DETERMINATION OF THE REFRACTIVE INDEX OF THE BLACK SHEET FOR THE T2K EXPERIMENT

by

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ABSTRACT

“Black Sheet” is a thin layer of polyethylene terephthalate mixed with carbon. It is used as a light absorber surface in the inner structure of the Super-Kamiokande (SK) detector for the T2K experiment. To make a full event reconstruction in the SK detector, knowing the optical properties of the Black Sheet is required. Specifically, it is very important to know the complex refractive index as a function of the wave length of the incident light. In this thesis, the experimental results leading to the calculation of the refractive index of the Black Sheet for three different wave lengths are provided. The accuracy of the experiments is validated by testing two control samples. The results show that the real and imaginary parts of the refractive index of the control samples can be determined with tolerances of about 0.02 and 0.1 with 90% confidence level, respectively.
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DEDICATION

To my wonderful parents
Chapter 1. INTRODUCTION

T2K is a long base-line neutrino experiment mainly designed to find the last unknown oscillation parameters, $\theta_{13}$. To obtain this unknown, T2K searches for the $\nu_\mu$ disappearance and $\nu_e$ appearance from an initial $\nu_\mu$ beam after traveling 295 km. The essential components for this search are two powerful detectors, one at the beginning and the other at the end of the muon neutrino path. The one at the beginning is a detector complex located 280 m away from the muon neutrino production target and is called ND280. The detector at the end is a huge water Cherenkov detector (called Super-Kamiokande) and is located 295 km away from the production target. The Super-Kamiokande (SK) detector mainly consists of a very huge cylindrical stainless-steel tank (containing 50,000 tons of water), 13,000 photomultiplier tubes (PMTs), and electronics. The neutrinos that enter the SK detector interact with the nuclei of water and produce charged leptons that move with a speed greater than the speed of light in water and therefore, emit Cherenkov light. The created Cherenkov light is detected by the PMTs. Most of the PMTs are installed on a 70 cm grid on a support structure located 2.5 m inside the tank and away from its walls. The space in between the PMTs is covered with a thin layer of a plastic polymer called the Black Sheet. The Cherenkov light that hits the Black Sheet may be reflected, transmitted, or absorbed. These behaviours are required to be known as accurate as possible; otherwise, the accuracy of the detector simulation will be affected. As the performance of the detector and its capability of reconstruction and classification of neutrino interactions are negatively affected, the systematic uncertainties in the oscillation analysis increase.
The reflection coefficients for the incoming beam with a specific incident angle and polarization can be calculated with the Fresnel equations as a function of the refractive index of the Black Sheet. In the most general case, the refractive index is a complex number and is a function of the wave length of the incoming light. Knowing the refractive index as a function of wave length for an object with a perfectly smooth surface, the amount of the reflected light for a beam with any incident angle, wave length, and polarization can be exactly predicted.

The main objective of this thesis is to obtain the complex refractive index of the Black Sheet as a function of the wave length of the incoming light. In Chapter 2, a review of the neutrino oscillation phenomenon is provided. In Chapter 3 the T2K experiment and its main instruments except for the Super-Kamiokande detector are reviewed. The SK detector is described in Chapter 4. In Chapter 5, the theory of reflection, the generalized Fresnel equations for an object with a complex refractive index, the design of our experiment setup, and the final experimental results for two controls sample are provided. The final results of the refractive index of the Black Sheet for three different wave lengths are presented in Chapter 6. The conclusive remarks and suggested future work of this thesis are described in Chapter 7.
Chapter 2. **NEUTRINO OSCILLATION**

2.1. **MASSIVE NEUTRINO**

Neutrinos represent six of the 61 elementary particles including 12 leptons, 36 quarks, 12 mediators, and one Higgs particle. They were first postulated by Pauli in 1930 [1] in order to solve the problem of missing energy in the energy spectrum of the electrons produced in a beta decay. Neutrinos belong to the lepton family. They possess no electric charge and have a negligible mass; therefore, they are only capable of interacting with other particles through the weak interactions. Neutrinos’ weak interactions are divided to two categories: Charged Current (CC) and Neutral Current (NC) interactions. In the CC interactions, neutrinos interact by exchanging a $W^\pm$ boson leading to the production of the corresponding charged lepton. In the NC interactions, they interact by exchanging a $Z^0$ boson which leads to the production of uncharged particles. Neutrinos’ CC and NC weak interaction vertices are shown in Figure 2-1. Until today, three types of neutrinos have been found. They are named as $\nu_e$, $\nu_\mu$, and $\nu_\tau$, which is based on the lepton they are associated with in the Charged Current (CC) weak interaction processes. For example, $\nu_e$ is the neutrino which is associated with an electron in the CC interactions. In all experiments carried out so far, all the flavour neutrinos, produced by the weak interactions, have been found to be left-handed (LH) and no right-handed (RH) flavour neutrino has been observed yet. This means that if RH neutrinos exist, they are much weaker than LH neutrinos in the weak interactions. Since the RH neutrinos do not interact at all in the Standard Model (SM), they are often left out. The LH neutrinos ($\nu_{LL}(x)$) are described by SU(2)$_L$ doublet with their corresponding leptons in the SM.
Neutrinos are massless particles in the SM; however, there is no obligation on this assumption and treating neutrinos as being massive does not have any contradiction with the SM. During the last decade, there have been very strong evidences that neutrinos are massive and their masses differ from one type to another. The first evidence of the massiveness of neutrinos was manifested by the problem of missing solar neutrinos. This was found when Raymond Davis Jr. and John Bahcall in 1964 tried to understand the reactions responsible for the creation of sunlight. Bahcall provided the theory and calculations while Davis’s group performed the experiments to measure the flux of the solar electron neutrinos on the earth. The experiment was conducted in the Homestake gold mine, South Dakota, USA. In the experiment, the electron neutrinos of different energies coming from the sun entered a tank filled with a chlorine-base fluid (C₂Cl₄) and produced radiative argon atoms (³⁷Ar). The number of these argon atoms was determined both experimentally and theoretically in order to find the number of neutrinos of different energies produced by the sun. Their first experimental results were released four years later in 1968 [2]. The experiment detected about one-third of the electron neutrinos that was predicted theoretically. To solve the mystery of missing neutrinos, three explanations were proposed at that time. First, there was a probability of a huge mistake in the theoretical calculations; second, there was a probability of an enormous mistake in the experimental procedure; and third, there was a probability of existence of some new unknown behaviour of
neutrinos during their passage from the sun to the earth. The latter guess was proposed by Bruno Pontecrovo in 1969 [3]. He suggested that if a specific type of neutrino travels a sufficiently long path, its type can change to another. This phenomenon is called “neutrino oscillation”. Neutrino oscillation can be explained within the SM; however, for the complete description of this phenomenon in the SM, neutrinos must have a non-zero mass. Otherwise, the probability of oscillation will be zero.

Since the Pontecrovo’s suggestion until now, many other experiments have been conducted to study the behaviour of neutrinos during their travel in long paths. As a result of the last two decades experiments, there are now strong evidences in hand that neutrino oscillations happen when they travel long enough distances. This phenomenon is explained comprehensively in the following sub-section.

### 2.2. Neutrino Oscillation

Similar to the observation of John Bahcall regarding the mystery of the missing solar neutrinos, many other experiments have observed a non-zero probability of transition between the flavour neutrinos. This phenomenon is known as neutrino oscillation and happens as a result of flavour mixing and massiveness of neutrinos.

To further clarify the neutrino oscillation phenomenon, consider a case in which a beam of a specific flavour neutrino (e.g. $\nu_e$) leaves a source and is to be detected with a detector capable of distinguishing all three types of flavour neutrinos. If the neutrinos travel a significantly long
path, at the destination the detector may detect other flavour neutrinos and the flux of the original flavour neutrino (here $\nu_e$) may be found to be less than its initial value. This implies that there is a non-zero probability of transition between different flavour neutrinos.

Having both non-degenerate masses and non-trivial mixing are the requirements for the neutrino oscillation to occur. Mixing between different flavour neutrinos is very similar to the mixing of quarks. Neutrino mixing is a consequence of the difference between flavour neutrinos, $\nu_l(x)$, i.e. the eigenstate terms of the weak interactions in the Lagrangian and the massive neutrinos, $\nu_j(x)$, i.e. the eigenstate terms of the mass terms in the Lagrangian. In the presence of mixing, LH flavour neutrino fields, $\nu_{lL}(x)$, can be written as a linear combination of the massive neutrino fields, $\nu_{jL}(x)$:

$$\nu_{lL}(x) = \sum_j U_{lj} \nu_{jL}(x) \quad , \quad l = e, \mu, \tau \quad j = 1, 2, 3$$

(2-1)

In the above formula, $U$ is the neutrino mixing matrix and is a unitary matrix that connects the flavour neutrinos to the massive neutrinos. This mixing matrix is similar to the quark mixing matrix (CKM) and is called the Pontecrovo-Maki-Nakagawa-Sakata (PMNS) mixing matrix.

Neutrinos have no electric charge; therefore, they can be either Dirac or Majorana particles [4]. If they are Dirac fermions, they will have distinctive anti-particles, but if they are Majorana fermions, they will be identical to their anti-particles. In principle, Majorana particles can be distinguished from Dirac particles by searching for the neutrino-less double beta decay. This
decay arises only if neutrino is a Majorana particle [5]. From the data in hand so far it is not possible to determine the neutrinos are whether Dirac or Majorana particles.

If there are n flavour neutrinos and n massive neutrinos, the resulting $n \times n$ PMNS matrix can be described by $n^2$ parameters. These parameters can be divided into $n(n-1)/2$ Euler angles and $n(n+1)/2$ phases. If the massive neutrinos are Dirac fermions, independent phases will be reduced to $(n-1)(n-2)/2$. But if they are Majorana fermions, there will be $n(n-1)/2$ independent phases in the PMNS matrix. Phases are responsible for CP violation, which occurs when the PMNS matrix is complex.

In the three neutrinos case, in general there will be three Euler angles and three CP violation phases (one Dirac phase and two Majorana phases). In this cases, the PMNS matrix can be described by a $3 \times 3$ unitary matrix.

$$
U = \begin{pmatrix}
C_{13}C_{12} & C_{13}S_{12} & S_{13}e^{i\delta} \\
-C_{23}S_{12} - S_{13}C_{12}C_{23}e^{i\delta} & C_{23}C_{12} - S_{13}S_{12}S_{23}e^{i\delta} & C_{13}S_{23} \\
S_{23}C_{12} - S_{13}C_{12}C_{23}e^{i\delta} & -S_{23}C_{12} - S_{13}S_{12}C_{23}e^{i\delta} & C_{13}C_{23}
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i\alpha} & 0 \\
0 & 0 & e^{i\beta}
\end{pmatrix}
$$

where $C_{ij} = \cos(\theta_{ij})$, $S_{ij} = \sin(\theta_{ij})$, $\delta$ is the CP violation Dirac phase, and $\alpha$ & $\beta$ are the CP violation Majorana phases which do not have any consequences for the neutrino oscillations.

