Robust CT Scanning of Logs with Feature-Tailored Voxels

by

Edward J. Angus

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Abstract

The greatest cost in sawmills is contained in uncut logs. Significant increases in profit stand to be made if logs are properly processed so that the amount of defect and feature-free products is maximized. This has driven research into internal sensing such as computed tomography (CT) for use in sawmills. Traditional CT systems from the medical industry are ill-suited for industrial use and provide distorted reconstructions when used with less than perfect or partially incomplete data. Industrial CT systems in areas such as quality control have embraced so-called Algebraic Reconstruction Techniques (ART) for robustness to errant data, but these systems are difficult to provide reconstructions rapidly enough for sawmill use.

A system is proposed here for an ART scanning arrangement specific for log-scanning use. The image space’s voxel pattern is defined to reflect the geometry of a log’s internal features. This greatly reduces the number of unknowns without a loss of information and allows for faster reconstruction. The geometry of the voxels makes exact calculation and storage of the series solution basis fast and practical for multi-slice scans. Data scaling and normalization eliminate unnecessary voxels in the reconstruction. It also makes the reconstruction scheme tolerant of rigid body motion radially and circumferentially about the cone beam source. The voxel numbering scheme means that knot features are contained in voxels that are numerically close for quick registration.

A camera-based detector system was implemented to collect radiographs of logs. Logs were translated and rotated in an x-ray cone beam and their position was monitored by an optical encoder. The detector was activated at the proper intervals to yield radiographic data. The series solution basis for this helical movement was constructed. An iterative solver and selective low-pass filtering was found to provide good reconstruction results. Segmentation was implemented to demonstrate use of the reconstructions for lumber processing. Eccentric spiral motion tests demonstrated the effectiveness of the data scaling and normalization process. Results are provided that point towards efficient automated registration of features.
Preface

All the work presented here was carried out at the Renewable Resources Laboratory at the University of British Columbia’s Point Grey campus.

I was the principal investigator and was responsible for all major areas of data collection and analysis. Dr. Gary Schajer, my supervisor, was responsible for the concept of a feature-specific timber algebraic CT algorithm. Dr. Anthony An was responsible for the seminal construction of the experimental apparatus and control system, as well as the rotational algorithms. Iterative solvers from AIR Tools were developed by Christian Hansen of Denmark Technical University. Production of the helical reconstruction algorithm and its synthesis with modified smoothing iterative solvers was my own.
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Nomenclature

\( \alpha \) pixel-ray column inclination to \( xy \) plane

\( \beta \) material x-ray mass attenuation coefficient

\( \Delta \theta \) angular sampling spacing of Radon Transform

\( \Delta d \) ray spacing of Radon sampling

\( \lambda \) relaxation parameter of Kaczmarz iterate

\( C^n \) matrix column rotation operator by \( n \) places

\( D_n \) Derivative operator with respect to the \( nth \) variable

\( H_n \) Hilbert Transform operator with respect to the \( nth \) variable

\( R \) Radon transform operator

\( R^{-1} \) Inverse Radon Transform operator

\( \Psi \) x-ray cone beam angle

\( \rho (x) \) one dimensiona density function

\( \rho_j \) density function value of voxel \( j \)

\( \theta' \) coordinate of polar input to Radon Transform

\( \bar{\rho} \) density vector for reconstruction

\( \tilde{y}_{total} \) Radon Transform samples for all views

\( \tilde{y}_{view} \) Radon Transform samples for one \( \theta' \)
fractional scaler of innermost annular voxel
angle between each equal-angular pixel ray
diagonal matrix made up of column sums of \( G \)
distance from x-ray source to log center
diagonal matrix made up of row sums of \( G \)
distance from x-ray source to detector screen
radius of domain of the Radon Transform in the image space
polar Radon transform operand
Cartesian Radon transform operand
threshold function for selective smoothing
path length basis
path length intersection of ray \( i \) with voxel \( j \)
basis sub component from pixel-column intersection with a given slice
basis component from one view of multi-slice reconstruction
the component of the path length basis from one view in spiral reconstruction
basis matrix component from one view
attenuated x-ray signal intensity
unattenuated x-ray signal intensity
Radon space ordinate
Radon space abscissa
number of Radon Transform views
number of samples per Radon Transform view
$N_{annuli}$ number of annular voxel elements

$N_{elem}$ number of vertical detector elements on EMCCD

$N_{sectors}$ number of sector voxel elements

$N_{slices\_view}$ number of slices within the cone beam in one data capture

$N_{total}$ total number of slices reconstructed

$N_{views}$ number of views around log

$O$ Radon space origin

$P_1, P_2$ furthest z coordinate extent of cone beam on log

$p_i$ hyperplane described by the $ith$ row of $G$

$P_{ang,orm}$ normalized log pixel-ray radius

$P_{ang}$ number of pixel-rays in fan beam

$P_{col}$ number of pixel-ray columns

$R^n$ Hilbert Space of degree $n$

$r_i$ coefficients of the $ith$ row of $G$

$R_{log}$ physical diameter of the log

$R_{norm}$ normalized log radius

$R_{pix}$ number of pixel rays subtended by the log

$t'$ length along Radon space abscissa

$v_i$ voxel value of $ith$ index in smoothing cluster

$w$ smoothing weight for selective regularization

$W_D$ height of the detector screen

$W_z$ new active detector width
$y_i$ sample of Radon Transform

$z'$ coordinate along $K$

t slice thickness value
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I would like to thank first and foremost my supervisor Dr. Gary Schajer, whose perspective and countenance were always sure to steer my efforts in the right direction and with meaning. Also Dr. Anthony An, without whom none of the work here would be possible. Finally, thanks to my lab-mates for their support and suggestions, insights and skill which made the task at hand that much easier.
Dedication

For my parents, whose wisdom is appreciated more by the year.
Chapter 1

Introduction

1.1 Overview of log grading and scaling

The sawmill industry currently operates on an economic model in which significant funds are spent on the raw material for both softwood and hardwood operations. In any sawmill the most valuable asset is contained in the form of unprocessed material or felled logs, which often account for more than three-quarters of all expenses [1]. As such, there is a significant economic motivation to maximize the amount of profit extracted from each individual log by optimizing the type and quality of product made from it. In order to make the most of the raw material, sawmills today employ persons known as graders to both grade and scale the logs [2]. Grading refers to the process by which features of the wood such as size, shape, knots, cracks, etc. are taken into account in order to assign a specimen with a certain grade reflecting its overall quality. The process of scaling is then performed in which the quantity of said product able to be extracted from the log is determined, and another label applied to the sample. Oftentimes, the grade of the wood may reflect certain minimum standards for scaling as well i.e. a log meant for siding panels of a certain dimension must have some minimum diameter.

Clearly, the downside of visual inspection is the variability between graders and the inability to detect features and defects that are not visible or indicated on the surface before the first cut. Rot, voids, and internal knots (limbs lost earlier in the tree’s life and grown over) are all difficult to detect and may cause both the grade and scale of the log to be overestimated. This in turn will lead to improper processing of the sample and maximum value will not be realized. Incorrect scaling and grading necessitates increased investment in quality assurance and monitoring so that inferior product does not make its way out of the operation. The manipulation of logs
via roll out does not fit with the industrial process flow of the modern sawmill in which timely processing is of the utmost importance [1, 3].

1.2 Sensing technologies in sawmills

It is estimated that the accurate identification of features within logs and subsequent smart processing may increase lumber yield value upwards of 15% [4]. The application of various nondestructive sensing technologies in log grading and scaling has accelerated over the past thirty years, with many technologies being developed and applied. These run the gamut from ultrasonic sensors to a wide variety of electromagnetic wave based systems, with the most successful implementations those that are both mechanically and computationally robust [2]. The need for reliable and economic operation is paramount in the function of an industrial log scanner, and several technologies have developed. Furthermore, the technologies must operate quickly and determine the ideal product and/or orientation of a log in almost real time, as the flow of material on line may be as high as 3m/s and cannot be impeded or profit is lost [5].

The sensing technologies currently in use in modern sawmills for cut and uncut wood may broadly be classified into external technologies (benefiting mainly the scaling process of logs) as well as internal technologies by which further information regarding the interior of pieces is obtained. Technologies that can tell something about the piece before the first cut maximize the profit potential from the log and have the most promise. Of the external technologies, laser based topological scanners have been widely studied and are implemented in industry to dimension logs and scale them from their overall form. These scanners are able to identify externally protruding knots from which inner information may be inferred and cut or grading decisions made, and are the most common sensors currently in sawmills [6].

The internal sensing technologies that have been implemented include ultrasound, interferometry vibrational sensing, microwave, nuclear magnetic resonance, x-ray and more. Of these, x-ray, and particularly x-ray computed tomography, has shown the most promise in being effectively implemented for feature detection [2, 7, 8]. This is due to several factors, not least of which is fresh lumber’s x-ray mass attenuation qualities being close to that of the human body, allowing for much of the work done in the medical realm to be implemented towards an industrial sawmill scanner. X-rays are also relatively inexpensive and easy to generate, and thus are a logical choice for a sawmill technology that is not only effective but affordable as well [2]. Lastly, in the extreme industrial environment of a sawmill, x-ray is attractive as no physical contact need be made (unlike ultrasound in which various gels are implemented), it need not
be implemented near the log surface, and it is comparatively noise robust when compared to
techniques such as interferometry. X-ray does have downside in the form of power consumption
as well as real safety concerns requiring shielding in an industrial setting.

1.3 X-ray sensing in sawmills

Much work has been put into x-ray technologies for application in sawmills, with a general
emphasis on application to hardwood operations in North America and softwood products in
Europe. The area of x-ray log defect scanning may be broadly partitioned into single direction
x-ray scanning, multiple source-detector arrangements (in effect partial computed tomography),
as well as full-field computed tomography (CT) technologies. Single direction x-ray scanning
involves 2D radiography in which a single x-ray source and detector are used to obtain informa-
tion regarding the interior of the log, and is used in some sawmills today in conjunction with
laser surface scanning to assemble a representation of the piece’s overall form [7]. Figure 1.1
shows the scanning arrangement for this case as well as what a representative data capture may
look like.

![Figure 1.1: Linear Radiograph](image)

It’s clear that this arrangement, while being rapid and low cost, also has some drawbacks.
The inability to differentiate depth and the superposition of features make accurate measurement
difficult. This lack of clear demarcation of feature orientation has been the impetus for both the
multi-source and CT systems. In larger systems, such as shown in Figure 1.2, multiple sources
and detectors are used yielding additional information about the locations of knots and the
like [5, 9]. These systems have implemented up to eight source-detector pairings, together with
the consequent increased cost, energy consumption, and safety considerations of many x-ray
sources. Furthermore, they are often incapable of resolving all the needed details about the internal log geometry. Seger et al. produced a two source-detector pair system comprising of multiple 1D detectors arrayed together and oriented at $90^\circ$ with respect to each other about the log’s longitudinal axis. The reconstruction results of that system are displayed in Figure 1.2(b) and are represented by a transverse cross section of the log. It shows that although knots may be displayed clearly, the sapwood heartwood details are lost as would be any other circumferentially dominant feature.

The remedy for the limitations of the previous two systems has been available for some time in the form of computed tomography, as introduced by Hounsfield and Cormack in the late 1960’s and early 70’s. The technology has matured profoundly since its first implementations in 1971, almost entirely due to research in the medical sector. An x-ray source is revolved about a region in space so that a detector is always in view, a minimum number of views are acquired as dictated by features such as the x-ray beam profile and reconstruction algorithm being used, and an inverse problem is solved so that a full reconstruction of the density variations is obtained. It is CT that has generated the most interest in the forestry research community for its 3-D capabilities. Figure 1.3(a) demonstrates this relative motion of the detector and source, and a CT reconstruction of the same transverse slice of 1.2 [5]. Here the heartwood sapwood boundary is clearly visible.
Still, there are many barriers-to-entry of CT technology to the conditions present and operational demands within a sawmill. Medical inspired log CT scanners require a great deal of precision and accuracy in reconstruction that causes them to be too slow for implementation in a real industrial line setting, too delicate for continued service (near continuous operation is a reality for most major mills), and far too costly for many small-medium sawmills to purchase [1, 6]. These considerations are not serious impediments in the medical setting, but the lumber industry is still waiting on a product that can fulfill these demands.

1.4 Motivation

Rotating source-detector gantry CT sawmill scanners developed from medical-style CT scanners are challenging for sawmill use. In these machines, the rotating x-ray source or rotating source-detector assembly maintains power and data transfer via complex transmission rings, which will require attention for continuous operation. They are subject to corruption of the reconstruction due to movement of the log within the scan area that may ensue from the relatively uncontrolled environment of an industrial sawmill. Although capable of reconstructions with ~1mm resolution, they struggle to meet the time requirement for implementation in a sawmill (estimated to be one minute to reconstruct a four meter long log buck) [7]. Furthermore, current offerings are large, standalone systems that are capital intensive and do not present much flexibility for incorporation into current lines without a large amount of production line reorganization.

With these factors in mind, it is the intention of this work to lead toward a computed tomography system that is well suited for use in sawmill-specific applications. It will accomplish this by being mechanically and computationally simple, robust, as well as devised with cost in mind. The ideal product will feature some immunity to perturbations of the log during scan time.
and deliver data packaged to make fast decisions on processing and cutting. Lastly, a premium will be put on the system’s adaptability to different sawmill environments and variations in raw material geometry, as it will require retro-fitting into existing mills.

There are several key differences in logs as a reconstruction object from other conventionally scanned bodies. First, because a log is a non-living rigid body, it may be manipulated in ways not possible for medical CT applications. The idea to rotate the log itself and not the CT equipment is a possibility. The log will not be harmed by higher ionizing radiation levels, which makes algorithms that sacrifice reconstruction speed for low dosage not necessary. Lastly, much is known about the way in which logs of a given tree species grow. This means logs’ overall shapes and the forms of their features don’t change much overall. That fact may be exploited to make for faster CT reconstruction. The work here will present a CT algorithm that is specifically constructed for scanning softwood logs based off of individual log’s commonality that will use this idea. It is first necessary, however, to introduce the fundamentals of how computed tomography works.
Chapter 2

Computed tomography overview

2.1 Basics of computed tomography

2.1.1 Computed tomography setup

Before formally introducing the computed tomographic reconstruction problem and its solutions, it is helpful to give a basic introduction. The purpose of CT is to determine the spatial locations of features inside a body by using the penetrative power of x-rays. A typical measurement system comprises an x-ray source, an object to be reconstructed, and an x-ray detector. The x-ray detector measures the x-ray intensity. The x-rays are partially absorbed when they pass through denser objects within the body, so in effect a 'shadow' is cast on the detector at those regions. The spatially distributed nature of the detector means that the relative positions of these shadows within the object are known. Figure 2.1 shows this by plotting the spatial signal of the detector looking at body A from the side. As can be seen, the detector picks up on the presence of the two objects B and C, but is unable to determine that there are two distinct bodies. It is necessary for another view, shown in Figure 2.2 to be taken to distinguish their positions. The rotation of body A means both objects are seen and block the rays. If the process is repeated, and the object rotated in the detector field for a number of intervals, the interior of object A may be completely reconstructed.

