ELECTRO-HYDRODYNAMIC MICRO PRISM WITH APPLICATIONS TO AUTOMULTISCOPIC DISPLAYS

by

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B.A.Sc., The University of British Columbia, 2011

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF APPLIED SCIENCE in THE FACULTY OF GRADUATE AND POSTDOCTORAL STUDIES (Mechanical Engineering)

THE UNIVERSITY OF BRITISH COLUMBIA

(Vancouver)

April 2015

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Abstract

The recent trend in display technology is to provide the viewer with an artificial three-dimensional (3D) experience using lenses or aides, however the number of viewers and resolution of the display is limited. To remedy these problems, an array of prisms can be placed over the display redirecting the projected light at specific angles in a time-multiplexed fashion and at full resolution. The difficulty in this approach is that the angle of the prism needs to be adjustable with accurate and fast control.

This thesis presents the theory, development, and analysis of a novel adjustable prism coined an electro-hydrodynamic micro prism (EHMP). An EHMP consists of an elongated conducive water droplet with pinned contact lines using hydrophobic surface patterning. By applying a voltage between the droplet and an offset electrode above it, the shape of the droplet is deformed into a triangular prism where the angle of the prism is dictated by the strength of the applied voltage.

A numerical model of an EHMP was developed using finite element analysis and smoothed particle hydrodynamics to model the electro-hydrodynamics of the system. The numerical model was qualitatively verified using the collapsing square and oscillating droplet tests, and then used to predict an operating voltage range of 400 – 550 V for a 200 µm droplet, and that the leading edge of the electrode dictates the final deformation of the drop.

To fabricate a prototype EHMP, a microcontact printing technique was developed to pattern polytetrafluoroethylene nanoparticles onto an indium tin oxide coated glass slide creating the hydrophobic patterning. A prototype 1 mm diameter prototype EHMP was fabricated and tested in the 1.5 – 2 kV range. It was found that there was minimal droplet deformation before
failure due to electrospray formation. Though not useful for 3D displays, the results from these large-scale experiments experimentally validate the numerical model. Model simulations showed ideal EHMP deformations can occur under the right conditions, however its performance under current conditions is limited due to dielectric breakdown failure and a fill factor of only 0.66 thus proving not to be a practical solution to automultiscopic displays.
Preface

The contents of this thesis represent work done solely by myself with the supervision of Dr. Stoeber. There was no collaboration with third parties, and none of the work has been published previously.
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Acknowledgements

I would like to pay special thanks to Dr. Stoeber for his supervision and patience throughout my project. His incredible eye for detail and ability to provide help regardless of my approach to a problem allowed for innovative solutions to a difficult project.

In addition, I would like to thank all of my labs mates who not only provided me with training and help operating equipment, but also thoughtful questions and feedback throughout our various group meetings.

Finally, I would like to thank my parents, family, and friends who without their continual support I could not have achieved what I have so far.

Thank you all.
Dedication

I would like to dedicate my work to my parents for their patience and support. I could not be the person I am today without you two, and I could not be more grateful.
Chapter 1: Introduction

1.1 Motivation

Three-dimensional (3D) displays use some combination of stereoscopic viewing and motion parallax to convince the brain that the planar image being displayed is in actuality a 3D scene [1]. Stereoscopic viewing is where each eye receives a different perspective of the same scene (figure 1.1a).

![Figure 1.1 Stereoscopic vs. Motion Parallax Viewing. a) Stereoscopic view where each eye receives separate views of the same scene, b) motion parallax where the user experiences different views of the same scene from different viewing areas.](image)

This provides the viewer with false depth perception creating a 3D experience for the viewer.

Conversely, motion parallax is the ability to experience changing perspectives of a scene as the viewer moves around the same scene (figure 1.1b). An important artifact of motion parallax is that objects closer to the viewer move faster than further away objects as the viewer traverses the scene [1]. Using these techniques 3D displays produce a more realistic rendering of a scene than a standard 2D display. This realistic rendering has been the basis for a push in the movie and personal entertainment industries to develop the best solution to creating a realistic 3D viewing experience. Though entertainment is the driving force behind developing 3D display technology, there are applications in the CAD design and medical imaging industries as well.
The simplest 3D displays only take advantage of stereoscopic viewing. One of the original methods of producing this effect is through color anaglyph [2]. Typically two images were projected from a single screen where each image was either partially or fully a specific color. The viewer would then wear a pair of glasses that had color filters to filter out one of the projected images. In this way, each eye would receive separate images creating the stereoscopic effect. However, one downfall of this is that each eye is receiving different color ranges [2].

To improve on the color anaglyph method, Infitec developed a method that rather than having two separate color filters, one for each lens in the glasses, each lens has three narrow bandwidth filters for each of the primary colors [3]. Each of the primary colors has a bandwidth of several hundred nanometers, so by choosing two narrow bandwidth filters, one for each eye, for each of the primary colors then two separate images can reach each eye while maintaining full color ranges. Though there are still slight color discrepancies between each eye, they are significantly less than traditional color anaglyph methods [3].

Another technique for producing stereoscopic imaging is to implement time sequential imaging [2]. In this technique, rather than displaying two images at the same time and later filtering out one image at the eye, the display switches between two different views sequentially. The glasses then provide some filter mechanism to allow only one view for each eye. The simplest implementation to conceptualize is to have fast electronic shutters on the lens of the glasses for the viewer [4]. These shutters are then synchronized with the alternating views from the display. If synchronized properly, only one shutter will be open at a time corresponding to a single view being projected from the display.

The most popular technique for stereoscopic viewing is a time sequential technique called RealD that is used in cinema [5]. As mentioned previously, the projector alternates between two
different views. However, rather than having active shutters to filter the images at the glasses, RealD uses polarization. By using polarizing optics in front of the projector, each of the projected views is either left or right circular polarized. The user then wears glasses with polarization filters to ensure only one of the polarized images reaches each eye [5].

The disadvantage of all of the stereoscopic techniques mentioned so far is that the viewer is required to wear a pair of glasses to ensure only one of the stereo images reaches a particular eye. To overcome this drawback, various techniques have been devised to create auto-stereoscopic or automultiscopic displays [2]. These displays have been coined ‘auto’ because they provide stereoscopic effects without the need for gasses or other filters. The general operation for these displays is that different views are projected at different angles from the display (figure 1.2).

![Automultiscopic display general operation](image)

*Figure 1.2 Automultiscopic display general operation. Multiple cameras collect different viewing angles from the same scene and automultiscopic display projects those multiple views in their respective angles.*

If the viewer stands in the correct zone, then separate images will reach each of the eyes providing the auto-stereoscopic effect. In addition, if more views are projected in the automultiscopic displays then the user can experience motion parallax.

Two common methods for achieving both auto-stereoscopic and automultiscopic displays are lenticular lenses and parallax barriers (figure 1.3a/b respectively) [6]. Lenticular lenses are
cylindrical lenses that are usually aligned in repeated arrays. Traditionally, to achieve auto-
stereoscopic viewing these arrays were aligned vertically over a display with each individual half
cylinder covering two pixels (figure 3a). The cylinder would redirect light from each of the two
pixels in separate directions. If positioned correctly, both horizontally and a specific distance
from the screen, the viewer would have the left projected pixels in one eye, and the right
projected pixels in the other. In contrast, parallax barriers are essentially transparent slits that are
aligned between the pixels. The slits are positioned so that light from a single pixel is blocked in
one direction while unobstructed in the other. In doing so this achieves the same stereoscopic
effect as the lenticular lens arrays described above [6].

![Figure 1.3 Commercial techniques for automultiscopic displays. a) Lenticular lens b) Parallax barrier [7]](image)

Both of these techniques have also been extended to automultiscopic displays by slanting
the orientation of the barriers or lenses with respect to the pixels [2]. In doing so, multiple views
are projected from a single display. For both techniques, slanting of the orientation has allowed
commercial products to have on the order of seven to nine different projected views [2]. The
newest development for lenticular lenses is to increase the width of the lenses producing a
prototype with 60 views [8], or to mechanically oscillate the lenses allowing all of the pixels to
be projected in each direction [9]. For parallax barriers, the newest developments have been
using double screens to create a dynamic parallax barrier display that is capable of projecting
every pixel on the screen in each viewing direction [10].
Both lenticular lens and parallax barriers have considerable drawbacks that hinder the 3D effect. In both of these technologies, there are very specific viewing positions where stereoscopic/multiscopic viewing is possible [2]. If the viewer is not in one of those positions the view in each eye is a mix of the views. In addition, only a portion of the pixels are being projected for each unique view so effective resolution of the display decreases with increasing number of views. There are ways to overcome this limitation, however advanced mechanical systems are required [9]. Finally, in the case of the parallax barriers the brightness of the display is greatly reduced since the barriers block transmitted light [2].

Though there are other techniques for producing automultiscopic displays such as head tracking [11], and multiple projectors [12], the ideal automultiscopic display would be a single full resolution screen that had no limitations on number of viewers and their positions.

1.2 Device Concept

To overcome the limitations of the techniques described above, this thesis proposes the technique outlined in figure 1.4. Half cylinder conductive water droplets are confined in an array on a surface using hydrophobic/hydrophilic patterning surrounded by oil (figure 1.4a). These droplets are grounded via electrodes on the substrate surface, while electrodes positioned above the droplets provide a method to deform the droplets into prisms (figure 1.4b). If there was no hydrophobic patterning, when a voltage is applied between the droplet and an electrode the droplet would slide along the substrate surface and center itself underneath the electrode to minimize the volume, and thus the energy, between the droplet and the electrode. However, due to the hydrophobic patterning the contact lines of the droplet are pinned and it cannot slide along the substrate surface. In this case, the droplet will deform into a shape similar to a triangular prism since it is still drawn towards the electrode and minimizing the volume/energy between
them (figure 1.4b). Using different ratios of voltages between the two electrodes different prism shapes can be achieved.

![Diagram of droplet deformation due to off center electrodes, confined half cylinder droplets on a surface using hydrophobic patterning, and array of prisms aligned vertically over display.](image)

Figure 1.4 Automultiscopic display concept. a) Droplet deformation due to off center electrodes. Blue: $V_1=V_2=0$ Green: $V_1=V_2>0$ Red: $V_1>0$ and $V_2=0$ b) Confined half cylinder droplets (blue) on a surface using hydrophobic patterning c) Array of prisms aligned vertically over display.

To create the automultiscopic display, the array of droplets would be positioned vertically over pixels in a display (figure 1.4c). The display would implement time sequential imaging for however many views are available in the video stream. The droplets would be synchronized with this sequential imaging so that the prisms would be actuated to redirect the light from the display in the direction corresponding to whatever view is being displayed. The advantages of this technique for an automultiscopic display are that it maintains full resolution while providing both stereoscopic as well as motion parallax cues for any number of viewers regardless of viewer...
position. In addition, since the system is analog the number of views is limited by the refresh rate of the screen, not the number of angles the prisms can form. In addition, as is the case with all automultiscopic displays there is a danger of cross talk between different projected views limiting the distance the viewer can be from the display. However, increasing the angular resolution of the projected images as well ensuring the display light is collimated can mitigate this. Therefore, the angular resolution, and thus the region of possible viewing, is limited solely by the number of camera angles and refresh rate of the screen.

To develop these arrays, coined electro-hydrodynamic microprism (EHMP) arrays, a numerical model of the electro-hydrodynamic system has been developed as well as a method to accurately pattern hydrophobic surfaces.

1.3 Background

1.3.1 Numerical Modeling

1.3.1.1 Numerical Modeling of Electro-Hydrodynamics

Having an accurate numerical model of the system allows for the identification of general trends between the influence of electrode and droplet geometries on the final prism shapes as well as confirm the driving forces of the physical system. To achieve this, the underlying physics of the electro-hydrodynamics needed to be investigated. Due to the length scale of the system the Reynolds number of the flow puts the hydrodynamics in the Stokes regime. In this regime, inertial forces are dominated by viscous forces and can be neglected allowing for the physics of the system to be simplified. In this case, the hydrodynamics reduce to a flow governed by pressure, surface tension, and viscous forces. Within microfluidics there is an area of research, electrowetting, that focuses on the electro-hydrodynamics within the Stokes regime. Initially, electrowetting was first described by Lippmann who was researching the effect of applying a
voltage across mercury electrolyte interfaces [13]. The difficulty with Lippmann’s approach is that the conductive fluids were directly exposed to electrodes so that the model he developed only worked for the few millivolts before current begins to flow [13].

To overcome this limitation, researchers have developed electrowetting on dielectrics (EWOD) that places a thin dielectric layer between the conductive fluid and electrode to prevent this current [13]. With applications to precision optics and digital microfluidics, precise translational control of microliter droplets using electrowetting, it was essential for an accurate model of electrowetting to be developed so that these applications could be optimized analytically rather than experimentally [13]. When analyzing an EWOD system, one can model it by assuming that the fluid is a perfect conductor so that the electrical energy of the system is determined by the capacitance between the fluid and the electrode [13]-[16]. By using this simplification the Young-Lippmann relationship can be used to describe the contact angle $\theta$ found in an EWOD system [13]-[16] (figure 1.5) as

$$\cos \theta = \cos \theta_Y + \frac{\varepsilon_r \varepsilon_o U^2}{2d}$$

where $\theta$ is the final contact angle, $\theta_Y$ is the zero voltage contact angle, $\varepsilon_r$ is the relative permittivity of the dielectric, $d$ is the thickness of the dielectric, and $U$ is the voltage applied between the droplet and electrode. As can be seen, this equation signifies that there is a local contact angle change that depends on the applied voltage. This implies that the surface energies themselves are voltage dependent [13].

Though this model accurately depicts the electrowetting phenomenon at low voltages it drastically looses this accuracy at higher voltages [13],[17]. This simple model shows that theoretically the contact angle will continue to decrease to zero as long as the voltage continues
to increase. However, experimentally it has been shown that there is a contact angle saturation after a certain threshold voltage [13],[17].

Figure 1.5 Basic EWOD system for a sessile droplet. The solid line represents an applied voltage $U=0$, and the dash line represents $U>0$.

After this threshold voltage the contact angle will not change regardless of whether or not the voltage is increased. Since the Young-Lippmann approach to electrowetting was not able to accurately predict the contact angle saturation researchers looked for a more accurate model of electrowetting. One approach is the electro-mechanical model using the Maxwell stress tensor [17]-[19]. This model focuses on the fact that a perfectly conductive fluid in an electric field will experience a local pressure across the fluid interface that is proportional to the local external electric field leading to [18]

$$\sigma \kappa + \frac{\varepsilon_r \varepsilon_0 E^2}{2} + \Delta p = 0,$$

where $\sigma$ is the surface energy of the fluid, $\kappa$ is the local surface curvature, $E$ is the electric field strength, and $\Delta p$ is the local pressure drop across the interface. Essentially this equation states that the Laplace pressure balances an electric pressure that is derived from the Maxwell stress tensor.
In this model, it states that the local curvature can be distorted by electric fields, however there is no change in surface energies due to the applied voltage [13],[17]. Therefore, for EWOD systems there is only an apparent contact angle change not a local contact angle change as described in the Young-Lippmann equation. This arises due to fringe field effects near the contact line that create non-negligible electric fields. These fields become balanced by an increase of curvature around the contact line that leads to an apparent increase in wetting of the liquid on the substrate surface. Mugele and Buehrle, [19], performed a series of experiments where they were able to directly confirm that there is an apparent contact angle change demonstrating that indeed this is the more accurate of the models. Though the Maxwell stress approach to modeling electrowetting seems to be the more accurate model of the physics, it still is not able to predict or explain the contact angle saturation. Though there have been some attempts to explain the contact angle saturation there has yet to be any conclusive models [17].

Using both the Young-Lippmann and Maxwell stress formalizations researchers have created various numerical modeling techniques for EWOD systems. There have been a number of numerical models using only the Young-Lippmann equation to model digital microfluidics and drop dynamics in those systems [20]-[25]. The simplest model used COMSOL Multiphysics [20] and Mathematica [21] to calculate the voltage across and capacitance of a drop in an EWOD device. After obtaining the voltage and capacitance, the Young Laplace equation was used to determine the direction and force on the drop. For more advanced modeling of the digital microfluidic systems, finite element [22], volume of fluid [23], and level set [24],[25] techniques have been implemented. In all these cases, the respective modeling techniques were used to model the fluid dynamics and electric field distribution of a 3D droplet. The electric field distribution calculated for each iteration was used to dictate the contact angle of the next iteration.
using a modified version of the Young-Lippmann equation. The draw back of these modeling techniques is that they are strictly for drop dynamics based on contact angle change and contact line travel. In the case of modeling EHMPs, the contact lines are pinned so contact angles will not be the main driving force behind the deformations.

For the modeling of EHMPs it is important to incorporate the interaction of the electric field along the entire fluid interface thus the Maxwell stress technique is more applicable. Using the Maxwell stress tensor formalism various models have been developed to incorporate this interaction. Initial research into the Maxwell stress model was done by numerically investigating the interface profile around the contact line of a droplet in an EWOD system in hopes of explaining contact angle saturation [26]. A differential equation was solved using numerical techniques that balanced the Laplace pressure and the Maxwell stress. Though the results were not conclusive in respect to contact angle saturation they demonstrated the idea that there was only an apparent contact angle change [26]. In similar fashion, a modified axisymmetric drop shape analysis was used to solve the differential equations to determine the interface of a standard sessile droplet electrowetting experiment [27]. The draw back of using direct numerical solutions to the differential equations is that there are limits to either have axis symmetry or only a partial fluid interface.

To implement the Maxwell stress model on a full droplet of arbitrary shape there have been attempts using volume of fluid [28] and level set [29] techniques. The volume of fluid technique was used to model the time variations in movement of a 2D digital microfluidic drop. Though capable of producing accurate results of droplet movement, the technique was limited in not being able to pin contact lines [28]. The level set technique was used to model the retraction of oil in an electrowetting display. Though the model was tuned to match experimental results,
the technique takes advantage of the smoothed interface between the two liquid phases when
calculating the Maxwell stress [29]. Since the width of this boundary is user defined it is
difficult to use this technique to predict voltages without having any experimental results to first
tune the model to.

1.3.1.2 Smoothed Particle Hydrodynamics

The numerical models presented so far have not been able to accurately model the interface
interaction with the electric field. Either there have been limitations with controlling the contact
line, or accurately representing the interface of the droplet. One numerical method that allows
direct control of contact line interactions, has strict interface representation for surface tension,
and intrinsically conserves volume is smoothed particle hydrodynamics (SPH) [30]. SPH is a
meshless particle modeling technique that can be used for various applications from
astrophysics, solid mechanics, and fluid dynamics. The general formalization of the technique is
to have a medium with continuous physical functions (pressure, velocity, etc.) throughout the
domain of interest. These functions are discretized using the standard SPH formulation

\[ f(x_i) = \sum_j \frac{m_j}{\rho_j} W(x_i - x_j, h) f(x_j) \]

where \( f \) is a continuous function, \( m \) and \( \rho \) are the mass and density of the particles respectively,
\( W \) is a smoothing function, and \( h \) is the smoothing length [30]. Essentially the method takes a
weighted average of the function \( f \) around a given point \( x_i \) by incorporating only the neighboring
particles within the smoothing length \( h \). \( W \), which typically takes the form of a Gaussian or
cubic spline curve that decreases to zero a distance \( h \) from the point \( x_i \), determines the weight of
each neighboring particle. Intrinsically SPH conserves the mass of the system by keeping the
number of particles and their masses constant. This conservation of mass can then be translated
into conservation of volume by implementing an equation of state with an exponential dependence between pressure and particle density. These equations of states are called pseudo incompressible and allow for efficient simulations of incompressible flow [30].

