An Indoor Optical Wireless Location
Comparison between an Angular Receiver and an Image Receiver

by

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Abstract

In this work, the positioning accuracies of two novel photoreceivers are demonstrated. The two photoreceivers, namely an angular receiver and an image receiver, estimate their position via triangulation by measuring the angle of arrival (AOA) of light from LED optical beacons. The angular receiver consists of three PDs assembled in a corner-cube structure, while the image receiver consists of a custom-made microlens over top of a CMOS array image sensor. The mean AOA accuracy of the angular receiver was found to be 2° whereas the mean AOA accuracy of the image receiver was found to be 0.5°. The effect of LED optical beacon and photoreceiver geometry was quantified in terms of Dilution of Precision (DOP). The position accuracy of the photoreceivers was quantified while static and in motion. In the static case, the mean position accuracy of the angular receiver was found to be 5 cm whereas the mean position accuracy of the image receiver was found to be 2.5 cm. While the photoreceivers were in motion, the mean position accuracy of the angular receiver was found to be on the order of 10 cm whereas the mean position accuracy of the image receiver was found to be 4 cm.
Preface

This work has been done under the supervision of Dr. Richard Klukas. Portions of this work have been published in the following journals, book chapter and conference papers.


Portions of this work have been submitted for publication in:
Preface

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<td>differential photocurrent</td>
</tr>
<tr>
<td>$\Delta \theta$</td>
<td>absolute difference between measured and true $\theta$</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>error in the $x$ coordinates</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>error in the $y$ coordinates</td>
</tr>
<tr>
<td>$\Delta z$</td>
<td>error in the $z$ coordinates</td>
</tr>
<tr>
<td>$d$</td>
<td>distance between transmitter and receiver</td>
</tr>
<tr>
<td>$d_0$</td>
<td>reference distance</td>
</tr>
<tr>
<td>deg</td>
<td>degrees</td>
</tr>
<tr>
<td>$D$</td>
<td>microlens diameter</td>
</tr>
<tr>
<td>$E()$</td>
<td>expected value operation</td>
</tr>
<tr>
<td>$E$</td>
<td>filter intensity</td>
</tr>
<tr>
<td>$\phi$</td>
<td>azimuthal angle</td>
</tr>
<tr>
<td>$\phi'$</td>
<td>azimuthal angle measured with respect to photoreceiver’s body frame</td>
</tr>
<tr>
<td>$\phi_{IS}$</td>
<td>image sensor azimuthal angle</td>
</tr>
<tr>
<td>$\phi_R$</td>
<td>azimuthal angle rotation</td>
</tr>
<tr>
<td>$f$</td>
<td>lens focal length</td>
</tr>
<tr>
<td>$f_H$</td>
<td>high frequency cut-off</td>
</tr>
<tr>
<td>$f_l$</td>
<td>low frequency cut-off</td>
</tr>
<tr>
<td>$f$-number</td>
<td>ratio of lens focal length to its diameter</td>
</tr>
</tbody>
</table>
List of Symbols

\( G \)  
colour green

\( h \)  
vertical height between receiver and LED grid

\( H \)  
Hue of a colour

\( H \)  
matrix of partial derivatives

\( \text{Hz} \)  
Hertz

\( i_n \)  
photocurrent generated by angular receiver

\( i_1 \)  
photocurrent generated by PD\(_1\)

\( i_2 \)  
photocurrent generated by PD\(_2\)

\( i_3 \)  
photocurrent generated by PD\(_3\)

\( I \)  
intensity function

\( k \)  
constant

\( K \)  
maximum number of LEDs

\( \mu \)  
micro

\( m \)  
metres

\( \text{mm} \)  
millimetres

\( \text{mW} \)  
milliwatts

\( M \)  
mega

\( \mathbf{M} \)  
vector of measured angle errors

\( n \)  
path loss exponent

\( \rho_{IS} \)  
focal spot length

\( \mathbf{P}^w \)  
coordinates defined with respect to an image plane

\( P_0(dB) \)  
average received signal power at \( d_0 \)

\( P_1 \)  
output power from PD\(_1\)

\( P_2 \)  
output power from PD\(_2\)

\( P_3 \)  
output power from PD\(_3\)

\( \mathbf{P}_c \)  
coordinates defined with respect to a camera coordinate system

\( \mathbf{P}_l \)  
coordinates defined with respect to a local coordinate system

\( P_r \)  
received optical power

\( P_t \)  
transmitted optical power

\( p \)  
LED grid spacing

\( \mathbf{P} \)  
vector of measured position errors

\( P'(dB) \)  
average received signal power

\( \theta \)  
polar angle

\( \theta' \)  
polar angle measured with respect to photoreceiver’s body frame

\( \theta_R \)  
polar angle rotation

\( r \)  
range

\( R \)  
colour red
List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$R_1$</td>
<td>buffer circuit resistor</td>
</tr>
<tr>
<td>$[R]$</td>
<td>rotational matrix</td>
</tr>
<tr>
<td>$\sigma_{\text{dB}}$</td>
<td>shadowing standard deviation</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>angle error measurement standard deviation</td>
</tr>
<tr>
<td>$\sigma_P$</td>
<td>position error standard deviation</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>$x$ error standard deviation</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>$y$ error standard deviation</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>$z$ error standard deviation</td>
</tr>
<tr>
<td>$s$</td>
<td>seconds</td>
</tr>
<tr>
<td>$S$</td>
<td>saturation of a colour</td>
</tr>
<tr>
<td>$\text{tr}()$</td>
<td>trace operation</td>
</tr>
<tr>
<td>$T$</td>
<td>transpose operation</td>
</tr>
<tr>
<td>$[T]$</td>
<td>translational vector</td>
</tr>
<tr>
<td>$u$</td>
<td>pixel shift in $x$ direction</td>
</tr>
<tr>
<td>$v$</td>
<td>pixel shift in $y$ direction</td>
</tr>
<tr>
<td>$V$</td>
<td>intensity of a colour</td>
</tr>
<tr>
<td>$V_1$</td>
<td>output voltage from PD$_1$</td>
</tr>
<tr>
<td>$V_2$</td>
<td>output voltage from PD$_2$</td>
</tr>
<tr>
<td>$V_3$</td>
<td>output voltage from PD$_3$</td>
</tr>
<tr>
<td>$w$</td>
<td>Least Squares weights</td>
</tr>
<tr>
<td>$W$</td>
<td>window function</td>
</tr>
<tr>
<td>$W$</td>
<td>watt</td>
</tr>
<tr>
<td>$\hat{x}$</td>
<td>Least Squares $x$ estimate</td>
</tr>
<tr>
<td>$x'$</td>
<td>$x$ coordinate of the angular receiver body frame</td>
</tr>
<tr>
<td>$x''$</td>
<td>$x$ coordinate of a pixel defined in an image plane</td>
</tr>
<tr>
<td>$x_R$</td>
<td>receiver $x$ position</td>
</tr>
<tr>
<td>$x_T$</td>
<td>transmitter $x$ position</td>
</tr>
<tr>
<td>$X_c$</td>
<td>$x$ coordinate defined in a camera coordinate system</td>
</tr>
<tr>
<td>$X_l$</td>
<td>$x$ coordinate defined in a local coordinate system</td>
</tr>
<tr>
<td>$\hat{y}$</td>
<td>Least Squares $y$ estimate</td>
</tr>
<tr>
<td>$y'$</td>
<td>$y$ coordinate of the angular receiver body frame</td>
</tr>
<tr>
<td>$y''$</td>
<td>$y$ coordinate of a pixel defined in an image plane</td>
</tr>
<tr>
<td>$y_R$</td>
<td>receiver $y$ position</td>
</tr>
<tr>
<td>$y_T$</td>
<td>transmitter $y$ position</td>
</tr>
<tr>
<td>$Y_c$</td>
<td>$y$ coordinate defined in a camera coordinate system</td>
</tr>
<tr>
<td>$Y_l$</td>
<td>$y$ coordinate defined in a local coordinate system</td>
</tr>
</tbody>
</table>
List of Symbols

\( \hat{z} \)  Least Squares \( z \) estimate
\( z' \)  \( z \) coordinate of the angular receiver body frame
\( Z_c \)  \( z \) coordinate defined in a camera coordinate system
\( Z_l \)  \( z \) coordinate defined in a local coordinate system
## List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>2-D</td>
<td>two-dimensional</td>
</tr>
<tr>
<td>3-D</td>
<td>three-dimensional</td>
</tr>
<tr>
<td>AOA</td>
<td>Angle of Arrival</td>
</tr>
<tr>
<td>CMOS</td>
<td>Complementary Metal Oxide Semiconductor</td>
</tr>
<tr>
<td>DAQ</td>
<td>Data acquisition</td>
</tr>
<tr>
<td>DOP</td>
<td>Dilution of Precision</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FOV</td>
<td>Field-of-View</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>HSV</td>
<td>Hue-Saturation-Value</td>
</tr>
<tr>
<td>HPF</td>
<td>High pass filter</td>
</tr>
<tr>
<td>IMU</td>
<td>Inertial Measurement Unit</td>
</tr>
<tr>
<td>INS</td>
<td>Inertial Navigation System</td>
</tr>
<tr>
<td>IR</td>
<td>Infrared</td>
</tr>
<tr>
<td>LabVIEW</td>
<td>Laboratory Virtual Instrument Engineering Workbench</td>
</tr>
<tr>
<td>LED</td>
<td>Light Emitting Diode</td>
</tr>
<tr>
<td>LOP</td>
<td>Line of Position</td>
</tr>
<tr>
<td>LPF</td>
<td>Low pass filter</td>
</tr>
<tr>
<td>MATLAB</td>
<td>Matrix Laboratory</td>
</tr>
<tr>
<td>MEMS</td>
<td>MicroElectroMechanical systems</td>
</tr>
<tr>
<td>NI</td>
<td>National Instruments</td>
</tr>
<tr>
<td>NOA</td>
<td>Norland Optical Adhesive</td>
</tr>
<tr>
<td>OWL</td>
<td>Optical Wireless Location</td>
</tr>
<tr>
<td>PD</td>
<td>Photodiode</td>
</tr>
<tr>
<td>PDOP</td>
<td>Position Dilution of Precision</td>
</tr>
<tr>
<td>PS3</td>
<td>Play station 3</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>RFID</td>
<td>Radio Frequency Identification</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Squared</td>
</tr>
<tr>
<td>RSS</td>
<td>Received Signal Strength</td>
</tr>
</tbody>
</table>
### List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEM</td>
<td>Scanning Electron Microscope</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to noise ratio</td>
</tr>
<tr>
<td>TDOA</td>
<td>Time difference of arrival</td>
</tr>
<tr>
<td>TOA</td>
<td>Time of Arrival</td>
</tr>
<tr>
<td>TOF</td>
<td>Time of Flight</td>
</tr>
<tr>
<td>TOT</td>
<td>Time of Transmission</td>
</tr>
<tr>
<td>UWB</td>
<td>UltraWide Band</td>
</tr>
<tr>
<td>VLC</td>
<td>Visible Light Communication</td>
</tr>
<tr>
<td>WiFi</td>
<td>Wireless Fidelity</td>
</tr>
<tr>
<td>WLAN</td>
<td>Wireless Local Area Network</td>
</tr>
</tbody>
</table>
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Dedication

To my Yasmin
Chapter 1

Introduction

Indoor positioning is concerned with navigating, tracking or monitoring people or objects inside buildings. Typical examples include monitoring patients in hospitals [1], locating assets stored in a warehouse [2], and robot [3], [4] and pedestrian [5] navigation.

The most well known positioning system, the Global Positioning System (GPS), works well outdoors as long as there is a clear view between the GPS receiver and the satellites. However, inside buildings or tunnels, heavy attenuation of the GPS satellite signals results in poor positioning accuracy or no position estimate at all. To remedy this, several indoor positioning techniques have been proposed in the literature such as positioning using a wireless local area network (WLAN) [6], infrared (IR) sensors [7], ultra-wide band (UWB) [8], ultrasound [9], vision analysis [10]-[11], and cellular systems [12]. These techniques vary in terms of cost, positioning accuracy, security and complexity.

Positioning based on WLAN relies on a mobile device measuring the received signal power from WiFi transmitters and uses the propagation model of the radio frequency (RF) channel to determine the position of the mobile device. Although WLAN positioning is simple, it is considered a coarse measure of position, with errors as large as a few meters. Positioning based on IR sensors requires installing a grid of IR transmitters on the ceiling, with each IR transmitter covering a certain region. A user with an IR receiver determines his or her position based on how close he or she is to any given transmitter. This positioning technique, called proximity detection, suffers from absolute power fluctuations and, therefore, requires the power of the transmitters to be synchronized. In UWB systems, distance information is extracted by measuring the propagation delay between several transmitters and the receiver to be positioned, i.e., a mobile device. Accuracies in the centimeter range can be achieved. However, this comes at the cost of expensive transceivers. RF angle of arrival (AOA) positioning requires an antenna array at the receiver. This is done by mounting two or more antennas on the receiver with known fixed locations relative to one another. By measuring the phase of the signal received by each antenna, the direction of the signal
1.1 Motivation

is calculated. The drawback of using AOA positioning is the cost and size of the antenna array. Hence it will not be practical for small, low complexity receivers. Ultrasound based positioning systems have strong dependencies on the environment such as temperature and, therefore, require continuous calibration to achieve accurate position estimates. Moreover, ultrasound systems are limited to ranges of a few meters. Vision based systems such as cell phone embedded cameras rely on comparing features in an image to a database of images. If the environment changes, the database needs to be updated increasing complexity. In addition, in order to identify features one must solve an optimization problem which may or may not converge, thus rendering it unreliable.

The above mentioned techniques suffer from at least one limitation such as high cost (UWB systems), multipath (WLAN), power issues (IR systems), or calibration issues (ultrasound and camera based systems). These limitation, and others, that will be described in Chapter 2, render these systems impractical for small, low-complexity receivers.

This thesis studies the feasibility of obtaining very accurate indoor positioning using optical frequencies, specifically those of visible light. A characterization and performance comparison of two photoreceivers is presented.

Chapter 1 is organized as follows. Section 1.1 discusses the motivation for the proposed work. Section 1.2 defines the thesis objectives. Section 1.3 shows the thesis outline.

1.1 Motivation

Recently, a growing interest has focused on using light-emitting diode (LED) optical beacons for indoor positioning [13] [14]. This concept called Optical Wireless Location (OWL) is based on work performed on visible light communication (VLC). VLC [15] [16] [17] [18] [19] [20] possesses several advantages over conventional RF systems such as less susceptibility to interference and increased security since light rays are blocked by walls. For VLC the optical transmitters used in the literature are either fluorescent light or LED optical beacons. LED optical beacons are a more favourable choice because they have a longer lifetime, smaller size and higher efficiency, compared to fluorescent light, and they can be modulated at high frequencies (MHz range) making them suitable for high data rate communications.

Using VLC is especially advantageous for low-complexity systems since suitable lighting infrastructure may already exist inside buildings. In a number of research papers, visible light positioning has been simulated and
1.1. Motivation

yielded centimeter-level accuracy [21], [22], [14]. However, the literature lacks a complete investigation of photoreceiver positioning, depending upon whether the receiver is a photodiode (PD) or an image receiver.

OWL systems in [13], [14] use Time Difference of Arrival (TDOA) with LED optical beacons modulated at MHz frequencies (wavelength on the order of 100 m). This results in very poor distance resolution since the photoreceiver needs to move on the order of the wavelength to detect a change in phase. Results shown in [13], [14] were either simulated or no comment on position accuracy was noted.

In this thesis an indoor angular receiver positioning system that employs a corner-cube PD structure to measure AOA of light emanating from LED optical beacons mounted on the ceiling is described. The angular receiver system proposed here aims to solve all of the above shortcomings of visible light positioning techniques. The proposed angular receiver structure provides a simple and effective way for position estimation, since it combines the simplicity of the proximity detection technique and the accuracy of AOA positioning systems. Each of the corner-cube photoreceiver sides generates a photocurrent proportional to the intensity of the light from a LED optical beacon source that strikes it. The geometry of the structure and the photocurrent intensity allows the AOA of LED optical beacons to be determined and hence the position of the corner-cube photoreceiver structure (angular receiver) to be estimated.

The angular receiver estimates AOA using a differential corner-cube PD receiver structure. The angular receiver can estimate AOA with average angular accuracies of $2^\circ$ which translates to positioning accuracies for indoor applications on the order of centimeters. Unlike the TDOA positioning systems in [13], [14], which require synchronization between the photoreceiver and LED optical beacons, the proposed system does not require synchronization. This is advantageous since the positioning accuracies of [13], [14] depend on the clock precision, and also require power for the clocks adding to the cost of the system. Comparing the proposed angular receiver system to [23] which is based on proximity detection, both systems are low-complexity. However, [23] requires that the photoreceiver see only one light source at a time, since all light sources emit at the same frequency. This is impractical. Another issue with proximity detection systems is that they require the LED optical beacons to be power synchronized. For the proposed angular receiver system, LED powers need not be synchronized, since the angular receiver uses the relative intensity on each of the corner-cube sides to determine an AOA.

Based on the above comparison, the proposed system is better in terms of
1.1. Motivation

practicality and complexity. In addition, the retroreflecting phenomenon of the corner-cube photoreceiver allows it to be used for optical communication as well [24].

A second indoor positioning technique using an image receiver (camera) is investigated in this thesis. The image receiver is made up of a CMOS sensor with a microlens (\(\mu m\) size). The image receiver estimates its position by way of measuring the AOA from different LED optical beacons focused on the CMOS sensor. Image receivers have been previously used for positioning [22], [21], [25]. However, these receivers have a long focal length of 2 cm and a narrow field-of-view (FOV). In the proposed image receiver a \(\mu m\) sized, electro-dispensed, polymer lens will be used to achieve a wider FOV and a shorter focal length than those in [22], [21], [25] in order to make the image receiver compact and, therefore, more practical from a user perspective.

In order to determine which LED optical beacon on the ceiling corresponds to which spot on the CMOS sensor, the authors in [22] proposed using different coloured LED optical beacons. A Hue, Sensitivity, and Intensity (HIS) algorithm was used in [22] to detect the colour of the LED optical beacons. Simulated positioning accuracy was on the order of half a meter. For the proposed image receiver system, a more practical LED optical beacon configuration is presented in which all LED optical beacons emit white light but with different modulation frequencies for each LED optical beacon.

Using the proposed image receiver, a more compact structure with a wider FOV than that of other receivers in the literature is proposed. The proposed image receiver is able to triangulate its position from LED optical beacons emitting white light with an AOA accuracy of 0.5°.

Therefore, in this thesis, two OWL receiver systems are investigated. The first system is the angular receiver (corner-cube photoreceiver) and the second system is the image receiver (camera). The two photoreceivers represent two extremes. The angular receiver consists of three orthogonal photodetectors, which is the bare minimum needed for AOA computation. On the other hand, the image receiver is made up of a large number of photodetectors (pixels). The angular receiver has low resolution, while the image receiver has high resolution. The performance of the two photoreceivers is investigated in terms of parameters such as AOA accuracy, positioning accuracy, optical beacon and receiver geometry, and speed and accuracy of obtaining reliable AOA measurements.

The specific contributions of this work compared to the state-of-the-art are as follows.
1.2 Research Objectives

The objective of this research is to develop and test two indoor positioning systems that use LED optical beacons as transmitters and two novel receivers, namely an angular receiver and an image receiver. A comparison between the angular receiver positioning system and the image receiver positioning system in terms of performance will be investigated. This primary objective will be achieved by pursuing the following secondary objectives.

1. Demonstrate how a novel corner-cube photoreceiver (the angular receiver), originally developed for OWC, can be used for indoor positioning at the cm-level using white light LEDs that can also serve as room lights.

2. This thesis also demonstrates how a simple image receiver, made to be extremely compact with wide-FOV microlenses can be used for indoor positioning at the cm-level using white light LEDs that can also serve as room lights.

3. Finally, this thesis also demonstrates that indoor positioning with the above novel receivers is possible while the receivers are in motion.

1.2 Research Objectives

1. Determine the feasibility of measuring the AOA of light with the angular receiver and the image receiver.

2. Characterize the angular accuracy of the measured AOA for both receivers.

3. Quantify the effect of optical beacon geometry on system positional accuracy.

4. Characterize the angular and position accuracy of both receivers while in motion for various measurement update frequencies and receiver speeds.

1.3 Thesis Outline

This thesis is organized as follows. Chapter 2 presents an overview and analysis of current indoor positioning techniques. Chapter 3 presents the angular receiver AOA measurements and characterization, whereas Chapter 4 presents the image receiver AOA measurements and characterization.
1.3. Thesis Outline

A performance comparison between the performance of the angular receiver and the image sensor is given in Chapter 5 for both receivers while in motion. Concluding remarks are made in Chapter 6.
Chapter 2

Indoor Positioning Techniques

Various types of indoor positioning techniques will be discussed in this chapter. The advantage and disadvantage of each of these techniques will be addressed. These techniques include WLAN positioning, Radio Frequency Identification (RFID), Ultrasound, IR positioning, positioning using a camera and positioning using visible light.

Indoor positioning techniques rely on measuring signal parameters such as the Received Signal Strength (RSS), the Time of Arrival (TOA), and the AOA of the propagating signal. Another common technique uses a camera and is commonly known as scene analysis. Positioning using inertial sensors is also discussed.

The aforementioned techniques will be discussed in the following sections. Section 2.1 explains the fundamentals behind RSS positioning. Positioning techniques based on TOA are discussed in Section 2.2. Camera based positioning is explained in Section 2.3. Positioning based on inertial sensors is shown in Section 2.4. Positioning based on visible light is demonstrated in Section 2.5. Section 2.6 provides a summary of the indoor positioning techniques discussed.

2.1 Received Signal Strength

Positioning based on RSS measurements can be achieved via trilateration, fingerprinting or proximity detection. Trilateration is the process of finding the position of an object by measuring ranges to three or more devices with known fixed positions. RSS positioning based on trilateration is as follows [26]. A receiver with unknown position in a wireless local area network measures the power of signals arriving from three or more WiFi access points with known position. RSS measurements depend heavily on the environment. This is described by a quantity called shadowing, which is the attenuation of signals due to objects between the transmitter and receiver.
2.1. Received Signal Strength

The average received signal power at the receiver is given by

$$P(dB) = P_0(dB) - 10n\log \frac{d}{d_0}$$ \hspace{1cm} (2.1)

where $P(dB)$ is the average received signal power at distance $d$ and $P_0(dB)$ is the received signal power at a reference distance $d_0$. The variable, $d$, represents the distance between the transmitter and receiver. The path loss exponent, $n$, depends on the environment between the transmitter and receiver. Typical values for $n$ for indoor non-light of sight environments range between 3 and 6 [27]. The difference between the measured received power and its average is modeled as a log normal distribution with mean $10n\log \frac{d}{d_0}$ and with a shadowing standard deviation of $\sigma_{dB}$, that ranges from 4 to 12 dB [27]. The relationship between the distance between the receiver and transmitter and their respective coordinates is

$$d = \sqrt{(x_T - x_R)^2 + (y_T - y_R)^2}$$ \hspace{1cm} (2.2)

where $(x_T, x_T)$ are the known transmitter coordinates and $(x_R, y_R)$ are the receiver coordinates to be estimated. Once the distance, $d$, is found from equation 2.1, $(x_R, y_R)$ can be found from equation 2.2. For the system demonstrated in [26], a positioning accuracy of 4 m was obtained.

