Abstract

This thesis investigates the effects of public policy intervention in the market for commercial mobile spectrum. Spectrum is a key input in the provision of commercial mobile telecommunications services (i.e. cellular mobile phone service).

In particular, the investigation centres on the regulatory framework of spectrum auctions that introduces set-aside spectrum or otherwise artificially discounted spectrum for entrant companies, for the purpose of increasing competition in the market for mobile telephony services.

This investigation is based on the development of a market model for the commercial mobile telephony market, using spectrum as an input. Suppliers, both incumbent and entrant, maximize profits by first choosing the amount of spectrum to purchase and then choosing the level of services afforded by the spectrum they own. We examine the results of the model in scenarios with both non-discounted and discounted spectrum for entrants.

We conclude that the successful introduction of entrants will result a lower equilibrium price, but that the extent of this price reduction is heavily dependent on the price elasticity of market demand. If entry is encouraged through discounted spectrum, then investment levels may be reduced in the long term.
# Table of Contents

Abstract ........................................................................................................................................................................... ii

Table of Contents ............................................................................................................................................................ iii

List of Tables ..................................................................................................................................................................... v

List of Figures .................................................................................................................................................................... v

Acknowledgements ............................................................................................................................................................. vi

Chapter 1 Introduction ......................................................................................................................................................... 1

Chapter 2 The Market for Spectrum .................................................................................................................................. 3

2.1 What Is Spectrum? ........................................................................................................................................................ 3

  2.1.1 Frequencies, Frequency Bands, and Channels ........................................................................................................ 3

  2.1.2 Channel Capacity and Network Capacity ................................................................................................................ 5

  2.1.3 Interference .............................................................................................................................................................. 7

  2.1.4 Allocation and Excludability .................................................................................................................................. 8

2.2 The Spectrum Market in Canada ................................................................................................................................. 9

  2.2.1 Spectrum Allocation Mechanisms in Canada ........................................................................................................... 10

  2.2.2 History of Spectrum Allocation Mechanisms in Canada .......................................................................................... 11

2.3 The Wireless Telecommunications Market in Canada .................................................................................................. 12

  2.3.1 Current Market Conditions ..................................................................................................................................... 12

  2.3.2 Market History ........................................................................................................................................................ 13

Chapter 3 Background (Literature Review) ...................................................................................................................... 16

3.1 The Cost Function .......................................................................................................................................................... 16

3.2 Cournot Analysis ........................................................................................................................................................... 18

3.3 Stackelberg Model ......................................................................................................................................................... 19

3.4 Demand for Telecommunications Services ................................................................................................................ 19

3.5 Fixed-line telecommunications .................................................................................................................................... 20

3.6 Cournot versus Bertrand Competition Model ............................................................................................................ 22

3.7 Monopolistic Competition, Collusion, and Competition .............................................................................................. 23

3.8 Market Structure ............................................................................................................................................................... 24

  3.8.1 Herfindahl-Hirschman Index .................................................................................................................................. 24

  3.8.2 Economies of Scale and Scope ............................................................................................................................... 24

  3.8.3 Impact of Investment ................................................................................................................................................ 25
List of Tables

Table 2.1: AWS Spectrum Plan................................................................. 10
Table 2.2: Key Events in the History of the Canadian Cellular Phone Industry.................................................. 14
Table 6.1: Bandwidth Holdings, 2010...................................................... 38
Table 6.2: Operators’ Total Operating Expenses...................................... 39
Table 6.3: Operators’ Network Operating and General Administrative Expenses................................................. 39
Table 6.4: AWS Auction Non Set-Aside Spectrum Prices.................................. 43
Table 6.5: AWS Auction Set-Aside Spectrum Prices..................................... 44
Table 6.6: 700 MHz Auction Spectrum Prices............................................. 44
Table 6.7: Operators’ Base Station Numbers............................................... 45
Table 6.8: Estimated Annual Growth Rate in Subscribers.................................. 47
Table 6.9: Slope of Demand Curve............................................................. 48

List of Figures

Figure 2.1: Example of Cellular Network – no frequency re-use.......................... 6
Figure 2.2: Example of a Cellular Network – frequency re-use............................ 7
Figure 2.3: Market Share by Provider......................................................... 12
Figure 6.1: AWS Auction Spectrum Prices.................................................. 42
Figure 6.2: Subscriber Counts and Revenues.................................................. 46
Figure 7.1: Quantity Result vs. Demand Curve Slope................................. 51
Figure 7.2: Price Result vs. Demand Curve Slope........................................ 52
Figure 7.3: Per-firm Profit Result vs. Demand Curve Slope............................... 53
Figure 7.4: Quantity Result vs. Demand Curve Slope – With Entrants.................... 54
Figure 7.5: Price Result vs. Demand Curve Slope – With Entrants....................... 55
Figure 7.6: Quantity Result vs. Entrants’ Operating Cost Level.......................... 56
Figure 7.7: Quantity Result vs. Entrants’ Spectrum Cost Level........................... 57
Acknowledgements

I would like to thank my supervisor Kenneth Carlaw for his dedicated support, guidance and advice throughout this research project.

I would like to thank the members of my supervisory committee, Ross Hickey, and John Janmaat, for their helpful comments and encouragement.

I owe special thanks to my husband Andrew for his understanding and patience.

All errors in this thesis are mine alone.
**Chapter 1  Introduction**

The resource of radio spectrum is a necessary input for the production of mobile telecommunications services. Wireless communication signals travel through a portion of the radio spectrum instead of through a wire, and the capacity of a telecommunications network is in part determined by the quantity of radio spectrum available. Wireless network operators continue to acquire additional spectrum in order to provide expanded services to a growing consumer base.

Throughout most of the world, access to radio spectrum is legally controlled by government. Government control is motivated largely due to the ability of spectrum users to interfere with one another’s use. The administration of spectrum usually consists of allocation of the spectrum by government to end users and service providers, such as government organizations or firms. In Canada, the Minister of Industry is charged with the management of spectrum, and most spectrum rights are issued as an exclusive licence defining a geographic area or location, and a portion (subset by frequency) of the radio spectrum.

In his first paper regarding radio spectrum, Coase (1959) argued for the use of auctions to assign spectrum, rather than direct allocation by the Federal Communications Commission, saying that:

“Certainly, it is not clear why we should have to rely on the Federal Communications Commission rather than the ordinary pricing mechanism to decide whether a particular frequency should be used by the police, or for a radiotelephone, or for a taxi service, or for an oil company for geophysical exploration, or by a motion-picture company to keep in touch with its film stars or for a broadcasting station. Indeed, the multiplicity of these varied uses would suggest that the advantages to be derived from relying on the pricing mechanism would be especially great in this case.”

and,

“An allocation scheme costs something to administer, will itself lead to a malallocation of resources, and may encourage some monopolistic tendencies - all of which might well make us willing to tolerate a considerable amount of imperfect competition before substituting an allocation scheme for market controls”
The idea of using the market mechanism to distribute spectrum was controversial at the time. It was not until 1994 that the U.S. Federal Communications Commission conducted auctions for radio spectrum; Industry Canada first conducted commercial mobile radio auctions in 2001.

Despite moving towards market-based allocation (i.e. auctions) for the commercial mobile spectrum, Industry Canada has, to a certain extent, manipulated the outcomes of these spectrum auctions through the imposition of various rules. This regulatory intervention has largely been undertaken with the goal of increasing competition in the mobile telecommunications market, in order to achieve lower price levels.

This paper examines how regulatory intervention in the market for radio spectrum affects competition and production in the wireless telecommunications industry. Specific attention is paid to the effect of the reservation of (suspension of competitive bidding for) portions of spectrum in the Canadian market, in order to encourage new entrants.

We expect that increased competition from entrant firms will lead to a lower equilibrium price. The extent of the increase in surplus depends on the initial state of the market (competitive or monopolistic), as well as the characteristics of the market. However, the overall welfare effect may be that outcomes other than changes in consumer surplus occur, such as reductions of investments by firms into quality and innovation.

The following chapter of this paper describes the nature of the spectrum resource and the history of spectrum allocation and the mobile telecommunications market in Canada. Chapter 3 presents a review of literature examining both spectrum and telecommunications. Chapter 4 develops a framework for building an economic model to describe the mobile telecommunications market using spectrum as an input. Chapter 5 develops this model, and Chapter 6 describes the parameter values used in simulations. Chapter 7 presents the results of model simulations. Chapter 8 proposes some implications of the model predictions, and Chapter 9 concludes.
Chapter 2  The Market for Spectrum

2.1  What Is Spectrum?

A mobile telecommunications network is constructed using two principal inputs: spectrum and base stations. These two inputs jointly determine the network capacity. The model described in this paper is a simplification of the complex interaction between an operator’s choices of base station quantity and spectrum quantity, and the resulting quantity of service that can be supplied. The emphasis of this model is on the choice of spectrum quantity.

This chapter describes in more depth the interaction of the spectrum choice with the base station choice. This interaction has been simplified in the final model, in order to focus on the effects of spectrum choice and the outcomes of various policy interventions in the market for spectrum.

This chapter also characterizes the spectrum resource. Though spectrum is allocated by government, spectrum should be considered a quasi-private or common good. The spectrum resource is limited (or scarce or finite), non-depletable, and rivalrous. Spectrum is not an excludable resource without a legal framework to prevent unauthorized use, and is therefore not a truly private good; most countries have a legal framework in place that makes spectrum excludable, and therefore functionally a private good.

The background provided in this Chapter 2 is intended to provide a better understanding of the underlying characteristics of the model structure. However, the background information is not strictly necessary for grasping the form and implications of the model.

2.1.1  Frequencies, Frequency Bands, and Channels

Radio frequency spectrum is the portion of the electromagnetic spectrum that is used to transmit analog or digital communications between a wireless radio transmitter and receiver. Examples of such communications include AM, FM and TV broadcasting, air and marine navigation systems, wireless in-home devices (routers, baby monitors, garage door openers), and commercial wireless telecommunications services (cell phone networks).

Radio spectrum is quantified in terms of frequencies, frequency bands, and channels, for the purpose of describing the spectrum. Frequency is a physical property of spectrum, and provides a precise quantification of
spectrum. Frequency bands and channels are constructs used by industry and governments in order to define portions of spectrum for allocation (i.e. sale or assignment).

A frequency is a particular point in the electromagnetic spectrum, where an electromagnetic wave oscillates at a particular speed measured in cycles per second or Hertz (Hz).

The frequency of a radio signal has a direct effect on the propagation of that radio signal, and therefore on the usefulness (or value) of the radio spectrum at that frequency. In the frequency range considered here, lower frequencies propagate further than higher frequencies. Radio signals above approximately 3 GHz do not propagate well through trees or buildings, and are therefore not generally considered useful for commercial mobile radio applications.

Conversely, radio frequency signals at lower frequencies can travel extremely long distances. Lower radio frequencies are used primarily for low data-rate applications that benefit from long propagation distances, such as marine and aeronautical navigation and radio broadcasting. Because of their long range, self-interference precludes lower frequencies from use in wireless telecommunications networks (interference is discussed in sub-chapter 2.1.3 below). Commercial mobile radio systems typically do not use frequencies that are below 500 MHz.

Radio spectrum is divided into channels, blocks, and bands for administrative purposes. A channel or block is a portion of the electromagnetic spectrum defined between two frequencies. For example, 100 MHz to 105 MHz defines a 5 MHz-wide channel or block. Digital or analog data is carried on a radio channel. A block (for example, a 5 MHz block) may be divided into a number of smaller channels (for example, five 1 MHz channels). A channel or block may be defined by its width (bandwidth or channel width) and its centre frequency (or carrier). For example, 100 MHz to 105 MHz could be described as the channel or block centred on 102.5 MHz, with a bandwidth of 5 MHz.

A frequency band refers to a large portion of radio spectrum that is used for a specific purpose. For example, the AM band includes the channels used for AM radio broadcasting, between 535 kHz and 1,705 kHz.

Commercial mobile radio bands typically fall between roughly 700 MHz and 2.5 GHz. These bands are given names based on commonly accepted nomenclature. Some of the frequency bands referred to in this paper are:
Due to both the conventions of assignment, and the propagation characteristics that permit certain frequency ranges to be useful for certain applications, radio spectrum is a limited resource. Since commercial mobile telecommunications has a high utility, and the spectrum that is suitable for this application is finite, this spectrum is a scarce and valuable input, and ownership of that input generates value.

It should also be noted here that spectrum is non-depletable. If a user of a frequency channel ceases to use that channel, it can be employed by another user without any persisting effects from the previous use.

2.1.2 Channel Capacity and Network Capacity

This chapter describes the fundamental principal of cellular network design: cellularization. This basic concept vastly increases network capacity. However, as discussed in the following chapter, it also introduces interference into the network, and the interaction between spectrum and base station quantity.

