Three Essays On Heterogeneity in Sectoral Price Flexibility

by

Mustafa Tugan

B.Sc., Middle East Technical University, 2004
M.Sc., Middle East Technical University, 2006

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

in
The Faculty of Graduate and Postdoctoral Studies
(Economics)

THE UNIVERSITY OF BRITISH COLUMBIA
(Vancouver)
February 2014
© Mustafa Tugan 2014
Abstract

This thesis focuses on the heterogeneity of price flexibility among sectors. For instance, does a multi-sector model in which the frequency of price changes differs among sectors predict radically different dynamics for aggregate variables following a monetary policy shock than a one-sector model in which sectors are assumed to be homogenous in their frequencies of price changes? Is there any relative price effect of the shocks to monetary policy in the United States? If there is, can this relative price effect be related to the heterogeneity of the frequency of price changes among sectors? What insights can be gained if dynamic stochastic open-economy models are elaborated by including the heterogeneity of price flexibility among sectors? These questions are among the types of the questions that I address in this thesis.

This thesis consists of three papers. The first paper studies the effects of a monetary policy shock on output, inflation and the real wage in the United States. Next, the dynamics of these aggregate variables as predicted by the one- and multi-sector dynamic stochastic general equilibrium (DSGE) models are compared. The main finding is that the dynamics predicted by the multi-sector model are quite similar to those predicted by the one-sector model.

The second paper focuses mostly on the effects of shocks to the federal funds rate on disaggregated sectors’ prices in the United States. The two main empirical findings in this chapter are the substantial heterogeneity in sectoral price responses to these shocks and
that the price responses in sectors are only weakly associated with their frequency of price changes.

The third paper, jointly written with Emek Karaca, is concerned with the effects of positive monetary shocks on output, the real exchange rate and the price level in developing countries which have adopted an inflation targeting regime. We find that such shocks are associated with a temporary rise in output; a temporary depreciation in the real exchange rate and a sizable contemporaneous increase in the price level in those economies.
Preface

The third paper of this dissertation is co-authored with Emek Karaca. The empirical models in this paper were primarily developed by Mustafa Tugan. Emek Karaca and Mustafa Tugan worked together to give a clear exposition of the findings in the empirical section. Both Emek Karaca and Mustafa Tugan were involved in developing the theoretical models in the paper and in the calibration of the theoretical models’ parameters. The econometric analysis for assessing the performance of the theoretical models considered in the paper were mainly performed by Mustafa Tugan.
# Table of Contents

Abstract .................................................. ii
Preface .................................................. iv
Table of Contents ........................................ vi
List of Tables ........................................... ix
List of Figures ........................................... x
Acknowledgments ......................................... xiii
Dedication ............................................... xiv

1 Introduction .......................................... 1

2 How Important is Sectoral Heterogeneity in Price Flexibility in Explaining the Effects of Monetary Shocks in a DSGE Framework? ........ 3
  2.1 Data ................................................. 7
  2.2 Model .............................................. 9
    2.2.1 The Household .................................. 11
    2.2.2 Firms .......................................... 15
    2.2.3 Equilibrium in the Frictionless Economy ......... 17
### Appendix to Chapter 3

B.1 The Bils, Klenow & Kryvtsov (2003) Model Reconsidered

- B.1.1 The Bils, Klenow & Kryvtsov (2003) Model
- B.1.2 Findings from the Bils, Klenow & Kryvtsov (2003) Model

B.2 Estimation of Confidence Intervals for Figure 3.3 Using a Block-Bootstrap Method

### Appendix to Chapter 4

C.1 The Empirical Strategy in Clarida & Gali (1994)

C.2 Aggregate Dynamics after Monetary Shocks

- C.2.1 Aggregate Dynamics in Empirical Model I after Monetary Shocks
- C.2.2 Aggregate Dynamics in Empirical Model II after Monetary Shocks in the United States

C.3 Calibration of Models' Parameters

- C.3.1 The Weak Link between the Level of Inflation and the Frequency of Price Changes
- C.3.2 Asymmetry in Currency Invoicing in International Trade between Developing and Advanced Economies

C.4 The One- and Multi-Sector Models' Dynamics with a Taylor-Type Rule
# List of Tables

2.1 Estimated Monetary Policy Rule: 1959Q1-2013Q1 ........................................ 30  
2.2 The Quarterly Frequencies of Price Adjustment over Different Percentiles of  
the Price Flexibility and Their Implied Durations ........................................ 32  
2.3 The Calibrated Parameters ................................................................. 34  
2.4 Estimates of Structural Parameters ..................................................... 37  
2.5 Estimates of Implied Parameters ........................................................ 38  
2.6 Calibration for Explaining Contrasting Findings in Carvalho (2006) ........... 50  
2.7 Cumulative Real Effects of Monetary Shocks and Their Persistence .......... 52  
3.1 Estimates of Structural Parameters ..................................................... 74  
3.2 Calibrated Parameters (The Multi-Sector Model with \textit{Asymmetric} Cost  
Structure) ................................................................................................. 91  
3.3 Estimates of Structural Parameters (Multi-Sector Model with \textit{Asymmetric}  
Cost Structure) ........................................................................................ 92  
4.1 Adoption Dates of Inflation Targeting in Developing Economies .......... 112  
B.1 Sectors with a Significant Response from the Federal Reserve .......... 176  
C.1 Calibration and Estimation ................................................................. 189  
C.2 Estimated Parameters of the Taylor Rule ............................................. 196
List of Figures

2.1 The Histogram of Quarterly Frequencies in *Entry Level Item* Categories . . . 5
2.2 Impulse Responses to an Unanticipated 1% Fall in $R_t$ . . . . . . . . . . . . 10
2.3 Impulse Responses to an Unanticipated 1% Fall in $R_t$ (One-Sector Model) . 40
2.4 Impulse Responses to an Unanticipated 1% Fall in $R_t$ (Multi-Sector Model) 41
2.5 The Model-Based Impulse Responses in the One- and Multi-Sector Models under Alternative Taylor Rules after an Unanticipated 1% Fall in $R_t$ . . . . 43
2.5 The Model-Based Impulse Responses in the One- and Multi-Sector Models under Alternative Taylor Rules after an Unanticipated 1% Fall in $R_t$ (cont.) 44
2.5 The Model-Based Impulse Responses in the One- and Multi-Sector Models under Alternative Taylor Rules after an Unanticipated 1% Fall in $R_t$ (cont.) 45
2.5 The Model-Based Impulse Responses in the One- and Multi-Sector Models under Alternative Taylor Rules after an Unanticipated 1% Fall in $R_t$ (cont.) 46
2.5 The Model-Based Impulse Responses in the One- and Multi-Sector Models under Alternative Taylor Rules after an Unanticipated 1% Fall in $R_t$ (cont.) 47
2.5 The Model-Based Impulse Responses in the One- and Multi-Sector Models under Alternative Taylor Rules after an Unanticipated 1% Fall in $R_t$ (cont.) 48
2.6 The One- and Multi-Sector Model-Based Impulse Responses of Output Gap after a Negative 1% $\epsilon^m$ Shock in (2.3.7) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 53
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 The VAR-Based Impulse Responses of Aggregate Variables to Monetary</td>
<td>62</td>
</tr>
<tr>
<td>Shocks</td>
<td></td>
</tr>
<tr>
<td>3.2 The Impulse Responses of the Price Levels of PCE Categories to an</td>
<td>64</td>
</tr>
<tr>
<td>Unanticipated 1% Increase in the Federal Funds Rate Shocks</td>
<td></td>
</tr>
<tr>
<td>3.3 Correlations of $\lambda_i$ with the Impulse Responses of $P_t$ to</td>
<td>68</td>
</tr>
<tr>
<td>an Unanticipated 1% Increase in $R_t$</td>
<td></td>
</tr>
<tr>
<td>3.4 Impulse Responses to an Unanticipated 1% Rise in $R_t$ (One-Sector</td>
<td>76</td>
</tr>
<tr>
<td>Model)</td>
<td></td>
</tr>
<tr>
<td>3.5 Impulse Responses to an Unanticipated 1% Rise in $R_t$ (Multi-Sector</td>
<td>77</td>
</tr>
<tr>
<td>Model with Symmetrical Cost Structure)</td>
<td></td>
</tr>
<tr>
<td>3.6 Model- and VAR-Based Correlations of $\lambda_i$ with $C_{1,20}^{\text{imp}}$</td>
<td>78</td>
</tr>
<tr>
<td>3.7 The Front-Loading Argument (The Multi-Sector Model with Symmetric</td>
<td>85</td>
</tr>
<tr>
<td>Cost Structure)</td>
<td></td>
</tr>
<tr>
<td>3.8 Model-Based $C_{2,3}^{\text{imp}}$ (The Multi-Sector Model with</td>
<td>86</td>
</tr>
<tr>
<td>Symmetric Cost Structure)</td>
<td></td>
</tr>
<tr>
<td>3.9 Impulse Responses to an Unanticipated 1% Rise in $R_t$ (Multi-Sector</td>
<td>93</td>
</tr>
<tr>
<td>Model with Asymmetric Cost Structure)</td>
<td></td>
</tr>
<tr>
<td>3.10 Model- and VAR-Based Correlations of $\lambda_i$ with $C_{1,20}^{\text{imp}}$</td>
<td>94</td>
</tr>
<tr>
<td>3.11 Inflation Dynamics in the Low and High Labor-Share Industries (The Multi-Sector Model with Asymmetric Cost Structure)</td>
<td>95</td>
</tr>
<tr>
<td>4.1 Impulse Responses to Monetary Shocks in Developing Economies (Empirical Model II)</td>
<td>113</td>
</tr>
<tr>
<td>4.2 Impulse Responses in Each Country to Monetary Shocks in Developing Economies (The VAR Model with Monthly Data)</td>
<td>115</td>
</tr>
<tr>
<td>4.3 Conditional Movements of the Real and Nominal Exchange Rates (Empirical Model II with Monthly Data)</td>
<td>116</td>
</tr>
</tbody>
</table>

xi
4.4 The Unconditional Co-movements of the Log-Changes in the Nominal and Real Exchange Rates .............................................. [117]

4.5 Model- and Panel VAR-Based Impulse Responses of $P$ and $\bar{y}$ to $\epsilon^z$ ........ [141]

4.6 Model- and Panel VAR-Based Impulse Responses of $\bar{e}$ and $Q$ to $\epsilon^z$ ........ [144]

4.7 Model- and Panel VAR-Based Impulse Responses of $P$, $\bar{y}$, $\bar{e}$ and $Q$ to $\epsilon^z$
(Without Investment) ........................................ [145]

A.1 Impulse Responses to an Unanticipated 1% Fall in $R_t$ (Efficient CMD Estimator) ........................................ [168]


B.2 Testing for the Significance of the Federal Reserve's Response for Sectoral Prices ........................................ [177]

C.1 Impulse Responses to Monetary Shocks in Empirical Model I ........ [187]

C.2 Impulse Responses to Monetary Shocks in the United States (Empirical Model II) ........................................ [188]

C.3 Median Inflation Rates in Developing Economies (1999M1-2012M9) ........ [191]

C.4 Consumer Prices Inflation and the Turkish Lira Share in External Trade in Turkey ........................................ [192]

C.5 One- and Multi-Sector Models with a Taylor-Type Rule ........ [197]
Acknowledgments

I am particularly indebted to my adviser, Paul Beaudry, for his excellent guidance throughout my thesis. Without such guidance, this thesis would not have been completed.

My experience during my doctoral studies can be divided into two different periods: the period before I met Paul Beaudry and the period after I met him. Before I met him, my experience in the Ph.D. program was not much different from a nightmare. I felt isolated from society and worked all the time without ever knowing where to go and mostly I was lost and lacking guidance at that time.

After I met him, on the other hand, I had a clear vision of where the research project was going. I had the relief that I could consult someone who could guide me when I faced a seemingly unsolvable problem. I am sure my view about his guidance is shared by all who have had the privilege of having him as his/her supervisor. His attentive behavior towards his students deserves much respect and gratitude.

I also specially thank Viktoria Hnatkovska and Yaniv Yedid-Levi for their careful reading of the draft of the thesis and their helpful suggestions. I would also like to thank Vadim Marmer as I benefited a lot from his comments and his long and insightful answers to the questions I asked via e-mail.

Last, but not least, I am thankful to Okan Yilankaya and Kadir Mercul. Their intimate and genuine attitude towards me in Canada reminded me of the beautiful people of my country, Turkey, when I was away a great distance from my home.
Dedication

To my mother and father, Zehra and Ali
Chapter 1

Introduction

Until recently, it has been standard practice in monetary economics to model the United States economy as consisting of identical firms which have the same frequency of price adjustment. After a proliferation of studies investigating the price frequencies at the disaggregated level, this practice is under question as sectors are found to differ largely regarding the frequencies of price changes in the United States.

Is this finding generalizable to the frequency of price changes among sectors in developing economies? It is true that higher inflation in developing economies is likely to lower the standard deviation of the frequency of price changes among sectors by causing a larger increase in the frequency of price changes in sectors where prices change infrequently compared with sectors where they change quite often. It is still, however, safe to argue that heterogeneity in price flexibility among sectors matters, to some degree, for developing countries. In my thesis, the implications of sectoral heterogeneity in price flexibility are investigated in terms of both the closed- and open-economy contexts.

In the first paper, I depart from the identical-firms economy by adding heterogeneity in price flexibility to the model and investigate the consequences. I find that the aggregate dynamics in the heterogeneous-firms economy do not deviate sharply from those in the identical-firms economy. This result contrasts with the finding in Carvalho (2006) and Nakamura & Steinsson (2008b) that monetary shocks induce larger and more persistent real effects in a heterogeneous-firms economy. These contrasting findings can be reconciled
with the fact that Carvalho (2006) and Nakamura & Steinsson (2008b) ignore the degree of wage rigidity in the United States. When this is taken into account, the dynamics of output after monetary shocks in the heterogeneous-firms economy and the identical-firms economy are similar.

The second paper studies sectoral price responses to an exogenous interest rate shock in the United States. It has two main findings. First, an interest rate shock causes strong relative price effects as price responses to such a shock differ largely among sectors. Second, sectoral asymmetries in the frequency of price changes are only weakly associated with sectoral price responses. I show that a multi-sector model where sectors differ not only in the frequency of price changes but also in the structure of production costs is capable of explaining these two findings.

The third paper investigates the movements of output, the price level, the bilateral nominal and real exchange rates with the United States following a monetary shock in developing economies with an inflation targeting regime. By means of an empirical model, we show that an expansionary domestic monetary shock in these economies causes a temporary increase in output, a temporary depreciation in the real exchange rate with the United States and a sizable contemporaneous increase in the price level. The multi-sector model where wage setting is staggered and there are asymmetries in the frequency of price changes among sectors proves equal to explaining these aggregate dynamics after such a shock. Contrasting with other staggered-wages models of Erceg, Henderson & Levin (2000) and Huang & Liu (2002), staggered wage setting in our model does not require complete financial markets. Since financial markets in developing economies are underdeveloped and lack sophistication, the incomplete financial market assumption adds realism to our model.
Chapter 2

How Important is Sectoral Heterogeneity in Price Flexibility in Explaining the Effects of Monetary Shocks in a DSGE Framework?

Until recently, it has been standard practice in the New-Keynesian tradition to model the United States economy as consisting of only one sector where firms are assumed to be homogeneous in regards to their ability to change their prices in each period. There are at least two justifications for this assumption. First, when frequencies of price changes do not differ significantly between sectors, one-sector models are easy to justify. Second, even when they differ substantially, if estimated structural parameters of the model are, in essence, invariant to allowing heterogeneous sectors with varying degrees of price stickiness, one-sector economy models can again be justified. As a matter of fact, if any of these reasons is strong and the gain from introducing sectoral heterogeneity into the model is limited, one-sector models should be preferred over multi-sector models as the latter models significantly
complicate the analysis. An analysis of the sectoral distribution of price change reveals little support for the first justification as the sectoral distribution of price changes in the United States economy is quite wide. Figure 2.1 illustrates this point with the help of a histogram of quarterly mean frequencies of price change, including sales in Entry Level Items (ELIs) in the CPI using the frequency of price changes in Nakamura & Steinsson (2008a)\textsuperscript{[1]} As shown in the figure, the United States economy features sectors with significantly differing price flexibility. The frequencies range from as low as 0.02% to 100%. More importantly, the distribution of sectoral frequencies of price change more closely resembles a uniform distribution than a tight normal distribution. Consequently, one-sector models are difficult to justify based on the fact that sectors have similar frequencies.

This leaves only one channel through which one-sector models can be justified, namely that inference from the structural DSGE models is insensitive to heterogeneity in frequencies of price change. Carvalho (2006) attempts to address this question within the context of the simple Calvo model. His main conclusion is that interest shocks have substantially larger and more persistent non-neutral effects on output in the multi-sector economy than in a one-sector economy when there is strategic complementarity between firms. Strategic complementarity refers to the phenomenon that firms have an incentive to raise their prices when other firms do the same. In order to explain the findings in Carvalho (2006), note that prices are by definition slow to adjust in low-frequency sectors. With strategic complementarity in price setting, the limited response of prices in these sectors gives a disincentive for firms in high-frequency sectors to change their prices when the shock happens. As a result, the behavior of prices in low-frequency sectors has a dominant effect on the behavior of overall prices. This dominant effect results in longer and larger effects of monetary shocks on output in the multi-sector economy compared to those in the one-sector model.

\textsuperscript{1}Entry Level Items are product categories that the Bureau of Labor Statistics uses to measure the CPI. Some examples of ELIs are “Girls’ Outwear” and “Parking Fees”.

4
Figure 2.1: The Histogram of Quarterly Frequencies in Entry Level Item Categories

Source: Own estimates based on data in Nakamura & Steinsson (2008a)

Note: To measure the CPI, the Bureau of Labor Statistics divides products into Entry Level Item (ELI).
Some examples of ELIs are “Girls’ Outwear”, and “Parking Fees”.

sector economy with the same average frequency of price change. Later, Schwartzman & Carvalho (2008) extend these findings for Taylor and sticky-information models. Lastly, Nakamura & Steinsson (2008b) find that stronger non-neutral effects of monetary shocks in multi-sector models can be generalized to menu-cost models. Indeed, they also find this is the case when the frequency of price changes in the one-sector model is calibrated as the mean frequency of price changes in the multi-sector economy.

In this paper, I also study the one- and multi-sector economies and compare the aggregate dynamics after monetary shocks between the former and the latter. However, unlike
Carvalho (2006) and Nakamura & Steinsson (2008b) who isolate the effect of heterogeneity in a simple model which performs unsatisfactorily in explaining what happens after a monetary policy shock. I work with a more complicated model which better explains these dynamics. I find that the dynamic properties of the model are largely insensitive to adding heterogeneity in the frequency of price changes to the model. This finding holds irrespective of whether the model is closed with the estimated or hypothetical interest rate rules.

How can the contrasting findings regarding the real effects of monetary shocks in the one- and multi-sector economies in this paper and in Carvalho (2006) and Nakamura & Steinsson (2008b) be reconciled? I show that the finding in Carvalho (2006) and Nakamura & Steinsson (2008b), that monetary shocks have larger and more persistent real effects in the multi-sector economy than those in the one-sector economy, is driven mainly by their flexible-wages assumption which is at odds with the data. For example, Barattieri, Basu & Gottschalk (2010) investigate the sluggishness of wage adjustment in the United States economy and find that the expected average duration of wage contracts in the United States is 5.6 quarters. When accounting for such a degree of wage rigidity, I find the dynamics of output in the one- and the multi-sector economies are alike for a given monetary shock.

I follow two steps for the systematic exposition of this finding. First, I do a replication exercise of Carvalho (2006) who considers a simple model that features Calvo-type nominal price contracts and flexible wages. In line with the finding in Carvalho (2006), I find that monetary shocks in the multi-sector economy have larger and more persistent real effects than those in the one-sector economy. Next, the model is elaborated by introducing

---

2For example, Woodford (2003) convincingly notes the simple Calvo (1983) model which features firms with solely forward looking behavior fails to account for the delayed effects of nominal disturbances on inflation. Indeed, when an expansionary monetary policy shock happens, the simple Calvo model predicts inflation to peak before real GDP peaks. Yet, evidence from structural VAR models indicates that the peak in inflation actually occurs much later than the strongest effect on real GDP.
staggered wage setting in the form of Erceg, Henderson & Levin (2000) type nominal wage contracts. I show that when costs are slow to respond to such shocks, the larger and more persistent real effects of monetary shocks in the multi-sector economy relative to those in the one-sector economy disappear.

The organization of the paper is as follows: Section 2.1 describes the data and specifies the VAR estimation. Section 2.2 develops one- and multi-sector economy models which explain the economy’s response following the shock. Section 2.3 compares the one- and multi-sector models using impulse response functions under estimated and hypothetical interest rate rules and discusses why the finding in this paper and in Carvalho (2006) and Nakamura & Steinsson (2008b) differ. The last section concludes the paper.

2.1 Data

I study the aggregate dynamics following an unanticipated 1% fall in the federal funds rate with the following vector autoregression (VAR) model,

\[ \mathbf{Y}_t = B_0 + \sum_{k=1}^{k_{max}} B_k \mathbf{Y}_{t-k} + A_0 \mathbf{\epsilon}_t \]  

(2.1.1)

where \( \mathbf{\epsilon}_t \) and \( k_{max} \) represent the vector of structural shocks that occur in the period and the number of lags included, respectively. A contemporaneous response matrix of the variables to these shocks is shown by \( A_0 \). Lastly, \( \mathbf{Y}_t \) denotes the vector of variables contained in the VAR and is given as:

\[ \mathbf{Y}_t = \begin{bmatrix} Y_t - Y^n_t \ , \ \pi_t \ , \ W_{real,t} \ , \ R_t \end{bmatrix} \]  

(2.1.2)

where \( Y_t - Y^n_t \) denotes the output gap. I follow Giordani (2004) and use capacity utilization in manufacturing as a measure of \( Y_t - Y^n_t \). The next variable, \( \pi_t \), denotes annualized
inflation, which is measured as the annual percentage change in the GDP deflator. Real wage is represented by $W_{\text{real},t}$, which is defined as the hourly earnings in manufacturing divided by the GDP deflator. Lastly, $R_t$ shows the quarterly federal funds rate. I use quarterly data that spans the period from 1959Q1 to 2013Q1 and the VAR contains four lags of each variable. The ordering of the variables in (2.1.2) implies that the Federal Reserve is assumed to observe disturbances in the output gap, inflation and the real wage ($k = 0$) and respond to them contemporaneously. For the least square estimates of the coefficients to be unbiased, the output gap, inflation and the real wage must be assumed to respond to monetary shocks only with a quarter lag. This is in line with the recursive identification of monetary shocks in Christiano, Eichenbaum & Evans (2005).

It is notable that it is conventional to use real GDP in place of the output gap in (2.1.2). However, if real GDP is used, “the price puzzle”, which refers to the counter-intuitive finding that an expansionary monetary policy shock lowers inflation, is observed. Giordani (2004) criticizes the practice of using real GDP in place of a measure of the output gap since using real GDP results in monetary shocks not being orthogonal to other structural shocks, which may be the cause of the puzzle. He shows no such problem exists when the VAR contains a measure of the output gap. My results support this conjecture. Indeed, when I use a measure of the output gap as opposed to real GDP, the puzzle

---

3Sims (1992) conjectures that the puzzle results from misspecification due to the failure of identifying an exogenous monetary policy shock. To explain this misspecification problem, it must first be noted that the overall prices in an economy respond only sluggishly to commodity price shocks. Now, consider a fall in commodity prices. Since a fall in future inflation is anticipated, it is natural for a central bank to react to this shock by lowering its interest rate. Despite the fall in interest rate, a fall in inflation may be observed since the effect of the fall in commodity prices on prices may outweigh the effect of lower interest rates on prices. Note that no such control for commodity prices are contained in the VAR system above. Hence, it is possible the finding that expansionary monetary policy causes a fall in inflation may reflect the effects of commodity prices on inflation and interest rates. In other words, what is regarded as an exogenous monetary shock in the VAR system above might in fact reflect the endogenous policy response of monetary authorities to the commodity price shock. In this case, an economic interpretation of impulse responses is difficult. To alleviate this problem, Sims (1992) includes a commodity price index in his VAR model. In contradiction to this conjecture, adding a commodity price index in (2.1.2) before the interest rate does not help alleviate the puzzle in our results when real GDP is used in place of a measure of the output gap.
is eliminated to a large extent. Figure 2.2 shows impulse responses for the aggregate variables in the VAR in (2.1.2) to a 1% expansionary interest rate shock. After this shock, the output gap shows a persistent increase. While the point estimates of impulse responses for the real wage indicate that the real wage rises, their confidence bands are not tight enough to conclude that the real wage shows a significant rise. After an insignificant fall in periods immediately following the shock, inflation rises. It is notable that while inflation responses to the expansionary shock are negative over some quarters, there is no puzzle in our results as these responses are not significant. Lastly, the federal funds rate stays below the pre-shock level for about three years after the shock.

It is notable that aggregate dynamics reported in Figure 2.2 are in line with those in Christiano, Eichenbaum & Evans (2005). In this regard, the aggregate dynamics I aim to match with those in dynamic stochastic models in Section 2.3.3 are similar to those reported in the literature.

### 2.2 Model

The model I employ for the analysis builds largely on the model in Giannoni & Woodford (2003). The model in Giannoni & Woodford (2003) departs from the simple textbook Calvo model in four ways. First, the model features staggered wage setting along the lines of Erceg et al. (2000). Second, when not optimized, prices and wages are set according to the backward-looking indexation rule. Third, consumer preferences exhibit habit persistence. Thus, an increase in today’s consumption increases today’s marginal utility, while it leads to a fall in tomorrow’s marginal utility. Fourth, while wages and prices are set one period in advance, the decision regarding real expenditure is made two periods in advance. My structural model is a modified version of this model. The model in Giannoni & Woodford (2003) include three modifications. First, firms have to pay their wage bill in advance,
Figure 2.2: Impulse Responses to an Unanticipated 1% Fall in $R_t$

(a) $Y_t - Y_t^*$

(b) $\pi_t$

(c) $W_{real,t}$

(d) $R_t$

Note: In the figure, the solid line indicates the estimated point-wise impulse responses. The area between the dashed lines shows the 95% confidence interval estimated with the method suggested by Sims & Zha (1999).

implying an increase in the opportunity cost of holding money leads to an increase in the cost of production, all things being equal. Second, the model is extended to allow sectoral heterogeneity in price flexibility. Third, as opposed to two periods in advance, the decision on real expenditure is made one period in advance. Such an assumption is consistent with
the VAR model presented in the last section where the output gap responds with a lag. Yet, the assumption in Giannoni & Woodford (2003), that real expenditure is predetermined for two periods, is debatable since such an assumption contradicts with their VAR model where the output gap response is delayed by only one period.

2.2.1 The Household

The objective of the infinitely lived household is to maximize its lifetime utility as specified by the following utility function:

\[
U_t = E_{t-1} \left\{ \sum_{s=0}^{\infty} \beta^s \left[ u(C_{t+s} - bC_{t-1+s}) - \mathcal{H}(h_{t+s}(i)) \right] \right\}
\] (2.2.1)

The household makes the decision for period \( t \) consumption one period in advance. The presence of habit formation in preferences implies that while an increase in today’s consumption increases today’s marginal utility, it leads to a fall in the marginal utility of the next period due to the presence of the \(-bC_{t-1+s}\) term in the utility function. The parameter \( b \) measures the degree of habit formation. In the standard utility function, \( b \) is taken as zero. When \( b \) is positive, the household is more intolerant to fluctuations in consumption, and thus, maintains a smoother consumption profile. It is well known that the presence of habit formation in a model results in hump-shaped dynamics for output after monetary shocks. Such dynamics are present in the VAR-based impulse response displayed in Figure 2.2.

The consumption aggregator, \( C_t \), is defined as:

\[
C_t = \left[ \sum_{j=1}^{J} n_j^{\theta_p/(\theta_p-1)} \right]^{\theta_p/(\theta_p-1)}
\] (2.2.2)

\[4\] Since each household supplies a differentiated type of labor, the hours of work supplied by each worker \( (h_{t+s}(i)) \) is indexed with \( i \) in the utility function.
where \( J \), \( n_j \) and \( \theta_p \) denote the number of sectors in the economy, the number of firms in the sector \( j \) and the elasticity of substitution between any two sectors in the economy, respectively. The aggregator consumption function for a sector, \( C_{jt} \) is defined as:

\[
C_{jt} = \left[ n_j^{-1/\theta_p} \int_0^{n_j} c_{jt}(j')^{(\theta_p-1)/\theta_p} dj' \right]^{\theta_p/(\theta_p-1)}
\]  

(2.2.3)

Here, \( c_{jt}(j') \) denotes the consumption of the differentiated good \( j' \) in sector \( j \). The elasticity of substitution of differentiated goods within sectors is \( \theta_p \).\(^5\) It is notable that the only source of asymmetry among sectors is the frequency of price changes in sectors. When the frequency of price changes is taken as equal among all sectors, the aggregator function in the economy reduces to the standard aggregator function for the one-sector model.\(^6\)

The representative household supplies a differentiated type of hours of work for firms. Let \( h_t(i) \) in (2.2.1) be the hours of work supplied by type \( i \). The function \( \mathcal{H} \) is assumed to be convex and increasing with \( h_t(i) \).

The Household’s Budget Constraint

Let \( w_t(i) \) denote the nominal wage demanded by the owner of the differentiated labor type \( i \) for an hour work. Then, the budget constraint of the household supplying that type of labor can be written as,

\(^5\)The purpose of this paper is to investigate the effects of introducing heterogeneity in the frequency of price changes among sectors on the deep parameter estimates. Thus, it is important that the only modification to the one-sector model is to drop the assumption of the same frequency of price changes in all sectors. If elasticities of substitution for the goods between the sectors in (2.2.2) and within the sectors in (2.2.3) are allowed to differ, this second modification to the one-sector model would make it impossible to isolate the effects of allowing different frequencies of price changes among sectors on the estimates of deep parameters.

\(^6\)It is notable that \( C_t \) is defined as the CES function of a finite number of sectors whereas \( C_{jt} \) is defined as the CES function of the consumption of a continuum of differentiated types of goods in a sector. In the case where I assumed a finite number of differentiated types of goods in each sector, the overall price in a sector would be affected by the price set by each firm in that sector. The continuum of differentiated types assumption is required to circumvent this complication.
\[ P_t C_t + B_{t+1} \leq w_t^S + w_t(i) h_t(i) + \sum_{j=1}^J \Pi_{jt} \]  

(2.2.4)

The household starts the period with a given wealth of \( w_t^S \). It has labor income from the supply of the hours of its differentiated labor type, \( w_t(i) h_t(i) \). In addition, the household has profit income from sectors in the economy shown by \( \sum_{j=1}^J \Pi_{jt} \) which will be specified later. The household allocates its total resources between holding a portfolio of assets and consumption. The price of the consumption is given by \( P_t \). The portfolio of assets that is acquired in period \( t \) and adds to the household’s wealth in period \( t + 1 \) is denoted by \( B_{t+1} \). Since both safe and risky assets are available in the economy, a typical portfolio may be written as:

\[
B_{t+1} = \left( \sum_{s_t^{t+1}|s_t^t} q(s^{t+1}|s^t) B_{t+1}^R (s^{t+1}|s^t) \right) + B_{t+1}^S
\]

(2.2.5)

where \( s_{t+1} \) and \( s^t \) denote the realization of the state of the world in period \( t + 1 \) and the history of the states realized until period \( t \), respectively. The risky and safe asset holdings of the household are represented by \( B_{t+1}^R \) and \( B_{t+1}^S \), respectively. Regardless of the realized state in \( t + 1 \), the safe asset pays a nominal interest of \( R_t \). Any risky asset pays one unit in nominal terms if the state is such that the risky asset is traded and pays zero otherwise. When the state in period \( t + 1 \) is realized as \( s_{t+1} \), the total \( \sum_{s_t^{t+1}|s^t} q(s^{t+1}|s^t) B_{t+1}^R (s^{t+1}|s^t) \) adds to the wealth in period \( t + 1 \) by an amount of only \( B_{t+1}^R (s_{t+1}|s^t) \). Accordingly, the wealth of the household for the next period can be written as:

\[
W_{t+1}^S (s_{t+1}|s^t) = B_{t+1}^R (s_{t+1}|s^t) + R_t B_{t+1}^S
\]

(2.2.6)

It must be emphasized that the presence of complete capital markets provides full insur-
ance for workers in an environment where they face idiosyncratic shocks. This enables the writing of a budget constraint for a representative agent in an economy with differentiated labor as noted in Woodford (2003, ch.3).

Optimal Consumption and Asset Holdings for the Household

The first-order conditions for consumption and the holding of safe and risky assets are given by (2.2.7), (2.2.8) and (2.2.9), respectively.

\[
E_{t-1} \left( \frac{\partial U(C_t - bC_{t-1})}{\partial C_t} - b\beta \frac{\partial U(C_{t+1} - bC_t)}{\partial C_t} \right) = E_{t-1}\lambda_t P_t \tag{2.2.7}
\]

\[
E_{t-1}\lambda_t = E_{t-1}\lambda_{t+1} R_t \tag{2.2.8}
\]

\[
E_{t-1}\lambda_{t+1} = E_{t-1}\lambda_t Q_{t,t+1} \tag{2.2.9}
\]

In (2.2.9), \(Q_{t,t+1}\) denotes the stochastic discount factor between the periods \(t\) and \(t+1\) when the state in period \(t+1\) is realized as \(s_{t+1}\) and is given by the ratio of the utility value of having an extra unit of money in the two periods for the given realization of the state in the period \(t+1\).

By definition, it must hold that:

\[
P_tC_t = \sum_{j=1}^{J} P_{jt} C_{jt}
\]

\[
P_{jt} C_{jt} = \int_{0}^{n_j} p_{jt}(j') c_{jt}(j') dj'
\]

where \(P_{jt}\) and \(p_{jt}(j')\) denote the price index of sector \(j\) and the price of the differentiated
good \( j' \) in the sector, respectively. Using these definitions and those in (2.2.2) and (2.2.3), the optimality conditions for \( C_{jt} \) and \( c_{jt}(j') \) can be written as:

\[
n_j \left( \frac{P_t}{P_{jt}} \right) \theta_p C_t = C_{jt} \quad (2.2.10)
\]

\[
c_{jt}(j') = n_j^{-1} \left( \frac{P_{jt}}{p_{jt}(j')} \right)^{\theta_p} C_{jt} \quad (2.2.11)
\]

Substituting these optimal conditions back into (2.2.2) and (2.2.3) gives the aggregate and sectoral price indices:

\[
P_t = \left[ \sum_{j=1}^{J} n_j P_{jt}^{1-\theta_p} \right]^{1/(1-\theta_p)} \quad (2.2.12)
\]

\[
P_{jt} = \left( n_j^{-1} \int_0^{\infty} p_{jt}(j')^{1-\theta_p} dj' \right)^{1/(1-\theta_p)} \quad (2.2.13)
\]

Note that each sectoral price index has a weight equal to the number of firms in a sector \((n_j)\) in the aggregate price index. In the next section, when the structural parameters are estimated, \( n_j \)'s are calibrated as the weights of the sectors in the CPI.

The optimality condition for hours of work will be discussed when the workers’ decision problem is considered below.

### 2.2.2 Firms

The production function of the firm producing the differentiated good \( j' \) in the sector \( j \) is given as:

\[
y_{jt}(j') = Z_t H_{jt}(j')^{\kappa} \quad 0 < \kappa < 1 \quad (2.2.14)
\]
where $Z_t$ is the technology level which is assumed to be common among all firms. $H_{jt}(j')$ denotes the firm’s demand for the hours of composite labor which will be specified below.

Each firm hires a continuum of differentiated types of labor with a mass of one. The hours worked by the continuum of differentiated types of labor is combined with the firm producing the differentiated good $j'$ in the sector $j$ to form the hours of composite labor, $H_{jt}(j')$, with the following technology:

$$H_{jt}(j') = \left( \int_0^1 h_{jt}(j', i)^{(\theta_w - 1)/\theta_w} di \right)^{\theta_w/\theta_w}$$

(2.2.15)

where $h_{jt}(j', i)$ denotes the demand for hours of work of the differentiated labor $i$ by the firm producing the differentiated good $j'$ in the sector $j$. Since each differentiated labor supplies hours of work for a continuum of firms, each firm has a negligible effect on the wage paid to the differentiated labor for hours of work, $w_t(i)$, and that to the composite labor, $W_t$, and takes $w_t(i)$ and $W_t$ as given. From the firm’s cost minimization, one can show the hours of work of each differentiated labor demanded by the firm producing the differentiated good $j'$ in the sector $j$ can be written as:

$$h_{jt}(j', i) = \left( \frac{W_t}{w_t(i)} \right)^{\theta_w} H_{jt}(j')$$

(2.2.16)

Substituting (2.2.16) in (2.2.15) gives the aggregate wage index for the composite hours:

$$W_t = \left( \int_0^1 w_t(i)^{1-\theta_w} di \right)^{\frac{1}{1-\theta_w}}$$

(2.2.17)

Let $h_t(i)$ denote the total demand for the differentiated labor $i$. One can write $h_t(i)$ as:
Before concluding this section, it is convenient to present the cost function of the producer of the \( j' \)th good in sector \( j \), \( TC_{jt}(j') \). Since the wage bill must be paid prior to production, \( TC_{jt}(j') \) is given by:

\[
TC_{jt}(j') = R_l W_t H_{jt}(j') = \left( \frac{y_{jt}(j')}{Z_t} \right)^\frac{1}{\theta} R_l W_t
\]  

(2.2.19)

### 2.2.3 Equilibrium in the Frictionless Economy

In this section, it is assumed in each period that households, firms and workers optimally decide on consumption, prices and wages, respectively. When prices are perfectly flexible, firms are able to set prices optimally in each period. The objective function of a firm can then be written as:

\[
max_{p_{jt}(j')} p_{jt}(j') y_{jt}(j') - TC_{jt}(j')
\]  

(2.2.20)

If we denote the potential outcome values of each variable with superscript \( n \), then, one can write the optimality condition for the price set by the firm under the frictionless market assumption as:

\[
y_{jt}^n(j') + p_{jt}^n(j') \frac{\partial y_{jt}^n(j')}{\partial p_{jt}^n(j')} = \frac{\partial TC_{jt}^n(j')}{\partial y_{jt}^n(j')} \frac{\partial y_{jt}^n(j')}{\partial p_{jt}^n(j')}
\]

(2.2.21)

Using (2.2.11) and the fact that output is demand determined, one can show that:

\[
p_{jt}(j')^n = \mu_p \frac{\partial TC_{jt}^n(j')}{\partial y_{jt}^n(j')}, \quad \mu_p = \frac{\theta_p}{\theta_p - 1} \geq 1
\]  

(2.2.22)
where \( \mu_p \) shows the markup over marginal output at the potential output due to product differentiation. Furthermore, using (2.2.19), one can write the marginal cost of the firm \( (S_{jt+s}(j')) \) as:

\[
S_{jt+s}(j') = \frac{\partial TC_{jt}^n(j')}{\partial y_{jt}^n(j')} = \frac{1}{\kappa} y_{jt}^n(j')^{\frac{1-\alpha}{\alpha}} Z_t^{\frac{1}{2}} R_t^n W_t^n
\]

(2.2.23)

This implies that one can write (2.2.22) as:

\[
\left( \frac{p_{jt}(j')^n}{P_{jt}^n} \right) \left( \frac{P_{jt}^n}{P_t^n} \right) = \frac{1}{\kappa} y_{jt}^n(j')^{\frac{1-\alpha}{\alpha}} Z_t^{\frac{1}{2}} R_t^n \left( \frac{W_t^n}{P_t^n} \right)
\]

(2.2.24)

Or equivalently,

\[
\left( \frac{Y_t^n}{y_{jt}(j')^n} \right)^{1/\theta_p} = \mu_p y_{jt}^n(j')^{\frac{1-\alpha}{\alpha}} Z_t^{\frac{1}{2}} R_t^n \left( \frac{W_t^n}{P_t^n} \right)
\]

(2.2.25)

Since the firms are assumed to have only negligible influence on aggregate variables, they take all the terms in (2.2.25) as given except \( y_{jt}(j')^n \). It can now be shown that the value of \( y_{jt}(j')^n \) which satisfies (2.2.25) is unique. Since the marginal cost is increasing in \( y_{jt}(j')^n \), the right-hand side is an increasing function of \( y_{jt}(j')^n \), but the left hand side of (2.2.25) is decreasing in \( y_{jt}(j')^n \). It can, thus, be concluded that \( y_{jt}(j')^n \) should be uniquely determined for given \( Y_t^n, R_t^n, W_t^n, P_t^n, \) and \( Z_t \). Consequently, the amount supplied by firms in the same sector must be equal to each other. This implies one can alternatively write \( y_{jt}(j')^n \) as \( y^n_{jt} \). Since there is a one-to-one relationship between the amount of the good supplied and the price set by firms when aggregate variables are given, the same supply of goods implies the same price set by the firms in the same sector. Thus, one can easily show from (2.2.11) that

\footnote{It is notable that the small-firms assumption is also used in writing (2.2.22).}
Furthermore, in a frictionless economy, sectors are homogeneous in every aspect. Thus, the quantity supplied by firms in different sectors must be the same since they face the same optimality condition given by (2.2.25). That is,

\[ y^n_{jt} = y^n_{j't} \text{ for } j \neq j' \]  

(2.2.27)

Hence, one can also drop the subscript \( j \) from \( y^n_{jt} \) and simply write \( y^n_t \). Lastly, using (2.2.2), (2.2.26) and (2.2.27), it is easy to show that:

\[ Y^n_t = y^n_t \]

(2.2.25) can, thus, be rewritten as:

\[ 1 = \mu_p \frac{1}{\kappa} Y^n_t \left( 1 - \kappa \right) Z^n_t \left( 1 - \kappa \right) R^n_t W^n_{\text{real},t}, \ W^n_{\text{real},t} = \frac{W^n_t}{P^n_t} \]  

(2.2.28)

where \( W^n_{\text{real},t} \) stands for the real wage paid to the composite hours of work. Let the log-deviations of a variable from its corresponding steady state be denoted with a hat over this variable. One can then write (2.2.28) in log-linearized form as:

\[ 0 = \omega_p Y^n_t - (1 + \omega_p) Z^n_t + R^n_t + W^n_{\text{real},t}, \ \omega_p = \frac{1 - \kappa}{\kappa} \]

(2.2.29)

where \( \omega_p \) is the elasticity of prices with respect to the supply of goods when the interest rate and wages paid for composite hours of work remain unchanged.

