FEASIBILITY STUDY OF INDUCTIVE HEATING COIL WITH DISTRIBUTED RESONANT CAPACITORS

by

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Abstract

Steam Assisted Gravity Drainage (SAGD) method is one of the most common methods used in the process of heavy oil recovery. However, the method is economically inefficient, inherently dependent of water availability and has some detrimental impact on the environment. Therefore the need for an efficient, economically feasible and environmental friendly solution to the existing problem is felt vehemently at the core of oil recovery industry. In the light of existing issue, electromagnetic heating technique has emerged as a promising solution and has received greater and greater attention of late. However, most of the studies (especially for inductive heating) have been limited to digital simulations and experiments primarily within the laboratory. In this thesis, preliminary investigations were carried out to confirm the feasibility of a new constructed capacitive compensated inductor coil. Design parameters of the inductor coil were computed and a test model was constructed in the lab for experimental verification of these parameters. The set up comprised a vertical drilling coil design with ferrite core inserted within, thus, creating an intensive electromagnetic field.

In this design, a distributed resonating capacitor was proposed to avoid capacitor breakdown due to high voltage. Different coil winding configurations were proposed, constructed and tested by Frequency Response Analysis (FRA) to identify the resonant frequencies. The step response tests and field tests were performed with a square wave supply and were contrasted with the expected induced magnetic field modeled in MatLab. The measurements of the designed coil parameters matched the theoretically computed parameters and the experimental setup verified the advantage of using distributed capacitors, resulting is low breakdown voltage requirement. However, double-layer winding has multiple resonant frequencies and the proposed coil designs
suffer from core losses core and winding copper losses. Thus, industrial application of this technology still requires further improvements.
Preface

The text of the thesis is original and unpublished work of the author, T. Xu.
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<th>Description</th>
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<tbody>
<tr>
<td>SAGD</td>
<td>Steam-Assisted Gravity Drainage</td>
</tr>
<tr>
<td>EM – SAGD</td>
<td>Electromagnetic Heating Assisting Steam-Assisted Gravity</td>
</tr>
<tr>
<td>IH</td>
<td>Inductive Heating</td>
</tr>
<tr>
<td>FRA</td>
<td>Frequency Response Analysis</td>
</tr>
<tr>
<td>EMF</td>
<td>Electromagnetic Force</td>
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<tr>
<td>ESR</td>
<td>Equivalent Series Resistance</td>
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<td>AC</td>
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To my family
Chapter 1: Introduction

As a fuel, petroleum stands out as one of the most important sources of energy in the world; it is also used in the manufacturing of a wide variety of materials. The huge consumption of petroleum (as much as 90 million barrels each day) and the depletion of its limited supply are of great concern [1]. Therefore, improving the manner of oil extraction in order to increase its productivity, has become a topic of urgent importance worldwide. In this thesis, the point of the work is that heating the sands in situ avoids disruptive mechanical mining of the sand in inductive method. Also, in energy terms the electrical load of inductive heating can be matched to the availability of intermittent sources such as wind.

1.1 Background of electromagnetic heating

Oil extraction is simply the removal of oil from the reservoir (oil pool). For heavy crude oil, especially for natural bitumen (the heaviest, thickest form of petroleum), thermal recovery methods are usually considered the most effective ways for lowering viscosity [2]. Steam injection, also called: Steam Assisted Gravity Drainage (SAGD) is the most common technique of heating, used to reduce extremely high viscosity. Figure 1. 1 shows the Steam-Assisted Gravity Drainage (SAGD) process. However, traditional thermal recovery methods are not consistently effective due to prohibitive heat loss from injection wells and reservoirs, steam leakage, large overburden heat loss, low reservoir injectivity and permafrost [3][4]. Also, these thermal extraction processes are considered not to be environmentally friendly. One problematic effect is the CO$_2$ emissions caused by the terrific energy consumption of the extraction process (which may include burning natural gas to heat and pressurize the reservoir to stimulate flow).
Another concern is that the great amount of water consumption necessary for steam heating methods causes damage to the hydrogeologic environment [5].

![Image of steam-assisted gravity drainage (SAGD) process](image)

**Figure 1.1. Steam-assisted gravity drainage (SAGD) process [41].**

The application of an electromagnetic heating technique in oil recovery methods has been recognized as an effective solution to improve energy efficiency and environmental impact. Recently, various electrical heating methods have been investigated, such as the low frequency resistive method and the high frequency method, as well as radio frequency range methods. With resistive heating systems, a resistor is required to provide more heat during electrical energy dissipation. One way of providing resistive heating is to use the reservoir to serve as the resistor, to realize the possible application of direct power transferred down from the grid. However, since electrodes must be used with this method, they will induce reversible electrode-to-reservoir contact loss. This will, as well, cause damaging surface site drilling impact to the environment [6]. Also, the tight vertical drilling pattern used with this method demands a limitation to a certain depth from the surface and there is a greater drilling accuracy needed for deeper
reservoirs. Using an electrical conductor as the heat source itself is another resistive method, but not an extremely effective one [7]. The concentrated heat source of this method limits the heat propagation.

Electromagnetic heating with radio frequency in the microwave range is also defined as dielectric heating, which causes friction by the vibration of molecules. Therefore, in heavy oil recovery fields, the microwave heating benefits basically from the natural reservoir water content and is generated from the friction of water dipoles at their resonant frequency. However, due to the limited skin depth of water at such high frequency, microwave heating gives only small volume effectiveness and requires a huge installation density. Due to the high loss density around the microwave source, temperature control must prevent thermal cracking and coking around the source. Also, the high reservoir power densities required need a great amount of borehole drilling, which increases the cost and environmental impact [8]. Nevertheless, in electromagnetic heating, the frequency is a compromise between fast heating and depth of penetration: the higher the frequency, the shallower the electromagnetic waves penetrate into the reservoir. Therefore, inductive heating at lower frequencies than microwave range is being investigated as a recommended solution among electromagnetic heating methods and has the potential to realize large effective areas in the contactless heating procedure. The principle and application of inductive heating in heavy oil extraction will be reviewed in more detail in Chapter 2 [9].
1.2 Thesis outline and objectives

On the basis of the inductive heating method applied by Siemens, the so-called ‘Electromagnetic Heating assisting SAGD’, EM – SAGD will be introduced in Chapter 2. A new configuration of coil design for inductive heating in a vertical direction (aided by the ferrite material) and the application of distributed resonant capacitors will be studied in this thesis.

This thesis consists of five chapters, beginning with the introduction of the research background and the motivation of this feasibility study. Chapter 2 will introduce the detailed principle of induction heating and the existing horizontal inductive heating method developed by Siemens. In Chapter 3, the new coil design formation will be proposed and the experimental findings surrounding the composed fundamental elements (i.e. capacitor and inductor) will be discussed. The frequency response analysis test is setup in Chapter 3 to analyze resonant frequency by sweeping the impedance and phases in a large frequency range. Also, two coil types will be presented and analyzed by means of the FRA test. After integrating the coil system with the power supply establishment, the theoretical result, as well as the tested one, of the electromagnetic field will be discussed in Chapter 4. More boundary conditions, such as the ferrite core saturation current and the break down voltage for the composed distributed capacitors will also be discussed. Additional alternative choices of coil design will be proposed and studied in Chapter 5. In conclusion, Chapter 6, will present more discussion on potential realms of future work.
Chapter 2: Literature Review

The Electro-Magnetic (EM) method is a non-conventionally developed technology aiming at heavy oil and in-situ bitumen production. EM is so novel that both the individual application of it in oil recovery or its use to assist the conventional heating method as a hybrid such as EM-SAGD are still at the theoretical level. Much research is tested by simulation models, laboratories and field trial all over the world, but few of them are extensively applied, especially for the inductive heating method [10]-[16].

2.1 Inductive heating principle

In inductive heating, the electric alternating current (AC) flowing through a set of conductors induces a magnetic field in the surrounding medium. The variation of the magnetic field, in turn, induces secondary currents which circulate in the medium to generate heat [17]. In the oil recovery, the medium will be the sand reservoirs around the conductors. The fundamental theory here is similar to the transformer theory, the difference being, that here substantial heat loss is productive. The prior mentioned inducing conductors work as a primary inductor coil, while the secondary load will be the oil sand around it.