When carrying a data stream, a radio channel has an associated channel capacity, and thus an associated property of spectral efficiency. For example, if a 5 MHz wide channel is used to carry a 5 Mbps stream of digital data, the capacity is 5 Mbps and the spectral efficiency is 1 bit per second per Hertz (1 bit/s/Hz). The spectral efficiency (and therefore the capacity) of a channel is determined by the equipment used to transmit and receive, and by the physical environment in which the channel operates.

In deploying a cellular network, channels are re-used in different geographic locations across a network area. Channel re-use is subject to the limitations of interference described in sub-chapter 2.1.3 below. This channel re-use serves to effectively increase the overall network capacity.
The following is a simplified example of how the capacity of a network is increased by cellularizing the network (hence the name, “cellular network”). Suppose an operator has access to three radio channels, each of which provides a capacity of 1 Mbps. The operator deploys a single base station with three radio transmitters, each operating on one of the three channels, as shown in Figure 1 below:

![Figure 1: Example of Cellular Network – no frequency re-use](image)

This network has three cells. Since each cell uses one channel, and each channel has a capacity of 1 Mbps, the network has an overall capacity of 3 Mbps. Each radio transmitter serves a geographic area called a cell.

In order to increase the overall network capacity, the operator reduces the cell size (or, the area served by each transmitter). This can be done by reducing the transmitter power. The operator deploys five base stations, each with three sectors (note that only two sectors are shown for one of the base stations). The same area is served, but with more cells, as shown in Figure 2 below:

![Figure 2: Example of Cellular Network – with frequency re-use](image)
Figure 2.2: Example of a Cellular Network – frequency re-use

In this case, the cells are smaller, so the network serving the target area has 14 cells. Again, since each cell uses one of the three channels, and each channel has a capacity of 1 Mbps, the network has an overall capacity of 14 Mbps.

In this way, the overall capacity of the network can be increased by increasing the number of base stations deployed, provided that the frequency re-use pattern is arranged to minimize intra-system interference.\footnote{For more information on radio network planning and frequency re-use see Mishra (2004).}

### 2.1.3 Interference

Interference introduces a limit on the extent to which a network can be cellularized, and also results in spectrum being a rivalrous resource.

Interference is a phenomenon that occurs when a radio receiver (the victim) is not able to fully distinguish between the desired signal of the corresponding transmitter and the undesired signal of another transmitter (the interferer). In analog systems, interference may be manifested in the audible presence of another signal (such as another conversation or radio station). In digital systems, interference causes the reduction of channel capacity, or throughput. Severe interference may result in the complete failure of the channel.
Interference may originate from any radio transmitting device. Interference becomes more harmful as the frequency of the interfering transmitter approaches the frequency of the victim system. As a result, other equipment operating in the vicinity of a potential victim receiver, either on the same channel as the victim, or on a nearby channel, produces interference. Since interference is often bi-directional, interference reduces the utility of the spectrum to both users.

As the network is sub-divided into increasingly smaller cells, the distance between base stations decreases, and therefore the potential for interference increases. If the operator has more channels available, then the distance between base stations using the same channel can be increased, and the cell size can be smaller. Alternatively, operators with more spectrum can use larger cell sizes, but use larger channels. Either of these two approaches will result in a higher network capacity.

This chapter has focused on the implications of interference on an operator’s network planning, in particular on the distance between base stations acting at the same frequency. However, the presence of another operator using the same frequencies can also compromise network capacity. We can therefore describe the spectrum resource as rivalrous. The use of spectrum by one party diminishes the spectrum’s usefulness for other parties.

### 2.1.4 Allocation and Excludability

The final attribute of spectrum is its excludability. Radio spectrum is not by its nature an excludable resource, but it has been designated as excludable by law in Canada through the *Radiocommunication Act*. Spectrum has been thus designated due to the ability of spectrum users to interfere with one another.

If one operator transmits using a particular channel, there is no physical impediment to another operator transmitting on the same channel, causing interference to the first operator. The ability of an interfering signal to impair channel capacity is a primary motivation for government involvement in spectrum management (Gow & Smith, 2006). Industry Canada’s Spectrum Policy Framework (Industry Canada, 2007, SGTP-001-07 Section 1) states that “Spectrum is a finite resource”. The Framework for Spectrum Auctions in Canada (Industry Canada, 2011) opens with the sentence, “Radio frequency spectrum is a finite public resource.” This presumption of scarcity is the foundation for spectrum allocation.

While there is no physical barrier to accessing spectrum, the Government of Canada, as with most governments, has reserved the power to permit use of radio spectrum. As stated in the Framework for Spectrum Auctions in Canada (Industry Canada, 2011),
The Minister of Industry has the statutory responsibility for Canada's radio frequency spectrum. The radio frequency spectrum is managed on the Minister's behalf by staff in the Program who ... obtain, plan, and authorize its use, and use sophisticated equipment and automated systems to ensure that harmful radio signals do not hamper its use by licensed and essential communications services. Without clear radio channels, all communications services would experience difficulty in carrying out their operations.

The exception to legislated excludability is the designation of frequency bands as licence-exempt. While the equipment used in these bands must conform to certain standards, no license is required, for example, to operate a wireless router, cordless telephone or FRS (Family Radio Service) radio (walkie-talkie). Commercial mobile radio does not operate in licence-exempt bands, and these bands are ignored for the purpose of this paper.

2.2 The Spectrum Market in Canada

Radio spectrum, as described in the previous chapters, is considered to be a finite (or limited or scarce) resource, that is rivalrous, non-depletable, and not excludable (except by law). Taking these characteristics into account, as a resource that is not physically excludable, governments administer the rights to use radio spectrum within their borders.

The Radiocommunication Act, Section 5 (1985) gives the Minister of Industry the authority to issue radio licenses, spectrum licenses, and plan the allocation and use of spectrum. This work is carried out through the Spectrum Management and Telecommunications branch of Industry Canada. All frequencies between 9 kHz and 275 GHz are allocated by Industry Canada, and Industry Canada's allocations generally conform to international standards established by the ITU (International Telecommunications Union). Industry Canada's allocations are listed in the Canadian Table of Frequency Allocations.

Radio frequencies are allocated and managed in a variety of ways. In the case of frequencies for commercial mobile applications (also referred to as cellular telephony or mobile wireless), a frequency band is divided up into a number of blocks. For example, the AWS (Advanced Wireless Services) band is 90 MHz wide, and was divided into three paired 10 MHz blocks and three paired 5 MHz blocks for auction, as shown in Table 1 below (Industry Canada, 2007):
Table 2.1: AWS Spectrum Plan

<table>
<thead>
<tr>
<th>Spectrum Block</th>
<th>Bandwidth (MHz)</th>
<th>Lower Frequency (MHz)</th>
<th>Upper Frequency (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20</td>
<td>1710 – 1720</td>
<td>2110 – 2120</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>1720 – 1730</td>
<td>2120 – 2130</td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>1730 – 1735</td>
<td>2130 – 2135</td>
</tr>
<tr>
<td>D</td>
<td>10</td>
<td>1735 – 1740</td>
<td>2135 – 2140</td>
</tr>
<tr>
<td>E</td>
<td>10</td>
<td>1740 – 1745</td>
<td>2140 – 2145</td>
</tr>
<tr>
<td>F</td>
<td>20</td>
<td>1745 – 1755</td>
<td>2145 – 2155</td>
</tr>
</tbody>
</table>

These spectrum blocks are allocated geographically. For example, a spectrum block may be allocated to an operator for use throughout Canada, or in a particular province or regional area.

The typical term of a radio licence in Canada is ten years, and these licences are normally renewed after expiry. From time to time, Industry Canada may retire an allocation by allowing radio licences to expire. This re-allocation normally follows an extensive public consultation process, and involves some compensation (monetary, or with alternative spectrum) to displaced licence holders.

2.2.1 Spectrum Allocation Mechanisms in Canada

Spectrum is allocated in Canada through a variety of mechanisms, including station licensing, geographical licensing, and license-exempt use.2

Commercial mobile radio spectrum used for mobile telecommunications is allocated on a per-block, per-licence-area basis. That is, once the license is awarded, the license holder may deploy any number of stations in the licensed geographical area, in the licensed frequency block.

2 License-exempt bands are heavily used by consumer devices, such as Wi-Fi devices, baby monitors, and garage door openers. These devices are subject to equipment design regulations (such as power limitations) set by Industry Canada, and the operation of these devices is not afforded any protection from interference.

A number of frequency bands are licensed on a per-station basis, including those bands designated for FM and TV broadcast systems and point-to-point microwave systems. In approving a license application for a station, Industry Canada performs an interference study to determine that the station will not cause interference to or receive interference from any existing stations.
2.2.2 History of Spectrum Allocation Mechanisms in Canada

The Canadian 1913 Radiotelegraph Act was based on the original act of 1905 (CRTC 2008), and established federal government authority over the licensing of all wireless equipment (Vipond, 1992, p.10). This effectively established control of the federal government over all radio spectrum. Radio and television broadcast operations are still licensed on a per-station basis.

The earliest allocation of commercial mobile spectrum in Canada was the 1985 allocation of the Cellular band. In this and the subsequent two allocation events, a block of spectrum was directly awarded to each of incumbent local telephone companies in their operating areas, and a block of nationwide spectrum was awarded to one or more new entrants who were chosen through a process of comparative selection (Industry Canada, 2004).

In 1994, Industry Canada conducted a review of the comparative selection process, and published the findings of this review and the public comments garnered through this process in February 1996. The report concluded that despite significant opposition by industry respondents, competitive bidding should be introduced to the spectrum allocation process (Industry Canada, 1996). In June of 1996, the Radiocommunication Act was amended to allow for the use of auctions as a method of spectrum assignment (Industry Canada, 1997).

Auctions, relying on market forces to allocate spectrum, are a common means of spectrum allocation in Europe and North America (Gow & Smith, 2006, pp. 18, 93,113). The Framework for Spectrum Auctions in Canada prescribes the use of a competitive licensing process such as an auction when the demand for spectrum exceeds the supply, and when policy objectives (such maximizing economic and social benefits for Canadians) can be met through an auction (Industry Canada, 2011, Sections 1&2).

In 2001, Industry Canada conducted its first spectrum auction to allocate commercial mobile spectrum. The auction resulted in winning bids totalling approximately CDN $1.5 billion.

In 2008, Industry Canada again auctioned commercial mobile spectrum, but with an important restriction. Several of the spectrum blocks being auctioned were set aside for non-incumbent carriers (wireless telecommunications service providers with less that 10% of the Canadian market). This set-aside resulted in a number of new service providers buying spectrum and launching mobile service.

In the recent 2014 spectrum auction, Industry Canada applied a spectrum cap. The spectrum cap is a limit to the amount of spectrum that any given bidder can win in the auction. These spectrum caps are once again intended
to encourage competition by ensuring that the three incumbent providers will not purchase the entire available spectrum in any geographical area (Industry Canada, 2012).

2.3 The Wireless Telecommunications Market in Canada

2.3.1 Current Market Conditions

The Canadian market is currently served by three large incumbent wireless telecommunications service providers: Telus, Rogers and Bell. In addition to these three major incumbents, there are two incumbent regional providers (MTS and Sasktel). A number of new entrants (Wind, Videotron, Mobilicity, Public Mobile and Eastlink) entered the market following the 2008 spectrum auction. Figure 3 below shows the market share of the carriers (by subscriber numbers) active in the market at Q3 2013 (CWTA 2013).

![Market Share by Provider](image-url)

Figure 2.3: Market Share by Provider
2.3.2 Market History

In 1985, the first cellular spectrum licences were issued. Each local telephone company was allocated one 20 MHz block in their trading area. A second 20 MHz block was made available to other applicants, and was awarded to Cantel (now Rogers). Four years later, an additional 5 MHz was awarded to all licence holders.

In 1995, Industry Canada allocated 80 MHz of PCS (Personal Communications Services) licenses. Both Clearnet and Microcell were both awarded 30 MHz nationwide licences. Rogers was awarded a 10 MHz nationwide licence, and the regional shareholders of Mobility Personacom were awarded 10 MHz licenses in their operating trading areas. Industry Canada set a spectrum aggregation limit (or cap) prohibiting any operator from owning more than 40 MHz of PCS spectrum or similar spectrum; this cap was raised to 50 MHz in 1999.

Between 1995 and 2001, there was significant re-organization among the local telephone companies that had been awarded cellular licenses, many of whom had subsequently become shareholders of Mobility Personacom. The firms TELUS and Bell were formed from the members of the Mobility Canada Association and the earlier Mobility Personacom group.


In 2008, Industry Canada auctioned 90 MHz of AWS spectrum, plus an additional 10 MHz of PCS spectrum. In this auction, Industry Canada set aside 40 MHz of AWS auction for new entrants. Rogers, Bell Mobility and TELUS were prohibited from bidding on these set-aside licences. The set-aside licenses were acquired by several new players, and five new companies launched service following their spectrum acquisitions: Globalive (operating as Wind); Bragg (operating as Eastlink); Quebecor (operating as Vidéotron); DAVE (operating as Mobilicity); and Public Mobile.

