Design, Modelling, Self-Testing and Self-Calibration of MEMS Accelerometers with Adaptive and Non-Linear Digital Control

by

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Abstract

My Ph.D. work revolves around proposing new methodologies for increasing the sensitivity of MEMS-based inertial sensors with a capacitive signal pick-off, namely accelerometers and gyroscopes for applications that require measurements of small accelerations (linear and rotational), such as inertial navigation, minimally-invasive surgery and oil explorations.

The sensors are fabricated by surface micro-machining of Silicon, a relatively cheap technology in comparison with the bulk micro-machining process. The latter has a great advantage in comparison to the first one which suffers from thin structures.

Our goal is to utilize the available resources in terms of cheap fabrication methodologies and the required parameters of the MEMS device, to increase its performance without an additional increase in cost.

Our major contribution is the application of sliding mode control (SMC) to the MEMS-based accelerometers. At first SMC was successfully applied as a closed-loop control scheme. Second, SMC was extended to a sensing scheme to extract the external acceleration exerted on the system using the chattering noise. Later, to increase the sensitivity, we exploited the non-linear mechanical coupling by operating at the "Border of Stability". As the accelerometer is in the metastable region, we achieved a sensitivity increase by two orders of magnitude. In addition, we applied the SMC as a band-pass SMC to drive resonators at their resonant frequency. As a result, the targeted MEMS-device showed a great stability, robustness, invariance to external disturbances and an increase in sensitivity. In addition, the proposed methods were evaluated in terms of hardware complexity on FPGA.

Besides this, we applied adaptive-based algorithms to the feedback control of an accelerometer. Inspired by the switching dynamics of SMC, we proposed a new adaptive algorithm based on high-frequency switching between one high step-size and a very small one. Last we proposed an adaptive methodology for self-testing/calibration of inertial sensors, tracking down the external variations which affect the stiffness and damping coefficients, and calibrate them in real time.
Preface

All chapters are based on work conducted in the Adaptive MEMS lab, under the supervision of Dr. Edmond Cretu.

A version of Chapter 1 (in particular Section 1.1 and Section 1.2.1) appeared at several Auto21 workshops [118, 125, 126, 128]. It also has appeared in a application note [114] that we wrote to CMC Microsystems ©.


Some figures (particularly Figure 1.2) and information presented on the poster combines my personal work and also work from my colleagues Mrigank Sharma, Jack Shiah and Hooman Rashtian, as we are all part of the "Inertial Sensors for Adaptive Path Prediction" project. Each one of us put on the
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poster materials related to his part of the project. In regards to the application note, I conducted all the simulations and the FPGA implementation in LabviewFPGA @ and wrote most of the manuscript.

Chapter 3 includes the characterisation results of the MEMS devices that I have designed. My colleague Mrigank Sharma helped in conducting these measurements. Figures from this chapter have been published in several papers, and appeared in several workshops [66, 118, 125] [126, 128].

The simulations results and hardware resources for using adaptive algorithms are presented in Chapter 4. A version of Chapter 4 will appear in IEEE Sensors journal.


The article is currently under review. I conducted all the analysis, simulations and the FPGA implementation of the proposed schemes, and wrote most of the manuscript. My colleague Mrigank Sharma helped in the hardware configuration.

Chapter 5 is divided into three major parts. A version of Chapter 5 (section 5.1-5.4) have been published in [113].


I conducted all the simulations and the FPGA implementation of the proposed schemes, and wrote most of the manuscript. My colleague Brian Cousins was responsible for the design of a closed-loop accelerometer based on ΣΔ control. Section 5.5 and Section 5.6 will appear in IOP Journal of Micromechanics and Microengineering.


I conducted all the analysis, and simulations of the proposed scheme, and wrote most of the manuscript. My colleague Mrigank Sharma helped in characterizing the micro-device. The readout circuit is designed by Ahmed Sharkia, and Siamak Moori helped in conducting the experimental tests and debugging the readout circuit.
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The third major part in this chapter (Section 5.7) was published in [116, 117].


I conducted all the analysis, simulations and the FPGA implementation of the proposed schemes, and wrote most of the two manuscripts. In this part I used the MEMS-based gyroscope of my colleague Mrigank Sharma which appeared in [65, 115, 123, 124, 127] and in [118, 125, 126, 128].

A version of Chapter 6 has been published in [65] and also appeared in CMC TEXPO Workshop in 2010 [66].


Ankit Kansal was part of our research team for two months working on PN Sequences for self-testing and self-calibration of MEMS accelerometers and gyroscopes. In addition to designing the accelerometer and the adaptive feedback loop, I conducted most the analysis and simulations of the proposed scheme, and wrote most of the two manuscripts. My colleague Mrigank Sharma designed the MEMS-based gyroscope, and the readout circuit (collaborative work between Mrigank Sharma and Andy Tsai).
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Several thousand years ago, King Solomon, son of King David wrote in the Book of Ecclesiastes (Ecclesiastes 7:8) "The end of matters is better than their beginning and a patient man is better than one haughty in spirit".

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In the Gospel of John (15:5), we read "I am the vine, you are the branches. He who abides in Me, and I in him, bears much fruit; for without Me you can do nothing". Therefore, ultimately, all thanks are due to the Beginning and End, the Alpha "A" and Omega "Ω", the True God, Jesus Christ.
Dedication

Saint Paul writes in his first Epistle to the Corinthians (I Corinthians 10:31) "So whether you eat or drink or whatever you do, do it all for the glory of God".

Therefore, to His Glory, and to my family
Chapter 1

Introduction

"I have much reservation about our earthy gravitation
That works so hard to keep us poor folk down
Yet there is some consolation in its’ grounding dedication
Since without it we’d be drifting all around"
— A poem by Stanley Cooper

1.1 Introduction to Accelerometers

An accelerometer is a sensor that is used to measure linear acceleration along single or multiple axes. In terms of annual volume production, micro electro mechanical system (MEMS) accelerometers (micro-accelerometers) are still among the top existing MEMS devices [64].

From a systematic standpoint, MEMS accelerometers are electromechanical devices, coupling between the electrical and mechanical domains, whereas the mechanical motion is sensed through an electrical readout circuit built depending on the chosen signal pick-off scheme. Capacitive, piezoelectric, piezo-resistive, magnetic, thermal and optical are the most common and known transduction schemes. They differ based on hardware integration complexity, output resolution and sensitivity and most importantly signal interference or parasitics.

Our work is based on the capacitive transduction schemes, since they offer several advantages, which makes the capacitive signal pick-off very attractive in our targeted applications. They have high sensitivity [25, 62, 72, 82, 137, 155, 158, 160], good linearity [25, 137], high bandwidth, good dc response [82, 155], high precision [62, 78] and noise performance, low drift [82, 158], low temperature sensitivity [25, 62, 72, 82, 158] low fabrication cost [25, 137], simple structure [72, 82] and wide range of applications [72, 160]. Since power consumption is a major concern in all micro-systems, capacitive-based micro-systems are one of the preferred solutions [72, 82, 129, 160].

1 A part of this chapter has appeared in a application note [114] that we wrote to CMC Microsystems ©: E. H. Sarraf and E. Cretu, "Closed-Loop Control of Micro-Accelerometers Using Labview-FPGA" CMC Application Note CMC-00200-01353, 2010.
1.1. Introduction to Accelerometers

Therefore, capacitive sensing offers a good trade-off between the advantages of small electronics [158, 160] on one side and the complexity of the readout electronics [78, 137, 160] on the other.

If an external acceleration is exerted on the system, it results in a proof mass displacement, which in turn induces a capacitance variation directly proportional to the seismic mass displacement; the differential capacitance variation is converted by the readout circuit to an output voltage, and this latter represents a direct measure of the external acceleration. It should be noted that in comparison to rival micro-technologies, Silicon MEMS sensors achieve much greater sensitivity and uniformity of performance [91].

From a physical standpoint, the mechanical side of an accelerometer can be modelled as a mass-spring-damper system. The dynamic behaviour of this lumped system can be described by a second order dynamic differential equation of motion.

\[ m \frac{d^2 x_m}{dt^2} + \gamma_d \frac{dx_m - x_f}{dt} + k_d (x_m - x_f) = F_{ext} \] \hspace{1cm} (1.1)

Figure 1.1: A mass-spring-damper model

Figure 1.1 illustrates the accelerometer structure where a proof mass is connected to the frame by a flexible spring [64]. Due to the mass inertia, a lag will result between the proof mass motion and the frame motion [64]. The difference in displacement between the frame \( x_f \) and the proof mass \( x_m \) is used to measure the acceleration. The noise-induced vibrations are usually damped by introducing gas or liquid (such as oil) inside the package, which results in preventing excessive ringing [64]. In the current context, the damping is represented by a dash-pot \( \gamma_d \). To analyse the mechanism of motion of the accelerometer, we start with the equation of motion for the proof mass given by [64]:
where \( x_m \) and \( x_f \) are the positions of the mass and frame, respectively, and \( F_{ext} \) is the external force acting on the mass, for example due to actuation or Brownian noise (discovered by Pollen [64]). \( k_d \) represents the stiffness coefficient. Subtracting \( m\frac{d^2x_f}{dt^2} \) from both sides of the equation leads to:

\[
m\frac{d^2(x_m - x_f)}{dt^2} + \gamma_d\frac{d(x_m - x_f)}{dt} + k_d(x_m - x_f) = F_{ext} - m\frac{d^2x_f}{dt^2} \tag{1.2}
\]

then substituting \((x_f - x_m)\) by \( x \) leads to the familiar one degree-of-freedom (DOF) damped resonator governed by:

\[
m\frac{d^2x}{dt^2} + \gamma_d\frac{dx}{dt} + k_d(x) = m\frac{d^2x_f}{dt^2} - F_{ext} = m\ddot{x}_f - F_{ext} = F_{total} \tag{1.3}
\]

where \( F_{total} \) is the sum of inertial and external forces. Solving this equation using Laplace transform and introducing the quality factor \( Q_f = \frac{\omega_0 m}{\gamma_d} \) and \( \omega_0 = \sqrt{\frac{k_d}{m}} \) as the natural resonant frequency, one gets the mechanical system response, defined as [64]:

\[
\frac{x}{\ddot{x}_f} = \frac{1}{s^2 + \frac{\omega_0}{Q_f} + \omega_0^2} \equiv H(s) \tag{1.4}
\]

where we have assumed that there are no external forces \((F_{ext} = 0)\) and \( \ddot{x} = \frac{d^2x_f}{dt^2} \) is the frame acceleration.

Acceleration measurement can indirectly provide information regarding other physical variables such as tilting angle, velocity, position, mechanical vibration spectrum, just to name a few. As a result, a plethora of commercial applications is on the market today, which in terms of annual volume production is still among the top existing MEMS devices [64].

Next, we will browse through some of the most common applications of accelerometers in the consumer market, where we will discuss the requirements for the targeted applications.

### 1.2 Industrial Applications, Trends and Challenges

*Yole Developpement ©*, one of the leading companies in MEMS market analysis [157], cites more than 25 applications in their latest reports; although
1.2. Industrial Applications, Trends and Challenges

Mr. Potin acknowledges that the opportunities are endless [69]. ST Microelectronics describes a world of applications for MEMS linear accelerometer in their brochure "MEMS Linear Accelerometers". Therefore it is very obvious that when it comes to accelerometers and the outcome benefits this micro-system is bringing into consumer space, "the sky is the limit" [69]. The aforementioned statements show dedication to MEMS technology on one side and to the consumer on the other side, assuring that the sensing of the aforementioned measurands (tilting angle, acceleration, etc.) is of utmost importance; they can be found useful in many sectors which include but is not limited to:

- The automotive industry dominates the highest portion in the market today, where common applications include triggering airbag deployment systems [52, 53, 145] and inertial navigation [64].

- In consumer electronic devices applications, micro-accelerometers found their use in horizontal and vertical sensing to determine portrait and landscape orientations in digital still cameras and in mobile phones [70], or in detecting motion in virtual reality headsets [121]. Applications include detecting free-fall [52, 53], protecting the hard drive from damage due to a sudden fall. In addition, the huge popularity in video games (WII®) has caused a real epiphany in 2007 [145] in the MEMS-based micro-systems world, and ever since the semiconductor world looks forward to the next big market in terms of micro-systems and related applications.

- The medical or health sector is not far from the success wave of these devices, where they can be used to provide positional information for the control of mobility in paraplegic patients and enable the ambulant monitoring of movements disorders of patients suffering from Parkinson’s disease. To name a few, medical applications include minimally invasive surgeries [8, 134], vibration sensing in machine health monitoring [121], monitoring the movements of patients [79], heart rate controllers [91], measurement of respiratory rate [80, 99], foot drop simulators [143], where they can detect the phase of a gait cycle and then stimulate a nerve in the leg to lift the foot at the appropriate time, hence helping the individuals suffering from neurologic conditions in lifting a drooping foot while walking.

- The aerospace sector [76], where for example, the micro-accelerometer can be mounted in the payload module of the satellite to measure the induced acceleration on the satellite during its mission [16].
1.2. Industrial Applications, Trends and Challenges

We have only mentioned few industrial sectors for micro-accelerometers. The variety of applications for micro-accelerometers is enormous and is too far from complete. For more information, the reader is referred to [52, 69, 70, 133, 145, 157]. Besides that, there is no doubt that the position of micro-accelerometers in the marketplace will remain one of the largest volume products among MEMS-based sensors and actuators; it is described as one of the largest killer industrial applications by [18], despite the slump for automotive demand imposed by the current financial crisis; this is due to ease of integration of these devices into a wide variety of industrial applications.

Certain applications are more demanding than others in terms of sensitivity, resolution, signal to noise ratio (SNR) and dynamic range, where a high sensitive, low-G micro-accelerometer is required. For example, some accelerometers are dedicated for measuring acceleration in milli-, micro- and nano-G ranges. Such accelerometers can be used in numerous applications that require high sensitivity, high precision and high resolution.

These accelerometers found their use in active suspension, adaptive brakes, alarm systems [137], motion detection for game control [53], tilt control [16, 72], tracking vehicles, intrusive detectors [17], vibration [4-6, 137], shock measurements [48], platform stabilization [4, 72], inertial measurement units [5], inertial navigation and guidance [71, 87, 132, 153, 155], machine control [3, 72, 137], microgravity measurements [6, 16, 72, 87, 100, 137, 153, 155], seismology [72, 137, 155] or geophysical sensing [6], and oil-field applications [71], earthquake detection [4, 5, 87], robotics [6, 137], underwater pressure gradient detection [17] and GPS-aided navigators [6, 72, 137, 153] on ground and in air (spacecraft guidance/stabilization) [6, 137].

1.2.1 Targeted Applications

In this research work we target applications in the low-G measurements category, taking into consideration the challenging requirements these applications demand in terms of sensitivity, resolution, cost, and system complexity. The targeted applications include: inertial sensors cluster for adaptive path prediction, minimally invasive surgery and oil explorations. For each application, we will analyse its system requirements and also its impact on the Canadian society, in terms of environment and safety.

A) Inertial Sensor Cluster for Adaptive Path Prediction

The application in question is related to adaptive path prediction. In other terms, deciding the lane position of a vehicle. Figure 1.2 illustrates the
1.2. Industrial Applications, Trends and Challenges

Figure 1.2: Adaptive path prediction
advantages of the application considering a hidden road at night time.

The system will make use of the information provided by the inertial sensors and tune the position of the lights such that they will illuminate the road in the direction of travel, as predicted by the rotation of the car. Another use of the system is where the inertial sensor needs to be complemented by extra systems, in order to provide a full functionality. An imminent collision warning system uses the inertial sensor cluster together with a radar system to be able to react and avoid the dangers of a dynamic traffic context. A path prediction algorithm is combined with the (radar) detected position of other cars, pedestrians or fixed objects, and able to warn the driver about an imminent collision, or to operate in an automatic break mode.

The benefit is an enhanced safety of the passenger, which is of primordial importance, especially on dangerous mountainous roads with poor external lights. The application of such a system will induce large potential for increasing the safety and comfort of the driving experience. A positive impact is also perceived from a pedestrian perspective, as the risk of an accident can be highly diminished.

It should be noted that present MEMS-based accelerometers and gyroscopes used in cars suffer from resolutions not completely suitable for inertial navigation. Their use was until recently limited to safety applications, like air-bags for front/lateral crash and roll-over protection \[26\]. Recent more advanced applications, like vehicle stability program, can make use of the present MEMS-gyroscope resolutions, around $0.1^\circ/sec$. These resolutions are still not sufficient for an accurate inertial navigation \[26\], and alternative expensive technologies have been used in the past for making inertial navigation platforms. Accelerometers with high sensitivity are crucial for implementing self-contained navigation and guidance systems \[158\].

Barbour \[13\] notes that high performance navigation grade ($0.01^\circ/hr$ and $25\mu G$) ring laser gyroscopes (RLG) and fiber optics gyroscope inertial measurement units (IMUs) are still expensive (>50k) and relatively large \[13\]. The same author continued to define the limitations in the current IMUs where he linked the current limitations by the gyroscope performance, which is now around $5 - 30^\circ/hr$, rather than by accelerometer performance, which has demonstrated $\sim 10\mu G$ or better. An IMU by Northrop \[142\] features a performance of $5^\circ/hr$ and $3mG$. Barbour states that in the future MEMS system with performance of around $1^\circ/hr$ and $\sim 100\mu G$ would be available.

The potential benefits of having a low cost inertial sensor cluster in a car are multiple, and mainly related to the safety and comfort of the vehicle occupants and pedestrians. Canada’s social development is improved by
incorporating in vehicles more safety features and early alert systems. The enhanced safety tremendously improves the quality of life on all Canadian drivers, passengers and pedestrians. The inertial navigation will become ubiquitous in every car in the future, and enhance the comfort and easiness of driving.

As a result, the use of MEMS sensors, compared with alternative sensing devices, will contribute to a reduction in the power consumption and miniaturization of the system, and an overall lower operation costs of the vehicles; in addition to that making the right turning decisions at the right time will also promote a lower fuel consumption [26]. Overall, the small size, high performance, low cost of the inertial sensor cluster will enable such advanced safety and comfort features in all the categories of vehicles, necessary in the present environment, characterised by an increasing traffic rate [26].

B) Inertial Sensor for Minimally Invasive Surgeries

Traditionally speaking, surgery refers to the performance of a series of operations (cutting and sewing of tissue [98]) inside the body, in order to treat diseases and other health disorders, usually accomplished by making some incisions on the patient’s body.

The open surgery techniques were developed first, where the surgeon had direct contact and interaction with the infected area, and manipulated them freely. On the other hand, such operation usually results in tremendous trauma to the patient, which eventually requires a long recovery time, known as post-operative pain [98].

The second generation of surgery, known as minimally invasive surgery (MIS), was born in 1985 with the aim of reducing harm to the patient, through the operation. This is done by inserting a small device through a 5 to 10mm port to reach and hence treat the diseased area [98]. The drawback in this type of surgery is that the surgeon does not have direct contact with the infected area, as in the open surgery case, and would have to control the device from the outside. The patient would have less harm and a shorter recovery time.

Robotic surgery, also known as computer-aided surgery, was introduced as the third generation of surgery [98] to solve some of the problems of the second generation such as the lack of natural hand-eye coordination and losing the sense of touch, in addition to 2D imaging. Developed and launched in 1998 by Intuitive Surgical ©, the Da Vinci surgical system is one of the most successful surgical robots and is still the most advanced surgical platform available [136]. It has many features to solve critical problems of
the second generation of surgery: for instance controllers [136] that simulate natural hand-eye coordination, and filter the movements of the surgeon, in addition to seven DOFs. While the Da Vinci system has brought the MIS to another level, there are still some unsolved problems that have to be addressed, which include [98] tactile feedback (as in the case of an open surgery), local information and tissue sensing, and tracking.

It is expected that MEMS will play a key role solving those issues. This technology will provide new opportunities for minimally invasive surgery or robotic surgery, since it induces the creation of small devices that are able to acquire and digitize information which can be integrated into endoscopic instruments [19].

According to [27], MEMS has the potential to play a major role in the development of new medical instrumentation and hence to have a considerable industrial impact in this field as well. Dr. Rebello [98] stated that MEMS will play a key role fueling the growth rate of the computer-aided surgery, with a large number where the estimate to grow by 2010 is $2.6 billion, comparing to $780 million in 2003.

B. Vigna already talked about an epiphany of MEMS devices in the automotive and consumer electronics [145], and the expectation of the next one are pointing to the medical and biological fields, where they seem very promising but at the same time most challenging for micro-mechanics and micro-system technologies [27]. However, in this field, it should be noted that there are many restrictions for MEMS. First of all, size would be a main concern; when the outer diameter of a standard guide wire is 0.36mm [53], sensors designed to be mounted on a catheter could not exceed that size. Other issues include biocompatibility, wire orientation, flexibility, and friction.

In such systems, inertial sensors are used for detecting the position and orientation of the surgical tool. The signal outputs can be integrated to determine or estimate the distance travelled by a surgical tool. Conventional MEMS accelerometers have accuracies in the milli-G range, which are not sufficient for measuring accurately with a high resolution the relatively small displacements made during surgery [98]. Additional specifications include resolution and accuracy < 1µm, and a system bandwidth > 15Hz [8]. Therefore the type of accelerometer chosen should exhibit a high sensitivity, good dc response, low drift, low temperature sensitivity, low-power dissipation. Thus, more accurate inertial sensors need to be developed before they can be integrated into surgical tools [98].

The authors in [9] propose an active hand-held instrument to sense and compensate physiological tremor and other unwanted movements during a
1.2. Industrial Applications, Trends and Challenges

*vitrectinal* micro-surgery. The proposed system comprises six inertial sensors to compute and control the motion of the tip. A CXL02LF3 tri-axial accelerometer from *Crossbow Technology* © [130] and three *CG-16* ceramic rate gyroscopes from *Tokin* © [140], have been deployed. The accelerometer is low-G, highly sensitive with a noise density of $70\mu G/\sqrt{Hz}$, and a sensitivity of $1V/G$. As a result, using the data from these sensors and the assumed knowledge of the instrument’s instantaneous center of rotation, appropriate kinematic calculations yields the three dimensional velocity of the instrument tip which is integrated to obtain tip displacement [9].

Another contribution is made by the authors in [8], where they proposed an all-accelerometer inertial measurement unit as a part of an intelligent hand-held micro-surgical instrument, which senses its own motion and distinguishes between hand tremor and the intended motion, and at the same time compensating the erroneous signals. Their inertial measurement unit consists of three dual-axis accelerometers *ADXL202E* from *Analog Devices* © [60]. This accelerometer is low-G, low cost, low power and features a digital output signal, with a $5mG$ resolution, $312mV/G$ as sensitivity and $500\mu G/\sqrt{Hz}$ as noise floor. The authors compared their proposed IMU with a conventional three gyroscopes and three accelerometers IMU, which resulted in reduction of the standard deviation of the estimates of translational displacements by 29.3%, and about 99% for the Euler orientation parameters. Later on, the same group [10] proposed a sensing unit that comprises a three-axis magnetometer *HMC-2003* from *Honeywell* ©, [55] and three dual-axis accelerometers *ADXL203* from *Analog Devices* © [60]. The accelerometer is characterised by high sensitivity accuracy, high precision with $1mG$ resolution, $1V/G$ sensitivity and $110\mu G/\sqrt{Hz}$ noise floor.

In [151], the authors designed an *arthroscopy* hook that includes an accelerometer and custom-designed actuator, as a tactile magnification instrument in order to measure the acceleration produced from the scratching of a surface, for *MIS*. This will allow for detection of small cuts in tissue-like materials. The authors used a $2G$ dual-axis accelerometer *ADXL311* from *Analog Devices* © [60]. The accelerometer is characterised by a high resolution, low power, with $300\mu G/\sqrt{Hz}$ noise floor, and a sensitivity of $174mV/G$. In [101], the authors design a *haptic* forceps for robotic surgery. These forceps are designed as a master device which is used to control one DOF surgical tools and later on accurately convey the sensations detected. The accelerometer used is an *ADXL210* from *Analog Devices* © [60] similar performance to *ADXL202*, with a sensitivity of $100mV/G$.

It should be noted that the placement of the inertial sensors on the surgical instrument (tip displacement to be tracked) can be tremendously
improved if the sensors are placed efficiently. This can be seen in a study conducted by [74] where the authors investigate the placement, location and orientation of accelerometers, in hand-held active tremor compensation instrument for micro-manipulations tasks, in order to obtain the least possible angular acceleration noise at the instrument tip. Their proposition improved the sensing resolution by 66.3% and 77.5% (if they added and correctly placed an additional accelerometer, total becomes seven).

As a result MEMS-based inertial sensors can add a brand new chapter to the development of surgery, ensuring more safety and less post-operative pain to the Canadian patient.

C) Inertial Sensor for Oil Explorations

Another challenge in high sensitivity and low-G measurements is found in the seismic sensing for geophysical and oil explorations applications. According to [119], oil companies are pushed to increase the investments in seismic exploration of new oil and gas due to the fluctuations in the price of crude oil. This requires measurement of extremely small acceleration signal, in order of $\sim nG/\sqrt{Hz}$ and at very low frequencies ($<100Hz$). The very high-sensitivity and low-bandwidth challenges imposed difficulties on the current development of chip-scale compact accelerometers that would meet these requirements [71].

The basic technique of seismic exploration can be summarized as follows [73]: "First a series of geophones are disposed on the ground. Second, seismic waves are generated and the time the waves require to travel from the source to the disposed series of geophones is measured. As a result, the measurement of different travel times and amplitudes of the reflected waves, belonging to different groups of geophones, permit the reconstruction of the elastic discontinuities in the earth subsurface conforming to the geology, which provides an indication of oil and gas presence". The challenge lies in measuring the very weak energy of reflected seismic waves (up to $10^{-7}$ of the emitted one). Such measurements at the surface require sensors to exhibit high sensitivity and very low noise floor.

The moving coil geophone is a conventional sensor that has been used for such applications. Large number of geophones in series leads to higher sensitivity [73]. Seismic exploration requires a large number (500–2000 nodes/km²) of sensors to be deployed in outdoor large areas ($> 20Km^2$) to measure backscattered wave fields [119]. As a result, a successful survey, which include large arrays of sensors deployed on the ground [80], would require a large number (up to half a million), which also complicates the transporta-
tion and the handling of such devices.

Thus, the MEMS technological breakthrough can definitely provide technical and economic benefits for seismic acquisition system users. In addition MEMS-based accelerometers associated with electronics can exhibit a perfect linear response down to 0Hz. As a result, there seems to be a growing trend in the industry to replace geophones with MEMS accelerometers [86], where large arrays of sensors tremendously increase the quality of seismic imaging. As a result, small size and weight of MEMS sensors can facilitate deployment of very large seismic surveys [86].

The authors in [73] report the usage of a 3D MEMS-based digital accelerometer, which contains two horizontal and one vertical accelerometers. A three-orthogonal MEMS-based device is mandatory for recording the full wave field (compression and shear). Such devices are easy to operate, and in addition they provide better data, thanks to their improved vector fidelity, defined as the ability to reconstitute the exact seismic wave-field based on the recording of the different components. All of these requirements are very hard to get with a conventional 3D analogue geophone. The device is made by Servel © [89], and has a noise floor of $0.4\mu G/\sqrt{Hz}$ and $120dB$ as system dynamic range at $0.004sec$.

To follow up with high sensitivity MEMS-based accelerometers, the authors in [71] designed an optical-based transduction Silicon accelerometer, with $36Hz$ as resonant frequency and $8nG/\sqrt{Hz}$ as noise floor. The high sensitive optical transduction scheme is based on multiple nano-grating arrays, symmetrically placed around the proof mass. This resulted in a sensitivity of $590V/G$ and noise floor corresponding to $17nG/\sqrt{Hz}$ at a frequency of $1Hz$. It should be noted that the proof mass is $18.9mg$ for the first and $33.6mg$ for the second version. The accelerations that were applied were in the order of $6.45\mu G$, corresponding to a sensitivity of $589V/G$. They measured displacements of $1nm$ and $1\mu m$ for accelerations of $10\mu G$ and $10mG$ respectively. The authors in [86] propose a highly sensitive capacitive-based accelerometer to meet the requirements for oil and gas exploration, in particular for imaging deep and complex subterranean features. The device features a noise floor of $10mG/\sqrt{Hz}$. It has a dynamic range of $120dB$ with $-80mG < a_{max} < 80mG$.

Within the same context, oil well construction begins with the drilling of a well bore. Then a steel casing is inserted into the bore hole and cemented in place. However, in order to allow the oil into the bore, perforations in the steel casing and cement by means of explosive charges are needed [21]. For safety reasons it is of utmost importance to know if all the explosives have detonated before the detonation apparatus is removed from the well. The
challenge lies in the small amplitude signals that indicate successful detonation; they are difficult to hear as the well bore is very deep (two to three miles). Therefore high sensitive accelerometers are required for such application. It should be noted that in many cases, the sensitive accelerometers also capture signals from generators, pumps and other equipment around the well head; therefore, these unwanted signals must be filtered. Halliburton © [21], a leading company in oil exploration, developed a filter that would separate the accelerometer signal from a combination caused by these ambient sounds. The developed filter is adaptive predictive and based on neural networks. In another work at the University of Calgary [39], the author use accelerometers from Analog Devices © (ADXL105 [60]) with noise floor of \(0.225mG/\sqrt{Hz}\) and a sensitivity of \(250mV/G\). They made use also of Kalman filtering technique to limit the error growth of inertial measurement units.

As we can see, the common denominator between the aforementioned targeted applications is high sensitivity, resolution, accuracy, in addition to low mechanical noise floor; the sensor must detect the slightest variations exerted on the system, which in turn are translated to significant decisions on the actuator side.

1.2.2 State-of-the-Art Accelerometers

For the last two decades, the demands of increasing accelerometer performance pushed the engineers for creativity in modelling, fabrication, tuning, characterisation, power consumption and digital closed-loop control of micro-accelerometers. We will analyse the availability of such devices on the market and discuss the challenges in terms of performance and cost.

It should be noted that it is not trivial to compare accelerometers just by the performance parameters such as noise floor, sensitivity and resolution, as each one is a function of many factors, such as the transduction scheme and the fabrication technique. Therefore, when it comes to comparison, many parameters should be taken into consideration, such as targeted application, cost, weight, signal pick-off and the fabrication process, which reflects on the thickness of the seismic mass.

The most common capacitive-based accelerometers are based on surface micro-machining processes, limiting the thickness of the proof mass around \(25\mu m\), which in turn affects the sensitivity and the mechanical noise floor. Large producers of such sensors are Analog Devices © [60] and Freescale © [54], incorporating a relatively thin mass along with interdigitated combs between the movable mass and the surrounding fixed structure. A noise floor
of $25 \mu G/\sqrt{Hz}$ is relatively common among surface micro-machined sensors. For high performance, the proof mass is increased ($>25 \mu m$) and this results in a lower noise floor, typically around $50 \mu G/\sqrt{Hz}$.

We point out to some of the state-of-the-art accelerometers on the market today. Table 1.1 summarizes the main characteristics of the state-of-the-art accelerometers.

Table 1.1: Technical specifications of some of the state-of-the-art accelerometers

<table>
<thead>
<tr>
<th>Device</th>
<th>Reference</th>
<th>Thickness</th>
<th>Sensitivity $mV/G$</th>
<th>Noise Floor $\mu G/\sqrt{Hz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMA7361LC</td>
<td>[54]</td>
<td>$&lt;25$</td>
<td>800</td>
<td>350</td>
</tr>
<tr>
<td>LIS344ALH</td>
<td>[133]</td>
<td>$&gt;25$</td>
<td>600</td>
<td>50</td>
</tr>
<tr>
<td>KXTH5-4325</td>
<td>[68]</td>
<td>$&gt;25 \mu m$</td>
<td>364</td>
<td>150</td>
</tr>
<tr>
<td>DSU3-428</td>
<td>[89]</td>
<td>$\sim 60 \mu m$</td>
<td>-</td>
<td>$\sim 100 e^{-3}$</td>
</tr>
<tr>
<td>SF3600</td>
<td>[122]</td>
<td>4 Wafers</td>
<td>1200</td>
<td>$300 - 500 e^{-3}$</td>
</tr>
<tr>
<td>CXLTG</td>
<td>[85]</td>
<td>3 Layers</td>
<td>833</td>
<td>20</td>
</tr>
<tr>
<td>131B-01/3</td>
<td>[120]</td>
<td>-</td>
<td>-</td>
<td>$2 - 500 e^{-3}$</td>
</tr>
<tr>
<td>-</td>
<td>[51, 146]</td>
<td>3 Wafers</td>
<td>25000</td>
<td>$100 e^{-3}$</td>
</tr>
</tbody>
</table>

According to [146], two vendors, Sercef (c) [89] and Colibrys (c) [122], have pushed the performance of MEMS inertial sensors, but this resulted in large and expensive devices.

Hewlett Packard (HP) (c) [51, 146] reports a very sensitive accelerometer. Three Silicon wafers were bonded in order to achieve wafer level packaging, and a surface electrode technology is used to create a later capacitance-based transducer. The mechanical noise floor is estimated to $100 nG/\sqrt{Hz}$ and $1.33 pF/G$ capacitance change per acceleration, along with a sensitivity of $25 V/G$. Recently HP [86] proposed a new accelerometer for oil and gas explorations with $10 nG/\sqrt{Hz}$ noise floor.

1.3 Conclusion and Dissertation Outline

MEMS inertial sensors have shown themselves to be an extreme enabling technology for new applications [13]. High performance, including low noise floor, high sensitivity/resolution/accuracy, very low bias drift, usually come with a large cost. In addition, when optical signal pick-off is employed, the cost factor is spanned by several orders of magnitudes, as it represents the most sensitive transduction scheme.
1.3. Conclusion and Dissertation Outline

It is therefore of utmost importance to push for performance increase with an affordable cost increase. This could include solutions at the sensor level on one hand, or at the system level (comprising the sensor) on the other hand, or ultimately it could be a combination of both. The goal is to fully use the available resources and features of the sensor system, and smartly deploy them to increase the performance of the device without increasing the system cost.

Such solutions requires contributions at different levels, ranging from the mechanical design, to closed-loop feedback control and hardware complexity. Thus, the work presented here started as a study of MEMS accelerometers performance and the advantages of combining MEMS devices with digital control for performance, flexibility, reconfigurability and cost issues. Later on, we addressed the issue of performance by proposing new non-linear techniques, providing good performance and low complexity, suitable for hardware implementation. This strategy is also reflected in the thesis structure, where the major points can be summarized as follows:

• Throughout the introduction chapter, we touched some of the most common applications that employ low-G accelerometers, and browsed through the targeted applications and their requirements in terms of performance and system complexity. We concluded that such applications require high performance, which comes with a high cost

• The second chapter presents possibilities of increasing the sensor performance, and the related performance/complexity ratio. It thoroughly discusses the limitations of open- and closed-loop architectures, including advantages of digital implementation on a field programmable gate array (FPGA). In addition, this chapter provides a static and a dynamic analysis for operating at the border of stability derived from [104] and using one of our designed devices. Such operation results in a very high sensitivity; however, many risks are involved. This will pave the way for our proposed novel work

• Any analysis or theory should be accompanied by some experimental testing. We designed four accelerometers which were fabricated using two different fabrication methodologies, SOI-MUMPS ® from Mems-cap © [84] (25µm thickness of the structural layer) and MEMS-SOI ® from Tronics © [29] (60µm). Chapter 3 presents the accelerometers characteristics and their experimental characterisations. Their extracted physical parameters will be used in the subsequent chapters for closed-loop control and testing
1.3. Conclusion and Dissertation Outline

- *Chapter 4* presents the proposed adaptive technique for closed-loop control of the accelerometers along with the appropriate results and the correspondent digital implementations on FPGA. No one has previously reported the usage of adaptive algorithms for closed-loop control of MEMS-based accelerometers. In this chapter we propose a novel adaptive algorithm based on switching of adaptation step-sizes.

