Abstract

This dissertation studies the incentives of economic agents to acquire information about financial assets when it must be acquired through time-consuming research. When information can not be obtained instantaneously, agents face a tradeoff between acting immediately and performing more thorough research. Better information allows for more informed trading decisions, but research is costly because other agents’ trades move prices adversely as time passes.

The first chapter develops a theoretical model in which agents sequentially trade a single financial asset. Each agent receives weak, private information when they arrive to the market and may trade immediately, or instead wait for additional information. Should they wait, other agents have an opportunity to trade before the first agent receives their additional information, which creates an endogenous cost to waiting. The analysis determines the conditions under which equilibrium behavior involves immediate trades (“panics”), and then studies the quantitative impacts of weakly informed trades on the ability of prices to aggregate information.

The second chapter experimentally tests the theoretical model in a laboratory setting, in order to determine whether or not subjects understand the tradeoff between better quality information and potential adverse price movements. Comparative static results establish that the theory broadly explains when panics, and the corresponding informational losses, occur. However, additional, “heuristic” panics are also frequently observed. Specifically, subjects exhibit a strong tendency to wait for more information when highly uncertain about asset values, but switch to trading as soon as possible once values become more certain.

Motivated by the findings of the second chapter, the third chapter ex-
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tends both the theory of the first chapter and the experimental results of the second chapter to a second, richer environment. Different from the sequential structure of the first model, agents may trade simultaneously in the richer model. Experimental results with the richer model produce trade clustering and serial correlations in returns, as predicted by the heuristic behavior identified in the second chapter. These phenomena are well-established features of real financial markets, suggesting that the heuristic subjects follow in the laboratory may provide a novel explanation for these phenomena.
Preface

This dissertation consists of original, independent, unpublished work by the author, Chad Kendall. The experimental work discussed in Chapters 2 and 3 is covered by UBC Behavioral Ethics Board Certificate number H13-01124.
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Chapter 1

Introduction

Information about financial assets fundamentally takes time to both acquire and to process into a trading decision. To uncover information that is not already publicly available, and hence not already incorporated into prices, one must perform time-consuming research. The large literature on incentives to acquire information in financial markets, however, assumes that one obtains private information by paying a monetary cost, so that information is acquired instantaneously. In this dissertation, I relax this assumption and ask what are the incentives to acquire information when it can only be acquired over time.

When information takes time to acquire, agents face a tradeoff between acquiring relatively weak information quickly, or performing more research to obtain better information. Better information allows one to make a more informed trading decision, but is costly if other agents may trade while one is researching the asset. This cost of research means that it may be rational to trade as soon as possible. If everyone rushes to trade immediately, a trading panic results. The existing literature on rational trading panics has emphasized the role of panics in explaining price crashes while I instead study how panics reduce incentives to perform time-consuming research, resulting in markets that aggregate only relatively weak information.

In the first chapter, I develop a stylized model in which agents sequentially trade a single financial asset. When a trader arrives to the market, they receive relatively weak, private information. They may trade immediately, or wait to obtain better information and trade then. I adopt an overlapping arrival structure so that, while one agent waits, the subsequent agent arrives to the market and has an opportunity to trade. The possibility that the next agent may trade creates an endogenous cost of waiting,
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because others’ trades move prices adversely in expectation, reducing one’s profits. Intuitively, others are more likely to buy the asset when you uncover information that the asset is currently undervalued. I characterize the equilibria of the model and determine the conditions under which rational panics (immediate trades) occur. Interestingly, the model predicts that better information is acquired only when uncertainty about the fundamental value of the asset is low, in stark contrast to existing models of information acquisition. I further quantify the effects of panics on the ability of prices to aggregate information, showing that panics increase the time it takes prices to converge to fundamental values. Finally, I develop several empirical predictions, some of which are confirmed by existing studies.

In the second chapter, I experimentally test the model of the first chapter in a laboratory setting. The purpose of the experiment is to determine the extent to which observed panics can be predicted by theory. Importantly, precise theoretical predictions allow me to categorize panics as either rational, equilibrium behavior, or the result of other trading heuristics. I find that equilibrium behavior explains the majority of observed panics, but that the use of a particular heuristic results in additional panics. The identified heuristic is closely related to momentum trading in which subjects wait for more information when uncertain about the asset’s fundamental value, but rush to trade in the direction of past price movements once the uncertainty is partially resolved.

Motivated by the identification of heuristic-based panics, the third chapter extends both the theory and experimental work to a richer environment in which agents can trade simultaneously. There, heuristic-based panics are predicted to result in trade clustering and serial correlation in returns. Both of these phenomena are found to occur in the experimental data, suggesting that the identified heuristic may play a role in explaining these two common features of real-world financial markets.

The theoretical work of the dissertation relates conceptually to two separate strands of the literature on financial markets: that on information acquisition and that on rational panics. The literature on information acquisition began with the work of Grossman and Stiglitz (1980). In their static
model, agents are less likely to acquire information when others do so, due to the fact that one can infer others’ information from prices. More recently, information acquisition has been studied in sequential models more closely related to the one considered here. Chamley (2007) considers the situation in which traders trade for short-term profits, unwinding their investments in the period after initial trade. He shows that, in this case, agents can be more likely to acquire information when others do: there is strategic complementarity in acquisition decisions, contrary to the strategic substitutability in Grossman and Stiglitz (1980). Strategic complementarity also arises in the models considered here, although its nature is quite different. Nikandrova (2012) and Lew (2013) consider sequential trading models in which agents are only present in the market for one period. In their models, information is only acquired when uncertainty about the fundamental value of the asset is high because it is at this time that information is most valuable. In all of these models, agents acquire private information instantaneously by paying a monetary cost. By considering the fact that information requires time-consuming research, I obtain very different results. Information is foregone during times of high uncertainty. Although it is most valuable at this time, the endogenous cost due to others’ trades is also largest at this time. Intuitively, when uncertainty is high, prices move a lot, making the cost of waiting higher.

The literature on rational panics, some of which shares the intuition that agents want to trade as soon as possible to avoid the price movements of others’ trades, has focused on explaining price crashes. The most closely related papers are Bulow and Klemperer (1994), Brunnermeier and Pedersen (2005), and Pedersen (2009).¹ In Brunnermeier and Pedersen (2005) and Pedersen (2009), rational panics occur when traders try to front-run each other. In their model, an exogenous shock to a trader’s holdings forces her to liquidate her position. Other traders then try to sell before the distressed trader pushes the price down, resulting in a price crash that may have have a ripple effect on other traders. A similar ripple effect occurs here, but it

¹For other papers on rational panics, see Romer (1993), Smith (1997), Lee (1998), and Barlevy and Veronesi (2003).
can happen in the absence of an exogenous shock. In Bulow and Klemperer (1994), a single seller sells $K$ objects to $K + L$ bidders through a series of sequential auctions. Bidders trade off waiting for a lower price against the probability that they will not receive a good. As a result, both clustering of trades and price crashes arise. While the models considered here share some of the underlying intuition of these papers, the focus is on studying the effects of panics on the quality of information aggregated by prices, not on price crashes (which do not arise here).

In terms of model structure, my work is related to the financial herding literature that utilizes the sequential trading model first introduced by Glosten and Milgrom (1985) (and, more broadly, to the herding literature that originated with Banerjee (1992) and Bikhchandani et al. (1992)). For a survey of the earlier financial herding literature, see Devenow and Welch (1996), and for more recent contributions, see Avery and Zemsky (1998), Park and Sabourian (2011), and Dasgupta and Prat (2008). In these papers, traders may ignore their private information to follow (herd) or trade contrary to their predecessors, which creates inefficiencies in information aggregation. Herding and contrarianism do not arise here, but convergence slows through another channel: not waiting to acquire information. Other related theoretical papers include those that allow for trade timing, such as Malinova and Park (2012), and Ostrovsky (2012), who extends the insider trading model of Kyle (1985) to multiple traders. In these papers, information is exogenously given, so information acquisition plays no role.

The experimental work extends a relatively small experimental literature on trade timing in financial markets. Importantly, the work extends previous studies by deriving precise theoretical predictions about when subjects should trade. Park and Sgroi (2012), in the most closely related paper, provide qualititative predictions guided by theory in an endogenous timing setting. They focus on the possibility of herding with exogenously given information. Shachat and Srinivasan (2011) study trading with sequential arrival of information, but in an environment in which a no trade theorem applies. Bloomfield et al. (2005) study the choice between market and limit orders in the absence of theoretical predictions. Several papers consider
Timing decisions in environments in which prices are fixed and theoretical predictions are known. See, for example, Sgroi (2003) and Ziegelmeyer et al. (2005) who each implement the irreversible investment model of Chamley and Gale (1994). In similar environments, see Ivanov et al. (2009), Ivanov et al. (2013), and Çelen and Hyndman (2012). Incentives to wait are quite different in these papers as observing others’ decisions can be beneficial when their information is not incorporated into prices. Finally, Brunnermeier and Morgan (2010) study timing decisions theoretically and experimentally in a game related to both preemption and war of attrition games.
Chapter 2

Informational Losses in Rational Trading Panics

2.1 Introduction

“Apple Stock Hits All-Time High As Investors Rush To Get In Early Before iPhone 5” - August 17, 2012, Cult of Mac

“Apple Stock Hit by Panic Selling: ’Someone Yelled Fire’” - November 15, 2012, CNBC

Informal commentary abounds with anecdotal evidence of “panic selling” and “panic buying” in financial markets. The two articles cited above claim to provide explanations for both booms and crashes in Apple’s stock. In panic buying, or “buying frenzies”, traders rush to buy a stock for fear of missing out on continued price increases. The first article quotes a fund manager as saying that “his biggest fear [...] is that he won’t get [the] chance to put all of his money into Apple before the share price skyrockets.” In the opposite direction, panic selling refers to traders rushing to liquidate their positions before prices fall further. The second article quotes David Greenberg of Greenberg Capital as saying “Someone yelled fire in the theater [...] and as traders do, they will trample you trying to be first to get to the exit.” These quotations illustrate the common perception that panics are driven by emotional or irrational investors who trade out of the fear of adverse price movements.

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This chapter shows that this behavior is not necessarily irrational: perfectly rational investors may optimally “panic”. Although the word “panic” in a financial market setting may have many different connotations, here I associate the term with the general meaning of the word: a sense of urgency to act as soon as possible. I define panics in a market setting as trading at the first opportunity, which does not necessarily imply irrational behavior or price movements in a single direction (which do not arise here). I show that panics are a natural consequence of unconditional correlation of information. Under the standard assumption that private signals are independent conditional on the asset’s true value, signals are *unconditionally* correlated: traders are likely to receive similar news. Consequently, others’ trades are likely to move prices adversely, rationalizing the fear of adverse price movements, and motivating one to preemptively trade. This rational fear of adverse price movements appears in previous papers, specifically Bulow and Klemperer (1994), Brunnermeier and Pedersen (2005), and Pedersen (2009). These papers have focused on explanations for price crashes, taking information as exogenously given. I instead consider the impact of panics on the informational content of trades, a topic that has not been previously addressed. Intuitively, if traders panic, they forgo the opportunity to acquire additional information through time-consuming research. This failure to “do one’s homework” can result in trades that are based on weak information, causing prices to more frequently deviate away from fundamental values and take longer to converge. As other researchers have noted (Vives, 1993), convergence speeds are important because knowing that prices converge to an asset’s fundamental value is not practically useful if convergence is so slow that, by the time it occurs, the value has changed.

I build upon the classic trading model of Glosten and Milgrom (1985) in which privately-informed agents sequentially trade an asset with a market maker. In the period in which she arrives, a trader may buy or sell short a single unit of an asset (with no restrictions on short sales). To this model I add an information acquisition decision in which traders can wait to obtain additional information before trading. Requiring traders to wait to obtain information reflects the fact that information is fundamentally gen-
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erated through time-consuming research and must be processed to generate one’s trading decision. It is this feature that sets the work here apart from the existing literature on information acquisition in financial markets, which instead assumes information can be acquired at a monetary cost. Equilibrium predictions are substantially different when information takes time to acquire: waiting to obtain information cannot be equivalently modeled as paying a monetary, opportunity cost.

To focus on the simplest possible timing decision, traders may trade only once in one of two periods: the period in which they arrive or in the subsequent period. Traders receive a private signal in the period in which they arrive and, should they choose to wait, an additional private signal in the subsequent period.\(^3\) In order to study preemptive trading, I adopt an overlapping timing structure such that if a trader waits, another trader may front-run her. The option value of waiting therefore depends upon the strategy of a trader’s successor. This payoff interdependence in a social-learning setting typically makes characterizing the set of equilibria difficult.\(^4\)

As in Glosten and Milgrom (1985), all trades are made with a market maker. In the first version of the model (the expected value model), I assume that the market maker sets a single price equal to the expected value of the asset conditional on all available public information.\(^5\) In this model, I abstract from the bid and ask prices that arise from the adverse selection problem in order to focus on the strategic interaction between traders. Under intuitive restrictions on off-equilibrium beliefs and prices, I obtain a

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\(^3\)The substantive assumption is that, should one trade upon arrival, one cannot also trade after receiving the additional signal. This assumption is meant to capture, in a tractable way, the idea that information arrives continuously, but trades are necessarily discrete (trading continuously, even if technically feasible, would be prohibitively costly). Therefore, a trader contemplating a trade at any point in time must decide how much information to obtain before trading. For tractability, I focus on one such decision, which eliminates incentives to manipulate prices or not reveal one’s private information (as in Kyle (1985)).

\(^4\)For examples of papers with interdependent payoffs, see Callander (2007), Ali and Kartik (1998), and Chamley (2007), which is discussed further below.

\(^5\)As in the financial herding literature, prices reflect all past information, so there is no incentive to wait to learn from others’ trades as there is in models of common investment opportunities, such as Chamley and Gale (1994), Gul and Lundholm (1995), and Chari and Kehoe (2004).
complete characterization of all equilibria. I then consider the zero-profit model, adopting the standard market microstructure setting in which competitive market makers post separate bid and ask prices, earning zero profits in expectation. Here, a complete characterization of the equilibria is complicated by the additional strategic interaction between the traders and the market maker. However, Section 2.5 demonstrates that the main insights of the expected value model extend to this case.

Panics (in which all traders buy or sell immediately upon arrival) occur when prices reflect uncertain public beliefs. Interestingly, because information is most valuable when uncertainty is high, this result implies that panics cause information to be forgone precisely when it is most valuable. This counter-intuitive finding results from the fact that the cost of waiting is endogenously determined by the price impact of others’ trades, which is largest when uncertainty is high. This result goes against standard intuition from the literature on information acquisition in financial markets. As first noted by Grossman and Stiglitz (1980), there is typically strategic substitutability between traders’ decisions to acquire information: when others acquire information, prices have strong informational content so that one has little incentive to do her own research. Conversely, here there is strategic complementarity: acquiring information is less costly when others are doing so. Intuitively, if others are not in a rush to trade, there is no need for you to rush either.

Strategic complementarity in traders’ timing decisions leads to a multiplicity of equilibria for some parameterizations. This multiplicity is reflected in the effects of market commentary on real asset markets, where news of panic can lead to further panic. It may also suggest a role for market intervention. For example, if a trading halt (or circuit-breaker) intervention is sufficient to change expectations of panic, panics can in fact be soothed.

In a related finding, Chamley (2007) considers a Glosten-Milgrom setup in

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6See Barlevy and Veronesi (2010) for an interesting counterexample.
7For an example of a successful trading halt, see http://www.ft.com/intl/cms/s/0/710240e6-1945-11e2-9b3e-00144feabd0.html#axzz2NCmc7u66: “Google shares fell as much as 10 per cent to $676 before trading was halted. Their partial recovery to $695 when trading resumed ...”
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which market participants trade for short-term profit, unwinding their positions in the period after they first trade. His setting also produces strategic complementarity in acquiring information and a multiplicity of equilibria. However, in his model, information is acquired at a monetary cost and trade timing is exogenous, so the nature of the strategic complementarity is much different. He also does not explicitly study the effect of information acquisition on the informational content of prices, instead focusing on its ability to produce discontinuous trader behavior and trading volumes.

Because information is forgone when it is most valuable, price convergence is much different than in sequential trading models with monetary costs of information, such as that of Nikandrova (2012) and Lew (2013). In their models, more information is acquired when uncertainty is high and information is most valuable. There, starting from a price reflecting high uncertainty, prices converge quickly to a price near the true asset value and then convergence slows (or stops completely if there is a fixed cost component), whereas here, convergence slows down immediately. Simulation results show that these slowdowns are quantitatively significant, as much as doubling the time required to converge under certain realistic parameterizations. Panics also cause prices to more frequently deviate away from fundamental values.

In a particularly counter-intuitive and perhaps troubling result, I show that increasing the quality of the information immediately available can actually slow convergence overall. This result can contribute to the debate regarding the impacts of high-frequency traders in the market (Biais et al. (2013) and Hoffmann (2013)). Technical innovations that make better quality information available more quickly can actually reduce the amount of information that is incorporated into asset prices.

The model can capture several existing empirical findings, including that the estimated probability of informed trading (Easley et al., 1996) and persistent price impacts of trades (Hasbrouck, 1991) are lower when trading volume is higher. Section 2.6 develops several testable predictions and discusses existing evidence that is suggestive of the novel predicted relationships between the informational content of trades and volume and uncertainty.
2.2 The Expected Value Model

The model is set in discrete time, \( t = 1, 2, \ldots, T \). A single asset of unknown value, \( V \in \{0, 1\} \), may be traded in each period. Its value is realized at \( T \).

For convenience, I assume \( T \to \infty \) but the results do not depend on this assumption.\(^8\) The prior belief that \( V = 1 \) is given by \( p_1 \in (0, 1) \). In each period, \( n \) new risk-neutral traders enter the market. In the remainder of the model description and the analysis of Section 2.2, I focus on the case of \( n = 1 \). In Appendix A.5, I consider the case of \( n > 1 \) to establish a comparative static prediction with respect to volume.\(^9\)

Upon arrival, each trader receives a private, binary signal, \( s_t \in \{0, 1\} \), which matches \( V \) with probability \( q = Pr(s_t = 1|V = 1) = Pr(s_t = 0|V = 0) \in (\frac{1}{2}, 1) \). I identify a trader by the period in which she arrives, \( t \). Given her signal, trader \( t \) may choose to buy, sell, or not trade, \( a_t \in \{B, S, NT\} \). If she trades, she leaves the market. If she chooses not to trade, she receives an additional, private, binary signal in period \( t+1 \), \( s_{t+1} \in \{0, 1\} \), which matches \( V \) with probability \( \eta = Pr(s_{t+1} = 1|V = 1) = Pr(s_{t+1} = 0|V = 0) \in (\frac{1}{2}, 1) \) and may then trade, \( s_{t+1} \in \{B, S, NT\} \). All signals are assumed to be independent conditional on \( V \). To allow for trades to occur during the time a trader waits to acquire the additional signal, each period is divided into two sub-periods with trader \( t + 1 \) trading prior to trader \( t \) if they trade in the same period.\(^{10}\) The timing of possible trades is shown in Figure 2.1. I abbreviate the time of trade as \( R \) for rush (\( a_t \in \{B, S\} \)) and \( W \) for wait (\( a_t = NT \)). With this overlapping arrival structure, a trader \( t \) that chooses to acquire additional information may face up to two intervening trades: one from trader \( t - 1 \) (if she chose to wait) and one from trader \( t + 1 \) (if she chooses to rush). The complete history of trades, timing decisions, and

\(^8\)For finite \( T \), the strategy of the final trader must be considered separately as no further traders arrive to the market.

\(^9\)The \( n > 1 \) results are also used in Section 2.4 to numerically assess the impacts of panics.

\(^{10}\)\( t \) is used to refer to the period containing the two sub-periods. A lower bar identifies values of variables (signals, actions, prices, etc.) in the first sub-period and an upper bar identifies values in the second sub-period.
2.2. The Expected Value Model

Figure 2.1: Trade Timing

Note: Arrows reflect the two periods in which each trader may trade. Trade identifier subscripts denote period, not trader identity.

prices, denoted $H_t$ and $\overline{H}_{t+1}$, are observed by all traders.\(^{11}\)

A risk-neutral market maker posts a single price equal to the public belief about the value of the asset, its expected value based upon all public information, $p_t = E[V|H_t] = Pr[V = 1|H_t]$ or $p_{t+1} = E[V|\overline{H}_{t+1}] = Pr[V = 1|\overline{H}_{t+1}]$. The expected payoffs to a trader who buys the asset at $t$ or $t+1$ are then, $E[V|H_t, s_t] - p_t$ or $E[V|\overline{H}_{t+1}, s_t, s_{t+1}] - p_{t+1}$, respectively. The expected payoffs from selling are identical but with opposite sign. $p_t$ plays a more prominent role in the analysis so I will generally refer to it as the price at $t$ and denote it $p_t$, unless a distinction must be made.

I emphasize that the market maker is not a strategic player: prices may be best thought of as being determined by an entity that is more concerned about information aggregation than profits (for example, the owner of a prediction market). Given the strategies of the traders, the market maker can compute the public belief on the equilibrium path, but at off-equilibrium histories some assumption must be made. In particular, if the equilibrium

\(^{11}\)Formally, the histories can be defined recursively as $A_t = \overline{A}_{t-1} \cup \overline{P}_{t-1}$, $P_t = A_t \cup \overline{p}_t$, and $P_{t+1} = P_t \cup P_{t+1}$, for $t = 2, \ldots$ where $A_1 = \emptyset$, $A_t = A_t \cup a_t$, and $P_1 = P_1 = p_1$. $H_t = A_t \cup P_t$ and $\overline{H}_{t+1} = \overline{A}_{t+1} \cup \overline{P}_{t+1}$ denote the joint action and price histories.
strategy is for traders with both $s_t = 0$ and $s_t = 1$ to rush, then the price that arises should a trader deviate to wait depends upon what information about $s_t$ is assumed to be revealed by the deviation.\textsuperscript{12} Fortunately, a natural assumption arises in the model. Lemma 2 shows that, independent of any particular assumptions about off-equilibrium prices, traders with $s_t = 0$ and $s_t = 1$ choose to wait with the same probability in any equilibrium. As a consequence, after a delayed trade, no information about $s_t$ is revealed and therefore the public belief is unchanged. Given this, I assume that if a trader deviates to wait, the public belief and price are also unchanged. I restrict the analysis to equilibria that satisfy this restriction on prices, providing further justification after Lemma 2 in Section 2.3.2.\textsuperscript{13}

Being a dynamic game of incomplete information, the appropriate solution concept for the model is sequential equilibrium (Kreps and Wilson (1982)).\textsuperscript{14} In addition to the restriction on price formation, I require that strategies be a function of only the payoff-relevant state (i.e. Markov strategies). While the sequence of past trades and/or prices could be used as coordination devices, I ignore such possibilities. The payoff-relevant state for a trader trading at time $t + 1$ is simply the price she faces. But, for a trader trading at time $t$, the price she faces and whether or not the previous trader, $t − 1$, rushed are both relevant. The price reflects any information revealed by the decision of $t − 1$ to wait, but when $t − 1$ waits, $t$’s expected profit from waiting is impacted by $t − 1$’s delayed trade. Formally, an equilibrium of the expected value model is defined as follows.

\textsuperscript{12}Other off-equilibrium histories are possible but it turns out that the assumption made about prices in these cases is not critical, so I make no particular assumption.\textsuperscript{13}The assumption can be thought of as a restriction on off-equilibrium beliefs as far as the other traders are concerned. Because the market maker is essentially a robot, however, formulating the restriction in terms of beliefs may lead to confusion. Under other assumptions about price formation, the general analysis is unaffected, with the main difference being that the cutoff prices in Theorem 2.1 are more difficult to compute.\textsuperscript{14}Sequential equilibrium, as opposed to weak Perfect Bayesian equilibrium, places restrictions on beliefs off-equilibrium which help to pin down the public belief, and therefore price. It is not, however, restrictive enough to completely pin down beliefs after a trader deviates to wait and thus an additional assumption is still necessary.
2.2. The Expected Value Model

**Equilibrium Definition:** An equilibrium of the expected value model consists of a set of behavioral strategies, \( \sigma_t : (H_t, s_t) \to \{B, S, NT\} \) and \( \sigma_{t+1} : (\overline{H}_t, s_t, s_{t+1}) \to \{B, S, NT\} \), a system of beliefs \( \nu \) and a sequence of prices, \( p_t \) and \( p_{t+1} \), such that:

1. \( \sigma_t \) and \( \sigma_{t+1} \) are sequentially rational given beliefs \( \nu \);
2. There exist a sequence of completely mixed strategies \( \{\sigma^k_t\}_{k=1}^{\infty} \) and \( \{\sigma^k_{t+1}\}_{k=1}^{\infty} \) with \( \sigma_t = \lim_{k \to \infty} \sigma^k_t \) and \( \sigma_{t+1} = \lim_{k \to \infty} \sigma^k_{t+1} \) such that \( \nu = \lim_{k \to \infty} \nu^k \) where \( \nu^k \) denotes the beliefs derived from \( \sigma^k_t \) and \( \sigma^k_{t+1} \) using Bayes’ rule;
3. Prices satisfy \( p_t = \mathbb{E}[V | H_t] \) and \( p_{t+1} = \mathbb{E}[V | \overline{H}_{t+1}] \) as derived from \( \sigma_t \) and \( \sigma_{t+1} \) using Bayes’ rule wherever possible;
4. At any history: (i) \( \sigma_t \) must specify the same behavioral strategy for any two traders who face the same price \( p_t \), the same timing decision of their immediate predecessor \( I(a_t = NT) \), and have the same signal \( s_t \); (ii) \( \sigma_{t+1} \) must specify the same behavioral strategy for any two traders who face the same price \( p_{t+1} \), and have the same signals \( s_t \) and \( s_{t+1} \);
5. At an off-equilibrium history \( \overline{H}_t \) reached by a trader waiting: (i) beliefs \( \nu \) are common among all traders and are such that the public belief does not change, \( \mathbb{E}[V | \overline{H}_t] = \mathbb{E}[V | H_t] \); (ii) prices are unchanged, \( \overline{p}_t = \overline{p}_t \).

Items 1-2 form the standard definition of sequential equilibrium. Item 3 specifies that prices are set equal to the public belief as derived from the traders’ strategies. Item 4 is the restriction of strategies to payoff-relevant states. Finally, item 5 is the restriction on prices and beliefs after a trader deviates to wait in the period she arrives. In the definition, \( I \) denotes the indicator function so that \( I(a_{t-1} = NT) \) is 1 if \( t \)'s predecessor waited and 0 otherwise.

Without loss of generality, strategies can be decomposed into a trading strategy (buy or sell) for each of the two periods in which a trader may trade and a timing strategy (rush or wait). When strategies are restricted
to payoff-relevant states, we can define the probability with which a trader observing a particular first period signal (which I refer to as her type) waits as \( \beta_x(p_t, I(a_{t-1} = NT)) \equiv Pr(a_t = NT|s_t = x, p_t, I(a_{t-1} = NT)) \). When it doesn’t lead to confusion, I drop the dependencies of \( \beta_x \).

### 2.3. Analysis of the Expected Value Model

To characterize the equilibria of the expected value model, I first prove several intermediate results that are of interest on their own. The first lemma determines the optimal trading strategy taking the timing strategy as given. The second lemma proves that both types of traders must follow the same timing strategy in any equilibrium. Proposition 2.1 establishes the motivating intuition of the model: the presence of other traders reduces the expected profit from waiting. Proposition 2.2 establishes the interesting fact that, as prices become certain, traders wait to obtain additional information.

#### 2.3.1 Optimal Trading Strategy

Lemma 2.1 provides the optimal trading strategy of a trader taking as given the timing strategies, \( \beta_1(p_t, I(a_{t-1} = NT)) \) and \( \beta_0(p_t, I(a_{t-1} = NT)) \). All proofs are contained in Appendix A.3.
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Lemma 2.1: In any equilibrium:

1. Any trader who rushes buys if $s_t = 1$ and sells if $s_t = 0$.
2. With timing strategies $\beta_1(p_t, I(a_{t-1} = NT))$ and $\beta_0(p_t, I(a_{t-1} = NT))$, at least one of which is strictly positive, for all $p_t$ and $I(a_{t-1} = NT)$, any trader who waits:
   (a) buys if $s_t = 1$ and $s_{t+1} = 1$; sells if $s_t = 0$ and $s_{t+1} = 0$
   (b) buys (sells) if $s_t = 0$, $s_{t+1} = 1$, and $g_0(q, \overline{q}) \equiv (1 - q)\overline{q}NT_0 - q(1 - \overline{q})NT_1 \geq (\leq)0$
   (c) buys (sells) if $s_t = 1$, $s_{t+1} = 0$, and $g_1(q, \overline{q}) \equiv q(1 - \overline{q})NT_0 - (1 - q)\overline{q}NT_1 > (\leq)0$

where $NT_0 = (1 - q)\beta_1(p_t, I(a_{t-1} = NT)) + q\beta_0(p_t, I(a_{t-1} = NT))$,
\[ NT_1 = q\beta_1(p_t, I(a_{t-1} = NT)) + (1 - q)\beta_0(p_t, I(a_{t-1} = NT)) \]
are shorthand for $Pr(a_t = NT|V = 0)$ and $Pr(a_t = NT|V = 1)$, respectively.

For rushed trades, the result of Lemma 2.1 is intuitive: traders with good signals buy and those with bad signals sell. For delayed trades, the optimal trading strategy is also determined by comparing the trader’s private belief to the price she faces, but the price she faces depends upon the timing strategies of the two types of traders, $\beta_0$ and $\beta_1$. If $\beta_0 \neq \beta_1$, some information about $s_t$ is revealed to the market when a trader waits and so her optimal trading strategy may depend upon these strategies. If $\beta_0 = \beta_1$, no information is revealed, so that the resulting optimal trading strategy is independent of the timing strategies (as can be seen by setting $\beta_0 = \beta_1$ in Lemma 2.1). By assumption, a deviation to wait does not reveal information and thus the optimal trading strategy in the continuation game after a deviation to wait corresponds to setting $\beta_0 = \beta_1$ in Lemma 2.1. Also, note that other traders’ strategies do not affect the delayed trading strategy of $t$ because any information revealed by their trades is both known to the trader and reflected in the price.

An implication of Lemma 2.1 is that, except in the case of indifference
2.3. Analysis of the Expected Value Model

when a trader’s signals oppose each other, a trader always trades.\textsuperscript{15} This simple fact is a consequence of the trader always having an informational advantage and hence a profitable trade.\textsuperscript{16} As a result, some private information is revealed by each trade. In Section 2.4, this fact will be used to establish that prices converge asymptotically to the true asset value.

2.3.2 The Benefit of Additional Information

To determine the optimal timing strategy of a trader, one must calculate the value of waiting to obtain additional information: the expected profit from trading at $t + 1$ less the profit from trading at $t$. When the benefit is positive (negative), a trader’s best response is to wait (rush). The benefit from waiting depends upon the timing strategies of $t$ because information may be revealed by her decision to wait. In equilibrium, the timing strategy for each type of trader must be consistent with the sign of the benefit.

The benefit from delaying also depends upon the timing and trading strategies of $t - 1$ and $t + 1$, because any trade that occurs while $t$ waits affects the price she will trade at. Initially, I use generic notation to denote the actions of the other traders because several properties of the benefit to waiting can be established for any possible strategies of others. I denote a generic event that occurs during the time $t$ waits as $\hat{a}$ and the set of possible such events as $A$, so that $\hat{a} \in A$. I also abbreviate $Pr(\hat{a}|V = y)$ as $\hat{a}_y$, for $y \in \{0, 1\}$, so that, when summing over all possible events, $\sum_{\hat{a} \in A} \hat{a}_0 = \sum_{\hat{a} \in A} \hat{a}_1 = 1$.\textsuperscript{17} In Appendix A.1, I derive a general form of the benefit from waiting, which I denote as $B_x(p_t, \beta_0, \beta_1)$ for traders with $s_t = x$. It is given by

\begin{align*}
\text{For concreteness, I assume a trader that is indifferent trades according to } \pi_{t+1} \text{ but the analysis is qualitatively robust to other assumptions, including that she does not trade.}
\end{align*}

\begin{align*}
\text{Another implication is that herding and contrarianism are precluded: a trader never ignores her private information and either copies or trades contrary to her predecessors.}
\end{align*}

\begin{align*}
\text{As an example, if } t - 1 \text{ and } t + 1 \text{ both rush, then } A = \{\xi_{t+1} = B, \xi_{t+1} = S\} \text{ is the set of possible events. From Lemma 2.1, } t + 1 \text{ buys if } \xi_{t+1} = 1 \text{ and sells if } \xi_{t+1} = 0, \text{ so } \sum_{\hat{a} \in A} \hat{a}_1 = \sum_{\hat{a} \in A} \hat{a}_0 = q + 1 - q = 1. \text{ If instead, both } t - 1 \text{ and } t + 1 \text{ trade during the time } t \text{ waits, then each possible (joint) event is more complex and may consist of, for example, } t - 1 \text{ buying and } t + 1 \text{ waiting, etc.}
\end{align*}
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\[ B_x(p_t, \beta_0, \beta_1) = \frac{p_t(1 - p_t)}{Pr(s_t = x)} \left[ \sum_{\hat{a} \in A} \hat{a}_0 \hat{a}_1 f(q, \bar{q}, \beta_0, \beta_1) \right] \left( \frac{Pr(\hat{a}_t = NT)}{Pr(\hat{a}_t = NT)} - (2q - 1) \right) \]  
(2.1)

where

\[ f(q, \bar{q}, \beta_0, \beta_1) = \begin{cases} 
(2\bar{q} - 1)(qNT_0 + (1 - q)NT_1) & \text{if } s_t = 1 \land g_1(q, \bar{q}) \leq 0 \\
qNT_0 - (1 - q)NT_1 & \text{if } s_t = 1 \land g_1(q, \bar{q}) > 0 \\
qNT_1 - (1 - q)NT_0 & \text{if } s_t = 0 \land g_0(q, \bar{q}) < 0 \\
(2\bar{q} - 1)(qNT_1 + (1 - q)NT_0) & \text{if } s_t = 0 \land g_0(q, \bar{q}) \geq 0 
\end{cases} \]

The formula for the benefit (2.1) is only defined when at least one type of trader waits with a positive probability. If both types rush with probability 1, then each of their benefits from deviating to wait depends upon the price set after such an off-equilibrium action. Without imposing any restriction on these off-equilibrium prices, however, Lemma 2.2 establishes that both traders must follow the same timing strategy in equilibrium.

