# How young children negotiate sociomathematical norms during inquiry-based learning in mathematics: A discursive psychological perspective.

by

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## Abstract

This dissertation uses a discursive psychological perspective to investigate how young children use the structure of spoken language to support their participation in mathematical discussion. There is a general consensus that participation in the practice of collective argumentation not only promotes, but actually constitutes the learning of mathematics (Cobb, Yackel & Wood, 1992a; Krummheuer, 2007). However, given the wide variety of forms that mathematical communication might take (Barwell, Leung, Morgan & Street, 2005), how to fruitfully define and document the practices of communication involved, especially for young learners, remains an open question. Through a series of related discourse analyses (three linked studies carried out on a common data set), I explore some of the discursive practices 5- to 7-year-olds use as they negotiate what will become a taken-as-shared understanding regarding mathematical validity: a sociomathematical norm regarding what it means *to know* in mathematics.

Discursive psychology (Edwards, 1997) affords an examination of how participants treat notions of knowing and understanding during interaction. Using corpus linguistic analysis in the first study here allows me to elaborate features of this group's culture of negotiation by illuminating patterns in the interactional sequences involving *doing knowing*. Conversation analysis in the second study affords an examination of the practices by which young children incorporate mathematical content within the social act of negotiation: *doing mathematical understanding*. Further analysis attending to multimodal aspects of communication in the third study shows how the participants used those previously noted discursive practices to develop and sustain a six week long investigation into the meaning of the square root symbol: *doing algebraic reasoning*.

These analyses show how the children are able to draw upon a range of sociomathematical norms as resources that enable them to participate in ways that co-ordinate with each other. The findings suggest that expanding our expectations for what *mathematical knowing* looks like with young children affords the development of learning environments that support all children's sustained, successful engagement with mathematics.

## Preface

- 100% of the research and writing contained in this dissertation was conducted by me.
- The activities of the N:Countr mathematics research study group were acknowledged by the UBC Behavioural Research Ethics Board certificate #H09-01315 first awarded in July 2009 and renewed annually thereafter.

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**This dissertation is dedicated to my children – I couldn't love you more.**

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You both and your children – **Ethan, Rueben, Mackenzie and Alyssa** – remind me each day of the importance of living life well and with an eye to the future.

# Chapter 1: Introduction

> Teaching and learning [need] not stand on opposite banks and just watch the river flow by; instead they [might] embark together on a journey down the water. Through an active, reciprocal exchange, teaching can strengthen learning and how to learn.
>
> – L. Malaguzzi (from an interview in Edwards, Gandini & Forman, 1998, p. 83)

## 1.1 A learning journey shared

In the summer and fall of 2009 a group of young children and I embarked on a teaching and learning journey into the realm of mathematics. I have been surprised by how much I have learned: both during the initial four months of the mathematics research study and continuing through the subsequent four-plus years of personal study. My learning, both *with* the children and *from* them, is the subject of the document you are beginning to read. Through the initial study and thematic analysis and into the secondary discourse analysis of the original data, this dissertation represents the “unfolding” of that journey.

My background in Early Childhood Education (ECE) prepared me well to participate in those “active, reciprocal exchanges” that constitute processes of learning, but I felt less certain of my ability to recognize and build on specifically *mathematical* learning. Therefore, I initiated the journey by conducting an ethnographic study that investigated the potential for the use of pedagogical documentation (Cadwell, 2003; MacDonald, 2007) as a tool for data collection in early years mathematics settings (McLellan, 2010). The research design included me as a researcher/participant observer and either pairs of children or a group of ten children meeting together on a regular basis. I called the study “N:Countr,” which is an acronym for “Numeracy: Children's own understanding of numbers and their representation.” It developed in two phases in a process I will outline in Section 1.3 of this introduction.

## 1.2 Identification of the problem and purpose of the study

Alongside literacy, the early development of mathematical competency remains one of the critical building blocks of learning. However, researchers' understanding of the development of that competence has recently undergone substantial change. The increased significance of sociocultural approaches to mathematics education research has influenced a rise in interdisciplinary scholarship and the concurrent development of what has come to be called reform-oriented classroom practice (Cobb & Hodge, 2002; Lerman, 2000). These new classroom practices require a change from the traditional roles of teacher providing knowledge and students acquiring it (Hunter, 2008; McCrone, 2005). As teachers facilitate inquiry-based learning, students are expected to participate in the *doing* of mathematics in order to *know* mathematics (Hiebert, 2003).

Scholars examining those highly interactive reform-oriented classroom practices have noted that the development of a taken-as-shared set of values concerning mathematical activities (*sociomathematical norms*) is crucial to creating environments that support participation and learning (Voigt, 1995; Yackel & Cobb, 1996). For example, a shared sense of what will count as mathematical validity facilitates communication.

A rich body of literature describes the discursive development of sociomathematical norms through negotiation (e.g., Cobb & Bauersfeld, 1995; Hershkowitz & Schwarz, 1999; Houssart, 2001). Throughout these studies, teacher contributions to and perspectives on the processes of negotiation are well-documented (McClain & Cobb, 2001; Perry, McConney, Flevares, Mingle & Hamm, 2011). However, less attention has been paid to the perspective of students (Levenson, Tirosh & Tsamir, 2009) despite the acknowledgement that active participation in classroom interaction promotes mathematical thinking and learning (Lerman, 2000; Seeger, 2001). Furthermore, the emphasis in recent research into sociomathematical norms has shifted away from students in early grades (Yackel & Cobb, 1996; Yackel, Cobb & Wood, 1991) to participants who are adolescent or older (e.g., Cheval, 2009; Kazemi, 1998; Williams, 2010) effectively minimizing our awareness of young children's capacity to contribute to classroom practices.

This study addresses those two areas of inadequacy (acknowledging the students' perspectives and the mathematical experiences of young children) by documenting the discursive strategies young children use during the negotiation of sociomathematical norms. The overarching research question is: *How do young children participate in the negotiation of sociomathematical norms in inquiry-based settings?* The position I take in answering the research question resonates with linguistic anthropological traditions, where the reflexive quality of communication is used to situate *norms* as ongoing features of discourse, implicated in every interaction rather than pre-established or prescriptive (Agha, 2007; Baker, 2000). This means the sociomathematical norms will be negotiated as participants interact within mathematical learning communities. The assumption is that each and every interaction will exhibit an orientation to those underlying normative assumptions.

In the chapters of the dissertation, I present a 3-step series of discourse analyses of small group interactions with and without adult involvement (Chapman, 1993; Yackel et al., 1991). I chose this research design because discourse analysis involves description of details of communication that are easily overlooked (such as use of vocabulary or tacit agreement) and it highlights participants' capacity to influence social processes (like negotiating norms) by focusing on what spoken discourse accomplishes (Cameron, 2001). The three steps evolved over time, allowing me to construct a rich, complex portrait of the children's experiences.

I examined transcripts based on video data from the N:Countr study to investigate how young children use the structure of spoken language to support their participation in mathematics. In doing so, I assumed 5- to 7-year-olds to be competent users of language in an adult world: in this case, demonstrating their social competence in the arena of the mathematical negotiation (Hutchby & Moran-Ellis, 1998). Therefore, I examined the discursive strategies they used (e.g., how they invoked sources of authority in their narratives) while participating in that social arena: “Paying attention, not merely to *what* people say but to *how* they say it, gives additional insight into the way people understand things” (Cameron, 2001, p. 14 italics in original). I see children’s participation in these negotiations as one aspect of *doing* mathematics in order to *know* mathematics (Lakatos, 1976; Schoenfeld, 2006) and therefore an act of learning (Lave & Wenger, 1991).

One of the common threads throughout the dissertation and the three sets of analyses is the data generated through the N:Countr mathematics research group. Therefore I will elaborate some features of this common data set before developing the rationale for the current study.

### 1.3 Research design of and data generated during the N:Countr I and II studies

I conducted the N:Countr study in the summer and fall of 2009. The research design drew from work in ethnomethodology (Garfinkel, 1967) and pedagogical documentation (Cadwell, 2003), an Early Childhood Education oriented method of ethnographic data collection (MacDonald, 2007). The setting included me as a researcher/participant observer and either pairs of children (Phase I) or a group of ten children (Phase II) meeting together on a regular basis in a room in my home that had previously served as a preschool playroom. I had a previous relationship with every participant and each of them had previous relationships with some of the others, a feature that facilitated and also influenced our social interactions.

The design of N:Countr Phase I was informed by an adaptation of ethnomethodology that included informal, unstructured interactions during mathematical play. This study was conducted with pairs of 5- and 6-year-olds during the month of July 2009 (McLellan, 2011). The purpose of this research was to investigate the potential for the use of pedagogical documentation (PD) as a tool for ethnographic data collection in early years mathematics settings.

PD as a tool for emergent curriculum development has become known in Canada through the practices of the educators in Reggio Emilia, Italy (Rinaldi, 2001). It sensitizes adults to the many different ways that children might communicate and can therefore highlight the otherwise unnoticed mathematical thinking in children's activities. It enjoys widespread popularity in the field of Early Childhood Education although some claim the approach is under-theorized (Grieshaber & Hatch, 2003). It includes features relating to both *content* (the collection of artifacts: photos, video, children's artwork, transcriptions of conversations) and *process* (collaborative revisiting of experiences to promote reflection, mutual respect between all partners, creating documentation panels to share learning with those outside the group) (Cadwell, 2003; Dalhberg, Moss & Pence, 1999).

Up to this point in 2009, PD had been used in research as formative assessment in kindergarten literacy programs (MacDonald, 2007) and to document the informal mathematics in preschool play (Perry, Dockett & Harley, 2007). This approach had proven fruitful, especially since children often express mathematical thinking in non-traditional ways (Baroody, 1987) and other methods of recording informal mathematics tended to under represent children's experience of it (Tudge, 2009). Some scholars had tried to address these issues by focusing on creative expression in mathematics (Worthington, 2005). Others had focused on collecting nonverbal data (Wolodko, 2005). However, I thought these foci still seemed to limit the possibilities available to children for communicating. They also privileged adult agency in determining the course of the experience. I hoped to overcome these limitations by capitalizing on the *processes* of PD to give greater agency to the children during the research study.

The first Phase of the study involved six girls aged 5 years 7 months to 6 years 10 months paired along friendship lines (see Table 1.1). I recruited my own former preschool students to be my participants, purposefully chosen based on incidents involving numeracy that had occurred during some literacy classes I taught them earlier. Each pair came twice, for a 30-minute session with me. My invitation to the children in each session was: "We can play, and while we play you can show me some of the things you know about numbers and math." In this way, we constructed together what I came to regard as informal, unstructured interviews supported by play and the use of materials – toys, art supplies etc (e.g., see McLellan, 2011).

**Table 1.1 N:Countr I participants by pairs**

| Group | Pseudonym | Age at first session | Grade in September | Gender |
|---|---|---|---|---|
| **CC** | **Carissa** | 5 years 7 months | 1 | Female |
| | **Carlyn** | 6 years 4 months | 1 | Female |
| **EK** | **Erin** | 6 years 3 months | 1 | Female |
| | **Kim** | 6 years 2 months | 1 | Female |
| **KC** | **Kendra** | 6 years 10 months | 2 | Female |
| | **Cleo** | 6 years 8 months | 2 | Female |

All sessions were videotaped with an unmanned camera. Activities were photographed at the request of the children or whenever I identified potential mathematical significance. As much as possible, I deferred to the children's interest in play in an attempt to authentically document their explorations. The data collected were mainly photographs of activities and child artwork (total number: 138) and video-recordings of the sessions (total recorded: 3 hours). The sessions with pairs of children generated rich data. However, after preliminary thematic analysis (McLellan, 2010), I felt that the social interactions of a group would more closely resemble a classroom, so the second phase of the study was developed to include a group of ten children.

N:Countr Phase II (September – November 2009) involved participants recruited more generally from the same pool as before. This time I had five girls and five boys, five of them were beginning Grade 1 and five were beginning Grade 2 (see Table 1.2). These participants ranged in age from 5 years 9 months to 7 years 7 months. Two children from Phase I returned (Carissa and Carlyn) while the other eight were new to the study.

**Table 1.2 N:Countr II participants**

| Pseudonym | Age at first session | Grade in School | Gender |
|---|---|---|---|
| **Carissa** | 5 years 9 months | 1 | Female |
| **Danica** | 5 years 11 months | 1 | Female |
| **Truman** | 5 years 11 months | 1 | Male |
| **Jimmy** | 6 years 1 month | 1 | Male |
| **Carlyn** | 6 years 6 months | 1 | Female |
| **Tara** | 6 years 9 months | 2 | Female |
| **Daniel** | 7 years 3 months | 2 | Male |
| **Nathan** | 7 years 3 months | 2 | Male |
| **Cormac** | 7 years 4 months | 2 | Male |
| **Anya** | 7 years 7 months | 2 | Female |

During N:Countr II, I developed a playgroup setting that reflected an inquiry-based or emergent mathematics approach (Baker, Semple & Stead, 1990; Lampert, 1990; Stoessiger & Wilkinson, 1991; Yackel & Cobb, 1996) that is consistent with the goals and values of my ECE play-based learning background. This included individual and small group explorations that emerged from a combination of the children's interests and my provocations, or questions. It also sometimes involved whole group discussions. The group came once a week for 11 weeks for one hour at a time and I collected data in the same manner as in Phase I, generating 378 photographs and 11 hours of videotape. I also collected child produced artifacts, including mathematics journal entries for five of the weeks (journal entries are *not* included in the number of photographs). It is important to recognize that we were not exploring mathematical concepts with manipulatives as one might in a clinical interview (Ginsburg, 1997): there were no formally preconceived mathematical goals and the mathematical content emerged from the interactions. This kind of setting has been described elsewhere as a "learning experiment" (Francisco & Hähkiöniemi, 2012) as compared to the "teaching experiment" of other design type studies (e.g., Cobb, Yackel & Wood, 1992b). The richness of the setting is that it incorporated aspects of both home and school, while foregrounding children's curiosity and agency. We were playing and talking and while we interacted, mathematics happened.

In total, video data were collected over a 5-month period (from July 2009 to November 2009) within the same physical setting but involving four different participant groups (three from Phase I and one from Phase II). During the same time period I also collected student created artifacts (drawings and stories) and compiled field notes, which will be used to support interpretation. In total 14 children participated (Phase I: 3 hours with 6 children total; Phase II: 11 hours with 10 children total – 2 children participated in both phases).

## 1.4 Ethical obligations

Interactions within the N:Countr mathematics research group produced rich data and the initial descriptive thematic analysis (McLellan, 2010) might have become the substance of my dissertation were it not for the "interruptions" of a growing sense of three ethical obligations. Each of them warned me of the potential for misrepresentation and each pointed to the need for a certain kind of interpretive framework. Above all, I wished to represent my participants well.

The first obligation concerned relational dynamics. An article by Osberg and Biesta (2008) highlighted for me the significance of power differentials between even benevolent, responsive adults and children. In traditional settings these are quite obvious but although less so, they also function in inquiry-based learning environments. While I had planned to enact an emergent mathematics curriculum (Baker, Semple & Stead, 1990; Stoessiger & Wilkinson, 1991), during initial data analysis I began to wonder how the environment I had created and worked to sustain may have privileged some kinds of knowledge over others. In other words, what kind of knowledge had *I allowed* to emerge? It became clear to me that even though my research question in the dissertation foregrounded children's perspectives, I would better serve my participants if my interpretive framework required an element of my own self-reflection.

The second ethical obligation emerged as a conflict between the deep-seated value regarding accuracy in mathematics and my desire to allow mathematical thinking to emerge as a learning process where inaccuracy might play a role in intermediate steps at least. The dilemma manifested itself when, in the research group, a child spoke aloud during a brainstorming session *"one hundred plus one hundred is one hundred and two"* and no one corrected him. In-the-moment I decided to let the comment stand and I copied it, with other ideas, onto a radial diagram that was displayed on the wall for the duration of the study (see Figure 1.1).

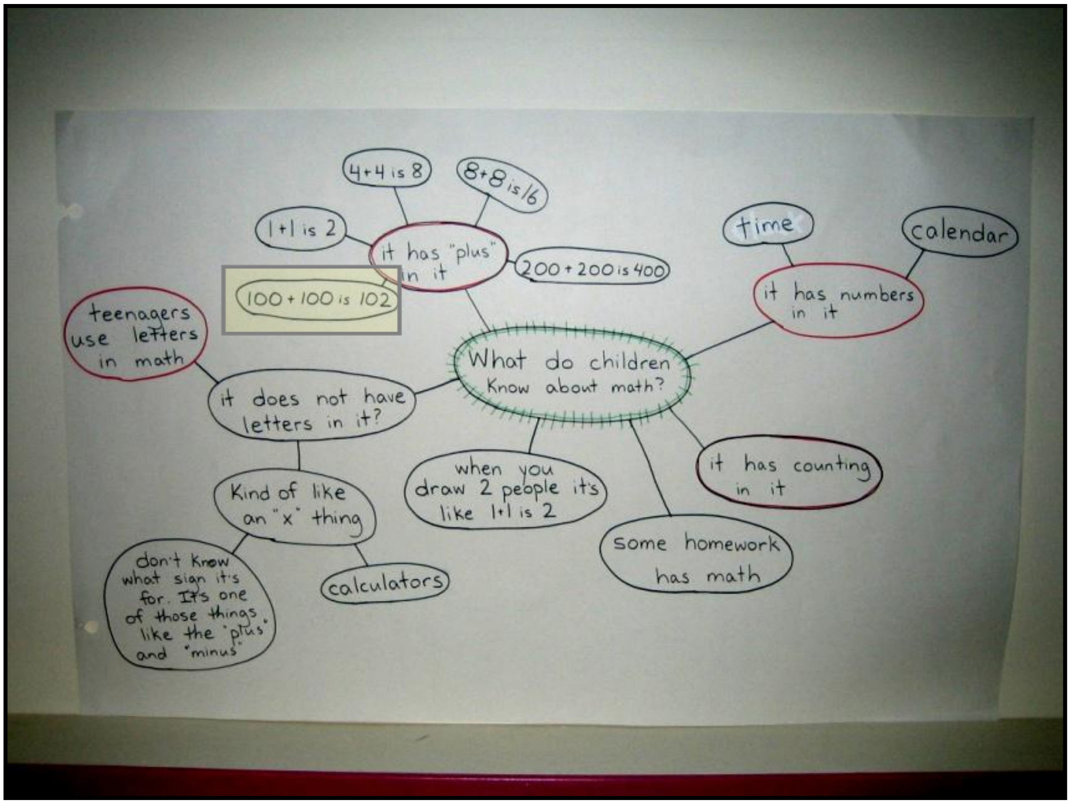


**Figure 1.1 Radial diagram from N:Countr II Week 1 with arithmetical inaccuracy highlighted**

As it happened, this ongoing representation of an arithmetical inaccuracy provided an opportunity for the same child to correct himself nearly ten weeks later: *"Actually, um, I was wrong – one hundred plus one hundred isn't a hundred and two, it's just two hundred"* (the incident is examined more closely in Chapter 4). The capacity for this kind of self correction, given enough time and an environment that made it relevant, drew my attention and caused me to focus more closely on *how* the children expressed their mathematical thinking. It became apparent to me that I might more readily recognize *mathematical learning* if my interpretive framework focused on the processes of meaning-making even when those processes were mathematically unconventional.

The third obligation accompanied the sense of entitlement that is connected to the role of teacher: “Who gets to define what for whom?” At the very beginning of the study I learned something substantial from a child as a consequence of having myself defied a normative expectation of what it means to be engaged with mathematics. While I was carrying on an extended discussion with the child regarding her understanding of the concept of “half” she listened to my question (*“How do you know it’s half?”*), then responded with an enthusiastic suggestion: *“I have an idea, let’s make paper airplanes!”* I followed her lead in this even though I expected she was going off-task. Her next words were spoken to instruct her friend in a paper-airplane-making technique: *“You have to fold it in half completely, so it’s right in half, so we know that that’s the middle.”* When I recognized the unexpected definition for “half” that she was enacting, I knew that I might miss something of what my participants were trying to communicate unless I employed an interpretive framework that could account for subtleties of communication in the moment of enactment.

My wrestling with this sense of ethical obligation through the preparation of course papers and conference presentations persuaded me to conduct secondary analysis from an interpretive perspective and eventually drew me to discourse analysis and discursive psychology. As an interpretive framework, discursive psychology (Edwards 1997) allows me to account for all three ethical obligations. I will outline the theoretical position in more detail in Chapter 2.

### 1.5 Positioning myself in the study

I came to mathematics education research through the back door, as it were. My 25 years of experience in Early Childhood Education (ECE) prepared me well for observing young children, teaching them in a generalist sense and responding to their interests and curiosities. Yet the world I am interested in researching is the world of the young child *after* preschool: the primary years. I see the value I bring to research on the mathematical thinking of primary aged children, based on my ECE learning framework. For instance, as I immersed myself in the research literature, it quickly became apparent to me that most mathematics education research with children is set in school classrooms. Two other lines of inquiry are conducted as individual interviews either in researchers' laboratories or in the children's own homes. A surprising number of studies rely solely on adult reports (either teachers or parents) to report young children's experiences with mathematics (Tudge, 2009). I looked for mathematics studies with groups of children set in alternate environments that foregrounded the perspective of participants older than preschool age and found none.

My ECE background informed my initial research design, using a familiar practice (pedagogical documentation) applied to a new setting (the N:Countr mathematics research group). I wondered: given a setting with *limited formal expectations* but *a clear invitation for mathematical activity*, how would the children respond? Due to my earlier experiences with these children I expected that they *would* actively explore mathematical concepts and processes. I was less clear about *how* they might explore them. In the end, I decided that a fruitful approach would be to trust the children to play, trust the processes of invitation and response based on our history of mutual respect and finally, trust myself to recognize what the children were communicating through their artifacts, actions and words. This would be a dynamic investigative process that would evolve as it happened.

I took the position of an "uninformed adult" during the research group, which the children oriented to as somewhat unexpected. Twice a child asked me *"Nanny, don't you know?"* to which I alluded to possibly knowing but not having the ability to fully communicate that knowing. I never corrected an inaccuracy and rarely performed any direct teaching, choosing instead to re-present what the children communicated and sometimes to ask questions. I almost never asked a question for which I already knew the answer. The role I played was central to the ways the interactions evolved and the positions available to children during negotiation.

My dual role as participant/analyst is uncommon in discursive psychology and I investigate that issue in more depth in Chapter 3. Furthermore, writing a multiple-study dissertation involves the self-imposition of sometimes conflicting expectations: those of an academic institution and those of scholarly journals. At some points in the writing process I was especially cognizant of one or the other audience. For example, the three results chapters are occasionally referred to as “manuscripts” here, but only in the sense that they take *the form* of a manuscript and I intend to publish those findings. As independent and stand-alone studies they are nevertheless too long to be publishable *as is*. However, the depth of findings presented here is warranted by the complexity of the data and so the presentation contributes to the rigour of the dissertation study. I therefore present the findings here as a series of three linked studies.

Even from the beginning of the N:Countr study I was curious to understand how these 5- to 7-year-olds *knew*, *experienced* and *represented* their mathematical thinking. By this I did not mean in the positivist sense that there might be a reality that I could somehow capture, but rather with the expectation that if I hoped to catch a glimpse of what it meant to learn mathematics through the children’s eyes I needed to experience the learning journey alongside them. Having noted the absence of their perspective in the mathematics education research literature, as a participant/observer I hoped to offer redress.

## 1.6 Significance of the study

The goal of this study is to document how young children participate in the negotiation of sociomathematical norms. However, qualitative data analysis is a recursive process, and as I progressed through the three steps intended to accomplish that documentation, I gradually became aware that in the *way* I was answering my research question, I was also addressing two other rather significant tasks.

First of all, in this study I particularize the processes involved in learning-as-participation, a foundational tenet of sociocultural theories of learning. During the months I spent analyzing the data for Chapter 6 (the third study) I recognized similarities between what my participants were doing with their algebraic reasoning and what Krummheuer's (2011) participants accomplished through their collective argumentation. When I slowed down to investigate, I found that I was able to trace that "learning-as-participation" motif back through all three studies. Methodologically, I foreground the messiness of early childhood participation in mathematical practice, paying close attention to the detail of talk. Therefore the analyses generated provide concrete examples of what learning by participating might actually look like. Through the three linked studies my data illuminates the interactive, argumentative and communicative processes of learning-as-participation, thereby contributing to the much needed operationalization of sociocultural theory (Krummheuer, 2011).

Secondly, in this study I illustrate, through a set of three related analyses, the potential inherent in applying discursive psychology to child interaction: thereby introducing a novel approach to research in early childhood education, a novel approach to the investigation of young children's mathematical thinking and extending the use of discursive psychology in mathematics education more generally. Transcriptions of the video data and subsequent discourse analyses provide interpretation of mathematical activities as they unfold turn-by-turn using the children's own words and gestures. This use of discursive psychology provides the means to focus on the social actions produced by the *ways* the children discursively construct their own positions and the positions of others during the negotiations (e.g., by doing convincing, explaining or refuting). The N:Countr research setting foregrounds child agency in determining the scope and sequence of the mathematics involved. The data include multiple forms of representation: video recording, child-generated art work and journals, photographs and field notes. Each of the three linked studies foregrounds some aspects of discursive psychology over others. This combination of setting, data generated and theoretical position affords the assemblage of examples of child mathematical argumentation that are distinct in mathematics education research.

## 1.7 Research design of the current study and how the dissertation is organized

This dissertation reports the findings of a series of three co-ordinated studies that I completed with secondary analysis on the previously generated N:Countr Phase II data. Each study was designed to answer one aspect of the research question: *How do young children participate in the negotiation of sociomathematical norms in inquiry-based settings?* The studies in sequence comprise a three-step investigation:

- Step one: What evidence do I find that sociomathematical norms are active in this setting?
- Step two: How do children display their understanding of the significance of norms during mathematical negotiation?
- Step three: How do children use sociomathematical norms as resources to support their participation?

My approach to answering these questions is driven by a discursive psychological framework. The answers I develop here will address a gap in the current research literature regarding the perspectives of students (Levenson, Tirosh & Tsamir, 2009) and in particular, young students.

These three investigative steps together comprise what I will call throughout the dissertation: *the current study*. Each step looks at one component of the research question and therefore findings attributable to each step represent drafts of publishable articles as well as chapters in the dissertation.

Chapter 2 in this dissertation provides a brief review of relevant literature surrounding the research question and the three investigative steps, since each results chapter includes its own review of relevant literature. With this overall review I establish a discursive psychological orientation as solid footing from which the whole issue of negotiating group norms can be approached. Chapter 3 outlines the research methodology for the current study. In it I provide a rationale for using secondary data analysis on previously generated data on the basis that it is essential to have a setting where the adult was not aware of the purpose of this research. In this sense, who I was when I collected the data is not who I have become through my ongoing academic scholarship. I recognize that using secondary data analysis is not *the only* way to ensure this "teacher blindness"; however, it is an ethical way to ensure that if I find evidence of negotiation (as opposed to the adult imposing norms on the children) then it is probably not because the adult was setting up opportunities for the children to participate.

Chapter 4 is the first of three chapters that present the findings of the current study and it corresponds to Step 1 in the research design. The chapter is titled "*How young children use discursive practices of knowing during mathematical negotiation."* The main findings of this study are to show first, how the children drew upon sociomathematical norms as resources to produce *ways of participating* that co-ordinated with each other and second, how sharing the authority for *mathematical knowing* between researcher and children afforded meaningful participation. Negotiating sociomathematical norms has implications for the negotiation of knowledge. In other words, what comes to count as mathematically valid affords, but also constrains the forms of knowing available and the ways in which members of a learning community might invoke those practices. The reverse also holds: negotiating knowledge has implications for the negotiation of sociomathematical norms. Thus as a first step in the current study I sought to elaborate features of the group's culture of negotiation by illuminating patterns in the interactional sequences that involved "doing knowing." Significantly, the analysis exemplifies some of the *interactive* processes involved in learning-as-participation.

In the study represented in Chapter 4, I use a form of discourse analysis called corpus linguistics, which is "a methodology for linguistic analysis that focuses on describing linguistic variation in large collections of authentic texts (the corpus), using automatic and interactive computer programs to aid in analysis" (Gray & Biber, 2011, p. 139). In Chapter 4, the data consists of whole word transcriptions of the entire 11 hours of videotape generated during N:Countr Phase II, resulting in a corpus of approximately 130,000 words. Discursive psychology affords an understanding of the tacit and often quite subtle sociomathematical norms that operate during interaction by examining their violations (Garnica, 1981) and the sanctions that arise from those violations (Boulima, 1999). Therefore, in this chapter I uncover evidence that sociomathematical norms are active within this group by showing when and how talk orients to those norms. Furthermore, I use discursive psychology to establish a locally meaningful distinction between *mathematical knowledge* and *mathematical knowing*.

Chapter 5 is the second of the three chapters that represent the findings of the current study, corresponding to Step 2 in the research design. This chapter is titled *"How young children display their understanding of mathematical content during negotiation."* The main findings of this study are first, to show how children drew upon the structure of production and recipient design to support their participation in mathematical argumentation and second, to illustrate how the children used the invoking of sociomathematical norms to demonstrate their interpretations of the meaning of mathematical content. The study here highlights the practices by which young children incorporate mathematical content within the social act of negotiation. It also shows how the *ways* the children discursively construct their participation displays their emerging mathematical understanding by examining in some detail how the group negotiated a taken-as-shared understanding of the meaning of the equals sign during week 4 of N:Countr Phase II. Significantly, the analysis exemplifies some of the *argumentative* processes involved in learning-as-participation.

In chapter 5, I draw upon a tradition of discourse analysis known as ethnography of communication (Hymes, 1964) to frame the negotiation as a communicative event and examine the discursive practices of "doing mathematical understanding." The tools of conversation analysis are useful to illuminate those discursive practices such as producing an example, drawing on identity, reporting a narrative or providing a justification that the children used as they referenced mathematical content throughout the negotiation. Thus, as a second step in the current study, I used sequential analysis to show how these children positioned themselves vis-à-vis the statements made by others and how they invoked six different sociomathematical norms by the *ways* they produced those positions. Therefore in this chapter, I provide evidence showing how children display their orientation to the significance of norms during mathematical negotiation (e.g., how they *acknowledge* or *resist* those normative expectations). Furthermore, I use discursive psychology to show how children used the invoking of sociomathematical norms to support the rhetorical organization of their talk.

Chapter 6 is the third of the three chapters that represent the findings of the current study, corresponding to Step 3 in the research design. The chapter is titled "*Algebraic reasoning as social practice in the experience of young children: A discursive psychological/ multimodal perspective."* The main findings of this study are first, to provide a rationale for redefining algebraic reasoning in the early years to include multimodal aspects of communication and second, to establish young children's capacity to draw on sociomathematical norms as resources to support their attempts to make meaning of unfamiliar mathematical content. Developing effective resources for algebraic reasoning during childhood is critical to gaining a clear understanding of more complex mathematical concepts later on (Kaput, 1999). However, it is widely recognized that transitioning from arithmetical to algebraic reasoning is a difficult process for many students. In this chapter I take the discursive psychological position that algebraic reasoning might be fruitfully considered a social rather than a cognitive practice, in order to provide insights into those difficulties. The data set considered here is a series of interactions from across six weeks of N:Countr Phase II while the children carried out their own inquiry into the function of the square root symbol. Significantly, the analysis exemplifies some of the *communicative* processes involved in learning-as-participation.

In Chapter 6, I take a broad view of discourse that includes multimodal features of mathematical language including verbal, gestural, visual and numerical aspects of communication (Clark, 2004; Kendrick, in press; Noss, Healy & Hoyles, 1997). This approach helps to make thinking visible, so that the social practices of the group are implicated in "doing algebraic reasoning." In contrast to the previous chapter where the negotiation examined lasted six minutes, the inquiry in this chapter played itself out over six weeks. Furthermore, the inquiry "ended" without the group ever reaching a consensus, perhaps due to time constraints or the challenging nature of the mathematics under consideration. Nevertheless, six children participated extensively and all the children contributed something to the square root inquiry. Therefore I uncover evidence for how children draw upon sociomathematical norms as resources to support their participation by showing how they use the familiar formulation of an argument to approach unfamiliar mathematical content as in inquiry. Furthermore, I use discursive psychology and multimodal analysis together to compare multiple versions of the same conjecture, thus strengthening the warrant for the resulting interpretation.

The final chapter (7) integrates the findings of the three results chapters, providing a comprehensive answer to the research question: *How do young children participate in the negotiation of sociomathematical norms in inquiry-based settings?* The main findings of the current study elaborate how young children use the structure of spoken language to support their participation in mathematical negotiation. Each of the three linked studies contributes something to that conclusion. During Chapter 7, I re-visit the milestones in the learning journey represented in the dissertation and finish with some implications for research and educational practice; showing how sharing the authority for *mathematical knowing* between researcher and children afforded meaningful participation, illustrating how increasing the visibility of unratified bystanders in transcriptions afforded a recognition of how children *built their participation upon previous contributions* through features of production and recipient design and highlighting multimodal expressions of algebraic reasoning as a *collective accomplishment*.

In Chapter 7, I also argue that it is essential that educators and researchers expand our current understanding of what it means *to know* mathematically, in order to support the meaningful participation of the younger learners and I highlight the ways the children in the

N:Countr mathematics research group made positive, significant contributions to the negotiation of sociomathematical norms during our sessions together.

To summarize the organization of the document: I seek to address two areas of inadequacy in the research literature with a three-step analysis of children's participation in an inquiry-based mathematics setting for the purpose of elaborating some details regarding *how* the children used the structure of spoken language to support their participation in mathematical negotiation (see Figure 1.2).

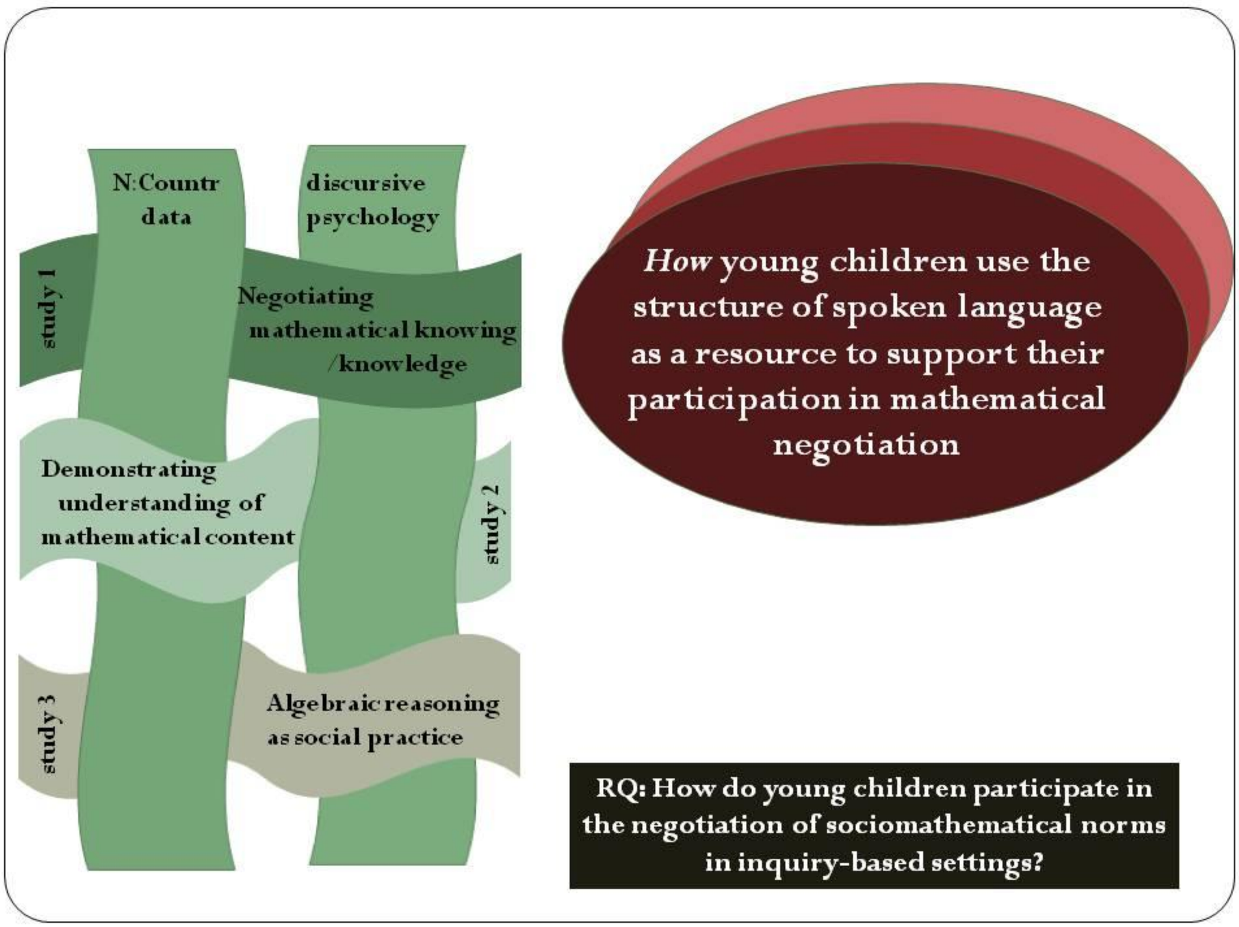


**Figure 1.2 Visual representation of the layers of meaning-making in the dissertation**

The three studies reported in Chapters 4, 5 and 6 use a common theoretical framework (discursive psychology) and investigate various aspects of a common data set (N:Countr Phase II mathematics research group). They provide evidence that young children have the capacity to:

1. draw upon sociomathematical norms as resources to produce *ways of participating* that co-ordinate with each other;
2. use the invoking of sociomathematical norms to produce their interpretation of the meaning of mathematical content during negotiation; and
3. draw upon sociomathematical norms as resources to support their attempts to make meaning of unfamiliar mathematical content.

In hindsight, I am able to retrace my steps through the data analysis and identify two co-ordinated motifs of significance that stand out from the background of the individual studies. These were operationalizing the notion of learning-as-participation and illustrating the potential for using discursive psychology to interpret the mathematical interactions of young children. They are illustrated in Figure 1.2 as layers of significance tucked behind the main finding. Although significant, these two areas deserve substantially fuller treatment than I can give them in the dissertation and therefore remain subordinated to the main conclusion. Nevertheless, while each of the reported studies highlights different aspects of significance, the co-ordination of all three studies provides a robust qualitative "triangulation" (Mathison, 1988) allowing me to present a rich and complex picture of the capacity of young children to contribute to the negotiation of sociomathematical norms.

Overall the dissertation is designed not just to re-present the learning journey the children and I shared, but to hold that journey up to scrutiny so that we might learn from it. To further that aim, in the writing here I take care to introduce those familiar with discursive psychology to the interactionally rich settings of inquiry-based learning in early childhood mathematics. I also foreground the discursive arenas of sociomathematical norms and negotiation for those more familiar with the informal interactions of early childhood education and I situate the analytic practices of applied linguistics within early childhood for those more familiar with mathematics education. These actions constitute my attempts to render that which might be unfamiliar nevertheless accessible.

Making the unfamiliar accessible represents the quintessential role of the teacher during an inquiry and is the ultimate role I take as author here. It is my hope that with a renewed recognition of young children's capacity to contribute to classroom practice, researchers and educators alike will be better equipped to develop learning environments that permit *all* children to participate more fully in mathematics.

# Chapter 2: Background on sociocultural theory, mathematics education reform and discourse

The three distinct studies that represent the results in the main body of this dissertation (Chapters 4, 5 and 6) as well as the overarching research question (*how do young children participate in the negotiation of sociomathematical norms in inquiry-based settings?)* are influenced by sociocultural approaches to mathematics education. They are set within the practices of mathematics education reform and employ linguistic analyses of mathematical discourse from a discursive psychological perspective in order to understand the ways in which norms are developed and deployed in learning environments. In this chapter, I consider each theme in turn. The three main sections might best be understood within a photographic metaphor of a lens (Lerman, 2000). Viewed first (Section 2.1) with a wide-angle lens, the investigation is broadly situated within sociocultural theory. Focusing next (Section 2.2) on mid-level social processes (groups and communities as opposed to either society or individuals) I describe the research landscape in view as it informs the research question. This relates to the confluence of social organization theories that provide vocabulary to describe social processes and mathematics education reform that foregrounds social participation for learning and a particular expression of "classroom" called "inquiry-based learning." Continuing to narrow the lens (Section 2.3), I focus on discourse as a social practice between individuals, outlining discursive psychology in some detail and providing a rationale for its use in approaching spoken mathematical discourse. I finish this chapter with a clarification of what I am *not* attempting to accomplish (Section 2.4).

## 2.1 Sociocultural theory

My use of the term ‘sociocultural theory’ here is broad and inclusive. Several research traditions use the term explicitly (cf. Forman, 2003) and others use the concepts implicitly within variations of what might be called sociocultural studies. These can be identified by their common focus on learning as participation in practice rather than acquiring knowledge (Ernest, 2010; Lave & Wenger, 1991; Sfard, 2006; Wenger, 1998). They also share a common commitment to remain mindful of the historical, cultural, and situational factors that influence learning (Forman, 2003; Kieran, Forman & Sfard, 2001).

The main concepts of the theoretical paradigm originate with the writings of Soviet psychologist Lev Vygotsky (see for example, 1978): “It was the work of Vygotsky that gave this area of study [sociocultural theory] its theoretical framework, its methodology, its unit of analysis, and basically its raison d'être” (Vasquez, 2006, p. 57). Many scholars have outlined Vygotsky’s influence on the development of sociocultural theory (see for example Forman, 2003; Lerman, 2000; Sfard, 2006; Vasquez, 2006; Wertsch, 1991) so my presentation here will be brief. Two situational factors influenced the way Vygotsky’s ideas have been developed. First, he died young, leaving many promising notions that were not fully explored and therefore remained to be developed by others (Lerman, 2000). Second, due to the isolation imposed on Soviet academia by various political regimes during the 20th century his writings did not become available to the West until long after his death (Forman, 2003). Depending on who you read, Vygotsky contributed two (Vasquez, 2006), three (Werstch, 1991) or four (Lerman, 2000) main ideas to theories on learning. I synthesize them here using Lerman’s (2000) framework: “The priority of the intersubjective; [processes of] internalization; [the role of] mediation [in learning]; and the zone of proximal development” (p. 34). The broad field of sociocultural studies exists as it does because many different scholars have taken up Vygotskian ideas, diverging in their applications and emphases (Sfard, 2006). Vygotsky's emphasis on the priority of the intersubjective is the main application of his work to the current study.

A number of summaries exist regarding the historical development of the use of sociocultural theory in educational research (cf. Ford & Forman, 2006) and in mathematics education research (cf. D'Ambrosio, 2004; Forman, 2003; Lerman, 2000; Schoenfeld, 2006). Each of them in some way refers to a remarkable confluence of five factors in the 1980s: (1) the influence of the growing practice of interdisciplinary scholarship (Steiner, 1985); (2) the importance of cross-cultural studies that used ethnography to engage with people within their own communities (Bishop, 1988a, 1988b; Carraher, Carraher & Schliemann, 1985; Kieran et al., 2001); (3) a sense of economic urgency regarding the weak mathematical abilities of students in American schools (National Commission, 1983); (4) the concurrent availability of a new theoretical framework with which to address the social context of learning (Vygotsky, 1978); and (5) Lakatos' (1976) reformulation of what it meant to know mathematics. For the purposes of this chapter, I focus on how the use of sociocultural theory in mathematics education research facilitates the conceptualization of school mathematics as discursive practice and as culture.

### 2.1.1 Mathematics as discursive practice

A rise in interdisciplinary scholarship during the 1980s meant that sociology, linguistics, cultural psychology and anthropology began to influence the design of studies in mathematical learning both theoretically and methodologically (Steiner, 1985).

Research findings in fields of specialization such as linguistic anthropology established the importance of communicative processes for learning, viewing language as "a system of symbolic resources designed for the production and interpretation of social and intellectual activities” (Ochs, 1996, p. 407). Scholars in this field observed that during everyday communication, speakers embed “messages about their messages” within their interactions, using indexical meanings that resonate with particular social or cultural histories (Agha, 2007). An example of this is when a parent says to a fussy child: “You’re ready for bed now.” Without actually saying as much, the adult has claimed both the expertise for interpreting behaviour and the authority to require compliance. Through such forms of indexical distinction, values and social order enter language, in this case, validating parental dominance and expecting child compliance. If a stranger were to make the same comment to our fussy child, our response (depending on our cultural context) might be quite indignant. Our reaction would then give evidence of our cultural understanding: in this case, the assumption would be that parents have the right to claim such a position of expertise and authority, strangers do not.

While linguistic anthropologists were affirming the linguistic construction of social realities, mathematics philosophers were reconceptualizing the learning of mathematics as *doing* rather than *acquiring* (Lakatos, 1976). This afforded radical changes in teaching practices. The National Council of Teachers of Mathematics (NCTM) produced a curriculum document concerning what *should be* valued in mathematics teaching and learning, in response to the perceived failure of mathematics instruction (NCTM, 1989). Its “social goals for education” drew heavily from the social and cultural concerns emerging from research and put more emphasis on mathematical process than content. The flurry of teaching experiments that followed these calls for reform uncovered the new significance of spoken language in classroom practices (Lampert, 1990; Wood, 1993; Yackel et al., 1991). In particular, scholars found that the spoken interactions of the classroom took on many aspects of everyday communication such as the indexical meanings cited above.

Researchers implemented a series of year-long classroom teaching experiments intended to support and document grade two and three teachers' revisions of their mathematics teaching (Wood, Cobb, Yackel & Dillon, 1993). They used the concepts of reflexivity and the interactive constitution of meaning from symbolic interactionism to account for the linguistic complexity enacted by teachers and students in these reform-oriented classrooms. Other scholars noted that conversants within a particular classroom negotiated meanings that came to be taken-as-shared (Herbel-Eisenmann, 2000). Taken-as-shared refers to aspects of uncertain intersubjective understanding (*do we mean the same thing when we use the term "half"?*) which even though those aspects are never verified are discursively oriented to *as if they were* true or at the very least as if they *could be considered true* until contradictory information is received. In other words, taking them to be true *(acting as if we* ***do*** *mean the same thing when we use the term "half")* is sufficient for the purposes at hand.

Voigt (1995) addressed the same issue from the specifics of mathematical negotiation. He based his definition of mathematical negotiation on the symbolic interactionist assumption that objects are ambiguous or open to various interpretations. It is through negotiation of those interpretations, Voigt argued, that classroom conversants come to share intersubjective meanings. However, it is not that speakers set out to negotiate in order to share knowledge. His position is the reverse: "Mathematical meanings are only taken as shared when they are produced through negotiation" (p. 172). He used transcripts of spoken interaction from classrooms to show how students working together and sometimes students with a teacher came to share a solution that could not be attributed to any one conversant alone. For him, these interactions exemplified negotiation. What he noticed, when looking at these examples more closely, is that typically the participants operated on the basis of a tacit agreement, without ever confirming during interaction that they actually shared common knowledge (Edwards & Mercer, 1987). These tacit agreements were organized around interactional patterns and what he came to call "sociomathematical norms."

Walkerdine (1988) exemplifies a commitment to seeing mathematical discourse as action in her conceptualizations of early mathematics learning. Outlining the relationship between signifiers, signified and signs, and observing regularities in "chains of signification," she noted that the transformation from home mathematics to school mathematics practices "appeared to be produced by a gradual series of shifts of signification" (p. 186). In Voigt's and Herbel-Eisenmann's terms, a set of values concerning what it meant to "do school mathematics" came to be taken-as-shared between the students, their parents and teachers. The *coming to be* describes what Voigt calls the process of negotiation and the *set of values* refers to the sociomathematical norms.

A traditional sociological approach to norms defines them as "statements that regulate behaviour" (Horne, 2001, p. 4). When individuals come to value the specified behaviour regardless of the absence of external sanctions, these assertions of the ideal are said to be internalized. In such an approach, the object of study might be exploring how children's understanding of social convention promotes behavioural self-regulation and how this, in turn, supports learning. This resonates with Yackel and Cobb's (1996) initial presentation of sociomathematical norms as regulating mathematical argumentation (although it is never clearly outlined in their paper exactly *how* this happens) and therefore influencing opportunities to learn.

In keeping with my discursive psychological framework, my use of the term "norm" in this paper differs from a sociological approach, aligning itself more with Voigt (1995) and an ethnomethodology of communication (Agha, 2007 ; Baker, 2000). Therefore, I do not concern myself with identifying ideal representations of mathematical norms or considering how they change through negotiation. Instead, I look for ways that norms are used by children (also called "deployed" or "invoked") during mathematical discussions in order to accomplish social actions.

This shift in research focus, away from seeing mathematical discourse as representation towards a commitment to seeing it as action, represents the core of understanding mathematics as discursive practice. Therefore, my analyses in this dissertation focus on the patterns and regularities in social interactions as they evolve during mathematical negotiation between and with young children. These regularities in discursive practice enhance the possibility for learning: "Sociomathematical norms are not obligations that students have to fulfill; they facilitate the students' attempts to direct their activities in an environment providing relative freedom for interpreting and solving mathematical problems" (Voigt, 1995, p. 196). During mathematical discussion a set of values regarding mathematical activities comes to be taken-as-shared. These "norms" then become available to analysts, just as they are available to participants, through the presence or absence of explicit markers or through the implicit discursive actions that enact, elaborate or violate them (Baker, 2000; Garnica, 1981).

Taken together, these sociocultural lines of inquiry support an understanding of mathematics as discursive practice. They also open that practice to examination, allowing us to seek evidence for the change implicated in *learning* (Seeger, 2001).

### 2.1.2 Mathematics as culture

In viewing mathematics practice as a cultural phenomenon, Bishop (1988a) noted, "The thesis is therefore developing that mathematics must now be understood as a kind of cultural knowledge, which all cultures generate but which need not necessarily 'look' the same from one cultural group to another" (p. 181). Cross-cultural ethnographic studies introduced a way of understanding mathematical learning outside the formal classroom setting, and contributed new vocabulary and concepts including ethnomathematics (D'Ambrosio, 1985) and mathematical enculturation (Bishop, 1988b). In time, these ethnographic research lenses were turned toward classrooms. This afforded an understanding of classrooms as cultural communities, bringing awareness that any investigation into learning must recognize the role that the microculture of the classroom plays in both offering and determining possible knowledge (Voigt, 1995). With that awareness, classrooms began to be framed in research as social settings where human beings were engaging with each other and/or materials and/or conceptual objects and making sense of what was happening: where students and teachers were *participating* together (Sfard, 2006; Wenger, 1998).

Looking through a sociocultural lens at classroom communities as microcultures afforded inferences about cognitive processes that were not available through other perspectives. For example, scholars were now in the position to consider alternatives regarding the function of rational thought. A position emerged in which it became simply one of several features of socially organized activity as opposed to its previously privileged position as the guiding principle of learning (Forman, 2003). From such a position, "thinking" functions as a cultural resource of classroom communities to which members may have differential access and mathematical conventions, such as the numbering system, become cultural tools (Cobb & Bauersfeld, 1995). In the same way, sociomathematical norms become resources of the discursive environment created within the classroom. Every interaction exhibits an orientation to those underlying normative assumptions (Herbel-Eisenmann, 2000) and their negotiation allows participants to establish a basis for recognizing activities as mathematics and then carrying out joint mathematical activity (Wood, 1993).

Thus, the 1980s and 1990s saw particular lines of inquiry develop in mathematics education research. They produced the complementary notions of mathematics as discourse and mathematics as culture. In response to this scholarship and in reflexive relationship with it, new school practices were developing that would become known as “reform” mathematics practices (Cobb & Hodge, 2002). This is the focus of the next section of background in this chapter and serves to focus our attention away from macro-level theories towards the more mid-level social interactions of communities and groups.

## 2.2 Mathematics education reform

The concept of sociomathematical norms arises from an exploration of mathematics education reform-oriented practice that emphasizes student participation and discourse in classroom communities (Yackel & Cobb, 1996). These inquiry-based learning communities are an educational context in which the negotiation of norms makes sense. Therefore, in this section, I will look at literature surrounding notions of inquiry-based mathematics settings, including broader sociocultural theories of learning as participation in practice (Lave & Wenger, 1991; Wenger, 1998) and research specific to norms in the mathematics classroom.

### 2.2.1 Learning as participation in practice

The Communities of Practice (CoP) theory (Wenger, 1998) is, above all else, a social theory of learning. It provides a vocabulary with which to describe the processes by which sociomathematical norms become classroom cultural resources.

#### 2.2.1.1 Sociocultural theories of learning

As an extension of situated learning theory (Lave & Wenger, 1991), Wenger (1998) developed CoP by explicitly examining learning outside of the educational institution. Intended as a way of understanding learning “from the bottom up” the theory provides a systematic vocabulary to inform discussions about lived experience. Wenger began with two basic observations: people generally come together in groupings to carry out activities and those groupings tend to be characterized by mutual engagement, joint enterprise, and a shared repertoire (Barton & Tusting, 2005). These are not equivalent in form to institutions such as school. Rather, he suggests using the vocabulary of CoP to explain the characteristics or function of a learning community. In other words, to the extent we are mutually engaged, we function as a community of practice. To the extent we share a common endeavor we function as a community of practice. And, to the extent we create a shared set of resources (e.g., language, style, routines) and use them to express our identity as group members, we function as a community of practice. It is important to recognize that these are not definitional criteria. Rather, they "characterize practice as a source of community coherence" (Wenger, 1998, p. 77).

Wenger (1998) identified the duality of participation in a community of practice and the reification involved in that practice as fundamental social processes of learning. Both are implicated in the negotiation of meaning and for our purposes here, the negotiation of sociomathematical norms. Within this perspective negotiation is considered to be a social process between human agents. The concept of reification is described as both a process and the product of that process:

> I will use the concept of reification very generally to refer to the process of giving form to our experience by producing objects that congeal this experience into 'thingness'. In so doing we create points of focus around which the negotiation of meaning becomes organized … Any community of practice produces abstractions, tools, symbols, stories, terms and concepts that reify something of that practice in a congealed form.
> (Wenger, 1998, pp. 58-59)

Reification involves treating abstractions as if they were concrete, in the same way that conversants operate within taken-as-shared meaning without ever confirming that shared understanding.

In the case of sociomathematical norms, the negotiation of meaning becomes organized around a taken-as-shared set of values concerning the ways in which mathematical objects can be properly used in discourse. For my purposes here, when an interaction shows evidence of an orientation to sociomathematical norms, then we can say the norms have been and are being reified through the interaction.

Borrowing the concept of a semiotic landscape of resources from literacy studies (Barton & Hamilton, 2005) I argue here that the reification of sociomathematical norms describes the process by which they become available to be used as resources by participants in mathematical discourse. Wenger (1998) identified four key features of reifications: succinctness, portability, durability and focusing effect. When norms are viewed as reifications, then I argue that these features serve to establish their usefulness and accessibility within a local context. This resonates with Voigt's (1995) treatment of the cultural tools, tasks and symbols of the mathematics classroom as points of focus for negotiation. For him, these are the "objects of the classroom discourse" (p. 163) and serve as resources for meaningful participation.

Wenger (1998) was referring to informal learning, so his examples of reifications are not intentional artifacts created by teachers or those representing formal domains of knowledge. Likewise, my examples represent reifications that are interactionally constituted by students and "teacher" participating in the practices of mathematics. A spoken utterance (e.g., "I do math everyday" given in support for mathematical validity – see Example 5.2 in Chapter 5) may not be highly durable, but *is* portable (i.e., the statement can travel with the speaker to many different contexts, unlike the workbook – another reification – that she uses to enact it). Also, because the speaker's reification refers to experiences that listeners can identify with, it serves to focus negotiation. Listeners have their own cultural resources from which to evaluate the inferred question (*"Is this satisfactory to denote a mathematically valid response?"*). The next utterances will be interpreted by all who hear them as if they orient to that first one. Therefore, "I do too" would accept the norm (valuing experience) but challenge the initial speaker's unique claim to authority based on it. On the other hand, "That doesn't mean it's right!" challenges the norm by invoking an over-riding characteristic of accuracy. Listeners would interpret that statement based on their understanding of how the world works – it *is* possible to do quite a lot of math, but incorrectly.

As this hypothetical interaction continues, I would interpret the ongoing discussion as a mathematical negotiation. What is at stake here is what this group will tacitly agree to value regarding a valid mathematical response, one based on experience or accuracy. This does not need to be a binary – multiple other possibilities may be presented through the interactions. The negotiation revolves around "whose claims concerning what issues will be temporarily honoured" (Goffman, 1959, p. 9f). This sort of interaction replicates the activities of mathematicians as they investigate and expand formal questions and conjectures, yet for learners, the negotiation is situated within a real-life problem, both of which point to elements of inquiry-based learning (Artigue & Blomhøj, 2013).

#### 2.2.1.2 Inquiry-based learning

Inquiry-based learning has been traced to philosophers that influenced Dewey (1938), although he is generally credited as formulating the pedagogical approach (Artigue & Blomhøj, 2013; Schoenfeld & Kilpatrick, 2013). It first found acceptance in the practices of science education and recently migrated to mathematics as well (Bruder & Prescott, 2013). It has been widely implemented in European educational contexts, and has experienced considerable interest in North America (Schoenfeld & Kilpatrick, 2013).

I first met inquiry-based mathematics as a comparison with Early Childhood approaches to literacy instruction, called "emergent mathematics" (Baker, Semple & Stead, 1990; Stoessiger & Wilkinson, 1991). This pedagogy foregrounds the meaning-making activities of students, learning as a process that happens over time and the importance of building on prior understanding. It requires teachers to provide contexts which support and facilitate inquiry. Researchers in an Australian context wondered "Could mathematics be taught by a process analogous to emergent literacy, and if so, would this lead to improvements in children's learning?" (Stoessiger & Wilkinson, 1991, p. 3).

These two perspectives came together for me only during the secondary data analysis used for the current study. Therefore, while some of my practice during N:Countr might seem unusual from a mathematics education perspective my actions at the time were informed by an Early Childhood pedagogical perspective. For example, at one point I introduced an internet printout that duplicated versions of the children's drawings of trajectory that evolved from their play with marbles, ramps and receptacles. This printout included mathematical representations of the quadratic formula as suggestions for how one might determine where the projectile would land, the exact real-life problem I saw the children dealing with in their play. I set up a classification activity for the children that might help them make sense of the mathematics (at the most basic level – organizing what we know and do not know in this representation). This created some conflict in the group and led to an extended negotiation regarding the children's understanding of the meaning of the "equals" sign (see Chapter 5). While unconventional, the introduction of that formal mathematics set-up the beginnings of an extended investigation initiated by the children into the function of the "square root" symbol (see Chapter 6). This is one way that the role I played as an Early Childhood Educator rather than a mathematics educator was central to the developing negotiations.

Inquiry-based learning environments make relevant the negotiation of sociomathematical norms precisely because they focus on children's learning as participation in practice. This is contrasted with traditional approaches to teaching mathematics, where learning focused on acquiring knowledge. Many researchers have contrasted these different pedagogies and suggested various implications for student learning (Bruder & Prescott, 2013). For example, Boaler (1999) compared the two approaches and argued that the sociomathematical norms were not the *context* for the students' learning, they *constituted* that learning. Thinking for oneself was afforded by one classroom environment and constrained by another. Students in the traditional learning environment had come to rely on a number of nonmathematical cues (Schoenfeld, 1985) to support their successful participation in textbook exercises. They had noticed that practice always followed demonstration. Also, subsequent exercises always required more difficult methods. The regularities in this informational system enabled the students to anticipate and choose an appropriate mathematical method.

Therefore, being attuned to these affordances in the classroom environment facilitated success. In fact, the more proficient a student became in noticing and using these cues the more "successful" he or she was, "…without developing any deep understanding of mathematics or an awareness of the connections within the broader mathematical domain" (Boaler, 1999, p. 266). As a result, when faced with a situation that deviated from the "norm", such as an examination, these otherwise "successful" students floundered. Even if they knew how to perform the mathematical procedures required, they were at a loss to choose an appropriate method. On the other hand, Boaler (1999) found that students from the inquiry-based learning environment had been taught few of the formalized mathematical methods required by the examination. However, the students were able to compensate when in the examination setting, because they had a deep understanding of the methods they knew, and they expected to be flexible and adapt.

### 2.2.2 Norms in the mathematics classroom

The studies that initiated an awareness of sociomathematical norms (Voigt, 1995; Yackel & Cobb, 1996) emerged at the intersection of the production of a philosophical document proposing a vision of what should be valued in mathematical learning (NCTM, 1989) and an interest in radical constructivist theories of knowledge (von Glasersfeld, 1984; 1987). The National Council of Teachers of Mathematics (NCTM) recommended drastic changes in teaching practices: "For the first time in curricular history, a major curriculum document gave as much attention to the *process* aspects of mathematical performance as it did to the mathematical content to be covered in the curriculum" (Schoenfeld, 2006, p. 488 italics in original). This focus on process resonated with the radical constructivism concurrently being explored (Cobb, Yackel & Wood, 1993) and led to the development of the Project classrooms. As previously mentioned, the setting developed by Yackel and Cobb (1996)[1] involved classroom teaching experiments intended to support and document grade two and three teachers' revisions of their mathematics teaching (Wood et al., 1993).