- *Chapter 5* presents the proposed non-linear sliding mode control (SMC) for micro-accelerometers. No one has previously reported the usage of SMC for micro-accelerometers. We apply the SMC to the feedback control of the device, and most importantly we take advantage of the *chattering* noise to measure the external acceleration exerted on the system. This noise has been a drawback in SMC applications, but it is exploited in our case as it contains information about the input acceleration. The advantages of the proposed method and the hardware implementations on FPGA are thoroughly discussed. The presented experimental results confirm the importance of the proposed technique. We further extend the application of SMC to a band-pass SMC for driving resonators at the desired resonant frequency, demonstrating the method to be insensitive to variations in temperature and uncertainties. The proposed method is novel and is an alternative approach to phase locked loops (PLL)s. In addition we propose the SMC to stabilize the accelerometer at the *border of stability*, to achieve high sensitivity, where it should be noted that current *state-of-the-art* controllers never operated in this region.

- After the MEMS design, fabrication and characterisation in *Chapter 3*, and improving the performance by proposing novel closed-loop control techniques in *Chapters 4 and 5*, the sixth chapter presents an adaptive method based on pseudo-random (PN) sequences for real-time self-testing and self-calibration of MEMS-based inertial sensors. The device stiffness and damping coefficients are strongly affected by environmental changes such as temperature and humidity, and others factors from the manufacturers side (packaging). The method is simple, was implemented on FPGA, and was experimentally tested.

- Finally, *Chapter 7* concludes this study with a summary and a description of thesis contributions. In addition some recommendations for future enhancements of the proposed schemes are proposed by the author.
1.3. Conclusion and Dissertation Outline

Figure 1.3 shows the thesis outline with the different interconnects and liaisons between the seven chapters.

Figure 1.3: Dissertation outline flowchart
Chapter 2

Motivations and Objectives

"What do you want to achieve or avoid? 
The answers to this question are objectives. 
How will you go about achieving your desire results? 
The answer to this you can call strategy"  
— William E. Rothschild  
"The pessimist sees difficulty in every opportunity. The optimist 
sees the opportunity in every difficulty"  
— Sir Winston Churchill

2.1 Introduction

MEMS technologies have revolutionized the conventional sensors and actuators in terms of concepts, fabrication methodologies, size, performance and cost. The first micro-accelerometer was brought to light in 1979 at Stanford University by Lee et al. [77] and the concept has evolved tremendously over the last decade using MEMS-based technology. MEMS-based devices became smaller and faster, but their real advantage is the low cost, using batch fabrication processes originally adapted from the manufacturing of integrated circuits [64]. Nevertheless, depending on the targeted application, this revolution in technology is constrained by many factors limiting the usage of the micro-devices. Some of these boundaries and limitations come from the sensor itself, such as pull-in voltages, non-linearities, dynamic range and sensitivity, while other boundary constraints could include packaging, surrounding elements such as integrated CMOS readout circuits and digital controllers.

To overcome these constraints and limitations \(^\text{2}\) one may target improvements at the sensor level; exploiting the latest fabrication advances,

\(^{2}\)A part of this chapter has appeared in an application note [114] that we wrote to CMC Microsystems ©: E. H. Sarraf and E. Cretu, "Closed-Loop Control of Micro-Accelerometers Using Labview-FPGA" CMC Application Note CMC-00200-01353, 2010.
2.1. Introduction

increasing sensitivity and lowering the noise floor, by incorporating a bulky proof mass for example, which requires "exotic" fabrication processes [64]. From a materials standpoint, the bulk micro-machined structures can be made of single-crystal Silicon in contrast to amorphous or polycrystalline thin films typically used for surface micro-machined structures. Dr. Kaajakari [64] notes that "the predictable and stable material parameters of crystalline Silicon are desirable for mechanical sensors". However this approach could be costly, and high aspect ratios are difficult to achieve. A second unusual approach includes operating in a closed-loop fashion. An extreme unusual approach pushes the sensor to its border of stability, where its sensitivity will reach a maximum (but with an operating mode more difficult to control) [102][104]. This last approach relies typically on a closed-loop implementation with a smart, robust controller. At the controller level and for system level efficiency, speed, resolution, parallelism and cost, the controller could be targeted on a flexible, reconfigurable technology such as FPGAs. This proposal is in line with the statement from Xiong et al. [150] expecting that very soon, MEMS-based devices will be fabricated on the same chip with digital FPGA and analogue circuits.

Figure 2.1: Intelligent micro-system block diagram

Figure 2.1 depicts the proposed system level architecture and it comprises, in addition to the MEMS accelerometer, an analogue readout circuit which converts the capacitance change induced by the seismic mass displace-
ment to an output voltage, proportional to the acceleration exerted on the system. At the output of the readout circuit, the signal is converted to a digital format and fed into the controller (adaptive or sliding mode control). This generates an output controlled voltage also proportional to the external acceleration, fed after conversion to analogue domain to the actuator combs of the transducer, for rebalancing and repositioning.

2.2 MEMS and FPGAs

FPGAs have historically been found in high-end professional broadcast systems, network surveillance cameras and medical imaging equipment [112]; their flexibility has made them very suitable for digital signal processing applications. The choice of FPGA as the digital signal processor (DSP) is motivated by the fact that they are relatively reliable, cheap, flexible [22, 23, 107, 148] and have longer lifespan [107]. The present FPGA-based design flow has been largely characterised by a hardware centric approach [112]. The main advantage is the exploitation of the high computational efficiency of FPGAs, matched by high bandwidth concurrent memory access and rich on-chip interconnectivity [112], combined with complete programmability. FPGAs can be effectively used for control purposes in processes demanding very intense loop cycle time [22]. These requirements make FPGAs well suited for high efficiency implementation of signal processing, packet processing and high performance computing applications.

FPGA-based digital signal processing is drawing lots of attentions to MEMS engineers lately. It shows great promise as a technology to replace the cumbersome, less flexible analogue version for large high precision data acquisition systems [67]. Having the ability to program and modify in real-time the control loop to adapt to the specificity of each particular microsystem boosts up the performance. Real time product verification includes responses to unexpected changes in the behaviour due to radiation, temperature shift, etc; a robust sensor is expected to cope with these changes in real time, and to calibrate, therefore FPGA-based control present the capability of in-situ programming [67].

The authors in [107] present an FPGA implementation of an automobile fuel control system, with a target to maximize fuel efficiency. The authors use an accelerometer with a proportional-integral-derivative (PID) control scheme and a Sigma-Delta (ΣΔ) modulator. A similar controller combination has been proposed by [81]. As a result the output signal of the accelerometer output signal after amplification and sampling is fed to the
2.3. Open-Loop Limitations

FPGA-based control hardware. In \cite{67}, the authors use the FPGA-based digital control to program and modify in real-time the control loop in order to adapt to the specificity of each particular gyroscope device, in addition to the change of the mechanical characteristic of the device during its lifetime. The use of a bias voltage, a graphical user interface (GUI) along with a digital implementation, made the tuning of the gyroscope easier and more accessible. In addition the FPGA has been used to control and adjust the digital filters and different gain blocks used in the drive and counterbalance loops. The authors in \cite{23} use an FPGA as a prototype for an integrated controller for a MEMS system array for air-flow planar micro-manipulation. The MEMS array was proposed to be integrated in a hybrid multi-chip module containing the FPGA-based controller.

As a result, the FPGA shows an effective interfacing with the MEMS-based devices, which was one of the driving forces in this work.

2.3 Open-Loop Limitations

Many of the accelerometers on today’s market operate in open-loop and are quite restricted in performance. The open loop sensitivity of a capacitive-based micro-accelerometer is proportional to the proof-mass size and capacitance overlap area and inversely proportional to the spring constant and air gap squared \cite{152}.

To directly state the problem, high performance (resolution and sensitivity) capacitive-based micro-accelerometers are made possible by incorporating complex fabrication methodologies using thick proof mass \cite{1, 53, 78} and high resolution readout electronics such as switch cap \cite{156}, capacitance to frequency converters \cite{41}, and ΣΔ modulators \cite{5, 34-38, 72, 131}. This opened the doors to many challenges in terms of packaging, noise-floor and most importantly cost. High performance inertial sensors are drawn to bulk micro-machining where the thickness of the proof mass can be made as thick as necessary in order to meet the performance requirements \cite{12, 135}, which we have witnessed in the previous chapter. The large proof mass overcomes the limited mechanical sensitivity and the mechanical noise floor imposed by thin structures (surface micro-machining), thus resulting in high resolution sensitivity and low cross-axis sensitivity \cite{144}. This bulk micro-machining approach for increasing sensitivity and overall performance can be referred to as the conventional approach, and it is very costly.

The real challenge lies in finding a trade-off between two conflicting requirements: design limitations forced by a cheap surface micro-machining
2.4 Closed-Loop Limitations

Within the aforementioned framework, one must justify the need for the second approach, defined by a system level performance in closed loop integration, by understanding the current performance limitations in the state-of-the-art micro-accelerometers when employed with other electrical components.

High sensitivity is achieved by high precision accelerometers that are typically operated in closed-loop, which in turns reduces or eliminates some recurrent problems in open-loop measurement systems, such as offsets and signal distortion due to non-linear elements, and potentially improves quantization noise shaping [131] to satisfy dynamic range, linearity and bandwidth requirements [45, 46, 48, 63]. Other advantages include minimizing non-linearity [48], better stability [48, 63], high sensitivity, low noise floor [155] and protection of accelerometer against strong electrical shock [45] which can happen at the realisation of the undesirable electrical contacts between the proof mass and the fixed electrodes.

The closing of the loop (negative feedback control) requires an opposing force to the external inertial force acting upon the movable seismic mass; electrostatic balancing forces are used in the present context. As a result, the proof mass is rebalanced to its central position, or close enough to it, by applying an electrostatic force to the feedback combs, that counteracts the input force exerted on the system; hence the proof mass displacement is minimized [131]. The feedback force balances the inertial force and also provides measurement information about the acceleration applied to the system [45].

When the inertial mass is deviated from the zero-equilibrium position, its displacement induces a (non-linear) capacitance change, due for instance to the gap variation between the fixed and movable plates [155]. Providing a negative feedback loop that counterbalances the induced motion will ensure that the net displacement is always near to the zero-position, minimizing the effects of the intrinsic non-linearities (e.g. electrostatic forces proportional
2.4. Closed-Loop Limitations

with the square of the voltage, the capacitance varies non-linearly with the gap, and the equivalent spring constant becomes non-linear for large displacements). The response of the system in closed-loop is much closer to the inputs signal’s form [45], and the electrostatic actuation control signal constitutes the sensor measurement signal [131]. As the accelerometer is linear in a small range, repositioning control loop have been designed to enable an extended linearity over a wide dynamic input range [48].

The problem that arises is the choice of an appropriate smart controller. Closing the loop requires a control strategy that takes into consideration the parameters of the system and the output of the plant (micro-accelerometer); the generated controlled feedback signal is mapped to an electrostatic force (through the actuating capacitors) that balances the proof mass, and brings it back to its central or rest position.

A problem with electrostatic feedback force is their attractive nature and the non-linear dependency on voltage and gap between the electrodes. Usually, a differential arrangement is used to linearise this relationship and to generate a force proportional to displacement. Special care must be taken concerning the design of the sensor loop architecture and in particular the controller [131], which must be able to adapt to operating in a real, time-varying environment, hence a smart controller solution would bring an advantageous alternative.

Conventional closed-loop architectures that include the physical sensing element, a readout circuit and a compensator in a \( \Sigma \Delta \) loop, have been one of the majors investigations in micro-accelerometers over the past years [81]. The direct digital output is the major advantage in this approach; the limit cycling property of the modulator, when subjected to zero input, can be used for self-testing purposes, which are vital in applications such as airbags release and inertial navigation. Another advantage is the constant magnitude and duration of the applied voltage; this solves the non-linearity problem (due to the square-law dependency of the electrostatic force on the voltage), and the net force becomes a linear function of the number of pulses per time period.

An example of an accelerometer balanced by means of \( \Sigma \Delta \) is proposed by Yazdi et al. [152] and includes an over-damped oversampling-based accelerometer with a dominant pole, employing either a lead-compensator or a lead-lag in the feedback path, or a lead-compensator in the forward path. The same group extends their work [153] with a second order electromechanical \( \Sigma \Delta \) to force-rebalance the proof mass, where the loop stability is ensured by integrating a digital lead compensator, realized by a pulse width modulation (PWM) of the electrostatic feedback bit. This approach is effec-
tive when a small phase shift is sufficient for stabilization. Repositioning the mass is made possible by making the chopping frequency higher than the resonant frequency. In further work [154, 155] they propose a folded-electrode approach with a $\Sigma\Delta$ modulation for closed-loop accelerometers.

High orders $\Sigma\Delta$ modulators push the noise outside the baseband, and achieve good performance in acceleration measurement. Extensive research in this area has been reported by Dong et al. [34-36]. The higher order of $\Sigma\Delta$ feedback achieves much better SNR comparing to a low order modulator, and a better motion cancellation. Other $\Sigma\Delta$ modulator approaches for highly sensitive micro-accelerometers were proposed by Amimi et al [5], Kulah et al. [72] and Soen et al. [131].

With the aforementioned research work one cannot argue with the popularity of $\Sigma\Delta$ modulators for inertial micro-systems. They have been investigated for accelerometers for the last two decades, and set as the state-of-the-art technology for closed-loop micro-accelerometers on the market today. Their major advantage, apart from the noise shaping and the resolution, is providing direct digital output and force-feedback control of the proof mass simultaneously over a wide dynamic range.

Despite the advantages, their design and analysis is based on extensive simulations or linearisation schemes of an intrinsically non-linear architecture. An attempt to linearise the non-linear problem by using separate linearised transfer functions for the signal and for the noise only results in adding more salt to the wound. While this works for lower-order control, high-performance systems require higher-order $\Sigma\Delta$ modulators, where the stability problems are dealt with mainly through time-consuming numerical simulations and optimizations cycles [113]; as a result, their design, which does not rely on a non-linear theory from first principles, becomes difficult.

\section{2.5 Border of Stability}

A major physical problem related to the nature of capacitive-based micro-accelerometers is the limitation to $1/3$ of the full gap of the travel distance of the capacitive transducer, due to pull-in voltage $V_{\text{pull-in}}$.

The pull-in voltage is defined as the maximum $dc$ bias voltage that can be applied on the capacitor [64], particularly considering a gap-varying geometry (illustrated in Figure 2.2) without taking the risk of entering into the unstable region [64]. It is shown that $V_{\text{pull-in}}$ for gap-varying combs always corresponds to a critical displacement:
2.5. Border of Stability

Here $d_0$ denotes the nominal gap of the capacitor, and $x_{\text{pull-in}}$ is the critical displacement, defining the border of stability, and is responsible for the limited dynamic range of capacitive-based micro-accelerometer. Beyond this point, the movable electrode will suddenly and catastrophically snap to the counter electrode.

As a result, considering a capacitive micro-accelerometer with relatively thin structure (25µm), we find ourselves cornered in terms of sensitivity and performance with a great deal of limited resources trying to meet the expectations imposed by the targeted application.

Another closed-loop attempt targeted extending the travel range of the capacitor beyond the border of stability defined in Eq 2.1, hence increasing the output capacitance. Such approaches include trajectory planning and robust non-linear control, geometry leverage and non-linear stiffness springs, current drive source or charge control, series capacitance, voltage control scheme and feedback linearisation [105, 161]. A handy comparison between these approaches in terms of performance, advantages and disadvantages is found in [105]. There the authors propose an interesting approach using two voltages, one is lower than the static pull-in voltage and the other one is higher to extend the travel range of the capacitor. The system switches between these two values depending on the current position of the proof mass. The result is an extension of the capacitor travel range to 100% if no mechanical stopper is deployed.

To overcome the fear of structure’s snapping due to pull-in voltages, the
2.5. Border of Stability

typical approach is to use low excitation voltages, much lower than pull-in, and to employ proper mechanical shock stops (Figure 2.2) and deflection limiters in order to protect the accelerometer. By adopting this approach, one may be on the safe side but loses in exploiting the high sensitivity close to instability border. Extensive literature review show the high sensitivity around the pull-in point defined at 1/3 of the gap as pointed in Eq 2.1. We will see in the following subsection how Rocha et al. [102, 104, 105] and Dias et al. [31-33] exploit operating in this strong non-linear unstable regime where they measure the acceleration based on the pull-in time.

Over the last two decades, countless efforts had targeted the analysis of the non-linear behaviour of electrostatic forces witnessed in gap-varying geometry capacitors (Figure 2.2) including the static pull-in phenomena first reported in 1967 by Nathanson et al. [88] as a property of the resonant gate transistor (RGT).

However, we should firstly identify the stability/instability points of the gap-varying comb drive shown in Figure 2.2. Identifying these equilibrium points would result in solidifying the novelty in our proposed methodology.

2.5.1 Static Analysis

At first glance, the variation of equilibrium points between electrostatic and elastic forces as a function of the applied voltage $V_{app}$ is observed. As energy can be transferred between the mechanical and electrical domains, the equilibrium points are defined by the intersection between the electrostatic and elastic forces, as shown in Figure 2.3. The applied voltage $V_{app}$ can be normalized with respect to the static pull-in voltage:

$$V_{app} = \alpha \times V_{pull-in}, \text{ where } \alpha > 0$$  \hspace{1cm} (2.2)

For $(V_{app} > V_{pull-in})$, $\alpha > 1$, the excitation voltage for instability based on Eq 2.2 can be formulated as follows:

$$V_{app} = \alpha \times V_{pull-in}, \text{ where } \alpha > 1$$  \hspace{1cm} (2.3)

As mentioned before, the comb drive used for exciting the device has a gap-varying capacitor characteristic (Figure 2.2). This parallel plate capacitor exhibits high non-linear forces with instability within displacement range and fits well in our design, as we later need to drive the device into instability as defined in Eq 2.3. The electrostatic attractive force used for exciting the device (putting it into motion) $F_{e,exc}$ generated by the capacitor due to an
2.5. Border of Stability

applied voltage $V_{app}$ is given by:

$$F_{e,exc} = \frac{1}{2} \frac{C_0 d_0}{(d_0 - x)^2} V_{app}^2$$  \hspace{1cm} (2.4)$$

where $C_0$ and $d_0$ denote the nominal capacitance and gap of the designated gap-varying comb drive respectively and $x$ represents the displacement of the proof mass.

After identifying the electrostatic force, we need to verify this equilibrium point by deriving the total forces at equilibrium. Making the assumption that the actuation voltage $V_{app}$ is changing slowly comparing to the resonant frequency, allows us to ignore the inertial and damping effects in this analysis [64].

At equilibrium, the total force acting on the mass (in quasi-static operation mode) is given by combining the electrostatic force and the elastic force [64]):

$$F_{total} = F_{e,exc} + F_k = \frac{1}{2} \frac{C_0 d_0}{(d_0 - x)^2} V_{app}^2 - k d x$$ \hspace{1cm} (2.5)

At the equilibrium point, the electrostatic force is balanced by the elastic one [64] and Eq 2.5 can be written as:

$$F_{total} = F_{e,exc} + F_k = \frac{1}{2} \frac{C_0 d_0}{(d_0 - x)^2} V_{app}^2 - k d x = 0$$ \hspace{1cm} (2.6)

Using Eq 2.6 we proceed to write the actuation voltage $V_{app}$ as a function of the mass displacement $x$. Defining the characteristic voltage as [64]:

$$V_C = \sqrt{\frac{2k d d_0^3}{\epsilon A}}$$ \hspace{1cm} (2.7)

Eq 2.6 is reformulated as [64]:

$$V^2 = V_C^2 \frac{x}{d_0} \left(1 - \frac{x}{d_0}\right)^2$$ \hspace{1cm} (2.8)

The stability of the equilibrium point can be analysed by deriving Eq 2.5 [64], to obtain the stiffness of the system:

$$\frac{\partial F_{total}}{\partial x} = \frac{\epsilon A}{(d_0 - x)^3} V^2 - k$$ \hspace{1cm} (2.9)

In order to obtain an expression for the stiffness at the equilibrium point, we substitute Eq 2.8 to Eq 2.9 and evaluate the expression for $x = x_{pull-in}$ as follows [64]:

27
2.5. Border of Stability

\[
\frac{\partial F_{\text{total}}}{\partial x} \bigg|_{x=x_{\text{pull-in}}} = \frac{\epsilon A}{(d_0 - x)^3} V^2 - k
\]  

(2.10)

Solving Eq (2.10) for:

\[
\frac{\partial F_{\text{total}}}{\partial x} = 0
\]

(2.11)

gives the pull-in displacement \( x_{\text{pull-in}} \) as defined in Eq (2.1) [64]. Substituting Eq (2.1) in Eq (2.8) yields the pull-in voltage \( V_{\text{pull-in}} \) [64]:

\[
V_{\text{pull-in}} = \sqrt{\frac{4}{27}} V_C
\]

(2.12)

Recalling Eq (2.5) for small voltages \( (V_{\text{app}} < V_{\text{pull-in}}) \), the electrostatic force \( F_{\text{e,exc}} \) is balanced by the elastic force \( F_k \). The equilibrium position is obtained by solving Eq (2.5). This results in third order polynomial order in \( x \) which reveals three equilibrium points corresponding to the zeros of the third order polynomial [106]. Two of them are in the gap range \([0, d_0/3]\) while the third one cannot be reached as it is situated beyond the achievable mechanical displacement [106]. All the accelerometers on the market operate within this range. It should be noted that out of the two equilibrium points, only the first one is stable assuming small perturbations on the displacement \( x \) around the equilibrium positions. Around the first equilibrium point, a small increase of displacement \( x \) due to a small perturbation for example, results in a larger restoring elastic force \( F_k \) comparing to the electrostatic force \( F_{\text{e,exc}} \) hence restoring the equilibrium position. On the other hand, any small perturbation around the second equilibrium point (unstable) makes the electrostatic force \( F_{\text{e,exc}} \) much larger than the restoring elastic force \( F_k \) pushing the displacement much further away from the initial equilibrium [106].

The intersection between the electrostatic and elastic forces for different applied voltages is depicted in Figure 2.3. Figure 2.4 shows the stability and instability regions for a gap-varying comb drive. Figure 2.3 and Figure 2.4 make use of comb A model of IMOBCEH$S0903$ device depicted in Figure 3.3.

As the voltage increases, the electrostatic force \( F_{\text{e,exc}} \) begins increasing at a higher rate than the elastic force \( F_k \) and the structure snaps; the movable structure hits the fixed structure. Thus, for \( (V_{\text{app}} = V_{\text{pull-in}}) \) the stable and unstable equilibrium points merge, and the resulting equilibrium becomes unstable. Only one equilibrium point is revealed at \( [d_0/3] \); finally, for \( (V_{\text{app}} > V_{\text{pull-in}}) \), no equilibrium can be found, as the system is dominated by the electrostatic force due to the high applied voltage, and eventually the
2.5. Border of Stability

In this work our main interest is Eq \(2.3\) in operating at the transition border between the second and third regimes.

This limitation results in a large sensitivity loss of the capacitive microdevice, for the travel distance is limited to less than \(1/3\) of the full gap. It should be noted that the equivalent stiffness of the system decreases with increasing \(dc\) voltage, hence the resonant frequency decreases as well, and it becomes zero at pull-in \([64]\). Our goal is not to increase the travel distance of the proof mass, but to exploit and take advantage of the high sensitivity available at the border of stability close to the pull-in state.

2.5.2 Dynamic Analysis

In contrast to the previous static analysis of the pull-in, this dynamic one will take into consideration inertial and damping forces, and the additional effect of an external acceleration which could significantly alter the pull-in threshold as stated by Rocha et al. \([102, 104]\). The dynamics on the pull-in threshold is of utmost importance, since it allows quantifying the external
2.5. Border of Stability

Figure 2.4: Verification of stability/instability points of the gap-varying capacitor (excitation), accelerometer used is IMOBCEHS0903 depicted in Figure 3.3
2.5. Border of Stability

forces exerted on the system as an attempt to change that equilibrium position.

Dynamic analysis is observed by depicting phase portraits which allow us to acquire some qualitative views on the behaviour of the targeted system [105], with respect to variation in the applied voltage and the quality factor. For the targeted micro-structure, one DOF resonator, the state variables are the displacement \( x_{p1} \) and velocity \( x_{p2} \). The equation of motion is given by:

\[
m\frac{d^2 x_{p1}}{dt^2} + \gamma \frac{dx_{p1}}{dt} + kx_{p1} = F_{e,exc} = \frac{1}{2} \frac{C_0 d_0}{(d_0 - x_{p1})^2} V_{app}^2
\]  

(2.13)

Using the state space formulations, Eq 2.13 can be rewritten as:

\[
\begin{align*}
\dot{x}_{p1} &= x_{p2} \\
\dot{x}_{p2} &= \frac{1}{m} \left( 1 - \frac{C_0 d_0}{(d_0 - x_{p1})^2} V_{app}^2 - \gamma x_{p2} - kx_{p1} \right)
\end{align*}
\]

(2.14)

For simplicity Eq 2.14 can be formulated into a more convenient [105], emphasizing on the main parameters that characterise the dynamics of the system:

\[
\begin{align*}
\dot{x}_{p1} &= x_{p2} \\
\dot{x}_{p2} &= \frac{1}{m} \left( 1 - \frac{C_0 d_0}{(d_0 - x_{p1})^2} V_{app}^2 - \omega_0^2 x_{p2} \right)
\end{align*}
\]

(2.15)

where \( \omega_0 \) is the mechanical resonant frequency, and \( Q_f \) is the quality factor.

Figure 2.5 and Figure 2.6 show the non-linear dynamic behaviour of the micro-accelerator considering three distinct excitation voltages corresponding to the aforementioned cases of \( V_{app} \) and two quality factors (under-damped and critically-damped cases). The plots were obtained using PPLANE developed by John C. Polking of Rice University.

The important parameters in this study are the quality factor \( Q_f \) and the applied voltage \( V_{app} \). In this analysis the damping force plays a significant role, as it affects the dynamic motion of the micro-device; we considered a critically-damped case \( Q_f = 0.5 \), as some damping is needed to allow enough time for the dynamic non-linear behaviour.

The parameters used in Figure 2.5 (a) \( Q_f = 5 \), \( V_{app} = 21 \) V (b) \( Q_f = 0.5 \), \( V_{app} = V_{pull-in} = 42.097 \) V and in Figure 2.6 (a) \( Q_f = 0.5 \), \( V_{app} = 21 \) V (b) \( Q_f = 0.5 \), \( V_{app} = 50 \) V. They are based on the analysis performed on comb A of IMOBCEHS0903 device illustrated in Figure 3.3.

Rocha et al. [102] asserts that "the unstable equilibrium point (pull-in) set the boundary for the basin of attraction, in other words it marks
2.5. Border of Stability

Figure 2.5: Phase portraits of the gap-varying capacitor (excitation) (a) $Q_f = 5$, $V_{app} = 21V$ (b) $Q_f = 0.5$, $V_{app} = V_{pull-in} = 42.097V$
Figure 2.6: Phase portraits of the gap-varying capacitor (excitation) (a) $Q_f = 0.5$, $V_{app} = 21V$ (b) $Q_f = 0.5$, $V_{app} = 50V$
2.5. Border of Stability

the limit between a stable and an unstable mode. The basin of attraction is determined by the nullclines corresponding to the set of curves of the proof mass motion considering a null time derivative of one component of the state variables" \[102\], such as \( \dot{x}_{p1} = \dot{x}_{p2} = 0 \). The basin of attraction decreases inversely to the applied voltage and the quality factor. For example considering the same magnitude of voltage \( V_{app} \) applied onto the excitation comb, a low quality factor results in a larger basin of attraction as shown in Figure 2.5(a) and Figure 2.6(a), since the oscillations strongly contribute to the loss of stability. On the other hand Figure 2.5(b), Figure 2.6(b) show the transition of equilibrium state to non-equilibrium state as a function of the applied voltage, where for a fixed damping coefficient, the basin of attraction is shrinking as a function of the applied voltage \( V_{app} \) since a transition to the non-equilibrium regime is taking place.

In addition Figure 2.5 and Figure 2.6 reveals that all trajectories (black arrows) with initial values within the basin of attraction seem to end at the equilibrium point, while all the others collapse at the counter electrode or the mechanical stopper placed just before \[102\].

The proposed methodology will aim to perform analysis and measurement in the unstable state (Figure 2.6(b)) while maintaining the structure (through feedback) on the border of equilibrium (Figure 2.5(b)).

As the applied voltage increases, the first equilibrium point (stable) gets closer to the second equilibrium point (unstable) and this reduces the basin of attraction. In addition all state points outside the basin of attraction result in a dynamic pull-in \[105\], even for voltages lower than the pull-in voltage \( V_{pull-in} \).

Thus stability is mainly guaranteed inside the basin of attraction around the stable equilibrium position, but as the voltage increases the system becomes more and more unstable. If the applied voltage \( V_{app} \) is greater than pull-in voltage \( V_{pull-in} \) reinstating that one-equilibrium-point condition becomes very difficult, and almost impossible considering conventional methods.

The static and dynamic analysis defined and highlighted the risks and difficulties of operation in the unstable regime, around the border of stability. That area features a high sensitivity; however operating beyond the pull-in point for engineers without a robust controller, is in many way as difficult as inexperienced sailors when sailing through uncharted waters.
2.5. Border of Stability

2.5.3 Pull-in Time Measurement

In response to the bulk micro-machining conventional approach, the measurement of the pull-in time concept for capacitive micro-accelerometers reported and extensively researched by Rocha et al. [102, 104, 105] and Dias et al. [31-33], has alleviated the limitation in sensitivity for such micro-devices. Their work makes extensive use of the high sensitivity at the pull-in point. They report that under the excitation of a dc voltage slightly higher than the static pull-in voltage, and just before the structure snaps, or hits the mechanical stopper placed before the counter electrode, it travels through a region of high sensitivity, known as the metastable region. This is due to non-linearities in the electrostatic force generated by the gap-varying comb drive due to the applied voltage, and the relatively high damping coefficient of the structure.

The region of $x_{\text{pull-in}}$ or border of stability defined in Eq 2.1 is characterised by a tight equilibrium between electrostatic and elastic forces. As a result, the magnitude of the external acceleration (measurand) is proportional to the amount of time spent in that region. This time is known as the pull-in time - this behaviour is only witnessed in low-$Q_f (< 1.2)$ devices, as stated by Rocha et al. [102, 104].

Such damping prevents the structure from directly hitting the counter electrode and will create a dynamic transition (metastable region) proportional to the magnitude of the damping coefficient. This latter is linearised since the displacement is small, and will be positioned around $x_{\text{pull-in}}$, as given in Eq 2.1.

This concept is advantageous since the pull-in time depends on force and not affected by the input-referred noise of the readout circuit [103]. The authors derive a formula for the pull-in time as a function of the physical characteristics and dynamics of the micro-structure.

The trend mentioned above suggests that the metastable region is of paramount importance, due to the unique equilibrium characteristic it exhibits. As a result, the duration of this region is sensitive to any external force, hence, sensitive to the smallest disturbance that could act on the system while the structure is travelling in this region. Rocha et al. [102, 104] continuously and repeatedly bring the structure to pull-in while measuring the time and comparing it against the nominal pull-in time assuming zero external acceleration. This approach has resulted in high sensitivity measurement. In our opinion this approach is not very effective on the long run due to the continuous non-steady application of actuation voltages, which is turned-off just before the structure hits the mechanical stopper [102, 104].
2.5. Border of Stability

The operating principle of the open-loop model (without any feedback control), characterised by the pull-in time measurement thoroughly investigated by [102-104] is very simple and its operating principle can be summarized as follows:

- A dc voltage slightly higher than the static pull-in voltage \( V_{app} > V_{pull-in} \) of the gap-varying comb drive is used for excitation (Figure 2.2), applied onto it as defined in Eq 2.3, which eventually will generate an electrostatic force as shown in Eq 2.4

- Consequently, since the applied voltage is higher than the pull-in voltage, following the analysis in the previous section, the structure will snap, and the movable capacitor will hit the counter fixed electrode or the mechanical stoppers usually placed right before closing the gap (Figure 2.2), and this will occur in a \( t_{pull-in} \) time

- At this point it should be noted that \( t_{pull-in} \) time is dependant on the damping coefficient of the micro-structure

- If the device is under-damped \( (Q_f > 1.2) \), the structure will have a short transient behaviour and eventually snaps very quickly

- In contrast, if the structure is over-damped \( (Q_f > 1.2) \) then the pull-in transition will define three distinct regions: "a first region where the structure moves fast until the near static pull-in displacement, a second metastable region where the movement is very slow and finally a third region that takes the structure to the counter electrode" [102]

- In the first region the initial imposed electrostatic force \( F_{e,exc} \) is compensated by the spring \( F_k \) and damping forces [102, 106]. Rocha et al. [102, 106] defines the behaviour in this region "in the beginning the damping force dominates but with deflection of the proof mass, the spring force balances the electrostatic one and the damping force reduces becoming almost negligible" [102, 106]

- The second one, known as the metastable region the system behaviour is characterized by an almost zero velocity [102, 106]. According to Rocha et al. [102, 106], in this region, the structure moves very slow with duration determined by the damping coefficient of the structure and the electrostatic force is almost the same as the elastic one, resulting in a metastable equilibrium [102, 106]. Therefore any small external disturbance would be able to alter this equilibrium, hence the
2.5. Border of Stability

high sensitivity that can be achieved in this region and is set to 1/3 of the gap. It should be noted that as more damping is introduced into the system in order to prolong the metastable region, it results in more mechanical noise.

- In the third region the elastic force can no longer compensate for the electrostatic force which leads the structure to catastrophically snap the counter or fixed electrode [102, 106]

- The pull-in time $t_{\text{pull-in}}$ in the presence of an external acceleration is shorter than the nominal pull-in time if the external acceleration is acting in the same direction of the pull-in and longer if it is acting in the opposite direction.

- A time-counting mechanism is used to measure the pull-in time $t_{\text{pull-in}}$ and hence to retrieve the acceleration that was exerted on the micro-system.

![Figure 2.7: Travelling trajectory of the proof-mass due to high input voltage](image)

Figure 2.7 shows the open-loop dynamic behaviour of the IMOBCEHS0903 accelerometer depicted in Figure 3.3. Three distinct regions can be identified with respect to differences in the dynamic behaviour of the proof mass.
2.5. Border of Stability

Figure 2.8: Behaviour of the electrostatic (excitation), damping and elastic forces in the open-loop model
2.6 Objectives

Figure 2.8 depicts the behaviour of the forces, damping, elastic and electrostatic (excitation) with respect to the dynamic transition of the proof mass. Initially, the imposed electrostatic force $F_{e,exc}$ is compensated by the damping and elastic spring forces. Although the metastable region is not perfectly established, the damping force is determining the behaviour in that region, while the elastic and electrostatic forces are quite equivalents until the third region where the elastic force can no longer counteract the gradually increasing electrostatic force.

One of the key advantages using the pull-in time as a sensing mechanism is reducing the circuit noise as a limiting factor, since pull-in depends on force and it is not strongly affected by the total input referred noise of the capacitive readout circuit [103]. On the other hand, repeatedly bringing the structure to pull-in [102], or applying on/off method [105], is not very intuitive, and a robust, smart controller is needed to take advantage of the sensitivity at the border of stability while maintain stability at the sensor level.

With the region of high sensitivity well identified, the aim should point to stabilize the structure in that region. This is very similar to somehow stabilizing a pencil on its tip, as it would become sensitive to the slightest force that would aim to alter its equilibrium.

With such stabilization, the pull-in time measurement will be replaced by the movement of the feedback force. However, due to the high non-linearity of the electrostatic force, stabilization at the border of stability becomes a big burden; current state-of-the-art controllers such as $\Sigma\Delta$ modulators attempts have failed to circumvent this dilemma, and all accelerometers producers on the market today warn the consumer about exceeding the recommended voltage, which is far below the static pull-in voltage. This feature will be thoroughly exploited in Chapter 5.

