Cooperative Beamforming for Cognitive Radio Systems in Presence of Asynchronous Interference

by

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

The Faculty of Graduate and Postdoctoral Studies

(Electrical and Computer Engineering)

THE UNIVERSITY OF BRITISH COLUMBIA

(Vancouver)

August 2015

Abstract

Cognitive radio (CR) is considered a key enabling technology to exploit the underutilized and non-utilized radio spectrum bands. On the other hand, cooperative communication among nodes in CR networks, can improve the overall performance of CR systems, in terms of increasing data rates, attainable coverage range and overall energy efficiency, providing some diversity against shadow fading, and having low deployment costs.

In a CR network, cooperative transmit beamforming can be achieved via a number of single antenna-based CR nodes organizing themselves in a virtual antenna array and focusing their transmission in the direction of intended CR receiver. However, deploying the beamforming in such a cooperative manner faces several implementation challenges. Therefore, in this thesis, we tackle some of these critical challenges facing the design and implementation of cooperative CR networks.

The first challenge is referred to as *asynchronous interference*, that results from asynchronous arrival of the same signal from the set of cooperating CR nodes at primary receivers. Next, we address the problem of *feedback overhead* needed for cooperative beamforming. Specifically, each cooperating CR node requires knowing global information including other nodes' locations, in addition to accurate and instantaneous knowledge of their channel state information (CSI). We also tackle the problem of *imperfect CSI estimation.*

Another important aspect of implementing cooperative CR networks is studied in this thesis, which is described as follows. Since the cooperating nodes can be located in different locations, they contribute differently to received signals at the CR receivers, as well as to interference signals at the primary receivers. Therefore, we propose different *cooperating CR node selection* strategies, to be applied in conjunction with cooperative beamforming. Finally, we study different *participation decision making* strategies that enable each CR user to independently decide whether to participate in the cooperative transmission or not, based on an offered incentive for cooperation and estimated cost of participation in cooperative transmission represented in transmit power.

At the end of each chapter, we present some numerical examples to show the implications of ignoring different implementation challenges in the design of cooperative CR networks, and to assess the performance of the proposed solutions.

Preface

In what follows, we present a list of the significant contributions to research and development that have resulted from work presented in this thesis.

- The work presented in Chapter 2 has resulted in the following list of publications:
  - M. H. Hassan and M. J. Hossain, "Cooperative beamforming for cognitive radio systems with asynchronous interference to primary user," IEEE Transactions on Wireless Communications, vol. 12, pp. 5468-5479, November 2013.
  - M. H. Hassan and M. J. Hossain, "Cooperative beamforming for CR systems with asynchronous interference to primary user," IEEE International Conference on Communications (ICC), pp. 4272-4276, June 2013, Budapest, Hungary.
- The work covered in Chapter 3 has resulted in the following list of publications:
  - M. H. Hassan and M. J. Hossain, "Cooperative beamforming in CR systems with asynchronous interferences to multiple primary users," IEEE Wireless Communications and Networking Conference (WCNC), pp. 1200-1205, April 2014, Istanbul, Turkey.
  - M. H. Hassan, M. J. Hossain, and V. K. Bhargava, "Cooperative Beam-Forming for Cognitive Radio Based Broadcasting Systems in Presence of Asynchronous Interference," Submitted for possible publication.
- The work proposed in Chapter 4 has resulted in the following list of publications:
  - M. H. Hassan, M. J. Hossain, and V. K. Bhargava, "Distributed beamforming and autonomous participation decision making in cooperative CR systems," IEEE International Conference on Communications (ICC), in press, June 2015, London, UK.

– M. H. Hassan, M. J. Hossain, and V. K. Bhargava, "Distributed beamforming and autonomous participation decision making in cooperative CR systems in presence of asynchronous interference," Submitted for possible publication.

In all the research contributions made in this thesis, I was the primary researcher. I came up with the idea of the research independently. My contributions included conducting the literature review and identifying the research problems. In addition, I formulated the research problems, and carried out the mathematical analysis. My contributions also included designing the proposed schemes, simulating the network performance, and analyzing the results. I also wrote the associated manuscripts for publication.

The co-authors of the resulting manuscripts included my Ph.D. supervisors, Prof. V. K. Bhargava, and Dr. Md. J. Hossain, who provided directions on identifying the research problems. They also provided valuable comments on my research progress, and helped me by providing technical and editorial feedback during the preparation of these manuscripts.

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Glossary

AWGN	Additive White Gaussian Noise
BS	Base Station
CBS	Cognitive Base Station
CCRN	Cooperating Cognitive Radio Node
CR	Cognitive Radio
CSI	Channel State Information
DSA	Dynamic Spectrum Access
JLS	Joint Leakage Suppression
LBF	Leakage BeamForming
LBS	Learning-Based Strategy
LCPA	Low Complexity Power Allocation
MILP	Mixed-Integer Linear Problem
MINLP	Mixed-Integer Non Linear Problem
MRT	Maximum Ratio Transmission
NE	Nash Equilibrium
NLP	Non Linear Problem
OSAM	Overlay Spectrum Access Mechanism

PU Primary User

PR Primary Receiver

QPSK Quadrature Phase Shift Keying

RF Radio Frequency

RLBF Robust Leakage BeamForming

RTS Regret Testing-based Strategy

SDMA Space Devision Multiple Access

SINR Signal to Interference plus Noise Ratio

SLNR Signal to Leakage plus Noise Ratio

SOPA Sub-Optimal Power Allocation

SU Secondary User

USAM Underlay Spectrum Access Mechanism

ZFBF Zero Forcing BeamForming

Acknowledgments

I would like to express my sincere gratitude to my supervisors, Prof. Vijay K. Bhargava and Dr. Md. Jahangir Hossain for their continuous support throughout my PhD program, for their patience, motivation, and immense knowledge. I am particularly thankful for their invaluable suggestions, helpful advice, and critical comments. Without their precious support it would not have been possible to conduct this research. Their guidance helped me in all the time of research and writing of this thesis. I could not have imagined having better mentors for my PhD study.

Besides my supervisors, I would like to thank Prof. David Michelson and Prof. Z. Jane Wang for serving as examining committee members during my PhD qualifying examination and my PhD departmental exam. I am very grateful for their insightful comments and suggestions that encouraged me to widen my research from various perspectives. My sincere thanks also go to Prof. Victor Leung and Dr. Ryozo Nagamune for serving as the University examiners during my PhD University exam, for their valuable suggestions and editorial corrections.

I would also like to express my heartfelt gratitude to my wonderful friend and colleague in the Electrical and Computer Engineering department in UBC, who also happens to be my husband, Ahmed ElTantawy. I am very fortunate to have him by my side. His unconditional support has always been my greatest asset. In many occasions, he has provided me with the technical support and innovative ideas that helped me carry out my research.

Last but not least, I would like to thank my colleagues in the "Information Theory and Systems" laboratory, for the stimulating discussions, also for all the fun we had in the last four years. I am especially thankful to Roya Arab Loodaricheh and Sudha Lohani for their friendship and genuine support during my PhD research. Many thanks go to Shankhanaad Mallick and Rindranirina Ramamonjison for their feedback and critical suggestions. I am also grateful to all my colleagues in the Electrical and Computer Engineering department in UBC for making my PhD study such a great experience.

Dedication

I dedicate this dissertation to my family and friends. A special feeling of gratitude to my loving parents, Hoda Ali and Hassan Mohamed, whose words of encouragement have always sustained me throughout my life. Many thanks to my sister, Amal, my niece, Mai, and my nephew, Ahmed, for their constant and unconditional love. I give my deepest expression of love and appreciation to my husband, Ahmed, for the encouragement that he gave me and all the sacrifices he made throughout my PhD program. This thesis is also dedicated to my baby boy, Mazin, who was born one week after my PhD departmental exam. His birth was the biggest encouragement and greatest blessing that I have ever received.

Chapter 1

Introduction

One of the main challenges facing wireless communications is the scarcity of the available radio spectrum. With the ever-increasing demand of wireless services and the limited nature of radio spectrum, the need to use the radio spectrum more effectively has become a significant aspect in the design of wireless networks. Therefore, the wireless industry has called on governments and regulators to refarm the underutilized spectrum [1]. In particular, refarming of spectrum resources refers to reassigning the government-regulated radio spectrum for services with higher value. Users of the existing spectrum are forced out, although they may be compensated in some manner. The frequency bands are then assigned to communications services that yield greater economic or social benefit. However, refarming the underutilized spectrum is both expensive and time-consuming.

An attractive and less expensive alternative to spectrum refarming is to maximize the use of underutilized and/or non-utilized bands through spectrum sharing, which is known as dynamic spectrum access (DSA) [2], or opportunistic spectrum access. Recently, DSA has been made possible by the availability of software-defined radio, thanks to the development of fast enough processors both at servers and at terminals that can enable the DSA capabilities. Cognitive radio (CR) has been proposed in the literature as one of the key enabling technologies of DSA to exploit both the underutilized and non-utilized radio spectrum bands [3], [4]. In particular, a group of potential users, referred to as CR users or secondary users (SUs), can be given an opportunistic access to the underutilized/non-utilized spectrum. In addition, CR has been considered as a promising technology to meet many of the challenges in future 5G networks, including exploding traffic volumes, emerging traffic types and data services to support applications such as smart grid, and machine-to-machine communications [5–7].

Different DSA techniques have been envisioned and studied in the literature [8]. To exploit the unused and/or underused spectrum bands, mainly two different approaches have been widely considered in the literature, namely, underlay spectrum access mechanism (USAM) and overlay spectrum access mechanism (OSAM) [8, Chapter 3]. According to the OSAM, the spectrum utilization can be increased by granting SUs to opportunistically exploit the unused frequency bands of the primary users (PUs), for whom the spectrum was originally assigned [9], [10]. On the other hand, as per the USAM, the primary and the CR users can co-exist in the same spectral band (see for example, [11]). In other words, the USAM allows simultaneous sharing of underutilized frequency bands by the SUs along with the PUs. For this scenario, the interference introduced by the SUs to the primary receivers should be kept below a certain threshold specified by the PU system or the regulatory authority, see for example [12] for details.

On the other hand, cooperative communication improves the overall performance of wireless networks, including CR networks [13–15]. The driving motivation of cooperative communications is to simultaneously and drastically increase data rates, attainable coverage range and overall energy efficiency. Cooperative communication can be realized through network densification by the deployment of small cells, resulting in a heterogeneous architecture [16], or coordination of the transmission of multiple individual base stations [17]. In a cost-efficient CR network, cooperative transmission can also be realized through a set of CR devices, that support one antenna element at their terminals. This set of CR devices cooperate to create a virtual antenna array and act as cooperating nodes[1]. Such a distributed antenna array could offer substantial path loss gains and would also provide some diversity against shadow fading, in addition to having low deployment costs.

Moreover, the distributed antenna array of CR devices can allow concurrent transmission of both the CR users and the PUs at a given channel [18, 19], which further improves the performance of CR systems in terms of data rates. In particular, the cooperating nodes emulate a large highly directional antenna array which is referred to as cooperative transmit beamforming. Then, the CR system can make use of such cooperative beamforming to mitigate the interference to primary receivers (PRs). Hence, in order to send a common message, a number of single antenna-based CR nodes organize themselves in a virtual antenna array and focus their transmission in the direction of the intended CR receiver.

[1]In general, a cooperating node is a node (either a CR relay or a CR source) that participates in cooperative transmission.

In fact, cooperative transmit beamforming can be very effective for an USAM scenario, due to the severe constraint on the transmission power of CR system [2]. For example, if a CR transmitter wants to broadcast a common message to a group of CR receivers, the transmitter may not be allowed to transmit sufficient power to cover all the receivers due to the interference restriction imposed by the nearby primary system. As such, cooperative transmit beamforming allows CR users to access a given frequency channel, with a limited or no interference to the PR [20, 21]. Another example where cooperative transmit beamforming proves to be very effective is when the direct links between a CR transmitter and its receivers are unavailable due to path loss and/or shadowing. For such situations, many cooperative transmit beamforming techniques have been proposed in the literature to be used in CR networks, and a flourish of works has been done in this area, see for example [19, 22–25], and the references therein.

In [26], it was shown that the transmit beamforming scheme that can achieve the channel capacity limit in a CR network involves dirty-paper coding, which is a non-linear scheme. The authors in [26] considered the extreme situation where the introduction of the CR should have no effect whatsoever on the PU's operation and performance, i.e. the primary system should be oblivious to the presence of the CR system. Despite the optimality of the dirty-paper coding technique, it is difficult to be implemented in practical systems due to its high computational burden, since it requires iterative nonlinear methods for successive encodings and decodings. On the other hand, zero forcing beamforming (ZFBF) [20] is a suboptimal linear strategy that can enable concurrent transmission of the CR and the PU systems on the same frequency channel, but with reduced complexity relative to DPC. In ZFBF, each CR user in the set of cooperating nodes encodes the transmitted data stream by a specific weighting factor. Careful selection of the weighting factors can eliminate the introduced interference towards the PR, by taking advantage of spatial separation between the PR and the intended CR receiver. This type of communication, which supports multiple users simultaneously, is called space-division multiple access (SDMA) [27].

In ZFBF, the beamforming weights are found by inverting the composite channel matrix of the users. Using an innovative orthogonal projection technique, the authors in [20] obtained the ZFBF weights of the cooperating CR nodes to null the interference at all PRs in the network. Using the ZFBF technique, the authors in [24] proposed a cross-layer optimization of the transmission rate and scheduling scheme of the data packets at the CR source and at the cooperating CR nodes in the CR network. In [25], power allocation for cooperative CR networks was studied, along with user selection under imperfect spectrum sensing. Despite its reduced complexity, the ZFBF technique has been shown to achieve a fairly large fraction of DPC capacity, and the sum rate of the beamformer approaches that of DPC as the number of users goes to infinity [28, 29]. Such beamforming technique has also been proposed and studied for traditional wireless communication networks e.g., wireless sensor networks, as it potentially offers large increases in energy efficiency, attainable range and transmission rate, see for example [15] and the references therein. However, the ZFBF technique suffers from a design limitation, in which it requires the number of antennas in the transmitter side to be greater than that in the receiver side. Otherwise, the interference to other users in the network cannot be theoretically forced to zero. So in the context of cooperative CR networks, the spatial diversity order of the ZFBF technique proposed in [20] equals the number of cooperating CR nodes minus that of PRs.

From a purely fundamental perspective, the ultimate limiting factor of the performance of any wireless network appears to be the availability of good enough channel state information (CSI) to facilitate the processing of data at the multiple antennas of the transmit beamformer [30]. Considering factors like mobility, Doppler shifts, phase noise, and clock synchronization, acquiring high-quality CSI seems to be easier with a collocated antenna array than in a system where the antennas are distributed over a large geographical area [31]. However, the deployment costs of a collocated antenna array is likely to be much higher than that of a distributed antenna system with single-antenna elements. Moreover, a distributed antenna system provides an increase in the attainable coverage range, in addition to offering substantial path loss gains and providing some diversity against shadow fading. In conclusion, the achievable data rate gain, with such cooperative beamforming, is quite compelling in spite of the costs associated with it, including synchronizing the cooperating nodes and the local exchange of cooperating nodes' information [32].

Therefore, in this thesis, we tackle some of the critical challenges facing the design and implementation of cooperative CR networks. The first challenge is referred to as *asynchronous interference*, that results from the asynchronous arrival of the same signal from the set of cooperating CR nodes at the PRs. To address this problem, we develop innovative beamforming techniques that can account for the effects of the asynchronous interferences at the PRs. Next, we address the problem of feedback overhead needed for cooperative beamforming. Specifically, each cooperative CR node requires to know global information including other cooperating nodes' locations, in addition to accurate and instantaneous information about the states of their channels towards the PR and towards the CR receiver. To address this problem we provide a distributed beamforming technique that requires minimal amount of feedback overhead. We also tackle the problem of imperfect CSI estimation, through considering the case of erroneous channel estimation, and the case of having only statistical knowledge of the CSI.

Another important aspect of implementing cooperative CR networks is studied in this thesis, which is described as follows. Since the cooperating CR nodes can be located in different geographical locations, they contribute differently to the received signal at the CR receivers, as well as to the interference signals introduced at the PRs. Therefore, we propose different cooperating node selection strategies, to be applied in conjunction with the cooperative beamforming, to enhance the overall performance of the CR network. Finally, we propose the use of a suitable incentive for the CR users to participate in the cooperative transmission. Moreover, we study different participation decision strategies that enable each CR user to independently decide whether to participate in the cooperative transmission or not, based on the offered incentive and the estimated cost of participation in the cooperative transmission.

In Section 1.1, we present further details of each of the design problems addressed in this thesis. We define the objectives of the proposed techniques and the motivation behind each of them. Next, in Section 1.2, we provide an overview of the presented chapters and an outline of the thesis organization.

1.1 Motivations, Objectives, and Contributions

Although cooperative beamforming in CR network can improve the radio spectrum utilization and enhance the network performance, it faces a number of challenging issues. In this thesis, we tackle five critical problems facing the implementation of cooperative CR networks. We propose different techniques to solve these problems and we evaluate the performance of the proposed techniques. These problems are introduced as follows.

1.1.1 Asynchronous Interference at Primary Receivers

As mentioned earlier, implementation of beamforming in a cooperative manner can lead to various design implications that are addressed throughout this thesis. The first challenge facing cooperative beamforming is presented as follows. Given the fact that in practice different cooperating nodes are usually located in different geographical locations, signals from different transmitting CR nodes arrive with different propagation delays at each PR and at each CR receiver in the network. While the received signal from different cooperating CR nodes can be synchronized at the intended CR receiver by using a timing advance mechanism, which is currently employed in GSM and 3G cellular networks (see for example [33]), the received signals at other CR receivers and at the PRs from different transmitting nodes can be not synchronized simultaneously. As such, simultaneous transmissions from the cooperating CR nodes can cause asynchronous interference at the different PRs which is discussed in details in Section 2.2. Such asynchronous interference can highly degrade the performance of existing cooperative beamforming techniques, as we will see later in Chapters 2 and 3.

The asynchronous interference issue has been studied in [33] for conventional cooperative multi-cell mobile networks, where multiple base stations (BS's) cooperate together for down-link simultaneous transmission of information to each mobile user in the network. The proposed leakage suppression approaches in [33] aim to maximize the signal to interference plus noise ratio (SINR) of each of the active mobile users concurrently without any restriction on the amount of induced interference to a particular mobile user from different BS's. However, in cooperative beamforming for CR systems, the design objective is different which is to maximize the SNR at the CR receiver while keeping the interference at the PRs below a certain threshold specified by the PU system or the regulatory authority. To the best of our knowledge, the asynchronous interference problem has not been considered in CR networks in the literature except in this thesis and our published papers.

The shortcomings in the existing literature motivate us to pursue the following objectives in the context of applying cooperative beamforming in CR networks.

- Formulating a mathematical model of the asynchronous interference problem in cooperative CR networks.

- Developing a closed form expression for an innovative beamforming technique that account for the effect of the asynchronous interference at the PR, in case of a cooperative CR network with a single CR receiver and a nearby primary system with single PR.
- Generalizing this beamforming technique to the case of having multiple PRs and multiple CR receivers in the case of broadcast-based CR networks.
- Proposing a lower complexity cooperative beamforming scheme to be used in the case of having multiple interference constraints due to the presence of multiple PRs in the network.

The first two objectives are tackled in Chapter 2. The last two objectives are addressed in Chapter 3. The contributions made regarding these objectives are clarified as follows. In this thesis, we address the asynchronous interference issue in two different settings.

- First, we consider a cooperative CR network with a single CR receiver and a nearby primary system with single PR in Chapter 2. In particular, each CR node in the set of cooperating nodes encodes the transmitted data stream to the intended CR receiver by a specific beamforming weight, to forward the information to the CR receiver irrespective of whether the PU is silent or not. We formulate the asynchronous interference model for this scenario, and we develop a convex optimization problem which aims to maximize the signal power at the CR receiver while it maintains the interference power at the PR below a target threshold, which in turn decreases the service interruption/outage of the SU. We provide a closed form solution for the cooperative beamforming vector, where we call the beamforming technique in this case, *leakage beamforming* (LBF) method.
- In Chapter 3, we consider a more generalized setup where a group of cooperating CR nodes uses cooperative beamforming to broadcast a common message to multiple CR receivers, using a wireless communication channel assigned to a primary transmitter to transmit information to multiple PRs simultaneously. It is also considered that these PRs have different interference constraints in general. For this generalized setup with multiple primary and CR receivers, transmission to a specific CR receiver introduces asynchronous interferences, not only at the PRs, but also at all other CR receivers. Due to these asynchronous interferences, the optimal beamforming technique developed in the first setting cannot be directly extended for

this generalized system. Therefore, we formulate the cooperative beamforming design as an optimization problem that maximizes the weighted sum achievable transmission rate of the CR receivers while it maintains the interference thresholds at the PRs below their respective target interference thresholds. However, the optimal beamforming technique for this case is indeed intractable due to the non-convexity and non-linearity of the problem. Hence, we use an approximation to transform the optimization problem into a convex and linear one. Then sub-optimally we obtain the beamforming directions and allocate power among different beamforming directions. We also show that the LBF technique developed in the first scenario is considered as a special case of the generalized beamforming technique developed in this setting. Due to the multiple interference constraints in the case of having multiple PRs in the network, the power allocation scheme can be of a high complexity as will be discussed later in Chapter 3. Therefore, we also propose a low complexity power allocation scheme to be used in the cooperative beamforming design of the generalized system.

1.1.2 Imperfect Channel Estimation

The proposed cooperative beamforming methods can be implemented if the channel fading gains between the cooperating CR nodes and the PRs as well as the channel fading gains between the cooperating CR nodes and the CR receivers are all known at the cooperating nodes. Different possible scenarios have been considered in the literature in order to estimate the CSI between the cooperating CR nodes and the PR (see for examples, [34], [35]). One possible scenario is that when the PR periodically transmits pilot signal to its own transmitter. If the cooperating CR nodes know the pilot signals, the channel between the cooperating CR nodes and the PR can be estimated by the CR system assuming the reciprocity of the channel. This scenario is considered in [35].

However in many scenarios, the cooperating CR nodes may not have the perfect CSI from the PRs in the network. During the design process of the cooperative beamforming technique, we must account for the effects of such imperfect CSI estimation, to ensure a robust protection of the primary system. Robust protection means that the resulting interference at the PR remains below a predefined threshold even for the worst case estimate of the CSI towards that PR. This motivates us to pursue the following objectives.

- Developing a robust cooperative beamforming method that account for the effect of having an erroneous estimation of the CSI between the cooperating CR nodes and the PR.
- Developing a robust cooperative beamforming method that account for the effect of having only statistical CSI of the channels between the PRs and the cooperative CR nodes at each cooperating node.

Therefore in Chapters 2 and 3, we develop robust cooperative beamforming methods that consider two forms of having imperfect CSI estimation, which are presented as follows.

- Chapter 2 considers the case of having an erroneous estimation of the CSI between the cooperating CR nodes and the PR. The robust cooperative beamforming method, developed in this case, instantaneously meets the interference threshold at the PR despite the presence of estimation error in the CSI between the PR and each cooperating CR node. However this robust design of the beamforming technique comes at the expense of having decreased received signal power at the CR receiver, as shown later in Chapter 2.
- Chapter 3 considers having only statistical CSI of the channels between the PRs and the cooperative CR nodes at each cooperating node. In this case, the asynchronous interferences at the PRs are guaranteed in a statistical sense [11], [36], as shown in Chapter 3. In the absence of a mathematically tractable expression of the distribution of the random interference powers at the PRs, we develop an upper bound on the probability of introducing asynchronous interference power at a given PR beyond a given threshold. Then this developed upper bound is used to design a robust cooperative beamforming technique. The proposed beamforming technique can protect the primary network's functionality by satisfying the probabilistic interference constraints at all PRs in the network.

1.1.3 Cooperating CR Node Selection

Another important aspect of applying cooperative beamforming has been studied in this thesis, which is described as follows. Since the cooperating CR nodes can be located in different geographical locations, it is intuitive that different cooperating nodes will contribute differently to the asynchronous interferences at the primary receivers. Also, different cooperating nodes may contribute differently towards the received signal power at the CR receivers. The joint design of cooperative beamforming and cooperating node selection has been studied before for conventional cooperative networks, see for example [37] and the references therein. In the case of CR systems, the cooperating node selection scheme should include the constraint of keeping the interference at the PRs below their respective desired interference thresholds. This motivates us to pursue the following design objectives.

- Proposing an optimal cooperating node selection strategy, to be applied in conjunction with the cooperative beamforming methods, and study its performance.
- Proposing a lower complexity sub-optimal cooperating node selection strategy with shorter convergence time for a more practical design of the cooperative CR network.

These objectives are tackled in Chapter 3, where three different cooperating CR node selection strategies are proposed to be used in conjunction with the cooperative beamforming techniques. As will be shown later in Chapter 3, an improved performance can be achieved using cooperative beamforming when applying these cooperating CR nodes selection strategies compared to the case when all the cooperating nodes participate in beamforming. The contributions made in Chapter 3 regarding these design objectives are summarized as follows.

- We formulate the optimization problem in case of joint cooperating node selection and cooperative beamforming design as a mixed-integer non linear problem (MINLP). The first proposed scheme optimally solves this problem through exhaustive search. Therefore, this proposed optimal cooperating node selection scheme yields the best performance when applied in conjunction with the cooperative beamforming technique, compared to the other two proposed cooperating node selection schemes.
- Despite the optimality of the first proposed scheme, it suffers from high implementation complexity that grows exponentially with the number of cooperative nodes in the network. Therefore, we propose a suboptimal scheme that limits the maximum number of cooperating CR nodes which are allowed to participate in the cooperative transmission to the CR receiver, while still applying exhaustive search among this set of cooperating nodes. This solution

decreases the computational complexity compared to the optimal scheme, at the expense of losing some received signal power at the intended CR receiver.

- We propose another simple low complexity cooperating CR node selection scheme, in which the set of cooperating CR nodes that are selected for cooperative transmission are heuristically chosen based on their channel fading coefficients towards the CR receivers and towards the PRs. The numerical results in Chapter 3 show that the proposed selection scheme in conjunction with beamforming can further increase the sum transmission rate of the CRs significantly compared to the case when all the cooperating CR nodes participate in the cooperative beamforming.

1.1.4 Feedback Overhead

Cooperative beamforming requires sharing of instantaneous CSI and location information among cooperative CR nodes or requires a master node that knows the global instantaneous CSI and location information. Both cases require a huge amount of feedback overhead. Exchanging such large amount of control traffic between the cooperating nodes requires additional bandwidth, and causes excessive power dissipation from the CR devices [38]. In addition, CSI estimation errors can become a bottle neck to the potential performance gain from CR nodes cooperation [39]. This motivates us to pursue the following objective.

- Proposing a distributed beamforming scheme that requires only minimal information sharing between the cooperating nodes, to enable the beamforming design in presence of asynchronous interference.

The contributions made in the thesis regarding this objective are clarified as follows.

- In Chapter 4, we propose a distributed beamforming method to be used in cooperative CR networks that requires only information sharing between cooperating CR nodes about their locations. Assuming each cooperating CR node knows its own CSI towards the CR receiver and towards the PR, and using the shared information about the other cooperating nodes' locations, each node can independently design its own beamforming weight. The location sharing is essential for each cooperating node to account for the effect of asynchronous interference it can cause, along with other cooperating nodes, to the PR in the network.

1.1.5 Participation Decision Making

The final problem of cooperative CR networks, addressed in this thesis, is finding a suitable incentive for CR users to participate in the cooperative transmission towards an intended CR receiver. In return for their cooperation, we propose that each assisted CR user can lease its own channel, for a certain amount of time, to the cooperating CR nodes for their opportunistic access. However, a particular CR user spends a certain amount of its limited battery power when acting as a relay for another CR user. Therefore, it becomes a critical decision for each user to decide whether to participate in the cooperative beamforming or not. We assume there is no cooperation among the participating CR users in the decision making, so each CR user does not know other users' decisions, and hence it cannot assess its own reward in case of participating in the cooperative transmission beforehand. Hence, this problem is considered as an example of unknown games, that can be tackled using a Bayesian game theoretic approach [40]. This motivates us to pursue the following design objectives.

- Proposing an optimal autonomous participation decision making strategy to help each CR user in deciding whether to participate in the cooperative transmission or not.
- Proposing a lower complexity sub-optimal autonomous participation decision making strategy that has a shorter convergence time, yet a good performance.

Therefore, in Chapter 4, we presented the following contributions in order to achieve these two objectives.

- Chapter 4 proposes an autonomous participation making strategy known as regret testing-based strategy (RTS) and is based on the well-known regret testing procedure [41]. We modify the regret testing procedure based on our model of CR networks. We prove that the proposed RTS can asymptotically achieve an approximate Nash equilibrium (ϵ-NE) state of the network within a certain convergence time. The NE state of a system is achieved when each CR user has chosen a strategy of participation decision making, and no CR user can benefit by changing its strategy as long as other CR users keep theirs unchanged [42].

- Despite the optimality of the proposed RTS, its complexity and slow convergence time stand against its practical implementation. Therefore, we propose a low complexity autonomous decision making strategy named the learning-based strategy (LBS), which has a shorter convergence time.

- The network model considered in Chapter 4 assumes that different frequency channels, originally owned by the primary system, are assigned each for one CR user only during the current scheduling frame. When more than one CR user are requesting cooperative transmission over their different scheduled frequency channels, it becomes an added challenge for the CR users that are willing to act as relays. Therefore, we extend the two proposed autonomous decision making strategies, namely the RTS and the LBS, to handle the case of having multiple CR users requesting cooperation simultaneously. Since each CR user can only participate in the cooperative transmission towards one CR receiver over its scheduled channel at a time, it becomes a critical decision for each CR user, not only to decide whether to participate in the cooperative transmission or not, but also to select which CR receiver to assist among the simultaneous requests that it receives. The decision making in this case is based on the best incentive provided by each CR user requesting assistance. The modified RTS and LBS are shown in Chapter 4 to provide enhanced performance of the CR network, despite the lack of cooperation among the participating CR users in making their decisions.

1.2 Thesis Outline

The rest of the thesis consists of four chapters, which are organized as follows.

- In Chapter 2, we describe the asynchronous interference effect, and provide its mathematical modeling. We also present the motivation for new beamforming techniques to be used in this context. We develop an innovative cooperative beamforming technique, named the LBF method, that enables the cooperating CR nodes to transmit data to the CR receiver with a certain limit on the interference introduced at the PR when the PU is active. Next, we address the effect of having imperfect CSI estimation on the performance of the proposed beamforming method. We also propose a robust cooperative beamforming method to account for the effect of having error in the channel estimation between the PR and each cooperating CR node. At the end of Chapter 2, we present some numerical examples to assess the performance of the proposed cooperative beamforming methods, and show the implications of ignoring the asynchronous interference problem in the design of cooperative CR networks.

- Chapter 3 considers a generalized scenario of a CR network with multiple PRs and multiple CR receivers. The cooperative beamforming design is formulated as an optimization problem of constrained weighted sum rate maximization. Due to the non-convexity and non-linearity of the formulated optimization problem, we use an approximation to convert it into a convex and linear one. Then sub-optimally we obtain the beamforming directions and allocate power among different beamforming directions. However, due to the multiple interference constraints, the power allocation scheme is computationally expensive as discussed in Chapter 3. Therefore, we also propose a low complexity power allocation scheme.

  Next, we extend the proposed cooperative beamforming technique for the case of having only statistical CSI of the channels between the PRs and the cooperating CR nodes at the cooperating nodes. In this case, the asynchronous interferences at the PRs are guaranteed in a statistical sense. In the absence of a mathematically tractable expression of the distribution of the random interference powers at the PRs, we develop an upper bound on the probability of introducing asynchronous interference power at a given PR beyond a given threshold. Then this developed upper bound is used to design a robust leakage beamforming technique. The proposed robust beamforming technique can protect the primary network's functionality by satisfying the probabilistic interference constraints at all PRs in the network.

  Moreover, we develop an optimal cooperating CR node selection scheme to be used in conjunction with the beamforming technique. Because of the exhaustive search, the optimal cooperating CR node selection scheme can be computationally expensive. Therefore, we also propose two sub-optimal selection schemes. Finally, we conduct comprehensive simulation experiments to show the performance of the different proposed beamforming schemes and the cooperating CR node selection schemes.

- As a solution for the problem of huge feedback overhead required for cooperative beamforming, in Chapter 4 we propose a distributed beamforming method to be used in cooperative CR networks with minimal amount of feedback overhead. Then, we define a suitable incentive for the CR users to participate in the cooperative transmission. In this chapter, we also propose two autonomous participation decision making strategies, namely the RTS and the LBS, to help each CR user in deciding whether to participate in the cooperative transmission or not, without any coordination among the participating CR users in decision making. The decision making strategies are based on the proposed incentive and the estimated cost of participation in the cooperative transmission.

  However, different cooperating CR nodes have different path loss values towards the intended CR receiver, as well as towards the primary receiver. In addition, as more CR nodes participate in the cooperative transmission, the individual reward value becomes smaller. Therefore, in case of having multiple CR users that are willing to participate in the cooperative transmission, a cooperating CR node selection scheme is important. As such, the received signal power value at the intended CR receiver is maximized. Therefore, we propose a cooperating CR node selection method in Chapter 4.

  Also in Chapter 4, we extend the two participation decision making strategies, the RTS and LBS, to handle the case of having multiple CR users requesting cooperation simultaneously. The numerical results in Chapter 4 reveal the effectiveness of our proposed distributed beamforming scheme and autonomous decision making strategies.

- Conclusions and future works are discussed in Chapter 5, where we provide possible future research directions.

Chapter 2

Cooperative Beamforming with Single Primary Receiver and Single CR Receiver

As mentioned earlier, the performance of CR networks can be improved using cooperation among different users in such network. In particular, in a CR network where a CR user/source wants to transmit information to a CR receiver that cannot be communicated directly by the source due to the path loss and/or shadowing, a group of SUs can act as relays and forward the message to the CR receiver. However, in such dual-hop cooperative CR networks, the availability of spectrum is more critical as it requires unused spectrum hole/slot for both hops' communications. As such it can lead to a very poor quality of communications, e.g., higher outage probability, for the CR users. In order to improve the quality of communications for such networks, beamforming methods [43] which enable concurrent transmissions of both PUs and SUs at a given channel can be exploited [22], [23], [19]. These beamforming methods require to have multiple antennas at the transmitters. However, for an implementation constraint, a CR source may not support more than one antenna element at its terminal.

For the above mentioned situations, a cooperative beamforming method as proposed in [20] can be used for the second hop i.e., CR relays to the CR receiver communication. The proposed cooperative beamforming method in [20] utilizes a virtual antenna array, which is created by a set of CR users that act as relays for the CR source and forward the information to the CR receiver. In particular, when the channel is not occupied by the PU, a single antenna based CR source transmits its information to a set of single antenna based cooperating SUs that serve as relays. In the next time slot, the cooperating relays which have correctly detected the transmitted information, forward this information to the CR receiver irrespective of whether the PU is silent or not. By carefully selecting the beamforming weight in each CR relay node, the interference from the CR relays can be efficiently suppressed or even thoroughly avoided.

Implementation of beamforming in such a cooperative manner, as explained in Chapter 1, can lead to an asynchronous interference at the PR. In [20], such asynchronous interference at the PR has been ignored. The ZFBF proposed in [20] cannot annul this asynchronous interference caused by the CR relays at the PR, except under a severe constraint, that the number of antennas in each relay node should be greater than the total number of antennas of all PUs [33]. This is explained in more detail in Section 2.2. In addition, the presented numerical examples in this chapter show that the ZFBF method introduces higher levels of interference at the PR, in such an asynchronous interference scenario. This leads to an increased service interruption/outage of the CR system when there is a certain limit on the introduced interference at the PR.

In this chapter, we provide the operating principles as well as the assumptions that we consider in our problem formulation. We also present a mathematical model for the asynchronous interference power introduced at the PR. Next, we develop a cooperative beamforming method called *leakage beamforming* (LBF) method that maximizes the signal power at the CR receiver while it maintains the interference power at the PR below a target threshold, which in turn decreases the service interruption/outage of the SU. Then, we develop a robust beamforming method that addresses the issue of having imperfect CSI estimation between each CR relay and the PR.

2.1 System Model

We consider an example of situations where cooperative beamforming proves to be very effective in CR networks. CR networks are usually low-power systems where the CR source may not be allowed to transmit enough power to cover the CR receiver due to the interference restriction imposed by the nearby primary system. In this chapter, we consider the case when the direct link between the CR source and its CR receiver is not available, which may result from high path loss. In this case, a group of CR users in the network can act as relays and forward the message to the CR receiver. In what follows we provide the operating assumptions and principles that we consider in our problem formulation.

2.1.1 Operating Assumptions

We consider a time-slotted system with a single primary link that is used to transmit information from a primary transmitter to a particular PR at a given time slot with probability p. This is so-called ON-OFF behavior of PUs [11, 44]. We also consider a CR network consisting of one CR source s, one CR receiver d and L other SUs that act as parallel relays as shown in Fig. 2.1. The CR relay nodes are denoted by r_l, $l = 1, 2, \cdots, L$. Due to the hardware constraint, it is assumed that the CR source s, the relays as well as the CR receiver d are equipped with single antenna each. The CR network is employed with a reliable sensing mechanism that can accurately senses the channel occupancy by the PU in a given time slot.

The channel path loss model used in the system is the log-distance path loss model presented in [45]. Generally, accurate path loss models can be obtained from complex ray tracing models or empirical measurements when tight system specifications must be met. However, for simplicity, we use a simple model that captures the essence of signal propagation without resorting to complicated path loss models, which are only approximations to the real channel [45]. Thus, the following path loss model is used for system design as a function of distance $D_{r,d}$ between relay r and the CR receiver d.

$$\mathrm{PL}_{r,d} = \kappa + 10\gamma \log_{10}\Big(\frac{D_{r,d}}{D_0}\Big), \tag{2.1}$$

where $\mathrm{PL}_{r,d}$ is the path loss in dB over the communication link between relay r and the CR receiver d, D_0 is a reference distance for the antenna far-field, and γ is the path loss exponent. κ is a unit-less constant which depends on the antenna characteristics of relay r and the average channel attenuation. The value of κ is set to the free space path loss at distance D_0 as follows

$$\kappa = 20 \log_{10}\Big(\frac{4\pi D_0}{\lambda_c}\Big), \tag{2.2}$$

where λ_c is the RF signal wavelength. Due to scattering phenomena in the antenna near-field, the model in eq. (2.1) is generally only valid at transmission distances $D_{r,d} > D_0$. We assume D_0 is equal to 10m. The value of γ depends on the propagation environment. We consider a propagation model that approximately follows a two-ray model, so the path loss exponent γ is set to 4.

