### **MIMO Backscatter RFID Systems:**

#### Performance Analysis, Design and Comparison

by

#### Chen He

M.A.Sc, University of British Columbia, 2009 B.Eng, McMaster University, 2007

### A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

#### **Doctor of Philosophy**

in

## THE FACULTY OF GRADUATE AND POSTDOCTORAL STUDIES

(Electrical and Computer Engineering)

The University Of British Columbia

(Vancouver)

January 2014

© Chen He, 2014

## Abstract

Backscatter Radio-Frequency Identification (RFID) systems are the most popular RFID systems deployed due to low cost and low complexity. However, they pose many design challenges due to their querying-fading-signaling-fading structure, which experiences deeper fading than conventional one-way channels. Recently, by simulations and measurements, researchers found that the multipleinput-multiple-output (MIMO) setting can improve the performance of backscatter RFID systems. These simulations and measurements were based on simple signaling schemes and no rigorous mathematical analysis has been provided. In this thesis, we explore querying, STC, and diversity combining schemes over the three ends of the backscatter RFID systems and provide generalized performance analysis and design criteria.

At the tag end, we show that the identical signaling scheme, which cannot improve the bit error rate (BER) performance in conventional one-way channels, can significantly improve the BER performance of backscatter RFID. We also analytically study the performances of orthogonal STCs, with different sub-channel fading assumptions, and show that the diversity order depends only on the number of tag antennas. More interestingly, we show that the performance is more sensitive to the channel condition of the forward link than that of the backscattering link.

In previous literature, the understanding of the query end is that the designs of query signals have no potential to improve the system performance. However, we show that some well-designed query signals can improve the system performance significantly. We propose a novel unitary query method in this thesis. Conventional measures of the physical layer performance cannot be obtained analytically in backscatter RFID channels with employing our unitary query. We thus provide a new performance measure to overcome the difficulty of conventional measures, and show that why the unitary query has superior performance.

The multi-keyhole channel is another type of cascaded channel. The backscatter RFID channel and the multi-keyhole channels look similar, but are essentially different and there difference has not been clearly studied in previous literature. In the final part of this thesis, by investigating general STCs and revealing a few interesting properties of this channel in the multiple-input-single-output (MISO) case, we show that the two channels achieves completely different diversity order and BER performance.

## Preface

This thesis is written based on a collection of manuscripts. The majority of the research, including literature survey, mathematical proofs, numerical simulations and report writing, are conducted by the candidate, with suggestions from Prof. Z. Jane Wang. The manuscripts are primarily drafted by the candidate, with helpful revisions and comments from Prof. Z. Jane Wang. In the manuscript "On the Performance of MIMO RFID Backscattering channels", Prof. Weifeng Su and Mr. Xun Chen helped on checking the mathematical derivations.

Chapter 2 is partially based on the following manuscripts:

- Chen He and Z. Jane Wang, "Closed-Form BER Analysis of Non-Coherent FSK in MISO Double Rayleigh Fading/RFID Channel," *IEEE Communications Letters*, pp. 848-850, Aug. 2011.
- Chen He, Xun Chen, Z. Jane Wang, and Weifeng Su, "On the Performance of MIMO RFID Backscattering channels," *Eurasip Journal on Wireless Communications and Networking*, vol. 11, pp. 1-15, 2012.

Chapter 3 is based on:

- Chen He and Z. Jane Wang, "SER of Orthogonal Space-Time Block Codes Over Rician and Nakagami-m RF Backscattering Channels," *IEEE Transactions on Vehicular Technology*, pp. 1-9, 2013.
- Chen He and Z. Jane Wang, "Gains by a Space-time-code Based Signaling scheme for Multiple-antenna RFID Tags," *Proc. of the 23rd Canadian Conference on Electrical and Computer Engineering*, pp. 1-4, 2010.
- Chen He and Z. Jane Wang, "Impact of the Correlation Between Forward and Backscatter channels on RFID System Performance," *Proc. of the 36th*

*IEEE International Conference on Acoustics, Speech and Signal Processing,* pp. 1-4, 2011.

• Chen He and Z. Jane Wang, "Unitary Query for backscatter RFID," in submitting to *the IEEE Transactions on Wireless Communications*.

And finally, Chapter 4 is partially based on the following manuscript:

• Chen He and Z. Jane Wang, "Analysis of General Space-time Codes in MISO Multi-keyhole Channels," submitted to *the IEEE Transactions on Vehicular Technology*, 2013.

# **Table of Contents**

Ał	ostrac	xt	ii			
Pr	eface		iv			
Ta	ble of	f Contents	vi			
Li	st of [	Fables	ix			
Li	st of l	Figures	x			
Li	st of A	Acronyms	xiii			
Notation						
Acknowledgments						
De	edicat	ion	xvii			
1	Intr	oduction	1			
	1.1	Background of RFID Technology	1			
	1.2	RFID Components and Standards	2			
	1.3	Backscatter RFID Principle: Reader Query and Tag Signaling	3			
	1.4	Motivations for MIMO Backscatter RFID	5			
		1.4.1 Comparison with the Multi-keyhole Channel	7			
	1.5	Fading Assumptions	8			
	1.6	Thesis Contribution and Organization	9			

2	Bacl	Backscatter RFID Systems with Uniform Query and Identical Sig-				
	naling					
	2.1	Mathe	matical Description of the MIMO Backscatter RFID	12		
	2.2	Unifor	rm Query and Identical Signaling for MIMO Backscatter RFID	14		
		2.2.1	Uniform Query at the Reader Transmitter End	14		
		2.2.2	Identical Signaling at the Tag End	17		
	2.3	BER F	Performance under Uniform Query and Identical Signaling .	18		
		2.3.1	Non-coherent Case	20		
		2.3.2	Coherent Case	22		
		2.3.3	Correlated Forward and Backscatter Links	26		
	2.4	Divers	sity Order and Performance Bottleneck	27		
	2.5	Conclu	usion	28		
3	Bacl	kscattei	r RFID Systems with Space-time Coding and Unitary Ouery	30		
	3.1	Space	-time Coding with Uniform Query	31		
		3.1.1	A Conditional Moment Generating Function Approach for			
			Orthogonal Space-time Block Codes (OSTBCs)	32		
		3.1.2	Diversity Order, Performance Bottleneck and Impact of the			
			Sub-channel Quality	37		
		3.1.3	PEP Lower Bound for General Space-time Codes and Max-			
			imum Achievable Diversity Order	46		
	3.2	Space	-time Coding with Unitary Query	52		
		3.2.1	New Measure for PEP Performance	53		
		3.2.2	Examples and Simulations	62		
	3.3	Conclu	usion	63		
4	Ana	lysis of	General Space-time Codes in MISO Multi-keyhole Chan-			
	nels	• • •		66		
	4.1	Multi-	keyhole Channels	67		
	4.2	Indepe	endent and Identical Transmission Antennas	69		
		4.2.1	Distribution of the Code Words Distance	71		
		4.2.2	Convergence to the Rayleigh Channel	75		
	4.3	Spatia	l Correlated Transmission Antennas	78		

		4.3.1	Case 1: $M \leq L$	80
		4.3.2	Case 2: $M > L$	82
		4.3.3	Examples and Simulations	85
	4.4	Conclu	ision	89
5	Sum	mary a	nd Future Work	91
	5.1	Summ	ary of Results	91
	5.2	Future	Work	93
		5.2.1	Explore the Time Diversity Brought by the Unitary Query	93
		5.2.2	Non-Coherent Schemes for the Unitary Query	94
		5.2.3	General Query for the Backscatter RFID	94
		5.2.4	Optimal Query Antenna Selection	95
Bi	bliogr	aphy .		97
A	Deri	vations		105
	A.1	Chapte	er 2 Derivations	105
	A.2	Chapte	er 3 Derivations	109
		A.2.1	Derivations for Rician Fading	109
		A.2.2	Derivations for Nakagami-m Fading	113

# **List of Tables**

Table 1.1	RFID frequency bands.	4
Table 2.1	Non-coherent case of the identical signaling scheme: Closed- form BER expressions for the $N \times L$ backscatter RFID channel	
	(equation (2.27))	23
Table 2.2	Coherent case of the identical signaling scheme: Moment Generating Functions $G_{N,L}(\theta)$ for the $N \times L$ backscatter RFID chan-	
Table 2.3	nel (equation (2.31)).Comparisons between the backscatter RFID channel and the	25
	Rayleigh Channel when both the channels employ the identi- cal signaling scheme.	28
Table 3.1	Diversity order comparisons between different fading channels when OSTBCs are employed.	42
Table 4.1	The effects of transmission correlations on the PEP performances of the multi-keyhole and Rayleigh channels in the asymptoti- cally high SNR regimes.	89
Table 4.2	Performance comparisons between the backscatter RFID and multi-keyhole channels for orthogonal space-time codes in the	
	MISO case.	89

# **List of Figures**

Figure 1.1	RFID antennas to track vehicles coming into and leaving a	
	gated community	3
Figure 1.2	The RF reader transmits an unmodulated (query) signal to the	
	RF tag and the RF tag scatters a modulated signal back to the	
	reader, where $\Gamma(t)$ is the reflection coefficient of the tag circuit	
	at time <i>t</i>	5
Figure 1.3	An illustration of the multi-keyhole channel, i.e., exhibiting a	
	signaling-fading-fading structure. TXs are trying to communi-	
	cate with RXs.	8
Figure 1.4	An illustration of the MIMO backscatter RFID channel, i.e.,	
	exhibiting a query-fading-signaling-fading structure. Tag an-	
	tennas are trying to communicate with RXs	8
Figure 2.1	An illustration of the general $M \times L \times N$ backscatter RFID	
	channel	16
Figure 2.2	BER performances (in (2.27) and (2.28)) of the MIMO backscat-	
	ter RFID channel using non-coherent identical signaling (BPSK	
	with EGC,)	24
Figure 2.3	BER performances (in (2.32) and (2.33)) of the MIMO backscat-	
	ter RFID channel using the coherent identical signaling (BPSK	
	with MRC) under perfect channel estimation.	26
Figure 2.4	The BER performances of the identical signaling scheme, with	
	different link correlations.	28

Figure 3.1	The SER performance of the backscatter RFID channel, the <i>K</i>	
	factors are $K_f = K_b = 0$ dB	38
Figure 3.2	The SER performances of the backscatter RFID channels, where	
	$K_f = K_b = 3 \text{ dB.}$	39
Figure 3.3	The SER performances of the backscatter RFID channel, with	
	$m_f = m_b = 1. \ldots \ldots \ldots \ldots \ldots \ldots$	40
Figure 3.4	The SER performances of the backscatter RFID channel, with	
	$m_f = m_b = 1.5. \ldots \ldots$	41
Figure 3.5	The SER performances of the backscatter RFID channel, with	
	$m_f = m_b = 2. \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$	42
Figure 3.6	Two receiving antennas are enough to capture most of the re-	
	ceiving side gain	43
Figure 3.7	The BER performance comparison between Alamouti's coding	
	scheme and identical signaling scheme.	44
Figure 3.8	The performance of the backscatter RFID channel is much	
	more sensitive to the <i>K</i> factor of the forward link	45
Figure 3.9	Illustration of the reason that the performance of the backscat-	
	ter RFID channel is much more sensitive to the forward link	46
Figure 3.10	The performance of the backscatter RFID channel is much	
	more sensitive to the m parameters of the forward link	47
Figure 3.11	PEP performance comparisons between the unitary query and	
	the uniform query for the $2 \times 2 \times 2$ backscatter RFID channel.	
	The unitary query can bring a large gain for the $2 \times 2 \times 2$ channel.	64
Figure 3.12	PEP performance comparisons between the unitary query and	
	the uniform query for the $2 \times 2 \times 1$ backscatter RFID channel.	
	The unitary query can only bring a small gain for the $2 \times 2 \times 1$	
	channel	65
Figure 4.1	The MISO multi-keyhole channel model.	69
Figure 4.2	Simulated PDFs of the code words distances for the MISO	
	multi-keyhole channel and the MIMO single-keyhole channel.	76
Figure 4.3	Asymptotic and simulated PEPs in the MISO multi-keyhole	
	channel	77

Figure 4.4	The PEP of the MISO multi-keyhole channel converges to that		
	of the MISO Rayleigh channel	78	
Figure 4.5	The effect of transmission correlations on the MISO multi-		
	keyhole channel for the case that $L < M$	87	
Figure 4.6	The effect of transmission correlations on the MISO multi-		
	keyhole channel for the case that $L \ge M$	88	

# **List of Acronyms**

- BER bit error rate
- CSI channel state information
- DPSK differential phase-shift keying
- EGC equal gain combining
- EM electromagnetic
- FSK frequency-shift keying
- IFF identification friend or foe
- IEC International Electrotechnical Commission
- ISO International Organization for Standardization
- ISM industrial, scientific, and medical
- LOS line-of-sight
- MGF moment-generating function
- MIMO multiple-input-multiple-output
- MISO multiple-input-single-output
- MRC maximum ratio combining
- NLOS non-line-of-sight

оок on-off keying

- OSTBC orthogonal space-time block code
- PDF probability density function
- **PEP** pairwise error probability
- **RF** radio frequency
- **RFID** Radio-Frequency Identification
- **SC** selection combining
- **SER** symbol error rate
- SIMO single-input-multiple-output
- SNR signal-to-noise ratio
- STC space-time code
- UHF ultra-high-frequency

# Notation

In this thesis, unless otherwise specified,  $\exp(\cdot)$ ,  $\Gamma(\cdot)$ , and  $\mathbb{Q}(\cdot)$  mean the exponential function, the Gamma function, and the Gaussian Q function, respectively;  $\mathbb{P}(\cdot)$ ,  $\mathbb{E}_X(\cdot)$ , X|Y,  $\|\cdot\|_F$ ,  $\|\cdot\|$ ,  $(\cdot)^T$ ,  $(\cdot)^H$ ,  $det(\cdot)$ ,  $R(\cdot)$ , and  $trace(\cdot)$  denote the the probability of an event, the expectation over the density of X, the conditional random variable of X given Y, the Frobenius norm of a matrix, the magnitude of a complex number, the transpose, the conjugate transpose, the determinant, the rank, and the trace of a matrix, respectively;  $A \doteq B$  means that A is equal to B in the limit,  $C \ll D$  means that C is much smaller than D, and  $X \sim Y$  means that X is identically distributed with Y.

## Acknowledgments

First and foremost, I would like to express my deepest gratitude to my supervisor, Prof. Z. Jane Wang, for her constant guidance, technical insight, and encouragement through this thesis work. I am greatly indebted to her for her commitment and understanding from the initial to final steps of my research. Without her academic and personal support, this thesis would be impossible.

I would also like to thank my supervisory committee members Prof. Dave Michelson and Prof. Vincent Wong, thesis examination committee members, Prof. Jiahua Chen and Prof. Victor Leung, and also Prof. Cyril Leung, for their valuable feedback and suggestions and also for their time and effort.

My best wishes go to my friends and colleagues at UBC for their help, support, and providing an open-mined environment for research. Special thanks to my labmates, Xiaohui Chen, Xun Chen, Joyce Chiang, Zhenyu Guo, Junning Li, Aiping Liu, and Xudong Lv, and my friends Haoming Li, Qiang Tang, Di Xu, and many others from the ECE department.

Finally, I am deeply indebted to my parents, for their endless and unconditional love, and their constant encouragement and support in all stages of my life.

This work was supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada.

# Dedication

To my parents

### **Chapter 1**

## Introduction

#### 1.1 Background of RFID Technology

Radio-Frequency Identification (RFID) is a wireless communication technology that allows an object to be identified automatically and does not require line-of-sight (LOS) transmission [1]. This technology has a long history and is evolved from several early prototypes. The earliest ones include identification identification friend or foe (IFF) transponder developed in the United Kingdom, which was routinely used by the allies during World War II to identify aircraft as friend or foe [2, 3], and the landmark work in 1948 by Harry Stockman in [4] which first described the backscattering principle. The first true ancestor [5] of the modern RFID, however, was established in the 1970's, known as Mario Cardullo's device, as it was a passive radio transponder with memory. Although RFID has been emerged for decades, it is in recent years that its practical applications and academic research have been proliferated significantly. The value of the RFID market in 2012 was 7.46 billion US dollars versus 6.37 billion in 2011, and the RFID world market is estimated to surpass 20 billion by 2014 [6].

RFID technology is used in many applications, such as inventory systems, product tracking, access control, libraries, museums, sports, and some other industries. The value added by RFID technologies is often significant. For instance, in *access control* applications, RFID tags are widely used in identification badges [7]. These RFID tags can be placed on vehicles, and allow the RFID reader to read

them in a distance, thus vehicles can enter controlled areas without having to stop. Fig. 1.1 shows RFID antennas to track vehicles coming into and leaving a gated community.

In *libraries* applications, RFID tags have been used to replace the barcodes on library items [8]. Since RFID tags can be read through an item, a book cover does not need to be opened for scanning, and a stack of books can be read simultaneously. Book tags can also be read while books are in motion on a conveyor belt, thus reduce processing time significantly.

In the *healthcare industry* applications, RFID technology is used to track patients/employees and facility assets [9, 10]. The active RFID technology is used to track high-value and frequently moved items, and the passive technology can be used to track smaller and lower cost items that only need room-level identification. In addition, some hospitals began implanting patients with RFID tags and using RFID systems for workflow and inventory managements. The use of RFID techniques to prevent mixups between sperm and ova in 'in vitro fertilisation' clinics is also being considered [11].

#### **1.2 RFID Components and Standards**

An RFID system includes the hardware known as readers (also known as interrogators) and tags (also known as labels), as well as RFID software or RFID middleware [7]. An RFID tag is a small electronic device that is allowed to have a unique ID. It transmits data over the air in response to interrogation by an RFID reader. The tags can be categorized into passive, active, and semi-active tags. An active tag utilizes its internal battery to continuously power its radio frequency (RF) communication circuitry, while a passive RFID tag has no internal power supply and relies on RF energy transferred from the reader to the tag. A semi-passive tag is powered by both its internal battery and RF energy from the reader.

RFID has various standards set by a number of organizations, which include the International Organization for Standardization (ISO), the International Electrotechnical Commission (IEC), EPCglobal, and several others. RFID operates at different frequent bands. At low-frequency and high-frequency bands, tags can be used globally without a license. At the ultra-high-frequency (UHF) band, the stan-



**Figure 1.1:** Radio-frequency identification antennas to track vehicles coming into and leaving a gated community. Figure by Larry D. Moore, CC BY-SA 3.0.

dards and regulations are different in different regions. In North America, UHF RFID can be used unlicensed for 902 to 928 MHz, with restrictions for transmission power. In Europe, UHF RFID operates from 865 to 868 MHz, with readers being required to monitor a channel before transmitting. This requirement has led to some restrictions on performance. Each application for UHF in these countries needs a site license, which needs to be applied for at the local authorities, and can be revoked. Table 1.1 summarizes the frequency band, operation range and data speed of different RFID standards [12].

### **1.3 Backscatter RFID Principle: Reader Query and Tag** Signaling

Most RFID applications deployed today use passive tags because they usually do not require internal batteries and have longer life expectancy. For the passive technology, to transmit the energy from the RFID reader to the tag, there are two fundamental principles for design approaches: magnetic induction and electromag-

Band	Regulation	Range	Data Speed
120-150 kHz, LF	Unregulated	10 cm	Low
13.56 MHz, HF	ISM band worldwide	10 cm to 1 m	Low to moderate
433 MHz, UHF	Short range devices	1 to 100 m	Moderate
902 – 928 MHz	ISM band	1 to 12 m	Moderate to high
(North America)			
868 – 870 MHz			
(Europe), UHF			
2.45 and 5.8 GHz,	ISM band	1 to 2 m	High
microwave			
3.1 – 10.6 GHz,	Ultra wide band	up to 200 m	High
microwave			

 Table 1.1: RFID frequency bands.

netic (EM) wave capture [1] [7]. The design based on magnetic induction is called near field, and that based on EM wave capture is called far field. Both design can transfer enough energy to power a remote tag. For near-field RFID systems, a reader passes a large alternating current through a reading coil, resulting in an alternating magnetic field in its locality. If a tag that incorporates a smaller coil is in this field, an alternating voltage will appear across it so that the tag chip is powered, while for far-field RFID systems, tags capture the EM waves propagating from a dipole antenna attached to the reader. A smaller dipole antenna in the tag receives the energy as an alternating potential difference which will result in an accumulation of energy to power its circuit. The far-field approach is also referred as backscattering [1]. In this thesis, we concentrate on the far-field approach. In physics, backscattering is the reflection of waves, particles, or signals back to the direction from which they came. A backscatter RFID signal modulation procedure is shown in Fig.1.2. The RF reader transmitter first broadcasts an unmodulated carrier signal, which is also called query signal, then the RF tag conveys information (i.e. the ID of the tag) to the reader by simply reflecting the query signal from the reader transmitter back to the reader receiver using load modulation [13] [14]. The ID information of the RF tag depends on the reflection coefficient of the tag antenna load which is changed by switching the RF tag antenna load between different states [14].



**Figure 1.2:** The RF reader transmits an unmodulated (query) signal to the RF tag and the RF tag scatters a modulated signal back to the reader, where  $\Gamma(t)$  is the reflection coefficient of the tag circuit at time *t*.

#### 1.4 Motivations for MIMO Backscatter RFID

At the physical layer, the backscatter channel, with a query-fading-signaling-fading structure, is radically different from conventional one-way wireless channels. In addition, real measurements in [13] and [14] showed that the backscatter RFID channel could be modeled as a cascaded channel with a forward sub-channel and a backscattering sub-channel, and both sub-channels can be Rayleigh, Rician or Nakagami-m distributed, depending on the radio propagation environment. This cascaded channel fades deeper than the conventional one-way channel and hence can reduce the data transmission reliability and reading range, which are two important performance metrics in RFID systems.

To overcome the challenges posed by deeper fading in the backscatter channel, researchers had to reconsider the design of backscatter RFID systems and many efforts have been made on improving the system performance [13–32]. These efforts include re-designing of tag circuits, antenna structures and tag modulations. Among those efforts, using multiple antennas for both tags and readers appears to be one of the practical and promising ways. Such multiple-input multiple-output (MIMO) systems had a great success in conventional wireless communications [33–37] and were found promising in RFID [14, 25–27, 30–32]. The MIMO system for RF backscattering radio was first explored by Ingram *et al.* in [25] for the

spatial multiplexing purpose. In [25] multiple reflection antennas were used by the RF tags to reflect according to different data streams, and multiple reader receiving antennas provided multi-stream detection capability. Simulations showed that the range can be extended by a factor of four or more in the pure diversity configuration and that backscatter link capacity can be increased by a factor of ten or more in the spatial multiplexing configuration. Later, [26, 27] provided a closed form probability density function of channel envelope for tags with multiple-antennas, with consideration of spatial correlation between forward and backscattering links. They showed that backscatter diversity can mitigate this fading by changing the shape of the fading distribution which, along with the increased RF tag scattering aperture, resulted up to a 10 dB gain at a bit error rate (BER) of  $10^{-4}$ . Simulations demonstrated that the above gain led to increased backscatter radio communication reliability and up to a 78 percent range increase. The measurements of using multiple antennas in the backscatter RFID channel were conducted in [14] and [31]. In [14] the measurement was conducted at 5.8 GHz, an unlicensed industrial, scientific, and medical (ISM) frequency band and in a non-line-of-sight (NLOS) environment. The measurements in [14] showed that gains are available for multipleantenna RF tags and the results matched well with the gains predicted using the analytic fading distributions derived in [26] and [27], while [31] proposed a novel measurement with a reduced number of measurement ports for MIMO backscatter RFID channels. More recently, [32] described how to overcome the extra path loss that RFID tags and RFID-enabled sensors experience at microwave frequencies as compared to UHF frequencies. They showed that additional antenna gains can be realized to mitigate or overcome extra path loss by using multiple antennas for narrowband signals centered at 5.8 GHz. In [30], researchers described a developed analog frontend for an RFID rapid prototyping system which allows for various realtime experiments to investigate MIMO techniques.

Exploring MIMO diversity of backscatter RFID channels is a relative new research area and all the above studies about the performance of MIMO backscatter RFID channels are based on measurement experiments and Monte Carlo simulations. However, no analytical studies have been provided yet in the literature, except [38], where only line-of-sight propagation and OSTBC were considered, and the result in [38] was developed in parallel and independent of our work in [39]. Additionally, only simple diversity mechanisms were considered, and the general understanding of the role of query signals is that they only play the role as the energy provider for tags (i.e., the reader transmitting antennas transmit the same query signal from M antennas over T time slots, as explained later in Chapter 2). To our best knowledge, there has been no investigation on other possible roles of query signals yet.

Therefore, to have a profound understanding of MIMO backscatter RFID channels, we need to have rigorous performance analysis studies beyond simulations to guide us in the design process of the MIMO RFID systems. Also, to achieve the full potential of MIMO settings for backscatter RFID systems, we need to explore all possible performance improvement mechanisms at the three ends, i.e., considering more complicated and generalized query schemes at the reader query end, investigating coding (signaling) schemes at the tag end, and employing optimal diversity combining schemes at the reader receiving end. The above are the motivations of this dissertation.

#### **1.4.1** Comparison with the Multi-keyhole Channel

There is another type of cascaded-like channel, the multi-keyhole channel, which has a signaling-fading-fading structure, as shown in Fig. 1.3. Recall that the backscatter RFID channel, on the other hand, has a query-fading-signaling-fading structure. Therefore, these two channels are essentially different. The comparisons of them are shown in Fig. 1.3 and Fig. 1.4. Another essential difference is that the multi-keyhole channel is still a conventional one-way channel, with the only difference from the Rayleigh channel being that its channel gain has a more complicated distribution. However, researchers sometimes are confused about the two types of the channels - the multi-keyhole channel and the backscatter RFID channel. Most recent research on the space-time code (STC) for the multi-keyhole channel gave a performance analysis only for the orthogonal space-time block code (OSTBC) [40]. This motivates us to make a further investigation on the performance of the multi-keyhole channel and make a comparison with that of the backscatter RFID channel.



**Figure 1.3:** An illustration of the multi-keyhole channel, i.e., exhibiting a signaling-fading-fading structure. TXs are trying to communicate with RXs.



**Figure 1.4:** An illustration of the MIMO backscatter RFID channel, i.e., exhibiting a query-fading-signaling-fading structure. Tag antennas are trying to communicate with RXs.

### **1.5 Fading Assumptions**

The work in this thesis is based on the model from the real measurements in [13] [14] of the backscatter RFID channel. More specifically, each sub-channel follows i.i.d complex Gaussian distribution, and the fading is quasi-static: i.e., the channel is constant over a long period of time and changes in an independent manner. This quasi-static assumption is valid as long as the transmitter and the receiver is not moving in high velocity, and it is one of the major assumptions for many wireless communication systems including many RFID systems.

#### **1.6 Thesis Contribution and Organization**

In this thesis, we investigate three main topics pertinent to MIMO backscatter RFID channels:

- Performance analysis of the identical signaling scheme (at the tag end) with the uniform query (at the reader query end) in backscatter RFID channels;
- Further exploration of diversity gains brought by the reader query end, the tag end, and the reader receiving end in backscatter RFID channels. Derivation of generalized methods for performance analysis and design criteria for space-time coded backscatter RFID systems with both the uniform query and the proposed unitary query;
- Derivation of the PEP of space-time codes in MISO multi-keyhole channels and comparisons between the backscatter RFID channel and the multikeyhole channel.

More specifically, Chapter 2 will address the first topic. We consider a specific diversity mechanism in the RFID system where the reader transmitter employs the uniform query and the tag employs the identical signaling scheme. The identical signaling scheme has been proved to be not useful for improving the BER performance in conventional one-way point-to-point wireless channels. However, the identical signaling scheme has been verified in [27] by Monte Carlo simulations that for some antenna settings in RFID, its BER performance improvement can be very significant, while for some other antenna settings, its improvement is insignificant. Yet, no literature has given an explanation why it happens and what is the underlying physical reason. To fill the gap, in Chapter 2, we will provide a rigorous mathematical analysis to reveal the performance behaviors of the identical signaling scheme for backscatter RFID channels.

In Chapter 3, we will address the second topic. First, we consider the case when the tags employ orthogonal space-time codes, meanwhile the reader transmitter still employs the uniform query. For this case, we will provide a general formulation for the performance analysis, and analytically study the symbol error rate (SER) performances for Rician and Nakagami-m sub-channels by providing

closed form SERs in asymptotic high signal-to-noise ratio (SNR) regimes. We will also show a few interesting properties of the SER performance for this case, and generalize the performance analysis to general space-time codes by providing a pairwise error probability (PEP) performance upper bound. Secondly, we will propose a novel query method at the reader transmitter end, referred as the *unitary* query. To our best knowledge, it is the first time that unitary query has been proposed in RFID. In previous studies for MIMO backscatter RFID, only the uniform query was considered, where the query signals played a role no more than an energy provider for the RFID tag and could not provide spatial diversity. In Chapter 3, however, we will show that in quasi-static channels, the query signals can provide time diversity through multiple reader transmitting antennas for some space-time codes. We will propose a new performance measure, which is based on the ranks of some random matrices, to overcome the difficulty that conventional measures (i.e. PEP and diversity order) cannot be obtained analytically for the unitary query with general space-time codes. Furthermore, we will analytically study the performance of the proposed unitary query with general space-time codes via the new performance measure.