The example shows that the arrangement is capable of finding the location of the interior objects, but does not explain how the densities of these objects and the 'shadow' are correlated. This connection is found in the Beer-Lambert law or Beer's law from classical optics, and is a fundamental building block of CT.
2.1.2 Beer’s law

When an x-ray beam encounters an object, it is partially absorbed. The Beer-Lambert law states that the ratio of the attenuated power intensity of a ray to that of the un-attenuated ray through the same space is equal to the exponent of the line integral of density along the
path divided by an attenuation coefficient $\beta$ intrinsic to the attenuating substance. This is represented in equation form by 2.1.

$$\frac{I}{I_0} = \exp\left(-\int \frac{\rho(x)}{\beta} dx\right)$$  \hfill (2.1)

Figure 2.3: Beer-Lambert Law

Figure 2.3 illustrates this relationship for a single slab of attenuating material with the properties $\rho$, $\beta$. The line integral of the density through the slab is what is of interest here, so that the density of the material may be found. In order to find this, Equation 2.1 is solved for this quantity by taking the natural logarithm of both sides and rearranging to yield Equation 2.2. Note that in this case, the coordinate $x$ is referring to the distance along the ray from its point of intersection in the direction of propagation.

$$\int \rho(x) dx = -\beta \ln \left(\frac{I}{I_0}\right)$$  \hfill (2.2)

It is the energy intensity of the rays that is measured by the detector. The constant $\beta$ may be determined by calibrating the system with a substance of known density and dimensions. Thus by taking a radiograph of the object in question to get $I$ and an un-attenuated 'reference' image to obtain $I_0$, an approximation of area density of the object is readily determined. This information can then be used to reconstruct the form of the object, as will be shown.

2.2 Overview of CT terminology

Computed tomographic reconstruction machines may be broadly classified based on the geometry of the x-ray source and construction of the detector assembly, as well as the relative motion
of the assembly to the object and the dimensional aspect of the detector used. When the terms “third” and “fourth generation” CT are used, it refers to the physical arrangement of the x-ray source with respect to the detector. A fourth generation assembly features a rotating source within a static detector ring, while a third generation scanner has both source and detector rotating together (Figure 2.4). For a third generation machine, the detector may be either flat (termed a ‘flat panel’ detector) or curved as shown.

![Figure 2.4: Typical CT measurement geometries](a) third generation (b) fourth generation

The earliest x-ray scanners used non-divergent x-rays and were called “parallel beam”. In both scanner arrangements of Fig. 2.4, the x-rays emanating from the source are divergent and form a “fan beam”. If the detector is multi-slice, a “cone beam” is made and the machine is called “7th generation”. The movement of the object being scanned along the transverse axis of the source trajectory indicates whether the reconstruction is rotational or helical (also known as “spiral”) CT. In rotational CT, there is no translation of the object, while helical features translation. For cases in which the speed of the scan is a factor, helical cone beam CT is the clear choice for quick data acquisition. The algorithms change for rotational vs. helical, single vs. multiple slice reconstruction, but broadly fall into either analytical techniques related to filtered backprojection (FBP) or series expansion techniques commonly called algebraic reconstruction techniques (ART) in the literature. Figure 2.5 contains three illustrations of the CT arrangements used in this research. Figure 2.5 (a) shows a single slice fan beam flat panel 3rd generation measurement system, the fan beam results from only one line of detectors being active. With all detectors active, the system becomes a 7th generation multi-slice cone beam scanner as shown in 2.5 (b). Finally, introducing log translation relative to the detector yields the 7th generation helical cone beam scanner arrangement of 2.5 (c).
2.3 The formal CT reconstruction problem and solution

2.3.1 The Radon transform and problem statement

This discussion will consider the case of parallel rays, as it is the most straightforward and the concepts can be applied to more advanced cases found in other references. The fundamental computed tomography inversion problem for x-ray attenuation has its foundation in what is known as the Radon Transform, introduced by Johann Radon at the beginning of the last century. Consider a function \( f(x, y) \) that represents some attribute about which more specific information is desired e.g. density in the case of standard x-ray computed tomography (Figure 2.6). It is the case that the spatial (\( x \) and \( y \) correspondence) form of the function is wholly unknown, and must be determined from information contained within its in-plane 'views'. In order to define these views, the function is first represented in polar coordinates as \( f(r, \phi) \), in which \( r = \sqrt{x^2 + y^2} \) and \( \phi = \arctan\left(\frac{y}{x}\right) \), \( x = r \cos \phi \) and \( y = r \sin \phi \). The density at a given point \((r', \phi')\) is then defined as \( f(r', \phi') \).
Consider now the line $L = \theta'$ in the plane. The angle $\theta'$ may be thought to correspond to a single view of the function $f$. Looking at Figure 2.7, it can be seen that a new coordinate system $(t, z)$ may be defined corresponding to this view, with $t$ denoting the length coordinate along $L$ from the origin to some orthogonal line $K$ (the intersection labeled as point $P$) and $z$ the length coordinate along $K$. 

Figure 2.6: Distributed density function

$\text{Figure 2.6: Distributed density function}$
These new arguments \((t, \theta)\) of the Radon transform are related to the coordinates \(r\) and \(\phi\) by the relation of Equation 2.3. It maps each point in \(\mathbb{R}^2\) into the appropriate Radon coordinate \(t\) for \(0 \leq \theta \leq 2\pi\). Figure 2.7 shows how for a given point \((r, \phi)\), the coordinate \(t'\) corresponds to \(K\) for selected view \(\theta = \theta'\), this in turn will determine the value \(z'\).

\[ t = r \cos (\theta - \phi) \quad (2.3) \]

Thus \(f(r, \phi)\) may be expressed \(f\left(\sqrt{t^2 + z^2}, \phi + \arctan \left(\frac{z}{t}\right)\right)\) and the Radon Transform denoted by the operator \(\mathcal{R}\) acting on \(f\), \(\mathcal{R}f(t, \theta)\) is given by Eqs. 2.4.

\[ \mathcal{R}f(t, \theta) = \int_{-\infty}^{\infty} f\left(\sqrt{t^2 + z^2}, \phi + \arctan \left(\frac{z}{t}\right)\right) dz \quad (2.4) \]

\[ \mathcal{R}f(0, \theta) = \int_{-\infty}^{\infty} f\left(z, \phi + \frac{\pi}{2}\right) dz \]

The physical interpretation of this formula is that for a single view, the Radon Transform yields the continuum of line integrals orthogonal to \(L\) (that is in the same direction as \(K\)) over the distributed function \(f(r, \phi)\). Figure 2.8 demonstrates this by plotting these line integrals from for the arbitrary density function in the plane for line \(L\).
The domain of the Radon Transform is limited by the physical detector size so that it is confined to a circle about the origin of radius $E$. The picture reconstruction area is thus traditionally held as the square of dimensions $\sqrt{2}E \times \sqrt{2}E$ inscribed within the circle. The domain may then be expressed as $-E \leq t \leq E$ and $0 \leq \theta \leq 2\pi$. It should be noted that the transform is periodic for one half-rotation for the case of parallel forward projected rays, and so $Rf(t,\theta) = Rf(t,\theta + \pi)$. The analytic form of the Radon Transform is continuous, but as with any physical system of a finite number of detectors the sampling is discrete. A total of up to $n$ rays at spacing $\Delta d = E/n$ may therefore be sampled per view for a total of $m$ views that are spaced $\Delta \theta$ apart. Figure 2.8 represents this by the discrete dots $y_i$ along the transform. The data from a single view $\vec{y}_{\text{view}} \in \mathbb{R}^n$ may then be concatenated with the samplings from all non-redundant views to form $\vec{y}_{\text{total}} \in \mathbb{R}^{m\times n}$. This leads to the formal problem statement:

Given the sampling $\vec{y}_{\text{total}}$, approximate $f(r,\phi)$

The actual samplings of the Radon Transform or the line integrals of $Rf(t,\theta)$ may be found using Beer’s law. Referring back to eq. 2.2, it is easily seen that the equation is simply stating the value of the Radon Transform for a given $t,\theta$ and that in this case the coordinate
is supplanted by the coordinate $z$ from Figure 2.7. As was presented in 2.2, the two most common methods of solving the problem are the backprojection family of methods and the series expansion methods.

### 2.3.2 Filtered backprojection overview

Understanding of the Filtered Back-Projection (FBP) method is important for differentiating and recognizing the advantages of the method of reconstruction described in this thesis compared with the industry standard, so a brief introduction is presented here. Exhaustive sources on analytical methods may be found by Natterer and Epstein [10, 11]. It has historically been the case that the medical community has relied on the filtered backprojection techniques for the majority of implementations, because it is computationally less burdensome than ART for the same reconstruction resolution.

In brief, the filtered backprojection techniques may be thought of as three separate steps. The first is the forward projection of rays through the image reconstruction circle as detailed previously. The Radon signal for each view is then convolved with a filter, often a ramp filter in the frequency domain. This filtered signal is then backprojected through the image space for the view, and the density function estimated at a point in $R^2$ by taking the summation of the backprojections from all pertinent views. Filtering is necessary in order to offset the 'blurring' effect resulting from the smearing that takes place during the backprojection. A graphical representation is presented here for visualization of the method.
Figure 2.9: Filtered Backprojection Steps Visualization

(a) Forward projection produces the radon transform (b) Radon transform is filtered (c) Filtered transform is back projected thru image space (d) & (e) Process is repeated for m views within $\pi$ to provide reconstruction

In the case of 2D beam geometries, this backprojection provides an analytically exact method for producing the inverse Radon Transform $R^{-1}$. The individual steps, presented without derivation, may be summarily described as first taking the derivative of the Radon transform (2.9 (a)) applied to $f(r,\phi)$ with respect to the first variable ($D$), then the Hilbert transform $H$ of this with respect to the first variable. This yields the filtered form of the forward projection, and it may be proved that this is equivalent to carrying out the aforementioned convolution (b). The backprojection operator $B$ of Equation 2.5 (shown for arbitrary function $q(r,\phi)$) may then be applied (c), and the entire operation multiplied by $-\left(\frac{1}{2\pi}\right)$ which is a step commonly referred to as normalization, which ensures that no single backprojected view is overrepresented. The operation is then repeated as $\theta'$ varies for $m$ views in steps (d) and (e).

$$Bq(r,\phi) = \int_0^\pi q(r \cos(\theta - \phi), \theta) \, d\theta \quad (2.5)$$

Altogether, this yields Equation 2.6. This states that for a function in the $xy$ plane that
is expressed in terms of its polar coordinates, the inverse Radon transform $R^{-1}$ may be determined such that $R^{-1}(Rf(r, \phi)) = f(r, \phi)$ and produce the desired estimation of the density function. These steps here are described in the spatial domain, but are usually executed in the Fourier domain in practice for computational convenience. Once the density distribution function $f(r, \phi)$ is determined for many $(r, \phi)$, it may be converted into the Cartesian coordinates $(x, y)$ of the pixel picture space $C \in R^2$.

$$B\mathcal{H}D q(r, \phi) = -\frac{1}{\pi} \int_\pi \int_{-\infty}^{\infty} \frac{\mathcal{P}(t, \theta)}{r \cos (\theta - \phi) - t} dtd\theta$$ (2.6)

The filtered backprojection method has historically been favored by the medical community for its ease of implementation and computational efficiency. As the reconstruction is essentially a summation of all backprojected views, easily calculable thanks in part to the simple natures of the Hilbert transform and backprojection formula, it is very computationally efficient. Discretization of the image space takes place ‘at the end’ of the method in the sense that the voxels (pixels of the image space with an associated thickness) are segmented and assigned value only after summation. It is therefore an attractive option for CT. It does have some limitations of flexibility in application, to be discussed shortly.

### 2.3.3 Series expansion methods

The series expansion or “algebraic” reconstruction technique is a well established method, indeed it was the method used by Hounsfield in the first generation scanner. It is simpler to conceptualize than the highly analytic FBP method. This method may be considered to ‘start’ with the discretization as opposed to ending with it as in the case of the filtered backprojection method. For the image space $C$ with an arbitrarily oriented ray through it, the Beer-Lambert law of equation 2.2 may be discretized so that the integrand on the left hand side may be represented as a summation of the segmented intersection path lengths $g_{ij}$ of ray $i$ with voxels $j = 1, 2, ... n^2$ multiplied by the densities $\rho_j$ of each respective voxel. This discretized version of 2.2 may then be written as equation 2.7. Here, $y_i$ is again the sampled Radon Transform of Figure 2.8. Figure 2.10 shows the ray passing through the reconstruction space, with its intersection path length with voxel $j$ emphasized.

$$\sum_{j=1}^{n^2} g_{ij} \rho_j = y_i$$ (2.7)
This can be done for all rays detected in order to setup the system of linear equations presented in 2.8. The matrix $G$ contains the path lengths of all intersections of each ray with all the voxels in the image reconstruction space. The right hand side is now the Radon transform sampling for all rays, $\vec{y}_{\text{total}}$. This large system of linear equations correspondingly has many ways to go about “solving” or optimizing the system in order to estimate a form of the density vector $\vec{\rho}$ that best approximates the real form of the solution. Another way to look at it is that the column space of the matrix $G$ represents a linearly independent basis for the image space that $\vec{\rho}$ acts upon to best approximate the reconstructed cross section.

$$G\vec{\rho} = \vec{y}_{\text{total}} \quad (2.8)$$

In practice, the calculation of $G$ for all rays at the prescribed directions $\theta$ is no small feat, and a significant amount of cost in computing power is incurred. The way that this has been handled in the past is by simply approximating the basis functions in $G$ using different schemes to get as close to the actual path lengths as possible. The most rudimentary of these is simply setting the value of all intersected voxels in the path length matrix as the length of one side of a voxel in the reconstruction space. More sophisticated techniques have been proposed, that take into account the changing aspect of the voxels based off of the change in viewing angle and the ray’s inclination and spacing in the plane. It is from these non-exact bases that the technique gets it’s given “series expansion” name, as the density function is being approximated via the series representation.

Figure 2.10: Series expansion representation
expression of these non-exact bases. Exact methods have been found to be computationally expensive and slow to implement [12]. In all cases, it is the nature of $G$ that it is highly sparse, overdetermined, and diagonally dominant. Equation 2.8 has been solved in practice applying a wide variety of solution schemes, including direct solvers such as least squares, QR factorization, or singular value decomposition, as well as iterative methods ranging from classical techniques such as Gauss-Seidel to more recent approaches like GMRES or the conjugate gradient method. To ensure an optimization close to the true form of the density vector, the column space must be sufficiently well-conditioned to allow for practicable inversion. This consideration often plays a large part in the determination of the solving system used.

### 2.3.4 Comparing methods

What the series expansion method benefits from in conceptual simplicity, it loses out on in computational cost. In modern medical CT scanners where sub millimeter resolution is necessary for successful prognoses, the size of $G$ can grow considerably, and the inversion can take a prohibitively long time to implement in an industrial scanner. This is in large part why the FBP derived methods have proven so resilient as the industry standard. Still, the backprojection methods also have concerns. It is necessary for the Radon transform to be taken regularly within an interval $0$ to $\pi$, i.e., the algorithm is highly sensitive to only a limited number of views being available. If there are not sufficient views, the reconstructed image will exhibit heavy artifacts. This is not as much of an issue with ART, it is much more resilient in working with less than $180^\circ$ coverage [13]. This is not a great concern for single slice reconstruction, but will be significant for fast helical scanning. Also, FBP is more sensitive to noise in sampling data than Algebraic Reconstruction.