Lemma 2.2: In any equilibrium, traders with \( s_t = 0 \) and \( s_t = 1 \) must follow the same timing strategy, \( \beta_1(p_t, I(\hat{a}_{t-1} = NT)) = \beta_0(p_t, I(\hat{a}_{t-1} = NT)) \forall p_t, I(\hat{a}_{t-1} = NT) \).

The intuition behind Lemma 2.2 is instructive and the proof by contradiction follows the intuitive argument closely. If, for example, a trader with a good signal were to wait more often than a trader with a bad signal, then the market maker would infer from the decision to wait that the first period signal is more likely to be good and thus raise the posted price. This increase in price benefits the trader with a bad signal (who is more likely to sell) more so than the trader with a good signal (who is more likely to buy). Thus, if the trader with a good signal is waiting with some positive probability, the trader with a bad signal must be waiting with certainty, contradicting the initial assumption that the trader with a good signal waits more often.

If both types of trader rush with probability 1, then their expected profit
from deviating depends upon the information that is assumed to be revealed by the deviation. But, under the assumption made about off-equilibrium price formation, no information is revealed, which corresponds to setting $\beta_0 = \beta_1$ in (2.1). Therefore, Lemma 2.2 and this assumption together imply that we can set $\beta_0 = \beta_1 = \beta$ in (2.1) for the remainder of the analysis.\footnote{Note that, were a different assumption to be made about the price and beliefs after a deviation to wait, the expected benefit from deviating would depend upon the beliefs and price assumed and therefore the benefit function would be discontinuous in the timing strategies of the two traders at $\beta_0 = \beta_1 = 0$. A different assumption would also introduce an asymmetry between the two types of trader that Lemma 2.2 establishes cannot be present on the equilibrium path. These reasons provide further justification for the assumption made.} As shown in Appendix A.1, with $\beta_0 = \beta_1$, the benefit function simplifies so that it no longer depends upon the timing strategies of the two types of traders. Therefore, I denote the simplified function $B_x(p_t)$.

Although Lemma 2.2 does not preclude mixed timing strategies, $\beta_1 = \beta_0 \in (0, 1)$, it proves useful to first consider a trader’s benefit when all other traders use pure timing strategies. Because both types of a trader must follow the same timing strategy, a pure timing strategy for a trader can be described simply as rush or wait. There are six possible cases for the (joint) timing strategies of others from trader $t$’s perspective. First, $t - 1$ is observed to either rush or wait. On the other hand, $t + 1$’s timing decision must be anticipated by $t$ and $t$ may expect it to depend upon the price $t + 1$ is expected to face. If $t - 1$ has rushed, $t$ knows that if she waits, $t + 1$ faces the same price as $t$. But, if $t - 1$ waits, $t$ may rationally expect $t + 1$’s decision to depend upon whether $t - 1$ buys or sells (which is observed by $t + 1$ but not $t$). The formulas for the benefit in each case are provided in Appendix A.1. The benefit is denoted $B_x^{u,v_1v_2}(p_t)$ where the superscript, $u,v_1v_2$, signifies the timing strategies, $R$ or $W$, of the other traders. $u$ corresponds to the observed timing decision of $t - 1$. When $u = R$, $v_1 = v_2$ corresponds to $t + 1$’s expected timing decision. When $u = W$, $v_1$ ($v_2$) corresponds to the expected timing decision of $t + 1$ if $t - 1$ buys (sells).

To determine the best response of a trader to the strategies of others, it is helpful to study the properties of the six benefit functions as a function of
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Figure 2.2: Example Benefit Functions

Note: \( s = 1, q = 0.75, \) and \( \bar{q} = 0.8. \) Only the zero-crossings for \( B_{2}^{R,RR}(p_t), 1 - \hat{p}_{R,RR} \) and \( \hat{p}_{R,RR} \) are labeled to reduce clutter.
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price. To illustrate, Figure 2.2 plots all six benefit functions for a trader with \( \gamma_t = 1 \), given parameters \( q = 0.75 \) and \( \bar{q} = 0.8 \). In general, each function is either always negative (not shown) or has one of the two shapes in the example: always positive or positive only near \( p_t = \{0, 1\} \). Propositions 2.1 and 2.2 prove two of the more interesting properties of the benefit functions, while the remainder are established in Lemma A.3.1 of Appendix A.3.

Proposition 2.1 establishes that each additional trade that may occur during the time a trader waits reduces her expected benefit from waiting.\(^{19}\) The unconditional correlation of signals rationalizes the fear of expected adverse price movements.\(^{20}\)

**Proposition 2.1:** Each additional, conditionally independent, informative, potential trade between \( t \) and \( t+1 \) strictly reduces the benefit to waiting, \( B_x(p_t, \beta_0, \beta_1) \), for all \( p_t \in (0,1) \) and all timing strategies, \( \beta_0(p_t, I(a_{t-1} = NT)) \) and \( \beta_1(p_t, I(a_{t-1} = NT)) \).

The implications of Proposition 2.1 can be observed in Figure 2.2. Specifically, the benefit, \( B_x^{R,WW}(p_t) \), is largest because, during \( t \)'s waiting period, neither \( t-1 \) nor \( t+1 \) trade. \( B_x^{R,RR}(p_t) \) and \( B_x^{W,WW}(p_t) \) are next largest because only one or the other trades. The remaining benefits are smaller because both trade, at least some of the time. From the formula for \( B_x^{R,WW}(p_t) \) in Appendix A.1, we see that it is positive for all \( p_t \in (0,1) \) if and only if \( \bar{q} > q \). When \( \bar{q} \leq q \), it is zero for all \( p_t \) because the additional signal never changes \( t \)'s trading decision and so is of no value. This fact, combined with Proposition 2.1, immediately implies that all of the benefit functions are (weakly) negative for all \( p_t \) when \( \bar{q} \leq q \). For the more interesting case of \( \bar{q} > q \), also note that \( B_x^{R,RR}(p_t) \) is greater than \( B_x^{W,WW}(p_t) \) at all prices due

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\(^{19}\) The focus here is on the potential trades by \( t-1 \) and \( t+1 \), but the lemma is actually broader in application. Any realization of a random variable, \( x_i \), (for example, a public earnings announcement) that is both informative (takes at least two possible values such that \( Pr(x_i|V = 1) \neq Pr(x_i|V = 0) \) for at least one value) and independent (conditional on \( V \)) similarly reduces the benefit to waiting.

\(^{20}\) Even when \( q = \frac{1}{2} \), such that the best prediction of the price at \( t+1 \) is the price at \( t \), informative trades by other traders reduce the benefit to waiting because others’ signals are unconditionally correlated with \( \pi_t \) so that they are likely to trade in the same direction as you.
to the weaker information revealed by rushed trades.

Proposition 2.2 captures a property of the benefit to waiting as prices become certain, \( p_t \to \{0,1\} \): the value of waiting approaches that of a trader that faces no possible trades between \( t \) and \( t + 1 \). This property arises because any intervening trade has a negligible impact on prices as they become certain. Importantly, however, it is not just the magnitude of the benefit that approaches that of a single trader. Proposition 2.2 also shows that the two benefits must have the same sign. Together, these properties ensure that the best response of a trader facing other traders approaches that of a trader who is alone in the market. Defining the benefit function when no other informative intervening trades are possible as \( B^{ST}_x(p_t, \beta_0, \beta_1) \), we have:

**Proposition 2.2**: For all timing strategies, \( \beta_0(p_t, I(a_{t-1} = NT)) \) and \( \beta_1(p_t, I(a_{t-1} = NT)) \), the benefit function, \( B_x(p_t, \beta_0, \beta_1) \), for any possible intervening informative trades satisfies:

1. \( \lim_{p \to \{0,1\}} |B_x(p_t, \beta_0, \beta_1) - B^{ST}_x(p_t, \beta_0, \beta_1)| \to 0 \)
2. \( \lim_{p \to \{0,1\}} \text{sgn} \left( B_x(p_t, \beta_0, \beta_1) \right) = \text{sgn} \left( B^{ST}_x(p_t, \beta_0, \beta_1) \right) \)

When \( \beta_1 = \beta_0 \) and \( \bar{q} > q \), I previously established that the benefit for a trader alone in the market, \( B^{RW}_x(p_t) \), is strictly positive for all \( p_t \in (0,1) \) and so an immediate consequence of Proposition 2.2 is that there exist prices sufficiently close to 0 and 1 such that all of the benefit functions are strictly positive. Therefore, in any equilibrium of the model with \( \bar{q} > q \), as prices become certain, panics cease to exist and all traders wait. This feature of the model may be surprising because, building on the intuition of Grossman and Stiglitz (1980), one may think that there would be a tendency to free ride off of the strong public information contained in prices as they become certain. While it is true that the private value of information decreases to zero as prices become certain, the difference here is that the cost of additional information also (endogenously) decreases to zero. As long as public beliefs are not perfectly certain, there remains a strictly positive benefit to obtaining the stronger information of the second signal.
Note that Propositions 2.1 and 2.2 did not impose the equilibrium restriction of Lemma 2.2 that the timing strategies of the two types of trader are the same. This fact becomes important in Section 2.5 where the robustness of results to the incorporation of bid and ask prices is analyzed.\footnote{In fact, Lemmas 3 and 4 would continue to hold if the restriction that information is revealed by a deviation to wait was replaced by some other reasonable assumption about what information is revealed by a deviation to wait (i.e. $\xi_t = 0$ deviates with a different probability than $\xi_t = 1$). It is in this sense that the fundamental nature of the equilibria are not altered by different off-equilibrium assumptions.}

In addition to the properties established by Propositions 2.1 and 2.2, Lemma A.3.1 establishes that each benefit function has, other than those at $p_t = \{0, 1\}$, at most two zero crossing points, $\hat{p}$ and $1 - \hat{p}$, which are symmetric with respect to $p_t = \frac{1}{2}$.\footnote{The zero-crossings for $B^{W,RW}_x(p_t)$ and $B^{W,WR}_x(p_t)$ are not symmetric, but they do have a symmetry property with respect to each other. If $B^{W,RW}_x(p_t)$ crosses zero at $p_t$, then $B^{W,WR}_x(p_t)$ crosses zero at $1 - p_t$, and vice versa.} When the additional zero-crossings exist, I denote the one for $B^{u,v_1,v_2}_x(p_t)$ that occurs for $p_t \in \left(\frac{1}{2}, 1\right)$ by $\hat{p}^{u,v_1,v_2}$. Under the restriction on prices and public beliefs on off-equilibrium histories, each $\hat{p}^{u,v_1,v_2}$ is a function of $\overline{q}$ and $\overline{q}$ only.\footnote{Under different assumptions, $\hat{p}^{u,v_1,v_2}$ would depend upon the assumed off-equilibrium beliefs.} In the following section, I in turn consider each of the price regions delineated by the zero-crossings, $\hat{p}^{u,v_1,v_2}$, to establish the fixed point of best responses within the region. In the discussion, I assume all of the price regions exist, as in Figure 2.2. However, Theorem 2.1 also applies when $q$ and $\overline{q}$ are such that one or more of the regions does not exist (see footnote 24).

2.3.3 Equilibrium Trading Behavior

In this section, I use the properties of the benefit functions to determine the fixed point of best responses in timing strategies at any price. For some price regions, the fixed point of best responses is particularly straightforward, while for others it is more complex because it involves considering the prices $t + 1$ is expected to face. Here, I work out two simple cases to provide the intuition, leaving the more complex cases to the proof in Appendix A.3. The discussion here is for the case of $\overline{q} > q$. When $\overline{q} \leq q$, the unique
2.3. Analysis of the Expected Value Model

equilibrium is for all traders to rush for the same reason that all traders rush at prices, \( p_t \in (1 - \hat{p}^{R,RR}, \hat{p}^{R,RR}) \), which is established below. First, however, I comment on the role of mixed timing strategies in equilibrium.

In order for a trader to mix between rushing and waiting, she must be indifferent between the two strategies. Because the timing decision of \( t-1 \) is observed, whether or not she is using a mixed strategy is irrelevant for \( t \). Therefore, if \( t+1 \) is playing a pure strategy, \( t \) can only mix if \( p_t \) happens to be at the zero-crossing of the appropriate benefit function. For generic parameters, however, a price equal to a zero-crossing is never reached. I ignore non-generic mixing possibilities in the analysis and assume that traders wait if indifferent. On the other hand, if \( t+1 \) is mixing between rushing and waiting with the appropriate probabilities, \( t \) can be induced to mix. When \( t+1 \) mixes, the benefit to waiting for \( t \) is a linear combination of the benefits that result when \( t+1 \) uses each of her pure strategies, with the weights being determined by the mixing probabilities. The possibility of mixed timing strategies in equilibrium is accounted for in what follows. Importantly, I show that one can still proceed by individually considering the price regions that are delineated by the zero-crossings of the pure strategy benefit functions.

The equilibrium timing strategy for a trader that faces a price, \( p_t \in (1 - \hat{p}^{R,RR}, \hat{p}^{R,RR}) \), nicely illustrates the idea of rational panic. If \( t-1 \) waits, \( t \)'s benefit is one of \( B^{W,WW}_x(p_t) \), \( B^{W,WR}_x(p_t) \), \( B^{W,RW}_x(p_t) \), or \( B^{W,RR}_x(p_t) \), all of which are negative in this price range, so she will rush regardless of what \( t+1 \) does. If, instead, \( t-1 \) rushes and \( t \) were to wait, \( t+1 \) would face a price of \( p_t \) and would rush because \( t \) waited. But, if \( t+1 \) rushes, \( t \)'s benefit is \( B^{R,RR}_x(p_t) < 0 \), so \( t \) must rush. Thus, we see that all traders rush because of the off-equilibrium threat of the next trader rushing if a trader were to wait. Mixing is not possible because \( t+1 \) can never mix when \( t \) waits. Panic ripples throughout the sequence of traders as long as prices remain in this range.

Next, consider \( p_t \in (1 - \hat{p}^{W,WW}, 1 - \hat{p}^{R,RR}] \cup [\hat{p}^{R,RR}, \hat{p}^{W,WW}) \). This price range differs from the previous one in that \( B^{R,RR}_x(p_t) > 0 \) so that \( t \) best responds by waiting if \( t-1 \) rushes, regardless of what \( t+1 \) does. But, as
in the previous case, if \( t - 1 \) waits, \( t \)'s benefit is negative, so she will rush regardless of what \( t + 1 \) does. Because \( t \) can condition her behavior on the observed timing decision of \( t - 1 \), we obtain what I call *conditional rushing* as the equilibrium timing strategy in this price range: if \( t - 1 \) waits, \( t \) rushes; if \( t - 1 \) rushes, \( t \) waits. Again, mixing is not possible because a trader’s decision does not depend upon what her successor does.

For \( p_t \in (0, 1 - \hat{p}^W,WR] \cup [\hat{p}^W,WR, 1) \), the proof of Theorem 2.1 establishes that all traders wait, reflecting Proposition 2.2. For the remaining price range, \( p_t \in (1 - \hat{p}^W,RR, 1 - \hat{p}^W,WW] \cup [\hat{p}^W,WW, \hat{p}^W,RR) \), the reasoning that determines equilibrium timing strategies is more complex. We can rule out rushing when \( t - 1 \) rushes because, in this case, the benefit is either \( B^R,RR \times (p_t) \) or \( B^R,WW \times (p_t) \), both of which are positive in this price range, so \( t \) will wait independent of what \( t + 1 \) does. Therefore, only conditional rushing or waiting are possible equilibrium strategies. When \( t - 1 \) waits, \( t \)'s best response depends upon what she expects \( t + 1 \) to do, which may in turn depend upon the two possible prices reached after \( t - 1 \) trades. Although \( t \) does not know whether \( t - 1 \) will buy or sell, she knows the resulting prices after each possible trade because they are functions of \( p_t \) and \( \overline{q} \) only. I denote the price reached after a buy decision as \( p_t^+ = p_{t+1} \big| \overline{a}_t = B \) and the price reached after a sell decision as \( p_t^- = p_{t+1} \big| \overline{a}_t = S \). If \( p_t^+ \) and \( p_t^- \) lie in price ranges for which it has previously been determined that \( t + 1 \) must follow a specific equilibrium timing strategy, then \( t \) can easily anticipate what \( t + 1 \) will do and thus best respond accordingly. However, for any given \( p_t \), \( p_t^+ \) and \( p_t^- \) don’t necessarily lie in any particular price range so there are many cases to consider. Also, if either \( p_t^+ \) or \( p_t^- \) lie again in a price range for which the best response of \( t + 1 \) is not known, what \( t \) expects \( t + 1 \) to do depends upon what she expects \( t + 1 \) expects \( t + 2 \) to do. Of course, this reasoning may extend indefinitely.

Progress can be made by realizing that \( t + 2 \) may in fact be in an identical situation to \( t \). If all of \( t - 1 \), \( t \), and \( t + 1 \) wait and the trading decisions of \( t - 1 \) and \( t \) turn out to be opposite, \( t + 2 \) faces an identical situation to \( t \) and, given the restriction that strategies must be the same in payoff-equivalent situations, \( t + 2 \) must follow the same timing strategy as \( t \). Therefore, rather
2.3. Analysis of the Expected Value Model

than thinking of the game between a trader and her successor, one can reframe the problem as a static game between the trader and the “neighboring” traders at prices which can be reached by delayed trades.

To describe this translation of the dynamic game into a static game formally, I introduce the concept of an unrestricted price chain, \( \mathbb{P}( \tilde{p} ) \): the set of all of possible prices that can be reached from delayed trades starting at some price, \( \tilde{p} \in [\frac{1}{2}, q] \).

\[
\mathbb{C}^U(\tilde{p}) \equiv \left\{ p \mid p = \frac{\tilde{p}q^k}{p_q^k + (1 - \tilde{p})(1 - q)^k}, k = -\infty \ldots \infty \right\}
\]

I also define a restricted price chain as the set of prices in \( \mathbb{C}^U(\tilde{p}) \) for which the equilibrium timing decision is not necessarily unique.

\[
\mathbb{C}^R(\tilde{p}) \equiv \mathbb{C}^U(\tilde{p}) \cap \left( (1 - \hat{p}^{WR}, 1 - \hat{p}^{WW}) \cup [\hat{p}^{WW}, \hat{p}^{WR}) \right)
\]

If \( \hat{p}^{WR} \) does not exist, I define the restricted price range to be the null set. The union of restricted price chains for all \( \tilde{p} \in [\frac{1}{2}, q] \) consists of all prices, \( p \in (1 - \hat{p}^{WR}, 1 - \hat{p}^{WW}) \cup [\hat{p}^{WW}, \hat{p}^{WR}) \) and no two restricted prices chains have any price in common. These facts together imply that one can partition this price range into price chains and describe the equilibrium strategies for each price chain.

Two examples of price chains are illustrated in Figure 2.3. The prices in an unrestricted price chain for \( k \) greater than some finite \( \hat{k} \) (and smaller than some finite \( -\hat{k} \)) must lie in a price region in which traders always wait due to Proposition 2.2. I loosely refer to this fact as the unrestricted price chain “ending” in a region in which traders wait, although strictly speaking there is no actual end to a price chain. The equilibrium timing decisions for those regions in which the equilibrium timing decision is unique are labeled accordingly.

In Case 1, the unrestricted price chain does not pass through \( p_t \in (1 - \hat{p}^{WW}, 1 - \hat{p}^{WW}) \) and therefore the only prices for which the equilibrium timing strategy is known are at its ends. In this case, multiple possible equilibrium timing strategies exist at prices in the associated restricted price
2.3. Analysis of the Expected Value Model

Figure 2.3: Equilibrium Timing Strategies and Sample Price Chains

Case 1

Case 2

Note: The infinite set of prices at each “end” of the price chain are not shown. The cutoff, \( p^{W,RW} \) (between \( p^{W,WW} \) and \( p^{W,WR} \)) is not shown because it does not directly play a role in determining the equilibrium timing strategies. Similarly, \( 1 - p^{W,RW} \) (between \( 1 - p^{W,WR} \) and \( 1 - p^{W,WW} \)) is also not shown.
2.3. Analysis of the Expected Value Model

chain. Intuitively, if each of a trader’s neighbors in the chain are waiting, a trader faces $B_{2}^{W,WW}(p_{t})$ and waits. If they are conditionally rushing, a trader faces a strictly negative benefit when $t - 1$ waits and conditionally rushes. The static game is essentially a coordination game between a trader and her neighbors, so as one might expect, there also exists an equilibrium in mixed strategies, as the proof in Appendix A.3 demonstrates.

In Case 2, the unrestricted price chain does pass through $(1 - \hat{p}^{W,WW}, 1 - \hat{p}^{W,WW})$, and the fact that the traders in this interval rush or conditionally rush forces all traders in the chain to conditionally rush. Theorem 2.1, the main result of the chapter, establishes these claims formally, characterizing the equilibria of the trading model.\footnote{When a price range in Theorem 2.1 does not exist, the next innermost price range specifies the timing strategies. For example, when $\hat{p}^{R,RR}$ does not exist, part 2b applies to all prices, $p_{t} \in (1 - \hat{p}^{W,WW}, \hat{p}^{W,WW})$ and when $\hat{p}^{W,WR}$ does not exist, traders at all prices wait.}
2.3. Analysis of the Expected Value Model

**Theorem 2.1:** All equilibria are characterized by the trading strategies of Lemma 2.1 and the following timing strategies, \( \beta_1(p_t, I(a_{t-1} = NT)) = \beta_0(p_t, I(a_{t-1} = NT)) \equiv \beta(p_t, I(a_{t-1} = NT)) \).

1. For \( \bar{q} \leq q \): \( \forall p_t \in (0, 1) \), a trader rushes
2. For \( \bar{q} > q \):
   (a) For \( p_t \in (1 - \bar{p}_R, \bar{p}_R) \), a trader rushes
   (b) For \( p_t \in (1 - \bar{p}_W, \bar{p}_W) \cup [\bar{p}_R, \bar{p}_R, \bar{p}_W, \bar{p}_W] \), a trader conditionally rushes
   (c) For \( p_t \in (1 - \bar{p}_W, \bar{p}_W) \cup [\bar{p}_W, \bar{p}_W, \bar{p}_W, \bar{p}_W) \), partition the interval into the restricted price chains \( \mathcal{C}(\bar{p}) \) for each \( \bar{p} \in [\frac{1}{2}, q) \). Then,
      i. \( \forall \mathcal{C}(\bar{p}) \) such that \( \mathcal{C}(\bar{p}) \cap (1 - \bar{p}_W, \bar{p}_W) \neq \emptyset \), a trader with \( p_t \in \mathcal{C}(\bar{p}) \) conditionally rushes
      ii. \( \forall \mathcal{C}(\bar{p}) \) containing only one price, \( p_s \), and such that \( \mathcal{C}(\bar{p}) \cap (1 - \bar{p}_W, \bar{p}_W) = \emptyset \), a trader at \( p_s \) waits
     iii. \( \forall \mathcal{C}(\bar{p}) \) not satisfying (i) or (ii), a timing strategy is part of an equilibrium if and only if it is one of: (A) all traders at prices \( p_t \in \mathcal{C}(\bar{p}) \) wait; (B) all traders at prices \( p_t \in \mathcal{C}(\bar{p}) \) conditionally rush; (C) two or more traders at prices \( p_t \in \mathcal{C}(\bar{p}) \) mix between conditionally rush and wait and the remaining traders conditionally rush or wait
   (d) For \( p_t \in (0, 1 - \bar{p}_W) \cup [\bar{p}_W, \bar{p}_W), 1) \), each trader waits

Here, rush means \( \beta(p_t, 0) = \beta(p_t, 1) = 0 \), conditionally rush means \( \beta(p_t, 1) = 0 \) and \( \beta(p_t, 0) = 1 \), and wait means \( \beta(p_t, 0) = \beta(p_t, 1) = 1 \).

Theorem 2.1 establishes that panics occur when uncertainty in the market is high, in accord with intuition. Panics are predicted to be more likely to occur, for example, in hot new technology stocks than in stocks for which the fundamental value of the company is well known, such as well-researched blue chip stocks. This result is driven by the feature of the model that the price impacts of other traders are largest when uncertainty is high. Without this effect, traders would be more likely to acquire more information
2.3. Analysis of the Expected Value Model

when uncertainty is high because information is most valuable at this time. But, the fact that panics occur when uncertainty is high also means that their impacts on price convergence are significant because when information is most valuable, it is not acquired. I study price convergence in detail in Section 2.4.

The multiplicity of Theorem 2.1 2c, part (iii), allows for expectations to affect optimal behavior and thus opens the door to the possibility that public information that influences expectations (such as media coverage, etc.) may shift equilibria, inducing or calming panics. The multiplicity exists over certain price ranges only, but, for some parameterizations, these price ranges can span almost the entire range of possible prices.\(^{25}\) On the other hand, for some parameterizations, the equilibrium is unique.\(^{26}\)

2c, part (iii), states that mixed timing strategies are possible but does not provide a complete characterization. I don’t pursue a complete characterization but, as Appendix A.2 shows, any equilibrium involving mixed strategies is unstable in the pseudo-dynamic sense: any small change in the strategy of one of the traders will tend to lead away from the mixed equilibrium towards one of the pure strategy equilibria. The pure strategy equilibria, on the other hand, are stable in that a small change in strategy will tend to reverse itself. Given their stability, the pure strategy equilibria are the focus in the following analysis of price convergence.

\(^{25}\)When \(\hat{p}^{W,WW}\) doesn’t exist and \(q < \hat{p}^{W,WR}\), then there are no restricted price chains in which the equilibrium timing strategy is unique. Then, the equilibrium timing strategy is only unique for \(p_t \in (0, 1 - \hat{p}^{W,WR})\) which may be very small.

\(^{26}\)When \(\hat{p}^{W,WW}\) exists and \((\hat{p}^{W,WW})^+ > 1 - \hat{p}^{W,WW}\), it is possible to show that all unrestricted price chains contain a price \(p \in (1 - \hat{p}^{W,WW}, \hat{p}^{W,WW})\) and therefore 2c, part (ii) ensures conditionally rushing is the unique equilibrium timing strategy for \(p_t \in (1 - \hat{p}^{W,WR}, 1 - \hat{p}^{W,WW})\) so that the equilibrium timing strategy is unique for all prices. If \((\hat{p}^{W,WR})^- < 1 - \hat{p}^{W,WR}\), the opposite is true: the unique equilibrium has traders waiting at every \(p_t \in (1 - \hat{p}^{W,WR}, 1 - \hat{p}^{W,WW})\) because all restricted price chains are singletons so 2c, part (i) applies to this entire range of prices. It is also possible to show that there is no parameterization in which both 2c, part(i) and 2c, part(ii) apply.
2.4  Effects of Panics

In this section, I discuss the impact of panics on the ability of prices to reflect fundamental values. In addition, I demonstrate the potential for panic cycles.

I begin with an illustrative example using $\underline{q} = 0.75$ and $\bar{q} = 0.80$. Under this parameterization, the equilibrium is to rush for all $p_t \in (0.18, 0.82)$ and conditionally rush for all $p_t \in (0.10, 0.18] \cup [0.82, 0.90)$. Figure 2.4 plots a randomly generated price path for the case of $V = 0$. For comparison, I consider a benchmark model in which all traders wait, ensuring all potential information is incorporated into prices. Figure 2.4 illustrates two (related) detrimental effects of panics on asset prices. First, because panics imply trading on weaker information, prices are more likely to diverge away from fundamental values, as seen at $t = 1, 2$. Second, although prices eventually converge to $V = 0$, they converge more slowly than in the benchmark model. While this example only illustrates the possibility of these negative effects, in the following subsections, I show that increasing deviations from fundamentals and longer convergence times also occur in expectation.

2.4.1 Price Convergence

A standard result from the literature, beginning with Glosten and Milgrom (1985), is that prices converge to the true value of the asset. This result is easily shown to extend to the model considered here. From Lemma 2.1, as long as prices are not equal to 0 or 1, there always exists a trader who is willing to buy and another who is willing to sell, and one of these traders must have a private belief about the value of the asset that is further from the true value than the public belief. These facts, as shown in Avery and Zemsky (1998), Proposition 2.4, imply that public beliefs, and therefore prices, must converge to the true value of the asset.

Of more interest here is the rate of convergence. Panics cause traders to trade on lower quality information than what they could have obtained, which can slow the rate of price convergence. I focus on the more interesting case of $\bar{q} > \underline{q}$ because when $\bar{q} \leq \underline{q}$, although the unique equilibrium is to panic
2.4. Effects of Panics

Figure 2.4: Sample Price Path

Note: \( n = 1, \ q = 0.75, \ \bar{q} = 0.80, \) and \( V = 0. \) Solid dots correspond to the periods in which a trader optimally panicked.
at every price, panics have no impact on convergence because all trades are based upon $s_i$ in both models.\textsuperscript{27} When $\bar{q} > q$, on the other hand, traders that wait trade according to $s_t$, revealing information of quality $\bar{q}$, but traders that panic trade according to $s_i$, revealing information of weaker quality, $q$.

In trading models with binary signals, no single number defines the rate of convergence because it varies with the public belief. Thus, convergence is typically measured by the expected number of periods it takes the price (or some convenient function of price such as the log-odds ratio) to reach some specified value, conditional on knowing the true value of the asset. As in Glosten and Milgrom (1985), one can use Wald’s lemma to derive an analytical expression for the expected number of periods, $\bar{T}$, for the log-odds ratio, $\log\left(\frac{p_0}{1-p_0}\right)$, to reach a particular value in the benchmark model.\textsuperscript{28} However, in the model with panics, there is no corresponding analytical expression. Knowing only the number of buys and sells is not sufficient to determine the price because the price ranges that are passed through, and therefore which traders rush or wait, depend upon the order of the buy and sell decisions. This complication makes it impossible to derive a closed form expression for the expected price (or log-odds ratio), making it difficult to perform comparative statics on the effects of changes in parameters on the rate of convergence.

An indirect approach allows me to obtain an interesting result: an increase in the first period signal strength, $q$, can actually \textit{slow} convergence. Basic intuition in a model with exogenous information would suggest that providing higher quality information would tend to increase the rate of convergence. But, this intuition does not apply here because increasing the quality of information in the first period induces traders to rush more often

\textsuperscript{27}Panics actually speed convergence through the mechanical effect of all trades being one period earlier in time. I do not emphasize this effect because it relies on the specific assumption of a single period of delay to acquire information and there being an initial period.

\textsuperscript{28}Specifically, the expected number of periods for the log-odds ratio to exceed $\log\left(\frac{b}{1-b}\right)$ for some public belief, $b \in (0,1)$, when the asset is worth $V = 1$ can be shown to be $E[\bar{T}] = \frac{\log\left(\frac{p_0}{1-p_0}\right) - \log\left(\frac{p_1}{1-p_1}\right)}{(2\bar{q} - 1)\log\left(\frac{\bar{q}}{1-\bar{q}}\right)}$ where $p_1$ is the initial public belief.
2.4. Effects of Panics

through two effects. First, it increases the profit from trading in the first period, making waiting less attractive. In the absence of other traders, this effect on its own is insufficient to cause traders to rush. With the fear of price movements, however, there is a second effect: if one waits and the subsequent trader rushes, her trade has a larger impact on the price, reducing one’s profits in expectation. Together these effects may induce a trader to trade on lower quality information. Proposition 2.3 formalizes this intuition by demonstrating that, for any second period signal strength, one can always find first period signal strengths such that convergence is slower when more information is available.29

Proposition 2.3: For all \( q \in (\frac{1}{2}, 1) \) and all \( p_1 \in (\frac{1}{2}, 1) \), there exist \( q_l, q_h \in (\frac{1}{2}, 1) \) with \( q_l < q_h \), and a price, \( \tilde{p} > p_1 \), such that the expected time for prices to converge to any cutoff price \( p \geq \tilde{p} \), conditional on \( V = 1 \), is strictly larger under \( q_h \) than under \( q_l \).

The reason Proposition 2.3 only guarantees convergence is slower to prices greater than or equal to some \( \tilde{p} \) is because panics initially speed up convergence mechanically by forcing trades to occur earlier in time. This caveat is relatively innocuous because one is normally interested in the time it takes prices to converge to a value close to the true asset value. The basic idea behind the proof is that, for \( q \) sufficiently close to \( \frac{1}{2} \), all traders wait in the unique equilibrium because all benefit functions are strictly positive. Thus, at this low value of \( q \) convergence proceeds at the rate of the benchmark model where all trades reveal signals of strength \( \overline{q} \). On the other hand, for \( q \) sufficiently large, any equilibrium involves price regions for which the unique trading strategy is to rush. Therefore, at least one trade must be based on lower quality information causing prices to take longer to converge in expectation.

To demonstrate that the quantitative impact of panics on convergence speeds can be substantial, in the absence of a closed-form expression for

29Proposition 2.3 is stated in terms of convergence when \( V = 1 \), but a symmetric proposition is easily proven for the case of \( V = 0 \).
2.4. Effects of Panics

convergence times, I rely on numerical simulation. Specifically, I simulate the time necessary to reach an average price of 0.99 when the true value of the asset is 1 for both the expected value model and the benchmark model, then calculate the percentage difference relative to the benchmark model. To understand how the quantitative effect of panics on price convergence varies with the signal strengths, \( q \) and \( \bar{q} \), Figure 2.5 provides a heat map of the simulated percentage slowdown for each combination of parameters, scaled such that black represents the largest slowdown (negative numbers) and white represents the largest speed up (positive numbers).

From Figure 2.5, we first note that slowdowns in convergence can be dramatic, with the expected value model taking more than twice as long to converge in the neighborhood of \( q \in [0.82, 0.84] \) and \( \bar{q} \in [0.96, 0.98] \). Also, there is clearly a complicated relationship between the parameters and the slowdown in price convergence, suggesting that no simple comparative static results exist. Convergence is noticeably discontinuous in the parameters at certain boundaries due to the fact that a small change in parameters can cause the discrete prices reached by trades to jump across a cutoff price that delineates panicking and waiting. Generally, convergence slows down as \( q \) increases for fixed \( \bar{q} \), as long as \( q < \bar{q} \), reflecting Proposition 2.3. Increasing \( \bar{q} \) for a fixed \( q \), on the other hand, has a non-monotonic effect. Although increasing \( \bar{q} \) intuitively increases the benefit to waiting, if one knows that by waiting, the asset value may be revealed with near certainty because of a delayed trade by your predecessor, it can cause one to rush to avoid getting almost zero profit at \( t+1 \). Thus, initial increases in \( \bar{q} \) raise the benefit but at higher values, the benefit begins to fall. This non-monotonicity carries over to the size of the ranges of prices for which panics occur and, therefore, the rates of price convergence.