In Chapter 4, we will address the third topic. We will consider general spacetime codes in the multi-keyhole channel, and prove that, for any pairs of code words in a space-time code, the code words distance, as a random variable in fading conditions, is identically distributed in MISO multi-keyhole channels and MIMO single-keyhole channels. Therefore the PEPs for a pair of code words in these two channel models share the same form and thus one can employ the design criteria in MIMO single-keyhole channels to design the codes for MISO multi-keyhole channels. We will further investigate the case when spatial correlations are present in transmission antennas and prove that, when the number of transmission antennas is greater than that of keyholes, depending on how the correlation matrix beamforms the code words difference matrix, the PEP can be either degraded or improved. The results in this chapter will clearly demonstrate that the backscatter RFID channel and the multi-keyhole channel have completely different performance behaviors.

Finally, in Chapter 5 we will summarize the results obtained in previous chapters. We also provide a number of potential topics for future work on the grounds of research presented in this dissertation.

### Chapter 2

# Backscatter RFID Systems with Uniform Query and Identical Signaling

In Chapter 1, we gave a brief introduction on the backscattering principle and the MIMO backscatter RFID channel. In this chapter, we first provide a full modeling of this MIMO structure. We can see that this MIMO structure has fading structure and signaling mechanism which are radically different from those in a conventional one-way point-to-point wireless channel, resulting in deeper fading and non-Gaussian statistical properties [27].

Then we consider diversity techniques for the backscatter RFID channel, and start from the simplest case of space-time coding: the reader transmitters employ the uniform query and the tag employs the identical signaling scheme. The identical signaling scheme has been proved to not be useful for improving the BER performance in conventional one-way point-to-point wireless channels. However, the identical signaling scheme has been verified in [27] by Monte Carlo simulations that, for some antenna settings, the BER improvement by the identical signaling scheme can be significant, while for some other antenna settings, the improvement is small. Yet, no literature has been able to provide an explanation on these observations and explain what is the underlying reason. In this chapter, we will provide a rigorous mathematical analysis to reveal the underlying behavior of the identical signaling scheme for backscatter RFID channels, and answer the question why the identical signaling scheme can sometimes improve the BER performance. We will also show that there is a performance bottleneck of identical signaling in backscatter RFID systems, and that is why the improvement by identical signaling is mild in some antenna settings in the backscatter RFID. The reported results can be useful for designing simple, effective MIMO backscatter RFID systems with high performance.

### 2.1 Mathematical Description of the MIMO Backscatter RFID

The backscatter RFID has three ends: the reader query end (i.e., the set of reader transmitting antennas), the tag end (i.e., the set of tag antennas), and the reader receiver end (i.e., the set of reader receiving antennas). These three ends can be mathematically modeled by an  $M \times L \times N$  dyadic backscatter channel which consists of M reader transmitter antennas, L RF tag antennas, and N reader receiver antennas. As shown in Fig. 2.1, the forward channel  $h_{ml}^{f}$  represents the propagation path from the *m*-th reader transmitter to the *l*-th RF tag antenna, while the backscatter channel  $h_{ln}^{b}$  represents the path in which the carrier signal is reflected by the *l*-th tag antenna to the *n*-th reader receiver. The forward and backscatter links that terminate or originate at the same tag antenna can be correlated, as indicated in Fig. 2.1, where  $\rho_{ml}^{ln}$  denotes the link correlation coefficient between the forward link  $h_{ml}^{f}$  and the backscatter link  $h_{ln}^{b}$ . The correlations between the links are caused by the separations and the angular spreads of the antennas. In a quasi-static wireless channel, this MIMO structure can be summarized by using the following matrices: More specifically,

$$\mathbf{Q} = \begin{pmatrix} q_{1,1} & \cdots & q_{1,M} \\ \vdots & \ddots & \vdots \\ q_{T,1} & \cdots & q_{T,M} \end{pmatrix}$$
(2.1)

is the query matrix, representing the query signals sending from the M reader transmitting antennas to the tag over T time slots.

$$\mathbf{H} = \begin{pmatrix} h_{1,1}^f & \cdots & h_{1,L}^f \\ \vdots & \ddots & \vdots \\ h_{M,1}^f & \cdots & h_{M,L}^f \end{pmatrix}$$
(2.2)

is the channel gain matrix from the reader transmitter to the tag, representing the forward sub-channels,

$$\mathbf{C} = \begin{pmatrix} c_{1,1} & \cdots & c_{1,L} \\ \vdots & \ddots & \vdots \\ c_{T,1} & \cdots & c_{T,L} \end{pmatrix}$$
(2.3)

is the coding matrix, where the tag transmits coded or un-coded symbols from its L antennas over T time slots, and

$$\mathbf{G} = \begin{pmatrix} h_{1,1}^b & \cdots & h_{1,N}^b \\ \vdots & \ddots & \vdots \\ h_{L,1}^b & \cdots & h_{L,N}^b \end{pmatrix},$$
(2.4)

is the channel gain matrix from the tag to the reader receiver, representing the backscattering sub-channels. Finally the received signals at *N* reader receiving antennas over *T* time slots, are represented by matrix **R** with size  $T \times N$ :

$$\mathbf{R} = \mathbf{Q}\mathbf{H} \circ \mathbf{C}\mathbf{G} + \mathbf{W} \tag{2.5}$$

where  $\circ$  means the Hadamard product, and the matrix **W** is with the same size as that of **R**, representing the noise at the *N* reader receiving antennas over *T* time slots. In this thesis, unless otherwise specified, both the forward and the backscattering sub-channels are modeled as i.i.d complex Gaussian random variables with zero mean and unity variance. In addition, in this thesis, it is assumed that the noise matrix is with independent and identically distributed (i.i.d.) standard complex Gaussian entries.

When compared with the conventional one-way MIMO wireless channel:

$$\mathbf{R} = \mathbf{C}\mathbf{G} + \mathbf{W},\tag{2.6}$$

the backscatter structure in (2.5) not only has one more layer of fading structure **H** but also one more signaling mechanism represented by the query matrix **Q**. In addition, the backscatter principle makes the received signals not a simple series of linear transformations of transmitted signals and channel gains, but actually there involves a non-linear structure in the backscatter RFID channel, which is the result from the Hadamard product in (2.5). Because it has such special and complicated signaling and channel structures, we expect completely different performance behaviors of the MIMO backscatter RFID channel when compared with the one-way channel. In this chapter, we concentrate on the simplest case, the MIMO backscatter RFID channel has interesting properties. In the next chapter, we will investigate MIMO backscatter RFID channels under more generalized query and signaling cases.

### 2.2 Uniform Query and Identical Signaling for MIMO Backscatter RFID

#### 2.2.1 Uniform Query at the Reader Transmitter End

We consider the simplest case of query signals, where the *M* reader transmitting antennas transmit the same query signal over *T* time slots, and the query matrix with size  $T \times M$  is thus given by

$$\mathbf{Q} = \frac{1}{\sqrt{M}} \begin{pmatrix} 1 & \cdots & 1\\ \vdots & \ddots & \vdots\\ 1 & \cdots & 1 \end{pmatrix}.$$
 (2.7)

We name this query scheme as the *uniform query*. The term  $\frac{1}{\sqrt{M}}$  is to ensure that the total transmission power from the reader transmitter end is fixed. In this case, since the query signals from the reader query antennas are identical over *T* time slots, if

the forward channels are independent Gaussian, the forward channel statistics are invariant for any *M*, i.e., the  $M \times L \times N$  channel is equivalent to a  $1 \times L \times N$  channel (or a  $L \times N$  for short).

Note that at a given time slot *t*,

$$\mathbf{Q}_{t}\mathbf{H}\circ\mathbf{C}_{t}\mathbf{G}\sim(h_{1}^{f},\cdots,h_{L}^{f})\circ(c_{t,1},\cdots,c_{t,L})\mathbf{G}$$

$$(2.8)$$

$$\begin{pmatrix} h_{1}^{f} \end{pmatrix}$$

$$= (c_{t,1}, \cdots, c_{t,L}) \begin{pmatrix} n_1 & & \\ & \ddots & \\ & & h_L^f \end{pmatrix} \mathbf{G}$$
(2.9)

$$= (c_{t,1}, \cdots, c_{t,L}) \begin{pmatrix} h_1^f h_{1,1}^b, h_2^f h_{2,1}^b, & \cdots, & h_L^f h_{L,1}^b \\ \vdots & \ddots & \vdots \\ h_1^f h_{1,N}^b, h_2^f h_{2,N}^b, & \cdots, & h_L^f h_{L,N}^b \end{pmatrix}^T$$
(2.10)

where  $\mathbf{Q}_t$  and  $\mathbf{C}_t$  are the *t*-th row of  $\mathbf{Q}$  and  $\mathbf{C}$  respectively. Therefore, in quasi-static wireless channels, for the uniform query, we have

$$\mathbf{QH} \circ \mathbf{CG} \sim \mathbf{CH}_{\text{uniform}},$$
 (2.11)

and, referred to the model in (2.5), the received signals at N over T time slots have an equivalent form as:

$$\mathbf{R} = \mathbf{C}\mathbf{H}_{\text{uniform}} + \mathbf{W},\tag{2.12}$$

where

$$\mathbf{H}_{\text{uniform}} = \begin{pmatrix} h_1^f h_{1,1}^b, h_2^f h_{2,1}^b, & \cdots, & h_L^f h_{L,1}^b \\ \vdots & \ddots, & \vdots \\ h_1^f h_{1,N}^b, h_2^f h_{2,N}^b, & \cdots, & h_L^f h_{L,N}^b \end{pmatrix}^T$$
(2.13)

is the equivalent overall channel matrix, in which the each (l, n) entry represents the overall channel gain between the the reader query antennas, the *l*-th tag antenna, and the *n*-th reader receiving antenna. When comparing Eqn. (2.5) and Eqn. (2.6), we can see that this equivalent overall channel matrix transforms the backscatter



**Figure 2.1:** An illustration of the general  $M \times L \times N$  backscatter RFID channel.  $h_{ml}^{f}$  represents the forward link from the *m*-th reader transmitter antenna to the *l*-th tag antenna and  $h_{ln}^{b}$  represents the backscatter link from the *l*-th tag antenna to the *n*-th reader receiver antenna.  $\rho_{ml}^{ln}$  means the correlation between the forward link  $h_{ml}^{f}$  and the backscatter link  $h_{ln}^{b}$ . For the case that all the forward channels are independent with each other, and are Rayleigh distributed, the  $M \times L \times N$  channel is equivalent to a  $1 \times L \times N$  channel. In the following parts of this chapter, we refer the later case as the  $L \times N$  channel for short.

RFID channel with the uniform query into the form of the conventional one-way wireless channel: the received signals are just a linear transformation of transmitted signals by the equivalent overall channel matrix. However, we also note that the equivalent overall channel matrix itself is non-Gaussian, and has statistically dependent entries, even if all the sub-channels are independent.

#### 2.2.2 Identical Signaling at the Tag End

By the backscattering principle, when the query signals arrive at the tag antennas, the antennas reflect a portion of energy from the query signals back to the reader, in this way the information symbols of the tag which are represented by the reflection coefficients of the tag circuits, can be conveyed to the reader. The reflection coefficient matrices (also referred as the tag signaling matrices) are given by [27]

$$\mathbf{S}(t) = \begin{pmatrix} \Gamma_1(t) & & \\ & \ddots & \\ & & \Gamma_L(t) \end{pmatrix}, \qquad (2.14)$$

for  $t = 1, \dots, T$ . Here  $\Gamma_l(t)$  is the load reflection coefficient of the *l*-th tag antennas at time *t*. In general the reflection coefficient matrices can have unequal load reflection coefficients based on specific tag circuit designs, and they are actually corresponding to the coding matrix **C**. Therefore space-time codes can be implemented via specifically designing the reflection coefficients in the tag circuit, while at a given time slot *t*, if the reflection coefficients  $\Gamma_l(t)$ 's are designed to be identical for all tag antennas, the tag signaling matrices **S**(*t*)'s take an identical form:

$$\mathbf{S}(t) = \Gamma(t)\mathbf{I}.\tag{2.15}$$

The above identical signaling scheme is the simplest space-time code. It has been proved that the BER performance in conventional one-way wireless channels cannot be improved by the identical signal scheme, while in [27], simulations showed that in the backscatter RFID channel, for some antenna settings the identical signaling scheme can improve the BER performance significantly, but for some other antenna settings the improvement is minor. Yet, there has been no analytical explanations on why this is the case. In this following section, we will analytically study the BER performance for the backscatter RFID channel with the uniform query and identical signaling scheme. We will use the equivalent overall channel model for the uniform query that we've derived in (2.13).

### 2.3 BER Performance under Uniform Query and Identical Signaling

Under the identical signaling scheme, the tag employs the identical signaling matrix in (2.15) in which each tag antenna transmits the same symbol at time *t*. From the channel matrix in (2.13), the instantaneous signal-to-noise ratio (SNR) at the *n*-th receiving antenna is given by

$$\gamma_n = \bar{\gamma} |\sum_{l=1}^L h_l^f h_{l,n}^b|^2, \qquad (2.16)$$

where  $\bar{\gamma}$  means the average SNR. Each of these instantaneous SNRs follows the following distribution [27]:

$$f_{\gamma_n}(\gamma_n) = \frac{2\gamma_n^{(L-1)/2}}{(L-1)!\bar{\gamma}^{(L+1)/2}} K_{L-1}\left(2\sqrt{\frac{\gamma_n}{\bar{\gamma}}}\right),$$
(2.17)

where  $K_{L-1}(\cdot)$  denotes the modified Bessel function of the second kind. Using the asymptotic approximations of the Bessel function [41], one can obtain the approximation of the probability density function (PDF) for high SNR (e.g. as  $\bar{\gamma} \to \infty$ ) as,

$$f_{\gamma_n}(\gamma_n) \doteq \begin{cases} -\frac{1}{\tilde{\gamma}} \ln\left(\frac{\gamma_n}{\tilde{\gamma}}\right), & \text{if } L = 1; \\ \frac{1}{(L-1)\tilde{\gamma}}, & \text{if } L > 1. \end{cases}$$
(2.18)

To derive the BER performance of the  $N \times L$  MIMO backscatter RFID channel where N > 1, since the *N* receiving branches at the reader are statistically independent as even for independent sub-channels, we cannot use the above distribution and its approximation to directly evaluate the performance of the MIMO channel by applying a widely used method as in [42] and [43] which requires independency of receiving branches. Alternatively, we consider evaluating the BER using the conditional probability approach. We will see later, to analytically study the BER performance, we first need to investigate the properties of  $G_{N,L}(\cdot)$ , a function defined by a multi-variate integration. The function  $G_{N,L}(\cdot)$  is defined as:

$$G_{N,L}(\bar{\gamma}) = \int_{\alpha_L=0}^{\infty} \cdots \int_{\alpha_1=0}^{\infty} \frac{1}{\left(1 + \bar{\gamma} \sum_{l=1}^{L} \alpha_l\right)^N} \exp\left(-\sum_{l=1}^{L} \alpha_l\right) d\alpha_1 \cdots d\alpha_L. \quad (2.19)$$

Here  $\alpha_l$  is the squared magnitude of the channel gain of the l-th receiving branch, N and L are the index of the function  $G_{N,L}(\bar{\gamma})$ , and we define  $\bar{\gamma} = \frac{g\bar{\gamma}}{\sin^2\theta}$ , where  $\bar{\gamma}$  is the average SNR and g is a constant which is modulation dependent. For the coherent transmission case, the function  $G_{N,L}(\bar{\gamma})$  is the moment-generating function (MGF) of the MIMO backscatter RFID channel with L tag antennas and N receiving antennas. For the non-coherent transmission case, the form of  $G_{N,L}(\cdot)$  is required in deriving the BER performance. The function  $G_{N,L}(\cdot)$  defined in (2.19) has the following recursive and asymptotic properties:

**Proposition 1.** 

$$G_{1,L}(\bar{\bar{\gamma}}) = \frac{e^{\frac{1}{\bar{\gamma}}}}{\bar{\bar{\gamma}}} E_L\left(\frac{1}{\bar{\bar{\gamma}}}\right) \doteq \begin{cases} \frac{\ln(\bar{\bar{\gamma}})}{\bar{\bar{\gamma}}}, & \text{if } L = 1;\\ \frac{1}{(L-1)\bar{\bar{\gamma}}}, & \text{if } L > 1. \end{cases}$$
(2.20)

**Proposition 2.** 

$$G_{N,1}(\bar{\tilde{\gamma}}) = \frac{e^{\frac{1}{\tilde{\gamma}}}}{\bar{\tilde{\gamma}}} E_N\left(\frac{1}{\bar{\tilde{\gamma}}}\right) \doteq \begin{cases} \frac{\ln(\bar{\tilde{\gamma}})}{\bar{\tilde{\gamma}}}, & \text{if } N = 1;\\ \frac{1}{(L-1)\bar{\tilde{\gamma}}}, & \text{if } N > 1. \end{cases}$$
(2.21)

**Proposition 3.** 

$$G_{N,L}(\bar{\gamma}) = \frac{1}{(-\bar{\gamma})^{N-1}(N-1)!} G_{1,L}(\bar{\gamma}) - \sum_{k=1}^{N-1} \frac{(k-1)!}{(-\bar{\gamma})^{N-k}(N-1)!} G_{k,(L-1)}(\bar{\gamma}). \quad (2.22)$$

**Proposition 4.** 

$$G_{N,L}(\bar{\gamma}) \doteq \begin{cases} \frac{1}{(L-1)\cdots(L-N)\bar{\gamma}^N}, & \text{if } N < L;\\ \frac{\ln(\bar{\gamma})}{(N-1)!\bar{\gamma}^N}, & \text{if } N = L;\\ \frac{1}{(N-1)\cdots(N-L)\bar{\gamma}^L}, & \text{if } N > L. \end{cases}$$

$$(2.23)$$
In the above propositions,  $E_N(\cdot)$  and  $E_L(\cdot)$  are the exponential integrals defined as  $E_N(x) = \int_{t=1}^{\infty} \frac{\exp(-tx)}{t^N} dx$  and  $E_L(x) = \int_{t=1}^{\infty} \frac{\exp(-tx)}{t^L} dx$  [44], where *N* and *L* are positive integers. The proofs of these propositions can be found in the appendix. With the above properties, we are now ready to derive the exact and asymptotic BER performances and study how the MIMO RFID backscattering channel behaves.

#### 2.3.1 Non-coherent Case

For non-coherent receivers, the carrier phase need not to be tracked, and this makes signal detections easier and less complex, while comparing with the coherent receiver, the non-coherent sacrifices a few dB for BER performance. The non-coherent receiver is usually preferred by low cost systems. One important diversity combining technique for non-coherent receiver at is called non-coherent equal gain combining (EGC), in which the received signal at each receiving branch is weighted by the same factor, irrespective of the signal amplitude. Modulation schemes that can incorporate with EGC include differential phase-shift keying (DPSK), frequency-shift keying (FSK) and on-off keying (OOK). In this section we analytically study the performance of the backscatter RFID channel that employs the uniform query at the reader query end, the identical signaling at tag end, and non-coherent EGC at the reader receiving end.

Note that the channel gain at the *n*-th receiving branch of the reader is given by

$$h_n = \sum_{l=1}^L h_l^f h_{l,n}^b.$$

When fixing the forward channel gains  $h_l^f$ 's, the channel gain  $h_n$  is a linear combination of i.i.d. complex Gaussian random variables, hence the conditional distribution of  $h_n$  on  $h_l^f$ 's is a complex Gaussian distribution with variance

$$\sigma_l^2 = \sum_{l=1}^L |h_l^f|^2.$$

Therefore by fixing  $h_l^f$ 's, the  $N \times L$  channel can be viewed as a single-input-multipleoutput (SIMO) channel in which each receiving branch is Rayleigh distributed and has power (or variance)  $\sum_{l=1}^{L} |h_l^f|^2$ . Consequently using the result of the SIMO Rayleigh channel [45], we can have the conditional (on  $h_l^f$ 's) BER for the  $N \times L$  backscatter RFID channel using non-coherent EGC as:

$$P_{N,L}(\bar{\gamma}|h_l^f) = \frac{1}{2^{2N-1}(N-1)!(1+g\bar{\gamma}\sum_{l=1}^L|h_l^f|^2)^N}$$
(2.24)  
$$\frac{N-1}{2^{2N-1}(N-1)!(1+g\bar{\gamma}\sum_{l=1}^L|h_l^f|^2)^N}$$

$$\times \sum_{k=0}^{N-1} b_k (N-1+k)! \left( \frac{g \bar{\gamma} \sum_{l=1}^L |h_l^f|^2}{1 + g \bar{\gamma} (\sum_{l=1}^L |h_l^f|^2)} \right)^k,$$
(2.25)

where

$$b_k = \frac{1}{k!} \sum_{n=0}^{N-1-k} {\binom{2N-1}{n}},$$

and g is a constant which is modulation dependent [46]. Note that

$$\begin{split} \left( \frac{g\bar{\gamma}\sum_{l=1}^{L}|h_{l}^{f}|^{2}}{1+g\bar{\gamma}\sum_{l=1}^{L}|h_{l}^{f}|^{2}} \right)^{k} &= \left( 1 - \frac{1}{1+g\bar{\gamma}\sum_{l=1}^{L}|h_{l}^{f}|^{2}} \right)^{k} \\ &= \sum_{i=0}^{k} (-1)^{i} \binom{k}{i} \frac{1}{(1+g\bar{\gamma}\sum_{l=1}^{L}|h_{l}^{f}|^{2})^{i}}, \end{split}$$

hence we have

$$P_{N,L}(\bar{\gamma}|h_l^f) = \frac{1}{2^{2N-1}(N-1)!} \sum_{k=0}^{N-1} b_k (N-1+k)! \sum_{i=0}^k (-1)^i \binom{k}{i} \frac{1}{(1+g\bar{\gamma}\sum_{l=1}^L |h_l^f|^2)^{N+i}}$$
(2.26)

Averaging the conditional BER over  $\alpha_l$ 's (where  $|h_l^f|^2 = \alpha_l$ ) yields the BER for the  $N \times L$  backscatter RFID channel as:

$$P_{N,L}(\bar{\gamma}) = \int_{\alpha_L=0}^{\infty} \cdots \int_{\alpha_1=0}^{\infty} P_{N,L}(\bar{\gamma}|\alpha_l) \exp\left(-\sum_{l=1}^{L} \alpha_l\right) d\alpha_1 \cdots \alpha_L$$
  
=  $\frac{1}{2^{2N-1}(N-1)!} \sum_{k=0}^{N-1} b_k (N-1+k)! \sum_{i=0}^{k} (-1)^i {k \choose i} G_{(N+i),L}(\bar{\gamma}).$  (2.27)

The closed-form of the above exact BER can be computed recursively using Proposition 3 with the initial knowledge  $G_{1,L}(\bar{\gamma}) = \frac{e^{\frac{1}{\bar{\gamma}}}}{\bar{\gamma}}E_L(\bar{\gamma})$  and  $G_{N,1}(\bar{\gamma}) = \frac{e^{\frac{1}{\bar{\gamma}}}}{\bar{\gamma}}E_N(\bar{\gamma})$ .

Table 2.1 shows a few examples under some antenna settings.

While the closed-form BER can be obtained, it involves complicated recursive forms and the behavior of the studied  $N \times L$  backscatter RFID channel is not easy to analyze, and we need to investigate an asymptotic form. Using Proposition 4, we can obtain an asymptotic BER of (2.27) as:

$$P_{N,L}(\bar{\gamma}) \doteq \begin{cases} \frac{\sum_{k=0}^{N-1} b_k (N-1+k)!}{2^{2N-1} (N-1)!} G_{N,L}(g\bar{\gamma}) \doteq \frac{\sum_{k=0}^{N-1} b_k (N-1+k)!}{2^{2N-1} (N-1)! (L-1) \cdots (L-N) (g\bar{\gamma})^N}, & \text{if } N < L; \\ \frac{\sum_{k=0}^{N-1} b_k (N-1+k)!}{2^{2N-1} (N-1)!} G_{N,L}(\bar{\gamma}) \doteq \frac{\sum_{k=0}^{N-1} b_k (N-1+k)! \ln(g\bar{\gamma})}{2^{2N-1} (N-1)! (N-1)! (g\bar{\gamma})^N}, & \text{if } N = L; \\ \frac{1}{2^{2N-1} \sum_{k=0}^{N-1} b_k (N-1+k)!} \\ \frac{1}{2^{2N-1} \sum_{k=0}^{N-1} b_k (N-1+k)!} C_{N,L}(\bar{\gamma}) = \frac{1}{2^{2N-1} \sum_{k=0}^{N-1} \sum_{k=0}^{N-1} \sum_{k=0}^{N-1} \sum_{k=0}^{N-1} \sum_{k=0}^{N-1} \sum_{k=0}^{N-1} \sum_{k=0}^{$$

$$\begin{cases} 2^{2N-1}(N-1)! \ \Delta k = 0 \ \delta k (V - 1 + N) \\ \times \sum_{i=0}^{k} (-1)^{i} {k \choose i} \frac{1}{(N+i-1)\cdots(N+i-L)(g\bar{\gamma})^{L}}, & \text{if } N > L. \end{cases}$$
(2.28)

We can see that the above asymptotic BER form depends on the relation of the values of *L* and *N*. Fig. 2.2 shows the BER performances of the  $N \times L$  RFID channels when employing the binary frequency-shift keying (FSK) with EGC. The asymptotic diversity order  $d_a$  can be obtained as

$$d_a = \lim_{\bar{\gamma} \to \infty} \left( -\frac{\log P_{N,L}(\bar{\gamma})}{\log(\bar{\gamma})} \right) = \min(N,L).$$
(2.29)

It means that the asymptotic diversity order of the  $N \times L$  backscatter RFID channel under non-coherent transmission schemes is determined by the smaller value of Nand L. For the case of L = N, compared with the case of  $L \neq N$ , it requires a higher SNR to achieve the diversity order N, because of the logarithm function in the numerator in (2.28) when N = L. This property means that even the diversity orders are the same the BER performances of the settings with N = L + 1 or L = N + 1are remarkably better than the performance of the setting with N = L. The BER performance improvements from N = L + 1 to N = L + 2, or from L = N + 1 to L = N + 2, is not significant.

#### 2.3.2 Coherent Case

Comparing with non-coherent receiver, coherent receivers need estimating the phase of the transmitted signals, and the hardware of the coherent receiver is usually more expensive than that of the non-coherent receiver. However the coherent receiver

**Table 2.1:** Non-coherent case of the identical signaling scheme: Closed-form BER expressions for the  $N \times L$  backscatter RFID channel (equation (2.27)).

	L = 1	L=2
N = 1	$e^{rac{1}{gar\gamma}}rac{E_1(rac{1}{gar\gamma})}{2gar\gamma}$	$e^{rac{1}{gar\gamma}}rac{E_2(rac{1}{gar\gamma})}{2gar\gamma}$
N = 2	$\frac{2e^{\frac{1}{87}}E_2(\frac{1}{8\tilde{\gamma}})}{s\tilde{\gamma}} +$	$\frac{2(-g\bar{\gamma}+e^{\frac{1}{S\bar{\gamma}}}E_1(\frac{1}{g\bar{\gamma}})+g\bar{\gamma})}{g\bar{\gamma}}+$
	$\frac{(g\bar{\gamma})^2 - 2g\bar{\gamma}e^{\frac{1}{g\bar{\gamma}}}E_1(\frac{1}{g\bar{\gamma}}) + g\bar{\gamma} - e^{\frac{1}{g\bar{\gamma}}}E_1(\frac{1}{g\bar{\gamma}})}{4(g\bar{\gamma})^3}$	$\frac{e^{\frac{1}{g\bar{\gamma}}}E_1(\frac{1}{g\bar{\gamma}})-3(g\bar{\gamma})^2+2(g\bar{\gamma})^2e^{\frac{1}{g\bar{\gamma}}}E_1(\frac{1}{g\bar{\gamma}})-g\bar{\gamma}+4g\bar{\gamma}e^{\frac{1}{g\bar{\gamma}}}E_1(\frac{1}{g\bar{\gamma}})}{4(g\bar{\gamma})^3}$

usually yields better performance than non-coherent receiver. The diversity combining techniques for coherent receivers include maximum ratio combining (MRC), EGC and selection combining (SC), among which MRC achieves the best BER performance. For MRC, the gain of each received signal is made proportional to the signal level and inversely proportional to the mean square noise level in that channel. In this section, we concentrate on MRC as it achieves the best BER performance among all the coherent diversity combining schemes .

