A New Approach for Modeling the Non-linear One Dimensional Consolidation Behaviour of Tailings

by

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ABSTRACT

The management of tailings largely depends on its consolidation behaviour. Extensive works on this sector have been performed as it plays a significant role in economic and environmental considerations of a tailings management facility. To resolve these issues, consolidation theories had been developed for one, two or three dimensional condition with numerical solutions for soft soils like tailings which behave differently from natural soils. Eventually, large strain consolidation tests have experienced at its advance level for precise determination of experimental data because non-linear behaviour of compressibility and hydraulic conductivity fits to a wide range of functions. This non-unique behaviour of tailings is believed to be an aftermath of the combination of flocculation, sedimentation, consolidation, segregation, deposition, freeze-thaw and desiccation phenomena. Similarly, a consequence of several factors combined called "apparent over-consolidation" is a mystery to the tailings industry and the reasons for this occurrence are not fully understood. It is believed to be the result of the combination of several contributing factors at low effective stresses. Previously, it was assumed that tailings are normally consolidated or consolidating under the load of mounting deposited materials and numerical modeling had been performed by different researchers based on this assumption. However, the apparent compressibility behaviour of tailings was noticed for different types of tailings at a wide range of solids content and various types of testing procedures. Conducting statistical analysis, a new compressibility function, one of the forms of Weibull distribution, is proposed to fit the void ratio-effective stress relationship considering pre-consolidation behaviour. A fully implicit model was developed by introducing that proposed compressibility equation to predict the

tailings long term consolidation behaviour. A case study was performed for different types of tailings to predict the consolidation behaviour followed by the sensitivity analysis of the developed model. Significant effects of apparent consolidation have been observed on void ratio, effective stress, excess pore water pressure and tailings settlement for a period of 50 years. The major outcome of this study is the consideration of apparent over-consolidation behaviour during the early stage of the deposition helps to formulate the model more precisely.

PREFACE

The research presented in this thesis is an original and independent work by the author. It consists of three major parts- a review on tailings consolidation behaviour, theoretical development of the model and prediction for one dimensional consolidation behaviour of tailings using the developed model.

Chapter 2: A review on tailings consolidation behaviour is an extensive study from the literature in search of a research gap from the past works. This chapter has already been submitted to "International Journal of Geotechnical Engineering" as a review paper (Co-author: Dr. Sumi Siddiqua) and currently under review process (Manuscript ID #IGE10). However, Figure 2.5 is reprinted from Proskin et al. (2010) with permission from Elsevier Limited (License Number: 3172070158182) and Figure 2.6 is reprinted from Sridharan and Prakash (1997) with permission, from Geotechnical Testing Journal, Vol. 24, No. 1, copyright ASTM International, 100 Barr Harbor Drive, West Conshohocken, PA 19428.

Chapter 3: Theoretical development of the model is conducted based on the new approach and part of this chapter along with validation, verification and sensitivity of the model from Chapter 4 was published (Paper ID 376) in proceedings of Canadian Society for Civil Engineers Annual Conference on May 29 - June 01, 2013 with the co-authorship of Dr. Sumi Siddiqua.

Chapter 5: Prediction for one dimensional consolidation behaviour of tailings was made by the developed model. Part of this chapter is submitted to a reputed journal in the field of geotechnical engineering, "Engineering Geology", and currently under review.

List of Publications:

1. Ahmed, S. I., Siddiqua, S., 2013. Statistical Analysis and Prediction for Consolidation Behaviour of Tailings. *In* Proceedings of CSCE 2013 General Conference, Montreal, May 29-June 01, 2013, GC 376-(1-9)
2. Ahmed, S.I., Siddiqua, S., 2013. A Review on Tailings Consolidation Behaviour. International Journal of Geotechnical Engineering. (Manuscript #IGE10)
3. Ahmed, S.I., Siddiqua, S., 2013. A New Approach to Predict the Consolidation Behaviour of Tailings. Engineering Geology. (Manuscript # ENGEO5348)

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LIST OF SYMBOLS

a, b, A, B, C, D, E, F and *Z*	Coefficients for the Hydraulic Conductivity and Compressibility Functions
e	Void Ratio
e_{exp}	Void Ratio at First Experimental Data Point
G_s	Specific Gravity
i	Material Coordinate Index
j	Time Index
k	Hydraulic Conductivity of Soil
n	Porosity
q	Applied Stress
t	Time
u	Excess Pore Water Pressure
x	One Dimensional Vertical Coordinate
z	Reduced Material Coordinate
Δz	The Material Coordinate Increment
γ_w	Unit Weight of Water (9810 kN/m^3)
γ_s	Unit Weight of Soil
σ'	Effective Stress

ACKNOWLEDGEMENT

At first I would like to convey my gratefulness to the Almighty Allah for the successful completion of my research work.

I would like to express my gratitude to my supervisor Dr. Sumi Siddiqua for her continuous supervision, dedicated encouragement, and valuable suggestions in every step of my research work. It has been a great honour for me to work with her. I am also grateful to Dr. Abbas S. Milani and Dr. Kasun N. Hewage, the University of British Columbia, Canada for their valuable thoughts and suggestions during the annual progress review. I am thankful to Dr. Carolyn Labun and Mrs. Amy Modahl from Centre for Scholarly Communication (CSC) of the University of British Columbia for their time and care in one-on-one consultation during the revision of the thesis.

I would like to thank my colleagues for providing such a friendly, helpful, and excellent environment during my research work period, particularly, to Ehssan Hosseini Koupaie for sharing his knowledge to develop my skill in statistical analysis. I would like to specially thank Lucas Calabrez for helping me in extracting data for numerical modeling at the very critical stage of my thesis completion. I would like to extend my admiration to Bangladeshi-Kelowna community to provide me such a nice homely atmosphere.

I would like to express gratitude to my dearest parents and sister for their support and encouragement during my Master’s program. I am also thankful to all of my friends and well-wishers in Bangladesh and Canada for their endless assistance. Above all, I am greatly thankful to my dearest wife, Shumsun Nahar Siddique, for her love, motivation, and delicious dishes which is truly a strong source of inspiration!

DEDICATION

To my lovely parents,

IMTIAZ AHMED

&

U.S. MAHMUDA KHATUN

1 INTRODUCTION

1.1 Problem Statement

Tailings, which consist of slurries in the form of water, sand and fine particles, are the by-product of surface mining operations. There is also bitumen residual in tailings when it originates from oil sands (Wong et al. 2008). Each year due to the high volume of water in fine tailings and the slow process of consolidation, a massive amount of tailings is produced by at least 3500 active tailings storage facilities (Davies and Martin 2000). Management of the vast volume of tailings results in one of the most challenging issues of mining operations. For example, Jeeravipoolvarn (2005) reported that for the surface mining technique, 40% of solids content produces 3.3 m^3 of tailings streams whereas 60% of solids content produces 1.9 m^3 during the extraction of the unit volume of in-place oil sands. However, the rate of settlement decreases with the increment of solids content since all other properties remain the same. Moreover, the new regulation of Energy Resources Conservation Board (ERCB) stated that from 2010 onwards, companies must implement plans to reduce growth in fluid tailings significantly by consolidating fluid tailings and forming deposits of consolidated tailings that are ready for land reclamation. Therefore, consolidation behaviour of tailings plays a significant role for proper management of tailings impoundments. The effective and economical disposal of waste requires the knowledge of basic physical properties as well as consolidation and desiccation behaviours of the tailings (Qiu and Sego 1998).

1.2 Research Background

In geotechnical engineering, consolidation may be defined as a process by which soils decrease in volume (i.e. change in void ratio, solids content, etc.) through dissipating excess pore water pressure and subsequently gaining effective stress. The action of consolidation is known as compressibility. The changes in these geotechnical properties of soil due to consolidation are referred to as the consolidation behaviour where hydraulic conductivity dominates the rate of consolidation over compressibility (Alpan 1970). However, this behaviour largely varies from tailings to tailings depending on their types, solids contents, extraction process, water quality, addition of coagulants and other physical and chemical properties. As a result, it is hard to predict consolidation behaviour of tailings with a single model. In the past several decades, research has been focused to understand these behaviours by both field and laboratory investigations followed by numerical simulations to assess the feasibility of long term disposal strategies (Consortium 1995).

Consolidation behaviour of tailings mainly depends on two constitutive relationships: compressibility and hydraulic conductivity of soil. It is difficult to obtain the exact hydraulic conductivity measurement during laboratory experiment due to several unavoidable factors such as undissolved air in the equipment and/or specimen, compliance in the equipment and time-dependent changes in the volume or distribution of pore pressure in the specimen (Olsen et al. 1985). Hence, power function is adapted widely for the void ratio-hydraulic conductivity relationship. On the other hand, the compressibility relationship varies with tailings types, amount of solids content, extraction process, deposition process and for other reasons too. Several researchers used power function, Weibull function, and logarithmic function to fit the compressibility curve accurately. Recent observation indicates most tailings show apparent pre-consolidation behaviour which is believed to be the gaining of thixotropic strength due to the consequence of the combination of several contributing factors during and after deposition. This over-consolidation behaviour leads the study towards developing a new compressibility function that would be applicable to any type of tailings by means of improved adjusted coefficient of determination as an indicator of goodness of fit.

Figure 1.1 illustrates a typical soil consolidation curve where effective stress lies in the horizontal axis in log scale and void ratio lies in the vertical axis in normal scale. In general, soils show two types of behaviour depending on their previous consolidation state. If soil experiences higher stress compared to its present state, that soil is known as over-consolidated soil. Conversely, if the highest consolidation occurs due to present effective stress, then it is called normally consolidated soil. Over-consolidated soil usually follows an S-curve while normally consolidated soil does not have any plot beyond to the left of pre-consolidation line. Here the pre-consolidation line is obtained by drawing a vertical line from the intersection of the extended virgin compression line and the bisector of the horizontal line and tangent at maximum curvature point.

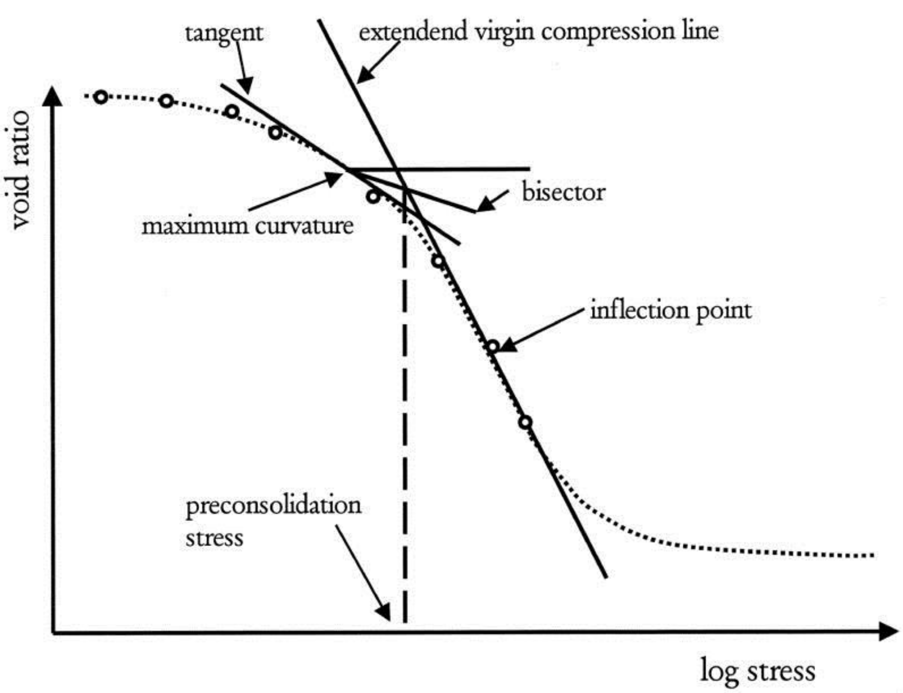

Figure 1.1: Description of soil consolidation

Up to the present time, it is assumed that tailings are normally consolidated or consolidating under the load of mounting deposited materials and numerical modeling has been performed by different researchers based on this assumption. However, the apparent over-compressibility or pre-compressibility behaviour of tailings has been noticed for different types of tailings at a wide range of solids content and by various types of testing procedures as well. The apparent over-consolidation behaviour of tailings is still an unknown phenomenon and the reasons for the occurrence are not fully understood yet (Friedel and Murray 2010). Miller et al (2010b) hypothesized that the gaining of thixotropic strength is responsible for this over-consolidation behaviour. However, it is believed to be the result of the combination of several contributing factors at low effective stresses. Mitchell and Soga (2005) reported that in both laboratory and field investigations, fine-grained soils had shown a tendency to increase strength and stiffness with time that may contribute to some over-consolidation.

A one dimensional consolidation model is suitable for containment ponds having small depth compared to the width and length since the direction of fluid flow and the settlement are primarily vertical. Therefore, the assumptions of one dimensional consolidation remain valid (Jeeravipoolvarn et al. 2008). However, two and three dimensional modeling is rarely attempted, not only for inadequate constitutive relationships, time and numerical difficulties, but also due to the fact that a one dimensional model satisfies most requirements (Ding et al. 2010). Additionally, Bromwell (1984) confirmed that two dimensional effects became significant when the width to height ratio of the impoundment is on the order of five or less (associating with side drainages), whereas most tailings storage facilities have much higher ratios.

1.3 Research Objectives

The main objective of this study was to predict the tailings settlement behaviour along with the effect on geotechnical properties of the tailings after the completion of the settling period by a consolidation model. The specific objectives were as follows:

- To perform statistical analysis of large strain consolidation test on available data from literature in search of factors that affect the response.
- To propose a universal compressibility equation based on the statistical analysis that will be applicable for all types of tailings.
- To develop a one dimensional consolidation model using the proposed compressibility equation to determine the tailings consolidation behaviour:
    - Model validation
    - Model verification
    - Sensitivity analysis of the model.
- To draw comparisons between the proposed model and the conventional model in terms tailings consolidation behaviour.

1.4 Thesis Outline

This introductory chapter is followed by Chapter 2 that delivers a review on tailings consolidation behaviour by familiarizing the one, two and three dimensional consolidation theories proposed by various researchers for both ordinary soils and soft soils like tailings. Next, the consolidation models developed based on those theories are described. Thereafter, the fundamental properties of the theory- volume compressibility and hydraulic conductivity- and also the laboratory investigations to obtain these parameters are described. Finally, the consolidation behaviour of tailings is presented in terms of the factors those affect tailings consolidation properties.

Chapter 3 represents the theoretical development of the model for this study. The mathematical background and constitutive relationships are stated followed by numerical solution. Prior to that, statistical analysis has been performed by significance test and regression analysis to develop a new compressibility equation that would be best fitted with laboratory investigated data. Finally, the computational scheme for numerical modeling has been illustrated to show the steps of the solution.

Chapter 4 deals with the validation, verification and the sensitivity of the model. This chapter begins with the validation and verification of the developed model. Then it explains the modeling results for sensitivity analysis of simulation, material and curve fitted parameters. Finally, the comparison of the proposed model with the conventional model in terms of long term consolidation behaviour is described.

Chapter 5 describes the prediction of consolidation behaviour obtained from present research and described graphically for a case study. At first, the material properties are listed and the statistical analyses of the constitutive relationships are presented. Then the validations of the model with the experimental results are illustrated. Finally, the results and discussion of the case study are described for 50 years of settling time.

Chapter 6 presents the conclusion and recommendation for future work on tailings consolidation behaviour. The conclusions have been drawn based on the results obtained from this research and future recommendations have been made to overcome the limitations of this model. These are followed by a bibliography and appendices.

2 A REVIEW ON TAILINGS CONSOLIDATION BEHAVIOUR

2.1 General

This chapter reviews the consolidation process and numerical models for slurries proposed by various researchers over time. Next, it presents the constitutive relationships of functional compressibility and hydraulic conductivity, the key components for any type of tailings. Laboratory test programs to develop theoretical relationships which significantly contribute to the history of mining industry are described. Because, the successful prediction of tailings impoundment capacity by numerical modeling primarily depends on the accuracy of large strain consolidation tests with hydraulic conductivity measurements. Finally, this review describes the ample works on consolidation modeling and behaviour for different types of tailings from scholars across the globe.

2.2 Consolidation Theory

Consolidation is a process by which soil gains effective stress through a dissipation of excess pore water pressure and decreases in volume. However, sedimentation is the prior stage of the settlement of soil where effective stress does not exist. These two phenomena are the fundamentals for proper understanding of the sedimentation and consolidation processes after the flocculation process in the containment. In fact, the void ratio, due to that effective stress, is controlled by the initial void ratio of the tailings (Bartholomeeusen 2003, Been 1980, Imai 1981, Sills 1998).

Figure 2.1 shows the settling of slurry versus elapsed time plot that is modified from Imai (1981). This plot contains three different zone or three different stages of tailings settlement. Starting from the deposition process, flocculation is the first stage when tailings remain in suspension with water and no effective stress exists during this time. At the end of flocculation, sedimentation and consolidation process starts simultaneously though consolidation of soil i.e. gaining of effective stress starts just after the sedimentation process by forming a solid-liquid interface and ends with complete soil formation line.

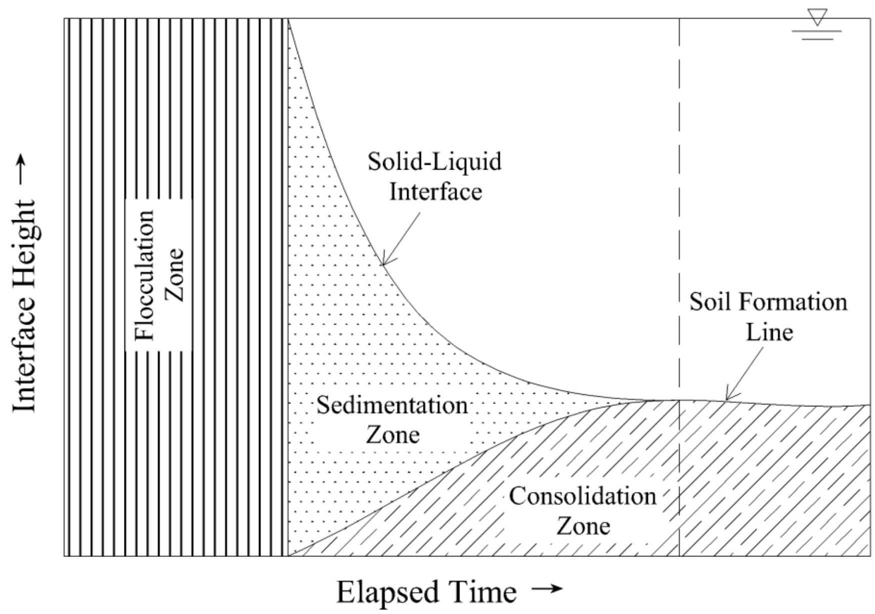

Figure 2.1: Settling of slurry versus elapsed time (modified from Imai, 1981)

Many researchers have studied and explained the sedimentation process (Coe and Clevenger 1916, Fitch 1966, Kynch 1952, Tan et al. 1988) and also have applied consolidation theory to soil sedimentation (Been and Sills 1981, McRoberts and Nixon 1976). Been (1980) found that slowed sedimentation could be derived from consolidation theory by setting the effective stress to zero. Later, Schiffman (1982) stated that self-weight is a key component for consolidation while Mikasa and Takada (1984) demonstrated that the process commenced after sedimentation. In general, large strain consolidation is associated with the process of sedimentation, when it is subjected to deposit below water (Koppula and Morgenstern 1982). However, the sedimentation is rapid due to sub-aerial deposition and not taken into account explicitly in the model (Seneviratne et al. 1996). Therefore, a selection of the end of sedimentation and the starting of consolidation is usually arbitrarily chosen.

2.2.1 One-Dimensional Consolidation Theory

The classical one dimensional small-strain consolidation theory was first defined by Karl Terzaghi in 1927 to calculate the settlement of soils in the field of geotechnical engineering. Combining fluid flow relationship, continuity equation and the principle of effective stress, Terzaghi's one dimensional consolidation theory was mathematically expressed as:

$$c_v \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad [2.1]$$

where, c_v is the coefficient of consolidation, u is the excess pore pressure, t is the time and x is a one dimensional vertical coordinate.

The theory was based on the assumptions of incompressible soil properties i.e. small strain, constant hydraulic conductivity and negligible self-weight (Terzaghi 1943) which are not applicable for soft materials like tailings. The compressibility and hydraulic conductivity of tailings are highly non-linear. As a result, significant changes occur in settlement when it is subjected to a stress increment by continuous deposition and cannot be considered as a small strain problem. Later, it was found that incompressible soil properties are inappropriate (Davis and Raymond 1965, Liu and Znidarčić 1991) and hydraulic conductivity has significant effects on changes to the void ratio. In addition, self-weight is an important factor to distinguish between sedimentation and consolidation phenomena of soft soils (Schiffman 1982). However, Terzaghi acknowledged the limitations of his proposed relationship when the secondary (i.e. time dependent) consolidation exceeds 20% of total settlement (Znidarčić et al. 1984).

In the context of large strain consolidation, a number of researchers developed theoretical relationships where changes of compressibility and hydraulic conductivity were considered as non-linear (Bardon and Berry 1965, Bromwell and Carrier III 1979, Davis and Raymond 1965, DeSimone and Viggiani 1976, Gibson et al. 1967, Gibson et al. 1981, Lee and Sills 1979, McNabb 1960, Mikasa 1965, Pane and Schiffman 1985, Schiffman 1958, Znidarčić and Schiffman 1981). Among them Gibson, England and Hussey in 1967 introduced the robust one dimensional finite strain consolidation theory with additional theoretical formulations. The changes in self-weight were also incorporated in this theory. The second order partial differential equation was first proposed by Gibson et al. (1967) in terms of void ratio and hydraulic conductivity stated as follows:

$$\pm\left(\frac{\gamma_s}{\gamma_w}-1\right)\cdot\frac{d}{de}\left[\frac{k(e)}{(1+e)}\right]\frac{\partial e}{\partial z}+\frac{\partial}{\partial z}\left[\frac{k(e)}{\gamma_w(1+e)}\cdot\frac{d\sigma'}{de}\cdot\frac{\partial e}{\partial z}\right]+\frac{\partial e}{\partial t}=0 \qquad [2.2]$$

where, γ_s is the unit weight of soil, γ_w is unit weight of water, e is void ratio, k is hydraulic conductivity, σ' is vertical effective stress, t is time and z is the reduced material coordinate.

The reduced material coordinate system eliminated one of the major problems, the use of Eulerian method where the rates of flux and soil movement are measured with respect to the fixed plane of reference (Gibson et al. 1981). The reduced material coordinates system (McNabb 1960), relates to the Lagrangian coordinate system (Schiffman et al. 1988), and was utilized to overcome the fixed reference system. However, the fixed reference system was incompatible to define the consolidation behaviour for large displacement on the top boundary of a consolidating layer in the finite strain theory (Jeeravipoolvarn 2010).

Lee (1979) derived the governing equation [2.3] of finite strain consolidation in terms of porosity, using the convective coordinate, but it became more complicated to program due to changes of settlement while consolidating.

$$-\frac{\partial}{\partial x}\left[\frac{k(1+e)}{\gamma_w}\frac{d\sigma'}{de}\frac{\partial n}{\partial x}\right]-\left\{(G_s-1)\cdot\frac{d\left[k(1-n)^2\right]}{dn}-\frac{\partial q}{\partial n}\cdot\frac{d}{dn}\left[\frac{k(1-n)}{\gamma_w}\right]\right\}\frac{\partial n}{\partial x}+\frac{k}{\gamma_w}\cdot\frac{\partial^2 q}{\left(\partial x^2\right)(1-n)}=\frac{\partial n}{\partial t}$$

[2.3]

where, n is the porosity, q is the applied stress and x is the one dimensional vertical coordinate.

In 1980, Somogyi reformulated the equation by using Koppula's (1970) rearrangement of continuity and fluid flow relationships to present it in terms of excess pore pressure, u. Later, Koppula and Morgenstern (1982) used the same approach to resolve problems where sedimentation had occurred.

$$\frac{\partial}{\partial z}\left[\frac{k(e)}{\gamma_w(1+e)}\right]\frac{\partial u}{\partial z}+\frac{k(e)}{\gamma_w(1+e)}\frac{\partial^2 u}{\partial z^2}+\frac{de}{d\sigma'}\cdot\frac{\partial u}{\partial t}-\frac{de}{d\sigma'}\left[(G_s-1)\cdot\gamma_w\,\frac{d(\Delta z)}{dt}\right]=0 \qquad [2.4]$$

where, G_s is the specific gravity.

Though several researchers updated and supported Gibson’s theory based on void ratio (Feldkamp 1989, Monte and Krizek 1976, Schiffman 1980, Schiffman and Cargill 1981), excess pore water pressure (Koppula and Morgenstern 1982, Krizek and Somogyi 1984, Somogyi 1980) and porosity (Lee 1979), they all acknowledged the usefulness of this theory to describe the consolidation characteristics of tailings more realistic manner (Caldwell et al. 1984, Scully et al. 1984, Townsend and McVay 1990). In 1984, Bromwell reported that the finite strain consolidation theory estimated smaller influence of height on time of consolidation rather than the conventional one; the theory also predicted slower dissipation of excess pore water pressure (Schiffman et al. 1984).

2.2.2 Two-Dimensional Consolidation Theory

The two-dimensional consolidation theory with sand drains was proposed by Carillo (1942) and Barron (1948). After a few decades, Somogyi et al. (1984) derived a quasi-two dimensional finite strain consolidation model parallel to the one dimensional derivation presented by Koppula (1970) providing an accurate estimation of the full-scale behaviour. Huerta and Rodriguez (1992) also presented a pseudo two dimensional extension of the one dimensional finite strain consolidation theory using the extended model to simulate the influence of the vertical drains. Bürger et al. (2004) described a two dimensional analysis of sedimentation and consolidation in various shapes of a thickener, primarily used for dewatering of slurries, assuming the volumetric solids concentration is constant across each horizontal cross section.

2.2.3 Three-Dimensional Consolidation Theory

In 1955, Biot proposed a three dimensional consolidation theory which was not used in the model because of numerical difficulties. Gjerapic et al. (2008) explained that "[t]he three dimensional models are rarely attempted due to lack of an appropriate constitutive relationship, excessive computational time requirements and numerical difficulties associated with nonlinearity of governing equations and material properties." Jeeravipoolvarn et al. (2008) used the one dimensional derivation, proposed by Schiffman (2001), to develop a quasi-multi-dimensional finite strain consolidation behaviour. The equations were numerically implemented in both two and three dimensional problems. Fredlund et al. also proposed a multi-dimensional consolidation theory in 2009.

2.3 Consolidation Model

Based on the one, two and three dimensional consolidation theories stated above, a number of numerical models have been developed worldwide. Some renowned models were FSCON 1-I and FSCON 2-I (Cargill and Schiffman 1980), ACCUMV (Schiffman et al. 1992), NFSCONSOL (Wu 1994), 1D CONDESO (Yao and Znidarcic 1997), CONSOL 2D (Jakubick et al. 2003), CC1 (Fox et al. 2005), 3D CONDESO (Coffin 2010). Besides these, Buscall and White (1987) explained settling behaviour in the context of fluid dynamics, including the consolidation process, and developed a mathematical formulation. In the same year (1987) Lewis and Schrefler applied finite element method to consolidation problems for further investigation. Recently Bo et al. (2011) proposed a detailed finite difference model to predict the rate of settlement in both low and high effective stress ranges for very soft soils with the help of time factor curves.