To derive the oscillation probability expressions, a case in which a flavour neutrino, $\nu_{IL}$, produced by the CC weak interactions, travels a long distance, L, toward the detector is
considered. The dimensions are given in Plank units (i.e. \(\hbar = c = 1\)). Let’s also assume the travel between the source and detector takes time \(T\) and the evolution of neutrino state, \(|\nu\rangle\), is considered in the lab frame (detector’s rest frame). As a result of lepton mixing, neutrino flavour states, \(|\nu_l\rangle\), can be expressed as a coherent superposition of the massive states, \(|\nu_j\rangle\):

\[
|\nu_l\rangle = \sum_j U_{lj} |\nu_j\rangle, \; l = e, \mu, \tau \quad j = 1, 2, 3
\]  

(2-3)

where \(\tilde{p}_j\) is the 4-momentum of the massive neutrino, \(\nu_j\). If the flavour neutrino travels a distance \(L\) in the time interval \(T\) in the lab frame, the amplitude of the probability that another flavour neutrino, \(\nu_{l'}\), will be detected can be written as:

\[
A(\nu_l \rightarrow \nu_{l'}) = \langle \nu_{l'} | D_l | \nu_l \rangle
\]  

(2-4)

where \(D_l\) describes the time evolution of the flavour neutrino. Using equation (2-4) along with equation (2-3), the amplitude of probability can be written as follows:

\[
A(\nu_l \rightarrow \nu_{l'}) = \sum_j U_{lj}^* D_j U_{l'j}, \; l,l' = e, \mu, \tau \quad j = 1, 2, 3
\]  

(2-5)

where \(D_j\) describes the time evolution of the massive neutrino between the source and detector. Therefore, \(D_j\) can be written as below [6]:

\[
D_j \equiv D_j(\tilde{p}_j; L, T) = - e^{-i(E_j T - \tilde{p}_j L)}
\]  

(2-6)

where \(E_j = \sqrt{p_j^2 + m_j^2}\) and \(p_j = |\tilde{p}_j|\). In order to calculate the flavour neutrinos transition probability, \(|A(\nu_l \rightarrow \nu_{l'})|^2\), only the interference factor, \(D_j D_j^*\), is important. This interference factor can be expressed as:
The neutrinos are relativistic; therefore, \( L \equiv T \). Considering this fact, the first term on the right-hand side of equation (2-7) will be negligible. Therefore, the interference factor can be expressed as follows:

\[
D_j D_k^* = e^{-i \frac{(m_j^2 - m_k^2)}{2p} L} = e^{-i \frac{\Delta m_{jk}^2}{2p} L} \quad (2-8)
\]

where \( \Delta m_{jk}^2 = m_j^2 - m_k^2 \). From equations (2-4), (2-6), and (2-8), the probability of oscillation can be found as follows [7]:

\[
P(\nu_i \rightarrow \nu_f) = \sum_j |U_{ij}|^2 |U_{kj}|^2 + 2 \sum_{j > k} |U_{ij} U_{kj}^* U_{ik} U_{jk}^*| \cos \left( \frac{\Delta m_{jk}^2}{2p} L - \phi_{tjk} \right) \quad (2-9)
\]

where \( \phi_{tjk} = \arg(U_{ij} U_{kj}^* U_{kj} U_{ij}^*) \). Also, it can be shown that the probability of oscillation of antineutrinos can be given by [7]:

\[
P(\bar{\nu}_i \rightarrow \bar{\nu}_f) = \sum_j |U_{ij}|^2 |U_{kj}|^2 + 2 \sum_{j > k} |U_{ij} U_{kj}^* U_{ik} U_{jk}^*| \cos \left( \frac{\Delta m_{jk}^2}{2p} L + \phi_{tjk} \right) \quad (2-10)
\]

From equation (2-9), it can be seen that the neutrino oscillation effect is only strong when:

\[
\left( \frac{\Delta m_{jk}^2}{2p} \right) L \geq 1 \quad (2-11)
\]

given \( j \neq j' \) for at least one \( \Delta m_{jk}^2 \).
From equations (2-9) and (2-10), it can be observed that CP invariance, \( P(\nu_i \to \nu_f) = P(\bar{\nu}_i \to \bar{\nu}_f) \), only holds if \( \varphi_{\nu_{ij}} = n\pi \), \( n = 0, 1, 2, \ldots \) i.e. if \( U \) is real. Also, it can be shown that the oscillation probabilities, \( P(\nu_i \to \nu_f) \) and \( P(\bar{\nu}_i \to \bar{\nu}_f) \), do not depend on the Majorana phases in the PMNS matrix [7, 8]. Therefore, only the Dirac phase in the PMNS matrix is responsible for the possible CP violation, i.e. \( P(\nu_i \to \nu_f) \neq P(\bar{\nu}_i \to \bar{\nu}_f) \).

If the number of massive neutrinos and flavour neutrinos is the same, the probability conservation equation can be written as follows:

\[
\sum_{l=e,\mu,\tau} P(\nu_i \to \nu_f) = 1, \quad l = e, \mu, \tau
\]  

(2-12)

Similar equations hold for \( P(\nu_i \to \nu_f), \ P(\bar{\nu}_i \to \bar{\nu}_f), \) and \( P(\bar{\nu}_i \to \bar{\nu}_f) \). However, there is a hypothesis about the possibility of the existence of an extra neutrino, called the sterile neutrino, that does not interact via the weak interaction and hence, has not ever been observed. If one sterile neutrino, \( \nu_{sL} \), exists, then the above conservation equations will change to

\[
\sum_{l=e,\mu,\tau} P(\nu_i \to \nu_f) < 1 \text{ if } P(\nu_i \to \nu_{sL}) \neq 0.
\]

In the case of three neutrinos mixing, two of the \( \Delta m^2_{ij} \) are independent and they relate to the third one with this equation: \( \Delta m^2_{12} + \Delta m^2_{23} + \Delta m^2_{31} = 0 \). Let’s assume a case in which one mass squared difference terms, \( \Delta m^2_{n1} \), is dominant. This assumption is made because it is understood from the experimental data that one of the two independent mass squared differences is much
smaller that the other one and consequently is negligible compared to the first one. The values of
\( \Delta m_{jj'}^2 \) suggested by the experimental data are [7]:

\[
\begin{align*}
|\Delta m_{21}^2| & \equiv 7.6 \times 10^{-5} \text{ eV}^2 \\
|\Delta m_{31}^2| & \equiv 2.4 \times 10^{-3} \text{ eV}^2
\end{align*}
\]  \tag{2-13}

In this case, from equations (2-9) and (2-10), by keeping only the terms that include \( \Delta m_{n1}^2 \), the following equation can be obtained:

\[
P(\nu_{jj'} \rightarrow \nu_{jj''}) \equiv P(\overline{\nu}_{jj'} \rightarrow \overline{\nu}_{jj''}) \equiv \delta_{jj'} - 2 \left| U_{ln} \right|^2 \left[ \delta_{jj'} - \left| U_{rn} \right|^2 \right] \left( 1 - \cos \left( \frac{\Delta m_{n1}^2}{2p} L \right) \right)
\]  \tag{2-14}

By neglecting the effect of term \( \Delta m_{12}^2 \), from equation (2-14), the following equations can be obtained [7, 9]:

\[
P(\nu_e \rightarrow \nu_e) \equiv P(\overline{\nu}_e \rightarrow \overline{\nu}_e) \equiv 1 - \frac{1}{2} \sin^2(2\theta_{13}) \left( 1 - \cos \left( \frac{\Delta m_{31}^2}{2p} L \right) \right)
\]  \tag{2-15}

\[
P(\nu_{\mu (e)} \rightarrow \nu_{e (\mu)}) \equiv \frac{1}{2} \sin^2(\theta_{23}) \sin^2(2\theta_{13}) \left( 1 - \cos \left( \frac{\Delta m_{31}^2}{2p} L \right) \right)
\]  \tag{2-16}

From equations (2-15) and (2-16), it can be observed that the oscillation probabilities depend on two terms: \( \left( \Delta m_{jj'}^2 / 2p \right) L \) (which itself depends on the distance and the neutrino energy) and \( \theta_{jj'} \).

As a result, the experiments looking for the neutrino oscillation are specified by the neutrinos average energy and the distance between the source and detector. From equation (2-11), the minimum value of \( \Delta m_{jj'}^2 \), to which the experiment is sensitive, can be determined:
\[
\min \left( \Delta m_{jj}^2 \right) = 2 \text{p/L}. \text{ The sensitivity of different neutrino oscillation experiments to the minimum value of } \Delta m_{jj}^2 \text{ is provided in Table 2-1.}
\]

Table 2-1: Sensitivity of different neutrino oscillation experiments to the minimum values of \( \Delta m_{jj}^2 \)

(Reproduced from [7]).

<table>
<thead>
<tr>
<th>Source</th>
<th>Type of neutrino</th>
<th>Average energy [MeV]</th>
<th>Source–Detector distance [km]</th>
<th>Min(( \Delta m^2 )) [eV^2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactor</td>
<td>( \bar{\nu}_e )</td>
<td>( \sim 1 )</td>
<td>1</td>
<td>( \sim 10^{-3} )</td>
</tr>
<tr>
<td>Reactor</td>
<td>( \bar{\nu}_e )</td>
<td>( \sim 1 )</td>
<td>10^2</td>
<td>( \sim 10^{-5} )</td>
</tr>
<tr>
<td>Accelerator</td>
<td>( \nu_\mu, \bar{\nu}_\mu )</td>
<td>( \sim 10^3 )</td>
<td>1</td>
<td>( \sim 1 )</td>
</tr>
<tr>
<td>Accelerator</td>
<td>( \nu_\mu, \bar{\nu}_\mu )</td>
<td>( \sim 10^3 )</td>
<td>10^3</td>
<td>( \sim 10^{-3} )</td>
</tr>
<tr>
<td>Atmospheric ( \nu )'s</td>
<td>( \nu_{\mu,e}, \bar{\nu}_{\mu,e} )</td>
<td>( \sim 10^3 )</td>
<td>10^4</td>
<td>( \sim 10^{-4} )</td>
</tr>
<tr>
<td>Sun</td>
<td>( \nu_e )</td>
<td>( \sim 1 )</td>
<td>1.5 \times 10^8</td>
<td>( \sim 10^{11} )</td>
</tr>
</tbody>
</table>

2.3. NEUTRINO OSCILLATION EXPERIMENTS

Neutrino oscillation experiments can generally be categorized into four groups depending on the source of the neutrino used in the experiment: (1) solar, (2) atmospheric, (3) accelerator, and (4) reactor. Up to present, a large number of neutrino experiments have been conducted and a comprehensive list of them is provided below (and each is ordered alphabetically):

1. Solar neutrino experiments:

   BOREXino, GALLEX, GNO, Homestake, KamLAND, SAGE, SNO, Super-Kamiokande.

2. Atmospheric neutrino experiments:

   Baksan, Frejus, IMB, Kamiokande, MACRO, MINOS, NUSEX, Soudan2, Super-Kamiokande.
3. Accelerator neutrino experiments:
   a. Short base-line (SBL):
      BEBC, BNL-E-776, BNL-E-816, CCFR, CDHSW, CERN-PS-191, CHARM, CHORUS,
      FNAL-E-0053, FNAL-E-0531, FNAL-E-0613, Gargamelle, IHEP-JINR, KARMEN,
      LAMPF-0645, LAMPF-0764, LSND, MiniBooNE, NOMAD, NuTeV, SciBooNE, SKAT.
   b. Long base-line (LBL):
      ICARUS, K2K, MINOS, OPERA, T2K.

4. Reactor neutrino experiments:
   a. Short base-line (SBL):
      Bugey, Gosgen, ILL, Krasnoyarsk, Rovno, Savannah River.
   b. Long base-line (LBL):
      CHooze, Double-CHooze, KamLAND, Palo-Verde.

More information regarding the above experiments can be found in [10].

Each of the above experiments deals with a specific type of neutrino, average neutrino
energy, and source-detector distance, as given in Table 2-1. Therefore, each neutrino
oscillation experiment is sensitive to a particular parameter of the probability equation, (2-15)
and (2-16), and the parameter could be different going from one experiment to another.
Correspondingly, all the above experiments are used in to determine different oscillation
parameters [7]. Acceptable regions for the neutrino oscillations parameters in some of the most important previous experiments along with their results are shown in Figure 2-2.

Figure 2-2. The regions of mass squared difference and mixing angle accepted or forbidden by the most important neutrino experiments.
Chapter 3. **T2K EXPERIMENT**

3.1. **INTRODUCTION**

The T2K (Tokai-to-Kamioka) experiment is a long base-line neutrino oscillation experiment [11]. It is mainly designed to measure the last unknown lepton sector mixing angle, $\theta_{13}$. This measurement is going to be done [11]by looking at electron neutrino appearance in a muon neutrino beam. T2K is designed to determine $\theta_{13}$ with a factor of 20 better sensitivity compared with its present best limit (which belongs to the CHOOZ experiment [12]). With the assumption of a zero CP violation phase and normal mass hierarchy, the precision of the T2K experiment for $\theta_{13}$ will be $\sin^2(2\theta_{13}) > 0.008$ at 90% confidence level. The T2K other goals include obtaining a more precise measurement values for the known oscillation parameters, $\Delta m^2_{23}$ values, and $\sin^2(2\theta_{23})$. This is going to be achieved by observing the muon neutrino disappearance (with the precision of $\delta(\Delta m^2_{23}) = 10^{-4}$ eV$^2$ and $\delta(\sin^2(2\theta_{23})) = 0.01$).

The experiment’s main components are: one neutrino beam line, one near detector complex (ND280), and one far detector (Super-Kamiokande) sited at 295 km from J-PARC (see Figure 2-2).