In order to show that monetary policy shocks are irrelevant in the determination of potential output, it is necessary to write each endogenous variable in (2.2.29) as a function
of potential output. First, $\hat{R}^n_t$ can be written as the log-linear form of equation (2.2.8):

$$\hat{R}^n_t = \varphi^{-1} \hat{x}_t - \varphi^{-1} E_t \hat{\pi}_{t+1} + \varphi^{-1} E_t \hat{\pi}_{t+1}$$

(2.2.30)

where $\hat{x}_t$ is given by:

$$\hat{x}_t = \left( \hat{Y}_t - b \hat{Y}_{t-1} \right) - b \beta E_t \left( \hat{Y}_{t+1} - b \hat{Y}_t \right)$$

(2.2.31)

and $\varphi$ represents the intertemporal elasticity of substitution given by:

$$\varphi = \sigma_u (1 - \beta b), \quad \varphi > 0$$

(2.2.32)

In the absence of habit formation, the intertemporal elasticity of substitution would be equal to $\sigma_u$ where:

$$\sigma_u = \frac{U_c}{U_c Y}$$

(2.2.33)

When there is habit formation in preference, the intertemporal elasticity of substitution is modified and given by $\varphi$ in (2.2.32).

In writing (2.2.30), steady state prices are normalized to one and price inflation is defined as $\pi_{t+1} = \log P_{t+1} - \log P_t$. In the frictionless-economy case, there is a Nash equilibrium with future price expectations equal to current prices. In this case, the percentage change in the price of future consumption relative to current consumption is given by $\sigma_u$. Consider preferences without habit formation. It can be shown that

$$\frac{\partial U_c}{U_c} = \frac{U_c(C_{t+1}) - U_c(C_t)}{U_c(C_t)} = \frac{U_c(C_{t+1})}{U_c(C_t)} - 1 = \frac{P_{t+1}}{P_t} - 1$$

Thus, $\frac{\partial U_c}{U_c}$ shows the percentage change in the price of next period’s consumption relative to that of today’s consumption. Hence, $\sigma_u$ indicates the percentage fall in next period’s consumption when the relative price of next period’s consumption increases by 1%. Hence, it must hold that $\sigma_u > 0$. 

---

8 (2.2.33) can be rewritten as:

$$\sigma_u = - \frac{\partial U_c}{U_c Y}$$

Consider preferences without habit formation. It can be shown that

Thus, $\frac{\partial U_c}{U_c}$ shows the percentage change in the price of next period’s consumption relative to that of today’s consumption. Hence, $\sigma_u$ indicates the percentage fall in next period’s consumption when the relative price of next period’s consumption increases by 1%. Hence, it must hold that $\sigma_u > 0$. 

20
equilibrium in which all prices respond fully and instantaneously to an aggregate shock. In other words, following an aggregate shock to the economy, prices reach their new levels immediately and remain there \( E_t \pi_{t+1}^n = 0 \). This suggests that the last term in (2.2.30) vanishes.

The last variable in (2.2.29), whose log-deviation from its steady state needs to be expressed as a function of the log-linearized output gap, is \( W_{real,t}^n \). To do this, I first specify the wage-setting environment. Workers have monopsonistic power over the hours they work. Hence, once they set their hourly wage, they are required to supply labor to satisfy all the demand from the aggregate output-producing firm. When workers are able to optimally set wages each period, the optimality condition for the wage set by the owner of the differentiated labor type \( i \) for hourly work \( w_t(i)^n \) is given by (2.2.34):

\[
\mathcal{H}^n_h \frac{\partial h_t(i)^n}{\partial w_t(i)^n} = \lambda_t^n \left( h_t(i)^n + w_t(i)^n \frac{\partial h_t(i)^n}{\partial w_t(i)^n} \right)
\]

Using (2.2.34) and (2.2.18), one can show that:

\[
w_t(i)^n = \mu_w \frac{\mathcal{H}^n_h}{\lambda_t^n}, \quad \mu_w = \frac{\theta_w}{\theta_w - 1}
\]

where \( \mu_w \) shows the markup over the marginal cost of supplying more hours imposed by the differentiated worker when setting his wage. To explain (2.2.35), note that \( \mathcal{H}_h \) is the utility cost of working an extra hour for the differentiated labor type \( i \). Dividing this by \( \lambda_t \), the utility gain of having an extra unit of money in period \( t \), results in a measure of the worker’s marginal cost in money terms. Hence, a worker’s wage is set by imposing a markup over the marginal cost of working an extra hour, where the marginal cost of working is expressed in money terms. When labor is not differentiated, workers demand the perfectly-competitive wage which is equal to the marginal cost of working an extra
hour without any markup over this cost.

Using (2.2.7) and (2.2.18), one can alternatively write (2.2.35) as:

\[ W^n_{\text{real},t} \left( \frac{H^n_t}{h_t(i)^n} \right)^{1/\sigma_w} = \mu_w E_t \left( \frac{\partial U(\nu)}{\partial Y^n_t} \right) + \beta b \frac{\partial U(Y^n_{t+1})}{\partial Y^n_t} \]

(2.2.36)

For given values of \( W^n_{\text{real},t}, h^n_t \) and \( Y^n_t \), it can be shown that the left-hand side of the above equation is a decreasing function of \( h_t(i)^n \), whereas the right-hand side is an increasing function of of the same variable. Thus, \( h_t(i)^n \) must be uniquely determined from this equation. Under the flexible-wages assumption, \( h_t(i)^n \) is the same for all differentiated labor types since all labor suppliers are alike. This implies \( h^n_t(i) = H^n_t \) and \( w^n_t(i) = W^n_t \).

Taking the log-linear approximation of (2.2.36) yields that:

\[ \hat{W}^n_{\text{real},t} = \sigma^{-1}_H \hat{H}^n_t + \varphi^{-1} \hat{x}^n_t \]

(2.2.37)

where \( \sigma_H \) denotes the elasticity of the number of hours with respect to real wage changes when the marginal utility of real income is constant (i.e., the Frisch-elasticity of labor supply). It is given as:

\[ \sigma_H = \frac{\mathcal{H}_h}{\mathcal{H}_{hh} H} > 0 \]

(2.2.38)

Using (2.2.14) and (2.2.37), it is easy to show that:

\[ \hat{W}^n_{\text{real},t} = \omega_w (\hat{Y}^n_t - \hat{Z}_t) + \varphi^{-1} E_t \hat{x}^n_t, \quad \omega_w = \frac{\sigma^{-1}_H}{\kappa} \]

(2.2.39)

where \( \omega_w \) denotes the output elasticity of the real wage for a constant marginal utility of real income.

---

9The right-hand side is increasing in \( h_t(i)^n \) as \( \mathcal{H} \) is assumed to be increasing and convex in \( h_t(i)^n \).
When \((2.2.30)\) and \((2.2.39)\) are substituted into \((2.2.29)\), one obtains:

\[
0 = \left( \omega_w + \omega_p \right) \hat{Y}_t^n - \left( \omega_w + 1 + \omega_p \right) \hat{Z}_t + \varphi^{-1} E_t \hat{x}_{t+1}^n \tag{2.2.40}
\]

Thus, monetary policy shocks are irrelevant as they do not appear in the equilibrium value of \(\hat{Y}_t^n\) in the frictionless economy which is determined by equation \(2.2.40\).

2.2.4 The Economy with Nominal Frictions

Sticky Prices

It is assumed that firms set prices one period in advance. In each sector, firms optimize their prices only when a price-change signal is received. The probability of receiving such a signal is different in each sector and is given by \(1 - \alpha_{pj}\) for sector \(j\). The fraction of firms in sector \(j\) which receive a price change signal in each period is also given by \(1 - \alpha_{pj}\).

When no such a signal is received, firms are assumed to set their prices according to the following partial adjustment backward-looking indexation rule:

\[
\tilde{p}_{jt}(j') = p_{jt-1}(j') \cdot \left( \frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_p} \tag{2.2.41}
\]

The tilde over \(p\) denotes the price set according to the backward-looking indexation rule. As noted by Woodford (2003), such a rule helps to explain that the peak effect of an interest rate shock on inflation occurs later than the peak effect of the shock on the output gap in the VAR.

When a firm is capable of setting an optimal price, it sets \(p_{jt}(j')^*\) to maximize

\[
E_{t-1} \left( \sum_{s=0}^{\infty} \alpha_{pj}^s Q_{t,t+s} \Pi_{jt+s}(j') \right) \tag{2.2.42}
\]

where \(Q_{t,t+s}\) is the stochastic discount factor between period \(t\) and \(t + s\) and is given...
by:

$$E_{t-1} \left( \frac{\lambda_{t+s}}{\lambda_s} \right) = E_{t-1}(Q_{t,t+s})$$  \hfill (2.2.43)

(2.2.42) implies that firms and households have the same stochastic discount factor.

The profit of firm $i$ in sector $j$ is given by:

$$\Pi_{jt+s}(j') = p^*_{jt+s,t} (i) y_{jt+s}(j') - TC_{jt+s}(j')$$

where $TC_{jt}(j')$ is the total cost of the firm, as given in (2.2.19), and $p^*_{jt+s,t}(j')$ is the price set in period $t + s$ by the firm that received a price-change signal in period $t$ and which does not have the opportunity to set an optimal price between $t$ and $t + s$. Due to the backward-indexation rule, one can write $p^*_{jt+s,t}(j')$ as:

$$p^*_{jt+s,t}(j') = p^*_{jt}(j') \chi^p_{t,t+s}$$  \hfill (2.2.44)

where

$$\chi^p_{t,t+s} = \begin{cases} 
\Pi^s_{k=1} \left( \frac{P_{t+k-1}}{P_{t+k-2}} \right)^{\gamma^p} & \text{if } s \geq 1 \\
1 & \text{if } s = 0 
\end{cases}$$  \hfill (2.2.45)

The optimality condition in (2.2.42) for $p^*_{jt}(j')$ can be expressed as:

$$E_{t-1} \left( \sum_{s=0}^{\infty} \alpha^s_{pj} Q_{t,t+s} \frac{d\Pi_{jt+s}(j')}{dp^*_{jt}(j')} \right) = 0$$  \hfill (2.2.46)

From this, one can show that (2.2.47) is true:

---

10It is notable that since firms are assumed to respond to monetary shocks with a one period delay, they have to condition their optimum price based on information until period $t - 1$ rather than period $t$. Correspondingly, $E_{t-1}$ appears in the objective function in (2.2.42), rather than $E_t$.  

24
and log-linearizing this equation yields:

$$E_{t-1} \sum_{s=0}^{\infty} \alpha_{pj}^s Q_{t+s} \chi_{t+s}^p \left( \frac{P_{t+s}}{P_{j}^{t+s}} \right)^{1+\theta_p} Y_{t+s} \times$$

$$\left( \frac{P_{j}^{t+s}(j')}{P_{jt+s}} - \mu_p \frac{S_{jt+s}(j')}{P_{jt+s}} \right) = 0$$  \hspace{1cm} (2.2.47)

and log-linearizing this equation yields:

$$E_{t-1} \sum_{s=0}^{\infty} \beta \alpha_{pj}^s \left[ \hat{p}_{jt}^s(j') - \hat{P}_{jt+s} + \hat{P}_{jt+s} - \hat{P}_{jt+s} + \chi_{t,s}^p \right]$$

$$- \left( \hat{R}_{t+s} + \hat{W}_{real,t+s} + \left( \omega_p \hat{y}_{jt+s}(j') - (1 + \omega_p) \hat{Z}_{t+s} \right) \right) = 0$$  \hspace{1cm} (2.2.48)

where $\chi_{t,s}^p$ is the log-deviation of $\chi_{t,s}^p$ from its steady state given by:

$$\chi_{t,s}^p = \begin{cases} 
\gamma_p \pi_t + \gamma_p \pi_t + 1 + \cdots + \gamma_p \pi_{t+s-1} & \text{if } s \geq 1 \\
0 & \text{if } s = 0 
\end{cases} \hspace{1cm} (2.2.49)$$

In Appendix [A.1] the dynamic equation for sectoral inflation ($\pi_{jt} = \hat{P}_{jt} - \hat{P}_{jt-1}$) is shown to evolve according to

$$\pi_{jt} - \gamma_p \pi_{t-1} = -\xi_{pj} (1 + \omega_p \theta_p) E_{t-1} \left( \hat{P}_{jt} - \hat{P}_t \right)$$

$$+ \xi_{pj} E_{t-1} \left( \hat{R}_t + \hat{W}_{real,t} + \left( \omega_p \hat{y}_t - (1 + \omega_p) \hat{Z}_t \right) \right) + \beta E_{t-1} \left( \pi_{jt+1} - \gamma_p \pi_t \right)$$  \hspace{1cm} (2.2.50)

where $\xi_{pj}$ is given by:

$$\xi_{pj} = \frac{1 - \alpha_{pj}}{\alpha_{pj} (1 + \omega_p \theta_p)} \hspace{1cm} (2.2.51)$$

Hence, sectoral inflation is a decreasing function of relative sectoral prices\footnote{This can be explained as follows, if the relative price of a sector was low in one period, the demand for its good would be high. This would induce firms in that sector to raise prices since their marginal cost is} an increas-
ing function of the log-deviation of real marginal cost from its potential output level and an increasing function of expected sectoral inflation in the next period.

It is notable that $\xi_{pj}$ is decreasing and convex in $\alpha_{pj}$, implying other things being equal, the relative sectoral price today matters most for the sector where prices change most frequently. This can be explained as follows: firms in sectors with a low frequency of price changes expect not to reoptimize their prices for a considerable period of time. One may argue that this leads them to more often consider relative prices in subsequent periods compared to firms in sectors where they are able to optimize their prices frequently. This leads the less-frequently optimizing firms to place greater weight on relative prices in subsequent periods and to place lower weight on relative prices today.

It should be noted that the one-sector model differs from the multi-sector model only in terms of the equation governing the aggregate inflation. When sectors are homogeneous in their price flexibility, (2.2.50) reduces to:

$$\pi_t - \gamma_p \pi_{t-1} = \xi_p E_{t-1} \left( \tilde{R}_t + \tilde{W}_{real,t} + \left( \omega_p \tilde{Y}_t - (1 + \omega_p) \tilde{Z}_t \right) \right) + \beta E_{t-1} \left( \pi_{t+1} - \gamma_p \pi_t \right)$$

(2.2.52)

where $\xi_p$ is given by[12]

$$\xi_p = \frac{1 - \alpha_p}{\alpha_p} \frac{1 - \beta \alpha_p}{1 + \omega_p \theta_p}$$

**Sticky Wages for Hours of Work**

The hours of work are differentiated among suppliers. There is a continuum of differentiated suppliers of hours of work in the economy with a mass of one. The owner of each type

convex and increasing in the output they supply.

[12] In (2.2.51), in place of $1 - \alpha_{jp}$, the weighted average of the frequency of price changes in sectors, which I denote with $(1 - \alpha_p)$, is used when writing $\xi_p$ above.
sets an hourly wage in the monopsonistically competitive market and is prepared to supply the number of hours demanded at this wage. Each supplier has a chance to optimize his wages only when a wage-change signal is received. The probability of receiving such signal is given by $1 - \alpha_w$. The fraction of the types receiving the signal is also given by $1 - \alpha_w$.

When such a signal is not received, workers set their wages by taking into account the change in inflation in the last period:

$$\bar{w}_t(i) = w_{t-1}(i) \left( \frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} \tag{2.2.53}$$

When the wage-change signal is received, the objective of the owner of the differentiated labor type $i$ is to set an hourly wage, $w_t(i)^*$, which lasts until the new wage-change signal is received. Indeed, the problem can be expressed as follows:

$$\max_{w_t(i)^*} E_{t-1} \left\{ \sum_{s=0}^{\infty} \alpha_w^s \left( -\beta^s \mathcal{H}(h_{t+s}(i)) + \lambda_{t+s} w_{t+s,t}^*(i) h_{t+s}(i) \right) \right\} \tag{2.2.54}$$

where $w_{t+s,t}^*(i)$ shows the wage set in period $t+s$ by the supplier of hours of differentiated labor type $i$ who optimized his wage at period $t$ and is unable to reoptimize between period $t$ and $t+s$, respectively. From (2.2.53), $w_{t+s,t}^*(i)$ is given as:

$$w_{t+s,t}^*(i) = w_t^*(i) \chi_{t,t+s}^{w} \tag{2.2.55}$$

where

\footnotesize
\begin{itemize}
\item Since the nominal wages are set one period in advance, the expectation operator is taken as $E_{t-1}$ in (2.2.54).
\end{itemize}

\normalsize
\[ \chi_{t,t+s}^w = \begin{cases} 
\prod_{k' = 1}^{s} \left( \frac{P_{t+k'-1}}{P_{t+k'-2}} \right)^{\gamma_w} & \text{if } s \geq 1 \\
1 & \text{if } s = 0 \end{cases} \] (2.2.56)

In Appendix A.4, the wage inflation for the composite hours of work is shown to evolve according to (2.2.57):

\[ \pi_w^t - \gamma_w \pi_{t-1} = \xi_w E_{t-1} \left( \frac{\sigma^{-1}_w}{K} (\hat{Y}_t - \hat{Z}_t) + \varphi^{-1} \hat{x}_t - \hat{W}_{real,t} \right) + \beta E_{t-1} \left( \pi_{t+1}^w - \gamma_w \pi_t \right) \] (2.2.57)

where \( \pi_t^w \) and \( \xi_w \) are defined as

\[ \pi_t^w = \log W_t - \log W_{t-1} \]

\[ \xi_w = \frac{(1 - \alpha_w)}{\alpha_w} \left( 1 - \alpha_w \beta \right) \frac{1}{1 + \theta_w \sigma^{-1}_w} \]

Hence, the wage inflation of hours worked is an increasing function of output and expected wage inflation and a decreasing function of the marginal utility from real income.

Lastly, it needs to be noted that the nominal wage for hours worked falls with the real wage paid per hour of composite labor.

### 2.2.5 The IS Equation

Log-linearizing (2.2.7) and (2.2.8) yields that:

\[ E_{t-1} \hat{x}_t = E_{t-1} \hat{x}_{t+1} - \varphi E_{t-1} \left( \hat{R}_t - \pi_{t+1} \right) \] (2.2.58)

where \( \hat{x}_t \) is defined in (2.2.31). This equation, (2.2.58), implies the presence of the
working capital channel produces a higher response of output to an expansionary shock during the periods directly following the shock. To see this, consider first the simple case without habit formation so that equation (2.2.58) reduces to a dynamic equation for real GDP. Since the working capital channel leads firms to set lower prices after an unanticipated fall in the interest rate, it can be seen from (2.2.58) that lower inflation with the working capital channel produces a larger increase in output due to the expansionary shock for given values of the model’s parameters and expected output in the next period. With habit formation in preferences, on the other hand, as a household avoids large changes in its consumption pattern, the effect of the working capital channel on output is likely to lessen. However, the model would still produce a larger output increase with the working capital channel.

2.2.6 Monetary Policy

The recursive assumption that is used to identify monetary policy shocks in the VAR above implies the current values of the output gap, inflation and real wage are predetermined. With this assumption, the OLS estimates become consistent. Using this and the fact that four lags of each variable are included in identifying the monetary policy shocks in the VAR model, I follow the standard practice and characterize the monetary policy with the following Taylor rule:

\[
R_t = c + \sum_{j=0}^{4} \phi^j_y (Y_{t-j} - Y_{t-j}^m) + \sum_{j=0}^{4} \phi^j_\pi \pi_{t-j} + \sum_{j=0}^{4} \phi^j_w W_{real,t-j} + \sum_{j=1}^{4} \phi^j_R R_{t-j} + \epsilon^m_t \tag{2.2.59}
\]

where \(R_t\) denotes the federal funds rate and \(c\) is a constant term which denotes the steady-state interest rate. \(\phi^j_y\), \(\phi^j_\pi\) and \(\phi^j_w\) show the response of the federal funds rate to the lagged \(j\) values of the output gap, price inflation and real wage, respectively. The response
### Table 2.1: Estimated Monetary Policy Rule: 1959Q1-2013Q1

<table>
<thead>
<tr>
<th>$j$</th>
<th>$\phi^y_j$</th>
<th>$\phi^\pi_j$</th>
<th>$\phi^w_j$</th>
<th>$\phi^R_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.36</td>
<td>0.06</td>
<td>-0.11</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.32</td>
<td>0.04</td>
<td>0.27</td>
<td>1.05</td>
</tr>
<tr>
<td>2</td>
<td>0.01</td>
<td>0.16</td>
<td>-0.11</td>
<td>-0.31</td>
</tr>
<tr>
<td>3</td>
<td>-0.01</td>
<td>-0.07</td>
<td>0.05</td>
<td>0.32</td>
</tr>
<tr>
<td>4</td>
<td>0.02</td>
<td>-0.10</td>
<td>-0.08</td>
<td>-0.11</td>
</tr>
</tbody>
</table>

**Note:** For the definitions of $\phi^y_j$, $\phi^\pi_j$, $\phi^w_j$ and $\phi^R_j$, see (2.2.59).

of the federal funds rate to its own lags are denoted by $\phi^R_j$. Lastly, $\epsilon^m_t$ is the monetary shock in the period which is introduced by the Federal Reserve and is uncorrelated across time and orthogonal to the explanatory variables of the policy rule. In Table 2.1 I report the estimates of $\phi^y_j$, $\phi^\pi_j$, $\phi^w_j$ and $\phi^R_j$.

### 2.3 Econometric Estimation

Let $g(\mathcal{P})$ and $f(\mathcal{P})$ be the model-based impulse responses for a given vector of model parameters ($\mathcal{P}$) in the one- and multi-sector models, respectively. The estimated vector of model parameters in the multi-sector model ($\mathcal{P}^M(\hat{A}_n)$) is given as the minimizer of the following classical minimum distance measure:

$$
\hat{\mathcal{P}}^M(\hat{A}_n) = \arg \min_{\mathcal{P}} (\hat{h}_n - f(\mathcal{P}))' \hat{A}_n' \hat{A}_n (\hat{h}_n - f(\mathcal{P}))
$$

where $\hat{A}_n$ and $\hat{h}_n$ are the weighting matrix used and the estimated VAR-based im-
pulse response of the output gap, inflation, the real wage and the federal funds rate to an unanticipated 1% reduction in the federal funds rate between the 1st and 20th quarters, respectively. Lastly, \( n \) stands for the sample size of the data used to estimate the VAR-based impulse responses. Since using different weighting matrices would yield a different estimator, \( \hat{P}^M \) is written as a function of \( \hat{A}_n \). Similarly, the estimated vector of parameters in the one-sector model \( \hat{P}^O(\hat{A}_n) \) are given as the minimizer of the following classical minimum distance measure:

\[
\hat{P}^O(\hat{A}_n) = \arg \min_p (\hat{h}_n - g(p))' \hat{A}_n' \hat{A}_n (\hat{h}_n - g(p)) \quad (2.3.2)
\]

As a weighting matrix, I use the diagonal matrix \( \hat{A}_n \) where the diagonal elements are given by the inverse of the standard deviations of the VAR-based impulse responses.\(^{14}\) This weighting matrix ensures that the deep parameters are estimated such that the more-precisely estimated impulse responses are given more importance.

### 2.3.1 Calibrated Parameters of the Model

There is information on price flexibility for approximately 270 ELI categories in Nakamura & Steinsson (2008a). Even if it is ideal to use all information about frequencies of price change for ELI in the CPI documented in Nakamura & Steinsson (2008a), adding such a large number of sectors to the model results in an excessive number of variables in the dynamic system in the DSGE model which is beyond the computation limit of today’s technology. As a practical solution, the number of sectors to be included in the multi-
Table 2.2: The Quarterly Frequencies of Price Adjustment over Different Percentiles of the Price Flexibility and Their Implied Durations

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>Frequency (%)</th>
<th>Duration (Quarters)</th>
<th>Weights (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-Sector Economy</td>
<td>40.0</td>
<td>1.95</td>
<td>100.0</td>
</tr>
<tr>
<td>Multi-Sector Economy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-10</td>
<td>10.0</td>
<td>9.49</td>
<td>14.9</td>
</tr>
<tr>
<td>10-20</td>
<td>19.8</td>
<td>4.53</td>
<td>18.4</td>
</tr>
<tr>
<td>20-30</td>
<td>28.8</td>
<td>2.94</td>
<td>7.0</td>
</tr>
<tr>
<td>30-40</td>
<td>37.5</td>
<td>2.13</td>
<td>6.1</td>
</tr>
<tr>
<td>40-50</td>
<td>45.6</td>
<td>1.64</td>
<td>4.2</td>
</tr>
<tr>
<td>50-60</td>
<td>53.5</td>
<td>1.30</td>
<td>4.6</td>
</tr>
<tr>
<td>60-70</td>
<td>59.4</td>
<td>1.11</td>
<td>5.3</td>
</tr>
<tr>
<td>70-80</td>
<td>67.3</td>
<td>0.89</td>
<td>11.6</td>
</tr>
<tr>
<td>80-90</td>
<td>74.9</td>
<td>0.72</td>
<td>9.0</td>
</tr>
<tr>
<td>90-100</td>
<td>90.9</td>
<td>0.42</td>
<td>19.0</td>
</tr>
</tbody>
</table>

Note: Weights indicates the weights \((n_j)\) in the 2000 CPI expenditures of Entry Level Items (ELIs) given in Nakamura & Steinsson (2008a). Frequency refers to the percentage of firms which adjust their prices in a quarter. The frequencies in the multi-sector economy above show the median of the quarterly frequencies for different percentiles of price flexibility. The median implied duration in quarters for percentiles of price flexibility in the multi-sector economy are computed from the median frequencies with the formula \((-\frac{1}{\ln(1-f^q)})\) where \(f^q\) refers to the median quarterly frequencies in each percentile. The frequencies in the one-sector economy \((f^q_{\text{One}})\) above show the weighted average quarterly frequencies defined as \((f^q_{\text{One}} = \sum_{j=1}^{J} n_j f^q_j)\) where \(f^q_j\) denotes the estimated quarterly frequency of ELIs. Lastly, the duration of price contracts in the one-sector economy is estimated as \((-\frac{1}{\ln(1-f^q_{\text{One}})})\).

The sector model is reduced to 10 in the following way. First, sectors are ordered according to their quarterly frequency of price changes from lowest to highest using the data given in Nakamura & Steinsson (2008a). Secondly, 10 percentile groups of price flexibility in the ELI categories are formed and all ELI categories are included in one of these ten groups. Table
2.2 gives some descriptive statistics about the price flexibility of the percentile groups.

It must be noted that the frequencies of price changes in Nakamura & Steinsson (2008a) are expressed as monthly percentages for the ELIs. Estimating quarterly frequencies involves two steps. First, the expected length of a price quotation in a sector \((d_k)\) in Nakamura & Steinsson (2008a) is estimated as \(\frac{1}{\ln(1-f^m_k)}\) where \(f^m_k\) denotes the monthly frequency of price changes in sector \(k\). Put differently, \(f^m_k\) can be rewritten as:

\[
f^m_k = 1 - e^{-\frac{1}{d_k}}
\]

To relate \(f^m_k\) to the Calvo (1983) model, let \(\delta_k\) be the constant hazard rate in sector \(k\) in the Calvo (1983) model. Note that of the total number of firms which set prices in period \(s < t\), a share,

\[
1 - e^{-\delta_k(t-s)}
\]

will again have received a price change signal between \(s\) and \(t\) in the Calvo (1983) model. If the unit of time is taken as a month \((t-s=1)\) and \(\delta_k\) is set equal to \(\frac{1}{d_k}\), one can relate the frequencies of price changes in the Calvo (1983) model to those reported in Nakamura & Steinsson (2008a).

Second, I estimate the quarterly frequency of price changes in the Calvo (1983) model \((f^q_j)\) by

\[
f^q_j = 1 - e^{-3\times\delta_k}
\]

where \(t-s\) in (2.3.3) is assumed to be three since unlike Nakamura & Steinsson (2008a) who estimate monthly frequencies, I estimate three-month (quarterly) frequencies.

In the one-sector economy, the weighted average of the quarterly frequency of price adjustment in the overall economy is estimated as \(\sum_{j=1}^J n_j f^q_j\) where \(n_j\)s are the shares in
Table 2.3: The Calibrated Parameters

<table>
<thead>
<tr>
<th>One-Sector Model</th>
<th>Multi-Sector Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 0.99$</td>
<td>$\beta = 0.99$</td>
</tr>
<tr>
<td>$\alpha_w = 0.83$</td>
<td>$\alpha_w = 0.83$</td>
</tr>
<tr>
<td>$\alpha_p = 60.0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sectors</td>
</tr>
<tr>
<td>Sector 1</td>
<td>$\alpha_{p1} = 0.90$</td>
</tr>
<tr>
<td>Sector 2</td>
<td>$\alpha_{p2} = 0.80$</td>
</tr>
<tr>
<td>Sector 3</td>
<td>$\alpha_{p3} = 0.71$</td>
</tr>
<tr>
<td>Sector 4</td>
<td>$\alpha_{p4} = 0.62$</td>
</tr>
<tr>
<td>Sector 5</td>
<td>$\alpha_{p5} = 0.54$</td>
</tr>
<tr>
<td>Sector 6</td>
<td>$\alpha_{p6} = 0.46$</td>
</tr>
<tr>
<td>Sector 7</td>
<td>$\alpha_{p7} = 0.40$</td>
</tr>
<tr>
<td>Sector 8</td>
<td>$\alpha_{p8} = 0.33$</td>
</tr>
<tr>
<td>Sector 9</td>
<td>$\alpha_{p9} = 0.25$</td>
</tr>
<tr>
<td>Sector 10</td>
<td>$\alpha_{p10} = 0.09$</td>
</tr>
</tbody>
</table>

the 2000 CPI expenditures of ELIs reported in Nakamura & Steinsson (2008a) and $f_j^q$ is measured using (2.3.4). The estimated value is about 40%. The implied duration indicates a typical price contract in the United States lasts less than 1.95 quarters. Such duration is lower than the value of the calibrated duration of price contracts in Giannoni & Woodford (2003). The calibrated duration in their paper is based on previous survey studies, such
as Blinder, Canetti, Lebow & Rudd (1998), which are not in line with the recent findings on price changes of ELI categories. Weights in the table correspond to the weights of the percentile groups computed using the weights given in Nakamura & Steinsson (2008a). The calibrated values for sectoral price rigidities ($\alpha_{pj}$s) in Table (2.3) correspond to $1 - f^q$, where $f^q$ shows the frequency of quarterly price adjustments for the percentiles in Table 2.2. It is evident in this table that the frequencies of price adjustment differ significantly among the percentile groups. More interestingly, the weights of the middle percentiles are generally less than those of the top and bottom percentiles. Thus, for the evolution of overall prices, it is more relevant to study the price dynamics in the top and the bottom percentiles than those in the middle percentiles.

Other calibrated parameters for the one- and multi-sector models in Table (2.3) are $\beta$ and $\alpha_w$. The calibrated value for $\beta$ is 0.99, which implies an annualized net real interest rate of 1%. The calibrated value for $\alpha_w$ is 0.83, which indicates an expected duration of 5.6 quarters for a typical wage contract as found in Barattieri et al. (2010).

The sectors’ frequencies still differ from their original frequencies in the multi-sector model since the frequencies in the same percentile group are equated to the median frequency of the group. However, the disparities in the sectoral frequencies are much higher in the one-sector economy where the frequencies of price changes for all sectors are equated to the weighted average of the frequency of price changes in the aggregate economy. In this

---

$^{15}$It is notable that were the correlation between the weights and the frequencies of price changes in sectors a strongly positive (negative), heterogeneity in price flexibility across sectors would only play a small effect on the aggregate price dynamics as it would result in such dynamics being largely determined by fast-adjusting (slow-adjusting) sectors and that slow-adjusting (fast-adjusting) sectors would have small effects on such dynamics. However, the correlation between sectoral weights and sectoral frequencies in Table 2.2 is almost zero. Hence, there is no a priori reason to expect that sectoral heterogeneity in price flexibility may only have trivial effects on the aggregate dynamics.

$^{16}$Assuming wage rigidity is the same across sectors in the multi-sector model may be of some concern since price dynamics across sectors following a monetary shock under this assumption may significantly differ from those under different wage rigidities across sectors. However, this concern is unwarranted since Barattieri et al. (2010) find little evidence of heterogeneity in the frequency of wage adjustment across industries in the United States.
respect, the multi-sector economy is much closer to reality than the one-sector economy.

2.3.2 Estimated and Implied Parameters

The calibrated parameters above reduce the parameters of the model to be estimated. In addition to these, one can further reduce the number of parameters to be estimated in the following way. Labor has a share of 0.66 at the steady state, where the price set by the firm producing the aggregate composite good must satisfy (2.2.22). Using this fact and (2.2.19), it is easy to show that the steady-state share of labor is given by:

\[
\frac{1}{\kappa R\mu p} = \frac{\kappa}{\mu p^{\beta-1}} = 0.66
\]

Thus, it must hold that:

\[
\kappa = 0.66 \frac{\theta_p}{\theta_p - 1} \beta^{-1}
\]

Hence, with the calibrated value of \( \beta \), \( \kappa \) can be obtained from (2.3.6) when \( \theta_p \) is estimated, and thus, it can be omitted from the list of variables which should be estimated.

The list of parameters (\( P \)) which need to be estimated includes \( P = [\varphi, \sigma, \theta_p, \theta_w, b, \gamma_p, \gamma_w] \).

Table 2.4 gives the estimated structural parameters for the one- and multi-sector models. The estimates of the intertemporal elasticity of substitution (IES) (\( \varphi \)) is quite close to zero. Although, there is contrasting findings regarding IES, the estimated values in the models are largely consistent with the empirical estimates of IESs in the literature.\(^{17}\) The

\(^{17}\) Apart from theoretical restrictions, the parameters are not restricted: \( \varphi > 0, \sigma > 0, \theta_p > 1, \theta_w > 1, 1 \geq b \geq 0, 1 \geq \gamma_p > 0 \) and \( 1 \geq \gamma_w > 0 \).

\(^{18}\) Hall (1988) estimates IES for the post-war period in the United States and finds that IES can be as low as 0.1 and may well be equal to zero. Hall (1988) attributes the substantial IES estimate reported in Summers (1984) for the post-war period to the invalid instrument used in the latter study. When a list of valid instruments is used for expected real returns, IES is estimated to be barely positive and not significantly different from zero. Similarly, the low values of IES given in Table 2.4 are also consistent with the empirical estimate of IES in Boldrin, Christiano & Fisher (2001) (0.09) for the sample period of 1964-I
Table 2.4: Estimates of Structural Parameters

<table>
<thead>
<tr>
<th></th>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>One-Sector Model</td>
<td>Multi-Sector Model</td>
</tr>
<tr>
<td>ϕ</td>
<td>0.011 (0.009)</td>
<td>0.011 (0.009)</td>
</tr>
<tr>
<td>σ_\text{H}</td>
<td>0.066 (0.645*)</td>
<td>0.001 (0.762*)</td>
</tr>
<tr>
<td>θ_\text{p}</td>
<td>67.232 (82.695*)</td>
<td>19.621 (20.948*)</td>
</tr>
<tr>
<td>θ_\text{w}</td>
<td>5.695 (6.453*)</td>
<td>5.192 (11.755*)</td>
</tr>
<tr>
<td>b</td>
<td>0.867 (0.249*)</td>
<td>0.886 (0.280*)</td>
</tr>
<tr>
<td>γ_\text{w}</td>
<td>0.999 (0.226*)</td>
<td>0.999 (0.184*)</td>
</tr>
<tr>
<td>γ_\text{p}</td>
<td>0.946 (0.120*)</td>
<td>0.925 (0.120*)</td>
</tr>
<tr>
<td><strong>Obj. Func.</strong></td>
<td><strong>14.267</strong></td>
<td><strong>11.795</strong></td>
</tr>
</tbody>
</table>

**Note:** The numbers in parentheses are standard errors. The asterisks next to the reported standard errors indicate the estimated parameter values are close to their theoretical limits, and thus, their standard errors may be unreliable.

estimated values of the Frisch-elasticity of labor supply (σ_\text{H}) in Table 2.4 are also close to zero. These values lie in the range of the estimates of the Frisch-elasticity of labor supply and 1988-II as it lies within one standard deviation of the point estimate. However, the estimated values for IES in Table 2.4 are not in line with those in Beaudry & van Wincoop (1996). Indeed, they estimate the value of IES to be close to one for non-durable consumption by using a panel of state-level data. However, the estimates for the entire United States economy are substantially lower even though they are found to be positive and statistically different from zero.
in the published literature reported in Browning, Hansen & Heckman (1999). Regarding the estimates of \( b \), \( \gamma_w \) and \( \gamma_p \), it can be noted that habit persistence in consumption and backward-looking indexation in both wages and prices are large, which are also in line with the estimated values in previous studies (See, for example, Giannoni & Woodford (2003)).

Table 2.4 also shows that the one- and multi-sector models differ to some extent in their capacity to account for the dynamics after the shock. The lower objective function value for the multi-sector model indicates that this model is, to some extent, more successful than the one-sector model in approximating the dynamic behavior of the variables after
the shock.

Three points should be noted about the estimates of implied parameters in Table 2.5. First, the values for $\mu_w$ and $\mu_p$ indicate the markups in wage and price settings are low. Second, $\omega_w = \frac{\sigma\kappa^{-1}}{\kappa}$ has a very large value. This suggests that when wages and prices can be set optimally in each period, an increase in output results in an unduly large increase in real wages because of a very low estimated value of the Frisch-elasticity of labor supply. However, given that wages are staggered and workers must supply any amount of hours demanded by firms at the set wages, the dynamic behavior of wages in the model is within the 95% confidence bands, despite a rise of a 0.5% in output following the shock. Third, the arithmetic mean of the slope coefficients in the inflation equations in the multi-sector model ($\xi_{pj}$) is much higher than the slope coefficient in the one-sector model ($\xi_p$) due to the fact that $\xi_{pj}$ is convex in $\alpha_{pj}$.