2.6 Objectives

Based on the previous sections, we have concluded that a high sensitivity is mainly dependent the proof mass (large for bulk micro-machining) and requires a reliable closed-loop operation. It should be noted that both technologies (bulk and closed-loop control) can be combined together as it is in the most of cases, for high performance. However, surface micro-machined accelerometers are sandwiched between low output capacitance on one side, and a small proof mass, on the other; this limits their performance, and as result their field of applications. In addition, we noted the complexity in
the analogue circuitry, where it takes a relatively large amount of time to fabricate and tune the electronics for testing and calibration. The objectives of the proposed Ph.D. work can be summarized in the following points:

- Propose and prove new adaptive and non-linear controller techniques applied to closed-loop micro-accelerometers. Problems to tackle include but not limited to: proof mass repositioning, acceleration measurement, high sensitivity/resolution, driving mode of resonators, self-testing and self-calibration, stability and robustness against external variations: we want to address the non-linear electrostatic pull-in problem by stabilising the proof mass of the accelerometer at in the metastable region, where high sensitivity may be achieved.

- Design and experimentally characterize accelerometers. The extracted parameters can be used for verification of the proposed techniques and also for experimental verification in case of readout circuit.

- Provide a low cost efficient solution by targeting the controllers on a flexible, reprogrammable technology such as FPGAs, as illustrated in Figure 2.1.

- Build an effective hybrid system which combines a MEMS device, a readout circuit with analogue circuitry, and an FPGA. One of the biggest difficulties is the synchronization between the different blocks. The closed-loop system should demonstrate the measurement of a small acceleration exerted on the system.

The outcome and the benefit of this Ph.D. work will result in proposing new types of strategies for micro-accelerometer closed-loop control, based on non-linear and adaptive schemes, which are suitable for hardware implementation. One expects that the proposed work will re-shape the current state-of-the-art closed-loop micro-accelerometers in terms of performance (sensitivity/resolution), and measurement techniques for the input acceleration.
Chapter 3

Design and Characterisation of MEMS Accelerometers

"Design is where science and art break even"
— Robin Mathew

"People think that design is styling. Design is not style. It’s not about giving shape to the shell and not giving a damn about the guts. Good design is a renaissance attitude that combines technology, cognitive science, human need, and beauty to produce something that the world didn’t know it was missing"
— Paola Antonelli

3.1 Introduction

The accelerometers on the market today do not offer the opportunity for independent or separate actuation schemes, and many would be packaged along the controller in a single die assembly. Therefore, a new custom MEMS device design is needed, which offers the possibility of separate comb drives for sensing and feedback actuation. To begin addressing the challenge, we have targeted four fabrication runs, three in SOI-MUMPs® technology [84] with 25µm thickness, and one in MEMS-SOI® technology [29] with 60µm thickness. The four accelerometers differ in geometry, characteristics and response and can be used for a wide variety of applications. The designed accelerometers are uni-axial, and were modelled and designed using Coventor® design suite.

3.2 Accelerometers Design in SOI-MUMPs®

Three categories of structures have been designed, simulated and fabricated based on this fabrication technology. The structures resemble in geometry but their physical characteristics differ, and can be targeted in different applications. For each run, the structures consist of a floating proof mass at-
3.2. Accelerometers Design in SOI-MUMPs®

Attached to the substrate by four identical beams placed symmetrically around the seismic mass. Each structure exhibits gap-varying geometry (Figure 2.2) and/or area-varying geometry comb drives (Figure 3.1); placed symmetrically along the proof mass to maintain its balance. Stoppers are placed on both sides of the proof mass to prevent damage to the structure from excessive shocks.

![Figure 3.1: Area varying comb drive](image)

The different steps in the SOI-MUMPs® fabrication process shown in Appendix A.

Experimental characterisations are carried out using a Polytec MSA-500 micro-system analyser equipment from Polytec® [83]. This state-of-the-art equipment combines laser Doppler vibrometry for out-of-plane displacements measurements and video stroboscopy for measuring in-plane dynamics. In this later case, the equipment can detect device motions down to 2nm resolution.

Figure 3.2 illustrates a testing scenario for the designed IMOBCESH0903 accelerometer. To define an area of evaluation we proceed by defining first a search pattern. A point on the movable part of the MEMS device is chosen (red frame) so that it coincides with a reference point (yellow frame). The red frame always show the position of the selected search pattern, while the yellow one is that of the identified search pattern. The software displays a second larger green frame all around the search pattern. The difference between the yellow and the red frame [83] is used by Polytec MSA-500® to
3.2. Accelerometers Design in SOI-MUMPs

Figure 3.2: Testing with Polytec MSA-500 equipment
measure the displacement for the chosen number of points.

### 3.2.1 IMOBCEHS0903

![Image of IMOBCEHS0903 accelerometer obtained with Polytec MSA-500®](image)

Figure 3.3: Image of IMOBCEHS0903 accelerometer obtained with Polytec MSA-500®

Figure 3.3 shows an image of the designed IMOBCEHS0903 accelerometer in SOI-MUMPs® [84]. In this structure eight comb drives are placed symmetrically around the proof mass. Gap-varying capacitors are placed on top and bottom (A and B) while longitudinal or area-varying capacitors C are placed on left and right of the micro-structure. Four guided beams suspend the structure. The combs A and B differ in the number of fingers, with A having a lower capacitance than B. Excitation, sensing and feedback comb drives are chosen based on the targeted application, where in most cases gap-varying combs are chosen for excitation and differential capacitive measurement, while area-varying combs are chosen for feedback control. The red arrows indicate the direction of movement, since the electrostatic forces are attractive forces regardless of the sign of the applied voltage to the combs drives.

The micro-device has been designed in Coventor®, packaged in 40-DIP holders, where it should be noted that the die comprises four identical accelerometers, two positioned in (0x) position and two in (0y) position as shown in the figure below (Figure 3.4).
3.2. Accelerometers Design in SOI-MUMPs

Figure 3.4: Image of four identical IMOBCEHS0903 accelerometers on the same die obtained with Polytec MSA-500®

The characteristics of the IMOBCEHS0903 accelerometer are shown in Table 3.1.

The proof mass value $m$ is determined by measuring the actual dimensions of the structure (using Polytec MSA-500®), and using the known Silicon density value. As for the stiffness coefficient, the seismic mass is suspended by four guided beam springs mounted in parallel, where each one can be modelled as two cantilevers in series. Therefore, the stiffness coefficient can be calculated as:

$$k_{total} = 4 \times \frac{12 \times E \times I_{beam}}{L_{beam}^3} = 4 \times \frac{12 \times E \times (T_{beam} \times W_{beam}^2)}{L_{beam}^3}$$

$$= 4 \times \frac{12 \times E \times (25 \times 10^{-6} \times (5 \times 10^{-6})^3 / 12)}{(350 \times 10^{-6})^3} = 49.56 \text{ N/m}$$ (3.1)

In the previous formula $L_{beam}, W_{beam}, T_{beam}$ represent respectively, the length, width and thickness of the beam. $I_{beam}$ is the moment of inertia, and $E$ is the Young's modulus of Silicon. Several samples have been characterized. The error variation percentage for the resonant frequency and
3.2. Accelerometers Design in SOI-MUMPs ®

stiffness coefficient are ~ 20% and ~ 37% respectively. This large variation is due to imperfections in the fabrication process in the SOI-MUMPs ® process (Table 3.1).

Table 3.1: Physical characteristics of the IMOBCEHS0903 accelerometer

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proof mass (mg)</td>
<td>$5.36 \times 10^{-2}$</td>
</tr>
<tr>
<td>Thickness (µm)</td>
<td>25</td>
</tr>
<tr>
<td>Natural resonant frequency (KHz)</td>
<td>4.989</td>
</tr>
<tr>
<td>Natural resonant frequency (Polytec ®) (KHz)</td>
<td>4.15</td>
</tr>
<tr>
<td>Stiffness coefficient (N/m)</td>
<td>49.56</td>
</tr>
<tr>
<td>Stiffness coefficient (Polytec ®) (N/m)</td>
<td>36.41</td>
</tr>
<tr>
<td>Quality factor (Polytec ®)</td>
<td>8.9</td>
</tr>
<tr>
<td>One overlap actuating finger C (µm³)</td>
<td>$175 \times 7 \times 25$</td>
</tr>
<tr>
<td>Total number of fingers (area-varying comb)</td>
<td>36</td>
</tr>
<tr>
<td>One overlap sensing finger A or B (µm³)</td>
<td>$375 \times 7 \times 25$</td>
</tr>
<tr>
<td>Total number of fingers (gap-varying comb)</td>
<td>88</td>
</tr>
<tr>
<td>Pull-In Voltage One Comb A (V)</td>
<td>42</td>
</tr>
<tr>
<td>Pull-In Voltage One Comb B (V)</td>
<td>26.82</td>
</tr>
<tr>
<td>Nominal Capacitance $4 \times A + 2 \times B$ (pF)</td>
<td>1.36</td>
</tr>
<tr>
<td>Noise Equivalent Acceleration $\ddot{x}_{\text{noise}}$ (µG/√Hz)</td>
<td>3.312</td>
</tr>
<tr>
<td>Input Referred RMS Acceleration $\ddot{x}_{\text{RMS,noise}}$ (µG)</td>
<td>862</td>
</tr>
<tr>
<td>Dynamic Range (G)</td>
<td>$[-125, 125]$</td>
</tr>
</tbody>
</table>

The value of the quality factor $Q_f$ is determined using a parametric fit of the experimentally measured spectral response with the response of a linear second order model. The first experimental characterisation was the frequency response. Unipolar voltages were applied onto the area-varying comb drives. We made use of a four-channel signal amplifier from Tabor Electronics ® [139] which exhibit a special unipolar mode for MEMS engine drivers [139].

Figure 3.5 shows the frequency response for different ac common voltages. This figure shows the useful feature the area-varying combs exhibit: the actuation force depends only on the voltage, but it is otherwise independent of the displacement. In contrast, the gap-varying combs exhibit a dependence of the electrostatic force on the displacement, leading to spring softening effects.

This limitation and non-linearity have been thoroughly discussed in the previous chapter, and Figure 3.6 clearly shows the risks of increasing the am-
3.2. Accelerometers Design in SOI-MUMPs

Figure 3.5: Frequency response for IMOBCEHS0903 with varying ac voltages (area-varying combs)

Figure 3.6: Frequency response for IMOBCEHS0903 with varying ac voltages (gap-varying combs)
3.2. Accelerometers Design in SOI-MUMPs

The amplitude of the applied voltage. Using a gap-varying comb for excitation along with a high applied voltage put the device in instability mode, and a robust mechanism would be required to stabilize the device in the unstable regime, which will be exploited throughout this work. The results have been processed in Mathematica® by Wolfram © [149]. As a result, the experimental Bode plot shown in Figure 3.7 yields a quality factor of $Q_f = 8.9$, relatively common for such surface micro-machined structures. It should be noted that the noise equivalent acceleration $\dot{x}_{\text{noise}}(\mu G/\sqrt{Hz})$ is computed based on the value of the quality factor, where at the resonance the structure exhibits low noise, while below resonance, which is the operating bandwidth of the micro-accelerometer, the noise is higher and is independent of the quality factor and is identified as input referred RMS acceleration $\ddot{x}_{\text{RMS,noise}}(\mu G)$ [64].

Figure 3.7: Measured Bode plot response of the IMOBCEHS0903 micro-accelerometer (using Polytec MSA-500®) and the parametric fit (using Mathematica®)

These two quantities will be used in Chapter 5 to demonstrate the sen-
3.2. Accelerometers Design in SOI-MUMPs

Sensitivity of the device and its performance measuring small signals.

3.2.2 IMOBCES21002

Table 3.2: Physical characteristics of the IMOBCES21002 accelerometer

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proof mass (mg)</td>
<td>$5.92 \times 10^{-2}$</td>
</tr>
<tr>
<td>Thickness ($\mu m$)</td>
<td>25</td>
</tr>
<tr>
<td>Natural resonant frequency ($KHz$)</td>
<td>2.14</td>
</tr>
<tr>
<td>Natural resonant frequency (Polytec ©) ($KHz$)</td>
<td>2.11</td>
</tr>
<tr>
<td>Stiffness coefficient ($N/m$)</td>
<td>10.7</td>
</tr>
<tr>
<td>Stiffness coefficient (Polytec ©) ($N/m$)</td>
<td>10.38</td>
</tr>
<tr>
<td>Quality factor (Polytec ©)</td>
<td>2.78</td>
</tr>
<tr>
<td>One overlap actuating finger ($\mu m^3$)</td>
<td>$285 \times 8 \times 25$</td>
</tr>
<tr>
<td>Total number of fingers (area-varying comb)</td>
<td>34</td>
</tr>
<tr>
<td>One overlap sensing finger ($\mu m^3$)</td>
<td>$285 \times 8 \times 25$</td>
</tr>
<tr>
<td>Total number of fingers (gap-varying comb)</td>
<td>112</td>
</tr>
<tr>
<td>Pull-In Voltage One Comb A (V)</td>
<td>9</td>
</tr>
<tr>
<td>Pull-In Voltage One Comb B (V)</td>
<td>7.853</td>
</tr>
<tr>
<td>Nominal Capacitance $4 \times A + 2 \times B$ (pF)</td>
<td>2.36</td>
</tr>
<tr>
<td>Noise Equivalent Acceleration $\ddot{x}_{noise} (\mu G/\sqrt{Hz})$</td>
<td>3.75</td>
</tr>
<tr>
<td>Input Referred RMS Acceleration $\ddot{x}_{RMS,noise} (\mu G)$</td>
<td>363.35</td>
</tr>
<tr>
<td>Dynamic Range ($G$)</td>
<td>$[-10, 10]$</td>
</tr>
</tbody>
</table>

Figure 3.8 depicts the structure of the IMOBCES21002 accelerometer. This second run design was a slight variation to the first one, due to some errors in the fabrication process at the foundry level (over-etching). Modifications include changing in the direction of some of the gap-varying combs of IMOBCEHS0903, to provide more symmetry for actuating the micro-device. In addition, the sensing gap, the pull-in voltage and the resonant frequency have been decreased. This architecture enhances the sensitivity of the accelerometer (in comparison with IMOBCEHS0903) and provides a better versatility in accommodating different capacitive sensors and actuators.

Similarly to the IMOBCEHS0903, the accelerometer is packaged in 40-DIP holders, where the die comprises four identical accelerometers, two positioned in (0x) position and two in (0y) position, as shown in Figure 3.9.

The error variation percentage for the resonant frequency and stiffness coefficient are $\sim 1.42\%$ and $\sim 3\%$ respectively. The physical characteristics
3.2. Accelerometers Design in SOI-MUMPs

![Image of the IMOBCES21002 accelerometer obtained with Polytec MSA-500](image1)

Figure 3.8: Image of the IMOBCES21002 accelerometer obtained with Polytec MSA-500

![Image of four identical IMOBCES21002 accelerometers on the same die obtained with Polytec MSA-500](image2)

Figure 3.9: Image of four identical IMOBCES21002 accelerometers on the same die obtained with Polytec MSA-500
of the *IMOBCE21002* structure are shown in Table 3.2. Following from Eq (3.1), the new stiffness coefficient is calculated as:

\[
k_{total} = 4 \times 12E \times \frac{Lh_{beam}^4}{Th_{beam} \times W h_{beam}^3 / 12} \\
= 4 \times 12E \times \frac{(25 \times 10^{-6} \times (3 \times 10^{-6})^3 / 12)}{(350 \times 10^{-6})^4} \\
= 10.7 \text{N/m}
\]

The experimental stiffness has only been shifted by –0.4N/m (Table 3.2). Experimental characterisations were also carried out using *Polytec MSA-500* equipment. The frequency response has been experimentally characterised, similarly to the previous accelerometer where differential voltages are applied to the area-varying combs.

Figure 3.10: Measured *Bode* plot response of the *IMOBCE21002* micro-accelerometer (using *Polytec MSA-500*) and the parametric fit (using *Mathematica*).

Figure 3.10 shows the frequency response *IMOBCE21002* with differential excitation using area-varying combs. The results have been processed in *Mathematica*. This structure exhibits lower quality factor \( Q_f \) than the *IMOBCEHS0903* accelerometer. The experimental *Bode* plot shown in Figure 3.10 yields a quality factor of \( Q_f = 2.78 \) with a maximum amplitude at
resonance $H_{DC} = 5.78 \times 10^{-8} m$.

### 3.2.3 IMOBCEH21103

The third run of SOI-MUMPs® accelerometers was designed with specific features in order to be mounted on the same die along with a CMOS-based readout chip, as part of a collaborative work with the system on chip Lab (SOC) at the University of British Columbia. The CMOS readout circuit is designed by my colleague J. Shiah, under the supervision of Dr. S. Mirabbasi. The size of bondpads connecting the mass and the area gap-varying used for capacitance sensing, are made similar in size and spacing to their counterpart on the CMOS chip (Figure 3.12), which minimizes errors while wire-bonding results in minimizing parasitics and enhancing the sensitivity of the microsystem (MEMS + CMOS).

Figure 3.11 depicts the structure of IMOBCEH21103 accelerometer. This structure, unlike the previous two runs, only exhibits gap-varying combs, for sensing, excitation and feedback control actuation. In addition, for high sensitivity and low cross-axis sensitivity, box-beams are used instead of single guided beams used in the case of IMOBCEHS0903 and IMOBCES21002 accelerometers. These beams eliminate any rotational movement that could be caused by an external acceleration, hence maintaining movement in the direction of interest. In addition, two pairs of area-varying combs used for capacitive sensing are connected to two bondpads, which in turn are connected to the input of the CMOS/PCB readout circuit.

The physical characteristics of the IMOBCEH21103 structure are shown in Table 3.3. In order to calculate the total stiffness coefficient, each box beam is divided in two parts (upper and lower), hence the total is four. The spring constant of each one is considered as the equivalent spring constant of two guided beams in series. As a result, the total spring constant of the movable structure is the equivalent of four considered in parallel. The total stiffness coefficient is calculated as:

$$k_{\text{total}} = 4 \times \left( \frac{1}{K_{\text{upper}}} + \frac{1}{K_{\text{lower}}} \right)^{-1}$$

$$= 4 \times \left( \frac{1}{12EI/Lh_1^2} + \frac{1}{12EI/Lh_2^2} \right)^{-1}$$

$$= 3.58 N/m$$

We have used $Lh_1 = Lh_2 = 400 \times 10^{-6} \mu m$, $Wh_1 = Wh_2 = 3 \times 10^{-6} \mu m$.

Experimental characterisations were also carried out using a Polytec MSA-500® equipment. The experimental stiffness has only been shifted by $1.57 N/m$ (Table 3.3). The error variation percentage for the resonant
3.2. Accelerometers Design in SOI-MUMPs

Figure 3.11: Image of the IMOBCEH21103 accelerometer obtained with Polytec MSA-500

Table 3.3: Physical characteristics of the IMOBCEH21103 accelerometer

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proof mass ( (mg) )</td>
<td>( 4.82 \times 10^{-2} )</td>
</tr>
<tr>
<td>Thickness ( (\mu m) )</td>
<td>25</td>
</tr>
<tr>
<td>Natural resonant frequency ( (KHz) )</td>
<td>1.375</td>
</tr>
<tr>
<td>Natural resonant frequency (Polytec ( (KHz) ))</td>
<td>1.65</td>
</tr>
<tr>
<td>Stiffness coefficient ( (N/m) )</td>
<td>3.5859</td>
</tr>
<tr>
<td>Stiffness coefficient (Polytec ( (N/m) ))</td>
<td>5.16</td>
</tr>
<tr>
<td>Quality factor (Polytec ( (\mu G)</td>
<td>2</td>
</tr>
<tr>
<td>One overlap sensing finger ( (\mu m^3) )</td>
<td>( 375 \times 20 \times 25 )</td>
</tr>
<tr>
<td>Total number of fingers (gap-varying comb)</td>
<td>48</td>
</tr>
<tr>
<td>Pull-In Voltage One Comb ( A ) ( (V) )</td>
<td>7.58</td>
</tr>
<tr>
<td>Nominal Capacitance ( 8 \times A ) ( (pF) )</td>
<td>1.3288</td>
</tr>
<tr>
<td>Noise Equivalent Acceleration ( \ddot{x}_{\text{noise}}(\mu G/\sqrt{Hz}) )</td>
<td>3.93</td>
</tr>
<tr>
<td>Input Referred RMS Acceleration ( \ddot{x}_{\text{RMS,noise}}(\mu G) )</td>
<td>258.63</td>
</tr>
<tr>
<td>Dynamic Range ( (G) )</td>
<td>(-7,7)</td>
</tr>
</tbody>
</table>
3.2. Accelerometers Design in SOI-MUMPs

Figure 3.12: Image of two identical IMOBCEH21103 accelerometers on the same die obtained with Polytec MSA-500®

Figure 3.13: Measured Bode plot response of the IMOBCEH21103 micro-accelerometer (using Polytec MSA-500®) and the parametric fit
3.3. Accelerometers Design in MEMS-SOI

The frequency response has been experimentally characterised, similarly to the previous accelerometers where differential voltages are applied to the gap-varying combs. The results have similarly been processed in Mathematica®. As a result, the experimental Bode plot shown in Figure 3.13 yields a quality factor of $Q_f = 2$ with a maximum amplitude at resonance $H_{DC} = 1.18 \times 10^{-7} m$. This structure exhibits the lowest quality factor among three structures designed in SOI-MUMPs®.

3.3 Accelerometers Design in MEMS-SOI

As mentioned earlier, only one design of accelerometers has been designed using the SOI-MUMPs® fabrication methodology. The different steps in the fabrication process shown in Appendix A.

3.3.1 IMTBCIMU

This accelerometer is designed to be integrated in an inertial measurement unit which comprises two identical micro-accelerometers (uni-axial) placed in $(0x)$ and $(0y)$ positions and one Yaw-rate micro-gyroscope. As noted earlier, the targeted technology is MEMS-SOI® [29], which offers a relatively thick (60µm) proof mass and is very advantageous in terms of sensitivity.

Figure 3.14 illustrates the inertial measurement unit die, containing two identical single-axis accelerometers placed orthogonally and one Yaw-rate gyroscope, yielding a three-DOF inertial measurement unit. A major challenge with the MEMS-SOI Yaw chip is that the cap is sealed and there is no direct access for an optical measurement, as in the case of the SOI-MUMPs devices.

The image shots shown in Figure 3.15 and Figure 3.14 were sent to us from the foundry. This structure has been designed in Coventor®, hermetically packaged in 40-DIP holders; as it cannot be tested with the Polytec MSA-500®, a readout circuit is required as the optical measurement is no longer valid. The IMTBCIMU accelerometer structure is illustrated in Figure 3.15 where crab-leg beams are used to suspend the proof mass. This geometry exhibits eight equivalent parallel plate capacitors comb drives (gap-varying geometry), which allows the usage of sensing and/or actuating schemes. Every two adjacent comb drives, from the left or right, are oriented in the same direction to provide symmetry. Proper stoppers are integrated to protect the proof mass from any damage in case of an excessive displac-
Figure 3.14: Image of the inertial measurement unit comprising two identical accelerometers in $(0x)$ and $(0y)$ and one yaw-rate gyroscope.
ment due to a high applied voltage or a high input acceleration beyond the dynamic range.

Table 3.4 summarizes the physical and geometrical characteristics of the IMTBCIMU micro-accelerometer. The crab-leg beams are used and tuned in order to guarantee high sensitivity in the direction of interest. Figure 3.15 shows the difference in thickness between the two part of the crab-leg beam. The upper part (attached to the anchor) is made with a large width ($L_{h_{\text{beam}}} = 65 \times 10^{-6} \mu m$, $W_{h_{\text{beam}}} = 55 \times 10^{-6} \mu m$) to cancel rotational movement while the lower part ($L_{h_{\text{beam}}} = 550 \times 10^{-6} \mu m$, $W_{h_{\text{beam}}} = 4 \times 10^{-6} \mu m$) attached to the proof mass is made with a much smaller width to enable movement in the direction of interest. The spring constant of each one is considered as the equivalent spring constant of two guided beams in series. As a result the total spring constant of the movable structure is the equivalent of four considered in parallel. Using the moment of inertia $I$ depicted in Eq 3.1 and following up from Eq 3.3 the total stiffness coefficient is cal-
3.3. Accelerometers Design in MEMS-SOI

Table 3.4: Physical characteristics of the \textit{IMTBCIMU} accelerometer

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proof mass ((mg))</td>
<td>0.1982</td>
</tr>
<tr>
<td>Thickness ((\mu m))</td>
<td>60</td>
</tr>
<tr>
<td>Natural resonant frequency ((KHz))</td>
<td>1.41</td>
</tr>
<tr>
<td>Stiffness coefficient ((N/m))</td>
<td>15.69</td>
</tr>
<tr>
<td>One overlap sensing finger ((\mu m^3))</td>
<td>562 \times 20 \times 60</td>
</tr>
<tr>
<td>Total number of fingers (gap-varying comb)</td>
<td>56</td>
</tr>
<tr>
<td>Pull-In Voltage One Comb (A (V))</td>
<td>9.76</td>
</tr>
<tr>
<td>Nominal Capacitance (8 \times A (pF))</td>
<td>4.77</td>
</tr>
<tr>
<td>Input Referred RMS Acceleration (\ddot{x}_{RMS,noise} (\mu G))</td>
<td>131.15</td>
</tr>
<tr>
<td>Dynamic Range ((G))</td>
<td>([-9, 9])</td>
</tr>
</tbody>
</table>

Figure 3.16: Frequency response of the \textit{IMTBCIMU} accelerometer in \textit{Coven-tor}
3.4. Conclusion

calculated as:

\[
 k_{total} = 4 \times (\frac{1}{K_{upper}} + \frac{1}{K_{lower}})^{-1} \\
= 4 \times (\frac{1}{12EI/Lh^3} + \frac{1}{12EI/Lh^3})^{-1} \\
= 15.69 \text{N/m} 
\]  
(3.4)

Figure 3.16 shows the frequency response in Coventor ®, for \((0x)\), \((0y)\), and \((0z)\) which reflects on the low cross-axis sensitivity of the designed accelerometer.

3.4 Conclusion

In this chapter we have presented four accelerometers fabricated in two different technologies, SOI-MUMPs ® [84] and MEMS-SOI ® [29]. The two technologies differ by fabrication recipes, in addition to the thickness of the Silicon structure which plays a great deal of importance for inertial sensors. We have seen in the first two chapters how a thick proof mass is always a major factor in increasing the open loop sensitivity of the device, which results in lowering the mechanical noise, making the device attractive for low-G measurements.

The devices fabricated in SOI-MUMPs ® technology [84] were optically characterised in order to determine the corresponding characteristics, such as the quality factor and the frequency response as function of the input voltage. The optical characterisation of the three micro-devices in SOI-MUMPs ® [84] yielded different quality factors which can be targeted into different applications. These measurements were obtained in open-air and at room temperature. It would be interesting to make such measurements in vacuum, but the vacuum chamber of the Polytec MSA-500 ® is not ready yet. In contrast to the SOI-MUMPs ® technology the IMTBCIMU fabricated in MEMS-SOI ® technology [29] do not offer the possibility of optical testing and a readout circuit is required to perform electrical measurements and characterisations.

The IMOBCEH21103 accelerometer has been also characterised with a PCB-based and CMOS-based readout circuits. The characterisation based on the PCB readout circuit is illustrated in Chapter 5. It will serve as a baseline for future experimental closed-loop control for the IMOBCEH21103 accelerometer, comparing to simulations results obtained with the other accelerometers.

The devices exhibit different geometries in terms of comb drives (area-and gap-varying) and also different beams to suspend the seismic mass. The
3.4. Conclusion

choice of the appropriate geometry and appropriate excitation and feedback control is related to the type of the targeted application. It should be noted that the four designed micro-accelerometers exhibit low cross-axis sensitivity; other resonance modes in the \((\theta y)\) and \((\theta z)\) directions are quite far and do not interfere with the main sensing mode.

Therefore, to achieve good sensitivity and resolution with surface micro-machined devices, a closed-loop control is put in place. This will be the goal of the next two chapters, where we present adaptive and non-linear feedback techniques.
Chapter 4

Closed-Loop MEMS Accelerometers with Adaptive Techniques

"You have to be fast on your feet and adaptive or else a strategy is useless"
— Charles De Gaulle

"What I try to do in the book is to trace the chain of relationships running from elementary particles, fundamental building blocks of matter everywhere in the universe, such as quarks, all the way to complex entities, and in particular complex adaptive system like jaguars"
— Murray Gell-Mann

4.1 Introduction

In this chapter we present the application of adaptive algorithms to the feedback control of micro-accelerometers. Besides simple stochastic gradient algorithms such as the least mean squares (LMS) and the normalized least mean squares (NLMS), we additionally propose two novel adaptive algorithms, modified set membership normalized least mean squares (MSMNLMs) and switch step least mean squares (SWSLMS). The proposed adaptive controllers are targeted on FPGA using rapid prototyping methodology with Labview®. The targeted device is VIRTEx-5 LX50 from Xilinx®. To the best of our knowledge, no one has previously reported an adaptive gradient descent control for a micro-accelerometer, despite its simplicity and common use in many applications. Hardware in the loop sim-

ulation results demonstrate the repositioning of the proof mass to its rest position in the presence of an external acceleration exerted on the system, while constantly measuring the external acceleration; the feedback increases linearity, sensitivity and dynamic range and decreases the cross coupling. Hardware resources estimation confirms the small area the proposed algorithm occupies on the targeted device.

4.2 MEMS and Adaptive Algorithms

In our work we place a great deal of interest in control schemes that can be easily implemented on FPGA, characterised by low mathematical complexity. Adaptive control is widely applied in many applications such as echo cancellation [96], beamforming smart antennas [42, 43], filtering [44] and channel equalization [49], just to name a few. In this work, we demonstrate the concept of applying adaptive controllers to closed-loop micro-accelerometers to reposition the proof mass to central position due to an external acceleration applied to the system.

In the literature we come across the use of adaptive filters for improving the performance of the piezo-resistive accelerometer for automotive applications as reported in [50], in order to cancel the noise that corrupts the relevant information coming from the accelerometer. Another adaptive noise canceller of motion artefact in stress electrocardiographic (ECG) signals using a commercial accelerometer (ADXL105) from Analog Devices [60] has been reported in [97], the algorithms used are LMS and recursive least squares (RLS). The same two algorithms have been used by [159] for eliminating the noise of the detection circuit for MEMS-based accelerometers, allowing for a noiseless measurement of the acceleration. In short, most of the applications of adaptive filtering techniques to accelerometers are targeted towards improvements at the system level. At that point the accelerometer could be in an open-loop fashion or in a closed-loop one employing ΣΔ. Our approach is to use the adaptive filtering techniques within the accelerometer subsystem entity as a feedback controller to replace the current state-of-the-art ΣΔ module.

Therefore, in this work, we apply four adaptive algorithms to the feedback control of the micro-accelerometer. These algorithms belong to the adaptive step size family of algorithms, where a fixed step size is used to adapt the weights and hence change the output accordingly and minimize the mean square error. Throughout this chapter, we demonstrate their usage in performance and hardware integration.
4.3 Proposed Adaptive Algorithms

Figure 4.1 illustrates the closed-loop system level architecture of the micro-accelerometer module, along with the readout electronics, ADC and DAC, and the adaptive controller targeted on FPGA. The closing of the loop requires an opposing force to the external inertial force; the most common and known actuating scheme for micro-accelerometers is via electrostatic actuation. As a result, the attractive electrostatic force applied to the actuation (feedback) combs will reposition the mass to its rest position and the accelerometer is rebalanced by the compensated signals provided to the feedback electrodes.

The change in displacement due to movement of the proof mass upon an external shock induces a differential capacitance variation that is measured and converted to a voltage signal $V_{out}$, as the output voltage of a readout circuit (capacitance to voltage converter). Each of the four proposed controllers requires (see Figure 4.1) as input two voltage sources: one would act as a bias voltage $V_{bias}$ (the voltage that is required to maintain the proof mass in a central position) and the other as reference voltage $V_{ref}$. This last one will act as the reference or desired signal that is required in most of the adaptive controllers. The difference between $V_{ref}$ and the output voltage $V_{out}$ is considered as the error signal and used as the error input to the adaptive controller. It should be noted that $V_{out}$ also represents the input signal to the digital controller. A constant is required as a step size. The feedback voltage $V_{feedback}$ is the output of the adaptive filter, and provides information about the input acceleration applied to the system. Once fed to the feedback capacitors, the feedback voltage results in an actuation force that balances and counteracts the inertial force, and cancels the movement caused by the external force exerted on the system.

Adaptive algorithms are based on computing the weight vectors; the literature behind this concept is abundant [49]. To simplify the number of equations and avoid redundancy, the proposed adaptive algorithms follow the same pattern for error and output signals, and differ in the weight adaptation method. For the given problem:

- The controller output is computed as:
  \[ y_m[n] = \hat{W}_m^H[n]P_m[n] \]  \hspace{1cm} (4.1)

- The estimated error can be computed as:
  \[ e_m[n] = y_m[n] - \hat{S}_m[n] \]  \hspace{1cm} (4.2)
4.3. Proposed Adaptive Algorithms

Figure 4.1: Block diagram of the proposed adaptive implementation.
It should be noted that in this work we only deal with real numbers, however the general equations of the problem and of the proposed method (LMS, NLMS, MSMNLMS, SWSLMS) are presented here for illustration purposes.

\( \hat{w}^H_m[n] \) denotes the transpose, \( n \) is the number of samples, \( P_m[n] \) is the input signal (which is equivalent to \( V_{out} \)), \( \bar{S}_m[n] \) is the reference signal (equivalent to \( V_{ref} \)) and \( \hat{W}^H_m[n] \) is the controller coefficient. The difference between the output voltage \( V_{out} \) and the reference voltage \( V_{ref} \) represents the error signal \( e_m[n] \).

The output of the controller denoted by \( y_m[n] \) is equivalent to the feedback voltage \( V_{feedback} \), is responsible for repositioning the proof mass to its rest position. \( m \) represents the number of DOF, in this work, we consider only one DOF, since the designed accelerometer is uniaxial (acceleration is measured along a single axis). A transversal finite impulse response implementation is assumed for all of the proposed controllers.

Figures 4.2 and 4.3 illustrate the FPGA implementation of the proposed adaptive algorithms considering a size of six taps. The number of taps can be easily increased for better performance, based on the requirements and the hardware resources available on the targeted FPGA, but a six-tap controller was found to have good performance and an average complexity, in case a closed-loop system with multiple DOF is required.

### 4.3.1 Least Mean Squares (LMS)

It is a widely used algorithm. The LMS method is a stochastic gradient algorithm and consists of two basic processes: the first one is filtering and involves (i) "computing the output of a transversal filter produced by set of tap inputs" [49], and (ii) "generating an estimation error by comparing this output to a desired or reference response" [49]; the second process is adaptation and involves the automatic adjustments of the tap weights of the filter according to the estimation error. For more information about this algorithm the reader is referred to [49]. The LMS weight adaptation process can be summarized as follows:

\[
\hat{W}^H_m[n+1] = \hat{W}^H_m[n] + \mu_{LMS} P_m[n] e^*_m[n]
\]  

(4.3)

Where \( * \) denotes the conjugate, and \( \mu_{LMS} \) is the adaptation step size. Figure 4.2 (a) shows the implementation of a six-tap LMS controller in Labview FPGA®.
4.3. Proposed Adaptive Algorithms

Figure 4.2: LabviewFPGA implementation of the proposed LMS (a) and NLMS (b) considering six taps
4.3. Proposed Adaptive Algorithms

Figure 4.3: *LabviewFPGA* ® implementation of the proposed *MSMNLMS* (a) and *SWSLMS* (b) considering six taps.

