DEVELOPMENT AND EVALUATION OF A MULTI-OBJECTIVE OPTIMIZATION MODEL FOR MULTI-RESERVOIR SYSTEMS

by

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B.Sc. (Civil Engineering), Universidad del Valle de Guatemala, 2008

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

in

THE FACULTY OF GRADUATE AND POSTDOCTORAL STUDIES

(Civil Engineering)

THE UNIVERSITY OF BRITISH COLUMBIA

(Vancouver)

January 2015

Abstract

The BC Hydro and Power Authority is the largest electric utility in the province of British Columbia, Canada. With a generating capacity of more than 12,000 MW, it serves almost 2 million customers in the province. It operates 31 hydroelectric facilities, most of them located in multi-reservoir systems. In order to facilitate the operation of these reservoirs, BC Hydro developed an in-house application called the Operations Planning Tool (OPT), a deterministic Linear Programming (LP) model that provides the optimal operation of the multi-reservoir systems considering multiple purposes. The objective of this research was to investigate, develop, incorporate and test additional modeling features that would expand the current capabilities of the OPT. This included developing a formulation for the analysis of units' maintenance outages and changing the optimization model to consider inflow uncertainty and avoid the use of weight coefficients and penalty functions.

The formulation developed for the analysis of units' maintenance outages is based on a two-stage algorithm. In the first stage, a pre-processor defines all the possible outage solutions given some initial configurations. In the second stage, a modified OPT model is run to find an outage solution that optimizes the objectives using a Mixed-Integer Linear Programming (MILP) algorithm. The formulation was tested using the *Bridge River* system in British Columbia, Canada.

An alternative OPT model was also developed to consider the uncertainty in the reservoir's inflow and modify the formulation of the objective function. It was desired to avoid the use of weight coefficients and penalty functions due to the limitations that they present. The proposed alternative was based on the development of a linear decision rule and the use of chance constraints. The linear decision rule is an operating rule that defines the spillway releases

and forebay elevation as a linear function of the inflow, the turbine releases and a deterministic decision variable. The chance constraints were used to consider the probability of the spillway releases and forebay elevation not being within a preferred range of values established by the user. The developed formulation was tested using the *Stave Falls* system.

Preface

The work presented in Chapter 3 is based on the development of new modeling features for the Operations Planning Tool (OPT). This is an application originally developed by the BC Hydro and Power Authority (BC Hydro) which is currently being expanded by a research team of the University of British Columbia. The author worked on the analysis and formulation of the two-stage algorithm and carried out the case study described in that chapter. Jiyi Zhou collaborated in the early stages of the algorithm formulation and tested it with other case studies. Professor Ziad Shawwash was the leader of the research team and was involved throughout the project. The author is currently working in the publication of this material in an academic journal.

The work presented in Chapter 4 is based on an alternative model for the Operations Planning Tool proposed and developed independently by the author. A version of this chapter was presented at the 11th International Conference on HydroInformatics in New York (August 2014). This material was reviewed by Prof. Shawwash and by Alaa Abdalla, Paul Vassilev, Gillian Kong and Vladimir Plesa, operation planning engineers from BC Hydro and current users of the OPT application.

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List of Abbreviations

AMPL	Applied Mathematical Programming Language
BC	British Columbia
BC Hydro	The British Columbia Hydro and Power Authority
DP	Dynamic Programming
GOM	Generalized Optimization Model
GP	Goal Programming
GPF	Generation Production Function
GS	Generation Station
GUI	Graphical User Interface
LDR	Linear Decision Rule
LP	Linear Programming
Mil.	Million
MILP	Mixed Integer Linear Programming
MS	Maintenance Scheduling
MW	Megawatt
MWH	Megawatt-hour
OPE	Operation Planning Engineer
OPP	Operation Planning Platform
OPT	Operation Planning Tool
PFQ	Penalty function for spillway releases
PFS	Penalty function for storage

Acknowledgements

I would like to express my sincere gratitude to my advisor, Professor. Ziad Shawwash, for the great opportunity of coming to UBC, working with him and learning from his vast experience.

I would like to acknowledge the BC Hydro and Power Authority (BC Hydro) and the Natural Sciences and Engineering Research Council of Canada (NSERC) for the technical and financial support received during the course of this research.

I am also grateful to all the people in BC Hydro who helped me in many different ways, but I extend my special thanks to Alaa Abdalla, Paul Vassilev, Amr Ayad, Susan Burton and Ziming Guan. I am very thankful to my colleagues and friends Jennifer Schaffer, Jiyi Zhou and Ryan Rasmussen. I also thank Professor Gregory Lawrence for the final revision of this thesis.

I extend a special thanks to all the friends that I found in Vancouver, especially in the Latin American community at UBC, in the Christian community at University Chapel and in St. Andrew's Hall residence.

Finally, I would like to thank my family. I was lucky to have my brother Manuel in Vancouver; things were much easier with him here. I also enjoyed the support, love and encouragement from my family in Guatemala: my parents Manuel and Martha, my sister Janet and my brother-in-law Pablo. The newest member of my family was also a great source of joy, thus I feel especially grateful for my beautiful niece Sara Gabriela, who I hope to meet in person soon.

To the only wise God our Saviour, be glory and majesty, dominion and power, both now and ever. Amen.

Jude 1:25

Chapter 1: Introduction

1.1 Background

Hydropower can be defined as the power derived from the energy of falling or running water. It was first used to generate electricity at the end of the nineteenth century and since then the hydropower generation capacity worldwide has been growing steadily: by the end of 2008 it was estimated that hydropower contributed 16% of world's electricity generation, becoming the largest renewable source of electricity (Kumar et al., 2011). In Canada, the construction of the first large hydroelectric projects dates back to the early 1890s but it was during the 1960s and 1970s when these constructions peaked. Today, Canada is one of the largest producers of hydroelectricity of the world, and more than 60% of the electricity produced in the country comes from hydropower generation (Helston, 2012).

1.2 Description of the Operations Planning Tool Model

For many decades, the single objective of hydropower generation was to satisfy the electricity demand in a sustainable manner. Nevertheless, a growing public concern over the impacts of hydroelectric projects has motivated electric utilities to incorporate additional objectives in its operation, including flood control, recreation, environmental protection, conservation of cultural and archaeological sites and the supply of water for human consumption and irrigation. The optimal operation of a reservoir serving multiple purposes is complicated when some of these objectives are in conflict with each other, especially when the uncertainty associated with future hydrologic conditions is considered. In addition, this operation becomes more complex in systems with multiple reservoirs located along the same water course (Labadie, 2004).

Such is the case of British Columbia in Canada, where almost 70 per cent of the installed generating capacity in the province comes from multiple reservoirs located in just two river basins, The Peace River and the Columbia River, while an additional 16% comes from smaller multi-reservoir systems located in other river basins (BCHydro, 2000). In 1996 the government of British Columbia approved the Water Act and it required the BC Hydro and Power Authority, the Crown Corporation in charge of generation and distribution of electricity in most of the province, to undertake a review process of the operation of all its hydroelectric facilities. In agreement with different government agencies, First Nations, local citizens and other stakeholders, BC Hydro developed a water use plan for each of the hydroelectric projects, which resulted in changes in the operations of their facilities, including new flow regimes for turbine and spillway discharges as well as in the forebay and tailwater elevation (BChydro, 2014).

In order to fulfill these agreements and the original objective of satisfy the growing electricity demand, BC Hydro developed an in-house computer application called the Operations Planning Tool (OPT). The purpose of this application is to aid the Operation Planning Engineers (OPEs) to make decisions regarding the operation of a multi-reservoir system. It consists of three main components: the graphical user interface (GUI), the optimization model and the solver software. The GUI allows the user to configure the optimization study, change model configurations, run the optimization and retrieve and display the output data. The optimization model is formulated in AMPL, a software package commonly used in mathematical programming, while the CPLEX solver is used to solve the optimization problem. The GUI is used to configure the study and prepare input data in the user's workstation; the problem is sent and solved at the server workstation where the AMPL and CPLEX solver resides and then the solution is sent back to the client workstation.

1.2.1 The OPT Optimization Model

The OPT optimization model has a deterministic Linear Programming (LP) algorithm. It is deterministic because it does not consider any uncertainty in the reservoir inflows and electricity prices; both parameters are inputs that must be specified by the user. The LP model can be divided into three basic components: (1) the decision variables, (2) the objective function and (3) the model constraints. A brief description of each of these components is presented in the following sections.

Decision Variables

The major decision variables of the model are the following:

$S_{p,t}$	Storage of reservoir *p* at the time step *t*
$spilQ_{p,n,t}$	Spillway release from reservoir *p*, through the release structure *n*, at time step *t*
$turbQ_{p,t}$	Turbine release from reservoir *p* at time step *t*
$turbQZone_{p,t,z}$	Turbine release from reservoir *p*, at time step *t* , during sub-time step *z*
$G_{p,t,z}$	Power generation at reservoir *p*, at time step *t* , during sub-time step *z*

It can be observed that there are two variables related to turbine releases. The first variable ($turbQ_{p,t}$) is the average turbine flow during the time step t, while $turbQZone_{p,t,z}$ is the average turbine flow during a shorter time step z (sub-time step) defined by the user. These sub-time steps depend on the variability of the electricity prices within a time step. For example, a typical day the user can define two price zones: Heavy Load Hours (HLH) for the heavy load hours, and Light Load Hours (LLH) for the rest of the day. The electricity prices in these two zones are different and therefore it is of interest to know the average turbine release in each of them.

The Objective Function

The OPT multi-objective problem is solved making use of an optimization method known as the Weighting Method of Multi Objective Optimization, where a grand objective is established adding all the individual objectives, each one multiplied by a weighting coefficient as follows:

$$Minimize\ \{W_S * \sum_{p,t} PFS\left[S_{p,t}\right] \ + \ W_Q * \sum_{p,n,t} PFQ\left[spilQ_{p,n,t}\right]$$
$$- W_R \ * \sum_{p,t,z} G_{p,t,z} * \Delta \mathrm{t} * priceZoneFraction_{t,z} * price_{t,z}\} \qquad (1.1)$$

The first two terms of the objective function refer to the minimization of the storage and spillway deviations from some preferred operating regimes, respectively. The minimization is accomplished through the use of “penalty functions” denominated PFS for storage and PFQ for spill. These functions, which must be defined by the user, are piece-wise linear curves where a penalty number is assigned to all the possible values that the elevation and the spillway releases can have. When the values are within the preferred range a penalty value of zero is produced, while those outside of the target range must result in a non-zero penalty. The farther a value is from the target, the greatest its penalty will be. The preferred ranges are based on the agreements of the Water Use Plans for each multi-reservoir system. The determination of the corresponding penalties and their unit changes (slopes) may be complicated and it could require a detailed analysis (Can and Houck, 1983).

The third term of the objective function refers to the maximization of the revenue from power generation. Due to the negative sign of this term, the model performs a maximization of the revenue even though the objective function as a whole is being minimized. In other words, because the model tries to make this term as negative as possible, the absolute value of the revenue will also be the highest possible. This revenue is computed for each sub-time step, multiplying the power generation, the number of hours in each sub-time step (Δt* *priceZoneFraction*) and the corresponding price.

The weighting coefficients W_S, W_Q and W_R are input parameters whose values should be based in the priorities of the decision makers. Nevertheless, it is a common practice in this optimization method to run the model several times varying these coefficients until the set of non-inferior solutions is generated (Revelle et al, 2004).

Model Constraints

The constraints in the OPT model are used to fulfill the following purposes:

- Satisfy the continuity equation (reservoir mass balance).
- Set limits to the optimization variables.
- Relate the total turbine release per time step ($turbQ_{p,t}$) with the turbine release in the sub-time steps ($turbQZone_{p,t,z}$).
- Calculation of the power generation.

1.3 Research Goals

There are two main goals in this research project.

- Develop and introduce analysis tools in the OPT application in order to assess the impact of units' maintenance outages in multi-reservoir systems. This includes allowing the user to specify fixed outages and letting the model define the optimal reservoir's operation. It is also desired to determine the outage schedule that optimizes the model's original objectives.
- Investigate, analyze and modify the OPT's AMPL optimization model in order to achieve the following specific goals: (1) consider the uncertainty in the reservoirs' inflows (2) avoid the use of penalty functions for the storage and spillway deviations and (3) avoid the use of weight coefficients in the objective function.

1.4 Organization of this Thesis

This thesis is organized into five chapters and one appendix. This chapter provides a brief description of the OPT application, the research goals and the thesis organization. **Chapter 2** presents a summary of the literature review carried out for this research. **Chapter 3** addresses the first research goal and describes the development of a two-stage algorithm for the analysis of maintenance outages in multi-reservoir systems. The algorithm is tested with the *Bridge River* system. The second research goal is considered in **Chapter 4** where *chance constraints* and *linear decision rules* are introduced in the model. The proposed modification is tested with the *Stave Falls* system. **Chapter 5** provides a summary of the results, conclusions and recommendations for future research work. **Appendix A** presents the derivation of the generalized linear decision rule for multi-reservoir systems that forms the basis of the model developed in Chapter 4.

Chapter 2: Literature Review

This chapter is divided in two parts. The first part provides an overview of formulations proposed by different authors for the maintenance scheduling of generating units. The second part reviews different optimization techniques that have been used in the operation of single reservoirs and multi-reservoir systems where multiple objectives were considered.

2.1 Optimization of Maintenance Scheduling in Generating Units

The optimal scheduling of preventive maintenance outages in generating units can constitute a complex and time-consuming problem. Several optimization techniques have been proposed for that purpose, including heuristic methods, integer and mixed integer programming, dynamic programming, goal programming, decomposition methods, among others. This section provides a review of several optimization models proposed by different authors using the techniques listed above.

Yamayee et al. [1983] addressed the optimal maintenance scheduling problem using a dynamic programming (DP) model that minimized operating costs and maintained an acceptable level of reliability in the system. They defined a total cost function that considered in each stage of the problem both the production costs and the cost of unreliability. The constraints included in the model considered crew and resource limitations, and also made sure that a minimum level of reliability for each stage was maintained. The authors pointed out that the major difficulty of DP is the "curse of dimensionality"; in order to overcome this they proposed the use of a DP successive approximation (DPSA). In this approach the problem was solved iteratively; in each iteration a different subset of variables were optimized while the remaining decision variables and their associated states were kept constant. They tested the proposed formulation in a predominantly coal burning utility comprised of 21 units.

Edwin and Curtius [1990] presented an optimization model that minimized the expected annual production costs using an integer programming algorithm. They considered a time horizon of one year subdivided into 52 increments of one week. The starting dates of the maintenance outages were the independent variables in the model. The annual production cost was defined as the sum of the weekly production costs, which were calculated by a stochastic simulation algorithm for power system operation. The model was first formulated as nonlinear integer programming model, but it was transformed into a linear model introducing a new binary variable denominated “combination variable”. A constraint was used to link these variables with the independent variables. Additional constraints were used to only allow one maintenance outage during the scheduling horizon, to restrict the allowed time for the occurrence of the outages and to avoid simultaneous outages. The model was tested using a thermal power system with 15 units.

Yellen et al. [1992] also proposed an optimization model that minimizes the total operating cost over the operational planning period, subject to unit maintenance and system constraints. The total operating costs consisted of two components, the maintenance costs and fuel cost over the planning period. The unit maintenance constraints considered the maintenance time interval, crew constraints and resource constraints. The systems constraints required that the available units for each week met the load requirements and the reliability of power supply for that week. The model was decomposed into a “master problem” and an operating subproblem. The master problem, which is modeled as a mixed integer programming problem, was solved to generate a trial solution for the decision variables. After these variables were fixed, the set of operating subproblems were solved using the fixed maintenance schedule obtained in the master problem. The model was run multiple times, and in each iteration the constraints generated in the subproblems were added to the master problem. The process was finalized when a feasible solution was found whose cost was sufficiently close to a lower bound previously established. The method was tested on a sample system consisting of five units over two different planning periods, a three months horizon and a one year horizon.

Kothari and Ahmad [1993] pointed out that in order to define an optimal maintenance schedule the operators' experience should be taking into account in the optimization. To achieve this, they developed a hybrid method where the scheduling output of a dynamic programming model was enhanced with a rule based expert system. The system guided the user adjusting the constraints, limiting their number and therefore ensuring a feasible solution within acceptable time limits. The rules included in the system were obtained from a survey of the literature and therefore are based on heuristics. The proposed method was tested in a thermal system with ten generating units of different capacities.

Carpentier et al. [1996] developed a stochastic decomposition method considered by the authors to be well-suited to deal with large scale unit commitment problems. They modeled future random disturbances through scenario trees, defined as scenarios paths organized in trees. The optimization consisted in minimizing the average generation cost over this "tree-shaped future". The authors then used an augmented Lagrangian technique to obtain a spatial decomposition algorithm. Therefore, for each generating unit, at each iteration, a stochastic dynamic programming problem had to be solved. In order to avoid combinatorial explosion of computations, the authors pointed out that the scenarios trees had to be kept simple. The proposed method was tested with a simplified model of the thermal generation system Électricité de France. This system consists of 50 generating units: 30 nuclear plants and 20 fossil fuel plants.

Muñoz and Ramos [1999] proposed a goal programming methodology to find the optimal maintenance scheduling of thermal generating units. The method was based on a sequential optimization process of economic and reliability objectives. In the first optimization run, the objective was the minimization of operating costs, which included fuel costs, startup costs, storage costs of fuel stocks and some penalties for non-served power, interruptibility and reserve margin defect. The second optimization minimized the sum of the differences between the thermal reserve margins of consecutive periods. The reserve margin was calculated dividing the available thermal capacity by the period peak load. The constraints considered in the model includes constraints to limit the maximum number of units in simultaneous outages, fuel scheduling constraints, reserve margin and generation-demand balance constraints. The methodology was tested with a simplified model of the Spanish electric power system.

Tang [2007] proposed a mixed integer linear programming model to solve the maintenance scheduling problem in large scale hydroelectric systems. A set of power and non-power constraints were introduced into an existing LP model named the Generalized Optimization Model (GOM). This model is used by operation engineers at BC Hydro to make planning and operational decisions in hydroelectric systems. The modified model reduced the number of binary variables in the model and simplified the computational process transforming nonlinear constraints into linear ones.

Perez [2007] addressed the maintenance scheduling optimization applying the Benders' decomposition method. The proposed model minimized three types of costs: start-up cost, which is the cost to put a generator into operation after being disconnected, production cost and maintenance cost. Four sets of constraints were included in the model: maintenance constraints, economic unit commitment constraints, maintenance and connection constraints and generating volume constraints. The model was decomposed into a master problem and a subproblem, and they were run iteratively until a convergence was reached. A high dimensioned power system was used to test the model, considering 75 power plants. The convergence was reached after 41 iterations.

Alhardi [2008] developed an integer linear programming model that optimizes the preventive maintenance schedule in a multi co-generation plant, which produced both electric power and desalinated water. The objective of the model was to maximize the available number of operational units in each plant. The constraints included in the model took into account the time that the equipment can be operating without maintenance, the amount of resources available and the maximum number of units that can be taken down for maintenance at the same time, due to manpower limitations. It was also considered that the outages had to be carried out without interruptions. The model was tested with two cogeneration plant from Kuwait, each one consisting of seven units, for a time horizon of 52 weeks.

2.2 Review of Optimization Models for Multi-Purpose, Multi-Reservoir Systems

2.2.1 Introduction

The use of mathematical programming in reservoir systems dates back to the early developments of the discipline. Since the early 1960s, many optimization models have been proposed for planning purposes and real time operations. The algorithm of each of these models depends on the characteristics of the reservoirs being analyzed, on the availability of information and on the constraints and the objectives being optimized (Yeh, 1985). The following sections review several techniques that have been proposed for the optimization of multi-purpose and multi-reservoir systems. The purpose of this review is not to conduct a detailed review of multi-objective optimization methods used in reservoir operations; rather, it aims is to find a formulation that could be incorporated into the current OPT model in order to address the inflow uncertainty and avoid the use of penalty functions and weight coefficients.