The series expansion methods also maintain an advantage in being easy to implement for any generalized source-detector geometry and configuration. This makes for a great deal of flexibility in defining the path length geometry as well as reconstruction geometry within the source-detector plane as will be seen later. In the case for filtered backprojection and its derivative analytic methods, new algorithms must be developed as the case is changed from parallel beam to divergent beam geometry, or if the detector shape changes (flat vs. curved). This is because the analytic form of the Radon Transform must be acquired to enable formulation of the backprojection operator. In ART, these changes are trivial and simple geometric adjustments may be done to enable Eq. 2.8 to still be used without major modification, so that the form of the Radon Transform need never be obtained. As the implementation of a log scanning system
into an existing production line necessitates a retrofitting that may take many forms in geometry and componentry, the robustness of ART’s adaptability is attractive [14]. Additionally, ART has shown promise in several studies at yielding not only accurate spatial feature detection, but also a superior ability to accurately find voxel density values when compared to analytic methods [14].

Both algorithms in conventional use are similar in that they are implemented with the image space \( C \) being referenced to an absolute physical space. That is to say, the algorithms reconstruct whatever passes through the reconstruction space in the radius \( E \) circle of Figure 2.8. As the material to be reconstructed moves through space, all the recorded Radon transform arguments \((r', \theta')\) correspond to the stationary world coordinates \((r, \phi)\) with the origin \( O \) in Figure 2.7 of both systems corresponding to the center of rotation of the source-detector pair. This allows the CT arrangement to be applicable to a great variety of body shapes and sizes, as anything may be constructed so long as it fits in the source-detector circle and does not move between views. Both algorithms have also been traditionally implemented with reconstruction space being defined to comprise a Cartesian grid. With a fine enough voxel pattern, this allows the algorithm to reconstruct irregular shapes to a high degree of accuracy, important in a medical setting.

2.4 Three dimensional reconstruction

Three dimensional CT, termed “7th generation” for both the pure rotational multi-slice and helical cases, may take one of two forms for both types of algorithm families. The first approach involves taking the two dimensional algorithms of the last section and applying them repeatedly in the transverse direction. This may be done by an actual translation “step” before rotation (effectively decoupling the translation and rotation from happening concurrently) or by translating and rotating simultaneously and interpolating the pitch of the source detector plane helix into single reconstruction planes periodically. This last technique, commonly referred to as 180LI or 360LI (LI here to designate “linear interpolation”) necessitates that pitch be tight enough for the interpolation along the helix to be usable. These algorithms are typically used with either single line detectors or two dimensional detectors with only a few lines arrayed together, as the two dimensional algorithms of Section 2.3 are only effective for small lateral angles. Multiple detector lines do help to relax the source-detector pitch required.

Clearly, the extended 2D algorithms are suboptimal for applications in which speed of reconstruction is a consideration, and the proliferation of wide-area x-ray detection technology
makes their use obsolete as they cannot function on these wide data sets. Also, the requirement for a pitch close to the size of a reconstructed slice puts significant mechanical limitations on the system in order to acquire data quickly. Fully volumetric formulations of the forward projection of lines for cone beam geometries (analogous to the Radon Transform of the previous section) and its inverse have been developed for both circular and helical source path cases. As is the situation in 2D reconstruction case, the 3D analytical projection-backprojection methods are computationally elegant, fast, and accurate when implemented correctly. Expectedly, the same problems are present as were in planar reconstruction. Limited number of views now causes artifacts in both the transverse and longitudinal directions, and changes in geometry or scanning scheme now require fundamental changes to the underlying backprojection mathematics [15]. This makes the algorithms highly suited to reconstructions in which the area being scanned and aspects of the reconstruction will be unaltered throughout service life and in different implementation.

The series expansion method for fully three dimensional reconstruction is fundamentally unchanged. The size of the path length matrix of eq. 2.8 is now much larger than before, but as long as a careful account of the geometry of the path lengths is kept, the inversion problem may be solved as was done previously. Thus a system implementing ART in volumetric reconstruction may be quickly adapted to a variety of detector shapes and ray-pixel binning arrangements without careful consideration of the effects on the quality of reconstruction. It is for this reason that the development of this work’s novel industrial scanner the simplicity of ART is best suited.

2.5 Summary

This chapter has briefly outlined the major ideas that have advanced CT in the past forty years. It is often the case that computational speed and robustness are at odds with accurate, clean reconstruction in an industrial setting. The work ahead aims to blend the best features of what has been discussed to achieve the present goals. The mechanical simplicity of using one detector and one source arrangement will be made feasible through manipulation of the log as opposed to the scanning hardware. The quick data acquisition of a multi-slice, helical cone beam setup will allow for fast reconstruction. Lastly, the noise resistant and simple nature of ART can be implemented rapidly with special considerations towards the shape of the features in the logs being scanned.
Chapter 3

The feature-tailed voxel approach

3.1 Overview of the method

3.1.1 Method description

The method proposed here is to implement a “7th generation” helical cone-beam flat panel CT scanner utilizing series expansion reconstruction to detect knots and features in logs. It will do this utilizing several key features that differentiate it from systems that have been widely implemented in the medical industry and make it specifically for use with logs. First, it implements an unconventional voxel pattern that closely follows the geometry of an actual log’s features of interest. It also uses a scheme to detect, normalize, and center detector data to allow for rigid-body motion in the reconstruction plane, as well as enabling fast basis construction and solving times. In order to work up to this helical scanner scheme, it is useful to first consider a single slice fan beam scanner and then graduate to a multi-slice rotational cone beam scanner. With these two cases established, the more complex helical scan can be developed.

3.1.2 Feature-tailed voxel geometries

Looking at the log cross section of Fig. 3.1, it is clear that the features affecting wood value in coniferous trees have some basic geometries. First, the logs are nominally circular in cross section and have very mild eccentricities compared to hardwood specimens [16]. This circular geometry is reflected in the form of the heartwood-sapwood boundary that separates the dense outer living part of the tree from the less dense mature heartwood. The knots are branches that grow outwards from the center of the tree over its lifetime. All knots start from the center of
the tree as the branches form successively at the upper part of the tree when it is still small. If a branch has been cut or broken off early during the tree’s life, the knot in that area will be grown over and become internalized. Thus it is never the case that a knot begins in the interior of the heartwood or sapwood and extends outwards without intersecting the center, but a knot may start at the center and be truncated in the interior.

![Medical grade CT reconstruction of softwood specimen from Szathmary](image)

Figure 3.1: Medical grade CT reconstruction of softwood specimen from Szathmary

The first and most obvious alteration to the standard reconstruction methods is to utilize the entirety of the reconstruction space as the image space. Instead of discarding the areas outside image space square, the image is simply defined to have a circular boundary of radius $E$. This means that there is no wasted space in the reconstruction as the circular image contains the log cross section entirely. The voxel geometries of the derivations of Sections 2.3.2 and 2.3.3 are Cartesian in both 2D and 3D with each voxel being square and cubic respectively. This means that it would take a large number of voxels to be able to reconstruct the shape of the heartwood sapwood boundary and knots within a log. If instead the voxels are concentric circles or a 'bulls-eye' arrangement, it takes only a few voxels to be able to demarcate the heartwood sapwood transition. A segmented circle pattern with sectors on the scale of the angular size of a knot means that relatively few sectors may accurately reconstruct the position of all knots. These arrangements are shown in Figure 3.2. The logical step is to use a superposition of both
patterns in order to get the benefits of both, yielding the "combined" geometry.

Figure 3.2: Voxel reconstruction cases

This polar voxel arrangement has been implemented and studied before in CT, by Jian et al. in the single slice case and Thibedau et al. for volumetric purely rotational reconstruction, but not for helical lumber CT which presents unique opportunities due to the way in which wood forms [17][18]. While retaining the same amount of feature resolution for a given log section, the spatial voxel resolution may be reduced by multiple orders of magnitude to make series expansion reconstruction practical, bringing all of its comparative advantages to bear to the system under consideration. Another big advantage is that the reconstructed image basis may be formed by calculating the path lengths of each ray’s intersection with the voxels. Because the voxel pattern is symmetric with respect to the origin, the basis calculation may be solved quickly for one view and then reused, as will be shown later.

3.1.3 Data binning, centering, and normalization

In order for the entire reconstruction space to be equal in size to the image space, the data collected must be manipulated so that no empty space is included in the reconstruction problem. This has the added benefit of making the system immune to small movement of the log in the detector view since the reconstruction space follows the log.
Consider the system shown in Figure 3.3, that of a flat panel 3rd generation fan beam scanner with a log in view. The distance from the x-ray source to the detector is taken to be $D_s$ and the distance from the source to the center of the log is taken to be $D_L$. The real, physical radius of the log in the fan beam arrangement is $R_{log}$ and the detector is taken to be of height $W_D$ divided into $N_{elem}$ detector elements. This detector subtends the fan beam at an angle of $2\Psi$. Each detector element may be thought of as corresponding to a “pixel-ray” that extends from the source straight to the detector. In the context of this research the detector elements are linearly spaced and present in a number in excess of what is needed by the voxel density pattern. It is therefore possible to re-bin the pixel rays that lie linearly along the detector into $P_{ang}$ pixel rays that intersect equal-angularly at an interval $d\Psi = 2\Psi/P_{ang}$ at the x-ray source. The equal-angularly spaced pixel rays are the arrangement that would be measured with a curved detector of radius $D_s$. The relation found in eq. 3.1 details the linear spacing of these equal-angular pixels along the flat detector, and the relation is shown in Figure 3.4. Binning the rays like this will allow for the correction of small displacements parallel to the detector as will be shown. This is important because the harsh nature of a sawmill makes it very difficult to manipulate and orient a log on the production line.
Before the log can be centered and the extraneous detectors eliminated, it is necessary to find the location of the center of the log in the plane and approximate the log’s size. The general scheme for locating the log center within the detector view is to first approximate the geometric center of the log by the centroid of its Radon Transform signal, then estimating the radius of the log by approximating its Radon signal as a semi-ellipse. This method uses all of the data and has been observed to be more robust in practice than edge detection methods, which involve just a few measurements near the outside surface. The method is best understood for the case of a uniform density log within the detector view. Consider the circle shown in figure 3.5 and its Radon Transform shown on the ordinate. Using the coordinates of 2.7, the density function bounded by the circle with radius $R_{\text{log}}$ and constant density $\rho'$ is eq. 3.1.3.

$$\rho (r, \phi) = \begin{cases} \rho' & r \leq R_{\text{pix}} \\ 0 & r > R_{\text{pix}} \end{cases}$$

(3.2)

For the view $\theta = 0$, it is clear that the parallel beam Radon Transform in this case is equal in
form to the equation of an ellipse as the line integrals parallel to the $y$ axis have the arc length values $z = 2 \sqrt{R_{\text{pix}}^2 - x^2}$. In this setup’s case this is an approximation as the non-parallel nature of the divergent fan-beam distorts the signal to be not exactly elliptical. Still, for small enough values of $\Psi$ the approximation holds well enough to give good results.

$$Rf(t,0) = \rho \left( \frac{y^2}{4R_{\text{pix}}^2} + \frac{x^2}{R_{\text{pix}}^2} - 1 \right) = 0$$

![Figure 3.5: Uniform density in a bounded circle and its Radon Transform](image)

The relation between the semi-ellipse’s centroid height and it’s major axis dimension is given by $C_y = \frac{4h}{3\pi}$, and its area by $A = \frac{\pi R_{\text{pix}}h}{2}$ from geometry. The signal may be thought of as series of pixel bins each containing a sample of the Radon Transform. Using the additive nature of second moment of area, the centroid may be determined as $C_y = \frac{\sum b_i^2}{2 \sum b_i}$; the area of course being $A = \sum b_i$. Manipulation of the equations leads to cancelation of $h$ and eq. 3.3, readily providing an estimate of the radius $R_{\text{pix}}$ of the log. This may then be related via simple geometry to an estimate of the log’s actual radius, $R_{\text{log}}$, by eq. 3.4.
Figure 3.6: Relating elliptical Radon Transform width to height

\[ R_{pix} = \frac{16 \left( \sum b_i \right)^2}{3\pi^2 \sum b_i^2} \]  \hspace{1cm} (3.3)

\[ R_{log} = D_L \sin \left( R_{pix} d\Psi \right) \]  \hspace{1cm} (3.4)

Figure 3.7 shows the log arrangement from before but now with a small vertical displacement of the cross section in the reconstruction plane. Accordingly, the measured Radon Transform is shifted on the detector as well by a small value \( e \). If the displacement is small, it can be approximated to be along the circumference of the circle of radius \( D_L \) with the source at its center. Because the pixel-rays are now equal-angular in nature, the Radon Transform of the log may be found in the detector field and “re-centered” simply by shifting the center by \( e = P_{ang}/2 - C_x \). In addition, the data outside of \( C_x \pm R_{pix} \) may be truncated, causing the signal to take up the entire reconstruction space.
This now leads to the consideration of horizontal motion as shown in Fig. 3.8. As the log moves closer to the source, its Radon projection will take up a greater amount of the detector area and vice versa for movement towards the detector. To accommodate this situation it is assumed that for small angles of the diverging x-ray beams, the path length basis column space will not change considerably. It is then possible to scale the truncated data to a normalized case for which the path lengths have already been computed for a user defined number of normalized angular pixels $P_{\text{ang\_norm}}$. This normalized case in turn has its own associated ‘real’ log, with a radius of $R_{\text{norm}}$. Since the basis weight (i.e. the Radon Transform sampling) has a direct relation to the linear size of the radius of the density function, expression 3.5 may be used to scale the data to the normalized case. The steps are shown in Figure 3.9. These steps are convenient in that they allow for a predetermined path length matrix to be used, so that no path lengths need to be calculated at runtime of the reconstruction inversion.
With the data conditioned so that the predetermined path length matrix may be used, the system of equations of eq. 2.8 may be constructed. The system of equations is highly overdetermined, with most of the work going into assembling the path length matrix $G$. The

$$\mathcal{R} f(\theta, t)_{\text{scaled}} = \mathcal{R} f(\theta, t) \cdot \frac{R_{\log}}{R_{\text{norm}}}$$

(3.5)
steps used to calculate $G$ change whether the reconstruction is single vs multiple slice, pure rotational vs helical, and depending on the voxel arrangement used.

### 3.2 Calculation of basis for a single slice

The calculation of the basis for each of the voxel arrangements follows a similar process. First, the path lengths for the intersection of each ray and each voxel must be determined and assembled into a path length matrix $G_{\text{view}}$. The matrix arrangement is determined by the numbering of the voxels and rays, with the voxel numbering patterns shown in Figs. 3.10-3.12 and the rays in the fan beam starting from the top, least to greatest as in Figure 3.4. The next step is to advance the voxel arrangement by a rotation $d\theta = 2\pi/N_{\text{sectors}}$. The rays in these additional views add rows to the path length matrix, and their intersections with the voxel pattern are concatenated onto the bottom of the previous view. This will be explored in more depth with each case.

