LISTENING TO STUDENTS:
A STUDY OF ELEMENTARY STUDENTS’ ENGAGEMENT IN MATHEMATICS
THROUGH THE LENS OF IMAGINATIVE EDUCATION

by

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Abstract

This dissertation investigates the problem of student engagement in elementary mathematics through the particular theoretical framework of imaginative education (IE) (Egan, 1997, 2005). The question at the centre of this study is what the use of IE and imaginative lesson planning frameworks means to children and for their engagement in elementary mathematics.

For this study, five intermediate-aged elementary students were tracked through a unit of shape and space (geometry). The unit, framed with the binary opposites of vision and blindness, asked students how they might come to understand shape and space as a sighted and as a visually impaired person. Thus a humanized perspective was brought to the learning of mathematics. After the unit the five focus students took part in an individual and a whole-group semi-structured interview with the teacher/researcher.

The study used qualitative instrumental case study methods; data sources included students’ mathematics journals, activity pages, transcripts of audio- and videotaped semi-structured individual and group interviews, a teacher/researcher diary, and a detailed unit overview and lesson plans. The study gathered rich descriptive data focused on bringing out the students’ perspective of their experience.

Results indicate that the students demonstrated positive engagement with mathematics and that the IE theory, which utilized the students’ imaginations and affective responses, allowed multiple access points through which to connect with the mathematical concepts. Three conclusions of the study were that the students expanded their mathematical awareness through making a variety of connections, they were able to develop self-confidence in their learning of mathematics by using emotions and imagination, and they were able to use cognitive tools, particularly a sense of wonder, to engage with mathematics. The dissertation concludes with a discussion of implications and recommendations in four areas, including the need for further research in different educational contexts, in the interaction of imagination and affective responses, and into characteristics of mathematical engagement such as self-confidence. Recommendations for how future pedagogical practice might use the IE theory and embrace the expansion of students’ perspectives in classroom practice bring the dissertation to a close.
Preface

This research received approval from the University of British Columbia (UBC) Behavioural Ethics Research Board (BREB) Human Ethics under certificate No HO8 – 00243 – Imagination and Mathematics
# Table of Contents

Abstract .................................................................................................................................................. ii  
Preface ................................................................................................................................................... iii 
Table of Contents ................................................................................................................................... iv  
List of Tables .......................................................................................................................................... ix  
List of Figures ......................................................................................................................................... x  
Acknowledgements ................................................................................................................................. xi  
Dedication ............................................................................................................................................... xii  

1 Chapter: Introduction ........................................................................................................................... 1  
1.1 Importance of Mathematics ............................................................................................................. 2  
1.2 Engagement in Learning .................................................................................................................. 3  
1.3 Disengagement in Mathematics Education ....................................................................................... 4  
1.4 Humanization of Mathematics Education ....................................................................................... 5  
1.5 Imaginative Education ..................................................................................................................... 6  
1.6 The Current Study ............................................................................................................................ 7  

2 Chapter: General Literature Review .................................................................................................... 10  
2.1 Importance of Mathematics ............................................................................................................. 10  
2.2 Student Engagement and Disengagement ....................................................................................... 11  
2.3 Mathematics Education and Student Disengagement ................................................................... 14  
2.4 Humanization of Mathematical Learning and Teaching - A Wider Perspective ......................... 17  
2.5 Use of the Affective Domain in Learning Mathematics ..................................................................... 20  
2.6 Conclusion ....................................................................................................................................... 24  

3 Chapter: Imagination and the Theory of Imaginative Education ....................................................... 25  
3.1 Overview of Imaginative Education and Cognitive Tools ............................................................... 25  
3.2 Five Phases of Understanding ......................................................................................................... 27
4 Chapter: Methodology .................................................................52

4.1 Research Design - Case Study .................................................52
4.2 The Role of the Teacher/Researcher ...........................................53
4.3 Research Context .....................................................................55
4.4 Unit Plan, Overview and Imaginative Lesson Planning Frameworks ....57
4.5 Data Collection Procedures ......................................................61
4.5.1 Data Sources: Students .........................................................61
4.5.1.1 One to One Semi Structured Interviews ..............................61
4.5.1.2 Group Interview ...............................................................62
4.5.1.3 Students' Mathematics Journals ...........................................62
4.5.1.4 Students' Classroom Activity Pages ....................................63
4.5.1.5 Student Work Samples .......................................................64
4.5.1.6 Student Assessment ..........................................................66
4.5.2 Data Sources: Teacher/Researcher .......................................66
4.5.2.1 Research Diary ...............................................................66
5 Chapter: Findings .................................................................................. 76

5.1 Courtney's Engagement .............................................................. 76

5.1.1 Theme 1: Drawing on Emotions .............................................. 77

5.1.2 Theme 2: Making Connections ................................................ 80

5.1.3 Theme 3: Developing Self-Confidence ...................................... 83

5.1.4 Theme 4: Cultivating Mathematical Awareness ....................... 85

5.1.5 Theoretical Findings - Cognitive Tools ...................................... 88

5.1.6 Courtney - Summary ............................................................. 89

5.2 Kee's Engagement ....................................................................... 90

5.2.1 Theme 1: Demonstrating a Sense of Wonder ......................... 91

5.2.2 Theme 2: Making Connections ............................................... 93

5.2.3 Theme 3: Developing Self-Confidence .................................... 95

5.2.4 Theme 4: Cultivating Mathematical Awareness ...................... 96

5.2.5 Theoretical findings - Cognitive Tools .................................... 98

5.2.6 Kee - Summary .................................................................... 99

5.3 Freddie's Engagement ................................................................ 100

5.3.1 Theme 1: Demonstrating a Sense of Wonder ......................... 100

5.3.2 Theme 2: Developing Self-Confidence ................................... 102

5.3.3 Theme 3: Making Connections ............................................... 105

5.3.4 Theme 4: Cultivating Mathematical Awareness ...................... 107

5.3.5 Theoretical Findings - Cognitive Tools .................................... 109

5.3.6 Freddie - Summary ............................................................ 110

5.4 Grace's Engagement .................................................................. 111
6 Chapter: Discussion ........................................................................................................141

6.1 Conclusions................................................................................................................141

6.1.1 Children Expand Mathematical Awareness........................................................141

6.1.2 Children Develop Self-Confidence from Using Emotions and Imagination ........142

6.1.3 Children Use Cognitive tools to Engage with Mathematics .................................144

6.2 Implications and Recommendations .........................................................................145

6.2.1 Further Research Different Contexts .................................................................145

6.2.2 Further Research into Imaginative Education and Affect in Mathematics .........146

6.2.3 Further Research into Mathematics Engagement .................................................147

6.2.4 Further Practices with Imaginative Education ......................................................148

Epilogue .........................................................................................................................150
References ..........................................................................................................................151

Appendix ..............................................................................................................................164

1. Unit Overview - G4/5 Shape and Space Unit.................................................................164
2. The Unit Romantic Planning Framework........................................................................165
3. Semi-Structured Interview Protocol .............................................................................166
4. Space and Shape - Geometry .........................................................................................167
5. Space and Shape - Geometry - Creating a Mathematical Vision Quilt ......................168
6. Student Engagement Observation Protocol .................................................................169
7. Performance Rubric - Shape and Space Unit ...............................................................170
8. Coding Scheme ..............................................................................................................172
9. Coding Scheme - Sample ..............................................................................................173
10. Examplar Case Coding .................................................................................................174
List of Tables

Table 3.1 Sets of Cognitive Tools .................................................................26
Table 3.2 Egan’s Cognitive Development Set Imaginative Education Frameworks ……29
Table 4.1 Unit Plan .........................................................................................58
Table 5.1 Courtney’s Use of Cognitive Tools .....................................................88
Table 5.2 Kee's Use of Cognitive Tools ............................................................98
Table 5.3 Freddie's Use of Cognitive Tools .......................................................109
Table 5.4 Grace's Use of Cognitive Tools .........................................................120
Table 5.5 Jason's Use of Cognitive Tools .........................................................132
Table 5.6 Cross-case Comparison of Emergent Themes .....................................135
Table 5.7 Summary Student Use of Cognitive Tools .........................................138
List of Figures

Figure 1.1 Lorna’s Cheque ........................................................................................................... 2
Figure 4.1 The Unit Mythic Planning Framework ........................................................................ 59
Figure 5.1 Courtney’s Quilt Design ............................................................................................ 81
Figure 5.2 Courtney’s Tessellation Template .............................................................................. 85
Figure 5.3. Courtney’s Tessellation Design ................................................................................ 86
Figure 5.4 Kee’s Mathematics Journal Entry for the Vision Walk ............................................. 92
Figure 5.5 Kee’s Quilt Design ..................................................................................................... 94
Figure 5.6 Kee’s Tessellation Design ........................................................................................... 96
Figure 5.7 Freddie’s Mathematics Journal Entry for the Vision Walk ........................................ 101
Figure 5.8 Freddie’s Mathematics Journal Entry for Properties of Shapes ............................... 101
Figure 5.9 Freddie’s First Challenge .......................................................................................... 102
Figure 5.10 Freddie’s Second Challenge .................................................................................... 103
Figure 5.11 Freddie’s Mathematics Journal Tessellation ........................................................... 103
Figure 5.12 Freddie’s Tessellation Design ................................................................................ 104
Figure 5.13 Freddie’s Quilt Design ............................................................................................ 108
Figure 5.14 Grace’s Quilt Design ............................................................................................... 112
Figure 5.15 Grace’s Pythagoras Illustration .............................................................................. 117
Figure 5.16 Grace’s Tessellation Design ................................................................................... 119
Figure 5.17 Jason’s Tessellation Design .................................................................................... 123
Figure 5.18 Jason’s Tessellation Template Drawing ................................................................... 124
Figure 5.19 Jason’s Quilt Design ............................................................................................... 125
Figure 5.20 Jason’s Angle Sketch .............................................................................................. 127
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Dedication

To my dear father Stanley Frederick Summerfield who sowed the seed for a love of learning early in my life with many evenings of answering questions, and who sadly was not here for the final stages of this educational journey,

To Rudy Zuyderhoff, who as an undergraduate started me on the university studies path when saying “If you think you can do better, why don’t you become a student!”, and who sadly was also not here for the final stages of this educational journey,

To my dear husband Paul, who was with me at every stage of the way in this particular educational journey,

To my two dear daughters, Jennifer and Courtney, who encouraged me to keep going even when the going seemed tough.
CHAPTER 1: INTRODUCTION

Some years ago, I was teaching a unit on place value to my Grade 4 and 5 class. Towards the end of a unit, I asked the students to write a letter to anyone they wished, describing what place value is and why it is important. I encouraged them to use their imaginations in choosing a recipient, but I gave no other specific instructions. This activity replaced the more usual end-of-unit test.

In the response, shown in Figure 1.1, Lorna (a pseudonym) was able to explain to Harry Potter the importance of the different positions represented in the place value of numbers with regard to money and sports. Lorna’s submission was more creative than her previous mathematics exercises and oral discussions. Her response included three examples of creatively expressing her understanding of place value, as represented by money, the place value of a certain digit in large numbers, and the variation of scores in sports events, as well as a sense of honest ironical humour (“you’d be happy, but that’s not you’re score is it?”). The self-drawn cheque suggested her strong connection with the activity. When given the opportunity to express her ideas in a more imaginative way than is traditionally expected in a mathematics lesson, Lorna responded with an enthusiasm I had not previously observed.

Dear Mr. Potter

My name is Lorna S. and I believe in magic and have been doing calculations and discovered that 1 galleon is equal to $3.50 Canadian, so I’m sending $1001.00 or 286 galleons. I hope that will be enough for a broom.

Well while I am here why don’t I talk about Place Value. Place value is the sequence of numbers that go into a bigger number. It’s the fact that the 3 in 7356 is equal to 300, stuff like that.

Money is a big example for using place value. Like you use it to figure out that four ninety nine is four dollars ninety nine and not four hundred ninety nine dollars. You use place value to make sure you get paid the right amount on your pay cheque. It’s important to know place value because if you were to play Quidditch and you scored 725 but the scorekeeper made it 72.5 or 527, or 7.25 x then you wouldn’t be to [sic] happy.

But if they gave you 7250, you’d be happy but that’s not you’re score is it?

yours sincerely  Lorna Summerfield
Lorna was not an isolated example in this regard. Over a period of approximately 3 years during which I was becoming familiar with and using aspects of imaginative education (IE; Egan 1997, 2005) in my teaching of mathematics, other students also responded in multifaceted ways to the invitation to use their imaginations. I gradually moved from trying out this theory in isolated opportunities to planning groups of lessons using the theory’s recommended imaginative lesson planning frameworks (ILPFs).

Based on my almost 20 years of teaching practice, it appeared that when I taught mathematics from an IE perspective, the subject became more appealing and engaging for students. The more I used this theory, the more my students’ reactions seemed to contradict popular opinion that most students do not like mathematics very much and that it is often a challenge to get them engaged with the subject. This raised some puzzling and deep questions for me. What was really going on when I used aspects of IE theory in my mathematics lessons? How could there be such a wide difference between what I was seeing in my students’ work and the widely held beliefs about difficulties in trying to engage students with mathematics?

1.1 The importance of mathematics

The significance of mathematics to our society and its citizens is echoed time and time again in the literature (see, for example, du Sautoy, 2010; National Council of Teachers of Mathematics, 2000). The National Research Council (2002), in a report on recommendations to improve mathematical proficiency of students in the elementary and middle grades, states the following:

Figure 1.1 Lorna’s letter explaining place value.
Mathematics is one of humanity’s great achievements. By enhancing the capabilities of the human mind, mathematics has facilitated the development of science, technology, engineering, business, and government. Mathematics is also an intellectual achievement of great sophistication and beauty that epitomizes the power of deductive reasoning. For people to participate fully in society, they must know basic mathematics. Citizens who cannot reason mathematically are cut off from whole realms of human endeavor. Innumeracy deprives them not only of opportunity but also of competence in everyday tasks. (p. 1)

Thus, being numerate is very important in the 21st century. However, engaging students in the learning of mathematics is generally considered to be a challenge (Bruce, 2007; Clarke, 2003). Bruce (2007) and Clarke (2003) both suggest that pedagogical issues dominate this challenge. In particular, Bruce (2007) discusses the challenges teachers face in teaching in a manner different than that used in their own school learning and in finding the time to both assist their students in becoming or staying engaged with mathematics and meet the demands of the curriculum. Similarly, Clarke (2003) concludes that the lack of mathematical content knowledge and pedagogical content knowledge underpin the problem of student engagement in mathematics education.

A dichotomy therefore exists between the recognition of how important mathematics is on the one hand and our ability to engage students in mathematics and to foster numeracy skills on the other. It is thus vital to examine the role pedagogy plays in student engagement in mathematics and to look for teaching methods that enhance engagement.

1.2 Engagement in learning

Before examining this dichotomy in the specific context of mathematics, it is important to consider engagement in more general terms. Over the last 30 years, numerous studies have presented a variety of interpretations of just what student engagement means, whether psychological (Appleton, Christenson, Kim, & Reschly, 2006; Marks, 2000; Newmann, Wehlage & Lamborn, 1992) or behavioural (Finn & Cox, 1992; Finn & Voekl, 1993). While some researchers (e.g. Carini, Kuh and Klein, 2006) caution the relationship between engagement and achievement is not as strong as might be thought, most educators and researchers accept that student engagement is an important part of learning (Furlong & Christenson, 2008; Parsons & Taylor, 2011). In reviewing the literature on student engagement, Parsons and Taylor (2011) reinforce the importance of engaging students in learning when they state that “student engagement has become one of the key concerns and key strategies for educational and social reform particularly in middle and high schools” (p. 7). Given this view, it seems
reasonable to infer that learners who are engaged in the learning process are more likely to absorb and understand the content of a lesson (Brophy, 2010). This applies to any subject, including mathematics.

1.3 Disengagement in mathematics education

The difficulty of engaging students in mathematics is illustrated by the striking amount of literature that examines disengagement from learning mathematics (Boaler, 2000; Boaler & Greeno, 2000; Ma, 1999; Nardi & Steward, 2002a 2002b, 2003; National Survey of Student Engagement. Archives, 2007; Ward-Penny, Johnston Wilder, & Lee 2011). As discussed in Chapter 2, several studies (see, for example, Boaler, 2000; Education Quality and Accountability Office [EQAO], 2010; Mullis et al., 2000; Mullis, Martin, & Foy, 2008) indicate that there is a large and disturbing drop in the number of children who express a liking for mathematics at the start of the intermediate grades when compared to earlier grades. Mullis et al. (2008) find this sentiment even among students who have performed well on mathematics tests. Therefore, there is a need for studies that seek to understand more about student engagement with mathematics in these formative years of school.

Such a lack of engagement has both short and long-term consequences. In the short term, disengaged students have less opportunity to develop conceptual understanding in this important subject, which forms a large part of the elementary school curriculum (EQAO, 2010; Mullis et al., 2008). In the long term, disengagement from mathematics continues to grow in the secondary grades (Boaler & Greeno, 2000; Nardi & Steward, 2003) and in post-secondary education (Boaler & Greeno, 2000; National Survey of Student Engagement, 2007). For students who lack positive experiences with learning mathematics, the subject may become the “curriculum of the dead” (Ball, 1993, p. 195), something without relevance, meaning, or utility to their own lives and the world at large.

A lack of engagement with mathematics I would suggest becomes part of a cycle that leads to negative societal attitudes towards mathematics. Many people suffer from mathematics anxiety (Arem, 2009; Burns, 1998; Ma, 1999). Popular opinion, bolstered through media that focus on how disengagement from math is the norm, is illustrated by comments such as, “Oh, I was never any good at math.” (National Survey of Student Engagement, 2007; O’Brien, 2012; Wieschenberg, 1994).

Teachers who are themselves disengaged from mathematics may pass on their negative perceptions or their math anxiety in their own classrooms (Brady & Bowd, 2005; Ma, 1999); thus, lack of engagement can be transmitted from one group, the teachers, to another group, the learners. Because teachers are the prime deliverers of mathematical curriculum, it is especially important that they
develop a greater awareness of student engagement—a key facet of learning—in order to prevent the development of a downward spiral into disengagement from and avoidance of mathematics.

As mentioned above, this cycle of disengagement from mathematics begins for many with negative experiences in elementary school. Thus, it is important for educators to gain a better understanding of how to teach mathematics in a more engaging way at the elementary level. If positive experiences with mathematics can be fostered in the early years, this will contribute to a more positive attitude towards this important subject.

1.4 Humanization of mathematics education

In order to address issues of disengagement from mathematics, there have been a number of attempts to reform curriculum and pedagogy and improve students’ experiences. Humanizing mathematics education is one approach. Falkenberg (2006) points out that the humanization of mathematics has taken a number of forms. For example, there have been attempts to infuse the human origins and utilitarian purposes of mathematics back into the subject which students sometimes see as an abstract entity encountered only in formal educational contexts and not connected with or relevant to their lived experiences. I argue that what is already known about the humanization of mathematics (Flewelling & Higginson, 2003; Higginson, 1980, 2000a, 2000b, 2006; Wheeler, 1975) is pertinent to the current study, in that like Imaginative Education (Egan, 1997, 2005), its focus is on the ways that individuals make sense of their world. In addition, the humanization of mathematics allows mathematics to portray the richness of its origins and evolution from the past, in the present and for the future.

Of course, other, more general reform initiatives have also considered a human response to the learning of mathematics. In particular, researchers have been trying to better understand the role of emotion and affect in learning mathematics, (see, for example, DeBellis & Goldin, 1993, 1997; Goldin, 1993, 1997; Hannula, 2002, 2006; Malmivouri, 2001, 2006; McLeod, 1988, 1992, 1994, McLeod & Adams, 1989), although not necessarily as factors in student engagement, per se. Affect and emotion are key components in Imaginative Education, as will be explained more fully in Chapter 3.

The increasing attention to students’ perspective (DeBellis & Goldin, 2006; Fielding, 2008; Hannula, 2006; O’Loughlin, 1992; Rudduck & Fielding, 2006) in the research literature is consistent with a humanistic perspective of mathematics education. In this dissertation, I argue that consideration of the students’ perspective is vital to fully understand factors that contribute to their engagement and
their disengagement in mathematics, and that therefore this needs to include consideration of the students’ affective response. With its focus then on a humanistic view of learning and teaching mathematics that includes the affective domain of learning as a fundamental element of human response, the IE framework is well suited to this study.

1.5 Imaginative education

Given the significant role mathematics plays in school curriculum, its relevance to further learning opportunities (British Columbia Ministry of Education, 2007; O’Brien, 2012), and the potential long-term implications of early disengagement from mathematics, it is especially important to understand students’ engagement, or lack of engagement, in mathematics. Most importantly, we need to understand what constitutes engaging mathematics experiences for our students. Despite a venerable and growing body of research related to the humanization of mathematics with particular reference to the affective domain, such an approach has not been implemented in mainstream mathematics teaching.

A variety of reasons have been put forward for this lack of implementation: consideration of different belief systems (Leder, Pehkonen, & Törner, 2002); difficulty in transference between the subject and researcher (Cabral, 2004); and an early focus on the negative orientation of affect rather than on its positive, facilitative potential (Evans, 2006). However, all of these researchers acknowledge that consideration of the affective domain is a crucial factor in ongoing mathematics education research. Therefore, research that addresses the affective element while bringing a new perspective of using imagination has rich potential to help facilitate the implementation of affect into the teaching of mathematics. Such is the case with this study, in which use of the IE theory combines consideration of both affect and imagination in the implementation of curriculum.

IE takes a humanized approach to learning, is grounded in a sociocultural perspective to education, and can be applied to any subject area (Egan, 1997, 2005; Judson, 2008, 2010), including mathematics (Jonker, 2009). It considers both a student’s emotive response and his or her imagination as important parts of the process of learning and developing understanding (Egan, 1997, 2005). As such, it is conducive to understanding how student engagement can be enhanced. Attending to human faculties that a learner already possesses, referred to as cognitive tools within the IE theory, may provide opportunity to enhance both learning and teaching at a critical juncture in students’ mathematics journey, the intermediate years.
The theory, which will be discussed in detail in Chapter 3, is framed by five kinds of understanding that reflect a developmental progression by which individuals make sense of the world around them (Egan 1997, 2005). In the pre-language phase, somatic understanding, individuals make sense of their world primarily through their senses. This is followed by a phase of mythic understanding, in which primarily oral language is used to make sense of experiences. As individuals gain more experience of their world with the addition of written language, they enter the third phase, romantic understanding. In the fourth phase, philosophic understanding, learners make increased connections between aspects of the world around them, becoming aware of associations among experiences. In the final phase, ironic understanding, individuals become aware of the limitations of logical thinking, which is now deemed insufficient to represent important aspects of life experiences.

In order to move from one phase of understanding to another, individuals utilize groupings of cognitive tools (Egan, 1997, 2005) to make increasing sense of their world. These tools, such as stories and metaphors, help individuals retain information by making it more relevant and engaging (Egan 1997, 2005). These are ways of thinking that have been developed culturally over long periods of time to help individuals come to know and interact within their sociocultural environment (Vygotsky, 1962, 1978). Further discussion of cognitive tools as human elements of functioning is found in Chapter 3.

As stated above, a human response to learning is a key aspect of engaging learners in education. Because the IE theory provides a framework for developing pedagogical methods that are humanized to consider the affective domain and the use of imagination, the role IE can play in enhancing student engagement warrants investigation. This is especially so when students themselves, the recipients of the learning experience, are given the opportunity to make a legitimate and valid contribution to the research enquiry.

1.6 The current study

My use of IE theory and ILPFs, the lesson planning frameworks recommended by Egan (1997, 2005) for implementation of the theory, in recent years has given my students opportunities to work on mathematical tasks in ways that seem to have allowed them to better connect with the concept being studied and to express their understanding in a broader and creative manner. My experience suggests that one means to gaining a better understanding of students’ mathematical engagement is to study lessons such as these, in which children explore and express mathematics beyond the conventional
textbook. Given the need to find alternative ways to engage children in learning mathematics, a study into what IE means to children’s mathematical engagement is warranted.

This study examines the use of IE in elementary mathematics teaching and its effects on student engagement. Using a qualitative case study design, this research examines a Grade 4 and Grade 5 unit on shape and space geometry that is framed using IE theory. Five students are tracked throughout the study. Seeking to understand more about how blending imagination and emotive responses can affect the learning of mathematics, the question at the heart of this study is this: What does the use of the theory of IE and its ILPFs mean to children and their engagement in elementary mathematics?

The value of this research is twofold. First, the infusion of imagination and affective responses to learning mathematics has not previously been considered in any substantive way in a humanized perspective of learning mathematics. Second, this research contributes to our knowledge about the use of IE theory. While there has been a significant amount of conceptual discussion and analysis of what an IE approach to learning entails (Buckley, 1994a, 1994b; Egan 1979, 1992, 1994, 2002; Pinar, 2005), no systematic examination of IE theory has been carried out from the student’s perspective (K. Egan, personal communication, September 7, 2005, June 20, 2010; M. Fettes, personal communication, September 6, 2005). This investigation, therefore, adds to and broadens a growing body of research about IE theory that has thus far focused on the teacher’s perspective (Fettes, 2007; Gadanidis & Hoogland, 2003; Jonker, 2009; Judson, 2010; Nicol & Crespo, 2005).

Chapter 2 begins with a discussion of the importance of mathematics as one way of developing understanding of our world, which is followed by consideration of the concept of engagement and its’ counterpart disengagement. Deliberation is also given to mathematics education and a tendency for students to disengage from the subject. Following this a wider perspective of a humanized view of learning mathematics is given, in which consideration is also made of research related to use of the affective domain in learning mathematics.

In Chapter 3, IE theory, which forms the theoretical framework for the study, is examined in detail. An overview of the theory is given, laying out the five phases in the development of understanding. Considerations of imagination found in Egan’s work, as well as in the work of other authors, are reviewed together with Egan’s conception of the development of knowledge. The chapter concludes with a critique of the theory.

Chapter 4 opens with a discussion of the methodological foundation for the study and then reviews the case study research design and the chosen role of teacher/researcher. The research context
is detailed as is the unit plan of lessons that will be the focus of the research study. Data sources from the students and the teacher/researcher will be explained before details of the process of data analysis is laid out. The chapter will finish with discussion of the credibility and limitations of the study.

In Chapter 5, details of the findings for each of the five students tracked in the study are given through discussion of emergent themes before being brought together in a collective summary. Here a cross case analysis of the emergent themes is given as is a cross case analysis of the students’ use of cognitive, or thinking tools, from the IE theory.

Finally Chapter 6 brings the dissertation to a close. Conclusions from the study are presented and discussed in three different areas. These are followed by consideration of implications and recommendations in three areas of research and an area of further practices with imaginative education (Egan, 1997, 2005). A reflective epilogue completes the chapter.
CHAPTER 2: GENERAL LITERATURE REVIEW

As shown in chapter 1, mathematics is both an important subject and a central part of the school curriculum. Mathematical knowledge is essential for individuals to become numerate, or mathematically literate citizens. In addition, engagement with the subject helps in becoming numerate.

It is therefore imperative to find methods for teaching mathematics that engage students, especially in the intermediate grades, when disengagement seems to begin. This study investigates the meaning that the theory of imaginative education (IE; Egan, 1997, 2005), a humanistic approach that is grounded in a sociocultural perspective and utilises emotions and imagination, has for children and what it means for their engagement in elementary mathematics.

In this chapter I review the literature related to key concepts of this study. First, I consider different perspectives which illustrate the importance of mathematics. I then consider different viewpoints of engagement in and disengagement from learning. Next I consider the topic of student engagement and disengagement from learning, followed by a tendency for students to become disengaged from mathematics at various points in their educational journey and at the factors that may influence this disengagement. Next I consider mathematics education from a more humanistic perspective and examine the role of the affective domain. Together the literature in this chapter is critically reviewed to lay a foundation for considering the issue of engaging students in mathematics.

2.1 Importance of mathematics

Since ancient times mathematics has been considered a subject that can bring great discovery and innovation in all facets of life (Pickover, 2009). Indeed, du Sautoy (2010) referred to it as “the language of the universe” (p. 18), a central human means of communicating and understanding the world.

It is not only mathematicians, however, who value competence in mathematics. Government bodies and national organisations throughout the world, such as the Canadian National Research Council (NRC) and the international Organisation for Economic Co-operation and Development (OECD) also acknowledge that numeracy plays a crucial role for citizens of the 21st century. A report prepared for the OECD by the Programme for International Student Assessment states, “This functionality [mathematical literacy] is an important survival skill for the citizen in today’s information and knowledge society” (p. 19).
Researchers and educators alike also acknowledge that the place where most formal learning about mathematics should take place is in schools, as part of an educational system (Kilpatrick, Swafford, and Findell, 2002; Reyna & Brainerd, 2007; Schoenfeld, 2002). This acknowledgement is not surprising when one considers that those who teach within schools have received training in pedagogical practice. Once trained, teachers are required to follow curriculum documents from governmental agencies such as provincial ministries of education, which specify the subjects that teachers are required to teach (British Columbia Ministry of Education, 2007).

Despite the recognition that mathematics is an important subject and that it should be learned within an educational system, helping students to be successful with the subject is a challenge. This is confirmed by Kilpatrick et al. (2002) when stating: “The concerns expressed by many Americans, from prominent politicians to the people next door, that too few students in our elementary and middle schools are successfully acquiring the mathematical knowledge, the skill, and the confidence they need to use the mathematics they have learned” (p. 1).

The risks of not being successful with learning mathematics, and thus not becoming numerate, are adroitly stated by the NRC: “Citizens who cannot reason mathematically are cut off from whole realms of human endeavor. Innumeracy deprives them not only of opportunity, but also of competence in everyday tasks” (p. 1). With this risk in mind, it is crucial that students be engaged with mathematics.

2.2 Student engagement and disengagement

Despite general agreement across educational researchers that student engagement is multifaceted, unfortunately, the specifics of what student engagement is and how this relates to students’ learning are not uniformly defined in the literature thereby creating a somewhat muddied field of enquiry. Indeed numerous terms have been used to describe student engagement over an almost 20 year period since Wlodkowski and Ginsberg (1995) stated that “engagement is a multifaceted concept and at its most basic level is a meaningful response to something on the part of the learner.” These terms have included psychological investment (Newmann et al., 1992), cognitive activity (Henningsen & Stein, 1997), participation (Willms, 2003), attitude and motivation (Robitaille et al., 1996; Wilms 2003) and behavioural engagement (Munns, 2007).

One example of this lack of uniformity and diversity shows Marks (2000) taking a psychological approach to student engagement, characterizing it as “the attention, interest, investment
and effort students expend in the work of learning” (p. 155). After surveying students in 24 schools identified as having made substantial progress towards restructuring, Marks asserts that the processes through which students engage in learning when provided with “challenging and compelling instructional work” (p. 154) affects their level of engagement. Indeed, Marks contends that paying attention to these areas and the “intellectual substance and quality in school” (p. 176) has lead to enhanced academic engagement and achievement in these schools. There is, therefore, every reason in the contemporary educational context to look at ways in which students can become engaged in learning. Importantly for the proposed study, Marks’ use of certain aspects of the student perspective further suggests that consideration of this perspective can contribute significantly to improvements in instructional activities.

A further example of the diversity in the characterization of student engagement has Munns (2007) taking a behavioural perspective by considering pedagogical practices that are effective in heightening levels of student engagement in suburban primary schools in Sydney, Australia. Munns (2007) argues that students, and their actions or behaviour, are very affected by teachers’ classroom practice. Furthermore classrooms in which students have the opportunity to be reflective and curious tend to show high levels of student engagement. Thus, Munns (2007) suggests that a more egalitarian mode of classroom interaction in which there are “Interruptions to the discourse of power.” (p. 5) promote student engagement more than in a traditional hierarchical structure.

Accepting that student engagement is a comprehensive term covering many aspects of involving students in learning, a review of these terms reveals that many of the variations in terminology are rooted in research on student disengagement. While some students are engaged in their learning, for others there can be an antithetical thread of disengagement, which Marks (2000) translates into “The ultimate act of disengagement, dropping out.” (p. 174).

Researchers such as Fullan (1991, 1993) and Goodlad (1984, 1994, 1997) have painted a disturbing picture of the level of disengagement found in many Canadian schools: “Taking a typical cohort, one-third will leave school before graduation, and approximately another third will lead lives characterized by uninspired passive learning” (Fullan, 1991, p. 182). Various provinces in Canada have adopted reform measures to address this disturbing trend (British Columbia Ministry of Education, 2009; Ontario Ministry of Education, 2008). In addition, the general public’s expectations of schools have risen. Provincial governments in Canada have responded in part with policies requiring increased accountability on the part of school districts and schools (Smith et al., 1998).
is, therefore, strong national and international support for further investigation into student engagement and disengagement in school and possible policy reform to address these issues.

Notwithstanding general agreement among the researchers cited above, I suggest tracing the etymology of words used to discuss contemporary ideas provides an opportunity to sharpen the focus of a line of reasoning. The original meaning of *engage*, “to bind by a pledge” (Skeat, 1993, p. 168), suggests not a casual connection, but rather a commitment with an underlying obligation. This is particularly relevant in education, where there is an obligation on the part of the educator to share his or her learning and knowledge with the learner, and an obligation on the part of the learner to participate in this process.

Appleton et al. (2012) provide a further point of refinement that strengthens the reasoning for a sharpened contemporary view of student engagement by suggesting that “Engagement is either the outcome, a process to other defined outcomes, or plays a dual role” (p.vii). In calling for more common agreement of terminology Appleton et al (2012) rightly contend that the ensuing consistency and reliability will have positive implications for any pedagogy, research or policy initiatives.

Notwithstanding the achievement of more common agreement through research and deep reflective enquiry as to what constitutes student engagement, I would suggest that a range of questions will still arise that need to be answered. These would and should include, for example, questions such as why pursuit of student engagement is a worthy goal, and whose responsibility it is to ensure that this is achieved.

Preliminary or contributory answers to questions such as the above two, I would contend are emerging in recent literature and research reviews. Trowler and Trowler (2010) provide a somewhat circular but reasoned view that “Student engagement in general improves outcomes, specific features of engagement improve outcomes, and that engagement improves specific desirable outcomes.” (p. 4) such that engagement itself is no longer questioned. In addition Trowler and Trowler (2010) also rightly state that “Responsibility for engagement is shared” (p.4). This latter point concurs with that of Taylor and Parson (2011) who suggest that a collaborative and respectful horizontal organisational model of peer-to-peer type relationships between students and teachers leads to successful classes in which students are engaged. Here Parsons and Taylor (2011) believe that a culture of learning is created where “Teachers are learning with students. Language, activities and resources focus on learning and engagement first and achievement second” (p.26).
It is within this type of culture of reciprocity, I would assert that there is a meaningful place for researchers and educators to consider what students can contribute towards greater understanding of matters that affect and involve them as participants and recipients of education. Parsons and Taylor (2011) point to a slowly increasing amount of research that is actively seeking the student perspective of learning and call for “Deeper research into student’s perspectives about what keeps them engaged in learning” (p.51). It is at this particular juncture that the current study offers a contribution to increased knowledge about the important topic of student engagement by asking students themselves about their engagement in learning.

2.3 Mathematics education and student disengagement

Regrettably students appear to be particularly prone to disengagement and alienation from mathematics (Boaler 2000; Nardi & Steward 2002a, 2000b, 2003). This is especially troubling in an era where citizens need to be mathematically literate to keep up with the ever-accelerating pace of technological change.

Boaler (2000) identified one possible cause for disengagement in a four-year study of teaching methods and ability groupings in upper intermediate and lower secondary grades in the United Kingdom. Boaler (2000) attributes the disengagement of students from mathematics to teaching methods and classroom practices that do not foster connection to the real world outside the classroom. Boaler was also able to determine that for students to be engaged with the mathematics of the classroom, they need to form an identity that fits with the norms of the classroom environment. If they did not, students began to see mathematics as having a “meaningless nature” (Boaler, 2000, p. 386).

A similarly bland experience of mathematics was found with students in Nardi and Steward’s (2002a, 2002b, 2003) research on disaffection from mathematics among young secondary students in England. This work focused on the students’ ability to engage in mathematical tasks; data included the students’ own assessments of their engagement. The single study reveals that, rather than engaging in mathematical tasks out of a desire to do a task or for enjoyment, the students were actually disinterested and disaffected. They took part in mathematical activities only “out of a sense of professional obligation or under school and parental pressure” (p. 349). Thus, Nardi and Steward (2003) emphatically warn mathematics educators in this regard: “The students plead for classroom mathematical experiences that are tailored to their individual needs. In the absence of such individualisation they grow alienated from the subject and even eventually wish to drop it” (p. 361).
Studies initially designed to report on student achievement in mathematics have also shown evidence of a disturbing trend towards disengagement. This is the case both in the Trends in International Mathematics and Science Study (TIMSS) (Mullis et al., 2000, 2008) and in a study carried out by Ontario’s Education Quality and Accountability Office (EQAO), 2010, 2012).

In the TIMSS research, an index of positive affect toward mathematics (PATM; Mullis, 2008) was used to analyse responses from fourth and eighth grade students to Likert-scale statements about their learning of mathematics, such as “I like mathematics.” As mentioned in chapter 1 there was a significant drop (18%) in the number of students who expressed positive feelings about their learning of mathematics in grade 8 when compared to the grade 4 students. A similar drop was measured by an index of students’ self-confidence in learning mathematics (SCM). Here students were asked to respond to statements about their perceptions of their mathematics ability, such as “I am just not good at mathematics.” More than half (57%) of students in grade 4 felt confident, compared to 43% in grade 8. Together, the PATM and SCM results indicate a worrying downward trend to students’ attitude towards and readiness to engage with mathematics, which in turn could negatively affect their achievement, as they move into higher grade levels.

