Essays in Operations Management

by

Sha Liao

B.Sc., Zhejiang University, 2009

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

The Faculty of Graduate and Postdoctoral Studies

(Business Administration)

THE UNIVERSITY OF BRITISH COLUMBIA

(Vancouver)

July 2015

Abstract

This dissertation addresses two topics in the domain of operations management. First we study a single utility's optimal policies under the Renewable Portfolio Standard, which requires it to supply a certain percentage of its energy from renewable resources. The utility demonstrates its compliance by holding a sufficient amount of Renewable Energy Certificates (RECs) at the end of each year. The utility's problem is formulated as a stochastic dynamic program. The problem of determining the optimal purchasing policies under stochastic demand is examined when two energy options, renewable or regular, are available, with different prices. Meanwhile, the utility can buy or sell RECs in any period before the end of the horizon in an outside REC market. Both the electricity prices and REC prices are stochastic. We find that the optimal trading policy in the REC market is a target interval policy. Sufficient conditions are obtained to show when it is optimal to purchase only one kind of renewable energy and regular energy, and others to show when it is optimal to purchase both of them. Explicit formulas are derived for the optimal purchasing quantities in each case.

In the second essay, we examine the interaction between a buyer (Original Equipment Manufacturer, OEM) and his supplier during new product development. A "white box" relationship is assumed: the OEM designs the specification of the product and outsources the production to his supplier. The supplier may suggest potential specification problems. Our research is motivated by the fact that the supplier may detect potential specification problems, and one cannot take for granted that the supplier would inform the OEM. We solve an optimization problem from the perspective of the OEM. We first prove that it is strictly better for the OEM to design the contract so that the supplier will inform the OEM should she detects any flaws. Then we characterize the optimal solutions for the OEM. We also perform some sensitivity analysis at the end.

Preface

Chapter 2 is co-authored by Tim Huh and Mahesh Nagarajan. Chapter 3 is co-authored by Hao Zhang and Yimin Wang. In both chapters I was the main contributor. I was responsible for developing the models, carrying out analysis and reporting the results, as presented in this dissertation. At the same time, co-authors for both chapters have devoted tremendous time discussing with me and contributed invaluable advice. The background story of Chapter 3 was inspired by Yimin Wang. Both chapters will be reformatted and submitted for publication in academic peer reviewed journals.

Table of Contents

Appendices

List of Figures

Acknowledgements

My years of pursuing Ph.D. at the University of British Columbia has been an immensely rewarding and satisfying journey. I am deeply indebted to my research advisers, Tim Huh, and Mahesh Nagarajan for their continuous support through these years, academically, mentally, and financially. They provided me with every bit of guidance and assistance that I needed during my first few semesters. When I was ready to venture into research, they gave me the freedom to choose my own topic, at the same time spent tremendous effort discussing with me, and contributed valuable feedback, advice and encouragement.

I own a great debt of gratitude to professor Hao Zhang, who collaborated with me on the second essay and served on my thesis committee.

I am very thankful to professor Yimin Wang at Arizona State University, who collaborated with me on the second essay.

I want to thank professor George Zhang, who served on my thesis committee and provided invaluable comments on my research works.

My thanks also go to other faculty members of the OPLOG division. I would particularly like to acknowledge professor Maurice Queyranne, who was my instructor for most of the fundamental courses in optimization and operations management.

Finally, I would like to thank Guido Alexander De Maeseneer Junior, for all his love and support.

Dedication

To my parents, Qionggao Liao and Shanlian Wang.

Chapter 1

Introduction

This dissertation presents two essays, each contributing to the fields of operations management by attempting to mitigate a gap identified in academic literature.

The first essay examines a newly initiated mechanism on renewable energy in the United States, known as the Renewable Portfolio Standard (RPS). The mechanism requires each obligated utility to supply a certain percentage of their energy from renewable resources. Different states may have different percentage levels, and the percentage levels will increase over time. For example, in California, the percentage level was 20% annually since 2012, and will be raised up to 25% starting from 2016, and eventually 33% starting from 2020 (Wiser and Barbose, 2008).

The RPS mechanism is similar to the cap-and trade program, which has been widely studied in operations management literature. The way for utilities to demonstrate their compliance is by holding Renewable Energy Certificates (REC). An REC is given to energy generators for every megawatt hour of renewable energy they generate. Utilities have the option of purchasing these RECs with or without the renewable energy that they came from. At the end of each year, the utilities with shortage in RECs are charged a penalty cost of \$50 -\$55 per unit of RECs they are short on (Wiser and Barbose, 2008).

Since RECs can be traded separately from the underlying energy, an REC market has been formed, where RECs can be sold or purchased. The trading is often private, and the trading price of RECs can vary widely over time and region. Utilities may trade in the REC market to speculate or hedge against the price volatility.

We study a single utility's optimal policies under the RPS. The utility's problem is formulated as a stochastic dynamic program. The problem of determining the optimal purchasing policies under stochastic demand is examined when two energy options, renewable or regular, are available, with different prices. Meanwhile, the utility can buy or sell RECs in any period before the end of the horizon in an outside REC market. Both the electricity prices and REC prices are stochastic. We find that the optimal trading policy in the REC market is a target interval policy. Some sufficient conditions are obtained to show when it is optimal to purchase only one kind of renewable energy and regular energy, and others to show when it is optimal to purchase both of them. Explicit formulas are derived for the optimal purchasing quantities in each case.

This paper will be the first one investigating the RPS in OM literature. In general, this paper will be the first one to examine the policy from the perspective of a utility. This may have to do with the fact that the RPS is a newly initiated program, and many states did not start their initial compliance year until 2010. Most of the existing literature is from economics and focuses on the efficacy of the RPS.

Our interest is not to discuss the efficacy of the RPS. The RPS has been rapidly expanding with increasing percentage requirements, and the very urgent question for those obligated utilities is how to deal with it. We feel it is needed to untangle the trade-offs in choosing energy sources and provide guidelines for utilities to comply with the RPS with minimum cost.

The second essay studies the interaction between a buyer (Original Equipment Manufacturer, OEM) and his supplier during new product development. We consider a "white box" buyer-supplier relationship: the OEM owns product specification and outsources the production to a supplier. The supplier may suggest potential specification problems. Our research is motivated by the fact that the supplier may perceive potential specification problems based on manufacturer alternatives, local market tastes, and different regulatory mandates, which the OEM firm may not be able to anticipate a priori.

One cannot take for granted that suppliers will always be willing to point out specification problems to OEM. Often the supplier's objective is not perfectly aligned with that of the OEM's. For example, specification flaws are often observed only after the volume production has begun, and hence the supplier may lose significant business if it points out the specification flaw but no immediate resolutions are available.

Given that suppliers may not always be willing to point out specification flaws, one possible solution could be integrating the supplier in project teams. There has been extant literature discussing the pros and cons of this solution. The discussions mainly focus on how the OEM should involve the suppliers, e.g., the timing and depth of supplier involvement, in new product development. An implicit assumption in these papers is that the supplier will share process knowledge with the OEM as long as they are "included". This implicit assumption, however, is not necessarily true. We feel the need to fill the gap in literature by examining whether or not the suppliers are willing to share their knowledge even when they are included in the project team.

We started by identifying potential levers that the OEM may use to motivate the supplier to voluntarily point out potential specification flaws. We focus on two levers in this essay, the ordering quantities and the contingent cancellation payment. We then solve the optimization problem from the perspective of the OEM. First we solve the optimal strategy of the OEM on the condition that he will design the contract so that the supplier will not inform even if she detects the flaws. Then we solve the optimal strategy of the OEM on the condition that he will design the contract to motivate the supplier to inform. We compare the optimal profit of the OEM in these two cases and prove that it is strictly better for the OEM to design the contract to motivate the supplier to inform. Having this principle in mind, we then provide full description of the optimal solutions of the OEM, and conduct sensitivity analysis on some parameters. In specific, we find that the cancellation payment provided by the OEM should decrease in his capability to fix the flaws in time, and also decrease in the spillover effect from the first period sales.

Chapter 2

Coping with the Renewable Portfolio Standard: A Utility's Perspective

2.1 Introduction

In the past decade, many countries have made enthusiastic efforts in the harnessing of renewable energy such as hydro, wind, solar and biomass power. In 2004, global investment in renewable energy was $40 billion, and the share of renewable energy in power capacity expansion was 10%. By 2012, these rose to $244 billion and 42% (McCrone (2013)), respectively. Ambitious targets were announced, followed by various mechanisms. China aims to provide 15% of its annual national power from renewable resources by 2020, EU 20% by 2020, and the U.S. 25% by 2025.

One new initiative in the U.S. to promote renewable energy production is the "Renewable Portfolio Standard" (RPS). This policy breaks down the responsibility for achieving the national target to state level, with many states in turn requiring utility companies to supply a specified percentage of their energy from renewable resources by a given date. There is no federal policy. The target for each state is set on an individual basis and is expected to increase over time. For example, California's requirement was 20% in 2012, but will rise to 25% by 2016 and 33% by 2020. New York's percentage level was 24% in 2013 and will rise to 30% by 2015 (Wiser and Barbose, 2008). The RPS has been adopted in 33 out of 50 states of the U.S. (Figure 2.1). Similar mechanisms have been adopted in many other countries.

Since the restructuring of the U.S. electricity market, most utilities no longer own power generators themselves, so purchase electricity from a wholesale electricity market and then supply their end-users in a retail electricity market. The RPS further stipulates a certain amount of the supply

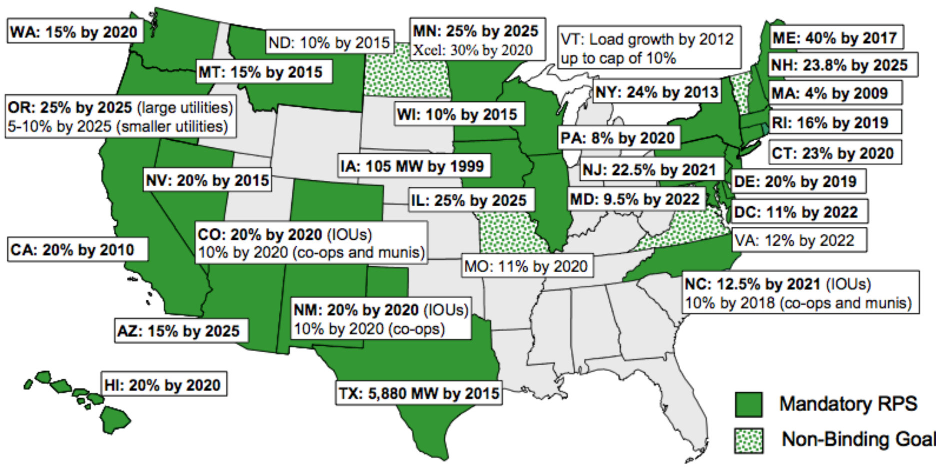

Figure 2.1: State RPS policies (Wiser and Barbose (2008)).

be renewable. Given that the RPS is unlikely to be abandoned, and given the highly competitive nature of the electricity market, the imperative for utility companies must be to comply in the most cost effective manner. This paper builds a theoretical model from the perspective of a utility company illustrating the tradeoffs of compliance options and suggest strategies enabling the utility company to comply with the RPS but at minimum cost.

How much will it cost companies to comply with their obligations under the RPS? Consider the case of a California utility in 2011 which supplied electricity at an average rate of \$132.2 per megawatt hour (Institute For Energy Research, 2011). The company was obliged to supply 20% of its energy from renewable resources, or face a penalty of \$50 per megawatt hour short of its target (Wiser and Barbose, 2008). Suppose the utility did not supply any renewable energy, the RPS would penalize the utility 8% ($\$50 \cdot 20\%/\$132.2$) of its revenue. One may consider this overestimation. After all, If a utility company could easily meet its obligation, then the RPS would not incur such a high additional cost. In reality, many utilities in California in 2011 failed to meet the required percentage level of the RPS and paid penalties. What's more, the California Public Utilities Commission placed a cap on RPS charges, so that no utility would pay more than \$25 million per year in penalties (Institute For Energy Research, 2011). The RPS has undoubtedly resulted in challenges to companies seeking to avoid incurring increased costs but the RPS does provide some flexibility in terms of compliance. It is the opportunity that this flexibility offers that is the subject of this paper.

The RPS works as follows. Renewable energy producers receive from the government one renewable energy certificate (REC) for each megawatt hour of renewable energy they generate. Utilities who do not generate renewable energy purchase RECs from renewable energy producers to demonstrate their compliance of the RPS. Utilities can purchase RECs with the underlying energy, termed “REC-bundled” energy, or purchase RECs without the underlying energy, termed “unbundled” RECs. At the beginning of each year, the Public Utility Commission (PUC) checks whether utilities have sufficient RECs to cover the required percentage of the output from the previous year, and charges those who do not a penalty. Most states set the penalty at between \$50 and \$60 per megawatt hour of renewable energy the utility is deficient or, equivalently, per unit of REC the utility is short (Wiser and Barbose, 2008).

Since RECs can be traded separately from the underlying energy, a secondary market has formed, where RECs are bought and sold. We refer this secondary market as “REC market”. Trading is often private and can be done bilaterally or assisted by an REC agent (U.S. Department of Energy, 2014). The REC trading price varies widely over time, from less than \$2 to \$50-\$55 per unit (essentially the RPS penalty cost) (Heeter and Bird, 2011). Since the PUC checks utilities’ REC amount only once a year, and since RECs can be banked without costs, utilities have incentive to trade in the REC market to speculate or hedge against price volatility.

Aside from acquiring RECs, utility companies need to purchase electricity from the wholesale electricity market in order to supply their customers. Because electricity is nonstorable, and because utilities are obliged to satisfy the customer demands instantly with exactitude though the customer demands are volatile and difficult to predict, utilities seek instantaneous supplies. This is provided by an Independent System Operator (ISO) through a“spot” market. In the spot market, parties bid on or offer electricity on a real-time basis. The ISO matches the aggregate supply and demand, announces the market-clearing price known as the “spot” price, and coordinates the transmission of electricity. Utilities are thus able to purchase electricity and supply their customers instantaneously. Inevitably, the spot price is highly volatile, as indicated by an empirical study of the spot price from 2000 to 2002 at the PJM market, the largest ISO in the United States

(Longstaff and Wang, 2004). During peak hours, from 1 pm to 6 p.m., the spot price rocketed to above $1000 per megawatt hour, nearly 21 times the average value during these hours. At the other extreme, during the night, the spot price could drop to zero.

In an attempt to mitigate the volatility of the spot price, ISOs run a "day-ahead" market before the spot market, where utilities can purchase electricity to be delivered during the subsequent day. The electricity price at the day-ahead market tends to be less volatile than the spot price, though not significantly so (Longstaff and Wang, 2004). To achieve greater price stability, utilities purchase most of their power directly from power generators in a "bilateral" market, not through ISOs. In Texas, for instance, 95-98% of the energy is traded bilaterally (Hortacsu and Puller, 2008).

Power trading occurs first in the bilateral market, then in the day-ahead market, and lastly on the spot market. The bilateral market and the day-ahead market are "forward" market, as the delivery times specified in the contracts are in the future. Spot markets, on the other hand, are instantaneous. Utilities purchase power from the forward market, and then adjust their output in the spot market so that they can meet the customer demands instantly. Specifically, if utilities electricity bought from the forward market is insufficient to meet the demand, they will purchase additional power from the spot market; otherwise they will sell redundant power to the spot market. ISOs only facilitates the trading of regular energy (non-renewable energy or renewable energy with the REC removed). Therefore if utilities want to acquire REC-bundled energy, they can only do so through bilateral contracts.

We formulate a utility company's problem as a stochastic dynamic program. We divide an RPS compliance year into multiple periods. During each period, the company purchases electricity from a wholesale electricity market to meet a random demand in a retail electricity market. At the same time, the company buys or sells unbundled RECs in an REC market. The RECs can be carried over. The wholesale electricity market is modeled as two settlements, first a forward market and then a spot market. In the forward market, the utility chooses between REC-bundled and regular energy. Then the demand occurs and the utility balances its output through the spot market in order to meet the demand. Periodic decisions made by the utility relate to trading actions in the REC market and the purchasing quantities from each source in the forward market. The utility seeks to minimize the total discounted cost during the planning horizon.

Intuitively, a utility should sell RECs when its REC amount is in excess of need and should purchase RECs otherwise. How exactly should a company reach its decision that its REC level is in excess of need? In a given period, even if a utility has decided that its RECs are in excess, and it encounters low priced RECs, it might still have an incentive to hoard RECs. In this way, if the REC price increases later, the utility has secured some RECs purchased at a lower price, and it can even sell RECs to gain profit. Similarly, how exactly should a company decide that the REC price is low enough? Our analysis shows that a utility company should follow a target interval policy in the REC market: it should purchase or sell RECs in order to adjust its REC level between two thresholds. These two thresholds depend on a series of state variables, including the REC price, the electricity price, the mount of RECs the utility has on hand and he amount of demand the utility has met. We show that the thresholds are monotonic in some of the state variables.

We assume there are only two energy options in the forward electricity market, REC-bundled energy and regular energy. One unit of REC-bundled energy is, essentially, one unit of regular energy plus one unit of unbundled REC. Comparing these two options is more subtle than simply comparing their market prices. This is because from the perspective of the utility, RECs hold different values depending on the situation. For instance, RECs are more valuable to a utility with an insufficient numbers of RECs and at the end of the horizon. We describe some conditions under which a utility company should purchase only one kind of renewable energy or regular energy, and present others that show when it is optimal to purchase both of them. In each case, we describe optimal purchasing quantities. We also analyze the monotonicity of these optimal purchasing quantities and explain the intuition behind a utility's optimal policies.

The remainder of the paper is structured as follows: We start with a literature review in Section 2.2. Then we present the model formulation in Section 2.3 and describe the optimal strategy of the utility company in Section 2.4. Finally we conclude with a discussion in Section 2.5.

2.2 Literature Review

As far as we know, this paper is the first one investigating the RPS from the perspective of a utility. This may have to do with the fact that the RPS is a newly initiated program, and many states did not start their initial compliance year until 2010. Most of the existing literature focus on the efficacy of the RPS. Wiser and Barbose (2008) conducted empirical studies to show that approximately 76% of new renewable capacity was contributed by the states with the RPS in 2007. They therefore concluded that the RPS was effective in promoting renewable energy. Other supporters of RPS include Menz and Vachon (2006) and Hailu and Adelaja (2008). Some critics, include Bushnell et al. (2007) and Michaels (2007), pointed out that the mechanism of RPS will cause unbalance development of different renewable resources. Yin and Powers (2009) asserted that the cross border trade of RECs can “significantly” weaken the renewable energy development in certain states with scarce renewable resources, since the utilities in these states can purchase RECs from other states, essentially they are “paying the fresh air in other states”.

Our interest is not to discuss the efficacy of the RPS. The RPS has been rapidly expanding with increasing percentage requirements, and the very urgent question for those obligated utilities is how to deal with it. This paper untangles the trade-offs in choosing energy sources and provides guidelines for utilities to act both in the electricity market and REC market, and so to comply with the RPS with minimum cost.

One important feature of our model is that the utility can trade unbundled RECs in an outside market. The information of the trading price is very limited. One reason might be that the REC market emerges as a byproduct of the RPS and thus has a short history. Another reason is that most of the REC tradings are private. Heeter and Bird (2011) provides a status report of the REC market in 2010, from which we know that the REC price is quite volatile.

The REC trading scheme resembles the allowances trading scheme under the cap-and-trade program. There is scant literature in Operations Research studying the REC trading scheme, but there is extant literature on the cap-and-trade program. Factories, who emit pollutants during production, are granted with an initial allocation of allowances from the government. The amount of the allowances represents the total amount of pollutants the factories can emit in the following year. The factories may purchase or sell allowances in an outside market. The factories who fail to hold sufficient allowances at the end of the year will face a fine.There are some similarities if we compare the problems faced by utilities under the RPS and factories under the cap-and-trade. Factories under the cap-and-trade program often face options involving technology choice, capacity planning, investment in pollution abatement equipment, pricing decisions, etc. In addition, they can bank and trade allowances in a secondary market.

In many papers in the cap-and-trade literature, the entities of interest are electricity generators. For instance, Subramanian et al. (2007) modeled a three-stage game in an oligopoly setting. Electricity generators make investment decisions, bid for allowances and then decides production quantities. The selling price of the products is represented as an inverse demand function. Drake et al. (2010) studied a single electricity generator's technology choice and capacity decisions in a two-period stochastic setting. The price is fixed. The demand is realized once with penalty for unmet demand. Zhao et al. (2010) discussed the long-term impacts of different emissions allocation schemes on electricity generator's investment and pricing decisions, with a demand function decreasing in price. Our paper differs from these papers in three ways. First, they did not include the electricity generators' option of trading allowances in an outside market. Second, In these papers, electricity generators face options with fixed and known costs, or they can internalize the costs as decision variables. They make independent decisions on the prices and quantities they produce. In our paper, utilities purchase electricity at exogenous and random prices, and then sell electricity at fixed prices. The demands are random, and need to be satisfied. Third, these papers suggest long-term decisions of electricity generators, such as technology or capacity choice. While in our paper, a utility makes periodic observations and decisions.

Gong and Zhou (2013) presented a muti-period model studying a factory's technology choice and production plan under the cap-and-trade program. The factory was not a electricity generator. They also captured the factory's option of trading allowances in a secondary market with stochastic prices. Two factors contribute to the differences between our model and theirs, the difference between the cap-and-trade program and the RPS mechanism, and the specialty of electricity market. Under the cap-and-trade program, a fixed amount of allowance is allocated to the factory at the beginning, and the factory can decide independently the producing quantity, and therefore the amount of allowances it needs. The amount of allowances the factories need is fixed and endogenous. Under the RPS, there is no initial allocation. The required amount of RECs is a percentage of the total demand, which is both random and exogenous. The amount of RECs the utility needs is random and exogenous. Furthermore, for factories under the cap-and-trade program, the technologies are with known and fixed emission levels and costs. For utilities, the electricity prices in the wholesale market are random. As a result, the utility needs to make periodic observations of the electricity prices, and keeps updating the cumulative demand it has supplied, and then makes periodic decisions. Most importantly, the products of factories under the cap-and-trade are regular products and can be carried on from period to period. On the other hand, electricity is prohibitively expensive to be stored. As a result, utilities face a unique multi-leveled wholesale electricity market. In general, a utility under the RPS and a factory under the cap-and-trade are facing different problems. The only resemblance is the trading scheme of certificates.

Another important feature of our model is the multiple settlement structure of the wholesale electricity market. The wholesale electricity market in different regions can be at different status and under different regulations. Many technical reports discuss the deregulation or the potential design of the wholesale electricity market, to name a few, see Stoft (2002), Trebilcock and Hrab (2004), Chao and Wilson (1999), and Boucher and Smeers (2001). Another stream of research on electricity market is from finance and economics. They focus on how electricity should be priced and how the price actually behaves in those centralized markets hosted by ISOs, including day-ahead market, the hour-ahead market and the real-time market. For example, Longstaff and Wang (2004) collected data from the PJM market, and did empirical work to study the relationship between the electricity prices in the day-ahead market and the real-time market.

Our paper contributes to the literature in both applied and methodological sides. We investigated the impact of the RPS on a single utility and have provided practical guidelines for the utility to purchase electricity and to trade RECs. On the methodological side, although some results of our paper look similar to those in Gong and Zhou (2013), the models are different.

2.3 Model Formulation

We model an RPS compliance year as a finite horizon of T periods, indexed as $1, \cdots, T$. At the beginning of each period t, a utility first observes the REC price and the electricity prices. The utility then buys or sells unbundled RECs in an REC market, and purchases electricity from a wholesale electricity market in order to supply a random demand in a retail electricity market. At the end of the horizon, the utility needs to hold sufficient RECs to cover a percentage, say α, of its total supply during the entire horizon. If the utility does not have sufficient RECs, it pays a penalty cost π per unit it is short on. In practice, the tradings in electricity markets and the REC market can be continuous and simultaneous, but we think of them as periodic decisions, and assume that in each period the utility trades in the REC market first, then the electricity markets.

The utility acts as a price taker in the REC market, the wholesale electricity market and the retail electricity market. Therefore, we assume these markets as exogenous, stochastic, and independent from each other. In this section, we start by describing the set-ups of these three markets in subsections 2.3.1, 2.3.2 and 2.3.3. We then discuss the objective of the utility in subsection 2.3.4. After that, we state the sequence of events in subsection 2.3.5. Lastly, we write the dynamic programming in subsection 2.3.6.

2.3.1 REC market

At the beginning of each period t, the utility observes the REC price in an REC market. We use two random variables, b_t and s_t, to represent the trading prices of RECs, with b_t being the per unit cost of buying RECs and s_t being the per unit revenue of selling RECs. Later we refer b_t as the buying price of REC and s_t as the selling price of REC. We allow b_t and s_t to be different and assume b_t is greater or equal to s_t. The gap between them can be resulted from the transaction cost and the bid-ask spread in REC trading (Holt et al., 2011; Gillenwater, 2008). We denote $R_t = (b_t, s_t)$ and assume $\{R_t = (b_t, s_t), 1 \leq t \leq T\}$ forms a Markov chain for trackability.

Theoretically, the trading prices of REC should not exceed the penalty cost of the RPS, as otherwise, the utility will have no incentive to buy any RECs, and will wait until the end of the horizon and pays the penalty cost. This is also observed in practice (Heeter and Bird, 2011). Therefore we assume that $Pr(s_t \leq b_t \leq \gamma^{T-t+1}\pi) = 1$, with γ being the one-period discount factor, $0 < \gamma \leq 1$.

We assume the utility can sell RECs even when it has zero or negative amount of RECs on hand. This assumption is made since the utility's REC amount will be checked only once at the end of the horizon.

Let x_t and $\bar{x}_t$ be the utility's REC level before and after the REC trading in period t, respectively. If the utility buys RECs, then $\bar{x}_t > x_t$, and the utility generates a cost of $b_t(\bar{x}_t - x_t)$. Otherwise $\bar{x}_t < x_t$, and the utility earns a revenue of $s_t(x_t - \bar{x}_t)$.

2.3.2 Wholesale electricity market

After trading RECs, the utility needs to purchase electricity from a wholesale electricity market in order to satisfy its end-users. We capture two flavors of the wholesale electricity market, the multiple settlements and the high volatility of the spot price. To incorporate the first flavor, we assume the utility purchases electricity from a forward market first and then balances its output (either buy or sell electricity) in a spot market. In the forward market, the utility purchases power directly from power producers. In the spot market, tradings are centralized through an ISO. To incorporate the second flavor, we impose some properties on the utility's expected cost function in the spot market to reflect the utility's tendency to avoid trading in the spot market. This tendency is driven by the high volatility of the spot price.

Forward Market

In the forward market, the utility company can purchase power from a variety of sellers through a variety of power purchase contracts. We simplify the pool of sellers as two power producers, one sells REC-bundled renewable energy and the other sells regular energy. In addition, we assume that the power purchase contracts are unit-price forward contracts and the specified prices are for only one period. Specifically, in period t, REC-bundled renewable energy can be purchased at p_{1t} per unit, and regular energy can be purchased at p_{2t} per unit. These prices are are only for period t. The prices for next period may be different. Since the electricity prices are exogenous for the utility, we assume the prices are random variables, and

$\{(p_{1t}, p_{2t}), 1 \leq t \leq T\}$ is a Markov chain for trackability. It is also reasonable to assume REC-bundled renewable energy should be valued more than regular energy, since one unit of REC-bundled renewable energy includes both one unit of regular energy and one unit of REC. Nevertheless, electricity prices can be unpredictable, and all the results in this paper hold with or without this assumption. The prices (p_{1t}, p_{2t}) in the forward market are referred to as forward prices. We denote $P_t = (p_{1t}, p_{2t})$.

We assume there is no capacity constraint on the two power producers. They can provide as much electricity the utility requests. We make this assumption because most of the contracts in the forward market are only "financially binding" (Stoft et al., 1998). Financially binding contracts ensure the utility will, in the end, receive the exact amount of electricity at the exact price specified in the contracts. If the power producer is not able to provide sufficient amount of electricity specified in the contract in any period, the utility will buy electricity from the spot market, and the power producer will compensate the expense.

Let y_{1t} be the amount of REC-bundled energy the utility purchases and y_{2t} be the total amount of electricity the utility purchases from the forward market, then $y_{2t} - y_{1t}$ is the amount of regular energy the utility purchases.

Spot Market

After the utility purchased electricity from the forward market, a random demand D_t is realized. If the demand is more (less) than the amount the utility had purchased from the forward market, the utility will need to buy additional (sell redundant) electricity in the spot market, incurring expenses (revenue). The buying or selling will be conducted at the spot price with no bid-ask spread.

The utility does not make decisions in the spot market. The demand has to be satisfied, the spot price is exogenous, and the expense or revenue will take place. However, the utility had to take into account the expense or revenue to be incurred in the spot market when it made decisions in the forward market. At that time, neither the demand nor the spot price was revealed, and the utility needed to make decisions in the forward market based on an expectation on the expense or revenue to be incurred in the spot market. The utility's expectation is two fold including the demand

and the spot price. It is appealing to write the utility's expectation as $\int_{y_{2t}}^{\infty} e_t(z - y_{2t}) f_{D_t}(z)\, dz - \int_{-\infty}^{y_{2t}} e_t(y_{2t} - z) f_{D_t}(z)\, dz$, where e_t is utility's expectation on the spot price, y_{2t} is the amount of electricity the utility had purchased in the forward market, and $f_{D_t}(\cdot)$ is the probability density function of the customer demand D_t. However, we do not use this function as utility's expectation because this function relies on an accurate prediction of the spot price, which is unrealistic, and leads to some far-fetched strategies of utilities. Under this function, if the utility expects the spot price e_t to be greater than the forward price of regular energy p_{2t}, it will purchase as much as it can in the forward market, sell the excessive amount into the spot market, and make profit on the price difference. On the other hand, if the utility expects $e_t < p_{2t}$, it will purchase nothing in the forward market, and relies entirely on the spot market to satisfy the customer demand. Both of these cases are very different from what we observe in practice, where the utility purchase most of the energy needed in the forward market and trading in the spot market only when necessary.

We define $G_t(y_{2t})$ as the utility's expectation on its expense or revenue to be incurred in the spot market, where y_{2t} is the amount of electricity it has purchased from the forward market. In the following, we use one example of $G_t(y_{2t})$ to present some desirable properties of $G_t(y_{2t})$. The specific form of $G_t(y_{2t})$ is not restricted to this example.

$$G_t(y_{2t}) = \int_{y_{2t}}^{\infty} G_t^{+}(z - y_{2t}) f_{D_t}(z)\, dz - \int_{-\infty}^{y_{2t}} G_t^{-}(y_{2t} - z) f_{D_t}(z)\, dz. \quad (2.1)$$

In equation (2.1), the first term represents the case when the demand is greater than the utility's energy on hand. Here z represents the realized demand. If $z > y_{2t}$, the utility would need to purchase $z - y_{2t}$ amount of additional energy. We assume the expense is a function of $z - y_{2t}$, and write it as $G_t^{+}(z - y_{2t})$. Therefore $\int_{y_{2t}}^{\infty} G_t^{+}(z - y_{2t}) f_{D_t}(z)\, dz$ is the utility's expectation on its expense of purchasing additional energy when the demand is greater than its energy on hand. Note that $G_t^{+}(z - y_{2t})$ is nonnegative. We make the following assumptions:

- $G_t^{+}(z - y_{2t})$ decreases in y_{2t}. This assumption simply means that the less electricity the utility has on hand, the more it needs to purchase, and thus the more it spends in the spot market.
- $G_t^{+}(z - y_{2t})$ is convex in y_{2t}. This assumption implies that the marginal cost of buying electricity is increasingly more expensive when the util-

ity purchases more. Hence the utility wants to purchase at little as possible in the spot market.