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[1] This study is cited far more than any others regarding sociomathematical norms. However, Voigt (1995) was the first to coin the term.

#### 2.2.2.1 The original studies on sociomathematical norms

A large number of studies have been published based on research carried out in those reform-oriented classrooms. Four assumptions encapsulate the change required of teachers and inform the interpretation of the research observations in those original studies:

1. Teachers should provide students with instructional activities that will give rise to problematic situations.
2. Children's actions, which are logical to them but may be irrational from an adult perspective, should be viewed as rational by the teacher.
3. Teachers should recognize that what seem like errors and confusions are children's expressions of current understandings.
4. Teachers should realize that substantive learning occurs in periods of conflict, confusion, surprise, over long periods of time, and during social interactions. (Wood, 1993, p. 16)

Wood (1993) describes the surprise the research team experienced when they observed the effect of the change in teaching behaviour on classroom interaction. Their theoretical perspective had not prepared them for the interactional complexity that emerged when teachers assumed child competency and agency. The research team was challenged to more intentionally incorporate social and discursive perspectives in their theory (Cobb et al., 1993) in order to account for that complexity.

In these contexts, it became apparent to the research observers that the intersubjective negotiation of meaning was fragile and continuously at risk of breaking down. Voigt (1995) noted that regularities produced through the interaction functioned to minimize the risk of that breakdown: those discursive regularities therefore facilitated participation. For example, he observed that at different times, discussion occurred at two different levels; there was talk about doing mathematics (e.g., “All right now, how did you figure this out?” p. 194) and there was talk about talking about doing mathematics (e.g., “When you’re working together what are you going to have to be doing with your partner?” Yackel et al., 1991, p. 399). In these interactions, teachers communicated differential expectations and responsibilities regarding student participation. Their students oriented to these expectations, with the resulting regularities produced as social norms (Yackel et al., 1991). The environment allowed students freedom to interpret and solve mathematics problems in ways that were meaningful to them. The students in turn were expected to, and held responsible for, explaining their solutions, sharing strategies, listening to each other and resolving conflict through respectful discussion.[2]

It was not long before the researchers began looking for what was mathematically significant in these interactions. The students were learning how to participate in inquiry-based settings, but were they learning mathematics? And if they were, what was the relationship between that learning and the social interactions? This was addressed in Voigt’s (1995) theoretical paper which was designed to investigate how intersubjectivity was specifically accomplished in mathematics discourse.

Using transcriptions of interactions in the Project classrooms, Voigt observed that teachers and students exhibited a thematic coherence in their discourse as they negotiated the meanings of mathematical abstractions. Along social lines he identified the *elicitation pattern* and the *discussion pattern* as relevant to the Project classrooms. In identifying what was specifically mathematical about the interactions, his analysis produced three thematic patterns of

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[2] A large number of studies came from this research and developed the concept of social norms. See Wood (1993 note 1) for a list of ten published up to that point.

interaction: *direct mathematization* (where “a story or a picture is interpreted as a specific calculation problem” p. 185), *counting materials* (where “signs are interpreted as representations of concrete materials” p. 188 which are then quantified) and *calculating* (where “signs are interpreted as representations of numbers … [with a subsequent] application of arithmetical rules” p. 188). He used the term “sociomathematical norm” to describe the set of values that become taken-as-shared during negotiation of specifically mathematical abstractions. Once these norms were constituted and stabilized within the classroom microculture, Voigt (1995) understood that they facilitated the students’ self-regulated direction of their own mathematical activities. In other words, the children were able to use them as resources to support their own participation. He described ways in which the interactional constitution of sociomathematical norms supported student learning and used the term *evolution of practice* in the same way others have since used *reification* (Wenger, 1998) to describe those processes of negotiation by which the norms became active within the classroom discourse.

It is these processes of negotiation that are important to this study. Voigt (1995) observed classrooms nearly twenty years ago, during a time when teaching practices were changing. One of the most illuminating facets of his findings is that he noticed old ways juxtaposed with new ways of teaching. Both the elicitation pattern (where teachers fish for the correct answer) and the thematic pattern of direct mathematization (where the process of solving word problems is understood to be translating them into number sentences) were problematized within his study, as belonging to the old ways of teaching. Furthermore, the thematic pattern of calculating (some might attribute this to the old ways of applying the correct formula) referred to *any* use of arithmetical rules within the conventions of the number system, not just the standard algorithm.

By paying close attention to how the practices of teaching were done, Voigt (1995) identified processes of change using the teachers' perspectives. In the current study, I do the opposite: I examine the "practices of learning" in order to identify processes of change using the children's perspectives. I am interested to know about how children participate in the negotiation of sociomathematical norms. Norms are situationally specific and I therefore do not expect to observe the same norms as others. However, the discursive processes of negotiation are enduring and documenting the practices of knowing, or specifically how knowledge claims are produced or negotiated over time may also highlight processes of change, this time from the children's perspectives.

#### 2.2.2.2 Current applications of the concept of sociomathematical norms

While Voigt's (1995) paper was a theoretical discussion, others have applied that theory in order to name specific sociomathematical norms. I focus on three of the most influential here.

Yackel and Cobb (1996) worked within the Project classrooms and identified interactions during which teachers and students came to share notions of what it means that a solution is *mathematically valid* (e.g., it is *not* valid to say an answer is true because I have lots of experience doing math, rational argument is valued over experience), *different* (e.g., how will we determine if 4x8 is a different solution than 8x4?), *efficient* (e.g., on what basis will we choose multiplication as more efficient than repeated addition?) or *elegant* (e.g., what does it mean that this represents a *simple* solution path?).

Kazemi (1998) compared the practices of 23 upper elementary teachers in reform-oriented classrooms in order to understand what it means to "press for learning" or encourage students to think conceptually. While she found similarities in the social norms observed in these inquiry-based classrooms, there were differences in the sociomathematical norms. Specifically, she identified four sociomathematical norms that supported students' conceptual thinking. They were:

1. Explanations consisted of mathematical arguments;
2. Errors afforded opportunities to learn;
3. Mathematical thinking involved understanding relations between strategies;
4. Collaborative work involved individual accountability.

In a third study, Herbel-Eisenmann (2000) defined norms in terms of the taken-as-shared values concerning the expectations, roles and rights of teachers and students. She compared the discourse in two reform oriented grade eight classrooms in order to investigate how norms are both embedded in and carried by the discourse. Like Kazemi (1998), she also found that while the social norms active within the two classrooms were comparable, the sociomathematical norms differed. Herbel-Eisenmann (2000) was able to elucidate the form and function of several sociomathematical norms, distinguishing between those associated with the teacher (e.g., *authority for knowledge*), those associated with the students (e.g., *the role of previously developed common understanding*) and those associated with mathematical content (e.g., *the elegance or simplicity of a solution*). In so doing, she highlighted the reflexive nature of language as communication in mathematics classrooms.

#### 2.2.2.3 The importance of the current study

When learning is understood as participation, teachers' assumptions vis-à-vis their own and students' expectations, roles and rights are critical to positive learning outcomes. Teaching experiments carried out in the Project classrooms (Yackel & Cobb, 1996) and other mathematics reform-oriented classrooms have done much to illuminate the relationship between teachers' perspectives and positive learning outcomes. In particular, these ethnographic studies on reform-oriented classroom practice identified the function of sociomathematical norms or the taken-as-shared set of values concerning mathematical activities as crucial to creating an environment that supports participation and therefore provides opportunities to learn.

As reform-oriented mathematics or inquiry-based learning becomes more prevalent in the practices of schooling, listening to and attempting to understand the perspectives of all participants (including young children) will be essential to creating spaces that acknowledge and engage every child. Five to seven year olds are verbally articulate while at the upper end of the period considered “early childhood” (Bredekamp, 1996) and still new to the practices and expectations of schooling (Mishler, 1972). Yet they come to school with a rich background of experiences that they draw upon as resources to help them make sense of what will become *mathematical practice* in their new experience (Walkerdine, 1988). Each new experience in school will influence that repertoire of resources. This study provides critical baseline data concerning young children’s participation in the negotiation of sociomathematical norms. This can inform the development of learning environments that promote all children’s sustained, successful engagement with mathematics.

If mathematics involves discursive practice and learning mathematics comes through participating in that practice, then a final element in this argument that requires clarification is the term “discourse.” Exactly how I am using this term *discursive practice* is at the core of the next section of this chapter and serves to narrow the focus onto social interactions between individuals.

## 2.3 Discursive psychology: discourse, data and mathematical negotiation

There are at least two linguistic approaches to the study of discourse in the context of learning mathematics (Sfard, 2012b). In one, language is generally considered to function as a tool in the making and sharing of meaning in socially constructed ways (MacMillan, 1998). In another, the particular vocabulary and grammar of mathematics is described as a register that students construct in the course of learning (Chapman, 1993). These could be loosely construed as the *language in mathematics* approach versus the *mathematics in language* approach. Discursive psychology (DP) takes the first approach, considering discourse to be a tool that affords the making and sharing of meaning in socially constructed ways. This makes relevant a linguistic analysis of the interactions involved in the negotiation of sociomathematical norms. DP also provides a position from which to examine discursive practice from the participant's frame of reference (Edwards & Potter, 1992), an over-the-shoulder perspective that presents an ecologically strong position for exploring learning environments.

DP combines the assumptions of ethnomethodology with the analytic tools of conversation analysis to study traditionally psychological topics (Barwell, 2003). In mathematics education research, these topics might include knowing, understanding, attitudes or beliefs. Any of these *may be* salient to a participant during a negotiation. However, it is difficult to determine which, if any, *are* relevant without *getting inside the participants' heads*, so to speak. The physical impossibility of this complicates analytic interpretation for cognitive psychologists, who attempt to interpret social interactions like negotiation through mental representations of notions such as understanding.

Garfinkel (1967) developed the main tenets of ethnomethodology in order to explore how topics are brought into play during everyday conversation. He demonstrated how participants understand each other during and through interaction, *how they display this understanding to each other* and therefore how they produce the shared social order in which they live. Discursive psychology draws upon the ethnomethodological commitment to spoken interaction and focuses attention on how psychological topics, such as understanding, are treated by participants. Analysts can avoid the subjective complications of interpreting participants' mental constructs of notions such as understanding by focusing instead on

1. how speakers display their recognition of a co-conversant's understanding,
2. how they respond to others' displayed assumptions of their own understanding and
3. how speakers construct their statements in ways that anticipate various potential interpretations by others.

The main distinction between cognitive psychology and discursive psychology then, is that the latter rejects the assumption that people "possess underlying cognitive representations that are expressed in talk" (Edwards, 1993, p. 208) that might be then amenable to being studied. Discursive psychologists assume these notions are constructed socially. Thus, rather than asking "what do children really think?" discursive psychologists study "what counts for participants as, for example, understanding, thinking, and remembering" (Edwards, 1993, p. 219). This is what I refer to as an over-the-shoulder analytic perspective.

According to DP, people construct their thinking as they talk (Edwards, 1993). This stands in contrast to other sociocultural theories, which assume that talk reveals underlying structures regarding either *what* or *how* people think.[3] DP takes psychological topics such as thinking, knowing or understanding and "respecifies" them as social actions (Edwards & Potter, 2005). Therefore, it affords a position from which to interpret discursive strategies used during

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[3] As a "theory of mind", DP distinguishes itself from other Vygotskian approaches by asserting a more radical epistemological constructionism as opposed to the usual ontological version (Edwards & Potter, 2005). For Vygotsky (1978), mind is real and is constructed socially. The focus of analysis is on how the mind is built within a cultural world. For discursive psychologists, notions of the reality of "mind" are bracketed or set aside in favour of analysis of how notions of mind are established, maintained and challenged in discourse (Edwards, 1997).

negotiation for the social actions they accomplish. It can also illuminate how children use the mathematical norms they negotiate as resources to support their participation in the classroom.

In this section, I will outline the main tenets of DP by exploring its use of the term "discourse" and, by so doing, construct an argument for the possibilities afforded by linguistic analysis of spoken mathematical discourse. Following that, I consider how DP frames what might count as evidence and compare that to what has previously counted as evidence in research studies regarding sociomathematical norms. I finish by laying out a DP understanding of the processes of negotiation.

### 2.3.1 Discourse

DP's use of the term *discourse* coincides with ethnographic and linguistic anthropological definitions (Wetherell, Taylor & Yates, 2001) and it is often used as an adjective, as in "discursive practice." This tradition focuses on language-in-use as a social act of communication between human agents. It highlights three principles concerning discourse: language both constructing and constituting meaning, discourse as performing specifiable social actions and the co-construction or ongoing negotiation of meaning by participants.

The first principle concerning discourse is that language both constructs and constitutes the meaning involved in any interaction. A common example of this is "indexes" (Hanks, 1999). In any interaction, words such as *it*, *this* or *here* will have different meanings depending on what they refer to, or what they *index*. Therefore, their use in context constructs their meaning. Another sense of indexing was given earlier in the example of the parental directive: *You're ready for bed now*. In that case, a sense of entitlement was embedded in the language as it was used in context. By using the term discourse in this way, those who work within DP foreground the importance of starting any investigation with the discourse, rather than the participants

(Edwards & Potter, 2005). Discourse becomes an object of study in and of itself, not for what it can reveal about reality. In other words, we do not need to know anything about the parents' intentions, motivations or their state-of-mind in order to make sense of this statement. In fact, we do not need to know anything about the relationship between the adult and child in order to make sense of it. We can draw inferences about that relationship based on the subsequent utterances, according to how speakers (including the child) accept, challenge or question the implicit entitlement.

This approach can therefore account for norms being *active* in discourse by noticing when and how talk orients to them. It does not imply that the individual becomes invisible or unimportant. Rather, it takes the position that participants become who they are *as they talk themselves into being* (Wetherell et al., 2001). A DP approach to discourse allows a perspective from which norms are not prescriptive, but rather considered as participant resources, used to make sense of talk while it happens (Edwards, 1997). When a participant presents a solution *as if it is* a mathematically valid solution, this instantiation becomes part of what defines validity in that context. Future speakers will orient to that definition in their subsequent turns at talk, by ignoring it, accepting it or resisting it.

The second DP principle concerning discourse is that at its heart discourse is dialogical. Therefore, discursive practices are constructed in such a way as to perform specifiable social actions, such as convincing, denying, claiming authority, or undermining the position of another (Edwards, 1993). One example of this is seen in transcripts taken from reform-oriented classrooms when teachers use *contextualization cues* to frame mathematical activity. Contextualization cues "represent speakers' ways of signaling and providing information to interlocutors and audiences about how language is being used at any one point in the ongoing exchange" (Gumperz, 2001, p. 221). It is how people communicate the messages within their messages. Thus, when the teacher says, "You're ready for the next one. All right…" (Voigt, 1995, p. 170), the message embedded in that statement claims authority, both for keeping the floor (maintaining dominance in the interactions) and making an evaluation of the students' readiness.[4] The underlying discursive message signals to students: "You should interpret what I say next as being the substance of a new mathematical activity." Another example of the dialogical nature of discourse is notable in teacher evaluations of solutions, especially the indirect versions. For example, upon hearing a student suggestion for dealing with the "leftovers" of a division problem: "We're going to throw [the extra apples] away. Well, we can throw them away, but that's kind of wasteful" (Voigt, 1995, p. 171). This teacher evaluation draws on moral/social norms from outside the classroom to account for why this should be viewed as a dispreferred solution (ten Have, 1999).

In any form of argument, including mathematical argument and the negotiation of sociomathematical norms, the functions of social actions like convincing or denying become especially salient. For example, negotiation requires a speaker to manage both the content of the disagreement and the underlying social relations necessary to maintain the interaction (Barwell, 2005; Sorsana & Musiol, 2005). In other words, while negotiating classroom norms, participants must manage the literal aspects of disagreement, like making oppositional utterances in response to another's claim (Maynard, 1985) while maintaining the implicit connections essential to social order (Danby & Baker, 1998). To my knowledge no scholar has attempted to show how young children co-ordinate these communicative strategies during negotiation. Therefore, I will

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[4] Compare this statement to the everyday version spoken by parents and examined earlier in section 2.1.1: "You're ready for bed now."

document the negotiation of norms within a peer based social context – small group mathematical interactions.

A third principle regarding a DP perspective on discourse is that meaning is co-constructed through the discursive actions of the participants. This principle may be understood through both macro and micro applications. A macro perspective would refer to the wider social and cultural histories that might be implicated in any particular social interaction. In order to interact successfully, people must share common understanding of what is happening and what is expected of them. Each participant draws on past experience to evaluate a current situation and uses that to make judgments about current participation (Horne, 2001). During interaction, features of those evaluations or assumptions surface as participants frame their contributions to account for them. In the previous example, the teacher called upon an assumed taken-as-shared value that *waste is not good* in order to account for taking a position that challenged a solution as being appropriate. From a DP perspective, conversants co-construct *positions*. Positions are not predetermined on the basis of the sociocultural context of the interaction as they are for Foucault (1977). The students in the example accepted the valuation and, by inference, the teacher's position by treating it as a given in their subsequent talk. They proceeded by offering other, presumably more mathematically acceptable solutions: "Split them in half… Split them in fourths…" (Voigt, 1995, p. 171). This principle that meaning is co-constructed through the discursive actions of participants allows for an interpretation in which one conversant may be acting as if social positions *are* a given and the other one is not. Such a dynamic has been observed in mathematics classrooms when students challenge teacher positions (Houssart, 2001), an act that might be implicated during the negotiation of sociomathematical norms.

A micro application of co-constructed meaning refers to the situated use of language. An assumption is made that interaction can be interpreted meaningfully only within its context. This includes the individual interests or stakes of participants, which may be contested through and by their negotiations. This may be demonstrated with the strategic use of narrative and the construction of "facts" during negotiation or argumentation (Edwards & Potter, 1992). Documenting the ways in which conversants manage the duality of maintaining friendship while disagreeing (Barwell, 2005; Sorsana & Musiol, 2005) or the ways in which speakers simultaneously express and protect an identity (Joffe, 2003) therefore affords a deeper understanding of *how* children participate in the negotiation processes.

### 2.3.2 Data

If language constructs and constitutes the meaning involved in any interaction, then the primary form of evidence in any DP framed study of sociomathematical norms must be based on naturally occurring spoken discourse. Furthermore, if discourse is dialogical at its heart, as DP assumes, then any claims made about contributions to negotiation will be framed around how participants orient to the presence (real or perceived) of others. In this way, the social actions produced through talk take precedence over its content (Edwards & Potter, 2005). DP respecifies traditionally cognitive concepts, such as thinking, knowing, attitudes or beliefs as being reflexively constructed social actions. There is an interest in the practical, everyday use of the psychological lexicon (phrases referring to mental states *as if they existed*: I know, you don't understand etc.) In this framework a systematic examination of the development of sociomathematical norms might consider the ways in which elements of mathematical negotiation (such as authority for knowing or the role of previously developed common understanding, Herbel-Eisenmann, 2000) are produced or made relevant by social interaction.

How speakers construct facts and how they report events is of particular interest, especially when they use a psychological lexicon. Both actions are implicated in the negotiation of sociomathematical norms and both are regularly produced during naturally occurring spoken discourse.

#### 2.3.2.1 Studies that diverge from discursive psychological assumptions

The studies that initiated an awareness of sociomathematical norms (Voigt, 1995; Yackel & Cobb, 1996) were based on transcripts and observations of spoken classroom interaction but were framed within sociology/cognitive psychology. A number of scholars refer directly back to those studies when they describe their choice of evidence. I give two examples here.

In a first example, Tatsis and Koleza (2008) used a direct quote from Yackel and Cobb (1996) to describe their own linguistic focus on interactional patterns in spoken interactions as evidence for norms. The participants in this study were pairs of university pre-service primary mathematics teachers completing a problem-solving task. They were instructed to think aloud and cooperate with each other. Interactions were audio-recorded by a researcher who was present but avoided interacting. Tatsis and Koleza (2008) took a theoretical approach that considered the mathematical discourse to be the learning – this is different from my intent to view discourse as a tool for learning. They described a certain pattern of interaction that they expected to be typical during negotiation (introduction – discussion – evaluation). Then, when the analysts noted changes to the expected pattern or significant delays in its accomplishment, they counted those exceptions as evidence for the function of sociomathematical norms.

In a second example, Williams (2010) also studied adult learners. She prepared an in-service teacher development program aimed at improving the quality of whole class discussions by increasing the cognitive demand of teachers' questions. Her study was intended to evaluate the programs' success, measured as an increase in the teachers' use of cognitively demanding questions in their subsequent practice. In William's program, she drew an explicit link between Yackel and Cobb's (1996) descriptions of sociomathematical norms and cognitive demand. There were many instances of talking about talking about mathematics, just as Voigt (1995) noticed. However, William's main source of evidence was written (i.e., teacher self-reports in the form of in-class reflections on practice, setting goals for future practice in journals and then providing written anecdotes as evidence that changes in teaching practice had occurred). In her study, sociomathematical norms served as a means to an end as opposed to being the focus of the investigation.

#### 2.3.2.2 Studies that resonate with discursive psychological assumptions

Discursive psychology affords an understanding of sociomathematical norms by noticing when they are violated (Garnica, 1981) and the sanctions that arise from those violations (Boulima, 1999). This is taken from the participants' perspective, however, and would not involve the analyst pre-emptively deciding which pattern of interaction to consider typical. It would require a close sifting of the data, first defining how you might notice "typicality" and then looking to see what patterns the group treated as typical during interaction.

Ben-Yehuda, Lavy, Linchevski and Sfard (2005) exhibit such an orientation to participants' perspectives. They conducted interviews (supported by classroom observations) independently with two 18-year-old students who struggled with mathematics and used audio-recordings from each source of data to develop arithmetical discourse profiles for each student. They noted two different forms of sociomathematical norms, *enacted* norms (which are the domain of the analyst) and *endorsed* norms, which rely on either direct or indirect participant remarks. While an interview setting does not provide naturally occurring talk *per se*, they treat the analysis as if both the interviewer and interviewee were co-constructing meaning during the process of the interview, thus resonating with discursive psychological assumptions.

A study by Hershkowitz and Schwarz (1999) exemplifies a second facet of a DP approach to what might count as data: the use of a traditionally psychological lexicon (words such as *know*, *understand* etc.) during naturally occurring talk. These researchers sought to extend Yackel and Cobb's (1996) original findings by investigating the role of cultural tools, such as computers, in the negotiation of norms.

Hershkowitz and Schwarz (1999) looked at grade seven classrooms where students were interacting in pairs using computers as an aid to problem solving. They generated transcripts of interactions supported by classroom observations of computer manipulations. This afforded an opportunity to consider how classroom norms may be negotiated without an adult present. They determined that since the computer software was designed to show parallels between certain mathematical representations, it also served to legitimize some forms of justification over others.

For my purposes what is interesting in their data is the way a pair of girls negotiate while problem solving. The problem involved exploring ratios between the ages of a mother and son (e.g., 25:1 would become 7:1 three years later when she is 28 and he is 4) and, specifically, finding out how old the two would be when the ratio was 1.5:1. Up to the point in question, the girls had used a spreadsheet program to guess-and-check. As the negotiation begins, one of the girls expresses dissatisfaction with this method, arguing against the acceptability of finding solutions through automated computer actions without conceptual understanding (basically just dragging and copying numbers from one cell to others). She seems to argue for an algebraic method of solving the problem but has difficulty communicating her position. After several moments of conversation she says, "You don't understand what I want to tell you" (p. 162). Two turns later and after the first student has used the computer to demonstrate an alternate approach, her friend replies: “I understand.”

In DP, these two uses of the word “understand” are called *mental state avowals* (Edwards & Potter, 2005). This use of psychological lexicon, or categories (e.g., know, think, understand, want) to convey a specific version of oneself or another, is common in conversation, particularly during disagreement or when something is at stake as in a negotiation. In the data Hershkowitz and Schwarz provide, almost every turn includes at least one mental state avowal. This highlights the potential significance of mental state avowals for negotiation, and points to what might be fruitfully explored in a study that seeks to document children’s participation in the negotiation of sociomathematical norms. It suggests that a first step in a search for evidence might be lexical: locating instances of the explicit use of psychological categories in talk. Therefore, this is the approach I take in Chapter 4.

Hershkowitz and Schwarz (1999) accept the statement “I understand” as if it were a representation of what the child is actually thinking: “Surprisingly, Matan understands now Shula’s concern through this non-verbal communication [referring to the computer demonstration]” (p. 163). In a later excerpt, we hear this child Matan explain her friend’s dissatisfaction to the teacher. In a DP approach, it would be her explanation and not her mental state avowal that displays, or produces her understanding (Edwards, 1993). Instead, “I understand” (taken to be a *claim* as opposed to a *display* of understanding) would be considered for the social action it accomplished at that moment with that friend. For us, it is impossible to interpret this action for its social implications with any certainty without a complete transcription of the conversation (Hershkowitz and Schwarz end the excerpt with that comment). However, given the heated nature of the preceding discussion, it is quite plausible that the statement “I understand" was produced in order to align herself with a friend, with or without actually understanding the mathematical details of the negotiation. The version of the explanation she eventually gives, out of all the possible versions, then produces her understanding in a way that is available to her teacher, to her friend and to us, as analysts. This highlights the possibility of using descriptions and explanations as evidence of participation in mathematical negotiation and points to the approach I take in Chapter 6. Discursive strategies become visible with a fine-grained analysis of how positions vis-à-vis interlocutors are negotiated.

### 2.3.3 Mathematical negotiation

All of the studies concerned with sociomathematical norms are either set in a classroom, where the teacher's pedagogical agenda (Yackel & Cobb, 1996) is both figuratively and literally defining the events of negotiation (e.g., "You're ready for the next one," Voigt, 1995, p. 170) or have the negotiation aspects of the setting articulated by the researchers (e.g., asking the participants to think aloud and cooperate with each other, Tatsis & Koleza, 2008). In these settings, the adults in charge prescribe the mathematical negotiation. It is only because I intend to investigate naturally occurring talk that I need to consider how to analytically define mathematical negotiation. To that end, I plan to use the participants' own orientations, a position supported by a discursive psychological perspective on negotiation and also supported by Voigt's (1995) conceptualization of mathematical negotiation as discussed in Section 2.1.1.

DP shares with ethnomethodology a conviction that both child and adult participants display their understanding of an interaction to each other and thereby produce the shared social order in which they live (Garfinkel, 1967). Therefore, two assumptions of both DP and this study are that children actively seek to define and orient to the meaning inherent in talk-in-interaction and that they also hold each other and adults accountable for doing so (Edwards, 1993). These assumptions become important in analyzing negotiation, where the position of participants, vis-à-vis the content of talk becomes an issue. During these types of interaction, aspects of the rhetorical organization of talk (discursive strategies intended to convince, refute or persuade others) become more salient. On that basis, I will consider contextualization cues (Gumperz, 2001) and other discursive strategies that mark interactions as events to identify mathematical negotiation.

As Voigt (1995) developed his theory of classroom microculture, he noted that interactions were produced in organized ways, including the elicitation and discussion patterns noted earlier. Remember that this refers to an analyst's perspective. It is possible to recognize patterns in hindsight within interactions that are produced step-by-step by participants. These are not rules that govern interaction; they are the way communication happens. Voigt noticed that in the elicitation pattern ("fishing for the answer", or traditional teaching methods) the solution is the main focus of the negotiation while during the discussion pattern (reform-oriented teaching methods) the solution is a starting point for an explanation. Although the discussion pattern was the norm in the project classrooms, elicitation patterns of interaction emerged when there was some kind of conflict between perspectives. It is possible that teachers occasionally fall back on more traditional ways of interacting when reform practices are new or unfamiliar to them. As reform practices became more widespread, researchers began to notice disagreements between students without teachers present (e.g., Hershkowitz & Schwarz, 1999).

In the N:Countr setting from which my data arises, there are many examples of the children and me co-constructing ideas that cannot be attributed to any one person alone. Transcriptions and video recordings of these negotiations form the text for analysis in the results chapters of this dissertation (Chapters 4, 5 and 6).

## 2.4 Conclusion

The strength of a discursive psychological approach to the understanding of sociomathematical norms is in the congruence between studying how the processes by which they are negotiated function as social actions from within a sociological framework (Barwell, 2003). It allows a focus on the meaning of norms in mathematics as children construct and orient to that meaning. This type of approach is missing from the research literature and thus produces a sense of inadequacy regarding our understanding of the young child's contribution to mathematical practices. My study addresses that inadequacy.

In this study, I consider the practice of mathematics as discourse and the experience of inquiry-based learning as situated within a particular setting: the N:Countr mathematics research study group. Remembering the assumptions behind ethnomethodology, interactions here are understood as part of a greater social situation, not just the function of a dyadic pair (Goffman, 2001). As discursive psychological analysts, we interpret based on the empirical evidence of the spoken interaction and any supporting evidence that participants decidedly orient to (this might include gestures or drawings). Therefore, the video recordings of the whole group are considered the definitive source of authority regarding the behaviours observed.

However, the unit of analysis in my study is the *discourse*, not the *child*. It is worth mentioning here again the significance of starting with the discourse rather than the participants (Edwards & Potter, 2005). Discourse becomes the object of study by examining how it is used by speakers to perform specifiable social actions, like the alignment mentioned above. The evidence for this dynamic process regarding sociomathematical norms would then consist of showing when and how talk orients to norms. Therefore, it is not necessary to know the background of the participants or speculate regarding their expectations for mathematical practice or the construction of knowledge. The only evidence that is needed to interpret conversational moves is that which is available to the participants themselves in real-time.

This is not to say that there is only one interpretation that is valid for a particular interaction. Multiple interpretations are possible, based on the research question being considered, the analysts relationship to the community being studied and various cultural or historical backgrounds of any of the participants. This was outlined as a macro perspective of the co-construction of meaning. The discourse analytic role requires taking a position referred to as "data-near" (Atkinson, Okada &Talmy, 2011) so that only those features of background made relevant by the participants are considered. In writing up my analysis, I will account for the existence of other possible interpretations by providing whole segments of transcribed spoken interaction in the form of examples. As the analysis proceeds, I will again include small segments within the text to illustrate points. This is one way that I start with the discourse rather than the participants. It affords alternate interpretations from readers and contributes to the kind of dialogue this analysis is intended to provoke.

During the N:Countr mathematics research group, the task of the children was to play. My role as a researcher was to draw out the mathematics in their play. As our roles evolved, I asked questions or "provocations" (e.g., *what do children know about zero?* Or *how can you show me math with your body?*) to which they responded in multiple ways. These rich interactions form the basis of analysis in the manuscript chapters of the dissertation. As educators and educational researchers, we do our best work when we operate from a position of being informed. Therefore I argue that with a renewed recognition of young children's capacity to contribute to classroom practice, we will be better equipped to acknowledge and engage each student.

# Chapter 3: Methodology

> Researchers' stories are seen as connected to the world the way clothes are related to our bodies: they are human-made and should thus not be confused with what they are only supposed to "cover"; they have many versions, and although not everything goes, more than one of these versions would usually pass as a good match; and finally, be the match as good as it may, none of the versions should be seen as the "ultimate one." This postmodern vision replaces millennia long tradition of treating research as the activity of documenting the world's own testimony. (Sfard, 2012a, p. 7)

The purpose of the current study is to document how young children participate in the discursive development of sociomathematical norms. However, the documentation produced should not be understood as a transparent representation of reality, but rather as a construct of mine. In this study, the children's participation is demonstrated through a 3-step process of discourse analysis of transcripts based on video data previously collected during a study that combined ethnography and pedagogical documentation (McLellan, 2010). Each step of analysis is designed to answer one aspect of the research question: *How do young children participate in the negotiation of sociomathematical norms in inquiry-based settings?* In Chapter 1 of this dissertation (Section 1.3) I outlined the research design of and the data generated during the N:Countr I and II studies. This included details about both the setting and the participants. Therefore, in this chapter I focus on the research design of the current study which is secondary data analysis on previously collected video recordings. This chapter outlines the research approach taken, the data generated and some methodological assumptions regarding the processes of transcription and the setting. These sections provide some detail regarding how ethical issues have been addressed and finish with a summary of the research methodology.

## 3.1 The research design

The question I am interested in exploring invites a focus on interactional processes and therefore suggests a qualitative interpretive approach. While the theoretical framework that informs my study is discursive psychology (Edwards & Potter, 1992), I will draw from corresponding work in discourse analysis (Cameron, 2001) in order to illustrate the discursive strategies young children use when they participate in the negotiation of sociomathematical norms. In Chapter 2, I outlined a position within which mathematics is understood to be a discourse and learning mathematics is about participation in that discourse. Reform oriented classroom practices encourage that participation and nurture a particular combination of obligation, expectation and collaboration. Mathematical negotiations which arise then are supported by the taken-as-shared understanding of sociomathematical norms, which conversants display to each other through the ways they construct their own participation. Therefore, in order to provide evidence in answer to my research question, it is reasonable to analyze the discourse of social interactions within an inquiry-based mathematics setting for young children. This methodology was selected for two reasons.

First, discourse analysis on naturally occurring interaction allows description of aspects of communication that typically get taken for granted, such as turn design (Schegloff, 2007), vocabulary or tacit agreement (Cameron, 2001). As Bauersfeld (1993) specifies, the core implications of classroom norms exist at the metalevel and arise from indirect learning: “The core of what is learned through participation is when to do what and how to do it” (p. 4). The very process of reification that allows interaction to function as smoothly as it does renders the individual processes of communication invisible (Wenger, 1998). It is the very taken-for-grantedness of sociomathematical norms that produces their interactional power. Therefore, discourse analysis foregrounds the assumed and provides a way of documenting processes.

Second, as a methodology discourse analysis can highlight the capacity for children's contributions to the negotiation of classroom norms by focusing on what spoken discourse might accomplish for them (Cameron, 2001). In particular, the rhetorical organization of talk to strategic ends holds great promise here. Scholars are just beginning to recognize young children's capacity to anticipate consequences to how others receive and may respond to their contributions (Sanders & Freeman, 1998). When I write about norms being deployed during social interaction, I am referring to these underlying discursive strategies regarding convincing others or rallying support for ones position (Edwards & Potter, 1992). Thus I argue that using discourse analysis on transcriptions of naturally occurring spoken interactions recorded in inquiry-based mathematics settings will both provide insights regarding children's contributions to the development of sociomathematical norms and provide a means to describe in detail the discursive strategies they use.

What is needed is quality data that is also complete. By quality I refer to video data that would therefore support transcription of sufficient linguistic detail. By complete I refer to including enough interactions over a period of time to provide examples of the processes involved. In Section 1.8, I described various aspects of the data generated during the five month N:Countr study. During my graduate study coursework, I selected several interactions from the total 14 hours of videotape and transcribed them in order to allow a more detailed discourse or, specifically, conversation analysis. It was during this time that I began to recognize the value of the data I had collected, especially its richness regarding child-to-child and child-to-adult interaction. For the purposes of this study, involving an inquiry-based group setting, I limit my use of the data to the N:Countr II mathematics research group. A closer look at the data generated in that portion of the study clarifies issues of quality and completeness for this secondary analysis.

From N:Countr II then, I have ready access to 11 hours of videotaped social interactions collected with young children in an inquiry-based mathematics setting. Although collected with an unmanned camera, the quality of video recording is sufficient to support transcription that includes most linguistic aspects of communication as well as gaze, gestures, body position, silences etc. As for completeness, the 11 hours collected over 3 months includes every minute these children were together for the study. Furthermore, pedagogical documentation includes the collection of various artifacts, such as drawings, photographs and journals kept by the children, all of which are then available to support interpretation. Since the voices of young children are missing from research on sociomathematical norms, this type of data has the capacity to provide necessary baseline information concerning children's contributions to their negotiation.

### 3.1.1 Secondary data analysis

There are several aspects of the N:Countr data that make it a strong candidate for secondary analysis in pursuit of the research question I pose. First, the five to seven year old range has been identified as critical for the development of skills in mathematical argumentation (Krummheuer, 2013). These children are verbally articulate while still being within the period of early childhood (Bredekamp, 1996). This offers potential to capture discursive processes while they are still new practices. Secondly, these young children are still new to the practices of schooling (Mishler, 1972). As Cheval (2009) indicated, at the macro level, the beginning of the school year is pivotal for developing classroom norms. I have generated data that covers not only the beginning of a school year, but also the beginning of formal schooling. Although the interactions here would be more strongly framed by the specific context of the playgroup, it seems reasonable to expect that the beginning of children's exposure to formal schooling presents a key opportunity to study the negotiation of classroom norms.

Third, there is evidence to suggest that the social processes of formal mathematics schooling can affect students negatively as early as kindergarten and grade one by influencing the ways they come to approach learning in older grades (DiPerna, Lei & Reid, 2007). I argue that the negotiation of group norms is one of those processes. Understanding how young children participate in the negotiation of sociomathematical norms will help educators develop learning environments that promote all children's sustained, successful engagement with mathematics. Fourth, Eriksson (2008) identified the primary years as being a time of transition for children's approach to arithmetic, a time when their own informally constructed notions bump up against the formal instruction of teaching. In particular, Eriksson warns against the routine-like application of concrete materials to promote conceptual understanding, since that practice may require the students to repress their own strategies in order to share the teacher's interpretation (cf. Walkerdine, 1988). This has wide-spread implications for primary teaching practices and makes the early years a productive period to examine processes by which children use multiple means to contribute to their environment.

On these bases, I argue that this data has sufficient capacity to provide answers regarding children's participation in the negotiation of sociomathematical norms in an inquiry-based mathematics setting.

### 3.1.2 A process of three steps

Transcriptions supported by video will comprise the main data for analysis in this study. Each of the three steps of discourse analysis has been designed to answer one aspect of the research question. To begin with, I need to establish evidence that sociomathematical norms are active in this setting. Claims regarding the negotiation of sociomathematical norms would be strengthened with initial indications of their relevance to the interactions. Having determined an active presence, I then turn my attention to detailing the ways participants orient to the meaning of those norms as they position themselves vis à vis a mathematical concept: the function of the equals sign. Finally I explore the children's discursive actions while they pursue a mostly independent investigation of the function of the square root symbol, an advanced topic for which their previous use of experience and example will not suffice. While not considered complete or perfect, these representations of the children's participation in the negotiation of sociomathematical norms provide a multi-faceted and robust portrayal of the children's experiences, from their own perspectives.

#### 3.1.2.1 Step one: Data generated and analysis

In Chapter 4, I address the question: *What evidence do I find for sociomathematical norms in this setting?* In that study, I explore the children's perspectives on the role of discursive practices in the negotiation of mathematical norms through an examination of how they use statements involving "knowing" to construct locally relevant versions of mathematical negotiation. I use Transana software (Fassnacht & Woods, 2012) to aid transcription and simple search features of Microsoft Word ® to support analysis.

My analysis in Chapter 4 centers on the children's production of 181 uses of the term "know" (in various tenses) during the N:Countr Phase II mathematics research group. Corpus linguistic analysis of those utterances highlights regularities regarding how the discursive practices of *displays of knowledge* and *claims of knowing* are produced and deployed (Koole, 2010; Sacks, 1992). One of the children invokes a sociomathematical norm regarding accuracy on three separate occasions, where it is marked as a discursive irregularity. Close examination of both the regularities and the irregularities highlights some of the discursive practices used and deployed by these children to produce their participation in mathematical negotiation. Therefore in Chapter 4 I provide evidence for sociomathematical norms by demonstrating when and how the children's talk orients to those norms.

#### 3.1.2.2 Step two: Data generated and analysis

In Chapter 5, I address the question: *How do children display their orientation to the meaning of norms during mathematical negotiation?* In that study, I focus on the ways in which the children incorporate mathematical content within the social practice of negotiation, as exemplified within a segment collected during Week 4 of N:Countr Phase II. The interactions involve a small group of participants arguing about the meaning – or function – of the equals sign: is "equals" for adding or for subtracting? I found that scholars had identified this particular subject, a clear understanding of *equivalence,* as a key factor in moving from arithmetical to algebraic forms of reasoning (Carpenter, Franke & Levi, 2003; Knuth, Stephens, McNeil & Alibali, 2006), so I wondered what was happening discursively at this mathematically significant moment.

Using the tools of conversation analysis I identify nine different discursive practices by which the children produce their understanding of the meaning of the equals sign, including four different mathematical justifications using examples: general or specific, hypothetical or experienced. I found that participants used words, gestures, silences, proximity and discursive markers (so, well, etc.) to produce their positions in that negotiation. A closer look at the *ways* the children constructed these positions gives evidence for their understanding of the meaning of sociomathematical norms: they invoke six different sociomathematical norms as they position themselves within the negotiation. Therefore in Chapter 5 I provide evidence showing how the children display their orientation to the meaning of norms during mathematical negotiation.

#### 3.1.2.3 Step three: Data generated and analysis

In Chapter 6, I address the question: *How do the children specifically contribute to negotiation?* In that study, I investigate a process of group meaning making that evidently bridges the practices of arithmetic and algebraic reasoning. This negotiation plays itself out over six weeks while the children carry out their own inquiry into the function of the square root symbol. Unlike the negotiation considered in Chapter 5, this one never reaches consensus in a conventional way, perhaps due to the difficulty of the task or time constraints. I wondered how the children would discursively approach a negotiation of challenging mathematical content. How would they proceed without access to the resources of experience or example that had served them well in the negotiation examined in Chapter 5? This analysis therefore presents an opportunity to construct an “extreme case formulation” of my own (Pomerantz, 1986): evidence of young children’s participation in the negotiation of sociomathematical norms under mathematically challenging conditions strengthens the warrant for my claims.

My analysis in Chapter 6 expands to include multimodal features of mathematical interactions, in this case verbal, gestural, visual and numerical aspects of communication and is completed with multiple passes through the data. A process of unitizing the 11 hours of interactions resulted in 136 clips, or “communicative events” that comprised the square root inquiry, including 33 clips coded for features of algebraic reasoning. A combination of narrative and conversation analysis highlights the structure of the collective argument the children construct over time. Therefore I demonstrate the children’s capacity to contribute to mathematical negotiation of sociomathematical norms by showing the communicative resources they draw upon in their attempts to make themselves understood.

## 3.2 Some issues regarding transcription: Processes of entextualization

Interactions here are considered as part of a greater social situation, not just the function of a dyadic pair (Goffman, 2001). Therefore, the video recordings of the whole group are considered the definitive source of authority regarding the behaviours observed. However, including videotapes is not always possible even with online journals, given the concern of both parents and university research ethics review boards concerning internet publication of child research subjects. Furthermore, video recordings invite multiple interpretations of the actions involved, depending on the observer's theoretical perspective and other considerations. So, in reporting research, discourse analysts produce texts that are endowed with "analytic utility" (Jones, 2011) – that is, transcriptions – yet we recognize that the act of transcribing *is itself* a form of interpretation: the "what" and "how" of transcription influences the analysis, interpretations and conclusions that can be made (Ochs, 1979). During the four months I spent transcribing the video recordings of N:Countr, I engaged in multiple processes of entextualization (Jones, 2011): *framing* the negotiations I was interested in, *selecting* which particular features I would use to represent those negotiations, *summarizing* or determining what level of detail was needed in each step of analysis, *resemiotizing* by translating my observations and awareness of various paralinguistic features into written text and finally *positioning*, by which I determined how to represent my own multiple roles (more about this issue in Section 3.3). The benefit of spending these long hours transcribing was a certain level of familiarity with the data. The challenge has been to maintain a suitable level of consistency in my representation of the events.

In keeping with the principles of an ethnomethodological approach to conversation analysis as first outlined by Sacks, Schegloff and Jefferson (1974) and Schegloff, Jefferson and Sacks (1977), data analysis in this paper demonstrates a reflexive journey with repeated reviewing of the videotape in light of current and new understanding provided by detailed conversation analysis. In other words, it took several runs through the video recordings to develop the final versions of the transcriptions shown here and this process will be illuminated in the results chapters. This recursive process led to a realization of one particular difficulty with regards to transcription: small group work within inquiry-based settings incurs multiple, simultaneous, overlapping conversations and the physical proximity of others has the potential to influence any interactional encounter. Nevertheless, participants present but not involved with the specific interaction under consideration remain invisible in the transcriptions.

### 3.2.1 Some challenges on transcribing small group work

How best to deal with this issue involves two parts concerning the amount of detail necessary in a transcript, one implicated during data collection and the other during data analysis. First, during video data collection, a single source of audio captures many different conversations during small group work in a confined setting such as a classroom. An obvious solution is to use multiple sources of audio, and when their focus allows it, many researchers do (e.g., Gresalfi, 2009; Stevens, 2000). However, while solving one problem, this response produces another for me, since it does not allow for a feature of group experience that is intuitively understood and acknowledged through the existence of common words such as bystander, overhearing and eavesdropping: even though the conversations of bystanders may not form part of any particular interaction, those conversations can be overheard by other participants. Furthermore, any participant might enter a conversation thus overheard, even if uninvited. Since it is impossible to know which if any of the discursive features of the environment are being overheard or impacting on the participants, rigour demands that every utterance captured by the source of audio must be included in a transcription, with the assumption that others present can also hear all these utterances. However, the inclusion of peripheral conversations can make it difficult to follow the interaction under analysis.

The second issue concerns data analysis. Interactional complexities in classrooms have been implicated in mathematics education research since the beginning of the current reform in the 1990s (Cobb et al., 1992b). Goffman's (2001) participation framework (a form of discourse analysis) allows us to account for these complexities by using the participants' own perspectives. For example, Figure 3.1 shows the participation framework developed for the N:Countr mathematics research group at a particular point in a transcript used in Chapter 5.

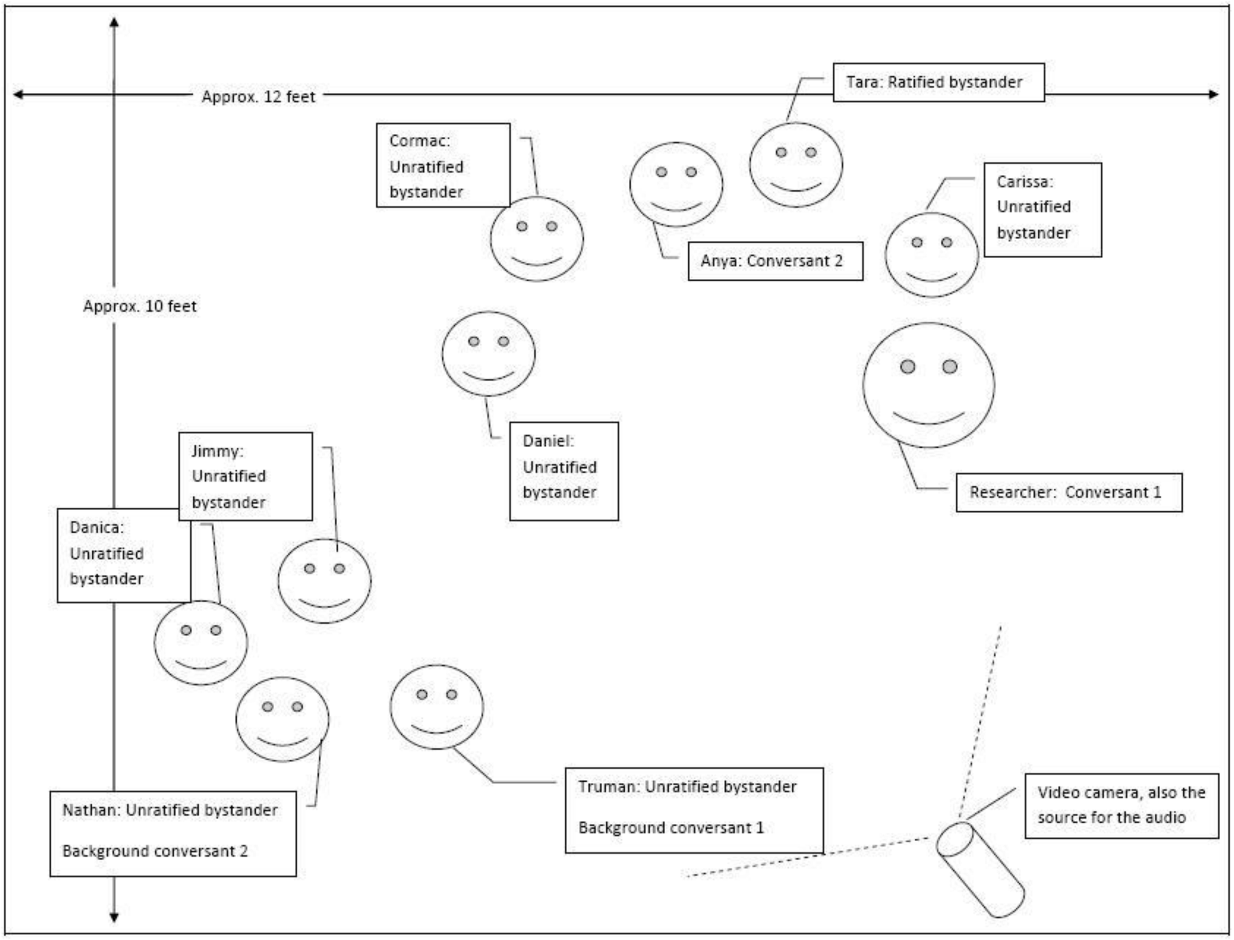


**Figure 3.1 Participation framework developed for line 15, Example 5.1 (see Chapter 5)**

This figure represents the physical location of all participants within the physical arena as well as their current social "situatedness" as indicated by analysis. The term "bystander" acknowledges a physical presence while "un/ratified" indicates whether the dominant conversants (as determined by the analyst, in this case Anya and me, numbers 1 and 2) refer to these bystanders or in any way orient to their presence. At the moment represented, Anya and I include Tara by briefly speaking to her and looking at the drawing she has produced. Therefore she is a ratified bystander. All others present remain unratified.

This commitment to the participant's perspective is an important feature of conversation analysis (van Lier, 1988) meaning that external or subordinate features of small group interactions are only included in analysis if the participants in the interaction under analysis themselves orient to those features of the environment. According to Goffman's (2001) understanding of speaker-hearer relations then, those he referred to as "unratified bystanders", by definition remain invisible in transcriptions, even though present in real time, since they are never acknowledged by the participants in the interaction under analysis (so in Figure 3.1, 7 out of 10 participants would conventionally remain invisible in any corresponding transcripts). However, in the collective development of an argument, as shall be seen in all three manuscript chapters, unratified bystanders can become ratified participants and in doing so, they can refer back to interactions overheard. Without a reference back to those earlier interactions, the utterances lose their significance within the development of the argument.

### 3.2.2 A proposed innovation

In order to strengthen the analytic utility of the transcriptions then, I propose a method of transcribing discursive background features (including the actions or words of other participants that are audible in the video recording), in a lighter font colour, like this: a lighter font colour. This proposed transcribing variation recognizes the possible implications of having small group participants overhear other conversations, allowing for analytic rigour, yet it facilitates analysis by foregrounding the conversation under inspection in black type. The current prevalence of online publishing facilitates this innovation – the cost of colour printing being prohibitive in an earlier print-dominated world.

As described in the results chapters of the dissertation, participants in the current study occasionally display their acknowledgement of overheard conversations, typically with gaze and attention. I argue that recognizing this is essential to socially situating their subsequent participation. However, in order to do so, I must include details in transcription that would otherwise seem superfluous to the interaction under analysis. By using a lighter font colour for those details, I am able to focus the analysis on the dominant interaction, yet draw connections between various threads in the developing argument that would not otherwise be noticeable.

### 3.2.3 Implications for analyzing mathematical negotiation

In much mathematics education research on spoken interactions the standard practice is to sanitize the transcription, cleaning up subordinate features in order to facilitate analysis of the mathematical content (Ryve, 2011). This also includes removing much of the communicative noise including *ums*, *ahs, false starts* and *cut-off words.* However, it is through that communicational noise that speakers display to each other (and to research analysts) their understanding of the content, for example of the social alignments being invoked or the mathematics under negotiation in the development of a collective argument. One key example involves the change-of-state token (Schiffrin, 1987) usually articulated as “ah” with a rising and falling intonation. This discursive marker has been identified in research to index a participant’s self-understanding: I know something new. Therefore those subordinate and background features of communication are vital to rigorous analysis.

As a special form of argumentation, negotiation requires a speaker to manage both the content of the disagreement and the underlying social relations necessary to maintain the interaction (Sorsana & Musiol, 2005). A mathematical negotiation is particularly complex, as speakers and hearers must also manage the mathematical meaning making that is essential to learning. Furthermore, in mathematics settings incorporating another’s position into your own argument through the actions of refuting, contesting or countering is a highly valued practice, essential in the development of a collective argument. As the process of transcription evolved through data analyses, I began to incorporate (in a lighter font colour again) the presence and behaviour of those participants who would later come to contribute significantly to the dominant interaction, even if they were unratified bystanders at the time they overheard conversations. Other subordinate features of communication, as indicated by the ratified speakers, are also transcribed in olive green in order to facilitate analysis.

For example, in Chapter 4 I examine an extended interaction between Carissa, Daniel and me during a group time (see Example 4.12). Anya raises her hand at line 149, indicating her interest in joining in the conversation. However, it is not until line 187 when I ask "Who else has an idea?" and Carlyn volunteers "I do" that I actually select Anya instead (line 189). It is only because I recorded Anya's earlier action (in olive green font) that my selection of a child who has not verbally indicated interest can be interpreted. Otherwise, Anya would have remained invisible in that transcript until I actually called her name. The same issue arises with Daniel in that transcript, where he is recorded (in olive green, line 159) as rising to his knees, squinting his eyes and nodding his head (as if counting) before he actually speaks in line 165, challenging Carissa's statements based on a miscount she performed in line 158. These actions take on new significance when seen in tandem.

My analytic approach juxtaposes the "emic" perspective of conversation analysis (Schegloff, 2007) alongside the "etic" perspective of footing (Goffman, 2001). While these have inherent tensions between them, researchers using video data have productively combined them (ten Have, 1999). In this, I am informed by the work of Goodwin (1990) who recognized the significance of gaze and other nonverbal aspects of face-to-face interaction functioning as social signals. Thus, I find no contradiction in referring to such gestural contextualization cues (Dorr-

Bremme, 1990) or gaze within a turn-by-turn sequential analysis. I also recognize that using membership categorizations such as “researcher” might imply pre-emptive analytic decisions, atypical of conversation analysis. However, I do so in the spirit of applied, rather than “pure” conversation analysis.

By providing this detailed accounting of my own analytic decision-making process, I hope to clarify the position I take towards my data: it is my construct, not intended to be taken as a re-presentation of reality. I have approached the video recordings through my theoretical lens and have constructed a “set of clothes” (Sfard, 2012a) that I expect contain some analytic utility and the potential to ultimately improve mathematics education.

## 3.3 Some issues regarding context

There are three issues that arise during the study regarding context. The first issue concerns a dilemma of how to represent my own multiple roles (participant/observer/analyst) in the study. The second involves the uniqueness of the setting (a “learning experiment”) and the third issue concerns the relatively short timeline for an ethnographic study. The first two are issues here because they potentially challenge the discursive psychological requirement to use only “naturally occurring” talk as data. The third requires an accounting for ethnographic assumptions in a study of only 11 hours over 12 weeks.

### 3.3.1 My role as participant/observer/analyst

My multiple roles as both participant and analyst potentially complicate data analysis as well as the writing of the dissertation (Ashmore, 1989). The approach I take here is to use first person throughout the data analysis, but identify myself in the transcription as “R” (for researcher). My participation at the time of data collection (in 2009) was uninformed by either discursive psychology or conversation analysis. Therefore, I find it reasonable to treat my own participation somewhat dispassionately during analysis (now, in 2014). Nevertheless, I recognize that my own participation influenced how the negotiation played out; therefore I find it necessary to analyze my own contributions. This is new territory for discursive psychology and conversation analysis and has important implications regarding the interpretation of the data (Antaki, personal communication, June 8, 2013). Therefore my base assumption is that, in keeping with the parameters of discursive psychology, my current analysis demonstrably does not depend on my memory of what the interactions involved. I include self-reflective sections in each chapter of the dissertation. By so doing I acknowledge that the person I am now and the experience of viewing myself as a participant has influenced how I have approached the secondary data analysis.

The complicating factor for discursive psychology is: *how "natural" is the interaction if I as a researcher was participating in it?* This potentially positions the current study within the debates about including talk-extrinsic data (Speer, 2002a, 2002b; Waring, Creider, Tarpey & Black, 2012) or more specifically, using video stimulated interviews to complement interactional analysis (Pomerantz, 2005). That debate however, involves researchers asking participants to comment on either their own intentions or the institutional implications of the discursive actions during subsequently viewed interactions. This is not specifically the same as analyzing my own participation at a considerably later date. Even Pomerantz's (2005) seminal research on involving ethnographic informants allowed for anyone who fit the medical category of "doctor" and might therefore have something to say regarding why this sort of interactional action might be useful in this moment. She was not relying on the particular doctor in the transcript to report supposedly private understandings (Antaki, 2012; Pomerantz, 2012).

My analytic aim here is to further our understanding of children's capacity to contribute to the negotiation of sociomathematical norms in inquiry-based settings. Therefore, my intentions at the time of interaction are only relevant insofar as they might impact on the children's capacity to contribute. As I have stated elsewhere (Section 1.7), in order to answer my research question it is essential to have a setting where the adult was not aware of the purpose of the study, and who I was when I collected the data is not who I have become through my studies nearly five years later. In 2009 I had never heard of sociomathematical norms and remained unaware of the significance their negotiation played in our interactions during N:Countr. Therefore, I find my position now in analyzing my own participation, an ethical way to ensure participant "blindness": I can state with some certainty that when I find evidence of the negotiation of norms (as opposed to the adult imposing norms on the children) it is not because I was "setting up" opportunities for the children to participate. Those negotiations emerged from the interactions as they unfolded. It is in this sense that I refer to "naturally occurring spoken interaction."

Discursive psychology demands a rigorous commitment to the data. So, I transcribe the communicative details and support analysis with other modes of communication such as the children's drawings and journal entries. I use and cite transcription conventions (see Appendix A: Atkinson & Heritage, 1984). I provide long examples of naturally occurring talk including paralinguistic features such as gaze and gesture and I attempt to make the sequential organization of that talk clear. My use of the term "from the participant's perspective" may seem to some like speculative insight more in line with cognitive psychology and completely at odds with my discursive position and its commitment to the data. I use the term "perspective" here as referring to taking a position of "looking-over-the-participant's-shoulder" rather than "getting-inside-her-head." In this way I limit my analysis and interpretation to relying on those social and discursive cues, and only those cues, that are available to the participants as they interact. This is how I establish my commitment to the data, and this is how I demonstrate that my current analysis does not depend on my memory of what the interactions involved: during analysis, the only aspects of my own participation that I draw upon are those features of communication that were available to my co-conversants at the time of interaction, as they *signal* that availability during interaction.

### 3.3.2 The setting: A learning experiment

When the object of discursive analysis is mathematical thinking and learning, the interactions studied more often than not take the form of co-ordinated task activities (Goffman, 2001; Seedhouse & Almutairi, 2009) as opposed to conversation. Therefore, in the examples that I consider in subsequent chapters, the interactions often function as a means to an end (the task at hand) while taking conversational regularities into account, rather than serving any purely conversational function per se. Nevertheless, the use of conversation analysis highlights the capacity of children to draw from discursive resources in the environment, such as overheard interactions, to support their subsequent participation in the processes of negotiation. Therefore, my claim here is that interpretation is supported by the tools of conversation analysis.

As a non-classroom teacher with no experience in school mathematics teaching and in a setting outside the school system I experienced no curriculum constraints. During the study, I hoped to document mathematical thinking during play (*any* mathematics: not that deemed *appropriate* to this age group). I had no formal responsibility for the learning of these children, neither was I required to evaluate either the children's learning or my own practice. Yet, as early conversation analysis of the interactions completed during coursework indicated, I clearly participated as an adult invested with authority within this setting – a teacher. The children oriented to my position as such by consistently referring to me as “Nanny” a title of respect referring back to our earlier relationships, raising their hands to get my attention and, on their own initiative, lining up in front of me to wait for a turn to talk. As I examined my own actions more closely still, I realized that they were informed by an Early Childhood Education perspective that includes a pedagogy of guiding, designing, facilitating and clarifying without ever directly teaching any mathematical content. Nevertheless, there is evidence that the children learned mathematically during our time together despite my disinclination to formally teach it. With no formal assessment it is difficult to measure exactly what or how much they learned, but the video includes examples of them talking about their learning (Carissa: If I never saw this, I wouldn’t never have known that – Example 4.10 line 398). Carissa had quantified the letters in an algebraic formula in an attempt to make sense of it. She constructed a connection between alphabetic order and quantity to decide that one letter was “higher” or bigger than another.

