Interference in Wireless Mobile Networks

by

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Abstract

Given a set of positions for wireless nodes, the interference minimization problem is to assign a transmission radius (i.e., a power level) to each node such that the resulting communication graph is connected, while minimizing the maximum (respectively, average) interference. We consider the model introduced by von Rickenbach et al. (2005), in which each wireless node is represented by a point in Euclidean space on which is centered a transmission range represented by a ball, and edges in the corresponding graph are symmetric. The problem is NP-complete in two or more dimensions (Buchin 2008) and no polynomial-time approximation algorithm is known. We show how to solve the problem efficiently in settings typical for wireless ad hoc networks. We show that if node positions are represented by a set P of n points selected uniformly and independently at random over a d-dimensional region, then the topology given by the closure of the Euclidean minimum spanning tree of P has $O(\log n)$ maximum interference, $O(1)$ average interference with high probability and $O(1)$ expected average interference. This work is the first to examine average interference in random settings. We extend the first bound to a general class of communication graphs over a broad set of probability distributions. We present a local algorithm that constructs a graph from this class; this is the first local algorithm to provide an upper bound on expected maximum interference. To verify our results, we perform an empirical evaluation using synthetic as well as real world node placements.

Preface

Part of Chapter 1 - 5 have been published in conferences and journals, as mentioned below.

- M. Khabbazian, S. Durocher, A. Haghnegahdar, and F. Kuhn, Bounding interference in wireless ad hoc networks with nodes in random position. Published in IEEE/ACM Transactions on Networking, 2014.
- M. Khabbazian, S. Durocher, and A. Haghnegahdar, Bounding interference in wireless ad hoc networks with nodes in random position. In Proceedings of the 19th international conference on Structural Information and Communication Complexity (SIROCCO'12).

The main problem was proposed and designed by Professor M. Khabbazian and S. Durocher. The author has had the main responsibility in developing the original ideas, analysis as well as writing the Chapter 3. Professor S. Durocher, M. Khabbazian and the author helped with the manuscript preparation for these papers which is included in Chapter 2. The author has implemented the software to perform all the numerical analysis and computer simulations for all of the aforementioned papers. This results are represented in Chapter 4. Furthermore, Professor F. Kuhn helped us with the proof of Theorem 2 presented in the journal paper.

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Glossary

SINR Signal to Interference plus Noise Ratio

MST Minimum Spanning Tree

GPS Global Positioning System

CBTC Cone Based Topology Control

UDG Unit Disk Graph

OPT Optimal Maximum Interference

OPTAvg Optimal Average interference

RWP Random Way Point

RW Random Walk

Acknowledgments

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Finally, a heartfelt and deep thanks go to my family, who have always encouraged me and given their unconditional support through my studies at the University of British Columbia. Without their support, I could not have completed this thesis.

Dedication

To my family...

Chapter 1

Introduction

1.1 Motivation

Establishing connectivity in a wireless network can be a complex task for which various (sometimes conflicting) objectives must be optimized. To permit a packet to be routed between any two nodes in a network, the corresponding communication graph must be connected. In addition to requiring connectivity, various properties can be imposed on the network, including low power consumption [1, 2], bounded average traffic load [3, 4], small average hop distance between sender-receiver pairs [5], low dilation (t-spanner) [5–11], and minimal interference; this latter objective, minimizing interference, is the focus of much recent research [2, 5, 12–28] and of this research.

The amplitude of a radio signal transmitted at a node p and received at a node q decreases as the distance between p and q increases. The signal from p must be sufficiently strong for q to receive it. That is, for a given transmission power level at node p, there exists some threshold, say $r(p)$, such that if q receives a message from p, then the distance from p to q can be at most $r(p)$. We model transmission in a wireless network by assigning to each wireless node p a radius of transmission $r(p)$, such that every node within distance $r(p)$ of p can receive a transmission from p, whereas no node at greater distance from p can. However, the distance between p and q alone is not sufficient to determine successful communication between p and q; even if q is within distance $r(p)$, signals sent from other nodes could interfere with the signal from p received at q. We adopt the interference model introduced by von Rickenbach et al. [12] which is related to the *geometric radio network* model of Dessmark and Pelc [29] and other early geometric models for wireless networks [30, 31].

We measure interference at node p by the number of nodes that have p within their respective radii of transmission. Given a set of wireless nodes whose positions are represented by a set of points P, we consider the problem of identifying a connected network on P that minimizes the maximum (respectively, average) interference. The problem of constructing the network is equivalent to that of assigning a transmission radius to each node (in general, $\Theta(n)$ distinct radii are assigned to a set of n nodes); once the transmission radius of each node is fixed, the corresponding communication graph and its associated maximum interference are also determined. Conversely, once a graph is fixed, each node's transmission radius is determined by the distance to its furthest neighbour.

In contrast to the algorithmic community which focuses on protocol models, the communication networks researchers are working with models where interference accumulation, attenuation, antenna height and orientation, terrain, surface reflection and absorption, and so forth is also taken into account [32, 33]. Physical model is one of the standard models for wireless networks. In this model, the energy of a signal fades with the power of the path-loss parameter. If the signal strength received at the receiver side divided by the strength of interference caused by concurrent transmitters plus noise (SINR) is above some threshold the receiver can decode the message successfully.

Beside the interference, channel (the link between nodes of our network) characteristics such as small scale fading and large scale shadowing will affect the signal quality at the receiver side. Small scale fading is often handled with diversity schemes and retransmit. Large scale shadowing on the other hand is very dependent on the location of receiver and cannot always be fixed. Considering these models for the links between the nodes will affect the shape of uniform disk that we consider earlier. For instance, in the existence of shadowing the uniform disk might have some holes depending on the location with respect to obstacles.

Clearly, every study does not need to use the detailed radio model and also explore every variation in the wide parameter space. The level of detail for a given study depends closely on the characteristics of that study. Disk model and, in particular, the interference minimization problem applied to this model are the focus of numerous publications and have generated significant recent research interest [5, 12, 13, 16–27]. Furthermore, a number of important algorithmic questions remain open with respect to interference minimization in this model. While models such as SINR arguably provide a more realistic physical representation of interference in a wireless network (e.g., [34–38]), algorithmic problems such as interference minimization are significantly more difficult to solve in the SINR model, motivating continued examination of interference minimization under both models. Finally, in some cases, a solution to a problem set in the model used in this work leads to an approximate solution to the corresponding problem under the SINR model (e.g., [23]). See Section 1.2 for a formal definition of the model and see Figure 1.1 for an example.

Given a set of points P in the plane, finding a connected graph on P that minimizes the maximum interference is NP-complete [17]. A polynomial-time algorithm exists that returns a solution with maximum interference $O(\sqrt{n})$, where $n = |P|$ [16]. Even in one dimension, for every n there exists a set of n points P such that any graph on P has maximum interference $\Omega(\sqrt{n})$ [12]. All such known examples involve specific constructions (i.e., exponential chains). We are interested in investigating a more realistic class of wireless networks: those whose node positions observe common random distributions that better model actual wireless ad hoc networks.

When nodes are positioned on a line (sometimes called the *highway model*), a simple heuristic is to assign to each node a radius of transmission that corresponds to the maximum of the distances to its respective nearest neighbours to the left and right. In the worst case, such a strategy can result in $\Theta(n)$ maximum interference when an optimal solution has only $\Theta(\sqrt{n})$ maximum interference [12]. Recently, Kranakis et al. [21] showed that if n nodes are positioned uniformly at random on an interval, then the maximum interference provided by this heuristic is $\Theta(\sqrt{\log n})$ with high probability.

1.2 System Model and Definitions

We represent the position of a wireless node as a point in Euclidean space, $\mathbb{R}^d$, for some fixed[1] $d \geq 1$.

For simplicity, we refer to each node by its corresponding point. Similarly, we represent a wireless network by its communication graph, a geometric graph whose vertices are a set of points $P \subseteq \mathbb{R}^d$. Given a (simple and undirected) graph G, we employ standard graph-theoretic notation, where $V(G)$ denotes the vertex set of G and $E(G)$ denotes[2] its edge set.

We say vertices u and v are *k-hop neighbours* if there is a simple path of length k from u to v in G. When $k = 1$ we say u and v are neighbours. The *k-hop neighbourhood* of a node u is the union of the sets of its k'-hop neighbours for all $k' \leq k$.

We assume each node has a range of communication that is equal in every direction (i.e., a *radius of transmission*), that different nodes can have different transmission radii, and we consider bidirectional communication links, each of which is represented by an undirected graph edge connecting two nodes. Specifically, each node p has some *radius of transmission*, denoted by the function $r : P \to \mathbb{R}^+$, such that a node q receives a signal from p (possibly interference) if and only if $\text{dist}(p, q) \leq r(p)$, where $\text{dist}(p, q) = \|p - q\|_2$ denotes the Euclidean distance between points p and q in $\mathbb{R}^d$. Similarly, a node q can communicate with p if and only if $\text{dist}(p, q) \leq \min\{r(p), r(q)\}$.

[1] In the majority of instances, two or three dimensions suffice to model an actual wireless network. Our results are presented in terms of an arbitrary d since this permits expressing a more general result without increasing the complexity of the corresponding notation.

[2] Note, $E(G)$ denotes the edge set of a graph G, whereas $\mathbb{E}[X]$ denotes the expected value of the random variable X.

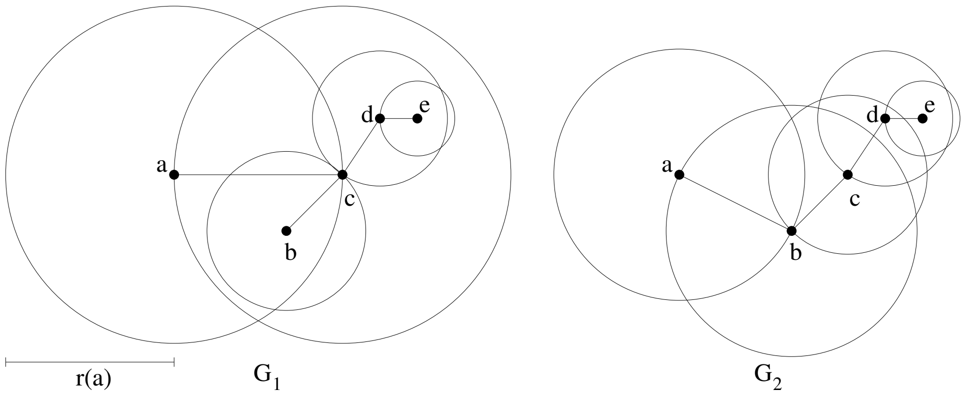

Figure 1.1: A set of points $P = \{a, b, c, d, e\}$ in $\mathbb{R}^2$ and two communication graphs on P, denoted G_1 and G_2, illustrating radii of transmission by their corresponding discs. Nodes b and c can communicate in G_1 and G_2 because $\text{dist}(b, c) \le \min\{r(b), r(c)\}$. Node b receives interference from node a in G_1 and G_2 because $\text{dist}(a, b) \le r(a)$, but the two nodes cannot communicate in G_1 because $\text{dist}(a, b) > r(b)$. The maximum interference in G_1 is 4, achieved at node c. The maximum interference in G_2 is 3, achieved at nodes b, c, and d. G_2 is an optimal solution for P, i.e., $\text{OPT}(P) = 3$.

Interference at a node q is defined by the number of nodes from which it can receive a signal, whereas connectivity in the communication graph is determined by the nodes with which q can communicate. For simplicity, suppose each node has an infinite radius of reception, regardless of its radius of transmission; that is, a node q can receive interference from any node p if $r(p)$ is sufficiently large, regardless of $r(q)$. See Figure 1.1 for an example.

Definition 1 (Communication Graph)**.** *A graph G is a* communication graph *with respect to a point set $P \subseteq \mathbb{R}^d$ and a function $r : P \to \mathbb{R}^+$ if*

1. *$V(G) = P$, and*

2. *for all vertices p and q in $V(G)$,*

$$\{p, q\} \in E(G) \Leftrightarrow \text{dist}(p, q) \leq \min\{r(p), r(q)\}. \tag{1.1}$$

Together, set P and function r uniquely determine the corresponding communication graph G. Alternatively, a communication graph can be defined as the closure of a given embedded graph. Specifically, if instead of being given P and r, we are given an arbitrary graph H embedded in $\mathbb{R}^d$, then the set P is trivially determined by $V(H)$ and a transmission radius for each node $p \in V(H)$ can be assigned to satisfy (1.1) by

$$r(p) = \max_{q \in \text{Adj}(p)} \text{dist}(p, q), \tag{1.2}$$

where $\text{Adj}(p) = \{q \mid \{q, p\} \in E(H)\}$ denotes the set of vertices adjacent to p in H. The communication graph determined by H is the unique edge-minimal supergraph of H that satisfies Definition 1. We denote this graph by H' and refer to it as the *closure* of graph H. Therefore, a communication graph G can be defined either as a function of a set of points P and an associated mapping of transmission radii $r : P \to \mathbb{R}^+$, or as the closure of a given embedded graph H (where $G = H'$).

Definition 2 (Interference)**.** *Given a communication graph G, the* interference *at a node $p \in V(G)$ or at a point $p \in \mathbb{R}^d$ is*

$$\text{inter}_G(p) = \left|\{q \mid q \in V(G),\ \text{dist}(q, p) \leq r(q)\}\right|,$$

the maximum interference *of G is*

$$\text{inter}(G) = \max_{p \in V(G)} \text{inter}_G(p),$$

and the average interference *of G is*

$$\text{interAvg}(G) = \frac{1}{|V(G)|} \sum_{p \in V(G)} \text{inter}_G(p).$$

In other words, the interference at p, denoted $\text{inter}_G(p)$, is the number of nodes q such that p lies within q's radius of transmission[3]. This does not imply the existence of the edge $\{p, q\}$ in the corresponding communication graph G; such an edge exists if and only if the relationship is reciprocal, i.e., q also lies within p's radius of transmission.

Given a point set P, let $\mathcal{G}(P)$ denote the set of connected communication graphs on P. Let $\text{OPT}(P)$ denote the optimal maximum interference attainable over graphs in $\mathcal{G}(P)$. That is,

$$\begin{aligned} \text{OPT}(P) &= \min_{G \in \mathcal{G}(P)} \text{inter}(G) \\ &= \min_{G \in \mathcal{G}(P)} \max_{p \in V(G)} \text{inter}_G(p). \end{aligned}$$

Similarly, let $\text{OPTAvg}(P)$ denote the optimal average interference attainable over graphs in $\mathcal{G}(P)$. That is,

$$\text{OPTAvg}(P) = \min_{G \in \mathcal{G}(P)} \text{interAvg}(G).$$

Thus, given a set of points P representing the positions of wireless nodes,

[3]In some definitions of interference a node cannot cause interference with itself. When p is a node in $V(G)$, the respective values of interference for the two definitions differ by an additive factor of one. We include p in the tally to allow a more general measure of interference whose definition applies consistently at any point p in $\mathbb{R}^d$, regardless of whether p coincides with a node in $V(G)$.

the *maximum interference minimization problem* is to find a connected communication graph G on P that spans P such that the maximum interference is minimized (i.e., its maximum interference is $\mathrm{OPT}(P)$). Similarly, the *average interference minimization problem* is to find a connected communication graph G on P that spans P such that the average interference is minimized (i.e., its average interference is $\mathrm{OPTAvg}(P)$).

We examine the maximum and average interference of the communication graph determined by the closure of $\mathrm{MST}(P)$, where $\mathrm{MST}(P)$ denotes the Euclidean minimum spanning tree of the point set P. Our results apply with *high probability*, which refers to probability at least $1 - n^{-c}$, where $n = |P|$ denotes the number of network nodes and $c \geq 1$ is an arbitrary fixed constant.

1.3 Literature Review

Selecting a proper interference and network model can affect analysis of wireless networks in terms of performance, accuracy and scalability. Burkhart et al. [39] proposed a *sender centric* measure for the graph-based interference model which evaluates the interference as the number of nodes affected by the communication over a particular edge in the network. Alternatively, *receiver centric* interference model, presented by von Rickenbach et al. [40], focuses on the interference at the receiving node. In this model, interference at node p is defined as the number of nodes disturbing the reception of a message at p. Our work is examining a node interference model similar to [40]. To analyze the network interference, existing articles typically consider the worst possible node placement. However, such node placements may be very infrequent or never appear in real world. Therefore, instead of the worst case analysis, we perform an average case analysis, where nodes are randomly placed in the network.

considering the *receiver centric* interference model, the network's interference is defined either as the maximum interference [40, 41] or the average interference [42, 43] over all the nodes in the network. Given a set of points P in the plane, finding a connected graph on P that minimizes the maximum interference is known to be NP-complete problem [44]. von Rickenbach et al. [40] proposed a polynomial time approximation algorithm to solve this problem with maximum interference $\Theta(n^{1/4})$ for nodes located on a line. Another polynomial-time algorithm exists that returns a solution with maximum interference $O(\sqrt{n})$, where $n = |P|$ in two (or more) dimensions [45]. Kranakis et al. [46] showed that if n nodes are positioned uniformly at random on an interval, then the maximum interference of Minimum Spanning Tree (MST) is $\Theta(\sqrt{\log n})$ with high probability. Khabbazian et al. [47] generalized this result to higher dimensions by showing that the maximum interference of a set of n points selected uniformly at random in a d-dimensional Euclidian space is $O(\log n)$, w.h.p. Devroye and Morin [26] strengthened this results by proving that the maximum interference of a set of n points selected uniformly at random is $\Theta(\sqrt{\log n})$, w.h.p.