While Figure 2.5 demonstrates substantial slowdowns in convergence speeds due to panics, when only a single trader arrives each period (\( n = 1 \))
2.4. Effects of Panics

Figure 2.5: Heat Map of Simulated Percentage Slowdowns in Price Convergence

Note: $n = 1$. The color reflects the percentage difference in average convergence times (to 99% of the asset’s true value) divided by the average convergence time of the benchmark model. Negative numbers correspond to the expected value model converging more slowly. Each average is over 100,000 simulated price paths.
2.4. Effects of Panics

Figure 2.6: Simulation of Average Price Paths

Note: \( n = 4, \bar{q} = 0.7, \bar{\bar{q}} = 0.80, \) and \( V = 1. \) Price paths are determined by averaging over 100,000 simulations. Each price reflects all trades in the period (both rushed and delayed).

the most substantial slowdowns are only observed when the second signal is very strong. By increasing \( n \) so that multiple traders arrive each period, waiting costs are increased so that traders may be more willing to panic. With \( n > 1 \), a full equilibrium characterization is difficult to obtain, but in Appendix A.5 I establish an upper bound on the amount of waiting that can occur in any equilibrium. Simulations then provide an upper bound on average prices at each point in time. To illustrate an example with \( n > 1 \), Figure 2.2 plots the average price over time when \( n = 4, \bar{q} = 0.7, \) and \( \bar{\bar{q}} = 0.8. \) Under this parameterization, traders must rush at all \( p_t \in (0.13, 0.87) \) and I assume they wait outside this range.

Figure 2.6 demonstrates that, although prices initially arise more rapidly in the model with panics due to the mechanical effect of trading earlier, panics cause an overall slowdown in convergence. In the benchmark model,
2.4. Effects of Panics

the average price takes about 4.5 periods to reach 99% of the asset’s true value, whereas in the expected value model, the same price level is only reached after 7.5 periods, a slow down of about 66%. Thus, by increasing the number of traders arriving in each period, one can demonstrate substantial slowdowns even with what might be considered more reasonable signal strengths. Furthermore, $n > 1$ seems realistic in real markets with many traders.

2.4.2 Increases in Mispricing

In addition to longer convergence times, Figure 2.4 illustrates that trading on weaker information during panics increases the probability that prices move away from fundamentals. Figure 2.7 plots the probability of observing a price that is farther away from the fundamental value ($V = 1$) than the initial price ($p_1 = 0.5$) for the same parameters for which average convergence was studied in Figure 2.6.\(^{31}\) Figure 2.7 clearly demonstrates that increases in mispricing are much more frequent when traders may panic and that this effect persists over time. In fact, at $t = 9$, the probability of observing $p_t < 0.5$ is more than an order of magnitude higher in the expected value model than in the benchmark model. Widening mispricing is important not only theoretically, but also may impact any arbitrage activity that attempts to correct for it. An arbitrageur that knows the true asset value and buys the asset at $t = 0$ would face considerably larger arbitrage risk under the expected value model where prices have a much higher chance of moving further away from fundamental value before they move towards it. Thus, arbitrageurs may be less active in attempting to correct for the mispricing (Shleifer and Vishny (1997)).

\(^{31}\)The simulated probability of observing a price, $p_t < 0.5$, is a lower bound because the simulations assume the most amount of waiting possible. Because $E[p_{t+1}|p_t, V = 1]$ is increasing in $q$, if traders actually rush more than assumed, mispricing would be more frequent.
2.4. Effects of Panics

Figure 2.7: Simulation of Probability of Increase in Mispricing

Note: $n = 4$, $q = 0.7$, $\eta = 0.80$, and $V = 1$. Probability of $p_i < 0.5$ is determined by averaging over 100,000 simulations.
2.4. Effects of Panics

2.4.3 Panic Cycles

The illustrative example of Figure 2.4 also shows that panic cycles can occur in equilibrium. During the initial period of uncertainty about the value of the asset, all traders panic, acting on relatively weak information about the value of the asset. As the value of the asset becomes more certain, a trader optimally waits to obtain additional information \((t = 3)\). When a strong signal is obtained, prices fall back into the region of uncertainty, generating further panic and oscillations in prices. Although eventually panics cease to exist, such panic cycles arise naturally during the early life of an asset.

Figure 2.4 also shows that there is a sense in which price crashes can arise endogenously in the model when prices reach the point at which traders find it valuable to do additional research.\(^{32}\) The price path in the figure could represent, for example, the stock issuance of a new technology firm, the value of which is initially uncertain. Imagine the technology is actually not viable such that the true value of the firm is low. Initial information, based on little research, happens to be favorable, so we observe an initial boom in the price of the stock as traders correctly infer from their positive information that others also have positive information. No one is willing to perform further research due to the rational fear of continued upward price movements. However, as the market becomes more certain that the firm is of good quality, traders are induced to perform further research because prices are no longer increasing as rapidly. When they do further research, they discover the firm’s value is actually low, and a large drop in the stock price results. Although it is unlikely that information acquisition is the sole source of price booms and crashes, the model demonstrates how endogenous information quality can play a role in their occurrence.

\(^{32}\)By crashes and booms, I simply mean large changes in the price as new (stronger) information is made available to the market. Crashes are often defined to be discontinuous changes in prices (Barlevy and Veronesi, 2003). No such discontinuities arise here.
2.5 The Zero-Profit Model

In the preceding analysis, I assumed that the price of the asset was given by a single value that does not incorporate the information contained in the current trade order. While this assumption allowed me to focus on the interaction between traders, it leads to a market maker who loses money in expectation. In this section, I show that the main results of the expected value model continue to hold when market makers instead earn zero profits in expectation.\textsuperscript{33} Formally, I modify the model as follows. I consider only the case of \( n = 1 \): a single trader arrives each period.

In each period, the trader that arrives at \( t \) is an informed trader with probability \( \mu \in (0,1) \) and is an uninformed, noise trader with probability \( 1 - \mu \).\textsuperscript{34} If a trader is informed, she receives information as in the expected value model, but if uninformed, she trades for reasons exogenous to the model, such as to meet liquidity needs. Specifically, a noise trader buys or sells in the period she arrives or the next period, with each possible trade having probability \( \frac{1}{4} \).\textsuperscript{35} Because the market maker takes into account the information contained in the current trade order, he posts separate prices for buy and sell orders. For a rushed trade, the ask price at which the market maker is willing to sell the asset is given by \( p_t^A = Pr(V = 1|H_t, a_t = B) \) and the bid price at which he is willing to buy the asset is given by \( p_t^B = Pr(V = 1|H_t, a_t = S) \). The bid and ask prices are set similarly for delayed trades. These bid and ask prices are easily shown to be optimal for a market maker that is constrained to earn zero profits.

To analyze the model with noise traders, I proceed in the same manner as in the analysis of the expected value model. The equilibrium definition remains as in Section 2.2 except that item 5, the restriction on off-equilibrium
prices and beliefs, is no longer required because noise traders ensure that beliefs and prices are pinned down after all histories. The main difference from the expected value model is that here, a trader’s timing strategy affects the prices she faces. Intuitively, if informed traders wait, then delayed trades will have a lot of informational content and the bid/ask spread (the difference between the bid and ask prices) will be large. Any rushed trade then must be due to a noise trader so that the bid/ask spread at that time will be zero. However, this difference in spreads reduces the value to waiting and encourages informed traders to rush. If informed traders always rush, the reasoning is reversed and waiting becomes profitable. Based upon this intuition, one can anticipate that equilibria generally involve mixed timing strategies on the part of the informed traders. The existence of bid/ask spreads also creates several possibilities in terms of which types of traders, based on their signals, will buy or sell when waiting. To focus on only one particular case, I set $q = q \equiv q$ in this section so that traders with contradictory signals do not trade because their expected value of the asset lies within the bid/ask spread.\footnote{Results for $q \neq q$ are qualitatively similar as long as the probability of a noise trader is not too high because traders with contradictory signals will still have expectations that lie within the bid/ask spread.}

The unique bid and ask prices are easily calculated using Bayes’ rule and are provided in Appendix A.4. For delayed trades, traders with $s_t = s_{t+1} = 1$ buy, those with $s_t = s_{t+1} = 0$ sell. For rushed trades, traders with $s_t = 1$ buy and those with $s_t = 0$ sell. These trading decisions can be shown to be optimal by comparing a trader’s private belief to the appropriate price, as in the proof of Lemma 2.1.

Given the optimal trading decisions, one can derive a general formula for the benefit to waiting for each type of trader just as in the expected value model. The formulas are provided in Appendix A.4. Unlike in the expected value model, the benefit function for each type of trader, $B_x(p_t, \beta_x)$, depends only on her own timing strategy because a delayed trade reveals both $s_t$ and $s_{t+1}$. One can show that each of the benefit functions is strictly decreasing in the timing strategy of the trader so that the equilibrium timing strategy
2.5. The Zero-Profit Model

is uniquely determined for fixed strategies of the other traders.

Comparing the formulas in Appendix A.4 for the benefit to waiting in the zero-profit model with those of the expected value model in (2.1), one can see that the general structures of the benefit functions are very similar. Because of this similarity, Propositions 2.1 and 2.2 are easily extended to the zero-profit model, as stated in Corollary 2.1.

Corollary 2.1: Replacing $B_x(p_t, \beta_0, \beta_1)$ with $B_x(p_t, \beta_x)$ for $x \in \{0, 1\}$, the statements of Propositions 2.1 and 2.2 hold in the zero-profit model.

From Corollary 2.1, we know that the benefit to waiting decreases when others trade during the waiting period and that this effect diminishes as prices become certain, as in the expected value model. There is, however, no counterpart to Lemma 2.2 of the expected value model because equilibria generally involve the two types of traders following different timing strategies. Also, as discussed previously, equilibria generally involve mixing due to the interaction between the market maker and the traders. These complications make determining the equilibria of the model more difficult than in the expected value case and I do not have a complete characterization. However, by solving for the simplest case of a single trader (i.e. a finite time version of the model with $T = 1$) and using Propositions 2.1 and 2.2, I am able to derive several necessary properties of the equilibria for $T \rightarrow \infty$.

Proposition 2.4 describes the equilibrium timing decisions of a single trader facing the market maker. Together with the optimal trading decisions at each point in time discussed previously, it characterizes the unique equilibrium for the single trader case.

Proposition 2.4: The unique equilibrium of the zero-profit model with a single trader is characterized by the following timing strategies for all $p_t \in (0, 1)$.

\[
\begin{align*}
\beta_{ST}^0(p_t) &= \frac{p_t(1-q) + (1-p_t)q}{p_t(1-q) + (1-p_t)q + p_t(1-q)^2 + (1-p_t)q^2} \\
\beta_{ST}^1(p_t) &= \frac{p_tq + (1-p_t)(1-q)}{p_tq + (1-p_t)(1-q) + p_tq^2 + (1-p_t)(1-q)^2}
\end{align*}
\]
2.5. The Zero-Profit Model

The optimal timing strategies given by Proposition 2.4 are interior for all \( p_t \in (0, 1) \) and all \( q \) so that each type of trader mixes in the unique equilibrium. Furthermore, the two types wait with the same probability only at \( p_t = \frac{1}{2} \), which is intuitive given the symmetry of the problem at this price. One can also show that \( \frac{\partial \beta_1}{\partial p_t} < 0 \) and \( \frac{\partial \beta_0}{\partial p_t} > 0 \) so that, when \( p_t > \frac{1}{2} \), the trader with \( s_t = 0 \) is waiting with a higher probability. Intuitively, when \( p_t > \frac{1}{2} \), the trader with \( s_t = 0 \) has the greater benefit from waiting due to having more unexpected information. By waiting more often, however, negative information about the quality of the asset is revealed, which reduces the price and thus decreases the benefit of type \( s_t = 0 \) relative to that of \( s_t = 1 \). In equilibrium, this reduction in price is just sufficient to ensure both types are indifferent between rushing and waiting. Also note that the equilibrium strategies of the traders in Proposition 2.4 are independent of the probability of an informed trader, \( \mu \). An increase in \( \mu \) increases the bid/ask spreads at both \( t = 1 \) and \( t = 2 \) in such a way as to keep each type of trader indifferent between rushing and waiting.

Using Propositions 2.1 and 2.4, Proposition 2.5 shows that the presence of other traders causes all traders to rush more often than they would if they were alone in the market.

**Proposition 2.5:** In any equilibrium of the infinite horizon zero-profit model, each type of each trader waits with probability \( \beta_0(p_t, I(\mathbf{a}_{t-1} = NT)) < \beta_{0ST}(p_t) \) and \( \beta_1(p_t, I(\mathbf{a}_{t-1} = NT)) < \beta_{1ST}(p_t) \) for all \( p_t \) and \( I(\mathbf{a}_{t-1} = NT) \), where \( \beta_{0ST}(p_t) \) and \( \beta_{1ST}(p_t) \) are given in Proposition 2.4.

From Proposition 2.2, we also know that the equilibrium timing strategies approach those given in Proposition 2.4 as prices become certain. Therefore, the main insights of the expected value model hold in the zero-profit model: panics cause traders to rush more often when uncertainty is high, but as prices become certain they acquire information as if the other traders were not present.
2.6 Empirical Implications

Theoretical analysis of the model has developed predictions with respect to the quality of information traders will trade upon relative to time, uncertainty, and volume. Because the quality of information traders possess is typically unobservable, in this section I develop several testable predictions in terms of observables. Some of these predictions have empirical backing and others suggest novel tests that could be used to validate (or falsify) the model.

The extension of the model to \( n > 1 \) traders arriving each period (see Appendix A.5) demonstrates that an increase in \( n \) leads to more panics and thus trading on lower quality information. An increase in \( n \) corresponds to an increase in volume, so we have the following prediction.

**Prediction 1:** In either a cross-sectional or time-series analysis, holding the level of uncertainty constant, order flows are more balanced and persistent price impacts are smaller when volume is higher.

Trading on lower quality information causes more balanced order flows because signal realizations are more likely to be incorrect when information is of lower quality.\(^{37}\) Beginning with Easley et al. (1996), empirical work has used the order flow imbalance to estimate the probability of informed trading (PIN). In the model of Easley et al. (1996), traders are assumed to be either uninformed or perfectly informed, so that more balanced order flows occur when the percentage of uninformed traders is higher (PIN is lower). In their model, the rates of arrival of informed and uninformed traders are exogenous, so any relationship between volume and the PIN can be captured. Here, because a lower estimated PIN results from more balanced order flows, the correlate of Prediction 1 is that the estimated PIN should be lower when volume is higher.\(^{38}\) In a cross-sectional study, Easley et al. (1996) in fact

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\(^{37}\)Order flow imbalance is defined with respect to those initiating the trade as in Easley et al. (1996).

\(^{38}\)Models that have been developed to explain intraday trading patterns in observables also make predictions regarding the relationship between volume and the PIN. In Admati and Pfleiderer (1988) and Malinova and Park (2012), the PIN increases with volume,
2.6. Empirical Implications

find that the PIN is lower in their sample of high volume stocks than in their sample of low volume stocks, consistent with Prediction 1. They do not control for the level of uncertainty among the stocks, which is important in this context because order flows are predicted to be more balanced when uncertainty is higher, as discussed below. However, unless their sample of high volume stocks happens to be those in which uncertainty is lower, their result can be considered a validation of Prediction 1.

When market participants trade on lower quality information, we would also expect the persistent price impact of trades to be lower. As first suggested by Hasbrouck (1991), informative trades should have longer lasting price impacts than trades for other purposes, such as inventory re-balancing, etc. Using the vector auto-regressive approach developed by Hasbrouck (1991) to measure the persistent price impact of trades, trades of high volume stocks have been found to have smaller price impacts in a sample of NYSE stocks (Engle and Patton, 2004) and in foreign exchange markets (Payne (2003) and Lyons (1996)). These results validate Prediction 1, again assuming that high volume is not directly correlated with low uncertainty. Thus, we see that lower informational content of trades when volume is higher is a robust empirical finding consistent with the model.

Proposition 2.2 and the equilibrium characterization of Theorem 2.1 predict that market participants trade on lower quality information when uncertainty is higher. As in Prediction 1, trades on lower quality information are expected to result in more balanced order flows, so we have the following prediction.  

and in Foster and Viswanathan (1990) either relationship is possible. The informational content of high volume stocks may also be reduced if high volume stocks attract noise traders due to their salience (Barber and Odean, 2008).

39The price impacts of trades are not necessarily predicted to be lower when uncertainty is higher due to two opposing effects. Holding the quality of information constant, price impacts are larger when uncertainty is higher. On the other hand, because market participants trade on lower quality information when uncertainty is higher, price impacts are lower. Thus, the prediction of the model is only that price impacts due to not necessarily increase as uncertainty increases, which would be the case in models with constant information quality, such as Glosten and Milgrom (1985). Similarly, the model makes no straightforward, monotonic prediction with respect to the relationships between volatility or the bid/ask spread and uncertainty. An important consequence of the model is that
2.6. Empirical Implications

**Prediction 2:** In either a cross-sectional or time-series analysis, holding volume constant, order flows are more balanced when uncertainty is higher.

Prediction 2 is perhaps the most novel prediction of the model as it is contrary to the predictions of models in which information is exogenous, such as Glosten and Milgrom (1985), and models with monetary costs such as Nikandrova (2012) and Lew (2013). With a fixed quality of information, there should be no relationship between order flows and uncertainty. With fixed monetary costs, when uncertainty is low, trading should stop completely rather than become more informative. Lew (2013) constructs a sequential trading model with increasing monetary costs of acquiring (perfect) information. His model delivers a prediction opposite to Prediction 2: the PIN is higher (and hence order flows are less balanced) when uncertainty is higher. Should Prediction 2 hold in the data, it would provide strong evidence for the mechanism suggested by the model, because it is the endogenous cost of acquiring information that delivers this prediction.\(^{40}\)

The relationship between the PIN and various measures of uncertainty has been explored (Kumar (2009) and Aslan et al. (2011)) with mixed results. However, the main difficulty in performing a test of Prediction 2 is in finding a suitable proxy for uncertainty. Uncertainty here refers to a flatter distribution over the possible fundamental values of the asset being traded. The ideal proxy would be uncorrelated with volume and would also not be likely to affect the information traders possess through any other channel. In Kumar (2009), firm age, firm size, monthly volume turnover, the way in which the informational content of prices is measured (PIN, bid/ask spread, persistent price impact) is important: not all measures need necessarily deliver the same results.\(^{40}\)

\(^{40}\)The model also predicts more frequent deviations from fundamental values during times of high uncertainty. Limits to arbitrage (see Shleifer and Vishny (1997) and Mitchell et al. (2002)) can also cause mispricing when volatility is high due to increased arbitrage risk. To the extent volatility is related to uncertainty, these two explanations provide similar predictions. However, volatility and uncertainty are not necessarily related (see footnote 39). Furthermore, the explanations can be distinguished by studying the informational content of trades when uncertainty is high.
and idiosyncratic volatility are used as proxies in a cross-sectional study, but firm age and size are likely to be directly correlated with the quality of information available, monthly volume turnover is closely related to volume, and volatility is an output of the model that is not necessarily related to uncertainty (see footnote 39). In Aslan et al. (2011), PIN is regressed on many accounting variables in a cross-sectional study. While some variables can arguably be interpreted as proxies for uncertainty (industry, volatility, volatility in earnings), these proxies suffer from problems similar to those in Kumar (2009).

Because of the difficulty in finding a suitable proxy for a cross-sectional study, a time-series study in which one can control for firm-specific effects may be better able to tease out the effect of uncertainty on the informational content of trades. Potential proxies that could be used include the implied volatility index of the market (VIX) as a whole or dispersion in analyst forecasts. Alternatively, one could look for particular news events that one can reasonably argue have increased (or decreased) uncertainty about firm valuations (for example, results of drug trials in the pharmaceutical industry). Certainly there is scope for interesting empirical work in this direction.41

Proposition 2.3 states that an increase in the initial quality of information can actually reduce the rate of price convergence: the increase, holding the second period information constant, can induce traders to panic. As panics result in more balanced order flows, we have:

**Prediction 3:** Access to higher quality initial information, holding the level of uncertainty and volume constant, results in more balanced order flows.

41For other cross-sectional evidence that is suggestive of less-informed trades when uncertainty is higher, it has been found that underreaction in stock prices is stronger in stocks with higher uncertainty (Zhang (2006) and Jiang et al. (2005)). One interpretation of underreaction, as summarized by Zhang (2006), is that underreaction is “more likely to reflect slow absorption of ambiguous information into stock prices than to reflect missing risk factors”. Under this interpretation, the fact the information appears to be more slowly absorbed into stocks of higher uncertainty is consistent with the model. However, strictly speaking, the model does not capture underreaction (prices follow a martingale). Extending the model to capture underreaction is an interesting avenue for future research.
2.6. Empirical Implications

For evidence related to Prediction 3, in a study related to that of Easley et al. (1996), Yan and Zhang (2012) provide a new algorithm to estimate the PIN. Interestingly, they find that estimates of the PIN on both the NYSE and the American Stock Exchange (AMEX) have significantly declined between the years of 1993 and 2004. They do not put forward a theory as to why this has occurred, but one possibility is that improvements in technology have made better quality information available sooner.\footnote{Recent increases in high-frequency trading as documented in Biais et al. (2013) and Hoffmann (2013) may have led to what Hoffman calls an “arms race” in which investors invest in better and better technology to obtain faster access to information.}

The novel empirical predictions of the model are driven both by the endogenous cost of acquiring information and the fact that weak information can result in misinformed traders that trade in a direction that moves prices away from fundamental values. Empirical work that estimates PIN or the persistence of price impacts assumes (explicitly or implicitly) instead that informed trades always move prices towards fundamental values. Should the predictions of the model be validated, it would suggest the importance of relaxing this assumption.
Chapter 3

Rational and Heuristic-Based Trading Panics in an Experimental Asset Market

3.1 Introduction

In real-world financial markets, traders decide not only what assets to trade, but also when to trade them. These timing choices are important and can fundamentally shape the nature of market outcomes. Perhaps most vividly, the ability to time trades can give rise to market panics - episodes in which traders rush to trade in order to avoid adverse price movements due to preemptive trades by others. Chapter 2 theoretically demonstrates that rational panics - defined as rationally rushing to buy or sell prior to receiving full information - have detrimental effects on the ability of prices to aggregate information, as traders rationally forgo acquiring additional information. It is therefore instructive to understand the nature of observed panics. Are they consistent with equilibrium behavior or do they result from other rules-of-thumb? Real-world evidence is difficult to interpret because we do not typically observe the information traders acquire nor when they acquire it.\footnote{Methods of inferring information from trades do exist. For example, Hasbrouck (1991) uses the persistence of price impacts and Easley et al. (1996) construct a structural model based on the arrival of news. Chapter 2 discusses the theoretical implications of rational panics for such indirect evidence.}

In this chapter, I conduct a laboratory experiment to understand the nature of panics and their consequences, benchmarking behavior to that predicted by the theory of Chapter 2.
3.1. Introduction

I study the model of Chapter 2 in which traders make only a single timing decision: they may trade immediately, based on their initial information, or wait to obtain more information before trading. Subjects trade sequentially in an overlapping sequence such that an additional trader arrives during the time it takes the first trader to acquire additional information. Thus, waiting is potentially costly because other market participants may trade in the interim, moving prices adversely in expectation. I design a pair of treatments within this framework, one in which it is rational to panic, and the other in which it is rational to wait for more information. Precise theoretical predictions allow me to assess the rationality of panics in a controlled laboratory setting, and allow for a significantly deeper understanding of subjects’ motivations. Comparing behavior across treatments, I provide the first available laboratory evidence that subjects rationally panic to avoid adverse price movements, suggesting that theory can be a useful guide to predicting panics in the field. Behavior is not completely captured by equilibrium theory, however, as a certain fraction of subjects exhibit non-equilibrium panic behavior.

Careful study of the conditions under which subjects panic out of equilibrium reveals a novel trading heuristic used by almost half of subjects. Specifically, they tend to buy (sell) when their beliefs exceed (fall below) a critical threshold, and wait otherwise. This heuristic, although not optimal in any of the treatments, is quite intuitive. Subjects trade in the direction that is more likely to provide a positive payoff. They fail to realize, however, that because prices already reflect all public information, trading in this way does not necessarily provide the highest expected payoff. At extreme prices, the heuristic prescribes trading against one’s private information, often referred to as herding. Looking at individual behavior, I find that these

Various definitions of herding and contrarianism are present in the literature. I follow the definitions of Avery and Zemsky (1998) and the subsequent experimental literature that tested their model (Cipriani and Guarino (2005) and Drehmann et al. (2005)). Herding refers to buying (selling) at a high (low) price when one’s information indicates to sell (buy). Contrarianism refers to selling (buying) at a high (low) price when one’s information indicates to buy (sell). In these papers and the models considered here, such behavior is not rational. For a survey on herding in financial markets, see Devenow and Welch (1996).
“herding” types are the most common in both treatments, so that they drive the majority of non-equilibrium behavior in the aggregate.

In addition to explaining behavior in this experiment, the heuristic can also explain the herding trades observed in past sequential trading laboratory experiments (Cipriani and Guarino (2005) and Drehmann et al. (2005)). Because trade timing is exogenous in this literature, it is not possible to relate herding to trade timing in order to detect the underlying heuristic traders are using. Following the heuristic, subjects herd as a function of their beliefs about the asset’s value, not due to conformity or beliefs about the mistakes of others. Finally, the heuristic also provides a novel explanation for behavior observed in experiments that study the information herding environment of Banerjee (1992) and Bikhchandani et al. (1992). Thus, it reconciles experimental findings across three different environments.

3.2 Model

The model is a finite-time version of the expected value model of Chapter 2. In order to distinguish it from the richer model developed in Chapter 4, I refer to this model as the Basic model. In each period \( t = 1, 2, \ldots, T \), a single new trader arrives to the market and may trade an asset of unknown value, \( V \in \{0, 1\} \), at a single price, \( p \). A market maker (the experimentalist) sets the price equal to the asset’s expected value based upon all public information. When the asset value is realized at \( T \), those who purchased the asset receive a payoff of \( V - p \) and those who sold (short) receive a payoff of \( p - V \). There is no discounting. The overlapping timing structure

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45Park and Sgroi (2012) also relate herding to trade timing, but in an environment with three states and signals where rational herd behavior is predicted. While they do not have precise theoretical timing predictions, they argue (and observe) that those who have signals that are more likely to herd move later than those with monotonic signals. In contrast, subjects that herd move earlier here.

46Cipriani and Guarino (2005) consider and reject the idea that herding is due to beliefs about mistakes by previous traders. Drehmann et al. (2005) consider and reject conformity. To my knowledge, no explanation of non-equilibrium herding that is supported by the data has been put forth in the literature.

47This literature originated with Anderson and Holt (1997). See also Kübler and Weizsäcker (2004) and Goeree et al. (2007).
is identical to that of Chapter 2 and all traders are informed with private signals as described there. Subjects are restricted to trade only once in the period they arrive, or the following period.

<table>
<thead>
<tr>
<th>Treatment Name</th>
<th>(q)</th>
<th>(\bar{q})</th>
<th>Subjects</th>
<th>Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic Rush (BR)</td>
<td>(\frac{17}{24})</td>
<td>(\frac{16}{24})</td>
<td>(n = 6)</td>
<td>(T = 6)</td>
</tr>
<tr>
<td>Basic Wait (BW)</td>
<td>(\frac{13}{24})</td>
<td>(\frac{17}{24})</td>
<td>(n = 6)</td>
<td>(T = 6)</td>
</tr>
</tbody>
</table>

In designing the two treatments for the Basic model, Basic Rush (BR) and Basic Wait (BW), the goal is to provide a strong comparative static test across treatments: subjects should rush in one, but wait in the other. To achieve this, I chose the parameters for the two treatments specified in Table 3.1. For both treatments, the initial prior, \(p_1\), is set to \(\frac{1}{2}\). The two treatments differ mainly in the quality of the initial information received, being higher in Basic Rush than Basic Wait. In both treatments, \(T = 6\) so that there are 6 subjects in each session of each treatment.

### 3.3 Theoretical Predictions

A detailed derivation of the equilibria of the Basic model for general parameters is given in Chapter 2. Here, I present only the results for the sets of parameter values used in the treatments. Equilibrium strategies can be decomposed into an optimal trading strategy (buy or sell), taking the time of trade as given, and an optimal trade timing strategy. The optimal trading strategies for the Basic model are provided in Proposition 3.1.

**Proposition 3.1:** In the Basic model:

a) traders who rush in either treatment and traders who wait in the BR treatment buy if \(\bar{s}_t = 1\) and sell if \(\bar{s}_t = 0\)

b) trades who wait in the BW treatment buy if \(\bar{s}_t = 1\) and sell if \(\bar{s}_t = 0\)

Intuitively, the trading strategies of Proposition 3.1 are optimal because
3.3. Theoretical Predictions

you ensure one buys when one’s private belief is greater than the public belief (which is equal to the price), and sells otherwise. Given the parameters chosen, no trader is ever indifferent between buying and selling.

Proposition 3.2 specifies the equilibria of the Basic model for the parameterizations used in the experiment. Appendix B.1 provides the details needed to apply the results of Chapter 2.

**Proposition 3.2:**

\[ a) \text{In any equilibrium of the Basic model with } q = \frac{17}{24} \text{ and } \bar{q} = \frac{16}{24} (BR) \text{ all traders rush at every history, except trader } T, \text{ who is indifferent when } T - 1 \text{ rushes.} \]

\[ b) \text{In the unique equilibrium of the Basic model with } q = \frac{13}{24} \text{ and } \bar{q} = \frac{17}{24} (BW) \text{ all traders wait at every history.} \]

*Equilibrium trading strategies are given by Proposition 3.1.*

In the BR treatment, the optimal timing decision is to rush at every price, \( p_t \), with the exception of trader \( T \) who is indifferent when \( T - 1 \) rushes. Intuitively, rushing is optimal because the second signal never changes one’s trading decision: if the two signals contradict one another, trades are made according to the first, stronger, signal. Thus, there is no benefit to waiting, but there is a cost due to trades by other traders. The final trader is an exception in that she is indifferent between rushing and waiting when \( T - 1 \) rushed, due to the fact that no intervening trades are possible. In the BW treatment, there is a strictly positive benefit to waiting at all \( p_t \), independent of whether or not \( t - 1 \) and \( t + 1 \) rush or wait, making the optimal timing strategy to wait.

Proposition 3.2 provides point predictions about subject behavior, but subjects are unlikely to behave rationally 100% of the time. A weaker hypothesis is that subjects rush more often in BR than in BW. I formally state this comparative static result as Corollary B1, as it follows immediately from Proposition 3.2.

**Corollary 3.1:** *Subjects rush more often in BR than in BW.*
3.4 Experimental Design

All subjects were recruited from the University of British Columbia student population using the experimental recruitment package Orsee. Subjects came from a variety of majors and no subject participated in more than one session. Four sessions of each treatment, for a total sample of 48 subjects, were conducted. New randomizations were performed for each session’s asset values, signals, and subject ordering, in order to avoid the possibility of a particular set of draws influencing the results. In each session, subjects first signed consent forms and then the instructions (provided in Appendix B.5) were read aloud. Subjects were allowed to ask questions while the instructions were read and then completed a short quiz. All quiz questions had to be answered correctly by each subject before the experiment began, and this policy was common knowledge. Once the experiment began, no communication of any kind between subjects was permitted.

In each session, 42 trials, preceded by two practice trials, were run. In each trial, subjects made their trading decisions via computerized interfaces, an example of which is provided in Appendix B.5. Past prices and trades were available on an intuitive graphical display. Software was developed using the Redwood package (Pettit and Oprea, 2013) which uses HTML5 to allow very rapid updating of the computer interface. This feature allows for many more trials than would have been possible otherwise, which is important because learning in this relatively complex environment is seen to play a role (see Appendix B.3). An additional benefit of a large number of trials is that it provides a sufficient number of observations to study individual behavior, something which has not been fully explored in previous trading experiments.

The trading environments were framed as such: subjects were told that they would trade a stock with the computer. It was emphasized that they could only trade once and that they had to trade (in each trial).

48 http://www.orsee.org/
49 Subjects were separated by physical barriers so that they could not observe each other’s information or decisions.
50 Both Cipriani and Guarino (2005) and Drehmann et al. (2005) allow subjects to
3.4. Experimental Design

Values were represented visually as bins containing different numbers of colored balls, with signals corresponding to draws from the appropriate bin.

Subjects earned payoffs as described in Section 3.2 in each trial. The asset value, $V$, and prices were scaled by a factor of 100 currency units. Subjects were endowed with 100 units with which to trade prior to each trial, for a maximum possible earning of 200 currency units per trial. In order to induce risk-neutrality, each currency unit represented a lottery ticket with a 1/200 chance to earn $1.00 Canadian. After all trials were completed, a computerized lottery was conducted for each paid trial and subjects were paid according to the results of the lotteries. In addition, each subject was paid $5.00 as a show-up fee. Average earnings were $28.67 (minimum $22.00, maximum $35.00) with a corresponding wage rate over an hour and a half of $19.11/hour.