If we fix the forward gains  $h_l^f$ 's, the MIMO backscatter RFID channel can be viewed as a SIMO Rayleigh channel in which the receiving branches are independent and have power (or variance)  $\sum_{l=1}^{L} |h_l^f|^2$ . Recall that the MGF for a Rayleigh fading channel is given by [42]:

$$\left(1+\frac{g\bar{\gamma}}{\sin^2\theta}\right)^{-1}$$

therefore the conditional MGF of the  $N \times L$  RFID backscatter channel is

$$M_{N,L}\left(g,\bar{\gamma},\theta|h_l^f\right) = \left(1 + \frac{g\bar{\gamma}\sum_{l=1}^L |h_l^f|^2}{\sin^2\theta}\right)^{-N}.$$
(2.30)

Integrating  $M_{N,L}\left(g,\bar{\gamma},\boldsymbol{\theta}|h_l^f\right)$  over  $\alpha_l$ 's (where  $\alpha_l = |h_l^f|^2$ ) leads to the MGF for



**Figure 2.2:** BER performances (in (2.27) and (2.28)) of the MIMO backscatter RFID channel using non-coherent identical signaling (BPSK with EGC,).

non-independent N receiving branches as:

$$M_{N,L}(g,\theta,\bar{\gamma}) = \int_{\alpha_L=0}^{\infty} \cdots \int_{\alpha_1=0}^{\infty} M_{N,L}(g,\theta,\bar{\gamma}|\alpha_l) \exp\left(-\sum_{l=1}^{L} \alpha_l\right) d\alpha_1 \cdots d\alpha_L$$
$$= \int_{\alpha_L=0}^{\infty} \cdots \int_{\alpha_1=0}^{\infty} \left(1 + \frac{g\bar{\gamma}\sum_{l=1}^{L} \alpha_l}{\sin^2 \theta}\right)^{-N} \exp\left(-\sum_{l=1}^{L} \alpha_l\right) d\alpha_1 \cdots d\alpha_L$$
$$= G_{N,L}(\bar{\gamma}), \qquad (2.31)$$

where  $\bar{\bar{\gamma}} = \frac{g\bar{\gamma}}{\sin^2\theta}$  and  $G_{N,L}(\cdot)$  is defined as in (2.19). Using the moment generating approach in [42], the BER of the  $N \times L$  backscatter RFID channel for the coherent

**Table 2.2:** Coherent case of the identical signaling scheme: Moment Generating Functions  $G_{N,L}(\theta)$  for the  $N \times L$  backscatter RFID channel (equation (2.31)).

	L = 1	L = 2
N = 1	$e^{rac{\sin^2 heta}{gar\gamma}}rac{E_1(rac{\sin^2 heta}{gar\gamma})}{gar\gamma}$	$\frac{e^{\frac{\sin^2\theta}{g\bar{\gamma}}}E_2(\frac{\sin^2\theta}{g\bar{\gamma}})\sin^2\theta}{g\bar{\gamma}}$
N=2	$\frac{e^{\frac{\sin^2\theta}{g\bar{\gamma}}}E_2(\frac{\sin^2\theta}{g\bar{\gamma}})\sin^2\theta}{g\bar{\gamma}}$	$\frac{-\bar{\gamma}\sin^{4}\theta + e^{\frac{\sin^{2}\theta}{g\bar{\gamma}}}E_{1}(\frac{\sin^{2}\theta}{g\bar{\gamma}})\sin^{6}\theta + \bar{\gamma}e^{\frac{\sin^{2}\theta}{g\bar{\gamma}}}E_{1}(\frac{\sin^{2}\theta}{g\bar{\gamma}})\sin^{4}\theta}{(g\bar{\gamma})^{3}}$

case can be expressed as:

$$P_{N,L}(\bar{\gamma}) = \frac{1}{\pi} \int_{\theta=0}^{\pi/2} G_{N,L}(\bar{\bar{\gamma}}) d\theta.$$
(2.32)

Since the closed form of  $G_{N,L}(\bar{\gamma})$  can be obtained recursively using Proposition 1 to Proposition 4, the BER  $P_{N,L}(\bar{\gamma})$  can be computed through the single integration in (2.32) respective to  $\theta$ . To have more insights on how the BER of the  $N \times L$  backscatter RFID channel behaves, we also derive an asymptotic form of this BER expression. Using Proposition 4, the asymptotic BER for (2.32) can be expressed as:

$$P_{N,L}(\bar{\gamma}) \doteq \begin{cases} \frac{1}{\pi} \int_{\theta=0}^{\pi/2} \frac{1}{(L-1)\cdots(L-N)(g\bar{\gamma})^N} d\theta = \frac{C_N}{(L-1)\cdots(L-N)(g\bar{\gamma})^N}, & \text{if } N < L; \\ \frac{1}{\pi} \int_{\theta=0}^{\pi/2} \frac{\ln(g\bar{\gamma})}{(N-1)!(g\bar{\gamma})^N} d\theta \doteq \frac{C_N \ln(g\bar{\gamma})}{(N-1)!(g\bar{\gamma})^N}, & \text{if } N = L; \\ \frac{1}{\pi} \int_{\theta=0}^{\pi/2} \frac{1}{(N-1)\cdots(N-L)(g\bar{\gamma})^L} d\theta = \frac{C_L}{(N-1)\cdots(N-L)(g\bar{\gamma})^L}, & \text{if } N \ge L, \end{cases}$$

$$(2.33)$$

where  $C_N = \int_{\theta=0}^{\pi/2} \sin^{2N} \theta d\theta = \frac{\Gamma(1/2+N)}{2\sqrt{\pi}\Gamma(1+N)}$  and  $C_L = \frac{\Gamma(1/2+L)}{2\sqrt{\pi}\Gamma(1+L)}$ . Here  $\Gamma(\cdot)$  means the Gamma function. Fig. 2.3 plots the BER curves of the  $N \times L$  RFID channels when employing BPSK with MRC at the reader receiver antennas. For the  $N \times L$  backscatter RFID channel under the coherent case, the asymptotic diversity order can be given by

$$d_a = \lim_{\bar{\gamma} \to \infty} \left( -\frac{\log P_{N,L}(\bar{\gamma})}{\log(\bar{\gamma})} \right) = \min(N,L).$$
(2.34)

As we can see that the asymptotic diversity order is still  $\min(N,L)$  in the coherent transmission case, and the BER behavior is similar to that of the non-coherent case.



**Figure 2.3:** BER performances (in (2.32) and (2.33)) of the MIMO backscatter RFID channel using the coherent identical signaling (BPSK with MRC) under perfect channel estimation.

#### 2.3.3 Correlated Forward and Backscatter Links

In previous sections, we assume that the sub-channels are independent. In real propagation environments, the forward and backscattering channel might be correlated (e.g. co-located reader transmitting antenna and receiving antenna), which introduces additional fading and therefore limits the diversity gain. In this section, we study the MIMO backscatter RFID channel with sub-link correlation  $\rho$  by simulations. We use the antenna setups:  $1 \times 2 \times 1$  and  $2 \times 2 \times 2$  and simulate the channels under different values of the link correlation, i.e.  $\rho_e = \{0, 0.2, 0.4, 0.6, 0.8, 1\}$ . Here,  $\rho_e \approx |\rho|^2$  means the link envelope correlation [26]. For the  $1 \times 2 \times 1$  channel, we assume  $\mathbb{E}(h_{11}^f h_{11}^b)/\sigma^2 = \mathbb{E}(h_{12}^f h_{21}^b)/\sigma^2 = \rho$ . For the  $2 \times 2 \times 2$  channel, it is assumed that  $\mathbb{E}(h_{11}^f h_{11}^b)/\sigma^2 = \mathbb{E}(h_{12}^f h_{21}^b)/\sigma^2 = \mathbb{E}(h_{21}^f h_{12}^b)/\sigma^2 = \mathbb{E}(h_{21}^f h_{12}^b)/\sigma^2 = \mathbb{E}(h_{12}^f h$ 

i.e.  $\rho_e = 1$ , which can occur only if the reader transmitter and the reader receiver are co-located and have the same antenna patterns [26]. Generally  $\rho_e < 1$ . To study the effect of the link correlation on the BER performance, we simulate the channels with different link correlation coefficient  $\rho$  and show the results in Fig. 2.4. We observe that for identical signaling scheme, the BER performance decreases as  $\rho_e$  increase in middle and high SNR regimes. For the  $1 \times 2 \times 1$  channel, at BER of  $10^{-4}$ , a 5 dB loss is observed from  $\rho_e = 0$  to  $\rho_e = 1$  for the identity signaling scheme. For the  $2 \times 2 \times 2$  channel, the loss is 3 dB for the identical signaling scheme.

## 2.4 Diversity Order and Performance Bottleneck

In conventional one-way wireless channels, the identical signaling scheme cannot improve the BER performance, since sending same signals through L transmitting antennas and combing the signals through N receiving branches will have a diversity of N, which means that the performance is invariant with the number of transmitting antennas and is only determined by the number of receiving branches, while for backscatter RFID channels, one interesting observation is that the diversity order under the identical signaling scheme, as shown in Eqn. (2.34), is  $\min(N,L)$ , which means that the diversity order is determined by both parameters N and L. Clearly for some antenna settings, having more tag antennas could bring significant performance improvements. For example, the antenna setting N = 3, L = 2 has diversity order of 2 and has much better performance than the setting N = 3, L = 1 which has diversity order of 1, while the diversity min(N, L) also implies that there is a performance bottleneck for the backscatter RFID channel: if N-L > 1, solely increasing the number of receiving antennas N does not enhance the BER performance significantly; similarly if L - N > 1, solely increasing the number of tag antennas L does not enhance the BER performance significantly either. The diversity orders and performance bottlenecks for the backscatter RFID channel and the one-way Rayleigh channel are summarized in Table 2.3.

**Table 2.3:** Comparisons between the backscatter RFID channel and the Rayleigh Channel when both the channels employ the identical signaling scheme.

Channel	Diversity Order	Bottleneck
backscatter RFID channels	$\min(L,N)$	increase N if $N - $
		L > 1; or increase L
		is $L - N > 1$
one-way Rayleigh channels	N	increase L



Figure 2.4: The BER performances of the identical signaling scheme, with different link correlations.

# 2.5 Conclusion

In this chapter, we provided a mathematical modeling of the MIMO backscatter RFID channel and showed that this channel has fading structure and signaling mechanism radically different from the conventional one-way point-to-point wireless channel. Then we considered the simplest diversity method for the backscatter RFID channel: the reader query antennas employ the uniform query and the tag employs the identical signaling scheme. We derived an equivalent overall channel matrix for the uniform query. By using the equivalent channel matrix, we showed that the achievable diversity order of the backscatter RFID channel for the identical signaling scheme is  $\min(N, L)$ , i.e. the minimum of the numbers of tag antennas and reader receiving antennas. This diversity order can also be used to explain why the identical signaling scheme, which has been proved to have no BER improvement in conventional one-way wireless channels, can improve the BER performance in the backscatter RFID channel significantly for some antenna settings, while the improvement can be minor for some other antenna settings. The analysis in this chapter can help us to better design simple, effective MIMO backscatter RFID systems with high performance.

# **Chapter 3**

# Backscatter RFID Systems with Space-time Coding and Unitary Query

In Chapter 2, we investigated the performance of the case when the reader transmitter employs uniform query, and the tag employs identical signaling scheme. In this Chapter we consider more complicated cases. First, we consider the case when the tag applies orthogonal space-time code, while the reader still applies uniform query. For this case, we provide a general formulation for performance analysis. This formulation is applicable to any sub-channels fading assumptions. Using this formulation, we analytically study the SER performances for Rician and Nakagami-m sub-channels, and derive asymptotic SERs in closed form. We also generalize the performance analysis to general space-time code by providing a performance upper bound that the backscatter structure may ever achieve. We find that the diversity order achieves L for Rician fading and achieves  $L\min(m_f, Nm_h)$ for Nakagami-m fading, where  $m_f$  and  $m_b$  are the m parameters of the forward and backscattering links, respectively. Two receiving antennas (N = 2) can capture most of the receiving side gain regardless of the number of tag antennas L for Rician fading, and this is also applicable to Nakagami-m fading if the two links of the cascaded structure have similar channel conditions. More interestingly, we show that the performance of the backscatter RFID channel is more sensitive to the channel condition (the K factor or the m parameter) of the forward link than that of the backscattering link.

Second, at the reader query end, we propose a novel scheme called *unitary query*. To our best knowledge, it is the first time that the unitary query has been proposed in RFID. In previous literature for MIMO backscatter RFID channels, only the uniform query was considered, and the understanding of query signals was that they only play a role as an energy provider for the RFID tag and thus cannot provide spatial diversity. In this chapter, however, we show that in quasistatic channels, the query signals can provide time diversity via multiple reader query antennas for some space-time codes, and hence improve the performance for the backscatter RFID significantly. We also analytically study the performance of the proposed unitary query. Due to the specific signaling and fading structure of the backscatter RFID channel, the PEP and even the diversity order are not trackable for the unitary query, we thus provide a new measure which can compare the PEP performance of the unitary query with that of the uniform query.

# 3.1 Space-time Coding with Uniform Query

In Chapter 2, we analytically study the identical signaling scheme which results from same reflection coefficients at each tag antenna load, while more complicated signaling schemes can also be implemented by designing unequal load reflection coefficients in the tag circuit. For example, the following reflection coefficients matrix

$$\mathbf{S}(t) = \begin{pmatrix} \Gamma_1(t) & & \\ & \ddots & \\ & & \Gamma_L(t) \end{pmatrix}, \qquad (3.1)$$

can result in space-time codes at the tag end. If we want to implement Alamouti's code, the circuit design will follow the diagonal signaling matrix at time slots t = 1 and t = 2 as

$$\mathbf{S}(1) = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \ \mathbf{S}(2) = \begin{pmatrix} -c_2^* \\ c_1^* \end{pmatrix}.$$
(3.2)

Suppose the tag is with 2 antennas and the RF tag ID is  $(c_1, c_2, c_3, c_4)$ . At the ID transmission layer, one way to implement the signaling scheme of (3.2) is to design

the reflection coefficients of one antenna loading to be  $(c_1, -c_2^*, c_3, -c_4^*)$ , and the coefficients of the other antenna loading to be  $(c_2, c_1^*, c_3, c_4^*)$ . In this design, the power consumption is roughly doubled but no computational capability is required. Nowadays design of RF backscatter circuit requires the power to be as low as 15.5 pJ/bit [21], which makes the implementation of space-time code possible.

### 3.1.1 A Conditional Moment Generating Function Approach for Orthogonal Space-time Block Codes (OSTBCs)

OSTBC is an attractive MIMO coding scheme and provides a solution for reliability of passive RFID systems at UHF [47] which allows good spacing between antennas. MIMO channels with OSTBC can achieve different diversity orders for different type of fading models: Full diversity of *LN* in i.i.d. Rayleigh fading [48] [34], and min(*L*,*N*) for the keyhole channel with i.i.d. Rayleigh sub-channels. In this section we study the performance of OSTBC when the sub-channels follow Racian fading and Nakagami-m fading (i.e. for cases that  $h_l^f$ 's and  $h_{l,n}^b$ 's are Racian distributed and Nakagami-m distributed respectively). Rician fading, a more suitable fading model for indoor ultra-wideband (UWB) channels [50], is also used to model sub-channels for UWB backscattering radio [51]. Our results can be easily narrowed down to Rayleigh fading which is a special case of Rician fading and Nakagami-m fading.

Due to the nested structure of the channel matrix in (2.13), it is difficulty to evaluate the SER of RF backscattering channels using the approach that was used for keyhole channels and other wireless channels: evaluating the distribution associated with the channel matrix first. Instead, we introduce a conditional MGF approach which transforms the nested structure into a nice form, and in general the approach can be used to find the SER with arbitrary fading assumption of subchannels  $h_l^f$ 's and  $h_{l,n}^b$ 's.

We assume that the channel is with quasi-static fading and the channel state information (CSI) is known at the reader. Because of the orthogonality property, OSTBC can be transformed from the MIMO fading channel to the following *M*  parallel SISO channels [35]:

$$\mathbf{y} = \sqrt{\|\mathbf{H}\|_F^2} \mathbf{x} + \mathbf{z},\tag{3.3}$$

where  $\|\mathbf{H}\|_F = \sqrt{\sum_{n=1}^{N} \sum_{l=1}^{L} |h_l^f h_{l,n}^b|^2}}$  is the Frobenius norm of  $\mathbf{H}, \mathbf{x} = (x_1, ..., x_M)^T$  represents the *M* incoming symbols and each element of  $\mathbf{z} = (z_1, ..., z_M)^T$  is complex Gaussian distributed with zero-mean and unit-variance.  $\mathbf{y} = (y_1, ..., y_M)^T$  represents the received symbols and can be detected based on a simple maximum likelihood method. Note that the channel gain was divided by *L* in [35] because the transmission power should be normalized to unity. In real passive RFID signal transmission, however, the transmission energy is from the reader and is proportional to the number of tag antennas when the reader querying energy is fixed, therefore (3.3) is a more appropriate modeling. Let  $E_b$  denote the average energy per bit and  $E_s$  denote the average energy per symbol, then  $E_s = E_b \log_2 K$  where *K* is the size of the signal constellation. The instantaneous SNR per symbol is therefore given by  $\gamma = \frac{\|\mathbf{H}\|_F^2 \log_2 K}{R} \frac{E_b}{N_0} = \frac{\|\mathbf{H}\|_F^2 \log_2 K}{R} \tilde{\gamma} = \|\mathbf{H}\|_F^2 g \tilde{\gamma}$ , where R = M/T means the rate symbol rate and we define  $g = \frac{\log_2 K}{R}$ . For the Rician RF backscattering channel, we assume that forward links  $h_l^f$  and backscattering links  $h_{l,n}^b$  are Rician distributed with *K* factors  $K_f$  and  $K_b$  respectively. The SER for OSTBC can be calculated by averaging the density of  $\|\mathbf{H}\|_F^2$  over  $Q(g \tilde{\gamma}\|\mathbf{H}\|_F^2)$ :

$$P_{\text{OSTBC}}(\bar{\gamma}) = \mathbb{E}_{\mathbf{H}}\left(\mathbb{Q}\left(\sqrt{g\bar{\gamma}}\|\mathbf{H}\|_{F}^{2}\right)\right) = \frac{1}{\pi}\int_{\theta=0}^{\pi/2} G(\bar{\gamma})d\theta.$$
(3.4)

Here we employ the alternative representation of the Q function as in [42] and we define  $\bar{\bar{\gamma}} = \frac{g\bar{\gamma}}{\sin^2\theta}$ .  $G(\bar{\bar{\gamma}}) = \mathbb{E}_{\mathbf{H}}\left(\exp\left(-\frac{g\bar{\gamma}\|\mathbf{H}\|_{F}^{2}}{\sin^2\theta}\right)\right)$  means the MGF of  $\|\mathbf{H}\|_{F}^{2}$ .

To find  $G(\bar{\gamma})$ , one approach which has been used in finding the SER of keyhole fading and other wireless channel is to find the PDF of  $\|\mathbf{H}\|_F^2$  first. However, for the structure in (2.13), evaluating the density of  $\|\mathbf{H}\|_F^2$  is not tractable. Instead, we define

$$\|\mathbf{H}\|_{F}^{2} = \sum_{l=1}^{L} \|\mathbf{H}_{l}\|_{F}^{2} = \sum_{l=1}^{L} \sum_{n=1}^{N} \alpha_{l} \beta_{l,n}$$
(3.5)

as the squared Frobenius norm of the *l*-th column of **H**, where  $\alpha_l = |h_l^f|^2$  and  $\beta_{l,n} = |h_{l,n}^b|^2$ . We can see that  $||\mathbf{H}_l||_F^2$ s' are independent random variables, therefore the MGF  $G(\bar{\gamma})$  can be represented as a multiplication of the MGFs of  $||\mathbf{H}_l||_F^2$ s':

$$G(\bar{\tilde{\gamma}}) = \prod_{l=1}^{L} G_l(\bar{\tilde{\gamma}}).$$
(3.6)

Note that if we fix  $\alpha_l$ , the random variable  $\|\mathbf{H}_l\|_F^2 = \alpha_l \sum_{n=1}^N \beta_{l,n}$  is exactly the same as the gain of an *N*-branch SIMO system with MRC at the receiver, with *N* branches  $h_{l,n} = \alpha_l \beta_{l,n}$  for n = 1, ..., N, and each branch has transmission power  $\alpha_l$ . So we have the MGF  $G_l(\bar{\gamma})$  as:

$$G_l(\bar{\bar{\gamma}}) = \int_0^\infty \prod_{n=1}^N G_{h_{l,n}|\alpha_l}(\bar{\bar{\gamma}}) f_{\alpha_l}(\alpha_l) d\alpha_l, \qquad (3.7)$$

and therefore

$$G(\bar{\gamma}) = \prod_{l=1}^{L} \left( \int_{0}^{\infty} \prod_{n=1}^{N} G_{h_{l,n}|\alpha_l}(\bar{\gamma}) f_{\alpha_l}(\alpha_l) d\alpha_l \right),$$
(3.8)

where  $f_{\alpha_l}(\alpha_l)$  is the PDF of  $\alpha_l$  and  $G_{h_{l,n}|\alpha_l}(\bar{\gamma})$  is the MGF of conditional distribution of  $h_{l,n}$  on  $\alpha_l$  (the squared magnitude of the *l*-th forward channel gain). The nice things for the form in (3.8) are: It involves only one scalar integral hence avoids lots of numerical difficulties; The PDF  $f_{\alpha_l}(\alpha_l)$  and the conditional MGF  $G_{h_{l,n}|\alpha_l}(\bar{\gamma})$ for known fading models are given in existing literature. In this chapter we focus on Rician fading and Nakagami-m fading.

#### **Rician Fading**

Here we evaluate the SER of OSTBC for the backscatter RFID channel with the assumption that  $h_l^f$ 's and  $h_{l,n}^b$ 's are Rician fading. For Rician fading, the PDF of  $\alpha_l$  is:

$$f_{\alpha_l}(\alpha_l) = (K_f + 1)e^{-K_f - (K_f + 1)\alpha_l} I_0\left(\sqrt{4K_f(K_f + 1)\alpha_l}\right),$$
(3.9)

where we use  $K_f$  to represent the *K* factor of the forward channels. Note that in Rician fading, the MGF of  $\beta_{l,n} = |h_{l,n}^b|^2$  is given by [42]

$$G_{\beta_{l,n}}(\bar{\bar{\gamma}}) = \frac{K_b + 1}{K_b + 1 + \bar{\bar{\gamma}}} \exp\left(-\frac{K_b\bar{\bar{\gamma}}}{K_b + 1 + \bar{\bar{\gamma}}}\right).$$
(3.10)

Therefore the conditional MGF  $G_{h_{l,n}|\alpha_l}(\bar{\gamma})$  can be given by multiplying the SNR of (3.10) by  $\alpha_l$ :

$$G_{h_{l,n}|\alpha_l}(\bar{\bar{\gamma}}) = \frac{K_b + 1}{K_b + 1 + \bar{\bar{\gamma}}\alpha_l} \exp\left(-\frac{K_b\bar{\bar{\gamma}}\alpha_l}{K_b + 1 + \bar{\bar{\gamma}}\alpha_l}\right),\tag{3.11}$$

where  $K_b$  is the *K* factor of the backscattering channel. Substitute  $f_{\alpha_l}(\alpha_l)$  and  $G_{h_{l,n}|\alpha_l}(\bar{\gamma})$  into (3.8), the exact form of  $G_l(\bar{\gamma})$  can be given as

$$G_{l}(\bar{\gamma}) = \sum_{m=0}^{\infty} \sum_{i=0}^{\infty} \frac{D_{1} D_{2}^{m} D_{3}^{i}}{i!(m!)^{2}} \bar{\gamma}^{i} K_{1}^{-N-i} K_{2}^{-m-i-1} e^{\frac{K_{1}K_{2}}{\bar{\gamma}}}$$

$$\times \left(\frac{K_{1}K_{2}}{r}\right)^{N'} \sum_{j=0}^{m+i} {m+i \choose j} \left(-\frac{K_{1}K_{2}}{\bar{\gamma}}\right)^{m+i-j}$$

$$\times \Gamma\left(j-N-i+1, \frac{K_{1}K_{2}}{\bar{\gamma}}\right)$$
(3.12)

where  $K_1 = K_b + 1$ ,  $K_2 = K_f + 1$ ,  $D_1 = K_2 e^{-K_f} K_1^N$ ,  $D_2 = K_f K_2$ , and  $D_3 = -NK_b$ . The above exact form is complicated and cannot provide a insight on how the channel behaves, therefore we provide an asymptotic form which is still a good approximation of the exact form but much more concise and can provide good insight on how the channel behaves:

$$G_{l}(\bar{\bar{\gamma}}) \doteq \begin{cases} C_{1}^{f}C_{2}^{b}\left(\ln(\bar{\gamma}) + C_{4}^{fb}\right)\bar{\gamma}^{-1}, & \text{if } N = 1; \\ C_{1}^{f}C_{3}^{b}(N-2)!\bar{\gamma}^{-1}, & \text{if } N > 1. \end{cases}$$
(3.13)

Substituting (3.13) into (3.6) then into (3.4) can yield asymptotic expression of SER for OSTBC:

$$P_{\text{OSTBC}}(\bar{\bar{\gamma}}, N, L) \doteq \begin{cases} C_L \left( C_1^f C_2^b \right)^L \left( (\ln(\bar{\gamma}) + C_4^{fb}) \bar{\gamma}^{-1} \right)^L, & \text{if } N = 1; \\ C_L \left( C_1^f C_3^b (N - 2)! \right)^L \bar{\gamma}^{-L}, & \text{if } N > 1, \end{cases}$$
(3.14)

where  $C_1^f = K_2 e^{-K_f}$ ,  $C_2^b = K_1 e^{-K_b}$ ,  $C_3^b = K_2 (-NK_b)^{-N+1} \left( e^{-NK_b} - \sum_{j=0}^{N-2} \frac{(-NK_b)^j}{j!} \right)$ ,  $C_4^{fb} = e^{K_f} - 1 - \ln(K_1K_2)$ , and  $C_L = \frac{\Gamma(\frac{1}{2}+L)}{2\sqrt{\pi}\Gamma(1+L)}$ . Since there are considerable volumes of derivations involved to arrive the expressions of MGF and SER, to give concise presentation, we put them into the Appendix for reference.

#### Nakagami-m Fading

For the Nakagami-m RF backscattering channel, we assume that forward links  $h_l^f$ and backscattering links  $h_{l,n}^b$  are Nakagami-m distributed with the parameters  $m_f$ and  $m_b$  respectively. Following the general approach given by in 3.1.1, a similar procedure as Rician fading can be applied to analyze the error rate performance of the Nakagami-m distributed channel. The PDF of  $\alpha_l$  is given by

$$f_{\alpha_l}(\alpha_l) = \frac{m_f^{m_f}}{\Gamma(m_f)} \alpha_l^{m_f - 1} \exp(-m_f \alpha_l), \qquad (3.15)$$

and the MGF of  $\beta_{l,n}$  in Nakagami-m fading is as

$$G_{\beta_{l,n}}(\bar{\bar{\gamma}}) = \left(1 + \frac{\bar{\bar{\gamma}}}{m_b}\right)^{-m_b},\tag{3.16}$$

hence conditional MGF  $G_{h_{l,n}|\alpha_l}(\bar{\gamma})$  can be obtained by multiplying the SNR of (3.16) by  $\alpha_l$ :

$$G_{h_{l,n}|\alpha_l}(\bar{\tilde{\gamma}}) = \left(1 + \frac{\bar{\tilde{\gamma}}}{m_b}\alpha_l\right)^{-m_b}.$$
(3.17)

Accordingly to Appendix B, we can have a closed form of  $G_l(\bar{\gamma})$ :

$$G_{l}(\bar{\gamma}) = \frac{e^{\frac{m_{f}m_{b}}{\bar{\gamma}}} \left(\frac{m_{f}m_{b}}{\bar{\gamma}}\right)^{Nm_{b}}}{\Gamma(m_{f})} \sum_{j=0}^{m_{f}-1} {m_{f}-1 \choose j} \left(\frac{m_{f}m_{b}}{-\bar{\gamma}}\right)^{m_{f}-1-j}} \times \Gamma\left(j-Nm_{b}+1,\frac{m_{f}m_{b}}{\bar{\gamma}}\right), \qquad (3.18)$$

for integer  $m_f$ , and the asymptotic form as

$$G_{l}(\bar{\bar{\gamma}}) \doteq \begin{cases} & \frac{m_{b}^{m_{b}}m_{f}^{m_{f}}}{\Gamma(m_{f})}(\ln\bar{\bar{\gamma}}-\ln m_{b}-\ln m_{f})\bar{\bar{\gamma}}^{-m_{f}}, & \text{if } m_{f}=m_{b}N; \\ & m_{f}^{b}m_{f}^{b}\frac{\Gamma(a-b)}{\Gamma(a)}\bar{\bar{\gamma}}^{-b}, & \text{if } m_{f}\neq m_{b}N. \end{cases}$$
(3.19)

Therefore, we can have an asymptotic SER as:

$$\begin{split} & P_{\text{OSTBC}}(\bar{\gamma}, N, L) \\ & \doteq \begin{cases} C_{Lm_f} \left( \frac{m_b^{m_f} m_f^{m_f}}{\Gamma(m_f)} \left( \ln \bar{\gamma} - \ln m_b - \ln m_f \right) \right)^L \bar{\gamma}^{-Lm_f} & \text{if } m_f = m_b N; \\ C_{Lb} \left( m_f^b m_b^b \frac{\Gamma(a-b)}{\Gamma(a)} \right)^L \bar{\gamma}^{-Lb} & \text{if } m_f \neq m_b N, \end{cases}$$
(3.20)

where  $a = \max(m_f, Nm_b)$ ,  $b = \min(m_f, Nm_b)$ ,  $C_{Lm_f} = \frac{\Gamma(\frac{1}{2} + Lm_f)}{2\sqrt{\pi}\Gamma(1 + Lm_f)}$ , and  $C_{Lb} = \frac{\Gamma(\frac{1}{2} + Lb)}{2\sqrt{\pi}\Gamma(1 + Lb)}$ . The exact and asymptotic forms are derived with the asymptotic that  $m_f$  is integer. We will see in simulations that the asymptotic form is also a good

approximation for non-integer  $m_f$ .

# **3.1.2** Diversity Order, Performance Bottleneck and Impact of the Sub-channel Quality

We perform Monte Carlo simulations to verify our analytical results. The simulations are based on BPSK and the OSTBC used is Alamouti's code [48]. Our derived expressions also generalize the SER expression for tags using one antenna, where no coding scheme is applied and MRC is applied at the receiver side. Thus for L = 1, no coding scheme is used; for L = 2 Alamouti's code is used. We can see from Figs. 3.1, 3.2 for Rician fading and Figs. 3.3, 3.4, 3.5 for Nakagami-m fading that our exact and asymptotic expressions match well with simulation results.