Figure 4.3: *LabviewFPGA* ® implementation of the proposed *MSMNLMS* (a) and *SWSLMS* (b) considering six taps.
4.3. Proposed Adaptive Algorithms

4.3.2 Normalized Least Mean Square (NLMS)

The NLMS algorithm is considered as companion of the ordinary LMS [49] algorithm and has been introduced to resolve some problems concerning the input vector in the LMS algorithm. The weight adaptation in the NLMS algorithm can be computed as:

$$\hat{W}_m^H[n + 1] = \hat{W}_m^H[n] + \frac{\mu_{NLMS}}{\alpha_{NLMS} + \| P_m[n] \|^2} P_m[n] e^*_m[n]$$ (4.4)

Where $\alpha_{NLMS}$ is a small positive constant and $\| P_m[n] \|$ is the Euclidean input vector norm.

Simon Haykin asserts that "the correction $\mu_{NLMS} P_m[n] e^*_m[n]$ applied to the tap-weight vector $\hat{W}_m^H[n]$ at iteration $[n + 1]$ is normalized with respect to the squared Euclidean norm of the tap input [49] vector $P_m[n]$ at iteration $[n]$. This normalization is important in particular if $P_m$ is large, when the LMS algorithm might experience a gradient noise amplification problem. For more about this algorithm the reader is referred to [49]. Similarly to the LMS, the LabviewFPGA® implementation of the NLMS was carried out for six taps and is shown in Figure 4.2 (b).

4.3.3 Modified Set Membership Normalized Least Mean Squares (MSMNLMS)

Set membership identification theory is "extended to the more general problem of linear-in-parameters filtering by defining a set-membership specification, as opposed to a bounded noise assumption" [44]. The set membership identification normalized least mean squares SMNLMS was proposed by Gollamudi et al. [44], for filtering applications where it outperformed the common adaptive NLMS. It is considered among the optimal bounding ellipsoids (OBE) algorithms.

However, when we applied this algorithm to MEMS-based accelerometers, the algorithm did not converge because it was experiencing a gradient noise amplification problem due to the input vector $V_{out}$. Therefore, to solve this problem a small positive constant $\alpha_{MSMNLMS} < 0.1$ is added to the Euclidean norm of the tap input vector in the denominator. We call this new algorithm the modified set membership NLMS or MSMNLMS. The weight vector can be adapted as follows:

$$\hat{W}_m^H[n + 1] = \hat{W}_m^H[n] + \beta_m[n] \frac{e_m[n] P_m^*[n]}{\alpha_{MSMNLMS} + P_m^H[n] P_m[n]}$$ (4.5)
4.4. Hardware in the Loop Simulation Results

Where
\[ \beta_m[n] = \begin{cases} 1 - \frac{\zeta}{|e_m[n]|} & \text{if } |e_m[n]| > \zeta_{MSMNLMS} \\ 0 & \text{otherwise} \end{cases} \]  

(4.6)

Where \( \zeta_{MSMNLMS} \) is a small constant between 0 and 1. The six-tap adaptive controller LabviewFPGA \( \text{®} \) implementation is shown in Figure 4.3 (a).

4.3.4 Switch Step Least Mean Squares (SWSLMS)

The proposed SWSLMS algorithm derives directly from the basic LMS algorithm with a step-size that switches between an upper and lower values of the feedback force depending on the actual proof mass position; in other words, the SWSLMS algorithm takes into consideration another input. In control design, it has been observed that "switching between two controllers can lead to higher performing control loops than if either of the controllers is used exclusively, and such switching can stabilize a loop that is proven to be unstable with either controller" [110].

The proposed SWSLMS uses two step sizes, a very small one \( \mu_{MIN} \) that will lead to slow convergence and a very large one \( \mu_{MAX} \) which leads to divergence. High speed switching between \( \mu_{MIN} \) and \( \mu_{MAX} \) stabilizes the system with optimal speed. The goal is to reposition the proof mass to its central position while measuring the acceleration, therefore the threshold for switching between the upper and lower values of the step size is null. The SWSLMS weight adaptation process can be summarized as follows:

\[ \hat{W}_m[n + 1] = \hat{W}_m[n] + \mu_{SWSLMS} P_m[n] e_m^*[n] \]  

(4.7)

Where
\[ \mu_{SWSLMS} = \begin{cases} \mu_{MAX} & \text{if } P_m[n] > 0 \\ \mu_{MIN} & \text{otherwise} \end{cases} \]  

(4.8)

Here \( \mu_{SWSLMS} \) is the adaptation step size. The algorithm can be further extended to include automatic adaptive adaptation of \( \mu_{MAX} \) and \( \mu_{MIN} \), similarly to [20]. It should be noted that the proposed SWSLMS is different than the two step sizes DLMS in [147] where the authors propose a dual step size algorithm behaving as two separate LMS algorithms. The LabviewFPGA \( \text{®} \) implementation of SWSLMS with six taps is shown in Figure 4.3 (b).

4.4 Hardware in the Loop Simulation Results

The system level architecture is illustrated in Figure 4.1. LabviewFPGA \( \text{®} \) is a rapid prototyping methodology that allows for quick verification of
the system implementation and hence cutting down the *time-to-market*. The *LabviewFPGA* module from *National Instruments* uses *Labview* embedded technology to extend *Labview* graphical development and target *FPGAs* on *NI* reconfigurable I/O (RIO) hardware [59]. *Labview* is distinctly suited for *FPGA*-based programming because it clearly represents parallelism and data flow, in many applications requiring high-speed hardware reliability and tight determinism.

The system consists of two virtual instruments (VIs), one is identified as the *Host* and comprises the models for the micro-accelerometer and readout electronics (capacitance to voltage converter), while the second is identified as the *Target* and comprises the adaptive signal processing unit targeted on *FPGA* (adaptive controller). The host is to be replaced by the physical micro-system (*MEMS* + capacitive readout) in the full implementation.

![Diagram](image)

Figure 4.4: *LabviewFPGA* implementation of the proposed adaptive algorithms considering six taps

Figure 4.4 illustrates the configuration of *Host VI*, and the communication between the *Host* and the *Target*. The latter is illustrated in Figures 4.2 and 4.3.

### 4.4.1 Proof Mass Repositioning

The application of adaptive algorithms to the closed-loop micro-accelerometer is validated using hardware in the loop fixed point numerical simulation,
4.4. Hardware in the Loop Simulation Results

which provides a baseline for the experimental implementations and future work. To validate the performance of the proposed adaptive algorithms, the closed-loop accelerometer employing a digital adaptive controller is compared to an open-loop one (with no feedback electrostatic force).

Table 3.1 depicts the physical parameters of the micro-accelerometer behavioural model used in the simulations. For simplicity, the damping analysis is linearised yielding a relatively high constant damping coefficient given by $\gamma_d = \mu_v A^2/d_0^3 = 0.603$, hence an over-damped accelerometer where $\mu_v$ is the air viscosity, $A$ is the capacitor area and $d_0$ is the initial gap [45]. Different scenarios based on variations of the input acceleration have been adopted for simulations. The amplitude of the acceleration is $1G$ unless otherwise specified, where $G = 9.81 \text{m/s}^2$ is the gravitational acceleration at sea level. The external parameters such as external acceleration and voltages are fixed. The purpose is to demonstrate the efficiency and the performance of the proposed algorithms taking into consideration the same input conditions.

The values of voltages and step sizes are fixed in all of the simulations schemes. We impose $\mu_{\text{LMS}} = 0.9$ for LMS, $\mu_{\text{NLMS}} = 0.5$ and $\alpha_{\text{NLMS}} = 0.1$, for NLMS, $\zeta_{\text{MSMNLMS}} = 0.1$ and $\alpha_{\text{MSMNLMS}} = 0.09$, for MSMNLMS, and $\mu_{\text{MAX}} = 10$ and $\mu_{\text{MIN}} = 0.01$ for the SWSLMS. In this work and for simplicity the reference $V_{\text{ref}}$ and bias $V_{\text{bias}}$ voltages are equal and set to $20V$, but they can be at different levels. It should be noted that in this design a longitudinal comb drive model (area-varying comb) is used as the actuator (feedback control). The accelerometer used is IMOBCEHS0903 illustrated in Figure 3.3. In such geometry, the electrodes move parallel to each other keeping the electrode gap constant but changing the capacitor area [64]. This geometry offers many advantages where the stroke distance is not limited by the pull-in and the capacitance changes linearly with the displacement [64].

The first scenario is for a step input with $0.01\text{msec}$ step time. Figure 4.5 depicts the results for the proposed adaptive approach, where the initial step in the input acceleration displaces initially the proof mass from the zero position. The controller action balances the action of the external disturbance and brings, after a short transient, the proof mass position back to the zero displacement. It should be noted that the proof mass position is not fully repositioned to null position due to the reference voltage signal.

The LMS algorithm is known for its slow convergence in comparison to other adaptive algorithms such as RLS. However in this particular application, the performance of the LMS is quite considerable, compared to the open-loop case. The SWSLMS which derives directly from the basic LMS with a switch step size mechanism, outperforms the others, and is the best candidate. At the time of the impact ($0.01\text{msec}$), the SWSLMS reacts by
4.4. Hardware in the Loop Simulation Results

limiting the displacement of the proof mass to a ~ 30% of the displacement in case of an open-loop accelerometer, and then adaptively proceeds to decreasing the proof mass position to a null position or close enough to that line.

Figure 4.6 shows the results of the second scenario of simulations where we apply a sine input acceleration defined by:

\[ a(t) = a_{MAX} \sin(\omega t) \quad (4.9) \]

Where \(a_{MAX}\) and \(\omega\) are the amplitude (1G) and frequency (90Hz) of the input acceleration respectively. The proposed adaptive approach reduces the amplitude of the proof-mass displacement similarly to the step input case, proving the effectiveness of the digital adaptive controller. There is a slight and negligible offset due to the reference voltage; never the less the proof mass position is balanced. As it shown in Figure 4.6, the MSMNLMS has a similar level of performance as the simple SWSLMS, which is still the best candidate.

In the third scenario, we apply a sequence of input pulses, each with a constant 0.01msec duration as follows:

\[ a(t) = 0G, 1G, 0G, 1G, 0G, 1G \quad (4.10) \]
4.4. Hardware in the Loop Simulation Results

Figure 4.6: Proof mass displacement after a sine input of $1G$ with a frequency of $90Hz$

Where $0$ corresponds to $a_{ext} = 0m/s^2$ and $1$ corresponds to $a_{ext} = 9.81m/s^2$. The results of this scenario are shown in Figure 4.7. In a similar manner with the previous scenarios, the proposed adaptive approach allows for a quick detection of any deviation from the zero position, followed by a compensation of the inertial force, hence forcing the mass to the rest position. Another interesting remark can be observed by analysing Figure 4.7.

We have mentioned earlier that at the time of the impact the adaptive controller reacts quickly and gives the maximum transient displacement of the proof mass of $5.7nm$ compared to $10nm$ for an open-loop accelerometer. However, in some applications, as in automotive and biomedical industries, the accelerometer is subject to several successive shocks at a constant or random time interval. In this case study the input accelerations have equal magnitudes $1G$, and the proof mass of an open-loop accelerometer shows the same displacement dynamics for the second input acceleration pulse as for the first one. However, using the proposed adaptive approach, the second input acceleration pulse saturates the proof mass position to the minimized position reached in the previous step; hence, even for a series of shocks, the proposed methodology guaranties reliability and long life span of the
4.4. Hardware in the Loop Simulation Results

Figure 4.7: Proof mass displacement after a sequence of pulses with 1G magnitude

accelerometer. In addition, as the proof mass displacement is being cancelled by means of the adaptive controller, the acceleration can be easily extracted and this is shown in the next subsection.

4.4.2 Acceleration Measurement

Accelerometers are designed for sensing external accelerations; the natural question that often arises is how to extract the information regarding the input acceleration from the dynamics of the employed controller, where the last could be analogue or digital. Conventional readout circuits use capacitive sensing schemes to map the position of the proof mass to a differential capacitance change, which is converted by the circuit into an output voltage [46]. The small value of the differential capacitive changes, in the range of $aF$ and $fF$, and the additional parasitic resistances and capacitances make this task very challenging. It should be noted that the response of the system in closed-loop is much closer to the inputs signal’s form [45] and the electrostatic actuation control signal constitutes the sensor measurement signal [131]. In our case, with a closed-loop system employing an adaptive controller, the proof mass displacement for 1G input acceleration is in the order
4.4. Hardware in the Loop Simulation Results

of \sim nm, which yields a differential capacitance in the order of \sim 10aF.

We have mentioned earlier that in closed-loop micro-accelerometers, the feedback electrostatic force, or the counterbalance force carries the information about the input acceleration exerted on the system, since it is mainly responsible for cancelling the motion and repositioning the proof mass to its central position. Generally speaking, the dominant trend in an acceleration sensing system is to minimize as much as possible the analogue electronics interface subsystem and to maximize the use of digital control. This statement is valid in our proposed approach for employing adaptive digital control schemes.

In the proposed approach the external acceleration can be measured digitally from the output of the controller on FPGA. In digital filtering the tap weights \( \hat{W}_m^n \) are multiplied by the input signal \( P_m^n \) to provide the feedback voltage \( V_{feedback} \) and the measurement of the external acceleration as well. The same scenarios of simulations considered in the previous subsections will be considered here, to demonstrate the measurement of the acceleration using the feedback voltage \( V_{feedback} \).

Figure 4.1 shows a product \( B = G \times A \), where \( A \) is the scaling factor (gain) that maps the external acceleration to the output voltage, and \( G \) is the earth gravitation at sea level; based on several simulations we found the optimal \( B \) product for each adaptive algorithm. For \( LMS B_{LMS} = 0.5556 \times G \), for \( NLMS B_{NLMS} = 0.4587 \times G \), for \( MSNLMS B_{MSNLMS} = 0.4613 \times G \) and last \( B_{SWSLMS} = 0.4562 \times G \) for the \( SWSLMS \).

In the first scenario, illustrated in Figure 4.8, where a step function with 0.01msec step time is applied, we noticed that the \( LMS \) has the largest error percentage for acceleration measurement, due to its slow rate of convergence followed by the \( NLMS \); in contrast the \( MSNLMS \) and \( SWSLMS \) show a very low error percentage. We mentioned and explained before that the \( SWSLMS \) derives directly from the \( LMS \); however, due to the upper step size, and the switching scheme between the upper and lower step size, it outperforms the \( LMS \) in terms of convergence rate.

The acceleration measurement results for a sine wave input are shown in Figure 4.9. The \( SWSLMS \) seems to reproduce the input acceleration with a smaller error percentage compared to the other implemented algorithms. In regards to the third scenario, we apply a sequence of input pulses, each with a constant duration, as in Eq 4.10. We mentioned that at the second and third pulse, the proof mass position does not respond in the same way as for the first impulse and the proof mass position is reduced tremendously. However, this does not affect the measurement of the acceleration, which can be seen clearly in Figure 4.10; again \( LMS \) shows the largest percentage
4.4. Hardware in the Loop Simulation Results

Figure 4.8: Acceleration measurement considering a step input of 1G magnitude

Figure 4.9: Acceleration measurement considering a sine input of 1G magnitude and a frequency of 90 Hz
4.4. Hardware in the Loop Simulation Results

Figure 4.10: Acceleration measurement considering a sequence of pulses with 1G magnitude

4.4.3 Hardware Resources and Trade-Offs

According to [159] "Adaptive signal processing plays an important role in small signal detection". As we mentioned before, it is important to be able to implement the desired functionality in a flexible and low cost manner, typically on FPGA hardware. For many researchers and hardware engineers, the bottleneck in FPGA signal processing is the optimization for area occupancy and/or speed.

Therefore the engineer has to be very careful when choosing the right word-length, to maintain a high performance over complexity ratio. The choice of the FPGA according to the number of available resources (slices, flip flops (FF), embedded multipliers, I/Os, etc.) played also a burden as they are related to cost.

An FPGA with a large number of resources is more expensive than the one with fewer resources. However, nowadays FPGA have a huge number of resources comparing to the ones made few years ago and they can accommodate more mathematical complexity, never the less, they remain a cheap, flexible solution for signal processing. The outcome is for the designer who has more area on the FPGA and can trade-off more for speed and resolu-
4.4. Hardware in the Loop Simulation Results

Therefore, we used the VIRTEX-5 LX50 from Xilinx © [93], which has large amount of hardware resources. In this work, we did not compromise on the resolution and a large word-length (integer=14, fractional=16) was used. The point is to demonstrate the concept of using adaptive controllers for the feedback loop of micro-accelerometers. Therefore assuming a worst case scenario where a large word-length is used, and considering an implementation of six-tap controller, the hardware resources estimation, and architecture speed and latency shown below are very encouraging, in terms of adopting this technique to closed-loop micro-accelerometers. Optimizations in terms of speed or area occupancy can be later optimized depending on the targeted applications and the available resources.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Total slices</th>
<th>Slice registers</th>
<th>Slice LUT</th>
<th>DSP 48s</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMS</td>
<td>1085 15.1</td>
<td>2990 10.4</td>
<td>2388 8.3</td>
<td>13 27.1</td>
</tr>
<tr>
<td>NLMS</td>
<td>1573 21.8</td>
<td>3753 13</td>
<td>3358 11.7</td>
<td>19 39.6</td>
</tr>
<tr>
<td>MSMNLMS</td>
<td>1831 25.4</td>
<td>3832 13.3</td>
<td>4392 15.3</td>
<td>41 85.4</td>
</tr>
<tr>
<td>SWSLMS</td>
<td>1300 18.1</td>
<td>3033 10.5</td>
<td>2411 8.4</td>
<td>13 27.1</td>
</tr>
</tbody>
</table>

The proposed implementations have very good concurrency and use fast hardware components. The degree of concurrency is determined by the underlying mathematical aspects of various components forming the closed feedback loop [113]. The hardware resources estimate includes number of slices, lookup tables (LUT), memory blocks (BRAM), embedded multipliers, FF, and tristate buffers (TBUF). In the previous subsections, we have seen that SWSLMS outperforms the rest of the proposed controllers, in terms of proof mass repositioning and acceleration measurement.

In addition it shows a simple hardware complexity comparable to LMS.
4.5 Conclusion

Good performance, comparable to \textit{SWSLMS} is achieved by the \textit{MSMNLM}, but however it exhibits large hardware resources. The proposed adaptive algorithms showed similar latency, and an acceptable achieved clock frequency (see Table 4.2). In general, adaptive algorithms present a good approach for closed-loop micro-accelerometers.

4.5 Conclusion

The proposed adaptive algorithms approach will pave the way for future digital closed-loop accelerometers where the current \textit{state-of-the-art} is based on $\Sigma\Delta$ modulators. Simple adaptive filtering techniques have been demonstrated for this purpose, where the proof mass displacement has been tremendously reduced to an almost null position. In addition to that we have demonstrated how the acceleration can be easily extracted in digital or analogue manner, right from the feedback voltage. The most attractive approach in terms of performance and hardware complexity was the \textit{SWSLMS} in terms of repositioning the proof mass to its central location, acceleration and hardware resources. The interesting part of this proposed method is that it relies on two step sizes, one large that would cause direct divergence and another small one that would cause slow convergence. By switching between the two at high frequency, the algorithm allows for fast convergence while maintaining stability of the system.

The drawback in this approach is the second input voltage that is required, where for simplicity was made equal to the bias voltage but it could change. The digital control of \textit{MEMS}-based transducers, through its potential for direct \textit{FPGA} implementation, reduces significantly the cost of developing and fabricating \textit{MEMS}-based micro-systems through single chip integration. The \textit{FPGA} implementation allows a fast design and optimization cycle.

The favourable results will pave the way for more real time integration which will include the \textit{MEMS}-based accelerometer chip and a readout circuit.
Chapter 5

Closed-Loop MEMS
Accelerometers with
Non-Linear Sliding Mode Control

"I've let whole days slip through my fingers,
Whole years, Decades squandered making money,
Buying stuff, Carefully packing it all into boxes,
Unpacking it again, Fixing things up, Throwing things away,
Going to different places And coming back again.
I'm sliding down hard ice, Faster, No meaning,
Faster, No feeling, Faster, No bottom in sight."
— Russ Allison Loar

5.1 Introduction

In this chapter we propose novel feedback control and sensing schemes based on sliding mode control (SMC)\textsuperscript{4} for closed-loop micro-accelerometers, as alternative digital control architectures to sigma-delta Σ∆-based approaches.

Designing a stable and robust sensor with very high measurements performance is considered a challenging problem for most of the measurands [131]. We have already mentioned how crucially the robustness of the control scheme and the adaptation to different operating environments contribute to the performance of the system. These environments could induce changes in controller structure, discrete jumps in parameter values, ambient temperature, etc.

As earlier stated, switching between two controllers can lead to higher performing control loops than if either of the controllers is used exclusively;

\textsuperscript{4}Several parts of this chapter have appeared in multiple conference and journal papers.
moreover, switching can stabilize a loop that is proven to be unstable with either controller [110]. Therefore switching between two reference voltages (or two actuation forces) has the potential of stabilizing the micro-accelerometer when employed in closed-loop, and this concept has been proven in control using $\Sigma\Delta$ modulators. We have also witnessed this in the previous chapter with switching between two adaptation step sizes ($\mu_{\text{MAX}}$ and $\mu_{\text{MIN}}$). The small step size leads to slow convergence while the large one may diverge the system, but by switching between the two values at high frequency the system was converging.

The first attempt is to apply SMC to the feedback control of the micro-accelerometer with a reference trajectory set to the null position $X_{\text{ref}} = 0$. We propose a novel scheme for extraction of the acceleration exerted on the system. An FPGA implementation using rapid prototyping methodology is proposed, and an estimation of hardware resources is provided. Simulation results demonstrate the effectiveness of the method in feedback control and acceleration measurement.

Second, given the advantages and the success of the first attempt, we extend the application of SMC to increase the sensitivity by operating at the border of stability, as defined in Eq 2.1. This is an excellent use of the SMC, tackling an intrinsically non-linear unstable problem. The importance of this methodology is verified with simulations and experimental results.

Third, the proposed SMC is applied to drive resonators at their resonant frequency. In this case the resonator becomes insensitive to changes in the environment (temperature) and is able to track the shift in the natural resonant frequency.

### 5.2 SMC and Application to MEMS

$SMC$ uses controlled structural variation (i.e. switching) as an integral part of a control mechanism, and is referred as variable structure control system. Basically, it uses different control laws depending on whether the state (in the phase space) is on one side or the other of a hyper-surface (also called switching-surface), as illustrated in Figure 5.1. In either case, the control causes the state to move towards the surface; once the surface is reached the same pair of control laws attempts to keep the state on it [110], hence stability will be achieved and maintained. In other words, the algorithm consists of enforcing the motions of the state trajectories on some manifolds in the system state-space [111].

It should be noted that the surface is chosen so that the state slides
5.2. SMC and Application to MEMS

naturally toward the target point. One of the advantages of the SMC is that the design process is decoupled into two stages [38, 110], with each involving a lower order system; the controller is designed based on a simple measure, that is, the distance from the sliding surface. As a result the controller offers robustness and invariance to uncertainties [38, 109-111], controller order reductions, superior disturbance rejection, and simple implementation by means of switches and comparators [111] - this last feature will be exploited in the FPGA implementation detailed in the following sections.

SMC is one of the promising control schemes for MEMS-based sensors operated in closed-loop systems. Sane et al. [111] demonstrate the application of sliding mode control to a two-axis gimbaled micro-mirror from Intellisense © [56]. This controller is characterised by controller order reductions, superior disturbance rejection, insensitivity to parameter variations and simple implementations by means of switches and comparators. They show that SMC meet the needs for a simple implementation - since the targeted implementation will contain capacitive sensing and control circuits, they mentioned that SMC reduced steady-state errors by 70 – 90%. SMC has been investigated theoretically for MEMS-based gyroscopes by Batur et al. [14], in order to maintain the proof mass oscillating in the desired direction at controlled amplitudes and frequencies, despite uncertainties in the

Figure 5.1: Sliding manifold in the state space
gyroscope parameters. In a subsequent paper, Fei and Batur [40] extend the work and propose an adaptive SMC to estimate the angular velocity, the linear damping and the stiffness coefficient of a MEMS-based gyroscope, where they adopted a proportional and integral sliding surface instead of the conventional sliding surface.

A robust SMC was targeted for a non-linear MEMS-based optical switch by Ebrahimi and Bahrami [38], to achieve a higher accuracy and robustness against disturbances, in addition to its ease of implementation in practical applications. Anderson et al. [7] presented a control approach based on SMC that reduces the effects of disturbances in the first few seconds of flight. This new control exhibits potential performance improvements for MEMS-based gyroscopes. Further applications of SMC to the MEMS field include a piezo-resistive nano-mechanical cantilever (NMC) for force tracking, with applications to imaging and nano-manipulation tasks, as proposed by Saeidpourazar and Jalili [109]. The piezo-resistive layer is used to measure the nano-scale forces at the tip of the NMC, instead of a bulky laser-based feedback. The modelling that was used for the NMC-based force sensor is based on lumped parameters, instead of the distributed parameter approach. By replacing the NMC with a linear mass-spring-damper model, the increased uncertainty and un-modelled dynamics have been addressed through the robust SMC-based controller, which has outperformed the conventional PID controller.

An interesting comparison of three controllers for an optical switch has been presented by Izadbakhsh and Rafiei [61] - according to their simulations, high-gain and model-free approaches showed more complex design, modelling, and increased simulation time compared to variable-structure method such as SMC. The three controllers demonstrated robustness against uncertainties, disturbances and modelled errors; however, for large uncertainty values, SMC becomes unstable. Nevertheless, in our opinion SMC is becoming more and more attractive for MEMS-based applications, due to its advantages in robustness, simplicity and stability, which will be demonstrated for micro-accelerometers. The robustness allows for instance achieving high performance even when using very simplified macro-models for the transducers in the design phase.

5.3 SMC Mathematical Framework

We already mentioned that the goal is to find a robust controller, with low complexity and high performance.
5.3. SMC Mathematical Framework

To begin addressing the challenge, we will proceed with a general derivation of the mathematical problem. The control could vary depending on the type of comb drive used for feedback actuation, which could be area-varying comb (Figure 3.1) or gap-varying comb (Figure 2.2).

Considering an area-varying comb geometry, the actuating electrodes move parallel to each other, keeping the electrode gap constant but changing the capacitor area \([64]\). The capacitance for one finger to finger electrode overlap is given by:

\[
C = \epsilon \frac{h(l + x)}{d_0} + C_f
\]  

(5.1)

Here \(h\) is the height of the finger, \(l\) is the initial overlap between the fingers, \(x\) is the displacement from zero-force position, \(d_0\) is the gap between the fingers and \(C_f\) is the fringe capacitance, which accounts for the non-overlapping portion of the fingers. The gradient of the capacitance from Eq 5.1 can be written as:

\[
\frac{dC}{dx} = \epsilon \frac{h}{d_0}
\]  

(5.2)

The electrostatic acting force on the proof-mass (connected rigidly to the movable plates) is therefore given by:

\[
F_e = \frac{1}{2} V_{app}^2 \frac{dC}{dx} = N_{o} h \frac{1}{2d_0} V_{app}^2
\]  

(5.3)

where \(N_{o}\) is the number of overlap fingers, and \(V_{app}\) is the applied voltage. Hence the electrostatic force can be written as:

\[
F_e = \eta V_{app}^2
\]  

(5.4)

The constant \(\eta\) is dependant on the geometry of the capacitor:

\[
\eta = \begin{cases} 
N_{o} \frac{h}{2d_0} & \text{for area-varying comb} \\
C_{0} \frac{1}{2d_0} & \text{for gap-varying comb, small displacement approximation}
\end{cases}
\]  

(5.5)

In this work, we consider a single degree of freedom accelerometer, whose dynamics is modelled by:

\[
m \frac{d^2x}{dt^2} + \gamma_d \frac{dx}{dt} + k_d x = F_e
\]  

(5.6)

In the previous equation \(m\) is the mass, \(\gamma_d\) is the damping factor, \(k_d\) represents the spring constant, and \(F_e\) represents the sum of the external forces.
(inertial and electrostatic). In this work, the control is applied through the electrostatic forces, while the external acceleration to be measured is treated as an external disturbance. This allows the use of the SMC formalism to demonstrate (via a Lyapunov second order function) that given a certain dynamic range, the effect of the external acceleration will be compensated by controller’s action. In SMC, the generic control variable $u$ has only two possible values, given by a switching law:

$$
\begin{align*}
+ F_0 & \quad \text{if } S(e, \dot{e}) \leq 0 \\
- F_0 & \quad \text{if } S(e, \dot{e}) > 0
\end{align*}
$$

(5.7)

Where $S(e, \dot{e})$ is the switching function or the sliding surface, and $e$ is the error variable (the difference between actual and desired position). The control law designed above forces the state trajectory to remain on $S$ possibly by means of infinite fast switching. In our case, the generic control corresponds to a net electrostatic force oriented toward the right $+F_0$ or left $-F_0$, respectively. Both actuation forces are generated by applying a reference voltage $V_{ref}$ to the right or left fixed plates of the capacitive actuators.

The first step in designing a variable structure controller is to choose an appropriate state vector function $S(e, \dot{e})$. Therefore, we wish to design a sliding mode controller so that $x$, the displacement of the proof mass follows closely $x_{ref}$, the desired, reference trajectory, which in our case is reduced to the zero (rest) position. For this second order system let the state variables be:

$$
\begin{align*}
x_{p1} &= x \\
x_{p2} &= \dot{x}
\end{align*}
$$

(5.8)

We have therefore the plant state equations:

$$
\begin{align*}
\dot{x}_{p1} &= x_{p2} \\
\dot{x}_{p2} &= \ddot{x} = \frac{1}{m} F e - \gamma_4 x_{p2} - \frac{k_d}{m} x_{p1} - \ddot{x}_{ref}
\end{align*}
$$

(5.9)

The tracking error is defined by:

$$
\begin{align*}
x_1 &:= x_{p1} - x_{ref} \\
x_2 &:= x_{p2} - \dot{x}_{ref}
\end{align*}
$$

(5.10)

The state equations can be mapped in terms of the error:

$$
\begin{align*}
\dot{x}_1 &= \dot{x}_{p1} - \ddot{x}_{ref} = \dot{x} - \ddot{x}_{ref} = x_{p2} - \ddot{x}_{ref} = x_2 \\
\dot{x}_2 &= \dot{x}_{p2} - \dddot{x}_{ref} = \ddot{x} - \dddot{x}_{ref} = \frac{1}{m} F e - \frac{2\gamma_4}{m} x_{p2} - \frac{k_d}{m} x_{p1} - \ddot{x}_{ref}
\end{align*}
$$

(5.11)
We consider in this work a first-order sliding mode function $S(e, \dot{e})$ of the state errors, defined as:

$$S(e, \dot{e}) = \dot{e} + \lambda e = x_2 + \lambda x_1 \Rightarrow$$

$$\dot{S}(e, \dot{e}) = \dot{x}_2 + \lambda x_1 = \frac{1}{m} F_e - \frac{\gamma_d}{m} x_p - \frac{k_d}{m} x_p - \ddot{x}_{ref} + \lambda x_1$$  (5.12)

The strategy of SMC is to firstly ensure a convergence of the trajectories in the phase space to the desired sliding surface (in this case defined by $S(e, \dot{e}) \equiv 0$) and then to maintain them on this manifold through the fast switching action of the same controller. The first phase, the convergence of the dynamics to the reduced dimensional sliding surface $S(e, \dot{e}) \equiv 0$, is ensured if a measure of the "energy" of the error, $W(s) = \frac{1}{2} S(e, \dot{e})^2$ (a second order Lyapunov function), is monotonously decreasing; in other words, the controller action leads to $\frac{d}{dt}(\frac{1}{2} S^2) < 0$. Following this short transient, the further dynamics of the system will evolve on the sliding surface defined by $S(e, \dot{e}) \equiv 0$. Hence using $S = x_2 + \lambda x_1$ and Eq [5.10] yields:

$$S = \dot{x} - \dot{x}_{ref} + \lambda (x - x_{ref})$$

$$= x_p - \dot{x} + \lambda (x_p - x_{ref})$$  (5.13)

Then we can write:

$$\dot{S} = \dot{x}_p - \ddot{x}_{ref} + \lambda (\dot{x}_p - \dot{x}_{ref})$$

$$= 0$$  (5.14)

Using Eq [5.11] the derivative of the sliding surface can be written as:

$$\dot{S} = \frac{1}{m} F_e - \frac{\gamma_d}{m} \frac{dx_p}{dt} - \frac{k_d}{m} x_p - \ddot{x}_{ref} + \lambda (x_p - \dot{x}_{ref})$$  (5.15)

- Considering an area-varying comb drive:

  Using Eq [5.3] Eq [5.15] can be written as:

$$\dot{S} = \frac{1}{m} N_{\text{comb}} \frac{\epsilon h}{2d} V_0^2 - \frac{\gamma_d}{m} x_p - \frac{k_d}{m} x_p - \ddot{x}_{ref} + \lambda (x_p - \dot{x}_{ref})$$  (5.16)

Therefore, with $\eta = N_{\text{comb}} \frac{\epsilon h}{2d}$ and $u = V_{app}^2$, the expression for the equivalent control is given by:

$$u_{eq} = \frac{1}{\eta} [k_d x_p + \gamma_d x_p + m(\ddot{x}_{ref} - \lambda (x_p - \dot{x}_{ref}))]$$  (5.17)
5.3. SMC Mathematical Framework

This equivalent control leads to a motion along the sliding surface assuming no uncertainties and disturbances, which is equivalent to the robust controller that will be described next. Therefore, giving this equivalent control, the sliding mode controller, defined as \( u_{SMC} \) (robust controller), can be written as:

\[
    u_{SMC} = \frac{1}{\eta} [k_d x_p 1 + \gamma_d x_p 2 + m(\ddot{x}_{ref} - \lambda(x_p 2 - \dot{x}_{ref} - \rho \text{sgn}(S)))] \quad (5.18)
\]

- Considering a gap-varying comb drive:
  using Eq 2.4, Eq 5.16 can be written as:

\[
    \dot{S} = \frac{1}{2} \left( \frac{C_0 d_0}{d_0 - x_p 1} \right) V_{\text{app}}^2 - \frac{\gamma_d}{m} x_p 2 - \frac{k_d}{m} x_p 1 - \ddot{x}_{ref} + \lambda(x_p 2 - \dot{x}_{ref}) = 0
\]

Therefore, with \( \eta = \frac{C_0 d_0}{2} \) and \( u = V_{\text{app}}^2 \), the expression for the equivalent control is given by:

\[
    u_{eq} = \frac{1}{\eta} [k_d x_p 1 + \gamma_d x_p 2 + m(\ddot{x}_{ref} - \lambda(x_p 2 - \dot{x}_{ref}))(d_0 - x_p 1)^2] \quad (5.20)
\]

Which yield a robust controller \( u_{SMC} \) defined as:

\[
    u_{SMC} = \frac{1}{\eta} [k x_p 1 + \gamma_d x_p 2 + m(\ddot{x}_{ref} - \lambda(x_p 2 - \dot{x}_{ref} - \rho \text{sgn}(S)))(d_0 - x_p 1)^2] \quad (5.21)
\]

\( \rho \) is a controller parameter, set to a constant value that depends on disturbances exerted on the system and the desired transient time (before reaching the sliding surface). This robust controller is applied not only to reach the sliding surface but also to stay on it, since the switching term forces the state to go back to the sliding surface in case it deviates from it. The control action switches the actuation between two constant values, \(+F_0\) and \(-F_0\). Applying either one as a constant input (open loop) does not stabilize the dynamics of the system, but by means of infinite switching between these two states (based on the switching law), the system trajectory converges exponentially fast [111] toward a stable state, with a convergence speed determined by \( \lambda \). The choice of \( \lambda \) depends on the trade-off between ensuring a high convergence speed, much faster than the time needed for the input acceleration signal to change, and avoiding reaching stability issues.
5.4 SMC for $x_{ref} = 0$

In this part we demonstrate the use of the SMC to reposition the proof mass or cancelling its movement after deviation from rest position $x_{ref} = 0$ due to an external acceleration exerted on the system.