Inductive heating (IH) is comprised of three basic factors: electromagnetic induction, penetration depth, and heat transfer. When the AC current flows into the coil, the primary side (inducing conductors) will produce a magnet field around the coil affecting the load (at the secondary side - oil-sands). According to Ampere’s Law, the magnetic field can be expressed as [18]

\[ \int Hdi = Ni = \mathcal{F} \]  \hspace{1cm} (2.1)

\[ \phi = \mu HA \]  \hspace{1cm} (2.2)
As Faraday’s law shows, the current generated on the surface of a conductive object has an inverse relationship with the current on the inducting circuit, as described in the equation below.

\[
E \frac{\partial \lambda}{\partial t} = N \frac{\partial \phi}{\partial t} \quad (2.3)
\]

As a result, the electric energy caused by the induced current and eddy current is converted into heat energy displayed by [18]

\[
P = \frac{E^2}{R} = i^2 R \quad (2.4)
\]

where, in this case, \( R \) is determined by the resistivity (\( \rho \)) and permeability (\( \mu \)) of the oil sand in this case; the current \( i \) is determined by the intensity of the magnetic field.

Heat energy will be in the opposite relationship to the penetration depth. Therefore it is necessary to define a penetration depth to determine the effect region around the coil. It is found that the intensity of the current will be larger if the frequency applied is higher. The induced current can be obtained:

\[
i_x = i_o e^{-x/d_o} \quad (2.5)
\]

where: \( i_x \) is the current density at the point \( x \)

\( i_o \) is the current density at \( x=0 \)

\( d_o \) is the penetration depth [18].

The penetration depth is dependent on the frequency applied, the current and the conductivity of the soil surrounding the inductor coils. Typical oil sand may have a specific resistivity of 100,…, 1000 \( \Omega \) m or 0.01S/m,…, 0.001S/m. The following equation shows the basis for calculation of
the penetration depth $d_o$ which means the distance from the surface of the solenoid deep into the oil sand

$$d_o = \sqrt{\frac{2}{\omega \sigma \mu}} = \frac{1}{\sqrt{\pi f \mu_0 \mu_r \sigma}} \quad (2.6)$$

where $\sigma$ is the conductivity [19].

To sum up, the current density at the position $x$ can be expressed as:

$$i_x = i_o e^{-x\sqrt{\pi f \mu_0 \mu_r \sigma}} \quad (2.7)$$

### 2.2 Induction heating system designed by SIEMENS

A first prototype of electromagnetic heating (inductive heating) that used an inductor cable and a high frequency converter was tested in Germany by Siemens Energy shows an excellent prospect. The technique involves the use of electromagnetic heating with horizontal wells, as shown in Figure 2.1. With Wadadar and Islam’s investigation of the possibility of electromagnetic heating with horizontal wells and the successful numerical simulation results found [17], Siemens has since concentrated their induction heating design on the single inductor loop in horizontal mode.

![Figure 2.1. Horizontal wells of inductor loop [21].](image-url)
As a source of electrical circuit, a capacitive compensated inductor is fed by the converter applied with typical frequency in the range of 10 kHz up to 200 kHz depending on reservoir conditions [16]. The inductor is installed horizontally in the reservoir as a loop as shown in Figure 2.1 and which generates an electromagnetic field in the reservoir due to electric conductivity of the water in the reservoir. The arising eddy current converts the energy into heat [21]. The inductor cable is constructed by multiple-coated litz wires twisting inside a plastic casing as displayed in Figure 2.2. At the same time, this application can effectively decrease the skin effect during conduction, which reduces the operating temperature [22].

![Inductor Cable Plastic Casing](Image)

**Figure 2.2. Inductor cable used by Siemens [20].**

As published in [16], in 2008, a small sandbox test, in the scale of 1:1000 has been successfully provided. The sandbox was determined by the mix of sand and the salt-water-solution to employ a defined conductivity close to the reservoir. The conductivity and the applied frequency have been adapted to cope with the test sand box in consideration of the penetration depth of the inductive heating. The inductor has been laid into a dry sand-filled plastic conduit (PVC tube),
which is used to heat the wet sand around the PVC tube rather than heating the inductor surface itself. The geometrical parameters are show as Figure 2.3.

![Geometrical parameters](image)

**Figure 2.3. Geometrical parameters of the small scale sand box test [16].**

The result of the scaled sand box test shows the feasibility of the electromagnetic heating method. However, Siemens’s design contains only one turn of inductor loop without any magnetic materials inside. The inductor loop can be considered as a one-turn air core inductor. Therefore, in order to increase the magnetic field, ferrite core material will be applied in this study and more turns of inductors will also be considered as discussed later in Chapter 3.2. Additionally, Siemens’s design is based on the horizontal wells, even though there are production disadvantages and problems unique to a horizontal well application in heavy oil reservoirs. The production rate declines very rapidly during the first several months of production. It is also possible that the entire length of the horizontal may not be productive. In some production systems, the limiting factor in producing the oil is the mechanical limitations of the pumping equipment, which is because of the very high viscosity of the oil [23]. In addition, horizontal drilling was much slower than vertical drilling due to the need to stop regularly and take time-consuming surveys, and due to slower progress in drilling itself (lower rate of penetration).
Especially for the wells with high inclinations, to prevent the sand influx into the well is less reliable and need higher effort, also more expensive equipment has to be mobilized to push tools down the hole.

Therefore, a vertical configuration will be investigated in this study which not only decreases the capital cost under the oil sand but also eliminates the difficulty of drilling horizontal wells. Moreover, in contrast to the traditional bulk capacitors applied with the inductor as used by Siemens, a new capacitive inductor coil will be designed as displayed in Chapter 3. The coil cable is constructed with a number of capacitors distributed inside and which formed a special cable winding outside of the ferrite core to obtain inductance. This design will distribute the voltage rating on the capacitors and thus assist in the decrease of possible breakdowns of the capacitor, as discussed in Chapter 4.3.2.
Chapter 3: Fundamental Coil design & FRA Test

This chapter introduces the characteristics of series RLC systems, the construction of the fundamental elements of coil design that comprises a capacitor and an inductor as well as theoretical calculating methods. The frequency response analysis (FRA) system was setup for resonant frequency measurement. Two types of CL coil design were integrated and the FRA results will be shown, presented and analyzed in this chapter.

3.1 Characteristic and brief analysis of series RLC system

Figure 3.1 shows a series RLC circuit. The total impedance in the circuit is given by,

\[ Z_{total} = R + Z_L + Z_C = R + j(X_L + X_C) = R + j(\omega L - \frac{1}{\omega C}) \]  (3.1)

In a series RLC circuit there is a frequency point where the inductive reactance of the inductor becomes equal to the capacitive reactance of the capacitor. In other words, \( \omega L = \frac{1}{\omega C} \), inductive reactance compensates capacitive reactance,

\[ \omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega_r = \frac{1}{\sqrt{LC}} \]  (3.2)

The frequency point at which the compensation occurs is called Resonant Frequency \( f_r = \frac{\omega_r}{2\pi} \) point and in a series RLC circuit, the resonance frequency produces Series Resonance. If the
circuit impedance is at the minimum at resonance, the circuit admittance must be at the maximum, because the circuit admittance is inversely proportional to the circuit impedance. The admittance of a series-resonant circuit is large, and this large admittance can be electrically dangerous because a very low value of resistance at resonance will cause a large amount of electric current to flow through the circuit.

The ratio of the voltage across the capacitor to the input voltage $V_{\text{step}}$ can be expressed as follows:

$$\frac{V_C}{V_{\text{step}}} = \frac{1}{1 - \omega^2 LC + j\omega RC}$$

(3.3)

The transfer function is

$$\frac{V_C(s)}{V_{\text{step}}(s)} = \frac{1}{1 + s^2 LC + sRC}$$

(3.4)

Therefore, for under damped cases ($\zeta < 1$), the capacitor voltage $V_C$ can be presented as:

$$V_C = V_{\text{step}}[1 - \frac{e^{-\zeta\omega_d t}}{\sqrt{1 - \zeta^2}} \sin(\omega_d t + \varphi)]$$

(3.5)

where $\zeta = \frac{R}{2\sqrt{CL}}$, $\omega_d = \omega_s \sqrt{1 - \zeta^2}$, $\tan \varphi = \frac{\sqrt{1 - \zeta^2}}{\zeta}$ [40].

The above formulae indicate that the output of capacitor will be charged up to 2 times of the supply voltage if the circuit series resistance is small as shown in Figure 3.2.
3.2 Capacitance

A capacitor is a device which stores electric charge. In induction heating system, capacitors are included to compensate with inductors to improve power factor. As discussed in last section, with a stepped-type power supply, the voltage of a charged capacitor can be 2 times as high as the supply voltage. At the resonant frequency, if the switching frequency of the step power supply is so fast that the power supply switches before the LC oscillation has died down, the capacitor will be damaged by overvoltage. Therefore, one of the main objectives of this thesis is to analyze the feasibility of applying distributed capacitors to the LC circuit to avoid capacitor breakdown in high voltage.
Capacitors may vary from very small, delicate trimming capacitors to large power metal-can capacitors, and the main difference between different types of capacitors are the dielectric material that is used between plates. Regular printing paper is used as the dielectric in this research for ease of material utilization for testing purpose.