The average interference minimization problem has also been studied in the literature. Moscibroda and Wattenhofer [48] showed that, when nodes are in general metric space, there is no approximation algorithm having an approximation ratio better than $\Omega(\log n)$ that can find the minimum average interference, unless it holds that $NP \in DTIME(n^{\log\log n})$. Lou et al. [49] proposed a polynomial-time, $O(n^3\Delta)$, algorithm to find the minimum average interference in one-dimensional line network, where Δ is the maximum node degree when each node is connected to every other node within its maximum transmission range. Later, they developed a more efficient algorithm that finds the minimum interference of a line network in time $O(n\Delta^2)$ [18]. For two-dimensional networks, Tan et al. [18] proposed an algorithm to compute minimum average interference in time $n^{O(m \log \lambda)}$, where m is the minimum number of parallel lines so that all the nodes are located on the lines. However, in general, the minimum number of parallel lines to cover all the nodes is linear in n, that is $m = \Omega(n)$. Therefore, the time complexity of this algorithm is exponential in general. Finally, Khabbazian et al. [43] studied the problem of minimizing the average interference in two-dimensional networks when nodes are distributed uniformly at random and showed that the expected average interference is $O(1)$. We strengthen this result in Chapter 3 by showing that when nodes are distributed uniformly at random the average interference is $O(1)$, with high probability.

1.4 Thesis Organization

In this thesis, we examine the corresponding interference minimization problems in two and higher dimensions. In Chapter 2, we generalize the nearest-neighbour path used in the highway model to the Euclidean minimum spanning tree (MST), and show that with high probability, the maximum interference of the MST of a set of n points selected uniformly at random over a

d-dimensional region $[0,1]^d$ is $O(\log n)$, for any fixed $d \geq 1$. As we show in Chapter 2, our results also apply to a broad class of random distributions, denoted $\mathcal{D}$, that includes both the uniform random distribution and realistic distributions for modeling random motion in mobile wireless networks, as well as to a large class of connected spanning graphs that includes the MST.

In Section 2.4 we present a local algorithm that constructs a topology whose maximum interference is $O(\log n)$ with high probability when node positions are selected according to a distribution in $\mathcal{D}$. Previous local algorithms for topology control (e.g., the cone-based local algorithm (CBTC) [1]) attempt to reduce transmission radii (i.e., power consumption), but not necessarily the maximum interference. Similarly, others attempt to minimize interference but do not guarantee connectivity (e.g., the k-neighbours algorithm [28]). Although reducing transmission radii at many nodes is often necessary to reduce the maximum interference, the two objectives differ; specifically, some nodes may require large transmission radii to minimize the maximum interference. Ours is the first local algorithm to provide a non-trivial upper bound on maximum interference. Our algorithm can be applied to any existing topology to refine it and further reduce its maximum interference. Consequently, our solution can be used either independently, or paired with another topology control strategy.

In Chapter 3 we consider the problem of minimizing the average interference and show that the expected interference of the MST over a set of n points selected uniformly at random in a d-dimensional region $[0,1]^d$ is $O(1)$. We also show that for any network size n, there exists a power level assignment such that the average interference of the resulting communication graph is $O(1)$, with high probability. In other words, irrespective of the number of nodes in the network, with high probability, the average interference is smaller than a constant when nodes are uniformly distributed in the network.

Chapter 4 presents the analysis of an empirical evaluation of our algorithm with a suite of simulation results on static, mobile, and real GPS track data. We consider the uniform distribution as the points random distribution and generate $100,000$ snapshot of this network. For the dynamic networks, we use the node distribution probability function resulted from Random Way Point mobility model to generate the node distribution. To analysis the performance of our network in a real world scenario, we use the GPS location of 500 moving vehicle and considered them as the location of nodes in our network. Finally, Chapter 5 concludes the thesis and presents possible directions for future work.

1.5 Publications

- A. Haghnegahdar, M. Khabbazian, Average Interference in Wireless Ad Hoc Networks with Nodes in Random Position. Submitted to The International Symposium on DIStributed Computing (DISC), 2014.
- M. Khabbazian, S. Durocher, A. Haghnegahdar, and F. Kuhn, Bounding interference in wireless ad hoc networks with nodes in random position. Published in IEEE/ACM Transactions on Networking, 2014.
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ference in wireless ad hoc networks with nodes in random position. In Proceedings of the 19th international conference on Structural Information and Communication Complexity (SIROCCO'12).

- A. Haghnegahdar, M. Khabbazian, and V.K. Bhargava, "Privacy Risks in Publishing Mobile Device Trajectories," IEEE Wireless Communications Letters, 2014.

- R. Ramamonjison, A. Haghnegahdar , V.K. Bhargava, "Joint optimization of clustering and cooperative beamforming in green cognitive wireless networks," IEEE Transactions On Wireless Communications , 2014.

Chapter 2

Minimizing Maximum Interference in Random Networks[4]

2.1 Generalizing One-Dimensional Solutions

Before presenting our results on random sets of points, we begin with a brief discussion regarding the possibility of generalizing existing algorithms that provide approximate solutions for one-dimensional instances of the maximum interference minimization problem (in an adversarial deterministic input setting).

Since the problem of identifying a graph that achieves the optimal (minimum) interference is NP-hard in two or more dimensions [17], it is natural to ask whether one can design a polynomial-time algorithm to return a good approximate solution. Although von Rickenbach et al. [12] give

[4]Part of this chapter has been published in *M. Khabbazian, S. Durocher, A. Haghnegahdar, and F. Kuhn, Bounding interference in wireless ad hoc networks with nodes in random position. IEEE/ACM Transactions on Networking, 2014.* And *M. Khabbazian, S. Durocher, and A. Haghnegahdar, Bounding interference in wireless ad hoc networks with nodes in random position. (SIROCCO'12).*

a $\Theta(n^{1/4})$-approximate algorithm in one dimension [12], the current best polynomial-time algorithm in two (or more) dimensions by Halldórsson and Tokuyama [16] returns a solution with maximum interference $O(\sqrt{n})$; as noted by Halldórsson and Tokuyama, this algorithm is not known to guarantee any approximation factor better than the immediate bound of $O(\sqrt{n})$. The algorithm of von Rickenbach et al. uses two strategies for constructing respective communication graphs, and returns the graph with the lower maximum interference; an elegant argument that depends on Lemma 1 bounds the resulting worst-case maximum interference by $\Theta(n^{1/4} \cdot \mathrm{OPT}(P))$. The two strategies correspond roughly to a) $\mathrm{MST}(P)'$ and b) classifying every $\sqrt{n}$th node as a hub, joining each hub to its left and right neighbouring hubs to form a network backbone, and connecting each remaining node to its closest hub. The algorithm of Halldórsson and Tokuyama applies ϵ-nets, resulting in a strategy that is loosely analogous to a generalization of the hub strategy of von Rickenbach et al. to higher dimensions. One might wonder whether the hybrid approach of von Rickenbach et al. might be applicable in higher dimensions by returning $\mathrm{MST}(P)'$ or the communication graph constructed by the algorithm of Halldórsson and Tokuyama, whichever has lower maximum interference. To apply this idea directly would require generalizing the following property established by von Rickenbach et al. to higher dimensions:

Lemma 1 (von Rickenbach et al. [12] (2005))**.** *For any set of points $P \subseteq \mathbb{R}$,*

$$\mathrm{OPT}(P) \in \Omega\left(\sqrt{\mathrm{inter}(\mathrm{MST}(P)')}\right).$$

However, von Rickenbach et al. also show that for any n, there exists a set of n points $P \subseteq \mathbb{R}^2$ such that $\mathrm{OPT}(P) \in O(1)$ and $\mathrm{inter}(\mathrm{MST}(P)') \in \Theta(n)$, which implies that Lemma 1 does not hold in higher dimensions. Consequently, techniques such as those used by von Rickenbach et al. do not immediately generalize to higher dimensions.

2.2 Randomized Point Sets

Although using the hybrid approach of von Rickenbach et al. [12] directly may not be possible, Kranakis et al. [21] recently showed that if a set P of n points is selected uniformly at random from an interval, then the maximum interference of the communication graph determined by $\mathrm{MST}(P)'$ is $\Theta(\sqrt{\log n})$ with high probability. Here we show that if points are selected randomly from $O(1)$-dimensional Euclidean space, with high probability, the maximum interference of $\mathrm{MST}(P)'$ is $O(\log n)$. We start with some basic definitions.

Definition 3 (Primitive Edge)**.** *Assume that a communication graph G is the closure of some embedded graph H. An edge $\{p, q\} \in E(G)$ is called* primitive *w.r.t. H if $\{p, q\} \in E(H)$ and $\min\{r(p), r(q)\} = \mathrm{dist}(p, q)$.*

Observe that because G is the closure of H, the radius of any node u is equal to the distance to its farthest neighbour in H and therefore, every node is incident to at least one primitive edge.

Definition 4 (Bridge)**.** *An edge $\{p, q\} \in E(G)$ in a communication graph G is* bridged *if there is a path joining p and q in G consisting of at most* *three edges distinct from* $\{p, q\}$ *such that for each of the three edges* $\{x, y\}$, $\text{dist}(x, y) < \text{dist}(p, q)$, *or* $\text{dist}(x, y) = \text{dist}(p, q)$ *and* $\{x, y\}$ *is primitive.*

Given a set of points P in $\mathbb{R}^d$, let $\mathcal{T}(P)$ denote the set of all communication graphs G with $V(G) = P$ such that G is the closure of some embedded graph H and such that no primitive edge in $E(G)$ is bridged.

Further, let $\mathcal{C}(R, r, d)$ be the minimum number of d-dimensional balls of radius r required to cover a d-dimensional ball of radius R. The following property holds since $\mathbb{R}^d$ is a doubling metric space for any constant d [50] (equivalently, $\mathbb{R}^d$ has constant doubling dimension [51, 52]):

Proposition 1. *If* $d \in \Theta(1)$ *and* $R/r \in \Theta(1)$, *then* $\mathcal{C}(R, r, d) \in \Theta(1)$.

For a given communication graph G, we define $d_{\max}(G)$ and $d_{\min}(G)$ as the lengths of the longest and shortest edges of G, respectively. That is, $d_{\max}(G) := \max_{\{s,t\} \in E(G)} \text{dist}(s, t)$ and $d_{\min}(G) = \min_{\{s,t\} \in E(G)} \text{dist}(s, t)$. Halldórsson and Tokuyama [16], Maheshwari et al. [27], and Lou et al. [18] give centralized algorithms for constructing graphs G, each with maximum interference $O(\log(d_{\max}(G)/d_{\min}(G)))$. As we show in Theorem 2, this bound holds for any graph G in the class $\mathcal{T}(P)$. In Section 2.4 we describe a local algorithm for constructing a connected graph in $\mathcal{T}(P)$ on any given point set P.

Theorem 2. *Let* P *be a set of points in* $\mathbb{R}^d$. *For any graph* $G \in \mathcal{T}(P)$,

$$\text{inter}(G) \in O\left(\log\left(\frac{d_{\max}(G)}{d_{\min}(G)}\right)\right).$$

Proof. We first normalize the scale of P to simplify the proof. Let $Q =$

$\{p \cdot \alpha \mid p \in P\}$ denote a uniform scaling of P by a factor of $\alpha = 1/d_{\min}(G)$ and let H denote the corresponding communication graph. That is, $\{u, v\} \in E(G) \Leftrightarrow \{u \cdot \alpha, v \cdot \alpha\} \in E(H)$. Similarly, scale transmission radii such that each node's transmission radius in Q is α times its corresponding node's transmission radius in P. Thus,

$$d_{\min}(H) = 1 \quad \text{and} \quad d_{\max}(H) = \frac{d_{\max}(G)}{d_{\min}(G)}. \tag{2.1}$$

Consider some node p and let U be the set of nodes that cause interference at p, i.e., $U = \{u \in V : \text{dist}(u, p) \leq r(u)\}$. Let $g = \lceil \log d_{\max}(H) \rceil$. We partition the set U into $g + 1$ subsets $U_0, U_1, \ldots, U_g$, such that for each $0 \leq i \leq g$, $U_i = \big\{u \in U : r(u) \in [2^i, 2^{i+1})\big\}$. We will show that for all $i \in \{0, \ldots, g\}$,

$$|U_i| \leq \mathcal{C}(2^{i+1}, 2^{i-2}, d) \cdot \mathcal{C}(2^{i+2}, 2^{i-2}, d). \tag{2.2}$$

Applying Proposition 1, this implies $|U_i| = O(1)$ and thus $|U| = O(\log(d_{\max}(H)))$, from which the claim of the theorem follows.

Let us therefore fix some $i \in \{0, \ldots, g\}$ and assume for the sake of contradiction that (2.2) does not hold. First recall that every node $v \in V$ is adjacent to some primitive edge of length $r(v)$. Hence, every node $u \in U_i$ is adjacent to some primitive edge of length $r(u) \in [2^i, 2^{i+1})$. We can thus define a mapping $\omega : U_i \to V$ such that for every $u \in U_i$, $\{u, \omega(u)\}$ is a primitive edge of length $r(u) \in [2^i, 2^{i+1})$. Also note that all nodes in U_i are contained in a ball with centre p and radius 2^{i+1}. By Proposition 1, this ball can be covered with $\mathcal{C}(2^{i+1}, 2^{i-2}, d)$ balls of radius 2^{i-2}. Thus, because we assume (for contradiction) that (2.2) does not hold, by the pigeonhole principle, there must be a ball B_i of radius 2^{i-2} that contains a set U'_i of at least $\mathcal{C}(2^{i+2}, 2^{i-2}, d) + 1$ nodes from U_i. We define $W_i := \{\omega(u) : u \in U'_i\}$. Because nodes in U'_i are in ball B_i (of radius 2^{i-2}) and for all $u \in U'_i \subseteq U_i$, $\text{dist}(u, \omega(u)) \geq 2^i$, we have

$$W_i \cap U'_i = \varnothing. \tag{2.3}$$

We consider two cases: i) there are two nodes $u_1, u_2 \in U'_i$ such that $\omega(u_1) = \omega(u_2)$, and ii) for any two nodes $u_1, u_2 \in U'_i$, $\omega(u_1) \neq \omega(u_2)$.

Case i. We define $w := \omega(u_1) = \omega(u_2)$. Without loss of generality, assume that $\text{dist}(u_1, w) \leq \text{dist}(u_2, w)$. Because u_1 and u_2 are both in ball B_i, $\text{dist}(u_1, u_2) \leq 2^{i-1}$ and therefore $\text{dist}(u_1, u_2) < \text{dist}(u_1, w) \leq \text{dist}(u_2, w)$. Since $\{u_1, w\}$ is a primitive edge, the edge $\{u_2, w\}$ is bridged. Because also $\{u_2, w\}$ is a primitive edge, this is a contradiction to the assumption that $G \in \mathcal{T}(P)$.

Case ii. We have $|W_i| = |U'_i| \geq \mathcal{C}(2^{i+2}, 2^{i-2}, d) + 1$. Since every node in W_i lies in a ball of radius $2^{i-2} + 2^{i+1} < 2^{i+2}$, and because a ball of radius 2^{i+2} can be covered with $\mathcal{C}(2^{i+2}, 2^{i-2}, d) + 1$ balls of radius 2^{i-2}, there must be a ball of radius 2^{i-2} that contains at least two nodes w_1 and w_2 from W_i. Assume that $u_1, u_2 \in U'_i$ such that $\omega(u_1) = w_1$ and $\omega(u_2) = w_2$. Without loss of generality, assume that either $\text{dist}(u_2, w_2) \geq \text{dist}(u_1, w_1) \geq 2^i$. Because $\text{dist}(u_1, u_2) \leq 2^{i-1}$, $\text{dist}(w_1, w_2) \leq 2^{i-1}$, and because $\{u_1, w_1\}$ is primitive, this implies that $\{u_2, w_2\}$ is bridged. Because $\{u_2, w_2\}$ is a primitive edge too, this is a contradiction to the assumption that $G \in \mathcal{T}(P)$.

A contradiction is derived in both cases. Therefore, (2.2) holds and thus the claim of the theorem follows. □

In the next lemma, we show that $\mathrm{MST}(P)'$ is in $\mathcal{T}(P)$. Consequently, in particular $\mathcal{T}(P)$ always includes a connected communication graph. In order to make $\mathrm{MST}(P)$ unique, assume that there is a global order $\prec$ on all the possible edges such that for any four nodes a, b, c, and d, $\mathrm{dist}(a, b) < \mathrm{dist}(c, d) \Rightarrow \{a, b\} \prec \{c, d\}$. We assume that $\mathrm{MST}(P)$ is the minimum spanning tree which is also minimal w.r.t. the global order $\prec$. Note that for every MST, there is such a global order $\prec$ such that this is the case.