**Figure 3.1:** The SER performance of the backscatter RFID channel, the *K* factors are  $K_f = K_b = 0$  dB. From the top to the bottom: (L = 1, N = 1), (L = 1, N = 2), (L = 1, N = 3), (L = 2, N = 1), (L = 2, N = 2), (L = 2, N = 3).

Below we discuss two important properties of this backscatter RFID channel: the diversity order and the effects of forward and backscattering links on the error rate performance.

One important property in a MIMO channel is the diversity order. From the asymptotic expression in (3.14), the diversity order for the MIMO RF backscattering channel under Rician fading is

$$d_a = \lim_{\bar{\gamma} \to \infty} \left( -\frac{\log P(\bar{\gamma})}{\log(\bar{\gamma})} \right) = L.$$
(3.21)

It is interesting that the diversity order does not depend on the number of receiving antennas, as also observed from Figs. 3.1 and 3.2, where it is clear that the slopes of the SER curves only depend on L. However, for one receiving antenna (i.e.



**Figure 3.2:** The SER performances of the backscatter RFID channels, where  $K_f = K_b = 3$  dB. From the top to the bottom: (L = 1, N = 1), (L = 1, N = 2), (L = 1, N = 3), (L = 2, N = 1), (L = 2, N = 2), and (L = 2, N = 3).

N = 1), it requires higher SNR to achieve diversity order of *L* than for the case  $N \ge 2$ , because of the logarithm function associated with SNR in Eqn. (3.14), and leads to a significant performance enhancement by increasing the number of receiving antennas from one to two. From Eqn. (3.14), we plot Fig. 3.6 to show the gains by increasing *N*. We can see that, for  $N \ge 2$ , the performance enhancement is not significant when using more receiving antennas. This is because, when  $N \ge 2$ , the SERs are only different by a coefficient  $C_3^b(N-2)!$  which cannot provide additional diversity gain. The above observations suggest a good trade-off between performance and hardware complexity: In the MIMO RF backscattering channel under Rician fading, since two receiving antennas can capture most performance enhancement by the receiving antenna diversity, it is good to have two receiving antennas regardless how many tag antennas the system has.



**Figure 3.3:** The SER performances of the backscatter RFID channel, with  $m_f = m_b = 1$ . From the top to the bottom: (L = 1, N = 1), (L = 1, N = 2), (L = 1, N = 3), (L = 2, N = 1), (L = 2, N = 2), (L = 2, N = 3).

For the Nakagami-m fading, the diversity order is

$$d_a = L\min(m_f, Nm_b). \tag{3.22}$$

For the case that the channel condition of the forward link is not significantly better than that of the backscattering link, i.e.  $m_f \leq Nm_b$ , the diversity order is reduced to

$$d_a = Lm_f. \tag{3.23}$$

This is consistent with the Rician fading case in the sense that the diversity order is not related with the number of receiving antennas, as observed in Figs. 3.3, 3.4 and 3.5 where the slopes of the SER curves are determined by L. For the case that the channel condition of the forward link is not significantly better than that



**Figure 3.4:** The SER performances of the backscatter RFID channel, with  $m_f = m_b = 1.5$ . From the top to the bottom: (L = 1, N = 1), (L = 1, N = 2), (L = 1, N = 3), (L = 2, N = 1), (L = 2, N = 2), (L = 2, N = 3).

of the backscattering link, the rule that two receiving antennas can capture most performance enhancement by the receiving antenna diversity is also applicable in Nakagami-m fading, as verified by checking the coefficients  $\frac{\Gamma(a-b)}{\Gamma(a)}$  in (3.20).

It is worth mentioning here that other types of cascaded channels generally achieve different diversity orders. For instance, the diversity order of the Rayleigh-Rayleigh keyhole channel is  $\min(L,N)$  [52]. This is due to the different cascaded structures of the channels, and we summarize the diversity gains in Table 3.1 for comparison.

# Performance Improvement by Employing OSTBC in Backscatter RFID Channels

In Chapter 2 we found that the diversity order for identical signaling with uniform query achieves (N,L). In this subsection, we investigate how much performance



**Figure 3.5:** The SER performances of the backscatter RFID channel, with  $m_f = m_b = 2$ . From the top to the bottom: (L = 1, N = 1), (L = 1, N = 2), (L = 1, N = 3), (L = 2, N = 1), (L = 2, N = 2), (L = 2, N = 3).

**Table 3.1:** Diversity order comparisons between different fading channels when OSTBCs are employed.

Cascaded form	Rician	Nakagami-m
RF backscattering	L	$L\min(m_f, Nm_b)$
keyhole channel	$\min(L,N)$ [53]	$\min(Lm_f, Nm_b)$ [54]
i.i.d cascaded channel	LN [55].	$LN\min(m_f, m_b)$

enhancement can be brought by employing OSTBC instead of identical signaling. Fig. 3.7 compares the BER performances of Alamouti's coding scheme and the identical signaling scheme in the  $N \times L$  backscatter RFID channel, where the RF tag is equipped with 2 antennas (i.e. L = 2) and the number of reader receiving antennas varies from 1 to 3. A significant performance improvement (about 10 dB) is observed by Alamouti's coding scheme for the setting N = 1. However, for the settings N = 2 and N = 3, the improvements by Alamouti's scheme are



**Figure 3.6:** The asymptotic form in (3.14) shows that two receiving antennas are enough to capture most of the receiving side gain: For  $N \ge 2$ , the receiving side gain is only brought by the coefficient  $(N-2)!C_3^b$ . We plot  $\frac{(N-2)!C_3^b|_{N=3}}{(N-2)!C_3^b|_{N=3}}$  and  $\frac{(N-2)!C_3^b|_{N=4}}{(N-2)!C_3^b|_{N=4}}$  which are the gains by increasing the number of receiving antennas from 2 to 3 and from 3 to 4, respectively, to compare with the gain from N = 1 to N = 2, at SNR = 20 dB.

not significant (i.e., 3 dB for N = 2 and 1.5 dB for N = 3). This observation can be explained by the derived asymptotic BER expressions in equations (2.34) and (3.21): Our analysis for the OSTBC scheme implies that for Alamouti's code the achievable diversity gain is L (L = 2 in this example) for any N in the  $N \times L$ backscatter RFID channel. Consequently for the settings with  $N \ge L$ , Alamouti's code yields the same diversity order as that of the identical signaling scheme in the  $N \times L$  backscatter RFID channel, and the BER performance improvement is limited. In other words, when  $N \ge L$  in a MIMO RFID system, OSTBC doesn't yield significant performance improvement over simpler signaling schemes.



Figure 3.7: The BER performance comparison between Alamouti's coding scheme and identical signaling scheme. A significant BER improvement by Alamouti's code is observed for N = 1, while the improvement is much smaller when  $N \ge 2$ . These properties can be explained by our analysis of the MIMO backscatter RFID channel under the OSTBC and identical signaling schemes.

#### **Impact of Forward and Backscattering Channel Conditions**

Another interesting property of the MIMO RF backscattering channel is that its performance is more sensitive to the channel condition of the forward link than to that of the backscattering link when  $N \ge 2$ .

For Rician fading, the following is observed from Fig. 3.8: with  $K_f$  being fixed, SER performances are almost remain the same when  $K_b$  changes. By contrast, with  $K_b$  being fixed, SER performances change significantly when  $K_f$  changes. This can also be observed from the asymptotic expression we derived in (3.14): the effect of forward channel is reflected by the coefficients  $C_1^f$ , and that of the backscattering



**Figure 3.8:** The performance of the backscatter RFID channel is much more sensitive to the *K* factor of the forward link. When  $K_f = 0$  dB is fixed, the variations of  $K_b$  (0,3,5 dB) do not affect too much on the SER, by contrast when  $K_b = 0$  dB is fixed, the variations of  $K_f$  (0,3,5 dB) change the SER significantly. Here N = 3, L = 1, and no coding scheme is used.

channel is reflected by  $C_3^b$ . A plot of the two coefficients is given in Fig. 3.9, which is consistent with what we note in the SER curves.

Fig. 3.10 shows that similar observations are true for the channel with Nakagamim distributed forward and backscattering links. This can also be inferred from the asymptotic form in (3.20), as  $m_f$  can change SER significantly if  $m_f < Nm_b$  which is highly likely to be true if we increase the number of receiving antennas N.



**Figure 3.9:** Illustration of the reason that the performance of the backscatter RFID channel is much more sensitive to the forward link. The asymptotic form in (3.14) shows that the error rate performance is much more sensitive to the channel condition of the forward links.  $C_1^f$  and  $C_3^b$  are the coefficients in the asymptotic SER related with the forward links and the backscattering links, respectively.  $C_3^b$  is almost constant as the *K* factor in the backscattering links increases, while  $C_1^f$  decreases significantly as the *K* factor in the forward links increases.

### 3.1.3 PEP Lower Bound for General Space-time Codes and Maximum Achievable Diversity Order

Although OSTBC is one of the most attractive MIMO schemes with a simple decoding process, we are still interested in the performance of general (non-orthogonal) space-time codes because we would like to investigate the performance limit of the backscatter RFID channel. The exact error-rate form of the backscatter RFID channel for the non-orthogonal space-time code case is not trackable due to the complexity of the channel matrix. In this section, instead of providing the exact



**Figure 3.10:** The performance of the backscatter RFID channel is much more sensitive to the m parameters of the forward link. With fixed  $m_f = 1$ , the variations of  $m_b$  (1,1.5,2) do not affect SER much; while for fixed  $m_b = 1$ , the variations of  $m_f$  (1,1.5,2) affect SER significantly. Here N = 3 and L = 2.

form of the PEP, we provide a PEP bound lower bound for the non-orthogonal code case. This bound helps us to understand the best performance this channel can ever achieve.

**Proposition 5.** With the assumption of ideal channel state information, let the probability of transmitting code word  $\mathbf{c}$  and deciding in favor of  $\mathbf{e}$  at the decoder be  $P(\mathbf{c} \rightarrow \mathbf{e})$ . This probability is lower bounded by

$$P(\mathbf{c} \to \mathbf{e}) \ge P_{OSTBC}(\lambda_{max}\bar{\gamma}, N, L), \qquad (3.24)$$

where  $\lambda_{max}$  is the largest eigenvalue of  $\mathbf{D} = (\mathbf{c} - \mathbf{e})(\mathbf{c} - \mathbf{e})^H$ .

This shows that the maximum achievable diversity order is *L* for the backscatter RFID channel under Rician fading and is  $L\min(m_f, Nm_b)$  for Nakagami-m fading, both with the maximum achievable coding gain as  $\lambda_{\max}^L$ .

Proof of Proposition 5. The conditional PEP on the channel gain H is

$$P(\mathbf{c} \to \mathbf{e} | \mathbf{H}) = \mathbb{Q}\left(\bar{\gamma} \sum_{n=0}^{N} \mathbf{H}_{n} \mathbf{D} \mathbf{H}_{n}^{H}\right)$$
(3.25)

where  $\mathbf{H}_n$ 's are the column vectors of  $\mathbf{H}$ . Since  $\mathbf{D}$  is Hermitian, it has an eigendecomposition as

$$\mathbf{D} = \mathbf{U}\mathbf{V}\mathbf{U}^H \tag{3.26}$$

where U is a unitary matrix and V is a diagonal matrix whose elements are the eigenvalues of D. Let the entries of V be replaced by the largest entries of V and we name the new matrix as  $V_{max}$ . It follows that

$$\mathbf{H}_{n}\mathbf{U}\mathbf{V}\mathbf{U}^{H}\mathbf{H}_{n}^{H} \leq \mathbf{H}_{n}\mathbf{U}\mathbf{V}_{\max}\mathbf{U}^{H}\mathbf{H}_{n}^{H}, \qquad (3.27)$$

because  $\mathbf{U}(\mathbf{V}_{\text{max}} - \mathbf{V})\mathbf{U}^{H}$  is positive-semidefinite. The equality holds for OSTBC.

Since the Q function is monotone decreasing, we have

$$P(\mathbf{c} \to \mathbf{e} | \mathbf{H}) = \mathbb{Q}\left(\bar{\gamma} \sum_{n=0}^{N} \mathbf{H}_{n} \mathbf{D} \mathbf{H}_{n}^{H}\right)$$
(3.28)

$$\geq \mathbb{Q}\left(\bar{\gamma}\sum_{n=0}^{N}\mathbf{H}_{n}\mathbf{D}_{\max}\mathbf{H}_{n}^{H}\right)$$
(3.29)

$$= \mathbb{Q}\left(\lambda_{\max}\bar{\gamma} \|\mathbf{H}\|_{F}^{2}\right)$$
(3.30)

for all **H**, where  $\mathbf{D}_{max} = \mathbf{U}\mathbf{V}_{max}\mathbf{U}^{H}$ . Taking the expectation of the above inequality, we have

$$P(\mathbf{c} \to \mathbf{e}) \ge P_{\text{OSTBC}}(\lambda_{\max}\bar{\gamma}, N, L).$$
(3.31)

A tighter bound given in Proposition 6 can be obtained with the assumption that the phases of forward channels and/or the phases of the backscattering are uniformly distributed over  $(-\pi, \pi]$  and are independent with their channel envelopes. The assumption is true for the forward-backscattering structures of Rayleigh-Rician, Rician-Rayleigh, Rayleigh-Rayleigh, Nakagami-Rayleigh, Rayleigh-Nakagami. It is also applicable to the Nakagami-Nakagami structure if the phases of the subchannels are uniformly distributed.

**Proposition 6.** If the phases of forward channels and/or the phases of backscattering channels are uniformly distributed, a tighter bound of PEP can be given as

$$P(\mathbf{c} \to \mathbf{e}) \ge P_{OSTBC}(\lambda_{avg}\bar{\gamma}, N, L), \qquad (3.32)$$

where  $\lambda_{avg}$  is the average of the eigenvalues of **D**.

This shows that in this case the coding gain can be further bounded by  $\lambda_{avg}^L$ .

*Proof of Proposition 6.* We first rewrite the pair-wise code distance  $\sum_{n=1}^{N} \mathbf{H}_n \mathbf{D} \mathbf{H}_n^H$  as

$$\sum_{n=1}^{N} \mathbf{H}_{n} \mathbf{D} \mathbf{H}_{n}^{H} = \sum_{n=1}^{N} \left( \sum_{l=1}^{L} d_{l,l} |h_{n,l}|^{2} + \sum_{l_{1} \neq l_{2}} d_{l_{1},l_{1}} d_{l_{2},l_{2}}^{H} h_{n,l_{1}} h_{n,l_{2}}^{H} \right)$$
$$= Z + X$$
(3.33)

where  $d_{i,j}$ s' are the entries of **D** and we define the real random variable *X* as  $X = 2\Re \left\{ \sum_{n=1}^{N} \sum_{l_1 < l_2} d_{l_1,l_1} d_{l_2,l_2}^H h_{n,l_1} h_{n,l_2}^H \right\}$ , and *Z* as  $Z = \sum_{n=1}^{N} \sum_{l=1}^{L} d_{l,l} |h_{n,l}|^2$ , where  $h_{n,l} = h_l^f h_{l,n}^b$  represents the entries of **H**.

*Case 1: The phases of forward channels are uniformly distributed* Our goal is to find  $\mathbb{E}_{\mathbf{H}} \left( \mathbb{Q} \left( \sum_{n=1}^{N} \mathbf{H}_{n} \mathbf{D} \mathbf{H}_{n}^{H} \right) \right)$ . Note that the *l*-th forward channel gain  $h_{l}^{f}$  can be written as  $h_{l}^{f} = |h_{l}^{f}| \cos \theta_{l}$ , where  $\theta_{l}$  is the phase of  $h_{l}^{f}$  and is uniformly distributed over  $(-\pi, \pi]$ . We first fix  $|h_{l}^{f}|$ 's (i.e. the magnitudes of the forward links) and  $h_{l,n}^{b}$ 's (i.e. the channel gains of the backscattering links). Note that right now we only leave the phase of the channel gains of the forward links  $h_{l}^{f}$ 's to be free to choose over the space in which  $|h_{l,n}^{b}|$ 's,  $h_{l,n}^{b}$ 's and  $|h_{l}^{f}|$ 's are all fixed. Since  $\theta_{l}$  is independent with  $h_{l,n}^b$  and  $|h_l^f|$ , it yields that the conditional distribution of X on  $h_{l,n}^b$ and  $|h_l^f|$  is identical with the conditional distribution of -X on  $h_{l,n}^b$  and  $|h_l^f|$ . This implies that the distribution of X is symmetric about zero, and therefore  $\mathbb{E}(X) = 0$ . Since the Q function is convex for positive arguments, using the conditional expectation and the fact the a convex function applied to the expected value of a random variable is always less or equal to the expected value of the convex function of the random variable, we have

$$P(\mathbf{c} \to \mathbf{e}) = \mathbb{E}_{Z,X}(\mathbb{Q}(g\bar{\gamma}(Z+X)))$$
  
=  $\mathbb{E}_Z(\mathbb{E}_{X|Z}(\mathbb{Q}(g\bar{\gamma}(Z+X))|Z))$   
>  $\mathbb{E}_Z(\mathbb{Q}\left(\mathbb{E}_{X|Z}(g\bar{\gamma}(Z+X|Z))\right))$   
=  $\mathbb{E}_Z(\mathbb{Q}(g\bar{\gamma}Z))$  (3.34)

or

$$\mathbb{E}_{\mathbf{H}}\left(\mathbb{Q}\left(g\bar{\gamma}\sum_{n=1}^{N}\mathbf{H}_{n}\mathbf{D}\mathbf{H}_{n}^{H}\right)\right) > \mathbb{E}_{\mathbf{H}}\left(\mathbb{Q}\left(g\bar{\gamma}\sum_{n=1}^{N}\sum_{l=1}^{L}d_{l,l}|h_{n,l}|^{2}\right)\right).$$
 (3.35)

Note that  $\mathbb{E}_{\mathbf{H}}\left(\mathbb{Q}\left(g\bar{\gamma}\sum_{n=1}^{N}\sum_{l=1}^{L}d_{l,l}|h_{n,l}|^{2}\right)\right) = \frac{1}{\pi}\int_{\theta=0}^{\pi}\prod_{l=1}^{L}G(d_{l,l}\bar{\gamma},N,1)d\theta$ . It follows that the coding gain is bounded by  $\prod_{l=1}^{L}d_{l,l}$  and the diversity gain is bounded by  $\bar{\gamma}^{L}$ . Now we claim that

$$\mathbb{E}_{\mathbf{H}}\left(\mathbb{Q}\left(g\bar{\gamma}\sum_{n=1}^{N}\sum_{l=1}^{L}d_{l,l}|h_{n,l}|^{2}\right)\right) > \mathbb{E}_{\mathbf{H}}\left(\mathbb{Q}\left(g\bar{\gamma}\sum_{n=1}^{N}\sum_{l=1}^{L}\lambda_{\mathrm{avg}}|h_{n,l}|^{2}\right)\right)$$
$$= P_{OSTBC}(\lambda_{\mathrm{avg}}\bar{\gamma}, N, L).$$
(3.36)

The proof is given as follows: Since  $||\mathbf{c} - \mathbf{e}||_F = \sum_{l=1}^L \lambda_l = L\lambda_{avg} = trace(\mathbf{D}) = \sum_{l=1}^L d_{l,l}$ ,

$$\mathbb{E}_{\mathbf{H}}\left(\mathbb{Q}\left(g\bar{\gamma}\sum_{n=1}^{N}\sum_{l=1}^{L}d_{l,l}|h_{n,l}|^{2}\right)\right) = \mathbb{E}_{\mathbf{H}}\left(\mathbb{Q}\left(g\bar{\gamma}(W+Y)\right)\right)$$
(3.37)

where we define

$$W = \sum_{n=1}^{N} \sum_{l=1}^{L} \lambda_{\text{avg}} |h_{n,l}|^2, \qquad (3.38)$$

and

$$Y = \sum_{n=1}^{N} \sum_{l=1}^{L} (d_{l,l} - \lambda_{\text{avg}}) |h_{n,l}|^2.$$
(3.39)

Moreover, by symmetry, the conditional distribution of  $|h_{l,n}|^2$ 's on *W* are identical, i.e.  $|h_{l,n}|^2|W$  ( $l = 1, \dots, L$ ) are identical (but not necessarily independent) r.v.s. Therefore their conditional expectations on *W* must be the same, i.e.,

$$\mathbb{E}_{|h_{1,n}|^2|W}(|h_{1,n}|^2|W) = \mathbb{E}_{|h_{2,n}|^2|W}(|h_{2,n}|^2|W) = \dots = \mathbb{E}_{|h_{L,n}|^2|W}(|h_{L,n}|^2|W).$$
(3.40)

It follows that

$$\mathbb{E}_{Y|W}(Y|W) = \sum_{n=1}^{N} \left( \sum_{l=1}^{L} (d_{l,l} - \lambda_{\text{avg}}) \right) \mathbb{E}_{|h_{1,n}|^2|W} \left( |h_{1,n}|^2|W \right)$$
  
= 0, (3.41)

since  $\left(\sum_{l=1}^{L} (d_{l,l} - \lambda_{avg})\right) = 0$ . Therefore we have

$$\mathbb{E}_{\mathbf{H}} \left( \mathbb{Q} \left( g \bar{\gamma}(W+Y) \right) \right) = \mathbb{E}_{W} \left( \mathbb{E}_{Y|W} \left( \mathbb{Q} (g \bar{\gamma}(W+Y)) | W \right) \right)$$
  

$$\geq \mathbb{E}_{W} \left( \mathbb{Q} \left( \mathbb{E}_{Y|W} (g \bar{\gamma}(W+Y) | W) \right) \right)$$
  

$$= \mathbb{E}_{W} \left( \mathbb{Q} (g \bar{\gamma} W) \right)$$
  

$$= P_{OSTBC} (\lambda_{avg} \bar{\bar{\gamma}}, N, L).$$
(3.42)

Again, this is followed by the fact the a convex function applied to the expected value of a random variable is always less or equal to the expected value of the convex function of the random variable.

For the case that the phases of the backscattering links are uniformly distributed, the proof is similar and we omit it here.  $\hfill\square$ 

### **3.2** Space-time Coding with Unitary Query

Recall that there are three ends in the backscatter RFID structure. In Section 3.1, the potential of diversity gain is fully explored at the tag end, by applying spacetime codes. Now we explore the potential diversity at the reader query end. In the previous literature [14, 26, 27], the understanding of the query end was that it only played a role as an energy provider, and since there was no information to be conveyed from the reader query end, query signals could not provide spatial diversity for the tag. However, in this section we reconsider the query signals and propose the *unitary query* the first time. We show that the proposed unitary query can improve the PEP performance of STC significantly, by providing the tag time diversity via employing multiple antennas at the reader query end. Due to the difficulty of obtaining the asymptotic PEP and the even diversity order for the proposed unitary query, we also provide a new measure for performance analysis. With the new measure, we do not need to exactly calculate the PEP but can still compare performances of different query and space-time coding schemes. Recall that in Chapter 2, the channel model of the  $M \times L \times N$  backscatter RFID can be characterized by

$$\mathbf{R} = \mathbf{Q}\mathbf{H} \circ \mathbf{C}\mathbf{G} + \mathbf{W},\tag{3.43}$$

where both the forward sub-channels (represented by  $\mathbf{H}$ ) and the backscattering sub-channels (represented by  $\mathbf{G}$ ) are modeled as i.i.d. complex Gaussian random variables with zero mean and unity variance.

In general, query signals can be designed followed by any arbitrary Q. For the so-called unitary query, the query matrix Q satisfies

$$\mathbf{Q}\mathbf{Q}^H = \mathbf{I}.\tag{3.44}$$

Since **Q** is unitary and the entries of **H** are i.i.d complex Gaussian, we have

$$\mathbf{Q}\mathbf{H} \sim \mathbf{X} = \begin{pmatrix} x_{1,1} & \cdots & x_{1,L} \\ \vdots & \ddots & \vdots \\ x_{T,1} & \cdots & x_{T,L} \end{pmatrix}, \qquad (3.45)$$

where  $x_{t,l}$ 's are i.i.d complex Gaussian. The resulting matrix **X** is with size  $T \times L$ , so the unitary query actually transforms the forward channel **H**, which is invariant over the *T* time slots, into a channel **X** which varies over the *T* time slots. We will show later that this variation over the *T* time slots is the fundamental reason that the unitary query can bring additional time diversity and significant performance improvement for some STCs in the backscatter RFID channel. Therefore the backscatter RFID channel with the unitary query has an equivalent channel model as

$$\mathbf{R} = \mathbf{X} \circ \mathbf{C}\mathbf{G} + \mathbf{W}. \tag{3.46}$$

Now we define the code words difference matrix for code words C and C' as,

$$\Delta = \mathbf{C} - \mathbf{C}' = \begin{pmatrix} \delta_{1,1} & \cdots & \delta_{1,T} \\ \vdots & \ddots & \vdots \\ \delta_{L,1} & \cdots & \delta_{L,T} \end{pmatrix}.$$
 (3.47)

The PEP can be obtained by

$$\operatorname{PEP}(\bar{\gamma}) = \mathbb{E}_{\mathbf{H},\mathbf{G}}\left(\mathbb{Q}\left(\sqrt{\bar{\gamma}Z_X}\right)\right). \tag{3.48}$$

where

$$Z_X = \|\mathbf{Q}\mathbf{H} \circ \mathbf{C}\mathbf{G} - \mathbf{Q}\mathbf{H} \circ \mathbf{C}'\mathbf{G}\|_F^2$$
  
 
$$\sim \|\mathbf{X} \circ \Delta \mathbf{G}\|_F^2$$
(3.49)

is the random variable which represents the distance between the code words C and C'.

#### **3.2.1** New Measure for PEP Performance

Diversity order is a conventional measure of the PEP performance for space-time codes. It has been used for performance analysis and is an important criteria for code construction. In conventional wireless fading channels, which have a simpler signaling and fading structure than that of the backscatter RFID channel, usually

the asymptotic PEP and the diversity order can be obtained in closed form, based on which the code design criteria can be derived accordingly. However, due to the query-fading-signaling-fading structure given in (2.5) of the backscatter RFID channel, the asymptotic PEP and diversity order for the general space-time code cannot be obtained in analytical form. In this section, we provide a new measure of the PEP performance in the backscatter RFID channel.

Recall the distance between two code words given in Eqn. (3.49). At each time slot *t*, the distance is given by

$$Z_X^t \sim \|(x_{t,1}, \cdots, x_{t,L}) \circ (\boldsymbol{\delta}_{1,t}, \cdots, \boldsymbol{\delta}_{L,t}) \mathbf{G}\|_F^2$$
  
=  $\|(x_{t,1}, \cdots, x_{t,L}) \Delta_t \mathbf{G}\|_F^2$   
(3.50)

where  $\Delta_t$  is defined as

$$\Delta_t \triangleq \begin{pmatrix} \delta_{1,t} & & \\ & \ddots & \\ & & \delta_{L,t} \end{pmatrix}, \qquad (3.51)$$

then over the T time slots we have

$$Z_X \sim \sum_{t=1}^T \| (x_{t,1}, \cdots, x_{t,L}) \Delta_t \mathbf{G} \|_F^2$$
  
=  $\sum_{t=1}^T \| (x_{t,1}, \cdots, x_{t,L}) \mathbf{E}_t \|_F^2,$  (3.52)

where  $\mathbf{E}_t$  is defined as

$$\mathbf{E}_t \triangleq \Delta_t \mathbf{G}. \tag{3.53}$$

We will see later that the ranks of the random matrices  $\mathbf{E}_t$ 's determine the performance for the unitary query.

We use  $Z_Y$  to denote the distance between code words when the backscatter

RFID channel employs the uniform query,

$$Z_{Y} \sim \sum_{t=1}^{T} \|(y_{1}, \cdots, y_{L})\mathbf{E}_{t}\|_{F}^{2}$$
  
=  $\|(y_{1}, \cdots, y_{L})(\mathbf{E}_{1}, \cdots, \mathbf{E}_{T})\|_{F}^{2}.$  (3.54)

Note that inside a  $\|\cdot\|_F$  operator, the columns of the matrix  $(\mathbf{E}_1, \dots, \mathbf{E}_T)$  are interchangeable, therefore we have

$$Z_Y = \|(y_1, \cdots, y_L)(\mathbf{D}_1, \cdots, \mathbf{D}_N)\|_F^2,$$
(3.55)

where  $\mathbf{D}_n$ 's are defined as

$$\mathbf{D}_n \triangleq \Delta \mathbf{G}_n, \tag{3.56}$$

and where  $G_n$ 's are defined as

$$\mathbf{G}_{n} \triangleq \begin{pmatrix} h_{1,n}^{b} & & \\ & \ddots & \\ & & h_{L,n}^{b} \end{pmatrix}, \qquad (3.57)$$

for  $n = 1, \dots, N$ . Also, we will see later that the rank of the random matrix

$$\mathbf{D} \triangleq (\mathbf{D}_1, \cdots, \mathbf{D}_N) \tag{3.58}$$

determines the performance for the uniform query.

Now we give the following two Lemmas about the ranks of the random matrices  $\mathbf{E}_t$ 's and the rank of the random matrix  $\mathbf{D}$ .