The EQAO (2010, 2012) study suggests that this downward trend appears to start somewhere between grade 3 and grade 6. The results of a Likert-scale survey in which students responded to statements about their attitudes toward mathematics show a disturbing drop of 14.5% between grade 6 students and grade 3 students who expressed liking mathematics (EQAO, 2010, pp. 8, 25), with consistently lower numbers of grade 6 students expressing a liking for mathematics some of the time, raising concerns about how to address disengagement from mathematics. Furthermore, the most recent EQAO data (2012), comparing students’ performance on standards test in grade 3 and later in grade 6, show that 6% of the students who did not meet the standards in grade 3 do meet them in grade 6. Furthermore, 18% of the students who did meet the standards in grade 3 do not meet them in grade 6. Therefore, for every one student who improved between grade 3 and grade 6, three students did not show evidence of improvement, thereby underscoring the need to engage students with mathematics in the intermediate grades to mitigate declines in achievement.

Unfortunately, disengagement from mathematics appears to continue to grow over time, as indicated by research with secondary students (Boaler & Greeno, 2000; Fairweather, 2007; Nardi & Steward 2003). In Boaler and Greeno’s (2000) study of mathematical confidence among American high school students, students were asked to describe their mathematics learning environment as part
of a “figured social world of mathematics education in which they participated as learners” (Boaler & Greeno, 2000, p. 173), in order to gain a better understanding of their experiences and beliefs. The findings reveal three thematic areas of concern: (a) the learning world the students experienced, (b) the students’ perceptions of their place in this “figured world of mathematics” (Boaler & Greeno, 2000, p. 175), and (c) students’ affective response and identity development with regard to their participation in mathematics learning both at the time of the research and subsequent to it. In the first thematic area, it was found that the mathematics learning was, for the most part, highly ritualized and took place largely through the receiving of knowledge, with far less opportunity for exploration and discussion. When it came to students’ place, in the more traditional classrooms, the student’s role was narrowly defined; there were many opportunities for practicing procedures but little opportunity to exercise a range of thinking practices. In the third thematic area, the students gave priority to their developing identities, together with the relationships they formed with other learners.

Boaler and Greeno (2000) conclude that for many students, their ultimate “rejection of mathematics” (p. 197) went much further than just liking or disliking the subject, involving deep concerns about their sense of self. In addition, an emphasis on rote procedures dramatically reduced the numbers of students considering mathematics as a field of further study. Furthermore, when learners were given more freedom to explore concepts, they found it easier to form their own identity as a learner.

Unfortunately, a lack of engagement with mathematics continues for many past secondary education (Ward-Penny, Johnston-Wilder & Lee, 2011; Wiliam, 2005), even for those who have chosen mathematics as a field of study at university. For example, Ward-Penny et al.’s (2011) thematic analysis of exit interviews with undergraduate students in England revealed that even students considered to be high achievers expressed very high levels of distancing themselves from mathematics. Even more worrying at a time when mathematical competency is essential for participation in an increasingly fast-paced world was the manner in which mathematics education was regarded in comparison to solely mathematics modules. Mathematics education was a means of increasing one’s separation from mathematics: “Many openly admit that they opt into the module [mathematics education] as a direct consequence of a growing disinclination for mathematics” (Ward-Penny et al., 2011, p. 21). This points towards an even greater need to understand how to engage students during the important intermediate years, where research suggests disengagement begins. Greater understanding of
student engagement with mathematics during this time may help to develop an inclination towards mathematics sustainable in later parts of the learning journey.

As we saw above, problems with the pedagogy of mathematics can cause problems for elementary students studying mathematics. Looking at later stages of the learning continuum, it can also hinder those learning to teach the subject, pre-service teachers. Brady and Bowd (2005) highlight research that shows first negative experiences of mathematics often begin at the start of the intermediate grades in elementary school. Brady and Bowd surveyed education students in Ontario, Canada, who were preparing to teach at the elementary level. Of particular note was the difference between responses when participants were asked about their enjoyment of learning mathematics in elementary and secondary school. Only 42.7% of participants expressed enjoyment of mathematics at the secondary level, whereas 60% of participants said they enjoyed mathematics at the elementary level. Thus, negative perceptions about mathematics began for many pre-service teachers somewhere between their elementary and secondary years.

This finding is confirmed by Ma’s (1999) meta-analysis of 26 studies on the relationship between mathematics achievement and anxiety about mathematics among elementary and secondary students. Looking at the development of mathematics anxiety across three grade-level groups of grades 4 to 6, grades 7 to 9 and grades 10 to 12, Ma confirms that “mathematics anxiety often originates during students’ early educational experiences” (p. 534). Also revealed was a consistent direct relationship between mathematics anxiety and achievement across the three grade level groups. This implies that once anxiety about learning mathematics begins in the early grades, achievement increasingly deteriorates in the secondary grades. Thus, it is argued that it is crucial for educators, in my opinion, to understand elementary students’ engagement in their learning of mathematics and to provide positive learning experiences during the elementary years.

2.4 The humanization of mathematical learning and teaching: A wider perspective

As discussed earlier, disengaging from learning mathematics is seen in this study as a human response to an unpleasant experience. With this in mind, this section critically examines the literature that has considered a humanized approach to mathematics, and thus a broader perspective of the subject itself.

Falkenberg (2006) provides a contemporary philosophical entry point into considering a humanized perspective for the learning of mathematics by suggesting that all education, including
mathematics, should have an element of purpose, a moral purpose, in order to prepare learners for different aspects of life. In pointing out that researchers have taken a number of different approaches to the humanizing of mathematics, Falkenberg suggests a broader vision of mathematical learning that considers humanizing the curricular teaching of mathematics where sight of the origins, purposes and uses of mathematics does not get lost to abstract conceptualization and prescriptive learning without relevancy to many.

It should be remembered; however, that humanization of mathematical learning has been found within education for many years. The humanization movement has generally seen mathematics as a result of human processes and activities, rather than as a decontextualised body of knowledge (Wheeler, 1975). Wheeler (1975) proposes that mathematics has two languages, one of laws and theorems and the other of a historical perspective. He argued that it was worth trying “to glue the languages back together, to give meaning, as it were, to the common edge of the two faces” (Wheeler 1993, p. 54), thus providing a blended outlook of the social sciences and mathematics.

Wheeler was not alone in calling for a humanized approach to mathematics. Seminal works such as Freudenthal’s (1983) Didactical Phenomenology of Mathematical Structures and Gattegno’s (1974) The Common Sense of Teaching Mathematics have both been utilised by numerous educators, such as Mason and Johnston-Wilder (2004), in attempts to provide a more humanized approach to the learning of mathematics. Within the mathematics education community, therefore, there is general acceptance that humanizing mathematical content is a positive step that can enrich students’ learning. The benefits are exemplified by Ward-Penny’s (2011) observation that humanized mathematics “can help pupils explore mathematical ideas in a more well-rounded way and reconnect many mathematical concepts to the exploration and enquiry from which they originally emerged” (p. 147).

One avenue spearheaded by Higginson (2000) and Flewelling and Higginson (2003) sees mathematics and creativity as coexisting, thus allowing for the human response and expression of mathematical understanding. Higginson (2000) suggests four conceptions through which a teacher may see mathematics and creativity intersecting. The first sees creativity in mathematics as a novelty, through which a teacher introduces concepts to students in an innovative manner. The second sees mathematical creativity coming from students’ physically building or creating artefacts, whereas the third sees creativity in the construction of symbolic systems. The fourth conception has the teacher structure the learning environment to have personal significance to the students. As Higginson (2000a) points out, “Humans are, by their nature, perceptive about patterns and the symbol systems which
evolve from them. This way of looking at the world from a mathematical/aesthetic perspective is potentially very powerful.” (p 10).

In a corresponding manner, Sinclair (2006) also sees creativity as part of a humanised view of the ‘doing’ of mathematics. Sinclair suggests an aesthetic domain is “Akin to that of mathematical production, which belongs to professionals and to children, both of whom are engaged in producing, doing and creating mathematics.” (p. 3).

Unfortunately, despite calls to regard mathematics as a creative subject, research carried out by Kaufman and Baer (2004) shows that many undergraduate students perceive the learning of mathematics to be devoid of any creativity. Their survey of undergraduate students from four universities, in which they asked participants to rate their own creativity in a range of subject areas, finds that students felt there was less opportunity to be creative in mathematics. In the types of situations reported by Kaufman and Baer (2004) and Nardi and Steward (2002a, 2002b, 2003), I suggest that students have not been presented with mathematical activities, lessons or programmes that had humanized curriculum content.

Regrettably, the move from research and theory to implementation in the classroom has not been widespread; discussion appears to have remained largely at the theoretical level. This assertion is based on my almost 20 years of teaching practice in elementary schools, writing curriculum documents with the British Columbia Ministry of Education, and working with publishers of mathematics curriculum programmes and resources. This is not entirely surprising considering the lack of agreement on just what the term humanized mathematics means. Even Wheeler (1975) had come to see that the name he chose for this field is wide ranging: “I see it now as a powerful and evocative title that covers far more than I dreamed” (p. 4).

Despite the relatively uncommon occurrence of humanised mathematics lessons in mathematics classroom, relatively recent examples illustrate what is possible. Percival (2001, 2003) has clearly shown that bringing a historical perspective of mathematics to young students engages them and stimulates their understanding of mathematics. In a series of “Time time-travel days,” grade 3 students were encouraged to imagine that they were travelling back in time to ancient civilisations. In this context students studied the mathematics of the period by engaging in a variety of activities. The young students demonstrated high levels of engagement, often expressing their excitement about the tasks they were performing.
Further, Cirillio (2007) has shown that learning something about the lives of the inventors of calculus helps secondary students appreciate the subject. In assigning the question “Who invented calculus?” Cirillio wanted the students to discover more of the field’s historical background. The use of facts and anecdotes about the lives of the inventors of Sir Isaac Newton and Gottfried Leibniz brought the mathematics alive for students and helped them to “see mathematics as a body of knowledge developed by humans” (Cirillio, 2007, p. 23).

Similarly, Cheung and Huang (2005) present a model of teacher professional development that portrays mathematics as a subject of human development for human use. They designed this model to assist teachers in primary and intermediate classes to learn a humanized perspective of mathematics. Teachers learned to use everyday contemporary problems to show that mathematics is a means of solving practical problems that could, in the future, be shared with their students. In this way, mathematics came to be seen by the teachers as “a human activity [that] is not only an activity of solving problems and looking for problems, but also an activity of organizing subject matter from reality and within the mathematics discipline” (Cheung & Huang, 2005, p. 4), that has an important place in the lives of teachers and students.

Although there is some inconsistency between the theoretical literature (Higginson, 2000, 2000a; Flewelling & Higginson, 2003; Sinclair 2006) and limited pedagogical literature (Percival 2001, 2003; Cirillio, 2007 Cheung & Huang, 2005), I argue that when a historical or creative perspective of mathematics is adopted, it can have benefits for students’ and teachers’ learning. Therefore it is reasonable to infer that a theoretical approach to learning mathematics that is infused with a systematic, humanized approach to learning, further supported by its’ grounding in a sociocultural perspective, provides opportunity to enrich the learning of mathematics. One such approach is the theoretical framework chosen for this study, imaginative education (Egan 1997, 2005), which will be discussed fully in the next chapter, chapter three.

2.5 Use of the affective domain in learning mathematics

Related to the issue of engagement with and humanization of mathematics is the affective domain as a necessary part of the ‘doing’ of mathematics. With the emotive response of the learner falling within an affective domain of learning, I will be using the term affective to include the manner in which we respond to situations emotionally; with particular emphasis on a learner’s response. This is in line with Egan’s (1997) use of emotions which are seen as central to engaging a learner with
knowledge, and Martin and Reigeluth’s (2013) use of affective when stating that “The affective domain may be equally if not more important than the cognitive domain in promoting student learning.” (p. 506).

Furthermore, because consideration of the affective domain in learning mathematics is an important juncture for considering the issue of engagement with and humanization of mathematics, I will be providing a critical review of research that has taken place which has considered the role of the affective domain in learning mathematics, with a specific focus on literature that considers an emotional response to learning. This in turn will help position the current study with respect to what is currently known and what remains to be examined with regard to engaging students in learning mathematics.

Research into the affective domain when learning mathematics is a growing and wide-ranging area of study. McLeod (1989, 1992, 1994) and Hannula (2002, 2006) show that active attention to the affective response of students is a vital part of research on student engagement with mathematics.

McLeod’s (1985, 1989, 1992, 1994) seminal work on the role of affect in learning mathematics has brought together several areas of research, including the use of affect in problem solving and the beliefs, attitudes and emotions involved in mathematics education. When considering a problem-solving context, where a goal or solution is not immediately attainable and there is no obvious algorithm for students to use, McLeod (1985) suggests that it is possible for learners to experience intense emotional responses, both positive and negative.

McLeod (1992, 1994) suggests that use of the affective domain of learning in mathematics education is beginning to focus more on the students’ affective responses rather than those of the teachers. In addition, McLeod (1994) proposes that qualitative methods may be more conducive to revealing the potential complexities of affective issues related to the learning of mathematics. Particularly significant to the current study is McLeod’s (1994) judicious comment that “students’ views of mathematics cannot be considered in isolation but must be analyzed in the context of an integrated approach that considers all the beliefs and motivating forces that influence the student” (p. 644). Thus a more integrated approach provides a wider outlook on students’ learning in which sociocultural influences are considered alongside affect. This is congruent with the overall sociocultural perspective taken in this study.

Hannula (2002, 2006) has continued to build on the work of McLeod in the area of affect and mathematics education. Hannula’s (2002) conception of an analytical framework for examining
attitudes towards mathematics is rightly based on the premise that attitude is connected to achievement and affective outcomes, of which emotion is just one part. Carrying out an ethnographic case study with a lower secondary school student, Hannula wanted to reconceptualise the idea of attitudes and their influence on learning mathematics. He suggests a framework with four evaluative considerations: (a) emotions aroused in a situation, (b) emotions associated with a stimuli, (c) expected consequences, and (d) relating a situation to personal values. Hannula (2002) suggests, “Emotions are more central to attitudes than cognition” (p. 28).


In suggesting the process of self-reflection, Malmivuori (2001, 2006) fittingly draws on the seminal body of work by Schoenfeld (1983, 1987, 1989, 1992) with regard to the central role of meta-cognition to affective issues in learning mathematics. By being more specific about aspects of affective responses, Malmivuori (2006) believes that it is possible for researchers and educators to create links between affective, behavioural and cognitive domains. With these links Malmivuori (2006) sees direct interaction between the “co-constructive and dynamic nature of affect and cognition” (p. 161). However, in Malmivouri’s (2001) theory, it is importantly the constructive character of and interaction between the affective and behavioural domains which lead to potential cognitive development. Affect influences behaviour, which may either promote or limit cognitive development.

Goldin (1988, 1998) and DeBellis and Goldin (1993, 1997) continue the consideration of self in mathematical learning situations. In an empirical longitudinal study with elementary-aged students on the development of children’s internal representations of mathematical tasks, DeBellis and Goldin (2006) focus on the role of meta-affect in learning mathematics, defined as “affect about affect, affect about and within cognition about affect, and the individual’s monitoring of affect through cognition (thinking about the direction of one’s feelings) and/or further affect” (p. 136). Thus, the feelings individuals have about their own learning influences their ability to engage with subject matter or in a learning context.
Consideration of meta-affect is therefore pertinent to the use of the IE (Egan 1997, 2005) theoretical framework as an affective response to learning is a key piece of both IE and meta-affect, as will be discussed in chapter 3. Indeed, there is further accord between IE and DeBellis & Goldin (2006): both recognise the importance of the overall sociocultural context of learning. DeBellis and Goldin (2006) state that “a supportive classroom culture provides a sense of safety in being ‘stuck’” (p. 137), and thus provides a place where students may persist in their engagement with a task in order to overcome an initially perceived difficulty.

DeBellis and Goldin (2006) also introduce the concepts of mathematical intimacy and mathematical integrity, which, they say, “influence the nature of learning and the knowledge attained” (p. 132). Mathematical intimacy as “deeply-rooted emotional engagement, vulnerability, and the building of mathematical meaning and purpose for the learner” (p. 137), whereas mathematical integrity is “an individual’s affective psychological posture in relation to when mathematics is ‘right,’ when a problem situation is satisfactory, when the learner’s understanding suffices, or when mathematical achievement deserves respect or commendation” (p. 137). These terms refer to the emotional experience of the learner and learners’ ability to judge the truth of their mathematical response, respectively; both are objectives towards which educators should strive in the growth of a students’ mathematical development.

Furthermore, mathematical intimacy and mathematical integrity fall well within the role of the affective component of learning mathematics. Students’ sense of truth and trust in the reliability of mathematics can greatly affect their emotional response to learning mathematics. DeBellis and Goldin (2006) further argue that in order to develop a personal sense of mathematical integrity where they have utilised their “deeply rooted emotional engagement” (p. 137) to develop their growing sense of meaning and understanding of mathematics, students must develop a personal sense of mathematical intimacy.

According to DeBellis and Goldin (2006), “The occurrence in learners of well-defined pathways of local affect has implications for the development of an individual’s global attitudes and beliefs towards mathematics, and conversely” (p. 145). Thus, a student can develop a great sense of self with mathematics, and be both part of a classroom culture for learning mathematics and part of a larger societal group of mathematics learners that can contribute towards greater understanding of what does engage students in learning mathematics.
2.6 Conclusion

This chapter has reviewed literature relevant to providing a context for the current study. First, literature on the importance of mathematics was considered. Student engagement (Appleton et al., 2006; Marks, 2000 Munns 2007; Parsons and Taylor, 2011) and disengagement from learning (Fullan, 1991, 1993; Goodlad, 1984, 1994, 1997; Smith et al., 1998) literature was considered next. The lack of uniformity in the concept of student engagement indicates a real need for clarity and a means by which student engagement can be observed, reviewed and analysed to provide greater understanding of this essential facet of student learning. For the purposes of this study where I use a particular theoretical framework of IE (Egan 1997, 2005) to examine the issue of student engagement in learning mathematics, I have chosen to define engagement according to both emotive and participatory (behavioural) responses individuals display in learning situations.

This review also highlighted the problem of many students’ disengaging from learning mathematics at a critical time in their learning journey due to a bland approach (Boaler, 2000; Nardi & Steward, 2002, 2003). Closer examination was given to stages of the learning journey where disengagement was particularly noted, such as between elementary and secondary grades (EQAO 2010; Mullis 2000, 2008) and to the perpetuation of disengagement from learning mathematics (Boaler & Greeno, 2000; Fairweather, 2007; Ma, 1999; Nardi & Steward, 2003) in the university years (Brady & Bowd, 2005; Ward-Penny et al., 2011).

Building on research in a humanised view of mathematics education (Falkenberg, 2006; Flewelling & Higginson, 2003; Higginson, 2000; Percival, 2001, 2003; Ward-Penny, 2011; Wheeler, 1975, 1993) and the affective domain of learning mathematics (DeBellis & Goldin, 1993, 1997; Goldin, 1988, 1998; Hannula 2002, 2006; Malmivuori 2001, 2006; McLeod, 1994, 1992, 1989, 1985), my study seeks to combine these two perspectives of teaching and learning mathematics through the adoption of an IE (Egan 1997, 2005) theoretical framework. While there are many avenues to combine affect and the humanization of mathematics, the emphasis on imagination and the emotions in IE for a unit of mathematics lessons provides the opportunity to gain further insight and understanding into what engages students when these areas are brought to the forefront, thereby diminishing the potential for disengagement from mathematics. Therefore in the next chapter, I provide an overview and discussion of the IE theoretical framework chosen for this study.
CHAPTER 3: IMAGINATION AND THE THEORY OF IMAGINATIVE EDUCATION

Having considered research related to the need to find alternative ways to engage children in learning mathematics, I now provide a review of imaginative education ([IE]; Egan, 1997, 2005), the theoretical framework chosen for this study. IE was chosen for this study to build upon previous work in mathematics education on affect in learning mathematics. IE also provides an opportunity to reconsider and possibly extend the creative element of a humanized view of mathematics education through the use of imagination. That is, bringing emotions and imagination together, as IE does, allows us to consider what can be done to ignite interest within students for mathematics.

The infusion of imagination and affective responses in learning mathematics has not previously been considered in any substantive way. In addition, this study listens to the students’ perspective of their experience, which has yet to be systematically researched within the context of IE.

The chapter first provides an overview of the theory and the five phases for the development of understanding, and then reviews two key concepts of the theory: emotion and imagination. I then discuss the development of knowledge through the use of the cognitive tools that are part of the five phases of understanding. Discussion of theoretical and philosophical considerations of the theory form part of its critique, together with discussion of three areas in which the theory has been scrutinised. Implementation and use of IE theory is considered, before the chapter concludes with a discussion of the contributions to education that are claimed for this theory.

3.1 Overview of imaginative education and cognitive tools

The theory of IE (Egan, 1997, 2005) involves a fundamental reconceptualization of education and its purpose, where the focus is on the development of understanding rather than on the acquisition of pieces of knowledge. In Egan’s (1997) view, this development occurs through five phases of gradually increasing internalisation, through the use of cultural tools such as language and communication systems. Over time, cultural tools become an individual’s cognitive tools and mediate within and between cultural and educational development. In this sense, a student is never just an individual or part of a societal or cultural group; rather he or she is someone who has a firm place in both spheres.

The central means used by humans to develop different forms of understanding is language (Egan, 1997, 2005). While language allows individuals to both express and receive understanding of
the world around them, language in Egan’s (1997) view is also a means of enlarging the mind. This is achieved through increasingly sophisticated use of language forms, such as oral, written and theoretic use of language, which leads to the acquisition of greater understanding of the world and all that it encompasses.

Table 3.1 Sets of cognitive tools (Tyers, n.d.)

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<th>Somatic Understanding</th>
<th>Mythic* Understanding</th>
<th>Romantic* Understanding</th>
<th>Philosphic Understanding</th>
<th>Ironic Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bodily senses</td>
<td>Story</td>
<td>Sense of reality</td>
<td>Drive for generality</td>
<td>Limits of theory</td>
</tr>
<tr>
<td>Emotinal responses and attachments</td>
<td>Metaphor</td>
<td>Extremes and limits of reality</td>
<td>Processes</td>
<td>Reflexivity and identity</td>
</tr>
<tr>
<td>Rhythm and musicality</td>
<td>Abstract binary opposites</td>
<td>Association with heroes</td>
<td>Lure of certainty</td>
<td>Coalescence</td>
</tr>
<tr>
<td>Gesture and communication</td>
<td>Rhyme, meter and pattern</td>
<td>Wonder</td>
<td>General schemes and anomalies</td>
<td>Particularity</td>
</tr>
<tr>
<td>Referencing</td>
<td>Jokes and humour</td>
<td>Humanizing of meaning</td>
<td>Flexibility of theory</td>
<td>Radical epistemic doubt</td>
</tr>
<tr>
<td>Intentionality</td>
<td>Forming images</td>
<td>Collections and hobbies</td>
<td>Search for authority and truth</td>
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<tr>
<td>Sense of mystery</td>
<td>Revolt and idealism</td>
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<tr>
<td>Games, drama and play</td>
<td>Context change and role play</td>
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</tbody>
</table>

*The two shaded columns, Mythic and Romantic, are the ones most in use during the elementary school years.

Egan (2005) uses the terms cognitive tools and tool to refer to aids that help us create understanding of our world. These terms will be used throughout this study to help maintain theoretical congruence. Table 3.1 presents Egan’s (1997, 2005) cognitive tools. Drawing on Vygotsky’s (1962, 1978) view of language development, Egan (1997, 2005) sees these tools as aids to thinking, which people gain through living and learning as part of a society and a cultural group. These tools, developed over long periods in cultural history, help with the development of understanding and thought such that people can make sense of what is going on in the world around them. Importantly, the set of learned cultural tools, such as language, become internalised and, once learned, become an individual’s own cognitive tools. For example, stories are very much part of oral cultures and in Egan’s (1997, 2005)
view they serve the purposes of transmitting information in a memorable way and orienting the listener’s feelings about what is being communicated.

3.2 Five phases of understanding

As indicated earlier, an important component of IE theory are the five phases of understanding (Egan, 1997, 2005), which are summarised in Table 3.2. Although ages for the appearance of the characteristics are suggested, these are guidelines rather than rigid or definitive limits. Indeed, children may well show some characteristics of one phase of understanding before they have fully entered a subsequent phase. Egan (1997) suggests that the phases of understanding are lenses or ways of seeing and understanding the world surrounding an individual.

3.2.1 Somatic understanding

Somatic understanding (Egan, 1997), the first phase of development, begins at birth. Children in this phase are pre-linguistic and make sense of their world through sight, sound, smell, touch, and taste. “The infant’s mind discover[s] its body” (Egan, 1997, p. 242) and infants acquire an increased sense of the world around them through tools such as gesture and communication and through emotional responses and attachments; thus an embodied sense of coming to knowledge and awareness develops. In this way, children discover a rudimentary sense of purpose through activity and the responses gained. They begin to understand effort and ease, hunger and satiety, pleasure and pain, and time and space. The starting place is in and with the body. Importantly, children also discover themselves to be social beings in a surrounding world with a variety of means of communication and an emotive response to stimuli. This all takes place within the larger sociocultural setting that is reflected in understandings garnered by children. For instance, infants will begin to realize that an emotional response of crying for more than a short period usually results in the presence of another person to check for the cause of the crying and then to comfort them.

3.2.2 Mythic understanding

As children evolve and develop the skills of spoken language, they move into the second phase, mythic understanding (Egan, 1997, 2005). This generally begins around the age of 2. In mythic understanding characteristics of the previous phase do not completely disappear but become somewhat less influential as other cognitive tools are acquired and developed. Some of the commonly used
cognitive tools during this phase of development, shown in Table 3.1, include language tools such as the use of metaphors, which enable children to comprehend one thing by seeing it in terms of another. Additionally, binary opposites such as good/bad, empty/full, and captivity/freedom help children to arrange and organize understanding and knowledge. The use of story makes content more appealing and therefore more easily remembered. Games, play, and drama provide an important opportunity for children to take part in activities outside of their immediate reality, to begin developing their ability to reflect upon events, and to develop a growing sense of the world. It is here that language starts to become a dynamic tool for children. In addition, during this phase, a sense of mystery enables children to begin to realize that there is more to the world than that which is immediately accessible. The world is becoming much larger and exciting as more is learned about it.

This is a period of incorporating previously garnered understandings with the new and growing sense of the world gained through spoken language. Children also become aware that written language, with which they are becoming familiar, can help them to further understand, represent, and converse with people and the world around them. Previously, children were limited to bodily senses, physical experience, and an understanding of spoken language. With a growing sense of independence and self by the end of the mythic phase, children develop awareness that the world is a more expansive one than that which they experienced as an infant.

3.2.3 Romantic understanding

The third phase of understanding, romantic understanding (Egan, 1997, 2005), occurs between approximately 8 and 15 years of age. Children become increasingly aware of and competent in the use of written language. Literature and stories progress from the fantasy world of fairy stories to those with more of a sense of reality; stories are based more in the real world and yet can powerfully bridge the gap between the real world and a fantasy world of magic, such as with the very popular Harry Potter series by J. K. Rowling.

It is during this phase of development that children become much more aware of their independence in an increasingly diverse and complex world. They move from focusing on themselves to becoming consciously aware of self in relation to the world. The cognitive tools of primary use in this phase include a strong sense of reality and the extremes of experience and reality, wherein children are able to see the world around them in new ways. For example, children become aware that while they are part of a family, they are also part of a community, which is part of a region, which is in turn
<table>
<thead>
<tr>
<th><strong>Somatic</strong></th>
<th><strong>Mythic</strong></th>
<th><strong>Romantic</strong></th>
<th><strong>Philosophic</strong></th>
<th><strong>Ironic</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Exploratory mode allows infant to gain notion of time, causality, effort and response, as well as sensation and feeling, pain and comfort, hunger and satiety.</td>
<td>Binary concepts and affective importance.</td>
<td>Organizing topic into narrative</td>
<td>(a) How can scheme be made vivid?</td>
<td>Insight into emancipation of self.</td>
</tr>
<tr>
<td>Discovery of social nature via communication, care and affection shaped into cultural patterns.</td>
<td>Organizing content into story form</td>
<td>(a) Extremes of reality or experience; can an image capture this aspect?</td>
<td>(b) What content exposes power to organize topic?</td>
<td>Balance between alienating irony (AI) and sophisticated irony (SI). SI preserves characteristics of other understandings.</td>
</tr>
<tr>
<td></td>
<td>(a) Content that most dramatically illustrates binary concepts. Access point to topic. Image that captures content and dramatic content. (b) Content that best articulates topic into clear story form.</td>
<td>(b) Content to articulate topic into a narrative of whole topic.</td>
<td>(c) What are most clear, powerful, and relevant theories, ideologies, etc.?</td>
<td></td>
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<td></td>
<td>Best way to resolve conflict in binary concepts. Degree of mediation? Degree of explicitness?</td>
<td>(c) Humanizing content. Hopes/fears and emotions to create sense of wonder. Ideals and revolts against convention.</td>
<td>Introducing anomalies. What content is anomalous? Begin with minor anomalies build to sensitive challenges.</td>
<td></td>
</tr>
<tr>
<td><strong>Evaluation</strong></td>
<td>Engagement/understanding with topic, its importance grasped and content learned.</td>
<td>Resolution and closure to story line of topic, conclusion.</td>
<td>Presenting alternative general schemes. What alternatives can now organize content? Which can be used to expose contingency?</td>
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<td><strong>Evaluation</strong></td>
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<tr>
<td><strong>Evaluation</strong></td>
<td>Engagement/understanding with topic, engagement and stimulation of imagination.</td>
<td></td>
<td>Conclusion</td>
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<tr>
<td></td>
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<td></td>
<td>Expose possible utility of variety of schemes. Work with disillusionment and alienation.</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Evaluation</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Content learned/understood. General scheme building.</td>
<td></td>
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</table>
part of a country. With extremes of experience and reality children often develop a fascination with peculiarities and facts that are noted in books like the *Guinness World Records* series.

In addition, humanization of meaning in learning and an association with heroes develops in this phase. Here the discoverers of knowledge, such as Pythagoras and Eratosthenes, can be introduced through inference of the emotions they may have experienced at the time of their discoveries, and can then be incorporated into learning opportunities. Children begin to understand and identify with the emotions and qualities that a role model or hero can embody without having to go deeply into abstract characteristics of topics being studied. Humanization of meaning can also play a larger role in developing understanding, by including stories from and about familiar people.

The sense of mystery that was experienced in the mythic understanding phase develops here into a sense of wonder, encouraging children to ask a range of questions about things they notice and experience. In the earlier stage, children began to recognize that something about what they were experiencing was unknown. With a growing sense of wonder, a specific cognitive tool in Egan’s (1997, 2005) view, in the romantic phase, they begin to speculate about the nature of the unknown entity, possibly trying to hypothesize potential answers. Egan (2005) asserts that this phase in particular “energizes the literate mind” (p. 92), and once energized in thought, children are able to become aware of another cognitive tool: revolt and idealism. Beginning with questioning of what they see as personal idealism, children may feel resentment towards perceived difficulties or obstructions to what they wish to do. This personal sense of resentment and revolt may grow to an appreciation and understanding of larger societal and cultural topics, which could include issues such as pollution and environmentalism. Here children, who are becoming youth, become more and more aware of extremes of reality contained in binary opposites such as peace/war and heroes/villains. Tools that may have been introduced in an earlier phase now take on more significance than when first experienced.

The latter part of the previous mythic understanding phase and the subsequent romantic understanding phase are those most in use in the elementary school age years, from approximately 5 to 11 years of age, and therefore inform this study. Hence, characteristics of these phases of understanding were incorporated into planning the unit of lessons used in the study.
3.2.4 Philosophic understanding

As youths move further into the teenage years, they enter the phase of philosophic understanding (Egan, 1997, 2005), which requires a more refined sense and use of language. Perhaps the most central tool of this phase is that of theoretic thinking. With this tool individuals begin to understand the cohesiveness of systems of beliefs and rules that may have previously been noticed as separate elements. It should be made clear that this understanding does not necessarily lead to agreement with a collective of rules, but rather leads to an assessment of the collective’s worth in a search for a sense of influence and truth, two further aspects of theoretic thinking.

A further important tool developed during the philosophic phase of understanding is that of a sense of agency on the part of individuals, helping them start to understand their place in the world (Egan, 1997, 2005). Through this sense of agency, the individual gains a better understanding of how we are all products of processes through which societies and cultures develop, and how each person’s development can be distinctive.

3.2.5 Ironic understanding

Through the appreciation that individuals develop in the philosophic understanding phase, a sophisticated sense of reflective thinking evolves, which allows the extremes of theories and ideas to be understood and appreciated. Individuals can now consider multiple views on an issue or idea, both the possible and impossible. This sophistication in thought is characteristic of ironic understanding, the final phase of understanding in the IE theory. Egan (1997, 2005) balances the development of two types of irony, alienating and sophisticated irony. Alienating irony casts an air of doubt and powerlessness by questioning the types of understanding gained earlier, whereas sophisticated irony is able to hold on to qualities from the previous four phases of understanding without denigrating the comprehension that was gained earlier. Thus, a meaningful open mindedness is developed.

3.3 Imaginative lesson planning frameworks

Egan (1997, 2005) has developed imaginative lesson planning frameworks (ILPFs) for the three phases that span a child’s school years, mythic, romantic, and philosophic. These
frameworks embody the principles of IE theory and the cognitive tools of the phases, described in Table 3.1, such as a sense of mystery, story, or metaphor.

The ILPFs are designed to aid an educator’s implementation of the theory by suggesting a structure that embodies the cognitive tools detailed earlier. This then allows the educator to present the content of a lesson in a manner that gives children the maximum opportunity to develop their understanding and to use their imaginations and emotions.

Since the ILPFs were first proposed by Egan (1992), they have undergone a number of transformations, the most recent of which can be viewed on the Imaginative Education Research Group (IERG) web site (www.ierg.net). As these relate more to the implementation of the IE theory, detailed discussion of these frameworks will be included in the methodology chapter.

3.4 Key concepts of imaginative education

Having given a brief overview of the theory, it is germane to begin consideration of two key concepts of the IE theory, imagination and emotions, which Egan (1997, 2005) believes help orient and establish the development of understanding. This allows us to judiciously reflect upon the previously reviewed research on affect in mathematics education and a possible extension to the creative element of a humanized view of mathematics and to consider what the IE theory may have to offer in these areas.

3.4.1 Imagination

To think of imagination as something that exists only in the mind is far too vague and simplistic. Dictionary definitions, such as “the faculty or action of producing ideas, especially mental images of what is not present or has not been experienced” (Hanks, 1986, p 764), seem insufficient also. There is no grounding in or explanation for what constitutes mental images, presence, or experience. We are thus left with an uninviting image that belies imagination’s rich potential to aid thought and achievement. It is therefore necessary to carefully unfold the concept of imagination if it is to be applicable and useful in educational research.

Egan (1992) defines imagination as

- the capacity to think of things as possibly so: it is the intentional act of mind; it is the source of invention, in the construction of all meaning; it is not distinct from rationality but is rather a capacity that greatly enriches rational thinking. (p. 43)
This definition creates the opportunity for openness in the exploration of ideas with the intention of enhancing the understanding of a concept or topic. Here, there is more room for creativity, growth, and the reaching of individual and collective potential. By incorporating this sense of imagination, the educational environment becomes one in which ideas are truly encouraged and celebrated in a supportive, nurturing manner, without limits to what can be achieved.

Imagination has often been regarded as either subjective (Garry, Manning, Loftus, & Sherman, 1996) or a fanciful entity (Van Sledright & Brophy, 1992), something that does not really have any relevance to the objective advancement of knowledge. Consequently, imagination has largely been relegated to childhood and play, where its use can be casually dismissed, without respect, honour, or value. Yet the vast majority of creativity in arts-related fields and advances in societal development, research, and science would not have happened if it were not for an individual and collective use of imagination.

Donald Coxeter, a Canadian mathematician, exemplifies the value of imagination in his field. Regarded as one of the leading geometers in the world, Coxeter has been credited with raising the profile of geometry in many mathematical and scientific areas (Roberts, 2007). To Coxeter, imagination is a necessary and fundamental part of inquiry and mathematical discovery: “As for the role of imagination, I should say that all discovery requires imagination” (personal communication, September 7, 2002). Thus, Coxeter strongly supports the inclusion of imagination in all scientific and mathematical activities.

It is helpful when examining IE’s use of imagination (Egan, 1997, 2005) to consider Eisner’s (1999) position that “imagination is the apotheosis of reason, not a diversion from the real business of thinking” (p. ix). This supports Egan’s (1992) sense of imagination as expanding what reason is about, rather than setting it up in dialectical opposition to reason. This more open, fluid sense of imagination is not restrained, and truly expands the notion of reason and logical thinking.

Children’s learning that involves using the imagination in this more expanded form, as is suggested by the IE theory (Egan, 1997, 2005), can open up the possibility for deeper understanding and knowledge development than learning that has been bounded by constraints, rules, theories, and principles (Egan, 1997; Jagla, 1994). With the use of imagination, it is possible to step beyond what is currently known. The broader conception of knowledge then includes not just what is known, but also what could be created in the present and future, as well
as expanding the understanding of what has taken place in the past. Including the use of imagination in children’s learning activities also offers educators tremendous potential to strengthen connections between the children and the subject content by providing additional points of access and support.

3.4.2 Emotions

Emotions are as central to the IE theory as imagination (Egan, 1997, 2005). The consideration of knowledge as a product of human minds generated from hopes and fears, joys and sorrow, love and hate leads to the acceptance that there is a human purpose to knowledge creation. There have been, and will continue to be, discoveries, inventions, and creations that began with someone’s emotional reaction to knowledge or to a situation. Our emotional response is a way of gauging the effect of that stimulus. Therefore, emotions are involved in both the creation of and the reaction to knowledge and must be considered as having an important effect on learning.