We assume that the channels between all nodes are additive white Gaussian noise (AWGN) channels with zero mean and two sided noise power spectrum density $N_0/2$. We also assume block fading channel model, that was used for example in [46], in which the channel fading is assumed to remain roughly the same over a time slot, but is independent of the fading in other time slots.

2.1.2 Operating Principles

There are two phases of transmission for the CR system as described below. Without loss of generality, let us assume that at time slot n, the communication channel is sensed as idle by the CR system. Therefore, the CR source s broadcasts its data to the CR relays. We assume that the source s sequentially transmits M symbols[2] during time slot n. At time slot $n+1$, the CR relays which can detect the source information correctly transmit the detected versions of the received symbols to the CR receiver d. At time slot $n+1$, if the channel is not occupied by the PU, the relays which have successfully detected the message, forward the message to the CR receiver d using the maximal ratio transmission (MRT) diversity scheme [47]. Otherwise these relays forward the message to the CR receiver using a beamforming method, such that the interference introduced to the PR remains below a target threshold specified by the PU system.

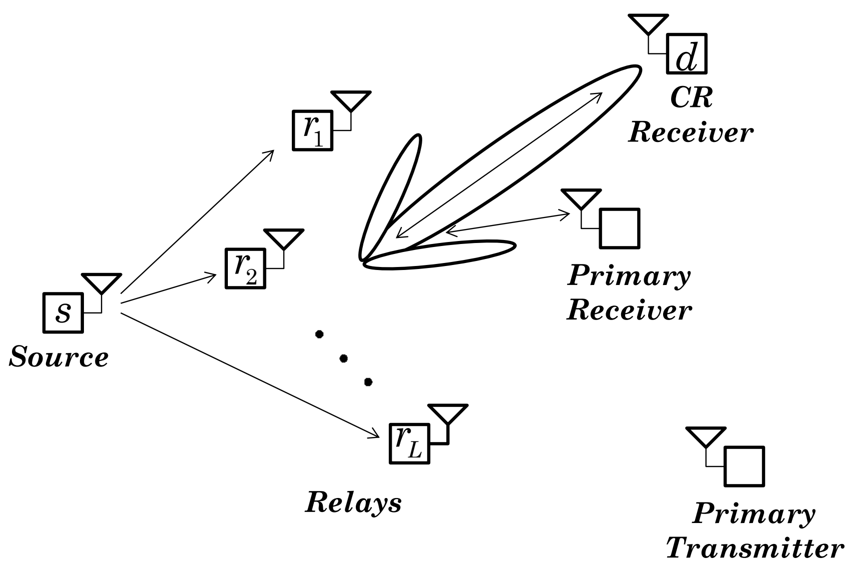

Figure 2.1: System model for cooperative beamforming with L cooperating CR users acting as relays.

[2]The problem can be formulated assuming $M = 1$, i.e., only one symbol is transmitted in each time slot. However, we consider a practical scenario where multiple symbols are transmitted in a given time slot.

We assume that P is the average power per symbol at the source s. The binary information bits are mapped into modulated symbols, and the data vector consisting of these M modulated symbols is denoted by $\mathbf{x}_s$. The received signal at a CR relay r at time slot n can be written as

$$\mathbf{y}_r[n] = h_{s,r}[n]\mathbf{x}_s[n] + \mathbf{z}_r[n], \tag{2.3}$$

where $\mathbf{z}_r$ is the AWGN vector at relay r and $h_{s,r}[n]$ is the channel fading gain between the source s and the relay r at time slot n.

Assume that at time slot n, K relays out of the L relays can successfully detect the message, where $K \leq L$. In particular, a relay r is considered to have successfully detected the message, if the instantaneous Shannon capacity of the channel between the CR source and relay r exceeds the value of a target spectral efficiency B [20]. Those successful relays are denoted by the set $\mathcal{K}_n$, where $|\mathcal{K}_n| = K$. Without loss of generality, the relays in set $\mathcal{K}_n$ can be denoted by $\{r_1, \ldots, r_K\}$. These relays forward their detected symbols to the CR receiver at time slot $n+1$. In particular, if the communication channel is occupied by the PU at time slot $n+1$, this set of relays forwards the message to the CR receiver d using a beamforming method which will be discussed latter. In this case, the received signal at d can be written as

$$\mathbf{y}_d[n+1] = \mathbf{h}\mathbf{g}\mathbf{x}_s[n] + \mathbf{w} + \mathbf{z}_d[n+1], \tag{2.4}$$

where $\mathbf{x}_s[n]$ is the transmitted data vector from the source at time slot n that has been successfully detected at the relays in $\mathcal{K}_n$ and $\mathbf{w}$ is the received interfering signal vector from the PU transmitter. $\mathbf{g} = [g_1, \ldots, g_K]^T$ is the beamforming weight vector, with each element g_r denoting the weight of the relay r. $\mathbf{h} = [h_{1,d}[n+1], \ldots, h_{K,d}[n+1]]$ is the the channel vector from the transmitting relays to the CR receiver d, where $h_{r,d}$ is the channel fading gain from relay r to receiver d. Our design goal is to obtain the beamforming vector $\mathbf{g}$ that maximizes the received signal power at CR receiver d while keeping the interference to the PR below a certain threshold.

If at time slot $n+1$, the PU is silent, the relays in set $\mathcal{K}_n$ directly forward the correctly detected symbols $\mathbf{x}_s[n]$ to the CR receiver using MRT [48] diversity scheme. In this case the received signal at the CR receiver d with MRT diversity scheme can be written as

$$\mathbf{y}_d[n+1] = \sqrt{\mathbf{h}\mathbf{h}^\dagger}\mathbf{x}_s[n] + \mathbf{z}_d[n+1]. \tag{2.5}$$

2.2 Modeling of Asynchronous Interference

In a practical scenario, the CR relays are usually located in different geographical locations[3]. Therefore, the received signals from different transmitting relays at the PR as well as at the CR receiver can experience different propagation delays. Although the received signal at the CR receiver d from different relays can be synchronized by using a timing advance mechanism which is currently employed in the uplink of GSM and 3G cellular networks, to compensate for the propagation delay from each user [33, 48], the received signals at the PR from different transmitting relays can be not synchronized simultaneously. As such, the PR will experience asynchronous interference. The asynchronous interference issue has been studied in [33] for the conventional cellular networks where multiple BS's cooperate with each other for downlink transmissions. Following this work, in what follows, we provide a mathematical model for the asynchronous interference introduced to the PR.

In order to maintain the synchronous reception of data symbols from all the transmitting relays in set $\mathcal{K}_n$ at the CR receiver, we consider that a timing advance mechanism is applied among those relays. In particular, the r^{th} transmitting relay advances its signal by $\Delta\tau_{r,d}$ which is calculated as

$$\Delta\tau_{r,d} = \tau_{r,d} - \tau^{(\min)}, \tag{2.6}$$

where $\tau_{r,d}$ is the propagation delay of r^{th} ($r \in \mathcal{K}_n$) relay signal to the receiver d and $\tau^{(\min)}$ is the minimum signal propagation delay of all the transmitting relays in the set $\mathcal{K}_n$.

Let us denote $\tau_r^{(\text{PU})}$ as the signal propagation delay from relay r ($r \in \mathcal{K}_n$) to the PR. Then the time delay of the received symbols at the PR from the r^{th} relay, $\Delta\tau_r^{(\text{PU})}$ can be written as

$$\Delta\tau_r^{(\text{PU})} = \tau_r^{(\text{PU})} - \Delta\tau_{r,d}, \tag{2.7}$$

where the received signal from r^{th} relay is advanced by its time advance, $\Delta\tau_{r,d}$, defined by eq. (2.6).

[3] We are considering a cooperative CR network where a set of randomly located cooperating CR users is acting as a set of relays.

Let us denote $\mathbf{i}_r[n+1]$ as the asynchronous vector of symbols received at the PR as shown in Fig. 2.2. This asynchronous vector of symbols $\mathbf{i}_r[n+1]$ is defined as

$$\mathbf{i}_r[n+1] = \mathbf{x}_s\left[nT_{\text{slot}} - \Delta\tau_r^{(\text{PU})}\right], \tag{2.8}$$

where T_{slot} is the time slot duration.

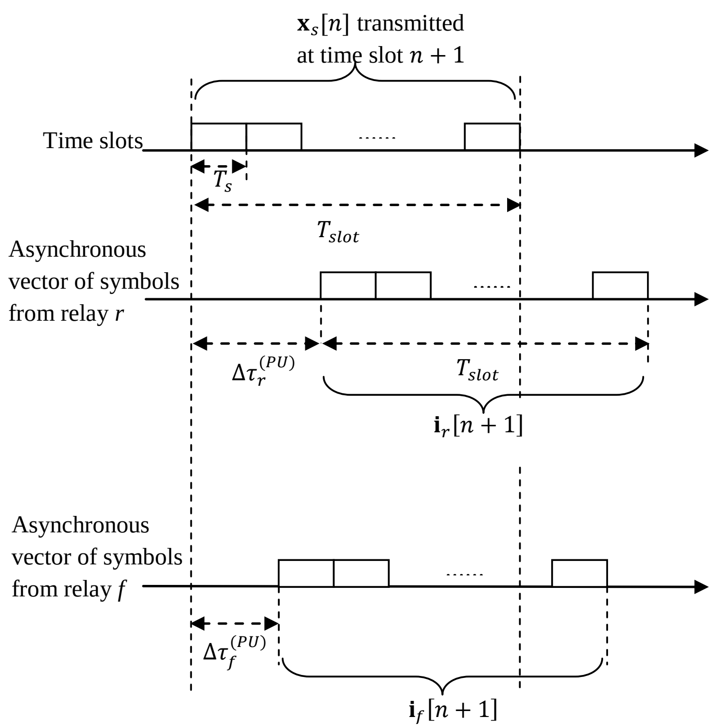

Figure 2.2: An example of the asynchronous interference at PR arising from a vector of M symbols, $\mathbf{x}_s[n]$, transmitted by relays r and f. T_s is the symbol duration and T_{slot} is the time slot duration.

The asynchronous received signal at the PR at time slot $n+1$ is given by

$$\mathbf{y}[n+1] = \sum_{r\in\mathcal{K}_n} h_r^{(\text{PU})}[n+1] g_r \mathbf{i}_r[n+1] + \mathbf{z}[n+1], \tag{2.9}$$

where $h_r^{(\text{PU})}[n+1]$ is the channel fading gain from relay r to the PR at time slot $n+1$, and $\mathbf{z}[n+1]$ is the AWGN vector at the PR with zero mean and two-sided PSD $N_0/2$.

In [20], such asynchronous interference at the PR has been ignored. From eq. (2.9) it is obvious that if ZFBF method, developed in [20], is used to force the asynchronous interference at the PR to be zero, the interference from each relay has to be annulled separately, i.e., the condition

$\left|h_r^{(\mathrm{PU})}g_r\right| = 0$ must be satisfied for every $r \in \mathcal{K}_n$, which cannot be fulfilled for single antenna based CR relays because it requires the number of PUs to be less than 1. Failing to satisfy this condition, the ZFBF method results in higher interference at the PR, as shown in Section 2.5. This leads to an increased service interruption/outage of the CR system when there is a certain limit on the introduced interference at the PR.

2.3 Problem Formulation and Optimal Beamforming Design

In this section, we develop a new cooperative beamforming method, called LBF method in order to address the problem of asynchronous interference at the PR. In this development, we use the same assumption as in [19, 20, 22, 23] that the channel fading gains between the CR relays and the PR as well as the channel fading gains between the CR relays and the CR receiver are known perfectly at the CR relays. Different possible scenarios have been considered in the literature in order to estimate the CSI between the CR relays and the PR (see for examples, [34], [35]). In the next section, we consider the case when the channel between the CR relays and the PR is not known perfectly.

The objective of the developed LBF method is to keep the leakage to the PR below a desired threshold, where leakage is the interference caused by the CR relays to the PR. The target threshold is imposed by the regulatory body, see for example [12] for details. The leakage signal at the PR due to CR relays' transmission at time slot $n+1$ can be written as

$$\boldsymbol{\Theta}[n+1] = \sum_{r=1}^{K} h_r^{(\mathrm{PU})}[n+1] g_r \mathbf{i}_r[n+1], \tag{2.10}$$

where $\mathbf{i}_r[n+1]$ is obtained from eq. (2.8). Now the leakage signal power at the PR can be written as

$$P_{\text{leak}} = \mathrm{E}(\boldsymbol{\Theta}[n+1]\boldsymbol{\Theta}^{\dagger}[n+1]), \tag{2.11}$$

where $\mathrm{E}(\cdot)$ is the expectation operation over the random data sequence, and $\boldsymbol{\Theta}^{\dagger}[n+1]$ is the conjugate transpose of $\boldsymbol{\Theta}[n+1]$.

For notational convenience, we will drop the time slot index n from now on. Using eq. (2.10) in eq. (2.11) and after some mathematical manipulations, the leakage power can finally be expressed as

$$P_{\text{leak}} = \sum_{r=1}^{K}\sum_{f=1}^{K} g_f^{\dagger}(h_f^{(\text{PU})})^{\dagger} h_r^{(\text{PU})} g_r \cdot \mathrm{E}\left(\mathbf{i}_r \mathbf{i}_f^{\dagger}\right). \tag{2.12}$$

Let us define a variable $\beta^{(r,f)} \triangleq \mathrm{E}\left(\mathbf{i}_r \mathbf{i}_f^{\dagger}\right)$, which is the correlation between the leakage symbols of the r^{th} and the f^{th} relays. Then the leakage power in eq. (2.11) can be written as

$$P_{\text{leak}} = \sum_{r=1}^{K}\sum_{f=1}^{K} g_f^{\dagger}(h_f^{(\text{PU})})^{\dagger} h_r^{(\text{PU})} g_r \beta^{(r,f)}. \tag{2.13}$$

The correlation between asynchronous symbols, $\beta^{(r,f)}$, can be obtained from one of the following two cases:

- Case 1: The propagation difference between the signals of relays r and f is larger than one symbol duration T_s. In this case, the correlation between $\mathbf{i}_r$ and $\mathbf{i}_f$ is equal to zero, i.e.,

$$\beta^{(r,f)} = 0, \qquad \text{if } \left|\Delta\tau_r^{(\text{PU})} - \Delta\tau_f^{(\text{PU})}\right| > T_s.$$

  This is because successive data symbols are assumed to be independent of each other with zero mean.

- Case 2: The propagation difference between the signals of relays r and f is less than one symbol duration T_s. In this case, the asynchronous symbols $\mathbf{i}_r$ and $\mathbf{i}_f$ are overlapping for a time duration of $T_s - \left|\Delta\tau_r^{(\text{PU})} - \Delta\tau_f^{(\text{PU})}\right|$, and hence their correlation is equal to the part of the transmitted symbol power in which they intersect, i.e.,

$$\beta^{(r,f)} = \frac{T_s - \left|\Delta\tau_r^{(\text{PU})} - \Delta\tau_f^{(\text{PU})}\right|}{T_s} P, \qquad \text{if } 0 < \left|\Delta\tau_r^{(\text{PU})} - \Delta\tau_f^{(\text{PU})}\right| < T_s.$$

The leakage power in eq. (2.13) can be rewritten in a matrix form as follows

$$P_{\text{leak}} = \mathbf{g}^{\dagger}\mathbf{R}_{\text{true}}\mathbf{g}, \tag{2.14}$$

where $\mathbf{g} = [g_1, \ldots, g_K]^T$ and

$$\mathbf{R}_{\text{true}} \triangleq \begin{bmatrix} \beta^{(1,1)}(h_1^{(\text{PU})})^\dagger h_1^{(\text{PU})} & \cdots & \beta^{(1,K)}(h_1^{(\text{PU})})^\dagger h_K^{(\text{PU})} \\ \beta^{(2,1)}(h_2^{(\text{PU})})^\dagger h_1^{(\text{PU})} & \cdots & \beta^{(2,K)}(h_2^{(\text{PU})})^\dagger h_K^{(\text{PU})} \\ \vdots & \ddots & \vdots \\ \beta^{(K,1)}(h_K^{(\text{PU})})^\dagger h_1^{(\text{PU})} & \cdots & \beta^{(K,K)}(h_K^{(\text{PU})})^\dagger h_K^{(\text{PU})} \end{bmatrix}. \tag{2.15}$$

$\mathbf{R}_{\text{true}}$ is the covariance matrix of the channel fading gains between the CR relays and the PR. This channel covariance matrix $\mathbf{R}_{\text{true}}$ can be calculated by the CR system using the known channel fading gain between each relay and the PR, $h_r^{(\text{PU})}$, as well as $\beta^{(r,f)}$ which can be calculated based on the locations of the PR and the CR receiver relative to the CR relays.

The received signal power at the CR receiver is given by

$$P_{\text{sig}} = P\mathbf{g}^\dagger \mathbf{h}^\dagger \mathbf{h}\mathbf{g}. \tag{2.16}$$

Our design goal is to maximize the received signal power at the CR receiver d, while keeping the leakage power at the PR below a certain threshold. This design goal can be formulated as an optimization problem, using eqs. (2.14) and (2.16), as follows

$$\begin{aligned} \mathbf{g}^{(\text{LBF})} &= \max_{\mathbf{g}} \left(P\mathbf{g}^\dagger \mathbf{h}^\dagger \mathbf{h}\mathbf{g} \right), \\ \text{subject to:} \quad & \mathbf{g}^\dagger \mathbf{R}_{\text{true}} \mathbf{g} \le \gamma_{\text{th}}, \end{aligned} \tag{2.17}$$

where γ_{th} is the maximum allowable interference at the PR.

The optimization problem in eq. (2.17) is a convex maximization problem, that maximizes a convex quadratic function under a convex quadratic constraint. The global optimality conditions of such optimization problem have been studied in [49]. According to [49], if the following three conditions are satisfied, the global optimal solution of such problem can be found by solving the Lagrangian dual problem, with zero duality gap. These three conditions are as follows:

1. The matrix $P\mathbf{h}^\dagger \mathbf{h}$ is a positive semidefinite matrix.

2. The matrix $\mathbf{R}_{\text{true}}$ is a positive semidefinite matrix.

3. There exists a vector $\mathbf{g}$, for which the inequality constraint is strictly satisfied, i.e., $\mathbf{g}^{\dagger}\mathbf{R}_{\text{true}}\mathbf{g} < \gamma_{\text{th}}$.

It can be easily shown that these three conditions are satisfied in the optimization problem in eq. (2.17). Therefore, the global optimum solution can be found by the Lagrange multiplier method, as follows

$$\mathcal{L}(\mathbf{g},\lambda) = P\mathbf{g}^{\dagger}\mathbf{h}^{\dagger}\mathbf{h}\mathbf{g} - \lambda\left(\mathbf{g}^{\dagger}\mathbf{R}_{\text{true}}\mathbf{g} - \gamma_{\text{th}}\right), \tag{2.18}$$

where λ is a Lagrange multiplier. The critical values of the Lagrange function $\mathcal{L}(\mathbf{g},\lambda)$ occur when its gradient is equal to zero. By taking the partial derivative of the Lagrange function in eq. (2.18) with respect to $\mathbf{g}$, and equating it to zero, we can easily write

$$P\mathbf{h}^{\dagger}\mathbf{h}\mathbf{g} = \lambda\mathbf{R}_{\text{true}}\mathbf{g}. \tag{2.19}$$

Defining $\psi \triangleq \mathbf{h}\mathbf{g}$, the beamforming vector in eq. (2.19) can be written as

$$\mathbf{g}^{(\text{LBF})} = \frac{\psi}{\lambda}P\mathbf{R}_{\text{true}}^{-1}\mathbf{h}^{\dagger}. \tag{2.20}$$

Equating the partial derivative of $\mathcal{L}(\mathbf{g},\lambda)$ with respect to λ, to zero, we find that

$$\mathbf{g}^{(\text{LBF})\dagger}\mathbf{R}_{\text{true}}\mathbf{g}^{(\text{LBF})} = \gamma_{\text{th}}. \tag{2.21}$$

Substituting eq. (2.20) into eq. (2.21), and after some mathematical manipulations, we can write $\frac{\psi}{\lambda}$ as

$$\left(\frac{\psi}{\lambda}\right)^2 = \frac{1}{P^2}\frac{\gamma_{\text{th}}}{\left(\mathbf{R}_{\text{true}}^{-1}\mathbf{h}^{\dagger}\right)^{\dagger}\mathbf{R}_{\text{true}}\left(\mathbf{R}_{\text{true}}^{-1}\mathbf{h}^{\dagger}\right)}. \tag{2.22}$$

Using eq. (2.22) in eq. (2.20), the optimum beamforming vector $\mathbf{g}^{(\text{LBF})}$ can finally be expressed in a desired closed form as follows

$$\mathbf{g}^{(\text{LBF})} = \sqrt{\frac{\gamma_{\text{th}}}{\mathbf{h}{\mathbf{R}_{\text{true}}^{-1}}^{\dagger}\mathbf{h}^{\dagger}}}\mathbf{R}_{\text{true}}^{-1}\mathbf{h}^{\dagger}. \tag{2.23}$$

2.4 Robust Beamforming Method with Imperfect Channel Knowledge

When the CSI between the CR relays and the PR $h_r^{(\mathrm{PU})}$ is perfectly known at the CR relays, our developed LBF method in the previous section can be used. However in some scenarios, the CR relays may have erroneous estimation of the channel between the PR and the CR relays. During the design process of the cooperative beamforming vector, we must account for the effects of such erroneous estimation, to ensure a robust protection of the PR. Robust protection means that the resulting interference at the PR remains below the predefined threshold even if the error in CSI estimation is maximum. In order to design a robust leakage beamforming (RLBF) method for such scenario, we adopt the following channel estimation uncertainty model.

If the channel estimation of $\mathbf{h}^{(\mathrm{PU})}$ is erroneous, the estimation error can be modeled as

$$\mathbf{h}_{\text{true}}^{(\mathrm{PU})} = \mathbf{h}_{\text{est}}^{(\mathrm{PU})} + \mathbf{e}^{(\mathrm{PU})}, \tag{2.24}$$

where $\mathbf{h}_{\text{true}}^{(\mathrm{PU})}$ is the actual instantaneous channel vector between the CR relays and the PR, $\mathbf{h}_{\text{est}}^{(\mathrm{PU})}$ is the estimated channel vector between the CR relays and the PR, and $\mathbf{e}^{(\mathrm{PU})}$ is the estimation error vector. Based on the accuracy of the estimation method used, the channel estimation uncertainty can be modeled by the so-called bounded uncertainty model[4].

Using the error model in eq. (2.24), the covariance matrix corresponding to $\mathbf{h}_{\text{true}}^{(\mathrm{PU})}$, $\mathbf{R}_{\text{true}}$ can be written as in eq. (2.25).

$$\mathbf{R}_{\text{true}}=\begin{bmatrix} \beta^{(1,1)}\big(h_{1,\text{est}}^{(\mathrm{PU})\dagger} h_{1,\text{est}}^{(\mathrm{PU})} + e_1^{(\mathrm{PU})\dagger} e_1^{(\mathrm{PU})}\big) & \cdots & \beta^{(1,K)}\big(h_{1,\text{est}}^{(\mathrm{PU})\dagger} h_{K,\text{est}}^{(\mathrm{PU})} + e_1^{(\mathrm{PU})\dagger} e_K^{(\mathrm{PU})}\big) \\ \vdots & \ddots & \vdots \\ \beta^{(K,1)}\big(h_{K,\text{est}}^{(\mathrm{PU})\dagger} h_{1,\text{est}}^{(\mathrm{PU})} + e_K^{(\mathrm{PU})\dagger} e_1^{(\mathrm{PU})}\big) & \cdots & \beta^{(K,K)}\big(h_{K,\text{est}}^{(\mathrm{PU})\dagger} h_{K,\text{est}}^{(\mathrm{PU})} + e_K^{(\mathrm{PU})\dagger} e_K^{(\mathrm{PU})}\big) \end{bmatrix}. \tag{2.25}$$

[4]The bounded uncertainty model is a well-accepted model that has been used in [35, 50–52]. It shows that the uncertainty in the channel estimation is described by a bounded region whose shape depends on the channel estimation method used. However, a spherical uncertainty region gives the worst case estimation error model [51]. In this case, the estimation error vector is bounded by $\left\|\mathbf{e}^{(\mathrm{PU})}\right\|^2 \leq \epsilon$.

Taking all the error terms into one matrix Δ_R, we get

$$\Delta_R = \begin{bmatrix} \beta^{(1,1)}(\mathbf{e}_1^{(\text{PU})})^\dagger \mathbf{e}_1^{(\text{PU})} & \cdots & \beta^{(1,K)}(\mathbf{e}_1^{(\text{PU})})^\dagger \mathbf{e}_K^{(\text{PU})} \\ \vdots & \ddots & \vdots \\ \beta^{(K,1)}(\mathbf{e}_K^{(\text{PU})})^\dagger \mathbf{e}_1^{(\text{PU})} & \cdots & \beta^{(K,K)}(\mathbf{e}_K^{(\text{PU})})^\dagger \mathbf{e}_K^{(\text{PU})} \end{bmatrix}, \tag{2.26}$$

which is a random matrix modeling the channel estimation errors. Now we can write

$$\mathbf{R}_{\text{true}} = \mathbf{R}_{\text{est}} + \Delta_R, \tag{2.27}$$

where $\mathbf{R}_{\text{est}}$ is the estimated covariance matrix of the channel fading gains between the set of CR relays and the PR. This matrix $\mathbf{R}_{\text{est}}$ can be calculated using $\mathbf{h}_{\text{est}}^{(\text{PU})}$ as well as $\beta^{(r,f)}$. Δ_R is the covariance error matrix and is bounded by $\|\Delta_R\| \leq \Omega_R$, where Ω_R is the bound of the uncertainty region of $\mathbf{R}_{\text{est}}$.

Since $\mathbf{R}_{\text{true}}$ is a covariance matrix, it can be factorized using Cholesky decomposition [53]. Therefore, we can write $\mathbf{R}_{\text{true}} = \mathbf{C}_{\text{true}}\mathbf{C}_{\text{true}}^\dagger$, where $\mathbf{C}_{\text{true}}$ is a lower triangular matrix. Similarly, we can write $\mathbf{R}_{\text{est}} = \mathbf{C}_{\text{est}}\mathbf{C}_{\text{est}}^\dagger$. Then, the relation between $\mathbf{C}_{\text{true}}$ and $\mathbf{C}_{\text{est}}$ can be written as

$$\mathbf{C}_{\text{true}} = \mathbf{C}_{\text{est}} + \Delta_C, \qquad \|\Delta_C\| \leq \Omega_C, \tag{2.28}$$

where Ω_C is the bound of the uncertainty region of $\mathbf{C}_{\text{est}}$.

Using eq. (2.28) in eq. (2.17), we can reformulate the leakage constraint as follows

$$\left\|\mathbf{g}^\dagger \mathbf{C}_{\text{true}}\right\|^2 \leq \gamma_{\text{th}}. \tag{2.29}$$

However, in order to ensure a robust design of the beamforming vector using $\mathbf{C}_{\text{est}}$, the above constraint must be satisfied for the worst case estimate of $\mathbf{C}_{\text{true}}$, i.e.,

$$\max_{\|\Delta_C\|} \left\|\mathbf{g}^\dagger \mathbf{C}_{\text{true}}\right\| \leq \sqrt{\gamma_{\text{th}}}. \tag{2.30}$$

Using the triangle inequality, we can write

$$\left\|\mathbf{g}^{\dagger}\mathbf{C}_{\text{true}}\right\| \leq \left\|\mathbf{g}^{\dagger}\mathbf{C}_{\text{est}}\right\| + \left\|\mathbf{g}^{\dagger}\Delta_{C}\right\|. \tag{2.31}$$

Now by applying Cauchy-Schwartz inequality, we can rewrite eq. (2.31) as

$$\left\|\mathbf{g}^{\dagger}\mathbf{C}_{\text{true}}\right\| \leq \left\|\mathbf{g}^{\dagger}\mathbf{C}_{\text{est}}\right\| + \|\mathbf{g}\| \|\Delta_{C}\|. \tag{2.32}$$

Using the maximum value of $\left\|\mathbf{g}^{\dagger}\mathbf{C}_{\text{true}}\right\|$ given in eq. (2.32) and substituting it in eq. (2.30), the design constraint now becomes

$$\left\|\mathbf{g}^{\dagger}\mathbf{C}_{\text{est}}\right\|^{2} \leq \left(\sqrt{\gamma_{\text{th}}} - \|\mathbf{g}\| \Omega_{C}\right)^{2}. \tag{2.33}$$

By using the relation between $\mathbf{R}_{\text{est}}$ and $\mathbf{C}_{\text{est}}$, the design constraint in eq. (2.33) can finally be expressed as

$$\mathbf{g}^{\dagger}\mathbf{R}_{\text{est}}\mathbf{g} \leq \left(\sqrt{\gamma_{\text{th}}} - \|\mathbf{g}\| \Omega_{C}\right)^{2}. \tag{2.34}$$

Therefore, our primal optimization problem for this RLBF method can be written as

$$\begin{aligned} \mathbf{g}^{(\text{RLBF})} &= \max_{\mathbf{g}} \left(\mathbf{g}^{\dagger}\mathbf{h}^{\dagger}\mathbf{h}\mathbf{g}P\right), \\ \text{subject to:} \quad & \mathbf{g}^{\dagger}\mathbf{R}_{\text{est}}\mathbf{g} \leq I_{\text{th}}, \end{aligned} \tag{2.35}$$

where $I_{\text{th}} = \left(\sqrt{\gamma_{\text{th}}} - \|\mathbf{g}\| \Omega_{C}\right)^{2}$. Following the same procedure as described in the previous section, we can find the optimal RLBF vector for the optimization problem in eq. (2.35). This optimal RLBF vector $\mathbf{g}^{(\text{RLBF})}$ can be written as

$$\mathbf{g}^{(\text{RLBF})} = \sqrt{\frac{I_{\text{th}}}{\mathbf{h}\mathbf{R}_{\text{est}}^{-1\dagger}\mathbf{h}^{\dagger}}}\mathbf{R}_{\text{est}}^{-1}\mathbf{h}^{\dagger}. \tag{2.36}$$

In order to find the RLBF vector $\mathbf{g}^{(\text{RLBF})}$ in eq. (2.36), we need to calculate I_{th} which in turn depends on $\|\mathbf{g}^{(\text{RLBF})}\|$. In what follows, we present the steps of finding the value of $\|\mathbf{g}^{(\text{RLBF})}\|$. By substituting $\mathbf{g} = \mathbf{g}^{(\text{RLBF})}$ in $I_{\text{th}} = \left(\sqrt{\gamma_{\text{th}}} - \|\mathbf{g}\| \Omega_{C}\right)^{2}$ and taking the norm of eq. (2.36), we can finally write

$$\left\|\mathbf{g}^{(\mathrm{RLBF})}\right\| = \frac{\sqrt{\gamma_{\mathrm{th}}}}{\Omega_C + \frac{\sqrt{\mathbf{h}\mathbf{R}_{\mathrm{est}}^{-1^\dagger}\mathbf{h}^\dagger}}{\left\|\mathbf{R}_{\mathrm{est}}^{-1}\mathbf{h}^\dagger\right\|}}. \tag{2.37}$$

Using eq. (2.37) in eq. (2.36), we can write the RLBF vector $\mathbf{g}^{(\mathrm{RLBF})}$ in a desired closed form as follows

$$\mathbf{g}^{(\mathrm{RLBF})} = \left(\sqrt{\gamma_{\mathrm{th}}} - \frac{\Omega_C\sqrt{\gamma_{\mathrm{th}}}}{\Omega_C + \frac{\sqrt{\mathbf{h}\mathbf{R}_{\mathrm{est}}^{-1^\dagger}\mathbf{h}^\dagger}}{\left\|\mathbf{R}_{\mathrm{est}}^{-1}\mathbf{h}^\dagger\right\|}}\right)\sqrt{\frac{1}{\mathbf{h}\mathbf{R}_{\mathrm{est}}^{-1^\dagger}\mathbf{h}^\dagger}}\mathbf{R}_{\mathrm{est}}^{-1}\mathbf{h}^\dagger. \tag{2.38}$$

2.5 Numerical Results

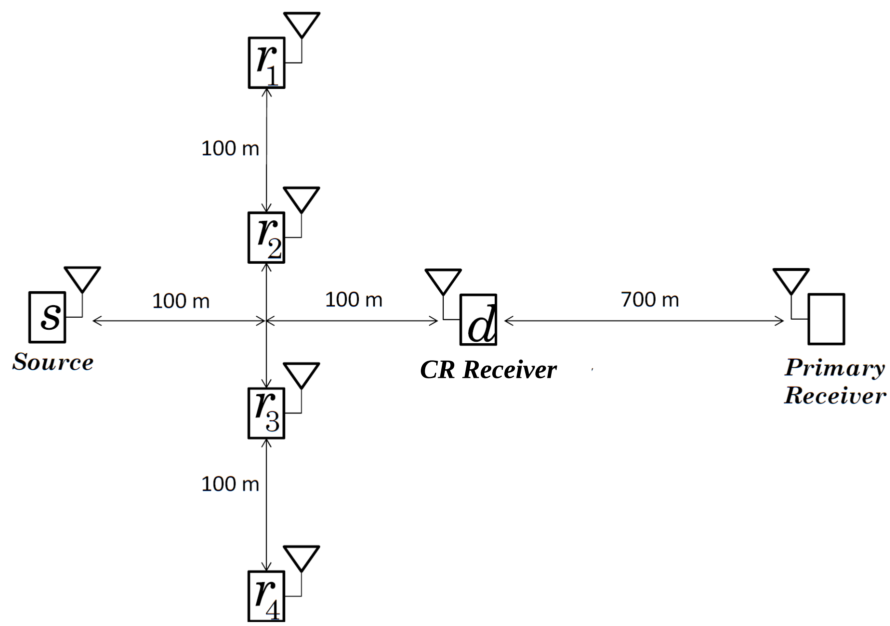

Figure 2.3: Simulated network topology.

In this section, we present some numerical examples in order to demonstrate the performances of various beamforming methods in the presence of asynchronous interference. For all the numerical examples presented in this section, we consider the network topology that is shown in Fig. 2.3[5]. We assume that all the channel fading gains are identically and independently distributed with Rayleigh distribution, and the log-distance path loss model with a path loss exponent value of 4 is considered. A slot duration of 0.4 msec is considered. The busy probability p of the PU,

[5]We simulated the performances of other network topologies as well. Similar performance trends have been obtained for other network topologies.

i.e., the channel occupancy probability, has a value of 0.7 unless other value is specified. To demonstrate the effectiveness of various beamforming methods, the interference threshold at the PR γ_{th} is set to be 4.1419×10^{-19} Watts which is 100 times of the noise power spectral density. The value of the target spectral efficiency B that is used in the simulations is equal to 1 bit/sec/Hz. When the PU transmitter is active, the received normalized average interference power from the primary transmitter to the CR receiver is assumed to be -10 dB.

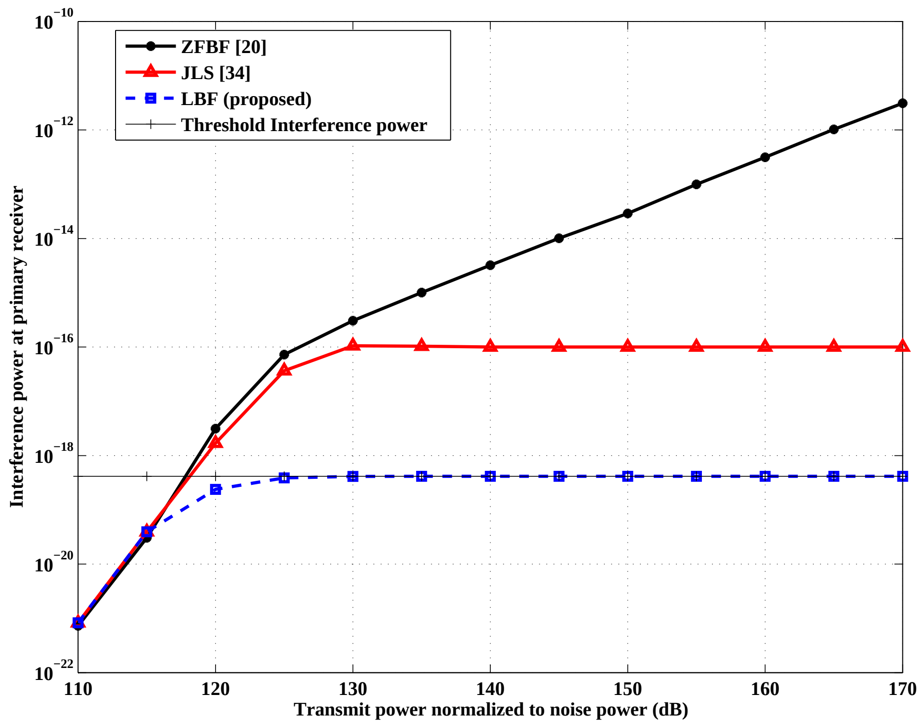

Figure 2.4: Asynchronous interference signal power at the PR with perfect channel information.

In Fig. 2.4 we plot the normalized transmit power versus the interference signal power at the PR with our proposed LBF method. In this figure we also plot the normalized transmit power versus the interference signal power at the PR with the ZFBF method proposed in [20], and the joint leakage suppression (JLS) method proposed in [33]. The JLS method is proposed in [33] for conventional cooperative cellular networks. Its design goal is to maximize the signal to leakage plus noise ratio (SLNR) of each of the active mobile users in the network concurrently without any restrictions on the induced leakage to a particular mobile user from different BSs. In the context of cooperative CR networks, the JLS method aims to maximize the SLNR of the CR receiver, without any restriction on the introduced leakage to the PR. From this figure we observe that the interference caused by the proposed LBF method increases as the transmit signal power increases at lower values of transmit power. However, as soon as the received interference power at the PR reaches the target interference threshold, it does not increase with the transmit power. This clearly shows that our proposed LBF method can maintain the interference threshold at the PR. On the contrary, the interference caused by the ZFBF method increases almost linearly with the transmit signal power and exceeds the interference target threshold for higher values of transmit power as expected. Also the JLS method causes interference power at the PR that exceeds the interference target threshold, since it aims to maximize the SINR at the CR receiver without any restriction on the introduced interference to the PR.