2.2.2 Linear Programming Models

Thomas and Revelle [1966] were one of the first to propose a Linear Programming model to optimize the operation of a multi-purpose reservoir. Their goal was to find the optimal reservoir policy for the Aswan High Dam reservoir in Egypt considering the water demand for hydropower generation and irrigation. Water released from the reservoir was used to generate electricity and then diverted to the irrigation system; nevertheless, while the demand for hydropower generation was relatively constant throughout the year, the amount of water needed for irrigation was higher during the warm months of the year. Assuming a mean annual inflow of 80 billion cubic meters, they developed a LP model where the variables were the monthly releases and the total water allocated to both demands per year. The objective function was to maximize the benefits from the two yearly allocations multiplying each by a preference coefficient value. They used coefficients values proposed by the Aswan Regional Development Project to run the model and find the optimal releases and yearly allocations.

Becker and Yeh [1974] developed an optimization model for the operation of multi-reservoir systems using a linear programming – dynamic programming (LP-DP) algorithm. The LP formulation determined the optimal reservoir releases and storage states for each period minimizing the potential energy losses. This was carried out for all the alternative paths from the storage vectors corresponding to the feasible incremental energy level of period i to any of the incremental energy levels of periods $i+1$. The DP algorithm was used to select from among these alternatives. The model was tested using the Shasta and Trinity sub-systems of the California Central Valley project.

Shawwash et al. [2000] developed a LP model called the Short Term Optimization Model (STOM) for BC Hydro. The objective function of the model had three terms: maximization of revenue from spot energy transactions, maximization of the added storage value in the reservoirs and minimization of the cost of thermal generation. The storage value was calculated multiplying the difference between the optimized storage and the target storage by the marginal value of water. The plant generation was calculated using a family of piecewise linear curves that calculate power generation as a function of the forebay level, turbine discharge and unit availability. These curves were defined using a computer application called Static Plant Unit Commitment (SPUC), which uses a Dynamic Programming (DP) algorithm to tabulate the optimal plant discharges for each increment in plant loading, forebay and unit availability. Additional constraints were added to make sure that the hourly domestic load was met, and that the total discharge from a reservoir was within the legal non power requirements.

Kiczko and Ermolieva [2012] proposed a multiple criteria decision support system for the Siemianówka reservoir on the Narew River, Poland.   The optimization of this decision support system considered different objectives, including wetland demands, flood protection, irrigation, demands for fisheries and energy production. The objective function aimed to minimize the weighted sum of several cost functions. The stochastic character of the downstream and upstream inflows was included in two different ways. In the first alternative the model was run for all forecasted sequences and the average weighted sum of the cost functions was minimized;

in the second alternative the averaging was directly applied to the forecast sequences. Both solutions were compared with the results of a “perfect” forecast (real discharge data) and the first alternative yielded better results than the second.

2.2.3 Linear Decision Rules Models

Revelle et al. [1969] formulated a Linear Programming model in order to find the optimal capacity of a multi-purpose reservoir. These purposes included water supply, recreation, flood control and waste dilution. They proposed the use of a “linear decision rule” (LDR), which is a linear equation that specifies the reservoir releases as the difference between the storage and a decision parameter called “*b*”. This rule can be interpreted as an aid for the reservoir operators to help them to fulfill storage and water release commitments with the different users of the reservoir. They combined the LDR with the continuity equation to express both the releases and storage in terms of the inflow and the decision parameter “*b*”. Then, they substituted these equations in the release and storage constraints of the model, while the objective was to minimize the capacity of the reservoir. The model was solved considering both deterministic and stochastic inflows. For the first approach, the inflow was considered to be known in advance. In the stochastic approach, the inflows in a particular period were not specified and were considered to be known with only some probability. This complication was solved making use of a chance-constrained formulation. In both solutions, the output of the model was the reservoir capacity and the value of the twelve decision parameters (one for each monthly time step).

Subsequently, many other authors have analyzed the use of LDR and chance constraints in reservoir systems optimization, including the operation of existing reservoirs. Revelle and Kirby [1970] applied the LDR to a reservoir with significant evaporation losses. Nayak and

Arora [1971] extended the LDR to a multi-reservoir system in the Minnesota River System. Similar to the original formulation developed by Revelle et al., their objective was to minimize the storage capacity of the reservoirs. They considered different requirements, including minimum and maximum water releases, minimum reservoir level and minimum freeboard capacity. Loucks and Dorfman [1975] proposed a general LDR formulation by introducing a coefficient called λ to indicate the extent to which the current inflow is considered in the water release during the same time step. Their objective was to estimate the minimum reservoir storage capacity and the summer storage volume required for various monthly release targets. They tested the formulation using values of 0 and 1 for λ and found that the model yields more conservative results when λ is equal to 1. Nevertheless, using a simulation program they concluded that both rules specified a storage capacity greater than was actually needed to meet the reliability levels specified by the chance constraints. Sreenivasan and Vedula [1995] applied a LDR to an existing reservoir in South India, aiming to find the maximum hydropower generation while meeting some irrigation demands. They used a linear approximation to overcome the nonlinearity in the power production function.

The LDR and chance constraints are intuitively appealing and simple to apply in practice. They consider the uncertainty in the inflow and can be applied to multi-reservoirs systems with multiple objectives; therefore this formulation can be a viable alternative for the OPT model. Nevertheless, some authors have been critical of the results obtained with LDR models. Stedinger et al. [1984] examined several of these models and suggested that the use of LDR policies does not result in efficient reservoir system operation. He pointed out that in highly constrained complex situations LDR models can indicate that some targets are not achievable when they really are.

2.2.4 Goal Programming Models

Goal Programming (GP) is an extension of Linear Programming that was first introduced by Charnes and Cooper [1961]. It aims to overcome the limited ability of LP models to effectively address problems involving multiple objectives. Given a target value for each of the multiple objectives, this optimization technique minimizes undesirable deviations from the set of targets (Ignizio, 1985). There are three major variants of GP: Lexicographic, Weighted and Chebysnev Goal Programming. The main characteristic of the Lexicographic variant, also termed preemptive GP, is the existence of a number of priority levels. The minimization of the deviational variables placed in a higher priority level is considered infinitely more important than that of deviational variables placed in lower priority levels. In the Weighted variant, also termed non-preemptive GP, a direct trade-off between all different deviational variables is allowed by placing them in a weighted, normalized single objective function. In the Chebysnev variant, the maximum deviation from any goal, as opposed to the sum of all deviations, is minimized. (Jones and Tamiz, 2010)

Several authors have studied the application of GP to reservoir operations. Can and Houck [1983] developed a preemptive GP model to optimize the operation of four reservoirs in the Green River Basin system in Kentucky, USA. In the preemptive GP approach, in addition to the target values the decision-maker is also required to assign priorities to the goals. They first presented an LP formulation where the objective was to minimize storage and flow penalties over the operating horizon; these penalties were defined by piecewise linear convex functions. Then, they proposed the GP alternative formulation using the penalty functions as a basis to rank the goals. Both optimizations were carried out, and the “quality” of the operation was compared using the penalties such that the operation that produced a smaller total penalty over an extended period would be considered the best. They concluded that the operations recommended by the two models were close to each other, although in some cases the GP model performed better than the LP model. The authors pointed out that the GP model has the advantage that the decision-makers are not forced to assign numerical weights to the flow and storage zones, although they pointed out that it may be necessary to assign weights to the goals with the same priority.

Loganathan and Bhattacharya [1990] analyzed several GP variants in the operation of the same multi-reservoir system studied by Can and Houck [1983]. In addition to the three main GP variants, they considered Fuzzy and Interval Goal Programming. In the Fuzzy GP alternative, membership functions were used to model the imprecision in the targets, while in Interval GP the model attempted to keep the objective function within some lower and upper bounds. In this study, they used a computer program called ADBASE to find the efficient corner points of the multi-objective problem. The program found 12 efficient points, and three of these points corresponded to the solutions of the Lexicographic, Weighted and Chebysnev formulations.

Al Mamun [2012] used a GP formulation to the Columbia River Treaty Model (CRTM) developed by BC Hydro, in order to consider fisheries requirements in the operation of the Columbia River reservoirs. The original CRTM considered only flood protection and hydropower generation as model objectives. The new fisheries requirements included flow augmentation to facilitate the migration of the salmon and to protect the spawning and hatching of whitefish and trout eggs from January to July. A Lexicographic GP formulation was introduced into the model giving a higher priority to the protection of salmon than whitefish, and Trout protection and the maximization of BC Hydro revenue. The results showed that it was possible to meet the fisheries requirements with a high level of satisfaction with a very little impact on the hydropower generation.

2.2.5 Chance-Constrained Goal Programming Models

Changchit and Terrell [1992] developed a model for a three reservoir system in Oklahoma using both chance constraints and a GP formulation. The objectives considered in the model included hydropower generation, flood control, drought control, recreation, releases for municipal and industrial water supply and releases for other uses. They classified these objectives in “deterministic goals” and “probabilistic goals”, and after defining deterministic equivalents for the probabilistic goals, the deviations from the targets were minimized. The deterministic equivalents used cumulative distribution functions for the inflows. The historical records for 36 years were used to define these distributions, and a Kolmogorov-Sminorv goodness-of-fit test showed that lognormal distributions were a good fit to the data.

Abdelaziz and Sameh [2001] also developed a chance constrained GP model for a multi-reservoir system in the north of Tunisia. The model objective was to determine the appropriate reservoirs releases in order to satisfy multiple conflicting objectives, including the minimization of the salinity at the Bejeoua reservoir and the minimization of pumping costs between the Echkel and Bejeoua reservoirs. In addition to the randomness in the inflow, they considered that the drinking water demand was also random. Therefore, they defined cumulative probability distributions for both the inflow and the water demand, and they used it in the deterministic equivalents of the probabilistic goals. The objective function was a weighted minimization of the deviational variables.

2.2.6 Implicit Stochastic Optimization

Some of the models reviewed in this literature survey consider the stochastic nature of inflow, while others assume that the inflows are known with certainty. Nevertheless, it is possible to incorporate the uncertainty of reservoir inflows in a deterministic model by introducing an Implicit Stochastic Optimization (ISO) formulation. The method consists in running the deterministic optimization model with a large number of equally likely inflow scenarios. A different optimal result is obtained for each inflow realization, and the total set of optimal results can be used to develop optimal operation rules by performing a multiple regression analysis on the model outputs. These rules provide the optimal release conditioned on observable information such as current storage levels or previous period inflows. Since ISO models can be extremely large, its application should be limited to the most efficient optimization methods (Celeste et al., 2009).

Labadie [2004] performed a state-of-the-art review of optimization methods for multi-reservoir systems, and he found that the ISO formulation has been adapted to many deterministic models, including linear programming models (Hiew et al., 1989), flow network algorithms (Lund and Ferreira, 1996), successive quadratic programming (Peng and Buras, 2000), dynamic programming (Young, 1967), (Karamouz et. al, 1992), among others.

Chapter 3: Optimization of Units Maintenance Scheduling in Multi-Purpose, Multi-Reservoir Systems.

This chapter presents the development of a mixed integer linear program (MILP) model formulation for the optimal operation of multi-reservoir systems, considering the preventive maintenance scheduling of the generating units. The proposed formulation finds the maintenance outage schedule that optimizes the model objectives: the maximization of revenue from power generation and the minimization of penalties resulting from deviations of reservoir elevations and spill releases from a preferred operating regime. The model is applied to the Bridge River system considering three different scenarios with fixed and optimized outages.

3.1 Introduction and Problem Definition

The optimal scheduling of preventive maintenance outages in generating units can constitute a complex and time-consuming problem. Several optimization techniques have been proposed for that purpose, including heuristic methods (Kothari and Ahmad, 1993; Shimomura et. al., 2002), integer and mixed integer programming (Edwin and Curtius, 1990; Takriti and Birge, 2000; Martin, 2000; Yuehao, 2007; Aghaei et al., 2013), dynamic programming (Yamayee et al., 1983; Georgakakos, 1997; Yi et al., 2003), goal programming (Muñoz and Ramos, 1999), decomposition Methods (Yellen et al., 1992; Carpentier et al., 1996) among others. Although some of these methods have been applied to hydroelectric projects, the maintenance scheduling (MS) in multi-reservoir systems has not been thoroughly investigated. The MS modeling in multi-reservoir systems is essential in utilities that largely rely on this type

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A version of this chapter will be submitted for publication.
Archila D. and Shawwash Z., “Optimization of Units Maintenance Scheduling in Multi-purpose, Multi-Reservoir Systems”

of systems. Such is the case of the British Columbia Hydro and Power Authority (BC Hydro), the largest electric utility in the province of British Columbia, Canada. More than 65% of BC Hydro's installed generating capacity comes from multiple reservoirs located in just two river basins, the Peace River and the Columbia River, while an additional 16% comes from smaller multi-reservoir systems located in other river basins (BCHydro, 2000). These multi-reservoir systems serve multiples purposes besides power generation, including flood control, recreation, environmental protection, conservation of cultural and archaeological sites and water supply for human consumption. The consideration of these multiple purposes makes the MS modeling even more complex: where the selected schedule for the maintenance outages might benefit one of the reservoir's objectives, it can also negatively affect others. Therefore, it is important for the reservoir's operators to understand the tradeoffs between the MS and the optimization of the different purposes. This paper presents the development of a mixed integer linear programming (MILP) model formulation that aims to achieve that. Based on the operator's modeling priorities, the proposed formulation establishes the optimal operation of a multi-reservoir system and the optimal MS of the corresponding generating units.

3.2 Proposed Formulation

The stages of the proposed MILP algorithm are described in Figure 3.1. Given the time step, length of the study period and the maintenance outage duration of all the generating units in the system, the first stage consists in the definition of all the possible outage alternative solutions.

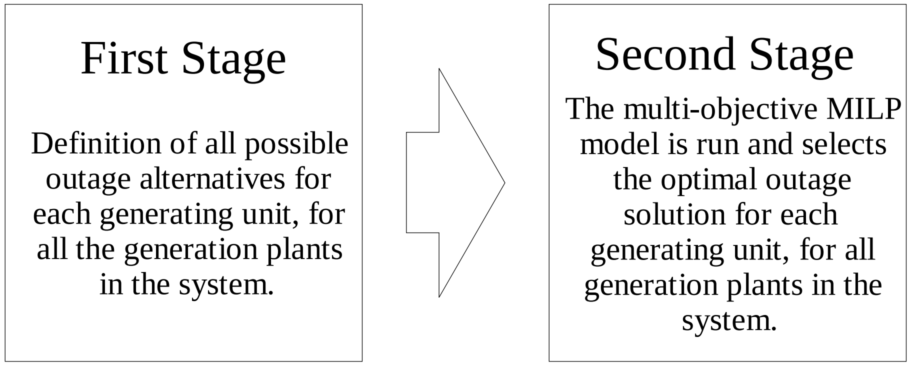

Figure 3.1 Stages of the proposed MILP Algorithm

As an example, Figure 3.2 presents all the possible alternative solutions for a 10-day maintenance outage of a single unit in a study period of 20 days and a daily time step. There is a total of 11 solutions, starting the earliest (S_1) in day 1 and finalizing in day 10, while the latest (S_{11}) starts in day 11 and ends in day 20. Similarly, all the possible MS solutions for the rest of the generating units in the system must be established. A preprocessor was developed in AMPL in order to define these outage alternatives and keep record of the units' status in each time step of every solution.

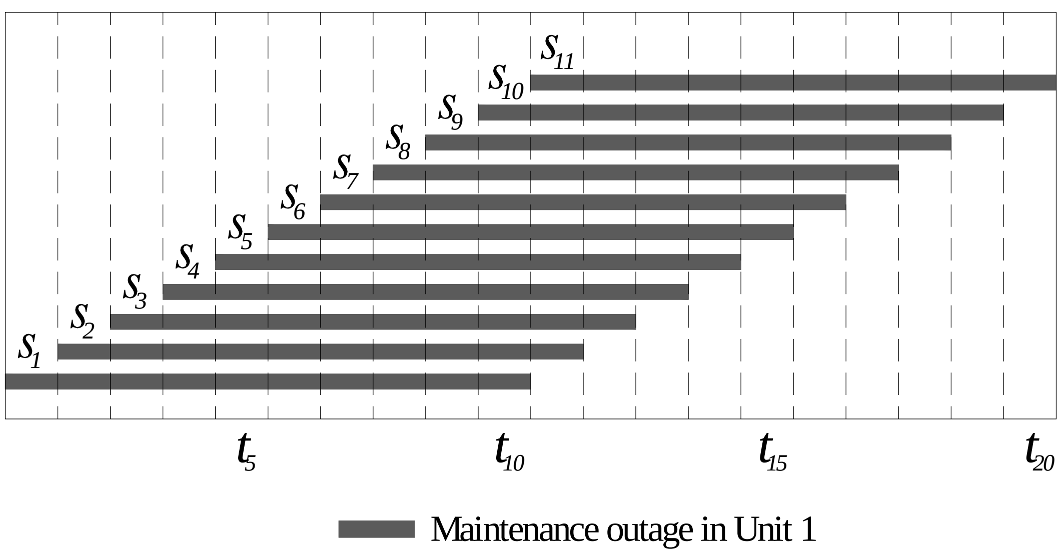

Figure 3.2 Alternative solutions for the MS of a single unit

In the second stage, a binary variable is indexed to each of the outage alternatives defined in the first stage, and the MILP optimization model assigns a value of one to a single solution for each generating unit. The selected solutions, which are those that optimize the model objectives, establish the generating units' availability combination for each time step of the study period. In turn, the units' availability combination determines the computation of the power generation in the model. The relationship between the selected solutions and the power generation calculation will be further explained in the formulation of the optimization model.

Several authors have pointed out the extensive computational resources that some integer and mixed integer programming models require, and they have suggested that this can be improved by reducing the number of variables (Guignard and Spielberg, 1981; Crowder et al., 1983; Savelsbergh, 1993; Babayev and Mardanov, 1994). Therefore, in order to accelerate the required solution time, a special emphasis is given to decrease the number of binary variables. This can be achieved by limiting the number of alternative solutions for the MS through the definition of sequential outages. For example, in a system with 3 generating units, there will be 33 alternative solutions for a maintenance outage of 10 days per unit in a study period of 20 days consisting of 11 alternatives for each generating unit, as it is shown in Figure 3.2. However, if sequential outages are specified in such a way that the outage in a unit begins after a fixed period of time since the start of another unit's outage, the number of solutions can be significantly decreased. Figure 3.3 presents an example of this assuming a lag of 3 days between the outages. It can be observed that the number of alternative solutions goes down from 33 to just 5. More significant reduction in alternatives is expected for longer study periods.

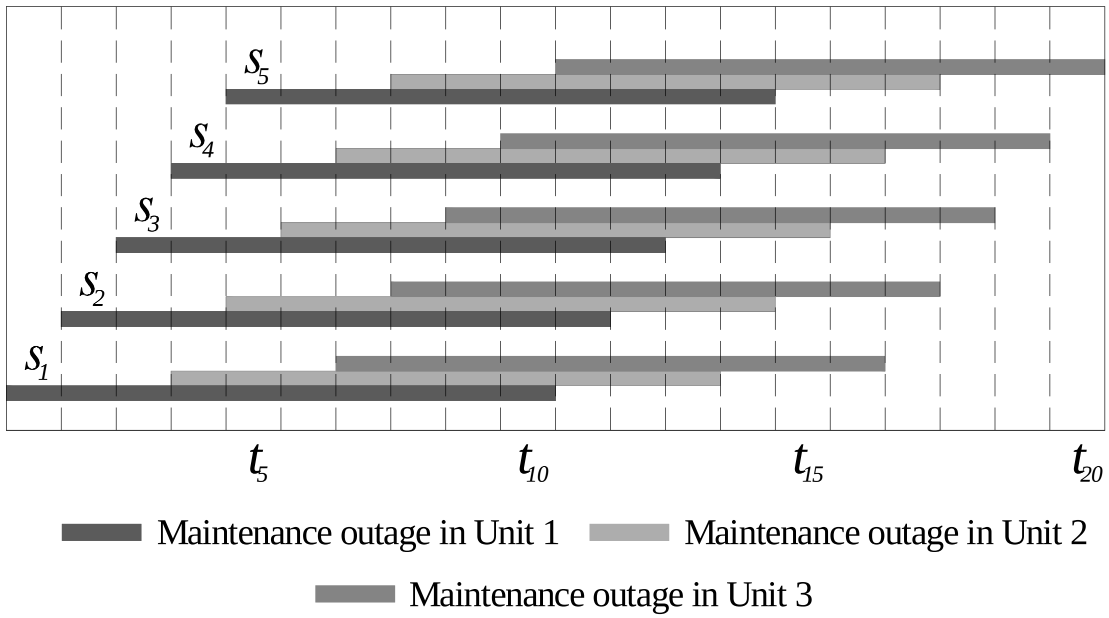

Figure 3.3 Alternative solutions for the MS of three units, assuming sequential outages.