- $G_t^+(z - y_{2t})$ is diffrentiable, and $|dG_t^+(z - y_{2t})/dy_{2t}| > p_{2t}$. Under this assumption, the per unit cost of buying electricity is always more expensive than the unit price of regular energy in the forward market. Hence the utility strictly prefers purchasing in the forward market.

The second term represents the case when the demand z is less than the utility's energy on hand y_{2t}. In this case, the utility would need to sell $y_{2t} - z$ amount of excessive energy. We assume the revenue is a function of $y_{2t} - z$, and write it as $G_t^-(y_{2t} - z)$. Therefore $\int_{-\infty}^{y_{2t}} G_t^-(y_{2t} - z) f_{D_t}(z)\, dz$ is the utility's expectation on its revenue of selling excessive energy when the demand is less than its energy on hand. Note that $G_t^-(y_{2t} - z)$ is also nonnegative. We make the following assumptions:

- $G_t^-(y_{2t} - z)$ increases in y_{2t}. This assumption simply means that the more electricity the utility sells to the spot market, the more revenue it earns.

- $G_t^-(y_{2t} - z)$ is concave in y_{2t}.

- $G_t^-(y_{2t} - z)$ is diffrentiable, and $d\, G_t^-(y_{2t} - z)/d\, y_{2t}$ approaches 0 as y_{2t} approaches infinity. These two assumptions suggest that the marginal revenue of selling electricity to the spot market decreases as the utility sells more, and will approach zero eventually as the selling amount approaches infinity.

- $d\, G_t^-(y_{2t} - z)/d\, y_{2t} < p_{2t}$. This assumption implies that the per unit revenue of selling electricity to the spot market is strictly less than the unit price of regular energy in the forward market. Hence there is no economic incentive for the utility to over purchase electricity from the forward market so as to sell into the spot market.

In aggregate $G_t(y_{2t})$ (2.1) is the utility's expectation on its expense or revenue in the spot market, with a positive value implying expense and a negative value implying revenue. Later we refer $G_t(y_{2t})$ as the "expected balancing cost" in the spot market. There are four properties of $G_t(y_{2t})$:

- $G_t(y_{2t})$ decreases in y_{2t};

- $G_t(y_{2t})$ is convex in y_{2t};

- $|dG_t(y_{2t})/dy_{2t}| > p_{2t}$ when $G_t(y_{2t}) \geq 0$, and $|dG_t(y_{2t})/dy_{2t}| < p_{2t}$ when $G_t(y_{2t}) \leq 0$;
- $\frac{dG_t(y_{2t})}{dy_{2t}} \to 0$ as $y_{2t} \to +\infty$.

The specific form of $G_t(y_{2t})$ should not be restricted to (2.1), but the four properties should preserve. As we will see, the first and second properties are important for deriving optimal strategies. The third and fourth properties exclude some unreasonable boundary solutions.

2.3.3 Retail electricity market

We assume the customer demands in different periods are independent random variables, and the demand in period t is distributed with a continuous and strictly positive probability density function $f_{D_t}(\cdot)$. In addition, we assume the customer demands are independent from the REC prices and the electricity prices. In practice, these random variables can be correlated. From the perspective of a utility, however, REC market and electricity markets are exogenous, and the correlation between them is not the focus of this paper.

2.3.4 Objective of the utility

We assume the utility's objective is to minimize the expected total discounted cost over the planning horizon. We ignore the utility's revenue from selling electricity to its customers since the utility has little control over its operations in the retail electricity market. The retail electricity rates are capped by the state government, and the demand elasticity is fairly small (Borenstein, 2009). We thus focus on utility's strategy in the wholesale electricity market and the REC market, not the retail electricity market.

2.3.5 Dynamic programming formulation

At the beginning of each period t, utility observes its REC amount on hand x_t as well as its cumulative demand u_t, cumulative demand being the demand the utility has supplied from from period 1 to period $t-1$. In addition, the utility also observes the buying and selling prices of unbundled RECs ($R_t = (b_t, s_t)$), as well as the prices of REC-bundled energy and regular energy ($P_t = (p_{1t}, p_{2t})$). The utility then begins its decision making process by trading in the REC market, adjusting its REC level from x_t to $\bar{x}_t$. After that, the utility decides how much energy to purchase from the forward market. The utility purchases y_{1t} amount of REC-bundled energy and $y_{2t} - y_{1t}$ amount of regular energy. Finally, the demand D_t realizes, the utility balances its output in the spot market, generating cost or revenue $G_t(y_{2t})$. At the end of period t, the utility's REC level updates to $x_{t+1} = \bar{x}_t + y_{1t}$, and its cumulative demand level updates to $u_{t+1} = u_t + D_t$. At the end of the horizon, if the utility does not have enough RECs, it will be charged a penalty cost.

Let $V_t(x_t,\, u_t,\, R_t,\, P_t)$ be the minimal expected cost of utility from period t to the end of the horizon given the utility's REC level x_t, the cumulative demand u_t, the REC prices R_t and the energy prices P_t. Then the utility solves the following dynamic program.

$$
\begin{aligned}
&V_t(x_t,\, u_t,\, R_t,\, P_t) \\
= &\min_{\substack{\bar{x}_t \\ y_{2t} \geq y_{1t} \geq 0}} \{ b_t(\bar{x}_t - x_t)^+ - s_t(x_t - \bar{x}_t)^+ + p_{1t}y_{1t} + p_{2t}(y_{2t} - y_{1t}) \\
&+ G_t(y_{2t}) + \gamma E_t[V_{t+1}(\bar{x}_t + y_{1t},\, u_t + D_t,\, R_{t+1},\, P_{t+1})]\}. \qquad (2.2)
\end{aligned}
$$

In the optimization equation (2.2), the decision variables are $\bar{x}_t$, y_{1t} and y_{2t}. These decision variables represent the three decisions the utility makes in each period, i.e., how many RECs to buy or sell, how much REC-bundled energy to purchase, and how much regular energy to purchase. There is no constraint on $\bar{x}_t$, because he utility is allowed to buy or sell RECs irresponsible of the amount of RECs it has on hand. The constraints on y_{1t} and y_{2t} make sure the utility purchases non-negative amounts of energy from power producers. On the right hand side of (2.2), $b_t(\bar{x}_t - x_t)^+ - s_t(x_t - \bar{x}_t)^+$ is the utility's expense or revenue from REC trading, $p_{1t}y_{1t} + p_{2t}(y_{2t} - y_{1t})$ is the utility's expense to purchase electricity in the forward market, $G_t(y_{2t})$ is utility's expectation on its expense or revenue in the spot market, and finally is the minimal expected cost from period $t + 1$ till the end of the horizon. We use $E_t[\cdot]$ to denote $E_{D_t}[E_{(R_{t+1},P_{t+1})}[\cdot|(R_t, P_t)]]$.

The value function at the end of the horizon is

$$
V_{T+1}(z_{T+1},\, u_{T+1}) = \pi(\alpha u_{T+1} - z_{T+1})^+. \qquad (2.3)
$$

If the utility does not have sufficient RECs to cover α percent of its total supply during the horizon (u_{T+1}), it will be charged π dollars per unit of RECs it is short. The salvage value of excessive RECs is set to be zero.

2.4 Optimal Policies

In this section, we give a complete characterization of utility's optimal policies in both the REC market and the electricity market. We divide the utility's decision making process in each period into two stages in time sequence. At stage one, the utility buys or sells RECs in the REC market. At stage two, the utility purchases electricity in the forward market.

The next theorem establishes the intuition that the utility should sell RECs when it has more RECs on hand, and should be buy RECs when it has less. Specifically, the utility should follow a target interval policy with two thresholds $L_t(u_t, R_t, P_t)$ and $H_t(u_t, R_t, P_t)$. If the utility's REC level is higher than $H_t(u_t, R_t, P_t)$, it should sell RECs; if the utility's REC level is lower than $L_t(u_t, R_t, P_t)$, it should purchase RECs; if the utility's REC level is between these two thresholds, it should not trade in the REC market. We will give more detailed description of these two thresholds later when we move on to the utility's optimal policies in the electricity market.

Theorem 2.4.1 *In each period t, $t = 1, \ldots, T$, given state (x_t, u_t, R_t, P_t), the utility's optimal REC trading policy is a target intervel policy with two state-dependent thresholds $L_t(u_t, R_t, P_t)$ and $H_t(u_t, R_t, P_t)$, with $L_t(u_t, R_t, P_t) \leq H_t(u_t, R_t, P_t)$. The optimal REC level after REC trading can be characterized as*

$$\bar{x}_t^* = \begin{cases} L_t(u_t, R_t, P_t) & \textit{if } x_t \leq L_t(u_t, R_t, P_t); \\ x_t & \textit{if } L_t(u_t, R_t, P_t) < x_t < H_t(u_t, R_t, P_t); \\ H_t(u_t, R_t, P_t) & \textit{if } x_t \geq H_t(u_t, R_t, P_t). \end{cases}$$

The threshold structure comes from the convexity of the cost-to-go function (Lemma A.0.1). There are two thresholds since the buying and selling price of RECs may be different. Because the buying price of RECs is more than the selling price of RECs, if the utility purchases RECs, it should purchase up to a lower target level than if it sells them.

Both of the thresholds depend on other state variables, including the cumulative demand, the REC prices and the electricity prices. In order to develop the monotonic property of the thresholds, we need the following lemma.

Lemma 2.4.2 *The value function $V_t(x, u, R, P)$ is submodular on (x, u).*

The submodularity of the value function implies that REC is a economic complement of the cumulative demand and leads to the following proposition that both of the thresholds increase in the cumulative demand. At the end of the horizon, the compulsory REC amount the utility needs to hold is a percentage of the cumulative demand it has supplied. Therefore if the utility has supplied more demand, it should be more conservative to sell RECs and more willing to buy RECs, which is reflected as higher target interval.

Proposition 2.4.3 *$L_t(u_t,\, R_t,\, P_t)$ and $H_t(u_t,\, R_t,\, P_t)$ increase in u_t.*

After the utility traded RECs in the REC market, it will purchase electricity from the forward electricity pmarket. There are two products to choose, REC-bundled energy and regular energy. REC-bundled energy will be separated as two parts upon purchase. One part is energy, which feeds into the power grids equivalently as regular energy. The other part is RECs, which can be stored for the RPS obligation till the end of horizon or sold for revenue before that. In some sense, these two products are substitutes.

The next proposition describes certain conditions under which it is optimal for the utility to purchase only one of the two products. We identify $\triangle_t = p_{1t} - p_{2t}$, the price difference between REC-bundled energy and regular energy, to be a critical value when compare these two products. In the remainder of this paper we refer $\triangle_t$ as the "intrinsic" REC price, for it is essentially the price of the RECs from REC-bundled energy. We define the optimal purchasing quantities of REC-bundled energy and regular energy in period t as y_{1t}^* and y_{2t}^*, respectively.

Proposition 2.4.4 *In each period t, $t = 1, \dots, T$, given state $(x_t,\, u_t,\, R_t,\, P_t)$, the utility's optimal energy choice in the forward market can be characterized as*

(a) If $\triangle_t \geq b_t$, it is optimal to purchase only regular energy, i.e., $y_{1t}^* = 0$;

(b) If $\triangle_t \leq s_t$, it is optimal to purchase only REC-bundled energy, i.e., $y_{2t}^* = y_{1t}^*$;

(c) If $s_t < \triangle_t < b_t$,

- *when $x_t \leq L_t(u_t,\, R_t, P_t)$, it is optimal to purchase only REC-bundled energy, i.e., $y_{2t}^* = y_{1t}^*$;*

- *when $x_t \geq H_t(u, R_t, P_t)$, it is optimal to purchase only regular energy, i.e., $y^*_{1t} = 0$.*

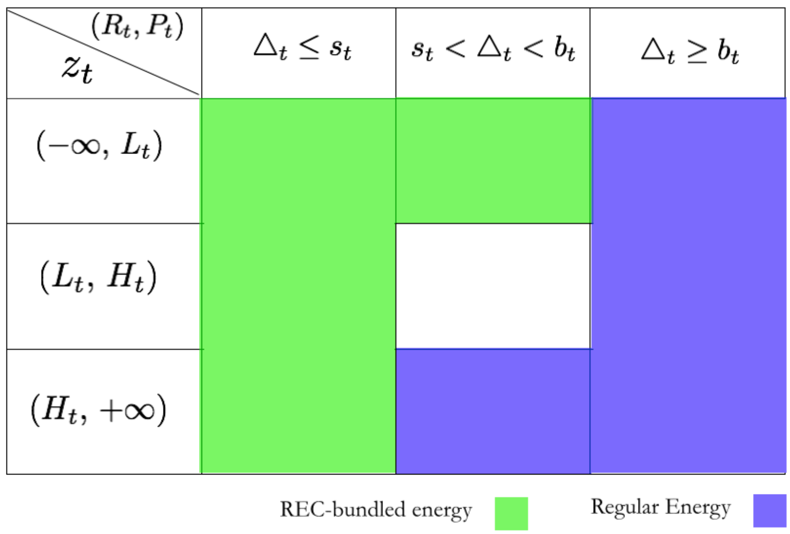

Figure 2.2: Optimal energy choice in the forward market.

Proposition 2.4.4 is presented with Figure 2.2. If $\triangle_t \geq b_t$, i.e., the intrinsic REC price is greater than the buying price of RECs, we claim that REC-bundled energy is dominated by regular energy in terms of price, so that the utility has no economic incentive to purchase REC-bundled energy. To explain this, write $\triangle_t \geq b_t$ as $p_{1t} \geq p_{2t} + b_t$. In this case, if the utility purchases one unit of regular energy and one unit of REC, and combine them together, it can get essentially the same product as one unit of REC-bundled energy, but at a cheaper price.

If $\triangle_t \leq s_t$, i.e., the intrinsic REC price is less than the selling price of RECs, we claim that regular energy is dominated by REC-bundled energy in terms of price, so that the utility has no economic incentive to purchase regular energy. To explain this, write $\triangle_t \leq s_t$ as $p_{1t} - s_t \leq p_{2t}$. In this case, by purchasing one unit of REC-bundled energy and selling the REC, the utility gets essentially the same product as one unit of regular energy, but at a cheaper price.

Proposition 2.4.4 (c) consider the case where $s_t < \triangle_t < b_t$, i.e, the intrinsic REC price is between the selling price and the buying price of RECs. In this case, we consider REC-bundled energy and regular energy to be competitive in price, and the optimal strategy of the utility is more subtle than one might expect. The utility should make its energy choice based on its REC level at the beginning of period t.

We explain the intuition of Proposition 2.4.4 (c) by contradiction. Write $s_t < \triangle_t < b_t$ as $p_{1t} < p_{2t} + b_t$ and $p_{2t} < p_{1t} - s_t$. When $x_t \leq L_t(u, R_t, P_t)$, by Theorem 2.4.1, a utility who follows the optimal policy in the REC market should have bought some unbundled RECs at stage one. If the utility purchases some regular energy on top of that, for one unit of regular energy it purchases together with a unit of REC it has already bought, it could have gotten them by purchasing one unit of REC-bundled energy at a cheaper price as $p_1 < p_2 + b_t$. Thus there is no incentive for the utility to purchase regular energy. Similarly, when $x_t \geq H_t(u_t, R_t, P_t)$, the utility should have sold some RECs at stage one. If the utility purchases some REC-bundled energy on top of that, then in aggregate it is purchasing regular energy, which is actually available at a cheaper price since $p_{2t} < p_{1t} - s_t$. Thus in this case, it is always better for the utility to purchase only regular energy.

There is one circumstance that is not included in Proposition 2.4.4, which is if $s_t < \triangle_t < b_t$ and $L_t(u_t, R_t, P_t) < x_t < H_t(u_t, R_t, P_t)$. In this case, so far by Theorem 2.4.1, we know that the utility should not trade in the REC market. We haven't talked about utility's optimal energy choice in the forward market. For that, additional analysis is required. However, before we come to this case, we use next two theorems to give a detailed description of utility's optimal purchasing quantities under the scenarios corresponding to Proposition 2.4.4 (a) and (b), when the utility is single sourcing.

Theorem 2.4.5 *In each period t, $t = 1, \ldots, T$, given state (x_t, u_t, R_t, P_t), if $\triangle_t \geq b_t$, then the optimal purchasing quantities of the utility in the forward market, (y_1^*, y_2^*), can be characterized as $y_{1t}^* = 0$, $y_{2t}^* = S_{2t}(p_{2t})$, where*

$$S_{2t}(p_{2t}) = \arg\min_{y \geq 0} \{p_{2t} y + G_t(y)\}. \tag{2.4}$$

Moreover, $S_{2t}(p_{2t})$ decreases in p_{2t}.

Theorem 2.4.5 specifies the optimal purchasing quantities for the case presented in Proposition 2.4.4 (a), and is presented in the fourth column of Figure 2.3. In period t, if the intrinsic REC price $\triangle_t$ is above the buying price of unbundled REC b_t, it is optimal for the utility to use the following strategy to comply with the RPS: purchasing regular energy to supply the end-users and buying unbundled RECs separately. These two activities does not interact with each other. As results, the optimal purchasing quantity depends only on the price of regular energy, and it decreases with the price.

From the definition of $S_{2t}(p_{2t})$ (2.4), we can see that the third assumption we made about $G_t(y)$ ensure a positive and finite value for $S_{2t}(p_{2t})$. When $y = 0$, the utility had purchased nothing from the forward market, thus $G_t(y)$ is positive, representing the utility's cost to purchase additional energy from the spot market. From the third assumption we made about $G_t(y)$, we know that the derivative of $G_t(y)$ at $y = 0$ is less than $-p_{2t}$, thus $p_{2t}y + G_t(y)$ is strictly decreasing at $y = 0$, thus $S_{2t}(p_{2t}) > 0$. On the other hand, assume that the demand is bounded, then $S_{2t}(p_{2t})$ will be a finite number. Consider when y is large enough so that the utility generates revenue by selling excessive energy to the spot market. In that case, $G_t(y)$ is negative, and the derivative of $G_t(y)$ is more than $-p_{2t}$. Therefore $p_{2t}y + G_t(y)$ is strictly increasing when y is large enough, thus $S_{2t}(p_{2t})$ is finite.

Theorem 2.4.6 *In each period t, $t = 1, \ldots, T$, given state $(x_t,\, u_t,\, R_t,\, P_t)$, if $\triangle_t \leq s_t$, then the optimal purchasing quantities of the utility in the forward market, (y_1^*, y_2^*), can be characterized as*

$$y_{1t}^* = y_{2t}^* = \begin{cases} S_{1t}^L(p_{1t}, b_t) & \text{if } x_t \leq L_t(u_t,\, R_t,\, P_t); \\ s_{1t}(x_t,\, u_t,\, R_t,\, P_t) & \text{if } L_t(u_t,\, R_t,\, P_t) < x_t < H_t(u_t,\, R_t,\, P_t); \\ S_{1t}^H(p_{1t}, s_t) & \text{if } x_t \geq H_t(u_t,\, R_t, P_t). \end{cases} \tag{2.5}$$

where

$$S_1^L(p_{1t}, b_t) = \arg\min_{y \geq 0} \{(p_{1t} - b_t)y + G_t(y)\}, \tag{2.6}$$

$$\begin{aligned} s_{1t}(x_t,\, u_t,\, R_t,\, P_t) = \arg\min_{y \geq 0} \{ & p_{1t}y + G_t(y) \\ & + \gamma E[V_{t+1}(x_t + y,\, u_t + D_t,\, R_{t+1},\, P_{t+1})]\}, \end{aligned} \tag{2.7}$$

$$S_{1t}^H(p_{1t}, s_t) = \arg\min_{y\geq 0}\{(p_{1t} - s_t)y + G_t(y)\}. \tag{2.8}$$

Moreover, the two thresholds

$$\begin{aligned} L_t(u_t,\, R_t,\, P_t) &= w_t^L(u_t,\, R_t,\, P_t) - S_{1t}^L(p_{1t}, b_t), \\ H_t(u_t,\, R_t,\, P_t) &= w_t^H(u_t,\, R_t,\, P_t) - S_{1t}^H(p_{1t}, s_t), \end{aligned} \tag{2.9}$$

where $w_t^L(u_t,\, R_t, P_t)$, $w_t^H(u_t,\, R_t, P_t)$ are the optimal REC levels at the end of period t when the utility buys or sells RECs in the REC market respectively.

Theorem 2.4.6 specifies the optimal purchasing quantities for the case in Proposition 2.4.4 (b), and is presented in the second column of Figure 2.3. If the intrinsic REC price $\triangle_t$ is less than the selling price of REC s_t, it is optimal for the utility to use the following strategy to comply with the RPS: supplying all of the customer demand with renewable energy. The optimal purchasing quantity is given by (2.5). If the utility has bought (sold) RECs in the REC market, the optimal purchasing quantity is $S_1^L(p_{1t}, b_t)$ ($S_1^H(p_{1t}, s_t)$). These two quantities are independent of utility's REC level or cumulative demand. If the utility has not done any REC trading, the optimal purchasing quantity is $s_{1t}(x_t,\, u_t,\, R_t,\, P_t)$, which depends on its REC level and cumulative demand.

Note that the third and fourth assumptions we made about $G_t(y)$ exclude some unreasonable boundary values for $S_1^L(p_{1t}, b_t)$ and $S_1^H(p_{1t}, s_t)$. From the third assumption of $G_t(y)$, we know that $dG_t(y)/dy$ is less than $-p_{2t}$ at $y = 0$. Therefore we have $p_{1t}-b_t+dG_t(y)/dy < p_{1t}-b_t-p_{2t} = \triangle_t-b_t$. Because the condition in Theorem 2.4.6 is $\triangle_t < s_t$, we have $\triangle_t < b_t$. Therefore, $(p_{1t} - b_t)y + G_t(y)$ is strictly decreasing when $y = 0$. Therefore from the definition we know that $S_1^L(p_{1t}, b_t) > 0$. Similarly we can show that $S_1^H(p_{1t}, s_t) > 0$. On the other hand, when y approaches infinity, from the fourth assumption of $G_t(y)$, we know that $dG_t(y)/dy$ approaches 0, so that the first derivative of $(p_{1t} - b_t)y + G_t(y)$ approaches $p_{1t} - b_t$. We did not make any assumption regarding the relationship between p_{1t} and b_t because the electricity prices can be unpredictable. However, in most cases, it would be reasonable to assume that in the same period, one unit of REC-bundled energy should be more expensive than one unit of unbundled REC, i.e., $p_{1t} > b_t$, then we have $S_1^L(p_{1t}, b_t)$ being finite. Similar argument can be made about $S_1^H(p_{1t}, s_t)$ if we assume $p_{1t} > s_t$.

In rare cases, if REC-bundled energy costs less than the buying price of unbundled RECs in period t, i.e., $p_{1t} < b_t$, intuitively the utility should never purchase any unbundled RECs. This is in line with the results in the Theorem. Given $p_{1t} < b_t$, according to the descriptions (2.6) and (2.9), the optimal purchasing quantity $S_1^L(p_{1t}, b_t) \longrightarrow +\infty$, and the lower threshold $L_t(u_t,\, R_t,\, P_t) \longrightarrow -\infty$. Since the lower threshold approaches minus infinity, the utility's REC level will never be lower than that, and thus will never purchase RECs. In other words, the case where $z \leq L_t(u_t,\, R_t,\, P_t)$ will not happen. Similarly, if $p_{1t} < s_t$, then according to the descriptions (2.8) and (2.9), the optimal purchasing quantity $S_1^H(p_{1t}, s_t) \longrightarrow +\infty$, and the higher threshold $H_t(u_t,\, R_t,\, P_t) \longrightarrow -\infty$. Since the higher threshold approaches minus infinity, the utility's REC level will be above that, and the utility should sell an infinite amount of unbundled RECs, since it can gain profit by selling unbundled RECs and then purchasing REC-bundled energy.

We also give another characterization of the two thresholds in the REC trading policy. We define $w_t = \bar{x}_t + y_{1t}$ as the REC level at the end of period t. Define $w_t^L(u_t, R_t, P_t)$ as the optimal REC level at the end of period t given that the utility had purchased RECs to increase its REC level to $L(u_t, R_t, P_t)$. Similarly, define $w_t^H(u_t,\, R_t,\, P_t)$ as the optimal REC level at the end of period t given that the utility had sold RECs to decrease its REC level to $H(u_t, R_t, P_t)$. If the utility has an REC level lower than $L_t(u_t,\, R_t,\, P_t)$, it would purchase some RECs to increase its REC level up to $L_t(u_t,\, R_t,\, P_t)$. Then since the utility will purchase $S_1^L(p_{1t}, b_t)$ amount of REC-bundled energy, the end-of period REC level $w_t^L(u_t,\, R_t,\, P_t)$ should be a sum of $L_t(u_t,\, R_t,\, P_t)$ and $S_1^L(p_{1t}, b_t)$. Therefore, we have $L_t(u_t,\, R_t,\, P_t) = w_t^L(u_t,\, R_t,\, P_t) - S_{1t}^L(p_{1t}, b_t)$. Similar analysis can be done to $H_t(u_t,\, R_t,\, P_t)$.

Next we develop some monotonic properties of the optimal purchasing quantities given in Theorem 2.4.6.

Proposition 2.4.7 *(a)* $S_{1t}^L(p_{1t}, b_t)$ *decreases in* p_{1t} *and increases in* b_t.

(b) $S_{1t}^H(p_{1t}, s_t)$ *decreases in* p_{1t} *and increases in* s_t.

(c) $s_{1t}(x_t,\, u_t,\, R_t, P_t)$ *decreases in* x_t *and increases in* u_t.

(d) $S_{1t}^L(p_{1t}, b_t) \geq s_{1t}(x_t,\, u_t,\, R_t, P_t) \geq S_{1t}^H(p_{1t}, s_t)$.

If $\triangle_t \leq s_t$, the utility should purchase only REC-bundled energy. As results, when the price of REC-bundled energy goes up, the purchasing quantities go down. Thus both $S^L_{1t}(p_{1t}, b_t)$ and $S^H_{1t}(p_{1t}, b_t)$ decrease in p_{1t}.

When $x_t \le L_t(u_t, R_t, P_t)$, the utility buys RECs. It also purchases the amount $S^L_{1t}(p_{1t}, b_t)$ of REC-bundled energy. If the buying price of REC increases, the utility would have an incentive to gain RECs through buying REC-bundled energy instead of buying unbundled RECs. This explains why $S^L_{1t}(p_{1t}, b_t)$ increases in b_t.

When $x_t \ge H_t(u_t, R_t, P_t)$, the utility sells RECs. It also purchases the amount $S^H_{1t}(p_{1t}, s_t)$ of REC-bundled energy. If the selling price of REC increases, the utility would have an incentive to purchase more REC-bundled energy so that it can sell into the REC market. This explains why $S^H_{1t}(p_{1t}, s_t)$ increases in s_t.

When $L_t(u_t, R_t, P_t) < x_t < H_t(u_t, R_t, P_t)$, the utility neither buys nor sells RECs. The only REC source is REC-bundled energy. If there is higher energy demand, i.e., higher u_t, the utility needs to buy more RECs as the compulsory level of REC is proportional to the energy demand. Thus $s_{1t}(x_t, u_t, R_t, P_t)$ increases in u_t. On the other hand, if the utility has more RECs to start with, i.e., higher x_t, the utility needs less RECs. Thus $s_{1t}(x_t, u_t, R_t, P_t)$ decreases in x_t.

Proposition 2.4.7 (d) implies that as the REC level increases, the amount of REC bundled energy the utility purchases decreases. This is reasonable because one purpose that the utility purchases REC-bundled energy is to gain RECs, if it has more RECs to begin with, then it will need less.

The following theorem demonstrates the optimal purchasing quantities under the scenario corresponding to Proposition 2.4.4 (c). If $s_t < \triangle_t < b_t$, when the utility buys or sells unbundled RECs, it should do single sourcing as well, and the optimal purchasing quantities are independent of its REC level and cumulative demand.

Theorem 2.4.8 *In each period $t = 1, \ldots, T$, given state (x_t, u_t, R_t, P_t), if $s_t < \triangle_t < b_t$,*

(a) When $x_t \le L_t(u_t,\ R_t,\ P_t)$, it is optimal to purchase only REC-bundled energy, and $y^*_{1t} = y^*_{2t} = S^L_{1t}(p_{1t}, b_t)$;

(b) When $x_t \ge H_t(u_t,\ R_t,\ P_t)$, it is optimal to purchase only regular energy, and $y^*_{1t} = 0$, $y^*_{2t} = S_{2t}(p_{2t})$.

Moreover, the two thresholds

$$L_t(u_t,\, R_t,\, P_t) = w_t^L(u_t,\, R_t,\, P_t) - S_{1t}^L(p_{1t}, b_t), \tag{2.10}$$

$$H_t(u_t,\, R_t,\, P_t) = w_t^H(u_t,\, R_t,\, P_t). \tag{2.11}$$

Theorem 2.4.8 is presented in the third column of Figure 2.3. The result is different from Theorem 2.4.5 and 2.4.6 in that the energy choice depends not only on the relationship between the intrinsic REC price and the REC prices, but also on the utility's REC level. If the utility is in need of RECs, REC-bundled energy is more attractive, the utility should purchase $S_{1t}^L(p_{1t}, b_t)$ amount of REC-bundled energy. In this case, the REC level at the end of period t should be a sum of the lower threshold for REC trading and the amount of REC-bundled energy the utility purchased, thus $w_t^L(u_t,\, R_t,\, P_t) = L_t(u_t,\, R_t,\, P_t) + S_{1t}^L(p_{1t}, b_t)$, thus $L_t(u_t,\, R_t,\, P_t) = w_t^L(u_t,\, R_t,\, P_t) - S_{1t}^L(p_{1t}, b_t)$. If the utility sells RECs, regular energy is more attractive, the utility should purchase $S_{2t}(p_{2t})$ amount of regular energy. In this case, the REC level at the end of period t should be the same as the higher threshold, i.e., $H_t(u_t,\, R_t,\, P_t) = w_t^H(u_t,\, R_t,\, P_t)$.

Finally, we show how the utility should choose the product when $s_t < \triangle_t < b_t$ and $L_t(u_t,\, R_t,\, P_t) < x_t < H_t(u_t,\, R_t,\, P_t)$. This will complete our characterization of the utility's optimal policy.

Theorem 2.4.9 *In each period $t = 1, \ldots, T$, given state $(x_t,\, u_t,\, R_t,\, P_t)$, if $S_t < \triangle_t < b_t$, there exists a pair of thresholds $(l_t(u_t,\, R_t,\, P_t),\, h_t(u_t,\, R_t,\, P_t))$ satisfying*

$$L_t(u_t,\, R_t,\, P_t) \le l_t(u_t,\, R_t,\, P_t) \le h_t(u_t,\, R_t,\, P_t) \le H_t(u_t,\, R_t,\, P_t), \tag{2.12}$$

such that the optimal purchasing quantities in the forward market, (y_{1t}^*, y_{2t}^*), can be characterized as

(a) When $L_t(u_t,\, R_t,\, P_t) < x_t \le l_t(u_t,\, R_t,\, P_t)$, it is optimal to purchase only REC-bundled energy, and $y_{1t}^* = y_{2t}^* = s_{1t}(x_t, u_t,\, R_t,\, P_t)$;

(b) When $l_t(u_t,\, R_t,\, P_t) < x_t < h_t(u_t,\, R_t,\, P_t)$, it is optimal to purchase both REC-bundled energy and regular energy, and $y_{1t}^* = w_t^{\triangle}(u_t,\, R_t,\, P_t) - x_t$, $y_{2t}^* = S_{2t}(p_{2t})$;

(c) When $h_t(u_t,\, R_t,\, P_t) \le x_t < H_t(u_t,\, R_t,\, P_t)$, it is optimal to purchase only regular energy, and $y_{1t}^* = 0$, $y_{2t}^* = S_{2t}(p_{2t})$;

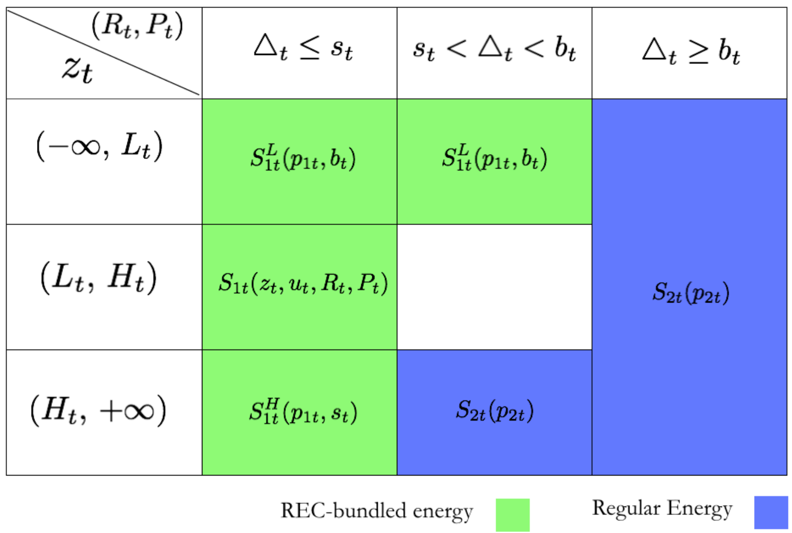

Figure 2.3: Optimal strategies in the forward market when single sourcing.