A summary of the history of the mobile telecommunications market in Canada is shown in Table 2 below. This table is taken from Munro (2010, Table 1, p. 5).
Table 2.2: Key Events in the History of the Canadian Cellular Phone Industry

<table>
<thead>
<tr>
<th>Year</th>
<th>Event Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1982</td>
<td>The federal Department of Communications announces that one 20 MHz block of spectrum for cellular phone services has been set aside for the local telephone companies and that a comparative review process (“beauty contest”) is now open for the second 20 MHz block of spectrum.</td>
</tr>
<tr>
<td>1983</td>
<td>The Department announces that Cantel (now Rogers Wireless) has won the second block.</td>
</tr>
<tr>
<td>1985</td>
<td>Cellular phone service commences in Canada.</td>
</tr>
<tr>
<td>1989</td>
<td>The incumbent cellular licensees are granted an additional 5 MHz of spectrum, bringing the total to 25 MHz each.</td>
</tr>
<tr>
<td>1993</td>
<td>The Department of Communications is disbanded and spectrum management responsibilities are moved to the newly formed Industry Canada.</td>
</tr>
<tr>
<td>1994</td>
<td>In <em>Telecom Decision CRTC 94-15</em> the Canadian Radio-television and Telecommunications Commission (CRTC) determines that it has the authority to regulate the cellular phone rates charged to customers, but also concludes that it is appropriate to forbear from exercising this authority, given the state of competition in the marketplace.</td>
</tr>
<tr>
<td>1995</td>
<td>Industry Canada awards second generation cellular licenses known as “personal communications services” (PCS) via a comparative review process (“beauty contest”). Newcomers Clearnet and Microcell each receive a 30 MHz license and the cellular incumbents – Rogers and the members of Mobility Canada (i.e., the wireless affiliates of the local telephone companies) – each receive a 10 MHz license. Another 40 MHz of PCS spectrum is held in reserve.</td>
</tr>
<tr>
<td>1998</td>
<td>The local telephone companies in Alberta (AGT), in British Columbia (BC TEL), and in certain areas of Quebec (Quebec Telephone) merge to form TELUS. TELUS leaves the Mobility Canada alliance; the remaining members are reconstituted as the Bell Wireless Alliance.</td>
</tr>
<tr>
<td>2000</td>
<td>TELUS acquires Clearnet.</td>
</tr>
<tr>
<td>2001</td>
<td>The remaining 40 MHz of PCS spectrum is assigned via an auction. Almost all of the licences up for bidding are won by cellular/PCS incumbents.</td>
</tr>
<tr>
<td>Year</td>
<td>Event</td>
</tr>
<tr>
<td>------</td>
<td>-------</td>
</tr>
<tr>
<td>2002</td>
<td>The local telephone companies in Atlantic Canada (NewTel, MT&amp;T, Island Tel and NB Tel) consolidate as Aliant.</td>
</tr>
</tbody>
</table>
| 2004 | Rogers acquires Microcell  
MTS and Bell Canada end their alliance. |
| 2006 | Aliant's operations are merged into Bell Canada. Bell Mobility takes over all wireless operations. |
| 2008 | Industry Canada's auction of Advanced Wireless Services (AWS) frequencies makes another 105 MHz of spectrum available for cellular services. Both incumbents and a number of newcomers win licenses across the country. |
| 2009 | Industry Canada issues *Consultation on the Renewal of Cellular and Personal Communications Services (PCS) Spectrum Licenses.*  
Wind Mobile, a winner of spectrum in the 2008 AWS auction, launches service and becomes the first new entrant to the wireless telephone industry in over a decade. |
| 2010 | More new entrants who won spectrum in the 2008 AWS auction – such as Mobilicity, Public Mobile, and Vidéotron – begin to roll out services. |

In the spectrum auction that concluded in February 2014, three of the recent entrants did not participate (Wind, Mobilicity, and Public Mobile). Two new operators, Vidéotron and Eastlink, won a considerable number of new spectrum licences. This outcome may be indicative of the successful business operations by the latter two companies, and by unsuccessful operations by the former three.
Chapter 3  Background (Literature Review)

This chapter provides a review of the existing literature that is relevant to the problem of modelling the mobile radio telephony industry. The overall structure of the model includes a demand curve that represents users’ demand for mobile telephony services, a supply curve that incorporates the various costs faced by service providers, and a mechanism of market interaction between suppliers.

Sub-chapter 3.1 describes the construction of the supply curve, and sub-chapter 3.4 describes the construction of the demand curve. The competition model is a combination of the Cournot and Stackelberg models, discussed in sub-chapters 3.2, 3.6, and 3.3. Sub-chapter 3.5 summarizes some of the existing literature on the subject of fixed-line telecommunications. Sub-chapters 3.7 and 3.8 review some literature that characterizes the mobile telecommunications market based on empirical market studies.

3.1 The Cost Function

This model uses the standard profit function of a firm, with the familiar form:

\[ \text{Profit} = \text{Total Revenue} - \text{Total Cost} \]  

The total revenue is given by the product of quantity \((Q)\) and price \((P)\):

\[ \text{Total Revenue} = P \times Q \]  

where quantity is a function of price, so that:

\[ \text{Total Revenue} = P \times Q(P) \]  

Since this is an imperfectly competitive market, there is an interaction between the quantity choice of firms, and the market price; a higher quantity supplied to the market results in a lower price. Each firms’ actions, then, will influence the market price.
The total cost function is given by (Tirole, 1994, p.19):

\[ C(q) = \begin{cases} C_F + \int_0^q C'(x) \, dx & \text{for } q > 0 \\ 0 & \text{otherwise} \end{cases} \]  \[\text{(4)}\]

where \( C_F \) is the fixed production cost, \( q \) is the quantity supplied and \( C(x) \) is the cost of production and \( C'(x) \) is the per-unit cost of production. The variable \( Q \) indicates the total quantity supplied to the market by all firms, where the variable \( q \) indicates the quantity choice of an individual firm. This paper uses the simplest version of the cost function, given by:

\[ C(q) = C_F + cq \quad \text{for } q > 0 \]  \[\text{(5)}\]

where \( c \) is the per-unit constant marginal cost of production. This formulation gives an average cost that is declining \((dC/dQ < 0)\) at a constant rate \((d^2C/dQ^2 = 0)\). This marginal cost can be attributed to the costs of serving an additional mobile user, such as additional network capacity and customer service.

This gives an overall profit function of:

\[ \Pi = P \cdot Q(P) - C_F - cq \]  \[\text{(6)}\]

where \( \Pi \) is the total profit.

The fixed and marginal cost components specific to the modelling exercise are described in Chapter 4.
3.2 Cournot Analysis

The model is developed using a traditional Cournot formulation described by Tirole (1994, pp. 218-221). In one-stage Cournot competition, each firm chooses a profit-maximizing quantity given the other firms’ quantity choices. That is,

\[ \Pi_i(q_i) = q_i P(Q) - C_i(q_i) - C_F, \quad \text{where } Q = \sum_{i=1}^{n} q_i \]

maximizing,

\[ \frac{\partial \Pi}{\partial q_i} = P(Q) + q_i P'(Q) - C_i'(q_i) = 0 \]  [7]

where \( q_i \) refers to the quantity supplied by firm \( i \), \( n \) is the total number of firms, \( C_i \) is the marginal cost faced by firm \( i \), \( C_F \) is the fixed cost of network operation, and \( P(Q) \) is the demand function. Each firm, \( i \), takes the other firms’ quantity choices as given, and equilibrium is reached when these choices stabilize.

Solving this equation gives a reaction function for each firm, \( q_i \), with respect to the quantities chosen by the other firms. In the case where \( P(Q) = 1 - Q \) and the marginal costs of each firm are equal to \( c \), the Cournot equilibrium is at:

\[ q_i = \frac{1-c}{n+1} \]  [8]

The Cournot equilibrium will produce a price level that is lower than the monopoly price. As the number of firms increases, the price level will approach the competitive price.
3.3 Stackelberg Model

The Stackelberg model is used in conjunction with the Cournot model to describe the interaction of incumbent and entrant firms in the modelling exercise.

The Stackelberg model, described by Tirole (1994, pp. 314-323), is a model of a two-firm industry where each firm chooses a level of capital, $K$, and the profits of the firms are given by:

$$\Pi_i (K_i, K_j) = K_i (1 - K_i - K_j)$$

[9]

In this two-stage game, firm 1 is the first-mover and has knowledge of firm 2's reaction function. Firm 1 maximizes profit given firm 2's reaction function. With symmetric profit functions, the first-mover advantage gives firm 1 more profit than firm 2.

This Stackelberg game depends on the irreversibility of the capital decisions. If firm 1 could reduce $K_i$ after firm 2's entry, it would do so in order to increase profit, and firm 2 would anticipate this, leading to a symmetric outcome.

3.4 Demand for Telecommunications Services

The demand curve that is used in the modelling exercise was chosen from a number of options found in the literature. This sub-chapter describes several demand curve formulations examined during the course of the literature review. These formulations vary in complexity and robustness, in terms of the datasets used and their statistical correlations to these datasets. For this modelling exercise, we have used the simplest formulation to aid in the tractability of the model. Although some accuracy may be lost in the demand curve, the purpose of this exercise is to examine effects of spectrum cost on the supply curve, so we accept a simplified demand curve in return for model tractability.

Tirole's Stackelberg-Spence-Dixit model (Tirole, 1994) is evaluated using a linear demand curve, $p = a - bq$. For the purpose of modelling simplicity, this approach is used in this paper. The reason for choosing this formulation is simplicity. Several other formulations are described below, presenting models that may more accurately describe demand in the mobile telephony market, but are less tractable within the final model. Since the focus here is more on the cost structure for firms in the market, we make this simplifying trade off in the choice of the demand curve.
Lee & Lee (2006) examine consumer surplus in the South Korean mobile telecommunications market. In their analysis, they use a logarithmic demand curve of the form $\log Q = \alpha \log P$. They include an analysis of the network externality associated with participation in a mobile network, that will further increase the consumer surplus.

Basalisco (2012) examines the demand for SMS markets in the UK. He examines the effects of both price and of the number of messages received on the number of messages sent, thus taking into account network effects. The empirical results are analyzed using a logarithmic demand curve with constant price elasticity. Basalisco (2012) estimates a long-run price elasticity of 0.9.

Agiakloglou & Yannelis (2006) examine the price elasticity of international calls in the Greek mobile telecommunications market. They consider demand characteristics of the form $\log Q = \alpha + \beta \log P$, and of the form $\log Q = \alpha + \beta P$. They find that the first form provides a better fit for long-term demand analysis, since the price elasticity is not constant in the long term.

Any of these more sophisticated demand curves could be incorporated into this model in future research.

### 3.5 Fixed-line telecommunications

The fixed-line telecommunications market has been a subject of a number of academic papers. The fixed-line market refers to the provision of basic phone services, as well as Internet services through the either copper telephone infrastructure or cable TV infrastructure. While this market is distinct from mobile wireless telecommunications, many aspects of these markets are similar, including the companies involved. The methods used to examine the fixed market can be applied to the mobile market.

This sub-chapter includes a brief overview of some papers that are relevant to this paper. These analyses of the fixed-line telecommunications market are not directly applicable to the formulation of a model to describe the mobile market. However, the overall methodology and the conclusions drawn regarding the impact of government price-setting regulations on investment and competitiveness are instructive in formulating an approach to investigating the effects of government intervention in the spectrum market.

Several reviewed papers examine unbundling, or the regulation that obliges infrastructure owners (i.e. incumbent local exchange carriers or ILECs) to provide competitive carriers (CLECs) with wholesale access to their...
infrastructure at set rates. Gayle & Weisman (2009) model the effects of unbundling on investment levels and market efficiency in the U.S. fixed-line telecommunications market. They model a market with an incumbent and with a competitor that may buy wholesale access to the incumbent’s network at a wholesale price.

Using a Bertrand model with differentiated products with symmetric cross-price effects, the demand functions are \( Q_i = \alpha - \beta p_i + \gamma p_e \) for the incumbent and \( Q_e = \alpha - \beta p_e + \gamma p_i \) for the entrant. The profit function for the incumbent will then be as follows:

\[
P_i = (p_i - c_{ui,i} - c_{di,i})(\alpha - \beta p_i + \gamma p_e) + (w - c_{ui,i})(\alpha - \beta p_e + \gamma p_i) - \delta I \tag{10}
\]

where the first addend is the profit from the incumbent’s direct sales, the second addend is the profit from the wholesale sales to the new competitor, and the third addend is the constant marginal cost of investment in the telecommunications plant. The profit function for the entrant will be:

\[
P_e = (p_i - w_i - c_{di,e})(\alpha - \beta p_e + \gamma p_i) \tag{11}
\]

where \( p_i \) and \( p_e \) are the prices of the incumbent and competitor; \( c_{ui,i}, c_{di,i}, \) and \( c_{di,e} \) are the upstream and downstream costs for the incumbent and competitor, and \( w \) is the wholesale price.