In order to provide the user more flexibility during the analysis of the MS, the proposed formulation allows different configurations of sequential outages. This includes setting the order of the outages, specifying gaps between outages, and scheduling simultaneous (paired) outages. Figure 3.4 presents an example that includes such configurations.

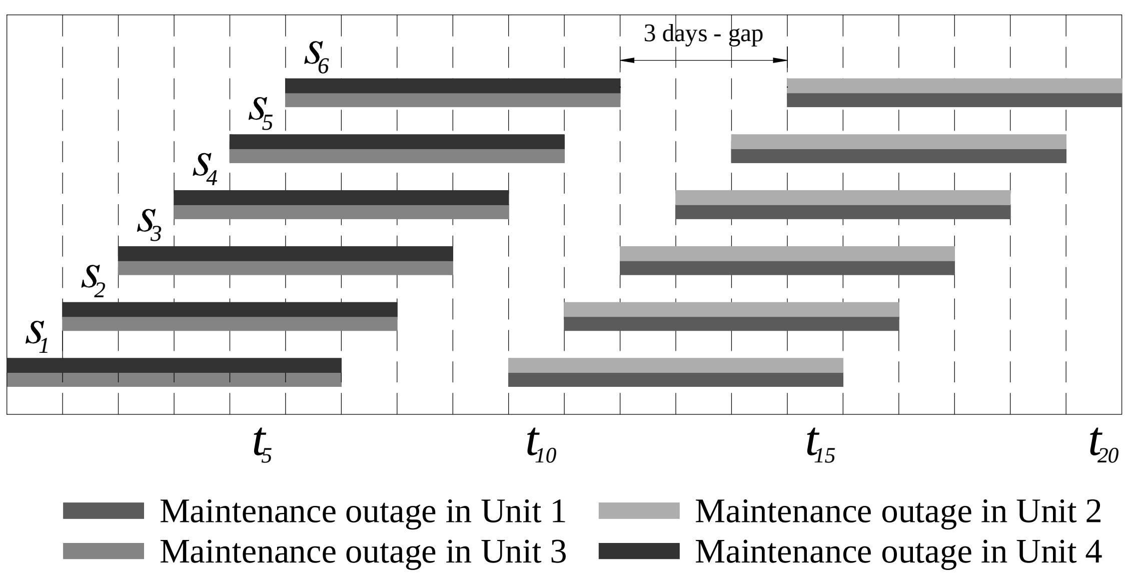

Figure 3.4 Alternative solutions for the MS of four units, assuming simultaneous sequential outages.

3.3 Formulation of the MILP Optimization Model

The deterministic optimization model used in the second stage of the algorithm was developed and solved using AMPL/CPLEX and it was divided into three basic components: (1) the decision variables, (2) the objective function and (3) the model constraints.

3.3.1 Decision Variables

The main decision variables of the model are the following:

$S_{p,t}$	Storage of reservoir *p* at the time step *t*
$spilQ_{p,n,t}$	Spill release from reservoir *p*, through the release structure *n*, during time step *t*
$turbQ_{p,t}$	Turbine release from reservoir *p*, during time step *t*
$turbQZone_{p,t,z}$	Turbine release from reservoir *p*, during time step *t* , during sub-time step *z*
$G_{p,t,z}$	Power generation at reservoir *p*, at time step *t* , during sub-time step *z*
$sCombo_{p,t,k}$	Binary variable indexed over reservoir *p*, time step *t* and the units' availability combination *k*
$sAlternative_{p,m,ns}$	Binary variable indexed over reservoir *p*, unit *m*, and the maintenance scheduling solutions *ns*

It can be observed that there are two variables related to turbine releases. The first variable ($turbQ_{p,t}$) represent the average turbine flow during the time step *t*, while $turbQZone_{p,t,z}$ is the average turbine flow for sub-time step *z* defined by the user. These sub-time steps depend on the variability of the electricity prices within a time step.

3.3.2 The Objective Function

The multi-objective problem is solved in the model making use of an optimization method known as the Weighting Method of Multi Objective Optimization, where a grand objective is established adding all the individual objectives, each multiplied by a weighting W coefficient as follows:

$$Minimize\ \{W_S * \sum_{p,t} PFS[S_{p,t}] \ + \ W_Q * \sum_{p,n,t} PFQ[spilQ_{p,n,t}]$$
$$- W_R \ * \sum_{p,t,z} G_{p,t,z} * \Delta\mathrm{t} * priceZoneFraction_{t,z} * price_{t,z}\} \qquad (3.1)$$

The first two terms of the objective function refer to the minimization of the storage and spillway deviations from some preferred operating regimes, respectively. The minimization is accomplished through the use of "penalty functions" for storage (*PFS*) and for spill (*PFQ*). These functions, which must be defined by the user, are piecewise linear curves where a penalty number is assigned for all the possible values of reservoir storage and the spill releases. The functions return a value of zero when the values are within the preferred range; otherwise it returns a penalty value. The farther a value is from the target, the greater the penalty. Figure 3.5 presents an example of a penalty function for spill releases.

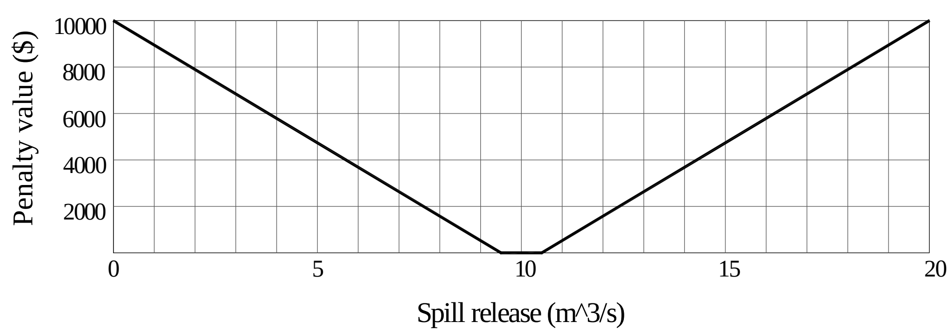

Figure 3.5 Example of a penalty function for spill release.

The third term of the objective function refers to the maximization of the revenue from power generation. The negative sign of this term results in maximization of the expected revenues of power generation. This revenue is computed for each sub-time step by multiplying the power generation, the number of hours in each sub-time step (Δt* *priceZoneFraction*) and the corresponding electricity price.

The weighting coefficients W_S, W_Q and W_R are input parameters whose values should be based in the priorities of the decision makers. Nevertheless, it is a common practice in this optimization method to run the model several times varying these coefficients until the set of non-inferior solutions is generated (Revelle et al, 2004).

3.3.3 Model Constraints

The following sets of constraints are used in the optimization model:

- General constraints as described below
- Constraints for the calculation of the power generation using Generation Production Functions.
- Constraints for the selection of the optimal MS solutions.

General constraints:

This set of constraints includes:

- Mass balance equation:

$$S_{p,t} = S_{p,t-1} + I_{p,t} - \sum_{n=1}^{N_P} spilQ_{p,n,t} - turbQ_{p,t} + \sum_{k=1}^{T}\sum_{n=1}^{N_k} A_{k,n,p} * spilQ_{k,n,t} + \sum_{l=1}^{T} B_{l,p} * turbQ_{l,t} \tag{3.2}$$

Where:

$I_{p,t}$ = deterministic inflow into reservoir *p* during time step *t*.

N_X = total number of spill release structures in reservoir *X*.

T = total number of reservoirs in the system.

$A_{k,n,p}$ = 1 if the spill from release structure *n* in reservoir *k* flows into reservoir *p*; 0 otherwise.

$B_{l,p}$ = 1 if the turbine release from reservoir *l* flows into reservoir *p*; 0 otherwise.

- Controlled spillways

$$spilQ_{p,\mathrm{n},t} \leq Spillfunc_{p,n}\left(S_{p,t}\right) \quad \{if\ contSpill[n] = 1\} \tag{3.3}$$

Where $Spillfunc_{p,n}$ is a piecewise linear function that defines the maximum spill flow that can be released from the release structure *n* in reservoir *p*, as a function of the reservoir volume. This constraint applies if the release structure *n* has been identified as a controlled release structure (i.e. if parameter *contSpill[n]* is 1).

- Non-controlled spillways

$$spilQ_{p,\mathrm{n},t} = Spillfunc_{p,n}\left(S_{p,t}\right) \quad \{if\ freeSpill[n] = 1\} \tag{3.4}$$

The only difference between Constraints 3.3 and 3.4 is that the latter is an equality constraint. It applies if the release structure *n* has been identified as a non-controlled release structure (i.e. if parameter *freeSpill[n]* is 1).

- Total turbine release

$$turbQ_{p,t} = \sum_{z=1}^{Z} priceZoneFraction_{t,z} * turbQZone_{p,t,z} \tag{3.5}$$

Where:

Z = Total number of sub-time steps defined by the user.

$priceZoneFraction_{t,z}$ = fraction of the sub-time step *z* in time step *t*.

- Limits on optimization variables

$$Smin_{p,t} \leq S_{p,t} \leq Smax_{p,t} \tag{3.6}$$

$$spilQmin_{pn,t} \leq spilQ_{pn,t} \leq spilQmax_{pn,t} \tag{3.7}$$

$$turbQmin_{p,t} \leq turbQ_{p,t} \leq turbQmax_{p,t} \tag{3.8}$$

$$Gmin_{p,t} \leq G_{p,t,z} \leq Gmax_{p,t} \tag{3.9}$$

Constraints for the calculation of the power generation using Generation Production Functions:

The Generation Production Functions (GPFs) are formulated as a family of tridimensional surfaces that provides the maximum power generation as a function of forebay elevation, turbine discharge and unit availability. Shawwash et al. [2000] developed a procedure to build the GPFs for the different hydropower generation plants in the BC Hydro system. This procedure uses the output of the Static Plant Unit Commitment (SPUC) (Smith, 1998) which uses a dynamic programming algorithm to determine the optimized turbine discharge for each increment of plant generation, forebay and tail-water elevation and turbine availability combination. Using this output, the procedure develops a set of two dimensional piecewise linear curves that accurately approximate the GPFs. The following constraints were included in the model in order to use the GPFs and overcome their dependency over the forebay elevation and the units' availability combination:

$$G_{p,t,z} \leq \sum_{k=1}^{K} GPF_{p,t,fb,k}\left(turbQZone\_Aux_{p,t,z,k}\right) \tag{3.10}$$

$$\sum_{k=1}^{K} sCombo_{p,t,k} = 1 \tag{3.11}$$

$$turbQZone\_AUX_{p,t,z,k} \geq sCombo_{p,t,k} * turbQmin_{p,t} \tag{3.12}$$

$$turbQZone\_AUX_{p,t,z,k} \leq sCombo_{p,t,k} * turbQmax_{p,t} \tag{3.13}$$

$$turbQZone_{p,t,z} \leq \sum_{k=1}^{K} turbQZone\_Aux_{p,t,z,k} \tag{3.14}$$

Constraint 3.10 is used to set an upper limit to the power generation variable, which is being maximized. This limit is the summation of the power generation calculated with the GPFs of all the different units' availability combinations in the system (*K*). Nevertheless, if the turbine release is zero for all the units' availability combinations except the one selected by the model, Constraint 3.10 will only consider the corresponding GPF as the limiting function.

This can be achieved introducing the variables *sCombo* and *turbQZone_Aux* in the model. The former is a binary variable indexed over reservoir, time step and all the units' availability combination, and Constraint 3.11 selects (assign a value of one) to a single units' availability combination for each reservoir and time step. Additional constraints are used to link this variable with the selection of the MS solution. The variable *turbQZone_Aux* is similar to *turbQZone*, but it is indexed over the units' availability combination. Constraints 3.12 and 3.13 forces *turbQZone_Aux* to become zero for all the combinations except the one selected in Constraint 3.11. Finally, Constraint 3.14 makes sure that the *turbQZone_Aux* value of the combination selected is equal to the original *turbQZone* variable, which is used in the rest of the model.

The GPFs are also indexed over the forebay elevation of the reservoir. In order to prevent the model becoming nonlinear, a forebay elevation is assumed for each time step of the study period. This elevation is used to calculate the GPFs coefficients that are updated with the actual forebay elevation calculated by the optimization algorithm. Therefore, the model is required to run iteratively several times until the forebay elevation converges.

Constraints for the selection of the optimal MS solutions.

Although constraint 3.11 is forcing the model to select a single units' availability combination for each time step of the study period, it is necessary to link it with the selection of the optimal MS solution. This is achieved through the following constraints:

$$\sum_{ns=1}^{NS} sAlternative_{p,m,ns} = 1 \tag{3.15}$$

$$\sum_{k=0}^{K} sCombo_{p,t,k} * Tag_k = \sum_{m=1}^{M}\sum_{ns=1}^{NS} sAlternative_{p,m,ns} * sUnitStatus_{p,m,\mathrm{t},ns} * 10^{unitType_{p,m}} \tag{3.16}$$

The binary variable *sAlternative* is indexed over the reservoir *p*, the total number of units in the reservoir *m*, and the number of solutions *ns* defined in the first stage of the algorithm. Constraint 3.15 is used to assign a value of one to a single MS alternative solution for each unit in the system, during the study period. Constraint 3.16 makes use of an identification number denominated *Tag* that is defined for each units' availability combination using equation 3.17:

$$Tag_k = \sum_{m=1}^{M} unitStatus_m * 10^{unitType_m} \qquad (3.17)$$

Where *unitStatus* indicates whether unit *m* is on (1) or off (0) and *unitType* is a parameter used to describe units with similar characteristics. Table 3.1 provides an example of the Tag number used in a system with three units.

Table 3.1 Example of tag numbers for a system with 3 units.

Units' availability combination	Unit 1 status (type 1)	Unit 2 status (type 1)	Unit 3 status (type 2)	Tag
000	off	off	off	0
001	on	off	off	10
010	off	on	off	10
110	on	on	off	20
100	off	off	on	100
101	on	off	on	110
110	off	on	on	110
111	on	on	on	120

There are seven possible units' availability combinations in a system with three units. It can be observed in the example of Table 3.1 that because both unit 1 and 2 share the same type unit (type 1) some combinations have the same tag number. For instance, combinations 001 and 010 represent the case where one unit type 1 is on and the single unit type 2 is off.

Constraint 3.16 is used to verify that the units' availability combination selected by the binary variable *sCombo* is consistent with the MS solution selected by the binary variable *sAlternative*. This is achieved making sure that the corresponding Tag number in both sides of the constraint is the same. In the right side of the constraint, the tag number for each time step is calculated considering the units status of the MS solution selected by *sAlternative*. The left side forces the model to assigns a value of 1 to the variable *sCombos* that represents the units' availability combination with the same tag number that is in the right side.

If the operator decides to consider sequential outage solutions (Figure 3.3), Constraints 3.15 and 3.16 must be replaced by the following constraints:

$$\sum_{ns=1}^{NS} sAlternativeSeq_{p,ns} = 1 \tag{3.18}$$

$$\sum_{k=0}^{K} sCombo_{p,t,k} * Tag_{p,k} = \sum_{ns=1}^{NS} sAlternativeSeq_{p,ns} * \sum_{m=1}^{M} sUnitStatus_{p,m,t,ns} * 10^{unitType_{p,m}} \tag{3.19}$$

These constraints use a different binary variable denominated *SAlternativeSeq* which is not indexed over the reservoir units. Therefore, this variable can be taken out from the summation over *m* in Constraint 3.19.

3.3.4 Additional Modeling Features

The MILP optimization model calculates the difference in revenue between the scenario specified by the user and a hypothetical scenario without any outages considered. The resulting difference is denominated the “outage cost” and it can be used as a reference to compare different outage scenarios.

Another feature of the proposed formulation is that it allows the user to specify the occurrence of fixed outages. In this scenario, the model does not perform any MS optimization and it uses the units’ availability combinations specified by the user to construct the GPFs. Constraints 3.11, 3.15 and 3.16 are dropped and the following constraint is introduced:

$$sCombo_{p,t,k} = 1 \ \{if\ Tag[k] = Tag[C_t]\} \tag{3.20}$$

Where the conditional statement {if Tag[k] = Tag[C_t]} forces the model to assign a value of one to the binary variable *sCombo* indexed over C_t, which is the combination selected by the user.

The formulation also let the user combine fixed outages with the MS optimization. After the user specifies the dates of the fixed outages and the configuration of the additional outages that will be optimized, the alternatives solutions established in the first stage of the algorithm are revised and those that overlap with the fixed outage are discarded. In the example presented in Figure 3.2, if a fixed outage of 3 days is specified in days 17, 18 and 19, the alternative solutions S_8 to S_{11} will be discarded. This is shown in Figure 3.6. This scenario does not require any modification in the optimization model.

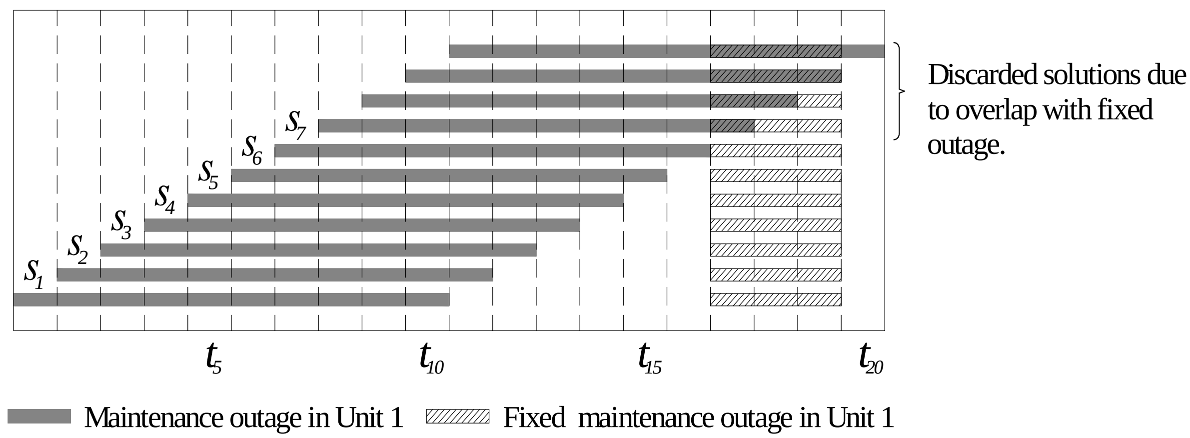

Figure 3.6 Fixed outage and alternative solutions for the MS of a single unit.

3.4 Case Study

The MS model was tested for a case study using the Bridge River hydropower project near Lillooet, British Columbia, Canada. This project consists of three dams and four generation stations. Figure 3.7 presents a schematic of the Bridge River project.