Moreover, the two new thresholds

$$l_t(u_t,\, R_t,\, P_t) = w_t^{\triangle}(u_t,\, R_t,\, P_t) - S_{2t}(p_{2t}),$$
$$h_t(u_t,\, R_t,\, P_t) = w_t^{\triangle}(u_t,\, R_t,\, P_t),$$

where $w_t^{\triangle}(u_t,\, R_t,\, P_t)$ *is the optimal REC level at the end of period* t *when the utility does not trade RECs in the REC market.*

Note that Theorem 2.4.8 and Theorem 2.4.9 together characterize utility's optimal policy in the forward market if $S_t < \triangle_t < b_t$, and these results are presented in the third column of Figure 2.4. These two Theorems together states that if $x_t \leq l_t(u_t,\, R_t,\, P_t)$, it is optimal to purchase only REC-bundled energy. If $x_t \geq h_t(u_t,\, R_t,\, P_t)$, it is optimal to purchase only regular energy. It is only when $l_t(u_t,\, R_t,\, P_t) < x_t < h_t(u_t,\, R_t,\, P_t)$, both regular energy and REC-bundled energy are purchased. The properties of the optimal purchasing quantities stated in Proposition 2.4.7 also applies here.

From the third column of Figure 2.4, we can see that as the REC level increases, the utility is in less need of RECs, thus it is gradually switching from REC-bundled energy to regular energy.

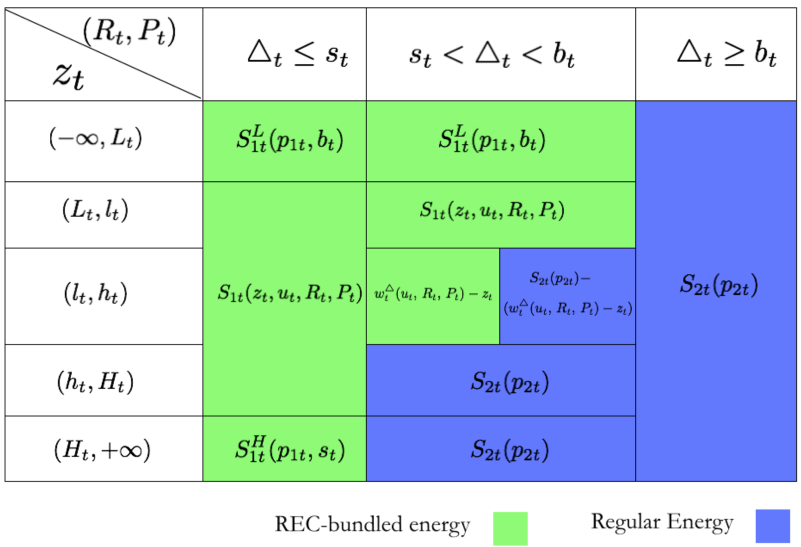

Figure 2.4: Optimal strategies in the forward market .

2.5 Conclusion

A utility under the RPS faces the challenge to comply with the regulation with minimum cost. The electricity prices and REC prices are stochastic and revealed at the beginning of each period. The utility observes these prices, as well as its REC level and cumulative demand level, and make periodic decisions in trading RECs and purchasing electricity in the forward market. In specific, the optimal REC trading policy is a target interval policy. If the utility's REC level is lower than the lower threshold, then it should purchase RECs to raise its REC level to the lower threshold. If the utility's REC level is higher than the higher threshold, then it should sell RECs to reduce its REC level to the higher threshold. If the utility's REC level is between these two thresholds, it should not do REC trading.

After the REC trading, the utility purchases electricity from the forward market. The optimal purchasing quantities are summarized in Figure 2.4. We identify the "intrinsic" REC price, the price difference between REC-bundled energy and regular energy, as a critical value for utility's energy choice. When the intrinsic REC price is less than the selling price of unbundled RECs, the utility should exclusively purchase REC-bundled energy (the second column of Figure 2.4). When the utility trades RECs, the optimal purchasing quantities are irrelevant with the utility's REC level or cumulative demand. While when the utility is not trading RECs, the optimal purchasing quantity would be based on the utility's REC level and cumulative demand. What's more, from the top to the bottom, the utility's REC level increases, and it is in less urgent need for RECs, thus the optimal purchasing quantity decreases. When the intrinsic REC price is more than the buying price of unbundled RECs, the utility should exclusively purchase regular energy (the forth column of Figure 2.4). In this case, the purchasing quantity is irrelevant with the utility's REC level, cumulative demand, or the REC prices, it only depends on the price of regular energy. When the intrinsic REC price is between the selling price and the buying price of unbundled RECs, the utility's energy choice should depend on its REC level (the third column of Figure 2.4). We show that only when the utility's REC level is between the two additional thresholds, it is optimal for the utility to purchase both kinds of energy. In this case, from the top to the bottom, the utility's REC level increases, the utility becomes less keen in RECs, thus the utility gradually switches from REC-bundled energy to regular energy.

We believe this paper can be a starting point for studying more sophisticated systems involve multiple utilities such as the market equilibrium with inelastic demand and the influence of REC price on the market equilibrium. One can also study a variation of our model with elastic demand and include pricing as a decision variable of the utility, since that is the vision of a future electricity market.

Chapter 3

Leveraging Suppliers to Calibrate Product Specification

3.1 Introduction

"In many cases, the supplier simply executes the design specifications from the manufacturer. If there is a design issue, a quality audit may not pick this up. It may be perfectly produced to a faulty design."
—Corporate Executive Board, (Gilligan, 2010).

Setting right product specifications is a vital function for any firm, and incorrectly or inappropriately set production specifications can significantly impact a firm's sales and reputation. As such, firms often closely examine their internal design and engineering processes to ensure specifications match what the market want. Specification flaws or mismatches, however, may persist even with best intentions from within the firm, and in this case the firm may benefit from tapping its suppliers' expertise to calibrate product specification.

Globalization has afforded suppliers ample opportunities to learn, develop, and accumulate unique and often tacit product and process knowledge. Such supplier-held knowledge can help firms calibrate and refine their product specifications to create successful products in the market. (Petrick, 2012) It is especially valuable when product specifications interact subtly with production process and technology choice: the supplier may perceive potential specification problems based on material/manufacturing alternatives, local/regional market tastes, and/or different regulatory mandates, which the OEM firm may not be able to anticipate a priori.

Petersen et al. (2005) noted that “suppliers, because of their product and process knowledge or expertise, may have more realistic information on the tradeoffs involved in achieving particular goals. Such goals are not limited to cost but often include product performance characteristics (such as weight, size, speed, etc.). The buying company will have the ultimate authority in goal setting, but the suppliers involvement can help in setting goals that are achievable.” Similarly, Ragatz et al. (2002) reported that ”using the knowledge and expertise of suppliers to complement internal capabilities may help reduce concept to customer cycle time, costs, quality problems,” and that “ interest in such efforts is growing.”

From an industry’s perspective, for example, suppliers (contract manufacturers) in chemical process industries can help to improve product specifications by suggesting “alternative chemical pathways” to a product. (Graff, 2014) Louis Assante, president of the Contemporary Cosmetic Group, noted that as a contract manufacturer it often “make suggestions for improvements or new technologies in personal care to our clients”. (Jeffries, 2004) Similar observations can be found in many other industries as well. In particular, Petrick (2012) noted that there is ample empirical evidence that supplier held knowledge can be important in creating successful products in the market place.

One cannot take for granted, however, that suppliers will always be willing to point out potential specification problems by sharing their product process knowledge with the OEM firm. Often the OEM firm’s objective is not perfectly aligned with that of the supplier’s, and hence the supplier may not be willing to suggest improvements or point out specification flaws. The supplier, for example, may notice that a particular ingredient may negatively impact the product’s fit to the local market’s taste, but it may know for sure whether the OEM could rapidly engineer an alternative specification. In addition, specification flaws are often observed only after the volume production has begun, and hence the supplier could lose significant business if it points out the specification flaw but no immediate resolutions are available. In 2007, for example, Dell had to discontinue its “pearl white” color specs with XPS notebooks when dust contamination problem was found with volume production runs not small test runs. (Cheng and Lawton, 2007) Here we focus on the incentive of the suppliers. We do not discuss ethic issues and reputation damage for suppliers.

Given that suppliers may not always be willing to suggest improvements or point out specification flaws, we seek to understand what factors may motivate the supplier to voluntarily help the OEM firm improve product specifications. The extant literature in supplier integration has examined the pros and cons of including suppliers in project teams (Ragatz et al., 2002; Hoegl and Wagner, 2005; Koufteros et al., 2005; Das et al., 2006; Parker et al., 2008). This stream of literature focuses primarily on how the OEM firm should involve the suppliers, e.g., the timing and depth of supplier involvement, in product development effort. An implicit assumption is that the suppliers will share tacit product and process knowledge with the OEM firm as long as they are "included". Relatively sparse attention has been paid, however, to whether the suppliers are willing to share their insights with the OEM firm even if they are included in the team.

We first examine when it is in the supplier's interest to voluntarily help the OEM to improve product specifications or pinpoint specification flaws. This question is particularly relevant when the OEM firm and its supplier form a "white box" relationship, where "buyer consults with supplier on buyer's design, discussion are held with suppliers about specifications/requirements but the buying company makes all design and specifications decisions. (Handfield and Lawson, 2007) Our research framework is in general not appropriate for the "black box" setting where "design is primarily supplier driven, based on buyer's performance specifications. As we focus on the "white box" type of relationships, we do not explore long term business contracts with commodity type of products. In other words, we are interested in industries with "fast clock speed".

Intuitively, the supplier would suggest to the OEM about potential specification issues only if doing so helps the supplier's current and/or near-future businesses. Given that the OEM controls product specification, the supplier cannot be faulted for any quality problems associated with specification problems. Nevertheless, the supplier may still be motivated to help the OEM improve product specifications if the supplier's current or near-future business is otherwise in jeopardy. Note that in this paper we ignore the supplier's outside options, which could either reinforce or diminish the supplier's incentive to improve product specifications. As the supplier becomes less dependent on the OEM firm's business, it becomes less concerned about losing the particular business with the OEM firm and therefore may have a stronger incentive to suggest specification problems. On the other hand, however, the supplier also becomes less interested in securing the OEM firm's business, which could dampen the supplier's incentives. The combined effect could be ambiguous and oftentimes are influenced by specific organizational culture and inter-firm relationships.

We then study the OEM's optimal strategy. The OEM's profit depends on the supplier's decision to inform or not inform. The OEM, however, does have the ability to design the contract to direct the supplier to choose to inform or not inform, whichever brings more profit to the OEM himself. If the supplier informs, the benefit for the OEM is that he might be able to rectify the flaw in time and thus will be able to satisfy the current period demand. In addition, the reputation of the products will bring a positive spillover effect to the demand in the future. The downside of the supplier informing is that the OEM might not be able to rectify the flaw in time. In that case, the OEM may need to cancel the order and pays a cancellation payment to the supplier. This cancellation payment might be expensive, especially given the fact that it might be just the incentive for the supplier to inform the OEM. What's more, the delay of releasing the products may hurt the OEM's market demand in the future.

If the supplier does not inform the OEM, the OEM will benefit by avoiding the cancellation payment. The OEM will be able to realize the flaw by collecting feedback and response from the customer. If the demand for the current period is fairly small and the cancellation payment is large, this may be better for the OEM. The downside for the OEM if the supplier does not inform is obvious. The OEM may lose the demand in the current period if the customers return the products. The defected products in the current period may cause reputation damage, which will negatively affect the demand in the future. Again, if the demand for the current period is fairly small, this might not be a sufficient incentive for the OEM to direct the supplier to inform. With all these factors entangled, the OEM's optimal strategy is ambiguous.

Therefore we solve two optimization problems from the perspective of the OEM. We first examine the optimal strategy for the OEM to maximize his profit given that he does not want the supplier to inform. We then explore the optimal strategy for the OEM to maximize his profit given that he wants the supplier to inform. After that, we compare the optimal profits of the OEM in these two cases. We prove that it is strictly better for the OEM to design the contract so that the supplier will inform if she detects any flaw. We give full description of the optimal solutions of the OEM and therefore provide guidelines for the OEM to design the contract.

3.2 Literature Review

Our paper is related to the supplier integration literature. Interestingly, this stream of literature has found conflicting evidences on whether supplier integration helps the OEM firm. Hoegl and Wagner (2005) empirically show that involving supplier in product development project can positively influence cost and schedule, but too much communication intensity may not benefit the project. Das et al. (2006) also discuss pros and cons of supplier integration. They find that too much investment in supplier integration is not productive. Koufteros et al. (2005) explores the relationship between internal integration and external integration, and find that internal integration positively influences external integration and product development outcomes. Parker et al. (2008) noted that cost of integrating suppliers across organizational boundaries imply that the benefit of integration must be significantly higher to justify the supplier's inclusion. From a general quality improvement perspective, Zhu et al. (2007) find that buyer's involvement plays a critical role in improving product quality and supply chain profits.

This study is also related to (but differs from) the extant literature on warranty services. In that stream of literature, the output quality is influenced by the supplier's effort (which may not be observable) and/or the OEMs effort. The OEM could motivate the supplier to produce higher quality product by sharing warranty cost with the supplier. Such shared warranty service is especially useful when the OEM cannot clearly disentangle the supplier's responsibility in the final product's quality problems. Reyniers and Tapiero (1995) study how price and warranty influence the supplier's quality effort in a game theoretical setting, where the quality decision is made by the supplier. Lim (2001) considers a similar problem, but incorporates information asymmetry where the buyer does not know the supplier's quality type and therefore must offer a menu of contract with appropriate warranty terms. Chao et al. (2009) explore warranty cost sharing contract based on selective or complete root cause analysis. Interestingly, they prove that both approaches could achieve optimal efforts (as compared with an integrated system), but cost sharing based on selective root cause analysis could achieve a higher profit for the supply chain. From a somewhat different angle, Dai et al. (2012) explore how the length of warranty influences the product quality and system profit. In their model, it is the supplier that determines the product quality level, but either party may determine the length of warranty period. They found that the party that bears a higher fraction of warranty cost should be delegated to set the warranty period.

Huang et al. (2008) study warranty service from an inventory management point of view, and they do not consider interactions between the supplier and the OEM. Note that the use of warranty service has also received extensive treatment in the new product development context, and we refer the interested reader to Murthy and Djamaludin (2002) for an excellent review of the literature on new product warranty.

Our research complements the above literature by considering how an OEM could motivate the supplier to voluntarily suggest potential specification issues, when the OEM is responsible for setting product specifications. In such a scenario, a shared warranty service would not be as effective because the supplier cannot be held responsible for mismatches between customer demand and product specification problems.

In closing, we note that Iyer et al. (2005) also explore the product specification problem, but with a very different focus. In contrast to our paper, they consider a "black-box" relationship where the supplier owns the product specification, but the OEM may allocate resources to help the supplier improve product specification. One can thus view our paper as complementary to theirs.

3.3 The Model

3.3.1 A descriptive overview

We consider an OEM firm sourcing a critical product (or component/subassembly) from an external supplier. The OEM firm determines the product features to be offered to the market and develops the corresponding specifications, whereas the supplier executes the OEM firm's order based on the OEM's specifications. Product specifications influence the market demand, and incorrect or misaligned specifications leads to lower demand.

The OEM firm may not be able to detect potential specification problems. That is, the OEM firm may perceive the specifications to fit the market taste well while in reality they may not. This could happen for several reasons. First, the distributed nature of production configuration often leads to dispersed product and process knowledge beyond the OEM's organizational boundary. Second, the supplier's process technology may interact subtly with product specifications (and performance), and it can be difficult for the OEM to access tacit supplier knowledge a priori. Third, the OEM may not have prior expertise in certain (new) markets whereas the supplier has accumulated unique and tacit knowledge in market tastes/trends through their relationship with other OEMs.

The supplier may recognize the OEM's specification problems, although the supplier may not may not be sure about whether the problems can be resolved timely. Such uncertainty exists if the supplier recognizes specification problems through its tacit production knowledge but does not have design and engineering capabilities to come up with a new set of specifications, or alternatively, if the supplier is unsure whether the OEM has the capability to re-engineer alternative specifications timely. Note that the OEM could reduce such supplier's uncertainty by sharing information about its engineering capabilities and/or collaborate more closely with the supplier.

Once the supplier recognizes the OEM's potential specification problems, it may either voluntarily point out the problem (and/or suggest improvements) to the OEM, or remain silent and simply execute the OEM's order to the print. In the former case, the OEM firm may or may not be able to fix the specification problems (or implement the suggested improvements) in time to satisfy current period market demand. If not, the OEM firm will cancel the order for the current period but will place another order for the next period. We assume that if the specification problems cannot be fixed in the current period, it will be fixed in the next period. Demand that cannot be satisfied in the current period is lost. We will consider a two-period model: such as model will best suit for fast-changing industries, and will serve as a starting point for other stable industries where the OEM firm has plenty of time to iron out winkles in its product specifications.

If the supplier remains silent and completes the OEM's order to the print, the OEM will subsequently recognize the specification problems through lower than expected market demand in the current period. We assume that the supplier's production production process does not introduce other defects and therefore the OEM cannot fault the supplier for the lower than expected product performance. Implicitly, this means that even though the supplier's production process may interact subtly with the OEM's specifications, the performance problem can be solely attributed to the OEM's specification flaws as opposed to production errors. This is quite different from the case where the root cause of the problem cannot be clearly disentangled (Kim and Tomlin, 2013).

Should the OEM discover its specification flaws through lower market demand, the OEM may also suffer from reputation damage from its second period demand. Such reputation damage is often referred to as product harm crises if the specification flaw is serious (Heerde et al., 2007). In such a case, even if the OEM is able to correct its specification flaws by the beginning of the second period, demand will still be lower due to customers' bad experience with its first period product offerings.

3.4 The Model Setup

Having sketched the general aspects of the model, we are now in a position to set up the model formally. In the following we refer the OEM as he and the supplier as she.

Let $t = 1, 2$ denote the time period. At the beginning of the first period, the OEM determines his product specification. We assume that with probability θ there are flaws in the specification. Note that the OEM is aware of this probability but is not sure about the existence of flaws. The OEM then offers a contract (q_1, T) to the supplier on a "take it or leave it" basis, with q_1 as the ordering quantity for the first period, and T as the cancellation payment if the OEM cancels the order in the first period.

The supplier receives the order and produces the products. If the OEM's specification is correct, then the supplier delivers to the OEM. After the OEM receives delivery from the supplier, he satisfies a market demand d_1 as much as possible. Unsatisfied demand is lost (with no penalty cost), but excess inventory can be carried over to the next period (with a holding cost h per unit). Note that the sale amount should be the minimum of d_1 and q_1. The OEM carries over $(q_1 - d_1)^+$ to the second period. The successful delivery of the products in the first period will bring a positive spill over effect. The market demand in the second period will be $d_2 + \beta(d_1 \wedge q_1)$, where d_2 is a base market demand in the second period, β is a positive spillover effect that is only effective on sales. At the beginning of the second period, the OEM will update his forecast on the demand in the second period accordingly, and will hence order $[d_2 + \beta(d_1 \wedge q_1) - (q_1 - d_1)^+]^+$. This scenario is presented as Case 0 in Figure 3.1.

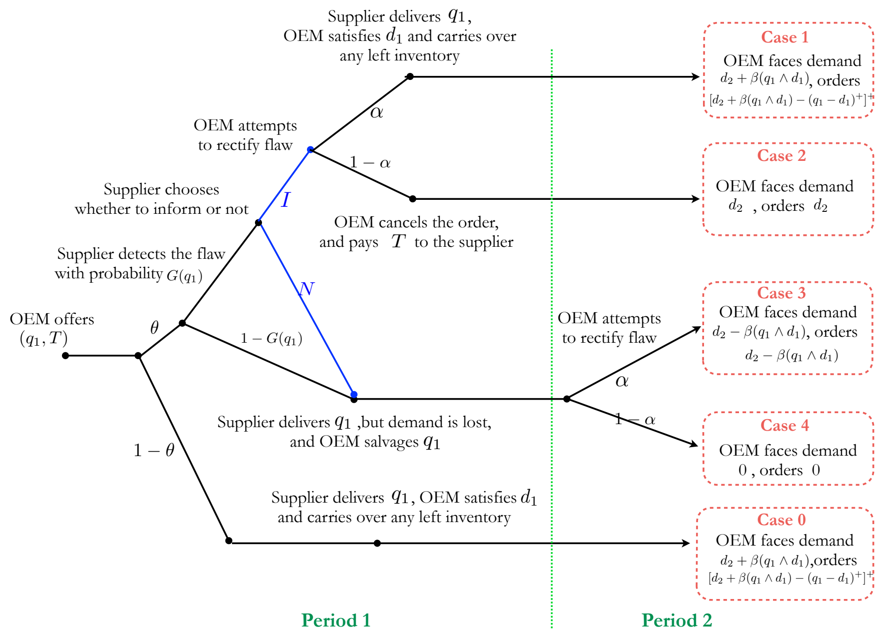

Figure 3.1: Sequence of events and cases

If the OEM's specification has flaws, the supplier may detect the flaws. We assume that the more the OEM orders, the better chance the supplier may detect the flaws. One explanation is that if the OEM orders more, the supplier may conduct a more refined and thorough preparation, and therefore has more chance to detect the flaw even before any production. Alternatively, imagine the supplier runs inspection on each individual unit as it produces. Assuming the passing rate for each individual unit is γ, then if the supplier has produced q_1 units, she can detect the flaw as long as one of these q_1 units failed the inspection. Therefore, the probability for the supplier to detect the flaw after producing q_1 unit is $1 - \gamma^{q_1}$. This function is concave and increasing in q_1. As q_1 increases, the probability for the supplier to detect the flaws increases. In addition, the marginal increase in the detecting probability decreases as q_1 increases. After certain value of q_1, the marginal increase is fairly small, and the detecting probability is very close to 1. We give an example of $1 - \gamma^{q_1}$ with $\gamma = 0.95$ in Figure 3.2(a), where the detecting probability is larger or equal to 0.98 when q_1 is at least 80. In fact, for any value of γ and an arbitrarily small value ϵ, there exists a threshold of q_1, such that for any q_1 larger than the threshold, the detecting probability is larger than $1-\epsilon$. In other words, after producing the amount of the threshold, the marginal increase in detecting probability if the supplier produces even more is negligible. In practice, the value of γ can vary depending on the property of the products and the OEM's specifications, and the threshold will vary accordingly. For the purpose of analysis, we normalize this threshold as 1, and use a piece wise function $G(q_1)$ (Figure 3.2(b)) to approximate the probability of the supplier to detect the flaws when she produces q_1, where

$$G(q_1) = \begin{cases} q_1, & q_1 \in [0, 1] \\ 1, & q_1 \in (1, +\infty), \end{cases}$$

so that when q_1 is less than 1, the detecting probability increases in q_1 linearly. After that, the detecting probability is 1. Producing more will not result in a higher detecting probability.

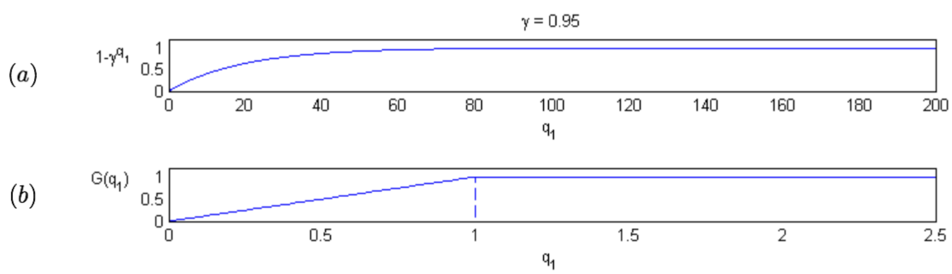

Figure 3.2: Probability of detecting flaws

We assume that $d_1 < 1$, so that producing the market demand in the first period will not be enough for the supplier to detect the flaws with certainty. We make this assumption to better examine under what condition the OEM will order more than the demand in the first period just for the sake of increasing the supplier's detect probability.

If the supplier detects any flaws, she may choose either to inform the OEM or remain silent. The supplier faces a dilemma because she is unsure about whether an immediate resolution is available to correct the specification flaws. Even the OEM himself is not sure about this, because he was not aware of the specification flaws and thus can not guarantee the problem can be resolved in time. We assume the OEM has communicated thoroughly with the supplier. As results, they have a common knowledge that with probability α the specification flaws can be corrected without significant delay (so that production can be completed in time for the current period's demand), and with probability $1-\alpha$ the specification flaw cannot be corrected until by the beginning of the next period. For the rest of the paper, we refer to α as the OEM's capability to correct any potential specification flaws. One could also alternatively interpret α as the supplier's engineering and process capability, but for exposition ease we use the former interpretation throughout the paper, with the understanding that all results developed in the paper could be adapted to the latter interpretation.

If the supplier detects the specification flaws and points out the specification issues to the OEM, there are two possible scenarios. One scenario is that the OEM immediately corrects the specification flaws, then the supplier produces the products and delivers to the OEM. The OEM then satisfies the demand d_1 and carries over any left over inventory. In the second period, the demand will be positively affected, the OEM will therefore order $[d_2 + \beta(d_1 \wedge q_1) - (q_1 - d_1)^+]^+$. This scenario is presented as Case 1 in Figure 3.1. Another scenario is that the OEM fails to find an immediate resolution. The OEM then cancels the first period's order and pays a fixed compensation fee T to the supplier. In this case, we assume that the OEM will find a resolution with certainty until the beginning of the second period. Because the first period's order is canceled, the demand in the second period will just be the base market demand d_2. The OEM will therefore order d_2 in the second period. This scenario is presented as Case 2 in Figure 3.1.

In contrast, if the supplier detects the specification flaws but remains silent, she finishes production with the OEM's original specifications. In this case, we assume the demand in the first period is lost. Customers return the defected products. The OEM discovers the specification flaws through the lost demand and salvages the products at s per unit. Until the beginning of the second period, the OEM has a chance of α again to find a resolution. If he does correct the flaws, there will be a market demand in the second period. The demand in the second period, however, will be negatively affected due to a reputation damage caused by the defected products from the first period. We assume the negative spillover effect is the same magnitude as the positive spillover effect. Therefore we can write the demand in the second period as $d_2 - \beta(d_1 \wedge q_1)$. We assume $d_2 \geq \beta d_1$, i.e., the base market demand in the second period is big enough so that the reputation damage from the first period will at most result in zero demand in the second period. Because there is no inventory carried over from the first period, the OEM will hence order $d_2 - \beta(d_1 \wedge q_1)$. This scenario is presented as Case 3 in Figure 3.1. On the other hand, if the OEM is not able to correct the flaws until the beginning of the second period, the demand in the second period will be lost. The OEM will update his demand forecast accordingly and hence order nothing for the second period. This scenario is presented as Case 4 in Figure 3.1.

On the other hand, it is possible that there are specification flaws but the supplier does not detect any. In this case, the supplier delivers the defected products. The first period demand is lost. The OEM salvages those defected products. In the second period, the OEM has a chance of α to correct the specification flaws. If he succeed, he will satisfy a demand $d_2 - \beta(q_1 \wedge d_1)$. Otherwise the second period demand will also be lost. Note that the OEM cannot differentiate this scenario from the scenario where the supplier detects the flaws but choose to remain silent, nor can the OEM blame the supplier for the specification flaws. This is precisely why we need to explore and disentangle the incentives for the supplier to inform the OEM should there be any flaws. Only having that clearly stated, we can then take the next step to discuss from the perspective of the OEM under what conditions it is beneficial for himself to design the contract so that the supplier will voluntarily inform should there be any flaws.

In the best scenario for the OEM, i.e., there is no flaw in the specification, the maximum total demand the OEM satisfies in two periods is $d_1 + d_2 + \beta d_1$. We assume that if the OEM orders this amount at the beginning of the first period, then the supplier can detect the flaws for sure. We write this assumption as $d_1 + d_2 + \beta d_1 \geq 1$ given the structure of the detecting probability $G(q_1)$.

The OEM pays the supplier a unit wholesale price of w for all units delivered, sells the product at a unit price of p, and the supplier incurs a unit production cost of c. We assume these prices are the same for two periods. In practice, these prices may be different for different periods. In addition, the supplier's unit production cost may change as the OEM corrects the specification flaws. To focus on the supplier's incentive to suggest potential specification flaws and the OEM's optimal strategy in designing the contract, we ignore the possible changes of wholesale price, selling price, and production cost. In addition, we assume $p > w > c > h \geq s$, and $w - c - h > 0$. Another thing to point out is that all of the parameters in our model is assumed to be common knowledge.

3.5 Problem Formulation

We first describe the supplier's decision problem, and then the OEM's decision problem.

3.5.1 The supplier's decision problem

The supplier needs to make a decision only when there are flaws in the specification and when she has detected the flaws. The supplier needs to decide whether to inform the OEM about the flaws. Let $a \in \{I, N\}$ denote the supplier's action of either informing or not informing the OEM. If the supplier informs the OEM, her expected profit is

$$\begin{aligned} S_I =& \alpha\{(w-c)q_1 + (w-c)[d_2 + \beta(q_1 \wedge d_1) - (q_1 - d_1)^+]^+\} \\ & + (1-\alpha)[T + (w-c)d_2]. \end{aligned}$$

We assume the supplier can fully salvage the production cost of those units that have already been produced using the faulted specification. The supplier has many ways to salvage those units which are still on the production line. She can use them for production with the corrected specification. There will be extra cost, but the extra cost can be negligible if the modification is minor. She can also sell those units to a secondary market or to other OEMs who may have different standards. She can at least disassemble those units and salvage them as raw materials.

Under this assumption, with probability α, the OEM resolves the specification flaws immediately. The supplier fully salvages the units that have already been produced. The supplier then produces with the correct specification. She incurs a production cost cq_1 for the first period order, and receives wq_1 from the OEM. The OEM satisfies a demand d_1, and carries over $(q_1 - d_1)^+$ amount of inventory to the second period. At the beginning of the second period, the OEM updates his demand forecast for the second period as $d_2 + \beta(q_1 \wedge d_1)$ and orders $[d_2 + \beta(q_1 \wedge d_1) - (q_1 - d_1)^+]^+$. Therefore the supplier receives $(w-c)[d_2 + \beta(q_1 \wedge d_1) - (q_1 - d_1)^+]^+$ in the second period. On the other hand, with probability $1-\alpha$, the OEM cannot revolve the specification flaws in time. He cancels the first period order, and pays a cancellation fee T to the supplier. The supplier fully salvages the units that have already been produced. In the second period, the OEM corrects the specification flaws, and update his demand forecast for the second period as d_2. Since there is no inventory carried over from the first period, the OEM then orders d_2 from the supplier. Therefore, the supplier receives T in the first period, and $(w-c)d_2$ in the second period.