2.4 Constitutive Relationships

The governing equation of finite strain consolidation was solved using two important nonlinear relationships: a void ratio-effective stress relationship known as compressibility and a hydraulic conductivity-void ratio relationship. These relationships, being a function of void ratio, were obtained from experimental data while numerical methods were used to determine the finite strain solution (Bartholomeeusen et al. 2002, Cargill 1982, Carrier III et al. 1983, Somogyi 1980).

Hydraulic conductivity is usually expressed by power function of the void ratio as shown in Equation [2.5] (Jeeravipoolvarn et al. 2008, Somogyi 1980, Townsend and McVay 1990, Yao and Znidarčić 1997). Another empirical equation was used by Carrier et al. (1983) to determine the hydraulic conductivity from void ratio for mineral waste as presented in Equation [2.6]. In addition, logarithmic function was developed by Bartholomeeusen et al. (2002) and is shown in Equation [2.7]

$$k = Ce^{D} \qquad [2.5]$$

$$k = \frac{Ee^{F}}{(1+e)} \qquad [2.6]$$

$$e = C\,ln\,(k) + D \qquad [2.7]$$

where, C, D, E and F are the curve fitted parameters and will be unique to each type of soil.

The relationship between void ratio and effective stress is crucial and proposed by researchers with different equations including power function [Equation 2.8] (Somogyi 1980, Townsend and McVay 1990), extended power function [Equation 2.9] (Liu and Znidarčić 1991), logarithmic function [Equation 2.10] (Bartholomeeusen et al. 2002) and Weibull function [Equation 2.11] (Jeeravipoolvarn et al. 2008). Weibull function is particularly applicable to oil sands fine tailings due to the presence of over-consolidation pressure (Jeeravipoolvarn 2005, Suthaker and Scott 1994).

$$e = A\sigma'^{B} \qquad [2.8]$$

$$e = A(\sigma' + B)^{C} \qquad [2.9]$$

$$e = A\,ln\left(\sigma'\right) + B \qquad [2.10]$$

$$e = A - B \cdot exp\left(-E\sigma'^{F}\right) \qquad [2.11]$$

where, A, B, C, E and F are curve fitted parameters.

Empirical relationships are also used to obtain compressibility and hydraulic conductivity from Atterberg limits, specifically by correlating data between liquidity index and effective stresses (Carrier III et al. 1983, Skempton and Jones 1944). Carrier and Beckman (1984) described compressibility and hydraulic conductivity for deposited slurries with respect to their liquid limit and plastic limit. Equations developed from the correlation, were combined with compressibility to obtain the finite strain consolidation parameters. The empirical correlations, proposed by Carrier et al. (1983), were only for the preliminary design of tailings impoundments and to estimate the time of consolidation by finite strain consolidation theory. Nagaraj et al. (1994) supported the previous findings by pointing out an important characteristic that despite the types of soil, it seemed to possess a relatively constant set of engineering properties at the liquid limit. This led to a reliable estimation of the consolidation behaviour by means of correlation, based on index properties of the tailings (Morris et al. 2000). Later, Morris and Lockington (2002) recommended the use of empirical correlations for constitutive relationships.

In most of the cases compressibility and hydraulic conductivity parameters were obtained from various consolidation tests conducted at different eras of tailings consolidation studies all over the world. A series of data are illustrated in Figure 2.2, Figure 2.3 and Figure 2.4, presenting effective stress-void ratio and void ratio-hydraulic conductivity relationships for different types of tailings at a wide range of solids content and various methods of laboratory investigation.

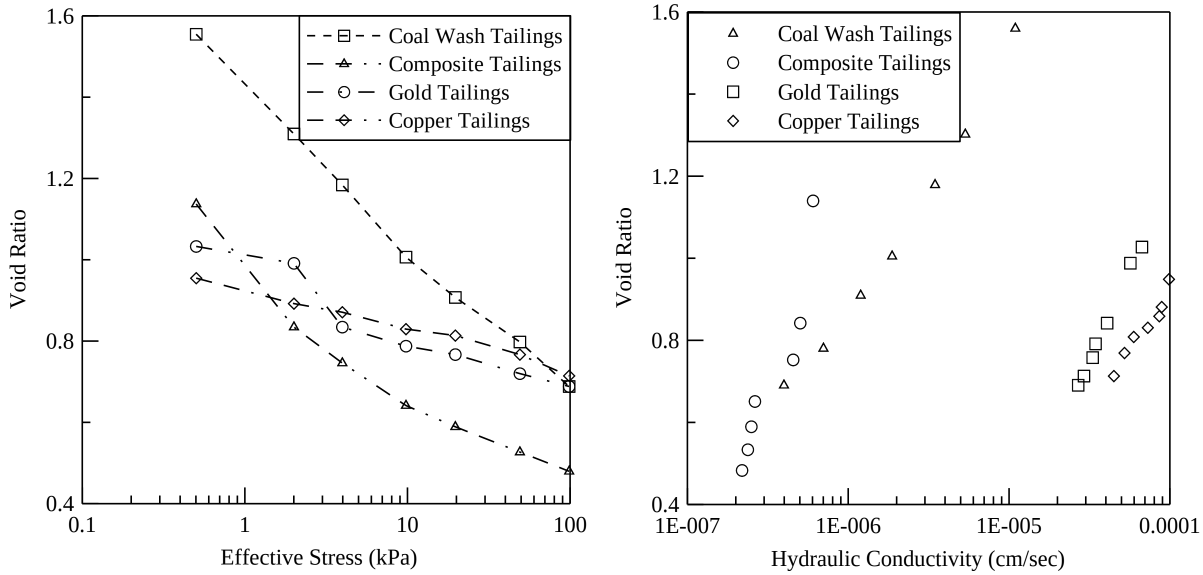

Figure 2.2: Compressibility and hydraulic conductivity of different types of tailings (modified from Qiu and Sego, 2001)

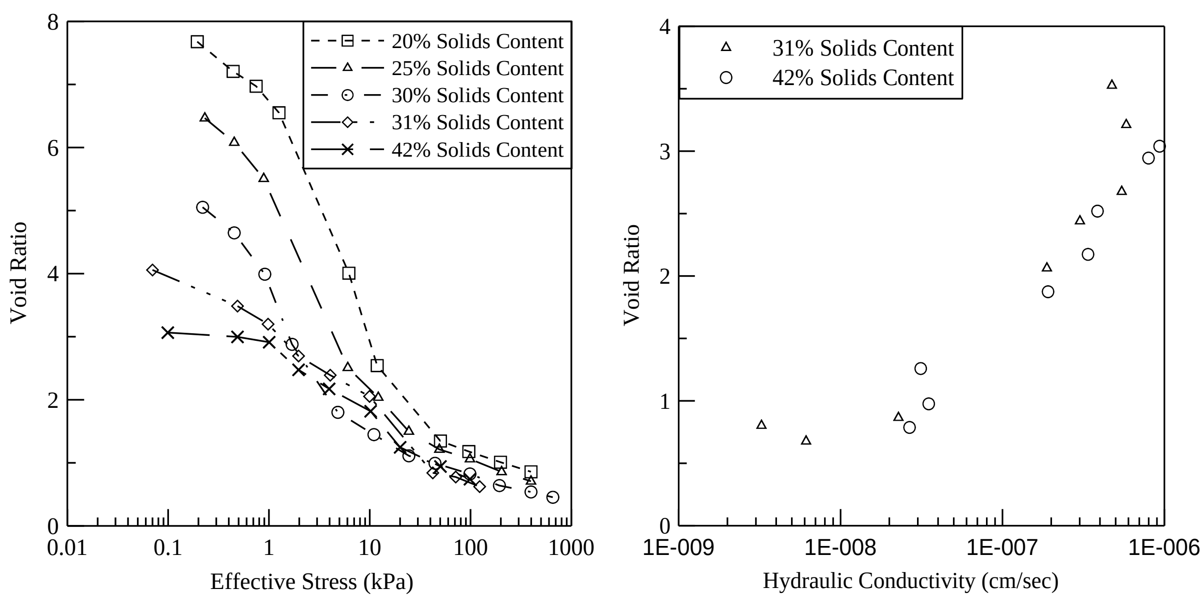

Figure 2.3: Compressibility and hydraulic conductivity of tailings at different solids content (modified from Proskin et al., 2010)

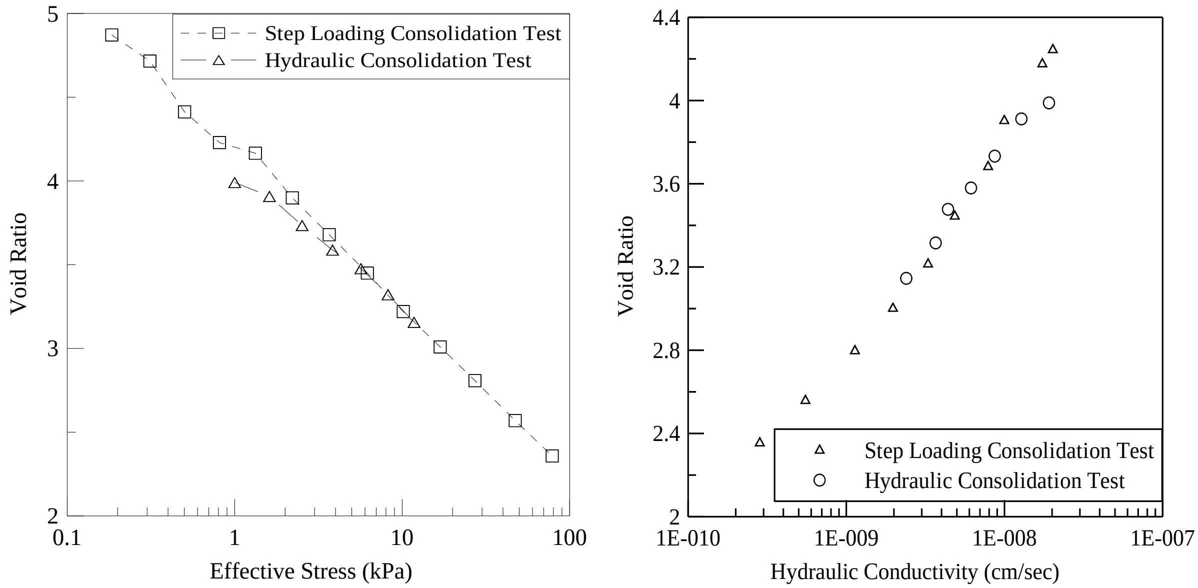

Figure 2.4: Compressibility and hydraulic conductivity of tailings by different testing procedure (modified from Fox and Baxter, 1997)

Hence, understanding of the consolidation behaviour primarily depends on the quality of the test performed by using similar types of tailings in the impoundment. The importance of laboratory investigations were highlighted by Jeeravipoolvarn (2010) and stated that, "[r]egardless of all the mathematical formulae available and empirical parameters in the literatures, ideally, constitutive relationships for detailed consolidation analysis of tailings should be determined from direct measurements and the mathematical form to be used should be decided by the experimental data to cover the range of void ratio that the material would experience in the field."

2.5 Consolidation Tests

The governing equation represents the consolidation theory to understand the consolidation process in a tailings management facility. It contains the hydraulic conductivity and compressibility of tailings, two fundamental properties of void ratio function (Znidarčić et al. 1984). Different types of consolidation tests, associated with hydraulic conductivity measurement, have been practiced to determine these properties and are listed below:

- Step loading test
- Constant rate of deformation test
- Controlled gradient test or constant hydraulic gradient test
- Constant rate of loading test
- Continuous loading test
- Seepage induced test
- Centrifuge test
- Relaxation test

Znidarčić et al. (1984), Pollock (1988) and Suthaker (1995) reviewed the consolidation test methods for determining consolidation properties of slurries by alternative mathematical approaches. Based on their reviews, the different types of large strain consolidation test have been summarized in the following paragraphs.

The standard oedometer test was first proposed by Terzaghi (1927) to determine one-dimensional consolidation properties of soils, where incremental loads were applied on the specimen and vertical deformations were monitored. The ASTM Standard oedometer test (D 2435) was not directly applicable for slurry type soils because of its high void ratio, low hydraulic conductivity and high compressibility during self-consolidation stages (Proskin et al. 2010). However, simple modification allowed it to follow the conventional large strain consolidation test (Monte and Krizek 1976, Sheeran and Krizek 1971).

A schematic diagram of the slurry consolidometer is illustrated in Figure 2.5. Though the vertical effective stress of about 1 kPa were possible with a conventional odometer apparatus, in the modified version, a low effective stress, for example 0.01 kPa, was desirable for mine tailings (Cargill 1984, de Ambrosis and Seddon 1986, Morris et al. 2000, Roma 1976). However, low effective stresses extend test duration and seepage induces consolidation during the hydraulic conductivity measurement for very soft soils. For these reasons, other testing techniques had been developed to determine consolidation parameters of the tailings in spite of their limitations.

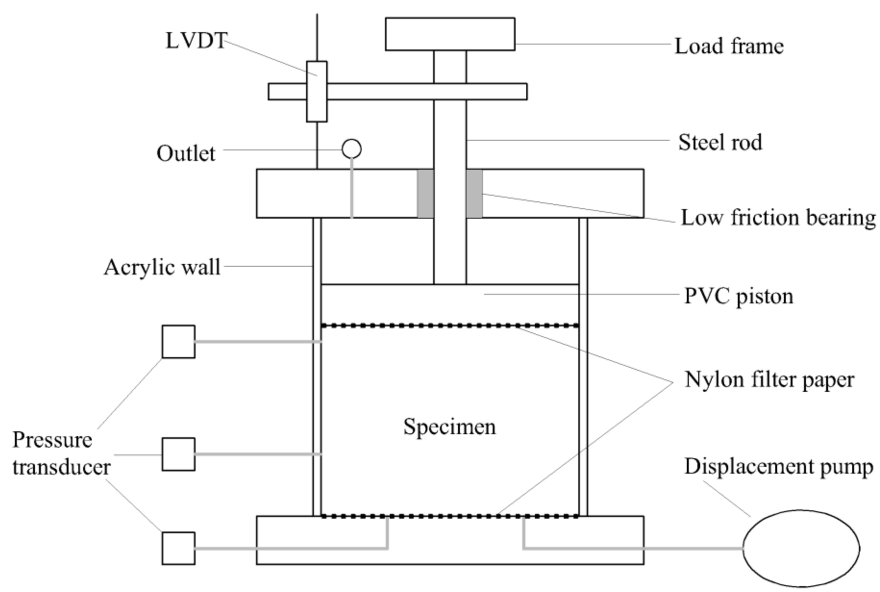

Figure 2.5: Schematic diagram of a slurry consolidometer (modified from Proskin et al., 2010)

In 1959, Hamilton and Crawford introduced the constant rate of deformation (CRD) test based on the assumptions of infinitesimal strain, constant permeability, linear compressibility relationship and constant void ratio (Znidarčić et al. 1984). This test overcame the drawbacks of step loading test by shortening the duration of the process and analyzing material under deformation rates that more closely estimated field deformation rates (Crawford 1964). Though in the experimental part, the authors were unable to measure the pore water pressure as well as effective stress, and they suggested for slower rate of deformation to determine these values by preventing the development of significant pore-water pressures. Later, Smith and Wahls (1969), Wissa et al. (1971), Umehara and Zen (1980), Lee (1981), Znidarčić et al. (1986) modified and updated the constant rate of deformation test for consolidation of tailings.

The controlled gradient test was proposed by Lowe et al. (1969) where the loading rate of the specimen was adjusted for the pore pressure to remain constant at undrained boundaries. This required a feedback mechanism and complicated the arrangement of necessary laboratory equipment. Meanwhile, Aboshi et al. (1979) suggested the constant rate of loading test to determine consolidation parameters of soil based on the theoretical work by Schiffman (1958), assuming the constant value of hydraulic conductivity and coefficient of consolidation. In 1981, Janbu et al. introduced some modifications on constant rate of deformation test, controlled gradient test and constant rate of loading test, by applying continuous loading on the specimen. The ratio of the applied load to excess pore-water pressure at the undrained boundary was kept constant and represented it as continuous loading test.

Usually laboratory tests were strain controlled to reduce the duration of the test. In addition, consolidation-void ratio relationships from the constant rate of strain test were higher than those from stress controlled tests (Cargill 1986, Gan et al. 2011). The described alternative methods of ASTM standard oedometer test were carried out by an inversion of Terzaghi's consolidation theory for the derivation of consolidation parameters from a set of test data and also restraining their application to finite strain materials (Proskin et al. 2010).

The basic idea of seepage technique, proposed by Imai (1979), was to use the effective stress difference along the specimen, caused by seepage force to induce the consolidation process. This technique has a drawback which is the sample rebound problem at the end of each test (Znidarčić et al. 1984). This is the only alternative technique which is independent of Terzaghi's theory for interpretation of test data despite the fact that careful sample handling is the key to minimize disturbances prior to void ratio determination (Proskin et al. 2010). Later, Fox and Baxter (1997) amended the procedure using some assumptions and developed a modified hydraulic consolidation test. The required time was 75% less than the conventional step loading test within the same effective stress range. A simplified seepage consolidation test was also proposed by Sridharan and Prakash (1999) for soft soils. The schematic of the experimental setup for simplified seepage consolidation test is shown in Figure 2.6.

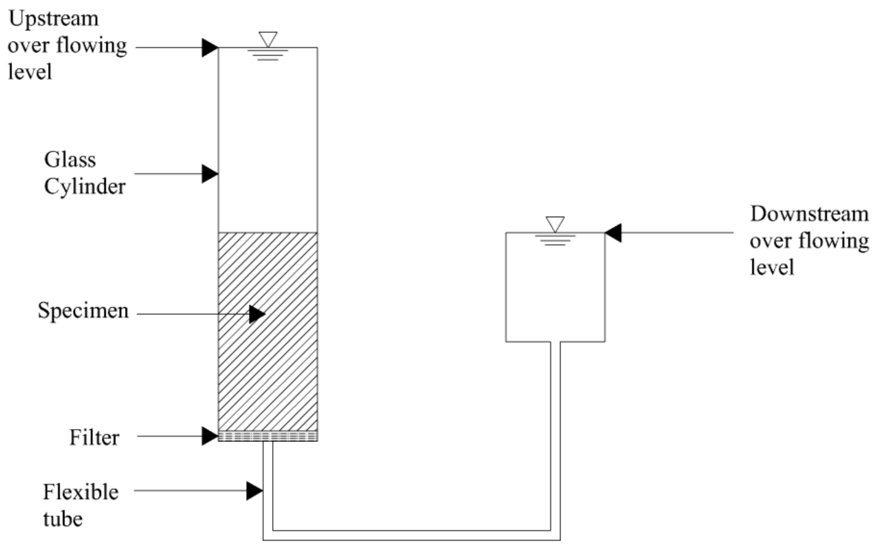

Figure 2.6: Schematic of the experimental setup for simplified seepage consolidation test (Reprinted, with permission, from Geotechnical Testing Journal, Vol. 24, No. 1, copyright ASTM International, 100 Barr Harbor Drive, West Conshohocken, PA 19428)

Hydraulic conductivity measurement is a crucial part of the large strain consolidation test. The most common method of the indirect determination of hydraulic conductivity was conducted by inverting Terzaghi's theory (Suthaker and Scott 1996). However, the method underestimated the measured values up to six times and the results were unacceptable particularly for highly compressible natural clays (Tavenas et al. 1983). The most reliable technique involved direct measurements through allowing a permanent flow within a specimen while monitoring the rate of flow or the hydraulic head changes induced.

2.6 Consolidation Behaviour

The purpose of the numerical modeling was to determine the representative material properties for a given type of tailings and to predict the degree of consolidation with respect to time. Usually two different methods have been practiced: small strain and finite strain consolidation method. Small strain method yields linear models such as those proposed by

Olson and Ladd (1979), Yong et al. (1983). In contrast, the models developed by Somogyi (1980), Gibson et al. (1981), Cargill (1982), Carrier et al. (1983), Schiffman et al. (1984) and Feldkamp (1989) were based on the finite strain consolidation method presented by Gibson et al. (1967). The models show non-linear behaviour and applicable to tailings that experience a large deformation during the initial stage of the consolidation at low effective stress. The introduction of finite strain consolidation approach reduces the computational complexity significantly (Krizek and Somogyi 1984).

Numerical solution of the finite strain theory could be done by the finite difference method with either explicit or implicit techniques. An explicit scheme has the advantage of known value of the dependent variable and corresponding material properties. On the other hand, an implicit method is computationally more complete and stable (Krizek and Somogyi 1984). Bromwell (1984) reported the comparison of two finite strain consolidation computer programs for field interface settlement, in explicit form (Schiffman 1958) and in implicit form (Somogyi 1980). Parameter lagging, in other words the stability of the implicit finite difference method, caused the error known as approximation of decay (Somogyi 1980). This resulted inconsistency of Somogyi's model directly from the assigned implicit finite difference scheme (Jeeravipoolvarn 2010) that yielded to a poor prediction of the implicit program compared to the explicit one. The theory and model had been advanced further after the critical statement of Krizek and Somogyi (1984) that "[e]ven the correct prediction does not necessarily validate the model or its input, because the complexity of the many factors involved allows the right overall behaviour to be obtained by using a fortuitous combination of erroneous material properties, boundary conditions, and mathematical formulation." The discrepancies between prediction and observation were stated for two practical approaches either by tuning the model or seeking improved material property relationships from sampling a program when the measured and calculated values did not agree.

Consolidation behaviour of tailings is affected by several factors which were reviewed by Suthaker (1995) and Jeeravipoolvarn (2005). Among them, compressibility is the most studied topic for its non-unique behaviour. It is not unique and depends on the initial void ratio as well as indicates over-consolidation behaviour due to thixotropic gain in strength from bonding. The gain of strength is a form of structural change between soil particles (Been and Sills 1981, Imai 1981; Jeeravipoolvarn 2010, Scully et al. 1984). The apparent pre-consolidation behaviour exits at a low effective stress of around 1 kPa but at relatively higher effective stress for example 10 kPa, all the compressibility characteristics start to converge (Bo et al. 2003, Jeeravipoolvarn et al. 2009, Qiu and Sego 2001, Scully et al. 1984, Sridharan and Prakash 2001, Suthaker 1995). The inclusion of time to the constitutive relationship in a form of void ratio, effective stress and rate of compression is the way to capture the non-unique compressibility behaviour in a numerical model (Bartholomeeusen 2003, Bjerrum 1967, Imai and Hawlader 1997, Kim and Leroueil 2001, Leroueil et al. 1985).

Hydraulic conductivity-void ratio relationship is another important constitutive relationship involved in the finite strain consolidation model for predictions. McVay et al. (1986) reported that at higher void ratios, the values of hydraulic conductivity influenced by the hydraulic gradient were not constant but decreased with time to a steady-state value (Suthaker and Scott 1996). Suthaker (1995) performed the hydraulic conductivity measurements and reported that lower hydraulic gradient would result in higher hydraulic conductivity. It contradicted the earlier findings described by Elnaggar et al. (1973) and

Scully et al. (1984) which might be due to non-Darcy flow behaviour. Jeeravipoolvarn (2010) suggested further research to understand the flow behaviour of fluid in tailings.

Thixotropy is an isothermal and reversible process of material. It is time dependent while stiffening at rest and softening or liquefying by remolding under constant composition and volume (Mitchell 2005). This is another factor that affects the consolidation and mainly depends on the size of the residual energy barrier (Van Olphen 1977). The apparent over-consolidation behaviour of tailings which has an adverse effect on dewatering (Somogyi and Gray 1977) was believed to be caused by thixotropy (Jeeravipoolvarn 2010). This phenomenon increases the shear strength (Banas 1991) as well as the true effective stress (Scott et al. 2004).

The tailings consolidation behaviour significantly varies due to the geological characteristics of the materials and also variations in the mineral extraction and disposal process (Morris et al. 2000). The hydraulic conductivity and the associated rate of consolidation are linearly proportional to the void ratio and decrease quickly from the initial void ratio in a small period of time. Thus it has little effect on the calculated storage capacity of a disposal facility. Suthaker and Scott (1994) reported that in terms of predicting the consolidation behaviour, the compressibility of tailings has less effect than the hydraulic conductivity. However, the measurement of hydraulic conductivity is more challenging and highly sensitive to changes in void ratio (Schiffman 1982). Therefore, the development of an improved void ratio-hydraulic conductivity relationship, to obtain precise curve fitting parameters, is paramount for a better prediction of tailings in-place consolidation behaviour.

2.7 Summary

The accurate prediction of consolidation behaviour is the key to proper controlling of the tailings management facility. Numerical modeling is the best way to estimate the condition of the impoundment area. Moreover, from the environmental point of view, the prevention of the long term seepage of toxic tailings and the reclamation of the tailings pond for revegetation, depends on understanding and improving the consolidation behaviour of slurries. This behaviour is primarily affected by the properties of tailings itself (Scott and Dusseault 1980). However, an accurate specification of material properties is the greatest obstacle for finite strain consolidation theory (Feldkamp 1989, Krizek and Somogyi 1984, Toorman 1996). This might lead to an erroneous result compared to the field observation. Despite that, quality laboratory work with precise determination of parameters for consolidation modeling, obtained from the compressibility and the hydraulic conductivity behaviour, are crucial for tailings performance studies. Future research on analysis of the combination of flocculation, sedimentation, consolidation, segregation, deposition, freeze-thaw and desiccation are required to assess the overall performance of tailings. An advanced numerical model incorporating all the essential factors will lead to a better prediction of the tailings consolidation.

3 THEORETICAL DEVELOPMENT OF THE MODEL

3.1 General

This chapter presents the research methodology in terms of theoretical development of the model carried out for the present study. The mathematical background of the large strain consolidation model is reformulated according to the proposed compressibility equation which is obtained from statistical analysis of the published data at first by the significance test followed by regression analysis. Then, a numerical solution of that governing equation of second order partial differential is derived. Finally, this chapter illustrates a computational scheme for the numerical modeling of the developed model.

3.2 Quiescent Condition

The model was developed for settlement and geotechnical properties calculation at quiescent condition. Quiescent condition is a condition where soil is consolidating under any loading condition without losing or gaining solids material or without changing applied stress. This case is generally applicable for soil or tailings that filled in a containment pond quickly enough so that the condition during filling can be neglected. In this chapter, a development of a quiescent finite strain consolidation model is presented.

3.3 Mathematical Background

For non-linear properties of soil that allow large deformation, the large strain consolidation theory introduced by Gibson et al. (1967), has been used in this study. The governing equation [Equation 2.2] was first expressed in terms of void ratio while Somogyi (1980) reformulated it in terms of excess pore water pressure [Equation 2.4]. The consolidation equation [2.4], which will be used in this thesis, can be recognized as a second order non-linear convection diffusion equation. A reduced coordinate (McNabb 1960) system is utilized to describe the consolidation behaviour of tailings because the use of a fixed reference system is inappropriate due to the large displacement of the top boundary of a consolidating layer. Conversely, the reduced coordinate describes the consolidating layer at any time in terms of volume of solids which is suitable for highly compressible material.

3.4 Constitutive Relationships

To solve the governing differential equation, many researchers apply correlations of void ratio-effective stress and hydraulic conductivity-void ratio into that equation. The most popular relationship was first introduced by Somogyi (1980) who used a power to handle the nonlinearity of void ratio-effective stress and hydraulic conductivity-void ratio relationships. Hydraulic conductivity measurement during the consolidation test is a difficult task and often the experimental data yield erroneous results. Hence, power function was used to express the nonlinearity for tailings hydraulic conductivity-void ratio relationship.

$$k = Ce^{D} \qquad [3.1]$$

$$e = a\sigma'^{b} \qquad (e < e_{\exp}) \qquad [3.2]$$

$$e = \frac{1}{A + Z\sigma'^{B}} \qquad (e \geq e_{\exp}) \qquad [3.3]$$

where, *A*, *B*, *C*, *D*, *Z*, *a* and *b* are curve fitted parameters.