### 3.2.1 Construction

Commercial types of capacitors are made from metallic foil interlaced with thin sheets of either paraffin-impregnated paper, Mylar or other materials as the dielectric. Some capacitors appear tube-like. This is because the metal foil plates are rolled into a cylindrical shape to form a small package containing the insulating dielectric material sandwiched in-between the plates. In this study, a simple, parallel-plate sandwich construction is applied. However, multiple layers of dielectric materials positioned in vertical direction will be adjusted to achieve the design goal. A lateral view configuration of an example with two layers of dielectric materials is shown in Figure 3.3 below. As demonstrated in the figure, each dielectric layer between two copper layers forms a capacitor (e.g., $C_{11}$) which have a parallel capacitor in vertical direction (e.g., $C_{12}$). The equivalent capacitance of $C_{11}$ and $C_{12}$ is $C_{1eq}$ as shown in the diagram. These equivalent capacitors are connected in series with each other to form chain which corresponds to distributed capacitance.
Physically, the capacity of storing electric charge for a given potential difference is measured as capacitance. Thus, it also could be defined as the constant charge-to-potential ratio of an isolated conductor, while the capacitance for the parallel plate type could be easily derived:

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon A}{d} = \frac{\varepsilon_o \varepsilon_r A}{d} \quad (3.6)$$

where $\varepsilon_o$ is the electric constant ($\varepsilon_o \approx 8.854 \times 10^{-12} \text{Fm}^{-1}$), $\varepsilon_r$ is the relative static permittivity (sometimes called dielectric constant) of the material between the plates, $A$ is the overlap area between conductor plates, $d$ is the thickness of the insulator.

According to definition of the capacitance, with the increase of the dielectric layers in vertical direction shown in Figure 3.3, the overlap area $A$ will be increased proportionally and thus will
enlarge the value of capacitance respectively. As the dimensions in Figure 3.3, if there are n layers of dielectric material in each capacitor and m such capacitors connected in series, the overall capacitance can be expressed:

\[ C = \frac{C_i}{m} = \frac{n\varepsilon_r\varepsilon_i A}{d} = \frac{n\varepsilon_r w\Delta l}{md} \]  

(3.7)

where \( w \) is the overlap width and \( \Delta l \) is the overlap length of a capacitor.

**Figure 3.4. Theoretical value and experimental values of capacitance plot.**

The theoretical capacitance values of one-layer dielectric cases are plotted with a blue line in Figure 3.4. In this experiment, 2cm width paper tape was used as the dielectric material having the relative static permittivity (\( \varepsilon_r \approx 2 \)). By varying the length of the paper tape up to 250cm, the
overlap area A varies respectively; thus varies the capacitance. Four cases of one-layer capacitors of varying length are measured, and the measurements are marked as red circles in Figure 3.4. It is noticed that there is some small and constant difference between the experimental and theoretical values. The diagram Figure 3.5 and calculations below show the most reasonable possibility of causing such difference is air gap between the layers.

![Diagram of Capacitor Construction with Air Gap]

**Figure 3.5. Capacitor construction with air gap.**

\[ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \]  \hspace{1cm} (3.8)

With \( C_1 = \frac{\varepsilon_1 \varepsilon_r A}{d_1} \), \( C_2 = \frac{\varepsilon_2 \varepsilon_r A}{d_2} \)

\[ \Rightarrow d_1 = \frac{\varepsilon_r \varepsilon_1 A}{C} \cdot \frac{d_2 \varepsilon_1}{\varepsilon_2} = 10\mu m \]  \hspace{1cm} (3.9)

where \( \varepsilon_1 = 1 \) for air, \( \varepsilon_2 = 2, d_2 = 0.15mm \) for paper.

It is calculated that the average thickness of air gaps is 10\( \mu m \) which is 15 times smaller than the thickness of paper insulator.

Air gaps (which enlarge the distance between the two electrodes thus decreases the capacitance) occurring between the conductor and insulator are inevitable in this experiment that utilized only hand making. In order to avoid a large accumulation error, only the one-layer dielectric capacitor was applied and will be discussed in this feasibility study.
3.2.2 Dielectric materials

A dielectric is an isolator filling placed between the conductors of a capacitor employed to maintain the separation of the conductors, as well as to increase the capacitance by the use of a dielectric constant. Dielectrics can be solid, liquid or gas. Table 3. 1 below lists several dielectrics showing their relative permittivity and dielectric strength. The values are given at under the room temperature and for static or low-frequency applied electric fields.

Table 3. 1. Relative permittivity and dielectric strength of selected materials [24][34].

<table>
<thead>
<tr>
<th>Material</th>
<th>Relative permittivity($\varepsilon_r$)</th>
<th>Dielectric strength(MV-m$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1</td>
<td>~3</td>
</tr>
<tr>
<td>Nylon</td>
<td>~3.6-4.5</td>
<td></td>
</tr>
<tr>
<td>Bakelite</td>
<td>~4.8</td>
<td>25</td>
</tr>
<tr>
<td>Mica(ruby)</td>
<td>5.4</td>
<td>200</td>
</tr>
<tr>
<td>Rubber</td>
<td>~2.4-3.0</td>
<td>25</td>
</tr>
<tr>
<td>Silicon(Si)</td>
<td>11.9</td>
<td>~30</td>
</tr>
<tr>
<td>Silicon nitride(Si3N4)</td>
<td>7.2</td>
<td>~1000</td>
</tr>
<tr>
<td>Paper</td>
<td>1.5-4</td>
<td>15</td>
</tr>
<tr>
<td>Mylar</td>
<td>3.2</td>
<td>295</td>
</tr>
</tbody>
</table>

In Table 3. 1, the right column shows the dielectric strength of the materials listed. It can be seen that the dielectric strength varies greatly among different types of materials with similar dielectric constants. However, dielectric strength changes, depending on different conditions. In the case of solids, microstructural defects, impurities, the shape of the dielectric, the manner in
which it was prepared, and its environment are all factors that may strongly affect the dielectric’s strength [24]. Theoretically, dielectric strength is the ability that a material can stand the influence applied by a high electric field, until it transfers from a good insulator to a good conductor. This transformation is called dielectric breakdown, which causes a substantial current to flow. For large plates and small separation, the electric field $E$ between the plates will be uniform over most of the area of the plates and is given by [25]:

$$E_{BR} = \frac{q_{enc}}{AE} = \frac{CV}{dAE} = \frac{V}{d} (V/m)$$

(3.10)

The maximum electric field that will cause the breakdown of a capacitor is dependent on the dielectric thickness. In this study, printing papers are used as experimental dielectric materials, which have $E_{BR} = 15MV/m$, with a thickness of 0.15mm. Thus the maximum voltage rating for this kind of capacitor is $2250V$.

$$V_{max} = E_{BR} \times d = 15000000 \times 0.15 \times 10^{-3} = 2250V$$

(3.11)

### 3.3 Inductance

Inductance is the description of the voltage induced by the current change in a conductor.

According to Lenz’s law, with a time-varying current, induces a change in the magnetic field, thus, a proportional voltage will also be induced by the conductor. However, a contributing fact is that electromagnetic force (emf) always opposes the change in current. Inductors are typically manufactured out of coils of wire. There are several types of inductor coils such as solenoid, toroid, coaxial cable, single loop of wire, as well as a pairing of parallel wires. In this specific design, solenoid will be considered.
3.3.1 Methods of inductance calculation

For infinite solenoid, the magnetic flux density B could be assumed to be constant, thus, there is a well-known way to calculate the inductance: \( L \approx \frac{\mu_0 \mu_r N^2 A}{l} \). This equation is for a current sheet, but is applicable to a wound coil if there are many turns close wound. The equation is approximate because there is leakage of the flux through the sides of the coil close to each end. If the coil is sufficiently as long as infinite, this leakage is a small percentage of the total and can be neglected. However, in this design, a finite solenoid will be considered, therefore the flux density B cannot be constant and the leakage cannot be dismissed. According to an example given by Umran S Inan and Aziz S Inan in [24], the axial B field at the ends of the solenoid will be half of that at the center. The distribution of the B field can be displayed as Figure 3. 6.

![Figure 3. 6. B field of a finite solenoid.](image)

There has been much research on the solenoid inductance calculation. In this section, the most familiar ones will be briefly discussed.

3.3.1.1 Nagaoka’s formula

Nagaoka’s formula first proposed that the inductance of a shorter solenoid as follows:
where \( N \) is the number of turns, \( A \) is the area of the coil cross-section, \( l_{coil} \) is the length of the coil, \( K_n \) is a correction factor to the formula, named as Nagaoka’s factor.