This is not a limitation in the sense that learning is not desired. It actually supports my argument regarding children as competent actors in an adult world and capable of agency in their own learning. It also speaks to Vygotsky’s (1978) indication that children learn from exposure to more competent others, be they adults or children. It is a limitation in the sense that five year olds typically do not have the opportunity to “play” with algebraic formulas. If these interactions are peculiar to settings including only a small group and where children’s agency is paramount, then given the constraints of formal schooling, this might limit my ability to generalize the findings.

However, there may be ways to use the peculiarity of the setting to advantage. Almost all of the studies currently published regarding children’s mathematical based inquiry rely on external (usually teacher driven) curricula expectations to determine the scope and sequence of the mathematics content involved. This limits the capacity of children to contribute, since their own curiosity is secondary to the intended content (Walkerdine, 1988). When provided with a setting that invites mathematical activity but presents few if any expectations of what that might look like then children have the opportunity to take the investigations to unexpected places (Anderson, 2010; Towers & Anderson, 1998). I argue that they might then have opportunity to participate in the negotiation of the sociomathematical norms in ways that might otherwise not be available to them. Therefore, the data generated during these studies optimizes children's *capacity* to contribute. So, although somewhat atypical, this study complements the work of others while providing insights that might guide future research into this phenomenon in diverse settings including formal mathematics classrooms.

Discursive psychology encourages an element of self-reflection that positions any participant present during an interaction as being significant. As already noted, this will help me to make sense of my own contributions, an analysis that will be facilitated by the amount of time that has passed since the data collection period. Nevertheless, it will be important to note any researcher participation in sequences of actions. Discursive psychology also requires that data be approached for what it is: in this case, a mathematics research study. These participants did not approach the setting as if it was *school*, we came together to explore *"What children know about numbers and math."* This basic orientation must be taken into account during any data analysis.

### 3.3.3 Time constraints

One final dilemma concerns the time constraints of the study, meaning that with 11 hours of video recordings collected over 12 weeks, there was less time than may be ideal for an ethnographic study. I will turn back to the research question in order to answer that critique. I am interested to know more about how children negotiate sociomathematical norms. Negotiation requires disagreement. If all members of a group agree, there is little to negotiate. This group incorporated three distinct characteristics that promoted amicable disagreement. The setting was "emotionally safe" – it was a familiar classroom with a familiar teacher and each child had some friends among the group. It was also cognitively challenging enough to be engaging – there were diverse outside experiences between our meeting times, including multiple school mathematics classrooms represented, leading to conflicting expectations of roles and identities. Moreover, it may have been socially confusing enough to maintain some disequilibrium – this became a setting wherein mathematical argument was encouraged, different from any previous setting we had experienced. The extended times between sessions may have functioned to slow down the processes of negotiating by prolonging the *newness* of the situation. Therefore the timeline of the study facilitates a more careful inspection of the negotiation process, in turn providing a reliable answer to the question.

## 3.4 Conclusion

Throughout this chapter I have outlined essential elements of the study's methodology, including addressing issues of transcription and context. The subsequent three manuscript chapters are framed as secondary data analysis carried out in three steps. This study represents an important contribution to our understanding of the development of sociomathematical norms within the experiences of young children, one that will complement without duplicating other studies. As inquiry-based settings become more prevalent in primary mathematics instruction, the perspectives of all participants will be essential to creating learning spaces that promote sustained, successful lifelong engagement with mathematics for each child.

# Chapter 4: How young children use discursive practices of knowing during mathematical negotiation

In this chapter I explore the role of discursive practices in young children's negotiation of mathematical norms through a discursive psychological lens. There is a general consensus that participation in the practice of collective argumentation not only promotes, but actually *constitutes* the learning of mathematics (Cobb et al., 1992a; Krummheuer, 2007) although how to fruitfully define and document the processes involved, especially for young learners, remains an open question. One of the challenges facing researchers interested in examining these processes is reasonably accounting for the reciprocal relationship between the negotiation of sociomathematical norms and the negotiation of knowledge within inquiry-based learning environments. I show here how a discursive psychological framing can elaborate features of a learning community's culture of negotiation by illuminating patterns in the interactional sequences that involve "doing knowing." Corpus linguistic analyses of young children's production of 181 uses of the term "know" (in various tenses) within a mathematics research group, highlights both regularities and irregularities in the interactional sequences involving the discursive practices of *displays of knowledge* and *claims of knowing* (Koole, 2010; Sacks, 1992). The data shows how the children draw upon sociomathematical norms as resources to produce *ways of participating* that co-ordinate with each other and how sharing the authority for *mathematical knowing* between researcher and children affords meaningful participation.

## 4.1 Introduction

Studies of inquiry-based learning have brought a renewed emphasis on the significance of student participation in classroom interaction for promoting mathematical thinking and learning. In these reform oriented mathematics environments, the negotiation of sociomathematical norms, or a taken-as-shared set of values concerning mathematical activities, has become a key practice (Walkerdine, 1988; Yackel & Cobb, 1996). Accordingly, negotiating sociomathematical norms has implications for the negotiation of knowledge. What comes to count as mathematically valid affords, but also constrains the knowledge available and the ways in which members of a learning community might interact with that knowledge. This understanding resonates with linguistic anthropological traditions, where the reflexive quality of communication is used to situate *norms* as ongoing features of discourse, implicated in every interaction rather than pre-established or prescribed (Agha, 2007; Baker, 2000). This means the norms will be negotiated as participants interact within mathematical learning communities. Each and every interaction will exhibit an orientation to underlying normative assumptions.

In this chapter, the first of three linked studies, I explore the role of discursive practices in the negotiation of mathematical norms through a discursive psychological lens using the words of the children themselves. Discursive Psychology (Edwards, 1997) affords the possibility of viewing mathematics as a discursive, social process. It assumes that meaning is co-produced by all conversants, and that discourse is fashioned in ways that perform social functions that are relevant to the local setting (Wetherell, 2001). This warrants a linguistic examination of how children use statements involving "knowing" to construct locally relevant versions of mathematical negotiation. It also affords an opportunity to document the children's participation in the practices of collective argumentation implicated in the learning of mathematics.

I have already stated that negotiating sociomathematical norms has implications for the negotiation of knowledge within any learning community: What comes to count as mathematically valid affords but also constrains the knowledge available and the ways in which members might interact with it. However, the reverse relationship also holds: negotiating knowledge has implications for the negotiation of sociomathematical norms. One way of accounting for this reciprocal relationship is to establish whether or not the distinction between *mathematical knowledge* and *mathematical knowing* is meaningful to those involved in the practice of collective argumentation, or mathematical negotiation as I call it here. The reciprocity between knowledge and knowing highlights one reason why the discursive practices of knowing are significant to this study of mathematical discussion: routine patterns of interaction that index states of knowing give evidence of participants' perspectives. Conversants embed messages that convey assumptions about their own and their partner's state of knowing: "The giving and receiving of information are normative warrants for talking, are monitored accordingly, and are kept track of minutely and publicly" (Heritage, 2012, p. 49). These are the discursive practices of knowing and provide an emphasis for the first research question: *How is knowing discursively produced and deployed in mathematical discussion?*

The concept of sociomathematical norms developed from an exploration of mathematics education reform-oriented practice that emphasized student participation and discourse in classrooms. Within these inquiry-based learning communities, sociomathematical norms are implied or interactionally deployed in each and every mathematical discussion, producing patterns of discursive regularities (Cobb et al., 1993; Voigt, 1995; Yackel & Cobb, 1996). It is these that provide an emphasis for the second research question: *How do patterns in these discursive practices relate to children's participation in mathematical negotiation?*

This chapter is the first of three stand alone studies that comprise the results chapters of the dissertation. In it I look at how young children use the structure of spoken language to support their participation in mathematical discussion. The data considered here include transcriptions supported by video recording of the N:Countr mathematical research group that met with me for eleven weeks in total (McLellan, 2010). In order to answer the research questions, I consider the same data from two different perspectives, looking for patterns in the sequence of interactions that might illustrate how norms are implicated in the negotiation of knowledge and how knowledge is implicated in the negotiation of norms. I propose that the regularities and irregularities in the sequences of interaction will provide insight into whether and how *mathematical knowledge* and *mathematical knowing* constitutes a meaningful distinction for this group.

This chapter then, initiates the first of the two tasks of significance in the dissertation: in it, I provide evidence that expands our understanding of the *interactional* processes involved in learning-as-participation. Defining and documenting young children's positive contributions to the negotiation of mathematical norms will strengthen the knowledge base regarding the function of norms in inquiry-based settings, renew our recognition of young children's capacity to contribute to classroom practice and provide a warrant to claim a meaningful distinction between *mathematical knowledge* and *mathematical knowing*.

## 4.2 Theoretical framework

Before examining the data for evidence, it will help to review from Chapter 2 the theoretical framework for the study as well as outline what it means to "do knowing" and my framing of the processes of mathematical negotiation. My intent is to provide just enough background research to situate this study within the dissertation.

### 4.2.1 Discursive psychology

Discursive psychology's use of the term *discourse* coincides with ethnographic and linguistic anthropological definitions (Wetherell, Taylor and Yates, 2001), a *language in mathematics* approach (Sfard, 2012b) where it is often used as an adjective, as in "discursive practice." This tradition focuses on language-in-use as a social act of communication between human agents. It highlights three principles concerning discourse: language both constructs and constitutes meaning, discourse performs specifiable social actions and meaning is co-constructed by participants during interaction. Within this theoretical framework then, the basis of understanding mathematics as discursive practice is a commitment to seeing mathematical discourse as action and not merely representation.

Discursive psychology uses the tools of conversation analysis to draw out the situationally relevant discursive markers around which interactions are organized. These can provide insights into participants' perspectives without needing to rely on analysts' inferences. Discerning the participant's perspective is a common concern for researchers who seek to understand the role of discursive practices in any negotiation (Tatsis & Koleza, 2008). Motivation, beliefs and intentions are not readily available to observers. Furthermore, even *mental state avowals* or explicit uses of a phrase such as "I know" can be interpreted in ways other than referring to an internal state of mind. Discursive psychology respecifies mental states as being social actions, producing an over-the-participant's-shoulder analytic gaze. This is what I refer to as the participant's perspective. The focus of research is on examining ways in which words such as "know" are used interactionally and rhetorically to contrast, or otherwise orient to, locally relevant alternatives (Edwards & Potter, 2005). It requires any inquiry to remain firmly grounded in empirical data in the form of spoken interaction.

In a similar vein, it is impossible to know for certain whether participants consider an interaction to be mathematical. In my analysis, I follow others in requiring an explicit reference to mathematical concepts, terms, symbols or processes (Moschkovich, 2003; Ryve, 2011). As an example, consider this brief interaction during N:Countr Phase II, between two children who sit at a table writing in their mathematics journals (Carissa 5y9m; Jimmy 6y1m):

**Example 4.1 Carissa and Jimmy Week 10**[5]

Carissa: *((chanting as she dances back to the table where Jimmy is sitting with two others))* and five. is an upside down two. guess what Jimmy?
Jimmy: *((without looking up))* what.
Carissa: (*(she plops her journal on the table))* five. is an upside down two.
Jimmy: (*(pleasant voice, without looking up))* **I know.**

When a child responds "I know" to a statement made by another, there are different traditions scholars might draw on in order to make sense of the interaction. In one, the child's response refers to a mental representation. Jimmy might be remembering a concept he learned earlier, or an observation he made previously (Muis, 2004; Schoenfeld, 1992). On the social plane, he might be agreeing with the observation of a friend, giving an affirmation of sorts. In either case, the reality of Jimmy's knowing is perceived by him and brought into the context of the interaction through memory and speech (Potter, 1998). The sequential unfolding of the utterances offers a third approach to analysis (Sacks, 1992; Schegloff, 2007). Carissa presents the same display of knowledge twice, the second time after nominating Jimmy as its hearer. He accedes to her nomination, after which she produces the repeated display. Her statement makes

[5] See appendix A for the transcription conventions used.

relevant a response which would be some version of an acceptance or challenge, or an accounting for not producing such a response. Jimmy is obliged to say something in response, so his “I know” is understood as acquiescence to the display of knowledge.

We might recognize several mathematical norms implied within this interaction. For example, the words *five* and *two* can refer to written symbols (as opposed to quantities); symbols can be abstractly manipulated in various orientations (e.g., upside-down) without losing their reference (e.g., an upside-down two is still a two) and metaphor can be used to construct relations between symbols. The difficulty for researchers looking to decipher participants’ perspectives in any tradition becomes deciding which interpretation comes closest to representing what Jimmy actually *means* by his statement *I know* (Edwards, 1993). In our case here, how can we decide which, if any, of those mathematical norms are particularly salient to the interaction?

My use of discursive psychology implies, among other things, a shift in research focus that bypasses this dilemma by concentrating on how language is used rather than on how it corresponds to reality. This provides a lens through which we might understand the meaning children assign to these practices by focusing on how they use them interactionally and rhetorically to accomplish specifiable social actions (Barwell, 2003). Therefore, as I seek to explore the children’s perspectives, I will not be interpreting what children *actually mean* by their statements. Instead, I will examine how the children use their statements to construct locally relevant versions of mathematical negotiation. In other words, I will retain an analytic over-the-shoulder gaze. In the example above, without making any guesses as to Carissa’s motivation we can see that she prefaces the second statement with a “Guess what?” simultaneously representing the upcoming information as newsworthy and assuring herself the following turn (Sacks, 1992).

Her pre-turn takes the form of a game, with apparently recognizable rules that Jimmy attends to: "What?" he responds, as opposed to actually making a guess. With this utterance, he produces his attention and invites her display of knowledge. Thereafter, his "I know" might function as a rebuttal, seen as a way of saying: this is not news to me. Notice that he is not refuting the object of knowledge itself, only its novelty, or newsworthiness.

By using analysis in this way, I can highlight children's *processes of participation* via their contributions to the negotiation of sociomathematical norms. In other words, I elaborate the processes of *learning-as-participation* by analyzing how the children use the discursive practices of knowing as interactional resources. This elaboration offers insights into the relationship between the negotiation of knowledge and the negotiation of norms. Jimmy contests neither the use of mathematical symbols, nor their representation, nor the use of metaphor, orienting only to the newsworthiness of the claim. With his response, Carissa's invoking of the norms remains unchallenged, producing an inferred or tacit agreement: the knowledge is taken-as-shared.

In the case of sociomathematical norms, the negotiation of meaning becomes organized around a kind of taken-as-shared set of values concerning the ways in which mathematical objects can be properly used in discourse. To summarize the analysis of the earlier example, I could say that norms regarding the use of linguistic referents to mathematical symbols (e.g., "five" and "two"), their abstract manipulation (e.g., "upside-down") and the use of metaphor to describe mathematical relationships (e.g., "five is an upside-down two") have been and continue to be reified through the interaction (Wenger, 1998). There is a taken-as-shared understanding of how these mathematical objects might properly be used within conversations. These norms are then available to support future interactions between Jimmy and Carissa as they participate in the practices of what it means to "do knowing" within this context.

### 4.2.2 "Doing knowing"

The perspective I take in order to answer the research questions is to focus on the discursive practices that people use to talk about, and therefore produce, mathematical thinking (Barwell, 2009). In this framing, "knowing" is not an internal state of mind but rather an interactional resource. Therefore, when I refer to the "negotiation of knowledge" I may be referring to the processes ("doing knowing") rather than any product. This distinction between a product (knowledge) and a process (knowing) is analytically meaningful, given my theoretical framework. However, while I maintain an over-the-shoulder participant's perspective, I cannot assume that what is analytically meaningful is also interactionally meaningful. I must look for evidence within the language use itself.

It is a central tenet of discursive psychology that the use of language constructs and constitutes the meaning involved in any interaction (Edwards & Potter, 1992). This reflexive relationship optimally frames "knowing" as something that is produced by and through interaction. For example, within this chapter I use the transitive verb "to produce" in four distinct ways, thus constructing and constituting the meaning in this chapter. First of all, I have used it in the sense of "generating",[6] as in producing a discursive form (e.g., in Example 4.1 Carissa *produced* two – identical – displays of knowledge – p. 87). It has also been used in the sense of "effectuating", or making available for public exhibit, as in a rhetorical verbal action (e.g., Carissa's statements make relevant an acceptance or challenge, "or an accounting for not *producing* such a response" p. 88) and "to effect or bring about", as in indexing a social action (e.g., Jimmy's attention was *produced* by his verbal response: "what?" (p. 89) In other words,

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[6] All definitions here come from *Merriam-Webster.com*. Retrieved January 4, 2014, from http://www.merriam-webster.com/dictionary/produce

although he never looked towards Carissa – the usual method of signalling attention – he signalled to her that he interpreted her question as a request for his attention and he gave it to her.) Finally, the term "produce" has also been used in the sense of "yielding" or "giving form, being or shape to" as in the upcoming methodology section where a way of organizing the raw data is described as *producing* a corpus of approximately 130,000 words. This nuanced use of language is an attempt to avoid jargon as well as a means of addressing my commitment to seeing mathematical discourse as action rather than representation. In keeping with a discursive psychological framework I foreground the social action in the discourse.

There are several discursive practices implicated in "doing knowing", the most obvious of which utilize the verb: *to know*. "Know" has been identified as one of the top 20 most frequent word choices in both spontaneous conversation and teacher-student interaction, by both teachers and students (Bishop, 2008). Therefore, it is reasonable to expect fairly frequent occurrences of the word within the data considered here. A discursive psychologically-framed investigation of spoken language can uncover ways in which statements such as "I know" are used interactionally and rhetorically to orient to locally relevant alternatives. This is what I refer to as interactional *deployment*. Ample empirical evidence challenges the simplistic relationship between linguistic function and form (Schegloff, 1984). That is, an utterance that takes the form of a question may not be "doing questioning." *Who knew?* is a classic non-questioning question (Clift, 2012). The rhetorical use of such a statement in speech (or in written text here) therefore constitutes a discursive deployment of "knowing." Accordingly, mental state avowels such as *I know* may or may not be "doing knowing." A number of studies are relevant here.

Heritage (2012) has delineated a sociology of knowledge based on the central role that knowledge asymmetries play in the organization of social interaction. He shows how the epistemic engine, called the driving force, propels communication through the giving and receiving of information. I previously interpreted Carissa's "Guess what?" from Example 4.1 as a pre-turn that presented the upcoming utterance as newsworthy. Heritage takes that interpretation farther. He uses the notion "territories of knowledge" to explain how conversants co-construct mutual knowledge as they work to equalize knowledge imbalances. An application of his work would suggest that Carissa's "guess what?" serves to mark an assumption that she holds: What she is about to say is something that Jimmy does not already know. Therefore, Heritage would say, she assumes to speak from the position of *more knowledgeable* (K+) to a recipient who hears from a position of *less knowledgeable* (K−). In the linguistic anthropological terms mentioned in the previous section, this embedded assumption is indexed through a linguistic marker: a question that is not actually "doing questioning." In Heritage's terms, the speaking of the question produces Carissa's interpretation of the relative epistemic *status* between her and Jimmy. She has formulated a K+/K− imbalance.

Heritage goes on to describe epistemic *stance* as the ways in which conversants position themselves vis à vis the epistemic status implied. This stance is produced through turn design, or the way subsequent turns are constructed. Therefore, when Jimmy responds with "I know", he dissembles the assumed status by claiming equality with Carissa, redressing the imbalance. The analytic outcome concerning the social actions performed remains the same: Jimmy challenges the newsworthiness of the statement. However, Heritage (2012) offers a way of understanding the discursive mechanism by which Carissa and Jimmy accomplish this.

Furthermore, Heritage (2012) offers a way to show how conversants organize *topic* (Sidnell, 2012). Carissa's "guess what?" and Jimmy's "I know" serve as discursive markers to indicate to each other their understanding of a topic opened and closed. This aspect of Heritage's

(2012) study is important for us here. It provides a rationale for identifying the brief four line interaction as a complete unit in the same way that Sinclair and Coulthard's (1975) seminal work provided the Initiation-Response-Feedback (IRF) organization of classroom talk. Being able to identify "complete units" from the participant's perspective is useful when looking at a large corpus of interactions. It is important to remember that Heritage (2012) is not referencing a real knowledge imbalance, but an assumed or constructed one. He concludes with a metaphor: "Underlying the conception of this article is a kind of 'hydraulic' metaphor, according to which any turn that formulates a K+/K− imbalance between participants will warrant the production of talk that redresses the imbalance" (p. 49).

In corresponding studies, scholars have investigated interactional uses of claims to insufficient knowledge: "I don't know." In their pioneering study, Beach and Metzger (1997) identified examples where free-standing instances of "I don't know" were used to disattend to a topic initiated by another, functioning to regulate what would be talked about and even for how long. Hutchby (2002) found this was especially true when children interacted with adults serving as an effective form of resistance to the adult's agenda. Scholars have also identified regularities in discursive practice regarding verbal and non verbal expressions of insufficient knowledge: Heritage (2012) noticed that verbal productions made from a position of K− invited bystanders to offer their own "displays of knowledge" (Koole, 2010) that redressed the imbalance. On the other hand, student silences and gaze aversions in response to teachers' questions were frequently addressed with an "epistemic status check": "you don't know?" (Sert, 2013).

In talk, there is a presumption, or a norm, that speakers base their reports on actually knowing (Pomerantz, 1984). Therefore, a person making a declarative assertion such as "I know" is accountable for being right. This norm was tacit between Jimmy and Carissa: she did not challenge his “knowing” but allowed the topic to close. Pomerantz (1984) found that sometimes claims of insufficient knowledge were followed by explanations or justifications. Using the discursive practices of knowing in that way allowed the speaker to account for “not knowing” in a situation where he or she might be properly expected to know.

This type of rhetorical organization, where talk is produced in ways that attend to others’ assumptions, has only rarely been investigated in young children. Two studies are relevant to this chapter; both considered the interactions of pairs of children. Sanders and Freeman (1998) showed young children’s competence in fashioning conversational turns that addressed potential interactional consequences pre-emptively. They videotaped 5- to 7-year-old dyads playing with Lego building toys. In every pair, at least one child repeatedly produced such utterances, “and all the children cooperated, in such a way as to avoid overt conflict in the local moment and achieve coordinated effort for sustained periods of time, despite generally, recurrently, facing each other with conflicting wants, and issues of parity and control” (p. 87). Barwell (2002) found similar results with pairs of 11 – 12 year olds working together to compose mathematics problems to be solved by their peers. His work with speakers of English-as-an-additional-language highlights the importance of restricting data analysis to what is actually spoken rather than speculating regarding what might be happening within children’s minds as they interact. This approach accounts for a requirement for rigour in data analysis that attempts to explore children’s perspectives on their own mathematical practice (Barwell, 2009).

### 4.2.3 Mathematical negotiation

My use of the term *mathematical negotiation* should not be confused with formal notions of mathematical argument or proofs, although they share certain aspects in common, such as an emphasis on rational argument. Krummheuer (1995, 2000, 2007, 2011, and 2013) has conducted considerable research on the development of formal mathematical argumentation during the primary years. He identified the age range of 5-to 7-years as critical for the development of skills in narrative argumentation as opposed to the more diagrammatic argumentation preferred by younger students (Krummheuer, 2013). That makes this age group a key period in which to document the discursive practices of mathematical negotiation.

I use the terms *negotiation of sociomathematical norms* and *mathematical negotiation* interchangeably here, in the sense of being reflexively related. Both terms represent processes of negotiating meaning and both processes function to facilitate communication, participation and learning. My interest in the negotiation of mathematical knowledge is broader than simply the structure of logic, incorporating the interactional accomplishment of subjectivity: "From this perspective, mathematical meaning is not taken as existing independently from the acting individuals and from their interaction, but is viewed as accomplished in the course of social interaction" (Cobb & Bauersfeld, 1995, p. 296).

In order to document the discursive strategies young children use when they participate in mathematical negotiation, I will draw on the analytic tools of conversation analysis (Cameron, 2001). As Bauersfeld (1993) specifies, the core implications of sociomathematical norms exist at the metalevel and arise from indirect learning: "The core of what is learned through participation is when to do what and how to do it" (p. 4). The very process of reification that allows communication to function as smoothly as it does also obscures the particular features of it (Wenger, 1998). In other words, it is the taken-for-grantedness of sociomathematical norms that produces their interactional power. Therefore, the tools of conversation analysis will support the interpretations here by foregrounding those aspects of communication that often get taken for granted, such as turn design (Schegloff, 2007) and tacit agreement (Cameron, 2001).

In the following sections, I explore how a group of young children use features of the discursive practices of knowing as interactional resources in an inquiry-based mathematics research setting. I am interested to examine how *knowing* is discursively produced and deployed in mathematical discussions and how patterns in those discursive practices might provide insights into children's participation in mathematical negotiation. Through this analysis, I hope to establish whether or not the distinction between *mathematical knowledge* and *mathematical knowing* is meaningful for those involved in the practice of mathematical negotiation.

### 4.3 Method

The data considered in this chapter was generated as part of the N:Countr study that explored representations that constitute young children's mathematical thinking. The inquiry-based mathematics research group was comprised of ten children (5- to 7-years-old) and me as a participant-researcher ("R" in the transcripts). I presented myself to the children as an interested, educated adult, but neither a mathematics teacher nor a parent. In fact, the children all had a previous multi-year teacher/student relationship with me. As a formal preschool learning community, we shared a history of exploring aspects of our mutual interests together. However, while we had certainly considered mathematical interests during the earlier years, they were never signified as such. On the other hand, the eleven-week-long research group was decidedly concerned with exploring "What children know about numbers and math" and began with explicit negotiations of what toys, equipment, and by extension activities, we would agree to call "doing math." The richness of the setting is that it incorporated aspects of both home and school, while foregrounding children's curiosity and agency.

After that first week of explicit negotiations, the children engaged in a variety of activities including play, art and construction work, individual and small group conversations with and without me and whole group discussions that I facilitated. Various child- and adult-produced artifacts became the focus of discussion at different times and are included here if relevant. The video data was transcribed using whole word conventions that involved the exclusion of all partial-words, hesitations, false starts and fillers (Bishop, 2008) to produce a partially idealized corpus of approximately 130,000 words generated over eleven hours of interactions (Gray & Biber, 2011). Simple word searches were performed on the transcriptions to identify uses of the term *know* in its various forms: knowing, knew, you know, I don't know etc. Those findings are represented here in Table 4.1.

**Table 4.1 Number of times "know" is used, by week with researcher/child distinction**

| **Week** | ***know* total**[7] | ***I know*** | ***you know*** | ***don't know*** | ***knows*** | ***knew*** |
|---|---|---|---|---|---|---|
| **1** | 87 | 20 (R12, **C8**) | 31 (R28, **C3**) | 10 (R2, **C8**) | 6 (R5, **C1**) | 4 (R4) |
| **2** | 55 | 13 (R9, **C4**) | 21 (R17, **C4**) | 6 (R5, **C1**) | 1 (R1) | 2 (R2) |
| **3** | 66 | 12 (R3, **C3**) | 34 (R28, **C6**) | 6 (R3, **C3**) | 1 (R1) | 1 (R1) |
| **4** | 133 | 20 (R10, **C10**) | 45 (R43, **C2**) | 22 (R19, **C3**) | 6 (R4, **C2**) | 8 (R5, **C3**) |
| **5** | 53 | 15 (R9, **C6**) | 20 (R18, **C2**) | 10 (R5, **C6**) | 2 (R2) | 0 |
| **6** | 58 | 8 (R5, **C3**) | 28 (R27, **C1**) | 8 (R6, **C2**) | 2 (R2) | 13 (R8, **C5**) |
| **7** | 57 | 9 (R4, **C5**) | 28 (R24, **C4**) | 9 (R4, **C5**) | 2 (R2) | 2 (**C2**) |
| **8** | 75 | 14 (R9, **C5**) | 33 (R27, **C6**) | 15 (R10, **C5**) | 4 (R4) | 3 (**C3**) |
| **9** | 76 | 15 (R9, **C6**) | 34 (R28, **C6**) | 7 (R3, **C4**) | 3 (R2, **C1**) | 3 (R3) |
| **10** | 101 | 20 (R9, **C11**) | 34 (R31, **C3**) | 17 (R7, **C10**) | 0 | 2 (R1, **C1**) |
| **11** | 97 | 12 9R8, **C4**) | 26 (R21, **C5**) | 23 (R12, **C11**) | 0 | 1 (R1) |

[7] Total refers to number of times used this week and does not represent the mathematical total of the rows, due to unspecified uses of "know": I want to know, she doesn't really know etc.

The contents of each cell can be interpreted as the total number of uses of the term during that week and the relative number of uses by adult and child. Therefore, the third column, top cell which reads “20 (R12, **C8**)” should be understood as “There were 20 uses of the term ‘I know’ during the first week. I produced 12 and the children altogether produced 8.” It quickly becomes apparent that I produced most instances of the term *know*, in any form (about 79%). However, there were at least 181 uses of the term by the children (bolded in the chart and in the examples), and I focus on those utterances here. These became the data from which I discerned the linguistic regularities produced by the children as they contributed to *doing knowing*. Examples of interactions were extracted from the larger corpus to provide illustrations here and were then transcribed using simplified conventions of Atkinson and Heritage (1984, see Appendix A). In all of the examples, claims of knowing made by children and displays of knowledge produced by the children are marked with an arrow to the left of the line number. All uses of the verb “to know” are bolded (n.b., prosodic emphasis is marked with underline).

Weeks 4 and 10 stand out as having more productions of the word “know” than other weeks. These weeks produced negotiations that are particularly interesting from a mathematics point of view and will be examined more closely in Chapters 5 and 6 of this dissertation. They are also weeks in which the children produced as many or more instances of “I know” as did I, suggesting a rich resource to investigate children’s contributions to mathematical practices. The analysis in this chapter focuses on the overall linguistic variation evident in the corpus of interactions. In that way I am able to elaborate features of the group’s culture of negotiation while accounting for the reciprocal relationship between the negotiation of knowledge and the negotiation of mathematical norms. I begin with corpus analysis to identify and describe interactional regularities and then continue with conversation analysis to examine the small number of irregularities thus identified.

## 4.4 Results

Discourse analysis of the 181 child-spoken utterances highlights regularities regarding how the discursive practices of *displays of knowledge* and *claims of knowing* were produced and deployed (Koole, 2010; Sacks, 1992). Furthermore, in this setting, a sociomathematical norm regarding accuracy was invoked by one child on three separate occasions, where it is marked by a discursive irregularity. Close examination of both regularities and irregularities highlights some of the discursive practices used and deployed by children to produce their participation in mathematical negotiation and provide insights into a meaningful distinction between mathematical *knowing* and mathematical *knowledge*.

### 4.4.1 RQ1: How is *knowing* discursively produced and deployed in mathematical discussion?

The two most common discursive practices related to knowing produced by the children in this data were *claims of knowing* and *displays of knowledge* (Koole, 2010; Sacks, 1992). This is a function of my analytic decision to limit myself to uses of the verb "to know", but also offers a useful way to organize a complex data set. A number of relationships between the two practices emerged during the analysis. Due to the limited space of a dissertation, in this study I focus on displays of knowledge in order to highlight the regularities in practice (i.e., interactions which the participants treated as typical or unproblematic). I make this choice because there were four sequential regularities surrounding children's displays of knowledge that became significant with regards to interactional irregularities. Therefore, I focus on these four sequential regularities in order to answer the first research question of the study in Chapter 4 (see Figure 4.1). The columns in the figure represent turn sequencing (first turn, second turn, third turn) and therefore indicate different speakers.

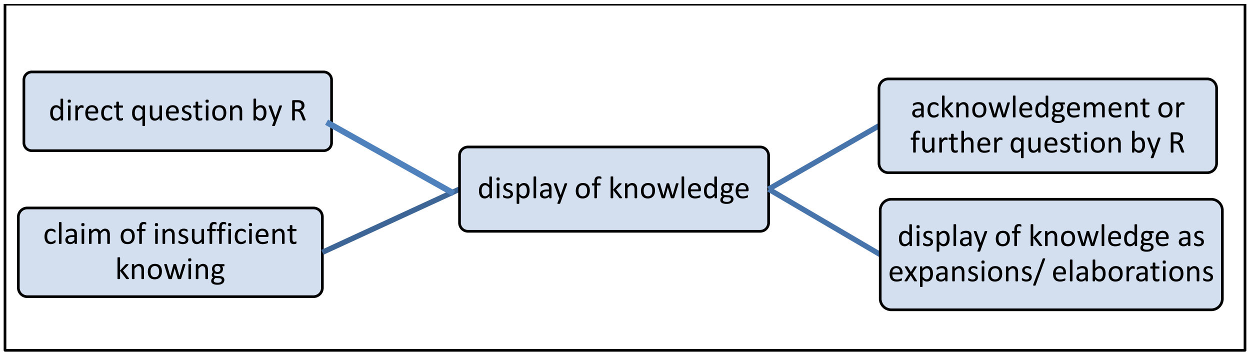


**Figure 4.1 Typical sequencing of the turn preceding and turn following a display of knowledge**

There were two typical sequences in the data regarding what might precede a child's display of knowledge and two for what might follow:

1. a child could offer a display of knowledge following a direct question by me; under these circumstances, the child's display of knowledge was almost always followed by an acknowledgement or further question by me;
2. a child could follow another child's claim of insufficient knowing (I don't know) with a display of knowledge; under these circumstances, the first display of knowledge was sometimes followed by a display of knowledge from another child that elaborated or extended the first display.

It is important to recognize that the figure includes only *some* of the interactional regularities. For instance, a child might respond to a direct question by me with a claim of insufficient knowing (e.g., "I don't know"), which was then followed by another child's display of knowledge. The figure is only intended to indicate the significance of what immediately preceded or immediately followed a child's display of knowledge.

#### 4.4.1.1 Displays of knowledge deployed following direct question by R

Throughout the eleven weeks, I continually asked the children to articulate their mathematical knowledge, with Epistemic Status Checks (Sert, 2013) such as "Do you know about...?" but also "What do you know about...?" and "How do you know...?" When a child produced a display of knowledge in response, the sequence was typically followed up with either an acknowledgement or a further question by me. This sequence is represented by following the path along the top row in Figure 4.1. This corresponds to the initiation – response – feedback (IRF) sequence identified by Sinclair and Coulthard (1975) and represents a classic indication of traditional classroom interaction, although the versions spoken here included acknowledging feedback by me rather than evaluations (i.e., I never responded with "good" or "correct"). For example, this interaction occurred while two boys were playing a board game (Daniel 7y3m):

**Example 4.2 Daniel Week 6**

577 R: I just want to ask, um, Daniel and Nathan the one question first.<br>
578 Daniel: eight nine ten.<br>
579 R: how did **you know** that was ten?<br>
580 Daniel: what? [because=<br>
581 R: [how do **you know**<br>
→582 Daniel: =six plus four equals ten?<br>
583 R: ah. oka:y. oka:y.

Immediately prior to the interaction, Daniel had thrown two dice and quietly moved his game piece. I asked him to articulate how he knew to move ten spaces. This is consistent with Pomerantz's (1984) observation that speakers can be held accountable for the authority behind their statements if that authority is not readily inferable. I asked Daniel to account for his action and he responded with a display of knowledge that referenced arithmetical facts as the authority.

Another form of acknowledgement by me included re-voicing the display of knowledge. For example, the interactions in Example 4.3 occurred during a whole group discussion when I was introducing the concept of zero by drawing its representation (Anya 7y7m; Carlyn 6y6m):

**Example 4.3 Anya and Carlyn Week 6**

750 R: *((holding open notebook on chest, facing the group seated on the floor in front of her))* okay, ready? here. I'm going to draw it for you.<br>
751 (0.8)<br>
752 R: this is <u>not a letter</u>.<br>
753 (1.0)<br>
754 R: it's not reading, this is math.<br>
755 (0.4)<br>
756 R: tell me, <u>do **you know**</u> what this means?<br>
757 (2.4) *((begins to draw on the page))*<br>
758 R: I [can't do it upside down.<br>
759 Anya: [hmh.<br>
760 Carlyn: why don't you turn the pi-<br>
761 (1.0)<br>
762 Carlyn: paper.<br>
763 (4.2) *((R finishes drawing and looks up. Carlyn and Carissa raise their hands))*<br>
→764 Anya: zero?<br>
→765 Carlyn: [[zero.<br>
766 R: [[zero.

In Example 4.3, I began with four different pre-turns oriented to drawing attention (line 750). I then referred to previous discussions regarding letters in math (lines 752 & 754). In both Examples 4.2 and 4.3, I initiated the interaction and then responded immediately to the displays of knowledge with positive feedback, thereby signifying sufficiency in the answer.

Example 4.3 is the closest example to the elicitation pattern of interaction (Voigt, 1995) in this data. It represents one of the few times where I seemed to know in advance what specific display of knowledge would satisfy. For Example 4.2, a response such as “I counted the dots on the dice” would have sufficed to answer the question “How did you know?” (Although it would have referenced a different source of authority: mathematical procedure as opposed to arithmetical facts). For Example 4.3, “zero” was the only response acceptable, as indicated by my orientation: I repeated the response, rather than simply acknowledging it.

Although Sert (2013) identified Epistemic Status Checks as common teacher responses to student gaze aversion or silence, the questions he observed were always spoken in the negative form: “You don’t know?” or “no idea?” Positively worded questions inviting displays of knowledge (as in this data: “Do you know about...?”) are rare in both classroom and everyday talk (Koole, 2010). Their prevalence here speaks to the way participants oriented to this setting. We were participating in *doing mathematics research group* as opposed to *doing classroom* or *doing friends/family*.

#### 4.4.1.2 Displays of knowledge deployed following a claim of insufficient knowing

When a child produced a claim of insufficient knowing in answer to a direct question (e.g., “do you know…? – “no” or “I don’t know”), it seemed to prompt other children within hearing to produce their own displays of knowledge. This corresponds with Heritage’s (2012) concept of K−/K+ imbalance redressing. Example 4.4 shows a group of children during an informal discussion based around Carissa’s explorations with the calculator. Previously Daniel and I considered what might be the largest number you could “do on a calculator.” The discussion here is incited by Carissa coming to show me what she calls “the biggest number you can get in this.” Although unclear in the video recording, it may have been 10,000,000 since the calculator allows for eight digits. The other children gather around the calculator to see and the following interactions occur (Truman 5y11m):

**Example 4.4 Carissa, Anya, Truman, Carlyn and Daniel Week 8**

→386 Carissa: this is the biggest number you can get, in this.
387 R: [[okay, so this is the biggest number you can get in this. and
388 Daniel: [[whoa.
390 (0.6)
391 R: do **you know** what the number is? Carissa?
392 (2.0)
→393 Carissa: [[I **don't know**.
→394 Truman: [[a thousand?
→395 Anya: one thousand.
396 R: is it one thousand?
→397 Truman: [[one thousand and beyond.
→398 Carlyn: [[one thousand and one.
399 R: one thousand and beyond, or [you think one thousand and one?
→400 Daniel: [if, if you, if you put one more it would be - it would be, a, a billion.
401 (0.6)
402 R: if you put one more zero, it would be a billion? how do **you know** all this stuff.
→403 Daniel: I **don't know**.?

A rapid succession of displays of knowledge (lines 394, 395, 397, 398, 400) followed Carissa's "I don't know" (line 393). This interactional sequence is represented in the bottom row of Figure 4.1. Example 4.4 includes two claims of insufficient knowing (lines 393 and 403). It is interesting to note that Carissa's version invited others to produce their own displays of knowledge, while Daniel's version closed the interaction (the topic changed immediately following line 403). I notice a slight difference between the two questions leading up to these claims: "do you know what the number is?" and "how do you know all this stuff?" and I wonder if the difference indexes knowledge (the first question) versus knowing (the second question). If that is so, this is the first indication that there was a meaningful distinction between the two.

It is important to remember not to interpret claims of insufficient knowing as reliable indicators of what the children really know (especially given Daniel's rising intonation in this example). The research focus here is on how language is used rather than how it might correspond to some version of reality. Therefore, these examples are meant to show only how displays of knowledge and claims of knowing were exploited as interactional resources during mathematical discussion.

When a child produced a display of knowledge within the hearing of other children, it seemed to invite other displays of knowledge. Example 4.4 illustrates this, as children elaborated and extended given displays by introducing displays of their own without contradicting or challenging the previous utterances (cf. lines 391-400).

#### 4.4.1.3 Regularities in the discursive practices of knowing

The previous examples illustrate some of the regularities in the ways *mathematical knowing* was produced and deployed in the data. Among several other possibilities, I focused on the discursive practices of displays of knowledge because they represent the form of “doing knowing” that is implicated in sequential irregularities. Four relationships in the sequencing surrounding displays of knowledge emerged during this analysis (see Figure 4.1). Children demonstrated their mathematical knowing with displays of knowledge that were produced in response to my questions or following indications of insufficient knowing made by other children. The children’s displays of knowledge were unproblematically followed by my acknowledgement or other children’s displays of knowledge that elaborated or extended the first display. These four relationships inform our understanding of the interactive processes involved in learning-as-participation. As regularities in the discursive practices, they provide a backdrop against which to notice irregularities and provide an answer to the first research question: *How is knowing discursively produced and deployed in mathematical discussion?*

The next section examines the children’s participation in the negotiation of sociomathematical norms as it is marked with irregularities in their discursive practices. Norms introduce culpability, or a sense of responsibility for which someone can be held accountable in the case of a violation. Invoking a norm or designing an interactional turn to account for that possibility can therefore provide a source for recognizing participants' assumptions. Therefore, in the following section, I identify four irregularities in the corpus and then narrow the analytic focus from the corpus to the conversations in order to answer the second research question: *How do patterns in these discursive practices relate to children's participation in mathematical negotiation?*

#### 4.4.2 RQ2: How do patterns in those discursive practices relate to children's participation in mathematical negotiation?

In earlier sections, I illustrated how displays of knowledge were typically preceded by my questions or a claim of insufficient knowing by a child and followed by my acknowledgements or elaborations by other children. However, within this group there were a few times when displays of knowledge were challenged, sometimes with other displays of knowledge. A form of challenge appeared in the sequential regularities: refuting the novelty or newsworthiness of a claim (cf. example 4.1 where Jimmy responded with "I know"). However, such refutations did not challenge the knowledge itself, but rather the nature of the claim to knowing.

There are four examples within the data where children challenged the knowledge itself. These challenges were produced as a counterclaim, a contrast, a probe and an objection. I examine each challenge in turn in order to ascertain the types of sequential irregularity that are implicated and the clues they might provide regarding a meaningful distinction between *mathematical knowing* and *mathematical knowledge*.

#### 4.4.2.1 A counterclaim

The first irregularity considered here comes from an interaction between Cormac and me, where he challenged a display of knowledge I had produced as a re-voicing of the children's ideas. (In the figures that follow, the irregularity in sequencing is marked with a saw tooth symbol superimposed over the connecting line):

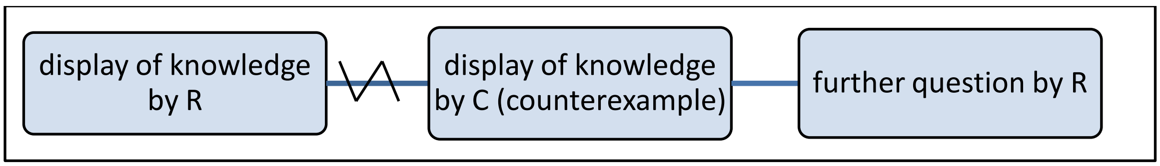


**Figure 4.2 First irregularity: Display followed by display produced as a counterexample**

The sequence of *question-response* (e.g., the first part of the interactional regularity discussed previously) is an adjacency pair, one of the most common forms of conversation organization. Displays of knowledge produced in this form were prevalent during a whole group discussion in the first week regarding "What do children know about math?" Initially the children produced a string of arithmetic facts in response: "1 plus 1 is 2", "2 plus 2 is 4", "4 plus 4 is 8" and so on. In this way, they displayed their knowledge. The following example (Example 4.5) comes from the same whole group discussion, after I revoiced the children's observation that there were no letters in math. Cormac's display of knowledge in line 156 is produced in contrast to the generalization made by me. I had noted "There are no letters in math" (line 147) as a revoicing of the children's negative response to the question: "Are there any letters in math?" (line 142 – Cormac 7y4m):

**Example 4.5 Cormac Week 1**

→156 Cormac: but there is,[kind of like an "x".  
157 R: [*((open handed, palm up points towards Cormac))*  
158 (1.0)  
159 R: but there is, kind of like an "x".  
160 (0.4)  
161 R: what do **you know** about the "x", that's in math.  
162 (0.8)  
163 R: wha - why is it kind of like an "x". do **you know**, Cormac?  
164 (1.0)  
165 Cormac: um  
166 (1.4)  
→167 Cormac: I **don't know** what sign it's for? but  
168 (1.2) *((R nods head))*  
→169 Cormac: it's one of the, things like, a plus and a minus?  
170 R: oka:y, it's one of the [things like a plus and a minus and-  
→171 Cormac: [only sort of.  
172 R: and it looks like an "x".  
173 (0.8) *((*Cormac *nods head))*  
174 R: oka:y.

Cormac challenged my generalization by producing a counterclaim (line 156: but there is, kind of like an "x"). By challenging me in this way and by responding the way I did, we make relevant a negotiation of mathematical knowing. His response to my question in lines 167 and 169 display his knowledge. Cormac's further response in line 171 qualifies my revoicing of his display of knowledge and refers to that aspect of his original claim that I questioned (line 163: Why is it kind of like, an "x." Do you know, Cormac?) His qualification is not explored further in the discussion. However, the uncertainty he introduced is represented in the radial diagram I drew to represent this conversation, in the form of a question mark written after a statement that was verbally produced as a declarative (cf. Figure 1.1, the written form *it does not have letters in it?* with the verbal negatives: R: Are there any letters in math? Multiple children: No. There's no letters in math).

Cormac's challenge references a sociomathematical norm regarding logic in mathematical argument: for a generalization to be true there must not be even a single counterexample. He challenges my display of knowledge (actually just revoicing on behalf of the group) not by saying "That's not true" or "yes there are." He simply *proves* it to be not true by producing the counterexample through what I have called a display of knowledge. I tacitly concur with the reference to that norm, and the example that disavows my generalization, by incorporating Cormac's display of knowledge into my response (lines 159-161: But there is kind of like an "x." What do you know about the "x" that's in math.)

This appeal to mathematical logic was the only occurrence that involved me directly, but it was not the only time it occurred. I present here three occasions where Daniel challenged Carissa's display of knowledge on the basis of a sociomathematical norm regarding accuracy. In Week 4, he exposed a discrepancy in her proposition by providing a logical continuation to a recitation of members of a set (alphabet letters) that invalidated the argument being made (I call this "exposing an inconsistency"). In Week 7, Daniel produced a probe to challenge the same type of mathematical argument but acceded to a clarification produced by Carlyn, and in Week 9, he objected to an explanation on the basis of a miscount that weakened the algebraic pattern represented. Each interaction unfolded differently.

#### 4.4.2.2 Exposing an inconsistency

The second irregularity considered here comes from an interaction between Carissa, Daniel and me during a small group classification activity. Daniel exposes an inconsistency in Carissa's logic by re-reporting an accurate version of her source of authority for knowing (see Figure 4.3).

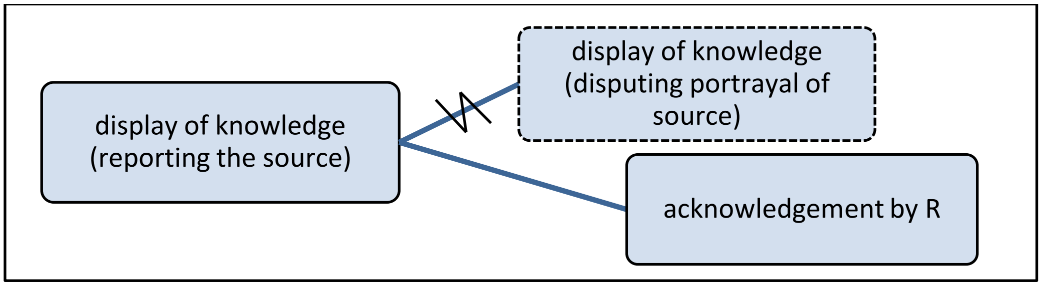


**Figure 4.3 Second irregularity: Display followed by display produced as a contrast**

Leading up to Example 4.6, Carissa and Daniel are working within a small group completing a classification activity. Carissa works quietly on the floor for several minutes beside the others as conversations continue around her. As the example begins, she interjects her comment within an ongoing conversation between Anya, Daniel, Nathan and me by producing an elaborated claim of knowing (called thus because she simply claims to know, without providing evidence in that turn). Elaborated claims of knowing were produced in the data as a preface to displays of knowledge. In this way the claims of knowing functioned to frame the following turn, suggesting to hearers how they ought to interpret the upcoming display of knowledge (the ratified hearer was always me in these examples). This speaks to recipient design, or ways in which speakers construct their turn to anticipate hearer interpretation.

After receiving my acknowledgement, Carissa produces her display of knowledge. Trying to make sense of the use of letters in mathematics was an ongoing theme for this group (cf. Example 4.5). The resolution Carissa proposes is to quantify the letters according to alphabetic order, thus her announcement that "this is a higher number for this" (line 386).

**Example 4.6 Carissa and Daniel Week 4**

384 Carissa: *((bringing her paper over to show R))* give me some. now **I know** something.
385 R: okay.
386 Carissa: look. *((pointing to the letters on her page))* this is a higher. number for this.
387 R: ah-hah. (*pointing to her paper*) so this "x" is a higher number, for the "v".
388 Carissa: yes.
389 R: Ah::?.
→390 Carissa: 'cause. ((*singing*)) A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R. *((pointing deliberately for each subsequent letter))* X. T, U, V.
391 (2.2)
→392 Daniel: [[W, X.
393 R: [[o:::h, look at that. she figured out where they were in the alphabet and she knows the "x" comes first. and the "v" comes later.
394 Carissa: yes.
395 R: you figured something out ((*Carissa nods*)) oka::y. well,
396 Jimmy: Nanny. where's Carlyn's other picture?
397 R: I **don't know**, [Truman **know**s. It must be right there beside.]
398 Carissa:*((holds her page up to R))*[if I never saw this.] if I never saw this, I wouldn't never have kno- **known** that.
399 *((Jimmy and Truman talk in background))*
400 R: You would have never thought of that idea, Carissa, but you did think of that idea. ((*Carissa nods and smiles*)) wow. well, can- can you cut out. around. this and decide which area you want to stick it in? do we **know** about it, do we not **know** about it, or, we do **know** about it but not in math. *((turning to look at Anya, who has remained beside R on the floor during the entire exchange with Carissa))* so: the equals.

When Daniel draws attention to Carissa's inaccuracy, it is offered after a 2 second silence (the only such silence in the interaction) and simultaneously with my utterance. My "oh" is a very strongly articulated (really lengthened) change-of-state token (Schiffrin, 1987) given by an authority figure. Nevertheless, Daniel overlays that response with a continuation of Carissa's explanation, carrying on where she left off, yet, in a contrasting way, stressing the second occurrence of the letter "x." This is an irregular pattern of interaction in this group, where explanations have typically been listened through, and accepted.

There was a high degree of indexicality in Carissa's hypothesis and I spoke her gestures aloud during line 387, making them available to others in the room as well as to the camera. Carissa affirms the revoicing (line 388: yes). I respond with an elongated change-of-state token (line 389: ↑↓Ah::. – signaling a change in my own epistemic status from K− to K+) and Carissa continues with a further explanation. This type of explanation is rhetorically designed to account for a challenge where the validity of an assertion is called into question or where a speaker assumes that the source of her knowing may not be inferable from the context (Pomerantz, 1984). Carissa attempts to "head off" these types of interactional problems by pre-emptively reporting the source of her knowing: alphabetic order. All other speech has stopped as everyone attends to Carissa's singing recitation of the alphabet. When she gets to the place usually reserved for "s", she recites a pronounced "x" while pointing to the same letter on her paper. This is followed with "t", "u" and finally Carissa points to and says "v." Then she stops.

I argue that Daniel's challenge is based on mathematical logic, because he spoke aloud an accurate continuation to her recitation, emphasizing the "x" in it by stopping there, rather than simply stating: *You said "x" instead of "s"*, finishing the alphabet or challenging my revoicing of a mistake. His challenge is based on two fundamental principles regarding accuracy:

1. In order for a mathematical claim in total to be valid, every part of it must be accurate.
2. When a set includes each member once (as the alphabet does) then a repetition of any one member is incorrect.

It is interesting to note that Carissa's original proposition fit with conventional alphabetic order. It was only her explanation that introduced the confusion and produced the resulting inaccuracy.

I reinforced that inaccuracy by ignoring Daniel's comment and indeed, revoicing Carissa's conclusion using the mistaken alphabetic order (cf. lines 387 and 393). So, in this case, Daniel's invoking of a sociomathematical norm regarding accuracy was overtly ignored with no apparent effect on the subsequent interactions. Functionally, it was invisible and inconsequential at the time (illustrated in Figure 4.3 with a dashed outline). A second case occurred three weeks later.

#### 4.4.2.3 A probing challenge.

The third irregularity considered here comes from an interaction between Carissa, Daniel, Carlyn and me during a whole group discussion in Week 7. Daniel again challenges Carissa's invoking of the alphabet as the source of her authority for knowing. However, this time he accedes to a clarification provided by Carlyn (see Figure 4.4).

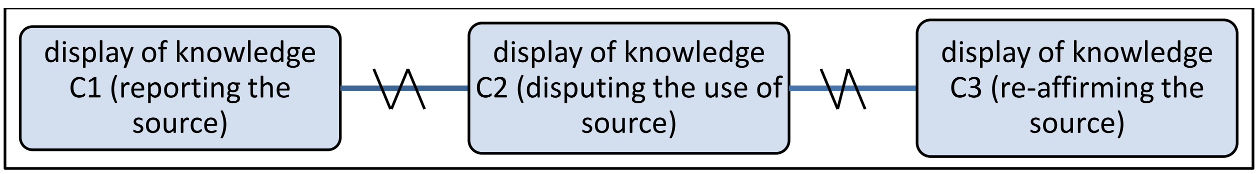


**Figure 4.4 Third irregularity: Display followed by display produced as a probe, followed by display from third child**

This discursive probing occurred during a whole group discussion concerning a carpentry book from the library. The children were examining the instructions beside the photographs of wooden toys, which included exploded drawings, measurements and letters. In order to produce a similar book of their own, they were trying to make sense of this configuration:

**Example 4.7 Carissa, Daniel and Carlyn Week 7**

100 R: *((sitting on small chair, holding a do-it-yourself carpentry book open in front of her to show the children who are seated on the floor))* there's this way, of drawing the pictures, that's called, an exploded drawing. can you see what they did? like, there's the photograph, of the toy when it's finished. and here's

the drawing they made. it's like, they pretended the pieces were not glued together they pretended the pieces were not nailed together?

101 (1.2) *((R looks around the group))*
102 R: and they drew it like that. *((continues to flip the pages))*
103 (1.0)
104 R: all of them have - well, and - they have [photographs,]
105 Carissa: *((rises to her knees))* [oh. ]
106 R: I can take the photographs [if you're making it]
→107 Carissa: *((raises her hand))* [**I know]**
108 R: if you can draw the pictures, but look at these kind of pictures, see what I mean? *((turns pages in the book))*
109 Daniel: whoa.
→110 Carissa: **I know** *((sitting on her knees, hand still raised))*
111 R: wh - like, they exploded it - all the pieces out. just for pretend, in the drawing. *((looks at Carissa))* yes, Carissa.
112 (0.8)
→113 Carissa: **I know** how to, *((stands up and walks towards the book))* make this. see the letters? *((points to page in the book))*
114 R: yes? *((Daniel stands up behind Carissa))*
→115 Carissa: you go to "A" fr - to "Zee". or [whatever,]
116 R: *((Nathan stands up beside Daniel))* [a:h?.]
117 Carissa: letter it's that.
→118 Daniel: well there's two, um, "A"s. *((points to book))* like, "A" - one "A" there [and two "A"s.]
→119 Carlyn: [yeah, you would have to do "A" first] and then ( )
120 Carissa: "A" to "Zee"
121 R: "A" to "Zee" or "A" to "Zed", right?
122 *((overlapping unintelligible voices))*
123 R: okay, can you sit? *((motioning to sit down))*it might be. Carissa and Daniel? it might be, because they're telling you, when - when you're going to - oh, *((points to Carlyn))*what were you going to say, Carlyn?
→124 Carlyn: what to, con- connect first.
125 R: ah ha:.
126 Danica: *((raises her hand))* um.
127 R: so, which one would you connect first?
→128 Carlyn: um, the "A"s.
129 R: okay, so if you see any "A"s it means connect these first, cause they're "A". and then what would you connect next?
→130 Daniel & others: "B"s.
131 R: then you can connect "B"s. And then what would you connect next?
→132 Carissa & others: "C"s.
133 R: okay, so it's a way of teaching someone how to make that kind of a toy.

The observation Daniel produced in line 118 functioned to probe Carissa's display of knowledge. She had, more explicitly this time, implied the set of alphabet letters (line 115: You go from "A" to "Z"). He challenged her reasoning, or the source of her authority for knowing, again by noticing double letters. This irregularity was not produced as simultaneous talk. Interactionally it looked more typical of this group. I call it an irregularity because it represents a display of knowledge followed by another that calls into question the use of a source of authority for knowing, rather than expanding on the first display (as in Example 4.4 or Figure 4.1). It also represents one time the group negotiated a solution to a challenge that could not be attributed to any one person alone (cf. Voigt, 1995). Daniel's challenge was answered, not by Carissa, but by Carlyn, whose first explanation was cut off (line 119: yeah, you would have to do "A" first and then). Finally, she completed an explanation that I began (line 123: they're telling you – line 124: what to connect first). With that explanation, Carlyn accounted for the sociomathematical norm of accuracy by allowing the sequence of alphabetic order without requiring the full set. Thus, she renamed the "set" as being only part of the whole alphabet and *requiring* doubles. Both Daniel and Carissa accede to this, producing a tacit agreement by vocally continuing the explanation (cf. lines 130 & 132).

The mathematics in this example is more subtle, since the discussion revolves around a diagram in a carpentry book. There are other perhaps more obviously mathematical aspects to the diagram (e.g., measuring the pieces or naming the geometrical shapes). Carissa's conjecture is neither precise nor complete, yet it introduces the possibility of using symbols (in this case letters) to represent cardinality. Her change of state token in line 105 and 107 (oh, I know) points to having constructed her knowing here-and-now (Koole, 2010) and she repeats her claims of knowing (line 110) until I acknowledge her (line 111: Yes, Carissa). Following Carissa's conjecture and Daniel's challenge, Carlyn expands Carissa's conjecture by including the cardinality term "first" in her procedural explanation (line 119: you would have to do "A" first). This accounts for a sociomathematical norm for precision, which seems to satisfy Daniel, since he drops the challenge. Carlyn's explanation allows the sequence of alphabetic order without requiring the entire set, and provides a reason for double letters.

#### 4.4.2.4 An objection.

The fourth irregularity considered here comes from an interaction between Carissa, Daniel and me during another whole group discussion, this time in Week 9. Daniel objected to an explanation Carissa produced based on a miscount (see Figure 4.5).

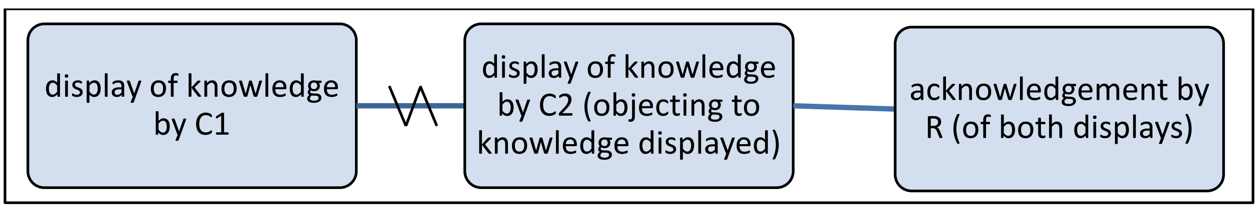


**Figure 4.5 Fourth irregularity: Display followed by display produced as an objection**

During the interaction in Example 4.8, the children were looking for patterns within a wall chart I had produced listing the results children had determined regarding square root using the calculators (see Figure 4.6). Since many of these utterances include gestures (mostly pointing) it is important to note that Carissa was sitting on the floor in front, within easy reach of the chart and Daniel was sitting at the back of the group and could only gesture towards the wall chart without actually touching it. This produced the effect of less precise gestures on his part.