### 2.3.3 Impulse Responses Predicted by the Models

Figure 2.3 and 2.4 show the predicted responses of the output gap, inflation, the real wage and the federal funds rate to an unanticipated 1% fall in interest rates over a five year period in the one- and multi-sector models, respectively. Assuming that the monetary policy shocks can be recovered correctly with the recursive assumption in the VAR, both the former and the latter models can be considered successful in explaining the reactions of the variables included in the VAR model above. As in all periods, the responses stay within the 95% confidence intervals.

Regarding aggregate dynamics, by construction, the contemporaneous impulse responses of the output gap, inflation and the real wage to the shock are zero. Starting with the first quarter, the output gap increases due to unexpectedly low real interest rates following the expansionary interest shock. Initial responses of inflation are negative due
Figure 2.3: *Impulse Responses to an Unanticipated 1% Fall in $R_t$*
(One-Sector Model)

(a) $Y_t - Y_t^*$

(b) $\pi_t$

(c) $W_{real,t}$

(d) $R_t$

Note: The solid lines represent the VAR-based impulse responses and the area between dashed lines indicate the 95% confidence intervals estimated using the method suggested by Sims & Zha (1999). The solid lines marked with circles show the dynamic responses of the variables as predicted by the model.

to the working-capital channel. Over time, inflation rises above its undistorted path due to an increase in output, interest rates and nominal wages which cause marginal costs to
Figure 2.4: Impulse Responses to an Unanticipated 1% Fall in $R_t$
(Multi-Sector Model)

Note: The solid lines represent the VAR-based impulse responses and the area between dashed lines indicate the 95% confidence intervals estimated using the method suggested by Sims & Zha (1999). The solid lines marked with circles show the dynamic responses of the variables as predicted by the model.

rise. The initial positive responses of the real wage result from the decrease in inflation and marginal utility of real income after the shock.
How does the one-sector model differ from the multi-sector model? In comparing Figure 2.3 and Figure 2.4, it is notable that even if the multi-sector model outperforms the one-sector model in matching the estimated VAR-based impulse responses, when uncertainty in the VAR-based impulse responses are considered, the success of the former model over the latter model fades. At the very least, it is difficult to distinguish the aggregate dynamics in the one-sector model from those in the multi-sector model in Figure 2.3 and Figure 2.4. It is therefore not possible to conclude the improvement in the multi-sector model relative the one-sector model is important.

2.3.4 A Comparison of Model-Based Impulse Responses under Alternative Taylor Rules

VAR-based impulse responses may provide a good explanation of the outcome of a monetary shock for a given Taylor rule assuming the estimated VAR-based policy shocks correctly identify the true shocks. VAR models, however, are unable to predict the dynamic behavior of variables after a shock to the interest rate when a new monetary regime is considered. This results from the fact that inference in VAR models is susceptible to the Lucas Critique.

DSGE models are useful as they may help in predicting the dynamic behavior of variables after a monetary policy shock when a new policy regime is implemented. Working with such models, however, necessitates making simplifying assumptions. In the context of this paper, for example, the assumption that all sectors have the same price flexibility represents a simplification assumption made in the one-sector model. Should the model-based impulse responses sharply differ between the models under different Taylor rules, this would call into question the inference drawn in the one-sector model since the simplifying assumption in this model is unrealistic. In this section, I investigate this by consider-
Figure 2.5: The Model-Based Impulse Responses in the One- and Multi-Sector Models under Alternative Taylor Rules after an Unanticipated 1% Fall in $R_t$

\[ \rho = 0.68, \ \phi_y = 0.27, \ \phi_p = 1.05 \]

\[ \rho = 0.68, \ \phi_y = 0.27, \ \phi_p = 1.60 \]

\[ \rho = 0.68, \ \phi_y = 0.27, \ \phi_p = 2.15 \]

Note: The solid lines show the multi-sector model-based impulse responses and the area between dashed lines indicate the 95% model-based confidence intervals for the multi-sector model. The dotted lines marked with asterisks display the dynamic responses of the variables as predicted by the one-sector model.

ing different sets of coefficients for the Taylor rule equation and document the resulting model-based impulse responses for the one- and multi-sector models. In this theoretical experiment, the interest rate rule corresponds to the following Taylor rule considered by Clarida, Gali & Gertler (1999):
Figure 2.5: The Model-Based Impulse Responses in the One- and Multi-Sector Models under Alternative Taylor Rules after an Unanticipated 1% Fall in $R_t$

(cont.)

\[ \rho = 0.68, \, \phi_y = 0.60, \, \phi_\pi = 1.05 \]

\[ Y_t - Y_t^n \]

\[ \pi_t \]

\[ W_{real,t} \]

\[ R_t \]

\[ \rho = 0.68, \, \phi_y = 0.60, \, \phi_\pi = 1.60 \]

\[ Y_t - Y_t^n \]

\[ \pi_t \]

\[ W_{real,t} \]

\[ R_t \]

\[ \rho = 0.68, \, \phi_y = 0.60, \, \phi_\pi = 2.15 \]

\[ Y_t - Y_t^n \]

\[ \pi_t \]

\[ W_{real,t} \]

\[ R_t \]

Note: The solid lines show the multi-sector model-based impulse responses and the area between dashed lines indicate the 95% model-based confidence intervals for the multi-sector model. The dotted lines marked with asterisks display the dynamic responses of the variables as predicted by the one-sector model.

\[ R_t^* = \alpha + \phi_\pi \left( E_t \pi_{t+1} - \bar{\pi} \right) + \phi_y \left( Y_{t-j} - Y_{t-j}^n \right) \]
Figure 2.5: The Model-Based Impulse Responses in the One- and Multi-Sector Models under Alternative Taylor Rules after an Unanticipated 1% Fall in $R_t$

(cont.)

$\rho=0.68$, $\phi_y=0.93$, $\phi_{\pi}=1.05$

$\rho=0.68$, $\phi_y=0.93$, $\phi_{\pi}=1.60$

$\rho=0.68$, $\phi_y=0.93$, $\phi_{\pi}=2.15$

Note: The solid lines show the multi-sector model-based impulse responses and the area between dashed lines indicate the 95% model-based confidence intervals for the multi-sector model. The dotted lines marked with asterisks display the dynamic responses of the variables as predicted by the one-sector model.

\[
R_t = \rho R_{t-1} + (1 - \rho)R_t^* \]

where $\alpha$ is the steady-state nominal interest rate and the bars over variables refer
Figure 2.5: The Model-Based Impulse Responses in the One- and Multi-Sector Models under Alternative Taylor Rules after an Unanticipated 1% Fall in $R_t$ (cont.)

<table>
<thead>
<tr>
<th>(ak) $Y_t - Y_t^n$</th>
<th>$\rho=0.79$, $\phi_y=0.27$, $\phi_\pi=1.05$</th>
<th>(al) $\pi_t$</th>
<th>(am) $W_{real,t}$</th>
<th>(an) $R_t$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>(ao) $Y_t - Y_t^n$</th>
<th>$\rho=0.79$, $\phi_y=0.27$, $\phi_\pi=1.60$</th>
<th>(ap) $\pi_t$</th>
<th>(aq) $W_{real,t}$</th>
<th>(ar) $R_t$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>(as) $Y_t - Y_t^n$</th>
<th>$\rho=0.79$, $\phi_y=0.27$, $\phi_\pi=2.15$</th>
<th>(at) $\pi_t$</th>
<th>(au) $W_{real,t}$</th>
<th>(av) $R_t$</th>
</tr>
</thead>
</table>

Note: The solid lines show the multi-sector model-based impulse responses and the area between dashed lines indicate the 95% model-based confidence intervals for the multi-sector model. The dotted lines marked with asterisks display the dynamic responses of the variables as predicted by the one-sector model.

to their steady-state values. $\rho$ is the interest rate smoothing parameter determined by the monetary authority. Under this policy rule, the target interest rate, $R_t^*$, is not set instantaneously. The sets of values for $\rho$, $\phi_\pi$ and $\phi_y$ are as follows:
Figure 2.5: The Model-Based Impulse Responses in the One- and Multi-Sector Models under Alternative Taylor Rules after an Unanticipated 1% Fall in $R_t$

(cont.)

$\rho = 0.79$, $\phi_y = 0.60$, $\phi_\pi = 1.05$

Note: The solid lines show the multi-sector model-based impulse responses and the area between dashed lines indicate the 95% model-based confidence intervals for the multi-sector model. The dotted lines marked with asterisks display the dynamic responses of the variables as predicted by the one-sector model.

$\rho \in \{0.68, 0.79\}$
Figure 2.5: The Model-Based Impulse Responses in the One- and Multi-Sector Models under Alternative Taylor Rules after an Unanticipated 1% Fall in $R_t$

$\rho = 0.79$, $\phi_y = 0.93$, $\phi_z = 1.05$

Note: The solid lines show the multi-sector model-based impulse responses and the area between dashed lines indicate the 95% model-based confidence intervals for the multi-sector model. The dotted lines marked with asterisks display the dynamic responses of the variables as predicted by the one-sector model.

$\phi_y \in \{0.27, 0.60, 0.93\}$
\( \phi_\pi \in \{1.05, 1.60, 2.15\} \)

These values for the coefficients reflect the estimates of the Taylor rule coefficients in [Clarida, Gali & Gertler (1999)](Clarida, Gali & Gertler (1999)) for the Pre-Volcker and the Volcker-Greenspan periods, as well as their means. 19

Figure 2.5 shows the model-based impulse responses for the one- and multi-sector models as well as the 95% confidence bands for the impulse responses of the variables in the multi-sector model under alternative Taylor rule coefficients. 20

It is notable that the estimated one-sector model-based impulse responses are always contained in the multi-sector model-based 95% confidence bands. This suggests when the monetary authority considers implementing a new policy rule, and the evolution of the economy in this new regime is a matter of interest, it would not be misleading to use the

---

19The estimate of \( \phi_\pi \) for the Pre-Volcker period is 0.83. As argued in [Clarida, Gali & Gertler (1999)](Clarida, Gali & Gertler (1999)), this makes the system of dynamic equations indeterminate. For this reason, I set the lowest value for \( \phi_\pi \) as 1.05.

20In estimating standard deviations for the multi-sector model-based impulse responses, the delta method is used. Indeed, by using the mean-value theorem, one can write

\[
\tilde{f}^A(\hat{\mathcal{P}}_M(\hat{A}_n)) = \tilde{f}^A(P^M_0) + \frac{\partial f^A(\tilde{\mathcal{P}}^M)}{\partial P} \left( \hat{\mathcal{P}}_M(\hat{A}_n) - P^M_0 \right)
\]

where \( \tilde{\mathcal{P}}^M \in (\hat{\mathcal{P}}^M(\hat{A}_n), P^M_0) \) and \( f^A(.) \) denotes the multi-sector impulse responses obtained with alternative Taylor-rule specifications.

It can be shown that

\[
\sqrt{n}(\tilde{f}^A(\hat{\mathcal{P}}_M(\hat{A}_n)) - f^A(P^M_0)) \sim N(0, V^f)
\]

where

\[
V^f = \left( \frac{\partial f^A(P^M_0)}{\partial P} A' A \frac{\partial f^A(P^M_0)}{\partial P} \right)^{-1} \frac{\partial f^A(P^M_0)}{\partial P} A' A \frac{\partial f^A(P^M_0)}{\partial P} \left( \frac{\partial f^A(P^M_0)}{\partial P} A' A \frac{\partial f^A(P^M_0)}{\partial P} \right)^{-1}
\]

This suggests that:

\[
\sqrt{n} \left( f^A(\hat{\mathcal{P}}_M(\hat{A}_n)) - f^A(P^M_0) \right) \sim N(0, \frac{\partial f^A(P^M_0)}{\partial P} V^f \frac{\partial f^A(P^M_0)}{\partial P})
\]

The plug-in method has been employed to obtain a consistent estimate of \( \frac{\partial f^A(P^M_0)}{\partial P} V^f \frac{\partial f^A(P^M_0)}{\partial P} \).
Table 2.6: Calibration for Explaining Contrasting Findings in Carvalho (2006)

(a): The Replication Exercise (Flexible Wages)

<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi$</td>
<td>$\sigma_H$</td>
<td>$\theta_p$</td>
<td>$\theta_w$</td>
<td>$b$</td>
<td>$\gamma_p$</td>
<td>$\gamma_w$</td>
<td>$\phi_Y$</td>
<td>$\phi_x$</td>
<td>$\rho_e$</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>11</td>
<td>21</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.33/4</td>
<td>1.24</td>
<td>0.92</td>
</tr>
</tbody>
</table>

(b): With Estimated Wage Rigidity

<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi$</td>
<td>$\sigma_H$</td>
<td>$\theta_p$</td>
<td>$\theta_w$</td>
<td>$b$</td>
<td>$\gamma_p$</td>
<td>$\gamma_w$</td>
<td>$\phi_Y$</td>
<td>$\phi_x$</td>
<td>$\rho_e$</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>11</td>
<td>21</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.33/4</td>
<td>1.24</td>
<td>0.92</td>
</tr>
</tbody>
</table>

one-sector model since the dynamic behavior of the variables as predicted by this model under the new regime does not differ sharply from the dynamic behavior of the variables as predicted by the multi-sector model where the degree of heterogeneity in price flexibility is close to the true degree of heterogeneity in price flexibility.

2.3.5 Explaining the Contrasting Findings

In order to isolate the effects of sectoral asymmetries in the frequency of price changes, both Carvalho (2006) and Nakamura & Steinsson (2008b) compare the response of the heterogeneous-firm multi-sector economy to that of the identical-firm one-sector economy. They find when parameters are realistically calibrated, monetary shocks have larger and more persistent real effects, which sharply contrast with the findings in this paper. Indeed, irrespective of the policy rule specified, a comparison between the dynamics of the output gap after the same shock in the one-sector and multi-sector economies in Figure 2.3, 2.4 and 2.5 reveals such dynamics in the one- and multi-sector economies are very similar.
In order to reconcile the findings in Carvalho (2006) and Nakamura & Steinsson (2008b) with my findings, I start with a replication of the findings in Carvalho (2006). The model in Carvalho (2006) deviates from the simple Calvo (1983) model in two ways. First, there is heterogeneity in the frequency of price changes among sectors. Second, rather than employing homogeneous labor, firm-specific labor is used. To replicate his finding, I retain the former but drop the latter feature of the model. Dropping the latter is inevitable since extending the analysis to staggered wage setting requires firm-specific labor to be replaced with homogeneous labor. The model presented in Section 2.2 lends itself to do such a replication. Indeed, in this replication exercise, I consider a simplified version of the model discussed in Section 2.2 where there is no habit formation ($b = 0$), no backward-looking indexation rule in non-optimized prices and wages ($\gamma_p = \gamma_w = 0$), and the working capital channel is not operational as firms pay the wage bill at the period that production takes place. Carvalho (2006) calibrates $\varphi$, $\sigma_H$, $\theta_p$ as 1, 0.5 and 11, respectively. Carvalho (2006) closes the model with a policy rule given by

$$ R_t = c + \phi_y (Y_t - Y^m_t) + \phi_{\pi} \pi_t + \epsilon^m_t \quad \text{where} \quad \epsilon^m_t = \rho_t \epsilon^m_{t-1} + \nu_t \tag{2.3.7} $$

To calibrate $\phi_y$, $\phi_{\pi}$ and $\rho_t$, Carvalho (2006) relies on the estimates reported in Rude-21

As extensively discussed in Woodford (2003), replacing specific labor markets with common labor markets results in a fall in the degree of strategic complementarity. However, in both the one- and multi-sector models, each firm owns a fixed amount of capital that never depreciates and may not be reallocated among firms, preventing the rental rate of capital from being equalized economy-wide. In addition, the production function is concave in labor input in both of the models. Under these assumptions, Woodford (2003) shows that there is strategic complementarity if

$$ \varsigma = \frac{\omega_w + \omega_p + \sigma_H}{1 + \omega_p \theta_p} < 1 $$

The calibrated parameters in the replication exercise gives $\varsigma = 0.81$, which still implies strategic complementarity in price setting so that fast-adjusting firms have a disincentive to strongly change prices when price responses of slow-adjusting firms are muted. It is notable that as a robustness check, I also calibrated the parameters in such a way that they imply a larger degree of strategic complementarity such as $\varsigma = 0.10$, the main result from the replication exercise, that heterogeneity in price flexibility plays only a small role under staggered wages, remains largely unchanged.
Table 2.7: *Cumulative Real Effects of Monetary Shocks and Their Persistence*

(a): The Replication Exercise (Flexible Wages)

<table>
<thead>
<tr>
<th>Half-life (Quarters)</th>
<th>Cumulative Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>The One-Sector Economy</td>
<td>10</td>
</tr>
<tr>
<td>The Multi-Sector Economy</td>
<td>20</td>
</tr>
<tr>
<td>Ratio</td>
<td></td>
</tr>
</tbody>
</table>

(b): With Estimated Wage Rigidity

<table>
<thead>
<tr>
<th>Half-life (Quarters)</th>
<th>Cumulative Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>The One-Sector Economy</td>
<td>11</td>
</tr>
<tr>
<td>The Multi-Sector Economy</td>
<td>11</td>
</tr>
<tr>
<td>Ratio</td>
<td></td>
</tr>
</tbody>
</table>

Note: The normalized cumulative real effects of monetary shocks are measured as $|\epsilon_{m,t}^{-1} \sum_{s=0}^{\infty} \hat{y}_{t+s}|$. The half-life refers to the first period in which the response of the output gap falls below the midpoint of its impact response.

Similarly, I use the same estimates and calibrate $\phi_y$, $\phi_{\pi}$ and $\rho_R$ as equal to $0.33/4$, $1.24$ and $0.92$, respectively. I again use the same calibrated values for $\alpha_j$ and $n_j$ reported in Table 2.3. In addition to such parameters, $\alpha_w$ and $\theta_w$ require calibration. I calibrate $\theta_w$ to $21$ based on the value in Christiano, Eichenbaum & Evans (2005). Accordingly, wage rigidities in these models can be adjusted solely by changing the fraction of workers who do not obtain a wage-change signal in the period ($\alpha_w$). Since wages are flexible in Carvalho (2006), to replicate his finding, I choose $\alpha_w = 0.001$. The calibrated parameters for the replication exercise is summarized in Panel (a) of Table 2.6.

Carvalho (2006) measures the size of the real effects of monetary shocks in the one- and multi-sector economies with normalized cumulative effects on the output gap defined as,
Figure 2.6: The One- and Multi-Sector Model-Based Impulse Responses of Output Gap after a Negative $1\%$ $\epsilon^m$ Shock in (2.3.7)

(a) With Flexible Wages

(b) With Estimated Wage Rigidity

Note: The dotted lines marked with asterisks and the solid lines show the model-based responses of the output gap to a negative $1\%$ $\epsilon^m$ shock in the one- and multi-sector economy, respectively.

$$|\epsilon_t^{m-1} \sum_{s=0}^{\infty} \hat{Y}_{t+s}|$$

(2.3.8)

where $\epsilon_t^{m-1}$ denotes the inverse of the size of the shock in (2.3.7). Carvalho (2006) measures the degree of persistence of monetary shocks on the output gap using the half-life of the output gap. This is defined as the first period in which the absolute value of the response of the output gap falls below that of the response of the output gap on impact. As evident in Panel (a) of Table 2.7, my finding under flexible wages conforms with the main finding in Carvalho (2006) since the cumulative real effects of monetary shocks in the multi-sector economy are larger than those in the one-sector economy by a factor of 2.74 (35.53 in the multi-sector economy compared to 12.99 in the one-sector economy). The half-life in the multi-sector economy is twice that in the one-sector economy, suggesting
that monetary shocks have more persistent real effects in the former. Panel (a) of Figure 2.6 helps visualize these findings.

Next, I consider that wage setting is staggered. Panel (b) of Table 2.6 reports calibrated parameters for the staggered wage-setting model with the calibrated wage rigidity parameter ($\alpha_w = 0.83$) in the United States. Panel (b) of Table 2.7 on the other hand, shows the degree of persistence and the size of the cumulative effects in the one- and multi-sector economies under this calibrated parameter for wage rigidity. Three findings are noteworthy. First, monetary shocks no longer have larger real effects in the multi-sector economy than the one-sector economy. This finding suggests that heterogeneity in the frequency of price changes in a staggered wage-setting model adds much less to the real effects of monetary shocks when compared with a flexible wage-setting model. Second, monetary shocks induce larger real effects in the identical-firms one-sector economy model with staggered wage setting than in the heterogeneous-firms multi-sector model under flexible wage setting (71.74 in the former and 35.53 in the latter). For this reason, if one aims to induce monetary shocks to have larger real effects, departing from the simple Calvo (1983) model by adding nominal wage contracts, as per Erceg et al. (2000), is more effective than adding heterogeneity in the frequency of price changes among firms. Third, when wage setting is staggered, it is no longer true that monetary shocks have more persistent real effects in the multi-sector economy than the one-sector economy. Panel (b) of Figure 2.6 displays the response of the output gap after a 1% $\epsilon^m$ shock for the staggered wage-setting models with the estimated degree of wage rigidity.24

---

22 The half-lives in the former and the latter are 20 and 10, respectively.
23 The half-lives in the former and the latter are 11 and 11, respectively.
24 As previously discussed, the degree of wage rigidity in the models is calibrated using the estimated degree of wage rigidity in Barattieri et al. (2010) who note their estimated degree of wage rigidity in the United States differs from the degree of wage rigidity chosen by some key papers which estimate DSGE models using macro data. For example, differently from the degree of wage rigidity in this paper (0.83), Christiano et al. (2005) and Giannoni & Woodford (2003) assume two thirds of wages are not optimized each quarter. As a robustness check, the one- and multi-sector models in this section are also estimated.
Next, I explain why a higher degree of wage rigidity leads monetary shocks to have similar output dynamics in the one- and multi-sector economies. Firstly, consider the explanation for the finding in Carvalho (2006) that monetary shocks have larger and more persistent real effects in the multi-sector economy than in the one-sector economy when wages are free to quickly respond to such shocks. In the multi-sector economy, firms with a broad spectrum of the frequency of price changes are present. That is, while some firms respond rapidly to monetary shocks, others are slower. In addition, the response of wages to monetary shocks is strong as wages adjust quickly in Carvalho (2006). Faced with such profound changes in their marginal costs, fast-adjusting firms tend to show strong responses. However, the existence of slow-adjusting firms in the multi-sector economy gives a disincentive for the fast-adjusting firms to respond strongly to monetary shocks in Carvalho (2006) since the calibrated parameters imply there is strategic complementarity in price setting. This results in an initial muted price response to monetary shocks. In the following periods, while a large fraction of firms obtain a chance to change prices at least once, the initial muted price responses restrain them to raise prices. In effect, the price level responses under strategic complementarity stay muted compared to the price level responses that would prevail if there were no such strategic interaction in price setting. Since the output gap responses are the direct opposite of the price responses, the output gap strongly responds to monetary shocks in the multi-sector economy. Nakamura & Steinsson (2013) note that if it were possible to transfer some price change signals from the fast-adjusting firms to the slow-adjusting firms, price adjustment would be much faster and the real effects of monetary shocks would be much smaller. Such transfers are made hypothetically in the one-sector economy. Indeed, by calibrating the frequency of price using 0.66 as the degree of wage rigidity. None of the results in this section is affected largely from this change. Indeed, I find that the ratio of the cumulative real effect of a monetary shock in the multi-sector model to that in the one-sector model is only 1.06 and that the half-life of output responses in both the one- and multi-sector models is 10 quarters.
changes in the aggregate economy as the weighted average of the frequency of price changes and treating all firms as identical, the price change signals are transferred implicitly from the fast-adjusting firms to slow-adjusting firms. Consequently, the one-sector economy induces much faster price responses and much smaller real effects from monetary shocks.

Why does a higher degree of wage rigidity lower the effects of heterogeneity in the frequency of price changes on output dynamics in the model? Note that for strategic complementarity to play a non-trivial role, fast-adjusting firms should at least have an inclination to give a strong price response in the first place. Such an inclination exists when wages are free to respond as fast-adjusting firms face profound changes in marginal costs. In the case of staggered wage setting, since monetary shocks initially only cause a muted response in marginal costs, fast-adjusting firms are not likely to be inclined towards giving a strong price response. As a result, strategic complementarity plays a small role under staggered wage setting which induces output to display similar post-shock dynamics in the one- and multi-sector models.

Do the findings in Table 2.7 contrast with the finding in Chari, Kehoe & McGrattan (2000) that sticky wages quantitatively have small effects on the extent to which monetary shocks induce persistent effects on consumption in their one-sector model? The finding of small effects from sticky wages in Chari et al. (2000) is similar to the finding in Table 2.7 that the half-life of shocks to consumption after monetary shocks in the one-sector model under flexible wages is virtually the same as that in the same model under staggered wages. However, such small effects from sticky wages do not hold if one considers the multi-sector model. Indeed, as reported in Table 2.7, I find the persistence of shocks to consumption after monetary shocks in the multi-sector model under staggered wages is

---

25 The half-lives in the former and the latter are 10 and 11 quarters, respectively. It is notable that output is simply given as consumption in the one- and multi-sector models since investment is excluded from both of the models. Consequently, the half-lives of shocks to consumption and output must be identical in the models.
drastically lower compared to that in the same model under flexible wages. This finding contrasts sharply with the finding in Chari et al. (2000).

2.4 Conclusion

In this paper, I have investigated the consequences of introducing heterogeneity in the frequency of price changes into a dynamic stochastic model which satisfactorily provides an explanation of the dynamics of the output gap, inflation, the real wage and the federal funds rate after an unanticipated interest shock. The main findings of this paper can be summarized as follows: first, the dynamic behavior of the variables after a monetary policy shock are similar in both models. This finding is robust to the alternative interest rules specified to close the model. Second, the finding that monetary shocks have similar real effects in the one- and multi-sector economies contrasts sharply with the finding in Carvalho (2006). I show that the staggered wage setting plays an important role in bringing the dynamics of output in the multi-sector economy close to those in the one-sector economy.

26The half-lives in the former and the latter are 11 and 20 quarters, respectively.
Chapter 3

Heterogeneity in Price Flexibility and Monetary Policy Shocks

This paper seeks to answer two questions: First, do shocks to monetary policy in the United States induce sectoral prices to exhibit common or divergent dynamics? Second, are such shocks the cause of different price dynamics in fast-adjusting sectors where prices change often, compared to the slow-adjusting sectors where prices change infrequently?

In regards to the first question, I find that monetary policy shocks in the United States lead to divergent sectoral price dynamics, which suggests there are relative price effects of the monetary shocks. Indeed, I find that while the price responses in some sectors to monetary shocks are muted, they are strongly positive or negative in others. This finding is in conformity with the finding in Balke & Wynne (2007). However, they surprisingly find that a contractionary shock to monetary policy preponderantly results in positive initial price responses. In contrast, I find that the initial price responses in sectors to such a shock are equally divided between negative and positive responses, resulting in the initial aggregate price responses staying muted.\footnote{This finding is similar to the finding in Boivin, Giannoni & Mihov (2009).}

The difference in the distribution of negative and positive price responses between Balke & Wynne (2007) and this paper can result from the fact that a measure of the output gap is missing in Balke & Wynne (2007). As argued in Giordani (2004), the absence of an output gap measure in a VAR model may result
in “the price puzzle”, which refers to the counter-intuitive finding that an unanticipated monetary tightening causes an increase in the price level. I use the capacity utilization rate as a measure of the output gap and find no evidence of the puzzle at the *disaggregated level* in the VAR model.

Next, I analyze the correlation between the frequency of price changes in a sector and its impulse response functions to a contractionary monetary policy shock over five years. I find the frequency of price changes in a sector does not play a decisive role in its price responses to monetary policy shocks as the correlations between the frequency of price changes and the price responses are weak and never significantly different from zero. This finding contrasts sharply with that in *Bils, Klenow & Kryvtsov (2003)* who find that a higher frequency of price change for a given sector is associated with a higher price response when a contractionary monetary shock occurs. I show the assumption in *Bils, Klenow & Kryvtsov (2003)*, that the isolated monetary shocks and sector-specific price shocks are orthogonal, is violated for a considerable number of sectors. The violation of such a critical assumption may drive the counter-intuitive finding in *Bils, Klenow & Kryvtsov (2003)*.

Lastly, I attempt to develop a DSGE model to explain two important findings in this paper: the interest rate shock causes strong relative price effects, and there is a weak association between the impulse response functions of sectoral prices and the frequency of price changes in sectors. Three DSGE models are considered: The first model is the *one-sector model*, in which it is assumed that the frequency of price changes is the same among all sectors. The second model is the *multi-sector model with symmetric cost structure* in which sectors differ only in regards to the frequency of price changes. Lastly, the third model is the *multi-sector model with asymmetric cost structure* in which sectors differ not only in regards to the frequency of price changes, but also in their production
costs’ structure. I show that while the one-sector model can explain the second finding successfully, this model may not explain the strong relative price effects of the interest rate shock at the disaggregated level. Quite the opposite, the multi-sector model with symmetric cost structure is successful in explaining relative price effects. Yet, this model fails to account for the low correlations of the frequency of price changes with sectoral price responses over five years following an interest rate shock. The last model, the multi-sector model with asymmetric cost structure, on the other hand, successfully explains both of the aforementioned two findings of the empirical section. Therefore, I conclude this model outperforms the other two models.

The organization of the paper is as follows. Section 3.1 presents the empirical strategy for isolating monetary shocks in the United States and studies the impulse response functions of sectoral prices to such shocks. Section 3.2 develops three theoretical models and evaluates the success of these models in explaining the strong relative price effects of the interest rate shock and the weak correlations between the frequency of price changes and sectoral price responses. The last section concludes the discussion.

3.1 The Empirical Section

This section develops my empirical strategy for analyzing sectoral price responses following a contractionary shock to the federal funds rate. Before investigating how sectoral prices change following an exogenous interest shock, it is first useful to study the aggregate dynamics.

3.1.1 Aggregate Dynamics after an Exogenous Shock in the Federal Funds Rate

In Tugan (2013), the following VAR model is considered,
\[ \Upsilon_t = B_0 + \sum_{k=1}^{k_{max}} B_k \Upsilon_{t-k} + A_0 \varepsilon_t \]  

(3.1.1)

where structural shocks and the number of lags included are denoted by \( \varepsilon_t \) and \( k_{max} \), respectively. \( A_0 \) stands for the contemporaneous response matrix of the variables to these shocks. Lastly, \( \Upsilon_t \) denotes the vector of variables contained in the VAR and is given as:

\[ \Upsilon_t = [y_t - y^n_t, \pi_t, w_t, R_t] \]  

(3.1.2)

I again consider the VAR model in (3.1.1) to study the aggregate dynamics in this paper. The variables included in \( \Upsilon_t \) are the capacity utilization rate in manufacturing as a measure of the output gap \( (y_t - \hat{y}^n_t) \), annualized inflation \( (\pi_t) \), the real wage \( (w_t) \) and the federal funds rate \( (R_t) \) (See Tugan (2013) for a detailed explanation of the variables). The VAR is quarterly and contains four lags of each variable. The sample spans the period of 1959Q1-2013Q1. The ordering of the variables in the VAR system implies that the capacity utilization, inflation and the real wage respond to the monetary policy shock with one quarter lag. This assumption is standard in the literature.

Figure 3.1 displays the impulse responses of the variables contained in the VAR system to an unanticipated 1% rise in the federal funds rate together with the 95% error bands estimated with the method proposed by Sims & Zha (1999). Regarding the effect of the expansionary policy shock, as shown in the figure:

- The point estimates suggest the output gap stays below its pre-shock level for four years after the shock. In addition, in the first two years, the 95% confidence bands indicate, the fall in the output gap is statistically significant with a trough occurring after about one and half years following the shock.

- Inflation is above its pre-shock level in the early periods following the shock but
Figure 3.1: The VAR-Based Impulse Responses of Aggregate Variables to Monetary Shocks

Note: In the figure, the solid line indicates the estimated point-wise impulse responses. The area between the dashed lines shows the 95% confidence interval estimated with the method suggested by Sims & Zha (1999).

rises over time below the pre-shock level. The trough realizes after about three years according to the estimated impulse responses. It is notable that my finding of a
delayed effect from the interest shock on inflation is a general finding for the United States economy as noted by Woodford (2003).

- The real wage falls after the shock, suggesting nominal wages fall relative to nominal prices following the expansionary shock.

- Lastly, the federal funds rate remains above its pre-shock level for about three years following the shock according to its point impulse response estimates.

### 3.1.2 Sectoral Price Responses after Interest Shocks

To study sectoral price responses following an unanticipated 1% increase in the federal funds rate, I consider again the same VAR in (3.1.1) but add the annualized percentage change in a sectoral price to the vector of variables(\(\Upsilon_t\)) in (3.1.2). This variable is denoted by \(\pi_{it}\) and is measured as \(4 \times (lnp_{it} - lnp_{it-1})\).

\[
\Upsilon_t = [y_t - y^n_t, \pi_t, \pi_{it}, w_t, R_t]
\] (3.1.3)

Two identifying assumptions for isolating exogenous interest shocks in (3.1.3) are worth mentioning. First, the Federal Reserve observes sectoral price movements before setting the federal funds rate. Second, there is at least a quarter lag in the response of sectoral prices to the federal funds rate shocks. The latter is consistent with with the aforementioned assumption that there is a quarter delay in the aggregate price level’s response to the federal funds rate shock. Were the sectoral prices assumed to respond contemporaneously to the shock while the aggregate price level was not, the analysis would be internally inconsistent.

Figure 3.2 shows the impulse responses of the sectoral and aggregate price levels to
Figure 3.2: The Impulse Responses of the Price Levels of PCE Categories to an Unanticipated 1% Increase in the Federal Funds Rate Shocks

Note: In the figure, the thick solid line marked with circles shows the aggregate price responses following an unanticipated 1% increase in the federal funds rate whose 95% confidence intervals are marked by the thick dashed lines. The thin solid lines, on the other hand, display the sectoral impulse responses to the same shock.

An unanticipated 1% increase in the federal funds rate\textsuperscript{28} Sectoral price level impulse responses represent the price level responses for 124 Personal Consumption Expenditure (PCE) categories for which I have an estimate of the frequency of price changes.

A crucial finding in Figure 3.2 is that an unanticipated change in the federal funds rate\textsuperscript{28} Since both \( \pi_t \) and \( \pi_{it} \) are measured as four times the difference in the log of price levels between two periods, to obtain impulse responses for sectoral and aggregate price levels (denoted with \( \ln p_{it} \) and \( \ln P_t \), respectively), cumulative impulse responses for \( \pi_t \) and \( \pi_{it} \) are obtained, which are then scaled down by four.
produces relative price effects in the United States. The existence of such effects requires only the lowest and highest sectoral price level responses to differ significantly. A stronger condition is met in Figure 3.2. Indeed, not only the highest sectoral price level responses differ radically from the lowest ones, but they are also outside the 95% confidence bands for the impulse responses of the aggregate price level.

It is also notable that initially, positive and negative sectoral responses are distributed evenly. This results in an initial muted response of the aggregate price level. Following this phase, the sectoral prices’ responses are predominantly negative. As a consequence, the aggregate price level shows a decline following the initial phase. It is worth mentioning that these findings are in conformity with the findings in Boivin, Giannoni & Mihov (2009) who use the factor augmented vector autoregression (FAVAR) approach to study the sectoral price responses to a federal funds rate shock. They advocate their method by showing that a contractionary interest rate shock results in a fall in most sectoral prices and that there is no evidence of a “price puzzle” when the FAVAR approach is used. However, Hanson (2004) finds that the “price puzzle” is mainly associated with the 1959-1979 sample period, and that evidence of a “price puzzle” is weak during the 1976-2005 period which Boivin, Giannoni & Mihov (2009) consider. Whether the FAVAR approach alleviates the puzzle or not is uncertain when the data sample is extended back to 1959. It is notable that even if my data sample covers the period in which Hanson (2004) finds strong evidence of the puzzle, my results do not indicate a “price puzzle”.

How strong is the association between sectoral price level responses in Figure 3.2 and sectoral frequency of price changes? Is a higher frequency of price change in a sector associated with a higher or a lower price level response in periods? To answer these questions, I first define the frequency of price changes in sectors and describe my data on the frequency of price changes. The frequency refers to the percentage of firms that adjust
their prices in a quarter. Our monthly frequency of price changes data for the United States comes from Nakamura & Steinsson (2008a). In contrast to the frequency of price changes in Bils & Klenow (2004), where only the frequency of price changes including sales are reported, using the frequency of price changes data in Nakamura & Steinsson (2008a) has the advantage that the frequencies in sectors are reported for both non-sales price changes and price changes including sales. Since Nakamura & Steinsson (2008a) find that the frequency of price changes including sales in some sectors differs radically from that of non-sales price changes to a great margin, it is important to check the robustness of my results in this paper for non-sales price changes and price changes including sales.

The frequency of price changes is estimated for entry level items (ELIs) of CPI in Nakamura & Steinsson (2008a). However, for many components of CPI, price series for the disaggregated sectors are not available for earlier periods, prior to 1970s. In contrast, sectoral price indexes can be obtained from 1959 for the bulk of the Bureau of Economic Analysis’ personal consumption expenditure (PCE) categories. For this reason, I use the PCE categories for estimation. However, since the frequency of price changes are not readily available for the PCE categories, the categories are mapped to the components of the CPI index in my analysis. To map ELIs in the CPI with the PCE categories, I use the mapping that Andrea Tambalotti made available. If a PCE category is matched with only one component of CPI, the frequency of price changes in that PCE category is taken as that of the CPI component. If there are multiple ELIs that map with a single PCE category, the frequency of price changes in this PCE category is measured as the weighted average of the frequency of price changes in these CPI components, with the weights given as the 2000 CPI expenditures of the ELIs reported by Nakamura & Steinsson (2008a).

Lastly, the frequencies of price adjustment within ELIs in Nakamura & Steinsson

---

29I am grateful to Andrea Tambalotti for sharing his mapping with me.
are reported as *monthly* percentages. The quarterly frequency of price changes in ELIs are estimated by using the method described in Tugan (2013). It is notable that the quarterly frequencies are found to differ substantially among the mapped PCE categories. The frequencies range from as low as 6.7% in intracity mass transit to 100% in net purchases of used motor vehicles.

Now, I study the association of the frequency of price changes with the impulse responses of sectoral prices to interest rate shocks. First, the impulse responses of sectoral prices to an unanticipated 1% federal funds rate shock have been obtained as in Figure 3.2. Next, for each period, the correlation between the frequency of price changes in sectors and their impulse responses is estimated. Figure 3.3 demonstrates these correlations and the corresponding 95% confidence bands. The correlations reveal that if prices change more frequently in a sector, it is more likely that prices in that sector increase after a contractionary interest rate shock during the first year following the shock. This finding holds whether or not the frequency of price changes includes non-sales price changes (See Panel (a) and Panel (b) of Figure 3.3). After one year following a contractionary monetary shock, on the other hand, a higher frequency of price change in a sector is associated with a lower impulse response. It is notable that the correlations of the impulse responses with the frequency of non-sales price changes and with the frequency of price changes including sales are rather similar. Lastly, the fact that the value of zero is contained in the confidence intervals for the correlations indicates that one may not reject the hypothesis of no correlation between the frequency of price changes and the impulse responses of sectoral prices following a contractionary federal funds rate shock. In other words, the price responses after a contractionary shock in sectors are only weakly associated with the fraction of firms in the sector that change their prices in a quarter.