A more sophisticated positioning technique that uses RSS is known as fingerprinting. It includes an offline and an online phase. A system known as RADAR [6] estimates position based on the 802.11 WLAN. Three base stations, with known positions, were deployed in an office environment with dimensions 43.5 m by 22.5 m. The offline phase entails building an extensive look up table containing the measured received power and the corresponding actual transmitter receiver separation. The online phase entails making RSS measurements and comparing these RSS values with the look up table to interpolate a position estimate. The main drawback of this technique is the high dependence on the environment. If the office environment changes (i.e. furniture rearranged) from one day to the next, then the entire calibration process needs to be redone. Accuracies of 2-3 m for stationary users and 3.5 m for mobile users were obtained.

Another positioning technique that uses RSS measurements is proximity detection. In proximity detection, the receiver with unknown position measures the received power from different transmitters. The receiver position is simply the position of the transmitter with the highest measured signal strength. In [7] a system that compares the received optical powers from an indoor infrared grid was introduced. The drawback of this technique is the
Position estimation based on TOA works by measuring the time it takes for a signal to travel from the transmitter (with known fixed position), to the receiver (the position of which is to be estimated) [31]. Geometrically, this provides a circle, centered on the transmitter and of radius equal to...
the range between the transmitter and receiver, on which the receiver must lie. This circle, consisting of possible locations of the receiver, is called a line of position (LOP). The intersection of three or more circles provides the receivers position in two dimensions (2-D). For three dimensional (3-D) positioning, the LOPs become spheres. The range or distance between the transmitter and the receiver is the difference between the TOA and the time of transmission (TOT), multiplied by the speed of propagation. The drawback of the TOA technique is that the transmitters must be synchronized with the receiver, and line of sight (LOS) propagation is required. TDOA systems overcome the need for synchronization between the transmitters and receiver, but still require the transmitters to be synchronized. The receiver measures the difference in travel time between signals from different transmitters. The difference in travel times between any two transmitters forms a LOP. This LOP is a hyperbola with a constant range difference from the two transmitters. The intersection of two hyperbolas gives the receiver position.

In [32], the position of a cellular phone was sought by measuring the TOA of signals from different cellular base stations. The standard deviation of the ranging errors reached 10’s of meters. Clearly using such a cellular system method to find position would be unsuitable when navigating inside a building. Other solutions include augmenting cellular positioning with WLAN signal strength data. Results show mean positioning accuracies of 5 m [33].

RF systems that employ TOA/TDOA are usually expensive or suffer from severe attenuation indoors. For instance UWB transceivers reduce multipath effects by spreading the signal energy over a wide range of frequencies. Positioning accuracies on the order of a few centimeters have been recorded for UWB systems [34]. However, UWB transceivers require precise clock synchronization and are, therefore, costly. The problem with computing position by measuring the TOA with RF systems is that to resolve a distance of 1 m, a clock with a resolution in the order of nanoseconds is required. This condition can be relaxed by using ultrasound systems for positioning which have lower clock resolution due to their low propagation speed [35]. Centimeter-level accuracies have been reported in [35]. However, the range of ultrasonic transmitters is limited to a few meters and ultrasound speed is correlated to temperature, which needs to be calibrated.

3-D cameras used in gaming applications utilize the Time of Flight (TOF) of a near IR signal to determine how far a user is away from the 3-D camera. The near IR signal is reflected from the user’s body and the reflected signal is captured by the camera’s CMOS sensor. Signal process-
2.3 Scene Analysis

Positioning based on scene analysis is most common in mobile robot positioning with a camera [3],[10] or in pedestrian positioning with embedded cell phone cameras [5]. It operates by first mapping features in captured images to stored images in a database. The stored image feature point coordinates in an image correspond to a known reference 3-D position with respect to a reference frame in the real world. By tracking the change in the positions of features in an image, rotational and translational transformations can be used to find the position and orientation of the robot/user. In [11], two ways have been employed to find position. The first is the naive approach. In the naive approach, a user takes an image of the surrounding and an algorithm matches the features in the image to find the closest match to images in the database. Once a match is found, the location of the matched image in the database with respect to a reference frame in the real world is now the location of the user. The second is called the hierarchical approach, where images corresponding to similar objects are grouped together. For instance, images in a particular room are grouped under one hierarchy and the algorithm compares the captured image with the database to see which room the user is in. The main advantage of this technique is that it speeds up the search process since the system has fewer images to search through. However, the algorithm will run into an infinite loop if it confuses the image with an image in a different room.

Positioning using a camera relies heavily on identifying features in an image. To identify features, various algorithms in the literature have been proposed such as colour histograms, shape matching, and the Harris corner detector [37]. Harris corner detection operates by dividing an image into 64 patches of fixed size and then searches for the best (i.e., most distinctive) patch. Features that do not change with the viewing angle are desirable. These features are usually the corners of a room. A filter intensity equation, $E$, is given by

$$E(u, v) = \sum W(x, y)[I(x + u, y + v) - I(x, y)]^2,$$

(2.3)

where $u$ and $v$ represent the pixel shift in the $x$ and $y$ directions respectively, $W$ is the window function, e.g., rectangle with pixel centre at $(x, y)$ and $I$
represents the intensity function. The filter is applied to each of the patches. For constant, but not very distinctive patches, \( E \) is minimum because the intensity difference, \( I(x+u, y+v) - I(x, y) \), does not change much. However, for a good feature in a distinctive patch the intensity equation will be a maximum. The match is that patch that maximizes the above function. Once the features are identified, the camera’s position and orientation are estimated with respect to a local reference frame.

Camera calibration takes place to determine the relationship between feature points in an image (camera coordinates) and where they are located in a local coordinate system (the positions of the features are assumed to be known before hand such as in [38]). The calibration phase involves finding the following:

- **Intrinsic parameters**: Finds the relation between image/pixel coordinates and camera coordinate system (uses a pin-hole camera model).

- **Extrinsic parameters**: Defines the location of the camera coordinates with respect to a local coordinate system, i.e., finds the position and orientation of the camera with respect to a local coordinate system.

Fig. 2.1 shows a 3-D camera coordinate system \((x, y, z)\) whose origin \(O\) represents the centre of projection, and \(z\) is along the optical axis. A point \(P_c\) at coordinates \((X_c, Y_c, Z_c)\) in the camera coordinate system will appear at point \(P'' (x'', y'', f)\) in the 2-D image plane defined with coordinates \((x'', y'')\). The relationship between the two coordinate systems is found from the pin-hole camera model using similar triangles, such that.

\[
x'' = f \frac{X_c}{Z_c}, \quad (2.4)
\]

and

\[
y'' = f \frac{Y_c}{Z_c}. \quad (2.5)
\]
The extrinsic parameters map the relationship between the camera coordinate system and the local coordinate system. Let point $P''$ be defined with respect to a local reference frame at $P_l = [X_l \ Y_l \ Z_l]^T$ and with respect to the camera reference frame at $P_c = [X_c \ Y_c \ Z_c]^T$. The relationship between the two frames is

$$[P_c]_{3 \times 1} = [R]_{3 \times 3} [P_l]_{3 \times 1} + [T]_{3 \times 1} \quad (2.6)$$

where $[R]$ is the rotational matrix, and $[T]$ is the translational vector along the $x$, $y$ and $z$ coordinates. By substituting equations 2.4 and 2.5 into equation 2.6, the orientation and position of the camera is computed from two or more point features with known positions.

In [39], [40] a mobile robot with a camera pointing toward the ceiling is tested for indoor positioning, where the light sources represent the distinctive features. An algorithm is devised to detect the features of the light sources and estimate the robot position. The light sources were spaced in close packed grids of 10 cm. Centimeter-level positioning accuracies were attained. The crux of using a camera for positioning relies on how efficiently the algorithm can detect features in an image. Factors such as a change in the camera’s view point and nonlinear changes in illumination in a room can make the feature detection algorithm fail and, therefore, result in no position estimate.


2.4 Inertial Navigation System

Positioning based on Inertial Navigation System (INS) uses accelerometers, gyroscopes, and magnetometers, to estimate a user/robot position with dead reckoning. Although tactical grade INS can be used to provide very accurate position estimates, they are heavy and expensive [38]. With advancements in Microelectromechanical systems (MEMS) technology, INS sensors are packaged into increasingly smaller Inertial Measurement Units (IMU) that are much smaller and cheaper. A typical use of MEMS INS is in adjusting screen-view orientation in cell-phones. INS sensors typically consist of triaxial gyroscopes, triaxial accelerometers and triaxial magnetometers. The choice of MEMS sensors for navigation depends on several factors such as bias stability, bandwidth and noise.

Accelerometers and gyroscopes provide relative measurements, while magnetometers provide absolute measurements. Gyroscopes are used to provide heading information by integrating the gyroscope’s angular velocity over time to estimate heading. Gyroscopes suffer errors due to temperature bias. Accelerometers determine translation by measuring the translational acceleration and double integrating it over time. Magnetometers are used to provide absolute heading information by measuring the earth’s magnetic field. However, magnetometers suffer from strong interference indoors from objects such as copiers making them non-benefical indoors. However, in outdoor environments, they suffer less interference [41].

After some time, due to biases in the sensor measurements, the INS system must be recalibrated. Since the INS is a relative positioning system, INS is often augmented with other absolute positioning systems to recalibrate the INS sensor position. In [28], RFID positioning was used to periodically correct a MEMS INS sensor measurement. Indoor positioning accuracies were 1-2 m. In robotics [42] and pedestrian [43] navigation, INS and vision-based positioning is a popular mix.

2.5 Visible Light

Indoor Optical Wireless Location (OWL) systems utilizing visible light were first proposed by [17]. An OWL system consists of light sources, typically LED optical beacons, and a photoreceiver. The photoreceiver can either be one or more PD [13] or an image receiver (camera) [17].

In [17], indoor positioning was simulated with three LED optical beacons mounted on the ceiling at known fixed positions. An image receiver, with a
2.5. Visible Light

FOV of 45°, captures an image of the LED optical beacons. By calculating the relative positions of the optical beacons on the image with respect to one another, and knowing the focal length of the lens, the position of the image sensor was inferred using similar triangles. The focal length of the lens was 2 cm.

In [21] indoor positioning was demonstrated with a dual camera (stereo camera) compared to the monocamera presented in [17]. Similar to [17] LED optical beacons were simulated to be on the ceiling, and the position of the stereo camera was calculated from geometry. Each of the cameras had a FOV of 45°. The focal length of the lenses of the cameras was 2.7 cm. Only simulated results were presented. The work presented in [17] and [21] used white LED optical beacons. In order to infer which LED optical beacon is being imaged, [22] proposes using different colour LED optical beacons.

Indoor OWL systems using PDs were presented in [13],[23] and [14]. In [13] 2-D positioning was investigated using four LED optical beacons and a PD. Two LED optical beacons were intensity modulated at 20 MHz, with one LED phase shifted from the other. A PD, acting as a receiver, estimates the TDOA of the two peaks corresponding to each of the two LED optical transmitters. A second TDOA is calculated from the second pair of LED optical beacons. The PD position is then estimated using hyperbolic trilateration (the intersection of two hyperbolas). A main factor in the accuracy of this positioning technique is the choice of wavelength of the signal used to modulate the LED optical beacons. At 2 MHz (150 m wavelength), the receiver has to move in the order of a wavelength to detect a phase difference. Also, the LED optical beacons need to be synchronized, adding to the cost of the OWL system.

In [14] a similar system employing TDOA is implemented. The authors used three LED optical beacons each modulated at a different frequency. The frequencies were 1, 3 and 5 MHz. The receiver is able to detect the three signals and compute their phase differences and, therefore, TDOA, and a position estimate. No experimental results are given in this paper. Similar to the work in [13], LED synchronization adds to the cost of the OWL system.

In [23], an indoor positioning system based on fluorescent lighting was introduced. The system offers 3-4 m accuracy. The transmitter is a modulated fluorescent light and the receiver is a single PD. The positioning system is based on proximity detection; the closer one is to a given transmitter, the more likely one’s position can be inferred to be that of the transmitter. A drawback of this system is that it can only resolve one modulated light signal at a time. Therefore, the transmitters need to be placed a fixed minimum
distance apart to avoid interference.

2.6 Summary

Table 2.1 provides a summary of previously published empirical positioning techniques. The accuracy of the proposed OWL systems will be compared to these empirical techniques. In conclusion RSS measurements are simpler than their TOA counterpart. However, they suffer from poorer positional accuracy due to environmental effects such as shadowing. INS systems suffer severely from sensor bias drift that must be periodically calibrated by other positioning techniques. Positioning using a camera requires building a map of positions where a user travels, and may not work properly under different lighting conditions. OWL systems are emerging as a strong competitor to other positioning techniques, with promising simulation positioning accuracies. Positioning systems such as UWB, ultrasound, and vision-based system have very good accuracy (cm level), but their disadvantages disqualify them from being the low complexity solution to the indoor positioning problem.
Chapter 3

Angular Receiver Positioning

This chapter presents the angular receiver. A detailed characterization of its angular and positioning performances is quantified. Section 3.1 introduces the angular receiver structure. Section 3.2 presents the angular receiver AOA empirical error characterization. Section 3.3 discusses the effect of Dilution of Precision (DOP) on positioning accuracy. Finally, Section 3.4 presents empirical positioning performance results.

3.1 The Angular Receiver

The angular receiver structure is made up of three orthogonal sides, each of which consists of a silicon PD. The sides form the interior of a corner-cube. The angular receiver is shown in Fig. 3.1.

![The angular receiver](image)

Figure 3.1: The angular receiver.

The angular receiver was developed in the Integrated Optics Lab at the University of British Columbia. The photoreceiver was initially developed for optical communication but has been adapted here for indoor positioning.
3.1. The Angular Receiver

Each PD is a Thorlabs FDS1010, with an active area of $9.7 \times 9.7 \text{ mm}^2$, operating over a range of 400 – 1100 nm and with a maximum responsivity of 0.65 A/W at 1000 nm. The field of view (FOV) of the device spans a solid angle defined by the azimuthal angle, $\phi$, and polar angle, $\theta$, ranging from $0^\circ$ to $90^\circ$ as shown in Fig. 3.2. The angular receiver is a retroreflector allowing light to reflect back to the light source and therefore can be used to provide bidirectional communication.

Figure 3.2: Schematic of the angular receiver showing azimuthal and polar angles and photodiode side numbers.

The angular receiver, first introduced in [44], consists of three orthogonal PDs. The PDs are defined as PD$_1$, PD$_2$, and PD$_3$ in the $y'$-$z'$, $x'$-$z'$, and $x'$-$y'$ planes, respectively, where the $x'y'z'$ coordinate system represents the angular receiver body frame (see Fig. 3.2).

Responsivity in amps (A) per watt (W) is defined as the ratio of the output photocurrent generated by a PD to the input optical power incident on that PD. Responsivity is wavelength dependent, and since the LED optical beacons (OPTEK Technology OVS5MxBCR4) used here have white light (broadband) spectral characteristics, as shown in Fig. 3.3, an effective or average responsivity needs to consider the wavelength-dependent responsivity of the PDs. With this in mind, the spectrum of the white light LEDs was recorded by a spectrometer giving the result shown in Fig. 3.3. The total area under the curve in Fig. 3.3 is normalized to correspond to an optical power of 1 W.
3.1. The Angular Receiver

The normalized spectrum is then multiplied by the known spectral responsivity curve for the PDs, to arrive at the photocurrent curve as a function of wavelength. The total area under the curve (which is a result of an optical power of 1 W) gives an effective responsivity of 0.27 A/W for the LED and PD configuration used in this work.

Sections 3.1.1 and 3.1.2 present the measurements done in order to characterize the angular receiver’s angular and intensity responses.

3.1.1 Angular Response

An incident beam from an LED optical beacon is characterized by an AOA in the body frame with an azimuthal angle $\phi$ defined in the $x'y'$ plane and a polar angle $\theta$ defined relative to the $z'$ axis as shown in Figure 3.2.

In [44], the relationship between the photocurrents, $i_1$, $i_2$, and $i_3$, as a function of the azimuthal angle, $\phi$ and polar angle, $\theta$ were derived and experimentally verified. These expressions are piece-wise functions. For positioning applications one seeks to estimate $\phi$ and $\theta$ given the photocurrent values of each PD. In order to solve for the AOA angles $\phi$ and $\theta$, one needs to invert the expressions in [44]. However, since explicit expressions are difficult to find, an approximation of the differential photocurrents, $i_1-i_3$ and $i_2-i_3$ are formed.

Since there are two unknowns ($\phi$ and $\theta$) and three known photocurrent values, $i_1$, $i_2$, and $i_3$, differential photocurrents are formed such that

Figure 3.3: White LED spectrum.
3.1. The Angular Receiver

\[
\Delta i_1(\phi, \theta) = i_1(\phi, \theta) - i_3(\phi, \theta) \\
\approx C_0 + C_1 \theta + C_2 \phi + C_3 \theta^2 + C_4 \theta \phi + C_5 \phi^2 + C_6 \theta^3 + C_7 \theta^2 \phi + C_8 \theta \phi^2, \tag{3.1}
\]

\[
\Delta i_2(\phi, \theta) = i_2(\phi, \theta) - i_3(\phi, \theta) \\
\approx D_0 + D_1 \theta + D_2 \phi + D_3 \theta^2 + D_4 \theta \phi + D_5 \phi^2 + D_6 \theta^3 + D_7 \theta^2 \phi + D_8 \theta \phi^2. \tag{3.2}
\]

The theoretical piece-wise expressions for \(\Delta i_1(\phi, \theta)\) and \(\Delta i_2(\phi, \theta)\) are approximated with polynomial distributions using a least-angle regression analysis with a root mean squared (RMS) fitting error of 0.2% for the numerical fitting parameters shown in Table 3.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_0)</td>
<td>-9.08 \times 10^{-1}</td>
</tr>
<tr>
<td>(C_1)</td>
<td>1.58 \times 10^{-2}(\circ)^{-1}</td>
</tr>
<tr>
<td>(C_2)</td>
<td>-2.44 \times 10^{-4}(\circ)^{-1}</td>
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<tr>
<td>(C_3)</td>
<td>1.51 \times 10^{-1}(\circ)^{-2}</td>
</tr>
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<td>(C_4)</td>
<td>-1.06 \times 10^{-4}(\circ)^{-2}</td>
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<tr>
<td>(C_5)</td>
<td>8.40 \times 10^{-6}(\circ)^{-2}</td>
</tr>
<tr>
<td>(C_6)</td>
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<tr>
<td>(C_7)</td>
<td>2.32 \times 10^{-6}(\circ)^{-3}</td>
</tr>
<tr>
<td>(C_8)</td>
<td>-2.51 \times 10^{-6}(\circ)^{-3}</td>
</tr>
<tr>
<td>(D_0)</td>
<td>-8.63 \times 10^{-1}</td>
</tr>
<tr>
<td>(D_1)</td>
<td>-1.40 \times 10^{-2}(\circ)^{-1}</td>
</tr>
<tr>
<td>(D_2)</td>
<td>-1.22 \times 10^{-3}(\circ)^{-1}</td>
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<td>3.60 \times 10^{-4}(\circ)^{-2}</td>
</tr>
<tr>
<td>(D_4)</td>
<td>5.58 \times 10^{-4}(\circ)^{-2}</td>
</tr>
<tr>
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</tr>
<tr>
<td>(D_6)</td>
<td>-1.17 \times 10^{-6}(\circ)^{-3}</td>
</tr>
<tr>
<td>(D_7)</td>
<td>-2.31 \times 10^{-6}(\circ)^{-3}</td>
</tr>
<tr>
<td>(D_8)</td>
<td>-2.52 \times 10^{-6}(\circ)^{-2}</td>
</tr>
</tbody>
</table>

To solve for the AOA, the photocurrent values \(i_1, i_2, \text{ and } i_3\) are measured, the differential photocurrent values \(\Delta i_1\) and \(\Delta i_2\) are formed, as shown in equations 3.1 and 3.2, and the two equations are solved simultaneously for \(\phi\) and \(\theta\). Fig. 3.4 shows a graphical representation of normalized \(\Delta i_1(\phi, \theta)\) and \(\Delta i_2(\phi, \theta)\) as a function of \(\phi\) and \(\theta\). The value \(\Delta i_3(\phi, \theta) = i_3(\phi, \theta) - i_3(\phi, \theta)\)
3.1. The Angular Receiver

represents the differential photocurrent zero plane. The intersection of the $\Delta i_1(\phi, \theta)$, $\Delta i_2(\phi, \theta)$, and $\Delta i_3(\phi, \theta)$ planes is the AOA solution. In this case, $\phi = 45^\circ$ and $\theta = 54.7^\circ$ is the solution. The AOA at $(\phi = 45^\circ$, $\theta = 54.7^\circ)$ is a result of balanced photocurrents $i_1$, $i_2$, and $i_3$ due to equal optical illumination on the three PDs. This is apparent in Fig. 3.4 where $\Delta i_1(\phi, \theta)$ and $\Delta i_2(\phi, \theta)$ are symmetric.

Figure 3.4: Analytical results are shown for normalized differential photocurrents $\Delta i_1(\phi, \theta)$ and $\Delta i_2(\phi, \theta)$ versus azimuthal $\phi$ and polar $\theta$ angles.

When an incident light beam from an LED optical beacon strikes the angular receiver, photocurrents $i_1$, $i_2$ and $i_3$ are generated. The amplitudes of these photocurrents are extremely small, in the order of nano-Amperes and the signal is noisy resulting in a poor signal to noise ratio (SNR). In order to increase the SNR, the circuit block diagram shown in Figure 3.5 is designed to first filter out the noise components and then increase the signal power.
3.1. The Angular Receiver

The photocurrents generated by each PD pass through a buffer with a feedback resistance $R_1 = 10\ \text{k}\Omega$. At this stage, the photocurrent is converted to a voltage signal. The high input impedance of the buffer and the large resistance $R_1$ are beneficial in that they decrease thermal noise. The signal then passes through a fourth–order Butterworth high-pass filter to attenuate low frequency components such as the photocurrent due to the 60 Hz room light and its harmonics. Similarly, a second–order Butterworth low-pass filter removes high frequency components such as microwave frequencies. The allowable frequency band for LED optical beacon operation is between 500 Hz and 3 kHz. After filtering out the noise components of the signal and, therefore, increasing the SNR, an amplifier with a gain of 1000 is applied to increase the level of the signal. Fig. 3.6 shows a schematic diagram of the circuit. The circuit was built with discrete components on a prototyping board.