The accelerometer used in this section is IMOBCEHS0903 depicted in Figure 3.3. For feedback control, the two left-right area-varying combs are used. In this case $\eta = N_{olcb} \frac{h}{2d_0}$.

So far only the action of the electrostatic forces has been considered. To demonstrate the robustness against disturbances, let us consider the external, unknown acceleration, as an equivalent bounded disturbance $a(t)$ introduced into the system dynamics:

$$\begin{cases} \dot{x}_{p1} = x_{p2} \\ \dot{x}_{p2} = \ddot{x} - \frac{1}{m} \gamma \dot{x}_{p2} - \frac{k_d}{m} x_{p1} + a(t) \end{cases}$$

(5.22)

We assume there is an upper, known limit, $|a(t)| \leq a_{max}$ set according to the dynamic range of the micro-accelerometer; $a(t)$ may also include other uncertainties, such as un-modelled dynamics and parametric uncertainties.

By using the second method of Lyapunov, let the Lyapunov function be $W = \frac{1}{2} S^2$. Differentiating $W(s)$ with respect to time, we obtain:

$$W = S[-\rho \text{sgn}(S) + a(t)] \leq S[-\rho \text{sgn}(S) + a_{max}]$$

(5.23)

If $\rho \geq a_{max}$ is satisfied, then $\dot{W} < 0$ is sufficiently ensured, i.e. the state moves toward the surface. The value of $\rho$ gives the magnitude of the equivalent electrostatic acceleration exerted by the control loop, so it depends on the magnitude of the actuation voltages. It is chosen so that the electrostatic forces will be able to compensate the desired dynamic range of the accelerometer. If $\rho \gg a_{max}$ the chattering noise will increase (large electrostatic forces trying to compensate through the averaging of their fast switching for a small input disturbance), and as a result the performance of the system will suffer. The trade-off is to choose $\rho$ large enough to compensate for the external acceleration, without any increase in the chattering noise; a good rule is to use $\rho \approx 5 \times a_{max}$, where in our case we took $a_{max} = 100G$. As previously mentioned, the second design parameter, $\lambda$, sets the rate of convergence or the speed of the transient regime - a very large $\lambda$ speeds up the convergence.

\textsuperscript{5}This part has appeared in a journal paper: E. H. Sarraf, B. Cousins, E. Cretu, and S. Mirabbasi. Design and implementation of a novel sliding mode sensing architecture for capacitive MEMS accelerometers. Journal of Micromechanics and Microengineering, 21:115033, November 2011.
5.4. $SMC$ for $x_{ref} = 0$

but however could lead to instability of the system. Therefore $\lambda$ has to be chosen to assure a good balance between a sufficient rate of convergence and a large bandwidth in tracking.

5.4.1 Repositioning of the Proof Mass

The micro-accelerometer in Figure 3.3 exhibit a quality factor of $Q_f = 8.9$, measured experimentally using Polytec MSA-500®. Therefore, the application of $SMC$ to the closed-loop micro-accelerometer system is validated on a macro-model whose parameters are identified from the experimental characterisation using Polytec MSA-500®; this provides a baseline for the experimental implementations and future work.

However, to verify the system architecture robustness, the application of $SMC$ is extended to an over-damped accelerometer model, where for simplicity a constant damping coefficient is considered, given by $\gamma_d = \mu_v A^2/d_0^3$ (where $\mu_v$ is the air viscosity, $A$ is the capacitor area and $d_0$ is the initial gap) [45,46]. In either case, the $SMC$ demonstrates the repositioning of the proof mass to null position; however the over-damped model exhibits shorter transition time (15msec for 1G acceleration) due to the increased damping.

To validate the performance of the proposed methodology, we therefore consider two cases. The first one uses an under-damped accelerometer model (the experimentally measured damping coefficient), while the second structure is an over-damped one. In each case, the performance of the closed-loop accelerometer is compared to that of the same accelerometer used in open-loop scheme (with no feedback electrostatic force).

Different scenarios based on variations of the input acceleration have been adopted for simulations. The amplitude of the acceleration is $1G = 9.81m/s^2$ unless otherwise specified. The first scenario is for a step input with 0.01msec step time.

Figure 5.2 depicts the results for the $SMC$-based approach, where the initial step (with a magnitude equal to 1G) in the input acceleration, displaces initially the proof mass from the zero position. The controller action balances fast the action of the external "disturbance" and brings, after a short transient, depending of the damping coefficient, the proof mass position back to the zero displacement (our sliding surface). The value of the external acceleration is encoded in the difference between the time intervals when the controller actuates with $(+F_0)$ and those corresponding to $(-F_0)$. In the under-damped case, a longer transient period and little ringing is observed before returning to rest position.

The proof mass peak displacement response to 1G step input acceleration
5.4. SMC for $x_{ref} = 0$

Figure 5.2: Proof mass displacement with/without SMC for 1G step input

Figure 5.3: Proof mass displacement with/without SMC for a sequence of ±1G input pulses
5.4. SMC for $x_{ref} = 0$

is 6nm for the under-damped accelerometer in closed loop, about twice as low than for the open loop response. The same reduction factor is about five times for the over-damped accelerometer. Figure 5.3 depicts the results of the second scenario of simulations. In this scenario we apply a sequence of input pulses, each with a constant 0.01msec duration, $a_{ext} = 0G, 1G, 1G, 0G, 1G$ similarly to Eq 4.10.

Figure 5.3: Proofs mass displacement with/without SMC for 1G sinusoidal input

In a similar manner with the previous case, the SMC-based approach allows for a quick detection of any deviation from the zero position, followed by a compensation of the inertial force, hence forcing the mass back to the rest position. The dynamics of the position can be interpreted from the SMC viewpoint as a fast exponential transition toward the sliding mode (zero displacement), followed by a continuous switching about it. Compared to the open-loop approach, SMC architecture reacts to minute variations in the external acceleration, leading to a high-sensitivity system and linearisation of the response. Figure 5.4 shows the results of a different scenario, where we apply a sine input acceleration (of 10Hz frequency) defined by Eq 4.9 with a magnitude equal to 1G. As shown in Figure 5.4, the proposed methodology can be used with both under-damped and over-damped accelerometers. However, the results for an over-damped accelerometer are more intuitive.
5.4. SMC for $x_{ref} = 0$

to interpret, due to the longer settling time exhibited by the under-damped accelerometer.

5.4.2 Acceleration Measurement

As noted in the previous chapter, accelerometers are designed for sensing external accelerations, but in the SMC-based architecture the measurand is treated as an external disturbance. The natural question is how to extract the information regarding the input acceleration from the dynamics of the controller. Traditional readout circuits use capacitive sensing schemes to map the position of the proof mass to a differential capacitance change, which is converted by the circuit into an output voltage [45, 46]. The small value of the differential capacitive changes, in the range of $\sim aF \sim fF$, and the additional parasitic resistances and capacitances make this task very challenging. The dominant trend in an acceleration sensing system is to minimize as much as possible the analogue electronics interface subsystem and to maximize the use of digital control.

In our case, with a closed-loop system employing SMC, the proof mass displacement for $1G$ input acceleration is in the order of $\sim nm$, which yields a differential capacitance in the order of $\sim 10aF$.

We have mentioned in the previous sections that in closed-loop microaccelerometers, the feedback electrostatic force, or the counterbalance force carries the information about the input acceleration exerted on the system, since it is mainly responsible for cancelling the motion and repositioning the proof mass to its central position. In the SMC approach this statement is valid as well. The control voltage is constantly switching back and forth between right and left comb drives at a very high frequency, depending on the value of the state on the sliding surface. In the classic SMC approach this is known as chattering and considered a negative phenomenon, like an added switching noise into the system. It could lead to low control accuracy, high heat losses in electrical power circuits and may excite un-modelled high frequency dynamics that could degrade system performance and even lead to instability [141].

In our case, the switching noise is treated as a digital signal that encodes the necessary information for recovering the value of the externally applied acceleration, in the same way $\Sigma \Delta$ controller schemes use the generated bit-stream for the recovery of the measurand.

While the external acceleration is typically well-below the mechanical resonance of the MEMS device, the frequency of the switching of the SMC electrostatic actuation happens at a much higher frequency, and is averaged
by the low-pass characteristic of the mechanical structure.

In a $\Sigma\Delta$ modulator, the output bit-stream encodes the information about external acceleration, which can be extracted by averaging it. This is typically performed using a low-pass filter at the output. Similar to $\Sigma\Delta$, the switching mechanism of the electrostatic forces between right and left comb drives in the $SMC$ controller generates a bit-stream that encodes the input acceleration; hence, a low-pass filter (an averaging operation) is required to extract the desired acceleration information. A first-order low-pass filter is used, which offers tremendous advantage in terms of decreasing complexity and reducing the cost. The transfer function of the filter is:

$$H(s) = \frac{H_0}{s + 2\pi BW}$$

(5.24)

$BW$ (set to $1KHz$ in our system) is a predefined operational bandwidth of the accelerometer, while $H_0$ is the scaling factor (gain) that maps the external acceleration to the output voltage.

For the $SMC$, the low-pass filter can be implemented either in analogue electronics or as digital filter, so can be easily targeted on FPGA following the implementation used for $\Sigma\Delta$ illustrated in Figure 5.7 and Figure 5.8 (next section). The cut-off frequency is chosen such that the chattering noise is eliminated and as a result, input acceleration signal is reconstructed. Therefore, the corner frequency of the filter needs to be beyond the bandwidth of the sensed external acceleration. The filter could have higher order, which makes it advantageous for large bandwidth accelerometers; however, in our case the separation between the clock frequency and the accelerometer bandwidth $1KHz$ is sufficiently large to get a good reconstruction with a first-order low-pass filter. When using a clock (synchronous sliding-mode control), the chattering noise will be beyond the clock frequency, which is set to the clock of the FPGA. These are the major factors that should be taken into account when choosing the low-pass filter, similar to the $\Sigma\Delta$ filter.

By applying a low-pass filter to the binary feedback voltage, information about the external acceleration is extracted from the so-called chattering signal. Figure 5.5 shows the extracted acceleration for a sequence of different input pulses, while Figure 5.6 shows the measured input acceleration for a sinusoidal input with $1G$ magnitude.

The proposed sensing mechanism is novel and is expected to extend the application domain of sliding mode approach into the domain that was previously covered by $\Sigma\Delta$ techniques. For decades, $SMC$ has been extensively and primarily used in non-linear control for many electro-mechanical applications, on a macro- [141] and micro-scale [38, 47, 111, 161] level.
5.4. SMC for $x_{ref} = 0$

Figure 5.5: Acceleration measurement from the sliding surface and the switching scheme, for a sequence of input pulses with $1G$ magnitude

Figure 5.6: Acceleration measurement from the sliding surface and the switching scheme, for a sinusoidal input with $1G$ magnitude
5.4.3 Proof of Concept FPGA Implementation

As we mentioned earlier, the trend for future accelerometers is to incorporate digital controllers as intrinsic modules on the chip. Hence our proposed methodology offers several advantages in terms of cost and performance. This proposal is in line with the statement from Xiong et al. [150] expecting that very soon, MEMS-based devices will be fabricated on the same chip with digital and analogue circuits.

Therefore it is important that the chosen controller is suitable for a hardware implementation, towards very large scale integration (VLSI) such as FPGA, which reflects directly into the overall performance/cost ratio. The advantage of this methodology is to cut down the time-to-market and to allow the designer to quickly test the functionality of the designed system, prior to a full chip implementation that contains, MEMS, CMOS and FPGA components. Besides that, non-linear control schemes are easier to implement in digital [138] than in analogue electronics. The hardware implementation of the controller is achieved in the present work through a rapid prototyping methodology that uses MATLAB/SIMULINK tools, together with Xilinx System Generator toolbox in Matlab/Simulink®. The proposed digital control algorithms, after being simulated and optimized from an algorithm perspective in Simulink®, are mapped to real-time hardware implementation into Xilinx FPGA devices. Among the advantages of this design methodology we can mention: a significant reduction of the time-to-market, quick incremental optimization and testing of the system functionality, together with a minimization of the number of iteration for the fabrication of a full chip implementation.

In order to make a fair comparison, we have designed and implemented on FPGA two distinct digital control alternatives: the first one is the proposed SMC structure, while the second one is a ΣΔ-based architecture. In this section, the closed-loop feedback controllers are denoted by $C_{cont}\{m, n\}$, where $C_{cont}$ indicates the name of the controller, $m$ is the order of the modulator and $n$ represents the total order of the feedback loop.

For example, the implemented feedback controller based on modulator includes a decimator filter (4 integrators stages) [92], a PID controller [22], a low-pass filter and a 1-bit first-order ΣΔ digital to analogue converter (DAC), and therefore, it is represented as $\Sigma\Delta\{1, 6\}$.

Figure 5.7 shows the overall architecture for the $\Sigma\Delta\{1, 6\}$ implementation, together with snapshots of the signals at various points within the feedback loop.

An external acceleration will tend to displace the proof mass from its rest
5.4. SMC for $x_{ref} = 0$

Figure 5.7: *Simulink*® model of a micro-accelerometer behavioural model employing $\Sigma\Delta$ targeted on FPGA

position, and this displacement is sensed by a differential change in the sensing capacitances. The change is converted to an output voltage which is fed into the oversampling converter. The resulting bitstream (a series of digital 0s and 1s) enters into the decimator block [92], which lowers the sampling rate to the Nyquist rate value, to increase the resolution. The decimation filter minimizes the amount of information for subsequent transmission, storage, or digital signal processing. The signal is then fed to a low-pass filter to remove the excess quantization noise. Afterwards, the signal enters the PID controller and is finally routed through the DAC, which generates two possible outputs. The analogue output voltage is fed to one of the capacitive actuation drives, creating an attractive electrostatic force that pulls the proof mass towards the left or to the right. The net action is to balance the external inertial force and bring the proof mass back to the rest position.

Figure 5.8 depicts the FPGA implementation of the $\Sigma\Delta\{1, 6\}$ feedback loop system. Part (a) of Figure 5.8 shows the decimator [92] and the low-pass filter, while part (b) shows the PID controller and the digital to analogue converter. Although $\Sigma\Delta$ are classified as high sampling 1-bit converters, signal processing with one bit from the oversampling converter (bit stream) is converted into $N$-bit pulse code modulation (PCM), which explains the necessity of these blocks at the digital level [15].
5.4. SMC for $x_{ref} = 0$

Figure 5.8: FPGA/Simulink® implementation of the feedback loop with $\Sigma\Delta$ modulator
5.4. SMC for $x_{ref} = 0$

Table 5.1: Arithmetic complexity of the feedback loop with $\Sigma\Delta$ modulator and $SMC$, considering one $DOF$ and one iteration

<table>
<thead>
<tr>
<th>Method</th>
<th>1st order $\Sigma\Delta$</th>
<th>1st order $SMC$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Additions</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>Subtractions</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>Multiplications</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Delays</td>
<td>25</td>
<td>25</td>
</tr>
</tbody>
</table>

The proposed closed-loop feedback controller uses a first order $SMC$ modulator denoted as $SMC_{\{1,1\}}$, which is also the order of the whole feedback loop. If we analyse the arithmetic equations in Table 5.1 we see that the $SMC_{\{1,1\}}$ requires fewer operations than the $\Sigma\Delta_{\{1,6\}}$ approach, and therefore less hardware resources. No special blocks are required, similarly to $\Sigma\Delta_{\{1,6\}}$ system - the FPGA implementation of $SMC_{\{1,1\}}$ uses only simple arithmetic operators, and this reflects on the simplicity of the algorithm. The proposed implementations have very good concurrency and use fast hardware components.

It should be noted that since the reference or desired displacement is the null position, the complexity of the FPGA implementation has been reduced tremendously comparing to the arithmetic operations depicted in Table 5.1. The Simulink® model of $SMC_{\{1,1\}}$, along with the FPGA implementation, are illustrated in Figure 5.9.

Table 5.2: Arithmetic complexity of the feedback loop with $\Sigma\Delta$ modulator and $SMC$, considering one $DOF$ and one iteration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1st order $\Sigma\Delta{12,8}$</th>
<th>3rd order $\Sigma\Delta{12,8}$</th>
<th>4th order $\Sigma\Delta{12,8}$</th>
<th>5th order $\Sigma\Delta{12,8}$</th>
<th>1st order $SMC{12,11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slices</td>
<td>555</td>
<td>1915</td>
<td>2331</td>
<td>2757</td>
<td>470</td>
</tr>
<tr>
<td>Flip Flops</td>
<td>493</td>
<td>1681</td>
<td>2111</td>
<td>2445</td>
<td>331</td>
</tr>
<tr>
<td>BRAMs</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>LUTs</td>
<td>800</td>
<td>2736</td>
<td>3344</td>
<td>3880</td>
<td>592</td>
</tr>
<tr>
<td>Emb. Mult.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TBUFS</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The degree of concurrency is determined by the underlying mathematical aspects of various components forming the closed feedback loop. The hardware resources estimate includes number of slices, $LUT$, $BRAM$ memory blocks, embedded multipliers, $I/O$ blocks, $FF$, and tristate buffers ($TBUF$).
5.4. SMC for $x_{\text{ref}} = 0$

Figure 5.9: FPGA/Simulink® implementation of the feedback loop with SMC
5.4. SMC for $x_{ref} = 0$

These estimates make it easy to determine how design choices affect hardware requirements.

The amount of FPGA hardware used is directly related to the data width (wordlength), so it is best, for reasons of cost, circuit speed and power dissipation, to request only the precision required for a particular application. Coupling different technologies based on control, MEMS and distributed computation is required to create a micro-smart system [23]. The challenge in this field is to combine all these elements in one functional system that will be integrated later in a single chip solution.

In this implementation, multiplications are done by bits shifting, hence no embedded multipliers are used. Table 5.2 shows the hardware resources for the proposed implementation. Within the notation (12,8), for instance, the first element of the pair indicates the total number of bits (in this case a 12-bit signed 2's complement value), while the second element of the pair (8 in this example) indicates the number of bits reserved for the fractional part.

Figure 5.10: SMC fixed-point vs. floating-point acceleration measurement

Our target is to provide a low cost solution, hence the trade-off between performance and area occupancy should respond to the constraints imposed
by the application. The major constraint imposed by the performance is the sensitivity. Therefore, the quantization error should be as low as possible in order to guarantee a good performance on the FPGA. Figure 5.10 illustrates a fixed-point implementation for \( SMC_{(1,1)} \) with \((12,11)\)-bit representation.

It should be noted that we have a latency of 7 and 16 for \( SMC_{(1,1)} \) and \( \Sigma\Delta_{(1,6)} \), respectively. The achieved clock frequency in both approaches is acceptable. For the \( SMC_{(1,1)} \) we have a minimum period of 14.31\( \text{ns} \) and a maximum frequency of 69.84\( \text{MHz} \), and for \( \Sigma\Delta_{(1,6)} \) the minimum period and the maximum frequency are 25.73\( \text{ns} \) and 38.86\( \text{MHz} \) respectively. The FPGA implementation of \( \Sigma\Delta_{(1,6)} \) system contains more arithmetical operations in terms of additions, subtractions, multiplications and delays than that of the \( SMC_{(1,1)} \). In terms of area occupancy, the proposed first-order modulator \( SMC \) technique barely uses (only 1\% Slices) of the FPGA device on board (\textit{VIRTEX-II Pro XC2VP100} of Xilinx ©).

Table 5.3: Hardware resources of the feedback loop with \( \Sigma\Delta_{(1,6)} \) modulator

<table>
<thead>
<tr>
<th>FPGA</th>
<th>% Slices</th>
<th>% FF</th>
<th>% LUT</th>
<th>Number of DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>XC2VP2</td>
<td>36</td>
<td>14</td>
<td>28</td>
<td>2</td>
</tr>
<tr>
<td>XC2VP7</td>
<td>10</td>
<td>4</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>XC2VP30</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 5.4: Hardware resources of the feedback loop with \( SMC_{(1,1)} \) modulator

<table>
<thead>
<tr>
<th>FPGA</th>
<th>% Slices</th>
<th>% FF</th>
<th>% LUT</th>
<th>Number of DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>XC2VP2</td>
<td>33</td>
<td>12</td>
<td>21</td>
<td>3</td>
</tr>
<tr>
<td>XC2VP7</td>
<td>9</td>
<td>3</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>XC2VP30</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

Hence, taking the number of slices as the main criteria for measuring hardware complexity, a smaller size and cheaper FPGA device such as \textit{VIRTEX-II Pro XC2VP7} (see Table 5.3 Table 5.4), could easily accommodate six degrees of freedom inertial measurement unit, decreasing the cost of the system.

In summary, higher order \( \Sigma\Delta \) modulators are necessary for high performance control, and they result in increasing the risk of instability, and a much larger hardware complexity. Perhaps even more important, from the design perspective, is the increasing difficulty in their robust design, for they do not employ a non-linear theory from first principles, in contrast to the
5.4. SMC for \( x_{\text{ref}} = 0 \)

SMC which is dedicated to non-linear control. SMC is simpler in its design methodology, because it is decoupled into two sub-problems of lower dimension - after a finite time interval preceding the sliding motion, the system will possess the desired dynamic behaviour.

5.4.4 SNR Comparison Between \( \Sigma\Delta \) and SMC

In the previous subsection, the comparison between SMC and \( \Sigma\Delta \) was based on the percentage of area occupancy of each method. Each one originally exhibits a first order modulator, where the digital implementation of feedback loop using \( \Sigma\Delta \) shows higher orders as shown in Figure 5.8.

In earlier chapters, we have discussed the popularity of high order \( \Sigma\Delta \) modulators due to their advantages in noise reduction and the direct digital output. However, most of \( \Sigma\Delta \)-based approaches tend to employ high-order modulators which push the noise outside the baseband, and achieve good performance in acceleration measurement [5,34-36,72,131]. The higher order achieves much better SNR comparing to a low order modulator, and a better motion cancellation.

In this subsection we address the comparison in terms of SNR between \( \Sigma\Delta \) and SMC of the same modulator order. To the best of our knowledge no work has reported this comparison yet. The theory of the SNR performance for \( \Sigma\Delta \)-based modulators is very well established in practise and in theory as well.

For this study we considered a \( dc \) input or a step input with 1G magnitude, and a sine input with similar magnitude and a frequency of 250Hz. In this study we used the IMOBCEHS0903 accelerometer model with characteristics depicted in Figure 3.3 and Table 3.1. The SNR is computed for each input (sine, step) while the oversampling rate (OSR) is varied between \{4,8,64,128,256\}. It should be noted that the OSR is a direct correlation of the switching frequency between \(+F_0\) if \( S(e,\dot{e}) \leq 0 \) and \(-F_0\) if \( S(e,\dot{e}) > 0 \) as shown in Eq 5.7.

For each OSR and for each input the SNR is computed as:

\[
t_{\text{settle}} = 20\log_{10}\left(\frac{V_{\text{eff}}}{\text{std}N}\right)
\]

Where \( V_{\text{eff}} \) is given by \( \max(\text{FFT} \text{(signal)})/\sqrt{2} \) the magnitude of The frequency of and \( \text{std}N \) is the standard deviation of the noisy signal. The low-pass filter at the output of the switching signal is fixed to 500Hz with a bandwidth of 4KHz.
Figure 5.11: SNR comparison between SMC and ΣΔ considering a first order for the modulator

Figure 5.11 depicts the SNR for each OSR and each input. As before, we considered a first order SMC. A first conventional observation is the increase of SNR along with the increase of OSR. The increase of the switching period increases the response and the stability of the system on one hand, and on the other it allows for a quick repositioning of the proof mass after detecting an acceleration exerted on the system. As the SNR increases with increasing OSR, the magnitude of the noise signal at the output of the low-pass filter decreases drastically.

References [15, 60] show an improvement of 9dB per octave (doubling of OSR) and 15dB per octave for a first and second order ΣΔ modulators respectively. Figure 5.11 shows an SNR improvements of 12.27dB and 12.20dB for sine and step inputs respectively (OSR between 4 and 32). As a result, in terms of SNR performance, a first order SMC can be compared to a second order ΣΔ modulator.

The slope of the two curves in Figure 5.11 is not quite linear in contrast to ΣΔ shown in [15, 60], and this is due to the magnitude of the input acceleration, the dynamic range of the accelerometer and the cut-off frequency of the low-pass filter. Figure 5.11 shows that for a fixed cut-off frequency,
and a fixed input acceleration, an OSR of $\{32, 64\}$ is enough to achieve good performance and a good noise level.

5.5 SMC for $x_{ref} = x_{pull-in}$

In the previous section, the excitation voltage was below the pull-in voltage (Eq 2.2), which guarantees a safe level of operation and linearity. In the second chapter, we thoroughly investigated the risks operating around the pull-in position (Eq 2.1).

In this section, we propose the SMC for robust stabilization, high sensitivity and resolution of the micro-accelerometer operating at the border of stability. The micro-device is excited with a voltage slightly higher than the capacitor pull-in voltage, which guarantees a relatively short-time metastable region, defined at 1/3 of the capacitor gap as defined in Eq 2.1 and characterized by a tiny equilibrium between the elastic and the excitation electrostatic force, hence highly sensitive to any small perturbation.

The proposed application is solidified by simulations and experimental results using IMOBCEHS0903 and IMOBCEH21103 accelerometers respectively.

5.5.1 Stability at the Border of Stability

There is a fundamental challenge in measuring small signals when making small sensor systems [64]. Therefore this section is dedicated to small acceleration measurement where the proposed methodology maintains the proof mass of the accelerometer in that sensitive region, by means of infinite switching of the feedback electrostatic forces, where this same switching is used to extract the acceleration exerted on the system [113].

Simulations results demonstrate the effectiveness of the proposed scheme by achieving small-acceleration measurements with magnitudes close to the mechanical noise floor of the micro-system while maintaining the proof mass at 1/3 of the gap. With such stabilization, maximum sensitivity is reached with an increased overall ratio by two orders of magnitude comparing to the conventional approach with reference trajectory set to null. This approach encompasses sensitivity in based approaches since they cannot operate at the border of stability.

---

5.5. SMC for $x_{ref} = x_{pull-in}$

In this section, the same path for SMC formulation has been followed as in the previous section. The values are extracted from the IMOBCEHS0903 accelerometer, with area-varying combs used for feedback control. Hence, by means of infinite switching between these two states (based on the switching law), the system trajectory converges exponentially fast \[111\] toward a stable state, with a convergence speed determined by $\lambda$.

As a result, and in comparison with the open-loop model, the proof mass is maintained at the pull-in position $x_{pull-in} = 1.33\mu m$ by means of infinite switching while the proof mass in the open-loop model hits the stopper since the elastic force can no longer counteract the electrostatic force.

![Figure 5.12: Comparison of proof mass displacement between an open-loop and closed-loop with SMC considering ($\alpha = 1.001$)](image)

Figure 5.12 illustrates the comparison between the two models. The open-loop model thoroughly discussed in Chapter 2 and proposed by Rocha et al. \[104\] uses time-based measurement, in contrast to our proposed approach which does not make use of the time, but uses the controlled structured variations. The equilibrium established in the SMC-based closed-loop model, allows for the measurement of any small signal, as it attempts to alter the equilibrium. This tight equilibrium is sensitive to any external disturbance, however the robustness of the proposed approach maintains the structure in the metastable region by means of infinite switching while al-
small external disturbance to be measured continuously.

Figure 5.13 shows the difference in forces equilibrium between the open-loop and closed models. As the electrostatic (feedback force $f_{e,feedback}$) is switching between two values, based on the dynamic of the sliding mode control, equilibrium is established between the electrostatic force (excitation force $f_{e,exc}$) and the elastic force $F_k$ ensuring damping force $F_γ$ is null. In other terms, the switching scheme guarantees stability in the metastable region in order to achieve high sensitivity.

We mentioned before that a considerable amount of damping is required in particular in the open-loop model, in order to guarantee the non-linear behaviour and enough travelling time in the metastable region, however, it is very difficult to obtain the required amount of damping in the structure, due to physical and mechanical constraints such as the type of liquid or gas used, and packaging (difficulty in obtaining a good absolute dimensional control [64]).

High damping guarantees a metastable region and a good travelling time of the micro-structure before hitting the fixed counter part. However, it introduces more noise into the system. On the other hand, low damping increases the response time but also risks instability due to ringing. The required quality factor according to [102, 104] is $Q_f < 1.2$. The experimental quality factor of IMOBCEHS0903 is $Q_f = 8.9$ as shown in Table 3.1, which in the open-loop case exhibits lots of ringing. The targeted applications early mentioned demand high reliability, accuracy and fast response. Small errors could result in catastrophic consequences if used in safety critical applications such as bridge monitoring, inertial navigation or non-invasive surgery.

The sliding mode control dynamics should act very fast and stabilize the proof mass. In this demonstration we will assume a critically damped case with $Q_f = 0.5$ to guarantee the fastest response without overshooting or ringing. However later on will we demonstrate experimentally the application of SMC for under-damped structures.

### 5.5.2 Acceleration Measurement

In this section we demonstrate the high sensitivity and resolution in acceleration measurement that can be achieved by stabilizing the proof mass at 1/3 of the gap, as given in Eq 2.1. In other terms, by exciting the micro-accelerometer, and pushing it to instability and by stabilizing the proof mass at the equilibrium position as shown in the proposed methodology, the sensor achieves maximum level of sensitivity and resolution that can be achieved with the given geometrical characteristics. As a result the proposed approach
5.5. SMC for $x_{ref} = x_{pull-in}$

Figure 5.13: Elastic, damping and electrostatic forces (excitation and feedback) for the closed-loop model
outperforms others that tend to extend the travel range of the capacitor and modulators which cannot operate in the unstable region. This will be proven by small signal measurements and sensitivity analysis.

In the dynamic behaviour of the micro-structure, subjected to a high voltage described in Eq. [2.3] we had identified the region of high sensitivity, defined as the metastable region, where its duration in the open loop model depends on the damping coefficient of the targeted micro-device.

We have already mentioned that no time measurement is needed as the structure never reaches the stopper due to the dynamics of the SMC maintaining the proof mass in the equilibrium position. Ideally the output of the accelerometer should follow the input (exerted acceleration), but deducing the acceleration based on the capacitance variation is not intuitive at all. The small value of the differential capacitive changes, in the range of \( \sim aF \sim fF \), and the additional parasitic capacitances make this task very challenging. The dominant trend in an acceleration sensing system is to minimize as much as possible the analogue electronics interface subsystem and to maximize the use of digital control [113].

The proof mass is maintained at \( d_0/3 \) making the capacitance measurement extremely challenging. In our previous work [113] we have used a 1G- acceleration magnitude in our simulations; the purpose was to prove the concept of applying SMC to micro-accelerometers in actuation feedback control and in sensing modes, where we demonstrated the extraction of the input acceleration from the switching signal or chattering by using a low-pass filter.

The same extraction technique is still valid in the current approach, but the target in this work is high sensitive accelerometer, therefore a 1G- acceleration magnitude would be considered quite large, since the targeted applications require low-G measurements. Therefore, we will target to measure accelerations slightly higher than the system noise level, which requires us to define a range for the smallest measurable acceleration.

In inertial systems, the input referred noise is of great importance in quantifying the sensor noise performance as it gives a direct measurement of the smallest measurable acceleration [64]. The noise equivalent acceleration is given by:

\[
\ddot{x}_{\text{noise}} = \sqrt{\frac{4K_BT\omega_0}{mQ_f}} \approx 14\mu G/\sqrt{Hz} \quad \text{for} \quad Q_f = 0.5 \quad (5.26)
\]

Were \( K_B \) and \( T \) denote the Boltzmann constant and the absolute temperature respectively, \( m \) represents the proof mass, \( Q_f \) denotes the quality factor and \( \omega_0 \) represents the natural resonant frequency. This equation suggests
that by increasing the proof mass and the quality factor, the noise can be reduced. Unfortunately, this equation does not represent the total mechanical noise present in the system. The approach of increasing the quality factor is purely "superficial" [64] since the total noise energy integrated over all frequencies is constant. As a result, the input referred root mean square (RMS) acceleration is a function of the RMS vibration magnitude $x_{RMS,noise}$ is independent from the quality factor and is given by:

$$
\ddot{x}_{RMS,noise} = x_{RMS,noise}\omega_0^2 = \sqrt{\frac{K_B T}{k_d} \omega_0^2} = \sqrt{\frac{K_B T}{m}} \omega_0^2 = 862 \mu G \tag{5.27}
$$

Therefore our aim is to measure small accelerations with magnitude slightly higher than the RMS acceleration $\ddot{x}_{RMS,noise}$. It should be noted that the difference $(\ddot{x}_{RMS,noise} - \ddot{x}_{noise})$ is quite large and can be reduced by increasing the proof mass (bulk micro-machining).

Since the magnitude of the acceleration to measure is small, the switching frequency has been increased. Therefore the measurand is very well covered within the chattering noise, thus a second order low-pass filter (Averaging operation) is used at the output of the switching signal. The transfer function of the filter is:

$$
H(s) = \frac{H_0}{s^2 + s + 2\pi BW} \tag{5.28}
$$

$BW$ (set to $1KHz$ in this work) is a predefined operational bandwidth of the accelerometer, while $H_0$ is the scaling factor (gain) that maps the external acceleration to the output voltage. The cut-off frequency is chosen such that the chattering noise is eliminated and as a result, input acceleration signal is reconstructed. Therefore, the corner frequency of the filter needs to be beyond the bandwidth of the sensed external acceleration.

Figure 5.14 and Figure 5.15 shows the limitation in the dynamic range using the approach presented in [113] with reference trajectory $x_{ref} = 0$. Several accelerations with different magnitudes have been applied to the system, while the measured acceleration is extracted from the switching scheme of the electrostatic forces, which is previously demonstrated in our work [113].

Figure 5.14 and Figure 5.15 suggests a limitation in sensitivity and resolution for the accelerometer with reference trajectory set to zero $x_{ref} = 0$. For magnitudes equal and/or higher than $100mG$ the signal is very well filtered from all the chattering (noise) and is a direct measure of the input acceleration exerted on the system as shown in Figure 5.14 (a) and (b). As the magnitude of the input acceleration goes below $100mG$, the extraction
Figure 5.14: Limitation of the open-loop model in terms of smallest measurable acceleration (a) applied acceleration = 1G (b) applied acceleration = 100mG
5.5. SMC for $x_{\text{ref}} = x_{\text{pull-in}}$

Figure 5.15: Limitation of the open-loop model in terms of smallest measurable acceleration (a) applied acceleration = $10mG$ (b) applied acceleration = $1mG$
of the measurand (acceleration) becomes challenging as the signal cannot be extracted properly from the noise around and hence noise dominates (Figure 5.15 (a) and (b)). At that level no low-G measurement can be achieved. As a result, a new scheme is required to measure acceleration forces below and even close enough to the mechanical noise floor in Eq [5.27].

This limitation demonstrates the importance of the equilibrium stabilization in the metastable region where the smallest perturbation, even in the range of 1mG could affect the tight equilibrium maintained between the electrostatic and elastic forces. For a system with $x_{ref} = 0$ shown in [113], such a small perturbation had no effect as shown in Figure 5.15 (b), since it was dominated by the noise in the system.

On the other hand, in our proposed methodology, the equilibrium position is set to be the point of operation and the proposed SMC-based approach will maintain the proof mass in that position by means of infinite switching of the electrostatic force as shown in the previous section. The same second order low-pass filter stage used in the previous case $x_{ref} = 0$, to extract the acceleration is used in this case.