On the other hand, this author reviewed a number of published data on void ratio-effective stress relationship and finally reported that some tailings show apparent over-consolidation pressure during the self-weight consolidation. Due to inherent properties of tailings, void ratio at very low effective stresses cannot be measured and hence, the effective stress-void ratio relationship for that range was assumed as power function [Equation 3.2]. However, a new approach for tailings compressibility has been proposed by developing a form of Weibull function [Equation 3.3] to fit the void ratio-effective stress plot from experimental data more precisely with an extensive applicability. The first point of experimental void ratio data is denoted by e_{exp} which is also the point of compressibility equation transformation. Figure 3.1 illustrates the use of constitutive relationships for this study that will be applicable for both types of tailings whether it possesses apparent over-consolidation behaviour or not. Statistical analysis has been performed on published data to support the newly developed equation which will be described in the following section 3.5.

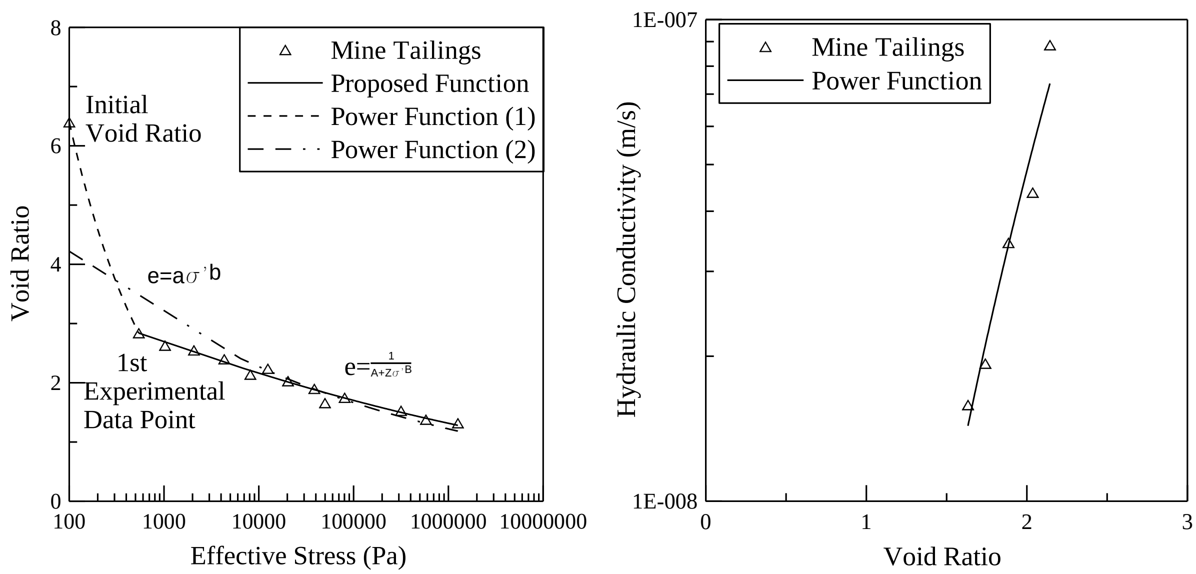

Figure 3.1: Compressibility and hydraulic conductivity function for conventional and present study (modified from Gan et al., 2011)

3.5 Statistical Analysis

Statistical analysis is an important part of scientific and engineering research to provide statistical support, and this analysis gives a way to quantify the confidence of random data. In this study, the significance test by analysis of variance (ANOVA) followed by regression analysis have been performed on published data from various researchers. This section describes the statistical analysis to develop a compressibility equation that will consider both normally consolidated and over-consolidated behaviour of tailings.

3.5.1 Analysis of Variance (ANOVA)

Analysis of variance (ANOVA) is one of the simplest statistical tools to observe the effect of factors over responses. To perform this analysis, the following large strain consolidation test data were chosen where void ratio was the only response and tailings type, amount of solids content, testing procedure and effective stress were considered as independent factors:

- Different types of tailings (from Qiu and Sego, 2001)
- At a wide range of solids content (from Proskin et al., 2010) and
- Various testing procedures (from Fox and Baxter, 1997)

Two different hypotheses were assumed with 5% level of significance for two-way or two-factor ANOVA without replication. The acceptance or rejection of these hypotheses depends on the P-value results, obtained from statistical analysis. Additionally, remoulded data from the laboratory experiment was not relevant for the field condition and hence no replication was available. Table 3.1 summarizes the two-factor ANOVA results for different cases and specified the effect of the factors on response for each case. At each case, for

ANOVA analysis based on different consolidation test data, except effective stress other factors remain constant. So, there is no interaction among the factors. ANOVA assumptions checking plots for all cases are illustrated in Appendix-A. The analysis shows that effective stress, tailings type and solids content has an effect on void ratio while testing procedure has no effect which is rational.

Table 3.1: Effect of factors on void ratio for tailings compressibility

Compressibility	Factor	P-Value	Remarks	Source of Data
Based on tailings type	Tailings type	<0.001	has effect	Qiu and Sego (2001)
	Effective stress	<0.001	has effect	
Based on solids content	Solids content	<0.001	has effect	Proskin et al. (2010)
	Effective stress	<0.001	has effect	
Based on tests type	Test type	0.080	has no effect	Fox and Baxter (1997)
	Effective stress	0.005	has effect	

3.5.2 Regression Analysis

Results of ANOVA led the statistical analysis to its next phase for regression analysis. A form of Weibull function [Equation 3.3] was selected to best fit the void ratio-effective stress experimental data point for all the cases especially at the low effective stress range to consider the apparent over-consolidation behaviour. Previously, power function, extended power function, Weibull function and logarithmic function were used to represent the compressibility behaviour of tailings. A comparison based on adjusted coefficient of determination has been made among the functions for consistency and applicability of a newly developed equation for all the cases and are illustrated in Figure 3.2, Figure 3.3 and Figure 3.4. It should be noted that extended power function does not yield any fit against all of these compressibility plots.

Therefore, proposed function has an improved adjusted coefficient of determination for most of the cases while Weibull function has similar values for some specific circumstances. Power function and logarithmic function produce almost identical values. It becomes an advantage for the proposed function over Weibull function for its consistency in terms of improved adjusted R^2 values. At the same time the proposed function has an overall applicability for each and every case. Confidence interval plots and curve fitting plots for all of these data are illustrated in Appendix-A.

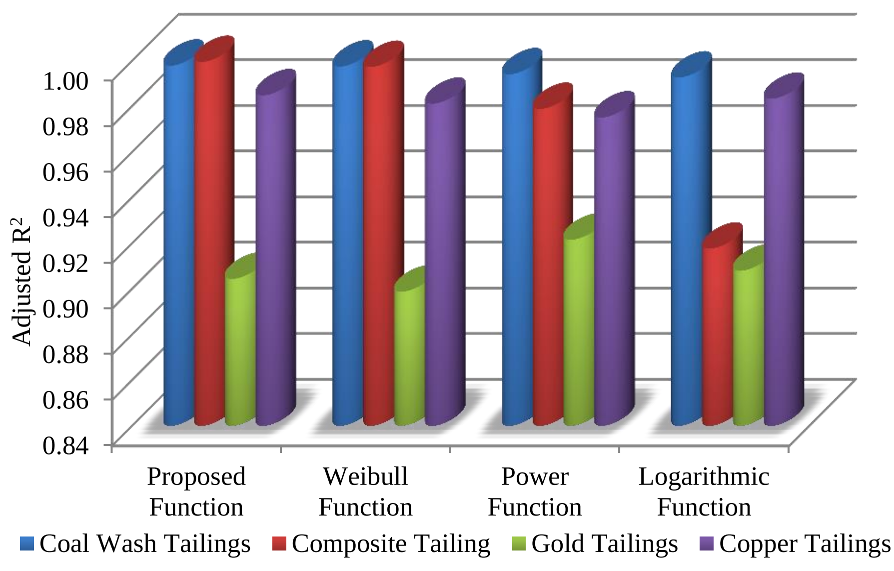

Figure 3.2: Comparison of adjusted R^2 for different types of tailings

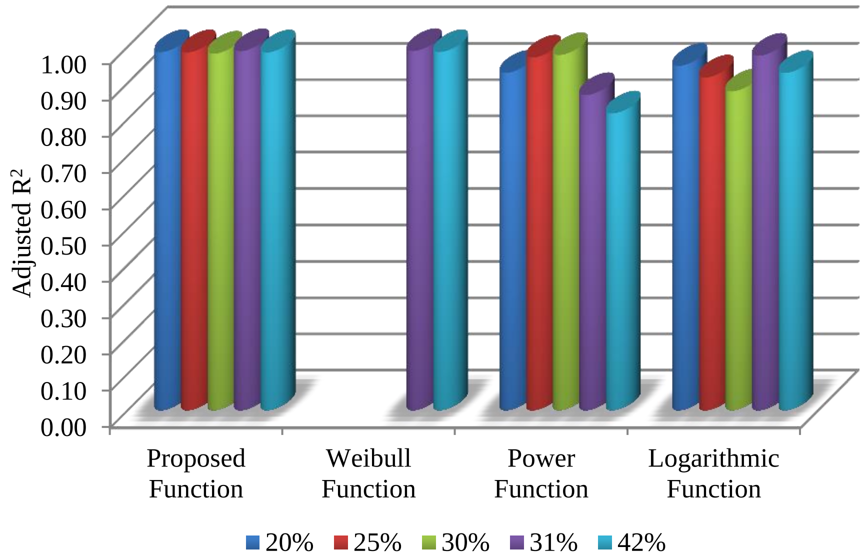

Figure 3.3: Comparison of adjusted R^2 for different solids content

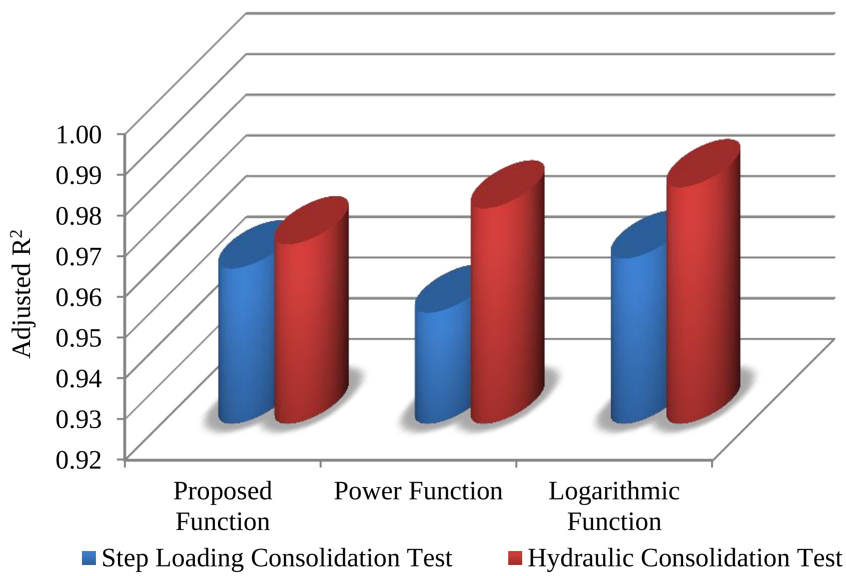

Figure 3.4: Comparison of adjusted R^2 for different testing procedures

From above statistical analysis, it can be concluded that the proposed compressibility equation has the advantages of improved coefficient of determination with consistent values to represent goodness of the fit. Also, the equation is applicable to any type of compressibility curve to represent the tailings consolidation behaviour.

3.6 Numerical Solution

A fully implicit, backward time central space, method was chosen for consolidation behaviour calculation due to its inherent stability and convergence for self-weight consolidation of tailings. The disadvantage of this numerical scheme is the unknown condition of material properties at the next time step in order to calculate the present time. This problem is overcome by setting up a small time increment to ensure that little variation will occur. Somogyi (1980) stated that the stability of the implicit finite difference method will cause the error introduced by this approximation to decay.

Equation 3.2 or Equation 3.3 was used to express the void ratio-effective stress relationships depending on value of void ratio compared to the first point of void ratio from experiment. Power function [3.2] is assumed for the ranges of void ratio where effective stress cannot be measured and Weibull function [3.3] produces the best fitted line for the experimental data.

The compressibility equation can be written in a differential form as

$$\frac{de}{d\sigma'} = ab\sigma'^{(b-1)} \qquad (e < e_{\exp}) \qquad [3.4]$$

or,

$$\frac{de}{d\sigma'} = -\frac{ZB\sigma'^{B-1}}{(A + Z\sigma'^{B})^2} \qquad (e \geq e_{\exp}) \qquad [3.5]$$

Substitute [3.4] and [3.5] in to the governing equation [2.4], Equation 3.6 and Equation 3.7 is obtained respectively.

$$\frac{\partial u}{\partial t}+\frac{\sigma'^{\beta}}{\alpha}\left(\frac{k}{1+e}\right)\frac{\partial^2 u}{\partial z^2}+\frac{\sigma'^{\beta}}{\alpha}\frac{\partial\left(\frac{k}{1+e}\right)}{\partial z}\frac{\partial u}{\partial z}=\gamma_b\frac{d(\Delta z)}{dt}\qquad (e<e_{\exp})\qquad [3.6]$$

or,

$$\frac{\partial u}{\partial t}-\frac{\sigma'^{\beta}}{\alpha\cdot e^2}\left(\frac{k}{1+e}\right)\frac{\partial^2 u}{\partial z^2}-\frac{\sigma'^{\beta}}{\alpha\cdot e^2}\frac{\partial\left(\frac{k}{1+e}\right)}{\partial z}\frac{\partial u}{\partial z}=\gamma_b\frac{d(\Delta z)}{dt}\qquad (e\geq e_{\exp})\qquad [3.7]$$

where,

$\gamma_b=\gamma_s-\gamma_w$

$\alpha=ab\gamma_w \qquad (e<e_{\exp})$ or, $\alpha=ZB\gamma_w \qquad (e\geq e_{\exp})$

$\beta=1-b \qquad (e<e_{\exp})$ or, $\beta=1-B \qquad (e\geq e_{\exp})$

From Figure 3.5, at any interior node (i, j), Equation 3.6 and 3.7 is approximated by

$$\frac{u_i^{j+1}-u_i^j}{\Delta t}+\frac{\sigma_i'^{\,j+1\beta}}{\alpha}\cdot\left(\frac{k_i^{j+1}}{1+e_i^{j+1}}\right)\cdot\left(\frac{u_{i+1}^{j+1}-2u_i^{j+1}+u_{i-1}^{j+1}}{(\Delta z)^2}\right)+$$

$$\frac{1}{4(\Delta z)^2}\cdot\frac{\sigma_i'^{\,j+1\beta}}{\alpha}\cdot\left(\frac{k_{i+1}^{j+1}}{(1+e_{i+1}^{j+1}}-\frac{k_{i-1}^{j-1}}{(1+e_{i-1}^{j-1}}\right)\cdot\left(u_{i+1}^{j+1}-u_{i-1}^{j+1}\right)=\gamma_b\frac{\Delta z}{\Delta t}\ (e<e_{\exp})\quad [3.8]$$

or,

$$\frac{u_i^{j+1}-u_i^j}{\Delta t}-\frac{\sigma_i'^{\,j+1\beta}}{\alpha\cdot e_i^2}\cdot\left(\frac{k_i^{j+1}}{1+e_i^{j+1}}\right)\cdot\left(\frac{u_{i+1}^{j+1}-2u_i^{j+1}+u_{i-1}^{j+1}}{(\Delta z)^2}\right)+$$

$$\frac{1}{4(\Delta z)^2}\cdot\left(-\frac{\sigma_i'^{\,j+1\beta}}{\alpha\cdot e_i^2}\right)\cdot\left(\frac{k_{i+1}^{j+1}}{(1+e_{i+1}^{j+1}}-\frac{k_{i-1}^{j-1}}{(1+e_{i-1}^{j-1}}\right)\cdot\left(u_{i+1}^{j+1}-u_{i-1}^{j+1}\right)=\gamma_b\frac{\Delta z}{\Delta t}\quad(e\geq e_{\exp})\ [3.9]$$

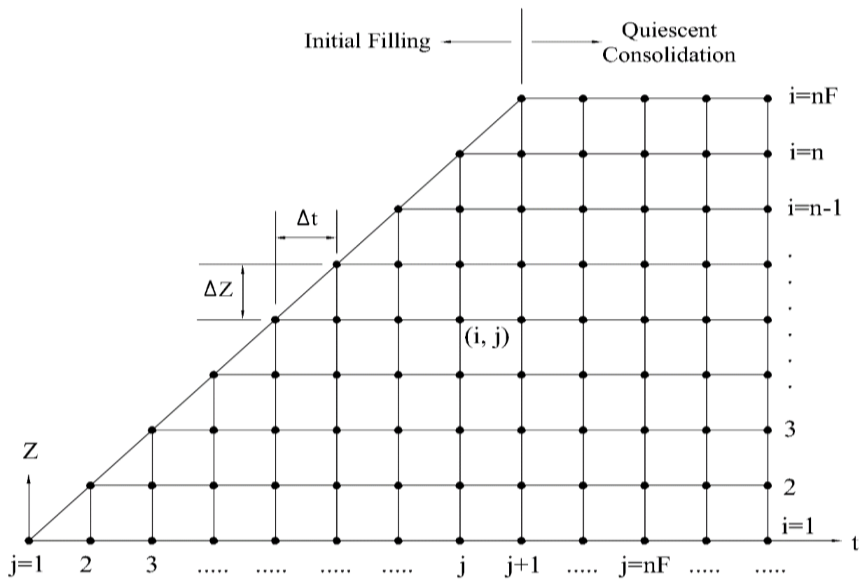

Figure 3.5: Domain discretization for quiescent consolidation

By replacing the values of σ', k and e at j+1 by their values at j, Equation 3.8 and Equation 3.9 becomes,

$$(u_i^{j+1}-u_i^j)+S_i^jK_i^j\delta(u_{i+1}^{j+1}-2u_i^{j+1}+u_{i-1}^{j+1})+S_i^jD_i^j\delta(u_{i+1}^{j+1}-u_{i-1}^{j+1})=\gamma_b(\Delta z)\quad[3.10]$$

where,

$$S_i^j=\frac{\sigma_i'^{\,j\beta}}{\alpha}\qquad(e<e_{\exp})$$

or,

$$S_i^j=-\frac{\sigma_i'^{\,j\beta}}{\alpha}\cdot\frac{1}{e_i^{\,2}}\qquad(e\geq e_{\exp})$$

$$K_i^j = \frac{k_i^j}{(1+e_i^j)}$$

$$D_i^j = \frac{1}{4}\left(\frac{k_{i+1}^j}{(1+e_{i+1}^j} - \frac{k_{i-1}^j}{(1+e_{i-1}^j}\right)$$

$$\delta = \frac{\Delta t}{(\Delta z)^2}$$

Rearranging Equation 3.10 yields

$$S_i^j \delta(K_i^j + D_i^j)u_{i+1}^{j+1} + (1-2S_i^j K_i^j \delta)u_i^{j+1} + S_i^j \delta(K_i^j - D_i^j)u_{i-1}^{j+1} = u_i^j + \gamma_b(\Delta z) \quad [3.11]$$

Due to self-weight consolidation, no excess pore pressure generates during the consolidation process and hence $\gamma_b(\Delta z)$ is zero. So, the Equation 3.11 becomes

$$S_i^j \delta(K_i^j + D_i^j)u_{i+1}^{j+1} + (1-2S_i^j K_i^j \delta)u_i^{j+1} + S_i^j \delta(K_i^j - D_i^j)u_{i-1}^{j+1} = u_i^j \quad [3.12]$$

where, i is the material coordinate index, j is the time index, Δt is the time increment, and Δz is the material coordinate increment.

In quiescent condition, the upper drainage boundary set is $u_{i-1}^{j+1} = 0$ and the impervious bottom boundary set is $u_{i+1} = u_{i-1}$. For double drainage problem, excess pore water pressure is set to zero at the bottom boundary. The initial condition is defined by the initial solids content or initial void ratio which is related to hydraulic conductivity and effective stress through Equation 3.1 and Equation 3.2 or Equation 3.3 respectively. The initial self-weight pressure is calculated from the height of the deposit multiplied by the buoyant unit weight.

3.7 Model Formulation

The one dimensional numerical model was developed based on the following underlying assumptions listed below:

- The model is developed for quiescent condition
- Only vertical deformation due to self-weight of tailings is considered for this one dimensional model; consolidation due to evaporation or applied stress and horizontal deformation are not incorporated
- Both single and double drainage problem can be solved since lateral drainage problem is not appropriate for a one dimensional model

The implicit method yields a system of linear equations which can be solved for excess pore water pressure for the next time step by a direct method, inverse matrix. After each time step the material parameters are updated as the void ratio changes due to consolidation, and the parameters are used for the next time step. With the initial and boundary condition, the rearranged governing equation [3.11] can be solved. The numerical modeling scheme used for this study is illustrated in Figure 3.6.

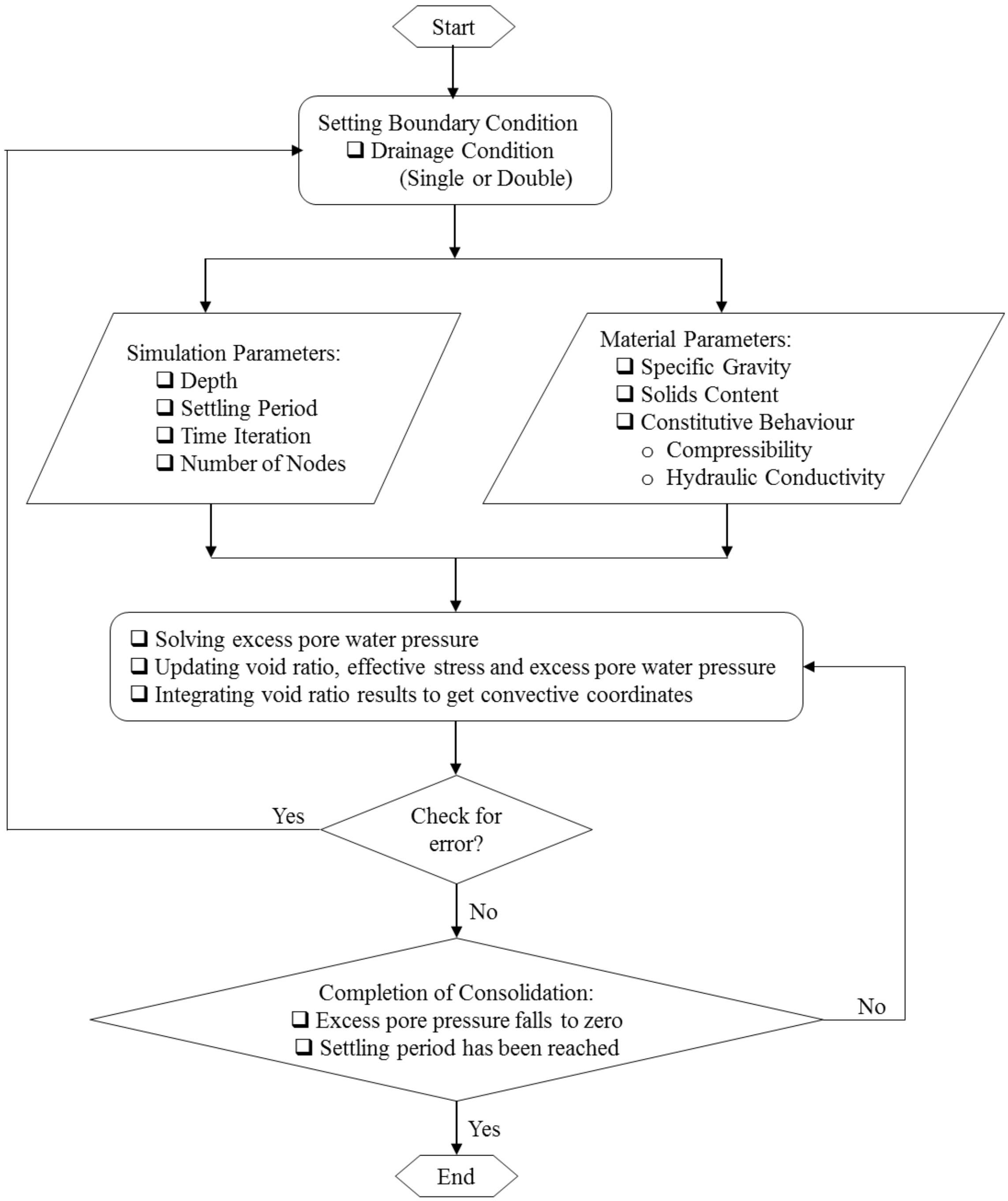

Figure 3.6: A computational scheme for numerical modeling

3.8 Summary

Non-linear finite strain consolidation theory is one of the most advanced theories in Geotechnical engineering for prediction of tailings consolidation. Though most of the finite strain consolidation models yield similar prediction, the use of the developed model based on the proposed compressibility equation improve long term prediction for both types of tailings, whether it possesses apparent pre-consolidation behaviour or not. Most importantly the proposed compressibility equation is good enough to fit both the normally consolidated line and the over-consolidated line because of its structure. Numerical solution of the finite strain consolidation theory was developed by using an implicit method to solve the second order partial differential governing equation. Strong mathematical background and stable numerical solution are used to formulate the model for calculation of the settlement and geotechnical properties of tailings.

4 MODEL VALIDATION, VERIFICATION AND SENSITIVITY

4.1 General

This chapter presents the prediction for consolidation behaviour of tailings from published data. First of all, it validates and verifies the model with standard benchmarks, set up by different scholars. Then, it incorporates with sensitivity analysis of simulation parameters, material parameters and modeling parameters using data from Proskin et al. (2010). A comparison between the conventional model and the proposed model has also been conducted for long term consolidation behaviour of the tailings. Finally, as a case study, the model predicts consolidation behaviour of different types of oil sand tailings (from Miller et al. 2010b) in terms of settlement profile, void ratio profile, excess pore pressure profile and effective stress profile for a period of 50 years under different drainage conditions.

4.2 Model Validation

The proposed model was validated with 7 days experimental data of large strain consolidation test from Bartholomueesen et al. (2002). The soil was collected as a dredged material at low tide from the river Schelde in Antwerpen, Belgium having a specific gravity of 2.72 and initial solids content of approximately 52%. The experiments of large strain consolidation test and hydraulic conductivity measurement were performed by settling column test. Curve fitted parameters, used as modeling parameters, have been obtained from hydraulic conductivity and compressibility plots after fitting the equations 3.1, 3.2 and 3.3. Input parameters for initial and boundary conditions and modeling parameters are summarized in Table 4.1 and Table 4.2.

Table 4.1: Input parameters for initial and boundary conditions for model validation

Model	**Initial condition**		**Boundary condition**		
	Specific gravity	*Solids content*	*Depth (m)*	*Settling time (days)*	*Drainage*
Proposed Model	2.72	52%	0.565	7	Single

Table 4.2: Modeling parameters for validation

Model	**Parameters**						
	A (Pa)	B	C (m/s)	D	Z	a (Pa)	b
Proposed Model	0.2897	0.3178	7.78×10^{-09}	6.33	0.0372	2.47	-0.0116

Figure 4.1 indicates that the model closely matches with the experimental data at the final stage of the experiment. It should be noted that initial settlement of the soil is mainly dictated by the compressibility behaviour at low effective stress ranges which is crucial to measure due to lack of a sophisticated apparatus.