Lemma 2. *For any set of points $P \subseteq \mathbb{R}^d$, $\mathrm{MST}(P)' \in \mathcal{T}(P)$.*

Proof. By definition, all primitive edges are edges of $\mathrm{MST}(P)$. Suppose there is a primitive edge $\{p, q\} \in E(\mathrm{MST}(P))$ that is bridged. Therefore, there is a path T from p to q in $\mathrm{MST}(P)'$ that contains at most three edges, such that for each edge $\{x, y\} \neq \{p, q\}$ of T, $\mathrm{dist}(x, y) < \mathrm{dist}(p, q)$ or $\mathrm{dist}(x, y) = \mathrm{dist}(p, q)$ and $\{x, y\}$ is primitive and thus also $\{x, y\} \in E(\mathrm{MST}(P))$. Assume that $\{p', q'\}$ is the largest edge in T w.r.t. the global order $\prec$. We must have $\mathrm{dist}(p', q') = \mathrm{dist}(p, q)$, as otherwise all edges in T will be smaller than $\{p, q\}$ w.r.t. the order $\prec$, hence a smaller MST can be constructed by replacing $\{p, q\}$ with one of edges of T. Since $\{p, q\}$ is bridged by T, the edge $\{p', q'\}$ has to be a primitive edge and therefore $\{p', q'\} \in E(\mathrm{MST}(P))$. However, this is a contradiction to the assumption that $\mathrm{MST}(P)$ is the minimal MST w.r.t. the order $\prec$ as it is possible to get a smaller MST by replacing $\{p', q'\}$ with another edge of T. □

Theorem 2 implies that the interference of any graph G in $\mathcal{T}(P)$ is bounded asymptotically by the logarithm of the ratio of the longest and shortest edges in G. While this ratio can be arbitrarily large in the worst case, we show that the ratio is bounded for many typical distributions of points. Specifically, if the ratio is $O(n^c)$ for some constant c, then the maximum interference is $O(\log n)$.

Definition 5 ($\mathcal{D}$)**.** *Let $\mathcal{D}$ denote the class of distributions over $[0,1]^d$ such that for any $D \in \mathcal{D}$ and any set P of $n \geq 2$ points selected independently at random according to D, the minimum distance between any two points in P is greater than n^{-c} with high probability, for some constant c (independent of n).*

Theorem 3. *For any integers $d \geq 1$ and $n \geq 2$, any distribution $D \in \mathcal{D}$, and any set P of n points, each of which is selected independently at random over $[0,1]^d$ according to distribution D, with high probability, for all graphs $G \in \mathcal{T}(P)$, $\text{inter}(G) \in O(\log n)$.*

Proof. Let $d_{\min}(G) = \min_{\{s,t\} \in E(G)} \text{dist}(s,t)$ and $d_{\max}(G) = \max_{\{s,t\} \in E(G)} \text{dist}(s,t)$. Since points are contained in $[0,1]^d$, $d_{\max}(G) \leq \sqrt{d}$. Points in P are distributed according to a distribution $D \in \mathcal{D}$. By Definition 5, with high probability, $d_{\min}(G) \geq n^{-c}$ for some constant c. Thus, with high probability, we have

$$\log\left(\frac{d_{\max}(G)}{d_{\min}(G)}\right) \leq \log\left(\frac{\sqrt{d}}{n^{-c}}\right). \tag{2.4}$$

The result follows from (2.4), Theorem 2, and the fact that $\log(n^c\sqrt{d}) \in O(\log n)$ when d and c are constant. $\square$

Lemma 3. *Let D be a distribution with domain $[0,1]^d$, for which there is a constant c' such that for any point $x \in [0,1]^d$, we have $D(x) \leq c'$, where $D(x)$ denotes the probability density function of D at $x \in [0,1]^d$. Then $D \in \mathcal{D}$.*

Proof. Let $p_1, p_2, \ldots, p_n$, be $n \geq 2$ independent random points in $[0,1]^d$ with distribution D. Let $c'' = 1 + \frac{\log c' + 2}{d}$ and let $\mathcal{E}_i$, $1 \leq i \leq n$, denote the event that there is a point p_j, $j \neq i$, such that $\operatorname{dist}(p_i, p_j) \leq n^{-c''}$. Let the random variable $d_{\min}$ be equal to $\min_{i \neq j} \operatorname{dist}(p_i, p_j)$. We have

$$\begin{aligned} \Pr(d_{\min} \leq n^{-c''}) &= \Pr\left(\bigvee_{1 \leq i \leq n} \mathcal{E}_i\right) \\ &\leq \sum_{1 \leq i \leq n} \Pr(\mathcal{E}_i), \end{aligned} \tag{2.5}$$

where the inequality holds by the union bound. To establish an upper bound on $\Pr(\mathcal{E}_i)$, consider a d-dimensional ball B_i with centre p_i and radius $n^{-c''}$. The probability that there is point p_j, $j \neq i$, in that ball is at most c' times the volume of $B_i \cap [0,1]^d$. The volume of $B_i \cap [0,1]^d$ is at most $(2n^{-c''})^d$.

Therefore, $\Pr(\mathcal{E}_i) \leq c'(2n^{-c''})^d$ for every $1 \leq i \leq n$. Thus, by (2.5), we get

$$\begin{aligned} \Pr(d_{\min} > n^{-c''}) &\geq 1 - \sum_{1 \leq i \leq n} \Pr(\mathcal{E}_i) \\ &\geq 1 - n \cdot c' \left(2n^{-c''}\right)^d \\ &= 1 - \frac{c' 2^d}{n^{d+\log c'+1}} \\ &\geq 1 - \frac{c' 2^d}{n \cdot 2^{d+\log c'}} \\ &= 1 - \frac{1}{n} \; . \end{aligned}$$

Therefore, $D \in \mathcal{D}$. Note, here $c = c''$ in Definition 5. □

Corollary 4. *The uniform distribution with domain* $[0,1]^d$ *is in* $\mathcal{D}$.

By Corollary 4 and Theorem 3, we can conclude that if a set P of $n \geq 2$ points is distributed uniformly in $[0,1]^d$, then with high probability, any communication graph in $G \in \mathcal{T}(P)$ will have maximum interference $O(\log n)$. This is expressed formally in the following corollary:

Corollary 5. *Choose any integers* $d \geq 1$ *and* $n \geq 2$*. Let* P *be a set of* n *points, each of which is selected independently and uniformly at random over* $[0,1]^d$*. With high probability, for all graphs* $G \in \mathcal{T}(P)$*,* $\text{inter}(G) \in O(\log n)$.

2.3 Mobility

Our results apply to the setting of mobility (e.g., mobile ad hoc wireless networks). Each node in a mobile network must periodically exchange information with its neighbours to update its local data storing positions and transmission radii of nodes within its local neighbourhood. The distribution of mobile nodes depends on the mobility model, which is not necessarily uniform. For example, when the network is distributed over a disc or a box-shaped region, the probability distribution associated with the random waypoint model achieves its maximum at the centre of the region, whereas the probability of finding a node close to the region's boundary approaches zero [4]. Since the maximum value of the probability distribution associated with the random waypoint model is constant [4], by Lemma 3 and Theorem 3, we can conclude that at any point in time, the maximum interference of the network is $O(\log n)$ with high probability. In general, this holds for any random mobility model whose corresponding probability distribution has a constant maximum value.

2.4 Local Algorithm

As discussed in Section 1.1, existing distributed algorithms for topology control attempt to reduce transmission radii, but not necessarily the maximum interference. By Lemma 2 and Theorem 3, if P is a set of n points selected according to a distribution in $\mathcal{D}$, then with high probability $\text{inter}(\text{MST}(P)') \in O(\log n)$. Unfortunately, a minimum spanning tree cannot be generated using only local information [53]. Thus, an interesting question is whether each node can assign itself a transmission radius using only local information such that the resulting communication graph belongs to $\mathcal{T}(P)$ while remaining connected. We answer this question affirmatively by presenting a local algorithm (LocalRadiusReduction), that assigns a transmission radius to each node such that if an initial communication graph $G_{\max}$ is connected, then the resulting communication graph is a connected spanning subgraph of $G_{\max}$ that belongs to $\mathcal{T}(P)$. Consequently, the resulting topology has maximum interference $O(\log n)$ with high probability when nodes are selected according to any distribution in $\mathcal{D}$. Our algorithm can be applied to any existing topology to refine it and reduce its maximum interference. Thus, our solution can be used either independently, or paired with another topology control strategy.

For the distributed algorithm, we assume that each edge e has a unique identifier $\mathrm{ID}(e)$. Such IDs can for example be obtained (locally) by using unique node identifiers. The edge IDs allow to define a global order $\prec$ on all the possible edges of the communication graph as follows. For any two edges $\{u_1, v_1\}, \{u_2, v_2\} \in E(G_{\max})$, we have $\{u_1, v_1\} \prec \{u_2, v_2\}$ if and only if $\mathrm{dist}(u_1, v_1) < \mathrm{dist}(u_2, v_2)$ or $\mathrm{dist}(u_1, v_1) = \mathrm{dist}(u_2, v_2)$ and $\mathrm{ID}(\{u_1, v_1\}) < \mathrm{ID}(\{u_2, v_2\})$.

Let P be a set of $n \geq 2$ points in $\mathbb{R}^d$ and let $r_{\max} : P \to \mathbb{R}^+$ be a function that returns the maximum transmission radius allowable at each node. Let $G_{\max}$ denote the communication graph determined by P and $r_{\max}$. We suppose that $G_{\max}$ is connected. Further, assume that $\mathrm{Adj}_{\max}(u)$ is the set of neighbours of a node u in $G_{\max}$. Algorithm LocalRadiusReduction assumes that each node is initially aware of its maximum transmission radius, its spatial coordinates, and its unique identifier.

The algorithm begins with a local data acquisition phase, during which every node broadcasts its identity, maximum transmission radius, and coordinates in a node data message. Each message also specifies whether the

```
LocalRadiusReduction(u)
r′(u) ← 0
for each v ∈ Adj_max(u) do
    if dist(u, v) > r′(u) and ¬Redundant(u, v) then
        r′(u) ← dist(u, v)
return r′(u)
```

Table 2.1: LocalRadiusReduction(u)

data is associated with the sender or whether it is forwarded from a neighbour. Every node records the node data it receives and retransmits those messages that were not previously forwarded. Upon completing this phase, each node is aware of the corresponding data for all nodes within its 2-hop neighbourhood. The algorithm then proceeds to a local transmission radius reduction phase, which does not require any additional communication.

We say that an edge $\{u, v\}$ of $G_{\max}$ is *redundant* iff there is a path at most 3 connecting u and v such that for every edge $\{x, y\}$ of the path, we have $\{x, y\} \prec \{u, v\}$. Let H be the graph consisting of all edges of $G_{\max}$ that are not redundant. The communication graph G is defined as the closure of graph H, i.e., $G = H'$. Consequently, each node u chooses $r(u)$ to be the largest distance to a neighbour $v \in \mathrm{Adj}_{\max}(u)$ such that $\{u, v\}$ is not redundant. The details are given in Algorithm 2.1.

Algorithm LocalRadiusReduction is 2-local, that is, each node only needs to learn about the initial state of nodes at distance at most 2 in $G_{\max}$. Further, the local computation time at each node is bounded by $O(\Delta^3)$, where Δ is the maximum vertex degree in $G_{\max}$. Each call to the subroutine Bridge costs at most $O(\Delta^2)$ time and there are at most Δ calls to Bridge from each node.

```
REDUNDANT(a, b)
result ← false
for each v ∈ Adj_max(a) do
    if v ∈ Adj_max(b) and
            {a, v} ≺ {a, b} and {v, b} ≺ {a, b} then
        result ← true
    for each w ∈ Adj_max(v) do
        if w ∈ Adj_max(b) and {a, v} ≺ {a, b} and
                {v, w} ≺ {a, b} and {w, b} ≺ {a, b} then
            result ← true
return result
```

Table 2.2: REDUNDANT(a, b)

Theorem 6. *The communication graph constructed by Algorithm* LOCALRADIUSREDUCTION *is in $\mathcal{T}(P)$ and it is connected if the initial communication graph $G_{\max}$ is connected.*

Proof. Let G denote the communication graph constructed by Algorithm LOCALRADIUSREDUCTION. We first prove that G is in $\mathcal{T}(P)$. By construction, G is the closure of the graph H consisting of all edges of $G_{\max}$ that are not redundant. An edge $\{u, v\}$ is redundant if it is the largest w.r.t. global order $\prec$ in some cycle of length at most 4 in $G_{\max}$. Consequently, H has no cycles of length less than 5. For contradiction, assume that there is an edge $\{u, v\} \in E(G)$ which is bridged and which is primitive (w.r.t. H). Then, there is a path T of length at most 3 that connects u and v such that for each edge $\{x, y\}$ of T, either $\operatorname{dist}(x, y) < \operatorname{dist}(u, v)$ or $\operatorname{dist}(x, y) = \operatorname{dist}(u, v)$ and $\{x, y\}$ is primitive (w.r.t. H). Hence, G contains a cycle of length at most 4 such that the longest edges of the cycle are all primitive (and thus also edges of H). This cannot be because from each such cycle of $G_{\max}$ the largest edge w.r.t. $\prec$ is not included in H.

It remains to prove that G is connected if $G_{\max}$ is connected. For contradiction, assume that G and therefore also H is not connected and consider a set $S \subset V, S \neq \varnothing$ such that H does not contain an edge between S and $V \setminus S$. Let $e = \{u, v\}$ be the smallest edge of $G_{\max}$ (w.r.t. $\prec$) over the cut $(S, V \setminus S)$. Edge $\{u, v\}$ cannot be redundant because every cycle of $G_{\max}$ that contains $\{u, v\}$ has to contain at least one other edge $\{u', v'\}$ across the cut $(S, V \setminus S)$ and by assumption $\{u, v\} \prec \{u', v'\}$.

□

More generally, since transmission radii are only decreased, it can be shown that $G_{\min}$ and $G_{\max}$ have the same number of connected components by applying Theorem 6 on every connected component of $G_{\max}$.

Chapter 3

Minimizing Average Interference in Random Networks[5]

3.1 Expected Average Interference of Random Networks

We now examine the problem of minimizing the average interference in a set of points whose positions are selected uniformly and independently at random over the unit square in the plane. This work is the first to examine average interference in a random setting.

We refer to the following lemma by Wan et al., where the *radius* of a set $P \subseteq \mathbb{R}^d$ is

$$\mathrm{rad}_P = \min_{p \in P} \max_{q \in P} \mathrm{dist}(p, q).$$

Consequently, there exists a point $p \in P$ such that a disc of radius rad_P

[5]Part of this chapter will be published in *A. Haghnegahdar, M. Khabbazian, Average Interference in Wireless Ad Hoc Networks with Nodes in Random Position. To be submitted to The International Symposium on DIStributed Computing (DISC), 2014.*

centred at p covers P [54].

Lemma 4 (Wan et al. [54] (2002))**.** *For any set $P \subseteq \mathbb{R}^2$ of points with radius one,*

$$\sum_{e \in E(\mathrm{MST}(P))} \|e\|^2 \leq 12.$$

In this section, we show that for any set P of points selected uniformly and independently at random in a unit square, $\mathbb{E}[\mathrm{interAvg}(\mathrm{MST}'(P))] \in O(1)$. Interestingly, this result does not hold for every distribution in $\mathcal{D}$ (see Definition 5). For clarity of the proofs, throughout this section, we assume that the distance between each pair of nodes is unique, i.e., that points in P are in general position.

Lemma 5. *For any set $P \subseteq [0,1]^2$ of n points selected uniformly and independently at random, with high probability, the longest edge in* $\mathrm{MST}(P)$ *has length $l_{\max} \in O\left(\sqrt{(\log n)/n}\right)$.*

Proof. Choose any real number $c \geq 2$. We divide $[0,1]^2$ into a square grid with $\left\lfloor \sqrt{n/(c \log n)} \right\rfloor^2$ square cells, each with side length $\left\lfloor \sqrt{c \log n/n} \right\rfloor \geq \sqrt{c \log n/n}$. We say two distinct cells are adjacent if they share a side. The distance between any two points in adjacent cells is at most

$$l = \sqrt{5} \left\lfloor \sqrt{c \log n/n} \right\rfloor \in \Theta\left(\sqrt{\log n/n}\right).$$

Assume each cell contains at least one node. If we select one representative node in each cell, connect every node in each cell to its representative node, and connect representative nodes in adjacent cells, the resulting graph will be connected with a maximum edge length of l. If every cell contains a node, then the longest edge in MST(P) has length at most l. Therefore, to prove the lemma it suffices to show that every cell contains at least one node with high probability.

The probability that a given cell does not contain any node is at most

$$\left(1 - \left\lfloor\sqrt{\frac{n}{c\log n}}\right\rfloor^{-2}\right)^n \leq \left(1 - \frac{c\log n}{n}\right)^n$$
$$\leq \exp\left(-\frac{c\log n}{n}\cdot n\right)$$
$$= n^{-c}.$$

By a union bound, the probability that every cell contains at least a node is at least $1 - n^{1-c}$, which completes the proof. □

Definition 6 (Communication Coverage)**.** *Given a communication graph G and any node $p \in V(G)$, the* communication coverage *of p is the region within the transmission range of node p, which we denote* $\mathrm{cov}_G(p)$*. That is* $\mathrm{cov}_G(p)$ *is the disc of radius $r(p)$ centred at p. We define the communication coverage of G as*

$$\mathrm{cov}(G) = \bigcup_{p\in V(G)} \mathrm{cov}_G(p).$$

The following lemma shows that for any set of points P, the average interference within cov(MST′(P)) is constant. Note that this result applies to a continuum of points in the plane, and not only to interference at discrete points in P.