The value of \( K_n \) is commonly tabulated because of the complicity of the underlying equation [26]. According to Welsby [35], a more empirical expression has been developed for \( K_n \) factors:

\[
K_n \approx 1/ \left[ 1 + 0.45 \left( \frac{d_{coil}}{l_{coil}} \right) - 0.0005 \left( \frac{d_{coil}}{l_{coil}} \right)^2 \right]
\]  

(3.13)

where \( d_{coil} \) is the coil diameter, \( l_{coil} \) is the coil length.

This equation agrees with the tabulated values to within ±1.5% for \( \frac{l_{coil}}{d_{coil}} \in [0.05, \infty] \)

For \( \frac{l_{coil}}{d_{coil}} < 0.05 \), an exact equation is given by Grover [36],

\[
K_n = \frac{2}{\pi} \frac{l_{coil}}{d_{coil}} \left( \ln \frac{4d_{coil}}{l_{coil}} - 0.5 \right)
\]  

(3.14)

3.3.1.2 Finite ferrite core solenoid inductance calculation

The finite solenoid inductance calculations provided above only considers air-core coils, while the inductance will be increased by a large factor when a ferrite core is inserted into an air coil. In this design, a hollow ferrite core is chosen, in order to facilitate the power supply system in the further application of oil exploitation. However, with the ferrite core inserted, the widely
quoted equations for predicting the new inductance are shown to be flawed since the inductance of the solenoid cannot be exactly increased by the factor of the ferrite relative permeability. This poor situation is recognized by many authorities and for instance Lo and Lee [37] say ‘It is noted that no satisfactory formulas for the inductance of ferrite antennae are available’, and Miron [38] says ‘there is no analytical help in determining inductance’. In fact, the permeability of the ferrite rods is difficult to measure, the manufacturers often state that by measuring the toroid of the same material instead. It is shown to be in divergence with the practical value for solenoid, which greatly affects the error since the forming pressures are very different.

According to a research of the Inductance of Ferrite Rod Antennas [28], a more accurate predictions based on the magnetic reluctance is given. For finite or even short coils, the reluctance outside the coil is much greater than that inside, and thus the magnetic core inside the coil will not have as great an effect on the inductance compared to that outside. However for high permeability ferrite material, which ‘steals’ most of the flux from within the coil. Thus the air reluctance mainly dominates. Moreover, for the hollow core, even if the hollowness is filled up with the ferrite completely, it is still in series with the much more reluctant path formed by air. Therefore, the hollowness of the core would not affect the sum of the reluctance very much.

Conventionally, inductance with ferrite $L_f$ will be determined by considering of the demagnetization theory such as the equation provided by Johnson & Jasik in [39],

$$L_f = \frac{\mu_{rod} * F_L * \mu_o * N^2 * A}{l_c}$$  \hspace{1cm} (3.15)
where \( N \) is the number of turns, \( A \) is the cross-sectional area of the coil and \( l_c \) its length.

However this equation gives values for inductance which are not very accurate, and \( F_L \) is an empirical factor derived from ‘averages of experimental data’. Therefore, rather than establishing the absolute value of the inductance when the core is introduced as above, the approach is to calculate the change in inductance when the core is inserted. It takes the advantage of the retaining knowledge for air coil inductance (\( L_{air} \)) calculations as presented in last section 3.3.1.1.

In this new approach, the inductance for ferrite-cored coil (\( L_f \)) is mainly determined by the increase in terms of \( \frac{L_f}{L_{air}} \) when the ferrite is introduced. Therefore, the inductance with ferrite is given by [28]:

\[
L_f = L_{air} \left( \frac{L_f}{L_{air}} \right) .
\]  

(3.16)

The increase factor \( \frac{L_f}{L_{air}} \) can be calculated as [28]:

\[
\frac{L_f}{L_{air}} = \frac{1 + x}{\frac{1}{k} + \frac{x}{\mu_{fe}}} .
\]  

(3.17)

where

\[
x = \frac{4.46 \left( \frac{l_c}{d_c} \right)}{1 + \frac{1.17 d_c}{l_c}} ,
\]
\[ k = \left( \frac{\varphi}{\varphi_{\text{max}}} \right)^* \frac{C_{\text{anf}}}{e_0} + 2d_f \]

\[ \frac{\varphi}{\varphi_{\text{max}}} \approx \frac{1}{1 + \left( \frac{l_f}{d_f} \right)^{1.4} \frac{5\mu_f}{\ln \left( \frac{2(l_f + d_f)}{d_f} \right) - 1}} \]

\[ C_{\text{anf}} = \frac{0.5\pi e_0 (l_f - l_c)}{\ln \left( \frac{2(l_f + d_f)}{d_f} \right) - 1} \]

\[ \mu_{fe} = 1 + (\mu_f - 1) \left( \frac{d_f}{d_c} \right)^2 \]

where \( l_c \) and \( d_c \) are the length and diameter of the coil, \( l_f \) and \( d_f \) are the length and diameter of the ferrite.

To sum up, the inductance for the solenoid with ferrite core can be calculated as:

\[ L_f = \left( \frac{L_f}{L_{\text{air}}} \right)^* K_n \frac{\mu_r N^2 A}{l_{\text{coil}}} = \mu_r \frac{\mu_r N^2 A}{l_{\text{coil}}} \]  

\[ (3.18) \]

where \( \mu_r = \left( \frac{L_f}{L_{\text{air}}} \right)^* K_n \).

With this method, the inductance values with different numbers of turns are calculated and plotted as a blue curve in Figure 3.7 below. In this experiment, the length of the ferrite core is fixed as 47.5cm. The red circle dots are the experimental results, which show up with quite high accuracy.
3.4 Resonant CL coil setup

The resonant circuit consists of capacitor, inductor, and resistor/source of resistance. Two types of resonant circuits are generally used: a series resonant circuit and a parallel resonant circuit. In this section, the selected series resonate circuit will be formed after setting up the frequency response analyzer (FRA) and explaining the CL design principle.

3.4.1 FRA test setup

The Venable model 3120 frequency response analyzer (FRA) combining with the Venable V3.1 software was developed to be a complete frequency response measurement system as shown in

![Figure 3. 7. Theoretical value and experimental values of solenoid inductance.](image)
Figure 3.8. It provides the most complete and accurate phase/gain and impedance measurement by sweeping the injected current frequency from 0.01Hz to 2.2MHz.

As explained before in RLC circuit analysis, at the resonant frequency the impedance of the circuit is entirely real. This is due to the cancellation effect of inductive and capacitive reactance. Also, there will be no overall change in the phase of current. Therefore, using frequency sweep it is possible to figure out the lowest impedance point and the corresponding zero crossing of phase of the circuit. The frequency at which the system has overall lowest impedance and zero phase shift will be the resonant frequency of the system. The FRA results of the designed CL circuits will be presented and analyzed in Section 0.
3.4.2 Effect of capacitor layers on overall inductance of system

Since the distributed capacitors are formed using two layers of conductors, the winding of these capacitors on the ferrite core has an impact on the overall inductance of the system. In order to investigate the impact of these added layers, a test case was considered which is shown in Figure 3.9. This configuration was used to explore the impact of added layers of conductor (to make a distributed capacitor). Using FRA analysis the length of the capacitor wound on the toroid was altered and the respective change in the system resonant frequency was observed. Using this approach, three cases were formed and the results of these cases are discussed as follows:

◆ Case 1

A single dielectric layer capacitor was used in this case. The capacitor length, 83cm, occupies the total length of the inductor winding. The capacitor before winding on the ferrite is shown in Figure 3. 10.

![Figure 3. 9. Winding configuration.](image)

![Figure 3. 10. Single-dielectric-layer capacitor.](image)
The fundamental elements values are listed in the following table:

Table 3.2. Fundamental element values for single dielectric layer case.

<table>
<thead>
<tr>
<th>Capacitance</th>
<th>Original Inductance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical value</td>
<td>Experimental value</td>
</tr>
<tr>
<td>2.098nF</td>
<td>1.65nF</td>
</tr>
</tbody>
</table>

In this case, with the measured capacitance and inductance, the resonant frequency can be estimated to be 297.2kHz. However, the frequency response analyzer result displayed in Figure 3.11 shows that the system resonates at around 190kHz which differs from the estimated resonate frequency by 36%. The capacitance is fixed and does not change by the winding, thus, the inductance was reckoned to be 425.7uH, which is 2.44 times greater than the measured inductance. In other words, assuming the affected inductance in the circuit was doubled by the extra layer of the capacitor electrode conductor, the new resonant frequency became 210kHz. This decreased the error to 10%. Therefore, it is reasonable to propose the fact that the extra capacitor conductor layer contributes to the overall inductance of the system.
Case 2

The capacitor length and winding turns are maintained as in Case 1. However, a double dielectric layer capacitor applied in this case is shown as Figure 3. 12.

