field of study. Potential applications of the research findings to the industry are also listed. Finally, this chapter closes with recommendations for additional research which could be developed from the work conducted as part of this thesis.
CHAPTER 2  LIMESTONE AND THE HISTORY OF ITS MINING

Underground limestone mining became popular in England during the Industrial Revolution because the limestone was close to or directly underlying seams of coal, iron ore and fireclays that were being heavily exploited as raw materials to make iron (Brook, 1991). Although extraction of sedimentary rocks has been recorded back to Neolithic flint mines over 10,000 years ago (Stocks, 1979), the Industrial Revolution marked the first boom of large scale room and pillar mining for limestone. The first North American underground limestone mine began in Kansas City, US in the late 1890’s producing aggregate as a construction material. This industry eventually took off, particularly in the northeastern states (West Virginia, Pennsylvania, Michigan, Ohio) and by 2002 there were 83 operating underground limestone mines producing 60Mt of material for the aggregate and construction industries in the US (Freas et al., 2006). Additionally, production grew from a few thousand tonnes per year to several million tonnes per year (Stocks, 1979).

Many other countries worldwide have at some time employed the room and pillar mining method for sedimentary rock extraction; most notably are large operations in Scandinavian countries and in Japan (Stocks, 1979). However, an economic situation which favours underground mining over surface quarrying for limestone has yet to present itself in Canada and therefore no underground limestone mines have ever been developed in the country.

This chapter discusses the role of limestone in the construction industry, the natural characteristics of limestone which control its feasibility as a mineral resource, room and pillar mining as the most common mining method for extracting the ore, and the studies conducted globally indicating that underground mining of limestone is becoming an economically viable option once again.

2.1  Limestone, Its Uses and the Importance of Its Impurities

Carbonate rocks comprise the basis of the construction industry producing aggregate, cement, lime and building stone. In the US, it is estimated that carbonate rocks comprise 75% of all stone quarried, and only next to sand and gravel, they are produced in the greatest quantities out of the mineral commodities (Carr et al., 1994). In industry, limestone (CaCO₃) and dolostone (CaCO₃-MgCO₃) composed mostly of the mineral calcite (CaCO₃) represent the majority of the carbonate rocks exploited due to their desirable physical and chemical properties.
Limestone and dolostone are sedimentary rocks formed by the deposition of material within bodies of water. Accordingly, these deposits are generally substantial in surface area and flat-lying. Most material is of a biologic origin (shells, fecal material, algae bi-products) and is deposited in shallow marine environments, where the energy during deposition can be quite variable (Carr et al., 1994). Deposits in high energy environments are generally composed of high purity carbonate material of larger grain size and are more economic than deposits formed from low energy sedimentation, which often are diluted by fine-grained, non-carbonate impurities. These impurities not only affect the economic viability of a deposit, but also substantially impact the rock mass quality and the ability of the rock mass to be self-supporting during mining.

The most common impurities are clay minerals – kaolinite, illite, chlorite, smectite and mixed-lattice types – precipitated in laminae or thin partings (due to their silica tetrahedral and alumina and/or magnesia octahedral structure) (Carr et al., 1994). These mineral concentrations are commonly referred to as shale interbeds amongst the carbonate rocks and due to their relatively weaker engineering properties form a natural plane of weakness within the rock mass. Consequently, shale interbeds are often the origin of rock mass instability on many scales when mining carbonate deposits.

Chert is another common mineral impurity. Chert is formed when a sedimentary rock undergoes mineral replacement as a result of diagenesis. Therefore, it tends to concentrate in lenses or nodules, instead of layers. Although they can act as the origin of local stress concentrations and expedite intact rock failure, chert inclusions rarely pose the same engineering concerns as shale layers during mining because they are so localized.

In summary, not all limestone or dolostone deposits are a resource. Their physical and chemical composition have to be desirable for aggregate, cement, lime and building stone production and any impurities with the rock mass not only degrade the resource but also have potential to increase the risk of geotechnical failure.

2.2 Mining Methodology

The selection of a mining methodology for extraction of a mineral involves understanding the nature of the deposit and the production requirements\(^1\) to ensure a profit can be made. From an economic

\(^1\) Production requirements in this context is meant to include all costs incurred by the mining company from construction of the mine to closing of the mine (including extraction of the mineral, processing costs, transportation costs and mine operating and maintenance costs).
perspective, the optimum mining methodology achieves the highest revenue from mineral extraction and minimizes capital and operating expenditures required to extract the resource. Since limestone and dolostone are low-value commodities, they need to be mined in large volumes to be profitable. Due to the high capital and operating costs of underground mining compared to surface mining, surface mining (quarrying) would be the preferred method of extraction for limestone and dolostone, assuming a near surface high purity carbonate formations existed near market.

However, due to a variety of restrictions and market requirements, underground mining may be the only economic option for some markets. As mentioned, limestone and dolostone need to be mined in large volumes to be profitable. Therefore, most underground limestone mines are room and pillar operations because the extraction ratio is generally the highest using this method when mining large volumes from laterally extensive, flat-lying deposits; where deposits are not laterally extensive but thick, long hole stoping is a preferred option even though it’s at a lower extraction ratio (Brown et al., 2010). Headings and cross-drifts in room and pillar mines are often developed using several benches to an overall excavation height of up to 30m (Figure 1); the excavation height is generally controlled by the thickness of the deposit and/or stability limitations.

Figure 1. Schematic of a Typical Room and Pillar Mining Operation (Hamrin, 2001)
It is common for a mine to develop adjacent headings in the same orientation and regularly blast cross-drifts between the development headings to maintain adequate ventilation of the workings. With this mining method which has multiple working faces, stress redistribution in the rock mass around the excavations (pillars and roof) is continuous and if excavations are sequenced improperly such that unsupported large spans or stress concentrations are created, roof or pillar failure can occur risking the safety of workers and potentially comprising the feasibility of the mining operation.

2.3 Underground Mining as a Viable Option for Limestone Extraction

Within Canada, limestone is currently only mined from surface to meet the aggregate and construction materials demand. However, many resource and feasibility assessments for subsurface limestone extraction have been conducted for the region of Southern Ontario by request of the Ontario government since the mid-1970s (Proctor & Redfern, 1974; Acres Consulting Services Ltd., 1976; Thompson, 1982; Grass and Dupak, 1986; Planning Initiatives Ltd. and Associates, 1992; Shinobe, 1997). These studies had fluctuant conclusions between marginally economical and marginally uneconomical depending on market prices at the time of the study, resource location and the study assumptions; the primary resource target for these studies was under Lake Ontario and in the Niagara Escapement, both located at close proximity to the GTA.

Canada is not the only country monitoring the economic potential of limestone mining since the 1970s. Even though the United Kingdom’s (UK) limestone mining declined throughout the 20th century and quarrying operations dominated as the iron industry subsided and the aggregate demand increased, over the past 35 years, there has been a growing interest in extraction of limestone via underground mining methods once again (Stocks, 1979). In 2010, the UK government funded a study to assess the feasibility of the underground mining of aggregates within their country (Brown, et al., 2010). This study was initiated because it was recognized that the surface hard rock resources suitable for aggregate are in the northern and western parts of England while the primary market demand is from the area around London and the south-eastern and eastern regions of the country. After considering factors including cost models for aggregate production, stone processing, haulage of product to market, environmental impact mitigation, health and safety, decommissioning and restoration, it was concluded that although the capital costs were 1.33 – 1.65 times higher than surface operations of comparable size, the cost per tonne of aggregate production by underground mining methods (£13.03 – £13.93 per tonne) is within the range of those costs reported by surface quarries (£10.95 – £16.48 per tonne) (Brown, et al., 2010).
However, the study also concluded that underground mines almost always have a greater carbon footprint than surface quarries on the basis of CO$_2$ eq/tonne produced, even though they can be much closer to the market, because the underground mines require continuous ventilation. As developed countries implement a form of carbon tax to encourage greenhouse gas reductions, having a greater carbon footprint could tip the scales back in the favour of surface quarrying.

Feasibility studies from many other countries worldwide arriving at similar conclusions as Canada and the UK provide a clear indication that the global aggregate industry is preparing to tread underground once again, but on a larger scale. Every study cited the driving factors as stricter environmental regulations and a deficit in available high quality aggregate deposits close to market. Greece’s study (Benardos, et al., 2001) looked beyond the immediate environmental and cost benefit to state that the underground space will be of critical importance in the future.

Publications released for the US aggregate industry also cite environmental restrictions and resource sterilization due to population growth as causing a shortage in limestone based products, such as Portland cement (US Geological Survey, 2012). This shortage is causing construction delays and requiring an increase in limestone product imports from countries including Canada. Given that the US remains the world’s top producer of aggregate in terms of number of operating mines and total tonnage of aggregate produced (Benardos et al., 2001; US Geological Survey, 1999) and they are struggling to meet the aggregate industry demand because of mining restrictions, the concept of Canada tapping into underground limestone resources is truly imminent.
CHAPTER 3  LIMESTONE MINING POTENTIAL IN SOUTHERN ONTARIO

Southern Ontario has been the focus of feasibility studies for limestone mining within Canada because some of the thickest limestone formations exist there. Also, Southern Ontario is densely populated containing over 30% of the country’s population (Government of Canada, 2015) and the GTA is the fastest growing region of Ontario (Ministry of Finance, 2011). Consequently Southern Ontario represents the largest domestic aggregate and construction materials market. These two factors make Southern Ontario the most likely region in Canada in which underground mining of limestone could be economically feasible.

This chapter discusses the geological events that lead to the sedimentary sequences in Southern Ontario, the characteristics of the sedimentary sequences as a result of the depositional environment, the current state of limestone extraction in Southern Ontario and the specific limestone and dolostone formations that have potential to be mined using underground mining techniques.

3.1  Regional Geology

Shallow epicontinental seas situated over Southern Ontario in the Paleozoic and Mesozoic eras resulted in an abundant sedimentary sequence of carbonates, siliciclastics and evaporites over Precambrian rocks of the Canadian Shield (Johnson et al., 1992; Derry et al., 1989). Two major regional tectonic events during these eras have given the sedimentary layers unique characteristics across the province. Primarily two basins – the Appalachian Basin and Michigan Basin – as well as two arches – the Frontenac Arch and Algonquin Arch – were formed tilting the initially flat-lying sedimentary deposits to dip at about 4 to 9m/km towards the basins (to the west north of the Algonquin arch and to the south of the Algonquin arch) (Johnson et al., 1992; Carmichael & Smith, 1978). As shown in Figure 2, Southern Ontario is centered on the Algonquin Axis and therefore sedimentary deposits tend to be thicker and better preserved towards the basins where deeper waters protected the deposits from the many erosional events since the late Cambrian as a result of eustatic sea level change. Most notably at the beginning of the Middle Ordovician much of the sediments of Southern Ontario were eroded leaving little evidence of late Cambrian and early Ordovician deposits, however, these sequences have been preserved in the basins. For perspective, the sedimentary deposits are preserved up to 1525m thick in some areas of Southern Ontario, and up to 13,000m thick in the Appalachian Basin. Uniquely, although the sedimentary sequences were deposited around major tectonic events, they show little evidence of structural disturbance; this is important for geotechnical design purposes because adverse structural
conditions often are the deciding factor when determining if a deposit is economical via underground mining for low-yielding commodities.

Figure 2. Principal Paleozoic and Mesozoic Tectonic Elements of Ontario (Johnson et al., 1992)
The tilted nature of the deposits and wide-spread erosion has resulted in surficial geology mimicking the stratigraphic sequence of the Southern Ontario sedimentary system with successively younger rock units outcropping from the northeast to the southwest of the region (Birchard et al. 2004), as shown on Figure 3. The surficial geology formations across Southern Ontario are of Ordovician, Silurian and Devonian age (359 – 488Ma) with the Ordovician exposed around the Kingston area (near the Frontenac Arch) and the Devonian exposed in the Detroit-Windsor area (near the Michigan Basin).
Figure 3. Paleozoic Geology of Southern Ontario (Birchard et al., 2004)
3.2 Current State of Limestone Extraction in Southern Ontario

There are approximately 50 active quarrying operations in Southern Ontario which supply much of the regions demand for construction materials and aggregates; no underground mines currently exist. Table 1 summarizes the formations in Southern Ontario which have been or are currently being quarried, their thicknesses, and their respective products (Derry, 1989). Due to more stringent government regulations, several of these operations have been denied expansion permits and few prospective quarry sites are being granted surface mining rights. As a result, many of the limestone producers are seeking new opportunities to extract stone since the market demand continues to increase.

Because of the tilt and lateral persistence of the sedimentary formations across Southern Ontario (particularly the Ordovician limestone units which are present at surface in Ottawa and exhibit similar characteristics when drilled at depth 500km away in Niagara Falls), limestone producers are looking at the potential of extracting similar rock formations by underground mining methods at suitable locations to the southeast of their current operations. Most notably, limestone producers with quarrying operations in between the towns of Lindsay and Bowmanville who mine the upper Ordovician formations are studying the feasibility of using underground mining to extract the middle Ordovician formations from properties closer to the major construction market (Greater Toronto Area), since they are anticipating only greater difficulties with obtaining quarry expansion permits in the future. Although this sort of study may seem a bit extreme given that underground mining is generally about three times the cost of surface mining, transportation costs are constantly increasing and therefore the reduction in distance to market (significantly reducing transportation costs) is making the underground mining option cost comparable with that for production and transport from traditional surface quarries located further from market.

In summary, quarries in Southern Ontario are already actively searching alternative ore bodies (either below their current quarries or to the southwest where the same ore body is at a suitable depth underground or closer to market) with the potential for extracting limestone via underground methods. As the aggregate industry looks underground to meet the limestone demand, the quarries of Southern Ontario provide key insight into the desirable geological formations which are likely to represent the target formations for underground extraction in the future since most formations exist at depth somewhere else in the province.
Table 1. Quarried Formations of Southern Ontario

<table>
<thead>
<tr>
<th>Formation</th>
<th>Age</th>
<th>Geology</th>
<th>Products</th>
<th>Thickness (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dundee</td>
<td>Middle Devonian</td>
<td>Limestone</td>
<td>Crushed stone, aggregate, cement, rock dust, armour stone</td>
<td>35 – 45</td>
</tr>
<tr>
<td>Lucas</td>
<td>Middle Devonian</td>
<td>Limestone/Dolostone</td>
<td>Aggregate, lime, cement, fillers, flux Anderdon Mb. – steel, cement and chemical industries</td>
<td>40 – 75</td>
</tr>
<tr>
<td>Amherstburg</td>
<td>Middle Devonian</td>
<td>Limestone/Dolostone</td>
<td>Aggregate, lime, cement, fillers, flux Formosa Reef Limestone – lime, cement, flux, building stone</td>
<td>40 – 60 (15)</td>
</tr>
<tr>
<td>Onondaga</td>
<td>Middle Devonian</td>
<td>Limestone</td>
<td>Crushed stone</td>
<td>30 – 33.5</td>
</tr>
<tr>
<td>Bois Blanc</td>
<td>Lower Devonian</td>
<td>Limestone/Dolostone</td>
<td>Crushed stone</td>
<td>3 – 50</td>
</tr>
<tr>
<td>Bertie</td>
<td>Upper Silurian</td>
<td>Dolostone</td>
<td>Crushed stone</td>
<td>&lt; 14</td>
</tr>
<tr>
<td>Guelph</td>
<td>Middle Silurian</td>
<td>Dolostone</td>
<td>Lime, crushed stone, aggregate, flux</td>
<td>4 – 100</td>
</tr>
<tr>
<td>Lockport</td>
<td>Middle Silurian</td>
<td>Dolostone</td>
<td>Crushed stone, lime, aggregate; Eramosa Mb. – building and landscape stone, flux</td>
<td>Up to 46 (20)</td>
</tr>
<tr>
<td>Amabel</td>
<td>Middle Silurian</td>
<td>Dolostone</td>
<td>Crushed stone, agricultural lime, armour stone; Wiarton/Colpoy Bay Mb – dimension stone</td>
<td>Up to 38 (25)</td>
</tr>
<tr>
<td>Decew</td>
<td>Middle Silurian</td>
<td>Dolostone</td>
<td>Cement - mined only in combination with the Lockport Fm.</td>
<td>Up to 4</td>
</tr>
<tr>
<td>St. Edmund</td>
<td>Middle Silurian</td>
<td>Dolostone</td>
<td>Aggregate, crushed stone</td>
<td>Up to 25</td>
</tr>
<tr>
<td>Manitoulin</td>
<td>Lower Silurian</td>
<td>Dolostone</td>
<td>Crushed stone, aggregate, building stone</td>
<td>Up to 25</td>
</tr>
<tr>
<td>Lindsay</td>
<td>Middle Ordovician</td>
<td>Limestone</td>
<td>Crushed stone, aggregate and cement</td>
<td>Up to 67</td>
</tr>
<tr>
<td>Verulam</td>
<td>Middle Ordovician</td>
<td>Limestone</td>
<td>Crushed stone, aggregate and cement</td>
<td>32 – 65</td>
</tr>
<tr>
<td>Bobcaygeon</td>
<td>Middle Ordovician</td>
<td>Limestone</td>
<td>Crushed stone, lime, building stone, aggregate</td>
<td>7 – 87</td>
</tr>
<tr>
<td>Gull River</td>
<td>Middle Ordovician</td>
<td>Limestone</td>
<td>Crushed stone, building stone, lime, aggregate</td>
<td>7.5 – 136</td>
</tr>
</tbody>
</table>

2 (Wolfe, 1993)
3 (Derry, 1989)
4 (Wolfe, 1993)
5 (Derry, 1989)
6 (Derry, 1989)
7 (Wolfe, 1993)
8 (Derry, 1989)
9 (Wolfe, 1993)
10 (Wolfe, 1993)
11 (Derry, 1989)
12 (Wolfe, 1993)
3.3 Potential Target Formations for Limestone and Dolostone Mining

In Southern Ontario, the succession of the Gull River, Bobcaygeon, Verulam and Lindsay Formations of the Middle Ordovician represent the most desirable targets for underground limestone mining due to their thickness and chemical composition. The deposits range from coarse-grained bioclastic carbonates to carbonate mudstones with interbedded calcareous and non-calcareous shales (Ontario Geological Survey, 1992). These units formed through continuous deposition as water levels generally decreased over the area (which was a marine reef or shelf at the time) resulting in gradational contacts between the formations. Consequently, field identification of individual formations is difficult which could prove a challenge for mine design. The interbedded shales not only impact the quality of the deposit but based on industry experience have major implications on the structural stability of the rock mass around underground excavations.

The upper Lindsay Formation gradates into highly bituminous shales in central Southern Ontario (graphical region extending from Toronto-Oshawa, east to Lake Huron, and north to the Bruce Peninsula and Manitoulin Island) (Ontario Geological Survey, 1992). In the literature, these shales have been specifically classified as the Collingwood Member of the Lindsay Formation or the lower gradational contact of the Blue Mountain Formation with the Lindsay Formation. These shales represent the upper limit of potentially feasible limestone units of the Ordovician age. From a geotechnical perspective, the shales of the upper Ordovician, although a poor quality rock mass to design within and may have some environmental concerns due to their bitumen content, are an effective water barrier and minimize vertical transmissivity of fluid. Therefore, situating a mine at a sufficient distance below these formations could reduce the influence of groundwater in the mine; water management is a significant cost in underground mining.

The Silurian age represents a gradation from interbedded sandstones and dolostones with shales to interbedded dolomitic limestones with shales to evaporites and dolostones in succession. Many Formations (Manitoulin, St. Edmund, Fossil Hill, Reynales, and Bertie) within the Silurian have been quarried in the past and/or continue to produce today, primarily for aggregate or crushed stone, but the sequence of the Amabel, Lockport and Guelph Formations of the Middle Silurian exhibit the greatest potential for further exploitation through underground mining.

The Amabel, Lockport and Guelph Formations were deposited as water levels decreased over the region creating a shallow shelf, and reef-interreef and barrier depositional environment. The resulting deposits
are generally thinly bedded to massive, coarse-grained, fossiliferous dolostones, containing a varying degree of bitumen. However, it should be noted that the deposits of the Lockport Formation are more complex than the Amabel Formation, although laterally equivalent\textsuperscript{13}, due to the deeper water levels in the Appalachian Basin. The more complex deposition conditions of the Lockport Formation resulted in higher content of chert and shale (Goat Island member) making the Formation a more difficult mining horizon than the Amabel Formation. Overall, the thicknesses and purity of these dolostone Formations (specifically the Wiarton/Colpoy Bay Member of the Amabel Formation and Eramosa Member of the Lockport Formation) make them desirable for a variety of applications including building stone, dimension stone, aggregate and crushed stone, and consequently potential underground mining targets.

The onset of the Acadian Orogeny in the latter part of the Middle Silurian altered the depositional environment over Southern Ontario by restricting circulation in the inland seas. Consequently, the Guelph Formation was overlain by a sequence of evaporites and dolostones (Salina Formation). Due to the low vertical permeability of the Salina Formation, the Guelph and Amabel/Lockport Formations act as an aquifer. Currently, these formations are the main water supply for the cities of Kitchener, Waterloo, Cambridge and Guelph. As such, regulations have been implemented restricting the exploitation of these Formations for any activity which has potential to disrupt or contaminate the water supply to these cities.

In 2011, The Highland Companies tested these regulations by submitting an application to the Ontario government to develop a limestone quarry in the Township of Melancthon (approximately 75km northwest of the GTA) targeting the Amabel Formation. Even though the proposed quarry location is an equivalent 75km northeast of Guelph (the closest city being supplied with water by the Amabel Formation), cooperation with the province and the local community could never be gained citing concerns about the potential for water contamination (VanDyken, 2011). The company ultimately withdrew its application in late 2012. Both supporters and protestors of the project have highlighted the key fact that the application was not officially denied by the government, so potential to target this formation in the future may exist (National Farmers Union, 2015; D'aliesio & Howlett, 2012); in the near future though, the Amabel Formation appears off-limits for mineral extraction within a feasible distance from the GTA market.