![Figure 3-1. Neutrino’s journey from J-PARC, through the near detector (ND280), to Super-Kamiokande (SK) detector located 295 km away from the production target.](image)
In order to produce the muon neutrino beam, T2K uses the J-PARC accelerator (Tokai-Japan). J-PARC produces 30 GeV protons which will later hit a target to create pions. Those pions will eventually decay into muons and muon neutrinos. The muon neutrino beam is intentionally generated off-axis [13]. The off-axis angle is set at 2.5 degrees which results in the muon neutrino beam having a peak energy of about 0.6 GeV. This value of peak energy maximizes the neutrino oscillation effect for a 295 km distance between the beam source and the far detector [11]. The muon neutrino beam, after travelling 280 meters from the production target, hits the ND280. The ND280 includes both on-axis and off-axis detectors to sample the beam at the beginning of its passage before oscillation. The on-axis detector (INGRID) is used to measure the direction and profile of the muon neutrino beam. The off-axis detectors (POD, TPC, FGD, ECal, and SMRD) measure the energy spectrum and flux of the muon neutrinos. They also measure the primary electron neutrino contamination in the muon neutrino beam which is very important for the characterization of signals and backgrounds reported from the far detector. After the detection at ND280, the muon neutrino beam travels a 295 km path toward the Super-Kamiokande (SK) detector. The SK is a very large water Cherenkov detector with well developed capability to identify muon and electron neutrinos. Its main components are a cylindrical stainless-steel tank filled with water (30 m in diameter and 42 m high with a capacity of 50,000 tons of water) and 13,000 photomultiplier tubes (PMTs).

A description of the J-PARC accelerator, neutrino beam line, and ND280 complex is presented in the following sections of this chapter. A detailed description of the Super-Kamiokande (SK) detector is provided in Chapter 4.
3.2. J-PARC ACCELERATOR

J-PARC, the newly constructed synchrotron, is located at Tokai, Ibaraki (look at Figure 3-2). It is composed of a linear accelerator (LINAC), a rapid-cycling synchrotron (RCS), and the main ring (MR).

![Figure 3-2. Aerial view of J-PARC, Tokai-mura.](image)

In order to produce the 30 GeV protons, first an $H^-$ beam is sent to the LINAC and is accelerated up to 400 MeV. At the RCS injection, the $H^-$ beam is converted to an $H^+$ beam by the use of charge stripping foils. Then, it is accelerated up to 3 GeV. Finally, protons are injected into the MR and accelerated up to 30 GeV, and are extracted toward the neutrino beam line.

3.3. NEUTRINO BEAM LINE

The neutrino beam line includes one primary and one secondary beam line. The primary and secondary beamlines are in tandem, so that one leads directly into the next. In the primary beam
line, the 30 GeV protons (eight circulating proton bunches) that are extracted from the MR are transported and bent directly toward Kamioka. Then, the proton beam continues its path in the secondary beam line where it hits a target to produce the secondary pions. The produced pions are focused by the use of magnetic horns and eventually decay into neutrinos. Figure 3-3 schematically shows the neutrino beam line.

![Figure 3-3](image)

Figure 3-3. An overview of the neutrino beam line.

3.3.1. Primary neutrino beam line

The primary neutrino beam line consists of three sections: preparation, arc, and final focusing (see Figure 3-3).

The preparation section is 54 m long and adjusts the proton beam with an array of 11 normal conducting magnets that direct the beam to the arc section. In the arc section, which is 147 m, the beam is bent toward Kamioka with the use of 14 doublets super-conducting magnets [14, 15]. In
the final focusing section, the proton beam is guided and focused onto the target with the aid of 10 normal conducting magnets.

In order to have high targeting efficiency, it is important to very carefully monitor the beam losses. In order to verify the intensity, position, profile, and loss of the proton beam, 95 monitors are used in the primary section. These monitors include five current transformers (CTs) to monitor the beam intensity, 21 electrostatic monitors (ESMs) to observe the beam position, 19 segmented secondary emission monitors (SSEMs) to monitor the beam profile, and 50 beam loss monitors (BLMs). Figure 3-4 schematically illustrates the primary beam line monitors.

![Figure 3-4](image)

Figure 3-4. The primary beam line and location of monitors.

3.3.2. Secondary neutrino beam line

The secondary neutrino beam line consists of three sections: target station, decay volume, and beam dump (see Figure 3-5).
The target station consists of a baffle, an optical transition radiation monitor (OTRM), the target, and three magnetic horns. The baffle is a collimator used to protect the horns. The OTRM is used to monitor the profile of the proton beam just before hitting the target. As the proton beam impinges on the target, pions are produced and focused by the three magnetic horns which produce a toroidal magnetic field with an axis along the beam direction. Then the pions enter the decay volume and decay mostly into muons and muon neutrinos.

The decay volume is a 96 m long steel tunnel filled with helium gas (used to decrease the pion absorption). At the end of the decay volume, there is a beam dump. This dump is composed of a core made of 75 tonnes of graphite contained in a helium vessel. The vessel is covered with 15 iron plates on the outside and two plates at the downstream end of the core on the inside. The beam dump absorbs everything except for muons with energies greater than 5 GeV/c and muon neutrinos. The muons that pass through the beam dump are monitored to characterize the neutrino beam.
3.3.3. Muon monitor

The muon monitor is placed behind the beam dump and is used to determine the intensity and direction of the neutrino beam [16, 17]. Using the measured muon distribution profile as well as considering the fact that muons are mostly produced together with neutrinos from the pion two-body-decay, the neutrino beam direction can be determined. The neutrino beam direction is determined with the precision of 0.25 mrad with the muon monitor. In order to measure the absolute flux and momentum distribution of the muon beam, a nuclear emulsion detector located right after the muon monitor is used. The emulsion is a temporary device that is inserted for the special runs and is removed during the normal runs.

3.4. Near detector (ND280)

To calculate the probabilities of transition, the energy spectrum and interaction rates of neutrinos before oscillation must be known. To achieve this goal, a detector complex called ND280 is placed 280 meters away from the production target.

All the ND280 detectors are located in a pit (17.5 m in diameter and 37 m deep) that has three floors (see Figure 3-6). The top floor which is 24 m below the surface houses the off-axis detectors including a pi-zero detector (PØD), two fine grained detectors (FGDs), and three time projection chambers (TPCs). These detectors are contained within an electromagnet that provides a 0.2 T dipole magnetic field, and are surrounded by an electro-magnetic calorimeter (ECal), as shown in Figure 3-7. The complex of the off-axis detectors is in the direction of a line connecting the target to the SK detector. The second floor, which is 33 m below the surface,
houses the horizontal modules of an array of iron/scintillator detectors. This array of detectors is called INGRID detector. The second floor also houses the electronics of the off-axis detectors. The bottom floor, which is 37 m below the surface, houses the vertical modules of the INGRID detector. The pit is covered with a building with an internal area of 21 m × 28 m; inside the building, there is a 10 ton crane.

Figure 3-6. The near detector complex (ND280). The off-axis detectors are located on the top floor; horizontal modules of INGRID are located on the second floor; and the vertical module of INGRID sits on the bottom floor.

Figure 3-7. The T2K off-axis detectors.
3.4.1. INGRID detector

INGRID (Interactive Neutrino GRID) is an array of iron/scintillator detectors centred on the neutrino beam axis. It is used to monitor the on-axis neutrino beam profile, direction, and intensity by measuring the neutrino interactions in iron.

INGRID is made of 16 modules. 14 identical modules are installed in a cross-shaped arrangement and two other modules are installed outside of the cross at off-axis direction (see Figure 3-8). Using this arrangement, INGRID samples the beam in a transverse section of 10 m × 10 m. There are two over-lapping modules at the centre of the INGRID. The center is at zero degree with respect to the direction of the initial proton beam.

Figure 3-8. The T2K on-axis detector; INGRID.

Each module is made of nine iron plates placed alternatively in between 11 scintillator planes (as depicted in Figure 3-9). There is no iron plate between the 10th and 11th scintillator planes because of the modules weight restriction. The iron scintillator sandwich is covered by veto scintillator planes. The veto planes are made of 22 scintillator bars and are used to refuse the
interactions outside of the module. The iron plates are 124 cm wide and 124 cm long and 6.5 cm thick. The entire nine iron plates have a total weight of 7.1 tonnes. Each of the tracking scintillator planes is made of two parallel plates. Each plate is composed of 24 scintillator bars. The bars on each plate are parallel to each other and perpendicular to the bars of the other plate. These two plates are both perpendicular to the beam direction. In total, there are 8448 \(= 24 \times 2 \times 11 \times 16\) scintillator bars in the INGRID. The dimension of each scintillator bar is 10 cm \(\times\) 50 cm \(\times\) 120.3 cm. The Wavelength shifting (WLS) fibers are Kurarary double-clad Y-11 1 mm in diameter with emission and absorption spectrums centered at 467 nm and 430 nm, respectively. One end of each fiber is glued to a connector and the polished surface of the connector is attached to a Hamamatsu Multi-Pixel Photon Counter (MPPC) with a sensitive area of 1.3 mm \(\times\) 1.3 mm and an operational voltage about 70 V. The MPPCs convert the light into an electrical signal which can be processed to determine the time and amplitude of the light pulse.

Figure 3-9. A module of INGRID. The blue planes in the left-hand side image show the tracking planes. The black planes in the right-hand side image show the veto planes.

A typical neutrino event in the INGRID module is illustrated in Figure 3-10.
Figure 3-10. A neutrino event in the INGRID module. The green cells, blue cells, and grey cells show the tracking scintillators, the veto scintillators, and the iron target plates, respectively. The red circles show the track of a charged particle produced by the incoming neutrino.

3.4.2. UA1 magnet

The off-axis sub-detectors are contained inside the old CERN UA1/NOMAD magnet (as shown in Figure 3-7). The UA1 magnet provides a 0.2 T dipole magnetic field. This magnetic field is required for the measurement of the momenta and identification of the sign of the charged particles created by the neutrino interactions. The UA1 magnet consists of water-cooled aluminium coils and a 850 tons flux return yoke. The coils produce a horizontal dipole field and are made of rectangular aluminium bars with a cross-section of 5.45 cm $\times$ 5.45 cm with a 23 mm diameter hole in their center for the flow of water. The inner and outer dimensions of the UA1 magnet are 7.0 m $\times$ 3.5 m $\times$ 3.6 m and 7.6 m $\times$ 5.6 m $\times$ 6.1 m, respectively. The UA1 magnet consists of two identical and independent halves. Each half consists of two coils that are mechanically supported by the yoke. Each of the two yokes consists of eight C-shaped parts each made of 16 low-carbon steel plates. These parts together form a ring that covers the four sides of the inner detectors.
3.4.3. **Pi-zero detector (PØD)**

The main function of the pi-zero detector (PØD) is to measure the neutral current single $\pi^0$ production. The $\pi^0$ events are the dominant sources of backgrounds to the $\nu_e$ appearance search at the SK detector. PØD measures the neutral current event on a water target similar to the filling matter of the SK detector.

The main structure of the PØD consists of sandwiches of scintillator planes, fillable water bags, and lead & brass sheets. To find the water target cross-section, the PØD is run with and without the water in bags. The PØD is divided into two central sections (upstream and central water targets) and the upstream and downstream of the central sections (upstream and central ECals); this is shown in Figure 3-11. The central sections consist of alternating scintillator planes, water bags, and brass sheets. The side sections consist of alternating scintillator planes and lead sheets. The arrangement of the Upstream ECal yields a veto region before and after the central water target area. This veto region rejects particles coming from the interactions upstream the PØD.

![Figure 3-11. The side view of the pi-zero detector (PØD).](image)

Figure 3-11. The side view of the pi-zero detector (PØD).
There are 40 scintillator modules in the PØD: 14 in the upstream and central ECAL and 26 in the central sections. Each scintillator module consists of two perpendicular arrangements of triangular scintillator bars. There are in total 260 scintillator bars in each module: 134 vertical and 126 horizontal. The WLS fibers pass through the scintillator bars and one end of each fiber is connected to a Hamamatsu MPPC. The PØD active target is a cube with dimensions 2103 mm × 2239 mm × 2400 mm in width, height, and length, respectively. The total mass of the PØD filled or empty is 16.1 or 13.3 ton, respectively.

