Modeling The Fluid-Structure Interaction Of The Upper Airway
Towards Simulation Of Obstructive Sleep Apnea

by

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Abstract

Obstructive Sleep Apnea (OSA) is a syndrome in which the human Upper Airway (UA) collapses during sleep leading to frequent sleep disruption and inadequate air supply to the lungs. OSA involves Fluid-Structure Interaction (FSI) between a complex airflow regime and intricate mechanics of soft and hard tissue, causing large deformation of the complicated UA geometry. Numerical simulations provide a means for understanding this complex system, therefore, we develop a validated FSI simulation, composed of a 1D fluid model coupled with a 3D FEM solid solver (Artisynth), that is applied to a parameterized airway model providing a fast and versatile system for researching FSI in the UA.

The 1D fluid model implements the limited pressure recovery model of Cancelli and Pedley [28] using a dynamic pressure recovery term, area function corrections allowing complete closure and reopening of fluid geometries, and discretization schemes providing robust behavior in highly-uneven geometries. The fluid model is validated against 3D fluid simulations in static geometries and simple dynamic geometries, and proves reliable for predicting bulk flow pressure. Validation of simulation methods in Artisynth is demonstrated by simulating the buckling, complete collapse, and reopening of elastic tubes under static pressure which compare well with experimental results.

The FSI simulation is validated against experiments performed for a collapsible channel (a “2D” Starling resistor) designed to have geometry and characteristics similar to the UA. The observed FSI behaviors are described and compared for both experiment and simulation, providing a quantitative validation of the FSI simulation. The simulations and experiments agree quite well, exhibiting the same major FSI behaviors, similar progression from one behavior to another, and similar dynamic range.

A parameterized UA model is designed for fast and consistent creation of geometries. Uniform pressure and dynamic flow FSI simulations are performed with this model for numerous parameters associated with OSA. Uniform pressure simulations compare well to clinical data. Dynamic flow results demonstrate airflow limitation and snoring oscillations. The simulations are fast, simulating 1 s of FSI in 30 minutes. This model is a powerful tool for understanding the complex mechanics of OSA.
Preface

The research topic of this thesis, numerical modeling of Obstructive Sleep Apnea, was offered to Peter Anderson by his supervisors Prof. Sheldon Green and Prof. Sidney Fels. With feedback from his supervisors, Peter Anderson performed a literature review and proposed a research agenda. The research proposal was reviewed by a supervisory committee and from resulting discussions further revised to use a simple 1D fluid model in the FSI system.

In general, Peter Anderson’s responsibilities included performing the literature review, writing the code necessary for FSI simulations and executing the simulations, performing data analysis, and writing the articles. He met regularly with his supervisors to present the results, discuss the meaning of the results, and discuss which research directions to pursue.

Chapters 1 and 2 were written by Peter Anderson, and reviewed by his supervisors. Portions of Chapter 2 are excerpts from the introductory material of journal/workshop articles 3,4, with the main body of those articles forming subsequent chapters.

Chapter 3 is derived from the published journal article “Implementation and Validation of a 1D Fluid Model for Collapsible Channels” by Peter Anderson, Sidney Fels, and Sheldon Green 3 (© ASME 2013, reproduced with permission). This article was published by the Journal of Biomechanical Engineering in 2013. Peter Anderson wrote the code for the 1D fluid model, derived the analytical test case (given in Appendix D), defined the driven-geometry viscous case, and performed all the simulations including the 3D CFD simulations. Data analysis was performed by Peter Anderson, while interpretation of data and decisions concerning what data is necessary and how to present it were the outcome of discussions with his supervisors. Peter Anderson wrote the initial draft of the article/chapter, but it was carefully edited by his supervisors.

Chapter 4 is derived from a workshop article “Simulation of Collapsible Tubes and Airways - Towards a Validated Upper Airway FSI Model” by Peter Anderson, Sidney Fels, and Sheldon Green 4. The workshop was the “1st International Workshop on Biomechanical and Parametric Modeling of Human Anatomy” in 2013. Peter Anderson performed all the simulations and the initial data analysis, while interpretation of data and decisions concerning what data is necessary and how to present it were the result of discussions with his supervisors. Peter Anderson wrote the initial draft of the article/chapter, but it was carefully edited by his supervisors.

The content of Chapter 5 has not yet been published. The intern students Matthieu Dupré, Pierre de Rancourt, and Kenneth Wang built the experimental rig and performed the initial experiments,
under the immediate supervision Peter Anderson and broader supervision of Sheldon Green and Sidney Fels. Due to findings from the initial experimental results, Peter Anderson made further modifications to the experiment (particularly the addition of the upstream plenum and narrowing the fluid channel from 20 cm to 5 cm) and recorded the final results that are presented in Chapter 5. Peter Anderson performed all the simulations and the initial data analysis, while interpretation of data and decisions concerning what data is necessary and how to present it were the result of discussions with his supervisors. Peter Anderson wrote the initial draft of the article/chapter, but it was carefully edited by his supervisors.

The content of Chapter 6 has not yet been published. Peter Anderson designed and implemented the parameterized upper airway model, performed the FSI simulations based on this model, and performed the data analysis. Decisions regarding necessary factors to study, interpretation of data, and presentation of results were the outcome of discussions with his supervisors. Peter Anderson wrote the initial draft of the chapter, which was edited by his supervisors.

Chapter 7 was written by Peter Anderson, and reviewed by his supervisors.

Throughout the main body of this thesis, the pronoun “we”/“our” is used to indicate the work of Peter Anderson, Sheldon Green, and Sidney Fels. When appropriate, “I” is used to refer to the individual work of Peter Anderson. In the Appendix, “we” is used to be inclusive of the reader following the background theory or derivations covered therein.
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1D 1-dimensional

3D 3-dimensional

AHI Apnea–Hypopnea Index or Apnoea–Hypopnoea Index

AP Anterior-Posterior or Anteroposterior

BC/BCs Boundary Condition(s)

CAD Computer Aided Design

CFD Computational Fluid Dynamics

CPAP Continuous Positive Airway Pressure

CSA Cross-Sectional Area

CT Computed Tomography

DNS Direct Numerical Simulation

FEM Finite Element Method

FFT Fast Fourier Transform

FSI Fluid-Solid Interaction or Fluid-Structure Interaction

Hex Hexahedral (referring to mesh element type)

Hz Hertz

KE Kinetic Energy

Lat Lateral

LES Large Eddy Simulation

OSA Obstructive Sleep Apnea

OSAS Obstructive Sleep Apnea Syndrome
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<td>Pa</td>
<td>Pascal (unit of pressure)</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds Number</td>
</tr>
<tr>
<td>RANS</td>
<td>Reynolds Averaged Navier Stokes</td>
</tr>
<tr>
<td>Tet</td>
<td>Tetrahedral (referring to mesh element type)</td>
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<td>UA</td>
<td>Upper Airway</td>
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Glossary

**Anterior** The direction towards the front (face-side) of the head.

**Anteroposterior** Describes the axis along the anterior-posterior directions.

**Apnea/apnoea** Temporary stoppage of breathing (literally “without breath”).

**Artisynth** An open-source biomechanical simulation toolkit used in this research.

**Compliant** Easily deforming.

**Constitutive relationship** Describes the deformation (strain) that a material undergoes in response to a stress.

**Coronal** The plane perpendicular to the anterior-posterior axis.

**Flow limitation** A fluid-solid interaction phenomenon in which an increased pressure drop results in a unchanged or decreased flow rate due to increased solid deformation.

**Inferior** The direction towards the bottom (neck-side) of the head.

**Laminar Flow** A flow regime in which the fluid flows in parallel layers.

**Lateral** Describes the left/right directions or axis of the head.

**Lateral recumbent** The body position in which the subject is lying on their side.

**OpenFoam** An open-source CFD software used for 3D fluid simulations.

**Oropharynx** The region of the pharynx at the level of the tongue.

**Patency** The condition of a passage, such as the airway, being open or unobstructed.

**Posterior** The direction towards the back of the head.

**Raphe** An anatomical term for a seam or ridge where two similar parts join.

**Retropalatal** The region of the upper airway behind the soft palate.

**Retroglossal** The region of the upper airway behind the tongue.

**Sagittal** The plane perpendicular to the lateral axis.
**Glossary**

**Superior** The direction towards the top of the head.

**Supine** The body position in which the subject is lying on their back. May also be called “dorsal recumbent”.

**Transitional Flow** A flow regime occurring between laminar and turbulent flow regimes.

**Transverse** The plane perpendicular to the superior-inferior axis (in the medical context).

**Turbulent Flow** A flow regime in which the flow is chaotic, with high mixing.

**Velopharynx** The region of the pharynx at the level of the soft palate (or velum).

**Velum** Soft palate.
Acknowledgements

I am indebted to my supervisors, Professor Sheldon Green and Professor Sidney Fels, for making this thesis possible, and for all the time and effort they have invested in me throughout this thesis. It has been an honor working with you and learning from you.

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I am grateful to Matthieu Dupré, Pierre de Rancourt, and Kenneth Wang who worked with me on portions on this research - it was truly a privilege working with you.

I would like to thank John Lloyd, PhD, for his assistance with Artisynth. I am grateful to Prof. Srikantha Phani for thoughtful discussions, to Dr. Ho Beom Kwan for thoughtful discussions and sharing data, to Prof. Barton Smith and Prof. Andrew Pollard for sharing data, and to Dr. Bill Pearson for helpful conversation.

I would like to thank NSERC for funding this research.
Dedication

To Livia, a new friend at the start of this project, and now my supportive and loving wife.

To my parents who have encouraged me to learn from a young age until now.

To God who gives purpose to my life and hope in all situations.
Chapter 1

Introduction

Obstructive Sleep Apnea is a dangerous, frustrating, and costly syndrome that affects many millions of people in North America and around the world. Obstructive Sleep Apnea (OSA) involves the partial or complete collapse of the Upper Airway (UA), which leads to disrupted sleep and inadequate air supply to the lungs, and occurs during sleep when the subject’s posture and relaxed airway muscles make the upper airway prone to collapse. OSA involves the Fluid-Structure Interaction (FSI) between air in a complex flow regime in the tortuous airway geometry and upper airway structures which are complicated by intricate musculature, neural activity, and great person-to-person variation. Factors that may lead to OSA include a constricted airway (resulting from obesity, in particular), diminished airway muscles (due to aging, for example), and improper muscle response/activation (due to drug usage, for example). These complexities make OSA very difficult to understand and to treat.

Understanding OSA requires an understanding of FSI, which is itself a topic of much academic research, but there is a need to move beyond simple FSI models to cases of complex geometry, large structural deformation and complete closure of the fluid domain in order to model OSA. Once FSI simulations are designed to meet the rigors of OSA modeling, and are applied in a trustworthy model of the upper airway, they can provide understanding of the complex mechanisms of OSA and improve diagnosis and treatment.

To address these requirements we develop a fluid-structure interaction simulation system, validate it against experiments, and apply it to a wide range of plausible upper airway geometries. OSA is an airflow problem in that it is defined by the lungs not receiving sufficient air and it is likely that airflow dynamics will bring a compromised airway to closure; therefore, an OSA model must include a description of the airflow. However, in another sense OSA is not an airflow problem in that it is caused by compromised UA structures, and these structures are little affected by small and rapid turbulent fluctuations, having much greater inertia than the air. Therefore, rather than attempting to use 3D fluid simulations in our FSI model, we estimate the bulk flow with a 1D fluid model. This proves to be robust in complex geometries and computationally fast, both deemed essential features for OSA simulations. In Chapter 3 we define our 1D fluid model in detail and demonstrate the validity of our approach through comparison with previous research, analytical models, and 3D simulations.

The solid model, however, should not be reduced to 1D or 2D as it needs to model the intricate 3D structures of the upper airway and reducing the dimensionality of the solid model would render
an accurate representation of the upper airway impossible. Therefore we use a 3D biomechanical modeling toolkit (Artisynth) to model the solid components. In Chapter 4 we simulate the buckling and collapse of a flexible tube under a uniform pressure, which is a simple case but related to that of upper airway collapse. Our results compare well with experimental results from literature. This demonstrates that we can properly simulate buckling and contact in Artisynth, as well as providing insight into the sensitivity of the solid model to assumptions and numerical parameters.

In Chapter 5 we validate the FSI system as a whole through directly comparable simulations and experiments. The experimental design is that of channel with a portion of one wall replaced with a flexible membrane (that is, a “2D” Starling Resistor). The fluid and solid characteristics are chosen to be similar to the human upper airway. The FSI simulation system replicates the steady and unsteady behaviors generally within 10% of the experimental results. The simulations are robust and fast. Beyond providing validation, these experiments/simulations provide first-of-a-kind results in a fundamental FSI case.

In Chapter 6 we apply our validated FSI system to OSA modeling. Due to the great difficulty of deriving an upper airway geometry from medical images of a real person, we define a parameterized airway model that is based on a simple yet flexible set of geometric parameters. This allows airway geometries and meshes to be consistently and quickly created, making a controlled sensitivity analysis feasible. We tested the sensitivity of numerous geometric and material parameters relevant to OSA in FSI simulations using both uniform pressures and dynamic flow scenarios. Our parameterized airway FSI model predicts numerous behaviors observed clinically, and demonstrates the OSA-related behaviors of snoring, airflow limitation, and airway collapse.

In our conclusions (Chapter 7), I summarize our key results. Our combination of a 1D fluid model with a 3D solid model is highly adept in OSA modeling. It is accurate, fast and robust, and thus a useful tool for the modeler to gain a wide-scale insight into FSI system they are simulating. Our parameterized airway model, in conjunction with the FSI system, provides a powerful system for understanding OSA.

I begin by reviewing the current understanding of OSA and FSI in Chapter 2. This background material does not delve into the theory and simulation methods of fluid, solid and FSI mechanics. However, I include a fundamental and theoretical description of the underlying principles in the Appendix: fluid mechanics and simulation (Appendix A), solid mechanics and simulation (Appendix B), and FSI theory and simulation (Appendix C).

1.1 Contributions

The main, novel contributions of this thesis are highlighted here.

Our implementation of the 1D fluid model works with an arbitrary area function, and the solution remains robust for highly uneven, largely deforming, and completely closed geometries. Through
1.1. Contributions

comparison with 3D CFD simulations, we demonstrate that this 1D model may accurately predict the bulk flow when the pressure-recovery term is properly defined, and we provide a definition for that term that is a function of Reynold’s number.

We demonstrate that our solid model accurately predicts the collapse of a 3D tube, including buckling and contact of the opposing walls, by validating against experimental results. We highlight the sensitivity of the tube collapse to physical and numerical parameters.

To validate our FSI simulation, we perform the first “2D” starling resistor experiment, and characterize the FSI behaviors when driving pressure, membrane tension, and side-wall gap are varied. Dynamic FSI behaviors are represented through space-time contour plots and global behaviors are demonstrated with spectrograms. We perform directly comparable simulations thus making quantitative validation possible, and demonstrate that our FSI model predicts all the major FSI behaviors observed experimentally. We include in our fluid model a term to account for air leakage at the side-wall gaps, which agrees with experimental observation.

Our parameterized airway model is an important tool in OSA modeling in that it provides a controlled airway-like geometry in which key factors to OSA can be controlled. Using our fast FSI simulation, we perform a wide sweep of uniform and dynamic flow simulations to illustrate the consequence of a parameter on the global behavior of the upper airway.
Chapter 2

Background

2.1 Obstructive Sleep Apnea Background

2.1.1 The Problem Of OSA

Obstructive Sleep Apnea (OSA) occurs when a person is sleeping, and the Upper Airway (UA) collapses, forming a partial or complete constriction that prevents adequate air supply to the lungs. When an apnea (breathing stoppage) occurs, the person awakens and regains control of their upper airway muscles and hence is able to breathe, but when this occurs numerous times each night the person gets a poor nights rest and reduced oxygen to the brain, and is at risk of diseases arising from the stresses these deprivations put on the body and accidents caused by drowsiness. OSA is a large economic burden because the the sleep-deprived OSA patient is less productive and more prone to accidents, as well as requiring increased medical care. In the USA, the estimated economic loss due to OSA is 16 billion dollars annually [18,40,97,135].

Approximately 9% to 24% of adults in the USA suffer from OSA [135], though estimates and definitions vary widely, and most cases remain undiagnosed [97,160]. The main risk factor for OSA is obesity, while other significant factors include: male gender, aging, and drug usage [37,97]. Being closely associated with obesity, OSA is a growing concern as obesity becomes more common and more severe.

OSA endangers life, diminishes life quality, and is a large economic burden. There is, therefore, a great need for understanding and effectively treating OSA. However, OSA is difficult to understand, as it involves the complicated structures of the upper airway, neurology, person-to-person variation, and intricate fluid mechanics, and fluid-structure interaction. Simulation provides a way to explore the mechanisms of OSA and therefore provide better treatment for those suffering from it.

2.1.2 Severity Measure: Apnoea-Hypopnoea Index

The severity of OSA is measured by the Apnea-Hypopnea Index (AHI), a measure of the number of apnea or hypopnea events per hour. An apnea is defined by complete cessation of breath for 10 s, while a hypopnea has a large reduction of airflow and is accompanied by decreased oxygen in the blood or the subjects arousal [97]. The severity of OSA can be defined as: mild (5 < AHI < 15), moderate (15 < AHI < 30), and severe (AHI > 30).
2.2. Upper Airway Mechanics

Even if a subject does not experience complete airway closure nor arousal from sleep, they may still be in danger of diseases related to decreased oxygen in the blood. Thus, some prefer classifying OSA according to oxygen desaturation in the blood rather than AHI. For example, Isono et al. classify OSA severity according to an Oxygen Desaturation Index (ODI) \[73,97\].

2.1.3 OSA Treatment

Many cases of OSA can be treated with a change of lifestyle, such as losing weight, reducing usage of alcohol or other drugs, or modifying sleeping posture \[74,97,119,148\]; however, compliance to these lifestyle changes is poor, nor are all cases of OSA correctable with a lifestyle change \[162\].

The most effective treatment is the use of a Continuous Positive Airway Pressure (CPAP) machine. The CPAP seals over the patient’s nose and supplies a positive pressure, typically using pressures between 500 Pa (5 cmH₂O) and 2000 Pa (20 cmH₂O)\(^1\) which acts as a pneumatic force resisting the collapse of the airway. Some patients find exhalation difficult against a large positive pressure and thus prefer Bilevel Positive Airway Pressure (BPAP), for which the positive pressure fluctuates between inhalation and exhalation \[27,75,97\]. Many find CPAP undesirable because it suppresses rather than cures the problem, it is uncomfortable, and it may lead to new problems such as central sleep apnea (sleep apnea that is related to dysfunctional respiration feedback controls) \[40,119,135\].

For less severe cases of OSA, an oral appliance can be used. These include devices that will draw the mandible forward, or hold the tongue forward using suction \[144\]. While CPAP is more effective, patients tend to prefer oral appliances, though still compliance is poor \[97\]. OSA may also be treated surgically. The most common surgery is uvulopalatopharyngoplasty (UPPP), which attempts to reduce excess soft tissue in the UA, thus increasing airway patency. The success rate of 41\% for this surgery is quite poor, and depends on the location(s) where the apnea occurs \[97,135\].

2.2 Upper Airway Mechanics

In general, OSA patients have a narrow and/or compliant upper airway as well as neurological issues \[69\]. However, the mechanics involved in OSA are exceedingly complex \[135\], being a fluid-structure interaction problem of non-linear fluids and structures, and wide patient-to-patient variability, as the causes of OSA and the locations and modes of OSA can vary. In the following section, I will review the key properties of the UA which play a role in OSA.

2.2.1 Upper Airway Geometry

When describing anatomical locations of the upper airway, I use the following conventions. Anterior describes the front/face-side of the head while posterior describes the back. The anterior-posterior

\(^1\)1 cmH₂O = 98.07 Pa, but 1 cmH₂O ≤ 100 Pa is a useful approximation
(or anteroposterior) axis describes the front-back axis and is divided by a “coronal” plane. “Superior”
describes the top of the head and “inferior” is the direction towards the neck/feet; this axis is called
the superior-inferior axis and is divided by a “transverse” plane. The left and right axis runs
in the “lateral” direction and is divided by a “sagittal” plane. The anterior-posterior direction is
abbreviated as AP and the lateral direction as Lat.

A sagittal view of the upper airway is shown in Fig. 2.1 with labels illustrating important land-
marks in OSA. Figure 2.1 approximately spans the pharynx, which is composed of the velopharynx
(or nasopharynx), oropharynx, and hypopharynx (or laryngopharynx). The velopharynx is the
retropalatal portion of the airway (that is, behind the soft palate). The oropharynx is the retroin-
guinal portion of the airway (that is, behind the tongue). The hypopharynx is the portion of
the airway behind the epiglottis and includes the upper regions of the esophagus after the larynx and
esophagus split. The esophagus is the passage food takes to the stomach while the larynx is the
“voice box” and the passage air takes to the lungs [14,64,149]. A typical length of an adult UA
from the posterior end of the nasal cavity to the base of the epiglottis is 10.5 cm [149].

The walls of the pharynx are largely defined by the superior, middle, and inferior pharyngeal con-
strictor muscles. These muscles originate posteriorly at the pharyngeal raphe, a stiff ridge running
in the superior-inferior direction along the posterior and mid-sagittal portion of the airway. From
the pharyngeal raphe the constrictor muscles form a tubo-like structure as they wrap anteriorly to
various termination points and thus define the posterior and lateral walls of the airway. These con-
strictor muscles partially overlap each other, but the superior constrictor is thin and not overlapping
in the superior portions that form the velopharynx [14,64].

Airway closure may occur in different, or multiple, locations: the velopharynx, the oropharynx,
or the hypopharynx [10,18,73,135]. The main collapse motion of the airway walls can occur in
either the anterior-posterior (AP) direction or in the lateral direction. The velopharynx is the most
common location of apnea, Barrera et al. [10] note that 96% of their patients have velopharyngeal
collapse. Isono et al. [73] confirm that the velopharynx is the primary location of collapse for most
subjects investigated. In an informative study, Soares et al. [135] find the velopharynx the most
common location of closure, where AP collapse is the most common and almost always forms a
complete closure while lateral collapse often forms a partial closure. Both AP and lateral collapse in
the oropharynx and hypopharynx are common. Importantly, the location(s) and mode of collapse
are correlated to the likelihood of surgical success. Schwab et al. [125], while studying the airway
response to positive nasal pressures, observed greater compliance in the lateral walls than the
antero-posterior direction.
The key risk factor in OSA is being overweight, which results in the UA being smaller due to increased fat in the surrounding structures \([37,40,97]\). A typical minimum cross-sectional area (CSA) is 106 mm\(^2\) in the velopharynx and 242 mm\(^2\) in the oropharynx for a normal subject. In contrast, a typical minimum CSA for an OSA patient is 64 mm\(^2\) in the velopharynx and 241 mm\(^2\) in the oropharynx \([149]\).

The cross-sectional shape of the airway is an important factor in the collapsibility of the UA. A more circular cross-section tends to be less-collapsible. Measured by the ratio of Anterior-Posterior diameter to lateral diameter \((AP/lat)\), both OSA and normal subjects have a similar velopharyngeal ratio \([33,82,148,149]\). In one study, Walsh et al. report that the ratio changes from \(AP/lat = 0.4\) to \(AP/lat = 0.6\) between supine and lateral recumbent postures \([148]\); however, in another study with supine patients they report \(AP/lat \approx 0.27\) for normal patients and \(AP/lat \approx 0.36\) for OSA patients \([149]\).

The tongue has a mass around 80 g, and as the subject changes orientation with respect to gravity, the rest position of the tongue can change significantly, thus altering the shape of the airway \([26,89,101,148]\). This may be reflected in that apnea occurrences decrease by half for most OSA subjects when sleeping in a lateral recumbent posture as opposed to supine posture \([148]\), and in the effectiveness of oral tongue-retention devices \([144]\). Additionally, sleep disordered breathing may be detectable in one posture but not another; for example, in the sitting posture, snorers and non-snorers may have similar airway pressures, but the difference between them becomes marked.
2.2. Upper Airway Mechanics

when supine [13]. However, Randerath et al. [119] do not recommend postural treatments as a general and effective therapy.

Soft palate length is correlated to OSA. Measured as the distance between the posterior nasal spine (that is, the posterior portion of the hard palate shown in Fig. 2.1) to the tip of the uvula, a normal soft palate has an average length of 38.6 mm while an OSA patient’s soft palate has an average length of 42.2 mm, and the vertical height of the palate relative to airway length is significantly larger for OSA subjects [132]. The palate has been measured as approximately 20 mm thick for normal subjects, mid-sagittally, at the thickest part of the palate [125]. However, one can observe significant variation in palatal thickness both in varying the sagittal plane and moving along the length of the palate [129].

The parapharyngeal fat pads, occurring on the lateral edges of the pharyngeal walls, are shown be significantly larger in OSA patients [107,151], and have been noted to have a more significant correlation to OSA than tongue or palate size [151].

2.2.2 Upper Airway Solid Properties

Not only do OSA subjects tend to have a smaller UA, but also a more compliant airway that is more prone to collapse. As will be shown in Chapter 6 there are numerous factors that can lead to a more compliant airway, but one key factor is the material properties of the upper airway soft tissues, and these properties may change with age, drug usage, and stage of sleep.

Muscle density is approximately $1000 < \rho < 1070$ kg/m³. Soft tissue has a non-linear stress-strain relationship in extension and compression [39,70]; however, describing it with a constant Young’s modulus (that is, linear stress-strain relationship) may still be useful as a first approximation. The Young’s modulus of a living and relaxed muscle at 10% strain may be around $E = 6200$ Pa but as high as $E = 110,000$ Pa for a contracted muscle, while $E = 1150$ Pa for muscles extracted from cadavers [26,39]. Muscle properties also change with age. Beyond the age of 20 years, the ultimate strength of muscle decreases by 0.5% each year [39]. For example, this is reflected by the observations that soft palate length increases with age [132]. Age is an important risk factor in OSA [97], which highlights the large contribution that poor muscle tone has in airway collapsibility.

A significant portion of soft tissue is composed of water, thus it is not surprising that soft tissue also exhibits fluid-like behaviors, including creep and other time-dependent behaviors that can be described with a viscoelastic material model, such as Fung’s quasi-linear viscoelasticity model [70].

Neurological activity changes the stiffness of muscles, and is closely linked with OSA. By definition, OSA is only a problem during sleep, when muscle activity is generally decreased [40,128], and OSA severity depends on the phase of sleep the subject is in, thus further linking OSA to neurological activity [37,75]. For example, the genioglossus muscle, which is a dilator muscle critical to the OSA problem, is estimated to reduce activity 10-30% during sleep [66-68]. The breathes leading up to an apnea may be accompanied by progressive narrowing of the airway before complete closure occurs,
again suggesting neural activity changing the mechanical properties of the airway [102,106]. We also see that drugs (such as alcohol and tobacco), which impair muscle activity, are linked with OSA [97,152]. Though a point of some debate, it is generally thought that neurological defects arise as a result of OSA rather than a cause of OSA [69,75,97,128].

Fat has properties similar to relaxed muscle, with $\rho \approx 900 \text{ kg/m}^3$ and $3250 \leq E \leq 5600 \text{ Pa}$ [39,49,124]. Bone density ranges approximately $1290 \leq \rho \leq 1750 \text{ kg/m}^3$ and $E \approx 12.0 \cdot 10^9 \text{ Pa}$ [39], and thus is many orders of magnitude more stiff than soft tissue and may be approximated as rigid in comparison with soft tissue [138]. Cartilage has properties between soft tissue and bone with $\rho \approx 1100 \text{ kg/m}^3$ and $E \approx 800,000 \text{ Pa}$ [39]. These material properties are summarized in Table 2.1.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ [kg/m$^3$]</th>
<th>$E$ [Pa]</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>muscle</td>
<td>$1000 \leq \rho \leq 1150$</td>
<td>$1150 \leq E \leq 110000$</td>
<td>[26,39]</td>
</tr>
<tr>
<td>fat</td>
<td>$900$</td>
<td>$3250 \leq E \leq 5600$</td>
<td>[39,49,124]</td>
</tr>
<tr>
<td>cartilage</td>
<td>$1100$</td>
<td>$80000$</td>
<td>[39]</td>
</tr>
<tr>
<td>bone</td>
<td>$1290 \leq \rho \leq 1750$</td>
<td>$12 \cdot 10^9$</td>
<td>[39]</td>
</tr>
</tbody>
</table>

Table 2.1: Summary of important properties of upper airway materials.

A simple estimate of the force produced by a muscle is $F = A_{cs} \cdot K \cdot f_{EMG}$, where $A_{cs}$ is the cross-sectional area of a muscle, $K$ is a constant (usually $40 \text{ N/cm}^2$ is used), and $0 \leq f_{EMG} \leq 1$ represents the muscle’s activation [50,109]. While this may be a crude approximation for airway muscles, it provides a plausible estimate that levator veli palatini, a small soft palate muscle, generates forces less than $1 \text{ N}$ [15,50]. In contrast, the genioglossus, a large tongue muscle, can produce forces of $25 \text{ N}$ [130].

### 2.2.3 Upper Airway Fluid Properties

The fluid mechanics of the UA, like the solid mechanics, are very complex. During sleep, a subject will typically breathe through the nose, taking approximately 1 breathe per every 4 seconds. The airway closure may occur during inspiration or at the end of expiration [31,37,102]. A wide range of flow rates are possible in the airway, varying from $5 \text{ liters/min}$ during quiet breathing to $120 \text{ liters/min}$ during extreme exercise; the corresponding Reynolds numbers are approximately $200 < Re < 13000$. For normal breathing, a peak flow rate of $30 \text{ liters/min}$ and $Re \approx 2000$ are plausible values [55,84,87,106,133,145,161]. This flow, including laminar, transitional, and turbulent flow regimes is difficult to model, especially in a highly irregular and deforming geometry such as the UA.

To give an idea of the scales involved in breathing, consider a hypothetical healthy, 30 year old male. He may take 17 breaths per minute when awake, each breath inhaling and exhaling $383 \text{ ml}$ [142]. His lungs may be able to produce $110 \text{ cmH}_2\text{O}$ ($10780 \text{ Pa}$) for inspiration or $150 \text{ cmH}_2\text{O}$ ($14700 \text{ Pa}$) for expiration [41], but such exertion is not needed because the pressure drop in his
2.2. Upper Airway Mechanics

Airway is only 264 Pa while asleep [13]. His upper airway will collapse under a negative pressure of 13.3 cmH₂O (1303.4 Pa) [127], which is safely beyond the 264 Pa loss that occurs in his airway.

In contrast, consider a hypothetical 50-year-old male with OSA. His lungs may be able to produce 100 cmH₂O (9800 Pa) for inspiration or 133 cmH₂O (12985 Pa) for expiration [41]. His airway is more constricted and has a pressure loss of 550 Pa [161] while breathing. His airway is more compliant and may collapse with a negative pressure of 3.1 cmH₂O (176.4 Pa) [75], and thus is prone to collapse with every breath.