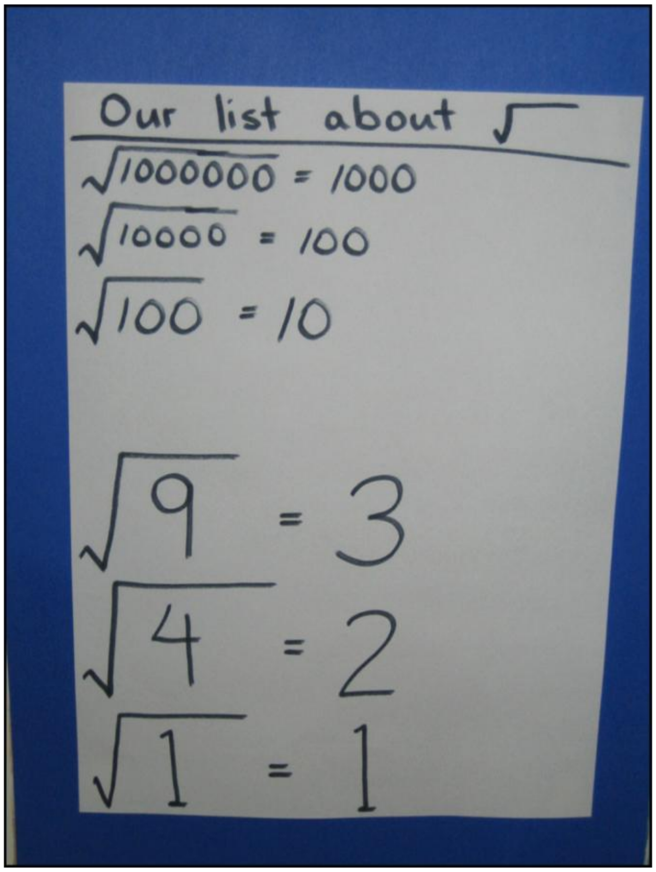


**Figure 4.6 Wall chart listing square root results to date Week 9**

**Example 4.8 Carissa and Daniel Week 9**

148 R: Carissa? *((gesturing towards Carissa))*[what's another one.]
149 Anya: [*((puts up her hand))*]
→150 Carissa: look. *((pointing to each of the zeros in 1,000,000 as she counts))*one two three four five six seven.
151 (1.0)
→152 Carissa: which. is, not, here, *((open handed gesture towards the chart))* but, *((pointing to the digits in 100))* one, two three
153 (1.2)
→154 Carissa: I mean, three?
155 (1.6)
156 Carissa: and
157 (1.0)
→158 Carissa: *((pointing to the lower 100))* three and, *((pointing to the 10))* ten
159 Daniel: *((rises on his knees and shuffles closer to squint at the chart))*

→160 Carissa: is not in there. *((open handed gesture towards chart))*

161 Daniel: *((drops back down to crouching but continues to squint toward the chart, nodding his head as if counting silently))*

162 (1.0)

163 R: mm. *((kneeling in front of her chair, hands folded across stomach, looking intently at chart))*

→164 Carissa: wait. that *((pointing to the upper 100))* one two three *((pointing to each digit in turn))*

→165 Daniel: *((rising again on his knees and pointing toward chart))* there's actually [six zeros there.

→166 Carissa: [one two three four. *((pointing to the digits in the 1,000))* there's one more than that. *((pointing to the upper 100))*

167 R: okay. *(Carissa sits down on her legs and looks up at R)* so you're comparing them this way, *((gesturing towards chart in an up and down motion))* saying that this one has one more than that. *((pointing to the 1,000 and 100 in turn))* and looking for ten in these ones *((points to the 10 and then gestures around the other numbers))*

168 Daniel: *((sits back down on his legs))*

169 Carlyn: *((tentatively raises hand and then slightly lowers it again))*

170 R: *((looks up at Daniel))* and you were mentioning that there's just actually six zeros in the top. *((pointing to the 1,000,000))*

171 Daniel: *((rises up on his knees again))*

172 Carlyn: *((finally lowers her hand completely))*

173 Danica: *((begins to twirl around on the floor on her hands and knees, moving away from group))*

174 R: hhh? I'm not sure if Carissa was trying to count the zeros, or, all the numbers.

175 (0.4)

176 R: *((looks down to Carissa))* which one

→177 Carissa: zeros.

178 R: oh, just the zeros, [yeah yeah

→179 Daniel: [there's six.

180 Carlyn: *((raises her hand again))*

181 R: then there's six in that one. *((pointing to the 1,000,000))*

182 (0.8)

183 R: and it turned [into three. *((pointing to the upper 1,000))*

184 Anya: [wh- um. *((raises her hand))*

185 R: so Carissa's making the comparison this way. *((gesturing up and down within the upper numbers))* and Nathan made this comparison. *((using two hands to gesture open handed to the upper and lower halves of the chart))*

186 Daniel: *((leans up against the wall with both hands))*

187 R: who else has an idea.

→188 Carlyn: I do.

189 R: Anya.

This time Daniel produces an objection (line 165: there's actually six zeros there), spoken in the middle of Carissa's extended turn. The many lines of silence (lines 151, 153, 155, 157 and 162) leading up to it speak to its significance as an irregularity in turn taking. Despite those five separate transition-relevance places (TRP) (Sacks, 1992) Carissa continues to hold the floor, as it were. She produces two displays of uncertainty in lines 152 (which, is, not, here) and 160 (is not in there). Her tone, the way the words are spread apart and her different hand gestures (open-handed as opposed to pointing) speak to that uncertainty. When Daniel speaks in line 165, Carissa interjects, without conceding to the usual conventions of TRP. Daniel's objection refers back to an early display of knowledge (line 150), and disputes the counting in it, calling to a mistake. His body postures and the time intervening indicate that he was counting before issuing the challenge.

Discussions involving zero had become a regular occurrence and the particular topic of how many zeros were in one million had been addressed earlier. During Week 8, Carissa had produced a claim of knowing ("and I already knew that") that referred to Daniel's comment that the symbol for one million had six zeros. This occurred just one week before the interactions in Example 4.8. Even though it is not clear that Carissa recognized the top number on the list as one million, it still highlights the difficulty of treating spoken discourse as if it represents reality.

During the interaction, I orient to Daniel's objection by acknowledging it in line 170 but then I offer Carissa a face saving gesture (Goffman, 1959) by suggesting that she might have been counting all the digits, not just the zeros. Carissa confirms Daniel's assumption in line 177 and he repeats the objection in the following turn, a move Carissa does not contest and I revoice in line 181.

This display of knowledge produced as an objection then represents another reference to a sociomathematical norm regarding accuracy. However, it is more than that. In Daniel's objection in line 165, he holds Carissa accountable for naming what she is counting by including the reference to "zero" in his display of knowledge. The confusion of Carissa's argument was, at least in part, brought about by the high degree of indexicality in her statements and the lack of reference to a unit. I orient to this same norm regarding precision when I voice the uncertainty in line 174 (I'm not sure if Carissa was trying to count the zeros, or all the numbers). This move invites Carissa to provide a clarification. When she does not, I specifically ask for one (line 176: which one).

The main purpose of this chapter is to examine how children participate in the negotiation of sociomathematical norms. What we see in these interactions is a high level of engagement, with each other and with the mathematics involved. The final section explores the functions of the irregularities noticed: what do they tell us about children's discursive participation in mathematical negotiation?

#### 4.4.2.5 The function of the irregularities

Conversation analysis turns around two foundational questions: *Why this?* and *Why now?* (Wetherell, 2001). One answer to these questions regarding the discursive irregularities highlighted in these results is that the invoking of sociomathematical norms in the form of challenges highlights instances where children interpreted that others were violating those norms. If Carissa had produced an accurate alphabetic order in Example 4.6 or an accurate counting of the zeros in "1,000,000" in Example 4.8, Daniel would have had no reason to challenge her. Yet, accuracy was important to the integrity of her arguments. Likewise, her explanation in Example 4.7 was not precise enough to make sense. Carlyn provided the precision necessary. Cormac's challenge of me regarding there being no letters in math (Example 4.4) also indicates assumptions he held regarding my own violations of a sociomathematical norm: a generalization can only be true if there is no counterexample.

However, another answer might be that these irregularities are about more than simply counting or alphabetical order. What these examples have in common is that they all revolve around *mathematical knowledge*. The normative references find significance in both the integrity of the argument and the source for mathematical validity, thereby speaking to the issue of mathematical *authority*. The negotiations involved deciding what this group would agree to value regarding a valid mathematical response. In Goffman's (1959) terms, we were negotiating "*whose claims* concerning *what* would be temporarily honoured" (p. 9f, emphasis added). The examples show that within the group, mathematical knowing (e.g., I know this…. I know that…. or other forms of displays of knowledge) was never problematized, unless the way the speaker produced that knowing involved an inaccurate (or potentially inaccurate, due to imprecision) portrayal of mathematical knowledge. The group co-constructed a taken-as-shared sense of what would come to count as mathematical knowing, but the authority for mathematical knowledge was invoked from some external source in the form of sociomathematical norms. I argue that this locally situated co-constitution of meaning indicates a meaningful distinction between mathematical knowing and mathematical knowledge. In other words, what comes to count as *knowing* within the group was something quite different than what comes to count as *knowledge*.

This interpretation is supported by a subsequent development in the group. In Example 4.8, Daniel's correction of Carissa's 7 to a 6 foregrounded the pattern of ratios, 6:3, 4:2 and 2:1 in the list on the wall chart (Figure 4.6). Later that same day Carlyn observed in her journal and while talking with me that this pattern was like taking away half. An inaccurate count would have precluded that mathematical observation. I am not referring to Daniel's intentions or saying he recognized that the challenges would play out in such a way. I am pointing out the social actions his utterances performed and the consequential mathematical functions they afforded. His challenge in Example 4.8 set up the possibility for Carlyn to make a conventionally valued mathematical observation regarding ratios.

Observations about negotiating whose claims would hold, points to the source for mathematical authority in the group. In this data, I documented a 7-year-old child initiating a norm regarding accuracy as being important in mathematical argument. It is worth noting that in this setting I was not invoking a norm regarding accuracy, since it would usually be the authority figure who would assert such a fundamental value. At the beginning of the results section, I noted that positively worded questions inviting displays of knowledge are uncommon in classrooms and in everyday talk. By using this form of question, I set up an expectation that metacognitive reflection might be appropriate. The way I phrased the questions made a thoughtful explanation relevant, even when a simple response might have sufficed. I argue here that this functioned to share the authority for *mathematical knowing* among the group and that when such authority is shared among a group of even young children, then the responsibility for holding each other accountable to principles such as accuracy and precision become a part of their mathematical practice. This constitutes meaningful participation.

Sharing the authority for mathematical knowing also allows for an environment where making mistakes or inaccuracies becomes part of learning, not something to impede participation. This interpretation is supported by an incident that occurred during the whole group discussion in Week 1 and was subsequently revisited. The radial diagram I drew to record a list of arithmetic facts produced by the children in Week 1 contained an arithmetical inaccuracy: “100 + 100 is 102” (this radial diagram was presented in Chapter 1 as Figure 1.1). The diagram itself remained on display for all eleven weeks and was the focus of explicit discussion at least three times. At no point did any of the children verbally notice or challenge that arithmetical inaccuracy. That is, until the very last minutes of the last session, when Jimmy (the originator of that particular display of knowledge in Week 1) was involved with a small group discussion at a table close by (Cormac, 7y4m):

**Example 4.9 Jimmy, Cormac and Truman Week 11**

998 R: when is math easy.
999 Cormac: when it's only one plus one.
1000 R: those kind of easy questions. and what's your idea Truman?
→1001 Truman: I **know** what a hundred plus a hundred is.
1002 R: is that easy for you? *((Truman nods once))* okay, what's a hundred plus a hundred?
→1003 Truman: two hundred.
1004 R: two hundred? *((Jimmy looks up at R))* a hundred plus a hundred is two hundred.
→1005 Cormac: [[it's almost like, one plus one.
→1006 Jimmy: [[actually, um, I (was wrong?) one hundred plus one hundred isn't a hundred and two, it's just two hundred.
1007 R: it is just two hundred. you match with Truman with your idea.
1008 *((Jimmy looks over to the wall and back to the table))*

After Truman supports his elaborated claim of knowing in line 1001 with the correct sum in line 1003, Jimmy produces his own correction, specific to the knowledge displayed on the chart. He alters his utterance away from a self-statement, to produce the correction as a mathematical object (line 1006: Actually, um, I was wrong – one hundred plus one hundred isn’t a hundred and two, it’s just two hundred) at the same time invoking a norm for precision by revoicing Truman’s and my own “a hundred” as “one hundred.” Whether the wall chart reminded Jimmy of his inaccuracy or whether he felt the significance of the inaccuracy because it was displayed in such a prominent way, or whether Truman’s sum brought his previous mistake to mind is impossible for us to know.

What we *can* say is that the way Jimmy produced his correction attends to a sociomathematical norm regarding accountability. By taking the responsibility for correctness onto himself the inaccuracy was presented in a more objective way, more like a conjecture, less like a mistake or something for which he might be held culpable. Furthermore, his self-correction attends to the change that might be involved in learning. This interpretation would hear his display of knowledge as saying "In the past, I knew that 100 + 100 was 102, but now I know that 100 + 100 is 200." Whereas Cormac was correcting me and Daniel corrected Carissa, Jimmy corrected himself, and in a way that allowed him to save face.

## 4.5 Discussion and conclusions

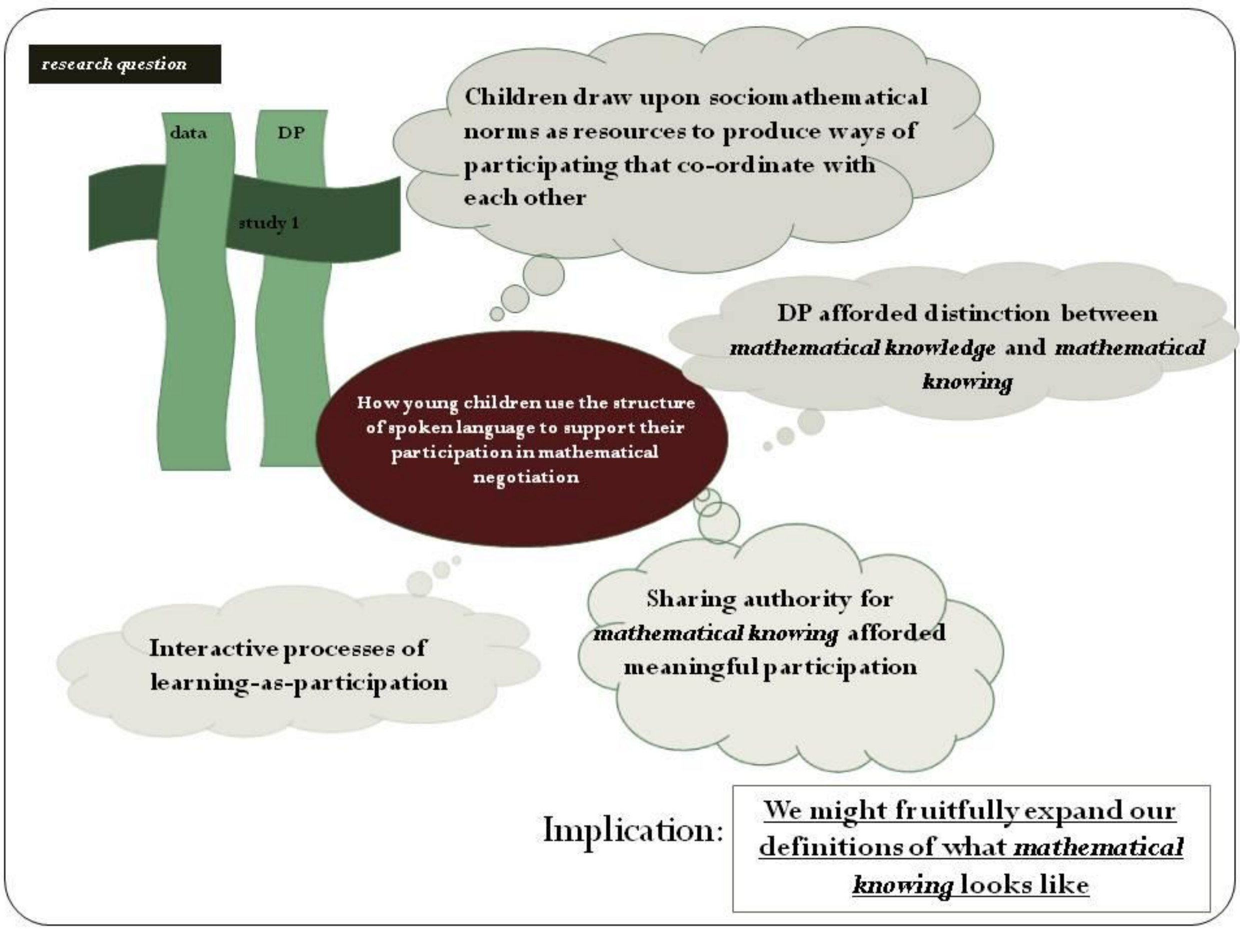


**Figure 4.7 Visual representation of main findings from Chapter 4**

This study has taken important first steps towards documenting and defining the processes involved in mathematical negotiation from the perspective of 5- to 7-year-olds by using discursive psychology to examine the complex relationship between the negotiation of knowledge and the negotiation of norms within inquiry-based learning environments. I have shown how regularities in the discursive practices called *displays of knowledge* served to support children's participation in mathematical negotiation by sharing the authority for mathematical knowing. I have shown how irregularities functioned to discursively mark challenges, objections and contrasts – all significantly valued practices within mathematical argumentation. I have also shown how those productions oriented to sociomathematical norms as a function of *mathematical knowledge*, thus establishing a locally situated meaningful distinction between knowing and knowledge: the group co-constructed a taken-as-shared authority for knowing, while the authority for knowledge was invoked from an external source (e.g., the larger domain of mathematics, arithmetical structure or the prescribed curriculum).

The point of this chapter has been to show that young children demonstrate a capacity to draw on sociomathematical norms as resources to produce coordinated ways of participating in mathematical discourse (see Figure 4.7). These results have significance for several points of inquiry and offer insights into the *interactional* processes involved in learning-as-participation. First of all, documenting children's discursive practices to this extent provides evidence of their capacity to participate in inquiry-based learning environments (Hutchby, 2005). Second, Koole (2010) found that 12- and 13-year-olds produced displays of knowing and displays of understanding depending on how their mathematics teachers phrased their questions. Although this study focused on displays of knowledge only, the findings here extend Koole's results, by showing how younger children used similar discursive practices regarding epistemic access in a less formal mathematics setting. These results also extend Heritage's (2012) epistemic engine theory by highlighting some of the knowledge redressing interactions that young children produce in mathematical negotiation.

The sociomathematical norms literature has developed along quite specific lines, promoting a juxtaposition of social and cognitive analytic perspectives (Cobb, 2007; Cobb & Yackel, 1996). While this has produced many promising results, it has also constrained the research questions that can be addressed. In particular, Voigt's (1995) patterns of thematic interaction have remained mostly unexplored. Voigt (1995) assumed the teacher as authority in the classroom, showing how teachers and students communicated together, but always considering the teacher as the mathematical authority. This study shows some of the possibilities afforded by sharing the mathematical authority, an area that has implications for classroom practices: we might fruitfully expand our definitions of what *mathematical knowing* looks like.

By focusing on the way language is used interactionally rather than on how it represents a version of reality, a discursive psychological approach to mathematical negotiation allows analysts to bypass significant dilemmas in interpretation while yet acknowledging them (Edwards & Potter, 2005). This has promise for studies of the relationship between the development of mathematical argumentation and the function of sociomathematical norms. Studies currently being published regarding children's mathematical inquiry-based learning tend to rely on external (usually teacher driven) curricula expectations to determine the scope and sequence of the mathematics content involved. This limits the capacity of children to contribute, since their own curiosity is secondary to the intended content (Walkerdine, 1988). When provided with a setting that invites mathematical activity but presents few if any expectations of what that might look like then children have the opportunity to take the investigations to unexpected places (Anderson, 2010; Towers & Anderson, 1998). I argue that they might then have opportunity to participate in the negotiation of sociomathematical norms in ways that might otherwise not be available to them, such as invoking a norm regarding accuracy. Therefore, the data generated in this study optimizes children's *capacity* to contribute and the findings give substance to that potential.

Finally, exploring the perspectives of young children regarding their experiences in meaning making has become an area of great interest. McTavish, Streelasky & Coles (2012) examined the practices of meaning making that young children valued, including multimodal meaning making (Narey, 2009). The findings of this study portray the practices of meaning making that young children produced as typical, and include several multimodal aspects, although for the most part those remained outside the scope of this chapter (however, note Example 4.8, where the wall chart and pointing gestures were important aspects of communicating meaning – Barwell, 2005).

The main limitation to this study rests with the method of transcription: having eliminated partial words, false starts and hesitations effectively served to eliminate data that might affect the resulting interpretations. I justify the practice due to the limitations of the computer word search features and note that patterns were still found, allowing a more focused choice of which interactions to fully transcribe. Nevertheless, the limitation is duly noted and further analyses on this data should not involve such regularization. This study represents a critical first step in investigating the positive contributions of children to the negotiation of sociomathematical norms. Defining and documenting that participation through a discursive psychological lens will strengthen the knowledge base regarding the function of norms in inquiry-based settings and renew our recognition of young children's capacity to contribute to classroom practice.

# Chapter 5: How young children display their understanding of mathematical content during negotiation.

In reform mathematics settings, children are expected to learn through meaningful participation in classroom discussion. In order to support that learning, educators require awareness of children's discursive capacity to display their understanding. There is limited research on the development of mathematical argumentation with young children, especially during the primary grades when reasoning skills are seen to progress from arithmetical to algebraic. In this chapter, I investigate how a small group of 5- to 7-year-olds negotiates a taken-as-shared understanding of the meaning of the equals sign, a key factor in emergent algebraic reasoning. Using a discursive psychological framework to document the learning showed that there were nine different discursive practices by which the children produced their understanding of the meaning of the equals sign. Further analysis of the *ways* the children produced their positions within the negotiation by invoking various sociomathematical norms provided evidence of the children's emerging mathematical understanding. These results provide valuable details regarding young children's capacity for mathematical argumentation, showing how they use the structure of production and recipient design to organize their participation.

## 5.1 Introduction

For several decades researchers have explored the relationship between mathematical learning and social interaction (Cobb et al., 1992b; Krummheuer, 2000; Lampert, 1990; Ryve, 2011). Previous research has outlined ways in which participants orient to each other during mathematical negotiation using the framework of sociomathematical norms (Yackel & Cobb, 1996; Voigt, 1995). Researchers have also explored how teachers orient to the mathematics involved (Pirie & Martin, 1997) but less is known about the students' perspectives, an essential component of the study of mathematical learning. Given the wide variety of forms that mathematical communication might take (Barwell, Leung, Morgan & Street, 2005), educators require increased awareness of how young children display their understanding of that which is specifically mathematical. Several studies have contributed insights into students' perspectives during mathematical negotiation, but most use adolescents or young adults as participants (Barnes, 2000; Pirie & Martin, 2000; Warren, 2006).

In this chapter, the second of three linked studies, I explore the perspectives of 5- to 7-year-olds as they participate in mathematical negotiation, an age range that has been noted as critical for the development of skills in mathematical argumentation (Krummheuer, 2013). Discursive psychology (Edwards, 1997; Edwards & Potter, 1992) affords an exploration of a mathematical negotiation from the participants' perspectives, that is, from an over-the-shoulder perspective. I use it in this chapter to investigate an example of a negotiation that emerged from an apparent mismatch between the adult and child perspectives on the content and purpose of an activity. The negotiation grew to include four other children who each produced explicit displays of understanding regarding the meaning of the equals sign. During the data analysis of Chapter 4 I was drawn to the interactions of this small group of participants arguing about the meaning – or function – of the equals sign: is "equals" for adding or for subtracting? I found that scholars had identified this particular subject, a clear understanding of *equivalence*, as a key factor in moving from arithmetical to algebraic forms of reasoning (Carpenter et al., 2003; Kieran, 1981; Knuth, et al., 2006), so I wondered what was happening discursively at this mathematically significant moment. Specifically, how did this group sustain a six minute long argument and what might the children's participation tell us about their mathematical learning?

Upon closer inspection I noticed an unexplainable disjunction between the mathematics and the linguistics: while the text of the negotiation seemed mathematically nonsensical, the social and linguistic features followed regular patterns, culminating in a consensus, or agreement. This provided some explanation for how the argument was sustained and resonated with the findings of Chapter 4 regarding the children's capacity to co-ordinate their contributions, but what about the mathematical learning? What might children be learning mathematically as they argue about whether the equals sign is for adding or subtracting? The mathematical symbol "=" refers to a concept called "equivalence", that is, a requirement that any quantities represented on either side of the symbol be equal. Therefore, the symbol does not *function* for either operation, neither are the operations included in its definition. However for this group, the text of the negotiation seemed to make sense. Furthermore, the accomplishment of the negotiation was signalled with a discursive social action: an agreement that the symbol could be used in adding and subtracting. It seemed that the participants were not as concerned with mathematical correctness as they were with developing a taken-as-shared understanding of what would be considered a valid response: might they be negotiating a sociomathematical norm? Such practice has become common in inquiry-based settings where students learn through meaningful participation in mathematical argumentation (Cobb, 1987; Krummheuer, 2000; 2013). Of course, another interpretation was also possible: the participants may have thought they were addressing issues of mathematical correctness. I wondered: could a discursive psychological focus on the rhetorical organization of talk provide an account that might distinguish between these two interpretations? Might it provide a warrant to claim one interpretation over the other? Furthermore, I wondered if my observations from Chapter 4 regarding a locally meaningful distinction between *mathematical knowing* and *mathematical knowledge* might somehow be implicated in this collective argument.

Another issue that an exploration of the negotiation had potential to address was the relationship between a child's mathematizing activities and the use of formal conventions: "Any student of mathematics has to accept the conventional symbols already in place; however, *acceptance of a symbol, in itself, is not always accompanied by its harbored meaning(s)* [emphasis added]" (Sáenz-Ludlow & Walgamuth, 1998, p. 153). Much of the interaction in inquiry-based mathematics involves interpreting the meanings or concepts that are attached to conventional symbols by using those symbols in various ways (Steinbring, 2006). Therefore, I wondered if discursive psychology might clarify how these children incorporated mathematical content in their talk assuming that their participation in the negotiation implied making meaning of the conventional symbols and operations involved.

To sum up, I am interested to know *what* is happening linguistically at these mathematically significant moments and I wonder *how* acknowledging that linguistic context might inform our understanding of mathematical learning. My inquiry here is guided by two research questions:

1. *What discursive practices do young children use as they incorporate mathematical content in their talk?*
2. *How do the ways the children discursively construct their participation display their emerging mathematical understanding?*

This chapter is the second of three stand alone studies that comprise the results chapters of the dissertation. In it I look at how young children use the structure of spoken language to support their participation in mathematical discussion by considering an interaction that exemplifies a concept introduced in Chapter 4: the negotiation of *mathematical knowing*. The data considered here include transcriptions supported by video recording of a six minute long segment of the N:Countr mathematics research group (McLellan, 2010) during Week 4. In order to answer the research questions, I explore the transcript using the tools of conversation analysis to look for indications that the children are referencing mathematical content and to understand how the ways they index mathematics provides evidence of their understanding. I propose that features of production and recipient design that emerged during analysis in Chapter 4 will both produce and constitute meaningful participation during this negotiation. I seek here to elucidate *how* those features do so.

This chapter then, continues the first of the two tasks of significance in the dissertation: in it, I provide evidence that expands our understanding of the *argumentative* processes involved in learning-as-participation. Defining and documenting young children's positive contributions to the development of a collective argument will strengthen the knowledge base regarding the significance of the structure of language in inquiry-based settings and renew our recognition of young children's capacity to contribute to mathematics classrooms.

### 5.2 Research framework

It is a basic tenet of both sociocultural theory and inquiry-based learning that children learn through participation. Therefore, I begin here by considering what we know about children's participation in mathematical meaning-making activities, including their displays of understanding and the negotiation of sociomathematical norms within inquiry-based settings.

Each contributes to our understanding of how children might integrate mathematical content within social interaction. I then briefly review again from Chapter 2 the main themes of a discursive psychological interpretive framework and specify its perspective on the relationship between learning as participation in mathematical practices and the rhetorical organization of social interaction. My intent again, is to provide just enough background research to situate this second study within the dissertation.

### 5.2.1 Making meaning in mathematics

Early observations in inquiry-based mathematical learning environments illuminated several features of children's ongoing attempts to make meaning during their participation. For example, it became apparent that the intersubjective negotiation of meaning within these settings was fragile and continuously at risk of breaking down. Voigt (1995) noted that regularities produced through the interaction functioned to minimize the risk of that breakdown: those discursive regularities therefore facilitated participation. He developed a definition of mathematical negotiation based on the symbolic interactionist assumption that objects are ambiguous or open to various interpretations. It is through negotiation of these interpretations, Voigt argued, that classroom conversants come to share understanding. However, it is not that speakers set out to negotiate in order to share knowledge. Voigt's position is the reverse: "Mathematical meanings are only taken as shared when they are produced through negotiation" (p. 172). He showed how students working together and sometimes students with a teacher came to share a solution that could not be attributed to any one conversant alone. For him, these interactions exemplified negotiation. What he noticed, when looking at these examples more closely, is that typically the participants operated on the basis of a tacit agreement, without ever confirming during interaction that they actually shared common knowledge. These tacit agreements were organized around interactional patterns and what he came to call "sociomathematical norms."

Scholars have built on those early studies, including children's perspectives by means of conducting fine-grained linguistic analyses of mathematics classroom talk. Koole (2010) found that 12- and 13-year-olds produced displays of knowing and displays of understanding differently depending on the interactional sequence. In particular, interactions where teachers told students how to proceed and then asked if the student understood the explanation relied on *whether or not* notions of understanding. In other words, this type of questioning served to ask the student to "confirm (or disconfirm) the adequacy of [the teacher's] proposal" (Heritage, 1984, p. 319). Koole found that students responded then to the question, not necessarily with indicators of mathematical understanding, even though sometimes teachers interpreted the students' responses as referring to the mathematics. On the other hand, interactions where teachers used a series of questions to have the student produce his or her own solution were typically followed by displays or demonstrations of knowledge, in other words relying on *how* notions of understanding. In these instances, students offered displays of how they understood the mathematics and teachers could then evaluate the correctness.

In Chapter 4 I argued that by using positively worded questions inviting displays of knowledge (e.g., What do you know about…?) during the N:Countr mathematics research group, I set up an expectation that made thoughtful responses relevant and functioned to share the authority for *mathematical knowing* among the group. In my analysis of students' displays of knowledge in this chapter, I will build on these findings by taking interactional sequencing into account, given the discursive psychological assumption that participants discursively co-construct the meaning of the negotiation (Edwards & Potter, 1992).

Krummheuer (2000) identified that such co-produced mathematical solutions had the linguistic features of a narrative. He elaborated the complexities involved in their constitution, especially for primary-aged students: “on the one hand, students have to understand the details of the presentation, and, on the other, they have simultaneously to infer the ‘sense’ of the whole story that is implicit because it is co-delivered” (p. 25). Krummheuer (2011) built on Garfinkel’s (1967) concepts of *recipient design* (i.e., a way of describing the adaptation of an utterance towards the particular person addressed) and Goffman’s (2001) notion of *production format* (i.e., a way of describing a speaker’s relationship to the story he or she is recounting) in order to describe the roles that children might assume vis-à-vis mathematical content during small group (i.e., polyadic) interaction. Krummheuer distinguishes between the syntactic formulation of an utterance – the word choice and form – and the semantics involved (i.e., the mathematical content portrayed). His identification of four roles of what he then calls “production design” are relevant to this study: an *author* presents an idea with original content and formulation, a *relayer* re-presents another’s idea using the same formulation as the original, a *ghostee* presents new content within someone else’s formulation and a *spokesman* paraphrases another’s content within an original formulation. This distinction offers a way of describing participants’ contributions to a negotiation and I will seek to determine its usefulness within an analysis of the negotiation of the meaning of the equals sign.

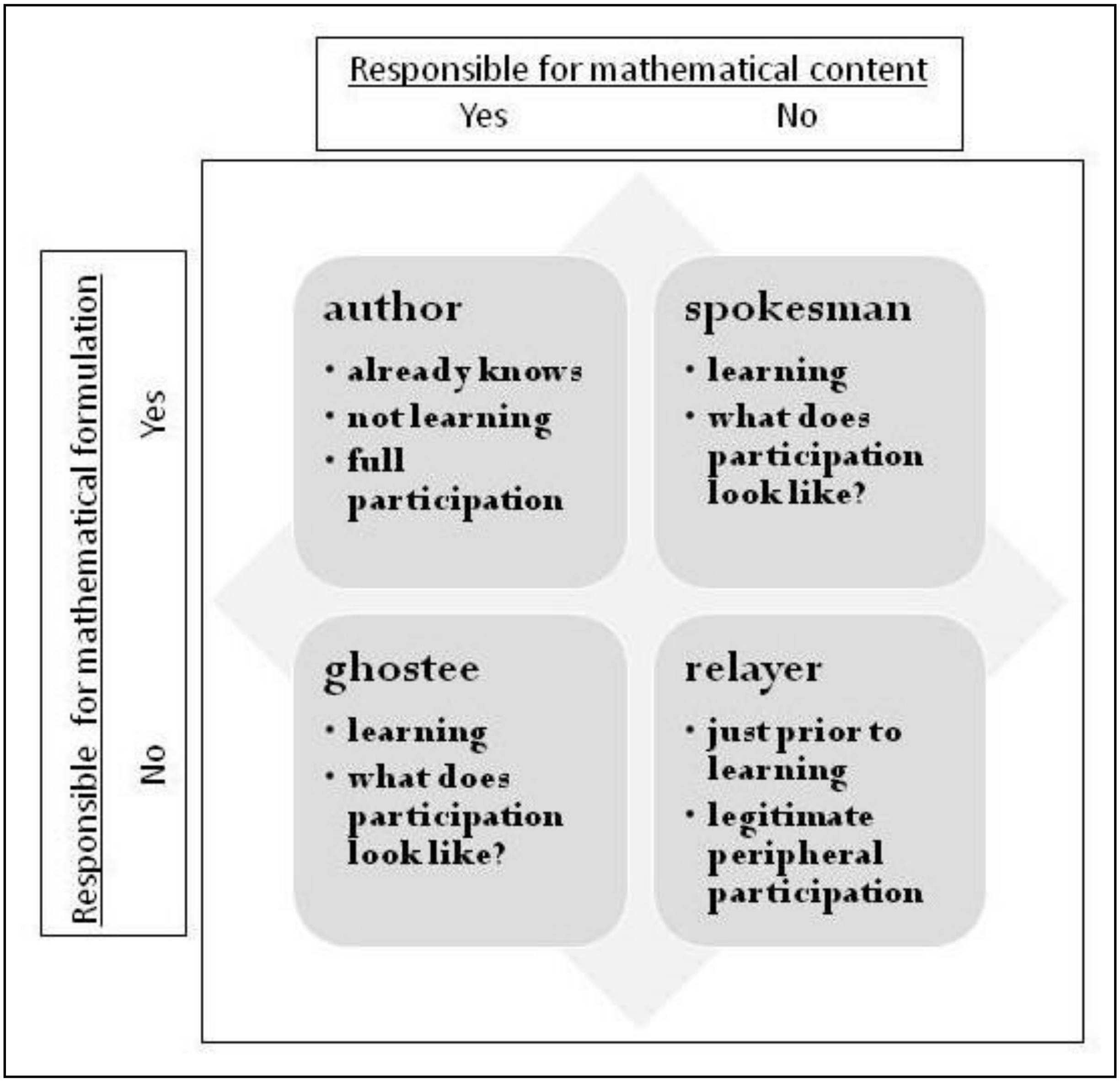


**Figure 5.1 Production design in mathematical polyadic interaction (generated from Krummheuer, 2011)**

Taken together, these studies highlight three interconnected aspects of our understanding of current mathematics education practice: inquiry-based learning, discourse and the collective negotiation of meaning. All are implicated in learning-as-participation and each contributes to our understanding of how children might integrate mathematical content within social interaction. Before continuing to the data however, I will briefly outline how my use of discursive psychology can account for their significance by illuminating *how* young children display their understanding of that which is specifically mathematical.

### 5.2.2 Discourse and discursive psychology

Of all the ways used to define discourse in research, I find those which focus on *language-in-use as a social action of communication between human agents* (Wetherell, Taylor and Yates, 2001) most amenable to inquiry-based learning, based on a shared commitment to understanding language as action and considering learner agency (Cobb, 1987; Krummheuer, 2000; 2013). This is the definition of discourse used in discursive psychology, an interpretive framework that combines the methodological approach of ethnomethodology with the analytic tools of conversation analysis to study traditionally psychological topics (Barwell, 2003). In mathematics education research, these topics might include knowing, understanding, attitudes or beliefs. Any of these *may be* salient to a participant during a negotiation. However, it is difficult to determine which, if any, *are* relevant without *getting inside the participants' heads*, so to speak. Garfinkel (1967) developed the main tenets of ethnomethodology in an effort to explore how topics are brought into play during everyday conversation. He demonstrated how participants understand each other during and through interaction, *how they display this understanding to each other* and therefore how they produce the shared social order in which they live. Discursive psychology draws upon the ethnomethodological commitment to spoken interaction and focuses attention on how psychological topics such as understanding are treated by participants. Analysts can avoid the subjective complications of interpreting participants' mental constructs of notions such as understanding by focusing instead on

1. how speakers display their recognition of a co-conversant's understanding;
2. how they respond to others' displayed assumptions of their own understanding and
3. how they construct their statements in ways that anticipate various potential interpretations by others.

During mathematical negotiation, understanding might be displayed through the social actions of explaining or convincing. However, the choice of whether to use explaining or convincing has implications for how hearers will receive and respond to the contribution and is therefore tied into point three above: the rhetorical organization of talk. Therefore, we can analyze what counts for participants as understanding by investigating in finer detail *how* these social actions are accomplished during interaction: this is what I refer to as an over-the-shoulder participant's perspective.

As Koole (2010) showed and I elaborated on in Chapter 4, answering a teacher's direct question is one way that students display their understanding. This is accomplished through the "adjacency-pair" feature of sequential organization: question – response is a regular form of interaction (Sacks, 1992; Schegloff, 2007). Participants can and do draw upon this regularity as a resource to manage the complexity of intersubjective meaning making in mathematics (Voigt, 1995). However, there are other features of sequential organization (the ways in which speakers organize their turns-at-talk) that may also provide a structure within which children display understanding. The tools of conversation analysis can attend to turn taking (implicit "rules" that determine relevancy for *who* talks *when*), preference design (an acknowledgment that interpersonal *alignment* is preferred) and repair (discursive tools available to address interactional problems). These are the discursive practices I will refer to as I explore the data here for how young children incorporate mathematical content in their talk.

Conversation analysis is also effective for exploring features of communication such as the deployment of discursive markers (the social function of words such as *so*, *well*, *oh*, etc – Heritage, 1984; Schiffrin, 1987) or the use of self-repair (the discursive action of stopping an utterance part way through in order to “change directions” as it were – Lee, 2006; MacBeth, 2004). Goffman’s (2001) conception of participation framework can illuminate the position of participants as solicited or bystanders and whether they are ratified or not (that is, whether speakers indicate an awareness of a bystander’s presence). Through these communicational features, children might display their understanding of who is allowed, expected or obligated to participate in a mathematical negotiation. Likewise, notions of facework (Goffman, 2006) can inform interpretation of speakers’ rhetorical uses of language such as doing convincing or claiming authority. Through the ways these practices are used, we can examine how young children display their emerging mathematical understanding through their own perspectives.

In the following sections, I explore how a group of young children use the structures of spoken language as interactional resources in an inquiry-based mathematics research setting. I am interested in examining the practices by which they incorporate mathematical content within the social act of negotiation and then to consider how patterns in those discursive constructions might provide insights into children’s emerging mathematical understanding. Through this analysis, I will use Krummheuer’s (2011) distinction between *mathematical content* and *mathematical formulation* to describe the roles participants take within the negotiation. With these principles in mind, I turn now to a setting, a group of participants and a problem to explore how this group negotiates a taken-as-shared understanding of the meaning of the equals sign.

## 5.3 Method

In chapter 4 I considered each of the two research questions separately during analysis, using corpus linguistics to illuminate interactional regularities and using conversation analysis to explore irregularities. In this chapter, I draw from the assumptions of ethnography of communication (Hymes, 1964) to focus on *a negotiation* as a speech event within mathematics practice, using the participants' signals to determine when that communicative event begins and ends. Therefore, I divide the analysis into two negotiations (the negotiation of the task and the negotiation of the meaning of the equals sign) and use conversation analysis to provide answers to the research questions in a more integrated fashion. The data were generated as part of a larger study that explored representations that constitute young children's mathematical thinking, the N:Countr mathematics research group as described in earlier chapters. The main data for analysis in this chapter consists of the transcription of a six minute long interaction during Week 4 supported by video recordings. There are three photographs related to the interaction and therefore included in this section: an internet printout that was shown to the children prior to their categorization activity, the drawings the children had completed the previous week which provided the impetus to show them the printout and the chart which was completed during the activity. During analysis I also include two images captured from the video to show the position and gaze of participants at different moments in time.

The relevant aspects of the larger data set and the analysis that are significant in this secondary analysis are the limited physical space which facilitated children observing other's activities and overhearing conversations, the task the group was engaged with at the time and the particulars of using conversation analysis.

### 5.3.1 Transcribing small group work

Small group work in a confined setting involves multiple, simultaneous, overlapping conversations and the physical proximity of others has the potential to influence any encounter. Recognizing this with analytic decisions regarding the level of detail to include in transcription is essential to socially situating any conversants participation. It may be particularly important in this chapter, since I am looking for how the participants position themselves vis-à-vis the stances towards the mathematical content that are taken by others: overhearing may be salient to a child's contribution, although he or she may be invisible in the transcription. Therefore, as outlined in Chapter 3, in this chapter I transcribe discursive features that seem to be secondary to the negotiation at hand in a lighter font colour, like this: a lighter font colour. This feature of transcription recognizes the possible implications of having participants overhear other conversations, yet it facilitates analysis by foregrounding the conversation under inspection.

### 5.3.2 The task: Mathematical symbols chart

During the N:Countr mathematics research group, the children engaged in a variety of activities including play, art and construction work, individual and small group discussions with or without me and whole group discussions that I facilitated. The negotiation considered here comes from Week 4, when the children were involved with a small-group activity that I had initiated. My instructions required the children to copy mathematical symbols from an internet printout (see Figure 5.2), cut them out and glue them onto a large poster size chart according to three categories:

- we know these symbols,
- we know these symbols but not in math, or
- we don't know these symbols.

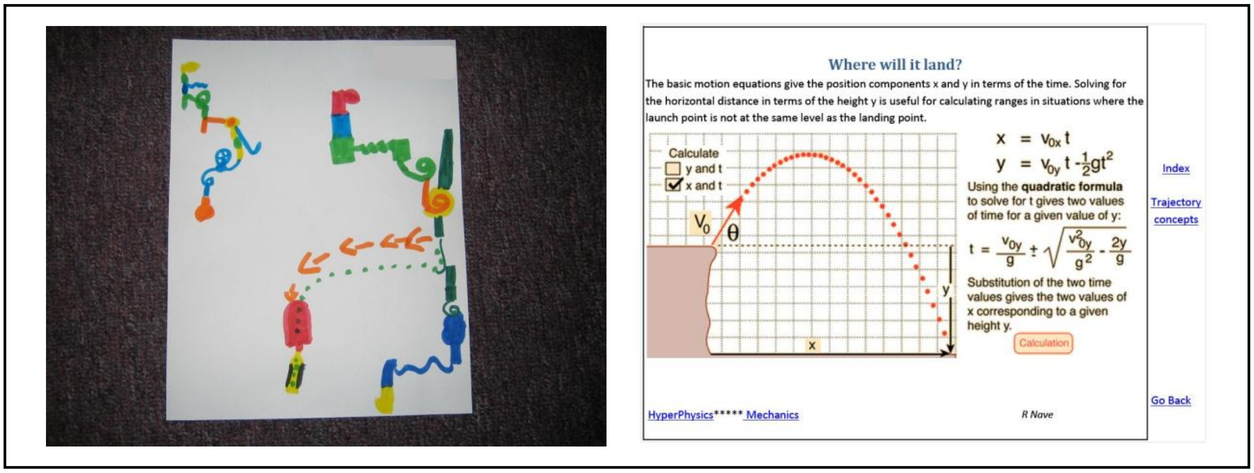


**Figure 5.2 Tara's trajectory drawing with similar internet-found illustration © n.d. R Nave, GSU. Reprinted with permission. Retrieved September 28, 2009 from http://hyperphysics.phy-astr.gsu.edu/hbase/traj.html**

The printout was introduced in the sense of creating a mathematically rich environment (Duckworth, 2001; Stoessiger & Wilkinson, 1991), not with any intent to explore the mathematics involved. It duplicated the drawings of trajectory that the children had produced as an extension of their marble play in a previous week. The activity provided a way to make sense of the mathematics in the printout, in the same way an adult might read a story to children when the language is beyond their capacity to read to themselves. Classification is a common early numeracy activity, so the task was reasonably within the children's capabilities.

By the end of the hour, the children had produced a version of the chart that speaks to some of the difficulties they encountered with the activity (see Figure 5.3).

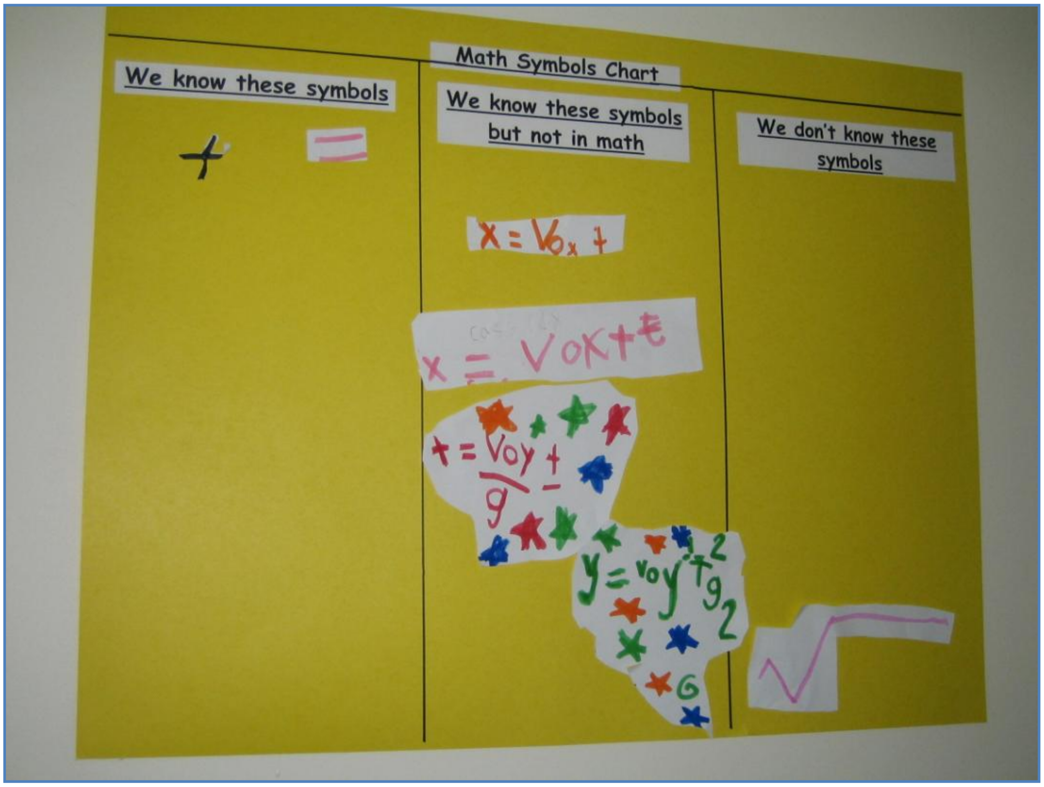


**Figure 5.3 The completed math symbols chart**

The middle column in particular reveals how the children and I may well have been operating under a different set of assumptions. On my part, the way the activity was set up assumed that a mathematical formula consisted of discrete symbolic representations. Thus, a formula such as $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ could be understood as a collection of numerical, variable and operational units: 2, =, $x$, √, + and so on. The task asked the children to select a discrete unit, copy it on paper and cut out around it so that it might be glued onto the chart in one of the three columns. An interesting response occurred wherein the children seemed to "see" the mathematical sentence as a whole. I assume these young children had no previous experience with alphabetic and numerical symbols juxtaposed and some of them treated the non-numeric variables as if they were words. These children copied and cut them out together, including the equals sign in these "words." This is what Anya (7y7m) was doing when I first spoke to her.

The interaction therefore opened with an implicit negotiation between Anya and me regarding what the task involved. From there, it developed into a dispute over the meaning of the equals sign between Anya and another child, Carissa (5y9m). Eventually, I invited other children to provide input and the negotiation came to include seven of the nine children present that day. The way the interaction plays out exemplifies how a group negotiates what will be accepted as a mathematically valid response. As children participated in the discussion, they presented various orientations to the mathematical content. They did so by the ways they constructed their explanations of the use of the equals sign in different arithmetical operations and sometimes by providing justifications. This particular interaction ended when I, as the adult in the group, signalled a satisfactory resolution and work on the task resumed.

### 5.3.3 Data analysis

Discursive psychology underlies all the analytic tools and will be used to examine the discursive practices of "doing mathematical understanding." The tools of conversation analysis will be used to illuminate those discursive practices such as producing an example, drawing on identity, reporting a narrative or providing a justification that the children use as they refer to mathematical content throughout the negotiation. In particular, I will focus on how children position themselves vis-à-vis the statements made by others and how they display an understanding of sociomathematical norms by the ways they produce those positions. These will provide an integrated answer to the research questions: *What discursive practices do young children use as they incorporate mathematical content in their talk? And how do the ways young children construct these statements display their emerging mathematical understanding?*

## 5.4 Results

There were two topics during the interaction that involved negotiation: the nature of the task and the meaning of the equals sign. Both of these negotiations included segments where children oriented to the mathematical content. The example opens with an exchange between Anya and I that indicates a difference in perspectives regarding both the content and the purpose of the activity. Anya displays her understanding by responding to questions sometimes with actions and sometimes with verbal answers. The negotiation around the meaning of the equals sign involves first two children (Anya and Carissa) then three others (see Table 5.1).

**Table 5.1 Nine discursive practices used by children during mathematical negotiation**

| Example # | Child's name | Discursive practice |
|---|---|---|
| **5.1** | Anya | Producing a label, with its definition |
| **5.1** | Carissa | Producing a contrasting definition |
| **5.2** | Carissa | Asserting mathematical experience |
| **5.3** | Anya | Providing a specific but hypothetical example from arithmetic |
| **5.4** | Carissa | Producing a narrative: A specific arithmetical example from experience |
| **5.6** | Jimmy | Producing an elaborated claim of not knowing |
| **5.7** | Nathan | Providing an abstract hypothetical example from arithmetic |
| **5.8** | Daniel | Producing a counterexample |
| **5.9** | Anya | Producing a narrative: A general arithmetical example from experience |

The participants use words, gestures, silences, proximity and discursive markers (so, well, etc.) to produce their understandings. I will address each negotiation in turn.

### 5.4.1 Negotiating the task

The children were sitting on the floor working on two different assigned tasks and I was circulating among them. As I approached the group working on the mathematical symbols chart, I knelt down on the floor beside Carissa and Tara (6y9m), opposite the chart from Daniel (7y3m), Cormac (7y4m) and Anya (the other members of this small group). The following interaction occurred:

**Example 5.1 Introducing the "text" to be negotiated**

```
1    R:((to Anya)) oh, you're copying a lot of details. when you know-
     okay, now, see, the thing about that [one right here=]((pointing))
2    Tara:                                [mine. Nanny.  ]
3    R: =Anya, is. some of those symbols we [know.=  ]
4    Anya:                                  [((nods))]
5    Truman:((to Nathan)) I need some.
6    R: =some of them we only know, [but not in math=       ]
7    Anya:                          [((Anya looks up at R))]
8    ((other children speaking in background))
9    R: =and some of them, well, maybe that's- that's it.
10   Tara: look. mine.
11   Anya: ((looks down and puts her cut out figure in the middle
     column: 'we know this symbol but not in math'))
12   R: so, you're going to have to put some of it in this one
     ((pointing to the cut out and the left column: 'we know this
     symbol' in turn)) and some of them in that one. ((pointing to the
     middle column)) you'll have to cut them [apart.
13   Tara:                                   [do you like mine?
14   R:((puts scissors on the floor in front of Anya and looks to
     Tara)) it looks very complicated.
15   (1.8) ((other children talking in the background. Anya remains
     still))
16   R: ((to Anya)) see. the thing about this, Anya=
17   Nathan: ((to Truman)) ask Nanny for some
18   R: =is. can you tell me what that symbol is? ((pointing to the
     equals sign on Anya's drawing))
19   Nathan: ((to Truman)) we need some more.
20   Anya: equals.
21   R: equals. you [know  ] what equals means-((looking up at Anya))=
22   Jimmy:         [Nanny.]
23   R: =do you [know what equals means?]
24   Anya:      [((nodding her head yes and sitting back on her
     knees))] it's um, it's [like, ] adding?
```

```
25    Tara:                   [Nanny?]
26    Carissa: or taking away.
27    (1.4)
28    Daniel: (um, [where's the glue.])
29    Tara:         [Nanny?]
30    Truman: ((comes over to stand in front of R)) [Nanny?]
31    Cormac:                                       [(    )]
32    R: okay. I'm. a:sking a question about equals. is it [adding,]=
33    Truman:                                                [Nanny?]
34    R: =or is it taking away.
```

I initiate the interaction by observing aloud something that is obvious (line 1: oh, you're copying a lot of details). This social action of "noticing" serves to ask for an accounting (Antaki, 2002). In this case, it implies the question "*why* are you copying a lot of *unnecessary* details?" This is the first signal of an interactional difficulty and it is quickly followed by two others: the beginning of a question that might explicitly require accountability and the action of cutting that off to produce a self-repair (line 1: when you know- okay, now see). Almost immediately there is overlapping talk, a bid for attention from the child sitting beside Anya (line 2: mine. Nanny). I verbally select Anya in my next turn by calling her name, a move that might be considered unnecessary given our close proximity. However, the selection signals to Tara a context boundary, to which Tara displays recognition by waiting until the end of the utterance before making her second bid for attention in line 10. Overlapping Tara's talk, Anya silently responds to my initiative by placing her figure on the poster. I provide follow-up to Anya's gesture with a mitigated declarative (line 12: so, you're going to have to put some of it in this one, and some of them in that one.) followed by a directive (line 12: you'll have to cut them apart). The change of pronoun mid statement (from "it" to "them") displays an assumption that the formula might be understood as a collection of parts rather than a whole. This interpretation is supported by the pronoun "them" in the directive (line 12: you'll have to cut them apart).

The still image captured from the video (see Figure 5.4) indicates the positioning of each child and the gaze, or attention they display around line 15 in the transcript.

**Figure 5.4 Positioning and gaze of each child during line 15**

Anya (on hands and knees, third from right) touches her cut out and looks at Tara’s drawing. (Tara’s drawing is shown within Figure 5.3: it contains a representation of the formula surrounded with coloured stars). Although this is an awkward body position to hold, Anya remains unmoving for several seconds. Cormac (five from the left) looks on as well and Carissa’s gaze is undetermined. (Her body is concealed in this photo because she is immediately behind me, but I mention her here because she speaks six turns later). All other children are engaged elsewhere, mostly with their own tasks. Anya’s immobility, occurring as it does immediately following a clear directive by an authority figure may be seen as resistance.

Following a noticeable pause, during which Tara and I exchange words, other children talk in the background and it is clear in the video that Anya has not picked up the scissors, I re-select Anya, perhaps to contest the background talk and request Anya's attention (line 16: see, the thing about this Anya). In sequence with the non-verbal gestures, my response is again produced as a self-repair (line 18: is. can you tell me what that symbol is?) According to Lee (2006) "wherever it is found, the repair organization routinely displays the speakers' analysis of the talk for its intelligibility and common understanding, for example, what it means, how it sounds and how it could be recognized" (p. 706). By responding in this way to Anya's resistance as opposed to taking a more didactic or authoritative approach, I demonstrate a commitment to the development of shared understanding. However, the question I ask is closed, meaning that Anya might have properly responded with a "yes." Nevertheless, Anya orients to the question as a request for a display of knowledge. As I speak, I point to the equals sign in Anya's drawing and Anya responds with a single word: "equals" (line 20). Having been implicitly asked for a label, she produces one.

I revoice Anya's response and display the meaning attributed to her gesture (putting the image in the middle column: "we know these symbols but not in math") with my emphasis on the word *know* (line 21: equals. you <u>know</u> what equals means). I then follow up with an epistemic status check (Sert, 2013; line 23: <u>do</u> you know what equals means?). At this point Anya sits back, puts her hand to her hair, re-establishes eye contact, nods and there are four children visibly attending to the conversation between her and I (see Figure 5.5).

In Chapter 4 I noted that a direct question by me followed by a claim of knowing (either positive: "yes", "I know" or negative: "no", "I don't know") was a regular discursive sequence between the children and me. Furthermore, this "direct question by R followed by claim of knowing by child" discursive sequence was typically treated as an invitation for other children to display knowledge, *if* the child answered "no" or "I don't know." In the sequence here, when I asked Anya "do you know?" all the other children heard the question as well. Perhaps in anticipation of an opportunity to contribute, several of them momentarily attended to Anya: they looked up from their own tasks.

**Figure 5.5 Positioning and gaze of each child during line 24**

It is essential to recognize this use of gaze by participants as a form of acknowledging an overheard conversation (Goffman, 2001): it allows us to reasonably account for the socially situated nature of teaching/learning interactions (Lave & Wenger, 1991). The participation framework created to represent this moment is re-copied here from Chapter 3. It shows that three of the four participants who signal their attention are nevertheless unratified in this moment and therefore invisible in the transcription (Cormac, Jimmy, Daniel: Figure 5.6).

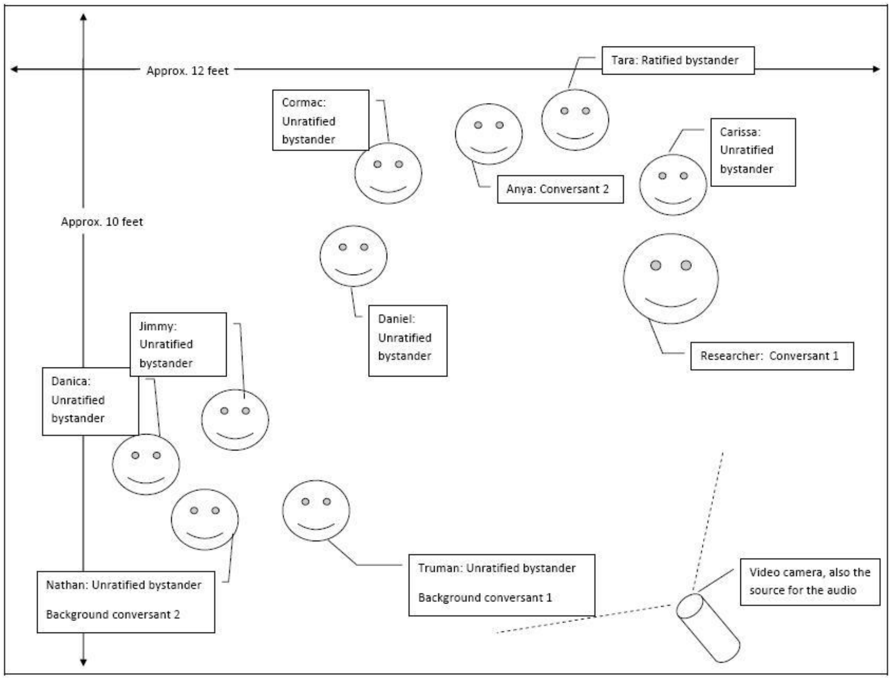


**Figure 5.6 Participation framework for line 15 in Example 5.1**

As Anya responds to my direct question, she nods and then produces a display of knowledge, albeit triply hedged, that is with marked uncertainty through the use of “um”, the hesitant “like” and the questioning format (line 24: it's u::m, it's like, adding?) It is at this moment that Carissa produces her statement (line 26: or taking away). Neither Anya nor I had previously indicated awareness that Carissa was present and could overhear (i.e., she was an unratified bystander); therefore, her interaction with us was not invited. Nevertheless, Carissa speaks as one who has authority to oppose, a position she supports with further assertions. Although her body is hidden in the video, her voice is loud and clear in the audio and both Anya and I turn to her.

The silence following it (line 27) discursively “marks” Carissa's statement: the up-to-this-point regular flow of conversation is interrupted. The silence thus serves multiple functions of producing a transition-relevance place (Sacks, 1992) and signaling a change in the participation framework (Boulima, 1999). In other words, the ensuing silence signals to all participants that they might take a turn at speaking and it also provides evidence of a different kind of interactional difficulty: a third party has entered the negotiation uninvited. Four other children initiate bids for control of the floor (Daniel, Tara, Truman and Cormac: lines 28-31). However, instead of responding to any of them, I produce a contextualization cue, using “okay” at the beginning of my statement to produce my authority (Dorr-Bremme, 1990). I then use multiple discursive markers (elevated volume, exaggerated enunciation) during my assertion in order to retain control of the floor (line 32 & 34: okay. I'm a:sking a question about equals. Is it adding or is it taking away.)