3.4.4. Time projection chambers (TPCs)

The TPCs fulfill three key functions. First, they are superior 3D imaging devices, capable of determining the number and orientation of the charged particles passing through them accurately. This is in contrast to the scintillator detectors where two 2D measurements must be combined to obtain a 3D track that might result in ambiguities in which XZ information gets matched with which YZ information. Second, TPC is unique because of its low density, which allows the track to be measured with minimal multiple scattering; this allows measuring the sing and momenta of the charged particles that are produced by the neutrino interactions elsewhere in the detector. With the aid of this measurement, the event rate before oscillation can be obtained as a function of neutrino energy. Third, they can measure the ionization left by each particle. This measurement together with the measured momentum yields to the identification of different types of charged particles and finally, the relative amount of electron neutrinos in the beam is found.
There are three TPCs in the ND280 complex. Each TPC consists of two boxes (one completely contained in the other). The inner box, made from composite panels with copper–clad G10 cover, holds an argon-based drift gas. This box together with a central cathode panel provides a uniform electric drift field cage. The outer box, made from composite panels with aluminium cover, hold CO₂ insulating gas. There are two readout panels at the end of the TPC (see Figure 3-12).

![Figure 3-12. A schematic figure of the TPC detector.](image)

When the charged particles pass through the TPCs, they produce ionization electrons in the inner box. These electrons are drifted away from the central cathode toward the readout panels. At the readout panels the ionization electrons are multiplied and captured by micromegas detectors. There are 12 micromegas modules at each end of the TPC. In total, 72 ( = 12×2×3) modules are used in the three TPCs. The dimensions of each TPC are 2.3 m × 2.4 m × 1 m; each TPC has about 3 m² active surface. The signal pattern together with the arrival time of the signals provides the 3D image of the path of the incoming charged particle.
3.4.5. Fine grained detectors (FGDs)

In addition to the TPCs, there are two FGDs used for tracking the charged particles. The FGDs also provide the neutrino interaction’s target mass. The two FGDs are placed alternatively in between the three TPCs.

The two FGDs are different from each other. The upstream FGD consists of 30 layers of scintillator planes. Each of these planes is made of 192 scintillator bars (as can be seen in Figure 3-13). The scintillator planes are oriented alternatively either horizontally or vertically (all perpendicular to the beam direction). The scintillator bars are made from polystyrene and their dimensions are 9.61 mm \times 9.61 mm \times 1864.3 mm (representing in width, length, and height, respectively) with a reflective coating that contains TiO$_2$. Along the length in the middle of each bar is a hole and a wavelength shifting (WLS) fibre passes through that hole. One end of each fiber is connected to an MPPC. In total, there are 5760 (= 30 \times 192) scintillator bars in the first FGD.

![Figure 3-13. An FGD, without the front cover. The green plate shows the scintillator modules.](image)
The downstream FGD consists of alternating seven double scintillator planes and six layers of water. The water layers are made from polycarbonate sheets and are 2.5 cm thick. Each double scintillator plane consists of a plane of vertical scintillator bars attached to a plane of horizontal scintillator bars; both planes are perpendicular to the beam direction. In total, there are 2688 ( = 2 × 7 × 192) scintillator bars in the second FGD. The outer dimensions of both FGDs are 2300 mm × 2400 mm × 365 mm (representing the width, height, and depth in the beam direction, respectively).

Using the information obtained from both FGDs and comparing the interaction rates reported from each one, the cross-section on carbon and on water can be found separately.

3.4.6. Electro-magnetic calorimeter (ECal)

In addition to the information obtained from the off-axis detectors, the energy, orientation, and identification of photons and charged particles is required to perform a full event reconstruction. For this purpose, the assembly of the off-axis detectors (i.e. PØD, TPCs, and FGDs) is surrounded by an electro-magnetic calorimeter (ECal). The role of ECal is particularly important in the reconstruction of π⁰ that is produced in neutrino interactions inside the tracking detectors.

ECal consists of 13 separate modules of three different types. There are six Barrel-ECal modules covering the assembly of TPCs and FGDs on the four sides parallel to the beam direction (see Figure 3-7). The downstream end of this volume is covered by one downstream module (DS-ECal). There are six PØD-ECal modules that cover the PØD on its four sides parallel to the beam direction. Each ECal module is made of adjacent scintillator planes glued to a sheet of lead
converter (as can be observed in Figure 3-14). The scintillator bars used in each scintillator plane have a cross-section of \(4 \text{ cm} \times 1 \text{ cm}\) and the WLS fibers run along a hole in the center of each bar. The bars in the beam direction are read by MPPCs at each end, and the bars in the directions perpendicular to the beam are read by a single MPPC at one end.

![Figure 3-14. An ECal module. The scintillator bars lie horizontally inside the module](image)

3.4.7. **Side muon range detector (SMRD)**

The side muon range detector (SMRD) key functions can be summarized as:

1) recording and measuring the momenta of the muons that escape from the ND280 detector with high angles with respect to the neutrino beam direction;

2) triggering on cosmic ray muons which go into the ND280 detector;

3) identifying the beam-related interactions that take place in the magnet iron.

The SMRD is installed in the 1.7 cm gap between the steel plates which make the yoke of the UA1 magnet. This detector is made of 440 scintillator modules which all fill the innermost gaps in the UA1 magnet to capture the particles that escape from the inner detectors. Since the gaps
have different sizes, the horizontal and vertical modules covering the gaps are different in size. The horizontal SMRD modules consist of four scintillation contours with dimensions 875 mm × 167 mm × 7 mm. The vertical SMRD modules consist of five scintillation contours with dimensions 875 mm × 175 mm × 7 mm. The scintillation contours are composed of extruded polystyrene dimethylacetamide mixed with POPOP and para-terphenyl. There is a white diffuse layer on the surface of the scintillation contour that behaves as a reflector. An S-shaped groove is machined into each scintillator. A 1 mm diameter wavelength shifting fiber is glued into each groove. An MPPC is coupled to the polished end of each wavelength shifting fiber.

![Figure 3-15. A view of SMRD scintillation counter.](image)

The complementary information about the ND280 complex including its electronics, DAQ, and MC simulation can be found in [11].
Chapter 4. **SUPER-KAMIOKANDE DETECTOR**

4.1. **INTRODUCTION**

The Super-Kamiokande (SK) is the world’s largest Cherenkov detector. It has been designed in due to the need for highly accurate detection of neutrino events and studies of neutrinos produced from various sources such as sun, atmosphere, reactors, and accelerators. The SK has played an important role in the neutrino oscillation studies since April 1996 until present. Currently it is being used for the T2K experiment as the far detector. As the T2K gathers data, the atmospheric, solar and, other astrophysical studies continue. In this chapter, an overview of the SK detector as used for the T2K experiment is provided.

4.2. **THE CHERENKOV LIGHT**

In the SK detector events are notified by the detection of the Cherenkov light. This light is an electromagnetic radiation emitted from a charged particle which travels in a medium with a velocity greater than the velocity of light in it. The Cherenkov light trajectories, as shown in Figure 4-1, form a cone which is described by a Cherenkov angle, $\theta_C$. The Cherenkov angle can be found by calculating the ratio of the distance travelled by the charged particle and the distance travelled by the emitted wave during the time $t$. These two distances are equal to $t\beta c$ and $tc/n$, respectively (where $\beta = v/c$, $n$ is the refractive index of the medium, and $v$ is the velocity of the charged particle). The Cherenkov angle can be calculated using the following equation:

\[
\cos(\theta_C) = \frac{ct/n}{\beta ct} = \frac{1}{\beta n} \quad (4-1)
\]
From equation (2-1), it can be observed that the Cherenkov radiation only occurs when $\beta \geq 1/n$.

The Cherenkov threshold energies of the main charged particles produced in the SK detector (i.e. $e^\pm$, $\mu^\pm$, and $\pi^\pm$) are provided in Table 4-1.

Table 4-1. The Cherenkov radiation threshold energy of $e^\pm$, $\mu^\pm$ and $\pi^\pm$

<table>
<thead>
<tr>
<th>Particle</th>
<th>Cherenkov threshold in total energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^\pm$</td>
<td>0.768</td>
</tr>
<tr>
<td>$\mu^\pm$</td>
<td>158.7</td>
</tr>
<tr>
<td>$\pi^\pm$</td>
<td>209.7</td>
</tr>
</tbody>
</table>

The Cherenkov light spectrum is wavelength dependant and the number of produced Cherenkov photons (dN) per unit wavelength (d$\lambda$) per unit track length (dx) can be written as follows:

$$\frac{d^2N}{dx d\lambda} = 2\pi\alpha \left(1 - \frac{1}{(n\beta)^2}\right) \frac{1}{\lambda^2}$$  \hspace{1cm} (4-2)

where $\alpha$ is the fine structure constant.
4.3. **THE SK DETECTOR**

Super-Kamiokande (SK) is located in Mozumi mine of the Kamioka Mining & Smelting Company, Gifu Prefecture, Japan (see Figure 2-2). The detector’s hall lies one kilometre under the Peak of the Ikenoyama Mountain. This distance is sufficient to stop the cosmic ray muons with energies smaller than 1.3 TeV that are a large source of background. The observed muon flux that does not represent a remarkable background for the experiment is $6 \times 10^{-8}$ cm$^{-2}$s$^{-1}$sr$^{-1}$.

The SK detector main component is a cylindrical stainless steel tank 39.9 m in diameter and 41.4 m high with the capacity of 50,000 tons of water. Two doors on the top of this tank and one door at its bottom provide the access into the tank. The top of the tank plays the role of a platform for the support of electronic huts, monitoring devices, equipment for calibrations, and other facilities (see Figure 4-3). Approximately, 2 - 2.5 m inside the walls of the tank is a framework of thickness 55 cm. This framework supports a large number of photomultiplier tubes (PMTs).
PMTs include 11,146 Hamamatsu R3600 50 cm diameter hemispherical PMTs facing inward at 2.5 m inside the walls and 1,885 Hamamatsu R1408 20 cm diameter hemispherical PMTs facing outward at 2.0 m inside the walls. A schematic view of a 50 cm PMT is shown in Figure 4-4. The water volume viewed by the inward facing PMTs is called the Inner Detector (ID), and the water volume viewed by the outward facing PMTs is called the Outer Detector (OD), see Figure 4-5. The inner detector is 33.8 m in diameter and 36.2 m high. The ID and OD spaces are isolated from each other by the use of two light-proof sheets on the all sides of the PMT’s support structure. The OD space is a veto counter for incoming particles such as neutrons and gamma rays that come from the surrounding rocks. The OD also can ensure that the event is contained. That is, for an interaction in the ID, the particles remain in the ID, which is important for estimating their energies. Otherwise, the OD will see light, indicating that not all the Cherenkov light from the particle is detected. The OD PMTs are distributed as follows: 1,275 on the side walls, 302 on the top, and 308 on the bottom. The ID PMTs are distributed as follows: 7,650 on the side walls, 1,748 on the top, and 1,748 on the bottom. The ID PMTs are mounted on a 70 cm grid.

Figure 4-3. A schematic view of the site of the SK detector.
Figure 4-4. A schematic view of the ID 50 cm PMT.

Figure 4-5. A side view of the SK detector.
Since the wavelength of the Cherenkov light in the experiment is about 390 nm, the material of the photocathode of 50 cm PMTs is chosen to be bialkai (Sb-K-Cs) that has a peak quantum efficiency of about 21% at 360-400 nm. This is illustrated in Figure 4-6.

![Figure 4-6](image)

Figure 4-6. The quantum efficiency of the bialkai (Sb-K-Cs) photocathode as a function of the wavelength of the incident light.

Opaque black polyethylene telephthalate sheets (called Black Sheets) cover the spaces between the PMTS in the ID volume, as shown in Figure 4-7.

![Figure 4-7](image)

Figure 4-7. A schematic view of the support structure of the ID PMTs.
4.3.1. **Black Sheets**

The Black Sheet is made from polyethylene terephthalate mixed with carbon. Polyethylene terephthalate, usually called PET, is a plastic polymer. The Black Sheet enhances the isolation between the inner and outer spaces of the SK detector by absorbing any light that hits it. The reflectivity of the Black Sheet varies with the wavelength of the incident light and similar to any other plastic polymer, it decreases as the wavelength increases.

Although in the T2K experiment the reflectivity of the Black Sheet has been measured for different wavelength, its refractive index as a function of wavelength has not been determined yet. This leads to the main objective of this thesis. Particularly, this thesis addresses the optical behaviour of the Black Sheet and the variations of its refractive index as a function of wavelength.