This is illustrated in Fig. 2.2 which shows a plausible scenario for a healthy airway (Fig. 2.2a) and an OSA airway (Fig. 2.2b) during inhalation. For the healthy airway, the negative air pressures created during inhalation are safely above the negative pressures which would cause that portion of the airway to collapse. For the OSA airway, the negative air pressure in both the velopharynx and oropharynx are below the pressure required for those regions to collapse, indicating that apnea is likely to occur there. The OSA airway is more constricted, thus more pressure is lost, requiring the lungs must exert more effort to receive the needed airflow (thus the blue curve is lower). The OSA airway is also more compliant and therefore more prone to collapse (thus the red curve is higher).

![Air pressure and airway collapse pressure](image1)

(a) Air pressure and airway collapse pressure for a healthy airway.

![Air pressure and airway collapse pressure](image2)

(b) Air pressure and airway collapse pressure for an OSA airway.

Figure 2.2: An illustration of plausible pressures for a healthy and OSA airway. The solid blue curve shows the air pressure function between the nose and the lungs, while the solid red curve shows the pressure which would cause that region of the airway to collapse. The dashed red line, termed $p_{\text{crit}}$, shows the pressure at which any part of the airway closes.

In addition to the flow complexity, the boundary conditions (BCs) are ill-defined. As the lungs
expand or contract they behave like a bellows (thus suggesting velocity/volumetric flow rate BCs). However, the lungs are limited in strength and have deformable walls. When the flow resistance upstream/downstream becomes too high (such as when the airway is closed or nearly closed), the lungs do not draw the intended flow rate. Instead, the flow rate becomes independent of the lungs effort, called flow rate limitation, and negative effort dependence may be observed in which flow rate decreases as lung effort increases \([41,106]\). In this limiting case the flow would be pressure driven. We demonstrate flow limitation and negative effort dependence in Ch \([6]\).

Therefore, a simulation of OSA places high demands on the fluid model used: one that may be coupled to a complex and dynamic geometry, one that allows for complete closure and re-opening of the geometry, one that predicts the fluid pressure loss with sufficient accuracy though the flow may range from laminar to turbulent, and one that includes boundary conditions able to characterize the full range of lung behaviors.

### 2.2.4 Airway Response To Static Pressure

The collapsibility of the UA is closely related to the existence and severity of OSA. This is commonly measured by the critical pressure, \(p_{\text{crit}}\), at which the airway will close (which is illustrated in Fig. \([2.2]\)). This can be measured by applying an increasingly negative nasal pressure until airway closure is detected. Healthy airways, having \(p_{\text{crit}} < -1000 \text{ Pa}\), do not collapse until a substantial negative pressure is reached. Snorers are in the range \(-1000 < p_{\text{crit}} < -500 \text{ Pa}\) and subjects with mild OSA experiencing flow limitation but not complete closure are in the range \(-500 < p_{\text{crit}} < 0 \text{ Pa}\). Subjects with severe OSA have \(p_{\text{crit}} > 0 \text{ Pa}\), indicating that their airway is naturally collapsed unless an interfering force, such as their waking UA muscles or the positive pressure of a CPAP, provides airway patency \([37,128,143]\). This is an approximate measure, and depends on the subjects consciousness (awake, stage of sleep, or under anesthetic) and posture \([73,76,126,127]\). For example, \(p_{\text{crit}}\) may vary by 200 Pa depending on the stage of sleep \([75]\).

Some researchers have measured the cross-sectional area (CSA) of the airway as a function of a static pressure applied at the nose. A selected portion of their (digitized) data measured in the velopharynx is given in Fig. \([2.3]\). The critical pressure \(p_{\text{crit}}\) may be observed as the pressure at which CSA is 0 for the curves that extend that far. Kuna et al. \([88]\) measured patients in 3 categories (normal, obese non-OSA, and OSA), with patients awake and sitting, applying positive pressure and measuring cross-sectional area (CSA). Their data was averaged, and a linear response was found. Schwab et al. \([125]\) measured normal patients, awake and supine. The data was averaged, and shows a fairly linear response. They note the high compliance of the lateral pharyngeal walls. Isono et al. \([73]\) measured patients in 3 categories (normal, mild OSA, severe OSA), with patients anesthetized and supine, applying positive and negative pressures. The data is not averaged, and the response is non-linear. This airway response data provides a useful measure for comparing simulation response in Chapter \([6]\).
2.2. Upper Airway Mechanics

![Figure 2.3: Clinical measurements of the velopharyngeal cross-sectional area in response to a uniform pressure. Data includes selected trends from Isono et al. [73] normal patients (solid black), Kuna et al. [88] normal patients (dotted red) and OSA patients (dash-dotted red), and Schwab et al. [125] normal patients (dashed green).](image)

2.2.5 Snoring

Snoring is a symptom of OSA, and another example of Fluid-Structure Interaction (FSI) in the UA. Snoring occurs when the airflow causes the UA to vibrate at an audible frequency [48,68,91]. As noted above, snoring is often considered an intermediate condition between a healthy airway and an OSA airway [75,76,143]. Like OSA, snoring depends on the stage of sleep and posture, and may occur at various locations [116].

A palatal snore typically has a well-defined fundamental frequency (<100 Hz) with clear harmonics, and most of the spectral energy is below 500 Hz. By contrast, snores coming from other parts of the airway are higher frequency and more noise-like (lacking distinct fundamental and harmonic frequencies). This has led researchers to attempt to distinguish between apneic and non-apneic subjects by their snore characteristics [114,116].

2.2.6 Studying OSA Using Simulation

Clearly OSA is a serious problem. It is a dangerous and difficult condition to have, yet many suffer from it as there are no effective treatments that they find acceptable. Those desiring to understand and treat OSA are confronted by a confounding number of factors, but gathering data is limited by what is ethical and convenient for patients. In this situation, the use of simulations to better understand the underlying mechanisms, guide research questions, and predict outcomes may be of great benefit [85].
Simulations provide a well-controlled and non-invasive way to explore a system. The influence of modeling assumptions and parameters may be systematically investigated, thus the modeler may gain insight into the key factors that drive the system’s behavior. Simulation design and results may bring to focus factors or questions not yet investigated by clinicians. And once simulation methods are well-trusted, they can be used to predict outcomes making possible the eventual goal of patient-specific treatment.

However, a comprehensive simulation of OSA remains illusive. For one, the simulation technology is still coming of age: a stable simulation involving fluid-structure interaction and multiple deformable bodies colliding with each other is a difficult task, and computationally expensive. But second, were the simulation technology adequate, a complete conceptual model of the UA and OSA is still lacking along with complete data sets describing the geometry and behaviors of the UA with which to build the model. Simulations, being an implementation of a conceptual model, provide a tool for developing a comprehensive OSA model.

2.3 Simplified Models

Fluid-structure interaction, even with simple geometries and materials, has a rich diversity of behaviors; therefore, it is useful to gain an intuition and theory for FSI behaviors in simplified models before examining the much more complex case of FSI in the UA. I will review the Starling resistor, an essential FSI model, as well as other relevant simplified models.

2.3.1 Starling Resistor

The Starling Resistor model, illustrated in Fig. 2.4a, involves flow driven through a flexible tube that is attached to rigid upstream and downstream segments. The conceptually simple geometry, fluid properties (typically water or air), and solid properties (typically an elastic material) make this a good case for theoretical, experimental, and numerical research. Though simple, this model exhibits a wide range of FSI behaviors. It also provides a good analogy for numerous biomechanical scenarios, such as airflow in the UA or blood flow through veins. Theoretical and numerical studies often consider a 2D analogy of the Starling Resistor, shown in Fig. 2.4b. Both 2D and 3D cases have been studied extensively and there exist many excellent reviews [18, 51, 58, 61, 86, 111].
2.3. Simplified Models

Figure 2.4: Diagrams of the 3D and 2D Starling Resistor models. These illustrate the rigid upstream/downstream portions (black), the deformable tube/membrane (dashed, green), and the external pressure chamber (red) used to generate the pressure $p_{\text{ext}}$ external to the deformable portion. These diagrams illustrate a flow, with velocity $u$, driven by a pressure gradient from $p_{\text{inlet}}$ to $p_{\text{outlet}}$.

Experimentally, Bertram and others have carefully characterized the behavior of this system [16, 17, 19-21, 23-25], controlling upstream pressure/flow rate, upstream/downstream resistance, and external pressure. The tube behavior may be a steady (completely collapsed, completely open, or with a constriction near the downstream end of the tube), or oscillating in regular or chaotic manners.

Theoretical and numerical studies of the system include lumped parameter models [22], 1D models [28, 78, 131], 2D models [80, 90, 94, 95], and 3D models [54, 56, 57, 60, 62, 98].

Modeling the flexible tube is an essential component of the Starling Resistor as the FSI behavior is sensitive to the choice of structural model [90]. Figure 2.5 shows the response of a tube to positive and negative pressures, in which the CSA (labeled $A$) is non-dimensionalized by the CSA of the undeformed tube (labeled $A_0$). This empirical “tube law”, characterizing CSA as a function of pressure, has been used as closure for lumped parameter and 1D fluid models [22, 28, 46, 110, 131, 155]. However, a generalized model of the full length of the tube is desirable.

If the wall thickness $h$ is much smaller than the inside radius $r_i$ (typically when $h/r_i < 0.05$), the tube may be considered thin walled and one may justifiably use Kirchhoff-Love shell theory [98] or theory describing expansion (positive internal pressure) [141]. However, most real-world tubes are
2.3. Simplified Models

thick-walled and such assumptions would be poor. Marzo et al. [98] simulated 1/4 of a thick-walled tube, predicting tube law behavior similar to experimental results; however, not handling collisions of opposing walls. More recently, Zhu et al. [163] simulated buckling of complete tubes, but not handling contact nor comparing with experimental results. A validated simulation of thick-walled tubes predicting buckling, closure, and reopening is a necessary step towards full FSI simulations of this system. We demonstrate this in Chapter 4.

In addition to the FSI system’s sensitivity to the structural model, it is sensitive to the fluid mechanics, particularly the flow resistance model which must be included to capture many of the experimentally observed behaviors. While the 2D and 3D simulations are able to model flow resistance directly from the Navier-Stokes equations using RANS or LES models (see Appendix A for details), the 1D simulations must resort to simpler models. Of particular interest here is the model defined by Cancelli and Pedley [28] that limits pressure recovery by the use of a parameter \( \chi \), where \( \chi = 0 \) implies no pressure recovery and \( \chi = 1 \) corresponds with complete pressure recovery, as shown in Fig. 2.6. Previous studies have used a constant value of \( \chi = 0.2 \) [78], but this simplification neglects the dependence of pressure recovery on Re and geometry [134]. One shortcoming of this model is that the only sources of unsteadiness arise from a changing cross-sectional area or flow fluctuations at the inlet/outlet, while the effects of detached flow are modeled in an averaged sense. Subsequent research has revealed that there is strong energy dissipation upstream of the

Figure 2.5: Tube law schematic. The simulated tube is shown with arrows indicating where on the “tube law” curve that geometry is found. From top to bottom: 1) tube inflated by positive internal pressure, 2) tube with zero pressure, 3) tube soon after buckling, 4) tube at the point of opposite wall contact, and 5) a mostly collapsed tube.
separation point as well as downstream in a rigid portion of the channel of constant cross-sectional area, neither of which are accounted for by this model \[94,111\]. Despite the simplifications, this model has predicted numerous behaviors observed in stabling resistor experiments \[28,61,78,79,81\], as well as clinical observations of snorers and non-snorers \[1\]. In previous implementations, the fluid equations are closed using a simple tube law relation between pressure and area, but in Chapter \[3\] we decouple the fluid solve from a solid solve allowing the solid model to be a black box and/or complex solver. Our implementation also allows for an uneven and completely closing geometry, with $\chi$ estimated as a function of $Re$.

\begin{align*}
\chi &= 1, \text{ complete pressure recovery.} \\
0 < \chi < 1, \text{ partial pressure recovery.} \\
\chi &= 0, \text{ no pressure recovery.}
\end{align*}

Figure 2.6: The meaning of $\chi$-values in the limited pressure recovery model (similar to illustration by Heil and Jensen \[61\]).

The behavior of the Starling Resistor varies according to the relative influence of solid inertia to fluid inertia. The ratio may be written as: $m = (\rho_{\text{solid}} h_{\text{solid}}) / (\rho_{\text{fluid}} h_{\text{fluid}})$ where $\rho$ is the density and $h$ is the thickness of the tube or fluid domain. When $m < 0.01$ the fluid inertia is the deciding factor, while $m > 10$ means that the solid inertia dominates \[18\]. Soft tissue is about 800 times more dense than air while the fluid and solid domains are of similar thickness, thus OSA is strongly dominated by the solid inertia. The dominance of the airway structure’s inertia over the air inertia in OSA is an important factor suggesting that a bulk flow fluid model may be sufficient to capture the FSI in OSA.

A central research topic in Starling Resistor literature is understanding the onset and mechanism for oscillations \[62,80,95,117,154\]. Bertram et al. \[25\] confirm that flow rate limitation and the onset of oscillations are separate events. Jensen and Heil \[80\] identify an important mechanisms valid for a membrane with high axial tension: oscillations develop when the upstream length is shorter than the downstream length because the lesser upstream fluid inertia causes fluid being displaced by an oscillating membrane to favor upstream displacement which causes energy extraction from
the mean flow as opposed to downstream displacement which encourages an outflow of energy through the outlet. They conclude that a long downstream length (or equivalently specifying a flux BC at the outlet) promotes instability, while a long upstream length (or equivalently specifying a flux BC at the inlet) promotes stability. While these provide partial explanations for the onset of oscillations, a general theory is still lacking [58].

Thus far, FSI simulations have been successful in predicting behaviors similar to experiments and providing insight into the mechanisms of various FSI behaviors. However, assumptions such as thin-walled solids [54,59] or not handling collisions [98], along with uncertainty of experimental methods, have prevented direct comparison of simulations and experiments. In Chapter 5, we bridge this gap by demonstrating simulations and experiments side-by-side in a 2d scenario.

2.3.2 Other Models

There are FSI models relating to OSA beyond the Starling resistor, while maintaining a simplified geometry. A series of simulations and experiments have been done which model the tongue as a deformable cylinder that forms a constriction with a rigid rear pharyngeal wall [31,32,120,145]. This research demonstrates the sensitivity of some fluid and solid modeling methods with directly comparable experiments, but is limited to steady behavior and relatively small deformations.

Balint and Lucey [7] perform 2D FSI simulations of a thin, flexible plate immersed in the fluid. This provides an analogy to soft palate oscillations when during inhalation through both the nose and mouth.

2.4 Upper Airway Models

Though upper airway simulations are challenging to design and execute, the need to better understand UA mechanics and predict outcomes has motivated numerous solid, fluid, and even FSI models of OSA and related UA phenomenon.

2.4.1 Solid Mechanics

Numerical models of the UA structures have been used in a diversity of applications, for example: study the consequences of jaw surgery [52,53,136], predict muscle activity [137], understand snoring [91], and predict soft palate and tongue motion [15,65,50,112]. Some of the main challenges when modeling these structures are deriving the proper geometry (typically from medical imaging), defining the appropriate material properties for the soft tissues, and defining the appropriate muscle fibers and activations.

UA simulations typically model the soft tissue using an FEM model, and bones as a rigid body [50,138] or a very stiff FEM [91]. The geometry of the upper airway structures is often derived
from medical images [26, 50, 66, 112, 138, 150]. While it is beneficial to have a “realistic” geometry, the geometry construction process is time-consuming and depends on user judgments, thus the modeler should be skilled in UA anatomy and mesh generation [30]. Other factors which may be unknown or difficult to account for are the posture of the subject when scanned, the state of the subject (awake, sleeping, deceased), if the subject had any UA disorders, and the tissue properties of the subject. For example, Pelteret and Reddy [112] attempt to remove the effects of gravity on their model derived from images of a deceased subject, a calculation that depends on the soft tissue properties. These numerous complications would make it extremely difficult to build a consistent and reliable set of UA geometries, which leads us to design a parameterized UA model in Chapter 6.

FEM muscle tissue is often modeled with a linear material model, using Young’s Modulus in the range of 6000 ≤ $E$ ≤ 25000 Pa for the soft palate and 4000 ≤ $E$ ≤ 15000 Pa for the tongue [50, 67, 68, 150, 157]. These are plausible values given the estimates in Section 2.2. However, hyperelastic constitutive models are often used to better represent the non-linear properties of muscles (as described by Duck [39]) [13, 26, 112, 138], or viscoelastic models are used to describe time-dependent viscous behaviors [91]. The tongue, being nearly incompressible, has been modeled with Poisson ratio 0.49 [66] or 0.499 [26, 91], while a Poisson ratio of 0.42 has been used for the soft palate [91, 150].

One common method of modeling muscles is to embed point-to-point muscle fibers within the soft tissue material. The behavior of the muscle as a whole will be a combination of the forces caused by the embedded muscles and the stresses resulting from the constitutive model. As a result, one needs to be careful that this combined behavior is in keeping with muscle properties measured from humans. Commonly, the embedded muscles are a Hill-type muscle, which estimates the passive force and the active force of the muscle as a function of the muscle length in relation to its rest length. The active force may range between 0, when there is no muscle activation, to the muscle’s maximum force when there is full muscle activation. In this way, the directional and time-dependent effects of muscle activity can be accounted for [26, 50, 66, 112, 138].

### 2.4.2 Fluid Simulations

To understand the fluid behavior in the airway, such as the pressure drop, numerous researchers have performed Computational Fluid Dynamic (CFD) simulations in non-deforming UA geometries [2, 6, 93, 105, 133, 147, 161]. Other researchers have performed simulations and experiments side-by-side, both in an idealized airway [8, 9, 55, 84], and in realistic airways [100, 104, 158]. Amongst the Reynolds Averaged Navier Stokes (RANS) turbulence models, the $k – \omega$ model performs the best, though still with significant errors [8, 104, 158]. Even Large Eddy Simulations (LES) on a coarse mesh [104], and Direct Numerical Simulations (DNS) using a Lattice Boltzmann method [9], have significant differences from experiments. Even though these experiments and simulations are in the
2.4. Upper Airway Models

much simpler case of a static geometry, they reveal the immense challenge of properly simulating airflow in the upper airway, both in terms of design and computational expense.

As presented in Sections 2.2.3 and 2.2.4, excessive pressure drop in the airway can lead to flow limitation and closure. As a simple approximation, some researchers estimate the pressure drop as function of the flow rate squared based on their CFD results [2,55,133,158]. Van Hirtum et al. [145] discuss some of the factors of the flow that have a large influence on the pressure drop, which largely occurs across the collapse-prone constriction. A change from laminar to turbulent flow leads to a pressure drop 40% larger. The force from fluid pressure can written as $F = \int_{\text{inlet}}^{\text{separation}} p(x) \, dA$, thus highlighting the effect that the location of flow separation beyond the constriction has. They also note that a small change in the geometry can lead to dramatic changes in the pressure drop; the turbulence and location of separation will naturally play a large role in this.

A single value for pressure drop is over-simplistic for OSA simulations, which require local pressure values to apply to the airway walls. However, proper 3D fluid simulations are very demanding in design and computational expense. Therefore, in Chapter 3 we implement the 1D fluid model defined by Cancelli and Pedley [28] (described in Section 2.3.1) and compare its performance with 3D fluid simulations in upper airway geometries to demonstrate the applicability of a 1D model in OSA simulations.

2.4.3 Fluid-Structure Interaction

There have been few attempts to simulate FSI in a realistic UA. Perhaps the most notable are the 2D [66,67] and semi-3D [68] simulations by Huang et al. They predict plausible $p_{\text{crit}}$ values and oscillations at a snoring frequency. They simulate the consequences of stiffening portions of the soft palate or tongue, as might be done surgically to an OSA subject; an example of the predictive power of simulations. Some limitations of these results are that the rear pharyngeal wall is not deformable, the side pharyngeal walls are not included, and the air flow is laminar.

Wang et al. [150] perform a nice FSI simulation comparing a pre- and post-operative 3D geometry using a $k - \omega$ turbulence model and linear muscle material. They demonstrate decreased pressure loss in the post-operative case for both inspiration and expiration. However, they only simulate a steady behavior with relatively small deformations and without complete closure, nor do they include dynamic pharyngeal walls.

These FSI simulations do investigate the sensitivity to some parameters, but mostly material and fluid parameters are adjusted rather than geometric parameters. However, as reviewed, a small UA geometry is a key factor in OSA. We address this concern in Chapter 6, in which we use a parameterized UA geometry to examine the sensitivity of numerous geometric and material parameters on FSI behaviors. We also predict plausible $p_{\text{crit}}$ values and snoring oscillations using a complete 3D geometry with dynamic pharyngeal walls.
2.5. Artisynth

Numerical stability is an important issue in FSI simulations. The best stability comes from a monolithic approach in which the fluid, solid, and coupling equations are solved together; but this makes the use of a dedicated fluid and/or structure solver difficult or impossible. Alternatively, one may solve the fluid and structure equations separately in a segregated approach, with the coupling enforced as input to each step. This allows one to leverage the power of specialized solvers (assuming they expose this functionality to the user), but suffers from instability, particularly for large-deformation FSI\textsuperscript{[36,57,59].} Causin et al.\textsuperscript{[29]} demonstrate that segregated approaches become unstable for slender geometries when the solid is thin with a density close to that of the fluid. Though the airway is a fairly slender geometry, its soft tissue structures are thick and are about 800 times more dense than air, therefore, segregated solvers are a viable option for OSA simulation.

2.5 Artisynth

The simulations described in this thesis are performed using Artisynth (with the exception of the CFD simulations, described in Chapter 3). Artisynth is an open-source biomechanical modeling toolkit, with capability to model systems including non-deformable solids, deformable solids, particles, springs, as well as various joints and connectors.

Artisynth includes features particular to biomechanics, such as Hill-type muscle models and non-linear material models; including a quasi-linear viscoelastic material model and hyperelastic material models (Saint Venant-Kirchoff, Mooney-Rivlin, Fung, Ogden, and Neo-hookean). Artisynth can handle contact between various objects as well as self-intersection. It also includes an inverse mechanics solver. Being open source, we implement the 1D fluid model and the fluid-solid coupling directly in the Artisynth framework, thus removing the overhead of communicating between two simulation packages. These features make it a useful system for OSA modeling\textsuperscript{[43,45,52,53,137,138].}

Artisynth contains a number of low-level unit tests to verify proper numerical implementation. It contains many simple demos that highlight the functionality of the software while providing situational testing and examples of proper usage. Artisynth has also been tested against ANSYS (ANSYS\textsuperscript{®} Academic Research, Release 13.0) and FEBio\textsuperscript{[96]}, to further ensure the reliability of the software\textsuperscript{[92].}

An overview of solid mechanics simulation is given in Appendix B, and a description of the Artisynth simulation engine is given in Fels et al.\textsuperscript{[43].} Here, we overview how nodal forces are updated, particularly highlighting the definition of damping parameters which are mentioned elsewhere in this thesis. The nodal forces are calculated by:

\[ \mathbf{F} = \mathbf{F}_{\text{ext}} + \mathbf{F}_{\text{g}} - \mathbf{F}_{\text{int}} - \mathbf{F}_{\text{damp}} \]

The four components in the force calculation are:
1. $F_{\text{ext}}$: external forces, user assigned forces.

2. $F_g = mg$: the force due to gravity $g$.

3. $F_{\text{int}}$: internal forces calculated according to the compressibility and material model assigned.

4. $F_{\text{damp}} = F_{\text{md}} + F_{\text{sd}}$: damping forces. Mass damping is defined by $F_{\text{md}} = d_{\text{mass}} m \ddot{u}$, thus increasing the mass damping coefficient $d_{\text{mass}}$ resists the momentum of the node. Stiffness damping is defined by $F_{\text{sd}} = d_{\text{stiff}} \sum K_n u_n$, where the summation is being performed for each neighboring node and $K$ is the node’s stiffness matrix, thus increasing the stiffness damping coefficient $d_{\text{stiff}}$ resists the stress-induced pull between the node and its neighbors. The damping in a spring is a simple analogy for stiffness damping.

### 2.6 Prospects

This background material illustrates the need for an enriched understanding of FSI behaviors and improved FSI simulation technology, which is then applied to OSA modeling to develop a comprehensive and trusted model of OSA. We make progress towards those ends in the following chapters.
Chapter 3

Fluid Model

3.1 Introduction

The airflow mechanics in the human Upper Airway (UA) are very difficult model, with Obstructive Sleep Apnea (OSA) being a particularly difficult case. The UA geometry is very complex and may deform greatly even to the point of complete closure, and during a breathing cycle the flow regime may range from laminar to turbulent. However, a fluid model must handle these complexities in a robust and reasonable manner in order to be useful in Fluid-Structure Interaction (FSI) modeling of OSA. In this chapter we present a numerical implementation of a 1D fluid model, which is designed for the purpose of modeling airflow in the human upper airway.

The speed and simplicity of the Cancelli and Pedley [28] 1D fluid model (presented in Section 2.3.1) make it an attractive option, if indeed it can reasonably predict the bulk pressure in the UA. However, to our knowledge this model has only been implemented as tightly-coupled with very simple solid models, which has limited it to rather academic cases. We hypothesize that this model is useful for FSI simulations of OSA, and therefore present an implementation that meets the following requirements:

- The fluid equations are solved separately from the structure equations, which permits the structural dynamics of the upper airway to be computed using a high quality, 3D simulation. In this formulation the area function is known prior to the solution of the fluid equations, and is treated as a known parameter.
- The area function derived from the upper airway may be very uneven, so the numerical methods must be robust to such an area function.
- To simulate OSA or snoring, the geometry is expected to close completely and perhaps reopen. This case must be handled carefully to be numerically stable and physically reasonable.
- Velocity boundary conditions may be appropriate for a relatively open geometry, but when the geometry nears closure they will lead to unphysical pressures and numerical instability,

This chapter is largely derived from our paper “Implementation and Validation of a 1D Fluid Model for Collapsible Channels” [3] (© ASME 2013, reproduced with permission).
and must be modified. The implementation must be robust to such a dynamic modification of boundary conditions.

- Airflow in the airway is at a relatively low Reynolds number, where viscous losses play an important role. These viscous losses must be modeled with simple values of $\chi$.

In the following section, the model and its numerical implementation is presented. Then, we present three test cases to examine the worthiness of the 1D model. First, an analytical case to verify our numerical methods. Second, a more demanding case where the geometry closes completely, which includes the viscous and closure models. And third, the 1D model is compared with two 3D simulations in a static airway geometry to examine the viscous model. Afterwards, we discuss the usefulness of the 1D model to FSI simulations of OSA.

### 3.2 1D Fluid Model

Following the ideas of Cancelli and Podley [28], we write the fluid continuity and momentum equations as:

$$\frac{\partial}{\partial t} A + \frac{\partial}{\partial x} Au = 0$$  \hspace{0.5cm} (3.1)

$$\rho u \frac{\partial}{\partial x} u + \rho \frac{\partial}{\partial t} u + \frac{\partial p}{\partial x} - \frac{s}{A} = 0$$  \hspace{0.5cm} (3.2)

$$\tau - \tau_{\text{fric}} - \tau_{\chi} = 0$$  \hspace{0.5cm} (3.3)

where $s$ is the perimeter around cross-sectional area $A$ (a circular perimeter is not assumed), $u$ is average velocity, $p$ is pressure, and $\rho$ is density. The term $\tau$ models viscous losses and contains the terms $\tau_{\text{fric}} = -2 \mu \left(\frac{s}{A}\right) u$, which describes laminar losses [153], and $\tau_{\chi} = \left(\frac{A}{s}\right) (1 - \chi) \rho u \left(\frac{\partial u}{\partial x}\right)$ which describes losses due to flow separation. The $\tau$ terms are separated into their own equation for clarity, but could easily be substituted into the momentum equation. Thus, the solution variables are: $u$, $p$, and $\tau$.

The $\chi$ term defines flow separation as occurring when $\partial p/\partial x = 0$, and limits downstream pressure recovery, such that:

$$\chi = \begin{cases} 
1 & \text{for } u \frac{\partial p}{\partial x} < 0 \\
\chi_{\text{min}} & \text{for } u \frac{\partial p}{\partial x} \geq 0 
\end{cases}$$  \hspace{0.5cm} (3.4)

where $u$ multiplies the pressure gradient to accommodate the case of flow in the $-x$ direction, and $\chi_{\text{min}}$ is a user-defined constant, which may be flow-dependent. We use the inviscid approximation $\partial p/\partial x \approx -\rho u \left(\frac{\partial u}{\partial x}\right) - \rho \left(\frac{\partial u}{\partial t}\right)$ for calculating $\chi$, which is highly advantageous because it removes $p$-dependency from the $\tau$ equation, and the $\tau$ term is generally small where $\partial p/\partial x = 0$.  

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3.2. 1D Fluid Model

3.2.1 Numerical Implementation

In the case of a velocity-driven flow, these equations can be solved sequentially: first, solve Eq. 3.1 for \( u(x) \) using the known velocity BC; second, solve Eq. 3.3 for \( \tau(x) \), which depends only on \( u \); and finally, use \( u \) and \( \tau \) to solve Eq. 3.2 for \( p(x) \).

However, the flow in the upper airway demands BCs more complex than a velocity driven flow, requiring coupled solution of Eqs. 3.1, 3.2, 3.3. In previous work, this was done using variations of MacCormack’s method [28]. However, with \( A(x,t) \) now a known quantity, these equations are not hyperbolic and such a method is not applicable. Therefore we use Newton’s method:

\[
J(X) \cdot \Delta X = -F(X)
\]

(3.5)

where \( X \) is the solution vector, \( J \) is the Jacobian matrix, and \( F \) is the residual vector. The requirement to know the Jacobian makes the coupled solution more complicated. Our solution procedure is:

1. Create an initial guess for \( X^{n+1}_{i=0} \). The superscript refers to simulation time, and the subscript refers to Newton-iterations.
2. Calculate the \( F(X^{n+1}_{i}) \) and \( J(X^{n+1}_{i}) \).
3. Solve the system (Eq. 3.5) for \( \Delta X \).
4. Calculate \( X^{n+1}_{i+1} = X^{n+1}_{i} + \Delta X \cdot 0.5j \) where \( j \) is iterated until \( \| F(X^{n+1}_{i+1}) \| < \| F(X^{n+1}_{i}) \| \).
   The line search is added to the standard Newton method to improve convergence robustness.
5. Iterate steps 2-4 until \( \| F(X^{n+1}_{i+1}) \| \) is below a specified tolerance.

Applying Eqs. 3.1, 3.2, 3.3 to the form of Eq. 3.5 yields:

\[
\begin{bmatrix}
\frac{\partial A}{\partial x} + A \frac{\partial}{\partial x} & 0 & 0 \\
\rho \frac{\partial u}{\partial x} + \rho u \frac{\partial A}{\partial x} & \frac{\partial}{\partial x} - \frac{s}{A} & 0 \\
-\frac{\partial \nu_{bc}}{\partial u} - \frac{\partial \tau_{\chi}}{\partial u} & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\Delta u \\
\Delta p \\
\Delta \tau
\end{bmatrix} = -
\begin{bmatrix}
\frac{\partial}{\partial x} \frac{\partial A}{\partial x} + \frac{\partial}{\partial x} Au \\
\rho u \frac{\partial u}{\partial x} + \rho \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} - \tau \frac{s}{A} \\
\tau - \tau_{bc} - \tau_{\chi}
\end{bmatrix}
\]

(3.6)

where \( \frac{\partial \nu_{bc}}{\partial u} = -2\mu (s/A) \), and \( \frac{\partial \tau_{\chi}}{\partial u} = \rho (A/s) [-u (\partial \chi/\partial x) + (1 - \chi) (\partial u/\partial x) + (1 - \chi) u (\partial/\partial x)] \).