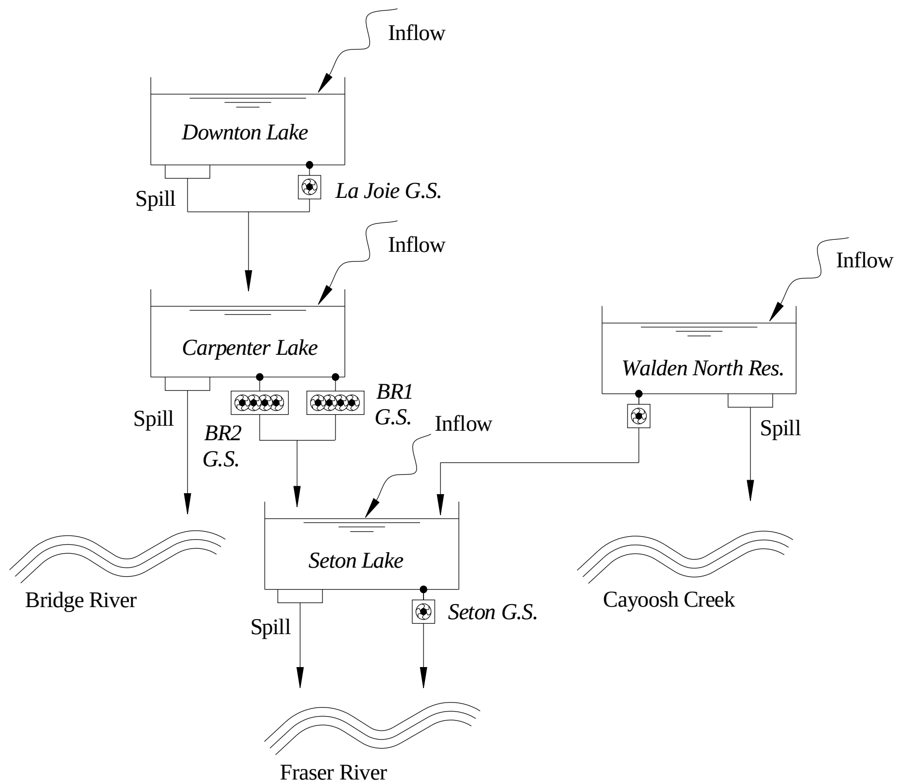

Figure 3.7 Hydraulic configuration of the Bridge River project.

The La Joie dam impounds the upstream portion of the Bridge River and forms Downton Lake reservoir. The La Joie generating station has a single unit with an operating range between 3 and 25 MW. Both spill and turbine discharges from La Joie flows into Carpenter Lake, impounded by Terzaghi dam. Water from this reservoir can be diverted to two different generating stations: Bridge River #1, with four units and operating range between 20 and 52 MW each, and Bridge River #2 with another four units capable of operating between 20 and 75 MW each. The spillway discharge from this reservoir flows into the Lower Bridge River, while the turbine discharges flows directly into Seton Lake. The Seton generating station has a single unit with operating range between 5 and 48 MW. The spill and turbine discharges from Seton Lake flows into the Seton River, a tributary of the Fraser River. An additional independent power producer is also considered in the system: the Walden North project is a reservoir with limited storage whose turbine discharge flows into Seton Lake. The spillway from this project discharges directly into Cayoosh Creek.

The model was run considering the inflow values of the year 1984 for the required outage durations presented in Table 3.2. The 4-days outages are required for regular maintenance, while longer periods are used for rehabilitation purposes. No outages were considered in the Seton generation station.

Table 3.2 Duration of maintenance outages in La Joie and Bridge River G.S.

	Duration of Maintenance Outage (days)								
Generation station	**La Joie**	**Bridge River #1**				**Bridge River #2**			
Turbine	**Unit 1**	**Unit 1**	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**
First outage	-	110	67	67	110	4	4	4	4
Second outage	25	4	53	53	4	4	4	4	4

The formulation was tested using three different scenarios and considering fixed and optimized outages, as listed in Table 3.3. The dates of the fixed outages for Scenarios 1 and 2 are presented in Table 3.4. All the scenarios were run considering a study period of 365 days and a daily time step.

Table 3.3 Model configuration for Sequences 1, 2 and 3

	Weight coefficients			Outage configuration	
	W_S	W_Q	W_R	First outage	Second outage
Scenario 1	0.30	0.30	0.40	fixed	fixed
Scenario 2	0.30	0.30	0.40	fixed	optimized
Scenario 3	0.30	0.30	0.40	optimized	fixed

Table 3.4 Fixed MS for Scenarios 1 and 2

G.S.	**La Joie**	**Bridge River #1**				**Bridge River #2**			
Turbine	**Unit 1**	**Unit 1**	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**
First outage	-	Feb 1st to May 21st	Feb 1st to Apr 8th	Feb 1st to Apr 8th	Feb 1st to May 21st	Aug 8th to Aug 11th	Aug 8th to Aug 11th	Aug 19th to Aug 22nd	Aug 19th to Aug 22nd
Second outage	Oct 5th to Oct 29th	Sep 9th to Sep 12th	Aug 16th to Oct 7th	Aug 16th to Oct 7th	Sep 9th to Sep 12th	Nov 1st to Nov 4th	Nov 1st to Nov 4th	Nov 12th to Nov 15th	Nov 12th to Nov 15th

Figures 3.8 to 3.10 show the resulting MS for each of the scenarios tested with the model.

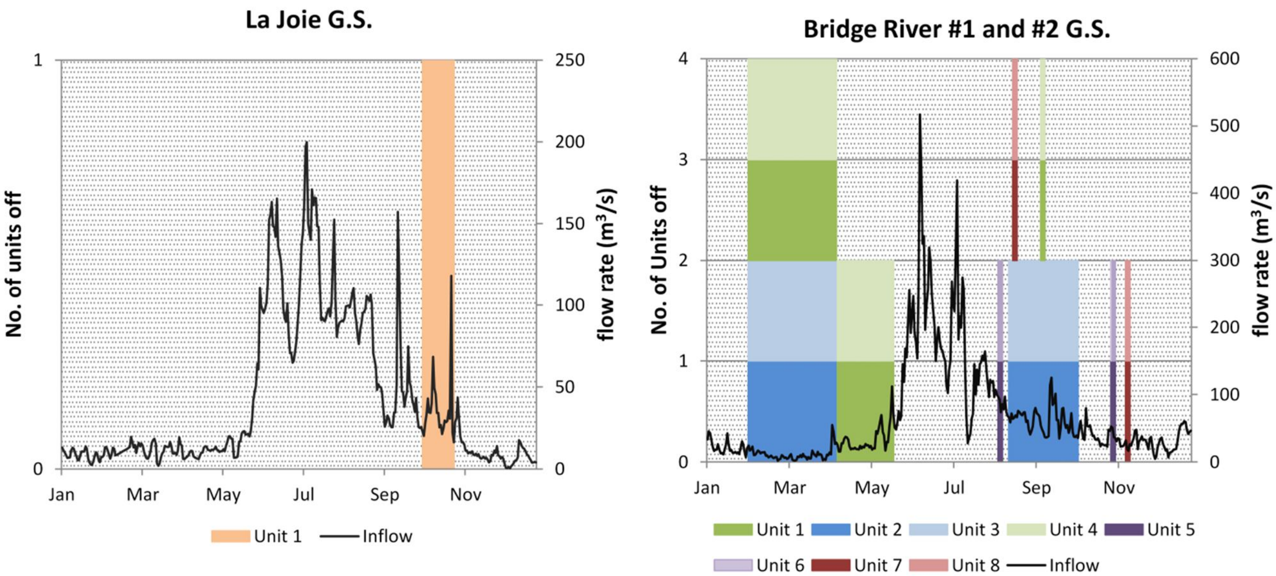

Figure 3.8 Maintenance Scheduling in Scenario 1

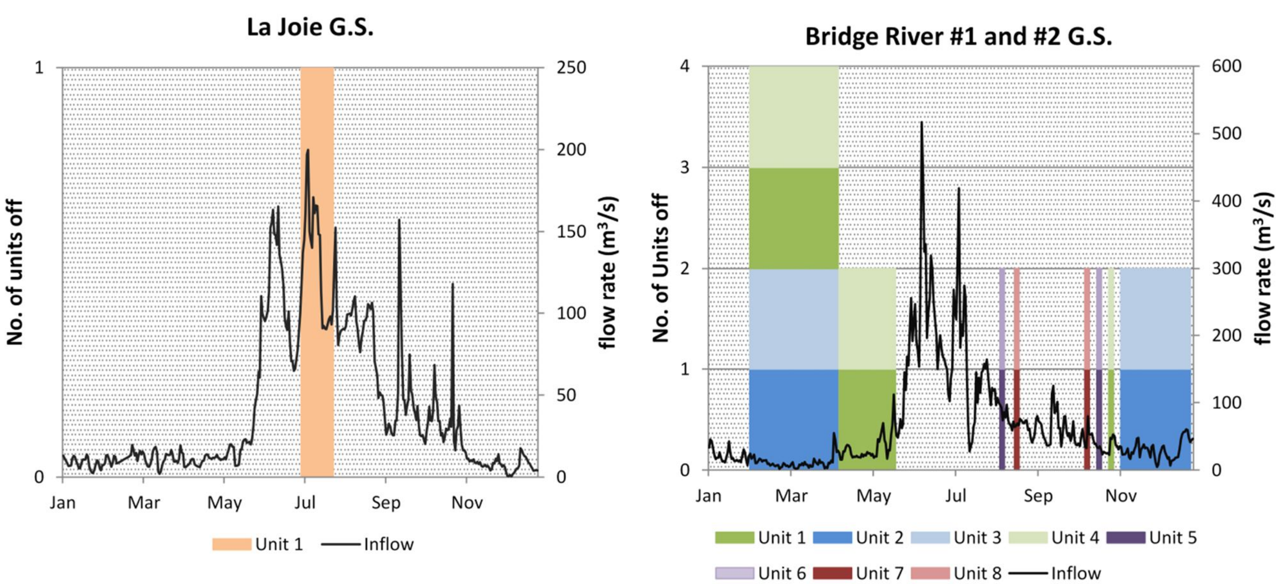

Figure 3.9 Maintenance Scheduling in Scenario 2

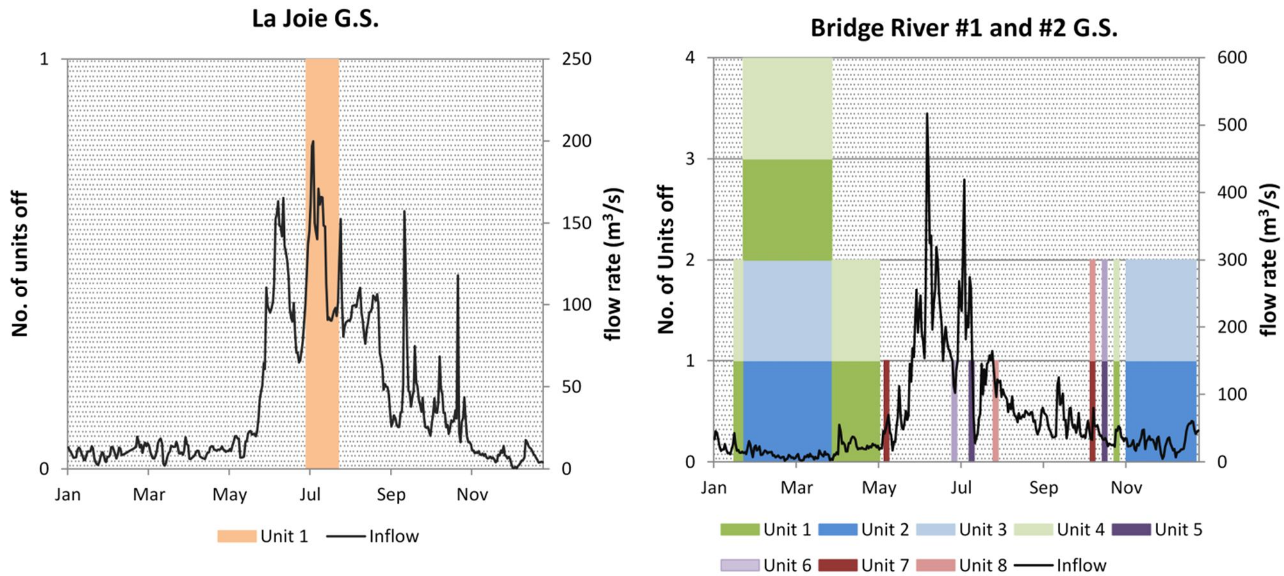

Figure 3.10 Maintenance Scheduling in Scenario 3

Table 3.5 presents a comparison of the results of the three different scenarios. The MS cost of each of the scenarios is shown in the second column. It can be observed that the cost decreases progressively from the first scenario, where both outages (per unit) were specified by the user, to the third scenario, where the model found the optimal schedule for the first outages. The fixed outages in Scenario 1 were schedule during the months of low inflows in order to reduce the spilled water during the freshet period. However, the scheduling optimization in Scenario 2 was able to reduce the outage cost without increasing the spill and forebay elevation penalties. This was achieved even though the first outage for each unit was not rescheduled. The optimization of the second outages was performed assuming sequential simultaneous outages with a gap of 5 days between outages. In Scenario 3, the optimized second outages from Scenario 2 were fixed, and the optimization derived the optimal schedule of the first outages, except for the La Joie G.S. which only had one outage specified. Owing to the extended length of these outages, they were not required to be sequential. The model was able to find a schedule with lower outage cost and lower spill penalties than that of Scenario 2. The forebay elevation penalties were the same in the three scenarios.

Table 3.5 Model results for Scenarios 1, 2 and 3

	Outage cost (Mil. $)	Total forebay elevation penalties (Mil. $)	Total spillway penalties (Mil. $)	Computing time[1] (min)
Scenario 1	4.20	0.13	91.40	1
Scenario 2	3.64	0.13	91.38	10
Scenario 3	3.56	0.13	91.22	19

3.5 Summary and Conclusions

This research introduced a two-stage algorithm for the analysis of maintenance scheduling in multi-reservoir systems. The first stage of the algorithm consists in establishing all the possible MS solutions for a given configuration. In the second stage, a MILP model selects the MS solution that optimizes three objectives: maximization of revenue from power generation and minimization of penalties resulting from deviations of reservoir elevations and spill releases from a preferred operating regime. The model makes use of Generation Production Functions which are piecewise linear curves indexed over forebay elevation and the unit's availability combination, and are used to calculate power generation as a function of turbine discharge. The formulation was tested in a case study of the Bridge River System, using inflow sequences recorded in 1984. The results demonstrated that the MS selected by the model has a lower cost and at the same time presented smaller spillway penalties than the original MS.

The proposed formulation provides a new, practical tool for the analysis of MS of generating units. The flexibility in the optimization component allows the user to analyze different scenarios, including the specification of sequential outages with different configurations. Additional work is required to determine how the scheduling optimization is

[1] The three scenarios were run on an Intel® Core™ I5 2.50 GHz. Processor with 8.00 GB of RAM

affected by the weight coefficients and the values of the penalty functions used in the objective equation.

3.6 Acknowledgment

The authors greatly acknowledge the BC Hydro and Power Authority and the Natural Sciences and Engineering Research Council of Canada (NSERC) for the technical and financial support received during the course of this research.

Chapter 4: A Multi-Objective Optimization Model for Operations Planning of Multi-Reservoir Systems

This chapter presents the development and evaluation of a multi-objective optimization tool for the operations planning platform (OPP) at BC Hydro. The optimization model incorporates two main objectives: (1) to maximize revenue from power generation; and (2) to minimize penalties resulting from deviations of reservoir elevations and spill releases from a preferred operating regime. We analyze the use of penalty functions in the objective function and propose an alternative formulation using Chance Constraints and Linear Decision Rules. We present results of a case study to illustrate the capabilities of the tool to provide decision makers with timely information on trade-off between different objectives and the impacts of using chance constraints in lieu of penalty functions.

4.1 General Overview of the Operations Planning Tool (OPT)

The Operations Planning Tool (OPT) is an in-house application developed by BC Hydro to aid the Operation Planning Engineers (OPEs) to make decisions regarding the operation of a multi-reservoir system. It consists of three main components: the graphical user interface (GUI), the optimization model and the solver software. The GUI allows the user to configure the optimization study, change model configurations, run the optimization and retrieve and display the output data. The optimization model is formulated in AMPL, while the CPLEX solver is used to solve the optimization problem. The GUI is used to configure the study and prepare

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A version of this chapter has been published.
Archila D., Shawwash Z., Zhou J., Vassilev P., Kong G., Plesa V. and Abdalla A. (2014), “A Multi-Objective Optimization Model for Operations Planning of Multi-Reservoir Systems”, in *11th International Conference on HydroInformatics, HIC 2014*, 17-21August 2014, New York,

input data in the user's workstation; the problem is sent and solved at the server workstation where the AMPL and CPLEX solver resides and then the solution is sent back to the client workstation. This process is illustrated in Figure 4.1. This paper will focus on the optimization model formulation.

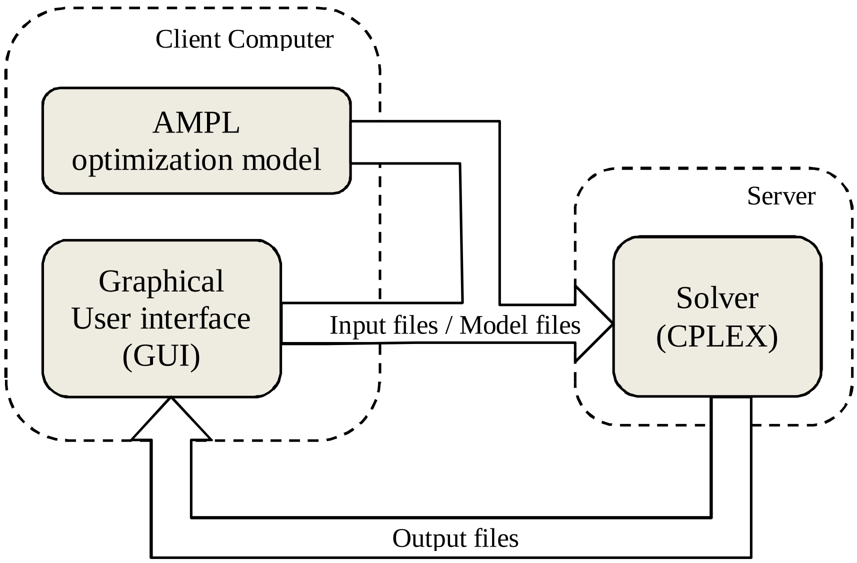

Figure 4.1 Process diagram of the Operations Planning Tool

4.1.1 The Optimization Model

The OPT is a deterministic Linear Programming optimization model and it can be divided into three basic components: (1) the decision variables, (2) the objective function and (3) the model constraints. A brief description of each of these components is presented in the following sections.

Decision Variables

The major decision variables of the model are the following:

$S_{p,t}$	Storage of reservoir *p* at the time step *t*
$spilQ_{p,n,t}$	Spillway release from reservoir *p*, through the release structure *n*, at time step *t*
$turbQ_{p,t}$	Turbine release from reservoir *p* at time step *t*
$turbQZone_{p,t,z}$	Turbine release from reservoir *p*, at time step *t* , during sub-time step *z*
$G_{p,t,z}$	Power generated at reservoir *p*, at time step *t* , during sub-time step *z*

It can be observed that there are two variables related to turbine releases. The first variable ($turbQ_{p,t}$) is the average turbine flow during the time step t, while $turbQZone_{p,t,z}$ is the average turbine flow during a shorter time step z (sub-time step) defined by the user. These sub-time steps depend on the variability of the electricity prices within a time step. For example, a typical day the user can define two price zones: Heavy Load Hours (hlh) for the heavy load hours, and Light Load Hours (llh) for the rest of the day. The electricity prices in these two zones are different and therefore it is of interest to know the average turbine release in each of them.