By utilizing emotions in this way, it is quite possible, and easier in Egan’s (1997) view, for students to have a better understanding of what they are studying. When students have this emotional connectivity with knowledge from the past and present, life and meaning are breathed into knowledge. Thus, there is a rich interaction between imagination and emotions. This can be seen in Egan’s (1997) suggestion for learning the geography of the Americas:

We might introduce it in the context of the emotions of the first discoverers and settlers. It is not just that we will see the landscapes and climate, the flora and fauna, through the eyes of those who first came across the Bering Strait land bridge, but we will feel those features through their emotional responses (as we can reasonably infer them). (p. 94)

Egan (2005) suggests that emotions and imagination are essential elements for meaningful learning: “Successful education does require some emotional involvement. . . . The best tool for doing this is the imagination” (p. xii). The blending of imagination and emotions can create a dynamic form of learning that clearly involves the affective domain, which includes our values, beliefs, and, importantly, our emotional responses to situations. Hannula (2002) believes that when active attention is given to the affective domain of learning and when affect is combined with cognitive learning, there is tremendous potential for even greater development of a learner’s understanding than there is with cognitive learning alone.
When cognitive learning is viewed as the development of knowledge and rational thought, there is an immediate sense of constraint and limitation, yet when emotional responses are considered, we can see the relevance of emotions: “Emotions are viewed as fulfilling important motivating and regulating functions” (Malmivuori, 2006, p. 4). Therefore, emotions and imagination becomes fundamentals and not mere niceties for meaningful learning.

Blending emotions with imagination, two fundamental human faculties, provides the opportunity to place mathematical learning in a humanized, sociocultural, and real-world context from which it has become detached for many individuals (Boaler, 2000; Nardi & Steward, 2002a, 2002b, 2003). Within an IE theoretical context, learning can become an educational experience in which there is opportunity for “the having of wonderful ideas . . . the essence of intellectual development” (Duckworth, 2006, p. 1).

3.5 Development of knowledge

While the development of understanding, rather than knowledge acquisition, is the key focus of the IE theory (Egan, 1997, 2005), knowledge is still a central concept of education, and it is therefore important to be clear about how this term is used in this research study. Egan (1997) sees knowledge as far more than information and facts represented in symbolic codes in books and acquired through the use of cognitive processes, “Everything we know is knowable through the lives of its inventors, discoverers, and we can have access to that knowledge through the hopes, fears, or intentions that drove them” (p. 93). Knowledge, therefore, becomes part of and present in the human mind and is a means of constructing understanding in the mind. Egan (1997) asserts that the knowledge gained in this way is more easily understood and remembered, and is significantly more meaningful to a learner.

Eisner (1999) concurs with Egan’s (1997) view of knowledge as an organic entity that evolves and develops to become understanding, rather than remaining as a closed and finite unit, and notes specifically that

- knowledge is not an object, it is a process. It is not a noun, it is a verb. It is not something that one discovers and ships to the four corners of the earth.
- Knowledge is something that leads to a life of change in the context of the human mind. (p. xii)

The decontextualised manner in which knowledge is often presented causes it to lose relevancy to many students’ lives (Fullan, 1991, 1993; Goodlad, 1984, 1994, 1997). However,
the IE theory offers a broad perspective on the development of knowledge and understanding that is in stark contrast to the more usual development of cognitive knowledge found in schools.

3.6 Theoretical considerations of imaginative education

In order to present a sound, reasoned argument for the use of the IE theory in this study, it is necessary to consider a number of perspectives. Therefore, in this section I consider other aspects of the theory, such as where it is situated in regard to other educational theories of learning.

IE is part of a sociocultural view of education. Egan (1997, 2005) draws on Vygotsky’s social constructivist theory in a number of areas. For example, Egan (1997) has built on the Vygotskian notion of language expansion (Vygotsky, 1962, 1978, 2004), having its development form a major part of the somatic, mythic, and romantic phases of development. Each of these phases includes an increasingly sophisticated use of language through particular characteristics or cognitive tools of language, such as metaphor in mythic understanding and a sense of reality in romantic understanding. In these phases of development, orality develops from an embodied sense of understanding the world in the somatic phase, and is then gradually refined through the development of thoughts, thinking processes, and cognitive constructs. This is congruent with Vygotsky’s (1962) belief that “the relation between thought and word is a living process; thought is born through words. A word devoid of thought is a dead thing, and a thought unembodied in words is a shadow” (p. 153).

Egan’s (1997) view of pre-language child development is based in children’s embodied sense of coming to know the world around them. Egan (1997) sees this as “how we first make sense with our distinctive human perceptions, our human brain and mind and heart and whatever else our bodies can deploy in orienting themselves” (p.166). This view of a sociocultural process resonates with the Vygotskian notion of the interpersonal and intrapersonal relationship of children to the world around them, which is explained by Vygotsky (1978) as follows:

Every function in the child’s cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological), and then inside the child (intrapsychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher functions originate as actual relations between human individuals. (p. 57)
Egan (1997, 2005) also agrees with the notion of sociocultural development and Vygotsky’s (1962, 1978, 2004) stance that the process of acquiring cultural tools, which then become cognitive tools of development, stems from a sociocultural base. Indication of cultural development, in Egan’s (1997, 2005) view, is manifested in various language forms, such as story and formation of images, which evolve into cognitive tools from which understanding is formed.

As no one is born to live in isolation, it is necessary for children to gain some sense of people, society, and the elements of life that surround them by acquiring a gradually increasing and more sophisticated set of experiences. Cultural tools such as language and mathematical systems are internalized by individuals in different ways to different degrees, becoming cognitive tools (e.g., a sense of story and a sense of wonder), which help individuals make sense of the world around them (Egan, 1997). Consideration of child development must take into account children in relation to others, to society, and to their culture, with culture creating the context into which an individual is born (Egan, 1997; Vygotsky, 1978).

With IE, the teacher becomes a pedagogical facilitator, part of the sociocultural process, who draws his or her principles of teaching from the larger cultural context. The teacher then moulds these principles into an expansive, outward-looking set of techniques that encompass a range of culturally developed tools of learning, such as language, counting systems, and artistic and musical expression, and that aim to develop understanding within students. By encouraging students to use these tools, the teacher enables them to move from being able to perform an activity on their own to being able perform a more advanced activity with the intentional, supportive aid of the teacher. For example, a teacher could encourage a student to research a leading mathematician such as Pythagoras and create contemporary and artistic examples of Pythagoras’ theories. Here Egan (1997) clearly draws on Vygotsky’s (1978) notion of the “zone of proximal development” (p. 86):

It is the distance between the actual development level as determined by independent problem solving, and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers. (p. 86)

However, Egan (1997) maintains his own general principle of the outwardly expanding nature of the development of understanding, in which cultural tools are drawn into the learning dynamic and ripple in many directions from a central starting point. The image of gradually
increasing concentric circles from a central node helps us understand the interrelatedness of these circles. This is an alternative to Vygotsky’s (1978) more linear zone where “the developmental process lags behind the learning process” (p. 90), which suggests an image of two points on some form of developmental continuum.

Egan (1997, 2005) suggests that with IE, all phases of understanding are available for use by an educator on behalf of students, and thus there is greater coalescence between the different levels. When teaching from an IE perspective, teachers may well focus their instruction in a particular phase of understanding, such as mythic understanding. Nonetheless it is possible to incorporate aspects of subsequent phases, such as romantic or philosophic. For example, young children often have a fascination with large numbers, and yet primary curriculum in many jurisdictions covers only the number facts to 10 (British Columbia Ministry of Education, 2007). This is likely based on constructivist principles of wanting children to work from the realm of what they already know to construct new knowledge. However, the IE theory allows educators to carefully include larger numbers to create a sense of wonder in young children. Thus, Egan (1997, 2005) contends, it is possible to further develop the outwardly expanding zone of understanding through a deliberate act of pedagogical instruction that sows a seed of advanced understanding towards which a student’s developing understanding can expand.

A casual comparison might also suggest parallels between Piaget’s (1959, 1977) stage theory of child development and Egan’s (1997, 2005) theory of IE. Both include phases of development that correspond to periods of childhood. However, closer examination nullifies deeper similarities. In Piaget’s (1959, 1977) stage theory, each of the four successive stages of cognitive development—sensorimotor, preoperational thought, concrete operations and formal operations—is completed before the child moves on to the next stage: “The next stage is achieved when the child succeeds in coordinating these classifications with each other both hierarchically and multiplicatively” (Gruber & Voneche, 1977, p. 360).

In Egan’s (1997, 2005) theory, the five phases of understanding are somewhat distinct and are progressive in nature, with a heightened degree of sophistication in each phase. However, elements of each phase can carry forth into the next phase, rather than being left aside or discarded. For example, being able to understand things through our bodily senses does not cease at the end of the somatic phase of understanding; it lessens in importance, but remains an influence even as other cognitive tools are brought into play. Egan (1997, 2005) feels that the
careful nurturing of later kinds of understanding can begin much earlier than is generally considered in Piaget’s (1959, 1977) stage theory. He believes that “we would be unwise to eliminate all interactions that involve later kinds of understanding until students are assumed to be ready for them” (Egan, 1997, p. 179). For example, acknowledging and carefully working with young children’s fascination with large numbers can be done without developing detailed formal conceptual understanding.

Another important difference between Egan’s (1997, 2005) theory and that of Piaget (1959, 1977) lies with Piaget’s focus on the epistemological origins of knowledge. Piaget focuses inward on the individual, with little consideration of the context in which knowledge development takes place (Donaldson, 1978). Egan (1997), however, focuses more globally on the process of education and knowledge development—the ‘how’ of knowledge development; these are two very different foci. This makes comparison between epistemology and educational development problematic. There are certainly commonalities in that both theories involve the use of knowledge, but as has been stated above, there are fundamental differences. A strength of IE and Egan’s (1997) perspective on Vygotsky’s (1962, 1978) notion of the sociocultural placement of knowledge is that the context of knowledge development is taken into account, as is done in this study. This occurs whether knowledge acquisition is in the past, present, or future, thereby providing further grounding or points of reference for the development of understanding.

3.7 Philosophical considerations of imaginative education

Just as it is important to consider the theoretical context in which IE theory sits, it is also important to consider its philosophical context in order to provide a reasoned argument for its use as the theoretical framework for a study of student’s engagement in elementary mathematics. This is especially important since the development of IE came from Egan’s dissatisfaction with existing educational philosophies and theories that were a legacy of the progressivist era.

Egan (1997) contends that there are three notable perspectives on education, which are mutually and seriously incompatible:

1. that the central role of education and schools is to socialize students to learn about their role in society and that students have the skills required to become a member of the said society and economy;
2. that education and schools are for inculcating particular forms of knowledge;
3. that the purpose of education and schools is to promote and develop the individual potential of each student on an experiential basis. (p. 3)

In particular, he considers that the aim of socialising students into society is at odds with the aim of developing the individual potential of students. Attempts to address both these aims in a balanced approach inevitably lead to a watering down of each. As well, emphasis on any single aim leads to serious weakening of the others. In addition, the current structure of much of conventional education practice appears to focus on the development of memory and the mastery and regurgitation of existing forms of knowledge for an increasingly faster paced economic world, rather than fostering the development of new ideas and understanding for the future. Egan (1997) has argued that these irreconcilable ideas give rise to “our modern social puzzle, the educational ineffectiveness of our schools” (p. 2).

What then, are the aspects of progressivism with which Egan does not agree? His critique is based not so much on its purported goals, but rather on some of the principles on which progressivism is built. Egan (1992, 1997, 1999, 2002) has consistently pointed towards what he considers to be the misguided errors of progressivism. Looking at the progressivist recapitulation principles of Herbert Spencer—which Egan (2002) feels heavily influenced the work and writing of John Dewey, often considered the father of progressivism—it is possible to begin appreciating where Egan’s (2002) concern lies:

The worse error I want to expose, in Spencer’s writing and today, is connected with the common belief that children’s minds have some preferred natural kind of learning and that if we can isolate and understand it we can then make the educational process more efficient and effective. (p. 39)

Egan (2002) believes that it is perilous to assume that the ability to learn lies dormant within a child’s mind, to be awakened by environmental stimuli, while at the same time requiring the mind to act as a magnet, drawing in caricatures of the outside world. Likening the development of the mind to the development of the body, as Egan (2002) feels was done within early progressivism, gives rise to circumstances that do not allow for the distinctive forms of children’s knowledge and understanding to be recognized. A young infant or child is not a being without any understanding of how his or her world works, but rather someone whose understanding is coming to be increasingly understood through studies of child development.
There is a clear sense from Egan (2002) that the early progressivist principles took the simile of the mind being like a body too literally.

A second error that Egan (1992, 1997, 1999, 2002) feels has arisen from progressivist principles is the generally accepted claim that “children will learn more effectively if what is taught is made relevant to their early experience” (Egan, 2002, p. 37). This comes, in Egan’s (1999) view, from Dewey’s (1916) notion of “material of ordinary acquaintance” (p. 258), which is seen in curricula commonly referred to as having an “expand[ing] horizons” focus (Dewey, 2010, p. 59). This focus is particularly noticeable in elementary social studies curriculum, in which young students first learn about themselves and their families and gradually progress to learning about local communities and then region and country.

Part of the progressivist rationale for the expanding horizons curriculum is a belief that it is easier to build on what a child already knows. What Egan (2002) brings forward for consideration is the awareness that young children already have understanding in many areas outside of the progressivist principle of the young child’s concrete experience. The social studies curriculum of the last paragraph focuses on the concrete, yet children are capable of understanding abstract factors such as freedom and captivity of various cultural groups, stemming from their interaction with stories, legends, and characters from their early years, such as the story of Hansel and Gretel.

This developed sense of understanding, in Egan’s (1997, 2002, 2005) view, includes abstract binary concepts such as fear and safety and love and hate, which children come to understand through embodying and internalising the concept of abstract binary opposites. In addition, Egan (1997, 2005) points out that young children are quite able to mediate between extremes of opposites along a continuum, such as small and large and hunger and a sense of fullness. Through this mediation children can develop a fascination for extremes, as does Moira in Robert Munsch’s (1992) popular story Moira’s Birthday. Moira organizes her own birthday party; rather than inviting just the six children recommended by her parents, she goes to the extreme of inviting her whole school, and then orders 200 pizzas.

While Egan (2002) very clearly believes that progressivist ideas “have been wrong from the start and haven’t become less wrong for a century and a half of assuming that they’re right” (p. 81), he does not dismiss progressivism completely. Rather, he appears to feel that some of the progressivist beliefs and principles should be considered in nontraditional ways. Indeed,
Egan (2002) has acknowledged and credited achievements of the progressivist era, and he clearly states that “Rousseau and Dewey have enriched our conception of education in important ways. We will not make educational progress by trying to cut away their contribution” (Egan, 1997, p. 31). Egan (n.d.) also notes, “One cannot deny the liberating effect of Dewey’s influence on modern conceptions of children’s mental life, and that the influence of progressivism in general has led to more careful attention to children’s intellectual activity” (p. 8). Therefore, despite Egan’s (2002) views about early progressivism, there are indeed areas of general agreement between Egan’s work and contemporary progressivism, such as a movement towards a more egalitarian and child-centred orientation for education, where each principle has the goal of critically examining issues within the field of education.

3.8 Critique of the theory of imaginative education

The theoretical and philosophical considerations discussed above show that there is a justifiable, meaningful place for the IE theory in education. However, in order to establish the validity of IE theory in this study, we must also examine how it has been received and critiqued more broadly. In this section I discuss both my own appraisal of IE theory and its critique by others.

Having worked with aspects of the theory for a number of years, I have been able to explore different features of the theory. This has included reflecting upon my use of the cognitive tools as I expanded my implementation of the theory from a single lesson to a small group of lessons, and becoming more aware of the role of the affective response in lessons with a purpose beyond solely fostering students’ use of imagination. What has not been possible to determine through informal use of the theory has become the research question at the heart of this study: What does the use of the theory of IE and ILPFs mean to children and their engagement in elementary mathematics? I have felt it important to investigate why there was such a contrast between student work when I used aspects of the theory and when I used more traditional methods of teaching.

While implementation of aspects of the IE theory in my own pedagogical practice have produced some successes, discussed in Chapter 1, personal experience alone is an insufficient and unreasoned basis for its adoption as a theoretical framework for the current study. Therefore, I have sought a broader appraisal from educational researchers and scholars. Critique
of the theory has largely revolved around three areas: firstly, whether Egan has given sufficient credit to the influential progressivist legacy; secondly, whether he minimizes the gains made in understanding children through the development of the progressivist movement; and lastly, whether or not the theory is implementable in the current context of education.

The first area of concern is Egan’s acknowledgement of the progressivist legacy. While Pinar, Reynolds, Slattery, and Taubman (1995) acknowledge Egan’s “elaborate development and sophisticated curriculum design” (p. 690), he is criticized for giving what Pinar (personal communication, November 3, 2005) considers insufficient credit to the legacy of progressivism. Pinar (2005) points to a shift away from Egan’s use of aspects of the progressivist movement in the early days of the development of the theory when he discusses “those who reject progressivism—Kieran Egan is, perhaps, the most startling example given his earlier reliance on it” (p. 9). However, I contend that this reorientation of focus is more a maturation of the theory, as explained in detail in Egan’s seminal work, *The Educated Mind: How Cognitive Tools Shape Our Understanding* (1997) and in Egan (2002, 2005, 2008). As discussed above, rather than rejecting progressivism, Egan (2002) is clear both about the parts of the progressivist legacy that are not supported—namely, some of the scientific principles on which progressivism is based—and, significantly, about the parts that he does support—learning that is centred around the learner.

Pinar (2001) cautions against overindulgent use of imagination. Acknowledging the important role that imagination can play in educational opportunities and its use by scholars such as Egan (1997) and Greene (2001), Pinar (20001) provides a sombre reminder about the macabre or negative uses to which imagination has been applied in acts of violence and racism, such as lynching and prison rape. Pinar (2001) reminds readers that no idea should be applied in education without rigorous and active ethical consideration and thought. This is not to say that imagination, and even a theory of IE that appears to expand educational opportunities, cannot be considered and utilized in educational activities, as many scholars such as Eisner (1985, 1988, 1999) and Bruner (1986, 1996) have shown, but rather that a moral and ethical standpoint should be maintained at all times. This is supported in an IE theoretical approach through its grounding in sociocultural theory with the Vygotskian belief in collaboration (Vygotsky, 1978).

Egan’s stance on the progressivist movement has also been regarded as somewhat dismissive by Buckley (1994a, 1994b), who noted “Egan’s rejection of Dewey and a curriculum
supposedly based on relevance to everyday life” (Buckley, 1994a, p. 34). However, this is refuted by Egan’s (1994) acknowledgement of the significant contribution that Dewey has made to education and by Egan’s (1997, 2005) stance on curriculum planning, discussed earlier, which offsets a Deweyan expanding horizons curriculum. I believe that Buckley (1994a, 1994b) is questioning whether the terms that Egan uses to describe periods of child development (mythic, romantic philosophic, etc.) are really appropriate to provide counterbalance to the progressivist view of learning as promoted by educators such as Dewey and Herbert Spencer.

There is also further difference of opinion between Buckley (1994a, 1994b) and Egan (1994) with regard to cultural recapitulation. Buckley (1994a, 1994b) interprets Egan’s stance on recapitulation as conservative, with a narrow and specific view of culture in which recapitulation has a constricted perspective that shuffles around existing educational ideals rather than standing for more general reformation. Buckley (1994a) states, “Egan’s system is in this sense too, more of a defence of the status quo than a plea for reform” (p. 40), which is not at all in line with the general recapitulation which Egan (1994, 1997) articulates. Rather, Egan (1994) believes two phases of recapitulation of educational ideas are needed. The first is in the set of implications for the kinds of understanding that evolve from learning oral language, and the second comes with the subsequent set of implications for curriculum design and learning, thus providing a reformation of educational ideas and the objectives of education.

I believe that Egan’s (2002) re-evaluation of progressivism and of Piaget’s work tries to move forward the refinement of pedagogical theory by proposing what he feels to be a more relevant and meaningful path for the development of education. Egan’s (2002) position is strengthened by O’Loughlin’s (1992) stance about the need to rethink Piagetian constructivism. O’Loughlin (1992), like Egan, acknowledges Piaget’s contribution to education but feels that Piaget’s constructivism is flawed because “it ignores the subjectivity of the learner and the socially and historically situated nature of knowing” (p. 791), and because it devalues “autobiographical experiences of the individuals” (p. 794). O’Loughlin (1992) puts forth a strong notion of the child’s being engaged in sociological and “dialectical interaction” (p. 810), whereby there is genuine empowerment and ownership of ideas and thoughts through which learning and transformation can occur. O’Loughlin’s (1992) sociocultural approach supports Egan’s (1997, 2005, 2008) view in regard to the importance of considering the social and historical nature of learning.
The second main area in which IE has been criticized is the claim that Egan minimizes the gains made in understanding child development through the progressivist movement and undervalues their contributions (N. Noddings, personal communication, April 8, 2006). Because the gains made through the work of scholars such as Dewey (1916, 2010) and Piaget (1959, 1977) have been generally accepted, Noddings’ prudence is fair. Part of Noddings’ caution, however, may be based on her considering different entities: Piaget’s psychological and genetic epistemological framework discussed earlier and Egan’s theory of educational development. The former is a very scientific framework coming from an educational theoretical framework that involves “attempts to isolate certain phenomena and generate theories that explain them” (Egan, 1979, p. 167), whereas the latter involves “educational development [that] brings knowledge to the forefront because it is the fuel of the process. . . . Without knowledge there is no education” (Egan, 1979, p. 156). These are two very different foci.

However, Egan (1979) has acknowledged the contributions made by the progressivist movement. Through his re-evaluation of Piaget’s (1959, 1977) theories, Egan (1997, 2005) has acknowledged the influence of Piaget, his theories of child development, and contemporary pedagogical discussions: “Piaget has contributed enormously to our understanding of children’s intellectual development” (p. 4).

Noddings’ (1992) credo towards an “ethic of care” (p. 21) and belief that “caring is the very bedrock of all successful education” (p. 27) further account for her apprehension about Egan’s view of the legacy of the progressivist era. Does Egan’s theory contain a sufficient ethic of care to truly nurture learning, particularly in the early stages of child development? The first of Egan’s (1997, 2005) phases, somatic understanding, relates to an embodied sense of development in understanding from the time of birth, but is the least developed of the five phases, and the one perhaps least related to school learning. When pointing out that a desire to be cared for is “almost certainly a universal human characteristic” Noddings (1992, p. 17) does provide a point of accord where her ethic of care meets Egan’s (1997, 2005) sociocultural theory of IE, since both provide an almost moral outlook from which educational endeavours can meaningfully move learning forward. This is supported, as stated earlier, by the Vygotskian (1962, 1978, 2004) notion of collaboration.

The third area of critique for the IE theory is pedagogical implementation. In particular, Frawley (1998) expresses concern about the practicability of the ideas presented in the theory
and implies that the suggested modes of planning are labour intensive: “I would hate to be the teacher who had to do all this... I wonder whether teachers have the time... [E]ven very sympathetic teachers will find it tough to do this kind of work” (p. 48). Frawley’s views are understandable, given that the majority of teachers have been trained and educated in predominantly Tylerian programmes of educational planning, different from the methods suggested by Egan (1979, 1992, 1997, 2005), and may need professional development opportunities to reorient their planning of lessons. In addition, Fullan’s (1993) work on many educators’ resistance to change and on the slow pace of educational change, mentioned in Chapter 1, adds to our understanding of Frawley’s concern.

Egan (1997, 2005) considers the perspective of the teacher as well as of the learner by providing suggestions for implementation of the IE theory with ILPFs. These frameworks were first suggested in *The Educated Mind* (Egan, 1997) and further refined in Egan (2005). In addition, the IERG assists teachers in learning more about the theory by providing contemporary resources, curriculum unit plans, and other support materials on its web site.

Indeed, Egan (2005) easily counteracts Frawley’s (1998) concerns about practicality with comments such as this: “Teachers who do find that emotional engagement typically find themselves energized rather than drained by the end of the day. And their classes have more children who are themselves imaginatively engaged, and that in turn energizes the teachers further” (p. 215). This quotation aptly captures my experience when implementing the IE theory in elementary mathematics, and is further supported by many comments in IERG newsletters available on the IERG site.

Notwithstanding the three areas of critique presented above, what Egan (2002) proposes with IE theory is that “the education of children today is a matter of ensuring that they make their minds most abundant by acquiring the fullest array of the cultural tools that can, through learning, be made into cognitive tools” (p. 184). It is the heritage of the cultural tools and the context in which these tools are used that will help us understand more fully the nature of a child, rather than empirical, fixed, scientific analyses of children. Egan 2002) states,

Whatever is the substratum of human nature is less accessible and less useful to the educator than understanding the cultural-cognitive tools that shape and mediate our learning, development, and everything else to do with the conscious world of educational activity. (p. 185)
Despite the somewhat gloomy picture of student engagement in contemporary education painted by Marks (2000), Fullan (1991, 1993), and Goodlad (1984, 1994, 1997) discussed in Chapter 2, judicious review of the critique of IE theory provides a sense of optimism that a light may be shed on the issues at the heart of this study, student engagement with mathematics learning. If, as suggested by Egan (2002, 2005), aspects of the larger sociocultural context can be included in the pedagogy of the study through use of IE, then a window into the learning mathematics with IE is possible. This may brighten the negativity about learning mathematics felt by many students and portrayed by Boaler (2000) and Nardi and Steward (2002a, 2002b, 2003).

3.9 Implementation and use of imaginative education

In recent years, further and more detailed consideration of the IE theory has moved away from the early conceptual and theoretical critique reviewed above towards research application and pedagogical implementation. Scholars such as Gardner have also provided review and commentary. In a review of one of Egan’s later works, The Future of Education: Reimagining Our Schools from the Ground Up, Gardner (as cited by Egan, 2008) states, “Egan is one of the most original ‘big picture’ thinkers in education. . . . Egan critiques both traditional and progressive education and puts forth his own provocative ideas on how change might be implemented” (back cover). Notwithstanding this review, an in-depth examination of the use of IE theory is needed to provide a more complete assessment of the theory and the issues that arise from its use.

A large amount of open critique and discussion of the IE theory has taken place at the international conference held by the IERG in Vancouver annually since 2003. This has allowed an important and necessary shift in focus away from conceptual analysis. In the early years of the conference, the vast majority of presentations were of a conceptual nature. However, some considered pedagogical implementation, as with a case study of 11 teachers by McKenzie and Fettes (2002), which found that a period of experience in order to become familiar with working with the theory aided its implementation in classrooms.

Other researchers have used elements of the IE theory in their investigations, such as Gadanidis and Hoogland (2003) and Nicol and Crespo (2005). Gadanidis and Hoogland applied aesthetic and storied aspects of IE theory to data collected as part of an investigation into
technology and the mathematical and pedagogical thinking of elementary mathematics teachers. Aesthetic elements of stories were found to form part of a teacher’s engagement with and views of mathematics for themselves, which they in turn could pass on to their students. However, aesthetic consideration of IE was not part of the research design and was applied after data collection had taken place.

Use of elements of the IE theory (Egan 1997, 2005) in data analysis is also the case with the research of Nicol and Crespo (2005). Here one imaginatively styled activity, “The Mayan Dresden Codex,” was carried out with a group of pre-service elementary teachers, and another, “Exploring Life in Flatland,” based on Edwin Abbott’s Flatland, took place with a Grade 6 and 7 class. Tasks assigned were deliberately outside of everyday experience in order to investigate how learners reacted to imaginatively styled tasks. “The Mayan Dresden Codex” activity challenged the pre-service teachers’ conceptions of number systems and provided them with emotionally and intellectually engaging tasks. Students in the Grade 6 and 7 class were very active in their investigations of how to recognize shapes in three dimensions. Nicol and Crespo (2005) conclude that imaginatively styled tasks created a sense of engagement in both the pre-service teachers and the students that they felt to be lacking from many school mathematics activities. Interestingly, Nicol and Crespo (2005) point towards the need to investigate the use of theoretical tools that examine characteristics of students’ engagement and, in particular, emotional engagement and intellectual elements, the former being very much a part of the IE theory.

The amount of research that uses the IE theory more intentionally and systematically continues to grow and includes relatively recent doctoral research projects. Takaya (2004) takes a historical and philosophical perspective, proposing a new standpoint for pedagogical implementation of the connections between imagination and education. Chodakowski (2009) suggests a model of pre-service teacher education which considers three areas: an educators’ imaginative understanding of the subject matter, pedagogical contexts, and educational goals. Judson’s (2008) research suggests that school-based ecological education programmes and curricula revisions emphasizing use of body, mind, and imagination can help develop students’ heightened awareness of the natural world. McKellar’s (2005) dissertation provides a systematic study with elementary students, as does the current study. McKellar (2005) finds that use of the theory in teaching history to Grade 6 students positively enhanced their learning of the subject,
especially when the students were specifically taught to identify and use some of the IE cognitive tools. In the current study, use of cognitive tools was not explicitly taught so that students would be free to demonstrate their understanding with whatever tools of understanding they might have and choose to use.

One further avenue of recent research focusing on the IE theory is a longitudinal five-year study by Fettes (2013) carried out in Haida Gwaii, British Columbia, between 2004 and 2008, entitled Learning for Understanding through Culturally Inclusive Imaginative Development, or LUCID. In this study, Fettes partnered with two school districts with high populations of aboriginal students to investigate ways of incorporating aspects of personal and cultural histories into education by utilizing students’ imaginations. Preliminary findings (Pearson, 2009) identify factors such as access to cultural expertise and resources within an aboriginal community as well as access to a source of expertise in IE as important factors that enhance the education delivered to students. It was also found that teachers had less difficulty motivating students to complete their work when an IE approach was used, and that within a one-year segment of the project a slight improvement in academic performance was noted compared with previous years when an IE approach was not used.

From the early, more conceptual theoretical and philosophical analysis and discussion of IE to the international IERG conference, research studies, and doctoral research, it is evident that a tide of consideration of the use and implementation of the IE theory is gradually growing. There is still a need for research that specifically examines the potential contributions of IE to the learning of young students.

3.10 Mind the gaps—Contributions to education

The primary contribution of IE to education and student learning comes through providing a strong educational theory that brings together the use of a learner’s imagination and affective response and is implementable in a pedagogical context.

The theory also embodies the Vygotskian notion that the creative process has two essential parts, without which expectations of students cannot be fully realized:
On the one hand, we need to cultivate creative imagination; on the other hand, a special culture is needed for the process of embodying the images created by imagination. . . . Only when both aspects are adequately developed can children’s creativity develop properly and provide the child with what we have a right to expect of him. (Vygotsky, 2004, p. 84)

Thus, in reconceptualising education as developing five kinds of understanding through the use of cognitive tools that invoke students’ emotions and imaginations, the IE theory holds the potential to address the ineffectiveness of schools and the failure to maximize students’ potential and development that is reported in the work on student disengagement by Fullan (1993) and Goodlad (1984). It also provides the opportunity to consider a model of implementation for a more humanized or creative view of learning mathematics, as discussed in Chapter 2.

IE allows children to demonstrate their understanding and knowledge, as well as to use cultural tools of reference that they already possess. By moving through the five multifaceted phases of development in this theoretical approach, learners can use their imaginations and emotions to develop increasingly sophisticated understanding, which enables them to contribute to their culture and society at large, and thus become participating members of a sociocultural approach to learning.

This study, which uses the IE theory from the initial planning of the research through its implementation, examines the meaning that an IE theoretical framework has for children and for their engagement in elementary mathematics. By considering the students’ comments about their engagement, valuable insights are gained about how IE theory and the ILPFs affect their engagement in mathematics. Greene and Hill (2005) provide a persuasive argument for the rightful inclusion of the student perspective: “It can be argued that without some kind of access to the content of a person’s experience, we have a very incomplete account, from a scientific perspective, of what it is that causes any person, adult or child, to act as they do” (p. 2). This points to the need, discussed in Chapter 2, to include this important dimension, which has been missing in previous work, in this study. Will we really know how effective the theory is if the very ones who are asked to learn in this way are not asked what they think?
3.11 Conclusion

By looking in depth at the theory of IE, building on the general context laid out in the preceding literature review, this chapter has provided a comprehensive review and critique of the theory. It showed how Egan (1997, 2005) draws on and refines Vygotskyian (1962, 1978) notions of language development through five phases of development of understanding with the use of cognitive tools. Two key aspects of the theory that help to ground it in a sociocultural perspective, imagination and emotion, were reviewed. Egan’s (1997, 2005) views on the development of knowledge as a way of constructing understanding that, he asserts, is more meaningful to a learner were clarified.

The position of IE in relation to other theories of education, such as Piaget’s (1959, 1977) stage theory, was given, highlighting how individuals draw on cultural tools to develop their own set of personally meaningful cognitive tools. Similarly, philosophical consideration was given to the development of the IE theory and its place within the discipline of education and, specifically, to IE’s relationship with progressivism.

A personal critique and a broader appraisal from the field of education allowed IE and its use to be critically assessed, from the early conceptual and theoretical critique to its implementation and use in more recent research. It was then possible to consider the contribution that IE can make to closing gaps in existing knowledge and understanding about the important issue of meaningfully engaging students in learning, and more specifically to engaging students with elementary mathematics. Having provided a comprehensive review of IE from a number of dimensions, I now turn to the methodology employed in this study to investigate the use of IE.
CHAPTER 4: METHODOLOGY

The research question addressed in this study is this: What does the use of the theory of imaginative education and imaginative lesson planning frameworks mean to children and for their engagement in elementary mathematics? A sociocultural perspective (Vygotsky, 1962, 1978, 2004) and Egan’s (1997, 2005) theory of IE frame the research. Within this framework, an instrumental case was constructed and qualitative case study methods selected to characterize the phenomenon of student engagement. This chapter begins with a discussion of the research design, which is followed by a description of the research context and the unit plan. Data collection and analysis procedures are described in detail.

4.1 Research design: Case study

This study reports on the work of five Grade 4 and Grade 5 students during a unit on shape and space forming part of the regular elementary mathematics curriculum; thus, the case, in this study is bounded by the number of students involved and a particular unit of curriculum. However, as the problem or issue being examined is that of student engagement in elementary mathematics, not the boundaries of the case itself, the study falls within an instrumental case study design (Creswell, 1998) where the relationships between elements of the case become important (Stake, 1995). The goal of examining the problem in this manner was to gain more understanding about what engages students in elementary mathematics, and to give readers a strong sense of what Van Wynsberghe and Khan (2007) describe as “being there” (p.83) by providing rich data arising from the case.

The study sought to address the research question from the perspective of the students as expressed in their semi-structured individual and group interviews, mathematics journal entries, and activity pages. As discussed in Chapter 2, this approach is supported by a number of researchers (Fielding, 2001a, 2001b, 2004a, 2004b; Rudduck et al., 1996); in addition, Tellis (1997) states that case studies are “designed to bring out the details from the viewpoint of the participants by using multiple sources of data” (p. 1). That said, an important feature, which supports and informs this instrumental case study is that I, as the teacher/researcher, am the facilitator of the study (Mills, 2000; Wood, Cobb, & Yackel, 1991), and act as a participant observer whose knowledge supports the chosen focus.
4.2 The role of teacher/researcher

Selecting which role to take in qualitative research endeavours requires careful, critical thought on the nature of the fundamental research question, on the balance in the power relationship between researcher and subjects, and on the context of the research itself (Ravitch & Wirth, 2007). After careful reflection, I chose to take the dual role of teacher/researcher for this inquiry.

The role of teacher/researcher, originating with action research (Altrichter, Posch, & Somekh, 1993; Masters, 1995) provides a privileged vista, one not regularly available to classroom teachers, to critically examine the work of the students from the position of a participant observer. Taking this role allowed me to use my multifaceted teacher-based knowledge of the classroom context and the students involved in the study. This included a general rapport with all the students in the class and a general awareness of their learning styles, abilities, and interests, both within school and, to some degree, outside of school. A further advantage of this role is that it allowed me to maintain the students’ regular learning environment to a high degree, thus providing an authentic research environment. Additionally, my knowledge of the research context made it possible to implement and critically examine IE in relation to practical details of the research study, such as the format, planning, and scheduling of the lessons. As the teacher, I used my professional training, experience, and ongoing professional development to assess and support students’ learning.

In the teacher role, however, I would not normally enquire into the deeper pedagogical, philosophical, and theoretical background behind students’ achievement that I explored as the researcher. It could be argued that the privileges afforded to the teaching role in such circumstances might challenge my research role. I did in fact have to renegotiate some of the values and expectations I had as a teacher when trying to balance a rigorous research study with teaching practice in order to carry out the study (Wong, 1995). For example, my primary role as a teacher was to facilitate students’ learning, whereas my primary role as a researcher was to understand the larger picture of students and their work in relation to the enquiry. I acknowledge, therefore, that an undeniable ebb and flow of consideration for the learner and learning between the roles of teacher and researcher exists, thus making a complete separation of roles challenging (Ravitch & Wirth, 2007).
As the researcher, I had the privilege of being a participant observer in the investigation, having an insider’s view, which required sensitivity towards the research subjects and their context of learning. I had an opportunity to reflect upon reasons and explanations for student behaviour and learning that is not often afforded teachers involved in the immediacy of a classroom context.

During the study I did experience a challenge to my researcher role, however, when I gained more insight during the post-teaching interviews than I would normally have as the teacher. This was in the later stage of the study and was related to what some of the students had intended to do when originally completing their work samples. I was left with ethical questions as to what I would have done if I had had this knowledge in my role as teacher at the time of marking the students’ work. For example, when explaining her quilt during the interview, Courtney had quiet moments where she seemed in a somewhat melancholy mood, which would have been regarded as being off-task during regular class time. However, after a few moments she explained to me the full scope of her design. Would this additional insight and knowledge have caused me to regard her work differently? I was convinced that I would not have had the time or opportunity to allow her to reflect on her work as I did in the interview context. I reconciled this dilemma by reminding myself that in separating the roles of teacher and researcher, I could use only the information I had at the time of the initial assessment, and that the role of researcher was an additional privileged role that I had for the purposes of the study.