\textsuperscript{13} The Lockport Formation is primarily located within the Appalachian Basin. The Amabel Formation is primarily located within the Michigan Basin. The two deposits are geographically separated by the topographical high of the Algonquin Arch. However, it is unclear as to whether these units were laterally gradational at one time and greater erosion over the Algonquin Arch eventually separated them.
Although the limestone and dolostone formations of the Devonian age (Bois Blanc, Onondaga, Amherstburg, Lucas, Dundee) are currently quarried extensively for aggregate, building stone, lime and cement in the southwest of Ontario, where the deposits have not been eroded, they are shallow (<200 m) and therefore it would be more economical to mine these units via surface extraction where permitting allows. Additional detailed investigation into the feasibility of subsurface extraction of these units might find it marginally economical near Sarnia where the formation depths and population density are slightly greater, however, for the purpose of the numerical analysis in this thesis, specific cases related to the underground extraction of Devonian age formations will not be modelled.

Depending on the location of market demand and existing infrastructure (such as an existing port on the Great Lakes), the discussed formations represent potentially viable limestone or dolostone resources. However, as summarized by Lee and White (1993), the Gull River of the Ordovician Formation is considered the most viable mining horizon for underground extraction because of the purity of the limestone and the structural competency of the rock mass. The numerical modelling for this thesis will be conceptually based on the extraction of the Gull River Formation below the GTA (an approximate depth of 150m below ground surface).
CHAPTER 4 MINING CONDITIONS IN SOUTHERN ONTARIO

Currently, underground mining in Southern Ontario is for the extraction of salt and gypsum. Both minerals are relatively soft in comparison to limestone and therefore present a unique set of geotechnical design challenges than would be anticipated for limestone. To predict the geotechnical challenges that may face an underground limestone mine, the mining conditions need to be understood. From a geotechnical perspective, the in-situ stress regime, structural nature of the rock mass and strength of the rock mass are the key aspects for input into a mine design and for geotechnical risk assessment; hydrogeology is another major aspect but this field of study has been omitted from this research because it does not contribute to the numerical analysis. In knowing the range in these mining conditions, an engineer can be proactive in assessing probably of failure via statistical or empirical analysis for the many potential failure modes, and consequently develop an appropriate ground control program (including ground support) to mitigate geotechnical risk.

This chapter discusses the in-situ stress regime in Southern Ontario, focusing on the GTA, as well as the regional structural characteristics of the sedimentary sequence in the same area. The rock mass strength characteristics and specific structural characteristics are detailed in Chapters 7 and 8. Finally, published experience from the US aggregate mining industry within similar mining conditions as anticipated for Southern Ontario is reviewed. Typical room and pillar mining geometries and a synopsis of the geotechnical stability issues plaguing the US limestone mines are presented.

4.1 Southern Ontario In-Situ Stress Regime

The existence of high horizontal stress in the sedimentary rocks of Southern Ontario has been well documented (Lee, 1981; Lo, 1978). Squeezing observations made during the construction of the wheel pits of the Canadian Niagara and Toronto Power Plants in 1903 first triggered the notion of high in-situ stresses. In the years following, heave of quarry floors, identification of “pop up” structures, buckling of canal floors, movement of bridge foundations and cracking of tunnel walls during or after construction provided further evidence. But it wasn’t until the 1970’s that the uniquely high horizontal stresses relative to the vertical stresses at shallow depths were characterized for the southern part of the province through in situ stress measurements performed in various limestone and shale formations (Lo, 1978).
The high horizontal in-situ stresses are a function of the current tectonic plate motions and glacial unloading. Ontario is situated on the North American plate which is currently moving in a WSW direction as it is pushed away from the Mid-Atlantic ridge creating a regional horizontal in-situ stress regime along an ENE trend (Nuclear Waste Management Organization and AECOM Canada Ltd., 2011; Esterhuizen et al., 2007). In the past, continental glaciers over the area increased the vertical stress resulting in a more isostatic stress field. As glacial unloading occurred, the relative difference in the horizontal and vertical stresses increased leaving the uniquely high horizontal stress (relative to vertical stress) conditions measured today.

Locally, the magnitude and orientation of the in-situ stress regime can deviate from the regional average due to rock mass quality and stiffness; stresses accumulate in massive, stiff rigid rock masses (granite or limestone) but not in weak, softer ones (soft shale or salt). Another possibility resulting in a unique local stress regime relative to the regional one is that the horizontal stresses could have been relieved over geological time by local events such as outcropping or folding (Iannacchione et al., 2001); pop-up structures in Southern Ontario limestones are a common sign of local stress relief.

As part of the studies conducted to assess the risk of developing a deep geologic repository for low and intermediate nuclear waste storage at the Bruce Nuclear Power Plant in Southern Ontario (Nuclear Waste Management Organization and AECOM Canada Ltd., 2011), published stress testing data from 26 sites within the Appalachian and Michigan basins which were plotted to empirically determine the stress ratios between the maximum and minimum horizontal stresses and the vertical stress (Figure 4). Key relationships from the in-situ stress data are as follows:

1) At depths of approximately 200m and 800m, there are distinct decreases in the horizontal to vertical stress ratios.

2) The maximum horizontal to minimum horizontal stress ratio remains nearly the same regardless of depth.

3) Near surface (<50m deep) stress ratios can be very high (k > 6) because of local concentrations of high horizontal stress are being compared to the low vertical stress induced by the weight of the bedrock. These uniquely high stress ratios were disregarded from this research because the minimum depth for mining was considered to be 100m.

The stress ratios as a function of depth are summarized in Table 2.
Table 2. Summary of Stress Ratios by Depth below Ground Surface

<table>
<thead>
<tr>
<th>Depth</th>
<th>$\sigma_H/\sigma_V$</th>
<th>$\sigma_H/\sigma_V$</th>
<th>$\sigma_H/\sigma_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0m – 200m</td>
<td>3.8</td>
<td>2.0</td>
<td>1.9</td>
</tr>
<tr>
<td>200m – 800m</td>
<td>2.0</td>
<td>1.1</td>
<td>1.8</td>
</tr>
<tr>
<td>&gt;800m</td>
<td>1.4</td>
<td>0.8</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Stress measurements taken at the Darlington Nuclear Power Plant, which is located about 30km east of the Greater Toronto Area and adjacent to one of the larger surface quarries in Ontario, concluded that the principal in-situ stress in the Ordovician limestone formations is horizontal and oriented at ENE as shown in Figure 5 (Haimson & Lee, 1980). Additionally, the horizontal stresses are relatively constant at approximately 13MPa for the maximum horizontal stress within the Ordovician limestone formations and not dependent on depth.
Figure 4. Regional In-Situ Stress Ratios as a Function of Depth: (a) $\sigma_H/\sigma_V$, (b) $\sigma_i/\sigma_V$, and (c) $\sigma_H/\sigma_h$ (Nuclear Waste Management Organization and AECOM Canada Ltd., 2011)
4.2 Structural Characteristics of Southern Ontario Sedimentary Sequences

Sedimentary sequences in Ontario often are characterized by a minimum of three major discontinuity sets: one flat-lying set representing the bedding plane, and two relatively orthogonal sub-vertical sets postulated to be the result of tensile fracturing as isostatic rebound occurred (Goudie, 2006). Bedding across Southern Ontario dips at approximately 0.5 degree towards the southwest. The strike of the sub-vertical sets can vary significantly on a local scale but regionally one set aligns with the direction of maximum horizontal stress and the other aligns with the direction of the minimum horizontal stress. A unanimous conclusion explaining the “coincidental” alignment of the fracture network with the maximum and minimum horizontal stress directions has yet to be reached, even though many scientists believe the current in-situ stress direction has regionally been constant since the Paleozoic and therefore could have resulted in the orthogonal fracture network mapped across Southern Ontario (Gross & Engelder, 1991; Holst, 1982). Geotechnical data collected from limestone formations at
various locations in Southern Ontario is presented in Figure 6 and shows this common structural relationship between bedding and two orthogonal joint sets in sedimentary rock formations.

(a) NWMO Outcrop Mapping   (b) Lincoln Quarry Wall Mapping

Figure 6. Structural Data from across Southern Ontario: (a) Kincardine, ON (Cruden, 2011), (b) Lincoln, ON (Gartner Lee Limited, 1996)

4.3 Geotechnical Experience from the US Aggregate Mining Industry

Room and pillar aggregate mining has flourished in the US over the past century. Aggregate mines located in the northeastern US exhibit similar stress conditions to those in Southern Ontario – near the Greater Toronto Area (as shown in Figure 7) – and therefore these mines provide a good basis for predicting the mining and geotechnical challenges that are likely to be encountered in the inevitable Southern Ontario aggregate mining operations.
After recognizing similar roof stability issues in underground limestone mines across the Eastern and Midwestern USA, Esterhuizen et al. (2007) conducted a survey of 34 mines collecting data pertaining to roof spans, rock mass properties, support practices and roof instabilities as part of a National Institute for Occupational Safety and Health research project. From the survey, there are three conclusions that provide useful insight into limestone mining under high horizontal stress conditions that can be translated back to the predictive research being conducted as part of this thesis for mining in Southern Ontario.

Firstly, Table 3 summarizes the mining dimensions measured during the survey which provide a basis for the excavation sizes likely to be employed in Southern Ontario.
Table 3. Summary of Underground Mining Dimensions from US Limestone Mines

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Literature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
</tr>
<tr>
<td><strong>Mining Height (m)</strong></td>
<td>11.6</td>
</tr>
<tr>
<td><strong>Pillar Width (m)</strong></td>
<td>13.8</td>
</tr>
<tr>
<td><strong>Room Width (m)</strong></td>
<td>13.5</td>
</tr>
<tr>
<td><strong>Intersection Diagonal (m)</strong></td>
<td>21.7</td>
</tr>
</tbody>
</table>

Secondly, small failures were observed in 30 of the 34 mines surveyed. Large failures were observed in 19 of the 30 mines that had small failures. These statistics indicate that the majority of mines under high horizontal stress in conditions similar to those expected in Southern Ontario are still not properly designed to prevent large failures and consequently the associated risk to human life would be considered unacceptable in today’s mining industry. Small and large failures were defined as rock falls and roof falls respectively based on their failure characteristics, as shown in Table 4.

Roof falls are the target of this thesis’ research. Esterhuizen et al. (2007) concluded that the main factors contributing to large roof falls are horizontal stress, large joints and insufficient thickness of the immediate roof beam. Of the roof falls, stress was the main contributing factor in 36% of them and these failures were observed at a variety of depths. Beam failures constituted 28% of roof falls and always truncated at the boundary between limestone and some overlying weak band or parting plane. Block failures governed by large discontinuities extending across entire rooms marked 21% of roof falls. Caving failures which represented only 15% of roof falls were all related to the collapse of weak shale exposed in erosion channels or progressive failure of weak roof rocks. According to these roof fall observations, weak layers or shales contributed to a minimum of 43% of failures (not including those classified as stress failures which may have had a weak layer or shale influence that could not be observed) in limestone mining environments and therefore these weak layer influences are critical to understand for safe mine design in the future. In the author’s opinion, Esterhuizen et al. (2007) adequately portray the detrimental risks to a mining company associated with roof falls through the following statement,

“*although large roof falls only make up a small percentage of total roof exposure, their potential impact on safety and mine operations can be very significant. Most cases of large roof falls require barricading-off or abandonment of the affected entry.*”
Based on the observation noted in this statement that roof falls typically result in a truncated mining header, one can easily infer that loss of equipment or life would occur if these entities were involved in the failure.

Table 4. Classifications of Roof Failure during Survey (Esterhuizen et al., 2007)

<table>
<thead>
<tr>
<th>Size of Failure</th>
<th>Type of Failure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>Rock Falls</td>
<td>Isolated rock fragments less than about 1m in length</td>
</tr>
<tr>
<td></td>
<td>Slabs</td>
<td>Thin slabs caused by weathering or stress spalling, less than about 30cm in length and about 25mm thick</td>
</tr>
<tr>
<td></td>
<td>Blocks</td>
<td>Blocky rock fragments caused by the intersection of joint planes, blasting fractures, bedding or stress fractures</td>
</tr>
<tr>
<td></td>
<td>Beams</td>
<td>Stepped roof or brow formed by fall propagation to bedding plane</td>
</tr>
<tr>
<td>Large</td>
<td>Roof Falls</td>
<td>Falls larger than about 1m in length, typically consisting of multiple rock fragments</td>
</tr>
<tr>
<td></td>
<td>Blocks</td>
<td>Large discontinuities and joints associated with fall</td>
</tr>
<tr>
<td></td>
<td>Beams</td>
<td>Bedded layers in the roof fail under gravity loading</td>
</tr>
<tr>
<td></td>
<td>Stress</td>
<td>Horizontal stress-related shearing and buckling of roof beds</td>
</tr>
<tr>
<td></td>
<td>Caving</td>
<td>Fall caused by progressive spalling, blocky roof or weathering of weak strata</td>
</tr>
</tbody>
</table>

The failure mechanisms leading to the rock and roof fall categories listed by Esterhuizen et al. (2007) are thoroughly discussed in Chapter 5.

Lastly, 25 of the 34 mines employed a mine design that accounted for a minimum thickness of limestone beam in the immediate roof. Mines that preserved a competent roof beam thickness of 2.25m or greater were able to mine without regular support, compared to those with average competent roof beam thickness of 1.3m required regular ground support in the roof.

This experience from aggregate mining in the Eastern and Midwestern US provides valuable insight into the potential geotechnical issues that engineers will need to account for when completing mine designs for similar mining conditions in Southern Ontario. However, the maximum in-situ horizontal stress in the US aggregate mines typically ranges between 2 to 3 times the in-situ vertical stress (Agapito & Gilbride, 2002). Therefore, the geotechnical issues observed may be exacerbated under the Southern Ontario stress regime where the maximum in-situ horizontal stress can be 3 to 4 times the in-situ vertical stress.
CHAPTER 5  FAILURE MECHANISMS

Rocks, and the conditions under which they fail, have been the subject of study since as far back as Ancient Egyptian times. More coordinated efforts targeted at understanding the driving forces and complex mechanisms leading to rock failure have dominated engineering publications since the 1960s, and reiterative analyses efforts continue as new technologies allow for more realistic representations of failure environments and loading conditions in experimental studies. Understanding of these complex relationships remains very difficult because many of the rock mass properties or loading conditions which appear to drive failure tend to have a coupled functionality, and subsequently are dependent on at least one other variable in the system when governing failure. Consequently, outlining of the boundaries that define the ranges in which each failure mechanism initiates is environment specific, and many failure mechanisms can initiate simultaneously. Over the years of experimentation, it has become clear that stress acting on a rock mass and structure within the rock mass in their varying degrees govern the mechanisms defining the different modes of failure possible around an underground excavation (Figure 8). Numerous modes of failure exist because many factors (physical properties, homogeneity and anisotropy of the rock mass, discontinuity properties, free face geometry, in-situ stress, etc.) affect the degree of impact in which the in-situ stress has on the rock mass. Therefore, in order to avoid the many forms of rock failure when performing engineering design, it is critical to have a thorough understanding of the rock failure mechanisms pertaining to the environment in which the design is for.
This chapter begins by outlining the general modes of stress and structurally controlled failure as well as failure modes governed by both stress and structural mechanisms interdependently. Following, a comprehensive review of the common mechanisms and modes of failure observed in stratified rock is given, highlighting the factors that have the greatest impact on the mechanisms of failure observed.

### 5.1 Types of Rock Failure

Rock failures are often described as either being stress controlled or structurally controlled based on the mechanism which appeared to drive failure. However, rock failure in reality is rarely that simple and a unique combination of applied stress and rock mass structure must exist to result in the failure

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**Figure 8. General Rock Failure Mechanisms for Underground Excavation (adapted from Martin et. al, 1999)**

<table>
<thead>
<tr>
<th>Mode</th>
<th>Massive $(RMR &gt; 75)$</th>
<th>Moderately Fractured $(50 &gt; RMR &lt; 75)$</th>
<th>Highly Fractured $(RMR &lt; 50)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low In-Situ Stress $(\frac{\sigma_1}{\sigma_c} &lt; 0.15)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear elastic response.</td>
<td>Falling or sliding of blocks and wedges.</td>
<td>Unraveling of blocks from the excavation surface.</td>
<td></td>
</tr>
<tr>
<td>Intermediate In-Situ Stress $(0.15 \leq \frac{\sigma_1}{\sigma_c} \leq 0.4)$</td>
<td>Brittle failure adjacent to excavation boundary.</td>
<td>Localized brittle failure of intact rock and movement of blocks.</td>
<td>Localized brittle failure of intact rock and unraveling along discontinuities.</td>
</tr>
<tr>
<td>High In-Situ Stress $(\frac{\sigma_1}{\sigma_c} \geq 0.4)$</td>
<td>Brittle failure around the excavation.</td>
<td>Brittle failure of intact rock around the excavation and movement of blocks.</td>
<td>Squeezing and swelling rocks. Elastic-plastic continuum.</td>
</tr>
</tbody>
</table>
mechanisms observed. This section discusses the spectrum of failure mechanisms from dominantly stress controlled to dominantly structurally controlled, as well as those with interdependent failure relationship.

5.1.1 Stress Controlled Failure

In homogeneous rocks, failure initiates when stress being applied to them exceeds either their compressive, tensile or shear strength. Tang and Hudson (2010) even suggest that at a micro-scale level, intact rock failure initiation is only a function of the tensile and shear components; since the tensile strength of rocks is about 10% of its compressive strength, $\sigma_C$ (Jumikis, 1979) the indirect tensile stresses induced in a compressive stress environment cause the rock to fail in tension before the compressive strength is exceeded. In-situ rock failure observations indicate that a combination of the three forms of stress application characterize the mode of failure observed. For example, during a uniaxial compression test on a homogeneous rock sample, failure usually initiates in tension in the middle of the sample. Once the tensile fractures coalesce under increased loading, failure becomes shear dominated until the sample yields at its maximum unconfined compressive strength.

Rock masses typically fail in two distinct manners, brittle or non-brittle, depending on the intact rock strength characteristics, and the stress conditions acting on them. Brittle failure through intact rocks commonly occurs around excavations in strong, massive rock masses (RMR$^{14}$ > 75) under moderate to high in-situ stress conditions ($\sigma_1^{15}/\sigma_C > 0.15$); i.e. Modes 4 and 7 shown in Figure 8. During brittle failure, instability often occurs in the form of axial splitting, and spalling or “bursting” (Li & Nordlund, 1993; Stacey & Page, 1986). As Martin (1997) has characterized, these failures are typically governed by the tensile strength of the intact rock and intact rock fracturing (i.e. spalling) initiates as a result of induced tensile stresses at grain-scale when the major principal stress ($\sigma_1$) equates to about 40% of the rock’s compressive strength. Non-brittle failure around excavations in massive rock masses under low stress conditions ($\sigma_1/\sigma_C < 0.15$); i.e. Mode 1 in Figure 8, are not common as the rock typically responds elastically to the stress and remains stable. In certain cases, where the rock mass is weak and under

$^{14}$ RMR (Rock Mass Rating) is a rock mass classification system develop by Bieniawski (1976) which has been refined and modified over the year; notable amendments are by Bieniawski (1989) and Laubscher (1990). RMR is a summation process that assigns a quantitative value to represent the quality of a rock mass. The RMR classification takes into account intact rock strength, rock quality designation (RQD), joint spacing, joint characteristics (roughness, alteration and aperture) and in-situ water conditions.

$^{15}$ In-situ stresses acting on a rock mass are analyzed using the magnitudes and orientations of their principal stress: $\sigma_1$ – major principal stress, $\sigma_2$ – intermediate principal stress, $\sigma_3$ – minor principal stress. The principal stresses are mutually orthogonal and characterize the three-dimensional stress field acting on a rock mass.
high stress conditions \((\sigma_3/\sigma_c < 0.4)\), the rock mass will continuously deform over time in the excavation in the form of squeezing or swelling, similar to Mode 9 in Figure 8. Therefore, a comprehensive understanding of the intact rock strength properties \((\nu, E, \sigma_c)\) and in-situ stress regime \((\sigma_1, \sigma_2, \sigma_3)\) is essential for assessing potential for instability in stress controlled environments.

### 5.1.2 Structurally Controlled Failure

Discontinuities from joints to large scale faults separate the rock mass into wedges or blocks of varying sizes, and interrupt the continuous nature of the mass by introducing boundaries with unique properties between the blocks changing the kinematics of failure. Structurally controlled failures are governed by the strength properties of the discontinuities. These failures occur in the form of gravity or sliding failures; i.e. Modes 2 and 3 in Figure 8. The orientations of the discontinuities will determine whether gravity or sliding failure is possible. The joint tensile strength governs failure initiation for gravity failures whereas the shear strength properties (effective angle of friction and joint cohesion) govern stability for sliding failures (Stacey & Page, 1986).

When the nature of rock mass is heavily fractured, disturbed or non-cohesive, failure typically occurs along particle boundaries when the cohesive and frictional properties between the particles of the rock mass are exceeded. Accordingly, failure is governed by the shear strength properties of the discontinuities. However, contrary to moderately fractured rock masses, heavily fractured rock masses often respond to stresses similar to weak, homogenous rock masses. For example, under high in-situ stress conditions \((\sigma_3/\sigma_c > 0.40)\), the small blocks composing the rock mass will continuously move and bridge over time in the excavation in the form of what is observed as squeezing; i.e. Mode 9 in Figure 8. For low and moderate in-situ stress conditions \((\sigma_3/\sigma_c < 0.40)\), failure occurs in the form of unravelling; i.e. Modes 3 and 6 in Figure 8.