With this statement I both define a new problem and elevate it as an issue that is significant to the business at hand. In other words, I frame the “text” that forms the problem which the negotiation is then intended to solve (Love, 2000). I adopt the stance of “one without knowledge” (Heritage, 2012) by implying that this is a question I cannot answer myself. It is apparent from the data that the students accept this position, since none ever challenges it (e.g., don't *you* know?) and from that point on, participants orient to the established parameters of the negotiation: is equals for adding or for taking away? This signals an end to the negotiation of task between Anya and me. The completed math symbols chart from Figure 5.3 indicates that it was Anya’s and not my understanding that came to be taken-as-shared for the activity, since the equals sign remains within the figures of the middle column. However, the equals sign also appears by itself in the left hand column, a result of the negotiation yet to come.

### 5.4.2 Negotiating the meaning of the equals sign

When Carissa produced her assertion in line 26 (or taking away) the meaning was ambiguous. Unlike the conjunction "and", which could have been used here and would have suggested a more definite interpretation, the conjunction "or" introduces uncertainty. It may be used to indicate an exclusive disjunctive (either this *or* that) but it may also be used to denote an inclusive proposition (as well as). In this case, Carissa's neutral intonation did not suggest one interpretation over the other and the interactional difficulty may be attributed to this ambiguity. When I revoice the two positions in line 34 (is it adding or is it taking away?) I foreground the disjunctive use, an action that both Carissa and Anya orient to in the contest that follows:

**Example 5.2 The original opposition and justification by experience**

```
32    R: okay. I'm. a:sking a question about equals. is it [adding,]=
33    Truman:                                                  [Nanny?]
34    R: =or is it taking away.
35    Anya:    [[adding.
36    Carissa: [[it's taking away.
37    Truman: ((pointing back to Nathan who is sitting on the ground))
      (    ) scissors.
38    Carissa: taking away.
39    R:((taking something from Truman and picking up scissors on the
             ground - handing them to him)) [[(          )
40    Anya:                                 [[no. adding.
41    Carissa: no, taking away.
42    Anya: it's adding.
43    Carissa: I did a [lot of math]
44    R:                 [okay, this ]is a tricky thing.
      ((Truman returns to sit on the floor with Nathan. Jimmy and Danica
      both stand up and approach R))
45    R:          is it adding, or is it [taking away?]
46    Daniel:((stands up in front of R))[where's the glue?]
47    ((Jimmy steps in front of Danica, giving the visual impression of
      a line up))
48    Jimmy:((handing R a piece of tape)) Nanny, can you fold this up
      for me?
49    Anya: it's [adding. ]
50    Carissa:   [I do math] everyday in my workbook.
```

The argument develops between Anya and Carissa for the most part while my attention is diverted elsewhere. Their oppositional stances construct the negotiation as divergent (Boulima, 1999): either adding or taking away can properly be included in the meaning for the equals sign, but not both. No one intervenes with this disagreement, although everyone is within sight and hearing. After six turns of blatant opposition (lines 35 – 42: adding/it's taking away, taking away, no adding, no taking away, it's adding) Carissa introduces a source of authority on the basis of being "active in a particular social capacity" (Goffman, 2001, p. 105; line 43: I did a lot of math. and line 50: I do math every day in my workbook). She *does* mathematics. This introduces the domain of mathematics as being relevant to an understanding of the meaning of the equals sign. Carissa's first attempt to support her position takes the form of co-opting the "text" of experience (line 43: I did a lot of math). By doing so, she invokes the value attributed to calling upon a source of authority beyond simply her own opinion, a sociomathematical norm. My overlapping statement weakens Carissa's claim, speaking as I do from a position of entitlement. Nevertheless, I am diverted again by the needs of other children and Carissa re-asserts her claim by specifying ongoing experience with a certain kind of mathematics: the kind done in workbooks (line 50: I do math everyday in my workbook). By doing so however, she takes the negotiation away from the abstract arena of mathematics and brings it into the personal realm. No longer is she defending a position (the equals sign is for subtraction); she is now defending her own capacity to argue. Anya acknowledges that switch in the pattern of negotiation by redirecting her efforts in a new way, also calling on a source of authority outside of her own opinion, but this time by giving a specific arithmetical example.

Fourteen seconds elapse between Carissa's claim in line 50 and Anya's response to her. During that time, I interact with both Jimmy and Danica (thus the gap in line numbers). While

Anya quietly looks on, Carissa's body remains hidden in the video recording. Then Anya attempts to re-engage Carissa in the argument:

**Example 5.3 Justification by specific hypothetical example from arithmetic**

```
70    Anya: ((to Carissa)) so::. if [let's say, one plus one         ]
71    R:                            [um::, yeah, we need to find out.]
72     ((Jimmy, Daniel and Danica are now wandering around the room.
      Jimmy is humming))
73    Anya: so one:::, and the:::n, plu::s,
74    R: but where's the equals?
75    Anya: and then equals. and then there's adding.
76    R: one plus one equa::ls, and then you put the answer?
77    Anya: yeah. so, it's adding.
```

The word "so" has been identified as a routine discourse marker, used to signal contrast, elaboration or transition (Norrick, 2009; Schiffrin, 1987). Anya uses the word three times within four turns here. As a way of managing complex interactions, the word *so* can implicitly signal topic resumption, especially after an interruption (Buysse, 2012). In line 70, it prefaces a suggestion (so, if, let's say, one plus one) that has the effect of both engaging the audience and taking the form of a highly valued mathematical trope: "Let $x = n$" which speaks to the authority within the mathematics itself. I signal my re-engagement with overlapping speech, apparently replacing Carissa as the ratified hearer. Anya's speech becomes highly protracted, every word is articulated slowly almost as if she were writing it down (which she is not). This serves to further draw me into the interaction, essentially co-producing the example.

The way I portray the equals sign in line 76 as being "the signal to put the answer" is rather problematic (see McNeil, 2008; Sáenz-Ludlow & Walgamuth, 1998) given the mathematical concept of equivalence that is so essential to developing algebraic reasoning. The action speaks to the interpretation mentioned in the introduction: the participants may have thought they were addressing issues of mathematical correctness. I will bracket this issue for now in order to focus on the children, but I will return to it in the discussion. Notwithstanding this difficulty, as the transcript in Example 5.3 shows, there is a fairly straightforward exchange between Anya and me. The exchange with Carissa (see Example 5.4) is far more protracted, occurring as it does in competition with rather loud humming sounds and including three different concurrent but unrelated interactions between Jimmy and me:

**Example 5.4 Justification by specific arithmetical example from experience**

77 Anya: yeah. [so, it's adding.]
78 R: *((to Carissa))*[and how do you think] it fits for subtracting. Carissa?
79 *((humming sound continues))*
80 Carissa: wha:ts?
81 R: like, she just said it fits for adding because you say one plus one equa::ls, and then you give the answer.
82 Anya: so it's [adding. ]
83 R: [how does it] fit for subtracting? for take away.
84 *((humming continues throughout))*
85 Carissa: Well, sometimes. what it does? when you- like. um. in. once when I was doing math? I noticed. then. I noticed that. I saw. this. then, I saw th- four.
86 Jimmy: *((to Nathan))* can I use that tape?
87 Carissa: I mean, like. five. say, I saw, like, five? and then. this? *((points to a symbol on her paper - equals sign?))*
88 R: [[equals ]
89 Multiple speakers:[[( )]
90 Carissa: [[and then.] I saw. four.
91 R: *((to Jimmy and Nathan))* help Danica, please.
92 Jimmy: Nanny?
93 R:*((to Carissa))* you saw five and then that. and then you saw four. you saw this equals? *((pointing to symbol on the paper))* and how did you know it was take away?
94 Carissa: 'cause I saw those two numbers. 'cause the. first? what you have to do. *((printing numerals on her paper))*'cause say this is a.
95 Jimmy: *((coming over to R and handing a piece of tape))* Nanny, this one( )
96 Carissa: and this?
97 R: *(( to Jimmy))* you made it longer ( )
98 Carissa: this is, the [same thing.
99 R: *((to Carissa))* [yes? un-huh?
100 Carissa: and this is a four.
101 R: so then you would have five. equals. four.
102 Carissa: yes.[so I knew

103 R: [and so you knew it was take away because? because why?
104 Carissa: because I saw [( )]
105 Nathan, Truman and Danica:[( )]
106 Jimmy:*((brings roll of masking tape and hands it to R))* Nanny. (can you help me with this?)
107 Carissa: and then a five and a four
108 R: *((speaking louder, as if addressing the whole group))* OKAY, WE HAVE A REAL MYSTERY HERE. WE NEED SOME HELP.

I produce my ongoing control of the negotiation by emphasizing the statements that contain inter-textually relevant information. We see this function at the beginning of Example 5.4 as I model a mathematically valued use of language, re-specifying Carissa's position as being relevant to subtraction (e.g., line 78: and how do you think it fits for subtracting, Carissa?), a move I repair in a subsequent turn in order to use Carissa’s own term (line 83: how does it fit for subtracting? for take away). I also model “example” as being relevant (line 81: like, she just said it fits for adding because you say one plus one equals, and then you give the answer; and line 83: how does it fit for subtracting? for take away).

Carissa's reply begins with many false starts and hedges. Then she repairs it to include an *example of her experience*, embedding herself in the narrative (Goffman, 2001; line 85: Well, sometimes. what it does? when you- like. um. in. once when I was doing math? I noticed. then. I noticed that. I saw. this. then, I saw th- four). It is difficult for a reader to understand the logic of Carissa’s explanation, given the multiple breaks and repairs. During the interaction, I display uncertainty concerning the meaning attributed to this narrative by probing for the relevance of Carissa’s example, twice (line 93: and how did you know it was take away? and line 103: and so you knew it was take away because, because why?).

This segment highlights some of the *messiness* of learning (Duckworth, 1996). Young children are just beginning their experience of formal schooling. The logic of their reasoning may not flow according to conventions, yet there is an acceptance of the messiness, although it seems not to move the negotiation further in this case. As an adult authority figure, and by giving this amount of time to drawing out Carissa's response, I display an orientation towards the development of a *process* (argumentative reasoning) rather than a *product* (the correct answer), a feature of the invisible pedagogy of inquiry-based learning (Love, 2000). As the negotiation continues, I proceed not by responding to Carissa’s final explanation, but by claiming the authority to convey the right of all those in attendance to join the negotiation:

**Example 5.5 Enlarging the participant pool: a projection of obligation and a restating of the problem**

108 R: *((speaking louder, as if addressing the whole group))* OKAY, WE HAVE A REAL MYSTERY HERE. WE NEED SOME HELP. *((to Daniel))* can you please not do that with the felts. thanks. Daniel. um. we have a real mystery. we don't know, what equals is, exactly.
109 *((Nathan and Danica come over to join the group around R))*
110 R: okay, so, well, it's not that we don't know. it's that, two people have two different ideas. [can anybody help us with an-
111 Carissa: [take away, and-
112 *((overlapping unintelligible voices))*
113 Carissa: take away and add them. or, and. not take away.
114 R: so. maybe you can help us with this Nathan, if you're finished over there, did you get your graph finished?
115 Nathan: *((nods))*
116 R: nice. *((addressing everyone again))* WE NEED YOUR HELP ON THIS. equals.

This use of ‘okay’ in a turn-initial position is often invoked by teachers as a framing move (Sinclair & Coulthard, 1975). By framing it as a mystery, I manage the accountability for the problem by placing agency on the students (Barwell, 2003), while mitigating the face threatening nature of the communication with fanciful language. Chouliaraki (1998) notes the combination of the plural pronoun ‘we’ with the modal ‘need’ creates a subtle form of obligation: “… although it presupposes the teacher as the source of authority, it leaves the responsibility for the accomplishment of the action to the [students]" (p. 19). In this case, it provides an opportunity to restate the original problem and therefore increase the stakes.

I adjust the initial statement describing the problem with a tag ("exactly") and a self-correction: the problem is not that people do not know, rather there are multiple, apparently conflicting ideas of what might be so. Carissa inserts her restatement of the problem in lines 111 and 113 (take away and – take away and add them. or, and. not take away). In doing so, she acknowledges a social norm for precision: she is describing the two positions that I just generally referred to. This is justifiably information that others might need to know in order to participate in the negotiation. However, she does not incorporate my revoicing to use the term subtraction, but persists with the everyday term, take away. Once again, Carissa's presentation is ambiguous. It is not clear from either the transcript or the video if she is producing a self-repair when she changes the connector from *and* to *or* and back to *and*, or if she is meaningfully referring to the substance of the negotiation as an exclusive disjunctive (it has to be either one proposition or the other) or an inclusive (we are negotiating whether it might be this and that proposition). Nevertheless, she produces her authority to restate the problem by persisting with her contribution even across overlapping talk, repeating what might not have been heard and completing the summary in line 113. Anya and I accede to her restatement by neither revoicing it nor repeating a different version of it. At this point, other children become actively involved:

**Example 5.6 The discursive production of "not knowing"**

116 R: *((addressing everyone again))* WE NEED YOUR HELP ON THIS. equals.
117 Jimmy: When can we [go, play-
118 R: [Jimmy? listen, we need your help. the equals sign. you know, the one that looks like this? *((points to the drawing on the paper))* two. small sticks on top of each other?
*119 ((seven of the nine children are physically oriented in towards this discussion now))*
120 Jimmy: yeah?

121 R: is it for adding? or is it for taking away?
122 Jimmy: I have. no clue.

In this example, Jimmy answers an epistemic status check regarding the symbol under negotiation (line 118: the equals sign. you know, the one that looks like this?) with a claim of knowing (line 120: yeah). Following that, I produce a direct question relating to the negotiation at hand and asking Jimmy to produce a position on the meaning of that symbol according to the two polarities. He responds to this question with an elaborated claim of not knowing (line 122: I have. no clue).

It is important not to interpret this statement as a reflection of what Jimmy actually may or may not know. A discursive psychological approach focuses on interpreting how statements of knowing are used interactionally to produce various social actions rather than treating statements as if they were unproblematically transparent (Potter, 2003). In this sequence, Jimmy had previously voiced an interest in playing (line 117: when can we go play). As a “non-questioning” question (Clift, 2012), this utterance produced his disinterest in the ongoing negotiation that I was in the process of extending to the whole group (implying an expectation for his participation). I respond by calling his name and indicating that his help in particular is needed (line 118: Jimmy? listen. we need your help). This selection explicitly positions him as a recipient of the obligation referred to earlier with the use of “we need” and holds him directly accountable for knowing. With this sort of high stakes accountability in mind, Jimmy’s elaborated claim of not knowing may be seen as producing a higher stakes version of his disattention (i.e., I don’t know and I don’t care to know). Beach & Metzger (1997) observed that these type of claims of insufficient knowing function as resistance and serve to regulate both the topic of conversation and the amount of time spent discussing it, a claim substantiated in this and other mathematics settings (Reis & Barwell, 2013).

In this example, I never follow through with Jimmy and he never again participates in the negotiation. If my discursive move is interpreted as an invitation, then Jimmy's discursive move can be understood as declining that invitation. Despite my position of entitlement, I leave that decline unchallenged, perhaps because Daniel and Nathan both immediately produce positions. This again substantiates the pattern of "direct question by R followed with a claim of insufficient knowing" as inviting the contributions of others seen in Chapter 4. In turn, I respond to Nathan:

**Example 5.7 Justification by abstract hypothetical example from arithmetic**

121 R: is it for <u>adding</u>? or is it for taking away?

122 Jimmy: I have. <u>no clue</u>.

123 Daniel: take away.=

124 Nathan: =like, if you, like, say, plus?

125 R: uh-huh?

126 *((Cormac speaks with Daniel in the background. Several overlapping unintelligible voices))*

127 Nathan: it's for. like. this plus this. equals. this. (*gesturing across the front of his body*)

128 R: ah::. so it's only for plus? this plus this equals the answer. that's what, Anya was saying too.

When Nathan provides his response he also gives an example, but this time an abstraction (line 127: it's for, like, this plus this, equals, this). The gesture he uses replicates that form of addition which might be found in workbooks. He motions sideways across his body, alluding to a horizontal orientation which includes the symbol for equals (e.g., $2 + 2 = 4$ as opposed to a vertical orientation: $\begin{array}{r} 2 \\ \underline{+\ 2} \\ 4 \end{array}$ where a single line replaces the equals sign.)

I acknowledge Nathan's response by repeating it (line 128). I then align his position with Anya thereby referring back to the original oppositional stance. The explicit interpretation is that of an exclusive disjunctive, although Nathan's statement does not require that interpretation (line 128: Ah, so it's only for plus?). My response may signal to the children that any example will be interpreted as disjunctive and if they seek a different interpretation, they must be more precise. This interpretation is supported with an interesting development: at the very end of the negotiation, almost two minutes later, I return to Nathan's example *as if he had just spoken*. I check with him that he may have actually intended the inclusive interpretation (line 173: was that what you were trying to say Nathan?) a suggestion he agrees to. This brings to mind Goffman's (2001) observation: "For often it seems that when we change voice ... we are not so much terminating the prior alignment as holding it in abeyance with the understanding that it will almost immediately be reengaged" (p. 109). This revisiting of Nathan's example marks it for significance, as we shall see. In the meantime, Daniel also takes a position in the negotiation, this time by using a specific example from arithmetic as a counterexample to "prove" his point:

**Example 5.8 Justification by counterexample**

128 R: ah::. so it's only for plus? this plus this equals the answer. that's what, Anya was saying too. s- Carissa thinks that she's seen it for take away.
129 Daniel:[yeah,
130 Anya: [hhh well, actually it could be for both.
131 Tara: Nanny? ( )
132 Daniel: it can be for take away.
133 R: *((looking at Daniel))* can it be for take away? *((looking at Anya))* can it be for both?
134 Anya: [yeah.
135 R: [please tell me about it.
136 Daniel:because, if it's, [five minus, like, equals four, you can't take four =
137 Anya: [like.
138 Daniel:[[=from five. because it'll equal zero.]
139 Tara: [[Nanny? ( ) You mind if I colour?]
140 R: ah-hah.
141 Anya: it's like, um. um. like, when I was in grade [one?
142 Carissa:*((bringing her paper over to show researcher))*[give me some. Now I know something.

Daniel's position here is interesting for several reasons. First of all, he tried earlier to enter the negotiation but was interrupted by Nathan, to whom I attended (see Example 5.7, line 123: take away). At that point, Daniel carried on an animated discussion with Cormac, on camera but out of hearing. He comes back into the negotiation after I signal the restatement of the original problem by revoicing Carissa's position (line 128: Carissa thinks that she's seen it for take away). Daniel aligns himself with Carissa (line 129: yeah). A second point of interest is that he uses both the everyday term "take away" and he introduces the more conventionally mathematical term "minus" into the negotiation. This is the first use of the term in this negotiation, signaling his fluency in the use of multiple terms to describe the same operation. Third, he directly refers back to Carissa's example given previously to support her position (see Example 5.4). This is the first time the negotiation has explicitly involved a loop back to revisit a previous position. Notice that Daniel appears nowhere in Example 5.4, indicating that this is another time when a participant overheard and was impacted by a statement, although that influence was not apparent at the time (the first time was indicated in video capture from Figure 5.4 when multiple participants changed their position to gaze towards Anya as she began to answer my question: do you know what equals means?).

Fourth, he produces an arithmetical sentence that is illogical according to conventions (lines 136 & 138: because, if it's, five minus, like, equals, four, you can't take four from five, because it'll equal zero). It is possible that Daniel might be referring to the equivalence concept of the equals sign, a more mathematically valued meaning than the process-oriented meaning being referenced so far ("=" means where you put the answer). However, it is impossible to interpret his sentence that way without further discursive support, which does not exist in the transcript. So, we can go no further here than to say that his position is expressed in a "messy" form, and particularly because he explicitly uses it to support a position saying that the equals sign *can* be used for subtraction. However mathematically messy Daniel's explanation may be, he is referencing a sociomathematical norm regarding logic by attempting to prove that a previously given example could not be correct. This suggests an attempt at algebraic reasoning or an early form of *mathematizing* in the broader sense (Freudenthal, 1991; van Oers, 2012). Nevertheless, I provide no scaffolding as I had for Carissa, instead simply acknowledging the contribution with a back channel (line 140: ah hah).

The negotiation is interrupted at this point by Carissa, who had retreated back to her own task within the math symbols chart activity and now comes to me in order to show a new understanding (on a different topic) that she just constructed (line 142: now I know something). Carissa's display of knowledge was included in Chapter 4 (Example 4.6) and deserves further examination, but the substance of it is outside the scope of this chapter. However, this analysis places it within the local context of the negotiation. As Carissa spoke, all participants including me stopped talking, thus displaying an expectation that her contribution might be pertinent to the negotiation, perhaps since she had been one of the original two children involved (thus, a gap in the line numbers). However, the new topic seems unrelated to the ongoing negotiation both now during analysis and apparently at the time as well. Therefore, after a little over one minute, I re-instate the negotiation by eliciting Anya's position and thereby inviting her to re-engage. Anya responds with an example of her experience, embedding herself in the story she tells:

**Example 5.9 The discursive production of change-of-knowing**

156 R: *((turning to look at Anya, who has remained beside R on the floor during the entire exchange with Carissa))* so: the equals.
157 Anya: so like, it can be, [um
158 Carissa: [I'm just going to write my name so (when it's time) you'll know about that one?
159 R: okay, you can write your name, okay. pardon me, Anya?

160 Anya: um. it's like. um. when I was in grade one, it could be- um. I did, like, math? and then, like. um. you: put, um. I ca:n't really think, um. the
161 (0.6)
162 R: the equals?
163 Anya: a:::. (*puts hand to her head*)
164 R: take away.
165 Anya: yeah.
166 R: subtract. okay. uh-huh?
167 Anya: and like, it was a take away,
168 R: yes?
169 Anya: so:: I noticed it oh:. so this can be a take away too.
170 R: [[so you had. the equals sign with the take away, too.
171 [[*((overlapping talk between Jimmy, Daniel and Danica))*
172 Anya: yeah.
173 R: yeah. *((looks over to Nathan))* was that what you were trying to say, Nathan?
174 Nathan: *((nods))*
175 R: it could be for take away. and it could be for addition.
176 Nathan: um-hm *((nodding))*
177 R: you could have equals in both of those.*((looking at Anya))* yes. it's true. so. we do know what equals sign is. so. (*as if to the whole group*) IF SOMEONE COULD PLEASE CUT OUT THEIR EQUALS SIGN-
178 *((everyone erupts almost simultaneously into overlapping talk))*

In this sequence, I select Anya twice, once by gesture (line 156: *turning to look at Anya*) and then verbally, prefaced by a politeness strategy that might seem extreme, coming from an adult to a child (line 159: okay. pardon me, Anya). Chouliaraki (1998) found that such uses of politeness strategies functioned to modify a teacher's authority. In the data concerning this negotiation, the only other politeness strategy evident is the use of the word please. In every case that word serves to mitigate a direct imperative produced by me (see lines 91, 108, 135 and 177). At this point in Example 5.9, I take the position of story listener with offers of assistance (line 162: the equals?) and continuers (lines 166 & 168: okay, un huh. yes?). Anya then continues the story that had been interrupted (cp. line 141 with line 160: um. it's like. um. when I was in grade one). As she recounts this narrative, Anya positions her actions as being in the past, something she *noticed* when she was in grade one, a possible alignment with Carissa. She also reports it as being new knowledge, constructed in the *here-and-now* (Koole, 2010) by using the change of state token "oh" (Heritage, 1984) in her self-report (line 169: so. I noticed it. oh. so this can be a take away too). By using the word "too", Anya implies an understanding of the equals sign as pertinent to subtraction *as well as* addition, the inclusive interpretation that she first spoke in line 130 (Example 5.8: well, actually it could be for both).

I confirm Anya's position (line 170: so you had. the equals sign with the take away, too) and then I ratify Nathan as a participant in the conversation with my gaze and a question seeking confirmation (line 173: was that what you were trying to say, Nathan?). When I reformulate the taken-as-shared understanding, I use both a specific and a general structure (line 175: it could be for take away. <u>and</u> it could be for addition – line 177: you could have equals in both of those). With this action, I invoke an authority by consensus of examples, raising the level of justification expected. Only after both children have confirmed my version of their understanding do I put the weight of my own (ultimate) authority behind the positive declarative (line 177: yes. it's true). It is important to note that I am not referencing a *mathematically correct* answer, the way a teacher might. Rather, I am making explicit the knowledge that will be taken-as-shared. By so doing, I answer the initial question from lines 32 and 34 (I'm asking a question about equals. is it adding or is it taking away?). My acknowledgement signals a conclusion to the negotiation. The children orient to that contextual cue by noisily resuming their work on the tasks (line 178).

This noisy resumption of tasks concludes the negotiation of the meaning of the equals sign that resulted in the appearance of that symbol in the left hand column of the math symbols chart ("we know these symbols") copied and cut out alone. During the negotiation Anya contributed repeatedly, while all the other children participated sporadically.

### 5.4.3 Summary of the negotiation – nine discursive practices

Figure 5.7 illustrates the essence and the flow of the negotiation of the meaning of the equals sign (n.b., several contributions are missing here in order to simplify the visual representation – they are nevertheless acknowledged in the summary below).

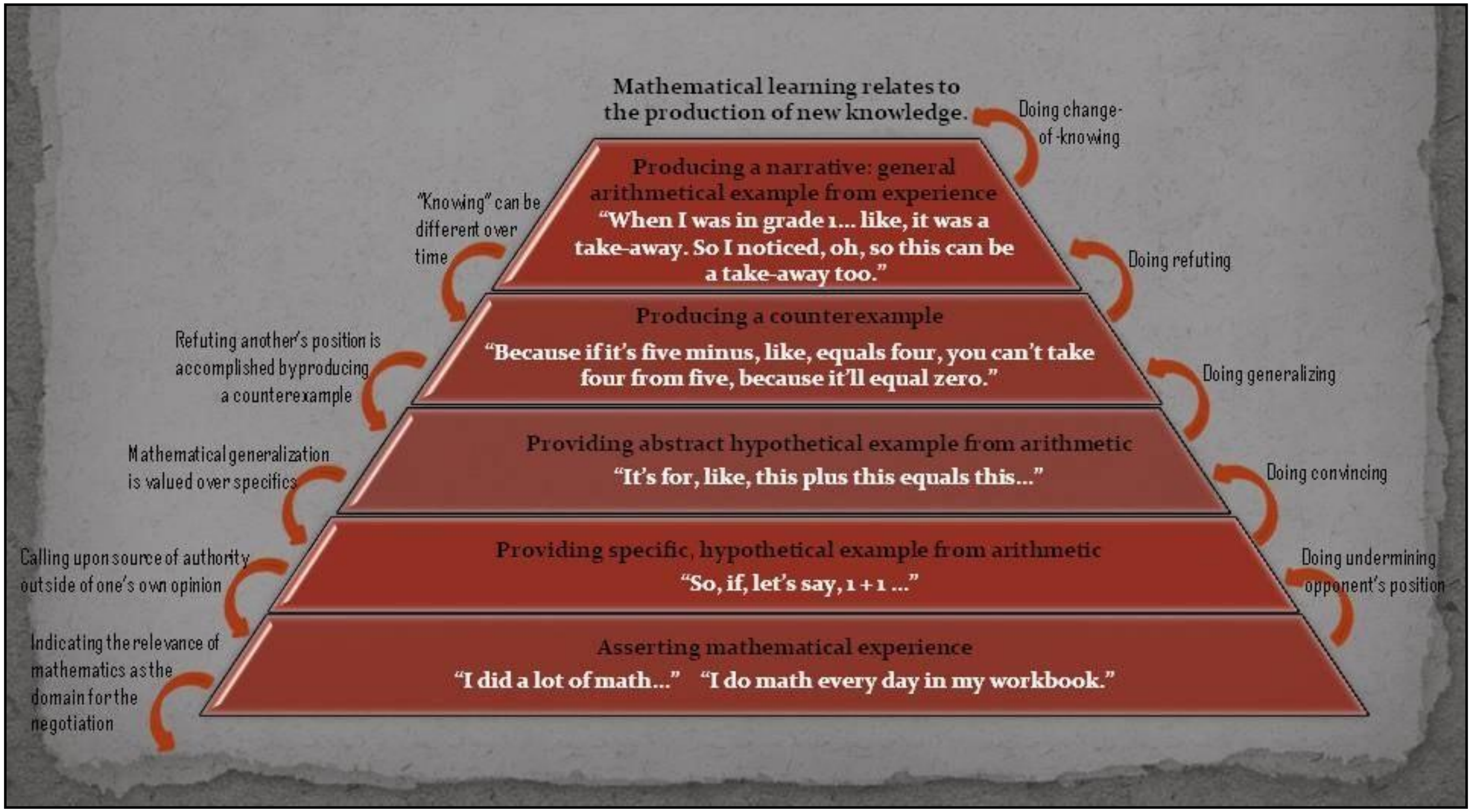


**Figure 5.7 How children positioned themselves vis-à-vis the statements made by others in the "equals" negotiation**

The first main finding of this chapter is that there were nine different discursive practices during this negotiation by which the children incorporated mathematical content, including four different mathematical justifications using examples: general or specific, hypothetical or experienced (see Table 5.1 and cf. Table 5.2 which also includes deployment). Figure 5.7 illustrates the relationships between the positions taken during the negotiation (e.g., the black and white texts within the trapezoids), by showing how children positioned themselves vis-à-vis the statements made by others (the arrows and text outside the main trapezoid).

Anya produced the original position that the equals sign related to the procedure of addition (the two initial positions are missing in the figure) and she went on to justify that position with a specific hypothetical example from arithmetic. She interacted with Carissa, who had taken the position that the equals sign was related to the procedure of subtraction or "take away." Carissa first justified her position with experience in doing mathematics and then, after being solicited by me, with a specific arithmetical example from her experience (this second contribution is also missing from the figure). Anya overheard a discursive production of "not knowing" (Jimmy – missing from the figure), a justification by abstract hypothetical example from arithmetic (Nathan) and a justification by counterexample (Daniel). It is possible that she overheard an animated conversation between Daniel and Cormac that concerned the same topic (see Example 5.7, line 126). Finally, after a fairly lengthy interruption, Anya produced a narrative that reported a change-of-state from "not knowing" to "newly knowing" (Heritage, 2012; Koole, 2010). She reported this as having occurred the year before; in grade one (the same grade that Carissa currently attended). In doing so, Anya accounted for an expectation that she might properly be expected to know something that a younger child knew. However, she extended that "knowing" to include both categories of addition and subtraction.

**Table 5.2 Discursive practices and how they were used by children during mathematical negotiation**

| Example # | Child's name | Discursive practice | How it was used |
|---|---|---|---|
| **5.1** | Anya | Producing a label | Responding to a direct question from R |
| **5.1** | Carissa | Producing a contrast | Injecting comment as an unratified bystander following Anya's display |
| **5.2** | Carissa | Asserting mathematical experience | Claiming authority to argue mathematically: doing undermining opponent's position |
| **5.3** | Anya | Providing a specific but hypothetical example from arithmetic | Following Carissa's assertion of experience: doing convincing |
| **5.4** | Carissa | Producing a narrative: a specific arithmetical example from experience | Responding to a direct question from R: doing explaining |
| **5.6** | Jimmy | Producing an elaborated claim of insufficient knowledge | Responding to a direct question from R: doing disinterest/ resistance |
| **5.7** | Nathan | Providing an abstract hypothetical example from arithmetic | Following R's invitation and Jimmy's claim: doing generalizing |
| **5.8** | Daniel | Producing a counter-example | Responding to a direct question from R by referring back to Carissa's specific example from arithmetic: doing refuting |
| **5.9** | Anya | Producing a narrative: a general arithmetical example from experience | Responding to R's invitation: doing change-of-knowing |

I find Krummheuer's (2011) roles within mathematical argumentation useful here. Within these nine discursive practices (see Table 5.2), the closest example of a spokesman position (paraphrasing someone else's mathematical content) occurs when Daniel refutes Carissa's example by producing his version of a counterexample (lines 136 and 138: you can't take four from five. because it'll equal zero). Each of the other positions includes new mathematical content, possibly constructed in a way that responds to a previous formulation (the ghostee position). Krummheuer (2011) noted a dearth of empirical evidence for this learning position, so such a negotiation provides rich testing ground for his theory as well as elaborating possibilities for the argumentative processes of learning-as-participation.

A closer look at the *ways* the children constructed these positions by invoking sociomathematical norms (deployment, fourth column, Table 5.2) gives evidence to their emerging understanding of mathematical content. Therefore, the second main finding of this chapter is that the ways the children invoked sociomathematical norms demonstrates the children's emergent understanding of the mathematical content in the negotiation as being *relevant to the production of knowledge* (see Table 5.3).

**Table 5.3 Mathematical understanding displayed via referencing sociomathematical norms**

| Example # | Sociomathematical norm referenced | Understanding displayed |
|---|---|---|
| **5.2** | Capacity to argue based on experience "doing" mathematics. | The relevance of mathematics as the domain for the negotiation. |
| **5.3** | Calling upon a source of authority outside of one's own opinion. | |
| **5.6** | Producing a position of "not knowing" is a tolerated practice during mathematical negotiation. | Disposition expressed. |
| **5.7** | Mathematical generalization is valued over specifics. | Arguing using the structure of formal mathematical logic. |
| **5.8** | Refuting another's position is accomplished by producing an arithmetical counterexample. | |
| **5.9** | "Knowing" can be different over time. | Mathematical learning relates to the production of new knowledge. |

The children invoked a number of sociomathematical norms as they positioned themselves within the negotiation. As previously mentioned, Carissa first brought the discussion explicitly within the domain of mathematical knowledge (Example 5.2). She accomplished this by calling into relevance her capacity to take a position vis-à-vis the content based on her experience in doing mathematics. In terms of the negotiation, her discursive actions functioned to *undermine her opponent's position*, a classic rhetorical strategy. Anya then acknowledged the importance of calling upon a source of authority outside of her own opinion: she used a specific example from arithmetic (Example 5.3). She supported her own position through the action of *doing convincing* by referencing a valued mathematical trope: so, if, let's say, one plus one (line 70). Through this action, Anya implied her acceptance of Carissa's positioning the argument within mathematics. They thus indicated to each other a shared understanding that the *ensuing discussion should be framed as mathematical*.

Through the invoking of these two sociomathematical norms: the relevance of mathematics as the domain for the discussion and the calling upon a source of authority outside of one's own opinion, these children affirm the position of mathematics education researchers who define mathematical discourse as that which includes mathematical content, processes or symbols (Moschkovich, 2003). It is as if the symbol of the equals sign indexes the domain of mathematics for the participants.

In the data, Carissa continued the negotiation by acknowledging the use of arithmetical example (Example 5.4). However, she called upon discursive resources of *doing explaining* to accomplish this as she reported an example of her experience (line 85: once when I was doing math? I noticed). After that, the negotiation expanded to include other participants (Example 5.5). In turn, Jimmy oriented to the mathematical content by refusing to take a position even though he was obligated to (Example 5.6). He accomplished a position of disinterest by producing an elaborated claim of insufficient knowing (line 122: I have. no clue). While we cannot interpret this action as referencing his mathematical understanding, we might consider it an indication of his emerging mathematical disposition, an important topic but outside the scope of this chapter.

Following Jimmy's claim, Nathan re-engaged the negotiation (Example 5.7) by producing a hypothetical example constructed as an abstraction (line 127: it's for, like, this plus this. equals. this). Taking the discussion into the realm of abstraction referenced a sociomathematical norm valuing generalization over specificity. Following that, Daniel provided a justification-by-counterexample (Example 5.8) constructed as an attempt to use mathematical logic to refute a previous argument (lines 136 & 138: you can't take four from five. because it'll equal zero). This referenced a sociomathematical norm regarding the production of mathematical argument and included an attempt at algebraic reasoning. These actions of generalizing and refuting represent an emergent capacity to argue in mathematically formal, algebraic ways.

Finally, Anya changed her position vis-à-vis the mathematical content by producing a narrative explanation – a story that referred to a specific earlier experience (Example 5.9). This was constructed as a report and included a discursive marker to signify a change-of-state (line 169: so I noticed it. Oh. So this can be a take away too). This production referenced a sociomathematical norm regarding learning in mathematics: it is possible to know something differently in the past than in the present. Considering that Anya had taken a different position at the beginning of this interaction, her report also acknowledged the value given to that sort of change. This was accomplished without questioning the value of an original position, even though it may be considered "not knowing" in light of the new knowledge (Heritage, 2012). In other words, in mathematics, it is okay to know something before that is different from what is known now. This interpretation is supported by Carissa's "interruption" of the negotiation with a report of newly constructed knowledge (line 142: now I know something).

Taken all together, the references to these sociomathematical norms demonstrate the children's understanding of the mathematical content in the negotiation as being *relevant to the production of knowledge*. These children were actively involved in their own learning by constructing new mathematically oriented knowledge: a process some call *mathematizing* in the broadest sense (Freudenthal, 1991; van Oers, 2012).

## 5.5 Discussion

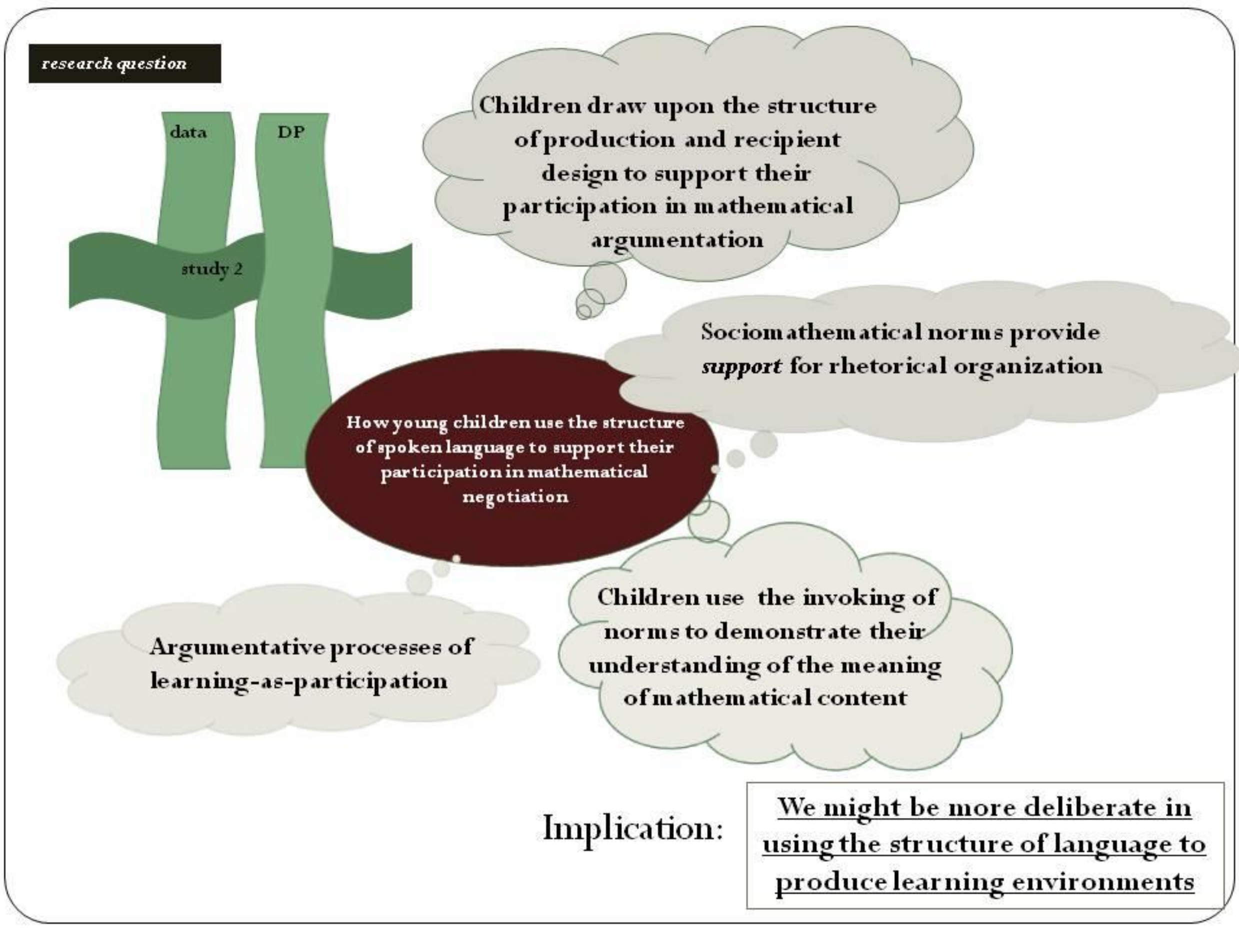


**Figure 5.8 Visual representation of main findings from Chapter 5**

This study has continued the task of documenting and defining the processes involved in mathematical negotiation from the perspective of 5- to 7-year-olds by using discursive psychology to examine some of the ways young children display their understanding of that which is specifically mathematical during negotiation. I have shown how, during the negotiation, the children used nine different discursive practices to incorporate mathematical content within their talk. I have shown how they used the invoking of sociomathematical norms to demonstrate their understanding of the meaning of the equals sign. I have also shown how the children's interactional orderliness gave evidence of their emerging capacity to mathematize and to negotiate meaning using more formal mathematical argument.

The point of this chapter has been to show how the children drew upon the structures of production and recipient design to support their participation in mathematical argumentation (see Figure 5.8). Through the set of interactions examined herein, an adult and a group of 5- to 7-year-olds negotiated a shared understanding of mathematical validity, realizing that understanding through the interactionally constructed features of talk. The negotiation involved what would become valued within this group as a taken-as-shared understanding concerning the meaning of the equals sign. The consensus was that the equals sign would be acknowledged as functioning within both addition and subtraction. This is a non-trivial accomplishment for a group this young and these findings speak to several areas of mathematics education research.

First of all, as an authority figure, I contributed to a setting that encouraged this mathematizing. However, I also produced a problematic version of the meaning of the equals sign, seen in the transcript of Example 5.3 (line 76: one plus one equals, and then you put the answer?). The prevalence of this understanding of the equals sign as being a command to act on numbers is well documented in research (McNeil, 2008; Sáenz-Ludlow & Walgamuth, 1998) even amongst educated adults (Cobb, 1987; Kieran, 1981). This study suggests one reason why that may be so, given Vygotsky's (1978) proposition that children learn from exposure to more competent others. What is at stake here is *who defines* "more competent"? If the adult authority figure in a classroom-like setting introduces incorrect understandings, do the children learn them *as if they were correct*? At the time of the original study I was no different from most elementary teacher candidates, who present themselves as mathematically naive and slightly anxious, based on their earlier negative or at best, mixed experiences with school mathematics (Sheats Harkness, D'Ambrosio & Morrone, 2007). If this is so, it suggests a need to specifically address this misconception regarding the teaching and learning of equivalence in elementary teacher education (Pirie & Martin, 1997).

There *is* a conventional answer to the meaning-of-the-equals-sign question; however it never entered the negotiation. In the introduction to this chapter, I wondered if the participants were not concerned with mathematical correctness because we were negotiating a taken-as-shared understanding of what would be considered a valid response or if the participants had actually thought we *were* taking correctness into account. Sáenz-Ludlow and Walgamuth (1998) provide a perspective on this issue. They note that during mathematical interaction, it can be important for an adult to keep the discussion within the children's use of terms in order to provoke the process of negotiation without introducing concepts to the children that might seem unrelated and therefore disrupt their participation. From this position one might argue that it was vital to the children's ongoing participation in the negotiation that I *did not* introduce the sense of equivalence into the argument, since the children themselves had positioned the equals sign on the basis of its use during arithmetical operations.

This speaks to the rhetorical organization of talk, an area keenly suited to a discursive psychological analysis. Within the example studied here I seemed to stay within the children's orientation. For example, I positioned the original negotiation as divergent (Boulima, 1999; either addition or subtraction would be acceptable, but not both) in keeping with Anya and Carissa's oppositional stances even though presumably I understood that the equals sign was relevant to both operations. Furthermore, I altered my own use of the term "subtract" to "take away" in keeping with Carissa's use (Example 5.4). I did the same with Anya during Example 5.9. However, there is a difference between passively not introducing seemingly unrelated concepts and actively introducing problematic concepts. Furthermore, I note that my introduction of the problematic meaning for the symbol occurred as my own production (see Example 5.3), not as a revoicing of a child's perspective. It was my contribution to a co-production with Anya. To me this illustrates the difference between staying within the children's construction of understanding and introducing one's own problematic understanding. Therefore, I still see the incident as troubling with regards to what children have the opportunity to learn (Hiebert, 2003).

Nevertheless, the incident warrants a claim regarding the distinction between mathematical knowing and mathematical knowledge first introduced in Chapter 4: The question we were negotiating was "Do you know what equals means?" This is substantially a negotiation of *knowing*, since it is debatable whether any one of the participants knows this or not, depending on whose definition of "equals" and "know" comes to be valued. This is different from asking for instance, "What does the equals sign do?" which would be a negotiation of *knowledge* rather than knowing. There is a conventional answer to that question as opposed to the one we were discussing. Future study might productively examine a negotiation of mathematical knowledge.

In a second area of mathematics education research, this study also speaks to theories of learning-as-participation or “the having of wonderful ideas” (Duckworth, 1996, title). Learning-as-participation in practice as opposed to learning as acquisition has been well-conceptualized as a major tenet of sociocultural theory (Ernest, 2010; Forman, 2003; Sfard, 2006). However, there are few concrete examples in the literature of what that might actually look like (see Krummheuer, 2011 for a notable exception). Duckworth (1996) noted that the practices of schooling may actually diminish children’s propensity to participate in practices of uncertainty. This has been observed in mathematics classrooms, where teachers and students alike typically orient to arithmetical correctness with its accompanying certainty (Cheval, 2009; DiPerna et al., 2007; Eriksson, 2008; Walkerdine, 1988). However, constructive mathematizing (Freudenthal, 1991; Lakatos, 1976) has both creative (Worthington, 2005) and playful (van Oers, 2012) elements to it, invoking features of uncertainty (how the activity might be experienced by individuals) and messiness (how it might look to others). “Messiness” was an observed feature of several examples here, highlighting the importance of studying the practice of mathematical negotiation with younger participants (Sáenz-Ludlow & Walgamuth, 1998; Warren, 2006). Therefore, these results serve to operationalize theories of learning-as-participation, with a particular emphasis on argumentative processes, and the results also support the suggestion that educators might be more deliberate in using the structure of language to produce learning environments (Duckworth, 2001).

Documenting the messiness of learning was foregrounded by the setting, a “learning experiment” that highlighted children’s capacity to contribute to mathematical interaction. However, it was also a function of the theoretical and analytic framework. Discursive psychology demands a rigorous commitment to data in the form of spoken interactions. This includes not only the substance of the interactions but also their messiness, or communicative "noise" (e.g., false starts, hedges, repairs). This feature of communication is missing from many studies of mathematical discourse that tend to sanitize the transcriptions in order to make them accessible to non-linguists (Ryve, 2011). Discursive psychology not only *allows* the communicative noise, but highlights it as the *focus of study* in order to consider participants' perspectives as they orient to each other and to the local context by the *ways* they construct their positions (Edwards, 1993). The findings here provide valuable details regarding young children's capacity for mathematical argumentation. Thus, discursive psychology offers great potential to improve the rigour of mathematics discourse research for answering these kinds of questions.

The main limitation in this study may have been its reliance on spoken language alone to illustrate the meaning-making practices of young children. I justify the practice according to the research question I was interested in answering and an acknowledgement that 5- to 7-year-olds are at an age where the narrative features of mathematical argumentation begin to take precedence over other forms of communication (Krummheuer, 2013). I note that the discursive practices of production and recipient design are almost exclusively recognized via their verbal indexes and they were not difficult to notice within the interactions here. Nevertheless, scholars who foreground the perspectives of young children emphasize the significance of non-verbal communication in meaning making (McTavish et al., 2012; Narey, 2009). Thus, the limitation is duly noted and I propose the inclusion of multimodal aspects of interaction in future studies.

In Chapter 4, I concluded that educators and researchers might fruitfully expand our definitions of what *mathematical knowing* looks like. I built on that suggestion in this chapter, by considering in detail a negotiation of mathematical knowing: *Do you know what equals means?* In this chapter, I noted that the mathematical substance of the negotiation represented a key factor in emergent algebraic reasoning. Indeed, the equals sign was not the only symbol that engaged the children's curiosity during N:Countr. The calculators in the room provided the children with opportunities to inquire into the function of the square root symbol, the existence of negative numbers and the representation of infinity. Features of algebraic reasoning emerged in Chapter 4, with Cormac's challenge of my generalization using a counterexample and Carissa's quantifying of letters in order to make sense of their inclusion in mathematics.

This study represents an essential second step in investigating the positive contributions of children to the negotiation of sociomathematical norms. The findings thus provide recognition of some of the ways young children display their understanding of that which is specifically mathematical during negotiation. Defining and documenting that participation through a discursive psychological lens will strengthen the knowledge base regarding the function of language in inquiry-based settings with young children and heighten the awareness of both researchers and practitioners regarding children's capacity to contribute to classroom mathematical practices.

# Chapter 6: Algebraic reasoning as social practice in the experience of young children: A discursive psychological/ multimodal perspective.

This chapter describes the combined research implementation of two approaches which are based in sociocultural theories of learning: discursive psychology and multimodal data analysis. My aim is to broaden our understanding of what constitutes algebraic reasoning in order that we may recognize earlier forms of it. The assumption is that if we can recognize earlier forms of algebraic reasoning we can design learning experiences to build on those, thereby addressing what has typically been a problematic transition for many students. For this purpose, I analyze the progression of a mathematical inquiry by ten children in an after school setting that integrated play and numeracy, through a series of interactions that occurred over six weeks. This serves to operationalize the relation between algebra and early mathematical thinking and offers discernment into the kinds of algebraic concepts young children can learn in a supportive setting. These examples provide insight into how the later learning of formal algebraic notation might build on earlier unconventional formulations.

## 6.1 Introduction

Algebra as a focal point for the early grades has been reconceptualized to incorporate experiences in algebraic reasoning rather than simply memorizing algorithms and formulas for the manipulation of symbols (Jacobs, Franke, Carpenter, Levi & Battey, 2007; Kaput, Carraher & Blanton, 2008). Elementary students express algebraic reasoning in various ways (Kaput, 1999) including the strategic use of generalizing, or formalizing patterns and regularities. Developing effective resources for algebraic thinking during childhood is critical to gaining a clear understanding of more complex mathematical concepts later on. However, it is widely recognized that transitioning from arithmetical to algebraic reasoning is difficult for students and research has provided many examples of classroom observations and teaching experiments aimed at understanding and overcoming that difficulty (Cai & Knuth, 2011; Goldenberg, Mark & Cuoco, 2010; Kieran, 2011; Lodholz, 1990; McNeil, 2008). Yet the difficulty persists for students (Blair & Razza, 2007; Boylan, 2010): "The transition from arithmetic to algebra is difficult for many students, even for those students who are quite proficient in arithmetic, as it often requires them to think in very different ways … [going] beyond mastery of arithmetic and computational fluency to attend to the deeper underlying structure of mathematics" (Cai & Knuth, 2011, p. viii-ix).

Central to the current understanding of early algebraic reasoning that is important for this study is a recognition that Vygotsky's (1986) identification of algebra as a higher level of thinking and arithmetic as a lower level has translated into an artificial pedagogical distinction between the two at least in North America: arithmetic is seen as a necessary precursor to algebra. Thus, the transition between the two is difficult for students precisely because they experience it as a transition in topics in school-based mathematics practice (Kieran, 2011; Radford, 2011). There is now general consensus among researchers that primary students can and do reason algebraically (Cai & Knuth, 2011; Kaput, 2008; Kieran, 2004; NCTM, 2000) and a set of teaching practices organized around "inquiry-based learning" has developed that foregrounds those kinds of meaning-making actions (Steinbring, 2006).

In this chapter, I address a difficulty in noticing algebraic reasoning in young learners by framing it as a *social rather than a cognitive practice,* expecting that this might offer new insight into students' experience. Most research into mathematical learning examines formal classroom or laboratory settings, despite robust evidence that the social practices of such settings contribute to enduring difficulties for individuals (DeCorte, Verschaffel & Depaepe, 2008; Yackel & Cobb, 1996). I take an alternate perspective here, assuming that the communicative medium of algebraic reasoning might be central to its expression rather than peripheral (Noss, Healy & Hoyles, 1997). Therefore, I use discursive psychology (Edwards, 1997) as a theoretical framework to explore data generated through a "learning experiment" (Francisco & Hähkiöniemi, 2012) grounded in the practices of inquiry-based learning and carried out with 5-to 7-year-olds. Furthermore, I take a broad view of discourse that includes multimodal features of mathematical language: verbal, gestural, visual and numerical aspects of communication (Clark, 2004; Kendrick, in press; Noss, Healy & Hoyles, 1997). This combination of theory, setting and analysis helps to make the children's "thinking" visible, in that the social processes of the group setting are understood to produce algebraic reasoning or make it a relevant action.

This chapter is the third of three stand alone studies that comprise the results chapters of the dissertation. In it I look at how young children use sociomathematical norms as resources to support their efforts to make sense of unfamiliar mathematical content. The data considered here include video tapes of a series of interactions from across six weeks of the N:Countr mathematical research group that met with me for eleven weeks in total (McLellan, 2010). The children encountered the square root symbol during a classification activity and were curious to explore what this symbol might mean. The presence of simple calculators in the room facilitated their inquiry as the children were able to use them to produce results lists of input and output values in a format very similar to a "guess-my-rule" game (Francisco & Hähkiöniemi, 2012).

The children used mathematics journals[8] to record their results and began to make verbal conjectures and generalizations, writing those conjectures in various forms at my suggestion to do so. Through analysis I will show how these students expressed their reasoning in ways that built on each others' contributions, thereby negotiating a shared understanding of the mathematics in the inquiry.

There are two ideas I intend to explore in this paper. The first is to describe what happened in the square root inquiry: *How do young children construct a mathematical inquiry?* The second idea I explore concerns a central process of algebraic reasoning: *How do young children express mathematical generalization?* By answering these questions, I hope to contribute to efforts to operationalize the relation between algebra and early mathematical thinking and also offer discernment into the kinds of algebraic reasoning young children can use in a supportive setting.

In the following sections, I briefly outline the discursive psychological framework that underlies this study as a review of concepts originally presented in Chapters 2 and 3. I then focus on the potential within multimodal data analysis for discerning nuances in the meaning-making practices of young children and introduce the research on early algebraic reasoning. Finally, I analyze examples from the data and explore what multimodal analysis from within a discursive psychological framework tells us about student participation in algebraic reasoning. This chapter then, formalizes an integration of the two tasks I set for myself in the dissertation: I use a discursive psychological perspective as a means to operationalize learning-as-participation.

[8] "Mathematics journals" in the sense I use it refers to a free form opportunity to make a written record of *anything* the children considered to be "mathematical."

## 6.2 Research context

Herein, I review from Chapter 2 what is known about inquiry-based learning, discursive psychology, and multimodal data analysis to provide just enough background research to situate this study. I then discuss what it might mean to "do algebraic reasoning."

### 6.2.1 Inquiry-based learning in mathematics

The type of interaction considered in this study is based in mathematics reform oriented practice that emphasizes student participation and discourse in classroom communities (Yackel & Cobb, 1996). These inquiry-based learning communities are an educational context in which the discursive actions of conjecturing and generalizing by students make sense. The pedagogy foregrounds the meaning-making activities of students, learning as a process that happens over time and the importance of building on prior understanding. It expects teachers to provide contexts which support and facilitate inquiry. Inquiry-based learning environments provide opportunities for reasoning algebraically precisely because they focus on children's learning as participation in practice. This is contrasted with traditional approaches to teaching mathematics, where learning is focused on acquiring knowledge (Bruder & Prescott, 2013).

Krummheuer (2011) observed four roles that children might assume vis-à-vis mathematical content during inquiry-based learning. His identification of four roles of production design are relevant to this study: an *author* presents an idea with original content and formulation, a *relayer* re-presents another's idea in the same form, a *ghostee* presents new content within someone else's formulation and a *spokesman* paraphrases another's content within an original formulation. While all of these roles exist in theory, Krummheuer found that less was known about the ghostee position in particular, a position that he identified as enacting learning-as-participation. Given the findings of Chapter 5, where I showed how the children built their negotiation upon previous positions, I wondered if a similar dynamic might be at work in this set of data generated over six weeks. My analysis in earlier chapters showed that I shared the authority for mathematical knowing and I offered space and time to allow children to participate in ways that were meaningful to them. I wondered how I might have foregrounded the children's meaning making activities in my practice.

Indeed, as I look back at the video data I collected, some of my practice during the research group seems unusual from a conventional mathematics education perspective. However, my actions at the time were informed by an Early Childhood pedagogical perspective that is well aligned with inquiry-based learning (Cadwell, 2003; Wien, 2008). For example, an Early Childhood perspective informed my introduction of the internet printout that duplicated Tara's drawing of trajectory that evolved from play with marbles, ramps and receptacles and prompted the negotiation examined in Chapter 5 (see Figure 6.1). Relevant to this study, the internet printout included a mathematical representation of the quadratic formula that introduced the square root symbol that became the focus of the children's inquiry here.

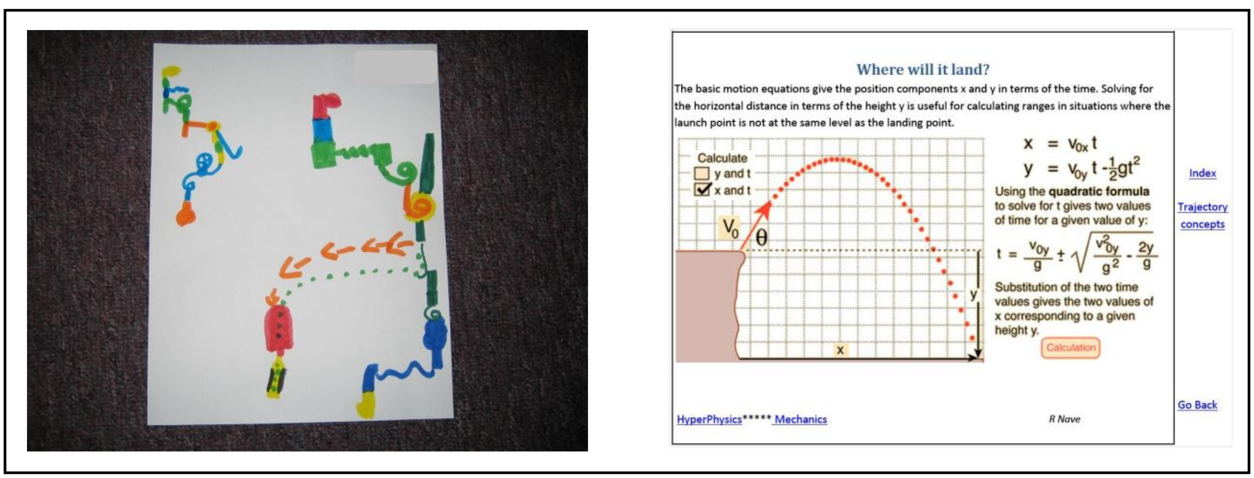


**Figure 6.1 Tara's trajectory drawing, with similar internet-found illustration © n.d. R Nave, GSU. Reprinted with permission. Retrieved September 28, 2009 from http://hyperphysics.phy-astr.gsu.edu/hbase/traj.html**

The mathematics in this study is centered on the children's attempts to make sense of that symbol. As it turns out, much of the interaction in inquiry-based mathematics involves interpreting the meanings or concepts that are attached to conventional symbols by using those symbols in various ways (Steinbring, 2006): "Any student of mathematics has to accept the conventional symbols already in place; however, *acceptance of a symbol, in itself, is not always accompanied by its harbored meaning(s)* [emphasis added]" (Sáenz-Ludlow & Walgamuth, 1998, p. 153). Symbols and notation are an essential part of mathematical communication and understanding at all ages (Cobb, Yackel & McClain, 2000) and encapsulate the power and efficiency of mathematical expression even at the primary education level (Hiebert, 1989). By studying the children's approach to a symbol for which *I* knew the function but *they* did not, I hoped to gain some understanding into the children's assumptions and the expectations that they brought to this mathematical experience (Potter & Hepburn, 2008).