These findings contrast with those in Bils, Klenow & Kryvtsov (2003) who find that
Figure 3.3: Correlations of $\lambda_i$ with the Impulse Responses of $P_i$ to an Unanticipated 1% Increase in $R_t$

Note: The solid lines display the correlations of frequency of price changes in a sector with the impulse responses of sectoral prices to an unanticipated 1% increase in the federal funds rate at each quarter. The 95% confidence intervals for these correlations are shown with dotted lines and are estimated using the block-bootstrap method explained in Appendix B.2.

there is an anomaly in the relative price movements following an unanticipated change in the federal funds rate. Indeed, the price of the flexible-price category rises significantly relative to that of the sticky-price category in the first eight months following a contractionary interest shock. They reason there are two possible explanations for this finding: either the sticky price models are incapable of explaining relative price movements following the exogenous monetary shocks or else inferred monetary shocks are not orthogonal to persistent price shocks in the flexible- and sticky-price categories. I discuss the Bils, Klenow & Kryvtsov (2003) model in detail in Appendix B.1 and offer an explanation for the contrasting findings in this paper and theirs.
In the next section, I aim to explain the empirical findings in this section with the three DSGE models. In the first model (one-sector model), the fraction of firms that may change their prices in a quarter in all sectors after an interest rate shock are assumed to be the same in all sectors. In this model, the frequency of price changes in the economy is approximated by the median frequency of price changes in all sectors. It is notable that the assumption of the same frequency of price changes among sectors in the one-sector model does not necessarily contradict with the finding in Bils & Klenow (2004) and Nakamura & Steinsson (2008a) that the distribution of frequency of price changes among sectors is wide in the United States. Indeed, while the frequency of price changes may differ largely among sectors for sector-specific and other types of shocks, they are the same for an interest rate shock. In the second model (multi-sector model with a symmetric cost structure), sectors are allowed to differ only in the frequency of price changes after an interest rate shock. In the last model, (the multi-sector model with an asymmetric cost structure), sectors differ not only in the frequency of price changes but also in the cost structure. The performance of these models in explaining the weak association of the frequency of price changes with impulse responses of sectoral prices after an interest rate shock in the economy is then evaluated. The findings are in favor of the multi-sector model with an asymmetric cost structure.

3.2 Theoretical Models

In this section, I consider a variant of the theoretical model in Tugan (2013). Since the model environment is discussed in detail in Tugan (2013), only a summary of main features of the model and the dynamic equations needed to solve the DSGE models are stated here:

- Price setting is staggered along the lines of Calvo (1983).
• Wage setting is staggered along the lines of Erceg, Henderson & Levin (2000).

• When the optimization signal is not received by firms and workers, wages and prices are set according to the backward-looking indexation rule.

• There is habit persistence in consumption.

• It is assumed that consumption decisions are made and prices and wages are set one period before observing the interest rate shocks.

• Firms are obliged to pay their wage bill in advance. As a consequence, when the monetary authority decides to introduce an unanticipated increase in the interest rate, real marginal costs may rise despite a fall in output accompanying the contractionary shock.

• The one- and multi-sector models differ only in the assumption regarding the frequency of price changes after an interest rate shock. In the one-sector model, the frequency of price changes in all sectors is assumed to be homogenous. In the multi-sector model, on the other hand, there is a heterogeneity in price setting among sectors. As a matter of fact, in some sectors, prices change more frequently than in others.

3.2.1 The Structural Equations in the Models

Now, I state the main equations of the model. Let the hat over variables, \( \hat{R}_t, \pi_{t+1} \) and \( \varphi \) denote the log-deviation of the variables from their corresponding steady states; the nominal interest rate; inflation in prices; and, the intertemporal elasticity of substitution, respectively. The IS equation is given by:

\[
E_{t-1} \hat{x}_t = E_{t-1} \hat{x}_{t+1} - \varphi E_{t-1} \left( \hat{R}_t - \pi_{t+1} \right) \tag{3.2.1}
\]
where \( \hat{x}_t \) is defined as:

\[
\hat{x}_t = \left( \hat{y}_t - b\hat{y}_{t-1} \right) - b\beta E_t \left( \hat{y}_{t+1} - b\hat{y}_t \right)
\]  \hspace{1cm} (3.2.2)

In (3.2.2), \( \hat{y}_t \), \( b \) and \( \beta \) denote the log-change in output; the habit formation parameter; and, the discount factor, respectively.

The second equation in the models is the wage inflation equation (\( \pi^w_t \)):

\[
\pi^w_t - \gamma_w\pi_{t-1} = \xi_w E_{t-1} \left( \omega_w \hat{y}_t + \varphi^{-1} \hat{x}_t - \hat{w}_t \right) + \beta E_{t-1} \left( \pi^w_{t+1} - \gamma_w\pi_t \right)
\] \hspace{1cm} (3.2.3)

In (3.2.3), \( \omega_w \) and \( \gamma_w \) denote the elasticity of real wages paid for the number of hours worked with respect to output changes for a constant marginal utility of real income and the backward-looking indexation parameter in wages, respectively. Lastly, letting \( 1 - \alpha_w \), \( \sigma_H \) and \( \theta_w \) denote the probability of receiving a wage change signal by a differentiated labor type; the Frisch-elasticity of labor; and, the wage elasticity of substitution among differentiated labor types, respectively, the wage stickiness parameter \( \xi_w \) in (3.2.3) can be written as:

\[
\xi_w = \frac{(1 - \alpha_w)}{\alpha_w} \frac{1 - \alpha_w\beta}{(1 + \theta_w\sigma_H)}
\]

The third equation in the models is the monetary policy rule. The monetary authority is assumed to control the interest rate and implements the following Taylor Rule to stabilize the economy:

\[
R_t = \rho_R R_{t-1} + [a_y \pi_t + a_y(y_t - y^n_t)] - \rho_R [a_y \pi_{t-1} + a_y(y_{t-1} - y^n_{t-1})] + \epsilon_t
\] \hspace{1cm} (3.2.4)
where \( y_t^n \) and \( \epsilon_t \) denote the potential output and the shock in monetary policy, respectively. The calibrated values for \( \rho_R, a_\pi \) and \( a_y \) are given as 0.92, 1.24 and 0.33, respectively. These calibrated values are based on Rudebusch (2002).

### 3.2.2 The Econometric Method

Since the number of sectors for which the frequency of price changes data is available is quite large, it is impractical to solve the DSGE models by considering each individual sector. To circumvent this problem, as in Tugan (2013), I reduce the number of sectors in the model to 10 by including each sector in one of the percentiles of the frequency of price changes and approximating the frequency of price changes in a sector with the median frequency in the percentile group where that sector is contained. It is notable that only the frequency of price changes including sales are considered in calibration. Since the calibration of \( f_k \), \( \alpha_{pk} \) and some other parameters of the models are extensively discussed in Tugan (2013), I skip describing the calibration method here and only discuss the estimation method for the free parameters of the models. Let \( \mathcal{P} \) denote the vector of free parameters to be estimated. In the models, \( \mathcal{P} \) contains 7 parameters:

\[
\mathcal{P} = [\varphi, \sigma_H, \theta_p, \theta_w, b, \gamma_p, \gamma_w]
\]

where \( \theta_p \) and \( \gamma_p \) denote the price elasticity of substitution for sectoral goods and the backward-looking indexation parameter in prices, respectively.\(^{30}\)

\[
\hat{\mathcal{P}}(\hat{A}_T) = \arg \min_{\mathcal{P}} (\hat{h}_T - f(\mathcal{P}))' \hat{A}_T' \hat{A}_T (\hat{h}_T - f(\mathcal{P}))
\]

(3.2.5)

where \( \hat{A}_T \) and \( f(\mathcal{P}) \) show the weighting matrix and the model-based impulse responses.

\(^{30}\)See (3.2.1) for the definition of \( \varphi \), (3.2.2) for the definition of \( b \), and (3.2.3) for the definitions of \( \sigma_H, \theta_w \) and \( \gamma_w \).
and correlations for a given parameter vector $\mathcal{P}$. Lastly, $\hat{h}_T$ stands for the vector of estimated VAR-based impulse responses and correlations and is given by:

$$
\hat{h}_T = \begin{bmatrix} C_{y_{1,20}}, y_{1,20}, C_{\pi_{1,20}}, C_{w_{1,20}}, C_{R_{1,20}}, \rho_{\ell_{p_{1,20}}}, \lambda_i \end{bmatrix}'
$$

(3.2.6)

where $\mathcal{C}_{1,20}$ denotes the impulse responses of the variable $Z$ to an unanticipated 1% rise in the federal funds rate between the 1th and 20th quarters following the shock as shown in Figure 3.1 and $\rho_{\ell_{p_{1,20}}, \lambda_i}$ represents the correlations of the frequency of price changes in sectors($\lambda_i$) with the impulse responses of sectoral prices between the 1th and 20th quarters ($\mathcal{C}_{1,20}$) as in Figure 3.3. Lastly, $T$ stands for the sample size of the data used to estimate the VAR-based impulse responses. As a weighting matrix, I use the diagonal matrix whose diagonal elements are given by the inverse of standard errors of each term in $\hat{h}_T$. This matrix ensures more precisely estimated VAR-based correlations and impulse responses have larger weights when choosing parameters in (3.2.5).

3.2.3 Results

Before presenting the aggregate and disaggregated model-based dynamics following an unanticipated 1% increase in the federal funds rate and comparing the outcomes in the models with the VAR-based dynamics, I first report the structural parameter estimates in the models in Table 3.1. Woodford (2003) shows that the backward-looking indexation rule in prices ($\gamma_p$) and the habit persistence in consumption ($b$) induce hump-shaped dynamics after a monetary shock. Hence, high estimates for such parameters in Table 3.1 can be related to the hump-shaped dynamics of consumption in Figure 3.1. Low estimates for the intertemporal elasticity of substitution ($\varphi$) and the Frisch-elasticity of labor ($\sigma_H$) are consistent with the estimates reported in Hall (1988) and Boldrin, Christiano & Fisher (2001). The estimated value for the backward-looking indexation rule in wages ($\gamma_w$) is close to its
### Table 3.1: Estimates of Structural Parameters

<table>
<thead>
<tr>
<th></th>
<th>(a) One-Sector Model</th>
<th>(b) Multi-Sector Model With Symmetric Cost Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ϕ</strong></td>
<td>0.012</td>
<td>0.025</td>
</tr>
<tr>
<td><strong>σ_H</strong></td>
<td>0.740</td>
<td>0.001</td>
</tr>
<tr>
<td><strong>θ_p</strong></td>
<td>147.215</td>
<td>8.723</td>
</tr>
<tr>
<td><strong>θ_w</strong></td>
<td>19.970</td>
<td>6.030</td>
</tr>
<tr>
<td><strong>b</strong></td>
<td>0.800</td>
<td>0.641</td>
</tr>
<tr>
<td><strong>γ_p</strong></td>
<td>0.999</td>
<td>0.915</td>
</tr>
<tr>
<td><strong>γ_w</strong></td>
<td>0.999</td>
<td>0.157</td>
</tr>
<tr>
<td><strong>Obj. Func.</strong></td>
<td>117.27</td>
<td>246.02</td>
</tr>
</tbody>
</table>

**Note:** Obj. Func. indicates the estimated value for the minimization problem discussed in (3.2.5). A lower value of Obj. Func. indicates a more successful model for accounting for aggregate dynamics and correlations between the frequency of price changes and sectoral price dynamics after an unanticipated 1% increase in the federal funds rate.

upper theoretical limit of one, as Christiano, Eichenbaum & Evans (2005) assume. Lastly, the calibrated values for the price elasticity of substitution for sectoral goods (θ_p) and the wage elasticity of substitution among differentiated labor types (θ_w) in the literature vary enormously. Our estimates lie within the range of those calibrated values.

### Aggregate Dynamics

Figure 3.4 and Figure 3.5 show the impulse responses of the output gap, inflation, the real wage and the federal funds rate over five years after a 1% contractionary shock in the federal funds rate in the one-sector model and the multi-sector model with symmetric...
cost structure. It is evident from these figures that the impulse responses of the aggregate variables are similar in these models. Output is constant on impact following the shock, by construction. Starting with the first period, output falls. This emanates from the fact that a higher interest rate reduces consumption by making saving more desirable. The one-sector model fails to account for increased inflation following the shock. The fall in real wage in the models is the product of two factors. First, a fall in output lowers nominal wage demand. To explain this, note that less effort is needed when output falls, and since the disutility from working is a convex function of effort, workers lowers their nominal wage if firms demand less effort. Second, an increase in prices contributes to a fall in real wage in earlier periods. Excessive fall in real wage in the multi-sector model with symmetric cost structure can be accounted for by the second factor.

Model- and VAR-Based Correlations

The dynamics displayed in Figure 3.4 and Figure 3.5 reveal the DSGE models have similar predictions regarding the impulse responses of aggregate variables. In what they differ quite substantially is their prediction of the correlations of the frequency of price changes with the impulse responses of sectoral prices ($\rho_{C_{1,20},\lambda_i}$).

Figure 3.6 shows the correlations of the frequency of price changes ($\lambda_i$) with the impulse responses of sectoral prices after an unanticipated 1% increase in the federal funds rate ($C_{1,20}$). In the one-sector model, the correlations have to be zero by definition since all sectors have the same price response to the interest rate shock. In the multi-sector model with symmetric cost structure, since it is assumed that sectoral prices are unresponsive

---

It is notable that in the one-sector model, it is assumed that the measured frequency of price changes differs among sectors for sector-specific and aggregate shocks, except the federal funds rate shock. For the federal funds rate shock, on the other hand, it is assumed that the fraction of firms in all sectors that change their prices are the same. This implies when a contractionary interest rate shock occurs, prices in all sectors respond in the same way. Consequently, by construction, the correlations between the measured frequency of price changes and $C_{1,20}$ are equal to zero in the one-sector model.
Figure 3.4: Impulse Responses to an Unanticipated 1% Rise in $R_t$
(One-Sector Model)

(a) $y_t - y_t^*$

(b) $\pi_t$

(c) $w_t$

(d) $R_t$

Note: The solid lines in panels show the VAR-based impulse responses and the area between dotted lines indicate the 95% confidence intervals estimated with the method suggested by Sims & Zha (1999). The solid lines marked with circles represent the dynamic responses of the variables as predicted by the model.

to the shock on impact, the impact correlation is zero as illustrated in Panel (b) of the figure. Following the impact period, the correlations are positive for two quarters. This is unconventional as it implies that a tightening of monetary policy shock leads to a higher
Figure 3.5: Impulse Responses to an Unanticipated 1% Rise in $R_t$  
(Multi-Sector Model with Symmetrical Cost Structure)

(a) $y_t - y_t^*$  
(b) $\pi_t$  
(c) $w_t$  
(d) $R_t$

Note: The solid lines in panels show the VAR-based impulse responses and the area between dotted lines indicate the 95% confidence intervals estimated with the method suggested by Sims & Zha (1999). The solid lines marked with circles represent the dynamic responses of the variables as predicted by the model.

price response in sectors where firms change prices frequently than sectors where firms change prices infrequently. Starting in the third quarter following the contractionary shock, the correlations become negative. This suggests that a higher frequency of price change in
Figure 3.6: Model- and VAR-Based Correlations of $\lambda_i$ with $e^{\text{lnp}_{1,20}}$

(a) One-Sector Model

(b) The Multi-Sector Model with Symmetric Cost Structure

Note: The solid lines show the VAR-based correlations and the area between dotted lines indicate the 95% confidence intervals estimated with the method suggested by Sims & Zha (1999). The solid lines marked with circles represent the dynamic responses of the variables as predicted by the model.

A sector is associated with a lower price response in these periods.

It is notable that compared to the multi-sector model with symmetric cost structure, the one-sector model performs much better in explaining the correlations. However, this model may not explain the rich set of sectoral price dynamics following the interest rate shock displayed in Figure 3.2. The multi-sector model with symmetric cost structure, on the other hand, can explain the wide distribution of the responses of sectoral prices to such a shock; yet, this model is unable to explain the correlations.

Why does the multi-sector model with symmetric cost structure fail to explain $\rho_{e^{\text{lnp}_{1,20}i},\lambda_i}$? To answer this question, I now detail the price-setting behavior of the firms in each model. It is assumed that firms set prices before observing shocks to the interest rate. In each
sector, firms optimize their prices only when a price-change signal is received. The fraction of firms which receive this signal is different in each sector and is given by \( \lambda_i = 1 - \alpha_{ip} \) for the sector \( i \). It is well known that this fraction is equal to the probability of receiving a price change signal in each period in the Calvo (1983) model.

Let \( p_{it-1}(i') \) and \( P_t \) denote the last period price of the good produced by the firm \( i' \) in Sector \( i \) in the last period and the price of the composite consumption good, respectively. When no such signal is received, firms are assumed to set their prices according to the following partial adjustment backward-looking indexation rule:

\[
\tilde{p}_{it}(i') = p_{it-1}(i') \left( \frac{P_t - 1}{P_t - 2} \right)^{\gamma_p}
\]  

(3.2.7)

where the tilde over \( p \) denotes the price set according to the backward-looking indexation rule and \( \gamma_p \) shows the backward-looking indexation parameter. If \( 0 < \gamma_p < 1 \), there is partial backward-looking indexation in the economy.

When a firm is capable of setting an optimal price, it sets \( p_{it}(i')^* \) that maximizes:

\[
E_{t-1} \left( \sum_{s=0}^{\infty} \alpha_{ip}^s Q_{t,t+s} \Pi_{it+s}(i') \right)
\]

(3.2.8)

where \( Q_{t,t+s} \) is the stochastic discount factor between the period \( t \) and \( t + s \) and is given by:

\[
\Pi_{it+s}(i'), \text{ on the other hand, shows the profit of the firm } i' \text{ in the sector } i \text{ and is given by}\]

\[
\Pi_{it+s}(i') = p_{it+s,l}(i)y_{it+s}(i') - TC_{it+s}(i')
\]

(3.2.9)

\footnote{It is notable that since the firms are assumed to respond the monetary shocks with a one period delay, they have to condition the optimum price based on the information till the period \( t - 1 \) rather than the period \( t \). Correspondingly, \( E_{t-1} \) appears in the objective function, rather than \( E_t \) in (3.2.8).}
where \( p_{it+s,t}^*(i') \) shows the price set in period \( t+s \) by the firm that received a price-change signal at the period \( t \) and does not have an opportunity to set an optimal price between \( t \) and \( t+s \). Due to the backward-indexation rule, one can write \( p_{it+s,t}^*(i') \) as:

\[
p_{it+s,t}^*(i') = p_{it}^*(i') \chi_{t,t+s}^P
\]

where

\[
\chi_{t,t+s}^P = \begin{cases} 
\prod_{k=1}^{s} \left( \frac{P_{t+k-2}}{P_{t+k-1}} \right)^{\gamma_p} & \text{if } s \geq 1 \\
1 & \text{if } s = 0 
\end{cases}
\]

\( TC_{it}(i') \) in (3.2.9), on the other hand, denotes the total cost of the firm. In the one-sector model and the multi-sector model with symmetric cost structure, all firms have the same total cost structure. Since firms are assumed to pay the wage bill in advance, \( TC_{it}(i') \) for these models can be written as:

\[
TC_{it}(i') = R_t W_t L_{it}(i')
\]

where \( W_t \) and \( L_{it}(i') \) denote the nominal aggregate wage and the labor demanded by the firm \( i' \) in the sector \( i \), respectively.

The optimality condition in (3.2.8) for \( p_{it}^*(i') \) can be expressed as:

\[
E_{t-1} \left( \sum_{s=0}^{\infty} \alpha_{p_t} Q_{t,t+s} \frac{d\Pi_{it+s}(i')}{dp_{it}^*(i')} \right) = 0
\]

Using (3.2.13), one can show the sectoral price inflation \( (\pi_{it}) \) in the multi-sector model with symmetric cost structure evolves according to the following equation:
\[ \pi_t - \gamma p \pi_{t-1} = -\xi ip (1 + \omega p \theta p) E_{t-1} \left( \hat{P}_t - \hat{P}_t \right) + \xi ip E_{t-1} \left( \hat{R}_t + \hat{w}_t + \omega p \hat{y}_t \right) + \beta E_{t-1} \left( \pi_{t+1} - \gamma p \pi_t \right) \]

(3.2.14)

where \( \xi ip \) is the stickiness parameter in each sector and is defined as:

\[ \xi ip = \frac{1 - \alpha ip}{\alpha ip} \frac{1 - \beta \alpha ip}{1 + \omega p \theta p}, \quad \omega p = \frac{1 - \kappa}{\kappa} \]

(3.2.15)

In (3.2.15), \( \omega p \) and \( \kappa \) denote the elasticity of prices with respect to the supply of goods when interest rate and wages paid for composite hours of work stay the same and the reciprocal of the output elasticity of labor demand, respectively. The aggregate inflation equation in the multi-sector model is a weighted average of sectoral inflation in the economy:

\[ \pi_t = \sum_{i=1}^{M} f_i \pi_{it} \]

(3.2.16)

where \( f_i \) is the sector’s share of aggregate consumption expenditure at the steady state.

The equation for \( \pi_t \) in the one-sector model, on the other hand, is given by:

\[ \pi_t - \gamma p \pi_{t-1} = \xi p E_{t-1} \left( \hat{R}_t + \hat{w}_t + \omega p \hat{y}_t \right) + \beta E_{t-1} \left( \pi_{t+1} - \gamma p \pi_t \right) \]

(3.2.17)

\[ \xi p = \frac{1 - \alpha p}{\alpha p} \frac{1 - \beta \alpha p}{1 + \omega p \theta p} \]

(3.2.18)

where \( 1 - \alpha p \) denotes the frequency of price changes in the model economy. It is measured as the median frequency of price changes in the United States.

Here, I aim to explain the strong positive correlations of the frequency of price changes with the sectoral price responses following the shock \( (\rho \omega^{in p}_{i,20} \lambda_i) \) in earlier periods and strong
negative correlations in later periods. I explain this by studying how firms in each sector set prices when they have a chance to optimize.

First, one can show from (3.2.13) that the following equation holds:

\[ E_{t-1} \left( \sum_{s=0}^{\infty} \alpha_{ip}^s Q_{t,t+s} \chi_{t,t+s}^p \left( \frac{P_{t+s}}{P_{t+s,t}(i')} \right)^{1+\theta_p} y_{t+s} \left( \frac{P_{it+s,t}(i') P_{it+s}}{P_{t+s}} - \mu_p \frac{S_{it+s}(i') P_{it+s}}{P_{t+s}} \right) \right) = 0 \] (3.2.19)

where \( \mu_p \geq 1 \) (i.e. \( \theta_p \geq 1 \)) shows the steady-state markup. \( S_{jt+s}(j') \) denotes the marginal cost of the firm. Letting \( y_{it}(i') \) be the output of the firms, \( S_{jt+s}(j') \) is defined as:

\[ S_{it+s}(i') = \frac{\partial TC_{it}(i')}{\partial y_{it}(i')} \] (3.2.20)

Log-linearizing (3.2.19) yields:

\[ E_{t-1} \sum_{s=0}^{\infty} \beta \alpha_{ip}^s \left[ \hat{p}_{it}(i') - \hat{P}_{it+s} + \hat{P}_{it+s} - \hat{P}_{t+s} + \hat{\chi}_{t,t+s}^p - \left( \hat{R}_{t+s} + \hat{w}_{t+s} + \omega_p \hat{y}_{it+s}(i') \right) \right] = 0 \] (3.2.21)

where \( \hat{\chi}_{t,t+s}^p \) is the log-deviation of \( \chi_{t,t+s}^p \) from its steady state and is given by:

\[ \hat{\chi}_{t,t+s}^p = \begin{cases} \gamma_p \pi_t + \gamma_p \pi_{t+1} + \cdots + \gamma_p \pi_{t+s-1} & \text{if } s \geq 1 \\ 0 & \text{if } s = 0 \end{cases} \] (3.2.22)

Using the approximation that \( \hat{P}_{t+s} = \hat{P}_t + \pi_{t+s} + \sum_{k=0}^{s-1} \pi_{t+k} - \pi_t \) for \( s \geq 1 \), (3.2.21) can be restated as:
\[
\hat{p}^*_it(i') = E_t-1\hat{P}_t + (1 - \beta\alpha_{ip})E_t-1\sum_{s=1}^{\infty}(\beta\alpha_{ip})^s(\pi_{t+s} + (1 - \gamma_p)\sum_{k=0}^{s-1}\pi_{t+k} - \pi_t)
+ (1 - \beta\alpha_{ip})E_t-1\hat{s}_{it} + (1 - \beta\alpha_{ip})E_t-1\sum_{s=1}^{\infty}(\beta\alpha_{ip})^s\hat{s}_{it+s}
\]

(3.2.23)

where \(\hat{s}_{it+s}\) shows the log-deviation of the real marginal cost of the firm from its steady state and is given by:

\[
\hat{s}_{it+s} = \hat{R}_{t+s} + \hat{w}_{t+s} + \omega_p\hat{y}_{it+s}(i')
\]

(3.2.24)

(3.2.23) can be rewritten as:

\[
\hat{p}^*_it(i') = E_t-1\hat{P}_t - \gamma_p\beta\alpha_{ip}E_t-1\pi_t + E_t-1\sum_{s=1}^{\infty}(\beta\alpha_{ip})^s((1 - \gamma_p\beta\alpha_{ip})\pi_{t+s})
+ E_t-1\hat{s}_{it} + E_t-1\sum_{s=1}^{\infty}(\beta\alpha_{ip})^s(\hat{s}_{it+s} - \hat{s}_{it+s-1})
\]

(3.2.25)

Two points must be emphasized regarding (3.2.25). First, as \(\gamma_p\) increases, firms give less importance to inflation in subsequent periods. When \(\gamma_p = 1\), they set prices such that importance given to inflation in subsequent periods is minimized. An intuitive explanation can be given for this: The fact that the prices are optimized only if the Calvo signal is received leads firms to take preemptive measures against expected inflation in subsequent periods. When the degree of backward-looking indexation is high in an economy, firms are able to change prices by taking into account inflation realized in the previous period even if prices are not optimized. This results in a decrease in the degree of firms’ preemptive measures against expected inflation in the subsequent periods.

Second, when expected real costs are higher in subsequent periods than today, the percentage increase in prices is higher than the percentage change in the current real marginal costs.
cost, holding fixed expected inflation in subsequent periods.\footnote{It is notable that since the effect of $\hat{p}_t(i')$ on $E_{t-1}\hat{P}_t$ is negligible, when setting prices, the firms treat $E_{t-1}\hat{P}_t$ as constant. Hence, whether the percentage change in the optimized prices outweighs that of the real marginal cost today depends entirely on the statement above.} Christiano, Eichenbaum & Evans (2005) refer to this as firms “front-load” for the expected real cost increases in subsequent periods in which the chance to optimize their prices is uncertain. The “front-loading” is most relevant for firms in a sector where the frequency of price changes is lower ($\alpha_{ip}$ is higher) since a higher frequency of price change discounts the importance of real marginal costs in subsequent periods on prices set by a firm when a Calvo signal is received. For example, consider an unanticipated rise in interest rate. A rise in today’s marginal costs is likely because of the working capital channel in the model. Yet, as interest rate returns to its undistorted level and output and wages decrease, marginal costs are bound to fall in subsequent periods. In the flexible-price sector where firms can optimize prices often, marginal costs today have a decisive effect on prices set. For firms in the sector where price flexibility is low, on the other hand, the extent that marginal costs in subsequent periods are taken into account in price setting is much larger. I illustrate the “front loading” argument in Figure [3.7]. In this figure, the sticky- and flexible-price sectors are defined as the percentile groups with the lowest and highest frequency of price changes among 10 groups in the model, respectively. It is evident from this figure that a contractionary monetary shock results in an initial fall in prices in the sticky-price sector and an initial rise in prices in the flexible-price sector. This results in a positive correlation between the frequency of price changes and sectoral price responses in the early periods following the contractionary monetary shock.

In subsequent periods, firms’ marginal costs fall markedly due to a persistent fall in the real wage, output and the interest rate. Because of a higher price flexibility in the flexible-price sector, prices in this sector fall more pronouncedly than those in the sticky-price
sector during these periods. This explains the finding in Figure 3.6 that a higher frequency of price change is associated with a lower price response in the multi-sector model with symmetric cost structure in the third period and onwards.

As evident in Figure 3.6, while the multi-sector model with symmetric cost structure explains the correlations successfully in qualitative terms, an undesirable feature of this model is that the correlations predicted by the model are too high compared to those found in the data. This is a natural consequence of the fact that sectors in the model are assumed to be identical apart from their frequency of price changes. With this assumption, price responses in sectors in any period are ordered to a large extent according to sectoral frequency of price changes after a contractionary interest rate shock. Figure 3.8 illustrates
**Figure 3.8: Model-Based $c_{2,3}^{lnp_t}$**
*(The Multi-Sector Model with Symmetric Cost Structure)*

**Note:** The points with numbers inside the figures show the model-based price responses in each sector to a 1% contractionary shock in the federal funds rate in the first and second quarters. Sectors in the figure are ordered according to the frequency of price changes from highest to lowest. For example, 1 in the figure denotes the price response of the sector with the highest frequency of price changes in the first and second periods.

this. In Panel (a) of the figure, the model-based sectoral price responses in the first and second periods are shown for the multi-sector model with symmetric cost structure. Sectors are indicated by numbers and are ordered according to their frequency of price changes from highest to lowest. For example, 1 in the figure denotes the price response of the sector that has the highest frequency of price changes among sectors. As evident from Panel (a) of the figure, price responses in sectors in the first and second periods are largely ordered according to sectoral frequency of price changes. This causes the predicted correlations between the frequency of price changes and the sectoral price responses in the model to be exceedingly high compared to those in the data.
Briefly, the discussion in this section notes that the strong positive correlations between the frequency of price changes and sectoral price responses in the initial periods and the strong negative correlations in subsequent periods are a direct consequence of the front-loading argument. However, such strong correlations conflict with the low VAR-based correlations shown in Figure 3.3.

3.2.4 The Multi-Sector Model with Asymmetric Cost Structure

In this section, I show that when there is asymmetry in the cost structure of firms in different sectors, not only is it possible to account for the low correlations in the data, but one can also explain the wide distribution of sectoral price responses to the shock which is evident in Figure 3.2.

I consider a multi-sector model with asymmetric output elasticity of labor demand where sectors differ not only in terms of price flexibility but also in terms of their cost structure since production functions used by firms differ among sectors in this model. This contrasts sharply with the multi-sector model with symmetric cost structure where firms use the same production function. To explain this model, it is useful to first write the production function that firms use to produce their output \( y_{it}(i') \) in the multi-sector model with symmetric cost structure,

\[
y_{it}(i') = Z_t H_{it}(i')^\kappa \quad 0 < \kappa < 1
\]  

(3.2.26)

where \( Z_t \) and \( H_{it}(i') \) denote the technology level and the demand of the firm for the composite labor, respectively. Lastly, \( \kappa \) denotes the reciprocal of the output elasticity of labor demand in sectors which is assumed to be identical among all sectors. In the multi-sector model with asymmetric cost structure, a differential \( \kappa_i \) for each sector is considered:
\[ y_{it}(i') = Z_t H_{it}(i')^{\kappa_i} \] (3.2.27)

Except the sectoral inflation equation \((\pi_{it})\) and the nominal wage inflation equation \((\pi_{it}^w)\), the structural equations are the same as in Section 3.2.1. The only change in the sectoral inflation equation is that \(\kappa\) in (3.2.15) should be replaced by \(\kappa_i\). \(\pi_{it}^w\), on the other hand, is now given as:

\[
\pi_{it}^w - \gamma_w \pi_{t-1} = \xi_p E_{t-1} \left( \sigma_{H^{-1}} \dot{H}_t + \varphi^{-1} \dot{H}_t - \dot{w}_t \right) + \beta E_{t-1} \left( \pi_{t+1}^w - \gamma_w \pi_t \right)
\] (3.2.28)

where \(\dot{H}_t\) denotes the total composite labor demand. Let \(\dot{Y}_{it}\) and \(n_i\) stand for sectoral output and the sectoral weight in total output, respectively. Then, \(\dot{H}_t\) can be written as:

\[
\dot{H}_t = \sum_{i=1}^{10} n_i \dot{Y}_{it}
\] (3.2.29)

where sectoral output is a function of sectoral relative price and total output:

\[
\dot{Y}_{it} = -\theta \left( \dot{P}_{it} - \dot{P}_t \right) + \dot{Y}_t
\] (3.2.30)

**Calibration for the Multi-Sector Model with Asymmetric Cost Structure**

It is notable that in the multi-sector model with symmetric cost structure, the number of sectors is reduced to 10 since it is impractical to solve the model if all disaggregated sectors, for which the frequency of price changes is available, are included. When grouping disaggregated sectors into 10 groups, sectors are ordered by their frequency of price changes, and they are included in one of the ten groups. The frequency of price changes in a sector
is then approximated by the median frequency of price changes in its group. Since sectors in the *multi-sector model with symmetric cost structure* only differ in their frequency of price changes and the frequency of price changes in all sectors contained in a group is approximated by the median frequency of price changes in that group, sectors within the same group must have the same sectoral inflation equation.

However, in the *multi-sector model with asymmetric cost structure*, grouping disaggregated sectors based only on the frequency of price changes may not be justified. This results from the fact that even when such sectors have a similar frequency of price changes, sectoral price dynamics following a monetary shock may be markedly dissimilar if their output elasticity of labor demand largely differs. Consequently, in the *multi-sector model with asymmetric cost structure*, both the frequency of price changes and labor shares in sectors are needed to solve the model. To calibrate these parameters, we first match 124 PCE categories, for which the frequency of price changes is available and whose price responses are shown in Figure 3.2, with the industries reported by Close & Shulenburger (1971). If an industry is matched with only one PCE category, the frequency of price changes in that industry is taken as the one in the PCE category. If there are multiple PCE categories that match with a single industry, the frequency of price changes in this industry is measured as the weighted average of the frequency of price changes in these PCE categories, with the weights given as the sum of the expenditure shares of the ELIs in 2000 that are mapped with the PCE categories in Section 3.1.2. Labor shares in industries are calibrated as those in 1948 reported by Close & Shulenburger (1971). Weights of each industry are calibrated as the sum of the weights of the PCE categories that match with the industry. However, some PCE categories may not be matched with an industry, caus-

---

34 It may be useful here to exemplify our matching. For example, the PCE categories “Tires” and “Accessories and parts” are matched with the industry of “Motor vehicles and equipment” in Close & Shulenburger (1971).
ing the sum of the industries’ weights to be less than one. Consequently, the weights of the industries need to be rescaled so that their sum is equal to one. Table 3.2 reports the calibrated values for industries’ labor share, the frequencies of price changes and weight. It is notable that while the petroleum and air-transportation industries have virtually the same frequency of price changes, they markedly differ in their labor shares.

In Table 3.3 I report the structural parameter estimates in the multi-sector model with asymmetric cost structure. It is notable that the value of the objective function in the multi-sector model with asymmetric cost structure is lower compared to the ones in the one-sector model and multi-sector model with symmetric cost structure as reported in Table 3.1 suggesting that the multi-sector model with asymmetric cost structure is the most successful in accounting for the aggregate dynamics and the correlations.

In Figure 3.9 I display the aggregate dynamics in the multi-sector model with asymmetric cost structure, after an unanticipated 1% increase in the federal funds rate, which are largely in conformity with the aggregate dynamics in the previous two models as shown in Figure 3.4 and Figure 3.5.

However, as evident in Figure 3.10 the correlations between the frequency of price changes and sectoral price responses in the multi-sector model with asymmetric cost structure differ markedly from those in the multi-sector model with symmetric cost structure. As a matter of fact, contrasting with the latter, the former can more successfully explain the low VAR-based correlations in the data.

As a matter of fact, contrasting with the latter, the former can more successfully explain the low VAR-based correlations in the data.

This can be attributed to the fact that when the asymmetric cost structure across industries is introduced in the multi-sector model, sectoral price responses may delink from sectoral frequency of price changes following the contractionary monetary shock. This point

---

35 The frequency of price changes in the petroleum and air-transportation industries are 0.97 and 0.94 in the former and latter, respectively

36 The labor shares in the petroleum and air-transportation industries are 0.32 and 0.88 in the former and the latter, respectively
Table 3.2: Calibrated Parameters  
(The Multi-Sector Model with Asymmetric Cost Structure)

<table>
<thead>
<tr>
<th>Industry</th>
<th>Labor Share</th>
<th>Frequency</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>0.74</td>
<td>0.66</td>
<td>0.143</td>
</tr>
<tr>
<td>Tobacco</td>
<td>0.52</td>
<td>0.69</td>
<td>0.019</td>
</tr>
<tr>
<td>Textiles</td>
<td>0.80</td>
<td>0.64</td>
<td>0.002</td>
</tr>
<tr>
<td>Apparel</td>
<td>0.87</td>
<td>0.66</td>
<td>0.064</td>
</tr>
<tr>
<td>Paper</td>
<td>0.64</td>
<td>0.58</td>
<td>0.003</td>
</tr>
<tr>
<td>Printing</td>
<td>0.79</td>
<td>0.20</td>
<td>0.013</td>
</tr>
<tr>
<td>Chemicals</td>
<td>0.56</td>
<td>0.39</td>
<td>0.030</td>
</tr>
<tr>
<td>Petroleum</td>
<td>0.32</td>
<td>0.97</td>
<td>0.060</td>
</tr>
<tr>
<td>Furniture</td>
<td>0.82</td>
<td>0.51</td>
<td>0.023</td>
</tr>
<tr>
<td>Fabricated metal</td>
<td>0.78</td>
<td>0.37</td>
<td>0.000</td>
</tr>
<tr>
<td>Electrical machinery</td>
<td>0.77</td>
<td>0.50</td>
<td>0.019</td>
</tr>
<tr>
<td>Transportation equipment and ordinance</td>
<td>0.89</td>
<td>0.21</td>
<td>0.035</td>
</tr>
<tr>
<td>Motor vehicles and equipment</td>
<td>0.63</td>
<td>0.60</td>
<td>0.049</td>
</tr>
<tr>
<td>Instruments</td>
<td>0.79</td>
<td>0.25</td>
<td>0.001</td>
</tr>
<tr>
<td>Miscellaneous manufacturing industries</td>
<td>0.75</td>
<td>0.41</td>
<td>0.011</td>
</tr>
<tr>
<td>Railroad trasportation</td>
<td>0.82</td>
<td>0.56</td>
<td>0.001</td>
</tr>
<tr>
<td>Local, suburban, highway passanger transportation</td>
<td>0.87</td>
<td>0.14</td>
<td>0.005</td>
</tr>
<tr>
<td>Water transportation</td>
<td>0.87</td>
<td>0.65</td>
<td>0.001</td>
</tr>
<tr>
<td>Air transportation</td>
<td>0.88</td>
<td>0.94</td>
<td>0.014</td>
</tr>
<tr>
<td>Trasporation services</td>
<td>0.80</td>
<td>0.29</td>
<td>0.002</td>
</tr>
<tr>
<td>Telephone and telegraph</td>
<td>0.80</td>
<td>0.65</td>
<td>0.037</td>
</tr>
<tr>
<td>Radio broadcasting and television</td>
<td>0.83</td>
<td>0.34</td>
<td>0.014</td>
</tr>
<tr>
<td>Electric, gas, and sanitary services</td>
<td>0.55</td>
<td>0.71</td>
<td>0.067</td>
</tr>
<tr>
<td>Wholesale trade</td>
<td>0.71</td>
<td>0.27</td>
<td>0.005</td>
</tr>
<tr>
<td>Retail trade</td>
<td>0.6</td>
<td>0.38</td>
<td>0.008</td>
</tr>
<tr>
<td>Hotels and other lodging places</td>
<td>0.69</td>
<td>0.75</td>
<td>0.040</td>
</tr>
<tr>
<td>Personal services</td>
<td>0.64</td>
<td>0.12</td>
<td>0.028</td>
</tr>
<tr>
<td>Miscellaneous business services</td>
<td>0.7</td>
<td>0.18</td>
<td>0.136</td>
</tr>
<tr>
<td>Automobile repair</td>
<td>0.62</td>
<td>0.42</td>
<td>0.016</td>
</tr>
<tr>
<td>Miscellaneous repair services</td>
<td>0.48</td>
<td>0.22</td>
<td>0.001</td>
</tr>
<tr>
<td>Motion pictures</td>
<td>0.77</td>
<td>0.44</td>
<td>0.001</td>
</tr>
<tr>
<td>Amusements</td>
<td>0.75</td>
<td>0.20</td>
<td>0.004</td>
</tr>
<tr>
<td>Medical and other health services</td>
<td>0.4</td>
<td>0.13</td>
<td>0.061</td>
</tr>
<tr>
<td>Educational Services</td>
<td>0.90</td>
<td>0.18</td>
<td>0.037</td>
</tr>
<tr>
<td>Nonprofit membership organizations</td>
<td>0.98</td>
<td>0.25</td>
<td>0.025</td>
</tr>
<tr>
<td>Miscellaneous professional services</td>
<td>0.58</td>
<td>0.15</td>
<td>0.013</td>
</tr>
</tbody>
</table>

Note: Labor shares in industries are calibrated from the labor shares in industries in 1948 as reported by Close & Shuilenburger (1971). See text for explanations related with the frequency of price changes in industries and the weight of industries.

is illustrated in Figure 3.11 where the sectoral inflation dynamics in the petroleum and air-transportation industries. As suggested by the frequency of price changes in Table 3.2.
Table 3.3: Estimates of Structural Parameters  
(Multi-Sector Model with Asymmetric Cost Structure)

<table>
<thead>
<tr>
<th></th>
<th>ϕ</th>
<th>σ_H</th>
<th>θ_p</th>
<th>θ_w</th>
<th>b</th>
<th>γ_p</th>
<th>γ_w</th>
<th>Obj. Func.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.014</td>
<td>0.001</td>
<td>81.4</td>
<td>12.68</td>
<td>0.78</td>
<td>0.97</td>
<td>0.87</td>
<td>53.59</td>
</tr>
</tbody>
</table>

Note: Obj. Func. indicates the estimated value for the minimization problem discussed in (3.2.5).

almost all firms in both industries optimize their prices each month. However, the former has a much lower labor share than the latter. For this reason, in Figure 3.11, the former and the latter are labeled as the low and high labor-share industries, respectively. As evident in the figure, inflation dynamics initially differ largely in the former and the latter despite having virtually the same frequency of price changes. Indeed, while inflation is almost unchanged in the former one period after the shock, inflation in the latter shows a strong increase one period after the shock, suggesting sectoral price responses may substantially differ across industries with a similar frequency of price changes in the multi-sector model with asymmetric cost structure, causing the association between the frequency of price changes and sectoral price responses in the sectors following a contractionary shock to be low compared to that in the multi-sector model with symmetric cost structure.  