### 5.1.3 Compound Failure Mechanisms

Much of the complexity associated with understanding the failure mechanisms acting in a rock mass has to do with the nature of the rock. A rock mass is rarely an ideal homogeneous, isotropic and elastic solid. The degree of heterogeneity, anisotropy, confinement, block geometry, boundaries and pre-existing defects affects the mechanisms and modes of failure which are able to develop. The heterogeneities in a rock mass can be a function of differential solidification processes, stratification differences or differential metamorphic processes as the rock formed. Because of the randomness usually associated with heterogeneities in a rock mass and spacing of discontinuity sets, it is common for
both structurally controlled and stress controlled modes of failure to be operating simultaneously. In these cases, the mechanisms leading to failure are a combination of localized fracturing of intact rock and an exceedance of the strength properties of a discontinuity. This is illustrated in Modes 5 and 8 in Figure 8 where in a moderately fractured environment (50 < RMR < 75) under moderate to high in-situ stress conditions ($\sigma_3/\sigma_c > 0.15$), failure will be a combination of localized brittle intact rock failure and sliding or tensile failure along the discontinuity.

Due to the nature of deposition of interbedded limestone and shale rock masses, semi-persistent occurrence of sub-vertical jointing and relatively weak discontinuity strength properties compared to the intact rock strength properties of the limestone, the modes of failure documented for room and pillar limestone mines are the result of compound failure mechanisms.

### 5.2 Modes of Failure and Failure Mechanisms in Stratified Rocks

Stratified rock masses, such as interbedded limestone and shale, are geometrically characterized by planar and persistent bedding which has low or zero tensile strength in the direction orthogonal to the bedding plane and low shear strength on the surfaces compared to the intact rock strength properties of the limestone. Consequently, upon excavating a room for the extraction of limestone, the intact rock between the bedding planes overlying the excavation becomes a self-supporting “beam”, and two different mechanisms of failure may initiate (Brady & Brown, 1985):

1) Surface spalling and internal fracturing of the intact rock comprising the beam or beams overlying the excavation (particularly at the ends and centre of the beam), and

2) Separation of the bedding planes due to the weight of the beam or indirect tensile stresses induced by compressive loads (such as in a horizontally dominated in-situ stress environment (Jumikis, 1979)) and shearing along the bedding planes as the beam or beams deflect into the excavation.

Depending on the strength of the intact rock relative to the in-situ stress conditions and the geometry of the beam (thickness to span or slenderness ratio), both mechanisms of failure are likely to initiate resulting in a compound rock failure.

In cases such as Southern Ontario where the cross-cutting joints are orthogonal to the bedding planes within a tolerance of one third to one half of the effective friction angle of the joints (Figure 9), then the
rock mass is considered to be a composition of a series of Voussoir beams\textsuperscript{16} of which Voussoir beam theory can be applied to predict failure in homogenous rock masses (Ran et al., 1994).

![Diagram](image)

**Figure 9. Voussoir Beam Criteria**

The modes of failure associated with Voussoir beams are slabbing, buckling, crushing or diagonal fracturing (Figure 10). These failure modes fall under Modes 2, 5 and 8 in Figure 8 depending on the in-situ stress conditions.

![Diagram](image)

**Figure 10. Specific Failure Modes in Stratified Rock Masses**

\textsuperscript{16} The Voussoir beam label for a beam of rock spanning an excavation was adopted from the Voussoir arch considered in masonry structures because of similar relationships between vertical deflection, lateral thrust and beam stability observed during Evans (1941) investigations of roof deformation mechanics.
Voussoir beam theory indicates that the maximum deflection that a beam can withstand prior to failure is a function of the specific weight ($\gamma$), Young’s modulus ($E$), uniaxial compressive strength (UCS), span and thickness of the beam (Diederichs & Kaiser, 1999; Stacey & Page, 1986; Brady & Brown, 1985; Sterling, 1980). Following a series of experiments using a base friction machine, Goodman (1989) extrapolated Voussoir beam theory for a single beam to calculate the stresses and deflections acting on the lower beam in a multiple beam failing system. Goodman’s equations were primarily functions of the beams’ geometries and their individual $\gamma$ and $E$. Figure 11 highlights the relative conditions under which each mode of failure is dominant. Division lines between failure modes in Figure 11 are hypothetical and based on published qualitative observations; they have not been validated with quantitative data. In general though, where the slenderness ratio is high (beam thickness is less than 1/10 of the span), shearing governs failure. In contrast, where the slenderness ratio is low (beam thickness is greater than 1/2 of the span), tensile fracturing – direct from beam deflection and indirect from compressive forces – governs failure.

Figure 11. Conditions Leading to Different Failure Modes in Stratified Rock Masses
It should be noted that in cases where additional moderately dipping (30 – 45 degree dip) rock mass structure exists, Voussoir beam theory does not apply because the stability of the beam is compromised by the moderately dipping structure; slip along these structures is likely to occur resulting in premature shear failure (Ran et al., 1994). Additionally, the Voussoir analogue is not applicable for poor rock mass conditions where there are more than 3 joint sets or the RQD is less than 50 (Diederichs & Kaiser, 1999). For the purpose of this research, only a rock mass composed of classic Voussoir beams was considered.

The following sections discuss the conditions which result in each of the Voussoir beam failure modes observed in stratified rock masses and how those conditions contribute to the development of the failure mechanisms that define each mode of failure.

5.2.1 The Arching Phenomenon

At a certain beam thickness to span ratio of a Voussoir beam, a phenomenon called “arching” occurs. Arching is the process of stress redistribution in the beam to its abutments when the beam is suspended under its own weight. If the beam’s thickness is enough to allow for complete redistribution of the stresses across the beam’s span to its abutments then stability can be achieved naturally. Brady and Brown (1985) mathematically characterized this phenomenon by calculating the stress distribution curve within a single deflecting beam. Their work has since been further developed by Diederichs and Kaiser (1999) so that an assessment of the stability of a Voussoir beam can be made quantitatively. The stress distribution curve in a deflecting beam is a function of the beam’s deflection as well as the lateral thrust generated by the force of the horizontal stress.

Although arching within a single beam has been mathematically explained in recent years, the arching phenomenon was discovered back in 1885 by Henri Fayol, a French mining engineer, while investigating the behavior of stacks of beams spanning a simple support system by observing the deflection of the lowest beam as successive beams were added on top (Brady & Brown, 1985). Fayol concluded that at a certain thickness of stack, the load applied by adding successive beams loaded onto the stack does not influence the deflection of the lowest beam. Having no mathematical proof for this phenomenon, he could only postulate that the additional load was transferred laterally to the beam supports instead of onto the lowest beam. The mathematics explaining arching in a stack of Voussoir beams – an accurate representation of a rock arch formed above a mining excavation in a stratified rock mass – remains estimated at best due to the number of complex interactions between the various beams forming the
arch. Figure 12 present a visual interpretation of how arching occurs within a single Voussoir beam as well as a stack of Voussoir beams.

![Compression Arch](image1)

**Figure 12. Compression Arches Composed of (a) Single Voussoir Beam, (b) Stack of Voussoir Beams**

The research being conducted as part of this thesis does not require the understanding of the mathematical relationships pertaining to Voussoir beams. However, an appreciation for the Voussoir beam properties directly contributing to the development of a multiple beam rock arch – span, thickness, Young’s modulus, specific weight and joint friction, cohesion and tensile strength – should be had since the arch controls the depth of failure in the stratified roof rock mass over an underground excavation and the purpose of this research is to assess how a shale layer influences the ability of an interbedded limestone-shale roof to create an arch and become self-supporting.

### 5.2.2 Slabbing Failure

Slabbing failure occurs when the beam thickness to span ratio is low to moderate and the in-situ horizontal stress is low. Under these conditions, the weight of the beam exceeds the tensile strength of the bedding plane with the overlying strata allowing separation from it and causing shear stresses to develop vertically at the abutments of the self-supporting beam. If the load of the beam or beams exceeds the rock mass shear strength, the beam will fail at the abutments, as shown in Figure 13. With very thin beams (thickness < 1/100 of the span), slabbing failure still occurs but shear stress
concentrations tend to form at local heterogeneities within the slab instead of at the abutments. Accordingly, very thin beams often fail in smaller slabs but have more frequent occurrences of failures. If the rock mass is much weaker than the weight of the slabs, slabbing failure can progress over time with successively higher beams failing resulting in a “chimney”-style failure. In cases where the mining excavations are near surface, the chimney failure can create surface subsidence or a sinkhole. This style of failure and corresponding sinkholes have plagued the UK in the recent past due to standard mining practices from the late 1800’s and early 1900’s for extracting high quality limestone beneath weak coal seams which left near surface mine workings unsupported and open after mine closure (Brook, 1991).

Figure 13. Slabbing Failure

5.2.3 Buckling Failure

Buckling failure commonly occurs when the bedding thickness is small in comparison to the beam length and the in-situ horizontal stress is moderate to high, or if the beam is jointed. Under these conditions, stress redistribution within the beam as shearing occurs along the bedding planes results in localized stress concentrations generating a new plane of weakness across the beam. For intact beams, fractures initiate in tension at the localized stress concentration points, which is often a result of a heterogeneity within the rock mass, and shear forces lead to coalescence of the micro-fractures into a diagonal fracture across the beam (Tang & Hudson, 2010). For jointed rock masses, stresses concentrate on existing discontinuities that extend across the beam. As shearing initiates along the diagonal fracture or existing discontinuity, induced lateral thrust results in tensile failure in the beam abutments. Ultimately, shearing continues along the diagonal fracture or existing discontinuity and corresponding tensile fracturing propagation continues in the abutments as the moment increases leading to inward failure of the beam into the excavation (Figure 14). In thin, strong (high modulus) beams, buckling can happen rapidly under its own weight (Diederichs, 1999).
5.2.4 Crushing Failure

Crushing failure commonly occurs when the bedding thickness is moderate to high in comparison to the beam length and the in-situ horizontal stress is high. Under these conditions, failure is governed by a combination of tensile and shear failure due to induced lateral thrusts from the beam's deflection (Tang & Hudson, 2010). As the beam undergoes flexural bending, tensile stresses develop mid-beam on its underside while conjugate compressive stresses concentrate on the top side, as shown in Figure 15. Simultaneously, tensile stresses develop in the abutments at the upper portion of the beam while compressive stresses concentrate on the lower side. Stress in the mid-span is approximately one half of that in the abutments and therefore the abutments fail first. Since rock is weak in tension – estimated at only about 10% of its compressive strength (Jumikis, 1979) – fracturing initiates at the top of the abutments and propagates downwards; this initial fracturing is invisible to an observer underground. Once the abutments begin to fail, the beam becomes essentially self-supporting which leads to subsequent tensile fracturing at the lower mid-span and crushing at the upper mid-span and lower side of the abutments as stresses are redistributed within the beam.
Diederichs and Kaiser (1999) developed a mathematical relationship to characterize crushing failure. They determined that fracturing initiates when beam deflection is approximately 10% of the beam thickness and ultimate failure is imminent when beam deflection reaches 25% of the beam thickness. These conclusions have strong practical application as alarm limits when monitoring deformation in underground excavations; particularly against crushing failure where initial fracturing is invisible to the eye.

5.2.5 Diagonal Fracturing Failure
Diagonal fracturing failure commonly occurs when the bedding thickness is high in comparison to the beam length and the in-situ horizontal stress is high. Under these conditions, when the underground excavation is created and a stable compression arch can nearly develop to resist beam deflection, discrete tensile fracturing occurs from the lower abutments to the upper mid-span following the inclined stress trajectories in the beam – parallel to the compression arch (Stimpson & Ahmed, 1992). Once the diagonal fractures reduce the ability of the beam to be self-supporting, an arched portion of the beam will fail under gravity into the excavation, as shown in Figure 16.

![Figure 16. Diagonal Fracturing Failure](image-url)

Stimpson and Ahmed (1992) characterized diagonal fracturing failure by conducting physical model tests in a laboratory and reproducing the fracturing propagation using finite element numerical analysis. During their study, they also concluded that this mechanism may be important where weak or broken material exists above the beam. The results of this thesis will be able to support or refute Stimpson and Ahmed’s conclusion since the shale interbed will be modelled much weaker than the limestone beam.
5.2.6 Effects of High Horizontal Stresses

High horizontal stresses alone do not necessarily indicate increased risk of failure. The geometric and boundary conditions of the structure of the rock mass, the strength of the intact rock and the stiffness of structures control the effects that the high horizontal stresses have on the rock mass. In massive rock masses, the horizontal stresses provide confinement which has proven through experimental evidence to significantly strengthen the rock mass (Tang & Hudson, 2010). Both the peak and residual frictional strengths increase with increasing confining pressure. In rock masses characterized by joint sets with dips greater than 60 degrees and a dip direction normal to the direction of the maximum horizontal stress, the horizontal stresses strengthen the rock mass by increasing the normal forces acting to resist shearing along the joint planes creating a “clamping” effect and ultimately resisting slabbing failure (Diederichs, 1999). In stratified rock masses, characterized by relatively flat-lying bedding (dip angles less than 30 degrees), stress concentrations along the bedding planes are exacerbated, encouraging greater extents of shear failure laterally and tensile failure vertically. Consequently, slabbing and buckling failures can develop in rock masses which under normal stress conditions ($\sigma_H/\sigma_V \approx 1$) would be considered stable. Furthermore, in underground excavations such as room and pillar mines, stress redistribution is constantly occurring as mine development persists generating stress concentrations and stress shadows within the roof beams and pillars (Figure 17). Large roof falls observed in room and pillar mines often have an elliptical shape with the long axis oriented perpendicular to the direction of maximum horizontal stress. Based on this observation, Iannacchione et al. (2001) recommended that mines under high horizontal stress conditions adjust their mine layouts and orient their headings parallel to the direction of maximum horizontal stress. Therefore, high horizontal stresses can significantly increase the risk of failure in stratified rock where bedding planes are inherently weaker and horizontal.
5.3 Failure Mechanisms from In-Situ Observations

Since the commencement of this research, the author has had the opportunity to visit two underground limestone mines in the US which experience high horizontal stress conditions and have had large roof failures; one in West Virginia (Mine A) and one in Pennsylvania (Mine B). For confidentiality reasons, specific details of the mines had to be withheld from this thesis, but the observations can provide valuable insight into the failure mechanisms at work and depths of failure under these conditions – which are expected to be similar to those present in Southern Ontario.

5.3.1 Mine A

Mine A was an underground limestone mine for high quality stone and aggregate which was interbedded with bands of competent shale, friable shale and marker beds of soft, weak green shale. Horizontal stresses at the mine were postulated at approximately 1.5 – 2 times the vertical. The mine had recurring issues with large roof falls, particularly at intersections where spans were greater. They did not have any system for monitoring deflections other than visual inspection. If deflection was visually observed by chance, the mine would use steel straps to attempt to contain the roof fall. Often
the roof fall presented no warning signs visible to the human eye since the mine workings were 12m in height and lighting in the mine was provided by head lamps or lights on mobile equipment; exceptions were at a working face or maintenance area where lighting was brighter. Following a roof fall, steel straps were installed to prevent further failure, as shown in Figure 18.

![Figure 18. Large Roof Fall which has been Supported with Steel Straps to Prevent Further Failure](image)

A supported roof failure was further observed to look for signs of its failure mechanisms. It was evident that the failure was comprised of a series of Voussoir beam failures, even though the large mass failed at once, which resulted in a stepped failure profile. The fractures across the beams were steep resembling those that are formed by the process of diagonal fracturing. However this is unusual according to the current understanding of failure in stratified rock masses because in this case the individual Voussoir beams were thin to moderate in thickness (thickness to span ratio equals 1/40 to 1/20) and the in-situ stress considered moderate which typically would have resulted in buckling or slabbing failure. Upon closer inspection of the steps, it could be seen that they corresponded to bedding planes or shale interbeds and the failure ultimately terminated at a weak green shale marker bed (Figure 19). Depth of failure was estimated at approximately 2.5m.
Figure 19. Weak Interbeds Controlling Failure Profile

A second failure in the mine was observed as shown in Figure 20. This failure also occurred at an intersection where the span was greater, and it presented a similar stepped failure profile as the first failure. However, where the first failure was more circular in nature, this failure was distinctly elliptical. Failure in the shortened axis orientation was observed to be prominently stepped on one side whereas failure in the elongated axis was observed to be steep with no obvious steps. Since the failure was unsupported, a closer observation of the steps within it was not possible. Depth of failure was estimated at approximately 3m for this roof fall.

Figure 20. Second Large Roof Fall in Mine A
The steep fractures are characteristic of diagonal fracturing which is not surprising if the failure was one 3m thick competent beam (beam thickness is approximately 1/5 failure span). However, the stepped failure is characteristic of a series of slabbing or buckling failures where individual thinner (beam thickness is approximately 1/20 failure span) competent beams fail. Assuming the failure truncated on a weak, shale interbed similar to the first failure observed, it would appear that the weak shale layer fails before the more competent interbeds closer to the excavation and therefore the failure initiates as a thick beam at the final depth of failure; as opposed to a series of thin beams successively failing upwards to the final depth of failure.

### 5.3.2 Mine B

Mine B was an underground limestone mine primarily for high quality lime production. Horizontal stresses at the mine were measured multiple times with varying results, but are considered to average between 2 – 3 times the vertical. The limestone formation mined was massive except for the upper 2m which was a transitional contact with the overlying shale formation. The shale formation was weaker than the limestone formation and therefore the transitional contact was comprised of weak and strong interbeds. The strata undulated across the mine with dips ranging from 5 to 15 degrees. The mine plan was to follow the limestone – shale contact and use this interface as the excavation roof. Unfortunately the undulation of the strata was impractical to predict and it was not uncommon for the interface to be lost while driving a header or drift. When the interface was lost and an intersection was created by blasting a cross-drift into an adjacent development header, near its mining face, a large roof fall would occur, as shown in Figure 21. The mine regularly installed “angels” – a monitoring instrument installed in the roof which releases a red visual (often a piece of flagging) when separation of the bedding in the roof is detected – as a warning system for unstable roof conditions. However, since the large roof falls occurred during development, the angels were only useful for indicating if a failure was progressing laterally. If an angel was triggered, then the mine would isolate that heading, drift or intersection.
Figure 21 and Figure 22 were two large roof falls observed by the author during a site visit. Although the failures could only be observed from the floor, the stepped failure profile and elliptical shape were clearly evident indicating that these falls were a series of Voussoir beam failures similar to the failures observed in Mine A. Since stress testing had been completed at the mine and the orientation of the principal stresses was generally known, it was confirmed that the elongated axis of the roof fall aligned with the minor horizontal in-situ stress orientation. Also, like Mine A, the failure mechanisms appear to be a combination of diagonal fracturing and slabbing or buckling. The roof fall shown in Figure 21 truncated at the overlying shale contact; the overlying shale is dark grey to black in colour making this observation visible from the floor. The truncating layer for the roof fall shown in Figure 22 could not as easily be discerned, however, the thick light coloured step (a thick limestone bed) suggests that the truncation was likely within the transition zone. Most notably, both roof falls had been previously supported by 8ft (2.4m) long rock bolts and the roof falls exceeded the supported depth. Accordingly, depth of failure was estimated to be approximately 4.5m and 4m respectively for the two roof falls.
The observations made at Mine B highlight the need to understand the mechanisms and depth of failure in these environments so that adequate ground support can be installed to prevent major roof falls and safeguard personnel.

5.4 Summary

Rock failure and the complex relationships of the mechanisms driving it continue to challenge the scientific community when it comes to precision engineering; unfortunately rock failure is not as simple as a stress controlled or a structurally controlled failure. When excavating in stratified rock masses, failure modes and mechanisms, although generically characterized by Voussoir beams theory, are uniquely defined by the beam geometries, rock mass strengths (or difference of strength) and in-situ stresses acting on the beam. These conditions can alter the amount of deflection before a beam will fail together with the depth of failure. Therefore, it is important to understand the many variables that contribute both through stress and structurally, as well as their interdependencies, before one can assess the potential for failure in stratified rock masses.
CHAPTER 6 NUMERICAL MODELLING

Numerical modelling methods have been employed in the fields of civil and mining engineering since the 1960s (Mukherjee, 2003) as a tool for better understanding of the mechanisms of failure in rock and soil on both small and large scales. The natural environment is often more complex than what can be practically represented by the mesh and equations governing the simulation of failure in numerical modelling. Given this limitation, a myriad of mathematical methods have been developed over the years which adopt certain simplifications best suited for specific environments. Each method is based on unique closed-form solutions or partial differential equations (PDEs) to most accurately represent the targeted environment of application; for example, representation of the ground conditions as a continuum or discontinuum. Simply, depending on the soil or rock strength, and presence of discontinuities, heterogeneities, anisotropy and plasticity in the rock mass, certain modelling methods are more appropriate to use than others.

This chapter provides a brief overview of numerical modelling methods, introduces UDEC and discusses why it was selected for conducting the numerical modelling carried out in this research.