Figure 3.5: Circuit block diagram stages of amplification and bandpass filter.

Figure 3.6: Butterworth bandpass circuit schematic to enhance the photodiode’s output SNR.
3.1. The Angular Receiver

After filtering and amplification, the output voltage corresponding to each PD is connected to a separate channel on the National Instruments (NI) wireless data acquisition (DAQ) unit that transmits the amplitudes of the captured signal values wirelessly to a computer. Using LabVIEW, a graphical programming language, the power spectral density of the captured signal from each PD channel is recorded. The power values $P_1$, $P_2$, and $P_3$ at the particular modulation frequency of the LED optical beacon of interest are recorded in dB, and converted to the corresponding output voltage values $V_1$, $V_2$ and $V_3$. Using the circuit diagram in Fig. 3.6, the corresponding photocurrents are computed by dividing the output voltages corresponding to each PD by the circuit impedance (10 MΩ). Differential photocurrents are then formed as shown in equations 3.1 and 3.2 to solve for the AOA values ($\phi$, $\theta$).

3.1.2 Intensity Response

Intensity independence is a significant distinction between optical positioning using proximity detection, also known as optical RSS, and optical AOA positioning using the angular receiver. Optical RSS-based positioning systems rely on measuring the incident optical power from LED optical beacons. A typical example of optical RSS-based positioning systems is illustrated in [7] where optical transmitters are mounted on the ceiling and a user carrying an optical receiver determines their position based on the signal strength of the optical signal measured by the receiver. The stronger the signal measured from an optical transmitter the closer one is to that transmitter. The major drawback of optical RSS-based systems is the inherent sensitivity to optical beacon grid powers. The system designer must make sure that all optical transmitters operate at the same power level. Any imbalances will render optical RSS-based systems inaccurate.

The proposed angular receiver’s AOA is independent of absolute optical powers being incident on the angular receiver PD₁, PD₂, and PD₃ sides, since the AOA calculation process uses normalized differences between these three powers. However, there exists a minimum intensity threshold, that is configuration-dependent, for optical AOA positioning using the angular receiver. In order to determine the minimum allowable intensity for the proposed positioning system to estimate the AOA reliably, an experiment is performed in which an LED optical beacon is incident along the central axis of symmetry of the angular receiver, at an AOA of $\phi = 45^\circ$ and $\theta = 54.7^\circ$ and with a separation distance of 0.5 m. At this orientation, an initial calibration of the PD responsivities is undertaken where the transimpedance
3.1. The Angular Receiver

amplifier gains connected to each of the three PDs are adjusted to yield balanced (i.e., equal) photocurrents. The AOA is then recorded and plotted versus incident optical intensity by varying the LED optical beacon power as shown in Fig. 3.7. Note that as the light intensity decreases, the AOA angles remain constant, until the light intensity reaches 0.2 $\mu$W/cm$^2$. At that point, the measured angles deviate from the true angles. This is due to large fluctuations in the received power on each of the three PD channels.

The minimum optical transmit power, $P_t$, that would be required to have the LED optical beacon achieve the minimum allowable received optical intensity of 0.2 $\mu$W/cm$^2$ can be found for a typical distance of 0.5 m between the LED optical beacon and receiver. The minimum optical transmit power for the LED optical beacon would be

$$P_t = (4\pi r^2) \times 0.2 \mu\text{W/cm}^2 = 6.3 \text{ mW}.$$ 

The knowledge of this value is critical for system designers to build optical beacon networks that are able to reliably estimate AOA for larger optical link distances such as those in [7].

![Figure 3.7: Direct intensity characterization of measured azimuthal $\phi$ and polar $\theta$ angles versus incident optical intensity.](image)

For the on-going analysis, the LEDs are operated with their maximum transmit power of 57 mW. With knowledge that the minimum received power intensity is 0.2 $\mu$W/cm$^2$, as shown in Fig. 3.7, it can be concluded that the system can operate with distances between the LED optical beacons
and receiver of up to \( r = \sqrt{\left( \frac{57 \text{ mW}}{0.2 \text{ µW/cm}^2} \right)} = 5.3 \text{ m}. \)

### 3.1.3 Multipath Response

In order to characterize the effect of diffuse reflections, from different objects in the environment, on the accuracy of the AOA measurements, an experiment is carried out where a variety of reflective surfaces (plywood, stainless steel, drywall) are set in the proximity of the angular receiver. Fig. 3.8 shows the diagram of the multipath characterization experiment.

![Figure 3.8: Multipath characterization experiment setup.](image)

The distance between the angular receiver and the LED optical beacon is fixed at 0.5 m. The angular receiver is oriented at \( \phi = 45^\circ \) and \( \theta = 54.7^\circ \), and the incident optical intensity is 1.2 µW/cm². The AOA is measured as the reflective surface distance is changed. The results are shown in Fig. 3.9 as a function of the reflective surface distance, with the angular receiver central axis of symmetry parallel to the reflective surface. Multipath effects, in the form of significant departure of measured \( \phi \) from the true value \( \phi \),
are apparent in the figure for each of the materials when the reflective surface distance is less than approximately 0.5 m. When the reflective surface distances increases above 0.5 m, $\phi$ converges to the true value of $45^\circ$ and $\theta$ converges to the true value of $\theta = 54.7^\circ$.

![Figure 3.9: Reflected intensity characterization of measured azimuthal $\phi$ and polar $\theta$ angles versus reflective surface distance for three surfaces (plywood, stainless steel, drywall).](image)

From Fig. 3.9 one observes that for the current configuration in Fig. 3.8, multipath reflections impact $\phi$ values more than $\theta$ values. Light reflected by the reflective surface will erroneously increase the incident optical intensity of PD$_1$ to a much greater extent than that of PD$_2$. Since $\phi$ is largely a function of $i_1-i_2$, whereas $\theta$ is largely a function of $i_2-i_3$, $\phi$ will be impacted by an inflated value of photocurrent much more so than will be $\theta$.

The total incident optical power on PD$_1$ is the linear sum of the LOS optical power and the NLOS or reflected power. Figure 3.10 shows the percentage of incident optical power that is reflected onto PD$_1$ due to multipath for the case of drywall. One observes that at approximately 20 cm reflective surface distance, 50% of the incident optical power is due to reflections. At approximately 40 cm, 30% of the incident power is reflected and this results in a $\phi$ error of $4^\circ$ compared to $2^\circ$ error at 50 cm reflective surface distance.

In conclusion, optical AOA positioning is most accurate if the reflected
light is approximately 20% or lower of the total incident light. In this case the effect of reflections will be negligible.

![Figure 3.10: Percentage ratio of reflected optical power to total incident optical power on PD1 for drywall.](image)

3.2 Angle-Of-Arrival Measurement Error Characterization

To characterize the angular receiver’s AOA accuracy, the angular receiver is mounted on two gyroscopes (with a precision of 0.5°). One gyroscope lies in the $x'$-$y'$ plane of the angular receiver body frame to adjust the incident $\phi$ angle, and the other gyroscope adjusts the incident $\theta$ angle (see Fig. 3.2). The angular receiver is illuminated by a single LED optical beacon at a fixed known position which ensures an intensity of 1.8 $\mu$W/cm².

The gyroscopes are adjusted such that the output photocurrents from PD₁, PD₂ and PD₃ are approximately equal. Since the three PDs have similar but not necessarily identical responsivities, the calibration of the PDs is carried out to ensure that the three photocurrents are equal (or balanced). This is done by adjusting the three preamplifier gains in the angular receiver electronics to yield balanced photocurrents. At this angular receiver orientation, the measured AOA is equal to the true AOA $\phi = 45^\circ$ and $\theta = 54.7^\circ$. The corresponding reference AOA (true AOA) is read from
3.2. Angle-Of-Arrival Measurement Error Characterization

the markings on the azimuthal and polar gyroscopes.

The gyroscopes are rotated and the AOA $\phi$ and $\theta$ angles measured and compared to the true angles. Each AOA ($\phi$ and $\theta$ pair) that is estimated is the result of 100 averaged power samples at a modulated LED optical beacon frequency of 2.5 kHz. Measured angle errors $\Delta\phi$ and $\Delta\theta$ are defined here as the absolute differences between the measured and true $\phi$ and $\theta$ angles respectively. The angle errors are shown in Figs. 3.11 and 3.12 as a function of $\phi$ and $\theta$. Error trends are apparent. The measured angle errors $\Delta\phi$ and $\Delta\theta$ are at their lowest level in close proximity to the $\phi = 45^\circ$ and $\theta = 54.7^\circ$ central axis of symmetry of the angular receiver. In moving away from this central axis, the errors increase in a way that reflects the structural symmetry. In Fig. 3.11, the measured angle error $\Delta\phi$ is roughly symmetric about the $\phi = 45^\circ$ line, as one would expect by the structures mirror symmetry about a $\phi = 45^\circ$ bisecting plane (see Fig. 3.2). At the same time, the measured angle error $\Delta\phi$ is disproportionately large for small $\theta$ angles, compared to those for large $\theta$ angles. This distinction is seen from illumination asymmetry for small or large $\theta$ angles. When the structure is illuminated near $\theta \approx 0^\circ$, both PD$_1$ and PD$_2$ yield negligible photocurrents and only PD$_3$ yields a high photocurrent. This gives way to large measured angle errors in $\Delta\phi$. When the structure is illuminated near $\theta \approx 90^\circ$, both PD$_1$ and PD$_2$ yield high photocurrents and only PD$_3$ yields a negligible photocurrent. This gives way to low measured angle errors in $\Delta\phi$.

In Fig. 3.12, the measured angle error distribution for $\Delta\theta$ is roughly symmetric about $\phi = 45^\circ$ when $\theta$ approaches $90^\circ$. This can be understood by examining Fig. 3.2, with its symmetry between PD$_1$ and PD$_2$. As $\theta$ is reduced, however, symmetry in the measured angle error $\Delta\theta$ diminishes, and the response becomes dominated by random error.

To deploy the angular receiver in OWL systems, the measured angle errors $\Delta\phi$ and $\Delta\theta$ must be kept below an acceptable level. For this investigation, a mean error of $2^\circ$ is deemed to be acceptable, and this is achieved, given the results in Figs. 3.11 and 3.12, by defining an operational cone of $\phi \times \theta = 40^\circ \times 40^\circ$ through the central axis of symmetry. LED optical beacons illuminating the angular receiver within this operational cone give measured mean angle errors $\Delta\phi$ and $\Delta\theta$ below $2^\circ$. For typical indoor optical link distances of 2 m, an azimuthal error of $2^\circ$ results in a 7 cm positioning error. For applications such as robot positioning an error of 7 cm would be sufficient for robot navigation without hitting obstacles, and for the robot to go through doorways and corridors.
3.2. Angle-Of-Arrival Measurement Error Characterization

Figure 3.11: Measured angle error \( \Delta \phi \) as a function of \( \phi \) and \( \theta \).

Figure 3.12: Measured angle error \( \Delta \theta \) as a function of \( \phi \) and \( \theta \).
3.3 Positioning Analysis Using Dilution of Precision

The term Dilution of Precision (DOP) quantifies the effect of optical beacon and angular receiver geometry on position error standard deviation. The lower the DOP number the lower the position uncertainty and, therefore, the better is the position estimate.

Dilution of precision is defined as the ratio of the position standard deviation \( \sigma_P \) to the measurement standard deviation \( \sigma_m \) as shown in

\[
DOP = \frac{\sigma_P}{\sigma_m}.
\]

(3.3)

DOP is commonly used as shown in equation 3.4 to quantify GPS positioning accuracy, where

\[
\text{Position error} \approx DOP \times \text{Range error}.
\]

(3.4)

The derivation of AOA DOP is presented next. Assume the angular receiver is positioned at \((x, y, z)\) with respect to a known reference frame. Measurements are made for AOA \(\phi_i\) and \(\theta_i\) angles for \(K\) LED optical beacons positioned at \((x_i, y_i, z_i)\), where \(i = 1, 2, \ldots K\). The relationship between the angular receiver position and the AOA angles is given by

\[
\tan \phi_i = \frac{x_i - x}{y_i - y}
\]

(3.5)

and

\[
\tan \theta_i = \frac{\sqrt{(x_i-x)^2+(y_i-y)^2}}{z_i - z}.
\]

(3.6)

The AOA angles \(\phi_i\) and \(\theta_i\) are defined for directions toward the \(i^{th}\) optical beacon.

Expressions 3.7 and 3.8 are fundamental relationships for linking the errors in the existing measured angles to the errors in the estimated position of the angular receiver. The measured angle errors are recorded in the measured angle error vector

\[
M = [\Delta \phi_1 \ \Delta \theta_1 \ \Delta \phi_2 \ \Delta \theta_2 \ \ldots \ \Delta \phi_K \ \Delta \theta_K]^T
\]

(3.7)

where \(\Delta \phi_i\) and \(\Delta \theta_i\) are the respective measured angle errors for \(\phi_i\) and \(\theta_i\) for the \(i^{th}\) optical beacon, and the T superscript denotes the transpose operation. Similarly, the position error vector is
3.3. Positioning Analysis Using Dilution of Precision

\[ P = [\Delta x \ \Delta y \ \Delta z]^T \]  

(3.8)

where \( \Delta x, \Delta y \) and \( \Delta z \) are the respective errors in the \( x, y, \) and \( z \) coordinates. The measured angle errors \( \Delta \phi \) and \( \Delta \theta \) can be linked to the absolute position errors by taking partial derivatives of the observation’s \( \phi_i \) and \( \theta_i \) with respect to the unknown angular receiver position \( x, y, z \). The resulting partial derivative matrix, \( H \), is defined by

\[
H = \begin{bmatrix}
\frac{\delta \phi_1}{\delta x} & \frac{\delta \phi_1}{\delta y} & \frac{\delta \phi_1}{\delta z} \\
\frac{\delta \phi_1}{\delta x} & \frac{\delta \phi_1}{\delta y} & \frac{\delta \phi_1}{\delta z} \\
\vdots & \vdots & \vdots \\
\frac{\delta \phi_K}{\delta x} & \frac{\delta \phi_K}{\delta y} & \frac{\delta \phi_K}{\delta z}
\end{bmatrix}_{2K \times 3}
\]

(3.9)

and the relationship between \( P, H, \) and \( M \) is

\[ M = HP. \]  

(3.10)

Equation 3.10 has \( 2K \) equations and three unknowns \( \Delta x, \Delta y \) and \( \Delta z \). To solve for the position errors, \( \Delta x, \Delta y \) and \( \Delta z \), the over-determined linear system is solved using the method of Least Squares [45] resulting in a position error equal to

\[ P = (H^T H)^{-1} H^T M. \]  

(3.11)

For this AOA-based system, the measured angle errors are assumed to be independent, zero-mean, Gaussian-distributed random variables with equal variance, \( \sigma_m^2 \) [17], [46]. The covariance of equation 3.11 can then be used to express the position error variance as

\[ \sigma_P^2 = \text{tr} \left[ \mathbb{E} \left( PP^T \right) \right] = \text{tr} \left[ \left( H^T H \right)^{-1} \right] \sigma_m^2 \]  

(3.12)

where \( \mathbb{E}(\cdot) \) and \( \text{tr}(\cdot) \) denote the expectation and trace operations respectively. Three-dimensional position DOP (PDOP) is then defined as the ratio of positioning error standard deviation \( \sigma_P \) to the standard deviation of the measured angle errors \( \sigma_m \) such that

\[ \text{PDOP} = \frac{\sigma_P}{\sigma_m} = \sqrt{\frac{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}{\sigma_m}} = \sqrt{\text{tr} \left[ \left( H^T H \right)^{-1} \right]}. \]  

(3.13)

Here, position standard deviations in \( x, y \) and \( z \) coordinates are denoted by \( \sigma_x, \sigma_y \) and \( \sigma_z \), respectively. Note that the PDOP in equation 3.13 is
an AOA-based quantity with units of meters per radian, unlike the unitless PDOP for range-based systems such as GPS. PDOP acts as a weighting factor on the measured angle standard deviation for the calculation of the position standard deviation. The effect of the angle standard deviation on the position standard deviation will depend on the angular receiver position with respect to observable optical beacons. Given two observable optical beacons in close proximity, for example, the angular receiver registers two AOAs with the corresponding LOPs being nearly parallel, which in turn yields a large PDOP and large position standard deviation. Given two observable optical beacons that are well separated, in contrast, the angular receiver registers two AOAs with the corresponding LOPs being nearly orthogonal, which in turn yields a small PDOP and small position standard deviation. For the present analysis, the 3-D positioning error standard deviation $\sigma_P$, results from the measured mean angular error of $\sigma_m = 2^\circ$ in the operational cone and can be found, for a particular OWL system, by calculating the PDOP from 3.13.

In order to visualize the effect of PDOP on positional accuracy, an OWL system is simulated to predict the position standard deviation, $\sigma_P$ in equation 3.13, using PDOP and assuming an AOA standard deviation, $\sigma_m = 2^\circ$. The OWL system is simulated for two and four LED optical beacon configurations as illustrated in Fig. 3.13.

![Figure 3.13: Schematic of the optical AOA positioning system. The $(x',y',z')$ represent the angular receiver’s body frame, while the $(x,y,z)$ represent the navigation reference frame.](image)

For the two optical beacon configuration, the optical beacons are positioned with LED A1 at $(x_1 = 15 \text{ cm}, y_1 = 0 \text{ cm}, z_1 = 50 \text{ cm})$ and with LED
3.3. Positioning Analysis Using Dilution of Precision

LED A₂ at \((x_2 = -15 \text{ cm}, y_2 = 0 \text{ cm}, z_2 = 50 \text{ cm})\), and with the angular receiver scanned across the \(z = 0\) plane. Results are shown as predicted position standard deviation in Fig. 3.14. The position standard deviation values range from 4 to 6 cm. The mean 3-D position error standard deviation is \(\sigma_P = 4.7 \text{ cm}\). Note that the largest position standard deviation occurs in the plane \(y = 0\). This is because LOPs from LEDs A₁ and A₂ in this region are parallel, resulting in larger DOP which subsequently gives rise to large position error standard deviation. The corresponding DOP plot for the two LED beacon configuration is shown in Fig. 3.15 with DOP values ranging from 2 to 3 cm/deg.

![Figure 3.14: The predicted 3-D positioning error standard deviation \(\sigma_p\) for optical AOA positioning with two LED optical beacons A₁ and A₂.](image)

Figure 3.14: The predicted 3-D positioning error standard deviation \(\sigma_p\) for optical AOA positioning with two LED optical beacons A₁ and A₂.
In order to improve the position standard deviation, four LED optical beacons are used in the OWL system. The addition of the two optical beacons improves the OWL system geometry (i.e., lowers DOP). The optical beacons are positioned with LED B1 at \((x_1 = 15 \text{ cm}, y_1 = 15 \text{ cm}, z_1 = 50 \text{ cm})\), LED B2 at \((x_2 = -15 \text{ cm}, y_2 = 15 \text{ cm}, z_2 = 50 \text{ cm})\), LED B3 at \((x_3 = -15 \text{ cm}, y_3 = -15 \text{ cm}, z_3 = 50 \text{ cm})\) and LED B4 at \((x_4 = 15 \text{ cm}, y_4 = -15 \text{ cm}, z_4 = 50 \text{ cm})\). The angular receiver is scanned across the \(z = 0\) plane. Figure 3.16 shows the predicted position standard deviation assuming \(\sigma_m = 2^\circ\). Position error standard deviations lower than those for the two optical beacon case (Fig. 3.14) are apparent. For the four optical beacon grid the position standard deviation values range from 2.7 to 2.8 cm. The mean 3-D position error standard deviation is \(\sigma_P = 2.8 \text{ cm}\). The corresponding DOP plot for the four LED beacon configuration is shown in Fig. 3.17 with DOP values ranging from 1.35 to 1.4 \text{ cm/deg}. The predicted mean 3-D position error standard deviation of \(\sigma_P = 2.8 \text{ cm}\) is a factor of two improvement over the mean predicted 3-D position error standard deviation of \(\sigma_P = 4.7 \text{ cm}\) for the two optical beacon grid.
Figure 3.16: The 3-D predicted positioning error standard deviation $\sigma_p$ for optical AOA positioning with four LED optical beacons B₁, B₂, B₃, and B₄.
3.4. Positioning Performance

For the 2-D and 3-D optical beacon configurations discussed above, one observes that the DOP values vary far less as a function of angular receiver location for the 4 LED optical beacons case than for the 2 LED optical beacons case. From Figs. 3.15 and 3.17 one observes that the PDOP and therefore $\sigma_P$ increases as the receiver nears an optical beacon. This leads to the conclusion that the angular receiver should not use the AOA for that particular beacon in the position estimate calculation. The angular receiver can detect its proximity to an LED optical beacon based on the AOA it measures (the angular receiver is directly below an LED if the measured AOA is $\phi = 45^\circ$, and $\theta = 54.7^\circ$), and the total photocurrent (the sum of $i_1$, $i_2$, and $i_3$) being greater at this location than the neighboring angular receiver locations.

3.4 Positioning Performance

In this section, the performance of conventional RSS optical positioning (which relies on proximity detection) is compared to that of the angular receiver AOA optical positioning technique. The performance analysis quantifies positioning error, which is defined as the Euclidean distance between
the measured and true 3-D positions. Each OWL system is tested with the four optical beacons B₁, B₂, B₃ and B₄ shown in Fig. 3.13. Section 3.4.1 presents the RSS measurement experiment, and Section 3.4.2 presents the AOA measurement experiment.

### 3.4.1 Optical RSS

Optical RSS positioning is tested with a single flat 9.7×9.7 mm² PD acting as the receiver. The PD is rastered across the xy plane at 25 different positions of (0, 0), (0, ±7.5 cm), (0, ±15 cm), (±7.5 cm, 0), (±7.5 cm, ±7.5 cm), (±7.5 cm, ±15 cm), (±15 cm, 0), (±15 cm, ±7.5 cm), (±15 cm, ±15 cm). The incident intensities measured by the PD are used to quantify the respective ranges to the four LEDs. The calibration and the process of finding range is performed as follows:

1. The received electrical power is measured using the PD connected to the circuit in Fig. 3.5 in section 3.1.1.
2. The equivalent photocurrent from the PD is calculated (µA).
3. Knowing the average responsivity of the PD, the received optical power \( P_r \) is calculated, where \( P_r = \text{photocurrent/responsivity} \).
4. Received optical power is related to transmitted optical power by \( P_r = kP_t/r^2 \) [17], where \( r \) is the range between the LED optical beacon and the PD. Each LED optical beacon will have a slightly different \( P_r \) due to power imbalances.
5. At the (0, 0) position, the value of \( kP_t \) is calculated for each LED optical beacon by multiplying the \( P_r \) by the known range squared, \( r^2 \). The value of \( kP_t \) is constant for a given LED optical beacon. This is the calibration process.
6. Given the received optical power at different locations, and knowing \( kP_t \), the range \( r \) is calculated such that \( r = \sqrt{kP_t/P_r} \).
7. A Nonlinear Least Squares algorithm takes the range values from the four LED optical beacons and determines the 2-D and 3-D coordinates of the PD via trilateration.