Within SPH fluid interfaces can either be modeled using two phase, or free surface flows. For two phase flows there are particles that belong to either of the phases and to distinguish them they are each assigned a color value of either zero or one depending on the phase they belong to [31],[32]. Using these color values, an intermediate weighted average color value for each particle is calculated using the smoothing function. This intermediate value is only used for calculating interface properties during a given time step, and does not replace the original particle color value of zero or one. Far from the interface this weighted average is either zero or one depending on which phase the particle is in. However, along the interface the weighted average experiences a smooth transition between zero and one. By calculating the gradient of this weighted average in that transition zone the normal to the interface as well as the curvature of the interface can be calculated. Once the curvature of the interface is found, it can simply be used to input into Young-Laplace’s equation as an additional force to model surface tension [31],[32]. Though capable of tracking the interface and modeling surface tension, the color field technique lacks accuracy due to the interface being calculated through the smoothing of the field.

The other option for fluid interfaces and surface tension is to model a free surface flow [33]-[36]. In these models it is unnecessary for the whole domain to be filled with particles. In this case there will be fluid particles that are not fully surrounded by other fluid or boundary particles. One difficulty with free surface flow is that the traditional method of calculating density
\[ \rho_i = \sum_j m_j W(x_i - x_j, h) \]

results in a decreased density near the free surface [33]-[36]. This is purely an artifact from the reduced number of neighboring particles within the smoothing length of a surface particle. There have been a number of solutions to overcome this artifact. One solution is to normalize the density of the boundary particles based on the densities calculated of the neighboring non-boundary particles [35]. In essence, this technique uses the average of the neighboring particles as the density for all boundary particles. This reduces the edge effect due to the free surface, however it does not completely remove it. Another solution that completely removes this artifact is to apply the SPH generalization to the continuity equation. This allows the density to be initialized at the start of a simulation, and then as the particles rearrange themselves the density is reduced or increased based on relative velocities [36]. That is to say, as particles move closer to each other due to pressure and other forces their density increases, and as they move apart their density decreases.

There are two main techniques for modeling surface tension in free surface flows, attractive forces [33],[34] and direct curvature measurement [35]. The attractive force method adds an additional force term into the momentum equation that is an attractive force between particles that is inversely proportional to distance between particles. Though this is an elegant method of modeling surface tension as it directly models the physical process of surface tension, the attractive forces need to be calibrated to accurately represent a specific surface tension. To overcome this calibration, a more direct approach directly measures the curvature of the free surface and applies Laplace’s equation to calculate surface tension [35]. Within the model, the surface particles are determined, and for each particle a polynomial is fit to the local surface
particles within its smoothing length. From this polynomial the curvature and normal can be determined. Though a more computationally intensive method, it is a direct representation of surface tension as defined by Laplace’s equation and is the most accurate method of modeling surface tension in free surface flows.

Though SPH has many benefits there are a few numerical instabilities that have proven troublesome for accurate results. The biggest instability has been coined the tensile instability and arises when using a non-absolute equation of state [37]-[41]. The tensile instability manifests itself by particles pairing up and aggregating together during the simulation. Initially the instability was researched by using perturbation analysis and traveling sound waves in an infinite array of particles [37]-[39]. From this analysis it was discovered that under compression the particles remained in a stable state, however under tension the particles would collapse together. The difficulty with this is that due to the nature of the derivative of the smoothing function, which usually has a negative region and is used to calculate the pressure field, as well as a non-absolute equation of state it is difficult to avoid a tensile state [37],[38]. To overcome this a short range artificial pressure term has been added to the momentum equation to create a sharp repulsive term to counteract the collapse [39],[40]. However, the problem with this is the function used for artificial pressure is arbitrary and it doesn’t completely remove the instability. A recent solution to the tensile instability was to create a hyperbolic smoothing function that had a positive derivative over the whole smoothing length [41]. This completely removes the tensile instability in Van der Waals simulations, however when used in SPH the relative equation of state still creates the tensile regions.

There has been one case of modeling electrowetting using SPH to study the oscillation of sessile droplets when exposed to periodic electric potentials [42]. In this study the electric field
was calculated by applying a moving least squares approximation to the Laplace equation. The surface tension of the model was calculated using the color field technique, and the coupling between the electric field and fluid flow was with the Maxwell stress tensor. Though the technique was successful it had limitations in requiring a large number of particles for the moving least squares as well as suffering the disadvantages of the color field technique described above [41].

1.3.2 Hydrophobic Patterning of Surfaces

For the EHMP arrays it is necessary to have a fast, reproducible, and accurate fabrication technique for patterning hydrophobic surfaces to pin the droplet contact lines. There are a number of techniques to pattern hydrophobic surfaces such as selective plasma treatment of Parylene C films [43], surface modification of silicon oxide and perfluoropolymer films [44], and UV treatment of titanium oxide fluoroacrylic composites [45]. The problems with these techniques are that they either result in non-transparent substrates, or are elaborate and time consuming. One technique that overcomes these issues is a nanocomposite of SU-8 photoresist and polytetrafluoroethylene (PTFE) nanoparticles [46],[47]. The nanocomposite maintains the photopatterningability of the photoresist while the surface energy and roughness of the nanoparticles provide a superhydrophobic surface. The nanocomposite is spray deposited onto a substrate and patterned like any other photoresist. The exposed substrate is then hydrophilic providing a sharp contrast in surface energy between itself and the nanocomposite. The main difficulty with this technique is that scattering from the nanoparticles limits the possible line resolution along features in the developed nanocomposite.

There are other techniques for patterning nanoparticles that have no limits on resolution, aside from the actual size of the nanoparticle, and are fast and reproducible. In particular,
microcontact printing has been shown to be an accurate technique for patterning nanoparticles for applications of selective nanotube growth [48]-[52]. In microcontact printing there are two important aspects to consider that will directly influence the transfer of the particles, the inking of the stamp and the surface affinity of the nanoparticles to the stamp and substrate. The inking of the stamp needs to leave a uniform layer of nanoparticles on the stamp so that when transferred the nanoparticles will create a uniform layer on the substrate. In addition, the surface affinity of the nanoparticles to the substrate needs to be higher than that of the stamp to facilitate the transfer of nanoparticles to the substrate [48]-[52].

Traditionally the inking step was performed by mixing the nanoparticles into ethanol and depositing the solution on to the stamp via a dropper [48],[49]. Ethanol was chosen as the suspending medium because it has a high evaporation rate and is readily absorbed into polydimethylsiloxane (PDMS), which can be used as the stamp material. This allowed for regions of uniform deposition of nanoparticles on the stamp. However, other inking techniques have been devised to provide uniform monolayers of nanoparticles over the entire stamp. In particular, gold nanoparticles were added to the surface of a water droplet where they formed a uniform monolayer [50]. The stamp was then passed over the droplet where the nanoparticles transferred to the stamp retaining their uniformity. Another inking technique that has single particle accuracy uses surface tension of a thin film to deposit the nanoparticles into features on the stamp [51],[52]. This technique provides the highest accuracy, down to single particle, of inking techniques, however it requires process specific equipment to deposit the nanoparticles on to the stamp.

PDMS has been adopted as the most common stamp material for two main reasons [51]. The first was mentioned previously in that ethanol is readily absorbed into PDMS allowing for
the ink solution to dry quickly leaving behind the nanoparticles. Secondly, when exposed to oxygen plasma PDMS drastically changes its surface energy. This allows for the surface affinity of the stamp to be tailored to the particular nanoparticle that is being used so that there is low affinity between the two. However, in some cases it is necessary to pattern nanoparticles on substrates that have low affinity with the particles. In these cases an adhesive such as Poly(methyl methacrylate) (PMMA) can be added to the substrate to help promote transfer [51].

1.4 Thesis Outline

This thesis covers the design and fabrication of an EHMP. In particular, the second chapter will focus on the design and implementation of a hybrid finite element SPH numerical model for predicting the deformations of the EHMP. The third chapter will introduce a microcontact printing technique that patterns PTFE nanoparticles on SU-8 coated indium-tin oxide (ITO) glass slides. The fourth chapter will focus on the testing and characterization of a single EHMP as well as the experimental validation of the numerical model. The fifth and final chapter will conclude the project with a review of the results as well as potential directions for future work.
Chapter 2: Hybrid FEM and SPH Numerical Model for an EHMP

When developing a prototype system for any application, it is important to have an analytical or numerical model that predicts the performance of the system. This allows for optimization of various parameters in the system to maximize the desired performance without having to continuously fabricate prototypes for testing. In the case of the EHMP, it is important to understand the influence of electrode geometry on the final deformation so that the electrodes can be optimized to create the optimal prism shape. In addition, the operation of the EHMP requires voltage potentials to deform the droplets into prism shapes. Depending on the magnitude of these potentials different equipment is required for operation and testing so it is desirable to be able to predict these voltages. From these two concerns of electrode geometry and voltage prediction, the first step in designing the EHMP was to develop a hybrid FEM and SPH numerical model that accurately modeled the system.

Figure 2.1 provides a schematic of the forces and regions that need to be modeled. In particular, there are three different regions that need to be modeled; the bulk flow within the droplet, the contact line, and the droplet interface. As mentioned in the introduction, due to the length scale of the system the Reynolds number of the flow puts the hydrodynamics in the Stokes regime. In this regime inertial forces are dominated by viscous forces and can be neglected allowing for the physics of the system to be simplified. In this case, the bulk flow hydrodynamics within the droplet reduce to a flow governed by pressure and viscous forces. In addition, the flow can be simplified further when considering the application of the problem; steady state equilibrium droplet deformations. Since the model is being designed to solve for equilibrium deformations viscosity is unnecessary since the time scale of the deformation
evolution is irrelevant. Therefore the bulk flow hydrodynamics can be simplified even further to only include pressure forces.

![Diagram of forces and regions in EHMP model](image)

Figure 2.1 Forces and regions that need to be taken into account when modeling an EHMP. P and \( \nu \) are pressure and viscosity within the bulk flow region of the drop respectively. \( F_e \) and \( F_s \) in the right inset represent the electric and surface tension forces along the drop interface region respectively. \( F_{gl}, F_{gs}, \) and \( F_{ls} \) in the left inset represent the liquid-gas, gas-solid, and liquid-solid surface energy forces acting around the contact line region respectively.

The contact line region of the droplet is an important region to model since the pinning of the contact line is crucial to ensure deformation into a prism shape. Without a pinned contact line, the droplet would simply shift along the substrate surface until it is centered under the electrode. Physically, the contact lines are pinned by a discontinuity in the surface energy along the surface of the substrate. When the contact line of the droplet becomes pinned at this discontinuity it must overcome the change in surface energy before it can be unpinned. This force of overcoming the surface energies manifests as the contact angle of the droplet. Both the hydrophilic and hydrophobic surfaces have a corresponding Young’s contact angle. If the contact angle of the droplet either exceeds the Young’s angle of the hydrophobic region, or decreases below the angle for the hydrophilic region then the contact line will become unpinned and the droplet will move [53].
To accurately model the physics of pinning the contact line, it would be necessary to model the surface energies along the substrate. However, the physical mechanisms that relate the surface energies to whether or not the contact line remains pinned is a complicated and difficult problem in numerical simulations and occupies its own field of research. In the case of the EHMP, if the contact line does not remained pinned then the device will fail. As discussed above, due to the hydrophobic surface patterning the contact line will remain pinned as long as the contact angle is between the corresponding contact angles on the two surfaces. That means that as long as the contact angle in the simulation is in this range, the contact line will be pinned. Using this knowledge, the model was simplified by manually forcing the contact lines to be pinned within the simulation regardless of the contact angle. If the final equilibrium contact angle lies outside the range of 18 – 152 °, the range created by the hydrophobic patterning technique described in chapter 3, then it can be determined that the device will fail due to contact line slipping.

The final region to model is the droplet interface. Along the interface there are two forces acting, surface tension and the electric force. It is well accepted that surface tension is modeled using the Young-Laplace equation

$$\Delta P = \kappa \sigma$$  \hspace{1cm} 2.1

where $\Delta P$ is the pressure drop across the interface, $\kappa$ is the surface curvature, and $\sigma$ is the surface energy of the liquid. The electric force is derived from the Maxwell stress tensor. When a perfectly conductive fluid is exposed to an external electric field, it experiences a stress along its interface of the form [29]

$$T_{ik} = \varepsilon_0 \varepsilon_i \left( E_i E_k - \frac{1}{2} \delta_{ik} |E|^2 \right)$$  \hspace{1cm} 2.2
where \( \varepsilon_0 \) and \( \varepsilon_r \) are the vacuum and relative permittivities respectively, \( i \) and \( k \) are the principle axes, \( E \) is the electric field strength, and \( \delta_{ik} \) is 1 when \( i = k \) and zero elsewise. Taking the divergence of this stress tensor gives the force acting along the interface of the fluid. This force can be simplified to a pressure of the form [29]

\[
P_e = \frac{1}{2} \varepsilon_0 \varepsilon_r |E|^2.
\]

Using this simplified electric pressure, the physics of the interface become a simple balance of pressures produced from surface tension and the electric field. When focused on a point along the interface these pressures become forces with surface tension acting normal to the interface towards the bulk of the droplet, and the electric force acting normal to the interface away from the droplet.

Knowing the forces within the model and what simplifications to take, an algorithm was developed that is outlined in figure 2.2.

![Figure 2.2 Numerical model algorithm flow chart.](image-url)
An initial geometry for the droplet is created and imported into COMSOL Multiphysics to calculate the initial electric pressure distribution along the interface. Once calculated, the electric pressure values are exported into Matlab where the SPH model was coded. Using Matlab, the droplet is deformed into an equilibrium shape using SPH and the initial electric pressure distribution. This equilibrium shape is then exported back to COMSOL where the electric pressure is recalculated. This cycle of calculating an electric pressure and finding a new equilibrium shape is continued until there is less than 0.5% change in the magnitude of the electric pressure between two cycles.

### 2.1 Calculating the Electric Pressure using COMSOL

To calculate the electric pressure it was necessary to numerically solve the electric field distribution generated by the electrode and grounded droplet. Though it is possible to use SPH for this calculation [42], it requires modeling the air domain between the droplet and the electrode making the problem a multiphase flow rather than free surface. To maintain a free surface model and simplify the coding process, COMSOL was used to calculate the electric field using FEM techniques based on the free surface interface geometry calculated using SPH.

Figure 2.3 outlines a flow chart of the process for calculating the electric pressure using the electrostatics module in COMSOL. Once an equilibrium shape is reached using SPH, the coordinates of the surface particles are exported to a text file in two columns of data, the x and y coordinates. Using the interpolation curve option in COMSOL, the x and y coordinates are read from the text files and recreate the equilibrium shape in the COMSOL model (figure 2.4). As will be explained in more detail later in this subsection, a second curve is imported from Matlab that is offset by the minimum node spacing in COMSOL, 1 µm, along the normal of the interface away from the droplet to aide in the calculation of the electric pressure (figure 2.4 inset).
With the geometry imported into COMSOL, the next step is to create the boundary conditions and mesh for the domain. Initially, all exterior boundaries in the COMSOL domain are set to the boundary condition of zero charge. The ground plate is created simply by using an electric potential boundary condition and selecting all of the bottom boundaries in the domain.
while setting the voltage to zero (figure 2.5 blue line). The electrode boundary condition is first created in the geometry by creating a single Bezier polygon segment along the top of the domain in the desired location (figure 2.5 red lines). Similar to the ground plate, a second electric potential boundary condition is created selecting the Bezier polygon segment and setting the voltage to any desired value.

With the boundary conditions set the materials for the domain are chosen. The interior of the equilibrium shape is set to water with a relative permittivity of 80. The top rectangle is set to glass with a relative permittivity of 4.7. The rest of the domain, including the space between the two interpolation curves is set to oil with a relative permittivity of 2.2. Finally, the domain is meshed using a free triangular extra fine mesh (figure 2.5). This allows for a minimum node distance of 1 μm.

Figure 2.5 COMSOL boundary conditions and mesh. Left: COMSOL model with boundary conditions. Right: COMSOL model mesh. Red represents the electrode boundary condition at voltage V, and blue represents the ground boundary condition at zero voltage.
Using the boundary conditions, and material properties explained above, the electric field distribution is calculated. Once the electric field is determined, the electric pressure as defined by equation 2.3 is calculated in the whole domain (figure 2.6). Using this calculation, the electric pressure is plotted along the length of the second interpolation line that is displaced 1 µm from the actual interface of the droplet (figure 2.6f). The electric pressure is measured along this curve instead of the actual interface because the electric field along the interface is essentially zero due to the boundary conditions and material properties. This electric pressure plot is exported to a text file and promptly imported back into Matlab where it is applied as a force to the interface particles in the SPH simulation (figure 2.7).

As can be seen in figure 2.6f, the initial calculation of the electric pressure is not as directional as desired (figure 4.20b/f). The directionality of the electric pressure is critical to achieve maximum deformation, and as evidenced by the blue curve in figure 2.6f the peak of the electric pressure is situated near the apex of the droplet. To redirect the electric pressure towards the side of the droplet, an additional ground electrode was added to the simulation centered above the droplet (figure 2.6c). This additional ground electrode drastically reduces the electric pressure directly above the droplet ensuring that the directionality is enforced. As can be seen in figure 2.6f, the directionality of the electric pressure distribution shifts towards the positive electrode side of the droplet as the width of the additional ground electrode increases. It was determined that a minimum width of 50 µm was required to achieve enough directionality in the electric pressure to ensure adequate deformation.

The consequence of the additional ground electrode, as evidenced by figure 2.6f, is that the maximum electric pressure is reduced as the electrode width increases. In the case of the
50 µm wide ground electrode (figure 2.6d), the decrease in electric pressure is only influenced by
the addition of the ground electrode.

Figure 2.6 Electrical model for the ideal EHMP with encapsulating glass slide above droplet. a) Initial
gometry with 50 µm wide positive electrode (red) above the droplet contact line and the only ground below
the droplet (blue). b) Electric pressure distribution for 50 µm wide positive electrode with no additional
ground. c) Updated electrode geometry with additional 50 µm wide ground electrode above droplet (blue). d)
Electric pressure distribution for the 50 µm wide positive and ground electrodes. e) Electric pressure
distribution for a 25 µm wide positive and 100 µm wide ground electrodes. f) Electric pressure distribution
along the surface of the droplets. Blue represents the no ground case in b, green represents the 50 µm ground
case in d, and red represents the 100 µm ground case in e.

However, as the ground electrode widens for the case in figure 2.6e, the positive electrode width
is reduced leading to a further decrease in the maximum electric pressure. This decrease in
electric pressure is counterproductive in terms of the deformation experienced by the EHMP since the amount of deformation is dependent on this magnitude. The only method to counter this reduction in electric pressure is to increase the applied voltage. However, increasing the applied voltage with the additional ground electrode introduces the potential of dielectric breakdown.

Figure 2.7 Electric pressure force acting on only surface particles in Matlab SPH simulation.

With the additional ground electrode is added, the shortest path between the applied voltage and ground is the distance between the positive and ground electrodes above the droplet rather than between the electrode and droplet. This shortened path significantly increases the gradient of the electric field and the chance of dielectric breakdown. Using COMSOL, the electric field gradient was calculated for different gaps between the positive and ground electrodes to test whether dielectric breakdown would occur. From figure 2.6, it is seen that a ground electrode of at least 50 µm is required to achieve directionality of the electric pressure. As the width is increased to 100 µm, the added directionality is limited in comparison to the decrease in maximum electric pressure. To remove the effect of reduced electric pressure due to the shortening of the positive electrode its width was fixed at 50 µm. The COMSOL simulation
was then run for a range of ground electrode widths between 50 and 80 µm, accounting for a
dielectric breakdown path length of 10 to 25 µm between the two electrodes.

Figure 2.8b-e presents the results for a 25, 20, 15, and 10 µm wide gap between the
ground and positive electrodes respectively at an applied voltage of 450 V. In these figures, the
blue line represents the ground electrode, and the red line represents the positive electrode.