Nonetheless, I would argue that the unique positioning and insights gained when taking this dual role offset the challenges presented. Keeping the primary, immediate goals of each enterprise in mind (Wong, 1995) allowed me to separate the roles more fully. For example, in the initial planning of the research study, I concentrated on the researcher role. During the teaching and classroom assessment of the students’ work from the unit, I focused on my teaching role. Once the unit was finished and all marking and assessment completed, I resumed the role of the researcher for the analysis phase. However, there was also a place where I could blend the two roles through recording observations and events in a research diary (Silverman, 2005). Here I noted events that affected the students’ typical engagement with lessons, such as school photographs or scheduling difficulties. Thus, the researcher’s role could be informed by that of the teacher, with the knowledge and observations of what had taken place in the class noted in
4.3 Research context

In order to maintain confidentiality, all names used in reporting this research are pseudonyms. The research setting was Hazelwood Elementary School, a suburban school located in a low-socioeconomic neighbourhood of a Lower Mainland school district in British Columbia, Canada. In the 2008/9 school year, this small school had an enrolment of 210 students. The diverse population included students who received support in English as a second language, gifted, counselling, resource room, aboriginal support and learning assistance programmes. While approximately 60% of the student population received support in one of these areas, the school was seen as a typical elementary school, one that could be found in any school district in British Columbia, Canada. The specific research site was an intermediate class with 13 students in Grade 4 (seven girls and six boys), and 15 students in Grade 5 (ten girls and five boys), for a total of 28 students. Eight students in the class were receiving support in one of the areas previously mentioned, and in this regard the class was considered a typical intermediate class.

The classroom context was a team-teaching situation in which I usually taught 40% of the time, two days per week, and my teaching partner taught the rest of the time. The mathematics programme we generally used was the relatively contemporary Quest 2000 programme from Pearson Education (Kelly, Wortzman, & Harcourt, 1998). However, I normally used the text series only for practice purposes, and supplemented it with a variety of resources which addressed the required Ministry of Education learning outcomes.

During the period of the research study, which took place in the third and final term of the school year, I taught all the mathematics lessons for the class, attending school in the mornings of the days on which I did not normally teach. I also taught the social studies curriculum, my teaching partner taught French and science, and we both taught the other areas of the regular curriculum. We had taught together in a team-teaching arrangement for two years prior to the research study and had established the learning environment of our classroom on the four guiding principles of respect, compassion, honesty, and fairness, in addition to the regular
school and district guidelines. This created a respectful environment for the students in which active learning and regard for all learning styles and situations were encouraged at all times.

In readiness for the study, consent letters describing the study were distributed to all 28 students in the class. The following criteria (Yin, 2003) for participation in the research were included in the consent letter:

1. students in the classroom during regularly scheduled lessons
2. students willing to take part in a one-to-one semi-structured interview
3. students willing to take part in a group discussion with the researcher and other participating students
4. students who had previously demonstrated the capacity to communicate about their work in mathematics in written, visual, or spoken format.

Seventeen responses were received; nine from Grade 4 students (three males and six females) and eight from Grade 5 students (three males and five females).

A representative sample (Patton, 2002) of five willing Grade 4 and 5 students was selected from the respondents for this research so that a range of grades, genders, abilities, and interests would be represented in the research group. The primary consideration in this selection was the students’ ability to communicate about their developing understanding of what they were learning. In making the selection of participants, I tried to embody a cross section of intermediate students that could be found in many elementary schools, and allow for different strengths to be utilised by the students in their communications, whether these be in a visual, textual or oral format.

Selecting students from each grade level was important, as this was the reality of the research context. Combined-grade glasses are common in elementary schools in British Columbia, so including this factor in the research provided external validity and application of the funds to other pedagogical contexts. Respondents for whom participating in the interviews could possibly cause confusion or undue stress because of a lower level of English language skills, such as international students or new immigrants to Canada, were not selected to take part in the study.

The participants chosen for the study were one Grade 4 girl, one Grade 4 boy, two Grade 5 boys and one Grade 5 girl. These students were selected prior to the start of the unit so that they could be tracked by the researcher and copies of work samples could made while the lessons were being carried out. In addition, prior selection was important to allow the critical friend to
focus her observations, which were to take place during the teaching of the unit. However, students and families were not informed of the selection until after all marks and grades for reporting had been submitted, because of the need for all students to be encouraged with their school learning in a usual manner. It was also important to normalize the learning environment so that all the students, including the research participants, would produce the sort of student work that they would ordinarily do, which I could later use for data analysis.

4.4 Unit plan and overview of imaginative lesson planning frameworks

The shape and space unit focused on two areas of the British Columbia Ministry of Education (2007) Grade 4 and 5 elementary mathematics curriculum: (a) transformations and (b) three-dimensional objects and two-dimensional shapes (see Appendix 1). These areas encourage students to develop their spatial sense in relation to the world around them. As the unit was to be taught from an IE perspective, I selected binary opposites from the mythic understanding phase, specifically the opposites of vision and blindness, as a focus for the unit to complement the mathematics curriculum, with a guiding question of “How does a blind person learn about space and shapes?”

Because the students were at different stages in their overall literacy development and ability to express understanding of the curriculum concepts, I used elements from both the mythic and romantic phases of IE to develop a detailed unit plan (Table 4.1). Although the basic outline of lessons (Figure 4.1) was focused around an ILPF for the mythic phase using the binary opposites tool, a second ILPF from the romantic phase was completed after the unit plan, but before teaching began, to ensure maintenance of the theoretical framework in the later stages of the unit (see Appendix 2). These measures allowed the theoretical framework to be linked with the classroom pedagogy and curriculum, and also with the expressions of understanding that the students would give in various stages of the unit.

The unit lasted approximately six weeks and comprised 15 planned lessons. At the beginning of the unit students were informed that we would be using a new theory of learning called imaginative education, which suggested that it was helpful for students to use their imaginations and feelings in their work. This was the only direct comment made to students about the theory during the research period. However, a small number of indirect references to imagination and feelings (e.g. prompts for the students to imagine, pretend, express how they
### Table 4.1 Unit plan for Grade 4/5 elementary mathematics: Shape and space

<table>
<thead>
<tr>
<th>Lesson No.</th>
<th>Title of Lesson</th>
<th>Description</th>
<th>Imaginative Characteristics</th>
<th>Resources</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Getting in Gear</td>
<td>Invite students to use prior knowledge. Suggest use of imagination. Let students know of AE. Ask them to think what they know and could learn.</td>
<td>Imagination, BO—vision/blindness heroes (AI Ein)</td>
<td>Helen Keller: Courage in the Dark (Huritz) + others</td>
<td>Student narratives, pictures in math journals</td>
</tr>
<tr>
<td>2</td>
<td>Vision Walk</td>
<td>Students to wear blindfold, taking turns and walking in pairs around school hallways. How do they get a sense of space and shapes?</td>
<td>Imagination, BO vision/blindness, empathy, role play</td>
<td>Scarves and blindfolds</td>
<td>Student narratives of experience in math journals</td>
</tr>
<tr>
<td>3</td>
<td>Mathematicians Are People Too!!</td>
<td>Move established context into thinking of historical background, some of the people of math. Ask students to think about taking role of one.</td>
<td>Imagination, story, role play, BO past/present</td>
<td>Mathematicians Are People Too, Historical Connections to Math (Reimer &amp; Reimer)</td>
<td>Student narratives, pictures in math journals</td>
</tr>
<tr>
<td>4</td>
<td>Shape of Me</td>
<td>Establish context. Invite students to become part of story, and later shapes! Morphing of shapes.</td>
<td>Imagination, BO, context, curiosity, ingenuity, form images, mystery, wonder, role play</td>
<td>The Shape of Me and Other Stuff (Dr. Seuss)</td>
<td>Student narratives and pictures</td>
</tr>
<tr>
<td>5</td>
<td>Shape Sorting</td>
<td>Students to start looking at set of shapes and group them according to their own classifications.</td>
<td>Imagination, BO known/unknown, curiosity, ingenuity</td>
<td>Copied set of shapes, regular, irregular, curved closed</td>
<td>Student narratives math journals, Teacher activity page</td>
</tr>
<tr>
<td>6a</td>
<td>Shape Hunt (a)</td>
<td>Students to go around exterior of school looking for shapes and objects in the environment</td>
<td>Imagination, patterns, BO, discovery, surprise/wonder</td>
<td>n/a</td>
<td>Student narratives math journals</td>
</tr>
<tr>
<td>6b</td>
<td>Shape Hunt (b)</td>
<td>Students to follow up prior activity with text, to link vocabulary with curriculum content. Done as homework.</td>
<td>Imagination, patterns, BO, discovery, ignorance to knowledge</td>
<td>Quest 2000 G4/5 student text (Pearson Educ.)</td>
<td>Student narratives math journals, text pages</td>
</tr>
<tr>
<td>7</td>
<td>Solids: 2D Shapes and 3D Objects</td>
<td>Connection of studies with curriculum/text materials. Nets prisms and solids</td>
<td>Imagination, BO, discovery, ignorance to knowledge</td>
<td>Quest 2000</td>
<td>Student narratives in math journals, text pages</td>
</tr>
<tr>
<td>8</td>
<td>Triangles and Shapes</td>
<td>Connection of studies with curriculum/text materials. Begin to look at variety of triangles and properties</td>
<td>Imagination, BO, discovery</td>
<td>Quest 2000</td>
<td>Student narratives math journals, text pages</td>
</tr>
<tr>
<td>9</td>
<td>Shapes /Imagination</td>
<td>Students to get used to variety of vocabulary, secure known information. Extend further</td>
<td>Imagination, BO, discovery, unknown to known and beyond</td>
<td>Teacher-created page</td>
<td>Student narratives math journals, activity pages</td>
</tr>
<tr>
<td>10</td>
<td>Different Properties of Different Shapes</td>
<td>Compare properties of shapes</td>
<td>Imagination, BO—ignorance/knowledge</td>
<td>Quest 2000</td>
<td>Student narratives math journals, text pages</td>
</tr>
<tr>
<td>11</td>
<td>Polyhedrons</td>
<td>Watch video on Platonic solids. Students to use polyhedrons to transfer/apply knowledge to practice.</td>
<td>Imagination, BO—surprise/expectation, wonder, curiosity</td>
<td>Platonic Solids (Key Curriculum), plastic polyhedron manipulatives</td>
<td>Student narratives math journals</td>
</tr>
<tr>
<td>12</td>
<td>Right Angle or Not?</td>
<td>Students become more familiar with properties of angles. Introduce protractors.</td>
<td>Imagination, discovery, known/unknown, ignorance, knowledge</td>
<td>Teacher-created page</td>
<td>Student narratives math journals, activity page</td>
</tr>
<tr>
<td>13</td>
<td>Tessellations</td>
<td>Students become aware of Escher and images created</td>
<td>Imagination, BO, wonder, creativity, ignorance, knowledge</td>
<td>Imagine a Day, Imagine a Night (Thomson &amp; Gonsalves)</td>
<td>Student narratives math journals, student tessellation</td>
</tr>
<tr>
<td>14</td>
<td>A Right Angle or Not? That Is the question!</td>
<td>Reading of text and student response to story and completion of teacher page. little direction, gentle guidance. What do you notice?</td>
<td>Imagination, story, BO, unknown to known, humanization of meaning</td>
<td>What’s Your Angle, Pythagoras? (Ellis), teacher-created page</td>
<td>Student narratives math journals</td>
</tr>
<tr>
<td>15</td>
<td>Sweet Clara—Prior unit application of content</td>
<td>Beginning of closure. Reading text, students take on role and use this to create their own quilt.</td>
<td>Imagination, BO—freedom/captivity</td>
<td>Sweet Clara and The Freedom Quilt (Hopkinson)</td>
<td>Math journals, quilt design explanation</td>
</tr>
</tbody>
</table>

*Note - First four lessons relate to the introduction of the unit*
The Unit Mythic Lesson Planning Framework

1. **Emotional engagement**—What is emotionally engaging about the topic? Why should it matter? How is it meaningful? What binary opposites best capture meaning and emotion?
   - Blindness/vision, captivity/freedom. Appreciation of what we do have (e.g., health and well-being). What is it like to have less? How do visually impaired people learn about space and shape? What does it feel like to be someone else?

2. **Central image of metaphor**—What image captures the heroic qualities of the topic?
   - Walking in someone else’s shoes, standing in their place.

3. **Organizing content into story form**—How can we organize the content into a developing story form?
   - How do we feel when we lose (temporarily) something we might have taken for granted? How did we come to know shapes and space? Record this. Do vision walk; come back and record narrative of experience. Now think about others, those less fortunate (e.g., Helen Keller, blind mathematician Euler). How did they manage?

4. **Images and metaphor**—What activities help students develop images, metaphors, or other forms of creative depiction?
   - Vision walk; sort shapes and examine properties; make tessellations; design quilts; play and work with protractor.

5. **Rhythm, rhyme, and pattern**—What activities help students experience and extend a sense of rhythm, rhyme, or predictability?
   - Making tessellations; sorting shapes; making solids from nets.

6. **Drama and role play**—How can students become characters in story? How can they be encouraged and supported to retell story using their own words, gestures and actions?
   - Take part in realistic activities (e.g., vision walk); use combination of emotions and imagination to take place of characters; record in their math journals.

7. **Towards further understanding**—How can the unit develop embryonic forms of romantic, philosophic and ironic understanding? What cognitive tools characteristic of literacy, the disciplines, or embodied self-awareness can be introduced here?
   - Mystery into wonder, forming images, using story, comparing and contrasting emotions and imagination of characters.

8. **Resolution**—How does the story end? How are the opposites mediated or resolved?
   - Can you help a blind person learn about shape and space (quilt)? Development of empathy for less fortunate, whatever life circumstance.

9. **Assessment**—How can one know whether the topic has been understood, its importance grasped, and the content learned? Student completion of narratives in math journals. Students to access prior understanding and knowledge that they might have about a concept. Regular assessment (i.e., performance standard—numeracy). Use ILPFs and characteristics of mythic and romantic categories.

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Figure 4.1. Outline of lessons.
feel and so forth) appeared on some activity pages used during the unit. An occasional comment was also made to the students to think about what it would have been like if they were in a situation similar to one of the characters in the unit lessons, such as artist M. C. Escher or a mathematician from the past.

The introductory unit lessons were organized around a vision walk, an activity in which students, working with a partner, took turns wearing blindfolds as they navigated their way around the school with the objective of moving back to their home classroom. The remainder of the lessons included a range of activities. For example, lesson 13 introduced the topic of tessellations with a piece of children’s literature, *Imagine a Day* (Thomson & Gonsalves, 2005). As they listened to the story, students were invited to imagine being a character from the story, after which they wrote in their mathematics journals and later began the planning and completion of a tessellation design. No journal sentence stems or questions were given. Other lessons included analyzing and constructing three-dimensional paper nets of a variety of Platonic solids and experimenting and becoming familiar with protractors to measure a variety of angles.

Some lessons required more than one 40-minute timetable block to introduce a new concept and allow students sufficient time to develop understanding and complete assignments. For example, this was the case with measurement of angles, where students were introduced to the concept of angle measurement through the story *What’s Your Angle, Pythagoras?—A Math Adventure* (Ellis, 2004). They then needed time to become familiar with protractors before completing activities requiring them to measure and begin classifying angles. Occasionally, certain lessons, such as the quilt and tessellation designs, were continued in the fine arts blocks. Two mathematics lessons each week were held in double blocks of about 80 minutes, allowing the students time for in-depth discovery and investigation; this was part of the regular classroom schedule from the start of the school year. An additional double-block lesson was added to the original plan to accommodate a visit from a students’ blind uncle, during which he shared his experiences of living as a blind person. Overall, approximately 36 lesson blocks, or 24 hours, were devoted to the unit, which was a typical amount of time for me to teach a unit of this nature.

Lessons usually began with a class meeting. For class meetings, I would sit on a stool at the front of the class, surrounded by all the students, to introduce the topic or idea for the lesson. If it was relevant to the lesson content, at this point in the lesson I mentioned leading individuals who had been involved with mathematical discoveries, thus giving some historical background to the mathematics and adding humanizing the content for the students.
In approximately six lessons I used a story or piece of literature to introduce the topic. For example, I used *What’s Your Angle, Pythagoras?—A Math Adventure* (Ellis, 2004), a fictionalized account for children of how Pythagoras made some of his mathematical discoveries, to introduce the topic of angles. Students were invited to imagine themselves alongside Pythagoras as he was making the discoveries. After the story and a class discussion, during which students were able to ask questions and clarify any thoughts they might have, they were asked to write in their mathematics journals about the discoveries that Pythagoras had made. They were also encouraged to comment on how they felt when making their own discoveries; their answers provide insight into their affective responses and their use of imagination. This type of lesson also allowed students to access prior understanding and knowledge that they might have about a concept.

### 4.5 Data collection procedures

Qualitative research methods provided scope to include a range of research tools that support children as “informants of their own lives” (Greene & Hill, 2005, p. 12) within a wider sociocultural framework. Data gathered fell into two categories: student research data and teacher research data.

#### 4.5.1 Data sources: Students

With this research examining the students’ perspective of their learning experience, it is important to have a multifaceted collection of student data to provide maximum opportunity for the students to demonstrate their understanding of the mathematical concepts.

##### 4.5.1.1 One-to-one semi-structured interviews

One-to-one interviews were conducted after all the lessons had been completed, students’ work marked and assessed, and final report cards for the year submitted to the school office for approval. All interviews were conducted within one week of the unit’s completion. The interviews occurred during the regular school day at a time of minimal intrusion, so that disruption to the students’ learning was minimized. There were no predetermined time limits for the interviews, rather allowance of time to enable the students to respond in an open and free manner as long as they had comments to contribute. For three of the students, Courtney, Kee,
and Grace, the interviews lasted close to 30 minutes, whereas Freddie’s interview was about 35 minutes and Jason’s was the shortest interview, at about 25 minutes.

The two Grade 4 students were interviewed on one day and the three Grade 5 students on the following day. These interviews were guided by an interview protocol, which had been piloted with a group of students of the same general age and grade as the research participants. Slight adjustments in wording and the number of questions had been made after the pilot session, and the revised protocol (see Appendix 3) was used with all five participants. Part of the one-to-one interview was focused on the students’ reflections on the four work samples chosen for detailed analysis, in order to gain a sense of their perspective of their engagement. In addition, open-ended questions were also included in the protocol to allow the students to freely add their own comments or questions at any time during the interview. This also allowed me to respond to students’ comments. Illustrative examples of my researcher responses are included in the transcript extracts in Chapter 5.

4.5.1.2 Group interview

The five participating students met as a group after all the one-to-one interviews had been completed to further discuss their individual thoughts and experiences during the shape and space unit. No student work samples were brought to the interview, in order to maintain openness for what might arise during discussion. Like the individual interviews, the group interview was conducted within one week of the unit’s completion. A group interview protocol was utilized, similar to the one used in the individual interviews (see Appendix 3), with the intent of providing an opportunity for open conversation among peers and possible stimulation of new ideas that might not have arisen in the one-to-one interview situation.

All the interviews were recorded in audio and video format. When transcribing the audio recording, I used digital versatile discs (DVDs) to which the video recordings had been transferred, to optimize accuracy of the transcriptions and reduce the inaudible sections.

4.5.1.3 Students’ mathematics journals

Students were asked to record their understanding and responses in their mathematics journals at a variety of points throughout the unit. The use of journals has become an accepted part of student representation of understanding in many mathematics classrooms, according to
Van De Walle (2005), who states they are “prized above all other sources of information for their usefulness in improving learning and instruction” (p. 78).

After the introduction of a lesson, students were often asked to make a note or comment in their mathematics journals. For example, during the viewing of the video *The Platonic Solids* (Key Curriculum Press, 1991), students made notes in their mathematics journals of points that interested or seemed important to them. The students then used their notes to create a more complete journal entry. The notes and subsequent journal entry were then available as a reference for subsequent activities, such as the building of the Platonic solids from nets and, later, describing the properties of the solids.

Students were encouraged to use both visual illustrations and narrative comments in their journal entries. In an informal classroom study prior to the current study, I had found that mathematics journal entries enabled students to incorporate an affective response to some of the activities in a narrative form, as well to express their mathematical understanding. As learners’ emotive connections and affective responses are key elements in IE theory, the journals were an important tool.

Although this research focused on five students, it was also important to consider all the students of the class and their range of abilities. The use of the mathematics journals provided all students an opportunity to demonstrate their developing understanding, whatever their level of ability or understanding. Normalizing the environment reduced research bias as much as possible and strengthened the external validity (Stake, 2007) and transferability of the study’s findings.

4.5.1.4 Students’ classroom activity pages

Although the teaching of this unit was informed by the IE theoretical framework and it was the first time I had taught a complete unit of lessons with this framework, it was important, as mentioned above, to normalize the learning environment of the students, maintaining as closely as possible the regular classroom context. Many teachers supplement published resources with additional resources, such as teacher-generated pages. Thus, in this study the students used a variety of activity pages from both published resources, such as *Quest 2000* (Wortzman, 1996) and *Elementary and Middle School Mathematics* (Van de Walle, 2007), and teacher-adapted or teacher-generated pages. These pages guided the activity as well as served as a record of students’ responses to the activity.
An example of a student activity page adapted for the unit is one involving a variety of shapes, such as polygons and curved-edge shapes, of different sizes for the students to cut out and organize into groups (see Appendix 4). Following the IE approach, I orally encouraged the students to think of themselves as one of the shapes that needed to be grouped. Students were then asked to describe and write about their choices and grouping of shapes. A class discussion about the attributes of shapes emerged from this exploratory activity. The inclusion of student activity pages in the data captured an aspect of the students’ cognitive engagement and understanding.

4.5.1.5 Student work samples

At the end of the data collection phase, after reviewing all the work samples from the five students, I selected four activities completed during the unit to code in detail and to use in the individual semi-structured interviews. These were selected to provide responses to a range of activities and resources at key points throughout the unit and, importantly, to reflect the range of learning styles of the 28 students in the class described in section 4.3.

The first selection was from one of the introductory lessons of the unit, the vision walk activity, which emphasized the binary opposites of vision and blindness. The lesson began with a discussion of the importance of understanding our general environment by learning about a variety of shapes and spaces. Students were then asked to go on a vision walk with a partner, during which they would wear a mask or blindfold and try to find their way from back to the classroom from a different place around the school, thus temporarily experiencing the world of a blind person. After returning to the classroom and changing roles with their partners, students returned to the classroom again to discuss their experiences. They were asked to imagine what it would be like to be blind all the time. Students then wrote in their mathematics journals about the experience of moving around in spaces and the objects they found.

I selected the students’ mathematics journal responses to the vision walk activity for detailed examination because this lesson established the context and tone of what was to come in the rest of the unit. This choice is supported by the widely accepted notion in pedagogical practice that the introductory phase of lessons or units is a key moment in student learning. Further, more specific support comes from the Vygotskian notion of the importance of the introductory phase of scientific concepts (Vygotsky, 1962). The use of the journal entries also provided the dimension of student voice (Fielding, 2001a, 2001b, 2004a, 2004b Rudduck,
Chaplain, & Wallace, 1996) used in this research, providing expression of response by the students during or immediately after an activity.

The second selected piece of work was related to the topic of angles and came approximately halfway through the unit. After they had listened to What’s Your Angle, *Pythagoras*?—A Math Adventure (Ellis, 2004) I asked the students to use a protractor to measure and classify a variety of angles printed on a conventional activity page. As this was the first time that many of the students had used a protractor, time was given for discovery and exploration with this mathematical tool. I selected this piece of work because it included traditional activity pages and required the use of a new mathematical tool.

The third selected work sample was the creation of a tessellation from the latter part of the unit. This selection related particularly to the humanization of mathematics, since the students came to know some of the mathematicians behind the formal mathematics they had previously experienced in textbooks. This activity required approximately four lesson blocks to complete; the introduction occurred during regular mathematics lessons and the remainder in the fine arts lessons. After investigating the properties of shapes, students were introduced to the work of Dutch artist M. C. Escher, as well as to H. S. M. Coxeter, a leading Canadian mathematician and geometer who was a close friend of Escher and with whom I had corresponded. In addition, tessellations allowed those students who preferred less text-based activities to demonstrate their understanding and knowledge of shapes visually. Students were given a square cut-out and shown how a shape could be morphed into another figure, which could be used as a template to make a pattern which was then to be coloured. A mathematics journal entry explaining the creation of the tessellation was also part of the lesson. During the individual interviews, students referred to both their design and their corresponding mathematics journal entry.

I selected the fourth activity from the end of the unit because it was one in which the students had the opportunity to represent the mathematical understanding they had garnered from the complete unit in both visual and written form. This lesson again incorporated a story to affectively orient the students (Egan, 1997), to assist them in connecting with the lesson task. After reading *Sweet Clara and the Freedom Quilt* (Hopkinson, 1995), a story in which a young plantation girl designs a quilt to be used by slaves to escape their captivity, the students were given an activity page that included completing a mathematics-journal-styled entry (see Appendix 5) in preparation for designing a quilt. Drawing on the binary opposites of vision and blindness that framed the unit, the students’ next task was to design a quilt that could help a blind
person learn about mathematics. Both the quilt design and the completed activity page were brought to the individual interview for further student comment.

Together, the one-to-one interview transcripts, the group interview transcripts, the students’ mathematics journals and their classroom activity pages provided a broad range of evidence about the students’ individual and collective perspective and their engagement with the IE lessons and the use of the ILPF.

4.5.1.6 Student assessment

During the study I, as the classroom teacher, used a performance rubric (see Appendix 7) to assess the students’ work for evidence of growth in understanding. Students were familiar with this form of assessment, as it had been in use since the beginning of the school year in a variety of subject areas. Although based on the British Columbia Ministry of Education (2002) performance standards rubrics, I had modified it to include a further category of assessment, exceeding expectations plus. I felt the extra category was necessary to recognize what could be expected from lessons that had a deeper and broader curriculum base. The unit added depth to the regular curriculum by including other dimensions, such as historical context and information about leading mathematicians of both past and present, to assist the students in accessing the regular content and learning outcomes. This category could also be used to recognize student achievement that fell within the child’s zone of proximal development (Vygotsky, 1978). I did not want to restrict what the students could demonstrate during the unit, and felt that use of the usual performance standard levels did not allow recognition of all they could potentially achieve. Here, the role of teacher informed the role of researcher through assessment and subsequent analysis.

4.5.2 Data sources: Teacher/researcher

A number of pieces of teacher/researcher data were also used in the research study to provide a comprehensive view of its implementation.

4.5.2.1 Research diary

The research diary (Silverman, 2005) served as a medium to record observations that, as the teacher, I would normally notice during regular class interactions and lessons, and to capture and record at the end of the day short verbatim comments from the students. For example, I was
able to record the exact words of one of the participating students when he requested additional resources after one of the lessons: “Mrs. H, do you have any chapter books about math?” A second research diary note recorded an interesting spontaneous action from the students: “Students spontaneously started to sing “P – L – A – T – O” based on a children’s song” [“Bingo,” a song about a dog in which his name is spelled out).

More importantly, the research diary allowed me to reflect on events that had occurred while teaching. An example of one of the reflection notes for May 24 states, “Could imagination be regarded as day dreaming at this point? **This could be good to explore in conclusion. Could it be that the use of emotions is not yet considered to be a valid tool of learning [by the students]?”

4.5.2.2 Critical friend observations and reflections

I included the use of a critical friend (Costa & Kallick, 1993; Foulger, 2010) in the data collection phase of the research study. Lesley (pseudonym), a teaching colleague in elementary school, fulfilled this role. As the critical friend she provided me with a welcome point of immediate reflection (Robinson & Carrington, 2002) on my researcher role. When recording her observations, Lesley occasionally asked questions that greatly assisted my later researcher reflections during data analysis. For example, during a lesson on angles, she observed one of the participant students writing in his mathematics journal and noted, “I wonder what it is that he discovered and that Pythagoras didn’t?” This then prompted me to examine in more depth the students’ mathematics journal entries and reflect on how many students seemed to be reacting positively to the inclusion of historical figures.

Lesley’s recorded observations conveyed a second perception on the students’ engagement, thus reducing potential bias in the research and augmenting the observations I could generate as a teacher. This additional perspective gave breadth to the data sources used, thereby adding strength to the findings.

Prior to the study, I designed and piloted a student engagement observation (SEO) protocol for Lesley to use (see Appendix 6), thereby allowing me to remain focused on teaching the unit. Adjustments were made to the original design and a revised version was given to Lesley to guide her recording of the behavioural, participatory, or affective responses the students exhibited. When using the SEO protocol, Lesley selected a discrete location in the classroom from which to observe the students. However, having the SEO protocol on a clipboard allowed
her to also move freely around the classroom to discretely monitor and record student actions and
comments as they occurred.

With my permission, Lesley audio-recorded her post-observation reflections on the same
day as the observations took place. In addition, we usually discussed her observations within 24
hours from when they had been made. For example, after a lesson on the properties of shapes, in
which students were asked to observe and try to analyze a ball that rotated internally and changed
colour when thrown in the air, she noted,

Freddie was listening very attentively and he got very [excited voice of Lesley] excited
when he saw that the ball had changed colour from pink/purple to turquoise; you could
just see it on his face—he was like ‘Whoa!’ sort of thing. (May 22)

Lesley observed 21 of the 36 lesson blocks. During four of these blocks, Lesley focused
on only two or three of the research students at a time to allow her to observe them more closely.
In the remaining observations, she observed all five research students within the context of the
entire class.

A challenge in the use of a critical friend that did occur during the study related to how
much interaction, if any, I could permit between Lesley and the students during her observations,
despite her efforts to be discrete. Students seeking additional help, and occasionally immediate
acknowledgement, would at times speak to her during lessons. This challenge required that I
maintain both the integrity of the research design and ethical expectations while respecting
Lesley’s individual personality and professionalism. Brief follow-up conversations with Lesley
after she had carried out a class observation allowed me to clarify what may have occurred
between her and the students. These were audio-recorded with Lesley’s permission, and added to
the transcript of her own audio recordings as noted above.

4.6 Data analysis

There were four stages to the analysis process. In each stage I kept in mind that, while the
use of multiple sources of data added to the credibility and validity of the results and allowed the
students to give their perspective of their learning experience, information found in one data
source might not be repeated in another. In addition, I acknowledge that at the data analysis point
in the study it is necessarily my interpretation of the student perspective that is given. In order to
present the reader with a window onto the analysis process, I provide three appendices.
Appendix 8 illustrates the cognitive tool codes, Appendix 9 provides a sample of a coded
transcribed interview extract, and Appendix 10 provides an exemplar case of extracts from all the
data sources for one student. It should be noted that at times more than one code was assigned to an excerpt of data, if the excerpt reflected use of more than one of the cognitive tools, (see appendix 9).

4.6.1 Phase 1: Transcription of interviews

In the first phase of data analysis, I transcribed all the audiotapes of the one-to-one and group interviews. I then verified and expanded the transcriptions during multiple viewings of the DVDs. I viewed and cross-referenced each of the one-to-one interviews and the group interview three times. The first time was to verify the audio transcription of the interviews. During the second viewing I made observations that I could not note when I had been focusing on conducting the interview. In the third viewing, responses from the students that would assist in addressing the research question, such as tone of voice or gestures like raising a hand in the group interview, were noted. These multiple viewings also allowed me to become very familiar with the students’ comments in order to prepare for the in-depth analysis and the emergence of possible themes. As the individual and group interviews were the most direct data source for accessing the students’ perspective of their learning experience, this first phase of data analysis was particularly important to answering the research question.

4.6.2 Phase 2: Participatory affective engagement

In order to understand more of what is involved in students’ engagement in tasks, I felt it was inadequate to simply state that a student is either engaged or not engaged. In current engagement literature, means of assessing student engagement tend to focus either on teacher observations of student behaviour, such as with the William and Mary Classroom Observation Scales Revised (Van Tassel-Baska et al., 2003), or on the cognitive focus of student engagement in school more generally (Appleton et al., 2006). Therefore, to describe characteristics of student engagement in the current study, that reflects students’ affective responses and their involvement in learning situations, I examined these students’ engagement in terms of both the affective and participatory (behavioural) domains of learning. That is I hypothesized that a student’s involvement may be active or passive and is combined with a degree of feeling or emotion that may be positive or negative. Thus four descriptors of Participatory Affective Engagement (PA engagement) (Hagen 2007; Hagen & Percival, 2009) were generated, namely active positive, passive positive, passive negative, and active negative.
Students’ passive engagement can be seen in minimal action or observation of an activity. Caution, however, needs to be exercised here; even though a student is not exhibiting noticeable behaviour, this does not necessarily mean that he or she is not engaged. For example, a student may be taking the role of an observer or listener in an activity before deciding what level of participation to take. Thus, the concept of passive engagement, while initially requiring more attentive and broadly based observations and enquiry by an educator, does allow us to consider more aspects of a learner’s experience. Active engagement is more easily observed. A student actively engaged will demonstrate some form of observable behavioural involvement with an activity or lesson. This could be voice, bodily action, or written or visual expression.

Likewise, when engaging with an activity, a student tends to experience some form of emotive response or connection with the activity, what Vygotsky (2004) describes as “internal expression” (p. 18); this could be in either a positive or a negative mode and may be exhibited in either a passive or an active manner. The example given above of a student observing before deciding on how to participate is a passive positive response, whereas non-participation or minimal compliance with lesson is a passive negative response. Once students have decided to pay no further attention to the task, their behaviour moves outside the spheres of PA engagement. The distinction between passive positive and passive negative engagement is less obvious; the educator may need input from the child to distinguish which category of participation should be assigned by the educator.

A student may also experience active positive and active negative engagement with learning opportunities. This type of response is much more discernible for the educator. Students’ active positive engagement, the most desirable form of engagement, is demonstrated through their active, willing involvement with the lesson or learning activity; the student may experience excitement and pleasure and therefore feel quite able to continue engagement with the activity. Conversely, active negative engagement, the least desirable form of engagement, would be demonstrated in unsettled, disturbed or disruptive behaviour that is directly related to the lesson or activity. An example of this type of response is a student demonstrating through body language, such as facial expressions and crossed arms that he or she is not willing to participate in a task. An even more extreme example would be a student who leaves the location of the activity to go to another area of the room. These 4 descriptors then were used to describe each students’ participation, or lack thereof, as they engaged in the learning activities during the Unit.
4.6.3 Phase 3: Cognitive tools

A coding scheme (Creswell, 1998; Hancock & Algozzine, 2006) for the cognitive tools of the mythic and romantic phases of the IE theory (Egan, 1997, 2005) was developed prior to beginning the analysis (see Appendix 8). In order to examine the meaning IE theory had for children and their engagement in mathematics, it was essential to note specific instances of when and how evidence of these cognitive tools emerged in the data sources. (See Appendix 9 for an illustrative sample).

An interrater (Armstrong, Gosling et al. 1997) coded a portion of a transcript from one of the one-to-one interviews to enhance the reliability of the codes. The interrater was knowledgeable about sociocultural theory, had some familiarity with Egan’s (1997) IE theory, and although she was not a mathematics teacher, she had some familiarity working with children. The piece I selected to be coded by the interrater contained more comments and conversation from a student and less researcher interaction in the interview process. In addition, the section of transcript was selected to be representative of the five students taking part in the study. In the chosen sample of transcribed data, 46 units were identified. There was immediate independent agreement on 33 items (72%). After discussion, agreement was achieved on 11 further items, for a total of 44 (95.6%). This provided validity to the coding scheme.

It is important to acknowledge that even though it was considered important to establish the meaning the IE theory had to the children in this study that by imposing theoretical codes on the data I may be imposing restriction and limitation on reporting the very perspective that the children are trying to illustrate and share. It is for these reasons that emergent theme analysis as the final and perhaps most important phase of the data analysis.

4.6.4 Phase 4: Emergent theme analysis

In the fourth and final phase of data analysis, I triangulated (Cohen, Manion, & Morrison, 2000; Mathison, 1988; Yin, 2003) all the data sources. Through triangulation I looked for areas of convergence, inconsistency, and contradiction in the data (Mathison, 1988) that would relate back to the original research question.

This triangulation process proved to be a very important for the data analysis. When all of the data sources were brought together, I was able to see a wealth of information coalescing and arising (Fereday & Muir-Cochrane, 2006) out of the initial analysis. Using the PA engagement descriptors had allowed me to begin to organize the information in the manner that
Boyatzis (1998) suggests, where a theme is seen as “a pattern in the information that at minimum describes and organizes the possible observations and at a maximum interprets aspects of the phenomenon” (p. 161). I also found an additional outcome of data triangulation, that of extension or emergence, that extends out from the original data sources, pointing towards a phenomenon that is tied to the original research question but could not be predicted prior to analysis.

In this study, patterns established in the data through repeated notation were classed as emergent patterns. I looked for representation of these patterns in the data and, because I was focusing on the students’ perspective of their experience, I also looked for comments by the students in the interview transcripts and their mathematics journals that were particularly revealing about the experiences and work being described.

As I began to look for emergent themes, I wrote brief note-form summaries of the data sources. (see Appendix 10 for an exemplar of coding). For example, when analyzing a student’s work samples I wrote, “Freddie—Motivated, emotionally/imaginatively engaged, kept going,” and from the interview transcripts I wrote, “Grace—articulate about how she learns and other children can learn and benefit, make connection/opportunity to real world, many connections to math and the real world, expression of enjoyment about experience.” Finally, from the group interview I made the following note:

Lots of volunteering of ideas; Maturity from G5’s to G4’s re doing work, ALL wanted to get rid of text books, see tb’s as more stressful, lacking in explanation, showed great u/s pedagogy practice, kids need connection to subject to learn.