Figure 3. 12. Double-dielectric-layer capacitor.
The fundamental element values are listed in the following table:

**Table 3.3. Fundamental element values for double dielectric layer case.**

<table>
<thead>
<tr>
<th>Capacitance</th>
<th>Original Inductance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Theoretical value</td>
<td>Experimental value</td>
</tr>
<tr>
<td>4.19nF</td>
<td>2.63nF</td>
</tr>
</tbody>
</table>

With the capacitance and inductance measured, the resonant frequency was estimated to be 235.4kHz. The FRA result is shown in Figure 3.13 demonstrating that this CL circuit resonated at around 178kHz. If considering the series connection theory of the extra capacitor conductor layers, the effective inductance in this CL circuit will be 519.5uH. Thus, the new resonant frequency of 136.2kHz will be closer to the FRA result.

![Figure 3.13. Frequency response test of case 2.](image)
Case 3

In this case, the capacitor length and dielectric layer numbers are all maintained as in Case 1. However, the number of winding turns is extended to 4 on each ferrite toroid. Thereby, the capacitor length only occupies 2/3 of the whole winding as shown in Figure 3.14.

![Figure 3.14. Single-dielectric-layer capacitor with 1/3 pure copper.](image)

The fundamental element values are listed in the following table:

**Table 3.4. Fundamental element value for case3.**

<table>
<thead>
<tr>
<th>Capacitance</th>
<th>Original Inductance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical value</td>
<td>Experimental value</td>
</tr>
<tr>
<td>2.098nF</td>
<td>1.65nF</td>
</tr>
<tr>
<td>263.7uH</td>
<td>300uH</td>
</tr>
</tbody>
</table>

With the original inductance and the capacitance value, the CL circuit should resonate at 223.4kHz. However the FRA test plot in Figure 3.15 displayed that the resonant frequency will be 175kHz. Considering of the 2/3 part of the double layered copper, an extra $\frac{2}{3}L$ inductance will be connected in series to the original $L$. Thereby the effected inductance will be 500uH which matches to the resonant frequency result.
Based on three cases discussed above, the extra conductor layers from the capacitor add up to the overall inductance of the system which is explained in the following equations.

Theoretically, inductance \( L \) could be expressed as: 
\[
L = \frac{\mu_0 \mu_r A N^2}{l_{coil}},
\]
where \( A \) is the coil cross area, \( N \) is the number of turns and \( l_{coil} \) is the coil length. The extra inductors have the same \( A \) and permeability as the original inductor. This extra inductance affected by each series capacitor can be expressed as:

\[
\Delta L = \frac{\mu_0 \mu_r A (\Delta N)^2}{\Delta l_{coil}}. \tag{3.19}
\]

Since the coil length is proportional to the coil turns, thus \( \Delta l_{coil} = \frac{\Delta N}{N} l_{coil} \). Therefore, the affected inductance can be demonstrated as:
where \( n \) is the extra conductor layer numbers which are also equal to the number of dielectric layer numbers (for instance \( n=1 \) for a single dielectric layer capacitor system and \( n=2 \) for a double dielectric layer capacitor system), \( m \) is the series capacitor number.

### 3.4.3 CL coil design integration & FRA results

Considering the practical needs of heating system in oil sand exploitation, the diameter of the ferrite core is determined while the resonated frequency is also specified around 100kHz. In this design, numbers of R material uncoated ferrite toroids are selected with 107mm outer diameter and 65mm inner diameter to stack together as a hollow core. As thin as 0.06mm thickness and 20mm width copper tape used in this design as capacitor conductor plates so will also wind as an inductor. As explained in Section 0 the 0.15mm thick printing paper is selected in this study for test purposes. Applying these selected materials, the inductance can be calculated as:

\[
L = \frac{L_f}{L_{air}} \times L_{air} = \frac{\frac{L_f}{L_{air}}}{1 + 0.45 \left( \frac{d_{coil}}{l_{coil}} \right) - 0.0005 \left( \frac{d_{coil}}{l_{coil}} \right)^2} \times \frac{\mu_0 N^2 A}{l_{coil}}. 
\]

As discussed in section 0 and Eq. 3.7, only one layer of selected printing paper will be used in this specific design, thus the capacitance can be presented:

\[
C = \frac{n\varepsilon_r \varepsilon_w \Delta l}{md} = \frac{2.36 \times 10^{-9} \Delta l}{m}. 
\]
Applying the selected $d_{\text{coil}} = 107\text{mm}$, the inductance can be simplified to be:

$$L = \frac{1.1288 \times 10^{-8} N^2}{l_{\text{coil}} \left[ 1 + 0.45 \left( \frac{0.107}{l_{\text{coil}}} \right) - 0.0005 \left( \frac{0.107}{l_{\text{coil}}} \right)^2 \right]^2 \left( 1 + \frac{41.6822 l_{\text{coil}}^2}{l_{\text{coil}} + 0.12519} \right)}.$$  \hspace{1cm} (3.23)

However, as discussed in the previous section, the inductance will be affected by the series capacitor as long as the CL circuit is integrated on the ferrite core. According to Eq.3.20, the affected overall inductance will be

$$L' = L \left( 1 + \frac{m\Delta N}{N} \right) = \frac{1.1288 \times 10^{-8} N^2}{l_{\text{coil}} \left[ 1 + 0.45 \left( \frac{0.107}{l_{\text{coil}}} \right) - 0.0005 \left( \frac{0.107}{l_{\text{coil}}} \right)^2 \right]^2 \left( 1 + \frac{41.6822 l_{\text{coil}}^2}{l_{\text{coil}} + 0.12519} \right) \left( 1 + \frac{m\Delta N}{N} \right)}.$$  \hspace{1cm} (3.24)

where $\Delta N = \left[ \frac{\Delta l}{0.107\pi} \right]$.

In order to realize the required resonant frequency, the capacitance and inductance are also related as:

$$f = \frac{1}{2\pi \sqrt{CL}}.$$  \hspace{1cm} (3.25)

where $f$ is the resonant frequency.

Therefore, in this design the number of turns and series number of capacitance can be related as:

$$f = \frac{1}{2\pi \sqrt{CL'}} = \frac{1}{2\pi \sqrt{\frac{2.36 \times 10^{-9} \Delta l}{2\pi m} \left[ \frac{1.1288 \times 10^{-8} N^2}{l_{\text{coil}} \left[ 1 + 0.45 \left( \frac{0.107}{l_{\text{coil}}} \right) - 0.0005 \left( \frac{0.107}{l_{\text{coil}}} \right)^2 \right]^2 \left( 1 + \frac{41.6822 l_{\text{coil}}^2}{l_{\text{coil}} + 0.12519} \right) \left( 1 + \frac{m\Delta N}{N} \right)} \right]}}.$$  \hspace{1cm} (3.26)
In this study, two single layer capacitors with 250cm long overlap copper tape are integrated, two different coils are designed as shown in Figure 3. 16 and Figure 3. 17. They differ in their coil lengths as well as the winding layers. The long solenoid with 90cm length in Figure 3. 16 is built with single layer winding, thus two terminals of the system are apart at both ends. With the provided condition, the number of winding turns can be settled down to 34 turns and each capacitor will occupy 7 turns of winding. (i.e. \( N = 34, \Delta N = 7, m = 2 \))

![Figure 3. 16. Integrated 90cm single layer winding solenoid CL system.](image1)

![Figure 3. 17. Integrated 47.5cm double layer winding solenoid CL system.](image2)

As designed, the two 250cm overlap capacitance values built in the circuit are listed in Table 3. It is obvious to see that there is a reasonable error as small as 5% between the practical and theoretical values.
Table 3.5. Integrated capacitance.

<table>
<thead>
<tr>
<th></th>
<th>Capacitor 1</th>
<th>Capacitor 2</th>
<th>Equivalent capacitance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Practical value</td>
<td>5.7nF</td>
<td>5.5nF</td>
<td>2.79nF</td>
</tr>
<tr>
<td>Theoretical value</td>
<td>5.9nF</td>
<td>5.9nF</td>
<td>2.95nF</td>
</tr>
</tbody>
</table>

However, due to the special integration, the inductance of this solenoid could not be measured directly. It can be computed as Eq. 3.24, which turns out to be 659 μH. Thereby, the resonant frequency of this 90cm design will be estimated at 117kHz, which is very close to the FRA result shown in Figure 3.18.

Figure 3.18. FRA result of 90cm single layer winding solenoid system.