Figure 2.8 Investigation of dielectric breakdown based on the gap between the positive (red) and ground
(blue) electrodes. Color scale clipped at 1.5x10^7 V/m to show breakdown path of oil. a) White rectangle
represents the region of interest between the electrodes that the following sub figure will focus on. b)
Gradient of electric field for a 25 µm gap. c) Gradient of electric field for a 20 µm gap. d) Gradient of
electric field for a 15 µm gap. e) Gradient of electric field for a 10 µm gap.

It was determined that dielectric breakdown would occur if a continuous path between the
electrodes could be drawn such that the gradient of the electric field along every point of this
path exceeded 15 MV/m, which is the dielectric breakdown of the oil. Any field gradients larger
than 15 MV/m were not important since breakdown would still occur. Therefore the color scale was set to clip at 15 MV/m to demonstrate the breakdown path in the oil. Since the focus of the test is on the breakdown within the oil, the paths could only be drawn below the electrodes since the region above is glass.

As can be seen in figure 2.8b, for a gap of 25 µm the gradient of the electric field does not exceed 15 MV/m along a continuous path between the electrodes allowing the conclusion to be drawn that dielectric breakdown would not occur. In figure 2.8c, for a gap of 20 µm again the gradient does not exceed 15 MV/m for a continuous path. However, there is only a small section (<1 µm) that does not exceed 15 MV/m so it would not be recommended to operate under these conditions since small fluctuations could easily cause dielectric breakdown. As seen in figures 2.8c/d, for any gap smaller than 20 µm the gradient exceeds $1.5 \times 10^7$ V/m and dielectric breakdown will occur. Therefore the optimal electrode configuration is 50 µm wide positive electrode and ground electrodes with a gap of 25 µm. This configuration maximizes the magnitude and directionality of the electric pressure without dielectric breakdown.

The final test is whether the electric pressure distribution is independent of the mesh size of the COMSOL model. The electric pressure was tested for both ‘extremely fine’ and ‘coarse’ mesh sizes in COMSOL with the results presented in figure 2.8. As can be seen in figure 2.9, there is less than 2 % decrease in electric pressure between the different mesh sizes. This demonstrates that indeed the electric pressure calculation is mesh size independent.

2.2 SPH Simulation of the EHMP Deformation

As explained in the beginning of this chapter, there are three main regions that need to be modeled; the bulk flow, contact line, and interface. Within these regions there are only three
forces that need to be explicitly modeled; electric pressure, pressure, and surface tension. Though many techniques can easily model the pressure forces within the bulk flow, SPH is well tuned for modeling the electric pressure and surface tension due to its ability to model free surface flows.

![Figure 2.9 Mesh size dependence of electric pressure](image)

### 2.2.1 Governing Equation in SPH

The basis for SPH is centered around the ability to find the value of a continuous function \( f(x) \) at \( x_i \) by [30]

\[
f(x_i) = \int_{-\infty}^{\infty} f(x) \delta(x - x_i). \tag{2.4}
\]

This can be approximated by [30]

\[
f(x_i) = \int_{-\infty}^{\infty} f(x) W(x - x_i, h), \tag{2.5}
\]

where the delta function has been replaced by a smoothing function \( W \) which has a smoothing length \( h \). The smoothing function \( W \) often has a bell curve shape and acts as a delta function in that when integrated over the whole domain it is normalized to 1. However, rather than having no width such as the delta function, it has a finite width that is defined as the smoothing length.
In a domain where the continuous function $f(x)$ is represented by a finite number $N$ of particles each with an individual mass and volume this calculation can be approximated by \[ f(x_i) \approx \sum_{j=1}^{N} \frac{m_j}{\rho_j} W(x_i - x_j, h) f(x_j), \] \[ 2.5 \]

where $m$ is the mass of each particle, and $\rho$ is the density of each particle calculated by \[ \rho_i = \sum_{j=1}^{N} m_j W(x_i - x_j, h). \] \[ 2.6 \]

Essentially, the value $f(x_i)$ is being approximated by a weighted average of all of the values $f(x_j)$ within a distance $h$ from $x_i$ where the weight is determined by the smoothing function $W$. In addition the derivative of $f(x)$ can be approximated by \[ \nabla \cdot f(x_i) = -\sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x_j) \cdot \nabla W(x_i - x_j, h), \] \[ 2.7 \]

where \[ \nabla W(x_i - x_j, h) = \frac{x_i - x_j}{r_{ij}} \frac{\partial W(x_i - x_j, h)}{\partial r_{ij}} \] \[ 2.8 \]

and $r_{ij}$ is the distance between $x_i$ and $x_j$. The approximations 2.5 and 2.7 are the basis for SPH, and how the governing equations for SPH numerical models are derived.

There are many different smoothing functions that have been used in SPH simulations. The forms range from Gaussian functions [30], to splines (cubic and higher) [30], as well as hyperbolic [41]. Each of these functions follows a few basic rules [30]:

1. The function must be normalized over the whole domain so that $\int_{-\infty}^{\infty} W(x - x_i, h) dx_i = 1$

2. For distances greater than the smoothing length, $h$, $W=0$
3. The function is always positive, smooth, decreases in magnitude as the distance from the particle of interest increases, and is symmetric.

4. The function obeys $\lim_{h \to 0} W(x - x_i, h) = \delta(x - x_i)$

Due to its wide use and simplicity, a cubic spline in the following form

$$W = \begin{cases} 
6q^3 - 6q^2 + 1 & 0 \leq q \leq \frac{1}{2} \\
2(1-q)^3 & \frac{1}{2} \leq q \leq 1 \\
0 & q > 1 
\end{cases}$$

is used as the smoothing function for the SPH simulations. Using 2.5 and 2.7 the following momentum equation was developed to model the EHMP

$$\frac{dv_i}{dt} = -\sum_{j=1}^{N} m_j \left( \frac{p_i}{\rho_i} + \frac{p_j}{\rho_j} \right) \nabla W_{ij} - \frac{\varepsilon \sigma E^2}{2} \hat{n}_i + \frac{\delta \kappa}{\rho_i} \hat{n}_i + \sum_{j=1}^{N} A e^{-\frac{B}{r_{ij}}} \hat{n}_i \hat{r}_{ij}$$

where $v$ is the particle velocity, $p$ is the particle pressure, $n$ is the surface normal, $\delta$ is the inverse of the particle spacing, $A$ and $B$ are scaling constants, and $r_{ij}$ is the distance between particles $i$ and $j$. Essentially, 2.10 equates the acceleration of particle $i$ to four different forcing terms that will each be described individually.

### 2.2.1.1 Pressure Force Term

The first forcing term in 2.10 is the SPH derivation of the pressure term in the Navier Stokes equation [30]. There are different forms of the derivation, however this particular representation is symmetric and conserves momentum between two particles. To calculate the pressure for a given particle the Tait equation is used as the equation of state.
\[ P_i = D \left( \left( \frac{\rho_i}{\rho_0} \right)^\gamma - 1 \right), \]  

where \( \rho_0 \) is a reference density, \( \gamma \) is a constant set to 7, and \( D \) is a scaling factor based on the speed of sound \( c \) in the fluid calculated by [33]

\[ D = \frac{\rho_0 c^2}{\gamma}. \]

By setting \( \gamma \) to 7, pressure becomes strongly dependent on the difference between \( \rho_i \) and \( \rho_0 \). Small fluctuations between the densities magnify into large fluctuations in the pressure field. These pressure fluctuations rearrange the particles so that particle densities remain constant and close to \( \rho_0 \) throughout the simulation mimicking an incompressible flow. Therefore, the Tait equation is commonly used in SPH hydrodynamic simulations to force pseudo-incompressibility with minimal computational expense [33].

### 2.2.1.2 Electric Pressure Force Term

The second term in 2.10 is the electric pressure force term. This term is only applied to surface particles since the droplet is assumed to be a perfect conductor and the Maxwell stress and thus the electric pressure only affects the interface. The magnitude of the force is calculated using COMSOL as explained in section 2.1, and the direction of the normal is calculated using the technique outlined in the following sub-section.

### 2.2.1.3 Surface Tension Force Term

The third term in 2.10 is the surface tension force term. As mentioned in the introduction there are many different techniques for modeling surface tension in SPH simulations. One of the more accurate techniques is the polynomial fitting technique described in [35]. In this technique the surface particles are found using a spoke technique outlined in figure 2.10. In this technique
each particle is individually tested using the spoke technique to see if it is in fact a surface particle. For each particle, a circle of radius $h$ is drawn around them, and then split into a finite number of spokes (blue lines in figure 2.10). For a given set of spokes, if there is a spoke that is further away than $h$ from all other particles in the simulation (red particle in figure 2.10) then the corresponding particle is labeled as a surface particle. For a given set of spokes, if each spoke has at least one particle within $h$ of it (green particle in figure 2.10) then the corresponding particle is an interior particle and does not get labeled.

![Figure 2.10 Spoke technique for finding surface particles. Left: Spoke technique centered around surface particle (red). Upper left spoke is further than $h$ away from all other particles. Right: Spoke technique centered around interior particle (green). All spokes are within $h$ of at least one other particle.](image)

Once the surface particles are labeled, each particle and its neighboring surface particles are fitted with a polynomial as described in figure 2.11. For each surface particle, the centroid of it and any other neighboring surface particles within a distance $h$ is calculated by

\[ x_c = \sum_{i} \frac{x_i}{N} \quad 2.13 \]
\[ y_c = \sum_{i} \frac{y_i}{N} \quad 2.14 \]
where $x_c$ and $y_c$ are the coordinates of the centroid, and $N$ is the number of surface particles within the smoothing length including the particle of interest.

$$
\alpha = \begin{cases} 
2\pi - \cos^{-1}(r_y / r) & \text{if } r_x > 0 \\
\cos^{-1}(r_y / r) & \text{if } r_x \leq 0
\end{cases}
$$

Figure 2.11 Surface curvature calculation technique. Left: Surface particle of interest (red) with all neighboring surface particles (blue) within a distance $h$ (circle). Right: Surface particle of interest and neighboring surface particles in new coordinate system with origin at center of mass and particle of interest on $y$-axis.

A new coordinate system is then defined with this centroid as the origin, and the particles are rotated about that so that the surface particle of interest is on the $y$-axis. The required angle of rotation is calculated by

$$
\alpha = \begin{cases} 
2\pi - \cos^{-1}(r_y / r) & \text{if } r_x > 0 \\
\cos^{-1}(r_y / r) & \text{if } r_x \leq 0
\end{cases}
$$

where $r_x = x_i - x_c$, $r_y = y_i - y_c$, $r = \sqrt{r_x^2 + r_y^2}$, and $(x_i, y_i)$ is the position of the surface particle of interest. Within the new coordinate system, a polynomial is fit to the particles using the Matlab function polyfit that produces the coefficients for a N-1 order polynomial $P(x)$. With $P(x)$ calculated, the local curvature and normal can be calculated by
\[ \kappa_i = \frac{|P''(0)|}{\left[1 + P''(0)\right]^\frac{3}{2}} \quad 2.16 \]

and

\[ \vec{n}_i = \begin{cases} 
\langle P'(0)\cos\alpha + \sin\alpha, P'(0)\sin\alpha - \cos\alpha \rangle & \text{if } P''(0) < 0 \\
\langle -P'(0)\cos\alpha - \sin\alpha, -P'(0)\sin\alpha + \cos\alpha \rangle & \text{if } P''(0) > 0 
\end{cases} \quad 2.17 \]

in the original coordinate system respectively. With the local curvature and normal calculated for every surface particle, the surface tension force can be calculated by using the modified Young-Laplace equation seen as the third term on the right hand side in 2.10. This modified equation is just the standard Young-Laplace equation seen in 2.1 with a scaling factor to counter changes in pressure and particle spacing.

### 2.2.1.4 Exponential Repulsive Force Term

The fourth and final term in 2.10 is a short-range repulsive force in the form of an exponential. It is included to combat the tensile instability. It has been found that while using a relative equation of state such as the Tait equation (2.11) that a numerical instability arises in the form of particles aggregating together [37]-[41]. When using a generalized momentum equation such as

\[ \frac{dv_i}{dt} = -\sum m_j \left( \frac{S_i}{\rho_i^2} + \frac{S_j}{\rho_j^2} \right) \nabla W_{ij} \quad 2.18 \]

where \( S \) represents the stress state of the fluid and is a summation of force terms that can include pressure and viscosity it was found that a stability criterion for the tensile instability is [41]

\[ W''S > 0. \quad 2.19 \]

Analyzing this stability criterion shows that the simulation can become unstable when \( W'' \) and \( S \) have opposing signs. As mentioned above, due to using a relative equation of state, \( S \)
can transition between negative and positive multiple times in a single simulation. In addition, figure 2.12 shows the form of a cubic spline smoothing function W and its first derivative.

![Cubic spline smoothing function W and its derivative W’ over a smoothing length h.](image)

From figure 2.12 it is apparent that inherently there is a transition distance where W” passes from negative to positive. Though figure 2.12 is a graph of the cubic spline used in this thesis, the same transition occurs in all Gaussian and higher order spline smoothing functions [41]. Due to the transitions in both W” and S it is difficult to avoid the tensile instability.

The first solution to combat the tensile instability was to include an additional term in the stress state S of the form [39]

\[
g(x_i) = \left( \frac{W(x_i - x_j, h)}{W(\Delta x, h)} \right)^n
\]

where \( \Delta x \) is the average particle spacing around \( x_i \), and \( n \) is a constant set to 4. By including \( g(x) \) in the summation of the stress state, it will act as an artificial short range pressure to counter the particle aggregating by forcing S to remain positive [39]. Though this solves half of the instability criteria, it still only works in the range of W” that is positive.
A more recent technique to combat the tensile instability uses a hyperbolic smoothing function that has a positive second derivative over the whole range of distances [41]. This solves the other half of the instability criteria, and works well in Van der Waal simulations where the equation of state is absolute and the tensile instability can be completely removed [41]. However, when using the standard SPH equations and its pseudo incompressible equation of state there is still the issue of a negative stress state causing the instability.

In the case of this thesis, the only force acting in the stress state $S$ is pressure thus giving the first term in 2.10. In equation 2.8, it is seen that $\nabla W$ is directly proportional to $W'$ so that the pressure term in 2.10 is directly proportional to $W'$. As shown in figure 2.10, $W'$ drops to zero as the distance between particles converges to zero causing unphysical pressure forces. It would be expected that as the distances between particles converge to zero the pressure force would become infinite rather than zero. In order to combat the tensile instability as well as unphysical pressure forces a short range repulsive force term was added outside of the stress state. Within numerical chemistry, it is common to use exponential terms to model short-range repulsive forces between molecules [54] so it was decided to use an exponential term to create a comparable physical force.

In essence, by adding the exponential repulsive term the pressure force has been broken up into two different terms, a macroscopic and microscopic term. At long ranges within the bulk flow of the fluid, the exponential term will have minimal effect and the macroscopic pressure force term as defined by the SPH representation of Navier Stokes will dominate the forces. However, if two particles come within the range that the SPH representation starts to become unphysical and converges to zero, the microscopic term in the form of the exponential repulsion becomes dominant. The now dominant microscopic repulsive force will then force the particles
away from each other back into a range where the macroscopic SPH representation is physical. In addition to making the pressure force more physical, the exponential repulsive force directly combats tensile instability by preventing particles from collapsing onto each other.

### 2.2.2 Time Advancement of the Particles

With the governing equations of the simulation identified the last step is to have a technique for advancing the particles through time. A common technique for time advancement of particles is the predictor corrector scheme that uses the following equations to advance the particles [38]

$$v^{1/2} = v^0 + \frac{\Delta t}{2} a^0$$ \hspace{1cm} 2.21

$$x^{1/2} = x^0 + \frac{\Delta t}{2} v^0$$ \hspace{1cm} 2.22

$$\rho^{1/2} = \rho(x^{1/2})$$ \hspace{1cm} 2.23

$$a^{1/2} = a(x^{1/2}, \dot{x}^{1/2}, \rho^{1/2})$$ \hspace{1cm} 2.24

$$v^{1/2} = v^0 + \frac{\Delta t}{2} a^{1/2}$$ \hspace{1cm} 2.25

$$x^{1/2} = x^0 + \frac{\Delta t}{2} v^{1/2}$$ \hspace{1cm} 2.26

$$x^1 = 2x^{1/2} - x^0$$ \hspace{1cm} 2.27

$$v^1 = 2v^{1/2} - v^0$$ \hspace{1cm} 2.28

where $a=dv/dt$ and $\rho$ are calculated using 2.10 and 2.6 respectively. The amount of time the simulation progresses through in a single time step, $\Delta t$, is calculated at the beginning of each step by [38]

$$\Delta t = 0.25 \min(\Delta t_c, \Delta t_f)$$ \hspace{1cm} 2.29
where

\[
\Delta t_{cv} = \min \left( \frac{h}{c + 0.6 \max \left( \frac{h(v_j - v_i) \cdot r_{ij}}{r_{ij}^2} \right)} \right) \tag{2.30}
\]

\[
\Delta t_f = \min \left( \frac{h}{|f_i|} \right) \tag{2.31}
\]

for all i and j. \( \Delta t_{cv} \) is a time step that measures the relative velocities of particles with respect to each other and ensures that the time step reduces as these relative velocities increase. \( \Delta t_f \) is a time step that measures the maximum forces of each particle and reduces the time step based on an increase in forces acting on the particles. Depending on whether the maximum forces or relative velocities are dominating, 2.29 takes this into account to ensure that the time step is small enough to allow convergence. Any time step greater than this will cause the simulation to diverge and have particles ejected from the droplet, while any time step smaller than this will produce the same results but at a cost of computational efficiency.

2.2.3 Boundary Conditions

The last consideration in forming the SPH model is the boundary conditions for the problem. In the case of modeling an EHMP, the boundary conditions are the pinned contact lines, and the interface forces. To pin the contact lines, an initial geometry for the system is formed as outlined in figure 2.13. A random distribution of N particles is disbursed at a minimum spacing of \( h_0/2 \) in a circular region where \( h_0 \) is an arbitrary starting smoothing length. This random distribution is run through the SPH simulation outlined above until it comes to an equilibrium distribution. This distribution is then linearly scaled and centered around the origin so that the radius of the
drop is as desired for the EHMP simulation (figure 2.11). The surface particles are searched for the pair with the greatest distance between them, and the droplet is rotated so that this pair aligns with the x-axis (green particles figure 2.13). If need be there is additional minor translations to ensure that this pair of particles is on the x-axis and will represent the contact lines in the simulation.

![Figure 2.13 Initial particle distribution. a) Initial random distribution of particles for simulation. b) Particles in equilibrium after being scaled to desired dimensions with the contact line particles highlighted in green.](image)

This particular technique of aligning a pair of particles at the contact lines provides an initial contact angle of 90°. To achieve other contact angles, the center of the droplet can be translated vertically with respect to the x-axis until the desired angle between the droplet and x-axis is achieved. Once translation is complete, it is necessary to determine the distance between the new contact lines along the x-axis, $x_{\text{cl}}$. With that distance, the same technique as above can be used to rotate and translate the droplet to ensure a pair of particles corresponds to the contact lines. However, instead of finding a pair separated by the furthest distance, a pair with a separation equal to $x_{\text{cl}}$ is used.

After the geometry is scaled, the constants within the model need to be scaled so the forces remain in equilibrium and physical. The smoothing length, $h$, is scaled linearly the same
amount as the geometry so that each particle keeps the same number of neighboring particles. The mass can be scaled by

\[ m = \frac{\sum W(x_{i0} - x_{j0}, h_0)}{\sum W(x_i - x_j, h)} m_0 \]

where the subscript 0 denotes values before the linear scaling. Using this scaling ensures that the density remains the same after the scaling so that the pressure force remains physical during the scaling. Finally, the scaling factor A in the repulsive term in 2.10 is scaled down by the same ratio of smoothing functions that is used to scale the mass to ensure that the repulsive force occupies the same fraction of a particle’s total force as it did in the initial simulation.