**Lemma 1.** For the matrices  $\mathbf{E}_t$ 's defined in (3.53), we have  $rank(\mathbf{E}_t) = \min(N, L_t^*)$ with probability (w.p.) 1 for all  $t \in \{1, \dots, T\}$ , where  $L_t^*$  is the number of non-zero elements of the t-th column of the code words difference matrix  $\Delta$ .

*Proof.* Let  $\mathbf{g}_1, \dots, \mathbf{g}_N$  denote the columns of  $\mathbf{G}$ . We consider a set of scalars
$\{a_1, \dots, a_N\}$  where  $a_n \in \mathbb{C}$ , for any linear combination of the set of vectors,  $\{\mathbf{g}_1, \dots, \mathbf{g}_N\}$ 

$$\mathbf{b} = \sum_{n=1}^{L} a_n \mathbf{g}_n \tag{3.59}$$

is a zero-mean complex Gaussian random vector with covariance matrix  $\sum_{n=1}^{L} ||a_n||^2 \mathbf{I}$ Therefore

$$\mathbb{P}\left(\mathbf{b}=\mathbf{0}\right)=0.\tag{3.60}$$

When  $N \leq L$ , (3.60) implies that

$$\mathbb{P}\left(\operatorname{rank}(\mathbf{G}) < N\right) = 0,\tag{3.61}$$

or

$$\mathbb{P}\left(\operatorname{rank}(\mathbf{G})=N\right)=1.$$
(3.62)

When N > L, by performing a linear combination of the rows of **G** and following a procedure similar to the case that  $N \le L$ , we can obtain

$$\mathbb{P}\left(\operatorname{rank}(\mathbf{G})=L\right)=1.$$
(3.63)

Hence the matrix **G** is of full rank with probability 1, i.e.

$$\mathbb{P}(\operatorname{rank}(\mathbf{G}) = \min(N, L)) = 1.$$
(3.64)

Now notice that  $\Delta_t$  is diagonal, therefore  $\mathbf{E}_t = \Delta_t \mathbf{G}$  has  $L_t^*$  non-zero rows. Because **G** is full rank w.p. 1, we have

$$\operatorname{rank}(\mathbf{E}_t) = \min(L_t^*, N) \tag{3.65}$$

w.p. 1.

**Lemma 2.** For the matrix **D** defined in (3.58), we have  $rank(\mathbf{D}) = min(N \times rank(\Delta), L)$  with probability 1, where *L* is the number of non-zero columns of the code words

#### difference matrix $\Delta$ .

*Proof.* Following similar steps to prove that G is of full rank w.p. 1, we can show that

$$\mathbb{P}(\operatorname{rank}(\mathbf{G}_n) = L) = 1, \tag{3.66}$$

i.e.,  $\mathbf{G}_n$  is also of full rank w.p. 1. Since

$$\mathbf{D}_n = \Delta \mathbf{G}_n, \tag{3.67}$$

we have

$$\mathbb{P}(\operatorname{rank}(\Delta \mathbf{G}_n) = \operatorname{rank}(\Delta)) = 1, \qquad (3.68)$$

i.e. the rank of  $\mathbf{D}_n$  is the same as the rank of  $\Delta$  w.p. 1.

Now let us consider the following two cases:

*Case 1:*  $N \times rank(\Delta) \leq L$ 

By Eqn. (3.68), clearly the columns of each of  $\mathbf{D}_n$ 's span a subspace of dimension rank( $\Delta$ ) in  $\mathbb{C}^L$  w.p. 1. Now consider a set of scalars  $a_{i,j}$ 's, where  $i \in \{1, \dots, N\}$ , and  $j \in \{1, \dots, T\}$ . If for  $i \in \{2, \dots, N\}$  and  $j \in \{1, \dots, T\}$ ,  $a_{i,j}$ 's are not all zero, it is not hard to verify that

$$\mathbb{P}\left(\sum_{j=1}^{T} a_{1,j} \mathbf{D}_{1,j} = \sum_{i=2}^{N} \sum_{j=1}^{T} a_{i,j} \mathbf{D}_{i,j}\right) = 0.$$
(3.69)

This implies that the rows of all  $\mathbf{D}_n$ 's span a subspace of dimension  $N \times \operatorname{rank}(\Delta)$  in  $\mathbb{C}^L$  w.p. 1, i.e. the rank of the block matrix  $\mathbf{D}$  is  $N \times \operatorname{rank}(\Delta)$  w.p. 1 in this case. *Case 2:*  $N \times \operatorname{rank}(\Delta) > L$ 

Following the similar procedure as in Case 1, it is easy to see that the dimension of the subspace spanned by the rows of all  $\mathbf{D}_n$ 's is *L*. i.e. the rank of the block matrix **D** is *L* w.p. 1 in this case.

With the results from Case 1 and Case 2, we have Lemma 2 hold.

Now we introduce the following theorem on the new measure for the unitary query and the uniform query.

**Theorem 1.** In asymptotic high SNR regimes, the PEP performances of space-time codes with the unitary query and the uniform query in the  $M \times N \times L$  backscatter *RFID* channel given in (2.5) can be measured by

$$R_{unitary} = \sum_{t=1}^{T} \min(N, L_t^*),$$
 (3.70)

and

$$R_{uniform} = \min(N \times rank(\Delta), L), \qquad (3.71)$$

respectively, where *L* is the number of non-zero columns of the code words difference matrix  $\Delta$ , and  $L_t^*$  is the number of non-zero elements of the *t*-th column of the code words difference matrix  $\Delta$ . In other words if

$$R_{unitary} > R_{uniform}, \tag{3.72}$$

we have

$$\lim_{\bar{\gamma}\to\infty}\frac{PEP_{Z_X}(\bar{\gamma})}{PEP_{Z_Y}(\bar{\gamma})}\to 0;$$
(3.73)

if

$$R_{unitary} < R_{uniform}, \tag{3.74}$$

we have

$$\lim_{\bar{\gamma}\to\infty}\frac{PEP_{Z_{Y}}(\bar{\gamma})}{PEP_{Z_{X}}(\bar{\gamma})}\to 0;$$
(3.75)

and if

$$R_{unitary} = R_{uniform}, \tag{3.76}$$

we have

$$\lim_{\bar{\gamma}\to\infty}\frac{PEP_{Z_X}(\bar{\gamma})}{PEP_{Z_Y}(\bar{\gamma})} = c > 0; \qquad (3.77)$$

where c is some positive constant.

*Proof of Theorem 1.* We consider singular value decompositions of  $\mathbf{E}_t$ 's and  $\mathbf{D}$ , i.e.

$$\mathbf{E}_t = \mathbf{U}_t \Lambda_t \mathbf{V}_t, \qquad (3.78)$$

and

$$\mathbf{D} = \mathbf{U}^* \Lambda^* \mathbf{V}^*. \tag{3.79}$$

Note that, for the unitary query, for a realization of **G** the distance between codewords can be given as

$$Z_{X}|\mathbf{G} = \sum_{t=1}^{T} \|(x_{t,1}, \cdots, x_{t,L})\mathbf{E}_{t}\|_{F}^{2}$$

$$= \sum_{t=1}^{T} \|(x_{t,1}, \cdots, x_{t,L})\mathbf{U}_{t}\Lambda_{t}\mathbf{V}_{t}\|_{F}^{2}$$

$$\sim \sum_{t=1}^{T} \|(x_{t,1}, \cdots, x_{t,L})\Lambda_{t}\|_{F}^{2}$$

$$= \sum_{t=1}^{T} \sum_{i=1}^{\operatorname{rank}(\mathbf{E}_{t})} \lambda_{t,i} \|x_{t,i}\|^{2}, \qquad (3.80)$$

where  $\lambda_{t,i}$ 's  $(i = 1, \dots, \operatorname{rank}(\mathbf{E}_t))$  are the non-zero eigenvalues of  $\mathbf{E}_t$ . Given a realization of  $\mathbf{G}$ , the conditional PEP on  $\mathbf{G}$  is given by

$$PEP_{Z_{X}|\mathbf{G}}(\bar{\gamma}) = \mathbb{E}_{Z_{X}|\mathbf{G}}\left(\mathbb{Q}\left(\sqrt{\bar{\gamma}\sum_{t=1}^{T}\sum_{i=1}^{rank(\mathbf{E}_{t})}\lambda_{t,i}||x_{t,i}||^{2}}\right)\right)$$
$$=\prod_{t=1}^{T}\prod_{i=1}^{rank(\mathbf{E}_{t})}\frac{1}{1+\lambda_{t,i}\bar{\gamma}}$$
(3.81)

Therefore the PEP for the unitary query can be obtained as

$$PEP_{Z_{X}}(\bar{\gamma}) = \mathbb{E}_{\mathbf{G}} \left( PEP_{Z_{X}|\mathbf{G}}(\bar{\gamma}) \right)$$
$$= \mathbb{E}_{\mathbf{G}} \left( \prod_{t=1}^{T} \prod_{i=1}^{\operatorname{rank}(\mathbb{E}_{t})} \frac{1}{1 + \lambda_{t,i}\bar{\gamma}} \right)$$
$$= \mathbb{E}_{\mathbf{G}} \left( \prod_{t=1}^{T} \prod_{i=1}^{\min(N,L_{t}^{*})} \frac{1}{1 + \lambda_{t,i}\bar{\gamma}} \right).$$
(3.82)

The last step of the above derivation is obtained by using the result from Lemma 1 and the fact that  $0 < \frac{1}{1 + \lambda_{r,i} \tilde{\gamma}} < \infty$ .

Similarly, for the uniform query, for a realization of G, the distance between codewords can be given by

$$Z_{Y}|\mathbf{G} = \|(y_{1}, \cdots, y_{L})\mathbf{D}\|_{F}^{2}$$
  
=  $\|(y_{1}, \cdots, y_{L})\mathbf{U}^{*}\Lambda^{*}\mathbf{V}^{*}\|_{F}^{2}$   
 $\sim \|(y_{1}, \cdots, y_{L})\Lambda^{*}\|_{F}^{2}$   
=  $\sum_{i=1}^{rank(\mathbf{D})} \lambda_{i}^{*}\|(y_{1,i})\|^{2}$ , (3.83)

where  $\lambda_i^*$ 's are the eigenvalues of **D**. For a realization of **G**, the conditional PEP is given by

$$PEP_{Z_{Y}|\mathbf{G}}(\bar{\gamma}) = \mathbb{E}_{Z_{Y}|\mathbf{G}}\left(\mathbb{Q}\left(\sqrt{\bar{\gamma}\sum_{i=1}^{\operatorname{rank}(\mathbf{D})}\lambda_{i}^{*}\|x_{i}\|^{2}}\right)\right)$$
$$= \prod_{i=1}^{\operatorname{rank}(\mathbf{D})}\frac{1}{1+\lambda_{i}^{*}\bar{\gamma}}$$
(3.84)

Therefore the PEP for the uniform query is given by

$$\begin{aligned} \text{PEP}_{Z_{Y}|\mathbf{G}}(\bar{\gamma}) &= \mathbb{E}_{\mathbf{G}} \left( \prod_{i=1}^{\text{rank}(\mathbf{D})} \frac{1}{1 + \lambda_{i}^{*} \bar{\gamma}} \right) \\ &= \mathbb{E}_{\mathbf{G}} \left( \prod_{i=1}^{\min(N \times \text{rank}(\Delta), L)} \frac{1}{1 + \lambda_{i}^{*} \bar{\gamma}} \right). \end{aligned} \tag{3.85}$$

The last step of the above derivation is obtained by using the result from Lemma 2 and the fact that  $0 < \frac{1}{1+\lambda_i^* \tilde{\gamma}} < \infty$ .

The expectations in (3.82) and (3.85) are quite difficulty to obtain for general  $\Delta$ , as the distributions of  $\lambda_{t,i}$ 's and  $\lambda_i^*$ 's are not traceable. We assume that

$$\mathbb{E}_{\mathbf{G}}\left(\prod_{t=1}^{T}\prod_{i=1}^{\min(N,L_t^*)}\frac{1}{\lambda_{i,t}}\right) < \infty,\tag{3.86}$$

$$\mathbb{E}_{\mathbf{G}}\left(\prod_{i=1}^{R_{\text{uniform}}}\frac{1}{\lambda_i^*}\right) < \infty, \tag{3.87}$$

Using the assumption in (3.86) and by applying Dominated Convergence Theorem (DCT) we have

$$\lim_{\bar{\gamma}\to\infty} \left(\bar{\gamma}^{R_{\text{unitary}}} \times \text{PEP}_{Z_X}(\bar{\gamma})\right) = \mathbb{E}_{\mathbf{G}} \left(\prod_{t=1}^T \prod_{i=1}^{\min(N,L_t^*)} \frac{1}{\lambda_{i,t}}\right),$$
(3.88)

and similarly, using the assumption in (3.87) by applying DCT we have

$$\lim_{\bar{\gamma}\to\infty} \left(\bar{\gamma}^{R_{\text{uniform}}} \times \text{PEP}_{Z_{Y}}(\bar{\gamma})\right) = \mathbb{E}_{\mathbf{G}} \left(\prod_{i=1}^{R_{\text{uniform}}} \frac{1}{\lambda_{i}^{*}}\right).$$
(3.89)

Case  $1:R_{unitary} > R_{uniform}$ 

In this case,

$$\lim_{\bar{\gamma}\to\infty} \frac{\operatorname{PEP}_{Z_{X}}(\bar{\gamma})}{\operatorname{PEP}_{Z_{Y}}(\bar{\gamma})} = \lim_{\bar{\gamma}\to\infty} \frac{\bar{\gamma}^{R_{\operatorname{uniform}}} \mathbb{E}_{\mathbf{G}}\left(\prod_{t=1}^{T} \prod_{i=1}^{\min(N,L_{t}^{*})} \frac{1}{\lambda_{i,t}}\right)}{\bar{\gamma}^{R_{\operatorname{uniform}}} \mathbb{E}_{\mathbf{G}}\left(\prod_{i=1}^{R_{\operatorname{uniform}}} \frac{1}{\lambda_{i}^{*}}\right)} \to 0.$$
(3.90)

Case 2: $R_{unitary} < R_{uniform}$ In this case,

$$\lim_{\bar{\gamma}\to\infty} \frac{\operatorname{PEP}_{Z_{Y}}(\bar{\gamma})}{\operatorname{PEP}_{Z_{X}}(\bar{\gamma})} = \lim_{\bar{\gamma}\to\infty} \frac{\bar{\gamma}^{R_{\operatorname{uniform}}} \mathbb{E}_{\mathbf{G}}\left(\prod_{i=1}^{R_{\operatorname{uniform}}} \frac{1}{\lambda_{i}^{*}}\right)}{\bar{\gamma}^{R_{\operatorname{uniform}}} \mathbb{E}_{\mathbf{G}}\left(\prod_{t=1}^{T} \prod_{i=1}^{\min(N,L_{t}^{*})} \frac{1}{\lambda_{i,t}}\right)} \to 0.$$

$$(3.91)$$

Case 3:  $R_{\text{unitary}} = R_{\text{uniform}}$ In this case, we have

$$\lim_{\bar{\gamma}\to\infty} \frac{\operatorname{PEP}_{Z_{X}}(\bar{\gamma})}{\operatorname{PEP}_{Z_{Y}}(\bar{\gamma})} = \lim_{\bar{\gamma}\to\infty} \frac{\bar{\gamma}^{R_{\operatorname{uniform}}} \mathbb{E}_{\mathbf{G}}\left(\prod_{t=1}^{T} \prod_{i=1}^{\min(N,L_{t}^{*})} \frac{1}{\lambda_{i,t}}\right)}{\bar{\gamma}^{R_{\operatorname{unitary}}} \mathbb{E}_{\mathbf{G}}\left(\prod_{i=1}^{R_{\operatorname{uniform}}} \frac{1}{\lambda_{i}^{*}}\right)} = \frac{\mathbb{E}_{\mathbf{G}}\left(\prod_{t=1}^{T} \prod_{i=1}^{\min(N,L_{t}^{*})} \frac{1}{\lambda_{i,t}}\right)}{\mathbb{E}_{\mathbf{G}}\left(\prod_{i=1}^{R_{\operatorname{uniform}}} \frac{1}{\lambda_{i}^{*}}\right)} = c.$$
(3.92)

#### 3.2.2 Examples and Simulations

In this section, we give a few examples and provide corresponding simulation results for Theorem 1. Consider an  $M \times L \times N$  backscatter RFID channel, and the following code words difference matrix:

$$\Delta = \left(\begin{array}{cc} 1 & -2\\ 1.5 & 2.5 \end{array}\right). \tag{3.93}$$

Suppose M = 2, L = 2 and N = 2. Based on Theorem 1, the PEP performance for the unitary query can be measured by

$$R_{unitary} = \min(2,2) + \min(2,2) = 4, \tag{3.94}$$

and the PEP performance for the uniform query can be measured by

$$R_{uniform} = \min(2 \times 2, 2) = 2.$$
 (3.95)

Therefore the PEP performance of the unitary query is expected to be much better than that of the uniform query. Simulations confirm this as we can see in Fig. 3.11: there is a large PEP performance gain by employing the unitary query for the  $2 \times 2 \times 2$  backscatter RFID channel.

In addition, we consider the  $2\times 2\times 1$  backscatter RFID channel, based on Theorem 1 we have

$$R_{unitary} = \min(1,2) + \min(1,2) = 2, \tag{3.96}$$

and

$$R_{uniform} = \min(1 \times 2, 2) = 2. \tag{3.97}$$

In this case, the performance of the unitary is expected to be similar to that of the uniform query, as we can see from the simulation results shown in Fig. 3.12.

#### 3.3 Conclusion

In this chapter, we considered more complicated reader query and tag signaling methods for the backscatter RFID channel. First, we investigated the case when the tag employs orthogonal space-time codes, while the reader still employs the uniform query. For this case, we provided a general formulation for performance analysis which is applicable to any sub-channels fading assumptions and studied the SER performances for Rician and Nakagami-m sub-channels. It was shown that the diversity order achieves *L* for Rician fading and achieves  $L\min(m_f, Nm_b)$  for Nakagami-m fading. Two receiving antennas (N = 2) can capture most of the



**Figure 3.11:** PEP performance comparisons between the unitary query and the uniform query for the  $2 \times 2 \times 2$  backscatter RFID channel. The unitary query can bring a large gain for the  $2 \times 2 \times 2$  channel.

receiving side gain regardless the number of tag antennas. More interestingly, we showed that the PEP performance in this case is more sensitive to the channel condition (the K factor or the m parameter) of the forward link than that of the backscattering link. Second, we proposed a novel reader query scheme called unitary query at the reader query end, and showed that in quasi-static channels, the unitary query can provide time diversity via multiple reader query antennas and thus can improve the performance significantly. Due to the difficulty of calculating the PEP and the diversity order directly for the unitary query, we suggested a new performance measure based on the rank of some random matrices. To our best knowledge, this was the first time that the unitary query was proposed in RFID.



**Figure 3.12:** PEP performance comparisons between the unitary query and the uniform query for the  $2 \times 2 \times 1$  backscatter RFID channel. The unitary query can only bring a small gain for the  $2 \times 2 \times 1$  channel.

## Chapter 4

# Analysis of General Space-time Codes in MISO Multi-keyhole Channels

In the previous two chapters, we investigated the performance and design of spacetime codes and reader query mechanism for the MIMO backscatter RFID channels. Recall that the backscatter RFID channel has a special query-fading-signalingfading structure, which is a cascaded form. The multi-keyhole channel is another type of cascaded channel, which also has two layers of fading, but with a signalingfading-fading structure. The multi-keyhole fading happens in propagation environments where each end has its own set of multipath components and is separated from the other end by a screen with a number of keyholes of small size (smaller than half a wavelength), as shown in Fig. 4.1.

From the structures of these two types of cascaded channels, we can see that they are indeed different. But these two channels look similar at the first impression, and researchers sometimes may get confused about the two types of channels. Therefore one purpose of this chapter is to give a brief introduction of the multikeyhole channel model and analytically study the performance for general spacetime codes in the MISO case, which is not done yet in the literature. The other purpose is that we want to make comparisons between the backscatter RFID channel and the multi-keyhole channel. We will show that the backscatter RFID channel has completely different performance behavior from that of the multi-keyhole channel.

#### 4.1 Multi-keyhole Channels

In conventional non-backscatter wireless channels, if the scattering environment is not-so-rich, it is demonstrated in [56] [57] that MIMO fading channels can experience keyhole conditions, where despite rich local scattering and independent transmitting and receiving signals, the system only has a cascaded channel structure. The early research for keyhole channels mainly concentrated on single-keyhole channels [52, 58–63]. In particular, in [58], a closed-form expression of the ergodic capacity for an uncorrelated single-keyhole channel was obtained. Later, [59] [60] examined the capacity of single-keyhole channels in the presence of spatial correlation. The space-time coding research for this channel included the analysis of orthogonal space-time codes in [61], [52], and the analysis and design of general space-time codes in [62] [63] investigated the symbol error rate of spatially correlated single-keyhole channels with orthogonal space-time block coding and linear precoding.

Later, researchers found that the single-keyhole channel is not often encountered in practice. Actually it was shown in [64] that the single-keyhole effect is difficult to observe. To include these scenarios and expand the keyhole channel model, a multi-keyhole channel model, which consists of a number of statistically independent keyholes, was introduced in [65] [66]. Fig. 4.1 shows a multi-keyhole channel. In this channel, each end has its own set of multi-path components and is separated from the other end by a screen with a number of keyholes of size smaller than half a wavelength. Some efforts have been taken to investigate the multi-keyhole channel recently. [65] showed that the asymptotic outage capacity of the multi-keyhole channel can be described by summing the capacities of individual keyholes. In [67], the approximated PDF of the eigenvalues of the channel correlation matrix was provided. In [68], a closed form of asymptotic diversitymultiplexing tradeoff was derived. More recently, [69] studied the outage capacity [70, 71] investigated the ergodic mutual information for this channel, and [72– 74] studied beamforming schemes the multi-keyhole MIMO systems with channel state information (CSI) at the transmitter. The analysis of space-time codes (STC) in multi-keyhole channels, however, is quite limited and only available for OST-BCs [40]. In [40], analytical expressions of the SER of the OSTBC were derived and using OSTBC as a pivot, it was shown that the achievable diversity order is  $n_T n_S n_R/max(n_T, n_S, n_R)$ , where  $n_T$ ,  $n_S$  and  $n_R$  mean the number of transmission antennas, the number of effective scatters and the number of receiving antennas, respectively. Although the results in [40] are of great importance, it is only the result for orthogonal code, and many STC schemes that have excellent performances are often not orthogonal ones [75–78]. This motivates us to investigate general space-time codes under the multi-keyhole conditions. In this chapter, we focus on communication systems that have multiple transmission antennas and one receiving antenna, i.e. multiple-input-single-output (MISO) systems, and provide a performance analysis of general space-time codes for multi-keyhole channels. We consider both the cases when the transmission antennas are spatially independent and are spatially correlated. The major results of this Chapter are as follows:

- We prove that for any pair of code words in a space-time code, the code words distance in the MISO multi-keyhole channel (with *M* transmission antennas and *L* keyholes) and that in the MIMO single-keyhole channel (with *M* transmission antennas and *L* receiving antennas) are identically distributed. Therefore the two types of channels share the same form of PEP, and one can employ the design criteria in MIMO single-keyhole to design the codes for MISO multiple-keyhole. We further show that the PDF of the code words distance asymptotically converges to that of the Rayleigh channel when *M* approaches infinity.
- 2. In the high SNR regime, when  $M \le L$ , the transmission correlations always degrade the PEP performance; when M > L (the number of transmission antennas greater than the number of keyholes), depending on how the correlation matrix beamforms the code words difference matrix, the correlations can either degrade or improve the PEP performance. Particularly we prove



**Figure 4.1:** The MISO multi-keyhole channel model: each end has its own set of multipath components and is separated from the other end by a screen with a number of keyholes of small size (smaller than half a wavelength).

that if there is an integer K,  $1 \le K \le M - L - 1$ , such that

$$\bar{\lambda}^{L} \left( \sum_{i=K+1}^{M} \frac{\lambda_{i}^{-1}}{M} \right)^{L} < \frac{\Gamma(M-L)\Gamma(M-K)}{\Gamma(M)\Gamma(M-K-L)},$$
(4.1)

we can always find certain correlation matrices that can improve the PEP performance. We also provide one form of such matrices.

#### 4.2 Independent and Identical Transmission Antennas

In this section, we investigate the PEP performance of general space-time codes when the transmission antennas are spatially independent. Consider a frequency non-selective quasi-static fading channel with M transmitting antennas and one receiving antenna that is shown in Fig. 4.1. In this MISO multi-keyhole channel, the signal model is given by

$$\mathbf{R} = \sqrt{\frac{\bar{\gamma}}{M \times L}} \mathbf{H} \mathbf{S} + \mathbf{W}, \tag{4.2}$$

where the  $1 \times T$  matrix **R** represents the received signal, *S* is the  $M \times T$  transmitted code words difference matrix,  $\bar{\gamma}$  is the average SNR, and **W** is the zero-mean additive circularly symmetric complex Gaussian noise matrix with size  $1 \times T$ , whose elements have unit variance per dimension. We use  $h_{m,n}$ 's (m = 1, ..., M, n = 1, ..., L)

to represent the normalized channel gains from the *M* transmitting antennas to the *L* keyholes, use  $g_n$ 's (n = 1, ..., L) to represent the normalized channels from the *L* keyholes to the single receiving antenna, and let  $\mathbf{h}_n \triangleq (h_{1,n}, h_{2,n}, \cdots, h_{M,n})^T$ . We further assume that

- The entries of **h**<sub>n</sub>'s are independent complex Gaussian distributed with zero mean and unit variance.
- $g_n$ 's are also Gaussian with zero mean and unit variance.
- The keyholes are statistical independent, i.e. the random vectors **h**<sub>n</sub>'s are independent and the random variables g<sub>n</sub> are independent.

Consequently, the channel matrix, which is actually a vector in the MISO channel, is given by

$$\mathbf{H} = \sum_{n=1}^{L} \mathbf{h}_n g_n. \tag{4.3}$$

To decode the received code word  $\mathbf{R}$  at the receiver side, the maximum likelihood (ML) decoder is employed. We assume that the CSI is perfectly known at the receiver and unknown at the transmitter.

PEP, the probability of transmitting code word  $\mathbf{c} = (\mathbf{c}_1, ..., \mathbf{c}_T)^T$  over T time slots and deciding in favor of another code word  $\mathbf{e} = (\mathbf{e}_1, ..., \mathbf{e}_T)^T$  at the decoder, generally serves as a design criterion for space-time codes. When signals transmit over a fading channel with channel matrix  $\mathbf{H}$ , the code words distance between  $\mathbf{c}$ and  $\mathbf{e}$  is defined by the random variable  $\|\Delta \mathbf{H}\|_F$ , where  $\Delta \triangleq \mathbf{c} - \mathbf{e}$  is the code words difference, and  $\|\cdot\|_F$  is the Frobenius norm. The PEP of a Gaussian noise channel can be evaluated by averaging the density of  $\|\Delta \mathbf{H}\|_F$  over the Q function as

$$P(\mathbf{c} \to \mathbf{e} | \mathbf{H}) = Q\left(\sqrt{\frac{\bar{\gamma}}{M \times L}} \|\Delta \mathbf{H}\|_F^2\right).$$
(4.4)

Using an alternative representation of the Q function, we have

$$P(\mathbf{c} \to \mathbf{e} | \mathbf{H}) = \frac{1}{\pi} \int_{\theta=0}^{\infty} \exp\left(-\frac{\bar{\gamma}}{M \times L} \frac{\|\Delta \mathbf{H}\|_{F}^{2}}{2\sin^{2}\theta}\right) d\theta.$$
(4.5)

For the Gaussian noise channel, to find the PEP we need to investigate the distribution of the code distance  $\|\Delta \mathbf{H}\|_F$ .