Figure 5.16 shows the measured acceleration for different scenarios based on variations of the input acceleration. It should be noted that the noise floor in the micro-accelerometer is in the range of $862\mu G \approx 1mG$, due to the physical characteristics of the sensor, but nevertheless the magnitude of the input acceleration exerted on the system is made almost equal to the equivalent noise floor. However, in the work reported by [31,33,102-104], on high sensitivity dynamic pull-in time, several micro-accelerometers have been employed with various noise floor characteristics such as, $\tilde{x}_{RMS, noise} = 1.747mG$ in [102], $\tilde{x}_{RMS, noise} = 4.89mG$ in [103] and $\tilde{x}_{RMS, noise} = 48.25\mu G$ in [31,33]. The common denominator is using a large input acceleration comparing to the aforementioned noise floor. In contrast, in this work, we demonstrate the effectiveness of our proposed approach by using an input acceleration with magnitude slightly higher than the noise floor.

The first scenario depicted in Figure 5.16 (a) is for a step input of magnitude $a_{ext} = 1mG$ applied at 0.01msec, which is equivalent to shock test. The proposed methodology allows the system to measure such small magnitude input acceleration, relatively close to the mechanical noise floor of the micro-accelerometer, due to the stabilization of the proof mass at 1/3 of the gap, i.e. in the metastable region. This stabilization and the resultant tiny equilibrium allows the system to be sensitive to such small magnitudes of perturbation, hence high sensitivity can be achieved. Such attempt to alter the equilibrium is compensated effectively, as seen before, by means of infinite switching between the electrostatic forces. Without this equilibrium,
Figure 5.16: Output Acceleration vs input acceleration (a) step input (b) pulse input (c) sine input
the system shows indifference to such small magnitudes input forces.

Figure 5.16(b) depicts the results of the second scenario of simulations where we apply a sequence of input pulses, each with a constant 0.01 msec duration, with the same magnitude $a_{ext} = 0 mG, 1 mG, 1 mG, 0 mG, 1 mG$ where $0 mG$ corresponds to $a_{ext} = 0 m/s^2$ and $1 mG$ means $a_{ext} = 9.81 \times 10^{-3} m/s^2$. The measurement scheme is reliable for positive and negative external acceleration exerted on the micro-system, following exactly the same pattern as the input signal. This is also proven in Figure 5.16(c) where we apply a sine input acceleration $250 Hz$ defined by $a_{ext}(t) = a_{max}\sin(\omega t)$, with a magnitude $a_{max}$ equal to $1 mG$.

Extracting the output acceleration from the switching signal has alleviated the limitation in the capacitance measurement, particularly in this work where the position of the proof mass is maintained at $1/3$ of the gap. Hence with such a small input acceleration close to the mechanical noise or, the variation in capacitance is almost negligible. As a result, applying a second order low-pass filter to the output of this switching signal allows retrieving the output acceleration or the measurand.

5.5.3 Sensitivity Analysis

Rocha et al. [102] state that two factors limit the practical application of using the sensitivity in the metastable region. The first one is related to high non-linearity of the sensitivity and the second is the small operating range. For improvements, the authors suggest defining an operating point with certain sensitivity, the applied voltage can be controlled using a feedback loop, in order to keep the operating point. As a result the sensor will have a high sensitivity no matter the range of the measured acceleration. In our work, we defined the targeted operation point as the pull-in position (Eq 2.1) which exhibits the highest sensitivity and we applied sliding mode control for as a feedback scheme for control, stabilization and acceleration measurement.

In this section the sensitivity of the proposed approach is compared against the aforementioned closed-loop model presented in [113] with reference position equal to null. Therefore, both models employ sliding mode control, the difference is in the reference trajectory ($0$ or $d_0/3$). However for consistency, the micro-accelerometer model and the external parameters are the same for both approaches. The purpose is to solidify the argument of the urge to stabilize the proof mass at $1/3$ of the gap.

In the previous subsection, we have demonstrated how stabilization at $1/3$ of the gap, allows for small signal measurements, while on the other hand, with the conventional approach (reference trajectory is set to null), the
small signal to be measured is dominated by the mechanical noise. Physically speaking, the performance of the accelerometer cannot exceed what can be achieved by stabilization at the border of the stability, since the smallest perturbation will risk driving the micro-device into instability.

Figure 5.17 (a), (b) and (c) illustrate the sensitivity gain factor \( G_s = a_{out}/a_{in} \) of both approaches, for different input accelerations. In this part we consider a ramp function for the input acceleration. The two models differentiate based on the reference trajectory, which could be set to null as seen in [113] or to the \( 1/3 \) of the gap (metastable region) which is the proposed approach.

Figure 5.17 (a) implies that the magnitude of the input acceleration has to be larger than \( 1G \), so that the SMC-based micro-accelerometer with reference trajectory set to null, is able to cope and track linear variations of the input acceleration. Such magnitude is much larger than the mechanical noise floor hence the measurement or the extraction of the exerted acceleration would not cause any problem. The input signal is much higher than the noise level. As the magnitude of the input gets smaller, towards the mechanical noise floor level, the extraction of the measurand becomes dramatic (Figure 5.17 (b)), and the sensitivity of the system is almost negligible as seen in Figure 5.17 (c) where noise dominates. On the other hand, setting the reference trajectory to the pull-in point at \( 1/3 \) of the gap maintains high sensitivity in tracking variations in the input acceleration where at the same time allowing small accelerations measurements, not achievable by conventional methods.

Thus, the comparison in sensitivity between the two approaches needs to be quantified in order to get an approximate sensitivity scaling factor between the conventional and the proposed approaches. The conventional approach shows a ripple effect as the magnitude of the input acceleration draws near the mechanical noise floor. One effective way to get this scaling factor for the sensitivity comparison would be to compare the standard deviation of the output acceleration taking into consideration different input interval of the input acceleration.

Figure 5.18 (a) shows that the standard deviation for the conventional approach \((x_{ref} = 0)\) decreases as a function of the input acceleration magnitude. One order of magnitude is witnessed between the two approaches, which reflects on the high sensitivity of the micro-accelerometer when stabilized at \( 1/3 \) of the gap. As the acceleration increases the standard deviation of the conventional approach could match that of the proposed approach \((x_{ref} = d_0/3)\), but this would actually deviate the micro-device from the targeted application which requires low-G measurements and high sensitivity.
5.5. SMC for $x_{ref} = x_{pull-in}$

Figure 5.17: Sensitivity analysis (a) $G_s$ for 1G ramp input acceleration (b) $G_s$ for 100mG ramp input acceleration (c) $G_s$ for 1mG ramp input acceleration
5.5. SMC for $x_{ref} = x_{pull-in}$

Figure 5.18: Standard deviation for (a) 100mG ramp acceleration (b) for 100mG ramp acceleration input (c) for 10mG ramp acceleration
5.6 SMC for $x_{ref} = x_{pull-in}$ Experimental Results

As the magnitude of the input acceleration decreases towards the mechanical noise floor, illustrated in Figure 5.18 (b) and Figure 5.18 (c), the standard deviation of the conventional approach increases and faulty measurements of the measurand result as the output signal is dominated by noise. With stabilization at the metastable region, the proposed approach outperforms the conventional approach by two orders of magnitude, driving the sensitivity to its maximum, making the micro-accelerometer suitable for low-G measurements.

5.6 SMC for $x_{ref} = x_{pull-in}$ Experimental Results

The successful application of SMC operating at the border of stability, in terms of robustness, stability and resolution in acceleration measurement, urged for experimental verification. Figure 5.19 depicts the system level block diagram. It comprises:

![System level block diagram]

Figure 5.19: System level block diagram with IMOBCEH21103

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5.6. SMC for $x_{ref} = x_{pull-in}$ Experimental Results

- The MEMS device which in this the IMOBCEH21103 accelerometer fabricated in SOI-MUMPs ® technology. The Characteristics of this accelerometer and the optical characterization are shown in Figure [3.11], Figure [3.13] and Table [3.3].

- A readout circuit which converts the differential capacitance induced by a proof mass displacement to voltage.

- An FPGA module, we mentioned earlier the advantages of targeting the controller on FPGA.

It should be noted that Figure [5.19] complies with the specification of Figure [2.1]. Usually the MEMS device is required to have a low noise floor, the readout circuit should have a minimum circuitry, and the proposed controller should have a low complexity for digital implementation.

### 5.6.1 Design of a Readout Circuit

The simulation system makes use of the current position and velocity of the proof mass in order to decide the next state of switching. As noted earlier, the sliding mode control dynamics estimate the distance between the current state and the defined sliding surface.

In order to get an estimate of the proof mass position, two readout circuits were developed for measuring the differential capacitance induced due to a certain external acceleration exerted on the system.

The first one is based on switch capacitors technique designed and implemented by our colleague J. Shiah. This readout circuit is fabricated in CMOS technology and integrated on the same chip along with the IMOBCEH21103 accelerometer. The description of CMOS-based readout circuit is beyond the scope of this chapter. The second readout circuit illustrated in Figure [5.20] is PCB-based and is developed by our colleague A. Sharkia.

The block diagram in Figure [5.20] describes the operating principle of the capacitive readout circuit. A clock signal (square wave) is applied to the common node between the two variable capacitors; this common node corresponds to the proof mass of the MEMS accelerometer. The top variable capacitor is part of a simple operational amplifier OP-AMP circuit which has the following transfer function:

$$\frac{V_{out1}}{V_{in}} = -\frac{C_s + \Delta C}{C_f}$$  \hspace{1cm} (5.29)
5.6. SMC for $x_{ref} = x_{pull-in}$ Experimental Results

The bottom variable capacitor is also part of an OP-AMP circuit with the following transfer function:

$$\frac{V_{out2}}{V_{in}} = -\frac{C_s - \Delta C}{C_f}$$  \hspace{1cm} (5.30)

Since the signal is differential, we get:

$$\frac{V_{out2} - V_{out1}}{V_{in}} = \frac{2\Delta C}{C_f}$$  \hspace{1cm} (5.31)

This means that after the first stage the clock input signal gets modulated (multiplied) by the change in capacitance. The second stage is a high pass filter with gain. It is necessary because if the mass has a $dc$ bias, that $dc$ bias is going to get modulated by the capacitance change, and is going to appear on the output of the first stage, which would distort the signal. Next, the signal is converted to single ended and it gets demodulated with the same clock signal which controls the analogue switch. Finally the signal gets low-pass filtered to remove the high frequency components which are caused by the clock signal.
5.6. SMC for $x_{\text{ref}} = x_{\text{pull-in}}$ Experimental Results

$$v_{\text{applied}} = V_{\text{applied}} \sin(\omega t)$$

Figure 5.21: Electrical connections between IMOBCEH21103 micro-accelerometer bondpads, the function generator and the PCB-based readout circuit.

Figure 5.22: Measured Bode plot response of the IMOBCEH21103 micro-accelerometer (using PCB-based readout circuit) and the parametric fit.
5.6. SMC for $x_{\text{ref}} = x_{\text{pull-in}}$ Experimental Results

Figure 5.23: Sensitivity of the readout circuit with IMOBCEH21103 micro-accelerometer in $\{-1G, 0G, +1G\}$ positions (using PCB-based readout circuit)

In the first measurement we started by acquiring the frequency response of the accelerometer, which is plotted in Figure 5.22. The connections between the different bondpads, the function generator and the readout circuit are shown in Figure 5.21. The output voltage is monitored as function of an input frequency sweep. The resultant quality factor and resonant frequency are 1.8 and 1.9 KHz respectively, where least squares fitting LSQ similarly to the previous devices is performed in Mathematica® [149].

Second, we targeted the sensitivity of the PCB-based readout circuit, in this measurement the output voltage is monitored while the accelerometer position is changed between $\{-1G, 0G, +1G\}$. The duration for each position was about eight minutes. The output voltage as function of the input acceleration is depicted in Figure 5.23. Based on the characterisation in Figure 5.23 which shows a quasi-linear variation of the output voltage in function of the input voltage, the sensitivity of the readout circuit can be computed as follows:

$$S_{\text{Readout}} = \frac{\Delta V_{\text{out}}}{\Delta G} = \frac{V_{\text{out},+1G} - V_{\text{out},-1G}}{2G} = 23.4 mV/G$$  \hspace{1cm} (5.32)
And the correspond linearity can be computed as:

\[ \text{Linearity}_{\text{Readout}} = V_{\text{out},0G} - \frac{1}{2}(V_{\text{out},+1G} + V_{\text{out},-1G}) \]

\[ = 0.0840 - \frac{1}{2}(0.1110 - 0.0669) \]

\[ = -1.75mV \]

\[ (5.33) \]

It should be noted that the sensitivity changed slightly (23mV/G to 25mV/G and 28mV/G) as the chip is changed. Therefore the repeatability is quite low and this is due to the imperfections in the fabrication process and the parasitics in the PCB-based readout circuit.

### 5.6.2 Pull-in Voltage Derivations

For operation at the border of stability we noted that a voltage slightly higher than the static pull-in voltage is required. First we monitored the output voltage as a function of a sequence of input voltages, applied to the two most left gap-varying combs of the IMOBCEH21103 accelerometer depicted in Figure 5.21. The input voltage is in the form of \( v_{\text{applied}} = V_{\text{applied}}\sin(\omega t) \), where the frequency is set to 1.9KHz and \( V_{\text{applied}} \) is the peak-to-peak amplitude of the input signal, the proof mass was biased at \( V_{\text{Bias}} = 5V \) and the accelerometer was put in \{0G\} position. It should be noted that the orientation of two most left combs (used for the input signal) is the same (see Figure 5.21). Therefore, the resultant force is the sum of the two forces generated by each comb.

The pull-in voltage for a single gap-varying comb drive is about 7V as it is showing in Table 3.3 However, as the proof mass is biased to 5V, the resultant electrostatic force responsible for moving the device changes, and consequently the pull-in voltage. Therefore, we need to solve analytically the new pull-in voltage. We have electrostatic forces acting in different directions, thus the characterisation of the resultant force is of great importance to understand the behaviour of the accelerometer under a certain applied voltage \( v_{\text{applied}} \).

The analysis for the net or resultant force will follow a dynamic system approach from Eq 2.4 and Eq 2.5 since the comb used has a gap-varying propriety. The net force can be written as:

\[ F_{\text{net}} = F_{e1} + F_{e2} + F_k \]

\[ = \frac{1}{2} \frac{C_{0d_0}}{(d_0-x)^2} V_1^2 + \frac{1}{2} \frac{C_{0d_0}}{(d_0-x)^2} V_2^2 - k_d x \]

\[ (5.34) \]

Where \( C_0 \) and \( d_0 \) denote the nominal capacitance and gap of the designated gap-varying comb drive respectively and \( x \) represents the displacement of the
5.6. SMC for $x_{\text{ref}} = x_{\text{pull-in}}$ Experimental Results

Proof mass. $C_0$ is the same for all combs since they are all identical. where $V_1$ is equal to $(V_{\text{applied}} - V_{\text{Bias}})$ and $V_2$ is equal to $V_{\text{Bias}}$. By substituting these two quantities, Eq[5.34] can be rewritten as:

$$F_{\text{net}} = 2 \times \left( \frac{1}{2} \frac{C_0 d_0}{(d_0 - x)^2} (V_{\text{applied}} - V_{\text{Bias}})^2 \right) - 2 \left( \frac{1}{2} \frac{C_0 d_0}{(d_0 + x)^2} V_{\text{Bias}}^2 \right) - k_d x$$

In order to find the voltage at which the stability is lost, the derivative with respect to displacement $x$ can be written as:

$$\frac{\partial F_{\text{net}}}{\partial x} = \frac{2C_0 d_0 (V_{\text{applied}} - V_{\text{Bias}})^2}{(d_0 - x)^2} + \frac{2C_0 d_0 V_{\text{Bias}}^2}{(d_0 + x)^2} - k_d \tag{5.36}$$

If we consider an equilibrium point $x_{\text{equilib}}$ which in our case is a quite symmetric mode ($x = 0$) as two electrostatic forces are pulling in opposite directions; we find that $x_{\text{equilib}}$ is stable under the following condition:

$$\frac{\partial F_{\text{net}}}{\partial x} < 0 \tag{5.37}$$

This results in:

$$\frac{\partial F_{\text{net}}}{\partial x} (0, V_{\text{applied}}) < 0 \quad |V_{\text{applied}}| < \sqrt{\frac{k_d d_0^2}{2C_0}} - \frac{V_{\text{Bias}}^2}{V_{\text{Bias}}} + V_{\text{Bias}} \quad \text{13.49V} \tag{5.38}$$

The interesting part about this case study is that it is symmetric drive analysis that includes two asymmetric drives where the connected combs act in the same direction. As a result the symmetric drives yields a larger pull-in voltage as compared to the asymmetric mode of operation. This is very useful in applications that operates near the pull-in where the pull-in voltage can be tweaked according to the specification in the desired mode of operation. As explained in the previous chapter, in case of an asymmetric drive, the spring force counteracts the electrostatic force until the movable mass snaps or hits the fixed part at the pull-in point. On the other hand, in the symmetric drive mode, the electrostatic fields are quite balanced as two electrostatic forces pulling in opposite directions. As a result, the spring force counteracts the resultant electrostatic force (difference between $F_1$ and $F_2$ shown in Figure 5.21) and this causes the pull-in to take place at higher values.

Figure 5.24 shows the pull-in voltage for the two gap-varying combs connected in series (Figure 5.21). These two combs are used to excite the microaccelerometer and drive it to instability by applying a dc voltage slightly higher than the static pull-in voltage. At pull-in the voltage drops because
5.6. SMC for $x_{ref} = x_{pull-in}$ Experimental Results

the mechanical stoppers are part of of the fixed structure and not separately placed as in the case of IMBOCEHS0903, IMOBCES21002 and IMTBCIMU accelerometers. It should be noted that the dc offset is $0.7V$, which is cancelled later on. Figure 5.24 shows that for $1V$ step increase at the input, the output voltage is larger as the proof mass is getting closer to pull-in position defined in Eq 2.1. This can be seen clearly when applying $7V$, $8V$ and $8.5V$. An important thing to note in Figure 5.24 is the high sensitivity as the applied voltage approaches the pull-in one.

5.6.3 Experimental Setup

The correspondent setup for testing the accelerometer along with the developed readout circuit is illustrated in Figure 5.25. We made use of a data acquisition system NI PXIe 1062Q from National Instruments \textcopyright\[59] which includes a Windows XP \textcopyright-based controller NI PXIe - 8133 and several instruments such as a digital multimeter, a dc power supply, a function generator and an FPGA board NI PXI 7854R with Virtex-5 LX110 FPGA from Xilinx \textcopyright\[93]. On the side a digital oscilloscope was used to continuously verify the output and input signals from/to the FPGA I/Os. The resolution of the on board ADC is 16 bits, and the voltage range of the I/Os module is $[-10, 10]V$, which affects the overall resolution of the system.

The readout circuit shown in Figure 5.20 is excited with the appropriate voltages $\{-5V, 0V, +5V\}$. No acceleration is applied at this stage. The readout circuit output signal with an offset of $0.7V$ is fed to one of the
5.6. SMC for $x_{ref} = x_{\text{pull-in}}$ Experimental Results

Figure 5.25: Experimental setup with PXI equipment

The controller is fully implemented on FPGA. Prior to this hardware, the controller was verified first with a floating point representation, to verify the different levels and ranges of input and output signals from and to the controller. At the FPGA level, the current position of the proof mass represented by the readout circuit output voltage and the derivative are estimated and fed to the sliding mode dynamics block as shown in Figure 5.19. As thoroughly explained throughout this chapter, the sliding mode control dynamics use the current position, velocity of the proof mass and also of the reference trajectory, wherein this case is set to the pull-in position (Eq 2.1). Besides that, an important determining factor in the SMC dynamics is the sliding surface defined in Eq 5.12. The controller determines the value of the switching voltages (binary actuation) $\{-V_{SMC}, +V_{SMC}\}$, and a switch determines between the two depending on the value of $S$.

It should be noted that the level of the output voltage is limited by the range of the I/Os module channels. In the current case, the output voltage and the switching frequency are automatically determined by the controller.
system. The switching depend on the levels of the output voltages and the magnitude of the input acceleration or perturbation. We refer to this mode of operation as "Asynchronous" SMC. On the other hand, the levels of switching voltages and the switching frequency can be set to a fixed value. We refer to such mode of operation by as "Synchronous" SMC. The later is very advantageous in terms of digital implementation, which can run at higher speeds.

\[ V_{app} = \alpha \times V_{pull-in}, \alpha > 1 \]

Figure 5.26: Electrical connections between IMOBCEH21103 micro-accelerometer bondpads, the SMC block and the PCB-based readout circuit

Figure 5.26 depicts the different connections between the accelerometer bondpads, the controller (SMC) and the readout circuit. The output voltages \{-V_{SMC}, +V_{SMC}\} are connected to two analogue output channels from the I/Os module which in their turn are connected to the comb drives responsible for feedback control as shown in Figure 5.26.

5.6.4 Experimental Results

In this subsection, the application of SMC to drive the accelerometer to the \textit{border of stability} is experimentally demonstrated. In addition we demonstrate the measurement of small accelerations that cannot be measured with conventional methods. We have previously proposed [113] the extraction of
5.6. SMC for $x_{ref} = x_{pull-in}$ Experimental Results

Figure 5.27: Operation at the border of stability beyond pull-in

the exerted acceleration by means of low-pass filtering of the switching and it will be demonstrated in experimentally.

In this section we followed an asynchronous implementation, as the system presented many limitations in terms of ADC resolution (16 bits), operating system (Windows) and I/Os voltages levels ($[-10, 10]V$).

Considering a zero input acceleration, the first task is to verify the stabilization of the proof mass at the pull-in position defined, in the metastable region (Eq 2.1). As mentioned earlier, as the proof mass is stabilized, the pull-in time measurement proposed by [102, 104, 105] and Dias et al. [31-33] will no longer be available. Figure 5.27 shows the operation beyond the pull-in position. As soon as the input voltage (higher than pull-in as shown in Eq 2.3) is applied, the SMC with the switching dynamics prevents the structure from snapping and maintains the proof mass at the position defined by $x_{ref}$ which is in this case, the pull-in position.

Once the proof mass reaches the reference trajectory $x_{ref}$, the proof mass becomes quasi-immobile. The switching voltages are responsible for this dynamic behaviour. As the proof mass position is constant, so is the output voltage. We have explained earlier the challenges related to measuring small capacitance variations with conventional method.

The goal of operating at the border of stability or at the metastable region is to measure small signals. With the proof mass position constantly maintained at the pull-in position; the change in the proof mass position due to a small external acceleration is not significant and is very challenging to be noticed.

A sine wave generator is implemented on FPGA along with the sliding
5.6. SMC for $x_{ref} = x_{pull-in}$ Experimental Results

mode controller block for synchronization purposes. The output represents the external acceleration and is added to dc bias. The magnitude of the input acceleration is governed by the hardware resolution. Given 16-bit ADC resolution and $[-10, 10]V$ at the I/O channels, the resolution of the low significant bit is $0.3mV$. Given a readout resolution between $23 \sim 28mV/G$, the minimum acceleration that can be applied is $10 \sim 13mG$.

Figure 5.28: (a) Measurement of the $200mG$ input acceleration (b) FFT of the measured acceleration

The input acceleration is defined in Eq 4.9. The frequency is set to $250Hz$ and the magnitude of the input signal is varied between $200mG$ and $10mG$. As we proposed in [113], a first order low-pass filter is applied at the output of the switching signal $Sgn(S)$. Figure 5.28 and Figure 5.29 show the measured acceleration considering an input acceleration with amplitudes of $200mG$ and $10mG$ respectively. An FFT block is added to check the frequency and amplitude of the measured signal. It should be noted that for high amplitude input acceleration of $200mG$ one low-pass filter is required to measure the signal. The shape of the signal can be improved by adding a second stage as in the case for $10mG$ input. On the FPGA level, and for measuring a small acceleration, the resolution of the hardware blocks is increased by increasing the number of bits reserved for the fractional part.
5.6. SMC for $x_{ref} = x_{pull-in}$ Experimental Results

The hardware implementation induces a delay between the measured and the input signal, which can be adjusted by tuning the gain of the two low-pass filters.

Two major advantages can be extracted when using SMC for operation at the border of stability comparing to the pull-in measurement time proposed in [102, 104, 105] and [31-33]. First, we demonstrated that by adjusting $\lambda$ and $\rho$, we can maintain the structure operating in the metastable region without the need for extra damping which by the way introduces more noise into the system. Slightly under-damped structure can operate at the metastable region. The voltages are constantly switching, with speed determined by the parameter $\lambda$ [113]. On the other hand high $Q_f$ accelerometer are not of preference as they could lead to the instability of the system. The second advantage is in terms of bandwidth, the method proposed in [102, 104, 105] and [31-33] require a small bandwidth, while in the proposed method, the bandwidth is large as the proof mass is maintained in that area.

Figure 5.29: (a) Measurement of the 10mG input acceleration (b) FFT of the measured acceleration
5.7 SMC for Driving Resonators

After applying the SMC to the feedback control of the micro-accelerometer, in stability control and sensing, in this section we demonstrate the application of a novel band-pass SMC to the driving mode of MEMS-based resonators and resonant sensors (e.g. vibrating gyroscopes)\footnote{This part has appeared in a journal paper: E. H. Sarraf, M. Sharma, and E. Cretu, "Novel band-pass sliding mode control for driving mems-based resonators", Sensors and Actuators A:Physical, 186:154-162, 2012.}. SMC architectures have been traditionally used for cancelling the disturbance-induced dynamics in systems, as seen in the previous subsection. We have recently extended the use of SMC\footnote{This part has appeared in a conference paper: E. H. Sarraf, M. Sharma, and E. Cretu., "Novel sliding mode control for MEMS-based resonators", Procedia Engineering, 25:1305-1308, 2011.} from the disturbance rejection to the measurement of an input excitation (in this case, the external acceleration).

The proposed technique relies on binary electrostatic actuation of the movable mass, dependent on the dynamics of the sliding/switching surface, in such a way that the generated actuation tracks possible changes in the mechanical resonant frequency induced by external factors (humidity, temperature, pressure, etc.).

The core of the proposed adaptive method is the implementation in hardware of an adaptive behavioural reference model used by the controller, able to track the sensed changes in dynamics. The architecture has a low hardware complexity and is suitable for VLSI implementation, in addition to a robust tracking behaviour for large variations in the parameters of the resonator (e.g. temperature-induced changes in the stiffness coefficient). Simulation results demonstrate the effectiveness of the proposed technique by maintaining the actuation of the system at its resonance frequency even in the presence of variations of the stiffness coefficient (step, linear) or of applied external forces (external acceleration and noise). Simulations results suggest that the novel SMC-based oscillator offers an advantageous alternative to more complex phase-locked loop (PLL) architectures, traditionally used for the driving of MEMS resonator structures.

5.7.1 MEMS Actuation

The resonant sensors are expected to maintain a calibrated and reproducible sensitivity in different environments, typically difficult to achieve due to the influence of external factors like temperature and pressure on certain physical parameters (e.g. stiffness and damping coefficients).
5.7. SMC for Driving Resonators

Figure 5.30: Image of the micro-gyroscope obtained with Polytec MSA-500® [12]

We can take for instance the case of a MEMS vibratory gyroscope [65, 123, 127], where the energy is pumped into the driven mode and coupled into the sensing mode by the non-inertial Coriolis force. A small mismatch of 100Hz (less than 2%) between the frequency of the actuation forces and the sensing mode resonance frequency led to a 50% drop in overall sensitivity to the external angular rate. The typical architecture for the driven mode actuation traditionally involves a closed-loop control that adaptively tracks the mechanical resonant frequency, for instance with a phase-locked loop or bandpass ΣΔ circuit [95].

MEMS-based resonators are traditionally driven by a combination of ac and dc voltages applied to the capacitive comb drives of the MEMS structure. As shown in the formula below, the resulting force has typically harmonic components, with an additional dc force component \((V_{dc}^2 + v_0^2/2)\) that could be used for self-actuation [64], in particular for RF resonators.

\[
F_e = \frac{1}{2}(V_{dc}^2 + v_0^2/2 + 2V_{dc}V_0 \cos(\omega t) + v_0^2/2 \cos(2\omega t)) \frac{dC}{dx} \tag{5.39}
\]

Here \(V_0\) is the amplitude and \(\omega\) is the frequency of the ac signal; the static equilibrium point for the transducer is set by \((V_{dc}^2 + v_0^2/2)\). The third and
the last terms in Eq 5.39 give harmonic forces at the excitation frequency and twice the excitation frequency, respectively.

We present in this section a novel alternative closed-loop control based on binary SMC, applicable to the excitation of the resonance modes in MEMS devices. SMC offers a promising theoretical framework for the efficient control of non-linear systems, due to its robustness against disturbances, low complexity and easy digital implementation [38, 141].

The present work proposes the use of a novel band-pass SMC for an oscillator architecture using MEMS resonant structures [116, 117], and applies these concepts for digitally actuating the driven mode of a MEMS gyroscope. The proposed SMC methodology is applied to the driven resonant mode of the vibratory gyroscope structure presented in Figure 5.30 and sets it into harmonic motion at its resonance frequency.

Table 5.5: Physical characteristics of the micro-gyroscope [124]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Actuation</th>
<th>Sensing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness Coefficient</td>
<td>122.28</td>
<td>132.23</td>
</tr>
<tr>
<td>Damping Coefficient</td>
<td>18.9µ</td>
<td>57µ</td>
</tr>
<tr>
<td>Proof Mass [nKg]</td>
<td>48.67</td>
<td>48.67</td>
</tr>
<tr>
<td>Quality Factor</td>
<td>128.8</td>
<td>43.63</td>
</tr>
<tr>
<td>Nominal Capacitance</td>
<td>0.36</td>
<td>0.6</td>
</tr>
</tbody>
</table>

The characteristics of the micro-gyroscope are illustrated in Table 5.5. The closed-loop architecture is therefore a digital control alternative to the phase-locked loop circuit traditionally used for the driving mode actuation. To the best of our knowledge, no previous work has reported SMC for driving resonators in controlled oscillator configurations.

5.7.2 Proposed Implementation

In order to move from the traditional sliding mode control for disturbance rejection to a bandpass SMC, a reference oscillator is implemented in FPGA, with an oscillating frequency tuned by the sensed dynamics of the MEMS device. The digital reference model provides a reference output displacement $x_{ref}$ that is used by the sliding mode controller for the binary electrostatic actuation. The driven mode dynamics of the vibratory gyroscope (or, in a more general context, that of a MEMS resonator) is modelled as a second order system; we have therefore considered a similar second order linear system for the reference model. The resulting equation of dynamics is given.
5.7. SMC for Driving Resonators

by:

\[ m \frac{d^2x}{dt^2} + b_d \frac{dx}{dt} + k_d x = F_{app} \]  (5.40)

Where \( x \) represents the position of the proof mass \( m \), \( b_d \) is the damping coefficient, \( k_d \) represents the equivalent spring constant, and \( F_{app} \) represents the electrostatic actuation force.

Figure 5.31: Complete system level block diagram of the proposed implementation along with the MEMS device and the readout electronics

The reference model and a sine wave generator excitation are implemented as digital blocks in FPGA, i.e. as subsystems of the digital SMC module (Figure 5.31).

The parameters of the reference model are based on the experimental characterisation of the fabricated gyroscope [14] depicted in Figure 5.30 using optical video-stroboscopy (Polytec MSA-500 \( \text{®} \) equipment).

The resonant frequency of the reference model is \( \omega_R = 2\pi f_R = \sqrt{k_{ref}/m} \), where \( k_R \) is adaptively adjusted based on the measured dynamics of the driven mode (and thus it tracks the variation of the stiffness coefficient \( k_d \) of the gyroscope).

The SMC controller is characterised by controller order reductions, superior disturbance rejection, insensitivity to parameter variations and simple implementations by means of switches and comparators [113]. As a result, the robustness allows for instance a high performance, even when using very simplified macro-models for the transducers in the design phase.
For low-cost system and hardware efficiency the controller was targeted on FPGA. Replacing or cutting down the cumbersome, less flexible analogue implementation with more flexible technology such as FPGAs follows the route envisaged by Xiong et al. [150] for the future of integrated circuits. The proposed implementation, using rapid prototyping methodology with Simulink ®/Xilinx ™ blockset, comprises two main function blocks, the SMC dynamics and the reference model. The proposed implementation of the SMC reflects the derived equations in the previous section. To facilitate the test, a macro-model of the fabricated gyroscope was firstly obtained from the experimental measurements performed using Polytec MSA-500 ® equipment. The macro-model was implemented in Simulink (floating point), while the sliding mode controller and the reference model were implemented in FPGA(fixed point).

Figure 5.32: Proposed FPGA implementation of the SMC dynamics using rapid prototyping technique in Simulink ®
5.7. SMC for Driving Resonators

Figure 5.32 illustrates the FPGA implementation of the proposed SMC dynamics, which translates the aforementioned equation into hardware blocks. The controller uses the current position and velocity for the actual device \( \{x_{p1}, x_{p2}\} \), in addition to the current position, velocity and acceleration of the reference model \( \{x_{ref}, \dot{x}_{ref}, \ddot{x}_{ref}\} \), in order to determine the proper dynamics of the sliding surface and hence the proper switching scheme of the electrostatic force.

Inertial sensors (e.g. accelerometers, gyroscopes) and resonators can be modelled as second order linear systems, or as two coupled second order systems in the case of vibratory gyroscopes. We are targeting driving a resonator (in this case a gyroscope) at its resonant mode hence our reference model uses a simple linear constitutive model given by [65][116]:

\[
m \frac{d^2 x_{ref}}{dt^2} + b_{ref} \frac{dx_{ref}}{dt} + k_{ref} x_{ref} = F_{exc}(t) \tag{5.41}
\]

Here \( x_{ref} \) is the displacement of the proof mass, \( b_{ref} \) is the damping coefficient, \( k_{ref} \) is the spring constant, and \( F_{exc}(t) \) is the excitation force. Based on the values of the parameter \( b_{ref} \), the system can be under-damped, critically-damped or over-damped.

![Diagram](image)

**Figure 5.33:** Proposed FPGA implementation of the reference model including the sine wave generator using rapid prototyping technique in Simulink®

The conversion of the lumped element behavioural blocks (Figure 5.33 in...
5.7. SMC for Driving Resonators

Eq. 5.41 such as integrators into FPGA hardware blocks is made possible by the efficiency of the rapid prototyping technique [65, 116].

The a priori values used for the damping and stiffness parameters of the gyroscope are $b_{ref} = 1.898 \times 10^{-5} \text{Kg/sec}$ and $k_{ref} = 122.8 N/m$, respectively. The resonant frequency of the MEMS resonator is measured and is equal to 7.99KHz. They have been determined from experimental measurements on a sample device using Polytec MSA-500® video-stroboscopy equipment and its planar motion analysis software. The reference model contains a sine wave generator whose amplitude is of the same magnitude as the electrostatic force $F_e$ generated by the capacitor; the output can be defined as:

$$x_R = F_e \sin(\sqrt{\frac{K_R}{m}} \cdot t)$$  (5.42)

Where $K_R$ represents the adaptive stiffness coefficient and $m$ is the proof mass.

5.7.3 Driving Mode of the Resonator

As mentioned before, SMC has always been applied in order to reject a perturbation, in a low-pass configuration, whereas in this work we demonstrate the effectiveness of the proposed methodology for setting and maintaining a constant oscillation at resonance in the driving mode of the micro-gyroscope.

We have mentioned earlier the conventional driving method for resonators by means of $ac$ and $dc$ voltages. In this section we demonstrate the application of SMC to the driving mode of the resonator, where the electrostatic force generated by the actuating comb drives (see Figure 5.30) is made possible by means of infinite switching of an applied dc voltage on the right or left comb drives, depending on the dynamics of the proposed controller. Following from Eq. 5.39, we conclude that if the frequency of last term (2\omega) is much higher than the mechanical resonant frequency of the resonator \omega_0, it can be ignored since it will only generate small displacements. Therefore Eq. 5.39 can be simplified to:

$$F_e = \frac{1}{2}(V_{dc}^2 + \frac{1}{2}v_0^2 + 2V_{dc}v_0 \cos(\omega t)) \frac{dC}{dx}$$  (5.43)

Since the targeted micro-device is a gyroscope with area-varying actuation combs, Eq. 5.43 can be simplified to [2]:

$$F_e = 2.28 \cdot v_0 \cdot V_{dc} \cdot N_{oltb} \cdot \frac{eh}{d} = 0.68\mu N, \text{ where } v_0 = V_{dc} = 10V$$  (5.44)
The reference model yields $x_{ref}$ and $\dot{x}_{ref}$ used by the dynamics of the proposed controller. The output electrostatic force is applied constantly to the actuation capacitors depending on whether the state (in the phase space) is on one side or the other of the predefined sliding surface.