After scaling, the final step before simulation is to apply the boundary conditions. To pin the contact lines a no slip boundary condition is applied by fixing the particles on and below the y-axis. In doing so, the contact lines (green particles in figure 2.13) are inherently pinned. This technique is not strictly physical in the sense that it does not take into account a half circle droplet on a planar substrate surface, which the EHMP is. Instead, it assumes the fixed substrate surface is curved and only exists below the droplet. However, if a planar substrate were to be used then there would be a sharp angle between the surface particles of the droplet and the surface particles of the substrate. This sharp angle would cause poor polynomial fitting when measuring the curvature of the droplet surface near the contact line. Instead, by continuing the curved interface below the contact line, the surface tension calculations will remain accurate along the whole surface of the droplet while maintaining a no slip boundary condition and pinning the contact lines. The final boundary condition is to apply the electric pressure acting around the interface of the droplet using the method outlined in section 2.1 and specifically figure 2.7.
2.3 Validation Tests

The first step in testing the model is to validate that it works as expected from the physics of the problem. When modeling free surface flows using SPH there are two standard tests to determine whether the model is working appropriately, a collapsing square [33, 35, 40, 41] and an oscillating ellipse [33, 34, 41]. In the collapsing square test (figure 2.14) the particles are evenly spaced on a square grid and released from rest.

![Sequence of collapsing square test through time.](image)

From the initial square position, the particles are allowed to oscillate and rearrange eventually coming to equilibrium in a circle. The flow is modeled as described previously with surface tension and pressure as the main forces acting on the particles. However, for the simulation to evolve into an equilibrium state viscosity needs to be implemented so particle kinetic energy can be dissipated. In the case of this test, a standard SPH viscosity term was added to equation 2.10 in the form of [34]

\[
\frac{dv_i}{dt} = 2\mu \sum_{j=1}^{N} m_j \frac{(v_i - v_j)}{\rho_i \rho_j (r_i - r_j)^2} (r_i - r_j) \cdot \nabla W(r_i - r_j, h)
\]

2.33
where \( \mu \) is the dynamic viscosity.

Though there are no quantitative results to draw from this test, there are a number of qualitative notes. The expected collapse of the square under surface tension is for the corners to initially round due to the high curvature, and for the particles to oscillate into a diamond shape. Though the dominant shape will be the diamond, the corners will have rounded due to viscous dissipation. As time progresses the overall shape of the drop will oscillate between a square and diamond all the while having the corners continue to round more due to dissipation. Eventually the oscillations will stop and the particles will come to rest in a circular shape. As can be seen from figure 2.14 this particular trend of oscillations is evident in our simulation showing that the physics of the model is working as expected.

Another conclusion to draw from figure 2.14 is in regard to the exponential term and tensile instability. At \( t=5.3 \) ms in the simulation it is quite evident that the corner particles have been shot into the inside of the drop due to high surface tension forces and have ended up very close to other particles. Due to the tensile instability these particles and their nearest neighbor should have remained physically close for the rest of the simulation unable to escape proximity to the other. However, as can be seen throughout the rest of the simulation the particles are forced apart and return to an even spacing due to the exponential term.

The final qualitative result from this test is the final equilibrium shape of the drop. Ideally for an incompressible fluid, the density would remain the same over the whole drop domain where the magnitude of the pressure is dictated by the strength of the surface tension. This constant density ensures that there are no internal flows due to a pressure gradient and that steady state equilibrium can be achieved. To accomplish this with a free surface simulation it is necessary to have a higher number of particles with tighter packing near the surface to ensure
there is no decrease in density near the surface. As can be seen in the final frame of figure 2.14, the final equilibrium shape of the simulation has a higher packing of particles near the free surface to ensure that the density remains the same across the whole drop to counter the surface tension force.

The final equilibrium drop shape from the collapsing square experiment leads directly into the oscillating ellipse drop test. When studying liquid streams, Rayleigh determined that an incompressible inviscid 2D oval drop would oscillate under surface tension with a period \[33\], \[34\], \[41\]

\[
\tau = 2\pi \sqrt{\frac{R^3 \rho}{6\sigma}}
\]

where \(R\) is the radius of a 2D droplet with the same area as the ellipse. This oscillation would occur due to the imbalance of curvature along the major and minor axes. The higher curvature along the major axis would draw the transverse diameter inward forcing the conjugate diameter outward along the minor axis. Eventually the axis would flip due to this movement so that the major axis would become the minor and vice versa. Due to no viscous dissipation this transition would eventually reverse creating the oscillation that Rayleigh predicted.

In SPH, it has become customary to test surface tension models using this technique. An equilibrium circular drop would be transformed into an ellipse via the transformation \[33\], \[34\], \[41\]

\[
\begin{pmatrix}
  x_i' \\
  y_i'
\end{pmatrix} = \sqrt{\frac{2\left(x_i^2 + y_i^2\right)}{\sin(e\pi)}} \begin{pmatrix}
  \sin\left(\frac{e\pi}{2}\right) \sin\left(\arctan\left(\frac{x_i}{y_i}\right)\right) \\
  \cos\left(\frac{e\pi}{2}\right) \cos\left(\arctan\left(\frac{x_i}{y_i}\right)\right)
\end{pmatrix}
\]

2.35
where $e$ is the eccentricity and is set to 0.55. This ellipse is then released from rest and the oscillation period is tracked. If the period calculated from the model is in agreement with the expected period from equation 2.35 then it is a good indication that surface tension has been modeled correctly.

The technique of polynomial fitting to model surface tension has already been shown to be accurate in [35]. In the case of this thesis, the oscillating drop is a test of whether the exponential term has an adverse effect on the movement of the particles beyond keeping them from collapsing onto each other. A number of different drops with different number of particles and radii were tested for their oscillation period. A sample oscillation can be seen in figure 2.15.

![Sample oscillations of the x and y radii. The ellipse radius had a radius of 0.9m, density of 1200 kg/m$^3$, and $\sigma=100,000$ N/m.](image)

Equation 2.34 predicts a period of 0.24 sec based on a radius of 0.9 m, density of 1200 kg/m$^3$, and $\sigma=100,000$ N/m. The unrealistic surface tension was chosen to ensure a fast oscillation for a roughly 1 m droplet for computing purposes. As can be seen in figure 2.15 the major and minor
axis oscillate between the x and y axes at a period of 0.255 sec that is within 6% of the theoretical prediction. All other particle concentration and radii tests fell within the same error as the above example. From these tests it is evident that the physics modeling the motion of the droplet is accurate. In addition, it shows that the exponential force has no impact on the motion of the droplet beyond keeping the particle ordered.

It should be noted that theoretically an inviscid drop should have no dissipation of energy meaning the amplitudes of the oscillations in figure 2.15 should not decrease. However, as seen in the figure there is a significant decrease in oscillation amplitude signifying a dissipation of energy. This effect is an intrinsic artifact in particle simulations and has been well studied before [34]. In short, the discretization of a continuous domain using a particle method creates an artificial viscosity within the simulation without a specific viscosity term in the governing equation. This viscosity will dissipate energy, which is what is seen in figure 2.15. Increasing the number of particles in the simulation can minimize the effect, however that comes at a cost of requiring more processing power [34].

2.4 EHMP Numerical Results

With the numerical model of the hydrodynamics validated, the final step is to simulate an EHMP. As mentioned in the introduction, the EHMP consists of an elongated droplet surrounded by oil with an electrode positioned lengthwise along the elongated contact line. For the numerical simulation, the cross-section of the droplet is used as the domain consisting of a half circle droplet with a surface tension of 0.031 N/m between the water and oil. In addition, the relative permittivity of the oil is 2.2 for the electric pressure calculations. Using the initial particle distribution and boundary conditions outlined in section 2.2.3 as well as the optimum
electrode geometry found in section 2.1 a 200 µm diameter droplet was tested over a wide range of voltages.

2.4.1 Low Voltage Results

Figure 2.16 reveals the results of the numerical simulation at low voltages.

![Graph showing droplet deformation](image)

**Figure 2.16** Surface deformations of a 200 µm diameter EHMP for different electrode voltages. The electrode is a line placed between -100 and -50 µm in the x direction and at 175 µm in the y direction (not shown).

In this case the positive electrode is a placed between -100 and -50 µm on the horizontal scale and at 175 µm on the vertical scale. The ground electrode is between -25 and 25 µm on the horizontal scale and 175 µm on the vertical scale. As can be seen, the deformation of the droplet is as expected with the droplet deforming into a triangular prism shape with the free vertex pointing towards the electrode. In addition, the prism edges become straighter and the vertex more defined as the voltage increases. In addition to testing various voltages, the input was
tested with a step and ramp input (figure 2.17). For the step input, the droplet started from the initial circular distribution, and a high voltage was applied from the start. From this voltage an equilibrium deformation was found (blue curve figure 2.17). For the ramp input, an equilibrium deformation was found at 25 V increments on the way to the final desired voltage. Theoretically, there should be no difference between a ramp and step input in the model, and this can be witnessed in figure 2.17. This test ensures that large initial deformations do not find a local equilibrium deformation.

Figure 2.17 Equilibrium shapes of step (blue) and ramp (red) inputs.

The final analysis of the EHMP numerical results is a prediction of the performance of the prism with respect to automultiscopic displays. In particular, there are two parameters that will be used to judge the performance, the fill factor and the average curvature along the interface. The ideal case for the EHMP is to have a high fill factor and a low average curvature. This shows that the prism is projecting a high amount of the incoming light at the correct angle with minimal blurring for a given voltage. These values will be unique for a given voltage and
equilibrium deformation, and it is expected the best results will be at the higher voltages due to the sharper deformation. Though there will be a range of values, the fill factor and average curvature will provide important information on the viability of the device with respect to automultiscopic displays.

The fill factor is a measure of what fraction of the display light is projected at the correct angle due to the prisms. This can be measured as the fraction of the interface that is at the correct prism angle for a given voltage. A simple calculation for this is to consider the horizontal position, \( x_p \), of the peak, tallest point, along the interface measured from the center of the droplet base. The fill factor, \( e_{ff} \), then can be calculated by

\[
e_{ff} = \frac{1}{2} + \frac{x_p}{x_d}
\]

where \( x_d \) is the cross sectional diameter of the droplet at the substrate. As the peak approaches the contact line and thus \( x_p \) approaches \( x_d/2 \), then \( e_{ff} \) approaches one or a perfect fill factor. For the 450 V profile in figure 2.16 a fill factor of 0.66 is achieved meaning approximately 66% of the display light will be projected at the desired angle. However, this calculation assumes that the angle of the interface is constant from the peak until the contact line. As seen in figure 2.14 this is not the case, and thus the fill factor is over calculated for a given profile.

In addition to reducing the actual fill factor, the curvature of the interface also acts as a lens blurring the projected image. This blurring is undesirable, and will affect the viability of the EHMP with respect to automultiscopic displays since the ability to maintain a displays resolution will be defeated if the projected image is blurry. Therefore, a second calculation is performed to determine the average curvature of the interface between the peak and contact line of the droplet.
As the average curvature increases, so does the blurriness, allowing this calculation to be a measure of the quality of the projected image.

In the case of the 450 V profile in figure 2.14, as mentioned before the fill factor is 0.66 and the average curvature is 6.2 ± 1.7 mm⁻¹. This indicates that though the interface curvature has reduced by almost a factor of two from its initial curvature of 10 mm⁻¹, there still is significant curvature along the interface. This leads to the conclusion that the actual fill factor will be less than predicted and the projected image will be blurred. The initial results from figure 2.14 indicate that the EHMP may not have much viability with respect to automultiscopic displays, however it is still essential to create and test a prototype to confirm these results with experimental data.

2.4.2 High Voltage Results

The low voltage simulations demonstrated that it is possible to deform the cross section of the droplet into a triangular shape by applying a voltage between the droplet and an anti-symmetric electrode. In addition, it was concluded that the leading edge of the electrode controls the direction and amount of deformation experienced by the droplet. However, as the voltage of the system was increased to 500 V it was found that the simulation fails to find an equilibrium deformation (figure 2.19). At 500 V it was found that the force from the electric pressure was too great for the surface tension to compensate for, and surface particles were pulled away from the bulk of the droplet. As holes formed behind the surface pressure forces could not react by drawing particles up from the bulk of the droplet.

Though no specific failure mechanism was designed in the model, it is understandable that there would be an electric pressure that would be too strong for the particle nature of the method to compensate for. As the vertex becomes more pronounced fewer particles can occupy
the space just inside of the vertex and thus the model will break down physically. This effect was present regardless of the size of the time step chosen. From these experiments it can be shown that at voltages below 500V for a 200 μm droplet the numerical model can produce physically reasonable results for the system and reach an equilibrium deformation. However, at higher voltages the model breaks down and should not be used to predict deformations.

Figure 2.18 Failure of SPH model at 500 V. Left: Simulation just before failure. Right: Simulation just after failure when surface particles pull away from the bulk drop.
Chapter 3: Hydrophobic Patterning of Surfaces using Microcontact Printing

One of the most important design considerations of an EHMP is how to pin the contact lines of the water droplet in order to allow for the deformation of the droplet into a prism shape. If the contact lines were not pinned and a single electrode was powered then the droplet would center itself under the electrode while maintaining an axisymmetric cross-section. Similarly, if two electrodes were powered, the droplet would center itself equidistance between the two electrodes to maintain a symmetric electric field distribution along the surface of the droplet. However, with pinned contact lines and an off center electrode for actuation the droplet may deform into a shape similar to a triangular prism.

As discussed in chapter 2, the difference between the contact angles of the hydrophilic and hydrophobic regions determines the strength of the contact line pinning. The larger the difference the stronger the pinning is. Therefore it is essential to ensure that the difference between the surface energies, and thus the contact angles, is maximum to ensure the contact line remains pinned during deformation. As detailed in the introduction, currently the methods for accurate, quick, and repeatable patterning of electrodes are limited for the required transparency and line resolution.

One technique that overcomes a number of these limitations is microcontact printing [48]-[51]. Microcontact printing is very similar to standard stamping techniques where a stamp with a pattern cut into it is evenly coated in ink. Once inked, the stamp is placed in uniform contact with the substrate to be stamped and the ink is transferred from the stamp to the substrate. However, wherever the pattern had been cut out of the stamp the ink does not transfer since the indented stamp surface does not come into contact with the substrate. Thus, a negative
pattern of the cut out pattern in the stamp is printed onto the substrate with ink. In microcontact printing the patterns that are being stamped typically are on the micrometer scale so the stamp is fabricated through micro fabrication techniques such as soft lithography [48]-[51].

Currently research in microcontact printing has looked at using nanoparticles as the ink in the stamping process [48]-[51]. This allows for nanoparticles to be precisely patterned onto a substrate for various applications such as controlled nanotube growth [48]-[51]. However, the implementation of an EHMP requires the patterning of hydrophobic surfaces. To achieve this a microcontact printing technique was developed to deposit polytetrafluoroethylene (PTFE) nanoparticles that create patterned hydrophobic surfaces (figure 3.1).

![Process flow chart](image)

Figure 3.1 Process flow chart. Prior to starting the ink is prepared by mixing dry 300 nm diameter PTFE nanoparticle powder into ethanol at a 1:50 weight ratio. a) Initial airbrush deposition onto PDMS (10:1 base to hardener) stamp, b) additional particle thickness added with additional coatings, c) stamp air dried, d) 1 \( \mu \)m SU-8 layer spun onto glass, e) SU-8 heated to 100\(^\circ\)C for 2 minutes, f) uniform contact made between stamp and SU-8 for 10 seconds, g) stamp is removed and device is cooled, h) SU-8 cured through backside exposure, i) sample is heated to 130\(^\circ\)C for 20 min to reflow the coating on the PTFE nanoparticles.

In detail, an airbrush is used to disperse an even coating of PTFE nanoparticles (Microdispers – 200, Polysciences Inc.) onto a PDMS (Sylgard 184) stamp by atomizing a solution of PTFE nanoparticles dispersed in ethanol (figure 3.1a). Multiple passes with the
airbrush allows for a thicker layer of particles to transfer to the substrate (figure 3.1b). Once the stamp is inked, a 1 µm thick layer of SU-8 (Microchem, SU-8 3005) is spin coated onto the desired substrate and heated to 100 °C (figure 3.1d/e). While the substrate is held at 100 °C, the stamp is brought into uniform contact with the SU-8 layer (figure 3.1f). No additional force is required to transfer the particles beyond slight pressure to ensure that there are no air pockets trapped between the stamp and substrate. The stamp is held in uniform contact for 10 seconds before removal after which the substrate is removed from the heat source and let cool (figure 3.1g). After cooling, the newly patterned surface is exposed to ultra violet light to crosslink the SU-8 increasing its durability, and subsequently heated to 130 °C to reflow the dispersion coating on each of the PTFE nanoparticles (figure 3.1h/i) [46],[47].

3.1 Inking PTFE Nanoparticles onto a PDMS Stamp

The inking step is crucial since the order and uniformity of the particles on the stamp directly controls the order and uniformity of the final particle layer on the substrate. As mentioned previously in the introduction, there have been various inking techniques devised to provide the highest packing density and uniformity of nanoparticles on the stamp [48]-[51]. However, in all of the previous applications of the transferred nanoparticle patterns it was desirable to have a single monolayer of particles. This desire led to highly specialized inking processes that are only able to produce single monolayers of particles [48]-[51]. Through initial testing of different inking processes it was determined that it is actually necessary to have multiple layers of PTFE nanoparticle to increase the hydrophobicity of the final nanoparticle layer. From this finding it was evident that previous inking techniques were not suitable for the application of PTFE nanoparticles to patterning hydrophobic surfaces.
To produce a uniform coating of PTFE nanoparticles that was multiple layers thick an airbrush was used to ink the PDMS stamp. The airbrush was fabricated in house that allowed for various parameters to be adjusted that controlled the atomization of the PTFE nanoparticle and ethanol solution (figure 3.2).

Figure 3.2 In house fabricated airbrush. a) Schematic of airbrush fabricated from a pressure chamber and concentric needle tips. The inner diameters are 0.2 mm and 0.8 mm for the inner and outer needles respectively. Stream atomization is controlled through the input air pressure ($P_{in}$), the distance between the inner and outer needle ends ($d$), ink flow rate ($Q$), and the distance from the substrate ($L$). b) In-house fabricated airbrush disassembled. c) In-house fabricated airbrush assembled.

By controlling the parameters outlined in figure 3.2 the size of the atomized droplets could be controlled so that the ethanol would be evaporated by the time the droplets reached the PDMS surface. With no pooling of ethanol on the surface the nanoparticles pack randomly along the PDMS surface while also being deposited multiple particle layers thick. Though the airbrush does not provide tightly packed layers of nanoparticles, it does rapidly deposit an average uniformly thick layer onto the stamp. In addition, the randomness of the nanoparticle deposition
increases the roughness of the nanoparticle surface that in turn increases the hydrophobicity of the final nanoparticle layer [46],[47].

3.2 Transferring PTFE Nanoparticles to an ITO Coated Substrate

When transferring nanoparticles from the stamp to a substrate using microcontact printing the biggest concern is the surface affinity between the nanoparticles and the different surfaces [48]-[51]. Ideally the surface affinity between the nanoparticles and the stamp is lower than the affinity between the nanoparticles and substrate. However, in general the surface affinity between the nanoparticles and the substrate is fixed based on the process and application. Therefore it is desirable to have a stamp material where the surface affinity can be tuned based on the nanoparticles and substrate being used. PDMS has become a standard stamp material because exposure to oxygen plasma can alter its surface energy. Initially PDMS has a low surface energy, however after exposure to oxygen plasma the surface energy drastically increases. Using this technique its surface affinity to nanoparticles can be tuned [48]-[51].