Due to the influence of constructivist theory in mathematics education research (Cobb et al., 1992a), studies concerning children's understanding of mathematical notation tend to focus on either the children's invented notation (Brizuela, 2004) or the ways in which they use conventional notation in their written mathematics (Gifford, 1990), or both. While this emphasis on written expressions is understandable, it has left a gap in the research regarding the ways in which children attempt to understand conventional symbols written by others. However, the failure to meaningfully appropriate conventional mathematical symbols remains a key problem for children (Hiebert, 1989; van Oers, 2000; Witherspoon, 1999). Therefore, I wondered if taking a discursive psychological approach in this study might offer insights into the children's algebraic sense-making activities, which included conjecturing, refuting and generalizing with regard to the meaning of a conventional symbol. By describing how the children developed and sustained the “square root inquiry”, as I came to call it, I will answer the first research question: *How do young children construct a mathematical inquiry?*

### 6.2.2 Discursive psychology

Discursive psychology (Edwards, 1997) affords the possibility of viewing mathematics as a discursive, social process. It assumes that meaning is co-produced by all conversants, and that discourse is fashioned in ways that perform social functions that are relevant to the local setting (Wetherell, 2001). This theoretical framework facilitates a focus on algebraic reasoning as a social action: what I call here *doing algebraic reasoning.*

Discursive psychology concerns itself with talk-in-action and/or the reporting of that talk based on an ethnomethodological assumption that participants display their understandings to each other as they communicate and thereby produce the social order within which they interact (Edwards, 1993; Garfinkel, 1967). The same assumption can be applied to a revoicing of something said earlier, either by oneself or by another (Barwell, 2003). Of all the various ways a re-statement might be formulated, the particular form it takes displays a speaker’s orientation to particular underlying assumptions. In this study, I will look for instances where children produce multiple forms of the same conjectures in order to understand the assumptions behind the ways in which the different forms were socially constructed or made relevant.

Discursive psychology takes traditionally psychological topics, such as thinking, knowing or understanding and “respecifies” them as social actions (Edwards & Potter, 2005). Therefore, it affords a position from which to interpret discursive strategies used during algebraic reasoning for the social actions they accomplish. In other words, within a discursive psychological framework *thinking* is seen as a social action. That means we can consider that the argument produced *is* the reasoning. In an earlier chapter (Chapter 4) I examined the discursive practices of *doing knowing*. Building upon that previous analysis one could imagine social actions in an algebraic sense such as *doing explaining*, *doing conjecturing* or *doing generalizing*. These social actions can form a basis upon which to distinguish communicative events within and between interactions (Hymes, 1964; Saville-Troike, 2003). Therefore, I use them as a basic unit of coding in this analysis.

### 6.2.3 Multimodal data analysis

Krummheuer (2013) noted that within mathematical argumentation, narratives are almost exclusively verbal, presenting some difficulties for young students who might not be able to comprehend the rationality of the whole plot. As Kaput (1999) noticed, in mathematical contexts young children often speak using unconventional terms and rely on gestures and other non-verbal modes of communication to relay their meaning. Rather than viewing this as a deficit, multimodality calls that the child's natural synaesthetic capacity (Cope & Kalantzis, 2009; New London Group, 1996). Therefore, the use of multimodal data analysis alongside discursive psychology here offers enhanced opportunities to include multiple modes of communication as features in children's meaning-making processes. This coincides well with the notion of *communicative event* from the ethnography of communication perspective (Hymes, 1964; Saville-Troike, 2003) and Halliday's (1978, 1994) *semiotic resources* that would include language, visual images and mathematical symbolism (O'Halloran, 2005).

Several studies of multimodal experiences of meaning making that include young children's own perspectives and interpretations inform an understanding of numerate thinking. Kendrick and McKay (2009) used both content analysis and children's interpretive explanations of their own drawings to develop a more expansive view of literacy than they were able to conceive when they considered the drawings alone. Concerning numeracy, Worthington (2005) noted how few teachers could provide specific examples of children's creative mathematics, in spite of an expectation that they existed. She concluded that this was at least in part due to a narrow view of numeracy as concerned solely with specific resources (e.g., sand or blocks) and activities (e.g., singing or painting). Kendrick and McKay (2004) also found that young children can visually express complex notions of literacy yet unverbalized. This has been noted in numeracy studies by Davis (1996), who conceptualized *unformulated understandings* as those mathematical meanings that a student might express through gestures or actions when unable to articulate them.

There are several ways this multimodal analytic approach coincides well with discursive psychology. For example, I use the term "mode" in this study to refer to the *nature of the action* of making meaning, that is, whether it is verbal, visual or gestural. This aligns with O'Halloran (2005) but is distinct from Kress and van Leeuwen (2001) who use *mode* to refer to semiotic resources such as narrative. Thus, a communicative event here can be described by its modality (e.g., verbal, gestural, visual) *and* its social action (e.g., doing explaining, doing agreeing). In this way, I argue that using multimodal analysis beside discursive psychology facilitates a new perspective on the ways children have available to them to *do algebraic reasoning*. For example, Barwell (2005), a mathematics education researcher working from within a discursive psychological perspective, describes the significance of indexing gestures within mathematical communication, especially when speakers are referring to written or symbolic representations. In this study I propose to extend that significance to non-indexing gestures as well, those gestures that do not coordinate specifically with spoken referents (e.g., this, that, it). So, while both discursive psychology and multimodality orient to the social action, multimodality extends the kinds of action available for analysis.

The socially constructed nature of participation in learning is another area of congruence between the two perspectives. In multimodality this is noted as "one person's designing becomes a resource in another person's universe of available designs" (Cope & Kalantzis, 2009, p. 177). In discursive psychology, one person's uncontested introduction of a type of social action (say theorizing or invoking a particular sociomathematical norm) into a conversation makes that action subsequently relevant for others. In other words, the first instance provides a warrant for the kind of participation subsequently produced. In Chapter 5 of the dissertation, I showed how the participants in the mathematics research group constructed a collective argument by building their participation upon previous arguments. In this chapter, I use the same approach to show how the participants construct a mathematical inquiry over several weeks by building their participation on previous positions taken by others. Both multimodality and discursive psychology support such analysis.

A third relation between the two perspectives is found in what is called *the pedagogy of design and multimodality* (Cope & Kalantzis, 2009). In both perspectives, representations (including language) are seen as dynamic processes constructed (DP) or designed (MM) for locally situated contexts: "Meaning makers are not simply replicators of representational conventions. Their meaning-making resources may be found in representational objects, patterned in familiar and thus recognizable ways. However, these objects are reworked" (Cope & Kalantzis, 2009, p. 175). Therefore, although we may recognize a child's reproduction of the square root symbol and even be able to make sense of his or her use of it in arithmetical number sentences, we must not do so without understanding that we are putting our own filter of understanding onto another's representation.

This highlights a significant difference between discursive psychology and multimodality. Conventional multimodal analysis does often foreground a disclaimer regarding analyst subjectivity: “critical to this process [of analysis] was a negotiated interpretation of the images, filtered through our own preconceived understandings and subjectivities as researchers” (Kendrick, in press, p.6). However, there are also times when representations are interpreted as if they provide a mirror into what-happens-in-the-mind, a position untenable to discursive psychology. For example, Kress (1997) observes a child silently organizing (classifying) six ordinary objects that he has drawn and then cut out. After being asked for an explanation, the child verbally responds. Kress then interprets, assuming that the action of classifying occurred within the mind of the child, in a visual/spatial mode and without language, until that mode was requested: “What seemed like silence was in fact a period of cognitive activity… language, acting as a universal communicational solvent, serve[d] to describe, *after the event*, what ha[d] taken place” (p. 42, italics in original). I argue that an analyst working from within discursive psychology must respond to this interpretation with a, “well, maybe” (Edwards, 1993).

After all, this is *not* the *only* interpretation possible, a position that multimodal analysts would certainly agree with. However, discursive psychology goes one step further, *requiring* a “putting aside” of this kind of interpretation in order to make a different kind of interpretation that might be less dependent on the subjectivity of the analyst. In other words, discursive psychology does not answer every possible research question, and does not even attempt to answer those involving *what children really think* (Edwards, 1993) or what might be going on in their minds. Rather, it takes a position that focuses on *the way the child produced his explanation* and how that production gives evidence of his assumptions regarding the situation he finds himself in. It is an *over-the-shoulder* perspective rather than *in-the-mind*. What is interesting to me is that both approaches come to the same conclusion regarding the child in Kress's (1997) observation and that child's action of classifying: the version where the child produced and then manipulated his own objects required more complex reasoning of classification than a traditional version that might require drawing lines to connect identical pictures. The conclusion is the same; the analytic approach differs. On that basis I argue that discursive psychology and multimodal analysis can fruitfully be combined despite some divergence in their perspectives: discursive psychology gains the ability to include other modes of meaning-making such as non-indexing gestures, and multimodal analysis gains a form of rigour in warranting for interpretations.

The participants in this study were engaged in a form of "generalized arithmetic" as they worked with numerical abstractions and used arithmetical operations to account for patterns within and across the numbers in their input/output lists (Kaput, 1999). Therefore, I present their mathematical activity in the square root inquiry as a socially constructed form of *doing algebraic reasoning*. Exactly what I mean by this phrase is the concern of the next section.

### 6.2.4 "Doing algebraic reasoning"

The perspective I take in order to answer the research questions is to focus on the discursive practices that people use to talk about, and therefore produce, mathematical thinking (Barwell, 2009). In this framing, "reasoning" is not an internal state of mind but rather an interactional resource, defined as "a flow of propositions within a discourse of reasoned argumentation" (Anderson et al., 2001). Inquiry-based mathematics foregrounds the meaning-making activities of students, facilitating this focus on the flow of propositions as collective, discursive activity. By assuming here that the discursive medium of algebraic reasoning might be highly salient to the form its expression takes (Noss et al., 1997), I can account for children's expressing generalizations in gestures as well as talk (Kaput, 1999). This new set of assumptions might then afford new insights into what the activity of algebraic reasoning means in the context of young children's learning.

A focus on the meaning-making role of representations is somewhat unusual in discursive psychology, "although not excluded in principle" (Potter & Edwards, 1999, p. 448). This is illustrated by my particular emphasis on using the auxiliary verb "do" in the present continuous form before a main verb extensively in this chapter and elsewhere in the dissertation (n.b., *doing knowing*, *doing agreeing*, *doing algebraic reasoning*) a rather distinctive practice even within the discursive psychology literature. Therefore the study here has the capacity to extend the influence of discursive psychology as well as provide insights into the relationship between algebra and early mathematical thinking, *as social processes*. However, when one constructs a notion by extension in this fashion, it is prudent to "establish that the notion itself is not conceptually contradictory" (Richards, 2006, p. 56) before introducing supposedly exemplary interactions. For that reason, I will examine how scholars have conceptualized early algebraic reasoning and identify those significant features of interaction that direct attention to its possibility as a social process. With this argument I will present my formulation of what it means to *do algebraic reasoning*.

A rich literature has developed concerning the practice of early algebraic reasoning. Kieran (2011) synthesizes multiple studies when she describes generalizing as both *a route into* and *a characteristic of* early algebraic thinking and conceptualizes algebraic thinking as reasoning *in certain ways* (Radford, 2011). However, I find Kaput's (1999) emphasis on the relationship between generalizing and symbolic representation best suited to inform this study since the students here were engaged with the practices of generalization in the pursuit of understanding the function of the symbol: "√."

> Generalization involves deliberately extending the range of reasoning or communication beyond the case or cases considered, explicitly identifying or exposing commonality across cases, or lifting the reasoning or communication to a level where the focus is no longer on the cases or situations themselves, but rather on the patterns, procedures, structures and the relations across and among them … but expressing generalizations means rendering them into some language, whether in a formal language, or, for young children, in intonation and gesture. (Kaput, 1999, p.136)

Relevant to this study, Kaput's formulation speaks to the significance of multimodal expressions of communication in the experiences of young children.

Kaput, Carraher and Blanton (2008) include several different perspectives in their seminal work on conceptualizing early algebra. For example, Smith and Thompson (2008) consider quantitative reasoning to form an essential bridge between arithmetical and algebraic reasoning. Others in Kaput et al. (2008) consider algebraic reasoning as mathematical sense making (Schoenfeld, 2008), the practice of symbolizing (Kaput, Blanton & Morino, 2008) and representational thinking (Smith, 2008). Within empirical studies of children generalizing, Lannin, Barker and Townsend (2006) foreground the connection between symbolizing and meaning-making and propose "generalizing numeric patterns is viewed as a potential vehicle for transitioning students from numeric to algebraic thinking because it offers the potential to establish meaning for algebraic symbols by relating them to a quantitative referent" (p. 3). In their study of the factors influencing student strategy choice, they include social interaction as a possible contributing factor alongside cognitive and task features. Indeed, they find that social interaction is implicated in every one of the 18 sessions they examine, but conclude that the "unpredictable nature" of the influence renders further analysis untenable. They therefore focus on cognitive and task features in their analysis. I propose that a sociological framing such as the one I describe here might have increased the analytic power available to these researchers.

Within the literature I find three other studies particularly relevant to defining algebraic reasoning as discursive activity. Noss et al. (1997) consider algebraic reasoning as a way of "layering meanings on each other, rather than as a way of replacing one kind of meaning with another. The emphasis is on *connections* between ways of knowing and seeing, rather than on the *replacement* of one by another" (p. 226, emphasis in original). They highlight the extent to which their participants' expressions of mathematical meaning are bound up in the medium, be it language, action or symbolic formulation. In a follow-up study (Mavrikis, Noss, Hoyles & Geraniou, 2011, p. 2) they derived three key ways of reasoning algebraically with respect to generalization in particular:

- perceiving structure and exploiting its power;
- seeing the general in the particular, including identifying variants and invariants;
- recognizing and articulating generalizations, including expressing them symbolically.

A similar list comes from Francisco & Hähkiöniemi (2012), who determined generalization by noting "whether students saw a pattern emerging, whether they were able to see that the pattern held for all numbers and [whether they] provided a rule in recursive, explicit or other form" (p. 1008).

I adapt Schegloff's (1987) comment on the pre-emptive determination of context (*any* context) in support of my position to view reasoning as a social action: "it is the talk of the parties that reveals, in the first instance *for them,* whether or when the [context of algebra] is relevant" (p. 219). This claim from a prominent conversation analyst suggests that algebraic reasoning is *not necessarily* outside the realm of possibility for young children, *provided we can* *justify a way of recognizing their interactional signals or "indexes."* This becomes then essentially a methodological problem: how to recognize algebraic-reasoning-like behaviour in young children who may be unfamiliar with sociomathematical conventions. Pedagogical resources for elementary teachers (Goldenberg et al., 2010; Small, 2009) have identified simple versions of generalization that have potential to address this issue:

- representing mathematical relationships,
- explaining relationships among quantities and
- analyzing change.

These are introduced as strategies that young children might be taught in classrooms. My study concerns children's propensity to reason algebraically in an informal context. Therefore, I propose that if we can identify the discursive practices implicated in the actions of representing mathematical relationships, explaining relationships among quantities or analyzing change in that setting, we can highlight something of the children's capacity to do so.

Building on the findings of previous exploratory studies with this group (Chapters 4 and 5), I suggest that the practices of generalizing might be identifiable when children either respond to those of my direct questions that invoke practices of algebraic reasoning or build on their own or others' previous examples of algebraic reasoning. These practices then formulate my conception of *doing algebraic reasoning* and as they involve generalization, can answer the second research question: *How do young children express mathematical generalization?*

## 6.3 Method

### 6.3.1 Participants and setting.

This chapter draws from data generated during a larger study that explored representations that constitute young children's mathematical thinking. The eleven week after-school mathematics research group met weekly for one hour in a room in my house in a suburban community in Western Canada. The group included ten children (5- to 7-years-old: five boys and five girls) and myself as a participant-researcher ("R" in the transcripts). I chose five to seven year olds as participants for the original study, expecting that they would be articulate while still within the early childhood period considered central to the transition between arithmetical and algebraic ways of reasoning (Linchevski & Herscovics, 1996). As a participant-researcher I presented myself as an interested adult, but neither a mathematics teacher nor a parent. The children all had a previous multi-year relationship with me in a child care setting, consistently referring to me as "Nanny", a title of respect alluding to those earlier days. We shared a history of exploring aspects of our mutual interests together during their pre-K years.

While we had certainly considered topics of mathematical interest during the earlier years, they were never signified as such. On the other hand, the eleven week long research group was decidedly concerned with exploring "What children know about numbers and math." Activities were organized around basic learning types that were familiar to the participants: circle time (a whole group discussion), small groups that were assigned specific tasks, journal writing time and play time (both guided and free-choice). The children displayed a physical orientation to the setting as being a classroom, by enacting such practices as asking for permission to use the washroom, raising their hands to gain my attention and spontaneously lining up in front of me to wait for a turn to talk. However, it is important to recognize that we were not exploring mathematical concepts with manipulatives as one might in a clinical interview (Ginsburg, 1997): there were no formally preconceived mathematical goals and the mathematical content emerged from the interactions. This kind of setting has been described elsewhere as a “learning experiment” as compared to the “teaching experiment” of other design type studies (Francisco & Hähkiöniemi, 2012; cf. Cobb et al., 1992b). The richness of the setting is that it incorporated aspects of both home and school while foregrounding children’s curiosity and agency. We were talking and playing: while we interacted, mathematics happened.

### 6.3.2 The inquiry

During the weeks leading up to the inquiry as formalized here, the group encountered the square root symbol during an adult-initiated extension of their play. This experience has been reported elsewhere (Chapter 5) so I summarize as such: the square root symbol emerged as the only representation categorized as “we don’t know this symbol” during a classification activity in Week 4 (see Figure 6.2).

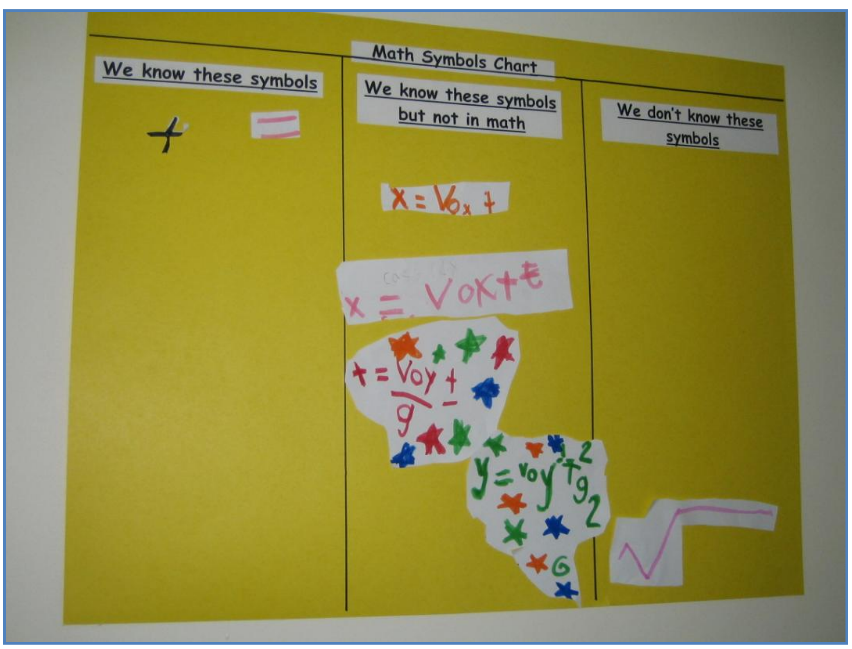


**Figure 6.2 Completed math symbols chart with the square root symbol**

From this initial exposure, the children's engagement with the symbol was especially influenced by Carissa's observation that it was represented on calculators. Three (later, four) simple calculators, such as those used in earlier studies such as CAN (Calculator-Aware Number Project, Duffin, 1997) or the Calculators in Primary Mathematics project (Stacey & Groves, 1994) were available in the room as a tool to explore with. The children discovered that on a calculator, the √ key "does" something. In fact, it is the *only* key on a simple calculator that produces a change when pushed in sequence following a numeral (with the exception of the % key, which transforms every numeral into "zero" – this was discovered and quickly abandoned, perhaps considered uninteresting). In contrast, the √ key "changes" *every* numeral into a *different* numeral.[9] It was these "results of an unknown process" that seemed to engage the children's curiosity, and they began to organize those results and notice patterns in them. The inquiry included up to six children during any one week. At one time or another, every child in the group contributed to it. There were three children who participated most consistently in the inquiry: Carissa (5y9m), Daniel (7y4m) and Nathan (7y4m). Carlyn (6y6m), Cormac (7y5m) and Anya (7y8m) occupied occasional but significant roles. Nevertheless, it never took up an entire hour for any one child. Rather, it represented one activity amongst several that the child would engage with during the hour.

### 6.3.3 Data collection and analysis

Each week, the group interactions were video recorded with a single unmanned camera (also the source for audio) which was set up at the side of the room. I photographed activities at the request of the children or whenever I felt they had potential mathematical significance. The

[9] With the exceptions of "1" and "0." It is interesting that the children recognized "√1 = 1" as part of the series, recording it in their lists even though the numeral had apparently not changed. This speaks to an assumption that the √ key consistently performed the same operation.

children engaged in a variety of activities including play, art and construction work, individual conversations with and without me, small group discussions with and without me and whole group discussions that I facilitated. Various child- and adult-produced artifacts became the focus of discussion at different times and are included here if relevant to the square root inquiry. I completed field notes at the end of each hour while re-viewing the video in order to note actions that may have been missed due to camera angle and if possible, to recall inaudible statements.

The main data for analysis consists of the video recording and transcripts of the Weeks 4 and 6 – 11 of N:Countr Phase II, supported by photos, my field notes and the children's journal entries. Initially the data was digitized and then imported into Transana, computer software designed to aid discourse analysis (Fassnacht & Woods, 2012). A transcript was generated using whole word conventions (see Chapter 4) then the weeks of video and transcripts were analyzed over five passes roughly in line with the suggestions made in Duncan (n.d.).

During the first pass, I coded the transcripts at the stanza level (Gee, 2005), identifying principal incidents or "communicative events" relating to the inquiry (Kendrick, in press), a non-trivial task considering the group nature of the interactions. The criteria for determining stanza boundaries (called "clips" within the software program) were set as the initiation of a new idea relating to the inquiry and limited to one over-riding social action. Clips included both the speaker and the recipient if both exhibited a shared orientation (e.g., through adjacency pairs like question/answer, revoicing an identical statement or unproblematic turn taking like the giving and receiving of information). It was possible for the same speaker to initiate multiple social actions – and one "conversation" therefore, might be coded with multiple clips. On the other hand, some clips included me alone. These were removed from the data to allow a focus on the actions of the children. Clips were enclosed with video time codes that coordinated the transcript with the video to which it referred. This allowed repeated viewing of the clips with the accompanying transcript and facilitated tracking of speech alongside movement.

A focus on video as opposed to transcripts alone led to a discovery that some interactions regarding the inquiry occurred between children alone, without me. Oftentimes these child-only interactions were not well represented in the transcript (due to inaudibility) but the actions in the video were clear. If there was reason to believe the interaction involved the square root inquiry (e.g., it happened while children were waiting to show me something related, or some words were discernible), I included these clips during the first pass, coded as “without R.” During this phase of data analysis it also became clear that some ideas, in the form of journal entries, had never been presented publicly, therefore they did not readily position themselves within the clips. Nevertheless they clearly involved the mathematics under investigation; therefore these representations were included as analytic notes attached to the week in which they were written. There was only one representation of the inquiry in Week 11 that involved a child (journal entry by Daniel). Since it was never referred to in the video, it was removed from the data, assuming it could not influence further explorations. On that basis, I determined that the inquiry proper finished in Week 10.

The second pass through the video data tracked the background of the inquiry: coding the basic activity that served to organize the hour. Basic activities included circle time, (assigned) small groups, play time (guided and free-choice) and journal writing time (unstructured except for the explicit expectation that the children would engage in some form of what they considered to be “mathematics”). The order of the activities varied by week and not all activities occurred every week. Furthermore, the transitions between these activity periods were sometimes lengthy, but were not indicated in this coarse grained analysis. This pass resulted in each week receiving

3 – 4 codes (clips) that encompassed the whole hour. During this pass I also corrected notable transcription errors and fine tuned some of the coding to address consistency across the weeks.

During the third pass, I took a multimodal discourse perspective (Nathan, Eilam & Kim, 2007) by coding each clip with the mode of communication used by the child(ren) in it – identifying five modes (demonstration, verbal, gestural, numerical or written – see Table 6.1).

**Table 6.1 Communication codes: definitions**

**Keyword Summary Report**

*demonstration*

This keyword will be assigned to a clip that includes a physical demonstration of some sort, either using a tool (e.g., calculator, snap cubes etc.) or parts of one's body. This does not include gestures given in an indexical or semiotic sense, but requires a clear demonstration of some action being performed.

*gestural – indexing*

This keyword will be assigned to a clip that includes gestures that involve a discursive "indexing" function. This includes nodding while saying yes or no and pointing while using the words this, it or that. Pointing and counting will be coded as indexical if the symbol being pointed to replicates the number being counted.

*gestural – non-indexing*

This keyword will be assigned to a clip that includes gestures that perform functions other than indexing. This may include semiotic functions, where the gesture is essential to convey meaning but does not include demonstrations, even if a demonstration is used to convey meaning. Pointing and counting will be coded non indexical if the symbol being referenced does not replicate the number being spoken. This does include hand-raising to signify that a child has something to contribute or other actions that might signify the same meaning (e.g., standing up and approaching me, walking over and waiting beside me etc.)

*numerical*

This keyword will be assigned to a clip that includes any form of numerals (equations spoken or written, numbers, mathematical symbols etc.) The clips will simultaneously be coded either verbal or written to indicate the medium. This will include incidents where the child speaks aloud a number while writing it down, even if that written document no longer exists. It will not include clips where the only person using a number word or symbol is me.

*verbal*

This keyword will be assigned to a clip that includes verbal communication on the part of the child - even if it is unintelligible in the audio. This does not include clips where only I spoke and the child responded in a way other than verbally (pointing, shrugging etc.) However, even an *Mm hm* will be considered verbal.

*written*

This keyword will be assigned to a clip that includes any form of writing, including words, numerals or symbols represented in journals or written on the white board or posters. This includes clips where journal entries are being actively referred to. If a journal entry includes numerical or symbolic forms, it will be simultaneously coded numerical.

The distinction between indexing and non-indexing gestures was also made in order to allow a more nuanced analysis of gestures. Analytic notes were attached to most clips describing the rationale behind the coding, especially if there were gestures involved.

After coding the clips for the mode of communication used, patterns were generated using Transana's keyword mapping feature. This report showed the distribution of the clips across the time line of the inquiry and helped to identify several episodes of particularly intense activity. These episodes could also then be located within the type of basic activity involved (e.g., circle time, small groups, play or journal writing). These three initial passes and the corresponding analysis produce thick description of the square root inquiry, necessary to situate practices of algebraic reasoning.

The fourth pass coded the clips for the particular features of algebraic reasoning they exhibited, using Kaput's (1999) category of generalizing and formalizing patterns and regularities. The three specific designations used were informed by the work of Small (2009) and Goldenberg, Mark and Cuoco (2010): representing mathematical relationships, explaining relationships among quantities and analyzing change. This pass focused on the communicative events within the already identified clips, in order to illustrate how the children produced algebraically significant social actions such as *doing recognizing similarities* and *doing discovering relations* (de Lange, 1999). Not all of the clips were expected to exhibit features of algebraic reasoning, so exploring patterns among those which did might then prove fruitful.

The final pass specifically looked within the algebraically significant clips at how multimodality analytically afforded me a different kind of meaning-making; focusing on those clips and pairs of clips with related modal pairings (e.g., when a child spoke a theory and then wrote it in the journal, or when a child incorporated non-indexing gestures within an explanation). These examples can then provide insight into how the later learning of formal algebraic notation might build on existing understanding.

## 6.4 Results

There are four sections here: the first presents the broadest view of the contours of the square root inquiry and the second foregrounds multimodal analysis to highlight the communicative modes. These two sections provide answers to the first research question of the study (*How do young children construct a mathematical inquiry?*) and as part of the dissertation, offer insights into the *communicative* processes involved in learning-as-participation.[10] The third section here finds focus in discursive psychology in order to isolate instances of algebraic reasoning from within the initial clips and the fourth section integrates the two perspectives to show how doing so enables a different kind of analytic meaning-making. These third and fourth sections provide answers to the second research question of the study (*How do young children express mathematical generalization?*) and as part of the dissertation, illustrate the potential in using discursive psychology with child-based group interactions.

### 6.4.1 Broad overview of the square root inquiry

I addressed the first research question by initially exploring the shape and social features of the inquiry as it was situated in the larger mathematics research group and then examining the modes of communication used by the children.

The unitization process produced 150 clips which represented the entire inquiry from Week 4 through Week 11 (excluding Week 5, where no activity related to the inquiry occurred).

[10] This builds on the examples previously discussed regarding some of the interactional (Chapter 4) and argumentative (Chapter 5) processes evidenced in learning-as-participation.

The total time covered by the clips was approximately 60 minutes (total time possible: approx. 455 minutes) indicating that the inquiry potentially incorporated up to 13% of the hour for some children at least. The process of removing the clips which included me alone and the extra two clips from Week 11 resulted in 136 clips which then comprised the main data for analysis, supported by 23 artifacts (5 produced by me, 18 produced by the children – see Figure 6.3).

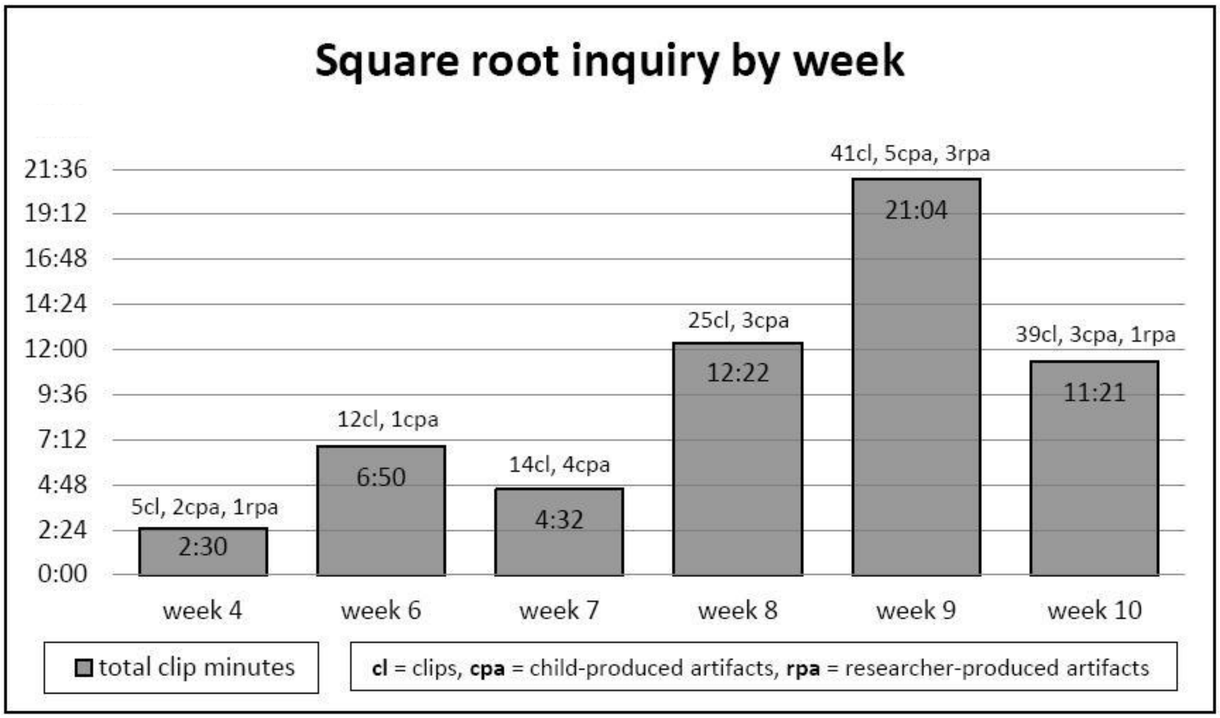


**Figure 6.3 Square root inquiry showing time per week (in minutes and seconds), number of clips and number of artifacts produced**

This bar graph indicates that time spent on the square root inquiry generally increased over the weeks, with some discrepancy regarding Weeks 7 and 10. However, the interpretation is supported by considering the numbers of clips per week (i.e., 5, 12, 14, 25, 41, and 39). In that case only Week 10 indicates anomaly. The correlation between basic activity and the clips is enlightening. This is illustrated in the keyword maps of Figures 6.4 and 6.5 by comparing the position of the clips each week against the backdrop of the basic activity. (The colours here in the electronic version are only significant in that they distinguish neighbouring clips from each other.)

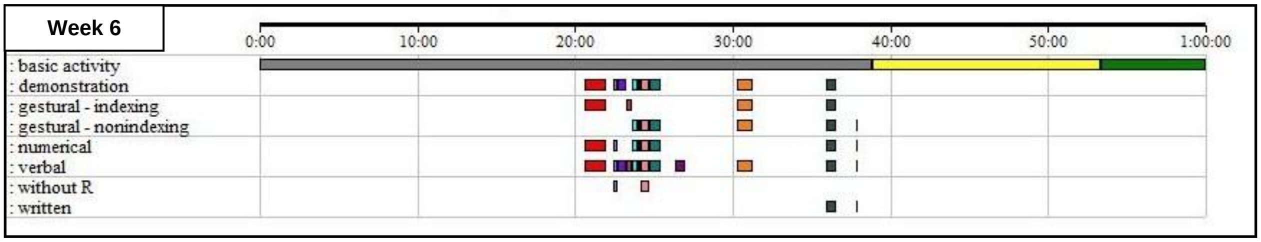


**Figure 6.4 Keyword map of the square root inquiry showing Week 6 – social action clips (n=12) identified by communicative modes and basic activities (n=3: play time, circle time, guided play) over the whole hour**

As Figure 6.4 shows, all 12 clips coded during Week 6 (see Figure 6.3) occurred within a 17 minute time window that comprised the second half of free-choice play time. By comparison, in Weeks 7, 8, and 9 the clips were more generally spaced across the hour and occurred within all the basic activities (see the keyword maps in Figure 6.5, next section). In Week 10, all 39 clips occurred within the first 32 minutes of the hour with most falling within the first 10 minutes that comprised circle time. This pattern and that of Figure 6.3 suggests that the inquiry progressed in three stages: Weeks 4 and 6 appear to be different in some way from Weeks 7, 8, and 9. Week 10 appears distinct. This compelled me to look more closely for possible distinctions between the three, as sections.

Each clip (distinguished in the keyword maps as a column of single coloured "bars") represents a communicative event that was identified based on social actions during the first and second passes through the data. These events included the speaker and the recipient if there was a shared orientation between them. Various codes with illustrative clips are represented in Table

6.2. The clips are labelled for the week in which they occur (i.e., “wk10#3” should be understood as referring to the third clip identified during the activities of Week 10).

Table 6.2 Name and frequency of social action code with illustrative clips

| Code | *N* | Example |
|---|---|---|
| ***showing*** | 39 | (wk10#3) Truman: um, two like this, and two like this. three like this and three like this. and four like this and four like that. |
| ***reporting*** | 16 | (wk8#7) Carissa: I wanted to say something about what she just noticed about it. what she noticed is that if you put three zeros here and one, and three more, it takes it away, the other three zeros |
| ***knowing*** | 25 | (wk6#6) Carissa: I know. I knew that. |
| ***explaining*** | 14 | (wk9#11) Anya: it’s like, this is a really, like, small number, this is a little higher and this is a little higher. |
| ***representing*** | 7 | (wk9#40) Cormac: *((dictating to me while I write it down))* it means infinity and, um, there's too many numbers, that's why it can't show them. |
| ***evaluating*** | 5 | (wk6#5) Cormac: this is kind of weird, ‘cause it goes from nine *((pushing square root key))* to three. |
| ***suggesting*** | 8 | (wk7#4) Carlyn: maybe because that means “take away”. |
| ***agreeing*** | 11 | (wk9#4) R: because it had too many numbers? Daniel: un huh. |
| ***idiosyncratic*** | 11 | e.g., volunteering, giving an account (non-math), inviting, etc. |

As the results indicate, *showing*, *reporting* and *knowing* comprised nearly 60% of all the clips, illustrating the general nature of the inquiry: it involved children sharing their learning with others (usually but not always me). At the start of the inquiry, Carissa had produced a definition for the square root symbol when it was introduced as part of the classification activity in Week 4: (wk4#1 coded under idiosyncratic as “defining”) “Oh that means, um, in the, calculator, that means, like, another, and you check what other ones you have done before.” By speaking aloud her *understanding of the meaning* of the symbol, she made relevant the reporting of any such understanding by other children. She also introduced the calculator as a cultural tool. Her description introduces the actions of "doing" math (*what other ones you have done before*) and "checking", perhaps implying accuracy. Explicitly sharing understandings with others was a regular feature of the square root inquiry.

Explaining and representing comprised 15% of the clips and refers to incidents where children were expressing their understanding in more nuanced ways, using various forms of representing (sometimes but not always written) and elaborating or refining upon ideas. Many of these clips became significant during the analysis of algebraic reasoning. The actions called here evaluating (4%) were of the kind that communicated "this is not what I expected" and tended to refer to surprising results on the calculators. The example in Table 6.2 for "evaluating" refers to the first time Cormac (or anyone) had tried a number that was a perfect square. In this case, the result was a single digit, quite a contrast to all previous results that had filled the display window to the seventh position past the decimal point. Without understanding the concept of decimal numbers (it became of interest during the inquiry) the children viewed the imperfect square results as "bigger numbers" and the perfect squares "smaller numbers." These discoveries played a role as the children began recording their key strokes (in order to re-demonstrate these at will).

The type of social action (column 1 in Table 6.2), while interesting, was not the focus of this study so I did not investigate these codes further. Nevertheless, I am aware that a response like a revoicing by an adult has a different social function regarding authority than a simple acknowledgement. However, for my purposes here the orientation to shared social action functioned primarily to provide boundaries around clips in order to stay as close as possible to the participants' perspectives.[11] Therefore, something like a challenge, while a separate social action, was included within a clip coded *explaining* if all participants acknowledged and responded unproblematically to it (e.g., see Example 4.12 from Chapter 4 where Daniel challenged Carissa's accuracy in counting the zeros on the wall chart. Everyone accepted the substance of that challenge by incorporating it into Carissa's explanation. Therefore, that incident was coded as a single clip "wk9 #10" in this analysis). Likewise, there is a very small and arguably disputable difference between "reporting" and "showing." For the purposes of this study, I did not pursue that distinction except for its practical implications while coding.

However, in anticipation of Section 6.4.3, during the fourth pass I noted some interesting relationships between the categories of social actions and algebraic reasoning. I have already mentioned that most of the "explaining" clips were implicated in algebraic reasoning (10 of the 14). In another case, 4 of the 8 clips coded for "suggesting" included some kind of mathematical conjecturing. Carlyn's suggestion "Maybe because that means take away" (see Table 6.2) was spoken as she overheard Daniel showing me a list of calculator results he had recorded. Her suggestion made relevant the action of *finding a meaning within the patterns* and was followed by other attempts to do so: both Daniel and Nathan also then made conjectures that day.

### 6.4.2 Modes of communication

In this section I explore the communicative processes used by the children using keyword maps and "timelines." Over the course of the six weeks, children communicated in various ways. In part, this was related to the resources available in the setting. For example, Carissa introduced the use of calculators. Although the calculators had always been present in the room, her explicit

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[11] In keeping with the parameters of discursive psychology, this refers to an over-the-shoulder type of analytic viewpoint, not in-the-mind.

use of the tool made relevant its contribution to the inquiry. After that point calculators were used extensively, every week. I introduced journals in Week 7 and blocks in Week 9. These cultural tools (Cobb, 1995) were incorporated alongside wall-sized radial diagrams, lists and three-dimensional objects that I produced, as well as journal entries produced by the children.

#### 6.4.2.1 Keyword maps

The inquiry did not unfold in a balanced way during any one week. Periods of intense activity in the square root inquiry were interspersed with periods of other activity. After the third pass through the data, the keyword mapping feature of Transana was used to show the ebbs and flows of the inquiry through the weeks by mapping the clips onto the "timeline" of the hour. These maps then illustrated how the basic activities for the hour correlated with periods of square root inquiry (see Figure 6.5).

In the fourth week of the mathematics research group (the first week of the square root inquiry), clips denoting the inquiry occurred during the small group activity that was introduced during circle time as mentioned previously. During Week 6 of the mathematics research group (the second week of the inquiry), all of the clips occurred during play time (the first basic activity of the hour), with a cluster near the beginning as Carissa initiated a calculator game. Her ideas were taken up by Cormac who then conducted his own calculator explorations, ultimately listing his findings. (The box for Week 6 is copied into this figure from Figure 6.4).

By Week 7, most clips occurred during journal writing time (the second basic activity of the hour). The final three clips involve a conversation about a journal entry and Danica's demonstration with a calculator during play time.

During Week 8 about half of the clips identified, occurred during circle time in the first 14 minutes. This cluster was initiated by Daniel, who arrived telling about a theory he and his

Mom had discussed (the children never took their journals home, but apparently they spoke about the mathematics group activities with family members).

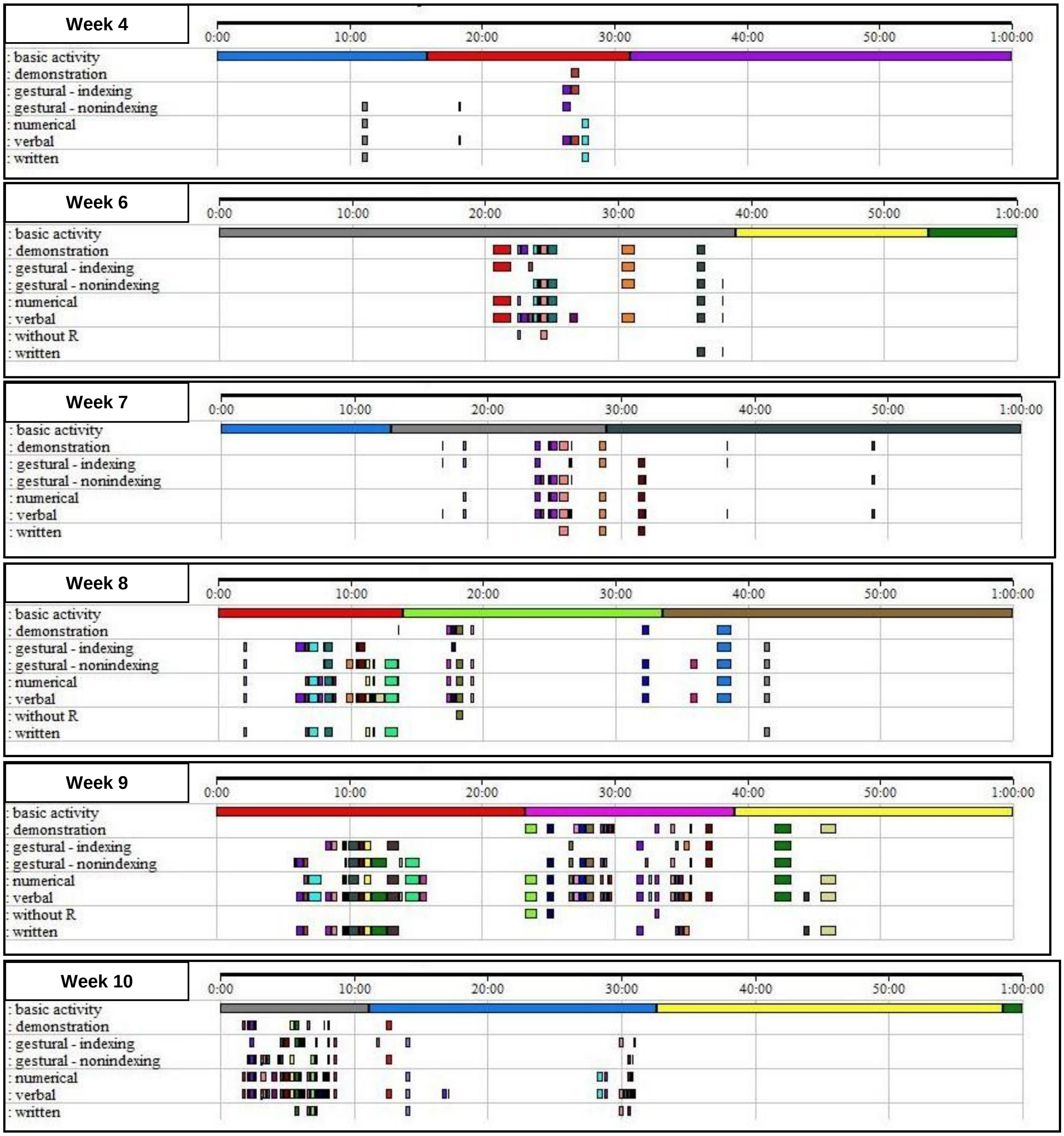



**Figure 6.5 Keyword map of square root inquiry by week – clips, modes of communication and basic activity**

During circle time, Daniel used his journal entry from the previous week to explain the theory to everyone. The other half of the clips that week come from journal time. The final three clips involved Anya who had (like Danica the week before) had to wait until play time to get use of a calculator. She wrote in her journal during play time of Week 8, recording her key strokes regarding a single eight-digit calculation.

In some ways Week 9 seems to represent the peak of the inquiry for the children. It produced nearly double the number of clips as the previous week and more artifacts than any other week (41 clips, 8 artifacts total, see Figure 6.3). Again, about half the clips occurred at circle time and half during journal time. This week it was Cormac who needed to wait until play time to explore with the calculator and record the results in his journal. Week 10 rendered almost the same number of clips as Week 9, but most of them (26/39) occurred during circle time. Circle time that week represented a very intense cluster of clips and it is explored further here (the keyword map for this ten minute period is re-presented in the next section as Figure 6.11). A few clips occurred during journal time and the final ten occurred when I attempted to re-engage a small group with the discussion from circle time.

This broad overview of the modes of communication drew my attention to the journals and the use of artifacts. Circle time also included significant inquiry activity and I wondered if there might be a difference between those explorations that were and were not directly prompted by me (i.e., during circle time vs. other times). Weeks 6, 8 and 9 all contain clips that were coded "without R" indicating that children were sustaining the inquiry by their own initiative during those weeks. Certainly the circle time discussions in Week 8 had been driven, initially at least, by Daniel's contributions regarding his mother's theory. However, I wondered about the circle times of Weeks 9 and 10, where nearly half of the clips appeared in what was essentially a whole group discussion led by me. While these results foregrounded potential activity clusters of interest, I wanted to highlight the children's participation in the inquiry. I needed another way to present the findings from these first three analytic passes.

#### 6.4.2.2 Time lines of the "flow" of the inquiry

Figure 6.5 illustrates that most clips received multiple codes for communication. This is not surprising, given the age of participants and the nature of inquiry-based learning. However, in order to illuminate the processes of learning-as-participation, it is important to examine those modes of communication more closely, especially as they relate to journals and the production of artifacts. Given the complex interaction between the role of resources and people, the context of time and the development of the line of inquiry, I have chosen a form of visual representation here that attempts to preserve the narrative of the square root inquiry. By using this form of representation, I am able to follow the flow of the inquiry over six weeks in much the same way I followed the flow of the collective argument over six minutes in Chapter 5 of the dissertation. In this section I maintain a multimodal analytic focus, but keep discursive psychology in mind, in order to render the original data as transparent as possible. Given the space restraints of a dissertation, I acknowledge that there are other valid interpretations possible besides this one.

In keeping with the practices of multimodal analysis, I present the resources available and the modes of communication used by children as "timelines" in Figures 6.6, 6.7 and 6.10. The figures are meant to portray the general flow of the inquiry and communication as organized around the artifacts, so several "clips" are often combined into one entry. The data presented in this form correlates with that previously presented in Figures 6.3 and 6.5. (e.g., Week 6 shows one artifact produced by a child in Figure 6.3 and that journal entry produced by Cormac is presented in Figure 6.6; likewise, the modes of communication represented as coloured bars in

Figure 6.5 are identified in brackets following the child's pseudonym). The order of presentation within any one week is determined by space alone and is not analytically significant.

#### 6.4.2.2.1 Weeks 4, 6, and 7

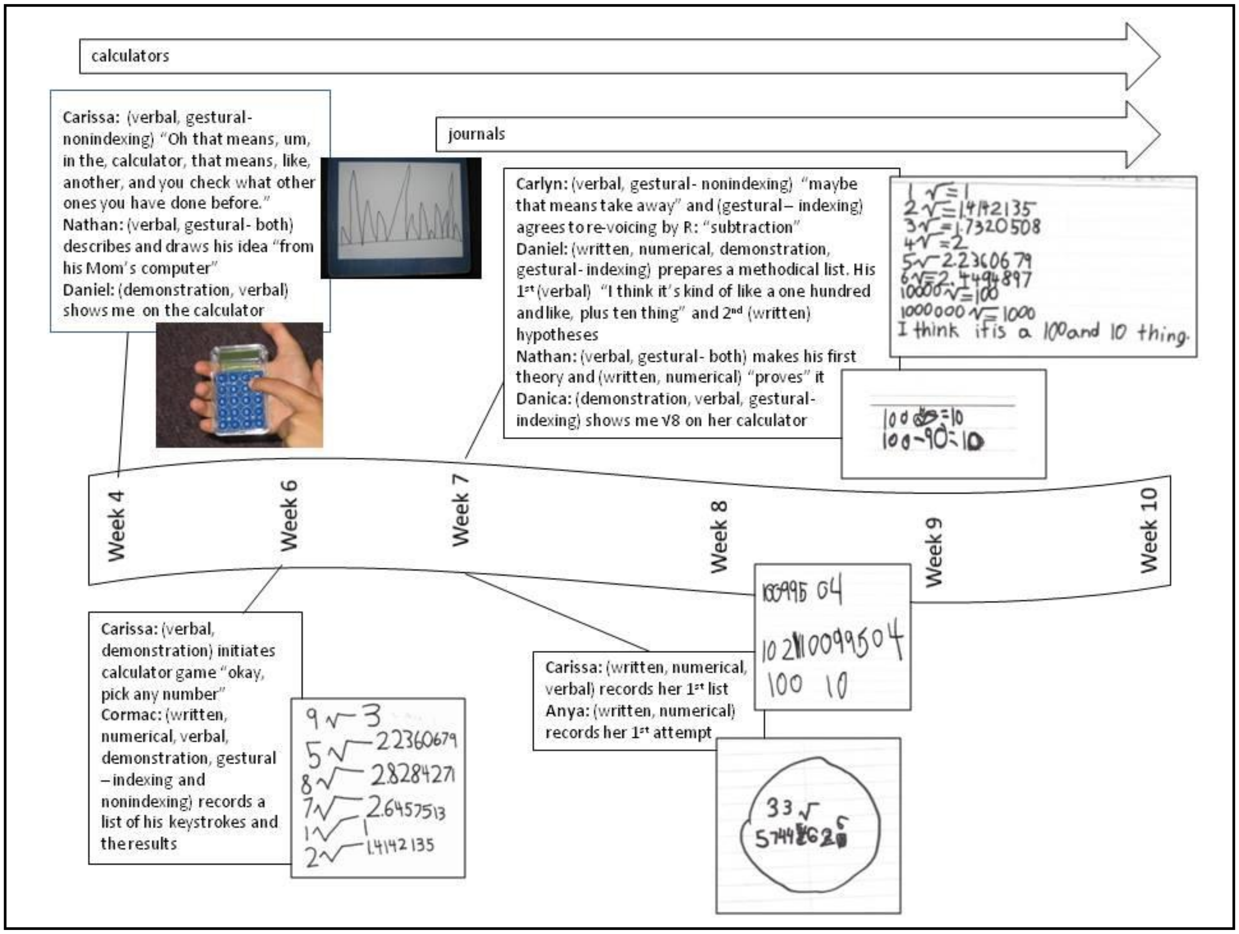



**Figure 6.6 Timeline of inquiry Weeks 4, 6, and 7 – children, modes of communication, artifacts produced and resources available/used**

The initial activity of the square root inquiry (during Week 4) has been adequately portrayed in previous sections. Carissa's noticing of the symbol on the calculator is visually presented in Figure 6.6. Nathan offered a contrasting idea that might have been taken up, but was not. It was recorded in Week 4, and then dropped (see Figure 6.6).

Two weeks after that initial exposure Carissa (5y9m) brought the calculator to show me something that she "already knew." After asking me to "pick any number" she demonstrated using keystrokes alone, with very few words, that pressing the 3 (the number I chose) followed by subsequent repeated pressing of the square root symbol changed the numeral displayed. Her confidence in asking me to choose the number she would demonstrate suggests some previous experience with this activity – she expected that any number I chose would produce an interesting result. After 22 repeated keystrokes, the calculator finally showed "1" and her demonstration ended. By this time, numerous children had gathered around to watch. During a subsequent interaction with me, she restated her own definition of "square root" from two weeks earlier, using almost identical words: "Yeah, and it shows you what you already have done" (wk6#6). In this context, her definition seems to reference memory more than mathematics, but it may be inferring an accuracy of remembering. At the time this was simply acknowledged with an exact revoicing (by me) and then dropped for another conversation with a different child.

However, Carissa's demonstration gained relevance when Cormac (7y5m) picked up on her idea and began to explore his own numbers on the calculator, eventually coming upon the square root of nine – seemingly at random – that he presented to me as "this is kind of, weird. Cause it goes from nine, to three" (wk6#5). At my suggestion he began to record his keystrokes in the form of a list so that we could remember what number turned into what number. He spent the rest of play time (about 15 minutes) engaged in his investigation, trying out different numerals and recording the results (see Figure 6.6). As he sprawled on the floor in the middle of the room, Carissa and Daniel each briefly looked over his shoulder, but most children continued their noisy play around him.

The following week (Week 7), Daniel (7y3m) began to explore with the calculator and discovered something that he wanted to show me but was unable to replicate. This prompted the suggestion to write down a list as Cormac had. Daniel then began writing in his mathematics journal thus producing his own methodical list. While Cormac's list began with his notable result, it continued without apparent organization although it eventually included six single digit numerals. In contrast, Daniel used the natural order of numbers to organize his attempts and by extension his findings (see Figure 6.6). He also introduced the use of the equals sign to represent "where you put the answer" a problematic understanding of the equals sign, but one that this group had actually ratified during a previous negotiation in Week 4 (this negotiation was covered in some detail in Chapter 5). Cormac's list was a more accurate recording of his key strokes since Daniel never needed to press the equals sign in order to get a result. However, Daniel's notation indicates that he recognized that he was performing an operation on a number when he pressed the "√."

Carlyn's (6y8m) initial conjecture ("maybe because that means take away" – wk7#4) was not produced as an artifact, but made relevant the act of theorizing, which both Daniel and Nathan oriented to. For example, later in the conversation Daniel produced his first conjecture about the function of the square root symbol: "I think it's kind of like a one hundred and like, plus ten thing" (wk7#10). At my suggestion to write his conjecture down, he did. Later that session I mentioned Daniel's conjecture to Nathan who responded with a theory of his own: "Or minus, ninety" (wk7#11). I will explore all three of these conjectures in Section 6.4.3 on algebraic reasoning.

That same week (Week 7) Carissa and Anya also represented square root results in their journals, although Anya never spoke with me about it. When analyzing the data I did not immediately recognize Carissa's entry as being relevant, even though she had spoken with me about it, since she did not accurately represent the square root symbol or include the decimal point in her answer. Nevertheless, the number 10.099504 is an accurate result for the square root of 102 so I include it in Figure 6.6 as her first attempt at representing square root. Danica demonstrated to me "√8" on the calculator.

#### 6.4.2.2.2 Weeks 8 and 9

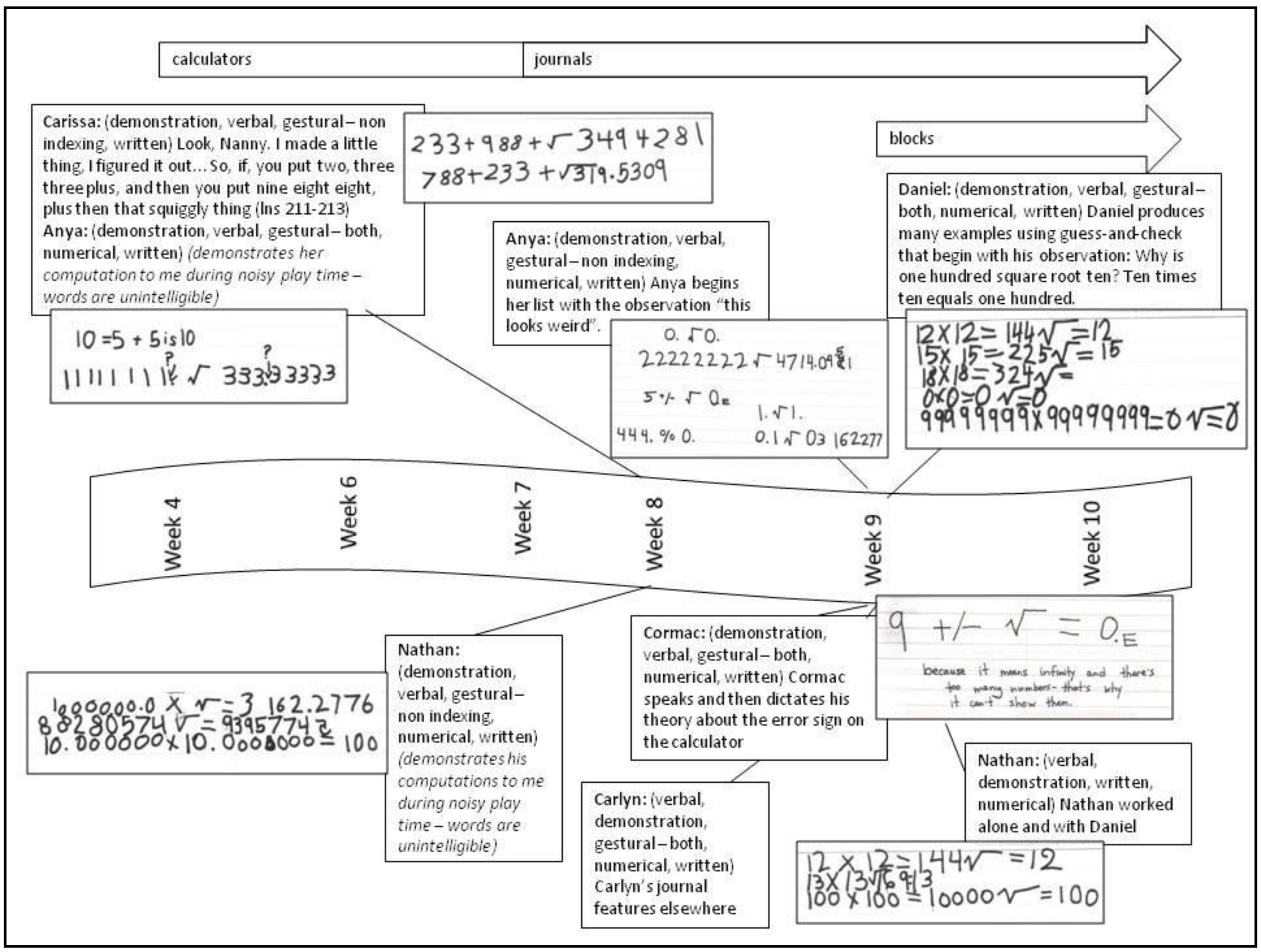



Figure 6.7 Timeline of inquiry Weeks 8 and 9 – children, modes of communication, artifacts produced and resources available/used

The next week (Week 8), Daniel came into the room ready to tell us about a theory that he had discussed with his Mother. During circle time he stood at the front of the group holding his journal open to the previous week's list and explained to us all as he pointed to the entry: "Um, it's like one million that - and that sign equals, um, one thousand, because my Mom says it takes, like half – half " (wk8#3). After revoicing the theory and drawing the numbers on a large piece of paper, I asked if anyone could repeat what Daniel had told us. Anya repeated the computation, and I asked for more: what had his Mother suggested as a reason why? In response Carissa came up to the front and pointed to my drawing[12] as she spoke: "I wanted to say something about what, she just noticed about it. What she noticed is, if you put three zeros, here, and one - and three more, it takes it away. The other three zeros." (wk8#7 – coded as "reporting" see Table 6.2)

During a short general discussion that followed I introduced the name "square root" as given by "my teacher" and Nathan and Daniel noticed aloud that the symbol looked like half a square. Cormac had been quietly looking at his own list during those interactions, but then put up his hand and responded to my acknowledgement with: "Um, but, um, it's not minus a thousand, 'cause um, the nine, equals three" (wk8#12), referring back to Daniel's original conjecture. I had him repeat that observation from the front of the group which he did, in a modified version (more about that in Section 6.4.3.2 on algebraic reasoning). This was the first time anyone had refuted another's theory.

During journal time that week Carissa, Anya and Nathan all made entries relevant to the square root inquiry (see Figure 6.7). Carissa recorded two multi-digit multi-step computations,

[12] Her specific gestures were lost to the camera due to the camera angle.

this time including the symbol for square root. Her first entry missed the decimal point and the second entry included it but incorrectly. She correctly recorded the key strokes since the result would be different without the second "+." Carissa recited her first number sentence to me while demonstrating it on the calculator, she also repeated the recitation to Daniel while they sat together at the table, and directed him as he reconstructed the computation on his own calculator (wk8#20). Throughout journal writing time Carlyn, Carissa and Daniel associated regularly while sitting around the table: talking, gesturing to their journals and demonstrating with the calculators to each other.