4.3.2. **Event reconstruction**

The Cherenkov light cone emitted from the charged particles forms a circular projection pattern on the surface of the PMTs. The PMTs convert the incident light into an electric signal. This signal is finally read by DAQ systems. Using event reconstruction algorithms, the type, direction, and distribution of the energies of the charged particles can be determined, which are eventually used to calculate the neutrinos information. The reconstruction procedure includes vertex fitting, ring fitting, particle identification, MS vertex fitting, momentum reconstruction, decay electron finding, and ring number correction.
In the vertex fitting, the position of the vertex is estimated with a single ring assumption. After reconstructing the vertex position and finding the orientation of one dominant ring, a ring fitter algorithm searches for the other possible rings. The particle identification algorithm uses the Cherenkov ring pattern and the Cherenkov angle to determine the particle type of each ring. The MS vertex algorithm finds the vertex position of the multi-ring events mainly by using timing information. Also, the vertex position of each ring that is determined by the vertex fitter algorithm is reconstructed more accurately by using this algorithm. The momentum reconstruction program estimates the momentum of the charged particles. This estimation is performed by using the total number of photoelectrons detected inside a 70 degrees half-opening angle in the direction toward the reconstructed ring. In the decay electron finding, the decay electrons following the primary events are identified. This is important in order to reject the atmospheric neutrino background while accepting the proton decay signal. Finally, in the ring number correction, the final ring momenta are verified and the very low momentum rings are rejected.

4.3.3. Detector simulation

A number of detailed Monte Carlo programs have been written so far. These programs are capable of simulating the propagation of the charged particles, the Cherenkov radiation, and the response of the PMTs and the electronics. These customized simulation programs have been developed based on the GEANT package [18]. GEANT is a simulation program that has been used to describe the propagation of the elementary particles in the matter. An example of simulated event pattern that can be detected in the SK detector is shown in Figure 4-8.
Figure 4-8. A simulated event created by 500 MeV/c muons in the SK detector. Small circles indicate the PMTs that detect photons.

4.3.4. Detector calibration

The main calibrations of the SK detector include the PMT timing calibration, the PMT’s relative and absolute gain calibrations, and the light attenuation length measurement. In the timing calibration, the time offset and the time walk are measured. The time offset comes from the transit time of the PMT and its cable. The time walk shows the dependence of the signal height on the timing information. For the timing calibration, the light source of a diffuser ball is placed in the detector tank. The light is produced by a nitrogen laser and is guided to the diffuser ball by an optical fibre (see Figure 4-9). By making the light source flash, the time offset and the time walk are measured. In the final physical analysis, the timing information is corrected by considering these calibrations. In the PMT’s relative gain calibration, the gain of the PMTs is adjusted to obtain a uniform response of the SK detector. For this calibration, the high voltage
value of the 20 inch PMTs is adjusted by using the light source in the tank and comparing the observed photoelectrons in each PMT and its neighbouring PMTs. For the PMT’s absolute gain calibration, an experiment is designed to measure one photoelectron distribution. These photoelectrons are provided by using a low-energy gamma emission (6-9 MeV). The attenuation length measurement is performed using a nitrogen laser and a CCD camera (which is located on the top of the SK detector), see Figure 4-10. A diffuse ball connected to the laser through an optical fibre is installed inside the tank. Using this setup, the attenuation length can be obtained by measuring the intensity of the ball images at different distances and for different wavelengths.

Figure 4-9. The timing calibration experimental set up.

Figure 4-10. The attenuation length measurement experimental set up.
Chapter 5. **MEASUREMENT OF A COMPLEX REFRACTIVE INDEX**

5.1. **INTRODUCTION**

In this chapter, the theory of reflection of an incoming electromagnetic wave from the boundary between two dielectric media, whose optical properties can be summarized as an index of refraction, is presented. The theoretical expressions for the reflection coefficient with a polarized incoming beam are provided as a function of the refractive index of the second medium. Then, by comparing the theoretical expression of the reflection coefficient and its experimental values, the refractive index of two test samples is calculated.

5.2. **THEORY OF REFLECTION FROM A MEDIUM WITH A REAL REFRACTIVE INDEX**

When an electromagnetic wave passes through a boundary between two media, it will be reflected, transmitted, and absorbed. In order to calculate the contribution of reflection, the boundary conditions should be imposed on both electric and magnetic components of the incoming wave. To do this, assume an incoming plane wave from a medium with a real refractive index, $n_1$, that hits the surface of another medium with a real refractive index, $n_2$, at the boundary plane of $z = 0$ (See Figure 5-1).

In Figure 5-1, $\vec{n}$ is the unit vector perpendicular to the boundary, $\theta_1$ and $\theta_2$ are the incident and refraction angles with respect to normal vector ($\vec{n}$), respectively, and $\vec{k}_i$, $\vec{k}_r$ and $\vec{k}_t$ are the incident, reflected, and transmitted wave vectors, respectively. The plane made by
\( \vec{k}_i \) and \( \vec{n} \) vectors is called the incident plane. All the mentioned waves can be considered as a composition of two polarizations: P and S. The P-polarized component is defined as the one parallel to the incident plane. The vectors \( \vec{E}_1', \vec{E}_1' \) and \( \vec{E}_2 \) shown in Figure 5-1 are all the P-polarized components. The S-polarized component is perpendicular to the incident plane and is not shown in the figure.

![Figure 5-1. Passage of an electromagnetic wave through the boundary of two dielectric mediums.](image)

For the P-polarized components, as shown in Figure 5-1, the Fresnel reflection (\( r_{12P} \)) and transmission (\( t_{12P} \)) coefficients can be given by:

\[
\begin{align*}
\mathcal{r}_{12P} &= \frac{n_2\cos(\theta_1) - n_1\cos(\theta_2)}{n_2\cos(\theta_1) + n_1\cos(\theta_2)} \\
\mathcal{t}_{12P} &= \frac{2n_1\cos(\theta_1)}{n_2\cos(\theta_1) + n_1\cos(\theta_2)}
\end{align*}
\] (5-1) (5-2)
The detailed derivation of these two coefficients can be found in [19]. The reflection ($R_P$) and transmission ($T_P$) coefficients can be written as:

$$R_P = \frac{\vec{n} \cdot \vec{S}'_{1P}}{\vec{n} \cdot \vec{S}_{1P}} = r_{12P}^2 \quad (5-3)$$

$$T_P = \frac{\vec{n} \cdot \vec{S}_{2P}}{\vec{n} \cdot \vec{S}_{1P}} = \frac{n_2 \cos(\theta_2)}{n_1 \cos(\theta_1)} t_{12P}^2 \quad (5-4)$$

where $\vec{S}_1$, $\vec{S}_i$, and $\vec{S}_2$ are the incident, reflected, and transmitted Poynting vectors.

If the first medium is vacuum, i.e. $n_1 = 1$, the reflection coefficient can be written in terms of $\theta_1$ and $n_2$, as follows:

$$R_p(\theta_1, n_2) = \left( \frac{n_2 \cos(\theta_1) - \cos(\theta_2)}{n_2 \cos(\theta_1) + \cos(\theta_2)} \right)^2 \quad (5-5)$$

where $\theta_2$ can be found from the Snell’’s law:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \quad (5-6)$$

Equation (5-5) can be used as a good approximation for cases where the first medium is air, i.e. $n_1 = 1.0003$, and can also be written in the following form:

$$R_p(\theta_1, n_2) = \left( \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)} \right)^2 \quad (5-7)$$
From equation (5-7), it can be interpreted that when: \( \theta_1 + \theta_2 = \pi/2 \), the P-polarized reflection coefficient becomes zero. The incident angle \( \theta_1 \) which satisfies this condition is called the Brewster angle, \( \theta_B \). Using equation (5-6) and knowing that now \( \theta_2 = \pi/2 - \theta_B \), the Brewster angle can be found as a function of \( n_1 \) and \( n_2 \), as follows:

\[
n_1 \sin(\theta_B) = n_2 \sin\left(\frac{\pi}{2} - \theta_B\right) = n_2 \cos(\theta_B)
\]

\[
\Rightarrow \tan(\theta_B) = \frac{n_2}{n_1}
\]

(5-8)

5.3. Theory of Reflection from a Medium with Complex Refractive Index

If the second medium has a complex refractive index, \( \hat{n}_2 \), the refraction angle, \( \hat{\theta}_2 \), will be complex too. Consequently, the Snell’s law can be written as:

\[
n_1 \sin(\theta_1) = \hat{n}_2 \sin(\hat{\theta}_2)
\]

(5-9)

where \( \hat{n}_2 = n_2 + ik_2 \), and \( n_2 \) & \( k_2 \) are the real and imaginary parts of \( \hat{n}_2 \), respectively. The real part, \( n_2 \), represents the phase speed, and the imaginary part, \( k_2 \), indicates the absorption loss of the propagating electromagnetic wave inside the second medium.

Let’s introduce two new parameters, \( p \) and \( q \) as follows:

\[
n_1 \sin(\theta_1) = \hat{n}_2 \sin(\hat{\theta}_2)
\]

(5-10)
\[ p^2 - q^2 + 2i pq = \hat{n}_2 \cos^2(\hat{\theta}_2) = n_2^2 - k_2^2 + 2im_2k_2 - n_1^2 \sin^2(\theta_1) \]  
\[ \text{(5-11)} \]

Comparing the real and imaginary parts of the right- and left-hand sides of equation (5-11), the following can be obtained:

\[ pq = n_2 k_2 \]  
\[ \text{(5-12)} \]

\[ p^2 - q^2 = n_2^2 - k_2^2 - n_1^2 \sin^2(\theta_1) \]  
\[ \text{(5-13)} \]

Solving the coupled equations (5-12) and (5-13), parameters \( p \) and \( q \) can be obtained in terms of \( n_1, n_2, k_2, \) and \( \theta_1 \):

\[ p = \sqrt{\frac{1}{2} \left[ (n_2^2 - k_2^2 - n_1^2 \sin^2(\theta_1)) + \sqrt{(n_2^2 - k_2^2 - n_1^2 \sin^2(\theta_1))^2 + (2n_2k_2)^2} \right]} \]  
\[ \text{(5-14)} \]

\[ q = \sqrt{\frac{1}{2} \left[ - (n_2^2 - k_2^2 - n_1^2 \sin^2(\theta_1)) + \sqrt{(n_2^2 - k_2^2 - n_1^2 \sin^2(\theta_1))^2 + (2n_2k_2)^2} \right]} \]  
\[ \text{(5-15)} \]

Here the P-polarized reflection coefficient is defined similar to the case with real refractive index and can be written as:

\[ R_P = \left| \hat{r}_{12P} \right|^2 \]  
\[ \text{(5-16)} \]

where the Fresnel reflection coefficient, \( \hat{r}_{12p} \), is now given by:

\[ \hat{r}_{12p} = \frac{\hat{n}_2 \cos(\theta_1) - n_1 \cos(\hat{\theta}_2)}{\hat{n}_2 \cos(\theta_1) + n_1 \cos(\hat{\theta}_2)} \]  
\[ \text{(5-17)} \]
Knowing that \( \hat{n}_2 = n_2 + ik_2 \) and substituting equation (5-10) in equation (5-17) results in:

\[
\hat{r}_{12P} = \frac{(n_2 + ik_2)\cos(\theta_1) - n_1((p+iq)/(n_2 + ik_2))}{(n_2 + ik_2)\cos(\theta_1) + n_1((p+iq)/(n_2 + ik_2))}
\]  \hspace{1cm} (5-18)

Finally, by substituting equation (5-18) in equation (5-16), the reflection coefficient \( (R_P) \) can be written in terms of \( p, q, n_1, n_2, k_2, \) and \( \theta_1 \):

\[
R_P(\theta_1, n_1, n_2, k_2) = \hat{r}_{12P} \times \hat{r}_{12P}^* = \frac{\left((n_2^2 - k_2^2)\cos(\theta_1) - n_1p\right)^2 + \left(2n_2k_2\cos(\theta_1) - n_1q\right)^2}{\left((n_2^2 - k_2^2)\cos(\theta_1) + n_1p\right)^2 + \left(2n_2k_2\cos(\theta_1) + n_1q\right)^2}
\]  \hspace{1cm} (5-19)

where according to equations (5-14) and (5-15), \( p \) and \( q \) are also function of \( n_1, n_2, k_2, \) and \( \theta_1 \). In conclusion, the reflection coefficient is only a function of \( n_1, n_2, k_2, \) and \( \theta_1 \). If the first medium is air \( (n_1 = 1) \), the reflection coefficient will only be a function of \( n_2, k_2, \) and \( \theta_1 \):

\[
R_P(\theta_1, n_2, k_2) = \frac{\left((n_2^2 - k_2^2)\cos(\theta_1) - p\right)^2 + \left(2n_2k_2\cos(\theta_1) - q\right)^2}{\left((n_2^2 - k_2^2)\cos(\theta_1) + p\right)^2 + \left(2n_2k_2\cos(\theta_1) + q\right)^2}
\]  \hspace{1cm} (5-20)

This equation is the generalized version of equation (5-6) for a case where \( n_2 \) is a complex number.
5.4. DESIGN OF THE EXPERIMENTAL SETUP

So far, two definitions for the reflection coefficient have been provided: 1) equation (5-20) provides the theoretical expression of the reflection coefficient for any incident angle as a function of $n_2$ and $k_2$; and 2) according to equation (5-3) our reflection coefficient is the ratio of the reflected Poynting vector over the incident Poynting vector. Let’s focus on the second definition first. Knowing that the Poynting vector is always in the direction of the wave vector and its unit is power per unit area, equation (5-3) can equivalently be written as:

$$R_p(\theta) = \frac{P_{\text{ref}}(\theta)}{P_{\text{inc}}}(5-21)$$

where $P_{\text{ref}}(\theta)$ is the power of the reflected light and $P_{\text{inc}}(\theta)$ is the power of the incident light. The polarization index, $P$, is not included in these parameters because the incident light is always $P$-polarized in this work. $R_p(\theta)$ in equation (5-21) can be calculated given the measured values of $P_{\text{ref}}(\theta)$ and $P_{\text{inc}}(\theta)$ for each incident angle.