This analytical representation does not include the BCs. Also, discretizing the \( \partial/\partial t \) operator requires that known terms be added to \( \Delta X \) (for example, \( \Delta u^n \) and \( \Delta u^{n-1} \) are needed for a 2nd order solution).

The \( \chi \) model is discontinuous, and we find better behavior using a discrete step function rather than a smoothed transition function. Conveniently, the Jacobian only requires \( \partial \chi/\partial x \), which is calculated numerically, rather than an analytical expansion of \( \chi \) and its step-function (a strong advantage to writing Eq. 3.4 only as a function of \( u \)).
With this coupled solution method, it is easy to switch between velocity driven flow \((u_{\text{inlet}}, p_{\text{outlet}})\) defined), pressure driven flow \((p_{\text{inlet}}, p_{\text{outlet}})\) defined), or a more complex BC. The BC for \(\tau\) is \(\partial \tau / \partial x|_{\text{inlet}} = 0\), i.e. a fully developed inlet flow. To account for the finite strength of the lungs (or whatever is driving the flow), we define pressure-limited BCs:

\[
BC_{\text{inlet}} = \begin{cases} 
  u(0, t) = u_{\text{inlet}} & \text{for } p(0, t) \leq p_{\text{threshold}} \\
  p(0, t) = p_{\text{threshold}} & \text{for } p(0, t) > p_{\text{threshold}} 
\end{cases}
\] (3.7)

where \(p_{\text{threshold}}\) is the maximum pressure the lungs can produce.

For temporal discretization, the 2\textsuperscript{nd} order central scheme (centered at \(n\) rather than \(n + 1\)) is chosen for stability, though 2\textsuperscript{nd} order backwards Euler would be more accurate. For spatial discretization, we use a 4\textsuperscript{th} order asymmetrical scheme: \(\partial f / \partial x = (-2f_{i-1} - 3f_i + 6f_{i+1} - f_{i+2}) / (6 \cdot \Delta x)\), which behaves robustly in a ragged geometry, transitions smoothly to the one-sided schemes at the boundary, and has smooth errors. (See Appendix \(\text{□}\) for review of derivative schemes.)

The case of area approaching zero needs special consideration. As can be seen in Eq. \(\text{3.6}\) there are terms divided by \(A(x, t)\) which cause numerical instabilities for near-zero areas. To handle this, we define a “safe” area function:

\[
A_{\text{safe}}(x, t) = A(x, t) + A_{\text{closed}} \cdot w(A(x, t))
\] (3.8)

with the transition function:

\[
w(A(x)) = \begin{cases} 
  0 & \text{for } A(x) > A_{\text{small}} \\
  \frac{A(x) - A_{\text{small}}}{A_{\text{closed}} - A_{\text{small}}} & \text{for } A_{\text{closed}} \leq A(x) \leq A_{\text{small}} \\
  1 & \text{for } A(x) < A_{\text{closed}} 
\end{cases}
\] (3.9)

and where \(A_{\text{closed}}\) is the minimum allowable area, and \(A_{\text{small}}\) is a larger area at which the transition begins. Thus, \(A_{\text{safe}}(x, t) = A(x, t)\) where \(A(x, t) > A_{\text{small}}\), but as \(A(x, t)\) approaches zero, \(A_{\text{safe}}(x, t)\) approaches \(A_{\text{closed}}\). \(A_{\text{closed}}\) should be chosen to be close to zero while remaining numerically stable. \(A_{\text{small}} = 2.5 \cdot A_{\text{closed}}\) is a good choice to ensure that the transition is smooth and semi-local. At the same time, to ensure that the flow rate through the closure is small, we add an artificial viscous term \(\tau_{\text{small}} = -0.5\rho u^2 \cdot w(A(x, t))\) and \(\partial \tau_{\text{small}} / \partial u = -\rho u \cdot w(A(x, t))\) is added to the Jacobian. Using this simple area correction, we have found the simulation to proceed stably and smoothly, even though \(A(x, t)\) contains zeros.

### 3.3 Results

The 1D fluid model is applied to 3 test cases to verify the numerical implementation and analyze its behavior.
3.3.1 Analytical Case

For the sake of code validation, an analytical solution was derived assuming the area function \( A(x,t) = A_0 - A_m \sin(\pi x) \sin(\pi t) \) where the initial area \( A_0 \) and the magnitude of the collapse \( A_m \) are constants. We define \( \alpha_{\text{min}} = \min(A(x,t))/A(0,0) \) and note that \( A_m = A_0 \cdot (1 - \alpha_{\text{min}}) \). We also assume that \( u(0,t) = u_0, p(0,t) = p_0, \) and \( \tau(x,t) = 0 \). For these constraints the solution to Eqs. 3.1 and 3.2 is:

\[
\begin{align*}
  u(x,t) &= \frac{1}{A}(u_0 A_0 + A_m \cos(\pi t)(1 - \cos(\pi x))) \\
  p(x,t) &= \pi \rho A_m \int \sin(\pi t)\frac{\sin(\pi x)}{A}(1 + (u^2 - 1) \cos(\pi x)) \, dx - \rho u^2 + c_p
\end{align*}
\]

(3.10) (3.11)

The derivation of these solutions is given in Appendix E. The integration of Eq. 3.11 and definition of constant \( c_p \) according to the BCs is performed numerically. Though ignoring the viscous effects and using simple BCs, this analytical solution is useful for verifying the numerical implementation of the 1D model, particularly considering the case where the minimum area approaches zero (when \( A_m \) approaches \( A_0 \)).

To compare with the numerical solution, we set \( u_{\text{inlet}} = 1, p_{\text{inlet}} = 0, \rho = 1.2, \) and \( A_0 = 0.2 \) for the domain \( 0 \leq x \leq 1 \) and \( 0 \leq t \leq 2 \). The numerical and analytical solutions are compared in Fig. 3.1 in terms of a non-dimensional pressure error, \( p^* = \max(|p(x,t) - p_a(x,t)|)/\max(\rho \cdot u(x,t)^2) \), where \( p_a(x,t) \) is the analytical solution from Eq. 3.11. This is performed for \( 0.01 \leq \alpha_{\text{min}} \leq 0.99 \) with a coarse mesh (\( \Delta x = 0.1, \Delta t = 0.02 \)), a medium mesh (\( \Delta x = 0.05, \Delta t = 0.01 \)), and a fine mesh (\( \Delta x = 0.025, \Delta t = 0.005 \)).

Figure 3.1 shows that the error rapidly increases as \( \alpha_{\text{min}} \) approaches zero. This is expected, and illustrates the need to define a finite \( A_{\text{safe}}(x,t) \). The numerical error reduces by a factor of 4 when the mesh is refined by a factor of 2, demonstrating the 2nd order accuracy of the method (see [122] concerning mesh refinement). Finally, Fig. 3.1 may serve as a guideline for choosing a mesh quality and \( A_{\text{closed}} \) value according to an allowable error. For example, with the finest mesh one may set \( A_{\text{closed}}/A_0 = 0.05 \) (95% closed) and still constrain the pressure error to less than 2% of the dynamic pressure.
3.3. Results

![Graph showing mesh refinement study](image)

Figure 3.1: Mesh refinement study, showing numerical error $p^*$ as a function of $\alpha_{\text{min}}$, for 3 levels of mesh quality: coarse (dash-dot red curve), medium (dashed blue curve), and fine (solid green curve).

### 3.3.2 Closing Geometry Case

The analytical case, however, is limiting in that it does not consider the viscous term, nor area closure handling, and only uses velocity driven BCs. To challenge the model further, we define the area function $A(x,t) = A_0 - A_0 \cdot e^{-((t-0.125)/(0.03))^2} \cdot \left[e^{-(x-0.03)/(0.015)^2} + 0.95 \cdot e^{-(x-0.11)/(0.03)^2}\right]$ for $A_0 = 2 \text{ cm}^2$ and $0 \leq x \leq 0.15 \text{ m}$ and $0 \leq t \leq 0.25 \text{ s}$, roughly modeling the airway dimensions. This closes rapidly in time and space, with two locations of constriction; the first closing entirely (even producing a negative “area”) and the second closing 95% of $A_0$, thus $A_{\text{safe}}(x,t)$ must be used (with $A_{\text{closed}} = 0.025A_0$). The viscous terms are included with $\chi_{\text{min}} = 0.25$, and pressure-limited BCs are used with $u_{\text{inlet}} = 2 \text{ m/s}$, $p_{\text{threshold}} = 200 \text{ Pa}$, and $p_{\text{outlet}} = 0 \text{ Pa}$.

The area and pressure functions are shown at selected times in Fig. 3.2. The curve at $t = 0.110 \text{ s}$ shows the pressure as the geometry is collapsing. At $t = 0.125 \text{ s}$ the geometry is closed and the inlet pressure has reached the threshold of 200 Pa at which point pressure-driven BCs are used to make sure it is not exceeded. The pressure function becomes very steep yet remains smooth across the closed section. At $t = 0.135 \text{ s}$, the geometry is reopening yet still at the $p_{\text{threshold}}$ limit. Finally, at $t = 0.136 \text{ s}$ the BCs are velocity-driven once again because $u(0,t) > u_{\text{inlet}}$ indicating that $p(0,t)$ has dropped below $p_{\text{threshold}}$. The switch from pressure-driven to velocity-driven BCs suddenly zeros the $\partial u(0,t)/\partial x$ term of Eq. 3.6, which contributes to the big change between $p(x,0.135 \text{ s})$ and $p(x,0.136 \text{ s})$. As the geometry rapidly reopens, the velocity becomes negative at the outlet (yet remains positive in most of the domain) to accommodate the expanding volume. Throughout this sequence, the solution remains both sufficiently smooth and qualitatively reasonable.

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3.3. Results

![Figure 3.2: Area and pressure function at selected times.](image)

3.3.3 3D Simulation Cases

To compare the 1D model with viscous flow in a complex 3D geometry and inform the choice of $\chi_{\text{min}}$, we performed 3D fluid simulations in two static geometries. The geometries are meshed with a regular hexahedral mesh; including coarse, medium, and fine meshes to study mesh dependence. The airflow is simulated with OpenFoam, including initial laminar simulations on the coarse mesh, RANS simulations using the $k - \omega$ SST turbulence model on all meshes, and LES simulations on the fine mesh (though too coarse for LES, this is still a useful comparison). The flow is assumed incompressible. All simulations are run on an 8-core 2.67GHz PC, with 12GB RAM.

To compare the 1D results with the 3D results, we must derive a centerline and area function from the 3D geometry. Calculating the Voronoi diagram of the mid-sagittal slice of the geometry reveals an obvious centerline, which we extract by hand. From this, the area function, which serves as input to the 1D simulation, is defined by taking cut-planes normal to the centerline at regular intervals. The 3D simulation is reduced to 1D velocity and pressure by performing spatial interpolation and temporal averaging along the centerline using the VTK library/Paraview.

3.3.3.1 Case 1: Velocity BC, Idealized Geometry

The first case considers an inspiratory velocity-driven flow through the idealized airway of previous research [8, 9, 55, 84], that was designed with the intent of researching airflow in a geometry composed of simple shapes while describing an “average” oropharynx. While their results are the benchmark, we performed simulations for the sake of having a complete data set to analyze and because our geometry varies in minor details from theirs.

The coarse, medium and fine meshes have 85,000, 202,000 and 679,000 cells respectively. The fine LES simulation, running at 7.18 s/step, was significantly faster than the fine RANS simulation,
which took 19.55 s/step; both fine cases were computed in parallel. The simulation time step was chosen to keep the Courant number below 1; in most cases 5 \cdot 10^{-5} s. At least 0.6 s of simulation time were calculated, providing an adequate amount of statistically steady flow for analysis. At the inlet, the velocity is 0.88 m/s and Reynolds number 1100.

The 3D geometry of the first case is shown in Fig. 3.3a, colored by pressure at an instant in time. The complexity of the flow can be seen in Fig. 3.3b, which shows velocity from the fine-mesh LES at an instant in time.

![3D geometry colored by pressure](image)

![Velocity from fine-mesh LES](image)

(a) A 3D view of the geometry, colored by pressure. The flow is driven by a velocity inlet (upper right corner), and develops through a uniform pipe before entering the oral cavity of the idealized airway. The outlet (bottom left corner) is in the trachea.

(b) A mid-sagittal view of velocities from a LES, illustrating the complexities of the flow. The 1D centerline with distance labels is included.

Figure 3.3: Case 1 simulations.

A comparison of the 1D model and 3D models is shown in Fig. 3.4. The $k – \omega$ SST simulations changed little with mesh refinement, indicating mesh independence. Even though the LES is quite under-resolved, it matches the $k – \omega$ SST closely.

The 1D model captures the 3D models quite well when using $\chi_{\min} = 0.25$, though from $0.18 < x < 0.23$ m there is a notable deviation. The results for $\chi_{\min} = 0$ (no recovery allowed) and $\chi_{\min} = 1.0$ (complete recovery allowed) are included to show the range of the $\chi$ model.
3.3. Results

Figure 3.4: A comparison of the case 1 simulations: a 3D RANS (solid blue curve), a 3D LES (solid green curve), and the 1D model with $\chi_{\text{min}} = 0.25$ (dashed black curve). The bounds of the model are shown with $\chi_{\text{min}} = 0.0$ (solid red curve) and $\chi_{\text{min}} = 1.0$ (dashed red curve), and the effects of a small variation are shown with $\chi_{\text{min}} = 0.20$ (thin solid black curve) and $\chi_{\text{min}} = 0.30$ (thin dash-dot black curve).

The velocity and pressure of the 1D model are averaged values, and can not account for turbulent fluctuations. To consider the significance of unsteady fluctuations, the LES centerline pressure with the temporal standard deviation of pressure is shown in Fig. 3.5. The pressure deviations are quite small along most of the centerline, though there are some larger fluctuations around $x = 0.2$ m where the flow has broken into turbulence. At this location, the pressure fluctuations are spread across a broad range of frequencies.

The pressure given for the 3D models is the temporal average along the centerline, but we can also consider temporal cutplane averages which take the average over the entire cutplane that is perpendicular to the centerline, rather than at the centerline itself. For example, at $x = 0.1$ m, $p = 5.75$ Pa with the 1D model, $p = 5.87$ Pa on the 3D LES centerline, and 5.91 Pa on the 3D LES cutplane. In comparison, at $x = 0.21$ m where pressure fluctuation is highest, $p = 0.13$ Pa with the 1D model, $p = 1.13$ Pa on the 3D centerline, and 0.83 Pa on the 3D cutplane, thus illustrating that the centerline and cutplane averages may differ significantly.
3.3. Results

![Graph showing pressure distribution](image)

Figure 3.5: LES results for case 1 (solid black curve), with ±1 standard deviation of pressure (dashed green curves).

### 3.3.3.2 Case 2: Pressure BC, Realistic Geometry

The second case uses a geometry that is reconstructed from CT scan data. The pressure-driven flow \( (p_{\text{inlet}} = 0, p_{\text{outlet}} = -30 \text{ Pa}) \) simulates inhalation through the nasal passage. The inlet is taken above the velum, rather than outside the nose, because the main interest of this case is the constriction behind the velum, where the passage narrows to 4.7% of the inlet area. In this area the velocity is greatly elevated and therefore most of the pressure loss occurs here. The geometry, colored by pressure from the fine-mesh LES, is shown in Fig. 3.6a, and the corresponding mid-sagittal velocity field is shown in Fig. 3.6b.
3.3. Results

![Image of 3D view of geometry and velocities from LES](image)

(a) A 3D view of the geometry, colored by pressure. The flow, which comes from the nasal cavity, enters the geometry at the pressure inlet (upper left corner) and exits in the upper trachea (bottom). The oral cavity (upper right) has no flow.

(b) Velocities from a LES showing flow through a narrow constriction. The 1D centerline with distance labels is included.

Figure 3.6: Case 2 simulations.

The coarse, medium and fine meshes have 222,000, 749,000 and 1,775,000 cells respectively. At least 0.1 s of simulation time was calculated; in most cases the time step was $1 \cdot 10^{-5}$ s. The Reynolds number ranges from approximately 300 to 1500.

The 1D and 3D models are compared in Fig. 3.7. Again, the RANS models converge with mesh refinement (though not as well as the previous case). Though still a good match, the LES and RANS do not agree as strongly as in case 1, with the most significant deviation around 0.06 m where the LES predicts the jet breaking into turbulence whereas RANS predicts that the highest turbulence intensity occurs around 0.045 m.

The 1D model agrees within 10% for $\chi_{\text{min}} = 0.20$. The most significant deviation occurs in the region $0.03 < x < 0.07$ m, where the area suddenly expands as the nasal passage joins the oral cavity (the sudden expansion at 0.03 m is more evident in the 3D geometry than the mid-sagittal slice). The 3D models predict a well-formed jet through this expanded area, and pressure recovery further downstream where the jet breaks up and the flow decelerates.

Comparing the 1D model with 3D centerline and cutplane averages, at $x = 0.025$ m in the constriction, $p = -33.30$ Pa with the 1D model, $p = -33.46$ Pa on the 3D LES centerline, and $p = -31.97$ Pa on the 3D LES cutplane. In comparison, at $x = 0.06$ m where LES predicts significant turbulence, $p = -29.86$ Pa with the 1D model, $p = -32.18$ Pa on the 3D centerline, and $p = -31.56$ Pa on the 3D cutplane.
### 3.4 Discussion

Concerning the model’s implementation, we offer the following comments and suggestions. First, use of the 4th order asymmetrical scheme for spatial discretization is beneficial. It is robust for geometries with a jagged area function and for unsmooth solution variables. While showing a bias in one direction, it is functional for both directions, and has errors both smaller and smoother than the jagged errors of the 2nd order central scheme. Second, for the sake of numerical accuracy, it is better to keep the term $\partial Au/\partial x$ as is, rather than writing it as its mathematical equivalent $A(\partial u/\partial x) + u(\partial A/\partial x)$. In the former case, the errors in $u$ and $A$ will, to some extent, counter-balance each other. Third, one weakness in this model is that it requires a Jacobian, and defining an analytical Jacobian can be tedious. A possible remedy is to calculate an approximate Jacobian (with the secant method) rather than providing an analytical Jacobian. Finally, switching abruptly between velocity-driven and pressure-driven BCs is not smooth. More smooth and realistic behavior may be gained by constraining the BC to a pressure-velocity curve, similar to a fan curve.

We now consider the usefulness of the 1D model. For OSA simulations, it is of great importance to accurately model the fluid pressure in a narrow and collapsing constriction, and thus capture the FSI of snoring or apnea. Though we do not have a quantitative comparison for a complete dynamical closure, the smooth and reasonable behavior of the model in small and collapsing constrictions is best seen in the closing geometry case of section 3.3.2 and the airway simulation of section 3.3.3.2. The 1D model performs well using $\chi_{\text{min}} = 0.25$ in the first case and $\chi_{\text{min}} = 0.20$ in the second.
In general, $\chi_{\min}$ will vary with both Reynolds number and geometry. This is demonstrated by Smith [134], whose results illustrate that pressure recovery varies most rapidly for $Re \lesssim 3000$, then more gradually for higher Re, though the distinctiveness of this trend is itself dependent on the sharpness of the geometry. In future work, a dynamic $\chi_{\min}$ model may be derived from the results of Smith 2004. The $\tau_\chi$ model does deviate significantly at locations of pressure recovery; 3D simulations predict a gradual recovery well downstream of the separation point, while the $\tau_\chi$ model predicts a rapid recovery. However, the $\tau_\chi$ model does capture the magnitude and location of the bulk flow pressure loss, even in a complex geometry. The $\tau_{fric}$ term is defined for fully-developed laminar flow in pipes, and as such is invalid for the flows considered here. It is included for the stability it provides in some cases, but otherwise contributes very little to the overall pressure loss.

While the 1D model does a good job predicting the average pressure, one must consider if the unsteady turbulent fluctuations, which the 1D model does not predict, are significant. Figure 3.5 demonstrates that the pressure deviations are quite small in most locations, though significant at locations of turbulence. Even those larger deviations are unlikely to be significant because the structures of the airway have much greater inertia than air, causing temporally rapid or spatially local pressure fluctuations to be effectively averaged into a bulk pressure acting on the structures. The fluctuations are spread across a broad spectrum, and are unlikely to excite a strong structural resonance. Also, the locations of narrow geometry are the ones prone to collapse, but the turbulent fluctuations mostly occur downstream of the narrow regions. Therefore, the unsteady turbulent fluctuations appear insignificant for OSA simulations.

Furthermore, in the highly complex flow of interest, even 3D simulations vary significantly and are very difficult to execute properly. The 1D model may only be marginally worse, while being much faster and more robust. And due to its simplicity of design and speed, a 1D model allows the modeler to run many more trials, experimenting with BCs, viscous settings, or other parameters of interest. Thus, the large-scale insight possible with a 1D model is another strong benefit. For these reasons, the 1D model is an excellent approach for OSA models.

3.5 Conclusions

We have presented a 1D fluid model, designed to meet the demands of OSA simulations, or other FSI problems involving complete collapse of complex geometries that may be reduced to 1D. The model is independent of the solid mechanics making it possible to use a very sophisticated and potentially black-box solids solver. This means that $A(x,t)$ and the fluid solution are computed sequentially. To solve the resulting non-linear system of partial differential equations, Newton’s method with an exact Jacobian is chosen. This solution strategy is deemed necessary due to the nature of the system of equations and the BCs required for OSA simulations.

Approximating the location of flow separation and the pressure losses thus incurred is done using the $\chi$ model of previous research. In our implementation, this works for positive or negative flow

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3.5. Conclusions

directions at multiple, dynamically inferred, locations of separation.

The model is discretized with 2\textsuperscript{nd} order temporal accuracy and a 4\textsuperscript{th} order accurate spatial scheme, which behaves well in a complex airway geometry. We also handle the case of a collapsing geometry, proving that the fluid model can smoothly transfer to and from a closed geometry.

In the validation cases presented, the discretization scheme is shown to be robust and the 1D model simulations nearly as accurate as 3D simulations. The dynamic test cases demonstrate that the model behaves in a stable and reasonable manner during abrupt closure. In Chapter [5] we present a complete FSI system with this fluid model 2-way coupled to a 3D finite element model of the solid mechanics, and validate with an experiment.
Chapter 4

Solid Model

4.1 Introduction

The human upper airway includes intricate muscles, fat, mucus, tendons, and bones; the combination of which forms a complex geometry with non-linear soft tissue behavior. The mechanics of the Upper Airway (UA) are poorly understood, and hence a large challenge to model properly with a numerical simulation. However, even with a complete understanding of UA mechanics, designing a proper simulation would be a challenge. In this chapter we validate our numerical methods on collapsible tubes, which have many similar numerical demands to a collapsible airway while being a much simpler and a better understood case. This provides guidelines for how to best simulate Fluid-Structure Interaction (FSI) in the UA, as well as giving insight into the sensitivity of the solid model to numerical methods and parameters.

In Section 2.3.1 I discussed the Starling Resistor model as a simple but useful FSI model. Even in this simple case, most simulations to date have modeled the tube using an empirical model or thin-shell approximation, nor do they handle contact. In this chapter we improve upon the existing simulations of collapsing tubes by modeling a 3D, thick-walled tube that completely collapses and reopens. A collapsing tube is simple yet analogous to the more complex case of the human UA, and the previous research of Bertram [17] provides a readily comparable experimental data set against which to validate our results.

In the next section we present our methods for designing a simulation of collapsing tubes in keeping with the experimental methods of Bertram for validation purposes. In the results we demonstrate that our tube simulations agree well with the experiments of Bertram. We show that they predict the same behavior as the tube buckles, collapses and then reopens, with the main discrepancy being that the simulated tubes are more resistant to collapse. We also present the sensitivity of the simulation to parameter variation. In the discussion we present the sources of error which lead to the differences between the simulation and experiment, and highlight the useful applications of this study. We conclude by reviewing our key findings.

This chapter is largely derived from our workshop paper “Simulation of Collapsible Tubes and Airways - Towards a Validated Upper Airway FSI Model” [4].
4.2 Methods

Our goal is to investigate the behavioral sensitivity of a collapsing tube to a range of physical and numerical parameters validated with experimental data. The main parameters of interest are: mesh type (hexahedral or tetrahedral), mesh quality, compressibility settings, material type (linear or Neo-hookean), and random perturbation of nodes.

The primary tube used in this study is designed according to the thinner tube in Bertram’s uniform pressure experiments [17]; the relevant details of these experiments are as follows. The tube had an unsupported length of 230 mm, inner radius of 6.35 mm, and thickness of 2.4 mm. Axial strains of 0.7%, 5.5%, and 12% were applied. Young’s modulus and Poisson ratio were measured as $E = 3.8$ MPa and $\nu = 0.423$ respectively. A water solution was maintained at constant pressure inside the tube, while the air pressure outside of the tube was adjusted to achieve the desired transmural pressure ($p_T = p_{\text{inside}} - p_{\text{outside}}$). Over a span of 360 s, $p_T$ was decreased (nearly linearly) from 65 kPa to -150 kPa, and returned to 65 kPa, and the internal tube CSA was presented as a function pressure.

We use the same tube dimensions, and apply a stretch of 1.61 mm to account for the 0.7% axial strain [98]. The same Young’s modulus and Poisson ratio are used, and because density was not given, we use $\rho = 1250$ kg/m$^3$ which is a reasonable value for silicone rubber. We set $p_T$ by keeping $p_{\text{outside}} = 0$ and adjusting $p_{\text{inside}}$. We define $p_T(t)$ in a similar way; going linearly from 0 kPa to -150 kPa and back to 0 kPa over a time span $T_{\text{dur}}$. The effects of varying $T_{\text{dur}}$ are demonstrated.

All the simulations are performed in Artisynth [43,44,138]. Artisynth includes functionality to auto-generate tubes of a given dimension, mesh quality, and mesh type. We consider 3 mesh qualities (written as number of nodes lengthwise, radially, and circumferentially): coarse mesh (25,2,25), medium mesh (35,3,45), fine mesh (45,4,65). The hexahedral (hex) and tetrahedral (tet) meshes contain the same number of nodes, but the hex meshes contain 600, 3060, 8580 elements respectively, while the tet meshes contain 3000, 15300, 42900 elements respectively. A tet mesh, therefore, is more fine and computationally slower than the corresponding hex mesh.

Due to the high mesh symmetry of a collapsing tube, the numerical tube is significantly more resistant to buckling than the physical tube. We introduce imperfections into the numerical tube by randomly perturbing each node in $x, y$, and $z$ directions. The consequences of this perturbation are presented below. All simulations use a time step of $dt = 0.005$ s. Stiffness damping is 0.005, and particle damping is 0.0.

Figure [4.1] shows the standard tubes used in these simulations, with a medium resolution mesh and perturbations applied. Figure [4.1a] displays a hex mesh while Figure [4.1b] displays a tet mesh.
4.3 Results

We now present the tube simulation results and compare with experimental data. Examples of medium-resolution hex and tet mesh tubes as they collapse are shown in Figure 4.2.
4.3. Results

Figure 4.2: Selected snapshots of simulated tubes as they collapse. A medium-resolution mesh is used. The rigid supports are shown in solid blue.

The effect of varying $T_{dur}$ is shown in Figure 4.3, in which the CSA (labeled $A$) is non-dimensionalized by the CSA of the undeformed tube (labeled $A_0$). A couple features of this (and following figures) should be pointed out. Buckling occurs approximately at $A/A_0 = 0.9$, and the opposite tube walls
come in contact around $A/A_0 = 0.2$ (we refer the reader to Fig. 2.5 for a schematic of this behavior). Because $p_T(t)$ is cycled from 0 to -150 kPa and back to 0, there is a decreasing pressure path and an increasing pressure path; the decreasing path is the lower of the two. When these curves are not coincident, as is the case in Figure 4.3, there is hysteresis or path-dependence occurring. We observe this hysteresis in the numerical simulations using a linear material property, which is likely an artifact of momentum history as well as numerical damping. However, for $T_{dur} = 200$ s, this effect is very small. Recall that Bertram used $T_{dur} = 360$ s, yet hysteresis is clearly exhibited in his data, indicating the non-linearity of the tubing material. From this point on, $T_{dur} = 20$ s is used in the simulation, as a trade-off between simulation runtime and accuracy to experiments.

![Figure 4.3: Effect of varying $T_{dur}$. The tubes are a linear, incompressible material on a medium, hex mesh. The durations of 10 s (dot-dash red), 20 s (dash blue) and 200 s (solid green) are compared with Bertram's data (solid black).](image)

Next we consider the effect of perturbations in Figure 4.4. The more perturbed mesh buckles more easily, though not greatly different from the unperturbed mesh. Though not illustrated here, it was observed that perturbations may have a much greater difference for coarse, hex meshes, which are highly symmetric, while perturbations are much less important for tetrahedral meshes which are less symmetric. It is noteworthy that the perturbations do not effect the return path of increasing pressure, an indication that the added imperfections have not changed the general behavior of the tube. A perturbation of 0.1 mm, which seemingly adds physically reasonable imperfections to the tube, is used in all tube simulations unless otherwise mentioned.
4.3. Results

Figure 4.4: Effect of adding perturbation. The tubes are a linear, incompressible material on a medium, hex mesh. The maximum perturbation magnitudes are 0.0 mm (dot-dash red), 0.05 mm (dash blue) and 0.1 mm (solid green), compared with Bertram’s data (solid black).

In Figure 4.5 we observe the effects of mesh refinement and compressibility. The quality of the mesh has a dramatic effect on the buckling point and the final collapsed area, illustrating the importance of using a high-quality mesh. The solution does not yet demonstrate mesh-independence; however, using the coordinates \((A/A_0, p)\) of the buckling point and contact point as distinct features, we observe that the solutions converge linearly as element length decreases (the points fit a line with \(r^2 \geq 0.98\), excepting the \(A/A_0\) estimate of the contact point which has a poor linear fit). Extrapolating to the “exact” solution for which elements are infinitesimal, we expect both compressible and incompressible simulations to predict the \(A/A_0\) at which the tube buckles within 2% of the experiments; however, the compressible simulation is expected to significantly underestimate the buckling and contact pressures by approximately 37% while the incompressible simulation would overestimate these pressures by 23% and 2% respectively. It is not surprising that the compressible simulation is expected to have greater error than the incompressible simulation, given that rubber is a largely incompressible material; however, it was observed that the compressible simulations are faster and slightly more stable.

A medium quality mesh was the default simulation setting, which is an acknowledged error, but allows for faster simulation runtimes making it possible to explore the parameter set more fully. Were an exact reproduction of the experimental results needed, we expect a mesh with 16 times more elements than the fine mesh (that is, 1/4 the element length) would yield a substantially better result.
4.3. Results

Figure 4.5: Effect of Compressibility and mesh refinement. The tubes are a linear material on a hex mesh. Only the decreasing-pressure path is shown. Incompressible (blue curves) are compared with Compressible (green curves), each showing a coarse (dash-dot), medium (dash), and fine (solid) mesh. They are compared with Bertram’s data (solid black).