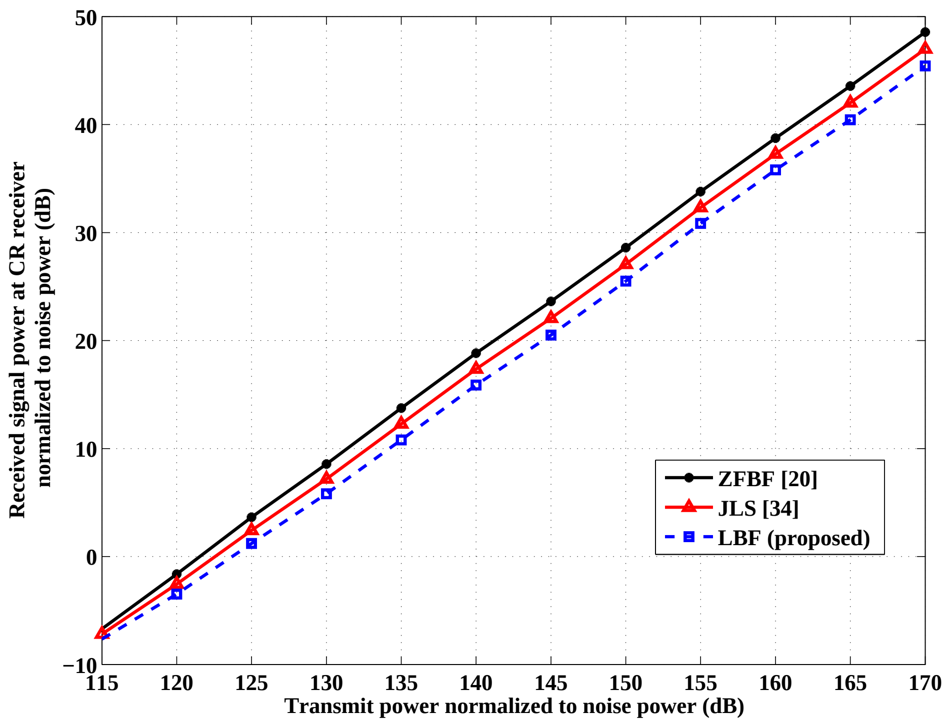

Figure 2.5: Received signal power at the CR receiver with perfect channel information.

The received signal power at the CR receiver for three beamforming methods is plotted in Fig. 2.5. It is observed from this figure that the received signal power at the CR receiver for our proposed LBF method is slightly less than that of the ZFBF and the JLS methods. The ZFBF and the JLS methods are both having higher received signal power at the Cr receiver at the expense of higher asynchronous interference at the PR.

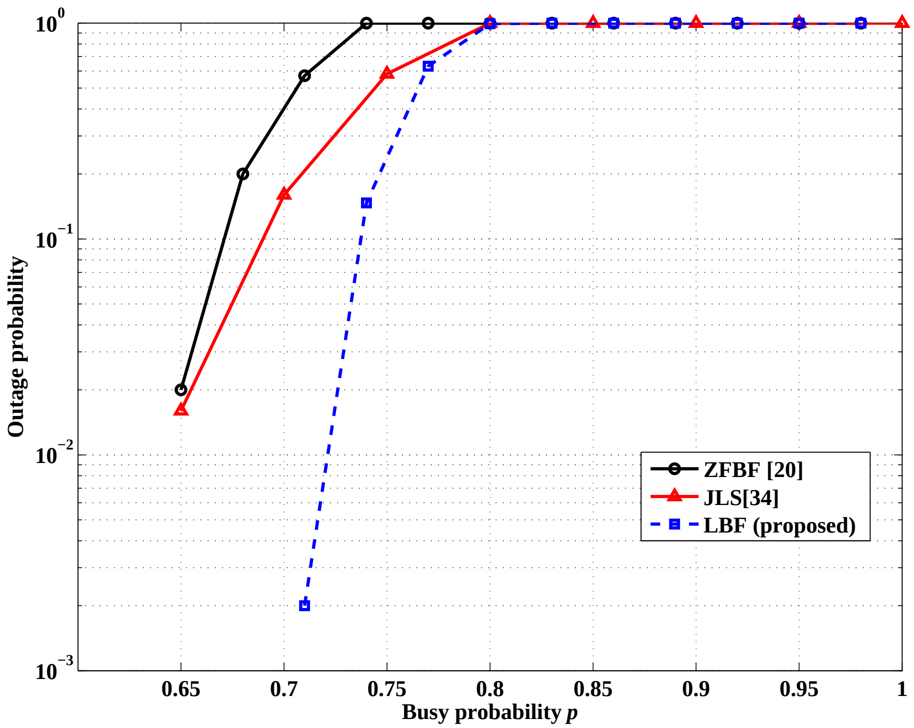

Figure 2.6: Outage probability of SU system versus busy probability of PU.

Another important performance metric is the outage probability which corresponds to the probability of having the instantaneous transmission capacity C in a given time slot below the target spectral efficiency B [54]. The outage events are determined as follows. If the number of successful relays K is zero, this is considered an outage event. Otherwise, the K CR relays either forward the data directly to the CR receiver when the PU is silent in the second hop or they cooperatively beamform the data stream to transmit the data to the CR receiver when the PU is active. The outage events in these two cases occur when the instantaneous transmission capacity C is below the target spectral efficiency B. When the PU is active, outage events also occur when the the instantaneous interference caused by the CR relays to the PR exceeds the target interference threshold γ_{th}. In this case, the CR relays are not allowed to transmit their data. The outage probability versus channel busy probability, p, is plotted in Fig. 2.6 for our proposed LBF method and that of the ZFBF method. Although we did not include the performance of the JLS method, JLS will have a higher outage probability than the LBF method as expected. It is also observed

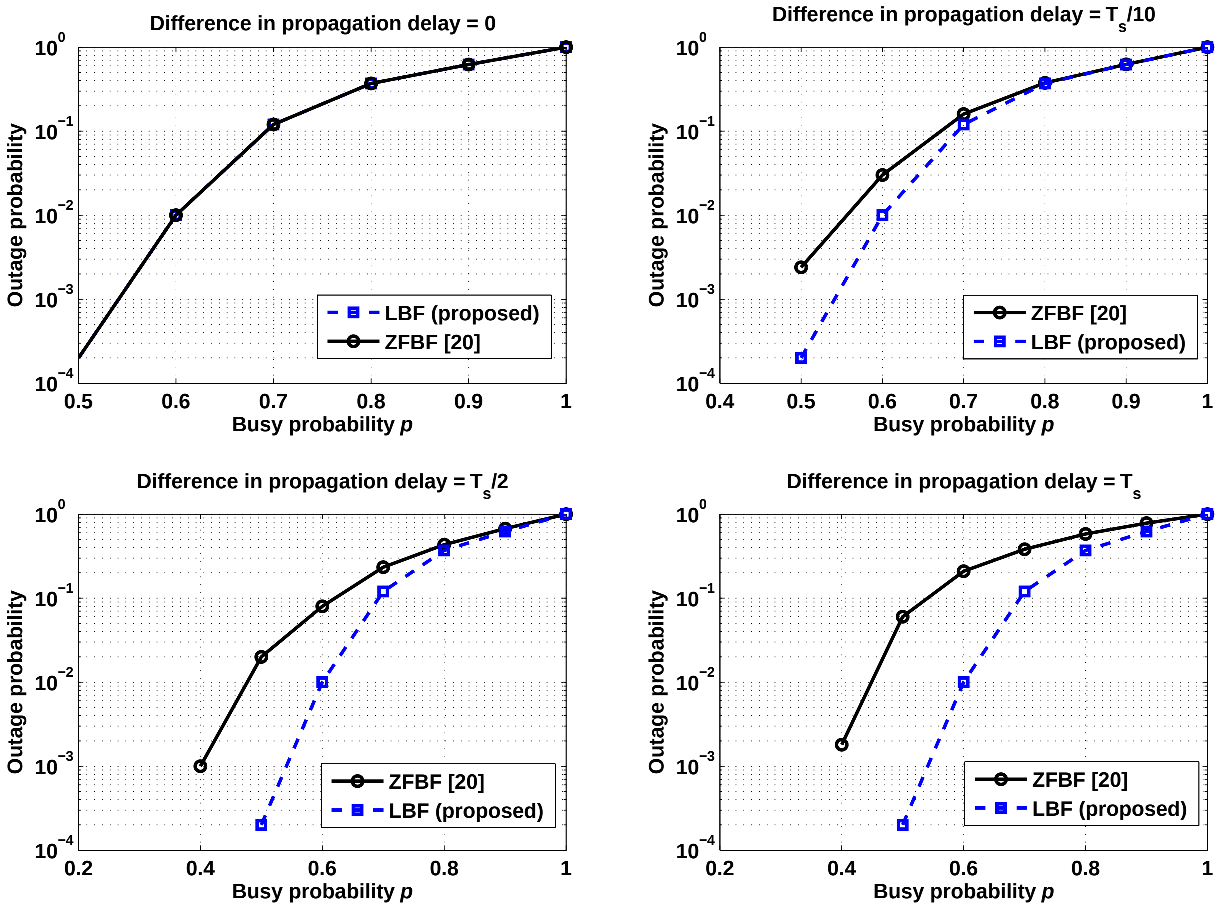

Figure 2.7: Outage probability of SU system versus busy probability of PU for different values of propagation difference.

from Fig. 2.6 that the outage probability of the LBF method is significantly less than that of the ZFBF method. This can be explained as follows. When the PU is silent in the second hop, the outage probabilities of the ZFBF method proposed in [20] and our proposed LBF scheme are equal. However, when the PU is active, the instantaneous interference introduced to the PR by the ZFBF method exceeds the target threshold γ_{th} most of the time. As such the CR relays cannot forward the data symbols and outage events occur frequently. On the other hand, our proposed scheme satisfies the target interference threshold instantaneously. Therefore our proposed LBF method has a lower outage probability. In particular, our proposed LBF method decreases the system outage probability up to 75% compared to the ZFBF method, for a PU's busy probability of 0.75.

Next, we investigate the effect of the difference in propagation delays between the cooperating CR relays on the performance of the wireless network. In Fig. 2.7, we consider having two CR relays in the network. In this figure, we plot the outage probability of the proposed LBF method and that of the ZFBF method in [20] versus the busy probability of the PU, p, for 4 different values of propagation difference between the two relays. When the propagation difference between the two relays at the PR is zero, the proposed LBF method has the same performance as that of ZFBF method. As the value of the propagation difference increases, the outage probability of the ZFBF method increases, while the LBF method continues to have the same outage probability values. Therefore, the performance gap between the two schemes increases with increasing the propagation difference between the cooperating relays, until it reaches its maximum value when the propagation difference is T_s.

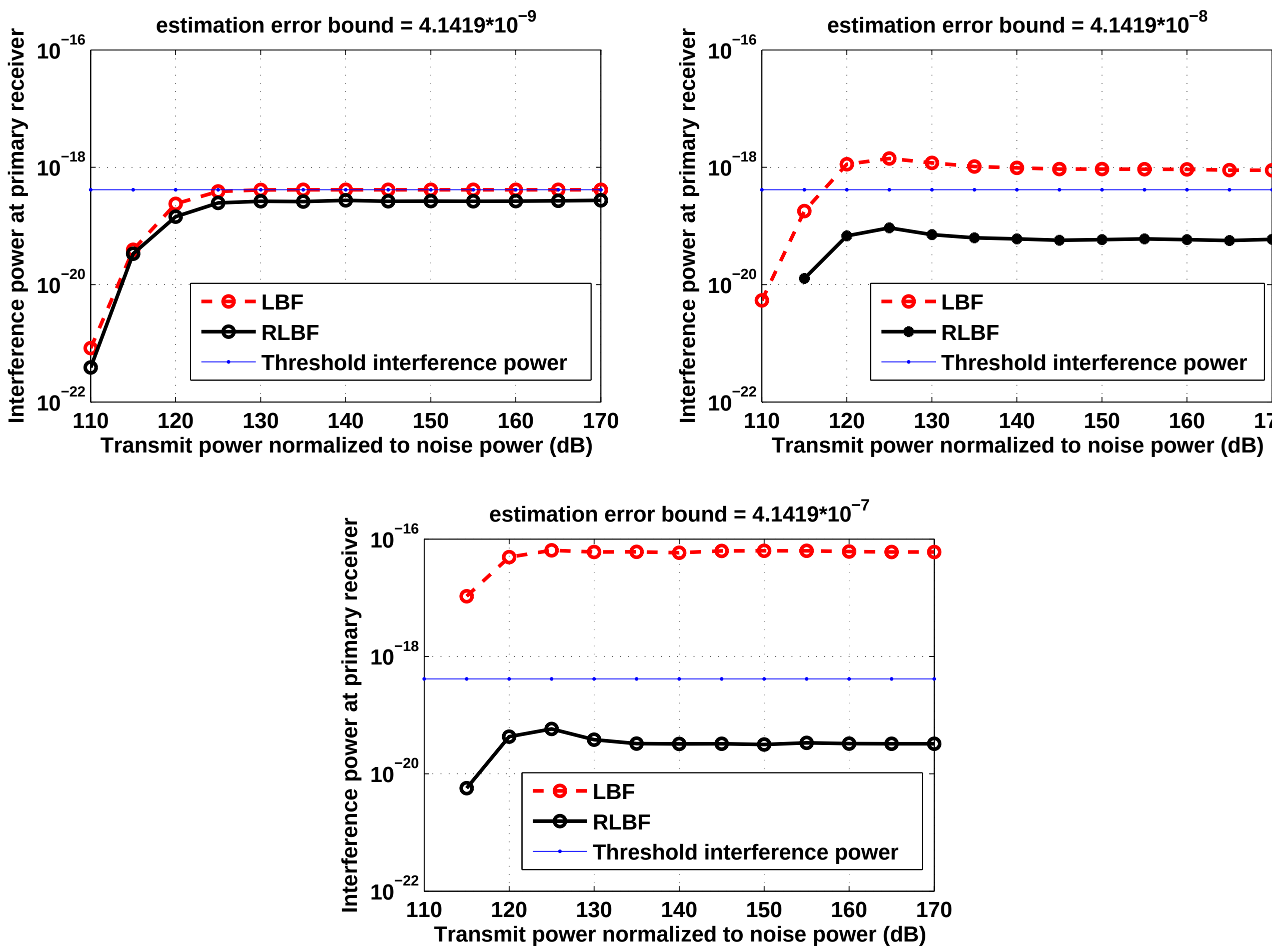

Figure 2.8: Transmit power versus interference power at the primary receiver for the RLBF method, for different values of estimation error bound, Ω_c.

Now we investigate the performances of the proposed RLBF and LBF methods for a transmission scenario when the CR relays have imperfect estimate of the CSI between the PR and the CR relays. In Fig. 2.8, we plot the normalized transmit power versus the interference signal power at the PR for both the LBF and the RLBF methods when the CR relays have imperfect estimation of channels between the CR relays and the PR, for different values of estimation error bound, Ω_c. From this figure, it is obvious that the introduced interference from the RLBF method is always well below the interference threshold even when the channel is not known perfectly at the CR relays, however the LBF method cannot meet the interference constraint at the PR for higher values of estimation errors. This is due to the fact that the designed RLBF method is a conservative design considering the maximum channel estimation error i.e., the worst case estimate of the channel. For the values of the estimation error bound $\Omega_C \geq 4.1419 \times 10^{-8}$, the LBF method violates the interference threshold of the PR. As the estimation error bound increases, the violation of the LBF method to the interference threshold becomes more severe.

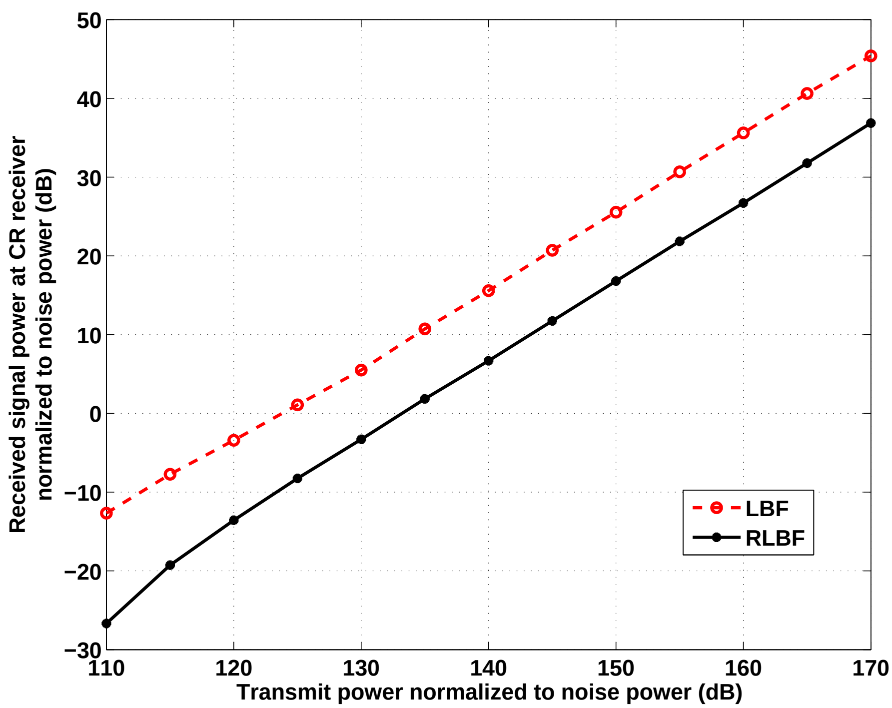

Figure 2.9: Transmit power versus received power at the CR receiver for the RLBF method.

In Fig. 2.9 we plot the normalized transmit power versus the received signal power at the CR receiver using both the LBF and the RLBF methods, using a value of estimation error bound Ω_C

equals to 4.1419×10^{-8}. Both of these methods exhibit similar behavior as the the transmit signal power increases the received signal power at the CR receiver increases. However with the RLBF method, the CR receiver receives less power compared to the LBF method. Again this is expected due to the conservative design of the RLBF method considering the worst case channel estimation as discussed above.

Chapter 3

Cooperative Beamforming with Multiple Primary Receivers and Multiple CR Receivers

As a follow-up of our work in Chapter 2, in this chapter we consider a more generalized setup where a group of CR nodes which are referred to as cooperating cognitive radio nodes (CCRNs) uses cooperative beamforming technique to broadcast a common message to *multiple* CR receivers. The cooperative CR network uses a wireless broadcast channel assigned to a primary transmitter to transmit information to *multiple* PRs simultaneously. This scenario is particularly important when a CR source wants to broadcast a common message to a group of CR receivers, the CR source may not be allowed to transmit enough power to cover all the CR receivers due to the interference restriction imposed by the nearby primary system. In such situation, a group of CCRNs can collaboratively use transmit beamforming to broadcast the common message to the CR receivers.

With multiple primary and CR receivers, transmission to a specific CR receiver introduces asynchronous interferences, not only at the PRs, but also at all other CR receivers. Due to these asynchronous interferences, the optimal beamforming technique developed in Section 2.3 cannot be directly extended for a generalized system with multiple primary and multiple CR receivers. In particular, when there is only one CR receiver in the system, there is no cross asynchronous interferences between CR receivers [55]. Then optimal beamforming design problem becomes a convex problem that can be solved optimally using the dual Lagrange function, as we have seen in Section 2.3. However, for the generalized setup with multiple primary and CR receivers considered in this chapter, the optimal beamforming technique is indeed intractable due to the non-convexity and non-linearity of the problem. In light of the intractability of the optimal beamforming design problem, in this chapter, we propose an approximation to design the beamforming directions and to allocate power among different beamforming directions. Even then development of a suboptimal beamforming technique is complex due to multiple interference constraints corresponding to multiple PRs which is discussed later in Section 3.3. Therefore, we also propose a low complexity power allocation algorithm.

In what follows, we present the overall system description and model the asynchronous interference signals at the PRs as well as at the CR receivers mathematically. Next, we develop the proposed beamforming techniques with perfect CSI at the CCRNs. Then, we extend the beamformong design problem for the case of having only statistical CSI of the PRs available at the CCRNs. Finally, we propose and investigate the performance of joint CCRN selection and cooperative beamforming.

3.1 System Model

In what follows, we provide the operating assumptions as well as the operating principles of the system model that we consider in our problem formulation.

3.1.1 Operating Assumptions

In this chapter, we consider a CR-based broadcasting system as the one shown in Fig. 3.1, where a group of L CCRNs uses a transmit beamforming technique to broadcast a common information to a group of K CR receivers. All nodes are assumed to be equipped with single antenna each. As we mentioned earlier in Chapter 1 that similar type of cooperative beamforming scenario has been considered for traditional wireless networks, e.g., wireless sensor networks, due to its compelling gain in the transmission rate, see for example [15], [32]. Using the USAM, the CR system shares a communication broadcasting channel with a primary transmitter, e.g., a primary BS that transmits information to J PRs simultaneously. For notational convenience, K CR receivers are denoted by $\mathrm{d}_k, k = 1, \cdots, K$, L cooperating CR relay nodes are denoted by $\mathrm{c}_l,\ l = 1, \cdots, L$ and J PRs are denoted by $\mathrm{p}_j,\ j = 1, \cdots, J$. We consider that both primary and CR systems work in a time-slotted fashion with a slot duration T_{slot} sec. We assume a block fading channel model, similar to the one we considered in Section 2.1.

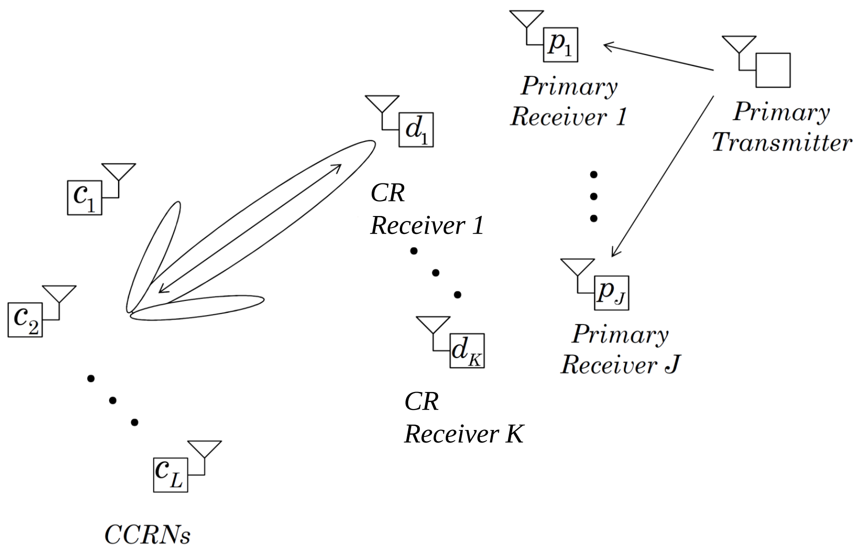

Figure 3.1: System model for cooperative beamforming with L CCRNs, K CR receivers and J primary receivers.

3.1.2 Operating Principles

At CCRN, c_l the information stream is mapped into modulated symbols, x_s which has average power P and the data vector consisting of these M modulated symbols is denoted by $\mathbf{x}_s$. The set of cooperative CCRNs uses K different beamforming vectors to transmit the data to K different CR receivers. The received signal at CR receiver, d_k can be written as

$$\mathbf{y}_k[n] = \mathbf{h}_k^{\mathrm{s}}[n]\mathbf{g}_k[n]\mathbf{x}_s[n] + \mathbf{I}_k[n] + \mathbf{m}_k[n] + \mathbf{z}_k[n], \tag{3.1}$$

where $\mathbf{x}_s[n]$ is common message symbols transmitted at time slot n and $\mathbf{h}_k^{\mathrm{s}}[n] \triangleq [h_{k1}^{\mathrm{s}}[n], \ldots, h_{kL}^{\mathrm{s}}[n]]$ is the channel vector from L transmitting CCRNs to CR receiver, d_k. The vector $\mathbf{g}_k[n] \triangleq [g_{k1}[n], \ldots, g_{kL}[n]]^T$ denotes the beamforming weight vector of the set of CCRNs corresponding to transmission to CR receiver, d_k with each element g_{kr} denoting the weight of the CCRN, c_r. $\mathbf{z}_k[n]$ is the AWGN vector at CR receiver, d_k with zero mean and two-sided power spectrum density $N_0/2$, and $\mathbf{m}_k[n]$ is the received interfering signal vector from the primary BS at CR receiver d_k. $\mathbf{I}_k[n]$ is the asynchronous interference signal at CR receiver, d_k resulting from the data transmissions to the other $(K-1)$ CR receivers[6]. The mathematical model of this asynchronous interference

[6]Even though the same message is transmitted to all CR receivers, the asynchronous arrival of the data stream intended to one CR receiver, at other CR receivers, is still considered a form of interference. This concept is similar to that of inter-symbol interference that takes place in multi-path environments.

signal is introduced in the following section. It is important to note that the proposed techniques in this chapter can be applied to unicast systems as well as broadcast systems without any change in the algorithms, as the data stream intended for each CR receiver is transmitted using this user's specific beamforming vector.

3.2 Modeling of Asynchronous Interferences in CR-Based Broadcasting Systems

Due to the difference in path lengths between the CCRNs, the received signals from different CCRNs at different PRs and at different CR receivers can experience different propagation delays. Although the received signal at a particular CR receiver e.g., d_1 from different CCRNs can be synchronized by using the timing advance mechanism, as mentioned in Section 2.2, or other mechanism [32], the received signals at the PRs p_j $(j = 1, \cdots, J)$ and at the other CR receivers, d_k $(k = 2, \cdots, K)$ cannot be synchronized simultaneously. As such, the signal transmissions from CCRNs will introduce asynchronous interferences at PRs p_j $(j = 1, \cdots, J)$ and at the other CR receivers, d_k $(k = 2, \cdots, K)$. In Section 2.2, we have modeled the asynchronous interference at the PR with one PR and one CR receiver in the system. However for the generalized scenario considered in this chapter, we need to model the asynchronous interferences not only at different PRs but also at different CR receivers. In what follows, we model these asynchronous interferences. For notational convenience, we will drop the time slot index n.

3.2.1 Asynchronous Interference at Primary Receiver, $\mathbf{p}_j$

Using eq. (2.13) from Chapter 2, the asynchronous interference power resulting from transmission to CR receiver, d_k at PR, p_j, $P_{\text{asynch}}^{(j,k)}$, can be written mathematically in the following form

$$P_{\text{asynch}}^{(j,k)} = \sum_{r=1}^{L} \sum_{f=1}^{L} g_{kf}^{\dagger} (h_{jf}^{\mathrm{p}})^{\dagger} h_{jr}^{\mathrm{p}} g_{kr} \beta_k^{j\,(r,f)}, \tag{3.2}$$

where h_{jr}^{p} is the channel fading gain from CCRN c_r to PR, p_j, $\beta_k^{j\,(r,f)}$ is the correlation between the asynchronous symbols of CCRNs, c_r and c_f at PR, p_j corresponding to the transmission to CR receiver, d_k. The value of $\beta_k^{j\,(r,f)}$ can be calculated for given propagation delays between CCRNs,

c_r and c_f to PR, p_j using the same technique described in Section 2.2. Now the total asynchronous interference power at PR, p_j can be expressed as

$$P_{\text{asynch}}^{j} = \sum_{k=1}^{K}\sum_{r=1}^{L}\sum_{f=1}^{L} g_{kf}^{\dagger}(h_{jf}^{\mathrm{p}})^{\dagger}h_{jr}^{\mathrm{p}}g_{kr}\beta_{k}^{j\,(r,f)}. \tag{3.3}$$

The asynchronous interference power at PR, p_j in eq. (3.3) can be rewritten in a matrix form as follows

$$P_{\text{asynch}}^{j} = \sum_{k=1}^{K} \mathbf{g}_k{}^{\dagger}\mathbf{R}_k^j\mathbf{g}_k, \tag{3.4}$$

where $\mathbf{R}_k^j$ is expressed as

$$\mathbf{R}_k^j = \begin{bmatrix} \beta_k^{j\,(1,1)}(h_{j1}^{\mathrm{p}})^{\dagger}h_{j1}^{\mathrm{p}} & \cdots & \beta_k^{j\,(1,L)}(h_{j1}^{\mathrm{p}})^{\dagger}h_{jL}^{\mathrm{p}} \\ \beta_k^{j\,(2,1)}(h_{j2}^{\mathrm{p}})^{\dagger}h_{j1}^{\mathrm{p}} & \cdots & \beta_k^{j\,(2,L)}(h_{j2}^{\mathrm{p}})^{\dagger}h_{jL}^{\mathrm{p}} \\ \vdots & \ddots & \vdots \\ \beta_k^{j\,(L,1)}(h_{jL}^{\mathrm{p}})^{\dagger}h_{j1}^{\mathrm{p}} & \cdots & \beta_k^{j\,(L,L)}(h_{jL}^{\mathrm{p}})^{\dagger}h_{jL}^{\mathrm{p}} \end{bmatrix}. \tag{3.5}$$

The received signal power at CR receiver, d_k is given by

$$P_{k,\text{signal}} = P\mathbf{g}_k{}^{\dagger}(\mathbf{h}_k^{\mathrm{s}})^{\dagger}\mathbf{h}_k^{\mathrm{s}}\mathbf{g}_k. \tag{3.6}$$

3.2.2 Asynchronous Interference at CR Receiver, d_k

We denote the asynchronous interference signal at CR receiver, d_k resulting from the data transmissions to the other $(K-1)$ CR receivers, by $\mathbf{I}_k[n]$. This asynchronous interference signal can be written as follows

$$\mathbf{I}_k[n] = \sum_{i=1, i\neq k}^{K}\sum_{r=1}^{L} h_{kr}^{\mathrm{s}}[n]g_{ir}[n]\mathbf{i}_r^k[n], \tag{3.7}$$

where $\mathbf{i}_r^k[n]$ is the asynchronous vector of symbols received at the CR receiver d_k from the CCRN c_r, as shown in Fig. 3.2.

The asynchronous interference power at CR receiver, d_k resulting from transmission to CR receiver, d_i is given by

$$\mathrm{AI}_i^k = \sum_{r=1}^{L}\sum_{f=1}^{L}\beta_i^{k\,(r,f)}g_{if}^{\dagger}(h_{kf}^{\mathrm{s}})^{\dagger}h_{kr}^{\mathrm{s}}g_{ir}, \tag{3.8}$$

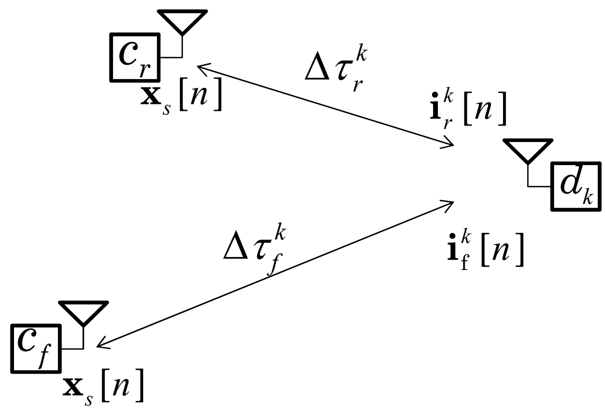

Figure 3.2: An example of the asynchronous vector of symbols received at the CR receiver d_k from two different CCRNs, with two different propagation delays.

where $\beta_i^{k(r,f)}$ is the correlation between the asynchronous symbols of CCRNs, c_r and c_f at CR receiver, d_i corresponding to the transmission to CR receiver, d_k where $k \neq i$. The value of $\beta_i^{k(r,f)}$ can be calculated for given propagation delays between CCRNs, c_r and c_f to CR receivers, d_i and d_k using the same method described in Section 2.2. Therefore, the total asynchronous interference power at CR receiver, d_k resulting from data transmission to the other $(K-1)$ CR receivers, AI^k can be written as

$$\mathrm{AI}^k = \sum_{i=1, i\neq k}^{K} \sum_{r=1}^{L} \sum_{f=1}^{L} \beta_i^{k(r,f)} g_{if}^{\dagger} (h_{kf}^{\mathrm{s}})^{\dagger} h_{kr}^{\mathrm{s}} g_{ir}. \tag{3.9}$$

Similar to eq. (3.6), AI^k can be written in a matrix form as follows

$$\mathrm{AI}^k = \sum_{i=1, i\neq k}^{K} \mathbf{g}_i^{\dagger} \mathbf{T}_i^k \mathbf{g}_i, \tag{3.10}$$

where $\mathbf{T}_i^k$ is written as

$$\mathbf{T}_i^k \triangleq \begin{bmatrix} \beta_i^{k(1,1)} (h_{k1}^{\mathrm{s}})^{\dagger} h_{k1}^{\mathrm{s}} & \cdots & \beta_i^{k(1,L)} (h_{k1}^{\mathrm{s}})^{\dagger} h_{kL}^{\mathrm{s}} \\ \vdots & \ddots & \vdots \\ \beta_i^{k(L,1)} (h_{kL}^{\mathrm{s}})^{\dagger} h_{k1}^{\mathrm{s}} & \cdots & \beta_i^{k(L,L)} (h_{kL}^{\mathrm{s}})^{\dagger} h_{kL}^{\mathrm{s}} \end{bmatrix}.$$

From eqs. (3.2) and (3.8), it is obvious that if ZFBF method is used to force the asynchronous interference power at each PR p_j and each CR receiver d_k to be zero, the interference from each CCRN has to be annulled separately, i.e. each CCRN must have a number of antennas $> (K+J)$.

So for single antenna based CCRN, failing to satisfy this condition, the ZFBF method results in higher interference at the PRs, as shown in Section 3.6.

3.3 Beamforming Design with Perfect Channel Knowledge

In this section, we reformulate the cooperative leakage beamforming (LBF) technique in order to address the problem of asynchronous interferences at the PRs and other CR receivers. In this reformulation, we use the same assumption as in Section 2.3 that the channel fading gains, i.e., instantaneous CSI between the CCRNs and the PRs as well as the instantaneous CSI between the CCRNs and the CR receivers are known perfectly at the CCRNs. In the next section, we consider the case when only the statistical CSI between the CCRNs and the PRs are known at the CCRNs.

3.3.1 Problem Formulation

Let us denote the achievable transmission rate of CR receiver d_k by r_k, which can be expressed using the ideal capacity formula as follows

$$r_k = \log_2\left(1 + \frac{P\mathbf{g}_k^\dagger(\mathbf{h}_k^{\mathrm{s}})^\dagger \mathbf{h}_k^{\mathrm{s}}\mathbf{g}_k}{\sigma^2_{\mathrm{n,i},k} + \sum\limits_{i=1, i\neq k}^{K} \mathbf{g}_i^\dagger \mathbf{T}_i^k \mathbf{g}_i}\right), \tag{3.11}$$

where $\sigma^2_{\mathrm{n,i},k}$ is the total interference (from the primary transmitter) and noise power at CR receiver d_k. The goal is to design K different beamforming vectors corresponding to K CR receivers that maximize the weighted sum rate of all CR receivers while keeping the interference to the PRs below their target thresholds. We consider maximizing the weighted sum rate of all the K CR receivers since it is more generalized (see for example [56], and the references therein). The design goal can be formulated as an optimization problem as follows[7]

$$\begin{aligned}
\mathbf{g}_1^{(\mathrm{opt})}, \mathbf{g}_2^{(\mathrm{opt})}, \cdots, \mathbf{g}_K^{(\mathrm{opt})} &= \max_{\mathbf{g}_1, \cdots, \mathbf{g}_K} \sum_{k=1}^{K} w^k \log_2 \left(1 + \frac{P \mathbf{g}_k^{\dagger} (\mathbf{h}_k^{\mathrm{s}})^{\dagger} \mathbf{h}_k^{\mathrm{s}} \mathbf{g}_k}{\sigma_{\mathrm{n,i},k}^2 + \sum_{i=1, i \neq k}^{K} \mathbf{g}_i^{\dagger} \mathbf{T}_i^k \mathbf{g}_i} \right), \\
\text{subject to:} \quad & \sum_{k=1}^{K} \mathbf{g}_k^{\dagger} \mathbf{R}_k^j \mathbf{g}_k \leq \gamma_{\mathrm{th}}^j, \text{for } j = 1, \cdots, J. \qquad (3.12)
\end{aligned}$$

where w^k is the weighting factor of CR receiver d_k, and γ_{th}^j is the required interference threshold for PR, p_j.

3.3.2 Development of Sub-Optimal Cooperative LBF Technique

The optimization problem in eq. (3.12) is a non-linear and non-convex optimization problem due to the presence of the interference power $\mathrm{AI}^k = \sum_{i=1, i \neq k}^{K} \mathbf{g}_i^{\dagger} \mathbf{T}_i^k \mathbf{g}_i$ in CR receiver d_k's transmission rate, r_k. The design problem of the beamforming vectors in eq. (3.12) can be converted into a joint beamforming and power allocation problem without the loss of optimality. In particular, we propose a cooperative bemforming technique that has two phases as described below. It is important to note that if the asynchronous interference terms at the PRs, as well as the cross asynchronous interference terms between the CR receivers, are neglected in eq. (3.12) , the ZFBF technique in [20] can provide the optimal beamforming vectors directly in a single step.

Phase I

In this phase, the direction of the normalized beamforming vector, $\bar{\mathbf{g}}_k$ is obtained. As such the received signal power at CR receiver d_k is maximized while minimizing the interference at all PRs and other CR receivers. This can be written as the following optimization problem

$$\bar{\mathbf{g}}_k^{(\mathrm{LBF})} = \max_{\bar{\mathbf{g}}_k} \frac{\bar{\mathbf{g}}_k^{\dagger} (\mathbf{h}_k^{\mathrm{s}})^{\dagger} \mathbf{h}_k^{\mathrm{s}} \bar{\mathbf{g}}_k}{\bar{\mathbf{g}}_k^{\dagger} (\mathbf{R}_k + \mathbf{T}_k) \bar{\mathbf{g}}_k}, \qquad \text{for } k = 1, \cdots, K, \qquad (3.13)$$

where $\mathbf{T}_k = \sum_{i=1, i \neq k}^{K} \mathbf{T}_k^i$ and $\mathbf{R}_k = \sum_{j=1}^{J} \mathbf{R}_k^j$. The signal-to-leakage power ratio in eq. (3.13) is in the form of a generalized Rayleigh quotient, that is maximized when $\bar{\mathbf{g}}_k^{(\mathrm{LBF})}$ is the normalized

[7]We do not consider transmit power constraint for the CCRNs as we develop the beamforming technique for the interference limited scenario.

eigen vector of the matrix $\left(\mathbf{R}_k + \mathbf{T}_k\right)^{-1}(\mathbf{h}_k^{\mathrm{s}})^{\dagger}\mathbf{h}_k^{\mathrm{s}}$ that corresponds to its maximum eigen value [57]. Therefore the normalized beamforming vector, $\bar{\mathbf{g}}_k^{(\mathrm{LBF})}$, can be expressed as follows

$$\bar{\mathbf{g}}_k^{(\mathrm{LBF})} = \frac{\left(\mathbf{R}_k + \mathbf{T}_k\right)^{-1}\mathbf{h}_k^{\mathrm{s}\dagger}}{\left\|\left(\mathbf{R}_k + \mathbf{T}_k\right)^{-1}\mathbf{h}_k^{\mathrm{s}\dagger}\right\|}. \tag{3.14}$$

Phase II

In this phase, power is allocated among different beamforming directions. As such the weighted sum rate of CR receivers is maximized while the interference thresholds at different PRs are met. In particular, given the normalized beamforming vector $\bar{\mathbf{g}}_k^{(\mathrm{LBF})}$ obtained in phase I, we obtain its allocated power $\alpha_k^{(\mathrm{LBF})}$ that satisfies the interference thresholds at all PRs simultaneously, where $\mathbf{g}_k^{(\mathrm{LBF})} = \sqrt{\alpha_k^{(\mathrm{LBF})}}\bar{\mathbf{g}}_k^{(\mathrm{LBF})}$.