In contrast, if the supplier does not inform the OEM, her expected profit is

$$S_N = (w-c)q_1 + \alpha(w-c)[d_2 - \beta(q_1 \wedge d_1)],$$

where the supplier carries out production as usual in the first period (the OEM cannot blame the supplier for his specification flaws), but in the second period the supplier carries out production only if the specification flaws can be rectified at the beginning of the second period. Note that the OEM's second period order $d_2 - \beta(q_1 \wedge d_1)$ is based on an updated demand forecast due to reputation damage caused by first period specification flaws.

Given the OEM's contract (q_1, T), the supplier's problem is to decide $a \in \{I, N\}$ to maximize $\{S_I, S_N\}$. Note that in the above formulation, we implicitly assume that the supplier knows the OEM's second period order at the beginning of the first period. This is reasonable because all the parameters in our model are assumed to be common knowledge. If, on the other hand, the supplier has to estimate the OEM's second period order in advance, then we need to replace the order quantity with supplier's expectation on the OEM's order quantity. If the supplier is risk neutral and her estimation is unbiased, then such as change will not affect our analysis. If, however, the supplier is risk averse or has biased estimation, then a different model that incorporates risk aversion or asymmetric information would be more appropriate.

3.5.2 The OEM's decision problem

The OEM needs to design the contract in the first period (q_1, T) to maximize his total profit over two periods, where q_1 is the order quantity for the first period, and T is the cancellation payment if he cancels the order in the first period. Note that the OEM does need to decide the order quantity in the second period as well. This decision, however, is straightforward. Once the OEM has the updated demand information, he just orders up to the market demand in the second period, as we stated in the previous sections. Therefore we focus on the OEM's decision problem in the first period.

We first write down the OEM's profit in each case (Figure 3.1). We use V^i to denote the OEM's profit in Case i, $i = 0, 1, 2, 3, 4$.

$$
\begin{aligned}
V^0 = V^1 &= -wq_1 + p(q_1 \wedge d_1) - h(q_1 - d_1)^+ \\
&\quad + \{-w[d_2 + \beta(q_1 \wedge d_1) - (q1 - d_1)^+]^+ + p[d_2 + \beta(q_1 \wedge d_1)]\}, \\
V^2 &= -T + (p - w)d_2, \\
V^3 &= -wq_1 + sq_1 + (p - w)[d_2 - \beta(q_1 \wedge d_1)], \\
V^4 &= -wq_1 + sq_1.
\end{aligned}
$$

Note that from the OEM's perspective, Case 0 where there is no specification flaw, is equivalent as Case 1 where there are specification flaws, the supplier detects and informs the OEM, and the OEM rectifies the flaws instantly. In Case 1 and 2, the supplier detects the flaws after a certain amount of production, she informs the OEM about the flaws and fully salvages the production cost of those units. In Case 3 and Case 4, the defected products have been delivered to the OEM and sold to the customers. It is the OEM who salvages the defected products. We assume it is much harder for the OEM to retrieve the cost. The salvage value is at s, lower than the supplier's production cost c.

The OEM's total expected profit in two periods will depend on the supplier's decision.

If the supplier decides to NOT inform the OEM even if she detects the flaw, the OEM's expected profit is

$$V_N(q_1, T) = (1 - \theta)V^0 + \theta\{\alpha V^3 + (1 - \alpha)V^4\}$$

If there is no flaw in the OEM's specification, then it is Case 0. If there is flaw in the OEM's specification, because the supplier has decided NOT to inform the OEM even if she detects the flaw, it does not matter whether or not the supplier detects the flaw. The supplier carries out the production according to the OEM's original specification and delivers to the OEM. The demand in the first period is lost. The OEM realizes the flaws through the lost demand and salvages the defected products. At the beginning of the second period, the OEM has a probability of α to rectify the flaw.

- If the OEM rectifies the flaws, the demand in the second period will be $d_2 - \beta(q_1 \wedge d_1)$, and the OEM will order $d_2 - \beta(q_1 \wedge d_1)$. (Case 3).

- If the OEM is not able to rectify the flaws, the demand in the second period will be lost, and the OEM will order nothing (Case 4).

In contrast, if the supplier decides to inform the OEM if she detects the flaw, the OEM's expected profit is

$$V_I(q_1, T) = (1-\theta)V^0 + \theta\{G(q_1)[\alpha V^1 + (1-\alpha)V^2] + [1-G(q_1)][\alpha V^3 + (1-\alpha)V^4]\}.$$

If there is no flaw in the OEM's specification, then it is Case 0. If there is flaw in the OEM's specification, then the supplier has a probability of $G(q_1)$ to detect the flaw.

- If the supplier detects the flaw, she then informs the OEM. The OEM has a probability of α to rectify the flaws in a timely manner.
  - If the OEM rectifies the flaw in time, the demand in the first period will not be affected, the OEM carries over inventory (if any) to the second period. The demand in the second period will be $d_2 + \beta(q_1 \wedge d_1)$, and the OEM will order $[d_2 + \beta(q_1 \wedge d_1) - (q_1 - d_1)^+]^+$. (Case 1).
  - Otherwise, the OEM fails to resolve the flaws in time, he cancels the order. At the beginning of the second period, the OEM rectifies the flaw. The demand in the second period will be d_2, and the OEM will order d_2. (Case 2).
- If the supplier does not detect the flaw, then the demand in the first period is lost. The OEM realizes the flaw through the lost demand and salvages the defected products. At the beginning of the second period, the OEM again has a probability of α to rectify the flaw.
  - If the OEM rectifies the flaw, the demand in the second period will be $d_2 - \beta(q_1 \wedge d_1)$, and the OEM will order $d_2 - \beta(q_1 \wedge d_1)$. (Case 3).
  - Otherwise, the OEM is not able to rectify the flaw, the demand in the second period will be lost again, and the OEM will order nothing (Case 4).

The OEM can design the contract (q_1, T) to direct the supplier to inform or not inform. Therefore we solve the OEM's decision problems as follows. We will characterize the optimal solutions for the OEM to maximize his expected total profit given that he directs the supplier to not inform or inform, respectively (section 2.4.1 and section 2.4.2). Then we will compare the OEM's optimal expected profit in these two cases (section 2.4.3). We will show that it is strictly better for the OEM to design (q_1, T) so that the supplier will inform should she detect any flaw. Finally, we will conduct sensitivity analysis on the optimal solutions to gain some managerial insights.

3.6 Analysis

3.6.1 Optimization problem N

In this section, we examine the optimal solutions of the OEM on the condition that the OEM directs the supplier NOT to inform when she detects flaws. The OEM's decision problem can be written as

$$\begin{aligned} \max_{q_1 \geq 0, T} & V_N(q_1) = V_N(q_1, T) \\ s.t. & S_I \leq S_N \\ & S_N \geq 0 \\ & T \geq 0, \end{aligned}$$

where $V_N(q_1)$ is the OEM's expected total profit if he orders q_1 in the first period given that the supplier does NOT inform even if she detects any flaw, $S_I \leq S_N$ is the incentive constraint for the supplier to NOT inform, $S_N \geq 0$ is the individual rationality constraint for the supplier, and $T \geq 0$ is the nonnegative constraint for the cancellation payment. Because we have assumed that $d_2 \geq \beta d_1$, the individual rationality constraint always holds.

Note that the objective function in optimization problem N does not depend on T. The OEM does not want the supplier to inform, thus he will never try to rectify the flaws, and thus will never cancel the order and pay the cancellation payment T. Cancellation payment serves as an empty threat in the contract. The only constraint on the cancellation payment is that it needs to be small enough so that the supplier will not have incentive to inform.

We can summarize the optimal solutions for the OEM in optimization problem N as follows.

Theorem 3.6.1 *The optimal solutions for the OEM in optimization problem N are as follows.*

- *When* $d_2 > \dfrac{1-\alpha(1+2\beta)}{1-\alpha}d_1$, $q_1^* = \alpha(1+2\beta)d_1+(1-\alpha)d_2$, *and* $T^* = 0$.
- *When* $d_2 \leq \dfrac{1-\alpha(1+2\beta)}{1-\alpha}d_1$, *the optimal solutions will depend on the value of* θ.
  - *When* $\theta \leq \dfrac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, $q_1^* = d_1$, *and* T^* *can be any value satisfying the incentive constraint. We set* $T^* = 0$.
  - *When* $\theta > \dfrac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, $q_1^* = \dfrac{1-\alpha}{1-\alpha-2\alpha\beta}d_2$, *and* $T^* = 0$.

We can demonstrate Theorem 3.6.1 on the (d_1, d_2) map, as shown in Figure 3.3 (a). Note that because we have assumed $d_2 \geq \beta d_1$, if $\dfrac{1-\alpha(1+2\beta)}{1-\alpha} \leq \beta$, theorem 3.6.1 degenerates to only the first case where $q_1^* = \alpha(1+2\beta)d_1 + (1-\alpha)d_2$ and $T^* = 0$, as shown in Figure 3.3 (b).

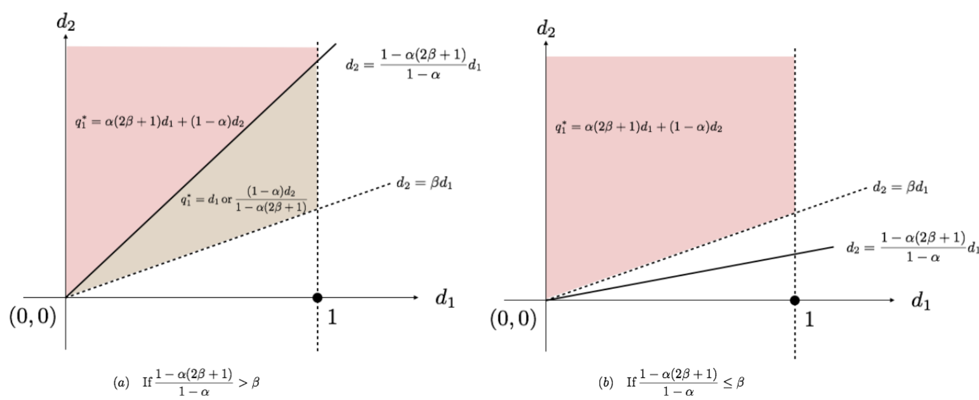

Figure 3.3: The optimal solutions for optimization problem N

3.6.2 Optimization problem I

In this section, we explore the optimal solutions of the OEM on the condition that the OEM directs the supplier to inform when she detects flaws. The

OEM's decision problem can be written as

$$\begin{aligned}\max_{q_1 \geq 0, T} & V_I(q_1, T) \\ s.t. & S_I \geq S_N \\ & S_I \geq 0 \\ & T \geq 0,\end{aligned}$$

where $V_I(q_1, T)$ is the OEM's expected total profit if he orders q_1 in the first period and sets T as the cancellation payment on the condition that the supplier chooses to inform should there be flaws, $S_I \geq S_N$ is the incentive constraint for the supplier to inform, $S_I \geq 0$ is the individual rationality constraint, and $T \geq 0$ is the nonnegative constraint for the cancellation payment.

One observation we make here is that if the incentive constraint holds, then the individual rationality constraint also holds. Specifically, because we have assumed $d_2 \geq \beta d_1$, then $S_N \geq 0$, therefore if $S_I \geq S_N$, then $S_I \geq 0$. Thus the individual rationality constraint is redundant. Optimization problem I is equivalent as

$$\begin{aligned}\max_{q_1, T} & V_I(q_1, T) \\ s.t. & S_I \geq S_N \\ & T \geq 0.\end{aligned}$$

There are two piece-wise functions in the objective function, one is

$$[d_2+\beta(q_1 \wedge d_1)-(q_1-d_1)^+]^+ = \begin{cases} d_2 + \beta q_1, & q_1 \leq d_1, \\ (\beta+1)d_1 + d_2 - q_1, & d_1 < q_1 \leq (\beta+1)d_1 + d_2, \\ 0, & q_1 > (\beta+1)d_1 + d_2, \end{cases}$$

the other is

$$G(q_1) = \begin{cases} q_1, & q_1 \in [0,1], \\ 1, & q_1 \in (1, +\infty). \end{cases}$$

We therefore divide the region of q_1 into four subregions: $[0, d_1]$, $(d_1, 1]$, $(1, (\beta+1)d_1 + d_2]$, and $((\beta+1)d_1 + d_2, +\infty)$, and examine the local optimal solutions in these four subregions receptively. After that, we will evaluate the local optimal solutions in these subregions to get the global optimal solutions for the OEM in optimization problem I.

Optimal Solutions in the subregion $[0, d_1]$

We first study the local optimal solutions for the OEM in optimization problem I when $q_1 \in [0, d_1)$.

Proposition 3.6.2 *In the subregion $q_1 \in [0, d_1]$, the optimal solutions for the OEM in optimization problem I are as follows.*

- *When $d_2 > \frac{1-\alpha(1+2\beta)}{1-\alpha} d_1$, $T^* = 0$.*
  - *When (d_1, d_2) is above Line 1, $q_1^* = d_1$.*
  - *Otherwise $q_1^* = 0$.*
- *When $d_2 \le \frac{1-\alpha(1+2\beta)}{1-\alpha} d_1$:*
  - *When (d_1, d_2) is above Line 2, $q_1^* = d_1$, and $T^* = \frac{w-c}{1-\alpha}[(1-\alpha-2\alpha\beta)d_1 - (1-\alpha)d_2]$.*
  - *Otherwise $q_1^* = 0$, and $T^* = 0$.*

The equations for the Lines are

$$\begin{aligned}
\textbf{\textit{Line 1:}} \quad d_2 =& -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{w-s}{(1-\alpha)(p-w)}]d_1 \\
&+ \frac{\theta(w-s) - (p-w)[(1+\beta)(1-\theta) - \alpha\beta\theta]}{\theta(1-\alpha)(p-w)}, \\
\textbf{\textit{Line 2:}} \quad d_2 =& -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{c-s}{(1-\alpha)(p-c)}]d_1 \\
&+ \frac{\theta(w-s) - (p-w)[(1+\beta)(1-\theta) - \alpha\beta\theta]}{\theta(1-\alpha)(p-c)}.
\end{aligned}$$

We can demonstrate Proposition 3.6.2 on (d_1, d_2) map in Figure 3.4(a). Note that if $\frac{1-\alpha(1+2\beta)}{1-\alpha} \le \beta$, Proposition 3.6.2 degenerates to only the first part, as shown in Figure 3.4(b). Another thing to note is that when $\theta \le \frac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, both Line 1 and Line 2 has nonpositive intercepts. In that case, the optimal solutions for the OEM in optimization problem I in the subregion $[0, d_1]$ are $q_1^* = d_1$ and $T^* = 0$.

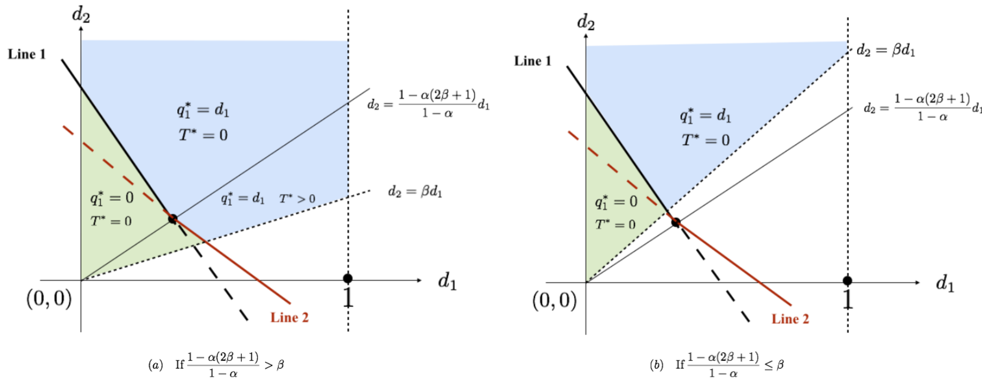

Figure 3.4: The optimal solutions for optimization problem I when $q_1 \in [0, d_1]$

Optimal Solutions in the subregion $(d_1, 1]$

We then explore the optimal solutions for the OEM in optimization problem I when $q_1 \in (d_1, 1]$.

Proposition 3.6.3 *In the subregion $(d_1, 1]$, the optimal solutions for the OEM in optimization problem I are as follows.*

- *When $d_2 \le \dfrac{1-\alpha(1+2\beta)}{1-\alpha}d_1$:*
  - *When (d_1, d_2) is above Line 6, $q_1^* = 1$, and $T^* = \dfrac{w-c}{1-\alpha}[1-\alpha(1+2\beta)d_1 - (1-\alpha)d_2]$.*
  - *Otherwise, $q_1^* = d_1$, and $T^* = \dfrac{w-c}{1-\alpha}[(1-\alpha-2\alpha\beta)d_1 - (1-\alpha)d_2]$.*
- *When $d_2 > -\dfrac{\alpha(1+2\beta)}{1-\alpha}d_1 + \dfrac{1}{1-\alpha}$, then $T^* = 0$.*
  - *When it is above Line 3, $q_1^* = 1$.*
  - *Otherwise, $q_1^* = d_1$.*
- *When $d_2 \le -\dfrac{\alpha(1+2\beta)}{1-\alpha}d_1 + \dfrac{1}{1-\alpha}$:*
  - *When (d_1, d_2) is above both Line 4 and Line 5, $q_1^* = 1$ and $T^* = \dfrac{w-c}{1-\alpha}[1-\alpha(1+2\beta)d_1 - (1-\alpha)d_2]$.*

- *When* (d_1, d_2) *is below both Line 4 and Line 5,* $q_1^* = d_1$ *and* $T^* = 0$ *.*
- *When* (d_1, d_2) *is above Line 4 and below Line 5,* $q_1^* = \alpha(1 + 2\beta)d_1 + (1-\alpha)d_2$ *and* $T^* = 0$*.*
- *When* (d_1, d_2) *is below Line 4 and above Line 5,* q_1^* *could be* d_1 *or* 1*. If* $q_1^* = d_1$*, then* $T^* = 0$*. If* $q_1^* = 1$*, then* $T^* = \frac{w-c}{1-\alpha}[1 - \alpha(1+2\beta)d_1 - (1-\alpha)d_2]$*.*

The equations of the lines are

$$\begin{aligned}
\textbf{\textit{Line 6:}} \quad d_2 = & -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{c-s}{(1-\alpha)(p-c)}]d_1 \\
& + \frac{\theta(w-c) + h(1-\theta+\alpha\theta)}{\theta(1-\alpha)(p-c)}. \\
\textbf{\textit{Line 3:}} \quad d_2 = & -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{w-s}{(1-\alpha)(p-w)}]d_1 \\
& + \frac{h(1-\theta+\alpha\theta)}{\theta(1-\alpha)(p-w)}. \\
\textbf{\textit{Line 4:}} \quad d_2 = & -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{w-s}{(1-\alpha)(p-\alpha h-s)}]d_1 \\
& + \frac{\theta(w-s) + h(1-\theta)}{\theta(1-\alpha)(p-\alpha h-s)}. \\
\textbf{\textit{Line 5:}} \quad d_2 = & -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{\alpha h}{(1-\alpha)(p-\alpha h-s)}]d_1 \\
& + \frac{\theta(w-c) + h(1-\theta+\theta\alpha)}{\theta(1-\alpha)(p-\alpha h-s)}.
\end{aligned}$$

Depending on the positions of these Lines, the division of (d_1, d_2) map can be of various forms. One representative case is shown in Figure 3.5(a). Note that If $\frac{1-\alpha(1+2\beta)}{1-\alpha} \leq \beta$, the first case in Proposition 3.6.3 does not exist, as presented in Figure 3.5(b).

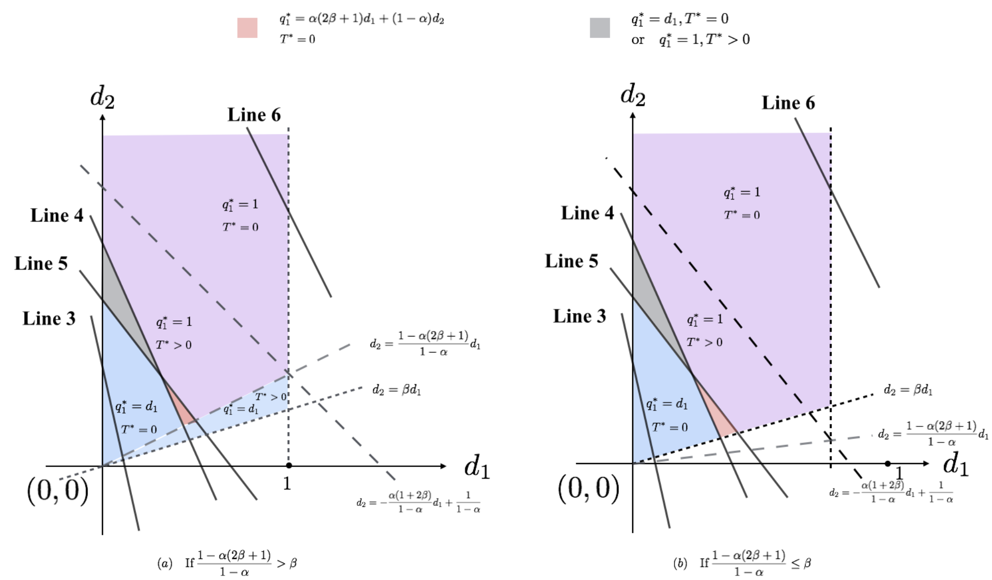

Figure 3.5: The optimal solutions for optimization problem I when $q_1 \in (1, d_1]$

Optimal Solutions in the subregion $(1, (\beta+1)d_1 + d_2]$

Next, we examine the optimal solutions for optimization problem I when $q_1 \in (1, (\beta+1)d_1 + d_2]$.

Proposition 3.6.4 *In the subregion $q_1 \in (1, (\beta+1)d_1 + d_2]$, the optimal q_1 for the OEM in optimization problem I is $q_1^* = 1$.*

- *When $d_2 \leq -\dfrac{\alpha(1+2\beta)}{1-\alpha}d_1 + \dfrac{1}{1-\alpha}$, then $T^* = \dfrac{w-c}{1-\alpha}[1-\alpha(1+2\beta)d_1 - (1-\alpha)d_2]$.*
- *Otherwise, $T^* = 0$.*

Optimal Solutions in the subregion $((\beta+1)d_1 + d_2, +\infty)$

Finally we study the optimal solutions for optimization problem I when $q_1 \in ((\beta+1)d_1 + d_2, +\infty)$. If there is no specification flaw in the OEM's design, or there is flaw but the supplier detects, informs, and the OEM corrects the flaw in time, then the total demand for two periods would be $(\beta+1)d_1 + d_2$. Thus $(\beta+1)d_1 + d_2$ is the maximum total demand for the

OEM in two periods. Intuitively, the OEM has no incentive to order more than $(\beta+1)d_1+d_2$ in the first period.

Proposition 3.6.5 *In the subregion $q_1 \in ((\beta+1)d_1+d_2, +\infty)$, the optimal solutions for optimization problem I are $q_1^* = (\beta+1)d_1+d_2$, and* $T^* = (w-c)(\beta+1)d_1 + \dfrac{\alpha}{1-\alpha}(w-c)(d_2-\beta d_1)$.

Optimal Solutions for optimization problem I

Now we are ready to state the optimal solutions for the OEM in optimization problem I. The possible optimal values of q_1^* are 0, d_1, $\alpha(1+2\beta)d_1+(1-\alpha)d_2$, and 1.

Theorem 3.6.6 *When $d_2 \le \dfrac{1-\alpha(1+2\beta)}{1-\alpha}d_1$, the optimal solutions for optimization problem I are as follows.*

(a) If $\theta \le \dfrac{(p-w)(1+\beta)+h}{(p-w)(1+\beta+\alpha\beta)+c-s+(1-\alpha)h}$, then

- *when (d_1,d_2) is above Line 6, $q_1^*=1$ and* $T^* = \dfrac{w-c}{1-\alpha}[1-\alpha(1+2\beta)d_1-(1-\alpha)d_2]$.
- *When (d_1,d_2) is below Line 2, $q_1^*=0$ and $T^*=0$.*
- *When (d_1,d_2) is above Line 2 and below Line 6, $q_1^*=d_1$ and* $T^* = \dfrac{w-c}{1-\alpha}[(1-\alpha-2\alpha\beta)d_1-(1-\alpha)d_2]$.

(b) If $\theta > \dfrac{(p-w)(1+\beta)+h}{(p-w)(1+\beta+\alpha\beta)+c-s+(1-\alpha)h}$, then

- *When (d_1,d_2) is above Line 2, $q_1^*=1$ and* $T^* = \dfrac{w-c}{1-\alpha}[1-\alpha(1+2\beta)d_1-(1-\alpha)d_2]$.
- *When (d_1,d_2) is below Line 6, $q_1^*=0$ and $T^*=0$.*
- *When (d_1,d_2) is above Line 6 and below Line 2, in the area that is above Line 7, $q_1^*=1$ and $T^*=0$. In the part that is below Line 7, $q_1^*=0$ and $T^*=0$.*

Theorem 3.6.6 is presented in Figure 3.6. Note that Line 2 can have negative intercept. Both Line 2 and Line 6 can have intercepts that is big enough so that they do not contribute to divisions.

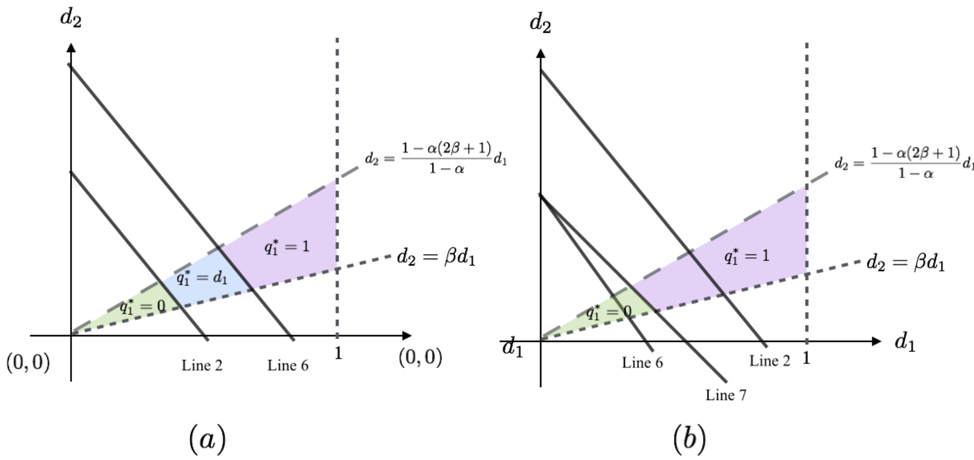

Figure 3.6: The global optimal solutions for the OEM in optimization problem I when $d_2 \leq \frac{1-\alpha(1+2\beta)}{1-\alpha}d_1$.

Theorem 3.6.7 *When $d_2 > \frac{1-\alpha(1+2\beta)}{1-\alpha}d_1$, the optimal solutions for the OEM in optimization problem I is as follows.*

(i) If $\theta \leq \frac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, then the optimal q_1 for optimization problem I is restricted on $[d_1, 1]$. The possible optimal value for q_1 is d_1, 1, and $\alpha(1+2\beta)d_1 + (1-\alpha)d_2$.

(ii) If $\theta > \frac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, then the optimal q_1 for optimization problem I is restricted on $[0, 1]$. The possible optimal value for q_1 is 0, d_1, 1, and $\alpha(1+2\beta)d_1 + (1-\alpha)d_2$.

- *When $q_1^* = 0$ or $\alpha(1+2\beta)d_1 + (1-\alpha)d_2$, $T^* = 0$.*
- *When $q_1^* = d_1$, $T^* = \max\{\frac{w-c}{1-\alpha}[(1-\alpha-2\alpha\beta)d_1 - (1-\alpha)d_2], 0\}$.*
- *When $q_1^* = 1$, $T^* = \max\{\frac{w-c}{1-\alpha}[1-\alpha(1+2\beta)d_1 - (1-\alpha)d_2], 0\}$.*

We shown a representative case of the optimal solutions for the OEM in optimization problem I in Figure 3.7. It is worth pointing out that under Line 1, the local optimal q_1 in the subregion $[0, d_1]$ is $q_1^* = 0$, and the local optimal q_1 in the subregion $(d_1, 1]$ is $q_1^* = d_1$, then the global optimal q_1 is $q_1^* = 0$. This is implied by the continuity of the optimization problem. In the next section, we will further discuss the division of (d_1, d_2) map.

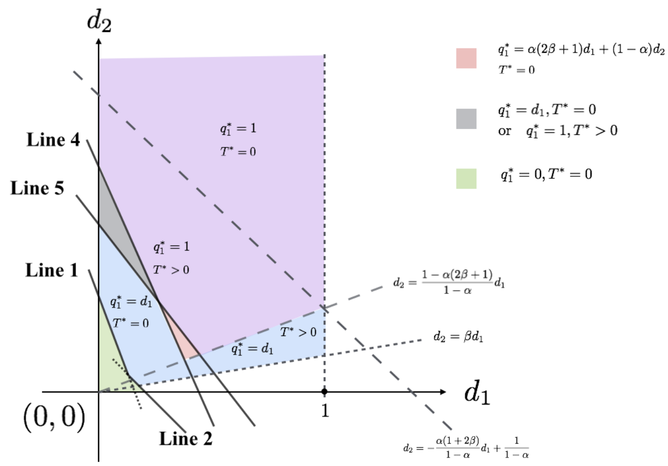

Figure 3.7: The global optimal solutions for optimization problem I.

3.6.3 Overall optimal solutions for the OEM

General Structure

Theorem 3.6.8 *The optimal objective value of optimization problem I is strictly bigger than the optimal objective value of optimization problem N.*

This is a strong conclusion. This theorem implies that it is always better for the OEM to design the contract so that the supplier voluntarily informs. The optimal solutions for the OEM are the same as the optimal solutions for the OEM in optimization problem I.

Having this theorem, next we focus on providing guidelines for the OEM to design the contract. In the following, we conduct sensitivity analysis on the parameters θ, α, and β, to gain some insights from previous results.

Sensitivity Analysis on θ

First, we want to isolate the influence of θ on the optimal solutions (q_1^*, T^*) of the OEM. We fix other parameters at reasonable values and explore how the optimal solutions change as θ changes.

Figure 3.8 presents the optimal solutions of the OEM in a case when the demands in two periods are comparable. When θ is small, meaning that there is a small chance the specification has flaws, the OEM should order only d_1. Because if the OEM orders more than d_1 in the first period, he incurs holding cost on the inventory. At this point, the supplier informs voluntarily without the OEM providing cancellation payment. However, as θ increases, once θ reaches a certain level, the benefits of ordering more which is the increased detecting probability exceeds the cost of ordering more which is the holding cost, and the OEM should order 1. At the same time, because the OEM orders a large amount in the first period, if the supplier informs, she might loose the first period order and only gets the second period demand, which is 0.5. Therefore, the supplier prefers to remain silent about the flaws unless the OEM provides a positive cancellation payment. Because if the supplier remains silent, she gets the order of 1 in total in any case.

Figure 3.9 presents the optimal solutions of the OEM in a case when the second period's demand is substantially larger than the fist period's demand, and the spill over effect is strong. In this case, the OEM does not want to risk losing the second period's demand. Therefore as long as there is a tiny probability that the specification has flaws, the OEM will order 1 in the first period so that the supplier can detect the flaw with certainty and informs the OEM. On the other hand, because the second period's demand is significantly larger than the first period's demand, the supplier wants to inform out of her own interest. Therefore the OEM does not need to provide a cancellation payment.