In Figure 4.6 we compare hex and tet meshes with a compressible simulation. The coarse tet mesh buckles at a smaller pressure than the coarse hex mesh, but the hex meshes improve more quickly with refinement, making the medium and fine hex meshes better behaved than their tet counterparts; however, the tet meshes do appear more compliant once buckled, collapsing to a smaller final area.

Using a linear extrapolation as before, the tet mesh is expected to overestimate the buckling and contact pressures by approximately 11%, in contrast to the hex mesh which is expected to underestimate by 37%. Tet meshes are particularly prone to mesh locking [5], which may account for the difference between the hex and tet solutions. The hex mesh is preferable because it approaches the solution more rapidly with refinement and also has a faster runtime (having significantly fewer elements than the equivalent tet mesh). However, in complex geometries a hex mesh is difficult to generate.
4.3. Results

![Graph showing results of mesh type and quality comparison.]

Figure 4.6: Effect of mesh type and quality. The tubes are a linear, compressible material. Only the decreasing-pressure path is shown. Tetrahedral meshes (blue curves) are compared with hexahedral meshes (green curves), each showing a coarse (dash-dot), medium (dash), and fine (solid) mesh. They are compared with Bertram’s data (solid black).

Bertram reports the dimensions 230x6.35x2.4 mm (length, inner radius, thickness) as the “nominal” dimensions of the tube. However, he notes that the tubes have a slightly oval-shaped cross-section rather than a perfect circle. His measured averaged dimensions are 230x6.75x2.4 mm. This “corrected” tube is simulated using a fine mesh, and the results are shown in Figure 4.7. Even though the $h/r_i$ ratio is only 6.3% smaller in the corrected tube, the tube buckles at a significantly smaller pressure, showing the sensitivity of buckling to $h/r_i$. With a finer mesh and longer $T_{dur}$, we expect this tube would fall very close to Bertram’s experimental results.
Figure 4.7: Best results. The tubes are a linear, compressible material with a fine, hex mesh. The standard tube (dash dot green) is compared with the corrected tube (dash red) and Bertram’s data (solid black).

Other parameters were tested, though not illustrated here. Notably, the Neo-hookean material was tested, but stability both in collapse and expansion were a problem. While linear and Neo-hookean models behaved similarly for small deformations, the difference becomes noticeable at larger deformations, though this is also where stability became a problem preventing a reliable comparison. Compared to the tube stiffness and pressures involved, the effect of gravity is very small, though it was used for these simulations (oriented perpendicular to the axial direction). For a softer material, the effect of gravity is more significant. Stiffness damping was found to be an important parameter. Too much stiffness damping can greatly inhibit the dynamics (though the steady solution should be the same); however, the proper amount adds stability and is realistic, providing a viscoelastic effect though the material is modeled as linear (no real-world material is perfectly linear).

4.4 Discussion

The simulation predicts tube buckling at a larger (negative) pressure than Bertram’s experiments, much like the Marzo et al. study [98]. Differences from the experiment are: how the tube is fixed at the ends, how \( p_T \) is created, the aging of tubes, and the exact shape and dimensions of the tube. We suspect the latter is the main source of error (see discussion in Marzo et al. [98]). Extrapolation suggests that further mesh refinement would significantly improve the agreement between simulation and experiment. However, our current simulation methods proved adequate to capture the collapsing and re-opening behaviors of the tube.

Unlike previous work, we have simulated the complete collapse and recovery of the tube, using
a $p_T(t)$ function similar to Bertram’s experiment. One positive result from this is that we can compare the shape of the entire curve with the experiments, including the area at the time of opposite wall contact. Bertram notes that $A/A_0 \simeq 0.17$ at the time of contact; a surprising result, being significantly smaller than $A/A_0 \simeq 0.27$ for thin-walled tubes. Our simulations have a slightly larger $A/A_0 \simeq 0.19$, but do confirm Bertram’s observation.

We find Bertram’s $p_T(t)$ method very useful for simulations in that it is easy to automate and produces a continuous tube response function beneficial for comparisons. It does not produce a strictly steady-state and path independent curve; a complicating factor, but a positive feature for those interested in hysteresis and time-dependent tube behavior. We use a similar method for comparing upper airway response in Chapter [4].

Another very positive result is the speed of these simulations in Artisynth. A fine, hex mesh simulates 1 s of physical time in 7.5 min. A medium, hex mesh is significantly faster. These speeds allow us to try many scenarios and variations, and quickly develop an intuition for the numerical and physical behavior of the system.

One area in which our simulations can be improved is stability. Some cases failed or showed unnatural behavior during the collision of the opposite walls, and it would be particularly beneficial to have stable simulations with nonlinear materials.

For future tube simulations, these results are very encouraging. We are able to replicate the behavior of Bertram’s experimental tube with a reasonably sized hex mesh and fast simulation times. This opens the door for examining the FSI dynamics of this system, which have already been carefully characterized experimentally. This leave many possibilities for validation and further exploration.

These results are also very useful for future UA simulations. Through the parameter study, the sensitivity of the solution to physical and numerical settings is revealed, giving a much better intuition for the magnitude and nature of errors various parameters may introduce.

4.5 Conclusions

We performed a parameter study on the relatively simple and well-researched case of a tube collapsing under pressure to examine the trustworthiness of our model parameters. We found a hex mesh to be faster, more accurate and stable than a tet mesh. This is preferred in almost every situation when possible. The high symmetry of a hex mesh (and to a lesser extent a tet mesh) resists buckling; random node perturbations are added to model an imperfect real-world tube. An overly coarse mesh can dramatically hinder buckling and collapse by a factor of 3 or more. These findings reinforce the importance of careful mesh design. Numerical stability hindered a thorough study of nonlinear materials, but they did not appear to make much difference in tube collapse (larger differences are expected for large deformations). Compressible simulations were both fast and stable,
but differed significantly from an incompressible simulations regarding buckling pressure.

Using a high-quality, perturbed hex mesh, our tube simulations closely match Bertram’s experimental results including the self-collision that occurs after collapse, all the while having a reasonable runtime. The $p_T(t)$ method used experimentally by Bertram is found to be a very useful method for comparing simulation trials.
Chapter 5

2D Channel FSI

5.1 Introduction

Obstructive Sleep Apnea (OSA) is a case of Fluid-Structure Interaction (FSI) in the human Upper Airway (UA) in which the airflow interacts with the soft-tissue structures, such as the soft palate and the tongue. Numerical models of OSA can be a powerful tool to investigate the mechanisms of OSA and consider treatments; however, a complete OSA model describing the 3D structural mechanics, under the influence of passive and active muscles, and coupled with fluid mechanics, is neither computationally feasible nor conceptually understood. To move towards a trustworthy OSA model, we ignore the intricacies of the UA geometry and neurological influences, and focus on modeling an FSI regime similar to that of OSA, but in the simplified geometry of a rigid channel with a compliant thick-walled membrane (i.e., the 2D Starling resistor). We perform directly equivalent simulations and experiments for this FSI system for the sake of 1) understanding the FSI behavior in this simplified model, and 2) validating our FSI simulation which, once proven in this simplified case, may prove a very robust and high-speed tool useful in OSA simulations.

We hypothesize that one may capture the key dynamics of OSA accurately using a 1D fluid model such as Cancelli and Pedley [28], focusing the modeling effort on the complex UA structures to include realistic geometries, muscle definitions and neurological models. To test this hypothesis, we perform side-by-side experiments and simulations on an FSI regime with fluid and structural properties comparable to those of OSA yet simple enough to readily measure and compare the results. The simulation uses the 1D fluid model described in Chapter 3 coupled with a 3D FEM solid model to provide a fast FSI model. In the following sections we define the model and our experimental and simulation procedures, present and analyze the results, and discuss the implications of the results in the context of starling resistor literature and OSA modeling.

5.2 Methods

We designed our experiment and simulation case to be an FSI system simple enough to allow for analysis and validation, yet analogous to the UA in terms of the fluid regime, the material properties, and the geometry. The conceptual geometry is illustrated in Fig. 5.1. The entire FSI system is 80 cm long (x-axis), 20 cm wide (y-axis), and 6 cm in depth (z-axis, aligned with gravity).
5.2. Methods

We define coordinates with $x = 0$ at the upstream edge of the membrane, $y = 0$ mid-width of the membrane, and $z = 0$ on the channel floor at $x = 0$. The inlet occurs at $x = -40$ cm, the channel converges from $-30 < x < -23$ cm, the membrane starts at $x = 0$, the membrane ends at $x = 20$ cm, and the outlet is at $x = 40$ cm. In the converging section the channel converges linearly, from a width of 20 cm to 5 cm, and from a height of 6 cm to 1 cm, yielding a channel contraction ratio of 24. The dimensions of the collapsible portion are 20 cm long, 5 cm wide, and 1 cm in depth.

The flow is driven by a pressure inlet at $x = -40$ cm, and exits to atmospheric pressure at $x = 40$ cm. The pressure above the membrane is also atmospheric pressure. This is a notable difference from most Starling resistor models, which allow the external pressure to be controlled and independent from the outlet pressure. We will now present our experimental and numerical implementation of this design.

![Figure 5.1: Schematic of the FSI geometry (not drawn to scale). The flexible membrane is shown with a dashed green line. The y-dimension goes into the page. All dimensions are given in centimeters.](image)

5.2.1 Experiment

The rigid channel is constructed from 1/2 inch (12.7 mm) polycarbonate according to the dimensions described above. However, the assembled experiment has deviations due to imperfection of the non-planar polycarbonate and machining precision. The experimental dimensions are $h = 1.06$ cm at $x = -23$ cm, $h = 1.155$ cm at $x = 0$ cm, $h = 1.235$ cm at $x = 20$ cm, and $h = 1.215$ cm at $x = 40$ cm. These deviations could have a significant effect on the flow, and are included in the simulation geometry.

The membrane material is an “ultra soft polyurethane” from McMaster-Carr (durometer 40 OO, 1/8 inch thick); the measured thickness is 2.92 mm. The unsupported membrane span is 20 cm, but we cut some membranes with shorter unsupported lengths to observe the effects of tension. Tension 1 has a 20 cm unsupported length and thus is untensioned; Tension 2 has a 19.5 cm unsupported length and thus is stretched 0.5 cm; Tension 3 has 19.0 cm of unsupported length and thus is stretched 1.0 cm. All membranes have 2 cm on either end of their unsupported section for attaching to the channel using a thin double-sided tape allowing us to mount the membranes flush with the channel walls. The membranes, being a very soft and sticky rubber, were coated with a very thin layer of cornstarch to prevent sticking to the channel floor.
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The membranes were laser cut to the appropriate size to allow a close fit with the side walls, thus minimizing air leakage (See Fig. 5.2b). However, leakage at the side-walls is inevitable, and as we will demonstrate, the FSI behavior is quite sensitive to the side-wall gap. Therefore, each time the side-walls are placed, we take a top-view photo (similar to Fig. 5.2b) including markers in the photo for calibration, and manually extract the area of the gap.

To drive the flow, we use three San Ace 60 computer fans, each capable of producing 84 CFM and approximately 350 Pa static pressure at our typical flow conditions. Initially these fans were mounted directly to the channel, but due to excessive vibrations that affected the results, we mounted them in a 40 cm long plywood plenum that mates with the main channel by a layer of weather stripping to minimize any vibrational coupling between the plenum and the main channel. Flow-straightening honeycomb is placed in the plenum. The fans are controlled by a variable power supply, allowing accurate control over flow rate/upstream pressure. Photos of our experimental apparatus are shown in Fig 5.2.

![Side view of apparatus.](image)

![Top view of Tension 3 membrane, illustrating fit between membrane and side walls.](image)

Figure 5.2: The experimental apparatus.

The pressure is recorded at 6 locations along the bottom of the channel at the centerline: \( x = -35 \) cm (the inlet pressure), \( x = -10 \) cm, 0 cm, 10 cm, 20 cm, and 30 cm. We also mounted a stagnation pressure tube at \( x = -10 \) cm, thus allowing us to infer the flow velocity. All pressure measurements
are taken using SensorTechnics LBA differential pressure sensors, with range -250 Pa to 250 Pa and error=0.015%(reading + full-scale-span), and response time 1.5 ms. Each sensor was calibrated individually, though some drift was observed in the calibrations. We record the pressure sensor data with SignalExpress, sampling at 1000 Hz.

For select trials, flow velocity was measured at the outlet using a Dantec Dynamics single sensor fiber film probe (wire at 45 degree angle and with a 0.5 μm coating), capable of measuring velocities 0.05 < u < 350 m/s up to frequencies of 175 kHz.

We have two experimental protocols: continuous recording and discrete recording.

5.2.1.1 Continuous Recording

In continuous recording, the operator sweeps through the oscillation range of the membrane while continuously recording the pressure. This allows us to create a spectrogram of the membrane dynamics, which provides a global view of the oscillation behaviors. The spectrograms are calculated from the pressure at x = 10 cm; the fft uses a Hann window function and a 2.5 s time-window (a 4 s time-window is used with Tension 1 due to low frequency). Hysteresis is observed in this system, therefore the sweeps are performed both in the increasing-p direction (from low $p_{\text{inlet}}$ to high $p_{\text{inlet}}$), and in the decreasing-p direction (from high $p_{\text{inlet}}$ to low $p_{\text{inlet}}$).

The inlet pressure is indirectly controlled by the fan voltage, which is manually incremented/decremented by the operator. However, we desire spectrograms that are plotted as a function of $p_{\text{inlet}}$ rather than t for the sake of direct comparison. To achieve this, we 1) smooth the $p_{\text{inlet}}$ function to remove sensor and turbulence noise thus creating a function that is strictly increasing/decreasing, and 2) interpolate the spectrogram that is a linear function of time to be a linear function of the smoothed $p_{\text{inlet}}$ function. This transformation, illustrated in Fig. 5.3, occasionally produces minor discontinuities in the spectrogram; however, the main features are preserved. We interpret the spectrograms in detail in Sections 5.3.2 and 5.3.3 and illustrate behaviors with unique spectral signatures in Section 5.3.5. The high-frequency content (near 150 Hz) that is unrelated to the harmonics and increases with $p_{\text{inlet}}$ comes from the fans that drive the flow and is found in all experimental spectrograms.
5.2. Methods

Figure 5.3: Spectrograms illustrating the mapping from a $t$-based spectrogram to a $p$-based spectrogram. The colorbar indicates spectral amplitude.

We performed multiple trials in which the operator only varied the rate at which they swept through the voltage/pressure. Varying this rate from 1.1 Pa/s to 4.1 Pa/s resulted in a difference of 3.0 Pa for the start of dynamics and 1.0 Pa for the end of dynamics on increasing-$p$ trials. Running two trials at an average sweep rate of 1.6 Pa/s yielded a consistent start of dynamics to within 0.1 Pa and end of dynamics to within 1.1 Pa. The decreasing-$p$ trials had similar repeatability. The start and end of dynamic regions was manually chosen from the spectrograms. Our average sweep rate was approximately 1.5 Pa/s.

5.2.1.2 Discrete Recording

In discrete recordings, the membrane motion and air pressures are synchronously recorded at a fixed $p_{\text{inlet}}$ value. This allows us to visualize a specific oscillation behavior and provides further comparison with the simulation. The membrane motion is recorded with a Phantom v12 or v611 high speed video camera. To synchronize the camera and sensors, a switch and light were added in series with a pressure sensor; turning the switch on resulted in a visible signal to the camera and a starting pressure signal in the recorded sensor data. All sampling was done at 1000 Hz. Due to the hysteresis in the system, we distinguish whether the voltage/inlet pressure was arrived at by increasing or decreasing the inlet pressure.

The high-speed camera records were exported to a sequence of images for processing and analysis. We perform a feature extraction algorithm for each high-speed camera image to ultimately calculate the CSA function $A(x, t)$ for the experiments allowing further quantitative comparison between simulation and experiment. The feature extraction algorithm is:

1. A background image ($\Psi_{\text{BG}}$) is calculated from a case where the membrane has large oscillations. We iterate through each image, keeping the maximum value at each pixel. The
background is bright (high pixel values), while the membrane is dark (low pixel values), thus an image is found that excludes the membrane in favor of the background. However, the background image does contain some fragments of the membrane at the edges where the membrane is nearly fixed.

2. For each image ($\Psi$) in the sequence, we calculate:

(a) An edge-emphasized image: $\Psi_{\text{edge}} = \partial \Psi / \partial y \cdot (\Psi - \Psi_{BG})^{10}$. The resulting image is an intersection of the high y-gradient region along the membrane edge with the (highly amplified) subtraction of the background image which would ideally leave only the membrane. In effect, this causes the upper and lower edges of the membrane to have large (yet oppositely signed) values.

(b) The upper edge, being well-defined more consistently than the lower edge, is used to define a function $y_u$, which is selected using high pixel intensities and excluding values that deviate markedly from the neighboring pixels. The endpoints are manually selected once for all images because neither the camera nor the apparatus moves.

(c) To reduce the discrete pixel effect, iterations are performed in which $y_u$ is nudged to the highest y-gradient of $\Psi_{\text{edge}}$, then smoothing the derivatives of $y_u$.

3. The initial estimate of the upper edge, $y_u$, is usually quite good; however, there often is a significant temporal jitter from image to image. Therefore, all $y_u$ estimates are compiled into a space-time surface, which is smoothed in two dimensions.

4. The lower edge, $y_l$, of the membrane is calculated by shifting $y_u$ in the direction of its normal by the membrane thickness (the membrane extensions are so modest that Poisson-ratio reduction of the thickness is negligible).

5. Finally, $y_l$ and $y_u$ are translated to world coordinates using known landmarks in the image, and $A(x, t)$ may be easily calculated from $y_l$.

A sample result of this algorithm is shown in Fig 5.4 This algorithm has a standard deviation of 0.32 mm, using 3 manual tracings as the gold standard. Camera distortion and perspective are not taken into account when translating the solution to world coordinates.

![Figure 5.4: A high-speed camera image, with the results of the edge detection, showing the initial jagged solution (solid yellow), the smoothed and translated lower edge (solid green) and the user-defined boundaries (dashed magenta).](image)

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5.2. Methods

5.2.2 Simulation

A fluid-structure interaction simulation requires a solid solver and a fluid solver that are coupled both spatially and temporally. Our system uses a 3D finite element (FEM) solid solver coupled with a 1D fluid solver. The solid and fluid are largely 2D in behavior (see Section 5.3.1), thus one could achieve similar results with a 2D solid model; however, given our end-goal of simulating FSI in the highly-3D human UA geometry, we chose to use our 3D solid solver.

5.2.2.1 Solid Solver

To solve the solid mechanics we use Artisynth, an open-source FEM toolkit specializing in biomechanical models \[43,44,138\]. In Artisynth one may create rigid or FEM bodies, and define collisions between them. The geometry described above, including geometric imperfections, is implemented in the simulation model.

The membrane is the only deformable (FEM) body in the FSI system. To determine the material properties of the membrane, we affixed a narrow strip of membrane (about 18 cm long and 2 cm wide) at one end and hung a weight from the other and then set it bouncing like a spring and recorded the motion. In Artisynth, we re-created this experiment using a compressible linear elastic material and modified \(E\), \(\nu\), and the stiffness damping until the simulation produced a similar frequency and decay of bouncing amplitude. The determined properties are: \(E = 100000\) Pa, \(\nu = 0.40\), and stiffness damping \(SD = 0.04\). The existence of a finite (but small) stiffness damping confirms that the material is not perfectly linearly elastic. The membrane density was measured to be \(\rho = 1250\) kg/m\(^3\).

5.2.2.2 Fluid Solver

We largely apply the 1D fluid solver described in Chapter 3\[3\], however, various modifications are necessary for this experiment, including a term to account for leakage in the side-wall gap. We review the model here:

\[
\frac{\partial A}{\partial t} + \frac{\partial}{\partial x} Au + \frac{q_{gap}}{\Delta x} = 0
\]  
(5.1)

\[
\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} - \tau \frac{s}{A} = 0
\]  
(5.2)

\[
\tau - \tau_{\text{fric}} - \tau_{\chi} - \tau_{\text{small}} = 0
\]  
(5.3)

\[2\] Also described in Anderson et al. \[3\]
where $s$ is the perimeter around area $A$, and $q_{\text{gap}}$ is the flow rate through a gap $g = g(x)$. The term $\tau$ contains the terms $\tau_{\text{fric}} = -2\mu(s/A)u$, which describes laminar losses \cite{153}, and $\tau_\chi = (A/s)(1 - \chi)\rho u(\partial u/\partial x)$ defines the limited pressure recovery from the Cancelli and Pedley \cite{28} model. To handle the case of area going to zero in a smooth and stable manner, we modify $A(x,t)$ between $A_{\text{small}} > A > 0$ to map to $A_{\text{small}} > A > A_{\text{closed}}$, where $A_{\text{closed}}$ is a user-defined value chosen to be as small as possible while maintaining simulation stability and $A_{\text{small}} \approx 2\cdot A_{\text{closed}}$ allows $A(x,t)$ to remain relatively smooth. We assign $A_{\text{small}} = 1.667\cdot A_{\text{closed}}$ in this study. At the same time (for $A_{\text{small}} > A > 0$), an artificial viscosity, $\tau_{\text{small}} = -0.5\rho u^2$, is smoothly introduced which will effectively reduce large flow velocities to near-zero even though the modified area function is non-zero.

We expect $\chi$ to depend both on geometry and Re, as is the case with turbulent flows. Therefore, we consider three possible definitions for $\chi = \chi(Re)$ which are later used to determine the sensitivity of this parameter:

1. $\chi_{\text{smooth}} = -0.3/15000^{2.2}\cdot(15000 - \text{Re})^{2.2} + 0.3$ for $\text{Re} \leq 15000$ and $\chi_{\text{smooth}} = 0.3$ for $\text{Re} > 15000$. This definition is inspired by the pressure recovery experiments of Smith \cite{134} for a smoothly expanding geometry.

2. $\chi_{\text{sharp}} = -1.479e + 22\cdot \text{Re}^5 - 1.291e - 17\cdot \text{Re}^4 - 4.517e - 13\cdot \text{Re}^3 + 7.588e - 09\cdot \text{Re}^2 - 5.153e - 05\cdot \text{Re} + 0.2033$ for $\text{Re} \leq 25000$ and $\chi_{\text{sharp}} = 0.2$ for $\text{Re} > 25000$. This definition is inspired by the pressure recovery experiments of Smith for a suddenly expanding geometry.

3. $\chi_{\text{const}} = 0.2$, a common value used in previous research.

These models are illustrated in Fig. 5.5. We expect $\chi_{\text{smooth}}$ to be the most applicable model because the oscillating membrane defines a smoothly changing geometry, hence this is our default definition for $\chi$. 

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Figure 5.5: The models for $\chi(Re)$ used in this study. The $\chi_{\text{smooth}}$ model (solid blue), $\chi_{\text{sharp}}$ model (dashed green), and $\chi_{\text{const}}$ model (dash-dot red).

We define air leakage through the side-walls by introducing a term which follows from orifice flow equations:

$$q_{\text{gap}} = \text{sign}(p - p_{\text{out}}) \cdot c_{\text{gap}} \cdot \Delta x \sqrt{\frac{2}{\rho}} \cdot |p - p_{\text{out}}|$$

(5.4)

where $c_{\text{gap}}$ is an experimentally determined constant taken to be $c_{\text{gap}} = 0.6$ in this study (see page 514). The pressure outside of the gap is atmospheric pressure, therefore $p_{\text{out}}$ is zero. However, this equation has a singularity when $p = 0$, therefore we approximate it with the function:

$$q_{\text{gap}} \approx c_{\text{gap}} \cdot \Delta x \sqrt{225} \tanh \left( \frac{2}{\rho} \cdot (p - p_{\text{out}}) \cdot \frac{2.0}{225} \right)$$

(5.5)

which remains within 20% of Eq. 5.5 for pressures $65 < p < 565$ Pa. The gap function is defined as:

$$g = \frac{G \pi}{0.2} \sin \left( \frac{\pi x}{0.2} \right)$$

(5.6)

that is, a sine-shaped function along the length of the membrane, having total area $G$. Upstream and downstream of the membrane $g = 0$.

When $g(x) = 0$, hence removing the pressure dependency from Eq. 5.1 we opt for a decoupled solve, first solving Eq. 5.1 for $u(x,t)$ given an $A(x,t)$ and $u_{\text{inlet}}$, next solving Eq. 5.3 for $\tau(x,t)$ and then Eq. 5.2 for $p(x,t)$. To achieve pressure BCs we iterate $u_{\text{inlet}}$ using a bounded secant method until $p_{\text{inlet}}$ is the desired value. This provides a simple and fast solution. A non-zero gap term requires the coupled solve described in Chapter 3 which solves more slowly than the decoupled
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model. The coupled system of equations can become poorly-conditioned for a large number of fluid points; however, re-scaling the system and writing as the normal equations provides a more stable solution than a direct solve (that is, solving $J^T J \cdot \Delta X = -J^T F$ rather than $J \cdot \Delta X = -F$ as in Eq. 3.5).

5.2.2.3 Spatial Coupling

To complete the FSI system, we must define coupling between the fluid and solid simulations, which involves spatial coupling of the geometries and forces, and defining how the simulations advance in time.

The geometry coupling involves reducing the 3D solid geometry to a 1D area function, which the fluid solver takes as an input. To do this we define a centerline and cutplanes that are perpendicular to the centerline at uniform steps along the centerline. At each cutplane the cross-sectional area is calculated from the geometry it intersects. In this case the centerline is a line at $y = 0, z = 0.5$ cm for $-40 \leq x \leq 40$ cm.

The force coupling involves projecting the pressures from the fluid solution back onto the 3D solid geometry. This is done by calculating the closest location on the centerline for each solid face involved in the FSI problem, performing a linear interpolation to get the pressure at that centerline location, and applying that pressure as a force on the FEM or rigid body.

To reduce computational expense, the intersection locations for each cutplane and the mapping between the centerline and the solid geometry faces are stored and used throughout the simulation, rather than being calculated at each time step. This introduces error when the solid deforms out of plane, which is minimal for this problem.

5.2.2.4 Temporal Coupling

We use an explicit time-advance scheme for the FSI problem. For each time step: 1) step the solid solver, 2) calculate the fluid geometry, 3) step the fluid solver, and 4) calculate the forces which are applied to the solid model at the next time step. This time advance scheme is desirable owing to its simplicity and speed; however, better stability may be attained by performing iterations within a time step or implementing a monolithic solve [29, 42, 57].

Following the experimental protocols, we perform simulations in which $p_{\text{inlet}}$ is both gradually increased/decreased to sweep the dynamic range, and simulations in which $p_{\text{inlet}}$ is held constant. For increasing/decreasing $p_{\text{inlet}}$ simulations, the simulation is given 1 s to settle to the initial steady solution before the sweep begins. For fixed $p_{\text{inlet}}$ simulations, the simulation is given 2 s to settle to the FSI behavior before the solution is recorded.

In Table 5.1 we summarize the numerical parameters used to determine the resolution of the simulations, including time step size ($\Delta t$), the number of elements used to discretize the solid mesh,
5.3. Results/Analysis

the number of points used to discretize the fluid centerline, the $p_{\text{inlet}}$ sweep rate (not applicable for fixed $p_{\text{inlet}}$ simulations), and $A_{\text{closed}}$.

<table>
<thead>
<tr>
<th>resolution</th>
<th>$\Delta t$ (s)</th>
<th>solid: # elems (nx,ny,nz)</th>
<th>fluid: # pts</th>
<th>sweep rate (Pa/s)</th>
<th>$A_{\text{closed}}/A_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>coarse</td>
<td>0.001</td>
<td>840 (140, 6, 1)</td>
<td>134</td>
<td>4</td>
<td>0.012</td>
</tr>
<tr>
<td>medium</td>
<td>0.0005</td>
<td>2400 (150, 8, 2)</td>
<td>268</td>
<td>3</td>
<td>0.006</td>
</tr>
<tr>
<td>fine</td>
<td>0.00025</td>
<td>4800 (160, 10, 3)</td>
<td>534</td>
<td>2.4</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Table 5.1: Summary of parameters used for a simulation resolution.

In Table 5.2 we demonstrate the sensitivity of fluid KE ($KE = \int_{x_{\text{in}}}^{x_{\text{out}}} \frac{1}{2} \rho u^2 dx$) to refinement of numerical parameters (from Table 5.1). Because KE may fluctuate rapidly, we perform time-averaging over a 3 s window. For all parameters, we see that the medium solution is within 1% of the fine solution. Therefore, the medium resolution is preferred, providing a balance between simulation speed and solution accuracy. The KE of medium-resolution coupled and decoupled solves differ by 0.34%.

Simulations are performed on an 8-core 2.67GHz PC with 12GB RAM (the simulations are not multi-threaded). Simulating 1 s of physical time requires approximately 2.7 minutes of simulation time at the coarse resolution (coarse dt, solid mesh, and fluid mesh), 12.6 minutes at the medium resolution, and 50 minutes at the fine resolution.

<table>
<thead>
<tr>
<th>Parameter refined:</th>
<th>$\Delta t$</th>
<th>solid mesh</th>
<th>fluid mesh</th>
<th>duration</th>
<th>$A_{\text{closed}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>% diff: coarse-medium resolutions</td>
<td>0.65</td>
<td>2.59</td>
<td>2.85</td>
<td>0.56</td>
<td>1.80</td>
</tr>
<tr>
<td>% diff: medium-fine resolutions</td>
<td>0.51</td>
<td>0.69</td>
<td>0.47</td>
<td>0.06</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Table 5.2: Sensitivity of fluid KE to numerical parameters. Row 1 reports the percent difference between coarse and medium solutions; row 2 between medium and fine solutions.

5.3 Results/Analysis

We now present the results from our simulations and experiments; first, considering the two-dimensionality of the FSI behavior, then presenting a global overview of the experimental and simulation results followed by a closer look at the typical steady and unsteady behaviors observed. We then examine the influence of the air leakage through the gap, and end by examining the sensitivity of the simulations to some model and physical parameters.

5.3.1 Two-Dimensionality

To examine how well the flow may be approximated as two-dimensional, we recorded the pressure and velocity profile in the transverse direction ($y$ direction) for a Tension 3 traveling oscillation occurring at $p_{\text{inlet}} \approx 145$ Pa (refer below for description of associated unsteady behavior), as shown
5.3. Results/Analysis

In Figure 5.6, Figure 5.6a shows pressure at \( x = 10 \) cm, with data collected at \( y = 0 \) cm, \( y = 1 \) cm, \( y = 2 \) cm, \( y = 2.5 \) cm and \( y = -2.35 \) cm. At \( y = 2.5 \) cm the pressure tap is centered at the edge of the side wall, and at \( y = -2.35 \) cm the pressure tap edge is flush with the side wall (pressure taps are 3 mm in diameter). The traces from \( y = 0 \) cm and \( y = 1 \) cm are nearly identical. At \( y = 2 \) cm the trace remains largely the same except for a minor deviation for \( 0.195 < t < 0.213 \) s as the membrane is flung upwards after closure. This deviation becomes more pronounced at \( y = -2.35 \) cm and \( y = 2.5 \) cm where the traces drop approximately 80 Pa below the \( y = 0 \) cm trace. The localized pressure deviations near the test section side walls are believed to be due to the strong streamwise vortices generated at the corners \[115\]. As shown in Figure 5.6, the pressure traces are uniform to within 10% over 80% of the span of the test section. Likewise, we observed a fairly uniform lateral distribution of pressure even at higher values of \( p_{\text{inlet}} \).