While at the same table, Nathan seemed to work more independently, seldom looking at other's journals or talking with them. During journal time he made several unsuccessful attempts to demonstrate a calculation to me that he had apparently performed on his calculator. I began to wonder if the problem was the incorrect positioning of the decimal point and so provided some direction. Both Nathan and Anya then included references to the decimal point in their journal entries. Nathan made three entries in his journal that week, including a representation of 10 x 10 = 100 that included six zeros after the decimal point (see Figure 6.7). He demonstrated and described that computation to Daniel and Carissa, who commented, "It takes away the zeros" (wk8#21) perhaps referring to the multiplication button, since pushing that button has the effect of eliminating the unnecessary zeros to the right of the decimal point (wk8#21).

Anya had to wait until journal writing time was finished before she was able to get a calculator, something she apparently wanted badly enough to wait for. During a noisy free play time she demonstrated a calculation to me and then copied it into her journal (see Figure 6.7). The first number sentence in the journal emerged from her activity with mathematical puzzle cards while she waited for a calculator. Her curiosity about the decimal point (represented with a question mark) emerged through the conversation with Nathan and me.

The following week (Week 9) began with a circle time during which I laid out two charts I had developed in order to summarize the square root inquiry to this point (see Figures 6.8 & 6.9).

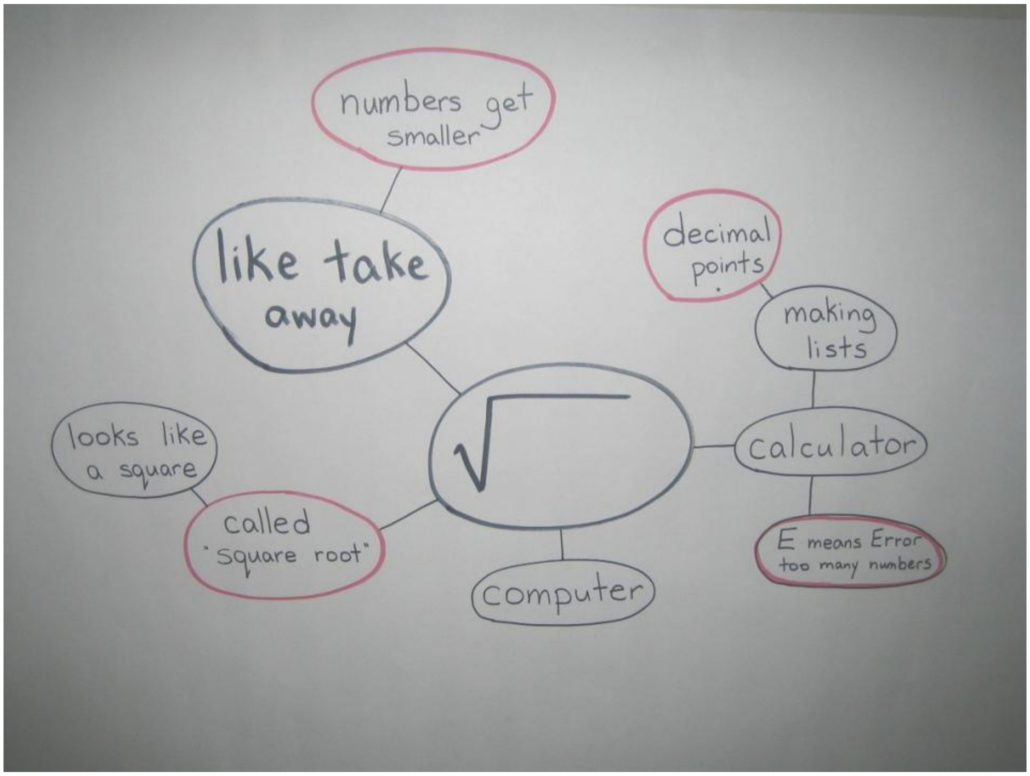



**Figure 6.8  Radial diagram summarizing the ideas**

The discussion concerning the radial diagram developed around the idea that square root was "like take away" perhaps because the numbers "got smaller." This second point was one I had inferred so I brought it to the group for a member check (Charmaz, 2006). Although Carlyn agreed with the inference, both Daniel and Cormac noted that sometimes the numbers got bigger. In Nathan and Cormac's words: "Um, sometimes it's take away, sometimes it's, um... [Nathan: plus.] Cormac: plus" (wk9#2). It took me a little while to understand their logic: they were not distinguishing between the decimal numbers and the whole numbers. To them, decimal number roots seemed to "get bigger" than their input areas because the results included more digits. At that point, I suggested working with a subset of our answers: just the simple squares.

In a corresponding move, I had previously coordinated the children's lists of results into a list that exploited the order of the natural numbers (see Figure 6.9). I initiated a discussion about this list during circle time in Week 9 by noting that I had listed the results from smallest to largest and then had recognized a pattern. This brief comment initiated a lively interaction involving Anya, Daniel, Nathan, Carlyn and Carissa who found patterns both horizontally and vertically within the chart. This interaction has been analyzed elsewhere (Chapter 4) and will be revisited here in the section on multimodal representations of mathematical relationships.

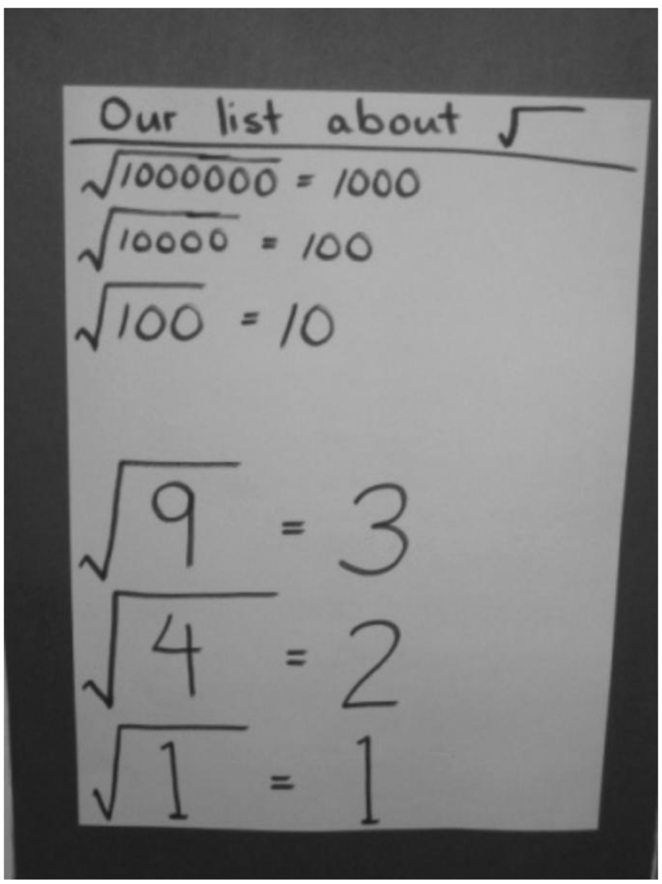



**Figure 6.9 Adult produced list consolidating the children's findings**

As we were finishing up at circle time I asked the children to focus on the results of the list and notice the gap between 3 and 10. I wondered if we could fill in that gap, a challenging task given that the children had never focused on the result before, but had only always worked from left to right, first entering an "area" and then finding the square root. From there I directed the conversation to the idea of "squares" since the symbol was called "square root" and asked what children knew about squares? In order, the following suggestions were verbalized:

> Anya: they have four corners
>
> Truman: it can be a picture frame
>
> Nathan: you can decorate squares
>
> Carlyn: they have four lines
>
> Carissa: they can also be a window
>
> Daniel: there are square blocks
>
> Jimmy: there are square shovels
>
> Nathan: you can build with squares

After a brief but noticeable silence, I explicitly connected Daniel's and Nathan's ideas and asked if we could build squares with square blocks? Everyone enthusiastically agreed and Nathan offered: "all you need is four squares" (wk9#17). The blocks came into use during play time, as the children built squares out of blocks to be photographed. I do not include those photographs or the interactions surrounding their depiction in this discussion or in Figure 6.7 since it has become apparent to me that none of the children equated activities with building blocks as being part of the square root inquiry. Until I made it more explicit during circle time in Week 10 this was my connection alone.

During Week 9 I had specifically tried to seat all six children who had been active in the inquiry together at the table (Daniel, Nathan, Cormac, Carissa, Carlyn and Anya). Anya actually ended up with others on the floor but Daniel and Nathan talked and worked together at the table and it was not long before Daniel noticed Nathan's 10 x 10 = 100 and connected it with the √100 = 10. As he and I talked about his discovery, he gestured towards the wall chart several times and then began to produce with the calculator, example after example of such related equations, using a guess-and-check type of strategy. Both boys spent a considerable amount of time to complete multiple entries in their journals concerning this new finding (see Figure 6.7). During the same time, Carlyn copied my list from the wall and developed her theory from circle time further by writing it down and then explaining it to me in two versions. This will also be explored in greater detail in Section 6.4.4.1 on multimodal representations of mathematical relationships.

Cormac and Anya had discovered the "+/−" key on the calculators and through that had noticed an error sign in the display of the results.[13] Connected to previous ideas about the error sign signifying too many numbers to display and questions about trying to replicate infinity in the calculator, Cormac speculated that he had found a way to do so. His theory, "because it means infinity and there's too many numbers – that's why it can't show them" (wk9#40) was dictated to me and I copied it into his journal, another way for young children to represent their ideas. Cormac's use of the pronoun *it* is ambiguous here, since it is not clear which symbol he is referring to: plus or minus, square root, or error.

[13] The square root of a negative number is an imaginary number – something these simple calculators were not prepared to identify. Therefore, they produced an error sign.

#### 6.4.2.2.3 Week 10

The final week of the inquiry (Week 10) saw an intense period of square root inquiry activity focused in circle time. These first ten minutes of the hour also most closely resembled classroom teaching, with its initiation-response-feedback discursive organization (Sinclair & Coulthard, 1975). It was far more directed by me than other segments of the inquiry. As the door opened that week, I was seated near the carpet area and holding a tray with four different models of the squares-made-from-squares from the previous week (see Figure 6.10).

**Figure 6.10 Adult produced representation of children's "squares-from-squares"**

Nathan walked in the door, came over to me and immediately spoke aloud “1, 2, 3, 4” while pointing to each of the square configurations in turn from left to right. At my insistence, he waited until everyone had come to the carpet area and then demonstrated again, with the same words but more precise actions: this time counting twice and spanning the width of each configuration and then the height. At my invitation, Truman (wk10#3 – see Table 6.2), Carissa (wk10#5) and Jimmy (wk10#11 and #12) replicated Nathan's demonstration with various refinements and elaborations. However, the children were unable to answer the question "what exactly is it that they're counting?" producing responses like "the squares?" (Anya wk10#6) or "the cubes?" (Daniel wk10#8). I tried to refine my frame of reference by showing that when I counted these configurations I did not get "1,2,3,4" I got "1,4,9,16." Even Nathan's more precise version "rows of four, like that and like that [gesturing for width and height]. Rows of three like that and like that [similar gesture] rows of two" (wk10#13) still confounded rows with columns.

The next question I asked was "do you see this anywhere on the lists that we made (the list from last week was on the wall immediately beside where the children were sitting on the floor – see Figure 6.9). Carlyn responded: "Hey, it matches it!" subsequently pointing to each representation in turn (the list and the cubes) and saying "the nine goes here and the one goes here [R: and the four?] goes there" (wk10#14). In another version, still apparently fishing for the answer I wanted (in a more traditional teaching way and certainly more directive than I had ever been with these children) I then asked "when you look at the chart, do you see 1,2,3,4 anywhere?" (wk10#18). Carissa pointed to the zeros in the top half of the list so I attempted a summary and re-question: "We have figured out how we can look at the cubes, the squares here and see one, two three four. I'm asking, when you look at the chart, do you see one two three four anywhere." (wk10#19) It is important to notice that this is barely seven minutes into the hour and I have already asked nine different questions that have received rapid-fire answers of various kinds. There were 19 different clips coded to this point. The intensity of the interactions had dramatically increased (see key word map for this first ten minutes, Figure 6.11 which is a magnification of a section previously represented in the lowest box of Figure 6.5).

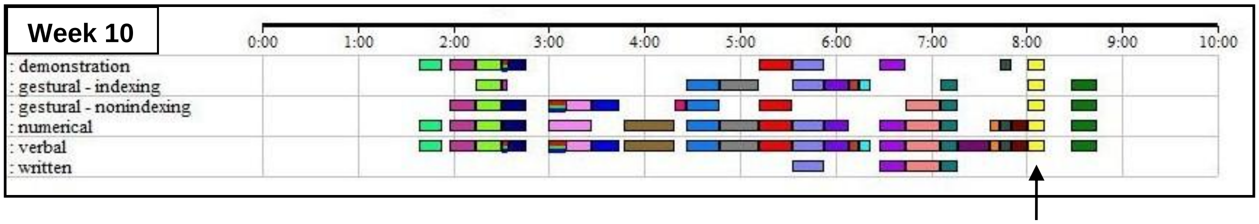



**Figure 6.11 Keyword map for Week 10 (first 10 minutes during circle time) – Communication codes for clips #1 – 26, with Carissa's demonstration of square root identified with an arrow (clip 25)**

As interactions continued at circle time, I again pointed out the importance of "filling in all the results between 3 and 10" and set up Daniel and Nathan to help with "doing [the square root of] 16" (wk10#21). When Daniel answered that he knew the result would be 4, I responded, "yeah, it makes sense, doesn't it?" (wk10#22) followed immediately by an epistemic status check (Sert, 2013): "does it make sense?" (wk10#23). This action on my part (a statement that is actually *doing questioning* immediately followed by an epistemic status check) is reminiscent of a previous time (Week 4) when this group had entered into a lengthy negotiation of the meaning of the equals sign (see Chapter 5, where the negotiation was launched with my statements: *"You know what equals means. Do you know what equals means?"*). In this case, when Cormac responded to my "does it make sense?" with another version of the "1,4,9, and 16" pattern and Daniel demonstrated on the calculator I then presented an imaginative twist to the question, objectifying the calculator and asking "how did the calculator know, that it [the result of √16] was going to be 4?" (wk10#24). Carissa interjected a change-of-state token (Heritage, 1984) "ahh" in the middle of my question, indicating a formulation of new knowledge. She stood up and reached towards the cube configuration of 4 x 4: "because, it took away all those, and, put it, four." (wk10#25 – indicated in Figure 6.11 with an arrow.) As she spoke she used a sweeping down gesture with her forearm to signify the bottom 12 cubes and demonstrate “taking away.” She then pointed to the top row as she said “four.”

This explanation, I believe, was the closest any child got to representing the mathematical concept of square root and therefore warrants a detailed presentation here. The clip was coded for five modes of communication: verbal for the statement, numerical for the “four”, indexing was for "all those" and pointing to the “four [top row]” as she gestured, non-indexing was the particular eight cubes in question, not just any eight cubes and demonstration was the sweeping down gesture to accompany "took away.” I present this example as indicating the strength of multimodal analysis in allowing us to recognize how a young child “thinks” mathematically, while retaining the discursive psychological requirement for an over-the-shoulder perspective and without needing to speculate about the workings of her mind. Carissa was producing a new understanding ("Oh!") but was not able to express it in conventionally mathematical ways (Davis, 1996).

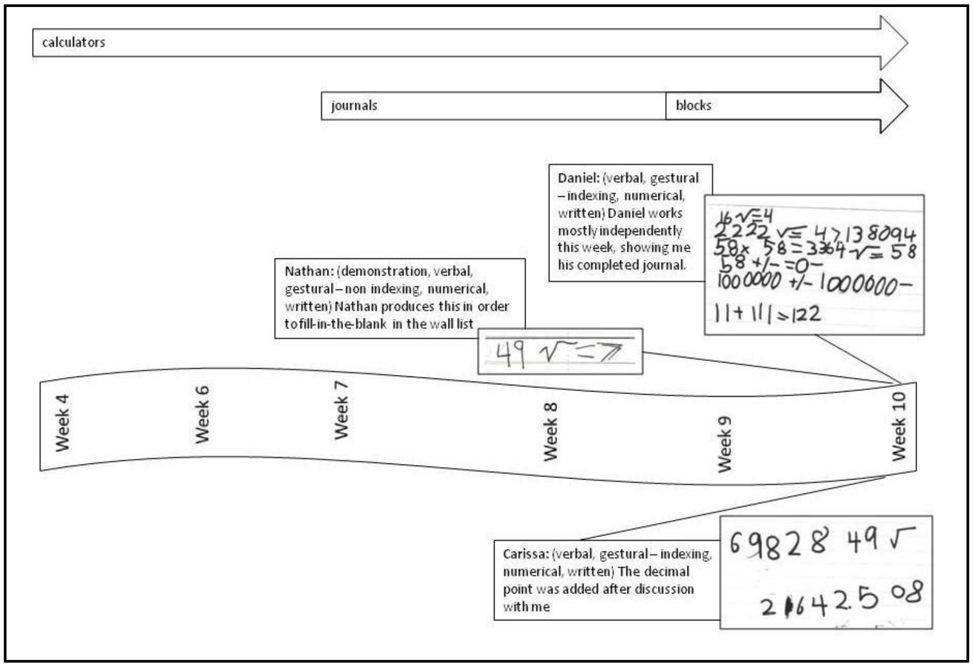



**Figure 6.12 Timeline of inquiry Week 10 – children, modes of communication, artifacts produced and resources available/used**

Despite the intensity of the circle time interactions (or perhaps because of it) no one drew squares during journal time. Daniel produced a list of two inverse operations (unsystematic) and other calculator explorations of various sorts (see Figure 6.12). Nathan produced a single result: "√49 = 7" and even Carissa, after her demonstration at circle time, only produced a single apparently unsystematic calculation (see Figure 6.12). Based on these results, I interpret the lack of connection between the square root inquiry and the building of squares. For the children, the inquiry peaked in Week 9.

### 6.4.3 Summary: How do children construct a mathematical inquiry?

Throughout these sections, I have produced thick description regarding how the children used verbal statements of differing kinds, spoken and written references to numbers, indexing and non-indexing gestures and demonstrations to produce their participation within the square root inquiry. In order to answer the first research question I will briefly consider the interaction within these data between mathematical content, the formulation of ideas and the role of environmental features (tools such as calculators or activity such as circle time), connecting these results to Krummheuer's (2011) roles of learning-as-participation.

In the first five weeks, the inquiry was driven primarily by the children's curiosity and sustained with their efforts. Carissa produced the first formulation of square root as pertinent to calculators and she sustained that idea with the calculator game. In these instances she operated as *author* of the idea and the way it was formulated, a position that Krummheuer (2011) considers full participation: not necessarily learning, but presenting what she knows. However, when Cormac picked up on Carissa's idea and presented novel content within her formulation by evaluating his results (this is kind of weird, the nine turns to three) he operated as *ghostee* according to Krummheuer: a position of learning. Likewise, Daniel continued Cormac's content by listing his results, but adapted the formulation when he organized those results by using the natural order of the numbers. In this action he was operating as a *spokesman* (Krummheuer, 2011) another position of learning. Krummheuer's operationalization of participation-as-learning is insightful, but it requires speculation that is unavailable to a discursive psychologist (e.g., did Daniel actually *know* that re-formulating Cormac's way of listing his results would be fruitful, enabling the finding of patterns and the making of generalizations or was his action simply fortuitous?) I require a way of talking about Daniel's learning without needing to speculate regarding his intentions.

When participants utilize particular words in their utterances we do not need to speculate. For example, when Carlyn prefaced her first conjecture with "maybe because" ("maybe because it means take away") we can recognize speculation as a social action both indicated and acknowledged with my revoicing ("well, maybe it does mean take away"). Likewise, with "I think" per Daniel's use ("I think it's a one hundred plus ten thing") and "or", used to produce a contrast, as Nathan offered ("or minus ninety"), we can recognize the social action of *doing suggesting* without needing to rely on analytic speculation of our own. This is the type of analysis I relied on in Chapter 4 to elaborate the practices of *doing knowing*: limiting myself to those utterances that contained the word *know*. However, this practice is constrained by the common use of language. No one ever says "I generalize" even in mathematical discussion.

How do children construct a mathematical inquiry? I have found that they initiate and sustain mathematical content and formulation, co-ordinating their actions using multiple modes of communication. They build their participation upon the contributions of others, thereby jointly contributing to an outcome that cannot be attributed to any one participant alone. In short, they construct a negotiation that evolves over weeks rather than minutes. I hesitate here, however, since my methodology involved examining timelines of the inquiry so that I could recognize its flow as I had the collective argument that developed in Chapter 5. Is my result then simply a function of my methodology? My warrant is based on analysis without speculation, so I feel confident that it is not so, but questions remain.

For one, the actions of participation on the part of the children offer opportunities to recognize learning-as-participation but they do not entirely transfer onto Krummheuer's (2011) categories, especially when I consider my own participation. For example, by organizing the children's results into a logical list, (their mathematical content formulated after Daniel's idea) I operated as a *relayer*, according to Krummheuer, another position of learning. But was I learning? I contributed towards the opportunities to theorize and justify. Perhaps the role of *relayer* when enacted by an authority figure promotes learning-as-participation. When I took the role of *author* on the other hand, introducing the building-of-squares-with-squares as relevant to the inquiry, I also took over some control of the direction of the inquiry and thereby constrained the opportunity for the children to learn-by-doing. These results speak to the affordances and constraints of teacher mediation in learning and merit future consideration.

Another unresolved issue concerns the apparent *phases* of the inquiry (Weeks 4 & 6, Weeks 7, 8 & 9, Week 10). Nothing like this appeared in the construction of the negotiation. I identified my own role in the distinction of Week 10, but what was different between Weeks 6 and 7? And how was that difference sustained and developed in Weeks 8 and 9? In both the negotiation of Week 4 and the square root inquiry, the children were investigating a mathematical symbol. One difference was that they *did* resolve the function of the equals sign within six minutes and *did not* resolve the function of the square root symbol, even after six weeks. Therefore, and in order to answer the second research question (*How do young children express mathematical generalization?*), I determined to pull back a level and examine the square root inquiry from a macro-perspective: positioning it within the landscape of algebra, the realm of mathematical symbols. As I began the fourth analytic pass through the data, looking to identify and separate out those clips that involved *representing mathematical relationships*, *explaining relationships among quantities* and *analyzing change* (Goldenberg et al., 2010; Kaput, 1999; Small, 2009) I began to notice interesting patterns within the categories of social actions that emerged in the new, smaller set of clips. I expected these patterns would provide insight into how the children produced their algebraic reasoning and provide some answers to these questions.

### 6.4.4 Algebraic reasoning

The results so far might be compared to a pointillist painting, which when seen through the detail (the interactions) has the potential to lose its message (Nathan et al., 2007). In order to situate the actions of the children here in a mathematical context and answer the question of significance (what are the processes of learning-mathematics-as-participation?), it will be productive to examine the inquiry from a more coarsely grained level: situating the inquiry within the mathematical language of algebra. In this section I foreground discursive psychology, a perspective that assumes meaning is co-constructed through the discursive actions of the participants (Wetherell et al., 2001). This principle may be understood through macro and micro applications where the macro version would refer to the wider social and cultural histories that might be implicated in any particular social interaction, in this case the broader realm of mathematics and algebra.

Previously in the dissertation I have focused on micro applications of the principle, showing how young children *do knowing* by highlighting the regularities in their interactional sequences and showing how they draw upon those regularities as resources to support their participation (Chapter 4). Micro applications of the co-construction of meaning also figured in Chapter 5, where I showed how the children drew upon those discursive practices of knowing to produce their understanding of the meaning of the equals sign and how they invoked several different sociomathematical norms to produce locally relevant positions during the negotiation. In this chapter I extend those findings by including multimodal analysis alongside discursive psychology and seek to position my work within the broader field of mathematics education. Therefore, I now turn to macro applications of the principle that meaning is co-constructed by participants.

The assumption behind the macro application of the principle is that in order to interact successfully, people must share common understanding of what is happening and what is expected of them. Each participant draws on past experience to evaluate a current situation and uses that to make judgments about current participation (Horne, 2001). During interaction, features of those evaluations or assumptions surface as participants frame their contributions to account for them. This perspective will inform my "looking" as I extend the analysis here to position the inquiry in the realm of algebra and thereby provide an answer to the second research question: *How do young children express mathematical generalization?*

As mentioned earlier, during a fourth pass through the data, the clips were coded with three codes, based on the work of Kaput (1999), Small (2009) and Goldenberg et al. (2010):

- representing mathematical relationships (19 clips),
- explaining relationships among quantities (11 clips) and
- analyzing change (3 clips)

There were 33 clips from the larger pool of 136 that were designated as incorporating features of algebraic reasoning. Their total time was just shy of 12 minutes, meaning that the average duration of any one clip was about 22 seconds. To put it in perspective, the clip considered above where Carissa performed her understanding of square root with a taking away gesture occupied 11 seconds. Remember that most clips included at least two participants, a speaker and recipient. These interactions are short. They occur in four different weeks (7 – 10) and appear in clusters (see keyword maps in Figure 6.13).


**Figure 6.13 Keyword maps of algebraic reasoning clips by week – modes of communication and basic activity**

If algebra is broadly considered a language with which to describe what we know and to derive what we do not know (Goldenberg et al., 2010) then the three codes used here focus on the first aspect, an appropriate application for young learners who have an amazing ability to learn language (in this case the square root function) from context. In the inquiry, that ability manifested itself in instances of recognizing similarities and discovering relations (de Lange, 1999), making generalizations and conjectures and refuting the conjectures of others. In order to recognize these events as algebra, however, we must broaden our understanding of "representation" to include the multimodal aspects that we have already been considering. If we do so, we can increase our capacity to recognize earlier forms of algebraic reasoning than are usually recognizable, due to the unconventional nature of the written or numerical representations. In this section, I examine two clusters of algebraic reasoning (from Weeks 7 and 8). I note that there were more intense clusters in Weeks 9 and 10, but those occurred within the context of teacher-led discussions as opposed to these first examples, which more strongly foreground the children's contributions.

#### 6.4.4.1 Week 7 – methodical list affords conjecturing and a reason to justify

It was Carlyn during Week 7 who initially introduced the action of *finding a meaning within the patterns.* As previously noted, she overheard Daniel telling me about the results he was getting on the calculator and interjected her speculation in our conversation: "maybe because that means take away" (wk7#4). Her suggestion was accompanied by a slight head wagging during the words "take away", giving emphasis to them. I immediately revoiced Carlyn's conjecture verbatim. A little later, I reworded it to use the term "subtraction" and Carlyn agreed to the rewording. Within minutes Daniel had offered a conjecture of his own and it was elaborated by Nathan. This was the first time someone had tried to generalize the square root operation and it makes this cluster of algebraic reasoning an interesting case.

Transana software allows a re-positioning of the clip within the episode ("episode" refers to the "week" in this study, both video and transcription). This allows a look at the broader context and what actions led up to Carlyn's suggestion. As it turns out, I had just indicated to Daniel an inability to provide a reason for why the calculator was producing the results it was.

**Example 6.1 R, Daniel and Carlyn Week 7**

410 R: it's another one to add to your list. This is a mystery.
411 Carlyn: [two hundred and ( )? ]
412 R: [I'm going to ask my teacher] - no, he pushed one hundred on the calculator and then he pushed that squiggly line?
413 Daniel: *((pointing to the square root key on the calculator))* this? and it turned to ten.

414 R: and it turned to ten.
415 Danica: Nanny?
416 Carlyn: maybe because that [means take away.
417 R: [this is a mystery.
418 Danica: Nanny?
419 R: well, it may be because it means take away.

The use of the everyday term “take away” was ubiquitous with several of the younger children in this research group (see Chapters 4 and 5) and brings to light some of the benefits and challenges of using everyday terms in a mathematical context for young learners. On the one hand, everyday terms can render the subject of mathematics accessible for young children. Carissa used “take away” during her demonstration of square root explored above. However, those terms that have other different, specifically mathematical, meanings like “take away” which refers to subtraction but might involve comparing or finding a missing addend, can be confusing. To borrow a notion from computer programming, the terms function like an “overloaded constructor” in the experience of the children. In programming, there are terms that are overloaded with more than one meaning. However, the meaning is never intended to be ambiguous but rather is discernible from the context. In the experience of young learners, terms like “take away” may need to be contextualized within the language of mathematics.

In this case, by suggesting that the meaning of the square root sign might be “take away”, I propose that Carlyn was analyzing the change in the results Daniel produced and thereby extending her understanding from subtraction (where in her experience numbers got smaller), to square root, which seemed to exhibit the same quality. It is important to remember that Carlyn was dealing with an abstract symbol, not the conventional process of finding the square root. She had never “learned” that there was an exclusive relationship between mathematical symbols and operational processes (i.e., that “ – ” was the *only* symbol that could mean “take away”) so that symbol and its referential process was available to her. Nothing about the square root symbol suggested this connection; it was her analysis of the *results of an unknown process* that led her to make the conjecture, in light of my apparent inability to provide a reason. In other words, she was labelling the process the calculator was performing: maybe it was *taking away*.

With this suggestion, Carlyn invoked a sociomathematical norm regarding quantity: it is reasonable to expect that a change in quantities follows regular, consistent, repeatable and thereby *nameable* processes. By revoicing her conjecture with emphasis on the word “may”, I oriented to the same sociomathematical norm and to the same expectations regarding her opportunity to participate in this way. Furthermore, we both oriented to a social norm concerning who had the authority to “know” in this situation: as the adult in the group, I had already invoked a “teacher” (line 412: I’m going to ask my teacher) yet ignored my own statement when Carlyn produced her suggestion. I authenticated her idea instead. With my response, I shared the authority for *mathematical knowing*. In discursive psychological terms, Carlyn interpreted the situation as warranting her participation since I had indicated an inability to propose a reason for Daniel’s unexplainable results by referring to seeking the help of an external authority. This interactional sequence follows the pattern noticed in Chapter 4 when a claim of *not knowing* by a child functioned as an invitation to other children to display their knowledge. One interpretation possible is that based on Carlyn’s prior experience with numbers and operations, she recognized conditions similar to subtraction and she therefore proposed that this “take away” may be an arithmetical operation relevant to Daniel’s situation.

It is tempting to speculate that Carlyn was generalizing, but discursive psychology requires a rigorous commitment to the data, so I must acknowledge that she made the suggestion based on a single example ($\sqrt{100} = 10$). She had experienced other examples in earlier weeks, but this interaction concerns a single example only. It also involves mainly verbal communication (with the simple non-indexing gesture that served to emphasize “take away”).

This interpretation is supported with the way the interaction unfolded, when later in the conversation Daniel produced his first conjecture about the function of the square root symbol: “I think it’s kind of like a one hundred and like, plus ten thing” (Daniel, Week 7, line 503). He interpreted the situation as warranting his participation based on many previous experiences with this group – however, it was Carlyn’s previous conjecture that provided a warrant for the *kind* of participation he produced. At my request to write his conjecture down, he made a similar, less precise but more certain written claim in his journal (Figure 6.14).

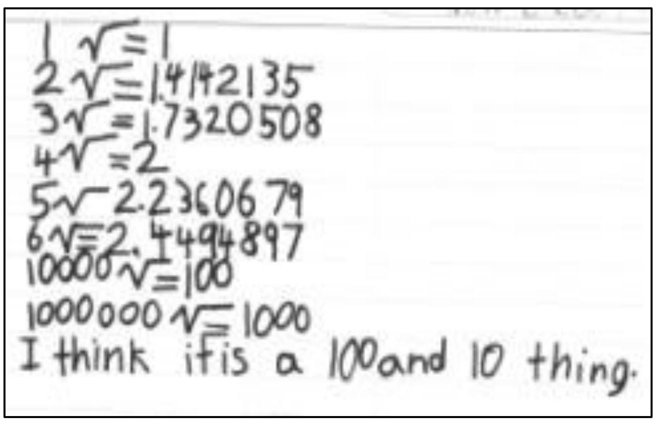



**Figure 6.14 Daniel's conjecture in written form (journal entry Week 7)**

This example speaks again to the power of multimodality as we can recognize some of his assumptions based on the differences between his two versions: written language has less opportunity for face saving gestures like hedges (n.b., “kind of like” and “like” in his spoken version) but the same function can be accomplished in mathematics by being less precise and writing “and” instead of the verbal “plus.” Notice this is another version of using everyday language in a mathematical context; however in this case the term is not overloaded, it is ambiguous. From the written version alone, there is no way of knowing if Daniel was referring to a three digit number ("110") or some other meaning. Juxtaposing the written with the spoken strengthens the warrant for an interpretation of each.

Later that session I mentioned Daniel's conjecture to Nathan who responded with an elaboration of his own: "Or minus, ninety" (Nathan, Week 7, line 534). This can be understood as a contrast to Daniel's "plus ten" because it begins with the conjunction "or." Thus he builds his participation on Daniel's. I asked Nathan to write his conjecture down as well, so we would not forget. Instead of writing words, as Daniel had, Nathan used symbols, listing the two arithmetical sentences that both "equaled" 10 (see Figure 6.15): Daniel's and then his own. Note that he did not compare Daniel's conjecture with his own, he used Daniel's example alongside his own.

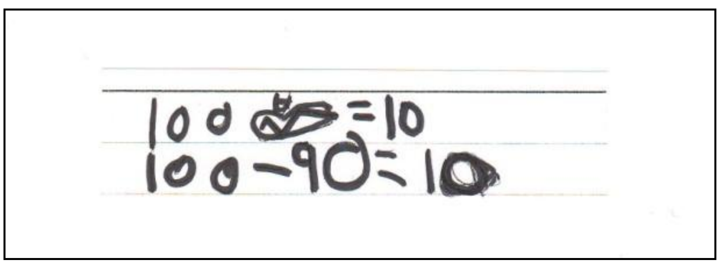



**Figure 6.15 Nathan's list "proving" his conjecture with a pseudo-form of equivalence (journal entry Week 7)**

Nathan followed Daniel's format of using the equals sign. However, by listing a square root with a subtraction sentence, Nathan altered the typical pattern of listing all the same operations together. It is possible that he used this format in a form of pseudo "proof", calling upon the principle of equivalence. This interpretation may be surmised based on his presentation of the verbal conjecture juxtaposed with the written. However, it is clearly not the only interpretation possible, so this example speaks to the constraints of teacher mediated learning. In this case, teacher mediation never occurred since I did not interact with Nathan on the basis of his journal entry. It therefore represents a missed opportunity to have qualified an interpretation of rather significant algebraic reasoning, given his young age.

It is not surprising that these young children referred to arithmetical operations when they first began to generalize numerical patterns. Arithmetic was the familiar domain of mathematics. In discursive psychological terms, the children were drawing upon their earlier experiences to evaluate and warrant both the occasion and the nature of their participation in the square root inquiry. In Krummheuer's (2011) terms, they were incorporating previous participant's formulations with novel content: acting as *ghostees*, a position of learning-as-participation.

#### 6.4.4.2 Week 8 – generalization disproved by counterexample

The next week at circle time, Daniel formally presented a generalization that he had reportedly discussed with his Mother that was different from his previous conjecture. He stood at the front of the group holding his journal open to the previous week's list (Fig. 6.14) and explained to us all as he pointed to the entry: "Um, it's like one million that - and that sign equals, um, one thousand, because my Mom says it takes, like half – half " (Daniel, Week 8, line 79). Although he never verbalized the other result in his list that also fit the generalization ($\sqrt{10{,}000} = 100$), it was visible to all as he held up his journal. I have already outlined (in Section 6.2.1) the interactions that occurred between Daniel's formal presentation and Cormac's refutation, so I will skip those here. However, I note that five minutes of discussion passed before Cormac's observation and all the while he looked at the list in his journal which was open on his lap.

During a lull in the interactions (discursively, a transition relevant place) Cormac put up his hand and responded to my acknowledgement with: "Um, but, um, it's not minus a thousand, ‘cause um, the nine, equals three” (Week 8 – line 121). I had him repeat that observation from the front of the group, at which time he produced a slightly different version: “So, um, instead of minus a thousand, um, well the nine is just, um, minus six” (Week 8 – line 127). This was the first time anyone had refuted another’s conjecture so it makes this cluster of algebraic reasoning clips another interesting case to consider here.

First of all, it is interesting mathematically to note that no one had ever mentioned a conjecture regarding “minus a thousand” and although *I* recognized the action as a refuting (or at least, mathematically significant, based on my asking him to re-present his idea formally from the front), it is not clear whether any of the children did. Cormac had inferred “minus a thousand” from a combination of Daniel’s “take half” and Carissa’s consequential re-wording: “it takes it away, the other three zeros.” Cormac built his participation on theirs. This shows a solid understanding of place value and exactly what is meant by “taking away three zeros.” It also invokes a sociomathematical norm that a generalization can only be valid if it works for every case.

I propose that in this interaction Cormac bridged arithmetical and algebraic reasoning. This occurred when he repeated the verbal statement as he concluded that for the case he was noting it was only minus six. With this proposition, I am drawing attention to the difference between saying that “the [case of the square root of] nine equals three” and “the [number sentence generated when one applies the operation of subtraction to the results of the square root of] nine is just minus six.” This is complex logical reasoning that bridges two different forms, algebraic and arithmetical: “to understand arithmetic is to think relationally about arithmetic. What could feed transition to more formal algebra would be explicit experience of relational thinking" (Mason, 2011, p. 565). As interactions continued for us, it was the "minus six" point that became relevant, in a way distracting us from the significance of the earlier statement.

The flow of information through the examples in these two sections on algebraic reasoning provides a sense of the more global dimensions of the inquiry:

- Daniel produced the first methodical list, facilitating the action of noticing numerical patterns;
- Carlyn overheard a verbal presentation of that list and verbally conjectured about the nature of the process involved;
- Daniel built on that conjecture by producing a tentative conjecture of his own in two versions (verbal and written);
- Nathan verbally elaborated Daniel's conjecture by offering an alternative and may have numerically represented a pseudo-proof of his own version;
- Daniel verbally presented another conjecture, this time a generalization, based on his Mother's authority;
- Cormac challenged Daniel's generalization by producing a counterexample, given in two versions (both verbal).

I propose that this structure of collective argumentation, if it occurred with older students and involved more formal algebraic language and symbols would be readily recognizable as algebraic reasoning. My proposition in this paper is that this group of young children was engaged in essentially the same behaviour, building their novel approach to the mathematical content on the formulation of previous participants. To strengthen that interpretation, I performed one more pass through the data, focusing on a cluster of clips designated algebraic reasoning that also involved related modal pairings and showing how discursive psychology and multimodal analysis afford a different kind of analytic meaning-making.

### 6.4.5 Multimodal representations affording a different kind of meaning-making

One more cluster of clips designated as exhibiting features of algebraic reasoning occurs in Week 9 before the events of circle time in Week 10 that led up to Carissa's demonstration of square root. During this cluster, Carlyn produced a verbal generalization followed up with a written version in her journal, and this time I talk with her on camera about her journal entry, twice. This has the effect of producing six different versions of her conjecture. I show here how integrating two analytic perspectives provides a warrant to say that Carlyn used the familiar formulation of sociomathematical norms to support her efforts to make sense of the unfamiliar mathematical content.

#### 6.4.5.1 Week 9 – representations of mathematical relationships revisited

As previously noted, I began circle time in Week 9 with the introduction of two different visual representations I had produced in order to summarize the investigation of square root up to this point: a radial diagram (Figure 6.8) and a numerical list (Figure 6.9). This prompted a lively interaction involving Anya, Daniel, Nathan, Carlyn and Carissa describing various patterns between the numbers both horizontally and vertically within the list. The interaction was analyzed in Chapter 4, so is simplified here in order to focus on Carlyn's observation as she compared the top three entries with the bottom three. I start the clip by responding to Anya's previous observation. Carlyn remains with her hand in the air.

**Example 6.2 R, Carlyn and Daniel Week 9**

160 R: *((Carlyn is sitting on her knees in the group, holding her hand in the air))* okay, so it's like, the order of the numbers were in. that's the smallest, and a little bit bigger and a little bit bigger than that? is that what you mean? unh, huh. and Carlyn?

161 Carlyn: um, *((she rises to her knees and reaches towards the wall chart. Her specific gestures are lost due to the camera angle, but she is pointing to various places in the chart as she speaks))* maybe you have - you take away three, the number what was there because if you take away three, it'll be still be that number, you take away two it'll be that num - unh the same number,

162 R: *((nods slightly and puts her hand to her chin))*

163 Carlyn: take away one zero it'll be, ten *((immediately looks up to R and puts her hands in her pockets))*.

164 R: o::h. are you - are you making a connection between this *((pointing to the numeral "3"))* and this ((*pointing to the numeral "1000" at the top of the list))*? *((repeats gesture connecting "3" with "1000"))*[because you take away three?

*165* Carlyn: [yeah, and then the <u>two</u> *((pointing to the numeral "2"))*

166 R: *((hand back to her chin))* yeah, mm hm.

167 Carlyn: then you take away two *((gesturing towards "100"))*, and one, you take away one *((gesturing towards the second "100"))*. and that'll be ten *((gesturing towards the "10"))*.

168 R: *((still looking at the list, hands on hips))* look at that. that's a pattern that I never even saw.

169 Daniel: what.

170 R: okay *((looking out to the group))*, that's even something different again.

I again used Transana's ability to re-locate the clip in the episode in order to determine what led up to this generalization by Carlyn. What question was she answering and how had others answered the question before her? As it turned out, there was no specific question, only a *"doing noticing"* on my part, a social action that has been identified as calling for an accounting (Antaki, 2002 – line 139 R: so I put it [my wall list] in an order. and I decided to start with the smallest number that we knew, and I went up to the biggest number that we knew. And I noticed a pattern). In this case, the accounting was interpreted by the children as an opportunity to display the patterns they noticed: immediately Anya, Carissa and Nathan verbalized patterns of "1, 2, 3" with overlapping talk. I acknowledged their contributions, but minimized them:

**Example 6.3 R, Nathan and Anya Week 9**

143 R: yeah, look at that. and then. I - I noticed there was a
different pattern - there's a different set of patterns which –
144 Nathan:[oh, oh, I know I know.]
145 R: [which was to do with all those zeros, right?]*((pointing to the top half of the chart))*all the zeros [what's your idea Nathan?
146 Anya: [less, more, more.
147 Nathan: it has three zeros, two zeros, one zero *((gesturing towards the chart))*. oh, like three, two one *((gesturing specifically at the bottom half of the chart))*.
148 R: well now, look at that. you're comparing the three zeros two
zeros and one zero with the three two one, it's another one.

This time I had framed the accounting as asking for a "different set of patterns", and it seemed to call forth more complex patterns of relationships between the numbers listed on the chart (see Figure 6.9). In the first case, Nathan drew attention to the *results* on the right hand side of the chart. This brought a specific relevance to Carlyn's subsequent set of patterns which named the *process between the initial number and the result it produced*. As she described her "pattern" (Example 6.2) in this context (in real time it *followed* the interactions in Example 6.3) Carlyn made use of Nathan's one-to-many correspondence (Nunes, 2013), again using the numerals in the bottom half of the chart to correspond to digits (literally, zeros) in the top half of the chart. This set of generalizations is reminiscent of Cormac's refutation produced at circle time the week prior, where he contrasted the emphasis on the solution "the 9 equals 3" with the process, "the 9 is just minus 6." Carlyn's representation of the mathematical relationships she observed was the fifth and final clip coded for algebraic reasoning in circle time of Week 9.

During journal time, Carlyn sat with Daniel, Nathan and Carissa at the table and copied the list from the wall chart. Then she developed her idea more fully with a written representation (see Figure 6.16). In her journal entry I suggest that she is using the equals sign in the sense of "equivalence" rather than "this is where you put the answer", since there are no words to the right of it (i.e., she might have written "takaway [sic] 3 zeros").

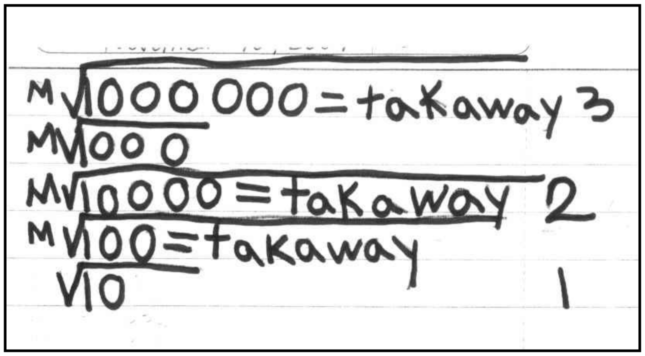



**Figure 6.16 Carlyn's list and theory**

I note that Carlyn's verbal representation began with the hesitant, face-saving "maybe." I never asked her at the time, so I can only speculate now on the purpose or function of the "M" at the beginning of her number sentences. At this point I see the correlation with her verbal "maybe." After a number of other interactions – all of which Carlyn overheard although she is invisible in the transcript – I knelt beside her at the table and initiated a conversation.

**Example 6.4 R and Carlyn Week 9**

*433* R: okay, Carlyn. what are you - what have you done here? *((leans on the table with her elbows))*
434 Carlyn: I did the, square root?
435 R: yes?
436 Carlyn: take away three and it will still be, this number *((pointing to journal with her pencil))*. and if you take away two, it'll be this number *((pointing to journal with her pencil))* and take away, like there's two zeros and then take away one zero ( )
437 R: right, so you're finding the pattern between zeros, right?
438 Carissa: Nanny?
439 R: this is the pattern of three two one.
440 Carlyn: they're all those numbers that were the three two one *((looks towards and points to the wall chart))*.
441 Carissa: Nanny? Nanny. Nanny.
442 R: yes, I see that. I see that.

The second time I interact with Carlyn about her journal entry comes a few seconds later, after several brief conversations with other children spoken from my position on the floor beside Carlyn (line 457 R: okay, so you wrote down your patterns. When Daniel found a pattern, he was able to go backwards. You know what I mean?). There was little contribution from Carlyn and it is not clear whether she recognized a connection between what she had told me and what I then told her. However, two minutes later, after multiple other interactions but while staying beside Carlyn, I re-voiced her representation. During this interaction, she produced an elaboration of my version by emphasizing the result over the process:

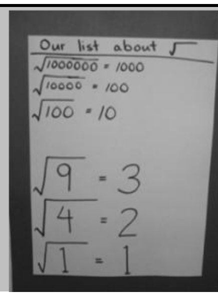
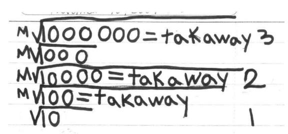
**Example 6.5 R and Carlyn Week 9**

502 R: did you figure this out Carlyn, using the calculator (?), or did you figure it out using your head.
503 Carlyn: I just figured it out using my head and that. *((points towards wall chart))*
504 R: yeah, using your head and looking and seeing the patterns up there. *((gesturing towards the wall chart and nodding))* uh huh. I think this *((pointing to Carlyn's journal))*, has a little bit to do with, what Carissa found out *((pointing to Carissa's journal))* about this one with the zeros in it? there's so many zeros? *((pointing back and forth between the two journals))* Carissa said it took away one zero and added the four?
505 Daniel: Nanny?
506 R: and what you're finding is it's taking away three zeros
507 Carlyn: and then that's going to be three *((pointing to her journal))*
508 R: yes. and this is taking away two zeros
509 Carlyn: and it's still two. *((pointing to her journal))*
510 R: and that's taking away one zero.
511 Carlyn: and it's still one. *((pointing to her journal))*
512 R: and it's still one. yes, which is another way of saying it's taking away half of them. *((gesturing two flat hands in front of her body and making a breaking apart motion))*
513 Carlyn: unintelligible

These interactions occur during a relatively noisy time and some of Carlyn's words are unintelligible so we cannot be certain if she specifically indicated a connection between the mathematical relationship she voiced at circle time, the version written here and the generalization about "taking away half of them" which I speak aloud in line 512. The production is situated like a revoicing, which was a standard discursive practice of mine, presumably to make certain the children's statements were available to the camera. It was not, however, an immediate revoicing, so it would be speculative to infer it here.

If this is what we *cannot* say, what *can* we say about Carlyn's different versions of her representation? We have five different spoken versions and two written versions (one produced by me), referred to during the explanations (see Table 6.3). These represent another form of generalization: "Generalization involves deliberately extending the range of reasoning or communication beyond the case or cases considered [or] explicitly identifying or exposing commonality across cases" (Kaput, 1999, p. 136).

**Table 6.3 Carlyn's algebraic representation of mathematical relationships in Week 9 [my words bracketed]**

| No. | source | Carlyn's words |
|---|---|---|
| | Wall chart "list" produced by me Week 9 |   |
| **1** | Example 6.2 (lines 161 – 163). | maybe you have - you take away three, the number what was there because if you take away three, it'll be still be that number, you take away two it'll be that num - unh the same number, take away one zero it'll be, ten. |
| **2** | Example 6.2 (lines 164 – 167). | [because you take away three?] yeah, and then the <u>two</u> *((pointing to the numeral "2"))* then you take away two *((gesturing towards "100"))*, and one, you take away one *((gesturing towards the second "100"))*. and that'll be ten |
| | Carlyn's journal entry Week 9. |  |
| **3** | Example 6.4 (lines 434 – 436). | I did the, square root? [ yes?] take away three and it will still be, this number *((pointing to journal with her pencil))*. and if you take away two, it'll be this number *((pointing to journal with her pencil))* and take away, like there's two zeros and then take away one zero (      ). |
| **4** | Example 6.4 (line 440) | [this is the pattern of three two one.] they're all those numbers that were the three two one *((looks towards and points to the wall chart)).* |
| **5** | Example 6.5 (lines 506 – 512). | [it's taking away three zeros] and then that's going to be three *((pointing to her journal))* [yes. and this is taking away two zeros] and it's still two. *((pointing to her journal))* [and that's taking away one zero.] and it's still one. *((pointing to her journal))* [and it's still one.] |

During these clips Carlyn's gestures were either indistinct due to camera angle or coded as indexical. In other words, without the video and the written representations referred to, these explanations would be difficult to understand. Another point to note: every single version progresses in the same numerical direction – from the largest example to the smallest. There is remarkable consistency between the various versions (remember the differences evident in other children's revoicing). The consistency is remarkable considering that the versions were produced for different audiences: public (the group as a whole) and private (to me at the table). This shows her assumption that the same frame of reference held in both situations. It is possible that this consistency was influenced by the presence of the written versions, but it is notable that previous variants were also supported with written versions (although only written numerically). There is however, a progression to be noticed in the use of referents. The distinction occurs in the way Carlyn presents the first and second parts of her propositions in each example and where the emphasis is placed.

The first example in Table 6.3 includes specific references (take away three; take away two) in the first part of each proposition and non-specific referents in the second part of each proposition (that number, that numb – unh, the same number). During the third proposition, the form produced accounts for mathematical precision in both parts of the proposition (take away one zero, it'll be ten). The exact same pattern happens in the third example, which is the first time Carlyn refers to her own written work. The second example shows Carlyn prefacing her second proposition with the short descriptor (*and then the two*. then you take away two.) This discursive pre-turn serves to frame her second proposition as a *case* (and then *the [case of ] "2"* – pointing to the "2"), implying that she views all three propositions as cases (based on the definition of case). In this interpretation, the list of 3, 2, 1 at the bottom of the chart indicates labels for the cases, not simply the amounts that represent the process each time. Carlyn supports that interpretation with her reference in Number 4 (Table 6.3). As I revoiced her representation she maintained that the relationship was more specific than just any 3, 2, 1 pattern: her proposition referred to the specific 3, 2, 1 from the chart. In Number 5, the last example of Table 6.3, Carlyn puts the emphasis on the result of the process (similar to Nathan's original representation at circle time). This contrasts with her version at circle time, where the emphasis was on the process instead of the result. As I oriented to it in my response at the time, her emphasis foregrounded the generalization of taking away "half" by reminding me that when one took away three zeros, there were still three zeros left, and so on, possibly referring back to Daniel's generalization.

At the time of these interactions, I was dismayed by Carlyn's reasoning, afraid that the way I had configured my chart influenced her thinking in a very unproductive manner. (I recognize that this type of speculation is unavailable to me in discursive psychology, wherein my analyses must demonstrably *not* rely on my memory of what happened. However, it is available in multimodal analysis, so I call upon my field notes to warrant its inclusion here.) My points in taking this aside, is that if I had not been a researcher, I may well have intervened to stop this line of reasoning. Mathematically, Carlyn was confounding place value, treating digits as if they were tallies and straying far from the conventional concept of square root. It is possible that this experience fueled my determination the following week (Week 10) to forge a connection between the lists and the arrays of cubes, an action that may have shut down the inquiry.

In any case, as a researcher I did not "intervene" and therefore I have access to these multiple versions of Carlyn's conjecture. By using discursive psychology and multimodal analysis together in examining different versions of Carlyn's "reasoning" I am able to show how she exploited the power of the underlying structure of mathematics, using the arithmetical operation of "take away" – a very algebraic process (Kieran, 2004, Mavrikis et al., 2012).

#### 6.4.5.2 Summary: How do young children express mathematical generalization?

Throughout these sections I have generated interpretation regarding the children's actions of representing mathematical relationships, their attempts to make sense of and explain relationships among quantities and their ideas regarding change between and within lists of numbers. In order to answer the second research question I will briefly consider the development of the inquiry in three phases and my expectation that I might recognize *indexes* to algebraic ways of reasoning through the *ways* the children constructed their participation.

After completing the five passes through the data, it became clear that the difference between Weeks 6 and 7 was the introduction of processes of algebraic reasoning. Earlier, there had been other types of noticing, but no examples of reasoning algebraically occurred until Carlyn presented her first conjecture ("maybe because it means take away"). She was articulating a meaning regarding mathematical process and indexing the particular process of subtraction. The function of indexing was oriented to by each of the other participants: me as I revoiced her suggestion and gave credence to it, Daniel and Nathan as they produced conjectures of their own, both including mathematical processes (but, unlike Carlyn, adding specific numbers to their operations: "plus 10" and "minus 90"). With these actions, Daniel and Nathan used Carlyn's formulation and elaborated the mathematical content (Krummheuer, 2011). They were acting as *ghostees* according to Krummheuer's operationalization of learning-as-participation.

The mathematical meaning assigned to "√" as being "the answer to the square problem" or "how-to-find-the-length-of-the-side-of-a-square-when-you-only-know-the-area" is strictly a sociomathematical convention. Young children can and do invent their own sociomathematical "conventions" (e.g., see Chapter 5, where Carissa tried to claim her capacity to argue mathematically based on her experience of *doing math* regularly, at home) and sometimes those inventions approach societal forms (Enyedy, 2005; DiSessa, Hammer, Sherin & Kolpakowski, 1991). However, that is because of the logic behind their necessity, unlikely to be re-produced in a situation like this one, where children were interacting with the abstract symbol first and the concept second.

The interpretation that the children were interacting primarily with the abstract symbol is supported by the actions of Daniel and Cormac as I attempted to re-engage them near the end of Week 10 with the configurations of cubes and Daniel's representation of inverse operations. I asked Daniel "how do you look at this square [the 4 x 4 configuration] and tell me 4 times 4 is 16?" His answer was simply "I don't know." Cormac offered "we haven't done times yet" and Daniel added "I don't know times (except in the calculator)" (wk10#39). The irony of these statements was not lost on me and at the time I responded by simply noting "this is something that we have to be curious about …." After all those weeks of producing lists of square roots and inverse operations just *what did they think they had been doing?* I concede here that it is possible their actions were no more mathematically meaningful to them than simply "doing calculatoring." It is possible that by the third month of second grade, these boys had already determined that multiplication was something that must be taught and thereby acquired. On the other hand, Carissa and Carlyn, whose actions had instigated the inquiry in the first place and then brought it into the realm of algebra, were near the beginning of first grade. It is possible they had not yet stopped expecting to make meaning of the activities they were doing.

In previous chapters I have shown how the children in this study were familiar with and used several different sociomathematical norms concerning arithmetic, in a form that Kieran (2004) calls meta-level mathematical activities. The general nature of these activities (e.g., noticing structure, studying change, analyzing relationships) makes them nevertheless available to children who are not yet familiar with the mathematical conventions of algebra. As Kieran points out, you can participate in these activities without doing algebra, but you cannot participate meaningfully in algebraic thinking unless you do them.

### 6.5 Discussion

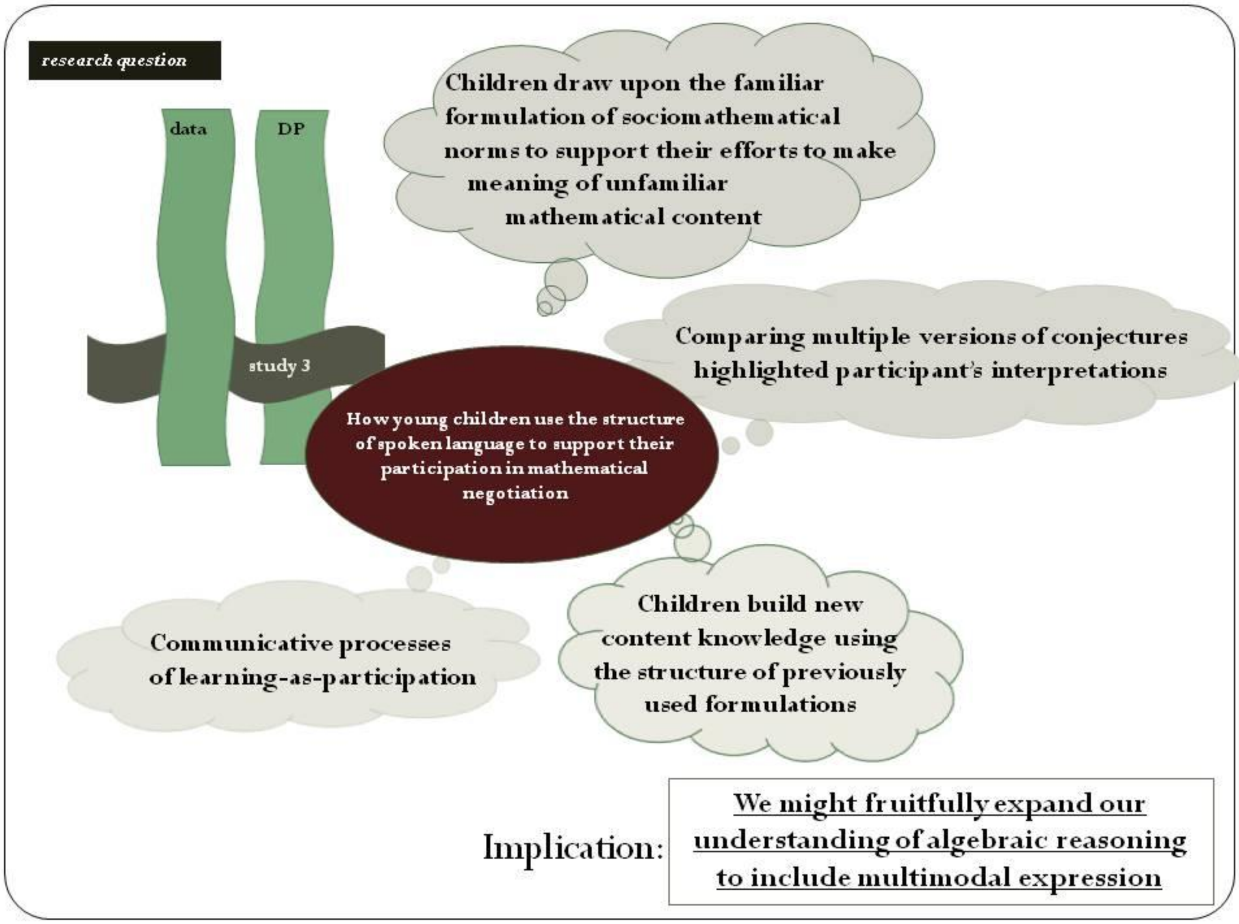



**Figure 6.17 Visual representation of main findings from Chapter 6**

This study completes my third step in the task of documenting and defining the processes involved in mathematical negotiation from the perspective of 5- to 7-year-olds by using discursive psychology and multimodal analysis to explore the ways children constructed a mathematical inquiry. I have shown how, during the inquiry, children were able to bridge arithmetical and algebraic reasoning, by comparing multiple versions of the same conjectures. I have shown how children used gestures and other modes of communication to meaningfully demonstrate the sense they made of challenging mathematical content. I have also shown how the flow of algebraic reasoning reveals a structure to the collective argument they developed in answer to their own question: *What does the square root symbol do?*

The point of this chapter has been to show that young children have a capacity to draw on sociomathematical norms as resources to support their attempts to make meaning of unfamiliar mathematical content (see Figure 6.17). They use what they know about arithmetic and quantities to approach unknown mathematical symbols. They use what they know about mathematical argument and negotiation to attend to the collective processes of meaning-making. They build their participation on the contributions of their classmates and teachers, if they are given the opportunity. It is possible to gain new insights into students' experiences by considering children's algebraic reasoning as a social rather than purely cognitive practice.