37 This can be explained as follows: Since the frequency of price changes in both of the industries is almost one, it is reasonable to assume firms optimize their prices each period. Under this assumption, it can be shown that firms set prices relative to the aggregate price by imposing some constant mark up over real marginal costs \(s_{it+s}(i')\) which can be written in its log-deviation as:

\[
\delta_{it+s}(i') = \frac{1 - \kappa_i}{\kappa_i} \tilde{y}_i(i') + \tilde{R}_t + \tilde{w}_t
\]

A contractionary shock has two effects on prices which work in opposite directions. The first is that marginal costs increase due to the working-capital channel in the model and an increase in \(\tilde{R}_t\). The second is that a fall in output results in a fall in marginal costs. The second effect is more decisive in the low labor-share industry since the real marginal costs faced by firms in the low labor-share industry would fall much more markedly compared to those in the high labor-share industry for a given fall in their output as the former
Figure 3.9: Impulse Responses to an Unanticipated 1% Rise in $R_t$
(Multi-Sector Model with Asymmetric Cost Structure)

(a) $y_t - y_t^n$

(b) $\pi_t$

(c) $w_t$

(d) $R_t$

Note: The solid lines show the VAR-based impulse responses and the area between dotted lines indicate the 95% confidence intervals estimated with the method suggested by Sims & Zha (1999). The solid lines marked with circles represent the dynamic responses of the variables as predicted by the model.

In addition to bringing the correlations closer to those found in the data, the multi-sector model has much lower $\kappa_i$. This explains why prices in the low labor-share industry fall, while they increase strongly in the high labor-share industry one period after the shock.
Figure 3.10: Model- and VAR-Based Correlations of $\lambda_i$ with $C_{1,20}$

Note: The solid lines show the VAR-based $\rho_{c_{1,20}, \lambda_i}$ and the area between dotted lines indicates the 95% confidence interval for $\rho_{c_{1,20}, \lambda_i}$ that is estimated using the block-bootstrap method described in Appendix B.2. The solid lines marked with circles represent $\rho_{c_{1,20}, \lambda_i}$ predicted by the model.

sector model with asymmetric cost structure can also explain the wide distribution of sectoral price responses to a contractionary interest rate shock displayed in Figure 3.2. My findings in this section suggest adding asymmetries in the cost structure is crucial in explaining the low correlations between the frequency of price changes and the sectoral price responses to an interest rate shock.

3.3 Conclusion

In this paper, the implications of heterogeneity in price flexibility at the disaggregated level are studied. I have found that price responses to an unanticipated change in the interest
rate differ substantially among sectors. Based on this finding, it is safe to claim that interest rate shocks have strong relative price effects at the disaggregate level. Next, I have investigated whether this differential price response across sectors can be associated with the wide distribution of the frequency of price changes in the United States. The findings in this paper indicate that the association is weak. Lastly, the performances of three DSGE models are evaluated in explaining the aforementioned findings in the empirical section. It has been shown that the one-sector model may not explain the wide distribution of sectoral price responses following the shock. It is possible to account for this finding by using a multi-sector model where sectors differ only in their frequency of price changes. However, contrary to the weak association of the frequency of price changes with sectoral price responses in the data, this model predicts a strong correlation between these variables. For this reason, an alternative multi-sector model has been considered. In this model,
sectors differ not only in the frequency of price changes but also in their cost structure. Such sectoral asymmetries in the cost structure and the frequency of price changes have proved important in successfully explaining the strong relative price effects of the interest rate shock and the weak association of the frequency of price changes with sectoral price responses in the data.
Chapter 4

Which Type of Model Best Captures the Effects of Monetary Shocks in Developing Countries?

There are many developing countries that have been exploring alternative monetary regimes after years of high and variable inflation. However there remains considerable debate regarding the appropriate framework for analyzing monetary policy in such an environment. In particular, these economies are different on many fronts from those of more developed countries, and therefore monetary models appropriate for the most developed countries are unlikely to be appropriate for developing countries.

The goal of this essay is to develop a model which is appropriate for monetary policy analysis in developing economies. Obviously, among many candidate models, the models in which key variables respond very differently than they do in actual economies to the same type of shock studied are not appropriate for such an analysis. To reduce the number of candidate models, Christiano, Eichenbaum & Evans (1998) suggest applying the Lucas program. Here, we follow this advice and apply the Lucas program using monetary shocks. This involves three steps. First, we attempt to isolate monetary shocks in developing economies which adopted an inflation targeting regime. In the second step, we study the dynamic behavior of output, the price level, the real and exchange rates in developing
economies following an expansionary monetary shock that results in a 1% increase in the price level in the long-run. In the last step, the same experiment is conducted in two different model environments and the outcomes in these models are compared with those in actual economies. The model in which the outcomes fit best with those in actual economies is nominated as a viable candidate for monetary policy analysis in developing economies.

Now, we elaborate on each of these steps. In the first step, for identifying monetary shocks in developing economies under inflation targeting, we make two assumptions. The first is that monetary shocks have no effect on the level of real variables in the long-term. This assumption is consistent with a broad class of models where monetary shocks have no long-run effect on real variables. The second identifying assumption is that monetary shocks in developing economies do not affect the aggregate price level in the United States in the long-term. This assumption is in conformity with the small-country assumption for developing economies which is often made in the literature. With these assumptions, we show monetary shocks can be isolated.

Alternatively, the recursive assumption may be invoked to isolate monetary shocks. This method requires placing short-run restrictions on the contemporaneous response of variables as opposed to the long-term restrictions in our method. At this point, it is useful to review the recursive assumption and discuss the reasons why adopting the recursive assumption may be unsuitable for isolating monetary shocks in developing countries. In the recursive assumption, a monetary authority is assumed to set its operating instrument by observing movements in two different sets of variables. The first set of variables contains variables that may respond only with a lag to monetary policy shocks and whose current values are known to the monetary authority before a decision on its operating instrument.

\footnote{A 1% increase in the price level in the long-run is just a normalization. Indeed, it is by assumption in the empirical model that monetary shocks must result in an increase in the price level. We simply normalize the shock so that it induces a 1% increase in the price level in the long-run.}
is made. The second set of variables, on the other hand, consists of variables that may contemporaneously respond to monetary policy shocks and whose current values are unknown to the monetary authority before setting its operating instrument. The necessity of including variables in one of these sets lies at the root of the controversy over the recursive assumption for identifying shocks to monetary policy in developing economies. For example, in which set should the price level be included? Including it in the first set implies prices are sluggish in responding to monetary policy shocks. Such an assumption would be in conflict with the fact that a considerable share of prices change in a typical month in developing economies. Additionally, because of the fast response of exchange rates to monetary policy shocks and the strong pass-through of exchange rates into import prices in developing countries, it is plausible to assume that monetary shocks affect prices contemporaneously through their effect on exchange rates. Consequently, including the price level in the first set of variables is questionable. Including it in the second set of variables, however, necessitates the assumption that central banks do not observe current values of the price level before setting their operating instrument. However, they collect data on a large volume of prices and are likely to predict the general trend in prices over any period. For this reason, including the price level in the second set of variables is questionable, too. In our view, the price level in developing economies belongs to neither the first nor the second set of variables. Yet, that it has to be included in either of the two sets if the recursive assumption is adopted for isolating monetary shocks in developing countries lead us to abandon this strategy.

In the next step, we characterize our experiment. We study how output, the bilateral real and nominal exchange rates with the United States and consumer prices move in developing countries under the inflation targeting regime after an expansionary domestic monetary shock that results in a 1% long-run increase in the price level. We find this
shock is characterized by a temporary rise in output, a short-lived depreciation in the real exchange rate, a sizable overshooting of the nominal exchange rate and a 0.5% contemporaneous increase in the consumer prices in these countries.

Our findings of short-lived effects from monetary shocks on output and the real exchange rates in developing economies contrast sharply with the long-lasting and persistent effects of monetary shocks on such variables in advanced economies. For example, while Christiano et al. (2005) find the effect of a monetary shock on output in the United States dissipates in about three years, we find the effect from a monetary shock on output becomes negligible in less than one year in developing economies. Similarly, Rogoff (1996) finds shocks to the real exchange rates in advanced economies have a half-life of three to five years, whereas we find in this study that shocks to the real exchange rate in developing economies have a half-life of less than a year. In addition, the speed of price adjustment is different between advanced and developing economies. In fact, while the inertial character of inflation results in a slow price adjustment in advanced countries, we find price adjustment is fast in developing economies. Moreover, the extent to which inflation has inertia is greatly limited in developing economies. As a matter of fact, prices adjust half-way, or more, within the same period as the shock and the full price adjustment occurs in only one year. We show such short-lived real effects and faster price adjustment following a monetary shock in developing economies can be traced to the higher pass-through of exchange rates into import prices, the fact that import prices are largely denominated in the foreign currency and the fact that prices change more frequently in developing economies.

In the last step, we turn to assess the ability of two dynamic stochastic general equilibrium models to explain these findings. Before describing their differences, we discuss their six common features. First, there are Calvo-type nominal price contracts. Second, the frequency of price changes differs between the home and foreign countries. Third,
less of being domestic or foreign, if a firm sets prices in the home (foreign) currency, it is subject to the price rigidity in the home (foreign) country. Fourth, insurance is incomplete as households in both domestic and foreign countries only have access to the non-state contingent foreign asset. Fifth, in regards to the real side of the models, both of the models maintain that acquiring new capital is subject to adjustment costs and capacity utilization can be variable. Sixth, they incorporate staggered wage setting.

Next, we describe how these models differ. The first model is a one-sector model with identical firms that have the same frequency of price changes. In contrast, the second model is a multi-sector model with heterogeneous firms which have different frequencies of price changes. Indeed, while prices remain unchanged for long durations in some sectors, they change frequently in others. We then compare the outcomes in these models to those in the actual economies after the monetary shock that causes a 1% long-run increase in the price level. We find the latter is particularly accurate in accounting for the aggregate dynamics in the actual economies.

The organization of the paper is as follows: Section 4.1 presents our empirical strategy for isolating monetary shocks in developing economies and reports our findings on the consequences of monetary shocks in developing economies with the inflation targeting regime. Section 4.2 develops two dynamic stochastic sticky price small-open economy models. Section 4.3 describes the estimation and calibration of the models’ parameters. Section 4.4 evaluates the success of the models in accounting for the outcomes of a domestic monetary shock in the actual economies that are reported in Section 4.1. The last section concludes.

4.1 Empirical Section

In this section, we develop an empirical model for studying the dynamics of output, the real exchange rate and the price level in developing countries under inflation targeting following
a positive monetary shock. In the next section, we consider an empirical model for isolating monetary shocks that closely follows the strategy in Clarida & Gali (1994). However, since monetary shocks in developing countries and the United States are not identified separately, we argue in our second empirical model that this strategy is questionable. Next, we develop an empirical model which enables us to study monetary shocks in developing countries and the United States separately.

4.1.1 Empirical Models

Empirical Model I

By employing a Blanchard & Quah (1989) type decomposition, Clarida & Gali (1994) identify various structural shocks in four developed countries. In contrast to their concentration on developed countries, our focus is on developing economies. We first consider an empirical model based on the strategy in Clarida & Gali (1994). However, as opposed to estimating a VAR model for each country as in Clarida & Gali (1994), we estimate the following panel VAR model for the group of developing countries under inflation targeting,

\[ X_{i,t} = \sum_{p=1}^{p_{\text{max}}} B_p X_{i,t-p} + \mu_i + u_{i,t} \] (4.1.1)

where \( \mu_i \) is the time-invariant country-specific fixed-effect term and \( p_{\text{max}} \) denotes the number of lags included in the panel VAR regression. We use both quarterly and monthly data to estimate (4.1.1) with the lag lengths chosen to be four and twelve, respectively. The endogenous variables in the panel VAR system of (4.1.1), \( X_{it} \), consist of three variables,

\[
X_{i,t} = \begin{bmatrix}
\Delta y_{i,t} - \Delta y^*_{i,t} \\
\Delta Q_{i,t} \\
\Delta P_{i,t} - \Delta P^*_{i,t}
\end{bmatrix}
\] (4.1.2)
where \( \Delta y_{i,t} - \Delta y_t^* \) is the difference between the log-changes in economic activity in the country of interest and the United States. For the quarterly data, we measure \( \Delta y_{i,t} - \Delta y_t^* \) with real GDP differences in Economy \( i \) and the United States as in Clarida & Gali (1994). For the monthly data, on the other hand, we measure it with the differences in industrial production indexes between Economy \( i \) and the United States. The second variable in (4.1.2), \( \Delta Q_{i,t} \), denotes the percentage change in the bilateral real exchange rate of the country of interest with the United States. \( Q_{i,t} \) is defined as the cost of the consumption basket in the United States relative to that in the country of interest in the same currency. Lastly, \( \Delta P_{i,t} - \Delta P_t^* \) denotes inflation differences in consumer prices between the country of interest and the United States.

Clarida & Gali (1994) assume there are three different structural shocks which account for the movements of the variables in \( X_{i,t} \). These are: supply difference shocks in the country of interest and the United States \( (\epsilon_{i,t}^p - \epsilon_t^p^*) \); demand difference shocks in the United States and the country of interest \( (\epsilon_t^d - \epsilon_{i,t}^d) \); and, money difference shocks in the country of interest and the United States \( (\epsilon_{i,t}^m - \epsilon_t^m^*) \). Demand shocks can be regarded as government spending shock or any other demand shock apart from money shocks.

The identification of structural shocks is achieved by placing restrictions on the long-run response matrix. To explain the identification method, let \( u_{i,t} \sim N(0, \Omega) \) where \( \Omega \) is the non-diagonal variance-covariance matrix of \( u_{i,t} \). Also, suppose that \( u_{i,t} \) is related to the structural shocks in the following way,

\[ u_{i,t} = \epsilon_{i,t}^p - \epsilon_t^p^* + \epsilon_t^d - \epsilon_{i,t}^d + \epsilon_{i,t}^m - \epsilon_t^m^* \]

---

\(^{39}\)Where data for seasonally adjusted series are available, we used these series. Otherwise, we obtained seasonally adjusted series from non-seasonally adjusted series by using the Demetra + program from Eurostat.

\(^{40}\)Let \( E_{i,t} \) be the home currency price of the United States dollar in economy \( i \). Also denote \( P_t^* \) and \( P_{i,t} \) as indexes of the consumption basket in the United States and Economy \( i \), respectively. We measure \( Q_{i,t} \) as \( \frac{E_{i,t}P_t^*}{P_{i,t}} \). Hence, a rise in \( Q_{i,t} \) is associated with a depreciation of the real exchange rate vis-a-vis the United States.

\(^{41}\)Since our empirical approach is related to the empirical strategy in Clarida & Gali (1994), we give a review of their method in Appendix C.1.
where $C_0$ is a $3 \times 3$ matrix of the contemporaneous responses of the variables to shocks.

It is notable that due to the assumption of independence among different type of structural shocks, the variance-covariance matrix, $C_0^{-1}\Omega C_0^{-1'}$, is diagonal. Furthermore, under the normalization that the variance-covariance matrix of structural shocks is an identity matrix, the following equality has to hold:

$$C_0 C_0' = \Omega$$  \hfill (4.1.4)

Clarida & Gali (1994) identify structural shocks by imposing restrictions on the effects of these shocks on the level of the output difference, the real exchange rate and the price level difference in the long-run. Denoting the matrix of the long-run impulse responses by $\mathcal{D}$, Clarida & Gali (1994) isolate structural shocks by assuming that $\mathcal{D}$ is lower triangular,

$$\mathcal{D} = \begin{vmatrix}
  d_{11} & 0 & 0 \\
  d_{21} & d_{22} & 0 \\
  d_{31} & d_{32} & d_{33}
\end{vmatrix}$$  \hfill (4.1.5)

The ordering of the variables in (4.1.2) implies only supply shocks influence the level of the output difference in the long-run. Neither demand nor money shocks have a permanent effect on the level of the output difference. Regarding the real exchange rate, its level is affected permanently by supply or demand shocks. Lastly, all three shocks have a long-run impact on the level of the CPI difference.

In order to uniquely recover structural shocks, in addition to the lower triangularity of
the long-run matrix, it is necessary to impose sign restrictions on $D$. A larger supply and monetary shock in Economy $i$ compared to the United States are assumed to increase the long-run levels of GDP and CPI in Economy $i$ relative to the United States, respectively ($d_{11} > 0, d_{33} > 0$). In addition, a larger demand shock in Economy $i$ compared to the United States is assumed to appreciate the long-run level of the real exchange rate of Economy $i$ relative to the United States ($d_{22} > 0$). This can happen if government spending mostly fall on non-traded goods.

Some restrictions on the long-run impact matrix in Clarida & Gali (1994) are debatable. For example, the sign restriction that an expansionary fiscal shock in Economy $i$ appreciates the real exchange rate in the long-run should necessarily be taken with a grain of salt (For example, see Ravn, Schmitt-Groh & Uribe (2007) for counter evidence). Similarly, the exclusion restriction in Clarida & Gali (1994), that the fiscal shocks have no long-run effect on the level of output, is subject to criticism as it is quite likely that fiscal shocks such as spending shocks on education and infrastructure impact the long-run output level in a country. Based on these considerations, we slightly modify the long-run impact response matrix. Indeed, as in Clarida & Gali (1994), we assume monetary shocks have a long-run impact on neither output level nor the real exchange rate level. Yet, we do not place any restriction regarding the long-run impact of productivity and demand shocks on the level of any of the variables. Let $\tilde{D}$ denote the modified long-run impact matrix of structural shocks with the above noted restrictions on the level of the variables. This matrix can then be written as

$$\tilde{D} = \begin{bmatrix}
\tilde{d}_{11} & \tilde{d}_{12} & 0 \\
\tilde{d}_{21} & \tilde{d}_{22} & 0 \\
\tilde{d}_{31} & \tilde{d}_{32} & \tilde{d}_{33}
\end{bmatrix} \quad (4.1.6)$$
In addition to the restrictions in (4.1.6), in Appendix C.1 we show in (C.1.13) that \( \vec{D} \) must also satisfy

\[
\vec{D}\vec{D}' = \left( I - \sum_{p=1}^{p_{\text{max}}} B_p \right)^{-1} \Omega \left( I - \sum_{p=1}^{p_{\text{max}}} B_p \right)^{-1}
\]  

(4.1.7)

The modified long-run impact matrix of structural shocks, \( \vec{D} \), has seven free parameters whereas \( \vec{D}\vec{D}' \) is symmetric so it has only six independent elements. Hence, it is not possible to uniquely recover all the parameters of the \( \vec{D} \) matrix. In particular, an analysis of the dynamic responses of the variables following productivity and demand shocks necessitates knowing the elements in the first and second columns of (4.1.6), respectively. Yet, such an analysis is not feasible as the elements in these columns are unidentifiable given the structure of \( \vec{D} \). However, the third column can be uniquely recovered. This allows us to investigate dynamic responses of the variables to monetary shocks. To prove this, note first that since the model is not uniquely identified, there are many matrices satisfying (4.1.7). Letting \( \vec{D} \) and \( \vec{D}^A \) be two of such matrices (i.e. both \( \vec{D} \) and \( \vec{D}^A \) are block lower-triangular as stated in (4.1.6) and satisfy (4.1.7)), we can always find a square block lower-triangular orthonormal matrix \( \vec{\omega} \) such that

\[
\vec{D}^A = \vec{D}\vec{\omega}
\]  

(4.1.8)

One can show the reason for \( \vec{\omega} \) matrix to be block lower-triangular and orthonormal in three steps. First, we show \( \vec{\omega} \) is orthonormal. Since \( \vec{D} \) and \( \vec{D}^A \) satisfy (4.1.7), the following equation has to hold:

\[
\vec{D}\vec{\omega}\vec{\omega}'\vec{D}' = \vec{D}\vec{D}'
\]  

(4.1.9)

Multiplying both sides with \( \vec{D}^{-1} \) from the left and with \( \vec{D}^{-1}' \) from the right yields
\( \vec{\omega} \vec{\omega}' = I \) where \( \vec{D} \) is invertible by assumption. The implication being that \( \vec{\omega} \) has to be an orthonormal matrix.

Second, note that \( \vec{\omega} = \vec{D}^{-1} \vec{D}^A \). Since the product of two block lower-triangular matrices has to be block lower-triangular, \( \vec{\omega} \) has to be block lower-triangular, as well. Hence, one can write \( \vec{\omega} \) as

\[
\vec{\omega} = \begin{vmatrix}
\omega_{11} & \omega_{12} & 0 \\
\omega_{21} & \omega_{22} & 0 \\
\omega_{31} & \omega_{32} & \omega_{33}
\end{vmatrix}
\] (4.1.10)

Third, multiplying both sides of (4.1.8) with \( \vec{\omega}' \) and using the fact that \( \vec{\omega} \) is orthonormal yields \( \vec{D}^A \vec{\omega}' = \vec{D} \). Since \( \vec{D}^A \) and \( \vec{D} \) are block lower-triangular, \( \vec{\omega}' = \vec{D}^{-1} \vec{D} \), \( \vec{\omega}' \) must also be block lower-triangular. This implies \( \omega_{31} \) and \( \omega_{32} \) are equal to zero as well. Furthermore, since \( \vec{\omega} \) is orthonormal, \( \vec{\omega} \) has to be in the form of one of two matrices:

\[
\vec{\omega} = \begin{vmatrix}
\omega_{11} & \omega_{12} & 0 \\
\omega_{21} & \omega_{22} & 0 \\
0 & 0 & -1
\end{vmatrix} \quad \text{or} \quad \vec{\omega} = \begin{vmatrix}
\omega_{11} & \omega_{12} & 0 \\
\omega_{21} & \omega_{22} & 0 \\
0 & 0 & 1
\end{vmatrix}
\] (4.1.11)

Lastly, the final step in uniquely identifying the monetary shock requires the assumption that an expansionary monetary shock results in a permanent rise in price level differences between the developing economies and the United States.\textsuperscript{42} This sign restriction uniquely identifies the third column by ensuring \( \omega_{33} = 1 \). Therefore, even if there are many matrices satisfying both (4.1.7) and (4.1.9), their third column must be the same. Identifying the elements of the third column this way enables us to analyze dynamic responses of the variables to monetary shocks.\textsuperscript{43}

\textsuperscript{42}Therefore, \( \vec{d}_{33} \) is positive in (4.1.6)

\textsuperscript{43}Here, it is natural to ask whether structural monetary shocks can be identified by placing restrictions
Empirical Model II

Clarida & Gali (1994) employ their strategy for isolating structural shocks in developed economies. In comparison to developed countries, an analysis of the dynamic responses of variables to structural shocks in developing countries may require more demanding assumptions. In particular, note that Clarida & Gali (1994) isolate differences in structural shocks between the country of interest and the United States, $\epsilon_{i,t}^m - \epsilon_t^m$, rather than isolating them separately, $\epsilon_{i,t}^m$ and $\epsilon_t^m$. When only differences in shocks are isolated, a 1% expansionary monetary shock in the country of interest is implicitly assumed to induce the same dynamics as a 1% contractionary monetary shock in the United States. Under the symmetric-country assumption, this may be a plausible assumption if one studies the movements in $Y_{i,t} - Y_t^*$, $Q_{i,t}$ and $P_{i,t} - P_t^*$ between a developed economy and the United States. Yet, it is not realistic to maintain the symmetric-country assumption for a developing economy and the United States. For example, the coefficients of exchange rate pass-through into import and consumer prices in developing economies and the United States are markedly dissimilar. Furthermore, the frequencies of price changes among sectors in developing economies contrast with those in the United States. These asymmetric features may cause the dynamics of $Y_{i,t} - Y_t^*$, $Q_{i,t}$ and $P_{i,t} - P_t^*$ between developing economies and the United States after a 1% expansionary monetary shock in developing economies to differ significantly from those after a 1% contractionary monetary shock in the United States.

44 Apart from these asymmetric features, a difference in the monetary shock process between developing economies and the United States [44] only on the long-run responses matrix to monetary shocks. By writing the equation for the structural shock explicitly in (4.1.12), we show that this is not possible:

$$\epsilon_{i,t} = C_{\eta}^{-1}u_{i,t} = \vec{D}^{-1}\left(I - \sum_{p=1}^{p_{max}} B_p\right)u_{i,t}$$

(4.1.12)

Since monetary shocks are ordered as the third element of $\epsilon_{i,t}$, recovering them requires the third row of the inverse of $\vec{D}$ in (4.1.12). Yet, the third row cannot be identified by placing restrictions only in the long-run effects of monetary shocks on the level of output differences and the real exchange rate between the United States and the developing country. Consequently, structural monetary shocks are unidentifiable in Empirical Model I.
For this reason, we believe it is more plausible to study the consequences of monetary shocks in developing economies and the United States separately. To achieve this, we consider the same panel VAR model in (4.1.1), yet the vector of variables, $X_{i,t}$, is now given as

$$X_{i,t} = \begin{bmatrix} \Delta y^*_t \\
\Delta y^i_{1,t} \\
\Delta Q_{i,t} \\
\Delta P^*_t \\
\Delta P^i_{t} \\
\end{bmatrix} \tag{4.1.13}$$

Here, $\Delta y^*_t$ ($\Delta y^i_{1,t}$) and $\Delta P^*_t$ ($\Delta P^i_{t}$) denote the log-change in output and the consumer price level in the United States (the country of interest), respectively.

Fluctuations in the vector of variables in Empirical Model II are assumed to be driven by five structural shocks in the following order:

1. Supply shocks in the United States ($\epsilon_t^p$)
2. Supply shocks in developing economies ($\epsilon^p_{i,t}$)
3. General preference shocks ($\epsilon^d_{i,t}$)
4. Monetary shocks in the United States ($\epsilon_t^m^*$)
5. Monetary shocks in developing economies ($\epsilon^m_{i,t}$)

Our goal is to analyze dynamic responses of the variables to monetary shocks in the United States and developing countries separately. This can be achieved if the following economies and the United States may also result in the dynamics of $y_{i,t} - y^*, Q_{i,t}$ and $P_{i,t} - P^*$ between developing economies and the United States after a 1% expansionary monetary shock in developing economies differing significantly from those after a 1% contractionary monetary shock in the United States.
assumptions are made regarding the $\tilde{D}$ matrix which shows the long-run level responses of the variables in developing economies to each shock in Empirical Model II:

$$
\tilde{D} = 
\begin{bmatrix}
   d_{11} & d_{12} & d_{13} & 0 & 0 \\
   \tilde{d}_{21} & \tilde{d}_{22} & \tilde{d}_{23} & 0 & 0 \\
   \tilde{d}_{31} & \tilde{d}_{32} & \tilde{d}_{33} & 0 & 0 \\
   d_{41} & d_{42} & d_{43} & d_{44} & 0 \\
   \tilde{d}_{51} & \tilde{d}_{52} & \tilde{d}_{53} & \tilde{d}_{54} & \tilde{d}_{55}
\end{bmatrix}
$$

(4.1.14)

In the structure of (4.1.14), monetary shocks in the United States have been constrained to have no impact on the long-run level of output in both economies and the real exchange rate. In addition to these constraints, monetary shocks in the developing economies are restricted to have no permanent impact on the price level in the United States. This assumption is consistent with both the small-country assumption for developing economies and the standard practice of modeling the United States as a closed economy in the literature. In fact, our maintained assumption in Empirical Model II regarding the effect of domestic monetary shocks in developing economies is weaker than the small-country assumption in our theoretical models presented in Section 4.2. Indeed, while the assumption in Empirical Model II constrains domestic monetary shocks in developing economies to have no long-term impact on the price level in the United States, the small-country assumption in our theoretical model imposes that they have a negligible impact on the price level in the United States in the short- and long-terms.

Now, we aim to separately analyze the dynamic responses of the variables to monetary shocks in the United States and developing economies. This can be achieved if the elements of the fourth and fifth columns of (4.1.14) are known. By following the same arguments in Section 4.1.1, it can be shown that Empirical Model II is unidentified and there are many
matrices satisfying (4.1.14) and (4.1.15),

\[ \mathcal{D}' \mathcal{D}' = \left( I - \sum_{p=1}^{p_{\text{max}}} B_p \right)^{-1} \Omega \left( I - \sum_{p=1}^{p_{\text{max}}} B_p \right)^{-1} \tag{4.1.15} \]

By following exactly the same arguments in Section 4.1.1, it is easy to show that any two such matrices \( \mathcal{D} \) and \( \mathcal{D}' \) have the same fourth and fifth columns. This results from the fact that the orthonormal square matrix, \( \mathcal{\omega} \), linking these two matrices must be in the following form:

\[
\mathcal{\omega} = \begin{bmatrix}
\hat{\omega}_{11} & \hat{\omega}_{12} & \hat{\omega}_{13} & 0 & 0 \\
\hat{\omega}_{21} & \hat{\omega}_{22} & \hat{\omega}_{23} & 0 & 0 \\
\hat{\omega}_{31} & \hat{\omega}_{32} & \hat{\omega}_{33} & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix} \tag{4.1.16}
\]

Having identified the fourth and fifth columns of (4.1.14) this way, an analysis of the dynamic responses of the variables to the monetary shocks in the United States and the developing economies is straightforward.

### 4.1.2 Empirical Results

This section presents our findings on the responses of domestic economic activity, the bilateral real exchange rate with the United States and prices after domestic monetary shocks in developing countries under an inflation targeting regime. Since the adoption dates of the inflation targeting regime were not the same among the countries in our sample, we have an unbalanced panel data. As stated in Arellano & Bond (1991), this does not fundamentally change our analysis since we only require the assumption that observations are independently distributed in the initial cross-section and that subsequent
Table 4.1: Adoption Dates of Inflation Targeting in Developing Economies

<table>
<thead>
<tr>
<th>Country</th>
<th>Monthly Data</th>
<th>Quarterly Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poland</td>
<td>1998-M10</td>
<td>1998-Q4</td>
</tr>
<tr>
<td>Brazil</td>
<td>1999-M6</td>
<td>1999-Q2</td>
</tr>
<tr>
<td>Chile</td>
<td>1999-M9</td>
<td>1999-Q3</td>
</tr>
<tr>
<td>Colombia</td>
<td>1999-M9</td>
<td>1999-Q3</td>
</tr>
<tr>
<td>South Africa</td>
<td>2000-M2</td>
<td>2000-Q1</td>
</tr>
<tr>
<td>Thailand</td>
<td>2000-M5</td>
<td>2000-Q2</td>
</tr>
<tr>
<td>Mexico</td>
<td>2001-M1</td>
<td>2001-Q1</td>
</tr>
<tr>
<td>Hungary</td>
<td>2001-M6</td>
<td>2001-Q2</td>
</tr>
<tr>
<td>Peru</td>
<td>2002-M1</td>
<td>2002-Q1</td>
</tr>
<tr>
<td>Philippines</td>
<td>2002-M1</td>
<td>2002-Q1</td>
</tr>
<tr>
<td>Guatemala</td>
<td>2005-M1</td>
<td>2005-Q1</td>
</tr>
<tr>
<td>Indonesia</td>
<td>2005-M7</td>
<td>2005-Q3</td>
</tr>
<tr>
<td>Romania</td>
<td>2005-M8</td>
<td>2005-Q3</td>
</tr>
<tr>
<td>Turkey</td>
<td>2006-M1</td>
<td>2006-Q1</td>
</tr>
<tr>
<td>Serbia</td>
<td>2006-M9</td>
<td>2006-Q3</td>
</tr>
</tbody>
</table>

Source: Roger (2009)

additions and deletions occur randomly. Table 4.1 reports the adoption dates of inflation targeting in the developing countries contained in our sample for which we have quarterly or monthly data.

Our source of data on the level of economic activity, bilateral nominal exchange rates with the United States and consumer prices in our sample of countries is the IMF’s International Finance Statistics data. Our data spans the post-inflation targeting period for each country until March, 2013. Due to data limitations on industrial production index for some developing countries at the monthly frequency, Chile, Colombia, Guatemala, Indonesia, Peru, Philippines and South Africa are dropped from the sample at the monthly frequency. Instead of the industrial production index, real GDP is used at the quarterly fre-
Figure 4.1: Impulse Responses to Monetary Shocks in Developing Economies (Empirical Model II)

Quarterly

(a) $y_{i,t}^*$
(b) $y_{i,t}$
(c) $\Omega_{i,t}$
(d) $P_{i,t}^*$
(e) $P_{i,t}$

Monthly

(f) $y_{i,t}^*$
(g) $y_{i,t}$
(h) $\Omega_{i,t}$
(i) $P_{i,t}^*$
(j) $P_{i,t}$

Note: Our calculations are based on the IMF’s International Finance Statistics. The solid lines indicate the estimated point-wise impulse responses. The area between the dashed lines shows the 90% confidence interval estimated using the Bayesian method suggested by Sims & Zha (1999).

Before presenting our results, it is essential that logged real exchange rates of developing economies compared to the United States, $\Omega_{i,t}$, the logged real GDP and CPI in Economy $i$ and the United States (denoted by $y_{i,t}$, $y_{i,t}^*$, $P_{i,t}$, and $P_{i,t}^*$, respectively) all have unit roots. For the series pertaining to developing economies, we estimated a panel autoregressive model with country-specific fixed effects containing four and twelve lags for the quarterly and monthly data, respectively. With the level specification, we perform the augmented Dickey-Fuller test. The unreported results indicate that one cannot reject the null that all five series contains a unit-root at the 5% significance level. With the growth specification, on the other hand, the null is rejected strongly at the 5% significance level. Hence, we conclude that all five series have unit roots.

quency. Since the series of real GDP are available for most sample countries, our quarterly data contains a larger sample of economies.45

45Before presenting our results, it is essential that logged real exchange rates of developing economies compared to the United States, $\Omega_{i,t}$, the logged real GDP and CPI in Economy $i$ and the United States (denoted by $y_{i,t}$, $y_{i,t}^*$, $P_{i,t}$, and $P_{i,t}^*$, respectively) all have unit roots. For the series pertaining to developing economies, we estimated a panel autoregressive model with country-specific fixed effects containing four and twelve lags for the quarterly and monthly data, respectively. With the level specification, we perform the augmented Dickey-Fuller test. The unreported results indicate that one cannot reject the null that all five series contains a unit-root at the 5% significance level. With the growth specification, on the other hand, the null is rejected strongly at the 5% significance level. Hence, we conclude that all five series have unit roots.
We now study the aggregate dynamics after an expansionary domestic monetary shock in developing economies using *Empirical Model II*. These aggregate dynamics are displayed in Figure 4.1. It is evident from this figure that an expansionary monetary shock in developing economies

- causes a modest, short-lived impact on output in the United States;
- induces an increase in the level of output in developing countries relative to its undistorted path which lasts for about one year;
- depreciates the real exchange rate on impact, implying that the goods from the developing economies is worth less in terms of the goods from the United States;
- leads to either a small, temporary increase or no change at all in the price level of the United States; and,
- results in a permanent increase of the price level in the developing economies.

Such findings only show the *average* impulse response functions for the group of developing countries which adopted an inflation targeting regime. However, the impulse response functions of the variables to an expansionary domestic monetary shock in each country in the group differ radically from the average impulse response functions. Figure 4.2 illustrates this point. The impulse response functions of output, the real exchange rate and the price level to an expansionary monetary shock in each country is obtained

---

46 We study aggregate dynamics following monetary shocks in *Empirical Model I* and following monetary shocks in the United States in *Empirical Model II* in Appendix C.2.

47 When the quarterly data is considered, output in the United States shows modest but significant responses on impact and in the first period. This unusual finding may result from the fact that some of the countries in our sample are large economies as they are included in the G20.

48 It is notable that the real exchange rate attains its highest level *on impact*, and falls afterwards. This contrasts with the finding in Section C.2.2 that the real exchange rate exhibits hump-shaped dynamics after monetary shocks in the United States. Hump-shaped dynamics following monetary shocks in the United States are also found by Clarida & Gali (1994) and Eichenbaum & Evans (1995) for the bilateral real exchange rates between the United States and other developed countries.
Note: Our calculations are based on the IMF’s International Finance Statistics. The solid line marked with circles indicates the median of the estimated point-wise impulse response functions in the group in each period. The dot-dashed line shows the country-specific impulse response functions separately estimated for each country in the group using the VAR version of Empirical Model II. separately by considering the country-specific VAR model version of Empirical Model II with monthly data. The size of the shock in each country is normalized to induce the same long-run response in the price level. It is evident from this figure that the impulse response functions of all three variables in the individual countries differ radically from the median impulse functions in the group.

The Conditional and Unconditional Co-movements of the Real and Nominal Exchange Rates

Next, we show the co-movements of the real and nominal exchange rates conditional on the domestic monetary shock in the Empirical Model II with monthly data. The impulse
Figure 4.3: Conditional Movements of the Real and Nominal Exchange Rates
(Empirical Model II with Monthly Data)

Note: Our calculations are based on the IMF’s International Finance Statistics. The dotted lines marked with circles in Panel (a) and Panel (b) indicate the log-change and the level impulse response functions of the real exchange rate to the domestic monetary shock, respectively. The dot-dashed lines marked with asterisks in Panel (a) and Panel (b) show the log-change and the level impulse response functions of the nominal exchange rate to the domestic monetary shock, respectively.

Response functions of the nominal exchange rates ($\hat{E}$) are obtained as $\hat{Q} + \hat{P} - \hat{P}^*$. It is evident from Panel (a) of Figure 4.3 that conditional on the domestic monetary shock, the deviation (in percent) of the log-change in the nominal and real exchange rates from their undistorted path follow a similar pattern. Such co-movements are also noticeable from the common pattern of the impulse response functions of the level nominal and real exchange rates in Panel (b).

Lastly, the co-movements of log-changes in the real and nominal exchange rates are analyzed unconditionally for each developing economy that has adopted an inflation targeting regime using monthly data in Figure 4.4. Again, we find movements in the real exchange
Figure 4.4: The Unconditional Co-movements of the Log-Changes in the Nominal and Real Exchange Rates

Note: Our calculations are based on the IMF’s International Finance Statistics. The dotted line marked with circles indicates the log-change in the real exchange rates for each country. The dot-dashed lines marked with asterisks show the log-change in the nominal exchange rates for each country. Rates closely follow those in the nominal exchange rates.
4.2 Theoretical Models

In this section, we present two small-open economy DSGE models. In the next section, we study the consequences of a monetary shock that causes a 1% increase in the price level in the long-term for each model, and compare the outcomes in these models with those in the actual economies to the same shock. We start by presenting models with the problem of Home and Foreign households.