Lemma 6. *For any set of points $P \subseteq \mathbb{R}^2$, the average interference in*

$\text{cov}(\text{MST}'(P))$ *is* $O(1)$, *i.e.,*

$$\frac{1}{|\text{cov}(\text{MST}'(P))|}\iint_{\text{cov}(\text{MST}'(P))} \text{inter}_{\text{MST}'(P)}(x,y)\ dx\ dy \in O(1).$$

Proof. Choose any set of points $P = \{p_1, \ldots, p_n\}$ in $\mathbb{R}^2$. Let q be a point selected uniformly at random in $\text{cov}(\text{MST}'(P))$. It suffices to show that $\mathbb{E}[\text{inter}_{\text{MST}'(P)}(q)] \in O(1)$. Without loss of generality, suppose that P has radius one. We partition $\text{cov}(\text{MST}'(P))$ into n disjoint regions $\mathcal{R}_1, \ldots, \mathcal{R}_n$ such that for each i, $\mathcal{R}_i$ includes all the points in $\mathbb{R}^2$ that are within the transmission ranges of exactly i nodes in P. For each i, let e_i denote the longest edge in $E(\text{MST}(P))$ incident to the point $p_i \in P$. Note that e_i and e_j are not necessarily distinct for $i \neq j$. However, for every i, there is at most one $j \neq i$ such that $e_i = e_j$.

$$\begin{aligned}
\mathbb{E}[\text{inter}_{\text{MST}'(P)}(q)] &= \frac{\sum_{i=1}^{n} i|\mathcal{R}_i|}{|\text{cov}(\text{MST}'(P))|} \\
&= \frac{\sum_{i=1}^{n} i|\mathcal{R}_i|}{\sum_{i=1}^{n} |\mathcal{R}_i|} \\
&= \frac{\sum_{i=1}^{n} |\text{cov}_{\text{MST}'(P)}(p_i)|}{\sum_{i=1}^{n} |\mathcal{R}_i|} \\
&= \frac{\sum_{i=1}^{n} \pi\|e_i\|^2}{\sum_{i=1}^{n} |\mathcal{R}_i|} \\
&\leq \frac{2\sum_{e\in E(\text{MST}(P))} \pi\|e\|^2}{\sum_{i=1}^{n} |\mathcal{R}_i|} \qquad (3.1)\\
&\leq \frac{2\sum_{e\in E(\text{MST}(P))} \pi\|e\|^2}{\sum_{e\in E(\text{MST}(P))} \frac{\|e\|^2}{4\sqrt{3}}} \qquad (3.2)\\
&= 8\pi\sqrt{3}.
\end{aligned}$$

(3.2) follows from (3.1) by the fact that the lozenges with e_i as their largest diameters (with angles $\pi/3 - \epsilon$) do not overlap [54]. □

Lemma 7. *Let l be a positive real number, let $\mathcal{S}$ be a square of size $l \times l$, and let P a set of n points in $\mathcal{S}$. If the transmission range of nodes is determined by* $\mathrm{MST}(P)$*, then the average interference in $\mathcal{S}$ is $O(1)$, that is*

$$\frac{1}{l^2} \iint_{\mathcal{S}} \mathrm{inter}_{\mathrm{MST}'(P)}(x, y)\ dx\ dy \in O(1).$$

Proof. Choose any set of points $P = \{p_1, \ldots, p_n\}$ in $\mathcal{S}$. Let q be a point selected uniformly at random in $\mathcal{S}$. It suffices to show that $\mathbb{E}[\mathrm{inter}_{\mathrm{MST}'(P)}(q)] \in O(1)$. Without loss of generality, suppose $l = 1/\sqrt{2}$. Note that the radius of P is at most one as the diameter of $\mathcal{S}$ is one. We partition $\mathcal{S}$ into n disjoint regions $\mathcal{R}_1, \ldots, \mathcal{R}_n$ such that for each i, $\mathcal{R}_i$ includes all the points in $\mathbb{R}^2$ that are within the transmission ranges of exactly i nodes in P. For each i, let e_i denote the longest edge in $E(\mathrm{MST}(P))$ incident to the point $p_i \in P$. We have

$$\begin{aligned}
\mathbb{E}[\text{inter}_{\text{MST}'(P)}(q)] &= \frac{\sum_{i=1}^{n} i|\mathcal{R}_i|}{|\mathcal{S}|} \\
&= 2\sum_{i=1}^{n} i|\mathcal{R}_i| \\
&= 2\sum_{i=1}^{n} |\operatorname{cov}_{\text{MST}'(P)}(p_i) \cap \mathcal{S}| \\
&\leq 2\sum_{i=1}^{n} |\operatorname{cov}_{\text{MST}'(P)}(p_i)| \\
&= 2\sum_{i=1}^{n} \pi\|e_i\|^2 \\
&\leq 4 \sum_{e \in E(\text{MST}(P))} \pi\|e\|^2 \\
&\leq 4 \cdot 12 \qquad (3.3) \\
&= 48,
\end{aligned}$$

where (3.3) is by Lemma 4.

□

Lemma 8. *For any set of points $P = \{p_1, \ldots, p_n\} \subseteq \mathbb{R}^2$ and any $p_x \in P$,*

$$\text{inter}_{\text{MST}'(P)}(p_x) \leq \text{inter}_{\text{MST}'(P \setminus \{p_x\})}(p_x) + 6. \qquad (3.4)$$

Proof. Without loss of generality, we show (3.4) holds when $p_x = p_i$. For any pair $\{p_i, p_j\} \subseteq \{p_2, \ldots, p_n\}$, we show that if the edge $\{p_i, p_j\} \in E(\text{MST}(P))$, then $\{p_i, p_j\} \in E(\text{MST}(P \setminus \{p_1\}))$. By contradiction, suppose there exists a pair $\{p_i, p_j\} \subseteq \{p_2, \ldots, p_n\}$ such that $\{p_i, p_j\} \in E(\text{MST}(P))$ and

$\{p_i, p_j\} \notin E(\mathrm{MST}(P \setminus \{p_1\}))$. By removing $\{p_i, p_j\}$, $\mathrm{MST}(P)$ is divided into two connected components, C_1 and C_2. Since $\mathrm{MST}(P \setminus \{p_1\})$ is connected, there must an edge $\{p_k, p_l\}$ such that $p_k \in C_1$ and $p_l \in C_2$. Note that $\{p_k, p_l\} \neq \{p_i, p_j\}$ by our assumption that $\{p_i, p_j\} \notin E(\mathrm{MST}(P \setminus \{p_1\}))$. Also, $k \neq 1$ and $l \neq 1$ since p_1 is not in $V(\mathrm{MST}(P \setminus \{p_1\}))$. Furthermore, $\mathrm{dist}(p_k, p_l) < \mathrm{dist}(p_i, p_j)$ because otherwise by replacing the edge $\{p_k, p_l\}$ with $\{p_i, p_j\}$ we can reduce the sum of the lengths of edges in $\mathrm{MST}(P \setminus \{p_1\})$. This derives a contradiction, however, as we can reduce the sum of length of edges in $\mathrm{MST}(P)$ by replacing $\{p_i, p_j\}$ with $\{p_k, p_l\}$. The above argument shows that the transmission range of any non-neighbour of p_1 determined by $\mathrm{MST}(P \setminus \{p_1\})$ is not more than its transmission range determined by $\mathrm{MST}(P)$. We conclude the proof by noting that for any set of points Q, the maximum degree of $\mathrm{MST}(Q)$ is at most six, hence p_1 has at most six neighbours in $\mathrm{MST}(P)$. □

Theorem 7. *Let n be a positive integer. Let $\mathrm{interAvg}(\mathrm{MST}'(P))$ be a random variable equal to the average interference of a set P of n points distributed uniformly and independently at random in $[0, 1]^2$. Then*

$$\mathbb{E}[\mathrm{interAvg}(\mathrm{MST}'(P))] \in O(1).$$

Proof. Let $P = \{p_1, \ldots, p_n\}$ be a set of n points selected uniformly at random in $[0,1]^2$.

$$\mathbb{E}[\text{interAvg}(\text{MST}'(P))] = E\left[\frac{1}{n}\sum_{i=1}^{n}\text{inter}_{\text{MST}'(P)}(p_i)\right] \tag{3.5}$$

$$= \frac{1}{n}\sum_{i=1}^{n} E\left[\text{inter}_{\text{MST}'(P)}(p_i)\right] \tag{3.6}$$

$$= E\left[\text{inter}_{\text{MST}'(P)}(p_1)\right],$$

where (3.5) holds because the expected value of a sum of random variables (independent or not) is equal to the sum of the individual expectations, and (3.6) holds by the fact that, due to symmetry, for every $\{p_i, p_j\} \subseteq \{p_1, \ldots, p_n\}$,

$$E\left[\text{inter}_{\text{MST}'(P)}(p_i)\right] = E\left[\text{inter}_{\text{MST}'(P)}(p_j)\right].$$

Since p_1 is selected uniformly and independently at random, by Lemma 7, we get

$$\mathbb{E}[\text{inter}_{\text{MST}'(P\setminus\{p_1\})}(p_1)] \in O(1).$$

Furthermore, by Lemma 8, we have

$$\text{inter}_{\text{MST}'(P)}(p_1) \leq \text{inter}_{\text{MST}'(P\setminus\{p_1\})}(p_1) + 6.$$

Thus,

$$\mathbb{E}[\text{inter}_{\text{MST}'(P)}(p_1)] \leq \mathbb{E}[\text{inter}_{\text{MST}'(P\setminus\{p_1\})}(p_1)] + 6$$

$$\in O(1),$$

which completes the proof. □

3.2 Average Interference of Random Networks

Before delving into the details of our proof, in this section, we provide some intuition about our approach and the main ideas used. We divide the network (a unit square) into a square grid with $c \cdot \frac{n}{\log n}$ square cells, for some constant c. We carefully select c such that, with high probability, i) every square cell contains $\theta(\log n)$ points, and ii) the cell side is at least a few times of the length of the largest edge of $MST(P)$.

Next, we focus on the "local interference" at each cell $\mathcal{C}$, that is $\text{interAvg}(H'_{\mathcal{C}})$, where $H_{\mathcal{C}} = MST(P)[P_{\mathcal{C}}]$, $P_{\mathcal{C}} \subseteq P$ is the set of points in the cell $\mathcal{C}$, and $MST(P)[P_{\mathcal{C}}]$ is the subgraph of $MST(P)$ induced by $P_{\mathcal{C}}$. We show that, for every cell $\mathcal{C}$,

$$\text{interAvg}(H'_{\mathcal{C}}) \leq \text{interAvg}(MST'(P_{\mathcal{C}})).$$

Therefore, by [43] we conclude that the expected local average interference in any cell is constant,$\mathbb{E}[\text{interAvg}(H'_{\mathcal{C}})] = O(1)$. Assume that there is the same number of nodes in each cell. In this case, each cell has a fix number of nodes distributed uniformly at random. Therefore, all cells have the same average interference distribution. Also, by [43], the expected average interference in each cell is constant. Therefore, by the law of large numbers, for large enough n, the mean of average interferences over all cells is almost surely constant.

There are two major problems to achieve the desired final result: 1) although the order of the number of nodes in each cell is the same, the exact number can differ from one cell to the other, hence we cannot directly apply the law of large numbers. 2) we have not accounted all the inferences as there are crossing edges between cells that have not been considered.

To deal with the first problem, we divide cells into $O(\log n)$ groups such that, in each group, every cell contains the same number of nodes. If there is large enough number of cells in a group, we can apply the law of large numbers on that group individually. For other groups (with lower number of cells), we show that their effect on the overall average interference of network is limited by an additive constant.

We approach the second problem by considering 16 different square grids and show that any edge in $MST(P)$ causing interference at a node, together with that node, will fall into one cell in at least one of these 16 grids. Since for each grid, the average of all local interferences is constant, with high probability, using a union bound, we conclude that the average of all interferences occurred in at least one of these grids is constant, w.h.p., hence the overall average interference of network is constant, w.h.p.

Let $L_{\max}$ denote the length of the largest edge in $MST(P)$. When n points are distributed uniformly at random in a unit square, it was shown in [43] that

$$L_{\max} \in O\left(\sqrt{\frac{logn}{n}}\right),$$

with high probability.

Definition 7. *($Grid^d_{a,b}$) We define the grid $Grid^d_{a,b}$ as the set of all squares* *(simply called cells) with side length d centred at points $(id+a, jd+b)$, where $i, j \in \mathbb{Z}$ and $0 \le a, b < d$.*

Definition 8. *(Non-complete/complete cell) A cell $\mathcal{C}$ in a gird $Grid^d_{a,b}$ is called non-complete if $\mathcal{C} \cap [0,1]^2 \neq \mathcal{C}$. Otherwise, it is called complete. In other words, a cell is complete if it entirely falls inside the network, and is non-complete, otherwise.*

Definition 9. *($\overline{Grid}^d_{a,b}$) We define $\overline{Grid}^d_{a,b}$ as*

$$\overline{Grid}^d_{a,b} = \{\mathcal{C} | \mathcal{C} \in Grid^d_{a,b} \wedge \mathcal{C} \cap [0,1]^2 = \mathcal{C}\},$$

that is, the set of all complete cells in $Grid^d_{a,b}$.

Let $\eta = \theta\left(\sqrt{\frac{\log n}{n}}\right)$ be a real number such that i) $\eta \ge \frac{2}{1-\frac{\sqrt{2}}{4}} L_{max}$, and ii) $\frac{1}{\eta}$ is an integer. Note that we can always find such a number η as $L_{max} = O\left(\sqrt{\frac{\log n}{n}}\right)$. Also, the network is fully covered by the grid, $\overline{Grid}^{\eta}_{\frac{\eta}{2},\frac{\eta}{2}}$. Figure 3.1 illustrates two examples of these grids.

The following lemma shows that when η is large enough, every cell in $\overline{Grid}^{\eta}_{\frac{\eta}{2},\frac{\eta}{2}}$ will contain $\theta(\log n)$ nodes, w.h.p.

Lemma 9. *Let us divide the network into a gird such that the side length of each cell of the grid is $\sqrt{\frac{2c\log n}{n}}$, for some constant $c \ge 1$. Fix any cell, and let X be a random variable equal to the number of nodes in that cell. Then, we have*

$$Pr(c\log n \le X \le 3c\log n) \ge 1 - n^{-\frac{c}{5}} - n^{-\frac{c}{4}}.$$

Proof. Since nodes are distributed uniformly at random, the probability that a node falls into a given cell is equal to the area of the cell, which is

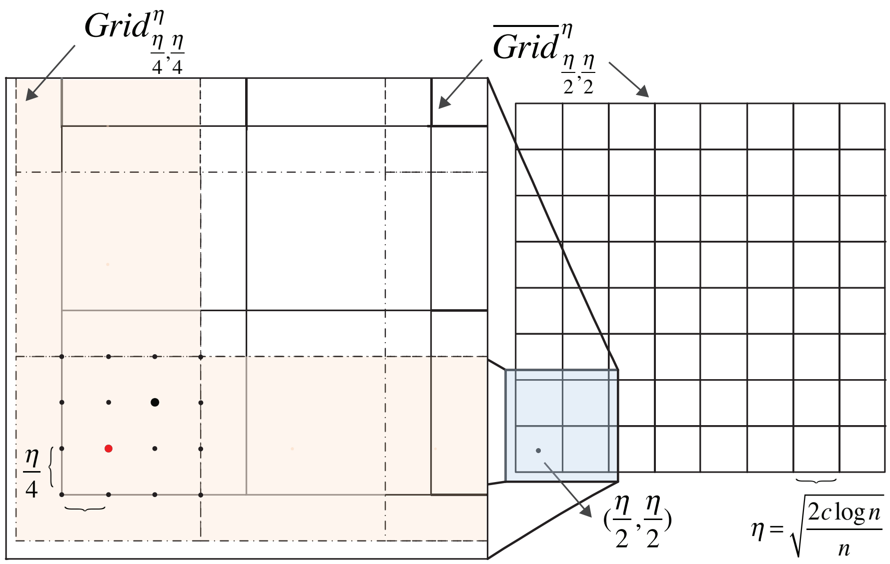

Figure 3.1: The right side of above figure shows the entire network which is divided into square cells by $\overline{Grid}^{\eta}_{\frac{\eta}{2},\frac{\eta}{2}}$ (solid lines). The left side shows a small part of both entire network and $Grid^{\eta}_{\frac{\eta}{4},\frac{\eta}{4}}$ (dotted lines). The highlighted cells in $Grid^{\eta}_{\frac{\eta}{4},\frac{\eta}{4}}$ illustrate the non-complete cells which do not fall completely in $[0,1]^2$ region.

