THREE-DIMENSIONAL MODELING OF METAL PLATE CONNECTED WOOD TRUSS JOINTS

by

Xiaoqin Liu

B. A. Sc., University of Electronic Science and Technology of China, China, 1999
M. A. Sc., Chongqing University, China, 2003

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

The Faculty of Graduate Studies

(Forestry)

THE UNIVERSITY OF BRITISH COLUMBIA
(Vancouver)

June 2013

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Abstract

This thesis presents theoretical and experimental studies of metal-plate-connected (MPC) wood truss joints under uni-directional tension or out-of-plane bending.

A theoretical computer program, SAMPC, was developed based on finite element method (FEM). MPC joint models were constructed using SAMPC, to evaluate the three-dimensional nonlinear performance of the joints.

Experimental studies were carried out on MPC truss joints under tension. The joint failure modes were discussed, and the potential reasons for the failure were explored. Data processing techniques were applied to obtain the specific load-displacement relationships, which were in turn used as reference for model calibration and verification. Based on the experimental results, optimized model parameter calibration and model verification were discussed.

The program application of MPC joints subjected to out-of-plane bending was investigated. Comparisons of the results from the joint bending test and model verified the applicability of the program for evaluating the out-of-plane rotational stiffness of MPC joints.

A reliability analysis was conducted to evaluate the critical buckling load and lateral bracing force of single- and double-braced wood truss web systems. The probability characteristics of a number of variables that affect the performance of braced truss web system were investigated. Based on the results, a factor relating the ratio of the
lateral restraining force and axial load was established. This factor with adequate reliability was recommended as a web/bracing design amendment to Canadian Code on Engineering Design of Wood.

For the investigated truss joints, SAMPC appears to be superior in terms of its ability to simulate MPC joints in elaborate detail. This detailed model can aid in developing a better understanding of joint behavior under realistic joint configurations and loading conditions. The ability of the model to accurately predict the behavior of the designed MPC joints brings up the potential of modeling joints composed of different wood species and truss plate types featuring more complex joint configurations and loading conditions. The body of information from modeling results can be used to evaluate the adequacy of a given structural design, to facilitate truss plate, truss joint and overall truss design.
Preface

This thesis focuses on the development and verification of a computer program to simulate the MPC wood truss joints under different loading conditions (e.g. tension or out-of-plane bending).

The theoretical program, SAMPC, was compiled by me, following the FEM concepts and techniques described in Chapter 3. For all the other three Chapters of 4, 5 and 6, I conducted the experimental studies, constructed MPC models using the SAMPC, and compared the model-derived results with the database collected from the corresponding experiments. Under the guidance of Professor Lam, I undertook a reliability analysis of MPC truss bracing system, and wrote the Chapter 6.
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Chapter 1. Introduction

1.1. Light frame metal-plate-connected wood truss connection

Ever since the prefabricated metal-plate-connected (MPC) wood truss system was first introduced in the 1950s as a replacement for the nailed plywood gusset board truss system, it has rapidly gained popularity in North America in residential, industrial and low-rise commercial buildings due to its versatility, cost effectiveness and ease of installation.

In MPC wood trusses, dimension lumber with thicknesses of 38 mm are connected by specially designed metal plate connectors, also called truss plates, to form a truss system. Generally in North America, metal plates are made of galvanized steel sheets of 16-, 18-, or 20-gauge, measuring approximately 1.6mm (0.062 in.), 1.27mm (0.05 in.), and 0.9mm (0.038 in.) in thickness, respectively, and featuring die-punched teeth in a regular pattern across the plate. Truss joints are often considered one of the critical links of a truss system, because if they have less strength or stiffness than the wood members, failure in the joints can induce the failure of the whole truss system. Therefore, the strength and stiffness properties of truss joints are extremely important in truss design.