The competitor faces a make-or-buy decision. If the price is below a certain threshold, the competitor will buy from the incumbent; if above, the competitor will construct their own network. The latter choice would be inefficient, since the incumbent should be more efficient at constructing networks. However, the incumbent’s incentive to invest is less if the competitor buys, and higher if the incumbent makes. The government, then, faces a trade-off in setting the wholesale price - either increase the incentive to invest or increase the overall efficiency of the market outcome.

The Gayle & Weisman (2009) paper describes a scenario that is analogous to the situation described in this paper, in that a government pricing decision affects firms’ investment decisions.

Both Bauer (2010) and Inung, Gayle & Lehman (2008) look at the effect of unbundling on competition using empirical data. Bauer shows that lower wholesale prices reduce an incumbent's incentive to invest in infrastructure, shifting the incumbent's investment into differentiated services. Inung et al. (2008) look at competition and investment using data before and after the U.S. telecommunications act of 1996, which imposed
unbundling. They show that higher investment by the ILEC (incumbent local exchange carrier) is associated positively with the market share of CLECs (competitive local exchange carriers), and negatively with the total number of CLECs, implying that a few powerful competitors are more effective competition than many weak competitors. Overall, however, Inung et al. (2008) note a high level of persistence in ILEC’s investment habits, suggesting that the overall effect of unbundling on investment may be small in practice.

3.6 Cournot versus Bertrand Competition Model

As described in sub-chapter 3.2 above, this modelling exercise uses a Cournot approach to describe the interaction between firms. This sub-chapter describes the reason for using this approach, rather than the Bertrand model.

Five reviewed articles included models that described competition. Three papers used Cournot models (Tardiff 2009, de Mesnard 2010, Konan & Van Assche 2007, Manenti & Scialà 2013, and Gruber 2001), and two used Bertrand models (Gayle & Weisman 2007).

Under Bertrand competition, price is driven down to marginal cost at equilibrium. In order for firms to realize economic profits in this scenario, resulting from prices above marginal costs, the products must be differentiated. Mobile telecommunications providers present a bewildering array of packages that change regularly, making a comparison of products that would evaluate the homogeneity (or differentiation) of these products difficult. As de Mesnard (2010, pp. 186-187) concludes:

“Obviously, in real conditions, the service is not completely homogeneous but it is handy to pose this hypothesis because complexity of operator’s supply and their dozens of different contracts (tariffs, number of hours subscribed, annex services, etc.) would prevent any operational analysis of interaction between them.”

De Mesnard (2010) justifies his choice of Cournot over Bertrand on the basis that it is the most common choice, that it is symmetric across firms, and that no price war is observed. De Mesnard (2010) proceeds to predict changes to consumer surplus and profits where three firms are operating in either three-way Cournot competition or a monopolistic cartel of three, and the addition of a fourth firm results in four-way Cournot competition, a monopolistic cartel of four, or a monopolistic cartel of three plus a competitive fringe. De Mesnard (2010) concludes that if the firms are indeed currently operating in Cournot competition, price reductions may be less
than two percent; however, if the firms are operating as a cartel it is unlikely that the regulator can bring about Cournot competition. In de Mesnard’s model (de Mesnard, 2010), each firm’s profit is:

\[ \Pi_i(q(n)) = p(n)q(n) - q(n)c(n) - F_i, \]

where \( n \) is the number of firms and \( F_i \) is the fixed costs of the \( i \)th firm.

For the purpose of this paper, we assume that mobile telecommunications services are undifferentiated products. We also recognize that the provision of mobile telecommunications has a high fixed cost, in the form of both network construction and spectrum purchase. Finally, the choice of spectrum investment is a key determinant in the quantity of services that can be provided. The purchase of spectrum is, to a large degree, lumpy and irreversible. Though the framework exists in Canada for a secondary market in spectrum, in practice the secondary market is not widely used, except in the case of acquisitions.

The outcome of Cournot competition, based on quantity choices, is that a small number of firms can realize economic profits. The modelling exercise here uses a Cournot model in order to permit these profits, which will justify both high fixed costs and ongoing investment.

### 3.7 Monopolistic Competition, Collusion, and Competition

The state of the telecommunications market is a subject of some rhetoric in the current popular media. While operators assert that they operate in a competitive environment, there is considerable media attention given to the claim that the three major operators are collusive and monopolistic in their behaviour. This sub-chapter reviews several papers that investigate the level of competitiveness in the mobile telecommunications markets of various countries.

Several papers were reviewed that investigate monopolistic and collusive behaviour in the telecommunications market. Parker & Röller (1997) perform an empirical analysis of data from US cellular companies during the 1990’s, when the market moved from one local provider (monopoly) to two. They determined that with only two companies, the market behaviour was more likely to be collusive than competitive.

Konan & van Assche (2007) investigate the specific case of liberalization in Tunisia using Computational General Equilibrium (CGE) methods to describe scenarios where a foreign entrant is permitted into the
telecommunications model. They compare outcomes under Cournot competition and collusion, showing welfare improvements if competition can be induced and potential reductions in welfare if not.

Gruber (2001) explicitly addresses license fees as artificial barriers to entry. Gruber (2001) argues that the open auction process provides an incentive for firms to collude to increase the license fees, and that the auction process itself permits this behaviour. So, if there is perfect competition, spectrum prices will be driven up to the point where profits are zero. If there is collusion, profits will be driven above the competitive rate but below the price that would drive monopoly rents to zero. If this collusion is not maintained following the auction, then firms will be driven out of the market since oligopoly rents will not be realized. Gruber (2001) asserts that technological progress will increase the number of firms that can participate in the market, and that competition in the market will increase as a result.

The investigation in this sub-chapter demonstrates that the telecommunications market is probably not perfectly competitive, but might not be fully monopolistic either. The decision to use a Cournot competition model is further supported following this investigation.

3.8 Market Structure

In order to gain a better understanding of the overall characteristics of the mobile telecommunications market, a number of papers were reviewed that investigate market concentration, economies of scale, and the impact of investment. While not directly relevant to the formulation of the model, this portion of the literature review provides a better understanding of the market in general.

3.8.1 Herfindahl-Hirschman Index

Prasad & Sridhar (2009) use the Herfindahl-Hirschman Index (HHI) to measure the concentration of suppliers in the market. This is not a proxy for monopoly power, but nonetheless is used to provides a measure of how competitive the industry is, and can be useful for comparing across markets. Tardiff (2009) also uses the HHI to measure market concentration.

3.8.2 Economies of Scale and Scope

Economies of scale in the telecommunications industry have been investigated by a several authors, in both fixed-line markets (Guldman 1991; Majumdar & Chang 1996) and the wireless market (Prasad & Sridhar 2009). These authors generally concur that some degree of scale economies exist, though Guldman (1991) finds that scale economies are exhausted at a relatively small size.
This paper investigates the effect of entrants facing a higher marginal cost than incumbents. An incumbent may have lower marginal costs than a new entrant as a result of economies of scale, learning-by-doing, and economies of scope. Economies of scope may result from resources that can be shared across divisions, for example support staff that might offer services to both wireline and wireless customers. No explicit review of economies of scope in mobile telecommunications networks was found.

Early literature refers to land-line telecommunications as a natural monopoly (Posner 1969), however no recent articles were found that describe mobile telecommunications as a natural monopoly.

3.8.3 Impact of Investment

The impact of investment was reviewed in both the context of the telecommunications market and more generally. Gayle & Weisman (2007), and Kováč, Vinogradov & Žigić (2010) show that investment decreases operating cost by an amount proportional to the square root of the investment in telecommunications firms. Tardiff (2009) shows that overall welfare increases with investments into quality enhancement, but that the overall welfare impact of investments into cost reduction is ambiguous. Grossman & Helpman (1993) discuss at length the effects of investment (R&D) into product variety versus product quality.

The research discussed in this chapter completes a picture of a mobile telecommunications market that is highly concentrated and not perfectly competitive. In this modeling exercise, we create a static model in which profits are invested into research and development. This investment will result in a long-term welfare gain.
Chapter 4 Market Characteristic Parameterization

This paper presents the model for a system with incumbent mobile telecommunications service providers, and entrant service providers that face different costs in the Canadian market.

4.1 The Demand Curve

The demand curve is defined to be linear:

\[ P = a - b Q \]  \[12\]

where \( P \) is the price level, \( Q \) is the quantity supplied, and \( a \) and \( b \) are free parameters that characterize the demand curve.

4.1.1 Demand Units

Consumers' demand for mobile telecommunications services could be measured in a number of ways. Many phone plans conduct billing based on voice minutes used, long distance voice minutes used, text messages sent or received, or megabytes (MB) of network traffic used. Other features are offered as fixed-price options, such as bundles of text messages, unlimited long-distance, call display, and voicemail. Land-line based Internet connectivity services are priced based on their data transfer rate in Mbps (megabits per second), and mobile Internet connectivity may eventually follow that model.

From the wide variety of mobility plans and packages available, we can further surmise that consumers have a wide variety of demand levels, and have preferences that include a varying mix of services.

In order to create a simple model of a market that can be manipulated in a standard way, this paper uses an arbitrary unit of demand, where a single user represents one unit of demand.

Though the proposed demand unit is not representative of the multi-dimensional nature of consumer demand, this choice permits the gathering of some useful empirical data. Publicly-traded telecommunications operators publish annual results that include their total number of subscribers, so that expenses and revenues can be measured on a per-subscriber basis. Using this simplified demand measurement, the model can be parameterized using values that can be compared to observable data.
4.2 Quantity-Spectrum Relationship

In order to enter the mobile telecommunications market, an operator must first obtain spectrum. Spectrum will affect the capacity of a network to provide service. As described in sub-chapter 2.1.2 above, network capacity can be expanded by installing more base stations in the coverage area, increasing the cell density.

In this paper, the relationship between network capacity and bandwidth is modelled as a linear relationship. Since network capacity is measured, as described in sub-chapter 4.1 above, in units of demand, this gives a linear relationship between the quantity that a network can supply and the bandwidth available to that network. The relationship is modelled as:

\[ q_i = \Phi B_i \]  

where \( q_i \) is the quantity supplied by firm \( i \), \( B_i \) is the bandwidth (quantity of spectrum) held by firm \( i \), and \( \Phi \) is the amount of quantity that can be supplied per unit of bandwidth.

This model has the advantage of simplicity, effectively imposing cells of equal geographical size on the network so that spectrum quantity is the only choice that will affect the quantity supplied. Under this model, a network operator will not adjust the density of cells in order to increase the quantity supplied.

4.3 The Cost Function

The profit function of a mobile telecommunications service provider takes the familiar form (see sub-chapter 3.2 above):

\[ \Pi_i(q_i) = q_i P(Q) - C_i(q_i) - C_F, \quad \text{where} \quad Q \equiv \sum_{i=1}^{n} q_i \]  

that is, revenues less cost, where the cost has a marginal component and a fixed component.

This model is formulated without any truly fixed costs. Three cost components are included, each being variable to a different degree. These components are the marginal cost, the base station cost, and the spectrum cost.
4.3.1 Base Station Cost

In this paper, the term “base station” is used to refer to an entire installation that may include a tower or real estate asset, multiple radio transceivers, a connection to the operator's network, and any other hardware and labour costs incurred to deploy a cell site.

This paper adopts a fairly simplistic assumption that any base station can support a set quantity of demand units, given by $\frac{1}{\eta}$. As a result, the total number of base stations required is given by $\eta Q$, where $\eta$ can be described as the base station efficiency.

If each base station has a cost given by $C_B$, then the total cost of all base stations in the profit function will be given by:

$$C_B \eta Q$$  \[15\]

As described in sub-chapter 2.1.2, a base station operates on a particular channel or set of channels, and each channel has a channel capacity that depends on both the technology used and on the bandwidth of the channel (or, the spectrum occupied by the channel). So, doubling the channel bandwidth of a base station will double the quantity that the base station can supply, thus halving the number of base stations required to supply a given quantity. As described in sub-chapter 4.2 above, we assume that base station density is fixed, so that additional bandwidth has the effect of increasing the quantity that can be served without changing the base station density choice.

4.3.1.1 User Density

In reality, neither users nor user demand are evenly distributed across a geographic area. A market will have areas of dense demand and sparse demand, and demand patterns will change throughout a day.

In areas of very sparse demand, such as rural areas, the quantity (amount of customer demand) supplied by a base station will not be limited by the amount of bandwidth available and the base station efficiency. The amount of demand served will instead be limited by the maximum propagation distance of the base station equipment. In these cases a base station will serve its maximum serving area and will not be used to capacity. The total marginal cost associated with base stations will be higher than described above. In Canada, incumbent operators have substantial networks covering many sparsely populated areas and road corridors. These network segments
are not considered in this model. Investment in rural / highway coverage could be considered as an investment in network quality, or could be considered in a model that includes variation in the spatial distribution of users.

In areas of very dense demand, for example a busy subway station during rush hour, the total quantity of service supplied in an area will be limited by a minimum distance between base stations imposed by the constraints of intra-system interference. In other words, base stations cannot be installed at a very high density, since the noise level in that area would severely degrade the performance of these base stations. In these cases, the total marginal cost described above will be valid, but the quantity supplied will be less than the quantity that the operator might wish to supply due to capacity constraints. In reality, this might be experienced as dropped calls.

