INVESTIGATION OF STOCHASTIC OPTIMIZATION METHODS FOR OPERATING RESERVOIRS WITH SNOWMELT-DOMINANT LOCAL INFLOWS AND LIMITED STORAGE CAPABILITY IN BRITISH COLUMBIA DURING THE SPRING FRESHET

by

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Abstract

The reservoir operations model developed in this thesis is a stochastic dynamic programming decision support tool for the optimization of the operation of snowmelt-driven reservoirs with small storage flexibility hydropower systems during the spring freshet. The model operates under the objective of maximizing the value of electricity generation through electricity trading over a short-term planning period. Project and watershed data, stochastic inflows, and estimated electricity prices are used to calculate optimal expected turbine release policies over a short-term planning period. Results are used to provide decision support to operators in the form of a daily expected optimal turbine release volume and marginal value of energy of the reservoir. Including stochasticity in the model allows for inflow probabilities, which may not be easily evaluated by an operator, to be reflected in an operation decision. A combination of forecast, historical, and current state of the system data is included in the model to reflect the most up-to-date view of uncertain conditions. Case studies indicate that although operators may deviate from the expected optimal policy to meet other interests and requirements in real-time, the model provides an optimal expected policy during the freshet period and has shown in a case study to increase the value of a single reservoir's operations by 6% during one three-month freshet period.

Preface

The research presented was fully funded by BC Hydro and a Collaborative Research and Development (CRD) grant from the Natural Sciences and Engineering Research Council (NSERC) to Dr Ziad Shawwash of the University of British Columbia. All modeling, analysis, and writing was performed by Ryan Rasmussen under the supervision and mentorship of Dr Ziad Shawwash and many BC Hydro employees, as detailed in the Acknowledgements section.

This thesis includes one published and one manuscript to be submitted, both relating to the research subject described in the abstract. The first manuscript, presented in Chapter 2, was written early in the research and development process and was published by PennWell Publishing in August 2014 as a conference paper for the 2014 HydroVision International conference under the title “Optimizing the Operation of Small Storage Reservoirs in British Columbia During the Spring Freshet” as in Rasmussen et al., 2014. This manuscript does not include significant details on results or any conclusions regarding the model, as testing and validation was still in progress during its submission. The second manuscript, presented in Chapter 3, was written at the end of the research and development process and presents a complete view of the model’s development and use. The paper is to be submitted to be published to a technical journal relating to water resources management.

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List of Abbreviations

BC	British Columbia
BC Hydro	British Columbia Hydro and Power Authority
DP	Dynamic Programming
ESO	Explicit stochastic optimization
ESP	Ensemble Streamflow Predictions
GOM	Generalized Optimization Model
GS	Generating Station
GWh	Gigawatt-hour
HLH	Heavy Load Hours
HYSIM	Hydro Simulation Model
IPP	Independent Power Producer
ISO	Implicit stochastic optimization
LLH	Light Load Hours
MW	Megawatt
OASIS	Open Access Same-time Information System
SDP	Stochastic Dynamic Programming
SSDP	Sampling Stochastic Dynamic Programming
STOM	Short-term Optimization Model
TPM	Transition Probability Matrix
VBA	Visual Basic Application

Glossary

Unit	Hydropower turbine
Spilled water	Reservoir water that is discharged without passing through a turbine and generating electricity
Forebay	Water level of a reservoir with respect to a datum
Draft	Discharge water from reservoir and lower forebay through turbine discharges or spills
Freshet	A large increase in streamflow caused by heavy rain or melting snow
Heavy load hours	A time of day (typically from hours 7:00 to 23:00) from Monday through Saturday, when electricity demand is relatively high
Light load hours	A time of day (typically from hours 23:00 to 7:00) from Monday through Saturday, and all day Sunday, when electricity demand is relatively low
Stage	A term used in dynamic programming to denote a time step at which a decision is required
State	A term used in dynamic programming to denote important variables used to determine the value of a decision
Planning Period	A term used in dynamic programming to denote the total number of stages solved by the objective function.
Rough Load Zone	Generation ranges which are inefficient and cause increased wear on the turbine.
Project	A hydropower development, including the reservoir and generating station

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First and foremost I would like to thank my graduate studies supervisor, Dr Ziad Shawwash, for introducing me to the BC Hydro –UBC Grant-in-Aid program and mentoring me throughout my stay. I am truly grateful to have been given the opportunity to immerse myself not only in the theory but also the practice of hydropower engineering.

The experience and practical knowledge of Mr Jim Gaspard, PEng, a Principle Engineer at BC Hydro, significantly shaped the final model created in this research. His understanding of the BC Hydro system and its operation was essential in creating a usable operations model. Although Mr Gaspard had no direct involvement with the BC Hydro-UBC Grant-in-Aid program, he never hesitated to take time out of his busy schedule to lend his expertise.

I would also like to thank Dr Alaa Abdalla, who managed the BC Hydro – UBC Grant-in-Aid program at BC Hydro. Through Dr Abdalla, I was able to assist BC Hydro planning engineers in problem solving and gain valuable exposure to real-world hydropower engineering. He also offered feedback during regular meetings, helping all students to stay on track.

In addition, I would like to acknowledge the help of other BC Hydro staff, In particular the support of Ms. Renata Kurschner, Director, Generation Resource Management, Mr. Kelvin Ketchum, Manager of System Optimization (now retired), Mr. Amr Ayad, System Optimization engineer, and Dr. Dave Bonser, Principle Engineer, System Optimization. Many others at BC Hydro provided administrative and technical help to myself and the UBC research team working at BC Hydro which was greatly appreciated.

Dedication

To those in my academic and personal life who have believed in, supported, and inspired me.

Chapter 1: Introduction

Although British Columbia (BC) is rich with healthy water resources, care must be taken to preserve their integrity and prevent overconsumption. Since many entities depend on the province's water resources, water use planning is used across BC to provide limits for water license holders and ensure adequate flow for all interests in a given water source. These water resources are used almost exclusively to meet the electricity needs of British Columbians through hydropower. Providing suitable flows for interests such as recreation, wildlife, irrigation, and drinking water often reduces the production of hydropower projects. Therefore, optimizing BC's system of rivers, dams, reservoirs, generation stations, and transmission lines is crucial in providing a fair balance between conflicting water license holders and interests. This chapter gives a brief overview of BC's integrated electricity system and how its operators aim to produce electricity as efficiently as possible.

1.1 Electricity Generation in BC

The majority of BC's electricity generation and demand is managed by the British Columbia Hydro and Power Authority (BC Hydro), a crown corporation. BC Hydro owns and operates 31 of BC's largest hydropower projects and 3 natural gas powered projects amounting to 12,000 MW of installed capacity and an average annual electricity production of approximately 60,000 GWh (Integrated Resources Plan, 2013). Figure 1-1 shows a map of BC Hydro's major infrastructure.

Figure 1-1: Major BC Hydro Generating Facilities and Transmission System

As seen in the above figure, BC Hydro also handles the transmission and distribution of electricity to BC residents. Many smaller private independent power producer (IPP) projects across the province use hydropower, thermal generation, and wind power projects to produce and sell electricity into BC Hydro's system. Hydropower generation projects use a variety of reservoir sizes in many different watersheds across the province and produce the majority of BC's electricity. Thermal generation projects in BC provide valuable auxiliary support to the system and are fueled by natural gas. BC Hydro also buys and sells electricity with producers outside of BC through interconnections, in order to create a secure and reliable electricity supply.

1.1.1 Generation and Demand

Since electricity cannot be feasibly stored on a large scale, BC's electricity supply must match demand at all times to prevent electricity shortages. Electricity demand follows yearly, weekly, and daily trends that depend on factors such as temperature, daylight hours, and the habits of the province's population. BC's climate creates a trend of relatively high electricity demand in the winter and relatively low demand in the summer as seen in Figure 1-2, which shows domestic electricity demand for 2013. A major factor in this trend is the large amount of heating used by residents during BC's cold winters compared to the lack of climate control required by residents during BC's mild summers.

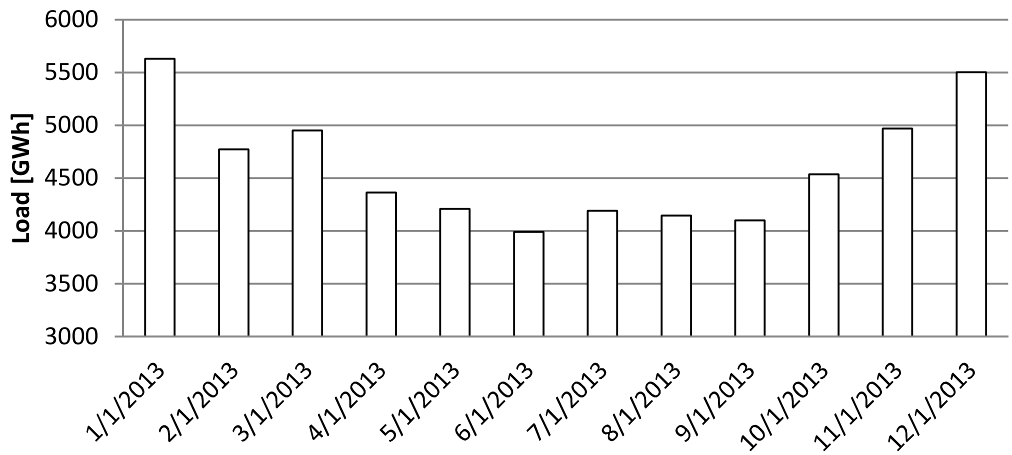

Figure 1-2: Domestic Electricity Demand on BC Hydro Grid for 2013 (Adapted from BC Hydro Database)

Electricity demand is also dependent on the population's weekly and daily schedules. Lower demand periods during weekends, holidays, and nights are known as light load hours (LLH) whereas higher demand periods during business days and daylight hours are known as heavy load hours (HLH).

BC Hydro estimates its future generation capacity by using historical inflows and current capacity data to produce an expected generation sequence for future years. This method allows BC Hydro to predict periods where demand may exceed available generation resources and prepare in advance. It also gives BC Hydro an indication of when they may have excess electricity to export through interconnections to Alberta or the western United States. A contingency capacity must also be included for unit outages, irregular inflows, and other unexpected events.

1.1.2 Hydropower

BC's private and public electricity producers use hydropower to meet approximately 90% of the province's electricity demands. Private and public electricity producers operate a combination of hydropower projects with no usable storage (run-of-the-river), limited storage flexibility (sub-yearly storage), and large storage reservoirs (yearly or multi-year storage). This diverse range of hydropower projects are operated together with other resources to deliver sustainable and dependable electricity to the province.

As mentioned in Section 1.1.1, electricity demand in BC is greater in the winter months as compared to the summer months. Conversely, most of BC reservoirs' inflows are higher in the spring freshet when snow begins to melt and much lower in the winter. In order to shape the availability of water into the winter where it is needed most, large reservoirs are used. These large reservoirs aim to fill during the spring and summer season using the large freshet inflows from approximately May through July when electricity demand is low. The reservoirs then release the stored water throughout the rest of the year in order to meet electricity demand even with small amounts of local inflow.

Although hydropower provides a suitable solution to BC's electricity needs, its supply is less certain than other conventional generation methods. Inflows vary season to season and system operators must plan for droughts and other unexpected events to protect BC residents from electricity shortages.

1.1.3 Thermal Generation

While the majority of BC Hydro's generation capacity is contained in hydropower projects, thermal generation also plays an important role in providing secondary capacity. Burrard Generating Station is a natural gas powered thermal plant that can provide 913 MW of auxiliary electricity to help meet BC's electricity demand in peak seasons. Another advantage of this thermal plant is its proximity to Greater Vancouver and Vancouver Island, where close to 70% of BC's electricity demand is located. Many of BC Hydro's interconnections and large hydro projects are located hundreds of kilometers away from areas of demand, increasing the risk of interruption by a large transmission line failure. Prince Rupert and Port Hardy have combustion turbine generation stations totaling 90MW nearby to provide reserve and emergency electricity during interruptions.

1.1.4 Environmental and Social Considerations

Besides aiming to use BC's water resources to optimally produce hydropower, BC Hydro also considers drinking water, flood control, water transportation, recreation, and aquatic habitat during their operations. Accounting for other values such as these often results in decreased electricity production, but is crucial in order to sustain BC's natural resources and ecosystems. An example of a common environmental consideration is providing a minimum flow downstream of dams in order to maintain water levels. This preserves marine ecosystems and recreation activities in the water course. Similarly, reservoir forebay levels are sometimes bounded by environmental and social objectives. The rates of change of water levels can also be a constraining factor in reservoir operations. BC Hydro consults with many different stakeholders such as private consultants, First Nations groups, government and non-government organizations, and the general public to develop these system constraints.

1.2 BC Hydro System

BC Hydro generates electricity using hydropower almost exclusively, with any remaining needs being met by thermal facilities or purchased electricity (Making the Connection, 2000). BC Hydro's 31 hydropower projects are located in four different regions and are connected through a province-wide transmission and distribution system as described in the following sections. Each project is composed of a reservoir, dam, and generating station (GS).

1.2.1 Peace Region

The Peace River in the north-eastern area of BC contains two of BC Hydro's largest hydropower projects which account for approximately 37% of BC Hydro's total generation capacity. The 177000 hectare Williston Reservoir, 183m high WAC Bennett Dam, and 2,730MW GM Shrum GS accounts for 80% of the Peace Region's capacity and provides a large amount of the province's electricity. The 890 hectare Dinosaur Reservoir, 61m high Peace Canyon Dam, and

700MW Peace Canyon GS is located only 23km downstream of the GM Shrum GS and provide the remaining 20%. The Peace Canyon project is used to provide additional generation for water stored in Williston Reservoir and often drafts the same volume of water as the GM Shrum GS in order to maintain a near-constant forebay level. A large portion of the runoff in the Peace River watershed comes from snowmelt.

The cold climate of the Peace Region presents a challenging scenario for operators of the two hydropower projects located there; ice that forms on the Peace River during winter can cause ice jams and flooding during spring when ice begins to melt and flows increase. To reduce the risk of flooding in the spring, operators keep flows high during ice formation to increase the hydraulic capacity of the river.

1.2.2 Columbia Region

BC Hydro has 13 hydropower projects on the Columbia River and its tributaries that provide close to half of BC Hydro's total hydropower generation capacity, with the majority being provided by the Mica and Revelstoke projects. The 42,500 hectare Kinbasket reservoir, 244m high Mica dam, and 1,792MW Mica GS and the 11,530 hectare Revelstoke Reservoir, 175m high Revelstoke dam, and 2,570MW Revelstoke GS share a similar relationship to the GM Shrum and Peace Canyon projects. The Revelstoke project is located only 130km downstream of the Mica project, and receives close to 70% of its yearly inflows from Mica's discharges. Also, the Revelstoke reservoir maintains a small forebay range and is mainly used to provide additional generation from water stored in the Kinbasket Reservoir. Both reservoirs' inflows come primarily from snowmelt.

Since the Columbia River runs from BC into the western United States, a contract between the two countries was created. The Columbia River Treaty, signed in 1961, covers water allocation and flood control and entitles BC to payment for half of the estimated additional electricity generated in the US as a result of storage operations in BC. Three dams (Mica, Hugh Keenleyside, and Duncan) were constructed in BC as a means of providing this storage. Additional plans between BC and the US cover environmental issues such as operational policies to protect fish and other wildlife.

1.2.3 Lower Mainland

This region is contains 11 of BC Hydro's hydropower plants with 44% of the region's generation capacity contained in the 480MW Bridge River project. The Cheakamus and Clowhom projects are located in the Lower Mainland region containing small reservoirs with a generation capacity of 157MW and 33MW, respectively. Since the majority of BC's population is located in the lower mainland, many hydropower projects share their water resources with other interests such as drinking water supply and recreational use. A large portion of these projects' inflows are provided by snowmelt.

1.2.4 Vancouver Island

The majority of hydropower generation capacity on Vancouver Island is located in three projects on the Campbell River System. The Upper Campbell Lake reservoir feeds the 64MW Strathcona GS, which discharges into the Lower Campbell Lake reservoir feeding the 47MW Ladore GS, which discharges into the John Hart Reservoir feeding the 126MW John Hart GS. Although the system contains three stations and reservoirs, Upper Campbell Lake provides the majority of the active storage for all three stations. Due to the large recreation and fisheries culture along Campbell River, a lot of work is put into keeping reservoirs debris-free and providing protection for salmon.