#### 3.2.1 Path length basis function geometry by voxel arrangement view

The simplest case is the annular model. First, the annular voxel sizes must be determined in a standard way for a user chosen number of annuli, $N_{\text{annuli}}$. This is done by specifying a starting innermost voxel radius as a percentage $cR_{\text{norm}}$ of the normalized radius $R_{\text{norm}}$ of the standard log view. From there equation 3.6 may be used to ensure that the remaining annular voxels are equal in area, which is a convenient way to standardize the method for determining their sizes for a desired number. Intersection path lengths are determined by considering a ray $i$ intersecting with the annular voxel $j$. These intersections are shown in Fig. 3.10 for a given ray by the dots along the pixel-ray line. The path length of a given ray in a voxel may be calculated by calculating the path lengths within the outer circles and then subtracting the path lengths from within the inner circles. Doing this for all rays $P_{\text{ang}}$ in the fan beam for one view is necessary to establish the path length matrix for the first view’s worth of equations.
Figure 3.10: Annular Case (a) Pixel intersection (b) $G_{view}$ for annular case

$$R_j = \sqrt{(j-1) \frac{R_{norm}^2 (1-c^2)}{(N_{annuli} - 1)} + c^2 R_{norm}^2}$$ \hspace{1cm} (3.6)

For the sector model, the path length intersections of each ray in a view with the voxel boundaries must again be determined. The total number of sectors is constrained here to be even, as well as the number of ray pixels in the fan beam. This ensures that no pixel will fall along a voxel boundary and means that only half the path lengths must be calculated, and then their symmetric path lengths can be simply inserted into $G_{view}$. Figure 3.11 shows the intersection of an arbitrary ray with this voxel pattern. Note that the voxel numbering goes counterclockwise, with the first sector taken to be that one in the near vertical position. Calculation of the intersection path lengths is accomplished by defining a point $O$ on the ray that intersects the origin orthogonally. For each sector, the line comprising the leading edge of each sector is recorded, giving the intersection point $p_j$. The differences between the lengths $l_{ij}$ gives the entries into the path length matrix.
The combined model algorithm is expectedly a combination of the path length combinations of the previous two, with each ray now having many more intersections than was seen previously. The voxel indexing now starts from the center and goes outwards from the innermost near-top sector voxel and continues outwards in the counterclockwise direction. Now path lengths in inner voxels are determined by subtracting the path lengths of the smaller inner sectored circles from the outer larger ones. The voxel numbering means that the view matrix is a horizontal concatenation of grouped sector matrices, with each submatrix having \( N_{\text{annuli}} \) columns, or one column for each annulus.
3.2.2 Path length permutations by view

In the case of the annular voxel pattern, there is no shifting of voxels as the log rotates. This means that the path length matrix $G$ for $N_{views}$ is simply $G_{view}$ stacked on top of itself $N_{views}$ number of times (Figure 3.13).
The case of the sector model is more complicated. The nature of the rotation is shown in 3.14. After each data sample, the log is rotated away from the detector by one sector. This means that the path lengths that were determined to construct $G_{\text{view}}$ can still be used, but the indices of the values must change before the matrix can be concatenated on. The rotation in the physical log corresponds to a rotation of the columns of $G_{\text{view}}$ by one column to the left. This may be done for as many views as desired to fully constrain the system. This is shown in Figure 3.15, in which $C^n$ represents this matrix operation to the $n$th rotation. It has been shown by Turbell et al. that for a fanbeam pixel arrangement, the CT data becomes redundant after $\gamma = \pi + 2\Psi$ angular coverage [15]. More views than this do not introduce new information, but will serve to simply constrain the system more.
For the combined log model, it is the submatrices that are rotated, that is each additional concatenation has a rotation to the left by $N_{annuli}$ columns. This is shown in Figure 3.16. Again, only $\gamma$ views/sector rotations are needed to get a fully redundant data set. Still, more views do not hurt the reconstruction and are easily accommodated into the path length matrix.
for the single slice view as size is less of an issue and speed of scan is not yet a factor.

Figure 3.16: Combined single slice total basis

3.3 Calculation of the basis functions for multiple slices

3.3.1 Multi-slice basis function geometry

The case of multi-slice reconstruction in which path length calculations carried out on a 3D model instead of on a single plane is more complicated. The largest difference here is that now the pixel-rays may pass through more than one slice at a time, meaning that the slices are interconnected in the subsequent matrix inversion. This makes the inversion much larger and susceptible to instabilities.
There are now two dimensions that the pixel rays extend into, as shown in Figure 3.17. The coordinate space is arranged with the origin at the x-ray source and the x, y, and z axes as shown. The detector is now taken to be a 2D plane at a distance $D_s$ along the y axis. The total number of pixels $P_{view}$ in a given image capture is now equal to $P_{col}$ pixel columns in the z direction of the log and $P_{ang}$ pixels that are in the x direction. While the angular pixels are again cylindrically distributed, the longitudinal pictures are linearly spaced in the z direction, so that the pixel columns may be thought of as extending outwards like pages of an open book and form a pyramidal shape with the detector screen. The total number of log slices that are encountered by the cone beam in a view is taken to be $N_{slice\_view}$. The height of the detector is still $W_D$, but the z dimensional width is now $W_z$, which is less than or equal to $W_D$. This dimension is dependent on the reconstructed slice thickness $t$ and $N_{slice\_view}$. The dependency is shown in Figure 3.18. As can be seen, the cone-beam intersects the log (taken here to be a regular cylinder of radius $R_{log}$, estimated from a view as detailed in Section 3.1.3) with the furthest z coordinates being located at points $P_1$, $P_2$ on the detector side of the log. Using similar triangles, the value of $W_z$ may be determined. Note that for the extreme reconstructed slices on
either side of the log there are areas in which the cone beam will never touch, shown in red. The slices that contain untouched voxels are shown in light orange. These conical regions will not be included in the inversion, and must be kept track of in order to eliminate the corresponding voxels from eq. 2.8.

![Diagram of cone beam](image_url)

Figure 3.18: Voxels intersected in multi-slice reconstruction

The algorithm for the intersection of each pixel column with each slice makes use of the same process as the single slice case, but with some key differences in its operation. Symmetry is used by necessitating that that $P_{ang}$, $P_{long}$, and $N_{slice\_view}$ are all even so that only one ‘quadrant’ of path lengths must be calculated, and then mirrored across the x and z axes for the other paths. First, a logical matrix $G_{log}$ is constructed that records whether each pixel column intersects a given slice. If a slice is intersected, the parametric coefficients of the lines that correspond to the pixel-rays in that column are solved for so that the x and y coordinates of both entrances and exits of the rays with the slice are found (Fig. 3.19). These point’s coordinates are then stored in an array. The “block” in $G_{view}$ that corresponds to the pixel column’s intersection with the slice is then determined by calling the corresponding path length algorithm from the
previous section, now with an input that keeps track of the ray’s new entrance and exit into the slice and alters the path lengths accordingly. The ray’s inclination $\alpha$ with regards to the x-y plane is then found and the path lengths found previously are scaled by $G_{ij\_new} = G_{ij}/\cos \alpha$ to account for their oblique nature.

![Multiple slice ray entrance and exit](image)

Figure 3.19: Multiple slice ray entrance and exit

It is the case that the size of $G$ becomes prohibitively large in multi-slice reconstruction when stored in full. It is necessary then to utilize sparse matrix storage in order to minimize computational waste. This scheme is implemented with MATLAB 2013b sparse tools in this work. Here, all zero elements in the path length matrix are not stored, but only non-zero elements. The non-zero’s have three associated arrays, one with a registry of row indices, one with column indices, and another containing the path length in double precision. This cuts down greatly on the amount of storage required and keeps the program from unnecessary zero multiplications.

### 3.3.2 Permutations by view for multi-slice rotational reconstruction

The path length matrix permutations for purely rotational multi-slice reconstruction are similar to that of single slice. The main difference is that now the path length matrix $G_{view\_MS}$ for
one data capture is made up of numerous submatrices $G_{sub}$, one for every pixel column-slice intersection. Each $G_{sub}$ for the annular, sector, and combined cases is identical in size to the respective case of $G_{view}$ from the single slice cases in Figs. 3.13, 3.15, and 3.16 and is organized similarly. Figure 3.20 shows how the $G_{sub}$’s make up the total view matrix $G_{view\_MS}$. The value $N_{\text{voxels, slice}}$ is $N_{\text{annuli}}$ for the annular voxel case and $N_{\text{sector}}$, $N_{\text{sector}} \times N_{\text{annuli}}$ for sector and combined reconstruction respectively.

![Diagram](image)

Figure 3.20: Path length matrix for 3D view $G_{\text{view\_MS}}$

After $G_{\text{view\_MS}}$ is constructed, it may again be reused in such a way as to make assembly of $G$ quick and memory friendly. $G$ for the annular case is just a vertical repetition/concatenation of the $G_{\text{view\_MS}}$ matrix. The rotational operations of the single-slice sector and combined voxel cases are again used, but are now enacted on the sub-matrices $G_{sub}$ that make up $G_{\text{view\_MS}}$. 
For all voxel cases, the entire path length matrix is constructed as though every voxel in the \( N_{\text{slice}_\text{view}} \) set were intersected, paying no attention to those voxels that are within the red zone of 3.18. After it is assembled, the zero columns of \( G \) are truncated, as well as the corresponding voxels in the vector \( \vec{\rho} \). Those intersected voxels in the incomplete slices (the orange slices of Fig. 3.18) are included in the system inversion but are discarded afterwards.

\[
G_{\text{view}_{\text{MS}}} \quad \text{Concatenate for } k = N_{\text{views}}
\]

\[
G_{\text{view}_{\text{MS}}} \quad k = N_{\text{views}}
\]

\[
rot_k = \begin{cases} 
0 & \text{annular} \\
k & \text{sector} \\
kN_{\text{annul}} & \text{combined}
\end{cases} \quad k = \{0, 1, \ldots, N_{\text{views}}\}
\]

\[
m = P_{\text{ang}} \times P_{\text{col}} \times N_{\text{views}}
\]

\[
n = N_{\text{slices}_\text{view}} \times N_{\text{voxels}_\text{slice}}
\]

Figure 3.21: Multi-slice rotational \( G \) construction
3.3.3 Permutations by view for multi-slice helical reconstruction

In helical multi-slice reconstruction, the total number of slices reconstructed $N_{\text{total}}$ is greater than the $N_{\text{slices\_view}}$ contained in one image. The value of $N_{\text{total}}$ is determined by the physical length of the log $L$ and the slice thickness $t$, so that $N_{\text{total}} = \frac{L}{t}$. Instead of thinking of each of the $N_{\text{slices\_view}}$ positions of 3.18 as a real, physical part of the log in space, it is easier now to consider it as a “register” into which a given slice of the log can enter and exit between data captures. In order to keep track of the log ends as it moves into the cone beam, it is taken that only the first register is filled in the first image, leaving the rest empty. From there, the log advances in the $-z$ direction one slice thickness at a time and one sector’s rotation at a time. The last image is taken when slice $N_{\text{total}}$ is in the last register, meaning that all slices have received $N_{\text{slices\_view}}$ number of views. Figure 3.22 illustrates the manner in which the log moves into and out of the cone beam. The helical reconstruction complicates the computation of the path length matrix as there are pixel columns on both ends that will never intersect a reconstructed voxel, shown in red, as well pixel columns that intersect the log as well as excess space in the view area. This buffer space is shown in orange in Figure 3.22, and is handled as was done in the rotational multi-slice where it is computed but then discarded. This must be taken into consideration in constructing $G$, but as was done with the path length matrix of pure rotational multi-slice, it is first assembled as though every pixel-ray captured by the detector is to be included in the inversion.
Figure 3.22: Motion of the log in view registers
(a), (b), & (c) motion of the log into the viewspace (d) the log in view (e) & (f) log exiting view

The memory saving technique of reusing path lengths from $G_{\text{view}_MS}$ can be extended
into another dimension now that the log translates one slice per image. The first step is to
construct the first data capture’s matrix $G_{\text{view}_\text{spiral}}$. This is essentially a matrix of dimensions $P_{\text{ang}} \times P_{\text{col}}$ rows by $N_{\text{total}} \times N_{\text{voxels\_slice}}$ columns that is identical in the first $N_{\text{slices\_view}}$ columns to the values of $G_{\text{view}_\text{MS}}$ for the same geometry. The rest of the matrix is taken as zero because none of these slices are in view; the form of the matrix is presented in Figure 3.23.

![Figure 3.23: Representation of $G_{\text{view}_\text{spiral}}$](image)

Now for each additional view of the log, two nested rotations are enacted, one that acts on the entirety of the column space of $G_{\text{view}_\text{MS}}$ with form $G_{\text{rot}_k}$ for $n$ slice registers translation between data captures on the $k$th concatenation, and one that acts only on the $G_{\text{sub}}$’s as before with column rotation $\text{rot}_k$ corresponding to the sector and combined cases as for multi-slice rotational reconstruction. The nature of this matrix manipulation is shown in Figure 3.24. After the “full” path length matrix $G$ is formed, it is necessary to truncate the empty pixel columns. A script is run taking $G_{\text{logic}}$ as input and from it determines which pixel indices are not contacting a reconstructed slice for the entrance and exit of the log into the cone beam. The rows corresponding to these columns are eliminated, making for a large part of the column space of $G$ to be null. This column space is trimmed away to bring the correct number of voxels into the inversion problem. After the inversion is completed, voxels that contain data
outside the actual log (the “orange” slices) may finally be discarded.

Figure 3.24: Helical basis matrix $G$ construction
3.4 Advantages of the method

The traditional method of ART takes the complete path matrix as a logical matrix containing some average length dimension in those voxels encountered by rays. It lacks accuracy and is undesirable in a system already subject to reduced resolution. Exact path length ray tracing schemes developed by Siddon and improved upon by Jacob have been available for some time, but are costly for large cubic voxel projections [12, 15]. Fortunately, the symmetric nature of the voxel pattern helps to make accurate basis calculation fast and relatively simple. Jian and Thibedau have shown that utilizing polar series expansion methods has led to speedups by a factor ranging from five to twenty in basis construction times [17, 18]. Since the basis may be stored prior to runtime, this is not a critical factor but is most welcome.

One of the major advantages of the system is how the data manipulation enables an accurate reconstruction even with log motion in the cone beam. Traditional CT systems have no such mechanism and thus will present a blurring artifact in any region that was displaced even slightly. This in turn will lead the reconstruction greatly distort the reconstructed shape and location of any features of interest or lose them. In the uncontrolled environment of a sawmill, vibrations will abound and mechanical manipulation of the log will not always be optimal. Differences between log sizes will make the data location scheme valuable as no physical adjustment of the source or detector is needed to compensate for a bigger or smaller log. The centering method presented here is robust when compared to standard edge detection methods (Sobel, Canny, etc.) as the entirety of the CT data is used to determine the geometric center quickly and with less processing. This is important in a sawmill, as loose bark/detritus, bulging knots, etc will lead to ill-defined edges of the actual log in frame.

The other half of CT reconstruction is the problem of segmentation, or the identification of features within a three dimensional data set in order to perform analysis or act. This in many cases may be just as large of a problem as the CT reconstruction itself. In a traditional 3D data set, features may occupy voxels in a haphazard way so that the indexing of voxels containing parts of the feature are not numerically convenient. As an example, consider the case of Figure 3.25 in which a reconstruction is contained in Cartesian 3D with the voxels numbered by column and then by slice extending into the page. The log contained in the reconstruction space spans multiple rows, columns, and slices of voxels in a way that is unknown. Thus any segmentation software must look through data that may be large indices of separation apart from each other and recognize that this is the same feature of interest. This requires special algorithms that at the very least must rearrange any data vector of density voxel values into slices as a starting
point, locate the feature from there, and then use the directionality of the detected feature to place it in reference to the reconstructed body’s overall orientation.

For knot detection and segmentation from this work’s process, the problem is less daunting. The voxel indexing of Figure 3.12 means that a knot will occupy a space of closely numbered voxels for a thoughtfully scaled pattern (meaning the sectors are close to the physical size of an average knot). This will prove useful in allowing for quicker identification and orientation of the knot in the log, making decisions of how to rotate the log before cutting faster. The advantageous ordering of voxels closely within features will be explored later in full for several real reconstruction cases. The cropped knot image from before is shown in Figure 3.26, now with an overlaid polar voxel pattern into which the same density data would be reconstructed. Clearly, it will be easier to identify areas of higher density in these larger numerically adjacent bins.