The resulting positioning errors for the optical RSS positioning technique are shown as a best fit plot in Fig. 3.18. Note that at the centre of the grid (\( x = 0, y = 0 \)), the ranges from the LED optical beacons have been calibrated.
such that they are all equal, since the PD is equidistant from all four LED optical beacons, so this centre position gives the lowest positioning error. A mean RSS positioning error of 20 cm is found for positions across the $xy$ plane.

![Figure 3.18: The 3-D positioning error for optical RSS positioning.](image)

Motion away from the centre increases the positioning error due to two factors. The first factor is range DOP as illustrated in Fig. 3.19 and the second factor is range error. In order to validate the theoretical DOP in equation 3.13 (illustrated in Fig. 3.19), the empirical DOP is calculated using equation 3.3 as the ratio of the position standard deviations to the range standard deviations of the 25 PD test points in Fig. 3.18.

The average theoretical DOP is computed from Fig. 3.19 and is compared to the average empirical DOP from the 25 PD test points in Fig. 3.18. It is valid to do such a comparison since the range DOP in Fig. 3.19 changes by a maximum factor of 0.1 across the 25 PD test points. The mean theoretical DOP of Fig. 3.19 is equal to 1.35. The mean empirical DOP is calculated as
3.4. Positioning Performance

follows. Assuming there is no bias in the range measurements the standard deviation of the range measurements is equivalent to the range error. Therefore, the range standard deviation of the 100 range measurements (4 range measurements for each of the 25 PD locations) is calculated and is equal to 5.8 cm. Similarly, assuming there is no bias in the position estimates, the standard deviation of the position estimates is equivalent to the position error shown in Fig. 3.18. Therefore, the position standard deviation of the 25 position errors is calculated and is equal to 8.8 cm. Therefore, the empirical DOP = $\frac{\sigma_P}{\sigma_m} = \frac{8.8}{5.8} = 1.5$, approximates the average theoretical DOP value of 1.35. For a range standard deviation equal to 5.8 cm, the difference between the empirical (1.5) and theoretical (1.35) DOP translates to position errors of 0.9 cm. Since the setup was measured by a tape measure with mm resolution, it is conceivable that one would have up to 1 cm of error.

For the second factor, the position error plot in Fig. 3.18 can be verified by finding the standard deviation of the range error, $\sigma_m$ at a test position and multiplying it by the corresponding DOP value in Fig. 3.19. For instance at position ($x = -15, y = 15$), $\sigma_m = 20.0$ cm, and the corresponding DOP value is 1.45, this results in $\sigma_P = 20.0 \times 1.45 = 29$ cm. The value of 29 cm approximates the position error value of 30 cm in Fig. 3.18 at ($x = -15, y = 15$).

Figure 3.19: Simulated 3-D range DOP for optical RSS positioning setup.
3.4. Positioning Performance

3.4.2 Optical AOA

The optical angular receiver AOA positioning setup is illustrated in Fig. 3.13 with LED optical beacons B\textsubscript{1}, B\textsubscript{2}, B\textsubscript{3} and B\textsubscript{4}. The angular receiver is oriented as shown in Fig. 3.20 such that PD\textsubscript{1} is normal to

\[
(x, y, z) = (\cos(54.7^\circ) \cos(45^\circ), \cos(54.7^\circ) \cos(45^\circ), \sin(54.7^\circ))
\]
\[
= (1/\sqrt{6}, 1/\sqrt{6}, \sqrt{2}/\sqrt{3}), \tag{3.14}
\]

PD\textsubscript{2} is normal to

\[
(x, y, z) = (-\cos(54.7^\circ) \sin(45^\circ), \cos(54.7^\circ) \cos(45^\circ), \sin(54.7^\circ))
\]
\[
= (-1/\sqrt{6}, 1/\sqrt{6}, \sqrt{2}/\sqrt{3}), \tag{3.15}
\]

and PD\textsubscript{3} is normal to

\[
(x, y, z) = (0, \sin(54.7^\circ), \cos(54.7^\circ))
\]
\[
= (0, -\sqrt{2}/\sqrt{3}, 1/\sqrt{3}). \tag{3.16}
\]

Figure 3.20: Angular receiver orientation. \((\theta_R, \phi_R)\) represent the angular receiver body frame \((x', y', z')\) rotation with respect to the reference frame \((x, y, z)\).

The angular receiver is rastered across the \(xy\) plane at the same 25 test points as the RSS experiment. The AOA is measured from LED optical
beacons B₁, B₂, B₃ and B₄ (see Fig. 3.21), where the beacons are 0.5 m above the angular receiver.

At each of the 25 test points, the power values \( P₁, P₂ \) and \( P₃ \), corresponding to the generated photocurrents \( i₁, i₂ \) and \( i₃ \), are recorded for a given LED optical beacon modulated at a particular modulation frequency using the procedures in Section 3.1.1. A total of one thousand power measurements are collected for each LED optical beacon. The one thousand power measurements are recorded in a time span of 3.4 minutes. For each of the one thousand \( P₁, P₂ \) and \( P₃ \) power values measured from a given LED optical beacon, the corresponding mean power values are used to calculate the mean AOA \( \phi \) and \( \theta \) using equations 3.1 and 3.2. These AOAs are measured with respect to the angular receiver body frame \( x'y'z' \) axis. Knowing the orientation of the body frame with respect to the reference frame (i.e., \( \phi_R = 45^° \) and \( \theta_R = 54.7^° \)), the AOA angles are then computed with respect to the \( xyz \) navigational frame.

The AOA values from each of the four LED optical beacons are used in a Least Squares triangulation algorithm to calculate an estimate of the angular receiver 3-D position with respect to the known LED optical beacon positions.

The Least Squares algorithm is based on rearranging equations 3.5 and 3.6 such that

\[
\tan \phi_i (y_i - y) - (x_i - x) = 0, \tag{3.17}
\]

and

\[
\tan \theta_i (z_i - z) - \sqrt{(x_i - x)^2 + (y_i - y)^2} = 0. \tag{3.18}
\]

An \( A \) matrix is defined containing the partial derivatives of equations 3.5 and 3.6 with respect to the three unknowns \((x, y, z)\). Given an initial guess of the angular receiver position \((\hat{x}, \hat{y}, \hat{z})\), the weights, \( w \), are computed by substituting \((\hat{x}, \hat{y}, \hat{z})\) for \((x, y, z)\) in equations 3.17 and 3.18, respectively. The estimated corrections, \( \delta = [\delta x \ \delta y \ \delta z] \) for the \( x, y, \) and \( z \) position estimates are computed with

\[
\delta = (A^T A)^{-1} A^T w \tag{3.19}
\]

and the new \( x, y, \) and \( z \) estimates are equal to the initial estimates plus the delta corrections. The new \( x, y, \) and \( z \) are fed into an iterative algorithm such that they form the new guess values, from which new weights, \( w \), are computed and then a new \( \delta \). The process continues until \( \delta \) converges to a negligible value. The resultant \((\hat{x}, \hat{y}, \hat{z})\) position becomes the Least Squares estimate.
3.4. Positioning Performance

The resulting 3-D positioning error is shown as a best fit plot in Fig. 3.21. Position error depends on the AOA accuracy measured from the four LEDs at each of the 25 angular receiver positions. For instance, a large position error of 8 cm is calculated at angular receiver position \((x = -15 \text{ cm}, y = 15 \text{ cm})\) primarily due to the fact that the measured \(\phi\) from LED_4 has a large error of \(8^\circ\) error. This large \(\phi\) error is due to the fact that the light from LED_4 strikes the angular receiver at \((\phi_4 = 4^\circ, \theta_4 = 30^\circ)\) which is outside the operational cone.

Note that Fig. 3.11 shows that an AOA at \((\phi_4 = 4^\circ, \theta_4 = 30^\circ)\) has an error of approximately \(8^\circ\) in \(\phi\) which agrees with the error in the measured \(\phi\) for LED_4 in this test. As the angular receiver moves along the \(x = -15 \text{ cm}\) line, from \(y = 15 \text{ cm}\) to \(y = -15 \text{ cm}\), the average AOA error from the four LED optical beacons decreases, resulting in a lower positioning error.

At angular receiver position \((x = 15 \text{ cm}, y = 15 \text{ cm})\) an AOA of \((\phi_3 = 88^\circ, \theta_3 = 35^\circ)\) is measured from LED_3. Since the AOA is outside the operational cone, the \(\phi_3\) error is \(5^\circ\). This error in \(\phi\) is confirmed in Fig. 3.11 which shows that at this AOA the \(\phi\) error is approximately \(5^\circ\).
3.4. Positioning Performance

Others factors that contribute to the position error are biases in the angular receiver position and orientation. This is due to the fact that the angular receiver has to be physically moved to each of the 25 test locations, which results in biases in the true AOA.

The shape in Fig. 3.21 can also be explained by looking at Figs. 3.11 and 3.12. Angular receiver positions with significant illumination of two PDs (at large and negative y values in Fig. 3.21 with \( \theta \) approaching 90° in Figs. 3.11 and 3.12) have improved accuracy over orientations with significant illumination of only one PD (at large and positive y values Fig. 3.21 with \( \theta \) approaching 0° in Figs. 3.11 and 3.12). This is because more light flux is captured by the angular receiver at large and negative y values and, therefore, results in more accurate AOAs.

Overall, the mean positioning error of the optical AOA positioning is 5 cm, a result which is four times lower than that of the optical RSS positioning.
3.4. Positioning Performance

3.4.3 Optical AOA Precision

For the calculation of each of the 25 position estimates on which Fig. 3.21 is based, the average AOA value of 840 \( \phi \) and 840 \( \theta \) measurements is used. In order to quantify the precision of these instantaneous AOA values, the \( \phi \) error is defined as the difference between the measured instantaneous \( \phi \) values for one of the 25 test points and the average \( \phi \) value for the same test point. Similarly, the \( \theta \) error is defined as the difference between the measured instantaneous \( \theta \) values for one of the 25 test points and the average \( \theta \) value for the same test point.

A sample size of 25 test points is necessary to achieve a 95% confidence level with a standard deviation of 0.01° [47]. The relationship between the sample size, confidence level (z-score), confidence interval (margin of error), and measurement standard deviation is given by equation 3.20

\[
\text{Sample size} = (z\text{-score})^2 \sigma (1 - \sigma) / (\text{margin of error})^2. \tag{3.20}
\]

Figs. 3.22, 3.23, 3.24 and 3.25 show the histograms of the AOA \( \phi \) and \( \theta \) precision for the AOA measured from LED optical beacons B\(_1\), B\(_2\), B\(_3\) and B\(_4\) respectively. Note that in each histogram there are 21,000 measurements (25 test points with 840 measurements each).

![Histograms of AOA precision](image)

Figure 3.22: AOA measurement precision histograms for \( \phi_1 \) and \( \theta_1 \).
3.4. Positioning Performance

Figure 3.23: AOA measurement precision histograms for $\phi_2$ and $\theta_2$.

Figure 3.24: AOA measurement precision histograms for $\phi_3$ and $\theta_3$. 
3.4. Positioning Performance

Table 3.2 shows the standard deviation for the $\phi$ error. The maximum standard deviation is $\approx 0.03^\circ$. Similarly, Table 3.3 shows the standard deviation for the $\theta$ error. The maximum standard deviation is $\approx 0.01^\circ$.

Table 3.2: $\phi$ error precision

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$\phi_3$</th>
<th>$\phi_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>standard deviation ($^\circ$)</td>
<td>0.0278</td>
<td>0.0154</td>
<td>0.0185</td>
<td>0.0075</td>
</tr>
</tbody>
</table>

Table 3.3: $\theta$ error precision

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>standard deviation ($^\circ$)</td>
<td>0.0139</td>
<td>0.0088</td>
<td>0.0060</td>
<td>0.0079</td>
</tr>
</tbody>
</table>

The distribution of the error histograms in Figs. 3.22, 3.23, 3.24, and 3.25, and the precision results in Tables 3.2 and 3.3 suggest that the instantaneous AOA measurements are very precise. Consequently, there is very little random error in the AOA measurements of the angular receiver. An AOA error equal to $0.03^\circ$ (the maximum AOA standard deviation seen in Tables 3.2 and 3.3) results in a translation error of just 1 mm over an optical beacon/angular receiver separation of 2 m.

In order to validate the theoretical 3-D DOP in equation 3.13 (illustrated...
3.4. Positioning Performance

in Fig. 3.17), the empirical 3-D DOP in equation 3.3 is calculated as the ratio of the position standard deviations to the AOA standard deviations in Fig. 3.21. The overall AOA standard deviation in Tables 3.2 and 3.3 is calculated to be \( \sigma_m = 0.0149^\circ \). The position standard deviation of the 21,000 position estimates at the angular receiver 25 test points is calculated to be \( \sigma_P = 0.0193 \) cm. Therefore, the empirical DOP = \( \frac{\sigma_P}{\sigma_m} = \frac{0.0193}{0.0149} = 1.30 \) cm/deg. Note that the empirical 3-D DOP value approximates the theoretical 3-D DOP in Fig. 3.17 which has an average value of 1.37 cm/deg. It is safe to quote an average 3-D DOP value, since the 3-D theoretical DOP changes by as little as 0.1 cm/deg as shown in Fig. 3.17. Also, the difference between the empirical and theoretical DOP is negligible since for a \( \sigma_m = 0.0149^\circ \), the difference in position standard deviation \( \sigma_P \), for a 3-D DOP difference of 0.07 cm/deg would be \( 1.1 \times 10^{-3} \) cm.

3.4.4 Optical AOA Accuracy

In order to quantify the accuracy of the instantaneous AOA measurements for each of the 25 test points, the \( \phi \) error is defined as the difference between the measured instantaneous \( \phi \) values for one of the 25 test points and the true \( \phi \) value for the same test point as determined from the orientation of the angular receiver body frame with respect to the reference frame, and the geometrical position of the angular receiver with respect to the LED optical beacons. Similarly, the \( \theta \) error is defined as the difference between the measured instantaneous \( \theta \) values for one of the 25 test points and the true \( \theta \) value for the same test point as determined from the orientation of the angular receiver body frame with respect to the reference frame, and the geometrical position of the angular receiver with respect to the LED optical beacons. Figs. 3.26, 3.27, 3.28 and 3.29 show the histograms of the AOA \( \phi \) and \( \theta \) error for the AOA measured from LED optical beacons B₁, B₂, B₃ and B₄ respectively. Again, each histogram contains 21,000 measurements (25 test points with 840 measurements each).
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Figure 3.26: AOA measurement accuracy histograms for $\phi_1$ and $\theta_1$.

Figure 3.27: AOA measurement accuracy histograms for $\phi_2$ and $\theta_2$. 
3.4. Positioning Performance

Table 3.4 shows the mean and standard deviation for the $\phi$ error. The maximum standard deviation is $\approx 3^\circ$. Similarly, Table 3.5 shows the mean
3.4. Positioning Performance

and standard deviation for the $\theta$ error. The maximum standard deviation is $\approx 2.4^\circ$.

Table 3.4: $\phi$ error mean and standard deviation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$\phi_3$</th>
<th>$\phi_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean ($^\circ$)</td>
<td>-0.1688</td>
<td>-1.0103</td>
<td>0.8720</td>
<td>2.8116</td>
</tr>
<tr>
<td>standard deviation ($^\circ$)</td>
<td>0.8973</td>
<td>1.4380</td>
<td>2.4199</td>
<td>3.1935</td>
</tr>
</tbody>
</table>

Table 3.5: $\theta$ error mean and standard deviation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean ($^\circ$)</td>
<td>0.7445</td>
<td>1.7750</td>
<td>0.9979</td>
<td>-1.1102</td>
</tr>
<tr>
<td>standard deviation ($^\circ$)</td>
<td>2.4083</td>
<td>1.4886</td>
<td>1.4755</td>
<td>1.7431</td>
</tr>
</tbody>
</table>

There are two factors that determine the shape of the histograms. The first factor is systematic errors due to the nature of the experiment, where slight errors in the angular receiver orientation with respect to the LED optical beacons would result in a bias in the true AOA. The second factor is AOA errors due to angles outside the cone of acceptance which give rise to large errors as shown in Figs. 3.11 and 3.12.

Biases are apparent in Table 3.4 where the mean $\phi_2$, $\phi_3$, and $\phi_4$ errors are $-1.0^\circ$, $0.9^\circ$, and $2.8^\circ$ respectively, and in Table 3.5 where the mean $\theta$ errors $\theta_1$, $\theta_2$, $\theta_3$, and $\theta_4$ are $0.7^\circ$, $1.8^\circ$, $1.0^\circ$, and $-1.1^\circ$.

In particular, the $\phi_4$ error in Table 3.4 has the largest mean error compared to the rest of the $\phi$ values due to the fact that $\phi_4$ registers the largest percentage of angles outside of the acceptance cone of operation. A maximum $\phi_4$ error of $8.1^\circ$ (left side plot in Fig. 3.29) occurs when the angular receiver is at ($x$ = -15 cm, $y$ = 15 cm) (see Fig. 3.21). At this location, the angular receiver measures an AOA from LED$_4$ at ($\phi_4$ = $4^\circ$, and $\theta_4$ = $30^\circ$). The $\phi$ error characterization in Fig. 3.11 shows that at this particular AOA the error, $\Delta \phi$, is approximately $10^\circ$.

Note that the $\phi_1$ error histogram is centered about approximately zero as shown in the left side plot of Fig. 3.26. This is due to the fact that the $\phi_1$ angle registers the highest percentage of AOAs, at the 25 angular receiver locations, that are within the acceptance cone and, therefore, would yield AOA errors less than $2^\circ$.

The $\theta_3$ error shown in the right side of Fig. 3.28 has a maximum $\theta$ error of $3.2^\circ$ at angular receiver location ($x$ = 7.5 cm, $y$ = -15 cm) in Fig. 3.21. At this location, the angular receiver measures an AOA from LED$_3$ at ($\phi$
3.4. Positioning Performance

= 72°, and θ = 60°). From Fig. 3.11, (φ = 72°, and θ = 60°) corresponds to a \( \Delta \phi \) error of approximately 3°. The \( \theta_3 \) error is approximately zero at angular receiver locations \((x = -15 \text{ cm}, y = 0 \text{ cm})\), and \((x = -15 \text{ cm}, y = 15 \text{ cm})\), where the \( \theta_3 \) values are within the operational cone.

In conclusion, the angular receiver has demonstrated 3-D position accuracies on the order of a few centimeters as shown in the empirical results in Section 3.4.2. A minimum intensity threshold of 0.2 \( \mu \text{W/cm}^2 \) must be maintained for reliable AOA estimates. In order to triangulate the angular receiver position, at least two optical beacons must lie within the angular receiver FOV defined from \((\phi = 0° \text{ to } 90° \text{ and } \theta = 0° \text{ to } 90°)\). Note that the full angular receiver FOV will be used for the analysis below instead of the FOV defined for the operational cone. Although this may sacrifice some AOA accuracy, it will allow the angular receiver to view more distant LEDs, and, therefore, improves DOP. Since the LED optical beacons are mounted on the ceiling, it is prudent that the angular receiver be oriented upwards as shown in Fig. 3.20 such that \( \phi_R = 45° \) and \( \theta_R = 54.7° \) to capture a greater number of LED optical beacons.

Several factors such as the LED separation distances (grid size) and separation height between the angular receiver and LED optical beacons, \( h \), must be designed to maintain at least two optical beacons within the angular receiver FOV. The relationship between the grid spacing and the separation height is outlined as follows.

Assume that the angular receiver is at the origin of the \((x,y,z)\) navigation frame as shown in Fig. 3.20 and is pointing upwards towards the ceiling such that \( \phi_R = 45° \) and \( \theta_R = 54.7° \). The maximum distance that the LEDs can be placed with respect to the angular receiver position will be investigated. These distances are a function of the angular receiver FOV.

In order to determine the maximum distance an LED can be placed along the \( y \) direction and still be visible by the angular receiver, consider Fig. 3.30 which shows the angular receiver of Fig. 3.20 when looking at the \( yz \) plane. From Fig. 3.30, the maximum distance along the positive \( y \) direction is \( y_1 = h/\tan(\theta_R) \). Assuming the height separation between the LED and angular receiver is \( h = 2 \text{ m} \), this implies that \( y_1 = 2/\tan(54.7°) = 1.4 \text{ m} \). Similarly, the maximum distance an LED can be placed in the negative \( y \) direction is \( y_2 = h/\tan(\theta_c) \), and for \( h = 2 \text{ m} \), \( y_2 = 2/\tan(35.3°) = 2.8 \text{ m} \).
3.4. Positioning Performance

In order to determine the maximum distance an LED can be placed along the $x$ direction while maintaining visibility by the angular receiver, an incident angle along the $x$ direction $\phi = 0^\circ$, and $\theta = 39^\circ$ measured with respect to the $z$ axis will result in $\phi' = 0^\circ$, $\theta' = 63^\circ$, where $\phi'$ and $\theta'$ are the angles measured with respect to the angular receiver body frame. Note that $\phi' = 0^\circ$ represents the extreme angle when the light is incident along the $x'$ axis shown in Fig. 3.20. When $h = 2$ m, the maximum distance along the $x$ direction is $h/\tan(90^\circ - \theta) = 2/\tan(90^\circ - 39^\circ) = 1.6$ m. Similarly the maximum distance along the negative $x$ direction is 1.6 m.

Next, the maximum distance an LED can be placed in the positive $y$ direction is determined such that the LED has equal $x$ and $y$ components from the angular receiver position i.e., $\phi = 45^\circ$ or $\phi = 135^\circ$ when measured with respect to the $x$ axis shown in Fig. 3.20. When an LED is incident at $\phi = 45^\circ$, and $\theta = 45^\circ$ this results in $\phi' = 15^\circ$, and $\theta' = 90^\circ$. A $\theta' = 90^\circ$ represents the maximum FOV in the positive $y$ direction. Therefore, for $h = 2$ m, the maximum 2-D distance of the LED from the angular receiver is $h/\tan(90^\circ - \theta) = 2/\tan(45^\circ) = 2$ m. For equal $x$ and $y$ LED positions, $x = y = \sqrt{2^2/2} = 1.4$ m.

Next the maximum distance an LED can be placed in the negative $y$ direction is determined such that the LED has equal $x$ and $y$ positions from the angular receiver position i.e., $\phi = 315^\circ$ or $\phi = 225^\circ$ when measured with respect to the $x$ axis shown in Fig. 3.20. When an LED is incident at $\phi$
3.4. Positioning Performance

= 225°, and θ = -35° this results in φ' = 90°, and θ' = 36°. A φ' = 90° represents the maximum FOV in the negative y direction. Therefore, for \( h = 2 \text{ m} \), the maximum 2-D distance of the LED from the angular receiver is \( h / \tan(90° - |\theta|) = 2 / \tan(55°) = 1.4 \text{ m} \). For equal \( x \) and \( y \) positions, \( x = y = \sqrt{1.4^2 / 2} = 1.0 \text{ m} \).