After I triangulated all the data through a within-case analysis (Creswell, 1998; Yin, 2003) of all the five individual student cases, laying out the findings and themes in a spread-sheet like format, I carried out a cross-case analysis of the five cases. This allowed identification of similarities, differences, and patterns to be recognized across all the cases and new knowledge to be generated that was more substantive than by looking at individual cases (Khan & VanWynsberghe, 2008). A word table (Yin, 2003) was used to summarise and interpret the emergent themes, and is reported with supporting details in Chapter 5.

4.7 Credibility and limitations

To maintain the research focus on the meaning of IE theory and IPLFs for the children, it was essential to document the students’ perspective in as many appropriate formats as possible.
In doing so, I had to be mindful of possible limitations to the chosen methods, the need for credibility (Denzin & Lincoln, 2000; Guba & Lincoln, 1989) and the need to separate the teacher and researcher roles. I also had to be mindful of potential theoretical bias and took the following measures to mitigate both these concerns.

First, utilizing the general idea of student voice (Fielding, 2001a, 2001b, 2004a, 2004b; Rudduck & Fielding, 2006) in the research design of the data collection and data analysis phases strengthened the student perspective of the experience of using IE-framed lessons. It minimized the bias I might have had towards the choice of this theoretical framework by foregrounding the students’ experience in relationship to my own. It was what the students said and thought about their experiences throughout the unit—their perspective—that was profiled when analysing and reporting the results of this study.

Second, allowing the students to comment upon their work as the basis of the one-to-one semi-structured interviews, rather than having them respond to more questions written and posed by the researcher, also minimized bias towards the theoretical framework. My job as the researcher was to make sure the student perspective remained in the forefront, not my own. This also strengthened the validity of the findings by reducing research bias.

Third, having a critical friend observe the participants provided a necessary check and served to reduce any undue bias, which might arise in the observations. For example, while supportive, the critical friend made challenging statements and observations, which required reflection, such as questioning the appeal of the tasks to the wide range of ability levels in the class. In addition, I could confer with the critical friend during data collection to help ensure I maintained a balanced perspective throughout the study.

Fourth, in the initial planning of the unit I was able to blend both parts of the teacher/researcher role in order to plan a meaningful unit of lessons that would address the research question. However, when actually teaching the unit, I became the teacher in order to maintain the appropriate ethical distance, between myself and the students. For example, although I needed to do my regular teacher assessment once the unit of lessons was complete, I did not begin any of the more formal analysis until after complete marks and report cards for the term had been submitted to the school office. Once this was done, I could resume the role of the researcher, carry out an initial analysis of student data, and then conduct the interviews and full research analysis. Thus, the separation of roles as needed in the different phases of the study added to the credibility of the research findings.
Fifth, although the semi-structured interviews were the primary source of the student perspectives, triangulation between all sources of student data added strength and credibility to the research findings (Creswell, 1998; Patton, 2002; Yin 2003). Spreading the collection of data over the six-week-long unit, rather than allowing just one or a few pieces of student work to dominate, further supported the integrity of the findings. This process allowed the students’ understanding to occur over the time frame of the unit; the assessment of understanding was not dependent on any one sampling of student data.

Finally, having an interrater code a portion of the data to evaluate the internal consistency and reliability of the coding scheme and my coding proved to be another measure by which I could subjugate my own biases and maintain the credibility of the coding.

By taking these measures and reporting thick description of the context and participants, I aimed to establish credibility in the research (Denzin & Lincoln, 2000) and minimize its limitations. That said, I also acknowledge that in my analysis I interpret and synthesize much of what was said and reduce each student’s contribution to representative excerpts.

4.8 Conclusion

The question guiding this study is this: What does the use of the theory of imaginative education and imaginative lesson planning frameworks mean to children and for their engagement in elementary mathematics? This study served to examine the meaning that use of this theory had for children and how this affected their engagement with mathematics.

The study viewed the development of understanding and learning in a sociocultural context (Vygotsky, 1962, 1978, 2004) in which learning is achieved through a process of internalisation and transformation where knowledge is redefined and renegotiated within an individual who is part of the wider community and world. IE theory, particularly the key elements of imagination and emotions, was chosen as the framework from which to explore student engagement in a unit about shape and space.

A case study approach was selected to provide a focused, in-depth examination of the issue of student engagement in a particular unit of elementary mathematics, with a purposefully selected group of students to provide maximum variation (Creswell, 2007) that would allow for the differences that could be found in any elementary school. Thus the case was bounded (Hancock & Algozzine, 2006) by the number of students and a unit of curriculum. Having five
Individual cases within the overall case study approach allowed rich details and particulars to be brought out from the perspective of each of the research participants (Tellis, 1997).

Individual semi-structured interviews and a group interview were the primary sources for data analysis regarding the students’ perspectives of their learning experience. These data were supported by individual work samples from four specific activities that allowed the students to investigate mathematical concepts, which encouraged them to use their imagination and emotions to communicate development of understanding. These activities included experiential role play, art and activity pages, mathematical manipulatives, children’s literature, and videos.

The PA engagement descriptors (Hagen 2007; Hagen & Percival, 2009) allowed consideration of both the affective and behavioural responses of students to learning situations. These descriptors were used by me and the critical friend to generate an initial analysis of each student’s engagement.

The analysis methods sought to make sense of the gathered data corpus. As this was an interpretive study, I had to continually make judgments, principally through emergent themes that arose from the student data. I also used a coding scheme based on the IE theoretical framework to identify aspects of the theory that the students were using which might illustrate their engagement. In doing this analysis I needed to consider whether there were inconsistencies and contrary evidence within the data from each student and whether the results were consistent with the data collected (Guba & Lincoln, 1989).

Having given a picture of the methodology used in this study and wanting to remain true to the students’ perspectives and honour their individual and group contributions, in the next chapter I report the findings from each of the five research students. This includes excerpts from their individual work samples, their semi-structured interviews, and the group interview.
CHAPTER 5: FINDINGS

This chapter presents the analysis findings for each of the five students to address the research question: What does the use of the theory of imaginative education (Egan, 1997, 2005) and imaginative lesson planning frameworks (ILPFs) mean to children and their engagement in elementary mathematics? For each student I present the findings thematically in order to give readers an opportunity to come to know the students and the characteristics of their engagement as closely as possible.

As a new theme is introduced I demonstrate for the reader how this is evidenced in the data for each student. Themes will be given in an order that allows me to build an overall portrait of the students’ engagement over the course of the unit work and the interviews, rather than in frequency of occurrence.

For each student, this chapter also includes a section that relates to the cognitive tools of the IE theoretical framework of the study, in an effort to distinguish which aspects of the theory may be particularly meaningful for that student’s learning and engagement. A cross-case analysis presented as a collective summary synthesizes the data to lay a foundation for the discussion of the significance of the findings in the next chapter.

Reporting the findings in this manner allows a degree of individualization in relating each student’s perspective about the development of his or her understanding. I could not predict either what they would say about their learning experiences or how they would respond to the unit of lessons. Thus, refraining from corralling the themes too early allows a richer insight to emerge to address the research question. While I acknowledge Creswell’s (1998) view that at a certain point it is my own interpretation of the students’ words and experiences that has to be stated, and therefore the themes must be brought together, I did not want to lose the students’ voice of their own experience, as cautioned by O’Loughlin (1992), by doing this too soon.

5.1 Courtney’s engagement

Courtney was a Grade 4 girl whom, as her classroom teacher, I regarded as a typical Grade 4 student of average ability. She seemed very comfortable talking with her peers but appeared to hold back somewhat when expressing herself in written work and in class discussions. During the interview, however, Courtney was a willing and active participant,
responding to all questions asked. She was able to speak to all aspects of her work, at times in significantly more detail than what she had written at the time of the lessons.

After assessing her lesson assignments as the classroom teacher, but prior to the interview, my initial impression was that Courtney exhibited passive positive engagement (Hagen, 2007; Hagen & Percival, 2009). This was based on my end-of-unit assessment and researcher observations throughout the unit, as noted in the research diary. In particular, Courtney had not demonstrated as much completed work as expected for a typical G4 student during lessons and her early math journal entries and activity pages for the unit did not contain much mathematical detail or fully completed work. I noted in my research diary that Courtney appeared at times not to start activities with the other students and that on more than one occasion she was off-task for short periods of time. On such occasions, she seemed to be looking around at what other students were doing before beginning her own work. However, since Courtney had always submitted at least some work for the lessons, I described her engagement in the unit as passive positive.

Independently of my own assessment, the critical friend, Lesley, regarded Courtney as being less engaged than expected for a G4 student, thus concurring with my initial impression. Lesley observed that Courtney “was quite slow to begin today’s assignment” and she “did very little writing for this assignment.” She also noted that on a number of occasions Courtney sought clarification before starting assignments. These additional observations confirmed the passivity with which Courtney appeared to undertake tasks.

To further examine Courtney’s engagement, beyond this overall impression, I analysed Courtney’s individual interview, her group interview comments, and work samples in detail. The themes, which emerged from this analysis, were (a) drawing on emotions, (b) making personal and family connections, (c) developing self-confidence, and (d) cultivating mathematical awareness and understanding.

5.1.1 Theme 1: Drawing on emotions

The predominant theme to emerge for Courtney was one of emotive engagement. It seems that drawing on her emotions provided access, connection, and support to her understanding the mathematics being studied. This was demonstrated in a variety of ways during the unit and the interview process.
It soon became clear early in the interview that Courtney’s use of emotions with and in her work had grown during the unit. When asked to describe her learning in the unit, she confidently commented, “This term actually gave me some emotion to put into my math.” Such acknowledgement, without being prompted about emotions, indicated her self-awareness that emotions and learning mathematics could be linked.

When asked to describe her connection with the unit, Courtney reinforced the link between her emotions and the mathematics learning by responding, “I felt connected with this sort of math; like I was saying earlier, you can bring emotions and feelings and your description of math and stuff.” Here Courtney signalled that working “with this sort of math” allowed for interaction between her emotions and the mathematical content of the unit and, consequently, she “felt connected” to the mathematics. It appears that being able to express her feelings and have them validated allowed Courtney to engage with the mathematics in the unit.

During the unit Courtney expressed a range of emotions. She made a negative comment early in the unit, when drawing on her emotions to express a somewhat obligatory, involvement with the lesson. After the vision walk, the first activity, she wrote in her mathematics journal,

What was it like moving in space
- not knowing where you where [sic]
- wondering if your guide is going to trick you
- scarey [sic] because
- walking into people—tripping [sic] over things

As shown, in this entry she described her general thoughts about her activity experiences, which were somewhat uncertain, together with a somewhat negative affective response, “scarey.”

In a lesson midway through the unit, when experiencing a challenge constructing a solid from its net, Courtney did not hesitate to express irritation in her mathematics journal: “It is impossible to make, because it’s hollow and it represents water, i [sic] felt frustrated when I made it. It has lots of triangles and snowflakes.” This expression of perceived impossibility and frustration again points to Courtney drawing on her emotions within this lesson.

Most indicative of Courtney’s overall comfort with drawing on her emotions during her experiences in the Unit was her response when asked to reflect on the vision walk activity and journal entry during her individual interview. Courtney answered, “Yeah, so well I mean, I feel kind of, I don’t know, I have this weird feeling right now, that sort of feels sort of natural that comes to you in the middle of happy, sad, okay.” She was describing a place of comfort that she was feeling, situated somewhere between happy and sad, in response to viewing the work that she had completed.
Thus, the emotive theme running throughout Courtney’s work samples was further developed during the interview, showing her awareness that emotion plays a role in learning. When asked if there was anything to be added to the work samples, in general, now that she was seeing them again, Courtney responded,

Well I tried like always, like I tried to add more and as much detail and feeling as I can but sometimes I get kind of bad, oh I should have added to it, because you think of something the next day. And then you are like “Oh, but I already handed it in” and sometimes I just write little things to go with something because I forget it and stuff, so adding to things and I try to put in detail and feeling as much as I can. For me it is hard to feel feelings because I can’t really describe it but [pause].

Courtney’s sentiment “I try to put in detail and feeling as much as I can” implies that she was aware that her emotive responses were involved with her learning. In addition, Courtney stated that she continued to reflect on her work after handing it in for assessment. The closing sentence, “For me it is hard to feel feelings because I can’t really describe it but [pause],” suggests that Courtney was aware of and reflective of her own affective processes, although she had difficulty explaining her feelings.

This self-assessment was somewhat overshadowed when Courtney revealed a mature perceptiveness of affective reasoning, towards the end of her one-to-one interview:

Well, I sort of like, you don’t try to think when you are actually feeling that emotion. Your facial and body expression can sometimes explain it for you, so [pause] because your emotions, I think your emotions are connected to everything in your body, so.

When stating “You don’t try to think when you are actually feeling that emotion,” Courtney was describing her interpretation that physiological reactions can show affective responses without conscious awareness of the affective thought. Interestingly, her belief that “your emotions are connected to everything in your body” corresponds to an awareness of somatic understanding (Egan, 1997), an embodied kind of understanding and communication gained through non-language-based means of using our bodily senses.

The range of emotions Courtney experienced suggests that she was both receptive of and strongly responsive to the emotive element of the IE theoretical framework, which had been carefully included in both the pedagogical fabric of the lessons and in the selection of activities and resources. The evidence, illustrated by the examples given, shows that emotive engagement was a particularly meaningful opportunity for Courtney to engage with this unit. It also suggests that as the unit progressed, and also later in the interview, Courtney became increasingly aware of the value of drawing on her emotions to help her learning.
5.1.2 Theme 2: Making connections

A second theme arising for Courtney was one of making connections between what she was learning in the unit, herself, and her family. Perhaps the most telling example came from a comment in her mathematics journal in the early stages of the unit: “My uncle lernt [sic] about space and shape by touch at first, he would walk around the house and memorize the corners then each object.” Although she mentioned a family connection in this journal entry, she did not explicitly state that her uncle was blind. When, as her teacher, I read the early journal entry, I did not ascertain that she had a blind uncle, nor was I aware of this when planning a unit using vision and blindness to represent the primary cognitive tool of binary opposites. However, about halfway through the unit during a regular class discussion, Courtney told me about her uncle and we subsequently arranged for him to visit the class.

During the interview, the significance of this personal connection for Courtney resurfaced when she was asked to describe her engagement with the unit: “I sort of felt connected because, um you know the vision part with the blind walk, well I felt connected ’cause my uncle, he’s blind.” Thus, there had been, unbeknownst to me at the time, a personal connection for Courtney from the very first activity.

This special family connection was further exemplified during the interview when Courtney provided a full explanation about the quilt she had made (see Figure 5.1). In a monotonic, somewhat negative voice, she indirectly referenced this special connection between the unit, her uncle, and his blindness:

Researcher: Now that you really look at your quilt again, I was wondering how you feel generally about the work that you did now that you are seeing it again?

Courtney: It kind of makes me feel sort of down because a, it’s this makes a weird feeling inside when you feel it with your hands, because it is a mixture of feeling and sense. Like [pause]. It sort of makes me feel sort of in the Dark Ages [laugh] because of the black and white and then the jagged sort of again because of points.

In designing her quilt Courtney demonstrated an appreciation of her blind uncle’s world by not including any colour and by giving a Braille-like effect to the design through reverse pin pricking the outlines of shapes. Her comment likening her sadness to the Dark Ages and jagged points also illustrates the depth of her feelings about the reality of her uncle’s world.
Courtney drew other personal connections between herself, her family, and the mathematics unit during the interview; this time referring to her brothers:

[serious facial expression throughout] On my quilt I was, uh, feeling sort of lonely ’cause I was missing my 22-year-old brother, so I came up with putting the number of my, the children in my family, and that lead to four numbers. And then, um, I took those numbers and chose some, uh, numbers that would, like 1 plus 6 is 7 and that is how I got 7, and 7 plus 10 is 17, and 17 plus 12 is 29, and it goes on like that.

Although Courtney was somewhat inarticulate in explaining that the included numbers were the ages of members of her family, the inclusion of family information and the non-random
style in which the ages of family members were used strengthens the personal meaning these had for Courtney. The seriousness with which she said “I was, uh, feeling sort of lonely ’cause I was missing my 22-year-old brother” reinforced a deep family connection.

The somewhat negative emotion in this comment was contrasted in the comment that followed, about her brother and a place she had previously lived:

And you might have noticed that I put a sun. ’Cause, um, it right here is where my brother K lives, and I always was like more happy in Penticton because there was more sun, and just brought up feelings, so I put the sun there to match Penticton, where my brother lives, and the stars represent the night which is, uh, sort of a down-feeling emotion.

The importance of the family connection to Courtney, and the varied ways in which this occurred for her, was further illustrated when she talked about her father during the interview:

Researcher: Do you think this unit helped you become engaged with mathematics?

Courtney: [Smiling] And now you’ve got me thinking about math all around me, me and my dad going to Father’s Day dinner and there was this lamp, and there was this big oval lamp, and then [laughing] I went, “Oh, Dad, look this is an oval lamp,” and he’s like “Yeah, okay,” and then I am like, “It’s a big part of math.” . . . Yeah, and then his food came and it was in the shape of a square [laughing] and I was like, “Hey, Dad, your food is a square,” and he was like, “Why are you saying all these mathematical terms?” and I am like, “Mrs. Hagen taught me all those” and then he was like, “Oh, okay.”

Here Courtney was not only illustrating her family relationship, she was also indicating the information she felt happy to share with her father about the mathematics she was learning at school. The connection between family and mathematics was now reversed; mathematics was now a leading feature of a family discussion. Thus making connections for Courtney included bringing her family knowledge into the classroom and bringing her classroom learning into her family environment.

The close and personal connection that Courtney was building during the unit was also shown in mathematics journal entries written after completing work on the angle activities. After hearing the story What’s Your Angle, Pythagoras? A Math Adventure (Ellis, 2004) and analyzing some drawn angles, Courtney wrote,

When my friend Pythagoras matched all the tiles into red and blue squares and when he discovered the right angle i [sic] thought it was amazing because when you discover something you don’t know if your [sic] rong [sic] or right, but the thing is you can’t be rong [sic].

Rather than just name Pythagoras, Courtney referred to him as a friend, suggesting that she imagined a personal connection with this ancient mathematician, as indeed she expressed with
other literary characters in the unit, such as Clara from *Sweet Clara and the Freedom Quilt* (Hopkinson, 1995).

As noted in my research diary, imagining herself to be with a hero of mathematical discovery and referring to him as a friend seemed to help Courtney work through using a protractor for the first time by allowing her to see that discovery required an element of the unknown. It appeared that she did not feel alone when working with the protractor; she had a friend, albeit an imaginary friend. Thus, during both the unit and the interview, Courtney made connections between herself and the lessons, which were personally meaningful.

5.1.3 Theme 3: Developing self-confidence

A third theme that emerged for Courtney concerned her developing self-confidence. From her journal entry after the first activity, the vision walk, which was printed using bullet points, incomplete sentences, and without the expected cursive handwriting, Courtney’s growing self-confidence was increasingly evidenced in her work samples. For example, the journal entry for the quilt activity later in the unit was 10 lines of handwriting, in full sentences, such as “On my quilt starting at the left side that goes to the right side diagonley [sic] and then theres [sic] a 6 then a ☼, a 10, a ☼, a 12, then a, 22.”

During the one-to-one interview, Courtney’s developing self-confidence was apparent when she deflect questions back to me, something that I had not experienced from her in class. Courtney: For me it is hard to feel feeling because I can’t really describe it but [pause].

Researcher: You are doing a great job.

Courtney: The big feelings, like if you emotionally feel bad or emotionally feel happy, you can sense that, but if you’re feeling like sort of, how do you think you [researcher] are feeling right now?

Researcher: Quite excited.

In asking “How do you think you are feeling right now?” and trying to find a descriptive balance between the opposites of feeling emotionally bad and happy, Courtney showed the confidence to seek an additional perspective by reversing the researcher–student relationship. While it needs to be acknowledged that this might be explained by Courtney’s general familiarity with me as her regular classroom teacher, Courtney did not exhibit such confidence in conversations with me (as teacher) at the start of the unit, approximately five to six weeks prior to the interview.
Courtney continued to demonstrate self-confidence during the individual interview when she strongly critiqued textbooks, the choice and use of which is typically directed by educators rather than by students:

Researcher: When you think about this whole unit of lessons, would you add anything to it, like resources or books or other lessons?

Courtney: Because the textbook really doesn’t put it out there, it doesn’t really explain anything. . . . It just leaves you hanging there, like there’s something [pause].

Researcher: So if I listen carefully to what you are saying, I think we are talking about the textbook doesn’t give you all the explanation you think you need. Is that what you are saying?

Courtney: No [the textbook doesn’t].

Courtney was confident in stating her opinion to me as the teacher–researcher that mathematics textbooks do not give sufficient explanation of concepts to make the mathematics accessible and understandable. That she elaborated on her personal opinion in the group interview provided further evidence of her developing self-confidence:

Courtney: The textbook is a little bit more stressful.

All students: Yeah!

Researcher: In what ways?

Courtney: They do not explain really what you are supposed to do, than what the teacher says in person because the book just puts it out there, and sometimes you can’t just wonder. . . . Yeah, I think kids could learn better without textbooks, and just because it is sort of like some kids could be some page ahead of you and you would be like, “Oh no, I am way behind” and you would speed up and do your work wrong.

As previously indicated, Courtney progressively used more cursive writing, expanded the answers she gave to questions, and wrote journal entries of greater length as the unit progressed. She was therefore demonstrating far more willingness to become engaged with the activities in the mid- to later stages of the unit than she had at the beginning. In addition, by the time of the one-to-one and group interviews, the general comfort exhibited in Courtney’s later work transferred into the interview context. Indeed, her satisfaction was exemplified in an interview comment, “So I think I’m doing better in math,” thereby confirming her new self-confidence.
5.1.4 Theme 4: Cultivating mathematical awareness

The final theme that emerged from data analysis for Courtney was one of cultivating mathematical awareness. After the end of the unit Courtney was able to speak with an expanded perception of mathematics.

Researcher: Do you think this unit helped you become engaged with mathematics?
Courtney: Yes a lot, because like I said earlier, shapes and words are also a big part of math and before this term I didn’t really use words and shapes in math. I thought it was just numbers, boring old math, but then when I came to this term, I went “Oh, math is really fun” and then you showed me the Math Curse book and now you’ve got me thinking about math all around us.

Thus, from the IE infused Shape & Space unit (i.e. “this term”), Courtney appeared to have gained a sense of the breadth of mathematics wherein it’s —more than the “just numbers, boring old math” she had previously experienced. That is, Courtney was now aware of different mathematical concepts and seeing this in the world around her. Indeed, Courtney’s explanation for her formerly narrow view, “Before this term I didn’t really use shapes and words” suggests she cultivated this mathematical awareness not only through the inclusion of additional topics (“shapes”) but also through an emphasis on narrative forms (i.e. “words”), a central tenet of the IE theory. In addition, for Courtney, this new found mathematical awareness, meant a change in her perceptions of mathematics such that now, she expressed enjoyment (“Oh, math is really fun”) for the subject she had seen as “boring old math.”

Evidence that Courtney was cultivating a mathematical awareness was also apparent in her mathematics journal, as exemplified by this entry midway through the unit, where she described the tessellation she created during a mathematical art lesson:

![Courtney’s tessellation template](Image)
My tesleation \textit{sic} is a pattern of 2 squares that are different colors pink and purple it is a three x three so that the square has one extra square in the middle witch \textit{sic} would be the other colour. The actual\textit{sic} shape its self is a jagged shape I cut out the bottom and slid it to the top and i \textit{sic} cut out one side and it looks like this.

Of note here is that Courtney opens with specific mathematical details of the form of the ‘repeating’ pattern piece (‘two 3x3 squares’) and their alternating colours, thereby drawing our attention to their arrangement in the larger colour pattern (see Figure 5.3) In addition, she described how she created the template (Figure 5.2), moving sections for both sides (e.g., “slid it to the top”), and the shape of the edging of the tessellation piece. Interestingly, when looking at the tessellation design during the one-to-one interview, Courtney’s additional comments about her artistic process provided further insight into her developing mathematical awareness.

\textit{Figure 5.3.} Courtney’s tessellation design.
She began:

When I was doing it I kind of felt a happy feeling because the day was going so well because I chose two colours that felt happy to me, pink well, magenta—and purple, and I was just colouring around I didn’t even notice that I was making a three by three square, and then I noticed that, so I decided to colour the magenta inside because it has one leftover block inside, and I did the same with the magenta only switched it around, and I put purple squares on the outside of the square, I mean purple, three purple squares and three squares around the magenta.

Thus we hear how she was not initially aware of the pattern she was developing (i.e. “I chose two colours that felt happy … and I was just colouring around …”) and when she became aware of the mathematics (i.e. “… I was making a 3x3 square … noticed that, so I decided to colour and did the same …only switched around) she purposely created the overall design with a fairly elaborate repeating pattern. This is a point she reiterates as she continued to describe her tessellation, before turning her attention again to the template:

The pattern of two colours that are slightly different can make up a big picture if you do opposite colours, and I was feeling kind of frustrated there as well as happy, so I made jaggedy and when I was making this shape I was feeling kind of happy and frustrated so I made a jaggedy swervy kind of shape and I didn’t realize that it was, what you said just slide it up and it matches, the first time I tried doing it myself, it didn’t really match because I slid it to one side [laughing] and then it’s a yeah, it didn’t match, so I brought it to you and you helped me, and [pause] I can’t really find a way to describe it. . . . I think points and, like, points on a square fit with it so, well, squares make, and circles kind of feel of calm to me, because they’re original and stuff, and jagged points kind of make me, well, kind of stand out to frustrated or anger, and happiness is sort of like swerves and, um, you know just like a sort of like that roundy.

As Courtney describes her initial attempts at creating the template for her tessellation, she again illustrates how her mathematical awareness was cultivated through the experience. As Courtney indicated, her entry into the activity was on an emotional level (e.g. “I was feeling kind of frustrated … so I made a jaggedy swervy kind of shape …) and she was not initially aware of the mathematical properties involved (i.e. “I didn’t realize that it was … just slide it up and it matches”). Courtney attributes her increasing awareness of the mathematics in the lesson to the additional help she received from the teacher, after her first attempt failed to tessellate. Thus, this example illustrates Courtney’s cultivating mathematical awareness both within the lesson she recounts and through the reflections on mathematics she shares.

Therefore, from the ways in which Courtney was able to talk about her experiences as well as her work over the course of the unit, it seems that Courtney’s mathematical awareness
and understanding had grown, when mathematical concepts and knowledge were presented within an IE framework.

### 5.1.5 Theoretical findings: Cognitive tools

As the research question seeks to understand what the use of the IE theory and the ILPFs means to children and their engagement in elementary mathematics, it is important to consider how key aspects of the theory, the cognitive tools—how children create understanding—were manifested in the student data. These findings will be discussed in relation to the four principle work samples of the vision walk, angle activities, tessellation, and quilt design and are summarised in Table 5.1.

#### Table 5.1 Courtney’s use of cognitive tools (see coding scheme in Appendix 8)

<table>
<thead>
<tr>
<th>Activity</th>
<th>Mythic Cognitive Tools</th>
<th>Romantic Cognitive Tools</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vision walk</td>
<td>Mystery</td>
<td>Wonder, humanizing</td>
</tr>
<tr>
<td>Angle activities</td>
<td></td>
<td>Hero association, wonder, narrative, context change</td>
</tr>
<tr>
<td>Tessellation</td>
<td>Pattern, mental imagery</td>
<td></td>
</tr>
<tr>
<td>Quilt design</td>
<td>Pattern</td>
<td>Humanizing, hero association, wonder, context change and role play</td>
</tr>
</tbody>
</table>

Courtney’s math journal entry for the vision walk reflected her engagement and the cognitive tools of mystery (“What was it like moving in space”), wonder (“wondering if your guide”), both expressing a sense of enquiry and a fascination when taking part in the activity, and the humanized, practical aspect of the activity (“walking into people”).

For the angle activities Courtney’s work samples and math journal entries reflected the cognitive tools of hero association, wonder, narrative, and context change. She was able to engage with the narrative presentation (Egan, 1997, 2005) of a series of events in *What’s Your Angle, Pythagoras?* (Ellis, 2004) relating to the properties of angles, and to shift the context of her learning to place herself in ancient Greece with Pythagoras as a friend.

Within the tessellation activity, Courtney used the cognitive tools of pattern and mental imagery. In doing so she was able to connect with and utilize the Escher images that were shown in class to form her own mental image (Egan, 1997, 2005) of a tessellation and to create a unique
design that included a color/shape based pattern. Interestingly, she consciously associated aspects of her pattern with emotions (e.g., circles with calm, jaggedy points with frustration and anger).

The cognitive tools displayed in Courtney’s quilt activity, including the design and her math journal entry, were humanization, hero association, wonder, context change and role play, and pattern. Courtney formed an association both with the heroine, Clara, of the story that concluded the unit, *Sweet Clara and the Freedom Quilt* (Hopkinson, 1995), and with her uncle, thereby humanizing the mathematical concepts of the unit. To Courtney, both individuals were heroic. In addition, she was able to change the context of her learning from a sighted classroom to the visually impaired environment of her uncle. She reflected a sense of wonder in juxtaposing light and dark and extended this to use a Braille-like effect in her quilt patterns and design.

While I had initially questioned, through my participant observations, whether her level of development of understanding was in the mythic phase (Egan, 1997, 2005), the detailed coding and subsequent thematic analysis suggested that her understanding and engagement were becoming more sophisticated and were beginning to move into the romantic phase. She was able to understand her learning experiences and readily make connections between them, characteristic of romantic understanding.

5.1.6 Courtney: Summary

Overall, then it appears that the use of key aspects of IE, imagination and emotion, resonated with Courtney. The strongest theme emerging from the analysis of Courtney’s work samples and interview comments was her emotive engagement. She increasingly drew on her emotions throughout the unit and made repeated references to them during both the individual and the group interviews. In addition, her unprompted opening comment in the individual interview explicitly stated their importance: “This term actually gave me some emotion to put into my math.”

On the other hand, although imagination did not emerge as a theme in Courtney’s data, when asked in her one-to-one interview whether she had anything to say about IE, she commented, “Kids really want to imagine things.” This sentiment of almost yearning for freedom of thought and expression with imagination was reinforced during the group interview when Courtney stated, “Well, imaginative education, it helped kids, I find, because they like to imagine, most kids do, and I do, so!” Thus, it seems that being able to use imagination as a
learning tool was at the very least child friendly, and because it resonated with what children like
to do, it proved useful.

The connections that Courtney made between herself, her family, and the mathematical
concepts of the unit were especially meaningful and relevant to her. These extra connections that
Courtney built for herself allowed her to develop stronger self-confidence in her own abilities.
By the end of the unit she had grown in her ability to express herself in written form. As her
connection to and understanding of mathematics grew, she no longer hesitated to use either the
mathematical language introduced in earlier grades or the new math language of the shape and
space unit. This was reflected in her description of the differences between the properties of
straight, circular lines, and jagged lines in the tessellation activity and of the need for matching
squares in the angle activities related to the Pythagoras story.

Remembering that the cognitive tools of the IE theory are used to help create
understanding of the world around us and our living within it, identifying the cognitive tools that
Courtney utilized in the activities helps us understand which of the IE theory were meaningful for
her. She was able to connect with the overall presentation of the unit through the binary opposites
of vision and blindness, which fostered her personal engagement, and in turn, Courtney
frequently used wonder, hero association, humanizing, and pattern in her work.

As indicated at the outset of this section, my initial impression at the end of the unit, but
before the interview, was that Courtney was demonstrating characteristics of passive positive
engagement (Hagen, 2007; Hagen & Percival, 2009). However analysis of her work samples and
the opportunity to speak with Courtney in the interviews provided additional insight to me as the
researcher. I now recognized that the passive engagement, I perceived, was because Courtney
needed time for quiet thought and reflection to understand her feelings and to internalize the
developing understanding that she was getting with the mathematics. This “wait” time allowed
her to begin to produce work of increasing quality.

5.2 Kee’s engagement

Kee was a Grade 4 boy with slightly below-average overall achievement whom I had
selected to be part of the study because he had orally expressed a keen interest in mathematics.
He had transferred to the school in the final term of the year; information from his previous
school indicated an aptitude towards the subject.
My initial impression of Kee was that he exhibited characteristics of passive positive engagement (Hagen, 2007; Hagen & Percival, 2009). He frequently raised his hand to participate in classroom discussion but was slow to get started with activities and, on occasion, I regarded him as being off-task despite the appearance of listening to classroom instructions. Although Kee usually completed something for each activity, at times little written work was submitted. He needed frequent reminders and varied forms of encouragement.

Lesley, the critical friend, also assessed Kee as less engaged than other students, noting in her observations that he was “not really on task too much.” However, she also noted Kee’s frequent desire for oral involvement in class discussions, commenting on one occasion, “He was the first one, of course, to have his hand up; whenever you ask a question, up goes his hand.”

To examine this general impression of Kee’s engagement, I now turn to four themes that emerged from analysis of his classroom activity and interviews: (a) demonstrating a sense of wonder by using imagination, (b) making everyday connections, (c) developing self-confidence, and (d) cultivating mathematical awareness.

5.2.1 Theme 1: Demonstrating a sense of wonder

The first theme to emerge for Kee was demonstrating a sense of wonder. It became clear in the individual interview that Kee’s sense of wonder was far more expansive than I (as teacher) had realized. Early in the interview, when asked about his participation in the vision walk activity, Kee quickly responded with a question:

I feel really good about what I did. . . . Can I tell you about how I learn? Because I wanted to know a bit then about what was going on, then I think that what I was explaining here [his journal entry] was the differences and the, um, how it happened, and, like how it [vision/blindness] happened, how it didn’t happen, the ideas in the world about math, fractions, the perimeter.

Although this wide-ranging comment indicated an interest in contrasts ("how it happened, how it didn’t happen"), I was uncertain whether Kee was referring to the vision walk. So I asked for confirmation, which he gave, saying, “Yeah, if you really thought about it and you were walking down [the hallway] you could have just been pondering about different things.”

Indirectly then, these comments suggest that when carrying out the Vision Walk, he was wondering about a variety of things, “the differences . . . how it [blindness] happened, . . . how it didn’t happen, the ideas in the world about math, fractions, the perimeter.” This appears to take on added significance when we recognize that he prefaced his comments with the question, “Can
I tell you about how I learn?” Thus, he seemed to imply that he learned by “pondering about different things.” Indeed, Kee continued to demonstrate this sense of wonder as he shared a vision walk journal entry that he had not included for class assessment: “Imagineing [sic] is the key of math, I learnt [sic] fact is false compared to imagineing [sic]. Imagine a world with only fact and imagine a world only with imagination. Now morph them together, morphing is imagining? [sic].” He included a Venn diagram to illustrate his thoughts (Figure 5.4), explaining that he had written this at home when thinking about a world made up of only facts. As he contextualized this additional entry, “I just took a break to think about if what I was actually saying was right,” he indicates how the assigned journal entry triggered his sense of wonder. Thus, for Kee, engagement in this IE infused Shape & Space Unit meant time to ponder, to ask himself questions about the validity and merits of thoughts. Interestingly, within IE theory the purpose of wonder is seen as “encourag[ing] us to ask questions” (Egan, 2005, p. 91), and it appears opportunity for Kee to use his imagination grew from his emerging sense of wonder. In the group interview Kee further elaborated on the importance of imagining (i.e. wondering):

Kee: To build on like you [Jason] said, like imagination, Einstein’s imagination, I am using Einstein’s facts and building things, like he [Jason] said over there, that is absolutely true. . . . Imagination is more important than knowledge [knowledge was collectively said by all five research students at the same time].

Researcher: Can you [Kee] explain what you mean there?

Kee: OK, if you don’t really have imagination what’s going to go against knowledge? Build on your imagination. Knowledge and imagination balance each other out.
By questioning the relationship about imagination and knowledge, Kee seems to be supporting his view that demonstrating a sense of wonder, considering possibilities, which using his imagination enabled him to do, is needed in order to know.

5.2.2 Theme 2: Making connections

The second theme emerging for Kee was making connections between his learning and everyday things beyond the classroom. Kee’s conversation in both the one-to-one and the group interviews was peppered with words and comments that were not in his class work, but that illustrated he was making a variety of connections.

When asked during the individual interview whether there were resources or lessons that could be added to the unit, Kee commented,

Kee: Um, well, let me, I am trying to look out of the window to see what inspires me to say something. How do you say basically a book of life.

Researcher: Can you tell me about a book of life, would it help if I added it to this unit?

Kee: Because if you absolutely think about it, and you if close your eyes and walk around this room and felt every single thing, well that is reality but it would be space to you, ’cause you would not be able to see. So if I was to do this [Kee closed his eyes, held out his hands], I wouldn’t know if this [bar on a filing cabinet] was a bar or not, or was a thin narrow planet if I was like in space.

Researcher: Do you think there is a book of life out there somewhere that I could add?

Kee: Yes, there is lots of books about explaining it. Helen Keller, the Helen Keller book is actually very close to it, because it is like explaining her life and how she learned.

From his comment about looking out the window for inspiration, and his thoughts about how we come to know reality and space, whether sighted or not, Kee made connections to ways of living in the world. As he elaborated on his recommendation for a book of life, Kee made specific connections with the biographies available in the classroom (e.g., *Helen Keller: A Photographic Story of a Life* [Garrett, 2004]) to “lots of books” explaining the lives of others. It appears that Kee’s book of life was one that he felt would help other students come to appreciate their own lives more, through the lives of others. In building these connections between everyday situations and real-world people on the one hand and the learning he was doing in class on the other, Kee was engaged in the unit of lessons by making connections in a way that was personally meaningful.
Kee’s use of something from his everyday world to make a connection with his learning was again demonstrated when he described his quilt design (Figure 5.5.) during the one-to-one interview:

Very proud, very, very proud because at first when I heard the talk about space I started to think about planets, but then as you can see in this picture, I couldn’t have enough planets and I started thinking about adding, and this number that represents a 9, and my uncle he has his birthday on the 19th and I couldn’t fit in a 19th, so I put in a 9 for him, and the 5 represents my neighbour Jonathan.