6.1 Selection of a Suitable Model Method

Numerical modelling methods generally differ in how the way that blocks or elements within blocks are allowed to interact with one another while the modelled material deforms leading to failure or explicit movement during failure. The most common types of numerical modelling methods can be grouped as continuum, discontinuum or a hybrid of the two. The fundamental concepts, advantages and disadvantages of each method are summarized in Table 5.
Table 5. Overview of Modelling Method Groups

<table>
<thead>
<tr>
<th></th>
<th><strong>Continuum Methods</strong></th>
<th><strong>Discontinuum Methods</strong></th>
<th><strong>Hybrid Methods</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fundamental Concept</strong></td>
<td>Represents the system as a continuous material through interconnected elements. Discontinuities are implicitly represented by degraded material properties through a homogenization process.¹ Best suited for scenarios where joints have little influence on the failure mechanisms.</td>
<td>Represents the system as a set of interacting blocks with unique contact strength properties governing block movement. Discontinuities are explicitly represented, but may also be implicit within larger blocks. Best suited for scenarios where joints influence the failure mechanism.</td>
<td>Represents the system as a combination of interacting blocks that are further divisible. Intact blocks are assigned a brittle fracture criterion. Best suited for scenarios involving brittle fracturing and fragmentation.</td>
</tr>
<tr>
<td><strong>Advantages</strong></td>
<td>Geometrically simple and generally fast computing because adjacent elements remain connected during deformation and yield making computational processes streamlined.</td>
<td>Separation, displacement and rotation of the blocks is permitted. Discontinuities can be explicitly represented using laboratory joint properties to show directional weakness.</td>
<td>Combines the continuum and discontinuum methods to reduce limitations of each method independently.</td>
</tr>
<tr>
<td><strong>Disadvantages</strong></td>
<td>Separation, displacement and rotation of regions are not permitted; solution becomes unstable with large deformations. Continuity of adjacent boundaries must be maintained.</td>
<td>Requires more input parameters to be calibrated and constrained (i.e., both intact blocks and joint properties). More computationally demanding as block contacts and interactions must be tracked. Individual blocks are continuous and cannot be fractured during failure.</td>
<td>Potential for continuity and compatibility issues at interfaces between regions of different methods; particularly where adjacent elements employ different assumptions.</td>
</tr>
</tbody>
</table>
| **Common Methods**   | • Boundary Element Method  
• Finite Element Method  
• Finite Difference Method | • Distinct Element Method  
• Discrete Element Method  
• Particle Flow Method | • Boundary Element / Finite Element Methods  
• Distinct Element / Finite Element Methods |

¹Discontinuities can be explicitly modelled as small strain contacts or as narrow zones of elements assigned properties to simulate joint strength if desired; however, still no separation, displacement or rotation of those boundaries can occur.
In an effort to verify his theories pertaining to bedded roof rock behavior over mined spans, Sterling (1980) conducted a series of laboratory experiments. A key conclusion from these experiments was that roof beds cannot be simulated by continuous, elastic beams or plates since their behaviour is dominated by the rock mass blocks created by natural cross joints or induced transverse fractures. The author’s observational experience in US limestone mines as discussed in Section 5.3 concurs with Sterling’s conclusion because the author observed that the failure mechanisms in room and pillar limestone mines subject to high horizontal stresses have a distinct and important structural component controlling failure. The role of structure in the failures observed was evident because the stepped failure profile indicated that a release of blocks on bedding planes had occurred extending to an ultimate depth of failure controlled by a notably weaker shale interbed. Accordingly, a discontinuum method was chosen to numerically model the anticipated limestone mining conditions for Southern Ontario so that the structural component of the failure mechanism could be adequately simulated. In particular, the distinct element method using the commercial software UDEC was selected because this method permits modelling of the block separation, translation and rotational movements that occur during beam failure.

6.2 UDEC

The Universal Distinct Element Code (UDEC) is a two dimensional discontinuum numerical modelling code which models the quasi-static or dynamic response to loads applied to a simulated rock mass containing multiple, intersecting discontinuity planes (Itasca Consulting Group, Inc., 2015). The rock mass is represented as an assembly of discrete blocks (either rigid or deformable) and the discontinuities are treated as boundary conditions between blocks. Material strength and stiffness are applied to both the intact blocks as well as the contacts between the blocks themselves. UDEC assesses failure within blocks against defined failure criteria and calculates displacements by converting stresses acting on each block relative to adjacent blocks through the bulk and shear moduli to the associated strains. Deformations occur through both squeezing and bending of blocks, as well as through shearing and separation of the blocks along their contacts. Additionally in UDEC, the blocks are free to delaminate and separate from adjacent blocks during failure.

UDEC has a built-in scripting language called FISH which allows the user to customize or automate many of the programs operations. An automatic Voronoi tessellation generator is also available to create multidirectional contacts that can be bonded together to simulate intact rock fracturing. Of particular value, the Voronoi network of potential intact rock fracture pathways can be superimposed with a joint.
network representing the geological structures present in the rock mass. UDEC was specifically chosen for conducting this research because of these features.

Since investigating the depth of failure was going to involve iterative modelling under various geometrical and in-situ stress conditions, the automation ability of built-in scripting increased the efficiency of the modelling by reducing the number of manual adjustments required to carry out a series of simulations. Secondly, since the observed failures in room and pillar limestone mines under high horizontal stress conditions show stepped failures indicating a combination of structural and stress controlled failure, the Voronoi tessellation would allow for the simulation of intact rock failure between the bedding planes as observed in the US limestone mines and expected under Southern Ontario mining conditions.

The FISH script written to semi-automate the modelling is presented and discussed in Appendix C. Details of the Voronoi tessellation generator and a discussion of its use in the modelling are presented in Section 6.2.1 and in Chapters 7 and 8 respectively.

6.2.1 Voronoi Tessellation Generator
The Voronoi tessellation generator creates a series of randomly oriented breaks in an existing block to form polygons ranging from perfectly shaped and evenly sized hexagons to irregularly shaped and sized blocks. The polygons are formed by using the Voronoi algorithm to randomly distribute points throughout a defined region (which act as the centroids to the polygons), then constructing lines bisecting the distance between each point and its adjacent points (Itasca Consulting Group, Inc., 2015). When the contacts are given suitable micro-mechanical intact rock properties, degrading to joint properties after fracture, the Voronoi tessellation is useful for simulating the brittle fracture of intact rock. The only criteria maintained is the fracture length (edge length). The user defines an average edge length for the Voronoi joints in which the program ensures that this average length is achieved over the region in which the Voronoi tessellation is used.

The user can also control the degree of irregularity of the polygons by setting the iteration number. The iteration number controls an iteration procedure which shifts the polygon centroids resulting in variable lengths and orientations of the bisecting lines (Itasca Consulting Group, Inc., 2015). To obtain a uniform distribution of hexagons over the region being model using the Voronoi tessellation, a higher iteration number is used (greater than 1000). Although, utilizing regularly shaped polygons creates a directional bias in simulating the fractures because the majority of edges are aligned with the upward direction of
the model. By using a low iteration number (such as 1), the polygons are comprised of edges in a variation of directions which reduces fracture generation bias created by the use of the Voronoi tessellation generator. The concept of directional bias due to iteration number is shown in Figure 23. Accordingly, the modelling carried out as part of this research, utilized a low iteration number so that fracturing could develop more “naturally”.

![Figure 23. Visual of Directional Bias due to Iteration Number: (a) Iteration Number = 1, (b) Iteration Number = 100, and (c) Iteration Number = 10,000.](image)

Finally, where the Voronoi tessellation generated contacts are used to simulate fracturing within intact rock, the Voronoi contacts cannot simply be assigned rock properties because these are size dependent and can underrepresent the strength and stiffness of intact rock. Therefore, they must be assigned micro-mechanical properties which are calibrated to reproduce the strength and stiffness of the intact rock being modelled (Wang et al., 2013). The micro-mechanical joint calibration process is discussed in detail in Chapter 7.

### 6.3 Two Dimensional Versus Three Dimensional Modelling

A key question in numerical modelling is whether a two-dimensional plane strain assumption is valid or whether the problem being analyzed is truly three dimensional. For the context of observing the
increased maximum depth of failure as a result of high horizontal stresses in a stratified environment, the greatest impacts of the high stress can be captured in a linear alignment with the direction of the high stress. As a result, it is not necessary to have the added complexity and computing demands of a three dimensional modelling program (such as 3DEC - UDEC’s three dimensional modelling companion) to understand the operating mechanisms leading to the change in ultimate depth of failure in high horizontal stress environments. By opting to employ UDEC for this modelling, a relatively detailed Voronoi tessellation (described in Section 6.2.1) could be used without simplification. This would have not been possible in 3DEC because the computing demands required to handle a Voronoi tessellation with the equivalent amount of detail in the third dimension would have exceeded computing capabilities at the time of modelling.
CHAPTER 7  MICRO-MECHANICAL PROPERTIES CALIBRATION

In order to monitor the generation of the brittle fracture driven mechanisms developed in the haunches of the beams that lead to the beam failure observed in-situ, the expected failure area (plus suitable additional area to eliminate boundary effects) was modelled using a Voronoi tessellation. By using the Voronoi tessellation in this region, intact rock failure can be explicitly modelled by allowing displacements and separations along the Voronoi polygons representing the limestone beam and interbedded shale layer. However, invoking a synthetic rock mass using the Voronoi tessellation comes with its challenges; the contact properties given to the edges of the Voronoi polygons must simulate intact rock and respective strength properties to achieve an unbiased/valid representation. Many papers (Wang et al., 2013; Scholtes et al., 2011; Koyama & Lanru, 2007) have reported that Mohr-Coulomb strength properties (cohesion, friction and tensile strength) derived from laboratory testing of intact rocks when used directly as the contact cohesion, contact friction and contact tensile strength in a numerical model analysis do not accurately simulate an equivalent intact rock strength; the synthetic rock mass would perform weaker than desired. Therefore the laboratory derived strength properties need to be calibrated to find the equivalent micro-mechanical strength properties so that the intact synthetic rock strength can be properly modelled with the Voronoi tessellation. Determining the equivalent micro-mechanical strength properties to be used in the Voronoi tessellation is not a trivial task since the amount of adjustment of these properties is directly related to the Voronoi edge length used in the model. The edge length is user defined and based on the scale of the model and the mechanisms that are the target of the modelling exercise. Needless to say, the user must know the modelling capacity of the computer/software and have a basic understanding of the scale and type of movement required to be observed to achieve the goals of the modelling in order to select an appropriate Voronoi edge length and begin determining the equivalent micro-mechanical strength properties. This process can be iterative and time consuming depending on the level of preciseness required for the numerical model.

This chapter outlines the unconfined compressive strength (UCS) test simulations conducted to determine equivalent micro-mechanical strength properties to be used as the Voronoi contact properties in the numerical modelling analysis. In addition, it will discuss the importance of obtaining a suitable Voronoi polygon size (via edge length input) and the associated adjustment of contact properties required to develop a valid synthetic rock mass model in which the strength (and associated failure mechanisms) of an intact rock beam is represented adequately by the Voronoi tessellation.
7.1 Intact Rock Mass Properties of Limestone and Shale in Southern Ontario

A review of published rock mass properties for the Ordovician limestone formations in Southern Ontario, the target units for underground limestone mining in the region, was conducted to establish a range of strength and deformation properties (UCS, σT, c, E) to be represented in the numerical modelling analysis. The Lindsay, Bobcaygeon and Gull River Formations exhibit similar strength properties (average UCS of 110MPa) whereas the Verulam Formation, as might be expected given that it is characterized by thin limestone strata interbedded by equally thick shale, exhibits noticeably weaker strength properties (average UCS of 23MPa) (Lo, 1989). Due to the high percentage of shale content in the Verulam Formation resulting in a more uniform strength distribution between the limestone and shale layers, and therefore outside the focus of this research (i.e. the influence of a weak layer is absent), the Verulam strength properties were not be considered. In addition, the Cobourg Formation is the Michigan Basin equivalent nomenclature of the Appalachian Basin Lindsay Formation (Damjanac, 2008; Ontario Geological Survey, 1992). Since a low to intermediate level nuclear waste repository is planned for construction in the Cobourg Formation in Southern Ontario about 220km northwest of Toronto, considerable amount of public domain strength data exists for this limestone unit and therefore its values were also incorporated.

In summary, for this study, a range of the reported Lindsay, Bobcaygeon, Gull River and Cobourg Formations values were analyzed to establish a representative range of intact rock mass strength properties for the most economical mineable limestone units. These intact properties will be the target range for identifying correlating joint properties to be used in the Voronoi tessellation of the numerical analysis. The established average, minimum and maximum intact rock mass strength properties for the limestone are presented in Table 6.

<table>
<thead>
<tr>
<th></th>
<th>UCS (MPa)</th>
<th>Tensile Strength, σT (MPa)</th>
<th>Young’s Modulus, E (GPa)</th>
<th>Poisson’s Ratio, ν</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limestone</td>
<td>110 (72 - 143)</td>
<td>6 (3 - 10)</td>
<td>50 (32 - 63)</td>
<td>0.19 (0.12 – 0.26)</td>
</tr>
<tr>
<td>Shale</td>
<td>30 (20 - 44)</td>
<td>10 (5 - 14)</td>
<td>12 (2 - 27)</td>
<td>0.10 (0.02 – 0.17)</td>
</tr>
</tbody>
</table>

Strength properties for the weak shale layer are a range of public domain values published for laboratory testing conducted on the Ordovician shale units (Queenston, Georgian Bay and Blue Mountain Formations). Although an erosional contact exists between the middle Ordovician limestones and upper Ordovician shales, it is expected that a range of these shale strength properties will suitably
represent those of the interbedded shale layers for the purpose of this study. The established average, minimum and maximum intact rock mass strength properties for the shale are also presented in Table 6.

7.2 Equivalent Mohr-Coulomb Failure Criterion Parameters

Numerical modelling programs assess rock failure by comparing the stress applied to an element in the model to a user-defined failure criterion for the material. Hoek-Brown and Mohr-Coulomb are the most common failure criteria for intact rock available in most numerical modeling software. A standard approach in rock engineering is to use measured intact rock mass strength properties from laboratory testing and observable rock mass classification parameters to define the rock mass strength. This process involves carrying out a series of mathematical calculations derived from empirical relationships which are unique to the failure criterion targeted. RocLab, a software distributed by RocScience, streamlines this process for engineers because it accepts the user defined strength inputs and outputs the equivalent Hoek-Brown and Mohr-Coulomb intact rock failure criterion parameters. The relationships used are based on those published by Hoek et al. (2002).

Accordingly, intact rock mass properties compiled from literature (presented in Table 6) were input into RocLab to establish baseline cohesion and friction angle values; the basis of the Mohr-Coulomb failure criterion. The methodology RocLab follows requires the input parameters to be in the form of Hoek-Brown developed rock mass classification properties (UCS, GSI\(^{17}\), \(m_i\)^{18}, \(D\)^{19}). The inputted Hoek-Brown properties and calculated equivalent Mohr-Coulomb failure criterion parameters are shown in Table 7.

<table>
<thead>
<tr>
<th>Table 7. Corresponding Mohr-Coulomb Properties from RocLab</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hoek-Brown Properties</strong></td>
</tr>
<tr>
<td><strong>USC (MPa)</strong></td>
</tr>
<tr>
<td><strong>Limestone</strong></td>
</tr>
<tr>
<td><strong>Shale</strong></td>
</tr>
</tbody>
</table>

\(^{17}\) Geological Strength Index (GSI) is a rock mass characterization system developed to assist with converting reliable rock mass property data into failure criterion input data for numerical analysis (Marinos et al., 2007). For good quality rock masses (RMR > 40), GSI is considered equivalent to Bieniawski’s RMR.

\(^{18}\) \(m_i\) is an intact material constant analogous to frictional strength which can be derived from triaxial testing of intact rock (Eberhardt, 2012).

\(^{19}\) \(D\) is a disturbance factor which accounts for blast damage and stress relaxation (Eberhardt, 2012).

\(^{20}\) \(\sigma_3 \text{ max.}\) was calculated using a tunnel depth of 150m and a unit weight of 26 kN/m\(^3\) .
The Mohr-Coulomb failure parameters were used to define the intact rock mass strength of the limestone throughout the numerical modeling calibration exercise to establish corresponding micromechanical contact properties for the Voronoi tessellation contacts.

7.3 Developing a Properties Range using a UCS Simulation

The UCS of the many limestone formations in Southern Ontario is quite well defined from laboratory tests conducted for the design of large quarrying operations, building foundations, nuclear facilities and waste repositories. Much of this data is in the public domain, particularly from the nuclear waste repository project. Subsequently, a model representing a rock sample (51mm dia.) with a length to width ratio of 2.5 being compressed at a rate of 0.02 m/s – a standard UCS laboratory test as shown in Figure 24 – was used to identify a range of Voronoi edge lengths and corresponding adjusted Voronoi contact properties to represent the synthetic rock mass comprising the rock beam over the mine excavation.

The steel platens were modelled at 2.5 times the width of the rock sample to prevent axial stress concentrations at the edges of the rock sample and ensure even loading on the rock-platen interface.

Figure 24. Example of UCS Test Model for Voronoi Contact Property Calibration (Voronoi Edge Length = 0.005; 4% of sample length).
Given that UDEC is a two-dimensional program, this simulation is actually representing an infinitely long beam with cross-sectional dimensions shown in Figure 24 and not a cylindrical core sample that is used in laboratory UCS experiments. Therefore identifying exact Voronoi contact properties to mimic a specific UCS value is extraneous and the best that can be achieved from this study is to obtain a range in model properties which correlate to the range in typical limestone strength properties presented in Table 6 and Table 7.

To determine suitable adjusted Voronoi contact properties for numerical analysis, a systematic approach was carried out increasing the contact friction, contact cohesion and contact tensile strength values in a variety of combinations until the simulated UCS fell within the range of laboratory measured UCS. The maximum axial strength detected at the rock-platen interface in the UCS simulation was compared to the reported UCS data available to determine if the tested Voronoi properties of the synthetic rock mass simulated intact rock strength properties from laboratory testing. The following sections present the model set-up, systematic approach to modelling, a thorough discussion of the results and concludes with the equivalent range in Voronoi contact properties to be used in the limestone mining numerical study.

**7.3.1 UCS Modelling Conditions**

Figure 24 displays the dimensions of the UCS simulation where a rock specimen is situated between two steel platens; the bottom platen is fixed in both the x- and y- directions and the upper platen is fixed in the x-direction but has a constant y-velocity of -0.02m/s; this rate of compression aligns with the loading rate required to meet ASTM International D7012-14 standard (2014) for conducting UCS tests on intact rock specimens (Park, 2004). The contacts between the steel platens and the rock specimen are fixed in the x-direction to eliminate sliding along that interface which could be encouraged simply by Voronoi geometry bias. If such slippage occurred in a laboratory UCS test, the sample failure would be considered an invalid test and that strength measurement discarded. Both the steel platens and the rock mass were considered deformable, elastic blocks and modelled with the strength properties presented in Table 8.
Table 8. Model Elastic Properties

<table>
<thead>
<tr>
<th></th>
<th>Steel Platens</th>
<th>Intact Rock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (ρ)</td>
<td>7750 kg/m³</td>
<td>2650 kg/m³</td>
</tr>
<tr>
<td>Bulk Modulus (K)</td>
<td>160 GPa</td>
<td>30.6 GPa</td>
</tr>
<tr>
<td>Shear Modulus (G)</td>
<td>79.0 GPa</td>
<td>22.9 GPa</td>
</tr>
</tbody>
</table>

The edges of the Voronoi polygons representing potential fracture pathways through intact rock were given the strength properties presented in Table 9. These values remained constant while the Voronoi contact properties (referred to as joint properties in UDEC) were varied to determine suitable micro-mechanical properties. These constants were considered appropriate for intact rock and the fresh surfaces created during the failure of the intact rock in the simulated UCS test.

Table 9. Constant Voronoi Contact Properties (Referred to as Joint Properties in UDEC)

<table>
<thead>
<tr>
<th>Voronoi Property</th>
<th>Constant Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint Normal Stiffness (kn)</td>
<td>1600 GPa/m</td>
</tr>
<tr>
<td>Joint Shear Stiffness (kS)</td>
<td>160 GPa/m</td>
</tr>
<tr>
<td>Joint Residual Friction</td>
<td>31˚</td>
</tr>
<tr>
<td>Joint Residual Cohesion</td>
<td>0 MPa</td>
</tr>
<tr>
<td>Joint Residual Tensile Strength</td>
<td>0 MPa</td>
</tr>
<tr>
<td>Joint Dilation</td>
<td>0˚</td>
</tr>
</tbody>
</table>

7.3.2 Strength Overestimation by Using Elastic Blocks

Elastic properties were chosen for the Voronoi polygons because the purpose of introducing a Voronoi tessellation into the modelling was to allow for explicit failure of the intact rock mass through contact separation of adjacent blocks. It should be emphasized that failure through brittle fracture is restricted to the Voronoi contacts only and not through the Voronoi polygons themselves. Unlike plastic blocks in UDEC, elastic blocks do not yield internally nor deform appreciably. As a result, at greater edge length ratios, there is a higher possibility of “pseudo-bridging”\(^{22}\) which will cause the maximum axial stress required to ultimately fail the simulated intact rock to be overestimated.

---

\(^{21}\) The bulk and shear moduli are calculated from the Young's Modulus (E) and Poisson's ratio (ν) using the formulas:

\[
K = \frac{E}{3(1-\nu)} \quad \text{and} \quad G = \frac{E}{2(1+\nu)}
\]

\(^{22}\) Pseudo-bridging is the terminology used here because the bridging is a result of acute-angled corners of blocks within the rock mass preventing movement of much larger blocks. In-situ these acute-angled corners would be crushed. However, given that fractures cannot be generated through blocks in UDEC, this ‘pseudo-bridging’ is able to develop.
7.3.3 Measuring the Numerically-Simulated UCS

Axial stress versus axial strain plots were generated to monitor the sample’s strength as it deformed with the maximum axial stress representing the UCS of the sample. The axial stress was calculated in the numerical simulation by averaging the axial stress measured at 23 elements across the top of the sample as shown in Figure 25.