Ex-ante assumptions about behavior must be made in order to set prices in the experiment. I assume that no information is revealed by the decision to wait, which must be the case in equilibrium as proven in Lemma 2.2 of Chapter 2. Therefore, if no trade occurs, $p_t = p_{t-1}$. After a trade, the price is updated according to Bayes’ rule, assuming that traders follow equilibrium buy and sell strategies. In the case of a trade that occurs after an off-equilibrium timing decision, the price is set assuming traders make the optimal buy or sell decision according to their private information after the deviation.\footnote{As shown in Section 3.5, the second assumption is valid the vast majority of the time, but the first assumption is violated frequently in the data. In Section 4.5, I discuss the consequences of traders believing that other traders may not be making optimal trading or timing decisions when prices are set assuming that they do.}

Subjects were told that prices reflect the mathematical expected value of the value of the asset, conditional on all public information. They were also explicitly told that prices would increase (decrease) after buy (sell) decisions. The exact amount of each increase or decrease was not communicated. However, subjects participate over many trials so that they can learn the

\footnote{not trade in some treatments, finding that a considerable fraction do so. Given that not trading is never optimal and the additional complexity of the environment here, I chose to require subjects to trade in order to eliminate one potential source of noise.}
possible price movements over time.

3.5 Results

Table 3.2: Basic Environment Trading Results

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Rational</th>
<th>Herding</th>
<th>Contrarian</th>
<th>Irrational</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR</td>
<td>84.5% (284)</td>
<td>5.7% (19)</td>
<td>6.5% (22)</td>
<td>3.3% (11)</td>
</tr>
<tr>
<td>BW</td>
<td>83.9% (282)</td>
<td>6.3% (21)</td>
<td>6.9% (23)</td>
<td>3.0% (10)</td>
</tr>
</tbody>
</table>

Notes: Results reported for last 14 trials. Number of observations in parentheses: 336 total observations per treatment.

In all reported results, I focus on the last third of trials (14 trials), resulting in a total of 336 trading and 336 timing observations per treatment. Because the trading environment is relatively complex, focusing on the latter third of trials eliminates some of the noise associated with early behavior as subjects have had time to learn about their environment. Evidence of this learning is provided in Appendix B.3.

Table 3.2 begins the analysis of the Basic model by reporting the trading results. Behavior is rational if it corresponds to that of Proposition 3.1. In this case, traders reveal their private information through their trades, contributing to market efficiency. Behavior is classified as herding if a subject trades against their private information but in the direction indicated by the current price (i.e. buying at a price, \( p_t > 0.5 \), with a signal or signals that indicates one should sell, or the converse). Behavior is classified as contrarian if a subject trades against their private information and also in a direction opposite to the price. Finally, behavior is classified as “irrational” if one faces a price of 0.5 and trades contrary to one’s signal or signals. In Table 3.2, we see that a relatively high percentage of behavior is rational, 84.8% when pooled across treatments. This finding suggests that, at least along this dimension, traders have a good understanding of their environment. Finding 3.1 summarizes the trading behavior.
3.5. Results

Finding 3.1 (Proposition 3.1) In the Basic treatments, almost 85% of subjects rationally reveal their private information through their trades.

Table 3.3: Basic Environment Timing Results

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Frequency of Rush</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic Rush</td>
<td>47.6% (147)</td>
</tr>
<tr>
<td>Basic Wait</td>
<td>31.3% (105)</td>
</tr>
</tbody>
</table>

Notes: Results reported for last 14 trials. Number of observations in parentheses: 309 (336) total observations in BR (BW). Rushing corresponds to rational behavior in BR, but not in BW.

Turning to the timing decisions, Table 3.3 provides the percentage of rush decisions observed in each treatment, aggregated across sessions. In the BR treatment, I omit the timing observations of the final trader when the previous trader rushed, leaving 309 observations. A t-test comparing the average frequency of rushing across treatment rejects the null of equal means at the 5% level (p-value = 0.046), supporting Corollary 3.1. We conclude that subjects respond to the difference in equilibrium forces across treatments in a predictable way. This finding provides evidence of rational panics in a theoretically-understood laboratory setting: traders understand that waiting for others to trade is costly. Interestingly, behavior is not statistically different across treatments in the first third of trials (difference in means is 9.0%, p-value = 0.24), indicating that subjects learn to avoid adverse prices movements as they acquire experience. I summarize the comparative static result in Finding 3.2.

Finding 3.2 (Corollary 3.1) In the Basic treatments, subjects rush more often in the treatment where the equilibrium prediction is to rush (Basic Rush).

Equilibrium panics in treatment BR do not result in informational losses,

\[ 52 \text{When the final trader knows the previous trader rushed, she is indifferent. Of the 27 times traders are indifferent, they wait 66.7\% of the time.} \]
3.5. Results

because even if subjects were to wait to acquire additional information, their second signals would not be revealed by their trades: in equilibrium, they trade according to their first signal. However, non-equilibrium panics in treatment BW result in informational losses over and above those predicted by theory, evidence of which is provided in Appendix B.4.

Although subjects respond as predicted by Corollary 3.1, behavior is not perfectly rational. In particular, subjects both wait too much (Basic Rush) and rush too much (Basic Wait), depending on the treatment. It is tempting to attribute excess rushing to irrational fear and excess waiting to a cautious over-gathering of information.\(^{53}\) However, it turns out that a more subtle heuristic lies behind both behaviors. I first describe the intuitive heuristic that captures behavior and then provide strong evidence that subjects in fact use it.

A large proportion of subjects appear to make decisions based upon maximizing their chances of earning a positive profit, rather than maximizing their expected profit.\(^{54}\) In doing so, they place value upon public information that provides additional certainty about the asset value, even though such information is fully reflected in prices. Thus, they are willing to wait to observe others’ trades even when not optimal. When they become sufficiently certain of the asset value (based upon all public and private information), they then trade to maximize their chances of earning a positive profit. In the most extreme cases, subjects that follow this intuition herd, trading contrary to their private information. Because of this fact, I refer to this heuristic as the \(\tau\)-herding heuristic, where \(\tau\) is the sufficiently high belief threshold beyond which subjects immediately buy. More formally, I define the \(\tau\)-herding heuristic as follows. In the definition and throughout the analysis of timing decisions, I assume subjects treat the asset values of 1

\(^{53}\)Eliaz and Schotter (2010) provide experimental evidence of over-gathering of information. Subjects are willing to pay for “instrumental” information that is of no real value in order to be more certain about their decisions.

\(^{54}\)When there are no prices, or prices are fixed, the two criteria are the same, but when prices reflect all public information, they often differ. To take a simple example, consider \(p = 0.9\) and one’s private belief is 0.8. Buying the asset is more likely to result in a positive profit than selling, but selling maximizes one’s expected profit.
3.5. Results

and 0 symmetrically. Under this assumption, the price range can be transformed to \( p' \in [0.5, 1] \) where \( p' \equiv \max(p, 1 - p) \), and similarly for beliefs. For convenience, I refer to \( p' \) as the price.

**Definition 1:** A trader uses the \( \tau \)-herding heuristic if she has a threshold private belief, \( \tau \in [0.5, 1] \), and uses the following strategy.

1. With a private belief, \( b \in [1 - \tau, \tau] \), always wait for more information, if possible. If not possible, trade according to private information.

2. With a private belief \( b > \tau \), buy immediately. With a private belief, \( b < 1 - \tau \), sell immediately.

Use of the heuristic, while non-optimal, implies that subjects are reacting to their environment in a sophisticated way. Moreover, their behavior is completely predictable, providing alternative predictions to those based on equilibrium theory. I test the aggregate predictions first, and then show that almost half of individual subjects follow the \( \tau \)-herding heuristic.

---

55 Given the neutral framing of their environment, there is no reason to think subjects would favor trading in one direction over the other. Formally, I assume symmetric behavior around \( p = 0.5 \): a trader facing a price, \( p \), with private information, \( I \), makes the same timing decision as a trader facing price, \( 1 - p \), and complementary private information, \( I^C \). To test this assumption, I partition trials into those in which \( V = 1 \) and those in which \( V = 0 \). If there is an asymmetry between rising and falling prices, one would expect the determinants of panics to be different in these two samples. However, the interaction terms are insignificant if one interacts a dummy for \( V = 1 \) with the other covariates in the regression of Table 3.4 that follows.
3.5. Results

Table 3.4: Determinants of Rushed Trades in the Basic Treatments

<table>
<thead>
<tr>
<th></th>
<th>Basic Rush</th>
<th>Basic Wait</th>
<th>Basic Rush</th>
<th>Basic Wait</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(τ-herding types)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private</td>
<td>0.30</td>
<td>1.69***</td>
<td>0.76**</td>
<td>0.95***</td>
</tr>
<tr>
<td>Belief</td>
<td>[0.27]</td>
<td>[0.25]</td>
<td>[0.23]</td>
<td>[0.22]</td>
</tr>
<tr>
<td>Previous</td>
<td>-0.12</td>
<td>-0.04</td>
<td>-0.07*</td>
<td>-0.09**</td>
</tr>
<tr>
<td>Rushed</td>
<td>[0.07]</td>
<td>[0.07]</td>
<td>[0.04]</td>
<td>[0.04]</td>
</tr>
</tbody>
</table>

Notes: Dependent variable is a dummy variable: 1 indicates a rushed trade. 308 (294) observations in BW (BR). 168 (126) observations when restricted to τ-herding types. Logit marginal effects reported. Subject and trial fixed effects are included. Robust standard errors in brackets. Significance at the 10% level is represented by *, at the 5% level by **, and at the 1% level by ***.

In the Basic environment, the aggregate prediction is that as prices (and beliefs) become more extreme, so that more traders' threshold beliefs are surpassed, we should observe more rushed trades. Table 3.4 presents the results of a logit regression of the probability of a rushed trade on a trader's private belief for each treatment. In the regressions, subject and trial fixed effects are included (not reported), as is an indicator dummy that indicates whether or not the previous trader has rushed. Results are also shown for the subsample of individuals classified as τ-herding types. I postpone discussion of these results until after the classification exercise later in this section.

In Table 3.4, we see that, consistent with the τ-herding heuristic, the probability of a rushed trade increases with traders' beliefs. This effect is statistically significant and much stronger in the BW treatment: moving from complete uncertainty (belief is 0.5) to completely certainty (belief is 1), the probability of panic increases by almost 0.85, a very large effect.

56 The results are robust to clustering standard errors at the session level rather than using heteroskedastic robust standard errors. Because there are only a small number of clusters (4) and because the clustered standard errors are typically smaller, I choose to report robust standard errors instead.

57 The difference between the BW and BR treatments can be explained by considering the expected losses from deviating from the optimal strategies. These losses are largest at prices near one half, and they diminish towards zero as prices become certain where
Table 3.4 also shows that the probability of panic, although not significant, is reduced when the previous trader rushed, as would be expected because there can be no price impact from the previous trader in this case.

In addition to predicting when rushed trades occur, the $\tau$-herding heuristic predicts their direction. Because I have sufficient data for each individual, I explore this dimension of behavior at the individual level, providing a more detailed picture than an aggregate analysis. I attempt to classify each individual into one of four possible types: rational, $\tau$-herding, $\tau$-contrarian, and simplistic. A trader is $\tau$-contrarian if she follows a strategy identical to a $\tau$-herding type except that she trades against her belief, rather than with it. Simplistic types rush when they should wait, or vice versa, and follow their signal when they trade. To classify subjects, I impose a rather stringent criterion: their actions must be consistent with those of a particular type in $13/14$ of the last trials.\textsuperscript{58} I allow subjects to be classified into more than one type in order to get a sense of the robustness of the classification, but I also assign a unique type based upon the following prioritization scheme (high to low): rational, simplistic, $\tau$-herding, and $\tau$-contrarian. Table 3.5 provides the resulting classifications for each treatment, including the percentages of exact matches and subjects that have ambiguous types. Overall, the classification scheme is quite successful, with about 60% exactly matching a single type and only 23% remaining unclassified. $\tau$-herding types are the

\textsuperscript{58}For the rational and simplistic classifications, I only consider timing decisions. In order to be identified as $\tau$-herding ($\tau$-contrarian), it must be the case that whenever one rushes, one trades with (against) one’s signal. Whenever one waits, I ignore the subsequent trade direction because identifying a match would require an additional assumption about the subject’s belief threshold. Furthermore, any combination of waiting and rushing is allowed because a high (low) frequency of waiting is consistent with a high (low) threshold belief, $\tau$.\n
there is no profit from trading. In the BW treatment, the differences in losses reinforce the heuristic because the largest penalties from deviating from waiting are at prices near one half where the heuristic also predicts waiting. In the BR treatment on the other hand, the expected loss from deviating from rush is strongest where the heuristic specifies waiting. Thus, one would expect behavior to be more noisy in the BR treatment where the heuristic and payoff incentives oppose each other. Furthermore, the fact that the heuristic is moderated or strengthened by the cost of implementing the heuristic suggests that it is not itself driven by changes in payoff incentives but is instead an innate tendency that subjects bring to the lab.
3.5. Results

most common type, making up 45% of the population. In fact, about 40% of
the population matches the \( \tau \)-herding type exactly.\(^{59}\) The proportion
of subjects classified as \( \tau \)-herding types is very similar across treatments,
providing some evidence of the robustness of the heuristic.

Table 3.5: Frequency of Subjects of Each Type in the Basic Treatments

<table>
<thead>
<tr>
<th>Type</th>
<th>Basic Rush</th>
<th>Basic Wait</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(multiple)</td>
<td>(prioritized)</td>
</tr>
<tr>
<td>% Rational</td>
<td>8.3%</td>
<td>8.3%</td>
</tr>
<tr>
<td>% Simplistic</td>
<td>4.1-8.3%</td>
<td>8.3%</td>
</tr>
<tr>
<td>% ( \tau )-Herding</td>
<td>45.8-54.2%</td>
<td>50%</td>
</tr>
<tr>
<td>% ( \tau )-Contrarian</td>
<td>4.1-8.3%</td>
<td>4.1%</td>
</tr>
<tr>
<td>% Unclassified</td>
<td>29.2%</td>
<td>29.2%</td>
</tr>
</tbody>
</table>

Notes: 32 subjects for each of BR and BW. Results of assigning a single type through the
prioritization scheme are shown in blue.

I perform two additional checks to ensure that \( \tau \)-herding individuals ac-
tually follow the prescribed heuristic, focusing on those that match the type
exactly.\(^{60}\) First, if a subject follows the heuristic, then the beliefs at which
they trade should be higher on average than the beliefs at which they wait.
This criterion is satisfied for 78% of the \( \tau \)-herding types. Second, I return to
the regression results in Table 3.4 for the subsample of subjects classified as
\( \tau \)-herding. We observe that the previous relationship between the probabil-
ity of rushing and beliefs is now stronger and significant in treatment BR, as
expected.\(^{61}\) When the previous trader rushes now significantly reduces the
effect of panic, consistent with \( \tau \)-herding types being more sensitive to price
changes than the average subject. I summarize the results of the subject

\(^{59}\) 40% is almost certainly an underestimate. A large proportion of the rational types
in BW are likely herding types with high threshold beliefs and are classified as rational
only because I prioritize rational over herding.

\(^{60}\) There are 14 subjects in BW and 9 in BR that match the \( \tau \)-herding type exactly.

\(^{61}\) In the BW treatment, it is actually lower among \( \tau \)-herding types, but running a single
regression with interaction terms establishes that the difference is insignificant.
classification exercise as Finding 3.3.

Finding 3.3: Although subjects exhibit heterogeneity in their strategies, the proportion of subjects that can be described as $\tau$-herding are most common, making up about 45% of the population. On the other hand, $\tau$-contrarian subjects are the least frequent, making up only 4% of the population.

3.6 Discussion

Endogenizing trade timing allowed the the $\tau$-herding heuristic to reveal itself. Here, I argue that it provides an explanation for behavior in past experiments in which use of the heuristic was not easily identifiable. Cipriani and Guarino (2005) and Drehmann et al. (2005) study trading behavior in an exogenous timing version of the Basic model. Both papers document herding and contrarian trades, neither put forth a satisfactory explanation for herding. Cipriani and Guarino (2005) consider the possibility that subjects believe previous subjects made mistakes, but show that this explanation can only explain contrarian trades. Drehmann et al. (2005) instead consider, but reject, conformity: subjects trade in the direction of previous subjects, regardless of signals or beliefs. My explanation for herding in these previous papers is the $\tau$-herding heuristic: subjects herd when their beliefs become extreme, as they do here. In Appendix B.2, I show how the existence of $\tau$-herding types can also explain the lack of rational herding in yet another related environment, the informational herding environment of Banerjee (1992) and Bikhchandani et al. (1992). Thus, the $\tau$-herding heuristic reconciles behavior across at least three related laboratory environments.

Previous literature (Long et al. (1990) and Cipriani and Guarino (2005))

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62 I similarly reject this explanation as a source of herding in the data here (see the discussion in Chapter 4).

63 This finding has been demonstrated by numerous papers beginning with Anderson and Holt (1997). See also Kübler and Weizsäcker (2004) and Goeree et al. (2007).
3.6. Discussion

points out that contrarian trades tend to stabilize markets by causing prices to become less extreme, while herding trades do the opposite. Thus, the finding that $\tau$-herding types are much more frequent than $\tau$-contrarian types suggests that behavioral types may tend to destabilize markets overall. This conclusion stands in contrast to Cipriani and Guarino (2005) who, based a higher frequency of contrarian trades in their data (Drehmann et al. (2005) find the same), conclude that contrarianism is more frequent. I suggest two reasons their data may have led to the wrong conclusion. First, in the data here, the overall frequency of contrarian trades declines over time. As these two previous studies did not allow for as many learning opportunities, they picked up behavior only in non-experienced subjects. Second, part of the reason contrarian trades can be more frequent is that there are more opportunities to act contrarian: signals are more likely to go in the direction of the price trend rather than against it. Conditioning on the opportunity for herding or contrarian behavior, I find higher rates of herding behavior (21% vs. 14%).

Results at both the aggregate and individual levels strongly suggest that a large proportion of subjects follow the $\tau$-herding heuristic. If traders in actual financial markets follow such strategies, what would we expect to see? One prediction is clustering of trades as prices cross the common threshold belief of multiple traders.\textsuperscript{64} In addition, when trades cluster, they should all be in the same direction and in the direction of the initial price move. In this case, asset returns would exhibit short-term positive correlation. As both trade clustering and correlation in returns are robust features of actual financial markets, one wonders whether or not the $\tau$-herding heuristic might provide an explanation for their existence.\textsuperscript{65} The Basic environment has

\textsuperscript{64} In real markets, asset values are not binary, so rather than having a threshold belief, one can think of a threshold expected value. Beliefs and expected values are equivalent in the stylized models here.

\textsuperscript{65} Dufour and Engle (2000) provide empirical evidence of trade clustering. Short-term correlation in returns is often attributed to underreaction and has received considerable attention both theoretically and empirically. For theoretical explanations, see Barberis et al. (1998), Daniel et al. (1998), and Hong and Stein (1999). For empirical evidence, see Daniel et al. (1998) for a review. For an experimental paper demonstrating underreaction in a double-auction environment in which asset values follow a stochastic process, see
3.6. Discussion

sufficed to demonstrate rational panics in the laboratory, but its simplicity also prevents it from demonstrating clustering and correlation in returns due to the fact it doesn’t allow subjects to trade simultaneously. This limitation motivates an additional trading environment where these predictions can be tested. Such an environment is developed in Chapter 4.

Kirchler (2009).
Chapter 4

Panics with Simultaneous Trading: Theoretical and Experimental Extensions

4.1 Introduction

Chapters 2 and 3 have developed and studied experimentally a simple model in which trading panics lead to informational losses. However, when one thinks of panics, one typically thinks of many agents acting at (or nearly at) the same time. The stylized model of Chapters 2 and 3 captures the main intuition behind the tradeoff agents face, but precludes simultaneous trading in order to provide a tractable setting. In this chapter, a slightly richer model which allows simultaneous trading, the *Extended* model, is developed. Due to its additional complexity, only partial theoretical results are available, but they are sufficient to characterize the equilibria for the experimental treatments considered.

In the Extended model, all traders receive information simultaneously and are then given several opportunities to trade before receiving additional information in the final trading period. As in Chapter 3, I construct two treatments. Developing new theoretical results, I establish that subjects should theoretically rush to trade immediately in the first treatment, but should wait to trade in the final period in the second treatment. Comparing behavior in these treatments allows me to directly test and contrast the heuristic and theoretical (rational) predictions.

When panics are rational, they lead to severe informational losses: sub-
jects forgo acquiring perfect information about the asset’s value 99.7% of the time, instead rationally choosing to trade immediately with only weak information. 75% of trades occur in the period of arrival, leading to extreme clustering of trades as subjects rationally rush to be the first to trade. This treatment provides a robust demonstration of rational panics and their potential consequences.

In the treatment in which subjects should wait for additional information, the heuristic-based predictions made at the end of Chapter 3 are validated: both trade clustering and short-term positive correlation in returns are observed. Knowing the information traders possess, I can show that the positive correlation in returns is not driven primarily by herding. Instead, the novel explanation for correlation is that it is driven by traders with different private information choosing to trade at different times. Given the large proportion of subjects following the heuristic and the emergence of distinctive naturally-occurring patterns in the laboratory data, it seems likely that this heuristic is a potentially important driver of behavior in the field.

4.2 Extended Model

The asset structure, common prior, and payoffs are identical to the Basic model of Chapters 2 and 3. Time is discrete with trading periods $t = 1 \ldots T$, but rather than having a single trader arrive each period, $n$ traders are present from the first trading period. Each trader, identified by $i \in n$, receives a private signal before the first trading period, $s_i \in \{0, 1\}$, which has a correct realization with probability $q \in (\frac{1}{2}, 1)$. Each trader may trade only once in any of the $T$ trading periods. If a trades waits until time $T$ to trade, she receives an additional private signal, $\bar{s}_i \in \{0, 1\}$, immediately prior to $T$, which has a correct realization with probability $\bar{q} \in (\frac{1}{2}, 1]$. Note that I allow for the second private signal to reveal the true asset value perfectly. Denote the action of trader $i$ in period $t$ as $a_{i,t} \in \{B, S, NT\}$.

---

66Herding, whether information-based or due to conformity, has has long been considered an explanation for correlation in returns. See Hirshleifer and Hong Teoh (2003) for an extensive review.
4.2. Extended Model

Immediately after each trading period, there is a public announcement period in which a binary public signal, \( s_{P,t} \in \{0,1\} \), is revealed. It has a correct realization with probability \( q_P \in \left(\frac{1}{2}, 1\right) \). A single price, \( p_t \), equal to the expected value of the asset conditional on all publicly available information (including both trades and public signals), is set by a market maker prior to each trading period. Prior trades, timing decisions, and prices are observed by all traders.

This model can be thought of as a simplified version of the situation arising around a firm’s earnings announcement. Prior to the announcement, traders receive independent (weak) private information about the upcoming revision in the firm’s valuation. As the announcement approaches, public information slowly becomes available through a series of public signals and, finally, at the announcement date, the firm’s revised valuation becomes publicly known.\(^67\),\(^68\)

<table>
<thead>
<tr>
<th>Table 4.1: Extended Model Treatments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment Name</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Extended Rush (ER)</td>
</tr>
<tr>
<td>Extended Wait (EW)</td>
</tr>
</tbody>
</table>

Specifications for the two treatments of the Extended model, Extended Rush (ER) and Extended Wait (EW), are provided in Table 4.1. The experimental protocol for the Extended treatments is virtually identical to that of the Basic treatments discussed in Chapter 3. Four sessions of each treatment, for a total sample of 64 subjects, were run. Each session consisted of 30 trials plus two practice trials. Average earnings were $21.53 (minimum $12.00, maximum $30.00) with a corresponding wage rate over an hour and

---

\(^67\)The slow release of public information, while not unreasonable, has the additional benefit of providing a unique equilibrium prediction.

\(^68\)The theoretical model is not limited to public information at the announcement date. If one instead assumes the announcement generates further private information because traders have heterogeneous abilities to process information, or have different models of a firm’s dividend-generating process (Kandel and Pearson (1995)), theoretical predictions are still available.
4.3. Theoretical Predictions

A half of $14.35/hour. The instructions and an example of the trading interface are provided in Appendix C.4. Prices are set equal to the asset’s expected value by the market maker (experimentalist). If no trade occurs, the change in price reflects the public signal only. In addition to being told how prices change after trades, subjects were told that when a public signal suggests $V = 1$ ($V = 0$), the price would increase (decrease).

4.3 Theoretical Predictions

The optimal trading strategies for the Extended model are provided in Proposition 4.1. They ensure one buys when one’s private belief is greater than the public belief (which is equal to the price), and sells otherwise. All details and proofs are in Appendix C.1.

**Proposition 4.1:** In the Extended model:

a) traders who trade prior to period $T$ buy if $s_t = 1$ and sell if $s_t = 0$

b) traders who trade in period $T$ buy if the true asset value is 1 and sell if the true asset value is 0

Due to the richness of the strategy space of the Extended model, a complete characterization of all equilibria for general parameters is tedious to derive, although not conceptually difficult. I focus on the particular sets of parameter values used in the two treatments. Proposition 4.2 provides a necessary condition for any equilibrium of treatment ER and a statement of the unique equilibrium of treatment EW.

**Proposition 4.2:**

a) In any equilibrium of the Extended model with $q = \frac{3}{4}$, $q_P = \frac{17}{24}$, and $\bar{q} = 1$ (ER), all trades occur in the first trading period.

b) In the unique equilibrium of the Extended model with $q = \frac{13}{24}$, $q_P = \frac{17}{24}$, and $\bar{q} = 1$ (EW), all trades occur in the final trading period.

Equilibrium trading (buy or sell) strategies are given by Proposition 4.1.

---

69 Proposition 4.2 part a) does not specify the off-equilibrium timing strategies, but the proof in Appendix C.1 ensures that all trades occur immediately regardless of these strategies.
A weaker comparative static prediction follows directly from Proposition 4.2: subjects trade earlier in treatment ER than in treatment EW.

**Corollary 4.1:** Subjects trade earlier in ER than EW.

The difference between ER and EW lies solely in the initial signal strength. In ER, the stronger initial signal increases the profit from trading immediately. Furthermore, it increases the price impacts of others’ trades, should they trade before you. Together, these effects are expected to produce robust equilibrium behavior in which all trades occur immediately. On the other hand, in EW, one’s initial signal strength is so weak that it rationally pays to wait to learn the asset value in the final period before trading. However, given the many opportunities to trade before additional information is received, one expects use of the heuristic to be tempting.

### 4.4 Results

As in Chapter 3, I report data from only the last third of trials in order to focus on behavior after subjects have gained experience (320 trading observations per treatment). Evidence of learning is provided in Appendix C.2.

**Equilibrium Behavior and Informational Losses**

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Rational</th>
<th>Herding</th>
<th>Contrarian</th>
<th>Irrational</th>
</tr>
</thead>
<tbody>
<tr>
<td>ER</td>
<td>93.8% (300)</td>
<td>3.4% (11)</td>
<td>1.9% (6)</td>
<td>0.9% (3)</td>
</tr>
<tr>
<td>EW</td>
<td>77.2% (247)</td>
<td>15.9% (51)</td>
<td>5.9% (19)</td>
<td>0.9% (3)</td>
</tr>
</tbody>
</table>

Notes: Results reported for last 10 trials. Number of observations in parentheses: 320 total observations per treatment.
Figure 4.1: Cumulative Distribution of Trades Occurring at $t \leq t'$

Table 4.2 first demonstrates that trading behavior in the Extended model is highly rational, as in the Basic model results of Chapter 3. In fact, the proportion of each type of trade is very similar to that in the Basic model (Table 3.2), with the exception of the increased frequency of herding in the EW treatment.\(^70\)

To compare timing behavior across the ER and EW treatments, Figure 4.1 plots the empirical cumulative distribution functions (cdfs) of the fraction of trades that occur in periods $t \leq t'$ as a function of $t' \in 1\ldots8$. The rational prediction is that all trades occur at $t = 1$ in the ER treatment and at $t = 8$ in the EW treatment, corresponding to step function cdfs that transition from 0 to 1 at $t' = 1$ and $t' = 8$, respectively.

Figure 4.1 provides clear evidence that trades occur earlier in treatment ER relative to treatment EW. The median trading periods in the four ses-

\(^70\)This increased frequency of herding, relative to the Basic environment, is a result of $\tau$-herding subjects having many more opportunities to trade: their beliefs have more opportunities to cross their threshold belief, providing more opportunities for herding.
4.4. Results

Sions of ER and EW are \{1,1,1,1\} and \{2,3,3.5,4\}, respectively. Applying a non-parametric Mann-Whitney U test, one rejects the null of equal median trading periods (test statistic = 0, p-value = 0.05), supporting Corollary 4.1. I capture these trading and timing results in the following two findings.

**Finding 4.1** *(Proposition 4.1)* In the Extended treatments, approximately 85% of subjects rationally reveal their private information through their trades.

**Finding 4.2** *(Corollary 4.1)* In the Extended treatments, subjects trade earlier in the treatment for which the equilibrium prediction is to trade at \( t = 1 \) *(Extended Rush)*.

Behavior in the ER treatment provides a vivid demonstration of predictable, rational panics and their consequences. Traders rationally rush to trade in the first period almost 75% of the time, and over 90% of trades occur in the first two trading periods. Extreme clustering of trades is observed as subjects scramble to be the first to trade in all four sessions of the treatment. However, this result does not contradict the fact that subjects use the \( \tau \)-herding heuristic. In early trials of the ER treatment, there is evidence that subjects use the heuristic (results available upon request). But, the fact that it causes large decreases in expected payoffs means that it dies out in later trials.

Equilibrium behavior in ER produces large informational losses due to traders acting on weak information. Here, when all traders rush to be the first to trade, each trades with information that is correct only 75% of the time, forgoing the opportunity to obtain perfect information. In fact, only one subject in one trial (out of 320 subject-trials) ever waits to obtain the asset value at \( t = 8 \), meaning prices reflect the true asset value perfectly

---

71 A Kolmogorov-Smirnov test also rejects the null of equal empirical cumulative distributions (p-value = 0.01).

72 When all others are trading at \( t = 1 \), if one delays one’s trade, the price at \( t = 2 \) is likely to be very close to the true asset value, so that trading produces almost zero profit in expectation. Trading immediately instead results in an expected profit of \( 0.75 \times 1.50 + 0.25 \times 0.50 = 1.25 \) in each trial (relative to the endowment profit of $1.00).
4.4. Results

only 0.3% of the time.

Non-equilibrium behavior produces informational losses over and above those predicted, evidence of which I provide in Appendix C.3. Both too frequent rushing and trading against one’s private information contribute to these additional losses. Finding 4.3 summarizes the results related to informational losses.

Finding 4.3: Rational panics result in considerable information losses, as predicted (Extended Rush). Non-equilibrium panics and non-equilibrium trading behavior can, in some environments, severely exacerbate these losses, even after acquiring substantial experience.

Heuristic-Driven Behavior (Extended Wait)

Because behavior in the ER treatment is captured well by the rational prediction, I focus on the EW treatment to study the heuristic predictions. If subjects follow the $\tau$-herding heuristic, we’d expect to see trading at intermediate periods when beliefs cross traders’ threshold beliefs, and this is exactly what is observed: 62% of trades occur in trading periods other than $t = 1$ and $T = T$.

To investigate these trades in more detail, consider what we would expect to see with a population of $\tau$-herding subjects with a distribution of threshold beliefs. It is difficult to predict a simple relationship between beliefs and the probability of trade at a particular price (or period) because of selection issues: traders with low threshold beliefs exit earlier in time. However, a simple prediction is that when beliefs exceed a particular level for the first time, we should observe all traders with threshold beliefs below that level trading in the following period. I use the exogenous public signals, which drive most of the changes in prices (and therefore beliefs), as a coarse predictor of when belief thresholds are crossed. Specifically, I

\footnote{Trading at intermediate periods is difficult to reconcile with rationality. If one were to overestimate the precision of one’s signal, one should trade immediately at $t = 1$. Waiting for public information has an associated cost, but provides no benefit. Lemma C.1.1 in Appendix B.1 formalizes this intuition by showing that it is never optimal to wait until a period that produces no new private information.}
4.4. Results

construct a set of indicator variables, each of which is set to 1 when the absolute value of the difference in public signals reaches a new level *for the first time* in the period prior to the trading period of interest (and is zero otherwise). I denote these dummy variables PubDiff1 through PubDiff6 (6 being the largest absolute difference in public signals arising in the data). One expects to see a higher probability of trade in the period immediately after a new level is reached, as long as there are traders with threshold beliefs between adjacent levels.

Table 4.3: Determinants of Rushed Trades in the Extended Wait Treatment

<table>
<thead>
<tr>
<th></th>
<th>Extended Wait (τ-herding types)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PubDiff1</td>
<td>0.14***</td>
<td>0.38**</td>
</tr>
<tr>
<td></td>
<td>[0.05]</td>
<td>[0.17]</td>
</tr>
<tr>
<td>PubDiff2</td>
<td>0.12*</td>
<td>0.50**</td>
</tr>
<tr>
<td></td>
<td>[0.07]</td>
<td>[0.24]</td>
</tr>
<tr>
<td>PubDiff3</td>
<td>0.02</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>[0.07]</td>
<td>[0.29]</td>
</tr>
<tr>
<td>PubDiff4</td>
<td>-0.08*</td>
<td>-0.17</td>
</tr>
<tr>
<td></td>
<td>[0.04]</td>
<td>[0.23]</td>
</tr>
<tr>
<td>PubDiff5</td>
<td>-0.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.12]</td>
<td></td>
</tr>
<tr>
<td>PubDiff6</td>
<td>-0.06</td>
<td>-0.10</td>
</tr>
<tr>
<td></td>
<td>[0.09]</td>
<td>[0.34]</td>
</tr>
<tr>
<td>Price ($p'$)</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>[0.15]</td>
<td>[0.98]</td>
</tr>
<tr>
<td>Extreme Signal</td>
<td>0.03</td>
<td>0.25***</td>
</tr>
<tr>
<td></td>
<td>[0.03]</td>
<td>[0.08]</td>
</tr>
<tr>
<td>Period</td>
<td>0.08***</td>
<td>0.22***</td>
</tr>
<tr>
<td></td>
<td>[0.01]</td>
<td>[0.07]</td>
</tr>
</tbody>
</table>

Notes: Dependent variable is a dummy variable: 1 indicates a rushed trade. 1077 observations (229 when restricted to τ-herding types). Logit marginal effects reported. Subject and trial fixed effects are included. Robust standard errors in brackets. Significance at the 10% level is represented by *, at the 5% level by **, and at the 1% level by ***.