Here, then, Kee connects the term space in the Shape and Space Unit with outer space (i.e. “the planets”) and connects certain numerals with information about friends and family (i.e. “… my uncle … birthday … so I put in a 9 for him and 5 … my neighbour …”). Indeed Kee drew from his personal interest in astronomical space in various ways. As seen in Figure 5.5, he placed a sun at the centre of a universe surrounded by many planets, which he emphasized by placing a yellow card behind his design. In talking further about the quilt design he discussed ideas he had about a
particular astronaut and potential for new discoveries in space:

Well I was thinking to myself that if Neil Armstrong could go back and go on to the moon again and explain something different, when he just came back that he made a discovery, then other people could make discoveries.

Of significance in this remark about Neil Armstrong was the fact that Kee had considered Armstrong returning to the moon and making a new and different discovery that could be extended to further breakthroughs. In doing so, it appears he connected his everyday interest and his imagination to extend what had already been discovered to the potential to make new discoveries. Thus, by making connections with everyday things such as a book of life, the world of blind people, and his personal interest in astronomical space, Kee found personally relevant ways to begin engaging with the mathematical concepts of the unit.

5.2.3 Theme 3: Developing self-confidence

Kee’s work samples and interview comments evidenced the self-confidence that he developed during the unit. For example, when asked to comment on his engagement with the unit of lessons, Kee stated,

Kee: Very fun [emphasis in his voice] and it could be like, [pause] I wouldn’t be as good at math, if I didn’t learn about this stuff.

Researcher: Can you tell me in what way you think it helped you become engaged with math?

Kee: Well everything, every single way because I really started liking it and all the projects we started to do, it just really made me happy.

Kee expressed not only enjoyment about what he had been doing, but also that he felt he was becoming better with mathematics. In doing so, he exuded a sense of self-confidence about what he was learning. When Kee discussed his tessellation design (Figure 5.6) this developing self-confidence was expressed in terms of pride:

Um, I feel very proud of myself [small snigger]. Well I was really surprised and very proud of myself, ’cause with this first concept which turned this pyramid, ’cause I had this card game right, and it spelt Ancient Egypt and stuff like that. Well I found myself, when I was talking about that, I was like picturing a pharaoh and then I kept up with pharaoh and I kind of made like a triangle shape.

Thus, Kee’s strong satisfaction (i.e. “I feel very proud of myself”) with the product and the way Kee comfortably explained the inspiration for his design, seemed to indicate a strong confidence in himself and his work. This strong sense of pride (i.e. very proud, very, very proud) was also
echoed when Kee described his quilt. Thus, Kee demonstrated a developing self-confidence during the unit and after its completion.

5.2.4 Theme 4: Cultivating mathematical awareness

The theme of cultivating mathematical awareness was evident for Kee with some of his work samples during the unit. For an activity early in the unit related to observing an internally rotating three-dimensional plastic ball, Kee wrote in his mathematics journal, “I noticed that the 2-D ball turned 3-D in the air and canged [sic] and I noticed that there was a rektangle [sic] when the ball canged [sic] and as it canged [sic] and by pressure.” Although Kee was incorrect in first stating that the ball was two dimensional, it was clear through his repeated use of the word noticed that he was become increasingly aware of mathematics in the lessons of the unit. Further mathematics journal entries contained details of the mathematics about which he was becoming
more aware. For example, in an activity related to triangles, Kee wrote, “I noticed that the three triangles turned into a big one.”

In a similar vein, Kee’s note on an activity page related to Euclid and building cubes stated, “I felt that there was a pattern starting to happen. If you times the number of faces time the length of the edge you get the number of cubes used.” (If the length of a cube is 2 and the number of small faces on one face of a larger cube is 4, then the number of small cubes to make the larger cube is 8). With this activity students were building increasingly larger cubes from small cubes and trying to see if there were any patterns emerging in the numbers. Kee’s developing mathematical awareness was becoming more established as the unit progressed.

The theme of cultivating mathematical awareness also emerged for Kee during the one-to-one interview. After the end of the unit, Kee was able to speak to how his current perception of mathematics differed from those he held earlier in the year. For example, when asked to describe his learning in the unit, Kee replied,

Well actually lots. Well about the shapes parts I learned like about angles, that’s the first time, this was actually the first time I did angles. It was the first time we used a protractor, and also I know that [few inaudible words] than the shapes that were done, about the angle, I wrote how I felt about what I saw, and that’s about it.

Thus, it appears that Kee’s awareness of properties of shapes related to their angles stood out for him, due to the novelty of using a new mathematical tool (i.e. “… the first time we used a protractor …”) and due to the way in which he recorded his experiences (i.e. “I wrote how I felt about what I saw”). Similarly, when asked whether the unit might have helped him learn math, Kee spoke confidently about his awareness of differences:

Kee: [excited tone of voice] Oh a lot!
Researcher: Tell me more about that.
Kee: Well it helped me learn about math because in my old class, I only really did numbers . . . but I didn’t really get like multiplication, and stuff to do with 360º angles, artistic ways of explaining math, different types.

Such comments indicate a positive attitude towards his recent learning and a growing awareness of additional mathematical topics (i.e. “in my old class, I only did numbers”). Furthermore, he was aware of additional ways he could explain his mathematical understanding (i.e. “artistic ways of explaining math”) after experiencing an IE-framed unit.
5.2.5 Theoretical findings: Cognitive tools

When considering how Kee used the cognitive tools of the IE theory (Table 5.2), his work samples provide further insight into his engagement with the lessons.

<table>
<thead>
<tr>
<th>Activity</th>
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<th>Romantic Cognitive Tools</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vision walk</td>
<td>Mental imagery</td>
<td>Wonder, reality</td>
</tr>
<tr>
<td>Angle activities</td>
<td></td>
<td>Wonder</td>
</tr>
<tr>
<td>Tessellation</td>
<td>Mental imagery</td>
<td>Reality, wonder</td>
</tr>
<tr>
<td>Quilt design</td>
<td>Pattern</td>
<td>Context change, reality, narrative, role play</td>
</tr>
</tbody>
</table>

With the vision walk activity, Kee demonstrated the cognitive tools of wonder, reality, and mental imagery. In forming an association between facts and imagination, Kee blended the tools of wonder and reality based on his own exploratory experience of being a blind person. He formed an early foundational association between the mathematical concepts being studied and the humanized context of blindness, forming images of what it would be like to be blind in the present day context and what it would have been like to be Helen Keller.

Kee was absent for some of the individual angle activities. However, the work he did complete in this area illustrated the cognitive tool of wonder. This was found in particular in Kee’s consideration of the properties of angles of various shapes.

During the tessellation activity, Kee utilized the cognitive tools of mental imagery, reality, and wonder. He was clearly able to utilize visual images of a pyramid from Ancient Egypt in designing his tessellation, and yet took this into the reality of a fish-like design. In his mathematics journal Kee marvelled at the transformation of a basic square into a pyramid and then into the fish design.

During his quilt work Kee utilized the cognitive tools of context change, reality, narrative, and role play from the romantic phase of understanding and pattern from the mythic phase. Kee found that he could put himself in a different context from the classroom and continue to absorb new knowledge. He had a clear sense of reality, being able to focus on notions of astronomical space to help learn about geometric space, as seen with the inclusion of astronomical and geometrical shapes in his quilt design to form irregular patterns, although the quilt was simply drawn and coloured. In addition, Kee engaged with the humanistic aspect of the activity by
adopting Helen Keller’s role in his exploration of objects in the room. This stemmed, I believe, from his understanding and general engagement with the narrative manner in which information about Helen Keller had been given.

Looking at all the data related to Kee’s use of the cognitive tools, I consider his level of understanding in the unit to be moving from the mythic phase of understanding towards greater romantic understanding.

5.2.6 Kee: Summary

Although at times during the one-to-one interview Kee’s lengthy comments required particular attention to garner their essence, it became clear that the initial impression of Kee’s passivity, when his engagement was evaluated as passive positive (Hagen, 2007; Hagen & Percival, 2009), was due to the amount of time spent thinking about or, to use Kee’s own term, pondering, the mathematical concepts and ideas that had been introduced. He would then try to make discoveries about the task at hand, and only after his sense of wonder had been satisfied would Kee begin the active phase of his work.

It also seems that through his sense of wonder Kee had connected with the binary opposites used in the unit. His interview comments indicated that he did think about multiple perspectives of situations, as indicated by his comments about the vision walk activity (“how it happened, . . . how it didn’t happen”) and about the contrasts he saw between imagination and knowledge.

Kee was garnering meaning of the mathematical concepts of the unit by making connections between the concepts, himself, and things of interest or importance to him in his life. He clearly connected with the ideas of vision and blindness, speculating how he might, if blind, manage in a sighted world. He also drew connections to his personal interest in astronomical space, speculating about a return to the moon for astronaut Neil Armstrong for further discoveries and including astronomical space in his quilt design.

From the thematic analysis I could see that Kee’s developing self-confidence was fostered in the unit in ways that were personally meaningful for him. As Kee experienced more lessons in the unit, he gained a sense of confidence, which developed and allowed him to feel proud about the work he was doing.

Through data analysis I gained a greater understanding of Kee’s engagement with the unit and his learning style. It was only during this stage of the research, however, that I realized the
significance of a comment Kee had written in his mathematics journal in the early stages of the unit: “Before you mark, talk to me if you need clear.” Kee had asked for the extra opportunity to demonstrate his understanding in an oral manner, confirmed in the interview to be a way of communicating that was important to him. This then raised questions for me about a fundamental thread of the IE theory (Egan, 1997, 2005) where children become more literate as part of a sociocultural world, and how much their level of print literacy affects their ability to be successful learners.

5.3 Freddie’s engagement

Freddie was a Grade 5 boy who was also an English as a Second Language student with a proficient level of oral language. I selected him to be part of the research study because he was interested in a variety of activities, and thus seemed a typical elementary student.

Initially I assessed Freddie as exhibiting characteristics of active positive engagement (Hagen, 2007; Hagen & Percival, 2009). He actively completed work for each assignment and showed a high degree of general participation. The critical friend, Lesley, also assessed Freddie’s engagement as active positive, noticing on one particular occasion that he was “listening very attentively and he got very [emphasis in the voice] excited,” and on another that “he was very quiet, listening attentively, got involved with the task.”

To further examine this first impression of Freddie’s engagement, several themes which emerged from the analysis of Freddie’s interview, group interview, comments, and work samples are considered in detail. These are (a) demonstrating a sense of wonder, (b) developing self-confidence, (c) making connections, and (d) cultivating mathematical awareness.

5.3.1 Theme 1: Demonstrating a sense of wonder

Freddie’s work samples and interview comments showed that he demonstrated a strong sense of wonder as the unit progressed. An early indication of this sense of wonder was evidenced in Freddie’s vision walk mathematics journal entry and sketch (Figure 5.7) at the beginning of the unit:

It felt like whoa am I going to run (walk) into the wall, so you like stick out your hands but can you amagine [sic] being blind and you don’t now [sic] where you’re going, but can you amagine [sic] just close your eyes and try to walk around somewhere, try to amagine [sic] that you are like Helen Keller, can’t see can’t hear, can’t talk. So I felt like wow! How can she survive but what if you can’t hear or talk?
The recurring questioning in Freddie’s journal entry at this early stage of the unit (“But can you imagine [sic],” “How can she survive,” “But what if you can’t hear or talk?”) indicated that Freddie was already beginning to raise queries about a focus of the unit.

![Figure 5.7. Freddie’s math journal entry for the vision walk.](image)

Partway through the unit, Freddie continued to raise questions that demonstrated his sense of wonder. After constructing nets of the Platonic solids and looking at the properties of their shapes, Freddie described one of the shapes in his mathematic journal summary (see Figure 5.8): “I think that’s how you draw it. Well that’s what I think the shape is going to turn into but I’m not sure [sic].” This illustrated a desire to propose an answer for what he had done but also a degree of uncertainty as to what the shape could look like.

![Figure 5.8. Freddie’s math journal entry for properties of shapes.](image)

After hearing the story *What’s Your Angle, Pythagoras?* (Ellis, 2004) Freddie continued to demonstrate his sense of wonder and enquiry when writing about angles in his mathematics journal: “If I was walking near Pythagoras I would stop to look at what he was doing and I would try to figure out what he was doing, but like it would be hard but i’ll try.” This suggests that Freddie was willing to both observe a situation and deliberate about what someone else was doing, even if it might seem difficult at first.
Working through a situation was not a problem for Freddie. A desire to understand his own sense of wonder was also noted in Freddie’s unit work when he openly speculated about a connection he felt to the book *Imagine a Day* (Thomson & Gonsalves, 2005). This book’s artistic illustrations started with a tessellation-like shape, which then morphed into a three-dimensional perspective of an everyday scene. In his mathematics journal Freddie wonders about why he feels the way he does about the book:

> I felt what it would be like if you were [sic] in the picture. Mrs. Hagen have you had a connection about this book, because I felt Happy but sad. I don’t know why but I can’t understand why? Did I feel like that, but I know it was a mathematice [sic] thought.”

Evidently Freddie had begun querying and representing ideas that occurred to him as he worked through different lessons of the unit. As the unit progressed, Freddie moved from simply posing questions, as with the vision walk journal entry, to speculating about potential answers, to then querying his own thought processes. Exercising and portraying his sense of wonder were comfortable for Freddie and seemed to help him engage with the unit in a personally relevant way.

### 5.3.2 Theme 2: Developing self-confidence

A second theme emerging from Freddie’s comments and work was a developing sense of confidence in his work during the unit. For example, when working on a sorting activity to discover properties of shapes, Freddie did something extra. He drew an example of a large square with many lines (Figure 5.9), with the caption “A regular square if you put little squares into littler squares. Can you count that. He He I can’t. This is imagination.” That Freddie would present this as a challenge to me, his teacher, seemed to illustrate his developing self-confidence in IE and mathematics.

![Figure 5.9. Freddie’s first challenge.](image)
Not long after, Freddie went on and provided a second challenge for others in his mathematics journal with a drawing of a very large oval-like shape (Figure 5.10), with a caption of “Know [sic, recte now] try to fill this in, p.s. if you do WOW, this IS a problem for you.”

His inclusion of such challenges, this one written just eight days after the unit had started, suggested that Freddie’s self-confidence with the work he was doing was developing quickly.

That Freddie’s confidence continued to grow throughout the unit is supported by the work he did for the tessellation design. Combining sketches with written description, Freddie created a step-by-step explanation over two pages of how he had made his starting square into an intricate design. He also sought further clarification early in his writing, which was not something he usually did by asking “Mrs. H. if you have any time would you spare a minute and tell me what’s the vertex?”, thereby illustrating a confident desire to develop his understanding and knowledge.

The mathematics journal entry included a design explanation in which the caption for Freddie’s four-piece sketch (Figure 5.11) announced “Ta ta, ther [sic] it is beauty.” This too points to Freddie’s confidence in and satisfaction with what he had done.

When reviewing this work during the interview, Freddie commented,
Well I feel like, I put lots and lots of effort into this, there’s like a pattern. It starts with felts, felts in the rainbow and then it keeps on building and building to pencil crayons and the same thing for the rainbow of colours, and, well, yeah, it is kind of like with reflecting to you off of the black.

Freddie was clearly pleased with his efforts and confidently explained details of the colour patterns at the top and bottom of his design (Figure 5.12), shifting from darker felt marker colours to lighter pencil crayon colours. He also used the metaphor of a rainbow to further describe his pattern.

![Figure 5.12. Freddie’s tessellation design.](image)

As the interview progressed, Freddie’s confidence grew. When I asked him if this unit had helped him become engaged with mathematics, he replied:

I think so yes, because now when I was like Grade 4, I didn’t really like mathematics. I wasn’t very good at it but now, very good at it and lots of description, and lots of marks, good marks and yeah. So like in Grade 4 I wasn’t cheering myself on to do it.

Drawing a contrast between his Grade 4 and Grade 5 experiences, Freddie expressed that he now felt “very good at it”, indicating a greater self-confidence about what he was achieving (i.e. “lots
of marks, good marks”) and his ability (i.e. “cheering myself on to do it”) to succeed. Freddie’s developing self-confidence also extended to his thinking about the learning of others and how they might benefit from this unit. When asked if he thought the unit would help other students, he commented,

I think so, yes, because like, um, lots of people catch on to stuff, like right away, some people need some time or to explain it to somebody, or something like that. Sometimes you’ve just got to explain it to them mentally. . . . You can keep working and keep asking questions and don’t ever be, like, don’t ever think that a question is perfect, because every question deserves to be asked.

Thus, in addition to having the confidence to explain that students have different learning styles requiring different amounts of time and different strategies to support their learning, Freddie readily offered advice regarding perseverance and inquiry. Clearly Freddie valued effort (i.e. “You can keep working”) and the opportunity to ask questions (i.e. “every question deserves to be asked) and felt confident in sharing those insights. Further, it appears that, as Freddie suggested, “Because lots of people were asking questions and not, like um, the questions were like easy to answer, but they really helped.”, the discovery-based orientation of the IE infused unit, contributed to his heightened self-confidence.

5.3.3 Theme 3: Making connections

A third theme arising from Freddie’s interview remarks and unit work was one of making personal and historical connections with various aspects of the unit. At the end of the individual interview, I asked Freddie if there was anything else he wanted to say. He asked to return to a piece of work that I had not selected for discussion.

Can I go back to this [mathematics journal entry on morphing shapes]? Well this one about morphing the shapes, you know morphing shapes? . . . And you know the optical illusion, Imagine a Day or Imagine a Night, you know those? I thought that the book was really cool because it had lots of mathematical pictures standing out to you and yeah, and I always look at, I had a connection of the book, because like every single, well not, practically every single day I like to go sometime on the computer with my mum or my family, and we look at illusions, and yeah I saw that book on the Google and we just read it, and yeah I was kind of excited and now I know how the book goes and stuff like that.

We had read the books Imagine a Day (Thomson & Gonsalves, 2005) and Imagine a Night (Thomson & Gonsalves, 2003) in class and had discussed how basic shapes were morphed to create the illustrations. Here, Freddie voiced his own connections with the reading and how this in-class experience was connected to an out-of-school activity he shared with his family.
Interestingly Freddie had also written about his connection with the book in his mathematics journal: “Mrs. Hagen have you had a connection about this book, because I felt Happy but sad. don’t know why but I can’t understand why? Did I feel like that, but I know it was a mathematice [sic] thought.” This journal excerpt, and his more explicit interview comments, suggests that his connections with these activities were particularly meaningful to him.

Freddie also formed connections with artistic and historical individuals discussed in class. When writing a response to Imagine a Day (Thomson & Gonsalves, 2005), Freddie formed connections with Dutch graphic artist M. C. Escher and mathematician and geometer Donald Coxeter: “I noticed that usher (I think that’s how you spell it) [Escher] How could he draw that but how could that other mathematic guy could figure that picture?” Through his questions, Freddie first expressed fascination with how Escher could design his images and then referred to how Coxeter (“that other mathematic guy”) could have mathematically interpreted many of Escher’s works (“figure that picture”). The connections between the lessons taking place in class and these leading individuals appeared to help Freddie to become engaged with the activity.

Freddie also made connections with historical figures featured in the unit. Although referred to in a fictionalized manner, the historical background of Pythagoras seemed particularly interesting to Freddie. Following the reading of What’s Your Angle, Pythagoras? (Ellis, 2004), Freddie wrote in his mathematics journal,

I would notice is the triangles that he’s making are all straight, and not with a ruler Also like if he got the calculations [wrong] his father would die (ex.) he won’t survive so its’ a risk because people, well the people that sail only take food for one trip.

Freddie had put himself in an imaginary situation with Pythagoras to review mathematical discoveries, and in doing so raised real-world considerations of nautical safety and survival, albeit in an imaginary setting. To Freddie, connecting to historical figures seemed to mean that both he and Pythagoras had experienced these mathematical challenges.

A final example of the personal and historical connections that Freddie made came towards the end of the one-to-one interview. Freddie’s fascination with notable mathematicians surfaced again when he asked, “Was Euclid the geometry king or something?” It appeared that Freddie wanted to understand the significance of Euclid and his ideas and discoveries to contextualize the understanding and knowledge that he was developing himself. For Freddie, making connections between his work and the human origins and development of mathematics seemed to reinforce his engagement and developing understanding and knowledge.
5.3.4 Theme 4: Cultivating mathematical awareness

Aided by the personal and historical connections that he was making with the mathematical concepts throughout the unit, Freddie grew in his mathematical awareness. For example, one of the early lessons involved hunting for shapes as part of three-dimensional objects in and around the classroom. Freddie had little difficulty filling a page with his observations, such as “The protractor is a cimy [sic, recte semi]circle,” “The window is a rectinagle [sic],” “The back table is a circle.” Not only was Freddie beginning to see mathematics in the classroom, he completed two pages of a similar shape hunt for homework at home.

Freddie was also increasingly adding mathematical details to his work. In an activity distinguishing and naming a variety of angles, Freddie wrote in his mathematics journal, “When I was figuring out the angles I noticed that acute is small and obtuse is BIG, also 90º digrees [sic].” Freddie also drew a minute right angle drawing and labelled it correctly.

In the final activity related to properties of shapes and angles, in which students had to tear off the four vertices from a paper square and the three vertices from a paper triangle and then reassemble the torn-off sections, Freddie was particularly pleased with his discoveries. His journal entry stated,

I noticed that when you were adding up the 90º, you’ll get (□) = 360 and I thought of the first triangle △ and looked at it, then I took the square □ and put the square and created a right angle.

The process of working with his hands and manipulating vertices appeared to foster his developing mathematical awareness, including properties of the right angle.

Fittingly, it was the concluding activity of the unit, the quilt design (Figure 5.13), that affirmed that Freddie had most certainly cultivated mathematical awareness. His lengthy mathematics journal description, worth quoting in full, reinforced this:
My quilt is for a blind person to understand and learn space and shape for a blind person, and my quilt is all about touch and like one picture is like rough the other one is soft, and like it’s like Braille a little and it present lots of mathematic desription [sic]. It kinda represents piece [sic, recte peace] happiness. I honestly don’t know why. Also the quilt represents me a little because I love the 5 (five) stones [photographs of Neolithic stone models at the Ashmolean Museum were shown in class], I don’t know why but they look very cool or mathematic and their duels [sic] of the shape... My quilt has a right angle 90º degrees [sic], an acute, and a obtuse. There fractions like ¼, ½ ¾ and 1 whole. It has perimeter, area. Also it has addition, multiplication, subtraction and times table. My cuilt [sic] will be like standing out numbers and mathematic thought, but the numbers will be like felt [tactile] numbers, there’s going to be 2D shapes, 3D shapes, have some Geometry, like Parallel lines, volume, perimeter, ray, place value, addition, subtraction, multiplication [sic], Division, octahedron, detoctahedron [sic], Icosahedron, cube, tetrahedron, fractions. Properties of some of the mathematical shapes, some stone (5 stones). Right angles, acute obtuse, 90º degrees [sic] angles. It will be very neat there’s going to be congruent figures, perpendicular lines, intersecting lines, area, perimeter.
In his quilt design and description of it, Freddie clearly represented many mathematical terms and objects. He also included tactile ideas ("the numbers will be like felt numbers"), illustrating his empathy for blind people. What caught my attention, however, was his metaphorical representation of peace and calm ("It kinda represents piece [sic, recte peace] happiness"), although he was uncertain about the reasoning, in the midst of the mathematical features, as if showing the ease with which feelings and math arose in such an activity. Although in his interview Freddie reiterated only some of the mathematical ideas he had incorporated into his design: "I feel like it has lots of mathematical thoughts in it. Um, there’s like degrees and like 90º right angle and all kinds of stuff. My quilt is practically like Braille, a little" this culminating activity clearly demonstrated that by the end of the unit Freddie had cultivated mathematical awareness of an extensive array of ideas about shapes and space.

5.3.5 Theoretical findings: Cognitive tools

Freddie’s use of the cognitive tools of the IE theory was exemplified in the four activities, as shown in Table 5.3, helps us consider the meaning this theory had for his engagement in elementary mathematics.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Mythic Cognitive Tools</th>
<th>Romantic Cognitive Tools</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vision walk</td>
<td>Mental imagery</td>
<td>Hero association, reality, wonder</td>
</tr>
<tr>
<td>Angle activities</td>
<td></td>
<td>Wonder, context change, hero association, reality</td>
</tr>
<tr>
<td>Tessellation</td>
<td>Mental imagery, pattern</td>
<td>Wonder</td>
</tr>
<tr>
<td>Quilt design</td>
<td>Mystery</td>
<td>Wonder, humanizing</td>
</tr>
</tbody>
</table>

During the vision walk activity, Freddie utilized the cognitive tools of mental imagery from mythic understanding, and hero association, sense of reality, and sense of wonder from romantic understanding (Egan, 1997, 2005). Freddie was able to demonstrate an understanding of the context of being blind that both Helen Keller and the class guest, Uncle D, had experienced and used this to expand his own understanding of shape and space. He now appreciated space as a grey tone rather than black. For Freddie, this understanding developed into a fascination about the world of a blind person.
With the angle activities, Freddie utilized the romantic cognitive tools of wonder, context change, hero association and sense of reality in developing his increased understanding of angles. As has been shown, he had no difficulty forming a psychological, association with a historic figure, namely Pythagoras and used this to assist his wondering about reality of the life of the historical figure.

In the tessellation activity, Freddie demonstrated use of the cognitive tools of mental imagery and pattern from the mythic phase and wonder from the romantic phase (Egan, 1997, 2005). Clearly, the tools of mental imagery and pattern worked together to give Freddie a deep understanding of his own design and a developing appreciation of the mathematics behind his design, as evidenced by the work he completed during the unit. His interview comments further testified to a developing sense of wonder that helped him to wonder more about further aspects of tessellations, as detailed above.

In his quilt design, Freddie used the mythic cognitive tools of mystery and pattern, and the romantic cognitive tools of wonder and humanization (Egan, 1997, 2005). Freddie began the activity at a mythic level using the cognitive tool of mystery, discovering elements of the design and then enquiring further, going into a deeper level of wonder and understanding. He became intrigued with the historical and human elements of mathematical discoveries, and utilized this to develop his own understanding and geometric knowledge.

Taking the four activities as a whole, Freddie used more cognitive tools from the romantic phase of understanding than from the mythic phase, and was therefore more firmly grounded in romantic understanding (Egan, 1997, 2005).

5.3.6 Freddie: Summary

The depth of Freddie’s enthusiasm and engagement in the unit was something I had not anticipated when I selected him to take part. This continued in the interview, frequently illustrated by an excited tone of voice and a smiling face.

What emerged from the analysis of Freddie’s interviews and class work was, firstly, a very strong sense of wonder. He asked an increasing number of questions, both in his work and in the interviews, that, as time went by, he began to answer for himself. At the beginning he speculated about how someone like Helen Keller could manage in a sighted world. However, by the time of the group interview, Freddie’s general sense of wonder combined with the self-confidence he had developed during the unit; he made a profound statement for a Grade 5 student
to the others: “Keep asking questions and don’t ever be, like, don’t ever think that a questions is perfect, because every question deserves to be asked.”

Freddie made a number of meaningful connections between what he was studying and the historical figures and mathematicians mentioned in class, such as Euclid, Pythagoras, Coxeter, and Escher. He seemed to appreciate the historical and human perspective of the discoveries of these individuals and the connections to what he was studying. These connections seemed to develop further; Freddie then wanted to share his developing mathematical awareness with his family at home, regularly researching further information about mathematical illusions.

Clearly, Freddie’s early experiences continued in a positive way and helped him to become an active engaged participant, finding a place of comfortable learning for himself through the IE theoretical framework and sociocultural orientation of the unit. My analysis confirmed my initial impression of his engagement as showing characteristics of active positive engagement (Hagen, 2007; Hagen & Percival, 2009).

5.4 Grace’s engagement

Grace was a Grade 5 girl who had shown keen interest in mathematics and yet did not seem to be performing to her full potential. She seemed attentive and generally quiet, but preferred not to join in open class discussions. I selected her to be part of the study as representative of the underachieving students seen in many elementary classrooms.

Based on my participant observations and regular teacher assessment, my initial impression of Grace’s engagement, before the interviews and full analysis, was that it showed characteristics of active positive engagement (Hagen, 2007; Hagen & Percival, 2009). The critical friend, Lesley, agreed with this assessment, noting several times that Grace was “very quiet today, listened attentively, became actively involved” or that she occasionally “just wanted reassurance that she had discovered something and that she was right about her discovery.”

Although she had a quiet personality, Grace appeared to relish the opportunity to demonstrate her understanding of questions orally in both interviews, especially the one-to-one interview. She was a willing participant, answering all questions with longer answers as the interviews progressed.

Four themes emerged for Grace: (a) drawing on imagination, (b) making everyday connections, (c) cultivating mathematical awareness, and (d) developing self-confidence.
5.4.1 Theme 1: Drawing on imagination

A theme that became increasingly evident through analysis for Grace was that of drawing on her imagination. Grace’s use of her imagination strongly came through in her quilt design, the final product of the unit. Her mathematics journal entry about her quilt (Figure 5.14) clearly indicated her views about imagination:

My quilt symbolizes learning about Math. It is a math map, it teaches blind people about math. It has digrees [sic], circles and squares and other shapes. This quilt has a treasure. The treasure is, if you follow the path it will spell the word imagination. This map is made from imagination. My quilt represents to me special learning for the blind.

Figure 5.14. Grace’s quilt design.
Grace included a great variety of mathematical elements in the design, such as patterns, line segments, triangles, and circles. Of particular note was Grace’s representation of imagination as a treasure. This suggests that she placed a high value on imagination as a tool of learning, and also that she enjoyed the imaginative intrigue of leading her readers along a path towards treasure.

During the one-to-one interview Grace extended her mathematics journal comments:

Well, it had lots of colours in it and has a pattern around the border that is interesting, um [small laugh], and a path leading to the finish, and along the way there is also some math stuff like intersecting lines and 90º angles and here is a river made of circles and yeah.

Researcher: There was something when I looked at your quilt I noticed was very, very interesting on the pathway, could you talk about that, and the pathway on your quilt?

Grace: If you follow the path, if you collect all the letters it will spell imagination and I thought about it because like this quilt is made from my imagination, so why not just like patterns and shapes, and [pause].

Here, Grace points to imaginative ways she designed the quilt, with a patterned border (i.e. repeating the 4 operation signs), winding pathways and rivers of circles as well as typical math features like intersecting lines and right angles. Of note however is Grace’s recognition that since she “made the quilt from her imagination” she would embed the word “imagination” into the quilt. As the one-to-one interview continued, Grace provided further details when asked how she felt about seeing her design again:

I feel pretty happy with myself, if I didn’t have the quilt and like to solve from my imagination I might have come up with the same thing. If you put it on paper then you, like, can remember it. It’s like a piece of paper when you write on it, it is like a memory on a piece of paper, so. . . . When I thought about, like a 90º angle, so why not make it fancy, so I thought about it [laugh]. . . . I don’t know why there are lots of fours, this is square fields and lines and like one fraction and some questions and answers.

Grace’s comments suggest she felt free to use her imagination to develop mathematical components, such as a 90º angle, into a more aesthetically pleasing element of the overall design. The representation of imagination as a “memory on a piece of paper” further illustrates her imaginative ideas and the value she attributed to drawing on imagination.

However, it was her concluding comments for this segment of the one-to-one interview that illustrated the depth and significance of her drawing on imagination: “It’s like this page is like all full, full of my imagination, just like splattered on there.” The opportunity to “splatter”
her imagination on a piece of paper, to illustrate what she understood and learned about mathematics, apparently gave Grace great pleasure.

The value Grace placed on using her imagination was further illustrated when I asked her whether she would like to say anything about IE:

[With emphasis and a laugh] It is awesome! It [IE] feels like a gigantic building, well there should also be like a building of the museum of shapes, so like kids can go there and there is like a touch and hear room, so like they can like, look inside of the shapes and feel what it feels like inside of a circle, like the rough edges inside. So [IE] it is like, and there is also like programmes, and like magic of shapes and how it [IE] can help you through education and stuff.

Grace’s association of IE with a real-world, physical object (i.e. a gigantic building) seems in itself to be an imaginative way to think about Imaginative Education. And the way in which she imagined the sort of experiences appropriate for creating a sensorial environment, through which shapes could be understood, aligns with the somatic experiences of the first phase of understanding in IE. When Grace raised a question during the group interview: “Well if we didn’t have imagination what would our clothes look like and what would be inside rocks?” I asked her to clarify what she meant. Grace’s response, “Yeah, because like if there wasn’t any imagination there, inside rocks there would be all like nothing.” implies that drawing on her imagination to form mental images of geological properties, thereby bridging our understanding of things we cannot see, seems to suggest Grace believes that there is more than an aesthetic and creative aspect to imagination. Finally, of particular significance was Grace’s ability to see the value that the use of imagination and IE had outside this classroom: “[It] can help you through education and stuff.” It was not just in the immediate context of an elementary classroom and the learning of mathematics where imagination and its general purpose had value; for Grace, it appeared there was a life purpose to imagination.

5.4.2 Theme 2: Making connections

A second theme to emerge for Grace was the connections she made between what she was studying in class and the everyday world around her. These included connections with and understanding for those who were younger, her peers, and people who were blind. In addition, she made connections with the general environment around her.

Grace’s initial connection to the experiences of blind people appeared in her mathematics journal entry for the vision walk activity:
When I put on the mask and pretended to be blind, this is how it felt like, scary, weird, hard to find direction, don’t know where things are. This is how I think blind people will learn about shape: by feeling a shape like there [sic] face, wooden shape or box.

Here Grace not only described her affective response to the role-play activity, but also showed that she had thought about how blind people would learn what she was learning (i.e. “… I think blind people learn about shape: by feeling shape”). When I asked Grace how she felt about her vision walk work, early in the one-to-one interview, Grace again expressed empathy for blind people:

Well, I feel bad for the blind people because you can’t really see anything and you feel, you were not blind from the beginning and you don’t really know the shapes and how they look like, but like you can just feel them.

As such it was apparent that Grace connected well with the ways in which blind people could learn with the sense of touch. Indeed, in additional comments she attempted to elaborate how learning through touch might assist in determining colour.

Um well, I think if like a cube was hot then you would know what colour it was and like red, and stuff. If it’s like a cube is like freezing take it out and it is feeling cold and a blind person would know what colour it is, exactly that colour and that would be really cool . . .

Interestingly, by suggesting an association of colour and temperature (red with hot, possibly blue with cold), Grace appears to provide a glimpse into connections she had between these senses of touch and sight.

In an early unit activity in which students were asked to think about becoming a shape and choosing a colour for their shape, Grace made connections with the world around her, writing in her mathematics journal:

If I were a shape I would be a tree, because it would help the environment. I would be a maple tree, I would live near Hazelwood Elementary [pseudonym], I would be a very strong tree.

Thus, for her shape, Grace chose a living object, a Canadian symbol, and spoke of the environmental functionality of the tree. At first glance this appears unrelated to the more conventional shapes (i.e. cubes, octagon) that might be expected in a mathematics activity. However, when asked in the one-to-one interview if the unit helped her to learn mathematics, Grace returned to this connection to the everyday world around her, to demonstrate her understanding:
Well, it helped me, um, in lots of ways, because like a cube, you can measure it and that will teach me about numbers and degrees and if you like there, I didn’t know about degrees and angles, well, if there wasn’t no degrees and angles on earth there would be nothing on earth, so. Like trees, they have a certain shape, like sort of like a cylinder shape for the trunk, but if, like, they didn’t have any degrees or angles or something, they would be gone [laugh].

Grace’s comments, then, illustrate that she was applying what she learned in the unit (e.g. angles) to objects in her environment (i.e. trees). In another example, Grace not only connected mathematical shapes with man-made structures but used her imagination, to extend the possibilities:

Um, well ok, you have a building. If you didn’t have any building then, um, like all buildings, then like all buildings are made from different shapes. But they are mostly made up of triangles, rectangles and squares, but inside them there are more different things. It is like, um, like inside of a cube and there’s different stuff that goes inside. And the school inside has doors, or buildings have doors, boxes, windows and different shapes. If it was like an imaginary building, the inside could have triangle windows and circle doors, with square knobs, it is like an opposite.

The awareness and understanding Grace developed during the unit about mathematical properties and shapes within buildings had been combined and connected to an everyday structure. Not content with this connection, Grace had gone on to propose an imaginary building that could incorporate differences to what was normally experienced (“the inside could have triangle windows and circle doors, with square knobs”).

5.4.3 Theme 3: Cultivating mathematical awareness

The third theme arising from the analysis of Grace’s work and interview comments was the cultivation of her mathematical awareness through a process of discovery. When asked to review her work for the angle activities in the one-to-one interview, Grace commented,

Well, Pythagoras was, like, trying to find a way to help his dad in his workplace, but then he, like, discovered the right angle shape, so he went to go and measure with tiles, then he got into trouble but then he didn’t get in trouble. [laugh] So he measured it and, like, made sure and just looked around at what discoveries he made and it was like when he got home he looked at the math and he saw the right angle with the islands. So he measured that and he told his dad and then he lived happily ever after [laughing].
Grace’s happy tone of voice and body language in the interview indicated the angle activities and the story of Pythagoras had been particularly pleasurable. That she remembered many details of the story seemed to indicate that Grace had connected with the discovery process that the character of Pythagoras had followed. That she provided her own illustration (Figure 5.15) and description in her mathematics journal pointed toward the mathematical awareness she was cultivating here:

This is sort of an example of what Pythagoras did with the tiles: 3, 4, and 5. Some right angles are bigger or smaller, maybe even thinner or fatter. If I were Pythagoras and no one believed me about the island then I’d feel really sad or mad.