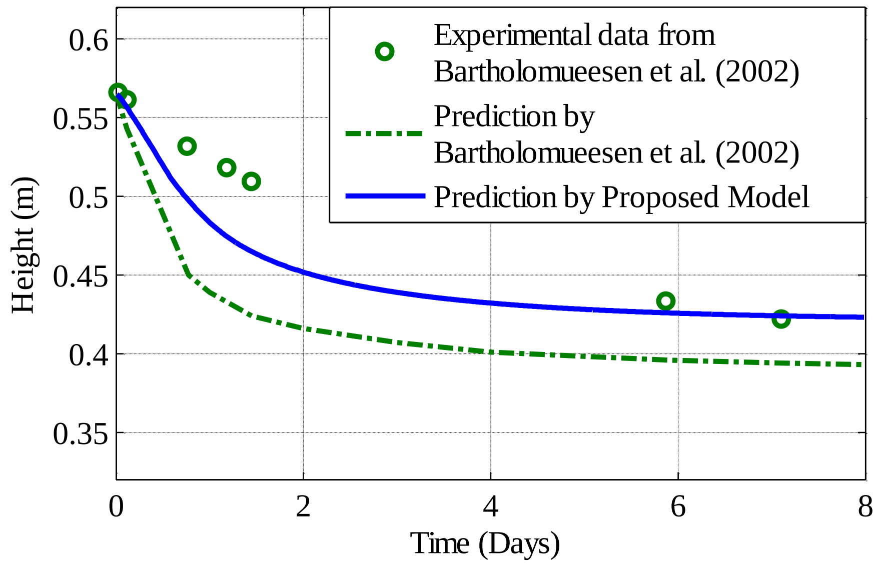

Figure 4.1: Model validation using experimental data from Bartholomueesen et al. (2002)

4.3 Model Verification

The widely used benchmark scenario A for quiescent consolidation of soft soils from Townsend and McVay (1990) was selected to verify the model. With a depth of 9.6 m and an initial void ratio of 14.8 (results in 16% solids content), the soft soils were consolidated under its self-weight stresses for single drainage boundary condition. The specific gravity of the soil was 2.82. However, equations 3.1 and 3.2 had been used as given in the paper for void ratio-hydraulic conductivity and effective stress-void ratio relationships instead of the proposed compressibility equation because the large strain consolidation test data were not available for this material. Though the number of nodes and time increment were not stated at the original model, it was assumed to have 50 nodes with a 10 day time increment. Input parameters for initial and boundary conditions and modeling parameters are summarized in Table 4.3 and Table 4.4 respectively.

Table 4.3: Input parameters for initial and boundary conditions for model verification

Model	Initial condition		Boundary condition		
	Specific gravity	*Solids content*	*Depth (m)*	*Settling time (years)*	*Drainage*
Proposed Model	2.82	16%	9.6	1, 10	Single

Table 4.4: Modeling parameters for verification

Model	Parameters			
	a (Pa)	b	C (m/s)	D
Proposed Model	35.29	-0.22	2.931×10^{-12}	4.65

The verified results are illustrated in Figure 4.2 and Figure 4.3 for settlement after 10 years and void ratio, excess pore pressure profiles after 1 year respectively. For all the cases, results from the present model, using conventional equations, were compared with the previous models from Bromwell and Carrier, Inc., Lakeland, Florida; University of Connecticut, Storrs, Connecticut; University of Florida, Gainesville, Florida; and USAE Waterways Experiment Station, Vicksburg, Mississippi. Almost all the results, except Bromwell and Carrier, show identical performance for void ratio, excess pore pressure profiles and settlement.

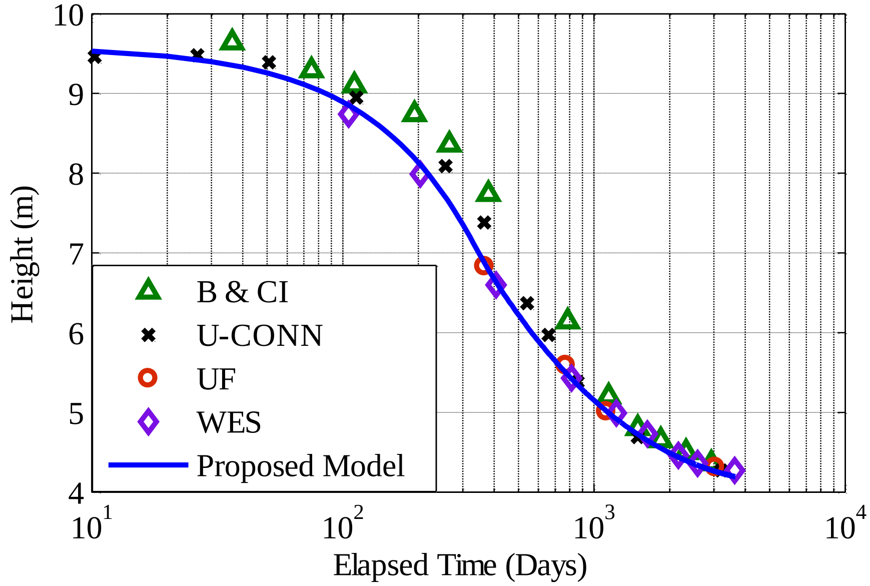

Figure 4.2: Interface settlement comparison (Modified from Townsend and McVay, 1990)

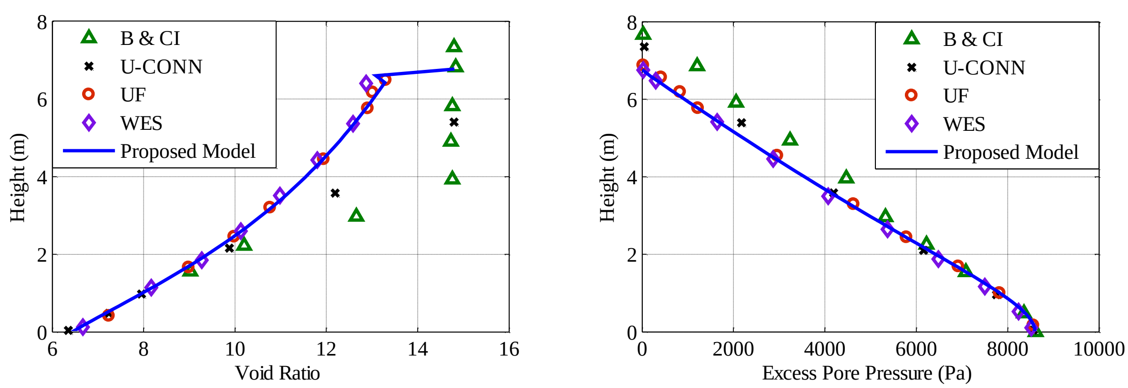

Figure 4.3: Void ratio and excess pore water pressure profile after 1 year (Modified from Townsend and McVay, 1990)

4.4 Sensitivity Analysis

Sensitivity analysis is an important part of numerical modeling. The sensitivity of the model related to various parameters are determined through this analysis and at the same time the parameters, requiring more care to achieve the accuracy of the prediction, are identified. This section demonstrates the sensitivity analysis for simulation parameters, material parameters, modeling parameters and comparison of models using the published data from Proskin et al. (2010) for two different types of Mature Fine Tailings (MFT), a special form of oil sand tailings.

4.4.1 Material Properties

Two types of specimen from Suncor mature fine tailings (MFT) were subjected to large strain consolidation tests and considered for this analysis. Specimen SU-3 was derived from a sample of MFT from a deeper part of the pond (2/3 depth) which has a higher solids content of 42%, and specimen SU-9 is representative of an average MFT specimen with an initial solids content of 31%. Specific gravity for both types of specimen was assumed to be 2.60. The material properties are summarized in Table 4.5.

Table 4.5: Material properties of oil sands tailings (modified from Proskin et al., 2010)

Tailings Type	**Specimen ID**	**Specific Gravity**	**Initial Solids Content (%)**
1	SU-9	2.60	31
2	SU-3	2.60	42

Figure 4.4 shows the compressibility and hydraulic conductivity plots for these two types of tailings having different initial solids content where they show over-consolidation behaviour at low effective stress ranges. So, the goodness of the fit for power function and proposed Weibull function becomes significant here. On the other hand, power function was assumed for void ratio-hydraulic conductivity relationship. Sensitivity analysis of simulation parameters, material parameters and curve fitted modeling parameters was done only for the tailings containing 31% solids content and for single drainage condition. Comparison of

models incorporates with both types of tailings having single and double drainage problems. It should be noted that for this study, initial void ratio is assumed to be at the effective stress of 1 Pa.

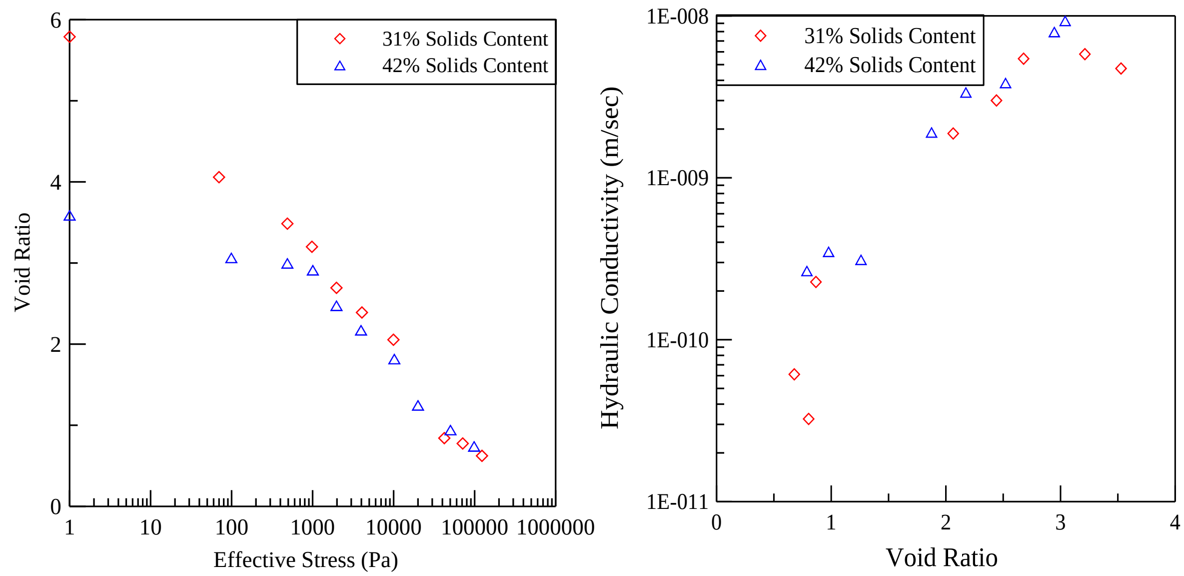

Figure 4.4: Compressibility and hydraulic conductivity of tailings contain 31% and 42% solids content (modified from Proskin et al., 2010)

4.4.2 Simulation Parameters

At first the sensitivity analysis of simulation parameters was conducted. There are two types of simulation parameters incorporate with the model: number of nodes and time of iteration. Both these parameters increase the accuracy of the model though the higher the accuracy lowers the pace of the calculation. That's why an optimization is required to set the suitable simulation parameters that would not compromise with the accuracy rather than decrease the time needed for solving the problem.

Figure 4.5 shows the effect of the number of nodes on tailings settlement profile. When the number of nodes is set as 10 it predicts higher settlement and decreases with the increment of node numbers. Over 50 nodes show quite similar results and there is not that much difference between selecting the number of nodes as 50 or 500. As a result, 50 nodes have been chosen for this study as an effective number of nodes for calculation.

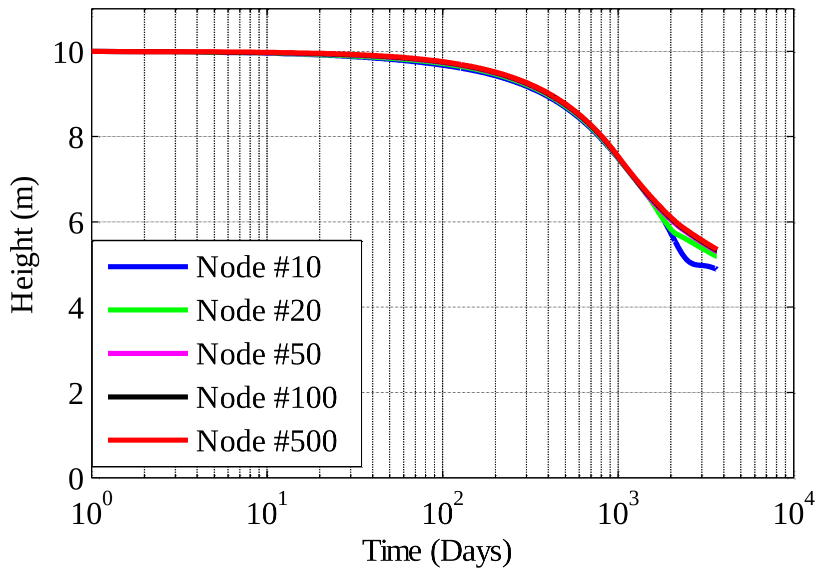

Figure 4.5: Effect of number of nodes on tailings settlement for 10 years

Effect of time iteration on tailings settlement was performed for both short-term and long-term consolidation periods. Figure 4.6 shows the effect of different time iteration on model accuracy for long-term consolidation which is a 10 years settling period. All the iteration that is less than 10 days shows similar prediction indicating the iteration time does not have any significant effect on the model unless it is higher than 10 days.

Figure 4.7 illustrates the identical behaviour for short-term (1 year settling period) consolidation as well. Any number of time iteration, between 0.01 day and 10 days, yields a similar settlement profile. From above sensitivity analysis for time iteration, for this study, time iteration of 1 day is chosen as it serves both the better accuracy and smaller time requirement for calculation of tailings consolidation behaviour.

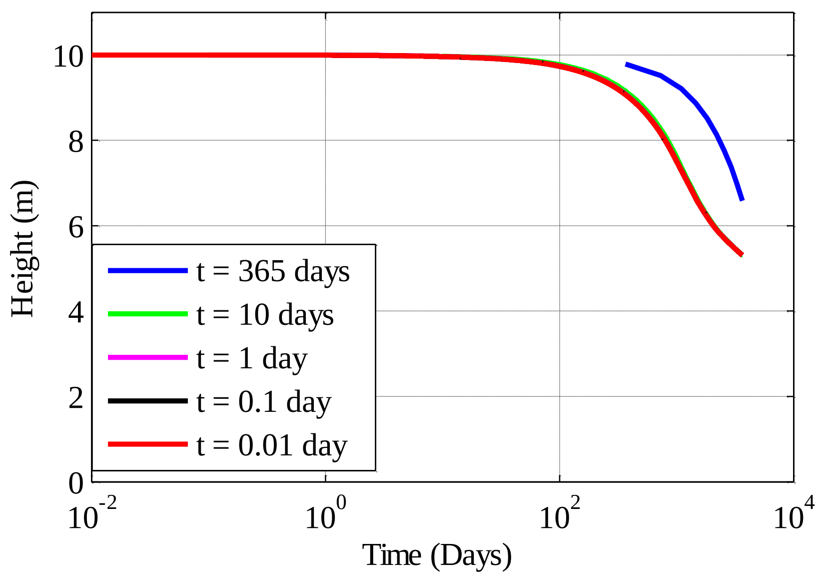

Figure 4.6: Effect of time iteration on tailings settlement for 10 years

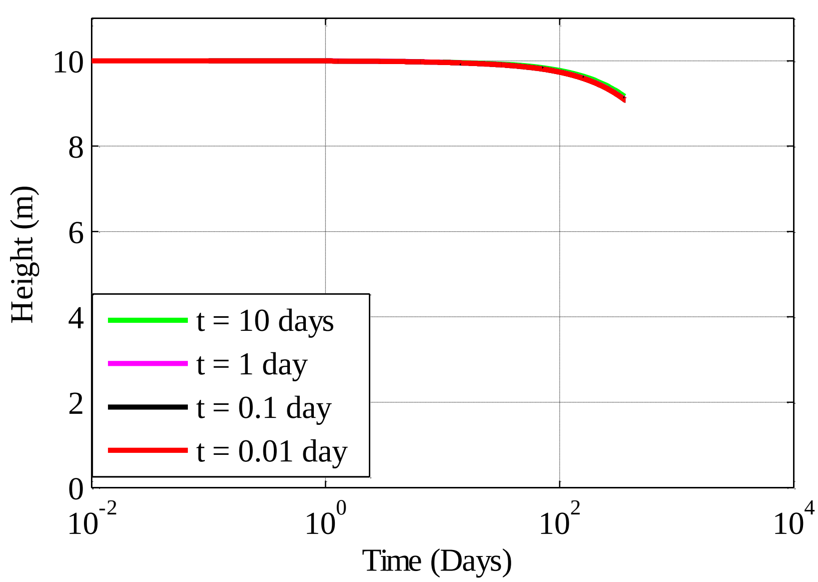

Figure 4.7: Effect of time iteration on tailings settlement for 1 year

4.4.3 Material Parameters

Material parameters such as specific gravity and initial solids content have significant effect on the model. This section demonstrates the effect of changes of specific gravity and initial solids content on tailings settlement for 10 years.

Usually, the specific gravity for mine tailings falls in between 2.6 and 4.4 (Dimitrova and Yanful 2012). Hence, the sensitivity analysis of the effect of specific gravity had been conducted within that range. Figure 4.8 shows that the higher the specific gravity, the higher the settlement is because specific gravity with a higher number indicates heavier material. This model is for self-weight consolidation only where specific gravity is directly linked with the consolidation behaviour of the tailings, which dominates the settlement rate.

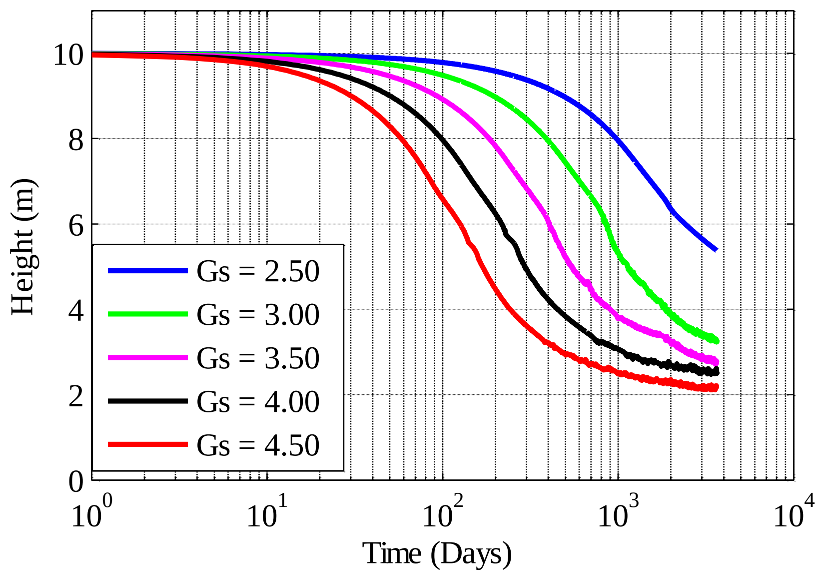

Figure 4.8: Effect of specific gravity on tailings settlement for 10 years

Initial solids content is another important material parameter as an initial condition for the consolidation model. Effects of initial solids content on the model was obtained for 10 years tailings settlement calculation. The settlement is high with low initial solids content and decreases with the increment of it.

Figure 4.9 graphically represents the effect of initial solids content and having solids content higher than 60% actually indicates a very small amount of settlement during the period of 10 years. This analysis could be used to design an efficient tailings stream for long term disposal.

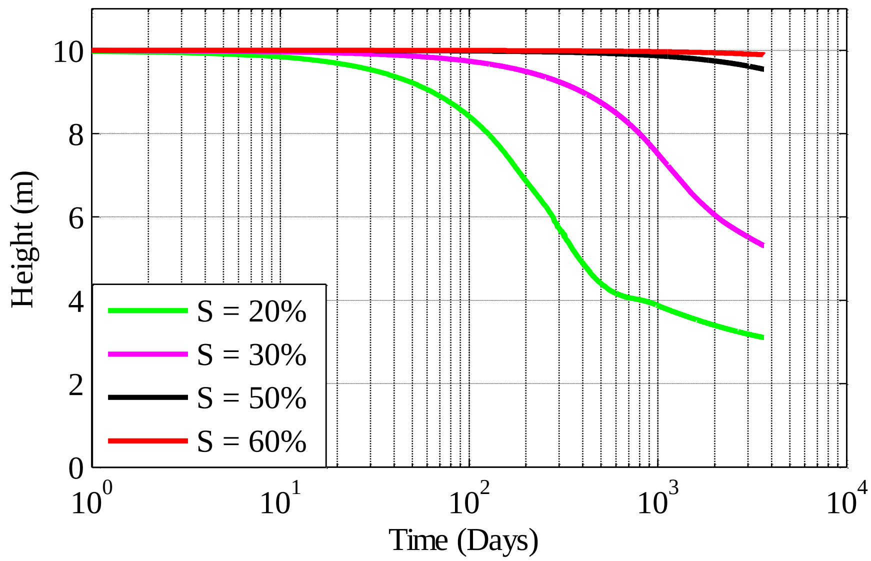

Figure 4.9: Effect of initial solids content on tailings settlement for 10 years

4.4.4 Curve Fitted Parameters

Curve fitted parameters obtained from laboratory experiments are important features of the model as they control the consolidation behaviour more than anything else. Sensitivity analysis on values of the parameters has been conducted by changing the order of magnitude of the curve fitted values against each type of parameter. The values are listed against curve fitted modeling parameters in Table 4.6.

Table 4.6: Sensitivity analysis of curve fitted parameters

Multiplier of Curve Fitted Value	**Parameters**						
	A	B	C (m/s)	D	Z (Pa)	a (Pa)	b
10^{-2}	2.5×10^{-3}	6.5×10^{-3}	1.74×10^{-12}	3.05×10^{-2}	8×10^{-6}	5.79×10^{-2}	-8.4×10^{-4}
10^{-1}	2.5×10^{-2}	6.5×10^{-2}	1.74×10^{-11}	3.05×10^{-1}	8×10^{-5}	5.79×10^{-1}	-8.4×10^{-3}
10^{0}	2.5×10^{-1}	6.5×10^{-1}	1.74×10^{-10}	3.05×10^{0}	8×10^{-4}	5.79×10^{0}	-8.4×10^{-2}
10^{1}	2.5×10^{0}	6.5×10^{0}	1.74×10^{-09}	3.05×10^{1}	8×10^{-3}	5.79×10^{1}	-8.4×10^{1}
10^{2}	2.5×10^{1}	6.5×10^{1}	1.74×10^{-08}	3.05×10^{2}	8×10^{-2}	5.79×10^{2}	-8.4×10^{0}

Figure 4.10 represents the results of sensitivity analysis of settlement calculation for different modeling parameters at a settling period of 10 years. Higher orders of magnitudes than curve fitted values for parameters *a* and *b* do not affect the settlement profile while lower orders decrease the rate of settlement proportionately. Among the parameters of the proposed compressibility equation, only *A* is highly sensitive to the model for higher orders of magnitudes of the curve fitted value. Parameters *B* and *Z* and also lower orders of magnitude of parameter *A* have negligible effect on the overall settlement profile. The hydraulic conductivity parameters *C* and *D* are highly sensitive to the model and increment of magnitudes increase the rate of settlement for both parameters. As stated earlier, the measurement of hydraulic conductivity is quite difficult compared to consolidation test.

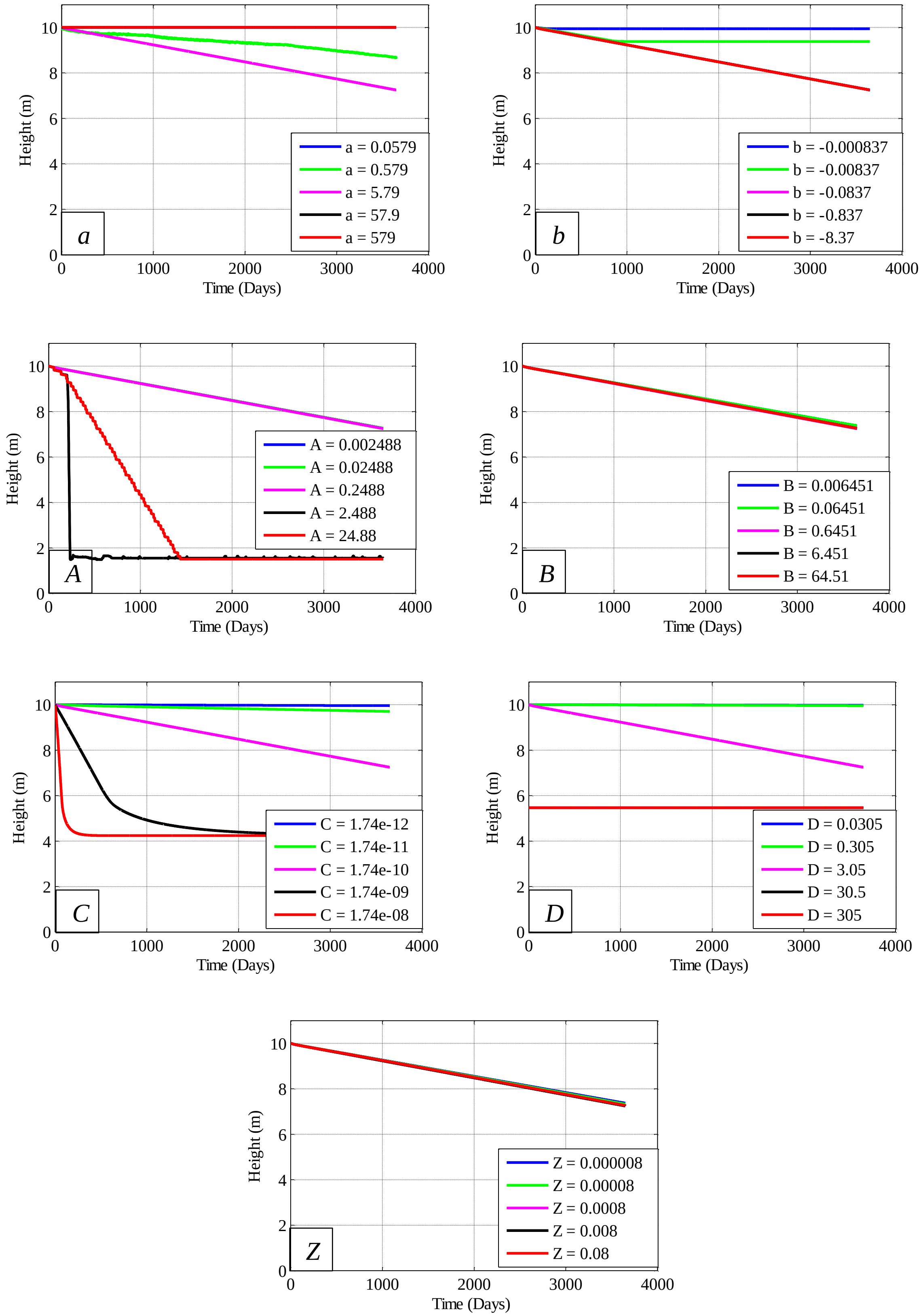

Figure 4.10: Sensitivity analysis of curve fitted parameters *a*, *b*, *A*, *B*, *C*, *D* and *Z*

4.4.5 Comparison of Models

Two types of tailings based on different solids content were compared between the conventional model and the proposed model. The conventional model uses power function [Equation 3.2] as the compressibility equation while the proposed model uses both the power function and a form of Weibull function [Equation 3.3] as the compressibility equation. Power function is applicable for the ranges of effective stress which cannot be measured during laboratory experiment and the proposed compressibility function is considered for data points obtained from large strain consolidation test. Both models use power function [Equation 3.1] to express the void ratio-hydraulic conductivity equation. The prediction was analyzed for a 50 years settling period in both single and double drainage condition at a depth of 10.0 m. Input parameters for initial and boundary conditions and modeling parameters are summarized in Table 4.7 and Table 4.8 respectively.