4.2.1 The Problem of Home and Foreign Households

There is a continuum of infinitely lived households in each country with a mass of one and indexed with $h$. Each household is comprised of two members. They aim to maximize their joint lifetime discounted utility with the discount factor given by $\beta$. In period $t$, the members of the $h^{th}$ household in the Home country have to make a sequence of decisions. First, they have to choose how much to consume from the home-country non-traded final consumption good ($C_t$). Second, they optimally choose how intensively they supply their capital ($u_t$) in each period. Third, they decide on the amount of investment ($I_t$), and therefore, on the next period's capital stock ($K_{t+1}$). Fourth, they have to decide on the amount of optimal holdings of a one-period risk-free foreign bond ($B_{t+1}$) which pays a gross nominal return of $R_t^B$. Lastly, only one of the household members obtains a chance to renegotiate its wage contract each period. The wage contract made in any period lasts for two periods and has to be signed before observing the shock. The problem of the Home household can be put more compactly as follows:

$$\max_{C_t, u_t, I_t, K_{t+1}, B_{t+1}, x_t} E_t \sum_{s=0}^{\infty} \beta^{t+s} \left( \frac{C_{t+s}^{1-\sigma_c}}{1-\sigma_c} - \frac{\tilde{n}_{t+s,i}^{1+\sigma_n}}{1+\sigma_n} - \frac{n_{t+s,i}^{1+\sigma_n}}{1+\sigma_n} \right)$$  \hspace{1cm} (4.2.1)

where $\tilde{n}_{t,i}$ and $n_{t,i}$ are the hours worked by the members of the household whose wage contract lasts for two periods.
contracts are signed in period $t$ and $t-1$, respectively. $\sigma_c$ and $\sigma_n$ stand for the reciprocal of the intertemporal elasticity of substitution and the Frisch-elasticity of substitution, respectively. In solving (4.2.1) the household has the following budget constraint:

$$
P_{t+s}\left(C_{t+s} + I_{t+s} + a(u_{t+s})K_{t+s}\right) + \mathcal{E}_{t+s}B_{t+1+s} = x_{t+s,i}n_{t+s,i} + x_{t-1+s,i}n_{t+s,i} + R^k_{t+s}u_{t+s}K_{t+s} + R^B_{t,s,t-1+s}\mathcal{E}_{t+s}B_{t+s} + \Pi_{t+s}
$$

(4.2.2)

In writing (4.2.2), we follow Christiano et al. (2005) and assume that increasing capacity utilization ($u_t$) involves real costs in units of the final good denoted by $a(u_t)$\footnote{49Let the bar symbol over the variables show the steady-state values of these variables. At the steady state, capital is fully utilized, $\bar{u} = 1$. The function $a(u)$ has the following properties: $a(1) = 0$, $a'(u) > 0$ and $a''(u) > 0$.} The price of the home non-traded final good is denoted by $P_t$. $\mathcal{E}_t$ stands for the nominal exchange rate between the currency of the home country ($\varepsilon$) and the foreign country ($\varepsilon^*$). $R^k_t$ denotes the rental rate of capital paid to the owners of capital stock. The gross nominal return on the holdings of last period’s foreign risk-free bonds is shown with $R^B_{t,s,t-1}$. $x_{t,i}$ and $x_{t-1,i}$ in (4.2.2) represent the hourly wage earnings of the household member who negotiates his wage in period $t$ and $t-1$, respectively. Lastly, $\Pi_t$ shows the profits of firms which belong to the household. In sum, the representative household earns wage, capital, profit and interest income. The household uses its resources to finance purchases of the final consumption good, investment, the cost associated with varying $u_t$ and purchases of foreign bonds.

The law of motion for capital in the home country is given as:

$$
K_{t+1} = (1 - \delta)K_t + \phi\left(\frac{I_t}{K_t}\right)K_t
$$

(4.2.3)
where $\phi\left(\frac{I_t}{K_t}\right) K_t$ shows the additional capital stock which new investment in the current period makes available for the next period.\(^{50}\)

The problem of the foreign household is similar. Her optimization problem and flow budget constraint can be written as:

$$
\max_{c^*_t, u^*_t, \bar{I}^*_t} E_t \sum_{s=0}^{\infty} \beta^{t+s} \left( \frac{c^*_{t+s} - 1}{1 - \sigma_c} - \tilde{n}^{1+\sigma_c}_{t+s,i} - n^{1+\sigma_c}_{t+s,i} \right)
$$

(4.2.4)

$$
P^*_{t+s} \left( C^*_{t+s} + I^*_{t+s} + a(u^*_{t+s})K^*_{t+s} \right) + B^*_{t+1+s} = x^*_{t+s,i} \tilde{n}^*_{t+s,i} + x^*_{t-1+s,i} n^*_{t+s,i}
$$

$$
+ R^*_t u^*_{t+s} K^*_{t+s} + R^B_{\mathbb{F},t-1+s} B^*_{t+s} + \Pi^*_{t+s}
$$

(4.2.5)

where the variables denoted with the superscript * represent the foreign counterparts of the home variables. It is notable that the gross nominal return pertinent to the holdings of the risk-free bond in the foreign country in (4.2.5), $R^B_{\mathbb{F},t-1}$, may differ from $R^B_{\mathbb{H},t-1}$ in (4.2.2). Following Devereux & Smith (2005), we assume that countries face a debt-elastic interest rate. Let the net position of the home country in the risk-free bond be given as $B_t$. The debtor country has to pay a higher interest rate than the lender country due to upward-sloping bond supply in international financial markets. The differential between $R^B_{\mathbb{F},t-1}$ and $R^B_{\mathbb{H},t-1}$ depends on the net bond holdings of the countries in the following way:

$$
R^B_{\mathbb{H},t} = \Theta \left( B_{t+1} - \bar{B} \right) R^B_{\mathbb{F},t}
$$

(4.2.6)

where $\Theta \left( B_{t+1} - \bar{B} \right)$ satisfies $\Theta \left( 0 \right) = 1$ and $\Theta' \left( . \right) < 0$. Since there is a continuum of

\(^{50}\)At the steady state, $\bar{I} = \delta \bar{K}$. The function $\phi\left(\frac{I_t}{K_t}\right)$ has the following properties. $\phi\left(\delta\right) = \delta$, $\phi'\left(\delta\right) = 1$, $\phi''\left(\delta\right) > 0$ and $\phi''\left(\delta\right) < 0$. The last assumption implies that $\phi''\left(\delta\right)$ is concave that emanates from the fact that new investment is subject to adjustment costs.
households in both countries, bond holdings of any individual household \((B_{t+1})\) has only a negligible effect on the net position of countries’ bond holdings \((B_{t+1})\). Thus, households do not internalize the interest rate country faces.\(^{51}\)

The optimality conditions for the Home household with respect to \(C_t, u_t, I_t, K_{t+1}\) and \(B_t\) are given as:

\[
C_t^{-\sigma} = \lambda_t P_t \tag{4.2.7}
\]

\[
a'(u_t) = r^k_t, \quad r^k_t = R^k_t/P_t \tag{4.2.8}
\]

\[
\lambda_t P_t = \mu_t \phi' \left( \frac{I_t}{K_t} \right) \tag{4.2.9}
\]

\[
\mu_t = \beta E_t \left[ -\lambda_{t+1} P_{t+1} a(u_{t+1}) + \lambda_{t+1} R^k_{t+1} u_{t+1} + \mu_{t+1} \left( 1 - \delta \right) - \phi' \left( \frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} + \phi \left( \frac{I_{t+1}}{K_{t+1}} \right) \right] \tag{4.2.10}
\]

\[
\lambda_t \epsilon_t = \beta E_t \lambda_{t+1} R^B_{\epsilon t} \epsilon_{t+1} \tag{4.2.11}
\]

where \(\lambda_t\) and \(r^k_t\) are the marginal utility of nominal income and the real rental price of capital in the home country, respectively. \(\mu_t\), on the other hand, stands for the shadow value of having one more unit of next period’s capital stock. In other words, it shows the

\(^{51}\)Assuming a debt-elastic differential in the two countries’ interest rates is a standard way to circumvent the problem of multiple steady states in imperfect financial markets. Without such an assumption, stationarity of the model would not be ensured as when a shock is introduced into the model, the model oscillates between different steady states without ever reaching a stable equilibrium. For a more complete description, see [Schmitt-Grohe & Uribe] (2003) and [Boileau & Normandin] (2008) who describe the problem of multiple steady states in the small- and large-open economy models with imperfect financial markets, respectively. They also evaluate different methods to circumvent this problem.
amount of the final good the household is willing to forgo in the current period to have
one more unit of capital stock in the next period. The condition (4.2.7) states that the
household equates the marginal utility of consumption with its marginal cost. As well, the
condition (4.2.8) implies that incremental variations in \( u_t \) would cost \( a'(u_t)K_t \) in resources
but since it allows the household to supply more capital services in the current period,
the real income of the household rises by \( r^k_tK_t \). At the optimal \( u_t \), these two should be
equal. In (4.2.9), the left-hand side is the opportunity cost of investing an incremental
amount. At optimum, this is equated to the utility gained from making that incremental
investment as it allows the household to have \( \phi' \left( \frac{I_{t+1}}{K_{t+1}} \right) \) more capital in the next period.
The condition (4.2.10) indicates that the marginal utility of having an extra unit of capital
stock in the next period is the sum of three terms. \(-\beta\lambda_{t+1}P_{t+1}a(u_{t+1})\) is the utility cost
associated with the deviation of the capacity utilization rate in the next period from its
steady state. The second term, \( \beta\mu_{t+1}R^k_{t+1}u_{t+1} \), indicates that having an extra unit of
capital stock in the next period would increase nominal income by \( R^k_{t+1}u_{t+1} \). The third
term, \( \beta\mu_{t+1} \left( (1-\delta) - \phi' \left( \frac{I_{t+1}}{K_{t+1}} \right) \right) \) denotes the utility gain of retaining
the extra unit of capital in period \( t+2 \). Lastly, the optimal bond holdings equation in
equation (4.2.11) states that purchasing an extra unit of foreign risk-free bonds would cost
\( \lambda_t \) in period \( t \) and would yield \( R^e_{t+1}E_{t+1} \) of nominal income in period \( t+1 \). Regarding
the equivalent problem of households in the foreign country, all of the first-order conditions,
except that of the bond holdings, are similar. The optimality condition for the foreign-
household’s bond holdings, on the other hand, can be written as follows:

\[
\lambda^*_t = \beta E_t \lambda^*_{t+1} R^e_{t+1} \quad (4.2.12)
\]

Using (4.2.7) and (4.2.11) along with their counterparts for the foreign household, the
equation for the real exchange rate between the home and foreign countries ($Q_t$) can be written as:

$$\sigma_c \left[ E_t \left( \hat{C}_{t+1} - \hat{C}_t \right) - E_t \left( \hat{C}^*_t + 1 - \hat{C}_t^* \right) \right] = E_t \left( \hat{Q}_{t+1} - \hat{Q}_t \right) + \Theta'(0) \bar{Y} \hat{B}_{t+1} \quad (4.2.13)$$

where $Q_t = \frac{\bar{C}_t}{\bar{Y}}$. In our paper, the bars and hats over the variables stand for the steady-state values and the log-deviation of the variables from their steady states, respectively. The only exception is $\hat{B}_{t+1}$ which is defined as $B_t - \bar{B}$ where $\bar{Y}$ is the steady-state value of the aggregate final-good output. Defining $\hat{B}_{t+1}$ this way makes it convenient to take a log-linear approximation of the domestic budget constraint.

**Aggregate Wage Equation**

Before describing our wage setting environment, we discuss two main methods discussed in the literature for incorporating staggered wage setting into models. The first is the Erceg, Henderson & Levin (2000) model where households supply differentiated hours of work and the chance to optimize their wages in each period is random with some given probability. The second is the Huang & Liu (2002) model where households are again assumed to supply differentiated hours of work, yet, the duration of wage contracts are non-random as these contracts stay in effect for a specified duration of time.

While these models are widely used, they are not particularly suitable for studying developing countries since they assume complete financial markets. Yet, financial markets in these economies are infant and lack sophistication. Based on this, we maintain households may hold only non-state contingent bonds. However, writing a single budget constraint when a household consists of only one member is not possible due to the assumption of incomplete financial markets in a staggered wage setting environment. A non-degenerate
income distribution of households due to incomplete insurance accounts for this fact as noted by Huang & Liu (2002). Indeed, since workers renew their wage contracts in different periods under the staggered wage setting, their wage income differs with incomplete insurance. Consequently, the problem of households in the economy with incomplete insurance might not be reduced to that of the “representative” household as their incomes would not be alike. Solving such a model involves the difficult task of following the non-degenerate income distribution period-by-period which can be computationally demanding.

Erceg et al. (2000) and Huang & Liu (2002) circumvent this problem by assuming complete financial markets. Under complete insurance, state-contingent assets are traded to eliminate idiosyncratic risks among households. In staggered wage setting environments, these risks are associated with uncertainty about the timing of wage contract renewals. For example, when an expansionary monetary shock happens, in the absence of full insurance, workers whose contracts are renewed soon may be in an advantageous position compared to workers whose contracts are renewed late. However, under complete financial markets, these idiosyncratic risks are eliminated since income transfers through state-contingent bonds among households exactly offset wage income differences among households so that households have the same income in all periods. In other words, income distribution is degenerate under complete insurance.

To the best of our knowledge, what is left unexplored in the literature is that idiosyncratic risks under staggered wage setting can be eliminated even when insurance is incomplete. To achieve this, we modify the Huang & Liu (2002) model. We now explain this. Note that households contain two members in our models, the wife and the husband, who negotiate their wages with employers in even and odd periods, respectively. Clearly, the wages of wives and husbands will be dissimilar after an expansionary monetary shock with incomplete insurance. However, given that the wage income of households is given by
the sum of wives’ and husbands’ wages, even in the absence of income transfers through financial assets, households’ income will be alike after such a shock. Consequently, the income distribution of households is degenerate as they all have the same income. Thus, we can consider the problem of a “representative household” instead of household-specific maximization problems. Achieving staggered wage setting without sacrificing the incomplete financial market assumption in developing countries adds realism to our model.

Now, we describe the home wage setting environment in detail. There is a continuum of employment offices with a mass of one in the home economy. They combine the differentiated hours of work supplied by the members of households ($\tilde{n}_{t,i}$ and $n_{t,i}$) into a composite labor of ($N_t$) and sell it to the firms. The employment offices use the following technology to form the composite of labor:

$$N_t = \left( \int_0^1 \tilde{n}_{t,i}^{(\theta_w-1)/\theta_w} di + \int_0^1 n_{t,i}^{(\theta_w-1)/\theta_w} di \right)^{\theta_w/(\theta_w-1)}$$

(4.2.14)

The optimization problem of employment offices can be written as:

$$\max_{\tilde{n}_{t,i}, n_{t,i}} W_t N_t - \int_0^1 x_{t,i} \tilde{n}_{t,i} di - \int_0^1 x_{t-1,i} n_{t,i} di$$

(4.2.15)

where, because of the assumption of a continuum of employment offices, individual offices do not have an effect on the aggregate wage ($W_t$) and the wages set by the owners of the differentiated labors in period $t$ and $t-1$ ($x_{t,i}$ and $x_{t-1,i}$). Employment offices’ demand for differentiated labor of workers whose wages are set in period $t$ and $t-1$ are given by:

$$\tilde{n}_{t,i} = \left( \frac{x_{t,i}}{W_t} \right)^{-\theta_w} N_t \quad ; \quad n_{t,i} = \left( \frac{x_{t-1,i}}{W_t} \right)^{-\theta_w} N_t$$

(4.2.16)

52For definitions of $\tilde{n}_{t,i}$ and $n_{t,i}$, see (4.2.1).
From (4.2.16), it is clear that $\theta_w$ is the wage elasticity of substitution among differentiated hours. In period $t$, one member of the households sets his wage before observing the shock that will remain fixed in period $t$ and period $t+1$. Hence, his optimality problem can be written as:

$$\max_{x_{t,i}} E_{t-1} \left[ \left( -\frac{n_{t,i}^{1+\sigma_n}}{1+\sigma_n} + \lambda_t x_{t,i} n_{t,i} \right) + \beta \left( -\frac{n_{t+1,i}^{1+\sigma_n}}{1+\sigma_n} + \lambda_{t+1} x_{t,i} n_{t+1,i} \right) \right]$$

(4.2.17)

Having renegotiated his wage in period $t$, the household member must supply differentiated hours of work as demanded by the employment offices due to the binding wage contract in period $t$ and period $t+1$. Due to the continuum of differentiated hours supplied, each individual worker has negligible effect on the aggregate wage. Using this and the fact that households’ budget constraints are identical, the contracted wage in period $t$ for all workers is the same, allowing us to drop the subscript $i$ in $x_{t,i}$ and write $x_t$:

$$x_t^{1+\theta_w\sigma_n} = \frac{\theta_w}{\theta_w - 1} \frac{E_{t-1} W_t^{\theta_w+\theta_w\sigma_n} N_t^{1+\sigma_n} + \beta E_{t-1} \left( W_{t+1}^{\theta_w+\theta_w\sigma_n} N_{t+1}^{1+\sigma_n} \right)}{E_{t-1} \left( \lambda_t W_t^{\theta_w} N_t \right) + \beta E_t \left( \lambda_{t+1} W_{t+1}^{\theta_w} N_{t+1} \right)}$$

(4.2.18)

By using (4.2.14), (4.2.16) and the fact that all of the contracted wages are equal, one can show that the aggregate wage equation is given by:

$$W_t = \left( x_{t-1}^{1-\theta_w} + x_t^{1-\theta_w} \right)^{\frac{1}{1-\theta_w}}$$

(4.2.19)

---

53It is notable that in our notation, the hours supplied by the workers who do not renegotiate their wages are shown without a tilde over $n$. Since it is not possible to renegotiate the wage in period $t+1$ once wage is set at period $t$, the hours supplied by the worker in the next period who set a wage at period $t$ is shown with $n_{t+1,i}$ not with $\tilde{n}_{t+1,i}$.
The wage-setting behavior of the owners of differentiated labor types in the foreign country is the same, yielding similar equations for the contracted and aggregate wages.

### 4.2.2 The Objective of Firms in the Home and Foreign Countries

#### Firms Producing the Final Good in the Home and Foreign Countries

The non-traded final goods in both of the countries are produced by a continuum of perfectly competitive firms. Firms produce the final goods by using the following technology which involves combining goods from different sectors:

\[
Y_t = \left( \sum_{k=1}^{k_{\text{max}}} f_k^{1/\eta} Y_{k,t}^{(\eta-1)/\eta} \right)^{\eta/(\eta-1)}
\]  

(4.2.20)

where \(Y_t\) and \(Y_{k,t}\) denote the amount of the final good produced by firms and the output of Sector \(k\), respectively. \(f_k, \eta\) and \(k_{\text{max}}\) denote the sectoral weight, constant elasticity of substitution for sectoral goods in the final good production and the total number of sectors in the home country, respectively. It is easy to show that the demand for sectoral goods and the aggregate price index \((P_t)\) are given by:

\[
Y_{k,t} = f_k \left( \frac{P_{k,t}}{P_t} \right)^{-\frac{\eta}{\eta-1}} Y_t
\]  

(4.2.21)

\[
P_t = \left( \sum_{k=1}^{K} f_k P_{k,t}^{1-\eta} \right)^{\frac{1}{1-\eta}}
\]  

(4.2.22)

where \(P_{k,t}\) denotes the aggregate price index of sector \(k\). Since the final-good firms in the foreign country solve a similar problem, for the sake of brevity, we omit writing the
equations for the sector-specific foreign demand \(Y^*_k,t\) and the foreign aggregate price \(P^*_t\).

**Firms Producing Sector \(k\) Output in the Home and Foreign Countries**

In both countries, sectoral goods are produced by an infinitely large number of perfectly competitive firms. The home firms producing sectoral goods combine domestic goods \((Y^*_{H,k,t})\) and import goods \((Y^*_{F,k,t})\) to produce sectoral output \((Y_{k,t})\) with the following technology:

\[
Y_{k,t} = \left(1 - \psi\right)^{\frac{1}{\rho}} Y^*_{H,k,t} + \psi^{\frac{1}{\rho}} Y^*_{F,k,t}
\]

(4.2.23)

where \(\psi\) and \(\rho\) represent the steady-state weight of the import good in the home country and the elasticity of substitution between the domestic and import goods, respectively. It is straightforward to show the demands for the domestic goods and those imported by the home country in sector \(k\) are given as:

\[
Y^*_{H,k,t} = \left(1 - \psi\right)^{-\rho} Y_{k,t}\quad;\quad Y^*_{F,k,t} = \psi^{-\rho} Y_{k,t}
\]

(4.2.24)

where \(P^*_{H,k,t}\) and \(P^*_{F,k,t}\) denote domestic and import price indexes in sector \(k\) in the home country, respectively. Using (4.2.23) and (4.2.24), one can write the sector \(k\) price index in the home country \((P_{k,t})\) as the weighted average of domestic and import price indexes in sector \(k\):

\[
P_{k,t} = \left(1 - \psi\right)^{1-\rho} P^*_{H,k,t} + \psi^{1-\rho} P^*_{F,k,t}
\]

(4.2.25)

Sector \(k\)’s good in the foreign country is again produced by perfectly competitive firms.
Yet, the technology combining home and foreign goods in sector \( k \) to produce its output may involve a lower steady-state share of imports in the foreign country than in the home country. Indeed, the foreign technology is given by:

\[
Y_{k,t}^* = \left( \left( 1 - \frac{\psi}{\tau} \right)^\frac{1}{\rho} Y_{F,k,t}^{\rho} \right) \left( \frac{\psi}{\tau} Y_{H,k,t}^{\rho} \right)^{\frac{\rho - 1}{\rho - 1}} \tag{4.2.26}
\]

It is clear from (4.2.26) that the steady-state import share in the foreign country is \( \left( \frac{\psi}{\tau} \right) \), which is smaller than the steady-state import share in the home country \( \psi \) when \( \tau \geq 1 \). This assumption is convenient since it allows us to study small- and large-open economies within the same model. Indeed, for a large economy, one can take \( \tau = 1 \). For a small economy, on the other hand, one can assume \( \tau \) is arbitrarily large as the size of its trading partners is much larger compared to its size.

We also give sector \( k \)'s price index and the demands for the home and foreign goods in sector \( k \) in the foreign country as:

\[
P_{k,t}^* = \left( \left( 1 - \frac{\psi}{\tau} \right) P_{F,k,t}^{1 - \rho} + \left( \frac{\psi}{\tau} \right) P_{H,k,t}^{1 - \rho} \right)^{\frac{1}{1 - \rho}} \tag{4.2.27}
\]

\[
Y_{F,k,t}^* = \left( 1 - \frac{\psi}{\tau} \right) \left( \frac{P_{k,t}^*}{P_{F,k,t}} \right)^{-\rho} Y_{k,t}^* \quad ; \quad Y_{H,k,t}^* = \left( \frac{P_{k,t}^*}{P_{H,k,t}} \right)^{-\rho} Y_{k,t}^* \tag{4.2.28}
\]

where the variables denoted with asterisks (*) show the foreign counterparts of the home variables.
The Invoice Currency and Pricing of Internationally Traded Goods

The home-import good in sector $k$ ($Y_{F,k,t}$) is produced by perfectly competitive home-import firms. Producing the home-import good involves combining intermediate foreign goods which are invoiced in different currencies. Indeed, while some intermediate goods are invoiced in the home currency ($c$), others are invoiced in the foreign currency ($c^*$). In producing the home-import good in Sector $k$, the home-import firm combines output from the foreign firms which set prices in the home and foreign currencies (denoted by $Y_{F,c,k,t}$ and $Y_{F,c^*,k,t}$, respectively) with the following technology:

$$Y_{F,k,t} = \left(1 - \omega_{c^*}^* \right) \frac{\theta_p}{1 - \theta_p} Y_{F,c,k,t}^{(\theta_p-1)/\theta_p} + \omega_{c^*}^* \frac{\theta_p}{1 - \theta_p} Y_{F,c^*,k,t}^{(\theta_p-1)/\theta_p}$$

(4.2.29)

where $\theta_p$ stands for the elasticity of substitution between intermediate foreign goods invoiced in different currencies and $\omega_{c^*}^*$ denotes the steady-state weight of the foreign-currency-invoiced intermediate foreign goods in the home-import price index of sector $k$. It is easy to show that the price index for the home-import good (denoted by $P_{F,k,t}$ and expressed in the home currency) and the demand for the intermediate foreign goods are given as:

$$P_{F,k,t} = \left(1 - \omega_{c^*}^* \right) P_{F,c,k,t}^{1-\theta_p} + \omega_{c^*}^* \left( E_t P_{F,c^*,k,t} \right)^{1-\theta_p}$$

(4.2.30)

$$Y_{F,c,k,t} = (1 - \omega_{c^*}^*) \left( \frac{P_{F,c,k,t}}{P_{F,k,t}} \right)^{-\theta_p} Y_{F,k,t} ; \quad Y_{F,c^*,k,t} = \omega_{c^*}^* \left( \frac{E_t P_{F,c^*,k,t}}{P_{F,k,t}} \right)^{-\theta_p} Y_{F,k,t}$$

(4.2.31)
where \( P_{F,e,k,t} \) and \( P_{F,e^*,k,t} \) represent the prices set for the intermediate foreign goods that are invoiced in the home and foreign currencies, respectively.

The home-export good is produced similarly. Indeed, perfectly competitive foreign importers in sector \( k \) combine output from the home firms which set prices in the home and foreign currencies (denoted by \( Y_{H,e,k,t}^* \) and \( Y_{H,e^*,k,t}^* \), respectively) with the following technology:

\[
Y_{2(k),k,t}^* = \left( \omega_e \frac{1}{E_t} P_{H,e,k,t}^{1-\theta_p} + (1 - \omega_e) \frac{1}{E_t} P_{H,e^*,k,t}^{1-\theta_p} \right)^{\theta_p} \tag{4.2.32}
\]

where \( \omega_e \) is the steady-state share in sector \( k \)'s foreign-import price index of the home-currency-priced intermediate home-export goods. The foreign-import price index (denoted by \( P_{3(k),k,t}^* \) and expressed in the foreign currency) and the demands for the intermediate home-export goods can be written as:

\[
P_{3(k),k,t}^* = \left( \omega_e \left( \frac{1}{E_t} P_{H,e,k,t} \right)^{1-\theta_p} + (1 - \omega_e) P_{H,e^*,k,t}^{1-\theta_p} \right) \tag{4.2.33}
\]

\[
Y_{H,e,k,t}^* = \omega_e \left( \frac{P_{H,e,k,t}}{P_{3(k),k,t}} \right)^{-\theta_p} Y_{2(k),k,t}^* ; \quad Y_{H,e^*,k,t}^* = (1 - \omega_e) \left( \frac{P_{H,e^*,k,t}}{P_{3(k),k,t}} \right)^{-\theta_p} Y_{3(k),k,t}^* \tag{4.2.34}
\]

where \( P_{H,e,k,t}^* \) and \( P_{H,e^*,k,t}^* \) denote the prices set for the intermediate home-export goods whose prices are invoiced in the home and foreign currencies, respectively.
Home and Foreign Firms Producing Varieties for Intermediate Goods

The intermediate domestic and import goods in both the home and foreign countries are composite goods composed of a variety of goods produced by firms engaging in monopolistic competition. The production technology used in the production of intermediate domestic goods is given as:

\[ Y_{\gamma,k,t} = \left( \int_0^1 Y_{H,k,j,t} \left( \frac{\theta_p - 1}{\theta_p} \right) dj \right)^{\frac{\theta_p}{\theta_p - 1}} \]  

(4.2.35)

where \( Y_{H,k,j,t} \) denotes demand for variety \( j \) of the firm producing the domestic intermediate good in the home country in sector \( k \). One can show that \( Y_{H,k,j,t} \) and the price index for the domestic intermediate good in the home country in sector \( k \) (\( P_{\gamma,k,t} \)) can be written as:

\[ Y_{H,k,j,t} = \left( \frac{P_{H,k,j,t}}{P_{\gamma,k,t}} \right)^{-\theta_p} Y_{\gamma,k,t} \]  

(4.2.36)

\[ P_{\gamma,k,t} = \left( \int_0^1 P_{H,k,j,t}^{1-\theta_p} dj \right)^{-\frac{1}{\theta_p}} \]  

(4.2.37)

where \( P_{H,k,j,t} \) is the price set by the monopolistically competitive firm producing variety \( j \) of the domestic intermediate good. When producing variety \( j \), the firm employs the composite labor (\( N_{H,k,j,t} \)) together with capital (\( K_{H,k,j,t} \)) and uses the following production function:

\[ Y_{H,k,j,t} = K_{H,k,j,t}^{1-\chi} N_{H,k,j,t}^{\chi} \]  

(4.2.38)
where $\chi$ is the steady-state share of labor in the home country. In each period, only a fraction of the firms producing different varieties in sector $k$ obtains a price-change signal. When firms obtain such a signal, they set prices with their intermediate domestic-goods suppliers. These prices remain constant until a new price-change signal is obtained. During this time, firms are obliged to supply any quantity demanded of their varieties. In the one-sector model, it is assumed sectors have the same frequency of price changes which is given by the weighted average of the frequencies of price changes in sectors. In the multi-sector model, on the other hand, the probability of receiving such a signal differs by sector. For the varieties of domestic sector $k$’s good in the home country, let $1 - \alpha_k$ indicate the probability of receiving the price-change signal in each period. Then, the objective of the firm producing variety $j$ which obtains a price-change signal in period $t$ can be written as:

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha^s_k (X_{H,k,j,t} Y_{H,k,j,t+s} - W_{t+s} N_{H,k,j,t+s} - R_{t+s}^k K_{H,k,j,t+s})$$ (4.2.39)

where $X_{H,k,j,t}$ denotes the contracted price for the home variety $j$ in sector $k$’s domestic good in the home country. Let $\Lambda_{\gamma,k,t}$ be defined as:

$$\Lambda_{\gamma,k,t} = \left( \frac{1}{P_{\gamma,k,t}} \right)^{-\theta_p} \left( \frac{P_{\gamma,k,t}}{P_{k,t}} \right)^{-\rho} \left( \frac{P_{k,t}}{P_t} \right)^{-\eta} Y_t$$ (4.2.40)

Then, from the first-order condition of (4.2.39), $X_{H,k,j,t}$ can be written as:

$$X_{H,k,j,t} = \frac{\theta_p}{\theta_p - 1} \left( \frac{1}{1 - \chi} \right)^{1 - \chi} \left( \frac{1}{\chi} \right)^{\chi} \frac{E_t \sum_{s=0}^{\infty} \beta^s \alpha^s_k (W_{t+s} P_{k,t}^{1 - \chi} \Lambda_{\gamma,k,t+s})}{E_t \sum_{s=0}^{\infty} \beta^s \alpha^s_k \Lambda_{\gamma,k,t+s}}$$ (4.2.41)
Since the objective function is identical across the firms that produce differentiated goods in sector $k$ and obtain a price-change signal in the same period, their contracted prices are the same ($X_{H,k,j,t} = X_{H,k,t}$). This, together with the Calvo-type randomization assumption, implies that $P_{\gamma,k,t}$ can be rewritten as:

$$P_{\gamma,k,t} = (1 - \alpha_{H,k})X_{H,k,t} + \alpha_{H,k}P_{\gamma,k,t-1} \quad (4.2.42)$$

Similar to the domestic intermediate good, the home-export goods are composite goods made up of a continuum of varieties produced by monopolistically competitive firms:

$$Y_{H,e,k,t}^* = \left( \int_0^1 Y_{H,e,k,j,t}^* \left( \frac{1}{\theta_p} \frac{1}{\theta_p} \right) \frac{1}{\theta_p} \frac{1}{\theta_p} \right) \frac{1}{\theta_p} \frac{1}{\theta_p} \quad (4.2.43)$$

where the demand for the home-export variety of $j$ priced in the home currency (the foreign currency) is denoted by $Y_{H,e,k,j,t}^* (Y_{H,e^*,k,j,t}^*)$.

The monopolistically competitive firm producing variety $j$ and the aggregator firm demanding this variety invoice in the same currency. It is also notable that while the varieties produced for the home-export firms are allowed to be invoiced in different currencies in the model, the demand elasticity between any two home-export varieties is not affected by the invoice currency. Indeed, the demand elasticity between any two home-export varieties is equal to $\theta_p$, regardless of whether they are priced in the same or different currencies.$^54$

Next, we write the maximization problem of the firm that produces variety $j$ for the home exporters and that set prices in the home currency (the foreign currency) as (4.2.44)

$^54$See Equation (4.2.29) and (4.2.43).
where $1-\alpha_k^*$ is the constant probability of receiving a price-change signal in the foreign-sector $k$, which is allowed to differ from that in the home-sector $k$ ($1-\alpha_k$). Since inflation influences the frequency of price changes, the assumption of a constant probability of receiving a price change signal may be considered strong due to variable inflation in our sample of developing economies. We discuss this in Appendix C.3.1 and conclude that over stable inflation periods, the frequency of price changes in sectors can be considered stable.

In writing (4.2.44) and (4.2.45), we make an important assumption that the invoice currency of monopolistically competitive home-export firms also determines the price rigidity which the firms face. Indeed, while the prices set in the home currency remain fixed with the probability of $\alpha_k$ in each period, those set in the foreign currency are subject to the price rigidity in the foreign sector $k$ and remain fixed with the probability of $\alpha_k^*$. We also make an analogous assumption for the monopolistically competitive home-import firms.

One can show that the optimal prices set for the home-export varieties $j$ which are invoiced in the home currency ($X_{H,c,k,j,t}^*$) and the foreign currency ($X_{H,c^*,k,j,t}^*$) can be written as:
\[ X_{H,e,k,j,t}^* = \frac{\theta_p}{\theta_p - 1} \left( \frac{1}{1 - \chi} \right)^{1 - \chi} \left( \frac{1}{\chi} \right) \frac{E_t \sum_{s=0}^{\infty} \beta^s \alpha_k^s \left( W_{t+s}^t P_{t+s}^k \right) \Lambda_{H,e,k,t+s}^*}{E_t \sum_{s=0}^{\infty} \beta^s \alpha_k^s \Lambda_{H,e,k,t+s}^*} \] (4.2.46)

\[ X_{H,e^*,k,j,t}^* = \frac{\theta_p}{\theta_p - 1} \left( \frac{1}{1 - \chi} \right)^{1 - \chi} \left( \frac{1}{\chi} \right) \frac{E_t \sum_{s=0}^{\infty} \beta^s \alpha_k^s \left( W_{t+s}^t P_{t+s}^k \right) \Lambda_{H,e^*,k,t+s}^*}{E_t \sum_{s=0}^{\infty} \beta^s \alpha_k^s \Lambda_{H,e^*,k,t+s}^*} \] (4.2.47)

where \( \Lambda_{H,e,k,t+s}^* \) and \( \Lambda_{H,e^*,k,t+s}^* \) are defined as:

\[ \Lambda_{H,e,k,t+s}^* = \left( \frac{1}{P_{H,e,k,t+s}^*} \right)^{-\theta_p} \left( \frac{P_{H,e,k,t+s}^*}{P_{H,e^*,k,t+s}^*} \right)^{-\theta_p} \left( \frac{P_{H,e^*,k,t+s}^*}{P_{H,e,k,t+s}^*} \right)^{-\rho} \left( \frac{P_{e^*,k,t+s}^*}{P_t^*} \right)^{-\eta} Y_t^* \] (4.2.48)

\[ \Lambda_{H,e^*,k,t+s}^* = \left( \frac{1}{P_{H,e^*,k,t+s}^*} \right)^{-\theta_p} \left( \frac{P_{H,e^*,k,t+s}^*}{P_{H,e,k,t+s}^*} \right)^{-\theta_p} \left( \frac{P_{H,e,k,t+s}^*}{P_{H,e^*,k,t+s}^*} \right)^{-\rho} \left( \frac{P_{e^*,k,t+s}^*}{P_t^*} \right)^{-\eta} Y_t^* \] (4.2.49)

The maximization problem of foreign firms can analogously be written.

### 4.2.3 Closing the Model

Our first approach to close the model is to assume the growth of nominal spending follows an exogenous process in both countries:
\[
\begin{align*}
\log Z_t - \log Z_{t-1} &= \rho z (\log Z_{t-1} - \log Z_{t-2}) + \epsilon^z_t \quad \epsilon^z_t \sim N(0, \sigma^z_e) \\
\log Z_t^* - \log Z_{t-1}^* &= \rho z (\log Z_{t-1}^* - \log Z_{t-2}^*) + \epsilon^z_t \quad \epsilon^z_t \sim N(0, \sigma^z_{e^*})
\end{align*}
\]

(4.2.50)

where \( Z_t = P_t Y_t \) and \( Z_t^* = P_t^* Y_t^* \) denote nominal spending in the home and foreign countries, respectively.

### 4.3 Calibration and Estimation

This section discusses calibration of the models’ parameters. It should be noted that since monthly frequencies of price changes are readily available, whereas quarterly frequencies are not, we assess the ability of the models by comparing the outcomes from the models with those in the actual economies using monthly data. In Table C.1 of Section C.3 of the appendix, we present calibrated parameter values along with a source on which we base our calibration for these parameters. We start with \( \theta_p \). It is taken to be equal to 11, implying an average markup of 10%, which is the estimated markup rate for the auto industry of the United States in Bresnahan (1981). We set \( \delta = 0.008 \), implying an annual rate of depreciation of 10%, which is the estimated annual rate of depreciation in the United States in Christiano & Eichenbaum (1992). We calibrate the values for \( \sigma_c, \sigma_n, \sigma_a, \sigma_\phi, \Theta^\hat{Y}, \rho, \eta \) and \( \chi \) directly from the sources outlined in Table C.1. \( \beta \) is set to 1.03112, which implies an annual real interest rate of 3%.

Next, we calibrate the frequency of price changes in each sector. It is noteworthy that since the main trading partners of developing economies are advanced countries, the price-stickiness parameters and sectoral weights in the foreign country (denoted by \( \alpha_k^* \) and \( f_k \)) need to be calibrated as those in advanced countries when we study aggregate dynamics following monetary shocks in developing economies in our model. When calibrating these
parameters, we rely on the estimates reported in Carvalho & Nechio (2011). They estimate the weighted average of the frequency of price adjustments ($\sum_{k=1}^{67} f_k(1 - \alpha_k^*)$) in the United States as 0.21. Based on this, we take the foreign price stickiness, $\alpha_k^*$, in the one-sector model as 0.79.

The home frequency of price changes, $1 - \alpha_k$, in the one-sector model is calibrated as 27.2%. That is, on average, 27.2% of prices change in each month in developing economies, which is in line with the estimates of the mean frequency of price changes in Mexico in Gagnon (2009) when inflation remained between 4% and 14%. We do not have estimates of sectoral frequency of price adjustments in developing economies. In calibrating sectoral price stickiness in developing economies for the multi-sector model, we ensure that $\sum_{k=1}^{67} f_k(1 - \alpha_k) = 0.272$. We also assume that the expected duration of price contracts in a home sector is shorter than that in its foreign counterpart by some factor, say by $D$. If $D$ is taken as 1.45, we find that the aforementioned condition is met. That is, if sectoral prices in these economies change 1.45 times more frequently than those in the United States, the condition that $\sum_{k=1}^{67} f_k(1 - \alpha_k) = 0.272$ is met. With such an assumption, the sectoral frequency of price changes in the home country can be calibrated using the following steps. First, estimate the expected duration of price contracts in a sector in the United States with the following formula,

$$d_k^* = \frac{1}{ln\alpha_k^*}$$

Second, estimate the expected duration of sectoral price contracts in the home country

\footnote{It is notable that while Carvalho & Nechio (2011) use the data from Nakamura & Steinsson (2008a) who report the frequency of price changes and the expenditure share for 271 categories of goods and services in the United States, to make their model computationally manageable, Carvalho & Nechio (2011) only include 67 sectors in their model by aggregating some sectors.}
by assuming that it is 1.45 times shorter than that in the United States,

\[ d_k = \frac{d_k^*}{1.45} \]

In the last step, estimate sectoral price stickiness in developing economies with,

\[ \alpha_k = e^{\frac{-1}{\Delta_k}} \]

Even if the frequency of price changes is calibrated for 67 sectors, we only include 3 sectors in our multi-sector model. The reason is that we have to estimate some parameters using minimum distance estimation in our paper and it is not computationally feasible to do estimation with 67 sectors. In reducing the number of sectors to three, we first order the sectors according to their frequencies of price changes. Next, we include the sectors whose frequency of price changes lies in \([0, 33],[34, 66]\) and \([66,100]\) percentiles of frequencies of price changes in the first, second and third group, respectively. The frequency of price changes that represents each group is approximated by the median frequency of price changes in each group. The expenditure share of each group \((f_k)\), on the other hand, is taken as the sum of the expenditure shares of the sectors forming the group.