Figure 3.25: Cartesian segmentation
Lastly, the helical scan system is very flexible in its construction when compared to that of commercially available machines. The nature of the 'dual' rotation of the matrices in $G_{view\_spiral}$ along with the robustness of ART means that the total path length matrix may be modified for any amount data capture intervals, so long as the pitch length of the log’s movement remains unchanged and the amount of translation and rotation is accounted. This aspect will be helpful in future work taking on the challenges of log manipulation.
Chapter 4

Experimental setup

The setup used to implement these algorithms is an upgraded version to the one used by An, and consists of a sample of log approximately one meter long mounted to a railed carriage assembly with shafts, which allows for both rotation and translation of the sample within an x-ray cone beam. The cone beam projects thru the log onto a detector box that is parallel to the track and located at its center. This means that the source and detector can remain stationary and only the log carriage assembly needs to move. The detector interfaces with a PC that triggers data capture by monitoring log rotational position via an optical encoder. The PC is also able to drive the log’s rotation in the cone beam via a stepper motor and timing belt connected to a pulley on the log shaft. A different configuration using two stepper motors allows for translation, or translation along with rotation. These may be made to stop on data captures for the exposure time. This PC as well as the x-ray generator, cooler, and controller are located behind lead shielding to protect the operator during tests. Figure 4.1 presents a plan view of the assembly, showing the overall layout of the equipment in the room for helical scanning. Figure 4.2 shows photos of the setup, with the various components labeled.
Figure 4.1: Experimental Setup Layout
4.1 Detector assembly

The x-ray detector for this research is the same used by An. et al. in their experimental trials [8, 19]. The central part of the x-ray detector used in this research is an Andor iXon 897 electron multiplying charge coupled device (EMCCD) camera. This camera uses a quantum “cascading” of electrons in its pixel registers that make it much more sensitive than a standard CCD camera. It is capable of detecting single photon counts on the sensor surface while operating at very low temperatures. The sensor itself comprises 512 by 512 pixels. The camera is contained in a detector box that is carefully fitted to block entry of any ambient light into the detector box. The system is novel in comparison to conventional CT systems because it makes use of a static detector and accordingly a static source. This will necessitate rotation of the log as will be discussed, and presents some advantages. An image of the camera setup is shown in Figure 4.3 [19].
On one side of the box there is a 60cm by 60cm gadolinium oxysulfide scintillator screen from Kasei Optonix that is in the path of the X-ray cone beam. This screen was selected from a number of options in the research of An and was found to be the brightest for a given X-ray energy level, while minimizing blur [19]. The camera is oriented at 90 degrees to the scintillator screen and inside a lead-shielded box so that no X-rays may directly enter the aperture and create optical noise. A mirror is used so that the camera gets a central view of the scintillator screen. Figure 4.4 shows the detector box with its door opened as well as the scintillator screen, Fig. 4.5 shows a diagram of the box interior.
Due to spatial constraints, the camera is very close to the mirror and a wide angle lens is used in order to capture the entirety of the scintillator. This leads to barrel distortion of the captured image, which is easily corrected after data capture. A large black and white
checkerboard was placed centrally and normal to the screen’s lateral and vertical dimensions. The highly contrasting corner points of these squares are easily located via software, and their undistorted locations solved for based off of the screen’s size. A standard barrel correction algorithm was then be implemented to correct the image.

![Figure 4.6: Barrel correction](image)

(a) before (b) after barrel correction

The detector assembly is crude when compared to medical-grade flat panel systems used today, but presents some advantages. Apart from the obvious advantage of cost, a detector of this type presents interesting possibilities due to its ability to scale. By using a different scintillator and setup geometry, the system can be made larger to accommodate logs that are of a greater diameter. This in turn makes the detector more cost advantageous as scaling a flat panel system requires the purchasing of multiple detectors. This is mainly due to the lack of large area flat panel detectors currently available on the market. Finally the fact that the lateral dimension of the detector is ‘wide’ (capable of capturing many slices in a single image’ lightens the mechanical burden on the system by making very fast rotation of the log (or a very small pitch in the case of helical reconstruction) less necessary for rapid scans. This is important as manipulation of a full sized log is a challenge unto itself. The static detector is desirable though because it greatly cuts down on the mechanical complexity of the CT hardware. Traditional 3rd and 4th generation derived systems are often in need of maintenance in order to upkeep their power and data transmission components. This is not necessary if it is the reconstructed body
that is rotating.

4.2 X-ray source specifications and shielding

The x-ray system used consists of a Phillips MG-160 x-ray generator/control panel combination, a Comet MXR 160 x-ray tube, and a Bernard water based cooling system. The generator is capable of producing x-rays up to 160kVp while x-ray tube current is adjustable up to 10mA. The x-ray tube is specified to work in this range as well. The focal spot of the x-ray tube is specified as 0.4 - 1.5mm, with emanating cone beam subtending an angle of twenty degrees. The tube is shown in Figure 4.7.

![Figure 4.7: X-ray tube](image)

Aperture and shutter visible

For the basis determined in Section 3.3 to be valid, it is necessary to get the x-ray tube aligned as closely as possible to normal to the plane of the scintillator and centered on its surface. It is also critical that the x-ray cone beam be normal to the longitudinal axis of the log’s translation, and so there are two alignments that are necessary. Alignment with the detector was done roughly by first making a small jig to hold a laser pointer that would interface with the tube and could then be placed flat on the front of the detector box in its middle (Fig 4.8). When the laser coincided with the same source points on the tube and box, the tube was
close to being in alignment. Further centering of the tube on the detector as well as adjusting the main axis of the track was accomplished via trial and error. A 'hooped phantom' centered on the detector and held on the log carriage’s shafts made exaggerated perspective errors in images apparent. Between images, the source could be manipulated slightly until satisfactory alignment was obtained (Fig 4.9).

Figure 4.8: Rough alignment via laser
(a) source (b) detector
The samples used in the experiments are of a density that necessitates voltages in excess of 100kVp and currents over 5mA. In order to protect from radiation back scatter, the setup was assembled in a basement room in which measurements were made and a shielding plan was designed. Calculations outlined by National Council on Radiation Protection document 147 on the safe construction of x-ray systems led to the setup of shown in part (b) of Figure 4.2. Three of the four walls of the room were solid concrete of sufficient thickness to require no shielding. The fourth wall was composed of non-structural gypsum board, and so seven panels were constructed of lead shielding measuring 1.6mm in thickness. University Radiation Risk Management safety personnel were consulted and implemented appropriate instrumentation to confirm that the safety shielding was sufficient.

4.3 Log carriage and locomotion

4.3.1 Log carriage

The rotating log is mounted on shafts attached to two carts that ride along a wheeled track, shown in Figure 4.10. The log sample must be a circular cylindrical object approximately one meter in length and around 30cm in diameter. The track is made up of an MDF base on which
two 5m aluminum u-channels run. The x-ray detector and source are placed transversely to this track arrangement so that the shafts are roughly in the center of the scintillator view area. The log is mounted to the shaft via two MDF plates that have four 5/16” mounting holes that may be used to attach the log sample via lag screws or bolts. To account for misalignment between the center of the log and the two shafts, low speed universal joints are used between the MDF plates and the shaft. These joints are capable of handling up to 15 degrees of misalignment and ensure that the log does not bind. Figure 4.10 shows the arrangement of the log carriage system and track as well as the mounting blocks and u-joints.

![Figure 4.10: Log carriage](image1.jpg)

(a) carts visible (b) u-joint and mounting plate

4.3.2 Belt drive rotation and helical movement

The setup uses stepper motors, pulleys, and timing belts to rotate or rotate plus translate the log sample in rotational and helical movement respectively. In the case of purely rotational movement, one stepper motor directly drives the pulley shown in 4.10(b). The carriage assembly is moved so that the features of interest are within the detector view for purely rotational multi-slice and centered if single slice reconstruction is desired.

For the case of helical motion, two 1.8 degree resolution size 34 stepper motors are employed to translate the log as well as provide rotational movement. This is done by linking the cart to two separate timing belts, one that provides simple translation and the other imparting rotation
to the shaft pulley. These components are shown in Figure 4.11. The locomotion timing belt is highlighted in the dashed green line, and the rotational belt in the solid red line. The rotational timing belt is not connected to any motor directly, but simply rides along the pulley system highlighted in the figure. This mechanically links the amount that the log rotates to how far it translates, eliminating the necessity for a control system to monitor this. Stepper motor control is handled via an ATmega 103L 8-bit microcontroller that provides constant speed options from $\omega = 1.1$ rev/s to $1.6$ rev/s on 18 tooth 3/8” pitch pulleys [19]. The diameter of the shaft pulley $D_p$ determines the helical pitch of the log’s motion to be its circumference $p = \pi D_p$ and when coupled with the resolution of the sector voxel arrangement the reconstruction slice thickness can be determined by $t = \frac{2\pi}{N_{\text{sectors}}} D_p$. In this research, two pulleys were used to study the effects changing the log pitch and thus voxel resolution - one 63 tooth pulley and another 36 tooth pulley. Both pulleys had standard 3/8” pitches to interface with the 1” HTD timing belt properly.

![Figure 4.11: Timing belt conveyors](image)

(a) far side (b) helical belt (c) near side

### 4.3.3 Control scheme, hardware, and sensing

Figure 4.12 shows the control diagram implemented in linking the locomotion, sensing, and data capture systems [19]. A US Digital S1 shaft encoder with 720 count/rev resolution was used to monitor how much the log rotates (Fig 4.13). The user is able to specify an angular interval upon which the camera is triggered for a data capture, the camera exposure time and electron multiplying gain of the EMCCD, and whether or not the system should delay log movement during capture and for what length of time. This angular value corresponds with the desired angular dimension of the sector voxels that the log will be reconstructed into. This rotational
based control scheme eliminates the necessity to monitor the log’s movement in time as it is entirely position dependent, a large upside for simplifying the control scheme. Data acquisition is via a US Digital USB4 DAQ that reads the encoder and provides control of the ATmega 103L. All tests were run via a Desktop PC featuring an Intel Core 2 CPU at 2.89 GHz with 8GB RAM.

Figure 4.12: Log scanning control scheme from An

Figure 4.13: Encoder on shaft
4.4 Test specimens

4.4.1 Physical phantom

Initial scanning was done utilizing a physical “phantom” or a body of known density distribution to provide a known entity against which to compare reconstructions. This physical phantom was constructed to have features of interest nominally resembling those that would be found in a log. The phantom consists of a doubled 12” (32.5cm) diameter concrete forming tube that is reinforced by fiberglass strapping tape and approximately 90cm long. Inside this tube is a concentric 6” (16.2cm) diameter concrete forming tube that has runs the entire length. The cardboard tube material has a density of approximately 700kg/m$^3$. The center of the phantom consists of a 1.5” square wooden shaft that is imbedded into two endcaps comprised of doubled 3/4” ply, providing the phantom with structural strength resistance to torsional shear. This leaves 84cm of length in which to insert features. Dried and compacted wheat kernel was used in the innermost cylinder as a filler, having a nominal density of 820kg/m$^3$. In the annular cavity between the outer and inner tubes, damp cedar sawdust was incrementally poured and tamped until it filled the entire area with a density averaging about 322kg/m$^3$. These filler materials were packed around a wedge shaped cylinder of blue housing insulation foam, meant to emulate a radial ’crack’ running the length of the log and with a density of 25.5kg/m$^3$. Along with the crack feature, three separate planar ’knot cluster’ features were inserted into the tubes at separate locations. These clusters are made of sector shaped pieces of 1” medium density fiberboard (density 760kg/m$^3$) subtending on average 15 degrees and are arranged as is shown in Figure 4.14. The radial coordinates are given in Table 4.1. The images in Fig. 4.15 shows one such knot cluster as it was laid into the phantom as well as the phantom tube. These were measured by marking the outside of the tube as the knots were laid into it, and then measuring the markings circumferentially from a reference on the outside (the positive x axis in the diagram).
Figure 4.14: Phantom dimensions

<table>
<thead>
<tr>
<th>Knot</th>
<th>Slice A</th>
<th>Slice B</th>
<th>Slice C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>99</td>
<td>0</td>
<td>212</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>78</td>
<td>318</td>
</tr>
<tr>
<td>3</td>
<td>240</td>
<td>198</td>
<td>353</td>
</tr>
<tr>
<td>4</td>
<td>332</td>
<td>229</td>
<td>//</td>
</tr>
<tr>
<td>5</td>
<td>//</td>
<td>300</td>
<td>//</td>
</tr>
</tbody>
</table>

Table 4.1: Angular $\theta$ coordinates of phantom knots
All values in degrees from x axis
4.4.2 Tree specimens

Two real tree specimens were scanned and reconstructed as well. The samples included one dry specimen as well as a “green” specimen which is representative of what would be encountered in a sawmill. Both specimens are of the Douglas fir species. The dry specimen measured 90cm long and ~27cm in diameter, while the “fresh specimen” measured 80cm long and ~28cm in diameter. The dry specimen featured several knot clusters as well as a longitudinal crack from rapid drying, unlikely to be present in an actual sawmill log except at the very ends. The green log was sealed completely upon cutting using PVA wood glue to ensure that no such splitting occurred. The dry log was much less dense on average and was used to simply confirm the experiment’s effectiveness at identifying log features, as good data was easy to collect. The wet log was used to show the setup is still valid for highly dense fresh logs as would be seen in sawmills. Figure 4.16 contains a picture of both log samples, clearly showing the crack in the dry log and the externally visible knots on both samples.
Figure 4.16: Douglas fir samples (left) dry log (right) green log
Chapter 5

Single slice reconstruction

5.1 Direct solution via the normal equations

In the case of single slice reconstruction, the path length basis matrix is relatively small (on the order of $10^3$ equations to solve). It is the case that it is still well conditioned enough that direct methods may be utilized to solve equation 2.8. Here, the choice has been made to utilize the traditional least squares approach. It may be shown that the vector $\vec{\rho}'$ that minimizes the error of the residuals squared of a model $[\vec{y}_{\text{total}} - G\vec{\rho}]^2$, that is the least squares solution, is the solution of the normal equation for the system:

$$G^T G \vec{\rho}' = G^T \vec{y}_{\text{total}},$$  \hspace{1cm} (5.1)

As is clear, the normal equation is simply constructed by taking both sides of the system equation and left-multiplying by the transpose of the basis matrix, $G^T$. It is the case that $G^T G$ is a diagonally dominant positive-definite sparse matrix of size $N_{\text{voxels_slice}}$. To solve for the vector $\vec{\rho}'$, Cholesky factorization is employed as it is applicable to this type of matrix. Equation 5.1 may be written as $LL^*\vec{\rho}' = G^T \vec{y}_{\text{total}}$, in which $L$ is a lower triangular matrix and $L^*$ is its conjugate transpose. The system may then be readily solved in $O(n^2)$ steps and the density vector found. For rapid solution, the factorization $L$ may be saved for the normalized parameters and used for further CT data at a later time.
5.2 Synthetic phantom reconstruction

Tests of the algorithm were conducted using synthetic data to evaluate the center finding scheme of Section 3.1.3 and the subsequent inversion. Density vectors for several phantom log cross sections were built so that they had distributions similar to that of what would be found in real logs, similar in concept to the ubiquitous Shepp-Logan used for brain imaging. Some examples are shown in Figure 5.1. Different cases were tested including purely annular/circular features (a and b), origin-symmetric sector cases (c), and asymmetric cases with combinations of features (c and d). These grayscale images correspond to density distributions with $N_{\text{sectors}} = 36$ and $N_{\text{annuli}} = 18$. Voxels with high intensity are shown in lighter shades, and less dense regions are darker. The images shown here correspond to realistic log densities ranging from $350\text{kg/m}^3$ to $550\text{kg/m}^3$ and are scaled on a 255 grayscale as a fraction of $650\text{kg/m}^3$. The synthetic log physical radius was taken to be $0.17\text{m}$, to reflect a realistic value close to the physical specimens.
These density vectors were multiplied by an appropriate path length matrix $G$ corresponding to the voxel case, $P_{\text{ang}} = 92$, $R_{\text{norm}} = .1667m$, $D_s = 2.1m$ and $D_l = 1.67m$ to form a realistic synthetic Radon transform vector $\tilde{y}_{\text{total}}$. This vector was segmented into $N_{\text{views}} = N_{\text{sectors}} = 36$ vectors of length $P_{\text{ang}}$ (values that were set by the size of $G$). These vectors were then inserted into zero arrays of size $128 \times 1$, simulating a 4x1 binning of the 512 element sensor to be used in real trials. The values $R_{\text{norm}} = R_{\log}$ and $P_{\text{ang}} = 92$ were chosen as they made it unnecessary to reverse the steps of Figure 3.9 (it was determined that a log of radius .1667m would subtend 92 pixel-rays in a 128 equal-angular fanbeam). These synthetic data arrays were then fed back into...
the center-finding/normalization algorithm and assembled into a vector $\vec{y}_{syn}$. The inversion was then completed using the method as discussed in Section 5.1.