Table 4.7: Input parameters for initial and boundary conditions for models comparison

Model	**Initial condition**		**Boundary condition**		
	Specific gravity	*Solids content*	*Depth (m)*	*Settling time (year)*	*Drainage*
Conventional Model	2.60	31%, 42%	10.0	50	Single Double
Proposed Model	2.60	31%, 42%	10.0	50	Single Double

Table 4.8: Modeling parameters for model comparisons

Model	ID	Parameters						
		A	B	C (m/s)	D	Z (Pa)	a (Pa)	b
Conventional Model	SU-9	-	-	1.74×10^{-10}	3.05	-	32.09	-0.3298
	SU-3	-	-	3.43×10^{-10}	2.79	-	19.13	-0.2741
Proposed Model	SU-9	0.2488	0.6451	1.74×10^{-10}	3.05	0.0008	5.79	-0.0837
	SU-3	0.2747	0.5789	3.43×10^{-10}	2.79	0.0015	3.59	-0.0344

Results from the numerical model using the conventional compressibility function and the proposed function for 31% and 42% solids content are shown in Figure 4.11 and Figure 4.12 respectively. In these figures the comparison for tailings-water interface settlement is illustrated for a period of 50 years settling time by both models.

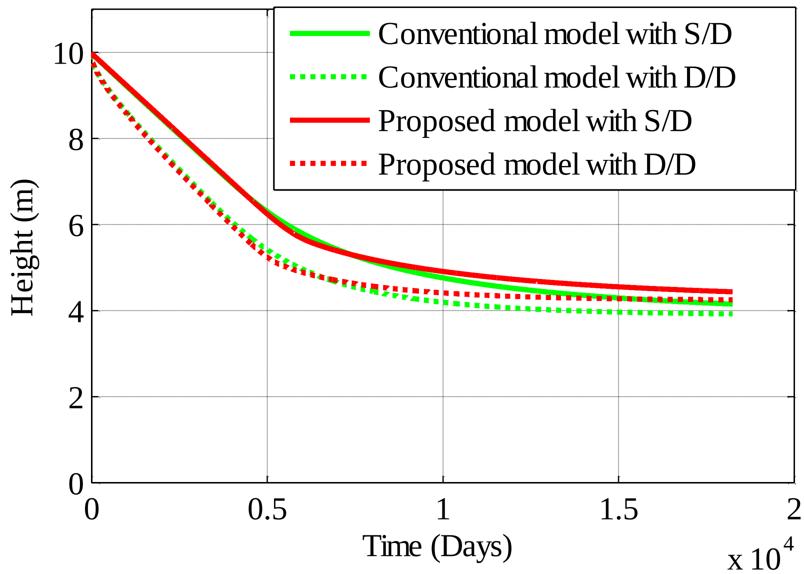

Figure 4.11: Comparison between conventional model and proposed model for settlement calculation of tailings having 31% solids content

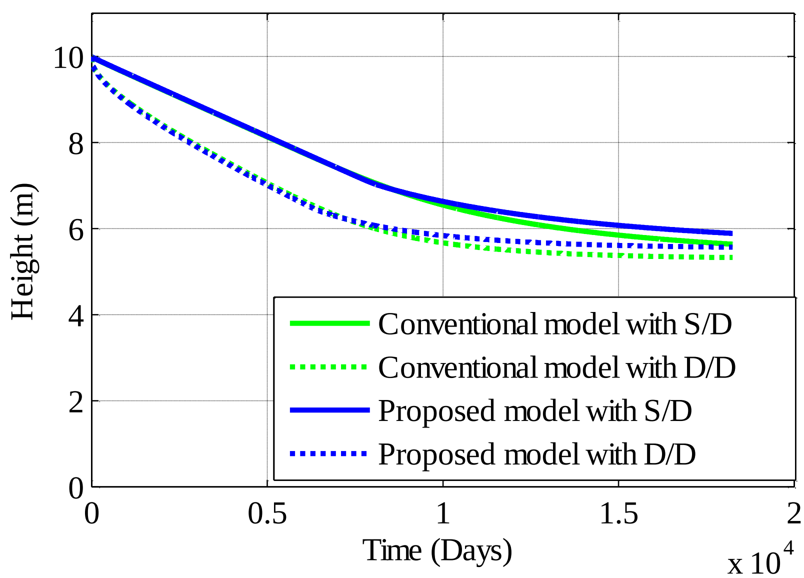

Figure 4.12: Comparison between conventional model and proposed model for settlement calculation of tailings having 42% solids content

Up to around 20 to 25 years, both models predict similar settlement while from 25 to 50 years; the conventional model predicts approximately 0.25 m higher settlement than this study. The governing equation chosen for this study as stated in Equation [3.2] has two main components- hydraulic conductivity and compressibility. The effect of settling time and material properties are directly related to the compressibility behaviour. That's why altering compressibility function affects the settlement for a longer period of time. Otherwise, hydraulic conductivity function, which remains the same for both models, has a greater effect on consolidation behaviour compared to compressibility (Azam et al. 2009). Comparison of models for settlement due to single and double drainage condition illustrates that the double drainage condition initially occurs with larger settlement compared to single drainage though at the final stage this difference becomes negligible.

Figure 4.13 and Figure 4.14 illustrate the prediction of both models on void ratio profile for a 50-year time span. These outputs actually reflect the properties of tailings and overall settling characteristics in specific years. There are significant variations on void ratio profile in terms of height for both models with single and double drainage problem though they show similar trend for both types of tailings.

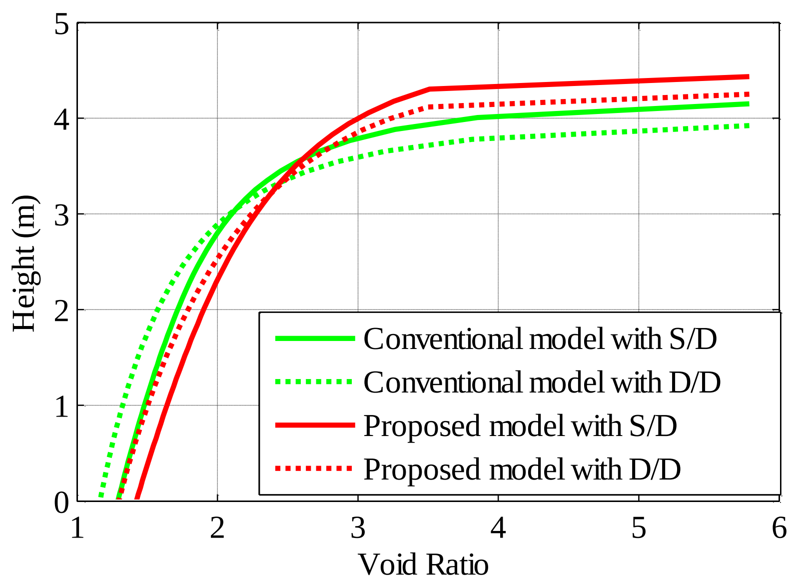

Figure 4.13: Comparison between conventional model and proposed model for void ratio profile of tailings having 31% solids content

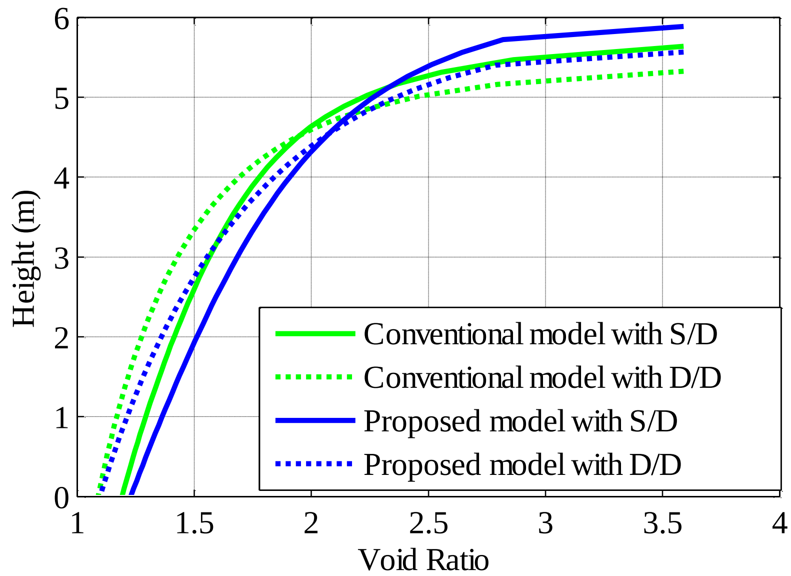

Figure 4.14: Comparison between conventional model and proposed model for void ratio profile of tailings having 42% solids content

It can be concluded that for lower solids content the void ratio near the tailings-water interface varies within a larger range than that of tailings having higher solids content. Additionally, the usual patterns of void ratio profiles are slightly higher near tailings-water interface for the proposed model which signifies lower settlement than the conventional model. For both models the higher void ratio exists in a very thin layer of soils.

Figure 4.15 and Figure 4.16 represent the excess pore pressure profile obtained from both models for tailings containing 31% and 42% solids content respectively. After the end of the settling period, the conventional model yields significantly higher excess pore pressure at the bottom of the tailings impoundment compared to the proposed model for both cases. Double drainage condition improves consolidation behaviour as the dissipation rate of pore water is faster for the proposed model than the conventional model with similar trends.

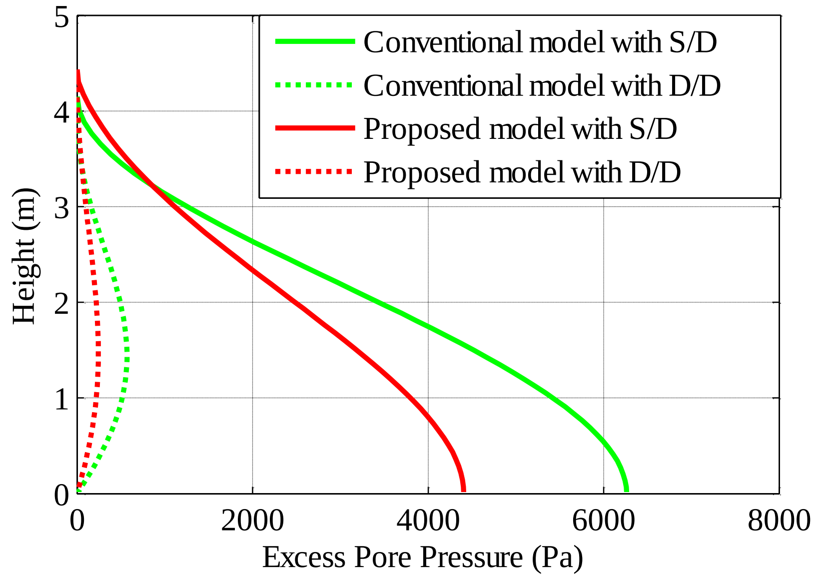

Figure 4.15: Comparison between conventional model and proposed model for excess pore water pressure profile of tailings having 31% solids content

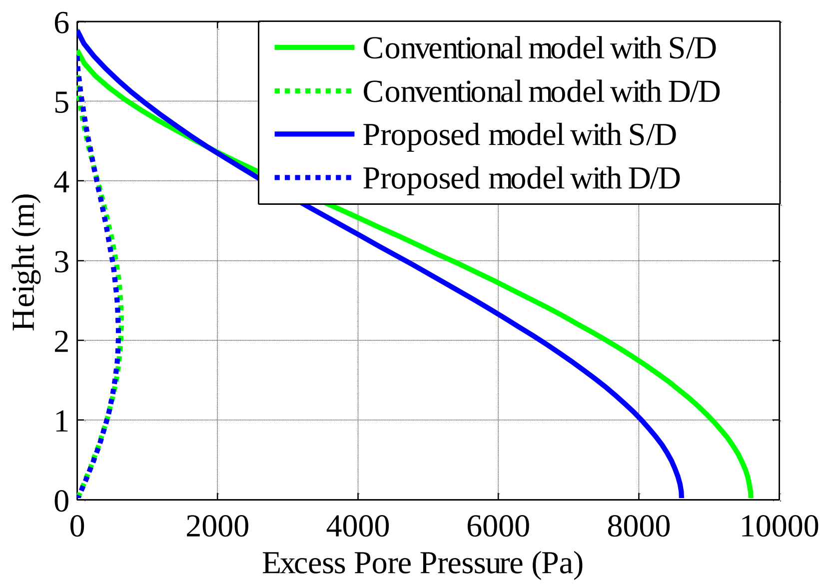

Figure 4.16: Comparison between conventional model and proposed model for excess pore water pressure profile of tailings having 42% solids content

From the excess pore pressure profile, it can be predicted that the selected tailings have reached near to the end of 100% consolidation by double drainage condition while it requires a lot more time even after the period of 50 years to achieve that by single drainage condition. The higher the solids content, the longer the time required in reaching the fully consolidated stage for tailings.

The effective stress profile shows similar behaviour because total stress remains the same for self-weight consolidation. Figure 4.17 and Figure 4.18 illustrate the comparison of models in terms of effective stress for both types of tailings where they show a similar trend. For double drainage condition, both models predict the same effective stress at the bottom of the impoundments with higher values compared to single drainage. As a result, the consolidation rate of tailings increases for double drainage condition and higher solids content yields more effective stress than the lower solids content though they settle less compared to the lower one.

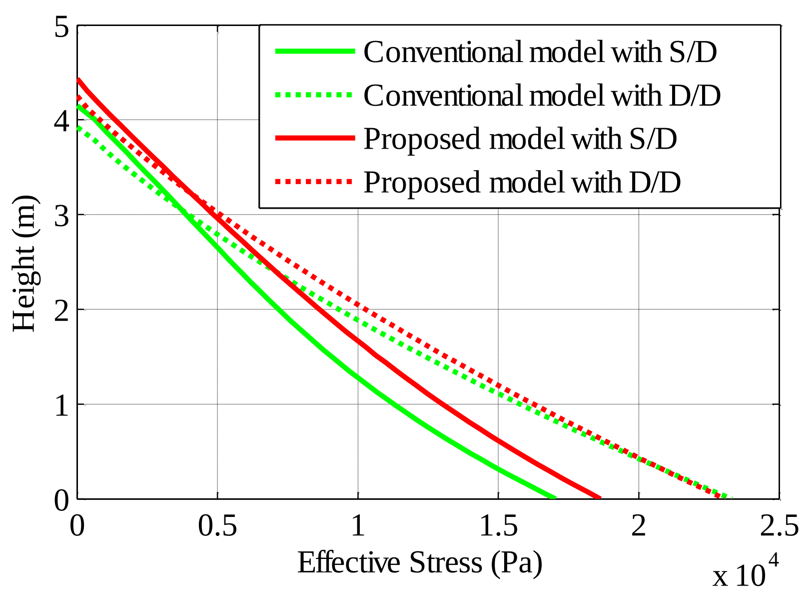

Figure 4.17: Comparison between conventional model and proposed model for effective stress profile of tailings having 31% solids content

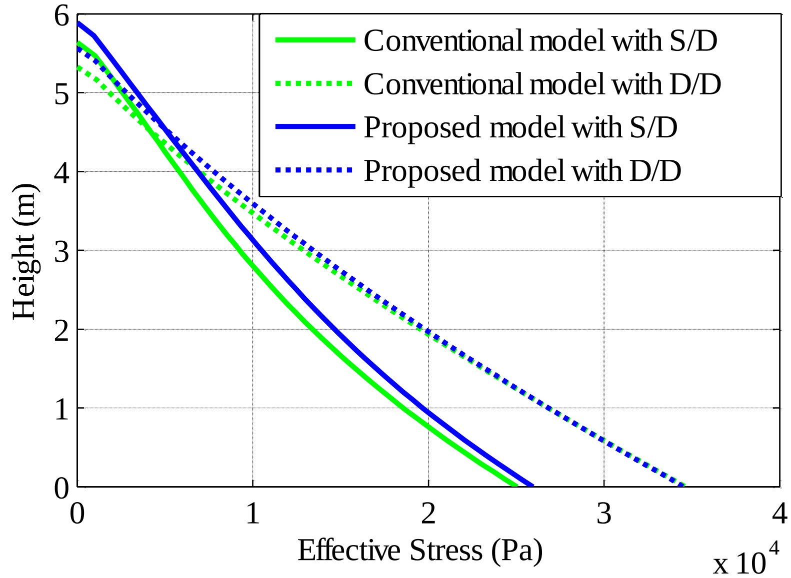

Figure 4.18: Comparison between conventional model and proposed model for effective stress profile of tailings having 42% solids content

From the above figures, it can be concluded that the proposed model significantly differs from the long term consolidation behaviour prediction in terms of settlement, void ratio, excess pore pressure and effective stress profile than the conventional model.

4.5 Summary

Model validation and verification described in this chapter aid to gain its reliability and widen its acceptance while sensitivity analysis demonstrates the change in prediction by altering simulation parameters, material parameters and curve fitted modeling parameters. However, comparison of models illustrates the significant difference between the conventional and the proposed one dimensional model.

5 PREDICTION FOR ONE DIMENSIONAL CONSOLIDATION BEHAVIOUR OF TAILINGS

5.1 General

This chapter describes the prediction for consolidation behaviour of tailings from published data. First of all, it provides the background of the case study from Miller et al. (2010a and 2010b) followed by the material properties. Then, the statistical analyses are presented for large strain consolidation test data. Validations of the experimental results are illustrated in the following section. Finally, the model predicts consolidation behaviour of different types of oil sand tailings in terms of settlement profile, void ratio profile, excess pore pressure profile and effective stress profile for a period of 50 years under different drainage conditions.

5.2 Case Study

In the oil sands industry, the caustic process was used successfully to recover bitumen from surface-mined oil sands ore. However, the caustic process results in extremely low consolidation rates due to the creation of dispersed high void ratio fine tailings. To overcome this problem by reducing fines dispersion, non-caustic process of extraction was introduced. Though fine tailings are unique and complex in nature, an evaluation from a single perspective is inadequate to fully understand their consolidation behaviour. There are many factors that influence the behaviour of these materials. In this case study, the comparison of fine tailings consolidation behaviour that results from the type of bitumen extraction process (caustic versus non-caustic), the choice of process water (treated versus untreated) and oil sand ore (Ore A versus Ore B) had been described.

5.3 Material Properties

For this case study, five different oil sands tailings, derived from two types of ores, were chosen for predicting the one dimensional consolidation behaviour in the long term using the proposed model. The tailings were differed based on the extraction process (caustic and non-caustic), the process water (Syncrude Recycled Pond Water, Treated Athabasca River Water and Untreated Athabasca River Water) and ores (Ore A from Syncrude Canada Ltd. and Ore B from Suncor Energy Inc.). Both the Syncrude and the Suncor oil sands mine are located in Northern Alberta, Canada. The material properties of these tailings are summarized in Table 5.1 from Miller et al. (2010a).

Table 5.1: Material properties of oil sands tailings (modified from Miller et al., 2010a)

Tailings Type	Ore Type	Extraction Process	Process Water	Specific Gravity	Initial Solids Content (%)
1	A	Caustic	Syncrude Recycled Pond Water (SRPW)	2.55	22.00
2	A	Non-Caustic	Treated Athabasca River Water (TARW)	2.51	22.00
3	B	Caustic	Syncrude Recycled Pond Water (SRPW)	2.48	22.00
4	B	Non-Caustic	Treated Athabasca River Water (TARW)	2.45	22.00
5	A	Non-Caustic	Untreated Athabasca River Water (UARW)	2.50	22.00

Figure 5.1 shows the compressibility and hydraulic conductivity behaviour of tailings produced from Ore A and Ore B. The laboratory investigations were carried out by Miller et al. (2010b) to determine the consolidation properties of fine tailings. In this study, they were subjected to statistical analysis to obtain the best fitted curve parameters for numerical modeling and predicting the tailings long term consolidation behaviour.

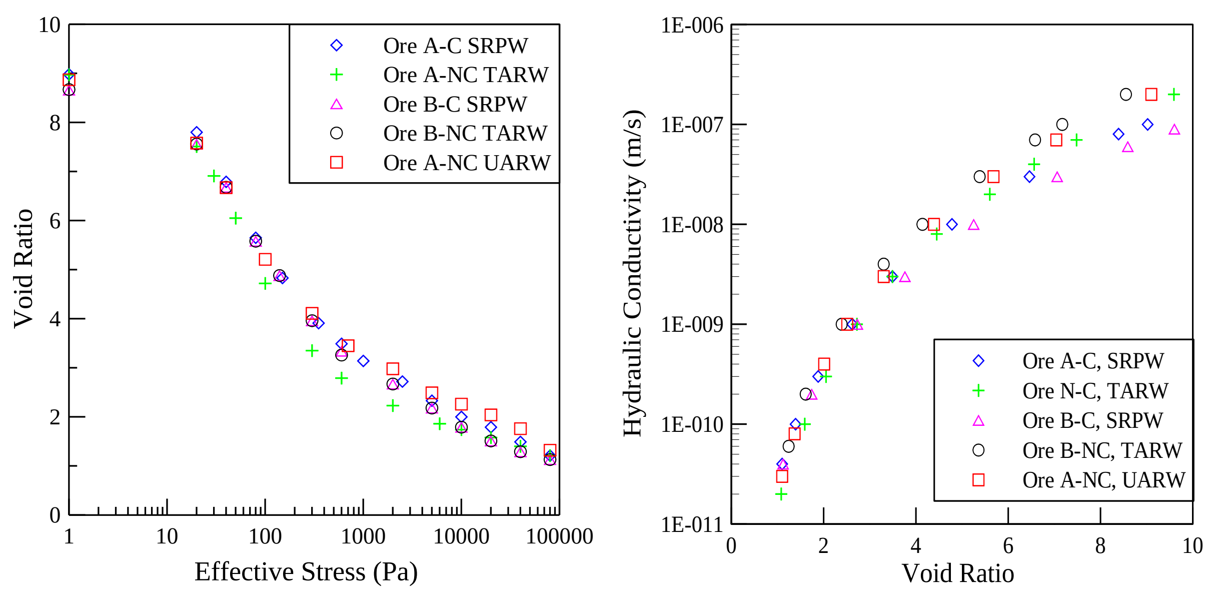

Figure 5.1: Compressibility and hydraulic conductivity of Ore A and Ore B tailings

5.4 Statistical Analysis

5.4.1 Analysis of Variance

At first the consolidation test data were involved in the significance test by analysis of variance to observe the effect of factors on responses. As stated earlier, types of tailings and effective stress are considered as factors while void ratio is the only response. Level of significance was assumed to be 0.05. There were no recurrence of the laboratory test data and hence two-way ANOVA without replication was conducted. From ANOVA results with significantly small p-values, summarized in Table 5.2, it can be concluded that both the tailings type and effective stress have effects on the void ratio and this outcome led to its next phase as regression analysis. The plots representing the assumptions validity for this ANOVA are illustrated in Appendix-B.

Table 5.2: Results of two-way ANOVA without replication

Compressibility	Factor	P-Value	Remarks	Source of Data
Based on extraction process and process water type	Tailings type	<0.001	has effect	Miller et al. (2010b)
	Effective stress	<0.001	has effect	

5.4.2 Regression Analysis

The results of regression analysis of the proposed function, power function and logarithmic function are shown in Figure 5.2 for effective stress-void ratio data obtained from laboratory tests of different types of oil sands tailings.

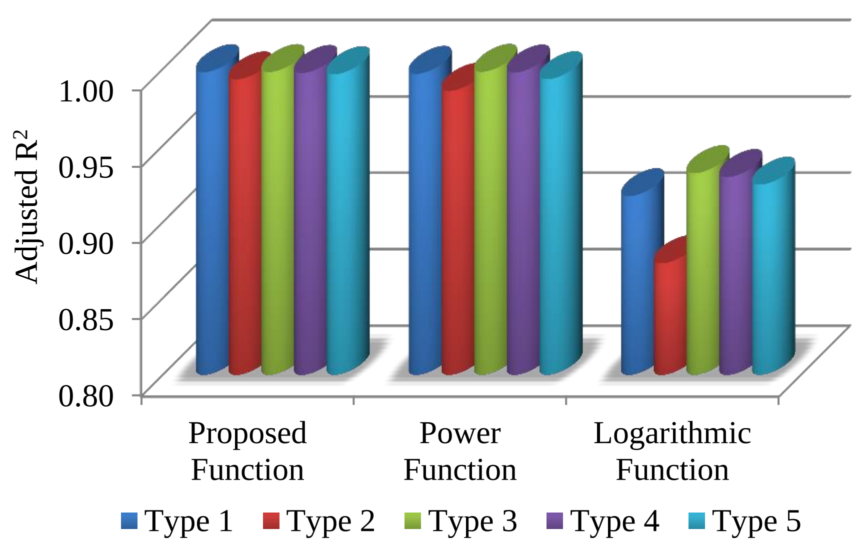

Figure 5.2: Comparison of adjusted R^2 for different types of tailings

The result shows better goodness of fit of the proposed function over power and logarithmic function. However, the Weibull function [Equation 2.11] used by Jeeravipoolvarn for oil sands tailings does not yield any fit for the obtained consolidation test data. Confidence interval plots and curve fitting plots for all of these data are illustrated in Appendix-B.

5.5 Model Validation

This section provides the validation of proposed model with experimental results from literature for different types of tailings based on extraction process, process water and ores from Figure 5.3 to Figure 5.7. Duration of the settling time is listed in Table 5.3 for each type of tailings. All the experimental results for different types of tailings from Ore A and Ore B perfectly matched with the prediction for settlement by the proposed model at the final stages. In this validation part, normalized height was used instead of actual height to compare the 1m standpipe and 2m standpipe tests results.