Nevertheless, considering the convenience of the power supply, a double layer winding idea with shorter size coil has been designed and integrated as shown in Figure 3.17. The main advantage of this design is that two terminals of the system are placed at the same end, which, in terms of
the power supply, is very practical and expedient. The coil length is 47.5cm, containing 18 turns on each winding layer. Since the capacitor used in this case is the same as the ones integrated in the 90cm coil system, \( \Delta N = 7, m = 2 \) the affected inductance is 647 \( \mu \) H and this CL circuit system will resonate at 118kHz. This matches with the resonant point as shown in the FRA result plot in Figure 3. 19. However, it should be noted that the double layer winding configuration causes a second resonant point with very low impedance at extremely high frequency (e.g. at 1MHz). This is because at an extremely high frequency, the skin effect will be much greater.

![Figure 3. 19. FRA result of 47.5cm double layer winding solenoid system.](image)

On the other hand, it is essential to see that the lowest impedance of the coil in both cases is as high as 100ohm during resonant frequency from the result plot in Figure 3. 18 and Figure 3. 19. The impedance analysis will be discussed in the next chapter.
Chapter 4: System Integration and Result Analysis

Using the coil systems designed in Chapter 3, an easy coil heating system was developed by supplying a DC signal converted by an H-Bridge converter. The impedance analysis discussed in this chapter was based on the step response test. The induced EMF test result will be presented and analyzed in this chapter, as well as the core saturation analysis.

4.1 Power supply establishment

In order to measure the field induced by the coil systems designed in Chapter 3 at resonant frequency, a power supply with sufficient input current is required. In this particular design, an AC input current with high frequency is an essential. However, due to the limitations of the experimental facilities, only DC power supply and small signal generators were available. For testing purposes, an H bridge circuit was built, as shown in Figure 4.1, realizing DC-AC conversion.

![Equivalent circuit of Induction Coil](image)

Figure 4.1. Equivalent circuit of the coil system.
According to the voltage rating of the FET converter board, a maximum ±50V square wave with different frequency signals were introduced to the induction coil system. The output of the H bridge circuit with 15V DC input is plotted in Figure 4. 2.

![H bridge output](image)

**Figure 4. 2. H bridge circuit output.**

**4.1.1 Step response test**

Using the circuit integrated in Figure 4. 1, a step response test with small signal at 10kHz were inserted in to the two coil type systems described in the last chapter. The step response test results are recorded in Figure 4. 3 and Figure 4. 4 respectively. In the plots, the purple waves are the current responses tested by the current probe. Though neither of the systems had many damping cycles, the single-layer winding system exhibited much smoother ringing cycles. Each ringing cycle induced by the 47.5cm double-layer winding system contained many small, noisy oscillation cycles. As the FRA test result showed in Figure 3. 19, the double-layer winding
configuration introduces an extra extreme high resonant point, which affects more harmonics. This is visible in Figure 4. 4.

![Figure 4. 3. Step response of 90cm single layer winding coil system.](image1)

![Figure 4. 4. Step response of 47.5cm double layer winding coil system.](image2)

According to the characteristics of the typical series RCL circuit, the neper frequency $\alpha$ and damped frequency $\omega_d$ are determined as following:

$$\alpha = \sqrt{\omega_0^2 - \omega_d^2} = \frac{R}{2L}.$$  \hspace{1cm} (4.1)

$$\omega_d = \frac{2\pi}{\Delta t}.$$  \hspace{1cm} (4.2)
where $R$ is the effective loss resistance [29].

From the step response plots, the time cycle may be measured, therefore the effective loss resistance at 10kHz could be evaluated as 513ohm for the 90cm coil system and 420ohm for the 47.5cm coil system. More detailed impedance analysis will be discussed in the following section.

4.1.2 Impedance analysis

The limited damping cycle shown from the step response indicates a relatively large loss in the coil system. In order to measure the pure equivalent series resistance (ESR) of the coil, the ferrite core is removed from the 90cm coil system. However, due to the extremely high resonant frequency of this air core coil which exceeds the limitation of the FRA measurement system, a step response test was used instead to test the ESR of the coil, by feeding the air coil system with a small signal at 10kHz. The result is plotted in Figure 4.5. While at this frequency, the loss of ESR was evaluated by the data derived from the plot to be 188ohm. This includes the copper loss from the coil as well as the dielectric loss from the capacitors.

**Figure 4.5. Step response of 90cm single layer winding air core coil.**
The DC resistance of a wire is \( R = \rho \frac{l}{A_w} \), where \( A_w \) is the bare cross-sectional area of the wire, \( l \) is the length of the wire, and \( \rho \) is the resistivity of the wire material. In order to minimize the ESR of the coil system, thicker copper tape was needed. Firstly, a 90 cm long, original copper tape with the thickness of 0.06mm was tested for its ESR, as shown in Figure 4.6. Another 23 times thicker copper tape with the same length and width was tested for its ESR, and is displayed in Figure 4.7. Though the ESR of the thicker copper tape is half of the thinner one at low frequency range, it is evident that the difference between these two types is negligible at high frequencies. The so called ‘skin effect’ is when the electric current flows mainly at the ‘skin’ of the conductor. The higher the frequency, the greater the skin effect will be. There is a limit depth between the outer surfaces to measure the effective resistance of a conductor, which is called the ‘skin depth’. The skin depth can be calculated by \( \delta = \frac{2\rho}{\sqrt{\omega \mu}} \). Using this method, copper will have the skin depth of 192um at the estimated resonant frequency 115kHz. In other words, the maximum effective thickness of the copper tape could be increased to 192um to reduce the ESR.
Figure 4.6. ESR measurement of thin copper tape.

Figure 4.7. ESR measurement of thick copper tape.

On the other hand, reviewing the FRA results in Figure 3.18 and Figure 3.19, the loss of resistance of both coil types of systems was almost 100ohm. This differs from the effective loss of resistance evaluated from the step response test in last section. The main difference was caused by the ferrite core loss, which was proportional to the input frequency.
4.2 Electromagnetic field test

For the induction heating design that was developed in the experiment, the main receiver was the bitumen around the heating coil. Therefore, the magnetic field produced by the designed heating coil, as well as the effective area around it, were the main concerns. As for solenoid, the static magnetic field produced by each individual coil loop is added to that of its neighbor creating a combined magnetic field. The resultant static magnetic field, with a north pole at one end and a south pole at the other, is uniform, as shown in Figure 4.8. Although, for long solenoid, the magnetic flux density \( B \) is assumed to be constantly at zero outside the coil. In this section, a more accurate method of calculating magnetic flux density for finite solenoid will be introduced which will be applied to this particular coil design. Thus the experimental result will be referenced to the estimated values.

![Figure 4.8. Magnetic field of finite solenoid [30].](image)

![Figure 4.9. Schematic diagram of a solenoid [31].](image)
4.2.1 Flux density evaluation

According to the magnetic field study published by A. K. Al-Shaikhli [31], a finite solenoid of N turns and length L, the schematic can be considered as that in Figure 4.9. Therefore, the axial and radial magnetic field strength can be described respectively [31]:

\[
B_z = \frac{\mu n i}{2\pi} \left[ \delta \cos \theta (a - r \cos \theta) \cos \theta d\theta \right] \frac{\delta +}{\delta -}. \tag{4.3}
\]

\[
B_r = -\frac{\mu n i}{2\pi} \left[ \frac{\cos \theta d\theta}{\sqrt{\delta^2 + r^2 + a^2 - 2ar \cos \theta}} \right] \frac{\delta +}{\delta -}. \tag{4.4}
\]

where \( a \) is the radius of the solenoid, \( r \) is the distance from the local point on the coil to the field testing point, \( n \) is the number of turns in unit length. If \( \delta = z - l, \delta_{\pm} = z \pm L/2 \) and \( l \) is the axial distance from the origin to the solenoid [32].

In this coil design, the effective heating area is the middle section close to the solenoid rather than the edge section, therefore only axial magnetic field is considered here. The method of evaluating the axial magnetic field strength of the designed coils is programmed on the basis of the equation provided above. The Matlab code is attached in Appendix A.

The magnetic flux density of the 90cm and 47.5cm coil systems is computed as shown in Figure 4.10. This is estimated for the case in which the coils are excited by 0.2A current and the x axis in Figure 4.10 represent the radial distance from the local point on the coil to the field testing point. In this estimation, the magnetic field strength is computed for at mid in axial direction by dictating \( z = 0 \) in the program. Note that in this plot, it is clear to see that the increase of the coil
length decreases the radial variation of the axial field. However, the shorter coil shows greater flux density in closer positions of radial distance.

![Distance Vs B field graph](image)

**Figure 4.10. Magnetic flux density evaluation result for two coil types.**

### 4.2.2 EMF measurement

While based on Faraday’s law, an electric field will be established by a changing magnetic field [33]. From Maxwell’s equations, the induced electromotive force could be expressed as

\[ e = N \frac{d\Phi}{dt} \]

For a sinusoid signal, the induced flux is represented as \( \Phi = \Phi_m \sin \omega t \). Thus, the induced EMF will be \( e = N\Phi_m \cos \omega t \), while the RMS value is normally derived as

\[ E = \frac{2\pi f N\Phi_m}{\sqrt{2}} = 4.44 f N \Phi_m = 4.44 f N B A. \quad (4.5) \]
where $N$ is the induction coil turns, $f$ is the signal frequency, $B$ is the flux density in the induction coil and $A$ is the flux cross area of the induction coil.