It was found that for the phantoms not displaying symmetry with respect to the origin, the center finding algorithm would incorrectly displace the center of many views by a small amount (on the order of 1-2% in the worst cases). Although small, this center displacement was enough to cause problems in reconstruction. For the direct solution of the combined model of an asymmetric phantom, this error presented itself as alternating intensities of light and dark sectors strongest at the origin. This 'starburst' artifact propagates through the sectors to the outer annuli of the reconstruction. This is shown for phantom (d) of Figure 5.1 in Fig. 5.2. The reconstruction of (a) exhibits the artifact due to center miscalculations, while (b) is a reconstruction with the center explicitly defined at the correct position.

![Figure 5.2: Center artifact in reconstruction](image)

(a) artifact (b) forced center

5.2.1 Modified centered finding

The error in finding the center of the log arises when the centroid of the Radon Transform does not coincide exactly with the geometric center of the log cylinder. This error occurs when the cross-section contains a substantially non-symmetric density distribution. The non-symmetry can be reduced (but not entirely eliminated) by penalizing those parts of the radon signal that are larger in magnitude while increasing the relative size of the lower parts of the signal. A
simple approach is to take the square root of the Radon Transform, which in effect biases the centroid away from the areas of high area density. In practice, it was found that results were improved even further by taking the cube root, subsequent gains were not readily observable for higher order roots. Figure 5.3 shows an example of this effect. The initial Radon Transform signal for the synthetic phantom (e) of figure 5.1 taken at $\theta = \theta'$ as shown in part (a). The larger, high density 'knot' region causes the ordinate centroid to lie at 63.7, slightly less than geometric center which is represented by the vertical dashed green line at pixel 64. Taking the cubed root of the transform as shown in (c) effectively makes the predicted centroid lie at precisely pixel 64. Symmetric views are unaffected by the process, and on the whole it serves to provide a more accurate center prediction.

![Figure 5.3: Center finding example](image)

(a) Ground truth $f$ with $\theta = \theta'$ marked (b) $R f (t, \theta')$ (c) cubed root of $R f (t, \theta')$
5.2.2 Tikhonov regularization

The modified center-finding algorithm is effective at biasing the center in the correct direction for most views, but it is not perfect. The reconstructions still feature faint traces of alternating light-dark sector elements about the center due to very small misalignment. In an effort to alleviate these areas of high contrast, Tikhonov “smooth” regularization in the circumferential direction was applied to the center voxels in the path length matrix before inversion. This technique works to make the form of the intensity’s curvature among adjacent sectors adjustable in order to allow for varying degrees of “smoothness” in the reconstruction [20]. Smoothing the voxels’ curvature allows for mild regularization in order to not lose the features of interest.

The “smoothness” of the voxels in the central circle is analogous to the second derivative of the density function in the circumferential direction. This second derivative may be represented in matrix form for a numerically sequential arrangement of elements by the matrix in Figure 5.4. In the case of taking the density function in sequence about the central annuli for the arrangement in Figure 3.16, this matrix must be adjusted for the non-sequential numbering in this direction. This effectively yields a very sparse tri-diagonal $N_{voxels} \times N_{voxels}$ matrix with “2’s” located at entries $(p \times N_{annuli}, p \times N_{annuli})$, $p = 0, 1, 2, 3...$ and “-1’s” at $(p \times N_{annuli}, (p \times N_{annuli} + 1) \mod N_{sectors})$ and $(p \times N_{annuli}, (p \times N_{annuli} + (N_{sectors} - 1)) \mod N_{sectors})$.

\[
C = \begin{bmatrix}
-1 & 2 & -1 \\
-1 & 2 & -1 \\
-1 & 2 & -1 \\
-1 & 2 & -1 \\
\end{bmatrix}
\]

Figure 5.4: Tikhonov regularization matrix for curvature

Following the method, this derivative matrix may be pre-multiplied by it’s transpose to be dimensionally consistent with $G^T G$ and multiplied by a coefficient $\alpha$ in order to modulate its
effects on the inversion. This yields the altered system of normal equations contained in eq. 5.2.

\[ G^T G \tilde{\rho}' + \alpha c^T c = G^T \tilde{y}_{total} \]  

(5.2)

There are many schemes for optimization of the parameter \( \alpha \), to achieve adequate noise reduction while avoiding distortion of the solution by excess smoothing. For this case, a rough neighborhood of acceptable values was determined by starting with a very low coefficient \( 10^{-6} \) and raised by orders of magnitude while observing changes in the mean-2 norm relative error of \( \tilde{\rho}' \). When excessive blurring caused the reconstruction to degenerate, the coefficient was reduced. A coefficient of \( \alpha \sim 10^{-3} \) was ultimately found to be sufficient to introduce adequate smoothing without causing significant distortion. Figure 5.5 displays the synthetic ground truth phantom (e) of Fig. 5.1 with the initial artifact in (a) and corrected using the modified center finding and regularization in (b). The mean-2 norm error in this case was reduced from approximately 11% to 4%. This example may be extrapolated to the multi-slice case, and shows the positive effect that regularization may have on the ART method of reconstruction.

![Figure 5.5: Phantom Artifact Correction](image)

(a) artifact present (b) Corrected center (c) Regularization and corrected center

5.3 Real data reconstruction

5.3.1 Calibration of the detector

In order for the inversion to give accurate density values for the reconstructed \( f (r, \phi) \), the x-ray detector must be calibrated to correlate the CT data, \( -\ln \left( \frac{I}{I_0} \right) \), with the sampled Radon Transform, \( \sum g_{ij} \rho_j \). Looking back at equation 2.2, it is apparent that this amounts to finding the mass attenuation coefficient \( \beta \) that corresponds the reconstructed log’s wood. For this work,
the method of An performed at 100kVp, 5mA x-ray power was closely followed [19]. Low density fiberboard, a wood-based material of known density and dimension, was used to perform this calibration. The fiberboard samples were square blocks measuring approximately 15cm per side and 2cm thick, with an approximate density of 500kg/m$^3$. A table was set up in front of the detector screen so that the blocks could be centrally located in the most intense part of the cone beam. Starting with one block a sequence of images was taken, incrementing the number of blocks by one between images. This was done from one to twenty-five blocks in sequential order. Figure 5.6 demonstrates the manner in which these blocks were stacked for calibration.

![Figure 5.6: Fiberboard calibration blocks](image)

A reference image was taken with no objects in view to provide values for $I_0$, and the corresponding mean value of pixels within the block stack were used to find $I$. In order for the calibration to provide the correct curve for the relationship between the CT data and the Radon Transform value of the attenuated rays, the same power of x-rays must be used for the calibration and the reconstruction data images. It is critical though that the detector not be saturated for calibration, and so for a given power of x-ray (determined by the power needed to penetrate the log being scanned) the EM gain of the camera was adjusted until saturation was not present. Figure 5.7 demonstrates the un-attenuated signal in (a) along with the attenuated signal for 5 blocks in (b) and 14 blocks in (c).
The calibrations were performed at 140kVp, 5mA and 160kVp, 5mA, which were the two energy levels used for this work. It is the case that the x-ray intensity of the source used fluctuates by as much as 5% over the course of several seconds, and so for each attenuation step ten images were taken and their values averaged. The corners of the image outside of the central cone beam were then used to scale the data sets to be 1:1 with the reference image in intensity. It was found that past fifteen blocks at both energy levels the resolution of the detector limited any further calibration. A polynomial curve was fit in this case to the first fifteen data points to yield a correspondence between CT data and the Radon sample.

### 5.3.2 Real log data sample and reconstruction

In order further to assess the center finding scheme and solution regularization approach, a single slice reconstruction of the data taken by An for the same log (the dried Douglas fir) was performed, using An’s detector calibration. The data corresponded to a capture time of 1s at 100 kVp and with a camera gain of 1. The Radon transform vector was taken as the pixel column corresponding to a four knot cluster as is shown in Figure 5.8 by a dashed line [19]. The data set consisted of 36 images (equal to $N_{sectors}$), each taken at a rotation of 10 degrees from one another and the log axis of rotation remaining unchanged in the detector view. The reconstructions are presented in Figure 5.9. It was found that the annular and sector cases were consistent with the reconstructions of An, but the combined case presented much less prominent artifacts and a more stable reconstruction, pointing to the advantages of the techniques of Section 5.2.
Figure 5.8: Single Slice data sample from An
Figure 5.9: Single Slice reconstruction

(a) annular reconstruction $N_{\text{annuli}} = 18$ (b) sector reconstruction $N_{\text{sectors}} = 36$ (c) combined reconstruction (d) combined reconstruction from An
Chapter 6

Multiple slice reconstruction

6.1 Computational considerations

Careful management of the computational requirements of storing the basis and the robustness of the solution becomes a necessity when the reconstruction extends into multiple slices. The single slice method of storing the basis in its complete form as well as implementing a direct solver would prove to be unfeasible due to the much greater size of the mathematical solution.

6.1.1 Storage of the basis

Although it is the case that $G$ is highly sparse, this does not necessarily ensure that $G^TG$ is similarly sparse. This makes the implementing the Cholesky solver of Chapter 5 very computationally burdensome and slow due to memory writing and rewriting on a standard system (8GB RAM) as the number of slices gets larger. Full storage of $G$ is also impossible, as the size of the path length matrix considering null values is much larger than $G^TG$. For this work, Matlab’s sparse matrix format was used in order to store $G$ in a more efficient manner, ensuring good runtime performance and easy manipulation of the basis. In this scheme, each nonzero element of $G$ is stored together with its matrix coordinate values $(i, j)$.

6.1.2 Ill-posedness of the basis

Calculation of the normal equations also increases the condition number of the basis, effectively squaring it [21]. It is therefore undesirable to use the method in the rotational and helical reconstruction cases as the condition number increases very quickly with additional slices. It was
found that the normal equation basis became effectively singular when the modest resolution sector, 18 annuli reconstruction method of An was extended into multiple slice reconstruction for a full-field view dataset (45 slices of .71cm thickness each to fill the detector). This necessitated utilization of more conventional iterative solvers that have been used for some time in ART.

6.2 Iterative methods

Two iterative methods were explored in this work for the solution of the inversion problem, Randomized Kaczmarz and the Simultaneous Algebraic Reconstruction Technique (SART).

6.2.1 Randomized Kaczmarz’s method

Kaczmarz’s method is a solution belonging to the Projection onto Convex Space (POCS) family of solvers, and has an established history as the preeminent solver for conventional iterative ART CT. It is preferred for its generality and adaptability, as the basis need not be square or positive definite, as is the case for other iterative methods (Gauss-Seidel, Conjugate Gradient, etc.) The algorithm works as follows: the solution of the system of equations $G\vec{\rho}' = \vec{y}_{total}$, in which again $A$ is a matrix of dimension $m \times n$, may be thought of as the intersection of quantity $m$ hyperplanes $p_i = r_i \cdot \vec{\rho}$ in $n$-dimensional space, in which $r_i$ is the $ith$ row of $G$. A starting 'guess' for the solution vector $\vec{\rho}'_0$ is made in $\mathbb{R}^n$. The first row of the basis matrix, $r_1$ is taken into the algorithm and an orthogonal projection of the solution vector is made onto $p_1 = r_1\vec{\rho}'_0$. This projection process is then repeated for the entirety of the $m$ constraints, at which point it is considered that the algorithm has gone through one iteration and the process is repeated from a new starting point, $\vec{\rho}'_0$. Thus the first subscript therefore corresponds to the 'sub-iterative' step of a projection and the second subscript to the outer algorithm iterative step. It may be shown that the iterative step, the updated solution vector $\vec{\rho}'_{i|k+1}$ for the $(k+1)th$ iteration, may be given by

$$\vec{\rho}'_{i|k+1} = \vec{\rho}'_{i|k} + \frac{y_i - p_i}{\|r_i\|^2} r_i$$  \hspace{1cm} (6.1)

Where $i = 1, 2, ... m$ denotes the sub-iterative projections. The method may be graphically represented in the case of $n = 2$ as the intersection of multiple lines in a plane (Figure 6.1). The highly overdetermined system of equations is represented by quantity $m$ lines $\{l_1, l_2, ... l_m\}$ that intersect at the solution point $\vec{\rho}$. As the algorithm operates, the initial guess projects along the path shown by the dotted lines until it coincides with $\vec{\rho}$.
In the case of this research, in which the basis is ill-conditioned and overdetermined, many solutions may be equivalently “correct” according to the standard algorithm. This can be visualized in the plane as the case in which multiple constraints of the system do not coincide at a single solution point but rather have a “zone of intersection”. Intuitively, it seems reasonable that in this case the best choice is the geometric center of such a set of points, and convergence towards this best choice is made with the use of a relaxation parameter, $\lambda$. Equation 6.2 shows the relaxation parameter incorporated into equation 6.1. Its effect is graphically illustrated in Figure 6.2. The solution vector in this case will make its way towards $\vec{\rho}$, the best solution for the system.

$$\vec{\rho}^i|_{k+1} = \vec{\rho}^i|_k + \lambda \frac{y_i - P_i}{\|r_i\|^2} r_i$$  \hspace{1cm} (6.2)
Vershynin and Strohmer have improved upon the standard Kaczmarz algorithm by using a version that selects the next hyperplane for projection based on a weighting probability distribution over all rows proportional to the Euclidean norm squared of each row, that is $\|r_i\|^2_2$. It has been shown to converge at an exponential rate to the equivalent least squares solution independent of the number of constraining equations based on the work by Strohmer and Vershynin and confirmed by Needell [22, 23]. They conclude that the weighted selection of projections has the benefit of convergence to the true solution in fewer iterations than possible with conventional Kaczmarz.