Table 5.3: Duration of settling time for experiments (modified from Miller et al., 2010b)

Tailings Type		**Type 1**	**Type 2**	**Type 3**	**Type 4**	**Type 5**
Settling Time (Days)	1m Standpipe Test	674	641	641	446	340
	2m Standpipe Test	715	715	722	458	707

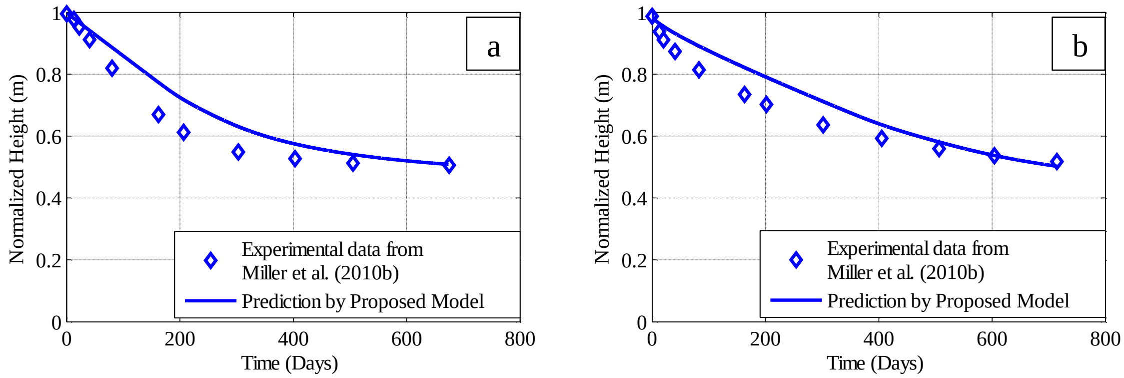

Figure 5.3: Model validation for 1m standpipe (a) and 2m standpipe (b) test of Type 1 tailings

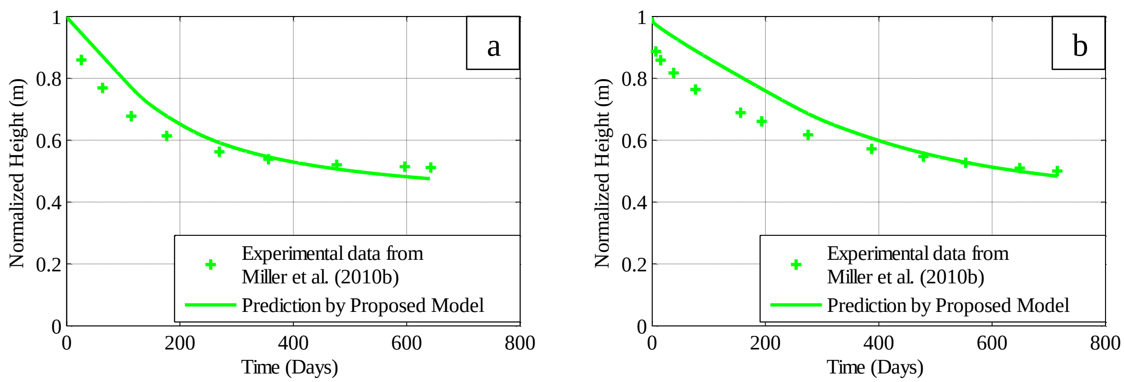

Figure 5.4: Model validation for 1m standpipe (a) and 2m standpipe (b) test of Type 2 tailings

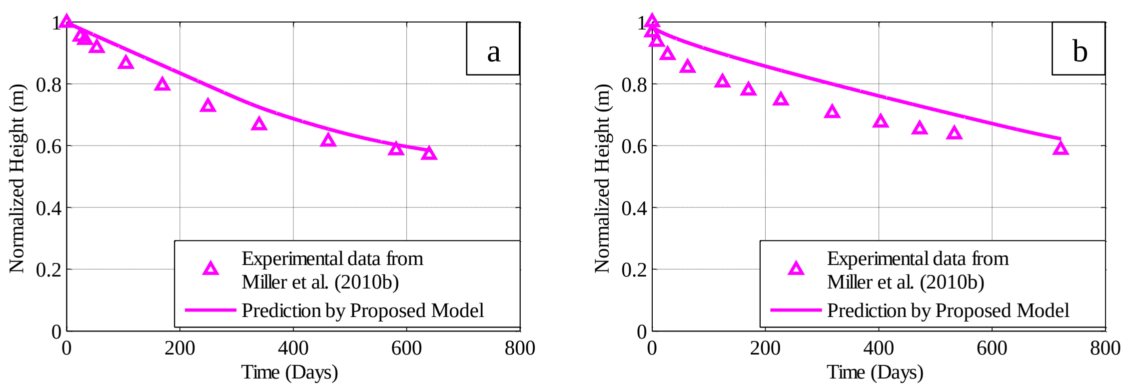

Figure 5.5: Model validation for 1m standpipe (a) and 2m standpipe (b) test of Type 3 tailings

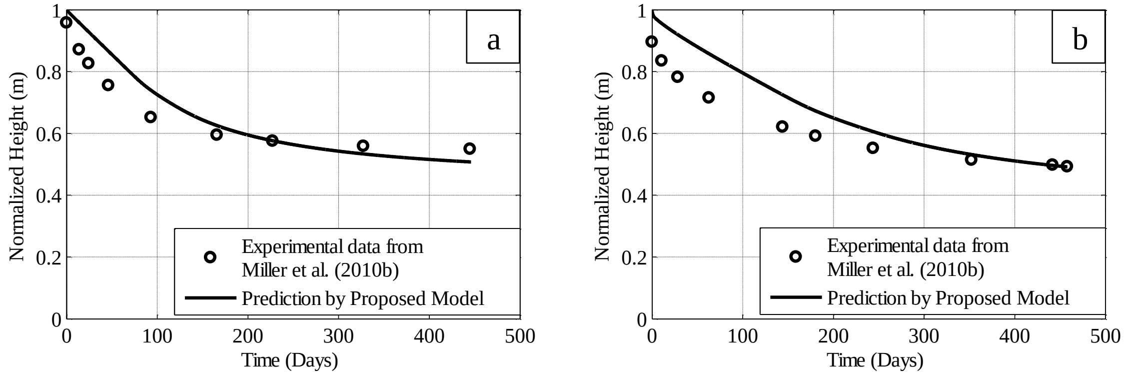

Figure 5.6: Model validation for 1m standpipe (a) and 2m standpipe (b) test of Type 4 tailings

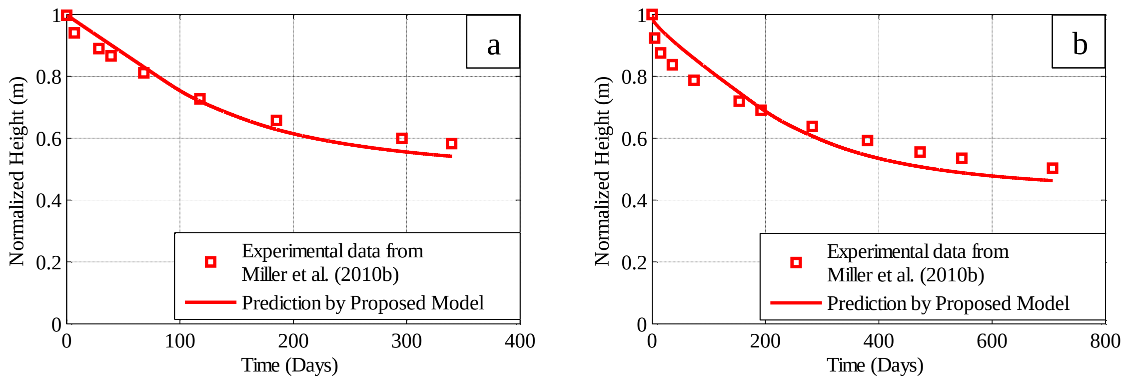

Figure 5.7: Model validation for 1m standpipe (a) and 2m standpipe (b) test of Type 5 tailings

5.6 Numerical Modeling

Numerical modeling for five types of oil sands tailings was performed for this research as a case study to observe the consolidation behaviour after a certain period of time. The modeling parameters are summarized in Table 5.4. The initial conditions have already been listed in Table 5.1. The drainage condition for this study was assumed both single and double to compare the effect of drainage condition on the consolidation behaviour as well. The best fitting curves of compressibility and hydraulic conductivity for all types of tailings are illustrated in Appendix-C.

Table 5.4: Modeling parameters for case study

Tailings Type	Parameters						
	A	B	C (m/s)	D	Z (Pa)	a (Pa)	b
Type 1	-0.0456	0.1873	2.85×10^{-11}	3.73	0.0984	8.98	-0.0470
Type 2	-0.1836	0.1463	1.45×10^{-11}	4.22	0.2019	8.97	-0.0593
Type 3	0.0008	0.2343	2.69×10^{-11}	3.58	0.0643	8.66	-0.0453
Type 4	-0.0021	0.2323	2.59×10^{-11}	4.19	0.0661	8.67	-0.0457
Type 5	-0.0895	0.1471	2.13×10^{-11}	4.16	0.1417	8.87	-0.0525

5.6.1 Settlement Profile

The settlement profiles for all types of tailings produced from Ore A and Ore B are illustrated in Figure 5.8 and Figure 5.9 respectively. Type 1, 2 and 5 belonging to Ore A have identical behaviour in tailings settlement. The types of tailings, originated from non-caustic extraction process (Type 2 and Type 5) settle faster in early stages but become slower with time compared to the tailings having the caustic extraction process (Type 1). However, the double drainage condition slightly improves the settlement profile by consolidating more amounts of tailings within the same period of time than the single drainage condition.

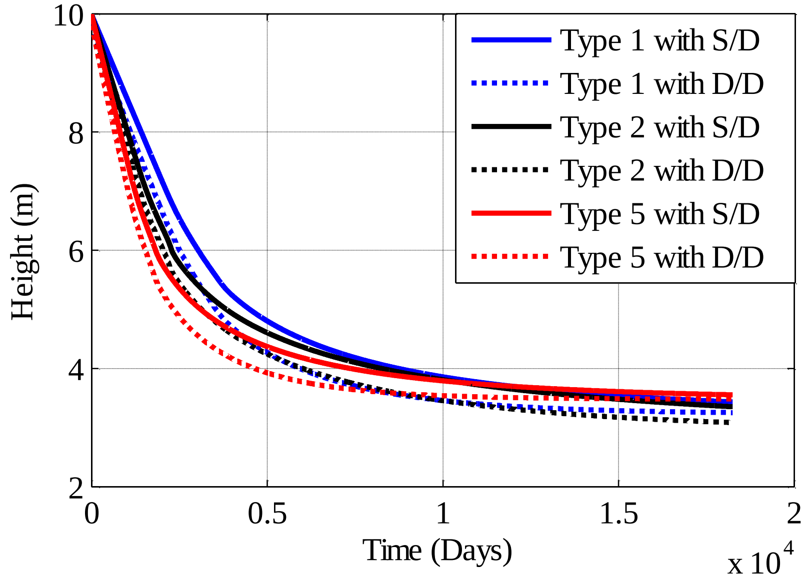

Figure 5.8: Tailings settlement profile of Ore A for 50 years with both single and double drainage condition

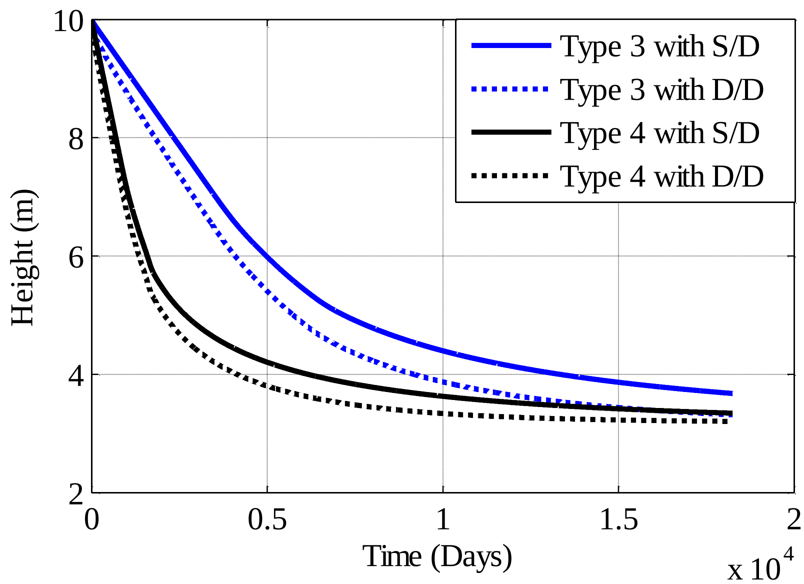

Figure 5.9: Tailings settlement profile of Ore B for 50 years with both single and double drainage condition

Similar findings have been observed for tailings from Ore B, despite that 50 years consolidation period are not sufficient for these tailings to achieve 100% consolidation. Additionally, in this case, the tailings from non-caustic extraction settles much quicker than the tailings from caustic extraction compare to those from Ore A. It should be noted that all the tailings have identical initial solids content with a small variation in specific gravity that yields similar initial void ratio as well as initial condition and affect the settlement at early stages.

Settlement profile is the primary indication of tailings consolidation behaviour and it has influence on all the geotechnical properties as well. From the above observations, it can be concluded that the extraction process and the process water have significant effect on the tailings settlement profile for Ore B than for Ore A.

5.6.2 Void Ratio Profile

Figure 5.10 and Figure 5.11 show the void ratio profiles for all types of tailings. The tailings from both types of ore have small change in void ratio from bottom up to approximately 3 m height for both drainage conditions. Eventually, the void ratio profile for all types of tailings show similar trend though their values are directly linked with initial void ratio and the drainage condition. Similar to the settlement profile, extraction processes affect the void ratio profile more for Ore B compared to Ore A. However, the untreated process water yields the higher void ratio profile than the treated water on tailings consolidation behaviour in terms of void ratio for Ore A.

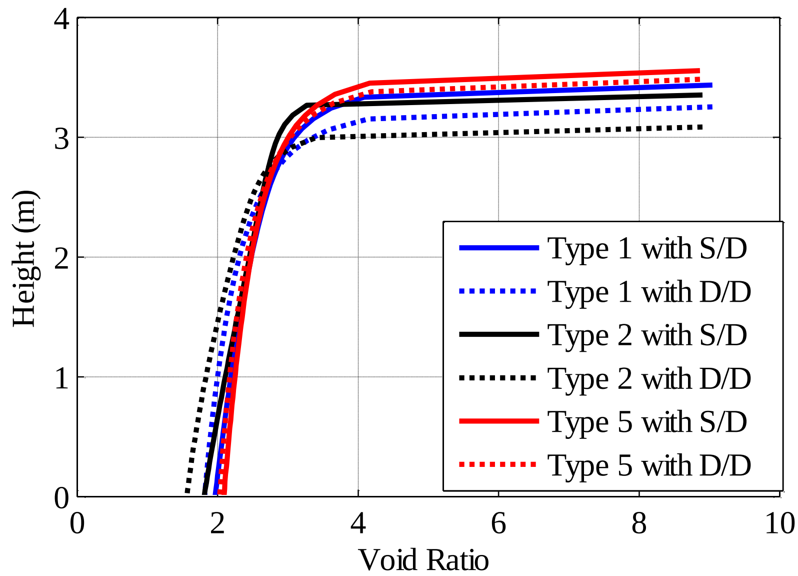

Figure 5.10: Tailings void ratio profile of Ore A for 50 years with both single and double drainage condition

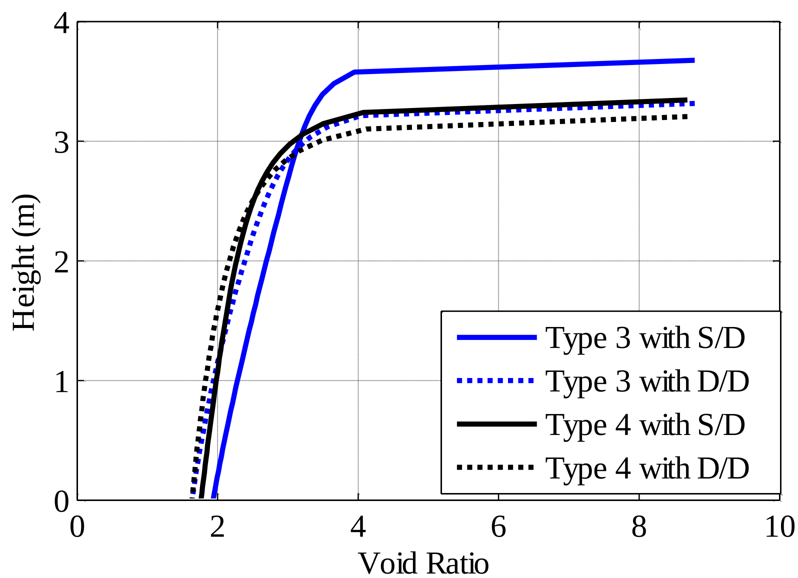

Figure 5.11: Tailings void ratio profile of Ore B for 50 years with both single and double drainage condition

From above analysis it can be concluded that the tailings void ratio profile for both single and double drainage condition predicts the similar trend despite their extraction process or process of water and types of ore.

5.6.3 Excess Pore Water Pressure Profile

The excess pore water pressure profile is the best way to understand whether the consolidation process has been completed or not. It represents the line of the total pore pressure minus the hydrostatic pore pressure. When the excess pore pressure falls to zero through the deposited material, the self-weight consolidation is considered to be completed.

Figure 5.12 and Figure 5.13 present the excess pore pressure profiles after 50 years for tailings originated from Ore A and Ore B having single and double drainage boundary condition. For tailings from Ore A, only double drainage of Type 5 tailings achieved the fully consolidated state and there will be no further settlement in this case. Other than that, all tailings are still in a process of consolidation and there is no significant effect of extraction process. However, dissipation rate is faster in both drainage conditions for Untreated Athabasca River Water (UARW) compared to other process waters of tailings from Ore A.

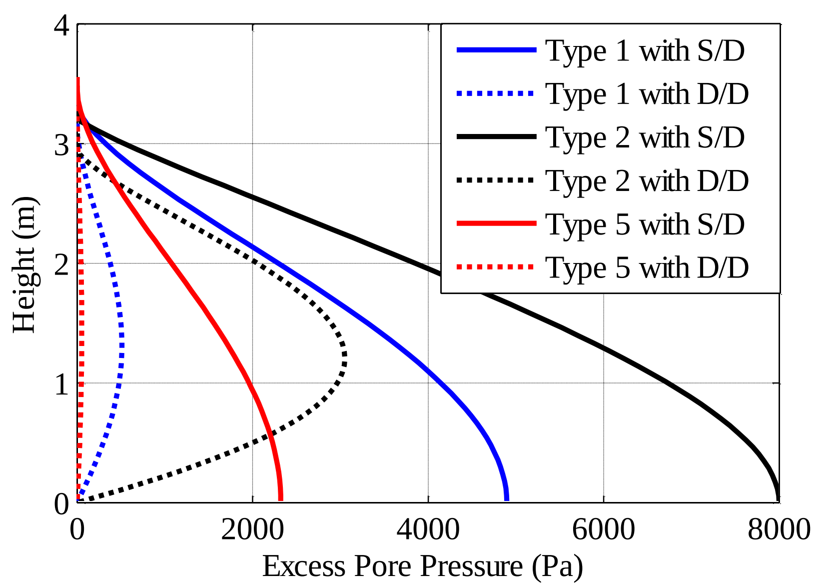

Figure 5.12: Tailings excess pore water pressure profile of Ore A for 50 years with both single and double drainage condition

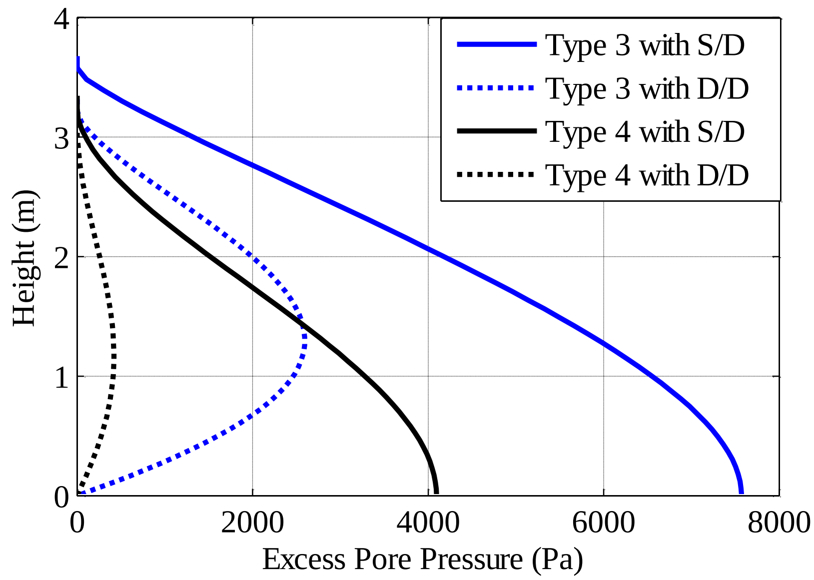

Figure 5.13: Tailings excess pore water pressure profile of Ore B for 50 years with both single and double drainage condition

Similar findings have been observed for tailings from Ore B. In this case, caustic extraction process yields the slower dissipation rate than the non-caustic extraction process in both drainage conditions.

5.6.4 Effective Stress Profile

The effective stress profiles for this case study are illustrated in Figure 5.14 and Figure 5.15 for tailings originated from different ores; Ore A and Ore B. Like other geotechnical properties, all types of tailings show the similar trend for effective stress calculation using this one dimensional consolidation model. In fact, the effective stress profile represents the opposite characteristics of excess pore water pressure profile as the total stress remains the same after the end of consolidation at each level due to the consideration of self-weight consolidation.

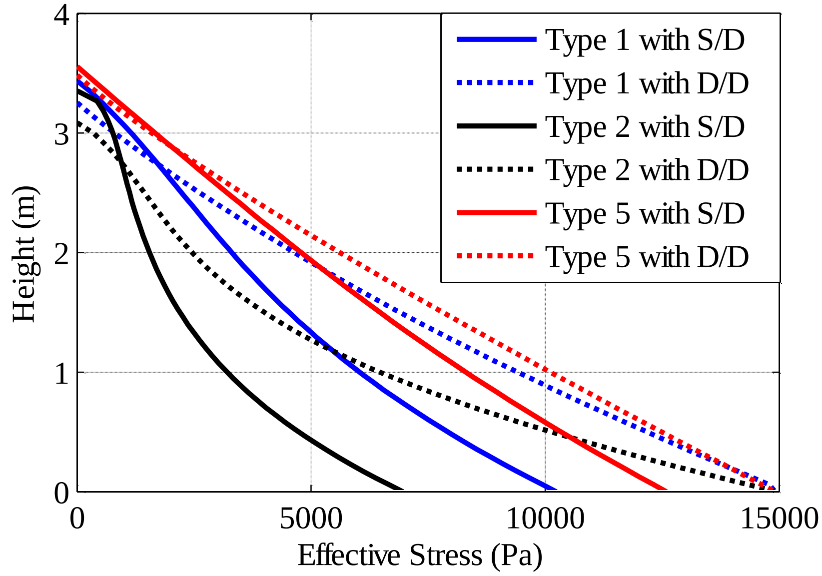

Figure 5.14: Tailings effective stress profile of Ore A for 50 years with both single and double drainage condition

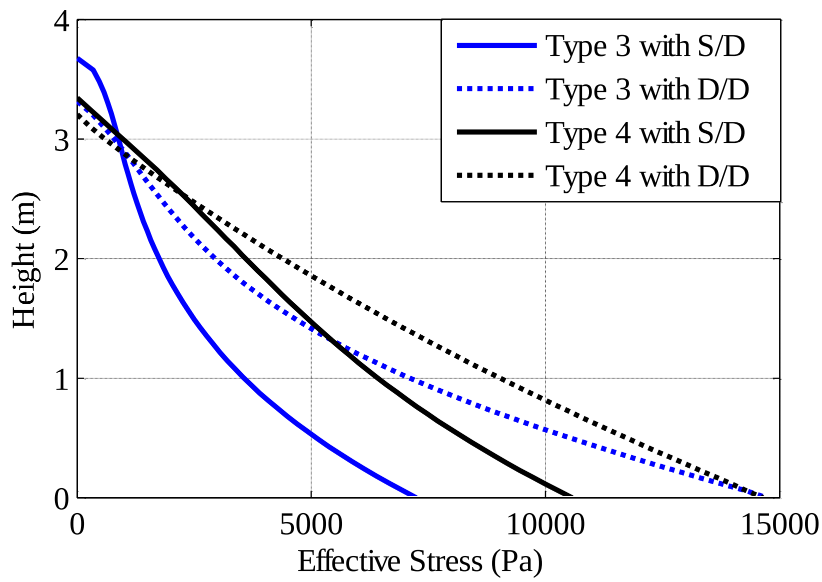

Figure 5.15: Tailings effective stress profile of Ore B for 50 years with both single and double drainage condition

There is no significant effect of extraction process for tailings from different ores though in both cases for both drainage conditions each of the tailings, Type 2 for Ore A and Type 3 for Ore B, shows drastic change in effective stress profile with respect to height while other types of tailings show constant change along the depth of the tailings impoundments. This may become a concern during the land reclamation operation.

5.7 Summary

The results and discussions in this chapter provide strong confidence to predict tailings consolidation behaviour using the proposed model. The case study for different types of tailings with both single and double drainage condition presents the tailings consolidation behaviour in terms of settlement, void ratio, excess pore water pressure and effective stress at a settling period of 50 years. These predictions conclude with similar findings from Miller et al. (2010b) by laboratory investigations.

6 CONCLUSIONS AND RECOMMENDATIONS

6.1 General

Tailings are produced in vast amounts each year and possess a serious threat to the environment. Effective handling of tailings by applying proper consolidation techniques can improve the scenario. Prediction of tailings consolidation behaviour could be an important tool for ensuring appropriate management of the tailings impoundment. It can also serve for tailings stream design to accelerate the rate of consolidation as well as the process of land reclamation. The focus of this study was the long term consolidation prediction using a one dimensional consolidation model. The model seems to be sound both technically and statistically by screening through validation, verification and statistical analysis during the theoretical development of the model. Finally, two case studies demonstrate the application of the model with the help of sensitivity analysis, comparison between models and comparison among different types of tailings.

6.2 Conclusions

This study was carried out for quiescent condition by developing a strong mathematical background and a solid statistical foundation. The combination of these two important parts of scientific research provides significant impacts on tailings settlement calculation. In-depth reviews on past works aid to obtain an adequate scope of research in this field of study. Several conclusions can be drawn from the overall work as listed below:

- Large strain consolidation theory introduced by Gibson et al. (1967) laid the basic foundation to calculate the consolidation behaviour for soft soil like tailings.

- A one dimensional model is adequate enough to predict the tailings consolidation behaviour for most of the tailings impoundment. A two and three dimensional model mainly assists to understand the mechanism in multidimensional aspects which was not the objective of this study.
- The research primarily focuses on the self-weight consolidation where self-weight of tailings is the key component for consolidation process.
- Non-linearity of tailings compressibility occurs at a low effective stress zone and a sophisticated apparatus is the pre-requisite to obtain the void ratio data at the low effective stress zone as well as to measure hydraulic conductivity precisely.
- Predictions of consolidation behaviour by different models yield similar behaviour though the present model improves the prediction quality by improving the goodness of fit of the compressibility function of tailings.

6.3 Contributions

This study has significant contributions to the field of geotechnical engineering. Consolidation behaviour plays an important role not only for the tailings, also with other soft soils like dredged material or landfill waste where non-linear behaviour of high compressibility involves. The contributions can be listed as:

- The proposed compressibility equation is applicable to any type of tailings to express the compressibility behaviour.
- The developed one dimensional consolidation model can be used as a preliminary investigation tools by the tailings management facility.

- Other than tailings, the developed model could be suitable for the prediction of consolidation behaviour of soft soils like dredged material due to its high compressibility behaviour.

6.4 Recommendations for Future Works

This study creates the opportunity for future works by adding more features to the model to increase acceptance among the tailings industries. Some of the recommendations for future works have been summarized below:

- Detailed study on apparent pre-consolidation will help to improve the prediction of tailings consolidation behaviour.
- Interpretation of curve fitted parameters with material properties will offer a new approach to predict without having laboratory investigations.
- This model is developed only for quiescent condition. Future research on the staged filling condition will support increasing the applicability of the model.
- Single drainage and double drainage problems can be solved using this model due to the consideration of the one dimensional effect. A further development for lateral drainage condition can be made for the multi-dimensional model.

6.5 Summary

Consolidation behaviour prediction for the long-term is a challenging task because of the effect of several factors. Development of this proposed model incorporates those effects by best fitting curves of the constitutive relationships of hydraulic conductivity and compressibility plots. A few input parameters with simplified initial and boundary condition make this model a useful tool for preliminary investigation by tailings industry for predicting the in-situ consolidation behaviour over a large number of settling periods. The proposed model shows significant variations over conventional model in terms of physical and geotechnical properties of tailings though the quality experimental and field data are required to match the model with the real world. Other than that, considering apparent over-consolidation behaviour during the early stage of the deposition helps to formulate the model more precisely.