Figure 3.31 represents a summary of the LED positions, discussed above, with respect to the angular receiver (assumed to be at the origin with orientations as shown in Fig. 3.20). The figure shows a top view of the LED/angular receiver separation assuming a \( h = 2 \text{ m} \).

![Figure 3.31: Maximum square grid side-length capability for the angular receiver.](image)

Note the asymmetry between the positive \( y \) and negative \( y \) directions (which is a result of the asymmetry in \( θ \)), and the symmetry between the positive \( x \) and negative \( x \) directions (which is a result of the symmetry in \( φ \)). For square optical beacon grids the maximum LED grid spacing should be at most 2 m when the height of the grid is 2 m above the angular receiver. The square grid is attractive because it facilitates tessellation.
Chapter 4

Image Receiver Positioning

This chapter presents the image receiver. A detailed characterization of its angular and positioning performance is quantified. Section 4.1 introduces the image receiver. The empirical AOA error characterization is shown in Section 4.2. Section 4.3 discusses the effect of Dilution of Precision (DOP) on positioning accuracy. Finally, Section 4.4 presents empirical positioning performance results.

4.1 Image Receiver

Section 4.1.1 describes the components of the image receiver, namely the image sensor and microlens. Section 4.1.2 describes the algorithms used to differentiate between the LED optical beacon transmitters.

4.1.1 Image Sensor and Microlens

In this work, the image sensor used for positioning is that of a Sony Play Station 3 (PS3) eye webcam. This webcam was chosen due to its widespread availability, cheap cost of only $10, and its high frame rate of 187 frames per second. The original lens that comes with the webcam is removed and is replaced with a custom-made microlens that is much more compact and has a wider field of view. This is advantageous as a wider FOV microlens can image more LED optical beacons.

The microlens fabrication process and apparatus was developed in the Integrated Optics Laboratory at the University of British Columbia. The process consists of electro-dispensing a polymer of type Norland Optical Adhesive (NOA 68), onto a glass substrate immersed in glycerol. The volume of the microlens is determined by the pressure control system, and the tuning of the microlens shape is accomplished by the voltage applied to the needle tip. Tuning the shape of the microlens produces microlenses with varying contact angles which affects the microlens FOV. After the polymer is dispensed on the glass substrate and has been electro-tuned to give the desired shape, it is solidified by curing it with ultra-violet light [48].
4.1. Image Receiver

Figure 4.1 shows an illustration of the image receiver consisting of a micro lens and a CMOS sensor, where θ is the polar angle, h is the LED grid/micro lens separation distance, f is the micro lens focal length, and α is the contact angle. A CMOS sensor is made up of thousands of pixels (photoreceiver elements). The Sony PS3 webcam CMOS sensor is made up of pixels where each pixel has a size of $6 \times 6 \, \mu\text{m}^2$. Microlenses are made with different diameters that range from 500 $\mu\text{m}$ to 800 $\mu\text{m}$, and with various contact angles that range from 30° to 120°. Higher contact angle microlenses have a larger FOV and, therefore, are able to image more distant beacons (see Fig. 4.1).

Another important parameter for lens design is the f-number defined as, $f$-number = $\frac{f}{D}$, where $f$ is the focal length and $D$ is the lens diameter. The f-number is inversely proportional to the lens FOV, such that $f$-number = $\frac{f}{D} \propto \frac{1}{\text{FOV}}$. Conventional microlenses [49] achieve $\mu\text{m}$ scale diameters and low curvature (low contact angles), which implies large focal length (cm scale), resulting in a large f-number, and, therefore, narrow FOV microlenses. Unlike conventional microlenses, the proposed microlens has $\mu\text{m}$ scale diameters, and high curvature (high contact angles), which implies short focal lengths (mm scale), resulting in a low f-number, and, therefore, wide FOV microlenses.
4.1. Image Receiver

4.1.2 Colour and Frequency Detection

Indoor positioning utilizing the image receiver works by capturing an image of the ceiling and computing the AOA of the LED optical beacons in the image as will be shown in Section 4.2. The image receiver position is then found by triangulating with the AOAs.

In order for the image receiver to distinguish between the LED optical beacons in an image, each LED optical beacon emits light at a specific wavelength of the visible spectrum (i.e., each LED has a different colour usually red, green, or blue) and/or is pulse modulated at a specific frequency usually in the kHz range.
4.1. Image Receiver

A common colour detection algorithm is the Hue Saturation and Value (HSV) algorithm [50],[51]. The HSV algorithm assigns each colour an \( H \) value for Hue, an \( S \) value for Saturation, and a \( V \) value that describes the brightness or luminance. The HSV colour representation is shown in Fig. 4.2. The \( H \) value describes the colour tone, and is described by an angle, whose reference is the colour red. The equation for calculating \( H \) is [51]

\[
H = \arccos \frac{0.5(2R-G-B)}{\sqrt{(R-G)^2-(R-B)(G-B)}}. \tag{4.1}
\]

where \( R \), \( G \), and \( B \) represent the normalized red, green and blue colours of a given pixel computed via MATLAB\textregistered. The \( S \) value represents the distance to the \( V \)-axis. The nearer a colour is to the \( V \)-axis the more diluted it is, and the further away it is from the \( V \)-axis the more saturated the colour gets. The equation for calculating \( S \) is [50]

\[
S = \frac{\max(R,G,B) - \min(R,G,B)}{\max(R,G,B)}. \tag{4.2}
\]

The \( V \) value represents the brightness or intensity of the colour such that \( V = \max(R,G,B) \). The colour black would have a \( V \) value of zero.
4.1. Image Receiver

Figure 4.2: HSV colour representation.

Figure 4.3 shows an image of four LED optical beacons, where each LED has a different colour. The image was captured by a 30° contact angle microlens and a Sony PS3 CMOS sensor using a Sony PS3 code laboratories.
4.1. Image Receiver

application that runs on a laptop computer. To differentiate between the different LED optical beacons, an HSV algorithm is implemented that uses the representation of Fig. 4.2. Once the algorithm detects a colour, it draws a circle on each of the coloured spots as shown in Fig. 4.4. The algorithm works by setting a range for the H, S, and V values corresponding to each colour as shown in Fig. 4.2. Depending on the ambient light and the particular wavelength of the LED optical beacons, the HSV values are tuned such that for the colour red, $H$ ranges from 0 to 0.005 and $S$ is greater than 0.3, for green $H$ ranges from 0.300 to 0.470 and $S$ is greater than 0.3, for blue $H$ ranges from 0.620 to 0.670 and $S$ is greater than 0.3, and for white $H$ ranges from 0.078 to 0.470 and $S$ is greater than 0.3.

Once the algorithm detects the colour, it draws a circle centered on the the pixel coordinates of the LED being imaged with a radius equal to 4 pixels. As will be shown later, the pixel coordinates are necessary to compute the AOA.

Figure 4.3: An image of four different colour LEDs (red, green, blue and white) appearing as red, green, blue and white spots.
4.1. Image Receiver

Figure 4.4: Colour discrimination (implemented using an HSV algorithm) detects each coloured spot in Fig. 4.3 and draws a circle around it.

Another common colour model is the RGB model illustrated in Fig. 4.5. The RGB model will be used in conjunction with frequency modulation later in this chapter (see Section 4.4). The RGB model is used instead of the HSV algorithm due to the fact that MATLAB® reads video frames in the RGB format. The RGB model is based on the Cartesian coordinate system. The primary colours red, green and blue have coordinates (1, 0, 0), (0, 1, 0), and (0, 0, 1) respectively. From Fig. 4.5 the colour yellow would have an RGB value of (1, 1, 0).
4.1. Image Receiver

Figure 4.5: Colour discrimination implemented using the RGB model.

The modulation frequency of a coloured LED optical beacon, for example red, is found by using the RGB algorithm to detect the pixels in an image with the red colour and capturing the amplitude of those pixels for a given time period. Then the fast Fourier transform (FFT) algorithm is used to find the spectral frequency of those pixels. The pixel with the correct colour (which is typically at the centre of the image focal spot), and frequency value is chosen for AOA estimation and the rest discarded.

The PS3 webcam high frame rate of 187 frames per second (fps), allows LED optical beacons to be modulated up to 93.5 Hz. At least 200 frames (approximately 1 second) need to be captured for accurate spectral frequency calculation. This is best illustrated with an example where an LED emitting blue light and modulated with a frequency of 80 Hz is imaged. The FFT algorithm is applied on the amplitude of the corresponding pixel for 100 frames (0.5 s) and 200 frames (1.07 s). Figure 4.6 shows the spectral analysis for both 100 and 200 frames. The 100 frames FFT analysis shows frequency components at 70 Hz and 80 Hz. The 70 Hz peak is a result of interference of the LED green lead modulated at 70 Hz. The spectral
4.2 Angle-Of-Arrival Measurement Error Characterization

analysis is therefore ambiguous. Using 200 frames on the other hand, results in the 80 Hz peak being higher than the peak at 70 Hz and therefore, 200 frames represent a sufficient number of frames needed to accurately acquire the LED modulation frequency.

![Figure 4.6: FFT analysis performed for 100 and 200 frames on colour blue. The left plot shows interference at 70 Hz, and the right plot shows a reduction in interference.](image)

4.2 Angle-Of-Arrival Measurement Error Characterization

In this section, the performance of two different microlenses is investigated. The first microlens has a contact angle of $\alpha = 30^\circ$ and achieves a wide FOV of 95°, while the second microlens has a contact angle of $\alpha = 90^\circ$ and achieves an ultrawide FOV of 130°. The Scanning Electron Microscope (SEM) images of the wide- and ultrawide FOV microlenses are shown at the bottom of Figs. 4.7 (a) and (b), respectively [48]. The wide FOV microlens has a radius of $r = 400 \mu m$, while the ultra-wide FOV microlens has a radius of $r = 250 \mu m$. 
4.2. Angle-Of-Arrival Measurement Error Characterization

![Diagram of wide-FOV and ultrawide-FOV microlenses](image)

Figure 4.7: Schematic views and SEM images for (a) the image sensor with a wide FOV microlens, having an $\alpha = 30^\circ$ contact angle, and (b) the image sensor with a ultrawide FOV microlens, having an $\alpha = 90^\circ$ contact angle. The microlens radius is $r$. Incident AOAs on the image sensors are defined on the $(x', y', z')$ coordinates of the body frame. The focal spot location on the CMOS array is defined by its azimuthal angle, $\phi_{IS}$, and radial distance, $\rho_{IS}$.

LED optical beacon intensities ranging between a low of 0.03 $\mu$W/cm$^2$ and a high of 0.2 $\mu$W/cm$^2$ are used for the AOA characterization. Figure 4.8 shows images for the 0.03 $\mu$W/cm$^2$ LED intensity (top image) and
4.2. Angle-Of-Arrival Measurement Error Characterization

the 0.2 $\mu$W/cm$^2$ LED intensity (bottom image). The focal spot size for the low intensity LED is 18 $\mu$m, while for the high intensity it is 24 $\mu$m. Higher optical beacon intensities would increase the focal spot size which would hinder the AOA accuracy.

![Low intensity focal spot size image](image1)

![High intensity focal spot size image](image2)

Figure 4.8: Low intensity LED focal spot size image (top) and high intensity LED focal spot size image.

An LED optical beacon’s AOA is defined with the ($x'$, $y'$, $z'$) body frame coordinates of the image sensor, with the CMOS array in the $x'$-$y'$ plane (see Fig. 4.7). The AOA is defined in terms of the azimuthal angle, $\phi = \arctan(y'/x')$ and the polar angle, $\theta = \arccos \left[ z'/(x'^2 + y'^2 + z'^2)\right]^{1/2}$.

The AOA is measured from the focal spot location on the CMOS array, which is defined by its azimuthal angle, $\phi_{IS}$, from the $x'$-axis, and its radial distance, $\rho_{IS}$, from the microlens centre. An AOA characterization is carried out by imaging LED optical beacons placed on the ceiling and determining the linear transformation from the measured values of $\phi_{IS}$ and $\rho_{IS}$ to the true AOA angles $\phi$ and $\theta$. The true AOA angles are determined from the LED and image receiver geometry.

The image receiver is moved to different locations with respect to the LED optical beacons. The separation height between the image receiver and the LED optical beacons is decreased to determine the AOA at the extreme FOV.
4.2. Angle-Of-Arrival Measurement Error Characterization

The azimuthal characterization results are shown in Fig. 4.9. Fig. 4.9(a) shows $\phi$ versus $\phi_{IS}$ for the image sensor with the wide FOV microlens. Fig. 4.9(b) shows $\phi$ versus $\phi_{IS}$ for the image sensor with the ultrawide FOV microlens. Both image sensors show strong linearity in $\phi$ versus $\phi_{IS}$, due to negligible astigmatism in the microlenses. For both image sensors, the linear transformation from the focal spot location on the image sensor to the true AOA angle is simply $\phi \approx \phi_{IS} - 180^\circ$, as one would expect for the inverted image in the focal plane of a planoconvex lens. The mean azimuthal AOA error in this linear region is $\Delta\phi \approx 0.5^\circ$ for both image sensors.

![Figure 4.9: Azimuthal characterization results, showing the AOA angle $\phi$ as a function of the measured $\phi_{IS}$ angle, for image sensors with the (a) wide FOV microlens and (b) ultrawide FOV microlens.](image)

The polar characterization results are shown in Fig. 4.10. Fig. 4.10(a) shows $\theta$ versus $\rho_{IS}/r$ for the image sensor with the wide FOV microlens. Fig. 4.10(b) shows $\theta$ versus $\rho_{IS}/r$ for the image sensor with the ultrawide FOV microlens. The polar angle, $\theta$ represents the true, geometrical $\theta$ given the position of the image receiver and the LEDs. For the desired low-distortion linear transformation from $\rho_{IS}$ to $\theta$, imaging is restricted to the linear regime, seen as solid circles, with the angular FOV being twice the maximum $\theta$ value. Imaging beyond the linear regime, seen as hollow circles,
Positioning Analysis Using Dilution of Precision

distorts the focal spots and increase the AOA errors. Overall, the angular FOVs of the image sensors with the wide- and ultrawide FOV microlenses are $2 \times 47.5^\circ = 95^\circ$ and $2 \times 65^\circ = 130^\circ$, respectively, and the mean polar AOA error over the linear regimes is found to be equal to that of the azimuthal angle, $\Delta \phi \approx \Delta \theta \approx 0.5^\circ$.

![Figure 4.10: Polar characterization results](image)

Figure 4.10: Polar characterization results, showing the AOA angle $\theta$ as a function of the measured normalized $\rho_{IS}/r$ distance, for the image sensors with the (a) wide FOV microlens and (b) ultrawide FOV microlens.

4.3 Positioning Analysis Using Dilution of Precision

A DOP characterization is carried out for the optical wireless testbed. The overhead optical beacon grid has nine LED optical beacons arrayed across a plane with a height of $h$ and pitch of $p = 150$ cm. In the $(x, y, z)$ global frame coordinates, the LED optical beacons are at $(-p, p, h)$, $(0, p, h)$, $(p, p, h)$, $(-p, 0, h)$, $(0, 0, h)$, $(p, 0, h)$, $(-p, -p, h)$, $(0, -p, h)$, and $(p, -p, h)$. The DOP is calculated for image sensors positioned across the $z = 0$ plane of the $(x, y, z)$ global frame coordinates as shown in Fig. 4.11.
4.3. Positioning Analysis Using Dilution of Precision

Figure 4.11: Illustration of LED optical beacon geometry for DOP calculation.

The DOP characterization results are shown in Figs. 4.12 and 4.13. The DOP, in units of cm/deg, is shown as a function of \( x \) and \( y \) coordinates in the \( z = 0 \) plane. Fig. 4.12 shows DOP for the image sensor with the wide FOV microlens. Given that the grid spacing is fixed to \( p = 150 \) cm, and \( \theta = 47.5 \) ° (being the maximum allowable \( \theta \) for the LED to be within the FOV of the wide FOV microlens), the minimum allowable height of the optical beacon grid above the image receiver, \( h \) is defined as \( h = \sqrt{8 \frac{p}{\tan(\theta)}} \). At this height, \( h = 400 \) cm, the wide FOV microlens is just able to observe all optical beacons within its FOV. Fig. 4.13 shows DOP for the image sensor with the ultrawide FOV microlens. The height of the optical beacon grid above the image receiver is now decreased to \( h = 200 \) cm, since at this height the ultrawide FOV microlens is just able to observe optical beacons within its FOV.
A comparison of Figs. 4.12 and 4.13 illustrates the effects of DOP on
4.3. Positioning Analysis Using Dilution of Precision

optical wireless positioning. The first attribute to note relates to the numerical scale of the DOP. The DOP of the image sensor with the ultrawide FOV microlens is approximately two-times lower than that of the image sensor with the wide FOV microlens. At the centre of the \((x, y, z)\) global frame coordinates, the wide FOV microlens yields a maximum DOP of 6.6 cm/deg, while the ultrawide FOV microlens yields a minimum DOP of 2.8 cm/deg. This leads to a proportional (and desirable) decrease in position error for the image sensor with the ultrawide FOV microlens. The second attribute to note relates to the concavity of the DOP surfaces near the centre of the \((x, y, z)\) global frame coordinates. The DOP for the image sensors with wide- and ultrawide FOV microlenses are concave-down and (slightly) concave-up, respectively. This dissimilarity is due to the FOVs and the established lines of position. For the wide FOV microlens in Fig. 4.12 the lines of position are largely parallel in the centre. Translations to the perimeter lead to increasingly orthogonal lines of position and reduced DOP, as the directions from the image receiver to the optical beacons being approached and receded from become increasingly orthogonal. In contrast, for the ultrawide FOV microlens in Fig. 4.13 the lines of position are largely orthogonal in the centre. Translations toward the perimeter lead to increasingly parallel lines of position and increased DOP, albeit to a small extent. In this case, the directions to the optical beacons being approached remain largely orthogonal throughout the translation.

The above AOA and DOP characterizations can be merged to quantify the overall positioning accuracy for the image sensors with the wide- and ultrawide FOV microlenses. This is done according to equation 3.3, by calculating the position error, \(\sigma_P\), as the product of the measurement AOA standard deviation, \(\sigma_A\), and DOP. In this investigation, systematic AOA errors are zero as demonstrated in Section 4.2 as long as the LED lies within the microlens maximum FOV. In this case the AOA error is equivalent to the AOA standard deviation, and therefore, \(\sigma_A = 0.5^\circ\).

The resulting position error, \(\sigma_P\), for the same optical beacon grids that produced the DOP characterization of Figs. 4.12 and 4.13, is shown in Figs. 4.14 and 4.15 as a function of \(x\) and \(y\) global frame coordinates in the \(z = 0\) plane. Fig. 4.14 shows the position error for the image sensor with the wide FOV microlens. Fig. 4.15 shows the position error for the image sensor with the ultrawide FOV microlens. The two position error distributions mimic those of the DOP distributions, as expected, and the broad view of the ultrawide FOV microlens leads to reduced position errors. The ultrawide FOV microlens is better able to image distant optical beacons leading to enhanced localization along its lines of position and reduced
4.3. Positioning Analysis Using Dilution of Precision

DOP. For this distribution of optical beacons, the ultrawide-FOV microlens yields a position error of approximately 1.4 cm at the centre, compared to the corresponding position error of approximately 3.3 cm for the wide FOV microlens.

Figure 4.14: Positioning accuracy for the wide FOV microlens in \((x, y, z = 0)\) navigational frame.
4.4. Positioning Performance

Figure 4.15: Positioning accuracy for the ultrawide FOV microlens in \((x, y, z = 0)\) navigational frame.

4.4 Positioning Performance

In this section, the empirical positioning accuracy of the wide FOV (30° contact angle microlens) and ultra-wide FOV (90° contact angle microlens) sensors are quantified. Four LED optical beacons are mounted on the ceiling at a height of \(h = 100\ \text{cm}\) and a pitch of \(p = 17.5\ \text{cm}\) at \((x, y, z)\) navigational frame coordinates of \((-p, 2p, h)\), \((p, 2p, h)\), \((-p, -2p, h)\) and \((p, -2p, h)\) as shown in Fig. 4.16.
4.4. Positioning Performance

Each of the four LED optical beacons uses an RGB LED. The LED optical beacons emit white light by frequency modulating the red, green and blue LED leads such that LED\(_1\) has \(R_{\text{DC}}G_{\text{f1}}B_{\text{f1}}\), LED\(_2\) has \(R_{\text{DC}}G_{\text{f1}}B_{\text{f2}}\), LED\(_3\) has \(R_{\text{DC}}G_{\text{f2}}B_{\text{f2}}\), and LED\(_4\) has \(R_{\text{DC}}G_{\text{f2}}B_{\text{f1}}\), where DC refers to \(f = 0\), \(f_1\) is 70 Hz and \(f_2\) is 80 Hz.

The image receiver is rastered across the \(z = 0\) plane at 9 different locations at \((x, y, z)\) navigational frame coordinates of \((0, 0, 0), (-v, 0, 0), (v, 0, 0), (0, -v, 0), (0, v, 0), (v, -v, 0), (v, v, 0), (-v, -v, 0), (v, v, 0)\), where \(v = 28\) cm as shown in Fig. 4.16.
4.4. Positioning Performance

At each of the nine test locations, an image of the ceiling is captured using the wide FOV microlens and the ultra-wide FOV microlens. In one such image, shown in Fig. 4.17, the radial focal spot radius, $\rho_{IS}$, is measured as the euclidean pixel distance between the image pixel and the microlens pixel centre on the image sensor. Fig. 4.17 illustrates the radial focal spot radius, $\rho_{IS,1}$, for LED$_1$. The respective LED optical beacons are identified in the image by observing the frequency component of the red, green and blue layers using the RGB model shown in Fig. 4.5. Fig. 4.18 shows the normalized frequency spectrum of the red, green and blue channels for LED$_1$, LED$_2$, LED$_3$, and LED$_4$.

Figure 4.17: An RGB image showing the measurement of $\rho_{IS,1}$, the radial pixel distance between the microlens centre to LED$_1$ focal spot.
4.4. Positioning Performance

Once $\rho_{IS,1}$ is measured for LED$_1$, the value of $\theta_1$ is calculated for the wide and ultrawide FOV microlenses using Fig. 4.10(a) and (b) respectively. The azimuthal angle, $\phi_1$ is calculated from Fig. 4.9(a) and (b) for the wide and ultrawide FOV microlenses respectively. The above procedure is repeated for LED$_2$, LED$_3$, and LED$_4$ to compute $(\theta_2, \phi_2)$, $(\theta_3, \phi_3)$, and $(\theta_4, \phi_4)$. For all 36 AOA measurements, the mean $\phi$ error, $\Delta\phi$, is equal to the mean $\theta$ error, $\Delta\theta \approx 0.5^\circ$.