However, the behavior of MPC truss joints is highly complex and influenced by several variables, which makes it difficult to determine the behavior of connections through either testing or modeling. Quaile and Keenan (1979) listed the following factors that may affect the strength properties of connections: orientation of plate and wood, end
and edge distance, size and properties of plate, joint geometry, method of installation, species and specific gravity of wood, wood moisture content, sampling, and elapsed time between assembly and testing. Moreover, various joint configurations and loading conditions further complicate the theoretical study of MPC joint performance.

Over the past six decades, extensive research has been conducted to experimentally study and analytically model the behavior of MPC truss joints to facilitate the MPC joint design or truss system design. Foschi (1977) developed an effective modeling approach, in the form of a computer program called SAT (Structural Analysis of Trusses), which could account for the behavior of MPC connections. Triche (1984, 1988) refined Foschi’s approach such that the results could be used in truss system analysis (PPSA). Vatovec et al. (1995) used a commercially available finite element program to predict behavior of different types of MPC joints under in-service loading conditions, assuming the tooth-to-wood interaction to occur at one, non-dimensional point. Most of these models were developed on two-dimensional analog, which is based on simplified, but reasonable assumptions regarding connection behavior, that have proven to be adequate for truss applications. However, with the increased use of MPC trusses in applications involving longer spans with fewer redundancies, a more thorough understanding is required of the behavior of metal plate connections, particularly if design procedures are to be upgraded. For example, a more realistic three-dimensional (3D) model capable of accounting for the structural characteristics of joints in terms of strength capacity and stiffness. The high computational cost for 3D analysis was often cited as the reason for not employing the more sophisticated method. However, with
contemporary computer technology, the development of a sophisticated, detailed 3D modeling system based on a thorough understanding of MPC joints is now feasible.

Load transfer and resulting deformations within MPC joints are highly complex due to the composite nature of the system. Truss plates transferring loads from one wood member through the plate to another member depend primarily on the gripping capacity of the teeth and the shear and tensile capacity of the metal plate. Thus, characterizing the load-slip relationship of each tooth with respect to wood bearing is of significance in determining the stiffness of MPC joints, which partially affects the overall performance of the truss system. Under a load, a single tooth will deform laterally relative to the lateral forces on the wood-bearing interfaces, and tangentially associated with the withdrawal forces. A 3D tooth model was implemented to account for the force-carrying mechanism of each tooth. In doing so, nonlinearity is introduced into the MPC joint model by the load-slip relationship and withdrawal action of the teeth. Thus, these characteristics, and the way they are incorporated in the joint model, constitute its most important components.

Depending on the joint configuration and loading condition, metal plate connectors can fail in different modes, including tooth withdrawal from the wood, wood failure within the plated region, compression buckling of the plate in the gaps between wood members, and plate yielding in tension or shear. Different failure modes affect the strength and stiffness of MPC joints differently. For instance, as a tension splice joint is loaded solely in tension, the lateral load transfer between wood members is dependent mainly on the gripping of the teeth. Thus when teeth peel out from the wood, the stiffness of the joints will decrease, and tooth withdrawal failure criteria will determine the
strength of the joint. Moreover, when a metal plate connector is subjected to eccentric loading, the gap between wood members will close on the compression side, resulting in a potentially significant increase in joint stiffness. Therefore, to gain a full picture of the performance of MPC joints, one must include failure models in the connection modeling. As the major objective of this thesis, a failure model is developed, which is capable of interpreting the tooth withdrawal mechanism with the aid of the 3D tooth model.

This thesis focuses on the fundamental understanding of the performance of MPC truss connection, and aims at the development of a theoretical model which can more accurately predict the strength and stiffness of truss connections. The theory of the model is presented, and its application to simulate web-to-chord truss joints subjected to out-of-plane bending is shown and discussed. In addition, reliability studies of lateral bracing forces in MPC wood trusses are presented.