The Objective Function

The OPT multi-objective problem is solved making use of an optimization method known as the Weighting Method of Multi Objective Optimization, where a grand objective is established adding all the individual objectives, each one multiplied by a weighting coefficient as follows:

$$Minimize \ \{W_S * \sum_{p,t} PFS\left[S_{p,t}\right] \ + \ W_Q * \sum_{p,n,t} PFQ\left[spilQ_{p,n,t}\right]$$

$$- W_R \ * \sum_{p,t,z} G_{p,t,z} * \Delta t * priceZoneFraction_{t,z} * price_{t,z}\} \qquad (4.1)$$

The first two terms of the objective function refer to the minimization of the storage and spillway deviations from some preferred operating regimes, respectively. The minimization is accomplished through the use of "penalty functions" denominated PFS for storage and PFQ for spill. These functions, which must be defined by the user, are piece-wise linear curves where a penalty number is assigned to all the possible values that the elevation and the spillway releases can have. When the values are within the preferred range a penalty value of zero is produced, while those outside of the target range must result in a non-zero penalty. The farther a value is from the target, the greatest its penalty will be. Figure 4.2 presents an example of a penalty function for spillway releases.

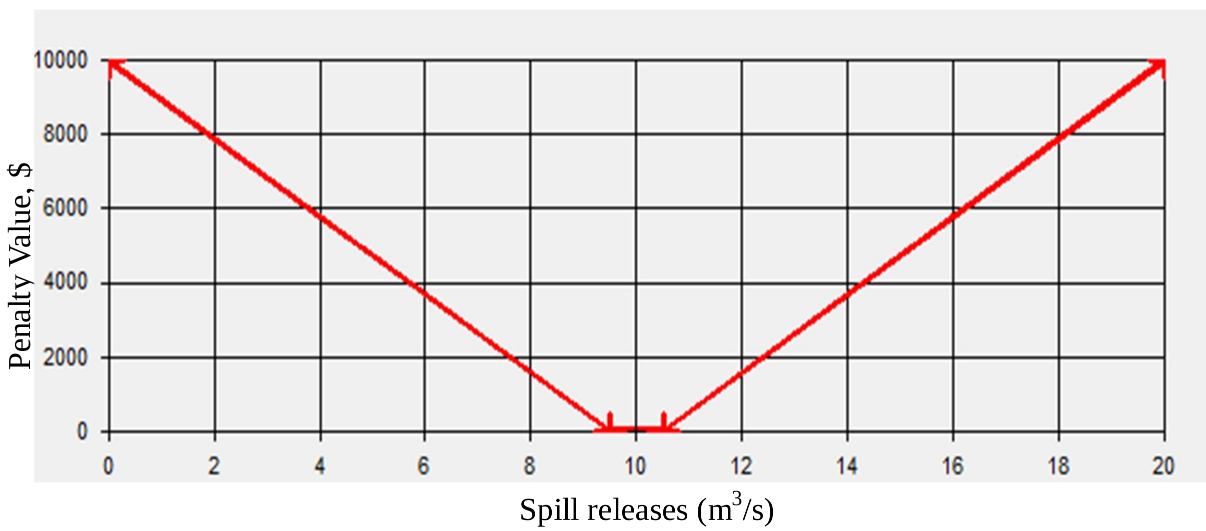

Figure 4.2 Example of a penalty function for spillway releases

The third term of the objective function refers to the maximization of the revenue from power generation. Due to the negative sign of this term, the model performs a maximization of the revenue even though the objective function as a whole is being minimized. In other words, because the model tries to make this term as negative as possible, the absolute value of the revenue will also be the highest possible. This revenue is computed for each sub-time step, multiplying the power generation, the number of hours in each sub-time step (Δt* *priceZoneFraction*) and the corresponding price.

The weighting coefficients W_S, W_Q and W_R are input parameters whose values should be based in the priorities of the decision makers. Nevertheless, it is a common practice in this optimization method to run the model several times varying these coefficients until the set of non-inferior solutions is generated (Revelle et al, 2004).

Model Constraints

The constraints in the OPT model are used to fulfill the following purposes:

- Satisfy the continuity equation (reservoir mass balance).
- Set limits to the optimization variables.
- Relate the total turbine release per time step ($turbQ_{p,t}$) with the turbine release in the sub-time steps ($turbQZone_{p,t,z}$).
- Calculation of the power generation through Generation Production Functions.

The Generation Production Functions (GPFs) are a family of tridimensional surfaces that provides the maximum power generation as a function of forebay elevation, turbine discharge and turbine availability. Shawwash et al. [2000] developed a procedure to build the GPFs for the different hydropower generation plants in the BC Hydro system.

4.2 Use of Chance Constraints in the OPT Model

The penalty functions used in the objective function are piecewise linear functions that allow the user to set targets for the forebay elevation and the spillway releases; nevertheless, these targets might be violated depending on the trade-off between the different objectives, which in turn is conditioned by the inflow scenario, the weight coefficients and the penalty values assigned to the targets. Penalty functions and piecewise linear functions have been used by different authors in reservoirs operations. Sigvaldason [1976] developed a flow network model for the Trent River System in Ontario. He used penalty functions in the objective function in order to consider the “operation perception of optimal operation”. Can and Houck [1984] proposed two optimization models for a multi-reservoir system in the Green River Basin, Kentucky. The first model made use of piecewise linear penalty functions, similar to the OPT model, while the second model used a preemptive Goal Programming approach. Oliviera and Loucks [1997] developed a Genetic algorithm (GA)-based methodology which identifies the system release rule and the reservoir balancing functions as piecewise linear functions.

The use of penalty functions in the OPT model presents some limitations: first, it is not an easy task for the user to assign the penalty values for the different targets. The x-axis of the two type of penalty functions use different units (e.g., meters for the forebay elevation, cubic meters per second for the spillway releases); therefore, in order to have a balanced trade-off in the optimization process, it requires a comprehensive analysis to decide which slope should be assigned to each segment of the piecewise linear functions. Second, the optimization method that is used in the model requires the multiplication of the objective terms by some weight coefficients. The selection of these coefficients might also require a detailed analysis. Marler and Arora [2004] carried out a survey of different approaches used to determine these weights, but they concluded that even varying the weights consistently and continuously may not result in an even and complete representation of the Pareto optimal set.

Therefore, it is desirable to find an alternative formulation for the OPT model that could provide the benefits of the penalty functions and at the same time overcome the limitations previously described. We have investigated the chance constraints method which could be a suitable alternative. Chance constraints act is a similar way to the penalty functions and they can be used as “soft constraints” allowing the establishment of targets but also considering that under certain conditions these targets may not be satisfied. They present the advantage that it can be easier for the user to establish reliability levels for the chance constraints compared to the construction of the penalty functions. An alternate formulation for the OPT model using Chance Constraints is proposed in Equations 4.2 to 4.6:

$$\textit{Maximize:} \quad \sum_{p,t,z} P_{p,t,z} * 24 * priceZoneFraction_{t,z} * price_{t,z} \tag{4.2}$$

Subject to:

$$P\left[S_{p,t} \geq \; STarget\_low_{p,t}\right] \geq \gamma_{p,t} \tag{4.3}$$

$$P\left[S_{p,t} \leq STarget\_up_{p,t}\right] \geq \delta_{p,t} \tag{4.4}$$

$$P\left[spilQ_{p,t} \geq spilQTarget\_low_{p,t}\right] \geq \alpha_{p,t} \tag{4.5}$$

$$P\left[spilQ_{p,t}. \leq spilQTarget\_up_{p,t}\right] \geq \beta_{p,t} \tag{4.6}$$

Where *STarget_low*, *STarget_up*, *spilQTarget_low* and *spilQTarget_up* are the lower and upper limits for the preferred storage and spillway release regimes. It can observed that in contrast to the objective function of the original model presented in Equation 4.1, the objective function using chance constraints consists of a single objective, which is the maximization of revenue from power generation. This change converts the OPT model into a single LP problem and the use of weight coefficients is no longer required. Therefore, the second limitation from the use of penalty function is also eliminated using chance constraints.

However, there are two main challenges that arise when using chance constraints. First, it is necessary to find a deterministic equivalent for constraints 4.3 to 4.6 and this requires the definition of probability distribution for the random variables used in the chance constraints. Since these are dependent on an operation policy, both the probability distributions of *S* and *spilQ* are unknown and they must be defined in terms of another random variable with a known distribution (Loucks and Dorfman, 1975). This can be achieved through the use of *linear decision rules* (LDR) which defines the storage and spill releases in terms of the inflow, another random variable whose probability distribution can be constructed based on historical records. Then a deterministic equivalent formulation can be derived and used in the optimization model. The second challenge is to determine the highest possible reliability levels of meeting the preferred storage and spillway releases regimes. This can be accomplished running the model several times for different reliability levels, each time with a higher reliability that in the previous run, until an infeasible operation is encountered. The increments in the reliability levels must be small enough in order to accurately find the highest possible level. The multiple running of the model in order to find these reliability levels is equivalent to the multiple runs required by the variation of the weight coefficients in the original OPT formulation. Nevertheless, it is easier for the user to increase the reliability levels than to take decisions about the variation of the weight coefficients.

4.2.1 The Definition of Linear Decision Rules

Linear decision rules have been used in reservoir system optimization to determine optimal operation policy rules in LP applications. Basically, they define storage and spillway releases in terms of the inflow and a deterministic variable. Loucks and Dorfman [1975] proposed the following general syntax for a LDR:

$$spilQ_{p,t} = (1-\lambda) * Inflow_{p,t} + S_{p,t-1} - b_t \qquad (4.7)$$

Where b is an unknown determinist variable defined for each time step of the study period and λ is a parameter with values between 0 and 1 that indicates how much of the inflow will be considered in the spillway operation rule. In the original linear decision proposed by Revelle et al. [1969] the λ parameter was equal to 1, and therefore the spillway releases relied only on storage during the previous time step. Loucks [1970] found that this assumption yielded conservative results, and hence he proposed a value of 0 for λ. Sreenivasan and Vedula [1996] also used the general LDR with λ equal to 0, but in addition, they incorporated the turbine releases as an additional deterministic variable in the rule. Several other authors have proposed different linear decision rules, but in this paper we perform that analysis using the general decision rule proposed by Loucks and Dorfman incorporating the turbine releases as suggested by Sreenivasan and Vedula as outlined in Equation 4.8:

$$spilQ_{p,t} = (1-\lambda) * Inflow_{p,t} + S_{p,t-1} - turbQ_{p,t} - b_t \qquad (4.8)$$

4.2.2 Development of Linear Decision Rules for a Multi-Reservoir System

Before the LDR can be used in the chance constraints, the spillway release must be defined in terms of deterministic variables or random variables with known distributions. This means that equation 4.8 must be modified in order to eliminate the dependence of $spilQ_{p,t}$ over $S_{p,t-1}$. This can be accomplished replacing the LDR into the equation of storage continuity. Equation 4.9 presents the continuity relationship for a single reservoir:

$$S_{p,t} = S_{p,t-1} + Inflow_{p,t} - turbQ_{p,t} - spilQ_{p,t} \qquad (4.9)$$

Using a λ value of 0 and substituting equation 4.8 into the continuity equation yields:

$$S_{p,t} = b_{p,t} \qquad (4.10)$$

Therefore, storage is set equal to the deterministic variable b. This applies to all the time steps of the study period, thus the spillway releases in the LDR can be expressed in terms of deterministic variables and random variables with known distributions:

$$spilQ_{p,t} = Inflow_{p,t} - turbQ_{p,t} + b_{p,t-1} - b_t \qquad (4.11)$$

Equations 4.10 and 4.11 can now be substituted into the chance constraints of the model presented in equations 4.2 to 4.6. Nevertheless, in a multi-reservoir system, the total inflow that enters a reservoir is the sum of the local inflow, the turbine discharge and spillway releases from upstream reservoirs. Equation 4.12 presents an extension of the LDR for a multi-reservoir system[1]:

$$spilQ_{p,t} = Upstream\_Inflows_{p,t} + \sum_{j=1}^{p-1} Link1_{j,p} * \left[inflow_{j,t} - turbQ_{j,t} + b_{j,t-1} - b_{j,t}\right] + \sum_{j=1}^{p-1} Link2_{j,p} * turbQ_{j,t} - turbQ_{p,t} + b_{p,t-1} - b_{p,t} \quad (4.12)$$

Where the parameters Link1 and Link2 are flags that indicate if there are some spillway and turbine connections between the reservoirs. These connections will be described with an example in the following section. After replacing Equations 4.10 and 4.12 into the original chance constraints, the deterministic equivalent constraints can be defined. In Equations 4.3 and 4.4 the storage random variable is replaced by the deterministic variable *b*, therefore the probability can be eliminated as it is shown in Equations 4.13 and 4.14.

$$b_{p,t} \geq STarget\_low_{p,t} \quad (4.13)$$

$$b_{p,t} \leq STarget\_up_{p,t} \quad (4.14)$$

In Equations 4.5 and 4.6, after replacing the spill variable with the linear decision rule shown in Equation 4.12, the deterministic equivalent can be found moving all the terms inside the probability to the right, except the inflow, and applying the inverse cumulative distribution function of the inflow to both sides of the external inequality. The resulting constraints are shown in Equations 4.15 and 4.16.

$$F^{-1}_{Upstream\_Inflows}\left[1 - \alpha_{p,t}\right] + \sum_{j=1}^{p-1} Link1_{j,p} * \left[inflow_{j,t} - turbQ_{j,t} + b_{j,t-1} - b_{j,t}\right] +$$

[1] The deduction of Equation 4.12 is presented in Appendix A

$$\sum_{j=1}^{p-1} Link2_{j,p} * turbQ_{j,t} - turbQ_{p,t} + b_{p,t-1} - b_{p,t} \geq spilQTarget\_low_{p,t} \quad (4.15)$$

$$F_{Upstream\_Inflows}^{-1}\left[\beta_{p,t}\right] + \sum_{j=1}^{p-1} Link1_{j,p} * \left[inflow_{j,t} - turbQ_{j,t} + b_{j,t-1} - b_{j,t}\right] +$$

$$\sum_{j=1}^{p-1} Link2_{j,p} * turbQ_{j,t} - turbQ_{p,t} + b_{p,t-1} - b_{p,t} \leq spilQTarget\_up_{p,t} \quad (4.16)$$

Where $F_{Upstream\_Inflows}^{-1}[.]$ is the inverse cumulative distribution function of reservoir p upstream inflows. It can be observed that the storage and the spillway releases are no longer variables in the new deterministic constraints. Instead, power generation, turbine releases and the deterministic parameter b would be the new outputs of the model.

4.3 Application and Results

The proposed model formulation was tested for a case study using the Stave Falls hydropower projects located near Mission, British Columbia. Figure 4.3 presents a simplified schematic of the Stave River system. Table 4.1 presents the values for the parameters *Link1* and *Link2* for the Stave Falls project. If the spillway from one reservoir is able to reach another reservoir going through the spillways of the intermediate reservoirs, the *Link1* flag between them is equal to 1. A similar criterion applies for the turbine discharge and *Link2*. If there are no intermediate reservoirs, *Link1* and *Link2* will be 1 if the spillway and turbine releases can flow from one reservoir into the other. The *Link1* and *Link2* parameters are used to define the deterministic equivalents of the chance constraints, using equations 4.13 to 4.16[2].

[2] Appendix A provides a more detailed explanation of the parameters *Link1* and *Link2*

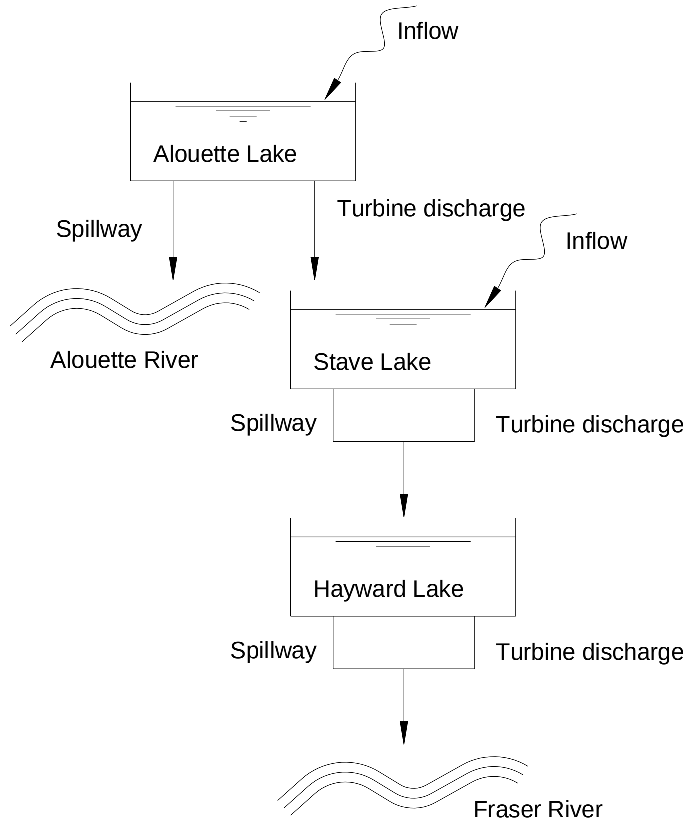

Figure 4.3 Hydraulic configuration of the Stave Falls project.

Table 4.1 Link1 and Link2 values for the Stave Falls project.

	Link1 flag (spillway discharge)			Link2 flag (turbine discharge)		
	Alouette L.	Stave L.	Hayward L.	Alouette L.	Stave L.	Hayward L.
Alouette L.	0	0	0	0	1	1
Stave L.	0	0	1	0	0	1
Hayward L.	0	0	0	0	0	0

Table 4.2 presents the preferred storage and spillway operating regimes for the Stave Falls project considering different demands such as flood control, recreation and environmental protection.

Table 4.2 Preferred elevation/storage and spillway regimes for the Stave Falls Project.

		Dates	Minimum	Maximum
			Storage (m^3) Spillway release (m^3/s)	
Alouette Lake	Storage	Jan 1 to Mar 31	697.74	1880.91
		April 1 to April 14	697.74	2274.13
		April 15 to June 14	1742.4	2274.13
		June 15 to July 15	1770.07	2274.13
		July 16 to Sep 5	1862.3	2274.13
		Sep 6 to Sep 15	1632.16	2274.13
		Sep 16 to Sep 30	697.74	2274.13
		Oct 1 to Dec 31	697.74	1880.91
	Spillway	Jan 1 to April 14	1.52	42.5
		April 15 to June 7	3	42.5
		June 8 to June 14	6	42.5
		June 15 to Dec 31	1.52	42.5
Stave Lake	Storage	Jan 1 to May 14	1207.5	6697.38
		May 15 to Jul 15	2743.51	6280.08
		Jul 16 to Sep 7	5231.15	6280.08
		Sep 8 to Dec 31	1207.5	6697.38
	Spillway	Jan 01 to Dec 31	0	500
Hayward Lake	Storage	Jan 1 to Feb 14	161.427	169.851
		Feb 15 to May 15	113.211	169.851
		May 16 to Oct 14	161.427	169.851
		Oct 15 to Nov 30	113.211	169.851
		Dec 01 to Dec 31	161.427	169.851
	Spillway	Jan 01 to Dec 31	0	340

The optimization model was run repeatedly by gradually increasing the reliability levels for equations 4.15 and 4.16 in each run. The selected LDR conditioned the model to keep the elevation within the preferred values throughout the optimization. For the spillway releases, two different combinations of reliability levels were tested. In the first alternative the reliability levels were increased as much as possible, giving priority to constraint 4.16. In the second alternative the reliability level corresponding to the minimum spillway in Alouette Lake was increased, but the reliability of spilling less than the maximum preferred value was decreased. Although it is possible to specify different reliability levels during the length of the study period, in both alternatives they were kept constant. The results are presented in Table 4.3.