1.2.5 Transmission and Distribution System

In order to transport electricity to its intended users, BC Hydro uses 17,800 km of transmission lines carrying voltages of 60kV and above and 52,600 km of distribution lines carrying voltages below 60kV. As seen in Figure 1-1, the 500kV lines connect major generating stations in the north of BC with the major electricity demand centers in the south. These 500kV lines are crucial to the entire BC Hydro system. They are planned and operated so that the lines can endure an outage of any single line and still transmit the expected peak electrical load. In addition, interconnection transmission lines exist between BC and Alberta and BC and the western United States. This allows for the buying and selling of electricity between the three systems. This relationship benefits all involved since all three regions can buy or sell electricity to make up any differences in supply and demand.

1.2.6 Electricity Trading Market

BC Hydro uses the Open Access Same-time Information System (OASIS) to buy and sell electricity with other registered users. Users can view and place orders on available transmission capacity or post available transmission capacities of their own with respective prices, days or months in advance. Registered users have the choice of purchasing these transmission services as a firm or non-firm service. These services cannot be interrupted for economic reasons; however, non-firm services are often cut or reduced first when generation is needed elsewhere. For large, long-term firm contracts that require an increased generation capacity, the buyer may be required to subsidize installation costs.

In order to manage the trading of electricity and transmission capacity, BC Hydro works with Powerex, a subsidiary company, to buy and sell wholesale electricity to the Western US and Alberta through interconnections. Since electricity resources and demand vary drastically from region to region, the ability to buy and sell electricity to meet any discrepancies benefits all electricity producers who are involved. BC Hydro has an advantage over many other electricity producers in the electricity market because of their large reservoirs. The reservoirs act like batteries, storing potential energy in the form of water when electricity demand or prices are low, and generating electricity by releasing water when electricity demand or prices are high.

In contrast, a large amount of generation in the western United States and Alberta is provided by thermal plants powered by natural gas or coal. These thermal plants are not as flexible as hydropower projects in ramping up and down generation to meet demand. Therefore, these plants may produce more electricity than is required during LLH which can be purchased inexpensively by BC Hydro in order to allow their reservoirs to recharge. Even if the purchased electricity is not required to meet demand in BC, the saved reservoir volume can be released several hours later during HLH and the resulting electricity can be sold for a profit. This practice is known as generation shaping. In contrast to BC's seasonal electricity demand peak in the winter due to heating loads, the western United States' seasonal peak is often during the summer due to air conditioning loads. Therefore, this strategy can also be used to leverage seasonal variations in wholesale electricity supply and demand in addition to daily variations.

1.2.6.1 Electricity Market Prices

One of the most unpredictable variables involved in the optimization of BC's hydropower system are electricity market prices. Many factors influence electricity market prices as shown in the simple influence diagram Figure 1-3 below.

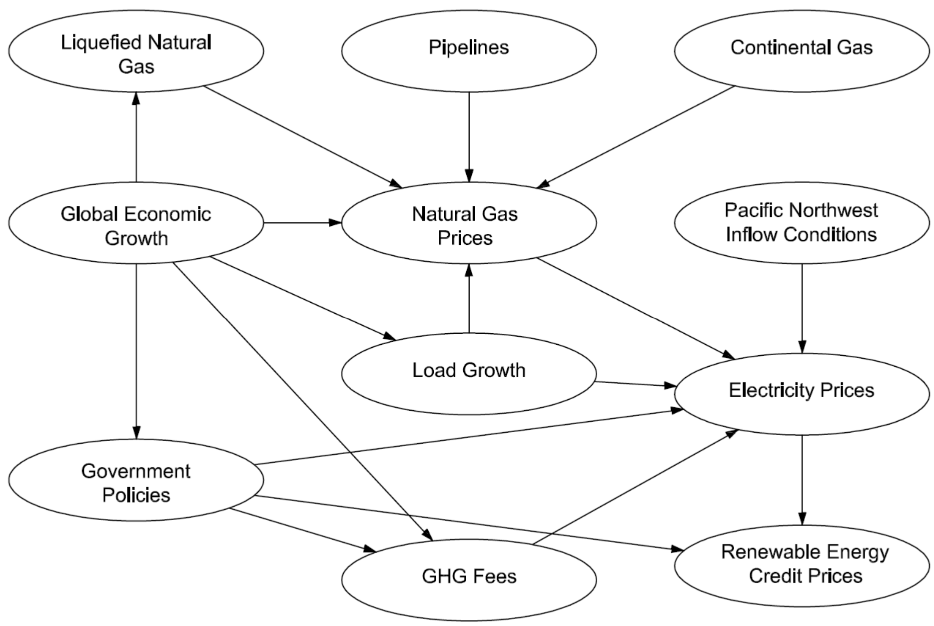

Figure 1-3: Simple Influence Diagram for Electricity Market Prices (Adapted from November 2013 IRP)

As seen in the influence diagram above, many uncertain factors influence or indirectly influence electricity prices for a given region. Two of the main drivers at work in this influence diagram are global economic growth and government policy. For example, Low economic growth may stall the development of greenhouse gas regulation and not affect electricity prices, while high economic growth is assumed to increase the development of greenhouse gas regulation and likely increase electricity prices.

1.3 Operations Planning

Operation plans are optimized based on forecasts of weather, water supply, electricity demand, and economic factors while following flood, generation, environmental, and physical constraints. With numerous constraints, objectives, and uncertainties to consider in the operation of BC's major reservoirs, it can be difficult for operators to make an optimal decision. Operators must plan well in advance to account for variables that may affect short or long term electricity supply. The planning of operations at BC Hydro is governed by safety, legal obligations, domestic electricity demand, and maximizing generation resources. To assist in operation decisions that may otherwise be decided heuristically, reservoir operators employ computer models to help achieve optimal operation of these systems. BC Hydro uses these computer models to perform long and short term planning.

1.3.1 Reservoir Operation

One of the main objectives of reservoir operators is to keep reservoir levels as high as possible without spilling water. Water that is spilled is directed through spillways and does not pass through the generating station to generate electricity. This may be done when a reservoir is nearing its storage capacity and inflows exceed the generation demand or generation capacity of the generating station. If inflows cannot be used to generate electricity for domestic use, electricity exports, or stored safely in the reservoir, they must be spilled. Spilling is undesirable for reservoir operators as it can be compared to wasting fuel. Reservoirs are often drafted prior to expected large inflows to reduce the risk of later high flow spills which may result in downstream flooding.

The value of maintaining high reservoir levels comes from the fact that one unit of water from a full reservoir creates more energy than one unit of water from a low reservoir. In other words, the hydraulic pressure created from high water levels creates more force behind the flow traveling into the generating station. Therefore, reservoirs are able to use their resources as efficiently as possible by maintaining a high reservoir level. A full reservoir also creates a more secure supply of water, as future inflows are always uncertain.

In years where BC's reservoirs receive below average inflows, BC Hydro meets electricity demand by drafting large reservoirs below normal temporal forebay levels, purchasing electricity from other producers, or running thermal plants. Extensive analysis is done to determine the most efficient method of meeting electricity demand with the least economic and environmental impact.

1.3.2 Medium to Long-Term Planning

Medium or long-term planning aims to ensure that resources and infrastructure will be available to meet electricity demand one or more years in the future. BC Hydro developed and now uses the Hydro Simulation Model (HYSIM) for long term planning. HYSIM is a simulation model that uses load and inflow data to determine if the reliability and operational order of a given system. This type of planning can also be used to study the effects of changing social constraints, economic factors, environmental restrictions, electricity demand, and the need for additional generation infrastructure.

1.3.3 Short-Term Planning

Short-term planning focuses on operations from the current day to one year in advance. These plans set general operating rules for hydropower generation and import/export schedules which are reviewed monthly and refined as more accurate date becomes available. Plans for operations are constantly adjusted until only hours before operations are made to meet changing meteorological, economic, or physical conditions. BC Hydro uses in-house software such as the Generalized Optimization Model (GOM), a modification of the Short-term Optimization Model (STOM) developed by Shawwash et al. 2000, to study short and longer-term plans. GOM is a deterministic simulation and optimization model that is used to optimize BC Hydro projects while accounting for IPP purchases and electricity market import/export schedules.

1.4 Problem Statement

BC Hydro's larger reservoirs, such as the Williston and Mica reservoirs, contain enough active storage to operation on a yearly or multi-yearly cycle. This means that the reservoirs often start at a near full position in late summer and discharge over the winter when domestic electricity demand is high. Once temperatures rise, snow begins to melt, and domestic electricity demand begins to decrease, the reservoirs replenish their storage with the help of the large freshet inflows. Throughout the freshet, discharge from these large reservoirs will often be minimal in order to support storage replenishment. During this time, generation from these projects is often not required since demand is largely supported by public and private hydropower projects using run-of-the-river reservoirs that operate at high generation levels using high freshet inflows.

While many run-of-the-river projects will operate at fixed generation levels across the freshet due to some combination of lack of storage or lack of generation capacity, there are plants within the system with sufficient hydraulic capacity and reservoir operating flexibility to economically shape generation. These small storage projects are strategically operated at higher generation levels during HLH to serve higher provincial electricity demand and stronger electricity prices, and reduced generation levels during LLH to refill the reservoirs. Depending on the operating flexibility, the storage/refill process may extend to a daily or weekly cycle.

This operational change occurring during the freshet is not explicitly optimized through computer operations models. Instead, reservoir operators make operational decisions heuristically based on forecasts of local inflows, electricity prices, and operational constraints. Although this method yields satisfactory results, there is potential benefit in using more explicit optimization. Operators also change posts frequently, causing specific projects to lose the benefit of learned intricacies of operation.

1.5 Goals

The objective of this research project was to develop a stand-alone, user-friendly computer optimization model to assists operators of reservoirs with limited storage in making optimal release and generation decisions during the freshet. This research has been performed closely with reservoir operators at BC Hydro to ensure that the end product meets their needs. The model will take account of forecasts and historical data that are available to the operator and will provide a base-line operational decision with the best expected value.

To achieve this goal, the model must:

- Be self-contained in a single or small number of easily transportable and run files compatible with software commonly found on most computers.
- Have a user interface easily understood by users who may be unfamiliar with computer modeling, with the inputs and outputs being clear, simple, and intuitive.
- Make use of electricity market price and inflow forecasts in order to reflect the expertise of electricity traders, hydrologists, and their models.
- Include physical, operational, and environmental constraints to reflect the limitations of the reservoir and its generating station.
- Provide operators with information needed to support short-term optimal expected operation decisions with respect to the value of immediate and future benefits for hydropower projects supported by reservoirs with small amounts of active storage relative to upstream major reservoirs during periods not accounted for in BC Hydro's existing computer models.

To reach these goals, the following processes were followed:

- Consult with operators about the current operational practice of hydropower projects with limited active storage during periods not accounted for in current computer models and identify areas that could benefit from optimization.
- Investigate existing constraints on these hydropower systems.

- Gain background knowledge on BC Hydro’s generation and import/export strategy.
- Identify a mathematical optimization modeling technique that can accurately simulate and optimize these hydropower project’s objectives and constraints.
- Formulate a stand-alone mathematical optimization model in a software package that is easily run using common software.
- Test and validate the model extensively to ensure proper behavior.
- Implement the model during the 2014 freshet for a case study of an appropriate hydropower project while providing decision support to BC Hydro operators.

Chapter 2: Optimizing the Operation of Small Storage Reservoirs in British Columbia during the Spring Freshet

The following paper was written early in the research and development process and was published by PennWell Publishing in August 2014 as a conference paper for the 2014 HydroVision International conference as described in the Preface. This manuscript does not include significant details on results or any conclusions regarding the model, as testing and validation was still in progress during its submission. The abstract, acknowledgements, and author's biographies sections were omitted from this chapter. All references cited can be found in the Bibliography section at the end of the thesis.

2.1 Hydropower in British Columbia

British Columbia's (BC) private and public electricity producers use hydropower to meet approximately 90% of the province's electricity demands. Private and public producers operate a combination of hydropower projects with no usable storage (run-of-the-river), limited storage flexibility (weekly storage), and large storage reservoirs (yearly or multi-year storage). This diverse range of hydropower projects are operated to deliver sustainable and dependable electricity. The majority of BC's electricity is produced by BC Hydro, a crown corporation which also owns and operates BC's largest reservoirs on the Peace and Columbia Rivers. Figure 1-1 shows a map of BC Hydro's major infrastructure.

Operating this system requires the management of dams, diversions, reservoirs, generation stations, and transmission lines all while balancing environmental, social, and economic factors.

Approximately 70% of BC Hydro's hydropower generation capacity is supported by two large storage reservoirs located on the Columbia and Peace Rivers. Natural inflows into these two reservoirs vary significantly depending on the time of year. During the freshet period, typically between May through August, large amounts of snowmelt result in large increases of inflow into BC's interior reservoirs as seen in Figure 2-1 below.

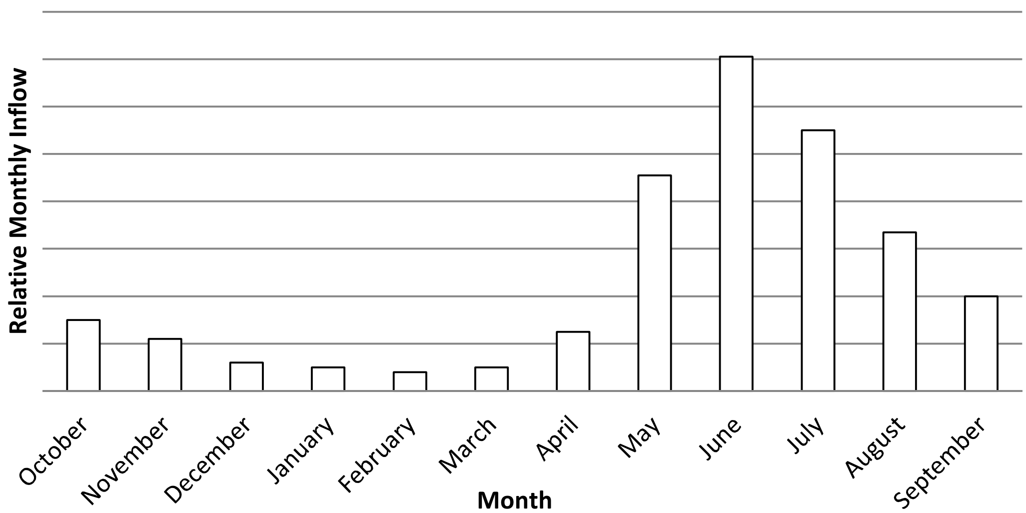

Figure 2-1: Typical Seasonal Inflow Pattern for British Columbia Interior Reservoirs (Adapted from BC Hydro Database)

Meteorologists are given the difficult task of forecasting future inflow patterns, but lack of climate information over the Pacific Ocean causes difficulties in forecasting more than several days in advance. The wholesale electricity trading market between Canada and the western United States also plays a large role in determining the operation of BC's hydropower system. Since electricity market prices vary significantly from hour to hour, operators must consider the current and forecasted electricity market price before release decisions are made.

With numerous constraints, objectives, and uncertainties to consider in the operation of BC's major reservoirs, it can be challenging for operators to make an optimal decision. To assist in operation decisions that may otherwise be decided heuristically, reservoir operators employ computer models to help achieve optimal operation of these systems.

2.1.1 BC Hydro System Operation

The Kinbasket Reservoir in the north end of the Columbia basin, created by Mica dam shown Figure 1-1, is considered to operate on an annual storage cycle. Williston Reservoir in the headwaters of the Peace River basin, created by WAC Bennett dam and containing the GM Shrum generating station as shown on Figure 1-1, is considered to operate on a multi-year storage cycle. The combined capacity of the generating projects supported by these two reservoirs is approximately 7,700 MW, or 70% of BC Hydro's system capacity.

Both large reservoirs will typically draft during the winter when electricity demand in BC is high and natural inflows into the reservoirs are low before refilling during the spring freshet when electricity demand is low and natural inflows are high. Throughout the freshet, discharge from these large reservoirs will often be minimal in order to support their replenishment. During this time, the province's electricity demand is supported by a large number of small projects with limited storage which take advantage of the high freshet inflows.

While some of these small projects are operated exclusively as non-storage, there are a number of projects within the system that will at times retain flexibility to utilize storage across a weekly basis. These small storage projects are strategically operated at higher generation levels on weekdays to serve higher provincial electricity demand and stronger electricity prices, and reduce generation levels on weekends to refill the reservoirs.