In addition, during the one-to-one interview, although not obvious when reviewing her class work, Grace revealed that she had made her own discovery as well:

Um, well, it was fun making new shapes and, um, I tried to stretch out the lines and the corner. . . . And I measured them but then it, they turned out to be like something just like, um, a 90º angle when it’s like the corner was longer or shorter, it seems to be like always 90º on some sheets.

Thus, Grace extended the angle measurement activity past simply measuring a variety of drawn angles with a protractor. She drew more angles than had been presented on the activity page and extended the lines of the drawn angles, through which she made her own mathematical discovery. With her comments Grace was trying to explain that the length of the lines of the angle drawings were irrelevant; the angle always stayed the same.

Grace was eager to give more examples of her mathematical awareness (i.e. geometrical development) as we continued the interview:

![Figure 5.15. Grace’s Pythagoras illustration.](image-url)
Grace: Well, like suppose in a box there is like a hollow space, but if you cover it up you cannot see the inside of it. But if you open it then you can see something, or if you cut it you can also see something, so like inside. Yeah, it is like doing surgery on a shape.

Researcher: Oh, interesting, I hadn’t thought of doing surgery on a shape. … Can you tell me a bit more about that? …

Grace: Like you can have scissors and just like cut them, but when you finish looking inside, like first, there’s lids on top, and you just like put it in, like put the stuff that you want inside, the place where you want them to, and then you shake them up and, first you put the lid on, and then shake them up, and cut it open, then you could see like where the shapes all are and stuff, so basically it is like inside a human body.

The spatial awareness and visualization that Grace used, here, was beyond that of a typical explanation of a net of a three-dimensional object, and her developing awareness of multiple perspectives of 2-D and 3-D shapes emerged when Grace described learning opportunities for different aged students:

Grace: It is like a picture book, you can like take some pictures of a cube, circle, triangle, like 2-D and 3-D, then like after that you could take pictures and cut them up and then take pictures of the inside so they could know what the inside and outside would look like.

To help children learn mathematics, then, Grace had a clear idea of a book which would include both the internal and external perspective of three-dimensional shapes, and photos of 2-D shapes, possibly pointing to an awareness that the visual representation of “slices of 3-D shapes are inevitably 2-D. For older students, Grace suggested other activities:

Grace: Um, maybe like kids could do assignments to study a certain shape, and then they can make a poster or report about what they have found out.

Researcher: That would be very interesting. So would you let students choose which shapes they were to study? What would you study, if you gave yourself this assignment?

Grace: Well, maybe a circle because it is the most interesting, because like a circle is not like a square where you can like divide and like a square where you can find triangles and stuff. It is just like round circle, and if you cut it up, you can find the angle or something. If you cut it, it would also be a semicircle.

Here again we get a glimpse of Grace’s cultivating mathematical awareness and the importance of discovery (i.e. “report what they found out”). When she speaks of her shape of choice (i.e. circle) she shares her knowledge of semicircle and indirectly her sense of a secant (i.e. cut it up, you can find the angle”) or at the very least a recognition that if she cut the circle (maybe as you would a pizza) an angle would be formed. Hence we see that in suggesting an appropriate
assignment for students of her own grade level she illustrates some of her learning about the properties of shapes.

5.4.4 Theme 4: Developing self-confidence

The final theme emerging from the analysis of Grace’s interview and work samples was one of developing self-confidence. In her mathematics journal for the tessellation design (Figure 5.16) she commented,

My template started off as a square but then I morphed it, I cutted out a semi-circle and Semi-squar [sic]. I put the sure [sic] across from the place I cutted out from and I did that with the semi-circle too. When I did my picture I did it diagonally across the page I Did it like a puzzle and I finished. The most challenging one was colouring it because I wanted to make a pattern, but I managed.

Thus Grace successfully completed the challenge she set herself and confidently provided further details in the interview:
It is bright and, yeah, sort of pattern, it has like some shapes on it and when I first thought about it I just, like, did a pattern that way, but when I looked at, there is also patterns going that way [pointing in different directions], so... When I looked at this part, the yellow part, I was thinking it was strange but then I looked up here with also part yellow, so then I counted like how many was out here, one, two, three, four, five, and then this one is the sixth one and if you go another six that way you will also be the same as this one.

Grace could recollect a lot about her thinking when she originally made the design with a variety of colouring patterns going in different directions. After explaining the title she had given her tessellation, she speaks of her confidence in her work overall:

Grace: Well, The Cheese, it’s just like this Cheddar cheese and yellow cheese, and I don’t well [pause] and darker Cheddar cheese and too bad I didn’t put macaroni shapes on it, then it would become macaroni and cheese [quiet laugh].

Researcher: [laughing] You could have, couldn’t you? ... If you think about all the work that you did for this unit, how do you feel about the work you did overall for this unit now that you are seeing it again?

Grace: Well, I am very proud of myself, and I am happy with my work because I put all my effort into it and like, used my mind and knowledge through what I know, so yeah. Hence, Grace’s self-confidence is evident in both the comfort with which she “jokes” about her design (e.g. “too bad I didn’t put macaroni shapes on it’) as well, the pride she expresses and her pleasure with the product. Of importance also is her recognition that both effort and knowledge contributed to her success, in an IE-framed Shape and Space unit.

5.4.5 Theoretical findings: Cognitive tools

As previously mentioned, an important aspect of this research question is how children create understanding when the cognitive tools of the IE theory are used. Grace’s use of the cognitive tools is exemplified in the four activities identified for detailed analysis.

**Table 5.4 Grace’s use of cognitive tools** (see coding scheme in Appendix 8)

<table>
<thead>
<tr>
<th>Activity</th>
<th>Mythic Cognitive Tools</th>
<th>Romantic Cognitive Tools</th>
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<tbody>
<tr>
<td>Vision walk</td>
<td></td>
<td>Wonder, reality</td>
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<tr>
<td>Angle activities</td>
<td>Mental imagery</td>
<td>Hero association, humanizing, context change</td>
</tr>
<tr>
<td>Tessellation</td>
<td>Mental imagery, pattern</td>
<td>Humanizing</td>
</tr>
<tr>
<td>Quilt design</td>
<td>Mental imagery</td>
<td>Humanizing, reality</td>
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</table>
With the vision walk activity, Grace demonstrated a strong sense of both wonder and reality from romantic understanding (Egan, 1997, 2005). Her responses also indicated aspects of somatic understanding; her own bodily experiences with this activity caused her to wonder how blind people and young children develop their understanding of the world around them.

While engaged with the work on angles, Grace used the cognitive tools of mental imagery from mythic understanding, and hero association, humanization, and changing context from the romantic phase (Egan, 1997, 2005). She particularly identified with the suggested emotions of Pythagoras as a child, shifting the context of learning from the present to ancient times. Grace was able to internalize her thoughts and emotions to further develop her understanding and knowledge of angles, including experimenting and making new discoveries.

For the tessellation activity, Grace utilized the cognitive tools of mental imagery and pattern from mythic understanding and humanizing from romantic understanding. Grace employed a strong sense of pattern in her tessellation design and colouring, and likened this to real-world and human objects of food and cheese. In addition, she was able to explain her metacognitive thought processes about the effort she put into this work and act further upon these ideas.

During the quilt activity, Grace utilized the cognitive tools of mental imagery from mythic understanding and humanizing and sense of reality from romantic understanding. This activity showed the strongest use of mental imagery by Grace, in her representation of imagination in her design. Grace also expressed a real connection with the human qualities and context of the characters in the book *Sweet Clara and the Freedom Quilt* (Hopkinson, 1995) used as the beginning of the activity that concluded the unit, when she called her mathematical design a “math map.” For Grace, it seems that using one’s imagination was a natural and usual process.

Grace’s use of the cognitive tools with the four activities indicates the ways in which she developed understanding of the shape and space unit, and the meaning it had for her. The thematic analysis showed more frequent use of tools from the romantic phase of understanding (Egan, 1997, 2005) than the mythic phase, especially with regard to the angle activities. Thus, the depth of development of her understanding was firmly based in the romantic phase.

5.4.6 Grace: Summary

It was evident that Grace became engaged with the imaginative element of the unit’s theoretical framework. The growth in her development and understanding of mathematics, from
the beginning with activities like the vision walk to the end of the unit when she completed her quilt, was evident. She found an avenue that allowed her to imaginatively explore a variety of topics related to shape and space and make mathematical discoveries about which she could express her understanding in multiple ways, such as through numbers, words, journal writing, and creative illustrations.

Grace also seemed to connect with the binary opposites of vision and blindness that grounded the unit, such that she demonstrated frequent connections between the mathematics she was learning and the everyday world around her, for example, environmental elements such as trees and geological properties. Thus Grace showed that she could apply her mathematical learning beyond the classroom.

During the unit, Grace also found a degree of self-confidence that allowed her, even though a quiet personality, to make suggestions for learning to others, both those younger than herself and her peers. She did this in both her one-to-one interview and the group interview. Her growing confidence also helped her to make suggestions for others that were not restricted to student matters; she was very clear in providing a suggestion to researchers that “if you find out what kids are more interested in, and make that into math, you would probably find out more.”

Using a variety of cognitive tools, Grace was actively engaged and participated throughout the unit. The analysis confirmed my initial assessment that Grace’s engagement was active positive (Hagen, 2007; Hagen & Percival, 2009).

### 5.5 Jason’s engagement

Jason was a Grade 5 boy that I selected to be part of the research because he seemed very comfortable with being assigned a variety of activities for his class work. Prior to beginning the interviews, I initially assessed Jason as representing active positive engagement (Hagen, 2007; Hagen & Percival, 2009). I arrived at this assessment using my participant observations and regular teacher analysis of Jason’s classroom work from the unit.

The critical friend, Lesley, also assessed Jason as showing active positive engagement in the unit, noting on one occasion that “J was also listening attentively; he went “H – O – L – Y” [expressive voice]. Anyway, he was definitely actively involved with all aspects of the lesson today,” and on another that “he [Jason] was the most active of all today; he had his hand up answering all the questions. There are many kinds of discoveries he had seen.”
The themes which emerged for Jason during the analysis are (a) cultivating mathematical awareness, (b) making connections, (c) developing self-confidence, and (d) developing a sense of wonder.

5.5.1 Theme 1: Cultivating mathematical awareness

The growth in Jason’s mathematical awareness was illustrated in a mathematics journal entry for the tessellation activity (Figure 5.17), approximately half way through the unit:

Tessellations—Morphing Shapes Template—My template used to be a hexagon. It used to have 6 60º angles, and 6 sides that are 2 cm long. After I morphed it, it became something else. This new shape has 4 sides that are 2 cm, 4 sides that are 1.2 cm, 2 sides that are .5 cm, 1 side that is .3 cm, and 1 side that is 6 cm. It has 6 60º angles, 2 55º angles, and 4 65º angles. Picture – My picture is a bit different. Since I used a hexagon like template, I could fit in much more than a square template. I changed my hexagon-like shapes into fish. It has many angles and sides, and many hexagon-like shapes make it up. My colour pattern goes red, purple, red etc. [sic] My hexagons are on an angle, and they all point north-east.

Jason’s work included lengthy descriptions indicating the depth of his engagement with the mathematics concepts. For example, in the template section he gave measurements for the

Figure 5.17. Jason’s tessellation design.
hexagon sides in centimetres, counted all the sides of the fish-like shape, and gave the angle measurements. In the picture section of his journal entry, Jason also noted the greater number of fish-like designs he fitted onto his piece of paper compared to other students, the pattern of the colouring design, the angle of orientation of the template placement and, finally, the northeast directional orientation. In creating his template, as shown in Jason’s sketch (Figure 5.18),

![Figure 5.18. Jason’s tessellation template drawing.](image)

he used his knowledge and experience with a protractor to transform the square of card into a hexagon and then a multi-sided figure.

With the new knowledge that Jason acquired in the unit he found that this task was an opportunity to challenge himself: “I wanted a challenge because the squares looked a bit too easy.” The extensiveness of Jason’s pre-interview description (journal entry) indicated the depth of his mathematical awareness.

A later indicator of growing and deepening awareness about and engagement with mathematical concepts was the journal entry Jason completed during the quilt lesson (Figure 5.19):

My quilt has a weird way of using stitches. The quilt is divided into 56 different rectangles that pop out because of the stitching. In each rectangle is a picture of a word that has something to do with math. These pictures and words pop out because of stitching too. That is how blind people feel it. My quilt isn’t just shape and space. It has numbers, fractions, and place value in it too. In shape and space my quilt has angles, 2-D shapes. 3-D shapes, lines parallel lines, intersecting lines and the 5 platonic solids. My quilt to me symbolizes how learning math can be fun, because I thought this shape and space unit was fun.
Jason’s comment “These pictures and words pop out because of the stitching too. That is how blind people feel it” indicates an awareness of the tactile needs of blind people. Both within the quilt design and his description Jason readily illustrates a breadth of mathematical ideas. In addition, Jason voiced his enjoyment and engagement (e.g. “My quilt to me symbolizes how learning math can be fun, because I thought this shape and space unit was fun”). Jason’s interview comments about his quilt indirectly provided further insight into his awareness of mathematics:

Well, I thought this was a pretty good piece of work. The only thing that I could have really done better on this was press a bit harder because I saw that some other kids had actually like made their lines bumpy and stuff like that, so if you closed your eyes you could sort of make a shape and then that is really colourful but blind people wouldn’t see that, so that is an improvement I could have made.

Here, Jason states a satisfaction with his work (e.g. “I thought this was a pretty good piece of work”) and thereby implying satisfaction with the mathematics it illustrated. This is further
reinforced when he suggests an improvement, saying he thought he could have done better by “press[ing] a bit harder,” that addresses the needs of a blind person but raises no such concerns or additions in the mathematics symbols.

Jason went on to clarify this distinction when discussing the colour of his quilt design:

Yeah, like I used a whole bunch of red and green and pink and stuff. If it were a blind person learning about space and shape, they couldn’t see that. But one thing that I felt was to be good about this piece, was that I did more than half of it about space and shape, but just to go through it a bit more, I did like numbers and place value and fractions up here, and then on top of all the 2-D and 3-D shapes I did the five Platonic solids in a row, and so if you remember we got one of the pages, and it said like each one had an element, like the tetrahedron represents fire, and the cube represents earth, so I coloured them accordingly, like the water one was blue, and the earth one was brown, and then I just wrote above it, so I thought that was pretty cool.

That Jason had included multiple representations of mathematics on his quilt, such as numeric and figural fractions, numbers, signs of operation, and base ten materials, as well as material from the shape and space unit, two-dimensional shapes, three-dimensional objects, and variations of line drawings, again indicates a deepening awareness of mathematical content, which also extended to Jason’s colouring and organization of the Platonic solids to match the classical elements of air, earth, fire, and water, for the sighted viewer of his work.

With the activities and lessons of the IE framed unit Jason cultivated his mathematical awareness, where with initially direct and then indirect support, he was able to extend his own understanding and confidently engage with the mathematics.

5.5.2 Theme 2: Making connections

Jason’s interest in the human aspect of mathematics was reflected numerous times in his work, mathematics journal entries, and interview comments. In one of the more text-based lessons, students were introduced to Euclid and his background of attending Plato’s academy in ancient Greece. In response to the white-faced cube activity (Reimer, 1992), in which students were asked to identify the number of white faces on the exterior face of large cubes built from smaller cubes, Jason wrote,

Back in ancient times, in Plato’s Academy learning math was tough. If we didn’t listen, Plato wouldn’t teach us anything. Of course when we were paying attention we learned a lot. The one thing Plato taught us a lot about is shape and space. We learn all about shapes, sizes and angles.
Jason imagined himself being a student in ancient times using contemporary notions of student learning responsibilities. That he also used the present tense of learning, “We learn all about,” further reinforces his connection to the historical context of Plato’s Academy.

In a shape hunt lesson, in which students looked for shapes in objects around the school, Jason again connected with the historical aspect of mathematics. In his journal he wrote,

When I was an ancient scholar in Greece I noticed something odd about shapes. When you really open your eyes and try to find them, shapes are everywhere. There are rectangles and squares and circles everywhere. When you aren’t looking, they seem like everyday objects, but they are all shapes.

Jason’s comments suggest that the additional context of the historical development of mathematics and imagining himself to be an ancient scholar provided a means to express seeing things that he had not previously discerned in contemporary settings.

Jason again adopted the persona of an ancient scholar in the lessons related to the measurement of angles. Following the reading of What’s Your Angle, Pythagoras? (Ellis, 2004), Jason drew Figure 5.20 in his journal and wrote,

If I were walking alongside Pythagoras I noticed that he discovered many different things that are important in math. One thing I noticed was the square theory. For example, when he saw a triangle he measured the triangle with squares, and he used a small 2 to describe the squares. I also figured out something that Pythagoras didn’t. While he was measuring the triangles, I noticed that each square had 4 90º angles of its own.

Through taking the role of an ancient scholar, Jason not only made a connection with the mathematics of Pythagoras, he used this to make his own discoveries, in which he took great pride: “I also figured out something that Pythagoras didn’t.”

![Figure 5.20. Jason’s angle sketch.](image)

The historical connection for Jason also surfaced early in the group interview, when he described his own learning:
Even though all the work we did was really fun, I think we learned a lot from it. We had fun learning about math. . . . Well, we learned about 2-D and 3-D shapes, like the Platonic solids and all the duals and we learned about angles and protractors and stuff like that but we learned about this stuff by looking back in the past about Plato and Euclid and stuff like that.

Even though it was encouraging that Jason recollected a number of mathematical aspects from the unit, of more significance was his view that students had been able to learn about mathematics “by looking back in the past.” This implies that the inclusion of historical contexts and people was helpful, allowing Jason to make connections to the mathematics he was learning in the contemporary context.

The connection that Jason developed within the unit was not just with the persona of ancient scholars; he formed connections with individuals in the contemporary context as well. In the one-to-one the interview, when Jason referred back to the work he had done for the first introductory lesson of the vision walk, he indicated the immediate personal connection he felt with the lesson:

For the vision walk it was fun, and just having Uncle D come in and teach us about stuff, how blind people learn and so stuff, it just, it must be really hard for them, because they have to do everything with their hands, like and their hands are their new eyes. . . . I remember when I was doing the vision walk with the blindfold over my head when you were walking, you start most of them with your hands out like moving around to see if there are any obstacles around you that you might trip, and, um, so I think that since we were using our hands maybe if you were, actually you would use your hands to feel stuff, and that is how you would learn about space and shape.

Jason’s comments indicate empathy towards blind people and suggest that he formed a connection to the guest, Uncle D, who had visited the class and talked about his life with blindness. In doing so, Jason was able to use his experiences to identify an important part of a blind person’s learning tools (“you would use your hands to feel stuff”) and to suggest a new sense of vision for blind people (“their hands are their new eyes”).

An extension of Jason’s personal connection with the unit involved sharing his work with others outside the class:

Jason: [smiling] I thought this was really neat. Like how the corners made circles and semi circles and um there were 180º and 360º. . . . I showed this to my parents, I took it home for homework one day, and I showed it to my parents and they didn’t even understand it, and then I remember when you were doing this piece of work, when J. wrote the Chinese on the board, like the straight angle and the weak angle, being 180º and 360 º and that helped me put a bit more details in my description.
Jason not only took great pride in describing his work on the angles of squares and triangles to his parents, his body language and tone of voice indicated he was very pleased to have knowledge that his parents did not have: “they didn’t even understand it.” The momentary reversal of the usual balance of knowledge in the child–parent relationship clearly appealed to him. Thus throughout the unit Jason made connections with fictional and historical characters, visitors and his parents, with mathematics bridging those relationships.

5.5.3 Theme 3: Developing self-confidence

Jason’s self-confidence was shown in some of his classroom work during the unit and gradually became more evident in the one-to-one interview, with comments such as “Well, I thought this was a really good piece of work,” and “Well, this is one of my favourite pieces of work.” His comment when asked about his own learning and whether this unit helped him learn mathematics affirmed his self-confidence: “Yes, all the acute angles, the right angles, obtuse angles, I learnt all about morphing shapes. I think I have everything I need to know now.” That Jason felt this way points to a genuine satisfaction with and confidence in his mathematical learning.

Later in the one-to-one interview, Jason’s comments further indicated his growing self-confidence. When describing his tessellation design, reviewed earlier (in section 5.5.1), Jason stated,

Well, this is one of my favourite pieces of work that we did in the space and shape unit, because it really took a lot of effort and hard work. And the thing that I liked about it was that I used hexagon tracers and I just chose that because I wanted a challenge because the squares looked a bit too easy.

Not only did Jason indicate a high degree of satisfaction with his work (“This is one of my favourite pieces of work”), he felt sufficiently self-confident to explain his rationale for giving himself a mathematical challenge of creating a hexagonal template, something different to other students. All the other students had kept their designs linked to the initial square of card, whereas Jason felt this was insufficient. It appeared that he wanted to apply more sophistication to his work and use his growing understanding of mathematics. Importantly, it was clear that during the unit Jason felt able and confident enough about what he wanted to do to fully engage with the mathematics.

Also indicative of Jason’s self-confidence in what he had done and experienced during the unit was the manner in which he made some general comments during the group interview.
Jason confidently responded to a comment from Freddie about people learning at different rates and in different ways:

I think we could learn all of the geometry in a fun way, then other kids could probably learn it. Like Freddie here said, we learn at different paces, but everyone will learn it eventually so why not just learn it in a fun way, instead of working in a textbook it can help.

Jason was clear about his preference for the unit lessons above traditional textbook work. He seemed to recognize some of the open opportunities provided by this unit of lessons and saw these as being better than the more usual textbook learning. With his comment “I think we could learn all of the geometry in a fun way, then other kids could probably learn it,” Jason comfortably reflected the enjoyment he felt the students had experienced in their learning into opportunities for other students to learn in similar ways.

The enjoyment and fun Jason experienced in the unit arose again in the group interview when I asked him to tell me more about learning to use imagination: “[confident tone] Even though all the work we did was really fun, I think we learned a lot from it, we had fun learning about math.” Jason recognized that while he wanted to have fun and enjoy learning there was, to him, an educational significance. His assuredness in making this comment indicated again that he—and in his view, others—had learned from their unit experiences. Such a tone of voice and sentiment were clearly indicative of Jason’s confidence about his learning.

5.5.4 Theme 4: Demonstrating a sense of wonder

A final theme emerging for Jason was demonstrating a sense of wonder. When considering Jason’s sense of wonder, it is helpful to remember the IE view of wonder, where questions are encouraged, are more focused, and are more directed beyond that of the general questioning of young children (Egan, 1997, 2005), and also the position taken in this study that wonder is part of imagination. A real indication of the sophistication Jason demonstrated in his sense of wonder was seen when in the one-to-one interview he continued to relay his thoughts on how he felt that IE allowed one to describe one’s learning:

Researcher: Can you talk about some of the ways you think it helps?

Jason: Yeah, I think you get better marks and them, hmm [small laugh], and even if you don’t like it, if you still put it down and making your description longer and you can say “This didn’t really help me that much” or “I have learnt a lot from this.” It [IE] gives you many different things to write down.
Researcher: Could you talk any more about how it helps you with more ideas?

Jason: Um, giving your ideas, well I noticed Grace’s tessellation, whenever I look at hers I always think she might have been happy, when she was feeling really good when she was doing it, because it’s all orange and yellow and bright red, and it stands out, but the some kids that might have been sad doing it, like they didn’t really like it, I saw some kids colour them dark blue and dark green and it didn’t really show up that much. . . . People were putting like their emotions into their work.

Jason’s perceptiveness and wondering went beyond a superficial and utilitarian desire to improve the marks one could receive on submitted work. He was able to appreciate and explain an affective association in the work of other students. Jason’s comments about the emotions he thought Grace had expressed in her work show an association of affective reactions to different colours, such as sadness with dark colours and happiness with bright colours. This illustrated a meta-understanding he developed about how the use of emotions can foster more expressive demonstration of learning.

During the one-to-one interview, when he was asked if he wished to add anything about IE, Jason commented,

I think that putting imagination into the work is like much better, because if you had fun learning this or if you didn’t like learning about shape and space, you can put that down and it [IE] just gives you a longer description and also like, even to write down that, like, “I thought this activity was really fun. It [using IE] took a while but it really paid off,” I think that like from what I have done makes a teacher give you better marks sometimes. Yeah, so um, I just think that adding imagination into your descriptions helps a lot in many different ways.

With these remarks Jason demonstrated he had understood it took some time to get used to learning with a different theoretical framework of IE and harnessing one’s imagination. However, he understood that once this was done it had a utilitarian value of assisting with demonstrating one’s understanding. He could also see and articulate, that based on his experience, one could get better marks if one could express better understanding through using some aspect of imagination in one’s work.

During the group interview, when asked what other students might think of this unit, Jason expanded his sense of wonder to include his own thoughts about the use of imagination:

I think that learning with imagination and stuff like that, and all the things that we have been doing with blind people and how they learn about space and shape, it is much better than the normal textbook work because I remember doing textbook work last year, and I basically forgot all of it but this year since we did this stuff in like a fun way and we did something new I can remember all of it.
With this comment Jason made a direct connection between use of his imagination and the mathematics he was learning in class. He then applied this perception to previous experiences he had when learning mathematics, rationalising his recent experience such that he now felt he could better remember the math he had learned in an IE manner. This was confirmed in a later remark from Jason in the group interview:

Because I remember in [another class] last year we just worked in the textbook and measured shapes and stuff like that, but this year . . . I find, like this way more interesting, way better and I have learned a bunch more.

Jason’s sense of wonder was indeed sophisticated and appeared to parallel what Egan’s (2005) definition of a sense of wonder: “Wonder can be an engine of intellectual enquiry. It is part of literate rationality’s persistent questioning.” (p. 91).

5.5.5 Theoretical findings: Cognitive tools

In the analysis of the ways Jason used the cognitive tools of the IE theory, his work samples provided further theoretical insights into the development of his mathematical understanding and the meaning this unit had for him. This piece of the findings will again focus on the four samples of work assessed and coded in detail.

With the vision walk activity, Jason utilized cognitive tools of wonder and sense of reality from the romantic understanding phase of the IE theory. Jason was able to immediately connect his experience of blindness, albeit a short and temporary one, with the reality of those who were truly blind. At the same time, he expressed wonder about a blind person’s world and their coping skills when he stated, “their hands are their new eyes.”

Table 5.5 Jason’s use of cognitive tools (see coding scheme in Appendix 8)

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</tbody>
</table>
While engaged with the work on angles, Jason utilized the cognitive tools of context change, hero association, humanization, narrative, and wonder, all from the romantic phase of understanding. This powerful combination of cognitive tools provided one of the deepest senses of engagement for Jason in this unit. He was able to imagine himself in ancient times yet comfortably carry on with classroom work. He also seemed to form a heroic association with the lives of ancient mathematicians such as Euclid and Pythagoras.

When working on the tessellation activity, Jason demonstrated use of the cognitive tools of pattern, sense of reality, and wonder. In choosing to challenge himself to make a template very different to others in the class, Jason showed a very strong sense of pattern, which he then used to make his templates into real-world fish-like shapes. Jason also developed a very strong sense of wonder about the mathematical aspects of the tessellation design.

With the quilt design, Jason used the cognitive tools of narrative when engaging with the literary introduction of the activity. At the same time he was able to exemplify wonder and context change by easily moving between the world of the main character of the story Sweet Clara and the Freedom Quilt (Hopkinson, 1995) and the reality of the world of blind people. Also included in his design was a strong sense of pattern, with the use of colours for the Platonic solids and classical elements.

The evidence related to Jason’s cognitive tool use shows more use of cognitive tools from the romantic phase than from the mythic phase of understanding. The depth of understanding shown by Jason firmly placed his development within the romantic phase.

5.5.6 Jason: Summary

Jason’s participation in the unit brought forward a high level of engagement. The growth in his mathematical awareness began early in the unit and continued throughout the lessons, which included numerous opportunities for Jason to explore mathematical concepts. The angle activities seemed to be particularly appealing to Jason, as he was able to explore a concept he already knew about, such as vertices, and use a variety of mathematical tools, such as protractors and compasses, to make new mathematical discoveries.

He demonstrated enjoyment in learning and great pride with what he achieved, thereby indicating significant growth in his level of self-confidence. This was further illustrated in his statement “I think I have everything I need to know now” and in the fact that he learned knowledge of shape and space that his parents did not have.
Jason seemed particularly keen on the historical associations that were part of the unit, which allowed him to make numerous connections with what was being studied in class. A personal connection also emerged in the analysis of Jason’s interview comments and work, illustrating interaction between him, his work, and a variety of individuals, such as the blind guest, Uncle D, and his own parents. He took great pride in sharing parts of what he was learning with his parents and then discovering that in some areas he was teaching them new knowledge.

Jason seemed a self-motivated student throughout the variety of lessons and activities, frequently giving himself extra challenges, such as with the tessellation design. Jason was the only student in the class to vary the basic tessellation template to be more than the square with which each student started. By the end of the unit Jason demonstrated wide-ranging and high levels of mathematical knowledge for students of his grade level.

Jason was articulate in expressing his understanding of IE and the engagement he found with the unit. Moreover, his additional explanation of learning being more memorable confirmed a connection Jason formed between the affective and imaginative threads of IE. The analysis confirmed Jason’s engagement with the unit as active positive engagement (Hagen, 2007; Hagen & Percival 2009).

5.6 Collective summary

Combining the findings for the five participants involved in this study provides the opportunity to consider conclusions that can be drawn from the data, and in turn to address the research question. It also aids in examining both the similarities and differences across the five cases.

5.6.1 Cross-case analysis of emergent themes

Table 5.6 gathers the thematic analysis together in a word table format (Yin, 2003) that illustrates characteristics of the students’ engagement. The emergent theme analysis was shared with my committee members, two of whom are experts in the field of mathematics education. It should also be remembered that the individuality of each student’s expression is an important aspect of the research (Duckworth, 1996; Harris, 2000), and will be noted where appropriate in the emergent themes analysis.
Table 5.6 Cross-case comparison of emergent themes

<table>
<thead>
<tr>
<th>Student</th>
<th>PA Engagement</th>
<th>Thematic Characteristics of Engagement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Descriptors</td>
<td></td>
</tr>
<tr>
<td>Grade Four</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Courtney</td>
<td>Passive</td>
<td>1. Drawing on emotions</td>
</tr>
<tr>
<td></td>
<td>positive engagement</td>
<td>2. Making connections</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. Developing self-confidence</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4. Cultivating mathematical awareness</td>
</tr>
<tr>
<td>Kee</td>
<td>Passive</td>
<td>1. Demonstrating a sense of wonder</td>
</tr>
<tr>
<td></td>
<td>positive engagement</td>
<td>2. Making connections</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. Developing self-confidence</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4. Cultivating mathematical awareness</td>
</tr>
<tr>
<td>Grade Five</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Freddie</td>
<td>Active</td>
<td>1. Demonstrating a sense of wonder</td>
</tr>
<tr>
<td></td>
<td>positive engagement</td>
<td>2. Developing self-confidence</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. Making connections</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4. Cultivating mathematical awareness</td>
</tr>
<tr>
<td>Grace</td>
<td>Active</td>
<td>1. Drawing on imagination</td>
</tr>
<tr>
<td></td>
<td>positive engagement</td>
<td>2. Making connections</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. Cultivating mathematical awareness</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4. Developing self-confidence</td>
</tr>
<tr>
<td>Jason</td>
<td>Active</td>
<td>1. Cultivating mathematical awareness</td>
</tr>
<tr>
<td></td>
<td>positive engagement</td>
<td>2. Making connections</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. Developing self-confidence</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4. Demonstrating a sense of wonder</td>
</tr>
</tbody>
</table>

When examining the evidence emerging from the data, three themes arise with sufficient similarity across all five students to form a focus of discussion. These themes are making connections, developing self-confidence, and cultivating mathematical awareness.

The evidence from the cross-case analysis demonstrates that all five students were able to form connections between themselves and the mathematical contents of the unit in a personally meaningful and relevant manner. Courtney, Jason and Freddie all drew connections between themselves and their families during the unit. Courtney found many opportunities to include her family in her work by referencing the ages of various family members in her quilt design. She also included symbols such as that of a sun to represent where she used to live with members of her family. Courtney had the added highly personal connection of having a family member in her home, Uncle D., who was blind and who was an invited guest during the unit. Courtney, Jason and Freddie took home part of what they were learning in school to share with their families.
Freddie involved his family on a regular basis with computer research at home, looking at the morphing of shapes and optical illusions after the activities that related to tessellations and artist, M.C. Escher. Jason replicated activities at home with his family related to the discovery of the properties of angles, and took great pride in teaching his parents something they did not already know. Courtney pointed out different shapes to her father during a meal, after having heard the story of the Math Curse (Scieszka, 1995) led her to seeing math everywhere.

For Kee and Grace the connections they formed with the unit related more to everyday and environmental connections. For Kee he wanted to understand more about life in general (e.g. "How things happen, how they didn’t happen"). Kee also frequently referenced and connected his own interest in astronomical space to what he was learning in class. Whereas for Grace some of the connection she formed were to trees in the environment around the school and to the geological properties of rocks and stones. Kee and Grace both suggested books to help others learn, Kee recommended a book life that would help explain things and Grace recommended a book of shapes.

For Jason and Freddie, they were forming personally meaningful connections between themselves and the historical contexts and individuals that were placed at various points in the unit. Jason had imagined himself as a scholar in ancient Greece, forming a connection with Pythagoras, when studying the measuring of angles. Freddie was amazed at the heroism of Helene Keller in managing her life with multiple disabilities including blindness.

All students were able to develop their self-confidence during the period of the study, again in a personally relevant manner. The students began to be more aware of their own style of learning and abilities, which was aided by becoming meta-aware of their emotive reaction to different learning situations. It is important to remember that the level of self-confidence was, appropriately, not commensurate across each of the students, but individual in nature reflecting the development of the students mathematical understanding during the unit. For example, Jason openly expressed in his one-to-one interview that he thought he had everything he needed to know, after the unit, Freddie was able to counsel Kee during the group interview as to what he might try to improve his marks, and Grace was able during her one-to-one interview to quietly reflect pride in splashing her imagination on the page with her quilt design.

Similarly, analysis of the emergent themes across the five cases showed cultivation of mathematical awareness for all the students, whereby there was a growing development of responsiveness to and interaction with mathematics. The IE unit of lessons included creative
artistic styled lessons, lessons that used mathematical tools and manipulatives, activity pages and mathematics journals where any form of notation such as pictures, words and numbers was accepted. Resources used also included video and DVD media and literature books. While all of the students showed growth in their mathematical awareness they did so in a way that suited their individual learning style. Evidence of their growing mathematical awareness was largely shown in their work samples and mathematics journals.

Courtney’s early math journals were rather obligatory in response, contained short incomplete sentences and little conceptual detail. However as the unit progressed it was possible to see a steady increase in length of a journal entry, the addition of drawings, more inclusion of conceptual details and the use of handwriting. Importantly, she began to ask questions in her journal entries, such as “What if the world could imagine a different shape of planets like earth being a sphere, it could be a cube, or Mars could be a triangle?” with the morphing of shapes activity about half way through the unit. At the end of the unit her quilt journal entry was 12 lines in length and contained more conceptual details.

A different perspective of the cultivation of mathematical awareness came from Jason who became engrossed in discovering facets of concepts that he had not previously known. In an activity related to properties of shapes Jason, started his journal entry with “When I was starting this I just used basic shapes like cubes and pyramids.” Through the process of discovery with this activity Jason finished with the following statement illustrative of the growth of his mathematical awareness in just one activity “This really helped my list of objects get longer.” He was now aware that there was more to how 2-D shapes made up 3-D objects than he had previously known. A comment from Jason in the group interview summarised the general consensus from the students about cultivation in their mathematical awareness, “Even though all the work we did was fun, I think we learned a lot from it, we had fun learning about math.” That he applied his own sense of mathematical learning to others, which was affirmed in the interview by Freddie, is indicative of the individual and collective growth in mathematical awareness that this group of students experienced.

The other themes which arose from the data for the students were more individual in nature. Courtney seemed to draw significantly on her emotions when considering her work during the unit, whereas Grace drew on her imagination when thinking about and completing her unit work. Both girls were drawing on affective personality qualities that were the most responsive for them when they were either working on activities or responding to tasks.
However, the evidence for the three boys, Kee, Freddie, and Jason, presented the development of a sense of wonder for these students, with some individual variation. The sense of wonder that these students demonstrated was very much in line with Egan’s (1997, 2005) view of wonder; whereby each of the boys speculated about a particular topic or aspect of the mathematical concepts being studied. That is, these three students tended to raise more questions about what they were doing rather than just responding to a task. This apparent gender grouping had me speculating as to whether there were any gender tendencies or bias that could be drawn from the evidence the students were providing.

### 5.6.2 Cross-case analysis of students’ use of cognitive tools

The students’ work samples indicated their depth of cognitive tool use and their adoption of cognitive tools. Table 5.7 summarizes the use of the cognitive tools by all students across the unit. This table was developed by counting the frequency of various cognitive tools exemplified in the students’ work samples, using Egan’s (2005) definitions of the cognitive tools.