For the purpose of describing a general model, this paper will consider that users are evenly distributed across a geographically homogeneous market area, and that the base station deployment is not constrained by high or low demand density.

Another simplification introduced to this model is the reduction of the base station cost to a marginal cost. In reality, base stations are a lumpy investment that must be purchased in discrete units. For the purposes of this model, we suppose that the network is large enough that the number of base stations is a continuous choice.

4.3.2 Spectrum Cost

As described in sub-chapter 4.2, a commercial mobile wireless network operator uses a particular quantity of spectrum, usually measured in MHz, to operate a mobile network.

In Canada, commercial mobile spectrum is sold in auctions run by Industry Canada. These auctions are scheduled in advance, though they are not regular, and each auction event releases a limited amount of spectrum.

The price paid for spectrum is commonly measured in $/MHz/pop. In this paper, we suppose a single market with a uniform demand characteristic that is analogous to population (see sub-chapter 4.1.1 above). Since the population, or demand faced by an operator, is uniform, we suppose that spectrum is valued in $/MHz.

The supply for spectrum is modelled as an upward-sloping curve. The price paid for spectrum is an outcome of the auction process, in which the price of a fixed amount of spectrum is set by a competitive bidding process. A brief investigation of the results of the most recent (2014 700 MHz) Industry Canada spectrum auction indicates that purchasers of higher quantities of spectrum paid a higher per-unit price than purchasers of smaller quantities.
This observation may be attributable to the fixed quantity of spectrum available in each auction event. In the long term, however, a company faces many opportunities to purchase spectrum. A model reflecting a rising per-unit spectrum cost is not strongly justified, nor would it be easily tractable.

In order to present a tractable model, the cost of spectrum is modelled as a parabolic function of a base price \( C_s \), the total spectrum bought by all market actors, and the spectrum purchased by the firm of interest. So, for firm “x”, the total spectrum cost is:

\[
\text{Spectrum Cost} = C_s \cdot \left( B_x + \sum_{i \neq x} B_i \right) \cdot B_x \tag{16}
\]

Where \( B_x \) is the bandwidth purchased by firm x. Since we have defined a direct relationship between bandwidth choice and quantity supplied, this brings the spectrum choice directly into the profit function.

This method has the drawback of not imposing any limitation on the amount of spectrum a firm may purchase. However, this model gives a spectrum price that increases quadratically with the amount purchased, imposing an increasing cost on spectrum purchases in the long term.

This method also permits different operators to pay different prices for spectrum, which may be an outcome of the auction process.
4.4 Summary of Model Components

Following the development of the formulations described in this sub-chapter, the profit function of firm \( x \) is described as:

\[
\Pi_x = (a - bQ_x) - C_M Q_x - \eta C_B Q_x - C_S (B_x + \sum_{i \neq x} B_i) B_x
\]

or

\[
\Pi_x = (a - b \Phi B_x - b \Phi \sum B_x) \Phi B_x - C_M \Phi B_x - \eta C_B \Phi B_x - C_S (B_x + \sum_{i \neq x} B_i) B_x
\]

where:

\( \Pi_x \) is the profit of firm \( x \);
\( Q_x \) is the quantity supplied by firm \( x \);
\( B_x \) is the amount of spectrum purchased by firm \( x \);
\( \sum B_x \) is the sum of the spectrum purchases by all firms other than firm \( x \);
\( 1/\eta \) is the quantity of demand that can be serviced using 1 base station;
\( \Phi \) is the amount of quantity that can be serviced using 1 MHz of spectrum
\( C_B \) is the cost of a base station
\( C_S \) is the base cost of spectrum
\( C_M \) is the marginal cost of supplying one unit of demand

[17]
Chapter 5 The Model

5.1 Model Construction

The complete model of the mobile wireless telecommunications market presented here consists of a two-stage Stackelberg-Cournot competition model in which firms ultimately compete based on their quantity choice, which is a direct result of their prior spectrum choice. In this interpretation, the incumbent firms have full knowledge of the entrant firms' cost functions, and therefore anticipate the entrants' reaction functions. In the absence of entrants, the incumbent firms engage in Cournot competition.

The equations set out below describe the profit function of telecommunications firms (the supply curve), and the demand characteristics of the market. The supply curve includes the revenue (a function of price and quantity), the marginal cost (a function of a constant marginal cost and quantity), and the cost associated with purchasing spectrum. We have defined the spectrum quantity to be directly related to the quantity supplied through the parameter $\Phi$.

As described in the previous chapter, the profit function for a firm, $x$, will be:

$$\Pi_x = P Q_x - C_M Q_x - \eta C_B Q_x - C_S \left( B_x + \sum_{i \neq x} B_i \right) B_x$$

$$= \left( a - b Q_x - b \sum Q_i \right) Q_x - C_M Q_x - \eta C_B Q_x - C_S \left( B_x + \sum B_i \right) B_x$$

$$= \left( a - b \Phi B_x - b \sum B_i \right) \Phi B_x - C_M \Phi B_x - \eta C_B \Phi B_x - C_S \left( B_x + \sum B_i \right) B_x$$

where $\sum B_i \equiv \sum_{i \neq x} B_i$

The firm maximizes its profit with respect to the quantity (directly related to the bandwidth) choice. In the standard Cournot model, each firm does not have information about the other firms' choices prior to the maximization process.

This process results in a bandwidth choice, $B_x$, that is a function of the model parameters.
In the case of an entrant \((e)\), the profit function will be dependent on both the entrant's quantity choice, the other entrant firms' quantity choices, and the incumbent firms' quantity choices, so that:

\[
\Pi_e = f(B_e, \sum B_e, \sum B_i)
\]  \[19\]

This process results in a bandwidth choice, \(B_e\), that is a function of both the model parameters and the incumbents' bandwidth choices, \(B_i\), which are yet not known by the entrant.

\[
B_e = f(\sum B_i)
\]  \[20\]

The incumbent's \((i)\) profit function will be dependent on both the other incumbents' quantity choices and the entrants' quantity choices.

\[
\Pi_i = f(B_i, \sum B_i, \sum B_e)
\]  \[21\]

Since the entrants' quantity choice \((\Phi B_e)\) is a function of the incumbents' choices \((B_i)\), this gives a profit function for the incumbent that depends only on the model parameters, and on the incumbents' quantity choice. The incumbent's maximization of profit with respect to quantity (or bandwidth) results in a quantity choice \((\Phi B_i)\) that is a function of the model parameters.

The full development of the model is shown in the appendix.
5.2 Overview of Results

5.2.1 Investigation

The investigation of the model presented here was conducted for the purpose of understanding the effects of certain model parameters.

The parameter that is most uncertain, given the limited availability of data, is the elasticity of the demand curve. Most of the simulations were conducted over a wide range of values of the parameter $b$ (the demand curve slope), so that the implications of policy decisions could be determined across a range of market conditions.

A survey of current packages available does not show any evidence of on-net discounts (i.e. reduced prices for calls to/from subscribers of the same network), however some social bias may incent consumers to choose more popular networks over others. The model does not include any modification of the demand curve for entrants to account for network effects, since no pecuniary network incentives are published by operators.

While reasonable guidance was available regarding the cost parameters (marginal cost and base station cost), no data was found to give guidance in the matter of the difference in costs faced by incumbents versus entrants. We assume that entrants would face slightly higher costs than incumbents, due to factors such as the necessity to obtain real estate in order to develop new sites whereas incumbents have an existing network of sites.

Government intervention in the mobile telecommunications market has occurred, recently, in the spectrum market. By designating set-aside spectrum, the government made it possible for entrants to purchase spectrum at lower cost than incumbents. The effects of differences in spectrum cost between entrants and incumbents were examined across a range of market characteristics (demand curve slope).
5.2.2 Analytical Results

A preliminary analysis of the model results show a few implications. All results presented here are discussed further in Chapter 7.

The first observation is that, for the incumbent firm (with or without the presence of entrants):

\[
\frac{\partial Q}{\partial b} < 0 \quad \frac{\partial \Pi}{\partial b} > 0
\]

and for the entrant firm:

\[
\frac{\partial Q}{\partial b} > 0 \quad \frac{\partial \Pi}{\partial b} > 0
\]

In both the case of the incumbent and the entrant, the analytical results show that:

\[
\frac{\partial P}{\partial b} < 0
\]

That is, as the market demand becomes more price-elastic \((b\) decreases), the price increases. However, the simulated results give us the inverse relationship, due to the choice of rotating the demand curve through a known point (so that, as the slope \(-b\) increases, the intercept \(a\) increases).

In varying the costs faced by entrants, we vary the ratio

\[
R = \frac{\text{entrant's marginal cost}}{\text{incumbent's marginal cost}} = \frac{\text{entrant's base station cost}}{\text{incumbent's base station cost}}
\]

If \(R = 1\), then the incumbent’s quantity choice does not change in the presence or absence of an entrant. However, the presence of an entrant will result in a decrease of the price level and of the incumbent’s profit. In the case of a single entrant, the entrant’s quantity supplied will be half of each incumbent’s quantity supplied (regardless of the number of incumbent firms).

Varying \(R\) in the range \(R > 1\), and holding all other variables constant, we observe that, for the incumbent:

\[
\frac{\partial Q}{\partial R} > 0 \quad \frac{\partial \Pi}{\partial R} > 0
\]
and for the entrant:

\[
\frac{\partial Q}{\partial R} < 0 \quad \frac{\partial \Pi}{\partial R} < 0
\]

and the price outcome is:

\[
\frac{\partial P}{\partial R} > 0
\]

Once the entrants' cost reaches a threshold level, the entrant's profit-maximizing quantity choice is zero.

In addition to varying the operating costs of the entrant relative to the incumbent, we also vary the ratio

\[
S = \frac{\text{spectrum cost faced by entrants}}{\text{spectrum cost faced by incumbents}}
\]

Varying \( S \) in the range \( S < 1 \), and holding all other variables constant, we find that, for the incumbent:

\[
\frac{\partial Q}{\partial S} < 0 \quad \frac{\partial \Pi}{\partial S} < 0
\]

and for the entrant:

\[
\frac{\partial Q}{\partial S} > 0 \quad \frac{\partial \Pi}{\partial S} > 0
\]

and the price outcome is:

\[
\frac{\partial P}{\partial S} < 0
\]
Chapter 6 Parameter Values

Parameter values for the model were selected using publicly available data. Extensive use was made of the Bell, Rogers and TELUS annual financial reports, which provide information regarding cost, revenue, and subscriber numbers. Among the new operators in Canada, several are not publicly traded, and those that are publicly traded do not show the results of their Canadian wireless divisions separately. As of Q3 2013 (CWTA, 2013), the market share of Bell, Rogers, and TELUS account for 92% of all subscribers. Data from new operators is not considered in the following investigation, as the data from Bell, Rogers, and TELUS is adequate to represent the characteristics of the market, and equivalent data from new entrants is not available.

In taking data from companies' annual reports, the data for each year is taken from the annual report of that year. In many cases, this data is modified in the subsequent year's annual report. Any modifications made to the published data is not reflected in the data used here.

Additional information was gathered from the Canadian Wireless Telecommunications Association (CWTA 2013), and Industry Canada. Both of these organizations publish information that is voluntarily provided for publication and self-reported.

6.1 Bandwidth-Quantity Relationship

In this modelling exercise, the quantity that an operator can supply varies directly with the amount of bandwidth held by the operator.

Since many spectrum licenses are not nationwide, but rather regional, we use average spectrum holdings across several markets in the parameterization of a bandwidth-quantity relationship.

Industry Canada (Spectrum Inventory, 2010) shows the spectrum holdings of various carriers in several major markets. The total number of subscribers of Bell, Rogers, TELUS, and Globalive are also given in this document. Looking specifically at these four providers, and their spectrum holdings in Toronto, Montreal and Vancouver, the bandwidth-quantity relationships are shown in Table 3 below.
### Table 6.1: Bandwidth Holdings, 2010

<table>
<thead>
<tr>
<th></th>
<th>Spectrum Holdings, Toronto (MHz)</th>
<th>Spectrum Holdings, Vancouver (MHz)</th>
<th>Spectrum Holdings, Montreal (MHz)</th>
<th>Spectrum Holdings, average (MHz)</th>
<th>Total Subscribers</th>
<th>Φ = Q/B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rogers</td>
<td>95</td>
<td>105</td>
<td>105</td>
<td>102</td>
<td>8,626,000</td>
<td>84,846</td>
</tr>
<tr>
<td>Bell</td>
<td>75</td>
<td>50</td>
<td>55</td>
<td>60</td>
<td>6,987,386</td>
<td>116,456</td>
</tr>
<tr>
<td>TELUS</td>
<td>50</td>
<td>65</td>
<td>60</td>
<td>58</td>
<td>6,699,000</td>
<td>114,840</td>
</tr>
<tr>
<td>Globalive</td>
<td>20</td>
<td>10</td>
<td>0</td>
<td>10</td>
<td>93,882</td>
<td>9,388</td>
</tr>
</tbody>
</table>

It is notable that Bell and TELUS share a remarkably similar quantity-bandwidth ratio (Φ). Rogers has a lower ratio, suggesting that perhaps there are some diminishing returns to spectrum in terms of network capacity. Globalive, with the lowest spectrum holding (as a new entrant) and the lowest number of subscribers, also has the lowest value of Φ. This result may be due to the newness of the operator, and the time required to deploy a network and mount an effective marketing campaign.