A Least Squares algorithm is then implemented to estimate the 2-D and 3-D position of the image receiver using the wide and ultra-wide FOV microlenses. The mean positioning error is shown in Table 4.1. For both microlenses the mean 2-D positioning error was on the order of 1 cm, while the 3-D positioning error was 2.5 cm. This result is expected since for the given LED and image sensor geometry the 3-D DOP is approximately 2.5 times the 2-D DOP.

In conclusion, the image receiver has demonstrated 3-D position accura-
4.4. Positioning Performance

Table 4.1: 2-D and 3-D positioning error results for the wide- and ultra-wide FOV microlenses.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>2-D</th>
<th>3-D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positioning error (cm)</td>
<td>1</td>
<td>2.5</td>
</tr>
</tbody>
</table>

...cies on the order of a few centimeters as shown in the empirical results in this section. This is brought about by operating within the image sensors' FOV, having AOA errors of $\approx 0.5^\circ$.

Although both microlenses have the same accuracy here, the ultrawide FOV microlens has a significant advantage over the wide FOV microlens. It can image more LED optical beacons due to its wider FOV. A greater number of LED optical beacons results in LED redundancy, and better DOP, due to the fact that the LED optical beacons will be more spaced out, and therefore positioning accuracy is improved.

Assuming the LEDs are placed at the edges of a square grid, the square grid length should be designed such that the LEDs lie within the image receiver FOV. For a separation height $h = 2$ m, the distance between the image receiver and LED should be $d = h \tan(\theta)$. For the wide FOV, $d = 2 \tan(47.5^\circ) = 2.1$ m. For an LED placed at equal $x$ and $y$ components from the image receiver, $x = y = \sqrt{(2.1^2/2)} = 1.5$ m. Similarly, for the ultra-wide FOV, $d = 2 \tan(65^\circ) = 4.3$ m. For an LED placed at equal $x$ and $y$ components from the image receiver, $x = y = \sqrt{(4.3^2/2)} = 3.0$ m. Figure 4.19 shows a top-view of the image receiver (at the origin) and LEDs placed on a square grid. Note that the ultrawide FOV microlens can image square grids with a side length equal to 6 m compared to 3 m for the wide FOV microlens.
4.4. Positioning Performance

Another factor that determines the accuracy of the positioning system is the incident intensity. It is advisable that the received LED optical intensity be between 0.03 $\mu$W/cm$^2$ and 0.2 $\mu$W/cm$^2$. Intensities below 0.03 $\mu$W/cm$^2$ are too faint, and, therefore, can not be imaged accurately, while intensities above 0.2 $\mu$W/cm$^2$ result in a large focal spot that is saturated at a larger size. Intensities outside of this range will increase the AOA errors above 0.5°.
Chapter 5

Receivers’ Performance while in Motion

In this chapter, the accuracies of the AOA estimates and the resulting position estimates are investigated for the angular receiver structure and the image receiver in motion. Motion is achieved by mounting each receiver on a robot platform. Section 5.1 presents the performance of the angular receiver structure, while Section 5.2 presents the performance of the image receiver.

5.1 Angular Receiver Positioning

For this analysis, the iRobot Create platform [52] shown in Figure 5.1 is the robot used to create motion. This particular platform is chosen due to its popularity as a research platform and the significant cargo bay space it offers to mount sensors on the robot. The robot can be controlled by a microcontroller that is programmed using the C language to transmit commands to the robot to move and turn in any direction.
5.1. Angular Receiver Positioning

Figure 5.1: iRobot Create platform.

In this work, an Arduino nano (ATmega 328) [53] is used to control the iRobot Create. The Arduino nano is shown in the bottom right hand corner in Fig. 5.2. The Arduino nano is used due to its compact size. The transmit and receive pins on the Arduino nano are connected to the receive (pin 1) and transmit pin (pin 2) respectively on the iRobot Create DB-25 connector [53].

As shown in Fig. 5.2, the angular receiver structure is mounted on the iRobot Create along with the amplifying circuit of Section 3.1.1 and a LabVIEW NI data acquisition device. The three output voltages corresponding to photocurrents $i_1$, $i_2$, and $i_3$ of PD1, PD2, and PD3, are each connected to the amplifying circuit and then to the LabVIEW data acquisition unit. The data acquisition unit wirelessly transmits the PD voltages over three separate channels via the NI LabVIEW wireless network to a Laptop computer, where the voltage values are used to compute the AOA estimates.
5.1. Angular Receiver Positioning

The iRobot Create powers both the Arduino nano and the LabVIEW DAQ. The iRobot Create 5 volt output (pin 8) is connected to the Arduino nano $V_{in}$ pin, while the iRobot Create 14 volt output (pin 10) is connected to the LabVIEW data acquisition device. The amplifying circuit op-amps are powered by two 9 volt batteries.

Four high-power (3 watt) LED optical beacons emitting warm white light [54] are used as the optical transmitters (warm white light has a yellowish colour light compared to cool white light which has white/blueish colour). Each LED optical beacon is connected to an amplifier of type Texas Instruments OPA552 [55] that is able to provide a power of $\approx 3$ watt to each of the LED optical beacons. The high power LED optical beacons are needed to operate the PDs at an intensity greater than the $0.2 \, \mu W/cm^2$ minimum intensity threshold described in Section 3.1.2. Each of the four LED optical beacons are frequency modulated at distinct frequencies, such that the LED$_1$ optical beacon has a frequency of 2.0 kHz, the LED$_2$ optical beacon has a frequency of 2.3 kHz, the LED$_3$ optical beacon has a frequency of 2.6 kHz, and the LED$_4$ optical beacon has a frequency of 2.9 kHz.

In order to recover the spectral components for each of the three PD channels with a FFT, LabVIEW is programmed to read 2048 voltage samples for each of the three channels at a rate of 10 kHz or 40 kHz. As a result,
the 10 kHz sampling rate captures a power reading at the appropriate LED modulation frequency every 0.2 s i.e., 5 Hz, while the 40 kHz sampling rate captures such a reading every 0.05 s, i.e., 20 Hz.

Figure 5.3 shows a top-view of the LED/receiver setup. The four LED optical beacons are positioned in an \((x, y, z)\) navigational frame coordinate system such that LED_1 is positioned at \((-35 \text{ cm}, 30 \text{ cm}, 100 \text{ cm})\), LED_2 at \((-35 \text{ cm}, 65 \text{ cm}, 100 \text{ cm})\), LED_3 at \((35 \text{ cm}, 30 \text{ cm}, 100 \text{ cm})\), and LED_4 at \((35 \text{ cm}, 65 \text{ cm}, 100 \text{ cm})\) as shown in Fig. 5.3. The \(z\) axis represents the vertical separation between the LED optical beacons and the angular receiver. The LED optical beacons are at \(z = 100 \text{ cm}\), while the angular receiver lies in the \(xy\) plane.
5.1. Angular Receiver Positioning

Figure 5.3: Illustration of LED optical beacon and receiver geometry as well as the trajectory of the robot from its start point to its end point. The point \( \mathbf{X} \) represents the start point of the robot carrying the receiver, and the point \( \mathbf{Y} \) represents the end point of the robot trajectory.

The angular receiver is oriented as shown in Fig. 3.20. The iRobot Create, hosting the angular receiver, is programmed to move along a straight line trajectory. The robot trajectory starts at the navigational frame coordinate \((0, 0, 0)\) and moves along the line \( x = 0 \) for a distance of 1.0 m, as shown in Fig. 5.3.

In order to determine the exact timestamp of the angular receiver data captured using LabVIEW and the location of the robot controlled by the Arduino, the timestamps for LabVIEW and Arduino are synchronized by connecting one of the output pins of the Arduino to one of the inputs of the
5.1. Angular Receiver Positioning

wireless DAQ. The Arduino nano is programmed such that the instant the iRobot Create starts moving the pin is set to high and when it stops it is set to low. The timestamps during the time period in which this pin is high corresponds to the angular receiver data collected during the robot motion along the line \( x = 0 \) for a distance of 1.0 m.

The following sections describe the angular receiver AOA and positioning accuracies for varying robot speed and varying LabVIEW sampling frequency. Section 5.1.1 presents the results for a robot speed of 10 cm/s. Section 5.1.2 presents the results for a robot speed of 50 cm/s, while Section 5.1.3 presents the results for when the robot moves at an average walking speed of 139 cm/s.

5.1.1 Low Speed 10 cm/s

As the robot moves along its path the voltage values corresponding to each LED optical beacon intensity on the three PDs are recorded using LabVIEW. For example, at a given instant LabVIEW registers three voltage values for LED optical beacon 1 (operating at frequency 2.0 kHz) for PD\(_1\), PD\(_2\), and PD\(_3\). Similarly, the voltage values for LED optical beacons 2, 3, and 4 operating at 2.3 kHz, 2.6 kHz, and 2.9 kHz respectively are recorded. The twelve voltage values are converted into photocurrent values by dividing the voltage values by the amplifying circuit impedance (10 MΩ) shown in Fig. 3.6. The photocurrent values corresponding to a particular LED optical beacon are used in equations 3.1 and 3.2 to solve for the AOA angles.

LabVIEW is programmed to read 2048 samples at a sampling frequency of 10 kHz, which results in an AOA measurement approximately every 0.2 s. Consequently, the AOA measurement update rate is 5 Hz. For a robot moving a distance of 100 cm at a speed of 10 cm/s, this results in approximately 49 AOA values. Figure 5.4 illustrates the process of measuring the AOA \((\theta_1, \phi_1)\) from LED\(_1\) with frequency \(f_1\) for the third instance along the robot trajectory. The photocurrents \(i_1\), \(i_2\), and \(i_3\) are generated as a result of sampling 2048 voltage values at a sampling frequency of 10 kHz, between the second and third time instances. The above process is repeated for LED\(_2\) at \(f_2\), LED\(_3\) at \(f_3\), and LED\(_4\) at \(f_4\).
5.1. Angular Receiver Positioning

Figure 5.4: Illustration of the process of measuring the AOA ($\theta_1$, $\phi_1$) from LED$_1$.

Figure 5.5 shows the photocurrents $i_1$, $i_2$ and $i_3$ corresponding to LED optical beacons 1, 2, 3 and 4 as the angular receiver is moving along its 100 cm path. Forty nine photocurrents are generated.
5.1. Angular Receiver Positioning

Figure 5.5: The photocurrents used to generate the AOA angles are shown versus iRobot Create distance traveled for LED$_1$, LED$_2$, LED$_3$, and LED$_4$. Photocurrents $i_1$, $i_2$, and $i_3$ are shown as the red (solid), black (dotted) and blue (dashed) curves respectively.

Note the trend for photocurrents $i_1$ and $i_2$. As the robot moves along its 1 m track, the angular receiver moves away from LED optical beacons 1 and 3, and therefore, $i_1$ (red-solid) and $i_2$ (black-dotted) decrease as shown for LED optical beacons 1 and 3. As the robot moves along its trajectory, the angular receiver first approaches LED optical beacons 2 and 4, resulting in the photocurrents $i_1$ (red-solid) and $i_2$ (black-dotted) increasing, and then recedes, causing photocurrents $i_1$ (red-solid) and $i_2$ (black-dotted) to decrease.

The trend for photocurrent $i_3$ is as follows. An increase in $i_3$ occurs for LED optical beacons 2 and 4 as the robot moves towards LED optical beacons 2 and 4. However, for LED optical beacons 1 and 3, $i_3$ increases until it reaches a maximum at around 60 cm. At this location more light is incident on PD$_3$ compared to the start position of the robot. Beyond 60 cm the $i_3$ decreases as the angular receiver moves further away from the LED optical beacons 1 and 3.

The smoothness of the photocurrent $i_1$, $i_2$, and $i_3$ curves is a direct result of the amplifier/filter design capability to suppress random noise effectively.
5.1. Angular Receiver Positioning

Knowing the rotation of the angular receiver $x'y'z'$ body frame with respect to the $xyz$ navigational frame as shown in Fig. 3.20, the 49 AOA measurements (with respect to the angular receiver body frame) are expressed in terms of an azimuthal angle, $\phi$, measured with respect to the $x$ direction, and a polar angle, $\theta$, measured with respect to the $z$ direction.

Figure 5.6 shows the 49 AOA values (expressed as $\theta$ and $\phi$) for LED$_1$ optical beacon as the robot moves along the line $x = 0$. The actual AOA, computed based on the robot to LED$_1$ optical beacon geometry, is shown for accuracy comparison. Similarly, Figs. 5.7, 5.8, and 5.9 show the measured AOA values for LED$_2$, LED$_3$, and LED$_4$ optical beacons respectively.

Figure 5.6: AOA ($\theta_1$, $\phi_1$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 10 cm/s at a 5 Hz AOA measurement rate.
5.1. Angular Receiver Positioning

Figure 5.7: AOA ($\theta_2, \phi_2$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 10 cm/s at a 5 Hz AOA measurement rate.
5.1. Angular Receiver Positioning

Figure 5.8: AOA ($\theta_3$, $\phi_3$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 10 cm/s at a 5 Hz AOA measurement rate.
5.1. Angular Receiver Positioning

Figure 5.9: AOA ($\theta_4$, $\phi_4$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 10 cm/s at a 5 Hz AOA measurement rate.

Note that in Figs. 5.6 - 5.9 $\phi$ changes by $\approx 100^\circ$ as the robot travels along its 1.0 m trajectory, compared to approximately only $20^\circ$ for $\theta$. The mean AOA is defined as the absolute difference between the measured AOA (data) and the AOA calculated based on the respective position of the robot to the LED optical beacons (actual). Table 5.1 shows the mean $\theta$ error for the 49 samples collected along the robot trajectory for $\theta_1$, $\theta_2$, $\theta_3$, and $\theta_4$. The overall mean $\theta$ error is $2.1^\circ$. Table 5.2 shows the mean $\phi$ error for the 49 samples collected along the robot trajectory for $\phi_1$, $\phi_2$, $\phi_3$, and $\phi_4$. The overall mean $\phi$ error is $2.4^\circ$.

Table 5.1: Mean $\theta$ error for 10 cm/s at 5 Hz AOA measurement rate.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean ($^\circ$)</td>
<td>2.4</td>
<td>3.0</td>
<td>1.9</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 5.2: Mean $\phi$ error for 10 cm/s at 5 Hz AOA measurement rate.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$\phi_3$</th>
<th>$\phi_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean ($^\circ$)</td>
<td>2.7</td>
<td>1.4</td>
<td>3.0</td>
<td>2.6</td>
</tr>
</tbody>
</table>
5.1. Angular Receiver Positioning

Even though the angular receiver is not operating strictly within its cone of acceptance, the AOA errors are bounded, and the mean AOA error is not greater than 3°. Using the AOA angles \((\theta_1, \phi_1), (\theta_2, \phi_2), (\theta_3, \phi_3), \text{ and } (\theta_4, \phi_4)\) a Least Squares algorithm is implemented to determine the 2-D and 3-D position estimates of the angular receiver. Figure 5.10 shows the 2-D and 3-D positioning error along the trajectory of the robot.

![Figure 5.10: The 2-D and 3-D positioning error for an angular receiver speed of 10 cm/s and a 5 Hz AOA measurement rate.](image)

Table 5.3 shows the statistics of the 2-D and 3-D positioning error shown in Fig. 5.10. The mean 2-D positioning error is 2.6 cm, compared to 11.4 cm for the 3-D case. The 3-D case also has higher position error standard deviation.

Table 5.3: The 2-D and 3-D error statistics for 10 cm/s at 5 Hz AOA measurement rate.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>2-D</th>
<th>3-D</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (cm)</td>
<td>2.6</td>
<td>11.4</td>
</tr>
<tr>
<td>standard deviation (cm)</td>
<td>0.8</td>
<td>4.0</td>
</tr>
<tr>
<td>minimum (cm)</td>
<td>0.6</td>
<td>3.0</td>
</tr>
<tr>
<td>maximum (cm)</td>
<td>3.8</td>
<td>17.3</td>
</tr>
</tbody>
</table>
5.1. Angular Receiver Positioning

In order to verify the shape of the 3-D position error plot, two investigations are performed. The first is an investigation of the AOA error, in particular the $\theta$ error. This is because the 2-D position estimate uses only equation 3.17 for Least Squares estimation of the $(x, y)$ angular receiver coordinates. Equation 3.17 is only a function of $\phi$. The resulting Least squares position estimate and the true position (from geometry) is used to define the 2-D position error. The 2-D position error in Fig. 5.10 is the Euclidean distance between the estimate and the true position. The 3-D position on the other hand, uses both equation 3.17 and 3.18 to estimate 3-D position. Since the 2-D position errors (which are solely a function of $\phi$) are fairly constant, and none of the AOA errors are dominating (i.e., magnitudes of the of the overall mean $\theta$ errors are approximately equal to the overall mean $\phi$ errors), the $\theta$ errors will therefore be investigated below to verify the 3-D position error plot in Fig. 5.10.

Figures 5.11 and 5.12 show the $\theta_1$ and $\theta_2$ errors respectively, as the absolute difference between the measured $\theta$ and the actual $\theta$. Note that the $\theta_3$ error distribution has the same error pattern as $\theta_1$, while the $\theta_4$ error distribution has the same error pattern as $\theta_2$, and are therefore not shown.

![Figure 5.11: Polar angle $\theta_1$ error versus distance traveled.](image)

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5.1. Angular Receiver Positioning

Figure 5.12: Polar angle $\theta_2$ error versus distance traveled.

Note from Fig. 5.11 that the $\theta_1$ error increases as the angular receiver travels from 0 cm to 45 cm, decreases as the angular receiver travels from 45 cm to 78 cm and increases again as the angular receiver travels from 78 cm to 100 cm. The AOA error increase or decrease can be explained by referring back to the $\theta$ error distribution shown in Fig. 3.12. When the angular receiver travels from 0 cm to 45 cm along the trajectory, the AOA changes from $(\theta = 65^\circ, \phi = 65^\circ)$ to $(\theta = 45^\circ, \phi = 69^\circ)$. This results in a $\theta_1$ error increase of 3.5°. When one observes the same change in AOA in Fig. 3.12, the theta error increases by 2.8°. When the angular receiver travels from 45 cm to 78 cm along the trajectory, the AOA changes from $(\theta = 45^\circ, \phi = 69^\circ)$ to $(\theta = 31^\circ, \phi = 80^\circ)$. This results in a $\theta_1$ error decrease of 5.0°. When one observes the same change in AOA in Fig. 3.12, the theta error increases by 4.7°. When the angular receiver travels from 78 cm to 100 cm along the trajectory, the AOA changes from $(\theta = 31^\circ, \phi = 80^\circ)$ to $(\theta = 23^\circ, \phi = 87^\circ)$. This results in a $\theta_1$ error increase of 2.5°. When one observes the same change in AOA in Fig. 3.12, the $\theta$ error increases by 1.4°.

For the results shown in Fig. 5.12, the $\theta_2$ error increases as the angular receiver travels from 45 cm to 78 cm and then decreases as the angular receiver travels from 78 cm to 100 cm. Similar to the Fig. 5.11 argument, the AOA error increase or decrease can be explained by referring back to the $\theta$ error distribution shown in Fig. 3.12. When the angular receiver travels...
from 45 cm to 78 cm along the trajectory, the AOA changes from \((\theta = 60^\circ, \phi = 64^\circ)\) to \((\theta = 45^\circ, \phi = 70^\circ)\). This results in a \(\theta_2\) error increase of 3.0\(^\circ\). When one observes the same change in AOA in Fig. 3.12, the theta error increases by 5.1\(^\circ\). When the angular receiver travels from 78 cm to 100 cm along the trajectory, the AOA changes from \((\theta = 45^\circ, \phi = 70^\circ)\) to \((\theta = 35^\circ, \phi = 74^\circ)\). This results in a \(\theta_2\) error decrease of 3.0\(^\circ\). When one observes the same change in AOA in Fig. 3.12, the theta error decreases by 2.5\(^\circ\).

The \(\theta\) error results in Figs. 5.11 and 5.12 can explain the general error trend for the 3-D position error in Fig. 5.10. From Fig. 5.10 one observes a 3-D position error increase from 0 cm to 40 cm along the trajectory. This is due to an increase in \(\theta_1\) and \(\theta_2\) errors, as seen in Figs. 5.11 and 5.12, along the same portion of the trajectory. The 3-D position error plateaus between 40 cm and 50 cm in Fig. 5.10 as a result of an equal decrease in \(\theta_1\) error and an equal increase in \(\theta_2\) error over the same region. Finally the 3-D position error decreases slowly between 50 cm and 80 cm in Fig. 5.10 as a result of a sharp decrease in \(\theta_1\) error compared to the increase in \(\theta_2\) error. The 3-D position error decreases sharply between 80 cm and 90 cm in Fig. 5.10 as a result of a sharp decrease in \(\theta_2\) error compared to the increase in \(\theta_1\) error.

The second investigation aims to validate the theoretical 3-D DOP in equation 3.13 using the empirical 3-D DOP in equation 3.3 defined as the ratio of the position standard deviations from the data of Fig. 5.10 to the AOA standard deviations from the data in Figs. 5.6-5.9. The empirical 3-D DOP is compared to the theoretical 3-D DOP as shown below.

The empirical 3-D DOP is calculated as follows. At distance 0 cm, the angular receiver measures 4 pairs of AOA \((\theta_1, \phi_1), (\theta_2, \phi_2), (\theta_3, \phi_3),\) and \((\theta_4, \phi_4)\) from LEDs 1, 2, 3 and 4 respectively. The AOA error is calculated as the difference between the measured AOA and the true AOA (calculated based on geometry). The standard deviation of those eight AOA errors is calculated and is denoted by \(\sigma_m\). The 3-D position estimate at distance 0 cm is computed using the measured AOAs in a Least Squares algorithm and results in an \(\hat{x}, \hat{y},\) and \(\hat{z}\) position estimate. The \(x\) error is calculated as the difference between the \(\hat{x}\) estimate and the actual \(x_R\) coordinate of the angular receiver. The \(y\) and \(z\) errors are computed in a similar fashion. The standard deviation of the \(x\) error, \(y\) error, and \(z\) error is calculated and is denoted be \(\sigma_p\). The ratio of \(\sigma_p/\sigma_m\) is the empirical DOP value at distance 0 cm. At the next point in the trajectory, empirical data is collected, and the process is repeated using the AOAs and position error at this particular distance. Fig. 5.13 shows a plot of the empirical DOP and the theoretical DOP along the entire trajectory.
5.1. Angular Receiver Positioning

Notice from Fig. 5.13 that the DOP increases at 30 cm and then decreases at 60 cm. This mirrors the behaviour of the 3-D position error in Fig. 5.10 over the same region of the trajectory. Therefore, this two-pronged investigation into the shape of the 3-D position error plot of Fig. 5.10 demonstrates that an increase in positioning error can be explained by an increase in the AOA error measurements, as well as an increase in PDOP. Figure 5.13 also verifies that the empirical DOP can be very well approximated by equation 3.13.