The difficulty in transferring PTFE nanoparticles to an ITO coated substrate is that the surface energy of the nanoparticles is quite low whereas the surface energy of ITO is quite high. Therefore there is little surface affinity between the nanoparticles and the ITO substrate. This poses a problem when using PDMS as the stamp substrate because neither energy state of the PDMS has significantly lower surface affinity with the nanoparticles than the nanoparticles have with ITO. In the case of the non-treated PDMS with low surface energy, the surface affinity with the nanoparticles is considerably higher than that of the ITO so the transfer percentage of nanoparticles is negligible. However, the transfer percentage of particles does not increase much with a plasma treated PDMS stamp because the increase in surface energy only matches that of ITO.
To overcome this difficulty PMMA has been used in the past as an adhesive to promote transfer of nanoparticles to surfaces with low affinity [50]. PMMA was chosen due to a low glass transition temperature so that it would retain some viscosity when heated to help promote the transfer of particles. However, PMMA is not a durable adhesive due to being readily dissolved in various liquids such as acetone [50]. To overcome the issue of durability, the current work substituted SU-8 photoresist for the PMMA as the adhesive. Once the SU-8 is cross-linked it is considerably more durable than PMMA both to physical and chemical exposure.

3.3 Process Optimization and Characterization

With the fabrication process developed it was necessary to characterize the process based on its repeatability. Ideally between different fabrication batches the process will be able to consistently pattern the desired features using the nanoparticles. In addition, it should be able to provide consistent particle film thickness, and thus contact angle, between the different fabrication batches. To ensure repeatability the process can be broken into two separate characterizations, inking and stamping.

3.3.1 Inking Process

3.3.1.1 Parameter Optimization

The first step in the characterization of the inking was to optimize the operational parameters of the airbrush. As mentioned in section 2.1 there are four different parameters that directly influence the deposition process; the input air pressure $P_{\text{in}}$, distance between the inner and outer needle ends $d$, ink flow rate $Q$, and the distance from the substrate $L$. In addition to these four parameters the concentration $C$ of PTFE nanoparticles in the ink solution impacts the deposition. Optimization of these parameters allows inking where the ethanol evaporates before the
nanoparticles reach the surface so there is no pooling of ink on the substrate. Without pooling of ink, the nanoparticles can deposit in multiple layers since there are no fluid flow fields and diffusion to rearrange them. To optimize the parameters a simple test was set up where the airbrush was held stationary above a substrate, and deposition occurred for five seconds. The input pressure, nozzle height, and flow rate were all changed in subsequent experiments, while the concentration was held constant at a weight ration of 1:50 PTFE to ethanol. Since both concentration and flow rate dictate the number of particles deposited in a given time frame it would be redundant to adjust both so concentration was held constant due to the ease of adjusting the flow rate. In addition, it was found that the distance $d$ influenced the size of the atomized droplets, which could also be controlled by varying $P_{in}$ and $Q$. Therefore, to simplify the optimization even further, the distance between needle tips is fixed at $d = 1$ mm. The results from the test can be seen in figure 3.3. $P_{in}$ was controlled via a standard pressure regulator attached to the building high-pressure air supply, $L$ was controlled by fixing the airbrush on an adjustable stand, and $Q$ was controlled using a syringe pump.

Each parameter influences the deposition process in distinct ways. $P_{in}$ influences the velocity of the atomized droplets traveling from the airbrush to the substrate. As can be seen in figure 3.3, if $P_{in}$ is increased for a given $L$ and $Q$ there is less time for the ethanol to evaporate increasing the chance of pooling. In similar fashion, as $L$ increases there is further distance to travel allowing more time for the ethanol to evaporate. Therefore, as see in figure 3.3, for a fixed $P_{in}$ and $Q$ there is less chance of pooling as $L$ increases. Finally, as $Q$ increases there is more liquid present in the air stream so for a given $P_{in}$ the size of the atomized droplets will be larger meaning more time is required for the ethanol to evaporate. Therefore, as $Q$ increases for a fixed $P_{in}$ and $L$ than there is a higher chance of pooling.
Figure 3.3 Pooling test results for $P_{\text{in}} = 5/10/15$ psi, $Q = 10/20/30$ mL/hr, and $L = 2/3.5/5$ cm. Filled circles represent pooling, and open circles represent no pooling.

Though there were a number of combinations that didn’t develop pooling, initially it was determined that $P_{\text{in}} = 15$ psi, $L = 5$ cm, and $Q = 20$ mL/hr was ideal. Beyond pooling, the amount of particles deposited as well as the inking area were also taken into account in the optimization. The larger $P_{\text{in}}$ allowed for a higher $Q$ meaning the particles could be deposited at a faster rate reducing inking time. In addition, the higher the nozzle was from the substrate increased the inking area also reducing the inking time. However, described in detail in the next chapter a linear stage is used to move the airbrush at a constant velocity along the stamp. This added velocity altered the optimization by reducing pooling by spending less time inking a given spot on the substrate. Increasing $P_{\text{in}}$ and $L$ while decreasing $Q$ compensated for this. It was found that $P_{\text{in}} = 20$ psi, $L = 5.5$ cm, and $Q = 15$ mL/hr were optimal while using the linear stage.
3.3.1.2 Inking Uniformity

To be a repeatable process the airbrush needed to be able to consistently ink a uniformly thick layer of nanoparticles over a large area. Assuming that all of the airbrush parameters are held constant as determined in section 2.3.1, a uniform layer can be deposited by ensuring that the airbrush spends equal time inking each region of the substrate. To achieve this, the airbrush would be attached to a linear stage that moves it in a straight line. Using a constant velocity on the linear stage ensures that there will be uniform inking along a line parallel to the direction of travel. To uniformly coat large areas it is then as simple as inking multiple parallel straight lines over the surface. If spaced properly then the final layer will be uniform over the whole area.

The first step in characterizing this inking technique is to ensure that the inking along the lines using the linear stage is indeed uniform. In addition, it is necessary to ensure that the thickness of the lines is repeatable over multiple inking runs. To test this the airbrush was attached to the linear stage and lines were inked separately onto multiple glass slides. The opacity, defined as the fraction of incident light on a surface that is either absorbed or reflected, of the particle layer was measured for each test line and was used as a representation of particle layer thickness. As the particle layer increased in thickness more light would get reflected and scattered increasing the opacity of the layer. Though the exact correlation between particle layer thickness and opacity of the film is unknown, it is unnecessary in terms of characterizing the repeatability and uniformity of the process. In particular, it is assumed that two layers with the same thickness will have the same opacity allowing repeatability and uniformity to be characterized by comparing relative opacity between the samples.

To measure the opacity of the samples a test apparatus was designed as outlined in figure 3.4. A white laptop screen was used as a white light source to backlight the sample while a
LaVision Imager Pro camera was used to capture an image of the sample. The sample was removed and a second image captured the background light distribution. Finally, a normalized opacity image (figure 3.5a) was created by taking the ratio of pixel values between the sample image and the background light distribution image. This normalized opacity image consisted of pixel values ranging from one for completely opaque, total light reflection/absorption, and zero for completely transparent. Using the normalized opacity images for each of the samples, cross section opacity profiles of the lines (figure 3.5b) were calculated by averaging the opacity of the lines along the direction of travel from the linear stage (along the arrow in figure 3.5a). It was found that for a given sample the standard deviation of opacity along the direction of travel was less than 10% for each sample. In addition, as can be seen from figure 3.5b the average maximum opacity for each trial is 0.19 ± 0.025, while the average full width half max (FWHM) is 7.67 ± 0.67 mm. Through this analysis it was shown that there was repeatable uniformity in layer thickness along the direction of travel of the linear stage within 13%.

![Opacity test apparatus](image)

Figure 3.4 Opacity test apparatus. White light produced from laptop screen passes through a sample and into a LaVision Imager Pro camera.

Having shown the repeatability of the airbrush spray, the final step in characterizing the inking process was to ensure that multiple parallel lines could provide a uniform particle layer over a larger area. Using the FWHM of the single line profiles calculated above as a starting point, four parallel lines were inked onto glass slides at 7, 8, and 9 mm spacing (figure 3.6).
As seen in figure 3.6, each of the samples provides a definite plateau in opacity between the parallel lines. In addition, as the spacing decreases the peaks from the individual lines becomes less defined, as expected. For each sample, the opacity variation remained below 10% along the length of the line continuing the repeatability of the inking defined above. From the above tests, a final spacing of 8 mm was determined as the ideal spacing because it optimized the fluctuations in opacity between the lines allowing for large areas of substrate to be covered in a uniform layer of PTFE nanoparticles.

Figure 3.5 Sample single pass opacity measurements. a) Sample normalized opacity image; the arrow indicates the direction of airbrush displacement. Black line represents profile cross-section that is plotted in (b). b) Multiple single coating straight-line opacity curves.

3.3.2 Stamping Process

The next step in characterizing the fabrication process is to analyze the stamping process. In particular, it is important to characterize the percentage of particles transferred from the stamp to the substrate, the contact angle of the final particle layer, and the resolution of the stamping process. As mentioned in section 2.1, it was determined that multiple layers of particles were necessary to create a patterned film with a high contact angle that led to the use of the airbrush to
ink the stamp. However, in the inking characterization all of the tests were run with only a single pass with the airbrush. To ensure that the final film does have enough particle layers to give an adequately high contact angle, all of the stamping characterization tests were run for different number of coatings, or passes, with the airbrush.

![Graph Image](image.png)

**Figure 3.6** Opacity curves of four single coating straight lines with spacing 7 mm (blue), 8 mm (green), and 9 mm (red) between them.

### 3.3.2.1 Particle Transfer

To analyze the percentage of particles transferred, five blank stamps without structure were inked each with a different number of coatings. These stamps were then run through the stamping process as outlined in figure 2.1. It was decided that again the actual thickness of the particle layers was unnecessary to know, so opacity measurements were made of the stamp after inking, the stamp after transfer, and the final deposited film (figure 3.7). For 100% particle transfer, the opacity of the stamp after inking would match the opacity of the deposited film. This would leave the stamp after transfer to have zero opacity since there would be no more particles
left on the stamp. As seen in figure 3.7, for a single coating the particle transfer is as expected for near perfect transfer. However, as the number of coatings increases a peculiar trend occurs where the opacity of the deposited film is higher than the initial inked stamp. In addition, the stamp after transfer has a non-negligible opacity so there are still particles left on the stamp.

![Opacity measurements of a stamp before and after a coating as well as the opacity of the deposited film vs. the number of coatings in the inking process.](image)

At first glance this might seem to indicate that there is an increase in the number of particles during the stamping process. However, the assumption of relative opacity only applies when the packing of the nanoparticles remains constant across the samples being compared. In the case of the stamping process, the particles initially have loose packing due to the random deposition process from the airbrush. It is hypothesized that when transferred via pressure the particles rearrange into tighter packing when applied to the film. This tighter packing has a higher opacity since light will be more readily scattered when travelling through the film even though not all of the particles transferred to the substrate.
This hypothesis is supported by looking at the error bars in figure 3.7 which represent the variability in opacity across a single sample. As can be seen in figure 3.7, in all cases the error bars of the deposited film are significantly less than those of the inked stamp. If the particles were indeed rearranging into tighter packing then it would be expected that the particle order in the film would be less random than when applied by an airbrush. This would cause more uniformity in the opacity across the sample resulting in smaller error bars as seen in the plot.

3.3.2.2 Contact Angle

In addition to characterizing the particle transfer percentage it is necessary to measure the contact angle of each of the deposited films. As seen in figure 3.8 the contact angle of the deposited film increases with more coatings and thus more particle layers as evidenced by the fact that the additional particle layers can be brushed off with a wipe or finger.

![Graph showing contact angle vs. number of coatings](image)

**Figure 3.8 Contact angle of water on deposited film vs. the number of coatings in the inking process.**

This is expected because the larger number of particle layers ensures that there is no exposed SU-8 that the water can wet. Needing multiple particle layers is a consequence for this particular method due to the randomness of the deposition by the airbrush. If the particles were perfectly
packed then only two layers would be needed to cover any exposed SU-8. However, the randomness of the airbrush deposition prevents perfect packing thus multiple layers are required to achieve maximum SU-8 coverage. However, one advantage of the multiple particle layers is that the randomness increases the roughness of the surface due to particle agglomerations. The increase in roughness of the surface will provide higher contact angles than what can be achieved with a flat PTFE surface [46],[47].

The final step of characterizing the patterned film is to compare the contact angle difference between exposed SU-8 and the PTFE nanoparticle film. As mentioned before, the magnitude of this difference corresponds to the strength of the contact line pinning. The greater the difference, the more force on the surface of the droplet is required to unpin the contact line. By exposing the sample to oxygen plasma the SU-8 drastically reduces its contact angle while the PTFE maintains it contact angle allowing for an even greater difference. As seen in figure 3.9, after plasma exposure it is possible to get a contact angle difference of 134°.

![Figure 3.9 Contact angle measurements. Water droplet on plasma exposed SU-8 (left) with contact angle 18° and microcontact printed PTFE nanoparticle surface (right) with contact angle 152°. Five coatings were used in fabricating the nanoparticle surface.](image)

### 3.3.3 Resolution Characterization

The final test of the fabrication process is to test the resolution of the stamping process. From the data collected in section 2.3.2.2 it was decided that five coatings was necessary to ensure a
maximum contact angle on the final film. As can be seen in figure 3.10, minimum features of 30 µm were readily achieved using five coatings during the inking process.

Figure 3.10 PTFE minimum features: a) line, b) spot, and c) hole respectively. Scale bar is 30 microns for all images.

These features match the corresponding mask and PDMS stamp in that their features were also 30 µm. However, as can be seen there is some roughness along the edges of the features seen in figure 3.10. This can be attributed to the randomness of the deposition as well as particles sticking to the PDMS stamp and not transferring.

As mentioned in the introduction, previous microcontact printing techniques have been able to have single particle resolution [50],[51]. However, due to the randomness of the inking from the airbrush, and the multiple particle layer requirement it is expected that the resolution of the PTFE nanoparticle process would be considerably lower. It has been shown experimentally that the ideal aspect ratio of the stamp features is on the order of 1:2 height to width [50],[51]. This is due to buckling of features in the stamp during the stamping process. Thus to have a resolution of 1 µm the depth of the features on the stamp would be 0.5 µm. However, the PTFE nanoparticles have a diameter of 200-300 nm so it would take on average three layers of particles to fill the features in the stamp thus preventing the features from being patterned. It is currently unknown how many particle layers a single coating creates; however it is known that a single
coating creates multiple layers. Therefore, it is understandable that due to this process requiring five coatings the limit for resolution would be around 30 µm.
Chapter 4: Results

Discussed in chapter two, a numerical modeling technique was developed using FEM and SPH to predict deformation voltages for an EHMP as well as determine the dependence of the deformation on electrode geometry. In the developed model, COMSOL is used to initially calculate the electric pressure along the surface of the droplet being modeled. Once calculated, this electric pressure is exported to Matlab where it is used as a forcing term in a SPH fluid dynamics simulation that deforms the droplet. After an equilibrium deformation is achieved based on the electric pressure distribution, the geometry is exported back to COMSOL completing a single iteration. Through multiple iterations, a final equilibrium deformation is achieved for a given initial geometry and voltage. The model was used to predict deformations for a 200 µm diameter droplet in the voltage range of 400-550 V, as well as determine that the position of the leading edge of the electrode influences the final deformation of the droplet.

Discussed in chapter three, a modified microcontact printing technique was developed to pattern hydrophobic surfaces using PTFE nanoparticles. An airbrush was used to deposit a mixture of PTFE nanoparticles and ethanol onto a PDMS stamp. By optimizing the parameters of the airbrush, the ethanol would evaporate shortly after deposition onto the PDMS surface leaving a dry uniform layer of PFTE nanoparticles. Due to surface affinities, a 1 µm thick layer of SU-8 was spun on to the substrate to promote the transfer of nanoparticles from the stamp. This printing technique was readily able to achieve 30 µm resolution as well as provide a 134° contact angle difference between the PTFE nanoparticles and oxygen plasma exposed SU-8.
4.1 Prototype Fabrication

The initial goal for the prototype fabrication was to create a single EHMP and test the feasibility of the concept for automultiscopic displays. Since the prototype was to be used for feasibility experiments, the design of the test apparatus was focused on simplicity and minimizing fabrication time. Based on this goal, it was essential to develop a simple droplet formation technique that does not require advanced tools. Using the microcontact printing technique a prototype 200 µm x 1.5 cm hydrophilic slit was fabricated on a 25 mm x 75 mm glass microscope slide to test various formation techniques of the elongated droplet in an EHMP (figure 4.1).

Figure 4.1 Elongated droplet formation techniques. a) Pipette technique. b) Dip technique. c) Rolling drop technique. d) Atomization technique.

The first technique tested was to use a pipette to dispense a small (<2 µL) droplet of water onto an area of the exposed hydrophilic slit (figure 4.1a) with the intention that the surface energy would draw the liquid into the hydrophilic area. However, it appeared that the surface tension of the droplet was too strong and the volume of the drop too large to allow the droplet to
draw into the hydrophilic area. Instead, the drop would maintain its spherical shape while being pinned on the substrate surface due to the hydrophilic slit.

The second technique tested for the droplet formation was the dip technique (figure 4.1b). In this technique, the sample is dipped into a beaker of water and drawn out slowly at a shallow angle. As the sample is drawn out of the beaker, ideally the hydrophobicity of the PTFE surface prevents water from wetting the surface wherever the PTFE is covering. In regions where there is no PTFE nanoparticles, such as the slit of exposed SU-8, water would wet the surface due to its hydrophilic nature. Once the sample is fully removed from the beaker, water will have only wetted the exposed SU-8 slit creating the elongated droplet.

In practice water would wet the PTFE surface as well as the exposed SU-8 creating a thin layer of water over the whole sample surface. Upon further testing, it was found that while the PTFE nanoparticle surface had a high hydrophobic contact angle it also had a high roll off angle. This means that as a horizontal PTFE surface with a water drop on it is tilted towards vertical a large angle of tilt is needed before the droplet rolls off of the surface. This is attributed to the randomness of the PTFE deposition technique using the airbrush. On a macro scale the deposited PTFE layer is quite uniform in thickness as demonstrated in chapter 3. However, on a micro scale the uniformity decreases dramatically creating small regions where there will be exposed SU-8 that the water can wet. As the sample is drawn out of the beaker in the dip technique, these small regions of exposed SU-8 provide various points of wetting for the water on the surface allowing a thin film of water to form over the whole sample surface preventing the formation of an elongated droplet.

In an attempt to overcome the steep roll off angle, a third technique was devised coined the rolling drop technique (figure 4.1c). In this technique, the sample is held at an angle steeper
than the roll off angle and a drop is applied to the surface using a pipette. Once released from
the pipette, the drop rolls down the surface without wetting the PTFE covered areas. As the drop
passes over the exposed SU-8 slit a small amount of water wets the SU-8 and is left behind while
the bulk of the droplet continues to roll off the sample. Though this technique successfully
wetted a small region of the SU-8 slit, it could not fully wet the slit with a single droplet. In an
attempt to fully wet the slit, multiple drops were rolled over the sample. However, as a rolling
droplet would make contact with an already wetted region of the slit it would actual draw the
water from the slit back into the droplet and reverse any wetting that was achieved previously.

The final technique that was tested was the atomization technique (figure 4.1d). In this
technique the airbrush that was built in chapter 3 was used to atomize water creating a region of
high-density mist that the sample was inserted into. The aim of the technique was similar to the
dip technique described above where the atomized water droplets in the mist would only wet the
exposed SU-8 surface and not the PTFE surface. However, similar to the dip technique the
atomization technique failed because the atomized water droplets would wet the small regions of
exposed SU-8 in the patterned PTFE nanoparticles leading to a film of water forming over the
whole sample.