For this modelling exercise, values of Φ between 50,000 and 120,000 are considered as this encompasses most of the variation in the limited dataset. The low value of the Globalive data point is not included since this operator is new to the market and represents a small market share. A value of 100,000 was assigned to the number of subscribers that can be served per unit of spectrum, as this value falls centrally in the range of investigation.

### 6.2 Supply Parameters

#### 6.2.1 Marginal Cost

In their published annual financial reports, Bell, Rogers and TELUS each provide certain information for their wireless and wireline divisions separately. The annual operating expenses for the wireless divisions from these companies' annual reports are as shown in Table 4 below (Annual Reports):
Table 6.2: Operators’ Total Operating Expenses

<table>
<thead>
<tr>
<th></th>
<th>Total Operating Expenses ($ Billion)</th>
<th>Total Operating Expenses per Subscriber ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bell</td>
<td>Rogers</td>
</tr>
<tr>
<td>2011</td>
<td>3.05</td>
<td>4.1</td>
</tr>
<tr>
<td>2010</td>
<td>3.19</td>
<td>3.8</td>
</tr>
<tr>
<td>2009</td>
<td>3.27</td>
<td>3.61</td>
</tr>
</tbody>
</table>

Rogers' annual reports break down the operating expenses into 1) the cost of sales, 2) sales and marketing expenses, and 3) operating, general and administrative expenses. This third category is used as a proxy for the marginal cost of servicing a customer. TELUS' annual reports break down the operating expenses into 1) equipment sales expense, 2) network operating expense, 3) marketing expense and 4) general and administration expenses. The sum of the second and fourth categories is used as a proxy for the marginal cost of serving a customer. These numbers are shown on a per-subscriber basis in Table 5 below.

Table 6.3: Operators’ Network Operating and General Administrative Expenses

<table>
<thead>
<tr>
<th>Per-subscriber Network Operating Expense for the year:</th>
<th>Rogers</th>
<th>Telus</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>287</td>
<td>220</td>
</tr>
<tr>
<td>2010</td>
<td>217</td>
<td>223</td>
</tr>
<tr>
<td>2009</td>
<td>226</td>
<td>233</td>
</tr>
</tbody>
</table>

For the modelling exercise, marginal cost values between $100 and $600 were considered. This range encompasses most of the range in two datasets above. A value of $200 was assigned to the annual marginal cost of serving one user (one unit of demand). This value reflects the trend in Tables 4 and 5 above that the network operating expenses are lower than the overall corporate operating expenses.
6.2.2 Base Station Cost

For the purpose of the modelling exercise, the cost of a base station is considered to be the full cost of installing a mobile wireless base station, including the cost of real estate (for example, tower construction or a rooftop lease), the cost of network connectivity (for example, a fibre connection to the network centre), and the cost of the radio equipment.

Several sources were located that referenced base station costs. Johansson, Furuskar & Zander (2004) estimate a total cost of €50,000 for a very small site, to €90,000 for a medium site, to €266,000 for a large site, taken from a study of western European operators. Using average 2002 exchange rates (1.4832) and core CPI inflation between December 2002 and December 2011 (118.2/100.7) this corresponds to $87,048; $156,686; and $463,094 respectively.

Sabat (2005, p.192) lists the cost of a new rooftop site as USD $105,000 and the cost of a new “indoor” site as $280,000. Using the 2005 US exchange rate (1.21) and core CPI inflation between December 2005 and December 2011 (118.2/106.2) this corresponds to $141,406 and $377,082 respectively.

Sabat (2005) also shows that the difference between building a new “outdoor” site and collocating on an existing outdoor site is significant. The new site cost is estimated at USD $240,000, and the collocated site cost is estimated at USD $51,000 (CDN $323,213 and $68,683 in 2011, respectively). If incumbents are able to build a network using existing sites, and new entrants must build their own sites, this indicates that entrants may face infrastructure costs up to 4.7 times higher than incumbents.

Lundborg, Reichl & Ruhle (2012) estimate base station cost as as €60,000 for site acquisition plus €40,000 for equipment, for a total of €100,000 per site ($128,500 using average the average 2012 exchange rate).

The annual cost of a base station is estimated by linearly amortizing the cost of the base station over ten years. Ten years is chosen as the lifetime of the base station in Johanssen et al. (2004), and ten years is a typical license term of spectrum licenses awarded by Industry Canada. However, an argument might be made for a shorter amortization period due to equipment obsolescence, as operators may upgrade their systems in response to new technologies.
For this modelling exercise, (annualized) base station cost values between $10,000 and $100,000 were investigated. This range encompasses most of the data points found in the literature. A value of $20,000 was assigned to the cost of constructing a base station, which is an approximate average of the data points collected.

This paper investigates the effect of entrants facing higher base station costs than incumbents. An incumbent may have lower base station costs due to buying power, or deploy a new network using existing sites that need not be acquired or constructed.

6.2.3 Spectrum Cost

Industry Canada publishes the final prices paid for commercial mobile spectrum following a spectrum auction. Spectrum prices for each license may be expressed as dollars per MHz per person ($/MHz/pop). There is a great deal of variation in spectrum prices by this measure. For example, each MHz-pop of spectrum in urban centres appears to be more valuable than spectrum in rural areas.

The prices paid for the 282 licenses (or spectrum blocks) sold in the 2008 Industry Canada AWS spectrum auction are shown in Figure 4 below, with the price/MHz/pop expressed in Canadian dollars ($). Each spectrum license or block is described by a specific amount of spectrum (in MHz), and a geographic area that encompasses a particular population. The chart on the left is a histogram of per-unit prices ($/MHz/pop) for each license or block; the chart on the right is a stripchart showing the distribution of prices on a per-block basis.
In this spectrum auction, some spectrum blocks were set aside for non-incumbent bidders. Other spectrum blocks were open to all bidders. Note also the spectrum blocks “G” and “I” were non-standard; that is, these blocks did not correspond to blocks in use in the United States. As a result, they were considerably less valuable than other blocks.

An ANOVA analysis of the prices paid in $/MHz/pop for blocks A through F shows that there is a difference between the prices paid for the set-aside blocks (mean price of $0.76/MHz/pop) and the open blocks (mean price of $1.12/MHz/pop).\(^3\)

The highest prices paid, in $/MHz/pop, were for the licenses in Toronto ($2.08/MHz/pop, by Rogers, A-block), Montreal ($3.38/MHz/pop by TELUS, E-block) and Vancouver ($2.68/MHz/pop, by Bell, E-E-block).

\(^3\) The ANOVA analysis was conducted using blocks A, E, and F (177 observations) versus blocks B, C, and D (87 observations). The test statistic $F=697$ is in the rejection region of the F-Distribution at $\alpha=0.005$, so we reject the null hypothesis that the means of the two groups are equal, with a level of significance of 0.5%.
Populations are taken from the Industry Canada lists in “Service Areas for Competitive Licensing”. These populations are based on 2001 census data.

In the modelling exercise, the price paid for spectrum, as described in sub-chapter 4.3.2, is a function of a base price times the total spectrum available times the quantity purchased. The parameter value for the base price, $C_S$, was estimated using the AWS auction results.

Looking at the auction results of the AWS auction, it appears as though two separate auctions took place – one for incumbents, and one for entrants. Due to the auction rules, no incumbents won spectrum in the set-aside blocks. Also, very few non-set-aside spectrum blocks were bought by non-incumbents (seven blocks out of 1,077). In determining a parameter value for $C_S$, we will investigate the AWS auction results as the results of two separate auctions, ignoring these seven blocks.

The results of the non-set-aside segment of the auction are shown in Table 6 below, based on 50 MHz of spectrum that was not set aside. The overall spectrum quantity is the mean of the spectrum winnings across all 59 Tier-3 geographical areas (Industry Canada, 2008). The parameter value $C_S$ from equation [16] is estimated by dividing the total paid by each firm by the total quantity of spectrum sold (50 MHz), and then by the quantity of spectrum purchased by that firm.

<table>
<thead>
<tr>
<th>Operator</th>
<th>Total amount paid</th>
<th>Overall Spectrum Quantity (MHz)</th>
<th>$C_S$ estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rogers</td>
<td>$ 999,367,000</td>
<td>20.0</td>
<td>$ 999,367</td>
</tr>
<tr>
<td>Bell</td>
<td>$ 740,928,000</td>
<td>13.4</td>
<td>$ 1,106,703</td>
</tr>
<tr>
<td>TELUS</td>
<td>$ 879,889,000</td>
<td>15.4</td>
<td>$ 1,140,955</td>
</tr>
</tbody>
</table>

The results of the set-aside segment of the auction are shown in Table 7 below, based on 40 MHz of spectrum that was set aside. The overall spectrum quantity is the mean of the spectrum winnings across all 59 Tier-3 geographical areas (Industry Canada 2008).
The recent 700 MHz auction (completed in February 2014) had published provisional winners and the prices paid as of the time of writing. The results of this auction are shown in Table 8 below, showing the mean of the spectrum winnings across the fourteen Tier-2 geographical areas (Industry Canada, 2014).

### Table 6.5: AWS Auction Set-Aside Spectrum Prices

<table>
<thead>
<tr>
<th>Winner</th>
<th>Total amount paid</th>
<th>Overall Spectrum Quantity (MHz)</th>
<th>( C_s = \text{total paid} / 40 / \text{overall quantity} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Globalive</td>
<td>$442,099,000</td>
<td>9.32</td>
<td>$1,185,629</td>
</tr>
<tr>
<td>9193-2962 Qc</td>
<td>$554,549,000</td>
<td>8.31</td>
<td>$1,669,306</td>
</tr>
<tr>
<td>Bragg</td>
<td>$25,628,000</td>
<td>6.61</td>
<td>$96,926</td>
</tr>
<tr>
<td>1380057 AB</td>
<td>$189,519,000</td>
<td>5.59</td>
<td>$847,093</td>
</tr>
<tr>
<td>SaskTel</td>
<td>$65,690,000</td>
<td>4.58</td>
<td>$358,862</td>
</tr>
<tr>
<td>DAVE</td>
<td>$243,159,000</td>
<td>4.41</td>
<td>$1,379,460</td>
</tr>
<tr>
<td>6934242 Can</td>
<td>$52,385,077</td>
<td>0.68</td>
<td>$1,931,700</td>
</tr>
<tr>
<td>Rich Telecom</td>
<td>$739,000</td>
<td>0.34</td>
<td>$54,501</td>
</tr>
<tr>
<td>Celluworld</td>
<td>$932,000</td>
<td>0.17</td>
<td>$137,470</td>
</tr>
</tbody>
</table>

The values for \( C_s \) that emerge from an analysis of the AWS auction, for those companies that bought appreciable amounts of spectrum, are in the range of \( 1 \times 10^6 \). The values for \( C_s \) that emerge from the 700 MHz auction vary widely across the principal winners, between \( 4.2 \times 10^5 \) and \( 2.6 \times 10^6 \). In this modelling exercise, spectrum base values between \( 1 \times 10^5 \) and \( 1 \times 10^7 \) (or, annualized over ten years, \( 1 \times 10^4 \) and \( 1 \times 10^6 \)) were investigated. This data range encompasses most of the values represented in the
results of these two auctions. A value of $1 \times 10^5$ was used in the simulation exercise, as this value is roughly in the centre of the range of data points.

6.2.4 Base Station Efficiency ($\eta$)

The efficiency of the base station represents the quantity of user demand that can be served by one base station.

Industry Canada's 2010 study (Industry Canada, 2010) provides the self-reported number of base station sites deployed by all mobile wireless carriers, as of August 2010. The data is summarized below by region and by operator, showing the number of sites deployed in each band. The total number of base stations deployed by each operator, as well as their subscriber count and the resulting value of $\eta$, are shown in Table 9 below.