$\left(\sqrt{\frac{2c\log n}{n}}\right)^2 = \frac{2c\log n}{n}$. Let fix a cell $\mathcal{C}$. Let X_i, $1 \le i \le n$, be a $\{0,1\}$ random variable such that $X_i = 1$ if node i falls into $\mathcal{C}$. Note that $X_1, X_2, \ldots, X_n$ is a set of independent Bernoulli variables with $Pr(X_i = 1) = \frac{2c\log n}{n}$ for every $1 \le i \le n$. Let $X = \sum_{i=1}^{n} x_i$. Therefore, we get $\mathbb{E}[X] = \mu = 2c\log n$. By a Chernoff bound, we have

$$Pr(X < (1-\delta)\mu) < \left(\frac{e^{-\delta}}{(1-\delta)^{(1-\delta)}}\right)^{\mu},$$

and

$$Pr(X > (1+\delta)\mu) < \left(\frac{e^{\delta}}{(1+\delta)^{(1+\delta)}}\right)^{\mu}.$$

where μ is the expectation of X and $\delta > 0$. Therefore, setting $\delta = 0.5$, we get

$$Pr(X < c\log n) < n^{-c(1+\ln(1/2))} < n^{-\frac{c}{4}},$$

and

$$Pr(X > 3c\log n) < n^{-c(3\ln(3/2)-1)} < n^{-\frac{c}{5}}.$$

Hence,

$$Pr(c\log n \le X \le 3c\log n) \ge 1 - n^{-\frac{c}{5}} - n^{-\frac{c}{4}}.$$

□

Using Lemma 9 and a union bound, we get that, when c is large enough, every cell of resulting grid will contain $\theta(\log n)$ nodes, w.h.p.

Lemma 10. *Let P be a finite set of points in $\mathbb{R}^2$, and $Q \subseteq P$ be a non-empty subset of P. Let $G = \mathrm{MST}(Q)$, and $H = \mathrm{MST}(P)[Q]$ be the sub-graph of*

MST(P) *induced by* Q. *Then*

$$interAvg(H') \leq interAvg(G')$$

Proof. To prove the lemma, we show that for every point $q \in Q$, we have

$$r_H(q) \leq r_G(q)$$

that is the transmission range of every node $q \in Q$ determined by H is not more that that determined by G. By contradiction, suppose that there is a $q \in Q$ such that $r_H(q) > r_G(q)$. Let $e = \{q, p\} \in E(H)$ be the link determining the transmission range of q. We must have $e \notin E(G)$ as otherwise $r_H(q) \leq r_G(q)$. Since G is connected, there is a sequence of distinct edges $e_i \in E(G)$, $1 \leq i \leq m$, connecting q to p. We have,

$$\forall 1 \leq i \leq m : \|e_i\| \leq \|e\|$$

because $e \notin E(G)$, and G is a minimum spanning tree. Since size of every edge is assumed to be unique we get

$$\forall 1 \leq i \leq m : \|e_i\| < \|e\|$$

Since $e \in H$, we get $e \in$ MST(P) since H is a subgraph of MST(P). This is a contradiction, because there is a sequence of edges from q to p (i.e. , $\{e_1, ..., e_m\}$) with edges whose size is less than $\|e\|$. □

Consider a cell $\mathcal{C}$ and assume that $Q = P_{\mathcal{C}}$ is the set of points in $\mathcal{C}$.

By [43], we know that $\mathbb{E}[\text{interAvg}(MST(Q)')] = O(1)$. Therefore, using Lemma 10, we get $\mathbb{E}[\text{interAvg}(H')] = O(1)$, where $H = \text{MST}(P)[Q]$ and P is the set of all nodes in the network.

Note that, although every cell contains the same order of nodes w.h.p., the actual number of nodes may differ from one cell to anther. Next, we divide cells into groups S_i, $1 \leq i \leq \theta(\log n)$, such that each group consists of all cells that contain the same number of nodes. Formally, $S_i = \{\mathcal{C} \mid |P_{\mathcal{C}}| = i\}$, where $|P_{\mathcal{C}}|$ denotes the number of nodes in cell $\mathcal{C}$.

We further classify groups S_i into two partitions. First partition, $\mathcal{P}_1$, includes all groups that have at most $\frac{n}{\log^3 n}$ cells, and the second partition, $\mathcal{P}_2$, includes the rest of groups. For any cell $\mathcal{C}$, let $H_{\mathcal{C}} = \text{MST}(P)[P_{\mathcal{C}}]$, where $P_{\mathcal{C}}$ is the set of nodes in $\mathcal{C}$. The following two lemmas show that the sum of interferences of nodes in each partition is $O(n)$.

Lemma 11. *For groups in the first partition, $\mathcal{P}_1$, we have*

$$\sum_{S \in \mathcal{P}_1} \sum_{\mathcal{C} \in S} \sum_{p \in P_{\mathcal{C}}} \text{inter}_{H'_{\mathcal{C}}}(p) = O(n),$$

w.h.p.

Proof. The average interference in each cell is at most equal to the number of nodes in that cell, which is $\theta(\log n)$ w.h.p. Therefore, the sum of all inferences in a cell is $\theta(\log^2 n)$, that is, for any cell $\mathcal{C}$,

$$\sum_{p \in P_{\mathcal{C}}} \text{inter}_{H'_{\mathcal{C}}}(p) = |P_{\mathcal{C}}| \cdot \theta(\log n) = \theta(\log^2 n),$$

w.h.p. Note that the total number of groups is $\theta(\log n)$, thus there are

$O(\log n)$ groups in the first partition $\mathcal{P}_1$, w.h.p. Also, by definition each group in the first partition has $O(\frac{n}{\log^3 n})$ cells in it. Consequently, the sum of interferences of all cells in the union of groups of the first partition is

$$\sum_{S\in\mathcal{P}_1}\sum_{\mathcal{C}\in S}\sum_{p\in P_\mathcal{C}} \text{inter}_{H'_\mathcal{C}}(p) = \frac{n}{\log^3 n}\cdot O(\log n)\cdot \theta(\log^2 n)$$
$$= O(n),$$

w.h.p. □

Lemma 12. *For groups in the second partition, $\mathcal{P}_2$, we have*

$$\sum_{S\in\mathcal{P}_2}\sum_{\mathcal{C}\in S}\sum_{p\in P_\mathcal{C}} \text{inter}_{H'_\mathcal{C}}(p) = O(n),$$

w.h.p.

Proof. Let S be any group in $\mathcal{P}_2$. By definition, all the cells in S contain the same number of nodes. Also, nodes are distributed uniformly in each cell. Therefore, $\text{interAvg}(H'_\mathcal{C})$, $\mathcal{C}\in S$, is a collection of i.i.d random variables, whose expected value is $O(1)$ by [43]. Thus, using the strong law of large numbers, we almost surely have

$$\frac{\sum_{\mathcal{C}\in S}\text{interAvg}(H'_\mathcal{C})}{|S|} = O(1)$$

when $|S|$ is sufficiently large (or, equivalently, when n is sufficiently large). In other words, when n is sufficiently large, we almost surely have

$$\sum_{\mathcal{C}\in S}\text{interAvg}(H'_\mathcal{C}) = O(|S|),$$

thus

$$\sum_{\mathcal{C}\in S}\sum_{p\in P_{\mathcal{C}}} \mathrm{inter}_{H'_{\mathcal{C}}}(p) = \Lambda(S)\cdot O(|S|), \tag{3.7}$$

where $\Lambda(S)$ denotes the number of nodes in any cell $\mathcal{C} \in S$ (by definition, every cell in group S contains the same number of nodes). Since (3.7) holds for any group $S \in \mathcal{P}_2$, we almost surely have

$$\begin{aligned}\sum_{S\in\mathcal{P}_2}\sum_{\mathcal{C}\in S}\sum_{p\in P_{\mathcal{C}}} \mathrm{inter}_{H'_{\mathcal{C}}}(p) &= \sum_{S\in\mathcal{P}_2} \left(\Lambda(S)\cdot O(|S|)\right)\\ &= O(n),\end{aligned}$$

when n is sufficiently large, which completes the proof. □

The next lemma simply combines the results of Lemma 11 and 12.

Lemma 13. *We have*

$$\sum_{S\in\mathcal{P}_1\cup\mathcal{P}_2}\sum_{\mathcal{C}\in S}\sum_{p\in P_{\mathcal{C}}} \mathrm{inter}_{H'_{\mathcal{C}}}(p) = O(n),$$

w.h.p.

An interference at node p caused by node q can be represented by a (not necessarily unique) tuple (p, q, r) such that i) $p = q = r$, or ii) $\{q, r\} \in E(\mathrm{MST}(P))$ and $\mathrm{dist}(p, q) \le \mathrm{dist}(q, r)$. Considering this representation of interference, for a given node $p \in P_{\mathcal{C}}$, so far we have considered only interferences at p that can be represented by a tuple (p, q, r) such that $q, r \in P_{\mathcal{C}}$. To count all the inferences at p at least once, we consider a set of 16 different grids $\mathcal{Q} = \{\overline{Grid}^{\eta}_{i'\frac{\eta}{4},j'\frac{\eta}{4}} | 0 \le i', j' \le 3\}$. We show that, with high probability, for every pair of points p and q such that q causes interference at p, there exists an interference representation tuple (p, q, r), and a cell $\mathcal{C}$ in at least one of the grids in $\mathcal{Q}$ that contains p, q, and r. Since the number of grids is constant, by applying Lemma 13 on every grid, and using a union bound, we get that the sum of all interferences is still $O(n)$, w.h.p. This implies that the average interference in the entire network is $O(1)$, with high probability.

Lemma 14. *Let p, q, r be three points in $\mathbb{R}^2$, and $m = (\frac{p_x+q_x+r_x}{3}, \frac{p_y+q_y+r_y}{3})$ be their average.*
Then, $dist(m,p) \leq L$, $dist(m,q) \leq L$, and $dist(m,r) \leq L$, where

$$L = max(\text{dist}(p,q), \text{dist}(p,r), \text{dist}(q,r)).$$

Proof. The proof is straight forward. Let consider a triangle with vertices p, q, r. The average of vertices, m, always falls inside triangle. Let Select one of the vertices, for instance p, then the distance between p and m can be at most equal to the longest edge that is incident to p. Therefore, by the above definition, we have $dist(m,p) \leq max\{dist(p,r), dist(p,q)\} \leq L$. The same result applies to q and r. □

We use the following lemma to show that if η is large enough, then among the cells in grids $\overline{Grid}^{\eta}_{i'\frac{\eta}{4}, j'\frac{\eta}{4}}$, $0 \leq i', j' \leq 3$, the one that has the closest centre to the average of p, q, r, contains all the points p, q, r.

Lemma 15. *Let p, q, r be three points in $\mathbb{R}^2$, and $L = max(\text{dist}(p,q), \text{dist}(p,r), \text{dist}(q,r))$. Let d be a real number and $Q_d = \{(id, jd) | i, j \in \mathbb{Z}\}$. Then, there is a point $t \in Q_d$ such that the square with side length of $\sqrt{2}d + 2L$ centered at t*

contains all three points p, q, r.

Proof. Let $m = (\frac{p_x+q_x+r_x}{3}, \frac{p_y+q_y+r_y}{3})$, $s = ([\frac{m_x}{d}]d, [\frac{m_y}{d}]d)$. We have

$$\begin{aligned} \text{dist}(m, s) &= \sqrt{(m_x - [\frac{m_x}{d}]d)^2 + (m_y - [\frac{m_y}{d}]d)^2} \\ &\leq \sqrt{\left(\frac{d}{2}\right)^2 + \left(\frac{d}{2}\right)^2} \\ &= \frac{\sqrt{2}}{2}d. \end{aligned} \tag{3.8}$$

By triangle inequality, we have $\text{dist}(s, p) \leq \text{dist}(s, m) + \text{dist}(m, p)$. Therefore, by Lemma 14 and inequality (3.8), we get $\text{dist}(s, p) \leq \frac{\sqrt{2}}{2}d + L$. Similarly, we can show that $\text{dist}(s, q) \leq \frac{\sqrt{2}}{2}d + L$. Therefore, any square centered at s with side length of at least $2\left(\frac{\sqrt{2}}{2}d + L\right)$ contains all three points p, q,and r. Note that by the definition of Q_d, $s \in Q_d$. □

Theorem 8. *Let P be a set of n random points selected independently and uniformly at random in the unit square $[0, 1]^2$. Then the average interference of the communication graph $MST(P)'$ is $O(1)$, with high probability.*

Proof. Let p and q be two distinct points in P such q causes interference at p. Let (p, q, r) be tuple representing this interference. Since (p, q, r) is an interference representative tuple, and p and q are distinct, by definition, we have $\{q, r\} \in E(MST(P))$ and $\text{dist}(p, q) \leq \text{dist}(q, r)$. Therefore, $\text{dist}(q, r) \leq L_{max}$ and $\text{dist}(p, q) \leq \text{dist}(q, r) \leq L_{max}$.

Let us set $d = \frac{\eta}{4}$, and $L = L_{max}$ in Lemma 15. By the assumption in Section 3.2, we have $\eta \geq \frac{2}{1-\frac{\sqrt{2}}{4}}L_{max}$, thus $\eta \geq \sqrt{2}\frac{\eta}{4} + 2L_{max}$. Therefore, by Lemma 15, there is a square $\mathcal{S}$ with side length η centred at a point

$s = (i\frac{\eta}{4}, j\frac{\eta}{4})$ for some $i, j \in \mathbb{Z}$, that contains all three points p, q, r. Let us define

$$\mathcal{E}(x) = \begin{cases} \frac{\eta}{2} & \text{if } x < \frac{\eta}{2} \\ x & \text{if } \frac{\eta}{2} \leq x \leq 1 - \frac{\eta}{2} \\ 1 - \frac{\eta}{2} & \text{if } x > 1 - \frac{\eta}{2}. \end{cases}$$

Since $\frac{1}{\eta}$ is an integer, then for every integer i, $\mathcal{E}(i\frac{\eta}{4}) = k\frac{\eta}{4}$ for some integer k, because $1 - \frac{\eta}{2} = (\frac{4}{\eta} - 2)\frac{\eta}{4}$. Let $s' = (\mathcal{E}(s_x), \mathcal{E}(s_y)) = (\mathcal{E}(i\frac{\eta}{4}), \mathcal{E}(j\frac{\eta}{4}))$, where s_x and s_y denote the x-coordinate and y-coordinate of point s. Note that $s' = (k\frac{\eta}{4}, h\frac{\eta}{4})$ for some integers k and h. Also, since $\frac{\eta}{2} \leq \mathcal{E}(x) \leq 1 - \frac{\eta}{2}$ for every x, the square $\mathcal{S}'$ with side length η centred at s' falls entirely in $[0,1]^2$. Every point in $\mathcal{S} \cap [0,1]^2$ is also in $\mathcal{S}'$. Therefore, $\mathcal{S}'$ contains all three points p, q, r. The square $\mathcal{S}'$ is a cell in the grid $\overline{Grid}^{\eta}_{k'\frac{\eta}{4}, h'\frac{\eta}{4}}$, where $k' = k \mod 4$ and $h' = h \mod 4$. This implies that for every interference representative tuple (p, q, r), there is a cell in a grid from $\mathcal{Q}$ containing points p, q, r.

We proved Lemma 13 when $\overline{Grid^{\eta}_{\frac{\eta}{2},\frac{\eta}{2}}} \in \mathcal{Q}$ is used. However, the lemma essentially holds for any grid in $\mathcal{Q}$. Consequently,

$$\begin{aligned} &\text{interAvg}(MST'(P)) \\ &\quad \leq \frac{\sum_{Grd \in \mathcal{Q}} \sum_{\mathcal{C} \in Grd} \sum_{p \in P_{\mathcal{C}}} \text{inter}_{H'_{\mathcal{C}}(p)}}{n} \\ &\quad = \frac{O(n)}{n} = O(1), \end{aligned}$$

with high probability.

□

Chapter 4

Empirical Evaluations[6]

We evaluated the performance of Algorithm LocalRadiusReduction in three settings (simulated static wireless networks, simulated mobile wireless networks, and real GPS track data) and compared it against four topology control algorithms: i) the cone-based local topology control (CBTC) algorithm [1], ii) the k-neighbour algorithm [28], iii) local computation of the intersection of the Gabriel graph and the unit disc graph (with unit radius $r_{\max}$) [55], and iv) fixed-radius topologies (unit disk graphs of radius $r_{\max} \in \{100, 200, 300\}$). Performance was evaluated by comparing average maximum interference, expected average interference, average physical degree, and average energy cost (the sum of the squares of the transmission radii [28]).

[6]Part of this chapter will be published in *A. Haghnegahdar, M. Khabbazian, Average Interference in Wireless Ad Hoc Networks with Nodes in Random Position. To be submitted to The International Symposium on DIStributed Computing (DISC), 2014.*, and *M. Khabbazian, S. Durocher, A. Haghnegahdar, and F. Kuhn, Bounding interference in wireless ad hoc networks with nodes in random position. IEEE/ACM Transactions on Networking, 2014.*

4.1 Topology Control Algorithms

4.1.1 Cone Based Topology Control

This algorithm is presented in [1]. Each node u tries to find at least one neighbor in every cone of degree α centered at u. In our Simulations, we set $\alpha = \frac{2\pi}{3}$ to ensure a connected resulting network [1]. Node u gradually increases it's transmission power starting from 0 to discover more neighbors. It checks whether each cone of degree α contains a node marked as neighbor. This process is as follows:

1. The nodes labeled as neighbor of node u are sorted according to their angles relative to the closest neighbor of u.

2. If there is a gap of more than α between the angles of two consecutive sorted nodes then u increases the transmission power to reach the next node surrounding it and considers that as a neighbor node.