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APPENDICES

Appendix-A

This appendix provides the statistical analysis of published volume compressibility data in the literature for tailings in the form of tables and figures.

Table A.1: Volume compressibility data for different types of tailings (Qiu and Sego, 2001)

Effective Stress (Pa)	**Void Ratio**			
	Tailings Type			
	Coal Wash	Composite	Gold	Copper
501.71	1.55	1.14	1.03	0.95
2002.45	1.31	0.83	0.99	0.89
3972.82	1.18	0.75	0.83	0.87
9781.84	1.01	0.64	0.79	0.83
19678.59	0.91	0.59	0.77	0.81
49076.19	0.80	0.53	0.72	0.77
98612.94	0.69	0.48	0.69	0.71

Two-way ANOVA: Void Ratio versus Effective Stress for Tailings Type

```
Source              DF       SS        MS      F      P
Effective Stress     6  0.80815  0.134692  12.37  0.000
Tailings Type        3  0.46382  0.154607  14.20  0.000
Error               18  0.19605  0.010891
Total               27  1.46802

S = 0.1044   R-Sq = 86.65%   R-Sq(adj) = 79.97%
```

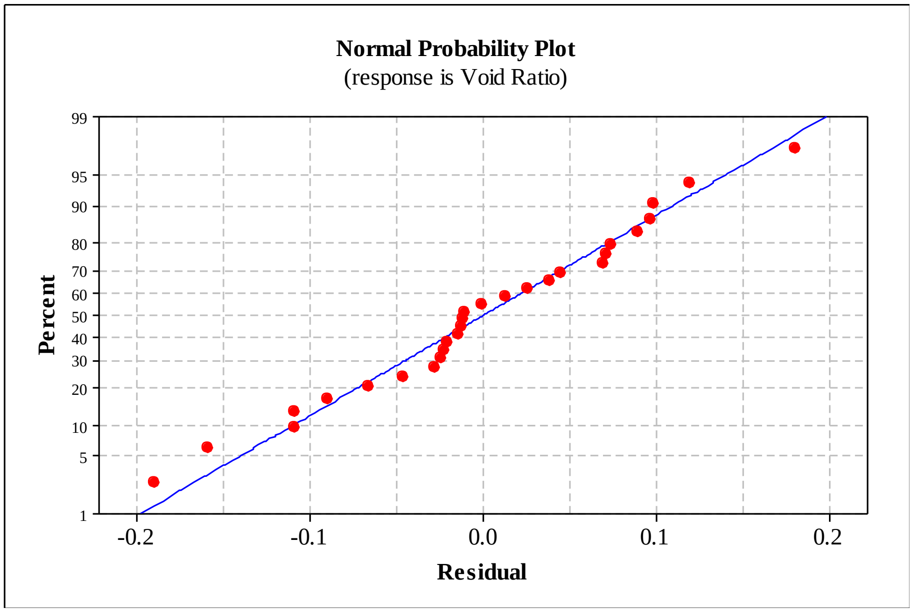

Figure A.1: Normal probability plot checking normality for assumptions of ANOVA of different types of tailings

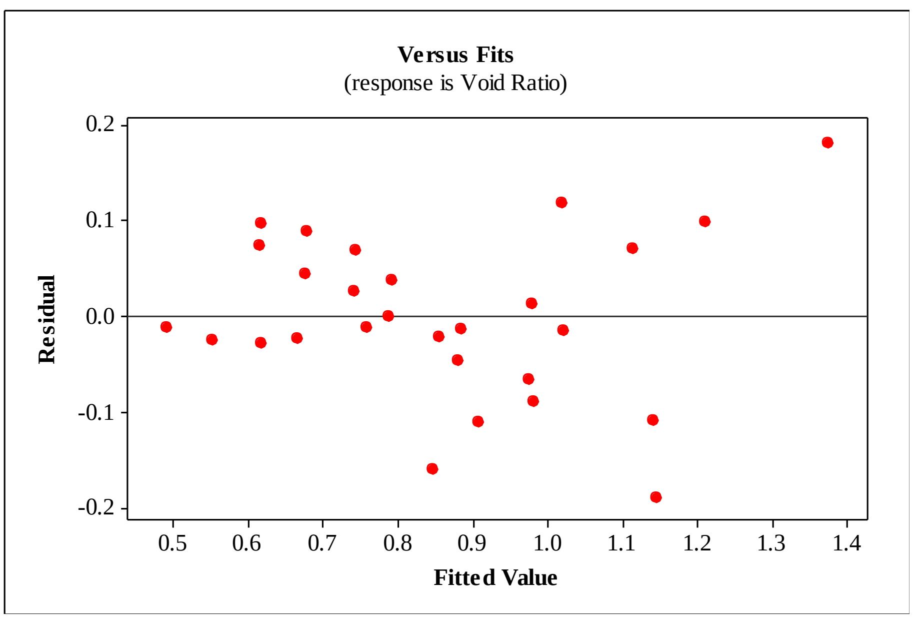

Figure A.2: Residual vs. observation value plot checking constant variance for assumptions of ANOVA of different types of tailings

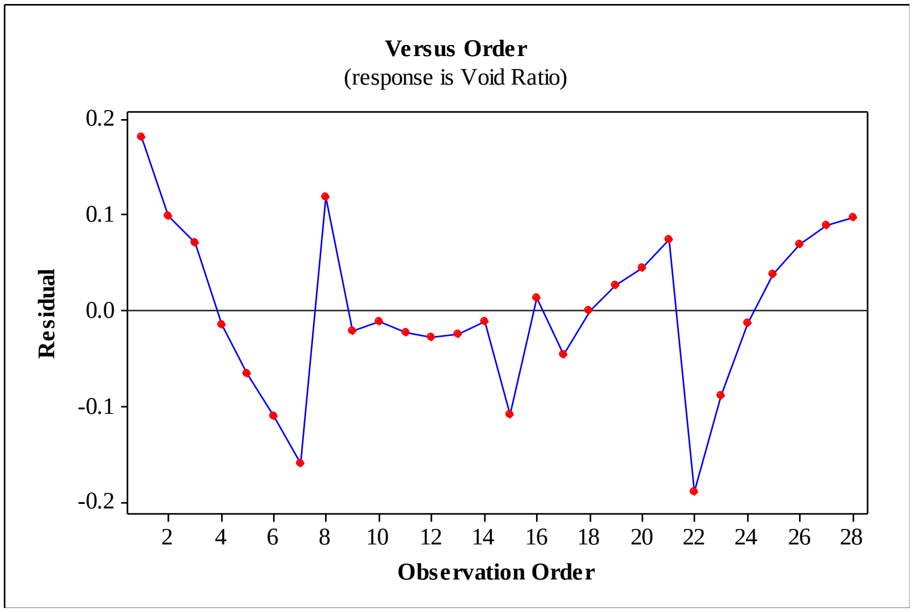

Figure A.3: Residual vs. run order plot checking independence for assumptions of ANOVA of different types of tailings

Table A.2: Volume compressibility data for tailings at different solids content (Proskin et al., 2010)

Effective Stress (Pa)	Void Ratio				
	Solids Content				
	20%	25%	30%	31%	42%
250	7.50	6.40	5.00	3.70	3.05
500	7.20	6.00	4.60	3.50	3.00
1000	6.80	5.30	3.90	3.20	2.90
2000	5.80	4.20	2.65	2.65	2.50
4000	4.65	3.05	2.00	2.40	2.15
8000	3.15	2.25	1.60	2.00	1.90
16000	2.25	1.90	1.35	1.75	1.50
32000	1.70	1.40	1.05	1.05	1.05
100000	1.15	1.05	0.85	0.75	0.80

Two-way ANOVA: Void Ratio versus Effective Stress for Solids Content

```
Source            DF       SS       MS      F      P
Effective Stress   8   99.860  12.4826  23.90  0.000
Solids Content     4   34.815   8.7037  16.66  0.000
Error             32   16.716   0.5224
Total             44  151.391

S = 0.7228   R-Sq = 88.96%   R-Sq(adj) = 84.82%
```

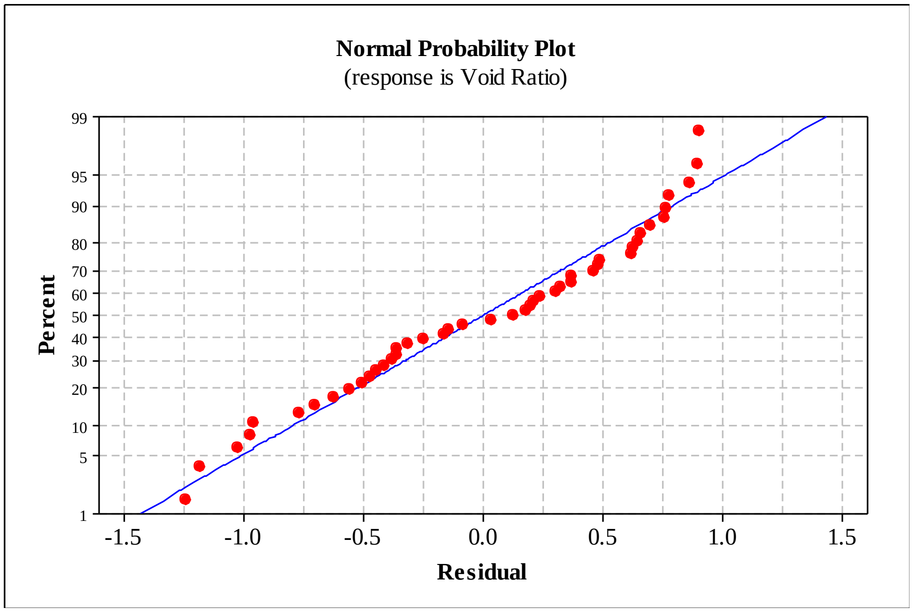

Figure A.4: Normal probability plot checking normality for assumptions of ANOVA of tailings at different solids content

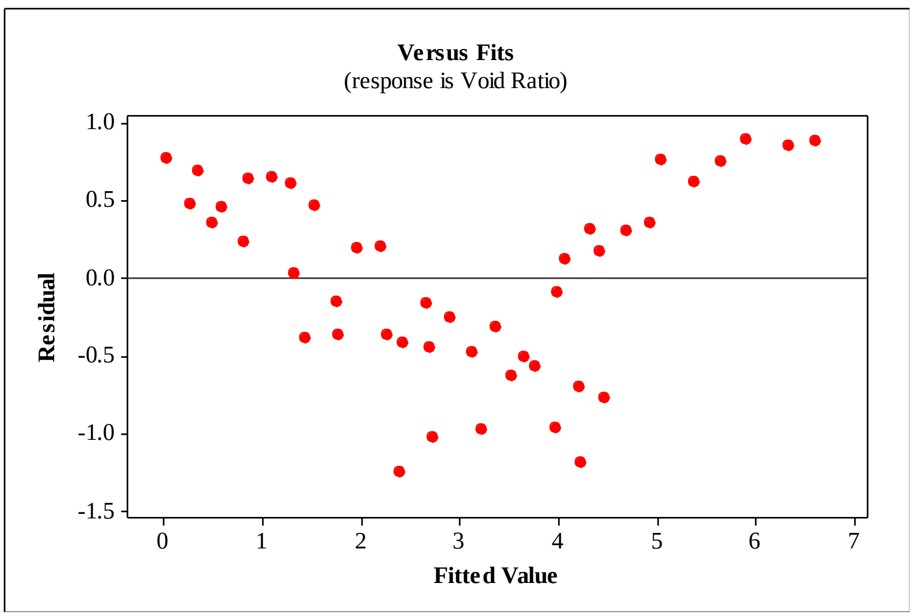

Figure A.5: Residual vs. observation value plot checking constant variance for assumptions of ANOVA of tailings at different solids content

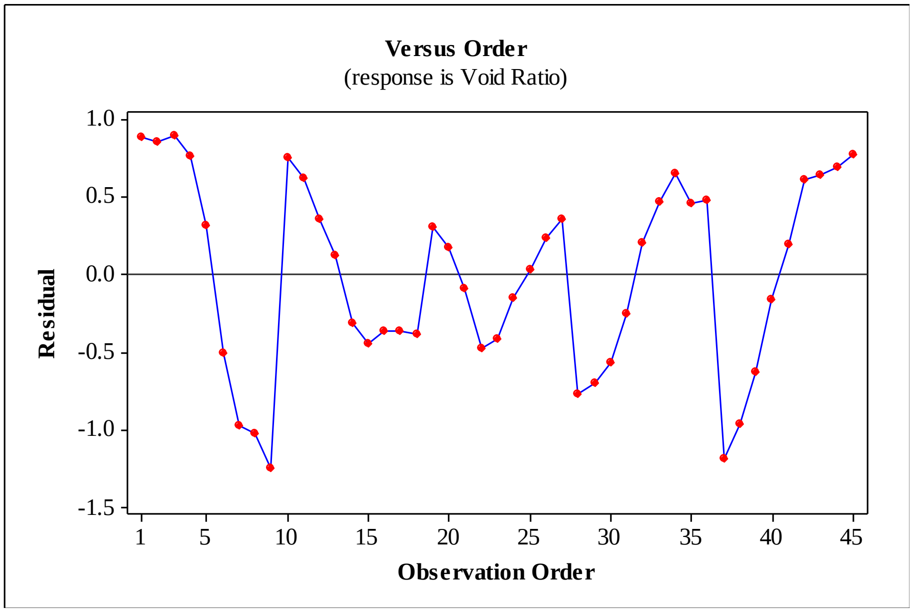

Figure A.6: Residual vs. run order plot checking independence for assumptions of ANOVA of tailings at different solids content

Table A.3: Volume compressibility data for different testing procedures of tailings (Fox and Baxter, 1997)

Effective Stress (Pa)	**Void Ratio**	
	Testing Procedure	
	Hydraulic Consolidation Test	Step Loading Consolidation Test
1000	4.00	4.20
2000	3.85	4.05
4000	3.55	3.65
8000	3.35	3.35

Two-way ANOVA: Void Ratio versus Effective Stress for Testing Procedures

```
Source            DF       SS        MS      F      P
Effective Stress   3  0.69000  0.230000  50.18  0.005
Test               1  0.03125  0.031250   6.82  0.080
Error              3  0.01375  0.004583
Total              7  0.73500
S = 0.06770   R-Sq = 98.13%   R-Sq(adj) = 95.63%
```

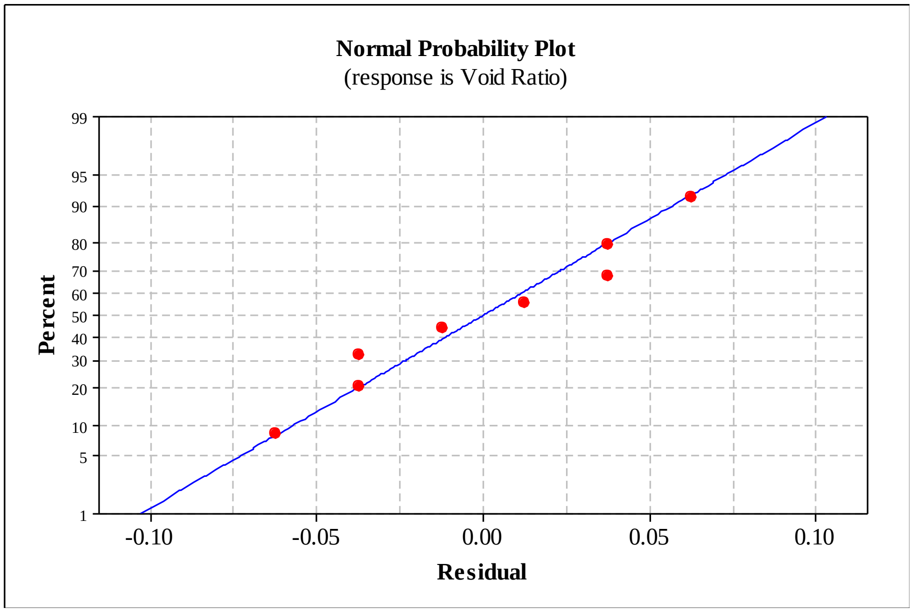

Figure A.7: Normal probability plot checking normality for assumptions of ANOVA of different testing procedures of tailings

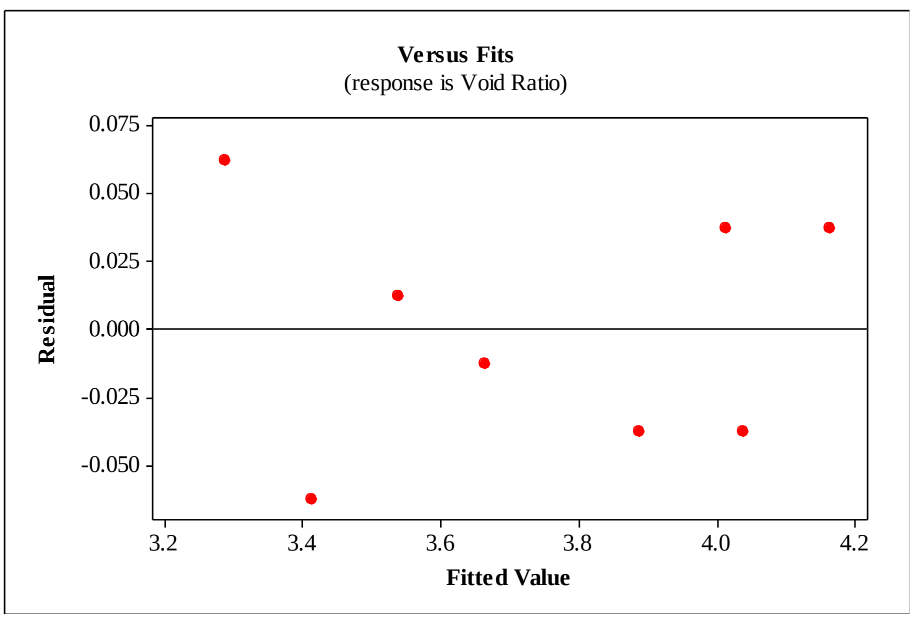

Figure A.8: Residual vs. observation value plot checking constant variance for assumptions of ANOVA of different testing procedures of tailings

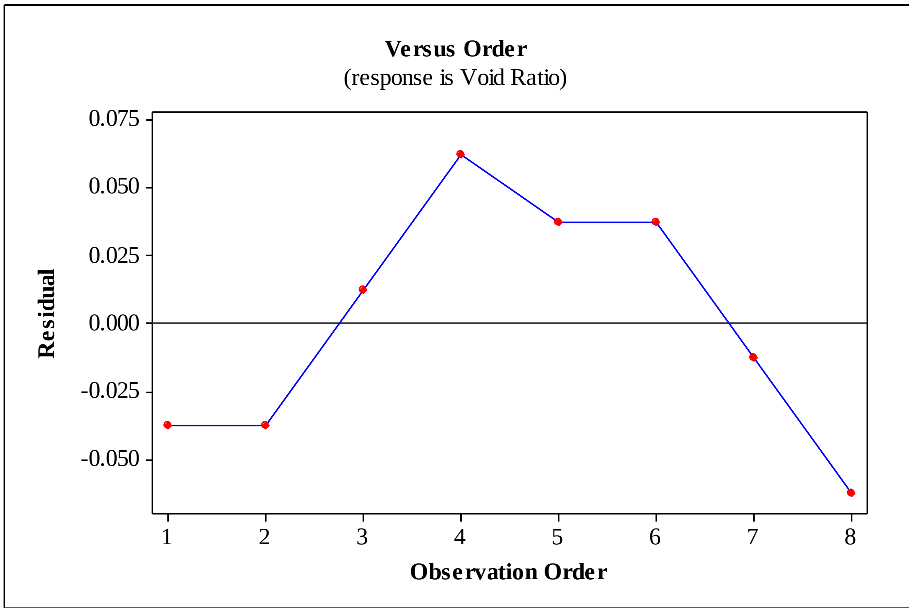

Figure A.9: Residual vs. run order plot checking independence for assumptions of ANOVA of different testing procedures of tailings

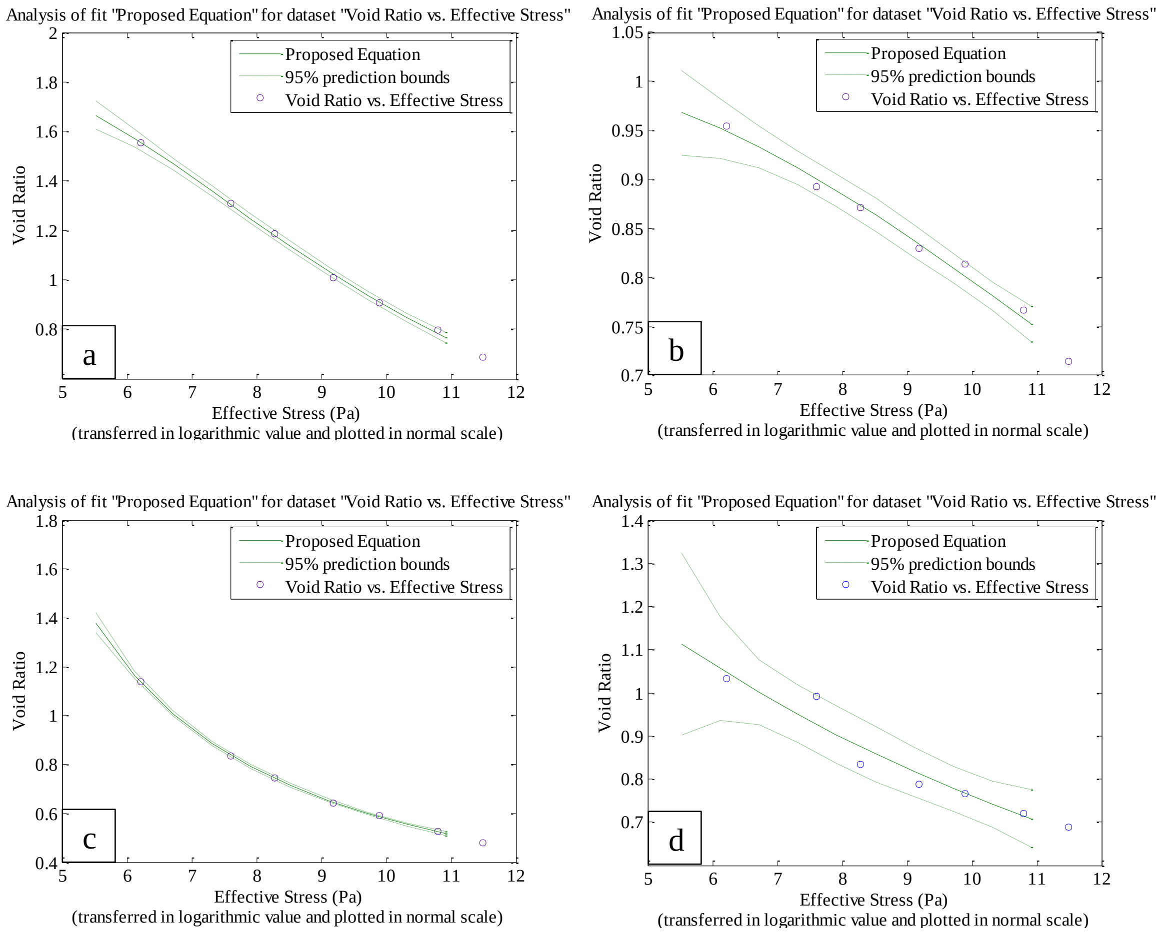

Figure A.10: Prediction bounds plot at 95% confidence of the proposed equation for Coal Wash Tailings (a), Composite Tailings (b), Gold Tailings (c) and Copper Tailings

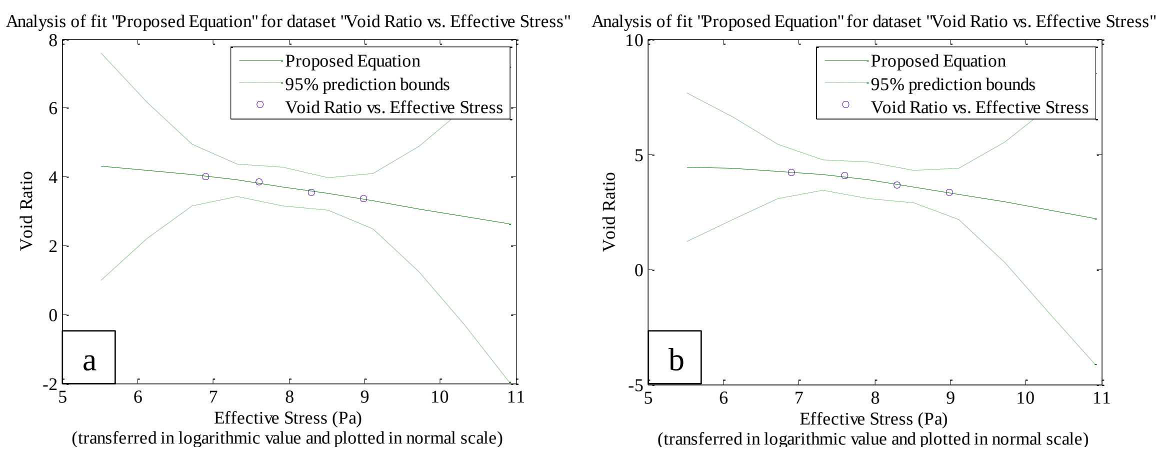

Figure A.11: Prediction bounds plot at 95% confidence of the proposed equation for tailings with Hydraulic Consolidation test (a) and Step Loading Consolidation Test (b)

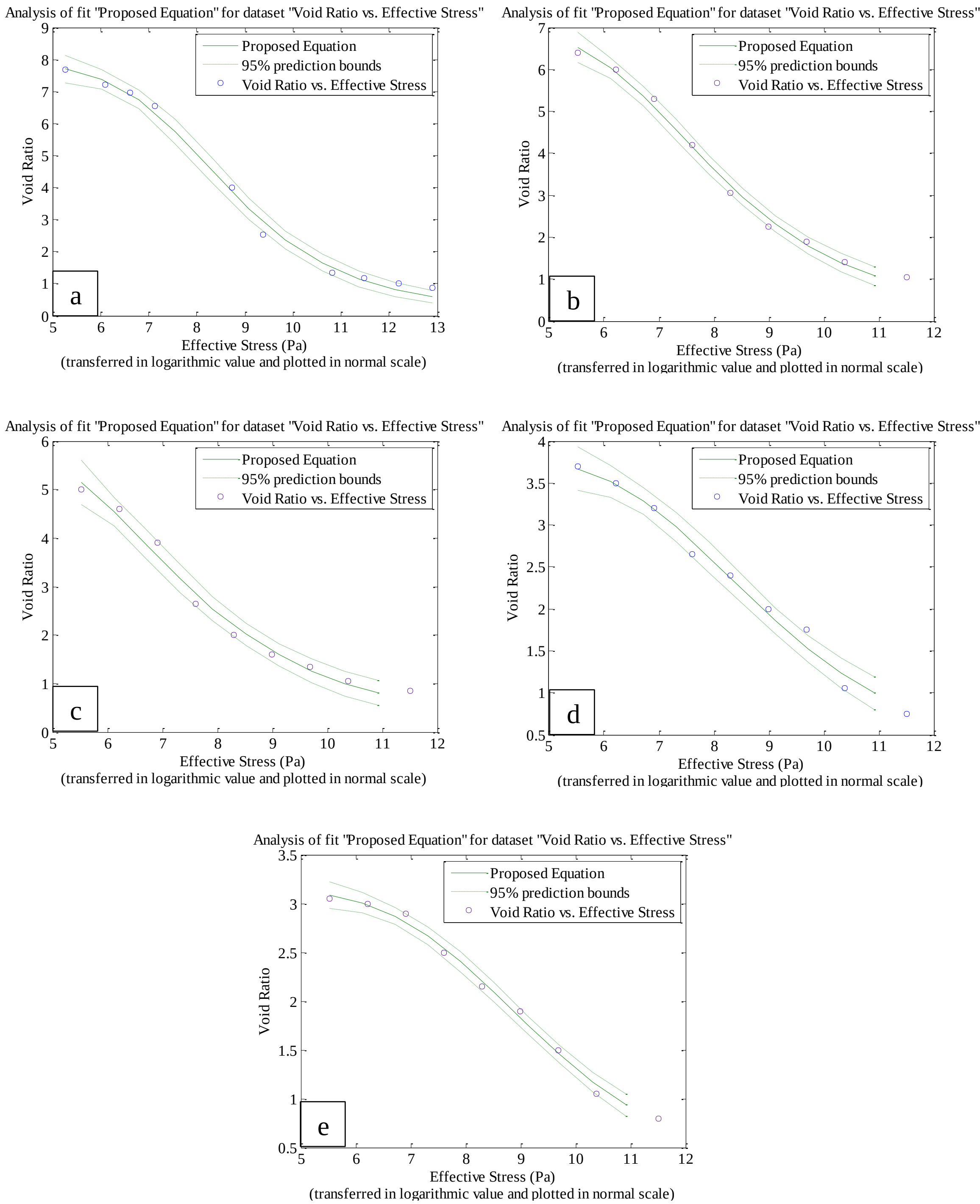

Figure A.12: Prediction bounds plot at 95% confidence of the proposed equation for tailings having solids content of 20% (a), 25% (b), 30% (c), 31% (d) and 42% (e)

Appendix-B

This appendix provides the statistical analysis for case study using published volume compressibility data in the literature for tailings in the form of tables and figures.