In calibrating the shares of final consumption \((s_c)\), investment \((s_m)\) and home imports \((\psi)\) in GDP, we use data for these series from the World Bank’s World Development Indicators in 2002. \(s_c, s_m\) and \(\psi\) are taken as the median values in the group. \(\tau\) which denotes the economic size of the foreign country relative to that of the home country is taken as 1000. \(\tau\) is set to be very high for developing economies, in line with the common small-country assumption for these countries in the literature. It is notable that setting \(\tau\) to a large value for developing economies, together with the assumption of no international

\[ ^{56} \text{This follows from } d_k = -\frac{1}{\ln \alpha_k} \]

139
borrowing at the steady state, requires that the steady-state shares of exports and imports in the foreign country be only $\frac{1}{\tau}$ as big as those in the home country. This is the essence of the small-country assumption in our model. The share of the home exports priced in the home currency ($\omega_e$) and the share of the home imports priced in the foreign currency ($\omega_e^*$) are calibrated based on the findings in Section C.3.2 for Turkey.

Lastly, in order to calibrate $\rho_z$, which represents the persistence in the exogenous nominal spending growth process in (4.2.50), the Panel AR(12) model for log changes in the monetary aggregates M1 and M2 are estimated for our sample using monthly data with country-specific fixed effects. The sum of AR coefficients for M1 and M2 are estimated as 0.35 and 0.29, respectively. Based on this, we set $\rho_z = 0.32$.

To study dynamics after nominal spending shocks, both models are log-linearized around the zero-inflation and zero-debt steady state.

4.4 Quantitative Results

In this section, our aim is to evaluate the ability of the one- and multi-sector models to account for the dynamics of output, the price level, the real and nominal exchange rates after monetary shocks in developing economies which adopted an inflation targeting regime.

4.4.1 Output and Price Level Dynamics

Figure 4.5 displays the model- and panel VAR-based impulse response functions of output ($\hat{Y}_t$) and the price level ($P_t$) in the home country.\footnote{It is notable that real spending (denoted by $Y_t$) differs from domestic output. We denote domestic output in the home country as $\hat{Y}_t$. $\hat{Y}_t$ can be written as:}

\[
\hat{Y}_t = \sum_{k=1}^{K} f_k (1 - \psi) \hat{Y}_{c,k,t} + \sum_{k=1}^{K} f_k \psi \omega_e \hat{Y}^*_H, c, k, t + \sum_{k=1}^{K} f_k \psi (1 - \omega_e) \hat{Y}^*_H, c^*, k, t
\]
Figure 4.5: Model- and Panel VAR-Based Impulse Responses of $P$ and $Y$ to $\epsilon_z$

Note: Our calculations are based on the IMF’s *International Finance Statistics*. The dotted lines with pentagrams and the dashed lines with squares indicate the model-based impulse response functions in the one- and multi-sector models, respectively. The solid lines show the estimated point-wise panel VAR-based impulse response functions. The area between the dotted lines shows the 90% confidence interval estimated with the method suggested by Sims & Zha (1999).

pentagrams and dotted line with squares show the impulse response functions to a domestic expansionary shock in the one- and multi-sector models, respectively. The panel VAR-based impulse responses of the variables in developing economies obtained in *Empirical Model II* with the monthly data are displayed with the solid lines. Lastly, the area between the dotted lines show the 90% confidence interval of the panel VAR-based impulse response functions estimated with the method suggested by Sims & Zha (1999). It is notable that for both the model- and panel VAR-based impulse response functions, we consider a monetary shock in developing economies that results in a 1% long-run increase in $P$.

We first discuss the price level dynamics. A striking observation in Figure 4.5 is that the price level responses in the multi-sector model stays muted compared to those in the
one-sector model. This point is explained succinctly in Nakamura & Steinsson (2013) for the case of no strategic interaction among firms. Suppose that an economy has two sectors. Let the first sector have a low frequency of price changes so that it takes quite a while for firms in this sector to respond to an aggregate shock (the sticky-price sector). Let the second sector have high price flexibility so that prices may respond fast to an aggregate shock in this sector (the flexible-price sector). It can be argued that firms in the flexible-price sector might have a chance to change their prices several times before firms in the sticky-price sector do so for the first time. However, apart from the period in which firms in the flexible-price sector obtain a chance to change their prices for the first time, the price adjustment in this sector in accompanying periods adds little to the aggregate price adjustment since firms adjust fully to the shock when they first obtain a chance to respond. In other words, apart from the first responses, all other price responses in the flexible-price sector are “wasted”. For the complete aggregate price adjustment, it is crucial that firms in the sticky-price sector obtain a chance to change their prices at least once after the shock. Nakamura & Steinsson (2013) note that if it were possible to have a more even distribution of the frequency of price changes among sectors, the aggregate price adjustment would be much faster. This conjecture is supported by our findings. Indeed, in the one-sector model, by taking the weighted average of the frequencies of price changes among sectors as the frequency of price changes in the economy, some price changes are implicitly reallocated from the flexible-price sector to the sticky-price sector. As a result, it is not surprising to observe a stronger contemporaneous response of the aggregate price level and faster price adjustment in the one-sector model than in the multi-sector model.

Regarding output, it is clear in Figure 4.5 that output shows less persistent dynamics in the one-sector model than the multi-sector model. This can be accounted for by a faster price adjustment in the former.
4.4.2 Real and Nominal Exchange Rate Dynamics

Figure 4.6 displays the dynamics of nominal and real exchange rate in the one- and multi-sector models along with their panel VAR-based dynamics. It is evident that the nominal exchange rate undershoots its new long-run level, which contrasts with a sizable overshooting of the nominal exchange rate in the actual economies shown in this figure. This mainly results from the muted initial impulse response functions of the real exchange rate.

Our findings regarding the models indicate that both the one- and multi-sector models are of limited ability in explaining the aggregate dynamics in developing economies following a monetary shock. Indeed, some impulse response functions stay out of 90% confidence intervals. Particularly, nominal exchange dynamics in the actual economies are poorly predicted by these models.

How can the predictions of the one- and multi-sector models be improved? We show in the next section that when adjustment costs of new capital are so large that they prohibit investment, the extent to which the exchange rate overshoots increases and the models’ performance improves to a certain degree.

4.4.3 One- and Multi-Sector Models without Investment

To understand the reason for the limited degree of exchange rate overshooting in the models, it is useful to consider the real exchange rate equation in the model. It can be shown from (4.2.13) that the % deviation of the real exchange rate ($\hat{Q}_t$) from its steady

\[ \frac{\%}{\text{deviation}} \text{ of the real exchange rate} \]

\[ \hat{Q}_t = Q^* - P - P^* \]

where $Q^*$, $P$, $P^*$ denote randomly generated impulse functions of the real exchange rate, the price level in developing economies and the United States, respectively. The area that stays within the 5th and 95th percentile of the distribution of randomly generated impulse response functions of the nominal exchange rate is reported in Figure 4.5 as the 90% confidence interval for the impulse response functions of the nominal exchange rate.

\[ 58 \text{ To obtain 90% confidence intervals for the impulse response functions of the nominal exchange rate, we first obtain 1000 randomly generated impulse response functions of the nominal exchange rate over 36 months (E) as E = Q^* - P - P^* where Q^*, P, P^* denote randomly generated impulse functions of the real exchange rate, the price level in developing economies and the United States, respectively. The area that stays within the 5th and 95th percentile of the distribution of randomly generated impulse response functions of the nominal exchange rate is reported in Figure 4.5 as the 90% confidence interval for the impulse response functions of the nominal exchange rate.} \]
Figure 4.6: Model- and Panel VAR-Based Impulse Responses of $\varepsilon$ and $Q$ to $\varepsilon^z$

(a) Nominal Exchange Rate ($E$)

(b) Real Exchange Rate ($Q$)

Note: Our calculations are based on the IMF’s *International Finance Statistics*. The dotted lines with pentagrams and the dashed lines with squares indicate the model-based impulse response functions in the one- and multi-sector models, respectively. The solid lines show the estimated point-wise panel VAR-based impulse response functions. The area between the dotted lines shows the 90% confidence interval estimated with the method suggested by Sims & Zha (1999).

state in the models is given by,

$$
\sigma_c \left[ E_t \left( \hat{C}_{t+1} - \hat{C}_t \right) - E_t \left( \hat{C}^*_t - \hat{C}^*_t \right) \right] = E_t(\hat{Q}_{t+1} - \hat{Q}_t) + \Theta'(0)\bar{Y}\hat{B}_{t+1} \quad (4.4.1)
$$

Since we maintain the small-country assumption, the impulse response functions of foreign consumption should be negligible after a monetary shock in developing economies ($\hat{C}^*_t \approx 0$). This, together with the small value of calibrated interest elasticity of foreign debt ($\Theta'(0)\bar{Y}$), implies that

$$
E_t(\hat{Q}_t - \hat{Q}_{t+1}) \approx \sigma_c E_t(\hat{C}_t - \hat{C}_{t+1}) \quad (4.4.2)
$$

144
Figure 4.7: Model- and Panel VAR-Based Impulse Responses of $P$, $Y$, $E$ and $Q$ to $\epsilon^z$ (Without Investment)

Note: Our calculations are based on the IMF’s *International Finance Statistics*. The dotted lines with pentagrams and the dashed lines with squares indicate the model-based impulse response functions in the one- and multi-sector models, respectively. The solid lines show the estimated point-wise panel VAR-based impulse response functions. The area between the dotted lines shows the 90% confidence interval estimated with the method suggested by [Sims & Zha (1999)](##).  

---

59Our models predict a strong correlation between relative consumption and the real exchange rate in conditional expectations. This does not necessarily contradict with the well-known weak *unconditional* correlation in the data. As shown by [Corsetti, Dedola & Leduc (2008)](##), persistent productivity shocks may
From (4.4.2), we conjecture that the weak contemporaneous response of the real and nominal exchange rates in the models can be traced to a weak contemporaneous response of consumption. Put differently, should the contemporaneous response of consumption have increased, the undesirable outcome of the nominal exchange rate undershooting in the models would be avoided. To this end, it is useful to consider the resource constraint in the home country:

\[
s_C \dot{C}_t + s_I \dot{I}_t + \frac{s_I}{\delta} \left( \frac{1}{\beta} - (1 - \delta) \right) \dot{u}_t = \dot{Y}_t
\]  

(4.4.3)

where \( s_C \) and \( s_I \) are the steady-state shares of consumption and investment in real spending in the home country, respectively. We conjecture that by increasing the contemporaneous response of \( C_t \) for some given \( Y_t \), excluding investment in the models may result in a more profound contemporaneous response of \( Q_t \), which may help the models to predict an overshooting of the exchange rates after monetary shocks.

Figure 4.7 offers supporting evidence for our conjecture that when investment is excluded from the models, \( Q_t \) gives a stronger contemporaneous response. This helps the models predict the nominal exchange rate overshoots its long-run level after the monetary shocks as found in the actual economies. Moreover, unlike the price dynamics in the one-sector model, the price dynamics in the multi-sector model never stay out of 90% confidence intervals of the impulse response functions of the aggregate variables in the actual economies when investment is too costly to make.

Lastly, one may argue that instead of excluding investment, the one- and multi-sector models without a variable rate of capacity utilization (\( u_t \)) would produce a higher exchange rate overshooting in the real and nominal exchange rates since the contemporaneous re-

lower the correlation by causing consumption and the real exchange rate to move in opposite directions on impact due to strong wealth effects under incomplete financial markets.
response of consumption would be stronger without a variable capacity utilization. However, we find excluding the variable $u_t$ has a negligible effect on the extent of overshooting. The reason is that when capacity is fully utilized in all periods ($\hat{u}_t = 0$), the rental rate of capital increases immediately when an expansionary monetary shock occurs, causing a stronger contemporaneous response of the price level and a weaker contemporaneous response of real spending. Consequently, when capital is assumed to be fully utilized in all periods, both $\hat{u}_t$ and $\hat{Y}_t$ fall, causing a small change in $\hat{C}_t$. This results in the nominal and real exchange rate overshooting being limited after the monetary shock (see (4.4.2)).

4.5 Conclusion

In this paper, we have studied what happens to output, the price level, the real and nominal exchange rates after a positive domestic monetary shock in developing economies under an inflation targeting regime. We have found such a shock causes a short-lived rise in output, a temporary real exchange rate depreciation, a sizable overshooting of the nominal exchange rate and an increase in the price level in the short- and long-terms in these countries. Then, we have compared these findings with the outcomes in the one- and multi-sector models under staggered wage setting. When adjustment costs of acquiring new capital is low, neither the former nor the latter can successfully account for the nominal exchange rate overshooting following domestic monetary shocks in the actual economies. Yet, when such costs are large, we have found that the multi-sector model successfully explains the aggregate dynamics following domestic monetary shocks in developing economies.
Chapter 5

Conclusion

This thesis considers the implications of heterogeneity in price flexibility among sectors. It consists of three papers. The first paper studies the extent to which allowing sectors to have different frequencies of price changes contributes to explaining the dynamics of inflation, output, the real wage and the federal funds rate following a monetary shock in the United States. The second paper analyzes the relationship between the frequency of price changes in sectors and the impulse responses of sectoral prices to an interest rate shock in the United States. Finally, the third paper addresses the effects on output, the price level, the real and nominal exchange rates of a monetary shock in developing countries which adopted an inflation targeting regime within the context of both the one- and multi-sector small-open economy DSGE models.
Bibliography


Carvalho, C. (2006). Heterogeneity in price stickiness and the real effects of monetary


153


Appendix A

Appendix to Chapter 2

A.1 Sectoral Inflation

A.1.1 Optimized Prices and Sectoral Inflation

All firms within the same sector which are capable of optimizing their prices have the same maximization problem and therefore set the same price. Using this fact and (2.2.41), one can rewrite (2.2.13) as:

\[ P_{jt}^{1-\theta_p} = (1 - \alpha_{pj})P_{jt}^{*1-\theta_p} + \left( \frac{P_{t-1}}{P_{t-2}} \right) \gamma_p(1-\theta_p) n_j^{-1} \int_0^{n_j\alpha_{pj}} p_{jt,t-1}(j')^{1-\theta_p} dj' \quad (A.1.1) \]

Each period, \( \alpha_{pj} \) represents the fraction of firms not receiving a price-change signal. \( p_{jt,t-1}(j') \) denotes the period \( t-1 \) prices of firms in sector \( j \) which are not able to optimize in period \( t \). Let \( k_j \) be the number of firms with the price \( p_{jt-1}(j') \) in the period \( t-1 \). Due to the randomization assumption in the Calvo model, there are exactly \( \alpha_{pj}k_j \) non-optimizing firms in period \( t \) with price \( p_{jt,t-1}(j') = p_{jt-1}(j') \). Note also that the space of \( p_{jt-1} \) is equivalent to the space of \( p_{jt,t-1} \) due to randomization. In other words, for an arbitrary \( p_{jt-1} \), there exists \( p_{jt,t-1} \) such that these two are equal. Consequently, the following must hold:
\[
n_j^{-1} \int_0^{n_j \alpha_{pj}} p_{jt,t-1}(j')^{1-\theta_p} dj' = \alpha_{pj} n_j^{-1} \int_0^{n_j} p_{jt-1}(j')^{1-\theta_p} dj' = \alpha_{pj} P_{jt-1}^{1-\theta_p}
\]

Hence, (A.1.1) can be rewritten as:

\[
P_{jt}^{1-\theta_p} = (1 - \alpha_{pj}) \hat{p}_{jt}^{*1-\theta_p} + \alpha_{pj} \left( \frac{P_{t-1}}{P_{t-2}} \right) \gamma_p (1-\theta_p) P_{jt-1}^{1-\theta_p} \quad (A.1.2)
\]

Log-linearizing this equation gives:

\[
\hat{P}_{jt} = (1 - \alpha_{pj}) \hat{p}_{jt}^* + \alpha_{pj} \left( \gamma_p \pi_{t-1} + \hat{P}_{jt-1} \right)
\]

or equivalently,

\[
\hat{p}_{jt}^* - \hat{P}_{jt} = \frac{\alpha_{pj}}{1 - \alpha_{pj}} \left( \pi_{jt} - \gamma_p \pi_{t-1} \right) \quad (A.1.3)
\]

where inflation in sector \( j \) is defined as:

\[
\pi_{jt} = \hat{P}_{jt} - \hat{P}_{jt-1}
\]
A.2 Sectoral Inflation

For the sectoral inflation equation, I work with a more general model where the elasticity of substitution of goods in different sectors is allowed to differ from that of goods in the same sector. Thus, while the sectoral aggregator consumption given in (2.2.3) is the same, the economy-wide consumption aggregator given in (2.2.2) is changed and given as:

\[
C_t = \left[ \sum_{j=1}^{J} t_j^{1/\eta} C_j^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}
\]

(A.2.1)

Then it can be shown that \( \hat{y}_{t+s(j')} = -\eta \left( \hat{P}_{jt+s} - \hat{P}_{t+s} \right) - \theta_p \left( \hat{p}_{jt+s,t}^{*}(j') - \hat{P}_{jt+s} \right) + \hat{Y}_{t+s} \), and one can rewrite (2.2.48) as:

\[
E_{t-1} \sum_{s=0}^{\infty} (\beta \alpha_p)^s \left[ (1 + \omega_p \theta_p) \left( \hat{p}_{jt+s}^{*}(j') - \hat{P}_{jt+s} \right) + (1 + \omega_p \eta) \left( \hat{P}_{jt+s} - \hat{P}_{t+s} \right) + (1 + \omega_p \theta_p) \hat{\chi}_{t,t+s}^p \right.

- \left( \hat{R}_{t+s} + \hat{W}_{real,t+s} + (\omega_p \hat{Y}_{t+s} - (1 + \omega_p) \hat{Z}_{t+s}) \right) \right] = 0
\]

(A.2.2)

Rearranging terms yields:

\[
\hat{p}_{jt}^{*}(j') = \frac{(1-\beta \alpha_p)}{(1+\omega_p \theta_p)} E_{t-1} \sum_{s=0}^{\infty} (\beta \alpha_p)^s \left[ (1 + \omega_p \theta_p) \left( \hat{P}_{jt+s} - \hat{P}_{t+s} \right) + (1 + \omega_p \eta) \left( \hat{P}_{jt+s} - \hat{P}_{t+s} \right) + (1 + \omega_p \theta_p) \hat{\chi}_{t,t+s}^p \right.

+ \left( \hat{R}_{t+s} + \hat{W}_{real,t+s} + (\omega_p \hat{Y}_{t+s} - (1 + \omega_p) \hat{Z}_{t+s}) \right) \right]
\]

(A.2.3)

(A.2.3) can be shown to be equal to:
\( \tilde{p}_t'(j') = (1 - \beta \alpha_{pj}) E_{t-1} \tilde{P}_t + \frac{(1-\beta \alpha_{pj})}{(1+\omega_p \theta_p)} \times \\
E_{t-1} \left[-(1 + \omega_p \eta) \left( \tilde{P}_{jt+s} - \tilde{P}_{jt+s} \right) + \tilde{R}_t + \tilde{W}_{real,t} + \left( \omega_p \tilde{Y}_t - (1 + \omega_p) \tilde{Z}_t \right) \right] + \beta \alpha_{pj} \frac{(1-\beta \alpha_{pj})}{(1+\omega_p \theta_p)} \times \\
E_{t-1} \sum_{s=0}^{\infty} (\beta \alpha_{pj})^s \left[ (1 + \omega_p \theta_p) \tilde{P}_{jt+s} - \tilde{P}_{jt+s} \right] - (1 + \omega_p \theta_p) \tilde{Y}_{t+1+s} + \tilde{Y}_{t+1+s} - (1 + \omega_p) \tilde{Z}_{t+1+s} \right] \right] \\
(A.2.4)

Using the fact that \( \tilde{\chi}_{t+1,t+1+s}^p = \left( \tilde{\chi}_{t+1,t+1+s}^p + \gamma_p \pi_t \right) \), (A.2.4) can alternatively be rewritten as:

\[ \tilde{p}_t'(j') = (1 - \beta \alpha_{pj}) E_{t-1} \tilde{P}_t + \frac{(1-\beta \alpha_{pj})}{(1+\omega_p \theta_p)} \times \\
E_{t-1} \left[-(1 + \omega_p \eta) \left( \tilde{P}_{jt+s} - \tilde{P}_{jt+s} \right) + \tilde{R}_t + \tilde{W}_{real,t} + \left( \omega_p \tilde{Y}_t - (1 + \omega_p) \tilde{Z}_t \right) \right] + \beta \alpha_{pj} \frac{(1-\beta \alpha_{pj})}{(1+\omega_p \theta_p)} \times \\
E_{t-1} \sum_{s=0}^{\infty} (\beta \alpha_{pj})^s \left[ (1 + \omega_p \theta_p) \tilde{P}_{jt+1+s} - \tilde{P}_{jt+1+s} \right] - (1 + \omega_p \theta_p) \left( \tilde{\chi}_{t+1,t+1+s}^p + \gamma_p \pi_t \right) \left( \tilde{R}_{t+1+s} + \tilde{W}_{real,t+1+s} + \left( \omega_p \tilde{Y}_{t+1+s} - (1 + \omega_p) \tilde{Z}_{t+1+s} \right) \right] \right] \\
(A.2.5)

For each step in the following equations, I rewrite the equation which comes before them in an alternative form:

\[ \tilde{p}_t'(j') = (1 - \beta \alpha_{pj}) E_{t-1} \tilde{P}_t + \frac{(1-\beta \alpha_{pj})}{(1+\omega_p \theta_p)} \times \\
E_{t-1} \left[-(1 + \omega_p \eta) \left( \tilde{P}_{jt+s} - \tilde{P}_{jt+s} \right) + \tilde{R}_t + \tilde{W}_{real,t} + \left( \omega_p \tilde{Y}_t - (1 + \omega_p) \tilde{Z}_t \right) \right] + \beta \alpha_{pj} \frac{(1-\beta \alpha_{pj})}{(1+\omega_p \theta_p)} \times \\
E_{t-1} \sum_{s=0}^{\infty} (\beta \alpha_{pj})^s \left[ (1 + \omega_p \theta_p) \tilde{P}_{jt+1+s} - \tilde{P}_{jt+1+s} \right] - (1 + \omega_p \theta_p) \left( \tilde{\chi}_{t+1,t+1+s}^p + \gamma_p \pi_t \right) \left( \tilde{R}_{t+1+s} + \tilde{W}_{real,t+1+s} + \left( \omega_p \tilde{Y}_{t+1+s} - (1 + \omega_p) \tilde{Z}_{t+1+s} \right) \right] \right] \\
(A.2.6)

158
Hence, (A.2.6) is given by:

\[ \hat{\beta} \alpha_{pj} E_{t-1} \hat{P}_{jt+1}(j') = (1 - \beta \alpha_{pj}) E_{t-1} \hat{P}_{jt} + \frac{(1 - \beta \alpha_{pj})}{(1 + \omega_p \theta)} E_{t-1} \left( - (1 + \omega_p \eta) \left( \hat{P}_{jt+1} - \hat{P}_{jt} \right) + \hat{R}_t + \hat{W}_{real,t} + \left( \omega_p \hat{Y}_t - (1 + \omega_p) \hat{Z}_t \right) - \beta \alpha_{pj} \gamma_p E_{t-1} \pi_t + \beta \alpha_{pj} E_{t-1} \hat{P}_{jt+1}(j') \right) \]  

(A.2.7)

Using (A.1.3), one can show that:

\[ \frac{\alpha_{pj}}{1 - \alpha_{pj}} \left( \pi_{jt} - \gamma_p \pi_{t-1} \right) = \frac{(1 - \beta \alpha_{pj})}{(1 + \omega_p \theta)} E_{t-1} \left( - (1 + \omega_p \eta) \left( \hat{P}_{jt+1} - \hat{P}_{jt} \right) + \hat{R}_t + \hat{W}_{real,t} + \left( \omega_p \hat{Y}_t - (1 + \omega_p) \hat{Z}_t \right) \right) \]

(A.2.11)
It is easy to show from (A.2.11) that:

\[
\pi_{jt} - \gamma_p \pi_{t-1} = -\xi_{pj}(1 + \omega_p \eta)E_{t-1} \left( \hat{P}_{jt} - \hat{P}_t \right)
+ \xi_{pj}E_{t-1} \left( \dot{R}_t + \dot{W}_{real,t} \right) + \beta E_{t-1} \left( \pi_{jt+1} - \gamma_p \pi_t \right)
\]  

(A.2.12)

where \( \xi_{pj} \) is given by:

\[
\xi_{pj} = \frac{1 - \alpha_{pj}}{\alpha_{pj}} \frac{1 - \beta \alpha_{pj}}{1 + \omega_p \theta_p}
\]  

(A.2.13)

In the models considered above, the maintained assumption is that \( \theta_p = \eta \). Substituting this assumption into (A.2.12) gives (2.2.50), which was the equation I aimed to show.
A.3 Percentile Group Inflation

In this appendix, the inflation equation for a percentile group is shown. First, it is notable that the frequency of price changes in sectors included in the same percentile is approximated by the median frequency in the percentile. Letting $\pi_{Ft}$ denote inflation in a percentile group, I define $\pi_{Ft}$ as the weighted average of inflation in the sectors included in the percentile group ($\pi_{ft}$):

$$\pi_{Ft} = \frac{\sum_f n_f \pi_{ft}}{\sum_f n_f}$$

where $n_f$ denotes the weight of the sector whose frequency is approximated by the median frequency of price changes in the percentile group, respectively. Apart from their frequencies of price changes, sectors are identical. This, combined with the fact that frequencies of the sectors in the same percentile group are assumed to be the same, implies that all sectors within the same percentile group have the same inflation equation. Hence, it must hold that

$$\pi_{Ft} = \pi_{ft}$$
A.4 Wages Set for an Hour of Differentiated Labor

Taking the first-order condition in (2.2.54) by using (2.2.18) and (2.2.55) yields:

\[
E_{t-1} \left\{ \sum_{s=0}^{\infty} (\alpha_w \beta)^s \chi_{t,s} \left( \frac{W_{t+s}}{\bar{w}_{t+s+s}(s)} \right) \theta_{w} \times \frac{H_{t+s}}{\sigma_{t+s}(t)} (\theta_{w} - 1)\times \left( \frac{W_{t+s}}{\bar{w}_{t+s+s}(s)} - \frac{\lambda_{t+s}}{\beta_{t+s}} \frac{\bar{w}_{t+s+s}(s)}{\hat{W}_{t+s}} \right) \right\} = 0 \tag{A.4.1}
\]

(A.4.1) can easily be log-linearized since only log-deviations of the terms in the square brackets remain and those of all other terms outside the square brackets vanish. To see this, note that the owner of the differentiated labor type \(i\) will set the wage by imposing a markup of \(\mu_w\) over the marginal cost of working as given by (2.2.35) in the steady state. Then, it is easy to confirm the steady-state value of the term given in square brackets is zero. Hence, the log-deviation of all terms outside the square brackets disappear. To show how overall wages evolve over time, it is convenient to define \(\lambda_{t+s}\) as \(\lambda_{t+s} = \frac{\lambda_{t+s}}{\beta_{t+s}}\).

(A.4.1) can then be rewritten as:

\[
E_{t-1} \left\{ \sum_{s=0}^{\infty} (\alpha_w \beta)^s \chi_{t,s} \left( \frac{W_{t+s}}{\bar{w}_{t+s+s}(s)} \right) \theta_{w} \times \frac{H_{t+s}}{\sigma_{t+s}(t)} (\theta_{w} - 1)\times \left( \frac{W_{t+s}}{\bar{w}_{t+s+s}(s)} - \frac{\lambda_{t+s}}{\beta_{t+s}} \frac{\bar{w}_{t+s+s}(s)}{\hat{W}_{t+s}} \right) \right\} = 0 \tag{A.4.2}
\]

A log-linearized version of (A.4.2) is given by:

\[
E_{t-1} \left\{ \sum_{s=0}^{\infty} (\alpha_w \beta)^s \left( \sigma_{t+s}^{\frac{1}{\gamma}} h_{t+s}(i) - \hat{\lambda}_{t+s} - W_{\text{real},t+s} - \hat{W}_{t+s} + \hat{W}_{t+s} \right) \right\} = 0 \tag{A.4.3}
\]

Using (2.2.18), (A.4.3) can be rewritten as:

\[
E_{t-1} \left\{ \sum_{s=0}^{\infty} (\alpha_w \beta)^s \left( \sigma_{t+s}^{\frac{1}{\gamma}} h_{t+s}(i) - \hat{\lambda}_{t+s} - W_{\text{real},t+s} - \hat{W}_{t+s} + \hat{W}_{t+s} \right) \right\} = 0 \tag{A.4.4}
\]

162
By inserting \( \hat{w}_{t+s,t}(i) = \hat{w}_t(i) + \chi^w_{t,t+s} \) and arranging terms in (A.4.4), it is easy to show

\[
\hat{w}_t(i) = \frac{1 - \alpha_w \beta}{1 + \theta_w \sigma_{\beta}} E_{t-1} \left\{ \sum_{s=0}^{\infty} (\alpha_w \beta)^s \left( \sigma^{-1}_{\beta} \hat{H}_t - \hat{\lambda}_t - \hat{W}_{\text{real},t+s} - (1 + \theta_w \sigma_{\beta}) \left( \chi^w_{t,t+s} - W_{t+s} \right) \right) \right\}
\]

Then, (A.4.5) can be restated as:

\[
\hat{w}_t(i) = (1 - \alpha_w \beta) \hat{W}_t + \frac{1 - \alpha_w \beta}{1 + \theta_w \sigma_{\beta}} E_{t-1} \left\{ \sum_{s=0}^{\infty} (\alpha_w \beta)^s \left( \sigma^{-1}_{\beta} \hat{H}_t - \hat{\lambda}_t - \hat{W}_{\text{real},t} - (1 + \theta_w \sigma_{\beta}) \left( \chi^w_{t,t+s} - W_{t+s} \right) \right) \right\}
\]

Using \( \tilde{\chi}^w_{t,t+1+s} = \tilde{x}^w_{t+1,t+1+s} + \gamma_w \pi_t \), (A.4.6) can be rewritten as:

\[
\hat{w}_t(i) = (1 - \alpha_w \beta) \hat{W}_t + \frac{1 - \alpha_w \beta}{1 + \theta_w \sigma_{\beta}} E_{t-1} \left\{ \sum_{s=0}^{\infty} (\alpha_w \beta)^s \left( \sigma^{-1}_{\beta} \hat{H}_t - \hat{\lambda}_t - \hat{W}_{\text{real},t} - (1 + \theta_w \sigma_{\beta}) \left( \tilde{\chi}^w_{t,t+1+s} - \hat{W}_{t+1+s} \right) \right) \right\}
\]

Hence, from (A.4.5), (A.4.7) can be restated as:

\[
\hat{w}_t(i) = (1 - \alpha_w \beta) \hat{W}_t + \frac{1 - \alpha_w \beta}{1 + \theta_w \sigma_{\beta}} E_{t-1} \left\{ \sum_{s=0}^{\infty} (\alpha_w \beta)^s \left( \sigma^{-1}_{\beta} \hat{H}_t - \hat{\lambda}_t - \hat{W}_{\text{real},t} - \alpha_w \beta \gamma_w \pi_t + \alpha_w \beta E_{t-1} \hat{w}_{t+1}(i) \right) \right\}
\]

\[60\text{Let the percentage change in overall wages be given as } \pi^w_t = \log W_t - \log W_{t-1}. \text{ Then, } \tilde{\chi}^w_{t,t+s} \text{ is given as:}
\]

\[
\tilde{\chi}^w_{t,t+s} = \begin{cases} 
\gamma_p \pi^w_t + \gamma_p \pi^w_{t+1} + \cdots + \gamma_p \pi^w_{t+s-1} & \text{if } s \geq 1 \\
0 & \text{if } s = 0
\end{cases}
\]

163
or as:

\[
\hat{\omega}_t^*(i) - \hat{W}_t = \frac{1 - \alpha_w \beta}{(1 + \theta_w \sigma_{\beta}^{-1})} E_{t-1} \left( \sigma_{\beta}^{-1} \hat{H}_t - \hat{\lambda}_t - \hat{W}_{real,t} \right) + \alpha_w \beta E_{t-1} \left( \hat{\omega}_{t+1}^* (i) - \hat{W}_{t+1} + \pi_{t+1} - \gamma_w \pi_t \right) \tag{A.4.9}
\]

By following the steps in Appendix [A.1.1] one can easily show that

\[
\left( \hat{\omega}_t^*(i) - \hat{W}_t \right) = \frac{\alpha_w}{(1 - \alpha_w)} (\pi_t^w - \gamma_w \pi_{t-1})
\]

Using this, it is easy to restate (A.4.9) as:

\[
\pi_t^w - \gamma_w \pi_{t-1} = \frac{(1 - \alpha_w)}{\alpha_w} \frac{1 - \alpha_w \beta}{(1 + \theta_w \sigma_{\beta}^{-1})} E_{t-1} \left( \sigma_{\beta}^{-1} \hat{H}_t - \hat{\lambda}_t - \hat{W}_{real,t} \right) + \beta E_{t-1} \left( \pi_{t+1}^w - \gamma_w \pi_t \right) \tag{A.4.10}
\]

I show in the following section that \( \hat{H}_t = \frac{1}{\kappa} (\hat{Y}_t - \hat{Z}_t) \). Further, it holds from (2.2.7) that \( E_{t-1} \hat{\lambda}_t = -\varphi^{-1} E_{t-1} \hat{x}_t \). Hence, (A.4.10) can be written as:

\[
\pi_t^w - \gamma_w \pi_{t-1} = \xi_w E_{t-1} \left( \frac{1}{\kappa} (\hat{Y}_t - \hat{Z}_t) + \varphi^{-1} \hat{x}_t - \hat{W}_{real,t} \right) + \beta E_{t-1} \left( \pi_{t+1}^w - \gamma_w \pi_t \right) \tag{A.4.11}
\]

where \( \xi_w \) is given as:

\[
\xi_w = \frac{(1 - \alpha_w)}{\alpha_w} \frac{1 - \alpha_w \beta}{(1 + \theta_w \sigma_{\beta}^{-1})}
\]

**A.4.1 Composite Labor Demand Equation**

I now aim to show that
\[ \dot{H}_t = \frac{1}{\kappa}(\dot{Y}_t - \dot{Z}_t) \quad (A.4.12) \]

First, it is easy to see from (2.2.10) and (2.2.11) that

\[ n_j \bar{C} = n_j \hat{c}_j(j') = C_j \quad (A.4.13) \]

This, together with (2.2.2) and (2.2.3), give

\[ \hat{C}_t = \sum_{j=1}^{J} n_j \hat{C}_{j,t} \quad (A.4.14) \]

\[ n_j \hat{C}_j = \int_{0}^{n_j} \hat{c}_j(j')dj' \quad (A.4.15) \]

Since investment is absent in the model and supply is demand determined, the following equalities must hold:

\[ \hat{c}_{jt}(j') = \hat{y}_{jt}(j'), \quad \hat{C}_{jt} = \hat{Y}_{jt}, \quad \hat{C}_t = \hat{Y}_t \quad (A.4.16) \]

where \( \hat{Y}_{jt} \) denotes sector \( j \)'s output. The total demand for the composite labor demand, \( H_t \), is given by

\[ H_t = \sum_{j=1}^{J} \int_{0}^{n_j} H_{jt}(j')dj' \quad (A.4.17) \]

where the composite labor demand of the firm producing the \( j^{th} \) type good in sector \( j \)
is denoted by $H_{jt}(j')$. Using (2.2.14), $H_{jt}(j')$ can be written as

$$H_{jt}(j') = \left( \frac{y_{jt}(j')}{Z_t} \right)^{\frac{1}{\kappa}} \tag{A.4.18}$$

Log-linearizing (A.4.18) yields

$$\hat{H}_{jt}(j') = \frac{1}{\kappa} (\hat{y}_{jt}(j') - \hat{Z}_t) \tag{A.4.19}$$

Furthermore, assuming all firms have access to the same technology $\bar{Z}$, one can show from (A.4.13), (A.4.17) and (A.4.18) that $\bar{H}_j(j') = \bar{H}$ for all $j$ and $j'$. Hence, the log-linear approximation of (A.4.17) is given by

$$\hat{H}_t = \sum_{j=1}^{J} \int_{0}^{n_j} \hat{H}_{jt}(j') dj' \tag{A.4.20}$$

Lastly, using (A.4.14), (A.4.15), (A.4.16), (A.4.19) and (A.4.20), one can show that

$$\hat{H}_t = \frac{1}{\kappa} (\hat{Y}_t - \hat{Z}_t) \tag{A.4.21}$$

(A.4.21) is what I have intended to show in this section.
A.5 Model-Based Impulse Responses with the Efficient Weighting Matrix

With the assumption that both models are correctly specified, the efficient classical minimum distance (CMD) estimator corresponds to $A_n' A_n = \hat{\Lambda}^{-1}$. In Figure A.1, I illustrate the model-based impulse responses with this choice of the weighting matrix. Using the efficient weighting matrix seems to hinder the stability of the minimization algorithm as shown in the figure. Note how well the VAR- and model-based first period impulse responses of the variables overlap with each other while they differ sharply in other periods. This may result from the fact that such a choice of the weighting matrix puts too much weight on first period impulse responses and too little weight on impulse responses in other periods. This can be explained with the fact the VAR-based first-period impulse responses are estimated much more precisely than those in other periods. It is true that the diagonal weighting matrix used in 2.3.1 and 2.3.2 also places more importance on the impulse responses with lower variances. However, in the efficient weighting matrix case, first-period impulse responses are assigned more weight, not only since they have low variances, but also since the covariances between any two first period impulse responses are much more precisely estimated. This leads the minimization algorithm to choose model parameters in such a way that matching the first period impulse responses has an undue weight and all other period impulse responses have little weight.

\footnote{For the definitions of $A_n$ and $\hat{\Lambda}$, see 2.3.2.}
Figure A.1: Impulse Responses to an Unanticipated 1% Fall in $R_t$
(Efficient CMD Estimator)

Note: The solid lines show the VAR-based impulse responses and the area between dashed lines indicates the 95% confidence interval estimated with the method suggested by Sims & Zha (1999). The lines marked with circles represent the dynamic responses of the variables as predicted by the model.
Appendix B

Appendix to Chapter 3

B.1 The Bils, Klenow & Kryvtsov (2003) Model Reconsidered

In this section, I aim to explain why the empirical strategy for studying the relative price effects of monetary shocks in the United States in Bils, Klenow & Kryvtsov (2003) produces different outcomes than those in Section 3.1.2. Before such an analysis, it is useful to review the Bils, Klenow & Kryvtsov (2003) model.

B.1.1 The Bils, Klenow & Kryvtsov (2003) Model

Bils, Klenow & Kryvtsov (2003) investigate sectoral price responses to a monetary policy shock using the following empirical method:

\[
\ln p_{it} = \lambda_i \sum_{k=k_{\min}}^{k_{\max}} \beta_k \epsilon_{t-k} + \mu_i + \tau_i t + \eta_{it} + \nu_{it} \tag{B.1.1}
\]

where \( \ln p_{it} \) denotes the logged price of sector \( i \). \( \lambda_i \) and \( \epsilon_{t-k} \) show the frequency of price changes in sector \( i \) and innovations in the monetary policy instrument, respectively. The error component in the panel data estimation of (B.1.1) is composed of sector- and time-specific terms. Sector-specific terms, which are included to allow each disaggregated serial to have a different intercept and a different trend, are denoted by \( \mu_i \) and \( \tau_i \), respectively.
Time-specific terms, on the other hand, are meant to capture factors unobservable to the researcher and are assumed to affect all prices by the same magnitude in period \( t \) and are denoted by \( \eta_t \). The maximum number of periods that a monetary policy innovation may have an impact on \( p_{it} \) is denoted by \( k_{max} \).