Table 4.3 Model Results for Alternatives 1 and 2

	Alternative no. 1			Alternative no. 2			Average Annual Generation 1990-2010 (MWh)
	α	β	Annual Generation (MWh)	α	β	Annual Generation (MWh)	
Alouette L.	0.45	0.80	45,000	0.60	0.70	23,500	39,620
Stave L.	0.60	0.85	193,100	0.60	0.85	181,900	322,790
Hayward L.	0.60	0.85	239,500	0.60	0.85	225,000	343,160

It can be observed in Table 4.3 that the annual generation in Alouette Lake decreases when the reliability level α is increased. Similarly, the generation in the Stave and Hayward Lake is affected even though the reliability levels corresponding to these reservoirs were kept constant. In both alternatives the annual generation is below the average generation during the 1990-2010 period.

4.3.1 Use of the Linear Decision Rules in a Simulation Model

Using the results of Alternative no. 2, a simulation spreadsheet was developed for the testing of the LDR. Table 4.4 presents an example of this spreadsheet for Alouette Lake reservoir. Similar spreadsheets were prepared for the Stave and Hayward Lake reservoirs. The input of the simulation model is the inflow (column A). The turbine release and the parameter *b* are the outputs of the optimization model (columns B and C respectively). The spillway release (column D) is computed using the linear decision rule presented in Equation 5.14, while the storage (column E) is calculated using the continuity equation.

Table 4.4 Simulation spreadsheet for Alouette Lake reservoir

ALOUETTE LAKE RESERVOIR					
	A	B	C	D	E
	Inflow	Turbine release (m^3/s)	Parameter "b"	Spillway release (m^3/s)	Storage (m^3)
1	19.51	10.71	1,822.22	10.98	1,822.22
2	24.35	10.71	1,822.08	13.78	1,822.08
3	21.24	10.71	1,821.40	11.22	1,821.40
4	23.19	10.71	1,820.10	13.78	1,820.10
5	19.86	10.71	1,819.06	10.19	1,819.06
6	19.73	10.71	1,817.97	10.11	1,817.97
7	24.95	10.71	1,817.94	14.27	1,817.94
8	23.31	10.71	1,819.14	11.40	1,819.14
9	23.50	10.71	1,820.68	11.25	1,820.68
10	26.43	10.71	1,822.05	14.35	1,822.05
11	25.52	10.71	1,823.73	13.13	1,823.73
12	29.06	10.70	1,827.52	14.57	1,827.52
13	29.21	10.70	1,831.53	14.50	1,831.53
14	44.67	10.69	1,843.12	22.39	1,843.12
15	40.57	10.68	1,853.23	19.78	1,853.23
16	30.48	10.68	1,858.41	14.62	1,858.41
17	29.15	10.68	1,862.16	14.72	1,862.16
18	31.62	10.67	1,865.87	17.23	1,865.87
19	30.70	10.67	1,868.98	16.91	1,868.98
20	24.49	10.67	1,869.04	13.76	1,869.04
21	19.14	10.67	1,868.63	8.87	1,868.63
22	17.21	10.67	1,866.48	8.68	1,866.48
23	23.69	10.67	1,866.93	12.56	1,866.93
24	23.79	10.67	1,868.16	11.88	1,868.16
25	18.18	10.67	1,866.50	9.17	1,866.50
26	20.80	10.68	1,865.17	11.45	1,865.17
27	23.88	10.67	1,866.47	11.90	1,866.47
28	22.86	10.67	1,867.32	11.34	1,867.32
29	28.60	10.67	1,870.78	14.47	1,870.78
30	32.16	10.67	1,876.37	15.90	1,876.37
31	22.43	10.67	1,876.98	11.15	1,876.98

The simulation spreadsheets were tested using two synthetic inflow sequences for the Alouette and Stave Lakes (The local inflows in Hayward Lake are considered negligible). One of the sequences had low inflow values, and the other one had high inflow values. These sequences were forced to be within the extreme high inflows and extreme low inflows sequences defined with the reliability levels α and β during the optimization model. These sequences are shown in Figure 4.4.

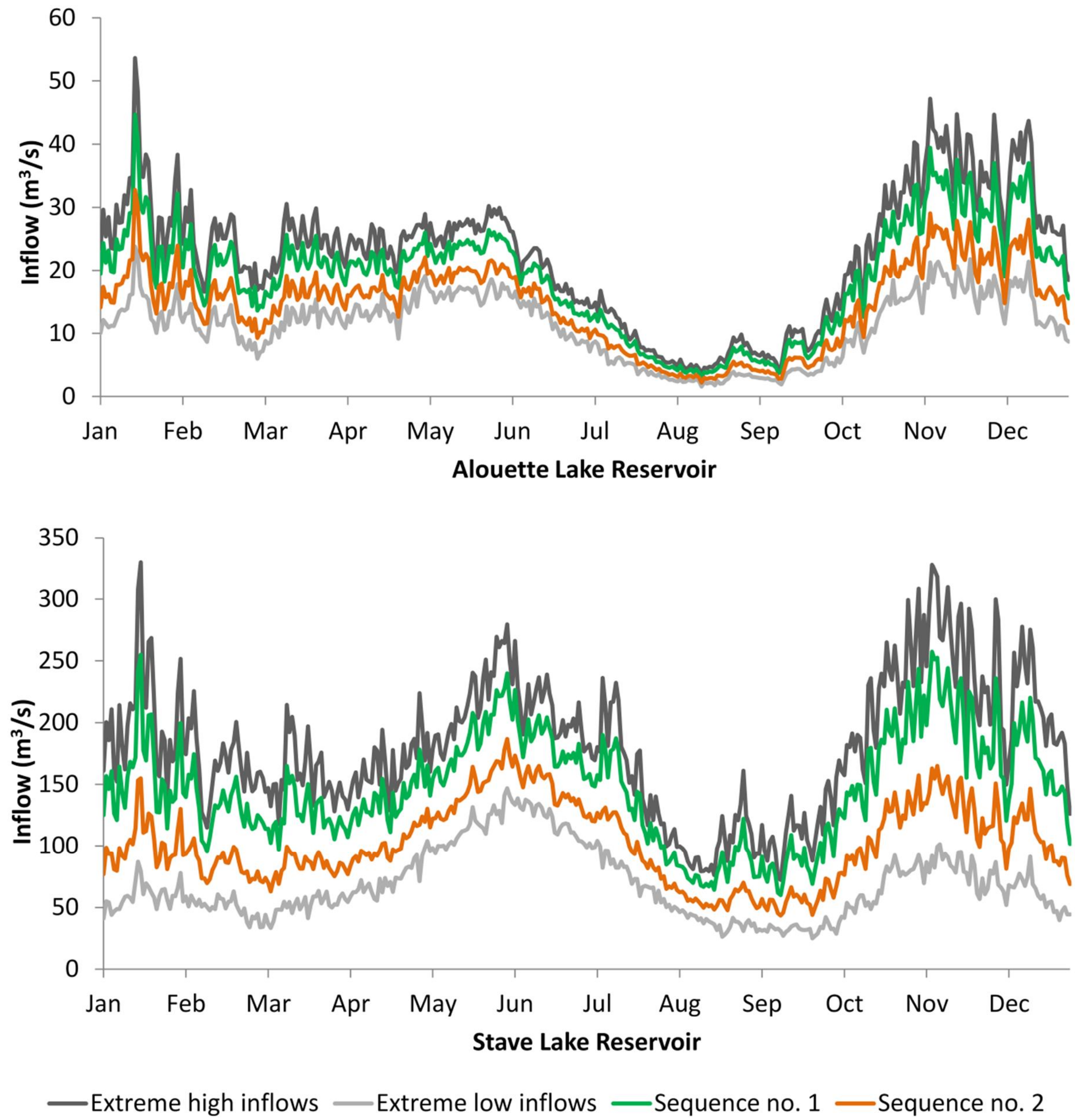

Figure 4.4 Synthetic inflow sequences for Alouette and Stave Lakes, used in the simulation spreadsheet

The data presented in Table 4.4 corresponds to the simulation results for January using the inflows of sequence no. 1. It can be observed that the computed storage values are equal to the corresponding decision parameter *b*; this is consistent with the relationship established in Equation 4.10. For the rest of the year and the other two reservoirs, the data is presented graphically in Figures 4.5 and 4.6. The spillway results for sequence no. 2 are presented in Figure 4.7. (The storage values for this sequence are also equal to the decision parameter *b*).

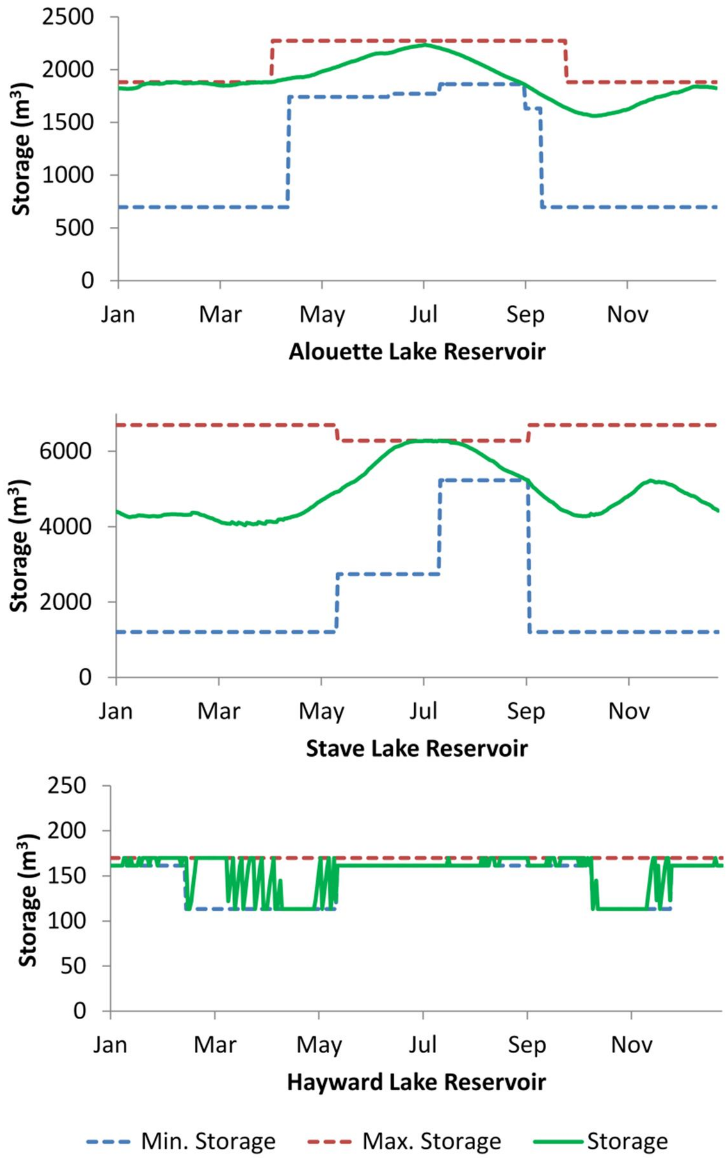

Figure 4.5 Storage results from simulation spreadsheet for inflow sequence no. 1

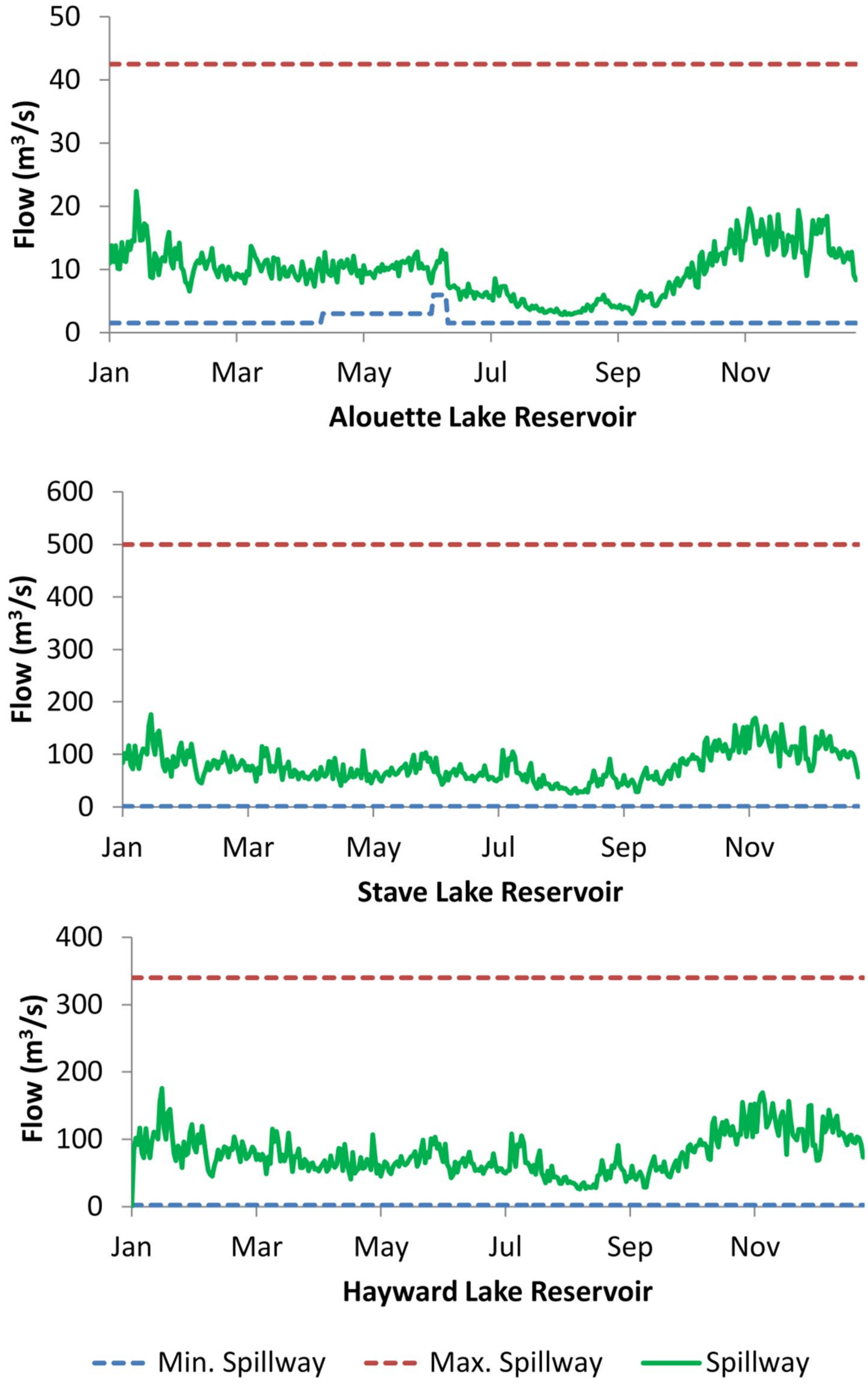

Figure 4.6 Spillway results from simulation spreadsheet for inflow sequence no. 1

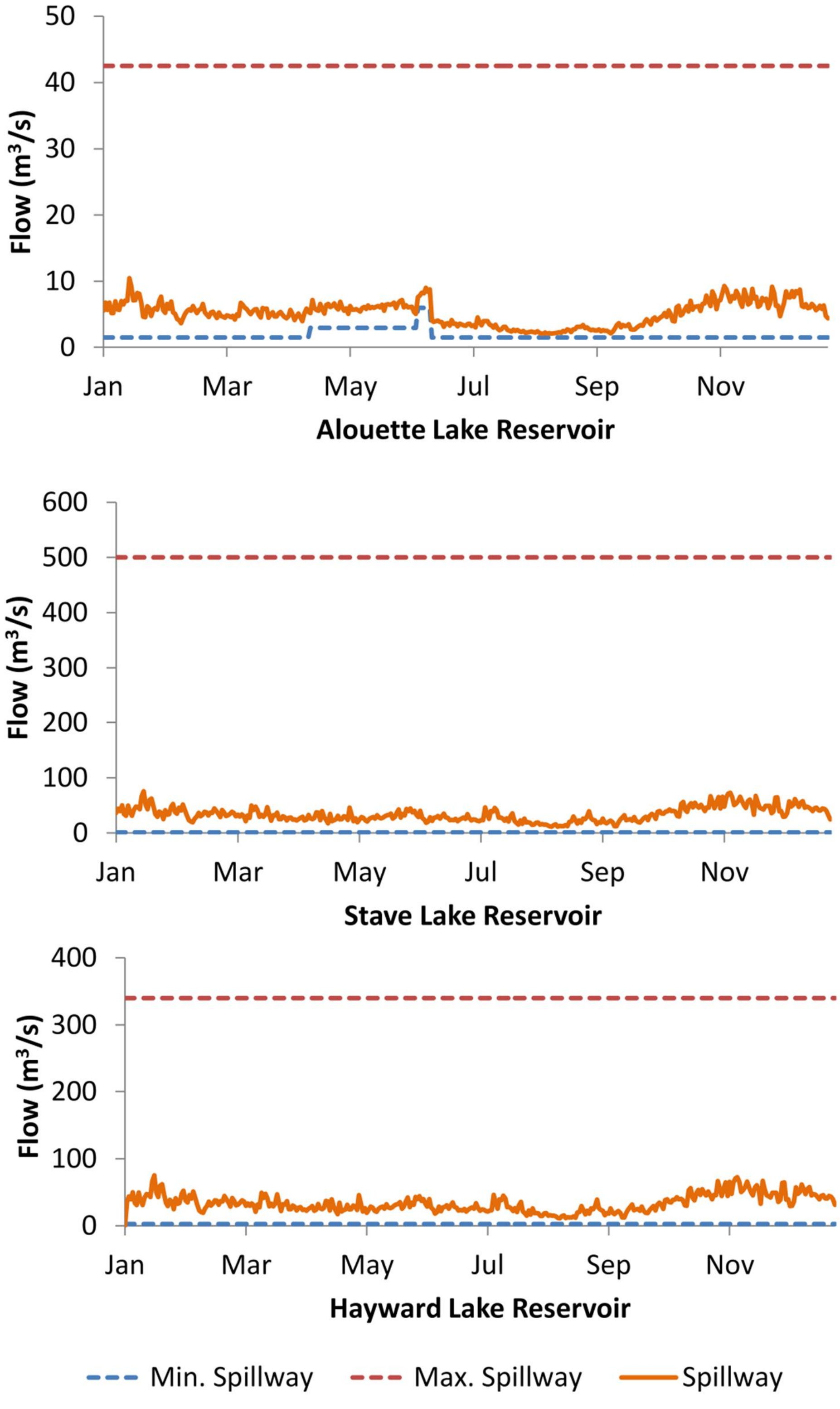

Figure 4.7 Spillway results from simulation spreadsheet for inflow sequence no. 2

Figures 4.5, 4.6 and 4.7 show that in the three reservoirs and for both inflow sequences, the spillway and storage values stay within the minimum and maximum targets. This is valid because the inflow sequences were forced to be within the extreme high and low inflow sequences used in the optimization model. Nevertheless, this will not always occur and in many occasions the inflows will be higher or lower than these extreme sequences. This is demonstrated in Figures 4.8 and 4.9 where the Alternative 2's extreme inflow sequences for Alouette and Stave Lakes are compared with the corresponding historical inflows from 1960 to 2010.