However, this pattern of flexible storage during the freshet is outside of the project's normal operating policy and is often not included in existing computer operation models. Currently, decision makers operating weekly storage reservoirs during the freshet use forecasted inflows and estimated electricity prices together with heuristics to determine a suitable operations policy. Although this method yields satisfactory results, there is room for more explicit optimization. Operators frequently change posts as well, losing the benefit of any learned intricacies of the operation process.

2.2 Computer Modeling

Mathematical simulation and optimization presents an effective method of accounting for uncertainties and constraints in a water resources system. In order to translate a reservoir operation problem into a computer model, a modeling technique must be chosen. Techniques such as linear programming, artificial neural networks, control theory, and stochastic dynamic programming (SDP) have been documented in simulating or optimizing real-world water resources problems with positive results. The freshet small storage model described in this paper, referred to as the Weekly Storage Model, uses SDP as its optimization method due to its performance record in modeling reservoir systems.

2.2.1 Documented Use of Stochastic Dynamic Programming in Reservoir Optimization

The SDP method has been proven to be well suited for reservoir operations problems. SDP can incorporate nonlinear and stochastic features common in reservoir operations as well as emulate its multistage decision nature. It has been used and enhanced for decades in reservoir optimization with positive results.

Little, 1955 was one of the first researchers to use the SDP method to optimize a reservoir operation policy for the Grand Coulee dam on the Columbia River. It was found that using even a simple model of the reservoir system, small scheduling improvements were made over previous rule-curve methods. Bras et al., 1983 employed an adaptive control technique in which the stream flow transition probabilities in their SDP model were updated continuously. This provided significant benefits to flood control as seen in their case study on the High Aswan Dam. Stedinger et al., 1984 used the current forecast local inflow as a hydrological state variable rather than the traditional SDP method of using the previous time step's local inflow in their SDP model. This resulted in substantial improvements in the simulated reservoir operations as compared with models using the traditional approach. Druce, 1990 implemented an SDP model of the BC Hydro system which established a marginal cost of generation while incorporating flood control on the Peace River. The model was able to produce an optimal monthly operating policy which included economic and physical data for decision support. Kelman et al., 1990 and Faber et al., 2001 took advantage of available inflow forecast sequences in a method known as sampling stochastic dynamic programming (SSDP). SSDP allows the model to capture the multi-period persistence of stream flows which is especially useful in streams fueled by snowmelt.

2.2.2 Notation in Stochastic Dynamic Programming

SDP addresses both the current and future time steps in reverse induction. Therefore, it must solve the current time step with reference to the subsequent time step before proceeding to the preceding time step. This is explained by using the following notation in Figure 2-2:

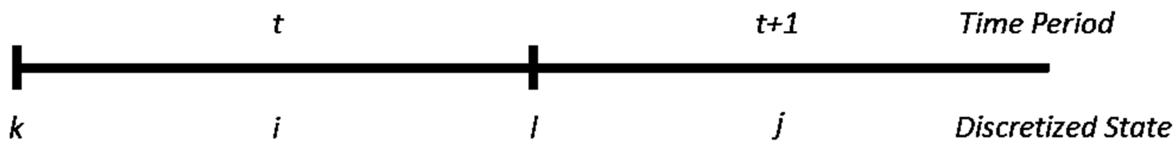

Figure 2-2: Subscript Notion used in SDP Equations

Where:

t = Current time step

t+1 = Subsequent time step

k = Discretized state of storage at start of period t

l = Discretized state of storage at start of period t+1

i = Discretized state of inflow during period t

j = Discretized state of inflow during of period t+1

The above notation is used in all subsequent equations to describe the time step and discretization associated with each variable as seen in Loucks et al., 1981.

2.2.3 State Variable Transitions

There are several different methods for transitioning between state variables in an SDP model. The Weekly Storage Model developed in this paper uses two state variables: a storage state variable which describes the amount of storage in the reservoir, and a hydrological state variable which describes the current inflows into the reservoir. Transitions between the storage state variables are governed by the deterministic mass balance equation as described Equation 2-1.

$$S_{lt+1} = S_{kt} + Q_{it} - R_{kilt} - E_{klt} \tag{2-1}$$

Equation 1 addresses the current and future storage state variables, S_{kt} and S_{lt+1} respectively, given the current inflow (Q_{it}), turbine release (R_{kilt}), and evaporation loss (E_{klt}). Evaporation losses are assumed to be negligible for BC's reservoirs.

The Weekly Storage Model uses two different methods to determine the probabilities of transitioning from one hydrological state variable to the next depending on if forecast data is available. A transition probability matrix (TPM) is used in both methods and gives the probabilities of transitioning from the current time step's discretized inflow interval (i) to the next time step's discretized inflow interval (j). The TPM has an equal number of rows and columns equivalent to the number of discretized inflow intervals, and must contain non-negative numbers which sum to 1 in each row as seen in Equations 2-2 and 2-3 below.

$$P_{ij}^{t} = \begin{bmatrix} p_{1,1} & \cdots & p_{1,j} \\ \vdots & \ddots & \vdots \\ p_{i,1} & \cdots & p_{i,j} \end{bmatrix} \tag{2-2}$$

$$\sum_{j} P_{ij}^{t} = 1 \; for \; all \; i \tag{2-3}$$

Where ${P_{ij}}^t$ is a TPM containing probabilities of transitioning from an inflow in time t to an inflow in time t+1. For stages in which forecast data is not available, hydrological state variable transition probabilities are determined using historical values in a first-order Markov chain. The first-order Markov chain method applied to an inflow transition scenario is shown in Equation 2-4 adapted from Loucks et al., 1981.

$$P[Q_t|Q_{t-1}, Q_{t-2}, \cdots] = P[Q_t|Q_{t-1}] \tag{2-4}$$

$$P_{ij}^t = P\left[Q_{t+1} = Q_j|Q_t = Q_i\right] \tag{2-5}$$

Equation 2-4 assumes that the conditional probability of the inflow in the current stage, Q_t, given the entire history of previous flows is assumed to be equal to the conditional probability of Q_t given only the previous stage's flow, Q_{t-1}. Equation 2-5 then applies this principle to create a TPM. This assumption was used since during the freshet, inflows for a given year may vary greatly, but on average follow an increasing or decreasing pattern for a given month. Figure 2-3 shows 78 yearly traces of daily average local inflows for a representative reservoir in BC's interior.

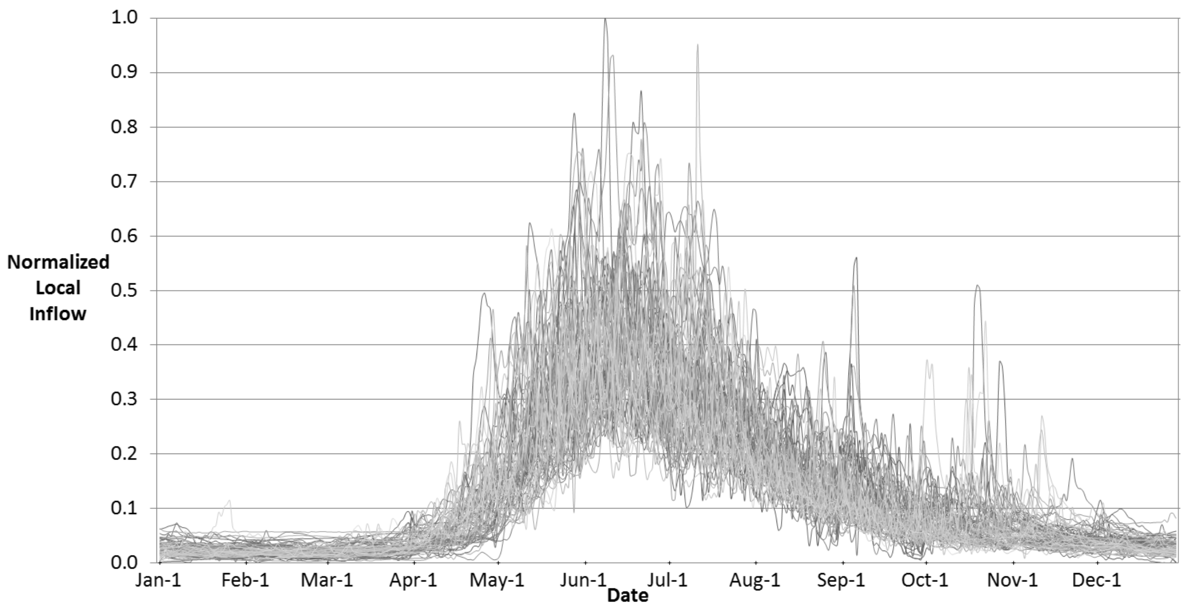

Figure 2-3: Representative Daily Average Local Inflows

As seen in the figure above, local inflows historically tend to increase from April to mid-June, and decrease from mid-June onward. Because of this temporal pattern, several historical TPMs are used depending on which time period the Weekly Storage Model is evaluating.

When reliable local inflow forecasts are available, typically for five days of the planning period, Equations 2-6 and 2-7 are used to create the TPM.

$$P[Q_t|F_t, F_{t+1}, \cdots] = P[Q_t|F_t] \tag{2-6}$$

$$P_{ij}^t = P\left[Q_{t+1} = Q_j|F_t = F_i\right] \tag{2-7}$$

Where F_n is the forecast inflow for staeg n. The probability distribution of the next stage's discretized inflow given its discretized forecast is derived from 12 years of historical forecast data and their respective actual inflows. The forecast's TPM does not depend on the current inflow as the first-order Markov chain TPM does, and therefore has identical rows whose values only depend on F_n.

The incorporation of forecast inflows into transition probabilities between hydrological states is beneficial in reflecting the hydrological state of the reservoir as calculated by an external watershed forecasting model. Short-term forecast inflows from a competent watershed model are assumed to be an improvement over inflow estimates from a historical first-order Markov chain relationship.

2.2.4 Objective Function

The characteristic feature of an SDP optimization model is the objective function. The objective function is also known as the Bellman Equation in discrete time step SDP after its creator, Richard Bellman. The function is used to apply Bellman's Principle of Optimality which states: "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" as referenced from Bellman, 1957.

In order to apply the Principle of Optimality in the Weekly Storage Model, the objective function includes all variables that hold value in the reservoir system. To perform an optimization, the objective function is either maximized or minimized depending on how the variables are presented in order to calculate the optimal value of each decision variable. For example, the objective function used in the Weekly Storage Model may be written as Equation 2-8:

$$f_t{}^n(k,i) = Max_{\{R_{kilt}\}}\left[B_{kilt} + \alpha \sum_j P_{ij}^t \cdot f_{t+1}^{n-1}(l,j)\right] \tag{2-8}$$

Where $f_t{}^n(k,i)$ is the optimal value, t is the current stage, n is the number of stages from the end of the planning period, R_{kilt} is the release decision, B_{kilt} is the release benefit, P_{ij}^t is the TPM of inflow i to inflow j, and $f_{t+1}^{n-1}(l,j)$ is the future value function of l and j. The future value function accounts for the value of storing water for future release. Without the future value function, the model would operate only to maximize its value in the current stage without considering any future benefits beyond t. α for $0 < \alpha \leq 1$ is the discount factor, which can lessen the significance of the future value function to account for time penalties such as monetary inflation. In the case of the Weekly Storage Model, α is set to 1 since the planning period is relatively small and benefits are assumed to be equal regardless of time.

2.2.5 Terminal Value Function

When using the backwards induction method when applying the objective function, n (as seen in Equation 2-8) increases from 1 to the N where N is the total number of stages in the planning period. Therefore in the first computation, in which n = 1, the future value function $f_{t+1}^{n-1}(l,j)$ is not defined. To account for this, a terminal value function is included to assign a value to each possible state variable combination at the end of the planning period for stage n = 0 beyond the end of the planning period. This value is approximated by calculating the value of the generation potential of the active storage resulting from each state variable combination. This method is approximate and does not give an exact value of the active storage, but gives a sensible method of creating a terminal value function as shown by Druce, 1990. The planning period should be long enough so that the terminal value function's influence on the optimal decision at stage 1 where $n = N$ is minimal.

2.2.6 Turbine Unit Availability

It is not always the case that all turbine units in a generating station are in service. Units are often taken out of service for planned maintenance or forced outages. To account for this, the Weekly Storage Model includes a deterministic input for each day of the planning period indicating the number of units in service.

The combinations of units in service also dictate the maximum turbine flow. Any amount of flow exceeding the maximum turbine flow in a given time step is counted as spill in the model and has no release value. In addition to the number of units in service, the Weekly Storage Model also has an input for minimum turbine release. Operating orders for the storage project may require a certain minimum flow under normal operation. Minimum releases are often required as an environmental constraint to maintain constant wetted perimeters and flow velocities downstream of the storage project.

2.2.7 Electricity Market Prices

One of the most unpredictable variables involved in the optimization of the operation of a hydropower project is the electricity market price. Many factors influence electricity prices such as the price of natural gas, inflow conditions in the Pacific Northwest, the demand of electricity in nearby regions, and government renewable energy or greenhouse gas emission policies. A simple influence diagram can be seen in Figure 1-3.

As seen in the influence diagram, two of the main drivers at work in this influence diagram are global economic growth and government policy. For example, Low economic growth may stall the development of greenhouse gas regulation, while high economic growth is assumed to increase the development of greenhouse gas regulation.

Electricity prices react very quickly to changes in any of the factors shown in Figure 1-3. Because of this, electricity prices very volatile, do not follow a trend, and cannot be predicted with the use of historical data with any accuracy. The Weekly Storage Model relies on the expertise of electricity trading specialists and their models for electricity price estimates. Operators of BC Hydro's generating systems work closely with these individuals and have electricity price estimates readily available to them. Since input electricity prices have a large influence on the behavior of the Weekly Storage Model, users are encouraged to try a variety of electricity price scenarios in order to perform a sensitivity analysis during periods of high electricity price volatility.

2.2.8 Marginal Value of Energy

Tilmant et al., 2008 describes a reservoir's marginal value as "the contribution of an additional unit of water to whatever public or private objective is under consideration". In reference to the Weekly Storage Model, the marginal value of energy is the dollar value associated with the inclusion of one additional unit of volume of stored water in the reservoir. As such, operators use these marginal water values to determine when to buy or sell electricity. This metric gives a means of comparing the current state of the reservoir with the current electricity market price in order to determine if it is more valuable to convert an additional unit of storage into electricity in the current time step or retain it for future use.

In a more technical sense, the marginal value of energy is the derivative of the objective function in Equation 2-8 with respect to a given storage state. The marginal value of energy decreases with increasing storage and hydrological states when water is plentiful, and increases with decreasing storage and hydrological states when water is scare. Reservoir operators will generate electricity as needed to meet domestic electricity demand and sell electricity when the marginal price of electricity in the reservoir is lower than the electricity market price, or buy electricity when the marginal price is greater than the electricity market price.

2.3 Weekly Storage Model Operation and Results

The Weekly Storage Model is a stand-alone, user-friendly, computer optimization program that assists operators of reservoirs with weekly storage flexibility to make optimal release and generation decisions during the freshet period while aiming to maximize the value of electricity generation. The model integrates forecast and historical inflow data and provides a base-line decision for the optimal expected-value operation.

To achieve this, the model:

- Is self-contained in a single, easy to run and transportable Microsoft Excel VBA file that is compatible with most computers' installed software packages. Inputs and outputs are clear, simple, and intuitive even to operators who may not be familiar with computer modeling.

- Makes use of up-to-date information on electricity market prices, unit availability, and local inflow forecasts.
- Includes operational constraints to reflect the limitations of the reservoir and its generating station.
- Provides operators with an optimal expected value release decision while considering the value of generating electricity in the present as well as the future. The marginal value of energy of the current state of the reservoir is also provided for decision support.

During testing, the Weekly Storage Model produced results similar to Figure 2-4 below.