Using the normalized beamforming vector, $\bar{\mathbf{g}}_k^{(\mathrm{LBF})}$, obtained in phase I into the optimization problem in eq. (3.12), the optimal power allocation problem among different beamforming directions can be rewritten as follows

$$\begin{aligned} \alpha_1^{(\mathrm{LBF})}, \cdots, \alpha_K^{(\mathrm{LBF})} \quad &= \quad \max_{\alpha_1, \cdots, \alpha_K} \sum_{k=1}^{K} w^k \log_2\left(1 + \frac{P\alpha_k \bar{\mathbf{g}}_k^{(\mathrm{LBF})\dagger}(\mathbf{h}_k^{\mathrm{s}})^{\dagger}\mathbf{h}_k^{\mathrm{s}}\bar{\mathbf{g}}_k^{(\mathrm{LBF})}}{\sigma_{\mathrm{n,i},k}^2 + \sum_{i=1, i\neq k}^{K} \alpha_i \bar{\mathbf{g}}_i^{(\mathrm{LBF})\dagger}\mathbf{T}_i^k \bar{\mathbf{g}}_i^{(\mathrm{LBF})}}\right), \\ \text{subject to:} \quad & \quad \sum_{k=1}^{K} \alpha_k \bar{\mathbf{g}}_k^{(\mathrm{LBF})\dagger}\mathbf{R}_k^j \bar{\mathbf{g}}_k^{(\mathrm{LBF})} \leq \gamma_{\mathrm{th}}^j, \text{for } j = 1, \cdots, J. \end{aligned} \tag{3.15}$$

The power allocation problem in eq. (3.15) is still a non-linear and non-convex optimization problem due to the presence of the asynchronous interference power AI^k. Non-convex optimization is significantly harder to be analyzed and solved, even by numerical methods. In particular, a local optimum may not be a global optimum and the duality gap can be strictly positive [58]. Several approximations, in different contexts, have been proposed to solve non-convex optimization problems, see for examples [59, 60] and the references therein. However, these approximations are not applicable to the optimization problem in our hand. On the other hand, power control problems for sum rate maximization for interference channels in conventional wireless communication systems have been studied in [61] and [62] under a sum power constraint. These power allocation schemes cannot be extended for the problem in eq. (3.15) due to the asynchronous interference power constraints at the PRs, which is essential for underlay CR networks. Moreover, the problem in our case is a weighted sum rate maximization problem which requires different handling. Therefore, we propose a different approximation for the power allocation problem in eq. (3.15) to relax it into a convex problem. In Section 3.6, the performance of the proposed approximation is compared with the numerical solution of eq. (3.15) obtained using active set method [63], for different noise plus interference power levels, $\sigma^2_{\mathrm{n,i},k}$. In fact, it is shown that the proposed approximation has a very close performance to that of the active set method in terms of achievable sum rate, but with much less computational complexity.

The proposed approximation is done by initially assuming that the power values, α_i ($i = 1, \cdots, K$) are equal, i.e.,

$$\alpha_1 = \alpha_2 = \cdots = \alpha_K = \alpha^{(0)}_{\mathrm{EQ}}, \tag{3.16}$$

where the initial value $\alpha^{(0)}_{\mathrm{EQ}}$ is obtained from the constraints of the interference thresholds at the PRs as follows

$$\alpha^{(0)}_{\mathrm{EQ}} = \min\left(\frac{\gamma^1_{\mathrm{th}}}{\sum_{k=1}^{K} \bar{\mathbf{g}}_k^{(\mathrm{LBF})\dagger} \mathbf{R}^1_k \bar{\mathbf{g}}_k^{(\mathrm{LBF})}}, \cdots, \frac{\gamma^J_{\mathrm{th}}}{\sum_{k=1}^{K} \bar{\mathbf{g}}_k^{(\mathrm{LBF})\dagger} \mathbf{R}^J_k \bar{\mathbf{g}}_k^{(\mathrm{LBF})}} \right). \tag{3.17}$$

The asynchronous interference power $\mathrm{AI}^k_{\mathrm{EQ}}$ corresponding to $\alpha^{(0)}_{\mathrm{EQ}}$ is given by

$$\mathrm{AI}^k_{\mathrm{EQ}} = (K-1)\alpha^{(0)}_{\mathrm{EQ}} \bar{\mathbf{g}}_i^{(\mathrm{LBF})\dagger} \mathbf{T}^k_i \bar{\mathbf{g}}_i^{(\mathrm{LBF})}, \quad k = 1, 2, \cdots, K. \tag{3.18}$$

Then using this asynchronous interference power $\mathrm{AI}^k_{\mathrm{EQ}}$, we propose to obtain power values α_k ($k = 1, 2, \cdots, K$) corresponding to different beamforming directions as follows

$$\begin{aligned} \alpha_1^{(\mathrm{LBF})}, \cdots, \alpha_K^{(\mathrm{LBF})} \quad &= \quad \max_{\alpha_1, \cdots, \alpha_K} \sum_{k=1}^{K} w^k \log_2\left(1 + \frac{P\alpha_k \bar{\mathbf{g}}_k^{(\mathrm{LBF})\dagger} (\mathbf{h}^{\mathrm{s}}_k)^\dagger \mathbf{h}^{\mathrm{s}}_k \bar{\mathbf{g}}_k^{(\mathrm{LBF})}}{\sigma^2_{\mathrm{n,i},k} + \mathrm{AI}^k_{\mathrm{EQ}}}\right), \\ \text{subject to:} \quad & \sum_{k=1}^{K} \alpha_k \bar{\mathbf{g}}_k^{(\mathrm{LBF})\dagger} \mathbf{R}^j_k \bar{\mathbf{g}}_k^{(\mathrm{LBF})} \leq \gamma^j_{\mathrm{th}}, \quad \text{for } j = 1, \cdots, J. \end{aligned} \tag{3.19}$$

Now the approximated optimization problem in eq. (3.19) is a convex optimization problem, that can be solved using the dual Lagrange function, with zero duality gap. Please note that the difference of updated AI^k corresponding to $\alpha_k^{(\mathrm{LBF})}$ ($k = 1, 2, \cdots, K$) in eq. (3.19) and $\mathrm{AI}^k_{\mathrm{EQ}}$ in eq. (3.18) is negligible compared to $\sigma^2_{\mathrm{n,i},k}$. This is due to the fact that the mutual asynchronous interference signals between the CR receivers, $\bar{\mathbf{g}}_i^{(\mathrm{LBF})\dagger}\mathbf{T}_i^k\bar{\mathbf{g}}_i^{(\mathrm{LBF})}$, have already been minimized by optimizing the beamformer directions in phase I. Therefore, the weighted sum rate obtained by solving the approximated optimization problem in eq. (3.19) is very close to the weighted sum rate that is obtained by numerically solving the original non-convex problem in eq. (3.15). This is verified in Section 3.6, where the values of $\alpha_k^{(\mathrm{LBF})}$ obtained by solving eq. (3.19) are shown to provide a weighted sum rate which is very close to that of the numerically obtained value in eq. (3.15) by using active set method [63], for different noise plus interference power levels, $\sigma^2_{\mathrm{n,i},k}$.

The Lagrange function of the above optimization problem can be written as

$$\mathcal{L} = \sum_{k=1}^{K} w^k \log_2\left(1 + \frac{P\alpha_k\bar{\mathbf{g}}_k^{(\mathrm{LBF})\dagger}(\mathbf{h}_k^{\mathrm{s}})^\dagger\mathbf{h}_k^{\mathrm{s}}\bar{\mathbf{g}}_k^{(\mathrm{LBF})}}{\sigma^2_{\mathrm{n,i},k} + (K-1)\alpha_{\mathrm{App}}\bar{\mathbf{g}}_i^{(\mathrm{LBF})\dagger}\mathbf{T}_i^k\bar{\mathbf{g}}_i^{(\mathrm{LBF})}}\right) - \sum_{j=1}^{J}\left(\lambda^j\big(\sum_{k=1}^{K}\alpha_k\bar{\mathbf{g}}_k^{(\mathrm{LBF})\dagger}\mathbf{R}_k^j\bar{\mathbf{g}}_k^{(\mathrm{LBF})} - \gamma^j_{\mathrm{th}}\big)\right), \quad (3.20)$$

where $\{\lambda^1, \cdots, \lambda^J\}$ are the Lagrange multipliers. Using KKT conditions, we can write

$$\frac{w^k}{\ln 2}\left(\alpha_k + \frac{\sigma^2_{\mathrm{n,i},k} + (K-1)\alpha_{\mathrm{App}}\bar{\mathbf{g}}_i^{(\mathrm{LBF})\dagger}\mathbf{T}_i^k\bar{\mathbf{g}}_i^{(\mathrm{LBF})}}{P\bar{\mathbf{g}}_k^{(\mathrm{LBF})\dagger}(\mathbf{h}_k^{\mathrm{s}})^\dagger\mathbf{h}_k^{\mathrm{s}}\bar{\mathbf{g}}_k^{(\mathrm{LBF})}}\right)^{-1} - \sum_{j=1}^{J}\left(\lambda^j\bar{\mathbf{g}}_k^{(\mathrm{LBF})\dagger}\mathbf{R}_k^j\bar{\mathbf{g}}_k^{(\mathrm{LBF})}\right) = 0 \quad \text{for } k = 1, \cdots, K, \quad (3.21)$$

$$\lambda^j\big(\sum_{k=1}^{K}\alpha_k\bar{\mathbf{g}}_k^{(\mathrm{LBF})\dagger}\mathbf{R}_k^j\bar{\mathbf{g}}_k^{(\mathrm{LBF})} - \gamma^j_{\mathrm{th}}\big) = 0, \qquad \text{for } j = 1, \cdots, J, \quad (3.22)$$

$$\sum_{k=1}^{K}\alpha_k\bar{\mathbf{g}}_k^{(\mathrm{LBF})\dagger}\mathbf{R}_k^j\bar{\mathbf{g}}_k^{(\mathrm{LBF})} - \gamma^j_{\mathrm{th}} \le 0, \qquad \text{for } j = 1, \cdots, J, \quad (3.23)$$

$$\lambda^1, \cdots, \lambda^J \ge 0, \quad (3.24)$$

According to eq. (3.21), the power allocation for beamforming direction corresponding to CR receiver d_k is given by

$$\alpha_k^{\text{(LBF)}} = \max\left(0, \frac{w^k}{(\ln 2)\sum_{j=1}^{J}\left(\lambda^j \bar{\mathbf{g}}_k^{\text{(LBF)}\dagger}\mathbf{R}_k^j \bar{\mathbf{g}}_k^{\text{(LBF)}}\right)} - \frac{\sigma^2_{\text{n,i},k} + (K-1)\alpha_{\text{App}}\bar{\mathbf{g}}_i^{\text{(LBF)}\dagger}\mathbf{T}_i^k\bar{\mathbf{g}}_i^{\text{(LBF)}}}{P\bar{\mathbf{g}}_k^{\text{(LBF)}\dagger}(\mathbf{h}_k^{\text{s}})^\dagger\mathbf{h}_k^{\text{s}}\bar{\mathbf{g}}_k^{\text{(LBF)}}}\right), \quad (3.25)$$

for $k = 1, \cdots, K$. The power allocation in eq. (3.25) is the cap-limited water-filling solution. In eq. (3.25), the power allocation values $\alpha_k^{\text{(LBF)}}$ are expressed in terms of Lagrange multipliers λ_j ($j = 1, \cdots, J$) which need to be evaluated.

In order to obtain the Lagrange multipliers and consequently $\alpha_k^{\text{(LBF)}}$, a recursive technique is used as described below. First, we assume that only one Lagrange multiplier is greater than zero, i.e., $\lambda^j > 0$, while $\lambda^i = 0$, for all i except $i \neq j$. This implies that the power allocation values $\alpha_k^{\text{(LBF)}}$, ($k = 1, \cdots, K$), satisfy the interference threshold with equality only at PR p_j. For this case, we can write

$$\sum_{k=1}^{K} \alpha_k^{\text{(LBF)}}\bar{\mathbf{g}}_k^{\text{(LBF)}\dagger}\mathbf{R}_k^j\bar{\mathbf{g}}_k^{\text{(LBF)}} - \gamma_{\text{th}}^j = 0. \quad (3.26)$$

Now the value of λ^j and the power allocation values $\alpha_k^{\text{(LBF)}}$, for all k are found by solving set of equations in (3.25) and (3.26) simultaneously. If these values of $\alpha_k^{\text{(LBF)}}$ satisfy the remaining $(J-1)$ interference constraints given by the set of equations in (3.23), then $\alpha_k^{\text{(LBF)}}$ for all k represent the optimum solution of (3.19). Otherwise, we set $\lambda^k > 0$ ($k \neq j$) while $\lambda^i = 0$, for all i except $i \neq k$, and so on until we find the power allocation values that satisfy all constraints simultaneously.

If no power allocation values that satisfy all constraints simultaneously is found, considering one constraint as equality constraint we consider the case when two constraints are met with equality. In other words, we set simultaneously $\lambda^j > 0$ and $\lambda^l > 0$ while $\lambda^i = 0$, for all i except $i \neq j, l$. Then, the following two slackness conditions in eq. (3.22) are satisfied as follows

$$\sum_{k=1}^{K} \alpha_k^{\text{(LBF)}}\bar{\mathbf{g}}_k^{\text{(LBF)}\dagger}\mathbf{R}_k^j\bar{\mathbf{g}}_k^{\text{(LBF)}} - \gamma_{\text{th}}^j = 0, \quad (3.27)$$

$$\sum_{k=1}^{K} \alpha_k^{\text{(LBF)}}\bar{\mathbf{g}}_k^{\text{(LBF)}\dagger}\mathbf{R}_k^l\bar{\mathbf{g}}_k^{\text{(LBF)}} - \gamma_{\text{th}}^l = 0. \quad (3.28)$$

The values of λ^j, λ^l and the power allocation values $\alpha_k^{\text{(LBF)}}$ for all k are found by solving the set of equations in (3.25), (3.27), and (3.28) simultaneously, where the Lagrange multipliers are found using the sub-gradient method. If these values of $\alpha_k^{(\mathrm{LBF})}$ satisfy the remaining $(J-2)$ interference constraints given by the set of equations in (3.23), then $\alpha_k^{(\mathrm{LBF})}$ for all k are the optimum power allocation values. Otherwise, we set another set of two constraints as equality constraint, i.e., $\lambda^m > 0$ and $\lambda^n > 0$ $(m, n \neq j, l)$ while $\lambda^i = 0$, for all i except $i \neq m, n$, and so on until we find the values of $\alpha_k^{(\mathrm{LBF})}$ that satisfy all constraints simultaneously. The worst case scenario in terms of complexity occurs when the J constraints hold with equality simultaneously.

This procedure is summarized below:

for $i = 1 \rightarrow J$ **do**

- Form $\binom{J}{i}$ different sets of λ's, such that each set $\mathcal{S}_k^i$ for $k = 1, \cdots, \binom{J}{i}$ is composed of i different λ's.

for $j = 1 \rightarrow \binom{J}{i}$ **do**

-Assume that $\lambda^m = 0$ for $\lambda^m \notin \mathcal{S}_j^i$, and that $\lambda^n > 0$ for $\lambda^n \in \mathcal{S}_j^i$, i.e., the interference constraints at i PRs are satisfied with equality simultaneously.

-Substitute these λ's in eq. (3.25), and in slackness conditions given in eqs. (3.22) to get the optimum power allocation, $\alpha_k^{(\mathrm{LBF})}$ $\forall k$.

-Check whether the total interference introduced due to the transmissions to K CR receivers satisfies the other $(J-i)$ interference constraints given in eqs. (3.23),

- if yes, exit. Otherwise, continue.

end for

end for

3.3.3 Special Case of Single PR and Single CR Receiver

In Chapter 2, a cooperative beamforming technique is developed for a CR network with one PR and one CR receiver. In what follows, we show that in presence of a single PR and a single CR receiver, the sub-optimal cooperative LBF algorithm proposed in this chapter provides the optimal LBF vector same as that in Section 2.3 as a special case. With only one CR receiver in the network, there are no cross asynchronous interference terms present i.e., $\mathbf{T}_k = 0$ in eq. (3.14). In this case, the optimum beamforming direction can be written as follows (c.f. eq. (3.14))

$$\bar{\mathbf{g}}_k^{(\mathrm{LBF})} = \frac{\mathbf{R}_k^{-1}\mathbf{h}_k^{\mathrm{s}\dagger}}{\left\|\mathbf{R}_k^{-1}\mathbf{h}_k^{\mathrm{s}\dagger}\right\|}. \tag{3.29}$$

When $\mathbf{T}_k = 0$, the optimization problem in eq. (3.15) is already a convex one that is solved optimally using the dual Lagrange function. Using the power allocation for the beamforming vector given in eq. (3.25), and the slackness condition in eq. (3.22), and after some mathematical manipulations, the optimum power allocation value is given by

$$\alpha_k^{(\mathrm{LBF})} = \frac{\gamma_{\mathrm{th}}}{\bar{\mathbf{g}}_k^{(\mathrm{LBF})\dagger}\mathbf{R}_k\bar{\mathbf{g}}_k^{(\mathrm{LBF})}}. \tag{3.30}$$

The optimum beamforming vector, $\mathbf{g}_k^{(\mathrm{LBF})}$ in case of a single PR, single CR receiver is given by, $\mathbf{g}_k^{(\mathrm{LBF})} = \sqrt{\alpha_k^{(\mathrm{LBF})}}\bar{\mathbf{g}}_k^{(\mathrm{LBF})}$,

$$\mathbf{g}_k^{(\mathrm{LBF})} = \sqrt{\frac{\gamma_{\mathrm{th}}}{\mathbf{h}_k^{\mathrm{s}}\mathbf{R}_k^{-1\dagger}\mathbf{h}_k^{\mathrm{s}\dagger}}}\mathbf{R}_k^{-1}\mathbf{h}_k^{\mathrm{s}\dagger} \tag{3.31}$$

which is identical to the cooperative beamforming technique developed in Section 2.3.

3.3.4 Low Complexity Power Allocation Scheme

The computational complexity of the power allocation scheme among different beamforming vectors proposed in Section 3.3.2 can, in the worst case scenario, is in the order of $\mathcal{O}\big(K(2^J-1)\big)$. The sub-optimal power allocation (SOPA) scheme[8] among beamforming directions obtained by solving approximated optimization problem in eq. (3.19), jointly finds all the K allocated power values which can, in the worst case, require solving the J interference constraints simultaneously. Therefore, we also propose a low complexity power allocation (LCPA) scheme as described below.

Rather than finding the power allocation value α_k by keeping all the J interference constraints simultaneously in eq. (3.19), we propose to find the power allocation value for only one interference constraint e.g., jth interference constraint at a time. For notational convenience let us denote, the

[8] Throughout this chapter, we refer to the solution of eq. (3.19) as sub-optimal one as we have approximated the original power allocation problem in eq. (3.15).

corresponding power value by $\alpha_k^{j,\mathrm{LCPA}}$ $(k = 1, \cdots, K)$ which can be written as follows

$$\alpha_k^{j,\mathrm{LCPA}} = \max\left(0, \frac{w^k}{(\ln 2)\lambda^j \bar{\mathbf{g}}_k^{(\mathrm{LBF})\dagger}\mathbf{R}_k^j \bar{\mathbf{g}}_k^{(\mathrm{LBF})}} - \frac{\sigma_{\mathrm{n,i},k}^2 + (K-1)\alpha_{\mathrm{App}}\bar{\mathbf{g}}_i^{(\mathrm{LBF})\dagger}\mathbf{T}_i^k\bar{\mathbf{g}}_i^{(\mathrm{LBF})}}{P\bar{\mathbf{g}}_k^{(\mathrm{LBF})\dagger}(\mathbf{h}_k^{\mathrm{s}})^\dagger\mathbf{h}_k^{\mathrm{s}}\bar{\mathbf{g}}_k^{(\mathrm{LBF})}}\right). \tag{3.32}$$

The value of λ^j is obtained from the following complementary slackness condition

$$\lambda^j\big(\sum_{k=1}^{K}\alpha_k^{j,\mathrm{LCPA}}\bar{\mathbf{g}}_k^{(\mathrm{LBF})\dagger}\mathbf{R}_k^j\bar{\mathbf{g}}_k^{(\mathrm{LBF})} - \gamma_{\mathrm{th}}^j\big) = 0, \qquad \text{for } j = 1, \cdots, J. \tag{3.33}$$

So, now for a given beamforming direction corresponding to a particular CR receiver d_k, we have J power values $\alpha_k^{j,\mathrm{LCPA}}$ $(j = 1, \cdots, J)$ corresponding to J interference constraints. Out of these J power values, the minimum power value is selected as the final power allocation value for kth beamforming direction, i.e.,

$$\alpha_k^{\mathrm{LCPA}} = \min\big(\alpha_k^{1,\mathrm{LCPA}}, \alpha_k^{2,\mathrm{LCPA}}, \cdots, \alpha_k^{J,\mathrm{LCPA}}\big). \tag{3.34}$$

The complexity of this proposed LCPA scheme varies linearly with the number of CR receivers, as well as with the number of PRs, given by $\mathcal{O}(KJ)$, compared to that of the SOPA scheme which is in the order of $\mathcal{O}\big(K(2^J - 1)\big)$ in the worst case. This lower complexity comes at the expense of sum transmission rate of CR receivers.

3.4 Extension of Beamforming with Statistical Channel Knowledge

In many scenarios, the instantaneous CSI of the channels between the CCRNs and the PRs may not be available at the CCRNs. In this section, we consider having only the statistical CSI[9] of the channels between the primary users and the CCRNs rather than the instantaneous CSI. For such scenario, the interference thresholds at the PRs can be guaranteed statistically. In absence of instantaneous CSI of the channel between PR and a CR transmitter, such statistical interference constraint to PRs has been used in [11], [36]. According to this statistical asynchronous interference

[9]Statistical CSI refers to the distribution of CSI which is assumed to be Rayleigh, and the corresponding parameter.

constraint, interference thresholds are met probabilistically as follows

$$\Pr\left(P^j_{\text{asynch}} \geq \gamma^j_{\text{th}}\right) \leq \epsilon^j, \tag{3.35}$$

where Pr denotes probability and ϵ^j is the maximum allowable probability of violating the interference threshold γ^j_{th} at PR p_j. Since the distribution of the random interference power P^j_{asynch} is not available in a closed-form, the probability in the left side of eq. (3.35) cannot be written in a closed-form in terms of average channel gains between the CCRNs and the PRs. In what follows we develop an upper bound on this probability value, i.e., $\Pr\left(P^j_{\text{asynch}} \geq \gamma^j_{\text{th}}\right)$, using the well-known Markov's inequality [64], in terms of average channel fading power gains between PR p_j and CCRNs.

According to Markov's inequality the probability that a nonnegative random variable X is greater than or equal to some positive constant a is upper bounded by the ratio of expected value of X and a, i.e., $\Pr(X \geq a) \leq \frac{\text{E}(X)}{a}$ [64], where $\text{E}(\cdot)$ denotes an expectation operator. Since the asynchronous interference power P^j_{asynch} is a non-negative function of the random variables h^{P}_{jr}, $r = 1, \cdots, L$, according to Markov's inequality, the probability $\Pr\left(P^j_{\text{asynch}} \geq \gamma^j_{\text{th}}\right)$ is upper bounded as follows

$$\Pr\left(P^j_{\text{asynch}} \geq \gamma^j_{\text{th}}\right) \leq \frac{\text{E}\left(P^j_{\text{asynch}}\right)}{\gamma^j_{\text{th}}}, \tag{3.36}$$

which leads to a limit on the average asynchronous interference power on PR p_j (c.f. eq. (3.35))

$$\text{E}\left(P^j_{\text{asynch}}\right) \leq \epsilon^j \gamma^j_{\text{th}}. \tag{3.37}$$

Since the total asynchronous interference power at PR p_j, P^j_{asynch}, is the summation of the interference powers corresponding to the transmissions of different CR receivers, the average value of the total asynchronous interference power at PR p_j can be written as

$$\text{E}\left(P^j_{\text{asynch}}\right) = \sum_{k=1}^{K} \text{E}\left(P^{(j,k)}_{\text{asynch}}\right). \tag{3.38}$$

The interference power at p_j resulting from transmission to CR receiver d_k, $P^{(j,k)}_{\text{asynch}}$ can be written in an expanded form as follows (c.f. eq. (3.2))

$$P_{\text{asynch}}^{(j,k)} = \sum_{r=1}^{L} \sum_{f=1, f\neq r}^{L} g_{kf}^{\dagger} (h_{jf}^{\text{p}})^{\dagger} h_{jr}^{\text{p}} g_{kr} \beta_k^{j\,(r,f)} + \sum_{r=1}^{L} g_{kr}^{\dagger} \left| h_{jr}^{\text{p}} \right|^2 g_{kr} \beta_k^{j\,(r,r)}. \tag{3.39}$$

Since the channel fading coefficients between different CCRNs and PR p_j are independent and have zero mean, the average value of the first term in eq. (3.39) is equal to zero. For the second term in eq. (3.39), it can be easily shown that for a Rayleigh fading channel, the the fading power gain, $\left| h_{jr}^{\text{p}} \right|^2$ has an exponential distribution with a mean value of Ω_r^j, where Ω_r^j is the path loss over the channel from the r^{th} CCRN to the j^{th} PR. The term $\sum_{r=1}^{L} g_{kr}^{\dagger} \left| h_{jr}^{\text{p}} \right|^2 g_{kr} \beta^{j\,(r,r)}$ is a summation of L independent and identically distributed (i.i.d.) exponential random variables, which is a hypo-exponential random variable, with a mean value of $\sum_{r=1}^{L} g_{kr}^{\dagger} \Omega_r^j g_{kr} \beta^{j\,(r,r)}$. Therefore the average value of $P_{\text{asynch}}^{(j,k)}$ is given by

$$\text{E}\left(P_{\text{asynch}}^{(j,k)} \right) = \sum_{r=1}^{L} g_{kr}^{\dagger} \Omega_r^j g_{kr} \beta_k^{j\,(r,r)}. \tag{3.40}$$

This average interference power at PR p_j can be rewritten in a matrix form as follows

$$\text{E}\left(P_{\text{asynch}}^{(j,k)} \right) = \mathbf{g}_k^{\dagger} \bar{\mathbf{R}}_k^j \mathbf{g}_k, \tag{3.41}$$

where

$$\bar{\mathbf{R}}_k^j = \begin{bmatrix} \beta_k^{j\,(1,1)} \Omega_1^j & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \beta_k^{j\,(L,L)} \Omega_L^j \end{bmatrix}. \tag{3.42}$$

Using eq. (3.41), eq. (3.37) can be written as

$$\sum_{k=1}^{K} \mathbf{g}_k^{\dagger} \bar{\mathbf{R}}_k^j \mathbf{g}_k \leq \epsilon^j \gamma_{\text{th}}^j. \tag{3.43}$$

Now the cooperative beamforming vector that maximizes the weighted sum rate of CR receivers while satisfying the new interference constraint in eq. (3.43) can be formulated as an optimization problem as follows

$$\begin{aligned}
\hat{\mathbf{g}}_1, \hat{\mathbf{g}}_2, \cdots, \hat{\mathbf{g}}_K &= \max_{\mathbf{g}_1, \cdots, \mathbf{g}_K} \sum_{k=1}^{K} w^k r_k, \\
\text{subject to:} \qquad & \sum_{k=1}^{K} \mathbf{g}_k^{\dagger} \bar{\mathbf{R}}_k^j \mathbf{g}_k \leq \epsilon^j \gamma_{\text{th}}^j, \ \text{for } j = 1, \cdots, J. \qquad (3.44)
\end{aligned}$$

The above optimization problem is again a non-linear and non-convex optimization problem. However, using the two steps procedure described in Section 3.3.2, a sub-optimal solution for the cooperative leakage beamforming method for the statistical CSI scenario can be obtained by substituting for γ_{th}^j by $\epsilon^j \gamma_{\text{th}}^j$ in eq. (3.12).

3.5 Joint CCRN Selection and Cooperative Beamforming

Contributions of different CCRNs vary significantly towards the interfering signals at the PRs, as well as the received signals at the CR receivers. This is due to the fact that they are located in different geographical locations, hence their signals experience different amount of path loss and fading conditions. Intuitively, a CCRN selection strategy can further improve the performance of the cooperative beamforming. The joint design of cooperative beamforming and relay selection has been studied before for conventional cooperative networks, see for example [37] and the references therein. In the case of CR systems, the CCRN selection scheme should include the constraint of keeping the interference at the PRs below their desired interference thresholds. In this section, we study CCRN selection strategies with multiple primary and multiple CR receivers. In particular, we develop the joint design of cooperative beamforming and optimal CCRN selection scheme. We also propose two sub-optimal CCRN selection schemes that have lower complexity, compared to the optimal scheme. In this development, we assume that the channel gains of the PRs are known perfectly at the CCRNs for simplicity of problem formulation. However, it is straightforward to extend the proposed CCRN selection schemes to the case where statistical channel knowledge of the channels between the PRs and the CCRNs is available at the CCRNs.

3.5.1 Development of Optimal CCRN Selection Scheme

To formulate the CCRN selection problem mathematically, we define a CCRN selection vector $\mathbf{S}$ of size $L \times 1$, where L is the number of CCRNs in the network. The elements of $\mathbf{S}$, s_j can take value of either 1 or 0, to indicate whether the CCRN c_j has been selected for transmission or not respectively. For notational convenience, we define $\mathbf{W}$ as a diagonal matrix having its diagonal elements equal to those of vector $\mathbf{S}$, as follows

$$\mathbf{W} = \mathrm{Diag}(\mathbf{S}). \tag{3.45}$$

With this CCRN selection matrix $\mathbf{W}$, the received signal power at CR receiver d_k, $P_{k,\mathrm{sig}}$, is given by

$$P_{k,\mathrm{sig}} = P\mathbf{g}_k{}^{\dagger}\mathbf{W}^{\dagger}(\mathbf{h}_k^{\mathrm{s}})^{\dagger}\mathbf{h}_k^{\mathrm{s}}\mathbf{W}\mathbf{g}_k. \tag{3.46}$$

Using this value of the received signal power in the optimization problem given in eq. (3.12), we can formulate the joint CCRN selection and beamforming problem as follows

$$\begin{aligned}
\mathbf{g}_1^{(\mathrm{S,opt})}, \mathbf{g}_2^{(\mathrm{S,opt})}, \cdots, \mathbf{g}_K^{(\mathrm{S,opt})} \quad &= \quad \max_{\mathbf{g}_1,\cdots,\mathbf{g}_K,\mathbf{W}} \sum_{k=1}^{K} w^k \log_2\left(1 + \frac{P\mathbf{g}_k^{\dagger}\mathbf{W}^{\dagger}(\mathbf{h}_k^{\mathrm{s}})^{\dagger}\mathbf{h}_k^{\mathrm{s}}\mathbf{W}\mathbf{g}_k}{\sigma_{\mathrm{n,i},k}^2 + \sum\limits_{i=1,i\neq k}^{K} \mathbf{g}_i^{\dagger}\mathbf{W}^{\dagger}\mathbf{T}_i^k\mathbf{W}\mathbf{g}_i}\right), \\
\text{subject to:} \qquad & \sum_{k=1}^{K} \mathbf{g}_k^{\dagger}\mathbf{W}\mathbf{R}_k^j\mathbf{W}\mathbf{g}_k \leq \gamma_{\mathrm{th}}^j, \quad \text{for } j = 1, \cdots, J.
\end{aligned} \tag{3.47}$$

The problem of joint CCRN selection and cooperative beamforming in eq. (3.47) is considered as a mixed-integer non linear problem (MINLP), since the elements of $\mathbf{W}$ can only take values of 0 or 1 each. Such MINLP can be solved, without losing optimality, by decoupling it into a non-linear problem (NLP) and a mixed-integer linear problem (MILP), as in [65, 66].

Using the value of the received signal power in eq. (3.46) for a given CCRN selection matrix

$\mathbf{W}$, the beamforming vector $\mathbf{g}_k^{(\mathrm{opt})}(\mathbf{W})$ can be found from the following NLP

$$\mathbf{g}_1^{(\mathrm{opt})}(\mathbf{W}), \mathbf{g}_2^{(\mathrm{opt})}(\mathbf{W}), \cdots, \mathbf{g}_K^{(\mathrm{opt})}(\mathbf{W}) \quad = \quad \max_{\mathbf{g}_1, \cdots, \mathbf{g}_K} \sum_{k=1}^{K} w^k \log_2\left(1 + \frac{P\mathbf{g}_k^\dagger \mathbf{W}^\dagger (\mathbf{h}_k^{\mathrm{s}})^\dagger \mathbf{h}_k^{\mathrm{s}} \mathbf{W} \mathbf{g}_k}{\sigma_{\mathrm{n,i},k}^2 + \sum_{i=1, i\neq k}^{K} \mathbf{g}_i^\dagger \mathbf{W}^\dagger \mathbf{T}_i^k \mathbf{W} \mathbf{g}_i}\right),$$

$$\text{subject to:} \qquad \sum_{k=1}^{K} \mathbf{g}_k^\dagger \mathbf{W} \mathbf{R}_k^j \mathbf{W} \mathbf{g}_k \leq \gamma_{\mathrm{th}}^j, \quad \text{for } j = 1, \cdots, J. \tag{3.48}$$

For a given CCRN selection matrix $\mathbf{W}$, the optimization problem in eq. (3.48) is non-convex and non-linear similar to the one in eq. (3.12). Therefore, for a given CCRN selection matrix $\mathbf{W}$, we can use the same two-phase sub-optimal approach described in Section 3.3.2 to find the sub-optimal $\mathbf{g}_k{}^{(\mathrm{LBF})}(\mathbf{W})$ for all k.

As explained before, having a single PR and a single CR receiver in the network is considered a special case of the more generalized optimization problem in eq. (3.48). For this special case, the optimization problem in eq. (3.48) is convex, and the beamforming vector $\mathbf{g}_k^{(\mathrm{S,LBF})}(\mathbf{W})$ can be optimally found in a desirable closed form expression. After some mathematical manipulations, the beamforming vector for a given CCRN selection matrix $\mathbf{W}$, $\mathbf{g}_k^{(\mathrm{S,LBF})}(\mathbf{W})$, can finally be written as

$$\mathbf{g}_k^{(\mathrm{LBF})}(\mathbf{W}) = \sqrt{\frac{\gamma_{\mathrm{th}}}{\mathbf{h}_k^{\mathrm{s}} \mathbf{W} (\mathbf{W}^\dagger \mathbf{R}_k \mathbf{W})^{-1\dagger} \mathbf{W}^\dagger \mathbf{h}_k^{\mathrm{s}\dagger}}} \cdot (\mathbf{W}^\dagger \mathbf{R}_k \mathbf{W})^{-1} \mathbf{W}^\dagger \mathbf{h}_k^{\mathrm{s}\dagger}. \tag{3.49}$$

Using the resulting beamforming vector $\mathbf{g}_k^{(\mathrm{LBF})}(\mathbf{W})$, the optimal CCRN selection matrix $\mathbf{W}^{(\mathrm{S,LBF})}$[10] is obtained from the optimization problem in eq. (3.47) by formulating the following optimization MILP

$$\mathbf{W}^{(\mathrm{S,LBF})} = \max_{\mathbf{W}} \sum_{k=1}^{K} w^k \log_2\left(1 + \frac{P\mathbf{g}_k^{(\mathrm{LBF})\dagger}(\mathbf{W}) \mathbf{W}^\dagger (\mathbf{h}_k^{\mathrm{s}})^\dagger \mathbf{h}_k^{\mathrm{s}} \mathbf{W} \mathbf{g}_k^{(\mathrm{LBF})}(\mathbf{W})}{\sigma_{\mathrm{n,i},k}^2 + \sum_{i=1, i\neq k}^{K} \mathbf{g}_i^{(\mathrm{LBF})\dagger} \mathbf{W}^\dagger \mathbf{T}_i^k \mathbf{W} \mathbf{g}_i^{(\mathrm{LBF})}}\right),$$

$$w_{l,l} \in \{0, 1\}, \sum_{l=1}^{L} w_{l,l} \leq K. \tag{3.50}$$

where $w_{l,l}$, for $l = 1, \cdots, L$ are the diagonal elements of the CCRN selection matrix $\mathbf{W}$. The

[10] Although the cooperative beamforming design for a given CCRN selection matrix is suboptimal, but we call the CCRN selection scheme an optimal one because the selection pattern is selected optimally via exhaustive search.

optimal solution for this problem is found via exhaustive search over all possible combinations of all possible subsets in the set of CCRNs. Finally, the corresponding $\mathbf{g}_k{}^{(\mathrm{S,LBF})}$ is obtained accordingly.

3.5.2 Development of Sub-Optimal CCRN Selection Schemes

The computational complexity of the optimal CCRN selection scheme when used in conjunction with the LCPA scheme is of the order of $\mathcal{O}\big(KJ\sum_{r=2}^{L}\binom{L}{r}\big)$, and its complexity order increases to $\mathcal{O}\big(K\big(2^J-1\big)\sum_{r=2}^{L}\binom{L}{r}\big)$, when it is used in conjunction with the SOPA method. The main complexity comes from the fact that CCRN selection scheme uses exhaustive search over all possible combinations of two or more CCRNs to find the best set of CCRNs for cooperative beamforming, and for every possible combination, the cooperative beamforming weight vectors need to be calculated for every combination. In particular, with L CCRNs, there are $\sum_{r=2}^{L}\binom{L}{r}$ different combinations of selecting at least two CCRNs for cooperative beamforming, then the corresponding beamforming weight vectors have to be calculated using the method proposed in Section 3.3 for every combination.