1.2. Objectives

This thesis focuses on an advanced 3D finite-element-method (FEM) based model that can adequately represent MPC wood truss joints and their behaviour in unidirectional tension and out-of-plane bending. This kind of numerical model can provide comprehensive information on the stiffness, deformation and load-carrying capacity of MPC joints. The specific objectives of the thesis were:

(1) To develop a theoretical model that can simulate MPC joints loaded in tension, and investigate the load transfer mechanism at individual tooth level, thus providing a numerical approach to predicting the strength and stiffness of MPC joints.
(2) To experimentally study the performance of specially designed MPC joints subjected to tension. The experimental results can be used to calibrate the model parameters and validate the model.

(3) To demonstrate the feasibility of the analytical model by predicting the axial strength and stiffness of MPC joints of different force-plate-grain orientations, with a limited amount of information obtained from the basic joint tests.

(4) To demonstrate the feasibility of the analytical model by evaluating the out-of-plane rotational stiffness of MPC joints. Based on the results, reliability analyses of the lateral bracing force in MPC wood trusses will be conducted.

This model incorporates the major variables that influence the performance of MPC joints: wood and steel properties, the load-slip relationship between plate tooth and wood members, which is dependent on plate-to-grain orientation, withdrawal criteria, withdrawal failure criteria, wood-to-wood contact, etc.

1.3. Organization of this thesis

The thesis was laid out in 6 distinct chapters: 1) background and a literature review; 2) formulation of the analytical model; 3) experimental studies of MPC joints; 4) model parameter calibration and model verification; 5) reliability analyses of lateral truss bracings; and 6) conclusions and recommendations.
Chapter 2. Literature Review

Introduction

This chapter reviews early research on the testing and modeling of MPC wood truss joints. Since MPC joint design is critical to truss design, a large amount of research has been undertaken to understand the structural behavior of MPC joints experimentally and theoretically. Based on the reviewed experimental studies, a solid basis has been established for explorations in this thesis. Moreover, the theoretical studies reviewed have been referenced to form the FEM based computer model.

2.1. Experimental studies

Conventional procedures for analyzing and designing of MPC trusses are based on the results obtained from experimental tests. Several researchers have recommended experimental testing of actual joints as a way to identify their failure modes and determine their stiffness and strength properties.

Lau (1986) studied the strength and stiffness properties of joints resembling the heel of a truss. Heel joint tests were conducted on four different truss designs and two plate sizes. Three failure modes were identified: wood splitting as the teeth withdrew at the wood-plate interface, tooth withdrawal, and plate buckling. Lau concluded that the failure modes and stiffness of the heel joints were affected by the joint configurations and the orientation of the plates. For example, the joints with the plate major axis lying parallel to the member in compression failed mostly due to plate buckling; the joints with the major axis lying parallel to the member in tension failed mostly due to tooth
withdrawal associated with wood splitting. Meanwhile the size effect of the plate length on the ultimate strength and stiffness of the joints were found to depend on the mode of failure. For example, an increase in the plate length significantly affected the joints that failed in tooth withdrawal, but had no significant effect on those that failed in plate buckling.

Wolfe (1990) performed several experiments to evaluate the load capacity of tension -splice joint connections that were subjected to five loading conditions. Test specimens of 38- by 89-mm southern pine lumber and 76- by 133-mm 20-gauge plates were tested under pure axial tension, pure bending, and at three levels of combined tension and bending. Joints were tested to failure, and their axial and/or moment capacity was determined. The results showed that the axial load capacity of the MPC tested was reduced significantly by the presence of a bending moment in conjunction with axial tension.

Gupta and Gebremedhim (1990) conducted destructive tests of actual truss joints to determine the stiffness and strength properties. Over 150 experiments were conducted on tension-splice joints, heel joints, and web at bottom chord joints. The research concluded that the most common mode of failure was tooth withdrawal associated with splitting in wood members. In another study, Gebremedhim et al. (1992) tested 21 truss joints in pure tension and 15 joints in shear. The tests included the four basic plate-wood grain configurations as specified by the CSA (1980), as well as other tests at intermediate orientations of 30°, 45° and 60°. Tooth withdrawal and wood failure in splitting were most prevalent in the tension tests, and only tooth withdrawal was observed in the 0°, 30°
and 45° tests. The stiffness values were calculated from the slope of the load-deflection curves.