![Figure 25. Axial Stress Measurement Locations along Platen-Sample Interface](image)

Every 500 cycles the axial strain was calculated by measuring the total vertical displacement of the upper platen and dividing by the length of the sample as well as the corresponding axial stress to produce stress-strain curves for each failure simulation. As shown in Figure 26, most failures occur after 500,000 cycles regardless of Voronoi edge length and therefore calculating axial strain and axial stress every 500 cycles provides more than sufficient detail to accurately identify the numerically-computed UCS to one decimal place.
7.3.4 Monitoring Shear and Tensile Fracture Development

Failure through intact rock initiates from local concentrations of tensile stresses exceeding the local tensile strength creating micro-fractures which through the continued application of the load and stress redistribution result in coalescence of the micro-fractures into macro-fractures along which shearing occurs (Tang & Hudson, 2010). A correct selection of Voronoi properties should allow for this fracture generation process to materialize. Ergo the numbers of tensile fractures and shear fractures as a percentage of total contacts created by the Voronoi tessellation were monitored to ensure that tensile fractures dominated in the early stages of the sample being compressively loaded, and as the sample approached its ultimate strength, the shearing mechanism dominated.

The tensile and shear fractures were determined by having UDEC scan every Voronoi edge contact at 500-cycle intervals and count the number of contacts that have normal displacements greater than 0.1mm for tensile fractures and shear displacements greater than 0.1mm for shear fractures. Tang and Hudson (2010) suggest that cracks initiate when the stresses on the intact rock sample reach 30-40% of the sample strength in discontinuity-free, homogenous rock. Therefore, monitoring of the development of shear and tensile fractures as the sample approaches failure was carried out by plotting the tensile and shear fracture counts against the axial stress as a percentage of the numerically-devised UCS.
The percentage of the UCS at which the sample begins to develop fractures and the relationship between the numbers of stress-induced shear versus tensile fractures were used as criteria for accepting or rejecting synthetic rock mass property results. For example, if a set of Voronoi contact properties resulted in a UCS value within the range of actual laboratory UCS values but tensile fractures initiated at 20% of the UCS, as shown in Figure 27a, then those properties were rejected as suitable equivalent model input properties. Similarly, as shown in Figure 27b, if fracture initiation was tensile dominant around at about 30% of the UCS value but the UCS value did not fall within the range of actual laboratory UCS values, then those properties were also rejected.
Figure 27. Rejected Failures: (a) UCS Test #32, (b) UCS Test #40
7.3.5 UCS Testing Results

A total of 77 numerical simulations were carried out with varying combinations of Voronoi edge lengths, contact friction angles, contact cohesions, and contact tensile strengths to establish suitable micro-mechanical properties. Each simulation was monitored to ensure that failure occurred through the modelled core sample, similar to a valid laboratory UCS test failure (Figure 28). Axial stress – axial strain curves were produced to ensure a relatively linear slope occurred prior to failure (Figure 29a). Finally, the percentage of peak strength in which fracturing initiated in the sample as well as the type of fracturing (shear or tensile) were monitored to ensure failure within the sample developed occurred accordingly to Tang and Hudson (2010) extensive research on fracture propagation within intact rocks (Figure 29b).

Figure 28. UDEC UCS Failure Simulation Compared to Laboratory Failed Core Samples
Figure 29. UCS Test #47 Failure Monitoring: (a) Axial Stress - Axial Strain Curve, (b) Contacts in Failure
### 7.3.6 Relationship between Voronoi Edge Length and Failure Mechanism Bias

Intuitively, by using a greater edge length (relative to the model size), UDEC perceives the Voronoi polygons as wedges moving amongst one another in which shear strength properties influence crack initiation and failure. At smaller edge lengths (relative to the model size), UDEC perceives the Voronoi polygons as individual grains in which the Voronoi contacts initially fail in tension until a macro-scale discontinuity develops and shearing has a dominant influence on failure. The latter is the preferred mechanism of failure for the purpose of establishing Voronoi contact properties that correspond to intact rock properties. However, the smaller the Voronoi polygons are, the greater the demands are on the modelling computer both in computing power and memory.

Accordingly, numerous simulations were conducted using a variety of properties over a range of edge lengths to determine the maximum Voronoi edge length that could be used while tensile fracturing still initiated cracking within the sample between 30 – 40% of the peak strength. The range of properties assessed covered expected and maximum adjusted values for each of friction angle, cohesion and tensile strength.

![Figure 30. Dependency of Mechanism Initiating Cracking on Voronoi Edge Length Ratio](image-url)
Figure 30 shows the results of the simulations indicating that at ratios of edge length to beam length\(^2\) of less than 8\(^%\)\(^2\), tensile stresses initiate cracking, and at greater than 17.5\%, shearing is the dominant mechanism for crack initiation. This boundary appears to exist for all cases independent of the Voronoi joint properties. Edge length to beam length ratios from 8\% to 17.5\% represent a transition region where the contacts dynamics (tensile separation and shearing) are mutually dominant. In summary, it can be postulated that a ratio of Voronoi edge length to a critical length measurement, in this case beam length, in a numerical model should always be less than 8\% when the desired mechanism is tension dominated. The orange highlighted region in Figure 30 represents the Voronoi joint properties and edge length combinations that meet the criteria of crack initiation occurring between 30-40\% of the maximum axial strength.

### 7.3.7 Dependency of Strength Properties on Voronoi Edge Length

Given that Voronoi edge length has a distinct impact on the mechanism of failure, numerous simulations were conducted to assess the sensitivity of the maximum axial strength to the edge length using the same range of Voronoi contact properties as inputs. As shown in Figure 31, a bi-modal strength curve appears to exist as the edge length ratio increases. The changes in the slope of the curves that define strength increase closely resemble the boundaries interpreted in Section 7.3.5 existing between governing failure mechanism within the model (shaded areas in Figure 30).

For the range of properties tested, a Voronoi edge length that is 7\% of the beam thickness would be equivalent to an intact UCS between 80 – 160MPa (more appropriately closer to 100MPa since the higher strength curves are those representing maximum friction angle and cohesion values) which is within a typical range of UCS values for Ordovician limestone units in Southern Ontario.

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\(^2\) In the discussion about Voronoi edge length ratio (edge length to beam length ratio), beam length is the UCS sample length in the micro-mechanical joint property analysis. Beam length was chosen as the normalizing dimension because ultimately in the depth of failure study, the high horizontal stress compresses the synthetic rock mass on the end of the Voussoir beam.

\(^2\) Since length of UCS sample is 128mm, Voronoi edge length is 0.01m for an edge length to beam length ratio of 8\%.  

66
Figure 31. Effect on Strength by Voronoi Edge Length

However, based on the curves, a Voronoi edge length to beam length ratio of 4% should be used to represent purely tensile failure. This is evident since the strength profiles are similar at ratios less than that indicating that the tensile strength is controlling failure – the desired mechanism of failure. Beyond 4%, the Voronoi contact property combinations which used the maximum values for friction angle and cohesion showed that these properties significantly contributed to the strength of the rock mass indicating that the Voronoi tessellation is sensitive to shear strength properties above this length ratio. However, at a 4% ratio, the UCS simulated is between 60 – 100MPa which is a lower range in strength than exhibited by typical Ordovician limestone units in Southern Ontario (as shown in Table 6). Therefore, additional simulations at a 4% ratio needed to be conducted (primarily increasing the tensile strength input) to identify suitable Voronoi contact property values which correspond to UCS values between 72 - 143MPa.

7.3.8 Suitable Baseline of Voronoi Contact Properties

In order to establish a suitable range in Voronoi contact properties that are equivalent to the target UCS value range for the Ordovician limestone units of Southern Ontario, an iterative process of increasing the tensile strength, friction angle, and cohesive strength inputs was completed. Since the Voronoi blocks represent an intact rock mass to be fractured in tension, it is imperative that the tensile strength is the limiting rock mass strength property. Figure 32 (adapted from Figure 30 to show the target crack initiation range, 30 – 40% of 72 – 143MPa) shows that within the tension-dominated range (Voronoi
edge length ratios < 8%), the tensile strength is the limiting input as desired. This is evident because out of the combinations simulated only three traces existed representing the three different tensile strengths tested, regardless of the cohesion and friction angle. However, it is clearly shown even though all combinations of properties are capable of producing the strengths equivalent to those within the lower range of the target zone (crack initiation pressure of 22 – 33MPa, light orange region), only those with tensile strengths of 25MPa meet the target range for average strength (crack initiation pressure of 33 – 44MPa, dark orange region). Therefore, a contact tensile strength of 25MPa was considered the minimum tensile strength for further numerical analysis.

![Figure 32. Contact Tensile Strength Required to Initiate Suitable Tensile Cracking](image)

Figure 32. Contact Tensile Strength Required to Initiate Suitable Tensile Cracking

A series of additional simulations at a Voronoi edge length ratio of 4% (0.005m) were carried out as a form of sensitivity analysis to refine a range in contact cohesion, contact friction angle and contact tensile strength for that edge length ratio. Since the combination of $\sigma_T = 25$MPa, $c = 50$MPa and $\phi = 50^\circ$ met the lower strength criteria for acceptable equivalent Voronoi contact properties, they were used as the basis for refine a suitable range.

Initially, the tensile strength was varied from 9MPa to 55MPa while maintaining a constant cohesion of 50MPa and friction angle of 50° to determine how much the tensile strength could be increased while still having the failure initiate in tension. As shown in Figure 33a, tensile strength can only be increased
to approximately 30MPa with the constants before failure initiates in shear instead of tension, which indicates that tensile strength is as high as it can be unless the shear properties (c, \( \phi \)) are increased. However, peak strength values are still at the very low range of acceptable UCS values.

Secondly, the cohesion was varied from 50MPa to 110MPa while maintaining a friction angle of 50° and tensile strength of 25 MPa. As shown in Figure 33b, additional rock mass strength was achieved through the increase in the cohesion and all failure initiated in tension. However, at cohesion values greater than 80MPa, failure initiation began below 30% of the peak strength so for a tensile strength of 25MPa, cohesion cannot be increased greater than 80MPa and additional strength must be gained by increasing the friction angle.

Figure 33. Sensitivity Analyses: (a) Tensile Strength, and (b) Cohesion
Lastly, the friction angle was varied from 30° to 70° while maintaining constant cohesion of 80MPa and tensile strength of 25MPa. As shown in Figure 34, a marginally strength increase occurred with the increase in friction angle. However, friction angles above approximately 54° resulted in failure initiation below 30% of the peak strength.

![Figure 34. Sensitivity Analysis on Friction Angle](image)

This iterative process was continued for a Voronoi edge length ratio of 4% (0.005m) and the refined range of baseline Voronoi contact properties representing intact rock mass values for the Ordovician limestone formations of Southern Ontario are presented in Table 10.

<table>
<thead>
<tr>
<th>Limestone</th>
<th>Friction Angle, $\phi$ (°)</th>
<th>Cohesion, $c$ (MPa)</th>
<th>Tensile Strength, $\sigma_T$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limestone</td>
<td>45 - 55</td>
<td>80 - 110</td>
<td>25 – 30</td>
</tr>
</tbody>
</table>

Results from all the UCS tests numerically modelled using the various combinations of contact friction angle, contact cohesion and contact tensile strength to establish suitable micro-mechanical contact properties for the synthetic rock mass are summarized in Appendix B.

### 7.4 Calibration Conclusions

From the 77 successful UCS test simulations completed to calibrate the micro-mechanical properties for use with the Voronoi tessellation generator in UDEC, some key conclusions became evident:
1) Micro-mechanical properties utilized in synthetic rock mass simulations must be calibrated to obtain a realistic model; using laboratory strength values directly for the Voronoi contacts will underrepresent the modelled intact rock strength.

2) Numerous combinations of micro-mechanical properties can achieve the desire intact rock strength. Without understanding the mechanisms of intact rock failure and monitoring the mechanism of failure during the calibration, it is possible to select a combination of micro-mechanical properties which do not accurately represent intact rock failure. Consequently, failure mechanisms in future modelling exercises employing these “calibrated” micro-mechanical properties will also be inaccurate.

3) The edge length of the Voronoi contact has a significant impact on the intact rock strength and the mechanisms of fracture initiation that can reasonably develop. Edge length ratios (edge length of the Voronoi contact normalized by a length under axial compression) less than 8% perform like grains within a rock mass and tensile strength initiates failure. In addition, small edge length ratios appear to require greater inflation of laboratory strength properties to achieve an equivalent synthetic intact rock strength. On the other hand, edge length ratios greater than 17.5% perform like a collection of wedges (or a blocky rock mass) and shearing mechanisms dominate failure. Accordingly, large edge lengths should not be used to represent intact rock failure. If large edge lengths are used to represent blocky ground, the Voronoi contact properties are anticipated to be similar to laboratory strength properties based on peak strength values observed during the calibration exercise. Since large edge lengths were not of interest for this study and not investigated further, conclusion about them are conceptual at best.

4) For failure through intact rock where fracture initiation occurs in tension, an edge length ratio less than 8% should be used. Contrarily, for intact rock failure where fracture mechanisms are predominantly shear, edge length ratios greater than 17.5% should be employed.

5) Edge length ratios between 8% and 17.5% do not appear to have any usefulness in numerical modelling. This range in edge lengths seems to be a transition zone where intact rock strength values are extremely sensitive to any change in input properties, which is not practical for an engineer performing analysis since input properties contain an element of uncertainty.
The conclusions from the micro-mechanical property calibration were considered when developing the engineer modelling for assessing the influence of a weak layer on roof failure in room and pillar limestone mines.
CHAPTER 8 ENGINEERING MODEL

The mechanisms contributing to the depth of failure in the roof of a room and pillar limestone mine were understood by developing a geometrically accurate scale model in UDEC of a typical room. In doing so, the scale effects of the numerical model can be minimized resulting in an increased level of confidence that the model is an accurate representation of in-situ conditions.

This chapter discusses the numerical model setup in UDEC including geometries, boundary conditions, rock mass and joint properties, variable and assumed input parameters as well as the modelling methodology employed to establish a suitable understanding of the influence of a competent, weak layer on the depth of failure in the roof within variable stress regimes and where the limestone is of different bedding thicknesses.

8.1 Model Geometry

A numerical model representing the geometries of the anticipated room conditions in a limestone mine was developed using ranges of in-situ stress conditions and bedding thicknesses likely to be encountered within Southern Ontario limestone mines. Since UDEC is a two-dimensional software, and observed failures reported in the literature and seen by the author were most commonly running diagonally at four-way intersections, a cross-section of the diagonal of a room in a typical room and pillar limestone mine was modelled. The dimensions considered typical for these mines were based on the research published by Esterhuizen et al. (2007) which summarized the mining geometries of 34 limestone mines located within the Eastern and Midwestern United States (Table 11). Accordingly, a 22m wide by 12m high excavation was selected for this research, as shown in Figure 35.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Literature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
</tr>
<tr>
<td>Mining Height (m)</td>
<td>11.6</td>
</tr>
<tr>
<td>Intersection Diagonal (m)</td>
<td>21.7</td>
</tr>
</tbody>
</table>

The external boundary for the model was extended five times the room size away from the area of interest; i.e. 110m wide (5 times the room width, 22m) on each side of the room and 60m high (5 times the room height, 12m) below and above the room. At this distance from the excavation, external boundary effects impacting the stress distributions and resulting strain around the room will be negligible. Since limestone mines will be at depths greater than 60m and the in-situ horizontal stresses
are dominant over the gravity driven vertical stresses within the mining environment, it is not necessary to represent the ground surface distance from the excavation for the upper external boundary in the model.

Within the external boundary, the model has been separated into 3 distinct areas of mesh generation, as shown in Figure 35, so that computational demands can be managed within the model and a more detailed analysis can be conducted in areas of greater interest. For the purpose of this study, mesh discretization includes the number of discontinuities represented within a comparatively sized area and the element size within the block.

![Figure 35. Geometry and Boundary Dimensions for the Numerical Modelling](image)

The coarsest mesh corresponds to the area of least interest and provides a buffer zone between the external boundary and the area where stress redistribution is expected to occur (shown as “Outer Area” in Figure 35). Within the Outer Area, only bedding at a 2m spacing is modelled. The purpose of this was so that any regional stress redistribution along bedding planes as a result of the high horizontal stresses could occur. Since this is not an area of primary interest, the bedding spacing was kept constant at 2m for all simulations; 2m spacing is twice that of the largest bedding spacing modelled during the analysis.
The “Inner Area” as shown in Figure 35 is the area in which stress redistribution as a result of the in-situ stresses will influence the mechanisms around the excavation leading to failure. Within the Inner Area, the bedding spacing is varied according to the simulation being modelled. Secondly, a randomly spaced, semi-persistent sub-vertical joint set is modelled as would be expected in most sedimentary formations. It is also important to note that the Inner Area provides a transitional change from the complex geometry modelled in the immediate roof rock above the excavation and the Outer Area so that boundary effects remain negligible between the areas of high and low geometric complexity.

![Image of Voronoi Tessellation Regions](image)

**Figure 36. Voronoi Tessellation Regions**

The densest mesh corresponds to the area of greatest interest to the research and incorporates the immediate roof and haunches (shown as “Roof Voronoi” in Figure 35). To explicitly show intact rock failure within the strata “beams” and the depth of failure in the roof, this area has been modelled using both the Voronoi tessellation generator and a discontinuity set representing the bedding thickness. The Roof Voronoi area extends 11m above the roof of the room, 3m along the sidewalls and 5m wide of the sidewalls of the room. To ensure that artificial boundary effects would not be generated by having small Voronoi blocks adjacent to large tabular blocks, the Roof Voronoi area was partitioned into 7 regions.
(shown in Figure 36) in which the Voronoi edge lengths were gradually increased in size. Since the research is focused on high horizontal in-situ stress environments, a gradual transition was not considered necessary in the vertical direction.

Lastly, a shale layer of constant thickness was included at some distance into the roof for simulations involving a shale layer. The shale layer extended across all areas of the model (as shown in Figure 36) to ensure that no unnecessary boundary effects were introduced.

8.2 Boundary Conditions
A few controls were implemented on the external boundary of the model in order to simulate in-situ conditions as much as possible. Firstly, the base of the model was restricted such that it cannot move vertically. This was required so that when gravity was included in the model, the model remained in place. All other boundaries were free to move both horizontally and vertically. Secondly, the in-situ horizontal and vertical stresses were applied to the external boundary to match those applied to the elements within the model. Finally, although the vertical stress increase due to gravity was considered negligible compared to the high in-situ stresses, it was applied so that detached blocks in the roof would fall towards the excavation void, either bridging or collapsing into the opening.

The controls implemented on the model are shown in Figure 35: F – Fixed velocity boundary, O – Free to move boundary, S – applied stress boundary.

8.3 Voronoi Edge Length
From the micro-mechanical property calibration completed (discussed in Chapter 7), a Voronoi edge length equivalent to 4% of the beam length in the roof was selected to be used in the model. Since the micro-mechanical property calibration was carried out via a UCS test simulation which compresses the synthetic rock mass along its long axis, and the in-situ stress regime being modelled will compress the strata parallel to the bedding, the length of the beam above the excavation was considered a suitable normalizing dimension. Accordingly, for a span of 22m, the Voronoi edge length utilized was 0.88m.

Table 12. Selected Micro-Mechanical Properties for Voronoi Contacts

<table>
<thead>
<tr>
<th></th>
<th>Friction Angle, ( \phi ) (°)</th>
<th>Cohesion, ( c ) (MPa)</th>
<th>Tensile Strength, ( \sigma_T ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Calibrated Range of Properties</strong></td>
<td>45 - 55</td>
<td>80 - 110</td>
<td>25 - 30</td>
</tr>
<tr>
<td><strong>Selected Properties</strong></td>
<td>45</td>
<td>80</td>
<td>25</td>
</tr>
</tbody>
</table>
Table 12 shows the range in Voronoi contact properties suitable for representing intact rock mass values for the Ordovician limestone units of Southern Ontario as established from the micro-mechanical calibration.

8.4 Rock Mass and Joint Properties for Numerical Modelling

The limestone blocks in the Outer Area and Inner Area and shale layer were modelled as plastic solids with the ability to internally fail and have their movements governed by residual properties. The limestone blocks in the Roof Voronoi area, however, were modelled elastically because the Voronoi joints provide an explicit fracture plane for failure to occur in this rock mass. The elastic and plastic rock mass properties given to the limestone and shale blocks were selected as the average rock mass properties summarized from literature (discussed in Section 7.1) and are presented in Table 13. Residual cohesion and residual tensile strength values were considered 0 because cohesion and tensile strength usually provide no significant resistance to failure once they have been exceeded.