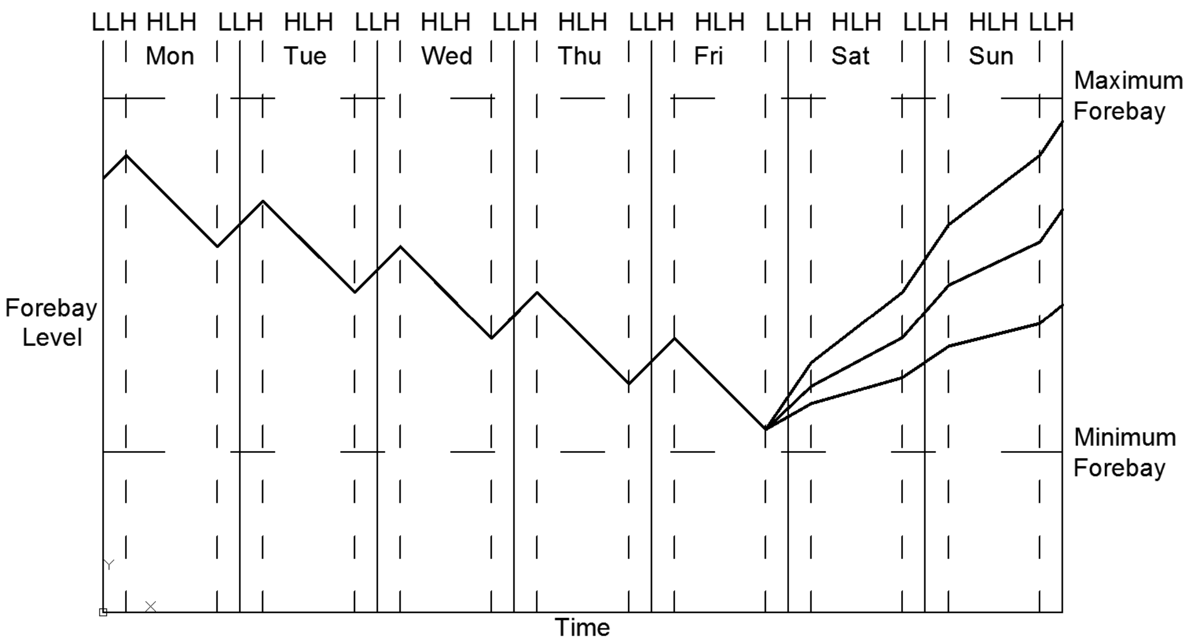

Figure 2-4: Example of Reservoir Drafting Path

Figure 2-4 represents a possible drafting pattern of the reservoir for one week given typical inflow, electricity price, and unit availability estimates. The forebay level is likely to begin relatively high on Monday morning due to low electricity prices and low electricity demand during the weekend as well as large inflows throughout the freshet period. The reservoir will then draft during HLH periods and store inflow during LLH periods. During the weekend when electricity prices and demand are generally lower than the weekdays, the reservoir will store inflow. The forebay level at the end of the week is dependent on forecasted and historical characteristics of the following week. For example, forecasted low electricity prices and high inflows in the following week will encourage the model to only partially refill and take advantage of relatively higher electricity prices in the current week. Conversely, forecasted high electricity prices and lower inflows in the following week will encourage the model to store a significant amount in the current week for later generation and an optimal total benefit for the full planning period. Operators of BC Hydro reservoirs plan to begin testing the Weekly Storage Model to assist in turbine release decisions during the 2014 freshet.

Chapter 3: Stochastic Optimization of Snowmelt-fed Reservoirs with Limited Storage during the Freshet

The following paper was written at the end of the research and development process and presents an overview of the model's formulation and use. In the interest of time, the reader may decide to only read this chapter for a summary of the thesis. It is planned that this paper or one similar will be submitted to be published to a technical journal relating to water resources management. The abstract, acknowledgements, and author's biographies sections were omitted from this chapter. All references cited can be found in the Bibliography section at the end of the thesis.

3.1 Introduction

The harnessing of running water for energy has been performed for centuries and continues to the present day. Hydropower projects, from simple run-of-the-river plants to more advanced pumped-storage and tidal technologies, contribute roughly 15% of the worldwide electricity production in over 100 countries (World Energy Resource: 2013 Survey, 2013). BC Hydro is the primary electricity supplier and distributor in BC, Canada, and uses hydropower to meet approximately 90% of the province's electricity needs. BC Hydro operates a combination of hydropower projects with no usable storage (run-of-the-river), limited storage flexibility (which operate on a sub-yearly fill-draft cycle), and large storage reservoirs (which operate on a yearly or multi-year fill-draft cycle). Although hydropower provides a suitable solution to BC's electricity needs, its supply is less certain than other conventional generation methods. Inflows vary from season to season and system operators must plan for irregular events to protect residents from electricity shortages while considering system and environmental constraints.

Because of this, a diverse range of hydropower projects are operated along with three auxiliary thermal plants and electricity imports and exports with the aim to deliver sustainable and dependable electricity to BC Hydro's ratepayers.

BC Hydro works with electricity traders to buy and sell wholesale electricity with the United States and Alberta through high-voltage transmission interconnections. Since the timing of electricity supply and demand varies drastically from region to region, the ability to buy and sell electricity benefits all electricity producers involved. BC Hydro has an advantage over many other electricity producers in the electricity market because of their large reservoirs. The reservoirs act like batteries, storing water when electricity demand or electricity prices are low, and releasing water when electricity demand or electricity prices are high (Making the Connection, 2000).

Electricity demand in BC is relatively greater in the winter due to increased electricity use for lighting and heating in the cold winter months as compared to the mild summer months. Conversely, many of BC's reservoirs' inflows are relatively high in the spring season when snow begins to melt and are much lower in the winter. In order to shape the availability of water into the winter where it is needed most, large reservoirs are used. These large reservoirs fill during the low demand spring season using the large freshet inflows from May through July. The reservoirs then release the stored water throughout the rest of the year to meet electricity demand with hydropower even with small amounts of local inflow.

These larger reservoirs are often located close to the headwaters of rivers with additional projects located downstream. The downstream projects are able to add additional generation capacity to the shaped discharges of the large reservoirs. Because of this, the reservoirs of these downstream projects often only use small amounts of active storage, relying on the large upstream reservoir for shaping the delivery of water. However, discharge from large reservoirs will often be minimal throughout the spring freshet in order to support reservoir refill. During this time, generation from these upstream storage projects is often not required since demand is largely supported by public and private hydropower projects using run-of-the-river and limited storage reservoirs that take advantage of high freshet inflows.

Therefore, hydropower projects directly downstream of these large reservoirs are receiving little to no upstream project flows. Instead, they may receive a large amount of local inflows from runoff, particularly in the case where the watershed is fed predominantly by snowmelt. The reduction in upstream releases and increase in local inflows results in a decoupling of operations between the upstream and downstream plants. Other projects that encounter this change include projects with available active storage that may not have been exercised during the winter due to lack of inflows. With the arrival of heavier freshet inflows, these projects may have the flexibility to utilize storage across a weekly or multi-weekly period depending on the project's active storage, inflow volumes, and maximum turbine flow. These downstream or single reservoir storage projects are strategically operated at higher generation levels during HLH weekdays to serve higher provincial electricity demand and stronger electricity market prices, and lower generation levels on weekends and nightly LLH to refill the small reservoirs.

For some hydropower projects, this short-term operational change is not included in BC Hydro's operation optimization models. Instead, reservoir operators prepare operational decisions heuristically based on operational constraints and forecasts of local inflows and electricity prices. Although this method yields satisfactory results, there is an opportunity to use more explicit and formal optimization. In addition, operators frequently change posts and the benefit of any learned intricacies of operating these projects may be lost.

This paper describes a stand-alone user-friendly stochastic dynamic programming optimization model built to assist BC Hydro operators in the management of reservoirs with limited storage, referred to as the Freshet Model. The Freshet Model provides guidance to operators in making optimal release and generation decisions specifically during the freshet period. The development of the Freshet Model was carried out in coordination with reservoir operators and optimization specialists at BC Hydro to ensure that the end product met their needs. Relevant uncertainties such as electricity market prices, unit availabilities, upstream and local inflows were addressed using historical and forecast data produced by electricity trading, operations, and hydrology experts and their respective models. The Freshet Model prepares optimal expected release policies and a marginal value of energy for these plants. These results can be referred to during the freshet period for decision support as an alternative to pure heuristics.

3.2 Literature Review

With the widespread availability of computing power in recent years, computer-run mathematical models offer a practical solution to optimizing water resource allocation problems. Many techniques have been created, implemented, and enhanced over the last several decades in order to obtain optimal or near-optimal policies for reservoir operation (Yeh, 1985; Labadie, 2004). In order to accurately describe a water resources system, the modeling method must address inherent nonlinear and stochastic characteristics in an appropriate manner. Two methods of achieving this include implicit and explicit stochastic optimization.

3.2.1 Implicit and Explicit Stochastic Optimization

Implicit stochastic optimization (ISO) involves the deterministic optimization of many historical or synthetically generated input sequences that reflect the statistical nature of the stochastic variable. Once a sufficient number of deterministic sequences are solved, a regression analysis can be performed to form a relationship between inputs, outputs, and decisions. An example of the ISO process is the Monte Carlo method. Iteratively running multiple sequences that match the system's typical behavior allows the model to implicitly include temporal and spatial correlations (Labadie, 2004). However, this method rests heavily on the historical sequences and synthetic generation methods used. It can also be computationally intensive as a large amount of solutions are required. Three popular types of deterministic optimization methods commonly used in the ISO procedure include linear programming, nonlinear programming, and deterministic dynamic programming.

In contrast to ISO, explicit stochastic optimization (ESO) uses probability distributions incorporated into a stochastic optimization model. ESO can therefore utilize more stochastic information than ISO by representing the stochastic variable in the form of conditional probabilities rather than a sample in the form of multiple sequences. This allows models to produce optimal policies without the need for interpreting outcomes from an iterative ISO method. However, ESO methods are often more computationally demanding relative to ISO methods. Because of this, ESO methods often are performed using discrete analysis.

For reservoir operations models, inflows are often a major uncertainty and are addressed as a stochastic variable. The probabilities of uncertain inflows can be determined using a spatial or temporal relation in the long term, and forecasting prediction error in the short term (Fayaed, 2013). Stochastic dynamic programming (SDP) and chance constraints are two common methods of ESO, with SDP being the most popular method of ESO in water resources optimization (Celeste, 2009).

3.2.2 Deterministic and Stochastic Dynamic Programming

A popular and successful technique in reservoir optimization is the dynamic programming (DP) method, which was formulated by Bellman, 1957. DP has a very broad area of application and can reach an optimal solution in systems where other optimization techniques may fail. DP aims to break a complex problem up into smaller sub-problems, solve the sub-problems, and then combine the solutions of the sub-problems to create an overall solution.

The DP technique involves performing a complete enumeration of all feasible decision variable combinations before identifying an optimal solution. Because of this, DP is able to produce optimal solutions regardless of the continuity, convexity, or concavity of the functions used. Problems must be decomposed as multi-stage decision procedures when being analyzed with DP using a series of stages, states, and decisions. In reservoir operations, stages are often time steps in which a decision is required and states are often water volume or other fundamental variables that summarize the information needed to identify the optimal decision for a given stage and state.

Although DP is not discrete by definition, discrete DP is the most popular method used in water resources as compared to methods such as differential DP and state incremental DP (Yakowitz, 1982). DP uses the Principle of Optimality which states “An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision” (Bellman, 1957). This principle is applied with the use of the objective function, which includes expressions for the current and future benefits of a given decision and is maximized or minimized to yield a policy of optimal decisions.

DP faces limitations when being applied to large scale systems because of the large number of stages and states required to solve the many sub-problems. This exponential increase in required computing power is known as the ‘curse of dimensionality’. However, DP is still attractive to smaller scale water resources problems due to its capability of solving highly nonlinear, non-continuous functions and constraints (Keckler, 1968).

Stochastic dynamic programming (SDP) is an ESO technique that enhances the DP method with the addition of a probabilistic description of the stochastic variables. The SDP method has been proven to be well suited for reservoir operations problems as it can incorporate nonlinear and complex features commonly found in reservoir operations such as uncertainty in streamflow and electricity market prices. The method has been used extensively for decades in reservoir optimization applications.

Little, 1955 was one of the first researchers to use the SDP method to optimize a reservoir operation policy for the Grand Coulee dam on the Columbia River. It was found that using even a simple model of the reservoir system, small scheduling improvements could be made over rule-curve methods. Bras et al., 1983 employed an adaptive control technique in which the stream flow transition probabilities in their SDP model were updated continuously. This provided significant benefits to flood control as seen in their case study on the High Aswan Dam. Stedinger et al., 1984 used the current forecast local inflow as a hydrological state variable rather than the traditional SDP method of using the previous stage's local inflow in their SDP model. This resulted in substantial improvements in the simulated reservoir operations as compared with models using the traditional approach. Druce, 1990 implemented an SDP model of the BC Hydro system which established a marginal cost of generation used in optimizing hydropower operations for projects on the Peace River. The model was able to produce an optimal monthly operating policy which included economic and physical data for decision support. Kelman et al., 1990 and Faber et al., 2001 took advantage of available inflow forecast sequences in a method known as sampling stochastic dynamic programming (SSDP). SSDP allows the model to capture the multi-period persistence of stream flows which is especially useful in streams fueled by snowmelt.

3.2.3 Summary

DP and SDP techniques have been successfully implemented to accurately model and optimize water resources system operations. The performance of these techniques depends largely on the characteristics of the system being modeled and amount of data available on the system. These methods are often altered slightly for each specific application to yield more accurate or efficient solutions.

This paper describes the development of an SDP optimization Freshet Model used to provide short-term decision support for single reservoir systems with limited storage during periods of large inflows, such as during the freshet. The project's important variables are strategically discretized to limit the size of the optimization problem. The objective was to maximize the value of electricity that can be generated while accounting for stochastic variables, which is well suited to the use of the SDP method. The 'curse of dimensionality', a major limitation of the SDP method, is avoided in large due to the Freshet Model considering only one reservoir of limited or moderate active storage for a relatively short planning period of up to 14 daily stages. Also, the use of SDP with a terminal value function does not depend on steady-state convergence and a corresponding increase in required computing power. This yields a reliable model for operators looking for quick and accurate decision support.

3.3 Freshet Model Methodology

The Freshet Model was developed with input from reservoir operators at BC Hydro with the goal of creating an operations planning tool that can be used as decision support for future operations for reservoirs with limited storage capability in the BC Hydro system during the freshet. A case study was then performed by applying the Freshet Model to a project in the BC Hydro system identified by operators as having the potential to receive benefits from increased optimization during the freshet period.

3.3.1 Stochastic Dynamic Programming Method

SDP enumerates all feasible solutions for a given problem and is based off of the deterministic DP method. An objective function is recursively applied to sub-problems composed by discrete stages and states in which Bellman's Principle of Optimality is used to account for future value.

Loucks et al., 1981 describes the deterministic DP method as follows: If a system has m discrete states, s_1, … ,s_m, and $NB_t(s_i, s_j, k)$ denotes the net benefits during period t when the system starts in state s_i and ends in state s_j when decision k is made, the resulting recursive equation is shown in Equation 3-1 (Loucks et al., 1981).

$$f_t(s_i) = max_k\left[NB_t(s_i, s_j, k) + f_{t+1}(s_j)\right] \tag{3-1}$$

Where $f_t(s_i)$ is the maximum net benefit obtainable in time period t onwards starting in state s_i in stage t. Since this is the deterministic case, the subsequent state s_j is simply a deterministic function of the initial state s_i and the decision k. Each recursive equation is dependent on the next stage's recursive equation's solution and represents an optimal decision for a given stage in each sub-problem of the overall problem. The recursive calculations often start at time period

$t = T$ and progress backwards in time, where T is the total number of stages in a process. This process is known as backwards induction.

The deterministic DP definition can be expanded to include stochastic variables in the SDP method: let $p_{\mathrm{ij}}^{\mathrm{t}}(k)$, known as the transition probabilities, equal the probability that the state in period t+1 is s_j, given that the state of period t is s_i and decision k is made as described in Equation 3-2 (Loucks et al., 1981) below.

$$p_{\mathrm{ij}}^{\mathrm{t}}(k) = \Pr\left[s^{t+1} = s_j | s^t = s_i \text{ and decision } k\right] \tag{3-2}$$

These transition probabilities are then included in the original deterministic DP recursive equation shown in Equation 3-3 (Loucks et al., 1981) below.

$$f_t(s_i) = \ max_k \left\{ \sum_{j=1}^{m} p_{\mathrm{ij}}^{\mathrm{t}}(k) \cdot \left[NB_t\left(s_i, s_j, k\right) + f_{t+1}\left(s_j\right)\right] \right\} \tag{3-3}$$

Using the SDP recursive equation above, the expected benefits for each decision k is calculated in a reverse induction method.