**Table 5.7 Summary of student use of cognitive tools**

<table>
<thead>
<tr>
<th>Phase of Understanding</th>
<th>Cognitive Tool</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mythic</td>
<td>Rhyme/metre/pattern</td>
<td>10</td>
</tr>
<tr>
<td>Mythic</td>
<td>Mental imagery</td>
<td>8</td>
</tr>
<tr>
<td>Mythic</td>
<td>Sense of mystery</td>
<td>2</td>
</tr>
<tr>
<td>Mythic</td>
<td>Role play</td>
<td>2</td>
</tr>
<tr>
<td>Romantic</td>
<td>Wonder</td>
<td>15</td>
</tr>
<tr>
<td>Romantic</td>
<td>Sense of reality</td>
<td>9</td>
</tr>
<tr>
<td>Romantic</td>
<td>Context</td>
<td>7</td>
</tr>
<tr>
<td>Romantic</td>
<td>Humanization</td>
<td>7</td>
</tr>
<tr>
<td>Romantic</td>
<td>Hero association</td>
<td>6</td>
</tr>
<tr>
<td>Romantic</td>
<td>Narrative</td>
<td>4</td>
</tr>
</tbody>
</table>

A key component of the IE theory (Egan 1997, 2005) that emerged as important in this study was the students’ use of the cognitive tool of wonder. For these students, wonder had acquired significance. A comment from Freddie at the end of his mathematics journal entry after the vision walk exemplifies the sense of wonder that the students collectively indicated had helped to lay a foundation on which they could build their cognitive understanding of the shape
and space concepts: “So I felt like wow! How can she survive but what if you can’t hear or talk?” In addition, Courtney’s comment in a later math journal entry, “What if the whole world could imagine a different shape for planets like earth being a sphere, it could be a cube or Mars could be a triangle,” confirmed that a general sense of wonder was increasingly becoming a way that students were mathematically connecting with their world.

This sense of open inquiry is evidenced in the students’ work samples illustrating their engagement with the unit of mathematics lessons. Thus, it appears that through experiencing mathematics in an IE-framed unit in which wonder is an important cognitive tool, the students acknowledged that the opportunity to wonder was important to them.

For these five students, noticing something mysterious about a shape or space concept and approaching it with a sense of wonder, in an IE manner, gradually increased their use of their imaginations. There were many instances where becoming imaginatively involved with their learning was purposeful daydreaming; it became the thing to do, and they became more and more aware that it could help them with their learning. This was especially so with Grace, whose imaginative engagement was amply represented in her quilt design and mathematics journal entry.

Overall, then it appears that all five students found ways to engage with the mathematics of the unit that was personally and meaningfully relevant. Using participatory affective engagement (Hagen, 2007; Hagen & Percival, 2009) to note characteristics across the students’ engagement allowed the opportunity to develop further understanding of the students’ engagement in the lessons. Two Grade 4 students, Courtney and Kee, assessed as demonstrating passive positive engagement, needed more time, for different reasons, to think about how they were going to engage in the lessons before becoming fully active. Courtney needed time to work through her affective response before becoming fully engaged with the lessons, while Kee wanted to satisfy his sense of wonder before becoming fully engaged. The other three students, Freddie, Grace, and Jason, all of whom were grade 5 students, became fully engaged with the lessons without any apparent hesitation, and their engagement was assessed as active positive engagement (Hagen, 2007; Hagen & Percival, 2009).

5.7 Conclusion

Based on the perspectives of the five participants in this study, various themes pertaining to their engagement with mathematics within an IE framed shape and space unit emerged. In
particular, a cross-case analysis pointed to all students in this research developing self-confidence, cultivating mathematical awareness and making connections. In addition, individual students drew on emotions or imagination while others demonstrated a sense of wonder as they engaged with the unit activities. Two of the Grade 4 students demonstrated passive positive engagement, while the three Grade 5 students demonstrated active positive engagement. It also appeared that most of the students used cognitive tools attributed to the romantic phase of understanding, when engagement with the four tasks under examination. In the next chapter, what the use of the IE theory and the ILPF’s (Egan, 1997, 2005) meant to these students and for their engagement in elementary mathematics is discussed and the significance of the findings is considered.
CHAPTER 6: DISCUSSION

When originally designing this research study, it was the contrast in students responses to lessons whenever I used the IE theory (Egan 1997, 2005) sporadically compared to when I used more traditional techniques to teach mathematics that began my journey of enquiry and which has led to this study. With this contrast in mind I wanted to find out more about my children’s engagement with mathematics when I used Imaginative Education to inform my teaching. As Duckworth (2006) suggests, “When teachers take a researcher’s stance in the classroom – engaging learners’ minds and hearing what they have to say – the students are not the only ones who learn.”(p. 171). Thus, I chose to focus the current study on the children’s perspectives of their experiences in an IE framed Shape and Space Unit. In turn, this case study serves to extend our current knowledge of ways in which children might engage with elementary mathematics (i.e. geometry) through Imaginative Education.

In this chapter, then, I address my research question - What does the use of the theory of IE and its ILPFs mean to children and their engagement in elementary mathematics? After discussing the conclusions from the study, implications and recommendations for future research and practice are addressed.

6.1 Conclusions

6.1.1 Children expand mathematical awareness through making connections

Boaler (2000) attributes students’ disengagement from mathematics to teaching practices that do not foster connections to the world outside the mathematics classroom. In the current study, two emerging themes - making connections and cultivating mathematical awareness - were prevalent across all five students. Throughout the unit and during the one-to-one interviews, participants began forming connections between themselves and the mathematical concepts they were learning in ways that were personally meaningful and relevant. As discussed in Chapter 5, these children formed connections with historical contexts and figures, their families and a blind community member, and their everyday interests such as astronomy, optical illusions, and the environment. In addition, as these students began to connect with the concepts of shape and space in these multiple ways, not only did they appear to cultivate a growing mathematical awareness, as shown in their work samples and mathematics journals, but, during interviews, they
explicitly spoke about their expanding views (e.g. “math includes shapes and words”; “math is fun”; “you can put emotions in math”; “math is all around us”). While some of this breadth could be attributed to the ‘new’ topic of shape and space (i.e. “not just numbers, old boring math”), these children consistently pointed to resources (e.g. books, videos,) and experiences (e.g. vision walk, designing a quilt, journal writing), which I (as teacher) had chosen to represent key tenets of the IE theory (Egan, 1997, 2005), as seminal to their growing awareness of mathematics in and beyond the classroom.

Likewise, these children’s strong connections to mathematicians and artists (e.g. Pythagoras, Coxeter, Escher) introduced through narratives of one form or another (i.e. a mainstay of IE theory, Egan, 1997, 2005) in this unit, concurs with Ward-Penny’s (2011) view that humanised mathematics “Can help pupils explore mathematical ideas in a more well-rounded way and reconnect many mathematical concepts to the exploration and enquiry from which they originally emerged” (p.147). These innovators of mathematical ideas and concepts in the past became people with whom these students felt a current human connection. Mathematics became a subject that had a rich historical past that was no longer inaccessible and clouded in obscurity, or stuck in a textbook. Mathematics also became a relevant and meaningful subject as part of the students’ current and contemporary lives. The students were now seeing mathematics in places that they did not see math before such as in their homes and within artistic images or creations they made, thus supporting the view of Higginson & Flewelling (2003) and Higginson (2000) that inclusion of creative mathematical tasks provides rich learning opportunities. Consequently, using IE theory (Egan, 1997, 2005) seemed to mean that for these children their engagement was such that mathematics gained a relevancy and connection to their everyday lives, which in turn expanded their awareness of what mathematics is and where it might be found.

6.1.2 Children develop self-confidence from using emotions and imagination

Nardi and Steward (2003) gave an emphatic warning to mathematics educators that, “In the absence of mathematical experiences suited to individual needs and consequent feelings of success and self-esteem, students become alienated from the subject and eventually choose not to study it.” (p. 5). In the current study, children reported feelings of pride in their work and in themselves and espoused their beliefs of doing better in mathematics. They reported that IE helps children “learn more mathematics” because “we usually do lots of cool stuff “ and “since we did stuff in like a fun way, … I [we] can remember all of it.” Indeed, a week after the unit had
finished, the confidence these five children exhibited within the focus group interview, through their frequent raising of hands, a plethora of voluntary contributions and a barrage of interconnected comments where the participants completed sentences for one another and actively referenced each other in their comments about imaginative education (Egan, 1997, 2005), was impressive. It was as if the self-confidence they were developing throughout the unit, albeit to varying degrees, had reached a crescendo in this context. To what extent this “sense of bravado” (i.e. as noted in my research diary at the time) was indicative of their role as informants in a research study (i.e. a role they took seriously) or of changes in their self-esteem attributable to IE is, of course, challenging, if not impossible, to untangle. That said, I argue the prevalence of the emergent theme ‘developing self-confidence’ across these five students suggests that IE was sufficiently implicated to warrant further discussion.

What appears unique to this study then is the way in which IE features of the “mathematical experiences” served to address children’s “individual needs and consequent feelings of success” (Nardi & Steward, 2003, p. 5). That is, the invitation to draw on their emotions or imagination or to demonstrate their sense of wonder (i.e. all emergent themes for individual children in this study), served to open up the ways in which each of the five children engaged with and recorded their mathematics. As stated during the group interview, “when we had to write about the description of things it helped me look more into the kinds of shapes …”; “with the textbook we had to be certain answers to certain questions and what we have been doing there [IE Unit] we could describe it in any way we wanted it to be…”. Thus, it appears children’s self-confidence was related to the more individual ways they could address the content and express their solutions. There was, therefore, a dynamic interplay between the use of imagination and emotions, which is congruent with both Egan’s (2005) view that successful learning requires emotional involvement, and that of Hannula (2002) that the combination of affect and cognitive learning provides tremendous potential for even greater development of a learner’s understanding than cognitive learning alone. Thus, the current study draws our attention to the ‘non-mathematical’ features of mathematical experiences, reaffirming the importance of both in our quest to optimize children’s engagement with mathematics.

Notwithstanding the important meaning that imagination, emotions and wonder had for these students and I argue for their self-confidence in learning mathematics, the data while indicating interactions between these characteristics is insufficient, except to serve as an impetus for further research. For as Egan (2005) claims- “the imagination is tied in complex ways to our
emotional lives” (2005, p. xii), which in turn, I argue, was tied in complex ways to these participants self-confidence. Of course, if we are to avail of the potential of IE to improve students’ levels of confidence in learning mathematics (Mullis, 2007), then we need to develop a more nuanced understanding of such complexities. With this caveat in mind, it appears using IE theory meant these children’s self-confidence developed through their use of emotions, imagination and wonder, when engaging with mathematics.

6.1.3 Children use cognitive tools to engage with mathematics

An important tenet of the Imaginative Education theory (Egan, 1997, 2005) is the need for educators to draw upon ways of thinking which are already familiar to learners (Egan, 1997), and which the students already use (often intuitively) to make sense of the world around them (Vygotsky, 1968, 1972). For example, this study’s unit of mathematics lessons was planned and designed around the cognitive tool, binary opposites, from the second phase, mythic understanding, of IE (Egan, 1997). The specific focus chosen was that of vision and blindness, with a guiding question of how a blind person learns about shape and space. By presenting the lessons to the students in this alternative way, different to what they had experienced before, it would require them to draw on and develop different ways to consider mathematics.

While not surprising, considering the explicit focus on IE, all participants in this study used various cognitive tools such as pattern, wonder, hero association, both in their work samples and in their interview comments. Thus, Egan’s premise that these cognitive tools resonate with ways in which children come to know the world was reaffirmed. It appears then, using the IE theory and IPLFs means children can and do draw on these cognitive tools to engage with mathematics, at least with shape and space. What this study adds to Egan’s theory, however, is that in this context, the cognitive tool of wonder was predominant and although planned within mythic understanding, these children used a greater variety of cognitive tools from the romantic phase of understanding (Egan 1997, 2005).

Egan (2005) defines wonder as “a key tool in our initial explorations of reality. It enables us to focus on any aspect of the world around us, or the world within us, and see its particular uniqueness” (p. 79). As the teacher/researcher, I could not predict or control what prior experiences the children would bring to the tasks of the unit, nor could I predict which aspects of the theory or specific cognitive tools, or ways of thinking, the students might independently choose to use. The students work and interview comments point to wonder as being the most
frequently used cognitive tool, and therefore more salient to them than other cognitive tools, whereas within the IE theory (Egan 1997, 2005) all the cognitive tools appear to have equal significance. Although not entirely clear from the data, it seems that the cognitive tool of wonder acquired such significance due to the overarching goal of understanding how a blind person might learn about shape and space.

In addition, the different ways in which individual children’s sense of wonder might be characterized resonates with definitions put forward by other researchers. For instance, Cookson (2004) sees wonder as an opportunity for students “to be creative and curious about their world.” (p. 10), Opdal (2001) describes wonder as “the state of mind that signals we have reached the limit of our present understanding, and that things may be different from how they look.” (p. 333) and Fisher (2003) defines wonder as “the hospitality of the mind to newness.” (p.49). As such, it appears more useful for researchers and educators to maintain an expansive view of wonder as captured by a collection of meanings (e.g. Cookson (2004), Fisher (2003), Opal (2001), and Egan (1997, 2005).

As indicated earlier, participants used a greater variety of cognitive tools from the romantic phase of understanding (Egan 1997, 2005). Indeed, each cognitive tool used from the romantic phase appeared more frequently than those in the mythic phase. Such a finding was somewhat expected, as this is commensurate with the general level of literacy development of Grade 4 and 5 students, who Egan (1997, 2005) suggests rely less on orally based forms of both receptive and expressive communication. However, what is of importance here is that using the mythic phase of understanding to plan the mathematics unit did not constrain the children’s responses, but rather appeared to provide a foundation for these children open-ended experiences through which they could express themselves at personally appropriate levels. The data as collected however remains inconclusive about the relationship between these two phases within the shape and space unit.

6.2 Implications and Recommendations

6.2.1 Further research in different contexts

While the current study provides some beginning insights into IE theory and children’s engagement with mathematics, the number of students, while appropriate for a case study, was small. There is a need, therefore, to continue to examine features of imaginative education
(Egan, 1997, 2005) and mathematics with different sized groups, so that both the breadth and depth of such experiences might be understood. For example, would themes, which appeared particular to one child in the current study, appear more prevalent if work samples from the whole class had been analysed. Furthermore, participants in the current study were students, at the beginning of the intermediate years of education (i.e. Grades 4 and 5). However, additional research with students in other grade levels could shed light on which aspects of the IE experiences resonate in particular grade bands. For example, what differences would exist in settings where there already exists a tendency to have more creative and imaginative play oriented learning, such as in the early years of learning (e.g. full-day kindergarten (BC Ministry of Education, 2010)), or with older groups of students where there is even more tendency for students to disengage from learning mathematics (Boaler & Greeno, 2000; National Archives, 2007; Ward-Penny, Johnston Wilder, & Lee 2011). Finally, it is acknowledged that the Shape and Space orientation of the unit, in the current study, may have been particularly conducive to the five students producing creative, artistic representations of their mathematics understanding. Therefore, further research about the use of an IE framed Unit in other strands of mathematics curricula, such as number concepts and operations, is recommended to capture how extensively Imaginative Education might be used to support children’s engagement with mathematics. Finally, since the current study focussed on student perspectives, as we continue to examine the use of IE theory in mathematics education, research into teacher perspectives are also warranted.

6.2.2 Further research into Imaginative Education and affect in mathematics

Within mathematics education much is already known about the importance of considering affective responses in mathematics problem solving, in particular what an affective response might involve (DeBellis & Goldin, 2006; Malmivuori, 2006). What remains unclear is how to situate imagination in this literature, so that we might better understand how children’s use of imagination, likely in combination with other affective responses, can contribute towards fostering mathematical engagement. The current study suggests that further research into the complementary characteristics of imagination and emotions, for example, is needed.

Since within the Imaginative Education theory (Egan, 1997, 2005) itself the relationship between emotions and imagination is yet to be fully explicated, both mathematics education researchers and IE researchers would likely benefit from further understanding of how these two concepts interrelate and under what circumstances they continue to foster positive engagement, as
was found in the current study. For instance, in the current study, Kee alerts us to the child whose imagination diverges such that the ‘having of too many ideas’ poses challenges in completing assigned tasks; and Courtney alerts us to the child for whom emotions are ‘felt in the moment’ and not readily written about. Therefore, it seems worthwhile to research IE theory based mathematics with children for whom key features of IE or mathematical affect are somewhat atypical. What might using IE in mathematics mean then for students who are highly anxious about mathematics or for those children who find it difficult to express their emotions?

### 6.2.3 Further research into mathematics engagement

Evidence from the current study pointed to self-confidence, connections, and mathematical awareness as important characteristics of these five children’s engagement with mathematics. However, in addition to identifying these characteristics, further research into how, and in what way, such characteristics influence students’ ability to engage with their mathematics learning or vice-versa is warranted. For example, is self-confidence an outcome of mathematical engagement, or a necessary pre-requisite for engagement? The current case study, while providing beginning insights into what IE might mean for mathematical engagement, merely opens a door of enquiry, and much remains for future researchers to discover.

A challenge posed by the results of the current study was the way in which mathematics learning appeared to yield at times to the influence of the IE theory (Egan, 1997, 2005). This was, in part, due to the design of the study to be open to the students’ perspectives of their experiences and their tendency during the study to i) reflect more on the IE features of their experiences and ii) make generic references (e.g. “it was fun”, “I learned lots”) to mathematics. In future research with data collected for the current study, a secondary analysis of the mathematics in the work samples and the perspectives of children (e.g. Jason) who fore grounded their mathematics, may serve to inform us further. In addition, as we continue to investigate the interconnections between IE and math engagement, it behoves us (researchers) to find ways to foreground the mathematics concepts and processes both in the practice (i.e. the intervention) and in the research.

Building from the current study whereby the data analysis occurred after the unit and interviews were completed, it is recommended that action research methodology might be used. This could include the teacher/researcher incorporating both on-going data analysis into the study design and throughout implementation to assist in highlighting the mathematics more, or the
inclusion of a form of pre-research assessment that considers issues teachers face in highlighting the mathematics when an IE theoretical framework is used.

### 6.2.4 Future Practice with Imaginative Education

Overall, the current study suggests using Imaginative Education theory and IPLFs meant these children engaged positively with mathematics in the Shape and Space Unit. Also, these students’ use of both their imagination and emotions seemed related to their developing self-confidence. And, making connections (often through stories) contributed to these children’s increasing awareness of mathematics around them. This study indirectly then implies, that while much remains to be understood about Imaginative Education in mathematics education contexts, teachers are encouraged to consider ways in which they might implement Imaginative Education, in part or whole, in their mathematics lessons.

With results from this study suggesting that cognitive tools were facilitative to the students’ engagement with mathematics, see Figure 5.7, I would recommend that teachers incorporate cognitive tools in regular mathematics lessons. The cognitive tools of wonder and rhyme, metre, pattern were shown to be most significant to students’ engagement during this study. Thus teachers are encouraged to consider ways they might inject inquiry into different topics and areas of the mathematics curriculum. For example, using the cognitive tool of wonder, within number concepts and operations, might entail the teacher posing questions such as “I wonder who invented the arithmetical signs of operation and how their use evolved in day to day life?” In turn, this would provide opportunities to incorporate the human and historical aspects of mathematics into an important area of mathematics curriculum. Using this type of questioning would not have to be restricted to the introductory phases of units or lessons, but can be posed at any time throughout a lesson or unit.

A further example would be to use resources that refer to historical characters in either a non-fictional or fictional manner, such as Kathryn Lasky’s (1998) The Librarian Who Measured the Earth, which portrays the life of the ancient Greek philosopher Eratosthenes, who closely estimated the circumference of the earth. Thus, the romantic cognitive tool of association with heroes is brought into play. By implementing either one or a small number of elements of the IE theory (Egan, 1997, 2005), such as the cognitive tools, into regular mathematics lessons, teachers can gradually become familiar with the overall theoretical framework and, as in the current study,
may choose to monitor their student learning to see how IE theory affects their students’
engagement with and understanding of mathematics.

Another contribution of the current study to future practice is drawn from the
methodology. The research focus on the student perspective allowed a richness and depth to be
added to what this IE Shape & Space unit meant to both the children and their engagement in
elementary mathematics. Therefore, in addition to joining Taylor and Parson (2011), in their call
for inclusion and expansion of students’ perspectives in research endeavours, I recommend the
inclusion and expansion of students’ perspectives in classroom practice. Thus just as these
children, were able to write about their experiences, and talk about their reflections on their work,
it is recommended that we continue to find ways in our practice to support other children in doing
so. Although the interviews were a method to collect research data, as a teacher/researcher, it was
striking, how these conversations served to extend and clarify my teacher knowledge, both about
the individual children but more generally about components of the Unit. Thus, in future practice,
I recommend we consider ways to provide opportunities for children to share their perspectives
(and I argue, to do so orally both one-to-one and in groups) on their uptake of the lessons as well
as their mathematical understanding. As such our practice is likely to be enriched by these
conversations and our reflections on the intended and experienced curriculum to which they will
inevitably lead.
Epilogue

*By enquiring into children’s experience we will come to know more about how they interpret and negotiate their worlds, material and discursive, past, present and future.*
*(Greene & Hogan, 2005 p. xii)*

This dissertation began with a discussion about the importance of mathematics. In a fast paced, 21st century, technological world students are aware that mathematics is an important subject and yet their ability and opportunity to become engaged with this subject that constitutes a large part of school curriculum is challenging.

Mathematicians and mathematics educators often see beauty, intrigue and wonder in their chosen subject contrasting with the often confusing, troubling and worrying perspective that many students come to know and experience. Why is this so? How could this gap exist?

Looking at mathematics in a broader manner as part of our living and sociocultural world that is not confined to formulas, rules, theorems and textbooks may help to begin to undo a negative image of mathematics that can get carried through life. Presenting mathematics to learners as part of this rich and dynamic world, opening up the locks of mystery surrounding the subject that some learners experience could help breathe life into the subject for young learners that establishes a foundation of interest and an open perspective towards mathematics.

Allowing learners the opportunity to express their responses to mathematics learning in respectful and appropriate ways can be enlightening for both the learner and the educator. It is my hope that what has been placed here for consideration may help to ignite some conversations about the richness that mathematics can have.
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162
Unit Overview

G4/5 Shape and Space Unit

This unit has a rich theoretical framework of Egan’s (1997, 2005) Imaginative Education (IE), and is planned with the use of Imaginative Lesson Planning Frameworks (ILPF’s). It will use the Mythic and Romantic levels of understanding from IE to shape the lessons for the G4/5 class.

The unit will incorporate the following Prescribed Learning Outcomes, from the BC Ministry of Education

**G4 - 2-D Shapes and 3-D Objects**

- Construct, analyse and classify triangles according to their side measurements.
- Build, represent and describe geometric objects and shapes.
- Classify and name polygons according to the number of their sides (e.g. 3, 4, 5, 6, 8).
- Cover two-dimensional shapes with a set of tangram pieces.
- Complete the drawing of a three-dimensional object on grid paper given the front face.
- Determine experimentally the minimum information needed to draw or identify a given two-dimensional shape.

**G4 – Transformations**

- Create and verify symmetrical two-dimensional shapes by drawing lines of symmetry.

**G5 – 2-D Shapes and 3-D Objects**

- Design and construct nets for pyramids and prisms.
- Relate nets to three-dimensional objects.
- Compare and contrast pyramids and prisms to describe a relationship.
- Identify and sort specific quadrilaterals, such as squares, rectangles, parallelograms, and trapezoids.
- Classify angles in a variety of orientations according to whether they are a right angle, less than a right angle, or greater than a right angle.
- Recognize, draw and name the following: point, line, parallel lines and intersecting lines.

**G5 – Transformations**

- Locate planes of symmetry by cutting solids.
- Cover a surface using one or more tessellating shapes.
- Create and identify tessellations using regular polygons.
- Identify regular polygons that can tessellate a plane.
The Unit Romantic Planning Framework

1. **Heroic Qualities** - *What “heroic” qualities or values are central to the topic? What makes the characters in this story full of wonder?*
   
   Blindness / Vision. What and how do we learn when we do not have all bodily senses? How do visually impaired people learn about Space & Shape? What does it feel like to be someone else?

2. **Heroic Image** - *What image captures the heroic qualities of the topic?*
   
   Walking in someone else’s shoes.

3. **Organizing Content into Story Form** - *What “heroic” narrative will allow us to integrate the content we wish to cover?*
   
   What do we know already about shapes and space? Record this. Do Vision Walk, come back and record narrative of experience. Now let’s think about others. After Vision Walk let’s think about others less fortunate, i.e. Helen Keller, Euler blind mathematician.

4. **Exploring Human Strengths & Emotions** - *How can students understand the human, hopes, fears, passions or struggles that have shaped our knowledge of this topic?*
   
   Experience it for themselves, hear how others cope. Fears, scared unknown, reasons for risk taking? Why do you think it helps us connect with these?

5. **Extremes of Reality** - *What extremes of reality are related to the topic – biggest, hottest, oldest, richest?*
   
   Vision Walk, sight / blindness. Need Mathematics to help explain some of these experiences.

6. **Collecting and Organizing** - *What parts of the topic can student’s best explore in exhaustive detail? How can students present their knowledge in some systematic form?*
   
   Sense of space, small and large. Sight and blindness. Students to create their written and oral narratives of their learning experiences through their activities, (i.e. math journals).

7. **Towards Further Understanding** - *How can the unit develop embryonic forms of Philosophic and Ironic understanding? What cognitive tools characteristic of the disciplines or embodied self-awareness can be introduced here?*
   
   Develop expression with increased literacy, speech, pictures, words. Oral expression increasingly sophisticated. Both lead to greater understanding and knowledge. Sense of agency (ourselves as related to others) see things in a new way.

8. **A Celebratory Ending** - *What is the best way of resolving the dramatic tension inherent in the lesson? What communal project or activity will enable the students to experience and share this resolution?*
   
   Sharing of Vision Walk experiences, after personal math journal written/drawn. What do you think about the learning of space both for a sighted person and visually impaired person.

9. **Assessment** - *How can one know whether the topic has been understood, its importance grasped and the content learned?*
   
   Student completion of narratives in math journals. Use regular assessment i.e. Performance Standard – Numeracy. Use Egan’s ILPF’s and characteristics of Mythic and Romantic categories.
Semi-Structured Interview Protocol

Structure: The purpose and goal of the interviews will be explained to each student, i.e. that their help is needed in answering some questions about student's engagement with elementary mathematics. Students will be asked if they are willing to proceed. Student engagement will be explained to the participating students as a connection between the student and the subject matter. It will be explained to students that in research activities there cannot be things that would publicly identify them, i.e. their own name, and therefore their work has been assigned a number. At this point the students will again be asked if they are willing to continue. Students will be invited to select their own pseudonym, if they wish. Each participant will be asked the same set of questions in the same order about the work they had individually completed during the unit; copies of the individuals work will be available as reminders, together with some of the resources used in the unit. Students will have samples of their work available for them as a stimulus to discussion.

Protocol (Teacher/Researcher):

1. Could you describe in as much detail as possible the learning that occurred for you during this unit? How did you feel about this?

2. How would you describe your engagement in this unit?

Students will see samples of their work and will be asked:-

3. Here is some work that you did for this unit, I was wondering how you felt about the work, now that you are seeing it again?

4. Is there anything you would want to add to this work now you are seeing it again?

5. Do you think this unit help you become engaged with mathematics? In what ways?

6. Do you think this unit would help other children become engaged with mathematics? In what ways?

7. Do you think this unit helped you learn mathematics? In what ways?

8. Do you think this unit you have studied would help other children learn mathematics? In what ways?

9. Would you add anything to this unit of lessons (i.e. resources, books, lessons)?

10. Would you take anything out of this unit?

11. Is there anything you would like to say about Imaginative Education?

12. Is there anything else you would like to say?
Now that we have had a chance to hear and see The Greedy Triangle, AND you have had a chance to feel what it would be like to become a shape, let’s sort some shapes by their properties!! Cut out the shapes below and group them in the way that you feel is right. Once you’ve decided stick them in that group in your math journal, have fun and ENJOY!
Space and Shape – Geometry

Creating a Mathematical Vision Quilt

We started off this unit of Space and Shape Geometry thinking about vision and blindness, and you taking part in a Vision Walk. Today you have heard the story of Sweet Clara and the Freedom Quilt and how a young girl made a quilt which contained a very powerful message.

**Your job is to create a quilt that will help a blind person learn about space and shape, in which you include as much mathematics as possible. You will also need to write a very rich and detailed math description of your quilt (there are two parts, the quilt design and the description).**

Using your imagination may well help you a lot. While listening to the story think about taking the role of a character from the story, try to “put yourself in their shoes”, as you hear the story. First you need to think about the story you have heard.

1) Which character did you choose?

2) Why did you choose this character?

3) What emotions do you think this character felt during the story?

4) What did it feel like to you to be this character in the story?

5) Anything else you would like to say about this story?

Now you have to think about designing a quilt for a blind person.

**Your job is to design a quilt that will help a blind person learn about space and shape, in which you include as much mathematics as possible. You will also need to write a very rich and detailed math description of your quilt (there are two parts, the quilt design and the description).**
## Student Engagement Observation Protocol

<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>Lesson</th>
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<table>
<thead>
<tr>
<th></th>
<th>Student 1</th>
<th>Student 2</th>
<th>Student 3</th>
<th>Student 4</th>
<th>Student 5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Student Participation</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Listens attentively</td>
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<tr>
<td>Shows evidence of an emotive response</td>
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<tr>
<td>Becomes actively involved, perseveres with task</td>
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<tr>
<td>Shows evidence of estimating, thinking, checking</td>
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<tr>
<td>Works independently and/or with others</td>
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<td></td>
</tr>
<tr>
<td>Talks for self-clarification and or communicates with others</td>
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<tr>
<td>Asks probing mathematical questions</td>
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<tr>
<td>Chooses and uses appropriate materials/calculator</td>
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<tr>
<td>Shows evidence of imagination</td>
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<tr>
<td>Shows evidence of discovery exploration/strategy</td>
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<tr>
<td>Makes connection to personal experiences</td>
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<tr>
<td>Completes task</td>
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</tbody>
</table>
Structure: The purpose and goal of the observations is to gather information that may shed light on possible answers to the research question. Students’ behaviours and participation will be observed during lessons. The features to be addressed during the observations are noted on the chart above. Comments should focus on when (i.e. beginning, middle, end of lessons), the emotive response/reaction (excited/bored, wonder, surprise, frustrated/calm) and how (i.e. orally, in writing, facial or physical expression) each category of behaviour is demonstrated.

*Original format of form was produced and utilised in landscape orientation.
Performance Rubric – Shape and Space Unit

Assessment tool to be used to assess students from where they start, to further along a continuum of learning. Important to remember individual student starting point.

Not Yet Within Expectations

The work does not meet what would normally be expected of a student of this age/grade level. None or very minimal evidence of progress toward the relevant curriculum content. Need to see what is going in, may need to provide variation or adapted alternatives.

Meets Expectations (Minimal)

Somewhat inconsistent, but generally in range of what could be expected at this age/grade level. Some progress or demonstration of understanding with regard to curriculum. Some support needed to move further forward.

Fully Meets Expectations

Generally have met expectation of assignment and curriculum. Good level of achievement in terms of age and grade level. May include variety of expressions. Can largely represent and communicate understanding. No support needed.

Exceeds Expectations

Work exceeds grade-level expectations in important ways. Includes a variety of expressions of understanding. Can represent and communicate understanding with ease.

Exceeds Expectations Plus

Works exceeds general age and grade level expectations in significant and variety of ways. Student has gone above and beyond what could normally be expected for particular assignment. Student has shown significant understanding and extension of inquiry of subject content.
Appendix 8

Coding Scheme

The focus of the coding scheme is to provide the means to analyse the data collected during the research project which was examining the question “What does the use of the theory of Imaginative Education (IE) and Imaginative Lesson Planning Frameworks (ILPF’s) mean to children and for their engagement in elementary mathematics?

Egan’s theory of Imaginative Education (IE)

<table>
<thead>
<tr>
<th>Mythic Understanding</th>
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<tbody>
<tr>
<td>MU</td>
<td></td>
</tr>
<tr>
<td>MUstory</td>
<td>- Story</td>
</tr>
<tr>
<td>MUMetaphor</td>
<td>- Metaphor</td>
</tr>
<tr>
<td>MUBinary</td>
<td>- Binary opposites</td>
</tr>
<tr>
<td>MUrhyme, meter, pattern</td>
<td>- Rhyme, meter, pattern</td>
</tr>
<tr>
<td>MUJokes/humour</td>
<td>- Jokes/humour</td>
</tr>
<tr>
<td>MUF形成的 images</td>
<td>- Formation of images</td>
</tr>
<tr>
<td>MUSense mystery</td>
<td>- Sense mystery</td>
</tr>
<tr>
<td>MUGossip, drama, play</td>
<td>- Gossip, drama, play</td>
</tr>
<tr>
<td>MUEmbryonic tool literacy</td>
<td>- Embryonic tool literacy</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Romantic Understanding</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>RU</td>
<td></td>
</tr>
<tr>
<td>RUSense of reality</td>
<td>- Sense of reality</td>
</tr>
<tr>
<td>RUHero association</td>
<td>- Hero association</td>
</tr>
<tr>
<td>RUWonder</td>
<td>- Wonder</td>
</tr>
<tr>
<td>RUCollections, hobbies</td>
<td>- Collections, hobbies</td>
</tr>
<tr>
<td>RUHumanizing meaning</td>
<td>- Humanizing meaning</td>
</tr>
<tr>
<td>RUNarrative understanding</td>
<td>- Narrative understanding</td>
</tr>
<tr>
<td>RURevolt and idealism</td>
<td>- Revolt and idealism</td>
</tr>
<tr>
<td>RUContext change</td>
<td>- Context change</td>
</tr>
<tr>
<td>RULiterate</td>
<td>- Rom. literate eye</td>
</tr>
<tr>
<td>RUEmbryonic theoretic thinking</td>
<td>- Embryonic theoretic thinking</td>
</tr>
</tbody>
</table>
C: “I can’t really describe it. Um…”

PH: “I was just curious about something you said, you were saying about the jagged edges and the frustration.”

C: “Oh well um, I think points and like points on a square fit with it so, well squares make and circles kind of feel of calm to me, because they’re original and stuff, and jagged points kind of make me, well kind of stand out to frustrated or anger, and happiness is sort of like swerves and um you know just like a sort of like that, roundy.”

PH: “Sort of smooth compared with the jagged edges.”

C: “Yeah, and I was having kind of a mixture there, so I decided to put swerves on two sides and jagged on the other two, getting really frustrated and um I was also happy.”

PH: “Um hm that is really interesting, do you want to talk, you talked a little about your quilt, now can we just go back and look at that again for a second? Now that you really look at your quilt again, um I was wondering how you feel generally about the work that you did, now that you are seeing it again?”

C: “It kind of makes me feel sort of down because a, it’s this makes a weird feeling inside when you feel it with your hands, because it is a mixture of feeling and sense. Like “

PH: “Touch?”

C: “It sort of makes me feel sort of in the Dark Ages [laugh] because of black and white and then the jagged sort of again because of points.”

PH: “So carry on”.

C: “No you go. [emotion in voice]”

PH: “I was thinking about what Uncle D. said when he said “It is like a white sheet being pulled over your head.”

C: “Yeah it is well he kind of describe it to me, what it’s like being blind and one time he made us play blind hide and go seek, so we all put blindfolds on white ones and that helped me see a bit, so we got all these white sheet covers and covered our eyes, and um we had our monitors as whoever was not allowed to take off their blindfolds, my Mum and my grandma, he knows his root so well that he climbed up onto his dresser and he was hiding there for about 15 minutes because none of us could find him so um, he finally said “Come here, find me already” so we came in his room, My Mum said “Get off your dresser.” And he is like I am just having fun with the kids.”

PH: “Ah so.”
Exemplar Coding Case – With Codes
Extracts Following Triangulation of Data Sources
Grace – Drawing on Imagination

Individual Interview

1) When reflecting during the interview on the Vision Walk math journal entry,

Grace: Well [pause] I really feel bad for the blind people because you can’t really see anything around you and you feel you were not blind from the beginning and you don’t really know the shapes and how they look like. But like you can just feel them”. :-

Grace: And if it was like if you were also deaf and blind and um you can learn like hand signs and stuff like that. :-

Grace: Um well [pause] I think if like a cube was hot you would know what colour it was, and like red, and stuff.” RUw

2) When reflecting on the quilt design

Grace: I feel pretty happy with myself if I didn’t have the quilt and like to solve from my imagination I might have came up with the same thing. Emotions

Grace: It’s like this pages is like all full, full of my math imagination just like splattered on there!”

Summary Notes after Transcription and Coding of Interview

Writing seen as representation of memory / imagination. Very creative and imagination, writing represents imagination; confirmation.

Group Interview

G: “The math work is fun!” enjoyment emotions

G: “I liked the quilt one because you got to use your imagination to like make the quilt and do like a map or yeah.” Imagination

G: “Yeah because like if there wasn’t any imagination there, inside books her would be all like nothing.” Imagination

G: “If you find out what kids are more interested in and make that into Math you would probably find out more.

Note at bottom of page after coding
An amazing amount of understanding about how students learn, their own personal, and learning in general.
Work Sample Summary

Note at bottom of handwritten summary page for four work samples.

G’s quiet personality belies the details given and attention paid to her work.

Critical Friend Observations

6th May

“If you can get their imagination for them to start thinking, they could solve mathematical problems too...and if you can, it really sucks them into math, you know what I am saying because then it becomes fun, not just a little button that you wear that math is fun, they really think it.”

4th June “Grace Lots of good discoveries with protractor getting what geometry was all about beginning to see connections.

9th June Tessellations
“Now Grace of course. Wow when I looked at hers I thoughts isn’t that nicely done, she had picked such nice bright colours and it took me a while to realise what looked so good about Grace’s and it really was the outlining and that is why I wanted to show the kids what it was that looked so good that she had outlined it in dark and it just made it stand out so much better. So not only had Grace done the actual drawing it she had worked on it at home and you could tell worked hard and she had it all beautifully coloured, brought in and everything done. Grace was able to put up her hand and answer the questions there to show evidence of discovery that she saw patterns with the letter and also that she I guess that she had counted the number of numbers. She said it was the same number of numbers in each square, so that was Grace.”

Grace: Big time bright colours discovery. Ap engagement

Research Diary

“21st May At the end of the day when I was trying to make sense of all the activities from the week, I came across two math journal entries which were really encouraging, one was from Grace where on her own she had given me some homework.

Note – Red print denotes researcher coding or comment