In order to calculate $n_2$ and $k_2$, the experimental reflection coefficients ($R_{\text{exp}}(\theta)$), given by equation (5-21), should be compared to its theoretical values ($R_{\text{th}}(\theta)$), given by equation (5-20). This can be written as:

$$R_{\text{exp}}(\theta) = R_{\text{th}}(\theta, n_2, k_2) (5-22)$$

On the right-hand side of equation (5-22), there are two unknown variables for any incident angle. Hence, in order to determine the value of these two variables, the reflection coefficients
for two incident angles at least must be known. Although two measurements could suffice for determining \( n_2 \) and \( k_2 \), by measuring the reflection coefficients for more than two incident angles, first the accuracy of results can be increased, and second it can be verified whether the simple dielectric model with a single interface is valid or not.

In the simplest case, to determine \( n_2 \) and \( k_2 \) of an object, the experimental setup should include an incoming beam, the experimental sample, and a detector (capable of reading the power of the reflected light). This is schematically illustrated in Figure 5-2.

Figure 5-2. A schematic diagram of the experimental setup used for measurement of the reflectivity of a sample.

5.5. EXPERIMENTAL SETUP

Our experimental setup is shown in Figure 5-3. In our setup, the incoming light is provided by a laser diode (Hamamatsu Picosecond Light Pulser – PLP 10 with a minimum pulse width of 10 ns and an output power of 10-100 mW). This laser works under four different wavelengths, i.e. 404, 445, 467 & 637 nm. These wavelengths are all in the visible region. The laser light is directed to the experimental setup by an optical splitter cable. In order to turn the partially
unpolarized laser light into P-polarized light, a laser Glan-Taylor polarizer is placed right at the end of the optical fibre cable. For the detection of the reflected light, one of the two silicon photodetectors (200 – 1100 nm with linearity ±0.5%) of the optical power meter (Newport – 2931C) is used. The photodetector’s active area is 1 cm$^2$. Both the splitter cable-polarizer assembly and the power meter photodetector are placed on a custom aluminium arc guide with radius 21 cm built for this experiment. The sample is placed at center of the arc. The entire experimental set up is placed in a light-tight box with dimensions 1 m $\times$ 1 m $\times$ 0.2 m that has a cap on its top surface.

In each experiment, the splitter cable-polarizer assembly is placed at 10-65 degrees (varying with 5 degrees intervals) on the left-hand side of the arc, aiming to the center of arc. The power of the
reflected light is read consequently at the same mirrored angle on the right-hand side of the arc. The power of the incident light ($P_{\text{inc}}$) is measured by placing the detector directly in front of the polarized laser light. To monitor the fluctuations of the incident light during the experiment, the laser light is directed into the splitter cable. While one branch of the splitter cable carries half of the laser light to the experimental setup, the other branch carries the other half of light into a free photodetector of the power meter. If any fluctuation of the incident laser intensity is observed during the experiment, it is recorded and the corresponding correction is used for $P_{\text{inc}}$ of that measurement. Through the entire process, $R_{\text{exp}}(\theta)$ (given by equation (5-21)) is calculated; then, with the aid of equation (5-22), $n_2$ and $k_2$ are determined.

5.6. RESULTS

For the results presented in this section, equation (5-22) is solved numerically (using a C++ code) to obtain $n_2$ and $k_2$. This solution leads to a region in $n_2$ - $k_2$ plane. To find the best $n_2$ and $k_2$, the least squares method (Chi-square test) has been used. The Chi-square, $\chi^2$, mathematically can be written as follows:

$$\chi^2(\theta) = \sum_{\theta = 10^\circ}^{65^\circ} \frac{(R_{\text{exp}}(\theta) - R_{\text{th}}(\theta))^2}{s^2(\theta)}$$  \hspace{1cm} (5-23)

where $s(\theta)$ is the theoretical error on $R(\theta)$ as $\theta$ is varied to $\theta + \delta\theta$. This error is calculated by varying $\theta$ in the expression of reflection coefficient, $R(\theta)$ (equation (5-5), which is written for real refractive index) and can be expressed as below:
\[ \delta R (\theta_i) = \\
2n_2 \sin(\theta_i) \delta \theta_i \left( \frac{(n_2 \cos(\theta_i) - n_1 \cos(\theta_2))(n_2 \cos(\theta_i) + n_1 \cos(\theta_2)) + (n_2 \cos(\theta_i) - n_1 \cos(\theta_2))^2}{(n_2 \cos(\theta_i) + n_1 \cos(\theta_2))^3} \right) \]  \hspace{1cm} (5-24)

where in our experiments, \( \delta \theta_1 \) equals 1 degree corresponding to a 3 mm error on the position of the cable-polarizer assembly on the rail, while the radius of the rail is 21 cm \( (\delta \theta_1 = \text{Arctan}(0.3/21) \approx 1^\circ) \). In equation (5-24), \( \delta \theta_2 = (n_1 \cos(\theta_i)/n_2 \cos(\theta_2)) \delta \theta_1 \) which can be derived from equation (5-6).

Using this method, the set of \( n_2 \) and \( k_2 \) that is within the confidence region are solutions to equation (5-22). The confidence region is determined with the following equation:

\[ \chi^2(n_2, k_2) \leq \chi^2_{\text{min}} + \Delta \chi^2 \]  \hspace{1cm} (5-25)

where \( \chi^2_{\text{min}} \) is the absolute minimum of Chi-square function, and \( \Delta \chi^2 = 4.61 \) for the confidence level of 90% for an experiment with two free parameters.

In total, 12 experimental data points are considered, i.e. \( R(10^\circ) \) to \( R(65^\circ) \) with 5 degrees intervals. Our experiments have been conducted for two known samples: 1) Fused Silica (FS) and 2) N-BK7 glass. The measured reflectivity of these two samples is shown in Figure 5-8 and Figure 5-9.

Our computations have been conducted for wavelength of 404 nm. Each experiment has been repeated three times and the values for \( n_2 \) and \( k_2 \) have been obtained by using the averaged
reflection coefficients (of these three experiments) as the input. The experimental values for \( n_2 \) and \( k_2 \), corresponding to the absolute minimum of the \( \chi^2 \) space are provided in Table 5-1. The table also includes the reference values of the refractive indices provided by the manufacturer for our samples.

Table 5-1: Experimental and actual values of the real and imaginary parts of the refractive indices of FS and N-BK7 test samples.

<table>
<thead>
<tr>
<th>Sample</th>
<th>( n_2 ) (experiment)</th>
<th>( k_2 ) (experiment)</th>
<th>( \chi^2_{\text{min}} )</th>
<th>( n_2 ) (reference)</th>
<th>( k_2 ) (reference)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS</td>
<td>1.46(^{+0.03}_{-0.02})</td>
<td>0.07(^{+0.06}_{-0.07})</td>
<td>6.18</td>
<td>1.470</td>
<td>0</td>
</tr>
<tr>
<td>N-BK7</td>
<td>1.51(^{+0.02}_{-0.03})</td>
<td>0.12(^{+0.10}_{-0.12})</td>
<td>4.06</td>
<td>1.524</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 5-4 and Figure 5-5 show the \( \chi^2 \) space as a function of \( n_2 \) and \( k_2 \) for each sample and Figure 5-6 and Figure 5-7 show the confidence region as a function of \( n_2 \) and \( k_2 \) for them. Note that in Figure 5-6 and Figure 5-7, the region of \( n_2 \) and \( k_2 \) with \( \chi^2 - \chi^2_{\text{min}} \leq 4.61 \) is acceptable with confidence level of 90%.

![Figure 5-4](image)

Figure 5-4. \( \chi^2(\theta) \) as a function \( n_2 \) and \( k_2 \) for Fused silica sample.
Figure 5-5. $\chi^2(\theta)$ as a function $n_2$ and $k_2$ for N-BK7 sample.

Figure 5-6. $\chi^2(\theta) - \chi^2_{\text{min}}$ as a function $n_2$ and $k_2$ for Fused silica sample.
Figure 5-7. $\chi^2(\theta) - \chi^2_{\text{min}}$ as a function $n_2$ and $k_2$ for N-BK7 sample.

**Figure 5-8 - Figure 5-9** show the comparison between $R_{\text{exp}}(\theta)$ and $R_{\text{th}}(\theta)$ using $n_2$ and $k_2$ of Table 4-1.

**Figure 5-8.** Comparison between the experimental and fitted values of the reflection coefficient, $R_P(\theta)$ for the Fused silica sample.
Figure 5-9. Comparison between the experimental and fitted values of the reflection coefficient, $R_p(\theta)$ for the N-BK7 sample.
Chapter 6. MEASUREMENT OF THE COMPLEX REFRACTIVE INDEX OF BLACK SHEET

6.1. INTRODUCTION

In this chapter, the results leading to measurement of the complex refractive index of the black sheet are presented and discussed. The experimental methodology is exactly the same as what was discussed for our control samples; it is thoroughly explained in Chapter 5; therefore, not repeated in this chapter. However, the computational procedure is different from the previous cases. This is mainly because for the control samples there is only specular reflection as the surface of those samples is extremely smooth. However, due to the surface roughness, both specular and diffuse reflections are present for the black sheet. As a result, both specular and diffuse reflections must be accounted for; hence, the computations are more complicated.

6.2. THEORY

As explained in the previous chapter, in order to calculate the complex refractive index of an object with only specular reflection, it is enough to measure the power of the incoming and reflected beams for every incident angle. The reflection coefficients can be calculated by dividing the power of the reflected beam over the power of the incoming beam. By comparing the reflection coefficients with theoretical expressions, equation (5-22), the values of the real and imaginary parts of the complex refractive index, n and k, can be determined.

\[ R_{\text{in}}(\theta) = R_{\text{exp}}(\theta) = \frac{P_{\text{ref}}(\theta)}{P_{\text{inc}}} \]  

(6-1)
where $R_{\text{exp}}(\theta)$ and $R_{\text{th}}(\theta)$ are the experimental value and the theoretical expectation for the reflection coefficient, respectively, and $P_{\text{inc}}$ and $P_{\text{ref}}(\theta)$ are the power of the incoming beam and the specular reflected beam, respectively.

For a partially rough surface, due to the existence of bumps and hills on the surface, a portion of the incoming light will see the surface at angles different from the main incident angle. Accordingly, that portion of incoming light will be scattered; this event is sometimes referred to as “diffuse reflection”. A schematic diagram of this phenomenon is shown in Figure 6-1.