Based on the limitations of the PTFE surface and the various droplet formation
techniques, the dimensions of the slit were increased to 1 mm x 1.5 cm so that it could readily be
wetted using a 2 µL pipette by hand. Though these dimensions for the prototype EHMP are
significantly larger than the 200 µm originally planned, they will still provide meaningful results
to demonstrate the concept of an EHMP and validate the numerical model.

The main worry of scaling the system is that as the forces scale a failure mechanism will
either appear or disappear after scaling. From the initial results in chapter 2, it became evident
that one of the major potential failure methods will be dielectric breakdown. The voltages required to achieve adequate deformation (500 – 550 V) are near the border of dielectric breakdown in the oil between the droplet and electrode. For the dimensions in the model the maximum potential gradient while deformed is around 14 MV/m, which is close to the 15 MV/m breakdown voltage of the oil. Therefore, it is important to know how scaling the prototype dimensions will affect the potential failure of dielectric breakdown.

As described in chapter 2, the balance of the electric pressure with surface tension dictates the deformation of the droplet. From equation 2.3, it is shown that \( P_e \sim E^2 \) where \( P_e \) and \( E \) are the electric pressure and electric field respectively. A simple scaling of the electric field is \( E \sim Vd^2 \) where \( V \) and \( d \) are the applied voltage and the distance between the electrode and droplet respectively. For surface tension, equation 2.1 shows that \( P_L \sim r^{-1} \) where \( P_L \) and \( r \) are the Laplace pressure across the droplet interface and radius of the droplet respectively. The scaling of \( d \) and \( r \) will be on the order 1:1, therefore we can see that as the dimensions of the EHMP are scaled up the ration of \( P_e \) to \( P_L \) will decrease. This means that as the dimensions are scaled up it will take a higher applied voltage to achieve the same amount of deformation as a smaller system. Since the breakdown voltage can be approximated as \( V_{bd} \sim Vd^{-1} \), it can be deduced that if dielectric breakdown does not occur in the 1 mm x 1.5 cm prototype than the smaller scale EHMP should not experience breakdown either.

4.2 Experimental Apparatus

With the droplet formation technique developed, the final step was to develop an experimental apparatus and test a prototype EHMP. One of the major difficulties in designing an experimental apparatus was how to image the deformation of the droplet. However, with the scaling to a 1 mm diameter droplet a stereoscope can provide the adequate magnification to capture the
deformation of the droplet. Using the stereoscope requires that the droplet be backlit so that its profile is easily distinguished. This creates a problem with the original EHMP design when using oil as a surrounding medium to the drop since the back lighting will have to travel through an oil and air interface on either end of the droplet. If this interface is not perpendicular to the plane of the PTFE patterned sample then it will cause distortions in the lighting and imaging. One way to overcome this problem is to create a completely enclosed sample with viewing windows for the droplet to be illuminated and imaged. However, this would require extensive design and fabrication work to get an apparatus built where, as stated before, the main goal of the prototype is to simply test the concept of the EHMP using a simple apparatus.

To simplify the prototype EHMP test apparatus air was used as the surrounding medium instead of oil. While solving the problem of imaging, using air as the surrounding medium causes a few changes in how the system will behave. As seen in equation 2.3, \( P \propto \varepsilon_r \). For the silicone immersion oil initially intended to be used as the surrounding medium, \( \varepsilon_r = 2.2 \). However, for air \( \varepsilon_r \) is essentially unity causing a decrease of about half in the electric pressure for a given voltage. This leads to higher voltages being required to get the desired deformation from the droplet. As mentioned in the previous section, there is concern that the voltages required to achieve adequate deformation are on the edge of dielectric breakdown while using oil. This increase of voltage, coupled with the fact that the dielectric breakdown of air is 5 time less (3 MV/m ) than oil, only increases that concern.

In addition to increasing the concern of dielectric breakdown, using air as the surrounding medium also introduces the problem of evaporation. In this configuration the surrounding air will be at the relative humidity of the lab that will certainly be less than 100 % relative humidity. In this case there will definitely be evaporation of the droplet meaning the elongated droplet will
have to be continually refilled to maintain a constant volume. This will cause error when using the experimental results to validate the numerical model since the cross sectional area of the droplet will be constantly changing due to evaporation. This error can be minimized since a 2 µL pipette can be used to reform the droplet for each experiment. Though these changes do bring drawbacks to using air, the benefit of having a simple system that can easily be tested outweighed the drawbacks mentioned above. If an open-air system is able to achieve deformation and show that the EHMP is a feasible concept than using oil will only improve the performance.

Figure 4.2 depicts the final design of the open-air system that was designed to test the prototype EHMP.

![Diagram of experimental apparatus](image)

**Figure 4.2** Front and side view of the experimental apparatus to test the prototype EHMP. V is the voltage applied between the electrode and droplet.

The system consists of two 25 mm x 75 mm ITO coated glass slides. One of the slides is patterned using the microcontact printing fabrication technique with the 1 mm x 1.5 cm slit. The fabrication process was altered slightly when the SU-8 was initially spun onto the ITO glass slide. A piece of Scotch tape was used to cover a small area of the ITO slide before the SU-8
was applied. Once spun, the scotch tape was removed revealing a region of uncoated ITO that is used later as the electric connection to ground the water droplet.

The second ITO glass slide is unchanged and used as the anti-symmetric electrode above the droplet. When a voltage is applied between this slide and the droplet deformation occurs. Each of these is attached to a 25 mm x 75 mm microscope glass slide using a cyanoacrylate adhesive (Krazy Glue, KG585). These microscope slides are arranged so that the ITO slides are facing each other as seen in figure 4.2. The spacing between the ITO slides is dictated by stacking 150 µm thick glass coverslips between the two 25 mm x 75 mm glass slides. Exact spacing between the two ITO slides is measured using image analysis techniques and is discussed later in section 4.3.3.

This apparatus is situated between a desk lamp and the stereoscope (Fisher Scientific, model 12-562-6). The desk lamp provides the back illumination while a camera (Nikon Digital Sight DS-U1) is used to capture images through the stereoscope. To provide the electric potential between the electrode and droplet, a 0-30 kV power supply (Gamma High Voltage Research, model ES30P-10W) is connected to the two ITO slides with the ground connector attached to the PTFE patterned slide and the positive connector attached to the blank ITO slide.

4.3 Results

In order to adequately test the feasibility of the EHMP, there are three main experimental tests that need to be performed. The first experiment is to test the low voltage regime where deformation occurs and characterize the deformation versus voltage. The second experiment is to test the high voltage regime where failure occurs to determine what the failure mechanism of the system is. The third experiment is to test how well the EHMP can deflect light to determine its feasibility for an automultiscopic display. In addition to the following experiments, it is
necessary to validate the numerical model with the low voltage deformation results. Finally, using the validated numerical model, and the knowledge gained from the different experiments, a final numerical simulation can be performed on an accurate to scale model of an EHMP that can be used as a final conclusion about the feasibility of the EHMP for an automultiscopic display.

4.3.1 Low Voltage Results

The first test of the EHMP was to test the deformation of the droplet in the low voltage regime. To test this regime, the EHMP was formed using a premixed 0.85 % weight/volume sodium chloride and water solution (LabChem Inc., Catalog number LC23450-2) using the pipette method. Using the testing apparatus described in section 4.2, the voltage between the droplet and electrode was slowly ramped from 0 V to 1800 V while the stereoscope and Nikon camera captured cross sectional images of the droplet. Figure 4.3 presents a selection of cross sectional images of the droplet deformed at various applied voltages.

![Image of droplet cross section](image)

**Figure 4.3 Images of the droplet cross section under different applied voltages. a) 0 V, b) 1500 V c), 1800 V.**

As can be seen in figure 4.3, the deformation of the droplet is minimal even at voltages up to 1800 V. This minimal deformation prevents the peak of the droplet from displacing far from the center of the base leading to a small fill factor. In addition, there is still significant curvature along the surface of the droplet meaning any projected light would experience significant divergence. The hydrophobic patterning using PTFE nanoparticles has successfully pinned the contact line of the droplet ever under significant electric fields. In addition, the deformation of
the droplet occurs such that the droplet is drawn towards the electrode where if the deformation were more pronounced it would create a prism shape.

4.3.1.1 Validation of Numerical Model

With the initial low voltage experimental results collected, it became possible to validate the numerical model developed in chapter 2. As discussed in section 4.1, there are a number of changes to the experimental EHMP system that differ from the initial numerical model presented in chapter 2. Most notably, the replacement of oil with air as the surrounding medium will cause the electric pressure to be reduced by a factor of 2.2 in the COMSOL calculation, and the surface tension will increase to 0.071 N/m. In addition, the size of the droplet has been scaled up to 1 mm in diameter from 200 µm. The factor of 2.2 can be easily changed in the electric pressure calculation set up of COMSOL, while the change in droplet size and surface tension can be corrected using the scaling technique that is described in section 2.2.3. Figure 4.4 presents the updated numerical results taking into account the new dimensions, relative permittivity, and surface tension for the applied voltages in figure 4.3.

4.3.1.1.1 Error Calculation

In order to accurately validate the numerical results, shape analysis is required to measure the similarity between the experimental profile of the droplet and the one calculated using the numerical model. Traditional shape analysis methods center around the Procrustes analysis where a shape in question is rotated, scaled, and translated to match a master shape [55]. To test the shape matching during the Procrustes analysis a comparison function such as

$$ p = \left( \sum_i \left[ (u_i - x_i)^2 + (v_i - y_i)^2 \right] \right)^{1/2} $$

(4.1)
is used where \((u_i, v_i)\) and \((x_i, y_i)\) are the coordinates of the shape in question and the master shape respectively, and \(p\) is a measure of the similarity between the two shapes. If \(p=0\), it can be said that through rotation, scaling, and translation the shape in question is identical to the master shape. As the two shapes diverge from being identical \(p\) becomes a relative measure of the similarity [54].

![Figure 4.4 Updated numerical results to match the experimental EHMP using air as the surrounding medium and a 1 mm diameter.](image)

In the case of comparing the EHMP experimental and numerical results, the traditional Procrustes analysis of rotating, scaling, and translating prevents the comparison of results from providing the desired information. As mentioned previously in chapter 2, the main purpose of the numerical model is to predict the deformation of a droplet for a given electrode geometry and applied voltage. If transformations are required to achieve accurate results with the numerical model, it shows that the model does not accurately represent the physical system and should not be used for predicting deformations. Therefore, in order to test the validity of the numerical model, and thus use it to predict deformations using different electrode geometry and voltage
combinations, the initial transformation techniques used in the Procrustes method are ignored and only the similarity calculation is used.

Though useful to optimize the transformations used in the Procrustes method and to compare multiple shapes with a master, the major drawback of using the similarity equation 4.1 is that the value calculated is an unintuitive measure of similarity. The value $p$ is similar to the root mean square of the system in that it gives a length representing the distance between the points in the master shape and the shape to be compared. However, rather than the mean it is a measure of the root of the total lengths between the two sets of points. A more intuitive and thus useful calculation is to calculate the error, $e$, in the cross-sectional areas via

$$e = \frac{\Delta A}{A_0}$$

(4.2)

where $\Delta A$ is the non-union area of the two droplet profiles and $A_0$ is the total area of the droplet. Figure 4.5 graphically shows the areas in equation 4.1, with the error being the ratio of the grey shaded regions in figure 4.5c to the black shaded region in figure 4.5b. Intuitively, this error represents the percentage of area that is mismatched between the numerical and experimental profiles. As the numerical profile approaches the experimental profile the $\Delta A$ will tend towards zero decreasing the error with it. Experimental Profile Extraction

For the technique of measuring the error between the numerical and experimental results in the previous subsection it is imperative that the cross-sectional areas of both profiles are the same. Without equal areas, the error would be increased significantly since $\Delta A$ would also include the difference in cross-sectional areas. To ensure equal area, the first step is to accurately extract the experimental droplet profile from the captured images so that the droplet
cross-sectional area can be measured. Figure 4.6 outlines the steps required to accurately extract the profile of the drop using image analysis techniques in Matlab.

Figure 4.5 Numerical model validation error calculation. a) Droplet profiles to be compared, b) Black shaded region is the total area $A_0$, c) grey shaded region is the non-union area $\Delta A$.

In detail, the original image of the droplet cross-section is converted into a grey scale image with pixel values ranging between 0 (black) and 255 (white) (figure 4.6a). An initial binary mask is calculated by applying a threshold Boolean operation on the grey scale image (figure 4.6b). Any pixel greater than 100 intensity was set to 0 and any pixel less than that was set to 1. In some instances, there was some light reflection in the upper right edge of the droplet that caused difficulty in thresholding the whole droplet. The areas of reflection would have pixel values greater than 100 causing those regions to be labeled 0 in the binary mask. In these cases, the droplet was separated into regions, those with reflections and those without, and separate thresholds were applied to each region. The region without reflection maintained the 100 pixel intensity threshold. However, the threshold for the regions with reflection was dropped to 80.
Figure 4.6 Technique to extract the experimental droplet profile from the captured images. a) Original image is converted to a grey scale image. b) Initial binary mask calculated using thresholds. c) Smoothing of binary mask using Gaussian kernel and convolution. d) Final binary mask achieved through second threshold calculation. e)-g) Images of 0 V, 1500 V, and 1800 V overlaid with extracted profiles (green) respectively.

After the initial binary mask is calculated, a 10 x 10 pixel Gaussian kernel with a standard deviation of 10 is created using the fspecial command in Matlab. This kernel is then convolved with the initial binary mask to smooth out the mask edges (figure 4.6c). The purpose behind this smoothing is to create a mask that maintains the overall shape of the profile while reducing single pixel discrepancies caused by the initial threshold calculation.

The smoothed mask is now a black and white image with brightness scaling from 0 to 1. A final binary mask is created from this smoothed mask by applying a second threshold calculation (figure 4.6d). The threshold is set so that any pixel in the smoothed mask greater than 0.3 is set to 1 and anything less than that is set to 0. Difficulty arose in the final threshold calculation due to the edge effects of the convolution. The function conv2 used in Matlab to
perform the convolution pads the edges of the image with 0 value pixels to be able to produce an output image that is the same size as the input image. As can be seen in figure 4.6c, the bottom edge of the droplet drops to zero in pixel values due to the padded edges. To overcome this drop in pixel values near the contact lines, the initial binary mask (figure 4.6b) is used to create the mask near the contact line regions while the smoothed mask threshold calculation is used to create the mask for the bulk of the droplet.

With the final binary mask produced (figure 4.6d) the profile was extracted by tracking the transition boundary between 0 and 1 valued pixels. To accomplish this, each column of pixels was examined individually and the position of the transition from pixel values of 0 to 1 was measured. With the transition position for each column measured, an outline of the profiles could be produced (green line figure 4.6e-g).

The final step in producing the experimental droplet profiles was to scale the profile curves from pixel units to millimeters. Knowing that the distance between the contact lines of the droplet are 1 mm due to the fabrication process and the pinning of the contact lines, the profiles were scaled simply by

\[
\bar{x}_{\text{mm}} = \frac{\bar{x}_{\text{pixel}}}{n}
\]  

(4.3)

where \(x_{\text{pixel}}\) is the profile data points in units of pixels, \(x_{\text{mm}}\) is the profile data points in units of millimeters, and \(n\) is the number of pixels between the two contact lines.

4.3.1.1.2 Numerical Results Area Matching

With the profiles of the experimental results extracted from the captured images, it is straightforward to measure the cross-sectional area of the droplets using the polyarea function in
Matlab. It was found that the cross-sectional areas of the different captured droplets varied by 10%. This was expected since inevitably there would be some variability in the areas due to evaporation during the image capturing and adjusting of the applied voltage. However, as mentioned above, the error calculation using equation 4.2 is highly dependent on discrepancies between the total areas of the experimental and numerical profiles. This poses problems for the initial conditions of the simulation since it had been planned to run one droplet configuration at the different voltages to produce the numerical results. Since the model does not take into account evaporation and is pseudo incompressible the area would be fixed between the different voltage results. Therefore, for a given initial droplet configuration in the model, the voltage results could only be compared to a single experimental profile.

Due to the varying cross-sectional areas in the experimental data, a different numerical simulation with independent initial conditions would be used for each of the applied voltages. This ensured the area of the numerical profile would always match that of the experimental profile it was being compared to. To accomplish this a technique needed to be devised so that the initial area of the droplet in the numerical simulation could be adjusted. As outlined in chapter 2, the numerical model has specific initial conditions where a full circular droplet is translated to center around the origin, and rotated so that there are particles at each of the contact lines. These initial conditions force a specific cross-sectional area for the droplet that does not equal any of the particular experimental results.

It was also mentioned in chapter 2 that the initial contact angle could be adjusted by translating the center of the droplet along the y-axis. In addition to changing the initial contact angle, this translation is also a simple solution to adjust the area of the numerical droplet. By controlling the distance the droplet is translated, the area of the simulation droplet can be tuned
to match the experimental area that it will be compared to. There are two problems that arise from this translation; there is no guarantee there will be particles at the contact lines after translation, and the initial contact angle of the droplet may not match between the simulation and experiments. Since the particles within the numerical simulation have arranged themselves into equilibrium through the hydrodynamics of the system, there will be no guarantee that as the droplet is translated there will be particles at the contact line. This proves detrimental to the simulation because the contact lines cannot be pinned without a particle representing the contact line.

In order to ensure the contact lines are pinned, the droplet not only needs to be translated but also rotated. This ensures that the areas of the numerical and experimental solutions will be matched while maintaining particles at the contact lines. Figure 4.7 outlines the technique to determine the amount of rotation and translation required to match the areas. In detail, the surface particles of the original centered droplet are combined into all possible pairs, and for each pair the area created by their circular segment is calculated. Based on the calculated areas, a pair of particles is found that has the closest circular segment area to the predetermined area from the experimental results (figure 4.7a). The droplet is then rotated and translated so that this pair of particles lie on the y-axis and is symmetric around the origin (figure 4.7b).

As mentioned above, the drawback of this particular method of translation and rotation for matching the areas is that the initial contact angle of the simulation is predetermined and may not match the experiment. In general this would not be a problem if the simulation technique allowed for freely changing contact angles during deformation. This would allow for the final deformation to be independent of the initial contact angle, which it should be. However, in the SPH technique developed for the numerical model, the contact angles cannot freely change over
a wide range due to the boundary conditions of the model as well as the method of measuring the surface curvature of the drop.

Figure 4.7 Simulation area matching technique. a) Pair of surface particles (green) found that provide the desired sector area. b) Circle is rotated and translated so that the pair of surface particles represent the contact lines of the droplet.

With the model described in detail in chapter 2, the main problem arises due to the boundary condition where the particles below the y-axis are fixed. Two consequences occur due to this boundary condition; the surface tension and pressure calculations are incorrect near the contact line during deformation. Since the particles below the substrate surface, x-axis, are fixed, during deformation the curvature remains the same below the substrate surface. However, above the substrate the curvature will either increase or decrease depending on the deformation. Outlined in figure 4.8, this creates a discontinuity in the physical surface curvature near the contact line. When the curvature is calculated for particles near the contact line, the fitted polynomial will encompass particles on either side of this discontinuity. In doing so, the calculated curvature will have a smooth transition rather than a discontinuity between the above
and below substrate regions. This smooth transition causes errors in the transition zone, which is a smoothing length distance on either side of the contact line.

In the case of the EHMP, during deformation the surface curvature near the contact lines is decreasing causing the calculated curvature to be greater than the physical curvature. Ultimately this leads to an unrealistically high surface tension force on the particles near the contact line. This unrealistic increase in force will prevent the curvature near the contact line from decreasing too much, thus introducing error into the deformation simulation.