3.3.2 State Variables

The Freshet Model uses a discrete deterministic storage state variable and a discrete stochastic hydrological state variable to describe the state of the reservoir and state of the watershed respectively. The discretization size was determined through trial and error and was dependent on the reservoir system to which the Freshet Model is being applied; there must be enough discrete states in each stage to accurately describe the state of the system without excessive precision and required computing power as investigated in Ayad et al., 2013.

3.3.2.1 Storage State

Transitions between the storage state variables are governed by the deterministic mass balance equation as described Equation 3-4.

$$S_{t+1} = S_t + Q_t - R_t - E_t \tag{3-4}$$

Equation 3-4 addresses the current and future storage state variables, S_t and S_{t+1} respectively, given the current stage's inflow (Q_t), turbine and spill release (R_t), and evaporation losses (E_t). Evaporation losses are assumed to be negligible for BC's reservoirs. Equation 3-4 assumes that the inflow in the current stage is known; the forecast inflow for stage t was used deterministically for this purpose.

3.3.2.2 Hydrological State

Two different methods are used to determine the probabilities of transitioning from one hydrological state variable to the next depending the available data at the given daily stage. A transition probability matrix (TPM) contains the probabilities used by the Freshet Model for transitioning from the current stage's discretized forecast inflow interval (indexed over i) to the next stage's discretized inflow interval (indexed over j). A TPM has an equal number of rows and columns (n) which are equivalent to the number of inflow discretization intervals and must contain non-negative numbers which sum to 1 in each row as seen in Equation 3-5 below referenced from Loucks et al., 1981.

$$\sum_{j=1}^{n} p_{ij} = 1 \quad \text{for all i} \tag{3-5}$$

Where p_{ij} is a TPM containing probabilities of transitioning from a given discretized inflow in the current to the subsequent stage.

3.3.2.2.1 First-Order Markov Chain

For stages in which forecast data is not available, the Freshet Model's hydrological state variable transition probabilities are determined using historical values in a Markov process. The Markov process assumes that future values depend on past values, and can be summarized by the current value or state. Equations 3-6 (Loucks et al., 1981) explains this assumption for the stochastic process $X(t)$ with k>0,

$$\mathrm{F}_X[X(t+k)|X(t), X(t-1), X(t-2), \dots] = F_X[X(t+k)|X(t)] \tag{3-6}$$

When a Markov process whose state $X(t)$ can only consist of discrete values, it is referred to as a Markov chain. When k = 1, the process is known as a first-order Markov chain. The first-order Markov chain applied to an inflow transition scenario is shown in Equation 3-7.

$$PH_{ij}^{t} = \Pr\left[Q_{t+1} = q_j | Q_t = q_i\right] \tag{3-7}$$

Equation 3-7 applies the Markov process using Equation 3-6 to create a TPM for a discrete Markov chain where Q_t is a discrete random variable in stage t, with $w < t \leq T$ where w is the total number of daily sequential streamflow forecasts available: This method of populating the TPM is only used for stages in which inflow forecasts are not available. The resulting values are denoted by q_i and q_t, discrete values in stage t and stage t+1 respectively.

The first-order Markov chain assumption is appropriate for the freshet situation since daily inflows between different year's freshets may vary greatly, but on average follow an increasing or decreasing pattern. Figure 2-3 shows 78 yearly traces of daily average local inflows for the case-study reservoir located in BC's interior.

It can be seen that, in general, local inflows historically tend to increase in April, peak in June, and then decrease in July. Because of this temporal pattern, several TPMs were created and are used by the Freshet Model depending on which calendar week is being evaluated. Using historical data in this way assumes a stationary stochastic process with no increasing or decreasing trends in statistical parameters such as variance and mean. To test this assumption, a standard-T test was used with the 78 years of inflow data to test for statistically significant correlation between inflow values and time. It was found that the historical data seen in Figure 2-3 shows no significant correlation with time and is assumed to be stationary with 95% certainty. An autoregressive analysis was also performed between sequential values of daily inflows to test first-order autoregressive correlation of the inflow values. It was also found that the correlation of the inflow values fall within the 95% certainty range, and are appropriate for use in a first-order Markov chain for forecasting future inflows.

The results of using this method during the freshet period to predict next-day local inflows for a reservoir in a snowmelt dominant watershed can be seen in Figure 3-1. Four sets of information are shown in the figure by manipulating the forecast value (F_t), actual inflow value (Q_t), and highest observed actual inflow value (Q_t^*). The normalized forecast error defined by $(F_t - Q_t)/Q_t^*$, the sorted historical inflows are defined by Q_t/Q_t^*, the Markov chain forecast is defined by F_t/Q_t^*, and the relative forecast error is defined by $(F_t - Q_t)/Q_t$.

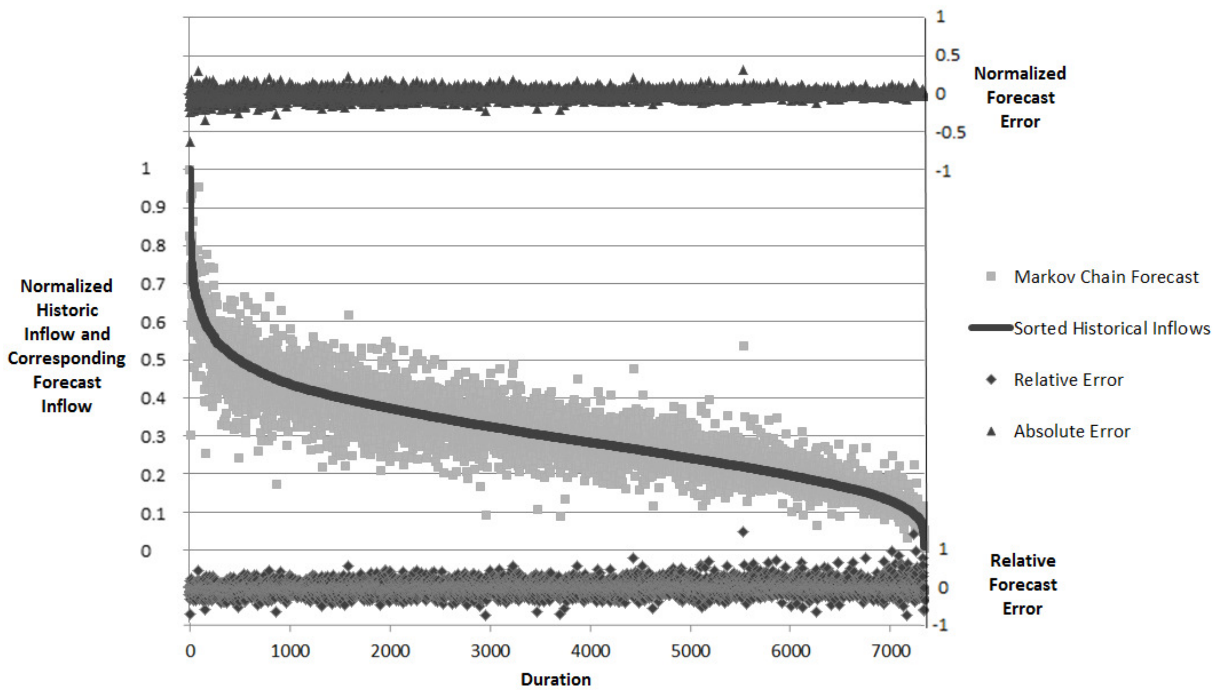

Figure 3-1: Performance of First-Order Markov Chain in Forecasting Local Inflows during the Freshet

As seen in the figure above, the first-order Markov chain forecasts correlate with the actual local inflows using historical records for the case study reservoir in a snowmelt-dominant watershed in the BC Hydro system during the freshet season. The normalized forecast error increases with actual inflow values while the relative forecast error decreases which shows that relatively large error are likely for low forecasts. The average absolute relative error for this method was calculated to be 11%. The distribution of forecast error is fairly uniform over the range of historical inflows.

3.3.2.2.2 Watershed Model Forecast

Although inflow forecasts often give a reasonable estimate of future local inflows into certain reservoirs, they are not without error. Climatologists and hydrologists are given the difficult task of attempting to predict future weather patterns and inflows, but lack of climate information over the Pacific Ocean can cause forecasts to become inaccurate only days after they are produced. Therefore, the Freshet Model includes this uncertainty along with the inflow forecasts. When local inflow forecasts are available, typically for five days of the planning period, Equations 3-8 and 3-9 are used to create the TPM.

$$\mathrm{Pr}[\mathrm{Q_t}|\mathrm{F_t}, \mathrm{F_{t+1}}, \cdots] = \mathrm{Pr}[\mathrm{Q_t}|\mathrm{F_t}] \tag{3-8}$$

$$PF_{\mathrm{ij}}^{\mathrm{t}} = \mathrm{Pr}\left[\mathrm{Q_{t+1}} = \mathrm{Q_j}|\mathrm{F_t} = \mathrm{F_i}\right] \tag{3-9}$$

Where F_t is the forecast inflow for stage t, with $1 \leq t \leq w$, where w is the total number of stages sequential streamflow forecasts are available for, starting with the earliest stage. The probability distribution of the next stage's discretized inflow given its discretized forecast was derived using 12 years of historical forecast data and their respective actual inflows. The forecast's TPM does not depend on the current inflow as the Markov process does, and therefore has identical rows whose values are conditional on F_t alone.

The incorporation of forecast inflows into transition probabilities between hydrological states is beneficial in reflecting the hydrological state of the reservoir as calculated in an external watershed forecasting model. Short-term forecast inflows from a validated watershed model have shown to be an improvement over inflow estimates from a historical lag-1 Markov chain relationship as seen in Table 3-1. Figures 3-2 and 3-3 below show sorted actual freshet local inflows and their corresponding same-day forecast and four day forward forecast for the case study reservoir.

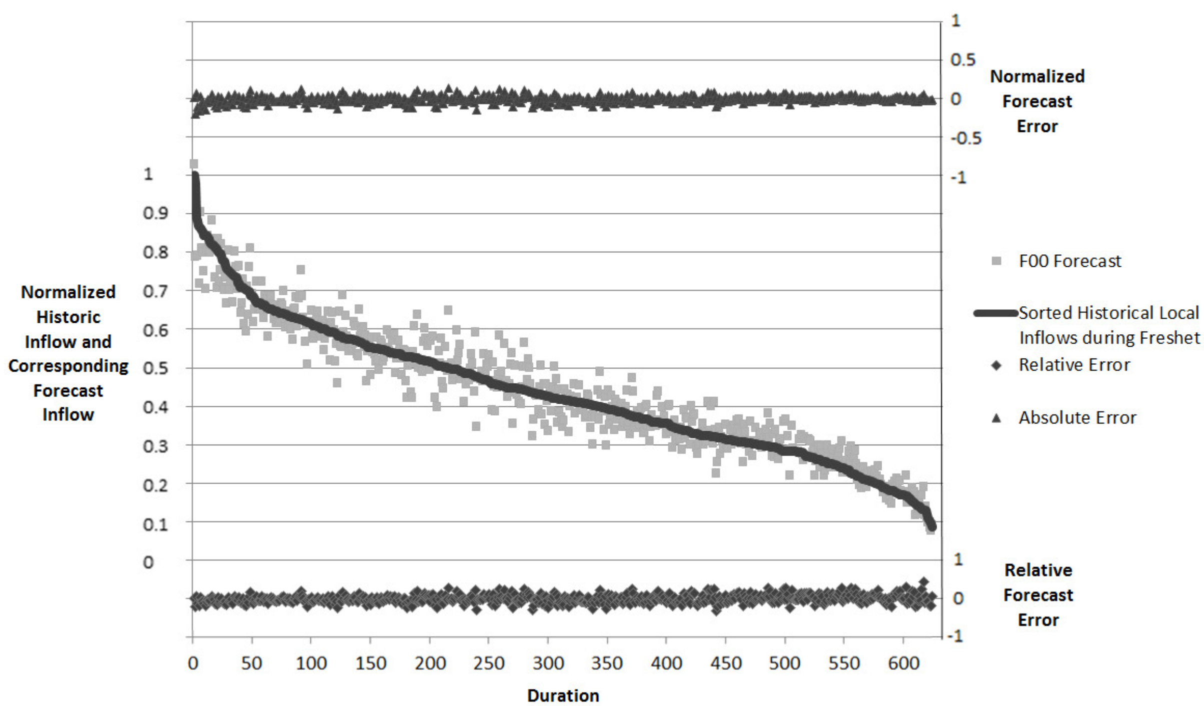

Figure 3-2: Performance of Same-Day Forecast in Predicting Local Freshet Inflows

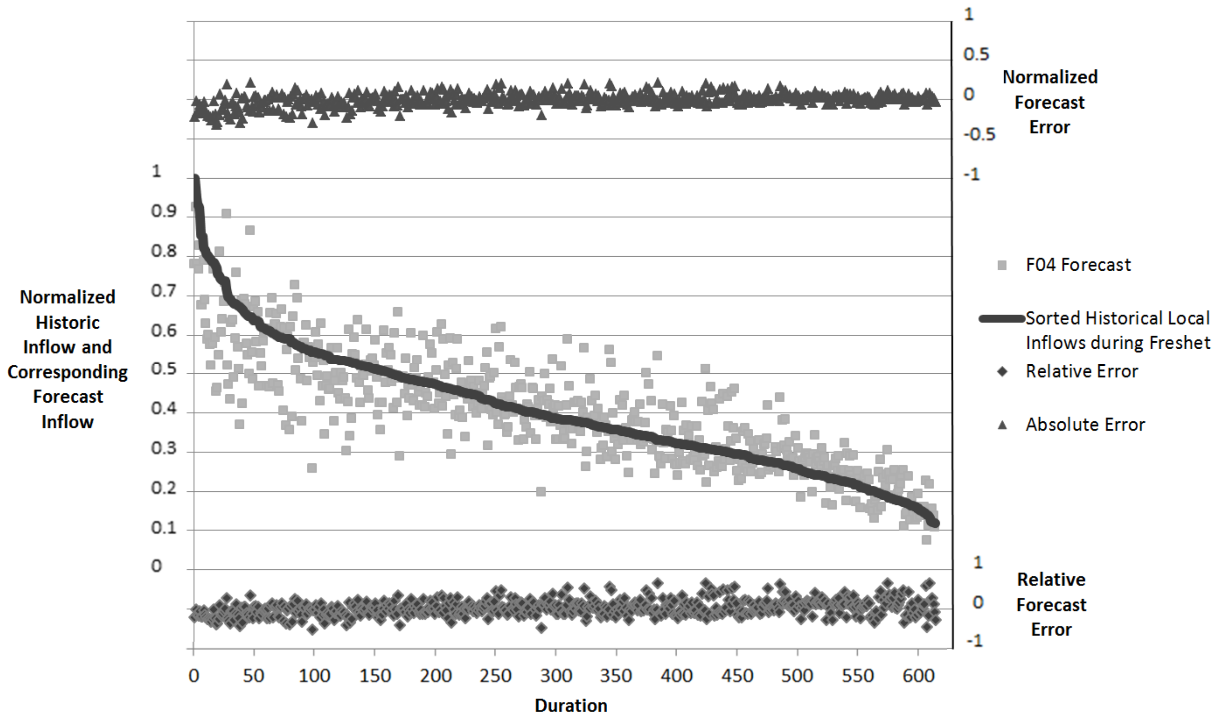

Figure 3-3: Performance of Fifth-Day Forecast in Predicting Local Freshet Inflows

In the figures above, the most recent 12 years of five-day forecast data was used in this analysis in order to reflect recent forecasting practices. As observed in the above figures, and in Table 3-1, inflow forecast accuracy generally decreases the greater the lead time is on the forecasts made as one would expect.

3.3.2.2.3 Forecast Accuracy and Limitations

The average absolute relative errors for each forecast type are shown in Table 3-1 below.

Table 3-1: Comparison of Prediction Error

Days in Advance Forecast	0		1	2	3	4
Prediction Type	Forecast	FMC	Forecast	Forecast	Forecast	Forecast
Average Absolute Relative Error	8%	11%	12%	13%	14%	16%

The first-order Markov chain forecast method showed an average absolute relative error of 11%. The watershed model forecast errors increase with lead time of forecasts, ranging from 8% to 16%. This result shows that the use of the first-order Markov chain is out performed by the watershed model forecast method when it is used to predict same-day inflows.