![Figure 6-1. A schematic diagram of diffuse reflection.](image)

In the reflection from the black sheet, a portion of light gets scattered; hence, equation (2-1) does not hold for it anymore. Due to scattering, the measured power of the specular reflected light is expected to be smaller than the power of the specular reflected light for the case with 100%
specular reflection. Considering this fact, a more general version of equation (2-1) can be written as follows:

\[
R_{th}(\theta) = \frac{P_{\text{ref}}(\theta) + P_{\text{scat}}(\theta)}{P_{\text{inc}}} \quad (6-2)
\]

where \( P_{\text{scat}}(\theta) \) is the power of light that would be reflected at angle \( \theta \) if the scattered portion of light was not scattered and instead was only reflected at angle \( \theta \). This quantity can generally be different from the total power of the scattered light, \( P_{\text{tot-scatt}}(\theta) \). However, by knowing \( P_{\text{tot-scatt}}(\theta) \) and the distribution of the scattered light, \( P_{\text{scatt}}(\theta) \) can be determined. This is explained in details in 6.4.

### 6.3. Calculating the Total Power of the Scattered Light (\( P_{\text{TOT-scatt}} \))

The total power of the scattered light, \( P_{\text{tot-scatt}}(\theta) \), is the summation of the powers of all the scattered lights at different angles when the specular reflected beam is detected at \( \theta \). \( P_{\text{tot-scatt}}(\theta) \) can be calculated by integrating the power of the scattered light per unit area over the surface of a half sphere with radius \( R \) with the specular reflected beam in the middle of it (see Figure 6-2). Here it is assumed that the scattered light is negligible in the other half-sphere. The radius of this sphere, \( R \), is the distance between the intersection point of the beam on the black sheet and the detector’s sensor. For our experiment \( R = 15.0 \) cm.
Given Figure 6-2, the total power of the scattered light can be calculated using the following equation:

$$P_{\text{tot-scatt}} (\theta) = \int_{0}^{\theta} \int_{0}^{90^\circ} G(\theta') R^2 \sin(\theta') d\theta' d\phi$$  \hspace{1cm} (6-3)$$

where $G(\theta')$ is the power of the scattered light per unit area at polar angle $\theta'$. In equation (6-3), it is required to exclude the specular reflected beam in the integration (see Figure 6-3). By measuring the radius of the spot made from the cross-section of the specular reflected beam and the integration half-sphere radius, $r$, and dividing this radius to the radius of the integration half-sphere, $R$, the angular border of the specular reflected beam on the half-sphere, $\theta_{\text{Spec}}$, can be calculated as given in bellow:
\[
\tan(\theta_{\text{Spec}}) = \frac{r}{R} \\
\rightarrow \theta_{\text{Spec}} = \arctan\left(\frac{r}{R}\right) = \arctan\left(\frac{5\text{ (mm)}}{150\text{ (mm)}}\right) \approx 2^\circ \quad (6-4)
\]

Figure 6-3. The excluded region of the specular reflection in the integration for calculation of \( P_{\text{tot-scatt}}(\theta) \).

From this calculation, it can be seen that the spectacular reflected beam is confined to 2 degrees around the angle \( \theta \). Therefore, in order to exclude the specular reflected beam from the integrations using equation (6-3), the lower bound of the polar angle is 2 instead of 0 degrees.

In this work, \( G(\theta') \) is determined by measuring the power of the scattered light (at 5 degree intervals) divided by the active area of the detector. Because of the experimental limitations in our setup, the above measurements are only carried out for the right-hand side of the specular reflected beam (\( \theta < \theta' < \theta + \pi/2 \)). The distribution of scattered light on the left-hand side (\( \theta - \pi/2 < \theta' < \theta \)) is assumed to be the same as the right-hand side. Since the distribution profile and the
amount of the scattered light are different for each incident angle ($\theta$), the function $G(\theta')$ is measured for every incident angle. The plots of the measured $G(\theta')$ for incident angles ranging from 10 to 65 degrees (with 5 degrees intervals) except for 60 degrees together with the fitting functions are provided in Figure 6-4 - Figure 6-14. Note that the incident angle of 60 degrees is very close to the Brewster’s angle of the black sheet; as a result the reflected power at this angle will be too small to be detected. For the fittings, a summation of two Gaussian functions was assumed by considering the shape of the $G(\theta)$ functions. Each fitted $G(\theta)$ function is described by 6 parameters and can be written as bellow:

$$G(\theta) = a e^{-\frac{(\theta - b)^2}{2c^2}} + a' e^{-\frac{(\theta - b')^2}{2c'^2}}$$ (6-5)

The 6 fitting parameters for incident angles 10 - 65 degrees (except for 60 degrees) is provided in Table 6-1.

Table 6-1. The fitting parameters of the $G(\theta)$ function for incident angles 10 - 65° (except for 60°).

<table>
<thead>
<tr>
<th>Incident Angle</th>
<th>$a$ (nW)</th>
<th>$b$</th>
<th>$c$</th>
<th>$a'$ (nW)</th>
<th>$b'$</th>
<th>$c'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 10$</td>
<td>450</td>
<td>10</td>
<td>4.8</td>
<td>20</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>$\theta = 15$</td>
<td>450</td>
<td>15</td>
<td>4.6</td>
<td>20</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>$\theta = 20$</td>
<td>450</td>
<td>20</td>
<td>4.6</td>
<td>19</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$\theta = 25$</td>
<td>360</td>
<td>25</td>
<td>4.8</td>
<td>18</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>$\theta = 30$</td>
<td>360</td>
<td>30</td>
<td>4.8</td>
<td>18</td>
<td>30</td>
<td>18</td>
</tr>
<tr>
<td>$\theta = 35$</td>
<td>270</td>
<td>35</td>
<td>4.8</td>
<td>21</td>
<td>35</td>
<td>16</td>
</tr>
<tr>
<td>$\theta = 40$</td>
<td>200</td>
<td>40</td>
<td>4.5</td>
<td>22</td>
<td>40</td>
<td>15</td>
</tr>
<tr>
<td>$\theta = 45$</td>
<td>130</td>
<td>45</td>
<td>4.6</td>
<td>28</td>
<td>45</td>
<td>11</td>
</tr>
<tr>
<td>$\theta = 50$</td>
<td>70</td>
<td>50</td>
<td>4</td>
<td>15</td>
<td>50</td>
<td>17</td>
</tr>
<tr>
<td>$\theta = 55$</td>
<td>16.97</td>
<td>55</td>
<td>15.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\theta = 65$</td>
<td>20</td>
<td>65</td>
<td>4</td>
<td>20</td>
<td>65</td>
<td>19</td>
</tr>
</tbody>
</table>
Figure 6-4. Variations of G(θ) with respect to θ for incident angle of 10 degrees.

Figure 6-5. Variations of G(θ) with respect to θ for incident angle of 15 degrees.
Figure 6-6. Variations of G(θ) with respect to θ for incident angle of 20 degrees.

Figure 6-7. Variations of G(θ) with respect to θ for incident angle of 25 degrees.
Figure 6-8. Variations of $G(\theta)$ with respect to $\theta$ for incident angle of 30 degrees.

Figure 6-9. Variations of $G(\theta)$ with respect to $\theta$ for incident angle of 35 degrees.
Figure 6-10. Variations of $G(\theta)$ with respect to $\theta$ for incident angle of 40 degrees.

Figure 6-11. Variations of $G(\theta)$ with respect to $\theta$ for incident angle of 45 degrees.
Figure 6-12. Variations of $G(\theta)$ with respect to $\theta$ for incident angle of 50 degrees.

Figure 6-13. Variations of $G(\theta)$ with respect to $\theta$ for incident angle of 55 degrees.
By substituting all the fitted equations for $G(\theta')$ in equation (6-3), the power of the total scattered light was obtained, as summarized in Table 2-1.

Table 6-2: Power of the total scattered light for different incident angles (with the power of the incoming light equal to 345 $\mu$W and wavelength equal to 404 nm).

<table>
<thead>
<tr>
<th>$\theta$ [degrees]</th>
<th>$P_{\text{tot-scat}}(\theta)$ [nW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>6731</td>
</tr>
<tr>
<td>15</td>
<td>6373</td>
</tr>
<tr>
<td>20</td>
<td>6230</td>
</tr>
<tr>
<td>25</td>
<td>5671</td>
</tr>
<tr>
<td>30</td>
<td>5306</td>
</tr>
<tr>
<td>35</td>
<td>4452</td>
</tr>
<tr>
<td>40</td>
<td>3477</td>
</tr>
<tr>
<td>45</td>
<td>2411</td>
</tr>
<tr>
<td>50</td>
<td>1479</td>
</tr>
<tr>
<td>55</td>
<td>1567</td>
</tr>
<tr>
<td>65</td>
<td>2877</td>
</tr>
</tbody>
</table>
6.4. Calculating $P_{\text{scat}}$

To calculate $P_{\text{scat}}(\theta)$ from $P_{\text{tot-scat}}(\theta)$, it is useful to consider a simple case where the entire incident light that is scattered (with the incoming power $P_{\text{inc-scat}}(\theta)$) stays in the same incident plane of the light that is reflected specularly (with the incoming power: $P_{\text{inc}} - P_{\text{inc-scat}}(\theta)$). The light that is scattered hits on the surface of the Black Sheet with angles other than $\theta$ i.e. $\theta \pm i \times \Delta \theta$. As a result, it will be reflected at the same angles $\theta \pm i \times \Delta \theta$ (for the future calculations in this work $\Delta \theta$ is chosen 5 degrees). Let’s assume that $a_i\%$ of the $P_{\text{inc-scat}}(\theta)$ is reflected at angle $\theta + i \times \Delta \theta$, and $a_i\%$ of the $P_{\text{inc-scat}}(\theta)$ is reflected at angles $\theta - i \times \Delta \theta$. This situation is depicted in Figure 6-15. In this situation, the $P_{\text{tot-scat}}(\theta)$ can be expressed by equation (6-6).
Figure 6-15. The division of the total incoming light into two branches, leading to the specular reflected beam and scattered beams.
\[ P_{\text{tot-scat}}(\theta) = \sum_{i=1}^{\lfloor 90^\circ/\Delta\theta \rfloor} \frac{a_i(\theta)}{100} \times P_{\text{inc-scat}}(\theta) \times (R_{\text{scat}}(\theta + i \times \Delta\theta) + R_{\text{scat}}(\theta - i \times \Delta\theta)) \]

and \[ \sum_{i=1}^{\lfloor 90^\circ/\Delta\theta \rfloor} 2a_i(\theta) = 100 \]  

(6-6)

Here it is assumed that the \( a(\theta) \) coefficients are the same for both \( \theta + i \times \Delta\theta \) and \( \theta - i \times \Delta\theta \). This means that the probability of getting scattered at \( \theta + i \times \Delta\theta \) is assumed to be equal to that of \( \theta - i \times \Delta\theta \).

If the scattered portion of the incoming light was instead reflected at angle \( \theta \), its detected power could be written as follows:

\[ P_{\text{scat}}(\theta) = P_{\text{inc-scat}}(\theta) \times R(\theta) \]  

(6-7)

In general, equation (6-6) can be written in the following form:

\[ \sum_{i=1}^{\lfloor 90^\circ/\Delta\theta \rfloor} \frac{a_i(\theta)}{100} \times P_{\text{inc-scat}}(\theta) \times (R_{\text{scat}}(\theta + i \times \Delta\theta) + R_{\text{scat}}(\theta - i \times \Delta\theta)) = \frac{b(\theta)}{100} \times P_{\text{inc-scat}}(\theta) \times R(\theta) \]  

(6-8)

Where \( b(\theta) \) is the percentage of \( P_{\text{inc-scat}}(\theta) \) that is reflected at angle \( \theta \) and produces the same reflected power as the total power of scattered light, \( P_{\text{tot-scat}}(\theta) \). In principle if all the \( a_i(\theta) \) coefficients are known, the \( b(\theta) \) coefficient can be found as:
From equation (6-8) together with equation (6-6), the following equation can be obtained:

\[
b(\theta) = \frac{100}{R(\theta)} \sum_{i=1}^{\left[\frac{90^\circ}{\Delta\theta}\right]} \frac{a_i(\theta) \times (R_{\text{scat}}(\theta + i \times \Delta\theta) + R_{\text{scat}}(\theta - i \times \Delta\theta))}{P_{\text{tot-scat}}(\theta)}
\]  

(6-9)

Finally, equation (6-10) along with equation (6-7) will lead to the relation between \( P_{\text{scat}}(\theta) \) and \( P_{\text{tot-scat}}(\theta) \):

\[
P_{\text{scat}}(\theta) = \frac{100}{R(\theta)} \sum_{i=1}^{\left[\frac{90^\circ}{\Delta\theta}\right]} \frac{a_i(\theta) \times (R_{\text{scat}}(\theta + i \times \Delta\theta) + R_{\text{scat}}(\theta - i \times \Delta\theta))}{P_{\text{tot-scat}}(\theta)}
\]  

(6-11)

The exact determination of \( a_i(\theta) \) coefficients is complicated due to the complex geometry of roughnesses and is beyond the scope of this work. In this thesis, the values for \( a_i(\theta) \) coefficients are assumed to be equal to the percentage of the scattered light at every outgoing angle \( \theta + i \times \Delta\theta \), i.e.:

\[
a_i(\theta) = \frac{\int_{\theta-i\Delta\theta}^{\theta+i\Delta\theta/2} \int_{\theta+(3i\Delta\theta/2)}^{\theta+(3i\Delta\theta/2)+2\Delta\theta/2} G(\theta') R^2 \sin(\theta') d\theta' d\phi}{P_{\text{tot-scat}}(\theta)}
\]  

(6-12)

A summary of the calculated \( a_i(\theta) \) coefficients using the above procedure is provided in Table 6-3.
Table 6-3. $a_i(\theta)$ coefficients for different incident angles (all angles are in degrees).