Table B.1: Volume compressibility data for different types of tailings based on extraction process, process water and ore (Miller et al., 2010b)

Effective Stress (Pa)	**Void Ratio**				
	Tailings Type				
	Type 1	Type 2	Type 3	Type 4	Type 5
20	7.80	7.51	7.56	7.56	7.58
40	6.79	6.91	6.68	6.68	6.67
100	3.91	4.72	4.88	4.88	5.21
300	3.49	3.35	3.96	3.96	4.11
700	3.14	2.79	3.34	3.26	3.45
2000	2.72	2.23	2.67	2.67	2.98
5000	2.33	1.86	2.18	2.18	2.49
10000	2.00	1.74	1.79	1.79	2.26
20000	1.79	1.58	1.51	1.51	2.04
40000	1.49	1.40	1.29	1.29	1.76
80000	1.21	1.21	1.13	1.13	1.32

Two-way ANOVA: Void Ratio versus Effective Stress for Different Ore Types

```
Source              DF       SS       MS       F      P
Effective Stress    10  232.731  23.2731  513.99  0.000
Ore Types            4    1.012   0.2531    5.59  0.001
Error               40    1.811   0.0453
Total               54  235.554
S = 0.2128   R-Sq = 99.23%   R-Sq(adj) = 98.96%
```

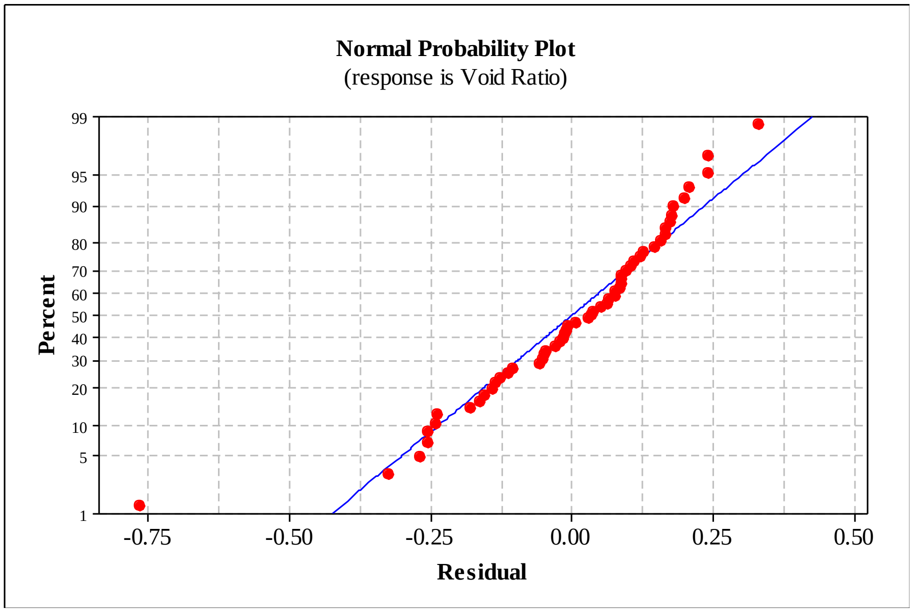

Figure B.1: Normal probability plot checking normality for assumptions of ANOVA of different types of tailings based on extraction process, process water and ore

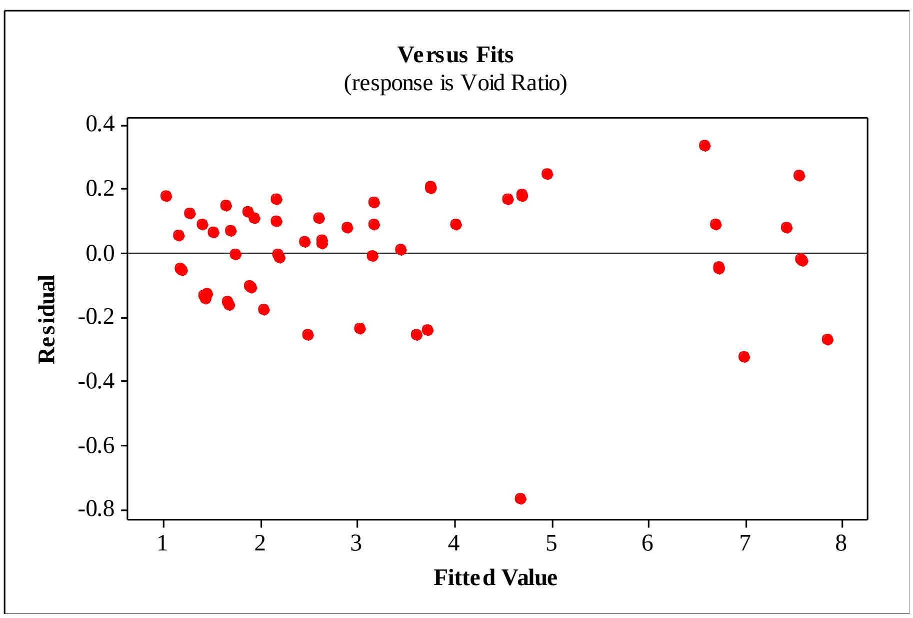

Figure B.2: Residual vs. observation value plot checking constant variance for assumptions of ANOVA of different types of tailings based on extraction process, process water and ore

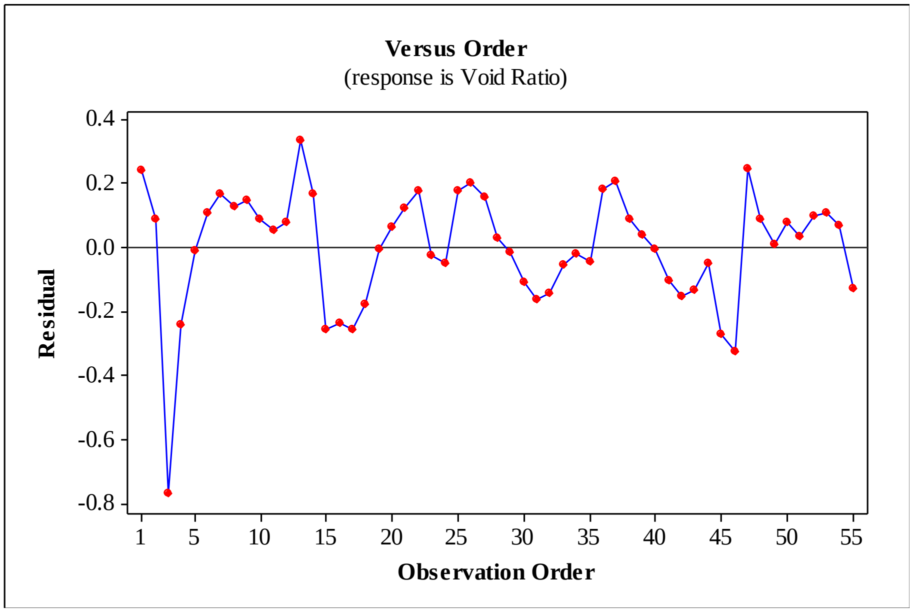

Figure B.3: Residual vs. run order plot checking independence for assumptions of ANOVA of different types of tailings based on extraction process, process water and ore

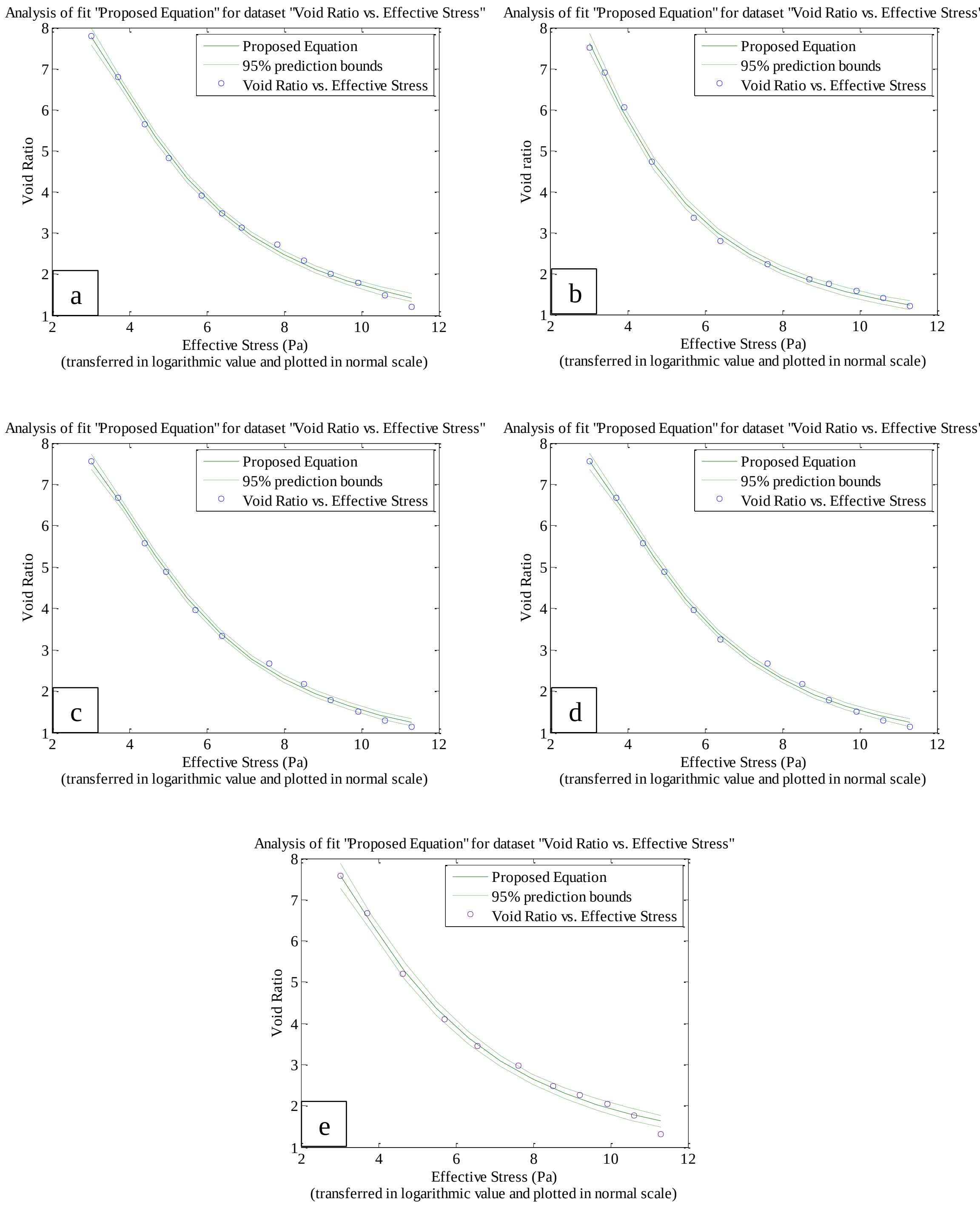

Figure B.4: Prediction bounds plot at 95% confidence of the proposed equation for tailings Type 1 (a), Type 2 (b), Type 3 (c), Type 4 (d) and Type 5 (e)

Appendix-C

This appendix provides the volume compressibility and hydraulic conductivity behaviour from literature for different types of tailings based on extraction process, process water and ores in the form of tables and figures.

Table C.1: Volume compressibility data in the literature for different types of tailings (Miller et al., 2010b)

Type 1		Type 2		Type 3		Type 4		Type 5	
Effective Stress (Pa)	Void Ratio	Effective Stress (Pa)	Void Ratio	Effective Stress (Pa)	Void Ratio	Effective Stress (Pa)	Void Ratio	Effective Stress (Pa)	Void Ratio
20	7.8	20	7.51	20	7.56	20	7.56	20	7.58
40	6.79	30	6.91	40	6.68	40	6.68	40	6.67
80	5.65	50	6.05	80	5.58	80	5.58	100	5.21
150	4.83	100	4.72	140	4.88	140	4.88	300	4.11
350	3.91	300	3.35	300	3.96	300	3.96	700	3.45
600	3.49	600	2.79	600	3.34	600	3.26	2000	2.98
1000	3.14	2000	2.23	2000	2.67	2000	2.67	5000	2.49
2500	2.72	6000	1.86	5000	2.18	5000	2.18	10000	2.26
5000	2.33	10000	1.74	10000	1.79	10000	1.79	20000	2.04
10000	2	20000	1.58	20000	1.51	20000	1.51	40000	1.76
20000	1.79	40000	1.4	40000	1.29	40000	1.29	80000	1.32
40000	1.49	80000	1.21	80000	1.13	80000	1.13		
80000	1.21								

Table C.2: Hydraulic conductivity data in the literature for different types of tailings (Miller et al., 2010b)

Type 1		Type 2		Type 3		Type 4		Type 5	
Hydraulic Conductivity (m/s)	Void Ratio	Hydraulic Conductivity (m/s)	Void Ratio	Hydraulic Conductivity (m/s)	Void Ratio	Hydraulic Conductivity (m/s)	Void Ratio	Hydraulic Conductivity (m/s)	Void Ratio
4.00E-11	1.1	2.00E-11	1.08	4.00E-11	1.12	6.00E-11	1.24	3.00E-11	1.1
1.00E-10	1.39	1.00E-10	1.59	2.00E-10	1.74	2.00E-10	1.61	8.00E-11	1.37
3.00E-10	1.88	3.00E-10	2.05	1.00E-09	2.73	1.00E-09	2.39	4.00E-10	2.01
1.00E-09	2.62	1.00E-09	2.72	3.00E-09	3.76	4.00E-09	3.3	1.00E-09	2.51
3.00E-09	3.49	3.00E-09	3.49	1.00E-08	5.25	1.00E-08	4.14	3.00E-09	3.3
1.00E-08	4.78	8.00E-09	4.45	3.00E-08	7.06	3.00E-08	5.38	1.00E-08	4.39
3.00E-08	6.46	2.00E-08	5.6	6.00E-08	8.59	7.00E-08	6.58	3.00E-08	5.68
8.00E-08	8.39	4.00E-08	6.56	9.00E-08	9.6	1.00E-07	7.17	7.00E-08	7.04
1.00E-07	9.02	7.00E-08	7.48			2.00E-07	8.55	2.00E-07	9.1
		2.00E-07	9.59						

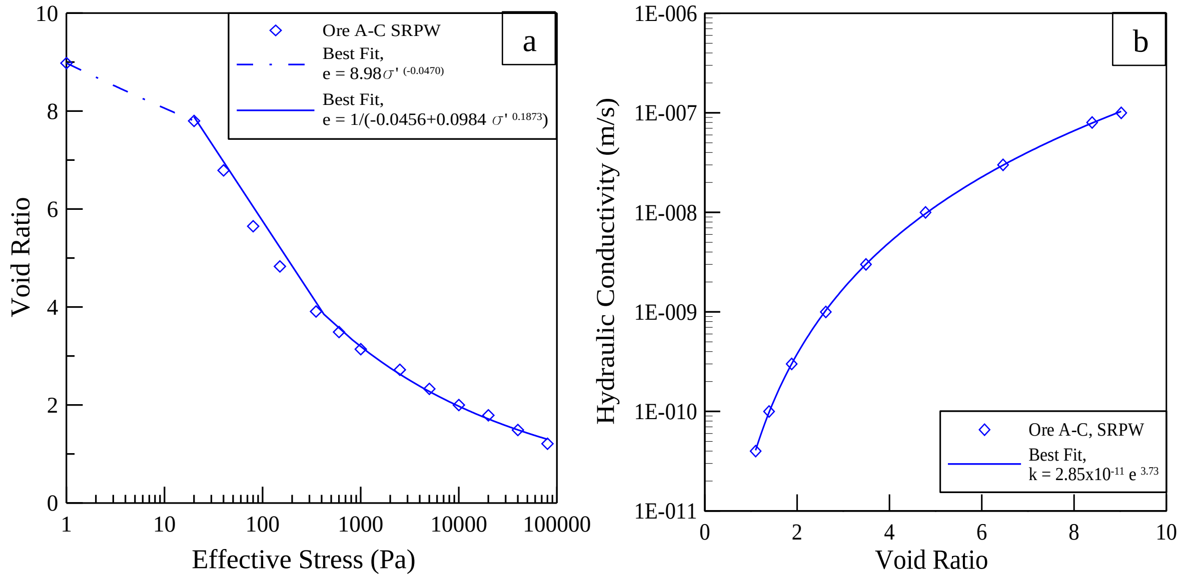

Figure C.1: Volume compressibility (a) and hydraulic conductivity (b) plot of tailings Type 1

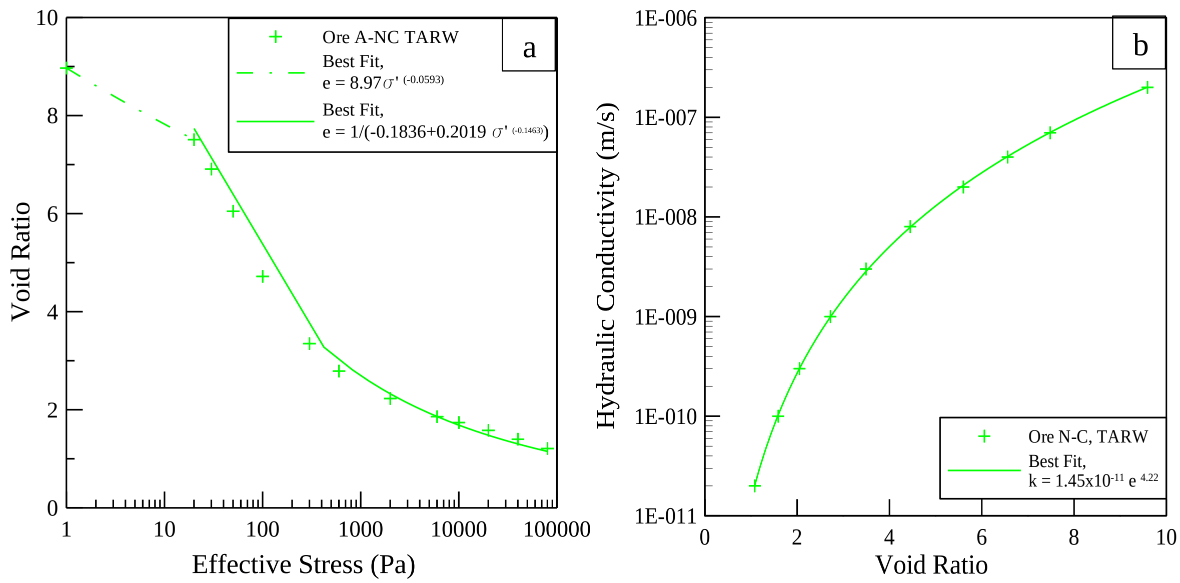

Figure C.2: Volume compressibility (a) and hydraulic conductivity (b) plot of tailings Type 2

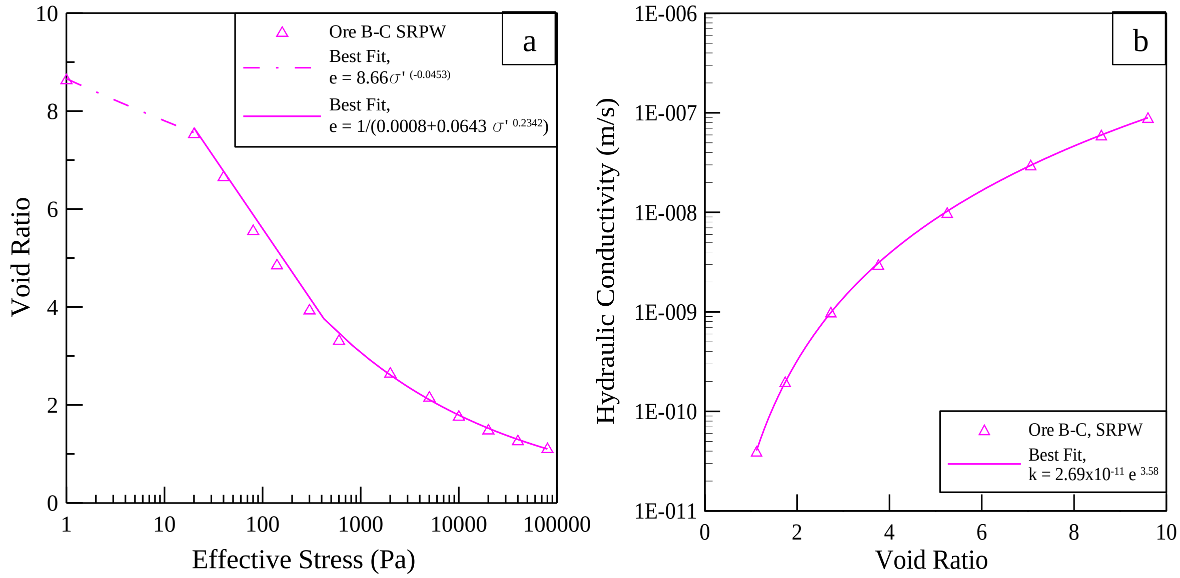

Figure C.3: Volume compressibility (a) and hydraulic conductivity (b) plot of tailings Type 3

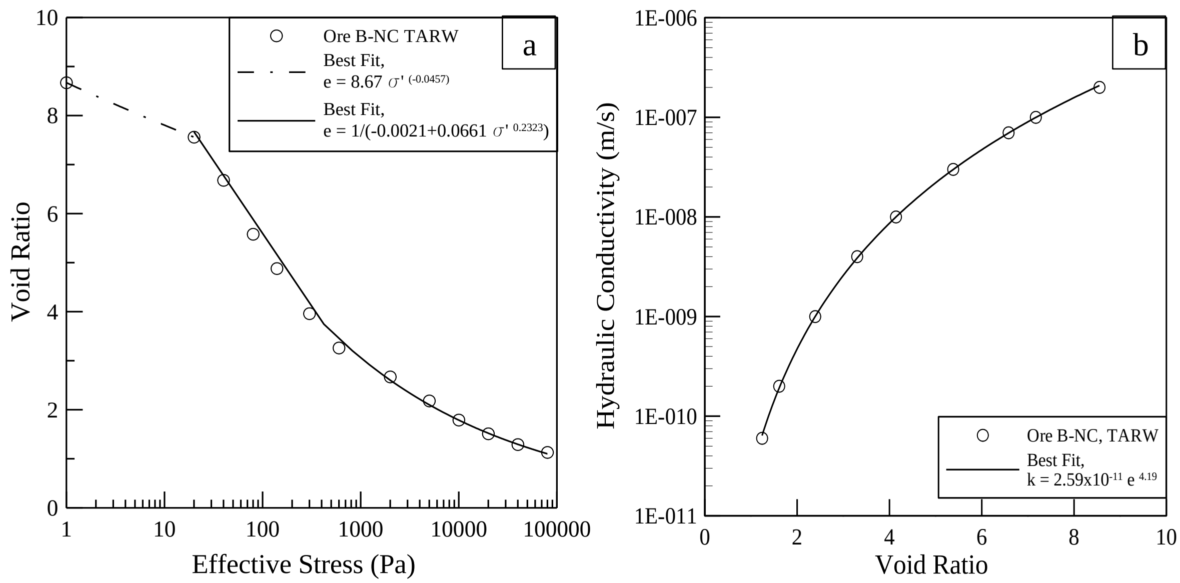

Figure C.4: Volume compressibility (a) and hydraulic conductivity (b) plot of tailings Type 4

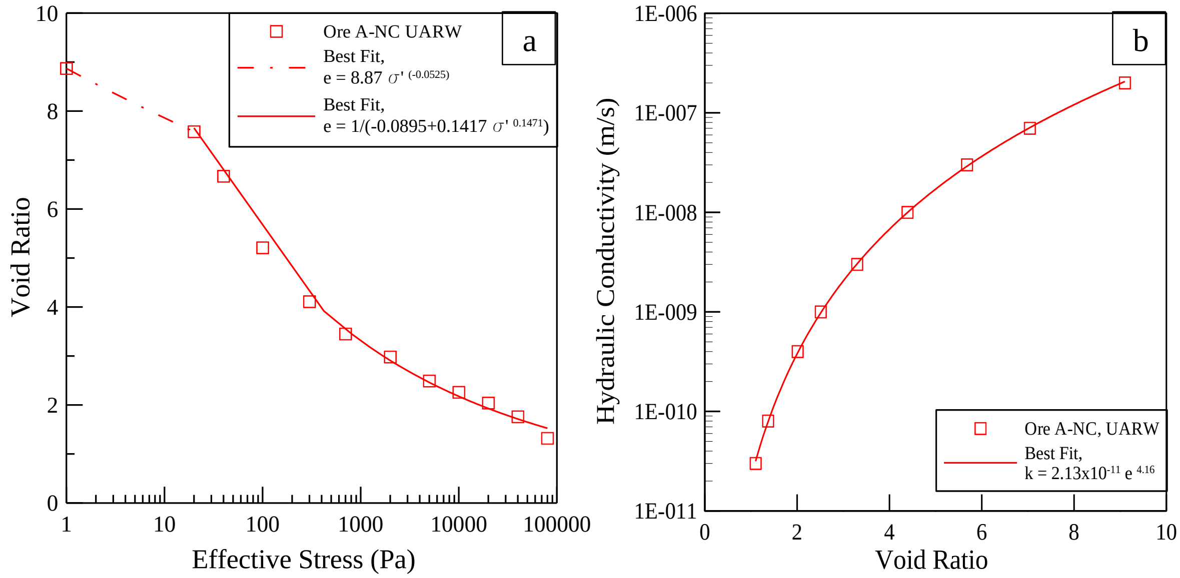

Figure C.5: Volume compressibility (a) and hydraulic conductivity (b) plot of tailings Type 5