Table 4.3 reports the results of a logit regression of the probability of
4.4. Results

trade on the public signal difference indicator variables described above. I exclude \( t = 8 \) because I am interested in the determinants of rushed trades. I also control for the price, \( p' \), whether a trader’s signal makes their beliefs more or less extreme (i.e. a positive signal makes beliefs more extreme when \( p \geq 0.5 \), but makes beliefs less extreme when \( p < 0.5 \)), and a linear time trend (i.e. trading period).\(^{74}\) Results are also reported when the subsample is restricted to only subjects identified as \( \tau \)-herding types. I postpone discussion of these results until later in this section.

Table 4.3 shows that when public signals first push prices to about 0.7 (PubDiff1) and about 0.85 (PubDiff2), the probability of trade jumps, suggesting that subjects with \( \tau \) values below \( \approx 0.7 \) and between \( \approx 0.7 \) and \( \approx 0.85 \) both exist in the data.\(^{75}\) The indicator variables corresponding to three or more public signals in the same direction are not significantly positive and one is in fact (marginally) significantly negative. Higher public signal differences are reached less often in the data so statistical power is low in these cases. For this reason, I cannot rule out other \( \tau \)-herding traders with threshold beliefs beyond 0.85.

The fact that a signal which makes one’s belief more extreme does not increase the probability of trade is surprising, although it is positive as one would expect.\(^{76}\) The positive coefficient on the time trend shows that traders are more likely to trade in later periods, all else equal.\(^{77}\)

The increased probability of trade when new levels of public signal differences are reached provide indirect evidence of trade clustering. As further evidence, consider the trading period (\( t = 2 \)) after the first public is revealed (PubDiff1), where the increase in trade probability is the largest.

\(^{74}\)Again, I choose to report heteroskedastic robust standard errors rather than standard errors clustered at the session level. The significance of results remains unchanged with clustered standard errors.

\(^{75}\)These characterizations are coarse because any trades by other traders, as well as one’s own private information, also affect beliefs (to a smaller degree).

\(^{76}\)Here, the effect of one’s signal is averaged across cases in which a new public signal difference level is reached and when it is not. In Table 4.5, we see that when we consider its effect conditional on reaching the first threshold, it is significant.

\(^{77}\)One possible explanation for this fact is that traders value public information more so than is reflected in the price, leading to beliefs that are too extreme. I discuss this possibility in more detail in Section 4.5.
81/320 = 25.3% of all trades occur in this second trading period even though it makes up only 12.5% of all trading periods. Put another way, there are an average of 2.03 trades at \( t = 2 \). If the eight traders were randomly choosing to trade in one of the eight periods, the expected number of trades at \( t = 2 \) would be given by a binomial distribution, so that we’d expect only \( 8 \times 0.125 = 1 \) trade, on average. Under the binomial distribution, the probability of observing more than 2.03 trades is only 6.25%. Thus, we see statistical evidence that trades cluster in time in a predictable manner.

So far I have provided evidence that the timing of trades is predictable, but have said nothing about their direction. When \( \tau \)-herding subjects rush to trade, they should trade in the direction of their private belief. This behavior should produce positive correlation in returns which I provide evidence of now.

Table 4.4: Correlation of Trading Returns in the Extended Wait Treatment

<table>
<thead>
<tr>
<th>Trading Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation</td>
<td>0.13</td>
<td>0.73***</td>
<td>0.05</td>
<td>0.27</td>
<td>-0.13</td>
<td>-0.03</td>
<td>0.15</td>
</tr>
<tr>
<td>Coefficient</td>
<td>[0.42]</td>
<td>[0.00]</td>
<td>[0.74]</td>
<td>[0.09]</td>
<td>[0.44]</td>
<td>[0.87]</td>
<td>[0.35]</td>
</tr>
</tbody>
</table>

Notes: Correlation is with respect to return due to first public signal. p-value of two-tailed t-test included in brackets. Each correlation is over 40 trials.

I continue to focus on the second trading period after the first public signal has just been revealed. At this time, 85% of trades are in the direction of both the trader’s belief and the first public signal. Table 4.4 provides the Spearman correlation coefficients between the return due to the first public signal and that due to trades in each of the first 7 trading periods, excluding the last where the asset value is known.\(^{78}\) We observe a very strong positive short-term correlation between the return due to the first public signal and that due to the trades in the following trading period. Were traders simply rushing and trading according to their signals, prices would form a martingale and such a correlation would be extremely unlikely. This finding provides solid evidence of trades being not only clustered, but

\(^{78}\) The results are very similar if the Pearson correlation coefficient is used, but there is no a priori reason to expect the correlations to be linear.
4.4. Results

also occurring in the same direction. Finding 4.4 summarizes the clustering and correlation results.

Finding 4.4: *Use of the \( \tau \)-herding heuristic produces clustering of trades and positive short-term correlation in returns, as predicted.*

I now dig deeper into the source of the correlation at \( t = 2 \), providing two pieces of evidence to show that it is primarily driven by traders with different private information choosing to trade at different times, and not by herding. Although herding has long been considered to be a source of correlation (Hirshleifer and Hong Teoh (2003)), this finding suggests a different explanation.

First, of the 81 trades that occur, 65% are by those whose signal agrees with the public signal even though only 51% have such signals. This fact is consistent with \( \tau \)-herding behavior because traders whose signals agree have more extreme beliefs so that their critical threshold for trading is more likely to be exceeded. Furthermore, all of those whose signals agree with the public signal trade in the direction of their beliefs, while only 57% of the others do (i.e. 57% herd). Therefore, on average, the herding trades contribute little to the price change.

\[ \text{79One may expect negative correlation in later trading periods, should those with signals that did not agree with the first public signal delay their trades and later trade in the opposite direction. However, any such negative correlation would be spread over the remaining trading periods as there is no salient time at which to trade. In addition, some of these traders may actually delay until } t = 8 \text{ when they learn the true asset value.} \]
### 4.4. Results

Table 4.5: Signals and Rushed Trades in the Extended Wait Treatment

<table>
<thead>
<tr>
<th></th>
<th>Simultaneous Wait</th>
</tr>
</thead>
<tbody>
<tr>
<td>PubDiff1</td>
<td>-0.00</td>
</tr>
<tr>
<td></td>
<td>[0.05]</td>
</tr>
<tr>
<td>PubDiff2</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>[0.08]</td>
</tr>
<tr>
<td>PubDiff3</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>[0.08]</td>
</tr>
<tr>
<td>PubDiff4</td>
<td>-0.10**</td>
</tr>
<tr>
<td></td>
<td>[0.04]</td>
</tr>
<tr>
<td>PubDiff1 * Extreme Signal</td>
<td>0.26**</td>
</tr>
<tr>
<td></td>
<td>[0.10]</td>
</tr>
<tr>
<td>PubDiff2 * Extreme Signal</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>[0.07]</td>
</tr>
<tr>
<td>PubDiff3 * Extreme Signal</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>[0.14]</td>
</tr>
<tr>
<td>PubDiff4 * Extreme Signal</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>[0.20]</td>
</tr>
<tr>
<td>Extreme Signal</td>
<td>-0.05</td>
</tr>
<tr>
<td></td>
<td>[0.05]</td>
</tr>
</tbody>
</table>

Notes: Dependent variable is a dummy variable: 1 indicates a rushed trade. 1077 observations. Logit marginal effects reported. Subject and trial fixed effects are included. Robust standard errors in brackets. Significance at the 10% level is represented by *, at the 5% level by **, and at the 1% level by ***.

As a second piece of evidence, I now show that the increased probabilities of trade at new public signal levels are driven by those whose signals make their beliefs more extreme than the current price. To do so, I interact each of the public signal indicator variables in Table 4.3 with a dummy that indicates a trader’s signal makes her belief more extreme. The results of Table 4.5, show that the increased probability of trade after the first public signal is solely due to traders whose signals make their beliefs more extreme.\(^{80}\) The

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\(^{80}\)For brevity, I do not report the coefficients on all variables. The price coefficient remains insignificant and the period coefficient remains positive and significant. In this case, with standard errors clustered at the session level, the coefficient on the interaction term of the first public signal indicator becomes insignificant (p-value = 0.219), but significance
4.4. Results

coefficient on the first public signal indicator, which now represents those with signals whose beliefs are less extreme than the price, is a fairly precise zero, indicating that there is no increased probability of a potential herding trade.

In summary, we have convincing evidence that the correlation in returns arises because traders choose when to trade based upon their private information. This finding provides a new explanation for the empirical puzzle of correlation in stock returns, specifically post-earnings-announcement drift, in which returns drift in the direction of the earnings surprise (here, the first public signal).81

Table 4.6: Frequency of Subjects of Each Type In Extended Wait

<table>
<thead>
<tr>
<th>Type</th>
<th>Percent of Subjects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(multiple) (prioritized)</td>
</tr>
<tr>
<td>% Rational</td>
<td>6.3% 6.3%</td>
</tr>
<tr>
<td>% Simplistic</td>
<td>25.0-43.8% 43.8%</td>
</tr>
<tr>
<td>% τ-Herding</td>
<td>31.3-50% 31.3%</td>
</tr>
<tr>
<td>% τ-Contrarian</td>
<td>6.3% 6.3%</td>
</tr>
<tr>
<td>% Unclassified</td>
<td>12.5% 12.5%</td>
</tr>
<tr>
<td>% Exact</td>
<td>53.1%</td>
</tr>
<tr>
<td>% Ambiguous</td>
<td>18.8%</td>
</tr>
</tbody>
</table>

Notes: 32 subjects total. Results of assigning a single type through a prioritization scheme are shown in blue.

As a final piece of evidence that subjects use the τ-herding heuristic in the Extended environment, I classify individuals as done in Chapter 3 for the Basic environment. Table 4.6 provides the results of this classification. Comparing with Table 3.5 of Chapter 3, we see remarkably comparable of the other coefficients remains unchanged.

81See Daniel et al. (1998) for a list of papers providing evidence of post-earnings-announcement drift. Although the Extended model was framed as one in which the earnings announcement occurs at the final time period, one can think of the earnings announcement generating private information in the first time period. The final time period may then refer to some future date when additional public information becomes available, such as the filing of the firm’s 10-K.
proportions of each type of subject, with $\tau$-herding types again making up a large proportion of subjects.\textsuperscript{82} As an additional check, among these types, the belief at which they trade should be higher than any previous belief at which they could have traded. In the 90 subject-trials in which $\tau$-herding types participate, there are only 5 violations of this prediction. Returning to the determinants of panic in Table 4.3, for the 9 subjects that match $\tau$-herding types exactly we see that the increased probabilities of trade at the new public signal difference levels are around three times as large as in the regression with all subjects. Although not significant, even the third public signal difference level has a large, positive coefficient, suggesting types with belief thresholds between $\approx 0.85$ and $\approx 0.92$ also exist. These results confirm that a fairly heterogeneous mix of belief thresholds are used among those identified as $\tau$-herding types. Also, among these types, a subject’s signal now has a significant positive effect, indicating that traders with signals that make their beliefs more extreme are significantly more likely to trade, consistent with the findings on return correlation. Given the evidence that subjects use the $\tau$-herding heuristic, we have Finding 4.5.

**Finding 4.5:** Use of the $\tau$-herding heuristic explains the majority of non-equilibrium behavior when it is observed.

### 4.5 Discussion

A critical question facing any laboratory study of markets is the degree to which behavior exhibited by university students can be extrapolated to real-world financial settings with experienced professionals. In terms of rational panics, Chapter 2 cites a variety of empirical evidence that is consistent with the fact that traders rationally rush to trade. Data from the laboratory backs up this evidence by providing a clear demonstration of rational panics in the Extended Rush treatment, where subjects clearly understand that

\textsuperscript{82}A subjective evaluation of the large number of subjects classified as simplistic suggests that most are likely $\tau$-herding types who made a single mistake too many to be classified as such.


4.5. Discussion

waiting to trade can be extremely costly. Although individually rational, the resulting rational panic is severely costly for the market as a whole due to large informational losses. It also produces acute trade clustering, which, interpreted in terms of an earnings announcement, is consistent with the fact that volumes spike around these announcements (Lamont and Frazzini (2007)).

The laboratory data can also provide insight as to when rational panics are more likely to occur in actual financial markets. Comparing observed behavior across treatments, we should expect to see rational panics when there are smaller differences in quality between initial private information and the information that can be acquired through research. Also, panics are more likely when many other traders are present in the market (as in the Extended environment) than when only one or two other traders may trade (as in the Basic environment of Chapter 3). Finally, I note that experience in the environments where rushing is optimal drives behavior towards rational panics. Given that equilibrium behavior involves preempting other traders, it seems likely that experienced professionals understand the benefit of acting quickly and do so.

Although heuristics found in the laboratory should generally be cautiously exported to the field, there are good reasons to suspect the $\tau$-herding heuristic operates in real-world markets. First, as already noted, the heuristic, although not optimal, is far from irrational behavior: it demonstrates a reasonable level of understanding of one’s environment. Second, the percentage of $\tau$-herding types in the data actually increases over time. As in the case of rational panics, this finding suggests that traders may actually learn over time that prices move against them in expectation, causing them to (rationally or heuristically) panic. Finally, the fact that such behavior generates the well-known phenomena of short-term positive correlation in returns and trade clustering in the laboratory data suggests that it is a good candidate explanation for these findings.

\footnote{The percentage of $\tau$-herding types increases from 6.3\% in the first third of trials to 31.3\% in the last third in the EW treatment. Similar increases also occur in the Basic environment of Chapter 3.}

\footnote{Although not detectable in the laboratory data, given that traders with signals that}
4.5. Discussion

As further suggestive evidence that traders may exhibit \( \tau \)-herding behavior in actual markets, consider the fact that one of the pillars of technical analysis is momentum trading: wait until prices appear to be moving in a particular direction and then trade in that direction.\(^{85}\) In the absence of private information, momentum trading and \( \tau \)-herding are identical, so, at least among those that subscribe to technical analysis, something akin to \( \tau \)-herding strategies is natural.\(^{86}\) Furthermore, \( \tau \)-herding strategies can potentially generate positive-feedback trading (Long et al. (1990)). As initial traders cause a trend in prices, those with more extreme threshold beliefs are encouraged to join in, reinforcing the trend and setting off further trading.

Given the explanatory power of the heuristic, two additional questions naturally arise about this behavior. Do traders understand that others use the heuristic? And, what is the underlying behavioral force that generates the use of the heuristic? Prices are set in the experiment assuming that observing a trader wait reveals no information and that traders always trade according to their private information. \( \tau \)-herding behavior causes violations of both of these assumptions so that prices are seen to be too extreme ex post.\(^{87}\) If traders understand this fact and the mispricing is severe enough, oppose the price trend delay their trades, it is not inconceivable that \( \tau \)-herding behavior could also be responsible for the longer-term negative correlation in returns (overreaction) observed in real markets (see Appendix A of Daniel et al. (1998) for a review of papers finding such evidence).

\(^{85}\) For a description of momentum trading, see http://www.investopedia.com/articles/trading/02/090302.asp.

\(^{86}\) Note, however, that there is a subtle difference between \( \tau \)-herding strategies and momentum strategies. In a momentum trading strategy, traders that observe a positive price change (or series of positive price changes) purchase the stock regardless of their private information, so there is no predicted relationship between traders’ signals and when they choose to trade. Instead, through the use of a threshold belief, \( \tau \)-herding behavior allows for differences in trade timing precisely because traders with different signals have different beliefs. As shown in Section 4.4, the correlation in returns in the data is primarily driven by differences in timing strategies and not by herding. If momentum strategies were instead the correct explanation, herding would be a more significant contributor to return correlation in the laboratory data. Several behavioral finance papers posit that traders follow momentum strategies in order to explain post-earnings-announcement drift (see Long et al. (1990) and Hong and Stein (1999)), but the data here suggests that \( \tau \)-herding behavior may be a better explanation.

\(^{87}\) For example, consider the case in which the price trend is upwards. A decision to wait reveals that a trader is more likely to have had a negative signal. Furthermore, a
then they may be induced to act contrarian and sell regardless of their signal (perhaps rushing to do so). Because such contrarian behavior is very infrequent, it would seem either that the majority of traders do not understand others are \( \tau \)-herding types, or that the drive to follow the \( \tau \)-herding strategy is so strong that traders follow it even after accounting for mispricing.

Can \( \tau \)-herding behavior be generated by alternative preferences or common behavioral game theoretical models? Without presenting a formal analysis, I argue informally that the answer is no. One possibility is traders have risk preferences different from than those induced (risk neutrality was induced by paying subjects in lottery tickets). However, if subjects are risk-averse, it can be shown that they would tend to trade in a contrarian manner. Risk-seeking behavior could potentially explain \( \tau \)-herding strategies, but it does not seem plausible that over 40\% of subjects are risk-seeking. Similarly, loss aversion also leads to contrarian behavior. Intuitively, at a high price, there is little to gain (relative to one’s endowment) from buying the asset, but a lot to lose. Finally, common behavioral game theoretic models also all lead toward contrarian behavior, rather than the more common herding behavior observed.\(^{88}\)

Two possible candidate theories can, at least intuitively, generate \( \tau \)-

\(^{88}\)Behavioral game theory has developed several theories that have proven successful at describing experimental behavior: quantal response equilibrium (McKelvey and Palfrey (1995, 1998)), level k reasoning (Stahl and Wilson (1994); Nagel (1995)), cognitive hierarchy (Camerer et al. (2004)), and cursed equilibrium (Eyster and Rabin (2005)). These theories are based upon models of subjects’ beliefs about other subjects behavior. Formally analyzing any of these theories in the environment here is a non-trivial task, but intuitively they all lead to contrarian behavior. Quantal response equilibrium assumes that subjects probabilistically take actions in proportion to the payoff of each action, and that this behavior is common knowledge. While there is some evidence of mistakes proportional to payoff differences (see footnote 57 in Chapter 3), if subjects believe others make such mistakes, then they would believe prices are too extreme. In this case, contrarian behavior would be prescribed. In level k and cognitive hierarchy, subjects believe other subjects are boundedly rational and best respond to this behavior. The lowest level of rationality is typically assumed to be random behavior, but this would again lead to prices being too extreme and contrarian behavior. Finally, in cursed equilibrium, subjects correctly predict the frequencies of actions, but believe that actions are not connected to underlying signals. Again, this belief leads to believing prices are too extreme and contrarian behavior.
herding behavior. The first is base rate neglect, a version of the representativeness heuristic. In base rate neglect, one underweights one’s prior. As such, one’s beliefs can become extreme relative to the price and herding may become optimal. Also, if one’s belief is too extreme, one would expect price movements to be greater in expectation, potentially inducing rushing. The second candidate explanation is a preference for certainty, as formulated by Eliaz and Schotter (2010). Intuitively, if one places explicit value upon being certain about the state before trading, one could be induced to wait when uncertain. If, in addition, there are diminishing returns to this preference for certainty, then subjects may choose to rush as beliefs become more certain. Detailed development of a psychologically-based model for \( \tau \)-herding behavior is an interesting direction for future research.

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89See Barberis and Thaler (2003) for a discussion of the representativeness heuristic in a financial context and for references to the psychological literature.

90To explain the fact that many traders herd after only a single public signal in the EW treatment, one’s belief must become extreme relative to the price after only a single signal. Models in which beliefs become extreme only after a sequence of signals (such as Barberis et al. (1998) and Rabin (2002)) cannot generate such behavior.

91A related concept is overconfidence in which one places too much weight upon one’s own signal. While similar, overconfidence can not generate \( \tau \)-herding behavior because a trader with a signal opposite to the price trend would always trade according to her signal. For a model of underreaction based on overconfidence, see Daniel et al. (1998). Overconfidence has many different interpretations. More precisely, overweighting one’s private information may be referred to as overprecision (Moore and Healy (2008)).
Chapter 5

Conclusion

This dissertation has studied informational losses due to trading panics, both theoretically and experimentally. The fear of adverse price movements due to others’ trades can induce traders to rush to trade as soon as possible. Because traders panic, they forgo doing research about the asset they are trading, resulting in trades based upon relatively weak information. As a consequence, prices take longer in expectation to converge to fundamental values and more frequent increases in mispricing are predicted.

The main theoretical result is that markets are susceptible to panics when uncertainty is high, so that information is forgone when it is most valuable. This finding stands in stark contrast to models with monetary information costs and suggests that future work which models information acquisition in financial markets should consider the time costs of acquiring information. The theoretical work also highlights the coordination aspect of trading panics. A multiplicity of equilibria exists due to strategic complementarity between traders in the market arises, allowing expectations of panics to become self-fulfilling. As such, there may be a role for market intervention to calm fears during times of panic.

The theoretical ideas may be used in future work on market design. Rushing to trade is only optimal when there is a rational fear of others trading before one has a chance to acquire information. In markets that are continuously open, this fear can be justified. On the other hand, in markets that are cleared through call auctions, as long as the auctions are not too frequent, such fears are alleviated. Thus, the model suggests that markets that are cleared through call auctions may be less prone to traders trading on weak information and the associated consequences.\footnote{For a recent paper on the debate about market design that summarizes previous}
the model justifies the practice of regulatory trading halts on stock exchanges when news is to be released. The temporary trading halt ensures traders have time to process information, which prevents rushing to trade on weak information and, assuming the accumulated information is sufficient, ensures no panics occur when trading resumes because prices will adjust to values near the fundamental value (where not panicking is optimal).

The experimental work investigates the sources and consequences of trading panics in a laboratory setting, using precise theoretical predictions as a guide. The theoretical results successfully predict the relative frequency of rational panics and allow non-equilibrium panics to be identified. Non-equilibrium behavior creates informational losses over and above those due to rational panics. I develop a simple heuristic to explain behavior observed in the first environment and use it to generate predictions that are confirmed in a second, richer environment. The heuristic also reconciles findings in previous experimental studies with related environments and puts forth a new explanation for the trade clustering and short-term correlation in returns observed in real financial markets.

A clear direction for future experimental research is to understand in more detail the trading heuristic used by almost half of traders. Of particular interest may be to determine whether or not subjects understand that others follow the heuristic and try to exploit such behavior to their advantage. For example, in settings in which multiple trades are possible, rational traders may buy in an attempt to push up prices such that they can then profitably sell the asset at an even higher price. If so, the heuristic may help to explain price bubbles, both in the field and in laboratory experiments (e.g. Smith et al. (1988)).

The trading heuristic has additional implications outside of the laboratory. In particular, it suggests that short-term correlation in returns should be more dramatic after earnings announcements that generate disagreement among traders, as opposed to those that are unambiguous in their implications. In the former case, there is more scope for traders with different beliefs to choose to trade at different times. Similarly, return correlation theoretical and empirical work, see Kuo and Li (2011).
should be more pronounced in stocks with higher volumes, where increased fears of adverse price movements cause traders to rush to trade on weaker information. Given these predictions, future empirical studies may be able to detect the use of the trading heuristic even without being able to observe traders’ private information.
Bibliography


Bibliography


Bibliography


Appendix A

Appendix to Chapter 2

A.1 Benefit Functions

Here, I show that the general form of the benefit to waiting can be written as (2.1). From Lemma 2.1, a trader trading at \( t \) with \( s_t = 1 \) always buys and so her profit is given by

\[
Pr(V = 1|s_t = 1, H_t) - p_t \iff \frac{p_t q}{p_t q + (1 - p_t)(1 - q)} - p_t \iff \frac{p_t (1 - p_t)(2q - 1)}{p_t q + (1 - p_t)(1 - q)}
\]

The profit for a trader with \( s_t = 0 \) is calculated similarly, using the fact that she always sells. The profit for both types of traders can be written

\[
Pr(s_t)
\]

The expected profit from waiting depends upon the timing strategies of the two types of trader and the strategies of the other traders. I calculate the profit explicitly for a trader with \( s_t = 1 \) who buys when receiving \( s_{t+1} = 1 \) and sells when \( s_{t+1} = 0 \). The other cases are calculated similarly and are omitted for brevity. The expected profit is

\[
\sum_{\hat{a} \in A} \left\{ Pr(s_{t+1} = 1 & \hat{a}|s_t = 1) (Pr(V = 1|s_t = 1, s_{t+1} = 1, \hat{a}) - Pr(V = 1|s_t = NT, \hat{a})) + Pr(s_{t+1} = 0 & \hat{a}|s_t = 1) (Pr(V = 1|s_t = NT, \hat{a}) - Pr(V = 1|s_t = 1, s_{t+1} = 0, \hat{a})) \right\}
\]

where all probabilities are also conditional on \( H_t \). Here, I sum over each possible generic combination of events that result from the timing and trading strategies of \( t - 1 \) and \( t + 1 \). The first term corresponds to the profit from buying the asset after receiving \( s_{t+1} = 1 \) and the second term from selling after receiving \( s_{t+1} = 0 \). Using Bayes’ rule and the independence of signals, the above becomes

\[
\sum_{\hat{a} \in A} \left\{ \frac{Pr(s_{t+1} = 1 & \hat{a}|s_t = 1)}{Pr(s_t = 1)} \left( \frac{p_t q Pr(\hat{a}|V = 1)}{Pr(s_{t+1} = 1 & \hat{a})} - \frac{p_t NT_1 Pr(\hat{a}|V = 1)}{Pr(\hat{a} = NT & \hat{a})} \right) + \frac{Pr(s_{t+1} = 0 & \hat{a}|s_t = 1)}{Pr(s_t = 1)} \left( \frac{p_t NT_1 Pr(\hat{a}|V = 1)}{Pr(\hat{a} = NT & \hat{a})} - \frac{p_t q (1 - \bar{q}) Pr(\hat{a}|V = 1)}{Pr(s_{t+1} = 0 & \hat{a})} \right) \right\}
\]
A.1. Benefit Functions

with \( NT_0 \) and \( NT_1 \) as in Lemma 2.1. Combining each of the terms in the first and second expressions, canceling and factoring out common terms gives

\[
\frac{p_t(1 - p_t)}{Pr(s_t = 1)} \sum_{\delta \in A} \frac{\hat{a}_0 \hat{a}_1}{Pr(q = \hat{s} \land NT)} \left( qNT_0 - NT_1(q(1 - q) + NT_1q - q(1 - q)NT_0) \right)
\]

\[
\leftrightarrow \frac{p_t(1 - p_t)}{Pr(s_t = 1)} \sum_{\delta \in A} \frac{\hat{a}_0 \hat{a}_1}{Pr(q = \hat{s} \land NT)} \left( (2\eta - 1)(qNT_0 + (1 - q)NT_1) \right)
\]

Finally, subtracting the profit at \( t \) from the expected profit and factoring out common terms gives the benefit formula stated in (2.1).

As discussed in the main text, we can set, \( \beta_0 = \beta_1 \), simplifying the benefit function. \( f(q, \eta, \beta_0, \beta_1) \) simplifies to \( \beta(2\eta - 1) \) when \( \eta > q \) and \( \beta(2q - 1) \) when \( \eta < q \), reflecting the fact that a trader follows her stronger signal when they are contradictory. The denominator simplifies to \( Pr(\hat{a} \land NT) = \beta Pr(\hat{a}) \) so that \( \beta \) cancels in the numerator and denominator. When \( \eta = q \), a trader with contradictory signals is indifferent between trading or not, but because I assume that such a trader follows her second period signal, \( f(q, \eta, \beta_0, \beta_1) = \beta(2\eta - 1) \) and \( \beta \) cancels in this case as well. Denoting \( q = \max(\eta, q) \), one can then write the general benefit function as

\[
B_x(p_t) = \frac{p_t(1 - p_t)}{Pr(s_t = x)} \left[ \sum_{\delta \in A} \frac{\hat{a}_0 \hat{a}_1(2q - 1)}{Pr(\hat{a})} - (2q - 1) \right]
\]

where the function no longer depends on \( \beta_0 \) or \( \beta_1 \).

For specific (pure) strategies of the other traders, one can substitute the information revealed by each possible event into the general formula to obtain a specific formula for those strategies. As shown in Lemma 2.1, any trader that rushes trades according to her first period signal and so, for a rushed trade by \( t + 1 \), \( Pr(\hat{s}_{t+1} = B|V = 1) = Pr(s_{t+1} = 1|V = 1) = q \) and \( Pr(\hat{s}_{t+1} = S|V = 1) = 1 - q \). For a delayed trade by \( t - 1 \), \( Pr(\hat{s}_t = B|V = 1) = q \) and \( Pr(\hat{s}_t = S|V = 1) = 1 - q \) where \( q \) denotes \( \max(\eta, q) \).

The unconditional probabilities of buy and sell decisions by \( t - 1 \) are then \( Pr(\hat{s}_t = B) = pq + (1 - p)(1 - q) \) and \( Pr(\hat{s}_t = S) = p(1 - q) + (1 - p)q \). The reason only the stronger signal is revealed comes from Lemma 2.1 when \( \beta_0 = \beta_1 \); if \( \eta > q \), a trader with \( \hat{s}_t = 1 \) (\( \hat{s}_t = 0 \)) buys (sells) regardless of \( s_t \), so that no information about \( s_t \) is revealed by her trade. Similarly, if \( q > \eta \), trades don’t depend on \( \hat{s}_t \). If the two signal strengths are the same, since I have assumed a trader follows her second period signal, only it is revealed. Substituting the information revealed, we get the following formulas, where \( Pr(\hat{s}_t = x \land s_{t+1} = y) \) is abbreviated \( Pr(x, y) \), \( S_x \equiv \frac{p_t(1 - p_t)}{Pr(s_t = x)} \) is a scale factor common to all of the benefit functions, \( Q \equiv q(1 - q) \), \( Q \equiv q(1 - q) \), and

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A.2 Stable of Equilibria

\( \bar{q} = \bar{q}(1 - \bar{q}). \)

\[
\begin{align*}
B_x^{R,WW}(p_t) &= S_x \left[ (2q - 1) - (2q - 1) \right] \\
B_x^{R,RR}(p_t) &= S_x \left[ (2q - 1)Q \left( \frac{1}{Pr(\tau_{t+1} = B)} + \frac{1}{Pr(\tau_{t+1} = 0)} \right) - (2q - 1) \right] \\
B_x^{W,WW}(p_t) &= S_x \left[ (2q - 1)Q \left( \frac{1}{Pr(\tau_t = B)} + \frac{1}{Pr(\tau_t = S)} \right) - (2q - 1) \right] \\
B_x^{W,RW}(p_t) &= S_x \left[ (2q - 1)Q \left( \frac{1}{Pr(\tau_t = S)} + q(1 - q) \left( \frac{1}{Pr(B, 1)} + \frac{1}{Pr(B, 0)} \right) \right) - (2q - 1) \right] \\
B_x^{W,RR}(p_t) &= S_x \left[ (2q - 1)Q \left( \frac{1}{Pr(B, 1)} + \frac{1}{Pr(B, 0)} + \frac{1}{Pr(S, 1)} + \frac{1}{Pr(S, 0)} \right) - (2q - 1) \right]
\end{align*}
\]

A.2 Stability of Equilibria

In this section, I argue that any possible mixed strategy equilibrium of the expected value trading model is unstable in the sense that pseudo-dynamics would tend to destabilize it, and that the pure strategy equilibria are instead stable. The discussion is relatively informal, as the purpose is to simply justify a focus on the pure strategy equilibria.

Consider first an equilibrium that involves mixed strategies. That such an equilibrium can exist was demonstrated in the proof of Theorem 2.1. In the equilibrium, two neighbors in the restricted price chain are mixing between conditionally rushing and waiting with probabilities such that the other is indifferent. Without loss of generality, label the two traders as the prices they face, \( p_t \) and \( p_t^+ \). No assumptions are made about the strategies of the traders at \( p_t^- \) and \( (p_t^+)^+ \): those traders may be mixing or playing pure strategies of either conditional rushing or waiting. \( p_t^+ \) is mixing such that \( p_t \) has a benefit of zero given the strategy of \( p_t^- \). If \( p_t^+ \) changes her strategy to wait slightly more often, the benefit of \( p_t \) then becomes strictly positive because her benefit is linear in the probability with which \( p_t^+ \) mixes and increases when \( p_t^+ \) waits more often. For example, if \( p_t^- \) is waiting, then \( p_t \) faces a benefit of \( B_x^{W,WW}(p_t) > 0 \) when \( p_t^+ \) waits and \( B_x^{W,RW}(p_t) < 0 \) when \( p_t^+ \) conditionally rushes. Therefore, her benefit is \( \beta B_x^{W,WW}(p_t) + (1 - \beta) B_x^{W,RW}(p_t) = 0 \) where \( \beta \) is the equilibrium probability with which \( p_t^+ \) waits. When \( p_t^+ \) waits more often, \( p_t \)'s benefit is strictly positive because more weight is placed upon \( B_x^{W,WW}(p_t) > 0 \) which will cause her to change...
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her strategy to wait with probability 1. But, when this happens, by the same argument, \( p_t \) will be induced to change her strategy to wait with probability 1. The small change to waiting more often by one trader leads both traders to wait with probability 1, so the equilibrium is unstable. A small change to rushing more often leads to both traders (conditionally) rushing with probability 1 by very similar arguments.

Consider instead what happens when the equilibrium is such that all traders in the restricted price chain wait. In this case, each trader faces a strictly positive benefit to waiting (ignoring the non-generic case of the price being exactly equal to one of the cutoff prices). Now, if a trader begins to conditionally rush with some small probability, since the other traders have strictly positive benefits, they will continue to wait. In addition, the trader who conditionally rushes is strictly worse off because any time she conditionally rushes, her profit is reduced given her positive benefit from waiting. Therefore, it is in her best interest to return to waiting with probability 1: the equilibrium is stable. A similar argument establishes that an equilibrium with all traders conditionally rushing is also stable, so that all pure strategy equilibria are stable.