![Diagram](image)

**Figure 4.8** The effect of fixing particles below the y-axis. 

a) Initial conditions where the curvature is continuous near the contact line where $\theta_1 = \theta_2$. 

b) Particles under deformation where the curvature is not continuous and $\theta_1 \neq \theta_2$

The second consequence of the boundary condition is that as the droplet is deformed and the contact angle changes, particles near the surface of the contact line are forced inward and downward towards the y-axis. This movement creates an increase in particle density, and thus pressure that should be countered by the particles and their neighbors rearranging themselves. However, since their movement is directed towards fixed particles the rearrangement is limited. Similar to the surface tension effect above, this limited rearrangement causes an increase in density and pressure that works to counter the change in contact angle introducing error in the deformation simulation.
These errors end up being magnified when area matching occurs since as discussed before a side effect of the translation is a decrease in initial contact angle, and the errors become more pronounced as the initial contact angle of the simulation decreases. A decrease in contact angle causes a larger discontinuity in curvature during large deformations increasing the error in the simulation. In addition, the decreased contact angle redirects the particle rearrangement more towards the fixed particles again increasing the error in the simulation. Though it is difficult to quantify how large of an error these effects will have on the results, they will manifest themselves by increasing the curvature of near the contact lines leading to an overall rounder shape.

4.3.1.1.3 COMSOL Electric Field Model Refinement

Since the experimental system is open air with the glass slide ending at the edge of the electrode, it was tested whether the system could be modeled with a no charge boundary condition in place of the air and glass side corner. Figure 4.9 outlines the change in electric field when the glass slides are taken into account. As can be seen, by modeling the additional air gap above the droplet the electric field distribution changes drastically. Figure 4.9c shows that the electric pressure distribution along the surface of the droplet not only increases in magnitude when modeling the glass, but also the peak of the magnitude is shifted towards the center of the droplet and its distribution is much wider. This reduces the deformation of the droplet into the desired prism shape since there will be a significantly higher fraction of the total forces as a vertical component. With this improved model of the electric field distribution, the accuracy from the numerical simulation improves dramatically, which can be seen in the next sub-section.
Figure 4.9 a) Electric field distribution while modeling only the air gap between the electrode and droplet. b) Electric field distribution while including the glass slides in the model. c) Comparison of the electric pressure along the surface of the droplet between including the glass slides (blue) in the model and not (green).

4.3.1.1.4 Validation Results

With the cross-sectional areas matched and the electric field distribution modeling improved, the final step in validating the numerical model is to run it for the different experimental results. For each experiment there are three variables that will affect the equilibrium deformation predicted using the model (figure 4.10a). In particular, the overlap of the electrode, $\Delta x$, over the droplet, the height, $h_e$, of the electrode over the ground plate, and the applied voltage will all affect the final equilibrium shape. Through numerical experiments, the effect of each of these variables on the final shape follows the expected trends. For a given applied voltage, increasing $\Delta x$ causes a decrease in the gap between the electrode and the nearest point on the droplet.
a) Geometric variables that will effect the outcome equilibrium shape in the numerical model. $h_e$ is the height between the electrode and ground plate, and $\Delta x$ is the overlap between the electrode and droplet. 

b) Effect the electrode overlap, $\Delta x$, has on the final equilibrium shape. 

c) Effect the applied voltage has on the equilibrium shape. 

d) Effect the height, $h$, between the electrode and ground plate has on the final equilibrium shape.

This decrease in distance will increase the divergence of the electric field and thus increase the electric pressure along the surface of the droplet causing greater deformation. In addition, as $\Delta x$ increases more of the droplet experiences significant electric pressure which will also increase the electric pressure. This trend in increased deformation can be seen in figure 4.10b when running tests with the numerical model.
Increasing the voltage and decreasing the gap, $h_e$, have similar effects on the final deformation. For a given geometry, an increase in voltage will increase the divergence of the electric field leading to a higher electric pressure and more deformation. For a given applied voltage, if $h_e$ is decreased then there is an increase in the electric field divergence again leading to an increase in electric pressure and deformation. Likewise, if the voltage is decreased or $h_e$ is increased there will be a decrease in the divergence of the electric field leading to less deformation. As can be seen in figure 4.10c and 4.10d these trends are again witnessed by running tests with the numerical model. The overall effect of a change in voltage or a change in $h_e$ is less than that of increasing $\Delta x$. However, each of these variables can affect the accuracy of the simulation.

Errors arise in each of these variables due to the inaccuracies in measuring them. The dimensions for $h_e$ and $\Delta x$ are both extracted from the experimental images using image analysis techniques similar to the scaling used in equation 4.3. The difficulty in using this particular technique is that the resolution of a single pixel is 20 microns. Therefore, an error of a single pixel in the measurement will already affect the accuracy of the numerical simulation. In addition, the power supply used only permitted manual voltage adjustment through an analogue dial. The minimum tick mark on the voltage control is 60 V leaving a potential 30 V error. These errors are inherent to the system leaving ranges of possible values for the different variables in the system. Within the simulation, these variables can then be altered within their ranges of error to ensure the best result within expected error.

It was found that for all cases the overlap of electrode over the droplet was 100 microns. However, the gap between the electrode and base varied between the different applied voltages. This change in gap is expected because there is an inherent electric force between the base plate
and the electrode since they essentially act as charged parallel plates. Flexibility in the glass slides and lack of support near the droplet allow this force to bend the slides towards each other reducing the gap between the electrode and base plate. This force will increase with the applied voltage again reducing the gap. This is evidenced in each of the experimental images where the gap, h, reduces from 730 µm to 710 µm and 690 µm for 0 V, 1500 V, and 1800 V respectively.

Table 4.1 presents the numerical validation results with the error calculated via equation 4.2 as well as the values for the variables described above used in the simulation. In addition, figure 4.1 visually presents the validation results. As evidenced by the calculation results in table 4.1, the error in the simulation is less than 5 % when under an applied voltage and is just over 6 % while at 0 V. It should be noted that the values for the applied voltage, electrode gap, and electrode overlap parameters in table 4.1 were determined through optimization using the simulation. For each droplet, simulations were run for gaps of 680 - 720 µm, overlaps of 50 - 150 µm, and ± 30 V around each of the experimental voltages. In each case, the final results presented in table 4.1 are the combination of the parameters that provided the smallest error. However, in each case the error from the least optimized parameters was no more than 8%. Therefore, the results were not significantly improved by choosing specific parameter values. The 0 V results are a bit redundant in that there is little useful information to help validate the model since the simulation does not need to be run to calculate them. However, the increase in error for the 0 V case, can be explained as an effect of the area matching technique.

As mentioned previously, the initial contact angle of the simulation is fixed for a given area. In addition, the initial curvature of the droplet is uniformly circular. These limitations in the initial conditions cannot compensate for the external factors in the experimental setup, in particular the hydrophobic patterning technique. As mentioned earlier, there are exposed areas
of SU-8 within the printed PTFE surface. In addition to creating a high roll off angle, these regions create a non-uniform edge along the printed slit causing the contact lines, and thus the contact angles along the slit to be non-uniform as well. Since the initial contact angles are not equal on either side of the droplet the initial cross sectional area is not a perfect circle like the simulation creating error between the experiment and simulation. However, as the drop deforms under an applied voltage, the main determining factor for the overall shape switches from the contact angles to the electric pressure. Since the electric pressure distribution is the same for both the experiment and numerical simulation it is expected that there would be a decrease in total error as the applied voltage increases, which is what is seen in table 4.1.

Table 4.1 Numerical model validation results and parameters

<table>
<thead>
<tr>
<th>Experimental Voltage (V)</th>
<th>Relative Cross Section Error</th>
<th>Numerical Voltage (V)</th>
<th>Electrode Gap (µm)</th>
<th>Electrode Overlap (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.062</td>
<td>0</td>
<td>730</td>
<td>100</td>
</tr>
<tr>
<td>1500</td>
<td>0.041</td>
<td>1500</td>
<td>710</td>
<td>100</td>
</tr>
<tr>
<td>1800</td>
<td>0.041</td>
<td>1800</td>
<td>690</td>
<td>100</td>
</tr>
</tbody>
</table>

Upon examination of figure 4.11b and c, the first trend noticed is that the maximum height of the numerical simulation is always less than that in the experiments. In addition, to compensate for this lack in height the outer edges of the numerical profile are always wider than the experiment. This arises because the overall shape produced through the simulation is more rounded than the experiment and the edges of the droplet cannot achieve the straighter edges seen in the experiment. As mentioned earlier in section 4.3.1.1.3, this is an artifact of the
boundary conditions of the model in that the surface tension and pressure forces resist the
deformation of the surface near the contact line.

![Graphs](image)

**Figure 4.11** Visual representation of experimental and numerical results validation. a)-c) Extracted experimental profile vs. numerical profile for 0 V, 1500 V, and 1800 V respectively. d)-e) Corresponding experimental images overlaid with numerical profile for 0 V, 1500 V, and 1800 V respectively.

Beyond the artifact described above, there are other sources of error that should be considered when analyzing the results. As mentioned previously, it was difficult to extract dimensions of the experimental setup from the images since a single pixel discrepancy in the measurement produces a 20 µm error. Compounding with this error is the orientation of the camera with respect to the experiment setup. To the best of the user’s ability, the camera was positioned perpendicular to the length of the droplet. However, any angle discrepancy in this alignment will cause additional error in the dimension measurements. Due to the design of the experiment, there is no quantitative way to test the alignment of the testing apparatus thus there
will be inherent error in the dimension measurements. These errors will manifest themselves as described by figure 4.10, however the magnitude of the error is immeasurable.

In similar vein, error is also introduced into the comparison through the extraction of the experimental profile from the collected images. As mentioned above, a single pixel inaccuracy will create a 400 \( \mu \text{m}^2 \) inaccuracy in the area measurement. Though a single pixel has little effect on the total area calculation, \(~0.001\) to be exact, there are factors that have the potential to cause multiple pixel errors. In addition to causing error in the dimension measurements, alignment issues can cause errors in the extraction of the droplet. The focusing plane of the stereoscope is ideally centered in the middle of the droplet, and perpendicular to the length of the droplet. Any angular misalignment will cause an increase in the measured cross-sectional area.

Beyond misalignment error, there is additional area error from the profile extraction technique described in figure 4.6. In that technique there are two thresholding steps where the threshold value was calculated by trial and error. Though each analysis was examined closely to ensure a good extraction, there was no quantitative measure of accuracy in the method. In addition, as can be seen in figure 4.11a-c the extracted profiles are considerably jagged. This is an artifact of the thresholding technique, and using pixels to represent a curve. The convolution smoothing step described in figure 4.6c was an attempt at minimizing this effect, however it was not able to remove it completely. This jagged profile adds additional area error to the comparison calculation.

The final source of error in the numerical simulation is that the layer of SU-8 between the droplet and ground electrode was not taken into account when calculating the electric field. This means that in the experiment the droplet was not fully grounded due to the layer of SU-8, where in the model it was directly grounded. Using COMSOL it was shown that the value of the
electric pressure varies less than 1% at any given point along the surface when including the layer of SU-8. Therefore, the error introduced by omitting the layer of SU-8 is negligible.

Considering the error discussion above, and the number of errors present in the modeling and comparison techniques, it is concluded that the numerical model can be validated using these results, and is thus an accurate simulation of the system. Calculation wise, the error between the numerical and experimental results is less than 5% and shows significant correlation between the two profiles. In addition, by visually analyzing the shapes of the profile there are two major sources of error in the profiles. The first noticeable error arises from a more rounded shape in the numerical profile. As discussed above, this is expected and is a known limitation of the modeling technique. The second noticeable error is in the jaggedness of the experimental profile. This is a limitation of the resources of the project, and could be corrected with more advanced equipment and techniques. It would be expected that a corrected smoothed profile would have little impact on the final results since the technique presented captures the bulk profile of the droplet.

4.3.2 High Voltage Results

As mentioned in section 4.1, it was initially thought that the failure mechanism would be dielectric breakdown of the surrounding medium since the electric potential required for adequate deformation was on the order of the breakdown voltage. Through dimensional analysis it was shown that if full operational deformation was achieved in the 1 mm EHMP prototype before dielectric breakdown, then it could be concluded that dielectric breakdown would not occur in the 200 µm EHMP. Using the same parameters and apparatus as the low voltage tests, it was found that as the voltage reached 2100 V small atomized droplets would eject from the
bulk droplet towards the upper electrode. With no spark or visible light emission characteristic of dielectric breakdown, the failure due to atomization displayed properties of electrospray.

The traditional tip-streaming electrospray apparatus is depicted in figure 4.12 with important parameters highlighted.

![Diagram of an electrospray apparatus with labels for distance between the emitter and the ground plate (h_e), radius of the emitter (r), applied voltage (V), and slope of the Taylor cone (θ).](image)

**Figure 4.12 Standard electrospray apparatus where h_e, r, V, and θ are the distance between the emitter and the ground plate, radius of the emitter, applied voltage, and slope of the Taylor cone respectively.**

An ionic fluid is supplied through a capillary tube with radius r called the emitter [56]. When an electric potential, V, is applied between the emitter and a base plate the ions travel to the liquid gas interface. As the applied voltage is increased, force on the ions cause the interface of the fluid to deform into a Taylor cone. If the applied voltage exceeds the threshold voltage

$$V_{spray} = \sqrt{\frac{r \sigma \cos(\theta)}{2\epsilon_0} \ln \left( \frac{4h_e}{r} \right)}$$

an electro spray forms and atomized droplets are ejected from the Taylor cone [56]. In equation 4.2, r is the radius of the capillary emitter, θ is the slope of the liquid interface, σ is the surface tension, and h_e is the distance between the emitter and the ground plate.
The values representative of the EHMP in figure 4.13 \( r = 0.5 \) mm, \( \sigma = 0.071 \) N/m, \( h = 0.69 \) mm, and \( \theta = 66^\circ \), yield a critical voltage of \( V_{\text{spray}} = 1586 \) V.

![Diagram showing electrospray parameters](image)

**Figure 4.13** Electrospray parameters used for the EHMP critical voltage calculation. \( r \) is the radius of the droplet, \( \theta \) is the contact angle of the droplet, and \( h \) is the distance between the ground plate and electrode. This result is considerably lower than the experimental results where electrospray on set was at 2100 V. However, it is expected that this calculation would produce a result that is lower than the actual critical voltage. In the electrospray apparatus in figure 4.12, the voltage is applied directly between the droplet and a ground plate significantly larger than the droplet. This would be equivalent to an EHMP where the electrode completely overlapped the droplet (figure 4.14b). In this case, the electric pressure along the surface of the droplet is significantly larger (figure 4.14c). As can be seen in figure 4.14c, the maximum electric pressure for a completely overlapping electrode is on the order of three times larger in magnitude than the electric pressure expected in the actual experimental geometry. Due to this significantly decreased electric pressure in the EHMP geometry, it would be expected that a higher voltage be required to induce electrospray.
Figure 4.14 a) EHMP electrode placement in experimental apparatus. Red represents the positive electrode, and blue represents the ground. b) Electrode placement equivalent for theoretical electrospray calculation. c) Electric pressure along the surface of a droplet for geometries in (a) (blue) and (b) (green). The applied voltage is the 1586 V that was calculated using the electrospray theory.

In addition, the EHMP geometry introduces other errors in the electrospray critical voltage calculation. The traditional electrospray apparatus in figure 4.12 has an axisymmetric droplet formed from a circular opening allowing for straightforward measuring of the droplet radius, \( r \), and fluid interface angle, \( \theta \). For the EHMP, due to the rectangular droplet base there is no strict definition of the drop radius due the rectangular base, and anti-symmetric droplet deformation. For the purpose of the calculation above, half the width of the droplet base was used for the radius. However, the other possibility to use for the drop radius, \( r \), is the curvature of the droplet closest to the electrode. In this case, the radius of curvature of the droplet closest to the electrode is 225 \( \mu \text{m} \) which gives \( V_{\text{spray}} = 1561 \text{ V} \). From this it is seen that though there is some dependence on the radius of the droplet, within the limitation of the EHMP deformations it will not affect the critical voltage as significantly as the overlapping electrode.

The deformation of the drop into an anti-symmetric triangular prism shape introduces another error into the calculation when measuring the liquid interface angle \( \theta \). In the case
presented in figure 4.13, the contact line closest to the electrode was used to measure $\theta$. However, if the other contact line is used, $\theta$ becomes $56^\circ$ and $V_{\text{spray}} = 1885$ V. Another possibility is to treat the peak of the droplet as the Taylor cone and $\theta$ will become the average of the two contact angles giving $\theta = 61^\circ$ and $V_{\text{spray}} = 1704$ V. As can be seen the critical voltage relies heavily on the angle of the interface, which will decrease as the droplet becomes more deformed just before failure.

Based on the analysis of the different parameters of the electrospray theory in the previous paragraphs, as well as the experimental evidence with atomized droplets, it is concluded that the failure mechanism for the prototype EHMP is becoming an electrospray and not dielectric breakdown. From the analysis in section 4.1, this result can be extrapolated to conclude that dielectric breakdown between the droplet and electrode will not occur in the desired micro scale EHMP. In addition, adjusting the parameters of the droplet to $r = 100 \mu\text{m}$, $h = 175 \mu\text{m}$, and $\sigma = 0.031$ N/m to match the desired micro scale EHMP dimensions presented in chapter 2 yields a critical voltage of $V_{\text{spray}} = 519$ V, or for $\theta = 61^\circ$ $V_{\text{spray}} = 567$ V. From the above analysis showing that the experimental critical voltage is significantly above the theoretical voltage, as well as the results in chapter 2 demonstrating a maximum operational voltage of 550 V, it is concluded that the ideal 200 $\mu\text{m}$ EHMP will not fail due to electrospray in its operational range.

4.3.3 Light Deflection Results

Having validated the numerical model using the experimental results, the final test for the 1 mm prototype EHMP is to experimentally test the viability of the device with respect to automultiscopic displays. These tests will determine whether there needs to be revisions to the
design of the EHMP to increase its viability, or if the current configuration is adequate. The viability will be critiqued via two different variables: the amount of blurring of the original projected light after passing through the EHMP, and the number of degrees that the light can be deflected. As mentioned in chapter 2, a curved surface will act as a lens for the projected light defocusing or blurring the projected light (figure 4.15a).

![Diagram of ideal and non-ideal blurring of projected light due to surface curvature.](image)

**Figure 4.15** a) Non-ideal and ideal blurring of projected light due to surface curvature. b) Experimental apparatus to test the image deflection and blurring of the prototype EHMP. $h$ is the height that the cardboard backing is suspended above the EHMP.

For the ideal automultiscopic display there would be minimal blurring of the projected light while being deflected at a particular angle (figure 4.15a). In addition, ideally the range of angles, as well as the precision of angles that the light is deflected is high to allow for a large display viewing area.

To test these two variables the prototype EHMP test apparatus was altered to what is seen in figure 4.15. A handheld laser pointer (McCoy, MC40004RP), was positioned below the EHMP so that the laser light would project through the prototype simulating the projected light of a pixel. A cardboard backing suspended a height $h = 6.5$ cm above the EHMP and at an angle so that the stereoscope and camera combination used to collect the low voltage results could
capture the projected image after traveling through the EHMP. If the camera position is fixed, the amount of deflection can be calculated by using image analysis to measure the physical distance the projected image has been displaced. The angular deflection can then be calculated by the arctangent of the ratio between the measured displacement and the height, h.

Initially the laser was unaltered so that the EHMP was illuminated using the full laser spot. However, as can be seen in figure 4.16 the blurring of the laser spot is significant. This result is expected since the cross-section of the EHMP is circular and acts as lens. Assuming the cross-section of the EHMP is a half circle of radius 0.5 mm, the EHMP will act as a lens with a focal length of 0.25 mm until deformed. This proves difficult to capture focused light due to the electrode ITO glass slide and supporting slide having a combined thickness of 2 mm above the EHMP. Therefore, the projected image is captured a significant distance beyond the focal point and is expected to have diverged considerably. The divergence is expected to only occur in the direction perpendicular to the length of the EHMP, which is the along the arrow in figure 4.16. As can be seen, the divergence of the projected images is acting as expected for the given shape of the EHMP.