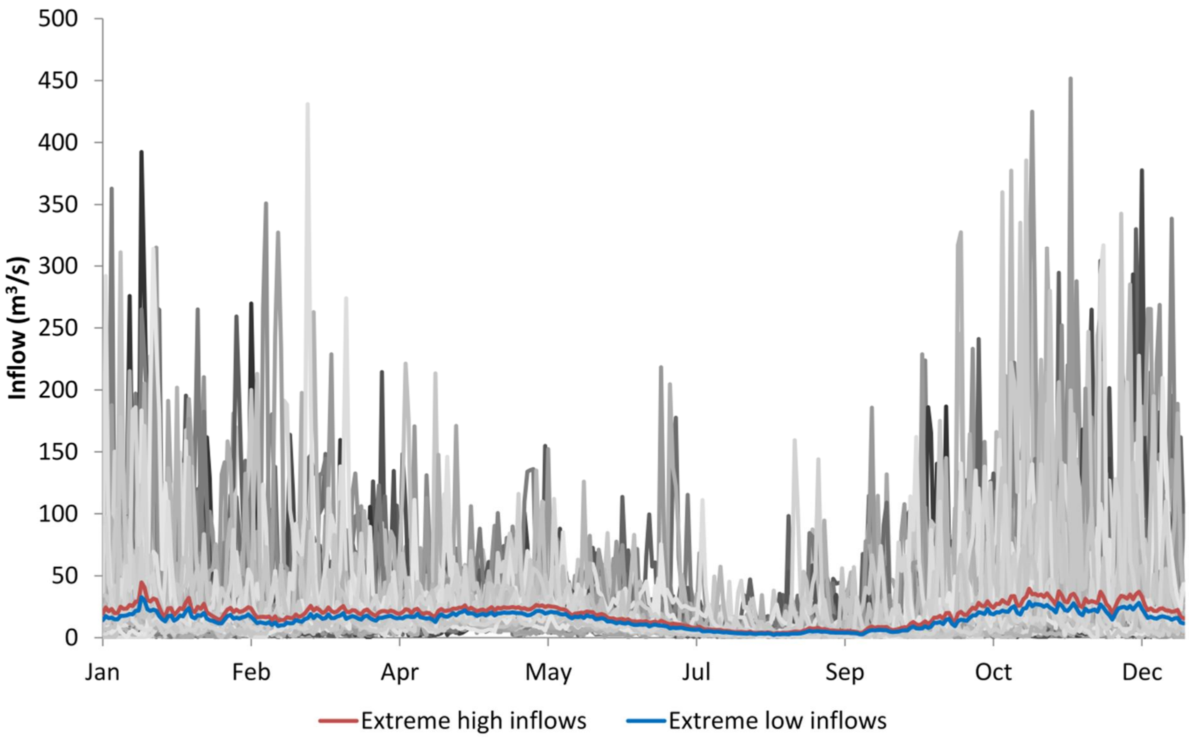

Figure 4.8 Comparison of Alternative 2's extreme inflow sequences and historical inflows in Alouette Lake

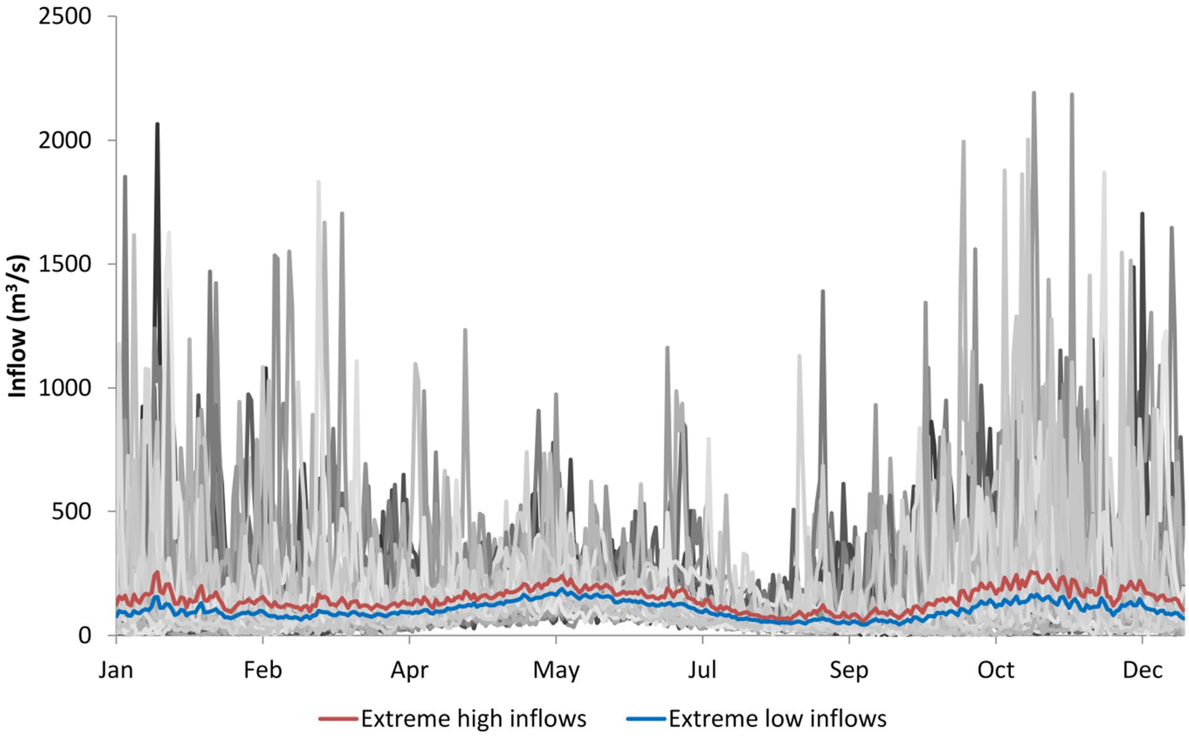

Figure 4.9 Comparison of Alternative 2's extreme inflow sequences and historical inflows in Stave Lake

Therefore, in order to successfully use the linear decision rules in a simulation spreadsheet it is necessary to establish additional "correcting rules" to deal with inflows that are unexpectedly high or low. These rules should be based on the experience and preferences of operators and decision-makers. Alternatively, the model can be run again aiming to increase the reliability levels (and hence pushing up and down the extreme inflow sequences); however, this requires revising and changing the target levels. It will be necessary to find a balance between desired minimum and maximum targets and reliability levels that are considered acceptable.

Chapter 5: Summary, Conclusions and Future Work

5.1 Summary

This thesis provided an overview of the main characteristics of the Operations Planning Tool (OPT), an optimization model used by planning engineers at BC Hydro for the operation of multi-reservoir systems. The model has a deterministic linear programming formulation: it assumes that the inflow is known with certainty before the optimization is performed. The review examined how the model considers multiple objectives in the optimization through the use of user-defined penalty functions. Two different formulations were incorporated into the OPT model in order to achieve the following goals: analyze and optimize the units' maintenance scheduling in multi-reservoir systems and modify the current OPT model to assess the inflow uncertainty and avoid the use of penalty functions and weighting factors. The first formulation was tested with the *Bridge River* multi-reservoir system while the second one was tested with the *Stave Falls* multi-reservoir system, both located in British Columbia, Canada.

The development of the formulation for the study of maintenance scheduling was based on the introduction of a two-stage Mixed-Integer Linear Programming algorithm in the OPT model. This formulation makes use of a binary variable that selects a Generation Production Function (GPF) for each time step of the study period. The GPFs, which are indexed over forebay elevation and the units' availability combination, calculates the power generation as a function of turbine discharge. Some constraints were introduced in the model in order to determine the schedule in which the maintenance outages specified by the user optimized the model's individual objectives. The optimization model selects the GPFs that correspond to the optimal outage schedule.

In the second formulation, the inflow uncertainty was considered in the optimization model through the use of linear decision rules (LDR) and chance constraints. The LDRs are operating rules that define the spillway releases and reservoir storage as a linear function of the inflow, the turbine releases and a deterministic decision parameter. The chance constraints are similar to traditional constraints, but they consider that certain percentage of the time they will not be satisfied. In this formulation, the chance constraints were used to consider the probability of the spillway releases and forebay elevation not being within a preferred range of values established by the user. This uncertainty was transmitted to the inflow through the LDRs. Therefore, the outputs of the model were the daily turbine releases and decision parameters that ensure that if the LDRs are used, the spillway releases and forebay elevation targets are met with the reliability level specified by the user. A simulation spreadsheet was developed to test the LDR with two synthetic inflow sequences.

5.2 Conclusions

The formulation developed for the analysis of maintenance scheduling in multi-reservoir systems aims to establish the outages' timing that optimizes the OPT's multiple objectives. This formulation is based in a two-stage process: in the first stage all the possible maintenance scheduling solutions are established given the duration of the outages, the time step and the length of the study period. These solutions are an input of the modified OPT-MILP model, which in the second stage of the formulation is run in order to find the optimal solution. The formulation provides the user with several configuration alternatives including the specification of sequential and simultaneous unit outages. This feature can be used to reduce the total number of solutions and therefore accelerate the solution time of the optimization model.

In the second stage the model computes the "outage cost" which is the difference in revenue between a scenario with outage scheduling and a hypothetical scenario without any outages; this cost can be used to compare the cost of different outage scenarios. In the case study using the Bridge River system the outage schedule optimized by the model presented a lower cost than other scenarios with fixed outages.

The second formulation introduced in the OPT aimed to modify the model in order to consider the inflow uncertainty and avoid the use of penalty functions and weight coefficients and proposed the use of Linear Decision Rules, which provides the optimal spillway releases as a function of inflow, a deterministic decision parameter and the turbines release, which is assumed to be deterministic (i.e. is not conditioned by the inflow conditions). The spillway releases and forebay elevation terms of the original OPT objective function were converted into chance constraints. The user was required to specify the minimum probability of these constraints being satisfied. The model was run and if a feasible operation was found, the user was required to increase the reliability levels and run the model again. This process was repeated until an infeasible operation was encountered. Similarly, if the model was not able to find a feasible operation in the first run, the user had to either decrease the reliability levels or modify the target levels. This formulation offers the advantage that the weight coefficients and the penalty functions are no longer used in the objective function. This was one of the research goals established in Chapter 1, but instead the new formulation requires running the model several times. Although every optimization is usually completed in just a few minutes, the process of changing the reliability levels and running the model again can become a tedious task. The user has no way to know which reliability level or target must be modified when an infeasible operation is encountered; hence this must be figured out on a trial-and-error basis. Some authors have reported that the use of LDRs and chance constraints in reservoirs operation yields conservative results. In the test with the Stave Falls project, two different combinations of reliability levels were analyzed (see Table 4.3) and in most of them the annual power generation was below the average generation in the 1990 to 2010 period. This might confirm the conservative nature of the LDRs. Finally, it is important to point out that this formulation took out some of the original OPT modeling features, including the definition of multiple release structures for one reservoir and the use of rating curves to relate the storage and the spillway releases. Therefore, the operator would be required to decide how to release the spill flow recommended by the model.

5.3 Future Work

The maintenance outages formulation introduced in the OPT in Chapter 3 can be used to perform different type of studies, including:

- Assess the impact of maintenance outages in the spillway releases and forebay elevation regulations established in the hydroelectric project's Water Use Plans. The developed formulation can also help the user to understand how the outages in one generation station affect the operations in upstream and downstream reservoirs.
- Analyze the advantages and disadvantages of having sequential or simultaneous (paired) maintenance outages.
- Using the fixed-outages feature of the formulation the user can analyze the impact of forced outages during high inflow events.
- Assess how the weight coefficients and the penalty values used in the objective function affect the maintenance scheduling optimization.

The MIP model of the maintenance outages algorithm can be improved in the following ways:

- One of the main challenges in the proposed model is the long running time that some optimizations take. Although using fixed outages or specifying sequential outages significantly decrease the running time, the current formulation can be revised in order to further reduce the number of binary variables used in the model and therefore accelerate the optimization process.
- The optimization component of the model determines the optimal schedule of a single maintenance outage per unit. If the user wants to analyze more than one outage, the model needs to be with the first outage, specify the optimal schedule of this first run as a fixed outage and run the model again with the second outage duration. The model formulation can be modified to perform this process automatically and hence allow the user to specify more than one outage per unit.
- It might occur that the optimal schedule defined by the model is in conflict with the interests of the user; for these situations the model should allow the establishment of periods when the outages cannot take place and the model should find the optimal timing out of these dates.
- The inflow and price energy uncertainty should be considered in the model formulation. The inflow uncertainty can be addressed using the chance-constraints formulation proposed in the second formulation. Another alternative is to use an Implicit Stochastic Optimization approach, which relies on running a deterministic model several times for multiple inflow scenarios.

The chance-constraints formulation presented in Chapter 4 could be further extended in the following ways.

- Several authors have proposed different Linear Decision Rules. The proposed model could be modified in order to test alternative rules and analyze its effects on the results. Different values for the parameter λ used in the general rule proposed by Loucks and Dorfman [1975] can be tested.
- The model can be modified in order to automatically look for the highest reliability level that provides a feasible operation. In the current formulation, this process has to be carried out manually by the user.
- The proposed formulation calculates a total spillway release per reservoir, nevertheless in the original OPT the model provided the release for each spillway structure. It can be analyzed which modifications can be performed in order to include the same feature in the new model.

Bibliography

Abdelaziz B. and Sameh M., “Application of Goal Programming in a Multi-objective Reservoir Operation Model in Tunisia", *European Journal of Operations Research,* Vol. 133, (2001), pp 352-361.

Aghaei J., Ahmadi A., Shayanfar H. and Rabiee A. “Mixed integer programming of generalized hydro-thermal self-scheduling of generating units”, *Electrical Engineering*, Vol. 95, No. 2, (2013), pp 109-125.

Al Mamun A., “A Goal Programming Algorithm to Incorporate the Columbia River Non-Power Flow Requirements in the Columbia River Treaty Model”, the University of British Columbia, Vancouver, BC, Canada (2012)

Alhardi M. and Labib A., “Preventive maintenance scheduling of multi-cogeneration plants using integer programming", *Journal of the Operational Research Society,* Vol. 59 (2008), pp 503-509.

Babayev D. and Mardanov S., “Reducing the number of variables in integer and linear programming problems", *Computational Optimization and Applications,* Vol. 3, (1994), pp 99-109.

BC Hydro, “Making the Connection: The BC Hydro Electric System and How It Operates”, Vancouver, BC, Canada (2000).

BC Hydro, “Water Use Plan Orders Review Program”, Vancouver, BC, Canada (2014).

Becker L. and Yeh W., “Optimization of Real Time Operation of a Multi Reservoir System", *Water Resources Research,* Vol. 10, No. 6, (1974), pp 1107-1112.

Can E. and Houck M., “Real Time Reservoir Operations by Goal Programming", *Journal of Water Resources Planning and Management ASCE*, Vol. 110, No. 3, (1984), pp 297-309.

Carpentier P., Cohen G., Culioli J. and Renaud A. “Stochastic optimization of unit commitment: a new decomposition framework”, *IEEE Transactions on Power System,* Vol. 11, No. 2, (1996), pp 1067-1073.

Celeste A., Curi W. and Curi R., “Implicit Stochastic Optimization for Deriving Reservoir Operating Rules in Semiarid Brazil", *Pesquisa Operacional*, Vol. 29, No. 1, (2009), pp 223-234.

Changchit C. and Terrel M., “A Multiobjective Reservoir Operation Model with Stochastic Inflows", *Computers & Industrial Engineering,* Vol. 24, No. 2, (1993), pp 303-313.

Crowder H., Johnson E. and Padberg M., “Solving large-scale zero-one linear programming problems", *Operations Research,* Vol. 31, No. 5, (1983), pp 803-834.

Edwin K. and Curtius F., “New maintenance-scheduling method with production cost minimization via integer linear programming", *Electrical Power & Energy Systems,* Vol. 12, No. 3, (1990), pp 165-170.

Georgakakos A., Yao H. and Yu Y., “Control model for hydroelectric energy-value optimization", *Journal of Water Resources Planning and Management,* Vol. 123, No. 1, (1997), pp 30-38.

Guignard M. and Spielberg K., “Logical reduction methods in zero-one programming", *Operations Research,* Vol. 29, No. 1, (1981), pp 49-74.

Helston C., "Large Hydro by the Numbers", *Energybc.ca,* University of Victoria, 2012, Retrieved 8 July 2014 from *www.energybc.ca/profiles/largehydro.html.*

Hiew K., Labadie J. and Scott J. "Optimal operational analysis of the Colorado-Big Thompson project", *Computerized Decision Support Systems for Water Managers*, (1989), pp 632-646.

Ignizio J., "Introduction to Linear Goal Programming", 1st edition, Sage Publications, (1985)

Jones D. and Tamiz M., "Practical Goal Programming", 1st edition, Springer, (2010)

Karamouz M., Houck M. and Delleur J., "Optimization and Simulation of Multiple Reservoir Systems", *Journal of Water Resources Planning and Management ASCE,* Vol. 118, No. 1, (1992), pp 71-81.

Kiczko A. and Ermolieva T., "Multiple-Criteria Decision Support System for Siemianówka Reservoir under Uncertainties", *Management Safety of Heterogeneous Systems, Decisions under Uncertainties and Risks*, Vol. 658, (2012), pp 187-201.

Kothari D. and Ahmad A., "An expert system approach to unit commitment problem", IEEE-TENCON, Beijing, China (1993)

Kumar, A., T. Schei, A. Ahenkorah, R. Caceres Rodriguez, J.-M. Devernay, M. Freitas, D. Hall, Å. Killingtveit, Z. Liu, 2011: Hydropower. In IPCC Special Report on Renewable Energy Sources and Climate Change Mitigation [O. Edenhofer, R. Pichs-Madruga, Y. Sokona, K. Seyboth, P. Matschoss, S. Kadner, T. Zwickel, P. Eickemeier, G. Hansen, S. Schlömer, C. von Stechow (eds)], Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

Labadie J., "Optimal Operation of Multireservoir Systems: State-of-the-Art Review", *Journal of Water Resources Planning and Management ASCE,* Vol. 130, No. 2, (2004), pp 93-111.

Loganathan G. and Bhattacharya D. “Goal-Programming Techniques for Optimal Reservoir Operations”, *Journal of Water Resources Planning and Management ASCE,* Vol. 116, No. 6, (1990), pp 820-838.

Loucks D. “Some Comments on Linear Decision Rules and Chance Constraints”, *Water Resources Research,* Vol. 6, No. 2, (1970), pp 668-671.

Loucks D. and Dorfman P. “An Evaluation of Some Linear Decision Rules in Chance-Constrained Models for Reservoir Planning and Operation”, *Water Resources Research,* Vol. 11, No. 6, (1975), pp 777-782.

Lund J. and Ferreira I., “Operating rule optimization for Missouri River reservoir system", *Journal of Water Resources Planning and Management ASCE,* Vol. 122, No. 4, (1996), pp 287-295.

Marler R. and Arora J., “Survey of Multi-objective Optimization Methods for Engineering", *Structural and Multidisciplinary Optimization,* Vol. 26, (2004), pp 369-395.

Martin Q., "Automated real-time hydropower scheduling for lower Colorado River, Texas", *Watershed Management and Operations Management 2000,* M. Flug, D. Frevert, and D. W. Watkins, eds., section 57, chapter 1, ASCE, Reston, Va.

Muñoz L. and Ramos A., “Goal programming approach to maintenance scheduling of generating units in large scale power systems", *IEEE Transactions on Power Systems,* Vol. 14, No. 3, (1999), pp 1021-1028.

Nayak S. and Arora S., “Optimal Capacities for a Multireservoir System Using the Linear Decision Rule”, *Water Resources Research,* Vol. 7, No. 3, (1971), pp 485-498.

Oliveira R. and Loucks D., “Operating Rules for Multireservoir Systems”, *Water Resources Research,* Vol. 33, No. 4 (1997), pp 839-852.

Peng C. and Buras N., “Practical Estimation of Inflows into Multireservoir System", *Journal of Water Resources Planning and Management ASCE*, Vol. 126, No. 5, (2000), pp 331-334.

Perez S., "Application of Benders’ decomposition to power plant preventive maintenance scheduling", *European Journal of Operational Research*, Vol. 184 (2008), pp 759-777.

Revelle C., Joeres E. and Kirby W. “The Linear Decision Rule in Reservoir Management and Design. 1, Development of the Stochastic Model”, *Water Resources Research,* Vol. 5, No. 4, (1969), pp 767-777.

Revelle C. and Kirby W. “Linear Decision Rule in Reservoir Management and Design. 2, Performance Optimization”, *Water Resources Research,* Vol. 6, No. 4, (1970), pp 1033-1044.