A.3 Omitted Proofs

**Proof of Lemma 2.1:**

The proof when rushing is similar to the proof when waiting, only simpler, so is omitted for brevity. Let \( \hat{a} \) denote any information that becomes public due to the decisions of traders other than \( t \) between \( t \) and \( t + 1 \) and abbreviate \( Pr(\hat{a}|V = y) \) as \( \hat{a}_y \), for \( y \in \{0, 1\} \). A trader, \( t \), who waits buys if her expected value of the asset is greater than the price she faces, \( \bar{p}_{t+1} = Pr(V = 1|\hat{a}, a_t = NT, H_t) \):

\[
\frac{Pr(V = 1|\bar{a}_t, \bar{s}_{t+1}, \hat{a}, H_t)}{p_t Pr(\bar{a}_t|V = 1) Pr(\bar{s}_{t+1}|V = 1) \hat{a}_1} > \frac{(1-p_t) Pr(\bar{a}_t|V = 0) Pr(\bar{s}_{t+1}|V = 0) \hat{a}_0}{p_t \hat{a}_1 Pr(\bar{a}_t = NT|V = 1)}
\]

\[
\iff Pr(\bar{a}_t|V = 1) Pr(\bar{s}_{t+1}|V = 1) NT_0 > Pr(\bar{a}_t|V = 0) Pr(\bar{s}_{t+1}|V = 0) NT_1 \tag{A.2}
\]

where the first equivalence follows from applying Bayes’ rule to each side of the inequality and using the fact that the public belief, \( Pr(V = 1|H_t) = p_t \).
A.3. Omitted Proofs

Using (A.2), a trader with \( s_t = 1, \bar{s}_{t+1} = 1 \) buys if

\[
q_0 NT_0 > (1 - q)(1 - \bar{q}) NT_1
\]

\[\iff q(1 - q)(2\bar{q} - 1)\beta_1 + (q^2\bar{q} - (1 - q)^2 (1 - \bar{q})) \beta_1 > 0\]

which is true \( \forall \bar{q}, \bar{q} \in (\frac{1}{2}, 1) \). Similarly, a trader with \( s_t = 0, \bar{s}_t = 0 \) always sells. Finally, the conditions in Lemma 2.1 under which traders with \( \beta > 1 \) and \( q, \beta \) be larger when \( B \) of difference between the bracketed terms in \( B \) is true using (A.2), a trader with \( s_t = 1, \bar{s}_{t+1} = 1 \) buys or sell are easily obtained by substituting for the appropriate probabilities in (A.2). \( \square \)

**Proof of Lemma 2.2:**

The proof is by contradiction. Assume there is an equilibrium in which \( \beta_1 > \beta_0 \) (the case of \( \beta_0 > \beta_1 \) similarly leads to a contradiction) at some \( \bar{p}_t \) and for some arbitrary strategies of the other traders. I first show that \( \beta_1 > \beta_0 \) and \( B_1(\bar{p}_t, \beta_0, \beta_1) \geq 0 \) together imply \( B_1(\bar{p}_t, \beta_0, \beta_1) > 0 \) for all possible strategies of the other traders and for all \( \bar{p}_t \).

One can see from the general form of the benefit in (2.1), that the sign of \( B_1(\bar{p}_t, \beta_0, \beta_1) \) is determined by the term in square brackets and that the only difference between the bracketed terms in \( B_0(\bar{p}_t, \beta_0, \beta_1) \) and \( B_1(\bar{p}_t, \beta_0, \beta_1) \) is due to differences in the function, \( f(q, \bar{q}, \beta_0, \beta_1) \). I show that this function is strictly greater in \( B_0(\bar{p}_t, \beta_0, \beta_1) \) than in \( B_1(\bar{p}_t, \beta_0, \beta_1) \) whenever \( \beta_1 > \beta_0 \) so that if \( B_1(\bar{p}_t, \beta_0, \beta_1) \geq 0 \), we must have \( B_1(\bar{p}_t, \beta_0, \beta_1) > 0 \) (because \( f(q, \bar{q}, \beta_0, \beta_1) \) is always positive). There are four possible cases depending upon the optimal strategies of buying and selling from Lemma 2.1.

Consider first the case of \( g_1(q, \bar{q}) \leq 0 \) and \( g_0(q, \bar{q}) \geq 0 \). The comparison of \( f(q, \bar{q}, \beta_0, \beta_1) \) relative to \( B_1(\bar{p}_t, \beta_0, \beta_1) \) is \( q NT_1 + (1 - q) NT_0 > q NT_1 + (1 - q) NT_0 \iff (NT_1 - NT_0)(2\bar{q} - 1) > 0 \iff (\beta_1 - \beta_0)(2\bar{q} - 1) > 0 \) so \( f(q, \bar{q}, \beta_0, \beta_1) \) is strictly greater in \( B_0(\bar{p}_t, \beta_0, \beta_1) \). When \( g_1(q, \bar{q}) > 0 \) and \( g_0(q, \bar{q}) < 0 \), we have \( q NT_1 - (1 - q) NT_0 > q NT_1 - (1 - q) NT_0 \iff NT_1 - NT_0 > 0 \iff (\beta_1 - \beta_0)(2\bar{q} - 1) > 0 \) so again \( f(q, \bar{q}, \beta_0, \beta_1) \) is strictly greater in \( B_0(\bar{p}_t, \beta_0, \beta_1) \).

When \( g_1(q, \bar{q}) \leq 0 \) and \( g_0(q, \bar{q}) < 0 \), I begin by noting that

\[ g_0(q, \bar{q}) < 0 \iff q NT_1 - (1 - q) NT_0 > (2\bar{q} - 1)(q NT_1 + (1 - q) NT_0) \]

This can be seen algebraically or simply by noting that \( B_0(\bar{p}_t, \beta_0, \beta_1) \) must be larger when \( t \) follows her optimal trading strategy at \( t + 1 \) instead of the optimal strategy for the other case, \( g_0(q, \bar{q}) \geq 0 \). But, then we have, 

\[ q NT_1 - (1 - q) NT_0 > (2\bar{q} - 1)(q NT_1 + (1 - q) NT_0) > (2\bar{q} - 1)(q NT_0 + (1 - q) NT_1) \]

where the second inequality was shown in the first case above, so \( f(q, \bar{q}, \beta_0, \beta_1) \) is strictly greater in \( B_0(\bar{p}_t, \beta_0, \beta_1) \). Similarly, when \( g_1(q, \bar{q}) > 0 \) and \( g_0(q, \bar{q}) \geq 0 \), \( g_0(q, \bar{q}) \geq 0 \) implies \( (2\bar{q} - 1)(q NT_1 + (1 - q) NT_0) > \]

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\( qNT_1 - (1 - q)NT_0 \) by optimality of the second period trading decision and \( qNT_1 - (1 - q)NT_0 > qNT_0 - (1 - q)NT_1 \) as shown in the second case above. Thus, \( f(q, \overline{q}, \beta_0, \beta_1) \) is strictly greater in \( B_0(p_t, \beta_0, \beta_1) \) in all cases and therefore \( B_1(p_t, \beta_0, \beta_1) \geq 0 \) implies \( B_0(p_t, \beta_0, \beta_1) > 0 \). Furthermore, with \( \beta_1 > \beta_0 \), we must have \( \beta_1 \in (0, 1) \) and therefore \( B_1(p_t, \beta_0, \beta_1) \geq 0 \). Then, we must have \( B_0(p_t, \beta_0, \beta_1) > 0 \) by the established implication. However, we must also have \( \beta_{0,t} \in (0, 1) \) and therefore \( B_0(p_t, \beta_0, \beta_1) \leq 0 \), a contradiction. \( \Box \)

**Proof of Proposition 2.1:**

To establish Proposition 2.1, I first prove the following mathematical claim:

**Claim:** The following inequality holds for any \( x, y \in \mathbb{R}^+ \), \( n \geq 1 \), and any \( a_i, b_i \in [0, 1] \) \( \forall i = 1 \ldots n \) satisfying \( \sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i = 1 \) and at least one of \( a_i \) or \( b_i \) greater than zero \( \forall i = 1 \ldots n \). Furthermore, it holds with equality if and only if \( a_i = b_i \neq 0 \forall i = 1 \ldots n \).

\[
\sum_{i=1}^{n} \frac{a_i b_i}{a_i x + b_i y} \leq \frac{1}{x + y}
\]

When \( n = 1 \), we must have \( a_1 = b_1 = 1 \) due to the constraints that each set of \( a_i \) and \( b_i \) must sum to one. In this case, the inequality is easily seen to be satisfied with equality. So, consider \( n > 1 \). I show that the lhs of the inequality reaches its global maximum at \( \frac{1}{x + y} \) when \( a_i = b_i \neq 0 \forall i = 1 \ldots n \) to show both that the inequality is always satisfied and that it only holds with equality when \( a_i = b_i \neq 0 \forall i = 1 \ldots n \).

I first show that \( f(a_i, b_i) = \frac{a_i b_i}{a_i x + b_i y} \) is concave and use the fact that the sum of any number of concave functions is also concave so that \( \sum_{i=1}^{n} \frac{a_i b_i}{a_i x + b_i y} \) is concave. With simple algebra, we have

\[
\frac{\partial^2 f(a_i, b_i)}{\partial a_i^2} = \frac{-2a_i b_i x y}{(a_i x + b_i y)^3} \leq 0, \quad \frac{\partial^2 f(a_i, b_i)}{\partial b_i^2} = \frac{-2a_i b_i x y}{(a_i x + b_i y)^3} \leq 0, \quad \text{and} \quad \frac{\partial^2 f(a_i, b_i)}{\partial a_i \partial b_i} = \frac{-2a_i b_i x y}{(a_i x + b_i y)^3}.
\]

Therefore, \( \left( \frac{\partial^2 f(a_i, b_i)}{\partial a_i \partial b_i} \right)^2 = 0 \). Thus, the Hessian of \( f(a_i, b_i) \) is negative semi-definite and therefore \( f(a_i, b_i) \) is concave.

If any \( a_i \) or \( b_i \) is equal to zero, the corresponding term in the summation of the lhs is zero and thus contributes nothing. So, consider the \( n^* \leq n \) non-zero terms of the summation. We must have \( \sum_{i=1}^{n^*} a_i = v \) and \( \sum_{i=1}^{n^*} b_i = w \) for some \( v, w \leq 1 \), due to the constraints. Consider the unconstrained maximization of the non-zero terms of the lhs of the inequality after using these constraints to substitute out \( a_{n^*} \) and \( b_{n^*} \). Because we are maximizing a concave function, the first-order conditions are necessary and sufficient...
for determining the global maximizer(s) of the function. We have, \( \forall i = 1 \ldots n^* - 1 \), the first-order conditions with respect to \( a_i \):

\[
\frac{b_{i}^2 y}{(a_i x + b_i y)^2} = \frac{b_{n^*}^2 y}{(a_{n^*} x + b_{n^*} y)^2} \iff \frac{b_{i}}{(a_i x + b_i y)} = \frac{b_{n^*}}{(a_{n^*} x + b_{n^*} y)} \iff a_i = a_{n^*}.
\]

The first-order conditions with respect to \( b_i \) result in the same set of equations. Substituting for each \( a_i \) in the constraint \( \sum_{i=1}^{n^*} a_i = v \), we have

\[
\frac{a_{n^*}}{b_{n^*}} \sum_{i=1}^{n^*} b_i = v \iff a_{n^*} = b_{n^*} \frac{v}{w} \text{ which then implies } a_i = b_i \neq 0.
\]

Using this relationship, we have

\[
\sum_{i=1}^{n^*} \frac{a_i b_i}{a_i x + b_i y} = \frac{vw}{x+y} \leq \frac{1}{x+y}, \text{ where the inequality holds with equality only when } v = w = 1 \text{ which requires no non-zero terms in the summation and implies } a_i = b_i \neq 0.
\]

Therefore, \( \sum_{i=1}^{n} \frac{a_i b_i}{a_i x + b_i y} \leq \frac{1}{x+y} \) is always satisfied and is satisfied with equality if and only if \( a_i = b_i \neq 0 \forall i = 1 \ldots n \), as claimed.

Now, to see that any additional informative potential trade reduces the benefit from waiting, one can apply the mathematical claim. In particular, looking at the general structure of the benefit functions, (2.1), one can see that an additional, independent, trade, \( c \), which results in one of \( n \) possible actions, modifies each contribution to the benefit from an event, \( d_j \), of trade \( d \) by replacing

\[
\frac{d_{j,1}d_{j,0}}{p_t NT_1 d_{j,1} + (1 - p_t) NT_0 d_{j,0}}
\]

with

\[
\sum_{i=1}^{n} \frac{d_{j,1}d_{j,0} Pr(c = c_i|V = 1) Pr(c = c_i|V = 0)}{p_t NT_1 d_{j,1} Pr(c = c_i|V = 1) + (1 - p_t) NT_0 d_{j,0} Pr(c = c_i|V = 0)} \leq \frac{d_{j,1}d_{j,0}}{p_t NT_1 d_{j,1} + (1 - p_t) NT_0 d_{j,0}}
\]

where \( d_{j,y} \) is shorthand for \( Pr(d = d_j|V = y) \), \( y \in \{0,1\} \). The inequality follows from applying the claim with \( a_i = Pr(c = c_i|V = 1) \), \( b_i = Pr(c = c_i|V = 0) \), \( x = p_t NT_1 d_{j,1} \), and \( y = (1 - p_t) NT_0 d_{j,0} \). Thus, the additional trade reduces the benefit to waiting, where the inequality is strict if the trade is informative \( (Pr(c = c_i|V = 1) \neq Pr(c = c_i|V = 0) \) for some \( i \in 1 \ldots n \) as long as \( p_t \neq \{0,1\} \).

Proof of Proposition 2.2:

By continuity of the benefit functions, the limit as \( p_t \to 1 \) must be the

\[93\text{If event } d \text{ were perfectly informative, the inequality would not be strict, but I have assumed signals (and therefore trades) are not perfectly informative.}\]
value of the function evaluated at 1. Therefore, consider the bracketed term in the general formula for the benefit function given in (2.1), evaluated at \( p_t = 1 \). We have

\[
\sum_{\hat{a} \in A} \hat{a}_0 \hat{a}_1 f(q, \overline{q}, \beta_0, \beta_1) \Pr(\hat{a} \& a_t = NT) - (2q - 1)
\]

\[
= \sum_{\hat{a} \in A} \frac{\hat{a}_0 \hat{a}_1 f(q, \overline{q}, \beta_0, \beta_1)}{p_t \hat{a}_1 NT_1 + (1 - p_t)\hat{a}_0 NT_0} - (2q - 1)
\]

\[
= \sum_{\hat{a} \in A} \frac{\hat{a}_0 \hat{a}_1 f(q, \overline{q}, \beta_0, \beta_1)}{\hat{a}_1 NT_1} - (2q - 1)
\]

\[
= \frac{f(q, \overline{q}, \beta_0, \beta_1)}{NT_1} - (2q - 1) \tag{A.3}
\]

where \( q = \max(\overline{q}, q) \) and using \( \sum_{\hat{a} \in A} \hat{a}_0 = 1 \). (A.3) is the benefit of a trader that faces no informative intervening trades, \( D_x^{ST}(p_t, \beta_0, \beta_1) \), evaluated at \( p_t = 1 \) (set \( \hat{a}_0 = \hat{a}_1 = 1 \) in the general formula for the benefit). At \( p_t = 1 \), and therefore, by continuity in the limit as \( p_t \to 1 \), we have then established that the sign of the benefit, which is determined by the bracketed term, is the same as when no informative intervening trades occur, establishing part 2 of the proposition. For part 1, it is easy to see that the magnitude of the benefit function approaches zero whether there are informative intervening trades or not because the part of the benefit outside of the bracketed term approaches zero as \( p_t \to 1 \) and the bracketed term remains finite, as shown above. The proof of both properties for \( p_t \) approaching 0 is identical.

\[\square\]

**Proof of Theorem 2.1:**

Before proving Theorem 2.1, Lemma A.3.1 establishes the remaining properties of the benefit functions exhibited in the example of Figure 2.2, specifically the number and locations of the zero-crossings. Theorem 2.1 assumes the restrictions on off-equilibrium public beliefs and prices and thus the simplified benefit functions given in (A.1) are the relevant ones for the theorem and lemma.
Lemma A.3.1: For $\bar{q} \leq q$, each of the benefit functions, $B_x^{u,v^1v^2}(p_t)$, is $\leq 0$ for all $p_t$. For $\bar{q} > q$, the benefit functions satisfy:

1. $B_x^{u,v^1v^2}(p_t) = 0$ for $p_t \in \{0, 1\}$
2. For $p_t \in (0, 1)$:
   
   (a) $B_x^{R,WW}(p_t) > 0$
   
   (b) $B_x^{R,RR}(p_t), B_x^{W,WW}(p_t)$, and $B_x^{W,RR}(p_t)$ are either $> 0$ over the full range or

   \[
   \begin{cases} 
   < 0 & p_t \in (1 - \hat{p}^{u,v^1v^2}, \hat{p}^{u,v^1v^2}) \\
   = 0 & p_t \in \{1 - \hat{p}^{u,v^1v^2}, \hat{p}^{u,v^1v^2}\} \\
   > 0 & p_t \in (0, 1 - \hat{p}^{u,v^1v^2}) \cup (\hat{p}^{u,v^1v^2}, 1) 
   \end{cases}
   \]

   (c) $B_x^{W,RW}(p_t)$ is either $> 0$ over the full range or

   \[
   \begin{cases} 
   < 0 & p_t \in (1 - \hat{p}^{W,WR}, \hat{p}^{W,RW}) \\
   = 0 & p_t \in \{1 - \hat{p}^{W,WR}, \hat{p}^{W,RW}\} \\
   > 0 & p_t \in (0, 1 - \hat{p}^{W,WR}) \cup (\hat{p}^{W,RW}, 1) 
   \end{cases}
   \]

   (d) $B_x^{W,WR}(p_t)$ is either $> 0$ over the full range or

   \[
   \begin{cases} 
   < 0 & p_t \in (1 - \hat{p}^{W,WR}, \hat{p}^{W,WR}) \\
   = 0 & p_t \in \{1 - \hat{p}^{W,WR}, \hat{p}^{W,WR}\} \\
   > 0 & p_t \in (0, 1 - \hat{p}^{W,WR}) \cup (\hat{p}^{W,WR}, 1) 
   \end{cases}
   \]

3. $1 - \hat{p}^{W,RR} < 1 - \hat{p}^{W,WR} < 1 - \hat{p}^{W,RW} < 1 - \hat{p}^{W,WW} < 1 - \hat{p}^{R,RR} < \hat{p}^{R,RR} < \hat{p}^{W,WW} < \hat{p}^{W,RW} < \hat{p}^{W,WR} < \hat{p}^{W,RR}$ where the zero crossings nearer to $p_t = \{0, 1\}$ exist if the next innermost zero crossing exists (but not necessarily the converse).

Proof of Lemma A.3.1:

With $\bar{q} \leq q$, the fact that all benefit functions are weakly less than zero follows from the fact that $B_x^{R,WW}(p_t) = 0$ for all $p_t$ and Proposition 2.1. So, consider $\bar{q} > q$. In this case, $Pr(\bar{a}_t = B) = Pr(\bar{a}_t = 1)$ and $Pr(\bar{a}_t = S) = Pr(\bar{a}_t = 0)$, so these substitutions can be made in equations

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Part 1 follows from evaluation of the benefit functions at $p_t = \{0, 1\}$. Part 2a follows from the fact that $B_{x,WW}^R(p_t) > 0 \iff \bar{q} > q$.

For part 2b, Proposition 2.2 ensures $B_{x,RR}^W(p_t), B_{x,WW}^W(p_t), B_{x,RR}^W(p_t) > 0$ as prices become close to 0 and 1 because each of these functions must have the same sign as $B_{x,WW}^R(p_t)$, which is positive from 2a. It remains to be shown that these functions cross zero at at most two additional prices, $\hat{p}_{u,v}^1$ and $1 - \hat{p}_{u,v}^1$. Considerable algebraic manipulation allows each of the three functions to be written

\[
B_{x,RR}^R(p_t) = S_x \left[ \frac{2(\bar{q} - q)Q - (2q - 1)^3 P_t}{Pr(s_{t+1} = 1)Pr(s_{t+1} = 0)} \right]
\]

\[
B_{x,WW}^W(p_t) = S_x \left[ \frac{2(\bar{q} - q)\overline{Q} - (2q - 1)(2\bar{q} - 1)^2 P_t}{Pr(s_t = 1)Pr(s_t = 0)} \right]
\]

\[
B_{x,RR}^W(p_t) = S_x \left[ \frac{(2q - 1)\overline{Q}A - (2q - 1)D}{D} \right]
\]

where $A = \overline{Q}Q + P_t(\overline{Q} + Q - 8\overline{Q}Q), D = P_t^2(\overline{Q} - Q)^2 + P_t\overline{Q}Q(1 - 2\overline{Q} - 2Q) + (\overline{Q}Q)^2$, and defining $P_t \equiv p_t(1 - p_t)$.

Given that the zeros of each of the three benefit functions can be written as a function of $P_t$, it follows immediately that any zero, $\hat{p}_{u,v}^1$, must be paired with another zero, $1 - \hat{p}_{u,v}^1$. For $B_{x,RR}^R(p_t)$ and $B_{x,WW}^W(p_t)$, we have a linear function of $P_t$ and thus there is at most one additional pair of zeros. Furthermore, we can see that there are cases in which such a pair of zeros exist by, for example, taking $\overline{q} \to 1$. For $B_{x,RR}^W(p_t)$, the additional zeros are determined by a quadratic function of $P_t$, so that there may exist two additional pairs of zeros. But I now show that one of the pairs always lies outside $p_t \in (0, 1)$ by showing that one of the solutions to the quadratic is always negative, $P_t^* < 0$, implying $p_t^* < 0$ or $p_t^* > 1$. The quadratic which determines the additional zeros of $B_{x,RR}^W(p_t)$ can be algebraically solved for $P_t$ to obtain $P_t^*$. For specific parameters, the two zeros can be equal, $\hat{p}_{u,v}^1, v^2 = 1 - \hat{p}_{u,v}^1, v^2 = \frac{1}{2}$. 

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\[94\] For specific parameters, the two zeros can be equal, $\hat{p}_{u,v}^1, v^2 = 1 - \hat{p}_{u,v}^1, v^2 = \frac{1}{2}$. 

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manipulated to obtain

\[(2q - 1)\overline{Q}Q A - (2q - 1) D = 0\]

\[\iff -(2q - 1) P_t^2 \left(\overline{Q}^2 - Q^2\right)^2 + P_t f(q, \overline{q}) + 2(q - q) (\overline{Q} Q)^2 = 0\]

where \(f(q, \overline{q})\) is a function which proves to be inconsequential. The solutions to the quadratic are, \(P_t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\) where \(a = -(2q - 1) \left(\overline{Q}^2 - Q^2\right)^2\), \(b = f(q, \overline{q})\), and \(c = 2(q - q) (\overline{Q} Q)^2\). Now, \(a < 0\) and \(c > 0\) for \(\overline{q} > q\) and therefore \(-4ac > 0\). But, this means that \(\sqrt{b^2 - 4ac} > |b|\) and therefore the root \(-b - \sqrt{b^2 - 4ac}\) is always strictly negative. Thus, there are at most two additional zeros of \(B_x^{W,RR}(p_t)\) and furthermore we know that there are exactly two roots in some cases because, by Proposition 2.1, \(B_x^{W,RR}(p_t) < B_x^{W,WW}(p_t)\) and I have already established that \(B_x^{W,WW}(p_t) < 0\) for \(p_t \in (1 - p_x^{W,WW}, p_x^{W,WW})\), for some parameterizations.

For parts 2c and 2d, I first note that Proposition 2.1 implies that

\[B_x^{W,RR}(p_t) < B_x^{W,RW}(p_t), B_x^{W,WR}(p_t) < B_x^{W,WW}(p_t)\]

for all \(p_t \in (0, 1)\). Because of this fact and the fact that the equation that determines the additional zeros of \(B_x^{W,RR}(p_t)\) and \(B_x^{W,WR}(p_t)\) (beyond those that occur at \(p_t = \{0, 1\}\)) is cubic, one can easily see graphically that at most two of the additional zeros lie within \(p_t \in (0, 1)\). That is, it is not possible to have either of these functions cross zero three times within \(p_t \in (0, 1)\) and still have it remain within the envelope of \(B_x^{W,RR}(p_t)\) and \(B_x^{W,WW}(p_t)\), given their established properties. It also implies that, for some parameter ranges these functions are always positive and for others they must cross zero exactly twice.\(^\text{95}\) Therefore, what remains to be shown is that if \(\tilde{p}_x^{W,RR}\) and \(\tilde{p}_x^{W,RW}\) are zeros for \(B_x^{W,WR}(p_t)\) and \(B_x^{W,RW}(p_t)\), respectively, then \(1 - \tilde{p}_x^{W,RR}\) and \(1 - \tilde{p}_x^{W,RW}\) are zeros for \(B_x^{W,RW}(p_t)\) and \(B_x^{W,WR}(p_t)\), respectively, as claimed in the lemma.

Combining terms in \(B_x^{W,RW}(p_t)\), we see that for it to be zero at \(\tilde{p}_x^{W,RW}\), we must have

\[(2q - 1)Q \left(Pr(1,1)Pr(1,0) + Q(Pr(1,0) + Pr(1,1)) \left(\tilde{p}_x^{W,RW}(1 - \overline{q}) + (1 - \tilde{p}_x^{W,RW})\overline{q}\right)\right)\]

\[-(2q - 1)Pr(1,1)Pr(1,0) \left(\tilde{p}_x^{W,RW}(1 - \overline{q}) + (1 - \tilde{p}_x^{W,RW})\overline{q}\right) = 0\]

where \(Pr(1,1) = Pr(\overline{s}_t = 1\&\overline{s}_{t+1} = 1)\) and \(Pr(1,0) = Pr(\overline{s}_t = 1\&\overline{s}_{t+1} = \frac{1}{2}\).

\(^{95}\) For specific parameters, the two zeros may be degenerate with both equal to \(\frac{1}{2}\).
0) are understood to be evaluated at the price, \( \hat{p}^{W,RW} \). We want to show that \( 1 - \hat{p}^{W,RW} \) is a zero for \( B_{x}^{W,WR}(p_t) \). Combining terms in \( B_{x}^{W,WR}(p_t) \), we see that for it to be zero at \( 1 - \hat{p}^{W,RW} \) requires

\[
(2q-1)\text{Pr}(0,0)\text{Pr}(0,1) + q(1-q)\text{Pr}(0,0) \text{Pr}(0,1)
\begin{align*}
&= (1-\hat{p}^{W,RW})q + \hat{p}^{W,RW}(1-q) \quad \text{where, here, } \\
&\quad \text{Pr}(0,0) = \text{Pr}(s_t = 0 & s_{t+1} = 0) \\&\quad \text{Pr}(0,1) = \text{Pr}(s_t = 0 & s_{t+1} = 1)
\end{align*}
\]

\[
(2q-1)\text{Pr}(0,0)\text{Pr}(0,1) \left(1 - \hat{p}^{W,RW} \right) + \hat{p}^{W,RW} \left(1 - q \right) = 0
\]

where, here, \( \text{Pr}(0,0) \) and \( \text{Pr}(0,1) \) are understood to be evaluated at the price, \( 1 - \hat{p}^{W,RW} \). Now, note that \( \text{Pr}(0,0) \) evaluated at \( 1 - \hat{p}^{W,RW} \) is equal to \( \text{Pr}(1,1) \) evaluated at \( \hat{p}^{W,RW} \) and \( \text{Pr}(0,1) \) evaluated at \( 1 - \hat{p}^{W,RW} \) is equal to \( \text{Pr}(1,0) \) evaluated at \( \hat{p}^{W,RW} \) so the two conditions are identical. Therefore, if \( \hat{p}^{W,RW} \) is a zero of \( B_{x}^{W,WR}(p_t) \), \( 1 - \hat{p}^{W,RW} \) is a zero of \( B_{x}^{W,WR}(p_t) \). The reverse is shown in the same manner.

For part 3, the properties established in parts 2b-d and Proposition 2.1 ensure that

\[
\hat{p}^{R,RR}, \hat{p}^{W,WW} < \hat{p}^{W,RW}, \hat{p}^{W,WR} < \hat{p}^{W,RR}
\]

so it remains to be shown that \( \hat{p}^{R,RR} < \hat{p}^{W,WW} \) and \( \hat{p}^{W,WR} < \hat{p}^{W,RR} \). The fact that

\[
1 - \hat{p}^{W,RR} < 1 - \hat{p}^{W,WR} < 1 - \hat{p}^{W,RW} < 1 - \hat{p}^{W,WW} < 1 - \hat{p}^{R,RR}
\]

then follows by the established symmetry properties of the zero-crossings.

To show \( \hat{p}^{R,RR} < \hat{p}^{W,WW} \), I show \( B_{x}^{R,RR}(p_t) > B_{x}^{W,WW}(p_t) \) for all \( p_t \in (0,1) \). For \( q > \bar{q} \), this inequality is equivalent to

\[
\frac{q(1-q)}{\text{Pr}(s_{t+1} = 1)} + \frac{q(1-q)}{\text{Pr}(s_{t+1} = 0)} > \frac{\bar{q}(1-\bar{q})}{\text{Pr}(s_t = 1)} + \frac{\bar{q}(1-\bar{q})}{\text{Pr}(s_t = 0)}
\]

Substituting for the probabilities of each signal and simple algebra shows that this inequality is equivalent to \( (\bar{q} - q)(1 - \bar{q} - q) < 0 \) which holds for \( \bar{q} > \bar{q} \) and \( \bar{q}, q > \frac{1}{2} \).

To show \( \hat{p}^{W,WR} < \hat{p}^{W,WR} \), I show that \( B_{x}^{W,WR}(p_t) > B_{x}^{W,WR}(p_t) \) for \( p_t >
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\[ \frac{Q}{Pr(\tilde{s}_t = 0)} + QQ \left( \frac{1}{Pr(1,1)} + \frac{1}{Pr(1,0)} \right) > \frac{Q}{Pr(\tilde{s}_t = 1)} + QQ \left( \frac{1}{Pr(0,1)} + \frac{1}{Pr(0,0)} \right) \]

\[ \iff \frac{Pr(\tilde{s}_t = 1) - Pr(\tilde{s}_t = 0)}{Pr(\tilde{s}_t = 0)Pr(\tilde{s}_t = 1)} + Q \left( \frac{Pr(\tilde{s}_t = 1)}{Pr(1,1)Pr(1,0)} - \frac{Pr(\tilde{s}_t = 0)}{Pr(0,1)Pr(0,0)} \right) > 0 \]

\[ \iff (Pr(\tilde{s}_t = 1) - Pr(\tilde{s}_t = 0)) Pr(1,1)Pr(1,0)Pr(0,1)Pr(0,0) \]

\[ + q(1 - q)Pr(\tilde{s}_t = 0)Pr(\tilde{s}_t = 1) Pr(\tilde{s}_t = 1) Pr(0,0) - Pr(\tilde{s}_t = 0)Pr(1,1)Pr(1,0) > 0 \]

Considerable algebraic manipulation of the second term allows one to factor out \( Pr(\tilde{s}_t = 1) - Pr(\tilde{s}_t = 0) \), resulting in this difference multiplied by a term which can be shown to be strictly positive. Thus,

\[ B^{W,RW}_x(p_t) > B^{W,WR}_x(p_t) \iff Pr(\tilde{s}_t = 1) - Pr(\tilde{s}_t = 0) = (2q - 1)(2p_t - 1), \]

which is strictly positive for all \( p_t > \frac{1}{2} \).

Given Lemma A.3.1, I now prove Theorem 2.1 by considering each of the price ranges in turn. The proofs for the cases of \( \bar{q} \leq q \) and for \( p_t \in (1 - \tilde{p}^{W,WW}, \tilde{p}^{W,WW}) \) have already been established in the main text, so the focus is on the case of \( \bar{q} > q \) and the remaining price ranges (parts 2c and 2d of the theorem). Also, as discussed in the main text, for the remaining price ranges, any equilibrium timing strategy involves waiting when \( t - 1 \) rushes, so I consider only the best response to \( t - 1 \) waiting here.

2c, part (i). When the unrestricted price chain passes through \( (1 - \tilde{p}^{W,WW}, \tilde{p}^{W,WW}) \), if there exist any traders in the associated restricted price chain and also in the price range, \( [\tilde{p}^{W,WW}, \tilde{p}^{W,WR}] \), then one of them, \( p \), must be such that the trader at \( p^- \) faces a price in \( (1 - \tilde{p}^{W,WW}, \tilde{p}^{W,WW}) \). Then, because the trader at \( p^- \) rushes or conditionally rushes, \( p \) must conditionally rush because her benefit is either \( B^{W,WR}_x(p_t) \) or \( B^{W,RR}_x(p_t) \), both of which are negative (ruling out \( p \) mixing as well). But then any trader in the restricted price chain at prices greater than \( p \) must also conditionally rush for the same reason: her neighbor at the next lowest price in the price chain conditionally rushes. Similarly, any trader in the restricted price chain and also in the price range, \( (1 - \tilde{p}^{W,WR}, 1 - \tilde{p}^{W,WW}) \), must conditionally rush because her neighbor at \( p^+ \) rushes or conditionally rushes, so her benefit is either \( B^{W,WR}_x(p_t) \) or \( B^{W,RR}_x(p_t) \), which are both negative in this price range.

2c, part (ii). When the unrestricted price chain does not pass through \( (1 - \tilde{p}^{W,WW}, \tilde{p}^{W,WW}) \) and the restricted price chain contains only one price, \( p_s \), then, by the definition of the restricted price chain, \( p^+_s \) and \( p^-_s \) must both be such that the trader at those prices waits. Thus, the trader at \( p_s \) has a
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benefit of $B_{z}^{W,WW}(p_t) > 0$ and must wait.  

2c, part (iii). When a restricted price chain contains more than one price and its associated unrestricted price chain does not pass through $(1 - \hat{p}^{W,WW}, \hat{p}^{W,WW})$, multiple possible timing strategies can be sustained in equilibrium. Note that only the two traders nearest the ends of the restricted price chain have neighbors that lie outside it and these neighbors wait. Consider first case A in which all traders facing prices in the restricted price chain wait. Then, each trader in the price chain has a benefit of $B_{z}^{W,WW}(p_t) > 0$ so that waiting is a best response and thus an equilibrium.

I next show that if any trader in the restricted price chain conditionally rushes and all use pure strategies, then they must all do so. This fact simultaneously proves that all of the traders conditionally rushing (case B) can be sustained in equilibrium and that no other combination of pure strategies is possible.