3. This continues until no-gap larger than α is found or u broadcasts with it's maximum power which is 300 here.

Finally as described in [1], we optimized the CBTC algorithm by considering the following optimization procedure : If there is an edge form u to v and from u to w such that $d(v, w) < \max(d(u, v), d(u, w))$, then remove the longer edge. Then update the transmission range again based on a new neighbor's list.

4.1.2 Gabriel Topology Control

Formally, Gabriel graph is a graph with vertex of point set P in which any points p and q in P are adjacent if a disc of which line pq is a diameter contains no other elements of P. For each node u, algorithm finds all the possible neighbors of u that fall on the disc with diameter equal to the distance of neighbor node and u and contains no other nodes. Algorithm for each node will stop the search after it checks all the possible neighbors or reaches the maximum possible transmission range of current node.

4.1.3 k-neighbour Algorithm

This algorithm is based on the research conducted in [28]. In this paper authors study the topology control problem with the goal of limiting interference as much as possible, while keeping the communication graph connected with high probability. Their method maintains the the number of physical neighbors of every node equal to or slightly below a specific value k.

The paper has two sections. First, they estimated the value of k that guarantees connectivity of communication graph with high probability. In the other section, they proposed k-neighbour protocol that "guarantees logarithmically bounded physical degree at every node".

In our simulations, for each node number, the value of k is selected from [28] such that guarantees the connectivity with probability greater than 0.95.

The k-neighbour Algorithm is :

1. Node u finds it's neighbors based on it's maximum transmission range and then sort them based on their distance.

2. u keeps the list of k nearest neighbors, L_u (if it has less than k neighbor L_u is the list of all neighbors of u.

3. Then nodes calculate the set of symmetric neighbors (Two nodes i ,j are symmetric iff $i \in L_j$ and $j \in L_i$) and store them in L_u^s. After this step we have a undirected graph and for any $(i, j) \in E$ let $P(i, j)$ denote the transmission power that i needs to reach j,

$$P(i, j) = d(i, j)^{\alpha=2}.$$

4. Node u sorts the list L_u^s according to increasing value of $P(u, j)$. Let $j_1, ..., j_k$ be the sorted list.

5. For $l = 2, ..., k$ do the following :

   (a) Check if j_l can be reached using a transmission range lower than $P(u, j_l)$ by routing through some $j_q,\ q < l$.

   (b) If $P(i, j_q) + P(j_q, j_l) \leq P(i, j_l)$ delete the link $P(i, j_l)$.

6. Set the transmission range equal to the power needed to reach the farthest immediate neighbor node in L_u^s.

4.1.4 Local Radius Reduction Algorithm

All the nodes start with the same maximum transmission range.

1. Each node u finds its one-hop neighbors (symmetric links) (the one can be reached by the max transmission range of node u) and sort them based on their distance

2. Node u picks it's furthest neighbor f and check the following steps

   (a) For all the other neighbor nodes $v \in Adj(u)$, if there exists v such that $\max\{d(u,v), d(v,f)\} < d(u,f)$ and $v \in Adj(f)$ then node u removes the f from it's neighbors and reduces the transmission range to it's new furthest neighbor. It then jumps to step 2. (skip (b , c)) if not check step (b) with the selected v.

   (b) For $w \in Adj(v)$ if $\max\{d(u,v), d(v,w), d(w,f)\} < d(u,f)$ and $w \in Adj(f)$ then u removes f from its neighbor list, reduces the transmission range and jumps to step 2.

   (c) If no w and v found terminate the algorithm for the current node u and set the maximum transmission range equal to the distance to it's furthest neighbor in the updated list of neighbors.

4.1.5 Fixed Range Algorithm

This algorithm assumes a fixed transmission range for all the nodes in the network. Using this basic algorithm, we assigned fixed transmission range of 100 m, 200 m, and 300 m to nodes and evaluated it's performance.

4.2 Simulation Parameters

We set the simulation region's dimensions to 1000 metres $\times$ 1000 metres. For both static and dynamic networks, we varied the number of nodes n from 50 to 1000 in increments of 50. We fixed the maximum transmission radius $r_{\max}$ for each network to 100, 200, or 300 metres. To compute the average maximum interference and the expected average interference for static networks, for each n and $r_{\max}$ we generated 100,000 static networks, each with n nodes and maximum transmission radius $r_{\max}$, distributed uniformly at random in the simulation region. In the RWP model, for each n and $r_{\max}$, we randomly generated 100,000 independent networks using the following approximation for nodes' spatial distribution [56]:

$$f(x, y) \approx \frac{9}{16x_m^3 y_m^3}(x^2 - x_m^2)(y^2 - y_m^2),$$

where $x_m = 500$, $y_m = 500$, $x \in [-x_m, x_m]$, and $y \in [-y_m, y_m]$. For a better approximation, we refer readers to [4]. To use the real mobility trace data of Piorkowski et al. [57], which includes GPS coordinates for trajectories of 537 taxi vehicles, we selected 500 vehicles with the largest trace samples, each has over 8000 sample points. We varied the number of nodes from 50 to 500 in increments of 50.

4.3 Maximum Interference Results

When simulating Algorithm LocalRadiusReduction, in both the static and mobile settings, each node collects the list of nodes in its 2-hop neighbourhood in two rounds, applies the algorithm to reduce its transmission radius and then broadcasts its computed transmission radius, allowing neighbouring nodes a final opportunity to eliminate asymmetric edges and further reduce their transmission radii while maintaining connectivity in the network.

We used the random waypoint model (RWP) [58] and real mobility trace data to simulate mobile networks. For the RWP model we applied the approximated probability distribution described by Bettstetter and Wagner [56] to position nodes independently at random across the network and generate independent snapshots in each simulation iteration. Using the mobility trace data, we estimated a probability density distribution which was used to generate independent snapshots.

Comparing the LocalRadiusReduction algorithm against other local topology control algorithms on a simulated static wireless network. Plots for the fixed-radius algorithm are omitted from Figures 4.2–4.13 to allow the remaining plots to be more easily differentiated.

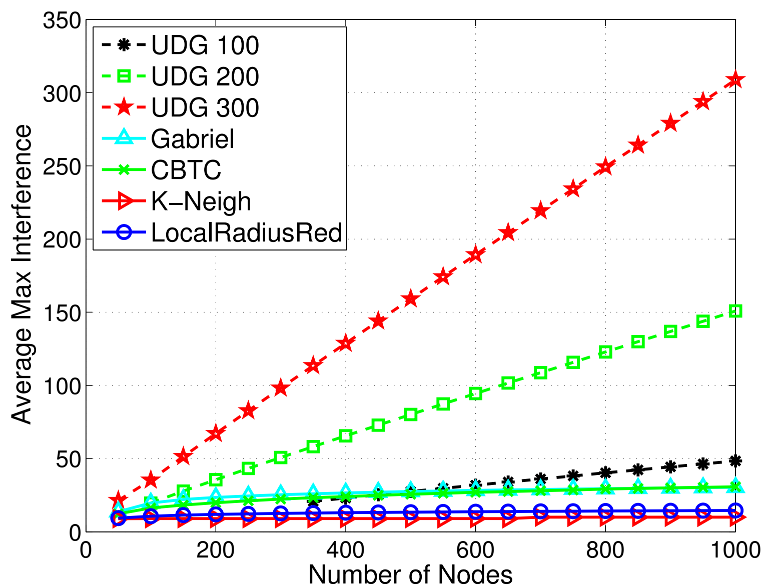

Figure 4.1: Maximum Interference in Static Networks

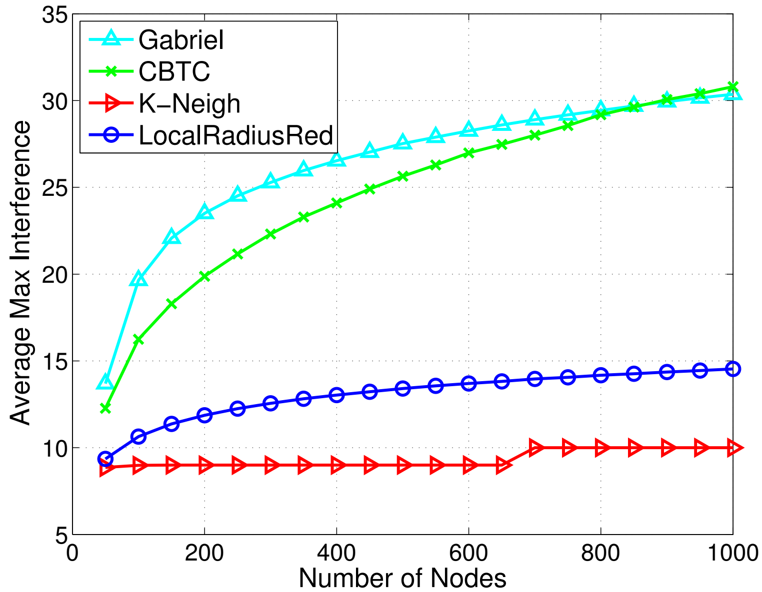

Figure 4.2: Maximum Interference in Static Networks

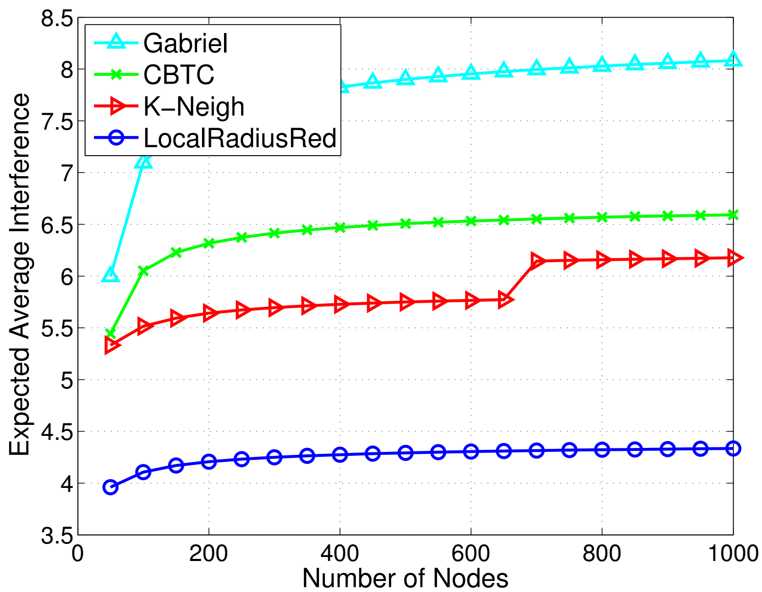

Figure 4.3: Expected Average Interference in Static Networks

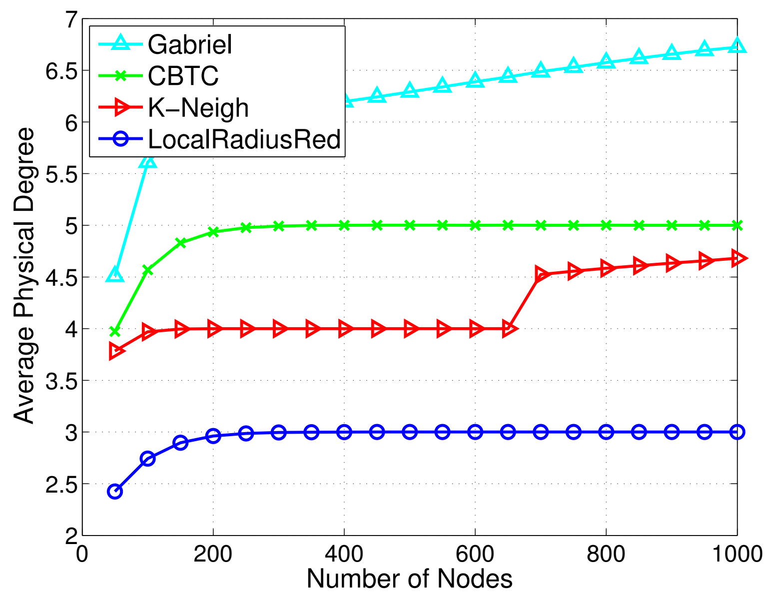

Figure 4.4: Average Physical Degree in Static Networks

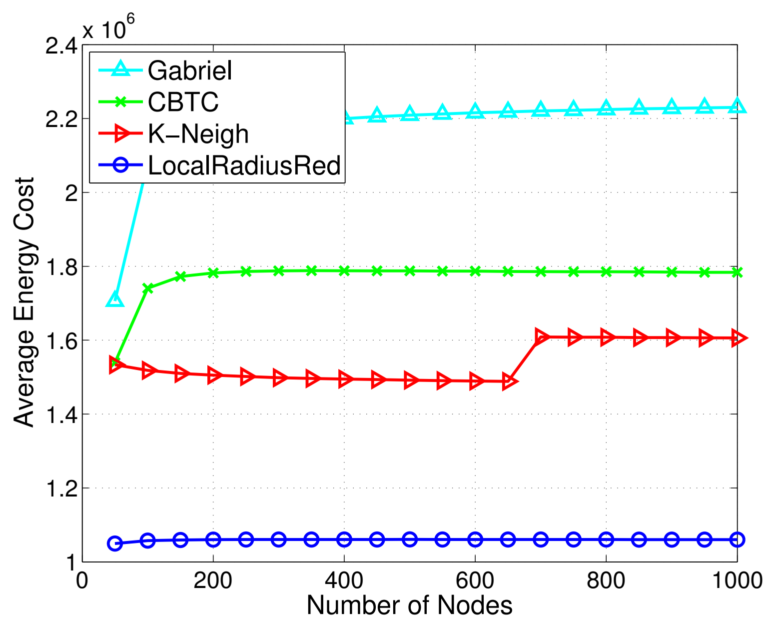

Figure 4.5: Average Energy Cost in Static Networks

As shown, the average maximum interference of unit disc graph topologies increases linearly with n (see Figure 4.1). Since these plots are significantly larger (i.e., they correspond to worse performance) than the other plots in all four evaluation criteria, unit disc graph plots are excluded from subsequent figures to permit more detailed comparison.

Although both the local Gabriel and CBTC algorithms performed significantly better than the unit disc graphs, the lowest average maximum interference was achieved by the LocalRadiusReduction and k-neighbour algorithms, for which the corresponding plots grow logarithmically with n, as seen in Figures 4.2, 4.3, and 4.10. Note that the LocalRadiusReduction algorithm reduces the maximum interference to $O(\log n)$ with high probability, irrespective of the initial maximum transmission radius $r_{\max}$. LocalRadiusReduction has average maximum interference slightly greater than k-neighbour (Figures 4.2, 4.3, and 4.10), but lower expected average interference, average physical degree, and average energy cost (Figures 4.3–4.5, 4.7–4.9, and 4.11–4.13).

Simulation results obtained using a RW model closely match those obtained on a static network because the distribution of nodes at any time during a random walk is nearly uniform [59]. The spatial distribution of nodes moving according to a RWP model is not uniform, and is maximized at the centre of the simulation region [4]. Consequently, the density of nodes is high near the centre, resulting in greater interference at these nodes. Figures 4.3–4.9 are comparing the LocalRadiusReduction algorithm against other local topology control algorithms on a simulated mobile network.