Figure 3.10 presents the optimal solutions of the OEM in a case when the second period's demand is small, and the spill over effect is weak. When θ is small, the OEM orders only d_1 to avoid the holding cost. At the same time, because the first period's demand is larger, the supplier has an incentive to remain silent, and the OEM needs to offer a positive cancellation payment so that the supplier would inform the OEM should she detects any flaws. As θ increases, once θ reaches a certain level, the OEM starts to order 1 so that the supplier can detect the flaws with certainty. The OEM needs to provide a larger cancellation payment to maintain the supplier's preference to inform.

The case in Figure 3.11 is to show that the OEM's order quantity does not necessarily jump among 0, d_1 and 1. Sometimes the OEM will order an intermediate amount between d_1 and 1, which is $q_1^* = \alpha(1+2\beta)d_1+(1-\alpha)d_2$. When he orders this amount, $T^* = 0$, meaning that the supplier will voluntarily inform without a cancellation payment.

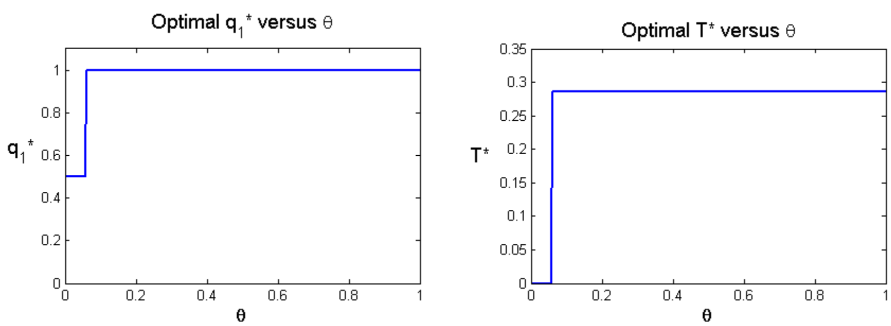

Figure 3.8: The optimal solutions versus θ when demands are comparable (with $d_1 = d_2 = 0.5$, $\beta = 1$, $p = 4$, $w = 2$, $c = 1$, $h = 0.1$, $s = 0.2$, and $\alpha = 0.3$).

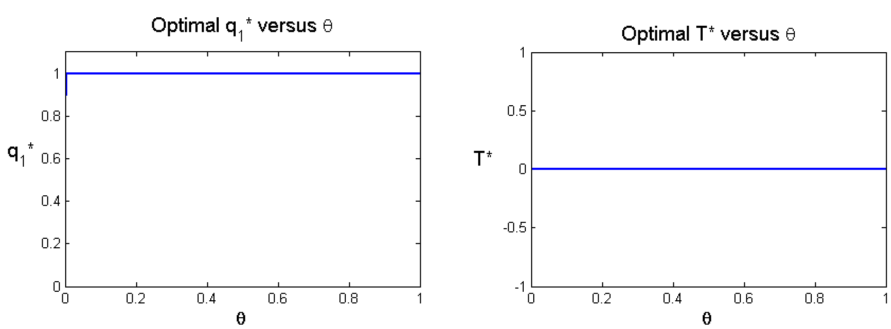

Figure 3.9: The optimal solutions versus θ when d_2 is large (with $d_1 = 0.8$, $d_2 = 30$, $\beta = 20$, $p = 4$, $w = 2$, $c = 1$, $h = 0.1$, $s = 0.2$, and $\alpha = 0.3$).

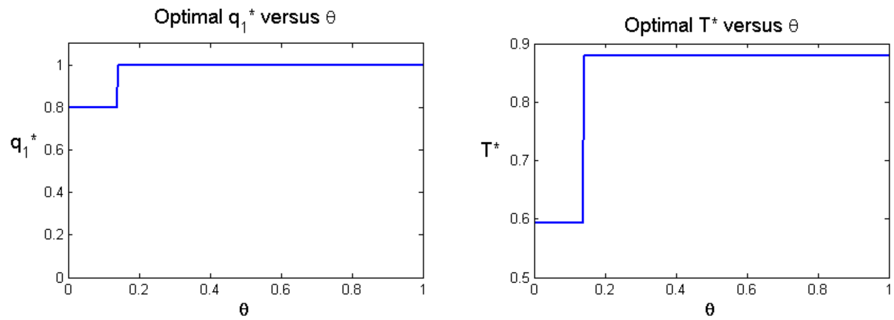

Figure 3.10: The optimal solutions versus θ when d_2 is small ($d_1 = 0.8$, $d_2 = 0.2$, $\beta = 0.01$, $p = 4$, $w = 2$, $c = 1$, $h = 0.1$, $s = 0.2$, and $\alpha = 0.3$).

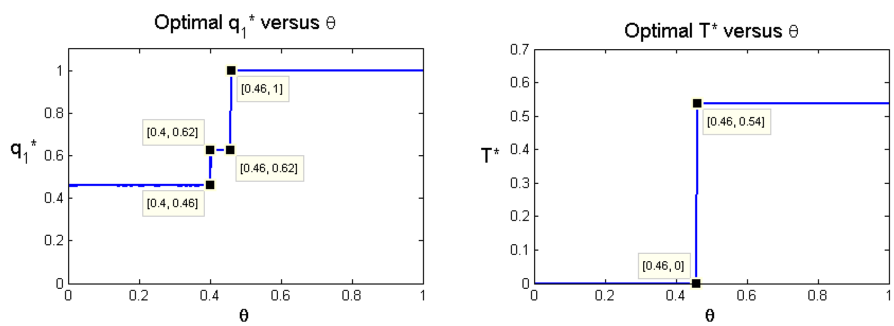

Figure 3.11: The optimal solutions versus θ with intermediate q_1^* presented (with $d_1 = 0.46$, $d_2 = 0.38$, $\beta = 0.8$, $p = 3$, $w = 2$, $c = 1$, $h = 0.3$, $s = 0.3$, and $\alpha = 0.3$).

Sensitivity Analysis on α

Secondly, we want to isolate the influence of α on the optimal solutions (q_1^*, T^*) of the OEM. We expect T^* to be decreasing in α. If α increases, the OEM has a better capability to rectify the flaws immediately, therefore should need less cancellation payment to motivate the supplier to inform.

Figure 3.12 presents the optimal solutions of the OEM in a case when the demands in two periods are comparable. As α increases, supplier becomes more confident in the OEM's capability to resolve the flaws in time. Therefore the OEM does not need to provide as much incentive for the supplier to inform. This explains why T^* decreases as α decreases.

Figure 3.13 presents the optimal solutions of the OEM in a case when the demand in the second period is substantially larger than the demand in the first period, and the spill over effect is strong. In this case, the OEM does not want to lose the demand in the second period. Therefore the OEM will always order 1 so that the supplier can detect the flaw. At the same time, because the second period's demand is much larger than the first period's demand, the supplier will inform without cancellation payment.

Figure 3.14 presents the optimal solutions of the OEM in a case when the demand in the second period is small, and the spillover effect is weak. In this case, when α is small, there is a big chance that the OEM is not able to rectify the flaw in time, and because the demand in the second period is very small, thus the OEM should order only d_1. When α increases, the OEM has more confidence to rectify the flaw in time, then he should order 1 so that the supplier can detect the flaw and inform him.

An interesting observation in this particular case is that when α approaches 1, T^* approaches infinity. When α is almost 1, meaning that the OEM in most cases can correct the flaws immediately. Suppose cancellation payment is zero, then the supplier would prefer not to inform. If the supplier informs, the OEM will correct the flaws, the supplier will get the first period's order, which is 1. In the second period, the demand will be $d_2 + \beta d_1$, but the OEM will order only βd_1, for he carries over some inventory from the first period. The supplier in total gets order $1 + \beta d_1 = 1.08$. In contrast, if the supplier does not order, she gets the first period demand 1. The OEM recognizes the flaws through lost demand. Defected products are salvaged, not carried over. In the second period, the demand will be $d_2 - \beta d_1$, and the OEM will need to order $d_2 - \beta d_1$. The supplier in total gets order $1 + d_2 - \beta d_1 = 1.12$. Therefore the supplier gets more order if she does not inform. In order to make the supplier inform, the OEM will need to fill this gap using the cancellation payment, so that the expected payment for the supplier to inform is better than not to inform. The cancellation payment is only effective when the OEM cannot rectify the flaws. Because this probability $1 - \alpha$ approaches zero, the cancellation payment approaches infinity.

The case in Figure 3.15 is to show that sometimes the OEM will order an intermediate amount between d_1 and 1, which is $q_1^* = \alpha(1+2\beta)d_1+(1-\alpha)d_2$.

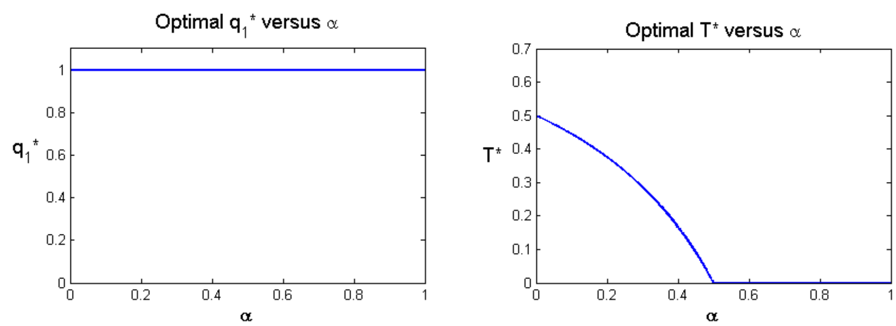

Figure 3.12: The optimal solutions versus α when demands are comparable (with $d_1 = d_2 = 0.5$, $\beta = 1$, $p = 4$, $w = 2$, $c = 1$, $h = 0.1$, $s = 0.2$, and $\theta = 0.5$).

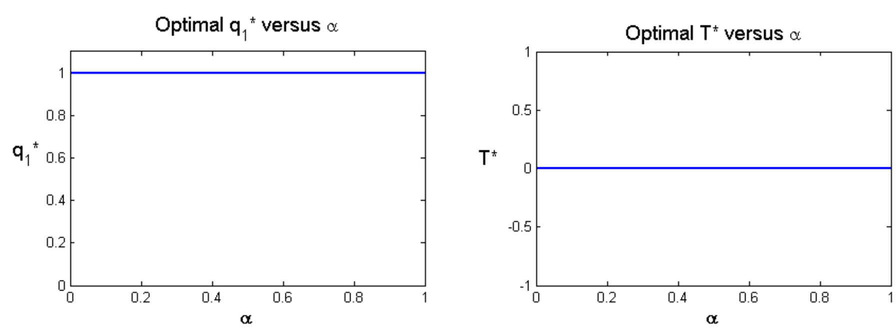

Figure 3.13: The optimal solutions versus α when d_2 is large (with $d_1 = 0.8$, $d_2 = 30$, $\beta = 20$, $p = 4$, $w = 2$, $c = 1$, $h = 0.1$, $s = 0.2$, and $\theta = 0.5$).

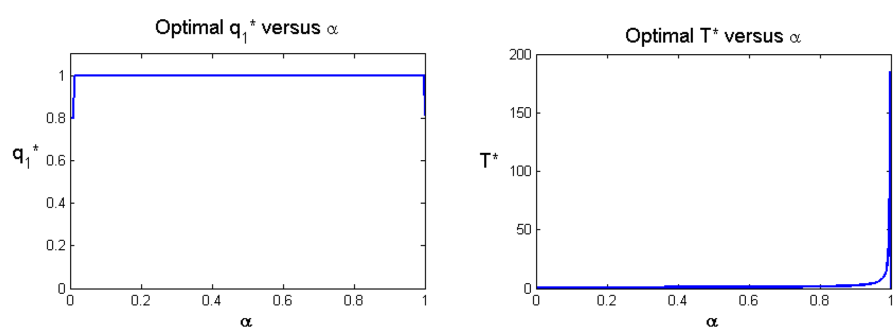

Figure 3.14: The optimal solutions versus α when d_2 is small (with $d_1 = 0.8$, $d_2 = 0.2$, $\beta = 0.01$, $p = 4$, $w = 2$, $c = 1$, $h = 0.1$, $s = 0.2$, and $\theta = 0.5$).

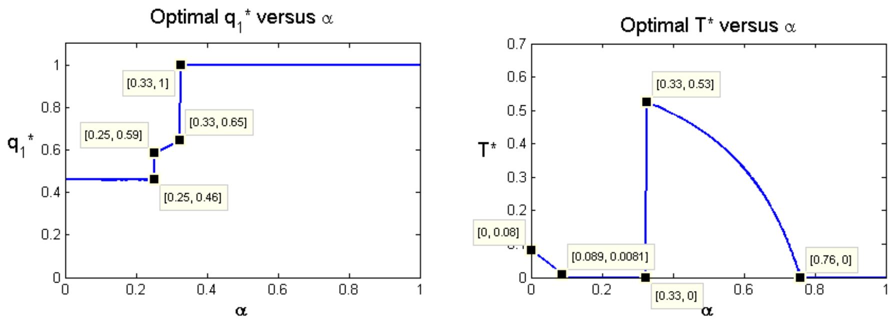

Figure 3.15: The optimal solutions versus α with intermediate q_1^* presented ($d_1 = 0.46$, $d_2 = 0.38$, $\beta = 0.8$, $p = 3$, $w = 2$, $c = 1$, $h = 0.3$, $s = 0.3$, and $\theta = 0.5$).

Sensitivity Analysis on β

Finally, we want to isolate the influence of β on the optimal solutions (q_1^*, T^*) of the OEM. Intuitively, as β increases, the spillover effect is stronger, the OEM has more incentive to direct the supplier to inform so that the potential flaws do not jeopardize the second period's demand. On the other hand, the supplier herself would have stronger incentive to voluntarily inform because she wants to secure the second period's demand. The interests of the OEM and the supplier are aligned. What's more, the optimal value of T^* should decrease in β. Because if β is larger, the supplier has stronger incentive to voluntarily inform and therefore does not need as much incentive provided from the OEM. Note that we have the assumptions that $d_2 \geq \beta d_1$, and $(\beta + 1)d_1 + d_2 > 1$. Therefore for different values of α, d_1, and d_2, the feasible value for β varies.

Figure 3.16 presents the optimal solutions of the OEM in a case when the demands in two periods are comparable. In this case, the OEM will order 1, and will need to provide positive cancellation payment so that the supplier will inform.

Figure 3.17 presents the optimal solutions of the OEM in a case when the second period's demand is substantially larger than the first period's demand, and the spill over effect is strong. In this case, the supplier will voluntarily inform without a cancellation payment.

Figure 3.18 presents the optimal solutions of the OEM in a case when the demand in the second period is small, and the spillover effect is weak. In this case, the demand in second period is not sufficient enough for the supplier to inform voluntarily, the OEM will need to provide a positive cancellation payment.

The case in Figure 3.19 is to show that sometimes $q_1^* = \alpha(1+2\beta)d_1 + (1-\alpha)d_2$. Note that T^* decreases in β. Because if the spillover effect is stronger, the supplier has a stronger incentive in inform voluntarily, so that the OEM does not need to provide as much incentive through T^*.

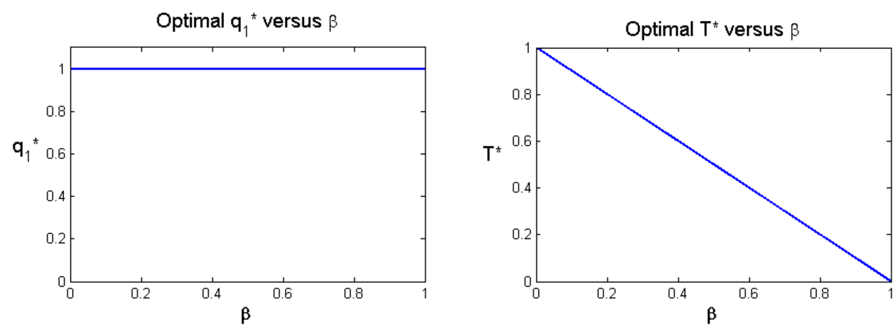

Figure 3.16: The optimal solutions versus β when demands are comparable (with $d_1 = d_2 = 0.5$, $p = 4$, $w = 2$, $c = 1$, $h = 0.1$, $s = 0.2$, $\theta = 0.5$, and $\alpha = 0.5$).

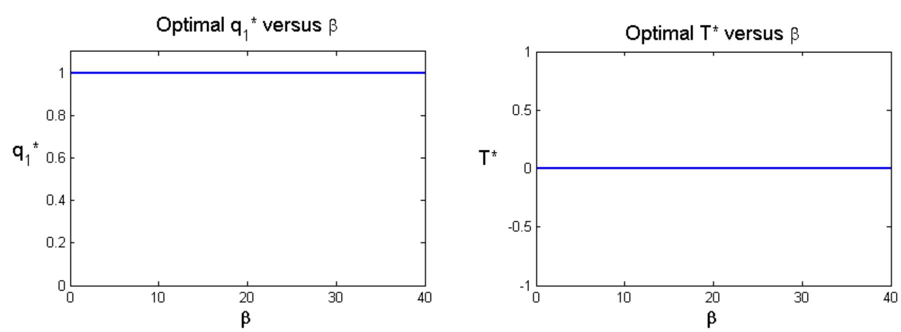

Figure 3.17: The optimal solutions versus β when d_2 is large (with $d_1 = 0.2$, $d_2 = 20$, $p = 4$, $w = 2$, $c = 1$, $h = 0.1$, $s = 0.2$, $\theta = 0.5$, and $\alpha = 0.5$).

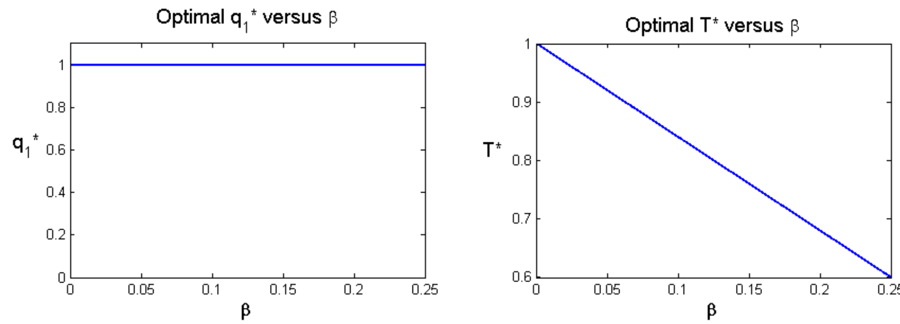

Figure 3.18: The optimal solutions versus β when d_2 is small (with $d_1 = 0.8$, $d_2 = 0.2$, $p = 4$, $w = 2$, $c = 1$, $h = 0.1$, $s = 0.2$, $\theta = 0.5$, and $\alpha = 0.5$).

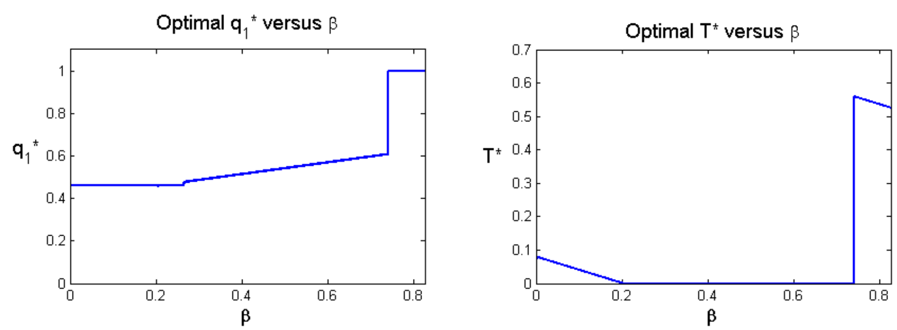

Figure 3.19: The optimal solutions versus β with intermediate q_1^* presented (with $d_1 = 0.46$, $d_2 = 0.38$, $p = 3$, $w = 2$, $c = 1$, $h = 0.3$, $s = 0.3$, $\theta = 0.5$, and $\alpha = 0.5$).

3.7 Conclusion

In this research, we first explore potential factors that may motivate the supplier to help the OEM improve product specifications by pointing out potential specification flaws and /or suggest improvements in product specifications. Our research is especially relevant when the supplier cannot be faulted for product quality issues arising from the OEM's specification flaws, where some common approaches such as shared warranty services may not be effective.

We then solve the optimization problem for the OEM. We prove that it is strictly better for the OEM to design the contract so that the supplier will inform if she detects flaws. This is a strong conclusion. With this principle in mind, we characterize the optimal strategy of the OEM, including order quantities in each period, and cancellation payment in the first period. We find that the optimal solutions are very sensitive with regard to some parameters. We perform sensitivity analysis on those parameters. One thing we show is that whenever the OEM is paying a positive cancellation payment to the supplier, then the cancellation payment should decrease as the OEM's capability increases, and decrease as the spillover effect for the demand increases.

Leveraging supplier's capabilities to improve product specifications and quality performance is an important area of research, and we hope that future research in this direction, either by us or others, will provide further insights in tapping the supplier's knowledge to create successful products.

Bibliography

S. Borenstein. To what electricity price do consumers respond? residential demand elasticity under increasing-block pricing. Working paper, University of California Energy Institute, 2009.

J. Boucher and Y. Smeers. Alternative models of restructured electricity systems, part 1: No market power. *Operations Research*, 49(6):821–838, 2001.

S. P. Boyd and L. Vandenberghe. *Convex optimization*. Cambridge Univ Press, 2004.

J. B. Bushnell, C. Peterman, and C. Wolfram. Local solutions to global problems: Policy choice and regulatory jurisdiction. Working paper, National Bureau of Economic Research, 2007.

G. H. Chao, S. M. R. Iravani, and R. C. Savaskan. Quality improvement incentives and product recall cost sharing contracts. *Management Science*, 55(7):1122–1128, 2009.

H. Chao and R. Wilson. Design of wholesale electricity markets. Technical report, Electric Power Research Institute, 1999.

R. Cheng and C. Lawton. Dell's consumer focus hits Snags; paint, supply woes plague more colorful laptops, causing shipping delays. *Wall Street Journal*, August 2007.

Y. Dai, S. X. Zhou, and Y. Xu. Competitive and collaborative quality and warranty management in supply chains. *Production and Operations Management*, 21(1):129–144, 2012.

A. Das, R. Narasimhan, and S. Talluri. Supplier integration–finding an optimal configuration. *Journal of Operations Management*, 24(5):563–582, 2006.

D. Drake, P. R. Kleindorfer, and L. N. Van Wassenhove. Technology choice and capacity investment under emissions regulation. *Faculty and Research*, 93(10), 2010.

M. Gillenwater. Redefining recspart 2: Untangling certificates and emission markets. *Energy Policy*, 36(6):2120–2129, 2008.

P. Gilligan. Corporate executive board review: Prepare for risk questions from senior leaders, February 2010. http://cebviews.com/2010/02/12prepare-for-risk-questions/.

X. Gong and S. X. Zhou. Optimal production planning with emissions trading. *Operations Research*, 61(4), 2013.

G. Graff. How to choose a contract manufacturer, October 2014. www.purchasing.com.

Y. G. Hailu and S. Adelaja. Effects of renewable portfolio standards and other state policies on wind industry development in the U.S. Working paper, Michigan State University, East Lansing, MI, August 2008.

R. B. Handfield and B. Lawson. Integrating suppliers into new product development. *Research-Technology Management*, 50(5):44–51, September 2007.

H. V. Heerde, K. Helsen, and M. G. Dekimpe. The impact of a product-harm crisis on marketing effectiveness. *Marketing Science*, 26(2):230–245, 2007.

Jenny Heeter and Lori Bird. Status and trends in us compliance and voluntary renewable energy certificate markets (2010 data). *Contract*, 303: 275–3000, 2011.

M. Hoegl and S. M. Wagner. Buyer-supplier collaboration in product development projects. *Journal of Management*, 31(4):530–548, August 2005.

E. Holt, J. Sumner, and L. Bird. The role of renewable energy certificates in developing new renewable energy projects. *Contract*, 303:275–3000, 2011.

A. Hortacsu and S. L. Puller. Understanding strategic bidding in multi-unit auctions: A case study of the texas electricity spot market. *The RAND Journal of Economics*, 39(1):86–114, 2008.

W. Huang, V. Kulkarni, and J. M. Swaminathan. Managing the inventory of an item with a replacement warranty. *Management Science*, 54(8): 1441–1452, 2008.

Institute For Energy Research. The Status of Renewable Electricity Mandates in the States. Technical report, January 2011.

A. V. Iyer, L. B. Schwarz, and S. A. Zenios. A principal-agent model for product specification and production. *Management Science*, 51(1):106–119, January 2005.

N. Jeffries. The evolving role of the contract manufacturing. *Global Cosmetic Industry*, 172(4):34–39, 2004.

S. H. Kim and B. Tomlin. Guilt by association: Strategic failure prevention and recovery capacity investments. *Management Science*, 59(7):1631–1649, 2013.

X. Koufteros, M. Vonderembse, and J. Jayaram. Internal and external integration for prouduct development: The contigency effects of uncertainty, equivocality, and platform strategy. *Decision Science*, 36(1):97–133, 2005.

W. S. Lim. Producer-supplier contracts with incomplete information. *Management Science*, 47(5):709–715, 2001.

F. A. Longstaff and A. W. Wang. Electricity forward prices: A highfrequency empirical analysis. *The Journal of Finance*, 59(4):1877–1900, 2004.

A. McCrone. *Global trends in renewable energy investment 2013*. Frankfurt School of Finance and Management Frankfurt, 2013.

F. C. Menz and S. Vachon. The effectiveness of different policy regimes for promoting wind power: Experiences from the states. *Energy Policy*, 34 (14):1786–1796, 2006.

R. J. Michaels. Intermittent currents: The failure of renewable electricity requirements. *Social Science Research Network*, 2007.

D. Murthy and I. Djamaludin. New product warranty: A literature review. *International Journal of Production Economics*, 79:231–260, 2002.

D. B. Parker, G. A. Zsidisin, and G. L. Ragatz. Timing and extent of supplier integration in new product development: A contingency approach. *Journal of Supply Chain Management*, 44(1):71–83, 2008.

K. J. Petersen, R. B. Handfield, and G. L. Ragatz. Supplier integration into new product development: Coordinating product, process and supply chain design. *Journal of Operations Management*, 23:371–388, 2005.

I. Petrick. Revitalizing the american dream: Putting america back on a prosperous footing. Forthcoming, 2012.

G. L. Ragatz, R. B. Handfield, and K. J. Petersen. Benefits associated with supplier integration into new product development under conditions of technology uncertainty. *Journal of Business Research*, 55:389–400, 2002.

D. J. Reyniers and C. S. Tapiero. The delivery and control of quality in supplier-producer contracts. *Management Science*, 41(10):1581–1589, 1995.

S. Stoft. Power system economics: Designing markets for electricity. *Jornal of Energy Literature*, 8:94–99, 2002.

S. Stoft, T. Belden, C. Goldman, and S. Pickle. *Primer on electricity futures and other derivatives*. Environmental Energy Technologies Division, Ernest Orlando Lawrence Berkeley National Laboratory, 1998.

R. Subramanian, S. Gupta, and B. Talbot. Compliance strategies under permits for emissions. *Production and Operations Management*, 16(6): 763–779, 2007.

D. M. Topkis. *Supermodularity and complementarity*. Princeton university press, 1998.

M. J. Trebilcock and R. Hrab. Electricity restructuring: A comparative review. *University of Toronto School of Law*, 2004.

U.S. Department of Energy. List of REC marketers, December 2014. http://apps3.eere.energy.gov/greenpower/markets/certificates.shtml?page=2.

R. Wiser and G. Barbose. Renewable portfolio standards in the united states-a status report with data through 2007. *Lawrence Berkeley National Laboratory*, 2008.

H. Yin and N. Powers. Do state renewable portfolio standards promote in-state renewable generation. *Energy Policy*, 38(2):1140–1149, 2009.

J. Zhao, B. F. Hobbs, and J. Pang. Long-run equilibrium modeling of emissions allowance allocation systems in electric power markets. *Operations Research*, 58(3):529–548, 2010.

K. Zhu, R. Q. Zhang, and F. Tsung. Pushing quality improvement along supply chains. *Management Science*, 53(3):421–436, 2007.

Appendix A

Appendix for Chapter 2

In each period t, $t = 1, \cdots, T$, the utility trades in the REC market first, then purchases electricity from the forward market. We write two cost-to-go functions for these two stages for ease of analysis.

In stage two, the utility's had adjusted its REC level to $\bar{x}_t$, the decision variables are y_{1t} and y_{2t}. Given state $(\bar{x}_t,\, u_t,\, R_t,\, P_t)$, the cost-to-go function after REC trading can be written as

$$W_t(\bar{x}_t,\, u_t,\, R_t,\, P_t) = \min_{\substack{y_{1t} \geq 0 \\ y_{2t} \geq y_{1t}}} \{p_{1t} y_{1t} + p_{2t}(y_{2t} - y_{1t}) + G_t(y_{2t}) \\ + \gamma E_t[V_{t+1}(\bar{x}_t + y_{1t}, u_t + D_t, R_{t+1}, P_{t+1})]\} \tag{A.1}$$

In stage one, the decision variable is $\bar{x}_t$, the cost-to-go function before REC trading can be written as

$$V_t(x_t,\, u_t,\, R_t,\, P_t) = \min_{\bar{x}_t} \{C_t(\bar{x}_t - x_t, R_t) + W_t(\bar{x}_t,\, u_t,\, R_t,\, P_t)\} \tag{A.2}$$

In the appendix, for simplicity, we use "increase (decrease)" to indicate"nondecrease (nonincrease)". Also, we omit the subscription t unless there is confusion. For instance, in the following Lemma, we write $W_t(x, u, R, P)$ instead of $W_t(x_t, u_t, R_t, P_t)$.

Lemma A.0.1 *$W_t(x,\, u,\, R,\, P)$ and $V_t(x,\, u,\, R,\, P)$ are jointly convex on $(x,\, u)$.*

Proof of lemma A.0.1:
We prove this lemma by induction in three steps:

(i) $V_{T+1}(x,\, u) = \pi(\alpha u - z)^+$ is jointly convex on $(x,\, u)$;

(ii) If $V_{t+1}(x,\, u,\, R, P)$ is jointly convex on $(x,\, u)$, then $W_t(\bar{x},\, u,\, R, P)$ is jointly convex on $(\bar{x},\, u)$;

(iii) If $W_t(\bar{x},\, u,\, R,\, P)$ is jointly convex on $(\bar{x},\, u)$, then $V_t(x,\, u,\, R,\, P)$ is jointly convex on $(x,\, u)$.

We show the proofs of these three steps in the following.

(i) Since πx^+ is convex in x, $V_{T+1}(x,\, u) = \pi(\alpha u - z)^+ = \pi((\alpha, -1)(u, z)^{\mathrm{T}})^+$ is jointly convex on $(x,\, u)$ by the preservation of convexity under composition with an affine mapping (Boyd and Vandenberghe, 2004).

(ii) According to equation (A.1), by the preservation of convexity under minimization, it is sufficient to show:

(a) $\{(y_1, y_2) : y_1 \geq 0, y_2 \geq y_1\}$ is a convex set;

(b) $p_1 y_1 + p_2(y_2 - y_1) + G_t(y_2) + \gamma E_t[V_{t+1}(\bar{x} + y_1,\, u + D, R_{t+1}, P_{t+1})]$ is convex on $(\bar{x}, u, y_1, y_2)$.