Figure 5.6a shows the velocity profile at the outlet, including the standard deviation of the velocity and the average velocity. At distances larger than 5 mm from the wall, the flow profile in the transverse direction is within 10% of the centerline velocity, so 90% of the channel width has velocity within 10% of the centerline velocity. In the \( z \)-direction, the measured flow profile is in excellent agreement with \( u(z) = u_{\text{max}}(1 - z/z_{\text{max}})^{1/6} \), an empirical profile for turbulent flow in a pipe when \( Re \approx 20000 \) \[103\]. Overlapping these \( y \)- and \( z \)-velocity profiles we approximate that \( \bar{u} = 0.777u_{\text{max}} \).

Figure 5.6: Flow characteristics in the transverse direction, illustrated by (a) pressure traces in the transverse direction, and (b) velocity profile at the outlet.

The membrane has sufficient flexural rigidity such that out-of-plane bending is minimal. Using a fine-resolution simulation at Tension 3, when the membrane is sagging under gravity, the simulation predicts 0.14 mm of out-of-plane deformation, or 2.3% of the total \( z \)-direction deformation from the membranes rest position. Experimentally, we did not observe significant out-of-plane bending or oscillations.
5.3.2 Global Behaviors (Experiments)

We now will illustrate the global behaviors observed experimentally by presenting the spectrograms of increasing and decreasing $p_{\text{inlet}}$ trials for Tensions 1, 2, and 3. All pairs of increasing and decreasing $p_{\text{inlet}}$ spectrograms are from the same experimental settings and are gathered sequentially.

Figure 5.7 illustrates characteristic spectrograms at Tension 1. To comprehend the meaning of the spectrograms, consider Figure 5.7a. For $0 \leq p_{\text{inlet}} \lesssim 130$ Pa the behavior is steady, hence there are no oscillations and no spectral content. For $130 \lesssim p_{\text{inlet}} \lesssim 185$ Pa there are oscillations that contain a fundamental frequency ($f_f$) with numerous harmonics of diminishing amplitude at higher frequencies. The fundamental frequency is the lowest band in the figure (here occurring around 4 Hz) while the harmonics are the bands which repeat at uniform intervals above (the diminished amplitudes are illustrated by the lighter blue color of the band). The type of oscillation that yields this spectral signature is termed a “traveling” oscillation, which will be illustrated in more detail in Section 5.3.3 and is the predominant oscillation type observed in our experiments and simulations. Above $p_{\text{inlet}} \simeq 185$ Pa the behavior is steady again.

Hysteresis is clearly visible between the increasing (Fig 5.7a) and decreasing $p_{\text{inlet}}$ (Fig 5.7b) trials. They both have an upper dynamic range around $p_{\text{inlet}} = 185$ Pa but the increasing $p_{\text{inlet}}$ trials have a lower dynamic border around $p_{\text{inlet}} = 133$ Pa and a $f_f = 3.3$ Hz, while the decreasing $p_{\text{inlet}}$ trials have a lower dynamic border around $p_{\text{inlet}} = 85$ Pa and $f_f = 5.4$ Hz. Where the increasing and decreasing $p_{\text{inlet}}$ trials overlap, they have the same $f_f$, which diminishes as $p_{\text{inlet}}$ increases until being close to 0 Hz at the upper dynamic border.

Figure 5.7: Spectrograms for experimental trials at Tension 1 (gap size of 4.39 cm²). Colors show spectral amplitude.

Figure 5.8 shows characteristic spectrograms for Tension 2 increasing and decreasing $p_{\text{inlet}}$ trials. As with Tension 1, these spectrograms exhibit traveling oscillations; however, for $90 < p_{\text{inlet}} < 120$ Pa the traveling oscillations are interrupted by an FSI behavior with a very noisy spectra. We
term this FSI behavior “complex” oscillations in response to the complex, multi-modal, oscillation patterns; it is also illustrated in more detail in Section 5.3.5

In comparison with Tension 1, the fundamental frequency is larger, starting at approximately $f_f = 10.7$ Hz at the lower dynamic border, but still diminishing to nearly 0 Hz at the upper dynamic border. This is in keeping with the results of Jensen et al. [80] who find oscillation frequency to be a balance of membrane tension and fluid inertia. At most locations the fundamental frequency diminishes continuously (as with Tension 1); however, across the complex oscillations the fundamental frequency steps down, suggesting that complex oscillations are a region of transition between modes.

The lower dynamic border for the decreasing $p_{inlet}$ trial is approximately 10 Pa lower than that of the increasing $p_{inlet}$ case, which is in contrast to the 50 Pa difference noted in Tension 1.

![Figure 5.8: Spectrograms for Tension 2 experimental trials (gap size of 4.39 cm²).](image)

We present the spectrograms for Tension 3 in Fig. 5.9. The first dynamic behavior from the increasing $p_{inlet}$ trial (Fig. 5.9a), which occurs around $p_{inlet} = 85$ Pa, is different from complex or traveling oscillations. It has a relatively high fundamental frequency, with $f_f = 11.7$ Hz, and only one or two distinct harmonics. We term this FSI behavior “mode-2” oscillations, and describe it in more detail in Section 5.3.5

Around $p_{inlet} = 95$ Pa a half-fundamental frequency band begins to appear before giving way to a noisy, “complex” oscillation for $100 < p_{inlet} < 110$ Pa. At $p_{inlet} = 110$ Pa, there is a distinct transition to traveling oscillations. The remainder of the spectrogram includes traveling oscillations which are interrupted by portions of complex oscillations. Unlike the Tension 2 spectrogram, the boundary between traveling and complex oscillations are indistinct.

The decreasing $p_{inlet}$ spectrogram is quite similar to the increasing case for $p_{inlet} > 110$ Pa, however, the mode-2 oscillations are not present except for a hint of them around $p_{inlet} = 80$ Pa when the dynamics fade. This was consistently the case with all decreasing $p_{inlet}$ experiments; they contained little to no evidence of mode-2 oscillations.
We note that here also, as with Tension 2, that the fundamental frequency steps down across regions of complex oscillations. However, the complex oscillations around \( 180 < p_{\text{inlet}} < 190 \) Pa appears to be an exception to this.

![Spectrograms for Tension 3 experimental trials](image)

(a) Increasing-\( p \) trial.  
(b) Decreasing-\( p \) trial.

Figure 5.9: Spectrograms for Tension 3 experimental trials (gap size of 2.80 cm\(^2\)).

### 5.3.3 Global Behaviors (Simulation)

We now present the spectrograms predicted by the simulations for Tensions 1, 2, and 3. In Figure 5.10 we show the spectrograms for Tension 1 trials. A gap size of 7 cm\(^2\) is used for this figure because the simulations do not predict dynamic behaviors for the gap of 4.39 cm\(^2\) used for Fig. 5.7. In agreement with the experiments, the simulations predict hysteresis between the increasing- and decreasing- \( p_{\text{inlet}} \) trials; the increasing \( p_{\text{inlet}} \) trial having a lower dynamic border at \( p_{\text{inlet}} = 145 \) Pa while the decreasing \( p_{\text{inlet}} \) trial has the border at \( p_{\text{inlet}} = 95 \) Pa. The simulations also predict traveling oscillations; however, the fundamental frequency is much lower, being \( f_f = 2 \) Hz at the lower border of the increasing \( p_{\text{inlet}} \) trials and \( f_f = 4 \) Hz at the lower border of the decreasing \( p_{\text{inlet}} \) trials, nor does \( f_f \) remain smoothly continuous across the dynamic range.
5.3. Results/Analysis

Figure 5.10: Spectrograms for Tension 1 simulation trials (gap size of 7.0 cm²). Experimental spectrograms from Figure 5.7 are reproduced in (c) and (d) to aid comparison.

Figure 5.11 illustrates spectrograms from Tension 2 simulations, with a similar gap size as the Tension 2 experiments. In agreement with the Tension 2 experiments, the increase- and decreasing-\( p_{\text{inlet}} \) show little sign of hysteresis, except that the decreasing \( p_{\text{inlet}} \) case has a lower dynamic border approximately 5 Pa lower than the increasing \( p_{\text{inlet}} \) case. Also in agreement with the experiments, the simulations predict the initial FSI behavior to be traveling oscillations, for \( 45 \lesssim p_{\text{inlet}} \lesssim 95 \text{ Pa} \). However, the fundamental frequency starts at \( f_f = 7.4 \text{ Hz} \) and they contain less high-frequency content than the experimental results.

Around \( p_{\text{inlet}} = 95 \text{ Pa} \), there is an abrupt halving of the fundamental frequency (from \( f_f \approx 6 \text{ Hz} \) to \( f_f \approx 3 \text{ Hz} \)). This is observed in the experiments around \( p_{\text{inlet}} = 90 \text{ Pa} \), but in the experiments this almost immediately gives way to complex oscillations. In the simulations, however, this lower frequency regime persists until giving way to complex oscillations for \( 130 \lesssim p_{\text{inlet}} \lesssim 150 \text{ Pa} \). The complex oscillations predicted by the simulation are not as noisy as the experiments, such that the fundamental frequency remains distinguishable amidst the noisy spectra; however, the fundamental frequency does step down across the complex oscillations, as observed experimentally.
The remaining oscillations, from $150 \lesssim p_{\text{inlet}} \lesssim 210$ Pa, are traveling oscillations in agreement with the experiments. However, the amplitude of these oscillations is significantly less than the experiment, nor are the high frequency harmonics as distinct.

![Figure 5.11](image.png)

**Figure 5.11:** Spectrograms for Tension 2 simulation trials (gap size of 4.0 cm²). Experimental spectrograms from Figure 5.8 are reproduced in (c) and (d) to aid comparison.

Finally, we present the spectrograms for the Tension 3 simulations in Fig. 5.12, using a gap size similar to the experiments presented in Fig. 5.9. The increasing $p_{\text{inlet}}$ trial predicts mode-2 oscillations for $70 \lesssim p_{\text{inlet}} \lesssim 95$ Pa and $f_f \approx 8$ Hz, before becoming complex oscillations for $95 \lesssim p_{\text{inlet}} \lesssim 105$ Pa at which point the behavior abruptly transitions to traveling oscillations. The decreasing $p_{\text{inlet}}$ trial only predicts traveling oscillations for $70 \lesssim p_{\text{inlet}} \lesssim 190$ Pa, with just a hint of the mode-2 oscillation at $p_{\text{inlet}} = 70$ Pa as the dynamics fade. These hysteresis observations between the increasing and decreasing $p_{\text{inlet}}$ trials are in excellent agreement with the Tension 3 experiments.

While the fundamental frequency does gradually change (for example, decreasing from $f_f = 5.6$ Hz at $p_{\text{inlet}} = 107$ Pa until $f_f = 4.6$ Hz at $p_{\text{inlet}} = 190$ Pa), it mostly changes in sudden steps. This is also in keeping with the Tension 3 experiments. However, as with the Tension 2 simulations, the
complex oscillations do not exhibit a spectra as noisy as the experiments, nor do those complex oscillations occur over as broad a region.

Figure 5.12: Spectrograms for Tension 3 simulation trials (gap size of 3.0 cm²). Experimental spectrograms from Figure 5.9 are reproduced in (c) and (d) to aid comparison.

In general, we see that the simulation spectrograms have the same essential features as their experimental counterparts; however, the main discrepancies are: 1) The experimental frequencies are consistently 1.5 times larger than the simulation predictions, and 2) The experiments are consistently more energetic, containing larger oscillation amplitudes and more high-frequency content.

5.3.4 Steady Behaviors

We now will examine some specific behaviors, first steady then unsteady in the following section. A selection of steady profiles are illustrated in Fig. 5.13. We classify the steady profiles as follows. The S1 profile ranges from the membrane sagging under gravity (Figs. 5.13a, 5.13b) until a bulge appears in the upstream portion of the membrane(Figs. 5.13c, 5.13d). The S2 profile ranges from the formation of the upstream bulge forms until the constriction at the downstream portion is
5.3. Results/Analysis

gone. Typically this form is interrupted by oscillations (not reaching the end form). An S2 profile, containing the upstream bulge and downstream constriction, are shown in Figs. 5.13c and 5.13d. The S3 profile, shown by Figs. 5.13e and 5.13f, describes a dilated membrane.

For Tension 1, all 3 steady profiles are observed, both in the experiments and in the simulations. For Tensions 2 and 3, only profiles S1 and S3 are observed. When presenting the effects of the side-wall gap in Section 5.3.6, we show the borders of these profiles as predicted by simulation and experiment. Steady solutions of this form are commonly observed in the Starling resistor literature (for a similar example and discussion of mechanisms, see Jensen [78].)

Figure 5.13: Selected steady profiles found by the experiment (a,c,e,g) and simulation (b,d,f,h) for Tension 1. The images illustrate: deformation under gravity alone (a,b), the border between S1 and S2 (c,d), characteristic S2 forms (e,f), and S3 forms (g,h).

5.3.5 Unsteady Behaviors

The unsteady FSI behavior of the system is more complex to classify, but we describe it with the approximate categories: mode-2 oscillations, traveling oscillations, and complex oscillations.\footnote{Selected videos of the unsteady FSI behaviors, both simulation and experiment, are posted at http://www.magic.ubc.ca/artisynth/pmwiki.php?n=OPAL.FluidStructureInteraction}
A traveling oscillation, thus named for the resemblance to a traveling wave on a string, is demonstrated in Fig. 5.14. In these oscillations, a constriction forms near the downstream end of the membrane and the membrane rapidly approaches the bottom wall, often forming a complete closure with it, before being flung open by the strong positive pressure that has developed upstream of the closure. All traveling oscillations we observed are accompanied by a pulse traveling up the membrane in response to the membrane being flung upwards. This pulse might reflect back and forth along the membrane before being damped out, depending on its strength. Traveling oscillations such as this were observed for all 3 tensions, in both simulations and experiments.

The pressure signals have a sharp spike when the membrane snaps shut, in which the upstream pressures greatly surpass $p_{inlet}$, while the downstream pressures drop far below $p_{outlet} = 0 \text{ Pa}$. These sharp spikes cause the repeating, high-frequency harmonics in the spectra. The experimental magnitude of these spikes is uncertain because the pressure sensors saturate around $\pm 250 \text{ Pa}$.

Note that the experimental pressure signal at $x = -35 \text{ cm}$ is quite stable and uninfluenced by the downstream oscillations. The average standard deviation at $x = -35 \text{ cm}$ is $\sigma = 4.7 \text{ Pa}$, while the highest standard deviation observed was $\sigma = 12.2 \text{ Pa}$. This small deviation suggests that using a constant pressure BC at $x = -40 \text{ cm}$ in the simulation is reasonable.
The motion of mode-2 oscillations is shown in Fig. 5.15 These oscillations are similar to mode 2 vibrations one might observe on a string, but there are some important differences. First, gravity causes the profile to be pulled downward such that at no point does it rise above $A_0$. Second, the profile forms a smaller area in the downstream direction due to viscous effects, as observed in the steady cases as well. Third, in ideal mode 2 oscillations there would be a fixed node at $x = 10$ cm, but in this case the node is not perfectly fixed and occurs upstream of $x = 10$ cm; in Fig. 5.15a,b this can be seen by the near-existence of a contour around $x/H = 9$. These mode 2 oscillations were observed in simulations and experiments of Tension 3, and occur at a low $p_{\text{inlet}}$ (being the first dynamic oscillations observed). In comparison with traveling oscillations, the mode-2 oscillations do not contain a strong spike in the pressure signal, and have few harmonics in the spectra.
5.3. Results/Analysis

Figure 5.15: A space-time contour plot of mode 2 oscillations for Tension 3 from (a) the experiment ($p_{\text{inlet}} = 83.3 \, \text{Pa}$) and (b) the simulation ($p_{\text{inlet}} = 80.0 \, \text{Pa}$), with the corresponding pressure signal in (c) and (d) and FFTs in (e) and (f).

Finally, “complex” oscillations are demonstrated in Fig. 5.16. They most resemble the traveling oscillations; however, there appear to be multiple modes of oscillation leading to constructive/destructive interference and hence complex oscillation patterns. For example, the downstream section of the membrane may have one or two shallow oscillations in which it does not contact the opposing wall followed by an oscillation in which it vigorously closes. If the oscillations remain periodic, though having a more complicated patterns, then the “half-fundamental” band will appear in the spectra which was observed over small regions in the experiments and more widely in the simulations. More commonly in the experiments, the oscillation pattern became aperiodic yielding a noisy spectra. Highly noisy spectra are not common in the simulation, though it may occur (as seen in Fig. 5.16, for example). Complex oscillations were observed for tensions 2 and 3 in both simulation and experiment.
5.3. Results/Analysis

Figure 5.16: A space-time contour plot of complex oscillations for Tension 3 from (a) the experiment ($p_{inlet} = 207.1$ Pa) and (b) the simulation ($p_{inlet} = 205.0$ Pa), with the corresponding pressure signal in (c) and (d) and FFTs in (e) and (f).

5.3.6 Side-Wall Leakage Effects

The side-wall gap, as illustrated in Fig. 5.2b, cannot be avoided given the experimental design. Here, we illustrate the consequences of varying the gap area, and the ability of the simple gap model in the simulations to account for the flow leakage through the gap. Experimentally, the minimum gap size was constrained by how close we could position the side walls without the membrane rubbing against them, while the maximum gap size was constrained by the need for the pressure sensors to still be in operation range when the membrane reached the dilated (S3) behavior. We were able to attain smaller gap sizes at Tensions 2 and 3 where the tension causes the membrane to have a smoother profile. For reference, a gap of 5 cm², distributed evenly on both sides of the membrane and along the 20 cm length of the membrane, equates to distance of 1.25 mm between the membrane and the side wall. All plots and data in this section come from increasing $p_{inlet}$
trials. The simulation results in this section come from coarse fluid mesh simulations, and thus have greater uncertainty.

To begin, we show the effects of modifying the gap size for Tension 1 simulations and experiments in Fig. 5.17. Figure 5.17a illustrates the borders between steady profiles and oscillations for the increasing $p_{\text{inlet}}$ trials of the simulations and experiments. The S1-S2 border exhibits little dependence on gap size, and within the experimental range, the simulation and experiments are in good agreement, both predicting this border around $p_{\text{inlet}} = 50$ Pa.

Experimentally, the lower and upper dynamic border rapidly converge as gap size decreases, suggesting that around gap area $G = 3.5$ cm$^2$ the dynamic range would vanish. While we were not able to make a gap small enough to observe this experimentally, the increasing $p_{\text{inlet}}$ trial with $G = 3.7$ cm$^2$ had an oscillation range of less than 10 Pa. In comparison, the simulations predict no dynamic range for $0 < G < 5$ cm$^2$. Dynamics appear above $G = 5$ cm$^2$, but the dynamic range does not expand as rapidly as the experiments demonstrate.

Figure 5.17b compares the average flow rate that is lost through the gap as compared to the outlet (flow rate is time-averaged over a 10s window). At low $p_{\text{inlet}}$, when the membrane is resting on the floor, the flow is entirely through the gap. When the gap is as large as $G = 9$ cm$^2$, the flow rate through the outlet is approximately equal the flow rate through the gap when $p_{\text{inlet}} = 150$ Pa which is the lower border between S2 and oscillations.

(a) The border between FSI behaviors as a function of gap size. Border between S1 and S2: simulations (dotted red) and experiments (blue triangles). Border between S2 and traveling oscillations: simulations (dashed red) and experiments (blue squares). Border between oscillations and S3: simulations (solid red) and experiments (blue circles). Note that simulations predict no oscillations when gap area is 5 cm$^2$ or less.

Figure 5.17: The effect of the side-wall gap at Tension 1 on (a) the transition between FSI behaviors, and (b) the outlet and gap flow rates.

Figure 5.18 shows the effect of gap on dynamic range (Fig. 5.18a) and comparative flow rate (Fig. 5.18b) for Tension 2. The simulations and experiments both predict that the border between S1
and oscillations has a relatively light dependence on gap area, and their predictions of that border are within 10 Pa. Concerning the upper (dynamics/S3) border, the simulation predicts a boundary that lies within experimental uncertainty; however, the widely scattered data makes it difficult to compare closely. The general trend that we observe, as with Tension 1, is that as gap area increases, the dynamic range is both shifted towards a higher $p_{\text{inlet}}$ and widened.

Figure 5.18b shows that, as $p_{\text{inlet}}$ increases, the flow through the outlet surpasses flow through the gap much more quickly than for Tension 1. However, even when $G = 2 \, \text{cm}^2$, the flow through the gap accounts for approximately 20% of the outflow throughout the dynamic range.

Figure 5.18: The effect of the side-wall gap at Tension 2 on (a) the transition between FSI behaviors, and (b) the outlet and gap flow rates.

Again, for Tension 3, Figure 5.19 shows the effect of gap on dynamic range (Fig. 5.19a) and comparative flow rates (Fig. 5.19b).

Included in Fig. 5.19a is the initial onset of traveling oscillations, this being a clearly distinguishable point present in all Tension 3 experiments and simulations. The simulation predicts the onset of mode-2 oscillations and the onset of traveling oscillations fairly well, though the simulation border is consistently lower than the experimental findings (between 5–25 Pa lower). The simulation border between oscillation and S3 appears to be shallower than the trend suggested by experimental data.

Unlike Tensions 1 and 2, at no point does the average flow rate through the gap exceed that through the outlet, as shown in Fig. 5.19b. However, the flow rate through the gap remains a substantial portion of the outflow, accounting for up to 10% when $G = 1 \, \text{cm}^2$ and up to 50% when $G = 5 \, \text{cm}^2$. We include experimental flow rate measurements in Fig. 5.19b. The flow rate is estimated from the centerline pressure assuming the velocity profile described in Section 5.3.1 for which $\bar{u} = 0.777u_{\text{max}}$, and is largely within 10% of the simulation prediction (between the $G = 3$...
cm² and $G = 4$ cm² curves). The error bars indicate the range possible for various flow profiles: the lower limit assuming a laminar profile in $y$- and $z$- directions, and the upper limit assuming a flat flow profile ($\bar{u} = u_{\text{max}}$).

(a) The border between FSI behaviors as a function of gap size. Border between steady and mode-2 oscillations: simulations (dashed red) and experiments (blue squares). Border between mode-2 and traveling oscillations: simulations (dotted red) and experiments (blue triangles). Border between oscillations and S3: simulations (solid red) and experiments (blue circles).

(b) Volumetric flow rates through the outlet (solid lines) and side-wall gap (dashed lines) as a function of gap size. Gap—0 cm² (black), Gap—1 cm² (red), Gap—2 cm² (green), Gap—3 cm² (blue), Gap—4 cm² (magenta), Gap—5 cm² (cyan). Includes experimental flow rate through the outlet for $G = 3.7$ cm² (short-dashed lines) with error bars indicating range possible with flow-profile assumptions.

Figure 5.19: The effect of the side-wall gap at Tension 3 on (a) the transition between FSI behaviors, and (b) the outlet and gap flow rates.

To illustrate the effect of the gap on the spectrograms, we present two spectrograms from Tension 3 simulations in Fig. 5.20, where Fig. 5.20a has $G = 0$ cm² and Fig. 5.20b has $G = 5$ cm². As already observed in Fig. 5.19a the dynamic range expands from a width of 100 Pa to a width of 220 Pa with the introduction of a gap of 5 cm². The fundamental frequency, however, does not change with gap size when comparing two equivalent oscillations regimes. The oscillation amplitude increases as $G$ increases; when $G = 0$ cm² the fundamental frequency has a maximum amplitude of 29 Pa (occurring when $p_{\text{inlet}} \approx 100$ Pa), but when $G = 5$ cm² the maximum amplitude is 45 Pa (occurring when $p_{\text{inlet}} \approx 170$ Pa).

Our gap model contains the constant $c_{\text{gap}}$, which we take to be $c_{\text{gap}} = 0.6$, but could reasonably have the range $0.6 < c_{\text{gap}} < 0.7$ [103]. Noting that $c_{\text{gap}}g = \text{const}$ in our model, we see that changing $c_{\text{gap}}$ from 0.6 to 0.7 is equivalent to assigning a new gap function: $g_{\text{new}} = 0.857 \cdot g_{\text{old}}$. Therefore, the effect of an alternate $c_{\text{gap}}$ value can be precisely predicted.
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![Figure 5.20: Spectrograms of Tension 3 simulations, increasing $p_{\text{inlet}}$ trials, with (a) no gap, and (b) a gap of 5 cm$^2$.](image)

5.3.7 Sensitivity To Model Parameters

In Table 5.3 we report the effect of modifying the simulation parameters, including the $\chi$ model, Young’s Modulus, and stiffness damping (SD). The fundamental frequency and amplitude at that frequency are measured at comparable oscillation regimes.

The only factor that significantly influences $f_f$ is Young’s Modulus. In previous results, we consistently observed that simulation fundamental frequency, oscillation amplitude, and onset of dynamics are lower than experimental data, all of which are consistent with a Young’s Modulus that is too low; therefore, it is likely that our measurement of $E$ is too low. However, $E$ alone would not account for the discrepancy between simulation and experimental $f_f$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$f_f$</th>
<th>Max amp. of $f_f$</th>
<th>Onset of dyn.</th>
<th>Dyn. range</th>
<th>Fluid KE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_{\text{smooth}} \rightarrow \chi_{0.2}$</td>
<td>0</td>
<td>$-11%$</td>
<td>0</td>
<td>+5%</td>
<td>$-1.6%$</td>
</tr>
<tr>
<td>$\chi_{\text{smooth}} \rightarrow \chi_{\text{sharp}}$</td>
<td>0</td>
<td>$-22%$</td>
<td>+10%</td>
<td>+1%</td>
<td>+2.5%</td>
</tr>
<tr>
<td>$E_{100000} \pm 20%$</td>
<td>±13%</td>
<td>±21%</td>
<td>±25%</td>
<td>+3.5%</td>
<td>±9%</td>
</tr>
<tr>
<td>$SD_{0.04} \pm 25%$</td>
<td>0</td>
<td>±12%</td>
<td>±3%</td>
<td>±2%</td>
<td>±4%</td>
</tr>
</tbody>
</table>

Table 5.3: The sensitivity of solution measures to some physical and model parameters. All values are given as the percent difference.

5.4 Discussion

The experimental results demonstrate a behaviorally-rich system. The FSI behaviors vary significantly depending on membrane tension, inlet pressure, side-wall gap size, and the history of $p_{\text{inlet}}$. We have demonstrated that a lightweight FSI system, using a 1D fluid model with a mass source/sink term added to account for flow leakage coupled with an FEM solid solver, can do a
5.4. Discussion

remarkably good job in capturing the global behaviors of the FSI system. The results presented in the study required simulating about 3600 s of FSI which can be accomplished in a matter of weeks using a powerful desktop computer; calculations of this extent are not feasible using an adequately resolved 3D fluid model.

The 2D starling resistor is a topic of much interest, being an essential FSI problem and a simplified analogy to the 3D Starling Resistor which has vast applications, particularly in the field of bio-fluids. The 3D Starling Resistor lends itself to experiments (see [23], for example), while the 2D Starling resistor is favorable for theoretical and numerical studies ( [28, 78, 80, 139], to name a few). Concerning the 2D Starling Resistor, there have been experiments performed in a piston-driven geometry [72,99], but experiments with unconstrained membrane motion have not been successful [18,71]. In this study; however, we have experimentally controlled for the effects of the leakage and modeled its effects in the simulations. Performing experiments and simulations side-by-side in an analogous design has allowed for direct comparison, which has not been published before.

While direct analogy to previous results is not possible, some comparisons can be drawn. Stewart et al. [139] (also see Xu et al. [159]) describe “slamming” motions in which the membrane, at relatively low tension, draws near to the opposing wall before vigorously re-opening, which appears similar to the “traveling” oscillations that we describe. As opposed to our experiments, Stewart et al. assumed a massless membrane without damping, and high Re flow. Bertram et al. [21,23,78], performing 3D Starling Resistor experiments, observed oscillations in which the constriction substantially moves in the flow-wise direction, in this way bearing similarity to our traveling oscillations, particularly those at low membrane tension.

Jensen [78], using a tube-law model to describe the solid coupled with the Cancelli and Pedley [28] fluid model, predicted mode-2 oscillations that have similar characteristics to our results. Jensen [78,79] also discusses, and simulates, how multiple unsteady modes can give rise to quasi-periodic/aperiodic oscillations; that is, motion with a noisy, broad band spectra such as our “complex” oscillations.

Our end goal is to use this FSI simulation system to model OSA. This experiment is not a direct analogy to snoring or OSA because 1) these phenomenon generally occur during inspiration while this flow is expiratory, and 2) the membrane is quite thin compared to the tongue, and the membrane remains connected at upstream and downstream ends as opposed to the soft palate. However, numerous features render these results useful. Like upper airway breathing, the membrane inertia is much larger than the air. The relative importance of fluid or solid inertia can have implications on the nature of the FSI, as well as the applicability of FSI simulation methods. Second, the experimental system presented here produced interesting FSI behaviors around $5000 < Re < 15000$. This Re is slightly higher than would be expected during sleep, but still plausible for upper airway airflow. The experimental membrane had $E \approx 100,000$ Pa, which is stiffer than relaxed tissue, though a plausible value for activated muscles [39]. Thus, the flow regime, solid mechanics, and
5.4. Discussion

FSI regime are reasonable for upper airway FSI. Also, the simulation system proves to be stable under sudden and complete closure. These are essential features if it is to be used for simulation of OSA and snoring.

From these results, we would not expect this system to reproduce OSA or snoring behaviors exactly, were perfect control data available for comparison. However, the uncertainties introduced by this lightweight FSI model are relatively small compared to the current uncertainties in OSA modeling. Specifically, models containing the complete soft-tissue structures of the UA are needed, including appropriate definitions of material properties and boundary conditions, and also including definitions of the internal musculature and innervation of those muscles. Until a comprehensive structural model of the UA comes of age, there seems to be little point in modeling the fluids in finer detail. Therefore, a FSI system, such as the one presented here, is an ideal candidate to examine the sensitivity of an OSA model to these components, and thus to focus our modeling efforts where they are most needed.

5.4.1 Experimental Error

There are a variety of sources of error in the experiments. Measurement of the side-wall gap area is calculated from photos without correcting for lens distortion or perspective. While the size of the gap is calculated, the shape of the gap is not taken into account. Given the sensitivity of the FSI behavior to the gap, this may contribute the experimental scatter in the gap-sensitivity plots (Figs. 5.17, 5.18, 5.19).

Initially, vibration from the fans was a significant problem that required that the experiment be modified. Once the fans were vibrationally isolated from the test section, fan coupling/resonance became imperceptible.

Considering the given accuracy of the pressure sensors, we can expect an error as big as 8 Pa for a reading of 50 Pa and an error up to 11 Pa for a reading of 250 Pa. While performing the experiments, we did observe drift in the sensors (we estimate 5 Pa or less). At times of membrane closure, the pressure spikes past the sensor limits of ±250 Pa, and therefore the magnitude of those spikes is inaccurate.

We observed that the fan head curves are not precisely stable for a given applied voltage, but rather oscillate around a setpoint. However, we expect this to be a very minor effect after the honeycomb, and especially after the channel narrows. The pressure sensor at $x = -35$ cm does show minor oscillations, but the fluctuations are minor and seemingly random ($\sigma \approx 5$ Pa), and could be flow or sensor noise.