I assume some delays may occur for prices in a sector when responding to monetary shocks. \( k_{min} \) denotes the number of lags in sectoral price responses. In estimating (B.1.1), I maintain there is a quarter delay in sectoral price responses to monetary shocks (\( k_{min} = 1 \)). Differently, Bils, Klenow & Kryvtsov (2003) assume monetary shocks have a contemporaneous impact on sectoral prices (\( k_{min} = 0 \)). However, such an assumption is at odds with the empirical strategy for isolating monetary shocks in Bils, Klenow & Kryvtsov (2003), which requires aggregate price level to respond to monetary shocks with a lag.

Lastly, it is notable that sector-specific errors (\( \nu_{it} \)) in the Bils, Klenow & Kryvtsov (2003) model are assumed to follow an AR(2) process and is given by:

\[
\nu_{it} = \rho_1 \nu_{it-1} + \varrho_1 \lambda_i \nu_{it-1} + \rho_2 \nu_{it-2} + \varrho_2 \lambda_i \nu_{it-2} + u_{it} \tag{B.1.2}
\]

The specification for sector-specific shocks in (B.1.2) implies that persistence in sector-specific shocks depends on the frequency of price changes in sectors. To explain why such an assumption is made in the Bils, Klenow & Kryvtsov (2003) model, consider first that a sector-specific shock emerges in the fully flexible-price sector. As all prices in this sector can adjust instantaneously to any type of shock, it is expected to have transitory effects on this sector’s price, \( p_{it} \). Next, consider a sector-specific shock hits a sticky-price sector where only a small fraction of firms can reset prices each period. Since it may take quite a while for firms in this sector to adjust fully to the shock, the shock is likely to have more persistent effects on this sector’s price. Lower persistence of sector-specific shocks in flexible-price categories are reflected in the Bils, Klenow & Kryvtsov (2003) model in the
conjecture that $\varrho_1$ and $\varrho_2$ have negative signs in (B.1.2).

The first step in Bils, Klenow & Kryvtsov (2003) is to obtain structural monetary shocks ($\varepsilon_t$). To do so, I assume that the Federal Reserve uses the federal funds rate as its policy instrument and uses the following interest rate rule:

$$ R_t = \theta_0 + \sum_{k=0}^{4} \theta_{y,y} y_{t-k} + \sum_{k=0}^{4} \theta_{\pi,\pi} \pi_{t-k} + \sum_{k=0}^{4} \theta_{w,w} w_{t-k} + \sum_{k=1}^{4} \theta_{i,i} R_{t-k} + \varepsilon_t \quad \text{(B.1.3)} $$

where $R_t$, $y_t$, $\pi_t$, and $w_t$ are defined in (3.1.2). After obtaining monetary shocks by using (B.1.3), the Cochrane-Orcutt procedure is used to estimate the coefficients in the Bils, Klenow & Kryvtsov (2003) model. In this model, the relative price effects of a contractionary monetary policy shock are given as:

$$ \beta_k (\lambda_{90} - \lambda_{10}) \quad \text{(B.1.5)} $$

where $\lambda_{90}$ and $\lambda_{10}$ show the frequencies of the sectors which lie on the 90th and 10th

---

62Since $\nu_{it}$ in (B.1.1) is autoregressive, the OLS estimates of the coefficients in (B.1.1) are inefficient. In addition, OLS standard errors of those coefficients are incorrect. If the true values of the autoregressive coefficients were known, (B.1.1) could easily be estimated by multiplying each side with the following term:

$$ 1 - (\rho_1 + \varrho_1 \lambda_i) L - (\rho_2 + \varrho_2 \lambda_i) L^2 \quad \text{(B.1.4)} $$

where $L$ is the lag operator. It needs to be emphasized that the transformation required for each sector is different as $\lambda_i$ varies across sectors. It is intuitive to transform (B.1.1) this way since multiplying $\nu_{it}$ with (B.1.4) yields errors of the transformed model($\zeta_{it}$) which are uncorrelated, and thus, the OLS with the transformed model is efficient. Yet, autoregressive coefficients are unknown and need to be estimated. In estimating these coefficients, Bils, Klenow & Kryvtsov (2003) employed the well-known Cochrane-Orkutt iterative procedure. In this procedure, parameters in (B.1.1) are first estimated with OLS. Then, estimated OLS residuals in (B.1.1) are used to obtain the first round estimate of $\varrho_1$, $\rho_1$, $\varrho_2$ and $\rho_2$ in (B.1.2). Then, both dependent and independent variables are transformed using these first round autoregressive estimates instead of the true autoregressive coefficients in (B.1.4). After both the dependent and independent variables are transformed this way, the second round coefficient estimates are obtained as well as the second round OLS residuals in (B.1.1). Using the second round OLS residuals, the second round autoregressive coefficients in (B.1.2) are obtained and the variables are transformed using these second round autoregressive coefficients once again. This iteration continues until the autoregressive coefficient estimates in two consecutive rounds differ no more than some threshold. As a convergence criteria, I chose estimates of $\rho_1$ from two consecutive rounds with a change of less than 0.01.
percentile of price flexibility, respectively. I call the categories which lie at these percentiles of price flexibility as the flexible- and sticky-price categories, respectively. The intuition for (B.1.5) is as follows. Note that when the Federal Reserve introduces an unanticipated 1% increase in the federal funds rate, the percentage change in $p_{it}$ of the flexible-price sector (the sticky-price sector) is given by $\lambda_{90}\beta_k$ ($\lambda_{10}\beta_k$). Accordingly, (B.1.5) measures the percentage change in $p_{it}$ of the flexible-price sector relative to that of the sticky-price sector in the $k^{th}$ period following the shock. Since the frequencies of price changes in the sticky- and flexible-price categories differ significantly, the dynamic behavior of $p_{it}$ in both sectors may show substantial differences unless the estimate of $\beta_k$s are too small.

**B.1.2 Findings from the Bils, Klenow & Kryvtsov (2003) Model**

Next, I study the relative price effects of monetary policy shocks in the Bils, Klenow & Kryvtsov (2003) model using my sample, which spans the period of 1959Q1-2013Q1. Figure B.1 shows the percentage change in $p_{it}$ of the flexible-price sector relative to that of the sticky-price sector following an unanticipated 1% increase in the federal funds rate in the Bils, Klenow & Kryvtsov (2003) method ($\beta_k(\lambda_{90} - \lambda_{10})$). In Panel (a) and Panel (b) of this figure, the dynamics of the relative price after the shock are estimated using the frequency of non-sales price changes and the frequency of price changes including sales, respectively. Two points are noteworthy regarding Figure B.1. First, the inclusion of price changes during sales in measuring the frequency of price changes in sectors has only a small effect on our results since the relative price dynamics after the shock are rather similar when the frequency of price changes includes or excludes price changes during sales. Second, our results are even more striking than the results in Bils, Klenow & Kryvtsov (2003). Indeed, following a contractionary monetary shock, I find the relative price stays above its undistorted path for about three years compared to only three quarters as found
Figure B.1: The Bils, Klenow & Kryvtsov (2003) Model-Based Impulse Responses of the Relative Price to Monetary Shocks

(a) With the Frequency of Non-Sales Price Changes

(b) With the Frequency of Price Changes Including Sales

Note: In the figure, the solid line indicates the estimated point-wise impulse responses. The area between the dashed lines shows the two standard deviation confidence intervals for the estimate of $\beta_k(\lambda_{90} - \lambda_{10})$ in (B.1.1). As in Bils, Klenow & Kryvtsov (2003), in estimating these confidence intervals, the uncertainty in estimating structural monetary shocks in (B.1.3) is not taken into account.

Barth & Ramey (2001) claim a rise in inflation after a contractionary monetary shock can be explained with the working-capital channel, which they show to be operative in the United States. Indeed, firms’ requirement to pay input costs in advance raises factor costs when the interest rate increases. Consequently, when there is an unanticipated rise in the interest rate, firms raise their prices for some periods after the shock despite the downward pressure on prices from a reduction in output following the shock. If the relative price responses in the flexible-price sector were positive only for a few periods, such a channel
could be invoked to explain the relative price puzzle. Yet, this channel may not explain the positive responses of the relative price for three years as found in this paper.


The reliability of the findings in Figure B.1 depends on whether monetary shocks are orthogonal to sector-specific errors in period $t$ in (B.1.1). To see this, note from (B.1.1) that the unbiasedness of coefficients requires $E(\nu_t | \epsilon_t) = 0$. That is, sector-specific shocks should be orthogonal to monetary shocks to have unbiased estimates. If this condition does not hold, the GLS estimates of the parameters in (B.1.1) will have bias and the results obtained with the Bils, Klenow & Kryvtsov (2003) model may be questionable. Next, I aim to test this hypothesis. Note, when obtaining monetary shocks, Bils, Klenow & Kryvtsov (2003) assume the Federal Reserve only responds to inflation in the general price level and does not take into account movements in any sectoral price. If this really holds, then, the sector specific shocks will be orthogonal to monetary shocks and the coefficients in the Bils, Klenow & Kryvtsov (2003) model can be estimated unbiasedly. One way to test whether the Federal Reserve responds to sectoral prices, apart from the general price level, is to incorporate a single sectoral price index in its policy reaction function in (B.1.3) and test whether the coefficients pertinent to the Federal Reserve’s response to sectoral prices are jointly zero. The following regression is considered for this test:

$$R_t = \theta_0 + \sum_{k=0}^{4} \theta_{y-y^n,k} (y_{t-k} - y^n_{t-k}) + \sum_{k=0}^{4} \theta_{\pi,k} \pi_{t-k} + \sum_{k=0}^{4} \theta_{\ln p_i,k} \ln p_{it-k} + \sum_{k=0}^{4} \theta_{w,k} w_{1-k} + \sum_{k=0}^{4} \theta_{i,k} R_{t-k} + \tilde{\epsilon}_t$$

(B.1.6)

Estimating (B.1.6) requires the assumption that the Federal Reserve observes sectoral
prices and may respond to them if this is desired. Also, it requires sectoral prices to respond
to monetary shocks with at least a quarter lag. The only difference between (B.1.3) and
(B.1.6) is the term $\sum_{k=0}^{4} \theta_{lnp_{i,k}} lnp_{it-k}$. The structural monetary shocks in (B.1.3) can be
associated with those in (B.1.6) in the following way:

$$\epsilon_{t} = \tilde{\epsilon}_{t} + \sum_{k=0}^{4} \theta_{lnp_{i,k}} lnp_{it-k}$$

(B.1.7)

The maintained assumption in the Bils, Klenow & Kryvtsov (2003) model is that
$\theta_{lnp_{i,0}} = \theta_{lnp_{i,1}} = \cdots = \theta_{lnp_{i,4}} = 0$. If this assumption does not hold, $E(\epsilon_{t} U_{it}) \neq 0$
and the GLS estimators in the Bils, Klenow & Kryvtsov (2003) model will be inconsistent.
One way to interpret the rejection of the null is that the response of the Federal Reserve to
price movements in these sectors is not confined to their marginal effects on the aggregate
price level. Apart from this, the Federal Reserve gives a statistically different response to
sectoral shocks in these sectors. By using (B.1.6), I perform an F-test for the null hypoth-
esis for each sector. I find among 125 sectors, the null hypothesis is rejected for 19 sectors.
Table B.1 lists the sectors for which the null is rejected. For the remaining sectors, the
Federal Reserve’s response is confined to its response to the change in the general price
level inflation caused by sector-specific price shocks in these sectors.

Next, I study the association between the likelihood that the Federal Reserve provides
a significant response to sectoral price shocks and the frequency of price changes. For this
purpose, a scatter plot of F-values and the frequency of non-sales price changes and a
scatter plot of F-values and the frequency of price changes including sales are displayed
with circles in Panel (a) and Panel (b) of Figure B.2 respectively. The 90% critical value for
an $F(5, T - 25)$ random variable, where $T$ indicates sample size, is graphically represented
by the thick solid line. If an F-value is greater than the critical value, the null hypothesis,
that the Federal Reserve is only concerned with the general price level inflation and does not respond to sectoral prices, should be rejected.

In Figure B.2 I also show the fitted line for the following regression:

\[ F_i = a_0 + a_1 \lambda_i \]  

A positive (negative) slope of the fitted line suggests the higher (lower) the frequency of price changes in a sector, the more (less) probable is the Federal Reserve to provide a significant response to sectoral price shocks. I estimate (B.1.8) with both the frequency of price changes including sales \( \lambda_{i}^{sales} \) and the frequency of non-sales price changes \( \lambda_{i}^{regular} \) and report the findings in (B.1.9).

\[ F_i = 1.19 + 0.18\lambda_i^{regular} \]
\[ F_i = 1.44 - 0.45\lambda_i^{sales} \]  

These findings indicate that while the likelihood of the Federal Reserve to provide a significant response to sectoral price shocks is positively associated with the frequency of non-sales price changes, it is negatively associated with the frequency of price changes.
Figure B.2: Testing for the Significance of the Federal Reserve’s Response for Sectoral Prices

(a) With the Frequency of Non-Sales Price Changes

(b) With the Frequency of Price Changes Including Sales

Note: In the figure, the circles show a scatter plot of the F-value for the null hypothesis and the frequency of price changes in sectors. The thick solid line indicates the 90% critical value for the F-test. The dotted lines marked with asterisks show the fitted line for the regression in (B.1.8).

including sales. However, both the positive and negative associations are weak. To see this point, note that the fitted lines in both cases are flat and they always remain below the critical value within the admissible region of the frequency of price changes in sectors. Hence, the frequency of price changes in a sector does not seem to be an important factor in the decision of the Federal Reserve once the effect of these shocks on the general price level inflation is controlled.

To sum up, the findings in this section indicate that structural monetary shocks that are needed to estimate the Bils, Klenow & Kryvtsov (2003) model ($\epsilon_t$ in (B.1.1)) are correlated with sector-specific price shocks ($\nu_{it}$ in (B.1.1)). Since this assumption is critical in the
Bils, Klenow & Kryvtsov (2003) model and is shown to be violated for a non-negligible number of sectors in our sample, it can be argued that the results in the Bils, Klenow & Kryvtsov (2003) model are questionable.
B.2 Estimation of Confidence Intervals for Figure 3.3 Using a Block-Bootstrap Method

The ‘block-of-blocks’ bootstrap method of Politis & Romano (1992) is used to estimate the confidence interval for the correlation between the frequency of price changes in sectors and sectoral price responses to a 1% increase in the federal funds rate. The following steps are followed to construct the confidence intervals.

1. Let $Z_t$ and $Z^t$ be defined as:

$$Z_t = \begin{bmatrix} y_t - y^n_t, \pi_t, w_t, R_t, \pi_{1t}, \pi_{2t}, \ldots \pi_{it}, \ldots \pi_{125t} \end{bmatrix}$$

$$Z^t = \begin{bmatrix} Z_t, Z_{t-1}, Z_{t-2}, Z_{t-3}, Z_{t-4} \end{bmatrix}$$

where $\pi_{it}$ in $Z_t$ represents the annualized percentage change in the price of the sector $i$.

2. Next, $T - b + 1$ overlapping blocks are formed where $T$ is the sample size and $b$ is the fixed length of blocks. The first contains observations $|Z_1, Z_2, \ldots, Z_b|^t$. The second contains observations $|Z_2, Z_3, \ldots, Z_{b+1}|^t$. The last contains observations $|Z_{T-b+1}, Z_{T-b+2}, \ldots, Z_T|^t$.

3. For the block length, the following values are considered $b \in \{6, 9, 12\}$. Since the results are similar, only the results for $b = 12$ are displayed in Figure 3.3.

4. A random sample of size $T$ is constructed by resampling from these blocks with replacement. Since $T/b$ is not an integer, the last block has been truncated.

5. Next, for each random sample, the correlation between the frequency of price changes
and sectoral price responses to a 1% increase in the federal funds rate is estimated by the method discussed in Section 3.1.2.

6. These steps are repeated 500 times. The confidence intervals displayed in Figure 3.3 give the area between the 2.5th and 97.5th percentiles of these randomly generated correlations.
Appendix C

Appendix to Chapter 4

C.1 The Empirical Strategy in Clarida & Gali (1994)

The following empirical model for quarterly data is used in Clarida & Gali (1994):

\[
X_{i,t} = B_{i,0} + \sum_{p=1}^{p_{\text{max}}} B_{i,p} X_{i,t-p} + u_{i,t} \tag{C.1.1}
\]

where the vector \(X_{i,t}\) is defined in (4.1.2). (C.1.1) in the companion form is given as:

\[
\begin{bmatrix}
X_{i,t} \\
X_{i,t-1} \\
\vdots \\
X_{i,t-p+2} \\
X_{i,t-p+1}
\end{bmatrix}
= \begin{bmatrix}
B_{i,0} \\
0 \\
\vdots \\
0 \\
0
\end{bmatrix}
+ \begin{bmatrix}
B_{i,1} & B_{i,2} & \cdots & B_{i,p-1} & B_{i,p} \\
I & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & I & 0 \\
0 & 0 & \cdots & 0 & 0
\end{bmatrix}
\begin{bmatrix}
X_{i,t-1} \\
X_{i,t-2} \\
\vdots \\
X_{i,t-p+1} \\
X_{i,t-p}
\end{bmatrix}
+ \begin{bmatrix}
u_{i,t}
\end{bmatrix} \tag{C.1.2}
\]

Or, rewrite (C.1.2) in a compact form as:

\[
Z_{i,t} = A_{i0} + A_{i1} Z_{i,t-1} + U_{i,t} \tag{C.1.3}
\]

where \(Z_{i,t}, A_{i0}, A_{i1}\) and \(U_{i,t}\) are defined in the following way,
\[ Z_{i,t} = \begin{bmatrix} X_{i,t} \\ X_{i,t-1} \\ X_{i,t-2} \\ \vdots \\ X_{i,t-p} \end{bmatrix} \]

\[ A_{i0} = \begin{bmatrix} B_{i0} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \]

\[ A_{i1} = \begin{bmatrix} B_{i1} & B_{i2} & \ldots & B_{ip_{\max}-1} & B_{ip_{\max}} \\ I & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & I & 0 \\ 0 & 0 & \ldots & I & 0 \end{bmatrix} \]

\[ u_{i,t} = \begin{bmatrix} C_{i0} \epsilon_{i,t} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \]

In the last part, we use \( u_{i,t} = C_{i0} \epsilon_{i,t} \) for writing \( U_{i,t} \) (For definitions of \( C_{i0} \) and \( \epsilon_{i,t} \), see (4.1.3)).

Suppose that \( \epsilon_{i,t} = I \). This corresponds to the study of dynamics of the variables in \( X_{it} \) to an unanticipated unit change in each of the three structural shocks, separately. In other words, each column of \( C_{i0} \) shows contemporaneous impulse responses of the variable to a one unit increase in one of the three structural shocks.

Let the impulse responses of \( Z_i \) in the \( j \)th period be defined as:

\[ E_t Z_{i,t+j} - E_{t-1} Z_{i,t+j} \]

Now, our goal is to calculate the cumulative responses over \( j \) periods. First, note that due to the AR(1) structure in (C.1.3), it is easy to show that the \( j \)th period impulse response of \( Z_i \) is given by:

\[ E_t Z_{i,t+j} - E_{t-1} Z_{i,t+j} = A_{i1}^j U_{i,t} \]

Denoting the cumulative impulse response of \( Z_i \) over \( j \) with \( Z_{i,t+j}^{cum} \), one can write:
\[
Z_{i,t+j}^{\text{cum.}} = \sum_{m=0}^{j} (E_t Z_{i,t+m} - E_{t-1} Z_{i,t+m}) = (I + A_{i1} + A_{i1}^2 + \cdots + A_{i1}^j)\mathcal{U}_{i,t}
\]

When \( j \to \infty \), it is easy to verify that the cumulative impulse response is given by,

\[
Z_{i,t+\infty}^{\text{cum.}} = (I - A_{i,1})^{-1}\mathcal{U}_{i,t} \Rightarrow (I - A_{i,1})Z_{i,t+\infty}^{\text{cum.}} = \mathcal{U}_{i,t} \tag{C.1.7}
\]

Using (C.1.4) and \( \epsilon_{i,t} = I \), one can write (C.1.7) as:

\[
(I - B_{i1})X_{i,t+\infty}^{\text{cum.}} - B_{i2}X_{i,t-1+\infty}^{\text{cum.}} - \cdots - B_{ip_{\max}}X_{i,t-p_{\max}+1+\infty}^{\text{cum.}} = C_0 \tag{C.1.8}
\]

Note that \( X_{i,t+\infty}^{\text{cum.}} = X_{i,t-1+\infty}^{\text{cum.}} = \cdots = X_{i,t-p_{\max}+1+\infty}^{\text{cum.}} \) since for any \( p > 0 \),

\[
X_{i,t-p+\infty}^{\text{cum.}} = \sum_{m=-p}^{-1} (E_t X_{i,t+m} - E_{t-1} X_{i,t+m}) + \sum_{m=0}^{\infty} (E_t X_{i,t+m} - E_{t-1} X_{i,t+m}) = X_{i,t+\infty}^{\text{cum.}} \tag{C.1.9}
\]

Thus, one can rewrite (C.1.8) as follows:

\[
\left( I - \sum_{p=1}^{p_{\max}} B_{ip} \right) X_{i,t+\infty}^{\text{cum.}} = C_0 \rightarrow X_{i,t+\infty}^{\text{cum.}} = \left( I - \sum_{p=1}^{p_{\max}} B_{ip} \right)^{-1} C_0 \tag{C.1.10}
\]

Note from (4.1.2) that the variables in \( X_{i,t} \) are in first differences. Then, the cumulative impulse responses of these differenced variables over \( j \) periods is the \( j \)th period level impulse responses. To see this, consider a differenced variable \( y \), one can show the equivalence of the cumulative impulse responses of \( \Delta y \) over \( j \) periods and the level impulse response of \( y \) in \( j \)th period in the following way:
\[
\Delta y_{t+\infty}^{cum.} = (E_t y_t - E_t y_{t-1}) - (E_{t-1} y_t - E_{t-1} y_{t-1}) + (E_{t} y_{t+1} - E_t y_t) - (E_{t-1} y_{t+1} - E_{t-1} y_{t+1}) \\
\quad + (E_{t} y_{t+2} - E_t y_{t+1}) - (E_{t-1} y_{t+2} - E_{t-1} y_{t+1}) \\
\quad \vdots \\
\quad + (E_{t} y_{t+\infty} - E_t y_{t+1+\infty}) - (E_{t-1} y_{t+\infty} - E_{t-1} y_{t+1+\infty}) 
\]

(C.1.11)

Hence, \((C.1.11)\) implies that the cumulative response of \(\Delta y_t\) over the long-run is the level impulse response of \(y\) to the shock in the long-run. Denoting the long-run level impulse response matrix of the variables in \(X_{i,t}\) with \(\mathcal{D}_i\), from \((C.1.10)\) and \((C.1.11)\), it follows that

\[
\mathcal{D}_i = \left(I - \sum_{p=1}^{p_{\text{max}}} B_{ip}\right)^{-1} C_{i0} \quad \text{(C.1.12)}
\]

Then, it is easy to show that

\[
\mathcal{D}_i \mathcal{D}_i' = \left(I - \sum_{p=1}^{p_{\text{max}}} B_{ip}\right)^{-1} \Omega_i \left(I - \sum_{p=1}^{p_{\text{max}}} B_{ip}\right)^{-1} \quad \text{(C.1.13)}
\]

Clara & Gali (1994) assume that \(\mathcal{D}_i\) satisfies the exclusion and sign restrictions in \((4.1.5)\). With such identifying assumptions, this matrix can be uniquely identified as the Cholesky decomposition of \((C.1.13)\). Further, it can be shown from \((C.1.12)\) that

\[
C_{i0} = \left(I - \sum_{p=1}^{p_{\text{max}}} B_{ip}\right) \mathcal{D}_i \quad \text{(C.1.14)}
\]

Moreover, if \(C_{i0}\) is assumed to be invertible, structural shocks can be recovered as

184
\[ \epsilon_{i,t} = C_{i0}^{-1} u_{i,t} \]  \hspace{1cm} (C.1.15)
C.2 Aggregate Dynamics after Monetary Shocks

C.2.1 Aggregate Dynamics in *Empirical Model I* after Monetary Shocks

Here, we report our findings on aggregate dynamics in *Empirical Model I* after monetary shocks.

Aggregate dynamic responses are displayed in Figure C.1 over five years at the quarterly and monthly frequencies. It is evident from this figure that a higher monetary shock in developing economies, relative to one in the United States, is associated with a short-lived increase in the level of output in the former relative to that of the United States. It quickly falls again to the level of the undistorted path. Similarly, the real exchange rate exhibits a temporary upward movement after the shock, indicating a temporary depreciation in the real exchange rate against developing economies. At the quarterly (monthly) frequency, our results for *Empirical Model I* suggest that the real exchange rate stays depreciated relative to its undistorted path for about six quarters (12 months) after the shock. It can also be seen that the real exchange rate exhibits hump-shaped dynamics after the shock. Lastly, a positive monetary shock causes the price level in developing economies to rise relative to the price level in the United States on impact.

C.2.2 Aggregate Dynamics in *Empirical Model II* after Monetary Shocks in the United States

We have discussed aggregate dynamics after an expansionary domestic monetary shock in developing countries in *Empirical Model II* in Section 4.1.2. This section, on the other hand, discusses our findings on aggregate dynamics after an expansionary monetary shock in the United States in *Empirical Model II*. The results are presented in Figure C.2.
Figure C.1: Impulse Responses to Monetary Shocks in Empirical Model I

**Quarterly**

(a) $y_{i,t} - y^*_t$

(b) $q_{i,t}$

(c) $p_{i,t} - p^*_t$

**Monthly**

(d) $y_{i,t} - y^*_t$

(e) $q_{i,t}$

(f) $p_{i,t} - p^*_t$

**Note:** Our calculations are based on the IMF’s *International Finance Statistics*. The solid lines indicate the estimated point-wise impulse responses. The area between the dashed lines shows the 90% confidence interval estimated using the Bayesian method suggested by Sims & Zha (1999).

Following a monetary shock in the United States:

- output in both the United States and developing economies stays above its undistorted level for about a year;
Figure C.2: Impulse Responses to Monetary Shocks in the United States (Empirical Model II)

Quarterly

(a) $y_t$
(b) $y_{1,t}$
(c) $\Omega_{1,t}$
(d) $P_t^*$
(e) $P_{1,t}$

Monthly

(f) $y_t$
(g) $y_{1,t}$
(h) $\Omega_{1,t}$
(i) $P_t^*$
(j) $P_{1,t}$

Note: Our calculations are based on the IMF’s *International Finance Statistics*. The solid lines indicate the estimated point-wise impulse responses. The area between the dashed lines shows the 90% confidence interval estimated using the Bayesian method suggested by Sims & Zha (1999).

- the real exchange rate appreciates on impact, and compared to the undistorted path, it stays appreciated for about 9 months; and,

- the price level in both the United States and developing economies contemporaneously rises.
### C.3 Calibration of Models’ Parameters

**Table C.1: Calibration and Estimation**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Values</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_p$</td>
<td>Price elasticity of demand for varieties within the same sector</td>
<td>11</td>
<td>Bresnahan (1981)</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>Wage elasticity of labor demand</td>
<td>4</td>
<td>Huang &amp; Liu (2002)</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Inverse of elasticity of intertemporal substitution</td>
<td>5</td>
<td>Hall (1988)</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>Inverse of Frisch-elasticity of labor supply</td>
<td>1</td>
<td>Carvalho &amp; Nechio (2011)</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>Inverse of the elasticity of capacity utilization with respect to the rental rate of capital</td>
<td>0.01</td>
<td>Christiano et al. (2005)</td>
</tr>
<tr>
<td>$\sigma_\phi$</td>
<td>The elasticity of the adjustment cost technology for investment with respect to $\frac{I}{K}$</td>
<td>-0.75</td>
<td>Devereux &amp; Hnatkovska (2011)</td>
</tr>
<tr>
<td>$\Theta\bar{Y}$</td>
<td>Elasticity of interest rate to net foreign assets</td>
<td>-0.01</td>
<td>Devereux &amp; Smith (2005)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Elasticity of substitution between the home and foreign goods</td>
<td>1.5</td>
<td>Carvalho &amp; Nechio (2011)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Elasticity of substitution between different sector goods</td>
<td>1</td>
<td>Carvalho &amp; Nechio (2011)</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Labor share in GDP</td>
<td>0.66</td>
<td>Christiano et al. (2005)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>1.03</td>
<td>Christiano et al. (2005)</td>
</tr>
<tr>
<td>$\alpha_k$ and $\alpha_k^*$</td>
<td>Price stickiness in sectors</td>
<td>See text</td>
<td>Carvalho &amp; Nechio (2011)</td>
</tr>
<tr>
<td>$f_k$</td>
<td>Expenditure share of sectors</td>
<td>See text</td>
<td>Carvalho &amp; Nechio (2011)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Monthly rate of depreciation on capital</td>
<td>0.008</td>
<td>See text</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>The persistence in nominal spending growth shocks</td>
<td>0.32</td>
<td>See text</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Relative size of the foreign country</td>
<td>1000</td>
<td>See text</td>
</tr>
<tr>
<td>$s_c$</td>
<td>% Share of final consumption expenditure in GDP</td>
<td>66</td>
<td>See text</td>
</tr>
<tr>
<td>$s_i$</td>
<td>% Share of investment in GDP</td>
<td>20</td>
<td>See text</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>Share of home exports invoiced in the home currency</td>
<td>0.05</td>
<td>See text</td>
</tr>
<tr>
<td>$\omega_i^*$</td>
<td>Share of home imports priced in the foreign currency</td>
<td>0.95</td>
<td>See text</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Share of home country imports in GDP</td>
<td>0.35</td>
<td>See text</td>
</tr>
</tbody>
</table>

189
C.3.1 The Weak Link between the Level of Inflation and the Frequency of Price Changes

A seemingly odd assumption in our dynamic stochastic sticky-price small-open economy models is that the frequency of price changes is independent of inflation. Indeed, regardless of changes in inflation over the period in the economy, it is assumed that a constant fraction of firms change their prices in each period. While it is odd to make this assumption for unstable inflationary periods, there is some empirical evidence in favor of the constant frequency of price changes in low and moderate inflation climates. For example, Nakamura & Steinsson (2008a) look at the relationship between the median frequency of price changes and inflation in the United States. During the low and stable inflation period of 1998-2005, even though their results indicate a positive association between inflation and the frequency of price changes, the estimated coefficient is not statistically different from zero. Similarly, Gagnon (2009) studies the association of the frequency of price changes and inflation in Mexico for both high and low inflation periods. He finds during moderate inflation periods, between 5-15% annual change in the price level, the frequency of price changes is largely dissociated with inflation. Indeed, while a rise in inflation in these stable periods results in a higher likelihood of observing price increases, it leads to a decline in the probability of observing price decreases, leaving the frequency of price changes largely unaffected with inflation in Mexico during low inflation periods. Higher inflation in these periods is accounted for through increases in the magnitude of average price adjustment resulting largely from the change in the distribution of price changes in favor of price increases in these periods. However, when inflation rises to high levels, as one would expect, Gagnon (2009) finds the frequency of price changes moves closely with inflation.

Hence, whether constant frequency of price changes is a reasonable assumption or not depends largely on the level of inflation which developing countries experienced in the
period. To evaluate this, we study the evolution of the median inflation rate in developing economies over our sample period in Figure C.3. The median inflation rate denotes the median of the twelve-month percentage changes in the CPI of the countries in each period. It is noteworthy that inflation has been stable in the inflation targeting countries. This result, together with the finding in Gagnon (2009) that the frequency of price changes is independent of inflation over stable inflation periods, offers supporting evidence for our assumption that the frequency of price changes is constant in developing economies for the period.
Figure C.4: Consumer Prices Inflation and the Turkish Lira Share in External Trade in Turkey

(a) TL Shares in External Trade

(b) Consumer Prices Inflation

Note: Our calculations are based on the Turkish Statistical Institute data. In Panel A, the dotted lines with circles and the dashed lines with multiplication signs indicate the share of TL-denominated exports in total Turkish exports and the share of TL-denominated imports in total Turkish imports, respectively.

C.3.2 Asymmetry in Currency Invoicing in International Trade between Developing and Advanced Economies

It is a well-known fact that there is an asymmetry between developing and advanced economies in regards to the currency in which exports and imports are denominated. Indeed, while exports and imports are largely denominated in home currencies in advanced economies, they are largely denominated in foreign currencies in developing economies. For example, in their study of pricing decision of the exports and imports in the United States, Gopinath & Rigobon (2008) report that 97% of exports and 90% of imports are priced in the United States dollar. To exemplify the pricing practices of exporters and importers in developing economies, we look at exports and imports by currency in Turkey.
In Figure C.4, we illustrate the share of exports (imports) priced in the Turkish Lira (TL) in total exports (imports) as well as the inflation in consumer prices between 1996 and 2012. Inflation is measured as the percentage change in CPI over the last twelve months. It is notable that the remarkable success in bringing down inflation has produced only a modest rise in the shares of TL denominated exports and imports over the recent years. Indeed, the shares of TL-denominated exports and imports have stayed at very low levels below 5% during this period. Our conjecture is that this finding holds generally for all developing economies and currency invoicing in international trade happens largely with the foreign currencies in this group.
C.4 The One- and Multi-Sector Models’ Dynamics with a Taylor-Type Rule

In this section, we analyze aggregate dynamics in the one- and multi-sector models without investment by considering a Taylor-type interest rate rule in the home and foreign countries instead of considering exogenous nominal spending growth. In doing so, we assume that in addition to the international foreign bond \(B_{t+1}\), there is a domestic bond \(D_{t+1}\) which is traded only domestically, supplied in zero net supply and pays a gross nominal interest of \(R_t\). The interest rates in the home country \((R_t)\) and in the foreign country \((R_{B,F,t}^*)\) are set according to the following rules:

\[
\hat{R}_t = \phi_\pi \times \pi_t + \phi_y \times \hat{Y}_t + \epsilon_t^r
\]

where \(\epsilon_t^r = \rho_r \epsilon_{t-1}^r + \eta_t\) and \(\eta_t \sim N(0, \sigma^2_\eta)\)

\[
\hat{R}_{B,F,t}^* = 0.79 \times \hat{R}_{B,F,t-1}^* + (1-0.79) \times 2.15 \times \pi_t^* + (1-0.79) \times 0.93 \times \hat{Y}_t^* + \epsilon_t^{r*}
\]

where \(\epsilon_t^{r*} \sim N(0, \sigma^2_{\epsilon_t^{r*}})\)

The coefficients for the foreign interest-rate rule reflect the estimates of the Taylor rule coefficients in Clarida et al. (1999) for the Volcker-Greenspan periods. The coefficients in the home interest-rate rule, on the other hand, have to be estimated since we do not have the estimates of the reaction function of the monetary authorities under inflation targeting in developing economies. Two cases are considered when estimating the parameters. The first is that \(\varepsilon_t\) is a white noise \((\rho_r = 0)\). The second is that the shock to the home interest rate can be persistent \((\rho_r > 0)\). In the first case, the estimated vector of parameters \((\mathcal{P})\) consists of \(\mathcal{P} = \begin{bmatrix} \phi_\pi & \phi_y \end{bmatrix}\). In the second case, it includes \(\mathcal{P} = \begin{bmatrix} \phi_\pi & \phi_y & \rho_r \end{bmatrix}\). Let \(f(\mathcal{P})\) denote the impulse response functions of the price level, output, the real exchange rate and the nominal exchange rate in developing economies for some \(\mathcal{P}\) between the 0\(^{th}\) and 12\(^{th}\)
months. We estimate \( \mathcal{P} \) as the classical minimum distance estimator and denote it with \( \hat{\mathcal{P}}(\hat{\mathcal{A}}_n) \):

\[
\hat{\mathcal{P}}(\hat{\mathcal{A}}_n) = \arg\min_{\mathcal{P}} (\hat{h}_n - f(\mathcal{P}))' \hat{\mathcal{A}}'_n \hat{\mathcal{A}}_n (\hat{h}_n - f(\mathcal{P}))
\] (C.4.2)

where \( \hat{\mathcal{A}}_n \) is the weighting matrix used. \( \hat{h}_n \) shows the impulse response functions of the price level, output, the nominal and real exchange rates in the actual economies between the 0\(^{th}\) and 12\(^{th}\) months. Lastly, \( n \) stands for the sample size of the data used to estimate the VAR-based impulse response functions. Since using different weighting matrices would yield different estimators, \( \hat{\mathcal{P}} \) is written as a function of \( \hat{\mathcal{A}}_n \). As a weighting matrix, we choose the widely-used diagonal matrix whose diagonal elements are given as the inverse of standard deviations of empirical impulse responses. (See, for example, Christiano et al. (2005) and Giannoni & Woodford (2003)). This weighting matrix ensures more precisely estimated impulse response functions are given more importance than the less precisely estimated ones.\(^{63}\)

Table C.2 shows the estimated parameters of the Taylor rule specified in C.4.1. Allowing persistence in the shocks to the interest rate in the home country significantly improves both the one- and multi-sector models’ performance as it leads to a sharp fall in the weighted distance between the model- and VAR-based impulse response functions (See Obj. Func. in the table). Figure C.5 visualizes this. In Panel A of this figure, the impulse response functions of the aggregate variables to an expansionary white-noise shock to the home interest-rate rule in (C.4.1) are illustrated. Both the one- and multi-sector models are incapable of explaining the aggregate dynamics when shocks to the interest rate in the home country are transitory.

\(^{63}\)To do the estimation, the lower and upper bounds for the parameters have to be entered in the computer program. For the parameters \( [ \phi_\pi \ \phi_y \ \rho_r ] \), we set the lower and upper bounds as \([1.00,2.14], [0,2.00], [0,0.99] \), respectively.
Table C.2: Estimated Parameters of the Taylor Rule

<table>
<thead>
<tr>
<th>Transitory Shocks</th>
<th>Persistent Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>One-Sector</strong></td>
<td><strong>Multi-Sector</strong></td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>1.00*</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Obj. Func.</td>
<td>619.36</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The numbers in parentheses denote estimated model-based standard errors. The numbers with an asterisk indicate that standard errors are not reported since the estimates of the parameters are close to either its lower bound or its upper bound as discussed in Footnote 63. Obj. Func. indicates the value of the objective specified in (C.4.2).

Next, we consider that the shocks to the interest rate in the home country are persistent. Panel B of Figure C.5 shows the model- and VAR-based impulse response functions with a persistent interest-rate rule. It is clear from this figure that with such a high persistence in the shocks, the dynamics of the price level, output and the nominal and real exchange rates after the monetary shock in both the one- and multi-sector models align quite well with those found in the data. While the model-based impulse response functions in both the former and the latter stay within the 90% confidence intervals for the panel VAR-based impulse response functions, it is evident that the latter is more successful than the former in explaining the movements of output, the real and nominal exchange rates in the actual economies following the monetary shock.
Figure C.5: One- and Multi-Sector Models with a Taylor-Type Rule

Panel A: Transitory Shocks ($\rho_r = 0$)

(a) $P$

(b) $Y$

(c) $\varepsilon$

(d) $Q$

Panel B: Persistent Shocks ($\rho_r > 0$)

(e) $P$

(f) $Y$

(g) $\varepsilon$

(h) $Q$

Note: Our calculations are based on the IMF’s *International Finance Statistics*. The dotted lines with pentagrams and the dashed lines with squares indicate the model-based impulse response functions in the one- and multi-sector models, respectively. The solid lines show the estimated point-wise panel VAR-based impulse response functions. The area between the dotted lines shows the 90% confidence interval estimated with the method suggested by Sims & Zha (1999).