Figure 5.34: Comparison between the driving modes of an open-loop resonator system and a closed-loop resonator driven by means of switching of electrostatic force (binary actuation)

Considering a non-variant stiffness coefficient, the SMC-based resonator (closed-loop) oscillates at the natural resonant frequency of the resonator, as shown in Figure 5.34. The oscillation magnitude depends on the electrostatic actuation force generated by the applied voltage.

The SMC dynamics forces the MEMS resonator to imitate or to follow the trajectory of the reference model, which is made possible by means of infinite switching of the constant applied electrostatic force between right and left comb drives. In this section we assumed that the stiffness coefficient is invariant and insensitive to variations, which is not the case in real-time applications. In the next section we demonstrate how the proposed controller can accommodate small and large variations of the stiffness coefficient such that the resonant system is always driven at resonance.

### 5.7.4 Stiffness Controller

In real time applications, the resonator is subject to sudden variation in temperature, humidity and pressure. As a result, these variations could lead
5.7. SMC for Driving Resonators

to changes in the equivalent spring constant of the resonator, which could lead to serious consequences in the application of the sensor. The proposed controller makes the sensor insensitive to any variation in the stiffness coefficient.

Depending on the operating conditions, we estimate the variations of the stiffness coefficient to be varying within the interval \([-\Delta k, +\Delta k]\), where \(\Delta k\) is chosen based on the maximum estimated stiffness variation in given operating conditions. Therefore the stiffness coefficient \(k_R\) of the reference model is set to the average given by the switching scheme between two possible values:

\[
k_R \rightarrow \left\{ \begin{array}{ll}
k_L &= k_d - \Delta k \\
k_H &= k_d + \Delta k \end{array} \right. \tag{5.45}
\]

Where \(k_d\) is the stiffness coefficient of the MEMS-resonator, experimentally characterised using Polytec MSA-500, under normal conditions. In this section we have analysed the tracking and tuning of the actuation when the mechanical resonance shifts, for instance when the equivalent spring constant suffers a ramp or step variation.

The variation of the stiffness coefficient can be estimated directly from the dynamics of the proposed SMC-based controller. Recalling Eq 5.13 yields:

\[
S(e, \dot{e}).\dot{S}(e, \dot{e}) \rightarrow 0 \tag{5.46}
\]

Then we can write:

\[
\dot{x}_2 + \lambda \dot{x}_1 = \frac{1}{m} F_{app} - \frac{b_d}{m} \dot{x} - \frac{k_{d,\text{var}}}{m} x - \ddot{x}_{\text{ref}} + \lambda x_2 \tag{5.47}
\]

\(k_{d,\text{var}}\) is the varying stiffness coefficient of the MEMS resonator. Eq 5.47 yields the equivalent stiffness coefficient:

\[
k_{d,\text{var}} = \frac{1}{x} \left[ \eta u - b_d \dot{x} - m (\ddot{x}_{\text{ref}} - \lambda (\dot{x} - \dot{x}_{\text{ref}})) \right] \tag{5.48}
\]

\(\eta = N_{\text{olp}} \epsilon (h/2d_0)\) where \(N_{\text{olp}}\) denotes the number of overlap fingers, \(h\) denotes the thickness and \(d_0\) denotes the gap. All the parameters of Eq 5.48 are known to the controller, including the position and velocity of the current device \((x, \dot{x})\) and of the reference model \((x_{\text{ref}}, \dot{x}_{\text{ref}}, \ddot{x}_{\text{ref}})\) which are tracked by the controller. Based on the dynamics of the stiffness coefficient variations, the new stiffness coefficient is fed to the reference model based on the switching scheme defined in Eq 5.45. The switching mechanism stabilizes the MEMS device despite the variations of the physical stiffness coefficient.

The same switching law controls both the electrostatic force applied to the mechanical resonator and making this control scheme simpler than [14].
and very robust to variations of the stiffness coefficient. The time-varying stiffness coefficient of the real resonator is estimated from the sliding mode dynamics, and the reference model adaptively tracks the new value, such that the resonant system is always driven at resonance.

The robustness and invariance of the proposed controller has been demonstrated by using three different variations schemes of the stiffness coefficient, from a constant value, to a step change in \( k \) and a linearly varying \( k \).

Figure 5.35 illustrates the comparison between the open loop driven MEMS resonator (with harmonic actuation at a constant frequency) and the SMC driving alternative for the case of a time-varying mechanical spring constant \( k(t) \), starting from a given constant value. It shows the response of both open and closed-loop systems as the spring constant \( k \) varies according to a predefined trajectory. The open-loop oscillation amplitude changes significantly with variations in \( k \), while the SMC approach makes the oscillation amplitude insensitive to such changes.

Figure 5.36 shows the variation in switching dynamics of the electrostatic forces for different schemes variations of \( k \). The magnitude and sign (left or right) of the electrostatic force can be seen clearly. In addition, as the variation scheme of the stiffness coefficient varies (from a step to a linear change for instance), the switching dynamics of the electrostatic forces change accordingly, which can be seen clearly in the variation of switching period. The magnitude of the electrostatic force remains the same, and only the periodicity of switching is influenced by changes of the stiffness coefficient \( k \).

### 5.7.5 Robustness Analysis

The action of the electrostatic forces has been considered so far in the absence of an external acceleration. To demonstrate the robustness against external forces, let us consider the external acceleration as an equivalent bounded disturbance \( a(t) \), introduced into the system dynamics by the following state equations:

\[
\begin{align*}
\dot{x}_p & := x_p \\
\dot{x}_p & := \ddot{x} = \frac{1}{m} F_e - \frac{b}{m} \dot{x} - \frac{k}{m} x + a(t)
\end{align*}
\]

We assume that there is an upper, known limit, \( |a(t)| \leq a_{\text{max}} \), set according to the dynamic range of the micro-resonator; in order to be able to balance electrostatically such an external perturbation, \( a_{\text{max}} \) can be set by:

\[
a_{\text{max}} = \frac{F_e}{m} = 14.1 m/s^2
\]
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Figure 5.35: The effect of variation of the stiffness coefficient on the driving mode of the MEMS resonator considering an open-loop and closed-loop (SMC) systems
5.7. SMC for Driving Resonators

Figure 5.36: SMC switching dynamics of the electrostatic forces between \{+F_{app}, -F_{app}\} for a fixed, step change and linearly varying stiffness coefficient $k$. We consider that the variations of $k$ start occurring at $0.02\text{sec}$.

In addition, $a(t)$ may also include other uncertainties, such as un-modelled dynamics and parametric uncertainties. By using the second method of Lyapunov, let the Lyapunov function be $W = (1/2)S^2$. Differentiating $W(s)$ with respect to time, we get:

$$\dot{W} = S[-\rho \text{sgn}(S) + a(t)] \leq S[-\rho \text{sgn}(S) + a_{\text{max}}] \quad (5.51)$$

If $\rho \geq a_{\text{max}}$ is satisfied, then $\dot{W}$ is sufficiently ensured, i.e. the state moves toward the surface. The value of $\rho$ gives the magnitude of the equivalent electrostatic acceleration exerted by the control loop, so it depends on the magnitude of the actuation voltages [113]. Since $a_{\text{max}}$ is set according to the magnitude of the electrostatic force, we have tested the robustness of the system in the presence of both noise and deterministic acceleration input components.

In addition to a band limited white noise acceleration with power spectral density (PSD) equal to $\bar{S}_a$ and a bandwidth of 400Hz, a sine wave input $a(t) = A_{app} \sin(\omega t)$ with frequency $\omega$ and an amplitude equivalent to $A_{app} \sin(\omega t)$ is applied at the input of the sensor. The PSD yields how much, on average, noise per unit bandwidth there is on a certain frequency [64]. To test the robustness of the system, we applied different levels of noise power.
and input accelerations.

Figure 5.37 illustrates the comparison of the proof mass displacement between three closed loop simulations, different in the magnitude of the external forces (white noise + input acceleration) which are applied to their inputs.

No external forces have been applied on the first model, which is considered as the ideal, noiseless model. An external force, with acceleration equal to \( A_{app} = a_{max}/4 \) and a PSD of \( S_a = 5 \times 10^{-11} V/\sqrt{Hz} \) is applied for the second micro-system. The third case corresponds to an external acceleration equal to \( A_{app} = a_{max}/2 \) and a PSD of \( S_a = 5 \times 10^{-7} V/\sqrt{Hz} \). Figure 5.37 shows that, as the external forces increase, so does the transient time, which increases by 6msec for the second model, and by 10msec, for the third one. The effect of the sine wave acceleration is dominant during the transient time and neglected at resonance. It should be noted that the SMC parameters such as \( (\rho = 10^2, \lambda = 1) \) are kept the same for all the three models. The transient time can be significantly reduced by tuning digitally (on FPGA) the appropriate values of \((\rho, \lambda)\) parameters for higher convergence speed and reduced transient regimes, considering the maximum estimated disturbances. The simulation results show that the proposed controller circumvent the variations and provide a robust and stable solution for the sensor.

5.8 Conclusion

In this chapter we have presented and effectively demonstrated the application of SMC to the feedback control of micro-accelerometers. We started addressing the challenge by applying the SMC to reposition the proof mass to its central position after the deviation caused by an external acceleration exerted on the system, in order to minimize the non-linearities, and to ensure system stability for a certain predefined dynamic range of operation. Switching between two values (electrostatic forces), based on the dynamics of the chosen sliding surface, yielded a robust control capable of repositioning the proof mass to its central position.

This proposed methodology is proven simple and suitable for a VLSI implementation. We targeted an FPGA implementation where we presented a comparison in terms of hardware complexity between the SMC and the conventional \( \Sigma \Delta \) approach. It should be noted that both systems exhibit a first-order modulator, however the degree of the feedback loop for the \( \Sigma \Delta \) approach is about six due the presence of some integrators in the decimator and the DAC.
Figure 5.37: Robustness against external disturbances considering different amplitudes of external forces (acceleration + white noise)
blocks. As a result, the proposed SMC does not occupy much resources on the FPGA.

For testing the application of SMC-based concept, the amplitude of the input acceleration is made equal to 1G. However the ultimate goal is to measure small signals for applications that require high sensitivity such as inertial navigation, non-invasive surgery and oil explorations. We started by identifying the limitation (smallest measurable signal) of SMC with reference trajectory set to null $X_{ref} = 0$. We extended the application to sensing, where we have demonstrated the extraction of the measurand (acceleration) from the switching scheme using a low-pass filter. We took advantage of the chattering noise in extracting the acceleration exerted on the system.

Our proposed approach was to drive the accelerometer to the border of stability, and apply the SMC to operate in the metastable region which is the region with high sensitivity. In this case the reference trajectory is made equivalent to the pull-in distance $X_{ref} = d_0/3$. Such operation surpasses the limitation in measuring small signals since the device is stabilized in the metastable region, where a tight equilibrium between the forces is maintained, hence any external force can alter the equilibrium, but the switching dynamics of the SMC prevent it and maintain the stability at the reference point or trajectory while providing a measurement of the external acceleration applied to the system, similarly as in the previous case with low-pass filtering of the switching scheme.

In the end, we extended the application of the SMC to driving resonators at their resonant frequency, hence beyond the conventional control problem, using a binary electrostatic actuation. The simplicity of the method makes it more attractive than the traditional analogue PLL loops. The proposed methodology through an FPGA-based behavioural model has proven to provide a robust reliable and stable solution for resonator behaviour, despite changes in environment that could affect the resonator parameter (for example a change in temperature could affect stiffness coefficient).
Chapter 6

Adaptive Self-Calibration and Self-Testing of MEMS-based Inertial Sensors Targeted on FPGA

"The only calibration that counts is how much heart people invest, how much they ignore their fears of being hurt or caught out or humiliated. And the only thing people regret is that they didn’t live boldly enough, that they didn’t invest enough heart, didn’t love enough. Nothing else really counts at all"
— Letters of Ted Hughes

6.1 Introduction

6.1. Introduction

micro-device, such that the induced response leads to the identification of the mechanical transfer function, and to the tuning of the associated digital behavioural model.

We have seen in Chapters 1 and 2, that in today’s MEMS’s world, highly sensitive MEMS-based micro-systems are made possible by incorporating complex fabrication methodologies and high resolution readout electronics, combined with costly packaging assemblies. The challenge for low-cost and increased functionality marks a growing interest in built-in-self-test (BIST) and built-in-self-calibration (BISC) for MEMS-based inertial sensors, where functional aspects can be verified and calibrated at rest and/or in operation. The advantages of BIST and BISC can be resumed in the following points:

- From the manufacturer’s perspective, BIST is an advantageous solution for an overall reduced fabrication cost and increased reliability. Failure-sources in the micro-manufacturing process (such as overetching, stiction and packaging-induced stress) may contribute to micro-systems malfunctioning, or induce changes in the operating characteristics. Functional testing costs less and is significantly faster than calibration. Therefore, the typical approach for large volume manufacturing is to perform functional testing before packaging (to avoid the high cost of packaging for known-bad sensors), followed by a calibration procedure after the micro-system has been packaged.

- From the user perspective, BIST is an essential component in crucial, usually safety applications, which require continuous monitoring, such as airbag deployment. BISC, on the other hand, is important for maintaining high accuracy in demanding applications, such as adaptive path prediction systems, in particular for recalibrating the mechanical performance when the equivalent stiffness and/or damping coefficients are affected by changes in the operating environment (temperature, humidity and pressure). This is also crucial in micro-G measurement applications such as oil explorations, earthquake detection, bridge monitoring and non-invasive surgery.

Testing provides only a yes/no answer to decide if the device satisfies basic functional threshold operating requirements. Moreover, according to Deb and Blanton [28], the testing process has limitations which sometimes can cause good micro-devices to be rejected (aspect known as the yield loss), or bad micro-devices to be accepted (aspect known as test escape). The testing scheme needs to provide the right compromise between reliability
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and overall cost. The BIST can help identify unstable defects [28], provide adequate mechanisms for improving device reliability and ensure its correct performance [24, 94].

Nevertheless, a simple functional test is not sufficient in many cases (e.g. for high sensitivity inertial sensors), where the specifications are so tight that a periodic re-calibration of the micro-system is necessary. Variations in the mechanical sensor characteristics can be compensated through the correlated tuning of the subsequent electronic stages. Difficulties arise when, once calibrated at the manufacturing site, the devices change their behaviour at the user side, while in operation. This happens due to large variations in the operating conditions, such as temperature and humidity. This aspect generates the need for low-cost calibration schemes applicable at the user side, to guarantee the continuous reliability and accuracy of the measured data. Failure in reliability and accuracy could lead to catastrophic consequences in the case of the aforementioned applications.

Most of the micro-accelerometers and micro-gyroscopes available on the market today, such as Analog Devices © [60] and ST Microelectronics © [133], have a self-testing feature: an applied test voltage generates an electrostatic force (testing stimulus) that causes a predetermined movement of the movable structure. This force should activate the device to its full working range, i.e. maximum output, and failure to demonstrate a full output operating range within a prescribed tolerance level reflects that the device is faulty, as stated by Xiong et al. [150].

Pseudorandom (PN), or maximum length sequences, have been popular for testing and calibration of electronic systems [90], and have been recently applied as well to the test and calibration of MEMS-based micro-structures. They offer several advantages, both theoretical and in terms of hardware implementation, as they are similar to white noise-like input stimuli [90], and avoid generation of complex and device-dependent test signals. PN sequences for MEMS testing have been employed for instance for on-chip measurement of both the thermal and mechanical dynamics of piezoresistive cantilever beams [30, 108]; one of the authors has further extended the methodology to accelerometers, and used it for instance to evaluate the static and dynamic response of ADXL103 [60] devices. Dumas et al. [37] used this methodology to ensure that the output test signal from the accelerometer was not degraded by noise - they were able to achieve of noise rejection and 2.55 sec test time.

This chapter reports the use of PN sequences as actuating voltages that induce small perturbation in the dynamics of the MEMS structures, in order to adaptively extract their time-varying damping and stiffness coefficients.
6.2. Proposed Adaptive Technique and FPGA Implementation

The whole calibration algorithm is efficiently implemented on FPGA. The method is an extension along the testing concepts presented in [30]. Xiong et al. [150] mentioned that very soon MEMS devices will be fabricated on the same chip with digital analogue, memory, and FPGA circuit technologies. An efficient, reliable and low cost system-level solution can be then obtained by integrating the sensor with BISC and BIST schemes targeted on cheap, reconfigurable and efficient technologies such as FPGAs.

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As time-varying operating environment conditions (temperature, pressure, humidity, etc.) modify the sensitivity of the MEMS sensors, maintaining a desired high-accuracy in the measurement process requires a periodic recalibration of the system sensitivity. An adaptive, on-board, real-time self-calibration scheme is able to track and adjust the changes in sensor characteristics. We propose in the present work an adaptive error-correction scheme based on a digital behavioural reference model, and demonstrate its implementation on FPGA.

Figure 6.1: System-level architecture

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The system level block diagram is illustrated in Figure 6.1 [65], [115]. It comprises the micro-device (fabricated in SOI-MUMPs® technology [84]), analogue readout electronics (either as a separate CMOS module or with discrete components), an ADC converter block to interface to FPGA, a DAC block for the configuration and actuation of the MEMS device, and the FPGA block (for the self-calibration controller).

![FPGA Implementation Diagram](image)

Figure 6.2: FPGA implementation of controller blocks (a) PN sequence generator, (b) damping controller, (c) stiffness controller

The FPGA implementation of the proposed technique using Xilinx® blocks is shown in Figure 6.2 and Figure 6.3. To minimize the physical...
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Figure 6.3: FPGA implementation of controller blocks (a) correlation cells, (b) DUT-R
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circuit area, the \( PN \) sequence generator was targeted on \( FPGA \) as part of
the controller block (see Figure 6.2 (a)) and its output is fed to both \( DUT-R \)
model (see Figure 6.3 (b)) and to the actual device-under-test (DUT), via a
simple one-bit digital-to-analogue converter.

A 1023-bit \( MLS \) is implemented by using 10 interconnected shift reg-
isters. The configuration shown corresponds to a generator polynomial
\([10, 7, 0]\). The input-output cross correlation function for a linear time in-
variant (LTI) system can be used to estimate its impulse response from the
response to an \( MLS \) input \([90]\). Assuming \( h[n] \) as the impulse response of the
LTI system, its output to an input signal \( x[n] \) is given by the convolution:

\[
y[n] = x * h[n] = \sum_{k=0}^{\infty} x[n-k] \cdot h[k]
\] (6.1)

In the case of using a maximum length sequence as input \( x[n] \), we can recover
the unknown impulse response from the cross correlation between the input
\( (x[n]) \) and measured output \( (y[n]) \) signals, \( \Phi_{xy}[n] = x \odot y[n] = E\{x[k] \cdot y[k+n]\} \), for it will be proportional to the impulse response \( h[n] \) of the system:

\[
x_{MLS} \odot y[n] = x_{MLS} \odot (x_{MLS} \ast h)[n] \\
= (x_{MLS} \odot x_{MLS}) \ast h[n] \\
= (\delta[n]) \ast h[n] = h[n]
\] (6.2)

This is due to both the associativity between cross-correlation and convolu-
tion operators, and to the fact that the autocorrelation function \( \Phi_{xx}[n] \) of a
\( MLS \) (PN) is, except for a small dc level (dependent on the length of \( MLS \)),
a good delta function \( \delta[n] \) approximation:

\[
\Phi_{xMLSxMLS}[n] \cong \delta[n]
\] (6.3)

As can be seen from Eq 6.2 if the input is an \( MLS \) sequence, the input-
output cross correlation is an estimate of system impulse response. For
the discrete time case, the cross correlation operator between two \( N \)-length
periodic sequences is defined as:

\[
\Phi_{xy}[n] = \frac{1}{N} \sum_{k=0}^{N-1} x[k] \cdot y[k+n]
\] (6.4)

In the corresponding \( FPGA \) implementation, for a binary input sequence
the multipliers can be replaced with a multiplexer, as shown in Figure 6.2.
In the case of a periodic \( MLS \) sequence as input, for every distinct delay \( k \)
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we get the associated signature \( h[k] \), representing the value of the impulse response at the sample time \( t = k \cdot T_s \). With corresponding added delays in the hardware, the coefficients can be identified from the post-processing of the output signal. For an MLS of length \( N = 2^M - 1 \), we have an \( N \)-length signature pattern \( (k = 0, 1, ... N - 1) \). When used for identifying the parameter values of a reference model, only the first coefficients will carry most of the relevant information in practical cases. The width of the significant portion is equal to \( F_s/BW \), where \( F_s = 1/T_s \) is the sampling rate (the clock rate of MLS here) and \( BW \) is the 3dB bandwidth of LTI system [90]. This relation reflects two important points:

- For a high value of \( F_s \) or a low value of \( BW \), the information is spread over a large number of redundant coefficients
- To standardize the proposed approach, we choose the MLS chip rate as a function of 3dB frequency of the LTI system

Using Nyquist criterion, the chip rate for MLS was chosen to be twice the 3dB frequency. From the simulation results it was observed that is usually sufficient to use only the first 10 – 15 coefficients in the signature. In fact, it has been shown [90] that for an LTI system with \( p \) poles and \( q \) zeros, only \( \{p + q + 1\} \) arbitrary impulse response terms are sufficient to characterise the system.

Inertial sensors (e.g. accelerometers) and resonators can be modelled as second order linear systems, or as two coupled second order systems in the case of vibratory gyroscopes. The linear constitutive equation of one degree of freedom resonant mode (valid for small displacement amplitudes) is given by [65]:

\[
\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + kx = f(t) \implies X(s) = \frac{F_s}{ms^2 + s\gamma + k} = \frac{F(s)/m}{s^2 + s\omega_0/Q + \omega_0^2} \tag{6.5}
\]

Here \( x(t) \) is the displacement of the proof mass (with \( X(s) \) as its Laplace Transform pair), \( \gamma \) is the damping coefficient, \( k \) is the spring constant, \( \omega_0 \) is the natural resonant frequency of the device, \( Q \) is the quality factor, \( C \) is the readout gain constant and \( f(t) \) (or \( F(s) \) in its Laplace form) is the excitation force. Based on the values of the parameter \( \gamma \) the system can be under-damped, critically-damped or over-damped. From Eq 6.5 we can see that under constant actuation (\( s = 0 \)) the output depends on the effective stiffness constant of the inertial sensor and on the applied dc force (static
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regime), while the effect of damping and inertia become noticeable only in dynamic mode (the structure is operated at atmospheric pressure), and at higher frequencies [123, 124, 127].

The linear model of the reference resonator (DUT-R) was implemented digitally in FPGA using a discretized state-space representation (Figure 6.3(b)), with velocity and displacement as internal state variables. In order to adjust the values of the parametrized reference model knowing the measured impulse response of the device, it is important to know how do the variations in \( m, k \) and \( \gamma \) reflect in changes of the \( h[n] \) samples. While the proof mass will remain constant in operation, \( k \) and \( \gamma \) may vary in time, reflecting environment changes.

![Figure 6.4: Sensitivity analysis](image)

The sensitivity analysis, illustrated in Figure 6.4, shows that the reconstructed digital impulse response \( h[n] = h_n \) depends indeed on both \( \gamma \) and \( k \) with different \( h[n] \) coefficients having distinct sensitivities to their independent variation. We have analysed the sensitivity of the impulse response terms \( (h_1, h_2, h_3, ...) \) to induced variations in the damping and stiffness coefficients, \( \frac{\partial h_n}{\partial \gamma}, \frac{\partial h_n}{\partial k} \), concluding that only the first samples \( h_i \) are significant.

While both the equivalent elastic and damping coefficients can vary in
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time due to changes in operating conditions (e.g. operating temperature),
the mass is taken as an invariant parameter for a given device. The self-calibration procedure has two distinct phases: the \( \text{dc} \) response is firstly used to determine the equivalent spring constant, while the dynamic response to the MLS excitation is used for recalibrating the value of the damping coefficient in the digital reference model.

The \textit{model-based recalibration technique} uses a replica of the behavioural model \( DUT-R \) of the device. It is stored on \textit{FPGA} and coupled to an adaptive algorithm that compares its response with the one of the \( DUT \). The differences are used to recalibrate the \( DUT-R \) parameters (initially set to the values estimated after the fabrication) to current operating values.

The efficiency of the rapid prototyping methodology for \textit{FPGA} implementation facilitated the conversion of the lumped element behavioural blocks (such as integrators) into hardware blocks. Two integrators, converting acceleration to velocity and velocity to position, have been implemented using \textit{Xilinx} \textcopyright\ equivalent blocks.

The a priori values used for the damping and stiffness parameters of the gyroscope are \( 5.57 \times 10^{-5}\text{Kg/sec} \) and \( 132\text{N/m} \), respectively, and \( 3.4 \times 10^{-5}\text{Kg/sec} \) and \( 49.56\text{N/m} \) for the accelerometer.

![Figure 6.5: Frequency response of the micro-gyroscope [124]](image)

They have been determined from experimental measurements on different sample devices using \textit{Polytec MSA-500} \textregistered\ video-stroboscopy equipment and associated planar motion analysis software, as illustrated in Figure 6.5 for the gyroscope and Figure 3.5 for the accelerometer.

The self-calibration block is the processing engine that adaptively com-
6.2. Proposed Adaptive Technique and FPGA Implementation

determines and stores the values of the stiffness and damping coefficients. It contains the correlation cells for damping control, along with the damping and stiffness controllers. The output of the PN sequence generator acts as test input for both the DUT and DUT-R (FPGA model). The proposed technique allows the use of the same binary generator, with proper initialization, as stimulus towards extracting both stationary and dynamic response (as quantified by the error in most sensitive impulse response coefficients) of the micro-device.

Compared to the functional testing methods for mixed signal systems based on PN the present work takes the concept further, and uses both the steady-state and dynamic responses of the device in a closed loop fashion to characterise and hence calibrate the micro-device in real time. The algorithm of operation is shown in Figure 6.6 and can be succinctly described by the following steps:

- Starting with the equivalent spring constant, a dc force is applied to the DUT. This is achieved by initializing the state of the test generator with all ones, stored as initial values in a linear feedback shift register (LFSR). The approximate settling time of DUT serves as a guide for choosing the order of the test PN sequence

- For the first 10 clock cycles (10 being the order of PN sequence used), the difference in static response between the DUT and the DUT-R is computed and fed to the stiffness controller. The binary actuation test signal will remain "1" during this time interval

- Using the steady state error (difference between the experimentally measured response and the output of the reference model), the stiffness parameter $k$ of the DUT-R is adjusted by the stiffness controller for a maximum matching between the responses of DUT and DUT-R

- The counter starts counting from the initial state, up to the total length of PN sequence

- For the damping coefficient adjustment, the complete responses $y[k], y_R[k]$ of the DUT and DUT-R respectively, corresponding to the entire length of PN sequence $x[k]$, are fed into the correlation cells. The algorithm computes through digital cross-correlation the significant coefficients of $\{h_{DUT}[n], h_{DUT-R}[n]\}$, in order to perform the adaptive correction on DUT-R
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Figure 6.6: Data flow of the proposed technique
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• The damping coefficient of the DUT-R $\gamma_{DUT-R}$, is being adjusted by the damping controller such that the error between the two signatures is minimized.

• The iteration cycle for the stiffness and damping coefficients ($k_{DUT-R}$ and $\gamma_{DUT-R}$) of the reference model repeats until the resulting error falls below a desired threshold, imposed by the accuracy constraints. At this point, the self-calibration is achieved, and the reference model parameters reflect the values of the real device.

The stiffness controller (see Figure 6.2(c)) (responsible for recalibrating the spring constant parameter) is active only during the first few clocks of every PN sequence cycle, and outputs a correction term proportional to the error between the stationary responses of DUT and DUT-R. Because for the physical DUT the MLS sequence maps to binary actuation voltages, while it can directly interface with DUT-R model, it is necessary to use a scaling constant $A$ for the actuation term on DUT-R. The proportionality constant $A$ is chosen to correspond to the equivalent displacement-to-voltage gain of the MEMS device response. A gain of $A = 7 \times 10^3$ is used in our case. The final output of the controller is the sum of the a priori stiffness coefficient and the output of the accumulator block.

The correlation block, illustrated in Figure 6.2(d), contains the cells which evaluate the cross-correlation expression $\sum x[n] \cdot y[n - k]$ between the binary PN input sequence $x[n]$ and the measured DUT or digitally computed DUT-R output $y[n]$. The PN sequence is associated in the mechanical domain with two actuation force levels, $\{\pm F_0\}$ (left or right actuation with forces of constant magnitude). The binary nature of the sequence means that the convolution sum reduces to a sequence of multiplications of $y[n]$ (or delayed versions) by $\{+1\} \text{ or } \{-1\}$, followed by result accumulation. A multiplexer selects $\{+y[n]\} \text{ or } \{-y[n]\}$ based on the input $x[n]$, followed by the accumulation of the partial result.

An adaptive scheme implements the error correction: the operating value of the damping coefficient to be stored in the behavioural model is iteratively adjusted, proportional with the accumulated difference (cumulated error) between the reconstructed impulse responses $\{h_{DUT}[n], h_{DUT-R}[n]\}$ of the DUT and DUT-R for a full PN cycle (every 1023 clock cycles).
6.3 Experimental Results and FPGA Hardware Resources

The proposed adaptive technique for self-calibration and self-testing has been experimentally verified for tuning the stiffness and damping coefficients of fabricated accelerometers, and for the sensing mode of a MEMS gyroscope. This is a complete system and comprises MEMS-based components, FPGA board, readout electronics, DAQ; each individual component has been verified separately, in order to decrease the probability of erroneous signals.

6.3.1 Calibration of the Reference Model DUT-R

The proposed scheme relies on the accuracy of the reference model, responsible for providing the appropriate time-varying damping and stiffness coefficients. To address the range of the quantization error, a fixed/floating point study was performed on the DUT-R model, to identify a suitable word length representation in binary format. For this, considering a single DOF system, the dynamic range has been fixed according to the targeted sensitivity. It should be noted that the chosen dynamic range also affects the limitation of the BIST/BISC, as the reference model should be tuned or programmed to match the DUT. Since we target high sensitivity and low-G measurements, the dynamic range has been fixed to \([-1G, +1G]\). In this section the model parameters (mass, stiffness and damping coefficients) are discussed for a MEMS accelerometer. The same study was separately performed for the MEMS gyroscope.

The digital implementations are made using a rapid prototyping methodology, where Matlab/Simulink® models are mapped to Xilinx® FPGA configurations. The DUT-R is firstly evaluated using a Simulink® floating point representation, and subsequently the regular Simulink® blocks are converted, using Xilinx System Generator module, to fixed point Xilinx blocks. The amplitude of the signal is monitored at the input and output of each arithmetic block for word-length verification. The same input acceleration signals are applied to both, floating-point (Simulink® regular blocks) and a fixed-point (Xilinx® system generator blocks, as shown in Figure 6.2 (e)) models.

In the first test case we apply a step-like input (Figure 6.7(a) at 0.05sec. A second case considered a sine-like input, as in (Figure 6.7(b) with a frequency of 90Hz. The acceleration magnitude is within the interval of \([-1G, +1G]\) in both test cases.

As the resolution of the reference model is of utmost importance, the
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Figure 6.7: Calibration of the DUT-R (accelerometer model) with floating-point vs fixed-point simulations
chosen word-length for the fixed point FPGA computation varies from one arithmetic block to another, depending on the magnitude of the input signal. The word-length is represented by two numbers, one for the number of bits representing the real part (left), and the other (right) for the number of bits representing the fraction part of the signal. For low magnitude input signals the used word-length is (32, 30) with a large amount of bits is reserved for the fractional portion of the signal, while for large magnitudes a word-length of (24, 6) is used, where the large portion of bits is reserved for the real part of the signal. Additionally, the resolution of the DUT-R strongly depends on the resolution of the two digital integrators (one for velocity and the other for position). Figure 6.7 shows that the proposed hardware fixed-point implementations, highly concurrent, match well the generic floating point models.

### 6.3.2 Experimental Setup

![Experimental setup diagram](image)

Figure 6.8: Experimental setup

Figure 6.8 illustrates the experimental setup. A capacitive readout circuit interfaces the micro-sensor with the FPGA-based controller. It consists of an oscillator generating a carrier signal at 100kHz, well outside the mechanical bandwidth, a transimpedance amplifier and a demodulator. The
output voltage is proportional with the differential change in the capacitance and thus implicitly with the relative displacement of the proof mass, for small displacement values. An ADC maps the voltage into a digital signal supplied as input to the FPGA module, where it is used for the tuning the digital reference model. The digital binary sequence generated in FPGA is converted into the two electrostatic actuation forces, \( \pm F_0 \), applied on a dedicated pair of actuation comb capacitors. This is accomplished using a multifunctional DAQ from Measurement Computing \( \odot \) [75] (USB-1208 Series), with 1.2 Ksample/sec to 1 Msample/sec sampling rate for DAC and ADC operations.

The DUT is excited with maximal length sequences of small amplitude, with a frequency within the mechanical bandwidth of the movable structure. The input-output cross correlation gives an estimate of the impulse response (whose digitized values \( h[n] \) will be referred to as coefficients) of the micro-device. Different coefficients have different sensitivities to variations in the equivalent stiffness \( k \) and damping \( \gamma \) values. The analysis and simulation results have indicated that knowing the static response to a constant actuation voltage and the few first \( h[n] \) coefficients is sufficient to tune the reference model parameters to match the DUT, and to subsequently perform the calibration of the micro-system.

Tests were carried out by imposing a dc voltage of 15 V and varying the ac signal amplitude for a fixed frequency of 8.5 KHz for the gyroscope, and a dc voltage of 20 V and varying the ac signal amplitude for a fixed frequency of 4.8 KHz for the accelerometer.

### 6.3.3 Self-calibration of Micro-Accelerometer and Gyroscope

Both micro-devices have been designed and simulated using Coventor \( \odot \) software tools, fabricated in SOI-MUMP@s \( \odot \) [84] technology (25 \( \mu \)m thickness for the structural layer), packaged in ceramic packages and experimentally characterised using a Polytec MSA-500 \( \odot \) equipment.

Actuation of the proof mass (mimicking an external acceleration input) is achieved by applying ac voltages between the moving and the fixed fingers of the actuators and a bias dc voltage to the proof mass of the sensor, in order to linearise the effect of the electrostatic forces. The experimental tests on the MEMS-FPGA closed loop self-calibration were carried out by actuating the MEMS-based accelerometer with the PN sequences. The essential parameters, such as resonant frequency, damping coefficient and quality factor, have been estimated from the experimental measurements.
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performed on the MEMS devices using Polytec MSA-500 ®. The measured frequency responses for the accelerometer and for the sensing mode of the micro-gyroscope are shown in Figure 6.5.

The value of the damping coefficient $\gamma$ is obtained by half power method. That is, we use $\gamma = \Delta f_0 / 2 f_0 = 1 / 2 Q$, where $\Delta f_0$ is 3dB-bandwidth in Hz and $f_0$ is the resonant frequency. The stiffness coefficient is given by $k = m(2\pi f_0)^2$, where the mass $m$ of the structure is a given data, estimated from the layout and material properties.

The resonant frequency and quality factor for each resonant mode are $\omega_0 = \sqrt{k/m}$ and $Q = \sqrt{\omega_0 m / \gamma}$ respectively, where the mass $m$ is assumed invariant to external variations, while $k$ and $\gamma$ can be time-varying parameters.