#### 4.2.1 Distribution of the Code Words Distance

For the MISO multi-keyhole channel, the squared code words distance is given by

$$\begin{split} \|\Delta \mathbf{H}\|_{F}^{2} &= \|\Delta \sum_{n=1}^{L} \mathbf{h}_{n} g_{n}\|_{F}^{2} \\ &= trace \left(\Delta \sum_{n_{1}=1}^{L} \mathbf{h}_{n_{1}} g_{n_{1}} \sum_{n_{2}=1}^{L} g_{n_{2}}^{H} \mathbf{h}_{n_{2}}^{H} \Delta^{H} \right) \\ &= trace \left(\Delta^{H} \Delta \sum_{n_{1}=1}^{L} \mathbf{h}_{n_{1}} g_{n_{1}} \sum_{n_{2}=1}^{L} g_{n_{2}}^{H} \mathbf{h}_{n_{2}}^{H} \right). \end{split}$$
(4.6)

Since  $\Delta^H \Delta$  is a Hermitian matrix, it has an Eigendecomposition as  $\Delta^H \Delta = \mathbf{U}^H \mathbf{V} \mathbf{U}$ , and the squared code distance can be written as

$$\|\Delta \mathbf{H}\|_{F}^{2} = trace\left(\mathbf{U}^{H}\mathbf{V}\mathbf{U}\sum_{n_{1}=1}^{L}\mathbf{h}_{n_{1}}g_{n_{1}}\sum_{n_{2}=1}^{L}g_{n_{2}}^{H}\mathbf{h}_{n_{2}}^{H}\right)$$
$$= trace\left(\mathbf{V}\sum_{n_{1}=1}^{L}\mathbf{U}\mathbf{h}_{n_{1}}g_{n_{1}}\sum_{n_{2}=1}^{L}g_{n_{2}}^{H}\mathbf{h}_{n_{2}}^{H}\mathbf{U}^{H}\right).$$
(4.7)

Note that

$$\sum_{n_1=1}^{L} \mathbf{U} \mathbf{h}_{n_1} g_{n_1} = \sum_{n_1=1}^{L} \alpha_{n_1} g_{n_1}, \qquad (4.8)$$

where  $\alpha_{n_1} = \mathbf{U}\mathbf{h}_{n_1}$  is the unitary transformed vector of  $\mathbf{h}_{n_1}$  by the transformation **U**. Given that  $\mathbf{h}_{n_1}$  is an i.i.d complex Gaussian random vector with zero mean and unit variance,  $\alpha_{n_1}$  is also a complex Gaussian random vector with zero mean, unit

variance i.i.d elements, and we have

$$\begin{split} |\Delta \mathbf{H}||_{F}^{2} &= trace\left(\mathbf{V}\sum_{n_{1}=1}^{L}\alpha_{n_{1}}g_{n_{1}}\sum_{n_{2}=1}^{L}g_{n_{2}}^{H}\alpha_{n_{2}}^{H}\right)\\ &= trace\left(\sum_{n_{1}=1}^{L}g_{n}^{H}\alpha_{n_{1}}^{H}\mathbf{V}\sum_{n_{2}=1}^{L}\alpha_{n_{2}}g_{n_{2}}\right)\\ &= \|\mathbf{V}^{1/2}\sum_{n=1}^{L}\alpha_{n}g_{n}\|_{F}^{2}\\ &= \sum_{i=1}^{R(\Delta)}\lambda_{i}\|\sum_{n=1}^{L}\alpha_{n,i}g_{n}\|^{2}, \end{split}$$
(4.9)

where  $\lambda_1, ..., \lambda_{R(\Delta)}$  are the non-zero eigenvalues of  $\Delta^H \Delta$ ,  $R(\Delta)$  is the rank of  $\Delta^H \Delta$ , and  $\alpha_{n,i}$  is the *i*-th element of  $\alpha_n$ . To investigate the distribution of  $\sum_{i=1}^{R(\Delta)} \lambda_i || \sum_{n=1}^{L} \alpha_{n,i} g_n ||^2$ , we derive the following Lemma:

**Lemma 3.** Let  $X = \sum_{i=1}^{R(\Delta)} \lambda_i \|\sum_{n=1}^{L} \alpha_{n,i} g_n\|^2$  and  $Y = \sum_{i=1}^{R(\Delta)} \lambda_i \sum_{n=1}^{L} \|g_n\|^2 \|\beta_i\|^2$ , where  $\lambda_i$ 's are some constants, and  $\alpha_{n,i}$ 's,  $\beta_i$ 's, and  $g_n$ 's are all i.i.d complex Gaussian *r.v.s* with zero mean and unit variance, then the random variables X and Y are identically distributed as well.

Proof. For presentation simplicity, we define

$$X_i \triangleq \sum_{n=1}^L g_n \alpha_{n,i}, \quad Y_i \triangleq \sqrt{\sum_{n=1}^L \|g_n\|^2} \beta_i, \quad (4.10)$$

hence

$$X = \sum_{i=1}^{R(\Delta)} \lambda_i \|X_i\|^2, \ Y = \sum_{i=1}^{R(\Delta)} \lambda_i \|Y_i\|^2.$$
(4.11)

It is clear that the conditional random variable  $X_i|g_1,...,g_L$  is complex Gaussian, with mean

$$\mathbb{E}(X_i|g_1,...,g_L) = \sum_{n=1}^L g_n \mathbb{E}(\alpha_{n,i}) = 0, \qquad (4.12)$$

and variance

$$\mathbb{E}(\|X_i\|^2 | g_1, ..., g_L) - \| (\mathbb{E}(X_i | g_1, ..., g_L)) \|^2$$
  
=  $\sum_{n=1}^L \|g_n\|^2 \mathbb{E}(\|\alpha_{n,i}\|^2) - \sum_{n_1 \neq n_2} g_{n_1} g_{n_2}^H \mathbb{E}(\alpha_{n_1,i} \alpha_{n_2,i}^H)$   
=  $\sum_{n=1}^L \|g_n\|^2.$  (4.13)

Further, it is easy to see that the conditional random variable  $Y_i|g_1,...,g_L$  is complex Gaussian, with zero mean as well. Therefore  $X_i|g_1,...,g_L$  and  $Y_i|g_1,...,g_L$  identically distributed. This implies that the conditional random variables

$$X|g_1,...,g_L = \sum_{i=1}^{R(\Delta)} \lambda_i ||X_i|g_1,...,g_L||^2,$$
(4.14)

and

$$Y|g_1,...,g_L = \sum_{i=1}^{R(\Delta)} \lambda_i ||Y_i|g_1,...,g_L||^2,$$
(4.15)

are also identically distributed. Consequently, the marginal distribution of X is same as that of Y.

Lemma 3 states that the squared code distance of the MISO multi-keyhole channel has the same distribution as that of the random variable Y. It provides a useful theory for studying the PEP of the MISO multi-keyhole channel, as Y is in a simpler form than X. More fortunately, since the random variable Y is also the squared code distance in the MIMO single-keyhole fading [62], we directly can have the following result:

**Theorem 2.** For any pair of code words in a space-time code, the code words distance  $\|\Delta \mathbf{H}\|_F$  as a random variable, is identically distributed in the MISO multi-keyhole channel (with M transmitting antennas and L keyholes) and the MIMO single-keyhole channel(with M transmitting antennas and L receiving antennas).

Therefore, for any space-time code, the MISO multi-keyhole and MIMO single-

keyhole channels have the same PEP. With Theorem 2 and the PEP result from [62], in the high SNR regime, for distinct  $\lambda_i$ 's, the PEP of general STC for the MISO multi-keyhole channel is given by

$$P_{\mathbf{I}}(\mathbf{c} \to \mathbf{e}) \doteq \begin{cases} C_1 \left( \prod_{i=1}^{R(\Delta)} \lambda_i \right)^{-1} \bar{\gamma}^{-R(\Delta)}, & \text{if } L > R(\Delta); \\ C_1 \left( \prod_{i=1}^{R(\Delta)} \lambda_i \right)^{-1} (\ln \bar{\gamma}) \bar{\gamma}^{-R(\Delta)}, & \text{if } L = R(\Delta); \\ C_3 \sum_{i=1}^{R(\Delta)} \frac{\ln \lambda_i}{\lambda_i^L} \prod_{j \neq i} \frac{\lambda_i}{\lambda_i - \lambda_j} \bar{\gamma}^{-L}, & \text{if } L < R(\Delta). \end{cases}$$
(4.16)

For identical  $\lambda_i$ 's, i.e.  $\lambda_i = \lambda$ , the PEP is given by

$$P_{\mathbf{I}}(\mathbf{c} \to \mathbf{e}) \doteq \begin{cases} C_1 \lambda^{-R(\Delta)} \bar{\gamma}^{-R(\Delta)}, & \text{if } L > R(\Delta); \\ C_1 \lambda^{-R(\Delta)} (\ln \bar{\gamma}) \bar{\gamma}^{-R(\Delta)}, & \text{if } L = R(\Delta); \\ C_2 \lambda^{-R(\Delta)} \bar{\gamma}^{-L}. \end{cases}$$

where

$$C_1 = \frac{\Gamma(\frac{1}{2} + R(\Delta))}{2\sqrt{\pi}\Gamma(1 + R(\Delta))} \times \frac{L^{R(\Delta)}\Gamma(L - R(\Delta))}{\Gamma(L)},$$
(4.17)

$$C_2 = \frac{\Gamma(\frac{1}{2} + L)}{2\sqrt{\pi}\Gamma(1 + L)} \times \frac{L^L \Gamma(R(\Delta) - L)}{\Gamma(R(\Delta))},$$
(4.18)

and

$$C_{3} = \frac{\Gamma(\frac{1}{2} + L)}{2\sqrt{\pi}\Gamma(1 + L)} \times (-1)^{L-1} \times \frac{1}{\Gamma(L)}.$$
(4.19)

Since the MISO multi-keyhole channel and MIMO single-keyhole channel share the same form of error probabilities except a normalization factor, we can follow the design criterion of the MIMO single-keyhole channel that has been studied in [62] to design the codes for the MISO multi-keyhole channel. For the case that  $L \ge M$ , the determinant criterion also applies to the MISO multi-keyhole channel, and STCs that have good performances for the Rayleigh fading channel will also have good performances for the MISO multi-keyhole channel. For the case that L < M, the code design criterion should be based on minimizing the expression

$$(-1)^{L-1} \sum_{i=1}^{R(\Delta)} \frac{\ln \lambda_i}{\lambda_i^L} \prod_{i \neq j} \frac{\lambda_i}{\lambda_i - \lambda_j}.$$
(4.20)

We consider a simulation example for Theorem 2. Fig. 4.2 shows the simulation results of the PDFs of the code word distance  $\|\Delta \mathbf{H}\|_F$  for the MISO multikeyhole channel and the MIMO single-keyhole channel. The simulation uses an code words pair for which

$$\Delta^{H} \Delta^{=} \begin{pmatrix} 2.16 & 0.23 - 0.23i & 0.84 \\ 0.23 + 0.23i & 1.68 & -0.23 - 0.23i \\ 0.84 & -0.23 + 0.23i & 2.16 \end{pmatrix},$$
(4.21)

which has eigenvalues of  $\lambda_1 = 1$ ,  $\lambda_2 = 2$  and  $\lambda_3 = 3$ . We assume that there are L = 2 keyholes in the channel. The PDFs are compared in Fig. 4.2, and we can observe that the distributions are identical. The simulations results on PEP are shown in Fig. 4.3 for this code words pair. We can see that the analytical result in equations (4.16) matches the simulated result very well for high SNR.

*Remark:* It is worth mentioning here that Theorem 2 holds for any distribution of  $g_n$ 's as long as the entries of  $\mathbf{h}_n$ 's are i.i.d complex Gaussian with zero mean. In some propagation cases, however, the distribution of the code words distance may not be identical in the MISO multi-keyhole channel and the MIMO single-keyhole channel in general. For example, if the entries of  $\mathbf{h}_n$ 's are Nakagami-m, Lemma 3 will not hold. In this chapter, we assume Rayleigh fading for the sub-channels.

#### 4.2.2 Convergence to the Rayleigh Channel

It has been verified from different aspects that the multi-keyhole channel which generalizes the Rayleigh channel and the single-keyhole channel becomes Rayleigh when the number of keyholes grows to infinity [69, 71]. Particularly, in [71] it is shown that for sufficiently large number of keyholes, the capacity of multi-keyhole MIMO channels approaches that of MIMO Rayleigh fading channels. In this section, instead, we exam the convergence of the multi-keyhole channel from the space-time code point of view, i.e. the distribution of code words distance and



**Figure 4.2:** Simulated PDFs of the code word distances for the MISO multikeyhole channel and the MIMO single-keyhole channel. We can see that the two PDFs are identically distributed. In the simulation,  $5 \times 10^6$ samples are used. Here M = 3 and L = 2 and the eigenvalues of  $\Delta^H \Delta$  in (4.21) are  $\lambda_1 = 1$ ,  $\lambda_2 = 2$  and  $\lambda_3 = 3$ .

the PEP performance.

As  $L \rightarrow \infty$ , the PEP expression in MISO multi-keyhole channel will take the first case of (4.16), where

$$C_{1} = \frac{\Gamma(\frac{1}{2} + R(\Delta))}{2\sqrt{\pi}} \times \frac{L^{R(\Delta)}}{(L-1) \times \dots \times (L-R(\Delta))}$$
$$= \frac{\Gamma(\frac{1}{2} + R(\Delta))}{2\sqrt{\pi}} \prod_{i=1}^{R(\Delta)} \frac{L}{L-i} \rightarrow \frac{\Gamma(\frac{1}{2} + R(\Delta))}{2\sqrt{\pi}},$$
(4.22)



**Figure 4.3:** Asymptotic and simulated PEPs in the MISO multi-keyhole channel for the code words pair for which  $\Delta^H \Delta$  is defined in (4.21). Here M = 3, L = 2, and the eigenvalues are  $\lambda_1 = 1$ ,  $\lambda_2 = 2$  and  $\lambda_3 = 3$ .

and

$$P(\mathbf{c} \to \mathbf{e}) \doteq \frac{\Gamma(\frac{1}{2} + R(\Delta))}{2\sqrt{\pi}} \left(\prod_{i=1}^{R(\Delta)} \lambda_i\right)^{-1} \bar{\gamma}^{-M}.$$
(4.23)

We note that (4.23) is also the asymptotic PEP for the MISO Rayleigh channel, so the MISO multi-keyhole channel converges to the MIMO Rayleigh channel in the sense of PEP. This convergence can also be seen by investigating the distribution of the code words distance: when *L* grows to infinity, the normalized squared code words distance is given by

$$\lim_{L \to \infty} \frac{\|\Delta \mathbf{H}\|_F^2}{L} = \lim_{L \to \infty} \frac{\sum_{n=1}^L \|g_n\|^2}{L} \sum_{i=1}^{R(\Delta)} \lambda_i \|\beta_i\|^2 \xrightarrow{d} \sum_{i=1}^{R(\Delta)} \lambda_i \|\beta_i\|^2, \quad (4.24)$$



**Figure 4.4:** The PEP of the MISO multi-keyhole channel converges to the PEP of the MISO Rayleigh channel when the number of keyholes grows from 1 to 28.

which has the same distribution as that of the squared code distance of the MISO Rayleigh channel [33]. The convergence in the sense of PEP is illustrated in Fig. 4.4, where M = 3 and the number of keyholes increases from L = 1 to L = 28.

### 4.3 Spatial Correlated Transmission Antennas

In the last section, we investigated the PEP performance of general space-time codes when the transmission antennas are spatially independent in the MISO multikeyhole channel. In reality, however, individual antennas could be correlated due to insufficient antenna spacing and lack of scattering [79–82]. For Rayleigh fading channels, it has been shown that in the asymptotically high SNR regime, the transmission correlations always degrade the PEP performance, while in the asymptotically low SNR regime, the transmission correlations may either improve or degrade the PEP performance[82].

For the multi-keyhole channel, the effect of transmission correlations in the multi-keyhole channel has been investigated in [40, 69, 71, 72]. In particular, the effect of correlations on the space-time codes has been investigated in [40]. Using majorization relations of the correlation matrices, [40] showed that for orthogonal space-time codes, the correlations will always degrade the PEP performance. The results in [40], however, is only valid for orthogonal codes. In this section, we study the PEP performance of general space-time codes when the transmission antennas are not independent. We will show that, very different from orthogonal codes, when the number of transmission antennas is greater than the number of keyholes, the PEP performance of general space-time codes *can be improved by the transmission correlations in multi-keyhole conditions, even in the high SNR regime*. This depends on how the correlation matrix beamforms  $\Delta$ .

Consider the following multi-keyhole channel model,

$$\mathbf{H} = \mathbf{A}^{\frac{1}{2}} \sum_{n=1}^{L} \mathbf{h}_n g_n.$$
(4.25)

Clearly  $A_{i,j}$  is the correlation between the overall propagation path from  $TX_i$  to RX and that from  $TX_j$  to RX, and A severs as the correlation matrix for transmission antennas because

$$\mathbb{E}(\mathbf{H}\mathbf{H}^{H}) = \mathbb{E}\left(\mathbf{A}^{\frac{1}{2}}\sum_{n=1}^{L}\mathbf{h}_{n}g_{n}\sum_{n=1}^{L}g_{n}^{H}\mathbf{h}_{n}^{H}\mathbf{A}^{\frac{H}{2}}\right)$$
$$= \mathbf{A}^{\frac{1}{2}}\left(\mathbb{E}\left(\|g_{n}\|^{2}\sum_{n=1}^{L}\mathbf{h}_{n}\mathbf{h}_{n}^{H}\right) + \mathbb{E}\left(\sum_{i\neq j}g_{i}g_{j}^{H}\mathbf{h}_{i}\mathbf{h}_{j}^{H}\right)\right)\mathbf{A}^{\frac{H}{2}}$$
$$= \mathbf{A}.$$
(4.26)

Then the squared code distance becomes

$$\|\Delta \mathbf{A}^{\frac{1}{2}} \sum_{n=1}^{L} \mathbf{h}_{n} g_{n} \|_{F}^{2} = \sum_{i=1}^{R(\Delta \mathbf{A})} \rho_{i} \| \sum_{n=1}^{L} g_{n} \alpha_{n,i} \|^{2}, \qquad (4.27)$$

where  $\rho_i$ 's are the eigenvalues of the matrix  $\mathbf{A}^{\frac{H}{2}} \Delta^H \Delta \mathbf{A}^{\frac{1}{2}}$ . Using the result from

Lemma 3, we have

$$\|\Delta \mathbf{A}^{\frac{1}{2}} \sum_{n=1}^{L} \mathbf{h}_{n} g_{n} \|_{F}^{2} \sim \sum_{i=1}^{R(\Delta \mathbf{A})} \rho_{i} \sum_{n=1}^{L} \|g_{n}\|^{2} \|\beta_{i}\|^{2}.$$
(4.28)

Consequently the asymptotic PEP when the transmission antennas are correlated can be obtained by replacing  $\lambda_i$ 's by  $\rho_i$ 's in Equations (4.16) and (4.17).

Although the asymptotic PEP for correlated transmission antennas has been obtained, our main question in this section, how a correlation matrix affects the PEP performance, is still not clearly answered. To investigate the effect of transmission correlations on the PEP performance, we first present the following facts and inferences about the correlation matrix **A** and the code difference matrix  $\Delta$ :

1.  $trace(\mathbf{A}) = M$ , or equivalently,

$$\sum_{i=1}^{M} v_i = M, \tag{4.29}$$

where  $v_i$ 's are the eigenvalues of **A**. This is because the total transmission power is fixed.

2.

$$\left(\prod_{i=1}^{M} v_i\right) \left(\prod_{i=1}^{M} \lambda_i\right) = \prod_{i=1}^{M} \rho_i.$$
(4.30)

This is from the fact that  $det(\mathbf{A}^{\frac{H}{2}}\Delta^{H}\Delta\mathbf{A}^{\frac{1}{2}}) = det(\mathbf{A}) \times det(\Delta^{H}\Delta)$ 

In this chapter, we assume that the code construction achieves full rank, i.e.  $R(\Delta) = M$ . We now start to analysis the effect of correlations on the PEP performance. We consider the cases that  $M \le L$  and M > L separately.

#### **4.3.1** Case 1: $M \le L$

We first consider the case that the number of transmission antennas is the same as or less than the number of keyholes:  $M \le L$ .

**Theorem 3.** In the MISO multi-keyhole channel, when  $L \ge M$ , the spatial correlations between transmission antennas always degraded the PEP performance in the high SNR regime.

*Proof.* If **A** is rank deficient, because we assume that  $\Delta^H \Delta$  is full rank, we have

$$R(\mathbf{A}^{\frac{H}{2}}\Delta^{H}\Delta\mathbf{A}^{\frac{1}{2}}) < R(\Delta^{H}\Delta), \tag{4.31}$$

from the PEP given in Equation (4.16), we can see that this will result in a reduction of the diversity order, hence the PEP performance is degraded. If **A** is of full rank, it means  $\prod_{i=1}^{M} v_i \neq 0$ . By the AM-GM inequality,

$$\prod_{i=1}^{M} \mathbf{v}_i \le \left(\frac{\sum_{i=1}^{M} \mathbf{v}_i}{M}\right)^M = 1,$$
(4.32)

therefore

$$\prod_{i=1}^{M} \rho_i \le \prod_{i=1}^{M} \lambda_i. \tag{4.33}$$

Note that the equality only holds when A is an identity matrix. Therefore the PEP is always degraded by transmission correlations for the case that  $M \le L$ .

It is worth to mention here that when the number of keyholes is greater than or equal to the number of transmission antennas, the correlation effect on the MISO multi-keyhole channel is similar to that on the MISO Rayleigh channel, since for MISO Rayleigh channel, the correlations always degrade the PEP performance in the asymptotic high SNR regime as well [82]. One intuitive explanation is that when the number of the keyholes is much larger than the number of transmission antennas, the MISO multi-keyhole channel behaves more like a Rayleigh channel. However, when M > L, the channel behaves more sophisticated and we will show in the next section that the correlations sometimes can improve the PEP performance.

#### **4.3.2** Case 2: M > L

To show how the transmission correlation matrix affects the PEP performance when the number of transmission antennas is larger than the number of keyholes, we first give the following Lemma:

**Lemma 4.** Let  $\lambda_i$  be real for  $i \in \{1, 2, ..., M\}$  and  $\overline{\lambda} = \frac{\sum_{i=1}^{M} \lambda_i}{M}$ . Let  $X_i$ ,  $i \in \{1, 2, ..., M\}$  be a set of i.i.d random variables, Y be another random variable which is independent with  $X_i$ 's, then we have

$$\mathbb{E}\left(f\left(Y\sum_{i=1}^{M}\bar{\lambda}X_{i}\right)\right) \leq \mathbb{E}\left(f\left(Y\sum_{i=1}^{M}\lambda_{i}X_{i}\right)\right),\tag{4.34}$$

where  $f(\cdot)$  is a convex function. The equality sign holds when  $\lambda_i = \overline{\lambda}$  for all  $i \in \{1, 2, ..., M\}$ .

*Proof.* To prove (4.34), we first prove that (4.34) holds for any fixed value of *Y*, i.e.

$$\mathbb{E}\left(f\left(y\sum_{i=1}^{M}\bar{\lambda}X_{i}\right)\right) \leq \mathbb{E}\left(f\left(y\sum_{i=1}^{M}\lambda_{i}X_{i}\right)\right),\tag{4.35}$$

where y is any possible value that the random variable Y can take. It is easy to see that (4.35) implies (4.34).

For presentation simplicity, let

$$X = y \sum_{i=1}^{M} \bar{\lambda} X_i, \qquad (4.36)$$

$$W = y \sum_{i=1}^{M} \lambda_i X_i, \qquad (4.37)$$

and

$$Z = X - W. \tag{4.38}$$

Based on the form of X, it is easy to see that the conditional random variables  $X_i|X$ ,

 $i \in \{1, ..., M\}$ , are identically distributed, which implies

$$\mathbb{E}(X_1|X) = \mathbb{E}(X_2|X) = \dots = \mathbb{E}(X_M|X). \tag{4.39}$$

Therefore

$$\mathbb{E}(Z|X) = \sum_{i=1}^{M} \bar{\lambda} \mathbb{E}(X_i|X) - \sum_{i=1}^{M} \lambda_i \mathbb{E}(X_i|X)$$
$$= \mathbb{E}(X_1|X) \left(\sum_{i=1}^{M} \bar{\lambda} - \sum_{i=1}^{M} \lambda_i\right) = 0.$$
(4.40)

Since  $f(\cdot)$  is convex, by Jensen's inequality we have

$$\mathbb{E}(f(X-Z)|X) \ge f(\mathbb{E}((X-Z)|X))$$
$$= f(X-0) = f(X)$$
(4.41)

Therefore

$$\mathbb{E}(f(W)) = \mathbb{E}(\mathbb{E}(f(X-Z)|X)) \ge \mathbb{E}(f(X)), \tag{4.42}$$

and consequently (4.34) holds.

Now we present the main result for the effect of correlations on the PEP performance when M > L:

**Theorem 4.** In MISO multi-keyhole channel, for any pair of code words, if we can find some integer K between 1 and M - L - 1, i.e.  $1 \le K \le M - L - 1$  such that

$$\bar{\lambda}^{L} \left( \sum_{i=K+1}^{M} \frac{\lambda_{i}^{-1}}{M} \right)^{L} < \frac{\Gamma(M-L)\Gamma(M-K)}{\Gamma(M)\Gamma(M-K-L)},$$
(4.43)

then there always exist correlation matrices that can improve the PEP performance in the asymptotic high SNR regimes. Here  $0 \le \lambda_1 \le \lambda_2 \le \cdots \le \lambda_M$  are the eigenvalues of  $\Delta^H \Delta$  in ascending order, and  $\bar{\lambda}$  is their average.

*Proof.* Referring back to Equation (4.5) and the squared code words distance for independent transmission antennas  $\|\Delta \mathbf{H}\|_F^2 \sim \sum_{i=1}^M \lambda_i \sum_{n=1}^L \|g_n\|^2 \|\beta_i\|^2$ , using the

result from Lemma 4, we have the PEP for independent transmission antennas been bounded as follows in the high SNR regimes:

$$P_{\mathbf{I}}(\mathbf{c} \to \mathbf{e}) \ge \frac{\Gamma(\frac{1}{2} + L)}{2\sqrt{\pi}\Gamma(1 + L)} \times \frac{L^{L}\Gamma(M - L)}{\Gamma(M)} \bar{\lambda}^{-L} \bar{\gamma}^{-L}.$$
(4.44)

Suppose the Eigendecomposition of  $\Delta^{H}\Delta$  is **UVU**<sup>*H*</sup>, we consider the following class of correlation matrices for which  $\mathbf{A}^{\frac{1}{2}}$  has singular value decomposition as

$$\mathbf{A}^{\frac{1}{2}} = \mathbf{U}\mathbf{S}^{\frac{1}{2}}\mathbf{D}^{H},\tag{4.45}$$

where **D** is a unitary matrix and the diagonal matrix **S** with  $\mathbf{S}_{i,i} = \mathbf{v}_i$  satisfies the power constraint:  $\sum_{i=1}^{M} \mathbf{v}_i = M$ . It follows that

$$\|\Delta \mathbf{A}^{\frac{1}{2}} \sum_{n=1}^{L} \mathbf{h}_{n} g_{n} \|_{F}^{2} = \|\Delta \mathbf{U} \mathbf{S}^{\frac{1}{2}} \mathbf{D}^{H} \sum_{n=1}^{L} \mathbf{h}_{n} g_{n} \|_{F}^{2}$$
$$= \|\mathbf{V}^{\frac{1}{2}} \mathbf{S}^{\frac{1}{2}} \mathbf{D}^{H} \sum_{n=1}^{L} \mathbf{h}_{n} g_{n} \|_{F}^{2}.$$
(4.46)

From the derivation for the independent case in Equation (4.8), it is easy to see that  $\mathbf{D}^{H} \sum_{n=1}^{L} \mathbf{h}_{n} g_{n}$  and  $\sum_{n=1}^{L} \mathbf{h}_{n} g_{n}$  are identically distributed. Therefore

$$\|\Delta \mathbf{A}^{\frac{1}{2}} \sum_{n=1}^{L} \mathbf{h}_{n} g_{n} \|_{F}^{2} \sim \sum_{i=1}^{R(\mathbf{A})} \rho_{i} \sum_{n=1}^{L} \|g_{n}\|^{2} \|\beta_{i}\|^{2}, \qquad (4.47)$$

where

$$\rho_i = \mathbf{v}_i \lambda_i \tag{4.48}$$

for all  $\rho_i$ 's. Now we can have a correlation matrix **A** such that

$$v_1 = v_2 = \dots = v_K = 0,$$
 (4.49)

and

$$v_{K+1}, \cdots, v_M > 0.$$
 (4.50)

If we pose another constraint on  $v_i$  such that the non-zero eigenvalues of  $\mathbf{A}^{\frac{H}{2}} \Delta^H \Delta \mathbf{A}^{\frac{1}{2}}$  are identical:

$$\rho_{K+1} = \rho_{K+2} = \dots = \rho_M,$$
 (4.51)

we have

$$\rho_{K+1} = \rho_{K+2} = \dots = \rho_M = M \left(\sum_{i=K+1}^M \frac{1}{\lambda_i}\right)^{-1}.$$
(4.52)

At the high SNR regime, the PEP respective to  $\mathbf{A}^{\frac{H}{2}} \Delta^{H} \Delta \mathbf{A}^{\frac{1}{2}}$  becomes

$$P_{\mathbf{A}}(\mathbf{c}\to\mathbf{e}) = \frac{\Gamma(\frac{1}{2}+L)}{2\sqrt{\pi}\Gamma(1+L)} \frac{L^{L}\Gamma(M-K-L)}{\Gamma(M-K)} M^{-L} \left(\sum_{i=K+1}^{M} \frac{1}{\lambda_{i}}\right)^{L} \bar{\gamma}^{-L}.$$
 (4.53)

Therefore  $P_{\mathbf{A}}(\mathbf{c} \to \mathbf{e}) < P_{\mathbf{I}}(\mathbf{c} \to \mathbf{e})$  if (4.43) holds, i.e. the correlation matrix  $\mathbf{A}$  defined in (4.45) improves the PEP performance.

#### 4.3.3 Examples and Simulations

In this section, we provide an example and perform Monte Carlo simulations for Theorem 3 and Theorem 4.