<table>
<thead>
<tr>
<th>Block Properties</th>
<th>UCS (MPa)</th>
<th>Density, ( \rho ) (kg/m(^3))</th>
<th>Young's Modulus, ( E ) (GPa)</th>
<th>Poisson's Ratio, ( v )</th>
<th>Bulk Modulus, ( K ) (GPa)</th>
<th>Shear Modulus, ( G ) (GPa)</th>
<th>Friction Angle, ( \phi )</th>
<th>Residual Friction Angle</th>
<th>Cohesion, ( c ) (MPa)</th>
<th>Residual Cohesion (MPa)</th>
<th>Tensile Strength, ( \sigma_T ) (MPa)</th>
<th>Residual Tensile Strength (MPa)</th>
<th>Normal Stiffness (GPa/m)</th>
<th>Shear Stiffness (GPa/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limestone</td>
<td>110</td>
<td>2680</td>
<td>36.0</td>
<td>0.19</td>
<td>19.4</td>
<td>15.1</td>
<td>51°</td>
<td>31°</td>
<td>2.16</td>
<td>0</td>
<td>-0.74</td>
<td>0</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Shale</td>
<td>32</td>
<td>2600</td>
<td>17.7</td>
<td>0.09</td>
<td>7.2</td>
<td>8.1</td>
<td>37°</td>
<td>31°</td>
<td>0.65</td>
<td>0</td>
<td>-0.12</td>
<td>0</td>
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<td>Joint Properties</td>
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<td>Bedding</td>
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<td>31°</td>
<td>31°</td>
<td>0.57</td>
<td>0</td>
<td>0.95</td>
<td>0</td>
<td>160</td>
<td>16</td>
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<td>45°</td>
<td>31°</td>
<td>0.57</td>
<td>0</td>
<td>0.95</td>
<td>0</td>
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<td>Micro-Mechanical</td>
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<td>Voronoi Contact</td>
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<tr>
<td>Limestone</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>45°</td>
<td>31°</td>
<td>80</td>
<td>0</td>
<td>25</td>
<td>0</td>
<td>160</td>
<td>16</td>
</tr>
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</table>

Itasca Consulting Group (2008) summarized a series of direct shear tests conducted on bedding planes of Ordovician limestones for the Ontario Power Generations’ Deep Geologic Repository for Low and Intermediate Level Waste project. Direct shear tests provide values of the friction angle and cohesion for the discontinuities tested. The bedding plane properties for the modelling were chosen to mimic the
Itasca reported friction angle and cohesion values. Based on these values, the associated tensile strength for the bedding planes was calculated using the Coulomb slip law,

$$\tau = c + \sigma_N \tan \phi$$

Where \(\tau\) - shear stress

\(c = \) cohesion

\(\sigma_N = \) normal stress (or negative tensile strength)

\(\phi = \) friction angle

Very little laboratory joint strength data was located for sub-vertical joint sets within the Ordovician limestone formations and therefore these discontinuities were given similar cohesion and tensile strength as the bedding planes. However based on the author’s experience, sub-vertical joints sets in these environments often have more irregular joint surfaces and therefore for modelling the joint friction angle was increased for this set to make these discontinuities more resistant to shear failure. The bedding plane and sub-vertical joint properties used for the simulations are presented in Table 13.

The lower bounds of the range of micro-mechanical properties considered reasonable to represent the intact properties of limestone were selected for the modelling so that an element of conservatism would exist in the results since there is no reasonable method to measure the factor of safety when the mode of failure is complex such as roof failures in stratified deposits under high horizontal stress conditions. Stiffness and residual strength properties were chosen the same as the other joint properties because once intact rock mass failure has occurred, failure resistance is primarily governed by the residual friction angle. The micro-mechanical Voronoi joint properties used for the simulations are presented in Table 13.

8.5 Variable Parameters

The in-situ stress ratio, bedding spacing within the limestone and the distance of the shale layer in the roof above the excavation were varied to represent the most probable range of mining conditions for Southern Ontario. The range in each parameter modelled as part of understanding the influence of the shale layer on the depth of failure in the roof are discussed in the following sections.

8.5.1 Stress Field

Within a horizontally dominated stress regime, there can be significant local variation in the in-situ stress ratio, \(k\) (horizontal stress, \(H\): vertical stress, \(V\)), as a result of local stress relief features or simply
changes in depth causing an increase in the vertical stress (Esterhuizen et al., 2007; Dolinar, 2003). Therefore, the research conducted simulated various stress regimes centred about the regional stress field discussed in Section 4.1. In recapitulation, within 200m of surface, the maximum horizontal stress averages 3.8 times the vertical stress. Below 200m depth, the average horizontal to vertical stress ratio halves to 2.0. Based on the geology of Southern Ontario, the Ordovician limestones with mining potential are situated between 100m and 300m below surface. Therefore, horizontal to vertical stress ratios, k, of 1H:1V, 2H:1V, 3H:1V and 4H:1V were simulated during the numerical modelling. A ratio of k=1H:1V was simulated for two reasons: 1) to compare results with those published in literature under isotropic stress conditions, and 2) to present a continuous understanding of the influence of the shale layer from isotropic (no dominant stress orientation) to anisotropic (horizontally dominant stress orientation) stress conditions.

The absolute value for the vertical stress used in the modelling was calculated for a mine situated at 150m depth and using a rock mass density of 2650kg/m³ for the overlying bedrock. The vertical stress was calculated using the formula,

\[ \sigma_V = \text{density} \times \text{gravity} \times \text{depth below surface} \]

A depth of 150m was chosen as the constant for stress calculations because literature suggests that stress ratios where the horizontal stress is greater than 3 times the vertical stress exist primarily above 200m depth and the Bobcaygeon Formation (Ordovician limestone formation in the eastern part of Southern Ontario considered an economic source of aggregate using underground mining techniques for extraction) is situated approximately 150m below ground surface. Accordingly, this depth equated to calculated stress values considered reasonable for many anticipated mining conditions in Southern Ontario.

The absolute horizontal stress values were calculated based on the desired stress ratio for the model and the vertical stress value. The absolute stress values simulated for each stress ratio are shown in Table 14.

<table>
<thead>
<tr>
<th>Stress Ratio, k</th>
<th>Vertical Stress, ( \sigma_V ) (MPa)</th>
<th>Horizontal Stress, ( \sigma_H ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1H:1V</td>
<td>4.0 MPa</td>
<td>4.0 MPa</td>
</tr>
<tr>
<td>2H:1V</td>
<td>4.0 MPa</td>
<td>8.0 MPa</td>
</tr>
<tr>
<td>3H:1V</td>
<td>4.0 MPa</td>
<td>12.0 MPa</td>
</tr>
<tr>
<td>4H:1V</td>
<td>4.0 MPa</td>
<td>16.0 MPa</td>
</tr>
</tbody>
</table>
As a form of calibration, the calculated absolute stress values were compared with absolute values obtained by Haimson and Lee (1980) from in-situ stress measurements conducted in a deep borehole in Bowmanville, Ontario using hydraulic fracturing methods, where the stress ratio is between k=2.5 and k=3.5. Results of the in-situ measurements indicated that the maximum horizontal stress within the Ordovician limestone formations varied between 10.6 and 15.4MPa, which is in good agreement for the respective stress ratios simulated.

8.5.2 Bedding Spacing
The bedding spacing in the strata above the excavation equals the thickness of beam in beam theory calculations when assessing beam deflection limits. As discussed in Section 5.2, beam theory states that a thicker beam has better flexural resistance than a narrow beam. Subsequently, a greater limestone bedding spacing is hypothesized to naturally reduce the influence that the shale layer has on the depth of failure in the roof.

Shale, on the other hand, has a bedding spacing which is often on the scale of millimeters; although a shale interbed may be thicker. However, as an interbed, shale does not form a competent beam that would provide any resistance to beam failure. Therefore, for the purpose of this research, the shale layer was modelled as a homogenous interbed without any internal bedding. Also, since the shale layer is relatively much weaker than the limestone and both literature and the author’s observations of these types of roof failures suggest that the entire shale interbed fails regardless of its thickness, the shale layer was modelled as having a constant thickness instead of a range of thicknesses.

In order to identify a suitable range of limestone bedding spacing and average thickness of shale interbed anticipated within the Ordovician limestone formations of Southern Ontario, a literature review was conducted. During the review, bedding spacing and interbed thickness were compiled into groups – very thin (0.1 – 5cm), thin (5 – 20cm), medium (20 – 50cm), thick (50 – 100cm), massive (100 – 300cm), very massive (>300 cm) – representing their respective dimensions. The results of the review are compiled in Figure 37.
Figure 37. Summary of Ordovician Limestone Bedding and Interbed Literature Review: a) Limestone Bedding Spacing, b) Shale Interbed Thickness
From the 107 limestone and 31 shale data, a range of bedding spacing and the average interbed thickness were selected for the numerical modelling simulations, as shown in Table 15. The range was selected by assuming the percentage of cases per formation having the same spacing/thickness correlates with the probability that the mining conditions in Southern Ontario will exhibit the same bedding spacings and interbed thickness.

Table 15. Bedding Spacing and Shale Layer Thickness for Numerical Modelling Simulations

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Limestone</strong></td>
<td></td>
</tr>
<tr>
<td>Bedding Spacings</td>
<td>0.1m, 0.2m, 0.5m, 1.0m</td>
</tr>
<tr>
<td><strong>Shale</strong></td>
<td></td>
</tr>
<tr>
<td>Average Interbed Thickness</td>
<td>0.2m</td>
</tr>
</tbody>
</table>

8.5.3 Distance of Shale Layer from Excavation Roof

Initially, baseline simulations were conducted without a shale layer to establish the minimum depths of failure for each stress regime and bedding spacing combination. Subsequent modelling then included a shale layer situated at varying distances above the roof of the excavation, starting at 2m above the excavation and increasing in 1m intervals to 8m above the excavation. A one meter interval for adjusting the shale layer between models was selected to match the largest bedding spacing modelled. Since the primary goal of the research is to understand how failure mechanisms change as the shale layer impacts the system and ultimately influences the depth of failure, as well as being cognizant of the limitations of the modelling, knowing the precise distance from the roof at which the shale layer ceases to influence the depth of failure is not necessary; or reasonably valid for that matter.

This research did not include numerical analysis of the influence the shale would have within 2m of the excavation because from the author’s experience, in high horizontal stress environments, the combination of blast damage with a weak layer in close proximity to the roof would result in excessive overbreak during excavation. Secondly, most mining operations utilize a mechanic scaler which would easily scale down loose slabs impacted by a weak layer within 2m of the roof. Finally, standard ground support for intersections involves a minimum of systematic bolting using 2.4m long rock bolts (and mesh for permanent travelways) which in most cases would be sufficient to retain any slabs remaining following excavation and scaling procedures.
8.6 Conditional Assumptions

Other environmental conditions were assumed negligible for the purpose of conducting this research because of two main reasons:

1) they are not mentioned in the literature as factors associated with high horizontal stress mining failure, or
2) they have such a strong influence on failure that they effectively change the mechanisms of failure preventing the influence of the shale layer from being detectable.

These conditions and their assumptions are discussed in the following sections.

8.6.1 Influence of Water

It is well-known that water has detrimental effects on the rock mass quality and joint conditions in sedimentary environments; particularly its ability to cause shales to swell and wash out. However, it is assumed that once mining has commenced that water would be controlled by drainage and pumping systems and therefore any flow transmitting through the roof strata would be minimal, if any, and have negligible impact on failure compared to the impacts of the high stress acting on the bedding planes.

Effects from changing air moisture content (humidity) are not negligible and they are discussed in detail in Section 9.5.

8.6.2 Pillar Stability

Although pillar stability is a concern for fully benched room and pillar limestone mining operations where horizontal stresses are less dominant (or anisotropically dominant), this aspect was not addressed in this research. The height of the excavation was chosen to align with the average room height from the survey conducted by Esterhuizen et al. (2007) of 34 aggregate mines in Eastern and Midwestern US. It is assumed that pillars of this height would remain competent over time and that the horizontally dominant stress regimes modelled mitigate pillar stress by concentrating the stress along the relatively horizontal bedding planes and creating stress shadows in the centre of the pillars.

8.7 Discretization of Discrete Blocks

The discrete blocks within the model were made deformable by discretizing them into zones using both the quadrilateral and triangular meshing function in UDEC. Movement of a zone relative to adjacent zones within a block will be calculated using finite-difference theory. The quadrilateral meshing function automatically generates diagonally opposed triangular zones in blocks with greater than 3 corners. Where possible, blocks were discretized using the quadrilateral meshing function because plastic flow
calculations are improved with this style of discretization (Itasca Consulting Group, Inc., 2015). Blocks which could not be discretized using the quadrilateral meshing function because they were defined by only 3 corners were discretized using the triangular meshing function. The triangular meshing function will work for all block shapes.

Regardless of the meshing function, an edge length characterizing the size of the discretized zone must be defined by the user. A fine mesh, created by using a small edge length, would allow for more detailed plastic yielding and deformation representation within the model. However, a finer mesh is more computationally demanding than a coarse mesh, created by using a larger edge length. Since the numerical model developed for this research employs the Voronoi tessellation generator resulting in computationally intense calculations, a coarse mesh that would properly discretize all blocks was chosen. To optimize the discretization within the model, coarser meshes were used at distal regions from the room excavation and finer meshes were used in the immediate vicinity of the excavation where movement is anticipated.

8.8 Establishing an Equilibrated Model

Prior to commencing any numerical simulations, the model must be brought to an initial equilibrium or static state under the boundary and in-situ stress conditions. In an effort to expedite this process, the model was first run through 30,000 cycles with all discrete blocks being modelled as elastic so that no permanent plastic deformations would be present before mining of the excavation was simulated. Secondly, the desired plastic properties were subsequently assigned to the discrete blocks after initialization (as discussed in Section 8.4) and the model was allowed to equilibrate for an additional 40,000 cycles.

After the combined 70,000 cycles, the number of unbalanced forces had exponentially decreased towards an asymptotic value considered negligible for modelling (as shown in Figure 38) and the displacement vectors depicting total displacement indicated that movements due to model initialization were less than 5mm (as shown in Figure 39). Accordingly, it was concluded that the initialized model had successfully reached an equilibrated state and that the model was suitable for continued use for conducting the required numerical simulations.
Figure 38. Unbalanced Forces Indicating Model Instability Negligible

Figure 39. Displacement Vectors Showing Movement Less Than 5mm (Within Acceptable Range for Model Equilibrium)
8.9 Validation Model against Current Literature

A final validation of the base model was conducted by simulating a homogenous limestone rock mass under isotropic in-situ stress conditions for a variation of bedding thicknesses (0.1m, 0.2m, 0.5m and 1.0m) and comparing the results to the minimum thickness of bedding predicted by Hutchinson and Diederichs (1996) in which stability would be achieved. Hutchinson and Diederichs (1996) prepared charts based on snap-thru (buckling) and crushing modes of failure which correlate thickness of lamination, rock mass modulus and UCS to a maximum stable span, as shown in Figure 40. Since the UCS of the limestone represented is 110MPa, under isotropic in-situ stress conditions, crushing failure should not occur according to Hutchinson and Diederichs (1996). However, for a span of 22m and the rock mass modulus of 36GPa – which remained constant throughout the research - the anticipated minimum bedding thickness for stability was back-calculated to be 0.67m; buckling failure is predicted when modelling thinner bedding thicknesses than this.
Figure 40. Span Charts for Preventing Buckling (Snap-Thru) and Crushing Failure (Hutchinson & Diederichs, 1996)
The equilibrated model under isotropic stress conditions showed failure in all models. For models where bedding spacing was less than 0.5m, multi-beam failure occurred to a maximum depth of failure of 0.4m. Where the bedding spacing was 0.5m or greater, failure did not occur. In all models, failure occurred by buckling (snap-thru) as shown in Figure 41. These results are considered consistent with Hutchinson and Diederichs (1996) failure chart.

Figure 41. Example of the Progression of Buckling Failure: (a) Failure in Motion, (b) Complete Failure.
CHAPTER 9  NUMERICAL MODELLING RESULTS

A total of 132 models representing various bedding thicknesses, in-situ stress regimes and distance of shale layer from the roof of the excavation were conducted to understand the influence that a weak layer has on the maximum depth of failure over rooms in room and pillar limestone mines.

This chapter presents the results of the numerical modelling and highlights observed trends in the data unique to each stress regime and bedding thickness. Also the mechanisms that govern failure as the shale layer influences the depth of failure are discussed.

The collected data for each of the 132 simulations is presented in Appendix A.

9.1 Base Case Models without a Shale Layer

Twenty simulations were completed for the range in bedding thicknesses and in-situ stress regimes modelled to act as the basis from which the shale layer models can be compared to. Figure 42 shows that depth of failure is under 1m except under unusually high horizontal stress conditions (k ≥ 3). All failures are less than 2m in depth.

![Figure 42. Base Case Simulations (without Shale Layer)](image)

The failure depths presented in Figure 42 align with the majority of failures observed in limestone mines. This is likely why the industry is having such difficulty understanding large roof falls;
conservatism in ground support design charts (including bolt length formulas) recommend ground support that will adequately support failures in all in-situ stress environments. For example, if ground support is designed using industry standard design curves published by Grimstad and Barton (1993), then for a conservative range in rock mass quality for the Ordovician limestone formations (poor to good rock), reinforcement between systematic bolting and 50 – 90mm of fibre reinforced shotcrete with bolting is recommended (Figure 43); these recommendations assume that the mine intersections modelled are permanent openings.

![Empirical Ground Support Design Chart](image)

**Figure 43. Empirical Ground Support Design Chart (Grimstad and Barton, 1993)**

Bolt length design in industry is guided by Barton et al. (1980) formula,

\[
\text{Bolt Length} = 2 + \frac{0.15(\text{span in m})}{\text{ESR}}
\]

For mine intersections of 22m span, the recommended bolt length is 4.1m, which is 2.1 beyond the greatest depth of failure. For practicality reasons, mines rarely use 4.1m long bolts due to challenges with installation and limited usefulness throughout the mining operation; they often use 2.4m or 3m long rock bolts and decrease the pattern spacing to make up for the shorter length. Therefore even systematic bolting with commonly used bolt lengths, provides adequate ground support in the vast majority of mining conditions. As such, large roof falls appear to be anomalies which are unpredictable
to the industry. However, when both stress and structure (interbedded weak layer) combined are analyzed, large roof falls are not as anomalous under certain mining conditions.

9.2 Shale Layer Models
A total of 112 simulations were carried out to observe the influence that a weak layer, such as an interbedded shale, has on the depth of failure as well as the failure mechanisms leading to failure. Figures 44 through 47 present the results of the modelling by stress regime and comparing the distance between the roof and the shale layer to the depth of failure. These graphs were further interpreted to identify the zone of shale influence and zone of no shale influence.

In general, the simulated data shows that beyond some critical distance that the shale layer is from the roof of the excavation, it does not influence the depth of failure. Also, as the bedding thickness decreases and stress ratio increased, the shale had a greater influence on the depth of failure.

![Figure 44. Modelling Results for k=1](image)

At a stress ratio of 1 (Figure 44), for the range in mining conditions modelled, the influence of shale is minimal and typical ground support design would have a high probability of preventing any large roof falls.

At a stress ratio of 2 (Figure 45), the influence of the shale becomes quite prominent for thinly bedded strata extending the depth of failure beyond the length of typical ground support installed in limestone.
mines. Under these stress conditions, a shale layer within 5m of the roof has potential to influence roof stability. Although, bedding thicknesses of 0.5m and 1m remain minimally influenced.

Figure 45. Modelling Results for k=2

Figure 46. Modeling Results for k=3
At a stress ratio of 3 (Figure 46), the entire range of bedding thicknesses modelled are influenced by the weak layer when the shale is less than 4m from the roof of the excavation. Thinly bedded strata can have failure depths exceeding 6m from the roof of the excavation if the conditions are right. Ground support design strategies need to change in these environments to mitigate large roof falls; thinly to moderately bedded strata will need to be supported with closely spaced cable bolts or a combination of cable bolts and shotcrete as opposed to systematic rock bolting.

Finally, at a stress ratio of 4 (Figure 47), the entire range of bedding is influenced by the weak layer when the shale is <5m from the roof. Thinly bedded strata can have failure depths exceeding 7m from the roof of the excavation. Ground support design strategies for all bedding thicknesses will need to change to mitigate large roof falls; a combination of cable bolts and shotcrete will likely need to be required as opposed to systematic rock bolting.

Figure 47. Modelling Results for k=4
9.2.1 Shale Layer Modelling Observations
Throughout the numerical modelling, a few general observations were made that were not intuitive and as such may have some significance in interpreting the data. Primarily, the depth of failure in the zone of no shale influence during the shale layer modelling only occasionally aligned with the depth of failure under the same mining conditions during the base case modelling; the depths of failure were similar but not identical. This observation provides an indication that even though the shale layer does not appear to have an influence on the depth of failure, there may indeed be some minor influence. From another perspective, the small variance (less than 0.20cm) between the depths of failure may simply be a result of model boundary conditions with the introduction of elements of differing properties through the area of interest.

Secondly, at the lower stress regimes, it was common for the depth of failure between bedding thicknesses of 0.1 and 0.2 to be similar, contrary to their significant differences at higher stress regimes. This could be an indication that the thickness of beam required to establish a compression arch is directly proportional to the in-situ stress regime; high stress regimes require greater thicknesses of bedding to achieve stability. If this observation has merit, then in-situ stress conditions will need to be incorporated into the mathematics defining compression arches (Diederichs and Kaiser, 1999) and defining stable beam thicknesses (Hutchinson and Diederichs, 1996).

9.3 Shale Influence Boundaries
A maximum depth of failure was interpreted for each stress regime and bedding thickness combination from the simulated data to develop a series of curves defining the boundary in which a shale layer significantly influences the depth of failure in the roof. These curves are shown in red in the Shale Layer Influence Chart (Figure 48) in addition to the curves representing the depth of failure curves when a shale layer does not influence the depth of failure (shown in purple).
This chart was developed as a risk management tool to assist geotechnical engineers with design of limestone mines under high horizontal stress conditions. For a given set of in-situ stress and strata structural conditions (bedding thickness), the geotechnical engineer can use this chart to determine the critical depth of competent limestone in which they must ensure exists over the roof of an intersection in order to mitigate the risk of large roof falls and minimize ground support costs. For example (as shown by the blue dashed lines in Figure 48), if the limestone bedding thickness is 0.5m and the in-situ stress ratio \( k \) is 3, then a shale layer within a distance of 5m from the roof will cause failure to the depth of the shale layer (i.e. if the shale layer is 4m from the roof than depth of failure would be 4m). If there is no shale layer within 5m, then the depth of failure expected will be approximately 1m. In order to minimize ground support requirements in this case, a geotechnical engineer would conduct investigation to determine if a weak shale interbed exists within 5m of the planned mining horizon, and if so, would make recommendations for adjustment to the mine plan or design a suitable ground support system for the conditions.
9.4 Mechanisms and Modes of Failure

The main failure in each mining condition simulated was classified based on the 4 primary failure modes in stratified rock (slabbing, buckling, crushing and diagonal fracturing). This data was gathered as justification to support or reject Stimpson and Ahmed’s (1992) conclusion that weaker layers influence the failure mechanisms within stratified rock masses. Furthermore, it will confirm the author’s observation from large roof falls in US limestone mines characterized by thinly to moderately bedded strata; in these environments, the author postulated that the failure profile resembled diagonal fracturing as opposed to a series of buckling or slabbing failures as current literature suggests should be the mechanisms of failure.