6.2.2 Simultaneous algebraic reconstruction technique

Another variant of the standard Kaczmarz algorithm is the Simultaneous Algebraic Reconstruction Technique or SART. In this implementation introduced by Andersen and Kak, the problem of an inconsistent overdetermined system is accommodated by taking into consideration the potential projections onto all hyperplanes of a given “view” of the system, and then taking the average of these projection vectors as the sub-iterate step [13, 24]. Looking back at the basis construction of Section 3.3, this means that the rows of $G$ are iterated down one at a time and the projected solution vectors $\tilde{\rho}'_{jk}$ are stored. When the sub-iterate index meets the condition
\( i = n \left( P_{\text{ang}} \times P_{\text{col}} \right), n = 1, 2, \ldots N_{\text{views}} \) the step of equation 6.3 is taken to update \( \vec{\rho}'_{k} \) (shown here with indexing corresponding to the first view). Once all views are considered, one iteration is taken to have been performed, that is when \( \vec{\rho}'_{k+N_{\text{views}}(P_{\text{ang}} \times P_{\text{col}})} \) is calculated.

\[
\vec{\rho}'_{k+1} = \vec{\rho}'_{k} + \frac{\sum_{i=1}^{P_{\text{ang}} \times P_{\text{col}}} \left( \vec{\rho}'_{i|k} - \vec{\rho}'_{k} \right)}{P_{\text{ang}} \times P_{\text{col}}}
\] (6.3)

By performing the updates in this manner a “smoothing” effect is obtained that greatly cuts down on high frequency noise in the reconstruction. This smoothing is not enforced regularization, but rather an integral aspect of the algorithm. It does lead to slower feature distinction than the randomized Kaczmarz algorithm, however.

Hansen et al. has shown that the algorithm for this scheme may be vectorized and implemented very simply, eliminating the need for looping [25]. It is presented here without derivation:

\[
\vec{\rho}'_{k+1} = \vec{\rho}_{k} + D_{r}^{-1} G^{T} D_{c}^{-1} \left( \vec{y}_{\text{total}} - G \vec{\rho}_{k} \right)
\] (6.4)

Here, \( D_{r} = \text{diag} \left( \| g_{i} \|_{1} \right) \) and \( D_{c} = \text{diag} \left( \| g_{j} \|_{1} \right) \) are diagonal square matrices of dimension \( m \times m \) and \( n \times n \) made up of the row and columnar sums of the coefficients in \( G \) respectively [25].

### 6.2.3 Selective regularization

The effectiveness of regularization on solution stability was demonstrated in the case of single slice reconstruction, and so an effort was made to incorporate a similar scheme into the multiple slice and helical reconstruction algorithms. It is stated by Herman that for ART reconstructions in which the form of the image function is made up of large distinct regions of different densities, good results may be accomplished through the implementation of so-called “selective smoothing”.

In this process, which is a 3-D expansion of the planar case presented by Herman, each voxel is examined upon iteration of the solver and compared to its nearest neighbors. The primary voxel \( v1 \) is surrounded in three dimensions by six voxels containing a common side \( v2 - v5, v10, v11 \) and twelve voxels containing a common edge \( v6 - v9, v12 - v19 \). This geometry and numbering is shown in Figure 6.3, with the primary voxel in yellow, its shared face voxels in blue, and its common-edge voxels in red.
Figure 6.3: Selective regularization voxel indexing

This voxel indexing is shown for a generic interior voxel that is not on the perimeter or center of the reconstructed log; it is surrounded on all sides by other voxels. In those cases where a voxel is not in contact with other voxels, a careful record must be kept to ensure that extra voxels are not included in the regularization.

Establishing a record of each voxel and its neighbors, a threshold value \( t \) is defined for the reconstruction which represents the difference in relative voxel values above which smoothing will take place. This threshold function is 'activated' via the relation

\[
f_i = \begin{cases} 
1, & \text{if } |v_i - v| \leq t \\
0, & \text{otherwise}
\end{cases}
\]  

(6.5)

The activated threshold function may then be inserted into equation 6.6 to provide a regularized value for \( v_i' \). The values \( w_1, w_2 \) and \( w_3 \) are the smoothing weights of the regularization that affect the directionality of the regularization and the degree to which small incongruities are blended. The first weight corresponds to the voxel under consideration, the second to the common-face voxels, and the third to the common-edge voxels.
\[
v'_1 = \frac{w_1 v_1 + w_2 \sum_{i=2}^{5,10,11} f_i v_i + w_3 \sum_{i=6}^{9,12-19} f_i v_i}{w_1 + w_2 \sum_{i=2}^{5,10,11} f_i + w_3 \sum_{i=6}^{9,12-19} f_i}
\]  \quad (6.6)

Herman notes that determining the relative values of the smoothing weights that yield the best results is a heuristic procedure and that it should be used carefully so as not to distort important features of the true solution. The threshold parameter enables smoothing to take place in areas of small differences, assuring that large important features are not lost in the process.

### 6.2.4 Algorithm comparison

Convergence optimization of projection solvers and the tuning of ending conditions is an intensive process, and has oftentimes been performed heuristically for specific applications [13, 26, 27]. For this work, the random Kaczmarz solver and vectorized SART solver developed by Christian Hansen et al. in the AIR Tools Matlab package were combined with a script implementing the regularization scheme of Section 6.2.3 to reconstruct the multi-slice rotational and spiral cases. Execution was carried out in Matlab 2013b on an Intel Core i7 2.8GHz CPU with 16 GB RAM.

A data set of ‘perfect data’ was constructed by constructing a synthetic density vector of a 100 slice, 10 annuli, 48 sector geometry of a desired ‘log-like’ distribution \( \vec{\rho}_{\text{syn}} \) interacting with a 92 \( P_{\text{ang}} \) by 42 \( P_{\text{col}} \) pixel rays to get \( G \), and then taking \( \vec{y}_{\text{syn}} = G \vec{\rho}_{\text{syn}} \). This data vector \( \vec{y}_{\text{syn}} \) is “perfect” in the sense that all the data are perfectly centered and scaled exactly so that only incongruities due to the algorithms used will be detected. Per recommendation by Herman, the iterated relaxation parameter in the random Kaczmarz was set to the under-relaxed value of .05 [28]. Herman heuristically found that good results for a noisy reconstruction was obtained using smoothing weights of \( w_1 = 9, w_2 = 4 \) and \( w_3 = 2 \), and so these values were used in the script attached to both solvers. It was found that the solvers converged on the ground truth solution with negligible error for both solvers with the number of iterations for the randomized, under-relaxed Kaczmarz at two and SART at twenty iterations.

Due to standard Matlab m-code being a non-compiled, looping is extremely inefficient in the environment. This means that even though fewer iterations were required for convergence from the randomized Kaczmarz solver, execution of it took dramatically longer than implementation of the vectorized expression in eq. 6.4. As such, almost all reconstructions carried out in the analysis of the multi-slice and helical reconstructions to be presented were carried out with SART as reconstruction quality of both was seen to be equivalent. If implemented in a compiled
script, it may well be the case that randomized Kaczmarz would prove to be faster. Optimization and tuning of the solvers described, including development of “smart” stopping criterion can be counted amongst further work that must be performed as the system matures.

6.3 Rotational multiple slice

6.3.1 Phantom reconstruction

Tests were constructed on the physical phantom tube of Section 4.4.1 in order to assess the effectiveness of the detector and reconstruction scheme. It was found that the dense grain-filled inner portion of the phantom necessitated an x-ray power of 140kVp and 5mA tube current. An EM camera gain of 30, 100ms exposure with a 200ms pause on capture was found to give the clearest picture. The reconstruction was set have 48 sectors, giving a sector angle of 7.5 degrees (half that of the average knot feature dimension), and 10 annuli starting from an innermost annulus equal to 16% of the outer dimension of the log. The normalized diameter of the reconstruction was set to have a value of .333m. The values of $D_L$ and of $D_S$ were measured to be 1.625m and 2.125m respectively after the source was aligned.

As with the single slice reconstruction, the number of views in the reconstruction was set to equal the number of reconstructed sectors. Although in the case of purely rotational reconstruction the value for the slice thickness $t$ is decoupled from the number of sectors and pulley size, .71cm was used to correspond to the slice thickness that would be obtained from the spiral case for the reconstruction geometry. Several data captures are shown in Figure 6.4. The central “knot” cluster in the image is the Slice B of Figure 4.14.
Looking at the data images, it is clear that at the lateral edges the intensity of the cone beam significantly decreases. It was found that reconstructions were affected when the multiple slice reconstruction contained data from these areas, and so the number of slices $N_{slices}$ was limited to be 42 in order to capture only the best quality region of the cone beam (roughly the middle 2/3rd of the detector). The number of angular pixels $P_{ang}$ in this case was heuristically chosen to be 100 and pixel columns $P_{col}$ to be 74 in order to provide an adequate sampling of all voxels in the inversion.

Using the altered SART-smoothed algorithm described previously yielded a reconstruction that could then be evaluated based on the dimensional positions of reconstructed features vs. the known phantom specifications. The reconstructed voxel values could also be observed vs the known density of the physical phantom to assess the calibration and scaling of data. Figure 6.5 presents two cross sections of the multi-slice reconstructed phantom.
Figure 6.5: Centered multi-slice reconstruction slices
(a) clear cross sectional region (b) knot cluster “B”

As can be seen, the algorithm effectively reconstructs the form of the physical phantom, showing the dense grain inner tube as well as the outer packed sawdust annulus. The grayscale indicates that the density values are reasonable based on the measured densities of the inserted features and is expectedly uniform. Unfortunately, the system is unable to differentiate clearly the inserted MDF knot features within the grain filled cylinder (the measured density difference between these two materials was 80kg/m$^3$). The innermost annulus is less dense than its neighbors due to the structural dried timber core of the phantom. The transition annulus between the grain and sawdust as well as the outermost annulus display a higher density than might be expected, but this is simply due to the denser cardboard tube being present in these locations. Still - the reconstruction is not perfect, and interestingly there are some artifacts present around the crack feature as well as the knots.

A script was written to measure the angular locations of the five knots of Slice B in the reconstructed log as they were presented in Table 4.1. Taking the median direction of each knot from its form in the outer sawdust annulus, the locations of the knots could be located relative to the coordinates of Fig 4.14. Detection of the knots was performed manually in this case, the direction vectors for the knots were determined by selecting their edges in the axes of Figure 6.6 (the dashed red lines) and then averaging each pair to define the knot directions (the solid red lines). Table 6.1 presents the knot locations of the reconstruction compared to measured values. They show to be accurate for several knots compared to the measured phantom, with two
showing discrepancies from the measured phantom on the order of one sector angle, $2\pi/N_{\text{sectors}}$. Given the relatively imprecise assembly of the phantom (the tamping was performed so as to not disturb the knot positions, but some displacement is likely) and the crudeness of measurement of their positions, the results are encouraging.

![PhantomSlice B](image)

**Figure 6.6:** Centered multi-slice knot directions

<table>
<thead>
<tr>
<th>Knot</th>
<th>Measured</th>
<th>Reconstructed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>78</td>
<td>78</td>
</tr>
<tr>
<td>3</td>
<td>198</td>
<td>192</td>
</tr>
<tr>
<td>4</td>
<td>229</td>
<td>230</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>300</td>
</tr>
</tbody>
</table>

**Table 6.1:** Slice B knot locations - measured and reconstructed

All values in degrees from x axis.

The purely rotational multi-slice algorithm was also tested in the case of rigid body displacement of the log within the cone beam. This was accomplished by attaching the log-off center on the carriage system, using two counterweights shown in Figure 6.7 to ensure that no backlash of the timing belt or stalling of the motors would occur. The geometry of the system-detector arrangement allowed for a maximum eccentricity of one inch of the log axis from the shaft. The
series of images in Figure 6.8 demonstrates the image set; the phantom can clearly be seen to drift up and down over the series of images as it rotates.

Figure 6.7: Off-center counterweights

Figure 6.8: Rigid body motion multi-slice data
(a) log at max displacement upwards (b) log rotated at max downward displacement
The cross sections presented previously are again shown in Figure 6.9. The reconstruction has experienced very slight degradation in some areas due to the discrepancies introduced into the small angle approximations of the cone-beam made in the normalization scheme. Still this level of distortion is not prohibitive and the knots are still visible, as well as their locations being comparable to the reconstruction for the centered multi-slice reconstruction. The knots of Slice B in Figure 6.9(b) again fall within one sector angle of their measured values.

![Image](Phantom Eccentric, Slice B Eccentric, Phantom Centered, Slice B Centered)

Figure 6.9: Off-center multi-slice reconstruction slices
(a) clear cross sectional region eccentric (b) knot cluster “B” eccentric (c) clear centered (d) “B” centered

The trials prove the efficacy of the multi-slice rotational algorithm for providing an accurate
density reconstruction of the phantom for the combined voxel pattern. As the detector is not wide enough to accommodate multiple knot clusters, lateral measurement from the phantom is not possible. It should be stated that minimum optimization was performed on the values of $P_{ang}$ and $P_{col}$, it was simply taken sufficient that these values provided in excess of two pixel-rays for even the smallest central voxels, and that each slice received at least two pixel columns of intersection. This ensured a sufficiently overdetermined system to give the best results given the relatively noisy data (as compared to commercial systems).

6.4 Helical scanning

Using the methods of Section 3.3.3, the helical algorithm may be readily implemented for reconstructing the physical phantom.

6.4.1 Effect of limited views

A key difference between purely rotational reconstruction in which $N_{views} = N_{sectors}$ and helical reconstruction is that the detector is not wide enough for this condition to be met (i.e. $t \times N_{sectors}$ is greater than the greatest possible $W_c$). This means that each voxel will not be able to receive the same level of overdetermination that was possible in the purely rotational case. Using Turbell’s criterion of $\gamma = \pi + 2\Psi$ being the minimum angle of rotation needed for complete data acquisition of a slice, it is found for the geometry under consideration that this amounts to ~200 degrees of rotation, or in the case of 48 sectors $N_{slices\_view} = 26$. With the pulley setting the slice thickness to be .71cm, this means that the log will rotate $26 \times (360/48) = 195$ degrees in a distance of $26 \times .0071$ meters, giving a helical wavelength of .34m.

6.4.2 Phantom reconstruction

The data set now consists of many more images, for the .71cm slice thickness set by the pulley diameter and sector geometry means that approximately $84/.71 = 118$ slices will be needed to reconstruct the entirety of the phantom. With $N_{slices\_view} = 26$, this means that in total $26 + 118 = 144$ images are required. Figure 6.10 demonstrates the nature of the data set by showing the far ends of the log as it enters and exits the “active” cone beam (marked by the vertical red lines). By definition only the first register is ‘filled’ in (a) and only the last in (c). It is possible for $N_{slices\_view}$ to be greater without data entering the dim region of the cone beam (giving a greater degree of coverage), but preliminary trials showed this additional
overdetermination to not help the solution’s convergence. As such, all helical tests in this thesis use the minimum set by Turbell’s rule to ensure the active detector width remains in the best quality region.