The principle of operation of the single axis one degree of freedom accelerometer can be derived from the resonator model in the previous subsection. The performance characteristics and the micro-structure geometry of the micro-accelerometer are illustrated in Table 3.1 and Figure 3.3 respectively. The micro-structure exhibits different sets of comb drives (longitudinal and lateral) for different actuating, sensing and feedback purposes. The basic structure of the micro-gyroscope is a proof mass suspended by crab-leg flexures with two planar DOFs. The micro-device features three sets of actuating and sensing comb drives.

Vibratory gyroscopes have two coupled modes, a driving and a sensing one. The driving mode is electrostatically excited, and in the absence of an external angular rate, the mechanical energy of its vibratory motion remains confined into the driven mode. A non-zero external angular rate will generate a non-inertial Coriolis force proportional to it, which couples the two resonant modes, and converts a part of the energy of the driving mode into an oscillation of the sensing mode. The external angular rate value is therefore determined by measuring the amplitude of the sensing mode vibration. The typical MEMS gyroscope structures use linear motions for the driven ($\theta_x$) and the sensing ($\theta_y$) coupled resonant modes [2].

In our case, the fabricated gyroscope has a driven mode resonant frequency of $8.5KHz$, with a forced amplitude of the ($\theta_x$)-driven displacement of $5\mu m$. The quality factors for the drive and sense mode are $Q_x$ and $Q_y$, respectively, while the stiffness coefficient in sense direction is $132N/m$. Table 5.5 [65], [115] illustrates the physical characteristics of the fabricated micro-gyroscope.

Assuming that we have matched modes (identical resonance frequencies for drive and sense modes), the Coriolis-induced displacement in the sense mode is given by $y = 2Q_y Q_x / \omega_x$, resulting in ($\theta_y$) (sense) displacement
6.3. Experimental Results and FPGA Hardware Resources

Figure 6.9: Decrease in sensitivity from $4.623\mu m$ to $1.791\mu m$ for $100 Hz$ frequency variation

amplitudes of $143.7nm$ for $1 \circ/sec$ external angular rate. For a $0.05rad/sec$ mismatch (see Figure 6.9) between the driven and sensing modes, the sensitivity will drop by $50\%$ (displacement will drop from $4.623\mu m$ to $1.791\mu m$). The tuning of the two resonant modes (driving, sensing) is crucial in high performance gyroscopes, where any mismatch could lead to a fast decrease in the sensitivity to external angular rates [123, 124, 127].

The basic structure of the micro-gyroscope is a proof mass suspended by crab-leg flexures with two planar degrees of freedom (DOFs). The microdevice features three sets of actuating and sensing comb drives. This sensor is designed to be actuated (driven) in $\{(0x)\}$ direction; when an external angular rate acts in $\{(0z)\}$ direction, due to Coriolis coupling, sensing is achieved by sensing the Coriolis-induced motion in the $(0y)$ direction. By measuring the amplitude of this induced motion, the external angular rate can be measured.

The dynamical behaviour of the gyroscope is modelled by a set of two coupled differential equations, for the driven and sensing mode, respectively:

\[
m \frac{d^2x}{dt^2} + \gamma_x \frac{dx}{dt} + k_x x = F_{\text{actuation}} + 2m\Omega_z \frac{dy}{dt} \tag{6.6}
\]

\[
m \frac{d^2y}{dt^2} + \gamma_y \frac{dy}{dt} + k_y y = -2m\Omega_z \frac{dx}{dt} \tag{6.7}
\]
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$F_{\text{actuation}}$ is the electrostatic force that puts the gyroscope in motion in the (0x) direction, directly proportional to the square of applied voltage. The Coriolis force $-2m\Omega_z \frac{dy}{dt}$ induces the motion in the (0y) direction; a reciprocal Coriolis force, $2m\Omega_z \frac{dy}{dt}$, will be induced by the (0y) motion of the sense mode, but this is negligible relative to the electrostatic actuation. The two resonant modes may have different equivalent spring constants and damping coefficients.

We have considered in the present chapter only the monitoring and adaptive recalibration of the sensing mode of the micro-gyroscope. PN sequences (converted into binary actuation voltages) are applied on the sensing mode (gap-varying) comb drives in order to trigger a response. If one considers only their action, in the absence of an external angular rate, then Eq 6.7 becomes:

$$m \frac{d^2y}{dt^2} + \gamma_y \frac{dy}{dt} + k_y y = \frac{1}{2} \frac{C_0}{(d - x)^2} V^2$$  \hspace{1cm} (6.8)

Here $C_0$ is the initial capacitance, $V$ is the actuation voltage, $d$ is the initial distance between the gap-varying plates, and $y$ is the displacement due to the applied electrostatic force. For convenience, the damping and stiffness coefficients are renamed from $\gamma_y$ to $\gamma$ and $k_y$ to $k$, respectively, since they are the only one to be determined in this case.

The experimental tests on the MEMS-FPGA closed-loop adaptive reference model tuning were carried out by actuating the MEMS-based device with the generator sequence, as described earlier. It should be noted that the capacitive readout electronics introduces its own noise, adding a strong limitation in terms of displacement measurement resolution.

The temporal traces for the damping and stiffness coefficients tuning are shown in Figure 6.10 for the accelerometer, and in Figure 6.11 for the gyroscope. The goal of the proposed methodology is to track, in an adaptive manner, changes in stiffness and damping coefficients due to external factors. The output of the PN sequence generator actuates both the DUT (MEMS accelerometer/gyroscope) and the DUT-R (FPGA-based model of accelerometer/gyroscope).

As a result, the difference in the impulse responses of the DUT and DUT-R is calculated and used for adaptive tuning of the DUT-R parameters, as previously described. For the first phase mode (tuning of the stiffness coefficient), Figure 6.10(a) and Figure 6.11(a) show the decreasing error as the stiffness coefficients for the accelerometer and the gyroscope are recalibrated. As a result, the adjusted stiffness coefficient is the sum of the a priori stiffness coefficient (accelerometer/gyroscope) and the output of the
6.3. Experimental Results and FPGA Hardware Resources

Figure 6.10: Experimental tuning results for the stiffness (a) and damping (b) coefficients of the micro-accelerometer
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Figure 6.11: Experimental tuning results for the stiffness (a) and damping (b) coefficients of the micro-gyroscope
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accumulator block. This tuning process of the adaptive scheme led to the new values of stiffness coefficients, 51.48\textit{N/m} and 137.7\textit{N/m} for the accelerometer and gyroscope respectively. The dynamic response to the MLS excitation is used for recalibrating the value of the damping coefficient in the digital reference model. The adaptation of the damping coefficient, as illustrated in Figure 6.10(b) and Figure 6.11(b) requires more clock cycles than the stiffness coefficient tuning. This adaptive tuning process led to the new values of damping coefficients, 4.57 \times 10^{-3}Kg/sec and 7.12 \times 10^{-5}Kg/sec for the accelerometer and gyroscope respectively.

6.3.4 Hardware Resources and Trade-Offs

The proposed adaptive scheme, including the PN sequence generator, is implemented on FPGA, leading to a reliable, cheap and flexible system [23, 107]. Replacing or cutting down the cumbersome, less flexible analogue implementation with more flexible technologies such as FPGAs follows the route envisaged by Xiong et al. [150] for the future of integrated circuits. The hardware resources estimate include numbers of slices, LUTs, flip-flops (FF) and embedded multipliers. These estimates make it easy to determine how design choices affect hardware requirements.

Table 6.1: Hardware resources of the proposed scheme (for the MEMS accelerometer)

<table>
<thead>
<tr>
<th>Block</th>
<th>Slices</th>
<th>FF</th>
<th>LUT</th>
<th>MULT</th>
</tr>
</thead>
<tbody>
<tr>
<td>PN Sequence Generator</td>
<td>12</td>
<td>11</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Stiffness Controller</td>
<td>248</td>
<td>58</td>
<td>484</td>
<td>0</td>
</tr>
<tr>
<td>Damping Controller</td>
<td>207</td>
<td>99</td>
<td>365</td>
<td>0</td>
</tr>
<tr>
<td>Correlation Cells</td>
<td>424</td>
<td>127</td>
<td>811</td>
<td>0</td>
</tr>
<tr>
<td>DUT-R</td>
<td>2618</td>
<td>1495</td>
<td>4865</td>
<td>21</td>
</tr>
<tr>
<td>Overall System</td>
<td>3547</td>
<td>1857</td>
<td>6461</td>
<td>21</td>
</tr>
</tbody>
</table>

Table 6.1 and Table 6.2 summarize the hardware resources required by the digital tuning algorithm for the micro-accelerometer.

Table 6.3 and Table 6.4 summarize the hardware resources required by the summarize the hardware resources required by the digital tuning algorithm. Mainly due to the large amount of high throughput multipliers (for increased accuracy), the high-resolution behavioural model used as reference for both the accelerometer and gyroscope cases occupies 75\% of the used FPGA implementation. The BISC for gyroscope occupies a slightly larger
### 6.3. Experimental Results and FPGA Hardware Resources

**Table 6.2: BISC percentage of occupation for accelerometers (%)**

<table>
<thead>
<tr>
<th>Device</th>
<th>Slices</th>
<th>FF</th>
<th>LUT</th>
<th>MULT</th>
</tr>
</thead>
<tbody>
<tr>
<td>XC2VP2</td>
<td>N/A</td>
<td>65.94</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>XC2VP7</td>
<td>71.97</td>
<td>18.84</td>
<td>65.55</td>
<td>47.72</td>
</tr>
<tr>
<td>XC2VP30</td>
<td>25.89</td>
<td>6.77</td>
<td>23.58</td>
<td>15.44</td>
</tr>
<tr>
<td>XC2VP70</td>
<td>10.71</td>
<td>2.8</td>
<td>9.76</td>
<td>6.4</td>
</tr>
<tr>
<td>XC2VP100</td>
<td>8.04</td>
<td>2.1</td>
<td>7.32</td>
<td>4.72</td>
</tr>
</tbody>
</table>

**Table 6.3: Hardware resources of the proposed scheme (for the MEMS gyroscope)**

<table>
<thead>
<tr>
<th>Block</th>
<th>Slices</th>
<th>FF</th>
<th>LUT</th>
<th>MULT</th>
</tr>
</thead>
<tbody>
<tr>
<td>PN Sequence Generator</td>
<td>12</td>
<td>11</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Stiffness Controller</td>
<td>252</td>
<td>58</td>
<td>492</td>
<td>0</td>
</tr>
<tr>
<td>Damping Controller</td>
<td>175</td>
<td>98</td>
<td>293</td>
<td>0</td>
</tr>
<tr>
<td>Correlation Cells</td>
<td>1694</td>
<td>2536</td>
<td>2827</td>
<td>0</td>
</tr>
<tr>
<td>DUT-R</td>
<td>2533</td>
<td>934</td>
<td>4646</td>
<td>35</td>
</tr>
<tr>
<td>Overall System</td>
<td>4686</td>
<td>3646</td>
<td>8275</td>
<td>35</td>
</tr>
</tbody>
</table>

**Table 6.4: BISC percentage of occupation for gyroscope (%)**

<table>
<thead>
<tr>
<th>Device</th>
<th>Slices</th>
<th>FF</th>
<th>LUT</th>
<th>MULT</th>
</tr>
</thead>
<tbody>
<tr>
<td>XC2VP2</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>XC2VP7</td>
<td>95.08</td>
<td>36.9</td>
<td>83.95</td>
<td>79.54</td>
</tr>
<tr>
<td>XC2VP30</td>
<td>34.21</td>
<td>13.31</td>
<td>30.2</td>
<td>26.73</td>
</tr>
<tr>
<td>XC2VP70</td>
<td>14.16</td>
<td>5.5</td>
<td>12.5</td>
<td>10.67</td>
</tr>
<tr>
<td>XC2VP100</td>
<td>10.62</td>
<td>4.13</td>
<td>9.38</td>
<td>7.88</td>
</tr>
</tbody>
</table>
area than the one for accelerometer, due to the different requirements between the two inertial micro-devices, in terms of dynamic range and physical characteristics, such as proof mass, damping and stiffness coefficients.

The proposed scheme is massively parallel and is suitable for fixed-point FPGA implementation. With the exception of the DUT-R hardware implementation, an average of (16,12) word-length has been used for the stiffness and damping controllers, and for the correlation cells. The implementation shown in Figure 6.2 and Figure 6.3 have been verified through end-to-end VHDL simulations along with actual hardware implementation, to determine the device resources and speed. All blocks have the same throughput, with quite similar latencies. The circuit has a maximum speed of 14.217 MHz for the BIST/BISC (accelerometer) and 17.727 MHz for the BIST/BISC (gyroscope). The achieved clock frequency in both implementations (accelerometer/gyroscope) is acceptable. Pipelining techniques can be used to increase the speed, if necessary.

The hardware resources illustrated in Table 6.1 to Table 6.4 show the percentage of occupation of the proposed adaptive methodology for BISC, for several FPGAs from the Xilinx © family. The one used for the experiments is a Virtex-II pro XC2VP100, which contains relatively small hardware resources compared to the state-of-the-art FPGA on the market today. Nevertheless, the proposed methodology has low hardware resources demands even in this case. Therefore, with the tremendous FPGA development following Moore’s law, area occupancy trade-offs has become less critical than other trade-offs, such as speed and resolution. In our work, resolution was not compromised in the reference model, due to its importance in the proposed methodology. We can conclude, based on these values, that the methodology is suitable for the self-calibration of a more complex 3-DOF inertial measurement unit, using a cheap and small FPGA like XC2VP30. In this case the PN sequence generator can be a shared resource, and three reference models, for the \((0x)\) and \((0y)\) accelerometers and the \((0z)\) angular rate sensors, respectively, will be implemented to operate concurrently on the same FPGA chip.

6.4 Future Work

BIST guarantees the safety and functionality of operation of the device via continuous testing. Adaptation of stiffness and damping coefficients due to a change in temperature, pressure or humidity change was proven in this work through an adaptive, off-line, calibration methodology. An iterative
process, as shown in Figure 6.12, is performed regularly, in order to adapt the reference model parameters and the related calibration values – a maximum accuracy is maintained to the user side in such a manner.

A future challenge is to be able to perform online self-calibration and self-tuning, when the test actuation with PN sequences superimposes over the natural mechanical actuation given by an external acceleration or angular rate.

6.5 Conclusion

In this chapter we have developed a novel, adaptive, self-testing and self-calibration scheme for MEMS inertial sensors and resonators. The algorithm relies on the use of a digital reference behavioural model implemented on FPGA, whose parameters (stiffness and damping coefficients) are tuned based on the accumulated error between the reconstructed impulse responses of the device under test and the reference model. The impulse responses are reconstructed using low amplitude PN sequences as actuation signals (so that the sensor remains within its linear operating regime), and subsequently computing the input-output cross-correlation. A low-cost capacitive readout circuit, together with ADC and DAC blocks, interface between the MEMS device under test and the digital module. The self-calibration procedure implemented on FPGA requires low hardware resources, and it is easy extensible to MEMS systems for multiple degrees of freedom.
Chapter 7

Conclusions, Thesis
Contributions and Future work

"I think and think for months and years. Ninety-nine times, the conclusion is false. The hundredth time I am right"
— Albert Einstein

"I came to the conclusion that the optimist thought everything good except the pessimist, and that the pessimist thought everything bad, except himself"
— G. K. Chesterton

7.1 Summary

The driving force behind this thesis was to propose an efficient system level trade-off between good performance and low complexity, for a surface micro-machined accelerometer. Far from the attractive, high cost bulky proof masses, the objective was to investigate and propose new methodologies for increasing the sensitivity of the device with an affordable increase in the cost of the system.

Generally speaking, the open-loop sensitivity of a bulk accelerometer is very high due to the thick proof mass, and yet in most of the reported work that we have encountered during our literature review, the reported authors pursued a closed-loop implementation along with other circuitry (mostly analogue components), in order to increase the sensitivity, and minimize the non-linearities by keeping the displacement around the null or rest position. System control engineering postulates the fact of advantages in employing closed-loop over open-loop systems in terms of sensitivity, linearity and bandwidth.

Throughout the first two chapters, we have seen that miniaturization through batch fabrication techniques is not the only challenge that MEMS-based devices are facing, the real challenge is in obtaining a micro-structure
that perform a useful function.

In order to accommodate the limitation of fabrication technologies which limits the geometry and the structure of the micro-device and inherently restrict the performance of the micro-accelerometer in terms of noise floor and sensitivity, control engineering kicks in and improve the overall performance, by employing the micro-structure in closed-loop fashion.

Several types of controllers have been put in place, where the most dominant one on the market today is the Sigma-Delta $\Sigma\Delta$ modulator. This oversampling-based $ADC$ converter is very attractive due to its noise shaping properties and the digital output it yields. On the other hand, the aforementioned reported work, use high orders of $\Sigma\Delta$ modulators, in order to guarantee good performance in sensitivity, resolution and dynamic range. We found, during our study that such approach is highly complex and does not tackle the highly non-linear problem encountered in the targeted capacitive-based accelerometers, in the area of pull-in voltages, in addition to risking instability at high orders.

As a result, the complexity/cost of the system is increased at several nodes of the system. First the bulk micro-machining process contributes enormously in this respect. Second, the analogue $CMOS$ electronics challenges in measuring small range capacitance is another factor, in particular when high-order modulators are involved. A third factor which is common to all types (surface and bulk micro-machined) is packaging. It should be noted that systems coefficients can easily vary due to fluctuations in the operating conditions (temperature, pressure, etc.); it is therefore imperative to monitor their value for a good performance, and use this information in a dynamic feedback that calibrates the system for accurate sensing.

Pull-in limits the dynamic range of capacitive accelerometers operating in open- and closed-loop. As a result the sensitivity of the micro-device is very limited. Previous work demonstrated that high sensitivity can be achieved at the pull-in point (1/3 of the gap).

As we accepted the challenge, we started a parallel investigation process of low-complex digital implementation techniques, such as $FPGAs$ to cut down the cost and have more flexibility, on one side and on the other side low-complex powerful algorithms to increase the performance of the micro-accelerometer in question.

The use of digital signal processing with $FPGAs$ yields a great ability to program and modify in real-time the control loop in order to adapt to the specification of each particular micro-system to increase the performance.

Our hard work resulted in proposing new methodologies that provide more stability, robustness, sensitivity/resolution, and most-importantly low-
7.2 Thesis Contributions

complexity making the proposed schemes attractive for a VLSI implementation. Due to the robustness, stabilization, high achieved sensitivity, we strongly believe that our proposed work will pave the way for new range of closed-loop micro-accelerometers operating at the pull-in points, in particular for applications that require high sensitivity such as inertial navigation, non-invasive surgery, oil explorations and earthquake detection. The following section summarizes our main contributions.

7.2 Thesis Contributions

Our work was very multidisciplinary, it involved MEMS design and characterisation, digital signal processing and FPGA implementation, where eventually it has resulted in multidisciplinary contributions. The highlights of the thesis contributions can be summarized as follows:

- **The application of adaptive algorithms to the feedback control of MEMS-based accelerometers**
  We have proposed the application of common adaptive step-size algorithms, such as LMS and NLMS for feedback control of micro-accelerometers due to their simplicity, and ease of integration. Adaptive algorithms have been used along with MEMS at a system level integration but never as a controller for repositioning. We have studied the tuning of different parameters to optimize the best convergence speed.

- **Proposing new adaptive algorithms to the feedback control of MEMS-based accelerometers**
  With the successful application of conventional LMS and NLMS, we proposed two new algorithms, one based on Set Membership Identification, and we called it the MSMNLMS and the other is based on a fast switching between two adaptive step-sizes, one low in value that leads to slow convergence if applied solely, and the other one is high in value that leads to divergence if applied solely. The controller switches between these two values at very high frequency which leads to a stable and a robust system. This proposed SWSLMS outperforms the others in terms of speed of convergence.

- **Hardware evaluation of the proposed adaptive algorithms**
  We have evaluated the hardware complexity of these proposed algorithms using LabviewFPGA®, where we evaluated the amount of
7.2. Thesis Contributions

hardware resources each one of them occupies on a given FPGA device, in addition to maximum speed. This evaluation is important as it gives a direct information on the number of DOF that can be accommodated on a given FPGA. This hardware study shows the new proposed adaptive algorithm SWSLMS has a comparable low-complexity as the conventional LMS.

• The application of sliding mode control to the feedback control of MEMS-based accelerometer
We mentioned how ΣΔ modulators are very attractive due to their digital output. We have proposed SMC as a replacement of ΣΔ modulators for closed-loop control of micro-accelerometers for reposition the mass in order to minimize the non-linearities by keeping the proof mass at zero position after an movement due to an external acceleration exerted on the system. SMC has never been applied to accelerometers. The proposed algorithm uses controlled structure variations as part of its mechanism, and its dynamics depend on whether the state is one side of the switching surface(sliding) or on the other. It should be noted that a first order SMC achieved very satisfying results.

• Sliding mode control as a sensing scheme for measuring the exerted acceleration
We extended the usage of SMC from feedback control to measuring the acceleration exerted on the system. SMC has never been reported for sensing. The literature around SMC complains about the chattering noise from the switching propriety in the dynamics of SMC, however we took advantage of this chattering where we used it to extract the measurand (acceleration) imposed on the system. We extracted the acceleration by applying a low-pass filter to the switching dynamics of the electrostatic forces. The proposed method is very advantageous in particular where small differential capacitance are required to be measured which imposes a great deal of challenge on CMOS-based readout circuits designers.

• Application of SMC for operation at the border of stability
During our study of the capacitive-based accelerometers, and with an extensive look at the reported work in the literature, we have identified the region of maximum sensitivity and the risks involved. In contrast to the conventional work of extending the range of the capacitor beyond the pull-in distance, we proposed the SMC to allow the accelerometer to operate at the border of stability in the region known
as the metastable region, where maximum sensitivity is achieved as any small force (close to the noise floor magnitude) will attempt to alter the equilibrium. However, the strong properties of the SMC maintains the proof mass in the metastable region. Operation in that region is highly non-linear due to the pull-in phenomena, however the robustness and the stability of the SMC makes it the ideal controller candidate for environment. Such application resulted in increase in the sensitivity of the micro-accelerometer by two orders of magnitudes. SMC architecture reacts to minute variations in the external acceleration, leading to a high-sensitivity system. In summary, SMC offers a sound theoretical framework for the non-linear control of inertial sensors. Therefore perturbations with magnitude close to the mechanical noise floor level are possible to extract. This allows for low-G measurements with applications that require high sensitivity and resolution. As a result, the micro-accelerometer is sensitive to any external small perturbation with magnitude just above the mechanical noise floor, since it will tremendously affect the delicate equilibrium state. The switching characteristic of the SMC maintain the micro-device in that equilibrium while this very signal is used to extract the input acceleration exerted on the system by means of low-pass filtering.

- **Experimental verification of SMC at the border of stability**  
  With a readout circuit, estimation of the current position and velocity of the proof mass is made possible. These parameters are required by the sliding mode controller dynamics in order to make the appropriate switching decision. The system level implementation was achieved, and it contains in addition to the MEMS device and the readout circuit an FPGA module. The effectiveness of the SMC-based closed-loop system is demonstrated by maintaining the proof mass at the border of stability where the measurement of small signals is made possible.

- **The application of SMC to driving resonators**  
  We demonstrated the application of a novel band-pass SMC to the driving mode of MEMS-based resonators. It relies on binary electrostatic actuation of the movable mass, dependent on the dynamics of the sliding/switching surface, in such a way that the generated actuation tracks possible changes in the mechanical resonant frequency induced by external factors (humidity, temperature, pressure, etc.). The proposed methodology exhibit a robust tracking behaviour for large variations in the parameters of the resonator (e.g. temperature-induced
changes in the stiffness coefficient) by maintaining the actuation of the system at its resonance frequency even in the presence of variations of the stiffness coefficient (step, linear) or of applied external forces (external acceleration and noise). The proposed scheme offers an advantageous alternative to more complex PLL architectures, traditionally used for the driving of MEMS resonator structures.

- **FPGA implementation of the proposed SMC controller**
  The proposed SMC applied for feedback control and for sensing has been evaluated in terms of hardware on FPGA and was compared with an FPGA implementation of a first-order ΣΔ modulator and the hardware resources were very close. However as mentioned before, most ΣΔ modulators operate at high orders, therefore the hardware complexity would increase. We had satisfying performance with a first-order SMC. The proposed SMC for driving resonators benefits from the implementation in hardware of an adaptive behavioural reference model used by the controller, able to track the sensed changes in dynamics. The reference oscillator is implemented in FPGA, with an oscillating frequency tuned by the sensed dynamics of the MEMS device.

- **SNR performance comparison between SMC and ΣΔ for the same order**
  For the first time, the SMC is being evaluated in terms of SNR as the switching frequency between the left and right electrostatic forces is increased gradually. As mentioned earlier, we considered a first order SMC. The performance has been compared to a first order ΣΔ [15] and it shows an improvement in the SNR by 3dB.

- **Adaptive self-testing and self-calibration with PN sequences**
  We proposed a novel adaptive technique based on pseudo-random PN sequences for self-calibration and self-testing of MEMS-based inertial sensors (accelerometers and gyroscopes). The advantages include reduced fabrication cost and increased reliability, safety, particularly in applications that require continuous monitoring, such as airbag deployment. In addition the proposed methodology will allow for maintaining high accuracy in demanding applications, such as adaptive path prediction systems, recalibrating the mechanical performance when the equivalent stiffness and/or damping coefficients are affected by changes in the operating environment (temperature, humidity and pressure).

- **FPGA implementation of the adaptive-based self-testing and**
7.3. Future Developments

**self-calibration**
Similarly to the SMC approach for driving resonators, the core of the proposed implementation is the implementation on FPGA of a replica (DUT-R) of the *behavioral* model of the device under test (DUT), whose parameters values are adaptively tuned, based on the response to pseudo-random actuation of the physical structure. The hardware resources are relatively low and can be accommodated on a cheap FPGA device.

---

### 7.3 Future Developments

Despite the intensive work we carried in new designs, new proposed adaptive and non-linear techniques for closed-loop control and self-testing and self-calibration of MEMS-based inertial sensors, the milestone is far from complete. A lot of work still need to be carried out on several sides. Some considerations for improvements can be summarized as follows:

- From the MEMS-based accelerometer design perspective, more care need to be taken in regards to the rules of the MEMS foundry, such as minimal gap, dimensions, Silicon ratio, etc. However sometimes we found ourselves cornered between the MEMS foundry rules and specification and the requested performance of the device in question. Incorporating several devices on the same die adds complexity in particular when specific tasks with a certain geometrical distribution are required such as measurements is ($\theta_1$) and ($\theta_2$). Besides that, targeting a fabrication process with a relatively thick seismic mass, would be extremely beneficial as this would span the sensitivity range of the device before extra increase with closed-loop techniques.

- From the signal-processing point of view, with respect to adaptive-based control, future work includes more tuning of the proposed techniques where accelerometers are subject to variations in their parameters such as damping and stiffness coefficients. Such improvements will provide a more solid framework for the application of adaptive step-size algorithms to the feedback control of accelerometers.

- We demonstrated the application of SMC for repositioning the proof mass to the rest position, and driving the proof mass to a certain predetermined position, in our case, it was the pull-in position. The difference lies in the the chosen reference trajectory, the values for
7.3. Future Developments

\{\lambda, \rho\} and the type of comb drives used for feedback control. Future work include an adaptive SMC where the input parameters adaptively change based on the desired mode of operation.

- In regards to the proposed SMC, for closed-loop control, driving mode of resonators and sensing scheme to measure acceleration, we anticipate that further investigations about the effect of increasing the order of the SMC should be done for the aforementioned applications. An interesting future work is to provide a solid mathematical framework to SMC similarly to \(\Sigma\Delta\) in regards to noise shaping due to oversampling rates (in the case of synchronous SMC), order and filtering.

- In the area of BIST and BISC a future challenge includes performing real-time self-calibration and self-tuning, when the test actuation with PN sequences superimposes over the natural mechanical actuation given by an external acceleration or angular rate. Such iterative process, will be performed regularly, in order to adapt the reference model parameters and the related calibration values a maximum accuracy is maintained to the user side in such a manner.

- For all the proposed methodologies, we accompanied them with a hardware evaluation on FPGA, to see the possibility of accommodating several DOFs. The simplicity of the proposed techniques and their inherent adaptive and non-linear nature means that they are also suitable for compact ASIC implementations in low-power micro-systems. An interesting future work is to develop an ASIC implementation for a proposed methodology (adaptive or non-linear) once all tuning is finalized, in different testing environments.
Bibliography


Bibliography


Bibliography


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Bibliography


Appendix A

Used Fabrication Processes

A.1 SOI-MUMPs Design Process

SOI-MUMPs® is a Silicon on insulator (SOI) micro-machining process with a simple 4-mask levels. The fabrication steps can be summarized as follows (Figure [A.1] [84]):

A1. An SOI wafer is used as the starting substrate. The wafer has three layers, Silicon (10 ± 1 µm micrometer or 25 ± 1 µm), an oxide layer with thickness of 1 ± 0.05 µm, deposited on a handle wafer (substrate) with a thickness of 400 ± 5 µm

A2. The Silicon layer is doped and patterned. It is etched all the way down to the oxide layer, with deep reactive ion etching (DRIE). This step is used to define resistor and mechanical structures, and/or electrical routing

A3. A front side protection material is applied to the top surface of the patterned Silicon layer. Reactive ion etching (RIE) is used to etch the bottom oxide layer, while DRIE is used to etch the substrate; this can be patterned and etched from the bottom side up to the oxide layer, which is later removed by a wet oxide etch process (only regions defined by the trench mask) and this results in through-hole structures

A4. Then a shadow-masked metal process is used to provide coarse metal features such as bond pads

A5. Last a second pad-metal feature, limited to areas not etched on he Silicon device layer, is applied, and results in finer metal features and more precision alignment

A.2 MEMS-SOI Design Process

The MEMS-SOI [29] technology is proposed on a Multi-Project Wafer (MPW) service as part of the Europractice program. The fabrication process can be
A.2. MEMS-SOI Design Process

Figure A.1: Design flow of SOI-MUMPs fabrication process [84]
A.2. MECS-SOI Design Process

summarized as follows (Figure A.2) [29]:

First the process is started with the fabrication of an SOI wafer [29]

B1. A bonded Silicon on insulator (BSOI) wafer, $P$ typed doped, $10 - 20mOhm.cm$, $60\mu m/2\mu m450\mu m$ is used followed by back etching of alignment marks

B2. The second step is metallization deposit and etch a metallic layer

B3. Then SOI patterning is achieved via Silicon deep dry etching with $SiO_2 SOI$ as the etch stop layer

B4. Then the sacrificial layer is etched with $HF$; a process of rinsing and drying is followed

The second step in the process is the $CAP$ wafer fabrication, which can be summarized as follows:

C1. A Silicon wafer, $P$ typed doped, $10 - 20mOhm.cm$, $300\mu m$ is used as starting point, followed by back etching of alignment marks

C2. The Silicon wafer suffers a thermal oxidation of $2.6\mu m$

C3. The next step is the patterning of the cavity, which is the area where the sealing will take place

C4. Then the oxide layer is back etched followed by metal $CAP$ deposition and patterning

C5. The $CAP$ wafer fabrication process is finalized with METAL2 deposition and patterning

The third and final step is the bonding of the (B.) and (C.) wafers, which can be summarized as follows:

D1. First the $CAP$ and the active SOI wafers are assembled together

D2. Finally etching of the via holes is followed, two masks ($100\mu m$ then $200\mu m$ steps) are used to etch through the $CAP$ wafer
A.2. MEMS-SOI Design Process

Figure A.2: Design flow of MEMS-SOI fabrication process [29]
Appendix B

Sigma-Delta $\Sigma\Delta$ Operating Principles

B.1 Introduction to $\Sigma\Delta$

We have thoroughly discussed in the second chapter of this thesis, the advantages and disadvantages of using $\Sigma\Delta$ modulators for closed-loop accelerometers. We noted that high orders $\Sigma\Delta$ modulators push the noise outside the baseband, and achieve good performance in acceleration measurement Dong et al. [34-36]. As a result, the higher order of $\Sigma\Delta$ feedback achieves much better SNR comparing to a low order modulator, and a better motion cancellation.

Despite the advantages of $\Sigma\Delta$ modulators (noise shaping, digital output, SNR), their design and analysis is based on extensive simulations or linearisation schemes of an intrinsically non-linear architecture.

High-performance systems require higher-order $\Sigma\Delta$ modulators, where the stability problems are dealt with mainly through time-consuming numerical simulations and optimizations cycles [113]; as a result, their design, which does not rely on a non-linear theory from first principles, becomes difficult.

In this appendix, we will briefly cover the principle of operation of $\Sigma\Delta$ modulators, in terms of analytical modelling, system level architecture, and SNR.

B.2 Definition of $\Sigma\Delta$ Modulators

A $\Sigma\Delta$ modulator is an interesting technique combining two approaches; a filtering approach where the information is hidden, needs to be extracted and an oversampling approach with an analogue to digital conversion. Generally, a $\Sigma\Delta$ system consists of the $\Sigma\Delta$ modulator which produces the bitstream and of the low-pass filter [15] as shown in Figure B.1. The bitstream is a one-bit signal with a bit-rate much higher than of the analogue-to-digital (ADC)
B.3. Architecture of $\Sigma\Delta$ Modulators

The major property of the bitstream is that its average level represents the average input signal level \cite{15}. For high resolution (greater than 12 bits) converters, $\Sigma\Delta$ modulation based ADC conversion technology is described as cost effective as it can be integrated on DSPs \cite{92}.

![Block diagram of a $\Sigma\Delta$ converter system](image1)

**Figure B.1:** Block diagram of a $\Sigma\Delta$ converter system

**B.3 Architecture of $\Sigma\Delta$ Modulators**

![Block diagram of a first order $\Sigma\Delta$ modulator](image2)

**Figure B.2:** Block diagram of a first order $\Sigma\Delta$ modulator

Figure B.2 depicts the system architecture of a $\Sigma\Delta$ modulator. It comprises a loop filter or loop transfer function $H(s)$, a clocked quantizer and a feedback digital-to-analogue converter (DAC). The output signal is a bitstream. The average level of this bitstream represents the input signal level \cite{15}.

From Figure B.2 the signal transfer function can be written as \cite{92}:

\[
Y(s) = \frac{X(s) - Y(s)}{s} \tag{B.1}
\]
where $X(s)$ is the input signal and $Y(s)$ is the output signal. It should be noted that in this case we consider that the quantised noise is negligible.

at the output of the low pass filter (Figure B.1) Eq B.1 can be written as [92]:

\[ \frac{Y(s)}{X(s)} = \frac{1}{\frac{1}{s} + \frac{1}{s}} = \frac{1}{s + 1} \]  

(B.2)

The comparator basically decides whether its input value is higher or lower than a certain threshold and puts out a single bit signal, the bitstream [15]. The low-pass filter at the output is required in order to gain the average signal level out of the bitstream signal [15]. Decimation takes place in ΣΔ modulators at the output of low-pass filter [15], this process can be used to provide increased resolution [92].

\section*{B.4 SNR}

Conventional converters require a sample rate of more than twice the highest input frequency. However ΣΔ converters require much more in order to produce a sufficient number of bitstream pulses. As a result the oversampling ratio (OSR) is much higher than two. For better approximation of the input signal by the average bitstream [15] more bitstream pulses are required. It should be noted that the average (low pass filtered) bitstream never exactly represents the input signal. It is always superimposed by some kind of noise [15]. To reduce this noise an increase in the clock rate is required [15]. The simple tutorial introduced by Maxim Integrated Inc © 57] shows that for a first order ΣΔ there is a 9dB per octave (doubling of the oversampling ratio), and 15dB per octave for a second order ΣΔ.

Due to the sampling theorem the sampling rate must be higher than twice the maximum input frequency. Any further increase is called "oversampling rate".