Gupta et al. (1992) tested tension-splice joints under six different loading conditions: pure axial tension, pure bending, and four different levels of combined (axial tension/bending) loading. All joints were fabricated using 38- by 89-mm No. 2 KD19 southern yellow pine lumber and 20-gauge truss plates. Combined loading tests showed that the axial load capacity of joints declined as the applied bending moment increased. Most of the joint specimens were tested to failure in tooth withdrawal, which indicates that tooth-grip capacity might govern the strength of the joint.

Stahl et al. (1994) carried out a series of tests to determine the feasibility of fabricating light-frame trusses using square-cut commodity webs, which can leave a significant gap between wood members. Due to the gap, compressive forces cannot be transferred by wood-to-wood bearing; at the same time there exists potential for plate buckling. Moreover, under a high shear force, web-to-chord connections would become distorted with one web in tension and the other in compression. A tentative test method was proposed for evaluating the strength and stiffness of square-end web-to-chord joints under concentric as well as eccentric loading, with respect to different plate-grain orientations and gap widths. The experimental results proved that the test method was applicable.

O’Regan et al. (1998) investigated the different failure modes tension-splice joints. The major mode of failure proved to be tooth withdrawal. Failure in the net cross section of the steel plate was also observed. This included yielding and rupturing of the steel due
to tension and bending occurring at the splice joints. According to this study, it appears that plate length plays a significant role in the failure mode of MPC connections. Based on his study, O’Regan developed a design procedure for determining truss-plate length for tension-splice joints; in addition he concluded that the minimum plate length required to prevent tooth withdrawal should at least be equal to or greater than the width of the truss plate.

Experimental research has shown tooth withdrawal to be one of the dominant failure modes for joints in tension, which indicates that tooth-grip capacity with respect to different force-plate-grain orientations may govern the strength and stiffness of the joint. Moreover, the configuration of joints and the size and tooth layout of the steel plates was found to played a major role in affecting the strength and stiffness of truss joints as well as their failure mode.

### 2.2. Theoretical studies

MPC joint modeling is highly complex because of the composite nature of joint systems. Researchers have proposed a variety of models to predict the load-slip behavior of MPC joints. In general, these models can be divided into three categories: Foschi’s tooth load-slip model, the beam-foundation model, and the fictitious-element model.

#### 2.2.1. Foschi’s model

Foschi (1977) was the first to use a theoretical model to represent the nonlinear load-slip relationship of connectors. At the same time, he developed the computer program SADT to analyze the behaviour of metal-plated trusses. A great deal of the
theoretical models subsequently developed to represent the nonlinear behavior of MPC joints were based on Foschi’s model.

In Foschi’s two-dimensional (2D) model, it assumed that within the plate-wood contact area, the wood and the metal plate remain infinitely rigid. Under this assumption, the relative displacement between plate and wood attributed solely to the deformation of the tooth-wood interaction. A nonlinear load-slip relationship in the form of a 3-parameter exponential was applied to represent the interaction between wood and tooth:

\[ P(\Delta) = (Q_0 + Q_1\Delta)(1.0 - \exp(-K\Delta/Q_0)) \]  

(2.1)

![Figure 2.1 The 3-parameter model for the load-slip relationship](image)

where \( P \) is the load, and \( \Delta \) is the displacement of the tooth. \( K \) is the initial or linear stiffness of the curve, \( Q_1 \) is the stiffness at a large displacement, and \( Q_0 \) the intercept of the asymptote with slope \( Q_1 \).

Employing Foschi’s model required determining the parameters for a given nonlinear load-slip curve on a per tooth basis. By assuming that all the teeth were evenly loaded over the plate, per tooth load-slip data could be obtained by dividing the load by
the total number of teeth engaged. The three model parameters could then be determined by model calibration.