Hence, the magnetic field can be tested by measuring the induced electromotive force (EMF) with the setup as shown in Figure 4. 11. In this experiment, a 520 turns, air-cored coil performs as the secondary test coil and the EMF can be measured by the multimeter.

![Figure 4. 11. EMF test.](image)

The test coil is placed in the middle of the ferrite coils axially and close to the coil in radial direction initially. The induced EMF is measured by separating the test coil in the radial direction, until a very weak signal is induced. One experimental record is plotted in Figure 4. 12 as well as the estimated result computed by Matlab for the 90cm coil system. Though it appears that there is some difference between the experimental result and the estimated one, the decreasing trend seen in both of them is totally the same. Moreover, the flux density calculated by the program is based on the dot positions (in actuality, the testing coil, which has a much larger cross area than
the coil that was used in calculation). This is the cause of the difference between the experimental results and the theoretical calculation results.

Figure 4. 12. Induced EMF result of 90cm single layer winding coil.

With the same exciting current, another experiment was produced to measure the magnetic field in axial direction parallel to the solenoid. The test coil was placed close to the primary ferrite coil and then the test coil was moved in the axial direction from one terminal of the ferrite core to the other, for induction. Both the 47.5cm coil and the 90cm coil were tested while they were fed by a constant current of 0.2A.

The induced EMF result demonstrated in Figure 4. 13 shows that both of the cases induce larger and more stable fields in the middle section rather than the terminal areas. Yet the shorter the
primary coil was, the larger the maximum induced EMF was. This will be an evaluation condition for the future coil design.

![Axial Distance Vs induced EMF @ I=0.2A](image)

**Figure 4.13.** Comparison of variation in induced EMF at axial direction for two length types of coil.

### 4.3 Boundary condition

There are two sides of limitations in this design. The first one is the saturation current and the other is the breakdown voltage of the capacitors.
4.3.1 Saturation current

In electromagnetic circuit, current will typically escalate and spike when the core of a magnetic component saturates. A magnetic material is saturated when there is an increase in the magnetizing force \( H \) and there is no longer an appreciable increase in the magnetic induction \( B \). Saturation flux density \( B_s \) is the maximum flux density with respect to the number of flux lines per unit area that it can efficiently conduct and which is a constant when the core arrives at saturation point. Faraday’s law predicts that [29]

\[
\nu(t) = NA \frac{dB_s}{dt} = 0 .
\] (4.6)

Therefore, if the core saturates, the inductance of the component decreases and all the other components of the circuit will fail due to the short circuit. The saturation current \( I_{sat} \) can be derived via Ampere’s law

\[
I_{sat} = \frac{B_l}{\mu N} .
\] (4.7)

According to the manufacturer’s data sheet on the ferrite material, the saturation flux density of the ferrite core used in this design is 0.47 \( T \). This is assuming the solenoid coil length is always the same as the ferrite core length. Following the calculation, the saturation current model is established in Matlab, as demonstrated in Figure 4.14. The program code is attached in Appendix B. The 3D plot visually implies that the saturation current is much more sensitive to the number of coil turns in this particular design. The saturation current is 326 \( A \) for the 47.5cm coil system and for the 90cm coil system it is 307 \( A \).
4.3.2 Breakdown voltage

In order to investigate the impact of distributed capacitors on the breaking voltage of capacitors a test circuit was designed. A 47.5cm long coil system which can be shown in a simplified fashion in the schematic circuit below, in Figure 4. 15 was constructed which contained two distributed capacitors in series. Supplying a square wave signal with different voltage magnitude, the voltage across the capacitance C1 and C2, as well as the voltage across the second layer winding solenoid V2 are recorded in Table 4. 1. From the measurement of the capacitance voltages, it is concluded that the distributed capacitance splits the voltage breakdown rating which helps to decrease the chances of breakdown of the capacitor.

Figure 4. 14. Saturation estimation versus solenoid length & winding turns.
Figure 4.15. Schematic of the test circuit for breakdown voltage analysis.

Table 4.1. Voltage recorded across distributed capacitors.

<table>
<thead>
<tr>
<th>Vin(V)</th>
<th>Vc1(V)</th>
<th>Vc2(V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.6</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>27.8</td>
<td>13.4</td>
<td>10.9</td>
</tr>
<tr>
<td>44.2</td>
<td>18.5</td>
<td>16.1</td>
</tr>
</tbody>
</table>
Chapter 5: Alternative Study

In this chapter, two alternative configurations are proposed. The preliminary test results show that there will be big room for improvement in the future.

5.1 Ferrite core increase

In the previous design, the coil systems are all assumed to be the same length as the ferrite core. While as an alternative study, more ferrite toroids will be added on to the 47.5CM double layer winding system which would make the ferrite core longer than the coil winding, as shown in Figure 5.1. The EMF test is processed with a 0.2 A exciting current and detected at the midposition axially in relation to the solenoid core. Simultaneously, at each terminal, one ferrite toroid was added on to either end of the coil, every time, until 5 extra toroids were added on both sides of the coil. The induced EMF is recorded in Figure 5.2, demonstrating each change introduced by the extra ferrite core. The induced EMF was improved (enlarged by 11%), but in comparison with the ferrite core (52% increase in length) the improvement was minimal.

![Figure 5.1. Configuration of new coil.](image-url)
From another aspect, the FRA test was applied on the new coil, whose ferrite core was elongated by 52%. The frequency swept of the impedance result in Figure 5. 3 shows that the resonant frequency was decreased to 80kHz, while the ferrite core loss was increased 25% consequently. Therefore, in future work increasing ferrite core length will be a trade-off.

Figure 5. 2. EMF by adding ferrite length.
Figure 5.3. FRA test of the new coil.

### 5.2 Intensive coil design

Instead of the coil winding axially on a long finite ferrite core, a new configuration can be formed with intensive radial windings on only one ferrite toroid, as shown in Figure 5.4 (a). Using a long single-dielectric-layer capacitor wind on a toroid radially, with numbers of turns, a capacitive inductor coil is constituted. However, in order to insulate the inner copper with the outer copper during the winding, one more layer of insulator is overlapped outside of the outer copper. Therefore, an extra capacitor layer is structured between each winding turn and thus, the overall capacitance is doubled after winding onto the toroid. The coil construction can also be considered as infinite small lump circuit elements, as shown in Figure 5.4 (b). According to the capacitance calculation method discussed in Chapter 3, the capacitance for this configuration can be presented as:

\[
C = 2\pi \varepsilon_0 \varepsilon_r \frac{w \Delta l}{d}.
\]  

(5.1)

Since the overlap length of the capacitor, in this type of winding is the same as the winding, thus
\[
\Delta l = 0.107 \pi N. \tag{5.2}
\]

where \( N \) is the number of winding turns.

Figure 5.4. (a) Intensive coil winding; (b) Equivalent circuit of partial intensive winding.

On the other hand, the inductance as expressed in Eq3.23:

\[
L = \frac{1.1288 \times 10^{-8} N^2}{l_{coil} \left[ 1 + 0.45 \left( \frac{0.107}{l_{coil}} \right) - 0.0005 \left( \frac{0.107}{l_{coil}} \right)^2 \right]^2 \left( 1 + \frac{41.6822 l_{coil}^2}{l_{coil}^2 + 0.12519} \right)}.
\]
Since the coil length is the same as one toroid length, therefore, $l_{coil} = 2.5\text{cm}$. Based on the expressions of the capacitance and inductance, the number of winding turns can be found using the desired resonant frequency through Eq.3.25.

Two cases of different winding turns are designed: one is comprised of 7 turns, the other is comprised of 12 turns. The FRA test results are plotted below in Figure 5. 5 and Figure 5. 6 respectively.

![Figure 5. 5. FRA result of intensive coil design with N=7.](image)

*Figure 5. 5. FRA result of intensive coil design with N=7.*
Using Eq. 3.23, Eq. 5.1 and Eq. 3.25, the theoretical values of inductance, capacitance and resonant frequency can be obtained. The designing parameters are summed up as presented in Table 5.1.