Therefore, in this section we develop two low complexity sub-optimal CCRN selection schemes to reduce the complexity of the optimal CCRN selection scheme.

Sub-Optimal CCRN Selection Scheme 1

One option to reduce the complexity of the optimal CCRN selection algorithm, is to limit the maximum number of CCRNs that can forward the vector of data symbols to CR receiver d_k. This solution decreases the computational complexity compared to the optimal one. For example, if only R relays are allowed to participate in the cooperative data transmission, where $R \leq L$, we need to compare $\sum_{r=2}^{R}\binom{L}{r}$ possible selections only. As an example, if we have 5 CCRNs in the network, and we have to select $R = 2$ CCRNs for cooperative transmission, we need to search only in 10 selections to determine the best 2 CCRNs, instead of having 26 possible selections to compare in case of applying the optimal CCRN selection algorithm in Section 3.5.1. The value of R, which trades off complexity with performance, is decided by the CR system under consideration.

Sub-Optimal CCRN Selection Scheme 2

Now, we develop another low complexity sub-optimal CCRN selection scheme in which R CCRNs are first selected heuristically based only on their channel fading coefficients towards the CR receivers and the PRs. Then these selected R CCRNs participate in cooperative beamforming and corresponding weight vectors for these selected CCRNs are calculated using the method described in Section 3.3.

In order to select R CCRNs out of the L cooperating CCRNs, we have $N_{\mathrm{S}} = \binom{L}{R}$ possible candidate sets and let us denote these sets $\mathcal{C}_1$, $\mathcal{C}_2$, $\cdots$, $\mathcal{C}_{N_{\mathrm{S}}}$ where the cardinality of $\mathcal{C}_i$ is R for all $i = 1, 2, \cdots, N_{\mathrm{S}}$. Since the goal is to increase the received signal power at each of the CR receivers, while reducing the interference to each of the PRs, heuristically we define a metric for each CCRN in a given set based on the channel fading gains between the CCRNs and the CR receivers and the PRs. In particular, the metric of CCRN r in a particular set, M_r, is directly proportional to the channel fading coefficients from CCRN r to all the CR receivers in the network. Also the metric of CCRN r, M_r is inversely proportional to the channel fading coefficients from CCRN r to all the PRs in the network. So, we define the metric M_r as follows

$$M_r = \frac{|h^{\mathrm{s}}_{1r}| + |h^{\mathrm{s}}_{2r}| + \cdots + |h^{\mathrm{s}}_{Kr}|}{|h^{\mathrm{p}}_{1r}| + |h^{\mathrm{p}}_{2r}| + \cdots + |h^{\mathrm{p}}_{Jr}|}. \tag{3.51}$$

Using the metrics for the CCRNs in a given set, the combined metric for that particular set is defined as the summation of the metrics of each of its constituting CCRNs. Without loss of generality, we express the combined metric for the first set of CCRNs, $\mathcal{C}_1$ as follows

$$M_{\mathcal{C}_1} = M_1 + M_2 + \cdots + M_R. \tag{3.52}$$

After calculating all the N_{S} combined metrics $M_{\mathcal{C}_i}$, corresponding to candidate sets $\mathcal{C}_i$, ($i = 1, 2, \cdots, N_{\mathrm{S}}$), the set of CCRNs that has the highest combined metric is selected. Finally the CCRNs of the selected set participate in cooperative beamforming and beamforming vectors for these selected R CCRNs are calculated using the method described in Section 3.3.

In Fig. 3.3, we have plotted the computational complexity of the optimal CCRN selection scheme and the two sub-optimal selection schemes when applied in conjunction with the LCPA

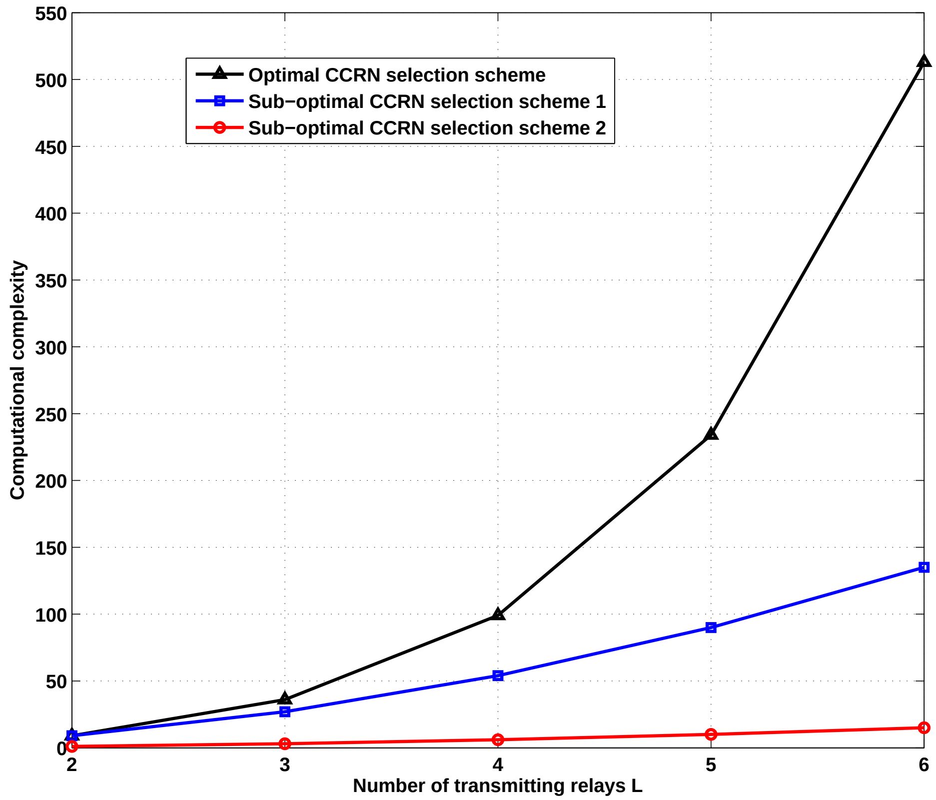

Figure 3.3: Comparison between the computational complexity of the optimal CCRN selection scheme and the low complexity sub-optimal CCRN selection schemes, when applied in conjunction with the LCPA scheme.

scheme, versus the number of CCRNs L in the system, given that the maximum number of CCRNs that can be used in case of the sub-optimal CCRN selection schemes is 2. As shown in Fig. 3.3, the sub-optimal schemes are less complex than the optimal one in Section 3.5.1, and the difference between their complexity increases for higher number of cooperating CCRNs in the system. It is also shown in this figure that the sub-optimal scheme 2 has lower complexity than that of scheme 1. Note that the gap between the computational complexities of the three selection schemes increases when the schemes are applied in conjunction with the SOPA method. This is because the complexity metric plotted in this figure is to be multiplied by $K\left(2^J - 1\right)$ in case of SOPA method, instead of multiplying by KJ in case of LCPA method.

3.6 Numerical Results

In this section, we present some numerical results in order to compare the performances of various beamforming techniques in the presence of asynchronous interference. For all the numerical examples presented in this section, we consider a network topology with $L = 4$ CCRNs denoted by $\{c_1, c_2, c_3, c_4\}$, $K = 3$ CR receivers denoted by $\{\mathrm{d}_1, \mathrm{d}_2, \mathrm{d}_3\}$, and $J = 3$ PRs denoted by $\{\mathrm{p}_1, \mathrm{p}_2, \mathrm{p}_3\}$. The distances between the set of CCRNs to each CR receiver, and to each PR are given in Table 3.1. The locations of the nodes in Table 3.1 are picked up arbitrarily. We assume that all the channel fading amplitude gains are independently Rayleigh distributed. We consider a slot duration $T_{\mathrm{slot}} = 1$ msec, during which a data frame of 1000 symbols is transmitted. We also consider a log-distance path loss model with a path loss exponent value of 4. Unless stated otherwise, the AWGN power used in the simulations is -110 dBm. The transmission bandwidth is 1 MHz. The normalized average interference power from the primary transmitter to the CR receivers, d_1, d_2 and d_3 are assumed to be $-10, -20$, and -15 dB, respectively. For simplicity, we consider weighting factors w_1, w_2 and w_3 are equal to one.

Table 3.1: Distances between nodes in the simulated CR-based broadcasting network.

CCRN	d_1	d_2	d_3	p_1	p_2	p_3
c_1	100m	120m	120m	2700m	2500m	1700m
c_2	110m	60m	43m	2400m	1800m	1400m
c_3	150m	110m	83m	2800m	2200m	1500m
c_4	170m	150m	130m	3200m	2700m	1880m

In Fig. 3.4, we plot the average normalized symbol power versus the total asynchronous interference signal power introduced at the PRs using our proposed cooperative LBF technique. We assume that interference thresholds at the PRs p_1, p_2, and p_3 are respectively, $\gamma_{\mathrm{th}}^1 = 0.02 \times 10^{-16}$, $\gamma_{\mathrm{th}}^2 = 0.05 \times 10^{-16}$, and $\gamma_{\mathrm{th}}^3 = 0.2 \times 10^{-16}$, which are in the order of the noise plus interference power from the primary transmitter, $\sigma^2_{\mathrm{n,i},k}$, value. In this figure we also plot the asynchronous interference signal powers introduced at the PRs when ZFBF technique [20] is used. This figure clearly shows that our proposed LBF technique can maintain the asynchronous interference thresholds at the PRs simultaneously. On the contrary, the interference caused by the ZFBF technique exceeds the interference target thresholds at the PRs. This is expected as ZFBF does not take asynchronous interferences into account in its design.

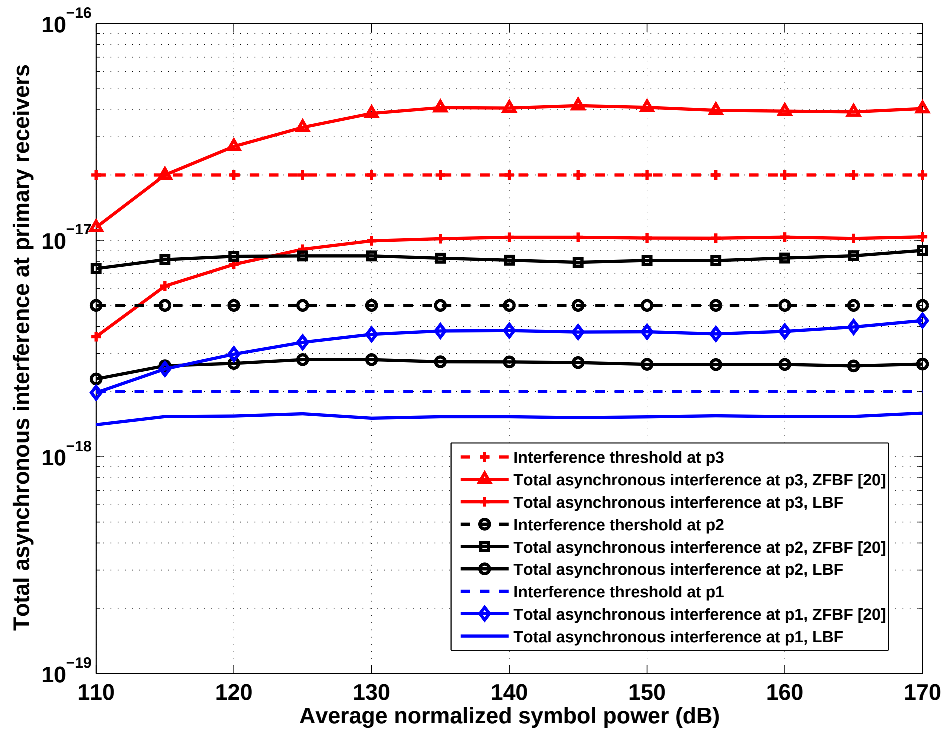

Figure 3.4: Total asynchronous interference power at the PRs with different interference thresholds ($\gamma_{\text{th}}^{1} = 0.1 \times 10^{-15}$ and $\gamma_{\text{th}}^{2} = 0.25 \times 10^{-15}$).

To assess the performance of the proposed approximations to design the LBF weight vector in Section 3.3.2, their achievable average sum rate is compared to that of the active set method [63] that numerically obtains the power values for the beamforming vectors. The active set method has the following structure. It first finds a feasible starting point, and computes the Lagrange multipliers for the active set, which is a set made up of the optimization constraints that are satisfied with equality at this starting point. Then the subset of constraints that have negative Lagrange multipliers are removed and a new feasible point is found and the algorithm is repeated until the solution is optimal enough, i.e., the constraints are satisfied with a tolerance factor, which we assumed to be equal 10^{-20}.

In Fig. 3.5, we plot the achievable average sum rate of CR receivers with the proposed cooperative LBF with SOPA scheme, the LBF with LCPA scheme, the active set method [63], and the ZFBF technique. For the sake of completeness in Fig. 3.5, we also plot the achievable sum

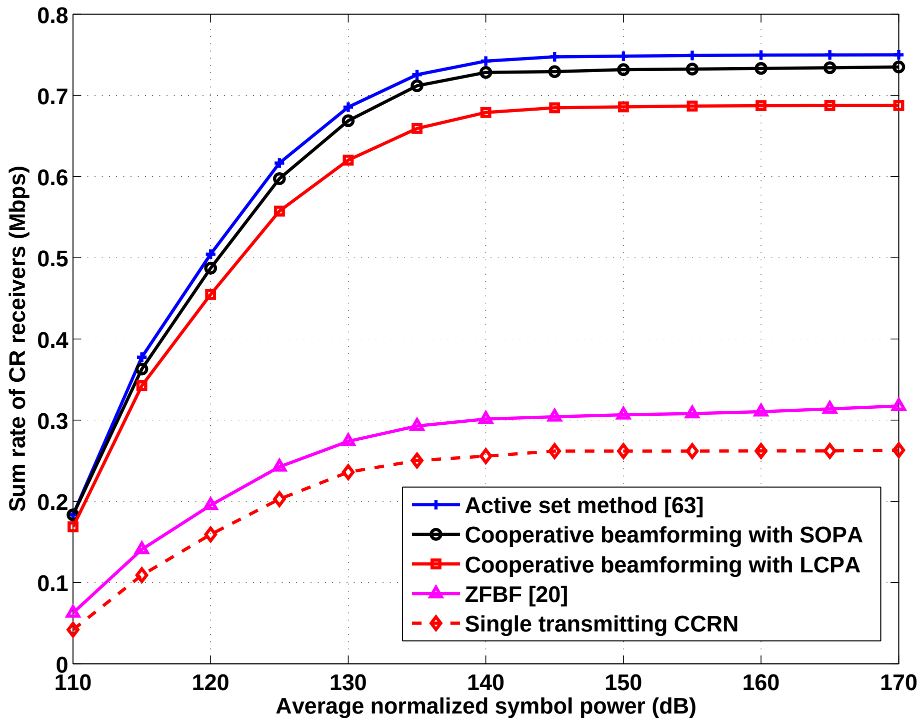

Figure 3.5: Achievable sum transmission rate with various beamforming techniques and single CCRN-based transmission.

transmission rate of CR receivers when a single CCRN is used for transmission to the CR receivers without applying any beamforming [11]. From this figure we can observe that the proposed LBF with SOPA method can achieve a sum transmission rate value that is very close to that of the active set method. It can also achieve a higher sum rate than the well-known ZFBF technique for the CR-based broadcasting system. In particular, the increase in sum transmission rate of CR receivers is about 150% with our proposed LBF technique compared to the ZFBF technique. This can be explained as follows. With ZFBF an outage is considered if the instantaneous interference caused by the CCRNs at any PR exceeds its corresponding target interference threshold. The ZFBF technique usually cannot satisfy the interference threshold(s), which leads to a frequent transmission outage events. In addition, the mutual asynchronous interference signals between the CR receivers are not optimized in the ZFBF method. As such the overall transmission rate of the CR receivers with ZFBF is degraded. From Fig. 3.5, we can also see that the proposed LCPA scheme that

[11] In this case we also consider an underlay spectrum access mechanism, in which we select the CCRN out of L CCRNs that offers the highest sum rate of all the CR receivers. The selected CCRN uses a transmit power value that satisfies all the primary interference constraints.

has a lower complexity suffers from a performance degradation compared to the SOPA scheme as expected, but still achieves a higher transmission rate compared to the ZFBF technique. We can also observe from this figure that the single CCRN-based transmission without beamforming offers the lowest possible sum transmission rate for the CR system. This can be explained by the fact that the single CCRN-based transmission scheme does not benefit from beamforming which improves the received signal power at the CR receivers while minimizing the effect of asynchronous interferences at the PRs.

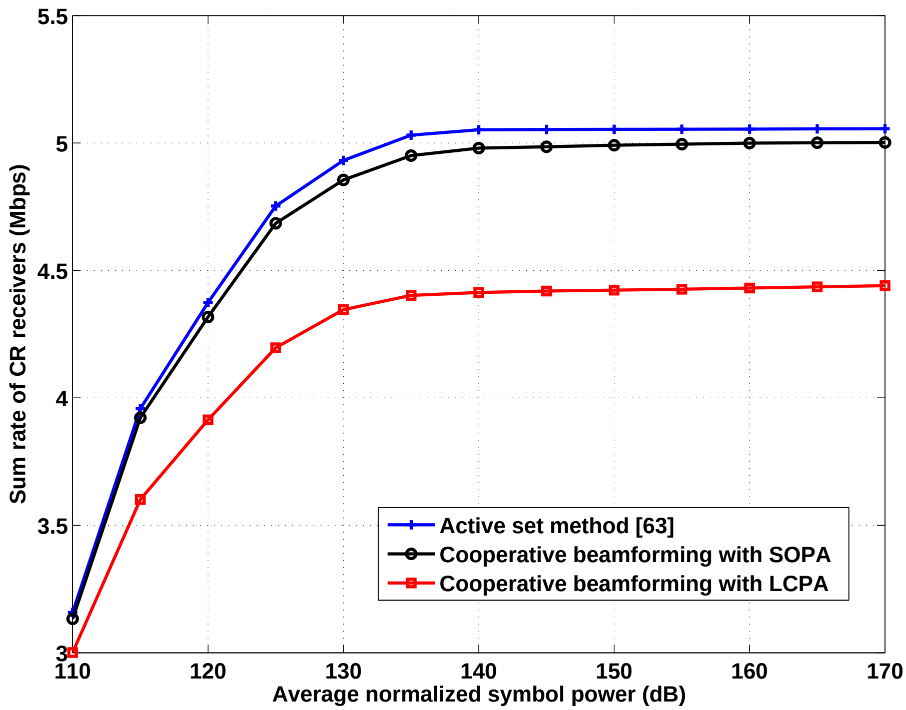

Figure 3.6: Achievable sum transmission rate with various power allocation schemes in low noise plus interference from the primary transmitter power environment.

For a thorough comparison between the proposed beamforming techniques and the numerically obtained power values using the active set method [63], the achievable sum rate of these methods are compared in a low noise plus interference power environment, assuming $\sigma^2_{\mathrm{n,i},k} = -130$ dBm. We can see from this figure that the performance of the LBF method with the proposed SOPA method is very close to the active set method. The LBF with the proposed LCPA offers a lower sum rate performance than the SOPA scheme, as expected but with much less computational complexity. In particular, as we mentioned earlier in this chapter, the worst case complexity of the SOPA method is linear with respect to the number of CR receivers, and exponential with respect to the number of constraints. The complexity of the LCPA scheme is linear with respect to both the number of CR receivers and constraints. On the other hand, the worst case complexity of the active set method is exponential with respect to both the number of CR receivers and constraints [67]. The average computational time needed to find the beamforming weights with the LCPA method, the SOPA method, and the active set method is 0.35 msec, 0.5 msec, and 25 msec, respectively using MATLAB on a Desktop computer with Intel Core i5-2410M processor and a clock frequency of 2.3 GHz. This clearly shows that the active set method requires much higher computational time.

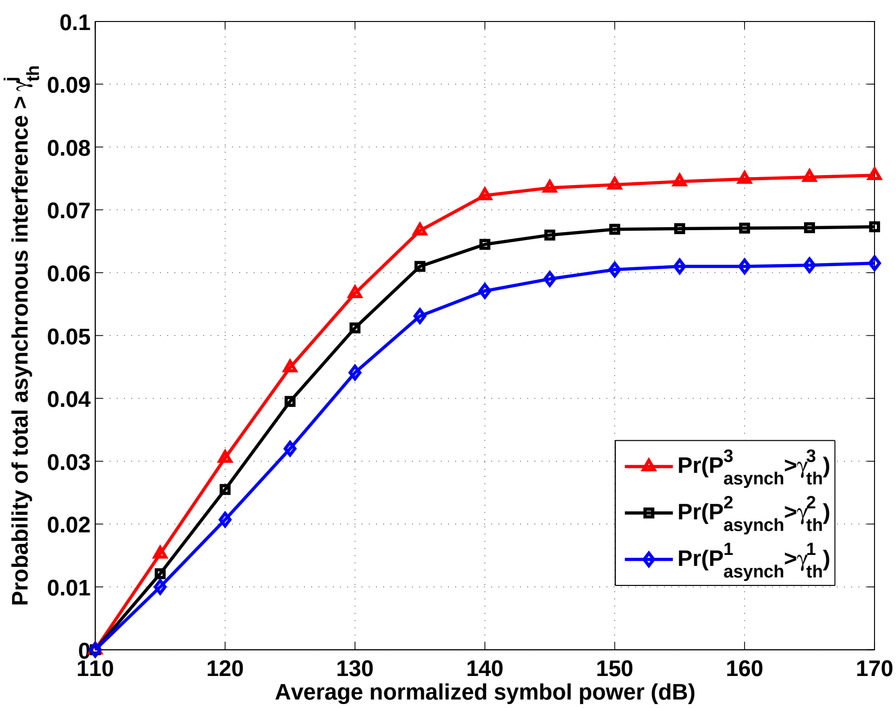

Figure 3.7: The probability that the total asynchronous interference at each PR is greater than γ^j_{th} with statistical CSI of the PRs at the CCRNs.

Next, we investigate the performance of the proposed cooperative leakage beamforming technique, in case of having only statistical CSI of the channels between the PRs and the CCRNs. In Fig. 3.7, we plot the probability of having the instantaneous asynchronous interference power at each PR greater than its target threshold. The value of the maximum allowable probability of violating the interference thresholds $\epsilon^1, \epsilon^2,$ and ϵ^3 are assumed to be 0.1. It is obvious from Fig. 3.7 that with our proposed cooperative LBF technique with statistical channel knowledge, the probability of violating the interference thresholds is maintained within the maximum allowable probability value.

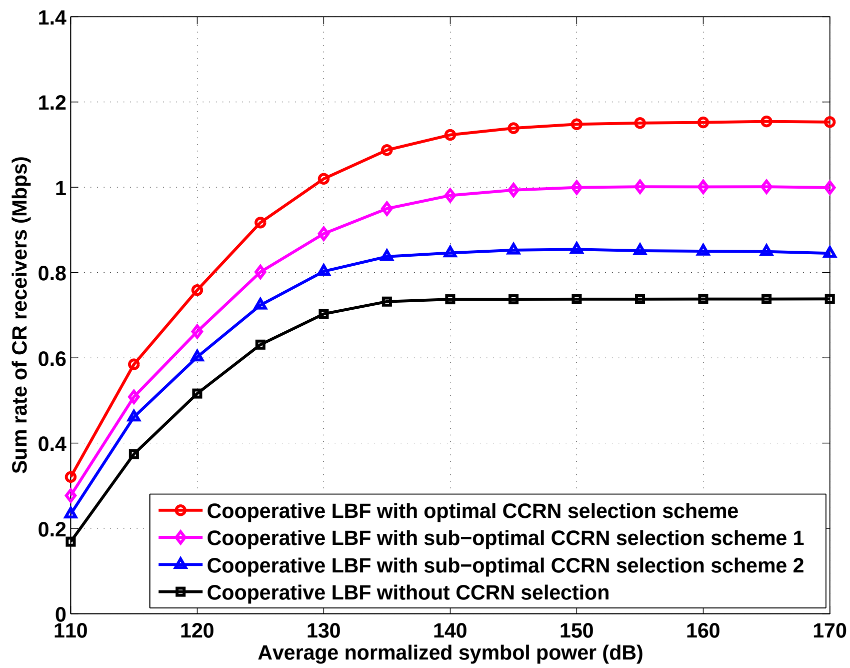

Figure 3.8: The sum rate of the CR receivers with the optimal CCRN selection scheme, sub-optimal CCRN selection schemes 1 and 2 and without CCRN selection.

Finally the sum rate performance enhancement that can be achieved by applying the CCRN selection schemes in conjunction with the cooperative LBF technique proposed in Section 3.5, is investigated in Fig. 3.8. In particular, in this figure we plot the normalized transmit power versus the average achievable sum rate of the CR receivers with cooperative beamforming technique and various CCRN selection schemes that are developed throughout the chapter. In this figure we also plot the achievable sum rate of the cooperative LBF technique assuming that all the CCRNs participate in beamforming (i.e., without applying any CCRN selection strategy). From this figure, it is interesting to see that the optimal CCRN selection scheme in conjunction with the LBF technique outperforms the LBF technique when no CCRN selection is employed. The increase in sum rate is about 45% and the reason can be explained intuitively as follows. When a CCRN selection scheme is employed, the CCRNs are selected judiciously considering their contributions towards the achievable sum rate at the CR receivers as well as the total interference power at the PRs. The proposed low complexity sub-optimal CCRN selection schemes suffer from performance degra-

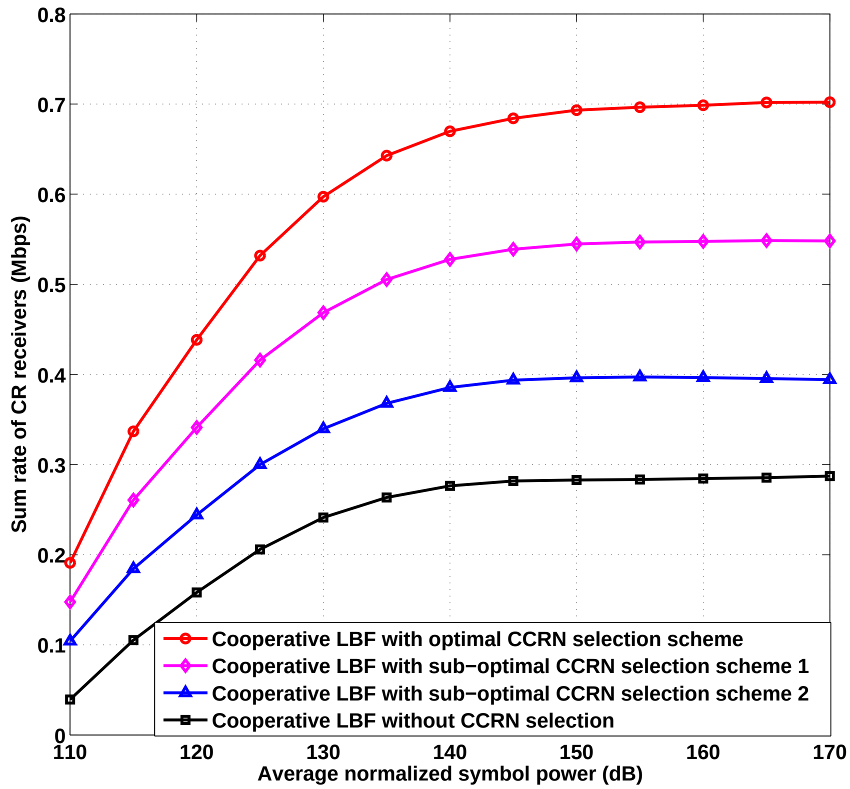

Figure 3.9: The sum rate of the CR receivers with the optimal CCRN selection scheme, sub-optimal CCRN selection schemes 1 and 2 and without CCRN selection with statistical CSI of the PRs at the CCRNs.

dation with respect to the optimal CCRN selection scheme as expected however they outperform the LBF method without any CCRN selection strategy. The sub-optimal CCRN selection scheme 1 has a better performance compared to that of the sub-optimal scheme 2, since its complexity is higher than that of sub-optimal scheme 2, as discussed in Section 3.5.

In Fig. 3.9, we plot the performance of the CCRN selection schemes in conjunction with the LBF method, in case of having only statistical CSI of the channels between the PRs and the CCRNs. Similar observations can be made from this figure, as we did from Fig. 3.8.

Chapter 4

Distributed Beamforming and Autonomous Participation Decision Making in Cooperative CR Systems

In this chapter, we address other challenges of applying cooperative beamforming in CR networks, which are the feedback overhead problem and the participation decision making issue. First, we address the problem of feedback overhead needed for cooperative beamforming. Cooperative beamforming requires sharing of instantaneous channel state information (CSI) and location information among cooperative CR relays or requires a master node that knows the global instantaneous CSI and location information. Both cases require a huge amount of feedback overhead.

Exchanging such large amount of information between the cooperating relays requires additional bandwidth, and causes excessive power dissipation from the CR devices [38]. In addition, CSI estimation errors can become a bottle neck to the potential performance gain from CR relays cooperation [39], as explained earlier. Therefore, in this chapter we propose a distributed beamforming method to be used in cooperative CR networks that requires only information sharing between cooperating CR relays about their locations. Assuming each CR relay knows its own CSI towards the CR receiver and towards the PR, and using the shared information about the other relays' locations, each relay can independently design its own beamforming weight.

As an incentive for the CR users to participate in the cooperative transmission, the assisted CR user can lease its scheduled channel, for a certain amount of time, to the cooperating CR relays for their opportunistic access. On the other hand, the CR users spend a certain amount of their limited life-time battery power when acting as relays for another CR user. Therefore, it becomes a critical decision for each user to decide whether to participate in the distributed beamforming or not. Since no cooperation is assumed between the participating CR relays in the decision making, each CR user does not know other users' decisions, and hence it cannot assess its own reward in case of participating in the cooperative transmission beforehand. Hence, this problem is considered as an example of unknown games, that can be tackled using a Bayesian game theoretic approach [40]. In this chapter, we propose two autonomous participation decision making strategies to help each CR user in deciding whether to participate in the cooperative transmission or not.

We also propose a relay selection method to enhance the performance of the cooperative CR network. On one hand, different cooperating CR relays have different path loss values towards the intended CR user, as well as towards the PR. Accordingly, they have different contributions towards the received signal at the intended CR user, as explained before in Section 3.5. On the other hand, as more CR relays participate in the cooperative transmission, the reward value represented in the amount of time during which the channel of the assisted CR user is leased to the cooperating CR relays is divided among more CR users. This means smaller reward value for each of them. Therefore, in case of having multiple CR users that are willing to participate in the cooperative transmission, we need to select some of the CR users to form the distributed beamformer. Hence, in this chapter, we also propose a relay selection method that only uses statistical CSI of the CR relays towards the intended CR user and towards the PR, to choose the best set of CR users that yields the maximum received signal power value at the intended destination. .

Finally, we extend the two proposed autonomous decision making strategies to handle the case of having multiple CR users requesting cooperative transmission simultaneously. Since each CR user can only participate in the cooperative transmission towards one receiver, it becomes a critical decision for each CR user to select such receiver among the simultaneous cooperation requests. The decision making in this case is based on the best incentive provided by each CR user requesting cooperation. The modified autonomous decision making methods are shown to provide enhanced performance of the CR network, despite the lack of cooperation among the participating CR users in making their decisions.

In what follows, we provide the system model that we consider in our problem formulation. We present the proposed distributed beamforming method, and the two proposed participation decision making strategies. Next we present the proposed relay selection mechanism. Finally, we extend the two proposed participation decision strategies to help each CR user decide whether to participate in the distributed beamforming or not and to which CR receiver, in case of receiving multiple cooperation requests simultaneously.

4.1 System Model

In what follows, we provide the operating assumptions and principles of the system model that we consider in our problem formulation.

4.1.1 Operating Assumptions

We consider a CR network with a central cognitive base station (CBS) and N CR users, and a primary network that has F non-overlapping frequency channels. A PU can randomly occupy its channel at a given time slot, which can be modeled by the so-called ON-OFF model (see for example [68] and the references therein). The CBS assigns these F channels to different CR users in the network. Usually, the scheduled CR user can use its assigned channel whenever the PU is idle. We consider a time-slotted system with slot duration T_{slot}. Both the primary network and the CR network operate in a time-synchronized manner. Channel assignment to the different CR users in the network takes place every scheduling frame, that has a duration of D time slots. The choice of the channel scheduling policy is beyond the scope of this thesis. We also assume that each of the F channels independently experiences a slow and frequency non-selective fading. In our system model, we assume that each CR user, as well as the CBS, is having only a single antenna element. We consider a downlink transmission scenario but the uplink case can be treated similarly.

4.1.2 Operating Principles

Sometimes, due to high path loss and/or shadowing, the channel between a specific CR user and the CBS gets weaker which degrades this user's quality of service, in terms of its data transmission rate. We denote such CR user by CR_d that is allocated with a frequency channel f_d during the current scheduling frame. Without loss of generality, we assume that the primary channel f_d is idle during the n^{th} time slot. In this case, the CBS can ask other CR users in the network to act as relays and forward the message to CR_d at the beginning of the n^{th} time slot. The set of CR

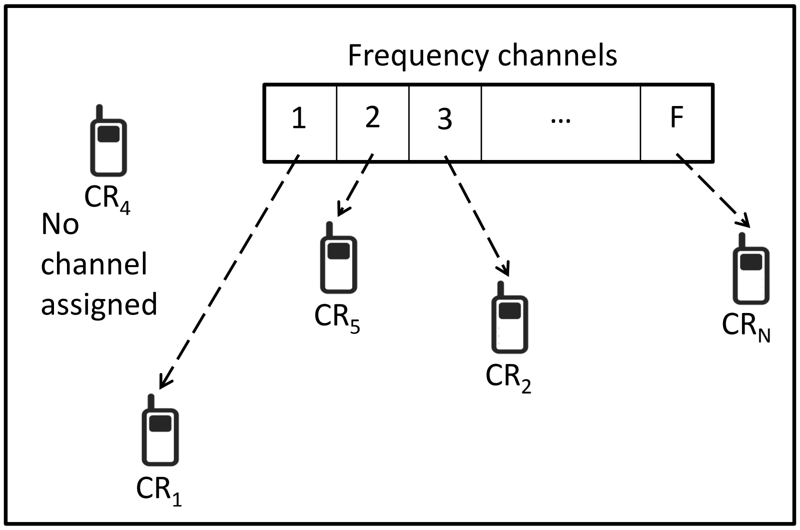

Figure 4.1: System model of a CR network with N CR users opportunistically accessing F frequency channels.

users that agree to act as relays inform the CBS with their decision during the same time slot [12]. The CBS broadcasts the first data packet to be forwarded to CR_d to all the cooperating CR relays over the channel f_d, during the n^{th} time slot. The set of cooperating CR relays can beamform in a distributed manner to forward the data to CR_d during the $(n+1)^{\text{th}}$ time slot, irrespective of whether the primary channel f_d is occupied or not. In particular, by carefully designing a complex weighting factor for relay r independently, denoted by g_r, a relatively higher signal power value can be achieved at CR_d, while limiting the interference introduced at the PR that uses channel f_d. The distributed design of the beamforming weights of different CR relays reduces the required CSI feedback in the network in terms of sharing the channel information between the cooperating relays. In addition to the data packets to be forwarded to CR_d, the CBS broadcasts the location of each of the CR relays participating in the cooperative transmission. Such location information sharing is essential to account for the effects of asynchronous interference at the PR, as will be seen later in Section 4.2.

As a suitable incentive for other CR users in the network to participate in the cooperative beamforming to CR_d, we consider leasing the scheduled channel of CR_d, f_d, to the cooperating CR relays for a certain amount of time. During this time, each of the CR users gets to opportunistically use either the frequency channel f_d or its own scheduled frequency channel, for its own data

[12]In this section, we only consider the case of having a single CR user requesting cooperative transmission. However, in Section 4.5, we extend the system model for the case of having multiple simultaneous cooperation requests.

transmission. As such, the probability of each of these CR users to find an idle time slot for its data transmission increases. Also, the CR users that are not allocated any frequency channels during the current scheduling frame can access the frequency channel f_d during such amount of leased time, whenever the PU in this channel is idle. Choosing a scheduling scheme for the cooperating CR relays to use the leased channel f_d is beyond the scope of this thesis.

The amount of time, during which channel f_d is leased for CR relay r, is proportional to the time difference gained from the cooperative transmission compared to the case of direct transmission from the CBS to CR_d, and is given by

$$\mathrm{LT}_r = \psi \frac{\Delta T}{K}, \tag{4.1}$$

where ψ is a proportionality constant, and K is the number of CR relays participating in the data forwarding to CR_d. ΔT is the time difference gained from cooperative transmission, and is given by

$$\Delta T = \delta_d \bigg(\frac{1}{R_{\mathrm{dir}}} - \frac{1}{R_{\mathrm{coop}}} \bigg), \tag{4.2}$$

where δ_d is the length of the data packet in bits to be transmitted to CR_d for one cooperation request by the CBS. R_{dir} is the data rate that can be achieved by direct transmission from the CBS to CR_d, and R_{coop} is the data rate that can be achieved by cooperative transmission. Later on in Section 4.7, we show that using this cooperation model, the overall performance of the CR system is improved. Assuming that the additive white Gaussian noise (AWGN) has a total power of σ_n^2, R_{dir} can be calculated as [45, Chapter 4]

$$R_{\mathrm{dir}} = B_d \log_2 \Big(1 + \frac{P h_{s,d_n} h^{\dagger}_{s,d_n}}{\sigma_n^2} \Big) \Big) \big(\mathrm{Pr}^d(0)|_n \big), \tag{4.3}$$

where B_d is the bandwidth of frequency channel f_d, h_{s,d_n} is the channel fading coefficient from the CBS to CR_d during the n^{th} time slot, and $\mathrm{Pr}^d(0)|_n$ is the probability of the primary channel f_d being idle, when used for direct transmission from the CBS to CR_d, during the n^{th} time slot. We consider a decode and forward relaying scheme, and R_{coop} can be calculated as [69]

$$R_{\text{coop}} = \frac{B_d}{2} \times \min\Bigg(\log_2\Big(1 + \frac{P\left(\sum_{r=1}^{N} |g_r h_{r,d_{(n+1)}}|\right)^2}{\sigma_n^2}\Big), \log_2\Big(1 + \frac{P\mathbf{h}_{s_n}\mathbf{h}_{s_n}^{\dagger}}{\sigma_n^2}\Big)\big(\Pr^d(0)|_n\big)\Bigg), \quad (4.4)$$

where $\mathbf{h}_{s_n}$ is the vector of channel fading coefficients from the CBS to the set of cooperative CR relays during the n^{th} time slot, and $h_{r,d_{(n+1)}}$ is the channel fading coefficient between relay r and CR_d during the $(n+1)^{\text{th}}$ time slot. g_r is designed according to the proposed distributed beamforming mechanism presented in the following section.