However, the combination of forecast inflows and estimates using a Markov chain comes with some complications as there may be a disconnect between the assumptions made in each method with regards to hydrological conditions. In the forecast method, generation of the TPMs within the first five days of the planning period is a reflection of the current meteorological conditions within the watershed. On the other hand, the historical method of generating TPMs is a reflection of the historical meteorological conditions within the watershed. Therefore, in extreme years large differences are observed in the generated forecasted and historical TPMs between the last forecast stage and first historical stage. For example, the watershed model may predict a large portion of snowpack will melt earlier than average as seen in Figure 3-4 Below.

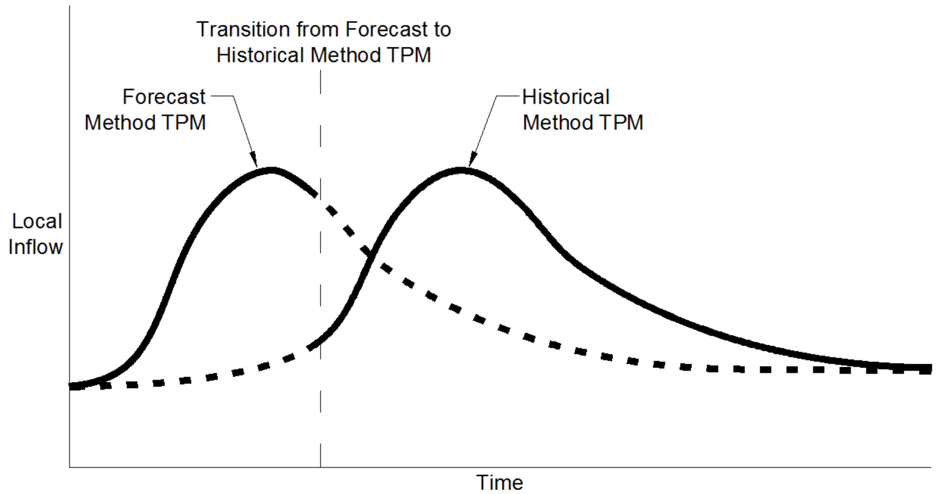

Figure 3-4: Disconnect Example between Extreme Forecast and Average Historical TPM Methods

If the majority of snowpack melts early in the freshet period, the snowpack will be depleted and inflows later in the freshet will likely be much less than average. However when the optimization algorithm transitions to the first-order Markov chain method later in the planning period, the algorithm assumes a historical average inflow and predicts inflows which may be physically impossible given the depleted snowpack used in the forecast method. The Freshet Model will now expect a second peak of inflows soon after transitioning to the historical method of calculating the TPM. However, any errors from this complication are somewhat mitigated since the Freshet Model's decisions are influenced more heavily by stages containing forecast inflows due to occurring sooner in the planning period.

3.3.3 Objective Function

The key feature of any SDP model is its objective function. In order to apply the Principle of Optimality in the Freshet Model, the objective function includes all variables that are defined by the operator to hold value and significantly influence the operation of the reservoir system. To perform an optimization, the objective function is either maximized or minimized depending on how the variables are presented to calculate the optimal value of each decision variable. The objective function used in the Freshet Model is written as shown in Equation 3-10 with the following variable definitions:

$$f_t^{\,n}(k,i) = \mathrm{Max}_{\{l\}} \left[B_{kit} + \sum_j p_{ij}^t \cdot f_{t+1}^{n-1}(l,j) \right] \qquad (3\text{-}10)$$

Where:

- $f_t^{\,n}(k,i)$= maximum sum of turbine release benefits given an initial discretized storage volume (indexed by k for time period t, and l for time period t+1) and discretized inflow volume (indexed by i for time period t, and j for time period t+1) time period t with n time periods remaining until the end of the planning period.

- B_{kit} = $QT_{kt} \cdot HK_{ki}(S_t, S_{t+1}, U) \cdot EP_t$ = immediate benefits from a release decision where QT_{kt} is the average turbine release flow over time period t, $HK_{ki}(S_t, S_{t+1}, U)$ is the efficiency factor between turbine flow and generation (which is a function of storage in time period t, and storage in time period t+1 and available turbine unites (U)) , and EP_t is the average electricity price time-weighted between HLH and LLH.

- p_{ij}^t = transition probability matrix for stage t. Using $PF_{\mathrm{ij}}^{\mathrm{t}}$ if $1 \le t \le w$ and inflow forecasts are available, and using $PH_{\mathrm{ij}}^{\mathrm{t}}$ if $w < t \le T$ and inflow forecasts are not available for t.

- $f_{t+1}^{n-1}(l,j)$ = maximum sum of turbine release benefits given an initial discretized storage volume for volume S_{t+1} and discretized inflow volume for flow Q_{t+1} in time period t+1 with n-1 time periods remaining until the end of the planning period.

The future value function, $f_{t+1}^{n-1}(l,j)$, accounts for the value of storing water for future releases. In the case of the Freshet Model, no discount factor is used since the planning period is relatively small and benefits are assumed to be equal regardless of time.

3.3.4 Constraints

Each reservoir and generating station has its own unique set of operational constraints. Some typical constraints that are included in the Freshet Model are shown in the following equations. Inequalities for the storage state are implemented to define the reservoir's active storage boundaries as seen in the equation below.

$$FB_{min} \leq FB \leq FB_{max} \tag{3-11}$$

Equation 3-11 states that the reservoir forebay (FB) must be between a minimum (FB_{min}) and maximum (FB_{max}) storage value at all times. The Freshet Model uses project-specific information regarding the case study reservoir's forebay to storage relationship, which is used to convert between each. Another constraint included in the Freshet Model gives a minimum release flow downstream of projects as seen in Equation 3-12.

$$QP_{min} \leq QP \leq QP_{max} \tag{3-12}$$

Where QP is the discharge from the project. The minimum plant flow limit helps to ensure the wetted perimeter downstream of the project is preserved for marine ecosystems in the river while the maximum prevents flooding in areas downstream of the reservoir. Similar constraints are used for turbine releases and spills to reflect the physical limitations of turbines and spillways.

3.3.5 Terminal Value Function

When applying the objective function in backwards induction, n (as seen in Equation 3-10) increases from 1 to the N where N is the total number of stages in the planning period. Therefore in the first computation, in which n = 1, the future value function $f_{t+1}^{n-1}(l,j)$ needs to be defined. In this study, a terminal value function is approximated by assigning a value to each feasible state variable combination for time period T+1 beyond the end of the planning period.

The terminal value function's values are approximated by calculating the value of the potential generation of the usable storage for each state variable combination. For each state variable combination, the Freshet Model calculates the sum of the active storage in the state variable and the volume of local inflow in the hydrological state variable. The total active storage value is then assigned a monetary value based on turbine efficiency data and user input electricity market prices, similar to the method used by Druce, 1990. This procedure is approximate and does not provide an exact value of the active storage, but gives a sensible method of creating a terminal value function. The longer the planning period is, the lower the terminal value function's influence will be on the optimal decision at stage 1, where n = N.

3.3.6 Turbine Variables

The amount of electricity generated by a certain turbine release decision depends on many factors such as head, flow, and the turbine's mechanical characteristics. In a generating station with multiple turbines, there are often multiple combinations of individual turbines that can generate a given megawatt value. BC Hydro uses in-house software and turbine-specific data from their in-house Static Plant Unit Commitment (SPUC) program to determine the optimal combination of turbines that will generate a given megawatt value using the least amount of flow.

An example of a normalized typical efficiency curve relating efficiency to turbine flow for a fixed forebay level that would be included in the Freshet Model for a generation station with six turbines can be seen in Figure 3-5 below.

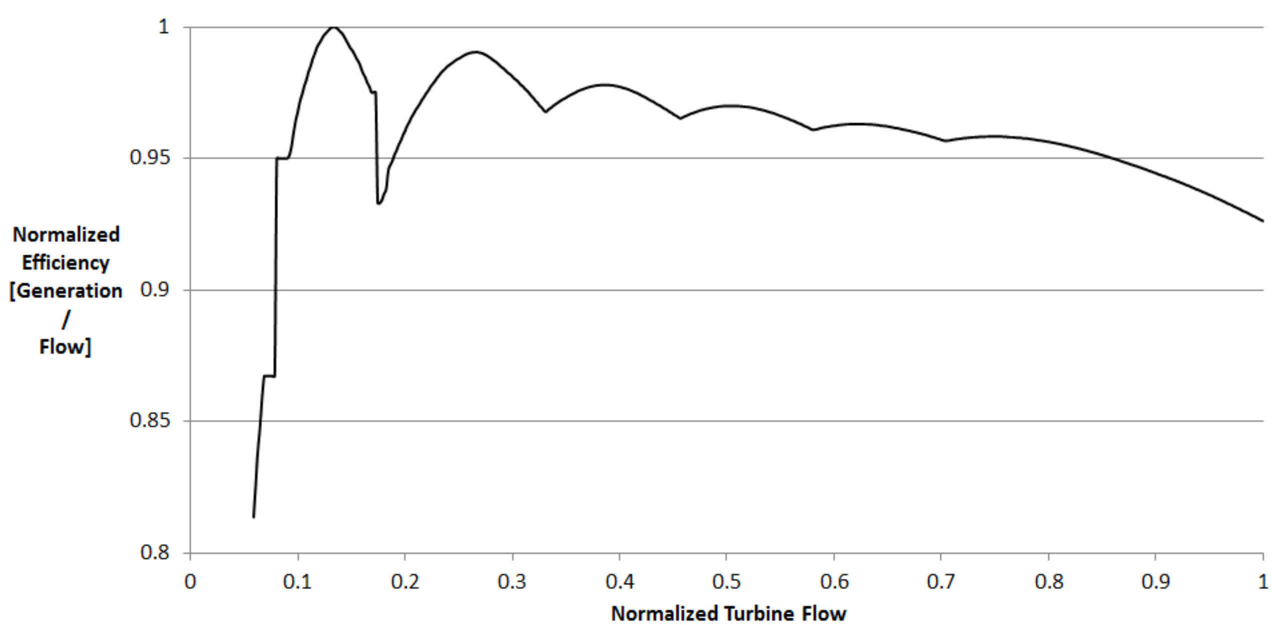

Figure 3-5: Normalized Efficiency Curve for a Generating Station with Six Units

As seen in the above figure, each additional local peak in increasing turbine flows represents the increase in efficiency of including an additional turbine while using the optimal combination of turbines. Therefore, six local peaks are observable for a generating station with six turbines. Several inefficient, or “rough load”, zones which contribute significant wear on the turbine are observable when only one turbine is active, but are eliminated once multiple turbines are active simultaneously. The curve in Figure 3-5 will rise and fall with the forebay level and net head of the reservoir, increasing the generation provided by a given turbine flow.

It is not always the case that all turbines in a generating station are in service. Turbines are often taken out of service for planned maintenance or forced outages. To account for this, the Freshet Model includes a deterministic input for each day of the planning period indicating the number of turbines in service. The number of units in service also dictates the maximum turbine flow. The efficiency curves and maximum turbine flow parameters are updated to reflect the number of turbines that are available for electricity generation. Any amount of flow exceeding the maximum turbine flow is categorized as spill in the Freshet Model and has no value. Although there is no direct penalty associated with a spill release in the Freshet Model, there in an indirect penalty since the resulting storage state will have less head and therefore less generation associated with a future turbine release decision.

In addition to the number of units in service, the Freshet Model also has an input for a minimum turbine release. Storage projects may require a certain minimum flow under normal conditions to avoid turbine rough load zones and/or provide minimum inflow requirements. Minimum releases are often required as an environmental constraint to maintain constant wetted perimeters to protect fish habitat and flow velocities downstream of the storage project.

3.3.7 Electricity Market Prices

Electricity prices depend on many different uncertain factors and are often difficult to predict. Therefore, the electricity price variable is certainly stochastic. However, in this study the electricity price variable was evaluated deterministically in the Freshet Model for the following reasons.

To begin, there is much less recorded historical and forecast data available on electricity prices as compared to local inflows. Electricity price estimate records are closely guarded and difficult to obtain as the electricity trading industry is very competitive. For this reason, an electricity price forecast accuracy analysis similar to the inflow forecast accuracy analysis performed in Table 3-1 was not possible. Actual electricity price data is also difficult to obtain and define since different buyers can pay different prices for various electricity services at various trading hubs. Instead, 14 years of estimated electricity price data from a third-party's utility survey was used in an analysis of electricity price forecasting using a first-order Markov chain method.

Upon evaluation, it was seen that electricity prices contain much more volatility compared to the inflow data shown in Figures 3-2 and 3-3. Since electricity prices react quickly to changes in the variables they depend on and values can rapidly increase or decrease, values are often less dependent on previous daily electricity price values. The relative error for HLH and LLH electricity prices for this data using a first-order Markov chain forecast is shown in Table 3-2 as compared to the watershed model relative error for freshet values.

Table 3-2: Error of First-order Markov Chain Forecasts for Inflows and Electricity Prices

Markov Chain Forecast Data Type	Inflows	HLH Prices	LLH Prices
Average Absolute Relative Error	11%	24%	54%

As seen in the table above, the prediction of daily freshet inflows using a first-order Markov chain yielded much more accurate predictions as compared to the same method used for daily freshet HLH and LLH electricity prices.

Because of lack of daily electricity price forecast record data and the inaccuracy of a first-order Markov chain method in predicting future electricity prices, the Freshet Model relies on the expertise of electricity trading specialists for future deterministic electricity price estimates. Reservoir operators at BC Hydro work closely with electricity trading specialists and have these estimates readily available to them. Since electricity price inputs have a large influence on the behavior of the Freshet Model, users are encouraged to perform a sensitivity analysis and evaluate a variety of electricity price scenarios during periods of high price volatility.

3.3.7.1 Marginal Value of Energy

Operators often compare electricity prices to the marginal value of energy of the reservoir being operated instead of basing operational decisions on electricity prices alone. (Tilmant et al., 2008) describes a reservoir's marginal value as "the contribution of an additional unit of water to whatever public or private objective is under consideration." In the Freshet Model, the marginal value of energy is the dollar value associated with the storage of one additional unit of volume of water in the reservoir. This metric gives a means of comparing the current state of the reservoir with the current electricity price of electricity in order to determine if it is more valuable to convert an additional unit of storage into electricity in the current stage or retain it for future use.

Technically, the marginal value of energy is the derivative of the objective function in Equation 3-10 with respect to a given storage state as defined in Equation 3-13 below.

$$MV_t = \frac{\partial {f_t}^n(k,i)}{\partial S_{it} \cdot HK_t} \tag{3-13}$$

Where MV_t is the marginal value of energy in stage t, $\partial {f_t}^n(k,i)$ is the total benefits calculated by the objective function in stage t, S_t is the storage in stage t, and HK_t is the efficiency value that provides a conversion between turbine flow and electricity generated in stage t. A normalized example of marginal value of energy plot is shown in Figure 3-6.

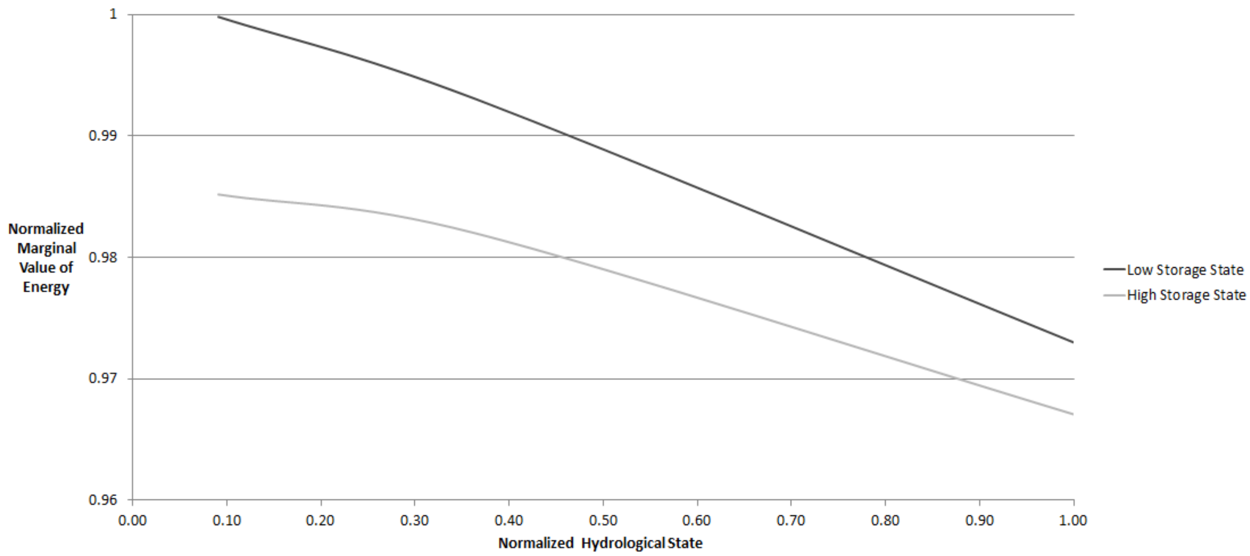

Figure 3-6: Normalized Marginal Value of Energy Plot

As seen above, the marginal value of energy decreases with increasing storage and hydrological states when water in plentiful and increases with decreasing storage and hydrological states in dry conditions. Operators of the reservoir will generate electricity as needed to meet the domestic load and sell electricity when the marginal value of energy in the marginal reservoir is lower than the electricity market price or purchase electricity when the marginal value of energy the marginal reservoir is greater than the electricity market price.