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5.4.2 Simulation Errors

The material properties of the membrane are approximate, and as Table 5.3 suggests, a material with higher stiffness may have been appropriate. Table 5.3 also reveals that $E$, varied within a plausible range, is more significant than the choice of $\chi$; therefore, a more careful material analysis, perhaps employing a non-linear material model, would likely improve the comparison between simulation and experiment.

The fluid viscosity model is known to be simplistic and cannot account for all the losses observed experimentally. One simplification is that we define $\chi = \chi(Re)$, but $\chi$ clearly has a dependence on the geometry as well as Re. Different $\chi$ models were shown to significantly effect the maximum oscillation amplitude and the $p_{\text{inlet}}$ at which FSI behaviors occurred. The limitations of this $\chi$ model, and the need for a strong 1D model, have been topics of previous research [94,140]. The transition between the converging and straight sections of the channel upstream of the membrane likely produces flow separation, which the $\chi$ model does not predict at all.

The gap-loss model is simplistic but appear to be effective. The shape of the gap was always modeled as sine-shaped, so an improvement would be to derive the shape of the gap from the experiments.

Stability is a delicate issue for these FSI simulations. As previously shown, some problems may be unconditionally unstable for explicit time stepping [29]. Because the membrane is somewhat thick and much more dense than air, we are within the stability range, but if the fluid and solid densities were closer, this would not be the case. It could be highly beneficial to implement tighter coupling between the fluid and solid solvers, allowing a broader range of parameters for which the simulation is stable.

Collisions occurring in the solid model are another potential source of instability. Artisynth has non-linear material models available, but these generally are more demanding in collisions, therefore we only use a linear material in this study. In the fluid model, locations of closure are another potential instability. The user-defined values $A_{\text{small}}$ and $A_{\text{closed}}$ are important to maintain stability, yet should be kept small to minimize unnecessary interference with the general fluid behavior. Varying $A_{\text{closed}}$ had minor consequences on the simulation results, but $A_{\text{closed}}$ and the corresponding $\tau_{\text{small}}$ model should be defined (or replaced) upon a theoretical basis.

5.5 Conclusions

We have presented a set of FSI simulations and experiments similar in design to the 2D Starling resistor. The FSI simulation system that we present is robust and powerful. The 1D fluid model, extending the Cancelli and Pedley [28] model, defines the pressure recovery term as a function of Re and uses a modified area function to allow complete closure in a smooth and realistic manner. We introduced a mass-sink term to account for leakage through the side-wall gap, an inevitable
5.5. Conclusions

artifact in the experiments. The solid model uses Artisynth, a 3D FEM solver capable of cutting-
edge biomechanical simulations of the upper airway, thus leaving this system poised to model OSA,
snoring, and other related upper airway FSI problems. The simulation time, being very fast, allows
the user to explore a wide parameter space and quickly develop an intuition for the system, which
is of huge benefit in this behaviorally-rich system.

The simulations predict the same key FSI behaviors, both steady and unsteady. The unsteady
behaviors, described as “mode-2”, “traveling”, or “complex” oscillations can be identified visually or
by their frequency content. The oscillation frequencies and amplitudes predicted by the simulations
are close to the experiments, though consistently lower, perhaps indicating that Young’s modulus
is too small. Globally, the simulations predict a very similar progression of FSI behaviors as $p_{\text{inlet}}$
and the side-wall gap is varied, including a hysteresis dependence on the direction that $p_{\text{inlet}}$ is
varied.

As the FSI simulations capture the experimentally observed behaviors, typically with errors around
10%, and the simplicity of the FSI simulations permit their ready application in more complex
geometries, including realistic upper airway models, we conclude that this FSI simulation system
is very useful for OSA research. Therefore, in future work, we plan to apply this FSI simulation
system to a simplified model of the human upper airway, and investigate important parameters
believed to be factors in snoring and OSA.
Chapter 6

Parameterized Upper Airway FSI

6.1 Introduction

The human Upper Airway (UA) is an exceedingly complex system, having dynamic and nonlinear muscle properties, intricate and dynamic geometry, great person-to-person variation, and even varies with a person’s posture or neural state. Hence, understanding dysfunctions of the upper airway, such as Obstructive Sleep Apnea (OSA), requires unraveling a vast number of interdependencies. There is a need for conceptually simple and highly controllable models of the upper airway that can rapidly and consistently explore a wide range of parameters, to better understand the key parameters involved in upper airway collapse.

Creating and using meshes derived from medical images is a very demanding task. Creating the geometry requires expertise in anatomy and image interpretation, as well as personal judgment regarding what defines the boundary of a muscle or some other UA component. Geometrical boundaries that are segmented from the medical images are used to construct a mesh. Due to the complex shapes of the UA geometry, the meshes are quite irregular, and often require hand-corrections to get a well-behaved simulation mesh that joins smoothly with neighboring UA components. Furthermore, a geometrically irregular mesh is difficult to re-mesh, making mesh refinement a challenge.

For Fluid-Structure Interaction (FSI) simulations, a watertight mesh describing the airway is desirable (though not required by the methods we use); however, the mesh segmentation typically results in gaps between UA components. For example, a segmented soft palate does not fit smoothly with the segmented pharyngeal walls, thus creating an ill-defined boundary for the fluid flow. The irregular nature of derived meshes also make them difficult for the modeler to experiment with; for example, should the modeler wish to apply a force to a specific region of the UA geometries, the irregular geometry often obliges them to hand select the vertices/faces involved rather than being able to do so algorithmically. In essence, the messiness of the realistic meshes hinders the modeler from determining proper model design and systematically exploring the UA behavior.

In this chapter we present a parameterized upper airway model that is designed to capture the essence of the UA in geometry and behavior, while having a mathematically-defined and controllable geometry, thus removing the complications associated with realistic geometries and allowing the modeler to focus on the fundamental questions of appropriate Boundary Conditions (BCs) and
6.2 Methods

6.2.1 Parameterized Model Design

The parameterized model is created in Salome, an open-source software that includes a Computer Aided Design (CAD) module for geometry creation and a meshing module capable of generating surface and volumetric meshes; both modules may be scripted using Salome’s Python interface. The parameterized model extends from the nasal cavity to the base of the tongue; however, the focus is on the velopharynx, arguably the most critical region for OSA, though certainly not the only location where closures occur. A sample geometry, viewed from an oblique angle, is shown in Fig. 6.1.
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The crucial soft tissue structures that need to be modeled are: the pharyngeal wall (mainly composed of constrictor muscles), the soft palate (composed of 5 muscles), and the tongue (composed of numerous muscles). We model these soft tissues as a single deformable body (FEM). This is reasonable because there are not clear boundaries between the pharynx, soft palate, and tongue as the various muscle groups weave together. The airway may be optionally extended below the base of the tongue as a simple trachea for dynamic fluid simulations, where the ratio of upstream-to-downstream lengths may play an important role in the behavior [22].

The soft tissues of the upper airway are largely encased and connected to bony structures (skull and vertebrae). These bony structures, being orders of magnitude more stiff than soft tissue, may be modeled as rigid bodies (see Sections 2.2.2 and 2.4.1). We model these structures as a single, encasing rigid body, in a manner akin to the bony enclosure model of Isono et al. [74]. We model the tongue as resting on a fixed platform, a simplifying assumption that diminishes the mobility of the tongue base, but used under the premise that this will be a minor factor in a model focused on velopharyngeal closure. We re-access this assumption later.

Our coordinate system is as follows: the $x$-axis lies along the anterior-posterior direction of the human head, with $x = 0$ occurring at the centerline of the airway and the $+x$-direction proceeding anteriorly; the $y$-axis lies along the lateral direction of the human head, with $y = 0$ occurring on the mid-sagittal plane and the $+y$-direction proceeding to the left; the $z$-axis lies along the superior-inferior direction of the human head, with $z = 0$ occurring at the base of the tongue and the $+z$-direction proceeding superiorly. Figure 6.1 illustrates the main design parameters in a mid-sagittal and transverse view of a sample geometry. The key controllable geometric parameters...
(along with mesh and simulation parameters) are summarized in Table 6.1. There are controllable parameters besides those shown in Table 6.1 (some of which are illustrated in Fig. 6.2), but these were not the focus of this study.

![Figure 6.2: Key controllable parameters labeled in (a) a mid-sagittal slice and (b) a transverse slice at the height of the velopharynx. The FEM is tan, the rigid body blue, and the centerline green.](image)

<table>
<thead>
<tr>
<th>parameter</th>
<th>symbol</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>velopharyngeal cross-sectional area</td>
<td>$A_{vp}$</td>
<td>50, <strong>100</strong>, 150 mm$^2$</td>
</tr>
<tr>
<td>velopharyngeal AP/lateral ratio</td>
<td>$\phi_{vp}$</td>
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</tr>
<tr>
<td>soft palate angle</td>
<td>$\theta_{pal}$</td>
<td>90, <strong>100</strong>, 110</td>
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<td>soft palate thickness</td>
<td>$t_{pal}$</td>
<td>3, 5, 7 mm</td>
</tr>
<tr>
<td>velopharyngeal wall thickness</td>
<td>$t_{vp}$</td>
<td>1, 2, 3 mm</td>
</tr>
<tr>
<td>mesh quality</td>
<td>-</td>
<td>mesh1, mesh2, mesh3</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>$E$</td>
<td>10000, <strong>15000</strong>, 20000 Pa</td>
</tr>
<tr>
<td>gravity</td>
<td>-</td>
<td><strong>none</strong>, $-x$ dir, $-y$ dir, $-z$ dir</td>
</tr>
</tbody>
</table>

Table 6.1: Key parameters for the parameterized upper airway model. Default values are bold.

Our choices for velopharyngeal CSA and ratio comes from Walsh et al. [149] (note that in another paper they report significantly different ratios of $0.4 \lesssim \phi_{vp} \lesssim 0.6$ [148]). Soft palate angles come from our estimates from anatomy books [14, 64]. The values for $t_{pal}$, while representing an average thickness, are estimated to describe the thinner portions of the palate which are expected to be more influential in allowing palate motion. The values for $t_{vp}$ are estimated from anatomy books [14, 64].

We use uniform, linear material properties for the FEM model, with Young’s modulus values chosen according to previous simulations and clinical data (see Sections 2.2.2 and 2.4.1). The gravitational directions represent the pull of gravity for a patient lying supine ($-x$ direction), lateral recumbent ($-y$ direction), and sitting ($-z$ direction). The default of no gravity, though not physically realistic,
6.2. Methods

is chosen for the sake of being position-neutral. As will be shown and discussed later, gravity has little consequence on these simulations.

6.2.1.1 Meshing

After a geometry is created in Salome’s CAD module, it must be meshed. The meshes are designed to concentrate mesh elements in the velopharyngeal region where accurate behavior is most desired, while the other regions (particularly the tongue) receive less meshing effort. The mesh quality is a controllable parameter; we use 3 mesh qualities in this study:

1. Mesh 1: a coarse mesh containing approximately 6000 elements. The thickness of the soft palate, $t_{\text{pal}}$, is resolved with 2 mesh layers and the thickness of the pharyngeal wall, $t_{\text{vp}}$, is resolved with 1 mesh layer.

2. Mesh 2: an intermediate mesh containing approximately 17000 elements; $t_{\text{pal}}$ is resolved with 3 mesh layers and $t_{\text{vp}}$ is resolved with 2 mesh layers.

3. Mesh 3: a fine mesh containing approximately 44000 elements; $t_{\text{pal}}$ is resolved with 4 mesh layers and $t_{\text{vp}}$ is resolved with 3 mesh layers.

We have automated the entire process of geometry and mesh creation using Salome’s Python interface. Creating the geometries/meshes for a simulation only requires setting the desired parameters before running the script, which typically executes in less than 1 minute in Salome. It is thus a quick and easy process to create a simulation-ready mesh.

6.2.2 Simulation Design

The FSI simulations of the parameterized airway are performed in Artisynth. In Artisynth, we read the meshes output from Salome (after a mesh format conversion), and define how the model is anchored, the material properties, and the FSI properties.

The soft tissue FEM model is anchored as follows. An FEM is attached to a rigid body at locations where they touch. Because the rigid bodies are not dynamic in our model, these locations serve as fixed anchor points. Specifically, the FEM anchors at the top of the pharyngeal wall (modeling where the pharynx attaches to the occipital bone), laterally and slightly anterior to the soft palate (modeling how the superior constrictor connects to the pterygomandibular raphe, but not extending as anteriorly as the constrictor muscle does), at the anterior edge of the soft palate (modeling where the soft palate connects to the hard palate), and at the base of the tongue.

The pharyngeal raphe is not solidly fixed to the vertebrae (indeed, during a swallow, it can move significantly), but generally images/videos of airway collapse show little motion by the raphe, perhaps due to the stiffness of the raphe, the weight of the airway system, and the suction pressure
that would be developed in the retropharyngeal space were a large gap created. Therefore, we define fixed nodes at the posterior, mid-sagittal region (±2 mm) of the pharynx.

We define the FEM to have uniform material properties, with density \( \rho = 1000 \text{ kg/m}^3 \) and Poisson ratio \( \nu = 0.45 \). The material is assumed linearly elastic, with a typical Young’s modulus of \( E = 15000 \text{ Pa} \).

Representing the UA with a single FEM has the disadvantage that it may self-intersect during simulation. To handle self-collisions Artisynth requires that potentially colliding sub-surfaces be defined. These sub-surfaces are the soft palate, tongue, positive-\( y \) portion of the pharynx and the negative-\( y \) portion of the pharynx, which are defined and exported from Salome and used in Artisynth to prevent self-intersection.

### 6.2.2.1 FSI Simulations

The FSI simulations require the definition of the airspace and the centerline which describes the path of the 1D air flow. Both are produced in Salome during the mesh creation process. The centerline starts at the nasal inlet (centered in \( y \) and \( z \) directions), and bends \( 90^\circ \) downward to pass through the center of the velopharynx (now centered in \( x \) and \( y \) directions) and continues to the base of the model, either at the base of the tongue for uniform pressure simulations or the base of the extended airway for dynamic fluid simulations. The fluid model’s area function is calculated by creating a cutplane at each centerline vertex that is perpendicular to the centerline at that point. The intersection of each cutplane with the solid bodies is calculated, and the portion that is visible to the centerline vertex is used to calculate the area associated with that vertex. The centerline contains 120 uniformly spaced vertices, resulting in a spacing less than \( \Delta x = 2.14 \text{ mm} \).

The airspace is the continuous space, neither filled by the rigid nor deformable bodies, through which air may flow. The shared boundaries of the airspace and solid bodies define the fluid-solid interface, and the pressures experienced at that interface are calculated from the fluid pressure at the closest portion of the centerline. From the nasal inlet to the velopharynx the geometry is safely 1D. However, the 1D assumption may break down where the nasal passage joins the oral cavity. The air dynamics in the oral cavity are expected to be minimal (as observed in Fig. 3.3b), but mapping the pressure in the oral cavity according to the nearest point on the centerline is a poor choice. This is further complicated by the position of the tongue. During sleep the tongue may rest against the anterior surface of the palate, especially when pulled by a negative pressure, thus the anterior portion of the palate may experience pressure from the tongue rather than air pressures. Due to these ambiguities we choose not to map fluid pressures to the oral cavity, starting at the anterior edge of the uvula. This assumption will be examined in more detail later.
6.2.2.2 Simulation Procedures

We perform two categories of FSI simulations in this study: uniform pressure simulations and dynamic flow simulations. In the first case, pressure that is spatially uniform but slowly varying in time is applied to the fluid-solid interface with the purpose of measuring the quasi-steady airway response to pressure and comparing with clinical measurements (see Fig. 2.3). The uniform pressure (in Pa) is defined as:

\[
p(t) = \begin{cases} 
2000 \frac{t}{0.5} & 0 \leq t \leq 0.5 \text{s} \\
2000 & 0.5 \leq t \leq 1.0 \text{s} \\
-4000 \frac{(t-1)}{12} + 2000 & 1.0 \leq t \leq 13.0 \text{s}
\end{cases}
\] (6.1)

The pressure initially rises to 2000 Pa and stabilizes there until \( t = 1 \text{ s} \). For \( 1 \leq t \leq 13 \text{ s} \) the pressure decreases steadily from 2000 Pa to -2000 Pa and the airway CSA function is recorded. This procedure is similar to that used for measuring tube response in Chapter 4. In these simulations, the time step \( \Delta t = 0.01 \text{ s} \) is used, thus requiring 1300 time steps. Using mesh 2, the simulation runs in approximately 25 minutes.

The dynamic flow simulations use the 1D fluid model, initially defined in Chapter 3 but revised in Chapter 5. We use the \( \chi_{\text{smooth}} \) model (illustrated in Figure 5.5), but note that the difference between this and the standard \( \chi = 0.2 \) is expected to be minimal (see Table 5.3). The nasal inlet is assigned a fixed atmospheric pressure, \( p_{\text{inlet}} = 0 \text{ Pa} \). Inhalation is modeled by applying a sub-atmospheric pressure at the outlet, that is, a pressure created by the lungs. We simulate a range of lung pressures, typically \( p_{\text{outlet}} = -200, -400, -600, -800, -1000 \text{ Pa} \). The simulation is initially run for 1 s, which we observed to be adequate time to reach a statistically steady behavior, and then the behavior is recorded for \( 1 \leq t \leq 1.25 \text{ s} \). The time step \( \Delta t = 0.0005 \text{ s} \) is used, thus requiring 2500 time steps for each \( p_{\text{outlet}} \) value used. Running a sequence of 5 \( p_{\text{outlet}} \) pressures takes approximately 3 hours. Note that an equivalent analysis using complete three dimensional unsteady airflow computations would have required months of computational effort.

6.3 Results And Analysis

We start by presenting the results of the uniform pressure sensitivity study, and then proceed to the results from dynamic fluid simulations.

6.3.1 Uniform Pressure Simulations

To demonstrate the response of our model to a uniform pressure (as defined in Eq. 6.1), we perform a sensitivity analysis for all the parameters listed in Table 6.1 using the default values as
the baseline simulation and varying one parameter at a time to the alternate values given in Table 6.1. All values given in Table 6.1 are within the range of physiologically reasonable values reported in the literature.

As an example of the model’s shape during collapse, we provide Figure 6.3 which shows a sequence of velopharyngeal cross-sections at selected pressures.

[Image: Figure 6.3: A sequence showing the velopharyngeal cross sections at important points in the airway response function.]

The main results of the uniform pressure study are illustrated in Figs. 6.4a through 6.7b where the consequence of varying each parameter is illustrated. We provide some curves from Isono et al. [73] as a reference, though we do not necessarily expect our model to respond like those cases because those subjects were supine and under anesthetic. Before looking at specific cases, we note that the compliance of the airway model is represented by the slope of the curve. Both a softer model and a (relatively) thinner model increase the compliance of the model. Most of our results have a fairly linear response without a distinctive buckling point as seen in the Isono et al. data.

The consequences of mesh refinement are given in Fig 6.4a. This refinement shows a generally small sensitivity to the mesh quality, with the minor exception that mesh3 (the finest mesh) shows a more distinctive buckling behavior than mesh1 or mesh2 in the region $-1000 < p < -500$ Pa.

The consequences of modifying the Young’s Modulus are given in Fig. 6.4b. Varying $E$ by $\pm 5000$ Pa causes a significant variation in the model response. Considering that much wider variations of Young’s modulus have been estimated for muscles ($6200 \leq E \leq 110,000$ Pa, see Section 2.2 for details), we conclude that this is a crucial parameter. This finding is in keeping with much OSA literature that regards airway compliance as a key factor.
6.3. Results And Analysis

Figure 6.4: Parameter sensitivity demonstrated by velopharyngeal cross-sectional area as a function of uniform pressure (red). Selections of Isono’s clinical measurements on normal patients is shown for comparison (solid black).

The consequences of varying the initial velopharyngeal CSA are shown in Fig. 6.5a. Not surprisingly, varying the initial CSA by ±50 mm² causes a significant change in the CSA response. The smallest geometry (CSA=50 mm²) is very resistant to pressure variations while the largest geometry (CSA=150 mm²) collapses more dramatically and exhibits some nonlinear buckling. Generally, a larger CSA makes an airway more compliant with a more distinct buckling behavior, which is also observable in the Isono et al. data (see complete data set in paper [73]).

The consequences of varying φvp (the ratio of AP diameter to lateral diameter) are shown in Fig. 6.5b. Our model predicts that a wider (more elliptical) geometry is slightly more collapsible. Clinical observations regarding this parameter are divided (see Walsh et al. [148] for review of this parameter). In the simulations we observe that a lower φvp encourages AP collapse; however, when airway collapse is dominantly in the lateral direction, a high φvp is expected to encourage collapse.

Figure 6.5: Continuation of Fig. 6.4

(a) Varying initial velopharyngeal CSA: \( A_0 = 50 \) mm² (dashes, red), \( A_0 = 100 \) mm² (solid, red), \( A_0 = 150 \) mm² (dots, red).

(b) Varying initial velopharyngeal AP/lat ratio: \( \text{ratio} = 0.25 \) (dashes, red), \( \text{ratio} = 0.30 \) (solid, red), \( \text{ratio} = 0.35 \) (dots, red).
6.3. Results And Analysis

The consequences of varying the palate angle $\theta_{\text{pal}}$ are illustrated in Fig. 6.6a. Our model finds very little sensitivity to this parameter.

The consequences of varying the thickness of the soft palate ($t_{\text{pal}}$) are shown in Fig. 6.6b. Not surprisingly, a thinner palate leads to a more collapsible airway. Varying $t_{\text{pal}}$ yields a very similar response as varying Young’s modulus (see Fig. 6.4b), implying that flexural rigidity would be an appropriate parameter to describe compliance, being a function of thickness $t$ and $E$.

![Figures](image)

(a) Varying palatal angle: $\theta = 90$ (dashes, red), $\theta = 100$ (solid, red), $\theta = 110$ (dots, red).

(b) Varying palatal thickness: $t_{\text{pal}} = 0.3$ cm (dashes, red), $t_{\text{pal}} = 0.5$ cm (solid, red), $t_{\text{pal}} = 0.7$ cm (dots, red).

Figure 6.6: Continuation of Fig. 6.4

The consequences of varying the thickness of the velopharyngeal wall thickness ($t_{\text{VP}}$) is shown in Fig. 6.7a. A very thin $t_{\text{VP}}$ leads to distinctive buckling occurring around $p \approx -100$ Pa. This is in contrast with most other cases where there is minimal observation of buckling. In comparison with varying the palate thickness (Fig. 6.6b), varying the velopharyngeal wall thickness has a smaller impact for positive pressures, but a larger impact for negative pressures. This illustrates the relative importance of the velopharyngeal walls on the airway collapse in this model.

The consequences of varying the model’s orientation to gravity are shown in Fig. 6.7b, which shows that gravity has almost no consequence. This is cause for concern, because numerous studies show variation in airway size, shape and collapsibility with regard to orientation to gravity. As will be presented in the discussion, this is likely a consequence of an over-simplistic definition of the tongue.
6.3. Results And Analysis

(a) Varying pharyngeal wall thickness: $t_{\text{phar}} = 1.0$ mm (dashes, red), $t_{\text{phar}} = 2.0$ mm (solid, red), $t_{\text{phar}} = 3.0$ mm (dots, red).

(b) Varying orientation to gravity: no gravity (solid, red), gravity in $-x$ direction (dashed, red), gravity in $-y$ direction (dash-dots, red), gravity in $-z$ direction (dots, red).

Figure 6.7: Continuation of Fig. 6.4

We demonstrate the difference between extending the FSI interface into the oral cavity or not in Fig. 6.8. When the FSI interface extends into the oral cavity, the pressure on the anterior surface of the soft palate counterbalances that of the posterior surface, with the result that the soft palate has reduced motion; in fact, a positive pressure draws the palate toward the centerline rather than away from the centerline as in the case of the FSI interface not extending into the oral cavity. The minor collapse observed at negative pressures is due to pharyngeal wall collapse rather than soft palate collapse. The plateau around $1000 < p < 2000$ Pa in the Isono et al. data may indicate that these pressures are sufficient to lift tongue from the soft palate and thus establish an approximately neutralized pressure balance on the palate, as occurs in the case with an FSI interface in the oral cavity. These cases differ significantly, but we expect that not extending the FSI interface into the oral cavity is the better approach for negative pressures when the tongue is expected to press against the soft palate.

Figure 6.8: Continuation of Fig. 6.4 Illustrates the difference between including the oral cavity in the FSI interface (dashes, red), and excluding it (solid, red).
6.3.2 Dynamic Simulations - Pressure Driven

We now turn our attention to the more interesting and complex situation of dynamic flow simulations. In these results we show the behaviors that arise from a pressure-driven flow (0 Pa at the inlet and variable negative pressures at the outlet, created by the lungs). We present these results in plots showing average flow rate at the outlet, $\dot{q}_{\text{outlet}}$, as a function of lung pressure at the outlet, $p_{\text{outlet}}$ (see Fig. 6.9 for an example). This is an instructive plot for OSA, where oxygen deprivation is a concern; points that have a low flow rate could be situations where oxygen deprivation is occurring. We include on the plot dashed curves showing ±1 standard deviation to show the magnitude of oscillations, if/when they occur.

Before looking at specific details, some general observations from Figs. 6.9a through 6.11 are:

- These plots illustrate flow limitation (see [18, 61]), where an optimal/maximal flow rate ($q_{\text{max}}$) exists for a negative lung pressure, and at a more negative pressure, though the lungs are making a stronger effort, the flow rate decreases because the geometry is more collapsed. That is, an increased respiratory effort leads to decreased air supply. When the optimal flow rate is below the flow rate required by the body, the respiratory needs cannot be satisfied.

- The source(s) of instability are a matter of much current research and are known to be a highly complex topic, depending on the inertia and tension of the solid, flow regime, and inertia of the fluid amongst other factors [58]. In our results, we observe that oscillations occur at/after $q_{\text{max}}$ has been reached (a possible exception is seen in Fig. 6.11). If snoring/oscillations occur only after flow rate limitation, then a snorer receives less air than the lungs pull for and oxygen deficiency is likely occurring. However, it is known that flow limitation and the onset of oscillations are separate events [25], and different snoring/oscillation types have different mechanisms, therefore further investigation is needed to know how broad this behavior is.

- As airway compliance increases, $q_{\text{max}}$ and the onset of oscillations occur at higher values of $p_{\text{outlet}}$. While the trends observed are reasonable, the maximum flow rates are quite large. A peak flow rate of 0.0005 m$^3$/s (30 l/min) is a typical value during sleep (see Section 2.2.3 for more details). In the discussion we consider the reasons for this error.

In Fig 6.9a we examine the consequence of varying $E$ in our dynamic simulations. Table 6.2 highlights some important trends from these simulations. We observe that increasing $E$: 1) admits a greater $q_{\text{max}}$, 2) causes $q_{\text{max}}$ to occur at a more negative $p_{\text{outlet}}$, and 3) causes the onset of dynamics to occur at a more negative $p_{\text{outlet}}$. Without extending the simulation range to $p_{\text{outlet}} < -1000$ Pa, we cannot conclude if the dynamic range has been decreased, or shifted to more negative $p_{\text{outlet}}$ values.
### 6.3. Results And Analysis

<table>
<thead>
<tr>
<th>$E$ [Pa]</th>
<th>$q_{\text{max}}$ [m$^3$/s]</th>
<th>$p_{\text{outlet}}$ at $q_{\text{max}}$ [Pa]</th>
<th>$p_{\text{outlet}}$ with unsteadiness [Pa]</th>
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<td>10000</td>
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<td>-800</td>
</tr>
<tr>
<td>20000</td>
<td>0.0031</td>
<td>-800</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6.2: Summary of trends as $E$ is varied.

In Fig. 6.9b we show the consequences of varying CSA. The smallest CSA (seemingly) allows the lowest flow rate, which occurs at $p_{\text{outlet}} = -1000$ Pa or below, and also exhibits no dynamics. In contrast, the largest CSA has a large maximum flow rate at $p_{\text{outlet}} = -400$ Pa, and unsteady behavior for $-400 < p_{\text{outlet}} < -1000$ Pa.

![Graph](image1)

(a) Varying Youngs Modulus: $E = 10000$ Pa (red), $E = 15000$ Pa (green), $E = 20000$ Pa (blue).

![Graph](image2)

(b) Varying initial velopharyngeal CSA: $A_0 = 50$ mm$^2$ (red), $A_0 = 100$ mm$^2$ (green), $A_0 = 150$ mm$^2$ (blue).

Figure 6.9: Average flow rate at the outlet ($q_{\text{outlet}}$) as a function of pressure at the outlet ($p_{\text{outlet}}$), when key parameters $E$ and CSA are varied. Dashed curves illustrate ±1 standard deviation.

In Fig. 6.10 we present the behavior that occurs when the palate and pharyngeal wall thickness is varied, hence varying the tendency between AP collapse (a relatively collapsible palate) and lateral collapse (a relatively collapsible pharyngeal wall). The effect of varying $t_{\text{pal}}$ is illustrated in Fig. 6.10a. A thinner, and hence more collapsible, soft palate causes $q_{\text{max}}$ to be diminished and occur at a higher $p_{\text{outlet}}$. As expected, dynamics begin at a higher $p_{\text{outlet}}$ for $t_{\text{pal}} = 0.3$ cm; however, at $p_{\text{outlet}} = -800$ Pa, very small oscillations occur for $t_{\text{pal}} = 0.7$ cm but not for $t_{\text{pal}} = 0.5$ cm, which is an exception to the trend of having dynamics appear at a higher $p_{\text{outlet}}$ for a more collapsible model.

In Fig. 6.10b we show the behavior that occurs when $t_{\text{VP}}$ is varied. The medium walled ($t_{\text{VP}} = 2$ mm) and thick walled ($t_{\text{VP}} = 3$ mm) cases are not very surprising. For $t_{\text{VP}} = 3$ mm the airway is slightly less collapsible, shifting the maximum flow rate to a higher value at a lower $p_{\text{outlet}}$, but not enough to significantly alter the dynamics. However, for $t_{\text{VP}} = 1$ mm the maximum flow rate occurs for $p_{\text{outlet}} \geq -200$ Pa, and by $p_{\text{outlet}} = -600$ Pa the geometry is largely as collapsed as it can be. This behavior is indicative of a highly collapsible geometry, in keeping with Fig 6.7a.

![Graph](image3)

Figure 6.10: Average flow rate at the outlet ($q_{\text{outlet}}$) as a function of pressure at the outlet ($p_{\text{outlet}}$), when key parameters $t_{\text{pal}}$ and CSA are varied.
6.3. Results And Analysis

(a) Varying palate thickness: $t_{\text{pal}} = 0.3 \text{ cm (red)}, t_{\text{pal}} = 0.5 \text{ cm (green)}, t_{\text{pal}} = 0.7 \text{ cm (blue)}$.

(b) Varying pharyngeal wall thickness: $t_{\text{phar}} = 1.0 \text{ mm (red)}, t_{\text{phar}} = 2.0 \text{ mm (green), } t_{\text{phar}} = 3.0 \text{ mm (blue)}$.