Figures 5.14, 5.15, 5.16, and 5.17 show the $\phi_1$, $\phi_2$, $\phi_3$, and $\phi_4$ errors respectively, as the absolute difference between the measured $\phi$ and the actual $\phi$ versus distance traveled. Using a similar argument to the 3-D case, the $\phi$ error results in Figs. 5.14, 5.15, 5.16, and 5.17 can explain the general error trend for the 2-D position error in Fig. 5.10.
5.1. Angular Receiver Positioning

Figure 5.14: Azimuthal angle $\phi_1$ error versus distance traveled.

Figure 5.15: Azimuthal angle $\phi_2$ error versus distance traveled.
5.1. Angular Receiver Positioning

Figure 5.16: Azimuthal angle $\phi_3$ error versus distance traveled.

Figure 5.17: Azimuthal angle $\phi_4$ error versus distance traveled.

Note from Fig. 5.10 that the 2-D position error starts to decrease at
40 cm. This can be attributed to the $\phi$ error patterns where at 40 cm the $\phi_1$ error in Fig. 5.14, the $\phi_3$ error in Fig. 5.16 and the $\phi_4$ error in Fig. 5.17 all decrease sharply. At 60 cm, the $\phi_2$ error in Fig. 5.15 also drops sharply, resulting in minimum 2-D position error at 60 cm. At 60 cm the $\phi_3$ error is increasing, with $\phi_1$ and $\phi_4$ errors increasing at 70 cm. As a result, the 2-D position error is increasing from 60 cm to 80 cm.

In order to determine the effect of AOA measurement rate on positioning performance while the angular receiver is in motion, the above experiment in which the iRobot Create again moves 1 m at a speed of 10 cm/s is repeated but with an AOA measurement rate of 20 Hz. This results in approximately 200 AOA samples recorded during the robot motion compared to 49 samples when the AOA measurement rate is 5 Hz.

Figure 5.18 shows the AOA measured with respect to the $x$-axis for LED$_1$ as the robot moves along the line $x = 0$. The actual AOA, based on the robot to LED$_1$ optical beacon geometry, is shown for accuracy comparison. Similarly, Figs. 5.19, 5.20, and 5.21 show the measured AOA values for LED$_2$, LED$_3$, and LED$_4$ optical beacons, respectively.

![Figure 5.18](image)

Figure 5.18: AOA ($\theta_1$, $\phi_1$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 10 cm/s at a 20 Hz AOA measurement rate.
5.1. Angular Receiver Positioning

Figure 5.19: AOA \((\theta_2, \phi_2)\) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 10 cm/s at a 20 Hz AOA measurement rate.
5.1. Angular Receiver Positioning

Figure 5.20: AOA ($\theta_3$, $\phi_3$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 10 cm/s at a 20 Hz AOA measurement rate.
5.1. Angular Receiver Positioning

Table 5.4 shows the mean $\theta$ error for the 200 samples collected along the robot trajectory for $\theta_1$, $\theta_2$, $\theta_3$, and $\theta_4$. The mean $\theta$ error is 2.4°. Table 5.5 shows the mean $\phi$ error for the 200 samples collected along the robot trajectory for $\phi_1$, $\phi_2$, $\phi_3$, and $\phi_4$. The mean $\phi$ error is 1.9°.

Table 5.4: Mean $\theta$ error for 10 cm/s at 20 Hz measurement update.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (°)</td>
<td>2.5</td>
<td>3.0</td>
<td>1.9</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Table 5.5: Mean $\phi$ error for 10 cm/s at 20 Hz measurement update.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$\phi_3$</th>
<th>$\phi_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (°)</td>
<td>1.9</td>
<td>2.0</td>
<td>1.8</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Figure 5.22 shows the 2-D and 3-D positioning error plots for the angular receiver at a speed of 10 cm/s and a 20 Hz AOA measurement rate.
5.1. Angular Receiver Positioning

Figure 5.22: The 2-D and 3-D positioning error for an angular receiver speed of 10 cm/s and a 20 Hz AOA measurement rate.

Table 5.6 shows the position statistics for the 2-D and 3-D position errors.

Table 5.6: The 2-D and 3-D error statistics for 10 cm/s at 20 Hz AOA measurement rate.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>2-D (cm)</th>
<th>3-D (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>3.2</td>
<td>9.9</td>
</tr>
<tr>
<td>standard deviation</td>
<td>1.2</td>
<td>4.0</td>
</tr>
<tr>
<td>minimum</td>
<td>0.3</td>
<td>2.7</td>
</tr>
<tr>
<td>maximum</td>
<td>6.3</td>
<td>17.2</td>
</tr>
</tbody>
</table>

From the above analysis, the AOA and positioning accuracies of the 5 Hz and 20 Hz measurements are similar, with no major improvements when the sampling rate, and hence AOA measurement rate, increases. This is valid for linear trajectories such as the one described in this analysis. However, in a more complex non-linear trajectory, a higher measurement rate will give a truer estimate of the trajectory.

5.1.2 Medium Speed 50 cm/s

This section determines the AOA and positioning accuracies while the angular receiver moves at a speed of 50 cm/s for 5 Hz and 20 Hz AOA
5.1. Angular Receiver Positioning

measurement rates. Results for the 5 Hz AOA measurement rate are shown first. As the angular receiver moves at a speed of 50 cm/s over 100 cm, 10 AOA samples will be recorded. Figure 5.23 shows photocurrents \( i_1 \), \( i_2 \) and \( i_3 \) corresponding to LED optical beacons 1, 2, 3 and 4 as the angular receiver moves along its 100 cm path. Note that the shapes of the photocurrents in Fig. 5.23 follow the same patterns as the photocurrents in Fig. 5.5 for the 10 cm/s speed, which asserts the performance of the amplifying/filter circuit to combat noise.

![Figure 5.23: The photocurrents used to generate the AOA angles are shown versus iRobot Create distance traveled for LED_1, LED_2, LED_3, and LED_4. Photocurrent \( i_1 \), \( i_2 \), and \( i_3 \) are shown as the red (solid), black (dotted) and blue (dashed) curves respectively.](image)

Figure 5.24 shows the 10 AOA values (expressed as \( \theta \) and \( \phi \)) for LED_1 optical beacon as the robot moves along the line \( x = 0 \). The actual AOA, computed based on the robot to LED_1 optical beacon geometry, is shown for accuracy comparison. Similarly, Figs. 5.25, 5.26, and 5.27 show the measured AOA values for LED_2, LED_3, and LED_4 optical beacons respectively.
5.1. Angular Receiver Positioning

Figure 5.24: AOA ($\theta_1, \phi_1$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 50 cm/s at a 5 Hz AOA measurement rate.
5.1. Angular Receiver Positioning

Figure 5.25: AOA ($\theta_2$, $\phi_2$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 50 cm/s at a 5 Hz AOA measurement rate.
5.1. Angular Receiver Positioning

Figure 5.26: AOA ($\theta_3$, $\phi_3$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 50 cm/s at a 5 Hz AOA measurement rate.
5.1. Angular Receiver Positioning

Figure 5.27: AOA ($\theta_4, \phi_4$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 50 cm/s at a 5 Hz AOA measurement rate.

Table 5.7 shows the mean $\theta$ error for the 10 samples collected along the robot trajectory for $\theta_1$, $\theta_2$, $\theta_3$, and $\theta_4$. The overall mean $\theta$ error is 3.2°. Table 5.8 shows the mean $\phi$ error for the 10 samples collected along the robot trajectory for $\phi_1$, $\phi_2$, $\phi_3$, and $\phi_4$. The overall mean $\phi$ error is 13.2°.

Table 5.7: Mean $\theta$ error for 50 cm/s at 5 Hz AOA measurement rate.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (°)</td>
<td>2.8</td>
<td>5.1</td>
<td>2.3</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Table 5.8: Mean $\phi$ error for 50 cm/s at 5 Hz AOA measurement rate.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$\phi_3$</th>
<th>$\phi_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (°)</td>
<td>10.7</td>
<td>14.0</td>
<td>13.0</td>
<td>15.4</td>
</tr>
</tbody>
</table>

Note that the AOA error for the 50 cm/s-5 Hz case has significantly increased compared to the 10 cm/s-5 Hz case in Section 5.1.1. Although the $\theta$ error changed very little, the $\phi$ error increased by a factor of five. This can be attributed to two factors: the first is the robot speed, which affects
the rate of change of the AOA measurements, and the second factor is the LED/receiver geometry.

The robot speed has increased from 10 cm/s in Section 5.1.1 to 50 cm/s in this experiment, resulting in the robot traveling the 100 cm path in a much shorter time (2 s) and, therefore, the distance the robot moves while collecting the 2048 voltage samples needed to produce one AOA measurement, ∆D, is significantly greater. This distance is defined as ∆D = speed (cm/s)/(sampling frequency-1 (Hz)). For the case of 50 cm/s-5 Hz, ∆D = 12.5 cm, while for 10 cm/s-5 Hz, ∆D = 2.5 cm. The larger the ∆D gets, the larger AOA errors become since the AOA changes significantly from the first voltage sample to the 2048th voltage sample needed to compute one AOA measurement. The second factor relates to the LED/receiver geometry. Since the change in θ in Figs. 5.24, 5.25, 5.26, and 5.27 along the entire trajectory is only 14° compared to 100° for φ, the φ angles vary more so than θ angles during the collection of the 2048 samples. Therefore, the true φ angle for the first sample is significantly different than the true φ for the 2048th sample resulting in a more erroneous φ for all 2048 samples.

Figure 5.28 shows the 2-D and 3-D position error as the robot moves 1 m at a speed of 50 cm/s with a 5 Hz AOA measurement rate. Note that the 2-D positioning error increases with increased distance moved. This is because the 2-D position error depends on φ, which increases with distance traveled, as shown in the right hand plots of Figs. 5.24, 5.25, 5.26, and 5.27. Note also that the φ error (responsible for the 2-D position accuracy) has an overall mean error of 13.2° that is much larger (four times) than the θ error, with overall mean error of 3.2°, as shown in Tables 5.7 and 5.8. This will result in the 3-D positioning error being dominated by φ error and, therefore, the 2-D and 3-D position plots will have similar error patterns.
5.1. Angular Receiver Positioning

Figure 5.28: The 2-D and 3-D positioning error for an angular receiver speed of 50 cm/s and a 5 Hz AOA measurement rate.

Table 5.9 shows the position statistics for the 2-D and 3-D position error.

Table 5.9: The 2-D and 3-D error statistics for 50 cm/s at 5 Hz AOA measurement rate.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>2-D</th>
<th>3-D</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (cm)</td>
<td>12.7</td>
<td>18.2</td>
</tr>
<tr>
<td>standard deviation (cm)</td>
<td>1.8</td>
<td>2.6</td>
</tr>
<tr>
<td>minimum (cm)</td>
<td>3.7</td>
<td>10.1</td>
</tr>
<tr>
<td>maximum (cm)</td>
<td>17.8</td>
<td>23.3</td>
</tr>
</tbody>
</table>

Now the performance of the angular receiver is investigated for the same speed but with a 20 Hz measurement update rate. Figure 5.29 shows the AOA measured with respect to the $x$-axis for LED$_1$ as the robot moves along the line $x = 0$ at a speed of 50 cm/s with a 20 Hz measurement update rate. Forty AOA measurements are collected along the 1.0 m trajectory. The actual AOA, based on the robot to LED$_1$ optical beacon geometry, is shown for accuracy comparison. Similarly, Figs. 5.30, 5.31, and 5.32 show the measured AOA values for LED$_2$, LED$_3$, and LED$_4$ optical beacons respectively.
5.1. Angular Receiver Positioning

Figure 5.29: AOA ($\theta_1$, $\phi_1$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 50 cm/s at a 20 Hz AOA measurement rate.
Figure 5.30: AOA ($\theta_2$, $\phi_2$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 50 cm/s at a 20 Hz AOA measurement rate.
5.1. Angular Receiver Positioning

Figure 5.31: AOA (θ₃, φ₃) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 50 cm/s at a 20 Hz AOA measurement rate.
5.1. Angular Receiver Positioning

Figure 5.32: AOA (\(\theta_4, \phi_4\)) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 50 cm/s at a 20 Hz AOA measurement rate.

Table 5.10 shows the mean error for \(\theta_1, \theta_2, \theta_3,\) and \(\theta_4\) for the 40 AOAs measured along the robot trajectory. The overall mean \(\theta\) error is 2.3°. Table 5.11 shows the mean error for \(\phi_1, \phi_2, \phi_3,\) and \(\phi_4\) for the 40 AOAs measured along the robot trajectory. The overall mean \(\phi\) error is 3.9°.

Table 5.10: Mean \(\theta\) error for 50 cm/s at 20 Hz AOA measurement rate.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(\theta_1)</th>
<th>(\theta_2)</th>
<th>(\theta_3)</th>
<th>(\theta_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (°)</td>
<td>2.6</td>
<td>3.3</td>
<td>1.8</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table 5.11: Mean \(\phi\) error for 50 cm/s at 20 Hz AOA measurement rate.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(\phi_1)</th>
<th>(\phi_2)</th>
<th>(\phi_3)</th>
<th>(\phi_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (°)</td>
<td>3.7</td>
<td>5.2</td>
<td>4.3</td>
<td>2.2</td>
</tr>
</tbody>
</table>

The Fig. 5.33 shows the 2-D and 3-D position error as the robot moves 1 m at a speed of 50 cm/s with a 20 Hz AOA measurement rate.
5.1. Angular Receiver Positioning

Table 5.12 shows the position statistics for the 2-D and 3-D position error.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>2-D</th>
<th>3-D</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (cm)</td>
<td>4.0</td>
<td>12.0</td>
</tr>
<tr>
<td>standard deviation (cm)</td>
<td>1.8</td>
<td>2.6</td>
</tr>
<tr>
<td>minimum (cm)</td>
<td>1.3</td>
<td>8.0</td>
</tr>
<tr>
<td>maximum (cm)</td>
<td>8.1</td>
<td>16.7</td>
</tr>
</tbody>
</table>

Comparing the 50 cm/s speed for the 20 Hz and 5 Hz measurement update rates, one sees a significant reduction in AOA and positioning error for the 20 Hz case. The average $\phi$ error has significantly reduced from 13.2° (50 cm/s-5 Hz) to 3.9° for the current case of (50 cm/s-20 Hz). Also, the AOA errors for the current case (50 cm/s-20 Hz) is comparable to the 10 cm/s-5 Hz case and the 10 cm/s-20 Hz case, and as a result, the 2-D and 3-D positioning errors in Fig. 5.33 follow the same shapes as in Figs. 5.10 and 5.22.
5.1. Angular Receiver Positioning

5.1.3 Average Walking Speed 139 cm/s

In this section, the AOA estimation accuracy and positioning performance of the angular receiver is quantified as it moves for 100 cm at an average speed of 139 cm/s with a 20 Hz AOA measurement rate. A speed of 139 cm/s (5 km/h) represents the average human walking speed [56]. A total of 14 AOA measurements are made along the 100 cm trajectory. Figure 5.34 shows photocurrents $i_1$, $i_2$ and $i_3$ corresponding to LED optical beacons 1, 2, 3 and 4 as the angular receiver is moving along its 100 cm path at a speed of 139 cm/s at 20 Hz AOA measurement rate.

Note that the photocurrents in Fig. 5.34 look more linear than the photocurrents for the 10 cm/s and 50 cm/s cases in Figs. 5.5 and 5.23 respectively. This is because for 139 cm/s at 20 Hz AOA measurement rate, the robot moves 100 cm in a much shorter time period of 0.7 s and collects only 14 AOA samples, compared to a 10 s time period with 49 AOA samples for the 10 cm/s robot speed with 5 Hz AOA measurement rate.

This phenomenon can be explained by observing the photocurrent $i_3$ (dashed blue) for LED2 in the top right hand plots of Figs. 5.5 and 5.34, and from the observed pattern of the robot motion at 10 cm/s and 139 cm/s. In the 10 cm/s case in Fig. 5.5 the robot moves along the 100 cm trajectory at a fairly constant speed. However, in the 139 cm/s case the robot experiences significant acceleration at the start of its motion to reach a velocity of 139 cm/s from an initial zero velocity. This means that the 2048 sample photocurrents collected to compute an AOA at time instant 0 for the 139 cm/s case in Fig. 5.34 is actually the average summation of photocurrents in Fig. 5.5 at time instances greater than and equal to 0. Since the photocurrent $i_3$ for LED2 in Fig. 5.5 is increasing from 0 cm to 60 cm, photocurrent $i_3$ for LED2 in Fig. 5.34 will be at a greater value at time instant 0 compared to that in Fig. 5.5. Similarly, towards the end of the robot motion, the robot must experience negative acceleration to reach zero velocity. This means that the 2048 photocurrent samples measured at a given time or distance shown in Fig. 5.34 (for instance at time $t = 0.7$ s or distance $d = 0.7 \times 139 \approx 100$ cm) is in fact the average summation of photocurrents in Fig. 5.5 at times less than or equal to 0.7 s or distances less than or equal 100 cm. Consequently, in the 139 cm/s case, photocurrent $i_3$ will be higher at the start of the trajectory than in the 10 cm/s case, and vice versa at the end of the trajectory. Since $i_3$ is increasing for LED2, the $i_3$ photocurrent plot for the 139 cm/s case appears linear.

The latency or delay in the angular receiver (PD and amplifying circuit) response is measured to be 0.15 ms, while the time needed for AOA mea-
5.1. Angular Receiver Positioning

measurement for the 139 cm/s-20 Hz measurement rate is \(0.7 \text{s}/(14-1) = 5.3 \text{ ms}\). This means that the angular receiver responds fast enough to changes in illumination levels as the robot moves at 139 cm/s at 20 Hz measurement rate, and, therefore, the linear behaviour of the photocurrents is only due to the non-constant velocity of the robot.

Figure 5.34: The photocurrents used to generate the AOA angles are shown versus iRobot Create distance traveled for LED\(_1\), LED\(_2\), LED\(_3\), and LED\(_4\). Photocurrent \(i_1\), \(i_2\), and \(i_3\) are shown as the red (solid), black (dotted) and blue (dashed) curves respectively.

Figure 5.35 shows the AOA measured with respect to the \(x\)-axis for LED\(_1\) as the robot moves along the line \(x = 0\) at a speed of 139 cm/s at a 20 Hz measurement update rate. The actual AOA, based on the robot to LED\(_1\) optical beacon geometry, is shown for accuracy comparison. Similarly, Figs. 5.36, 5.37, and 5.38 show the AOA measured from LED\(_2\), LED\(_3\), and LED\(_4\) optical beacons respectively.
5.1. Angular Receiver Positioning

Figure 5.35: AOA ($\theta_1$, $\phi_1$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 139 cm/s at a 20 Hz AOA measurement rate.
5.1. Angular Receiver Positioning

Figure 5.36: AOA ($\theta_2$, $\phi_2$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 139 cm/s at a 20 Hz AOA measurement rate.
5.1. Angular Receiver Positioning

Figure 5.37: AOA ($\theta_3$, $\phi_3$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 139 cm/s at a 20 Hz AOA measurement rate.
5.1. Angular Receiver Positioning

Figure 5.38: AOA ($\theta_4$, $\phi_4$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 139 cm/s at a 20 Hz AOA measurement rate.

Note that the AOA errors in Figs. 5.35, 5.36, 5.37, and 5.38 are much larger than the AOA errors at lower speeds of 10 cm/s and 50 cm/s. One possible reason for the large AOA error at the start of the robot motion is the fact that to reach a speed of 139 cm/s from standstill as quickly as possible, the robot must experience significant acceleration at the start of its trajectory, and also to stop, it must experience significant negative acceleration at the end of the trajectory. Therefore, the actual AOA is not representative of where the AOA measurements are made, and this will cause the difference between the actual and measured AOA to be large at the start and end of the trajectory, as shown in Figs. 5.35, 5.36, 5.37, and 5.38. For this particular experiment a longer track would likely yield more accurate AOA estimates since the time the robot experiences acceleration and negative acceleration will be less.

Table 5.13 shows the mean $\theta$ error for the 14 samples collected along the robot trajectory for $\theta_1$, $\theta_2$, $\theta_3$, and $\theta_4$. The overall mean $\theta$ error is 3.0°. Table 5.14 shows the mean $\phi$ error for the 14 samples collected along the robot trajectory for $\phi_1$, $\phi_2$, $\phi_3$, and $\phi_4$. The overall mean $\phi$ error is 19.5°.

Figure 5.39 shows the 2-D and 3-D positioning error versus distance traveled. Table 5.15 shows the position statistics for the 2-D and 3-D position.
5.1. Angular Receiver Positioning

Table 5.13: Mean θ error for 139 cm/s at 20 Hz AOA measurement rate.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>θ_1</th>
<th>θ_2</th>
<th>θ_3</th>
<th>θ_4</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (°)</td>
<td>3.9</td>
<td>3.3</td>
<td>1.8</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Table 5.14: Mean φ error for 139 cm/s at 20 Hz AOA measurement rate.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>φ_1</th>
<th>φ_2</th>
<th>φ_3</th>
<th>φ_4</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (°)</td>
<td>19.0</td>
<td>18</td>
<td>18.0</td>
<td>23.0</td>
</tr>
</tbody>
</table>

Note that the 2-D and 3-D positioning error plots shown in Fig. 5.39 have much larger position errors compared to the 10 cm/s and 50 cm/s cases in Sections 5.1.1 and 5.1.2 respectively. This is due to large φ and θ errors at the start and end of the robot trajectory. Note that, at the centre of the trajectory, around the 50 cm position, error drops to 5 cm for the 2-D case, and to 20 cm for the 3-D case. This is because the φ and θ errors are less towards the centre of the trajectory than at the start or end, as shown in Figs. 5.35, 5.36, 5.37, and 5.38, since the robot would have reached constant speed by this time.

Figure 5.39: The 2-D and 3-D positioning error for angular receiver speed of 139 cm/s and a 20 Hz AOA measurement rate.

Note that the φ error (responsible for the 2-D position accuracy) has
Table 5.15: The 2-D and 3-D error statistics for 139 cm/s at 20 Hz AOA measurement rate.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>2-D</th>
<th>3-D</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (cm)</td>
<td>20</td>
<td>27</td>
</tr>
<tr>
<td>standard deviation (cm)</td>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td>minimum (cm)</td>
<td>3</td>
<td>18.0</td>
</tr>
<tr>
<td>maximum (cm)</td>
<td>40</td>
<td>43</td>
</tr>
</tbody>
</table>

an overall mean error of $19.5^\circ$ that is much larger (6.5 times) than the $\theta$ error with overall mean error of $3^\circ$ shown in Tables 5.13 and 5.14. This will result in the 3-D positioning error to be dominated by $\phi$ error. Similar to the positioning results for the 50 cm/s at 5 Hz AOA measurement rate, in Fig. 5.28, the 2-D and 3-D position plots will have similar error patterns.