*Example* We consider a certain pair of code words for which

$$\Delta^{H}\Delta = \begin{pmatrix} 2 & -0.95 + 0.029i & -0.95 - 0.029i \\ -0.95 - 0.029i & 2 & -0.95 + 0.029i \\ -0.95 + 0.029i & -0.95 - 0.029i & 2 \end{pmatrix}, \quad (4.54)$$

the eigenvalues are  $\lambda_1 = 0.1$ ,  $\lambda_2 = 2.9$  and  $\lambda_3 = 3$ . Suppose L = 1, and we select K = 1, it appears that

$$\bar{\lambda}^{L} \left( \sum_{i=K+1}^{M} \frac{\lambda_{i}^{-1}}{M} \right)^{L} = 2 \left( \frac{1/2.9 + 1/3}{3} \right) = 0.45, \tag{4.55}$$

and

$$\frac{\Gamma(M-L)\Gamma(M-K)}{\Gamma(M)\Gamma(M-K-L)} = \frac{\Gamma(3-1)\Gamma(3-1)}{\Gamma(3)\Gamma(3-1-1)} = 0.5.$$
 (4.56)

By Theorem 4, there exist some correlation matrices that can improve the PEP performance. One of such matrices can be given by

$$\mathbf{A} = \mathbf{v}_1 \mathbf{u}_1 \mathbf{u}_1^H + \mathbf{v}_2 \mathbf{u}_2 \mathbf{u}_2^H + \mathbf{v}_3 \mathbf{u}_3 \mathbf{u}_3^H, \qquad (4.57)$$

where

$$v_1 = 0,$$
 (4.58)

$$v_2 = \frac{M\left(\sum_{i=K+1}^{M} \frac{1}{\lambda_i}\right)^{-1}}{\lambda_2} = 1.525,$$
(4.59)

$$v_3 = \frac{M\left(\sum_{i=K+1}^{M} \frac{1}{\lambda_i}\right)^{-1}}{\lambda_3} = 1.475,$$
(4.60)

and  $\mathbf{u}_1$ ,  $\mathbf{u}_2$ ,  $\mathbf{u}_3$  are the eigenvectors of  $\Delta^H \Delta$ . The correlation matrix that can improve the PEP is

$$\mathbf{A}_{1} = \begin{pmatrix} 1 & -0.5 - 0.0144i & -0.5 + 0.0144i \\ -0.5 + 0.0144i & 1 & -0.5 - 0.0144i \\ -0.5 - 0.0144i & -0.5 + 0.0144i & 1 \end{pmatrix}.$$
 (4.61)

In this example, we can see that the transmission correlations defined by  $A_1$  can bring about 1.5 dB gains for the PEP performance, which is illustrated by the square line in Fig. 4.5. We consider another correlation matrix  $A_2$  that has the same eigenvectors as that of (4.61) but different eigenvalues:  $v_1 = 1.8$ ,  $v_2 = 0.7$ and  $v_3 = 0.5$ . For this correlation matrix  $A_2$ , we can see that the transmission correlations degrade the PEP performance, as illustrated by the PEP curve (marked by circle) in Fig. 4.5. Further, with the same code words, but with different number of



**Figure 4.5:** The effect of transmission correlations on the MISO multikeyhole channel for the case that L < M. This figure demonstrates the example for Theorem 4: in the asymptotically high SNR regime, for the case that L < M, the correlation matrix can either improve the PEP or degraded the PEP, depending on how the correlation matrix beamforms the code words difference matrix. In this example, correlation matrix  $A_1$  improves the PEP performance, while the correlation matrix  $A_2$  degrades the PEP performance.

keyholes (L = 4), however, as shown in Fig. 4.6, we see that the PEP performance is degraded by both  $A_1$  and  $A_2$ .

It is worth mentioning here that when the space-time code is orthogonal (i.e. all the eigenvalues of  $\Delta^H \Delta$  are identical), Theorem 4 will never be satisfied since Lemma 4 implies that the transmission correlations always degrade the PEP performance for orthogonal codes, this is consistent with the result in [40], where majorization was used to show this property for orthogonal code in the MIMO multi-keyhole channel. The effects of transmission correlations on the PEP perfor-



**Figure 4.6:** The effect of transmission correlations on the MISO multikeyhole channel for the case that  $L \ge M$ . This figure demonstrates the example for Theorem 3: in the asymptotically high SNR regime, for the case that  $L \ge M$ , transmission correlations always degrade the PEP performance. In this example, when L = 4, both  $A_1$  and  $A_2$  degrade the PEP performance.

mance for the multi-keyhole and Rayleigh channels are compared in Table 4.1. We can see that transmission correlations play different roles on the PEP performances of the multi-keyhole channel and the Rayleigh channel.

In addition, we compare the OSTBC performance of the backscatter RFID channel with that of the multi-keyhole channel in Table 4.2. Clearly these two channels have entirely different performance behaviors.

**Table 4.1:** The effects of transmission correlations on the PEP performances of the multi-keyhole and Rayleigh channels in the asymptotically high SNR regimes.

Type of STC	Rayleigh	Multi-keyhole	
Orthogonal	Always degrades [82]	Always degrades [40]	
Non-orthogonal	Always degrades [82]	$M \leq L$ , always degrades (Our re-	
	sult, Theorem 3 in this chapter);		
		M > L, may either degrade or	
		improve (Our result, Theorem 4	
		in this chapter)	

**Table 4.2:** Performance comparisons between the backscatter RFID and multi-keyhole channels for orthogonal space-time codes in the MISO case.

	Backscatter RIFD	Backscatter RIFD	Multi-keyhole
	with uniform query	with unitary query	
Diversity Order or	L (diversity order)	$\sum_{t=1}^{T} \min(1, L_t^*)$	$\min(M,L)$ (diver-
the new measure in		(new measure)	sity order)
Theorem 1			

#### 4.4 Conclusion

In this chapter, we analytically studied the performance of STCs in multi-keyhole channels, and revealed a few interesting properties of this channel. We proved that, for any STC, the code words distances in the MISO multi-keyhole channel (*M* transmitting antennas, *L* keyholes, and one receiving antenna) and the MIMO single-keyhole channel (one keyhole, *M* transmitting antenna and *L* receiving antennas) have identical distributions. We also considered the case when spatial correlations are present between transmission antennas. We showed that, in the asymptotically high SNR regimes, when  $M \leq L$ , the transmission correlations always degrade the PEP performance; when M > L, depending on how the correlation matrix beamforms the code words difference matrix  $\Delta$ , the PEP performance can either be degraded or improved. Particularly, we proved that if the eigenvalues of  $\Delta$  satisfy certain conditions, there always exist correlation matrices that can improve the PEP. We provided one form of such correlation matrices. Our re-

sults in this chapter also showed that the backscatter RFID channel, which has a query-fading-signaling-fading structure, and the multi-keyhole channel, which has a signaling-fading-fading structure, have completely different performance behaviors.

## Chapter 5

## **Summary and Future Work**

In this chapter, we summarize the main results obtained in this dissertation and suggest a number of future topics based on the research in this dissertation.

#### 5.1 Summary of Results

In this dissertation, we have addressed a few main challenges that researchers encountered in the performance analysis and design of backscatter RFID channels. These challenges come from the unique query-fading-signaling-fading structure of the backscatter RFID channels. When compared with the signaling-fading structure of conventional point-to-point channel models, the performance analysis and design of MIMO backscatter RFID channels face more challenges.

In Chapter 2, we first provided a mathematical modeling of this specific MIMO structure which considered all aspects of the backscatter RFID channels at the physical layer: the query signals, the forward channels, the tag signaling, and the backscattering channels. This modeling shows that the backscatter RFID channel has radically different fading structure and signaling mechanism when compared with a conventional one-way point-to-point wireless channel. We then investigated the simplest mechanism: the reader transmitters employ the uniform query and the tag employs the identical signaling scheme. Different from the case of conventional one-way point-to-point wireless channel, it was shown by simulations that the identical signaling scheme can significantly improving the BER performance
in RFID for some antenna settings. For the first time in the literature, in Chapter 2, we gave a rigorous mathematical analysis to reveal the underlying structure of the identical signaling scheme for backscatter RFID channels, and answered the fundamental question why identical signaling can sometimes improve the BER performance in RFID channels by showing that there is a bottleneck for backscatter RFID channels. The results in Chapter 2 can be used to help the design of simple but still effective backscatter RFID systems.

In Chapter 3, we considered more complicated cases. First, we considered the case that the tags employ the orthogonal space-time code, while the reader still employs the uniform query. For this case, we provided a general formulation for SER performance analysis, and this formulation is applicable to any sub-channel fading assumptions. Using this formulation, we analytically studied the SER performances for Rician and Nakagami-m sub-channels, and derived asymptotic SERs in closed form. We also generalized the PEP performance analysis to general spacetime codes by providing a PEP performance upper bound that the backscatter RFID structure could ever achieve. For this case, we find that the diversity order is L for Rician fading and is  $L\min(m_f, Nm_b)$  for Nakagami-m fading. We suggest that using two receiving antennas (N = 2) is recommended in practice since N = 2 can capture most of the receiving side gain regardless of the number of tag antennas L. We also suggest that more design attention should be given to the forward links in RFID, because we found that the performance of the backscatter RFID channel is more sensitive to the channel condition (the K factor or the m parameter) of the forward link than that of the backscattering link. Second, we considered the case that the tags employ general space-time codes, while the reader employs the proposed *unitary query*. We proved that the proposed unitary query can improve the PEP performance of backscatter RFID systems significantly for some antenna settings and some space-time codes. We analytically studied the performance of the proposed unitary query with general space-time code. Different from the traditional uniform query in MIMO backscatter RFID, which cannot provide either time or spatial diversity, for the first time in RFID, we showed that in quasi-static channels, the unitary query can provide time diversity via multiple reader transmitting antennas for some antenna settings and some space-time codes, and hence improve the PEP performance significantly. Due to the query-fading-signalingfading structure of the backscatter RFID channel, the PEP and the diversity order are not analytically trackable for the unitary query with general space-time. We therefore provided a new measure for the performance analysis, and the proposed measure can compare the performance of the unitary query with that of the uniform query, for any space-time codes. Based on the results in this chapter, we can determine for which antenna settings the unitary query at the reader transmitter end can yield significant performance improvements. Such results could guide us to design high performance RFID systems at lower cost.

In Chapter 4, we analytically studied the PEP performance of space-time codes in multi-keyhole channels, which have a signaling-fading-fading structure. The main motivation for this chapter is to answer the question whether there is any performance behavior difference between the backscatter RFID and multi-keyhole channels since both types of channels have cascaded forms, which look similar. Particularly, we analytically studied the performance of general space-time codes for multi-keyhole channels in the MISO case. We also considered the case when spatial correlations are present between transmission antennas in multi-keyhole channels. The results in this chapter clearly showed that the backscatter RFID channel, which has a query-fading-signaling-fading structure, and the multi-keyhole channel, which has a signaling-fading structure, have completely different performance behaviors.

## 5.2 Future Work

In this section, we suggest a few future research directions based on the contents of this dissertation.

### 5.2.1 Explore the Time Diversity Brought by the Unitary Query

The fundamental reason that the unitary query proposed in this dissertation can significantly improve the BER performance for some antenna settings is that it can diversify the channel gains over time in a slow fading environment. Therefore, there is a huge potential for performance improvements by exploring this time diversity. An immediate idea is that simple repetition codes, which cannot help too much in conventional slow faded non-backscatter channels, are expected to yield superior performances when employed together with the unitary query in backscatter RFID channels. The joint design of unitary query and repetition code can be very attractive for RFID systems, which often prefer less complex hardware and lower cost. For example, suppose we have the following design requirements: only have one antenna at each tag; a low BER is required; and a lower transmission data rate is acceptable. For this case, a repetition code with factor 2 with a simple  $2 \times 1 \times 1$  channel can achieve a better performance (e.g., a measure of 2 based on the proposed new measure) with the unitary query, but the uniform query can never achieve a measure of 2 even if a large number of antennas are deployed at the reader. There are many time diversity techniques that we can explore for backscatter RFID systems with the unitary query at the reader transmitter end.

#### 5.2.2 Non-Coherent Schemes for the Unitary Query

For coherent detections, channel estimation poses a large overhead for the backscatter RFID channel when employing the unitary, as the channel has one more operational end. For the unitary query and general space-time coding, the reader has to estimate the channel state information for *MLN* branches, and this will decrease the system efficiency, which might be crucial for some RFID systems. Fortunately, we can consider alternatively schemes based on non-coherent transmission and detections [83] for the backscatter RFID with unitary query. As the fundamental reason for the superior performance of unitary query is that it diversifies the forward fading over time slots by using multiple reader transmitting antennas, the unitary query must also be able to bring similar performance enhancement for noncoherent transmissions. Non-coherent transmissions and detections can not only avoid the large overhead for channel estimation, but also requires low complexity and low cost tags and readers.

#### 5.2.3 General Query for the Backscatter RFID

In Chapter 3 we proposed unitary query and showed that there are significant performance improvement for some antenna settings. However, sometimes with hardware constraints, the unitary query may not be available. For instance, when T is larger than the number of query antennas, the time diversity cannot be fully exploited by the unitary matrix. Therefore it is necessary to give an analytical study of the PEP performance for any arbitrary query matrix. With hardware constraints, jointly design of space-time codes and query matrices can help to decide the tradeoff between the complexity of the hardware and the performance of the backscatter RFID system. Intuitively, the performance measure for general query matrix can be a linear combination of some form of the measure for the unitary query and that of the uniform query, while this still need to be confirmed by mathematical proof in future work.

### 5.2.4 Optimal Query Antenna Selection

We consider a query method for which if there exists a  $i \in \{1, \dots, M\}$ , such that  $\|h_{i,l}^f\| \ge \|h_{m,l}^f\|$  for all  $m \in \{1, \dots, M\}$  and for all  $l \in \{1, \dots, L\}$  over T time slots, then the reader allows only the *i*-th query antenna (we call the *i*-th query antenna the optimal query antenna) to send the query signals over the current T time slots, otherwise the reader still employs unitary query. It can be shown that this *optimal query antenna selection* method will yield even a better PEP performance than that of the unitary query. With the assumption that the forward channels are i.i.d., the probability that the optimal query antenna exists is given by

$$\frac{M}{M^L} = \frac{1}{M^{L-1}}.$$
 (5.1)

Except the case when L = 1, there is no guarantee that the optimal query antenna exists. So the PEP that for the above query method is given by

$$\operatorname{PEP}_{\operatorname{opt}} = \frac{1}{M^{L-1}} \operatorname{PEP}_{\operatorname{opt}}^* + \frac{M^{L-1} - 1}{M^{L-1}} \operatorname{PEP}_{\operatorname{unitary}},$$
(5.2)

and bounded by

$$1 \ge \frac{\text{PEP}_{\text{opt}}}{\text{PEP}_{\text{unitary}}} \ge \frac{M^{L-1} - 1}{M^{L-1}},$$
(5.3)

where  $PEP_{opt}$ ,  $PEP_{opt}^*$  and  $PEP_{unitary}$  are the PEP for the optimal query antenna selection, the PEP when the optimal query antenna exists and the PEP for the unitary query, respectively. It is expected that a few dB gains can be brought by the optimal query antenna selection method, compared with the unitary query method. The cost of this method is the overhead for the channel estimation, which is the same as that of the unitary query in the coherent detection case.

# **Bibliography**

- R. Want. An introduction to RFID technology. *IEEE Pervasive Computing*, 13:25–33, 2006. → pages 1, 4
- [2] M. Rieback, B. Crispo, and A. Tanenbaum. The evolution of RFID security. IEEE Pervasive Computing, 5:62–69, 2006. → pages 1
- [3] M. Roberti. The history of RFID technology. *RFID Journal*, 2005.  $\rightarrow$  pages 1
- [4] H. Stockman. Communication by means of reflected power. *Proc. I.R.E.*, 36:1196–1204, 1948.  $\rightarrow$  pages 1
- [5] M. Cardullo. Genesis of the versatile RFID tag. *RFID Journal*, 2013.  $\rightarrow$  pages 1
- [6] RFID World. World RFID market to reach 20 billion usd in 2014? 2013.  $\rightarrow$  pages 1
- [7] F. Klaus. RFID Handbook: Fundamentals and Applications in Contactless Smart Cards and Identification, 2nd edition. Wiley, Hoboken, N.J., 2003. → pages 1, 2, 4
- [8] S. Shepard. *RFID: Radio Frequency Identification*. McGraw-Hill, New York, 2004. → pages 2
- [9] F. Jill. Indoor Positioning and Digital Management: Emerging Surveillance Regimes in Hospitals. In T. Monahan (Ed), Surveillance and Security: Technological Politics and Power in Everyday Life. Routledge, New York, 2006. → pages 2
- [10] C. Swedberg. Group health reinvents patient care with RTLs. *RFID Journal*, 2012.  $\rightarrow$  pages 2

- [11] S. Bhattacharya. Electronic tags for eggs, sperm and embryos. *New Scientist*, 2010.  $\rightarrow$  pages 2
- [12] D. Sen, P. Sen, and A. Das. *RFID For Energy and Utility Industries*. PennWell, Tulsa, 2009.  $\rightarrow$  pages 3
- [13] D. Kim, M. Ingram, and W. Smith. Measurements of small-scale fading and path loss for long range RF tags. *IEEE Transactions on Antennas and Progagation*, 51:1740–1749, 2003. → pages 4, 5, 8
- [14] J. D. Griffin and G. D. Durgin. Multipath fading measurements for multi-antenna backscatter RFID at 5.8 ghz. *IEEE Transactions on Antennas* and Propagation, 58:3693–3700, 2010. → pages 4, 5, 6, 8, 52
- [15] U. Karthaus and M. Fischer. Fully integrated passive UHF RFID transponder ic with 16.7 uw minimum RF input power. *IEEE J. Solid-State Circuits*, 5:1602–1608, 2003. → pages
- [16] P. Nikitin, K. Rao, S. Lam, V. Pillai, R. Martinez, and H. Heinrich. Fully integrated passive UHF RFID transponder ic with 16.7 uw minimum RF input power. *IEEE Trans. Microwave Theory Tech.*, 9:2721–2725, 2005. → pages
- [17] F. Fuschini, C. Piersanti, F. Paolazzi, and G. Falciasecca. Analytical approach to the backscattering from UHF RFID transponders. *IEEE Antennas Wireless Propag. Lett.*, 7:3335, 2008. → pages
- [18] Y. Xi, H. Kim, H. Cho, M. Kim, S. Jung, C. Park, J. Kim, and Y. Yang. Optimal ASK modulation scheme for passive RFID tags under antenna mismatch conditions. *IEEE Trans. Microw. Theory Tech.*, 57:23372343, 2009. → pages
- [19] A. Bletsas, A. G. Dimitriou, and J. N. Sahalos. Improving backscatter radio tag efficiency. *IEEE Trans. Microwave Theory Tech.*, 58:1502–1509, 2010.
   → pages
- [20] R. Chakraborty, S. Roy, and V. Jandhyala. Revisiting RFID link budgets for technology scaling: range maximization of RFID tags. *IEEE Trans. Microwave Theory Tech.*, 59:496–503, 2011. → pages
- [21] S. Thomas, E. Wheeler, J. Teizer, and M. Reynolds. Quadrature amplitude modulated backscatter in passive and semipassive UHF RFID systems. *IEEE Trans. Microwave Theory Tech.*, 60:1175–1182, 2012. → pages 32

- [22] J. Kimionis, A. Bletsas, and J. Sahalos. Bistatic backscatter radio for tag read-range extension. In *IEEE International Conference on RFID*, page 356361, November 2012. → pages
- [23] C. Boyer and S. Roy. Coded qam backscatter modulation for RFID. *IEEE Transactions on Communications*, 60:1925–1934, 2012. → pages
- [24] D. Arnitz and M. Reynolds. Wireless power transfer optimization for nonlinear passive backscatter devices. In 2013 IEEE International Conference on RFID, pages 245–252, Penang, Malaysia, 2013. → pages
- [25] M. Ingram, M. Demirkol, and D. Kim. Transmit diversity and spatial multiplexing for RF links using modulated backscatter. In *International Symposium on Signals, Systems, and Electronics*, pages 24–27, 2001. → pages 5, 6
- [26] J. D. Griffin and G. D. Durgin. Gains for RF tags using multiple antennas. IEEE Communications Letters, 11:735–737, 2007. → pages 6, 26, 27, 52
- [27] J. D. Griffin and G. D. Durgin. Gains for RF tags using multiple antennas. *IEEE Transactions on Antennas and Propagation*, 56:563–570, 2008.  $\rightarrow$  pages 5, 6, 9, 11, 17, 18, 52
- [28] J. D. Griffin and G. D. Durgin. Complete link budgets for backscatter radio and RFID systems. *IEEE Antennas and Propagation Magazine*, 51:11–25, 2009. → pages
- [29] D. Kim, H. Jo, H. Yoon, C. Mun, B. Jang, and J. Yook. Reverse-link interrogation range of a UHF MIMO-RFID system in nakagami-m fading channels. *IEEE Transactions on Industrial Electronics*, 57:468–1477, 2010. → pages
- [30] R. Langwieser, C. Angerer, and A. Scholtz. A UHF frontend for MIMO applications in RFID. In *IEEE Radio and Wireless Symposium*, pages 124–127, New Orleans, LA, 2010. → pages 5, 6
- [31] E. Denicke, M. Henning, H. Rabe, and B. Geck. The application of multiport theory for MIMO RFID backscatter channel measurements. In *European Microwave Conference*, pages 523–525, Amsterdam, 2012. → pages 6
- [32] M. Trotter, C. Valenta, G. Koo, B. Marshall, and G. D. Durgin. Multi-antenna techniques for enabling passive RFID tags and sensors at microwave frequencies. In 2012 IEEE International Conference on RFID, pages 1–7, Orlando, FL, 2012. → pages 5, 6

- [33] V. Tarokh, N. Seshadri, and A. Calderbank. Sapce-time codes for high data rate wireless communication: Performance criterion and code construction. *IEEE Transactions on Information Theory*, 44:744–765, 1998. → pages 5, 78
- [34] V. Tarokh, H. Jafarkhani, and A. Calderbank. Space-time block coding for wireless communications: Performance results. *IEEE J. Select. Areas Commun.*, 17:451–460, 1999. → pages 32
- [35] S. Sandhu and A. Paulraj. Space-time block codes: A capacity perspective. IEEE Communications LettersS, 10:384–386, 2000. → pages 33
- [36] L. Zheng and D. Tse. Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels. *IEEE Transactions on Information Theory*, 49: 1073–1096, 2003. → pages
- [37] D. Tse and P. Viswanath. Fundamentals of Wireless Communication. Cambridge University Press, 2005. → pages 5
- [38] Colby Boyer and Sumit Roy. Space time coding for backscatter RFID. *IEEE Transactions on Wireless Communications*, 12:2272–2280, 2013. → pages 6
- [39] Chen He, Xun Chen, Z. Jane. Wang, and Weifeng Su. On the Performance of MIMO RFID Backscattering Channels. *EURASIP Journal on Wireless Communications and Networking*, 11:1–27, 2012. → pages 7
- [40] H. Shin and M. Win. MIMO diversity in the presence of double scattering. *IEEE Transactions on Information Theory*, 54:2976–2996, 2008. → pages 7, 68, 79, 87, 89
- [41] M. Abramowitz and I. A. Stegun. *Handbook of Mathematical Functions*, 10th ed. National Bureau of Standards, Washington, DC, 1972. → pages 18
- [42] M. Simon and M. Alouini. A unified approach to the performance analysis of digital communication over generalized fading channels. *Proceedings of The IEEE*, 86:1860–1877, 1998. → pages 18, 23, 24, 33, 35
- [43] D. G. Brennanr. Linear diversity combining techniques. Proceedings of The IEEE, 91:331–356, 2003. → pages 18
- [44] I. Gradsheteyn and I. Ryzhik. Table of Integrals, Series and Product, 7th ed. Academic Press, New York, 1994. → pages 20, 105, 106, 107
- [45] J. Proakis. *Digital Communications, 4th ed.* McGraw-Hill, New York, 2001.  $\rightarrow$  pages 21

- [46] C. Zhu, J. Mietzner, and R. Schober. On the performance of non-coherent transmission schmemes with equal-gain combining in generalized k-fading. *IEEE Transactions on Wireless Communications*, 9:1337–1349, 2010. → pages 21
- [47] D. M. Dobkin. The RF in RFID: Passive UHF RFID in Practice. Newnes, Burlington, MA, 2008. → pages 32
- [48] S. M. Alamouti. A simple transmit diversity technique for wireless communications. *IEEE Journal on Slected Areas in Communications*, 16: 1451–1458, 1998. → pages 32, 37
- [49] G. Stuber. Principles of Mobile Communication. Springer, Berlin, Heidelberg, 2001. → pages 32
- [50] D. Cassioli, Moe Win, and Andreas Molisch. The ultra-wide bandwidth indoor channel: from statistical model to simulations. *IEEE Select. Areas Commun.*, 20:1247–1257, 2002.  $\rightarrow$  pages 32
- [51] D. Dardari, R. DErrico, C. Roblin, A. Sibille, and Moe Z. Win. Ultrawide bandwidth RFID: The next generation? *Proceedings of the IEEE*, 98: 1570–1582, 2010. → pages 32
- [52] Y. Gong and K. Letaief. On the error probability of orthogonal space-timeblock codes over keyhole MIMO channel. *IEEE Transactions on Wireless Communications*, 6:2402–2409, 2007. → pages 41, 67, 110
- [53] A. Nezampour, A. Nasri, and R. Schober. Asymptotic analysis of space-time codes in generalized keyhole fading channels. *IEEE Transactions On Wireless Communications*, 10:1863–1873, 2011. → pages 42
- [54] H. Zhao, Y. Gong, Y. Liang Guan, and S. Li. Performance bounds of orthogonal space-time block codes over keyhole Nakagami-m channels. In *Proc. IEEE International Conference on Communications (ICC'2008)*, pages 4090–4094, Beijing, China, May 2008. → pages 42
- [55] P. Supnithi, W. Wongtrairat, and S. Tantaratana. Performance of m-psk in mobile satellite communication over combined ionospheric scintillation and flat fading channels with MRC diversity. *IEEE Transactions on Wireless Communications*, 8:3360–3364, 2009. → pages 42
- [56] D. Chizhik, G. J. Foschini, M. J. Gans, and R. A. Valenzuela. Keyholes, correlations, and capacities of multielement transmit and receive antennas. *IEEE Trans. Commun.*, 1:361–368, 2002. → pages 67

- [57] D. Gesbert, H. Bolcskei, D. A. Gore, and A. J. Paulraj. Outdoor MIMO wireless channels: Models and performance prediction. *IEEE Trans. Commun.*, 50:1926–1934, 2002. → pages 67
- [58] H. Shin and J. H. Lee. Capacity of multi-antenna fading channels: spatial fading correlation, double scattering, and keyhole. *IEEE Trans. Inf. Theory*,  $49:2636-2647, 2003. \rightarrow pages 67$
- [59] G. Levin and S. Loyka. Capacity distribution of a correlated keyhole channel. In *Proc. Canadian Workshop on Info. Theory*, pages 319–322, June 2005. → pages 67
- [60] A. Muller and J. Speidel. Characterization of mutual information of spatially correlated MIMO channels with keyhole. In *Proc. Int. Conf. Commun.*, pages 750–755, June 2007. → pages 67
- [61] H. Shin and J. H. Lee. Effect of keyholes on the symbol error rate of space-time block codes. *IEEE Commun. Lett.*, 7:27–29, 2003. → pages 67
- [62] S. Sanayei, A. Hedayat, and A. Nosratinia. Space time codes in keyhole channels: Analysis and design. *IEEE Trans. Wireless Commun.*, 6: 2006–2011, 2007. → pages 67, 73, 74
- [63] P. Yahampath and A. Hjorungnes. Symbol error rate analysis of spatially correlated keyhole MIMO channels with space-time block coding and linear precoding. In *Proc. IEEE Global Telecommunications Conference*, 2007., pages 5367–5371, November 2007. → pages 67
- [64] P. Almers, F. Tufvensson, and A. F. Molisch. Measurement of keyhole effect in a wireless multiple-input multiple-output (MIMO) channel. *IEEE Communications Letters*, 7:373–375, 2003. → pages 67
- [65] G. Levin and S. Loyka. Multi-keyhole MIMO channels: asymptotic analysis of outage capacity. In *Proc. IEEE Int. Symp. Info. Theory*, pages 1305–1309, July 2006. → pages 67
- [66] G. Levin and S. Loyka. On the outage capacity distribution of correlated keyhole MIMO channels. *IEEE Trans. Inf. Theory*, 54:3232–3245, 2008. → pages 67
- [67] M. Tsuruta, T. Taniguchi, and Y. Karasawa. On statistical distribution of eigenvalues of channel correlation matrix in MIMO multi-keyhole environment. *IEICE Trans. Commun.*, E90-B:2352–2359, 2007. → pages 67

- [68] S. Yang and J. C. Belfiore. Diversity-multiplexing tradeoff of double scattering MIMO channels. *IEEE Trans. Inf. Theory*, 57:2027–2034, 2011. → pages 67
- [69] G. Levin and S. Loyka. From multi-keyhole to measure of correlation and power imbalance in MIMO channels: outage capacity analysis. *IEEE Trans. Inf. Theory*, 57:3515–3529, 2011. → pages 67, 75, 79
- [70] S. Jin, M. R. McKay, C. Zhong, and K. K. Wong. Ergodic capacity analysis of amplify-and-forward MIMO dual-hop systems. *IEEE Trans. Inf. Theory*, 56:2204–2224, 2010. → pages 67
- [71] C. Zhong, S. Jin, K. Wong, and M. McKay. Ergodic mutual information analysis for multi-keyhole MIMO channels. *IEEE Trans. Inf. Theory*, 10: 1754–1763, 2011. → pages 67, 75, 79
- [72] S. Jin, M. R. McKay, K. K. Wong, and X. Gao. Transmit beamforming in Rayleigh product MIMO channels: capacity and performance analysis. *IEEE Trans. Signal Process.*, 56:5204–5221, 2008. → pages 67, 79
- [73] C. Zhong, S. Jin, and K. K. Wong. Optimum combining in MIMO double scattering channels with co-channel interference. *IEEE Trans. Commun.*, 57: 824–1835, 2009. → pages
- [74] C. Zhong, S. Jin, K. K. Wong, and M. R. McKay. Outage analysis for optimal beamforming MIMO systems in multi-keyhole channels. *IEEE Trans. Signal Process.*, 58:1451–1456, 2010. → pages 67
- [75] B. Hochwald and T. Marzetta. Unitary space-time modulation for multiple-antenna communications in Rayleigh flat fading. *IEEE Trans. Inf. Theory*, 46:543–564, 2000. → pages 68
- [76] H. Jafarkhani and N. Seshadri. Super-orthogonal space-time trellis codes. IEEE Trans. Inf. Theory, 49:937–950, 2003. → pages
- [77] J. Belfiore, G. Rekaya, and E. Viterbo. The golden code: A 2 by 2 full-rate space-time code with nonvanishing determinants. *IEEE Trans. Inf. Theory*, 51:1432–1436, 2005. → pages
- [78] P. Elia, K. Kumar, S. Pawar, P. Kumar, and H. Lu. Explicit space-time codes achieving the diversity-multiplexing gain tradeoff. *IEEE Trans. Inf. Theory*, 52:1432–1436, 2006. → pages 68

- [79] T.-A. Chen, M. P. Fitz, W.-Y. Kuo, M. Zoltowski, and J. Grimm. A spacetime model for frequency nonselective Rayleigh fading channels with applications to spacetime modems. *IEEE J. Select. Areas Commun.*, 18: 1175–1190, 2000. → pages 78
- [80] C. Chuah, D. Tse, J. M. Kahn, and R. A.Valenzuela. Capacity scaling in MIMO wireless systems under correlated fading. *IEEE Trans. Inform. Theory*, 48:637–650, 2002. → pages
- [81] Y. Xin, Z.Wang, and G. Giannakis. Spacetime diversity systems based on linear constellation precoding. *IEEE Trans. Wireless Commun.*, 2:294–309, 2003. → pages
- [82] J. Wang, M. Simon, M. Fitz, and K. Yao. On the performance of spacetime codes over spatially correlated Rayleigh fading channels. *IEEE Transactions* on Communications, 52:877–881, 2004. → pages 78, 79, 81, 89
- [83] G. Leus, W. Zhao, G. Giannakis, and H. Delic. Spacetime frequency-shift keying. *IEEE Transactions on Communications*, 52:346–349, 2004. → pages 94

# Appendix A

# **Derivations**

# A.1 Chapter 2 Derivations

*Proof of Proposition 1.* Let  $A = 1 + \bar{\gamma} \sum_{l=2}^{L} \alpha_l$ , then

$$\int_{\alpha_{1}=0}^{\infty} \frac{\exp(-\alpha_{1})}{A + \bar{\bar{\gamma}}\alpha_{1}} d\alpha_{1} = \int_{\alpha_{1}=0}^{\infty} \frac{\exp\left(-\alpha_{1} - \frac{A}{\bar{\gamma}}\right)}{\bar{\bar{\gamma}}\left(\alpha_{1} + \frac{A}{\bar{\bar{\gamma}}}\right)} e^{\frac{A}{\bar{\gamma}}} d\left(\alpha_{1} + \frac{A}{\bar{\bar{\gamma}}}\right)$$
$$= \frac{e^{\frac{A}{\bar{\gamma}}}}{\bar{\bar{\gamma}}} \int_{\alpha_{1}'=\frac{A}{\bar{\gamma}}}^{\infty} \frac{\exp(-\alpha_{1}')}{\alpha_{1}'} d\alpha_{1}' = \frac{e^{\frac{A}{\bar{\gamma}}}}{\bar{\bar{\gamma}}} E_{1}\left(\frac{A}{\bar{\bar{\gamma}}}\right). \tag{A.1}$$

where  $\alpha'_1 = \alpha_1 + \frac{A}{\bar{\gamma}}$  and  $E_1(x) = \int_{t=x}^{\infty} e^{-t}/t dt$  is a special function called the exponential integral [44].