4.2 Proposed Distributed Beamforming

The decision of every CR user in the network of whether to participate in the cooperative beamforming or not will be discussed in Section 4.3. For now, let us assume that a group of K CR users decided to act as relays and participate in the cooperative beamforming for CR_d. The design goal is to improve the signal power at CR_d, while limiting the amount of interference introduced to the PR at channel f_d. To achieve this goal, each cooperating CR relay independently calculates its beamforming weight defined as

$$g_r = \alpha_r \bar{g}_r, \quad (4.5)$$

where α_r is the magnitude of the beamforming weight of relay r, and $\bar{g}_r$ is the phase of such beamforming weight. For notational convenience, we will drop the time slot index n from now on. The phase of the beamforming weight, $\bar{g}_r$, is designed to compensate for the phase shift of the channel fading coefficient between relay r and CR_d, $h_{r,d}$. In particular, $\bar{g}_r$ is expressed as

$$\bar{g}_r = \frac{h_{r,d}^{\dagger}}{|h_{r,d}|}, \quad (4.6)$$

where $|h_{r,d}|$ is the magnitude of the channel fading coefficient from relay r to CR_d, and $(\cdot)^{\dagger}$ denotes the complex conjugate.

To find the magnitude of the beamforming weight, α_r, there are two cases. The first case is when the PU at the channel f_d is idle. In this case, α_r is set as follows

$$\alpha_r = \frac{1}{\sqrt{(K)}}, \qquad \text{for } r = 1, 2, \cdots, K. \tag{4.7}$$

The second case is when the PU is active. In this case, the cooperating CR relays use distributed beamforming, and the allocated power to each relay should result in a total asynchronous interference power at the PR below a target threshold value, γ_{th}. The total asynchronous interference power value at the PR at channel f_d, from all the relays participating in the distributed beamforming in the CR network can be easily derived from eq. 2.12 as follows

$$P_{\text{asynch}} = \sum_{r=1}^{K}\sum_{f=1}^{K} \alpha_f \alpha_r \bar{g}_f^{\dagger} \left(h_f^{(\text{PU})}\right)^{\dagger} h_r^{(\text{PU})} \bar{g}_r \beta^{(r,f)}, \tag{4.8}$$

where $h_f^{(\text{PU})}$ is the channel fading gain from the CR relay r to the PR. $\beta^{(r,f)}$ is the correlation between the asynchronous symbols of CR relays r and f at the PR, and is calculated for given values of propagation delays of the CR relays r and f to the PR, using the technique described in Section 2.3. So the value of $\beta^{(r,f)}$ only depends on the locations of the relays r and f.

To find α_r, the total asynchronous interference power at the PR in eq. (4.8) is set below a target threshold value, γ_{th}. We first consider the case of $K = 2$ relays for simplicity, then the general case of any K relays will be discussed later in this section. With $K = 2$, eq. (4.8) can be expanded as follows

$$\begin{aligned} P_{\text{asynch}} = \alpha_1^2 P \left|h_1^{(\text{PU})}\bar{g}_1\right|^2 &+ \alpha_1\alpha_2 \bar{g}_1^{\dagger}\left(h_1^{(\text{PU})}\right)^{\dagger} h_2^{(\text{PU})}\bar{g}_2\beta^{(1,2)} \\ &+ \alpha_2^2 P \left|h_2^{(\text{PU})}\bar{g}_2\right|^2 + \alpha_1\alpha_2 \bar{g}_2^{\dagger}\left(h_2^{(\text{PU})}\right)^{\dagger} h_1^{(\text{PU})}\bar{g}_1\beta^{(2,1)}, \end{aligned} \tag{4.9}$$

where P is the average power of the data symbols to be transmitted to CR_d. According to the definition of $\beta^{(r,f)}$ in Section 2.3, the values of $\beta^{(1,2)}$ and $\beta^{(2,1)}$ are equal. Therefore eq. (4.9) can be rewritten as

$$P_{\text{asynch}} = \alpha_1^2 P \left|h_1^{(\text{PU})}\bar{g}_1\right|^2 + \alpha_2^2 P \left|h_2^{(\text{PU})}\bar{g}_2\right|^2 + 2\alpha_1\alpha_2\beta^{(1,2)}\text{Re}\left\{\bar{g}_1^{\dagger}\left(h_1^{(\text{PU})}\right)^{\dagger} h_2^{(\text{PU})}\bar{g}_2\right\}. \tag{4.10}$$

By rearranging the terms, eq. (4.10) can be written as

$$P_{\text{asynch}} = \alpha_1^2 P \left| h_1^{(\text{PU})} \bar{g}_1 \right|^2 + \alpha_1 \alpha_2 \beta^{(1,2)} \text{Re}\left\{ \bar{g}_1^\dagger \left(h_1^{(\text{PU})} \right)^\dagger h_2^{(\text{PU})} \bar{g}_2 \right\}$$
$$+ \alpha_2^2 P \left| h_2^{(\text{PU})} \bar{g}_2 \right|^2 + \alpha_1 \alpha_2 \beta^{(2,1)} \text{Re}\left\{ \bar{g}_1^\dagger \left(h_1^{(\text{PU})} \right)^\dagger h_2^{(\text{PU})} \bar{g}_2 \right\}. \quad (4.11)$$

Note that the first and second terms in eq. (4.11) represent the interference power introduced due to transmission from the 1st relay, whereas the third and fourth terms represent the interference power introduced due to transmission from the 2nd relay. One possible power allocation scheme that can enable the CR relays to select their beamforming weights independently, is to limit the interference power from each relay separately to be below γ_{th}/K, where $K = 2$ in this case. Therefore, we can write the interference constraints for the two-relay case as follows

$$\alpha_1^2 P \left| h_1^{(\text{PU})} \bar{g}_1 \right|^2 + \alpha_1 \alpha_2 \beta^{(1,2)} \text{Re}\left\{ \bar{g}_1^\dagger \left(h_1^{(\text{PU})} \right)^\dagger h_2^{(\text{PU})} \bar{g}_2 \right\} \leq \frac{\gamma_{\text{th}}}{2}, \quad (4.12)$$

$$\alpha_2^2 P \left| h_2^{(\text{PU})} \bar{g}_2 \right|^2 + \alpha_1 \alpha_2 \beta^{(2,1)} \text{Re}\left\{ \bar{g}_1^\dagger \left(h_1^{(\text{PU})} \right)^\dagger h_2^{(\text{PU})} \bar{g}_2 \right\} \leq \frac{\gamma_{\text{th}}}{2}. \quad (4.13)$$

The second term in eq. (4.12) and eq. (4.13), $\text{Re}\left\{ \bar{g}_1^\dagger \left(h_1^{(\text{PU})} \right)^\dagger h_2^{(\text{PU})} \bar{g}_2 \right\}$, is a function of both relays' CSI. It can be rewritten as

$$\text{Re}\left\{ \bar{g}_1^\dagger \left(h_1^{(\text{PU})} \right)^\dagger h_2^{(\text{PU})} \bar{g}_2 \right\} = \left| h_1^{(\text{PU})} \bar{g}_1 \right| \left| h_2^{(\text{PU})} \bar{g}_2 \right| \cdot \cos\left(\angle \left(h_1^{(\text{PU})} \bar{g}_1 \right) + \left(\angle h_2^{(\text{PU})} \bar{g}_2 \right) \right). \quad (4.14)$$

From eq. (4.12) and eq. (4.13), we can find the following equality

$$\alpha_1^2 \left| h_1^{(\text{PU})} \bar{g}_1 \right|^2 = \alpha_2^2 \left| h_2^{(\text{PU})} \bar{g}_2 \right|^2. \quad (4.15)$$

Using eq. (4.15), eq. (4.14) can be rewritten as

$$\begin{aligned} \text{Re}\left\{ \bar{g}_1^\dagger \left(h_1^{(\text{PU})} \right)^\dagger h_2^{(\text{PU})} \bar{g}_2 \right\} &= \frac{\alpha_1}{\alpha_2} \left| h_1^{(\text{PU})} \bar{g}_1 \right|^2 \cos\left(\angle \left(h_1^{(\text{PU})} \bar{g}_1 \right) + \left(\angle h_2^{(\text{PU})} \bar{g}_2 \right) \right) \\ &= \frac{\alpha_2}{\alpha_1} \left| h_2^{(\text{PU})} \bar{g}_2 \right|^2 \cos\left(\angle \left(h_1^{(\text{PU})} \bar{g}_1 \right) + \left(\angle h_2^{(\text{PU})} \bar{g}_2 \right) \right). \end{aligned} \quad (4.16)$$

The value of $\cos\left(\angle \left(h_1^{(\text{PU})} \bar{g}_1 \right) + \left(\angle h_2^{(\text{PU})} \bar{g}_2 \right) \right)$ is a random number ranging from -1 to $+1$. So, the worst case interference value occurs when the $\cos(\cdot)$ is equal to $+1$. Using eq. (4.15) and considering the worst case interference value, we can rewrite eq. (4.14) as

$$\mathrm{Re}\left\{\bar{g}_1^\dagger \left(h_1^{(\mathrm{PU})}\right)^\dagger h_2^{(\mathrm{PU})}\bar{g}_2\right\} \leq \frac{\alpha_1}{\alpha_2}\left|h_1^{(\mathrm{PU})}\bar{g}_1\right|^2 = \frac{\alpha_2}{\alpha_1}\left|h_2^{(\mathrm{PU})}\bar{g}_2\right|^2. \tag{4.17}$$

Using eq. (4.17) in eq. (4.12) and eq. (4.13), the interference constraints can be rewritten as follows

$$\alpha_1^2\left|h_1^{(\mathrm{PU})}\bar{g}_1\right|^2\left(P+\beta^{(1,2)}\right) \leq \frac{\gamma_{\mathrm{th}}}{2}, \tag{4.18}$$

$$\alpha_2^2\left|h_2^{(\mathrm{PU})}\bar{g}_2\right|^2\left(P+\beta^{(1,2)}\right) \leq \frac{\gamma_{\mathrm{th}}}{2}. \tag{4.19}$$

From eq. (4.18) and eq. (4.19), the magnitudes of the beamforming weights, α_1 and α_2 can be calculated.

For the generalized case of K relays, using a similar approach we can write the interference constraint corresponding to relay r as

$$\alpha_r^2\left|h_r^{(\mathrm{PU})}\bar{g}_r\right|^2\left(P+\sum_{f=1,f\neq r}^{K}\beta^{(r,f)}\right) \leq \frac{\gamma_{\mathrm{th}}}{K}. \tag{4.20}$$

From this relation, one can easily obtain the magnitude of the beamforming weight α_r. Please note that, the asynchronous interferences from other relays are taken into account by relay r via $\beta^{(r,f)}$, which only requires knowledge of the locations of other relays.

4.3 Participation Decision Making Strategies

Upon participation in the cooperative transmission, each cooperating CR user is rewarded by a certain amount of time over the leased channel f_d, as explained in Section 4.1. On the other hand, the cooperative CR relay spends a certain amount of its limited battery power when acting as a relay for CR_d. Therefore, it becomes a critical decision for each user to decide whether to participate in cooperative beamforming or not. Therefore, in this section, we propose two autonomous participation decision making strategies that assist each CR user to decide whether to participate in the cooperative beamforming to CR_d.

The participation decision is made by each CR user, considering its acquired reward as well as its paid cost. The acquired reward by CR relay r is represented by the amount of its transmitted data over the leased channel f_d, which it gets in return for the cooperative transmission to CR_d. So, the reward for relay r, when assisting CR_d, can be written as

$$G_{rd} = \mathrm{LT}_r \cdot B_d \cdot \log_2\Big(1 + \frac{P_r h_{r,f_d} h^{\dagger}_{r,f_d}}{\sigma_n^2}\Big)\big(\mathrm{Pr}^d(0)|_{\mathrm{LT}_r}\big), \tag{4.21}$$

where P_r is the average symbol power of the transmitted data of relay r, and h_{r,f_d} is the channel fading coefficient of relay r to its destination on the leased channel f_d. $\mathrm{Pr}^d(0)|_{\mathrm{LT}_r}$ is the probability of the primary channel f_d being idle during the time it is leased to CR relay r.

On the other hand, CR relay r spends a certain cost for participating in the cooperative transmission to CR_d. Let us define $P_{r,d}$ as the transmission power of relay r required for cooperative transmission to CR_d. So $P_{r,d}$ is given by

$$P_{r,d} = P \cdot {\alpha_r}^2. \tag{4.22}$$

We define the cost of relay r as the amount of data that could have been transmitted by relay r, if it used the transmission power $P_{r,d}$ for its own transmission on its own channel, f_r, rather than in assisting CR_d. So, the cost function of relay r, when assisting CR_d, can be defined as

$$C_{rd} = T_{\mathrm{coop}} \cdot B_r \cdot \log_2\Big(1 + \frac{P_{r,d} h_{r,f_r} h^{\dagger}_{r,f_r}}{\sigma_n^2}\Big)\big(\mathrm{Pr}^r(0)|_{(n+1)}\big), \tag{4.23}$$

where B_r is the bandwidth of the channel f_r allocated to CR user r during the current scheduling frame, and h_{r,f_r} is the channel fading coefficient of relay r to its destination on the channel f_r. $\mathrm{Pr}^r(0)|_{(n+1)}$ is the probability of this primary channel f_r being idle during the $(n+1)^{\mathrm{th}}$ time slot, which is the time slot during which cooperative transmission to CR_d takes place. T_{coop} is the amount of time used for cooperative transmission to CR_d, and is given by

$$T_{\mathrm{coop}} = \frac{\delta_d}{R_{\mathrm{coop}}}. \tag{4.24}$$

Based on eq. (4.21), and eq. (4.23), CR user r can decide to participate in the cooperative transmission, if its reward value exceeds its paid cost. However, due to the lack of coordination between the CR users in decision making, as well as the lack of CSI exchange between them, each CR user does not know the other users' rewards and costs, and it does not even know its own reward and cost ahead. This behavior represents an unknown game, that can be solved from a Bayesian game perspective [40]. The optimal participation decisions of such Bayesian game are those that cause the system to reach its NE state. The NE state of the system is achieved when each CR user has chosen a strategy of participation decision making, and no CR user can benefit by changing its strategy as long as other CR users keep theirs unchanged [42].

To solve this unknown game, we propose two participation decision making strategies in this section. First, let us define some parameters that are used to describe the proposed decision making strategies. The decision of the CR user r to participate in the cooperative beamforming to CR_d is denoted by $A_{r,d}$, which takes value 1 if the decision is to participate, or 0 otherwise. From eq. (4.4) and eq. (4.23), the expected cost to be paid in the cooperative beamforming to CR_d depends on $h_{r,d}$. The better the condition of the channel state from CR user r to CR_d, the lower the cost paid in data forwarding to CR_d, and vice versa. Accordingly, the participation decision variable $A_{r,d}$ depends on the value of $h_{r,d}$, which is a continuous random variable. As an approximation, we divide the domain of the fading coefficient $h_{r,d}$ into N_r intervals, based on the average value of the fading coefficient magnitude in each interval. Accordingly, CR user r has N_r different participation decisions, which is expressed as follows, $A_{r,d} \in \{\bar{A}_{r,d}(1), \cdots, \bar{A}_{r,d}(N_r)\}$, where $\bar{A}_{r,d}(m)$ is the participation decision of CR user r in the m^{th} interval of the average fading coefficient $h_{r,d}$. We denote the vector $\bar{\mathbf{A}}_{r,d} = \{\bar{A}_{r,d}(1), \cdots, \bar{A}_{r,d}(N_r)\}$ as the action profile of CR user r when considering cooperative transmission to CR_d.

The set of all possible action profiles of the CR user r when transmitting to CR_d is denoted by the set $\pi_{r,d}$. We define the joint set of action profiles followed by all the K CR users as the mixed strategy profile of all the K CR users in the network that are considering either to participate or not in the cooperative transmission. We denote such mixed strategy profile by $\Pi = \{\bar{\mathbf{A}}_{1,d}, \cdots, \bar{\mathbf{A}}_{K,d}\}$. The set of all possible mixed strategy profiles is denoted by Σ. It is important to note that an ϵ-NE mixed strategy profile, Π_ϵ, is the one that causes the system to asymptotically converge to the ϵ-NE state. The design goal of the following two proposed decision making strategies is to search for Π_ϵ.

4.3.1 Regret Testing-Based Strategy (RTS)

We propose the first autonomous decision making strategy for the cooperative CR network that is based on the well-known regret testing procedure, which has been used in different contexts of game theory [70]. We modify the regret testing procedure based on our model of CR networks. The regret testing procedure represents one form of exhaustive search, and it can asymptotically converge to an approximate NE (ϵ-NE) state [41]. We prove that the proposed RTS converges to ϵ-NE state, within a certain convergence time.

In the search for the ϵ-NE mixed strategy profile, Π_ϵ, every T_{test} time slots are considered as a testing period. During this testing period, all CR users test their benefits or losses resulting from either participation or not in the cooperative transmission. During the testing period, T_{test}, CR user r follows a certain action profile, $\bar{\mathbf{A}}_{r,d}$, with probability p and follows its complement, $1-\bar{\mathbf{A}}_{r,d}$, with probability $1-p$. In either cases, each CR user calculates the profit, or loss, achieved from its decision. If the decision implied by the action profile, $\bar{\mathbf{A}}_{r,d}$, is to participate in the cooperative transmission, we define an indicator of the loss encountered by following such action profile as $l(\bar{\mathbf{A}}_{r,d})$, given by

$$l(\bar{\mathbf{A}}_{r,d}) = \begin{cases} 1, & \text{if } G_{rd} < C_{rd}, \\ -1, & \text{otherwise.} \end{cases} \tag{4.25}$$

CR user r keeps track of its encountered loss values to calculate its regret at the end of the testing period. We denote the regret of CR user r as $\Omega_{r,d}$. At the end of each testing period, if the CR user r achieves a regret value $\Omega_{r,d} \leq 0$, when it follows an action profile, $\bar{\mathbf{A}}_{r,d}$, it stays with the same action profile for the next testing period. Otherwise, it randomly chooses a new action profile from the set $\pi_{r,d}$. However, even if the CR user r achieves a negative regret when it follows a certain action profile, this action profile may not be the optimal one yet. So it has to keep exploring other action profiles, with some probability λ, known as the exploration factor.

To find the set of all possible action profiles of the CR user r when transmitting to CR_d, $\pi_{r,d}$, we make the following logical assumption. If the action for a certain channel state is set to 1, the action for all the higher channel states must be 1. Therefore, the action profile of the CR user r is in the form of $\bar{\mathbf{A}}_{r,d} = \{0, \cdots, 0, 1, \cdots, 1\}$. Hence, there are only $N_r + 1$ possible action profiles of the CR user r when transmitting to CR_d, constituting the set $\pi_{r,d}$. The proposed RTS can be described for the CR user r as follows.

Definitions:

- t_c is the cooperation inquiry slot count, incremented each time the CBS asks for cooperative transmission.

- $\bar{\mathbf{A}}_{r,d}$ is the action profile of the CR user r, initially selected randomly from $\pi_{r,d}$.

- $\Omega_{r,d}^{(t_c)}$ is accumulated regret value when the action profile $\bar{\mathbf{A}}_{r,d}$ is followed. The initial value $\Omega_{r,d}^{(0)} = 0$.

procedure

for $t_c = 1 \to \cdots$ **do**

- Find the interval, m, in which the channel fading coefficient $h_{r,d}$ lies in.

- With probability p, CR user r follows the m^{th} entry in its action profile, $\bar{A}_{r,d}(m)$.

  if $\bar{A}_{r,d}(m) = 1$ **then**

    - Accumulated regret when following the action profile is $\Omega_{r,d}^{(t_c)} = \Omega_{r,d}^{(t_c-1)} + l(\bar{\mathbf{A}}_{r,d})$.

  else

    $\Omega_{r,d}^{(t_c)} = \Omega_{r,d}^{(t_c-1)} + 0$.

  end if

- With probability $1-p$, CR user r follows the complement of this m^{th} entry, $1 - \bar{A}_{r,d}(m)$.

  if $\bar{A}_{r,d}(m) = 0$ **then**

    - Accumulated regret when not following the action profile is $\Omega_{r,d}^{(t_c)} = \Omega_{r,d}^{(t_c-1)} - l(\bar{\mathbf{A}}_{r,d})$.

  else

    $\Omega_{r,d}^{(t_c)} = \Omega_{r,d}^{(t_c-1)} + 0$.

  end if

  if $\mod (t_c, T_{\text{test}}) = 0$ **then**

    - Calculate the average regret value of following the action profile $\bar{\mathbf{A}}_{r,d}$, $\Omega_{r,d} = \dfrac{\Omega_{r,d}^{(t_c)}}{T}$.

    if $\Omega_{r,d} \leq 0$ **then**

      - With probability $1-\lambda$, choose $\bar{\mathbf{A}}_{r,d} = \bar{\mathbf{A}}_{r,d}$,

      - With probability λ, randomly select a new action profile, $\bar{\mathbf{A}}_{r,d}$, from $\pi_{r,d}$.

    else

      - Randomly select $\bar{\mathbf{A}}_{r,d}$, from $\pi_{r,d}$.

end if

- Reset the average regret value $\Omega_{r,d} = 0$.

end if

end for

end procedure

In what follows, we provide a convergence analysis of the proposed RTS. In particular, we prove that using the proposed RTS, the mixed strategy profile at time t_c, denoted by Π^{t_c}, asymptotically converges to ϵ-NE state, within a certain convergence time $t_c = T_{\text{con}}$. We start by introducing the following lemma.

Lemma 1 *The stochastic process Π^{t_c}, $t_c = 1, 2, \cdots$, defined by the RTS with $0 < \lambda < 1$, is a homogeneous, recurrent, and irreducible Markov chain with state space Σ. The transition probability of this Markov chain has a lower-bound given by*

$$p(\Pi^{t_c+1} = \Pi' | \Pi^{t_c} = \Pi) \geq \Big(\frac{\lambda}{N_r + 1}\Big)^K. \tag{4.26}$$

Proof of Lemma 1 *To prove that the process is a Markov chain, we note that at each $t_c = 1, 2, \cdots$, Π^{t_c} depends only on Π^{t_c-1}. It is irreducible since at each $t_c = 1, 2, \cdots$, the probability of reaching some mixed strategy profile $\Pi^{t_c} = \Pi'$ from any $\Pi^{t_c-1} = \Pi$ is strictly positive, when $\lambda > 0$. It is recurrent since all the states in Σ are guaranteed (with probability 1) to have a finite hitting time. The transition probability of this Markov chain is given as follows. In the proposed RTS, the probability of a CR user r choosing an action profile randomly is $\geq \lambda$. Then the probability of choosing a certain action profile $\bar{\mathbf{A}}_{r,d} \in \Pi'$ given the current action profile $\bar{\mathbf{A}}_{r,d} \in \Pi$ can be written as follows*

$$p(\bar{\mathbf{A}}_{r,d} \in \Pi' | \bar{\mathbf{A}}_{r,d} \in \Pi) \geq \frac{\lambda}{N_r + 1}, \tag{4.27}$$

where $N_r + 1$ is the number of all possible action profiles of CR user r in the set $\pi_{r,d}$. The probability of choosing a different mixed strategy profile is controlled by all the K CR users in the network.

Hence, the transition probability of the Markov chain is given as follows

$$p(\Pi^{t_c+1} = \Pi' | \Pi^{t_c} = \Pi) \geq \Big(\frac{\lambda}{N_r + 1}\Big)^K. \tag{4.28}$$

Since this Markov chain is irreducible and recurrent, therefore it has a stationary distribution Q [71]. Note that the Markov chain converges to the stationary distribution regardless of where it begins.

We define the subset in the state space of the mixed strategy profile, Σ, that leads to ϵ-NE state, as $\mathcal{N}_\epsilon$. We also introduce the notation $\bar{\mathcal{N}_\epsilon}$, where $\bar{\mathcal{N}_\epsilon} = \Sigma \backslash \mathcal{N}_\epsilon$, to denote the complement of the set of ϵ-NE state. To prove the convergence of the proposed decision making strategy, we need to prove that the probability that $\Pi^{T_{\text{con}}}$ is not included in the set of ϵ-NE state is at most ϵ, where $\epsilon \geq 0$ is a very small number. This is expressed mathematically as follows

$$p(\Pi^{T_{\text{con}}} \in \bar{\mathcal{N}_\epsilon}) \leq \epsilon. \tag{4.29}$$

For certain values of λ, T_{test}, we can prove that the proposed RTS asymptotically converges to an ϵ-NE state within a convergence time T_{con}. This result is summarized in theorem 1.

Theorem 1 *For the RTS, if the testing period* $T_{test} \geq \dfrac{-2}{\epsilon^2} \log C_2$*, and the exploration parameter* $0 < \lambda \leq \frac{C_2}{KC_2 - 1} - \frac{\epsilon(1-C_1)}{K(2-\epsilon)(KC_2-1)}$*, the probability of the mixed strategy profile being in an non ϵ-NE state, after a certain amount of time* $T_{con} \leq \dfrac{\log(\epsilon/2)}{\log\Big(1 - \Big(\frac{\lambda}{N_r+1}\Big)^K\Big)}$*, is bounded by*

$$p(\Pi^{T_{con}} \in \bar{\mathcal{N}_\epsilon}) \leq \epsilon. \tag{4.30}$$

Where C_1, and C_2 are constant values, $0 < C_2 < 1$, $0 \leq C_1 \leq 1$, and $\epsilon \geq 0$ is a very small number.

Proof of Theorem 1 *Let us denote the probability distribution of the mixed strategy profile at time $t_c = T_{con}$ as, $p(\Pi^{T_{con}})$. Then the probability that the mixed strategy profile $\Pi^{T_{con}}$ is in a non ϵ-NE state is denoted by, $p(\Pi^{T_{con}} \in \bar{\mathcal{N}_\epsilon})$. As explained before in Section 4.3.1, the mixed strategy profile at different values of t_c, Π^{t_c}, represents a Markov chain with a stationary distribution Q. Let us denote the stationary distribution of the non ϵ-NE state as $Q(\bar{\mathcal{N}_\epsilon})$. According to theorem* 16.2.4

in [72], the probability that $\Pi^{T_{con}}$ is in a non ϵ-NE state is related to the stationary distribution of such state through the following inequality

$$p(\Pi^{T_{con}} \in \bar{\mathcal{N}_\epsilon}) \leq Q(\bar{\mathcal{N}_\epsilon}) + \left(1 - \left(\frac{\lambda}{N_r + 1}\right)^K\right)^{T_{con}}. \tag{4.31}$$

From the definition of the stationary distribution [72], we can write

$$Q(\bar{\mathcal{N}_\epsilon}) = Q(\bar{\mathcal{N}_\epsilon})p(\Pi^{t_c+1} \in \bar{\mathcal{N}_\epsilon}|\Pi^{t_c} \in \bar{\mathcal{N}_\epsilon}) + Q(\mathcal{N}_\epsilon)p(\Pi^{t_c+1} \in \bar{\mathcal{N}_\epsilon}|\Pi^{t_c} \in \mathcal{N}_\epsilon). \tag{4.32}$$

Since the non ϵ-NE state is the complement of the ϵ-NE state, i.e., $\bar{\mathcal{N}_\epsilon} = \Sigma \backslash \mathcal{N}_\epsilon$, then $Q(\bar{\mathcal{N}_\epsilon}) = 1 - Q(\mathcal{N}_\epsilon)$. Therefore, eq. (4.32) can be rewritten, after some mathematical manipulations, as follows.

$$Q(\bar{\mathcal{N}_\epsilon}) = \frac{p(\Pi^{t_c+1} \in \bar{\mathcal{N}_\epsilon}|\Pi^{t_c} \in \mathcal{N}_\epsilon)}{1 - p(\Pi^{t_c+1} \in \bar{\mathcal{N}_\epsilon}|\Pi^{t_c} \in \bar{\mathcal{N}_\epsilon}) + p(\Pi^{t_c+1} \in \bar{\mathcal{N}_\epsilon}|\Pi^{t_c} \in \mathcal{N}_\epsilon)}. \tag{4.33}$$

The transition probability from an ϵ-NE state to a non ϵ-NE state, $p(\Pi^{t_c+1} \in \bar{\mathcal{N}_\epsilon}|\Pi^{t_c} \in \mathcal{N}_\epsilon)$ is equal to $1 - p(\Pi^{t_c+1} \in \mathcal{N}_\epsilon|\Pi^{t_c} \in \mathcal{N}_\epsilon)$. Hence, to calculate the value of $Q(\bar{\mathcal{N}_\epsilon})$ in eq. (4.33), we need to find the bound on the probability of staying in an ϵ-NE state. When the mixed strategy profile lies in the ϵ-NE state, the expected regret values $\Omega_{r,d}$ of all K CR users is at most ϵ. Since the regret value of CR user r, $\Omega_{r,d}$, is the average sum of T_{test} independent random variables taking values between $[-1, 1]$, then we can reach the following result using the generalization of Hoeffding's inequality [73].

$$p(\Omega_{r,d} \geq 0) \leq \mathrm{e}^{\left(-T_{test}\epsilon^2/2\right)}. \tag{4.34}$$

The probability that a CR user r keeps using the same action profile, $\bar{A}_{r,d}$, is given by $\bar{\mathbf{A}}_{r,d} = (1 - \lambda) \times p(\Omega_{r,d} \leq 0)$. Then the probability that a CR user r keeps using the same action profile, $\bar{A}_{r,d}$, is $\geq (1 - \lambda)\Big(1 - \mathrm{e}^{\left(-T_{test}\epsilon^2/2\right)}\Big)$. The probability of keeping the same mixed strategy profile, i.e., all K CR users keep using their action profiles, is bounded by

$$p(\Pi^{t_c+1} \in \mathcal{N}_\epsilon|\Pi^{t_c} \in \mathcal{N}_\epsilon) \geq (1 - \lambda)^K \left(1 - \mathrm{e}^{\left(-T_{test}\epsilon^2/2\right)}\right)^K. \tag{4.35}$$

Given that $\lambda \leq 1$, *and* $\mathrm{e}^{\left(-T_{test}\epsilon^2/2\right)} < 1$, *eq. (4.35) can be rewritten as*

$$p(\Pi^{t_c+1} \in \mathcal{N}_\epsilon | \Pi^{t_c} \in \mathcal{N}_\epsilon) \geq (1 - K\lambda)\Big(1 - K\,\mathrm{e}^{\left(-T_{test}\epsilon^2/2\right)}\Big). \tag{4.36}$$

Then the transition probability from an ϵ*-NE state to a non* ϵ*-NE state is bounded by*

$$p(\Pi^{t_c+1} \in \bar{\mathcal{N}_\epsilon} | \Pi^{t_c} \in \mathcal{N}_\epsilon) \leq 1 - (1 - K\lambda)\Big(1 - K\,\mathrm{e}^{\left(-T_{test}\epsilon^2/2\right)}\Big). \tag{4.37}$$

Let us assume that the probability of staying in a non ϵ*-NE state is below certain bound* C_1*, where* C_1 *is a constant value* $0 \leq C_1 \leq 1$*. This is expressed mathematically as follows* $p(\Pi^{t_c+1} \in \bar{\mathcal{N}_\epsilon} | \Pi^{t_c} \in \bar{\mathcal{N}_\epsilon}) \leq C_1$*. Using this assumption, and eq. (4.37), we can rewrite eq. (4.33) as follows*

$$Q(\bar{\mathcal{N}_\epsilon}) \leq \frac{1 - (1 - K\lambda)\Big(1 - K\,\mathrm{e}^{\left(-T_{test}\epsilon^2/2\right)}\Big)}{2 - C_1 - (1 - K\lambda)\Big(1 - K\,\mathrm{e}^{\left(-T_{test}\epsilon^2/2\right)}\Big)}. \tag{4.38}$$

Let us select the value of the testing period T_{test} *to be* $T_{test} \geq \frac{-2}{\epsilon^2} \log C_2$*, where* C_2 *is a constant value,* $0 < C_2 < 1$*. Let us select the value of the exploration parameter* λ *to be*

$$\lambda \leq \frac{C_2}{KC_2 - 1} - \frac{\epsilon(1 - C_1)}{K(2 - \epsilon)(KC_2 - 1)}. \tag{4.39}$$

For these selected values of λ, T_{test}*, we can rewrite eq. (4.38), after some mathematical manipulations, as follows*

$$Q(\bar{\mathcal{N}_\epsilon}) \leq \frac{\epsilon}{2}. \tag{4.40}$$

Let us assume that the convergence time of T_{con} *is bounded by*

$$T_{con} \leq \frac{\log(\epsilon/2)}{\log\Big(1 - \big(\frac{\lambda}{N_r+1}\big)^K\Big)}. \tag{4.41}$$

Using eq. (4.40) and eq. (4.41) in eq. (4.31) and after some mathematical manipulations, the upper bound on the probability of ending up in a non ϵ*-NE state, after the convergence time* T_{con}*, is given as follows*

$$p(\Pi^{T_{con}} \in \bar{\mathcal{N}_\epsilon}) \leq \epsilon. \tag{4.42}$$

4.3.2 Learning-Based Strategy (LBS)

The RTS proposed in Section 4.3.1 is one form of exhaustive search, and so it suffers from high complexity and slow convergence speed. It is worth noting that the convergence time T_{con} increases as the number of cooperative CR relays, K, increases (c.f. theorem 1). We propose another decision making strategy, namely LBS, which has lower complexity, yet a good performance. As stated before, for the CR user r to decide to participate in the cooperative transmission to CR_d, its expected reward value should exceed its expected paid cost. This condition is mathematically written as

$$E(G_{rd}) - E(C_{rd}) > 0, \tag{4.43}$$

where $E(\cdot)$ denotes the expectation operator.

In [74], the authors showed that for a number of observations of a random variable, the sample mean of these observations is an estimate of the true mean of the random variable. Hence, the expected reward and cost function of the CR relay r can be estimated by keeping track of the achieved reward and cost values in previous participations in the cooperative transmission to CR_d. The estimated values of the reward and cost function of relay r, $\tilde{G}_{rd}$ and $\tilde{C}_{rd}$, are given as follows

$$\tilde{G}_{rd} = \frac{1}{N_t} \sum_{i=1}^{N_t} G_{rd}^{(i)}, \tag{4.44}$$

and

$$\tilde{C}_{rd} = \frac{1}{N_t} \sum_{i=1}^{N_t} C_{rd}^{(i)}. \tag{4.45}$$

N_t is the number of observations of the reward and cost functions, G_{rd} and C_{rd}, during the previous participations in the cooperative transmission to CR_d. $G_{rd}^{(i)}$ and $C_{rd}^{(i)}$ are the achievable reward and cost of CR user r, respectively, when participating in the cooperative transmission during the i^{th} time slot. In the proposed LBS, as CR user r participates in more cooperative transmission, a better estimate of the expected reward and cost functions of CR user r towards CR_d is obtained. The proposed LBS for CR user r is described in the following algorithm.

Definitions:

- t_c is the cooperation inquiry slot count, incremented each time the CBS asks for cooperative transmission.

- $\bar{\mathbf{A}}_{r,d}$ is the action profile of the CR user r, with M entries all equal to 1 initially.

- $\tilde{G}_{rd}$ is the estimated reward of the CR user r when participating in the cooperative transmission, initially equals 0.

- $\tilde{C}_{rd}$ is the estimated cost of the CR user r when participating in the cooperative transmission, initially equals 0.

- T_p is the number of times CR user r participated in the cooperative transmission, initially equals 0.

procedure

for $t_c = 1 \rightarrow ..$ **do**

- Find the interval, m, in which the channel fading coefficient $h_{r,d}$ lies in.

  if $\bar{A}_{r,d}(m) = 1$ **then**

    - Increment the number of times CR user r participates in the transmission towards CR_d, $T_p = T_p + 1$.

    - Estimated reward of CR user r is given by $\tilde{G}_{rd} = \dfrac{1}{T_p}\sum_{i=1}^{T_p} G_{rd}^{(i)}$,

    - Estimated cost of CR user r is given by $\tilde{C}_{rd} = \dfrac{1}{T_p}\sum_{i=1}^{T_p} C_{rd}^{(i)}$,

  end if

  if $\tilde{G}_{rd} > \tilde{C}_{rd}$ **then**

    - Update action profile of the CR user r, $\bar{A}_{r,d}(m) = 1$.

  else

    - $\bar{A}_{r,d}(m) = 0$.

  end if

end for

end procedure

4.4 Relay Selection

In Section 4.2, we have considered that all CR users that are willing to participate in cooperative transmission, beamform in a distributed manner. However, since the amount of time the channel f_d is leased to the cooperating CR relays is divided among these relays, individual reward of the individual cooperating CR relays becomes smaller. This can be discouraging for the CR users from participating again in future requests of cooperative transmission. Moreover, different CR users have different channel gains towards the intended CR destination, as well as towards the PR. Therefore, we propose a relay selection strategy, such that only L of the CR users that are willing to participate in the cooperative beamforming to CR_d are selected for transmission, where $L \leq K$. The decision of selecting the CR relays is based on their channel qualities towards the PR at the channel f_d, as well as their channel qualities towards CR_d.