A summary of the main failure classification is provided in Table 16. These results strongly indicate that buckling failure is the dominant mode of failure when a shale layer does not have an influence on the failure. In all cases when a failure occurred when a shale layer was present but the failure did not extend to the shale layer, buckling was the main failure mode. Slabbing was only observed at low stress ratios (k = 0.5, 1) where a shale layer was not present. The most notable statistic though is that in all cases when failure extended to the shale layer, failure was by diagonal fracturing. This indicates that a weak layer inclusion in a rock mass not only influences the depth of failure in the roof but also influences the mechanisms that lead to failure, which fully supports Stimpson and Ahmed’s (1992) conclusions.

Table 16. Summary of Main Failure Mode Classification

<table>
<thead>
<tr>
<th>Condition</th>
<th>Slabbing</th>
<th>Buckling</th>
<th>Crushing</th>
<th>Diagonal Fracturing</th>
<th>No Failure</th>
<th>Total Simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>No shale layer</td>
<td>5</td>
<td>11</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>Failure extended to shale layer</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>42</td>
<td>0</td>
<td>42</td>
</tr>
<tr>
<td>Failure did not extend to shale layer</td>
<td>0</td>
<td>47</td>
<td>0</td>
<td>0</td>
<td>23</td>
<td>70</td>
</tr>
</tbody>
</table>

Examples of the dominant failure mode observed for the each condition are shown in Figure 49.
Figure 49. Examples of Failures Observed: (a) Failure to Shale Layer by Diagonal Fracturing, (b) Failure with No Shale Layer Present by Buckling, (c) Failure Not Extending to Shale Layer by Buckling
It should be noted that there could be a bias by using the Voronoi tessellation which results in buckling occurring as opposed to slabbing or crushing. Since UDEC does not allow for fracture generation through blocks and the Voronoi tessellation provides angular failure planes through intact rock, any crushing failures would present as buckling failures and some slabbing failures can present as diagonal fracturing failures. However, since slabbing and diagonal fracturing have distinctly opposite conditions under which their mechanisms develop, as shown in Figure 50, confusion between these modes of failure would only exist near their boundary (beam thickness to span ratio is moderate and stress conditions relative to the rock strength is also moderate).

![Diagram](image)

**Figure 50. Conditions Leading to Different Failure Modes in Stratified Rock Masses**

### 9.4.1 Shale Layer Influence on Stress Redistribution

The principal stress distribution around the room was observed to understand the role of stress redistribution on the mode of failure and the development of a stability arch. Intuitively, stress concentrations primarily occurred along bedding planes and the relatively weaker shale layer. In all cases, stability was achieved by developing an arch through multiple beams. However, between bedding planes, stress redistribution occurred differently depending on the mode of failure.

Figure 51 shows examples of different stress redistribution patterns observed in achieving a stable arch.
Figure 51. Stress Distribution around Room: (a) Stable Arch (No Shale Layer), (b) Shale Layer Failure with Stable Arch Attempting to Form Below Shale, (c) Shale Layer Failure
Figure 51a shows that when a stable arch developed over the excavation, stress vectors are shallowly angled. In contrast, Figure 51c shows that when failure extended to the shale layer, stress vectors were steeply angled from the abutments to the shale layer. Also, the stress vectors aligned with the steep failure profile observed during diagonal fracturing failures. It appears that the influence of the shale layer on stress redistribution is the reason that all failures extending to the shale layer failed by diagonal fracturing. Finally, Figure 51b shows conditions where a stable arch was attempting to form below the shale layer, but ultimately was unable to and failure occurred to the depth of the shale layer via diagonal fracturing. This stress distribution pattern was interpreted as failure conditions in which the shale layer was close to the critical distance from the roof in which the shale layer would not influence the depth of failure.

9.5 Limitations of Numerical Modelling Results

The major limitation of the research is that the modelling needed to be simplified to two dimensional analysis so that the Voronoi tessellation could be used to observe the failure mechanics that develop through intact rock. As a result any influences to stability in the third dimension, such as clamping effects that would be induced as a result of the secondary principal stress also being horizontal, would have been neglected during the study. As Agapito and Gilbride (2002) concluded in their research, anisotropic stress conditions, where the maximum horizontal stress was greater than 3 times the minimum horizontal stress, can result in roof failures depending on the orientations of the workings. In addition, they concluded that the shear strengths of the rock mass are often lower in anisotropic rock masses than isotropic ones. Although the measured stress regime in Southern Ontario indicates that the maximum to minimum horizontal stress ratio is approximately 2, there could be some merit in using three dimensional software to understand highly anisotropic horizontal stress conditions (maximum to minimum horizontal stress greater than 3) to confirm the conclusion of the two dimensional modelling.

Secondly, the influence of adjacent mine workings has not been captured in this research. Room and pillar mines have adjacent mine workings in every direction so these workings will control local stress concentrations and shadows. Both stress concentrations and stress shadows have potential to exacerbate the influence of the shale layer and reduce it.

Thirdly, water (including air moisture) and the element of time were not included within the modelling. Each simulation was run until unstable forces within the model became negligible. This is reasonable for the study performed. However, carbonate minerals are known to degrade through reactions with
water. Mine B visited by the author had reported increased roof falls during times of high humidity within the mine (summer months); so much that at one point they injected a water sealant into the rock mass over a critical intersection which was exhibiting signs of progressive roof failure to inhibit further failure. At the time of the site visit, they had concluded that it indeed slowed the development of failure. But the injection process had cost them over a million dollars to implement.

In addition, by not having a time reference for the modelled failures, the length of time between underground excavation and roof failure is unknown and therefore it is difficult to understand the risk of these roof falls. If a roof fall occurs during blasting, then the risk to people and equipment is minimal. There is just a financial implication with overbreak; this assumes that the roof stabilizes following the initial failure. But if the roof fall occurs 3 – 12 months later, then the risk that people and equipment are working under the unstable ground conditions is much higher.

It would be naïve to consciously omit the potential impacts of water and time, if computationally possible to include them in the modelling.

Finally, the research only focused on a narrow range of structural orientation and rock mass and joint properties. These input parameters have substantial influence on the performance of the model. As a result, the research conclusions are only valid for a specific rock mass environment. For example, if the bedding was dipping at 10 degrees, stress redistribution would occur differently in the roof and mechanisms of failure would in turn develop uniquely. Buckling failure is further encouraged in dipping strata so it can be anticipated that an even greater percentage of failures would be classified as buckling failure than were concluded in this research.
CHAPTER 10  CONCLUSIONS AND RECOMMENDATIONS

This chapter provides a final summary of the research conducted as well as outlines recommendations for future study.

10.1 Influence of a Weak Layer on the Depth of Failure in Stratified Rock within High Horizontal Stress Environments

Intuitively, high horizontal stresses are associated with clamping effects which result in greater stability with greater stress. However, this research shows that in stratified environments interbedded with a weak layer, there is greater instability with greater horizontal stress. This research concluded that the maximum depth of failure when a weak layer is present could increase as much as four times compared to that for a homogenous rock mass. The comparison in depth of failure from rock masses with and without the influence of a weak layer is summarized in Figure 48.

Another significant conclusion is that all failures observed where the shale layer has an influence on the depth of failure were via diagonal fracturing. This observation agrees with Stimpson and Ahmed’s (1992) conclusions from their laboratory and numerical study, which used discrete crack propagation finite element software. It appears that the shale layer having much weaker strength properties than the bedding planes between the limestones causes the roof between the excavation and the shale layer to act as one thick beam instead of multiple thinner beams. This often results in separation of the shale/limestone interface before separation of the limestone bedding planes and subsequent stress redistribution causing diagonal fracturing from the haunches to the shale layer causing failure.

Finally, in most cases, failure that extended to the shale layer truncated at the shale layer. It is possible that failure would progress beyond the shale when the shale layer is less than 2m from the roof because a compression arch providing stabilization in the new roof beam is less likely to be able to form over the remaining span.

Although not heavily discussed in this thesis, many of the research conclusions are dependent on a significant strength differential between the limestone and the shale (weak layer). The strength differential is the primary contributor to the large increase in depth of failure as well as the change in preferential failure mechanism to diagonal fracturing. The strength differential allows for unusually high stress concentrations to develop in the roof rock as well as it does not allow for stress transfer across the weak boundary.
10.2 Key Contributions

The key contribution from this research is the Shale Layer Influence Chart (Figure 48) which provides guidance for ground support and mine design. Also, it is the author’s hope that this dissertation presents justifiable reason to change the philosophy for ground support design in these mines under high horizontal conditions in the following ways:

1) There needs to be a greater importance put towards identifying the frequency of shale interbeds where the shale is significantly weaker relative to the limestone.
2) The range in bedding thickness within the limestone needs to be understood, as opposed to just using the rock mass classification used in empirical ground support charts.
3) Due to the depth of failure, cable bolts should be considered for intersections where the diagonal spans are greater as opposed to rock bolts because standard 2.4m or 3.0m rock bolts do not provide enough anchorage depth to support the weight of the slabs generated under these conditions.

Ultimately, the best approach to ground control would be to isolate the required depth of massive limestone such that a suitable compression arch can form and ground support requirements can be minimized. The Shale Layer Influence Chart (Figure 48) developed from the research can provide guidance for the thickness of competent beam that should be isolated to achieve this goal.

10.3 Possible Future Research

The conclusions of this research could be further explored to develop a greater understanding of the influence of a weak layer within high horizontal stress environments from a three-dimensional perspective as computing power increases, and to establish guidelines for ground support for underground aggregate mines with high in-situ stresses and interbedded weak layers. Finally the strength differential conditions between the shale interbed and limestone in order for weak layer influences to occur can be investigated. The following sections discuss these potential research directions.

10.3.1 Snaking Failures

In some aggregate mining operations in the US, snaking failures connecting multiple roof falls have formed suggesting that the failures simulated as part of this research can extend three dimensionally to distances of hundreds of meters. Figure 53 shows an example of a snaking failure which occurred as a series of failure over a 70 day period of time. The interesting part about these failures is that they progress opposite of the mining direction until they connect with a prior roof fall.
At this point, snaking failures have been attempted to be explained by simple three-dimensional stress redistribution models (Esterhuizen et al., 2008). However, since observations published in the literature and supported by the author concluded that many roof falls truncate at a weak layer such as a shale interbed, it may be possible that the lateral continuity of the weak layer allows for the migration of the roof failures over time as stresses do redistribute after a roof fall or a mining heading is extended. As such, there could be some merit in using three-dimensional discontinuous modelling software to back-analyze a snaking failure.

10.3.2 Ground Support Design Guidelines

From this research, the understanding of the increase in depth of failure as a result of the influence of a shale layer can be used to establish guidelines for ground support in mines subject to high horizontal stress conditions. Even though the results of this research are mostly conceptual, the trends relating to how the depth of failure could be used to provide guidance on the length and type of bolts that should be used in a ground support package for the mine at intersections, or the thickness of limestone that should be isolated between the roof and a known weak layer so that a less robust ground support
package is required. It should be noted that it is not just the length of bolt being optimized by isolating a competent beam of rock in the roof but also the robustness and frequency of ground support because with greater possible depths of failure comes the risk of larger volume slabs or wedges that need to be supported.

10.3.3 Strength Differential Requirements
The definition of a weak layer is not trivial. The relative difference in rock mass strength between the host rock and weak layer controls whether there is an increase in the depth of failure. If the strength differential is not beyond a critical threshold, then separation on the weak layer prior to intact rock failure is unlikely to occur. Consequently if separation of the bedding does not occur first, then failure will be in the form of a series of single beam buckling failures as opposed to a multi-beam diagonal failure. Therefore, a future research topic could be to investigate the critical strength differential required between the shale and limestone such that the influence of the weak layer on the depth of failure is important. From an engineering perspective, the merit in investigating the critical strength differential requirement is to understand under which rock mass conditions the interbeds need to be considered independently in the mine and ground support design.

An associated research topic may be to investigate the minimum thickness of shale required for it to perform as an independent weak layer. Intuitively, the strength differential requirement is likely an integral part of determining the minimum shale thickness requirements; a thin, very weak shale likely has the same influence as a thick moderately weak shale.
Bibliography


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Appendices

These appendices provide summary tables for each of the numerical models for both the calibration exercise and the shale layer influence modelling. The generic forms of the UDEC codes used to carry out the modelling are provided as well.

Appendix A  Summary of Roof Failure Numerical Model Results

This appendix provides a summary of the 132 numerical simulations of the varying combinations of in-situ stress, bedding thickness and distance the shale layer is from the roof of the excavation to assess the influence that a weak layer has on the depth of failure under horizontally dominant stress conditions. The conditions modelled, depth of failure and main mode of failure are provided in Table 17.

Table 17. Summary of Shale Layer Modelling Results

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Distance of Shale from Roof</th>
<th>Bedding Thickness (m)</th>
<th>Depth of Failure (m above roof)</th>
<th>Primary Mode of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stress Ratio = 0.5H:1V</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model_116</td>
<td>No shale</td>
<td>0.1</td>
<td>0.2</td>
<td>Slabbing</td>
</tr>
<tr>
<td>Model_115</td>
<td>No shale</td>
<td>0.2</td>
<td>0.2</td>
<td>Slabbing</td>
</tr>
<tr>
<td>Model_114</td>
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<td>0</td>
<td>No failure</td>
</tr>
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<td>1</td>
<td>0</td>
<td>No failure</td>
</tr>
<tr>
<td><strong>Stress Ratio = 1H:1V</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.4</td>
<td>Slabbing</td>
</tr>
<tr>
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<td>0.2</td>
<td>Slabbing</td>
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<td>No failure</td>
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<tr>
<td>Test ID</td>
<td>Distance of Shale from Roof</td>
<td>Bedding Thickness (m)</td>
<td>Depth of Failure (m above roof)</td>
<td>Primary Mode of Failure</td>
</tr>
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<td>----------------------------</td>
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<td>---------------------------------</td>
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**Stress Ratio = 3H:1V**

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| Model_109 | No shale | 0.2 | 1.3 | Buckling |
| Model_108 | No shale | 0.5 | 1   | Buckling |
| Model_2   | No shale | 1   | 1   | Buckling |
| Model_128 | 2         | 0.1 | 2   | Diagonal fracturing |
| Model_127 | 2         | 0.2 | 2   | Diagonal fracturing |
| Model_126 | 2         | 0.5 | 2   | Diagonal fracturing |
| Model_125 | 2         | 1   | 2   | Diagonal fracturing |
| Model_44  | 3         | 0.1 | 3   | Diagonal fracturing |
| Model_32  | 3         | 0.2 | 3   | Diagonal fracturing |
| Model_20  | 3         | 0.5 | 3   | Diagonal fracturing |
| Model_4   | 3         | 1   | 3   | Diagonal fracturing |
| Model_72  | 4         | 0.1 | 4   | Diagonal fracturing |
| Model_71  | 4         | 0.2 | 4   | Diagonal fracturing |
| Model_70  | 4         | 0.5 | 4   | Diagonal fracturing |
| Model_60  | 4         | 1   | 1   | Buckling |
| Model_48  | 5         | 0.1 | 5   | Diagonal fracturing |
| Model_36  | 5         | 0.2 | 5   | Diagonal fracturing |
| Model_24  | 5         | 0.5 | 2.5 | Buckling |
| Model_5   | 5         | 1   | 1   | Buckling |
| Model_59  | 6         | 0.1 | 6   | Diagonal fracturing |
| Model_58  | 6         | 0.2 | 2.8 | Buckling |
| Model_69  | 6         | 0.5 | 2.5 | Buckling |
| Model_68  | 6         | 1   | 1   | Buckling |
| Model_52  | 7         | 0.1 | 3   | Buckling |
| Model_40  | 7         | 0.2 | 2.8 | Buckling |
| Model_28  | 7         | 0.5 | 2.5 | Buckling |
| Model_16  | 7         | 1   | 1   | Buckling |
| Model_93  | 8         | 0.1 | 3   | Buckling |
| Model_92  | 8         | 0.2 | 2.8 | Buckling |
| Model_91  | 8         | 0.5 | 2.5 | Buckling |</p>
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Appendix B  Summary of UCS Test Numerical Modelling Results

This appendix provides a summary of the 77 UCS test models conducted to establish suitable micro-mechanical properties for the roof failure numerical study. The input parameters, peak strength, number of cycles to failure and tensile and shear failure initiation values are provided in Table 18.

Table 18. Summary of UCT Test Numerical Modelling Results

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<th>Joint Friction, φ (°)</th>
<th>Residual Joint Friction, φr (°)</th>
<th>Bulk Modulus, K (GPa)</th>
<th>Shear Modulus, G (GPa)</th>
<th>Max. Axial Stress/Peak Strength (MPa)</th>
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<th># of Joint Contacts</th>
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<td>Joint Cohesion, c (MPa)</td>
<td>Joint Tensile Strength, σ_t (MPa)</td>
<td>Joint Friction, φ (%)</td>
<td>Residual Joint Friction, φ_r (%)</td>
<td>Bulk Modulus, K (GPa)</td>
<td>Shear Modulus, G (GPa)</td>
<td>Max. Axial Stress/Peak Strength (MPa)</td>
<td># of Cycles to Failure</td>
<td># of Joint Contacts</td>
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<td>Shear Failure Initiation (% of strength)</td>
<td>Point when Shear Contacts exceed Tensile Contacts (% of strength)</td>
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Appendix C  Computer Code Generated Using the FISH Language

Coding using the FISH language within UDEC was required to adequately observe the displacements and stresses within the model and understand the mechanisms leading to failure. As part of this research, UCS test simulations were completed to ensure that micro-fracturing was simulated properly when using the Voronoi tessellation, and modelling of beam failure in the roof of classic room and pillar limestone mines in high horizontal stress environments was conducted to understand the impact of a weak layer on the depth of failure. This appendix discusses the codes and FISH language used during the research to simplify and automate the collection of data so that the numerical modelling was effective from both a time and computational perspective.