In order to limit the divergence of the projected light, the incoming laser spot needs to be reduced in diameter to be on a scale smaller than the radius of the EHMP. In order to achieve this, a 0.6 mm diameter hole was punched in a thin cardboard card using a Harris Uni-Core 0.5 hole punch. This card was used to block the majority of the laser spot, reducing its diameter to the order of the EHMP. As can be seen in figure 4.16, the projected laser spot is still blurred along the horizontal direction, however the full spot is still visible. Though ideally for an automultiscopic display the whole EHMP will be illuminated, using the decreased spot size allows for easier measurement of deflections once a voltage is applied.
Figure 4.16 Divergence of laser spot after traveling through the EHMP with the arrow representing the direction perpendicular to the length of the EHMP and the direction of expected divergence. a) Full laser spot without the EHMP. b) Full laser spot with the EHMP. c) 0.6 mm laser spot without the EHMP. d) 0.6 mm laser spot with the EHMP.

To test the deflection capabilities of the prototype EHMP, the gap between the electrode and base plate was increased to 800 microns to allow for an increased range of applied voltages before the onset of electrospray. Figure 4.17 presents the results of the deflection test in the voltage range of 1.5 – 2.4 kV. As can be seen, there is minimal deflection of the laser beam with a max deflection of 2.44° achieved. In section 4.3.1, it was shown that little deformation of the EHMP was achieved before failure due to electrospray. The deflection results coincide with those results since large deflections would require significant deformation.

The next test was to use two electrodes to deform the EHMP into a rectangular cross section similar to the ideal case in figure 4.15a. As explained in chapter 1, for two electrodes positioned with one over each of the contact lines, it was theorized that the EHMP would deform into a rectangular shape when equal voltages are applied between each of the electrodes and the
droplet. This rectangular shape would produce a fluid interface that would be parallel with the plane of the display preventing any blurring or divergence of the projected light.

Figure 4.17 Deflection of the laser spot through the EHMP for applied voltages of a) 0V, b) 1500 V, c) 1800 V, d) 2100 V, e) 2400 V with the vertical lines representing a 1° displacement. f) Plot of the angular deflection vs. applied voltage.

As seen in figure 4.18, there is some deformation at 2400 V, however it is opposite of what was theorized. Instead of deformation into a rectangular shape, it is deformed into a triangular shape. The electric pressure distribution for the double electrode configuration was investigated using COMSOL, and the results explain what is seen in the experiment. As can be seen in figure 4.18c, the intensity of the electric pressure does not dissipate between the two
electrodes but forms a bridge between them. This creates a semi parabolic electric pressure along the surface of the droplet (figure 4.19d).

Figure 4.18 Equal voltage double electrode results for a) 0V and b) 2400 V. c) COMSOL simulation of the electric pressure distribution for double electrodes. d) Electric pressure profile along the surface of the droplet for double electrodes.

Though there is a reduction of the electric pressure near the apex of the droplet, the reduction is not enough to allow the horizontal forces of the electric pressure to dominate the deformation. Instead, the net vertical force has a larger magnitude pulling the apex of the droplet upward towards the electrodes. Since the cross-sectional area must be conserved the droplet edges deform into a triangular shape.
The results presented in this section offer little viability for the 1 mm prototype EHMP with respect to automultiscopic displays. The inability of the drop to deform into a shape with flat interfaces prevents light to be projected through the EHMP without significant divergence. Therefore an image projected through an array of EHMPs would be blurry. Even the theorized electrode placement, double electrodes, to provide the least divergence proved to be wrong, and instead made the divergence worse by increasing the surface curvature. In addition, with a voltage range up to 2400 V before failure, the maximum deflection achieved was only 2.44°. This small deflection range would make it difficult to produce an autostereoscopic display and near impossible to provide the motion parallax for an automultiscopic one.

Though the overall results are negative from the experiments, there are a few positive ones to be found. Firstly, though the viability of the 1 mm droplet is low, the experimental results have proven to be quite useful in validating the numerical model of the EHMP. With a validated model it is easy to test different geometry and material configurations without the need for fabricating another prototype for each combination. In addition, when considering the discussions in sections 4.2 and 4.3.2, the experimental results show promise for the concept of the EHMP in general. The ability for the 1 mm EHMP to significantly deform into a prism is hindered by the use of air as a surrounding medium and increasing the diameter of the drop to 1 mm. In particular, the magnitude of the electric pressure is decreased by a factor of 2.2 requiring a higher voltage to achieve deformation. Coincidentally, this higher voltage induces failure by electrospray due to the instability along the surface of the droplet. The relative voltage for this failure to occur decreases as the diameter of the droplet increases. Therefore, higher relative voltages and stronger electric pressures could be used if the prototype was returned to its
intended diameter of 200 µm with oil as a surrounding medium. These effects would allow for greater deformation potentially increasing the range of angular deflection.

Looking at the images in figure 4.17 in detail, it is seen that as the applied voltage increases the width of the laser spot in the horizontal direction decreases. This is a promising result since this shows the peak of the droplet that contains the highest curvature is deformed out of the path of the laser light. Since the laser is being projected through less curvature, the divergence is decreased. As discussed in the previous paragraph, it is expected that the amount of deformation will increase by using a smaller EHMP surrounded by oil. Extrapolating the results seen from figure 4.17, it can be concluded that the increased deformation will also reduce the amount of divergence experienced by the projected light while being deflected.

The final promising result is seen in figure 4.17f. It can be seen that there is a roughly linear relationship between the applied voltage and angle of deflection. This is the ideal case for the EHMP since this would reduce the complexity of the system when trying to calculate the required voltage for a desired angle of deflection. At the moment this is merely speculation based on the data points available. There are many factors that could influence this curve and it is unwise to try and extrapolate the relation. However, the initial results show promise for future testing.

4.3.4 Prediction of Miniaturized System Performance

As discussed in the previous section, the experimental results of the prototype 1 mm EHMP have little viability for an automultiscopic display. However, this can largely be attributed to the size of the droplet and the surrounding medium. Through the analysis of the results and theory it has been shown that there is still potential for the EHMP concept if the droplet cross-sectional size was reduced and the surrounding medium was switched to oil. Having validated the numerical
model in section 4.3.1.1.5, it is possible to use it to predict the performance of the miniaturized ideal EHMP.

Using the results from the numerical model presented in section 2.1, the ideal EHMP parameters are a 200 μm diameter droplet surrounded by oil, a 50 μm wide positive electrode aligned above the contact line of the droplet, a 50 μm wide ground electrode centered above the droplet, and a gap of 25 μm between the ground and positive electrodes. The maximum achievable applied voltage before dielectric breakdown occurs is 450 V. Using these parameters the numerical simulation was run to predict the performance of the ideal EHMP with the results presented in figure 4.19. Figure 4.19a demonstrates the deformation of the ideal EHMP at 450 V.

As discussed in section 2.1, it was determined that 450 V was the maximum voltage possible before failure due to dielectric breakdown. Therefore the deformation seen at 450 V in figure 4.22a is the maximum possible. Analyzing both the experimental results and the ideal 200 μm results using the fill factor and average interface curvature as described in section 2.4.1, it is evident that in its current configuration the viability of the EHMP with respect to automultiscopic displays is low. For the experimental results the fill factor and average curvature are 0.59 and 1.60 ± 0.19 mm\(^{-1}\) respectively. Where as the fill factor and average curvature for the ideal 200 μm EHMP are 0.66 and 6.2 ± 1.7 mm\(^{-1}\) respectively. As can be seen there is an increase in fill factor that improves its viability for an automultiscopic display. However, even with the improvement, the fill factor shows that 34% of the display’s light will be projected at the wrong angle, which is still not very viable.
Figure 4.19 a) Maximum deformation of the ideal EHMP using a single 50 µm wide electrode and ground electrode. b) Maximum deformation of the ideal EHMP using two 50 µm wide electrodes and one 50 µm ground electrode.

In addition, as can be seen from the calculations there is a significant increase in average surface curvature in the ideal case. This can be expected since the starting curvature is significantly higher in the ideal case, 10 mm⁻¹, compared to the experimental case, 2 mm⁻¹. For the ideal EHMP this means the interface will need to deform significantly more to account for the initial curvature. However, even at the maximum deformation, the ideal EHMP will distort the image significantly more than the 1 mm experimental EHMP.

The final simulation run introduced an additional positive electrode above the second contact line to test the EHMP’s ability to create a flat horizontal interface equivalent to the experimental tests in section 4.3.3. The results from the simulation can be seen in figure 4.19b. In an attempt to quantify the change in the interface, the average curvature between -50 and 50 µm as seen in the plot was calculated for the before and after deformation curves. Before the deformation, the average curvature is 10 mm⁻¹ where as after deformation it is 9.3 ± 1.4 mm⁻¹. From these values, it can be seen that there is not a significant decrease in surface curvature and the projected image will still be significantly distorted.
Chapter 5: Conclusions and Future Work

The thesis presented in the previous chapters focuses on the work performed to develop an EHMP with applications to automultiscopic displays. In theory, the EHMP is a controllable water droplet micro prism that when positioned over a light source can deflect incoming light at a desired angle. The deflection angle is determined by the shape of the prism, which is controlled by applying a voltage between the droplet and an electrode positioned above it. The strength of the applied voltage dictates the amount of deformation in the droplet, thus increasing the deflection of the incoming light. If multiple electrodes are used, the prism can deflect incoming light with high accuracy and a large deflection range.

5.1 Conclusions

There are a few key potential advantages the EHMP has over other techniques for creating automultiscopic displays. At the moment, there are three major limitations in current research; in passive techniques the deflected images reduce the effective resolution of the display, and the number of viewers is limited since there are only particular regions where the 3D cues are active. In active techniques, the resolution is maintained while allowing for unlimited viewers, however the mechanisms are complicated to operate. Like other active techniques, the design of the EHMP inherently solves both of the passive limitations. However, it significantly simplifies the mechanism to project the images over other active techniques. In particular, if an array of the EHMPs are positioned over a display, all of the display’s pixels are deflected in the same direction allowing the native resolution of the display to be used for the 3D images. In addition, the EHMP is able to provide stereoscopic and motion parallax cues to unlimited viewers in any position within the deflection range. The main limiting factor of the
EHMP is that the display that synchronizes with the array, needs to have a fast enough refresh rate to accommodate all off the projection angles.

In order to develop a prototype EHMP, a numerical model needed to be developed for the system as well as a method of patterning hydrophobic surfaces. Current modeling techniques for the electro-hydrodynamics of the system focus generally on two governing equations: the Young-Lippmann equation and the Maxwell stress tensor. The Maxwell stress tensor approach is a more accurate representation of the physics in the system since it incorporates the effect of the electric field on the interface of the droplet rather than just the contact angle. In addition, current research has only focused on the effect of the Maxwell stress tensor near the contact line of droplets, and does not model a complete droplet.

It was found that many of the current modeling techniques make use of FEM to model the hydrodynamics of the system. The disadvantage of using FEM is that there is not a strictly defined droplet interface causing difficulty when calculating the magnitude of the Maxwell stress tensor. The modeling technique developed in chapter two, builds on the theory of current Maxwell stress tensor methods by using SPH to simulate a complete droplet with a strictly defined interface. This is possible to achieve since SPH can model free surface flows that provide strict interfaces of the droplet.

The difficulty with SPH is that there is a common instability called the tensile instability that causes accuracy problems. Current research on fixing the tensile instability has focused on adding artificial pressure terms to the stress state of the governing equations. However, these techniques leave the system with an unphysical pressure response when two particles near each other. A novel technique was implemented in this research to counteract the tensile instability while still maintaining a physical pressure response. To achieve this an exponential repulsive
force was added to the governing equation to mimic short-range molecular forces. This allowed
the Navier-Stokes macroscopic pressure to govern the fluid flow on large scales, however when
two particles came close to each other the repulsive force would become dominant forcing the
particles apart.

Since the SPH code was developed in house using Matlab, COMSOL was used to model
and calculate the Maxwell stress tensor for the system simplifying the SHP code. Using these
techniques a hybrid FEM and SPH model for the EHMP was developed. The hydrodynamics of
the model including the exponential term was validated using qualitative results from a
collapsing square test, and quantitative results from an oscillating droplet test. Using the
validated model the performance of the EHMP was predicted and it was determined that the
leading edge of the electrode would determine the final shape of the deformation. In addition, it
was shown that though deformation occurred the fill factor and interface curvature would be
limiting factors on the performance of the EHMP.

The next step in fabricating a prototype EHMP was to develop a technique to pattern
hydrophobic surfaces. Current techniques focus on etching deposited hydrophobic films, or
altering the surface energy of the film using plasma or ultra violet light. In all of these
techniques there was issues with not meeting the requirement for transparent surfaces or the
techniques were complicated. To meet the requirements of the EHMP fabrication process, a
microcontact printing technique was developed to pattern PTFE nanoparticles. The advantage of
this technique is that once the initial PDMS stamp is fabricated the process is fast and simple to
fabricate a patterned hydrophobic surface.

Currently there are many techniques to pattern nanoparticles using microcontact printing.
However, in all of the current applications in research it is desirable to have a single layer of
nanoparticles deposited in a pattern. To achieve this, many different stamp-inking techniques were developed. Through initial tests of these inking techniques it was found that a single layer of PFTE nanoparticles did not provide adequate hydrophobicity for the EHMP application. Therefore, an in-house fabricated airbrush was used to ink the stamp.

The advantage of the airbrush for inking purposes is that if the parameters of the airbrush are adjusted correctly then the particles are deposited dry onto the stamp. This allows for multiple layers of particles to be transferred during the printing step increasing the hydrophobicity of the final deposited film. The technique was tested for its ability to repeatedly produce uniform films of PTFE nanoparticles. Using opacity measurements it was shown that the technique can repeatedly produce uniform films with a contact angle of 152° and a contact angle contrast between the pattern PTFE and exposed substrate of 134°.

Using the patterning technique stated above, a prototype EHMP was fabricated. To simplify the test apparatus as well as the droplet formation, the surrounding medium was switched to air as well as the droplet diameter increased to 1 mm. The initial tests proved to be less than promising in that there was minimal deformation before failure of the droplet occurred. However, they were used to successfully validate the numerical model. Using the theory of electrospay it was demonstrated that the failure mechanism was electrospay and not dielectric breakdown of the air. The minimal deformation was able to deflect a laser beam 2.44°, and using two electrodes did not provide a flat interface, but rather drew the droplet up between the electrodes. From these results it was evident that the initial prototype EHMP was not well suited for use in an automultiscopic display.

Though the initial prototype is not a viable solution for automultiscopic displays, it was shown using dimensional analysis that if oil were used as the surrounding medium and the
The diameter of the droplet was reduced to 200 µm then failure due to electrospray would not occur, and it would be expected that the deformations would be more pronounced. Using the experimentally validated numerical model, the ideal EHMP was simulated in an attempt to quantify the improvement in the performance. Immediately problems with enclosing the EHMP in glass became apparent and a ground electrode was added to provide directionality to the electric field. The addition of this ground electrode reintroduced the failure mechanism of dielectric breakdown in the oil limiting the applied voltage to a maximum of 450 V.

Upon the analysis of the 450 V ideal EHMP simulation results it was evident that there was little improvement in performance over the experimental results. The fill factor improved by about 0.1, however even with this improvement 35% of the incoming light is not redirected at the desired angle. In addition, the reduction in droplet size means that the deformation would have to be significantly more to compensate for the higher initial interface curvature. The limitation of 450 V prevents adequate deformation to compensate for this, so the ideal EHMP will actually diverge the projected light more than the 1 mm prototype. This means that an imaged projected through an array of 200 µm EHMPs would be blurrier than the same image projected through an array of 1 mm EHMPs.

To summarize the contributions of this thesis, it presented:

- A new numerical modeling technique that combined both FEA and SPH to accurately model the electro-hydrodynamics of the EHMP.
- A new short range repulsive force into the SPH governing equation to counter the effect of the tensile instability.
- A modified micro contact printing technique that utilized an airbrush to ink the PDMS stamp to pattern PTFE nanoparticles creating hydrophobic patterning.
• A demonstration of the theory and experimental results of a new electro-hydrodynamic micro prism.

Considering everything presented in this thesis, it can be concluded that an EHMP in its current design of either a 1 mm or 200 µm elongated droplet surrounded by air or oil respectively is not viable with respect to automultiscopic displays. In addition, due to its inability to deform into a shape with straight interfaces, these EHMPs also have little applicability to beam deflection applications.

5.2 Future Work

As discussed previously, it has been concluded that the EHMP in its current design has little viability with respect to automultiscopic displays. To improve on the design, it is necessary to alter the geometry and materials of the device to maximize the ratio of electric pressure to surface tension force. If the geometry is kept, a simple technique to achieve this is to find two immiscible fluids to act as the droplet and surrounding medium that have a large difference between their relative permittivities with the surrounding medium having the greater of the two. This will maximize the electric pressure acting on the surface. In addition, the two fluids need to have a small interfacial surface tension constant between them. This will ensure that the surface tension force will be minimized. Finally, the dielectric breakdown voltage of the surrounding fluid needs to be high to minimize the risk of dielectric breakdown.

If two fluids cannot be found that achieve this, it is recommended to investigate ultra low voltage electrowetting [57] with respect to the EHMP. In the case of ultra low voltage electrowetting, there are immiscible ions in both the water and surrounding oil. This provides a double double-layer that has been shown to significantly decrease the required voltage for electrowetting applications [57]. This decrease in applied voltage has potential to allow the
EHMP to deform more before failure increasing its fill factor and decreasing average curvature. However, first the effect of the additional ions in the oil needs to be investigated on how it affects the dielectric breakdown voltage of the oil as well as the potential for electro spray formation.

If either of these options can be achieved then it would be worthwhile to fabricate the ideal EHMP with a diameter of 200 µm. If fabrication does occur, it should be noted that a new droplet formation technique needs to be developed. As mentioned previously, one of the reasons for adopting a 1 mm diameter droplet was because all of the formation techniques tried were not able to create a 200 µm droplet. For this formation to occur it is recommended to try using a micro printer that is capable of depositing 100-200 µm diameter droplets.

To create the actuation electrodes for the EHMP, the ITO can be patterned using hydrochloric acid as a wet etchant and SPR 7.0 photoresist as a etch mask. In addition, SU-8 2075 can be used to create a wall around the array device to provide accurate spacing between the droplet and electrodes to contain the surrounding medium. Using these techniques it should be possible to fabricate the ideal EHMP if it is deemed appropriate by the research into ideal fluids or ultra low voltage electrowetting.

If neither of these options are viable then it will be necessary to devise a new geometry for the EHMP. The elongated shape is the ideal droplet geometry since the curvature of the surface is reduced due to the infinite radius of curvature along the length of the droplet. For a given length scale of a droplet, this will minimize the surface tension force acting on the surface. Therefore, the redesigning should be focused on the electrode geometry. The most important consideration for the electrode geometry is to maintain directionality of the electric pressure along the surface of the droplet. As shown in chapter 4, this can be achieved by strategic
placement of ground electrodes. However, as also shown in chapter 4 this raises the risk of dielectric breakdown in the oil.

The simplest planar electrode, the one chosen for the 200 µm EHMP simulations, was demonstrated to suffer from failure due to dielectric breakdown. More complicated planar electrode patterns would suffer from the same drawbacks as the design presented in this thesis. One potential solution is to decrease the distance between the electrode and ground plate. This would allow a lower voltage to provide the same electric pressure decreasing the chance of dielectric breakdown between electrodes.
References