Revelle C., Whitlatch E. and Wright J. “Civil and Environmental Systems Engineering”, 2nd edition, Pearson Prentice Hall, (2004)

Savelsbergh M., “Preprocessing and probing techniques for mixed integer programming problems", *ORSA Journal on Computing,* Vol. 6, No. 4, (1994), pp 445-454.

Shawwash Z., Siu T. and Russell D. “The BC Hydro Short Term Hydro Scheduling Optimization Model”, *IEEE Transactions on Power Systems,* Vol. 15, No. 3, (2000), pp 1125-1131.

Shimomura K., Ishitoku H., Sakurai S. and Hirose F. “Advanced technologies of preventive maintenance for thermal power plants”, *Hitachi Review,* Vol. 51, (2002), pp 137-142.

Sigvaldason O., “A Simulation Model for Operating a Multipurpose Multireservoir System”, *Water Resources Research,* Vol. 12, No. 2 (1976), pp 263-278.

Smith G., “The BC Hydro Static Plant Unit Commitment Program”, Vancouver, BC, Canada (1998)

Sreenivasan K. and Vedula S. “Reservoir operation for hydropower optimization: A chance-constrained approach”, *Sadhana,* Vol. 21, No. 4, (1996), pp 503-510.

Stedinger J., “The Performance of LDR Models for Preliminary Design and Reservoir Operation", *Water Resources Research,* Vol. 20, No. 2, (1984), pp 215-224.

Tang Y., “A Mixed Integer-Linear Programming Model for Solving the Hydroelectric Unit Maintenance Scheduling Problem”, the University of British Columbia, Vancouver, BC, Canada (2007)

Takriti S. and Birge J. “Using integer programming to refine lagrangian-based unit commitment solutions”, *IEEE Transactions on Power Systems*, Vol. 15, No. 1, (2000), pp 151-156.

Thomas H. and Revelle R., “On the Efficient Use of High Aswan Dam for Hydropower and Irrigation", *Management Science,* Vol. 12, No. 8, (1966), pp 296-310.

Yamayee Z., Sidenblad K. and Yoshimura M. “A computationally efficient optimal maintenance scheduling method”, *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-102, No. 2, (1983), pp 330-338.

Yeh W., "Reservoir Management and Operations Models: A State-of-the-Art Review", *Water Resources Research,* Vol. 21, No. 12, (1985), pp 1797-1818.

Yellen J., Al-Khamis T., Vemuri S. and Lemonidis L. “A decomposition approach to unit maintenance scheduling”, *Transactions on Power Systems,* Vol. 7, No. 2, (1992), pp 726-733.

Yi J., Ahmadi A., Labadie J. and Stitt S. “Dynamic optimal unit commitment and loading in hydropower systems”, *Journal of Water Resources Planning and Management*, Vol. 129, (2003), pp 388-398.

Young G., " Finding reservoir operating rules", *Journal of Hydraulic Engineering ASCE*, Vol. 93, No. 6, (1967), pp 297-321.

Appendix A: Derivation of a General Linear Decision Rule for a Multi-Reservoir System

This appendix describes the derivation of a generalized decision rule for a multi-reservoir system from Equation 4.11. This expression is presented again in Equation A.1 as a reference.

$$spilQ_{p,t} = Inflow_{p,t} - turbQ_{p,t} + b_{p,t-1} - b_t \quad \text{(A.1)}$$

This linear decision rule (LDR) for a single reservoir system defines the spillway release as a function of the reservoir inflow, the turbine releases and the deterministic variable *b* in the present and previous time step. In order to extend this LDR to a multi-reservoir system, the inflow that enters a single reservoir must consider the release and turbine discharges from upstream reservoirs as it is shown in Equation A.2:

$$Total\_Inflow_{p,t} = Inflow_{p,t} + \sum_{j=1}^{m} A_{j,p} * spilQ_{j,t} + \sum_{j=1}^{m} B_{j,p} * turbQ_{j,t} \quad \text{(A.2)}$$

Where $A_{j,p}$ is equal to 1 if the spillway releases from reservoir j flows into reservoir p, otherwise is 0, and *m* is the total number of reservoirs in the system. Similarly, $B_{j,p}$ is used to indicate whether the turbine releases from reservoir *j* are flowing into reservoir *p*. Equation A.3 results from substituting *Total_Inflow* from Equation A.2 into the *inflow* of Equation A.1:

$$spilQ_{p,t} = Inflow_{p,t} + \sum_{j=1}^{m} A_{j,p} * spilQ_{j,t} + \sum_{j=1}^{m} B_{j,p} * turbQ_{j,t} - \mathrm{turbQ}_{\mathrm{p,t}} + \mathrm{b}_{\mathrm{p,t-1}} - \mathrm{b}_{\mathrm{t}} \quad \text{(A.3)}$$

Nevertheless, now the spillway release from reservoir *p* is dependent on the spillway releases from upstream reservoirs and they must be replaced by the same LDR defined in Equation A.3. This process requires the multiple substitution of the LDR and it will be finalized when the equation is no longer a function of upstream spillway releases. An example of this is provided in the following section.

A.1 Example of the LDR Extension into a Multi-Reservoir System

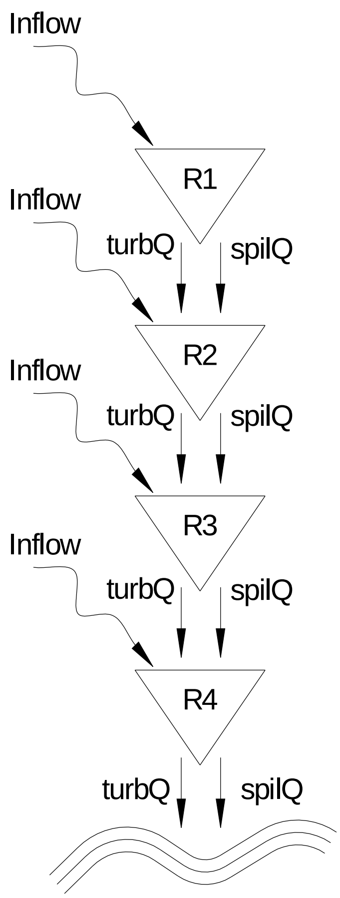

Figure A.1 Example of a multi-reservoir system configuration.

The configuration of a simple four reservoir system is presented in Figure A.1. It can be observed that the four reservoirs, denominated R1, R2, R3 and R4, receive natural inflow. Both the spillway and turbine releases from R1, R2 and R3 flow into R2, R3 and R4 respectively, while the releases from R4 discharge into a watercourse. The spillway release from reservoir R1 can be defined using the original LDR shown in Equation A.1:

$$spilQ_{1,t} = Inflow_{1,t} - turbQ_{1,t} + b_{1,t-1} - b_{1,t} \tag{A.4}$$

For reservoir R2, the spillway can be defined from the extended LDR defined in Equation A.3:

$$spilQ_{2,t} = Inflow_{2,t} + A_{1,2} * spilQ_{1,t} + B_{1,2} * turbQ_{1,t} - turbQ_{2,t} + \mathrm{b}_{2,t-1} - \mathrm{b}_{2,t} \quad (A.5)$$

The spillway and turbine release summations included in Equation A.3 were expanded in Equation A.5, but those expressions with parameters A and B equal to 0 were excluded from the equation (e.g. $A_{2,2}$, $A_{3,2}$). If the spillway release from R1 (Equations A.4) is substituted in A.5, the spillway from R2 becomes:

$$spilQ_{2,t} = Inflow_{2,t} + A_{1,2} * \left[Inflow_{1,t} - turbQ_{1,t} + b_{1,t-1} - b_{1,t}\right] + B_{1,2} * turbQ_{1,t} - turbQ_{2,t} + \mathrm{b}_{2,t-1} - \mathrm{b}_{2,t} \quad (A.5)$$

Similarly, Equation A.3 is used to define spillway from reservoir R3:

$$spilQ_{3,t} = Inflow_{3,t} + A_{2,3} * spilQ_{2,t} + B_{2,3} * turbQ_{2,t} - turbQ_{3,t} + \mathrm{b}_{3,t-1} - \mathrm{b}_{3,t} \quad (A.6)$$

After replacing the spillway release from R2, Equation A.6 becomes:

$$\mathrm{spilQ}_{3,t} = \mathrm{Inflow}_{3,t} + \mathrm{A}_{2,3} * \left(\mathrm{Inflow}_{2,t} + \mathrm{A}_{1,2} * \left[\mathrm{Inflow}_{1,t} - \mathrm{turbQ}_{1,t} + \mathrm{b}_{1,t-1} - \mathrm{b}_{1,t}\right] + \mathrm{B}_{1,2} * \mathrm{turbQ}_{1,t} - \mathrm{turbQ}_{2,t} + \mathrm{b}_{2,t-1} - \mathrm{b}_{2,t}\right) + \mathrm{B}_{2,3} * \mathrm{turbQ}_{2,t} - \mathrm{turbQ}_{3,t} + \mathrm{b}_{3,t-1} - \mathrm{b}_{3,t} \quad (A.7)$$

In Equation A.8 the multiplications of Equations A.7 are expanded:

$$\mathrm{spilQ}_{3,t} = \mathrm{Inflow}_{3,t} + \mathrm{A}_{2,3} * \mathrm{Inflow}_{2,t} + \mathrm{A}_{2,3} * \mathrm{A}_{1,2} * \mathrm{Inflow}_{1,t} - \mathrm{A}_{2,3} * \mathrm{A}_{1,2} * \mathrm{turbQ}_{1,t} + \mathrm{A}_{2,3} * \mathrm{A}_{1,2} * \mathrm{b}_{1,t-1} - \mathrm{A}_{2,3} * \mathrm{A}_{1,2} * \mathrm{b}_{1,t} + \mathrm{A}_{2,3} * \mathrm{B}_{1,2} * \mathrm{turbQ}_{1,t} - \mathrm{A}_{2,3} * \mathrm{turbQ}_{2,t} + \mathrm{A}_{2,3} * \mathrm{b}_{2,t-1} - \mathrm{A}_{2,3} * \mathrm{b}_{2,t} + \mathrm{B}_{2,3} * \mathrm{turbQ}_{2,t} - \mathrm{turbQ}_{3,t} + \mathrm{b}_{3,t-1} - \mathrm{b}_{3,t} \quad (A.8)$$

The same process is repeated for the spillway releases from reservoir R4. The resulting expression is presented in equation A.9.

$$\mathrm{spilQ}_{4,t} = Inflow_{4,t} + A_{3,4} * \mathrm{Inflow}_{3,t} + A_{3,4} * \mathrm{A}_{2,3} * \mathrm{Inflow}_{2,t} + A_{3,4} * \mathrm{A}_{2,3} * \mathrm{A}_{1,2} * \mathrm{Inflow}_{1,t} - A_{3,4} * \mathrm{A}_{2,3} * \mathrm{A}_{1,2} * \mathrm{turbQ}_{1,t} + A_{3,4} * \mathrm{A}_{2,3} * \mathrm{A}_{1,2} * \mathrm{b}_{p,t-1} - A_{3,4} * \mathrm{A}_{2,3} * \mathrm{A}_{1,2} * \mathrm{b}_{t} + A_{3,4} * \mathrm{A}_{2,3} * \mathrm{B}_{1,2} * \mathrm{turbQ}_{1,t} - A_{3,4} * \mathrm{A}_{2,3} * \mathrm{turbQ}_{2,t} + A_{3,4} * \mathrm{A}_{2,3} * \mathrm{b}_{2,t-1} - A_{3,4} * \mathrm{A}_{2,3} * \mathrm{b}_{2,t} + A_{3,4} * \mathrm{B}_{2,3} * \mathrm{turbQ}_{2,t} - A_{3,4} * \mathrm{turbQ}_{3,t} + A_{3,4} * \mathrm{b}_{3,t-1} - A_{3,4} * \mathrm{b}_{3,t} - B_{3,4} * turbQ_{3,t} - turbQ_{4,t} + \mathrm{b}_{4,t-1} - \mathrm{b}_{4,t} \qquad \text{(A.9)}$$

It can be observed that the different *inflow*, *turbQ* and *b* terms are multiplied by two different configurations of the flag parameters *A* and *B*. The first configuration is simply the multiplication of the parameters A that "connects" reservoir R4 with the upstream reservoirs. For example the third term of Equation A.8 is the multiplication of the inflow that enters R2 and the *A* parameters between R2 and R4, e.g. $A_{3,4}*A_{2,3}$. The second configuration is similar, but the "connection" between the first reservoir and the following downstream reservoir is specified by the parameter *B* instead of *A* (e.g. $A_{3,4}*B_{2,3}*turbQ_{2,t}$). These configurations will be denominated *Link1* and *Link2*, and they will be defined as follows:

- If the spillway releases from reservoir *j* can reach reservoir *p* going through the spillway releases of the reservoirs between them, *Link1* will be 1. Otherwise, it will be 0.

  $$\mathrm{Link1}_{j,p} = \mathrm{A}_{j,j+1} * \mathrm{A}*_{j+1,j+2}*\ldots*\mathrm{A}_{p-1,p}$$

- If the turbine releases from reservoir *j* can reach reservoir *p* going through the spillway releases of the reservoirs between them, *Link2* will be 1. Otherwise, it will be 0.

  $$\mathrm{Link2}_{j,p} = \mathrm{B}_{j,j+1} * \mathrm{A}*_{j+1,j+2}*\ldots*\mathrm{A}_{p-1,p}$$

After replacing these new parameters in Equation A.9, it becomes:

$$\mathrm{spilQ}_{4,t} = Inflow_{4,t} + Link1_{3,4} * \mathrm{Inflow}_{3,t} + Link1_{2,4} * \mathrm{Inflow}_{2,t} + Link1_{1,4} * \mathrm{Inflow}_{1,t} - Link1_{1,4} * \mathrm{turbQ}_{1,t} + Link1_{1,4} * \mathrm{b}_{p,t-1} - Link_{1,4} * \mathrm{b}_{t} + Link2_{1,4} * \mathrm{turbQ}_{1,t} - Link1_{2,4} * \mathrm{turbQ}_{2,t} + Link1_{2,4} * \mathrm{b}_{2,t-1} - Link1_{2,4} * \mathrm{b}_{2,t} + \mathrm{Link2}_{2,4} * \mathrm{turbQ}_{2,t} - Link1_{3,4} * \mathrm{turbQ}_{3,t} + Link1_{3,4} * \mathrm{b}_{3,t-1} - Link1_{3,4} * \mathrm{b}_{3,t} - Link2_{3,4} * turbQ_{3,t} - turbQ_{4,t} + \mathrm{b}_{4,t-1} - \mathrm{b}_{4,t} \quad \text{(A.10)}$$

In Equation A.11 the terms with similar *Link* parameters are associated:

$$\mathrm{spilQ}_{4,t} = Inflow_{4,t} + Link1_{3,4} * \mathrm{Inflow}_{3,t} + Link1_{2,4} * \mathrm{Inflow}_{2,t} + Link1_{1,4} * \mathrm{Inflow}_{1,t} - Link1_{1,4} * [\mathrm{turbQ}_{1,t} + \mathrm{b}_{p,t-1} - \mathrm{b}_{t}] + Link2_{1,4} * \mathrm{turbQ}_{1,t} - Link1_{2,4} * [\mathrm{turbQ}_{2,t} + \mathrm{b}_{2,t-1} - \mathrm{b}_{2,t}] + \mathrm{Link2}_{2,4} * \mathrm{turbQ}_{2,t} - Link1_{3,4} * [\mathrm{turbQ}_{3,t} + \mathrm{b}_{3,t-1} - \mathrm{b}_{3,t}] - Link2_{3,4} * turbQ_{3,t} - turbQ_{4,t} + \mathrm{b}_{4,t-1} - \mathrm{b}_{4,t} \quad \text{(A.11)}$$

This equation can be simplified defining a new parameter called *Upstream_Inflows*, which will be equal to:

$$Upstream\_Inflows_{4,t} = Inflow_{4,t} + Link1_{3,4} * \mathrm{Inflow}_{3,t} + Link1_{2,4} * \mathrm{Inflow}_{2,t} + Link1_{1,4} * \mathrm{Inflow}_{1,t} \quad \text{(A.12)}$$

If this new parameter is used in Equation A.11 and a summation over the different *Link* parameters is performed, the following equation can be obtained:

$$spilQ_{4,t} = Upstream\_Inflows_{4,t} + \sum_{j=1}^{3} Link1_{j,4} * [inflow_{j,t} - turbQ_{j,t} + b_{j,t-1} - b_{j,t}] + \sum_{j=1}^{3} Link2_{j,p} * turbQ_{j,t} - turbQ_{4,t} + b_{4,t-1} - b_{4,t} \quad \text{(A.13)}$$

The equation between brackets is the original LDR presented in Equation A.1. The cumulative inflows *Upstream_Inflows* and the value of the parameters *Link1* and *Link2* can be easily defined in any multi-reservoir system; therefore this new equation can be used as a general multi-reservoir LDR.

A.2 Additional Examples of Parameters *Link1* and *Link2*

Figure A.2 and Tables A.1 and A.2 are included in order to provide additional examples of the definition of the Link parameter values.

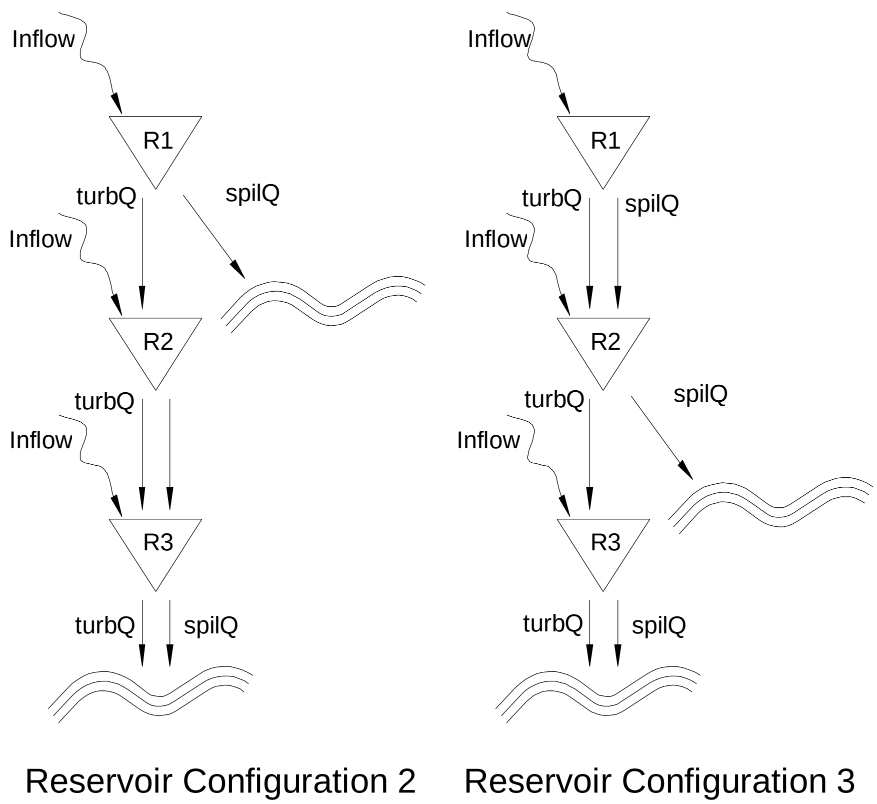

Figure A.2 Additional examples of multi-reservoir system configurations.

Table A.1 Link1 and Link2 values for multi-reservoir configuration 2 in Figure A.2

	Link1			Link2		
	R1	**R2**	**R3**	**R1**	**R2**	**R3**
R1	0	0	0	0	1	1
R2	0	0	1	0	0	1
R3	0	0	0	0	0	0

Table A.2 Link1 and Link2 values for multi-reservoir configuration 3 in Figure A.2

	Link1			Link2		
	R1	**R2**	**R3**	**R1**	**R2**	**R3**
R1	0	0	0	0	1	0
R2	0	0	0	0	0	1
R3	0	0	0	0	0	0