3.4 Freshet Model Use and Analysis

The Freshet Model is a stand-alone, user-friendly, computer optimization program that assists BC Hydro operators of reservoirs with weekly storage flexibility to make optimal release and generation decisions during the freshet period to maximize the value of electricity generation. Since operation plans are often made on business days only, the Freshet Model can be used to plan up to four days in advance. It was tested using data in the 2014 freshet for real-time operations as well as for past operations during a recent historical freshet using recorded data.

3.4.1 Model Programming Structure

The Freshet Model uses SDP to contain the components used to process its inputs and produce meaningful results to the user. A flowchart of the Freshet Model is show in Figure 3-7 below.

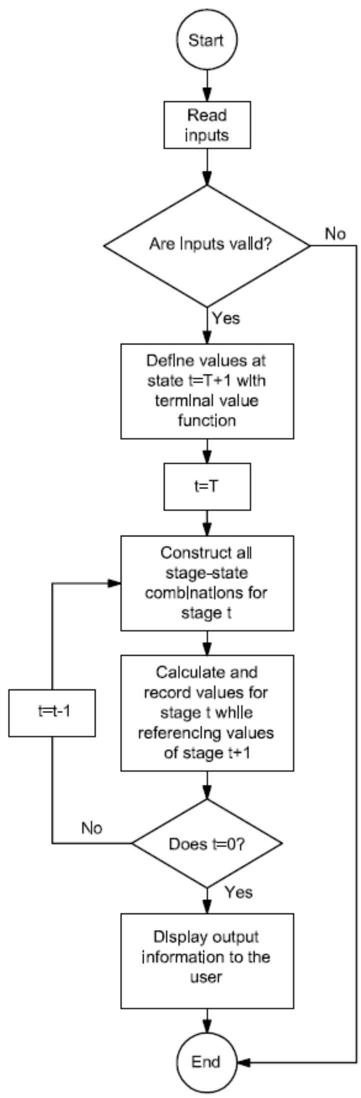

Figure 3-7: Simple Flow Diagram of the SDP Freshet Model

The modeling starts by validating inputs needed for creating the SDP structure such as the minimum forebay to determine the number of storage variables or the current date to determine the TPM to apply later in calculations. If any input is invalid, the Freshet Model will terminate and issue a warning message alerting the user of the origin of the error. If all inputs are valid, the Freshet Model then uses the terminal value function to create the values for each feasible state variable combination to represent stage $t = T + 1$ before looping through each stage of the planning period from the last stage to the first stage. The objective function is then used to calculate the values of each state variable combination in a given stage, which are then referenced by the next loop until $t = 0$. Decision support information is then retrieved and displayed to the user.

3.4.2 Freshet Model vs Historical Operations

The main function of the Freshet Model is to provide decision support to operators during periods in which the current operation decisions are developed heuristically. To evaluate the Freshet Model's performance, a comparison was performed between the Freshet Model and the past freshet policies for the case-study reservoir. This was done by running the Freshet Model using relevant historical data during a past freshet. To provide a reasonable comparison of policies, a recent freshet period was chosen that contained moderate inflows and electricity prices.

Historical forebay levels, HLH and LLH electricity prices, upstream release volumes, unit outage schedules, and forecast local inflow values for the characteristic reservoir for use with the Freshet Model were compiled. The historical forebay level at the start of the freshet was used as a starting point for the Freshet Model before using model-calculated data; after the model produced a decision, the resulting forebay level was used in the next run since it was assumed to be the only input variable dependent on previous operational decisions. The Freshet Model was run with the latest input data only on business days in order to simulate the historical scenario as closely as possible. Figure 3-8 below shows several plots which depict a summary of the freshet scenario.

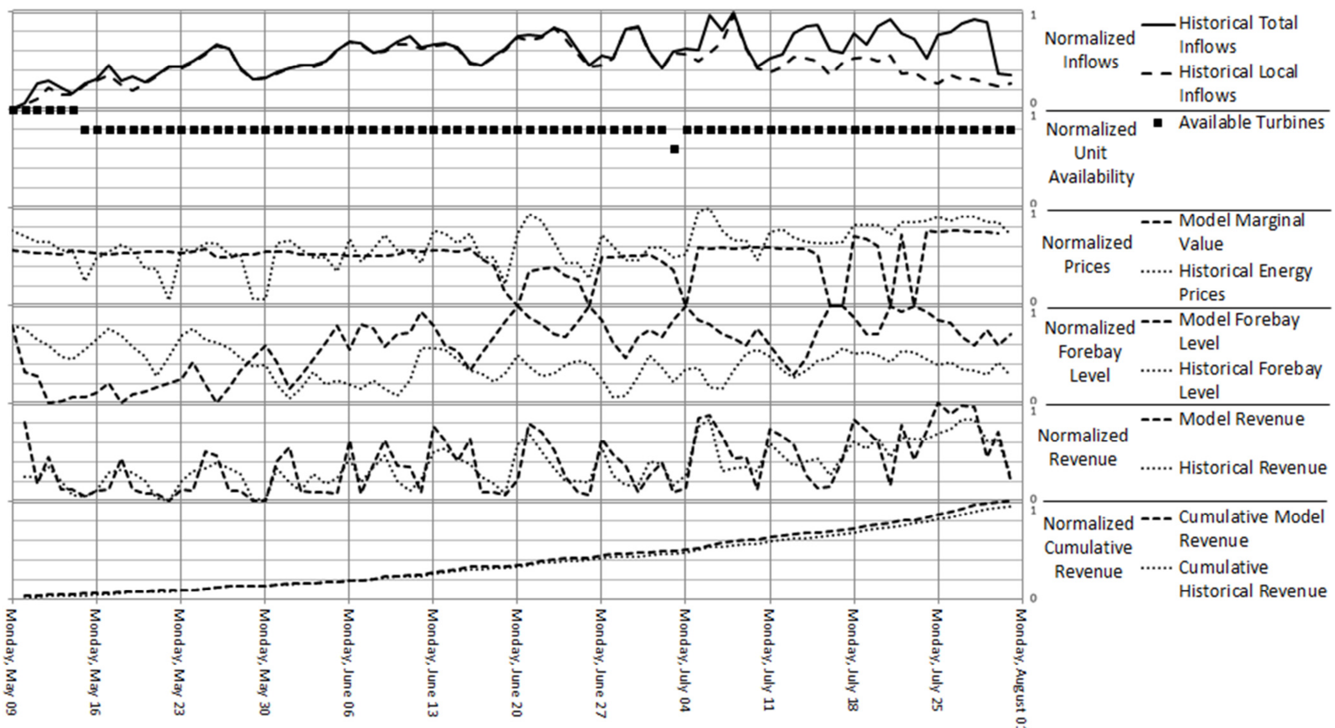

Figure 3-8: Historical versus Freshet Model Operation Results of the Case Study Reservoir

Starting from the top plot in the above figure, historical total inflows approach historical local inflows as the freshet begins and the upstream project reduces its discharges. The next plot shows outage activity for the project, which is fairly consistent in this case. The case-study reservoir historically has a low amount of unit outages during the freshet to take full advantage of the large inflow volumes.

Below the unit availability plot is the normalized electricity prices and Freshet Model's calculated marginal value of energy plot. As the marginal value of energy for the project dips below electricity market prices, the Freshet Model will recommend higher generation since the water in the reservoir is less valuable than the electricity on the market on a dollar per megawatt-hour basis. This is reflected in the normalized forebay level plot which shows the Freshet Model and historical forebay level.

The last two plots show the daily and cumulative revenue of the historical and Freshet Model's policies. Overall, the comparison shows that the Freshet Model's policy resulted in a 6% increase in revenue over the three month period and an ending forebay level 40% higher than historical operations.

The Freshet Model's policy suggests more variability in forebay operations as being more optimal than the historical operations. This is further illustrated by sorting and plotting the normalized values of historical and Freshet Model forebay levels of the case study reservoir from the freshet periods in 2010 to 2013 as seen in Figure 3-9 below.

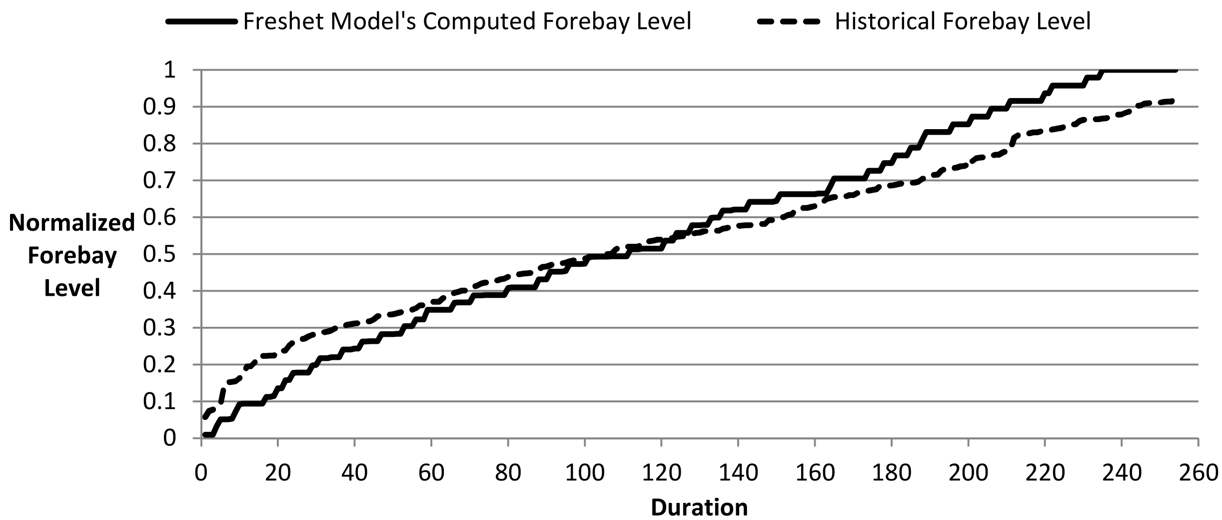

Figure 3-9: Case Study Reservoir's Sorted Normalized Historical and Model Forebay Levels for Freshet Periods 2010-2013

Figure 3-9 further demonstrates that operator would typically hold reservoir levels in a mid-range for longer durations than would be considered optimal under the Freshet Model operation. This may be explained given that operators may tend to prefer to retain the option the both import and export, given the volatile nature of electricity market prices. Although there is an added benefit from the additional head in a full reservoir, it is often outweighed by the benefit of having marketing flexibility and the ability to hedge unexpected changes in inflows or electricity market prices. If operators are uneasy with following a policy that can implement such high and low forebay levels, maximum and minimum forebay levels can be input in the Freshet Model to keep reservoir levels more moderate.

Another aspect that may explain the difference between the historical and Freshet Model recommended operations is different types of electricity markets. The Freshet Model is based off of day-ahead electricity prices and operations. However, operations are also influenced by the real-time market specifically when the reservoir is the marginal resource.

3.5 Areas of Future Study

The creation and application of a stochastic computer model is a practice in multi-objective optimization in itself; the advantages of one method are weighed against the advantages of another and model precision is balanced against required computation power. This was the case for many aspects of the Freshet Model.

3.5.1 Alternative Optimization Methods

Many pieces of literature support stochastic dynamic programming in presenting a reasonable solution to reservoir operation problems. However, the more recently developed method of sampling stochastic dynamic programming (SSDP) has the potential to more accurately incorporate the hydrological variable in BC's reservoirs since it specializes in snowmelt-dominated watersheds (Kelman et al., 1990). SSDP can use ensemble streamflow predictions (ESP), which are sets of possible inflow sequences. In the ESP process, a separate streamflow trace is simulated for each historical recorded year given the current state of the basin. Instead of only considering a group of probabilities as in the first-order Markov chain forecasting process, each trace is considered. The ESP traces would replace both the five-day forecast and first-order Markov chain method of predicting future inflows and avoid any disconnect problems associated with transitioning between the two forecasting methods.

One of the main reasons that prevented the use of the SSDP optimization method in the Freshet Model was the lack of availability of ESP traces. The Freshet Model uses a daily five-day inflow forecast which is calculated and available every business day to match the needs of reservoir operators. At BC Hydro, ESP traces are generated on demand for high inflow events, but are typically used for longer term models with monthly time steps and have therefore typically been generated only once per month. If ESP traces could be generated and provided at a frequency similar to the five-day forecast, their use would be more feasible for a short-term model such as the Freshet Model and the SSDP method could be considered. In addition, evolutionary computation, ISO, or other ESO methods could also be considered for use in creating a similar model and testing its performance against the SDP Freshet Model.

3.5.2 Electricity Price Uncertainty

One of the variables that could benefit the most from including stochasticity is the electricity price variable. As seen in the operational policy in Figure 3-8, the policy of the Freshet Model is very sensitive to electricity market prices. There are many different factors that influence electricity market prices which results in difficulty in predicting future prices. Because of the volatility and unpredictability of electricity prices, the electricity price variable input is included deterministically and based off of electricity trading specialist's estimates.

However, there may be a benefit to further investigating electricity price forecasting techniques or metrics to include in the Freshet Model and provide more structure to the inputs of this important variable. Furthermore, there may be value in including a second stochastic variable in the Freshet Model which relates to the electricity price state of the system. A procedure could be performed using historical and electricity price indicator data similar to the hydrological state transitions procedure performed with historical and forecast inflow data. BC Hydro already uses in-house programs to predict long-term electricity prices. If these models could be altered to provide daily electricity price probabilities then they may improve the function of the Freshet Model and reduce the influence of input data. Also, if short-term estimates from electricity traders appear to be more accurate than short-term estimates from a stochastic variable, the inputs could be divided into deterministic and model-generated for a given planning period to reflect this.

3.6 Conclusion

The Freshet Model addresses key information used by operators in heuristic operations decisions such as forecast inflows, estimated electricity market prices, and project specific data to value operational decisions and determine a daily operations policy and marginal value of electricity for an appropriate reservoir. The Freshet Model was developed using SDP due to its ability to accurately and efficiently represent the nonlinearities and stochastic features of a reservoir with limited flexible storage. Local inflow, as the hydrological state variable, was addressed as a stochastic variable in the Freshet Model due to the volatile nature of local inflows during the freshet. Records of actual and forecast inflows were used to create probability distributions for the hydrological state variable.

The Freshet Model excelled at optimizing the expected value of operational policies for a single reservoir which operates as the marginal resource in the system. However, operators will deviate from this policy to retain the option to import and to export into the real-time market. Operators may also make operational decisions based on factors external to the reservoir being modeled if a different reservoir system becomes marginal. Nevertheless, operators are able to reflect on the expected optimal decision and marginal value of electricity for the reservoir being modeled before decisions are made.

Chapter 4: Conclusion

4.1 Further Areas of Future Study

The creation and application of a stochastic computer model is a practice in multi-objective optimization in itself; the advantages and disadvantages of one method are weighted against the advantages and disadvantages of another. This was the case for many aspects of the Freshet Model. Some variations that may have yielded a more optimal operational model were not investigated in the interest of time. Therefore, in order to ensure the best possible formulation of the Freshet Model the following topics should be investigated.