Now we have

$$\begin{split} &\int_{\alpha_{2}=0}^{\infty}\cdots\int_{\alpha_{L}=0}^{\infty}\frac{e^{\frac{1}{\bar{\gamma}}}}{\bar{\bar{\gamma}}}E_{1}\left(\frac{1+\bar{\bar{\gamma}}\sum_{l=2}^{L}\alpha_{l}}{\bar{\bar{\gamma}}}\right)d\alpha_{L}\cdots d\alpha_{2} \\ &=\frac{e^{\frac{1}{\bar{\gamma}}}}{\bar{\bar{\gamma}}}\int_{\alpha_{2}=0}^{\infty}\cdots\int_{\alpha_{L}=0}^{\infty}\int_{t=1}^{\infty}\frac{\exp\left(\frac{t}{-\bar{\gamma}}-t\sum_{l=2}^{L}\alpha_{l}\right)}{t}dtd\alpha_{L}\dots d\alpha_{2} \\ &=\frac{e^{\frac{1}{\bar{\gamma}}}}{\bar{\bar{\gamma}}}\int_{t=1}^{\infty}\frac{e^{-\frac{t}{\bar{\gamma}}}}{t}\left(\prod_{l=2}^{L}\int_{\alpha_{l}=0}^{\infty}\exp\left(-\alpha_{l}t\right)d\alpha_{l}\right)dt \\ &=\frac{e^{\frac{1}{\bar{\gamma}}}}{\bar{\bar{\gamma}}}\int_{t=1}^{\infty}\frac{e^{-\frac{t}{\bar{\gamma}}}}{t^{L}}dt=\frac{e^{\frac{1}{\bar{\gamma}}}}{\bar{\bar{\gamma}}}E_{L}\left(\frac{1}{\bar{\bar{\gamma}}}\right)\doteq\begin{cases}\frac{\ln(\bar{\gamma})}{\bar{\gamma}}, & \text{if } L=1; \\ \frac{1}{(L-1)\bar{\gamma}}, & \text{if } L>1. \end{cases} \end{split}$$
(A.2)

The last step is obtained by the asymptotic property of the generalized exponential integral  $E_L(\cdot)$  [44].

Proof of Proposition 2.

$$\int_{\alpha=0}^{\infty} \left(\frac{1}{1+\bar{\bar{\gamma}}\alpha}\right)^{N} \exp(-\alpha)d\alpha$$

$$= \int_{x=1}^{\infty} \frac{\exp\left(-\frac{x}{\bar{\gamma}}\right) \exp\left(\frac{1}{\gamma}\right) \left(\frac{1}{\bar{\gamma}^{N}}\right)}{\left(\frac{x}{\bar{\gamma}}\right)^{N}} \frac{dx}{\bar{\bar{\gamma}}}$$

$$= \int_{x'=\frac{1}{\bar{\gamma}}}^{\infty} \frac{1}{x'^{N}} \exp(-x') \exp\left(\frac{1}{\bar{\bar{\gamma}}}\right) \frac{1}{\bar{\bar{\gamma}}^{N}} dx'$$

$$= \frac{E_{N}\left(\frac{1}{\bar{\gamma}}\right)}{\bar{\bar{\gamma}}} \exp\left(\frac{1}{\bar{\bar{\gamma}}}\right) \qquad (A.3)$$

where  $x = 1 + \bar{\bar{\gamma}}\alpha$ , and  $x' = \frac{x}{\bar{\bar{\gamma}}}$ . The asymptotic expression is just like that in Proposition 2.

*Proof for Proposition 3.* Let  $A = 1 + \bar{\gamma} \sum_{l=2}^{L} \alpha_l$  and

$$\exp\left(-\sum_{l=2}^{L} \alpha_{l}\right) \int_{\alpha_{1}=0}^{\infty} \frac{1}{(A+\bar{\bar{\gamma}}\alpha_{1})^{N}} \exp(-\alpha_{1}) d\alpha_{1}$$
$$= \exp\left(-\sum_{l=2}^{L} \alpha_{l}\right) \frac{E_{N}\left(\frac{A}{\bar{\gamma}}\right)}{A^{N-1}\bar{\bar{\gamma}}} \exp\left(\frac{A}{\bar{\bar{\gamma}}}\right)$$
$$= \exp\left(\frac{1}{\bar{\bar{\gamma}}}\right) \frac{E_{N}\left(\frac{A}{\bar{\gamma}}\right)}{A^{N-1}\bar{\bar{\gamma}}}, \qquad (A.4)$$

where  $E_N(z) = \int_{t=1}^{\infty} \frac{e^{-zt}}{t^N} dt$  is the generalized exponential integral. Using the relation that  $E_N(z) = \frac{1}{N-1} (e^{-z} - zE_{N-1}(z))$  [44] we can prove that

$$E_N(z) = \frac{(-1)^{N-1} z^{N-1} E_1(z)}{(N-1)!} + \sum_{i=1}^{N-1} \frac{(-1)^{i-1} (N-1-i)! z^{i-1} e^{-z}}{(N-1)!}$$
(A.5)

then

$$\begin{split} G_{N,L}(\bar{\bar{\gamma}}) &= \int_{\alpha_{L}=0}^{\infty} \cdots \int_{\alpha_{2}=0}^{\infty} e^{\frac{1}{\bar{\gamma}}} \frac{E_{N}(\frac{A}{\bar{\gamma}})}{A^{N-1}\bar{\bar{\gamma}}} d\alpha_{2} \cdots \alpha_{L} \\ &= \int_{\alpha_{L}=0}^{\infty} \cdots \int_{\alpha_{2}=0}^{\infty} e^{\frac{1}{\bar{\gamma}}} \frac{(-1)^{N-1}(\frac{A}{\bar{\gamma}})^{N-1}E_{1}(\frac{A}{\bar{\gamma}})}{A^{N-1}\bar{\bar{\gamma}}(N-1)!} \\ &+ e^{\frac{1}{\bar{\gamma}}} \sum_{i=1}^{N-1} \frac{(-1)^{i-1}(N-1-i)!(\frac{A}{\bar{\gamma}})^{i-1}e^{-\frac{A}{\bar{\gamma}}}}{(N-1)!} d\alpha_{2} \cdots d\alpha_{L} \\ &= \frac{e^{\frac{1}{\bar{\gamma}}}}{\bar{\bar{\gamma}}^{N}} \int_{\alpha_{L}}^{\infty} \cdots \int_{\alpha_{2}}^{\infty} \frac{(-1)^{N-1}E_{1}(\frac{A}{\bar{\gamma}})}{(N-1)!} d\alpha_{2} \cdots d\alpha_{L} \\ &+ \int_{\alpha_{L}}^{\infty} \cdots \int_{\alpha_{2}}^{\infty} \sum_{i=1}^{N-1} \frac{(-1)^{i-1}(N-1-i)!e^{-\sum_{l=2}^{L}\alpha_{l}}}{(N-1)!\bar{\bar{\gamma}}^{i}(1+\bar{\bar{\gamma}}\sum_{l=2}^{L}\alpha_{l})^{N-i}} d\alpha_{2} \cdots d\alpha_{L} \\ &= \frac{1}{(-\bar{\bar{\gamma}})^{N-1}(N-1)!} G_{1,L}(\bar{\bar{\gamma}}) + \sum_{i=1}^{N-1} \frac{(-1)^{i-1}(N-1-i)!}{(N-1)!\bar{\bar{\gamma}}^{i}} G_{(N-i),(L-1)}(\bar{\bar{\gamma}}) \\ &= \frac{1}{(-\bar{\bar{\gamma}})^{N-1}(N-1)!} G_{1,L}(\bar{\bar{\gamma}}) - \sum_{k=1}^{N-1} \frac{(k-1)!}{(-\bar{\bar{\gamma}})^{N-k}(N-1)!} G_{k,(L-1)}(\bar{\bar{\gamma}}). \quad (A.6) \end{split}$$

The last step is obtained by changing the index, i.e. k = N - i.

Proof of Proposition 4. Case 1: N > L

We apply mathematical induction to prove this property. It is easy to show that  $G_{N,1}$  is valid for (2.23), suppose for L = j the argument is valid and our goal is to show for L = j + 1 < N it is still valid. We have

$$G_{N,(j+1)}(\bar{\bar{\gamma}}) = \frac{1}{(-\bar{\bar{\gamma}})^{N-1}(N-1)!} G_{1(j+1)}(\bar{\bar{\gamma}}) - \sum_{k=1}^{N-1} \frac{(k-1)!}{(-\bar{\bar{\gamma}})^{N-k}(N-1)!} G_{kj}(\bar{\bar{\gamma}})$$

$$\stackrel{\doteq}{=} -\frac{1}{(-\bar{\bar{\gamma}})^{N}(N-1)!(j+1)} - \sum_{k=1}^{N-1} \frac{(k-1)!}{(-\bar{\bar{\gamma}})^{N-k}(N-1)!} G_{kj}(\bar{\bar{\gamma}})$$

$$\stackrel{\doteq}{=} -\frac{1}{(-\bar{\bar{\gamma}})^{N}(N-1)!(j+1)} - \frac{(N-2)!}{(-\bar{\bar{\gamma}})^{1}(N-1)!} G_{(N-1)j}(\bar{\bar{\gamma}})$$

$$\stackrel{\doteq}{=} -\frac{1}{(-\bar{\bar{\gamma}})^{N}(N-1)!(j+1)} + \frac{(N-2)!}{\bar{\bar{\gamma}}(N-1)!} \times \frac{1}{(N-2)\cdots(N-1-j)\bar{\bar{\gamma}}^{j}}$$

$$\stackrel{\doteq}{=} -\frac{1}{(-\bar{\bar{\gamma}})^{N}(N-1)!(j+1)} + \frac{1}{(N-1)\cdots(N-1-j)\bar{\bar{\gamma}}^{j+1}}$$

$$\stackrel{(A.7)}{=} \frac{1}{(N-1)\cdots(N-1-j)\bar{\bar{\gamma}}^{j+1}}.$$

Therefore (2.23) is valid for N > L.

Case 2: N = L

For N = 1 and L = 1 it is easy to show (2.23) is valid for N > L. Now we need to show that (2.23) is still valid for N = L = j + 1.

$$G_{(j+1),(j+1)}(\bar{\bar{\gamma}}) \doteq \frac{1}{(-\bar{\bar{\gamma}})^j j!} G_{1,(j+1)}(\bar{\bar{\gamma}}) - \sum_{k=1}^j \frac{(k-1)!}{(-\bar{\bar{\gamma}})^{j+1-k} j!} G_{k,j}(\bar{\bar{\gamma}})$$
$$\doteq -\frac{1}{(-\bar{\bar{\gamma}})^{j+1} j! j} - \sum_{k=1}^j \frac{(k-1)!}{(-\bar{\bar{\gamma}})^{j+1-k} j!} G_{k,j}(\bar{\bar{\gamma}}). \tag{A.8}$$

Since for k < j,  $G_{k,j}(\bar{\gamma}) \propto \frac{1}{\bar{\gamma}^k}$  and for  $k = j \ G_{k,j}(\bar{\gamma}) \propto \frac{\ln(\bar{\gamma})}{\bar{\gamma}^j}$ , we have

$$G_{(j+1),(j+1)}(\bar{\gamma}) \doteq -\frac{1}{(-\bar{\gamma})^{j+1}j!j} - \frac{(j-1)!}{(-\bar{\gamma})^{j+1-j}j!}G_{j,j}(\bar{\gamma})$$
$$\doteq \frac{\ln(\bar{\gamma})}{j!\bar{\gamma}^{j+1}}.$$
(A.9)

Case 3: N < L

A similar approach as that of Case 1 can be obtained for this case, therefore we omit the details here.  $\hfill \Box$ 

# A.2 Chapter 3 Derivations

## A.2.1 Derivations for Rician Fading

The PDF of the forward channel which follows the Rician distribution (normalized channel energy) is

$$f(\alpha_l) = (K_f + 1)e^{-K_f - (K_f + 1)\alpha_l} \sum_{m=0}^{\infty} \frac{(K_f(K_f + 1)\alpha_l)^m}{(m!)^2}$$
(A.10)

where the equality is given by the Taylor expansion of the modified Bessel function of the first kind (i.e.  $I_0(\cdot)$ ). We can expand the conditional MGF  $G(\bar{\gamma}|\alpha_l)$  as

$$G_l(\bar{\bar{\gamma}}|\alpha_l) = \left(-\frac{K_b + 1}{K_b + 1 + \bar{\bar{\gamma}}\alpha_l}\right)^N \sum_{i_1=0}^{\infty} \frac{1}{i!} \left(\frac{NK_b\bar{\gamma}\alpha_l}{K_b + 1 + \bar{\gamma}\alpha_l}\right)^i$$
(A.11)

Therefore averaging  $G_l(\bar{\bar{\gamma}}|\alpha_l)$  over the density of  $\alpha_l$  gives

$$\begin{aligned} G_{l}(\bar{\tilde{\gamma}}) &= \int_{\alpha_{l}=0}^{\infty} f(\alpha_{l}) G(\bar{\tilde{\gamma}}|\alpha_{l}) d\alpha_{l} \\ &= \int_{\alpha_{l}=0}^{\infty} \left( \frac{K_{b}+1}{K_{b}+1+\bar{\tilde{\gamma}}\alpha_{l}} \right)^{N} \sum_{i=0}^{\infty} \frac{1}{i!} \left( -\frac{NK_{b}\bar{\gamma}\alpha_{l}}{K_{b}+1+\bar{\tilde{\gamma}}\alpha_{l}} \right)^{i} \\ &\times (K_{f}+1) e^{-K_{f}-(K_{f}+1)\alpha_{l}} \sum_{m=0}^{\infty} \frac{(K_{f}(K_{f}+1)\alpha_{l})^{m}}{(m!)^{2}} \\ &= \sum_{m=0}^{\infty} \sum_{i=0}^{\infty} \frac{D_{1}D_{2}^{m}D_{3}^{i}}{i!(m!)^{2}} \bar{\tilde{\gamma}}^{i} \int_{\alpha_{l}=0}^{\infty} \alpha_{l}^{m+i} (K_{1}+\bar{\tilde{\gamma}}\alpha_{l})^{-(N+i)} \\ &\times \exp(-K_{2}\alpha_{l}) d\alpha_{l} \\ &= \sum_{m=0}^{\infty} \sum_{i=0}^{\infty} \frac{D_{1}D_{2}^{m}D_{3}^{i}}{i!(m!)^{2}} \bar{\tilde{\gamma}}^{i} F(\bar{\tilde{\gamma}},N',m') \end{aligned}$$
(A.12)

where  $D_1 = (K_f + 1)e^{-K_f}(K_b + 1)^N$ ,  $D_2 = K_f(K_f + 1)$ ,  $K_1 = K_b + 1$ ,  $K_2 = K_f + 1$ and  $D_3 = -NK_b$ . For presentation simplicity we define the function

$$F(\bar{\gamma}, N', m') = \int_{\alpha_l=0}^{\infty} \alpha_l^{m+i} (K_1 + \bar{\gamma} \alpha_l)^{-(N+i)} e^{-K_2 \alpha_l} d\alpha_l$$
  
=  $\frac{(m'-1)! K_1^{-N'} K_2^{-m'}}{(m'-1)!}$   
 $\times \int_{y=0}^{\infty} \left(1 + \frac{\bar{\gamma}}{K_1 K_2} y\right)^{-N'} e^{-y} y^{m'-1} dy$  (A.13)

where we use change of variable to obtain the second line:  $y = K_2 \alpha_l$ , m' = m + i + 1, N' = N + i where the function  $M(\frac{K_1K_2}{\bar{\gamma}}, N', m') = \frac{1}{(m'-1)!} \int_{y=0}^{\infty} \left(1 + \frac{\bar{\gamma}}{K_1K_2}y\right)^{-N'} e^{-y} y^{m'-1} dy$  was well studied in [52] and has a closed form of

$$M(\bar{\bar{\gamma}}, N', m') = \frac{e^{\frac{K_1K_2}{\bar{\gamma}}} \left(\frac{K_1K_2}{\bar{\gamma}}\right)^{N'}}{(m'-1)!} \sum_{j=0}^{m'-1} {m'-1 \choose j} \times \left(-\frac{K_1K_2}{\bar{\bar{\gamma}}}\right)^{m'-1-j} \Gamma\left(j-N'+1, \frac{K_1K_2}{\bar{\bar{\gamma}}}\right)$$
(A.14)

where  $\Gamma(.,.)$  is the incomplete gamma function. Substitute (A.14) back to (A.13) and (A.12) we obtain (3.12).

#### Proof of the asymptotic form:

The asymptotic form can be obtained when only considering the terms associated with the lower terms of *m* in the exact form. This is because the lower order of the PDF of  $\alpha_l$  determines the asymptotic performance when SNR is large.

### *Case 1:* N > 1

 $M(\bar{\gamma}, N', m')$  has an asymptotic form for large  $\bar{\gamma}$  [52]

$$M(\bar{\bar{\gamma}}, N', m') \doteq \begin{cases} & \frac{\ln\left(\frac{\bar{\bar{\gamma}}}{K_{1}K_{2}}\right)}{(m'-1)!\left(\frac{\bar{\gamma}}{K_{1}K_{2}}\right)^{m'}} & \text{if } m' = N'; \\ & \frac{(a-b-1)!}{(a-1)!\left(\frac{\bar{\gamma}}{K_{1}K_{2}}\right)^{b}} & \text{if } m' \neq N'; \end{cases}$$
(A.15)

where  $a = \max(m', N')$  and  $b = \min(m', N')$ . With m = 0 we have  $a = \max(m', N') =$ 

N+i,  $b = \min(m', N') = i+1$  and

$$F(\bar{\bar{\gamma}}, N', m') = (m'-1)! K_1^{-N'} K_2^{-m'} M(\bar{\bar{\gamma}}, N', m')$$

$$\stackrel{=}{=} \frac{(a-b-1)!}{(a-1)! \left(\frac{\bar{\bar{\gamma}}}{K_1 K_2}\right)^b} (m'-1)! K_1^{-N'} K_2^{-m'}$$

$$= \frac{(N-2)!}{(N+i-1)! \left(\frac{\bar{\gamma}}{K_1 K_2}\right)^{1+i}} i! K_1^{-N'} K_2^{-i-1'}$$

$$= \frac{(N-2)!}{(N+i-1)! \bar{\bar{\gamma}}^{i+1}} i! K_1^{1-N}$$
(A.16)

Substitute the asymptotic form of  $F(\bar{\gamma}, N', m')$  back to (A.12), we have the asymptotic form of  $G_l(\bar{\gamma})$  as

$$G_{l}(\bar{\bar{\gamma}}, m = 0) \doteq \sum_{i=0}^{\infty} \frac{D_{1}D_{3}^{i}}{i!} \bar{\bar{\gamma}}^{i} i! \frac{(N-2)!}{(N+i-1)!} K_{1}^{1-N}$$
$$= \sum_{i=0}^{\infty} D_{1}D_{3}^{i} \frac{(N-2)!}{(N+i-1)!\bar{\bar{\gamma}}} K_{1}^{1-N}$$
(A.17)

multiplying  $G_l(\bar{\bar{\gamma}},m=0)$  by  $\frac{D_3^{N-1}K_1^N\bar{\bar{\gamma}}}{(N-2)!D_1}$  yields

$$\frac{D_3^{N-1}K_1^N \bar{\tilde{\gamma}}}{(N-2)!D_1} G_l(\bar{\tilde{\gamma}}, m=0) \doteq \frac{\sum_{i=0}^{\infty} D_3^{N+i-1}}{(N+i-1)!}$$
$$= \sum_{j=N-1}^{\infty} \frac{D_3^j}{j!} = \sum_{j=0}^{\infty} \frac{D_3^j}{j!} - \sum_{j=0}^{N-2} \frac{D_3^j}{j!} = e^{D_3} - \sum_{j=0}^{N-2} \frac{D_3^j}{j!}$$
(A.18)

therefore,

$$G_{l}(\bar{\gamma}, m=0) \doteq \frac{\sum_{i=0}^{\infty} D_{3}^{N+i-1}}{(N+i-1)!} = \sum_{j=0}^{\infty} \frac{D_{3}^{j}}{j!} - \sum_{j=0}^{N-2} \frac{D_{3}^{j}}{j!}$$
$$= (N-2)! \left( e^{D_{3}} - \sum_{j=0}^{N-2} \frac{D_{3}^{j}}{j!} \right) D_{1} D_{3}^{-N+1} K_{1}^{1-N} \bar{\gamma}^{-1}.$$
(A.19)

For m > 0,  $M(\bar{\bar{\gamma}}, N', m') = o(\bar{\bar{\gamma}}^{-b}) \le o(\bar{\bar{\gamma}}^{-2})$ , the terms for m > 0 can be ignored

since  $G_l(\bar{\gamma}, m=0) = o(\bar{\gamma}^{-1}) >> o(\bar{\gamma}^{-2})$  for large SNR. Therefore we have

$$G_{l}(\bar{\tilde{\gamma}}) \doteq G_{l}(\bar{\tilde{\gamma}}, m = 0) \doteq \frac{\sum_{i=0}^{\infty} D_{3}^{N+i-1}}{(N+i-1)!} = \sum_{j=0}^{\infty} \frac{D_{3}^{j}}{j!} - \sum_{j=0}^{N-2} \frac{D_{3}^{j}}{j!}$$
$$= (N-2)! \left( e^{D_{3}} - \sum_{j=0}^{N-2} \frac{D_{3}^{j}}{j!} \right) D_{1} D_{3}^{-N+1} K_{1}^{1-N} \bar{\tilde{\gamma}}^{-1}.$$
(A.20)

*Case 2:* N = 1

With m = 0 since m' = N', we have a = b = i + 1 and

$$F(\bar{\bar{\gamma}}, N', m') \doteq \frac{\ln\left(\frac{\bar{\bar{\gamma}}}{K_1 K_2}\right)}{\left(\frac{\bar{\bar{\gamma}}}{K_1 K_2}\right)^{i+1}} K_1^{-i-1} K_2^{-i-1} = \frac{\ln\left(\frac{\bar{\bar{\gamma}}}{K_1 K_2}\right)}{\left(\bar{\bar{\gamma}}\right)^{i+1}}.$$
 (A.21)

substituting it back to  $G_l(\bar{\gamma}, m = 0)$  yields

$$G_{l}(\bar{\bar{\gamma}}, m = 0) = \sum_{i=0}^{\infty} \frac{D_{1} D_{3}^{i} r^{i}}{i!} F(\bar{\bar{\gamma}}, N', m')$$
  
$$\doteq D_{1} \ln(\bar{\bar{\gamma}}) \bar{\bar{\gamma}}^{-1} \sum_{i=0}^{\infty} \frac{D_{3}^{i}}{i!}$$
  
$$= D_{1} e^{D_{3}} (\ln \bar{\bar{\gamma}} - \ln(K_{1} K_{2})) \bar{\bar{\gamma}}^{-1}.$$
(A.22)

For m > 1, we have a = m + i + 1 and b = N + i = 1 + i and

$$G_{l}(\bar{\bar{\gamma}}, m > 1) = \sum_{m=0}^{\infty} \sum_{i=0}^{\infty} \frac{D_{1} D_{2}^{m} D_{3}^{i}}{i! (m!)^{2}} \bar{\bar{\gamma}}^{i} F(\bar{\bar{\gamma}}, N', m')$$
(A.23)

$$F(\bar{\bar{\gamma}}, N', m') = (m+i)! \frac{(a-b-1)!}{(a-1)! \left(\frac{\bar{\bar{\gamma}}}{K_1 K_2}\right)^b} K_1^{-b} K_2^{-a}$$
$$= m! K_2^{-m} \bar{\bar{\gamma}}^{-i-1}, \qquad (A.24)$$

substitute it back to  $G_l(\bar{\bar{\gamma}},m>1)$  we have

$$G_{l}(\bar{\bar{\gamma}}, m > 1) \doteq \sum_{m=1}^{\infty} \sum_{i=0}^{\infty} \frac{D_{1} D_{2}^{m} D_{3}^{i} \bar{\bar{\gamma}}^{i}}{i!(m!)^{2}} F(\bar{\bar{\gamma}}, N', m')$$
  
$$= \sum_{m=1}^{\infty} \bar{\bar{\gamma}}^{-1} K_{2}^{-m} D_{1} D_{2}^{m} \sum_{i=0}^{\infty} \frac{D_{3}^{i}}{i!}$$
  
$$= D_{1} (e^{K_{f}} - 1) e^{D_{3}} \bar{\bar{\gamma}}^{-1}.$$
(A.25)

Combining (A.22) and (A.25) yields

$$G_{l}(\bar{\bar{\gamma}}) \doteq D_{1}e^{D_{3}}\left(\ln(\bar{\bar{\gamma}}) - 1 + e^{K_{f}} - \ln(K_{1}K_{2})\right)\bar{\bar{\gamma}}^{-1}.$$
 (A.26)

## A.2.2 Derivations for Nakagami-m Fading

Let  $y = m_f \alpha_l$ , the MGF can be written as

$$G_{l}(\bar{\bar{\gamma}}) = \int_{y=0}^{\infty} \left(1 + \frac{\bar{\bar{\gamma}}}{m_{f}m_{b}}y\right)^{-m_{b}N} \times \frac{m_{f}^{m_{f}}}{\Gamma(m_{f})}y^{m_{f}-1}(m_{f})^{-m_{f}+1}\exp(-y)d\left(\frac{y}{m_{f}}\right)$$
$$= \int_{y=0}^{\infty} \left(1 + \frac{\bar{\bar{\gamma}}}{m_{f}m_{b}}y\right)^{-m_{b}N}\frac{y^{m_{f}-1}\exp(-y)}{\Gamma(m_{f})}dy, \qquad (A.27)$$

by using the result of (A.14), we obtain (3.18). The asymptotic form (3.19) can be obtained by using (A.15).