(a) is easy to verify. Now we prove (b). Since $V_{t+1}(x,\, u,\, R, P)$ is jointly convex on $(x,\, u)$, $V_{t+1}(\bar{x} + y_1,\, u + D, R_{t+1}, P_{t+1})$ is jointly convex in $(\bar{x}, u)$ and (y_1, u), also jointly convex in $(\bar{x}, y_1)$ by the preservation of convexity under composition with an affine mapping. Thus, $V_{t+1}(\bar{x}+y_1,\, u+D, R_{t+1}, P_{t+1})$ is convex in $(\bar{x}, u, y_1)$. Then $E_t[V_{t+1}(\bar{x}+y_1,\, u+D, R_{t+1}, P_{t+1})]$ is convex in $(\bar{x}, u, y_1)$ by preservation of convexity under nonnegative weighted sums (Boyd and Vandenberghe, 2004). Moreover, $G_t(y_2)$ is convex in y_2. Thus, we've proved (b), and we've proved (ii).

(iii) According to equation (A.2), by the preservation of convexity under minimization, it is sufficient to show $C_t(\bar{x} - x,\, R) + W_t(\bar{x},\, u,\, R,\, P)$ is convex in $(x, u, \bar{x})$. Since $bx^+ - s(-x)^+$ is convex in x, by the preservation of convexity under composition with an affine mapping, $C_t(\bar{x} - x, R) = b(\bar{x}-z)^+ - s(z-\bar{x})^+ = b((1,-1)(\bar{x}, z)^{\mathrm{T}})^+ - s(-(1,-1)(\bar{x}, z)^{\mathrm{T}})^+$ is jointly convex on $(\bar{x}, z)$. Meanwhile, $W(\bar{x},\, u,\, R,\, P)$ is convex in $(\bar{x}, u)$ by assumption. Thus, $C_t(\bar{x} - x,\, R) + W(\bar{x},\, u,\, R,\, P)$ is convex in $(x, u, \bar{x})$.

Lemma A.0.2 *$W_t(x,\, u,\, R,\, P)$ and $V_t(x,\, u,\, R,\, P)$ are submodular on $(x,\, u)$.*

Proof of lemma 2.4.2:
We prove this lemma by induction in three steps:

(i) $V_{T+1}(x, u)$ is submodular on (x, u);

(ii) If $V_{t+1}(x, u, R, P)$ is submodular on (x, u), then $W_t(\bar{x}, u, R, P)$ is submodular on $(\bar{x}, u)$;

(iii) If $W_t(\bar{x}, u, R, P)$ is submodular on $(\bar{x}, u)$, then $V_t(x, u, R, P)$ is submodular on (x, u).

We show the proofs of these three steps in the following.

(i) Since πx^+ is convex in x, $V_{T+1}(x, u) = \pi(\alpha u - z)^+$ is submodular on (x, u) (Topkis, 1998), Lemma 2.6.2.

(ii) According to equation (A.1), by the preservation of submodularity (Topkis, 1998), Theorem 2.7.6, it is sufficient to show:

(a) *(a)* $\{((\bar{x}, u), (y_1, y_2)) : u \geq 0, y_1 \geq 0, y_2 \geq y_1\}$ forms a lattice;

(b) *(b)* $p_1y_1+p_2(y_2-y_1)+G_t(y_2)+\gamma E_t[V_{t+1}(\bar{x}+y_1,\ u+D, R_{t+1}, P_{t+1})]$ is submodular on $((\bar{x}, u), (y_1, y_2))$.

(a) is easy to verify. Now we prove (b). Note $p_1y_1 + p_2(y_2 - y_1)$ is linear in (y_1, y_2) and $G_t(y_2)$ is a function of a single variable, thus, $p_1y_1 + p_2(y_2 - y_1) + G_t(y_2)$ is submodular on (y_1, y_2). If we can show $V_{t+1}(\bar{x}+y_1,\ u+D, R_{t+1}, P_{t+1})$ is submodular on $((\bar{x}, u), y_1)$, then $p_1y_1+p_2(y_2-y_1)+G_t(y_2)+\gamma E_t[V_{t+1}(\bar{x}+y_1,\ u+D, R_{t+1}, P_{t+1})]$ is submodular on $((\bar{x}, u), (y_1, y_2))$, because the sum of two submodular functions is also submodular. We know that $V_{t+1}(\bar{x}+y_1,\ u+D, R_{t+1}, P_{t+1})$ is submodular on $(\bar{x}, u)$ and (y_1, u) by the assumption. Also, because $V(x, u, R, P)$ decreases in z, $V_{t+1}(\bar{x}+y_1,\ u+D, R_{t+1}, P_{t+1})$ is submodular on $(\bar{x}, y_1)$. Thus we've proved (b), and we've proved (ii) .

(iii) Since $bx^+ - s(-x)^+$ is convex in x, $C_t(\bar{x} - x, R) = b(\bar{x} - z)^+ - s(z - \bar{x})^+$ is submodular on $(\bar{x}, z)$ (Topkis, 1998), Lemma 2.6.2. Because $W_t(\bar{x}, u, R, P)$ is submodular on $(\bar{x}, u)$ by assumption, $C_t(\bar{x} - x, R) + W_t(\bar{x},\ u,\ R,\ P)$ is submodular on $(x, u, \bar{x})$. By the preservation of submodularity (Topkis, 1998), Theorem 2.7.6, $V_t(x, u, R, P)$ is submodular on (x, u).

Proof of Theorem 2.4.1:

We can write equation (A.2) as

$$\begin{aligned}
V_t(x,u,R,P) &= \min_{\bar{x}}\{C_t(\bar{x}-x,R)+W_t(\bar{x},u,R,P)\} \\
&= \min_{\bar{x}}\{\min_{\bar{x}\geq z}\{b\bar{z}-bz+W_t(\bar{x},u,R,P)\}, \\
&\quad \min_{\bar{x}\leq z}\{-sz+s\bar{z}+W_t(\bar{x},u,R,P)\}\} \\
&= \min_{\bar{x}}\{-bz+\min_{\bar{x}\geq z}\{b\bar{z}+W_t(\bar{x},u,R,P)\}, \qquad \text{(A.3)} \\
&\quad -sz+\min_{\bar{x}\leq z}\{s\bar{z}+W_t(\bar{x},u,R,P)\}\}
\end{aligned}$$

Define

$$\begin{aligned}
L_t(u,\,R,P) &= \arg\min_{\bar{x}}\{b\bar{z}+W_t(\bar{x},\,u,\,R,P)\} \\
H_t(u,\,R,P) &= \arg\min_{\bar{x}}\{s\bar{z}+W_t(\bar{x},\,u,\,R,P)\}
\end{aligned}$$

Since $W_t(\bar{x},\,u,\,R,P)$ is convex in $\bar{x}$ (Lemma A.0.1) and $b\geq s$, we have $L_t(u,\,R,P)\leq H_t(u,\,R,P)$. Consider the two sub-optimization problems in (A.4). We refer to $-bz+\min_{\bar{x}\geq z}\{b\bar{z}+W_t(\bar{x},u,R,P)\}$ as optimization problem I and $-sz+\min_{\bar{x}\leq z}\{s\bar{z}+W_t(\bar{x},u,R,P)\}$ as optimization problem II.

(i) If $z\geq H_t(u,\,R,P)$, then $z\geq L_t(u,\,R,P)$. Let's consider optimization problem I first. From the definition of $L_t(u,\,R,P)$ and convexity of $W_t(\bar{x},\,u,\,R,P)$ on $\bar{x}$, $b\bar{z}+W_t(\bar{x},u,R,P)$ is increasing in $\bar{x}$ when $\bar{x}\geq z$. Thus, for optimization problem I, the optimal solution is $\bar{x}=z$, which gives an optimal value $-bz+bz+W_t(x,u,R,P)=W_t(x,u,R,P)$. Now let's consider optimization problem II. Note the optimal solution for optimization problem I, $\bar{x}=z$, is a feasible solution for optimization problem II. However, according to the definition of $H_t(u,\,R,P)$, $\bar{x}=H_t(u,\,R,P)$ is the optimal solution to optimization problem II, which gives an optimal value $-sz+sH_t(u,\,R,P)+W_t(H_t(u,\,R,P),u,R,P)$. From the definition of $H_t(u,\,R,P)$, we have $sH_t(u,\,R,P)+W_t(H_t(u,\,R,P),u,R,P)\leq sz+W_t(x,u,R,P)$. Therefore $-sz+sH_t(u,\,R,P)+W_t(H_t(u,\,R,P),u,R,P)$ $\leq W_t(x,u,R,P)$, meaning that the optimal value of optimization problem II is less than the optimal value of optimization problem I. Thus, for optimization problem (A.4), $\bar{x}^*=H_t(u,\,R,P)$.

(ii) If $z\leq L_t(u,\,R,P)$, then $z\leq H_t(u,\,R,P)$. Let's consider optimization problem II first. From the definition of $H_t(u,\,R,P)$ and convexity of

$W_t(\bar{x}, u, R, P)$ on $\bar{x}$, $s\bar{z}+W_t(\bar{x}, u, R, P)$ is decreasing in $\bar{x}$ when $\bar{x} \leq z$. Thus, for optimization problem II, the optimal solution is $\bar{x} = z$, which gives an optimal value $-sz+sz+W_t(x, u, R, P) = W_t(x, u, R, P)$. Now let's consider optimization problem I.

Note the optimal solution for optimization problem II, $\bar{x} = z$, is a feasible solution for optimization problem I. However, according to the definition of $L_t(u, R, P)$, $\bar{x} = L_t(u, R, P)$ is the optimal solution to optimization problem I, which gives an optimal value $-bz+bL_t(u, R, P)+ W_t(L_t(u, R, P), u, R, P)$. From the definition of $L_t(u, R, P)$, we have $bL_t(u, R, P) + W_t(L_t(u, R, P), u, R, P) \leq bz + W_t(x, u, R, P)$. Therefore $-bz + bL_t(u, R, P) + W_t(L_t(u, R, P), u, R, P) \leq W_t(x, u, R, P)$, meaning that the optimal value of optimization problem I is less than the optimal value of optimization problem II . Thus, for optimization problem (A.4), $\bar{x}^* = L_t(u, R, P)$.

(iii) If $L_t(u, R, P) \leq z \leq H_t(u, R, P)$, $b\bar{z}+W_t(\bar{x}, u, R, P)$ is increasing in $\bar{x}$ when $\bar{x} \geq z$, $s\bar{z} + W_t(\bar{x}, u, R, P)$ is decreasing in $\bar{x}$ when $\bar{x} \leq z$. Thus, for both optimization problem I and II, the optimal solution is $\bar{x}^* = z$. Therefore, for optimization problem (A.4), $\bar{x}^* = z$.

Lemma A.0.3 *In period t, $t = 1, \cdot, T$, given state (x, u, R, P), any feasible action $(\bar{x}, y_1, y_2)$ with $y_2 \geq y_1 \geq 0$ can be categorized into two types according to either $\bar{x} \geq z$ or $\bar{x} \leq z$. We can represent these two types as follows.*

Type one: $(-x, A, B)(x \geq 0, A \geq 0, B \geq 0)$, which means selling x units of unbundled RECs, buying A units of REC-bundled energy and B units of regular energy.

Type two: $(+y, A, B)(y \geq 0, A \geq 0, B \geq 0)$, which means buying y units of unbundled RECs, buying A units of REC-bundled energy and B units of regular energy.

We make two observations as follows:

(i) When $\triangle_t \geq b_t$, if the optimal action is type one $(-x, A, B)$, then $x \geq A$.

(ii) When $\triangle_t \leq s_t$, if the optimal action is type two (y, A, B), then $y \geq B$.

Proof of Lemma A.0.3:
We prove this Lemma by contradiction.

(i) When $\triangle_t \geq b_t$, suppose $\mathbf{a} = (-x, A, B)$ is the optimal action and $x < A$. Consider another action $\mathbf{a}' = (0, A - x, B + x)$. By taking either action $\mathbf{a}$ or $\mathbf{a}'$, the utility obtains $A - x$ units of RECs and $A + B$ units of electricity in period t. The costs of these two actions, however, are different. Note

$$
\begin{aligned}
cost(\mathbf{a}) - cost(\mathbf{a}') =& - s_t x + p_{1t}A + p_{2t}B - [p_{1t}(A - x) + p_{2t}(B + x)] \\
=& - s_t x + p_{1t}x - p_{2t}x \\
=& x(\triangle_t - s_t) \geq 0.
\end{aligned}
$$

Thus $\mathbf{a}'$ is a better action than $\mathbf{a}$, contradicts with the optimality of $\mathbf{a}$.

(ii) When $\triangle_t \leq s_t$, suppose $\mathbf{b} = (y, A, B)$ is the optimal action and $y < B$. Consider another action $\mathbf{b}' = (0, A + y, B - y)$. By taking either action $\mathbf{b}$ or $\mathbf{b}'$, the utility obtains $(A + y)$ units of RECs and $(A + B)$ units of electricity in period t. The costs

$$
\begin{aligned}
cost(\mathbf{b}) - cost(\mathbf{b}') =& b_t y + p_{1t}A + p_{2t}B - [p_{1t}(A + y) + p_{2t}(B - y)] \\
=& b_t y - p_{1t}y + p_{2t}y \\
=& y(b_t - \triangle_t) \geq 0.
\end{aligned}
$$

Thus $\mathbf{b}'$ is a better action than $\mathbf{b}$, contradicts with the optimality of $\mathbf{b}$.

Proof of proposition 2.4.4:
We prove this proposition by sample path and contradiction.

(i) In period t, if $\triangle_t \geq b_t$, we want to show that it is optimal to purchase only regular energy. Suppose the optimal action involves purchasing some REC-bundled renewable energy. Note $\triangle_t \geq b_t$ implies $p_{1t} \geq p_{2t} + b_t$. This means for every unit of REC-bundled energy, the utility can get the equivalent product by combining one unit of regular energy and one unit of REC, but at a cheaper price. Therefore if $\triangle_t \geq b_t$, it is always better to purchase only regular energy.

(ii) In period t, if $\triangle_t \leq s_t$, we want to show that it is optimal to purchase only REC-bundled energy. Suppose the optimal action involves purchasing some regular energy. Note that $\triangle_t \leq s_t$ implies $p_{2t} \geq p_{1t} - s_t$.

This means for every unit of regular energy, the utility can get the equivalent product by purchasing one unit of REC-bundled energy and then selling the REC comes with it. The resulting price is cheaper than purchasing regular energy directly. Therefore if $\triangle_t \leq s_t$, it is always better to purchase only REC-bundled energy.

(iii) In period t, if $s_t < \triangle_t < b_t$,

when $z \leq L_t(u, R, P)$, according to Theorem 2.4.1, it is optimal to purchase RECs. Therefore we can write the optimal action as type two $\mathbf{b_1} = (y, A, B)$. By Lemma A.0.3 we know that $y \geq B$. Consider another action $\mathbf{b_2} = (y-B, A+B, 0)$. By taking either of action $\mathbf{b_1}$ or $\mathbf{b_2}$, , the utility gains $y + A$ units of RECs and $A + B$ units of electricity in period t. Compare the costs of these two actions, we have

$$\begin{aligned}
&cost(\mathbf{b_1}) - cost(\mathbf{b_2}) \\
=&b_t y + p_{1t}A + p_{2t}B - [b_t(y - B) + p_{1t}(A + B)] \\
=&b_t B + p_{2t}B + p_{1t}(-B) \\
=&B(b_t - \triangle_t) \geq 0.
\end{aligned}$$

Thus action $\mathbf{b_2}$ is better than action $\mathbf{b_1}$, contradicts with the assumption that action $\mathbf{b_1}$ is optimal. Therefore it is optimal to purchase only REC-bundled energy.

When $z \geq H_t(u, R, P)$, according to Theorem 2.4.1, it is optimal to sell RECs. Therefore we can write the optimal action as type one $\mathbf{a_1} = (-x, A, B)$. By Lemma A.0.3 we know that $x \geq A$. Consider action $\mathbf{a_2} = (-(x - A), 0, A + B)$, By taking either of action $\mathbf{a_1}$ or $\mathbf{a_2}$, the utility gains $-x + A$ units of RECs and $A + B$ units of electricity in period t. Compare the costs of these two actions, we have

$$\begin{aligned}
&cost(\mathbf{a_1}) - cost(\mathbf{a_2}) \\
=&- s_t x + p_{1t}A + p_{2t}B - [-s_t(x - A) + p_{2t}(A + B)] \\
=&- s_t A - p_{2t}A + p_{1t}A \\
=&A(\triangle_t - s_t) \geq 0.
\end{aligned}$$

Thus action $\mathbf{a_2}$ is better than action $\mathbf{a_1}$, contradicts with the assumption that action$\mathbf{a_1}$ is optimal. Therefore it is optimal to purchase only regular energy.

Proof of Theorem 2.4.5:
If $\triangle_t \geq b_t$, by Proposition 2.4.4, it is optimal to purchase only regular energy, i.e., $y_1^* = 0$. We can write the cost-to-go function after the REC trading (A.1) as

$$W_t(\bar{x}_t,\, u_t,\, R_t,\, P_t) = \min_{y_2 \geq 0}\{p_2 y_2 + G_t(y_2) \\ + \gamma E_t[V_{t+1}(\bar{x},\, u + D_t, R_{t+1}, P_{t+1})]\}$$

Thus

$$y_2^* = \arg\min_{y_2 \geq 0}\{p_2 y_2 + G_t(y_2)\} \overset{\triangle}{=} S_{2t}(p_2)$$

Further, the convexity of $G_t(y)$ implies that $S_{2t}(p_2)$ decreases in p_2.

Proof of Theorem 2.4.6:

If $\triangle_t \leq s_t$, by Proposition 2.4.4, it is optimal to purchase only REC-bundled energy, i.e. , $y_1^* = y_2^*$. We consider three cases based on the utility's REC level at the beginning of period t. For ease of analysis, we write $w = \bar{x} + y_1$ as the REC level at the end of period t.

(i) When $z \leq L_t(u, R, P)$, by Theorem 2.4.1, it is optimal for the utility to purchase RECs to increase its REC level up to $\bar{x}^* = L_t(u, R, P)$. Therefore, we can write the cost-to-go function as

$$\begin{aligned} & V_t(x, u, R, P) \\ = & \min_{\substack{y_1 \geq 0 \\ w}}\{b_t[(w - y_1) - z)] + p_1 y_1 + G_t(y_1) \\ & + \gamma E_t[V_{t+1}(w, u + D, R_{t+1}, P_{t+1})]\} \\ = & - b_t z + \min_{\substack{y_1 \geq 0 \\ w}}\{(p_1 - b_t)y_1 + G_t(y_1) \\ & + b_t w + \gamma E_t[V_{t+1}(w, u + D, R_{t+1}, P_{t+1})]\} \end{aligned}$$

The objective function in the bracket is a separate convex functions on (y_1, w), thus

$$w^* = \arg\min_{w}\{b_t w + \gamma E_t[V_{t+1}(w, u + D, R_{t+1}, P_{t+1})]\} \overset{\triangle}{=} w_t^L(u_t, R_t, P_t), \tag{A.4}$$

$$y_1^* = \arg\min_{y_1 \geq 0}\{(p_1 - b_t)y_1 + G_t(y_1)\} \overset{\triangle}{=} S_{1t}^L(p_1, b_t), \tag{A.5}$$

and

$$L_t(u,R,P) = \bar{x}^* = w^* - y_1^* = w_t^L(u_t,R_t,P_t) - S_{1t}^L(p_1,b_t).$$

(ii) When $z \geq H_t(u,R,P)$, by Theorem 2.4.1, it is optimal for the utility to sell RECs to decrease its REC level to $\bar{x}^* = H_t(u,R,P)$. Therefore, we can write the cost-to-go function as

$$\begin{aligned}
&V_t(x,u,R,P)\\
=&\min_{\substack{y_1\geq 0\\ w}}\{-s_t[z-(w-y_1)] + p_1y_1 + G_t(y_1)\\
&+\gamma E_t[V_{t+1}(w,u+D,R_{t+1},P_{t+1})]\}\\
=&-s_tz + \min_{\substack{y_1\geq 0\\ w}}\{(p_1-s_t)y_1 + G_t(y_1)\\
&+s_tw + \gamma E_t[V_{t+1}(w,u+D,R_{t+1},P_{t+1})]\}
\end{aligned}$$

Thus

$$w^* = \arg\min_{w}\{s_tw + \gamma E_t[V_{t+1}(w,u+D,R_{t+1},P_{t+1})]\} \triangleq w_t^H(u_t,R_t,P_t),$$

$$y_1^* = \arg\min_{y_1\geq 0}\{(p_1-s_t)y_1 + G_t(y_1)\} \triangleq S_{1t}^H(p_1,s_t),$$

$$H_t(u,R_t,P) = \bar{x}^* = w^* - y_1^* = w_t^H(u_t,R_t,P_t) - S_{1t}^H(p_1,s_t). \tag{A.6}$$

(iii) When $L_t(u,R,P) < z < H_t(u,R,P)$, by Theorem 2.4.1, it is optimal for the utility not to trade RECs, i.e., $\bar{x}^* = z$. Therefore, we can write the cost-to-go function as

$$\begin{aligned}
&V_t(x,u,R,P)\\
=&\min_{y_1\geq 0}\{p_1y_1 + G_t(y_1) + \gamma E_t[V_{t+1}(z+y_1,u+D,R_{t+1},P_{t+1}]\}.
\end{aligned}$$

Thus

$$\begin{aligned}
y_1^* =& \arg\min_{y\geq 0}\{p_1y + G_t(y) + \gamma E_t[V_{t+1}(z+y,u+D,R_{t+1},P_{t+1}]\}\\
=& s_{1t}(x,u,R,P).
\end{aligned}$$

From the definitions above, $s_{1t}(L_t(u,R,P),u,R,P) = S_{1t}^L(p_1,b_t)$ and $s_{1t}(H_t(u,R,P),u,R,P) = S_{1t}^H(p_1,s_t)$.

Proof of proposition 2.4.7:

(a) Since $f(x, y) = xy$ is supermodular on (x, y), and $(p_1 - b_t)y + G_t(y)$ is supermodular on (p_1, y) and submodular on (b_t, y), $S^L_{1t}(p_1, b_t)$ (A.5) decreases in p_1 and increases in b_t by Topkis (1998) (Theorem 2.8.2).

(b) Similar with (a) we can prove (b).

(c) Define a function $g(x, u, R, P, y)$ as

$$\begin{aligned}&s_{1t}(x, u, R, P)\\ =&\arg\min_{y\geq 0}\{p_1 y + G_t(y) + \gamma E_t[V_{t+1}(z + y, u + D, R_{t+1}, P_{t+1}]\}\\ =&\arg\min_{y\geq 0} g(x, u, R, P, y).\end{aligned}$$

In order to show $s_{1t}(x,\, u,\, R, P)$ decreases in z, it is sufficient to show that $g(x, y, u, R, P)$ is supermodular on (x, y) (Topkis, 1998), Theorem 2.8.2. Since $\{(x, y) : y \geq 0\}$ is a sublattice of $\mathbf{R}^2$, and $V_{t+1}(x, u, R, P)$ is convex on $z \in \mathbf{R}$, thus, $V_{t+1}(z + y, u, R, P)$ is supermodular on (x, y) (Topkis, 1998), Lemma 2.6.2. Thus, $g(x, y, u, R, P)$ is supermodular on (x, y). In order to show $s_{1t}(x, u,\, R, P)$ increases in u, it is sufficient to show that $g(x, u, R, P, y_t)$ is submodular on (u, y), which is true since $V_{t+1}(x, u, R, P)$ is submodular on (x, u). Thus we've proved (c).

(d) From the convexity of $G_t(y)$ and the assumption $s_t \leq b_t$, we have $S^L_{1t}(p_1, b_t) \leq s_{1t}(x, u, R, P) \leq S^H_{1t}(p_1, s_t)$.

Proof of theorem 2.4.8:
If $s_t < \triangle_t < b_t$,

(a) when $x_t \leq L_t(u_t, R_t, P_t)$, we know $\bar{x}^*_t = L_t(u_t, R_t, P_t)$ (Theorem 2.4.1), and it is optimal to purchase only REC-bundled energy (Proposition 2.4.4), i.e., $y^*_{1t} = y^*_{2t}$. Denote $w_t = \bar{x}_t + y_{1t}$, we can write the cost-to-go function as

$$\begin{aligned}&V_t(x_t, u_t, R_t, P_t)\\ =&\min_{\substack{y_{1t}\geq 0\\ w_t}}\{b_t[(w_t - y_{1t}) - x_t] + p_{1t}y_{1t} + G_t(y_{1t})\\ &+ \gamma E_t[V_{t+1}(w_t, u_t + D_t, R_{t+1}, P_{t+1}]\}\\ =&-b_t x_t + \min_{\substack{y_{1t}\geq 0\\ w_t}}\{(p_{1t} - b_t)y_{1t} + G_t(y_{1t})\\ &+ b_t w_t + \gamma E_t[V_{t+1}(w_t, u_t + D_t, R_{t+1}, P_{t+1}]\}\end{aligned}$$

Thus $y_{1t}^* = S_{1t}^L(p_{1t}, b_t)$, $w_t^* = w_t^L(u_t, R_t, P_t)$. Moreover, we have

$$L_t(u_t, R_t, P_t) = \bar{x}_t^* = w_t^* - y_{1t}^* = w_t^L(u_t, R_t, P_t) - S_{1t}^L(p_{1t}, b_t).$$

(b) when $x_t \geq H_t(u_t, R_t, P_t)$, we know $\bar{x}_t^* = H_t(u_t, R_t, P_t)$ (Theorem 2.4.1), and it is optimal to purchase only regular energy (Proposition 2.4.4), i.e., $y_{1t}^* = 0$. We can write the cost-to-go function as

$$\begin{aligned}
&V_t(x_t, u_t, R_t, P_t) \\
=& \min_{\substack{y_{2t} \geq 0 \\ \bar{x}_t}} \{-s_t(x_t - \bar{x}_t) + p_{2t}y_{2t} + G_t(y_{2t}) \\
&+ \gamma E_t[V_{t+1}(\bar{x}_t, u_t + D_t, R_{t+1}, P_{t+1}]\} \\
=& - s_t x_t + \min_{\substack{y_{2t} \geq 0 \\ \bar{x}_t}} \{p_{2t}y_{2t} + G_t(y_{2t}) \\
&+ s_t\bar{x}_t + \gamma E_t[V_{t+1}(\bar{x}_t, u_t + D_t, R_{t+1}, P_{t+1}]\}
\end{aligned}$$

Thus $y_{2t}^* = S_{2t}(p_{2t})$, $H_t(u_t, R_t, P_t) = \bar{x}_t^* = w_t^H(u_t, R_t, P_t)$.

Proof of theorem 2.4.9:
If $s_t < \triangle_t < b_t$, when $L_t(u_t, R_t, P_t) < x_t < H_t(u_t, R_t, P_t)$, $\bar{x}_t^* = x_t$ (Theorem 2.4.1). We can write the cost-to-go function as

$$\begin{aligned}
&V_t(x_t, u_t, R_t, P_t) = W_t(x_t, u_t, R_t, P_t) \\
=& \min_{\substack{w_t \geq x_t \\ y_{2t} \geq w_t - x_t}} \{\triangle_t(w_t - x_t) + p_{2t}y_{2t} + G_t(y_{2t}) \\
&+ \gamma E_t[V_{t+1}(w_t, u_t + D_t, R_{t+1}, P_{t+1}]\} \\
=& \min_{\substack{w_t \geq x_t \\ y_{2t} \geq w_t - x_t}} \{-\triangle_t x_t + p_{2t}y_{2t} + G_t(y_{2t}) \\
&+ \triangle_t w_t + \gamma E_t[V_{t+1}(w_t, u_t + D_t, R_{t+1}, P_{t+1}]\} \\
=& \min_{\substack{w_t \geq x_t \\ y_{2t} \geq w_t - x_t}} f(x_t, u_t, R_t, P_t, w_t, y_{2t}).
\end{aligned} \tag{A.7}$$

Denote $(w_t^*, y_{2t}^*) = \arg\min_{\substack{w_t \geq x_t \\ y_{2t} \geq w_t - x_t}} f(x_t, u_t, R_t, P_t, w_t, y_{2t})$ as the optimal solutions to this optimization problem.
Note $f(x_t, u_t, R_t, P_t, w_t, y_{2t})$ is a separate convex function on (w_t, y_{2t}). Define

$$w_t^{\triangle}(u_t, R_t, P_t) = \arg\min_w \{\triangle_t w + \gamma E_t[V_{t+1}(w, u + D, R_{t+1}, P_{t+1})]\},$$

then $(w_t^{\triangle}(u_t, R_t, P_t),\, S_{2t}(p_{2t}))$ is the global minimum of f on $\mathbf{R}^2$ plane. W.l.o.g, we assume this global optimum is unique.

As x_t increases from $-\infty$ to $+\infty$, the feasible area $\{(w_t, y_{2t}) \in \mathbf{R}^2 : w_t \geq x_t,\, y_{2t} \geq w_t - x_t\}$ is moving towards right. In the following we divide the region of x_t $(L_t(u_t, R_t, P_t) < x_t < H_t(u_t, R_t, P_t))$ into three sub-regions, so that we can discuss whether or not the sub-region has the global minimum as an interior point.

To this end, define

$$\begin{aligned} l_t(u_t,\, R_t, P_t) =& w_t^{\triangle}(u_t,\, R_t, P_t) - S_{2t}(p_{2t}), \\ h_t(u_t,\, R_t, P_t) =& w_t^{\triangle}(u_t,\, R_t, P_t). \end{aligned}$$

First we show when $s_t < \triangle_t < b_t$,

$$L_t(u_t, R_t, P_t) \leq l_t(u_t,\, R_t, P_t) \leq h_t(u_t,\, R_t, P_t) \leq H_t(u_t, R_t, P_t),$$

so that we can divide $L_t(u_t, R_t, P_t) < x_t < H_t(u_t, R_t, P_t)$ into three sub-regions, $L_t(u_t, R_t, P_t) < x_t \leq l_t(u_t,\, R_t, P_t)$, $l_t(u_t,\, R_t, P_t) < x_t < h_t(u_t,\, R_t, P_t)$, and $h_t(u_t, R_t, P_t) \leq x_t < H_t(u_t, R_t, P_t)$.
From the definition of $l_t(u_t,\, R_t, P_t)$ and $h_t(u_t,\, R_t, P_t)$, we know that it is equivalent to show

$$L_t(u_t,\, R_t,\, P_t) \leq w_t^{\triangle}(u_t,\, R_t, P_t) - S_{2t}(p_{2t}) \leq w_t^{\triangle}(u_t,\, R_t, P_t) \leq H_t(u_t,\, R_t,\, P_t).$$

Let us start with the first inequality. When $s_t < \triangle_t < b_t$, we have $L_t(u_t,\, R_t, P_t) = w_t^L(u_t,\, R_t, P_t) - S_{1t}^L(p_{1t}, b_t)$. Thus in order to prove the first inequality, it is sufficient to show $w_t^L(u_t,\, R_t, P_t) \leq w_t^{\triangle}(u_t,\, R_t, P_t)$ and $S_{1t}^L(p_{1t}, b_t) \geq S_{2t}(p_{2t})$. From the definition of $w_t^L(u_t,\, R_t, P_t)$, $w_t^{\triangle}(u_t,\, R_t, P_t)$, and the submodularity of $V(x, u, R, P)$ on (x, u), we know that $w_t^L(u_t,\, R_t, P_t) \leq w_t^{\triangle}(u_t,\, R_t, P_t)$. On the other hand, from the definition of $S_{1t}^L(p_{1t}, b_t)$, $S_{2t}(p_{2t})$ and the convexity of $G_t(y)$, we know that $S_{1t}^L(b_t, p_{1t}) \geq S_{2t}(p_{2t})$. Thus we've proved the first inequality.

The second inequality is obvious since $S_{2t}(p_{2t}) \geq 0$.