4.1.1 Stochastic and Deterministic Variables

There are many factors that may influence a reservoir system. The Freshet Model considers variables applicable to the objective function such as inflows, electricity prices, generating station performance, and project limitations. However, it is possible that the Freshet Model's performance may benefit from the addition or alteration of certain stochastic or deterministic variables as described in the following sections.

4.1.1.1 Hydrological State Variable

The Freshet Model uses historical and forecast inflow data to estimate hydrological state transition probabilities. Although this method works well, other metrics may be observed, analyzed, and used to compute transition probabilities. For example, snowpack depth and temperature are popular metrics used in predicting inflows in snow-melt dominated watersheds. Snowpack depth gives an indication of the amount of available water that may melt and flow into the watershed and may be a more reliable long-term predictor of inflows than historical inflows. Using this method may also solve the discontinuity problem described in Section 3.3.2.2.3 regarding the forecast and historical methods of calculating the hydrological state transition probabilities, assuming sufficient data is available.

4.1.1.2 Energy Trading Constraints

The Freshet Model makes the assumption that all electricity generated can readily be used or exported. This assumption is reasonable since there is often a large demand for export from the Western United States during their relatively warmer spring season and the interconnection export capacity is a large 3150MW (Making the Connection, 2000). However, if there is a large reduction in export capacity and low demand in the province, generation may not be required. Therefore, a temporal constraint that reflects demand or fixed constraint for export capacity may be useful in the application of the Freshet Model.

4.1.1.3 Discretization

The choice of a stage or state variable discretization size directly affects the precision as well as the computations required by the Freshet Model. Further study could be performed on the benefit of increasing each variable's precision with respect to the corresponding increase in required computational power required. For example, increasing the precision of the hydrological state variable would introduce more variability in the inflows, increase the precision of future inflow predictions, and in turn increase the precision of turbine release decisions. If this could be done with minimal increase in required computational power, it would increase the Freshet Model's performance.

4.1.2 Alternate Spill Strategies

The policy of the Freshet Model is to meet all minimum downstream flow requirements with turbine flow in order to produce as much benefit as possible from project discharges. It assumes that operators will only consider spilling to mitigate extreme inflow events or to possibly take advantage of negative electricity prices. However, some turbine units require a much higher minimum turbine flow to avoid rough load zones as compared to the minimum downstream flow requirement of the project to which they belong. In these cases, downstream flow requirements are exceeded by projects.

Because of this, it may be beneficial to meet the minimum downstream flow using spills in periods in which it is undesirable to generate. Electricity prices are generally much lower during LLH periods as compared to HLH periods. If operators were to spill a significantly smaller minimum downstream flow requirement as compared to a significantly larger minimum turbine flow during a LLH period, the volume difference could be later discharged using a higher head during a HLH period and potentially yield a greater net benefit.

4.2 Conclusion

Through the development and application of the Freshet Model, all goals in Section 1.5 were met. The Freshet Model was programmed using the Visual Basic Application (VBA) extension in Microsoft Excel, software that is familiar to many of the operators. The data, algorithms, and simple user interface are self-contained in a single macro-enabled Microsoft Excel file. The algorithm is easily edited to reflect the physical, operational, and environmental conditions for a given project and provides operators with decision support for making reservoir operations decisions one to four days in advance for reservoirs with a five-day inflow forecast.

The Freshet Model excels at calculating the expected optimal operational decision and marginal value of energy for a single reservoir with available forecasts for inflows and electricity prices. However, as mentioned in Section 3.4.2, operators are often hesitant to rely on an expected value decision since uncertainty is involved. Because of this, operators may be unwilling to forego certain immediate benefits in exchange for expected future benefits. This theory is consistent with historical forebay records of the case study reservoir in Section 3.4.2 which shows a much more moderate and consistent reservoir level as compared to expected optimal forebay levels computed by the Freshet Model. However, it is possible that as operators become more familiar with the Freshet Model and its calculations, they will increase their reliance on in.

Bibliography

Andrieu, L., R. Henrion, and W. Romisch. "A Model for Dynamic Chance Constraints in Hydro Power Reservoir Management." *European Journal of Operational Research* 207 (2010): 579-89.

Arunkumar, R., and V. Jothiprakask. "Optimal Reservoir Operation for Hydropower Generation Using Non-Linear Programming Model." *Journal of the Institution of Engineers (India)* 93.2 (2012): 111-20.

Ayad, Amr, Ziad Shawwash, Alaa Abdalla, Ziming Guan, and Joel Evans. "Assessing the Impact of Storage State and Decision Space Discretization on Solution Efficiency and Precision of a Stochastic Dynamic Programming Algorithm in a Multireservoir Operations Planning Model." *HydroVision International 2013* (2013).

Barros, M. T. L., F. T-C. Tsai, S. Yang, J. E. G. Lopes, and W. W-G. Yeh. "Optimization of Large-scale Hydropower System Operations." *Journal of Water Resources Planning and Management* 129 (2003): 178-88.

BC Hydro. *Making the Connection: the BC Hydro Electric System and how it is Operated*. Vancouver: Canadian Cataloguing in Publishing Data, 2000.

BC Ministry of Energy and Mines "Transmission & Distribution System." *BC Ministry of Energy and Mines*. BC Government, n.d. Web. Aug. 2014. <http://www.gov.bc.ca/ener/>.

Bellman, R.E. *Dynamic Programming*. Princeton, New Jersey: Princeton University Press, 1957.

Bras, R. L., R. Buchanan, and K. C. Curry. "Real Time Adaptive Closed Loop Control of Reservoirs with the High Aswan Dam as a Case Study." *Water Resources Research* 19.1 (1983): 33-52.

Brooks, S., A. Gelman, G. L. Jones, and X-L. Meng. *Handbook of Markov Chain Monte Carlo*. N.p.: Chapman & Hall,

Cai, X., D. C. McKinney, and Leon S. Lasdon. "Solving Nonlinear Water Management Models Using a Combined Genetic Algorithm and Linear Programming Approach." *Advances in Water Resources* 24 (2001): 667-76.

Celeste, A.B., K. Suzuki, and A. Kadota. "Integrating Long- and Short-Term Reservoir Operation Models via Stochastic and Deterministic Optimization: Case Study in Japan." *Journal of Water Resources Planning and Management* 134.5 (n.d.): 440-48.

Conn, A. R., N. I. M. Gould, and Ph. L. Toint. *LANCELOT: A Fortran Package for Large-Scale Nonlinear Optimization (Release A)*. Berlin: Springer-Verlag, 1992.

Deka, P. C., and V. Chandramouli. "Fuzzy Neural Network Modeling of Reservoir Operation." *Journal of Water Resources Planning and Management* 135.1 (2009): 5-12.

Dorigo, M., V. Maniezzo, and A. Colorni. "Ant System: Optimization by a Colony of Cooperating Agents." *IEEE Transactions on Systems, Man, and Cybernetics* 26.1 (1996): 29-41.

Drobny, N. L. "Linear Programming Applications in Water Resources." *Water Resources Bulletin* 7.6 (1971): 1180-193.

Druce, D. J. "Decision Support for Short Term Export Sales from a Hydroelectric System." *Computerized Decision Support Systems for Water Managers, American Society of Civil Engineers* (1988): 490-97.

Druce, D.J. "Incorporating Daily Flood Control Objectives into a Monthly Stochastic Dynamic Programming Model for a Hydroelectric Complex." *Water Resources Research* 26.1 (1990): 5-11.

Faber, B. A., and J. R. Stedinger. "Reservoir Optimization Using Sampling SDP with Ensemble Streamflow Prediction (ESP) Forecasts." *Journal of Hydrology* 249 (2001): 113-33.

Fayaed, S. F., A. El-Shafie, and O. Jaafar. "Reservoir-system Simulation and Optimization Techniques." *Stochastic Environmental Research and Risk Assessment* 27 (2013): 1751-772.

Gablinger, M., and D. P. Loucks. "Markov Models for Flow Regulation." *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers* 96 (1970): 165-81.

Hall, W. A., and N. Buras. "The Dynamic Programming Approach to Water-resources Development." *Journal of Geophysical Research* 66.2 (1961): 517-20.

"Integrated Resources Plan: Meeting BC's Future Electricity Needs." *BC Hydro*. BC Hydro, Nov. 2013.

Jothiprakash, V., and R. Arunkumar. "Multi-reservoir Optimization for Hydropower Production Using NLP Technique." *KSCE Journal of Civil Engineering* 18.1 (2014): 344-54.

Kangrang, A., S. Compliew, and R. Hormwichian. "Optimal Reservoir Rule Curves Using Simulated Annealing." Water Management 164.WM1 (2010): 27-34.

Karamouz, M. "Annual Monthly Reservoir Operating Rules Generated by Deterministic Optimization." *Water Resources Research* 18.5 (1982): 1337-344.

Karamouz, M. "Bayesian Stochastic Optimization of Reservoir Operation Using Uncertain Forecasts." *Water Resources Research* 28.5 (1992): 1221-232.

Keckler, W. G., and R. E. Larson. "Dynamic Programming Applications to Water Resource System Operation and Planning." *Journal of Mathematical Analysis and Applications* 24 (1968): 80-109.

Kelman, J., J.R. Stedinger, L.A. Cooper, E. Hsu, and S.Q. Yuan. "Sampling Stochastic Dynamic Programming Applied to Reservoir Operation." *Water Resources Research* 26.3 (1990): 447-454.

Kirkpatrick, S., C. D. Gelatt, Jr., and M. P. Vecchi. "Optimization by Simulated Annealing." *Science* 220.4598 (1983): 671-80.

Kumar, D. N., and M. J. Reddy. "Ant Colony Optimization for Multi-purpose Reservoir Operation." *Water Resources Management* 20 (2006): 879-98.

Li, Y. P., and G. H. Huang. "Interval-parameter Two-stage Stochastic Nonlinear Programming for Water Resources Management under Uncertainty." *Water Resources Management* 22 (2008): 681-98.

Li, Y. P., G. H. Huang, and X. Chen. "Multistage Scenario-based Interval-stochastic Programming for Planning Water Resources Allocation." Stochastic Environmental Resources Risk Assessment 23 (2009): 781-92.

Little, J. D. C. "The Use of Storage Water in a Hydroelectric System." *Journal of Operations Research Society of America* 3.2 (1955): 187-97.

Liu, P., J. Zhao, L. Li, and Y. Shen. "Optimal Reservoir Operation Using Stochastic Dynamic Programming." *Journal of Water Resource and Protection* 4 (2012): 342-45.

Loucks, D. P., and L. M. Falkson. "A Comparison of Some Dynamic, Linear and Policy Iteration Methods for Reservoir Operation." *Journal of American Water Resources Bulletin* 6.3 (1970): 384-400.

Loucks, D. P., and P. J. Dorfman. "An Evaluation of Some Linear Decision Rules in Chance-Constrained Models for Reservoir Planning and Operation." *Water Resources Research* 11.6 (1975): 777-82.

Loucks, D. P., J. R. Stedinger, and D. A. Haith. *Water Resource Systems Planning & Analysis*. New Jeresy: Prentice-Hall, 1981.

Loucks, D. P., E. Van Beek, J. R. Stedinger, J. P. M. Dijkman, and M. T. Villars. *Water Resources Systems Planning and Management: An Introduction to Methods, Models, and Applications*. Paris: UNESCO, 2005.

Mamun, A. A. "A Goal Programming Algorithm to Incorporate the Columbia River Non-power Flow Requirements in the Columbia River Treaty Model." Diss. U of British Columbia, 2012.

Mantawy, A. H., S. A. Soliman, and M. E. El-Hawary. "An Innovative Simulated Annealing Approach to the Long-Term Hydroscheduling Problem." *Electrical Power and Energy Systems* 46th ser. 25.41 (2003): 41-46.

"Map of the Columbia/Kootenay River System." *BC Ministry of Environment*. April 2014. Web.

Marino, M. A., and B. Mohammadi. "Reservoir Operation by Linear and Dynamic Programming." *Journal of Water Resources Planning and Management* 109.4 (1983): 303-19.

Mohan, S., and D. M. Raipure. "Multiobjective Analysis of Multireservoir System." *Journal of Water Resources Planning and Management* 118.4 (1992): 356-70.

Murtagh, B. A., and M. A. Saunders. *MINOS 5.51 User's Guide*. Stanford: Stanford U, 2003.

Nelson, M. L., and D. M. Rockwood. "Flood Regulation by Columbia Treaty Projects." *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers* 97 (1971): 143-61.

Oron, G., A. Mehrez, and G. Rabinowitz. "Forecasting in Optimizing Dual System for Energy Generation and Irrigation." *Journal of Water Resources Planning and Management* 117.3 (1991): 287-300.

Pan, P. *Linear Programming Computation*. N.p.: Springer-Verlag Berlin Heidelberg, 2014. Print.

Ranjithan, S. R. "Role of Evolutionary Computation in Environmental and Water Resources Systems Analysis." *Journal of Water Resources Planning and Management* 131 (2005): 1-2.

Rasmussen, Ryan, Ziad Shawwash, and Jim Gaspard. "Optimizing the Operation of Small Storage Reservoirs in British Columbia During the Spring Freshet." *HydroVision International 2014* (2014).

Ratnayake, U., and R. Harboe. "Deterministic and Stochastic Optimization of a Reservoir System." *Water International* 32.1 (2009): 155-62.

Reddy, M. J., and D. N. Kumar. "Optimal Reservoir Operation Using Multi-Objective Evolutionary Algorithm." *Water Resources Management* 20 (2006): 861-78.

Russell, S. O., and P. F. Campbell. "Reservoir Operating Rules with Fuzzy Programming." *Journal of Water Resources Planning and Management* 122 (1996): 165-70.

Shabani, N. "Incorporating Flood Control Rule Curves of the Columbia River Hydroelectric System in a Multireservoir Reinforcement Learning Optimization Model." Diss. U of British Columbia, 2009.

Shapiro, A. "Analysis of Stochastic Dual Dynamic Programming Method." *European Journal of Operational Research* 209 (2011): 63-72.

Shawwash, Z. K., T. K. Siu, and S. O. D. Russell. "The BC Hydro Short Term Hydro Scheduling Optimization Model." *IEEE Transactions on Power Systems* 15.3 (2000): 1125-131.

Simonovic, S. "Influence of Different Downstream Users on Single Multipurpose Reservoir Operation by Chance Constraints." *Canadian Journal of Civil Engineering* 15.4 (n.d.): 596-600.

Sniedovich, M. "Dynamic Programming and the Principle of Optimality: A Systemic Approach." *Advances in Water Resources* 1.1 (1978): 183-90.

Stedinger, J. R., B. F. Sule, and D. P. Loucks. "Stochastic Dynamic Programming Models for Reservoir Operation Optimization." *Water Resources Research* 20.11 (1984): 1499-505.

Teegavarapu, R. S. V., and S. P. Simonovic. "Optimal Operation of Reservoir Systems Using Simulated Annealing." *Water Resources Management* 16 (2002): 401-28.

Tejada-Guibert, J. A., S. A. Johnson, and J. R. Stedinger. "Comparison of Two Approaches for Implementing Multireservoir Operating Policies Derived Using Stochastic Dynamic Programming." *Water Resources Research* 29.12 (1993): 3969-980.

Tilmant, A., D. Pinte, and Q. Goor. "Assessing Marginal Water Values in Multipurpose Multireservoir Systems via Stochastic Programming." *Water Resources Research* 44 (2008): 1-17.

Trosset, M. W. "What Is Simulated Annealing?" *Optimization and Engineering* 2 (2001): 201-13.

Wang, Y. C., J. Yoshitani, and K. Fukami. "Stochastic Multiobjective Optimization of Reservoirs in Parallel." *Hydrological Processes* 19 (2005): 3551-567.

"World Energy Council." *World Energy Resources: 2013 Survey*. World Energy Council, Web. Sept. 2014. <http://www.worldenergy.org/publications/2013/world-energy-resources-2013-survey/>.

Yakowitz, S. "Dynamic Programming Applications in Water Resources." *Water Resources Research* 18.4 (1982): 673-96.

Yeh, W. W-G. "Reservoir Management and Operations Models: A State-of-the-Art Review." Water Resources Research 21.12 (1985): 1797-1818.

Zadeh, L. A. "Fuzzy Sets." *Information and Control* 8 (1965): 338-53.

Zhang, Z., Y. Jiang, S. Zhang, S. Geng, H. Wang, and G. Sang. "An Adaptive Particle Swarm Optimization Algorithm for Reservoir Operation Optimization." *Applied Soft Computing* 18 (2014): 167-77.