5.1.4 Summary

Table 5.16 summarizes the positioning error results for the angular receiver in motion. $\Delta D$ is the distance traveled while collecting the total number of voltage samples required to find one AOA measurement. For instance when the robot moves at 10 cm/s at a 5 Hz AOA measurement rate, AOA measurements are made every $\Delta D = 10/(5-1) = 2.5$ cm. The 5 Hz measurement rate is a result of LabVIEW capturing 2048 samples at a rate of 10 kHz. Therefore, in this case, the time over which voltage samples are collected to produce one AOA measurement is 0.2 s.

Table 5.16: Summary of 2-D and 3-D error statistics.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>mean 2-D error (cm)</th>
<th>mean 3-D error (cm)</th>
<th>$\Delta D$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 cm/s-5 Hz-2048</td>
<td>2.6</td>
<td>11.4</td>
<td>2.5</td>
</tr>
<tr>
<td>10 cm/s-20 Hz-2048</td>
<td>3.2</td>
<td>9.9</td>
<td>0.53</td>
</tr>
<tr>
<td>50 cm/s-5 Hz-2048</td>
<td>12.7</td>
<td>18.2</td>
<td>12.5</td>
</tr>
<tr>
<td>50 cm/s-10 Hz-1024</td>
<td>8.3</td>
<td>13.0</td>
<td>5.5</td>
</tr>
<tr>
<td>50 cm/s-20 Hz-2048</td>
<td>4.0</td>
<td>12.0</td>
<td>2.6</td>
</tr>
<tr>
<td>139 cm/s-20 Hz-2048</td>
<td>20</td>
<td>27</td>
<td>7.3</td>
</tr>
</tbody>
</table>

The first column in Table 5.16 shows the robot speed in cm/s, the AOA measurement rate in Hz, and the number of samples read by LabVIEW. From Table 5.16 one observes that lowest positioning error occurs for the
5.2. Image Receiver Positioning

In this section, the performance of the image receiver is studied while in motion. Similar to Section 5.1, iRobot Create is used to host the image receiver. Figure 5.40 shows an image of the image receiver mounted on iRobot Create. An Arduino nano is used to control the robot motion. Figure 5.41 shows a magnified view of the image sensor and the microlens.
5.2. Image Receiver Positioning

Figure 5.40: Image receiver mounted on iRobot Create.

Figure 5.41: A magnified view of the microlens and the image sensor.
Four LED optical beacons are positioned in an \((x, y, z)\) navigational frame coordinate system as shown in Fig. 5.3 such that \(\text{LED}_1\) is positioned at \((-35\ \text{cm}, 30\ \text{cm}, 100\ \text{cm})\), \(\text{LED}_2\) at \((-35\ \text{cm}, 65\ \text{cm}, 100\ \text{cm})\), \(\text{LED}_3\) at \((35\ \text{cm}, 30\ \text{cm}, 100\ \text{cm})\), and \(\text{LED}_4\) at \((35\ \text{cm}, 65\ \text{cm}, 100\ \text{cm})\). Fig. 5.3 shows a top-view of the LED/image receiver setup. The \(z\) axis represents the vertical separation between the LED optical beacons and the image receiver. The LED optical beacons are at \(z = 100\ \text{cm}\), while the image receiver lies in the \(xy\) plane.

Similar to Section 4.4 the four LED optical beacons emit white light by frequency modulating the R, G and B LED that make up each optical beacon, such that \(\text{LED}_1\) has \(R_{\text{DC}}G_{f_1}B_{f_1}\), \(\text{LED}_2\) has \(R_{\text{DC}}G_{f_1}B_{f_2}\), \(\text{LED}_3\) has \(R_{\text{DC}}G_{f_2}B_{f_2}\), and \(\text{LED}_4\) has \(R_{\text{DC}}G_{f_2}B_{f_1}\), where DC refers to \(f = 0\), \(f_1\) is 70 Hz and \(f_2\) is 80 Hz. In order to match each LED optical beacon to its corresponding focal spot on the image, an FFT algorithm is run, before the image receiver starts moving, to detect the frequency components on the R, G, and B layers.

The iRobot Create is programmed to move along a straight line trajectory. The robot trajectory starts at the navigational frame coordinate \((0, 0, 0)\) and moves along the line \(x = 0\) for a distance of 100 cm. For this analysis, the microlens with the ultra-wide FOV (90° contact angle) is utilized. As the robot starts moving a video application starts. This application records the images captured by the image receiver on a video. By analyzing the video frames one can determine the exact times at which the robot starts and stops. Also, by knowing the image receiver’s frame rate, the exact frame corresponding to a translated position of the image receiver is calculated.

The AOA accuracy and the resulting positioning accuracy are studied when varying the image receiver speed. Section 5.2.1 shows the AOA and positioning results when the image receiver is moving at a speed of 5 cm/s, while Section 5.2.2 shows the AOA and positioning results when the image receiver is moving at a speed of 10 cm/s.

### 5.2.1 Very Low Speed 5 cm/s

The AOAs are computed at 10 cm intervals. At each interval the AOA is measured from each of the four LEDs which results in a total of 44 AOAs. Figures 5.42, 5.43, 5.44, and 5.45 show the AOA computed using the method described in Section 4.4 and the actual AOA based on geometry, for \(\text{LED}_1\), \(\text{LED}_2\), \(\text{LED}_3\), and \(\text{LED}_4\) respectively. The mean \(\theta\) and \(\phi\) errors, for all AOAs across the entire trajectory and for all four optical beacons, is approximately 1°. Note that the image receiver mean AOA error is approximately
5.2. Image Receiver Positioning

1°-3° which is more accurate than the angular receiver mean AOA error when operating at \( \Delta D \leq 2.5 \) cm.

Figure 5.42: AOA \((\theta_1, \phi_1)\) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 5 cm/s.
5.2. Image Receiver Positioning

Figure 5.43: AOA \((\theta_2, \phi_2)\) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 5 cm/s.
Figure 5.44: AOA ($\theta_3$, $\phi_3$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 5 cm/s.
5.2. Image Receiver Positioning

Figure 5.45: AOA (θ₄, φ₄) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 5 cm/s.

Figure 5.46 shows the 2-D and 3-D positioning error as the image receiver moves for 1 m. Table 5.17 shows the statistics of the 2-D and 3-D positioning error shown in Fig. 5.46. Note that in stark contrast to the 2-D and 3-D positioning error for the angular receiver, the 2-D and 3-D positioning errors for the image receiver are relatively constant over the entire trajectory.
5.2. Image Receiver Positioning

Figure 5.46: The 2-D and 3-D position error versus distance traveled for the image receiver at 5 cm/s.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>2-D</th>
<th>3-D</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean (cm)</td>
<td>0.9</td>
<td>4.9</td>
</tr>
<tr>
<td>standard deviation (cm)</td>
<td>0.17</td>
<td>0.18</td>
</tr>
<tr>
<td>minimum (cm)</td>
<td>0.7</td>
<td>4.6</td>
</tr>
<tr>
<td>maximum (cm)</td>
<td>1.3</td>
<td>5.3</td>
</tr>
</tbody>
</table>

In order to verify the 3-D positioning results, a 3-D DOP investigation is carried out to validate the theoretical 3-D DOP in equation 3.13 using the empirical 3-D DOP in equation 3.3 defined as the ratio of the position standard deviations from the data of Fig. 5.46 to the AOA standard deviations from the data in Figs. 5.42-5.45. The empirical 3-D DOP is compared to the theoretical 3-D DOP below.

The empirical 3-D DOP is calculated as follows. At distance 0 cm, the image receiver measures 4 pairs of AOA \((\theta_1, \phi_1), (\theta_2, \phi_2), (\theta_3, \phi_3),\) and \((\theta_4, \phi_4)\) from LEDs 1, 2, 3 and 4 respectively. The AOA error is calculated as the difference between the measured AOA and the true AOA (calculated based on geometry). The standard deviation of those eight AOA errors is
5.2. Image Receiver Positioning

calculated and is denoted by $\sigma_m$. The 3-D position estimate at distance 0 cm is computed using the measured AOAs in a Least Squares algorithm and results in an $\hat{x}$, $\hat{y}$, and $\hat{z}$ position estimate. The $x$ error is calculated as the difference between the $\hat{x}$ estimate and the actual $x$ coordinate of the image receiver. The $y$ and $z$ errors are computed in a similar fashion. The standard deviation of the $x$ error, $y$ error, and $z$ error is calculated and is denoted be $\sigma_p$. The ratio of $\sigma_p/\sigma_m$ is the empirical DOP value at distance 0 cm. At the next point in the trajectory, empirical data is collected, and the process is repeated using the AOAs and position error at this particular distance. Figure 5.47 shows a plot of the empirical DOP and the theoretical DOP along the entire trajectory. Note the very close agreement between the theoretical and empirical DOP validates equation 3.13.

Figure 5.47: Theoretical 3-D DOP versus empirical DOP calculated along the image receiver trajectory.

The next section studies the image receiver AOA and positioning accuracy when moving at a speed of 10 cm/s.

5.2.2 Low Speed 10 cm/s

The above test is repeated but with the image receiver moving at 10 cm/s. AOA measurements are again made at 10 cm intervals. Figures 5.48, 5.49, 5.50,
and 5.51 show the AOA computed using the method shown in Section 4.4 and the actual AOA based on geometry, for LED₁, LED₂, LED₃, and LED₄ respectively. The mean θ and φ errors of the data points is again approximately 1°.

Figure 5.48: AOA (θ₁, φ₁) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 10 cm/s.
Figure 5.49: AOA ($\theta_2$, $\phi_2$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 10 cm/s.
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Figure 5.50: AOA ($\theta_3$, $\phi_3$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 10 cm/s.
Figure 5.51: AOA ($\theta_4$, $\phi_4$) versus robot travel distance computed using both the AOA measurement (data) and geometry calculation (actual) when moving at a speed of 10 cm/s.

Figure 5.52 shows the 2-D and 3-D positioning error as the image receiver moves for 1 m. Table 5.18 shows the statistics of the 2-D and 3-D positioning error shown in Fig. 5.52. Again, as in the case of the image receiver speed of 5 cm/s, the 2-D and 3-D positioning errors are relatively constant.
5.2. Image Receiver Positioning

Figure 5.52: The 2-D and 3-D position error versus distance traveled for the image receiver at 10 cm/s.

Table 5.18: The 2-D and 3-D error statistics for the image receiver moving at a speed of 10 cm/s.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>2-D (cm)</th>
<th>3-D (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>1.5</td>
<td>4.1</td>
</tr>
<tr>
<td>standard deviation</td>
<td>0.15</td>
<td>0.12</td>
</tr>
<tr>
<td>minimum</td>
<td>1.3</td>
<td>4.0</td>
</tr>
<tr>
<td>maximum</td>
<td>1.7</td>
<td>4.4</td>
</tr>
</tbody>
</table>

Comparing the 2-D and 3-D results in Table 5.18 with the angular receiver 2-D and 3-D results for the same speed of 10 cm/s in Table 5.3 one observes that the 2-D and 3-D mean positioning errors for the angular receiver are greater than the 2-D and 3-D mean positioning errors for the image receivers by factors greater than approximately two in some cases.

Moving at speeds greater than 10 cm/s introduced AOA errors due to slight motion of the actual microlens with respect to the CMOS sensor which caused the focal spot to move. Therefore, 10 cm/s represents the maximum speed the image receiver can travel utilizing the setup in Fig. 5.40. To achieve accurate position results at higher speeds it is essential that the microlens and image sensor platforms be mounted as one platform to ensure
5.2. Image Receiver Positioning

In summary, comparing the AOA and positioning results for the angular and image receivers in motion, one observes that the AOA error characteristics of the angular receiver are highly dependent on the particular incident AOA as shown in the results of Figs. 3.11 and 3.12. As a result, the 2-D and 3-D position accuracies varied accordingly as the incident AOA changed while the angular receiver moved along its trajectory and this resulted in high position variances. In the image receiver case, the AOA errors, as shown in Fig. 4.9 and 4.10, are largely independent of the incident AOA within the angular receiver FOV. As a result the image receiver 2-D and 3-D position accuracies were fairly constant with little variations over the entire trajectory.
Chapter 6

Conclusions and Recommendations

This thesis presented a thorough assessment of indoor optical positioning using two receivers, namely an angular receiver and an image receiver. In this chapter all the key conclusions of this research are outlined. This is followed by recommendations for future work.

6.1 Conclusions

In order to evaluate the performance of the angular and image receivers for indoor optical positioning the following objectives were completed.

1. Determine the feasibility of measuring the AOA of light with the angular receiver and the image receiver.

2. Determine the accuracy of the AOA measured by the angular receiver and the image receiver.

3. Determine the effect of optical beacon geometry on the angular receiver and image receiver systems positional accuracy.

4. Determine the positional accuracy when both receivers are static and while in motion.

The conclusions drawn with respect to each of the objectives are given below starting with those associated with the angular receiver.

6.1.1 Angular Receiver

1. The angular receiver is a corner-cube structure with interior sides made up of PDs. When light from an LED strikes the angular receiver, each of the three PD sides produces a photocurrent proportional to the intensity of the incident light that strikes it. The three generated photocurrents are normalized with respect to the maximum photocurrent.
and are subtracted with respect to one photocurrent to form two differential equations. These two differential equations are then solved simultaneously to compute the AOA of the incident light beam. The precision of the AOAs was found to be very high with a maximum standard deviation of 0.03°.

2. Given the large bandwidth of the angular receiver and LED optical beacons, different LEDs were modulated at different frequencies which necessitated designing a band-pass and amplifier circuit to filter ambient noise, such as 60 Hz room light, so that the angular receiver can accurately detect and distinguish between different LEDs based on their modulation frequencies. A Butterworth bandpass filter was designed and tested with the angular receiver for this role, and proved to be effective in attenuating ambient light, since the AOA computed in the presence of ambient light was equal to the AOA computed in the absence of ambient noise.

3. The effect of diffuse multipath was investigated for various materials such as drywall, plywood and stainless steel. The effect of multipath reflections on AOAs measured with the angular receiver was found to be negligible when the percentage of reflected light was at most 20% of the total incident light.

4. For LOS environments, a configuration-dependent minimum optical intensity for accurate AOA estimation was determined to be 0.2 μW/cm². Light incident at the angular receiver below this minimum intensity yielded erroneous AOA measurements. The minimum intensity is unique to the particular PDs used in the angular receiver tested. An angular receiver constructed from different PDs would likely have a different minimum intensity.

5. The accuracy of the angular receiver AOA measurements was investigated for various LED incident angles. Least accurate AOAs were found to be at the edges of the angular receiver, when \( \theta \) approaches 0° or 90° or when \( \phi \) approaches 0° or 90°, while more accurate AOAs were found to be close to the angular receiver axis of symmetry \( \theta = 54.7° \) and \( \phi = 45° \). An operational cone of 40° \( \times \) 40° about \( \theta = 54.7° \) and \( \phi = 45° \) was found to provide a mean AOA error of 2°. For typical LED/angular receiver separation distances of 2 m, the 2° error translates into position errors in the order of a few centimeters. Such an
6.1. Conclusions

Accuracy is necessary for applications such as guiding robots through narrow areas such as doorways.

6. In optical RSS positioning systems, the position of a receiver with respect to a fixed grid of LED beacons with known positions is computed by first measuring its range to each of the LEDs using received signal strength and a channel model, and then estimating the receiver position using trilateration. The angular receiver is essentially an optical RSS system, since it measures the RSS on each of its PD sides. However, it uses normalized PD differences to get an AOA. Angular receiver position is then estimated using triangulation. It was essential to compare the positioning performance of the angular receiver to an optical RSS-based system. Such a comparison showed that the position error using the angular receiver was 75% lower than that of optical RSS.

7. Typically, the angular receiver will be mounted facing upwards to capture within its FOV the maximum number of LEDs mounted on the ceiling. For a square LED grid having four LEDs one at each corner, and for LED/angular receiver vertical distances of 2 m, the square grid side-length must be at most 2 m for the four LEDs mounted on the ceiling to be within the angular receiver FOV defined for $\theta$ between $0^\circ$ and $90^\circ$ and $\phi$ between $0^\circ$ and $90^\circ$.

8. The angular receiver position accuracy was investigated while the angular receiver was in motion. This was done by mounting the angular receiver on a robot platform. The robot moved at speeds of 10 cm/s, 50 cm/s and 5 km/h (average human walking speed). During motion, the angular receiver collected AOA readings from four optical beacons mounted on the ceiling. At each of the above speeds the AOA measurement rate was varied. For the given geometry of LEDs and angular receiver, it was found that to achieve a 3-D positioning error on the order of 10 cm, the AOA measurement rate in Hz must be at least half the angular receiver speed in cm/s.

6.1.2 Image Receiver and Angular Receiver Performance Comparison

1. The performance of the image receiver, consisting of a custom-made microlens and an image sensor, was tested for AOA estimation from LEDs mounted on the ceiling. The custom-made microlens presents
6.1. Conclusions

a significant advantage over current microlenses since it achieves an especially wide FOV (130°) and a very short focal length of a few millimeters. This facilitates the integration of this image receiver in emerging technologies such as cellular cameras.

2. An AOA was determined by first recording a video of the LEDs with the image sensor, and then finding the pixel position of a spot corresponding to a particular LED. Based on the LED pixel position with respect to the microlens pixel position, θ and φ were determined.

3. In order to test the maximum FOV of the microlens, several LEDs were scattered on the ceiling at various positions with respect to the image receiver, and their AOAs measured. Two different microlenses were used for this investigation having different curvatures and, therefore, different FOV characteristics. The maximum FOV, in terms of θ defined with respect to the vertical axis, was found to be 95° for the wide FOV microlens and 130° for the ultra-wide FOV microlens.

In comparison to the angular receiver FOV in Section 6.1.1, for a square LED grid having four LEDs one at each corner, and for vertical distances of 2 m between the LEDs and image receiver, the image receiver with the ultra-wide FOV microlens can image the four LEDs with a square grid side-length that is three times that of the angular receiver.

4. The mean AOA for angles within the FOV of the image receiver was found to be 0.5° which is 75% lower than the 2° AOA error for the angular receiver within its operational cone. This mean AOA was determined when the optical intensities of the LEDs being imaged were between 0.03 µW/cm² and 0.2 µW/cm². Note that the image receiver can determine an AOA at dimmer optical intensities compared to the angular receiver (having a minimum intensity of 0.2 µW/cm²).

5. Unlike the angular receiver which consists of only three PDs, the image receiver consists of thousands of PDs (pixels) and, therefore, is slower being limited by frame rate. The frame rate of the image receiver used in this thesis was 187 frames/s which allows it to distinguish between LEDs modulated up to approximately 90 Hz. In order for the image receiver to distinguish between different LEDs, a novel colour and frequency multiplexing scheme was implemented utilizing Red Green Blue (RGB) LED optical beacons. All LEDs appear to emit a white colour. However, each of the R, G, and B pins of an individual LED
6.2. Recommendations

are modulated at unique frequencies. An FFT algorithm operating on the image pixel data is utilized to determine the R, G, and B spectral components. One drawback of this technique is that it requires the camera to be stationary. This is needed to accurately acquire LED modulation frequencies.

6. While in motion the image receiver achieved an approximately constant position error. This was due to the constant nature of the AOA error which was largely independent of the incident AOA. For the same geometry, the mean 3-D position error of the image receiver was 4 cm which is at least 50% lower than the mean 3-D position error of the angular receiver in motion (having an AOA measurement rate in Hz that is half the angular receiver speed in cm/s).

6.2 Recommendations

The work presented has shown that both optical receivers, namely the angular receiver and image receiver can provide position errors on the order of a few centimeters for typical indoor positioning scenarios. However, several factors exist that need to be further investigated as future work for this research.

1. In order to make the angular receiver a more practical system, several design challenges need to be overcome. The current bandpass and amplifier circuit spans a significant amount of space (20 cm × 10 cm). Using a printed circuit board to miniaturize the angular receiver circuitry would make the angular receiver more practical. Also, each PD of the corner-cube angular receiver is approximately 1 cm × 1 cm. Smaller PDs of approximately 3 mm × 3 mm can be used to make the angular receiver more compact and, therefore, unobtrusive.

2. The field of view of the angular receiver can be increased by building a quadruplet angular receiver. Since one angular receiver (corner-cube) spans a quarter of a hemisphere, this limits its FOV to a few LEDs. It would be advantageous to increase the corner-cube angular receiver FOV so that it sees more LEDs. This in effect introduces redundancy and should, therefore, result in a better position estimate. A typical positioning scenario would be attaching the angular receiver to a box to be monitored in a warehouse. If the worker responsible for carrying the box tips the box, the current angular receiver may not
face the ceiling and, therefore, will not see any of the LEDs mounted on the ceiling. Having a quadruplet receiver will reduce the risk of this happening, allowing continuous positioning to be maintained. To do this, four corner-cubes could be assembled back to back.

3. Bidirectional communication between the network and the angular receiver should be implemented. In a typical communication system, the receiver is resource constrained. Therefore, the receiver will send the measured data (photocurrents in the case of an angular receiver) back to the network which will perform the heavy computational load. In order to do this, two way communication between the LED transmitters and the angular receiver needs to be established. The corner-cube structure of the angular receiver causes it to act as a retroreflector and, therefore, allows the LED light to be reflected back to the network. The network will have a PD beside each LED transmitter that will capture the light signals transmitted back from the corner-cube angular receiver. The corner-cube angular receiver will need to modulate the retroreflected light to carry information on the relative photocurrent values generated back to the network where the AOA will be calculated. One possible way to do this is by using a liquid crystal modulator [57]. However, liquid crystal modulators are limited to speeds of 150 Hz. To mitigate this, a switching technique with higher frequencies, known as multiple quantum well [58], can be utilized. This technique is used in free-space optical communications. The signal is encoded using on-off keying modulation of the carrier.

4. A more robust image receiver structure in which the microlens and image sensor are mounted together in one platform should be designed in order to overcome vibrations witnessed in the motion tests. This is especially difficult since the image receiver focal length is only a few millimeters. This would be essential to test the image receiver AOA and positioning performances at higher speeds than those tested in this thesis.

5. In this work, the orientation of both the angular and image receivers’ body frames are known with respect to a reference frame. In a typical system, the body frame orientation, defined by yaw, pitch and roll angles, is unknown and needs to be estimated. To solve for the three orientation angles a greater redundancy of LED beacons is required. At least six LED beacons would be needed to solve for 3-D position and receivers’ orientation.
6. The angular receiver and the image receiver represent two extreme cases in terms of the number of photodiodes used for positioning applications. The angular receiver has only three photodiodes, while the image receiver has thousands of photodiodes and, therefore, has a higher resolution compared to the angular receiver. The angular receiver, on the other hand, has a faster response and is less computationally intensive compared to the image receiver. A device that has a fast response yet achieves sufficient resolution needs to be designed to enhance indoor optical wireless positioning performance.
Bibliography


Bibliography


Bibliography