First, note that any trader in $(1 - \hat{p}^{W,RW}, 1 - \hat{p}^{W,WW}] \cup [\hat{p}^{W,WW}, \hat{p}^{W,RW})$ must conditionally rush if either of her neighbors in the restricted price chain conditionally rushes because she then faces either $B_{z}^{W,RW}(p_t)$ or $B_{z}^{W,WR}(p_t)$, both of which are negative in this price range. Second, any trader in $[\hat{p}^{W,RW}, \hat{p}^{W,WR})$ must follow the same timing strategy as her neighbor at the next lowest price because if this neighbor conditionally rushes, she faces either $B_{z}^{W,WR}(p_t)$ or $B_{z}^{W,WW}(p_t)$ which are both negative and, if this neighbor waits, she faces either $B_{z}^{W,RW}(p_t)$ or $B_{z}^{W,WW}(p_t)$ which are both positive. Similarly, any trader in $(1 - \hat{p}^{W,RW}, 1 - \hat{p}^{W,WW}]$ must follow the same timing strategy as her neighbor at the next highest price. Therefore, it is immediate that if any trader in $(1 - \hat{p}^{W,RW}, 1 - \hat{p}^{W,WW}] \cup [\hat{p}^{W,WW}, \hat{p}^{W,RW})$ conditionally rushes, then all traders in the price chain must conditionally rush and that this set of timing strategies can be sustained in equilibrium. The remaining possibility is that a trader in either $(1 - \hat{p}^{W,RW}, 1 - \hat{p}^{W,WW}]$ or $[\hat{p}^{W,RW}, \hat{p}^{W,WW})$ conditionally rushes but some other trader in the restricted price chain waits. I now show this is impossible. Assume that the trader that conditionally rushes is in $[\hat{p}^{W,RW}, \hat{p}^{W,WR})$. Then, by the second property above, all traders in the restricted price chain at higher prices must conditionally rush too, so the only remaining possibility is that a trader at a lower price in the restricted price chain waits. But, by the second property, her neighbor at the

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The conditions for this part of the theorem are only satisfied if $(\hat{p}^{W,WW})^{-} < 1 - \hat{p}^{W,RW}$. In this case, it can be shown that no non-empty restricted price chain has an associated unrestricted price chain with a price contained in $(1 - \hat{p}^{W,WW}, \hat{p}^{W,WW})$. Therefore, there is no parameterization for which 2c, parts (i) and (ii) both apply.
next lowest price must be conditionally rushing, otherwise she herself would wait. This argument can be repeated until the neighbor at the next lowest price is at a price lower than \( \hat{p}_{W,RW} \). This neighbor must be conditionally rushing. If this neighbor lies within \((1 - \hat{p}_{W,RW}, 1 - \hat{p}_{W,WW}) \cup [\hat{p}_{W,WW}, \hat{p}_{W,RW})\) then again, all traders must be conditionally rushing. If this neighbor is instead at a price less than \( 1 - \hat{p}_{W,RW} \), then all traders less than \( 1 - \hat{p}_{W,RW} \) must conditionally rush because they must all follow the same timing strategy as their neighbor at the next highest price. Finally, if the neighbor at the next lowest price is outside the restricted price chain, she can’t be conditionally rushing which creates a contradiction. In all cases then, all traders in the restricted price chain must be conditionally rushing if any trader in \([\hat{p}_{W,RW}, \hat{p}_{W,WR})\) conditionally rushes. A symmetric argument establishes that if any trader in \((1 - \hat{p}_{W,WR}, 1 - \hat{p}_{W,RW})\) conditionally rushes, then all must do so.

To demonstrate the possibility of mixed timing strategies in equilibrium (case C), consider the simplest case of only two prices in the restricted price chain, \( p_1 < \frac{1}{2} \) and \( p_2 > \frac{1}{2} \). The trader facing \( p_1 \) mixes between conditionally rushing and waiting such that the trader at \( p_2 \) has a benefit which is a linear combination of \( B_{W,WR}(p_2) < 0 \) and \( B_{W,WW}(p_2) > 0 \). The trader facing \( p_2 \) mixes such that the trader at \( p_1 \) has a benefit which is a linear combination of \( B_{W,WR}(p_1) < 0 \) and \( B_{W,WW}(p_1) > 0 \). It is easy to see that such mixing probabilities can always be found and therefore, there exists an equilibrium in which traders at both prices mix.

2d. For \( p_t \in (0, 1 - \hat{p}_{W,RR}] \cup [\hat{p}_{W,RR}, 1) \), no matter what \( t - 1 \) and \( t + 1 \) do, \( t \) has a dominant strategy to wait because all of the benefit functions are strictly positive. Thus, the only equilibrium timing strategy in this range is for all traders to wait (mixing is again precluded). Also note that if \( \hat{p}_{W,RR} \) does not exist, then all benefit functions are strictly positive so that the unique equilibrium consists of all traders waiting at every price.

Next, consider \( p_t \in [\hat{p}_{W,WR}, \hat{p}_{W,RR}) \). For prices in this range, when \( t - 1 \) waits, \( t \)'s best response is to rush if \( t + 1 \) rushes after both a buy and sell by \( t - 1 \) and to wait otherwise. Thus, knowing that \( t + 1 \) waits if \( t - 1 \) buys is sufficient to ensure \( t \)'s best response is to wait. Mixing is also precluded in this case because even if \( t + 1 \) mixes after a sell decision, \( t \) faces a linear combination of \( B_{W,WR}(p_t) \) and \( B_{W,WW}(p_t) \) which is always strictly positive in this price range, so \( t \) won’t mix. Now, note that when \( t - 1 \) buys, the price increases so that, for some \( p_t \in [\hat{p}_{W,WR}, \hat{p}_{W,RR}) \), \( p_{t+1} \) exceeds \( \hat{p}_{W,RR} \) so that \( t + 1 \) waits, as already established. Thus, the trader at this \( p_t \) waits. The following algorithm extends this reasoning to prove that the unique equilibrium timing strategy for \( p_t \geq \hat{p}_{W,WR} \) is to always wait.
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1. Define $\hat{p}_i$ such that $\hat{p}_i^+ = \hat{p}_{i-1}$ for $i = 1 \ldots k$ where $k > 1$ is the smallest integer for which $\hat{p}_k < p^{W,WR}$. Also, define $\hat{p}_0 = p^{W,RR}$.

2. If $k = 1$, jump to step 4. Otherwise, when $i = 1$, we have $p_t \geq \hat{p}_1$ is such that $p_t^+ \geq \hat{p}_0 = p^{W,RR}$ so that $t + 1$ waits after a buy by $t$. Therefore, $t$ must wait in equilibrium $\forall p_t \in [\hat{p}_1, \hat{p}_0]$.

3. Repeating step 2 for $i = 2 \ldots k - 1$, at each step we establish that waiting is the unique equilibrium $\forall p_t \in [\hat{p}_i, \hat{p}_{i-1}]$ because for $p_t \geq \hat{p}_i$ we know from the previous step that $t + 1$ waits when $t + 1$ buys $(p^+_t \geq \hat{p}_{i-1})$.

4. At step $k$, for $p_t \geq p^{W,WR}$, $t$ must wait because $p^+_t \geq \hat{p}_{k-1}$ and step $k - 1$ established $t + 1$ waits for $p_{t+1} \geq \hat{p}_{k-1}$. Thus, after $k$ steps, we have established that each trader facing $p_t \in [\hat{p}^{W,WR}, \hat{p}^{W,RR})$ must wait.

The fact that $k$ is finite follows from the fact a finite number of price increases are sufficient to move the price from $\hat{p}^{W,WR}$ to $\hat{p}^{W,RR}$. For $p_t < \hat{p}^{W,WR}$, the argument no longer goes through because knowing that $t + 1$ waits after a buy by $t - 1$ is not sufficient for $t$ to wait. A symmetric argument for $p_t \in (1 - \hat{p}^{W,RR}, 1 - \hat{p}^{W,WR}]$ proves that the unique equilibrium timing strategy in this price range is also for $t$ to wait. Here, a sufficient condition for $t$ to wait is to know that $t + 1$ will wait after a sell decision by $t - 1$ because $B_x^{W,WR}(p_t) > 0$ for $p_t < 1 - \hat{p}^{W,WR}$. Finally, note that if $\hat{p}^{W,WR}$ does not exist, the argument above can be applied to all $p_t \in (1 - \hat{p}^{W,RR}, \hat{p}^{W,RR})$ so that the timing strategy in any equilibrium must be to wait over this range. $\square$

Proof of Proposition 2.3:

The proof consists of three parts. In the first part, I establish that, for all $q \in (1/2, 1)$, there exists a $q_h \in (1/2, q)$ such that the unique equilibrium involves traders waiting at all prices. In the second part, I show that there exists a $q_h \in (1/2, q)$ such that at least some traders rush for any $p_1 \in (1/2, 1)$. Finally, using these facts, I show that there must exist a $\tilde{p} > p_1$ such that prices take strictly longer to converge in expectation to $\tilde{p}$ under $q_h$ than under $q_i$.

Fix $\eta$ to be any value in $(1/4, 1)$. By Theorem 2.1, part 2d, when the parameters are such that $\hat{p}^{W,WR}$ does not exist, then all traders must wait in any equilibrium. Given the shape of $B_x^{W,WR}(p_t)$ as determined by Lemma A.3.1, $\hat{p}^{W,WR}$ does not exist if $B_x^{W,WR}(1/2) > 0$. So, it suffices to show that for any $\eta$ we can find a value of $\eta$ such that $B_x^{W,WR}(1/2) > 0$. From the formula for $B_x^{W,WR}$ in (A.1), one can easily show that $B_x^{W,WR}(\eta) > 0$ at $\eta = 1/2$ for any $\eta \in (1/2, 1)$. Also, Lemma A.3.1 established that $B_x^{W,WR}(p_t) < 0$ at $q = \eta$.

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for any $p_t \in (0, 1)$ which includes $p_t = \frac{1}{2}$. Therefore, because $B^{W,WR}_x(\frac{1}{2})$ is continuous in $q$, there must exist a $q_j \in (\frac{1}{2}, \bar{q})$ such that $B^{W,WR}_x(q_j) > 0$.

A sufficient condition for at least one trader to rush is $B^{R,RR}_x(p_1) < 0$ so that the first trader rushes in any equilibrium. From Lemma A.3.1, $B^{R,RR}_x(p_t) < 0$ at $q = \bar{q}$ for any $p_t \in (0, 1)$, including $p_t = p_1$. Therefore, because $B^{R,RR}_x(p_1)$ is continuous in $q$, there must exist a $q_h \in (\frac{1}{2}, q)$ such that $B^{R,RR}_x(p_1) < 0$. Furthermore, $q_h$ must be strictly greater than $q_j$ identified in the previous step. To see this, we have $B^{R,RR}_x(\frac{1}{2}) > B^{W,WR}_x(\frac{1}{2}) > 0$ when the benefits are evaluated at $q_j$ and the first inequality follows from Proposition 2.1. We also have $B^{R,RR}_x(\frac{1}{2}) < 0$ when the benefit is evaluated at $q_h$. But, $B^{R,RR}_x(\frac{1}{2})$ is monotonically decreasing in $q$ so $q_h$ must be strictly greater than $q_j$:

$$
B^{R,RR}_x(\frac{1}{2}) = \frac{1}{2} [4(2q - 1)q(1 - q) - (2q - 1)]
$$

$$
\Rightarrow \frac{\partial B^{R,RR}_x(\frac{1}{2})}{\partial q} = \frac{1}{2} [4(2q - 1)(1 - 2q) - 2] < 0
$$

where I have used $q < \bar{q}$ in evaluating the benefit function and its derivative.

At the value of $q_h$, we have established that all traders wait and therefore convergence is as in the benchmark model. At the value of $q_j$, (at least) the first trader rushes, implying that the expected value of the price after the first trade is strictly less than the in the benchmark case because it is easily shown that $E[p_{t+1}|p_t, V = 1]$ is increasing in $q$. Therefore, if the number of trades were equal after some amount of time, the expected price would be lower with $q_h$ than with $q_j$. However, due to the first trader rushing, there is initially one more trade with $q_h$ than with $q_j$, and so the expected price is initially higher with $q_h$. But, if we set $\tilde{p} > p_1$ sufficiently close to 1, traders must begin to wait before reaching $\tilde{p}$ because Lemma 4 established that, for $q < \bar{q}$, at sufficiently high prices, all traders wait. Once traders wait, the number of trades becomes equal under $q_j$ and $q_h$ for any realized signals. Thus, there is no price path reaching $\tilde{p}$ for which the number of trades is higher under $q_j$ and so the effect of any trades being earlier disappears. But then convergence to $\tilde{p}$ must be slower in expectation under $q_h$ than $q_j$. □

Proof of Proposition 2.4:

I prove the proposition for the case of $s_j = 1$. The other case is proven similarly. With only a single trader in the market, her benefit can be written (set $\tilde{a}_0 = \tilde{a}_1 = 1$ in the general form of the benefit function):
A.3. Omitted Proofs

\[ B_1(p_t) = \frac{p_t(1-p_t)(2q-1)(1-\mu)}{Pr(\overline{a}_t = 1)} \left[ \frac{1}{Pr(\overline{a}_{t+1} = B)} - \frac{1}{Pr(\overline{a}_t = B)} \right] \]

The term outside the brackets is always strictly positive, so the bracketed term determines the sign of the benefit. Cross-multiplying the terms inside the brackets, we see that that sign is determined by

\[ Pr(\overline{a}_t = B) - Pr(\overline{a}_{t+1} = B) \]

\[ \iff p_t \left( \frac{1-\mu}{4} + \mu q (1-\beta_1) \right) + (1-p_t) \left( \frac{1-\mu}{4} + \mu q (1-\beta_1) \right) \]

\[ - p_t \left( \frac{1-\mu}{4} + \mu q^2 \beta_1 \right) + (1-p_t) \left( \frac{1-\mu}{4} + \mu q^2 (1-\beta_1) \right) \]

\[ \iff \mu \left( (pq + (1-p_t)(1-q)) - \beta_1 (pq^2 + (1-p_t)(1-q)^2 + pq + (1-p_t)(1-q)) \right) \]

From the above expression, we see that the benefit is strictly positive when \( \beta_1 = 0 \) and strictly negative when \( \beta_1 = 1 \) so that neither corner may be an equilibrium. The unique mixed strategy equilibrium is then obtained by setting the expression to zero, which corresponds to the strategy given in Proposition 2.4. \( \square \)

Proof of Corollary 2.1:
The proof of Proposition 2.1 in the case of the zero-profit model is identical to that in the expected value model except that, in applying the mathematical claim, \( NT_x = Pr(\overline{a}_t = NT|V = x) \) is replaced with either \( Pr(\overline{a}_{t+1} = B|V = x) \) or \( Pr(\overline{a}_{t+1} = S|V = x) \), depending on which type of trader’s benefit is being considered.

The proof of Proposition 2.2 in the case of the zero-profit model is identical to that in the expected value model. \( \square \)

Proof of Proposition 2.5:
By the application of Proposition 2.1 to the zero-profit model, if there is any possibility of either type of trader \( t - 1 \) or \( t + 1 \) trading between \( t \) and \( t + 1 \), then each type of \( t \)'s benefit must be strictly less than when alone in the market, \( B_x(p_t, \beta_x) < B_x^{ST}(p_t, \beta_x) \) \( \forall \beta_x \in [0,1] \) and \( \forall p_t \). But, given that \( B_x(p_t) \) is strictly decreasing in \( \beta_x \), this implies that \( t \) must be waiting with \( \beta_x(p_t, I(\overline{a}_{t-1} = NT) < \beta_x^{ST}(p_t) \) in equilibrium. The inequality is strict because \( \beta_x^{ST}(p_t) \) is interior for all \( p_t \). It then follows immediately that \( \beta_x(p_t, 1) < \beta_x^{ST}(p_t) \) for both types of \( t \) because, in this case \( t - 1 \) has waited and will potentially trade between \( t \) and \( t + 1 \). Therefore, it remains only to show that \( \beta_x(p_t, 0) = \beta_x^{ST}(p_t) \) is impossible for either type of \( t \) and,
A.4. Formulas for the Zero-Profit Model

by the above argument, it is only possible if both types of \( t + 1 \) always wait. But, if we assume \( \beta_x(p_t, 0) = \beta_x^{ST}(p_t) \) for either type of \( t \) and both types of \( t + 1 \) always wait, then we arrive at a contradiction: the type of \( t \) that is using \( \beta_x(p_t, 0) = \beta_x^{ST}(p_t) \) is waiting with strictly positive probability and, when she does, each type of \( t + 1 \) must rush with positive probability by the same argument as above, so both cannot be always waiting. □

A.4 Formulas for the Zero-Profit Model

The unique bid and ask prices for delayed trades are:

\[
p_{t+1}^A = Pr(V = 1|H_t, a_{t+1} = B) = \frac{p_t \left( \frac{1-\mu}{4} + \mu q^2 \beta_1 \right)}{p_t \left( \frac{1-\mu}{4} + \mu q^2 \beta_1 \right) + (1 - p_t) \left( \frac{1-\mu}{4} + \mu(1-q)^2 \beta_1 \right)}
\]

\[
p_{t+1}^B = Pr(V = 1|H_t, a_{t+1} = S) = \frac{p_t \left( \frac{1-\mu}{4} + \mu(1-q)^2 \beta_0 \right)}{p_t \left( \frac{1-\mu}{4} + \mu(1-q)^2 \beta_0 \right) + (1 - p_t) \left( \frac{1-\mu}{4} + \mu q^2 \beta_0 \right)}
\]

For rushed trades, the prices are:

\[
p_t^A = Pr(V = 1|H_t, a_t = B) = \frac{p_t \left( \frac{1-\mu}{4} + \mu q(1-\beta_1) \right)}{p_t \left( \frac{1-\mu}{4} + \mu q(1-\beta_1) \right) + (1 - p_t) \left( \frac{1-\mu}{4} + \mu(1-q)(1-\beta_1) \right)}
\]

\[
p_t^B = Pr(V = 1|H_t, a_t = S) = \frac{p_t \left( \frac{1-\mu}{4} + \mu(1-q)(1-\beta_0) \right)}{p_t \left( \frac{1-\mu}{4} + \mu(1-q)(1-\beta_0) \right) + (1 - p_t) \left( \frac{1-\mu}{4} + \mu q(1-\beta_0) \right)}
\]

The benefit functions for generic actions of the other traders are given by:

\[
B_1(p_t, \beta_1) = \frac{p_t(1 - p_t)(2q - 1)(\frac{1-\mu}{4})}{Pr(a_t = 1)} \left[ \sum_{\hat{a}_t \in \tilde{A}} \frac{\hat{a}_t \hat{a}_1}{Pr(\hat{a}_t \& a_{t+1} = B)} - \frac{1}{Pr(\hat{a}_t = B)} \right]
\]

\[
B_0(p_t, \beta_0) = \frac{p_t(1 - p_t)(2q - 1)(\frac{1-\mu}{4})}{Pr(a_t = 0)} \left[ \sum_{\hat{a}_t \in \tilde{A}} \frac{\hat{a}_t \hat{a}_1}{Pr(\hat{a}_t \& a_{t+1} = S)} - \frac{1}{Pr(\hat{a}_t = S)} \right]
\]

The timing strategies of the informed traders, \( \beta_0 \) and \( \beta_1 \), enter the benefit functions through the probabilities of observing a buy and sell decision at
A.5. Multiple Arrival \((n > 1)\)

In this section, I extend the expected value model to one in which \(n\) traders arrive in each period. Each trader receives an independent signal in the period they arrive and in the next period if they choose to wait. With multiple traders arriving each period, waiting costs are increased such that traders may rush to trade with even greater differences in signal quality. This extension is used in the numerical simulations of Section 2.4 and to derive the comparative static prediction with respect to volume of Section 2.6. A full equilibrium characterization is difficult to obtain and I do not attempt to derive one here. Instead, I rely on a combination of theoretical results and numerical analysis to determine an upper bound on the average price reached and a lower bound on the probability of increasing mispricing as functions of time. Because numerical analysis is used, I focus on the specific case of \(n = 4, q = 0.7\) and \(q = 0.8\) used in Section 2.4. However, most of the analysis is general in nature and applies to a wide range of parameters.

I look for a symmetric equilibrium in which traders with the same signals follow the same strategies. Because traders who trade at a time \(t\) do not affect the price each other faces, the optimal trading strategies are identical to the \(n = 1\) case. Furthermore, the general form of the benefit function, (2.1) continues to apply because it was derived for any possible combination of trades while one waits. Therefore, analogous statements to Lemma 2.2 and Propositions 2.1 and 2.2 can be easily derived. In particular, Lemma 2.2 implies that both types of traders must follow the same timing strategy so that the simplified benefit function of Appendix A.1 applies. With the addition of the symmetry assumption, we then have that all traders arriving in the same period must follow the same timing strategy.

The main difficulty in obtaining a complete characterization of the equilibrium for \(n > 1\) is that when the \(t - 1\) traders wait, there are many possible
prices that the traders at $t+1$ may face and thus many possible benefit functions a trader at $t$ may face. Thus, the approach used for the $n = 1$ case, looking at each of the price ranges determined by the zero-crossings of each of the benefit functions, becomes extremely tedious. My approach is to instead characterize the equilibrium within a specific price range and assume all traders wait outside this range. I can then simulate an upper bound on the average price reached and a lower bound on the probability of widening mispricing as argued in Section 2.4.

More specifically, for the parameters given, I claim that there exists a price, $\hat{p}_{R,R}^{n}$, such that any equilibrium involves all traders rushing when $p_t \in (1 - \hat{p}_{n}^{R,R}, \hat{p}_{n}^{R,R})$. In particular, $1 - \hat{p}_{n}^{R,R}$ and $\hat{p}_{n}^{R,R}$ are the prices at which the benefit to waiting when all $t-1$ and $t+1$ traders rush, $B_{n}^{R,R}(p_t)$, is zero. The analytic formula for this benefit is easily obtained using the simplified general form of the benefit in Appendix A.1. However, establishing the properties corresponding to those of Lemma A.3.1 is more difficult so I rely on numerical simulation to show that it in fact crosses zero at exactly two prices which are symmetric around $p = \frac{1}{2}$, $1 - \hat{p}_{n}^{R,R}$ and $\hat{p}_{n}^{R,R}$, where $\hat{p}_{n}^{R,R} \approx 0.866$. The benefit function is positive near $p = 0, 1$ and negative otherwise.

The benefit function when all $t-1$ and $t+1$ traders wait, $B_{n}^{W,W}(p_t)$ also plays a role. Because $\bar{q} > q$, the price impact of a $t-1$ trader is greater than that of a $t$ trader, so $B_{n}^{W,W}(p_t) < B_{n}^{R,R}(p_t) < 0$ over $p_t \in (1 - \hat{p}_{n}^{R,R}, \hat{p}_{n}^{R,R})$. To establish the claim, I show that there exists an equilibrium in which all traders rush, independent of the number of $t-1$ traders that wait (with $n$ traders, the number of $t-1$ traders observed to have waited is payoff-relevant) and that there does not exist an equilibrium in which all traders wait on the equilibrium path. The proof is as follows.

Consider first the claim that there does not exist an equilibrium in which all traders wait on the equilibrium path. If all $t-1$ traders rush and the traders at $t$ were to all wait at some $p_t \in (1 - \hat{p}_{n}^{R,R}, \hat{p}_{n}^{R,R})$, then the traders at $t+1$ face the same price with a benefit of $B_{n}^{W,W}(p_t) < 0$ or less (depending on the strategies of the $t+2$ traders: by Proposition 2.1, if any $t+2$ trader is rushing, $t+1$’s benefit would be reduced from $B_{n}^{W,W}(p_t)$) and so each would have an incentive to deviate to rush, a contradiction. Alternatively, if all $t-1$ traders wait, then all $t$ traders have a benefit of $B_{n}^{W,W}(p_t) < 0$ or less, and so would rush, a contradiction. Thus, the only possible symmetric equilibria are those in which all traders rush on the equilibrium path. So, consider the

\[97\text{It can be generally shown that one’s benefit is strictly decreasing in the signal quality of any trade that occurs while one waits (proof available upon request). This fact ensures that a trade by } t-1 \text{ impacts } t \text{ more than a trade by } t+1 \text{ when } \bar{q} > q.\]
strategy profile in which all traders rush regardless of the number of \( t - 1 \) traders that wait. One must show that no single trader has an incentive to deviate to wait under this strategy profile. There are two possible cases.

In the first case, all \( t - 1 \) traders were observed to rush. In this case, if a single trader at \( t \) deviates to wait, the \( t + 1 \) traders face \( p_t \in (1 - \tilde{p}_n^{R,R}, p_n^{R,R}) \) and therefore rush under the specified strategy profile. Thus, the trader at \( t \) that is considering deviating faces the price impacts of \( 2n - 1 \) traders. Given that the price impact of \( n \) traders gives her a benefit of \( B_n^{R,R}(p_t) < 0 \) and that Proposition 2.1 ensures additional traders reduce her benefit further, she will not deviate.

In the second case, one or more \( t - 1 \) traders were observed to wait. In this case, if \( t \) deviates, nothing can be said about the timing decisions of the \( t + 1 \) traders because the trades by the \( t - 1 \) traders may drive the price outside of \( p_t \in (1 - \tilde{p}_n^{R,R}, p_n^{R,R}) \). However, \( t \) still faces the price impacts of the \( n - 1 \) other \( t \) traders as well as those of the one or more \( t - 1 \) traders. Because \( \bar{q} > q \), the price impact of a \( t - 1 \) trader is greater than that of a \( t \) trader, so \( t \)’s benefit from waiting is strictly less than \( B_n^{R,R}(p_t) < 0 \), so she will not deviate.

In summary, there exists an equilibrium in which all traders rush and there does not exist an equilibrium in which all traders wait for all \( p_t \in (1 - \tilde{p}_n^{R,R}, p_n^{R,R}) \). Having established these facts, I can then simulate the model under the additional assumption that all traders wait outside \( p_t \in (1 - \tilde{p}_n^{R,R}, p_n^{R,R}) \). This simulation provides an upper bound for the average price reached and a lower bound for the probability of increases in mispricing. If in fact traders rush more often than assumed, the average price would in fact be lower and the probability of increases in mispricing higher. Simulation results are presented in Section 2.4.

Finally, the analysis for \( n > 1 \) justifies the claim in Section 2.6 that higher volume leads to more panics and therefore less informational content in trades. Proposition 2.1 guarantees that an increase in \( n \) decreases the benefit to waiting and thus makes the interval over which traders must panic, \( (1 - \tilde{p}_n^{R,R}, p_n^{R,R}) \), larger. It is in this sense that an increase in volume leads to more panics. As an example, when \( q = 0.7 \) and \( \bar{q} = 0.8 \), going from \( n = 1 \) to \( n = 4 \) changes the interval from not existing to about 73% of the entire price range.
Appendix B

Appendix to Chapter 3

B.1 Analysis Details

The equilibrium characterization for the Basic model is derived in Chapter 2. Here, I comment on the specific results for each of the two parameterizations. As in Chapter 2, define the probability of waiting for a trader with signal \( s_t = x \) at price \( p_t \) who has observed whether or not \( t - 1 \) traded or not as \( \beta_x(p_t, I(a_{t-1} = NT)) \), where \( I \) is the indicator function. To determine optimal timing decisions, I look at the net benefit to waiting (expected profit - current profit) for a trader with \( s_t = x \), \( B_x(p_t) \). One optimally rushes if this benefit is negative, and waits otherwise.

Proposition 3.1 is a simple restatement of Lemma 2.1 after applying \( \beta_1(p_t, I(a_{t-1} = NT)) = \beta_0(p_t, I(a_{t-1} = NT)) \) from Lemma 2.2.

Proposition 3.2 part a) is almost a direct application of Theorem 2.1 for the case of \( \bar{q} < q \) except that here \( T \) is finite. Finite \( T \) affects only the final trader who faces no potential trade from a subsequent trader if she waits. Because of this fact, if trader \( T - 1 \) rushes, rather than \( T \) rushing because the next trader will rush if she waits, trader \( T \) is indifferent between waiting and rushing. Thus, a multiplicity of equilibria exist, differing only in the behavior of \( T \). In the experimental results, I ignore the timing behavior of \( T \) when \( T - 1 \) rushes for this reason.

For treatment BW, Theorem 2.1 again applies, but it remains to be determined which case is relevant. To establish Proposition 3.2 part b), I proceed to establish that \( t \)'s net benefit to waiting is positive at all \( p_t \), no matter what the strategies of the other traders are. From Proposition 2.1, if we know that the benefit is positive when \( t - 1 \) waits and \( t + 1 \) rushes, then we know that it must be positive for all combinations of strategies because they result in less potential trades while \( t \) waits. Thus, consider the benefit for this set of strategies of the other traders, \( B_x^{W,RR}(p_t) \). Its formula from Appendix A.1 is
\[
B_{x}^{W,RR}(p_t) = S_x \left[ (2\eta - 1)\overline{Q} \left( \frac{1}{Pr(1,1)} + \frac{1}{Pr(1,0)} + \frac{1}{Pr(0,1)} + \frac{1}{Pr(0,0)} \right) - (2\overline{q} - 1) \right]
\]  

(B.1)

where \( S_x \equiv \frac{p_t(1-p_t)}{Pr(1,1)} \), \( \overline{Q} \equiv \overline{q}(1-\overline{q}) \), \( Pr(x,y) \equiv Pr(s_t = x & s_{t+1} = y) \), and I have used the fact that \( \overline{q} > q \) for the BW treatment. Lemma A.3.1 establishes that \( B_{x}^{W,RR}(p_t) \) crosses zero at at most two places which must be symmetric about \( p = \frac{1}{2} \). And, Proposition 2.2 establishes that, as prices approach 0 and 1 \( B_{x}^{W,RR}(p_t) \) must approach that of a trade alone in the market, who in this case has a positive benefit from waiting given \( \overline{q} > q \).

Together these facts imply that if \( B_{x}^{W,RR}(\frac{1}{2}) > 0 \), then \( B_{x}^{W,RR}(p_t) > 0 \) for all \( p_t \in (0,1) \) which means all traders must wait regardless of the strategies of other traders or price they face. Numerically evaluating (B.1) at \( p_t = \frac{1}{2}, q = \frac{13}{24}, \) and \( \overline{q} = \frac{17}{24} \) gives \( B_{x}^{W,RR}(\frac{1}{2}) \approx 0.130 > 0 \). Thus, Proposition 3.2 part b) is established.

**B.2 Rational Herding and the \( \tau \)-Herding Heuristic**

The informational herding environment of Banerjee (1992) and Bikhchandani et al. (1992) has been well-studied in experimental economics.\(^{98}\) In this environment, subjects sequentially take one of two actions after observing a private signal and the past actions of previous subjects. One can consider this environment a version of a sequential trading environment with exogenous timing in which prices are fixed: the cost of taking an action is independent of the previous actions taken (and is usually set to zero). Rational herding, in which one follows one’s predecessors instead of one’s signal is predicted to occur. Intuitively, after two previous subjects take the same action, their information outweighs one’s own, so it becomes rational to follow their actions. At this point, an informational cascade develops in which all subsequent subjects take the same action as the first two. In experimental evidence, rational herding occurs, but not as strongly as predicted. The two robust findings are that rational herds are frequently broken by subjects that instead follow their own signal, and that this “breaking” of a cascade

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\(^{98}\)Anderson and Holt (1997) provide the first study. See also Kübler and Weiszäcker (2004) and Goeree et al. (2007).
B.2. Rational Herding and the $\tau$-Herding Heuristic

decreases with the number of previous subjects that have taken the same action.

Goeree et al. (2007) show that quantal response equilibrium, in which subjects understand that previous subjects may have made mistakes when they chose their action, can explain both the fact that cascades break and that they break less frequently as the length of the cascade increases. Kübler and Weizsäcker (2004) have a similar explanation based on a limited depth of reasoning model. However, while these explanations explain behavior in the fixed price environment, the discussion in Section 4.5 shows that believing past trader’s made mistakes cannot explain behavior observed in the laboratory data obtained here. Cipriani and Guarino (2005) and Drehmann et al. (2005) also show that these explanations are inconsistent with herding in their sequential trading environments.

Having shown that the $\tau$-herding heuristic can explain both the behavior in the environment considered here and in the sequential trading environment with endogenous prices, one wonders whether it can also explain behavior in the fixed-price environments. Under the assumption that subjects understand that other subjects may be $\tau$-herding types, I show here that the answer is yes. The discussion is informal, but suffices to show the reasoning by which the heuristic can explain the observed behavior.

Consider the standard informational herding environment in which there are only two actions, $A$ and $B$, each of which is equally likely ex ante. Subjects receive private binary signals which indicate the true state ($A$ or $B$) with some probability, typically $0.6 - 0.75$. If a subject chooses an action corresponding to the true state, she receives a payoff of 1, otherwise she receives 0. Subjects observe all past actions. Now consider the behavior that would be observed if the majority of subjects follow the $\tau$-herding heuristic and understand that others do so. That is, subjects have threshold beliefs that the true value is $A$ above (below) which they will choose action $A$ ($B$) regardless of their private signal. For less extreme beliefs, they follow their signal.

The first subject simply follows her signal, regardless of her threshold belief because this is the only information she has. Without loss of generality, say the first subject took action $A$. The second subject will take action $A$ regardless of her threshold belief if her private signal indicates $A$. If, instead, her signal indicates $B$, she is indifferent. The typical assumption in this case is that she follows her own signal. If the third subject observes the past two subjects took action $A$, rationally she should choose $A$ regardless of her
private signal, since her belief will indicate A is more likely. However, if she follows the \( \tau \)-herding heuristic (which is a misnomer in this environment!), she will choose action B if her private signal indicates B and her threshold belief, \( \tau \), is greater than \( b^* \), where \( b^* \) is the belief corresponding to two signals indicating A and one indicating B. Therefore, if some subjects are \( \tau \)-herding types, we obtain the first experimental finding that informational cascades may break.

Now consider the belief of a fourth subject that observes three A choices in a row. If she believes that the third subject uses the heuristic, then she places a higher belief on state A than after observing only two A actions, because she realizes that the third subject would have chosen B if her threshold belief is above \( b^* \) and she observed a signal indicating B. If the fourth subject also uses the heuristic, she will take action A if her signal indicates A, but she may take action B if her signal indicates B and her belief does not exceed her own threshold belief. Importantly, she is less likely to take action B than the third subject because her belief that the state is A is higher than that of the third subject. Therefore, we obtain the prediction that, as the cascade lengthens, a subject is less likely to break the cascade, which is the second robust experimental finding.