Figure 6.10: Average flow rate at the outlet ($\dot{q}_{\text{outlet}}$) as a function of pressure at the outlet ($P_{\text{outlet}}$), when palate and pharyngeal wall thickness are varied. Dashed curves illustrate ±1 standard deviation.

Fluid-structure interaction may be profoundly affected by the flow upstream and downstream of the oscillating region. (See Bertram and Pedley [22] or Jensen and Heil [80] for nice examples). We examine the consequences of modifying upstream or downstream geometry in Fig. 6.11. Shortening the upstream airway segment by 8 cm destabilizes the behavior, causing a very strong oscillation in which the uvula strongly vibrates against the posterior pharyngeal wall at approximately 40 Hz. In contrast, shortening the downstream length by 8 cm has a stabilizing effect; no dynamics appear though $q_{\text{max}}$ has been surpassed. These behaviors are in keeping with Starling resistor literature which predicts that diminishing the upstream fluid inertia, relative to the downstream fluid inertia, has a destabilizing effect. Reducing the upstream cross-sectional area by 80% has a profound effect because it is now constricted enough that large viscous losses occur upstream, causing a greatly reduced flow rate. These examples demonstrate the need to carefully model upstream and downstream portions of the airway. Even though they are not directly oscillating or collapsing, the viscous losses and fluid inertia in these regions can strongly alter the flow rate and the existence of oscillations/snoring.

The oscillations resulting from a shortened upstream length appear plausible as a uvular snore. However, other oscillations primarily involved the pharyngeal wall motion with the uvula appearing floppy and oscillating lightly. For example, this was the case for the thin pharyngeal wall simulation ($t_{\text{VP}} = 0.1 \text{ cm}$) seen in Fig. 6.10b and the large CSA simulation ($CSA = 1.5 \text{ cm}^2$) seen in Fig. 6.9b.
6.4 Discussion

We find this parameterized airway model to be a powerful approach. The airway geometry can be quickly and consistently built and the simulation methods are fast and robust. This allows one to feasibly explore the vast parameter space and the rich behaviors related to OSA and snoring. Our parameter study, which examines the sensitivity of the model to numerous parameters commonly associated with OSA by clinical observations, agrees with clinical data in the trends observed for most cases, as well as predicting diminished airflow availability for parameters associated with OSA, and oscillations with plausible motions and frequencies for uvular snoring.

Validating an OSA model is a difficult task. The focus in medicine is caring for the needs of patients rather than gathering data, and those measurements that are taken are constrained by ethical considerations and concern for the convenience of the patient. Furthermore, the limited data that are gathered includes patient-to-patient variation, and variation of measurement protocols and equipment.

The primary role of these simulations is not to replicate a well-defined system, but to provide insight and thus better define a system that is complex and poorly understood. The simplified model provides intuition, revealing modeling decisions and parameters to which the system is sensitive, and therefore determining what is necessary for a comprehensive UA model of OSA. In the proceeding discussion we will point out the key shortcomings of this model, and hence the next steps towards a more complete OSA model.

In this model the tongue is a simple shape at a relatively large distance from the uvula/soft palate, and it is rigidly anchored at the base. This was done under the assumption that the tongue plays a minor role in velopharyngeal closure, which is our focus in this study. However, we now consider this an oversimplification. A tongue that is close to the palate will limit the palate’s anterior and

Figure 6.11: Average flow rate at the outlet ($\tilde{q}_{\text{outlet}}$) as a function of pressure at the outlet ($p_{\text{outlet}}$), when upstream and downstream characteristics varied. Full length centerline (green), upstream length decreased by 8 cm (red), downstream length decreased by 8 cm (blue), and upstream CSA decreased by approximately 80% (magenta). Dashed curves illustrate ±1 standard deviation.
inferior motion, but perhaps more significantly, gravitational and pressure forces may cause the
tongue to press into the palate, supplying a substantial force to the palate. Further, the locations
where the tongue is pressed against the palate should be excluded from the fluid domain. For
example, if the tongue is flush against the palate, then the oral cavity will be largely cut off making
the fluid domain much like a single tube, but the posture of the tongue in our model creates a
division in the fluid domain where the oral and nasal cavity separate and a poorly defined case for
a 1D model. Thus, a tongue that is close to the palate and more mobile may press against the
palate and have an important effect on the pressure/force experienced by the palate. This may be
an important reason why our model has little response to gravitational orientation (as seen in Fig.
6.7b), which is contrary to clinical observations.

We presented the static and dynamic results of airways with a small, medium, and large velo-
pharyngeal CSA. These results show the small airways to be less collapsible, both in uniform and
dynamic flow conditions. This is reasonable because the smaller airway models have a larger ratio
of wall thickness to inner radius. (For examples with tubes, this is seen in Fig 4.7). However,
clinical observations show that OSA patients generally have smaller but more collapsible geomet-
ries, which implies that collapse may be as much a result of details of the geometry, and not a
simple metric such as the CSA. The Isono et al. [73] data are consistent with this finding as they
show that a healthy but small geometry tends to be significantly less collapsible than an unhealthy
geometry of the same size. Walsh et al. [149] show that OSA geometries tend to be more circular in
cross-section, implying greater pressing in of the lateral walls than the AP walls. In keeping with
this, the parapharyngeal fat pads are correlated with OSA, the growth of which would pressure the
lateral dimensions to decrease [33,107]. Therefore, modeling an OSA geometry as merely smaller is
inadequate. The model should include mechanisms describing how a healthy geometry is deformed
and reduced to become unhealthy. For example, one such mechanism could be deforming the initial
geometry at the lateral walls to account for larger parapharyngeal fat pads.

While the trends predicted by the parameterized airway models are in keeping with clinical obser-
vation, the flow rates observed in these simulations are unreasonably large. For example, flow rates
of 0.002 m$^3$/s (120 L/min) are predicted for a relatively collapsed airway by the simulation. This
is largely due to the oversimplification of modeling the nasal cavity and trachea as smooth sections
of uniform $A(x,t)$. For example, at 30 L/min one can estimate that 40 Pa would be lost in the
nasal cavity from Riazuddin et al. [121] and that 50 Pa would be lost in the trachea from Johari
et al. [82]. In contrast, at 30 L/min our model predicts losses in the nasal cavity and trachea of
approximately 4 Pa and 0.2 Pa respectively. Resolving the complex geometry and flow of these
regions would be excessive when focusing on FSI in the pharynx, but to better account for the losses
occurring there, one could approximate the flow in that region with a BC: $p_0 - p_1 = \rho L \frac{\partial u}{\partial t} - RuL$,
where $p_0$ and $p_1$ are the pressure at the start and end of the region, $L$ is the length of the region,
and $R$ is a resistance assigned to that portion chosen from clinical or simulation data. The flow
rate does not go to 0 because the airway model resists complete collapse; for example, Fig. 6.3
shows $A \simeq 10$ mm$^2$ as the smallest CSA.
As reviewed in Section 2.4.1, the tongue and palate may significantly differ in stiffness, so splitting the FEM into sub-components which may take on different values of $E$ would be more realistic. Another improvement would be to use a non-linear material model that is more appropriate for soft tissue than a linear model. Defining muscle groups that are embedded into the FEM, and which may then account for passive and active muscle forces would be another step towards making this model more realistic.

The items above are our primary recommendations for future improvements to this model. Some other considerations for future work are:

- Use a more sophisticated shape for the soft palate. It is thick in the mid-sagittal plane at the location of musculus uvulae, but becomes thinner on either side. Also, it is substantially thicker where it bends (the location of $\theta_{\text{pal}}$ is our model) than at the location of the uvula (see [129] for 3D diagrams of the soft palate).

- Model the pharyngeal raphe and the retropharyngeal space carefully. In our model the region of the pharyngeal raphe is over-constrained by the use of fixed nodes.

- Simulate an entire breathing cycle. In our dynamic simulations, we simulated the statistically steady behavior that occurs during an inhalation; however, simulating a sequence of inhalation and exhalation would be more realistic and is quite feasible given our model’s simulation time.

## 6.5 Conclusions

In this chapter, we have presented the design of a parameterized upper airway model and performed an array of fluid-structure interaction simulations using that model with the goal of better understanding the mechanisms of OSA and snoring. The geometry and mesh are created in Salome, and the desired parameters can be defined in the script used to automate the process. This ensures quick and consistent mesh creation. Those 3D meshes are then coupled with a fast 1D fluid model for fluid-structure interaction simulations in Artsynth.

Uniform pressure FSI simulations were performed to determine the effect of parameter variation, and were compared with clinical data. We performed a sensitivity analysis on 8 parameters: mesh quality, Young’s modulus, cross-sectional area, anterior-posterior/lateral diameter ratio, palate angle, palate thickness, pharyngeal wall thickness, and orientation with gravity. In keeping with clinical observations, we found initial cross-sectional area to be an important parameter. Young’s Modulus, palate thickness, and pharyngeal wall thickness are the most important parameters in modifying the models compliance. The model does not respond significantly to gravitational orientation, likely because of over-simplification in the tongue model.

Dynamic flow FSI simulations are performed in which 5 parameters are varied: Young’s modulus, cross-sectional area, palate thickness, pharyngeal wall thickness, and upstream/downstream length.
We demonstrate that flow limitation occurs, in which the maximum flow rate and the pressure at which it occurs is dependent upon the parameters chosen in a manner in keeping with clinical OSA observations. “Snoring” oscillations are also predicted by our model, typically arising at pressures below the pressure at which the maximum flow rate is attained. The importance of carefully modeling portions of the airway upstream and downstream of the collapsible region is also demonstrated.

We conclude that using this simplified but representative parameterized airway model is a powerful approach. The quick and consistent mesh creation coupled with a fast and robust FSI simulation allows one to feasibly explore the vast parameter space and the rich behaviors related to OSA and snoring. This allows one to skip over the artifacts and high-order effects that comes with using “exact” models and examine the importance of modeling parameters and assumptions.
Chapter 7

Conclusions

This thesis presents the definition and validation of a numerical Fluid-Structure Interaction (FSI) system with application in Obstructive Sleep Apnea (OSA) modeling. Driven by our understanding that OSA is primarily a problem of the upper airway structures, we model the FSI with a 1D fluid model, thus only resolving the bulk flow, and couple this 1D fluid model with a 3D solid model. In Chapter 3 we present an implementation of the Cancelli and Pedley 1D fluid model that is robust in an uneven geometry that is deforming rapidly, as well as closing and reopening. We demonstrate that this fluid model usually predicts the bulk pressure within 10% of the values calculated by 3D CFD simulations using upper airway geometries and flow conditions.

In Chapter 4 we demonstrate that our solid mechanics simulation design, using Artisynth as the solver, is well-behaved by comparing collapsing tube simulations with experimental results from the literature. The simulations accurately capture the global behavior of the tube including buckling, contact of opposing walls, nearly complete closure, and reopening; however, the simulated tubes are more resistant to buckling than experimental tubes. The simulation results are quite sensitive to mesh quality and compressibility parameters.

We performed experiments and simulations using a “2D” Starling Resistor model with fluid and solid characteristics similar to those of the Upper Airway (UA), as presented in Chapter 5. These are the first “2D” Starling Resistor experiments that we are aware of, and are directly comparable with the simulations, making quantitative validation possible. The simulations and experiments agree well, typically having the same progression of FSI behaviors occurring within 10% of each other.

In Chapter 6 we presented a parameterized airway model used to generate UA geometries. The process is automated and fast, allowing the consistent creation of numerous geometries and meshes, which are used to demonstrate the sensitivity of FSI simulations to geometrical and material parameters. In most cases, our sensitivity analysis predicts the same trends as are clinically observed, particularly demonstrating that softer or deteriorated muscles make the UA more prone to OSA. Our simulations also predict snoring, flow limitation, and airway collapse.
7.1 Concluding Remarks For Present Study

Our results demonstrate that the unique approach of coupling a 1D fluid model with a 3D solid model is appropriate for OSA simulations. The fluid-structure interaction in OSA is dominated by the inertia of the solid, and therefore insensitive to minor fluid fluctuations (both spatial and temporal). However, accurate prediction of the viscous losses important for a fluid model. For example, for a pressure driven flow the viscous losses directly effect the predicted flow rate, hence predicting if the lungs will receive sufficient airflow requires a good estimate of viscous losses. Other examples are the cases where the Cross-Sectional Area (CSA) is sensitive to a small change in air pressure, such as the buckling points observed with a thin velopharyngeal wall thickness (Fig. 6.7a) or large initial CSA (Fig. 6.5a), along with the buckling behaviors observed by Isono et al. (see Fig. 2.3). It is not the fluid that causes a compromised geometry, but the fluid pressures will likely be the tipping-point which bring a compromised geometry to closure. Therefore, a fluid model that reasonably predicts the bulk flow, such as our 1D fluid model, is an appropriate choice for OSA modeling.

On the other hand, a 3D solid model is necessary. This is expected from the perspective of the fluid-solid inertial balance: the solid inertia is dominant, and therefore the FSI behavior is sensitive to the solid model. It is also expected from the perspective of literature that shows that OSA results from an unhealthy upper airway that is too small and/or too compliant. A specific example highlighting the necessity of a 3D solid model is that the mode of airway collapse, whether primarily in the AP or lateral direction, varies between patients and may also vary the optimal treatment. Our simulations confirm that the relative importance of AP walls vs lateral walls can be changed depending on which UA component is weakened/strengthened.

The use of a 1D fluid model is not only reasonable, but essential if a reasonable simulation runtime is needed. Our goal in OSA modeling is not so much to replicate clinical results exactly, but to gain an understanding of how a healthy and unhealthy UA are defined in a model and how those models respond to variations; this follows naturally from, and will feed into, a proper understanding of the UA and OSA. For this purpose a relatively light-weight model is necessary, allowing one to broadly observe the essential behaviors of the UA without being hindered by the immense computational expense and instability of 3D FSI simulations. Therefore a 1D fluid model combined with a 3D solid model is an ideal system for fast, robust, and accurate OSA simulation.

The parameterized upper airway model is recommended for upper airway simulations. It exposes the same modeling choices that are required for a realistic airway model, while providing well-controlled geometry and meshing parameters. This allows the modeler to focus on understanding the nature of the system without the immense effort of deriving the UA geometry from medical images and the complications that come from messy data.
7.2 Future Work

I have suggested potential improvements and future work at the end of each chapter, but now I would like to highlight a few directions from a global perspective. To improve our FSI modeling toolkit, I recommend implementing the 1D fluid model using a quasi-Newton algorithm if stability allows. This would remove the need for an analytical Jacobian, thus rendering the fluid model easier to modify and less prone to errors that come from a large and repetitive code base. I also recommend implementing a tightly-coupled or monolithic temporal coupling between the fluid and solid models to enhance simulation accuracy and stability. These improvements would promote extension of the FSI system to broader applications. For example, a readily extensible 1D fluid model may be applied to speech research, which is closely related to OSA in that it involves FSI in the UA. Enhanced stability would allow FSI simulation of scenarios in which the fluid inertia is similar to, or greater than, the solid inertia (assuming the 1D flow assumption remains valid and useful). This would allow application to blood flow, or replication of numerous Starling Resistor experiments.

The parameterized airway is a useful approach, and further pursuit in this direction is recommended. Implementing the suggested improvements at the end of Chapter [6] will improve the realism of the model and allow us to test our hypotheses, such as the role that the tongue plays in velopharyngeal closure. I also recommend working closely with OSA clinicians for further refinements of the model, thus tightening the connection between clinical experience and intuition gained from simulation. Nor is the parameterized airway limited to OSA modeling; it could be applied to other UA functions such as speech and swallowing.

To conclude, we have developed a unique fluid-structure interaction system well-suited for simulation of OSA, which we have applied to a parameterized upper airway model. Our results demonstrate that these OSA simulations are trustworthy and useful, and are recommended for future use to aid in the understanding and treatment of Obstructive Sleep Apnea.
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Appendix A

Fluid Mechanics Theory And Simulations

A fluid is a substance that will continually deform, or flow, under an applied shear stress \textsuperscript{[103]}. This is fundamentally different from a solid, and therefore should be studied and simulated differently. Before examining the equations of fluid flow, we should consider how we want to specify the fluid motion.

A.1 Eulerian And Lagrangian Flow Descriptions

An important concept in the formulation of fluid, solid, and FSI equations is the Lagrangian or Eulerian flow descriptions:

**Lagrangian:** The Lagrangian description follows a material point, as that point moves, the “observer” moves with it. In computational methods, this means the mesh will follow the motion of the material. For solid simulations this is usually fine, but if the material deforms greatly, as a fluid does, the mesh will also deform greatly and cause simulation failure. Lagrangian equations are often simpler and more intuitive to formulate.

**Eulerian:** An Eulerian description observes a fixed point in space and measures the physical values of the material as it flows through that fixed location. This is typically the preferred way to describe fluid motion and formulate simulations because it allows a mesh to remain static and of high-quality, and at the same time simulate a material that is deforming greatly.

To transform from a Lagrangian frame to an Eulerian frame, we apply the material derivative operator:

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla
\]  

(A.1)
A.2 Navier-Stokes Equations

The equations governing fluid flow can be derived from conservation principles:

\[ \frac{\partial (\phi)}{\partial t} + \nabla \cdot (\phi \mathbf{u}) = S \]  \hspace{1cm} (A.2)

where \( \phi \) is a conserved property that is being transported, \( \mathbf{u} \) is the fluid velocity, and \( S \) describes and sources or sinks in the fluid. Here, the conservation of mass, momentum, and energy describing fluid flow are written, emphasizing the conservation law from which they derive [12,38]:

\[ \frac{\partial}{\partial t} \begin{bmatrix} \rho \\ \rho \mathbf{u} \\ \rho E \end{bmatrix} + \nabla \cdot \begin{bmatrix} \rho \mathbf{u} \\ \rho \mathbf{u} \mathbf{u} \\ \rho E \mathbf{u} \end{bmatrix} - \begin{bmatrix} 0 \\ \sigma \\ \sigma \cdot \mathbf{u} - \mathbf{q} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{f} \\ \mathbf{f} \cdot \mathbf{u} + q_g \end{bmatrix} \]

<table>
<thead>
<tr>
<th>symbol</th>
<th>description</th>
<th>type</th>
<th>standard units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>density</td>
<td>scalar</td>
<td>( \frac{kg}{m^3} )</td>
</tr>
<tr>
<td>( \mathbf{u} )</td>
<td>velocity</td>
<td>vector</td>
<td>( \frac{m}{s} )</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>stress</td>
<td>tensor</td>
<td>( Pa )</td>
</tr>
<tr>
<td>( p )</td>
<td>pressure</td>
<td>scalar</td>
<td>( Pa )</td>
</tr>
<tr>
<td>( \mathbf{T} )</td>
<td>deviatoric stress</td>
<td>tensor</td>
<td>( Pa )</td>
</tr>
<tr>
<td>( \mu )</td>
<td>shear viscosity</td>
<td>scalar</td>
<td>( Pa \cdot s )</td>
</tr>
<tr>
<td>( \mathbf{f} )</td>
<td>body forces per unit volume</td>
<td>vector</td>
<td>( \frac{N}{m^3} )</td>
</tr>
<tr>
<td>( E )</td>
<td>specific energy</td>
<td>scalar</td>
<td>( \frac{J}{m^3} )</td>
</tr>
<tr>
<td>( \mathbf{q} )</td>
<td>heat flux</td>
<td>vector</td>
<td>( \frac{s \cdot m^2}{J} )</td>
</tr>
<tr>
<td>( q_g )</td>
<td>heat generation rate per unit volume</td>
<td>scalar</td>
<td>( \frac{J}{s \cdot m^3} )</td>
</tr>
</tbody>
</table>

Table A.1: Summary of quantities in the fluid flow equations.

We typically write: \( \sigma = -p I + \mathbf{T} \) where \( p \) is pressure (multiplying the identity matrix) and \( \mathbf{T} \) is the deviatoric stress tensor which exists due to fluid motion. A definition of \( \sigma \), which depends on the nature of the fluid being modeled, is needed to close this system of PDEs. We will make the following assumptions:

1. A Newtonian Fluid. This assumes that the stresses in the fluid are proportional to the rate-of-strain in the fluid, where \( \mu \) is the proportionality constant which describes shear viscosity. Air and water can usually be assumed to a Newtonian fluid.

2. An incompressible Fluid. A fluid may often be approximated as incompressible (\( \rho \) is constant)
when the fluid velocity is much less than the speed of sound in the fluid and one does not need to calculate the sound field (which propagates by compression waves).

These assumptions allow us to approximate the stress tensor as:

\[
\sigma = -pI + \mu \left( \nabla u + (\nabla u)^T \right)
\]

Keeping these assumptions, the Navier-Stokes equations can be written as:

\[
\begin{align*}
\nabla \cdot u &= 0 \quad \text{Conservation of Mass} \\
\rho \frac{\partial}{\partial t} u + \rho u \cdot \nabla u &= \nabla \cdot \sigma + f \quad \text{Conservation of Momentum} 
\end{align*}
\]

Alternatively, we can substitute in the material derivative operator \( \frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \), thus expressing the conservation of momentum from a Lagrangian point of view rather than an Eulerian one, and in a form suggestive of \( \mathbf{F} = m\mathbf{a} \) \( \text{[11 35 38]} \):

\[
\rho \frac{D}{Dt} \mathbf{u} = \nabla \cdot \sigma + \mathbf{f} \quad \text{(A.5)}
\]

### A.3 1D Fluid Models

#### A.3.1 1D Navier-Stokes

A 1D version of the Navier-Stokes mass and momentum equations are given in conservative form:

\[
\frac{\partial}{\partial t} \begin{bmatrix} \rho A \\ \rho_A u \end{bmatrix} + \frac{\partial}{\partial x} \begin{bmatrix} \rho A u \\ \rho_A u^2 \end{bmatrix} = \begin{bmatrix} 0 \\ -A \frac{\partial p}{\partial x} + \tau s \end{bmatrix}
\]

Written in non-conservative form, and making the incompressible assumption, we get:

\[
\begin{align*}
\frac{\partial}{\partial t} A + \frac{\partial}{\partial x} A u &= 0 \\
\rho \frac{\partial}{\partial t} u + pu \frac{\partial}{\partial x} u + \frac{\partial p}{\partial x} - \tau s \frac{s}{A} &= 0
\end{align*}
\]

The derivation of these equations from the 3D Navier-Stokes equation is given in Appendix D.
A.3.2 Bernoulli Equations

The well-known Bernoulli equation describes inviscid flow along a streamline, here written in common form for a steady and incompressible flow:

\[ p + \frac{1}{2} \rho u^2 + \rho gz = \text{const} \quad (A.8) \]

When used in conjunction with mass conservation \((Au = Q = \text{const})\), this can offer a quick estimate of the flow. However, the assumptions involved render the Bernoulli equation inaccurate for many flows. One common correction is to add an empirically-derived term to correct for viscous losses in a pipe, or similar well-classified geometry \([103, 145, 153]\). The limited pressure recovery model by Cancelli and Pedley [28] is an ad-hoc correction that can also be applied to approximate viscous losses.

A.4 Flow Properties

An essential dimensionless number in fluid mechanics is fluid mechanics is the Reynolds number, which is defined as:

\[ Re = \frac{\rho U L}{\mu} \quad (A.9) \]

where \(U\) is the characteristic velocity of the flow and \(L\) is the characteristic length. \(Re\) is a dimensionless number that describes the balance between inertial and viscous forces within the fluid. A large \(Re\) means that inertial forces are dominant, while a small \(Re\) means that viscous forces dominate. Flows with the same Reynolds numbers are comparable, even if the fluid or other flow characteristics differ.

For flow in a pipe, for \(0 < Re < 2300\) the flow is laminar, for which the fluid flows in parallel layers that do not mix. For \(Re > 4000\), the flow is turbulent, for which the flow is chaotic with high mixing. For \(2300 < Re < 4000\) the flow is transitional, containing characteristics of both laminar and turbulent flow.

A.5 Fluid Simulations

A.5.1 Finite Volume Method

The Finite Volume Method (FVM) is a numerical method for solving partial differential equations, and the primary method used in fluid mechanics simulations. The FVM arises from equations written in conservation form:

\[ \frac{\partial U}{\partial t} + \nabla \cdot F(U) = S \]
where $\vec{U}$ is a vector of conserved properties and $\vec{F}$ the flux of those properties. Note that this form is closely related to Eq. [A.2] Applying a control-volume integral:

$$\int \frac{\partial \vec{U}}{\partial t} dV + \int \nabla \cdot \vec{F} (U) \ dV = \int S \ dV$$

By divergence theorem we are able to write:

$$\int \frac{\partial \vec{U}}{\partial t} dV + \oint \vec{F} (U) \cdot \vec{n} \ dA = \int S \ dV$$

and by assuming that the mesh is not deforming (thus allowing us to pull the $\partial/\partial t$ out of the integral) we can integrate over the control volume to get:

$$\frac{d}{dt} \bar{U} \bar{V} + \oiint \vec{F} (U) \cdot \vec{n} \ dA = \bar{S} \bar{V}$$

where $V$ is the volume of the control volume. Finally, this is re-arranged to get:

$$\frac{d}{dt} \bar{U} = - \frac{1}{V} \oiint \vec{F} (U) \cdot \vec{n} \ dA + \bar{S}$$

Writing the fluid mechanics equations in conservative form makes application of Eq. [A.10] fairly straightforward. The flux of $F (U)$ across the surface of the control volume is calculated along with any source terms, and then the time advance is calculated to update $\bar{U}$.

### A.6 Turbulence Models

A critical challenge in many CFD applications is modeling turbulence. Following is an overview of 3 typical approaches.

#### A.6.1 Direct Numerical Simulation (DNS)

A DNS resolves all turbulent scales of the flow directly, using fluid dynamics equations such as Eq. [A.3] The computational power needed for a DNS renders it infeasible for all but the simplest geometries with a relatively low Reynolds number.

#### A.6.2 Reynolds-Averaged Navier-Stokes (RANS)

The Reynolds Averaged Navier-Stokes (RANS) equations are found by writing a flow variable $\phi (x, t)$ as a combination of time-averaged components, $\bar{\phi} (x)$, and fluctuating components, $\phi' (x, t)$:
\( \phi(x,t) = \overline{\phi}(x) + \phi'(x,t) \). Upon making this substitution into the Navier-Stokes Equations, averaging, and performing some algebra, one obtains the following equation (here written for incompressible flow) \[156\]:

\[
\rho \frac{\partial \overline{u}_i}{\partial t} + \rho \overline{u}_j \frac{\partial \overline{u}_i}{\partial x_j} = - \frac{\partial \rho}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \overline{T}_{ij} - \rho \langle u'_i u'_j \rangle \right]
\]

(A.11)

The troublesome term here, \( \rho \langle u'_i u'_j \rangle \), is called the Reynolds stress, and it arises from the fluctuating components of the flow itself rather than arising from the fluid properties. We will discuss 3 methods to close this equation:

1. Linear Eddy Viscosity
2. Nonlinear Eddy Viscosity
3. Reynolds Stress Equations

### A.6.2.1 Linear Eddy Viscosity

Linear Eddy Viscosity employs the Boussinesq approximation:

\[
-\rho \langle u'_i u'_j \rangle = 2 \mu_t \overline{S}_{ij} - \frac{2}{3} \rho \bar{k} \delta_{ij}
\]

In this equation, \( \mu_t \) is called eddy (turbulent) viscosity and is a scalar (thus implying isotropy). \( \overline{S}_{ij} \) is the mean strain rate:

\[
\overline{S}_{ij} = \frac{1}{2} \left[ \frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right] - \frac{1}{3} \frac{\partial \overline{u}_k}{\partial x_k} \delta_{ij}
\]

and \( \bar{k} \) is the mean turbulent kinetic energy:

\[
\bar{k} = \frac{1}{2} \left( \langle u'_1 u'_1 \rangle + \langle u'_2 u'_2 \rangle + \langle u'_3 u'_3 \rangle \right)
\]

At this point various turbulence models seek to close this equation in a physically meaningful way including algebraic closures, 1-equation closures, and two-equation closures \[47,108,123,156\].

Some specific RANS models using a linear eddy viscosity are:

1. One Equation Closures
   (a) Spalart-Allmaras: A fast model originally designed for aerofoil modeling.

2. Two-Equation Closures
   (a) \( k - \epsilon \): generally considered inaccurate for flows with large pressure gradients at the walls, but performs better for free-stream flows \[156\].
A.6. Turbulence Models

(b) $k - \epsilon$ variations for low Re: Contains wall modifications to adapt $k - \epsilon$. Due to wall modifications, these probably perform poorly for separated flows [77].

i. Launder and Sharma: Found to be a good model in a flat-plate flow [108].

ii. Chien: Consistently is mentioned as a notable model, though rarely seems to perform best [65,77,108,123].

iii. Lam and Brethorst: Is also consistently mentioned as a notable model, though rarely seems to perform best [65,77,108,123].

iv. Nagano and Tagawa: A good model as the flow moves from low-Re to high-Re [123].

v. Myong and Kasagi: One review found this to be the best model for pipe flow [65].

vi. Rodi, Mansour, Michelassi: This model appears to perform well at low-Re, but as the Re becomes higher the performance becomes worse (probably not suitable for flows with both regimes) [65,123].

(c) $k - \omega$: In general, a preferred model for boundary layers [108].

(d) $k - \omega$ SST: Uses $k - \omega$ near the wall and $k - \epsilon$ in the free stream to use the strengths of both models. Useful for low and high Re.

A.6.2.2 Nonlinear Eddy Viscosity

Nonlinear Eddy Viscosity models still model the eddy viscosity as a scalar, but now related to the Reynolds Stress in a nonlinear equation.

A specific RANS model employing nonlinear eddy viscosity is the $u^2 - f (\zeta - f)$ model, which is supposedly valid up to the wall, and thus good for low-Re turbulence modeling.

A.6.2.3 Reynolds Stress Equations

Rather than modeling Reynolds stress with a the scalar eddy viscosity, the Reynolds stress terms are solved for with the Reynolds Stress Transport equation which captures the directionality in the Reynolds stress term, and thus is capable of modeling anisotropy in turbulence. However, this introduces many more terms that must be modeled, making simulations more elaborate and slower.