Table 6.7: Operators’ Base Station Numbers

<table>
<thead>
<tr>
<th>Operator</th>
<th>Number of Base Stations</th>
<th>Number of Subscribers (Q)</th>
<th>$\eta = \frac{#BST}{Q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rogers</td>
<td>8,580</td>
<td>8,626,000</td>
<td>9.95E-004</td>
</tr>
<tr>
<td>Bell</td>
<td>4,111</td>
<td>6,987,386</td>
<td>5.88E-004</td>
</tr>
<tr>
<td>TELUS</td>
<td>2,824</td>
<td>6,699,000</td>
<td>4.22E-004</td>
</tr>
<tr>
<td>SaskTel</td>
<td>613</td>
<td>541,105</td>
<td>1.13E-003</td>
</tr>
<tr>
<td>MTS</td>
<td>224</td>
<td>469,744</td>
<td>4.77E-004</td>
</tr>
<tr>
<td>Public Mobile</td>
<td>244</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Globalive</td>
<td>691</td>
<td>93,882</td>
<td>7.36E-003</td>
</tr>
<tr>
<td>Mobilicity</td>
<td>172</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Videotron</td>
<td>464</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>17,923</strong></td>
<td><strong>23,417,117</strong></td>
<td><strong>7.65E-004</strong></td>
</tr>
</tbody>
</table>

In the modelling exercise, values of $\eta$ between $1 \times 10^{-4}$ and $1 \times 10^{-3}$ were investigated. This range encompasses most of the data points above. A value of $1 \times 10^{-3}$ was assigned to the efficiency, since this value is close to the average of all of the available data points.

6.3 Demand Parameters

The demand curve is challenging to characterize. Although service providers presumably have a great deal of data to describe the price sensitivity of their customers, this data is of course proprietary. An attempt is made here to gain some understanding of market demand in Canada.
6.3.1 Demand Information

The annual reports of Bell, Rogers and TELUS give subscriber counts and both total operating revenues and network / service revenues for their wireless divisions. Subscription-based and pre-paid subscribers are aggregated.

The network or service revenue can be divided by the total number of subscribers to give the Average Revenue Per User (ARPU). This number can be considered to be an annual price. The number of subscribers can be considered to be the quantity supplied.

As described in sub-chapter 4.3.1 above, real users can be supposed to have varying demand patterns. However, this paper uses an average demand level in order to provide a simplified basis for the purpose of analyzing the spectrum market.

The Figure 5 below shows the total quantities of subscribers and the ARPU for each of the three incumbents.

![Figure 6.2: Subscriber Counts and Revenues](image)
The number of subscribers appears to increase more or less linearly with time, while price levels fluctuate, with a generally increasing trend. (Note that the drop in ARPU for Rogers in 2004 corresponds to the takeover of Fido). We conclude that there is an annual rate of growth in demand for mobile telecommunications services over the ten year period measured. This does not necessarily imply perpetual growth; growth in demand for the same services will ultimately be limited by the consumer population. Demand growth may continue above the rate of population growth as long as service suppliers can provide innovative new products.

We note also that the average revenue per user shown on the right-hand side of Figure 5 embodies both a wide variety of product qualities (from basic packages to richly featured packages), and a changing product quality over time (for example, as data services have been introduced and more widely adopted). Isolating this variation in product quality from the observed results is not attempted here.

An attempt follows to separate the observed linear natural rate of growth from a change in subscriber numbers resulting from price changes. The slope of the demand curve \( \frac{dP}{dQ} \) is estimated using a series of steps. Since we do not have access to a single price level, but rather an ARPU that is different for each of three firms, the data has been treated on a per-firm basis.

First, a natural growth rate is estimated for each company individually using a basic linear model (least squares regression) of the number of subscribers versus the year over the ten-year period. The results of the linear modelling exercise are shown in Table 10 below.

<table>
<thead>
<tr>
<th>Company</th>
<th>Estimated annual growth rate, # subscribers per year, based on linear model of data</th>
<th>R-squared</th>
<th>( P ) (N=10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TELUS</td>
<td>479,245</td>
<td>0.98</td>
<td>2.7 E-9</td>
</tr>
<tr>
<td>Rogers</td>
<td>622,905</td>
<td>0.95</td>
<td>2.2 E-7</td>
</tr>
<tr>
<td>Bell</td>
<td>375,784</td>
<td>0.98</td>
<td>2.7 E-9</td>
</tr>
</tbody>
</table>

We suppose that the growth in mobile phone subscribers for each supplier has a natural rate of growth.

In the second step, the changes in price or ARPU (\( \Delta P \)) and the number of subscribers (\( \Delta Q \)) are noted for each year, with respect to the preceding year.
Third, the change in quantity (\(\Delta Q\)) is adjusted by subtracting the estimated annual growth rate, leaving the portion of the change in quantity that results from the change in price (\(\Delta Q'\)).

Finally, the slope of the implied demand curve, that is, the change in price divided by the adjusted change in quantity. The results of these calculations are shown in Table 11 below.

Table 6.9: Slope of Demand Curve

<table>
<thead>
<tr>
<th></th>
<th>Telus</th>
<th>Rogers</th>
<th>Bell</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta P)</td>
<td>(\Delta Q)</td>
<td>(\Delta Q')</td>
<td>(\Delta P/\Delta Q')</td>
</tr>
<tr>
<td>2003</td>
<td>20</td>
<td>428,500</td>
<td>-50,745</td>
</tr>
<tr>
<td>2004</td>
<td>22</td>
<td>512,400</td>
<td>33,155</td>
</tr>
<tr>
<td>2005</td>
<td>18</td>
<td>584,300</td>
<td>105,055</td>
</tr>
<tr>
<td>2006</td>
<td>-29</td>
<td>1,038,300</td>
<td>559,055</td>
</tr>
<tr>
<td>2007</td>
<td>71</td>
<td>8,900</td>
<td>-470,345</td>
</tr>
<tr>
<td>2008</td>
<td>-7</td>
<td>561,100</td>
<td>81,855</td>
</tr>
<tr>
<td>2009</td>
<td>-40</td>
<td>395,000</td>
<td>-84,245</td>
</tr>
<tr>
<td>2010</td>
<td>-12</td>
<td>447,000</td>
<td>-32,245</td>
</tr>
<tr>
<td>2011</td>
<td>21</td>
<td>369,000</td>
<td>-110,245</td>
</tr>
<tr>
<td>2012</td>
<td>18</td>
<td>330,000</td>
<td>-149,245</td>
</tr>
</tbody>
</table>

It is notable that 12 of the 30 results for \(\Delta P/\Delta Q'\) are greater than zero, and therefore not consistent with the characteristics of a normal demand curve. These 12 positive results cluster around the years 2005 and 2009-2010. As discussed in sub-chapter 2.3.2, the years 2009 and 2010 saw the entry of four new providers in the Canadian market. This new competition, together with the general economic recession around this time, may account for the decreasing prices and flat subscriber growth in these years. Several changes in the marketplace occurred in the 2004-2005 timeframe (restructuring of Bell into Bell Mobility, takeover of Fido by Rogers) which may account for changes to prices and customer numbers (as well as the number of customers reported by these entities) in this time period.

Ignoring these 12 positive results, the remaining negative results fall in the range between -7E-6 and -4E-3. In the modelling exercise, values between -5E-6 and -5E-3 are investigated. This range of values encompasses most of the negatively-valued data points.
The demand curve, as discussed above, is modelled as \( P = \gamma - bQ \), where \( b = \frac{dP}{dQ} \). In order to determine the value of \( \gamma \), the 2012 data point of price and quantity is used. In 2012, the total number of Bell, Rogers and Telus subscribers is 9,437,000 (Rogers) + 7,681,032 (Bell) + 7,670,000 (Telus) = 24,788,032. The average 2012 price level (ARPU) of Rogers ($712/year), Bell ($661/year) and Telus ($700/year) is $691/year.

Using the range of values described for \( b \) (or \( \frac{dP}{dQ} \)), and the data point of 24.8 million subscribers, and $691/year, the range of \( \gamma \) is between approximately $815 and $125,000.

These results give a very wide range of characteristics for the demand curve. A more thorough investigation of the demand curve might be undertaken using churn rates of service providers, or using comparisons with service providers in different countries.
Chapter 7  Results

A range of scenarios were investigated in order to simulate a variety of market conditions and interventions. First, we investigate the effects of varying the slope of the demand curve, since this parameter value is the most weakly supported. Following that, we investigate the effects of entry into the market, with an entrant facing costs equal to the incumbents’ costs; with an entrant facing marginal costs and base station costs greater than the incumbents’ costs; and with an entrant facing spectrum costs lower than the incumbents’ spectrum costs. New entrants to the market may indeed face higher operating costs than established incumbents. The purpose of examining the effect of discounted spectrum costs for incumbents is to model the effect of government interventions designed to encourage entry. The results of these scenarios are shown below.

7.1 Elasticity

In the parameterizations described in Chapter 6 above, the most uncertain parameter value is the slope of the demand curve \( b \). The initial investigation, therefore, examines the quantity and price outcomes of the model at a range of values for the parameter \( b \).

In varying the parameter \( b \), or the slope of the demand curve, the parameter \( a \), or the intercept of the demand curve, is also varied in order to maintain the average price and total quantity observed in the market in 2012 (CWTA, 2012). So, the demand curve is effectively rotated about this known point.

Figures 6, 7, and 8 below show the effect of varying \( b \) on quantity choice, price, and profit, respectively. These results are consistent with standard outcomes of Cournot competition models.
Figure 7.1: Quantity Result vs. Demand Curve Slope
Figure 7.2: Price Result vs. Demand Curve Slope
As elasticity increases ($b$ decreases), the quantity choice increases, price decreases, and the profit level decreases. This is consistent with standard Cournot model outcomes.

As the number of firms increases, the per-firm quantity choice decreases and the total quantity supplied increases; the price decreases, and the per-firm profit decreases. These effects are more pronounced as the market becomes more inelastic (higher values of $b$).

### 7.2 Entrant facing equal costs

Next, we leave the number of incumbent firms at 3, and we introduce entrants into the market. Again, we investigate the effects of varying the parameter $b$ on the quantity and price outcomes. Figure 9 below shows the effect of varying the number of entrants, at varying values of $b$. 
The incumbents' quantity choices are not affected by the presence or the number of entrants. As more entrants participate in the market, the market size increases. The market shares of the entrants and incumbents are not affected by the elasticity of demand ($b$). This result is consistent with the basic Stackelberg scenario; in effect, entrants are servicing the portion of the market that is not served by the incumbents.
Figure 10 below shows the effect on price of varying the slope of the demand curve ($b$). The analytical result of $\frac{\partial P}{\partial b} < 0$ is different from the apparent result below. The rising price level with $b$ results from the modelling choice of rotating the demand curve through the known 2010 ($P,Q$) data point, so that the price intercept of the demand curve changes as the slope changes.

![Price Level vs dP/dQ](image)

**Figure 7.5: Price Result vs. Demand Curve Slope – With Entrants**

The price level decreases as more entrants participate in the market. The difference in price is more prominent at more inelastic market conditions.
7.3 Entrant facing higher operating costs

Now suppose that the entrants face higher marginal costs and base station costs than the incumbents. The simulation is repeated. In this simulation, the parameter \( b \) is set at \( 5 \times 10^{-5} \), and we use the scenario with 3 incumbents and 2 entrants. Figure 11 below shows the effect of varying the ratio of the entrants’ variable costs (i.e. marginal cost and base station costs) to the incumbents’ variable costs.

![Figure 7.6: Quantity Result vs. Entrants’ Operating Cost Level](image)

Where the entrants’ operating costs are the same as the incumbents’ costs, the incumbents’ quantity choices do not change, as described in sub-chapter 7.2 above. As the entrants’ costs increase, the incumbents' quantity choice increases, and the entrants' quantity choice is reduced, until eventually the entrants make zero profit and do not enter the market, and the incumbents revert to the no-entrant quantities at no-entrant prices. This is consistent with the results of a standard Stackelberg analysis. At lower values of \( b \) (where the demand is more elastic), this pinch-off point occurs at lower cost ratios.

The varying of entrants' costs has a very small impact on the price level and the overall quantity supplied in the market.
7.4 Entrant facing lower spectrum costs

The next simulation examines the effect of a government intervention that provides discounted spectrum to entrants. Again, the parameter $b$ is set at $5 \times 10^5$, and we use the scenario with 3 incumbents and 2 entrants. Figure 12 below shows the effect of varying the ratio of entrants’ spectrum costs to incumbents’ spectrum costs.

![Chart showing quantity result vs. entrants' spectrum cost level](image)

The effect of reducing the spectrum cost faced by entrants is, qualitatively, the opposite of increasing operating costs faced by entrants. The more this cost is reduced, the lower the market share of incumbents and the higher the market share of entrants. Again, the varying of entrants' spectrum costs have a very small impact on the price level and the overall quantity supplied in the market.
Chapter 8 Investment and Innovation

In this chapter, we extend the modelling exercise to draw conclusions regarding the effects of the auction set-aside mechanism on the investment decisions of operators.

Investment can reduce the cost associated with production, for example reducing the marginal cost or base station cost. Investment can also increase the quality of service or the variety of services offered, shifting the demand curve. The exercise below demonstrates the effects of investment on production costs.

In this exercise, we suppose that investment is used to lower input costs for the purpose of increasing profits, and that this lowering of input costs is not used to deter entry. The investment decision here is made after the quantity choice has been made, and the market equilibrium price has been reached. This scenario could be expected if the model developed here is extended into multiple time periods, as is the case with spectrum distribution events occurring at irregular intervals, separated by one or more years.