The selection criterion of these CR users is to choose the set of relays that maximizes the received data rate at CR_d, given by eq. (4.48), while keeping the interference power at the PR at channel f_d below its target threshold. As discussed before, the value of α_r used in eq. (4.48) is given by

$$\alpha_r = \begin{cases} \alpha_{r,\text{id}} = 1/\sqrt{(L)}, & \text{if the PU at channel } f_d \text{ is idle in the } (n+1)^{\text{th}} \text{ time slot,} \\ \alpha_{r,\text{act}} = \dfrac{\sqrt{\gamma_{\text{th}}}}{\left|h_r^{(\text{PU})}\right| \sqrt{L\left(P + \sum_{f=1,f\neq r}^{K} \beta^{(r,f)}\right)}}, & \\ & \text{if the PU at channel } f_d \text{ is active in the } (n+1)^{\text{th}} \text{ time slot.} \end{cases} \tag{4.46}$$

The decision of the relay selection is made at n^{th} time slot, however, the cooperative transmission happens at $(n+1)^{\text{th}}$ time slot. Therefore, we use the expected achieved data rate as follows

$$\bar{R}_{\text{coop}} = R_{\text{id}} \cdot \Pr^d(0|0) + R_{\text{act}} \cdot \Pr^d(1|0), \tag{4.47}$$

where $\Pr^d(0|0)$ is the probability of channel f_d being idle at the $(n+1)^{\text{th}}$ time slot given that it is idle in the n^{th} time slot. Similarly, $\Pr^d(1|0)$ is the probability of channel f_d being active at the $(n+1)^{\text{th}}$ time slot given that it is idle in the n^{th} time slot. R_{id}, and R_{act} are the data rates achieved by cooperative transmission to CR_d when the primary user is idle or active respectively, and are given by

$$R_{\text{id}} = \frac{B_d}{2} \cdot \min\Bigg(\log_2\Bigg(1 + \frac{P\left(\sum_{r=1}^{L} \alpha_{r,\text{id}}|h_{r,d}|\right)^2}{\sigma_n^2}\Bigg), \log_2\Bigg(1 + \frac{P\mathbf{h}_s\mathbf{h}_s^\dagger}{\sigma_n^2}\Bigg)\Pr^d(0)|_n\Bigg), \tag{4.48}$$

and

$$R_{\text{act}} = \frac{B_d}{2} \cdot \min\left(\log_2\left(1 + \frac{P\left(\sum_{r=1}^{L} \alpha_{r,\text{act}}|h_{r,d}|\right)^2}{\sigma_n^2}\right), \log_2\left(1 + \frac{P\mathbf{h}_s\mathbf{h}_s^\dagger}{\sigma_n^2}\right)\Pr^d(0)|_n\right). \tag{4.49}$$

The set of L CR users that maximizes the data rate in eq. (4.47) is chosen via exhaustive search over the set of K CR users. However, the maximization of eq. (4.47) requires the instantaneous values of $|h_{r,d}|$ and $\left|h_r^{(\text{PU})}\right|$. Such instantaneous knowledge of the CSI requires a huge feedback from CR user r to the CBS, for $r = 1, \cdots, K$, which is impractical. Therefore, we propose to use the average values of $|h_{r,d}|$ and $\left|h_r^{(\text{PU})}\right|$ in eq. (4.48) and eq. (4.49), to calculate R_{id} and R_{act} respectively. Later, in Section 4.7, we show that the performance of the proposed relay selection scheme, using only the statistical knowledge of the CSI roughly yields a similar performance to the case of full CSI knowledge at the CBS.

The average values of the channel gains, $|h_{r,d}|$ and $\left|h_r^{(\text{PU})}\right|$, can be calculated based on the locations of the CR users and the PR. In particular, for a Rayleigh fading channel, $|h_{r,d}|$ has an average value of $\sqrt{\frac{\pi\Omega_{r,d}}{4}}$, where $\Omega_{r,d}$ is the path loss value over the channel from CR user r to CR_d [75]. Similarly, the average value of $\left|h_r^{(\text{PU})}\right|$ is given by $\sqrt{\frac{\pi\Omega_r^{(\text{PU})}}{4}}$, where $\Omega_r^{(\text{PU})}$ is the path loss value over the channel from CR user r to the PR at channel f_d.

It is important to note that the relay selection scheme proposed in this section is a centralized one, which is of a different nature compared to the rest of this chapter. However, we want to show the potential performance enhancement that takes place when a node selection scheme is applied in conjunction with the participation decision making strategies, previously proposed in this chapter. A fully-distributed relay selection scheme can be an interesting future research goal, which can use the centralized scheme proposed in this section as a baseline to be compared to.

4.5 Extension for Multiple CR Users Requiring Cooperation

So far we have considered the case where a single CR user CR_d requires assistance to receive its data message from the CBS. In this section, we consider the case where J CR users request cooperation. For convenience, we denote these CR users by a set $\mathcal{S}_{rc} = \{CR_{d_1}, CR_{d_2}, \cdots, CR_{d_J}\}$. We assume that these CR users are assigned with primary channels $\{f_{d_1}, f_{d_2}, \cdots, f_{d_J}\}$ respectively. In this case, the other CR users in the network, that do not belong to the set $\mathcal{S}_{rc}$, have to make a decision to either participate or not in the cooperative data forwarding to one of the CR users in the set $\mathcal{S}_{rc}$. In this generalized setup, each CR user has to decide whether to participate in the cooperative transmission or not, and which CR user from the set $\mathcal{S}_{rc}$ to assist. Therefore, we modify the two decision making strategies previously proposed.

Without loss of generality, we assume that the set of K_j CR relays that are willing to participate in the cooperating beamforming for CR_{d_j} in $\mathcal{S}_{rc}$ is denoted by the set $\mathcal{R}_j$, with cardinality $|\mathcal{R}_j| = K_j$. We define the amount of time during which the scheduled channel of CR_{d_j} is leased for CR relay r_j as LT_{rj}, for $r_j \in \mathcal{R}_j$, and is calculated according to eq. (4.1). The acquired reward by CR relay r_j when participating in the cooperative beamforming to CR_{dj} is denoted by $\mathcal{R}_{rd_j}$, and is defined according to eq. (4.21). We denote the cost of relay r_j when participating in the cooperative transmission to CR_{dj} as $\mathcal{C}_{rd_j}$, and is defined according to eq. (4.23). Based on the acquired reward values and the paid cost values, CR user r can decide to participate in the cooperative transmission to a given CR user $CR_{d_j} \in \mathcal{S}_{rc}$.

In what follows, we modify the proposed RTS and LBS to obtain the set $\mathcal{R}_j$. We define some variables that are used to describe the proposed algorithms. We denote the participation decision vector by $\mathcal{B}_r$, of size $J \times 1$. The elements b_{r_j} for $j = 1, 2, \cdots, J$ of this vector take value either 0 or 1. If CR relay r decides to participate in the cooperative transmission to CR user $CR_{d_j} \in \mathcal{S}_{rc}$, b_{r_j} takes value 1. Otherwise, it is 0. Since a CR relay r can only participate in the cooperative transmission to one CR user at a given time slot, one of the elements of vector $\mathcal{B}_r$ will be non-zero. The CR user CR_{d_j} that results in the highest accumulated difference between reward and cost is selected for cooperative transmission by CR user r.

4.5.1 RTS in Case of Multiple Simultaneous Cooperation Requests

In what follows, we present the RTS in case of having multiple cooperation requests.

Definitions:

- t_c is the cooperation inquiry slot count, incremented each time the CBS asks for cooperative transmission towards J CR users, where $J \geq 1$.

- $T_{p_{rj}}, \forall j$ is the number of times CR user r participates in the cooperative transmission to CR_{d_j},

- $\bar{\mathbf{A}}_{r,d_j}, \forall j$ is the action profile of the CR user r when considering transmission to CR_{dj}. Initially, it is selected randomly from π_{r,d_j}.

- $\mathcal{B}_r^{(0)}$ is the initial participation decision vector, with $b_{r_j} = 1$, and $b_{ri} = 0$, for $i \neq j$.

- $\Omega_{r,d_j}^{(t_c)}$ is accumulated regret of following the action profile $\bar{\mathbf{A}}_{r,d_j}$, with initial value $\Omega_{r,d_j}^{(0)} = 0, \forall j$.

procedure

for $t_c = 1 \rightarrow \cdots$ **do**

for $j = 1 \rightarrow J$ **do**

if $b_{r_j} = 1$ **then**

- Find the interval, m, in which the channel fading coefficient h_{r,d_j} lies in.

- With probability p, CR user r follows entry m in its action profile, $\bar{A}_{r,d_j}(m)$.

if $\bar{A}_{r,d_j}(m) = 1$ **then**

- Accumulated regret when following the action profile is $\Omega_{r,d_j}^{(t_c)} = \Omega_{r,d_j}^{(t_c-1)} + l(\bar{\mathbf{A}}_{r,d_j})$.

else

$\Omega_{r,d_j}^{(t_c)} = \Omega_{r,d_j}^{(t_c-1)} + 0.$

end if

- With probability $1-p$, CR user r follows the complement of this entry m, $1-\bar{A}_{r,d_j}(m)$.

if $\bar{A}_{r,d_j}(m) = 0$ **then**

- Accumulated regret when not following the action profile, $\Omega_{r,d_j}^{(t_c)} = \Omega_{r,d_j}^{(t_c-1)} - l(\bar{\mathbf{A}}_{r,d_j})$.

else

$\Omega_{r,d_j}^{(t_c)} = \Omega_{r,d_j}^{(t_c-1)} + 0.$

end if

else

$\Omega_{r,d_j}^{(t_c)} = \Omega_{r,d_j}^{(t_c-1)} + 0.$

end if

- Accumulated difference between reward and cost is $D_j = (\nabla_{r,d_j}^{(t_c)} + \bar{\nabla}_{r,d_j}^{(t_c)}) - (\gamma_{r,d_j}^{(t_c)} + \bar{\gamma}_{r,d_j}^{(t_c)})$.

end for

- Calculate the average regret values of following the action profiles $\bar{\mathbf{A}}_{r,d_j}$, $\Omega_{r,d_j} = \dfrac{\Omega_{r,d_j}^{(t_c)}}{T_{p_{rj}}}$.

- Find the CR user CR_{d_j} with the minimum regret value, Ω_{r,d_j}.

- For this user, set $T_{p_{rj}} = T_{p_{rj}} + 1$, $b_{r_j} = 1$, and $b_{r_i} = 0$ for $i = 1, \cdots, J, i \neq j$.

for $j = 1 \to J$ **do**

if $\mod(t_c, T) = 0$ **then**

if $\Omega_{r,d_j} \leq 0$ **then**

- With probability $1 - \lambda$, choose $\bar{\mathbf{A}}_{r,d_j} = \bar{\mathbf{A}}_{r,d_j}$,

- With probability λ, randomly select a new action profile, $\bar{\mathbf{A}}_{r,d_j}$, from π_{r,d_j}.

else

- Randomly select $\bar{\mathbf{A}}_{r,d_j}$, from π_{r,d_j}.

end if

end if

end for

end for

end procedure

It is important to note that if it is the first time a CR user CR_{d_j} requests cooperation, the CR relays in in the network have no history of the accumulated reward and cost values of this user. So, it has to be given higher priority to benefit from cooperative transmission by setting b_{r_j} equal 1 during the current time slot.

4.5.2 LBS in Case of Multiple Simultaneous Cooperation Requests

The modification of the LBS in case of having multiple cooperation requests follows similar steps to that of the RTS. In particular, the CR user CR_{dj} that results in the highest accumulated difference between the reward and cost values is selected for cooperative transmission by CR user r. However, if it is the first time a CR user CR_{d_j} requests cooperation, it must be given an opportunity to benefit from cooperative transmission, even if simultaneous requests are received at the CR user r. The modified LBS can be described using the following algorithm.

Definitions:

- t_c is the cooperation inquiry slot count, incremented each time the CBS asks for cooperative transmission towards J CR users, where $J \geq 1$,

- $\mathcal{B}_r^{(0)}$ is the initial CR users selection vector, with $b_{r_j} = 1$, and $b_{r_i} = 0$, for $i \neq j$.

- $\bar{\mathbf{A}}_{r,d_j}$, $\forall j$ is the action profile of the CR user r when considering transmission to CR_{d_j}, with M entries all equal 1 initially.

- $\tilde{\mathcal{R}}_{rd_j}$, $\forall j$ is the estimated reward of the CR user r when participating in the cooperative transmission to CR_{d_j}, initially equal 0.

- $\tilde{C}_{rd_j}$, $\forall j$ is the estimated cost of the CR user r when participating in the cooperative transmission to CR_{d_j}, initially equal 0.

- $T_{p_{rj}}$, $\forall j$ is the number of times CR user r has participated in the cooperative transmission to CR_{d_j}, initially equal 0.

- $G^{(i)}_{rd_j}$, $\forall j$ is the achievable reward of CR user r when participating in the cooperative transmission to CR_{d_j} during the i^{th} time slot.

- $C^{(i)}_{rd_j}$, $\forall j$ is the achievable cost of CR user r when participating in the cooperative transmission to CR_{d_j} during the i^{th} time slot.

- D_j, $\forall j$ is the difference between estimated reward and cost caused by CR_{d_j}.

procedure

for $t_c = 1 \to ..$ **do**

  for $j = 1 \to J$ **do**

    if $b_{r_j} = 1$ **then**

      - Find the interval, m, in which the channel fading coefficient h_{r,d_j} lies in.

      if $\bar{A}_{r,d_j}(m) = 1$ **then**

        - Increment the number of participation times of CR user r towards CR_d, $T_{p_{rj}} = T_{p_{rj}} + 1$

        - Estimated reward of CR user r is given by $\tilde{\mathcal{R}}_{rd_j} = \frac{1}{T_{p_{rj}}} \sum_{i=1}^{i=T_{p_{rj}}} G^{(i)}_{rd_j}$,

        - Estimated cost of CR user r is given by $\tilde{C}_{rd_j} = \frac{1}{T_{p_{rj}}} \sum_{i=1}^{i=T_{p_{rj}}} C^{(i)}_{rd_j}$,

      end if

      if $\tilde{\mathcal{R}}_{rd_j} > \tilde{C}_{rd_j}$ **then**

        - Update action profile of the CR user r, $\bar{A}_{r,d_j}(m) = 1$.

      else

        - $\bar{A}_{r,d_j}(m) = 0$.

      end if

end if

- Calculate $D_j = \tilde{\mathcal{R}}_{rd_j} - \tilde{C}_{rd_j}$.

end for

- Find the CR user CR_{d_j} with the maximum value of D_j.

- For this user, set $T_{p_{rj}} = T_{p_{rj}} + 1$, $b_{r_j} = 1$, and $b_{r_i} = 0$ for $i = 1, \cdots, J, i \neq j$.

end for

end procedure

4.6 Operation Protocol

In this section, we summarize the proposed techniques in this Chapter, by presenting a protocol that describes the operation of the proposed cooperative CR network, which is described as follows.

1. If the channel between a specific CR user, CR_d, and the CBS is weak such that CR_d cannot decode the data packet from the CBS, it feeds back a negative acknowledgment (NACK) packet to the CBS.

2. In this case, the CBS ask other CR users in the network to act as relays and forward the message to CR_d at the beginning of the n^{th} time slot.

3. The CR users in the network run a participation decision making strategy, proposed in Section 4.3, to decide whether to participate in the cooperative data forwarding to CR_d or not.

4. The CR users that decide to act as relays inform the CBS with their decision during the n^{th} time slot.

5. Using the relay selection scheme, proposed in Section 4.4, the CBS selects the final set of CR users that can act as relays for CR_d.

6. The CBS broadcasts the first data packet to be forwarded to CR_d to all the cooperating CR relays over the channel f_d. The CBS also broadcasts the location of each of the CR relays participating in the cooperative transmission.

7. The set of cooperating CR relays forward the data to CR_d, irrespective of whether the primary

channel f_d is occupied or not, using the distributed beamforming technique, proposed in Section 4.2.

4.7 Numerical Results

In this section, we present some numerical results to assess the performance of the proposed distributed beamforming technique, and the autonomous participation decision making strategies. Unless stated otherwise, for all the numerical examples, we consider a network topology with $N = 5$ CR users $\{CR_1, CR_2, CR_3, CR_4, CR_5\}$. There are $F = 4$ primary channels $\{f_1, f_2, f_3, f_5\}$, allocated by the CBS to the 4 CR users $\{CR_1, CR_2, CR_3, CR_5\}$ respectively for opportunistic spectrum access. It is assumed that CR_4 is not allocated any frequency channel. The CR user that requires cooperation from other CR users in the network is CR_5. The distances between the set of CR users to CR_5, and to the PR occupying the frequency channel f_5 are given in Table 4.1. The occupancy probabilities of the channels by the PUs, $\Pr^r(1)$, for $r = 1 : 4$, are also given in Table 4.1. We assume that all channel fading amplitude gains are independent and Rayleigh distributed. We consider a time slot duration $T_{\text{slot}} = 10$ μsec, and the scheduling frame duration is 1 sec. We also consider a log-distance path loss model with a path loss exponent value of 4. Unless stated otherwise, the AWGN power used in our simulations is -130 dBm, and the average transmit symbol power P is 1 mWatt. The bandwidth of each frequency channel is 25 KHz, and quadrature phase shift keying (QPSK) is used. The value of the interference threshold $\gamma_{\text{th}} = 1 \times 10^{-16}$ Watt.

Table 4.1: Simulated network topology in case of one CR user requiring cooperation.

CR user CR_i	distance between CR_i and CR_5	distance between CR_i and PR at f_5	$\Pr^r(1)$
CR_1	500m	760m	0.7
CR_2	225m	710m	0.5
CR_3	225m	710m	0.3
CR_4	500m	760m	1

The performance of the proposed distributed beamforming technique is verified in Fig. 4.2 and Fig. 4.3. In order to plot these figures, the primary channel f_5 is assumed to be occupied by the PU with probability $Pr(1) = 0.7$. It is also assumed that all the other 4 CR users in the network are acting as relays to forward the data message to CR_5. In Fig. 4.2, we plot the

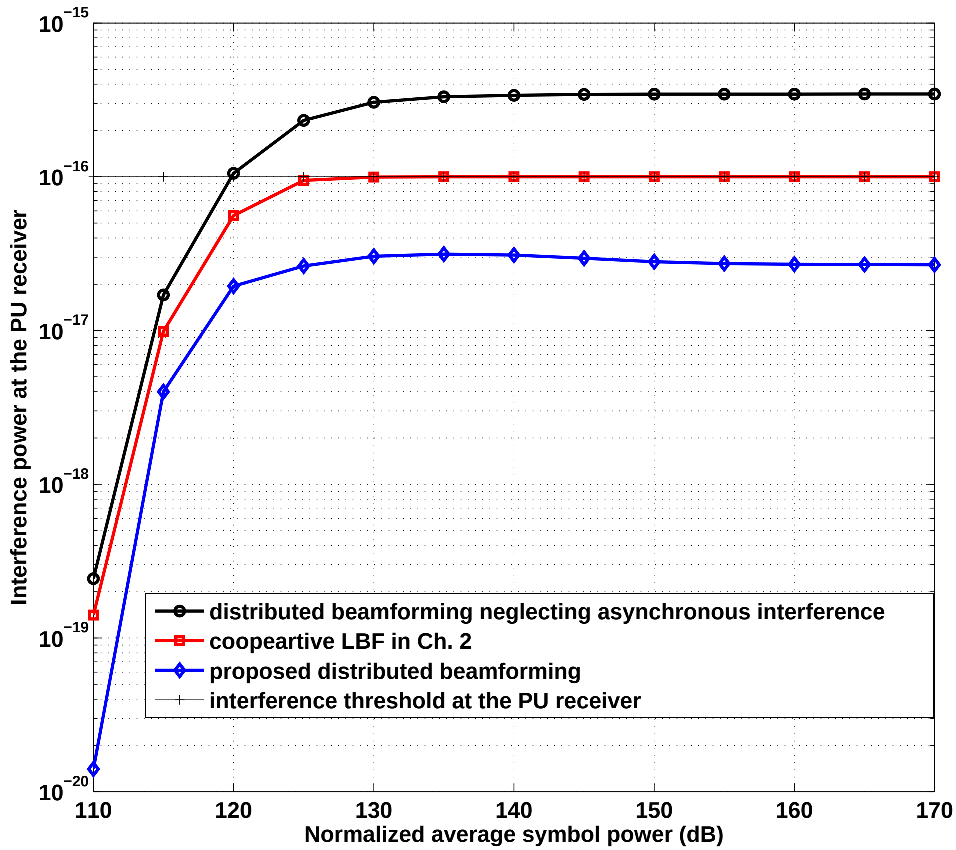

Figure 4.2: Asynchronous interference to the PR at the channel f_5.

asynchronous interference power at the PR resulting from our proposed distributed beamforming scheme versus the average transmit symbol power, normalized with respect to noise power. In this figure we also plot the performance of a distributed beamforming scheme that neglects the asynchronous interference resulting from the different cooperating CR relays. We compare the performance of both the proposed beamforming scheme and the distributed one that neglects the asynchronous interference, with the cooperative leakage beamforming (LBF) technique, proposed in Chapter 2. The LBF method requires full cooperation among all CR relays, including sharing their instantaneous CSI towards CR_5 and towards the PR at channel f_5. As shown in Fig. 4.2, the cooperative LBF technique in Chapter 2 meets the interference threshold at the PR. However, it requires a huge feedback overhead to exchange the CSI between all cooperating relays. It is also shown in the figure that the distributed beamforming scheme that neglects the asynchronous interference signals from different relays fails to meet the interference threshold imposed by the primary system. On the other hand, our proposed distributed beamforming keeps the asynchronous interference power values well below the interference threshold, due to its conservative design which considers the worst case scenario of asynchronous interference from the other cooperating relays.

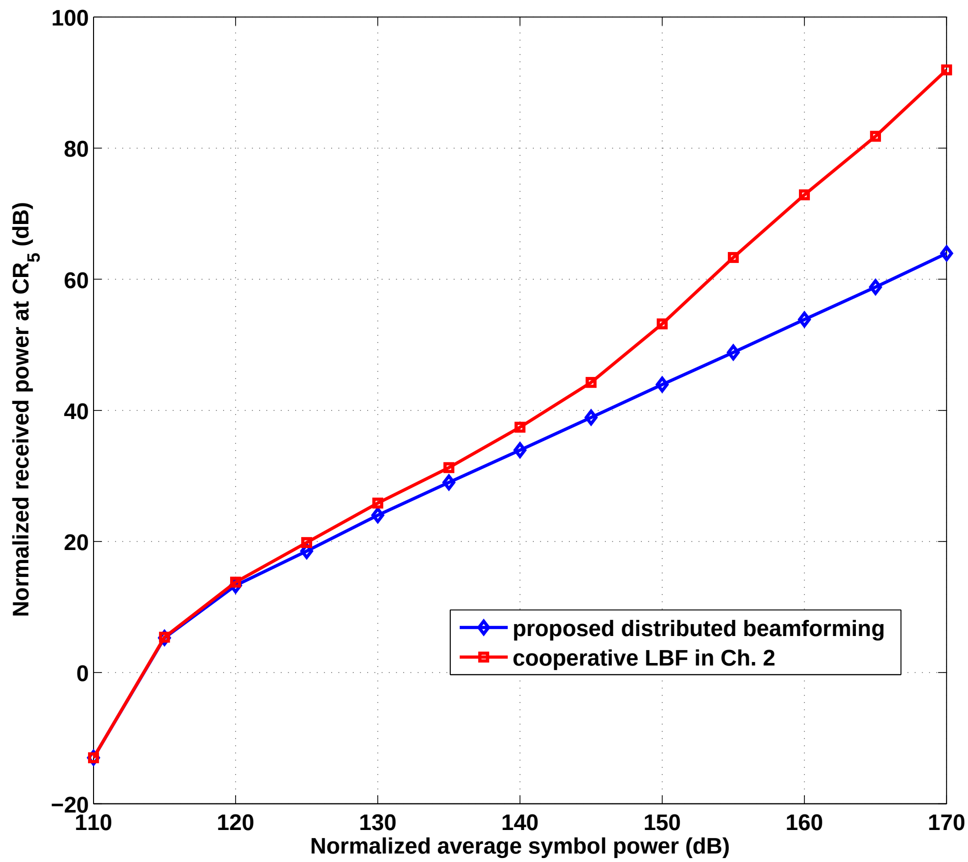

Figure 4.3: Received signal power at CR_5.

In Fig. 4.3, we plot the normalized received signal power at CR_5 versus the normalized average transmit symbol power. We compare the performance of our proposed method to the cooperative LBF technique in Chapter 2. As shown in the figure, full cooperation among the CR relays yields better performance compared to the distributed beamforming case, as expected. However, the performance degradation of the distributed beamforming is traded-off for having much reduced feedback overhead.

In the simulated performance of the decision making strategies, in Fig. 4.4 and Fig. 4.5, we consider a testing period of $T_{\text{test}} = 10$ time slots, and an exploration probability $\lambda = 1/3$. The

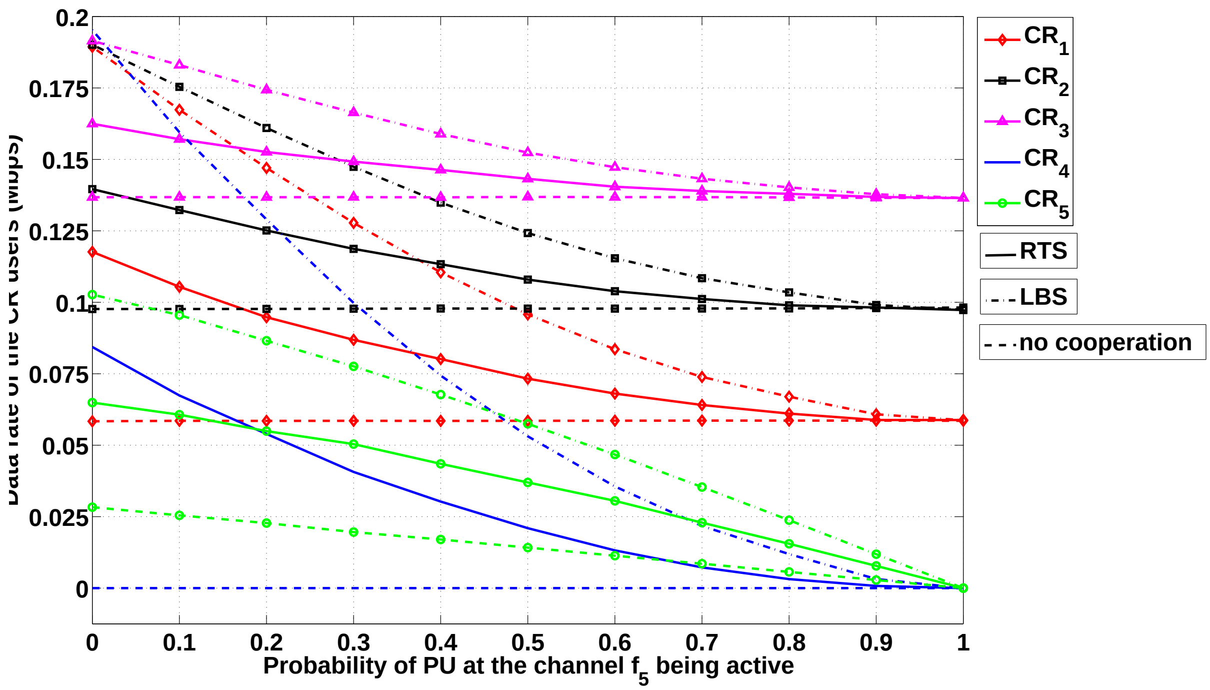

Figure 4.4: Data rates of CR users in case of participation and no participation.

results are shown for the case where the CBS asks for cooperative transmission 100 times, during 100 different time slots. In Fig. 4.4, we plot the achieved data rate by each CR user in the network versus the occupancy probability of the primary channel f_5, when the proposed RTS and the LBS are followed. In this figure, we also show the data rates of all CR users in case none of the CR users is participating in the cooperative transmission to CR_5. As shown in this figure, applying the autonomous decision making strategies results in increased values of the CR users' data rates, compared to the case of no participation at all. The increase in the CR users' data rates is degraded as the occupancy probability of the channel f_5 increases. When $\Pr^d(1)$ approaches one, the data rates of the CR users approach the values of the non cooperative transmission case. This figure also shows the data rate of CR_5, with cooperation and without cooperation. As expected, the cooperative transmission greatly enhances the data rate of CR_5 compared to the case of no cooperation, that is the direct transmission case from CBS to CR_d.

Figure 4.5 compares the two participation decision making strategies in terms of the paid costs by different CR users in the network. In this figure, we plot the data rate that could have been achieved by each CR user on its scheduled channel, if it used the transmission power $P_{r,d}$ for its

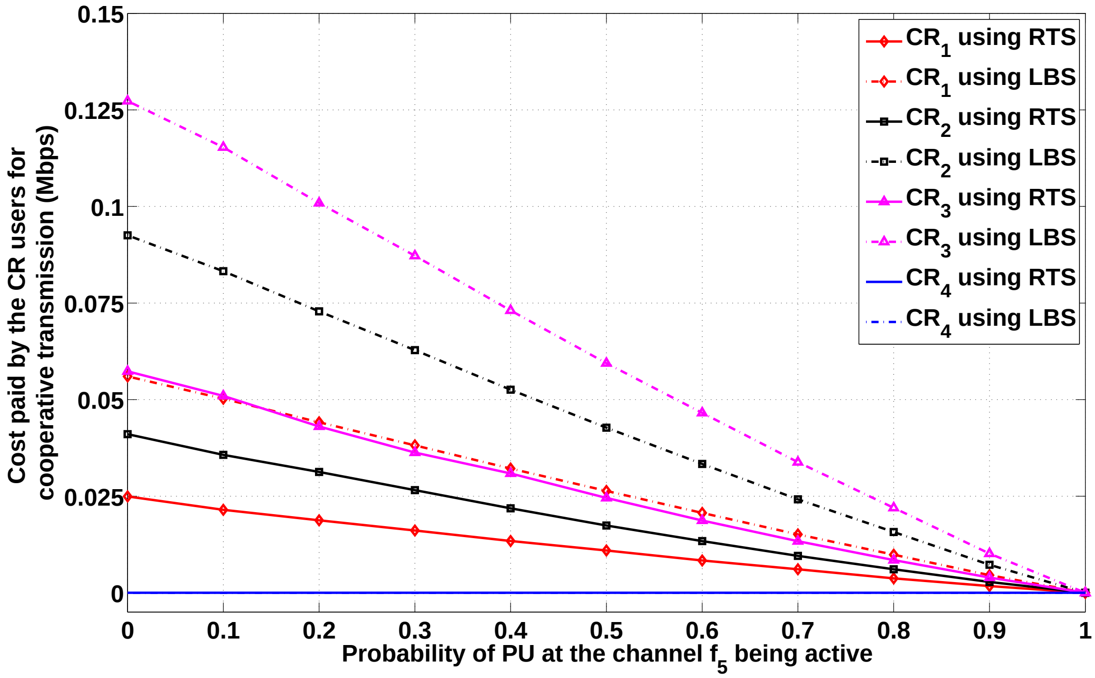

Figure 4.5: Cost of CR users with the RTS and the LBS.

own data transmission. As shown in this figure, the paid cost in terms of data rate of different CR users decreases as the occupancy probability of the channel f_5 increases. Also, for a given occupancy probability at channel f_5, the paid cost of the CR user r decreases as the value of $\Pr^r(1)$ increases. This can be explained as follows. As the value of $\Pr^r(1)$ increases, the chance of CR user r transmitting data over its scheduled channel decreases. Therefore, the amount of power $P_{r,d}$ cannot be efficiently utilized by the CR user r for its own data transmission, and its paid cost decreases. As shown in Fig. 4.4 the sum rate of the CR users is increased by up to 43% at $\Pr^d(1) = 0.3$, when using the RTS, while it increases by 98% in case of LBS. We notice the increase in the users' data rates is higher in case of LBS compared to RTS. Yet, the LBS requires higher cost, in terms of data rate, as seen in Fig. 4.5.

The convergence behavior of the two autonomous decision making strategies is shown in Fig. 4.6 assuming the occupancy probability of channel f_5 equals 0.6. In this figure, we plot the difference between the accumulated sum rate and the accumulated total cost of all the CR users in the network, when using the RTS and the LBS versus time slots. From this figure we notice that the RTS outperforms the LBF in terms of the difference between reward and cost values. But on the

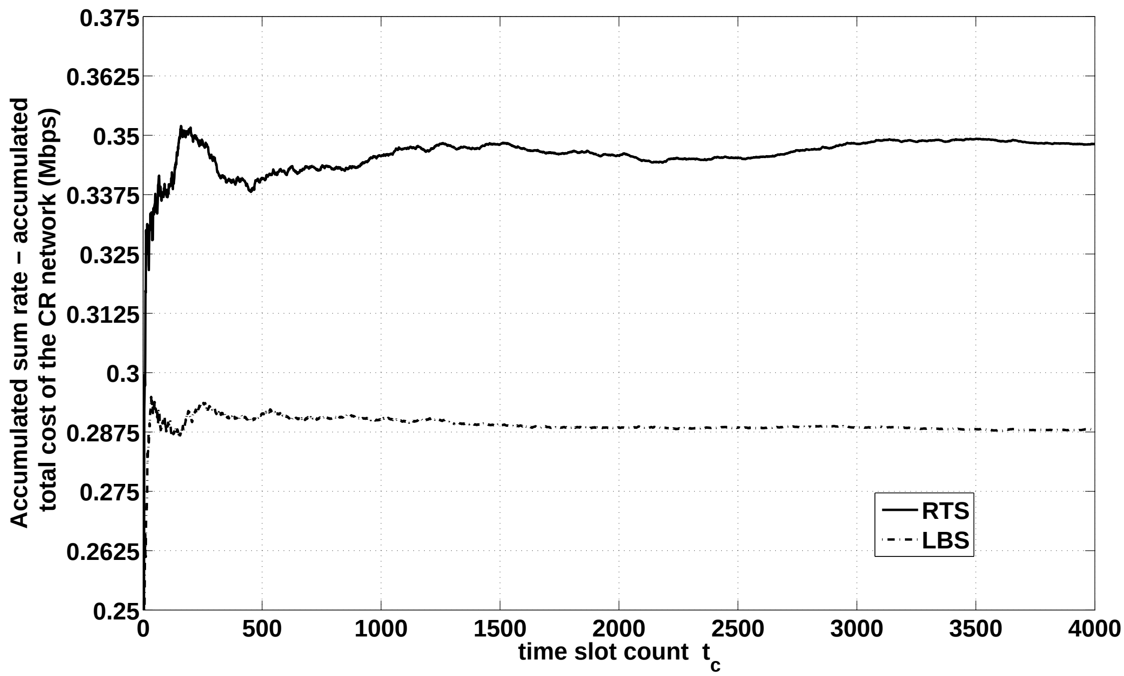

Figure 4.6: Convergence behavior of RTS and LBS.

other hand, we notice that the convergence time of the RTS is longer than that of the LBS, which is expected due to the higher complexity of the RTS.

The performance of the proposed relay selection scheme is shown in Fig. 4.7. In this figure, we plot the achieved data rate at CR_d versus the occupancy probability of the primary channel f_d, assuming that $\mathrm{Pr}^d(0|0) = 0.3$. As shown from the figure, applying the proposed relay selection scheme enhances the performance of the cooperative beamforming, compared to the case of not applying any relay selection schemes. It is also shown in the figure that the proposed relay selection scheme that requires only channel statistics has similar performance compared to the relay selection scheme that requires full CSI knowledge. This can be explained as follows. The statistical channel knowledge is only used by the CBS in the relay selection phase, while each CR relay designs its own beamforming weight using its instantaneous CSI towards the PR at f_5 and towards CR_5. This results in roughly the same average value of the data rate achieved in the two cases.

The performance of the modified autonomous decision making strategies in case of having multiple simultaneous cooperation requests is shown in Figs. 4.8−4.11. To plot these figures, we assume a network topology with $N = 6$ CR users $\{CR_1, CR_2, CR_3, CR_4, CR_5, CR_6\}$, and $F = 5$ primary

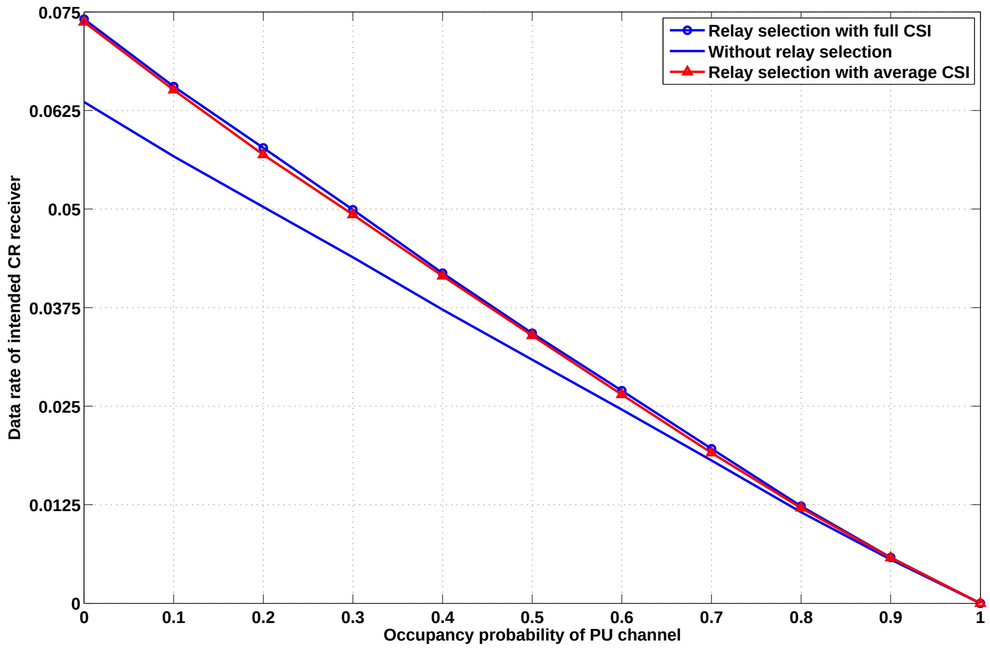

Figure 4.7: Achieved data rate at CR_d by cooperative transmission, in the cases of no relay selection, relay selection with full CSI knowledge at the CBS, and relay selection using average CSI.

channels $\{f_1, f_2, f_3, f_5, f_6\}$, allocated by the CBS to the 5 CR users $\{CR_1, CR_2, CR_3, CR_5, CR_6\}$ respectively, for opportunistic spectrum access. In these figures, the occupancy probability of f_6 is assumed to be 0.7. The CR users that require cooperative transmissions from the other CR users in the network are CR_5 and CR_6. The distances between the set of CR users to CR_6, and to the PR occupying the frequency channel f_6 are given in Table 4.2.

Table 4.2: Simulated network topology in case of two CR users requiring cooperation.