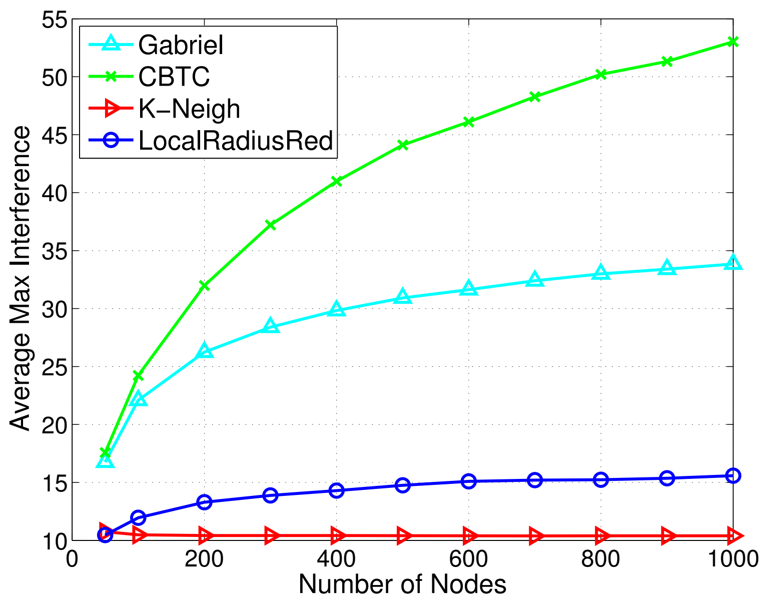

Figure 4.6: Average Maximum Interference in Mobile Networks

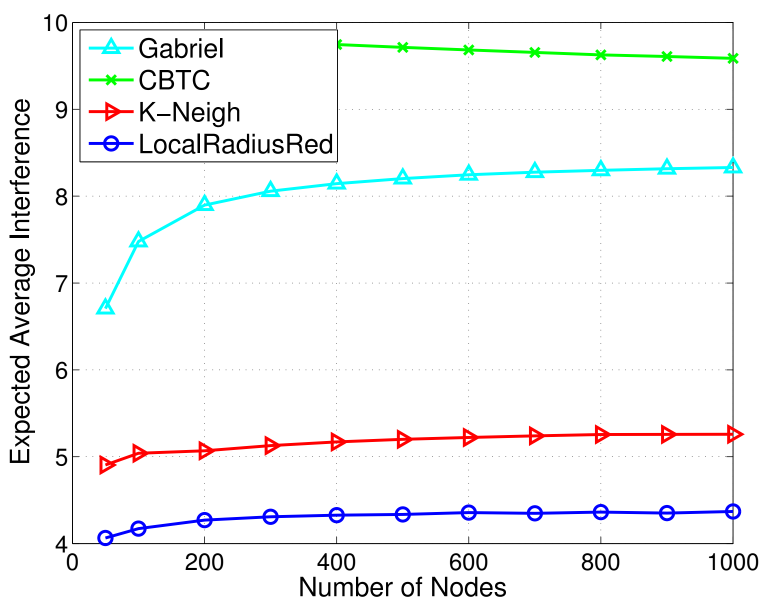

Figure 4.7: Expected Average Interference in Mobile Networks

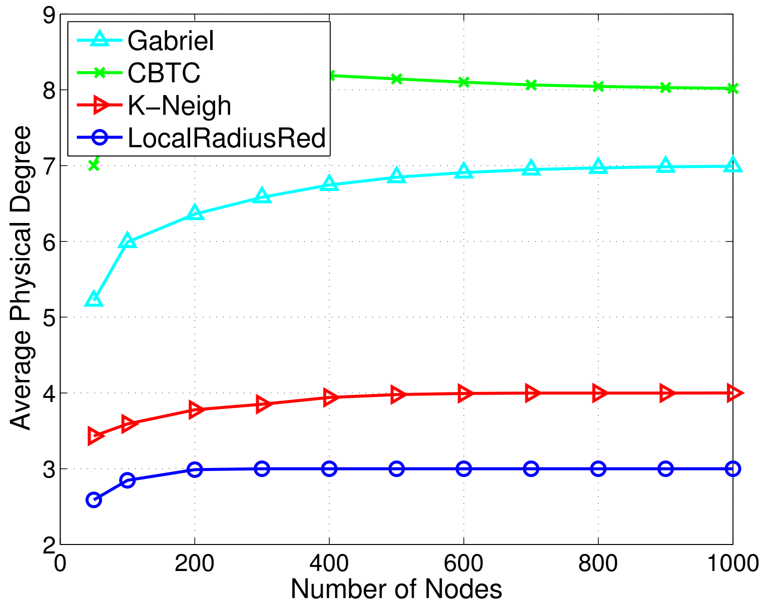

Figure 4.8: Average Physical Degree in Mobile Networks

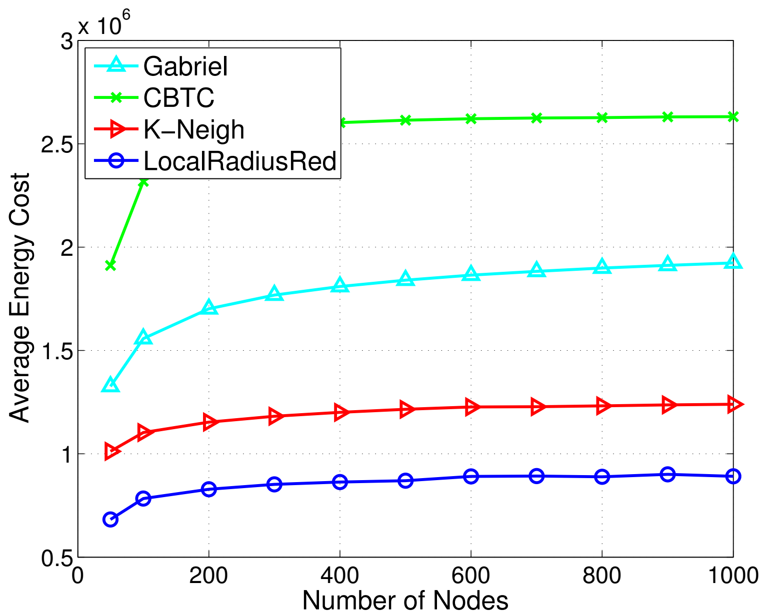

Figure 4.9: Average Energy Cost in Mobile Networks

Finally, we evaluated the algorithm LocalRadiusReduction using real mobility trace data of Piorkowski et al. [57], consisting of GPS coordinates for trajectories of 537 taxi vehicles recorded over one month in 2008, driving throughout the San Fransisco Bay area. We selected the 500 largest traces, each of which has over 8000 sample points. To implement our algorithm, we selected n taxis among the 500 uniformly at random, ranging from $n = 50$ to $n = 500$ in increments of 50. As seen in Figure 4.10, the results are similar to those measured in our simulation. The k-neighbours algorithm produced disconnected communication graphs (containing multiple connected components) in 2.5% of instances, even when the value of k was increased significantly (e.g., up to $k = 30$). This is likely explained by the highly non-uniform distribution of nodes in the track data. This difference is significant, however, because the k-neighbours algorithm does not guarantee that the returned topology is connected, failing to satisfy the primary objective of the interference minimization problem for some input instances.

Figures 4.10–4.13 are comparing the LocalRadiusReduction algorithm against other local topology control algorithms on recorded mobile vehicular GPS tracks.

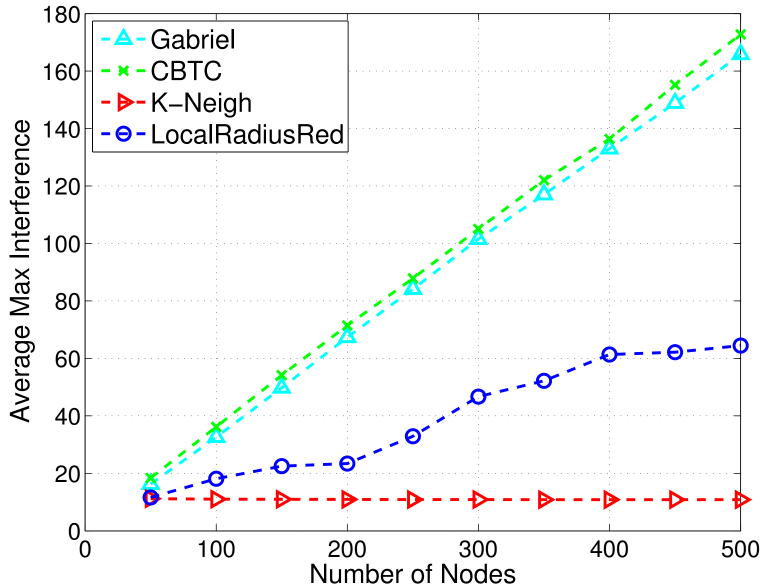

Figure 4.10: Average Maximum Interference in Real-world Networks

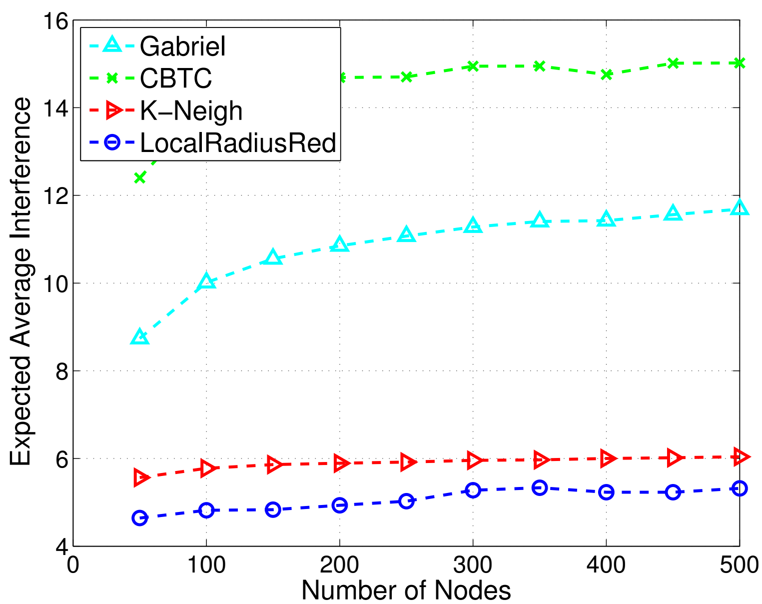

Figure 4.11: Expected Average Interference in Real-world Networks

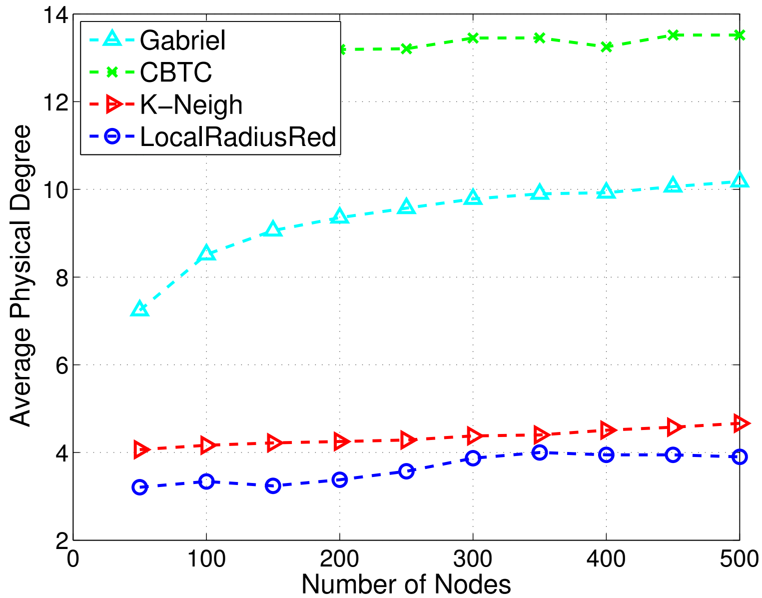

Figure 4.12: Average Physical Degree in Real-world Networks

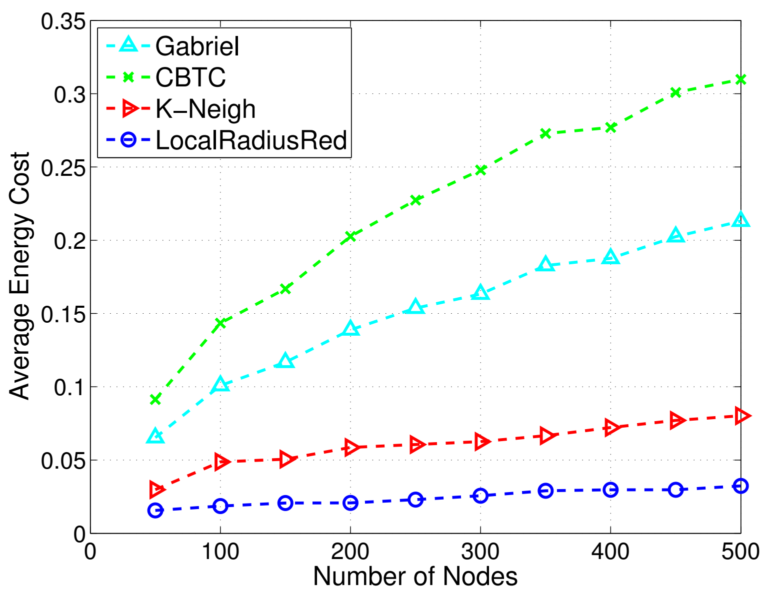

Figure 4.13: Average Energy Cost in Real-world Networks

4.4 Average Interference Results

In this section, we present the result of a simulation study carried out to verify our theoretical results over a network with nodes randomly distributed, and one with a real world node placement. For the first case, the simulation region is considered to be a square with the edge size of 1000 meters. The number of nodes, n, deployed randomly inside region varies from 100 to 1000 in increments of size 100. As the starting step, we construct a MST graph, and determine the transmission range of every node deployed inside the network plane. Then, we measure the corresponding interference at each node, and by that we compute the average interference of the entire network as defined earlier.

To have a better average case analysis, for each n, we generated one million networks, each with n nodes distributed uniformly at random in the region. For each generated network, we store the average interference. Figure 4.14 shows the mean, the maximum, and the minimum of these average interferences for each n.

As shown in figure 4.14, the average interference is bounded by a constant value as predicted by the theoretical analysis. To have a better insight about the distribution of the average interference, we plot its histogram for the case where $n = 500$ in Figure 4.15. This figure also suggests that, with high probability, the average interference is bounded by a constant value (e.g., here it is equal to 4).

We also evaluated the average interference using actual mobility trace data provided by Piorkowski et al. [60]. This data set consists of GPS

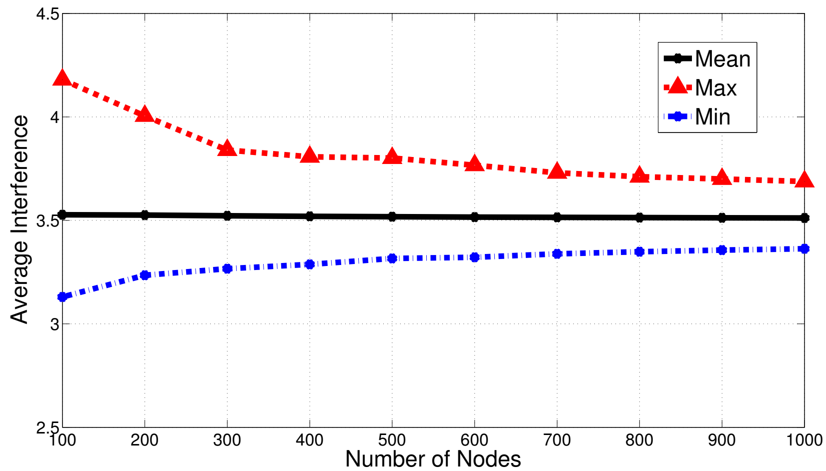

Figure 4.14: Average Interference

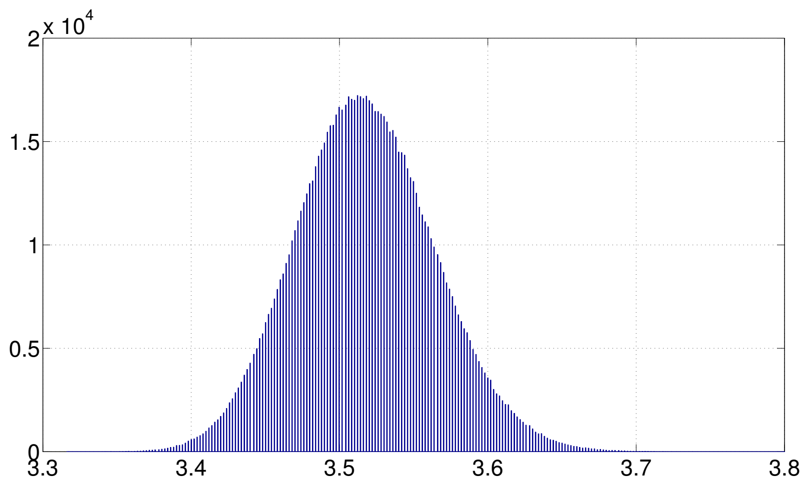

Figure 4.15: Average interference histogram for a network with 500 nodes

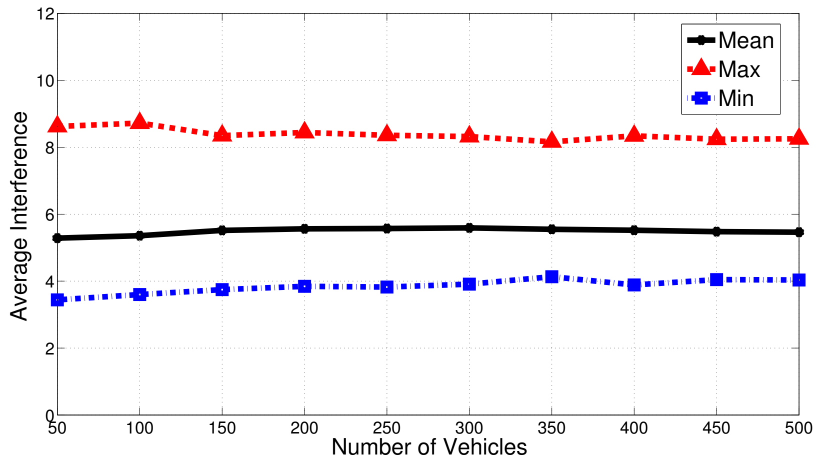

Figure 4.16: Average Interference

coordinates for 537 taxi vehicles recorded over one month in 2008, driving throughout the San Fransisco Bay area. We first select 500 vehicles with the largest trace samples, each of which has over 8000 sample points. To measure the average interference, we selected n taxis among the 500 uniformly at random, ranging from $n = 50$ to $n = 500$ in increments of 50. We considered each snapshot of vehicles placement (totally 8000 snapshots) and assigned a transmission range to each node using MST algorithm.

Next, we calculated the average interference for the nodes in each snapshot. Figure 4.16 shows the mean, the maximum and the minimum of the average interference over all snapshots for every given n. Note that the average interference is slightly higher when our real world placement is used instead of a uniform distribution.