Suppose that marginal cost, $C_M$ can be reduced by an investment, $I$. The characteristics of such a function might be:

$$\frac{\partial C_M}{\partial I} < 0 ; \quad \frac{\partial^2 C_M}{\partial I^2} \geq 0$$

If we use the exponential function as an example, we have:

$$C_M(I) = C_0 e^{-kl}$$

$$\frac{\partial C_M}{\partial I} = -k C_0 e^{-kl}$$

$$\frac{\partial^2 C_M}{\partial I^2} = k^2 C_0 e^{-kl}$$
In this case, the profit function will be:

\[
\Pi_x = \left( a - bQ_x - b \sum Q_x \right) Q_x - \eta C_B Q_x - C_0 e^{-kI} Q_x - C_S B_{TOT} B_x - I
\]

\[
\frac{\partial \Pi_x}{\partial I} = k C_0 Q_x e^{-kI} - 1 = 0
\]

\[
I = \frac{1}{k} \left[ 1 - \ln(k) - \ln(C_0) - \ln(Q_x) \right]
\]

\[
\frac{\partial I}{\partial Q} = \frac{1}{kQ} > 0
\]

This brief examination shows that as the quantity produced increases, the level of investment will increase.

In Chapter 7 above, we saw that the presence of an entrant facing the same costs as incumbents does not alter the incumbents’ quantity choices. However, where the entrant faces higher costs than the incumbents, the incumbents increase their quantity choices. As shown here, that increase in quantity would result in an increase in investment by incumbents.

Chapter 7 further shows that providing discounted spectrum to entrants has the effect of reducing the quantity supplied by incumbents. As shown here, that reduction in quantity would result in decreased investment by incumbents.
Chapter 9  Conclusions

This paper describes a dynamic mobile telecommunications market, in which current profits are used to invest in future innovation, using a static Cournot-Stackelberg model that generates economic profits. We investigate the effects of government intervention in the market for spectrum, which is a key input in the supply of mobile telecommunications services.

The investigations conducted in this paper point to several broad conclusions.

The model demonstrates that the presence of entrants does not necessarily affect incumbents' quantity choices, but will result in lower prices for consumers and reduced profits for incumbents. The magnitude of the price reduction is heavily dependent on the elasticity of demand in the market.

As entrants face higher operating costs with respect to those faced by incumbents, the market share of incumbent firms will increase, and the entrant firms' market shares will decrease. If entrants face significantly higher operating costs than incumbents, then entry may not be profitable and entry will not occur at all in the market.

As entrants face lower spectrum costs with respect to those faced by incumbents, the market share incumbent firms will decrease, and the entrants firms' market share will increase. In this way, the spectrum discount may offset the effect of higher operating costs faced by entrants.

In the case where operating costs are reduced by investment, the reduced quantity choice of incumbent firms resulting from discounting entrants' spectrum will result in reduced levels investment by incumbent firms.

If we suppose that incumbent firms are responsible for most of the investment in the market, then discounting spectrum, while having the near-term effect of reducing prices (where demand elasticity and operating costs permit entry to occur), will have the long-term effect of reducing overall investment into innovation.
9.1 Policy Implications

If the primary policy objective is to achieve lower price levels, then entry into the market will, according to this model, achieve that goal. If the market demand is sufficiently inelastic, or if entrants face significantly higher input costs than incumbents, then entry may not occur.

A government may try to encourage market entry by providing entrants with the opportunity to obtain discounted spectrum. According to this model, this may indeed encourage entry, but will have the effect of reducing the level of investment.

A key implication of this research is that the market elasticity characteristic has a strong influence on the effect of intervention in the spectrum market. The more inelastic the market, the more prominent the effects of cost differentials on price and quantity supplied. It is difficult to determine the market elasticity from publicly available data.

Without good knowledge of market elasticity, interventions by government to alter market outcomes may have unintended consequences, such as the reduction of investment levels in the market.
Bibliography


Appendix A  Mathematical Model

This appendix describes the full development of the mathematical model described in Chapter 5.

A.1 Development of Profit Function

From sub-chapter 4.4, we have the profit function of a firm:

\[ \Pi_n = P Q_n - C_M Q_n - \eta C_B Q_n - C_S \left( B_n + \sum B_n \right) \]

where:
- \( \Pi_n \) is the profit of firm \( n \);
- \( Q_n \) is the quantity supplied by firm \( n \);
- \( B_n \) is the amount of spectrum purchased by firm \( n \);
- \( \Sigma B_n \) is the sum of the spectrum purchases by all firms other than firm \( n \);
- \( 1/\eta \) is the quantity of demand that can be serviced using 1 base station;
- \( \Phi \) is the amount of quantity that can be serviced using 1 MHz of spectrum
- \( C_B \) is the cost of a base station
- \( C_S \) is the base cost of spectrum
- \( C_M \) is the marginal cost of supplying one unit of demand

The demand curve, from sub-chapter 4.1, is:

\[ P = a - b Q_{tot} \]

Where \( Q_{tot} \) is the sum of all quantities supplied by all firms, or:

\[ Q_{tot} = Q_n + \sum Q_n \]

So the demand function becomes:

\[ P = a - b Q_n - b \sum Q_n \]
Next, from sub-chapter 4.2, we have:

\[ Q = \Phi B \]

or,

\[ Q_n = \Phi B_n \]

So,

\[ P = a - b \Phi B_n - b \Phi \sum B_n \]

Now, the full profit function is as follows:

\[ \Pi_n = \left( P = a - b \Phi B_n - b \Phi \sum B_n \right) \Phi B_n - \Phi C_M - \Phi \eta C_B - \Phi C_S \left( B_n + \sum B_n \right) \]

A.2 Cournot Competition

Next, suppose there are \( N \) firms interacting through Cournot competition. Each firm maximizes profit through a quantity choice, which, in this case, is analogous to the choice of \( B \). We have:

\[ \frac{\partial \Pi_n}{\partial B_n} = a \Phi - 2 b \Phi^2 B_n - b \Phi^2 \sum B_n - C_M \Phi - \eta C_B - 2 C_S B_n - C_S \sum B_n = 0 \]

At this point we can impose the condition that all of the \( N \) firms are equal, giving:

\[ \sum B_n = (N - 1) B_n \]

The equation above then reduces to:

\[ a \Phi - 2 b \Phi^2 B_n - b \Phi^2 (N - 1) B_n - C_M \Phi - \eta C_B - 2 C_S B_n - C_S (N - 1) B_n = 0 \]

\[ 2 b \Phi^2 B_n + b \Phi^2 (N - 1) B_n + 2 C_S B_n + C_S (N - 1) B_n = \Phi \left( a - C_M - \eta C_B \right) \]

\[ B_n \left( b \Phi^2 (N + 1) + C_S (N + 1) \right) = \Phi \left( a - C_M - \eta C_B \right) \]

\[ B_n \left( b \Phi^2 + C_S \right) (N + 1) = \Phi \left( a - C_M - \eta C_B \right) \]

This gives an expression for \( B_n \).
A.3 Stackelberg Scenario - Entrant

Next, suppose these \( N \) incumbent firms \((i)\) interact with \( M \) new entrants \((e)\). In a Stackelberg scenario, these \( N \) firms have knowledge of the \( M \) entrants' profit functions. An entrant's profit function is as follows:

\[
\Pi_e = PQ_e - C_{Me} Q_e - C_{Be} Q_e - C_{Se} \left( B_e + \sum B_e + \sum B_i \right) B_e
\]

Entrants may face different costs than the incumbents, so that:

- \( C_{Be} \) is the entrant’s cost of a base station
- \( C_{Se} \) is the entrant’s base cost of spectrum
- \( C_{Me} \) is the entrant’s marginal cost of supplying one unit of demand

This time we have:

\[
Q_{\text{tot}} = Q_e + \sum Q_{\text{e}} + \sum Q_i
\]

where \( \Sigma Q_i \) is the sum of all the incumbents' quantities and \( \Sigma Q_e \) is the sum of the quantities supplied by all other entrant firms.

So the demand function becomes:

\[
P = a - b \Phi B_e - b \Phi \sum B_e - b \Phi \sum B_i
\]

The full profit function for the entrant is:

\[
\Pi_e = \left[ P = a - b \Phi B_e - b \Phi \sum B_e - b \Phi \sum B_i\right] \Phi B_e - C_{Me} \Phi B_e - C_{Se} \left( B_e + \sum B_e + \sum B_i \right) B_e
\]

Since the firms are interacting in Cournot competition, the entrant will maximize profit through a choice of quantity, or \( B_e \):

\[
\frac{\partial \Pi_e}{\partial B_e} = a \Phi - 2b \Phi^2 B_e - b \Phi^3 \sum B_i - b \Phi^3 \sum B_e - C_{Me} \Phi - \eta_e C_{Be} \Phi - 2 C_{Se} B_e - C_{Se} \sum B_i - C_{Se} \sum B_e = 0
\]

At this point we can impose the condition that all of the \( M \) entrant firms are equal, or:

\[
\sum B_e = (M - 1) B_e
\]
The equation above then reduces to:

\[ a \Phi - 2b \Phi^2 B_e - b \Phi^2 \sum B_i - b \Phi^2 (M-1) B_e - C_{Me} \Phi - \eta C_{Be} \Phi - 2C_{Se} B_i - C_{Se} \sum B_i - C_{Se} (M-1) B_e = 0 \]

\[ B_e \left( 2b \Phi^2 + b \Phi^2 (M-1) + 2C_{Se} + C_{Se} (M-1) \right) = \Phi \left( a - C_{Me} - \eta C_{Be} \right) - \left( b \Phi^2 C_{Se} \right) \sum B_i \]

or

\[ B_e = Z_1 - Z_2 \sum B_i \]

where

\[ Z_1 = \frac{\Phi(a - C_{Me} - \eta C_{Be})}{2b \Phi^2 + C_{Se} (M+1)} \quad \text{and} \quad Z_2 = \frac{1}{M+1} \]

A.4 Stackelberg Scenario – Incumbent

Now the incumbent firms, knowing the entrants' profit functions in a Stackelberg scenario, choose the quantity to supply by maximizing their profit functions. The incumbent's profit function is:

\[ \Pi_i = P Q_i - C_M Q_i - \eta C_B Q_i - C_S \left( \sum B_i + \sum B_e \right) \]

The total quantity supplied to the market is:

\[ Q_{tot} = \sum Q_i + \sum Q_e \]

where \( \sum Q_e \) is the sum of all the entrants’ quantities and \( \sum Q_i \) is the sum of the quantities supplied by all other incumbent firms.

So the demand function becomes:

\[ P = a - b \Phi B_i - b \Phi \sum B_i - b \Phi \sum B_e \]

The incumbent knows that the entrants are all equal, and that each entrant firm will choose:

\[ B_e = Z_1 - Z_2 \sum B_i \]

so,

\[ \sum B_e = M \left( Z_1 - Z_2 \sum B_i \right) \]

or

\[ \sum B_e = M \left( Z_1 - Z_2 B_i - Z_2 \sum B_i \right) \]
Now, the full profit function of the incumbent becomes:

\[
\Pi_i = \left[ a - b \Phi M Z_i - b \Phi (1 - M Z_2) B_i - b \Phi (1 - M Z_2) \sum B_i \right] \Phi B_i - \\
C_M \Phi B_i - \eta C_S \Phi B_i - C_S B_i \left[ M Z_i + (1 - M Z_2) B_i + (1 - M Z_2) \sum B_i \right]
\]

The incumbent maximizes profit through the choice of \( B \):

\[
\frac{\partial \Pi_i}{\partial B_i} = a \Phi - b \Phi^2 M Z_i - 2 b \Phi^2 (1 - M Z_2) B_i - b \Phi^2 (1 - M Z_2) \sum B_i - \\
C_M \Phi - \eta C_S \Phi - C_S M Z_i - 2 C_S (1 - M Z_2) B_i - C_S (1 - M Z_2) \sum B_i = 0
\]

At this point we can impose the condition that all of the \( N \) incumbent firms are equal, giving:

\[
\sum B_i = (N - 1) B_i
\]

The equation above then reduces to:

\[
a \Phi - b \Phi^2 M Z_i - 2 b \Phi^2 (1 - M Z_2) B_i - b \Phi^2 (1 - M Z_2) (N - 1) B_i - \\
C_M \Phi - \eta C_S \Phi - C_S M Z_i - 2 C_S (1 - M Z_2) B_i - C_S (1 - M Z_2) (N - 1) B_i = 0
\]

\[
B_i \left[ b \Phi^2 (1 - M Z_2) (N + 1) + C_S (1 - M Z_2) (N + 1) \right] = \\
a \Phi - b \Phi^2 M Z_i - C_M \Phi - \eta C_S \Phi - C_S M Z_i
\]

\[
B_i \left( b \Phi^2 + C_S \right) (1 - M Z_2) (N + 1) = \Phi \left( a - C_M - \eta C_S \right) - M Z_i \left( b \Phi^2 + C_S \right)
\]

This gives an expression for \( B_i \), and from it an expression for \( B_e \) in the Cournot-Stackelberg model.