While the above analysis is admittedly informal, it demonstrates how a more formal analysis that makes concrete assumptions about the threshold beliefs of subjects, and their beliefs about the threshold beliefs of other subjects, can generate behavior consistent with both experimental findings in the fixed price environment. In particular, as long as some subjects use the heuristic and also believe that some other subjects use the heuristic and have threshold beliefs above \( b^* \), the argument holds. In the experimental data here, evidence of considerable heterogeneity in threshold beliefs exists (see Section 4.4) and, in particular, some subjects use sufficiently high threshold beliefs for the above argument to hold. Therefore, the \( \tau \)-herding heuristic is capable of simultaneously explaining behavior across three related social learning environments, something which none of the other existing theories is capable of.
B.3 Learning

B.1 plots the evolution of perfectly rational behavior in each treatment over the full course of each session. There is considerable evidence of learning in the BW treatment, where subjects learn that an additional signal is valuable. In the BR treatment, however, learning is difficult because waiting is only costly if either the previous trader waited or the subsequent trader rushes and the trade(s) move prices adversely. If one’s predecessors mostly rush, it may take time to learn that prices on average move adversely. Learning may have perhaps been easier had subjects maintained the same position in the sequence in treatments BR and BW, but the concern in this case is that subjects would be learning about particular subjects rather than their environments.

\[99\text{For concreteness, I consider only the case in which all of one’s predecessors took the same action. However, the implications can be generalized to other cases.}\]
B.4 Non-equilibrium Informational Losses

Non-equilibrium behavior exacerbates the informational losses predicted by rational theory in both treatments. I measure informational losses by comparing final prices in each trial to actual asset values. B.2 plots the empirical cumulative distribution function (cdf) of the absolute value of the difference in these two values for each of the treatments. In each case, the difference between the asset value and the theoretical final price, had all traders acted rationally, is compared to the difference between the asset value and the actual final price in the data. In the treatment in which rushing is optimal, BR, we see that the theoretical and actual losses are very similar: any difference is due solely to trades not revealing private information. Therefore, when panicking is rational, almost all information losses are due to rational behavior. Importantly, non-equilibrium waiting does not cause additional information to be aggregated because waiting to obtain an additional signal does not reveal additional information.

In treatment BW, in which waiting is optimal, we observe large informational losses due to non-equilibrium behavior. Here, traders determine all information aggregation, and we observe the large losses due to non-equilibrium behavior. On average, the observed final price differs from the theoretical by 9.6%. The average difference in the BR treatment is 3.5%.

B.5 Instructions

The instructions for the BW treatment, are provided. The instructions for the BR treatment are identical except for the differences in parameters.
Figure B.2: CDFs of Difference Between Final Prices and Asset Values by Treatment (Basic)
Instructions

This is a research experiment designed to understand how people make stock trading decisions in a simple trading environment. To assist with our research, we would greatly appreciate your full attention during the experiment. Please do not communicate with other participants in any way and please raise your hand if you have a question.

You will participate in a series of 42 trials. In each trial, you and 5 other people will each trade a stock with the computer. The trades are sequenced so that each person arrives to the market one at a time. You will be paid $5.00 for completing all trials. In addition, in each trial you will earn lottery tickets as described next. The more lottery tickets you have, the more you may earn.

Before each trial, the computer will randomly select whether the stock's value, V, is 100 tickets or 0 tickets, with equal probability. When it is your turn, you will be given 100 tickets to begin and then you may choose either to buy or sell the stock. You will only trade once. If you buy the stock, you will gain its actual value minus the price, P, you pay for it. If you want to sell the stock, you must first 'borrow' it from the computer and later pay back its actual value. So, if you sell, you will gain the price you receive minus its value. In summary, your total profit is:

\[\text{Profit} = \begin{cases} 100 + V - P & \text{if you buy} \\ 100 + P - V & \text{if you sell} \end{cases}\]

So, for example, if you sell the stock at a price of 50 and it turns out to be worth 100, you would earn 100+(50-100)=50 tickets for that trial. But if it turns out to be worth 0, you would earn 100+(50-0)=150 tickets.

To help you guess the value of the stock the computer chose, you will get a First Clue when it is your turn. Specifically, there will be two possible bins: one that the computer will use if the value is 100 and another that the computer will use if the value is 0. Bins contain some number of blue and green marbles as shown below. The computer will draw a marble randomly from the bin and show it to you. Each marble in the bin is equally likely to be drawn. You can use this clue to get a better idea of what the stock's value is.

After observing your First Clue, you can choose to buy or sell the stock immediately. Alternatively, you can choose to wait and trade in the following period. If you choose to wait, you will get a Second Clue: the computer will draw a marble from another bin. The bin used will again depend on the stock's true value but the possible bins used for the Second Clue are different from the bins used for the First Clue as shown below. After observing the Second Clue, you may choose to buy or sell the stock. Note that you must trade in either the period you arrive or the next period and that you can only trade once.

<table>
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<tr>
<th>Bin 1</th>
<th>Bin 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>13 blue</td>
<td>11 blue</td>
</tr>
<tr>
<td>11 green</td>
<td>13 green</td>
</tr>
</tbody>
</table>

Importantly, if you decide to wait, other participants will have a chance to trade before you have your second chance to trade. Specifically, if the participant before you waited, they will trade before you. And, the next participant will arrive to the market and have a chance to trade immediately or to wait, just like you. If they trade immediately, they will also trade before you. The timing of trades is shown in the figures below. Each open circle indicates a possible trade time and the arcs indicate the two possible times at which a particular participant can trade. The red, highlighted arc corresponds to the participant whose turn it is (the second participant in this example).

When you arrive in the market, you will observe whether or not the person before you has already traded. In the first example to the right, the second participant has just arrived to the market and the first participant has already traded (they didn't wait). This is indicated by the arrow pointing to the time at which they traded. Also, the open circle contains a '+' sign (for a buy) or a '-' sign (for a sell) after a trade has completed. In this example, if the second participant waits, there is only one possible trade (by the third participant if they trade immediately) before the second participant gets a chance to trade again.

In the second example to the right, when the second participant arrived in the market, she saw that the first participant has waited (arrow to the right). In this case, the open circle where the first participant arrived disappears because no trade occurred. Also, a shaded circle appears where the first participant must trade. So, if the second participant were to wait, the first participant would trade for certain before the second participant has a chance to trade again. And, the third participant would also trade before the second, if they trade immediately.

All past trading decisions and prices will be displayed in a figure as shown below. In this example, the first participant waited and then bought the stock. The second participant sold the stock immediately. The third participant has waited and it is currently the fourth participant's turn. The price history is displayed in the graph along with the current price at which you can trade (67.27 in the example). The current time period is indicated by the dashed red line. After choosing buy or sell (or wait in the period you arrive), you will press the 'confirm' button (not shown).
The price of the stock will change over time based upon the trades that occur. Therefore, if you wait, and another participant trades, the price you will face when you have a second chance to trade will be different than the price you could have traded at if you traded immediately.

The initial price of the stock is 50 tickets, reflecting the fact that it is equally likely to be worth 100 tickets or 0 tickets. Similarly, after a trade, the price of the stock will be updated so that it continues to be equal to the stock’s expected value given the information that can be inferred from the trades, but no other information. The stock’s price will therefore increase after a buy and decrease after a sell, and the change will be smaller for immediate trades than for trades that take place after waiting (as you can see in the example above).

Summary

1. At the beginning of each trial, the computer randomly selects the stock’s value: 0 or 100.
2. The first participant is shown their First Clue and then chooses to buy, sell, or wait.
3. If the first participant chooses to buy or sell, they are done for this trial. If they choose to wait, the second participant arrives to the market, observes their own First Clue, and chooses to buy, sell, or wait.
4. If the first participant chose to wait, they will then buy or sell after observing a Second Clue.
5. Steps 2-4 are repeated for all six participants. Each participant observes their own First Clue (and Second Clue if they choose to wait) from the same bins as previous participants. Marbles are always replaced so later participants may see the same (or different) marbles.
6. At each point in time, all past prices and trading decisions are available to use to help guess the value of the stock (in addition to the Clues).

After all participants have traded, the trial is complete. The true value of the stock will then be revealed and you will be told how many tickets you earned for the trial. You will press ‘next trial’ to participate in the next trial. It is important to remember that, in each trial, the value of the stock is independently randomly selected by the computer — there is no relationship between the value selected in one trial and another. The order of participants in the sequence may be different from trial to trial: you may be participant 1 in one trial, participant 4 in another, and participant 6 in yet another. Which participant you are in each trial will be told to you before you make your trade.

After all 42 trials are complete, one lottery will be conducted for each trial. For each lottery, a random number less than 200 will be chosen by the computer. If the number is smaller than the number of lottery tickets you earned for the trial, you will get $1.00. Therefore, the more lottery tickets you earn in each trial, the more you can expect to make (partial tickets are possible and count too). For example, if you earn 100 tickets in each trial, you can expect to make 0.5*42*1=$21.00 over the 42 trials. But, if you earn 120 tickets in each trial, you can expect to make 0.6*42*1=$25.20.

Please try to make each decision within 15 seconds so that the experiment can finish on time. A timer counts down from 15 to help you keep track of time when it is your turn. Note, however, that if the timer hits zero, you can still enter your trading decision and will still have the same chance to earn money. Before beginning the paid trials, we will have two practice trials for which you will not be paid. These trials are otherwise identical to the paid trials except that they are not timed.

Quiz

Please answer the following questions. To ensure you understand the instructions, you must answer all of the questions correctly before we begin the experiment.

1. You are the first participant and have waited. Your First Clue is a green marble and your Second Clue is a blue marble. Based only on this information, What is the most likely value of the stock?
   - 100
   - 0

2. You are the second participant and observe the first participant
1. Why did you sell the stock immediately? What color marble are they most likely to have seen?
   - blue

3. You are the first participant and your first clue is a blue marble. What color marble is the second participant most likely to see?
   - green
   - blue

4. If you choose to sell the stock at a price of 80 and its value turns out to be 100, how many total tickets would you get for that trial?
   - 80
   - 20
   - 180

5. If you choose to buy the stock at a price of 25 and its value turns out to be 100, how many total tickets would you get for that trial?
   - 25
   - 75
   - 175

6. The stock's current price is 80. Which value of the stock is more likely?
   - 100
   - 0

7. You are the second participant and you observe that the first participant did not trade immediately. How many trades can occur before you trade again if you wait?
   - 0 or 1
   - 1 or 2
   - 1

Once you have completed the quiz, please press 'Check answers'.
Appendix C

Appendix to Chapter 4

C.1 Analysis Details and Omitted Proofs

In this section, I describe the theoretical analysis of the Extended model, proving Proposition 4.2. The details of the analysis borrow heavily from the results in Chapter 2, but do not immediately follow from the analysis there. Here, I focus on $q > q$. For $q \leq q$, it is easily shown that the unique equilibrium is for traders to trade immediately for the same reason as in the Basic model: the additional information, $s_i$, is of no value because it never changes one’s trading decision.

The solution concept is sequential equilibrium and I focus on Markov strategies that depend only upon the payoff-relevant state. Because the price is a sufficient statistic for all prior public information, a Markov strategy depends upon the price and not upon the specific realizations of past trades or public signals. The number of traders that have traded in prior periods, $\tilde{n} \in 0, \ldots, n - 1$, is also payoff relevant because, should one wait to trade, it determines the number of potential trades that could occur before one trades.

I restrict attention to off-equilibrium beliefs such that no information is revealed by the decision to wait to trade. This restriction is satisfied when one’s timing decision does not depend upon one’s private information. As shown below, any equilibrium of the Basic model must involve timing strategies which do not depend upon private information, so that it seems natural to assume that any deviation from the optimal timing strategy also does not.

A behavioral strategy for a trader in the Extended model is a mapping from her initial private signal, the number of other traders who have previously traded, and the current price, to an action: buy, sell, or not trade. Without loss of generality, this strategy can be decomposed into a trading strategy (buy or sell) and the probability of postponing one’s trade (waiting). Denote the probability of waiting, $\beta_x(p_t, \tilde{n})$, where $x$ denotes the realization of the trader’s initial signal, $s_i$. 

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C.1. Analysis Details and Omitted Proofs

Consider first the optimal trading strategy taking the timing strategies as given. The proof of Lemma 2.1 can be reproduced exactly to show that the optimal trading strategy for a trade in any period, \( t < T \), is to buy if \( s_i = 1 \) and sell if \( s_i = 0 \). In period \( T \), when \( V \) is known, it is simple to show that one must buy if \( V = 1 \) and sell if \( V = 0 \). These facts establish Proposition 4.1.

The next step in characterizing equilibrium strategies is derive the benefit to waiting to some future period and trading then. To do so, one must derive the optimal trading strategy after waiting. One can follow the derivation in Lemma 2.1 to determine whether buying or selling is optimal. As is the case there, events that occur during the waiting period are irrelevant because they affect both the trader’s private belief and the price in the same way. But, the optimal trading strategy does depend upon trader \( i \)’s timing strategies because, if \( \beta_0(p_t, \tilde{n}) \neq \beta_1(p_t, \tilde{n}) \), information is revealed by waiting and thus affects the price. The end result is that, if waiting until a period in which no further information is obtained, a trader with \( s_i = 1 \) buys and a trader with \( s_i = 0 \) sells, for all possible timing strategies. If waiting until \( T \) where an additional signal, \( s_t \), is obtained, the optimal trading strategy is given by Lemma 2.1, simply relabeling the timing strategies.

Given the optimal trading strategies, one can derive a general form of the benefit to waiting identical to that of Chapter 2. This general form does not depend on the particular types of events that occur while one waits, and so is equally applicable for trades by other traders or public signal events. The general form of the benefit is:

\[
B_x(p_t, \beta_0, \beta_1) = \frac{p_t(1 - p_t)}{Pr(s_i = x)} \left[ \sum_{\hat{a} \in A} \frac{\hat{a}_0 \hat{a}_1 f(q, \bar{q}, \beta_0, \beta_1)}{Pr(\hat{a} \& a_{i,t} = NT)} - (2q - 1) \right] 
\]  

(C.1)

where

\[
f(q, \bar{q}, \beta_0, \beta_1) = \begin{cases} 
(2\bar{q} - 1)(qNT_0 + (1 - q)NT_1) & \text{if } s_t = 1 \& g_1(q, \bar{q}) \leq 0 \\
qNT_0 - (1 - q)NT_1 & \text{if } s_t = 1 \& g_1(q, \bar{q}) > 0 \\
qNT_1 - (1 - q)NT_0 & \text{if } s_t = 0 \& g_0(q, \bar{q}) < 0 \\
(2\bar{q} - 1)(qNT_1 + (1 - q)NT_0) & \text{if } s_t = 0 \& g_0(q, \bar{q}) \geq 0 
\end{cases}
\]

\[
g_0(q, \bar{q}) \equiv (1 - \bar{q})qNT_0 - \bar{q}(1 - q)NT_1 \geq (\triangleq) 0, \quad g_1(q, \bar{q}) \equiv g(1 - \bar{q})NT_0 -
\]

Because in a finite game prices are always strictly different from 0 and 1, one is never indifferent.
C.1. Analysis Details and Omitted Proofs

\[(1 - q)\overline{q}NT_1 > (\leq)0, \quad NT_0 = (1 - q)\beta_1(pt, \bar{n}) + q\beta_0(pt, \bar{n}), \quad NT_1 = q\beta_1(pt, \bar{n}) + (1 - q)\beta_0(pt, \bar{n}), \quad \hat{a}_y \equiv Pr(\hat{a}|V = y), \quad \text{and } \hat{a} \text{ is a particular event in the set of possible events, } A, \text{ that can occur while one waits. This benefit function applies only when waiting to the final period where } \overline{s}_i \text{ is obtained. When waiting until any other period, } g_0(q, \overline{q}) < 0 \text{ and } g_1(q, \overline{q}) > 0, \text{ which pins down the formulas for } f(q, \overline{q}, \beta_0, \beta_1) \text{ that apply. Note that (C.1) implicitly depends on } \overline{n} \text{ by constraining the set of events, } A, \text{ that one must sum over.}

Because the form of the benefit function is identical to that in the Basic model, Lemma 2.2 also immediately applies, ensuring that traders with either realization of \(s_i\) must follow the same timing strategy in any equilibrium, \(\beta_0(pt, \bar{n}) = \beta_1(pt, \bar{n})\). To determine the benefit when one is considering a deviation to waiting (as opposed to waiting being an equilibrium strategy), the off-equilibrium beliefs of other traders and the expected price matter, so some assumption must be made as to what these are. I assume that beliefs about \(s_i\) after such a deviation are such that either signal is equally likely. This assumption seems natural since it implies traders with either signal follow the same timing strategy both in and out of equilibrium. Consistent with these beliefs, I also assume prices are unchanged after observing a timing deviation.

Under these assumptions, the benefit function can be simplified by setting \(\beta_0(pt, \bar{n}) = \beta_1(pt, \bar{n})\) in (C.1):

\[
B_x(pt) = \frac{pt(1 - pt)}{Pr(s_i = x)} \left[ \sum_{\hat{a} \in A} \frac{\hat{a}_0\hat{a}_1(2q - 1)}{Pr(\hat{a})} - (2q - 1) \right]
\]

where \(q = q\) if waiting to a period with no new private information and \(q = \overline{q}\) if waiting until \(T\) when \(\overline{s}_i\) arrives. Note that the benefit no longer depends on the timing strategies of trader \(i\). Propositions 2.1 and 2.2 also immediately apply given that the benefit function is identical. An immediate consequence of Proposition 2.1 is that waiting to trade in a future period in which no new information arrives can never be optimal. In such a case, Proposition 2.1 states that the (one or more) public signal(s) that arrive during the waiting time strictly decrease the benefit to waiting, but it is easily seen that (C.2) is identically zero when there are no intervening public signals or trades by others, and therefore negative when there are. Because there is at least one public signal between trading periods, if a trader waits, it must be until period \(T\). Intuitively, if one receives no new private information from waiting, one never waits because it is costly due to the public signals and potential trades by other traders which move prices against the trader in expectation (due to the unconditional correlation in
C.1. Analysis Details and Omitted Proofs

signals). This intermediate result is formalized in Lemma C.1.1:

**Lemma C.1.1:** At any history, an equilibrium behavioral strategy for the simultaneous model specifies either trading immediately or waiting until \( t = T \).

To determine whether trading at \( t \) or waiting until \( t = T \) is optimal, one must evaluate (C.2) at \( p_t \) and \( \tilde{n} \), given the strategies of the other traders. If positive, the trader waits, and, if negative, she trades immediately. An equilibrium is characterized by finding the fixed point in strategies. Rather than pursuing a full characterization here, I focus on two sets of sufficient conditions that allow the equilibria to be easily characterized.\(^{101}\)

I first derive a sufficient condition to ensure all trading occurs at \( t = 1 \). Define the benefit of waiting from \( t = 1 \) until \( t = T \) as \( B^W_x(p_1) \). Then, the cost of waiting is determined by the \( T - 1 \) public announcements only. I claim that if \( B^W_x(p_1) < 0 \) for \( x \in \{0, 1\} \), then any equilibrium involves all trades at \( t = 1 \). The proof is straightforward. When \( B^W_x(p_1) < 0 \) for \( x \in \{0, 1\} \), then even if all other traders were to trade at \( t = T \) so that their price impacts have no effect on trader \( i \), she faces a negative benefit of waiting until \( T \) and thus trades at \( t = 1 \). Then, by Proposition 2.1 of Chapter 2, if any other trader were to instead trade before \( t = T \), \( i \)'s benefit would be strictly less than \( B^W_x(p_1) \) and therefore negative. Thus, no matter what the strategies of the other traders are, \( i \) must trade at \( t = 1 \). Because this argument applies to all traders, \( i = 1, \ldots, n \), they must all trade immediately in any equilibrium. As noted in the main text, the claim does not completely specify the equilibrium strategies because it leaves open the strategy after off-equilibrium histories in which a trader waits until \( t > 1 \). In particular, it does not say that a trader must rush at every (off-equilibrium) history. In fact, by Proposition 2.2, as prices approach either 0 or 1, the benefit of waiting until \( T \) must become positive and so there are histories at which waiting becomes optimal, conditional on reaching such a history.

In general, the sets of parameters \((q, \overline{q}, q_P, p_1, T)\) that satisfy \( B^W_x(p_1) < 0 \) for \( x \in \{0, 1\} \) can be quite wide and, given the complexity of this benefit function, it is difficult to provide a simple characterization. However, it is a simple calculation to evaluate \( B^W_x(p_1) \) for any particular parameters, including the ones used in treatment ER, \( q = \frac{3}{4}, \overline{q} = 1, q^* = \frac{17}{24}, p_1 = \frac{1}{2} \), and

\(^{101}\)For parameter ranges that do not satisfy either of the sufficient conditions, my conjecture is that the game becomes a version of a coordination game with multiple symmetric equilibria.

\[ \text{132} \]
C.1. Analysis Details and Omitted Proofs

$T = 8$. Because only public announcements affect $B^W_x(p_1)$, we can rewrite it as

$$B^W_x(p_1) = \frac{p_1(1 - p_1)}{Pr(s_i = x)} \left[ \sum_{k=0}^{T-1} \frac{C_0(k)C_1(k)(2\bar{q} - 1)}{p_1C_1(k) + (1 - p_1)C_0(k)} - (2q - 1) \right] \quad (C.3)$$

where $C_0(k) = \frac{(T-1)!}{k!(T-1-k)!} (1-q)^k q^{T-1-k}$ and $C_1(k) = \frac{(T-1)!}{k!(T-1-k)!} q^k (1-q)^{T-1-k}$ are the probabilities of observing $k$ public signal realizations equal to 1, conditional on $V = 0$ and $V = 1$, respectively. Under the parameters of the experiment, we find $B^W_0(p_1) \approx B^W_1(p_1) \approx -0.075 < 0$. Therefore, the unique equilibrium is for all traders to trade immediately, proving Proposition 4.2 part a).

I now turn to a sufficient condition to ensure waiting until $T$ and trading then is the unique equilibrium. Define the benefit of waiting from $t = 0$ until $t = T$ when all others trade prior to $T$ as $B^R_x(p_t)$. I claim that if $B^R_x(p_t) > 0$ for $x \in \{0, 1\}$ and for all $p_t \in (0, 1)$, then all traders must wait to trade until $T$. The reasoning is as follows. If $B^R_x(p_t) > 0$ at some $p_t$, then even if all other traders will trade prior to $T$, trader $i$ would wait until $T$. Then, by Proposition 2.1, if some of the other traders have already traded ($\tilde{n} > 1$) or if we have reached a trading period where less than $T - 1$ public announcements remain, $i$’s benefit to waiting would be strictly greater than $B^R_x(p_t)$ and therefore greater than zero. Thus, no matter what the strategies of the other traders are or what trading period $i$ is in, $i$ waits until $T$. Because this holds for all $i$ at every history, the unique equilibrium is for all traders to wait to trade until $T$.

As with $B^W_x(p_1) < 0$, $B^R_x(p_t) > 0$ can be satisfied for many combinations of parameters, but deriving the set of such parameters is non-trivial. Thus, I proceed to evaluate $B^R_x(p_t)$ for the parameters of treatment EW, $q = \frac{13}{24}$, $\bar{q} = 1$, $q^* = \frac{17}{24}$, $p_1 = \frac{1}{2}$, and $T = 8$. To do so, I first rewrite the benefit when $n - 1$ trades and $T - 1$ public announcements occur while waiting as

\footnote{I believe the weaker condition, $B^R_x(p_1) > 0$, is actually sufficient because it appears to be a general property of the benefit function that it crosses zero at most twice at symmetric prices in the interval $(0, 1)$. For up to two events, this fact was established in Chapter 2, but I have not yet been able to extend the proof to a general number of events. If true, then $B^R_x(p_1) > 0$ is sufficient because Proposition 2.2 of Chapter 2 ensures the benefit function must be positive for prices sufficiently close to 0 and 1.}

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C.2 Learning

C.2. Learning

Figure C.1: Proportion of Rational Behavior as Trials Progress (Extended)

\[
B^W(p_t) = \frac{p_t(1-p_t)}{Pr(x_i = x)} \left[ \sum_{k=0}^{T-1} \sum_{j=0}^{n-1} C_0(k)C_1(k)D_0(j)D_1(j)(2q - 1) - (2q - 1) \right] 
\]

\[ (C.4) \]

where \( D_0(k) = \frac{(n-1)!}{j!(n-1-j)!} (1-q)^j q^{n-1-j} \) and \( D_1(j) = \frac{(n-1)!}{j!(n-1-j)!} q^j (1-q)^{n-1-j} \) are the probabilities of observing \( j \) buys by the \( n-1 \) other traders, conditional on \( V = 0 \) and \( V = 1 \) respectively, and \( C_0(k), \ C_1(k) \) are as above. Numerically evaluating (C.4) as a function of \( p_t \) shows that it is in fact positive everywhere except at \( p_t = 0 \) and \( p_t = 1 \), where it is identically zero. Therefore, for this parameterization, all traders must wait to trade until \( t = T \), establishing Proposition 4.2 part b).

C.2 Learning

C.1 plots the evolution of perfectly rational behavior in each treatment over the full course of each session. Clear evidence of learning is observed in the ER treatment, where robust convergence to rational rushing is observed. On the other hand, subjects do not appear to learn that the additional
C.3 Non-equilibrium Informational Losses

As in Appendix B.4, I measure informational losses due to non-equilibrium behavior by comparing final prices in each trial to actual asset values. C.2 plots the empirical cumulative distribution function (cdf) of the absolute value of the difference in these two values for each of the treatments. When rushing is optimal (ER), the theoretical and actual losses are very similar: almost all information losses are due to the robust rational panics. Non-equilibrium waiting to obtain an additional signal does not contribute informational gains because only one subject in one trial waited long enough to obtain additional information.

In the treatment, EW, in which waiting is optimal, we observe large informational losses due to non-equilibrium behavior. Because all traders should obtain perfect information, informational losses are particularly surprising because, should any one of the 8 traders wait to obtain perfect information, the final price would reflect the true asset value. Thus, in the trials in which information is lost, no trader waited. On average, the observed final price differs from the theoretical by 6.8% in EW versus only 2.6% in ER.

C.4 Instructions

The instructions for the EW treatment, are provided. The instructions for the ER treatment are identical except for the differences in parameters.
Figure C.2: CDFs of Difference Between Final Prices and Asset Values by Treatment (Extended)
Instructions

This is a research experiment designed to understand how people make stock trading decisions in a simple trading environment. To assist with our research, we would greatly appreciate your full attention during the experiment. Please do not communicate with other participants in any way and please raise your hand if you have a question.

You will participate in a series of 30 trials. In each trial, you and the 7 other participants will each trade a stock with the computer. You will be paid $5.00 for completing all trials. In addition, in each trial you will earn lottery tickets as described next. The more lottery tickets you have, the more you will earn on average.

Trading and Profits

Before each trial, the computer will randomly select whether the stock’s value, \( V \), is 100 tickets or 0 tickets. Each is equally likely to be selected. At the start of each trial, you will be given 100 tickets. You may choose either to buy or sell the stock in any one of 8 trading periods. You can only trade ONCE and MUST trade in one of the periods. If you buy the stock, you will gain its actual value minus the price, \( P \), you pay for it. If you want to sell the stock, you must first ‘borrow’ it from the computer and later pay back its actual value. So, if you sell, you will gain the price you receive minus its value. In summary, your total profit is:

\[
100 + V - P \text{ lottery tickets if you buy}
\]

\[
100 + P - V \text{ lottery tickets if you sell}
\]

So, for example, if you sell the stock at a price of 50 and it turns out to be worth 100, you would earn \(100 + (50-100) = 50\) tickets for that trial. But if it turns out to be worth 0, you would earn \(100 + (50-0) = 150\) tickets.

Clues about Value

To help you guess the value of the stock the computer chose, you will get a single Private Clue at the start of the trial. This Private Clue will be known only by you. Specifically, there will be two possible bins: one that the computer will use if the value is 100 and another that the computer will use if the value is 0. Bins contain some number of blue and green marbles as shown below. The computer will draw a marble randomly from the bin and show it to you as your Private Clue. Each marble in the bin is equally likely to be drawn. The color of the ball you see can give you a hint as to the stock’s value.

After observing your Private Clue, you can choose to trade immediately in the first trading period. Alternatively, you can choose to wait and trade in one of the following 7 trading periods. Between trading periods, there are public announcement periods. In each announcement period, a Public Clue will become available. The Public Clue, unlike your Private Clue, is seen by everyone. For the Public Clue, the computer will draw a marble from another bin. The bin used will again depend on the stock’s true value but the possible bins used for the Public Clue are different from the bins used for the Private Clue, as shown below. Note that the bins used for both clues are fixed throughout the trial - they depend only on the initially chosen random value of the stock. Also, marbles for both types of clues are always replaced before another is drawn.

<table>
<thead>
<tr>
<th>Clue</th>
<th>Contents of bin if value = 100</th>
<th>Contents of bin if value = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Clue</td>
<td>13 blue</td>
<td>11 blue</td>
</tr>
<tr>
<td></td>
<td>11 green</td>
<td>13 green</td>
</tr>
<tr>
<td>Public Clue</td>
<td>17 blue</td>
<td>7 blue</td>
</tr>
<tr>
<td></td>
<td>7 green</td>
<td>17 green</td>
</tr>
</tbody>
</table>

If you decide to wait to trade until the last trading period, the true value of the asset will be revealed to you before you trade. Otherwise, you will only learn the true asset value after the trial is complete. Importantly, however, each time you choose to wait, the price is likely to change before your next chance to trade, as described next.

Prices

The price of the stock is set by a Price Setter played by the computer. The Price Setter’s job is to set the price equal to the stock’s mathematical expected value given all of the public information available. Therefore, the price will always be between 0 and 100. The Price Setter can observe the trades made by you and the other participants and the Public Clues. However, she can not observe any of the Private Clues nor the true asset value (even when it is revealed in the final period).

Because the price changes with the available information, if you decide to wait, the price at which you can trade is likely to change. The initial price of the stock is 50 tickets, reflecting the fact that it is equally likely to be worth 100 tickets or 0 tickets. After a participant buys, the Price Setter will increase the price and after a participant sells, she’ll decrease the price. After a Public Clue is revealed, the price will increase if it suggests (on its own) that the stock is more likely to be worth 100 and decrease if it suggests it is more likely to be worth 0.

Trading Screen

The trading screen you will use to trade is as shown below. The eight trading periods are indicated by the numbers 1–8. Public Clues are revealed between trading periods at the times indicated by the megaphone symbol (the clues are not shown in the figure - they appear elsewhere on your trading screen). All past trading decisions and prices are displayed. Trading periods in which one or more trades occur are indicated by a solid dot. The number of buys is indicated by a “+” and then a number and the number of sells by a “-” and then a number. The current price at which you can trade (74.16 in this example) is displayed at the current time which is indicated by the dashed red line. The dashed red line will progress to the right
as we move through the periods.

In this example, one participant bought in the first trading period, so the price increased. In the second trading period, one participant bought and one sold, so the price did not change. In the third trading period, no one traded. The first and third Public Clues suggested the stock's value is 100 and the second that the stock's value is 0. It is currently the fourth trading period. Note that this is an example only and is not meant to suggest when you should trade.

In each trading period in which you haven't already traded, you must choose buy or sell or wait and then press the 'confirm' button. If you choose to wait, the red arc points to the next trading period in which you can trade. In trading periods after you have traded, you do not have to do anything - you will simply be notified that you have already traded. In the periods with Public Clues, you must press "OK" to acknowledge having seen the clue.

Summary
1. At the beginning of each trial, the computer randomly selects the stock's value: 0 or 100.
2. Each participant is shown their Private Clue from the same bin. Marbles are replaced so each participant may see the same (or different) marbles.
3. Each participant chooses to buy, sell, or wait in the first trading period. After all participants have made their decisions, a Public Clue is revealed and we move to the second trading period.
4. Step 3 is repeated until all 8 trading periods are complete. You must trade in one of the eight trading periods and may only trade once.
5. Just before the 8th trading period, the true value of the stock will be revealed to you if you have not already traded.
6. At each point in time, all past prices and trading decisions are available to use to help guess the value of the stock (in addition to the Clues).

After all participants have traded, the trial is complete. The true value of the asset will be revealed to all participants and you will be told how many tickets you earned for the trial. You will press 'next trial' to participate in the next trial. It is important to remember that, in each trial, the value of the stock is independently randomly selected by the computer — there is no relationship between the value selected in one trial and another.

After all trials are complete, one lottery will be conducted for each trial. For each lottery, a random number less than 200 will be chosen by the computer. If the number is smaller than the number of lottery tickets you earned for the trial, you will get $1.00. Therefore, the more lottery tickets you earn in each trial, the more you can expect to make (partial tickets are possible and count as well). For example, if you earn 100 tickets in each trial, you can expect to make 0.5*30*1=$15.00 over the 30 trials. But, if you earn 130 tickets in each trial, you can expect to make 0.65*30*1=$19.50.

Please try to make each trading period decision within 15 seconds so that the experiment can finish on time. A timer counts down from 15 to help you keep track of time. Note, however, that if the timer hits zero, you can still enter your trading or wait decision and will still have the same chance to earn money. Before beginning the paid trials, we will have two practice trials for which you will not be paid. These trials are otherwise identical to the paid trials.

Quiz
Please answer the following questions and press the 'Check answers' button to see whether or not you answered all questions correctly. To ensure all participants understand the instructions, everyone must answer all of the questions correctly before we begin the experiment.

1. Your Private Clue is a blue marble. Based only on this information, What is the most likely value of the stock?
2. It is the second trading period. You observe another participant sold the stock in the first trading period. What color marble is their Private Clue most likely to be? 
- green
- blue

3. Your Private Clue is a blue marble. What color marble is another participant's Private Clue likely to be? 
- green
- blue

4. If you choose to sell the stock at a price of 80 and its value turns out to be 100, how many total tickets would you get for that trial? 
- 80
- 20
- 180

5. If you choose to buy the stock at a price of 25 and its value turns out to be 100, how many total tickets would you get for that trial? 
- 25
- 75
- 175

6. The stock's current price is 80. Which value of the stock is more likely? 
- 100
- 0

Once you have completed the quiz, please press 'Check answers'.

Check answers