CR user CR_i	distance between CR_i and CR_6	distance between CR_i and PR at f_6
CR_1	650m	1200m
CR_2	200m	850m
CR_3	280m	800m
CR_4	1000m	750m

The performance of the modified RTS is shown in Fig. 4.8 and Fig. 4.9. In Fig. 4.8, we plot the achieved data rate by each CR user in the network with the modified RTS versus the occupancy probability of the primary channel f_5. In this figure, we also show the data rates of all CR users in case none of the CR users is participating in the cooperative transmission. As shown from the

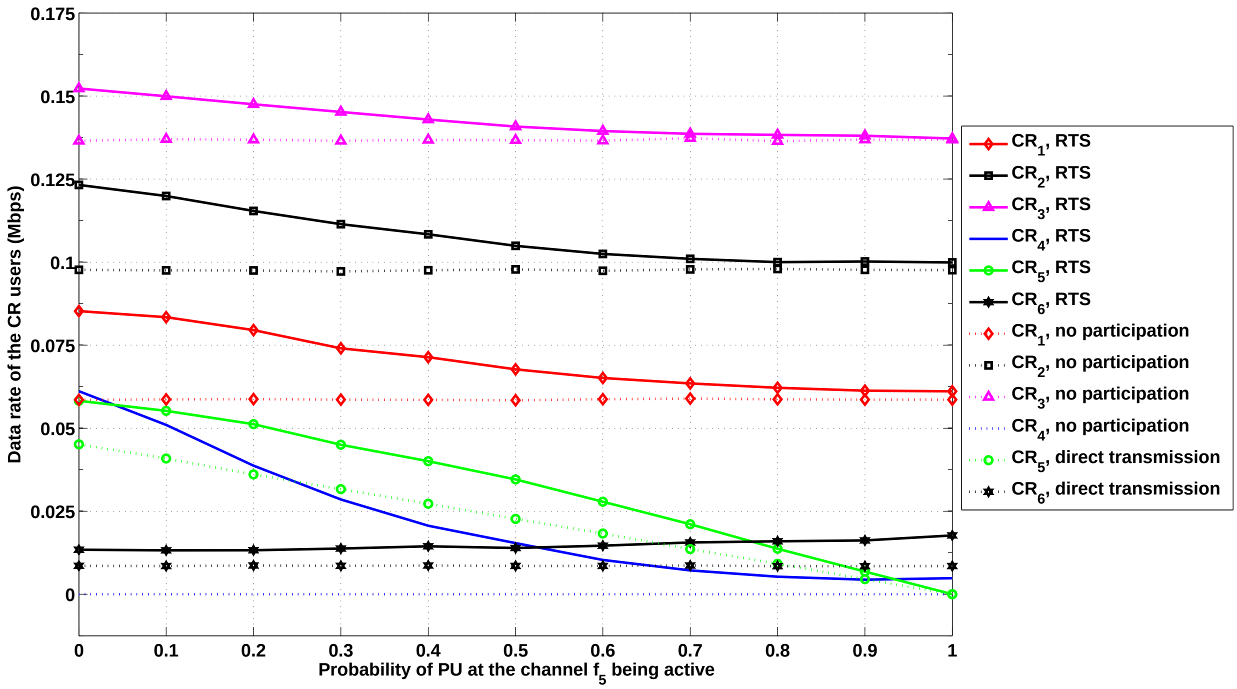

Figure 4.8: Data rates of CR users in case of no participation, and participation according to the modified RTS, in case of 2 CR users requesting cooperative transmission.

figure, the modified RTS results in increased values of the CR users' data rates, compared to the case of no participation at all. We also notice, from this figure, that even when the occupancy probability of f_5 approaches 1, the data rates of the CR users in case of cooperative transmission are still higher than those with no cooperative transmission in the network. This is due to the fact that the occupancy probability of channel f_6 is less than 1. We also notice that the data rate of the assisted user CR_6 starts to increase as the occupancy probability of channel f_5 approaches 1. This is because more cooperation opportunities are granted to user CR_6 in this case.

In Fig. 4.9, we plot the data rate that could have been achieved by each CR user on its scheduled channel, if it used the transmission power $P_{r,d}$ for its own data transmission, instead of cooperative transmission using the RTS. We observe from this figure, that the cost values of different CR users decrease as the occupancy probability of channel f_5 increases. However, the rate of this decrease slows down as the value of this probability tends to 1, due to the fact that the occupancy probability of channel f_6 is still less than 1. So the cooperation is now directed more towards user CR_6 instead, and does not entirely stop as the case in Fig. 4.5, even when the occupancy probability of f_5 is 1.

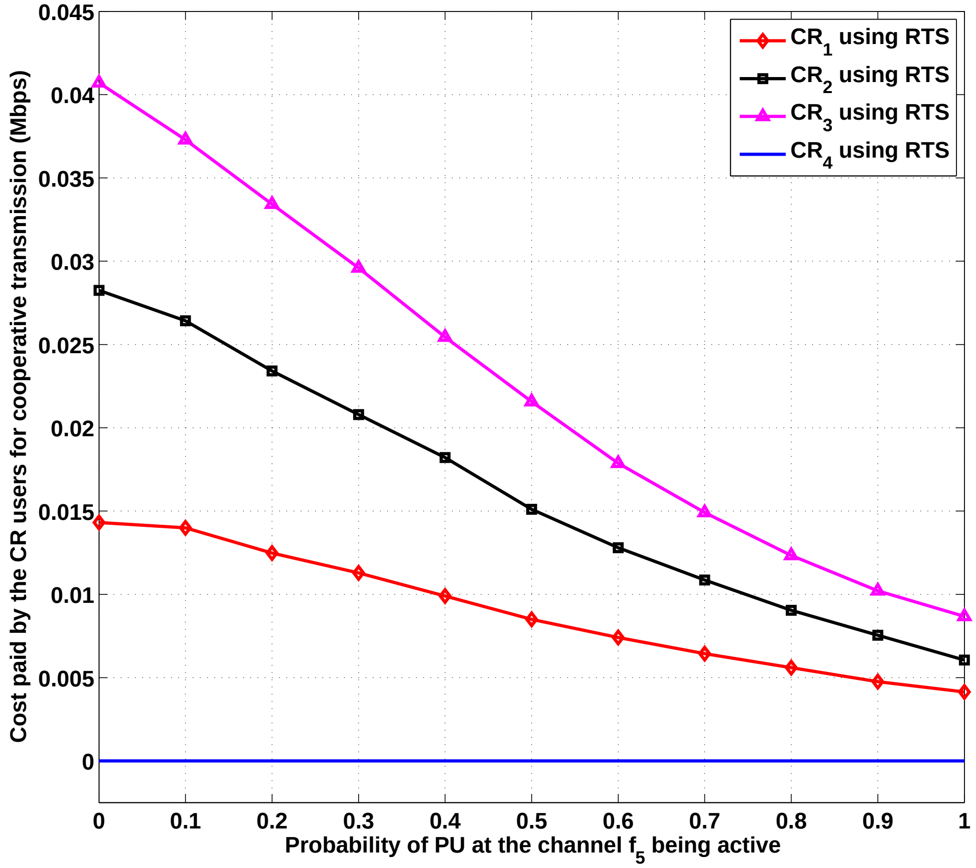

Figure 4.9: Cost of CR users when using modified RTS, in case of 2 CR users requesting cooperative transmission.

In Fig. 4.10, we plot the achieved data rate by each CR user in the network versus the occupancy probability of the primary channel f_5, when the modified LBS is followed. In this figure, we also show the data rates of all CR users in case none of the CR users is participating in the cooperative transmission. Similar observations to those from Fig. 4.8 can be driven from this figure. In Fig. 4.11, we plot the data rate that could have been achieved by each CR user on its scheduled channel, if it used the transmission power $P_{r,d}$ for its own data transmission, instead of cooperative transmission using the modified LBS. Similar observations can be made about the performance of the modified LBS compared to that of the modified RTS. However, it is noticed from Fig. 4.10 and Fig. 4.11 that the increase in the achieved data rates and the cost of the CR users, as the occupancy probability of channel f_5 approaches 1, is more noticeable than that in case of modified RTS. This is due to the fact that the convergence of the LBS is faster than that of the RTS, which means that it is less immune to the changes of the system parameters.

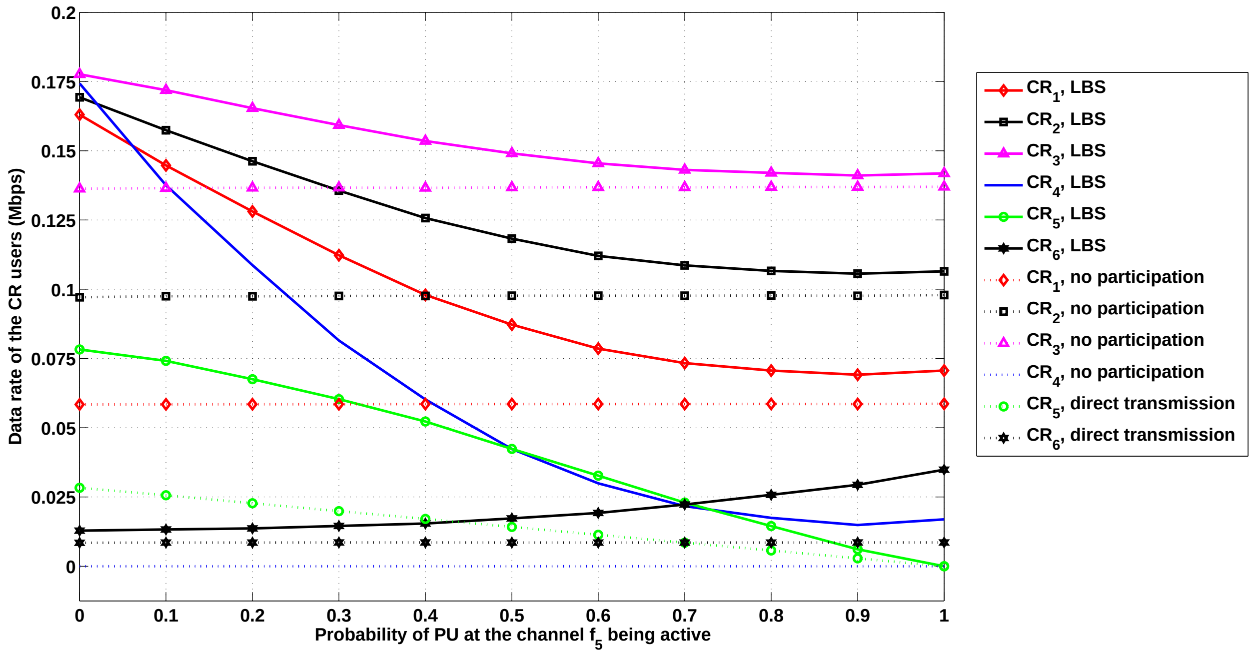

Figure 4.10: Data rates of CR users in case of no participation, and participation according to the modified LBS, in case of 2 CR users requesting cooperative transmission.

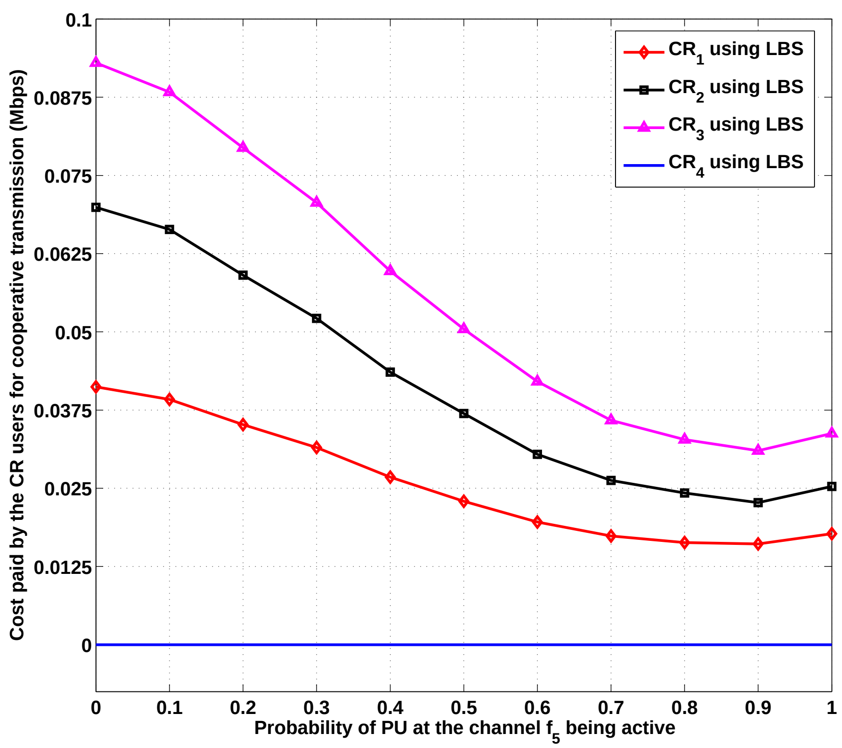

Figure 4.11: Cost of CR users when using modified LBS, in case of 2 CR users requesting cooperative transmission.

Chapter 5

Summary, Conclusion and Future Work

5.1 Summary and Conclusions

In this thesis, we have shown that even though cooperative beamforming in CR network can improve the radio spectrum utilization and enhance the network performance, it faces a number of challenging issues. Throughout the thesis, we have tackled five critical problems facing the implementation of cooperative CR networks, namely the asynchronous interference issue, the imperfect channel estimation problem, the need to apply relay selection schemes, the problem of feedback overhead, and the need for applying participation decision making strategies. We have proposed different techniques to solve these problems and we have evaluated the performance of these proposed techniques.

First, in Chapter 2, we have defined the asynchronous interference problem and have provided its mathematical modeling. We have also proposed a novel cooperative beamforming method, named the LBF method for cooperative CR systems. The LBF method accounts for the asynchronous interference at the primary receiver, and enables the CR relays to transmit data to the CR receiver with a certain limit on the interference introduced at the PR when the PU is active. In Chapter 2, we have also addressed the effect of having imperfect CSI estimation on the performance of the proposed beamforming method. We have proposed a robust cooperative beamforming method named the RLBF method to account for the effect of having an error in the channel estimation between the PR and each CR relay.

In conclusion, presented numerical results showed that the proposed LBF method has a superior performance compared to the ZFBF [20] and JLS [33] methods in the presence of asynchronous interference. In particular, the LBF method causes significantly less interference at the PR than the ZFBF and JLS methods. As such, our proposed LBF method decreases the CR system outage probability up to 75% compared to the ZFBF method, for a primary user's busy probability of 0.75. In the presence of an estimation error of the channel fading gains between the CR relays and the PR, the RLBF method proposed in Chapter 2 can satisfy the target interference threshold at the PR despite such estimation error.

Next, in Chapter 3, we have considered a generalized scenario of a CR-based broadcasting network with multiple PRs and multiple CR receivers. In order to address the asynchronous interference issue in this network model, we have proposed an innovative cooperative beamforming technique. In particular, the cooperative beamforming design has been formulated as an optimization problem of constrained weighted sum rate maximization. In light of the intractability of the optimal beamforming design problem, an approximation is used to design beamforming directions and to allocate power among different beamforming directions. Due to the multiple interference constraints corresponding to multiple primary receivers, the power allocation scheme, proposed in Chapter 3, still has high complexity. Therefore, in this chapter, we have also proposed a low complexity power allocation algorithm. Moreover, we have extended the proposed cooperative beamforming technique for the case of having only statistical CSI of the channels between the PRs and the CCRNs at the CCRNs. In this case, the asynchronous interferences at the PRs are guaranteed in a statistical sense. In the absence of a mathematically tractable expression of the distribution of the random interference powers at the PRs, we develop an upper bound on the probability of introducing asynchronous interference power at a given PR beyond a given threshold. Then this developed upper bound is used to design a robust leakage beamforming technique. In Chapter 3, we have also proposed three CCRN selection strategies for a generalized scenario with multiple primary receivers and CR receivers, to be used in conjunction with the beamforming technique.

To conclude the work accomplished in Chapter 3, the presented numerical results have shown that the proposed beamforming technique with SOPA can significantly reduce the interference signals at all PRs and can provide an increase up to 150% in the sum transmission rate of CR receivers compared to the ZFBF technique [20]. Moreover, it has been shown in the presented numerical results that our proposed robust design of the beamforming vector can statistically maintain the asynchronous interference constraints at the multiple PRs, when only statistical CSI knowledge is available at the CCRNs. In Chpater 3, we have shown that the optimal CCRN selection in conjunction with beamforming can further increase the sum data rate of CR receivers up to 45%.

As a solution for the problem of huge feedback overhead required for cooperative beamforming, in Chapter 4 we have proposed a distributed beamforming method to be used in cooperative CR networks with minimal amount of feedback overhead. Next in this chapter, we have defined a suitable incentive for the CR users to participate in the cooperative transmission. Then, we have proposed two autonomous participation decision making strategies, namely the RTS and the LBS methods, to help each CR user in deciding whether to participate in the cooperative transmission or not, by introducing cost and reward functions for different CR relays. We have assumed no coordination is present among the participating CR users in the decision making process. The performance of the proposed participation decision making methods has also been verified through the numerical results, and their convergence time has been compared. To further enhance the performance of the cooperative CR network, we proposed a relay selection method in Chapter 4 to resolve the competition taking place between multiple CR users that are willing to participate in the cooperative transmission. The selection criterion is to choose the best set of CR users that yields the maximum received signal power value at the intended CR receiver. The proposed relay selection method only requires statistical knowledge of the CR users' CSI, to decrease the amount of feedback overhead in the network. The provided simulation results showed the performance enhancement achieved by the proposed relay selection method. Finally in Chapter 4, we have generalized the proposed autonomous decision making strategies, the RTS and the LBS methods, to handle the case of receiving multiple simultaneous cooperation requests from different CR users. The decision making strategies have been designed to help each CR user, not only to decide whether to participate in the cooperative transmission towards a CR receiver or not, but also to select which CR receiver among the simultaneous requests that it receives. The modified autonomous decision making strategies are shown to provide better overall performance of the whole network compared to the case of no cooperative transmission.

To conclude Chapter 4, the presented numerical results have shown that the distributed beamforming scheme can meet the interference threshold at the PR, with no information exchange between the cooperating CR relays regarding their channel states towards the CR receiver and towards the PR. However this reduced feedback overhead comes at a certain expense of received signal power value at the CR receiver, compared to the cooperative LBF technique, which requires full cooperation between the CR relays in terms of sharing channel knowledge. The two developed autonomous participation decision making strategies, namely the RTS and the LBS, can increase the sum rate of the CR users by up to 43%, for a primary channel occupancy probability of 0.3, when using the RTS, and 98% when using the LBS, relative to the case of no cooperation.

In conclusion, cooperative beamforming can enhance the performance of CR networks under different operation scenarios. The work presented in Chapter 4 acts as the umbrella under which the proposed beamforming techniques in Chapters 2 and 3, as well as the distributed beamforming technique in Chapter 4, can be applied. In particular, the application of an autonomous participation decision making strategy enables the creation a cooperative CR network which outperforms conventional CR networks, in terms of the achievable data rates and overall energy efficiency.

5.2 Future Work

In our modeling of cooperative CR networks, throughout the whole thesis, we have considered different sources of channel impairments, including short term fading and path loss. In particular, we have considered the narrowband flat fading model of the channel and the simplified log-distance path loss model. In addition to these impairments, a signal can typically experience shadow fading effect, which represents a random variation about the path loss at a given distance, due to blockage from objects in the signal path. Changes in reflecting surfaces and scattering objects can also cause random variation about the path loss. Thus, a model for the random attenuation due to these effects is also needed for a more realistic representation of the cooperative CR network. For such modeling, we can use log-normal shadowing [76], which has been confirmed empirically to accurately model the variation in path loss in both outdoor and indoor radio propagation environments. In the log-normal shadowing model, the path loss is assumed random with a log-normal distribution.

Another channel impairment that can be examined in cooperative CR networks is the wideband multipath fading. The impact of multipath on the received signal depends on whether the spread of time delays associated with the line-of-sight and different multipath components is large or small relative to the inverse signal bandwidth. If the delay spread is large, the line-of-sight and all multipath components are typically resolvable, leading to the wideband fading model. While these multipath effects are captured in our modeling of narrowband fading channels throughout the thesis, wideband fading can be more convenient for urban areas modeling, where the delay spread takes large values. Generally, channel delay spread is highly dependent on the propagation environment. In indoor channels delay spread typically ranges from 10 to 1000 nanoseconds, in suburbs it ranges from 200-2000 nanoseconds, and in urban areas it ranges from 1-30 microseconds [77].

In addition to incorporating, into our future work, the generalized modeling of channel impairments, there are other interesting research directions, that can be built on top of the proposed work in this thesis, as described below. The first one is considering energy efficiency in the design process of the cooperative beamforming techniques. In the past few years, energy-aware communications have received a lot of attention from research and industrial communities due to the rising energy costs to operate future wireless networks, ecological, and environmental reasons. That said, designing energy-aware cooperative beamforming techniques is crucial to meet such requirements. The objective of the energy-aware cooperative beamforming techniques should consider the achievable data rates, transmit powers, and consumed circuitry powers at the transmitters. In our beamforming designs, we did not consider energy efficiency aspect. As a future research direction, energy efficient beamforming design in presence of asynchronous interference for CR systems can be considered.

Another interesting future research direction of the work done in this thesis could be to incorporate energy harvesting techniques into the design of cooperative CR networks. The ability to harvest energy, from ambient or dedicated sources, enables wireless charging of low-power CR devices and enhances the system design, usability, and reliability. Energy harvesting from the transmitted RF signal can even be used as an incentive or a form of compensation to be offered to the CR users that agree to act as relay nodes, in exchange for the energy that the CR relay node consumes for packet reception and retransmission to the intended CR receiver. However, energy causality constraints should be taken into account in the design process of the cooperative beamforming techniques [78], to indicate that the amount of energy that can be used in data transmission is that which has been harvested so far by the relay node.

Moreover, QoS is an important consideration in designing wireless communication systems. For example, CR users can have QoS requirement e.g., delay requirement for their data transmission. In our beamforming designs in Chapter 2 and Chapter 3, we have used Shannon capacity in the objective function however Shannon capacity does not put any restriction on the link layer delay performance and it is meaningful for the best effort traffic where delay bound is not a major concern. However, the concept of effective capacity has been introduced in the literature, that is defined as the maximum constant traffic arrival rate that a communication channel can support in order to guarantee a certain statistical delay constraint [79]. With a statistical delay constraint, the delay bound is guaranteed with a certain violation probability. Extension of our beamforming design with effective capacity maximization is quite interesting and can be pursued in the future.

Bibliography

[1] S. Han, Y. Liang, and B. Soong, "Spectrum refarming: a new paradigm of spectrum sharing for cellular networks," *IEEE Transactions on Communications*, vol. 63, no. 5, pp. 1895–1906, May 2015.

[2] Z. Qing and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Signal Processing Magazine*, vol. 24, no. 3, pp. 79–89, May 2007.

[3] J. Mitola and J. Maguire, G.Q., "Cognitive radio: making software radios more personal," *IEEE Personal Communications*, vol. 6, no. 4, pp. 13 –18, August 1999.

[4] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201 – 220, February 2005.

[5] X. Hong, J. Wang, C.-X. Wang, and J. Shi, "Cognitive radio in 5G: a perspective on energy-spectral efficiency trade-off," *IEEE Communications Magazine*, vol. 52, no. 7, pp. 46–53, July 2014.

[6] C.-X. Wang, F. Haider, X. Gao, X.-H. You, Y. Yang, D. Yuan, H. Aggoune, H. Haas, S. Fletcher, and E. Hepsaydir, "Cellular architecture and key technologies for 5G wireless communication networks," *IEEE Communications Magazine*, vol. 52, no. 2, pp. 122–130, February 2014.

[7] J. Mitola, J. Guerci, J. Reed, Y.-D. Yao, Y. Chen, T. Clancy, J. Dwyer, H. Li, H. Man, R. McGwier, and Y. Guo, "Accelerating 5G QoE via public-private spectrum sharing," *IEEE Communications Magazine*, vol. 52, no. 5, pp. 77–85, May 2014.

[8] V. K. Bhargava and E. Hossain, *Cognitive Wireless Communication Networks.* Secaucus, NJ, USA: Springer-Verlag New York, Inc., 2007.

[9] T. Weiss, J. Hillenbrand, A. Krohn, and F. Jondral, "Mutual interference in OFDM-based spectrum pooling systems," in *IEEE 59th Vehicular Technology Conference (VTC Spring)*, vol. 4, May 2004, pp. 1873–1877.

[10] G. Bansal, J. Hossain, and V. Bhargava, "Optimal and suboptimal power allocation schemes for OFDM-based cognitive radio systems," *IEEE Transactions on Wireless Communications*, vol. 7, no. 11, pp. 4710–4718, November 2008.

[11] D. I. Kim, L. B. Le, and E. Hossain, "Joint rate and power allocation for cognitive radios in dynamic spectrum access environment," *IEEE Transactions on Wireless Communications*, vol. 7, no. 12, pp. 5517 –5527, December 2008.

[12] H. Suraweera, P. Smith, and M. Shafi, "Capacity limits and performance analysis of cognitive radio with imperfect channel knowledge," *IEEE Transactions on Vehicular Technology*, vol. 59, no. 4, pp. 1811 –1822, May 2010.

[13] A. Nosratinia, T. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Communications Magazine*, vol. 42, no. 10, pp. 74 – 80, October 2004.

[14] T. Cover and A. Gamal, "Capacity theorems for the relay channel," *IEEE Transactions on Information Theory*, vol. 25, no. 5, pp. 572 – 584, September 1979.

[15] R. Mudumbai, D. Brown, U. Madhow, and H. Poor, "Distributed transmit beamforming: challenges and recent progress," *IEEE Communications Magazine*, vol. 47, no. 2, pp. 102–110, February 2009.

[16] Y. S. Soh, T. Quek, M. Kountouris, and H. Shin, "Energy efficient heterogeneous cellular networks," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 5, pp. 840–850, May 2013.

[17] J. Zhao, T. Quek, and Z. Lei, "Coordinated multipoint transmission with limited backhaul data transfer," *IEEE Transactions on Wireless Communications*, vol. 12, no. 6, pp. 2762–2775, June 2013.

[18] L. Zhang, Y.-C. Liang, and Y. Xin, "Joint beamforming and power allocation for multiple access channels in cognitive radio networks," *IEEE Journal on Seleted Areas in Communications*, vol. 26, no. 1, pp. 38–51, January 2008.

[19] S. Yiu, M. Vu, and V. Tarokh, "Interference reduction by beamforming in cognitive networks," in *IEEE Global Telecommunications Conference (GLOBECOM)*, November 2008, pp. 1–6.

[20] J. Liu, W. Chen, Z. Cao, and Y. J. Zhang, "A distributed beamforming approach for enhanced opportunistic spectrum access in cognitive radios," in *IEEE Global Telecommunications Conference (GLOBECOM)*, December 2009, pp. 1–6.

[21] A. Afana, V. Asghari, A. Ghrayeb, and S. Affes, "On the performance of cooperative relaying spectrum-sharing systems with collaborative distributed beamforming," *IEEE Transactions on Communications*, in press, 2014.

[22] G. Zhao, J. Ma, Y. Li, T. Wu, Y. Kwon, A. Soong, and C. Yang, "Spatial spectrum holes for cognitive radio with directional transmission," in *IEEE Global Telecommunications Conference (GLOBECOM)*, December 2008, pp. 1 –5.

[23] L. Zhang, Y.-C. Liang, and Y. Xin, "Joint beamforming and power allocation for multiple access channels in cognitive radio networks," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 1, pp. 38 –51, January 2008.

[24] J. Liu, W. Chen, Z. Cao, and Y. Zhang, "Cooperative beamforming for cognitive radio networks: a cross-layer design," *IEEE Transactions on Communications*, vol. 60, no. 5, pp. 1420–1431, May 2012.

[25] R. Xie, F. Yu, and H. Ji, "Joint power allocation and beamforming with users selection for cognitive radio networks via discrete stochastic optimization," in *IEEE Global Telecommunications Conference (GLOBECOM)*, December 2011, pp. 1–5.

[26] M. Uppal, G. Yue, Y. Xin, X. Wang, and Z. Xiong, "A dirty-paper coding scheme for the cognitive radio channel," in *IEEE International Conference on Communications (ICC)*, May 2010, pp. 1–5.

[27] T. Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 3, pp. 528–541, March 2006.

[28] M. Sharif and B. Hassibi, "A comparison of time-sharing, DPC, and beamforming for MIMO broadcast channels with many users," *IEEE Transactions on Communications*, vol. 55, no. 1, pp. 11–15, January 2007.

[29] B. Hochwald and S. Vishwanath, "Space-time multiple access: linear growth in the sum rate," in *40th Annual Allerton Conference on Communications, Control and Computing*, 2002.

[30] A. Lozano, R. Heath, and J. Andrews, "Fundamental limits of cooperation," *IEEE Transactions on Information Theory*, vol. 59, no. 9, pp. 5213–5226, September 2013.

[31] E. Larsson, O. Edfors, F. Tufvesson, and T. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Communications Magazine*, vol. 52, no. 2, pp. 186–195, February 2014.

[32] G. Barriac, R. Mudumbai, and U. Madhow, "Distributed beamforming for information transfer in sensor networks," *3rd International Symposium on Information Processing in Sensor Networks (IPSN)*, pp. 81–88, April 2004.

[33] H. Zhang, N. Mehta, A. Molisch, J. Zhang, and H. Dai, "Asynchronous interference mitigation in cooperative base station systems," *IEEE Transactions on Wireless Communications*, vol. 7, no. 1, pp. 155 –165, January 2008.

[34] L. Zhang, Y.-C. Liang, and Y. Xin, "Joint beamforming and power allocation for multiple access channels in cognitive radio networks," *IEEE Journal on Selected Areas in Comm.*, vol. 26, no. 1, pp. 38–51, January 2008.

[35] T. Al-Khasib, M. Shenouda, and L. Lampe, "Dynamic spectrum management for multiple-antenna cognitive radio systems: Designs with imperfect CSI," *IEEE Transactions on Wireless Communications*, vol. 10, no. 9, pp. 2850 –2859, September 2011.

[36] E. Dall'Anese, S.-J. Kim, G. Giannakis, and S. Pupolin, "Power control for cognitive radio networks under channel uncertainty," *IEEE Transactions on Wireless Communications*, vol. 10, no. 10, pp. 3541–3551, October 2011.

[37] C.-L. Wang, T.-N. Cho, and S.-J. Syue, "Cooperative beamforming with multi-relay selection for wireless ad hoc networks," in *IEEE 73rd Vehicular Technology Conference (VTC Spring)*, May 2011, pp. 1 –5.

[38] M.-F. Jian, W.-J. Huang, and C.-K. Wen, "Distributed beamforming with compressed feedback in time-varying cooperative networks," in *Signal Information Processing Association Annual Summit and Conference (APSIPA ASC), Asia-Pacific*, December 2012, pp. 1–4.

[39] N. Nonaka, Y. Kakishima, and K. Higuchi, "Investigation on beamforming control methods in base station cooperative multiuser MIMO using block-diagonalized beamforming matrix," in *Vehicular Technology Conference (VTC Fall), 2014 IEEE 80th*, September 2014, pp. 1–5.

[40] J. C. Harsanyi, "Games with incomplete information played by Bayesian players, I-III," *Management Science journal*, vol. 50, no. 12 Supplement, pp. 1804–1817, December 2004.

[41] N. Cesa-Bianchi and G. Lugosi, *Prediction, Learning, and Games*. New York, NY, USA: Cambridge University Press, 2006.

[42] P. Dutta, *Strategies and Games: Theory and Practice*. MIT Press, 1999.

[43] B. Van Veen and K. Buckley, "Beamforming: a versatile approach to spatial filtering," *IEEE Acoustics, Speech, and Signal Processing (ASSP) Magazine*, vol. 5, no. 2, pp. 4 –24, April 1988.

[44] M. Rashid, M. Hossain, E. Hossain, and V. Bhargava, "Opportunistic spectrum scheduling for multiuser cognitive radio: a queueing analysis," *IEEE Transactions on Wireless Communications*, vol. 8, no. 10, pp. 5259 –5269, October 2009.

[45] A. Goldsmith, *Wireless Communications*. New York, NY, USA: Cambridge University Press, 2005.

[46] L. Ozarow, S. Shamai, and A. Wyner, "Information theoretic considerations for cellular mobile radio," *IEEE Transactions on Vehicular Technology*, vol. 43, no. 2, pp. 359 –378, May 1994.

[47] T. Lo, "Maximum ratio transmission," *IEEE Transactions on Communications*, vol. 47, no. 10, pp. 1458–1461, October 1999.

[48] A. F. Molisch, *Wireless Communications.* Wiley-IEEE Press, Second Edition, 2011.

[49] J.-B. Hiriart-Urruty, "Global optimality conditions in maximizing a convex quadratic function under convex quadratic constraints," *Journal of Global Optimization*, vol. 21, no. 4, pp. 443–453, Dec. 2001.

[50] G. Zheng, K.-K. Wong, and B. Ottersten, "Robust cognitive beamforming with bounded channel uncertainties," *IEEE Transactions on Signal Processing*, vol. 57, no. 12, pp. 4871–4881, December 2009.

[51] Y. Zhang, E. DallAnese, and G. Giannakis, "Distributed optimal beamformers for cognitive radios robust to channel uncertainties," *IEEE Transactions on Signal Processing*, vol. 60, no. 12, pp. 6495–6508, December 2012.

[52] K. Phan, S. Vorobyov, N. Sidiropoulos, and C. Tellambura, "Spectrum sharing in wireless networks via QoS-aware secondary multicast beamforming," *IEEE Transactions on Signal Processing*, vol. 57, no. 6, pp. 2323–2335, June 2009.

[53] K. Tanabe and M. Sagae, "An exact Cholesky decomposition and the generalized inverse of the variance-covariance matrix of the multinomial distribution, with applications," *Journal of the Royal Statistical Society. Series B (Methodological)*, vol. 54, no. 1, pp. 211–219, 1992.

[54] J. Li, A. Bose, and Y. Zhao, "Rayleigh flat fading channels' capacity," in *3rd Annual Communication Networks and Services Research Conference (CNSR)*, May 2005, pp. 214 – 217.

[55] M. H. Hassan and M. Hossain, "Cooperative beamforming for cognitive radio systems with asynchronous interference to primary user," *IEEE Transactions on Wireless Communications*, vol. 12, no. 11, pp. 5468–5479, November 2013.

[56] L. Zhang, Y. Xin, and Y.-C. Liang, "Weighted sum rate optimization for cognitive radio MIMO broadcast channels," *IEEE Transactions on Wireless Communications*, vol. 8, no. 6, pp. 2950–2959, June 2009.

[57] B. Parlett, *The Symmetric Eigenvalue Problem*, ser. Classics in Applied Mathematics. Society for Industrial and Applied Mathematics, 1998.

[58] M. Chiang, "Nonconvex optimization for communication networks," in *Advances in Applied Mathematics and Global Optimization*, ser. Advances in Mechanics and Mathematics, D. Y. Gao and H. D. Sherali, Eds. Springer US, 2009, vol. 17, pp. 137–196.

[59] Z. Zhang, K. C. Teh, and K. H. Li, "Semidefinite relaxation based beamforming in clustered cooperative multicell MISO systems," in *IEEE International Conference on Communications (ICC)*, June 2013, pp. 5635–5639.

[60] A. Gershman, N. Sidiropoulos, S. Shahbazpanahi, M. Bengtsson, and B. Ottersten, "Convex optimization-based beamforming," *IEEE Signal Processing Magazine,*, vol. 27, no. 3, pp. 62–75, May 2010.

[61] A. Gjendemsj, D. Gesbert, G. Oien, and S. Kiani, "Binary power control for sum rate maximization over multiple interfering links," *IEEE Transactions on Wireless Communications*, vol. 7, no. 8, pp. 3164–3173, August 2008.

[62] N. Hassan, S. Saeed, C. Yuen, and Z. Zhang, "Power control for sum rate maximization on interference channels under sum power constraint," *IEEE Transactions on Vehicular Technology*, in press 2014.

[63] S. University, *IPSOL: An Interior Point Solver for Nonconvex Optimization Problems.* Stanford University, 2009.

[64] M. Mitzenmacher and E. Upfal, *Probability and Computing: Randomized Algorithms and Probabilistic Analysis.* Cambridge University Press, 2005.

[65] M. Duran and I. Grossmann, "An outer-approximation algorithm for a class of mixed-integer nonlinear programs," *Mathematical Programming*, vol. 36, pp. 307–339, 1986.

[66] A. Geoffrion, "Generalized Benders decomposition," *Journal of Optimization Theory and Applications*, vol. 10, pp. 237–260, October 1972.

[67] M. S. K. Lau, K. V. l. Yue, and J. M. Maciejowski, "Comparison of interior point and active set methods for FPGA implementation of model predictive control," in *European Control Conference (ECC)*, August 2009, pp. 156–161.

[68] M. Rashid, M. Hossain, E. Hossain, and V. Bhargava, "Opportunistic spectrum scheduling for multiuser cognitive radio: a queueing analysis," *IEEE Transactions on Wireless Communications*, vol. 8, no. 10, pp. 5259–5269, October 2009.

[69] M. S. Aloqlah and O. S. Badarneh, "Performance analysis of dual-hop systems with decode-and-forward relays over generalized fading channels," *Journal of Communications*, vol. 8, no. 7, pp. 407–413, 2013.

[70] D. P. Foster and H. P. Young, *Regret testing: A simple payoff-based procedure for learning Nash equilibrium.* Technical Report 04-12-034, Games and Economic Behavior, Santa Fe, New Mexico, NM, USA: Santa Fe Institute, 2004.

[71] R. Serfozo, *Basics of Applied Stochastic Processes.* Berlin, Heidelberg: Springer Berlin Heidelberg, 2009.

[72] S. Meyn and R. L. Tweedie, *Markov Chains and Stochastic Stability*, 2nd ed. New York, NY, USA: Cambridge University Press, 2009.

[73] W. Hoeffding, "Probability inequalities for sums of bounded random variables," *Journal of the American Statistical Association*, vol. 58, no. 301, pp. 13–30, 1963.

[74] K. Yao, F. Lorenzelli, and C.-E. Chen, *Detection and Estimation for Communication and Radar Systems.* Cambridge University Press, 2013, cambridge Books Online.

[75] J. Proakis and M. Salehi, *Digital Communications.* McGraw-Hill International Edition, 2008.

[76] A. F. Molisch, *Wireless Communications.* Wiley-IEEE Press, Second Edition, 2011.

[77] T. Rappaport, *Wireless Communications: Principles and Practice.* Upper Saddle River, NJ, USA: Prentice Hall PTR, Second Edition, 2001.

[78] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: Optimal policies," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 8, pp. 1732–1743, 2011.

[79] D. Wu and R. Negi, "Effective capacity: a wireless link model for support of quality of service," *IEEE Transactions on Wireless Communications*, vol. 2, no. 4, pp. 630–643, July 2003.