Let us look at the third inequality. When $s_t < \triangle_t < b_t$, $H_t(u_t,\, R_t, P_t) = w_t^H(u_t,\, R_t, P_t)$. Thus the third inequality is equivalent as $w_t^{\triangle}(u_t,\, R_t, P_t) \leq w_t^H(u_t,\, R_t, P_t)$. From the definition of $w_t^{\triangle}(u_t,\, R_t, P_t)$, $w_t^H(u_t,\, R_t, P_t)$, and the submodularity of $V(x, u, R, P)$ on (x, u), we know that $w_t^{\triangle}(u_t,\ R_t, P_t) \leq w_t^H(u_t,\ R_t, P_t)$. Thus the third inequality holds.

Now we've proved the legitimacy of dividing $L_t(u_t, R_t, P_t) \leq x_t \leq H_t(u_t, R_t, P_t)$ into three sub-regions with $l_t(u_t,\ R_t, P_t)$ and $H_t(u_t, R_t, P_t)$. In the following ,we discuss whether or not each of the three sub-regions has the global minimum as an interior point.

(a) When $L_t(u_t, R_t, P_t) < x_t \leq l_t(u_t,\ R_t, P_t)$, the feasible area is on the left-hand-side of the global minimum $(w_t^{\triangle}(u_t, R_t, P_t),\ S_{2t}(p_{2t}))$ and does not include it as an interior point. Since $f(x_t, u_t, R_t, P_t, w_t, y_{2t})$ is jointly convex on (w_t, y_{2t}), the optimal solution to optimization problem (A.7), (w_t^*, y_{2t}^*), is on the right boundary of the feasible set. Thus $y_{2t}^* = w_t^* - x_t$. Therefore $y_{2t}^* = y_{1t}^*$, the utility should purchase only REC-bundled energy. We have

$$\begin{aligned} & V_t(x_t, u_t, R_t, P_t) = W_t(x_t, u_t, R_t, P_t) \\ = & \min_{y_{1t} \geq 0} \{p_{1t} y_{1t} + G_t(y_{1t}) + \gamma E_t[V_{t+1}(x_t + y_{1t}, u_t + D_t, R_{t+1}, P_{t+1})]\}. \end{aligned}$$

Thus $y_{1t}^* = y_{2t}^* = s_{1t}(x_t, u_t, R_t, P_t)$.

(b) When $l_t(u_t,\ R_t, P_t) < x_t < h_t(u_t,\ R_t, P_t)$, the global minimum $(w_t^{\triangle}(u_t, R_t, P_t),\ S_{2t}(p_{2t}))$ is in the interior of the feasible set. Thus the global minimum is the optimal solution to optimization problem (A.7). In this case, the utility should purchase both REC-bundled energy and regular energy. We have $(w_t^*, y_{2t}^*) = (w_t^{\triangle}(u_t, R_t, P_t),\ S_{2t}(p_{2t}))$.
Thus $y_{1t}^* = w_t^{\triangle}(u_t, R_t, P_t) - x_t,\ y_{2t}^* = S_{2t}(p_{2t})$.

(c) When $h_t(u_t, R_t, P_t) \leq x_t < H_t(u_t, R_t, P_t)$, the feasible area is on the right-hand-side of the global minimum $(w_t^{\triangle}(u_t, R_t, P_t),\ S_{2t}(p_{2t}))$ and does not include it as an interior point. Since $f(x_t, u_t, R_t, P_t, w_t, y_{2t})$ is jointly convex on (w_t, y_{2t}), the optimal solution to optimization problem (A.7), (w_t^*, y_{2t}^*), is on the left boundary of the feasible set. Thus $w_t^* = x_t$. Therefore $y_{1t} = 0$, the utility should purchase only regular energy. We have

$$\begin{aligned} & V_t(x_t, u_t, R_t, P_t) \\ = & \min_{y_{2t} \geq 0, \bar{x_t}} \{p_{2t} y_{2t} + G_t(y_{2t}) + \gamma E_t[V_{t+1}(\bar{x_t}, u_t + D_t, R_{t+1}, P_{t+1})]\}. \end{aligned}$$

Thus $y_{1t}^* = 0,\ y_{2t}^* = S_{2t}(p_{2t})$.

Appendix B

Appendix for Chapter 3

B.1 Analysis for optimization problem N

There is a piece-wise function in the objective function

$$[d_2+\beta(q_1\wedge d_1)-(q_1-d_1)^+]^+ = \begin{cases} d_2+\beta q_1, & q_1 \le d_1 \\ (\beta+1)d_1+d_2-q_1, & d_1 < q_1 \le (\beta+1)d_1+d_2 \\ 0, & q_1 > (\beta+1)d_1+d_2. \end{cases}$$

This piece-wise function divides the region of q_1 into three subregions: $[0, d_1]$, $(d_1, (\beta+1)d_1+d_2]$, and $((\beta+1)d_1+d_2, +\infty)$. Therefore in our analysis, we consider these three subregions respectively. Although we will solve this optimization problem based on subregions of q_1, it is important to point out that because the objective function and the constraints are all continuous in q_1, the objective value at the division points are consistent regardless which subregion we include them in.
Let us start with the first subregion $q_1 \in [0, d_1]$.

Proposition B.1.1 *In the subregion $q_1 \in [0, d_1]$:*

- *when $\dfrac{d_2}{d_1} > \dfrac{1-\alpha-2\alpha\beta}{1-\alpha}$, there is no feasible solution in this subregion.*
- *when $\dfrac{d_2}{d_1} \le \dfrac{1-\alpha-2\alpha\beta}{1-\alpha}$,*
  - *when $\theta \le \dfrac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, then $q_1^* = d_1$, and T^* can be any value satisfying the incentive constraint. We set $T^* = 0$.*
  - *when $\theta > \dfrac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, then $q_1^* = \dfrac{1-\alpha}{1-\alpha-2\alpha\beta}d_2$, and $T^* = 0$.*

Proof of proposition B.1.1
With the restriction $q_1 \in [0, d_1]$, we can write the the first order derivative of the objective function w.r.t. q_1 as

$$\begin{aligned} V_N'(q_1) &= (1-\theta)(p-w)(1+\beta) + \theta[-w+s-\alpha\beta(p-w)] \\ &= (p-w)(1+\beta) - \theta[(p-w)(1+\beta+\alpha\beta) + w - s] \end{aligned}$$

Therefore $V_N(q_1)$ is linear, but the sign of $V_N'(q_1)$ is indeterminate.

- If $\theta \le \dfrac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, then $V_N'(q_1) \ge 0$.
- If $\theta > \dfrac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, then $V_N'(q_1) < 0$.

On the other hand, $S_I \le S_N$ can be written as

$$(1-\alpha)(w-c)d_2 + (1-\alpha)T \le q_1(w-c)(1-\alpha-2\alpha\beta),$$

Therefore the constraints $S_I \le S_N$ and $T \ge 0$ can be combined as

$$(1-\alpha)d_2 \le (1-\alpha-2\alpha\beta)q_1.$$

- If $(1-\alpha)d_2 > (1-\alpha-2\alpha\beta)d_1$, there is no feasible solution in this subregion.
- If $(1-\alpha)d_2 \le (1-\alpha-2\alpha\beta)d_1$, the optimal solutions will depend on the sign of $V_N'(q_1)$.
  - If $\theta \le \dfrac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, then $V_N(q_1)$ increases in this subregion, $q_1^* = d_1$, and T^* can be any value satisfying the incentive constraint, i.e., $0 \le T^* \le \dfrac{1-\alpha-2\alpha\beta}{1-\alpha}(w-c)d_1 - (w-c)d_2$.
  - If $\theta > \dfrac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, then $V_N(q_1, T)$ decreases in this subregion, $q_1^* = \dfrac{1-\alpha}{1-\alpha-2\alpha\beta}d_2$, and $T^* = 0$.

Next we look at the second subregion $q_1 \in (d_1, (\beta+1)d_1 + d_2]$.

Proposition B.1.2 *In the subregion* $q_1 \in (d_1, (\beta+1)d_1 + d_2]$,

- *when* $\dfrac{d_2}{d_1} > \dfrac{1-\alpha-2\alpha\beta}{1-\alpha}$, *then* $q_1^* = \alpha(2\beta+1)d_1 + (1-\alpha)d_2$, *and* $T^* = 0$.
- *when* $\dfrac{d_2}{d_1} \leq \dfrac{1-\alpha-2\alpha\beta}{1-\alpha}$, *then* $q_1^* = d_1$, *and* T^* *can be any value satisfying the incentive constraint. We set* $T^* = 0$.

Proof of proposition B.1.2
With the restriction $q_1 \in (d_1, (\beta+1)d_1 + d_2]$, we can write the objective function as

$$\begin{aligned}
&V_N(q_1)\\
=&-q_1[h(1-\theta)+\theta(w-s)] + d_1\{[(p-w)(\beta+1)+h](1-\theta) - \alpha\beta\theta(p-w)\}\\
&+d_2(p-w)(1-\theta+\alpha\theta)
\end{aligned}$$

Thus the first order derivative of the objective function is

$$V_N'(q_1) = -(1-\theta)h - \theta(w-s) < 0.$$

Therefore $V_N(q_1)$ decreases in q_1 in this subregion.
On the other hand, $S_I \leq S_N$ can be written as

$$q_1(w-c) \geq (1-\alpha)T + (1-\alpha)(w-c)d_2 + \alpha(w-c)(2\beta+1)d_1.$$

Therefore the constraints $S_I \leq S_N$ and $T \geq 0$ can be combined as

$$\begin{aligned}
q_1(w-c) &\geq (1-\alpha)(w-c)d_2 + \alpha(w-c)(2\beta+1)d_1\\
q_1 &\geq (1-\alpha)d_2 + \alpha(2\beta+1)d_1
\end{aligned}$$

Thus

- If $(1-\alpha)d_2+\alpha(2\beta+1)d_1 \leq d_1$, the optimal solutions are $q_1^* = d_1$, and T^* can be any value satisfying the incentive constraint;
- If $d_1 < (1-\alpha)d_2 + \alpha(2\beta+1)d_1 \leq (\beta+1)d_1 + d_2$, then the optimal solutions are $q_1^* = (1-\alpha)d_2 + \alpha(2\beta+1)d_1$ and $T^* = 0$;
- If $(1-\alpha)d_2+\alpha(2\beta+1)d_1 > (\beta+1)d_1+d_2$, there is no feasible solution in this region.

First let us compare compare $(1-\alpha)d_2 + \alpha(2\beta+1)d_1$ and $(\beta+1)d_1 + d_2$. We can show that $(1-\alpha)d_2 + \alpha(2\beta+1)d_1 \leq (\beta+1)d_1 + d_2$.

$$\begin{aligned}
&(1-\alpha)d_2 + \alpha(2\beta+1)d_1 \leq (\beta+1)d_1 + d_2\\
\Leftrightarrow&[\alpha(1+2\beta) - (\beta+1)]d_1 \leq \alpha d_2
\end{aligned}$$

Hence if $\alpha(1+2\beta)-(\beta+1)\le 0$, this condition always holds.
If $\alpha(1+2\beta)-(\beta+1)>0$, this condition is equivalent as $\frac{d_2}{d_1}\ge 2\beta+1-\frac{\beta+1}{\alpha}$.
Because

$$2\beta+1-\frac{\beta+1}{\alpha}-\beta=\beta+1-\frac{\beta+1}{\alpha}\le 0.$$

Thus from $\frac{d_2}{d_1}\ge\beta$ we know that $\frac{d_2}{d_1}\ge 2\beta+1-\frac{\beta+1}{\alpha}$. Therefore $(1-\alpha)d_2+\alpha(2\beta+1)d_1\le(\beta+1)d_1+d_2$ always holds.
Secondly, let us compare $(1-\alpha)d_2+\alpha(2\beta+1)d_1$ and d_1.

$$\begin{aligned}&(1-\alpha)d_2+\alpha(2\beta+1)d_1>d_1\\ \Leftrightarrow&(1-\alpha)d_2>[1-\alpha(2\beta+1)]d_1\end{aligned}$$

- when $\frac{d_2}{d_1}>\frac{1-\alpha-2\alpha\beta}{1-\alpha}$, then $q_1^*=\alpha(2\beta+1)d_1+(1-\alpha)d_2$, and $T^*=0$.
- when $\frac{d_2}{d_1}\le\frac{1-\alpha-2\alpha\beta}{1-\alpha}$, then $q_1^*=d_1$, and T^* can be any value satisfying the incentive constraint. We set $T^*=0$.

Finally we examine the third subregion $q_1\in((\beta+1)d_1+d_2,+\infty)$.

Proposition B.1.3 *In the subregion $q_1\in((\beta+1)d_1+d_2,+\infty)$, $q_1^*=(\beta+1)d_1+d_2$, and T^* can be any value satisfying the incentive constraint. We set $T^*=0$.*

Proof of proposition B.1.3
With the restriction $q_1\in((\beta+1)d_1+d_2,+\infty)$, we can write the objective function as

$$\begin{aligned}V_N(q_1)=&(1-\theta)q_1(-w-h)+(1-\theta)d_1(p+p\beta+h)+(1-\theta)d_2p\\&+\theta(-w+s)q_1+\theta\alpha(p-w)(d_2-\beta d_1).\end{aligned}$$

The first order derivative of the objective function is

$$V_N'(q_1)=-(1-\theta)(w+h)-\theta(w-s)<0,$$

thus $V_N(q_1,T)$ decreases in q_1 in this subregion.
On the other hand, $S_I\le S_N$ can be written as

$$(1-\alpha)(w-c)q_1\ge(1-\alpha)T+(1-2\alpha)(w-c)d_2+\alpha(w-c)\beta d_1.$$

Therefore the constraints $S_I \leq S_N$ and $T \geq 0$ can be combined as

$$q_1 \geq \frac{(1-2\alpha)d_2 + \alpha\beta d_1}{1-\alpha}.$$

Thus

- If $\frac{(1-2\alpha)d_2 + \alpha\beta d_1}{1-\alpha} > (\beta+1)d_1 + d_2$, then $q_1^* = \frac{(1-2\alpha)d_2 + \alpha\beta d_1}{1-\alpha}$ and $T^* = 0$.
- If $\frac{(1-2\alpha)d_2 + \alpha\beta d_1}{1-\alpha} \leq (\beta+1)d_1 + d_2$, then $q_1^* = (\beta+1)d_1 + d_2$, and T^* can be any value satisfying the incentive constraint. We set $T^* = 0$.

Next we show that $\frac{(1-2\alpha)d_2 + \alpha\beta d_1}{1-\alpha} \leq (\beta+1)d_1 + d_2$ always holds.

$$\frac{(1-2\alpha)d_2 + \alpha\beta d_1}{1-\alpha} \leq (\beta+1)d_1 + d_2 \Leftrightarrow [\alpha(1+2\beta) - (\beta+1)]d_1 \leq \alpha d_2.$$

If $\alpha(1+2\beta) - (\beta+1) \leq 0$, then this condition always holds.
If $\alpha(1+2\beta) - (\beta+1) > 0$, then this condition is equivalent as $\frac{d_2}{d_1} \geq (1+2\beta) - \frac{\beta+1}{\alpha}$.
Because

$$(1+2\beta) - \frac{\beta+1}{\alpha} - \beta = \beta + 1 - \frac{\beta+1}{\alpha} \leq 0.$$

Therefore from $\frac{d_2}{d_1} \geq \beta$ we know that $\frac{d_2}{d_1} \geq (1+2\beta) - \frac{\beta+1}{\alpha}$.
In summary, in the subregion $q_1 \in ((\beta+1)d_1 + d_2, +\infty)$, $q_1^* = (\beta+1)d_1 + d_2$, and T^* can be any value satisfying the incentive constraint. We set $T^* = 0$.

Proof of theorem 3.6.1
We have the following results from previous analysis.

- If $d_2 > \frac{1-\alpha(1+2\beta)}{1-\alpha} d_1$,
  - In the subregion $q_1 \in [0, d_1]$, there is no feasible solution (proposition B.1.1).
  - In the subregion $q_1 \in (d_1, (\beta+1)d_1 + d_2]$, $q_1^* = \alpha(1+2\beta)d_1 + (1-\alpha)d_2$, and $T^* = 0$ (proposition B.1.2).

- In the subregion $q_1 \in ((\beta+1)d_1 + d_2, +\infty]$, $q_1^* = (\beta+1)d_1 + d_2$, and T^* can be any value satisfying the incentive constraint. We set $T^* = 0$ (proposition B.1.3).

Therefore if $d_2 > \dfrac{1-\alpha(1+2\beta)}{1-\alpha}d_1$, the global optimal solutions for optimization problem N are $q_1^* = \alpha(1+2\beta)d_1 + (1-\alpha)d_2$, and $T^* = 0$.

- If $d_2 \leq \dfrac{1-\alpha(1+2\beta)}{1-\alpha}d_1$,
  - in the subregion $q_1 \in [0, d_1]$, according to proposition B.1.1,
    * If $\theta \leq \dfrac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, then $q_1^* = d_1$, and T^* can be any value satisfying the incentive constraint. We set $T^* = 0$.
    * If $\theta > \dfrac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, then $q_1^* = \dfrac{1-\alpha}{1-\alpha-2\alpha\beta}d_2$, and $T^* = 0$.
  - In the subregion $q_1 \in (d_1, (\beta+1)d_1 + d_2]$, $q_1^* = d_1$, and T^* can be any value satisfying the incentive constraint. We set $T^* = 0$ (proposition B.1.2) .
  - In the subregion $q_1 \in ((\beta+1)d_1 + d_2, +\infty)$, $q_1^* = (\beta+1)d_1 + d_2$, and T^* can be any value satisfying the incentive constraint. We set $T^* = 0$ (proposition B.1.3).

Therefore if $d_2 \leq \dfrac{1-\alpha(1+2\beta)}{1-\alpha}d_1$, the global optimal solutions for optimization problem N will depend on the value of θ.
If $\theta \leq \dfrac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, then $q_1^* = d_1$, and T^* can be any value satisfying the incentive constraint. We set $T^* = 0$. If $\theta > \dfrac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, then $q_1^* = \dfrac{1-\alpha}{1-\alpha-2\alpha\beta}d_2$, and $T^* = 0$.

B.2 Analysis for optimization problem I

For convenience of analysis, we define

$$f(q_1, T) = \alpha V^1 + (1-\alpha)V^2 - [\alpha V^3 + (1-\alpha)V^4],$$

and write optimization problem I as

$$\begin{aligned}\max_{q_1,T} V_I(q_1,T) =&(1-\theta)V^0+\theta[\alpha V^3+(1-\alpha)V^4]+\theta G(q_1)f(q_1,T)\\ &s.t. S_I \geq S_N,\\ &\quad T\geq 0.\end{aligned}$$

Proof of proposition 3.6.2
In the subregion $q_1 \in [0,d_1]$, $G(q_1)=q_1$, the optimization problem can be written as

$$\begin{aligned}\max_{q_1,T} V_I(q_1,T) =&q_1\{-\theta(w-s)+(p-w)[(1+\beta)(1-\theta)-\alpha\beta\theta]\}\\ &+d_2(p-w)(1-\theta+\theta\alpha)+\theta q_1^2[w-s+\alpha(p-w)(1+2\beta)]\\ &+\theta q_1 d_2(1-\alpha)(p-w)-(1-\alpha)Tq_1\theta\\ s.t.(1-\alpha)T \geq&(w-c)[q_1(1-\alpha-2\alpha\beta)-(1-\alpha)d_2],\\ T\geq&0,\\ q_1<&d_1.\end{aligned}$$

- If $d_2 > \dfrac{1-\alpha-2\alpha\beta}{1-\alpha}d_1$, then in the region $[0,d_1]$, the incentive constraint is always met, thus is redundant, and $T\geq 0$ is binding at the optimum. We plug $T=0$ into the objective function and get

  $$\begin{aligned}V_I(q_1,0) =&q_1\{-\theta(w-s)+(p-w)[(1+\beta)(1-\theta)-\alpha\beta\theta]\}\\ &+d_2(p-w)(1-\theta+\theta\alpha)+\theta q_1^2[w-s+\alpha(p-w)(1+2\beta)]\\ &+\theta q_1 d_2(1-\alpha)(p-w).\end{aligned}$$

  We derive the first order derivative and the second order derivative:

  $$\begin{aligned}\frac{dV_I(q_1,0)}{dq_1} =&-\theta(w-s)+(p-w)[(1+\beta)(1-\theta)-\alpha\beta\theta]\\ &+2\theta q_1[w-s+\alpha(p-w)(1+2\beta)]+\theta d_2(1-\alpha)(p-w),\\ \frac{d^2V_I(q_1,0)}{dq_1^2} =&w-s+\alpha(p-w)(1+2\beta)>0.\end{aligned}$$

  Thus $V_I(q_1,0)$ is convex in q_1.
  Define $q_{1,1}^1$ to be the root of the first order condition, then

  $$q_{1,1}^1=\frac{\theta(w-s)-(p-w)[(1+\beta)(1-\theta)-\alpha\beta\theta]-\theta d_2(1-\alpha)(p-w)}{2\theta[w-s+\alpha(p-w)(1+2\beta)]}$$

We compare $q_{1,1}^1$ with the midpoint of $[0, d_1]$ to get the optimal solutions in this subregion.

- If $q_{1,1}^1 \le \frac{d_1}{2}$, then $q_1^* = d_1$;
- If $q_{1,1}^1 > \frac{d_1}{2}$, then $q_1^* = 0$.

We can write

$$q_{1,1}^1 \le \frac{d_1}{2} \Leftrightarrow d_2 \ge -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{w-s}{(1-\alpha)(p-w)}]d_1 + \frac{\theta(w-s) - (p-w)[(1+\beta)(1-\theta) - \alpha\beta\theta]}{\theta(1-\alpha)(p-w)}.$$

- If $d_2 \le \frac{1-\alpha-2\alpha\beta}{1-\alpha}d_1$ (this implies that $1-\alpha-2\alpha\beta > 0$), then we need to divide $[0, d_1]$ into two parts:
  - In the first part $q_1 \in [0, \frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}]$, the incentive constraint is always met, thus is not binding at the optimum, and $T \ge 0$ is binding. We plug $T = 0$ into the objective function. We find that $V_I(q_1, 0)$ is convex in q_1, and the root of the first order condition is $q_{1,1}^1$.We compare $q_{1,1}^1$ with the midpoint of $[0, \frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}]$ to get the optimal solutions in this part.
    * If $q_{1,1}^1 \le \frac{1}{2} \cdot \frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}$, then the local optimal point in $[0, \frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}]$ is $\frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}$.
    * If $q_{1,1}^1 > \frac{1}{2} \cdot \frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}$, then the local optimal point in $[0, \frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}]$ is 0.

    We can write

$$\begin{aligned} & q_{1,1}^1 \le \frac{1}{2} \cdot \frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta} \\ \Leftrightarrow & d_2 \ge \frac{(1-\alpha-2\alpha\beta)\{\theta(w-s) - (p-w)[(1+\beta)(1-\theta) - \theta\alpha\beta\}}{\theta(1-\alpha)(p-s)} \end{aligned} \tag{B.1}$$

– In the latter part $q_1 \in (\frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}, d_1]$, the incentive constraint is binding, i.e., $(1-\alpha)T^* = (w-c)[q_1(1-\alpha-2\alpha\beta)-(1-\alpha)d_2]$. We plug T^* into the objective function and get

$$f(q_1, T^*) = q_1[(p-c)\alpha(1+2\beta)+c-s] + d_2(1-\alpha)(p-c),$$

$$\begin{aligned} V_I(q_1, T^*) =& q_1\{(p-w)[(1-\theta)(1+\beta)-\theta\alpha\beta] + \theta(-w+s)\} \\ & + d_2(1-\theta+\theta\alpha)(p-w) + \theta G(q_1)f(q_1, T^*). \end{aligned}$$

We derive the first order derivative and the second order derivative of $V_I(q_1, T^*)$ w.r.t. q_1,

$$\begin{aligned} \frac{dV_I(q_1, T^*)}{dq_1} =& q_1 2\theta[(p-c)\alpha(1+2\beta)+c-s] \\ & + \theta d_2(1-\alpha)(p-c) + (p-w)[(1-\theta)(1+\beta)-\theta\alpha\beta] \\ & + \theta(-w+s), \\ \frac{d^2V_I(q_1, T^*)}{dq_1^2} =& 2\theta[(p-c)(\alpha+2\alpha\beta)+c-s] > 0 \end{aligned}$$

This implies that $V_I(q_1, T^*)$ is convex.
Define $q_{1,1}^2$ to be the root of first order condition,

$$q_{1,1}^2 = \frac{\theta(w-s)-(p-w)[(1-\theta)(1+\beta)-\theta\alpha\beta]-\theta d_2(1-\alpha)(p-c)}{2\theta[c-s+(p-c)\alpha(1+2\beta)]}.$$

We compare $q_{1,1}^2$ with the midpoint of $(\frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}, d_1]$ to get the optimal solutions is this part.

- If $q_{1,1}^2 \le \frac{1}{2}[\frac{(1-\alpha)d_2}{(1-\alpha-2\alpha\beta)} + d_1]$, the local optimal q_1 in $(\frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}, d_1]$ is d_1.
- If $q_{1,1}^2 > \frac{1}{2}[\frac{(1-\alpha)d_2}{(1-\alpha-2\alpha\beta)} + d_1]$, the local optimal q_1 in $(\frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}, d_1]$ is $\frac{(1-\alpha)d_2}{(1-\alpha-2\alpha\beta)}$.

We can write

$$\begin{aligned}
q_{1,1}^2 \leq & \frac{1}{2}[\frac{(1-\alpha)d_2}{(1-\alpha-2\alpha\beta)} + d_1] \\
\Leftrightarrow d_2 \geq & -\frac{(1-\alpha-2\alpha\beta)[c-s+(p-c)\alpha(1+2\beta)]}{(1-\alpha)(p-s)}d_1 \\
& + \frac{(1-\alpha-2\alpha\beta)\{\theta(w-s)-(p-w)[(1+\beta)(1-\theta)-\theta\alpha\beta\}}{\theta(1-\alpha)(p-s)}
\end{aligned} \tag{B.2}$$

Now we can summarize the case when $d_2 \leq \frac{1-\alpha-2\alpha\beta}{1-\alpha}d_1$. Notice that inequality B.1 can imply inequality B.2.

- When $d_2 \geq \frac{(1-\alpha-2\alpha\beta)\{\theta(w-s)-(p-w)[(1+\beta)(1-\theta)-\theta\alpha\beta]\}}{\theta(1-\alpha)(p-s)}$, in the first part $[0, \frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}]$, the local optimal q_1 is $\frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}$; in the latter part $(\frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}, d_1]$, the local optimal q_1 is d_1. Therefore in the region $[0, d_1]$, the optimal q_1 is d_1.
- When

$$\begin{aligned}
d_2 \leq & -\frac{(1-\alpha-2\alpha\beta)[c-s+(p-c)\alpha(1+2\beta)]}{(1-\alpha)(p-s)}d_1 \\
& + \frac{(1-\alpha-2\alpha\beta)\{\theta(w-s)-(p-w)[(1+\beta)(1-\theta)-\theta\alpha\beta\}}{\theta(1-\alpha)(p-s)},
\end{aligned}$$

  in the first part $[0, \frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}]$, the local optimal q_1 is 0; in the latter part $q_1 \in (\frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}, d_1]$, the local optimal q_1 is $\frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}$. Therefore in the region $[0, d_1]$, the optimal q_1 is 0.

– When

$$-\frac{(1-\alpha-2\alpha\beta)[c-s+(p-c)\alpha(1+2\beta)]}{(1-\alpha)(p-s)}d_1$$
$$+\frac{(1-\alpha-2\alpha\beta)\{\theta(w-s)-(p-w)[(1+\beta)(1-\theta)-\theta\alpha\beta]\}}{\theta(1-\alpha)(p-s)}$$
$$< d_2 <$$
$$\frac{(1-\alpha-2\alpha\beta)\{\theta(w-s)-(p-w)[(1+\beta)(1-\theta)-\theta\alpha\beta]\}}{\theta(1-\alpha)(p-s)},$$

in the part $q_1 \in [0, \frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta})$, the optimal q_1 is 0; in the part $q_1 \in (\frac{(1-\alpha)d_2}{1-\alpha-2\alpha\beta}, d_1]$, the optimal q_1 is d_1. we need to compare the objective value at 0 and d_1.

Overall, when $d_2 \leq \frac{1-\alpha-2\alpha\beta}{1-\alpha}d_1$, the optimal q_1 is either $q_1^* = 0$ (with $T \geq 0$ binding) or $q_1^* = d_1$ (with the incentive constraint binding). We only need to compare the objective value at these two points to get the optimal solution.
The objective value at $q_1^* = 0$ (with $T \geq 0$ binding) is

$$V_I(0,0) = d_2(p-w)(1-\theta+\theta\alpha).$$

The objective value at $q_1^* = d_1$ (with the incentive constraint binding) is

$$\begin{aligned}V_I(d_1,T^*) =& d_1\{(p-w)[(1-\theta)(1+\beta)-\theta\alpha\beta]+\theta(-w+s)\}\\&+d_2(p-w)(1-\theta+\theta\alpha)+\theta d_1^2[(p-c)\alpha(1+2\beta)+c-s]\\&+\theta d_1 d_2(1-\alpha)(p-c).\end{aligned}$$

The difference of these two values

$$\begin{aligned}V_I(d_1,T^*)-V_I(0,0) =& d_1\{(p-w)[(1-\theta)(1+\beta)-\theta\alpha\beta]\\&+\theta(-w+s)\}+\theta d_1^2[(p-c)\alpha(1+2\beta)+c-s]\\&+\theta d_1 d_2(1-\alpha)(p-c).\end{aligned}$$

Thus

$$
\begin{aligned}
V_I(d_1, T^*) \geq & V_I(0,0) \\
\Leftrightarrow d_2 \geq & -\frac{(p-c)\alpha(1+2\beta)+c-s}{(1-\alpha)(p-c)}d_1 \\
& + \frac{\theta(w-s)-(p-w)[(1-\theta)(1+\beta)-\theta\alpha\beta]}{\theta(1-\alpha)(p-c)}.
\end{aligned}
$$

Thus we proved proposition 3.6.2.

Proof of proposition 3.6.3 :
In the subregion $q_1 \in (d_1, 1]$, $G(q_1) = q_1$, the optimization problem I can be written as

$$
\begin{aligned}
\max_{q_1, T} V_I(q_1, T) = & - q_1[(1-\theta)h + \theta(w-s)] + d_2(p-w)(1-\theta+\theta\alpha) \\
& + d_1\{(1-\theta)h + (p-w)[(1+\beta)(1-\theta) - \alpha\beta\theta]\} + \theta q_1 f(q_1, T) \\
s.t. (1-\alpha)T \geq & (w-c)[q_1 - \alpha(1+2\beta)d_1 - (1-\alpha)d_2] \\
T \geq & 0, \\
d_1 < & q_1 \leq 1,
\end{aligned}
$$

where

$$
\begin{aligned}
f(q_1, T) = & q_1(-\alpha h + w - s) + d_1\alpha[(p-w)(1+2\beta)+h] \\
& + d_2(1-\alpha)(p-w) - (1-\alpha)T.
\end{aligned}
$$

Note the right hand side of the incentive constraint is nonnegative if and only if $q_1 \geq \alpha(1+2\beta)d_1 + (1-\alpha)d_2$. Depending on the position of $\alpha(1+2\beta)d_1 + (1-\alpha)d_2$, we will have the following three scenarios:

- When $\alpha(1+2\beta)d_1 + (1-\alpha)d_2 \leq d_1$, then in the subregion $(d_1, 1]$, the right hand side of the incentive constraint is nonnegative, thus is binding at the optimum.

- When $\alpha(1+2\beta)d_1 + (1-\alpha)d_2 > 1$, then in the subregion $(d_1, 1]$, the right hand side of the incentive constraint is negative, thus $T \geq 0$ is binding at the optimum.