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$a_1(\theta)$</th>
<th>$a_2(\theta)$</th>
<th>$a_3(\theta)$</th>
<th>$a_4(\theta)$</th>
<th>$a_5(\theta)$</th>
<th>$a_6(\theta)$</th>
<th>$a_7(\theta)$</th>
<th>$a_8(\theta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20.5</td>
<td>11.5</td>
<td>4.5</td>
<td>4</td>
<td>3.5</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>21</td>
<td>10.5</td>
<td>4.5</td>
<td>4</td>
<td>3.5</td>
<td>3</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td>20</td>
<td>21</td>
<td>10.5</td>
<td>4.5</td>
<td>4</td>
<td>3.5</td>
<td>3</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td>25</td>
<td>20</td>
<td>11</td>
<td>4.5</td>
<td>4</td>
<td>3.5</td>
<td>3</td>
<td>2.5</td>
<td>1.5</td>
</tr>
<tr>
<td>30</td>
<td>21</td>
<td>11.5</td>
<td>5</td>
<td>4</td>
<td>3.5</td>
<td>2.5</td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td>35</td>
<td>19.5</td>
<td>12</td>
<td>6</td>
<td>4.5</td>
<td>3.5</td>
<td>2.5</td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td>40</td>
<td>18.5</td>
<td>11</td>
<td>6.5</td>
<td>5.5</td>
<td>4</td>
<td>2.5</td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td>45</td>
<td>20</td>
<td>14</td>
<td>8</td>
<td>4.5</td>
<td>2.5</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>15.5</td>
<td>11.5</td>
<td>11</td>
<td>10.5</td>
<td>1.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>55</td>
<td>5.5</td>
<td>9</td>
<td>10.5</td>
<td>9.5</td>
<td>7</td>
<td>4.5</td>
<td>2.5</td>
<td>1.5</td>
</tr>
<tr>
<td>65</td>
<td>5</td>
<td>6.5</td>
<td>8</td>
<td>8.5</td>
<td>8</td>
<td>6.5</td>
<td>5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

To write the expansion for $P_{\text{tot-scatt}}(\theta)$ as a function of $a_i(\theta)$ and $R(\theta + i \times \Delta \theta)$, it is important to note that when a scattered beam is reflected at angle of $\theta + 2 \times \Delta \theta$, it means that the incident beam has hit the surface with the incident angle of $\theta + \Delta \theta$ (see Figure 6-16). Therefore, the true expansion for $P_{\text{tot-scatt}}(\theta)$ can be written as:

$$
\frac{P_{\text{tot-scatt}}(\theta)}{P_{\text{inc-scatt}}(\theta)} = \sum_{i=1}^{\left[90^\circ/\Delta\theta\right]} a_i(\theta)(R_{\text{scatt}}(\theta + i \times \frac{\Delta \theta}{2}) + R_{\text{scatt}}(\theta - i \times \frac{\Delta \theta}{2}))
$$

(6-13)
Therefore, the corrected form of equation (6-11) will be:

\[
P_{\text{scat}}(\theta) = \frac{R(\theta)}{\sum_{i=1}^{[90^\circ/\Delta\theta]} a_i(\theta)(R_{\text{scat}}(\theta + i \times \frac{\Delta \theta}{2}) + R_{\text{scat}}(\theta - i \times \frac{\Delta \theta}{2}))} \times P_{\text{tot-sc}}(\theta)
\]  
(6-14)

Substituting equation (6-14) in equation (6-2), leads to the following:

\[
\frac{P_{\text{ref}}(\theta)}{P_{\text{inc}}} = R_{\text{th}}(\theta) - \frac{P_{\text{scat}}(\theta)}{P_{\text{inc}}}
\]

\[
= R_{\text{th}}(\theta) \times \left\{ 1 - \frac{1}{\sum_{i=1}^{[90^\circ/\Delta\theta]} a_i(\theta) \times (R_{\text{scat}}(\theta + i \times \frac{\Delta \theta}{2}) + R_{\text{scat}}(\theta - i \times \frac{\Delta \theta}{2}))} \times P_{\text{tot-sc}}(\theta) \right\} \]
(6-15)
6.5. RESULTS

For the results presented in this section, equation (6-15) is solved numerically (using a C++ code) to obtain \( n_2 \) and \( k_2 \). This solution leads to a region in \( n_2 - k_2 \) plane. Similar to what has been performed in the results section of the previous chapter, to find the best \( n_2 \) and \( k_2 \), the Chi-square fit has been used. Again the set of \( n_2 \) and \( k_2 \) that minimizes the Chi-square, \( \chi^2 \) (as given by equation (5-23)) is the answer to equation (6-15).

In order to calculate the summation term in equation (5-23), the incident angle of 60 degrees has not been included as this angle is very close to the Brewster’s angle of the black sheet and at this angle, the \( s_i \) (the theoretical error on \( R(60^\circ) \)) approaches zero. Consequently, at this angle \( \chi^2 \) will approach infinity for any \( n_2 \) and \( k_2 \).

In total, 11 experimental data points have been considered, i.e. \( R(10^\circ) \) up to \( R(65^\circ) \) with 5 degrees intervals except for \( R(60^\circ) \). Knowing that \( R_{th}(\theta) \) has two degrees of freedom, \( n_2 \) and \( k_2 \), the expected mean value of \( \chi^2 \) will be 9 ( = 11 - 2).

Our computations have been conducted for three different wavelengths of 404, 445 & 467 nm. Each experiment corresponding to one of the above wavelength has been repeated three times and the values for \( n_2 \) and \( k_2 \) have been obtained by using the averaged reflection coefficients (of these three experiments) as the input. The final results for \( n_2 \) and \( k_2 \), corresponding to the absolute minimum of the \( \chi^2 \) space are provided in Table 4-1.
Table 6-4: Experimental values of the real and imaginary parts of the refractive index of the black sheet for three different wavelengths.

<table>
<thead>
<tr>
<th>Wavelength [nm]</th>
<th>n&lt;sup&gt;2&lt;/sup&gt;</th>
<th>k&lt;sup&gt;2&lt;/sup&gt;</th>
<th>χ&lt;sub&gt;2&lt;/sub&gt;&lt;sup&gt;min&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>404</td>
<td>1.78 ±0.05</td>
<td>0.23 ±0.07</td>
<td>4.59</td>
</tr>
<tr>
<td>445</td>
<td>1.74 ±0.04</td>
<td>0.22 ±0.10</td>
<td>4.83</td>
</tr>
<tr>
<td>467</td>
<td>1.71 ±0.04</td>
<td>0.18 ±0.08</td>
<td>4.35</td>
</tr>
</tbody>
</table>

Figure 5-2 to Figure 6-19 show the χ<sup>2</sup> space as a function of n<sup>2</sup> and k<sup>2</sup> for each wavelength.

Figure 6-17. χ<sup>2</sup>(θ) as a function n<sup>2</sup> and k<sup>2</sup> for WL = 404 nm.
Figure 6-18. $\chi^2(\theta)$ as a function $n_2$ and $k_2$ for WL = 445 nm.

Figure 6-19. $\chi^2(\theta)$ as a function $n_2$ and $k_2$ for WL = 467 nm.
Figure 6-20 to Figure 6-22 show the confidence region as a function of $n_2$ and $k_2$ for each wavelength. Note that in these figures the region of $n_2$ and $k_2$ with $\chi^2 - \chi^2_{\text{min}} \leq 4.61$ is acceptable with confidence level of 90%.

![Figure 6-20](image1.png)

Figure 6-20. $\chi^2(\theta) - \chi^2_{\text{min}}$ as a function $n_2$ and $k_2$ for WL = 404 nm.

![Figure 6-21](image2.png)

Figure 6-21. $\chi^2(\theta) - \chi^2_{\text{min}}$ as a function $n_2$ and $k_2$ for WL = 445 nm.
Figure 6-22. $\chi^2(\theta) - \chi^2_{\text{min}}$ as a function $n_2$ and $k_2$ for WL = 467 nm.

Figure 5-8 to Figure 6-25 show the comparison between $R_{\text{th}}(\theta)$, $(P_{\text{ref}}(\theta) + P_{\text{scat}}(\theta))/P_{\text{inc}}$ and $P_{\text{ref}}(\theta)/P_{\text{inc}}$, using $n_2$ and $k_2$ of Table 4-1.
Figure 6-23. Comparison between the experimental and theoretical values of the reflection coefficient, $R_p(\theta)$ for WL = 404 nm.

Figure 6-24. Comparison between the experimental and theoretical values of the reflection coefficient, $R_p(\theta)$ for WL = 445 nm.
As a check point for the calculation of $n_2$, in addition to the main procedure that was described earlier in this chapter, it is also possible to use the definition of the Brewster’s angle (equation (5-8)). By finding the Brewster’s angle, $\theta_B$, from our experiment, using equation (5-8) and considering a 1 degree error in the Brewster’s angle, the acceptable region for $n_2$ is obtained, as summarized in Table 6-5, as shown.

Table 6-5: Acceptable values of $n_2$ calculated with the aid of the Brewster’s angle.

<table>
<thead>
<tr>
<th>Wavelength [nm]</th>
<th>$\theta_B$ [degrees]</th>
<th>Acceptable region of $n_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>404</td>
<td>60</td>
<td>1.66 &lt; $n_2$ &lt; 1.80</td>
</tr>
<tr>
<td>445</td>
<td>59</td>
<td>1.60 &lt; $n_2$ &lt; 1.73</td>
</tr>
<tr>
<td>467</td>
<td>59</td>
<td>1.60 &lt; $n_2$ &lt; 1.73</td>
</tr>
</tbody>
</table>
It can be seen that the acceptable regions for $n_2$ from the Brewster angle’s method (as given in Table 6-5) is in agreement with our previous results (as provided in Table 4-1).
Chapter 7. CONCLUSIONS

In this thesis, the complex refractive index of the Black Sheet has been calculated for three different wavelengths. It is achieved by first validating our calculations by comparing the experimental measurements of their reflectivity and the theoretical predictions of the reflectivity obtained with a simple dielectric model. The results show that the real and imaginary parts of the refractive index of the control samples can be determined with tolerances of about 0.02 and 0.1 with 90% confidence level, respectively.

While conducting the experiment in order to measure the reflectivity of the Black Sheet, it was noticed that a significant portion of the incoming light scatters off the Black Sheet. This is because the surface of the Black Sheet is not smooth. Note that the simple dielectric model (that was also used for the control samples) is only valid for cases with specular reflection. Therefore, this model in its original form cannot be used to determine the reflectivity of the Black Sheet unless the scattering is accounted for. A true characterization of the scattering phenomenon was beyond the scope of this work; as a result, a simplified approach is taken in this work. By making some assumptions, a compensation for the light lost by the scattering has been calculated. Using the measured power of the specular reflected light along with the equivalent scattered light, the reflectivity of the Black Sheet has been calculated for 11 incident angles and three different wavelengths. By comparing the measured reflectivities and their theoretical values and using a Chi-squared fit, the complex refractive indices of the Black Sheet have been found. The results show a decrease in both the real and imaginary parts of the refractive index as the wave length of the incident light is increased.
The accuracy of the results can be increased by using a more accurate experimental set-up. Also, a more accurate model accounting for the scattering phenomenon could lead to improved results. They both could be the scope of a future work of this thesis.
REFERENCES