Similar to Figure 4.15, we show the histogram of the average interference for $n = 50$ when real world data traces are used. This is an indication that,

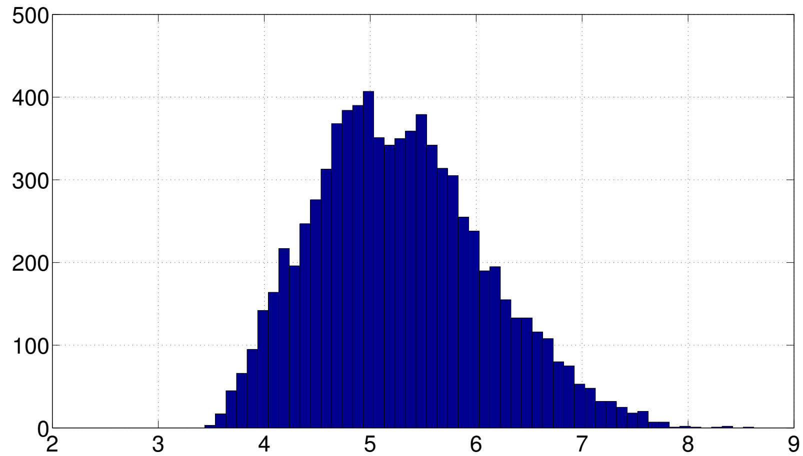

Figure 4.17: Average Interference histogram for 8000 snapshots and 50 Vehicles

in real world scenarios, the average interference can be very small.

Chapter 5

Conclusions and Future Work

We first studied the maximum interference problem in wireless ad hoc networks. Considering a broad set of probability distributions (called class $\mathcal{T}(P)$), we derived a bound for the maximum interference of resulting communication graphs. we also showed that, irrespective of the network size, the average interference in the communication graph corresponding to the minimum spanning tree of the nodes is smaller than a constant, with high probability. This is in contrast to the existing worst case analysis results in which the average interference can be arbitrarily large when nodes are artificially placed in the network.

Using Algorithm LocalRadiusReduction, each node determines its transmission radius as a function of its 2-hop neighbourhood. Alternatively, suppose each node could select its transmission radius at random using a suitable distribution over $[d_{\min}(G), d_{\max}(G)]$. Can such a strategy for assigning transmission radii ensure connectivity and low maximum interference with high probability? Similarly, additional topologies and local algorithms for constructing them might achieve $O(\log n)$ expected maximum interference. For example, our experimental results suggest that both the Gabriel graph and CBTC local topology control algorithms may provide $O(\log n)$ expected maximum interference. Since neither the Gabriel graph nor the CBTC topology of a set of points P is in $\mathcal{T}(P)$ in general, whether these bounds hold remains to be proved.

Several questions remain open related to the algorithmic problem of finding an *optimal* solution (one whose maximum interference is exactly $\mathrm{OPT}(P)$) when node positions may be selected adversarially. The complexity of the interference minimization problem in one dimension remains open; at present, it is unknown whether the problem is polynomial-time solvable or NP-hard [19]. While the problem is known to be NP-complete in two dimensions [17], no polynomial-time approximation algorithm nor any inapproximability hardness results are known. Another interesting future work is to analytically prove this using the existing distributions suggested for dynamic mobile networks. Extending the result of this work to three-dimensional networks, and finding an algorithm that can locally construct a communication graph with small expected average interference are other future work that we intend to study.

Bibliography

[1] L. Li, J. Y. Halpern, P. Bahl, Y.-M. Wang, and R. Wattenhofer, "A cone-based distributed topology-control algorithm for wireless multi-hop networks," *IEEE/ACM Transactions on Networking*, vol. 13, no. 1, pp. 147–159, 2005.

[2] P. Santi, "Topology control in wireless ad hoc and sensor networks," *ACM Computing Surveys*, vol. 37, no. 2, pp. 164–194, 2005.

[3] S. Durocher, E. Kranakis, D. Krizanc, and L. Narayanan, "Balancing traffic load using one-turn rectilinear routing," *Journal of Interconnection Networks*, vol. 10, no. 1–2, pp. 93–120, 2009.

[4] E. Hyytiä, P. Lassila, and J. Virtamo, "Spatial node distribution of the random waypoint mobility model with applications," *IEEE Transactions on Mobile Computing*, vol. 6, no. 5, pp. 680–694, 2006.

[5] M. Benkert, J. Gudmundsson, H. Haverkort, and A. Wolff, "Constructing minimum-interference networks," *Computational Geometry: Theory and Applications*, vol. 40, no. 3, pp. 179–194, 2008.

[6] G. Narasimhan and M. Smid, *Geometric Spanner Networks.* Cambridge University Press, 2007.

[7] M. Burkhart, P. von Rickenbach, R. Wattenhofer, and A. Zollinger, "Does topology control reduce interference?" in *Proceedings of the ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc)*, 2004, pp. 9–19.

[8] X.-Y. Li, G. Calinescu, and P.-J. Wan, "Distributed construction of a planar spanner and routing for ad hoc wireless networks," in *Proceedings of the Annual Joint Conference of the IEEE Computer and Communications Societies*, 2002, pp. 1268–1277.

[9] I. Kanj, L. Perković, and G. Xia, "Computing lightweight spanners locally," in *Proceedings of the Symposium on Distributed Computing (DISC)*, ser. Lecture Notes in Computer Science, vol. 5218. Springer, 2008, pp. 365–378.

[10] P. Bose, J. Gudmundsson, and M. Smid, "Constructing plane spanners of bounded degree and low weight," *Algorithmica*, vol. 42, no. 3–4, pp. 249–264, 2005.

[11] M. Damian, S. Pandit, and S. V. Pemmaraju, "Local approximation schemes for topology control," in *Proceedings of the ACM Symposium on Principles of Distributed Computing (PODC)*, 2006, pp. 208–218.

[12] P. von Rickenbach, S. Schmid, R. Wattenhofer, and A. Zollinger, "A robust interference model for wireless ad hoc networks," in *Proceedings of the IEEE Parallel and Distributed Processing Symposium (IPDPS)*, 2005, pp. 1–8.

[13] P. von Rickenbach, R. Wattenhofer, and A. Zollinger, "Algorithmic models of interference in wireless ad hoc and sensor networks," *IEEE/ACM Transactions on Networking*, vol. 17, no. 1, pp. 172–185, 2009.

[14] T. Locher, P. von Rickenbach, and R. Wattenhofer, "Sensor networks continue to puzzle: Selected open problems," in *Proceedings of the International Conference on Distributed Computing and Networking (ICDCN)*, ser. Lecture Notes in Computer Science, vol. 4904. Springer, 2008, pp. 25–38.

[15] T. Moscibroda and R. Wattenhofer, "Minimizing interference in ad hoc and sensor networks," in *Proceedings of the ACM Joint Workshop on Foundations of Mobile Computing (DIALM-POMC)*, 2005, pp. 24–33.

[16] M. M. Halldórsson and T. Tokuyama, "Minimizing interference of a wireless ad-hoc network in a plane," *Theoretical Computer Science*, vol. 402, no. 1, pp. 29–42, 2008.

[17] K. Buchin, "Minimizing the maximum interference is hard," *CoRR*, vol. abs/0802.2134, 2008.

[18] T. Lou, H. Tan, Y. Wang, and F. Lau, "Minimizing average interference through topology control," in *Proceedings of the International Symposium on Algorithms for Sensor Systems, Wireless Ad Hoc Networks and Autonomous Mobile Entities (ALGOSENSORS)*, ser. Lecture Notes in Computer Science. Springer, 2012, vol. 7111, pp. 115–129.

[19] H. Tan, T. Lou, F. Lau, Y. Wang, and S. Chen, "Minimizing interference for the highway model in wireless ad-hoc and sensor networks," in *Proceedings of the Conference on Current Trends in Theory and Practice of Computer Science (SOFSEM)*, ser. Lecture Notes in Computer Science. Springer, 2011, vol. 6543, pp. 520–532.

[20] M. Khabbazian, S. Durocher, and A. Haghnegahdar, "Bounding interference in wireless ad hoc networks with nodes in random position," in *Proceedings of the International Colloquium on Structural Information and Communication Complexity (SIROCCO)*, ser. Lecture Notes in Computer Science, vol. 7355. Springer, 2012, pp. 85–98.

[21] E. Kranakis, D. Krizanc, P. Morin, L. Narayanan, and L. Stacho, "A tight bound on the maximum interference of random sensors in the highway model," *CoRR*, vol. abs/1007.2120, 2010.

[22] M. Korman, "Minimizing interference in ad hoc networks with bounded communication radius," *Information Processing Letters*, vol. 112, no. 19, pp. 748–752, 2012.

[23] D. Kowalski and M. Rokicki, "Connectivity problem in wireless networks," in *Proceedings of the International Symposium on Distributed Computing (DISC)*, ser. Lecture Notes in Computer Science. Springer, 2010, vol. 6343, pp. 344–358.

[24] D. Bilò and G. Proietti, "On the complexity of minimizing interference in ad-hoc and sensor networks," *Theoretical Computer Science*, vol. 402, no. 1, pp. 42–55, 2008.

[25] A. Sharma, N. Thakral, S. Udgata, and A. Pujari, "Heuristics for minimizing interference in sensor networks," in *Proceedings of the International Conference on Distributed Computing and Networking (ICDCN)*, ser. Lecture Notes in Computer Science. Springer, 2009, vol. 5408, pp. 49–54.

[26] L. Devroye and P. Morin, "A note on interference in random point sets," in *Proceedings of the Canadian Conference on Computational Geometry (CCCG)*, vol. 24, 2012, pp. 201–206.

[27] A. Maheshwari, M. Smid, and N. Zeh, "Low-interference networks in metric spaces with bounded doubling dimension," *Information Processing Letters*, vol. 111, no. 23–24, pp. 1120–1123, 2011.

[28] D. Blough, M. Leoncini, G. Resta, and P. Santi, "The k-neighbors approach to interference bounded and symmetric topology control in ad hoc networks," *IEEE Trans. Mob. Comp.*, vol. 5, no. 9, pp. 1267–1282, 2006.

[29] A. Dessmark and A. Pelc, "Tradeoffs between knowledge and time of communication in geometric radio networks," in *Proc. SPAA*, 2001, pp. 59–66.

[30] E. Gilbert, "Random plane networks," *J. SIAM*, vol. 9, no. 4, pp. 533–543, 1961.

[31] W. K. Hale, "Frequency assignment: Theory and applications," *Proc. IEEE*, vol. 68, no. 12, pp. 1497–1514, 1980.

[32] D. Kotz, C. Newport, R. S. Gray, J. Liu, Y. Yuan, and C. Elliott, "Experimental evaluation of wireless simulation assumptions," in *Proceedings of the 7th ACM International Symposium on Modeling, Analysis and Simulation of Wireless and Mobile Systems*, ser. MSWiM '04. ACM, 2004, pp. 78–82.

[33] D. Kotz, C. Newport, and C. Elliott, "The mistaken axioms of wireless-network research," 2003.

[34] C. Avin, Y. Emek, E. Kantor, Z. Lotker, D. Peleg, and L. Roditty, "SINR diagrams: Convexity and its applications in wireless networks," *J. ACM*, vol. 59, no. 4, p. 18, 2012.

[35] O. Goussevskaia, Y. A. Oswald, and R. Wattenhofer, "Complexity in geometric SINR," in *Proc. MOBIHOC*, 2007, pp. 100–109.

[36] Z. Lotker and D. Peleg, "Structure and algorithms in the SINR wireless model," *SIGACT News*, vol. 41, no. 2, pp. 74–84, 2010.

[37] T. Moscibroda, R. Wattenhofer, and A. Zollinger, "Topology control meets SINR: the scheduling complexity of arbitrary topologies," in *Proc. MOBIHOC*, 2006, pp. 310–321.

[38] T. N. Nguyen, M. K. An, N. X. Lam, and D. T. Huynh, "The complexity of minimizing receiver-based and SINR edge interference," in *Proc. ICCCN*, 2011, pp. 1–7.

[39] M. Burkhart, P. von Rickenbach, R. Wattenhofer, and A. Zollinger, "Does topology control reduce interference?" in *Proceedings of the 5th*

ACM international symposium on Mobile ad hoc networking and computing, ser. MobiHoc '04, 2004, pp. 9–19.

[40] P. V. Rickenbach, S. Schmid, R. Wattenhofer, and A. Zollinger, "A robust interference model for wireless ad-hoc networks," in *In Proc. 5 th IEEE International Workshop on Algorithms for Wireless, Mobile, Ad-Hoc and Sensor Networks (WMAN*, 2005.

[41] M. Fussen, R. Wattenhofer, and A. Zollinger, "Interference arises at the receiver," in *Wireless Networks, Communications and Mobile Computing, 2005 International Conference on*, vol. 1, 2005, pp. 427–432 vol.1.

[42] M. Li, K. Moaveni-Nejad, W.-Z. Song, and W.-Z. Wang, "Interference-aware topology control for wireless sensor networks," in *Sensor and Ad Hoc Communications and Networks, 2005. IEEE SECON 2005. 2005 Second Annual IEEE Communications Society Conference on*, 2005, pp. 263–274.

[43] M. Khabbazian, S. Durocher, A. Haghnegahdar, and F. Kuhn, "Bounding interference in wireless ad hoc networks with nodes in random position," *submitted to Networking, IEEE/ACM Transactions on.*

[44] K. Buchin, "Minimizing the Maximum Interference is Hard," *ArXiv e-prints*, Feb. 2008.

[45] M. M. Halldórsson and T. Tokuyama, "Minimizing interference of a wireless ad-hoc network in a plane," *Theor. Comput. Sci.*, vol. 402, no. 1, pp. 29–42, Jul. 2008.

[46] E. Kranakis, D. Krizanc, P. Morin, L. Narayanan, and L. Stacho, "A Tight Bound on the Maximum Interference of Random Sensors in the Highway Model," *ArXiv e-prints*, Jul. 2010.

[47] M. Khabbazian, S. Durocher, and A. Haghnegahdar, "Bounding interference in wireless ad hoc networks with nodes in random position," in *SIROCCO'12*, 2012, pp. 85–98.

[48] T. Moscibroda and R. Wattenhofer, "Minimizing interference in ad hoc and sensor networks," in *Proceedings of the ACM Joint Workshop on Foundations of Mobile Computing (DIALM-POMC)*, 2005, pp. 24–33.

[49] H. Tan, T. Lou, F. C. M. Lau, Y. Wang, and S. Chen, "Minimizing interference for the highway model in wireless ad-hoc and sensor networks," in *Proceedings of the 37th international conference on Current trends in theory and practice of computer science*, ser. SOFSEM'11. Springer-Verlag, 2011, pp. 520–532.

[50] J. Heinonen, *Lectures on analysis on metric spaces.* New York: Springer-Verlag, 2001.

[51] A. Gupta, R. Krauthgamer, and J. Lee, "Bounded geometries, fractals, and low-distortion embeddings," in *Proceedings of the IEEE Symposium on Foundations of Computer Science (FOCS)*, 2003, pp. 534–543.

[52] P. Fraigniaud, E. Lebhar, and Z. Lotker, "A doubling dimension threshold $\theta(\log \log n)$ for augmented graph navigability," in *Proceedings of the European Symposium on Algorithms (ESA)*, ser. Lecture Notes in Computer Science, vol. 4168. Springer, 2006, pp. 376–386.

[53] M. Khan, G. Pandurangan, and V. S. A. Kumar, "Distributed algorithms for constructing approximate minimum spanning trees in wireless sensor networks," *IEEE Transactions on Parallel and Distributed Systems*, vol. 20, no. 1, pp. 124–139, 2009.

[54] P. Wan, G. Călinescu, X. Li, and O. Frieder, "Minimum-energy broadcasting in static ad hoc wireless networks," *Wireless Networks*, vol. 8, no. 6, pp. 607–617, 2002.

[55] P. Bose, P. Morin, I. Stojmenović, and J. Urrutia, "Routing with guaranteed delivery in ad hoc wireless networks," *Wireless Networks*, vol. 7, no. 6, pp. 609–616, 2001.

[56] C. Bettstetter and C. Wagner, "The node distribution of the random waypoint mobility model for wireless ad hoc networks," *IEEE Trans. Mob. Comp.*, vol. 2, pp. 257–269, 2003.

[57] M. Piorkowski, N. Sarafijanovic-Djukic, and M. Grossglauser, "CRAWDAD data set epfl/mobility (v. 2009-02-24)," http://crawdad.cs.dartmouth.edu/epfl/mobility, 2009.

[58] D. B. Johnson and D. A. Maltz, "Dynamic source routing in ad hoc wireless networks," in *Mobile Computing*, T. Imielinski and H. Korth, Eds. Kluwer Academic Publishers, 1996, vol. 353.

[59] A. Das Sarma, D. Nanongkai, and G. Pandurangan, "Fast distributed random walks," in *Proceedings of the ACM Symposium on Principles of Distributed Computing (PODC)*, 2009, pp. 161–170.

[60] M. Piorkowski, N. Sarafijanovic-Djukic, and M. Grossglauser, "CRAWDAD data set epfl/mobility (v. 2009-02-24)," http://crawdad.cs.dartmouth.edu/epfl/mobility, 2009.