C.1 Roof Failure in Limestone Mine Simulation Code

In an effort to semi-automate the generation and re-generation of the model geometry, material properties (including the automatic calculation of equivalent Mohr-Coulomb properties from Hoek-Brown properties), voronoi regions and joint properties and locations of history points for quantifying deflections as variations in the bedding thickness, shale distance from the excavation and stress conditions were studied, a code was written using UDEC’s FISH language. The following datafile was called into UDEC once the input parameters were defined by the user under the section of code titled, “INPUT PARAMETERS” to carry out the basic numerical modelling for this research.

```
;------------------------------------------------------------------------
new
ro 0.001
set ovtol 0.1
;------------------------------------------------------------------------

;INPUT PARAMETERS

define inputs
h=12
w=22
inner_joint_spacing = 1
outer_joint_spacing = 2
beam_bedding_spacing = 1
beam_thickness = 3
shale_thickness = 0.2
impact_zone_above_shale = 8

;ROCK MASS PROPERTIES
```
;LIMESTONE
L_intact_UCS = 110e6
L_GSI = 70
L_mi = 10
L_Disturbance_Factor = 0.7
L_Modulus = 36.04e9
L_res_fric = 31
L_res_coh = 0
L_res_t = 0
L_dil = 0
L_poisson = 0.19
L_dens = 2.68E3
vor_jkn = 1.60e11
vor_jks = 1.60e10

;SHALE
S_intact_UCS = 30e6
S_GSI = 50
S_mi = 6
S_Disturbance_Factor = 0
S_Modulus = 17.65e9
S_res_fric = 31
S_res_coh = 0
S_res_t = 0
S_dil = 0
S_poisson = 0.09
S_dens = 2.6e6

;JOINT PROPERTIES
sh_stiff = 1.60e10
nor_stiff = 1.60e11
bed_fric = 31
bed_coh = 0.57e6
bed_tens = 0.95e6
bed_rfric = 31
vor_jfric = 45
vor_jcoh = 80e6
vor_jten = 25e6

;STRESSES
hstress = -12e6
vstress = -4e6

end
inputs
;-------------------------------------------------------------------------------------
;CALCULATIONS
define fish
  g1 = 7*h
  g2 = 7*w
  g3 = 3*w
  g4 = 3*h
  g5 = 4*w
  g6 = 4*h
  g7 = 3*w-3*h
  g8 = 4*w+3*h
  g9 = 1*h
  g10 = 6*h
  j1 = 5*inner_joint_spacing
  j2 = 1*inner_joint_spacing
  vor1 = g3 - 1.5*inner_joint_spacing
  vor2 = g5 + 1.5*inner_joint_spacing
  vor3 = g6 - 1*inner_joint_spacing
  vor4 = g3 - 3*inner_joint_spacing
  vor5 = g5 + 3*inner_joint_spacing
  vor6 = g6 - 3*inner_joint_spacing
  vor7 = g3 - 5*inner_joint_spacing
  vor8 = g5 + 5*inner_joint_spacing
end fish

;SHALE LAYER
define shale
  lower_shale_contact = g6+beam_thickness*inner_joint_spacing
  upper_shale_contact = lower_shale_contact+shale_thickness
  g19 = lower_shale_contact+impact_zone_above_shale*inner_joint_spacing
end shale

;GEOMETRY
bl 0.0,0.0,0.0,1 g1,1 g2,1 g2,0.0

;ROOM DEFINITION
cr g3,1 g4,1 g5,1

;INNER BOUNDARY DEFINITION
cr g7,1 g9,1 g7,1 g10,1

;JOINT REGION ID
jreg id 1 g7,1 g9,1 g7,1 g10,1 g8,1 g10,1 g8,1 g9,1
jreg id 2 vor7,1 g6,1 vor7,1 lower_shale_contact vor8,1 lower_shale_contact vor8,1 g6
jreg id 3 vor1,vor3 vor1,g19 vor2,g19 vor2,vor3
jreg id 4 vor7,lower_shale_contact vor7,g19 vor8,g19 vor8,lower_shale_contact
jreg id 5 vor4,vor3 vor4,g19 vor1,g19 vor1,vor3
jreg id 6 vor4,vor6 vor4,vor3 g3,vor3 g3,vor6
jreg id 7 vor2,vor3 vor2,g19 vor5,g19 vor5,vor3
jreg id 8 g5,vor6 g5,vor3 vor5,vor3 vor5,vor6
jreg id 9 vor7,vor6 vor7,g19 vor4,g19 vor4,vor6
jreg id 10 vor5,vor6 vor5,g19 vor8,g19 vor8,vor6
jreg id 11 g7,lower_shale_contact g7,upper_shale_contact g8,upper_shale_contact
g8,lower_shale_contact

;BEDDING DEFINITION
jset a 0 g 0 s outer_joint_spacing o 0,g6
jset a 0 g 0 s inner_joint_spacing o 0,g6 range jreg 1
jdelete

;RANDOM SUBVERTICAL JOINTS
jset a 90 5 g 1 j 2 s 3 2 t 2.9 o g7,g9 range jreg 1
jdelete

cr vor1,vor3 vor1,g19
cr vor2,vor3 vor2,g19
cr vor4,vor6 vor4,g19
cr vor5,vor6 vor5,g19
cr vor7,vor6 vor7,g19
cr vor8,vor6 vor8,g19

;VORONOI BEAM AND ABUTMENTS
vo e 0.88 i 1 ro 0.001 seed 850925 range jreg 3
vo e 1.2 i 1 ro 0.001 range jreg 5
vo e 1.2 i 1 ro 0.001 range jreg 6
vo e 1.2 i 1 ro 0.001 range jreg 7
vo e 1.2 i 1 ro 0.001 range jreg 8
vo e 2 i 1 ro 0.001 range jreg 9
vo e 2 i 1 ro 0.001 range jreg 10
jdelete

;SHALE LAYER
cr g7,upper_shale_contact g8,upper_shale_contact

;MONITORING JOINTS (ALL HAVE PROPERTIES OF INTACT ROCK)
jset a 0 g 0 s beam_bedding_spacing o vor7,g6 range jreg 2
jset a 0 g 0 s beam_bedding_spacing o vor7,lower_shale_contact range jreg 4

;-----------------------------------
;MESHING
gen quad 0.5 range jreg 3
gen edge 0.2 range jreg 3
gen quad 1 range jreg 5
gen edge .5 range jreg 5
gen quad 1 range jreg 6
gen edge .5 range jreg 6
gen quad 1 range jreg 7
gen edge .5 range jreg 7
gen quad 1 range jreg 8
gen edge .5 range jreg 8
gen quad 1.5 range jreg 9
gen edge .8 range jreg 9
gen quad 1.5 range jreg 10
gen edge .8 range jreg 10
gen quad 1.5 range jreg 1
gen edge .8 range jreg 1
gen quad 2
gen edge 1
gen edge

;---------------------------------------------------------------------------

;MATERIAL PROPERTIES

;ROCLAB PROPERTIES

define Roclab

;LIMESTONE
L_mb = L_mi*exp((float(L_GSI)-100)/(28-14*L_Disturbance_Factor))
L_s = exp((float(L_GSI)-100)/(9-3*L_Disturbance_Factor))
L_a = 0.5+(exp(-float(L_GSI)/15)-exp(-20/3))/6
L_sig_c = L_intact_UCS*L_s^L_a
L_RL_t = L_s*L_intact_UCS/L_mb
Temp1 = (L_mb+4*L_s-L_a*(L_mb-8*L_s))*(L_mb/4+L_s)^(-L_a-1)
L_sig_cm = L_intact_UCS*(Temp1)/(2*(1+L_a)*(2+L_a))
L_sig3max = L_sig_cm*0.47*(L_sig_cm/-hstress)^(-0.94)
L_sig3n = L_sig3max/L_intact_UCS
Temp2 = 6*L_a*L_mb*(L_s+L_mb*L_sig3n)^(-L_a-1)
Temp3 = (Temp2)/(2*(1+L_a)*(2+L_a)+6*L_a*L_mb*(L_s+L_mb*L_sig3n)^(-L_a-1))
L_RL_phi = asin(Temp3)*180/pi
Temp4 = (1+(6*L_a*L_mb*(L_s+L_mb*L_sig3n)^(-L_a-1)))/(1+L_a)*(2+L_a)
GAB = (1+2*L_a)*L_s+(1-L_a)*L_mb*L_sig3n
Temp5 = L_intact_UCS*(GAB)*(L_s+L_mb*L_sig3n)^(-L_a-1)
L_RL_coh = (Temp5)/((1+L_a)*(2+L_a)*sqrt(Temp4))
L_Bulk_Mod = L_Modulus/(3*(1-2*L_poisson))
L_Shear_Mod = L_Modulus/(2*(1+L_poisson))

;SHALE
S_mb = S_mi*exp((float(S_GSI)-100)/(28-14*S_Disturbance_Factor))
S_s = exp((float(S_GSI)-100)/(9-3*S_Disturbance_Factor))
S_a = 0.5+(exp(-float(S_GSI)/15)-exp(-20/3))/6
S_sig_c = S_intact_UCS*S_s*S_a
S_RS_t = S_s*S_intact_UCS/S_mb
Temp6 = (S_mb+4*S_s*S_a*(S_mb-8*S_s))*(S_mb/4+S_s)^((S_a-1)
S_sig_cm = S_intact_UCS*(Temp6)/(2*(1+S_a)*(2+S_a))
S_sig3max = S_sig_cm*0.47*(S_sig_cm/(hstress))^(-0.94)
S_sig3n = S_sig3max/S_intact_UCS
Temp7 = 6*S_a*S_mb*(S_s+S_mb*S_sig3n)^((S_a-1)
Temp8 = (Temp7)/(2*(1+S_a)*(2+S_a)+6*S_a*S_mb*(S_s+S_mb*S_sig3n)^((S_a-1)
S_RS_phi = asin(Temp8)*180/pi
Temp9 = (1+(6*S_a*S_mb*(S_s+S_mb*S_sig3n)^((S_a-1)))/((1+S_a)*(2+S_a))
SED = (1+2*S_a)*S_s+(1-S_a)*S_mb*S_sig3n
Temp10 = S_intact_UCS*(SED)*(S_s+S_mb*S_sig3n)^((S_a-1)
S_RS_coh = (Temp10)/((1+S_a)*(2+S_a)*sqrt(Temp9))
S_Bulk_Mod = S_Modulus/(3*(1-2*S_poisson))
S_Shear_Mod = S_Modulus/(2*(1+S_poisson))
end
Roclab

;ASSIGNING ELASTIC MATERIAL PROPERTIES TO ROCK MASS
group zone 'block'
zone model elastic density L_dens bulk L_Bulk_Mod sh L_Shear_Mod range group 'block'
group zone 'shale' range 7 8 lower_shale_contact upper_shale_contact
zone model elastic density S_dens bulk S_Bulk_Mod sh S_Shear_Mod range group 'shale'

;ASSIGNING JOINT PROPERTIES
group joint 'bedding'
joint model residual jks sh_stiff jkn nor_stiff jfric bed_fric jcoh bed_coh jrf bed_rfric jresc 0 jdil 0 jt
bed_tens jrt 0 range group 'bedding'
set jcondf residual jks sh_stiff jkn nor_stiff jfric L_res_fric jcoh L_res_coh jdil 0 jt L_res_t jrf L_res_fric

;ASSIGNING JOINT PROPERTIES TO VORONOI JOINTS EQUAL TO ROCK MASS PROPERTIES
group joint 'voronoi above excavation mass' range jreg 3 angle 5,175
joint model residual jks vor_jks jkn vor_jkn jfric vor_jfric jcoh vor_jcoh jrf vor_jten jrf L_res_fric jresc
L_res_coh jrt L_res_t range group 'voronoi above excavation mass'
group joint 'voronoi abumont mass 1' range jreg 5 angle 5,175
joint model residual jks vor_jks jkn vor_jkn jfric vor_jfric jcoh vor_jcoh jrt vor_jten jrf L_res_fric jresc
L_res_coh jrt L_res_t range group 'voronoi abumont mass 1'
group joint 'voronoi abumont mass 2' range jreg 6 angle 5,175
joint model residual jks vor_jks jkn vor_jkn jfric vor_jfric jcoh vor_jcoh jr jrf L_res_fric jresc
L_res_coh jrt L_res_t range group 'voronoi abumont mass 1'
group joint 'voronoi abumont mass 2' range jreg 7 angle 5,175
joint model residual jks vor_jks jkn vor_jkn jfric vor_jfric jcoh vor_jcoh jrf L_res_fric jresc
L_res_coh jrt L_res_t range group 'voronoi abumont mass 2'
group joint 'voronoi abumont mass 3' range jreg 8 angle 5,175
joint model residual jks vor_jks jkn vor_jkn jfric vor_jfric jcoh vor_jcoh jrf L_res_fric jresc
L_res_coh jrt L_res_t range group 'voronoi abumont mass 3'
group joint 'voronoi abumont mass 4' range jreg 9 angle 5,175
joint model residual jks vor_jks jkn vor_jkn jfric vor_jfric jcoh vor_jcoh jrf L_res_fric jresc
L_res_coh jrt L_res_t range group 'voronoi abumont mass 5'
group joint 'voronoi abumet mass 6' range jreg 10 angle 5,175
joint model residual jks jkn jfric jcoh jten jrf L_res_fric jresc
L_res_coh jrt L_res_t range group 'voronoi abumet mass 6'
group joint 'voronoi shale' range jreg 11
joint model residual jks jkn jfric jcoh jten jrf S_RS_phi S_RS_coh jten S_RS_t jrf 0 jresc 0 jrt 0
range group 'voronoi shale'

;---------------------------------------------------------------
;BOUNDARY CONDITIONS

set gravity=0.0 -9.81
define xx
g13 = g1-0.1
g14 = g1+0.1
g15 = g2-0.1
g16 = g2+0.1
end
xx
boundary stress hstress,0,0 range -0.1,0.1 0,g1
boundary stress hstress,0,0 range g15,g16 0,g1
boundary stress 0,0,vstress range 0,g2 g13,g14
boundary yvelocity 0 range 0,g2 -0.1,0.1
insitu stress hstress,0,vstress szz hstress

;------------------------------------------------------------------
;HISTORY POINTS

;DEFINITION OF HISTORY POINT IN BEAM CENTRE
define yy
g17 = 3.5*w-0.05
g_hist = g6+0.05
number_of_bedding = (lower_shale_contact-g6)/beam_bedding_spacing
loop n (1,number_of_bedding)
g18 = g_hist + (n-1)*beam_bedding_spacing
command
hist xdisp g17,g18
hist ydisp g17,g18
endcommand
end_loop
end
yy

;DEFINITION OF HISTORY POINT IN SHALE CENTRE
define history_shale
hist2 = lower_shale_contact+.01
dend
history_shale
hist ydisp g17,hist2
hist xdisp g17,hist2

;ARRAY OF HISTORY POINTS IN BEAM AND ABUTMENTS
define hist_array
horizontal_divisions = 10
vertical_divisions = 4
uppercontact = lower_shale_contact-0.05
lowercontact = g6+0.05
loop m (-1, horizontal_divisions)
    hist_pointx = g3+(m+1)*(w/horizontal_divisions)
loop n (2, vertical_divisions)
    hist_pointy = g6+(n-1)*(float(beam_thickness)/vertical_divisions)
command
    hist ydisp hist_pointx, hist_pointy
endcommand
end_loop
command
    hist ydisp hist_pointx, lowercontact
    hist ydisp hist_pointx, uppercontact
endcommand
end_loop
end
hist_array

;---------------------------------------------------------------------------------------------------------------------------------
;MODEL EQUILIBRIUM

cycle 30000

save 2015_Model_ID_elastic.sav

;---------------------------------------------------------------------------------------------------------------------------------
;PLASTIC PROPERTIES

zone model mohr density L_dens bulk L_Bulk_Mod sh L_Shear_Mod dil L_dil fric L_RL_phi coh L_RL_coh
ten L_RL_t range group 'block'
zone model elastic density L_dens bulk L_Bulk_Mod sh L_Shear_Mod range jreg 3
zone model mohr density S_dens bulk S_Bulk_Mod sh S_Shear_Mod dil S_dil fric S_RS_phi coh S_RS_coh
ten S_RS_t range group 'shale'

;---------------------------------------------------------------------------------------------------------------------------------
;MODEL EQUILIBRIUM 2

cycle 40000

save 2015_Model_ID_plastic.sav

;---------------------------------------------------------------------------------------------------------------------------------
;CHANGE PROPERTIES HERE IF NEEDBE (SEE PROPERTY CHANGE DAT.FILE)

;--------------------
;EXCAVATION OF ROOM

reset disp jdisp
delete range g3,g5 g4,g6

;--------------------
;STEPPING

cycle 100000
save 2015_Model_ID_100000.sav;

This stepping code would be defined to repeat as necessary by the User

;--------------------

C.2 UCS Test Simulation Code

The purpose of conducting the UCS test simulations was to determine suitable Voronoi joint properties for a defined Voronoi edge length such that the intact rock mass had an unconfined compressive strength comparable to those results measured during actual laboratory testing while ensuring that the failure through the simulated sample was initially tensile dominated and failure occurred at approximately 30 – 40% of the unconfined compressive strength. In order to validate the sample failure and properly understand the impacts of the failure mechanisms on the sample strength, it was imperative to produce an axial stress versus axial strain curve as the sample was loaded and ultimately failed. This was completed by using FISH language in UDEC to calculate the average axial stress acting across the compressing platen – sample interface and the corresponding axial strain at 500 cycle intervals. Since UDEC does not have an explicit time function in which the loading velocity can be calculated into axial strain, the axial strain corresponding to each 500 cycles was calculated by measuring the vertical displacement of the compressing platen and dividing it by the length of the sample. So that the measurement process could be automated during the simulation, FISH language was used to loop the loading procedure based on a 500 cycle loop and record the axial stress and axial strain values per loop in a table, which could be plotted following the completion of the simulation. The FISH language used to code the automated measurement of the axial stress/axial strain curve is presented in the datafile code below under “DEVELOPMENT OF THE AXIAL STRESS/AXIAL STRAIN CURVE”.


A similar loop sequence was carried out to observe the development of tensile and shear fractures during the failure of the simulated sample. In this loop, FISH language was used to measure the amount of shear and normal displacement on all Voronoi edge lengths in the model at 500 cycle increments. Displacements were considered tensile fractures in two ways: 1) normal displacement on the contact was greater than 0.1mm regardless of shear displacement along the same joint (as long as the shear displacement did not exceed the normal displacement, 2) normal displacement on the contact was greater than 0.1mm and shear displacement was less than 0.1mm. In doing so, the fractures in pure tensile failure at the cycle of measurement could be observed; this is of interest because proper simulation micro-fracture generation in tension using the Voronoi tessellation was the goal of this calibration exercise. For overall tensile fracture count, it was assumed that as long as the normal displacement exceeded the shear displacement then the fracture has a tensile origin. Contacts with shear displacements greater than the normal displacements were considered to have a shear origin. These fracture counts were recorded in tables with the corresponding axial stress for each 500 cycle measurement so that they could be plotted following the simulation to determine the percentage of peak unconfined compressive strength in which fracturing initiated and the dominant fracture initiation mechanism. The FISH language used to code the automated counting of shear and tensile fractures at 500 cycle increments during the UCS test simulation is presented in the datafile code below under “COUNTING SHEAR AND TENSILE FRACTURES”.

;--------------------------------------------------------------------------------------------------------------------------
new
ro 0.0001
set ovtol 0.01
;--------------------------------------------------------------------------------------------------------------------------

;GEOMETRY, VORONOI AND MESHING

bl -0.06505 -0.084 -0.06505 0.084 0.06505 0.084 0.06505 -0.084
cr -0.06505 0.064 0.06505 0.064
cr -0.06505 -0.064 0.06505 -0.064
cr -0.02505 -0.064 -0.02505 0.064
cr 0.02505 -0.064 0.02505 0.064
jdelete

delete bl range -0.05,-0.03 -0.01,0.01
delete bl range 0.03,0.05 -0.01,0.01
vo e 0.005 i 1 ro 0.0001 seed 050582 range -0.02505,0.02505 -0.064,0.064
gen quad 0.02 range -0.02505,0.02505 -0.064,0.064
gen edge 0.01 range -0.02505,0.02505 -0.064,0.064

table 1 -0.02505,-0.064 -0.02505,0.064 0.02505,0.064 0.02505,-0.064
;----------------------------------------------------------------------------------------------------------------------------------

;MATERIAL AND JOINT PROPERTIES

;ASSIGNING ELASTIC MATERIAL PROPERTIES TO ROCK MASS

group zone 'block' range -0.02505,0.02505 -0.064,0.064
zone model elastic density 2.65e3 bulk 3.056e10 sh 2.292e10 range group 'block'

;ASSIGNING ELASTIC MATERIAL PROPERTIES TO PLATENS

group 'platens' range outside table 1
gen quad 1 range group 'platens'
zone model elastic density 7.750e3 bulk 1.60e11 sh 7.9e10 range group 'platens'

;ASSIGNING JOINT PROPERTIES TO VORONOI JOINTS EQUAL TO ROCK MASS PROPERTIES

group joint 'rockmass'
joint model residual jks 1.60e11 jkn 1.60e12 jfric 50 jcoh 80e6 jt 25e6 jrf 31 jresc 0 jrt 0 range group 'rockmass'
set jcondf residual jks 1.60e11 jkn 1.60e12 jfric 31 jcoh 0 jdl 0 jt 0 jrf 31
;----------------------------------------------------------------------------------------------------------------------------------

;BOUNDARY CONDITIONS (FIX BOTTOM PLACE AND COMPRESS UPPER PLATEN)

bound yvel=-0.02 range bl 342
bound yvel=0.0 range bl 202
bound xvel=0.0 range group 'platens'
def temp
ntab = 1
end
temp

;AUTOSAVE FUNCTION
def autosave
name1 = 'UCS1_'+cycID+'.sav'
end

;DEVELOPMENT OF THE AXIAL STRESS/AXIAL STRAIN CURVE

;DEFINE STRESS-STRAIN CURVE
def slip_load
ntab = ntab + 1
tot_str = 0.0
n_z = 0.0
x_z = 0.0
loop n (1,23)
x_z = float(n)
Temp11 = -0.02505+x_z/455
iz = z_near(Temp11,0.064)
tot_str = tot_str+z_syy(iz)
n_z = n_z +1
endloop

;AXIAL STRAIN MEASUREMENT
agp_ydisp = gp_near(0,0.06401)
astrain_ydisp = gp_ydis(agp_ydisp)
ax_strain = -astrain_ydisp/0.128
;AXIAL STRESS MEASUREMENT
ave_astress = -tot_str/n_z

;RECORDING MEASUREMENTS
xtable (2,ntab) = ax_strain
ytable (2,ntab) = ave_astress
end

;COUNTING SHEAR AND TENSILE FRACTURES

define failurecount
	ten1_count = 0

ten2_count = 0
sh_count = 0

tot_count = 0
peak_strength = 138e6
Temp12 = contact_head
loop while Temp12 # 0

tot_count = tot_count + 1
if c_ndis(Temp12) > 1e-4

ten2_count = ten2_count + 1
if c_sdis(Temp12) < 1e-4

ten1_count = ten1_count + 1
endif
endif
if c_ndis(Temp12) < 1e-4
if c_sdis(Temp12) > 1e-4
sh_count = sh_count + 1
else
if c_sdis(Temp12) < -1e-4
sh_count = sh_count + 1
endif
endif
endif

norm_ten1 = float(ten1_count)/tot_count
norm_ten2 = float(ten2_count)/tot_count
norm_sh = float(sh_count)/tot_count
norm_astress = ave_astress/peak_strength

xtable (3,ntab) = norm_astress
ytable (3,ntab) = norm_ten1
xtable (4,ntab) = norm_astress
ytable (4,ntab) = norm_ten2
xtable (5,ntab) = norm_astress
ytable (5,ntab) = norm_sh

Temp12 = c_next(Temp12)
tbs = table_size(3)
endloop
end

table 3 (0,0)
table 4 (0,0)
table 5 (0,0)

;---------------------------------------------------------------------------------------------------------------

;APPLY LOAD
def loop_load
do
loop h (1,1700)
cyclID = h*500
command
cycle 500
endcommand
;---------------------------------------------------------------------------------------------------------------
do
loop sk1 (1,20)
if h = sk1*100 autosave
command
save name1
endcommand
endif
end_loop

slip_load
failurecount
end_loop
end

loop_load
;==================================================================================