![Centered spiral reconstruction data](image)

Figure 6.10: Centered spiral reconstruction data
Phantom entering and exiting the cone beam

The SART-smoothed reconstruction gives good performance on the inversion, performing it in ~8s. The three knot cluster slices obtained from the inversion are shown in Figure 6.11, again showing clearly demarcated and uniform areas of materials inside the phantom, along with the mean knot directions. The same script was run as before in order to determine the positions of the knots within the phantom, the values for these are given in Table 6.2. Again, the results seem reasonable given the likely error in measuring the phantom, and are close to the measured values. The adjacent slices in which the respective knot clusters A, B, and C appeared could be averaged in order to give a slice value for their position in the phantom. The number of slices from the end cap of the phantom to these could then be counted in order to assess the lateral positioning of these knot slices compared to the measured values. It was found that their mean positions occurred in slices 17, 55, and 92 respectively. Taking into account the slice thickness, this corresponds to locations for knot clusters A, B, and C at 12, 39, and 66cm respectively. Looking back at the measured values of Figure 4.14, this confirms that the algorithm is putting reconstructed objects back into their proper locations.
The same test was conducted on a set of spiral data in which the log was displaced off-center as was done in the rotational case. Again the maximum possible value of 1° eccentricity was used. The forms of the reconstructed slices for the off-center reconstruction are shown in Figure 6.12. It is the case that the center finding and normalization algorithm again prove effective at reconstructing the knot cluster positions, and analysis confirmed that the knot location values to be as accurate as was the case for the centered spiral reconstruction, as well as their lateral positions. It is clear that within the allowable eccentric limits of the setup, the small angle approximation for the path lengths holds and the reconstruction of the phantom is very minimally impacted.
The series of slice images for the reconstructions were converted into NIfTI file-format medical images, paying respect to their scaling and spacing to preserve real-dimension reconstruction geometry [29]. The centered and eccentric spiral reconstructed phantoms were visualized in the open-source segmentation software ITK-Snap, allowing for complete visualization parallel to the yz and xz planes of Figure 4.14 [30]. This showed the uniformity of the inner grain core along with outer sawdust annulus. Screen captures from the software for the central are presented in Fig 6.13 for the centered spiral reconstruction. It is clear from these images that the boundary between the dense center cylinder and the outer annulus is the noisiest location of the reconstruction. Using ITK-Snap, it is possible to produce a manual segmentation of the reconstructed phantom (the ambiguity of the grain cylinder prohibits use of automatic segmentation). A rendering of this segmentation is presented in Figure 6.14. With the algorithm appearing to supply accurate spatial reconstruction, real log reconstructions could be undertaken with confidence.

Figure 6.12: Off-center spiral reconstruction of physical phantom
Figure 6.13: Planar cross sections of centered spiral reconstruction
(a) yz plane (b) xz plane of Fig. 4.14

Figure 6.14: Segmentation rendering of physical phantom
Knots are shown in green, the crack object in pink. Grain and sawdust in translucent brown.
6.4.3 Real log reconstruction of dry log

The dried Douglas fir log reconstructed in the single slice case by An was the first natural log reconstructed. The specimen is a good case study for the algorithm because it contains many knots, and is easily penetrated by the cone beam at 140kVp and 5mA due to it being fully dried. There are several large knot clusters visible externally, as well as multiple smaller internal knots. The knot clusters in this particular tree are also nearly planar, that is they have an almost normal branch angle to the longitudinal dimension of the log. Examples of these are visible in the radiograph data sample of Figure 6.15.

![Figure 6.15: Knot clusters in dried douglas fir](image)

External knot cluster in red ellipse, internal knot cluster in yellow ellipse

In order to capture these small internal knots, the spiral script was run with a 72 sector, 15 annular reconstruction with $N_{\text{slices}}\_\text{view} = 40$. The geometry now dictates that the slices be .48cm thick, making 180 slices necessary to capture the entire log. This high resolution makes for the reconstruction to be $\sim 3\times$ as large as that of the physical phantom, and as expected the 20 iteration SART-smoothed script finishes in more time, approximately 30s. Several reconstructed knot clusters are presented in Figure 6.16. The scaling here is the same as that for the phantom reconstruction, and the color-bars indicate the density being within the range expected for dry
softwood (≈500kg/m$^3$).

Figure 6.16: Combined pattern dry log slices - centered spiral
(a) five large knot cluster (b) internal knot (c) four large knot cluster

The same reconstruction may be undertaken on the dry log using a lower resolution for the sake of speed. If it is the case that only large knots are of interest and that the heartwood-sapwood boundary is not critical, the same data set may be broken down into a sector-only case in which each slice is comprised of 72 sectors. This reduces the system of unknowns by a factor of 15, and as expected the convergence in this case takes much less time, <8s. In reality, a balance between speed and quality of detail is always the tradeoff of any CT system. Note from 6.17 that while larger externally visible knots are clearly recorded, smaller internal knots are lost in the clear wood (slice (b)). The optimization of this balance of resolution vs computational time must be varied according to the geometry of the reconstructed tree.

Figure 6.17: Sector pattern dry log slices- centered spiral
(a) five large knot cluster (b) “lost” internal knot (c) four large knot cluster
The dried Douglas fir log may be cut along its longitudinal major planes in the same way as was the phantom in order to examine the inter-slice boundaries for artifacts. The reconstructions presented in Figure 6.18 illustrate clearly two longitudinal orthogonal planes along the length of the sample. The heartwood sapwood boundary as well as elongated spike knots and captive face knots are visible.

![Figure 6.18: Dry log major planes](image)

(a) xz plane (b) yz plane

In this case, the discrete areas of high density in the knots make it possible to use the automatic level-set segmentation algorithm of ITK-Snap in order to extract the knots from the rest of the log without user input. This was performed, and the reconstructed knot structure is presented in Figure 6.19. The clear wood (both heartwood and sapwood) is presented in translucent yellow, the knots again in green, and the large twisting crack in pink. The planar nature of the knots is clearly evident.
6.4.4 Real log reconstruction of green log

A more challenging task is reconstruction of the green log, as it is of a much higher density. This makes x-ray penetration more difficult. It was found that the x-ray source need be run at its highest power for this, 160kVp and 5mA. A sample of the data is presented in Figure 6.20, shown to the same scaling as that of the dry log data (Fig 6.15).
The green Douglas fir sample differs from the dried one in that it has large distinct knots that extend to the surface. They are also less grouped than in the dry sample. Finally, the knots in this log have a large branch angle, that is they have grown at an inclination to the longitudinal (z) axis of the log. Also challenging with this log is an area that is not truly circular, but exhibits a locally flattened recess in which one of the knots is growing (visible in (b) of Fig 6.20). The system was put to the test by reconstructing the wet log in 60 sectors and 15 annuli. The geometry necessitated a slice thickness of .57cm and therefore ~140 slices for a complete reconstruction. Reconstruction takes ~15s. To handle the non circular region of the log, the estimated log radius $R_{pix}$ is scaled by 96% in order to truncate the outer most irregularities. The major planes of this fresh log are shown below in the captures from ITK-Snap. The intensity scale is the same as was used previously, which makes it evident that this log is more dense (it contains a mean density of 750kg/m$^3$, with sapwood averaging 900kg/m$^3$).
Note as well from Figure 6.21 the large branch angle, this makes showing the knots by slice less useful, as only a small part of each knot is observable in a planar transverse slice. As most coniferous trees grow with a consistent branch angle according to the geographical latitude at which the tree was harvested, this presents some opportunities that will be discussed. A transverse slice displaying a partial knot intersection is given in Figure 6.22. A rendering of a complete segmentation is given in Figure 6.23. The ambiguity of the knots from the sapwood in the outer rings of the tree make a distinct segmentation of this region via automatic methods impossible, and so the knots are truncated in this region in order to clearly show the sapwood boundary.
Figure 6.22: Wet log cross sectional slice  
Branch angle causes knot to pass through transverse slice.

Figure 6.23: Fresh log segmentation
Chapter 7

Segmentation of log internal features

7.1 Reconstruction of lumber surfaces

The inherent value of CT reconstructions of logs is accurate visualization and decision making about the processing that will optimize yield of knot and defect free lumber. The results for both the green and dried logs may be manipulated to demonstrate this. One of the most straightforward processing techniques in an industrial softwood mill is the plane-sawing of dimensional lumber and paneling. This is when a series of parallel cuts are made in a given direction at an interval to produce boards of varying widths. Take for example the production of siding of 1” thickness. Accounting for planing (i.e. adding half an inch to each thickness dimension yielding a rough cut of 1.5”) the dried and fresh logs yield seven boards each. They consist of two 1×6’s, two 1×8’s, and three 1×10’s. A diagram detailing the rough cut lumber’s arrangement within the log is presented in Figure 7.1. The corresponding boards produced from the dry log are presented in Figure 7.2 and those that come from the fresh log are shown in Figure 7.3. The images show clearly sapwood-heartwood transitions as well as the locations of spike and face knots. Defects are clearly shown in the case of the dry log.
Figure 7.1: Plane sawn boards

Figure 7.2: Dry log plane sawn boards
Visualizations of other lumber products are possible as well. In the case of quarter-sawn boards, boards are cut at alternating right angles to one another in each quadrant of the log. This makes it so the grain runs transversely to the thickness dimension of the board and reduces warping as the piece dries out. A diagram of five boards of one and a half inch rough cut thickness is presented below for the reconstructed real logs. The corresponding pieces from the dry log and green log are shown in Figs. 7.5 and 7.6. As expected, all boards present the transition from heartwood to sapwood characteristic of lumber produced in this manner.
Figure 7.5: Dry log quarter sawn boards

Figure 7.6: Fresh log quarter sawn boards
7.2 Feature identification via data plots

In the implementation of a log scanning package into an industrial softwood mill, there will not be time for a human operator to make decisions on what log orientation and combinations of cuts will yield the highest profit. Thus scanning and accurate reconstruction is only the first part of the problem. The various voxel values in the solution vector $\vec{\rho}$ must be assembled and the features contained recorded in space in order for machine decision making. Segmentation of a large number of disparate voxels based on thresholding alone is problematic. It is clear from Fig. 7.3 that individual voxels contained in the sapwood and knotty regions of the log may be of a high enough intensity so that knot features identified by simply their magnitude will be extremely distorted. This points to the necessity of more discerning “smart” algorithms to carry out the actual segmentation of the knots. Conventional segmentation algorithms are powerful tools for delineating the extents of features, but have traditionally required an operator to specify a smaller volume of interest and ’seed’ the body in a step known as registration. It is known that for medical applications, no machine has thus far been able to match humans in the process of registration, in large part due to the complexity of human organ shapes and their abnormalities [31].

Fortunately the a priori knowledge of the way in which knots are present in logs combined with the expedient indexing of the voxel pattern serves to make knots readily identifiable for segmentation seeding. Additionally, the data truncation and normalization ensures a volume of interest with no empty space. If the completed reconstruction vector is iterated through slice by slice and the annuli examined starting at the center and working outwards, it can be shown that the knots are present as local maxima in the density plots. This may be shown by looking at the dry log reconstruction slices of Figures 7.7 and 7.8, and then their corresponding density plots, presented here as surface plots over consecutive annuli within the slice. This is conveniently accomplished by simply taking the indexing of the reconstructed voxel density values as they are outputed by the algorithm and plotting them side by side by sector in ascending order. A cross section of the five knot cluster surface is presented in Figure 7.9, corresponding to the fourth annulus of voxels in that slice.
Figure 7.7: Surface corresponding to internal knot and crack

Figure 7.8: Surface corresponding to five knot cluster and crack
The clear local maxima that make up the knots may be recorded and their locations registered as the coordinates from which segmentations should be seeded. Delineation may then occur based in the reconstruction space from which convergence is sure to be accelerated due to numerically close neighbor voxels. The knot features may then be projected onto a single plane and a decision matrix implemented in order to orient the cut pattern so as to maximize value yield based off of predetermined factors. This implementation is of course a project unto itself.
Chapter 8

Conclusion

8.1 Limitations of the method

8.1.1 Assumptions made

The assumption of small displacements having a negligible effect on the series solution basis so that the log may simply be centered and scaled will clearly deteriorate for very large movements. An interesting case study could be made utilizing a smaller diameter physical phantom log so that the limits of this assumption could be evaluated.

The cone beam in this implementation was kept to a distinct central portion of the detector. This had two effects, one of keeping the CT data within the area of highest signal, and also minimizing the error due to small misalignment between the source and log and detector. The perspective alignment method of Figure 4.9 is sufficient for using only this smaller central region. Undoubtedly, the increased noise in reconstruction encountered when using the full detector width is in part to increasing perspective error.

8.1.2 Difficulty of integration into mill

The greatest challenge in this scheme’s implementation into a sawmill are the constraints of speed on the acquisition of data, reconstruction, and the preprocessing-registration-segmentation process. Given the constant operation nature of modern sawmills, the challenges of hardware to meet the constant service requirements of the CT scanner are formidable. Mechanisms must be designed in order to obtain data captures rotated around half of the log, and these in turn must be capable of long service life with minimal maintenance. Detector width, reconstruction
geometry, and physical detector selected must be coordinated to put a premium on speed to allow time for segmentation and log manipulation.

### 8.2 Future work and considerations

A major breakthrough in the method will be the development of non-planar slices that reflect the geometry of the branch angles in a given specimen. The essence of this is the extension of the feature tailored voxel concept into another dimension to provide the commensurate advantages as are available in the planar reconstruction case. The branch angle of a tree is determined by a great number of factors, including the environmental circumstances of the stand the tree was harvested from. Much research has been put into the environmental growth factors as they relate to branch angle, and these may be combined with sensing in order to optimize the conical planar geometry and spacing. Consider the image of Figure 8.1. It is clear that with altered slice geometries, fewer slices and thus voxels in total will be needed to capture an equivalent number of features in the green log.

![Figure 8.1: Green log reconstruction with superimposed conical slices](image_url)

For the hardware required for implementation, promising work has been done in the area of security CT scanning which has usage demands similar to those of sawmills. It is possible that
Electron Beam CT may be an attractive alternative to more conventional and fragile contact ring rotating gantry systems. Another possibility is a fixed cone beam orientation as used in this work’s experimental setup. This would require development of a system that may rapidly translate and rotate a log without any line down-time.

Finally, development of machine decision making schemes must be implemented in order to take the density vector output from the presented algorithm and make cut decisions. This must take into account not only the features of interest present in the log but also the desired product to be made and its quantity. The output must have rapid effect on log manipulation and orientation they are sent to different areas in the plant to be processed.

8.3 Conclusion

Raw material in the form of logs is the principal operating cost in modern sawmills. Research into the use of penetrative sensing in sawmills to maximize the profit from individual logs has been driven by this. Computed tomography has been explored as a possibility in realizing a system that can fully map out features of interest within logs and optimize orientations for cutting lumber. Implementations thus far have been costly, delicate, and overly precise for use in sawmills. The robust Algebraic Reconstruction Technique (ART) methods of CT reconstruction have gained some traction in industrial use, but are slower due to large numbers of generic Cartesian voxels.

In order to speed up ART, a customized “polar” voxel pattern geometry in which the shape and arrangement of the voxel structure reflects the features of interest within logs was developed. This greatly reduces the necessary resolution required for determination of features such as knots, the heartwood/sapwood boundary, and physical defects such as cracks. Careful manipulation of the log by requiring one sector rotation and one slice thickness translation in between data captures allows for efficient construction of the series solution basis by reusing the path lengths of just one view, saving computation time and allowing for rapid storage. A data scaling and normalization scheme allowed for precomputation of the basis before runtime and had the effect of making the inversion robust to the effects of log movement circumferentially and radially within the cone beam. Scaling also made it so no voxels within the reconstruction space would be wasted on empty space.

Synthetic data arranged so as to mimic real collected data pointed towards alteration of the center finding scheme as well as the need for solution filtering. Two iterative solving techniques, Randomized Kaczmarz and Simultaneous Algebraic Reconstruction Technique (SART), were
combined with a selective regularization scheme to provide fast reconstruction. Radiographic
data taken with an experimental setup was used in order to demonstrate the effectiveness of the
basis and solution scheme on a physical log phantom, a dried log, and a green log. The physical
phantom confirmed that no large scale distortion was introduced into the reconstructions. The
real log reconstructions and subsequent segmentations show the promise of the setup for lumber
quality sorting. The numerical indexing output of the reconstructions is shown to be convenient
for quick registration and segmentation of knot features.
Bibliography


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