- When $d_1 < \alpha(1+2\beta)d_1 + (1-\alpha)d_2 \leq 1$, then we need to divide the subregion $(d_1, 1]$ into two parts. In the first part $q_1 \in (d_1, \alpha(1+2\beta)d_1 + (1-\alpha)d_2]$, $T \geq 0$ is binding at the optimum. In the latter part $q_1 \in (\alpha(1+2\beta)d_1 + (1-\alpha)d_2, 1]$, the incentive constraint is binding.

Next we discuss these three scenarios respectively.

- When $\alpha(1+2\beta)d_1 + (1-\alpha)d_2 \le d_1$, i.e., $d_2 \le \frac{1-\alpha-2\alpha\beta}{1-\alpha}d_1$, then in the subregion $(d_1, 1]$, the incentive constraint is binding at the optimum. We find that $V_I(q_1, T)$ is convex, and $q^1_{1,2}$ is the root to the first order condition. We compare $q^1_{1,2}$ with the midpoint of $(d_1, 1]$ to get the optimal solution.

  – If $q^1_{1,2} \le \frac{1}{2}(d_1+1)$, $q_1^* = 1$.

  – If $q^1_{1,2} > \frac{1}{2}(d_1+1)$, $q_1^* = d_1$.

  We can write

  $$q^1_{1,2} \le \frac{1}{2}(d_1+1)$$
  $$\Leftrightarrow d_2 \ge -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{c-s}{(1-\alpha)(p-c)}]d_1 + \frac{\theta(w-c)+h(1-\theta+\alpha\theta)}{\theta(1-\alpha)(p-c)}.$$

  We define Line 6 as

  $$d_2 = -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{c-s}{(1-\alpha)(p-c)}]d_1 + \frac{\theta(w-c)+h(1-\theta+\alpha\theta)}{\theta(1-\alpha)(p-c)}.$$

- When $\alpha(1+2\beta)d_1 + (1-\alpha)d_2 > 1$, i.e., $d_2 > -\frac{\alpha(1+2\beta)}{1-\alpha}d_1 + \frac{1}{1-\alpha}$, in the subregion $(d_1, 1]$, $T \ge 0$ is binding at the optimum. We plug $T = 0$ into the objective function, and derive the first order derivative and second order derivative,

  $$\begin{aligned}\frac{dV_I(q_1,0)}{dq_1} =& -(1-\theta)h - \theta(w-s) + 2\theta q_1(-\alpha h + w - s)\\ &+ \theta d_1\alpha[(p-w)(1+2\beta)+h] + \theta d_2(1-\alpha)(p-w),\\ \frac{d^2V_I(q_1,0)}{dq_1^2} =& -\alpha h + w - s > 0.\end{aligned}$$

  Thus $V_I(q_1, 0)$ is convex. Define $q^2_{1,2}$ to be the root of the first order condition, then

  $$q^2_{1,2} = \frac{\theta(w-s)+h(1-\theta)-d_1\theta\alpha[(p-w)(1+2\beta)+h]-\theta d_2(1-\alpha)(p-w)}{2\theta[-\alpha h + w - s]}.$$

  We compare $q^2_{1,2}$ with the midpoint of $(d_1, 1]$ and get the optimal solution.

- If $q_{1,2}^2 \leq \frac{1}{2}(d_1+1)$, $q_1^* = 1$.
- If $q_{1,2}^2 > \frac{1}{2}(d_1+1)$, $q_1^* = d_1$.

We can write

$$q_{1,2}^2 \leq \frac{1}{2}(d_1+1)$$
$$\Leftrightarrow d_2 \geq -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{w-s}{(1-\alpha)(p-w)}]d_1 + \frac{h(1-\theta+\alpha\theta)}{\theta(1-\alpha)(p-w)}.$$

Define Line 3 as

$$d_2 = -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{w-s}{(1-\alpha)(p-w)}]d_1 + \frac{h(1-\theta+\alpha\theta)}{\theta(1-\alpha)(p-w)}.$$

- When $d_1 < \alpha(1+2\beta)d_1 + (1-\alpha)d_2 \leq 1$, i.e., $d_2 \leq -\frac{\alpha(1+2\beta)}{1-\alpha}d_1 + \frac{1}{1-\alpha}$.
  - In the region $q_1 \in (d_1, \alpha(1+2\beta)d_1 + (1-\alpha)d_2]$, $T \geq 0$ is binding. We plug $T = 0$ into the objective function. We find that $V_I(q_1, 0)$ is convex, and $q_{1,2}^2$ is the root of the first order condition. We compare $q_{1,2}^2$ with the midpoint of $(d_1, \alpha(1+2\beta)d_1 + (1-\alpha)d_2]$ to get the optimal solution.
    * When $q_{1,2}^2 \leq \frac{1}{2}[d_1 + \alpha(1+2\beta)d_1 + (1-\alpha)d_2]$, then $q_1^* = \alpha(1+2\beta)d_1 + (1-\alpha)d_2$.
    * When $q_{1,2}^2 > \frac{1}{2}[d_1 + \alpha(1+2\beta)d_1 + (1-\alpha)d_2]$, then $q_1^* = d_1$.

  We can write

$$\begin{aligned} q_{1,2}^2 \leq& \frac{1}{2}[d_1 + \alpha(1+2\beta)d_1 + (1-\alpha)d_2] \\ \Leftrightarrow d_2 \geq& -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{w-s}{(1-\alpha)(p-\alpha h-s)}]d_1 \\ &+ \frac{\theta(w-s)+h(1-\theta)}{\theta(1-\alpha)(p-\alpha h-s)} \end{aligned}$$

  Define Line 4 as

$$\begin{aligned} d_2 =& -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{w-s}{(1-\alpha)(p-\alpha h-s)}]d_1 \\ &+ \frac{\theta(w-s)+h(1-\theta)}{\theta(1-\alpha)(p-\alpha h-s)} \end{aligned}$$

- In the region $q_1 \in [\alpha(1+2\beta)d_1 + (1-\alpha)d_2, 1]$, the incentive constraint is binding. We plug the binding incentive constraint into the objective function.

$$\begin{aligned} f(q_1, T^*) =& q_1(-\alpha h + c - s) \\ &+ d_1\alpha[(p-c)(1+2\beta) + h] + d_2(1-\alpha)(p-c), \end{aligned}$$

$$\begin{aligned} V_I(q_1, T^*) =& -q_1[(1-\theta)h + \theta(w-s)] \\ &+ d_1\{(p-w)[(1+\beta)(1-\theta) - \alpha\beta\theta] + h(1-\theta)\} \\ &+ d_2(p-w)(1-\theta+\alpha\theta) + \theta q_1 f(q_1, T^*) \end{aligned}$$

We then derive the first order derivative and the second order derivative.

$$\begin{aligned} \frac{dV_I(q_1, T^*)}{dq_1} =& q_1 2\theta(-\alpha h + c - s) + d_1\theta\alpha[(p-c)(1+2\beta) + h] \\ &+ d_2\theta(1-\alpha)(p-c) - \theta(w-s) - h(1-\theta), \\ \frac{d^2V_I(q_1, T^*)}{dq_1^2} =& 2\theta(-\alpha h + c - s) > 0. \end{aligned}$$

Thus $V_I(q_1, T)$ is convex. We define $q_{1,2}^1$ as the root of the first order condition.

$$q_{1,2}^1 = \frac{\theta(w-s) + h(1-\theta) - d_1\theta\alpha[(p-c)(1+2\beta)+h] - \theta d_2(1-\alpha)(p-c)}{2\theta(-\alpha h + c - s)}.$$

- When $q_{1,2}^1 \le \frac{1}{2}[\alpha(1+2\beta)d_1 + (1-\alpha)d_2 + 1]$, then $q_1^* = 1$.
- When $q_{1,2}^1 > \frac{1}{2}[\alpha(1+2\beta)d_1 + (1-\alpha)d_2 + 1]$, then $q_1^* = \alpha(1+2\beta)d_1 + (1-\alpha)d_2$.

We can write

$$\begin{aligned} q_{1,2}^1 \le& \frac{1}{2}[\alpha(1+2\beta)d_1 + (1-\alpha)d_2 + 1] \\ \Leftrightarrow d_2 \ge& -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{\alpha h}{(1-\alpha)(p-\alpha h - s)}]d_1 \\ &+ \frac{\theta(w-c) + h(1-\theta+\theta\alpha)}{\theta(1-\alpha)(p-\alpha h - s)}. \end{aligned}$$

Define Line 5 as

$$d_2 = -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{\alpha h}{(1-\alpha)(p-\alpha h - s)}]d_1 + \frac{\theta(w-c) + h(1-\theta+\theta\alpha)}{\theta(1-\alpha)(p-\alpha h - s)}.$$

Therefore when $d_2 \leq -\dfrac{\alpha(1+2\beta)}{1-\alpha}d_1 + \dfrac{1}{1-\alpha}$:

- In the area that is above both Line 4 and Line 5, in the first part $(d_1, \alpha(1+2\beta)d_1 + (1-\alpha)d_2]$, $q_1^* = \alpha(1+2\beta)d_1 + (1-\alpha)d_2]$. In the latter part $(\alpha(1+2\beta)d_1 + (1-\alpha)d_2, 1]$, $q_1^* = 1$. Therefore in the subregion $(d_1, 1]$, $q_1^* = 1$ and $T^* = \dfrac{w-c}{1-\alpha}[1-\alpha(1+2\beta)d_1 - (1-\alpha)d_2]$.
- In the area that is below both Line 4 and Line 5, in the first part $(d_1, \alpha(1+2\beta)d_1 + (1-\alpha)d_2]$, $q_1^* = d_1$. In the latter part $(\alpha(1+2\beta)d_1+(1-\alpha)d_2, 1]$, $q_1^* = \alpha(1+2\beta)d_1+(1-\alpha)d_2$. Therefore in the subregion $(d_1, 1]$, $q_1^* = d_1$ and $T^* = 0$.
- In the area that is above Line 4 and below Line 5, in the first part $(d_1, \alpha(1+2\beta)d_1+(1-\alpha)d_2]$, $q_1^* = \alpha(1+2\beta)d_1+(1-\alpha)d_2]$. In the latter part $(\alpha(1+2\beta)d_1+(1-\alpha)d_2, 1]$, $q_1^* = \alpha(1+2\beta)d_1+(1-\alpha)d_2$. Therefore in the subregion $(d_1, 1]$, $q_1^* = \alpha(1+2\beta)d_1 + (1-\alpha)d_2$ and $T^* = 0$.
- In the area that is below Line 4 and above Line 5, in the first part $(d_1, \alpha(1+2\beta)d_1 + (1-\alpha)d_2]$, $q_1^* = d_1$. In the latter part $(\alpha(1+2\beta)d_1 + (1-\alpha)d_2, 1]$, $q_1^* = 1$. Therefore in the subregion $(d_1, 1]$, q_1^* could be d_1 or 1. If $q_1^* = d_1$, then $T^* = 0$. If $q_1^* = 1$, then $T^* = \dfrac{w-c}{1-\alpha}[1-\alpha(1+2\beta)d_1 - (1-\alpha)d_2]$.

Thus we proved proposition 3.6.3.

Proof of proposition 3.6.4:
In the subregion $q_1 \in (1, (\beta+1)d_1+d_2]$, $G(q_1) = 1$, the optimization problem can be written as

$$\begin{aligned}
&\max_{q_1,T} V_I(q_1, T)\\
s.t.(1-\alpha)T &\geq (w-c)[q_1 - \alpha(1+2\beta)d_1 - (1-\alpha)d_2],\\
T &\geq 0,\\
1 < &q_1 \leq (\beta+1)d_1 + d_2.
\end{aligned}$$

Note that the right hand side of the incentive constraint is nonpositive if and only if $q_1 \leq \alpha(1+2\beta)d_1 + (1-\alpha)d_2$.

- If $\alpha(1+2\beta)d_1 + (1-\alpha)d_2 > (\beta+1)d_1 + d_2$, then in the subregion $(1, (\beta+1)d_1 + d_2]$, the incentive constraint is always met, thus is not binding at the optimum. Because the objective function decreases in T, $T \geq 0$ is binding at the optimum. We plug $T = 0$ into the objective function,

$$\begin{aligned}V_I(q_1, 0) = &- q_1[(1-\theta)h + \theta(w-s)] + d_2(p-w)(1-\theta+\theta\alpha)\\ &+ d_1\{(1-\theta)h + (p-w)[(1+\beta)(1-\theta) - \alpha\beta\theta]\}\\ &+ \theta\{q_1(-\alpha h + w - s) + d_1\alpha[(p-w)(1+2\beta)+h]\\ &+ d_2(1-\alpha)(p-w)\}.\end{aligned}$$

We derive the first order derivative

$$\frac{dV_I(q_1,0)}{dq_1} = -(1-\theta+\alpha\theta)h < 0.$$

Therefore $V_I(q_1, 0)$ decreases in q_1^*, the optimal solutions are $q_1^* = 1$ and $T^* = 0$.

- If $\alpha(1+2\beta)d_1 + (1-\alpha)d_2 \leq 1$, then in the subregion $(1, (\beta+1)d_1 + d_2]$, the incentive constraint is binding at the optimum. We plug $(1-\alpha)T^* = (w-c)[q_1 - \alpha(1+2\beta)d_1 - (1-\alpha)d_2]$ into the objective function,

$$\begin{aligned}V_I(q_1, T^*) =& q_1[-\theta(w-s) - h(1-\theta)]\\ &+ d_1\{[(p-w)(\beta+1)+h](1-\theta) - \alpha\beta\theta(p-w)\}\\ &+ d_2(p-w)(1-\theta+\alpha\theta) + \theta\{q_1(-\alpha+c-s)\\ &+ d_1\alpha[(p-c)(1+2\beta)+h] + d_2(1-\alpha)(p-c)\}.\end{aligned}$$

  We derive the first order derivative

$$\frac{dV_I(q_1,T^*)}{dq_1} = -\theta(w-c) - h(1-\theta+\theta\alpha) < 0.$$

  Thus $V_I(q_1, T^*)$ decreases in q_1^*, the optimal solutions are $q_1^* = 1$ and $T^* = \frac{w-c}{1-\alpha}[1-\alpha(1+2\beta)d_1 - (1-\alpha)d_2]$.

- If $1 < \alpha(1+2\beta)d_1 + (1-\alpha)d_2 \leq (\beta+1)d_1 + d_2$, then we need to divide the subregion $(1, (\beta+1)d_1 + d_2]$ into two parts, the first part is $(1, \alpha(1+2\beta)d_1 + (1-\alpha)d_2]$, the second part is $(\alpha(1+2\beta)d_1 + (1-\alpha)d_2, (\beta+1)d_1 + d_2]$. In the first part, at the optimum $T \geq 0$ is binding, we have shown that the objective function decreases in q_1. In the second part, at the optimum the incentive constraint is binding, we have shown that the objective function also decreases in q_1. Therefore in the subregion$(1, (\beta+1)d_1 + d_2]$, the objective function decreases in q_1, the optimal solutions are $q_1^* = 1$ and $T^* = 0$.

Thus we proved proposition 3.6.4.

Proof of proposition 3.6.5:
In the subregion $q_1 \in ((\beta+1)d_1 + d_2, +\infty)$, $G(q_1) = 1$, the optimization problem can be written as

$$\begin{aligned} &\max_{q_1,T} V_I(q_1,T) \\ s.t.(1-\alpha)T &\geq (1-\alpha)(w-c)(q_1-d_2) + \alpha(w-c)(d_2-\beta d_1) \\ T &\geq 0 \\ q_1 &> (\beta+1)d_1 + d_2. \end{aligned}$$

Note that $(1-\alpha)(w-c)(q_1-d_2) + \alpha(w-c)(d_2-\beta d_1) \geq 0$, therefore the incentive constraint is binding at the optimum. We plug $(1-\alpha)T^* = (1-\alpha)(w-c)(q_1-d_2) + \alpha(w-c)(d_2-\beta d_1)$ into the objective function,

$$\begin{aligned} f(q_1,T^*) =& q_1[-\alpha h + (1-\alpha)c - s] + d_1[(\alpha + 2\alpha\beta)p + \alpha h - \alpha\beta c] \\ &+ d_2[(1-\alpha)w - (1-2\alpha)c], \end{aligned}$$

$$\begin{aligned} V_I(q_1,T^*) =& q_1[-w - h(1-\theta) + \theta s] + d_1[(p + p\beta + h)(1-\theta) - \alpha\beta\theta(p-w)] \\ &+ d_2[p(1-\theta) + \alpha\theta(p-w)] + \theta f(q_1,T^*). \end{aligned}$$

We derive the first order derivative,

$$\frac{dV_I(q_1,T^*)}{dq_1} = -w - h(1-\theta) - \theta\alpha h + \theta(1-\alpha)c < 0$$

Therefore $V_I(q_1,T^*)$ decreases in this subregion, and the optimal q_1 is $q_1^* = (\beta+1)d_1 + d_2$. Thus we proved proposition 3.6.5.

Proof of theorem 3.6.6:
When $d_2 \leq \dfrac{1-\alpha(1+2\beta)}{1-\alpha} d_1$, from proposition 3.6.4 and proposition 3.6.5, we know that the objective function of optimization problem I decreases when $q_1 > 1$. Thus the optimal q_1 for optimization problem I will be in the region $q_1 \in [0,1]$. From proposition 3.6.2, in the subregion $q_1 \in [0, d_1]$,

- When (d_1, d_2) is above Line 2, then $q_1^* = d_1$ and T^* makes the incentive constraint binding.
- When (d_1, d_2) is below Line 2, then $q_1^* = 0$ and $T^* = 0$.

The equation for Line 2 is

$$d_2 = -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{c-s}{(1-\alpha)(p-c)}]d_1 + \frac{\theta(w-s)-(p-w)[(1+\beta)(1-\theta)-\alpha\beta\theta]}{\theta(1-\alpha)(p-c)}.$$

From proposition 3.6.3, in the subregion $q_1 \in (d_1, 1]$,

- When (d_1, d_2) is above Line 6, then $q_1^* = 1$ and T^* makes the incentive constraint binding.
- When (d_1, d_2) is below Line 6, then $q_1^* = d_1$ and T^* makes the incentive constraint binding.

The equation for Line 6 is

$$d_2 = -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{c-s}{(1-\alpha)(p-c)}]d_1 + \frac{\theta(w-c)+h(1-\theta+\alpha\theta)}{\theta(1-\alpha)(p-c)}.$$

Notice that Line 2 and Line 6 has the same slope. Compare the intercept of Line 2 and Line 6,

$$\begin{aligned}
&\frac{\theta(w-s)-(p-w)[(1+\beta)(1-\theta)-\alpha\beta\theta]}{\theta(1-\alpha)(p-c)} \leq \frac{\theta(w-c)+h(1-\theta+\alpha\theta)}{\theta(1-\alpha)(p-c)} \\
\Leftrightarrow &\theta(w-s)-(p-w)[(1+\beta)(1-\theta)-\alpha\beta\theta] \leq \theta(w-c)+h(1-\theta+\alpha\theta) \\
\Leftrightarrow &\theta \leq \frac{(p-w)(1+\beta)+h}{(p-w)(1+\beta+\alpha\beta)+c-s+(1-\alpha)h}.
\end{aligned}$$

Note that we have

$$\frac{(p-w)(1+\beta)+h}{(p-w)(1+\beta+\alpha\beta)+c-s+(1-\alpha)h} > \frac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}.$$

Therefore we have the following results.

- If $\theta \leq \frac{(p-w)(1+\beta)+h}{(p-w)(1+\beta+\alpha\beta)+c-s+(1-\alpha)h}$, then Line 2's intercept is less than Line 6' intercept.
  - When (d_1, d_2) is above Line 6, in the subregion $q_1 \in [0, d_1]$, $q_1^* = d_1$ and T^* makes the incentive constraint binding. In the subregion $q_1 \in (d_1, 1]$, $q_1^* = 1$ and T^* makes the incentive constraint binding. Therefore in the region $q_1 \in [0, 1]$, $q_1^* = 1$ and T^* makes the incentive constraint binding.

- When (d_1, d_2) is below Line 2, in the subregion $q_1 \in [0, d_1]$, $q_1^* = 0$ and $T^* = 0$. In the subregion $q_1 \in (d_1, 1]$, $q_1^* = d_1$ and T^* makes the incentive constraint binding. Therefore in the region $q_1 \in [0, 1]$, $q_1^* = 0$ and $T^* = 0$.
- When (d_1, d_2) is above Line 2 and below Line 6, in the subregion $q_1 \in [0, d_1]$, $q_1^* = d_1$ and T^* makes the incentive constraint binding. In the subregion $q_1 \in (d_1, 1]$, $q_1^* = d_1$ and T^* makes the incentive constraint binding. Therefore in the region $q_1 \in [0, 1]$, $q_1^* = d_1$ and T^* makes the incentive constraint binding.

• If $\theta > \dfrac{(p-w)(1+\beta)+h}{(p-w)(1+\beta+\alpha\beta)+c-s+(1-\alpha)h}$, then Line 2 has a bigger intercept than Line 6. Then

- When (d_1, d_2) is above Line 2, in the subregion $q_1 \in [0, d_1]$, $q_1^* = d_1$ and T^* makes the incentive constraint binding. In the subregion $q_1 \in (d_1, 1]$, $q_1^* = 1$ and T^* makes the incentive constraint binding. Therefore in the region $q_1 \in [0, 1]$, $q_1^* = 1$ and T^* makes the incentive constraint binding.
- When (d_1, d_2) is below Line 6, in the subregion $q_1 \in [0, d_1]$, $q_1^* = 0$ and $T^* = 0$. In the subregion $q_1 \in (d_1, 1]$, $q_1^* = d_1$ and T^* makes the incentive constraint binding. Therefore in the region $q_1 \in [0, 1]$, $q_1^* = 0$ and $T^* = 0$.
- When (d_1, d_2) is above Line 6 and below Line 2, in the subregion $q_1 \in [0, d_1]$, $q_1^* = 0$ and $T^* = 0$. In the subregion $q_1 \in (d_1, 1]$, $q_1^* = 1$ and T^* makes the incentive constraint binding. Therefore in the region $q_1 \in [0, 1]$, we need to compare the objective value $V_I(0, 0)$ and $V_I(1, T^*)$.

$$\begin{aligned} V_I(0,0) =& d_2(p-w)(1-\theta+\theta\alpha), \\ V_I(1,T^*) =& -[(1-\theta)h+\theta(w-s)] + d_2(p-w)(1-\theta+\alpha\theta) \\ &+ d_1\{(1-\theta)h+(p-w)[(1+\beta)(1-\theta)-\alpha\beta\theta]\} \\ &+ \theta(c-s-\alpha h) + \theta d_1\alpha[(p-c)(1+2\beta)+h] \\ &+ \theta d_2(1-\alpha)(p-c). \end{aligned}$$

Therefore

$$\begin{aligned} & V_I(1,T^*) - V_I(0,0) \\ =& -[(1-\theta)h+\theta(w-s)] \\ &+ d_1\{(1-\theta)h+(p-w)[(1+\beta)(1-\theta)-\alpha\beta\theta]\} \\ &+ \theta(c-s-\alpha h) + \theta d_1\alpha[(p-c)(1+2\beta)+h] + \theta d_2(1-\alpha)(p-c). \end{aligned}$$

We have

$$V_I(1,T^*) > V_I(0,0) \Leftrightarrow d_2 > -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{\alpha h}{(1-\alpha)(p-c)} + \frac{(1-\theta)h + (p-w)[(1+\beta)(1-\theta) - \alpha\beta\theta]}{\theta(1-\alpha)(p-c)}]d_1 + \frac{\theta(w-c) + h(1-\theta+\alpha\theta)}{\theta(1-\alpha)(p-c)}.$$

Define Line 7 as

$$\begin{aligned} d_2 = -&[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{\alpha h}{(1-\alpha)(p-c)} \\ &+ \frac{(1-\theta)h + (p-w)[(1+\beta)(1-\theta) - \alpha\beta\theta]}{\theta(1-\alpha)(p-c)}]d_1 \\ &+ \frac{\theta(w-c) + h(1-\theta+\alpha\theta)}{\theta(1-\alpha)(p-c)}. \end{aligned}$$

Recall that the equation for Line 6 is

$$d_2 = -[\frac{\alpha(1+2\beta)}{1-\alpha} + \frac{c-s}{(1-\alpha)(p-c)}]d_1 + \frac{\theta(w-c) + h(1-\theta+\alpha\theta)}{\theta(1-\alpha)(p-c)}.$$

Note that Line 7 and Line 6 has the same intercept. Compare the slope of Line 7 and Line 6, we have

$$\begin{aligned} &\frac{\alpha h}{(1-\alpha)(p-c)} + \frac{(1-\theta)h + (p-w)[(1+\beta)(1-\theta) - \alpha\beta\theta]}{\theta(1-\alpha)(p-c)} \\ &< \frac{c-s}{(1-\alpha)(p-c)} \\ \Leftrightarrow &\theta > \frac{(p-w)(1+\beta) + h}{(p-w)(1+\beta+\alpha\beta) + c - s + (1-\alpha)h}. \end{aligned}$$

Therefore Line 6 is steeper than Line 7. Line 7 divides the region between Line 6 and Line 2 into two parts. In the part that is above 7, $q_1^* = 1$ and $T^* = 0$. In the part that is below Line 7, $q_1^* = 0$ and $T^* = 0$.

We proved theorem 3.6.6.

Proof of theorem 3.6.6:
When $d_2 > \frac{1-\alpha(1+2\beta)}{1-\alpha}d_1$, from proposition 3.6.4 and proposition 3.6.5, we know that the objective function of optimization problem I decreases when $q_1 > 1$. Thus the optimal q_1 for optimization problem I will be in the region $q_1 \in [0,1]$.

- If $\theta \leq \dfrac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, both Line 1 and Line 2 has negative intercepts, therefore according to proposition 3.6.2, in the subregion $q_1 \in [0, d_1]$, the optimal $q_1^* = d_1$. Therefore the optimal q_1^* for optimization problem I is in the region $[d_1, 1]$, and the possible values are d_1, 1, and $\alpha(1+2\beta)d_1 + (1-\alpha)d_2$(proposition3.6.3).

- If $\theta > \dfrac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, according to proposition 3.6.2 and proposition3.6.3, the possible values for optimal q_1^* are 0, d_1, 1, and $\alpha(1+2\beta)d_1 + (1-\alpha)d_2$.

We proved theorem 3.6.7.

B.3 Analysis for overall optimal solutions for the OEM

Proof of theorem 3.6.8:
First, let us compare the optimal expected profit for the OEM in optimization problem N and optimization problem I when $d_2 > \dfrac{1-\alpha(1+2\beta)}{1-\alpha}d_1$. From theorem 3.6.1, we know that the optimal q_1 for optimization problem N is $q_1^* = \alpha(1+2\beta)d_1 + (1-\alpha)d_2$ and $T^* = 0$. At the optimum, the supplier is indifferent between inform or not inform. Next we show that given $q_1^* = \alpha(1+2\beta)d_1 + (1-\alpha)d_2$ and $T^* = 0$, the OEM can do strictly better if the supplier informs. Therefore the OEM strictly prefers the supplier to inform. Note that $d_1 < \alpha(1+2\beta)d_1 + (1-\alpha)d_2 \leq (\beta+1)d_1 + d_2$. Because

$$\begin{aligned} V_I(q_1, 0) = &- [h(1-\theta) + \theta(w-s)]q_1 + d_1\{(p-w)[(1+\beta)(1-\theta) - \alpha\beta\theta] \\ &+ h(1-\theta)\} + d_2(p-w)(1-\theta+\alpha\theta) + \theta G(q_1)f(q_1, 0). \end{aligned}$$

The difference between this two

$$V_I(q_1, 0) - V_N(q_1) = \theta G(q_1)f(q_1, 0),$$

where

$$\begin{aligned} f(q_1, 0) = &q_1(-\alpha h + w - s) + d_1\alpha[(p-w)(1+2\beta) + h] \\ &+ d_2(1-\alpha)(p-w). \end{aligned}$$

Because $G(q_1) > 0$ and $f(q_1, 0) > 0$, thus $V_I(q_1, 0) > V_N(q_1)$. Thus it is optimal for the OEM to direct the supplier to inform when $d_2 >$

$\frac{1-\alpha(1+2\beta)}{1-\alpha}d_1$.

Secondly, let us compare the optimal expected profit for the OEM in optimization problem N and optimization problem I when $d_2 \le \frac{1-\alpha(1+2\beta)}{1-\alpha}d_1$. From theorem 3.6.1 and theorem 3.6.6, we have the following results.

- If $\theta \le \frac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, the optimal solutions for the OEM in optimization problem N is $q_1^* = d_1$ and $T*$ can be any value satisfying $0 \le T^* \le \frac{w-c}{1-\alpha}[(1-\alpha-2\alpha\beta)d_1-(1-\alpha)d_2]$. The optimal objective value for optimization problem N is

$$
\begin{aligned}
V_N^* =& d_1\{(p-w)[(1+\beta)(1-\theta)-\theta\alpha\beta]+\theta(-w+s)\} \\
&+ d_2(p-w)(1-\theta+\alpha\theta).
\end{aligned}
$$

  At $q_1^* = d_1$ and $T* = \frac{w-c}{1-\alpha}[(1-\alpha-2\alpha\beta)d_1-(1-\alpha)d_2]$, the supplier is indifferent between inform or not inform. If we can show that when $q_1^* = d_1$ and $T* = \frac{w-c}{1-\alpha}[(1-\alpha-2\alpha\beta)d_1-(1-\alpha)d_2]$, the objective value of optimization problem N is less than the objective value of optimization problem I, then it is optimal for the OEM to direct the supplier to inform.

$$
\begin{aligned}
V_I(d_1,T^*) =& d_1\{(p-w)[(1+\beta)(1-\theta)-\theta\alpha\beta]+\theta(-w+s)\} \\
&+ d_2(p-w)(1-\theta+\alpha\theta)+\theta d_1^2[(p-c)\alpha(1+2\beta)+c-s] \\
&+ \theta d_1 d_2(1-\alpha)(p-c).
\end{aligned}
$$

  Apparently $V_I(d_1,T^*) > V_N^*$. Therefore it is optimal for the OEM to direct the supplier to inform.

- If $\theta > \frac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$, the optimal q_1 for optimization problem N is $q_1^* = \frac{1-\alpha}{1-\alpha(1+2\beta)}d_2$ and $T^* = 0$. The optimal objective value for optimization problem N is

$$
\begin{aligned}
V_N^* =& \{(p-w)[(1+\beta)(1-\theta)-\theta\alpha\beta]+\theta(-w+s)\}\frac{1-\alpha}{1-\alpha(1+2\beta)}d_2 \\
&+ d_2(p-w)(1-\theta+\alpha\theta).
\end{aligned}
$$

The optimal solutions (q_1^*, T^*) for optimization problem I is $(0,0)$, $(d_1, \max\{\frac{w-c}{1-\alpha}[(1-\alpha-2\alpha\beta)d_1-(1-\alpha)d_2], 0\})$, or $(1, \max\{\frac{w-c}{1-\alpha}[1-\alpha(1+2\beta)d_1-(1-\alpha)d_2], 0\})$. In order to prove optimization problem I dominates optimization problem N, it is sufficient to prove that at one of these points, the objective value of optimization problem I is bigger than the optimal objective value of optimization problem N. Next we show $V_I(0,0)$ is bigger than V_N^*.

$$V_I(0,0) = d_2(p-w)(1-\theta+\theta\alpha).$$

$$V_N^* - V_I(0,0) =$$
$$\{(p-w)[(1+\beta)(1-\theta)-\theta\alpha\beta]+\theta(-w+s)\}\frac{1-\alpha}{1-\alpha(1+2\beta)}d_2 < 0.$$

The inequality comes from the condition that $\theta > \frac{(p-w)(1+\beta)}{(p-w)(1+\beta+\alpha\beta)+w-s}$. Therefore it is optimal for the OEM is to direct the supplier to inform where $d_1 \leq \frac{1-\alpha(1+2\beta)}{1-\alpha}d_1$. We've proved theorem 3.6.8.