A.6.3 Large Eddy Simulation (LES)

The RANS equations describe a time-averaged flow, in which the unsteadiness arising from turbulence has been modeled. An alternative approach is to filter the Navier-Stokes equations based on spatial scales, in which the large scales are directly resolved by the simulation but the small scales are modeled. A key reason to do this is that the small scales of turbulence are isotropic and therefore easier to model while the large scales are strongly affected by the geometry of the problem.
A.6. Turbulence Models

To do this, we split a flow variable $\phi$ into filtered (overbar) and unfiltered (prime) components $\phi(x,t) = \overline{\phi}(x,t) + \phi'(x,t)$. Plugging this into the Navier-Stokes momentum equation and filtering the resultant equation yields:

$$\rho \frac{\partial \overline{u}_i}{\partial t} + \rho \frac{\partial}{\partial x_j} (\overline{u}_i \overline{u}_j) = -\frac{\partial \rho}{\partial x_i} + 2\mu \frac{\partial}{\partial x_j} \overline{S}_{ij} - \rho \frac{\partial \tau^r_{ij}}{\partial x_j} \quad (A.12)$$

where $\overline{S}_{ij}$ is the strain-rate tensor and $\tau^r_{ij}$ represents the components that need to be modeled. A practical assumption is that the simulation mesh is the filter. All scales smaller than the mesh cells need to be modeled with a sub-grid model, and the scales larger than the mesh cells are directly resolved by the simulation. The sub-grid scales are typically modeled employing the Boussinesq approximation \[47\[118\]. A proper LES should directly resolve 80% of the flow’s kinetic energy \[118\], which is computationally demanding, but various studies have found LES simulations to perform comparably to RANS simulations when performed on the same, relatively coarse mesh \[3\[55\[100\].
Appendix B

Solid Mechanics Theory And Simulations

B.1 Solid Mechanics Equations

A solid has stronger molecular bonds than a fluid and does not continually deforms under a shear stress. For this reason, it is usually preferred to model the solid mechanics from a Lagrangian point of view, because the solid generally will not lose its structure. Therefore, we can write conservation of momentum with the material derivative:

$$\rho \frac{Du}{Dt} = \nabla \cdot \sigma + f$$  \hspace{1cm} (B.1)

but these properties now are solid properties: $\rho$ is density, $u$ is velocity, $\sigma$ is the stress tensor, and $f$ describes the body forces. In form, this is the same as Eq. [A.5] however, applying them tends to be quite different for solids. For solids, just like for fluids, the constitutive law must be defined by the material characteristics. The simplest material is a linear elastic material, described by:

$$\sigma = E \varepsilon$$

where $E$ is Young's modulus and $\varepsilon$ is the strain. A simple model of strain is:

$$\varepsilon = \frac{1}{2} \left[ \nabla \mathbf{d} + (\nabla \mathbf{d})^T \right]$$

where $\mathbf{d}$ is displacement. Note that $\sigma$ is calculated from strain for solids, as opposed to fluids where we relate stress to rate-of-strain $^{[35,38,43,63]}$.

B.2 Finite Element Method

The Finite Element Method (FEM) is typically used when simulating complex solid mechanics problems. There are numerous variations of the FEM, but the unifying concept is to divide the domain into elements, each element having a simple function that approximates the governing
equation. If $\Psi(x)$ is the solution we are seeking, we can represent that solution for an element as:

$$\Psi^e(x) = \sum_{n=1}^{N} a_n \psi_n(x)$$

where $a$ are the unknown coefficients needed to solve the problem, $\psi$ are the shape functions, and $N$ are the number of nodes that compose the element. The shape functions are usually a linear or quadratic function, though higher order functions are also possible.

To find the solution $\Psi(x)$, the problem is written as an optimization problem, in which we solve for the coefficients $a$ which minimizes the residual, or error, that the solution $\Psi(x)$ produces in the governing equations.

For example, one could write Eq. [B.1] as:

$$\rho \frac{D}{Dt} u - \nabla \cdot \sigma - f = 0 = R$$

where $R$ is the residual that we wish to minimize when we represent the solution $u$ with simple shape functions of unknown coefficients. The choice of appropriate shape functions, and the phrasing of the optimization problem (including casting the problem into integral form) are not details we will delve into here [113].
Appendix C

Fluid-Structure Interaction Theory And Simulation

C.1 Theory

Conceptually, fluid-structure interaction theory is quite simple. The fluid equations (for example, Eq. [A.2]) and the solid equations (for example, Eq. [B.1]) are solved, but the fluid and solid domains are coupled, because of the requirements at the fluid-solid interface:

\[
\begin{align*}
    d_f &= d_s \quad \text{kinematic requirement} \\
    F_f &= F_s \quad \text{kinetic requirement}
\end{align*}
\]

where \(d\) is the displacement of the interface, \(F\) is the force at the interface, and the subscripts “f” and “s” represent the fluid and solid domains. This maintains conservation of energy because: \(F_f d_f = F_s d_s \rightarrow W_f = W_s\). While simple in concept, the consequence is that the fluid and solid behaviors are coupled, resulting in a more complex system.

C.2 Simulation Approaches

Simulating FSI is a significant challenge. Typically, solid simulations are done with a Lagrangian reference frame, and handle deformations at the boundary naturally. However, simulating a deforming fluid domain is difficult. Methods for the fluid simulation include:

**Eulerian** The fluid mesh remains fixed and the fluid equations are solved using the typical Eulerian reference frame. The deforming domain creates constraints that are placed on the fluid solve. The advantages to this approach are solving the fluid equations in the preferable Eulerian method and not needing to account for mesh motion; however, the mesh is not specialized according to the needs of the problem and defining the constraints can be difficult.

**Lagrangian** The fluid equations are solved in Lagrangian form, and hence the mesh deforms with the flow. This quickly leads to a highly distorted mesh, requiring frequent re-meshing.
Arbitrary Lagrangian-Eulerian (ALE) This method casts fluid equations in a form that allows for arbitrary mesh motion, neither requiring that the mesh follow the fluid nor remain fixed. This is achieved by writing the material derivative (Eq. A.1) as:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (u_{\text{material}} - u_{\text{mesh}}) \cdot \nabla$$

where $u_{\text{material}}$ is the velocity of the fluid (or whatever material is being simulated) and $u_{\text{mesh}}$ is the velocity of the mesh. This allows the mesh motion to be solved separately; for example, a simple mesh motion can be calculated by simulating the mesh as a ball-and-spring network. For large domain/mesh deformations an automated re-meshing procedure will probably be required, a particular challenge if for a domain with changing topology (as is the case for airway closure).

Mesh-Free Method To avoid the complications of mesh handling in FSI problems, mesh free methods such as Smoothed Particle Hydrodynamics (SPH) may be used. However, these methods typically have a weaker theoretical foundation and still require careful spatial analysis to initialize and update the simulation.

Simple Fluid Model One may also apply a simple fluid model, such as the Bernoulli equation (Eq. A.8) to avoid the complexities of mesh motion. This provides a quick, though low-accuracy, fluid solution. Note that the Bernoulli equation is Eulerian, and though not requiring a mesh, requires that the fluid domain can be approximated with an area function.

The spatial coupling requirements are challenging to model. Typically, a fluid domain requires a much finer discretization, and applying the same discretization in the solid domain will immensely increase the computational expense. Therefore, one may define a mapping, requiring fluid and solid nodes to share the same surface, but not requiring fluid and solid nodes to be identical. This may be expressed as $d_t = Hd_s$ where $H$ is the transformation matrix which depends on the interpolation method chosen. Radial Basis Functions (RBFs) are found to be a very good interpolation method for this problem. This mapping may also be used to apply the fluid forces on the solid: $F_s = H^T F_f \cdot [34, 146]$.

The manner in which the fluid and structure simulations are temporally coupled is very important to the accuracy and stability of the solution. The two approaches are:

Monolithic: In a monolithic solve, the fluid and solid equations are solved simultaneously. The monolithic solve can be written as:

$$F(X) = 0 = \begin{bmatrix} F_f(X_f, X_s) \\ F_s(X_s, X_f) \end{bmatrix} = \begin{bmatrix} \frac{\partial F_f}{\partial X_f} & \frac{\partial F_f}{\partial X_s} \\ \frac{\partial F_s}{\partial X_f} & \frac{\partial F_s}{\partial X_s} \end{bmatrix} \begin{bmatrix} \Delta X_f \\ \Delta X_s \end{bmatrix}$$

where $X$ is the solution vector and $F(X)$ are the discrete equations. This provides the most stable and accurate solution. However, formulating the off-diagonal Jacobian terms in Eq.
C.2 is difficult, and defining this solve using specialized/black-box solvers may be impossible. This tends to scale poorly to large problems.

**Partitioned:** In contrast, a partitioned solve uses separate fluid and solid solvers. A typical step involves: advancing the solid model, applying the solid deformations to the fluid domain, advancing the fluid solve, applying the fluid pressures as external forces on the solid domain. For loose coupling this is done just once during a time step, but a loosely coupled solution tends to be unstable because energy is not conserved at the boundary, especially when there are large deformations. Alternatively, one may achieve tight coupling by iterating this procedure within each time step until the solution is converged. While this helps with stability, it is slower and can encounter convergence problems.
Appendix D

Derivation Of 1D Fluid Model

Starting from the very general, integral form of conservation of mass and momentum, we will derive a 1D fluid model.

\[
\frac{\partial}{\partial t} \iiint \rho \, dV + \iint \left( \rho \bf{u} \cdot \nabla \right) = 0 \quad \text{Mass Conservation}
\]

\[
\frac{\partial}{\partial t} \iiint \rho \bf{u} \, dV + \iint \left( \rho \bf{u} \cdot \nabla \right) \bf{u} = \iiint \rho \bf{f}_{\text{body}} \, dV - \iint \bf{f}_{\text{surface}} \, dS \quad \text{Momentum Conservation}
\]

It is assumed that flow is approximately 1D in the \( x \)-direction, thus allowing us to describe the geometry/flow using cross-sectional area function \( A = A(x,t) \) and perimeter \( s = s(x,t) \), as shown in Fig. D.1. Our assumptions will be many:

1. Incompressible flow \((\rho = \text{const})\)
2. Pressure, velocity, and shear are uniform across each cross section.
3. A circular cross-section, so \( A = \pi r^2 \), \( s = 2\pi r \). However, a circular cross-section in not assumed when estimating viscous losses.

So our goal is to integrate each term along the cross-section, such that the mass and momentum equations are a function of \( A \) and \( s \) rather than \( y,z \).

![Figure D.1: Schematic of a 1D fluid flow.](image)

D.1 Conservation Of Mass

Deriving the 1D conservation of mass equation is fairly simple. Rewriting our initial equation:

\[
\frac{\partial}{\partial t} \iiint \rho \, dV + \iint \left( \rho \bf{u} \cdot \nabla \right) = 0
\]
The 1D assumption means that the fluid only flows in the $x$-direction. This allows us to rewrite the velocity vector $\mathbf{u}$ as the $x$-component $u$:

$$\frac{\partial}{\partial t} \int \int \rho \, dV + \oint (\rho \mathbf{u} \cdot \mathbf{n} \, dS) = 0$$

The first term becomes:

$$\frac{\partial}{\partial t} \int_{x-\Delta x}^{x} \int_{y_0}^{y_1} \int_{z_0}^{z_1} \rho \, dz \, dy \, dx = \frac{\partial}{\partial t} \int_{x-\Delta x}^{x} \rho A \, dx$$

With the second term, we want to include the possibility of mass being introduced or lost along the channel (as occurs in Chapter 5), so we define a gap $g(x)$ that has velocity $u_g$:

$$\oint \rho \mathbf{u} \cdot \mathbf{n} \, dS = \rho u_A |_{x-\Delta x}^{x} + \int_{x-\Delta x}^{x} \rho u_g (x) \, dx$$

Writing the entire mass equation, we have:

$$\frac{\partial}{\partial t} \int_{x-\Delta x}^{x} \rho A \, dx + \rho u_A |_{x-\Delta x}^{x} + \int_{x-\Delta x}^{x} \rho u_g (x) \, dx = 0$$

Being unable to integrate with respect to $x$, we differentiate to get:

$$\frac{\partial}{\partial t} \rho A + \frac{\partial}{\partial x} \rho A u + \rho u_g (x) = 0$$

The flow is incompressible, so we can divide the constant $\rho$ out of the equation, and write the gap flow in terms of a flow rate $q_g$:

$$\frac{\partial}{\partial t} A + \frac{\partial}{\partial x} u A + \frac{q_g}{\Delta x} = 0$$

This is the desired result. Of course, when there is flow lost through a gap, a closure model for $q_g$ must be introduced.

**D.2 Conservation Of Momentum**

The conservation of momentum term is more involved. Rewriting the initial equation:

$$\frac{\partial}{\partial t} \int \int \int \rho \mathbf{u} \, dV + \oint (\rho \mathbf{u} \cdot \mathbf{d} \overrightarrow{S}) \mathbf{u} = \int \int \int \rho f_{body} \, dV - \oint f_{surface} \, d\overrightarrow{S}$$

Again, conservation of momentum can be understood to arise from $\frac{d}{dt} (\rho \mathbf{V}) = F$ where the right-hand terms describe the body and surface forces applied on the control volume of fluid.
D.2. Conservation Of Momentum

this down term-by-term:

Term 1: \( \iiint (\rho \mathbf{u} \cdot d\mathbf{S}) \mathbf{u} \)

- Assume that \( \mathbf{u} = u(x) = u \). Sometimes a 1D model will assume a laminar flow which has a parabolic profile or a turbulent flow which has a plugged-shaped profile. In either case \( \mathbf{u} = u(x,y,z) \), and the integral is described as: \( \int \rho \mathbf{u}^2 dA = \rho \beta Au^2 \) where \( \beta \) is the momentum flux term, which includes the effect of the velocity profile in \( y,z \) on the momentum. For a fully-developed laminar flow, \( \beta = \frac{4}{3} \); for a turbulent flow \( \beta \) varies, but approximately \( \beta = 1.02 \). In our situation the flow can be found in both flow regimes but is not expected to be fully developed. Properly accounting for \( \beta \) would be an unnecessary complication for it’s relatively small influence; therefore, we assume \( \beta = 1 \) allowing \( \mathbf{u} = u(x) = u \).

- At the side-walls \( \mathbf{u} \cdot d\mathbf{S} = 0 \).

\[ \iiint (\rho \mathbf{u} \cdot d\mathbf{S}) \mathbf{u} = \iiint (\rho \mathbf{u} \cdot dA) \mathbf{u} = \rho Au^2 |_{x=\Delta x} \]

Term 2: \( \frac{\partial}{\partial t} \iiint \rho \mathbf{u} d\mathbf{V} \)

- Applying the same assumption as above, we can perform the volume integral in the \( y,z \) planes; however, we do not know what \( u(x,t) \) is, so we cannot perform the \( x \)-integration:

\[ \frac{\partial}{\partial t} \iiint \rho \mathbf{u} dz dy dx = \frac{\partial}{\partial t} \int \rho Au dx \]

Term 3: \( \iiint \rho f_{body} d\mathbf{V} \)

- Gravity is the only body force one might consider for airflow in the upper airway, but even this will be incredibly small. Therefore we can ignore this term.

Term 4: \( \iint f_{surface} d\mathbf{S} \)

- This is the most complicated term to model. We need to consider two surface forces: pressure \( (p) \) and shear stress \( (\tau) \). So:

\[ \iint f_{surface} d\mathbf{S} = \iint p d\mathbf{S} + \iint \tau d\mathbf{S} \]

1. Pressure: \( \iint p d\mathbf{S} = Ap - \int p \cdot \sin(\theta) s(x) dx \).

   (a) Because we assume \( p = p(x) \), integrating with respect to the \( x \)-component of \( d\mathbf{S} \) is straightforward, giving \( Ap |_{x=\Delta x} \).
(b) However, integrating the pressure along the walls is more complex, and can not be performed without knowing both \( p(x) \) and \( s(x) \). The pressure acts normally to the wall, but because we are assuming a circular cross-section, all the \( y \)- and \( z \)-components will cancel each other completely. Only the \( x \)-component will remain, for which we must know the angle \( \theta (x) \) that the walls make with the \( x \)-axis (\( \theta \) is illustrated in Fig. [D.1]). This allows us to write the pressure contribution as \( p \cdot \sin (\theta) \). To calculate \( \theta \) we can write: \( \tan (\theta) = \frac {dr}{dx} \). Therefore:

\[
\sin (\theta) = \sin \left( \tan^{-1} \left( \frac {dr}{dx} \right) \right) = \frac {dr}{dx} \cdot \frac {1} {\sqrt \left( \left( \frac {dr}{dx} \right)^2 + 1 \right)} \approx \frac {dr}{dx}
\]

which we arrive at through a trigonometry identity and by dropping the \( (\frac {dr}{dx})^2 \) term which should be very small, thus linearizing the problem.

(c) Note that \( s = 2\pi r \), so \( \sin (\theta) \cdot s \approx 2\pi r \frac {dr}{dx} = \frac {dA}{dx} \)

(d) So, our result is: \( \oint p \, d\vec{S} = Ap^r_{x-\Delta x} - \int p \frac {dA}{dx} dx \). However, by applying integration by parts we can rewrite this as \( A \frac {dp}{dx} = \int A \frac {dp}{dx} dx \), which is more useful later on.

2. Shear: \( \oint \tau \, d\vec{S} = A \tau - \int \tau_w \cos (\theta) s \, dx \)

(a) The velocity is assumed to be uniform yet, contradictorily, dropping to 0 at the wall. Throughout the uniform portion there is no shear, yet having a significant \( \tau_w \) term at the wall. The first component, \( A \tau \), which comes from integrating with respect to \( x \), will be zero: \( A \tau = 0 \).

(b) While pressure acts normal to the wall, shear acts in the direction of the wall, hence \( \tau_w \cos (\theta) \) describes the \( x \)-component from shear (again, the \( y \)- and \( z \)-components cancel). Our analysis is similar to above:

\[
\cos (\theta) = \cos \left( \tan^{-1} \left( \frac {dr}{dx} \right) \right) = \frac {1} {\sqrt \left( \left( \frac {dr}{dx} \right)^2 + 1 \right)} \approx 1
\]

(c) For simplicity we write \( \tau_w \) as \( \tau \), thus getting the result: \( \oint \tau \, d\vec{S} = - \int \tau_w s \, dx \)

- So we can write the surface forces as: \( \oint f_{surface} \, d\vec{S} = \int A \frac {dp}{dx} dx - \int \tau s \, dx \)

Now we are ready to piece together the 4 terms that form the 1D model:

\[
\frac {\partial} {\partial t} \int \rho Au \, dx + \rho Au^2 = - \int A \frac {\partial p}{\partial x} dx + \int \tau s \, dx
\]

Our final step is to differentiate, thus removing the unknown \( x \)-integrals:

\[
\frac {\partial} {\partial t} \rho Au + \left( \frac {\partial} {\partial x} \rho Au^2 \right) = -A \frac {\partial p}{\partial x} + \tau s
\]
That is our key result. For an incompressible fluid, $\rho$ is constant and may be pulled out of the differentials. $A(x,t)$ is known (or modeled with equations describing the solid mechanics of the tube) and $u(x,t), p(x,t)$ are our solution variables. However, $\tau$ is unknown and must be modeled.

We can rewrite this in an alternative form with some manipulation:

$$\frac{\partial}{\partial x} \rho A u^2 + \frac{\partial}{\partial t} \rho A u = \rho u \frac{\partial}{\partial x} A u + \rho A u \frac{\partial}{\partial x} u + \rho A \frac{\partial}{\partial t} u + \rho A \frac{\partial}{\partial t} A$$

$$= \rho u \left( \frac{\partial}{\partial t} A + \frac{\partial}{\partial x} A u \right) + \rho A \frac{\partial}{\partial x} u + \rho A \frac{\partial}{\partial t} u$$

The term in brackets is the continuity equation, and thus is zero. So we write:

$$\rho A u \frac{\partial}{\partial x} u + \rho A \frac{\partial}{\partial t} u = -A \frac{\partial p}{\partial x} + \tau s$$

And divide by $A$ to reach the form that used in most portions of this thesis:

$$\rho u \frac{\partial}{\partial x} u + \frac{\partial}{\partial t} u = -\frac{\partial p}{\partial x} + \frac{\tau s}{A} \tag{D.1}$$

## D.3 Viscous Term

The viscous term is divided into two parts: $\tau = \tau_{fric} + \tau_x$.

### D.3.1 Friction

The term $\tau_{fric}$ describes simple pressure loss models based on theoretical and experimental results for pipe flow [153]. These results are often expressed in terms of a friction factor $f$ that is related to the shear term by:

$$\tau_{fric} = -\frac{1}{2} \rho u^2 \left( \frac{f}{4} \right)$$

For laminar flow, we have the theoretically derived term:

$$f_{lam} = \frac{64}{Re}$$

with $Re = (\rho \cdot u \cdot d_h) / \mu$ and $d_h$ is the hydraulic diameter $d_h = (4 \cdot A) / s$. This creates a discontinuity at $Re = 0$, but plugging into the $\tau_{fric}$ equation and reducing yields a simple term without discontinuity and valid for flow in the $-x$ direction:

$$\tau_{fric} = -2\mu \frac{s}{A} u$$
D.3. Viscous Term

For turbulent flow the friction factor is based on experimental results:

\[ f_{\text{turb}} = \frac{1.0}{\left[-1.8 \cdot \log \left(\frac{6.9}{Re} + \left(\frac{\epsilon/d_h}{3.7}\right)^{1.11}\right)\right]^2} \]

where \( \epsilon \) is the surface roughness. If \( \epsilon = 0 \), this term has a discontinuity at \( Re = 6.9 \), which is well outside of the range for which this term is valid (turbulent flow, \( Re > 4000 \)). Because \( f_{\text{turb}} \) is not functional for flow in the \(-x\) direction, and a highly complicating term to account for in a Jacobian, we do not use it. Instead, we turn our attention to an ad-hoc correction that proves much more functional in the complex flow and geometry of the upper airway.

D.3.2 Limited Recovery

The ad-hoc term given by Cancelli and Pedley \[28\] limits the extent of pressure recovery. In their paper, Cancelli and Pedley present the momentum equation as:

\[ \rho \frac{\partial u}{\partial t} + \chi \rho u \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} \]  \hspace{1cm} (D.2)

where \( \chi \) limits the pressure recovery according to (repeating Eq. \[3.4\]):

\[ \chi = \begin{cases} 1 & \text{for } \frac{\partial p}{\partial x} < 0 \\ \chi_{\text{min}} & \text{for } \frac{\partial p}{\partial x} \geq 0 \end{cases} \]

For the sake of placing the pressure losses into a separate term \( \tau_\chi \), we equivalently write Eq. \[D.2\]:

\[ \rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} + (1 - \chi) \rho u \frac{\partial u}{\partial x} \]

and fit it to the form of our 1D model (that is, Eq. \[D.1\]) by defining:

\[ \tau_\chi = \frac{A}{s} (1 - \chi) \rho u \frac{\partial u}{\partial x} \]

D.3.3 Final Viscous Term

Our final viscous term includes the laminar friction correction and the limited pressure recovery, given by:

\[ \tau = -2\mu \frac{s}{A} u + \frac{A}{s} (1 - \chi) \rho u \frac{\partial u}{\partial x} \]

In most cases the laminar friction term has a very minor influence, but in some situations adds a significant contribution or numerical stability and therefore we include it in the viscous term.
Appendix E

An Analytical Solution For The 1D Fluid Model

For the sake of validating the 1D fluid model, we present the derivation of an analytical solution for the 1D fluid model. Therefore, we define the area function:

\[ A(x, t) = A_0 - A_m \sin(\pi x) \sin(\pi t) \]

We drop the viscous term to make an analytical solution reasonable:

1. Conservation of Mass: \( c_t \frac{\partial}{\partial t} A + \frac{\partial}{\partial x} A u = 0 \)
2. Conservation of Momentum: \( c_t \rho \frac{\partial}{\partial t} A u + \rho \frac{\partial}{\partial x} A u^2 = -A \frac{\partial}{\partial x} p \)

We have multiplied the transient terms by a constant \( c_t \) allowing transient factors to be removed by setting \( c_t = 0 \), but in most cases \( c_t = 1 \). The boundary conditions are:

1. \( u(0, t) = u_0 \)
2. \( p(0, t) = p_0 \)

With these BCs, these equations are linear, first-order differential equations of the form:

\[ \frac{dy}{dx} + Py = Q \quad (E.1) \]

Which has the known solution

\[ y = e^{-I} \int Qe^I dx + c \cdot e^{-I} \quad (E.2) \]

where \( c \) is an integration constant and \( I \) is defined as:

\[ I = \int Pdx \quad (E.3) \]

We can fit the fluid equations to this form to find their solution.
E.1 Mass Equation

To fit the mass equation into the form of Eq. [E.1] we apply chain rule $(\frac{\partial}{\partial x} Au = A \frac{\partial u}{\partial x} + u \frac{\partial A}{\partial x})$ and write:

$$\frac{\partial u}{\partial x} + u \frac{1}{A} \frac{\partial A}{\partial x} = -\frac{c_t}{A} \frac{\partial A}{\partial t}$$

so $P = \frac{1}{A} \frac{\partial A}{\partial x}$ and $Q = -\frac{c_t}{A} \frac{\partial A}{\partial t}$. Plugging this into the general solution term given by Eq. [E.3], we write:

$$I = \int P \, dx = \int \frac{1}{A} \frac{\partial A}{\partial x} \, dx = \int \frac{dA}{A}$$

Here we need to remember that $\int \frac{dx}{x} = \ln |x| + c$ and keeping in mind that $A$ will always be positive (zero or negative area is not allowed), we can ignore the absolute value. So we find

$$I = \ln (A) + c$$

thus

$$e^I = e^{\ln(A) + c} = Ae^c \equiv Ac_1$$

We will define $c_1$ later. So now we can calculate the term from Eq. [E.2]:

$$\int Qe^I \, dx = \int \left[ -\frac{c_t}{A} \frac{\partial A}{\partial t} \right] [Ac_1] \, dx = -c tc_1 \int \frac{\partial A}{\partial t} \, dx$$

Calculating that $\frac{\partial A}{\partial t} = -A_m \pi \sin(\pi x) \cos(\pi t)$ we plug this in and integrate:

$$\int Qe^I \, dx = ctc_1 \int A_m \pi \sin(\pi x) \cos(\pi t) \, dx = -c tc_1 A_m \cos(\pi x) \cos(\pi t)$$

The solution according to Eq. [E.2] is:

$$u(x,t) = e^{-I} \int Qe^I \, dx + c \cdot e^{-I}$$

$$= -\frac{1}{Ac_1} ctc_1 A_m \cos(\pi x) \cos(\pi t) + \frac{c}{Ac_1}$$

$$= \frac{1}{A} (c_2 - c tc_1 A_m \cos(\pi x) \cos(\pi t))$$

Using the boundary conditions of $A(0,t) = A_0$ and $u(0,t) = u_0$ we find the integration constant $c_2$ (constant for $x$, not $t$):

$$c_2 = u_0 A_0 + c tc_1 A_m \cos(\pi t)$$

and the final solution is:

$$u(x,t) = \frac{1}{A} (u_0 A_0 + c tc_1 A_m \cos(\pi t) (1 - \cos (\pi x))) \tag{E.4}$$
E.2 Momentum Equation

Having a solution to \( u(x,t) \), we now turn our attention to the momentum equation. To fit conservation of momentum into the form of Eq. [E.1] we write:

\[
\frac{\partial p}{\partial x} = -\frac{\rho c_t}{A} \frac{\partial}{\partial t} Au - \frac{\rho}{A} \frac{\partial}{\partial x} Au^2
\]

such that \( P = 0 \) and \( Q = -\frac{\rho c_t}{A} \frac{\partial}{\partial t} Au - \frac{\rho}{A} \frac{\partial}{\partial x} Au^2 \). This is fortunate, as \( I = \int Pdx = c \) and \( e^I = e^c \equiv c_1 \). So the solution can be written as:

\[
p(x,t) = e^{-1} \int Qe^t dx + c \cdot e^{-1}
\]

Evaluating the first term, we find:

\[
-\rho c_t \int \frac{1}{A} \frac{\partial}{\partial t} Au \ dx = -\rho c_t \int \frac{1}{A} \frac{\partial}{\partial t} A \left[ \frac{1}{A} \left( u_0 A_0 + c_t A_m \cos(\pi t) \left( 1 - \cos(\pi x) \right) \right) \right] \ dx
\]

\[
= -\rho c_t \int \frac{1}{A} \left( -c_t A_m \pi \sin(\pi t) \left( 1 - \cos(\pi x) \right) \right) \ dx
\]

\[
= \pi \rho A_m c_t^2 \int \frac{\sin(\pi t)}{A} \left( 1 - \cos(\pi x) \right) \ dx
\]

And evaluating the second term:

\[
-\rho \int \frac{1}{A} \frac{\partial}{\partial x} Au^2 \ dx = -\rho \int \left( \frac{1}{A} \frac{\partial}{\partial x} u^2 + \frac{1}{A} u^2 \frac{\partial}{\partial x} A \right) \ dx
\]

\[
= -\rho u^2 - \rho \int \frac{u^2}{A} \frac{\partial A}{\partial x} \ dx
\]

\[
= -\rho u^2 + \rho \pi A_m \int \frac{u^2}{A} \cos(\pi x) \sin(\pi t) \ dx
\]

We compile this into a complete equation of \( p(x,t) \):

\[
p(x,t) = \pi \rho A_m \sin(\pi t) \int \left( \frac{c_t^2 (1 - \cos(\pi x)) + u^2 \cos(\pi x)}{A} \right) \ dx - \rho u^2 + c_2
\]

This integral is feasible, but very complicated. Therefore, we leave the equation in this form and perform the final integration numerically.
Appendix F

Derivative Schemes

The derivative schemes (optionally) used by the fluid model are summarized below.

1. Central schemes:
   
   (a) 2\textsuperscript{nd} order accurate: \[ \frac{\partial f}{\partial x} = \frac{f_{i+1} - f_{i-1}}{2 \cdot \Delta x} \]
   
   (b) 4\textsuperscript{th} order accurate: \[ \frac{\partial f}{\partial x} = \frac{f_{i-2} - 8f_{i-1} + 8f_{i+1} - f_{i+2}}{12 \cdot \Delta x} \]

2. One-Sided schemes:

   (a) 1\textsuperscript{st} order accurate
      
      i. forward: \[ \frac{\partial f}{\partial x} = \frac{-f_i + f_{i+1}}{\Delta x} \]
      ii. backward: \[ \frac{\partial f}{\partial x} = \frac{f_i - f_{i-1}}{\Delta x} \]

   (b) 2\textsuperscript{nd} order accurate
      
      i. forward: \[ \frac{\partial f}{\partial x} = \frac{-3f_i + 4f_{i+1} - f_{i+2}}{2 \cdot \Delta x} \]
      ii. backward: \[ \frac{\partial f}{\partial x} = \frac{3f_i - 4f_{i-1} + f_{i-2}}{2 \cdot \Delta x} \]

   (c) 3\textsuperscript{rd} order accurate
      
      i. forward: \[ \frac{\partial f}{\partial x} = \frac{-11f_i + 18f_{i+1} - 9f_{i+2} + 2f_{i+3}}{6 \cdot \Delta x} \]
      ii. backward: \[ \frac{\partial f}{\partial x} = \frac{+11f_i - 18f_{i-1} + 9f_{i-2} - 2f_{i-3}}{6 \cdot \Delta x} \]

   (d) 4\textsuperscript{th} order accurate
      
      i. forward: \[ \frac{\partial f}{\partial x} = \frac{-25f_i + 48f_{i+1} - 36f_{i+2} + 16f_{i+3} - 3f_{i+4}}{12 \cdot \Delta x} \]
      ii. backward: \[ \frac{\partial f}{\partial x} = \frac{25f_i - 48f_{i-1} + 36f_{i-2} - 16f_{i-3} + 3f_{i-4}}{12 \cdot \Delta x} \]

3. Mixed schemes:

   (a) 4\textsuperscript{th} order accurate:
      
      i. forward: \[ \frac{\partial f}{\partial x} = \frac{-2f_{i-1} - 3f_i + 6f_{i+1} - f_{i+2}}{6 \cdot \Delta x} \]
      ii. backward: \[ \frac{\partial f}{\partial x} = \frac{2f_{i+1} + 3f_i - 6f_{i-1} + f_{i-2}}{6 \cdot \Delta x} \]