**Table 5.1. Intensive coil design parameters.**

<table>
<thead>
<tr>
<th></th>
<th>N=7</th>
<th>N=12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical value</td>
<td>Experimental value</td>
<td>Theoretical value</td>
</tr>
<tr>
<td>C=11.1nF</td>
<td>C=10.1nF</td>
<td>C=19nF</td>
</tr>
<tr>
<td>L=12.4uH</td>
<td>L=13.9uH</td>
<td>L=36.5uH</td>
</tr>
<tr>
<td>f=428kHz</td>
<td>f=450kHz</td>
<td>f=190kHz</td>
</tr>
</tbody>
</table>

According to the FRA results demonstrated above, it is clear that the desired resonant frequency can be achieved within a small margin of error for intensive coil design. Also, it deserves notice that the impedance of the coil system will be smaller with more winding turns. This is because
the coil system can be equivalent to a parallel circuit, as demonstrated in Figure 5. 4 (b). The more turns, the more parallel branches there will be. Additionally, the step response of the two coil designs recorded in Figure 5. 7 shows a better damping cycle with more winding turns.

![Step response of two intensive coils](image_url)

**Figure 5. 7. Step response of intensive coils of N=7 & N=12.**
Chapter 6: Conclusion

In this chapter, a summary of all contributions of work done in this study is demonstrated and the possible future work is discussed.

6.1 Summary

Reviewing the current oil extraction methods, electromagnetic heating is taken into consideration in this present focus. The advantages and disadvantages of different types of electromagnetic heating applications in oil recovery were discussed; the inductive heating method was proposed to be the most prospective one. In Chapter 2, the principle of inductive heating was presented with an inductive heating study done by Siemens; the case was successful to a certain extent, and new ideas were proposed for its insufficiencies. A new configuration of the capacitive inductor coil was proposed in Chapter 2 as well. In Chapter 3, the fundamental elements constituting the coil were studied. Details of the capacitor design and calculations were presented. The inductance calculation for the finite solenoid with a hollow ferrite core was demonstrated in Chapter 3. Additionally, a new finding of the extra inductance increase that was made by the integrated capacitor was discussed.

Presenting a practical application, an H-bridge converter was designed to provide the required power supply. After the integration of two types of winding coils, step response tests, as well as the coil impedance analysis were performed. The effective heating area was studied by investigating the induced electromagnetic field around the coil. Subsequently, the magnetic flux density produced by the coil was calculated theoretically, and also verified by comparing it with the measurable EMF. Moreover, boundary conditions such as the saturation current for the ferrite
core and the distributed capacitor breakdown voltage were studied. The distributed capacitor’s voltage measurement showed the proposed benefit of the split of voltage ratings.

During the heating progress, the conductivity of the oil sand decreases with the temperature rising. According to Eq. 2.6, the penetration depth increases thus the heat will be propagated from the soil surrounding the inductor coils to further areas. For typical oil sand which has a specific resistivity of 100, …, 1000 \( \Omega \) m or 0.01S/m, …, 0.001S/m, the penetration depth of the heat will be in the range of 10m to 100m.

As a conclusion, the proposed coil design has shown the feasibility of vertical solution of inductor loops. However, the noisy oscillation cycles in the step response of the double-layer wound, 47.5cm-length ferrite cored coil showed a utilitarian concern. Single-layer winding configuration could be used because of its smoothing damping result in the step response as well as its specific resonant frequency according to the coil length. Therefore in order to demonstrate a capacitive coil which resonates around 100kHz, the single-layer winding coil can be designed as the parameters concluded in Table 6. 1. The parameters can be varying with different requirements to the coil length, capacitor conditions as well as winding turns.

**Table 6. 1. 90cm single-layer coil design parameters.**

<table>
<thead>
<tr>
<th>Coil/Core OD</th>
<th>Coil/Core ID</th>
<th>Coil/Core length</th>
<th>Series Capacitor Number</th>
<th>Capacitor overlap length</th>
<th>Dielectric thickness</th>
<th>Dielectric layers</th>
<th>Copper width</th>
<th>Winding turns</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.7cm</td>
<td>6.5cm</td>
<td>90 cm</td>
<td>2</td>
<td>250 cm/per capacitor</td>
<td>0.15mm</td>
<td>1</td>
<td>2 cm</td>
<td>34</td>
</tr>
</tbody>
</table>
6.2 Future work

In this thesis, there is a precondition that the coil design was based on the premise that the coil was the same length as the ferrite core. In the alternative study presented in Chapter 5.1, the investigation of the design that had a longer ferrite core length than the coil, showed improvement of the induced field. Though the increase was limited, it showed a positive outcome. In the future, shorter coils can be wound on a similarly long ferrite core, section by section, and each section of the coil can be separated with uniform distances. Individual inverters for each coil section can be designed to enlarge the effective heating area. Alternatively, another winding configuration as proposed in Chapter 5.2 may be considered. Rather than winding with a traditional finite solenoid, intensive coil winding can be applied on each ferrite toroid, forming a long ferrite core. In the future, each coil on the core can be provided with individual inverters and this construction may be tested to see whether or not it can induce a larger magnetic field in the radial direction. Moreover the boundary conditions of the integrated, long, intensive coil design should also be studied in the future to eliminate more of the risks that may destroy the heating system.

For testing purposes, the dielectric material used in this study was simple printing paper which may not have provided good permittivity. In future practical application, higher permittivity with much thinner materials, such as mylar (which has greater dielectric strength than paper and also better performance at high temperatures) will be considered. Also, the manual work of making capacitors and coil cables will be replaced by more professional manufacturing methods, to reduce the chance of human error and more effectively achieve multiple-layered dielectric capacitors.
Bibliography


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[41] “Control and optimization of steam injection for Steam-Assisted Gravity Drainage
Appendices

Appendix A: MatLab code of calculating electromagnetic density

```matlab
%======================================================================
%Program 1
%This program doest the job of calculating the electromagnetic density B.
%
%Written by T. Xu, on Jan. 27, 2015.
%Modified by T. Xu, on Jan. 27, 2015.
%======================================================================

clear all;
L = 0.9; %coil length
N = 34; %number of turns
n = N/L;
a = 0.107/2; %radius of the coil
i = 0.2; %input current amplitude
u0 = 4*pi*1e-7; %permeability of air
ur = 32; %relative permeability
z = 0; %center position
plus_s = z+L/2;
minus_s = z-L/2;
d_theta = 0.05;
theta = [0:d_theta:pi];
dr = 0.0001;
index = 1;

for r = 0.117:dr:0.55
    plus_Bz = a*n*i*u0*ur/(2*pi)*sum(plus_s.*(a-r.*cos(theta))*d_theta./(r^2+a^2-2*a*r.*cos(theta))./sqrt(plus_s^2+r^2+a^2-2*a*r.*cos(theta))));
    minus_Bz = a*n*i*u0*ur/(2*pi)*sum(minus_s.*(a-r.*cos(theta))*d_theta./(r^2+a^2-2*a*r.*cos(theta))./sqrt(minus_s^2+r^2+a^2-2*a*r.*cos(theta))));
    Bz(index) = plus_Bz - minus_Bz;
    index = index+1;
end

flux = -l*Bz*(pi*(107/2000)^2); %flux
emfr = flux*4.44*520*117000; %emf

figure(1)
r = [0.117:dr:0.55];
pplot(r,emfr,'r');
xlabel('Distance in m')
ylabel('EMF in V')
grid on
title('Distance Vs induced EMF @ I=0.2A')
```

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figure (2)
plot(r,-Bz,'r');
xlabel('Distance in m')
ylabel('B in T')
grid on
title('Distance Vs B field')
hold on
Appendix B: MatLab code of calculating saturation current

```matlab
%==================================
%Program 2: Plot of the saturation current I versus N & l.
%This program does the job of calculating the saturation current.
%
%Written by T. Xu, on Jan.3, 2015.
%Modified by T. Xu, on Jan.3, 2015.
%======================================================================

clear all;
clc;
d_coil=0.107;%diameter of the solenoid

for l_coil=2.5:2.5:100;%length of the solenoid
l=l_coil./100;
d_l=d_coil./l;
kn((l_coil-2.5)/2.5+1)=1./(1+0.45*d_l-0.0005*(d_l).^2);
l_d=1./d_l;
Lf_Lair((l_coil-2.5)/2.5+1)=1+(4.46*l_d./(1+1.17*d_coil./l));
ur((l_coil-2.5)/2.5+1)=Lf_Lair((l_coil-2.5)/2.5+1).*kn((l_coil-2.5)/2.5+1);%real permeability

for N=12:1:80;%number of turns
B=0.47;%saturation flux density
i((l_coil-2.5)/2.5+1, N-11)=B.*l./(N.*ur((l_coil-2.5)/2.5+1).*4*3.14*10^-7);%saturation current
end
end
l_coil=[2.5:2.5:100];
N=[12:1:80];
[xx, yy]=meshgrid(N, l_coil);
figure(1);
mesh(xx, yy, i);
xlabel('Number of Turns');
ylabel('Solenoid length(cm)');
zlabel('Saturation current(A)');
```