In this paper I have illustrated how a combination of discursive psychology and multimodal analysis is productive in facilitating a broader understanding of early algebraic reasoning. The results reported here speak to the affordance of group interaction in mathematics. It seems that the very action of participating in a collective promoted success – one may not have the skills to produce an entire generalization oneself, but collectively young children are able to do so by drawing on the resources of the discursive environment that support their participation. The children here spoke aloud, heard each other's statements and questions and built on those to construct their own position and understanding. In a sense the collective was both sufficient and necessary, operating almost as a form of collective zone of proximal development (Kendrick & Kakuru, 2012). These findings speak to three areas of research.

My description of the development of a collective argument here complements work done in literacy studies by Anderson et al. (2001) in describing what they call the "snowball phenomenon." In exploring the social influences on the development of reasoning these researchers looked at fourth grade students as they participated in small group discussions regarding various literatures. They identified 13 different *argument stratagems* that students used in their collective reasoning. I recognize six of their stratagems as potentially relevant to mathematical discussion, and three appear in my data here. I have accounted for these patterns of argumentation by invoking multimodal discourse analysis and discursive psychology. Anderson et al. (2001) refer to Rogoff's (1995) *participatory appropriation* and use cognitive notions of stratagems and schemas. The findings here support their results, yet without having to contend with interpretive subjectivity regarding what is going on in the children's minds. Future research extending discursive psychology to emergent literacy practice might provide insights into students' experiences in that field.

A second area of research this study speaks to involves Nathan's written representation in Figure 6.15 "proving" his conjecture with a pseudo-form of equivalence (journal entry Week 7). In that entry he noted "100 √ = 10" and underneath it he put "100 – 90 = 10." I called this a possible "pseudo-proof" based on a representation of equivalence but could take it no further. Collis (1974) and Kieran (1981) support this interpretation, noting that children between the ages of 6 and 10 are unable to hold an unevaluated operation in suspension, an act that would be necessary in order to represent Nathan's conjecture as an equivalence (√100 = 100 – 90). Collis noted that "the child needs literally to be able to 'see' a unique result before the operations on numbers mean anything to him" (Kieran, 1981, p. 320). Not until age 13 is a learner able to abstractly infer the operations necessary to represent this type of equivalence according to formal conventions. Nathan may have been using the square root symbol in the same way we would conventionally use an alphabetic variable. I note that if we replace his square root symbol with a variable (e.g., 100 $x$ = 10) – *without superimposing our adult understanding that 100 $x$ involves* *multiplying* – then the $x$ is replaced with " – 90 " in his second equation, a very logical representation of his conjecture. This strengthens my interpretation of his formulation as a "pseudo-proof", only calling it "pseudo" because it does not follow conventions.

This leads to the final area of research this study speaks to: the normative status of conventions at all levels of learning. In this chapter, I referred to Carlyn's conjecture in its various representations as "unformulated understanding" according to Davis (1996). I propose an amendment to that here: her understanding *was* formulated, only not according to mathematical conventions. If we are to understand the ways that young children "do mathematical thinking" and appreciate the resources they bring to school mathematics, we must re-formulate *our* understandings to allow for unconventional expressions during early learning.

There are aspects of the children's contributions that were quite conventional. I refer to the invoking of several sociomathematical norms in the examples presented here. These social actions may provide the clues necessary to warrant a claim that unconventional formulations nevertheless are still understood *by the children* to be aspects of the formal mathematics they are engaging with in school. Thus we must not discourage young children's early efforts at algebraic reasoning – no matter how unconventional – in order to promote "accurate" understanding of formal algebraic notation later on. Indeed, we can build on those early efforts, with an understanding of their purpose and place in the early learning of mathematics: as young children participate in mathematical activity, they use *what they know* to build on *what they do not know*.

This requires teachers to be both willing and able to share the authority for mathematical knowing within their classrooms, an action that has been supported by mathematics teacher education research for a while (Ball, 1993). Pre-service teachers are often encouraged to employ inquiry-based methods in teaching both mathematics and literacy, especially with young children. However, without examples of what that might look like, teachers are left imagining and perhaps falling back upon other more traditional methods. The square root inquiry as it was developed by the children here offers empirical evidence of young children’s *formulated but unconventional* expressions of generalization, valid as locally situated meaning. Teachers might fruitfully allow time and space for children to construct mathematical inquiries of interest to them; then use the significant learning opportunities that occur within even unconventional attempts at making meaning, as objects of discussion. This would afford a natural integration of arithmetic and algebra at early ages by not forcing formal conventions prematurely.

As reform-oriented mathematics or inquiry-based learning becomes more prevalent in the practices of schooling, listening to and attempting to understand the perspectives of all participants (including young children) will be vital for creating spaces that acknowledge and engage every child. 5- to 7-year-olds are verbally articulate while at the upper end of the period considered “early childhood” (Bredekamp, 1996) and still new to the practices and expectations of schooling (Mishler, 1972). Yet they come to school with a rich background of experiences that they draw upon as resources to help them make sense of what will become *mathematical practice* in their new experience (Walkerdine, 1988). Each new experience in school will positively or negatively influence that repertoire of resources. This study provides baseline data concerning young children’s participation in the communicative practices of *doing algebraic reasoning*. It builds on the findings reported in Chapter 4 regarding the interactional practices of doing knowing and on the findings reported in Chapter 5 regarding the argumentative practices of doing mathematical understanding. Taken together, these findings can inform the development of learning environments that promote children’s sustained, successful engagement with mathematics.

# Chapter 7: Conclusions and discussion

My research interest involves developing an empirically grounded understanding of young children's participation in the processes of negotiating sociomathematical norms within inquiry-based learning environments. As one possible approach to satisfying that interest, with this dissertation I sought to address two areas of inadequacy in the research literature (the students' own words, young children's experiences with mathematics) through a three-step analysis of children's participation in mathematical negotiation. Each of the linked studies presented herein highlighted one aspect of *how* young children used the structure of spoken language as a resource to support their participation (see Figure 7.1).

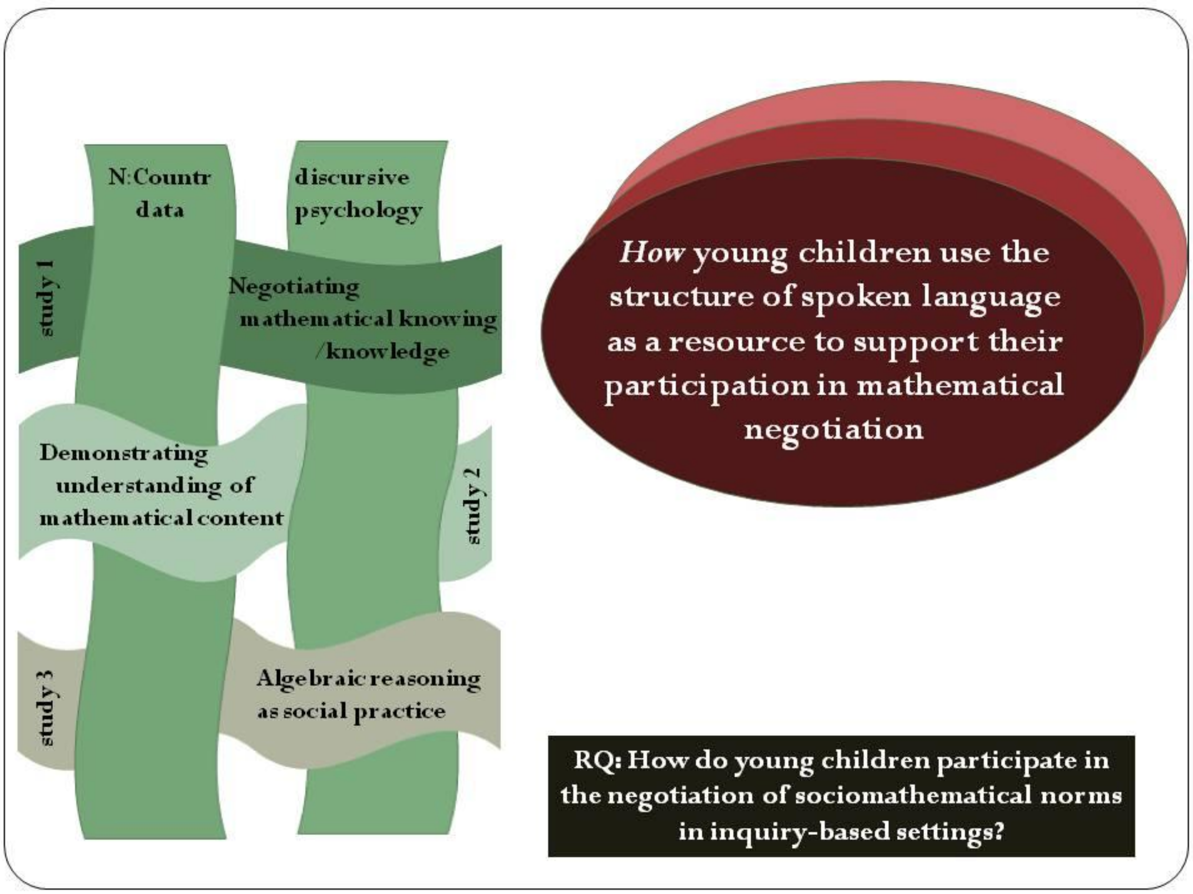



**Figure 7.1 Visual representation of the layers of meaning-making in the dissertation**

Through an extended set of discourse analyses, I made a substantive contribution to our understanding of the ways that young children use the structure of spoken language to support their participation in mathematical negotiation. The findings include:

- children draw upon sociomathematical norms as resources to produce interactional *ways of participating* that co-ordinate with each other;
- children draw upon the structures of production and recipient design to support their participation in mathematical argumentation;
- children use the invoking of sociomathematical norms to demonstrate their interpretations of the meaning of mathematical content;
- children use multiple modes of communication to express algebraic reasoning;
- children draw upon sociomathematical norms as resources to support their attempts to make meaning of unfamiliar mathematical content.

These results suggest that sharing the authority for *mathematical knowing* between adult and children afforded meaningful participation for these young learners.

The co-ordination of the three studies addressed two secondary tasks. The findings extend current research on sociomathematical norms by operationalizing the interactive, argumentative and communicative processes of learning-as-participation. Moreover, the results validate a novel approach to research in children's mathematical thinking by illustrating the potential in discursive psychology to elaborate the children's perspectives (these areas of significance are represented in Figure 7.1 as layers behind the summary of the main findings). Therefore, the dissertation provides a robust triangulation in the qualitative sense (Mathison, 1988) or a lamination of the three studies (Bloome et al., 2008) that allows me to present a rich and complex picture of mathematical learning from the perspective of its young participants.

From the very beginning of the dissertation, I have expressed the experience of designing, conducting and writing about my research as being a learning journey: a shared expedition wherein I learned *from* and *with* the children who were my research participants. When looking back on a journey, one often sees little of the detail but is able to see the mileposts more clearly than before. These mileposts become the source of *remembering,* of placing myself back in those experiences and revisiting the journey in a format now enriched with hindsight and a new understanding of self. In this chapter, I look backwards and then forwards in the sense of tying up loose ends. Looking backwards allows me to revisit the journey as well as consider the role I played as I influenced both the original events and the crafting of the dissertation. Looking forward, I consider the findings and how they might inform our future actions in research and education, thus presenting a systematic response to the question: *How do young children participate in the negotiation of sociomathematical norms in inquiry-based settings?*

In this chapter, "mileposts" provide the structure that allows me to revisit the journey. As I look back, I identify the mileposts in this dissertation as being

1. the initial research question,
2. the sense of ethical obligation that pushed me to refine my theory and methodology to incorporate discursive psychology and secondary data analysis
3. and the three studies that became my three results chapters.

I will revisit these points in time from a reverse perspective, looking back but taking the three studies as a whole, using general summaries to show how they informed each other and my understanding of each of them now, from the end of the writing.

## 7.1 An initial appraisal of the data: the biggest picture in Chapter 4

In the first results chapter, I used corpus linguistics in order to provide the broadest possible portrait of the data. I approached the question: *what evidence do I find that sociomathematical norms are active in this setting?* by illuminating patterns in the interactional sequences that involved "doing knowing." The main findings of this study were, first, showing how the children drew upon sociomathematical norms as resources to produce interactional *ways of participating* that co-ordinated with each other and second, how sharing the authority for *mathematical knowing* between researcher and children afforded meaningful participation (see Figure 7.2).

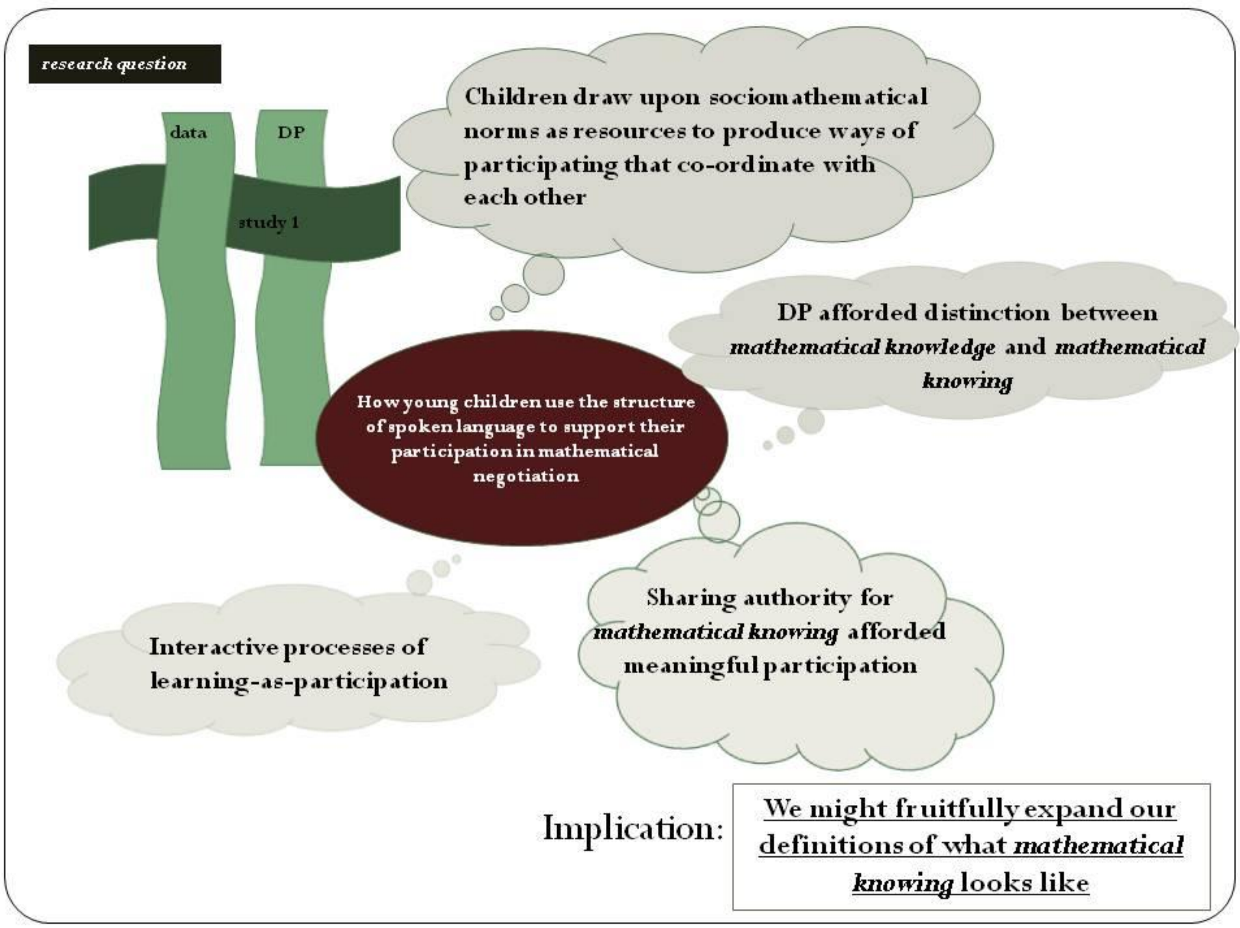



**Figure 7.2 Visual representation of the main findings from Chapter 4**

I produced empirical evidence for four relationships between the children's production of *displays of knowledge* and *claims of knowing* and I was able to show how regularities in those discursive practices served to support children's participation in mathematical negotiation. This corroborated Voigt's (1995) hypothesis and confirmed the active presence of sociomathematical norms in this setting with these participants. I showed how irregularities in the interactional patterns functioned to discursively mark challenges, objections and contrasts – all valued practices within mathematical argumentation. I provided evidence regarding how those productions of knowing oriented to sociomathematical norms. Moreover, I was able to use discursive psychology to establish a locally meaningful distinction between *mathematical knowing* and *mathematical knowledge* by showing how the participants interactionally treated knowing and knowledge differentially, thereby exemplifying some of the *interactive* processes involved in learning-as-participation. This study initiated the larger undertaking of documenting and defining the processes involved in mathematical negotiation from the perspective of 5- to 7-year-olds.

These findings uncovered features of the learning community's culture of negotiation which then played forward to influence the design of the studies that were outlined in Chapters 5 and 6. For example, Chapter 5 examined a negotiation that epitomized the complex relationship between the negotiation of knowledge and the negotiation of sociomathematical norms. I was drawn to that example while doing the analytic work of Chapter 4, in order to explore what was happening mathematically at a linguistically significant moment. Furthermore, the social actions of *doing knowing* were implicated in my choice of how to define communicative events in the analysis of Chapter 6: *knowing* was one discursive feature observed in algebraic reasoning alongside showing, reporting, explaining, representing, evaluating, suggesting and agreeing.

## 7.2 "Documenting discursive strategies" problematized through Chapter 5

In the second results chapter, I used conversation analysis in order to examine how the group negotiated a shared understanding of mathematical validity, realized through an extended negotiation of the meaning of the equals sign, a key factor in emergent algebraic reasoning. I approached the question *how do children display their understanding of the significance of norms during mathematical negotiation?* by focusing my analysis on the discursive practices by which the children demonstrated their understanding of the meaning of the equals sign. This highlighted the discursive practices of "doing mathematical understanding." The main findings of this study were first, to show how children drew upon the structures of production and recipient design to support their participation in mathematical argumentation and second, to illustrate how the children used the invoking of sociomathematical norms to demonstrate their interpretations of the meaning of mathematical content (see Figure 7.3).

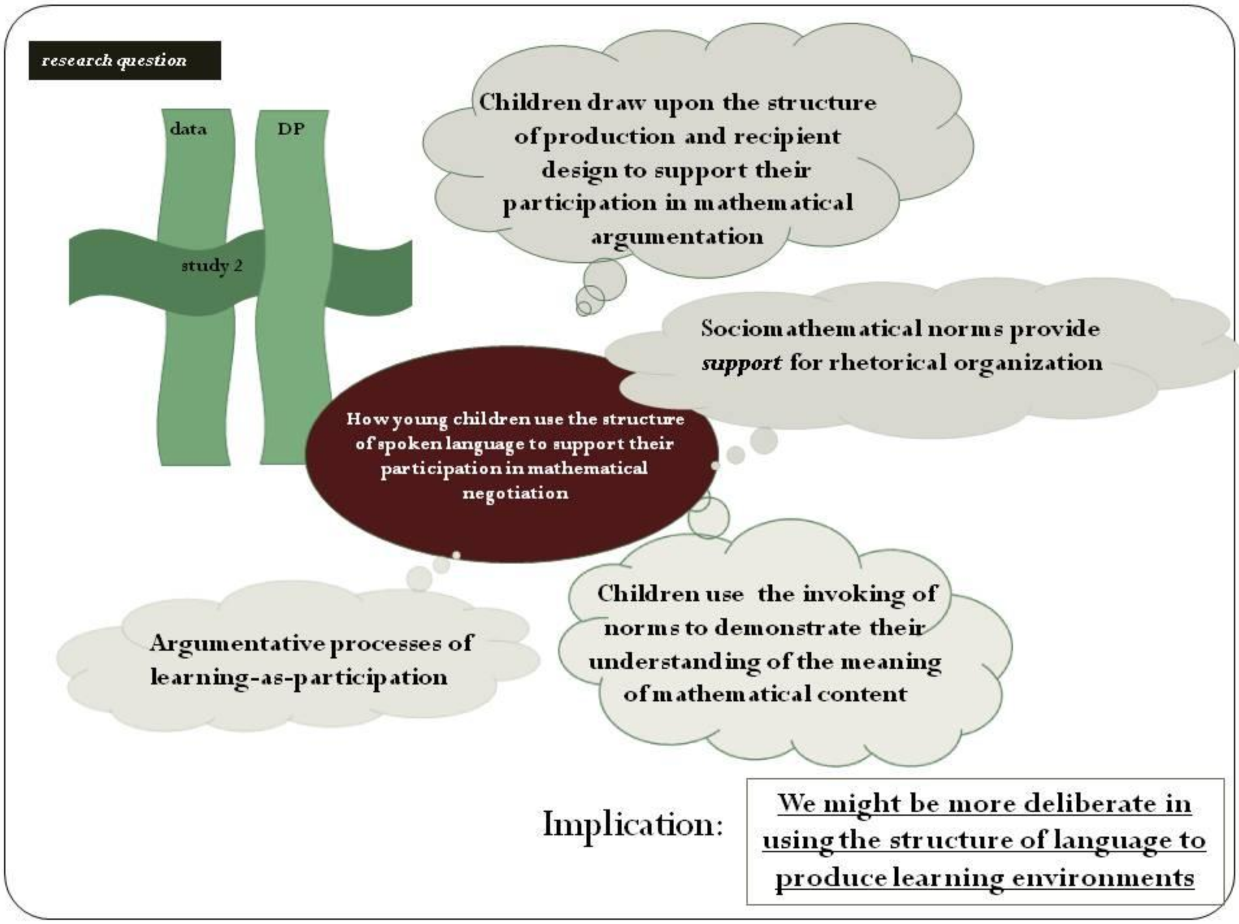



**Figure 7.3 Visual representation of the main findings from Chapter 5**

During the negotiation, the children used nine different discursive practices to incorporate mathematical content within their talk, drawing on the structure of production design (Goffman, 2001; Krummheuer, 2011) and recipient design (Garfinkel, 1967; Krummheuer, 2011) to produce ways of participating that co-ordinated with each other. They also referenced several sociomathematical norms and by so doing gave evidence of their emerging capacity to mathematize and to negotiate meaning using more formal mathematical argument. During analysis, I was able to use discursive psychology to show how the children used the invoking of sociomathematical norms to support the rhetorical organization of their negotiation, a valued aspect of meaning-making. Moreover, with the micro-analysis of the negotiation, I exemplified some of the *argumentative* processes of learning-as-participation.

The findings in Chapter 5 provided valuable details regarding young children's capacity for mathematical argumentation. However, scholars who foreground the perspectives of young children emphasize the significance of non-verbal communication in meaning making (McTavish et al., 2012; Narey, 2009). I began to wonder if I could strengthen the warrant for my claims by including multimodal aspects of communication in my analysis.

## 7.3 What multimodal research can tell us about "doing mathematics" in Chapter 6

In the third results chapter, I used a combination of discursive psychology and multimodal data analysis in order to highlight features of what it meant in the experience of these children to "do algebraic reasoning" as they worked together to determine the function of the square root symbol. I approached the question *how do children use sociomathematical norms as resources to support their participation?* by invoking an extreme case formulation of my own (Pomerantz, 1986). My assumption was that evidence of young children's participation in the negotiation of sociomathematical norms under mathematically challenging conditions would present a compelling argument. The main findings of this study were first, to validate the inclusion of multimodal aspects of meaning-making within a definition of algebraic reasoning in the early years and second, to establish young children's capacity to draw on sociomathematical norms as resources to support their attempts to make meaning of unfamiliar mathematical content (see Figure 7.4).

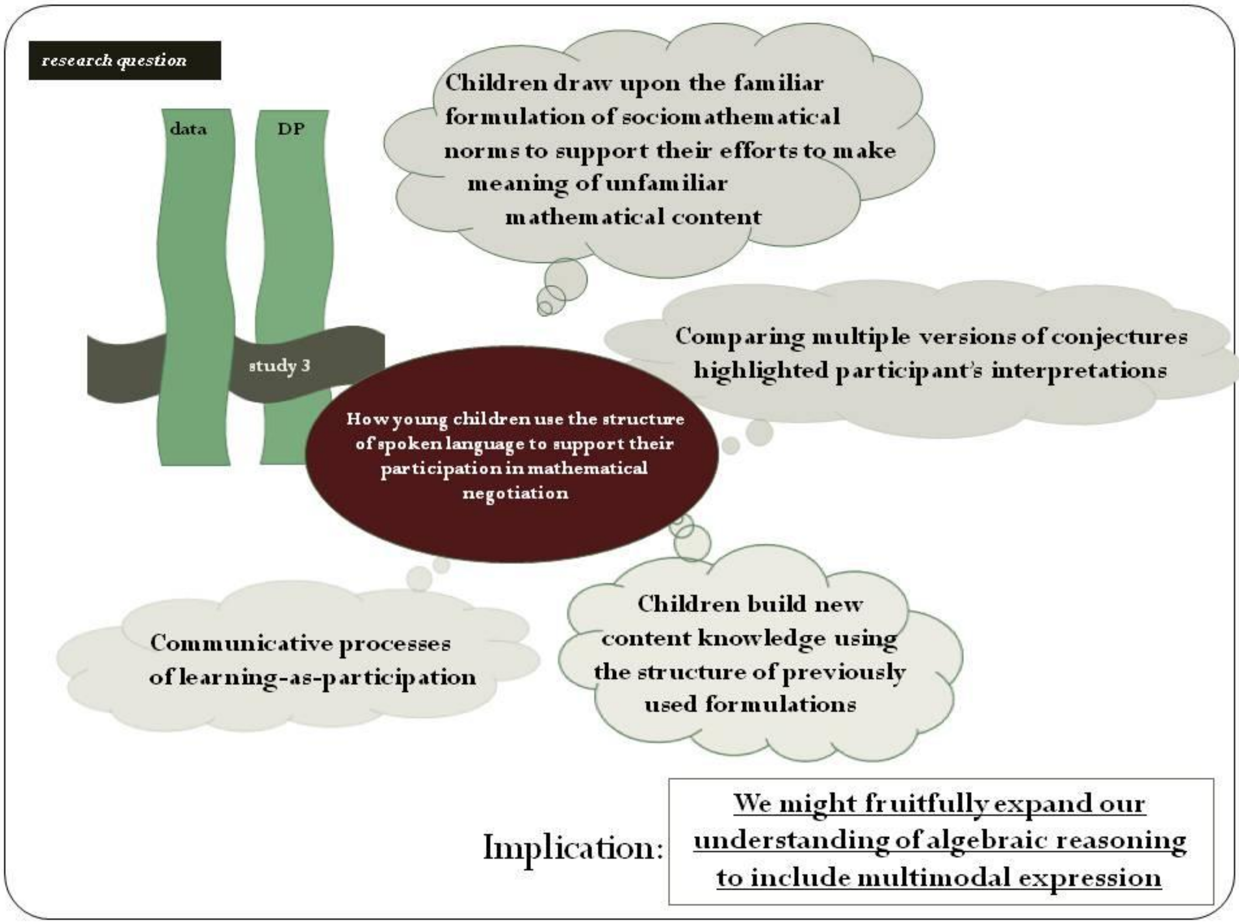



**Figure 7.4 Visual representation of the main findings from Chapter 6**

In the square root inquiry, the children used simple calculators to gain access to challenging mathematical content and I was able to provide evidence that they used the invoking of norms in a manner similar to that found in Chapter 5, but spread over the course of six weeks. The combination of discursive psychology and multimodal analysis afforded a different kind of analytic meaning-making, and I showed how the children collectively accomplished an extended argument of algebraic reasoning: a feat that may have been unavailable to any of them individually. In so doing, I exemplified some of the *communicative* processes of learning-as-participation.

The findings of the first two results chapters came to fruition in this third study. First of all, the inquiry involved the negotiation of conventional mathematical knowledge: what does this symbol do? Unlike the negotiation of mathematical *knowing* which can be open-ended (cf., do you know what equals means?), there *is* a correct (i.e., conventional) answer to that question, although the children's demonstrations taught me that there are multiple ways of expressing that understanding. Through their multimodal expressions, I was able to discern some of the complexities of the relationship between the negotiation of norms and the negotiation of knowledge as noted in the design of Chapter 4. Several of the key events identified in Chapter 4 as involving interactional irregularities played significant roles in the square root inquiry, such as Cormac's challenging Daniel's conjecture in Week 8 and Daniel's challenging Carissa's counting of the digits/zeros in Week 9.

The extended negotiation of Chapter 5 brought to my attention the conversations that occurred outside of the transcriptions, and the significance of overhearing and eavesdropping as (albeit passive, but potentially consequential) social actions in inquiry-based learning. Thus I was more inclined to return to the video data as my main focus of analysis during Chapter 6 and use the transcriptions to support it rather than vice versa. The resulting multimodal analysis of Chapter 6 allowed a fuller description of what it means to children to "do algebraic reasoning." In harmony with the pedagogical approach that drove the design of the mathematics research group N:Countr in the first case (Cadwell, 2003), we (together, the children as participant/*expressors* and I as participant/analyst) were able to make "thinking" more visible for those of us with adult, and therefore already somewhat narrow, perspectives regarding what it means to do mathematics.

## 7.4 Three ethical obligations revisited.

At an early point in the writing journey, I noted that in order to represent my participants well, I wanted an interpretive framework that:

- Required self-reflexive practice, especially regarding power differentials, no matter how benevolent: what kinds of knowledge had I *allowed* to emerge?
- Allowed a focus on meaning making, even when of the unconventional sort, even when it challenged the mathematical norm for accuracy.
- Allowed children and adults to operate on, at least the supposition of equal footing, in that they could each signal their interpretations to the other.

I claimed that using a discursive psychological framework allowed me to account for all three obligations. In hindsight, I wondered how, in having used the framework I might account for those obligations now, continuing my re-visiting in reverse order.

### 7.4.1 Third obligation: Evidence that we signalled our interpretations to each other.

Using a discursive psychological framework provided the capacity to notice mutual discursive signals if they were present. I found three different forms of such indicators. First, evidence that adult and children had signalled our intentions to each other in-the-moment emerged in the form of turn-by-turn discursive markers, or indexes. As expected, these emerged during conversation analyses in all three results chapters so will not be explored further in this chapter. Second, the negotiation examined in Chapter 5 included an example of Anya and me negotiating the task that she was to complete. During that analysis, I pointed out the completed math symbols chart (Figures 5.2 and 6.2) and used the placement of Anya's cut-out on the chart (it included the equals sign among other symbols) to indicate that Anya had "won" that negotiation. In other words, it had been her notion of validity that was represented in the column “we know these symbols but not in math.” However, I also noted that the equals sign had been cut out alone, and it was categorized as “we know this symbol” after the extended negotiation completed. I use this incident to illustrate that the enactment of the setting allowed all of us to indicate to each other our expectations for the interactions and our assumptions regarding who held the authority for what aspect of the mathematics. My use of discursive psychology required me to trace the interactional consequences of those indexes.

Finally, there was also evidence of formal, explicit signaling. For example, several times during our sessions in the mathematics research group I drew radial diagrams to represent the group discussions after listening to them in the video recordings (e.g., see Figure 1.1 for the diagram about “what do children know about math?” and Figure 6.8 for the diagram summarizing the square root inquiry). Each of these diagrams included ideas that were circled in red pen to indicate that they were my inferences. In subsequent weeks, I discussed those ideas with the children in the form of a member check (Charmaz, 2006). Likewise, but on other occasions, Cormac challenged my inferences twice. During Week 1 he challenged my generalization re-presenting the children’s comments that math had only numbers and not letters by pointing out that there was “kind of like an ‘x’ thing” (see Example 4.4). During Week 9 he challenged my inference that the children were conjecturing the square root function was like subtraction because the numbers got smaller by remembering that “sometimes the numbers got bigger” (Chapter 6, clip wk 9#2). Both of these challenges were consequential at the time and during analysis. Using a theoretical framework that resonated with the setting provided some coherence to the data collection and analysis.

However, there remains a limitation with regards to this obligation, from an ethnography perspective: the narrative I am currently writing for the dissertation has not been presented back to the participants for their response, removed in time as we are now four years after data collection. Thus I am missing a key piece of ethnographic analysis, contributing to an unfortunate silence and subsequent privileging of my own perspective. Nevertheless, with the discursive psychological framework, I am confident in having been able to account for some indications during analysis that the children had also been able to influence interactions during the original study.

### 7.4.2 Second obligation: Evidence for a foregrounding of meaning-making processes

Using a discursive psychological framework provided the capacity to notice the rhetorical organization of talk. For example, there is evidence within the results chapters that the form of questioning I used set up an expectation that metacognitive reflection might be appropriate. The way I phrased the questions made a thoughtful explanation relevant, even when a simple response might have sufficed. I made the argument in Chapter 4 that this would function to share the authority for *knowing* among the group and that when such authority was shared among a group of even young children, then the responsibility for holding each other accountable to principles such as accuracy and precision could become a part of their mathematical practice. I also pointed out that sharing the authority for knowing allowed for an environment where making mistakes or inaccuracies became part of learning, not something to impede participation. This played out with Anya's change-of-state-of-knowing at the end of the equals sign negotiation in Week 4 (see Example 5.9) as well as Jimmy's self-repair at the end of Week 11 (see Example 4.12).

However, I also noted a difference in my approach during the last week of the square root inquiry (Chapter 6, Week 10) when I reverted to a more traditional initiation-response-feedback pedagogy and began "fishing" for answers that I might recognize as being conventional. Even more startling to me in hindsight, is how quickly my actions of trying to connect the children's inquiry with the calculators to the geometric representations of square root seemed to shut down the children's efforts and interest. Even if I cannot say with certainty that the abrupt finish to the inquiry was caused by my actions, I can say that my actions did not lead the inquiry forward. The data in Chapter 6 shows that there was very little inquiry work done after I took over. My discursive psychological framework facilitated the positioning of those actions within the larger organization of the square root inquiry and showed how the children nevertheless responded to my actions with a quick succession of corresponding question-answer adjacency pairs, indicating their recognition of this form of interaction. I am confident in having been able to account for the participants' indications regarding the context of meaning-making vs. conventions.

### 7.4.3 First obligation: Evidence for the "sharing of power"

Finally, using a discursive psychological framework requires explicit self-reflection. It surprised me, no; it disappointed me to uncover the data regarding my own over production of speech in Chapter 4 (more than 80% of the uses of the term "know"). With ten other voices compared to my one, I really had thought the children did more speaking than that. Of course, this finding could be a result of using the transcripts as the main source of data for analysis – there were incidences of child-to-child talk that I uncovered when I revisited the video (in Chapter 6) that had not previously been transcribed because they were inaudible or fragmented. It may have also been a function of the methodology. As I laid out in Chapter 5, any conversations outside the "dominant interaction" (as defined by the analyst) remain invisible in transcriptions of this sort.

To me however, this remains the most significant limitation of the combination of methodologies as they inter-related. During the initial study, *I* was aware of the camera's presence in the room, because I was viewing the video at the end of each hour. In a few cases, it became apparent that the children were also aware (they asked, is it on? Or they came up close to make silly faces etc.) But mostly I believe *they* remained unaware. My awareness played itself out by my occasionally speaking louder than normal to make sure the video caught what was being said, or by repeating the children's comments verbatim for the same reason. During secondary analysis, I recognized that these actions on my part combined to produce a certain version of authority that we all oriented to.

In all of that, what kinds of knowledge *did I* allow to emerge? In hindsight now, I still recognize the effort I made to foreground child agency and curiosity. This was informed by the pilot study sessions in the N:Countr Phase I project where I documented the problem of superimposing adult definitions of engagement onto children's behaviour. I see the impact in several of my own actions, for example not taking a side in the "equals" negotiation in Chapter 5 and asking children to present their conjectures formally even if they did not make sense to me (e.g., Carissa in Chapters 4 and 5 or Carlyn in Chapter 6).

Nevertheless, in the presenting of the data, I chose particular interactions based on my own analytic decisions, which remained uninformed by the children's perspectives. The examples I used are not the only ones I might have chosen. They were, in my consideration, the clearest examples to use to elaborate the particular points I was trying to make. There were many "roads" into this data, and the particular road I took (and therefore, the answers I was able to construct) is a function of many small analytic decisions that accumulated over time. They were my own and in this way I say that the dissertation privileges my own researcher perspective.

That being said, by using discursive psychology and multimodal analysis together, I am confident that I was able to recognize learning in new ways. I point to the number of times my data here includes examples of children using a change-of-state token (e.g., “oh!” or, “ah!”) as they prefaced their explanations or observations to warrant that claim. It took the results of the previous two studies to emphasize for me the potential power of that combined methodological approach, through which I was not only alerted to the children’s indications of new knowledge, but was able to try and make sense of their expressions of it. Therefore, I use these results to suggest an extension of Edward’s (1993) discursive psychological premise that people construct their thinking *as they talk* by using instead the phrase “as they interact” for young children at least, who bring a rich array of experience to what we call “schooling” but who may not have access to the discursive resources we recognize as conventional, certainly in mathematics. So, I stand by my analytic decisions, while acknowledging that my conclusions are locally situated: they are my conclusions, warranted in this context and at this time.

### 7.5 Some discourse analysis on the research question

The research question for the current study was *How do young children participate in the negotiation of sociomathematical norms in inquiry-based settings?* One lesson I have learned from my participants and my experience of data analysis from within discursive psychology is that the way a question is framed affords and constrains the ways in which it can be properly answered. That lesson complicates the writing of this conclusion chapter since different framings of the question highlight several of the ways I *might have* answered the question but did not.

For example, supposing I focused on the word “How” in the question, as in “*How* do young children participate…?” An appropriate response might be “Very well, thank-you.” My participants showed me that they were very capable of participating; they even sometimes resisted my direction so that they could participate on their own terms. For instance the rhetorical deployment of “I don’t know” as it functions to resist the teachers agenda might have been fruitfully explored in this data, as it has been elsewhere (Reis & Barwell, 2013). However, I chose other foci.

Given an emphasis on "do": "How *do* young children participate…?" Well, this was a main research focus of Chapter 4 where I outlined seven discursive practices the children used in *doing knowing*. Not the only possible answer to this framing, but a valid one.

"How do *young* children participate…?" Since we operated as a group, I never distinguished between the actions of the 5 year olds over those who were older. However, I always reported the age of the participant the first time that child appeared in a study. In the conclusion of Chapter 6, I wondered if the younger children had not yet stopped expecting to make meaning of the activities they were doing. Given my theoretical perspective, that remains speculation. Other methodological approaches might be fruitfully employed to look at questions in this data regarding age. It has been one of my purposes to render the data transparent enough that others could use it in order to consider different research questions. For this reason I included long segments of transcription, and used Transana computer software (Fassnacht & Woods, 2012) in Chapter 6 to generate explicit connections between my video and my transcriptions.

"How do young *children* participate…?" Throughout the writing, I have chosen not to focus on gender as a possibly significant factor. It may have been. The group was comprised of an equal number of boys and girls, which was only *partly* intended. As it turned out it provided a balance of some sort, therefore offering opportunities for both boys and girls to inform our understanding of the negotiation of sociomathematical norms as if gender *were not* a factor.

Framing the question so that it emphasized the word *participate* was part of the rationale for Chapter 6, as I gave examples of eight different social actions that the children enacted during the square root inquiry (showing, reporting, knowing, explaining, representing, evaluating, suggesting and agreeing) and six different modes of communication that were used extensively to formulate algebraic reasoning (demonstration, gestural – indexing, gestural – nonindexing, numerical, verbal and written). These examples helped to empirically ground my understanding of the children's participation.

*Negotiation* was at the heart of my inquiry in Chapter 5: six minutes we spent deciding whether we would validate the equals sign as belonging to addition or subtraction. In the end we agreed that it could be used for both, a topic that was never re-visited again during the mathematics research group.

*Of* is a very small word to focus on but in English as a preposition it expresses the relationship between a part and a whole, in this case it highlights the association between the act of negotiation and its subject: here, sociomathematical norms. This speaks to an ongoing process rather than a completed act.

A focus on *sociomathematical* informed all three results chapters. I considered social norms as well, but only insofar as they were implicated in the negotiation of the mathematical norms. Table 7.1 summarizes the sociomathematical norms invoked by the children in each of the results chapters. These examples all together and my interpretation of them in the three results chapters provide empirical evidence to support my answers to the question *How do young children participate in the negotiation of sociomathematical norms in inquiry-based settings?*

**Table 7.1 Sociomathematical norms invoked by the children**

| Chapter 4 | | Sociomathematical norm invoked |
|---|---|---|
| | Daniel Week 4 | * regarding accuracy pointed out the inconsistency of the source of authority quoted (the alphabet) |
| | Daniel Week 7 | *challenged the source of authority for knowing by referring to imprecision (again the alphabet) |
| | Carlyn Week 7 | *accounted for above |
| | Cormac Week 8 | *for a generalization to be true there must not be even a single counterexample. |
| | Daniel Week 9 | *challenged imprecision in a public display of counting |
| | Cormac Week 10 | *knowledge is in some way developmentally sensitive, therefore different learning is appropriate for different ages – challenged R's violation of it. |
| **Chapter 5** | (all are from Week 4) | |
| | Carissa Example 2 | *capacity to argue based on experience "doing" mathematics |
| | Anya Example 3 | *calling upon a source of authority outside of one's own opinion |
| | Jimmy Example 6 | *producing a position of "not knowing" is a tolerated practice during mathematical negotiation. |
| | Nathan Example 7 | *mathematical generalization is valued over specifics. |
| | Daniel Example 8 | *refuting another's position is accomplished by producing an arithmetical counterexample. |
| | Anya Example 9 | *mathematical "knowing" can be different over time |
| **Chapter 6** | | |
| | Carlyn Week 7 | *it is reasonable to expect that a change in quantities follows regular, consistent, repeatable and thereby *nameable* processes |
| | Cormac Week 8 | * for a generalization to be true there must not be even a single counterexample (same interaction as in Chapter 4) |

Keeping within my primary discursive psychological framework, *norms* were defined here as ongoing features of discourse, implicated in every interaction rather than pre-established or prescriptive (Agha, 2007; Baker, 2000; Herbel-Eisenmann, 2000). They have been identified in research as being crucial to creating environments that support participation and mathematical learning (Voigt, 1995; Yackel & Cobb, 1996). Having observed and experienced this group negotiate sociomathematical norms, I conclude that this environment in its *enactment*, supported participation and learning.

## 7.6 My role

In Chapter 1, I noted that the role I played during the N:Countr mathematics research group was central to the ways the interactions evolved and the positions available to children during negotiation. Earlier in this chapter I acknowledged my own privileged role in the writing of the dissertation, given the institutional context of researching and writing for an academic committee. There are aspects of those points that warrant further inspection.

In my role as participant/observer, there were hints in the data that positions were available to some children and not others, yet there was never any discussion in the studies about this possibility. For example, Carissa, on her own initiative often stood up or came to the front to present her ideas during group discussions (e.g., Example 4.10 the discussion regarding the carpentry book and Clip wk10#25 in Chapter 6, the demonstration of square root using the block configuration). Her behaviour contrasts with the other children (e.g., Daniel, Cormac and Nathan only came to the front to present their conjectures in Chapter 6 after being invited by me. This functioned to frame their second presentations as "formal" as opposed to informal). Carissa's actions were generally not only tolerated but also attended to.

For example, consider the one minute interval in the midst of the negotiation about the equals sign, which we all oriented to as if it had been an interruption regardless that we listened mostly silently as Carissa sang the entire alphabet. In my role as analyst/writer of the dissertation, I frame that interaction slightly differently depending on the study. In Chapter 4 it appeared three times. Once it was accounted for as exemplifying the pattern of an elaborated claim of knowing followed by a display of knowledge (Example 4.9 transcription: "Now I know something. Look"). Next it was referred to as a present tense expression of knowing when Carissa followed up the entire display with an evaluation of learning (Table 4.2 #2: "If I never saw this I wouldn't never have know that"). Finally it was framed as representing an opportunity for Daniel to expose the inconsistency in her argument by disputing her portrayal of the source of her knowing (she had misquoted alphabetical order as Q, R, X, T, U, V). In Chapter 5, the interaction with Carissa appeared as an interjection in the midst of the equals negotiation, which we tolerated perhaps on the basis of her being one of the two originators of the argument. In Chapter 6 it was presented as an example of algebraic reasoning, since Carissa had quantified the letters in order to make sense of the quadratic formula.

This pattern becomes visible only when taking the three studies together. The participants did not experience that interaction as five separate events: we simultaneously and indivisibly constructed it collectively while experiencing it as individuals. Nevertheless, the act of following that interaction through five different presentations highlights some of the analytic decisions I made and therefore how I constructed the dissertation so that algebraic reasoning was a relevant topic for the third study.

In hindsight, I believe that Carissa enjoyed a privileged status within the group because she often introduced elements of mathematical significance: recall that it was her uninvited counter-position vis-à-vis Anya that began the equals argument and her initiative with the calculator that prompted the square root inquiry. There are several other significant examples within the N:Countr data that nevertheless do not appear in the dissertation. The questions of how she gained that status and how it was maintained through our actions and discursive indexes (or other mechanisms) remain unanswered: at no time was that position made explicit. This posits an opportunity for further exploration. Discursive psychology's focus is on the discourse, not the child, so these types of explorations never received attention here. The unit of analysis for the dissertation has been at the level of social interaction, an analytic given that afforded yet also constrained, my answers to the research question.

### 7.7 Implications for research.

In looking ahead, I want to consider some implications of these findings for research. My results find significance within three lines of inquiry: Herbel-Eisenmann (2000) and the research on sociomathematical norms, Krummheuer (2011) and the research on young children's participation in mathematical argumentation and Barwell's (2003) indications of the potential for using discursive psychology in mathematics education research.

My findings show how young children drew upon sociomathematical norms as resources to produce ways of participating that co-ordinated with each other, afforded participation in mathematical argumentation and supported their efforts to make meaning of unfamiliar mathematical content. These corroborate Voigt's (1995) observations that children and teachers together use thematic patterns of interaction to manage the fragility of intersubjective meaning-making, as already noted. They complement Cobb's (1987; Cobb & Bauersfeld, 1995; Cobb et al., 1992b; Yackel & Cobb, 1996) socio-cognitive perspective on sociomathematical norms by elaborating the social aspects as constituting, rather than representing mathematical learning. The results also speak to Herbel-Eisenmann's (2000) evidence that a sociomathematical norm regarding the authority for knowledge rested with the teacher. She contrasted that by attributing norms regarding the role of previously developed understanding to the students while attributing norms regarding the elegance or simplicity of a mathematical solution to the content itself. In my findings I recognized a meaningful distinction between the authority for mathematical knowledge and the authority for mathematical knowing; showing how the group co-constructed a taken-as-shared authority for knowing, while the authority for knowledge was invoked from an external source (e.g., the larger domain of mathematics, workbooks, arithmetical structure, the prescribed curriculum). Given that my participants were at least six years younger than Herbel-Eisenmann's, I suggest that these findings highlight a fruitful progression over time. Students with more years of formal schooling presumably have more access to the norms conventionally assigned to mathematical content. The children in my study were able to use their familiarity with the structure of language to support their meaning-making efforts. My point is that the participants in Herbel-Eisenmann's study may have been engaging in the same behaviour, although it looked different due to their experience with the conventions of formal mathematics practice.

Krummheuer (2011) considered third grade students and observed four patterns of participation in mathematical argumentation regarding assumed responsibility for the content and the formulation of the argument: author, spokesman, ghostee and relayer (see Figure 7.5). He suggested that the *ghostee* and *spokesman* positions represented stages in a learning process from *relayer* (just prior to learning) to *author* (fully knowing) and offered empirical evidence for what the *spokesman* position might look like within the everyday practices of mathematics classrooms. Working within discursive psychology, I choose to bracket the assumption that these positions might represent stages in a process of learning, but find that I can still productively contribute by providing evidence for what participation in a *ghostee* position might look like.

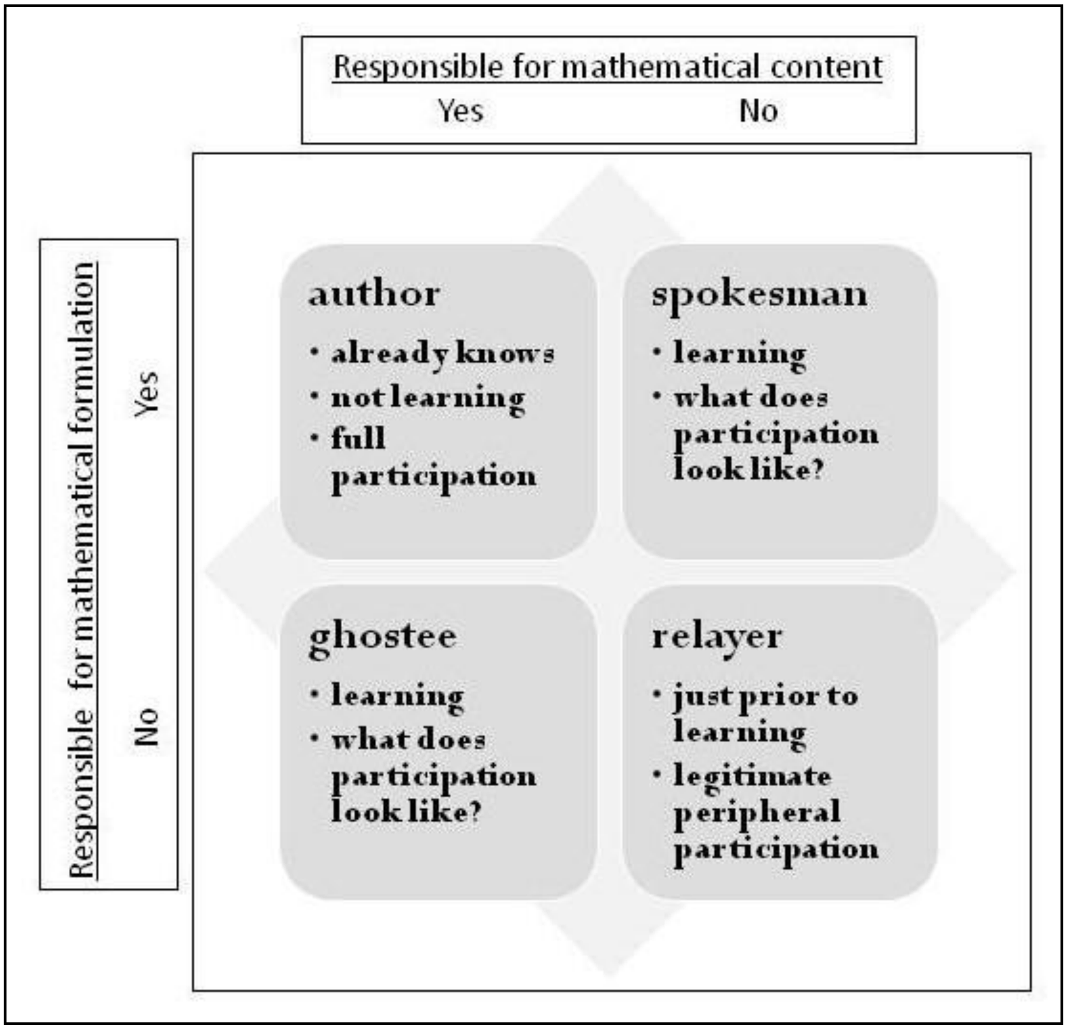



**Figure 7.5 Production design in mathematical polyadic interaction (generated from Krummheuer, 2011)**

My findings in Chapters 5 and 6 suggest that children who use the discursive formulation of recipient and production design[14] to co-ordinate their contributions to a mathematical argument are using a familiar formulation to carry novel mathematical content, in the sense of *mathematical knowing*. Thus when Anya responds to Carissa's calling upon her mathematical

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[14] It is important to recognize that while Krummheuer's development of the terms "production design" and "recipient design" are built upon Garfinkel's (1967) and Goffman's (2001) original ethnomethodological premises, he uses the terms in a novel way. In the current study, I use them according to Krummheuer.

experience as the authority for her knowing ("I do math every day in my workbook" Example 5.2) by saying "so, let's say one plus one" (Example 5.3) as an act of *convincing*, she builds her participation on Carissa's formulation of calling upon a source of authority outside of her own opinion but invokes the structure of arithmetic instead. Likewise, when Carlyn follows my claim of "insufficient knowing" with a conjecture ("maybe it's because [the square root symbol] means take away", Example 6.1) she follows the typical pattern of interaction noticed in Chapter 4 (i.e., "claim of insufficient knowing → display of knowledge") but uses her understanding of the number system to warrant the *way* she incorporates mathematics in her statement. The multitude of such examples within my data provides a reasonable case for operationalizing the notion of participation-as-learning within mathematics discussion.

Barwell (2003) originally drew my attention to the possibilities for using discursive psychology in mathematics education studies. He also noted some of the discursive complexities involved in negotiation as it required conversants to manage both the content of the disagreement and the underlying social relations necessary to maintain the interaction (Barwell, 2005). A self-reflexive piece on how researchers rhetorically construct their analyses so that the conclusions we come to are the most relevant of all other possible conclusions (Barwell, 2009) and a recent look at how "not knowing" is discursively produced through interaction (Reis & Barwell, 2012) have all informed my use of the theoretical framework in the dissertation. I already mentioned in Chapter 6 that I seek to extend his work on indexing gestures in mathematical explanation by including nonindexing gestures as well as other forms of multimodal expression. In the dissertation I seek to extend Barwell's studies mentioned here by extending the reach of discursive psychology into the realm of younger learners.

There are three ways that my use of discursive psychology was instrumental in being able to arrive at the conclusions I have drawn here. First, it afforded a distinction between mathematical knowledge and mathematical knowing, which I was then able to establish as a meaningful distinction for my participants in Chapter 4. Furthermore, an emphasis on rhetorical organization of everyday talk foregrounded the use of recipient and production design in the ways the children constructed their positions in the negotiation of the meaning of the equals sign in Chapter 5. Finally, discursive psychology provided the structure I needed to compare multiple versions of the same conjectures in Chapter 6, thereby showing how differences between the versions illuminated speakers' interpretations of what they were doing and the meanings they were making. Conversation analysts, with their focus on micro-analysis, are still trying to determine how to make use of longitudinal data (Zuengler, 2008). My results in Chapter 6 point to the "turn-by-turn" organization of a mathematical inquiry that emerged over six weeks. Therefore, this focus on the processes of meaning-making offers one possibility for using micro-analysis over time.

### 7.8 Implications for practice.

Robust research findings point to the significance of the early childhood years for future success. Given the role of success *in mathematics* as a gate-keeper to so many future opportunities educators must concern themselves with issues of equity and inclusion in mathematics. Understanding how young children participate in mathematical practices will help educators develop learning environments that promote *all children's* sustained, successful engagement with mathematics. I draw upon three implications for education that I generated in Figures 7.2, 7.3 and 7.4.

First, as educators we might fruitfully expand our definitions of what *mathematical knowing* looks like, especially with younger learners. An emphasis on social conventions during the primary years is understandable but these results suggest that this emphasis may be implicated in adult under-reporting of children's mathematical experiences (Tudge, 2009). In other words, teachers are not recognizing children's mathematical thinking because it does not appear in expected formulations. For example, consider the act of taking in written work without offering opportunity for explanation. The findings of Chapter 6 show that this act renders some work incomprehensible (e.g., consider Daniel's written conjectures in Figure 6.14, Nathan's unsubstantiated "proof" in Figure 6.15 and Carlyn's written theory in Figure 6.16). As a pedagogical practice it is based on assumptions that prioritize communication by writing alone (either numerical or prose). This case is particularly significant for young children who employ gestures as well as language to convey their meaning and are just beginning to use written language.

Second, teachers might be more deliberate in using the structure of language to create learning environments. This may seem counter-intuitive, given the cultural and linguistic diversity of classrooms today. However, I point out that the results in the current study show that children are using the structure of spoken language to support their participation, so relying less on any particular language because it is not yet a familiar domain may remove some of the sense-making opportunities that children naturally appropriate, such as the use of narratives (Curran, 2008). I noted here several instances of children positioning themselves vis-à-vis others using narratives (e.g., both Carissa and Anya produced small stories regarding their experiences in school mathematics during the negotiation "when I was in grade one, I did, like, math... and then I noticed it, oh, so this can be take away too" Example 5.9). This takes time and patience on the part of teachers, as well as possibly breaking long-standing patterns of interaction. These results suggest that a more self-conscious, deliberate awareness of language in the sense of *listening by teachers* would be fruitful for learners (Duckworth, 1996; 2001).

Finally, these results support the suggestion that teachers expand their understanding of algebraic reasoning to include multimodal expressions, especially for young learners (Kaput, 1999): "the success of early algebra instruction will depend in large measure on the implementation of a discourse-oriented pedagogy, in which students and teacher grapple with issues of sense making, and in which attempts to understand the mathematics are made public and reflected on" (Schoenfeld, 2008, p. 503). This resonates with practices of inquiry-based learning and the examples given here supply a much-needed source for teacher education purposes. In Chapter 5, I noted the same significance for teacher professional development regarding the use of "equals" for equivalence.

### 7.9 A final word on limitations and journeys down rivers

Throughout the drafting of this dissertation I have identified various limitations that need to be acknowledged. In Chapter 3 (Methodology), I discussed the limitations of the original setting, given that I included a sample of convenience in an informal learning environment and I was not subject to the usual constraints of formal schooling. I have tried to address this limitation in various ways throughout the secondary data analysis, but it should be noted. A second limitation regarding the original setting concerned the time constraints: 11 hours is not typically considered long enough for an ethnographic study. I counter that by suggesting that the amount of time required for data collection should be driven by the duration of the practice one is researching (Cole & Zeungler, 2008). In my case, mathematical negotiation was recorded here in two forms: six minutes and six weeks. Within a discursive psychological perspective such time frames are more than adequate and for the current study they would not, therefore, be perceived as a limitation. In Chapter 4, I noted another limitation in that during initial transcription the data was partially regularized with the exclusion of all partial-words, hesitations, false starts and fillers (Bishop, 2008). Excluding any parts of linguistic interaction results in a loss of data that might affect the resulting interpretations. I justified this procedure due to the word search limitations of the computer program and I note that patterns were found even with the idealization. Nevertheless, all examples explored in the second half of Chapter 4 and in subsequent chapters were fully transcribed.

At the beginning of the dissertation, I called upon a metaphor of teaching and learning entering a river from opposite banks to journey together, suggesting that my experiences learning with and from my participants resembled such a journey. In closing I would like to introduce a sailboat to the picture, as photographed through multiple lenses. As each version of the final three-dimensional portrait presents a somewhat identifiable image, it is only the layering or laminating (Bloome et al., 2008) of the versions, that suffices to represent some of the complexity in the experiences (cf. Figure 7.1). At the outset of Chapter 3 (methodology), I introduced Sfard's (2012a) metaphor of research being a set of clothing that in some way covers the experiences, without pretending to reproduce them. In that vein, I note that the act of sailing involves a complex relationship between understanding where and how the wind is blowing and being able to harness nature's power to good-enough effect. Thus, as the boat continues down the river and my teaching/learning journey continues beyond the dissertation, may I never underestimate the rich complexity of the relationship between those processes.

## 7.10 Summary

In this chapter, I have synthesized the main results of the current study and discussed some implications for research and practice. The key implication for education is that if we like what we see in the N:Countr children's participation, not only learning, but feeling positive and excited about that learning, then we must expand our expectations for what mathematical knowing and learning looks like. This will support the development of learning environments that invite all children's sustained, successful engagement with mathematics.

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## Appendix A: Transcription conventions

| | |
|---|---|
| (1.4) | the number in brackets indicates a time gap in tenths of a second |
| – | a dash indicates the sharp cut-off of the prior word or sound |
| : | colons indicate that the speaker has stretched the preceding sound or letter. The more colons the greater the extent of the stretching |
| ( ) | empty parentheses indicate the presence of an unclear fragment on the tape |
| (guess) | words within a single bracket indicate the transcriber's best guess at an unclear fragment |
| *((action))* | italicized words within a double bracket indicate movement, gaze or gesture |
| CAPS | capital letters indicate sustained elevated volume |
| . | a full stop indicates a dropping fall in tone, not necessarily a sentence |
| , | a comma indicates a continuing intonation |
| ? | a question mark indicates a rising inflection, not necessarily a question |
| <u>word</u> | underlined words indicate speaker emphasis |
| = | the 'equals' sign indicates contiguous utterances |
| [ ] | square brackets between adjacent lines of concurrent speech indicate the onset and end of a spate of overlapping talk |
| [[ | a double left-hand bracket indicates that speakers start a turn simultaneously |
| olive green | text in olive green font indicates talk *((or action))* that is secondary to the main analysis |
| **bold** | bolded text identifies uses of the verb "to know" (in Chapter 4 only) |
| → | arrow to the left of a line indicates a display of knowledge or claim of knowing produced by a child (in Chapter 4 only) |

*(These summarize the conventions used by the transcripts in this paper. For a more complete summary, please refer to the Atkinson & Heritage, 1984, book in the reference list.)*