In the SADT, Hankinson’s formula (1921) was used to obtain the parameters for the case of the arbitrary angles between loading direction, plate major axis with respect to grain orientation. This formula has been traditionally used in timber engineering to represent the dependence of mechanical properties on grain orientation.

The plate buckling, plate tensile strength and shear strength were also taken into account in Foschi’s model. The part of the plate over the gap between the two wood members was considered as a series of short columns between openings or slots. During the deformation, these columns might be in tension or compression, due to eccentric loading over the plates. If connected wood members moved relatively in the direction perpendicular to the plate’s major axis, the columns would undergo shear deformation. To form the finite element programming, the axial and shear stiffness were derived based on the properties of the steel plate sheet.

Lau (1986) conducted a series of tests on shear joints and heel joints to evaluate the accuracy of the SADT. It was found that while SADT could provide a reasonably good prediction of joint stiffness at low values, but that the ultimate load predicted for different failure modes was either much lower or higher than the test data. Lau concluded that SADT was unable to take into account possible failure modes and the friction effect on joints.

McCarthy and Wolfe (1987) conducted tests on MPC joints fabricated from 38-by 89-mm southern pine lumber and 76 mm by 127 mm 20 gauge truss plate. The joints
were assigned to one of three MOE categories. Foschi’s truss joint model was used to predict the lateral resistance of these MPC joints. The parameter values were obtained by fitting the test data for each of the four standard joint configurations (CSA S347-M1980 standard) with the connector load-slip curve for each joint type. Two non-standard configurations (with loads applied at 30° and 60° angles to the grain) were tested to evaluate the accuracy of the transformation methods (Hankinson’s interpolation). The best prediction of intermediate angle performance was obtained using Hankinson’s interpolation. This method showed that significant variations in the parameter values obtained could be due to the methods applied to fit the test data to Foschi’s model. Overall, however, a very close agreement was obtained between the experimental results and theoretical predictions.

In addition, factorial analyses were performed to assess the sensitivity of model parameters to the effects of MOE (McCarthy and Wolfe, 1987) and to the parameter determination methods (McCarthy and Little, 1988). The results from these analyses suggested that the lumber MOE did not appear to have a significant effect on joint performance, and that the parameters (K, Q₀, and Q₁) in the tooth load-slip model were sensitive to determination methods. Therefore, the researchers recommended standardizing the curve-fitting procedure with respect to both the regression fitting methods and the methods for taking into account the asymmetric joint failure.

Triche and Suddarth (1988) employed the mathematical load-slip relationship in Equation (2.1) to evaluate the relative displacement of the teeth; the per-tooth strength value was evaluated at the engineering design level. Refining and elaborating upon
Foschi’s nonlinear finite element program, Triche developed a more practical and dependable computer program called PPSA Foschi-Triche (PPSAFT) based on both SADT (1977) and PPSA (Purdue Plane Structures Analyzer, 1984). The program PPSAFT adopted the force and displacement on each tooth (per tooth design value) at one-third of the ultimate strength. To address the load-plate-grain orientation concern, the results from four basic CSA S347 MPC joints tests were modified to take into account grain angle and tooth considerations by an interpolation procedure (Hankinson’s). Good agreement between the model predictions and experimental results was reported.

These studies were testimonies to the efficacy of the Foschi’s model in predicting the strength and stiffness of MPC joints. Curve fitting the model predictions against MPC joint test results, demonstrated that the modeling parameters for lumber and truss plates were associated with particular material properties. The good prediction of intermediate angle MPC joints by Foschi’s model justified the use of Hankinson’s interpolation method over the modeling parameters. However, the assumptions, upon which the model was based, made it impossible to compute the stress and deformation within the connection itself. For example, the assumption of the rigid plate and wood member which neglected the elasticity and deformation of the material, and the assumption of an averaged per tooth strength value which was not consistent with the reality. Thus, although Foschi’s model is valuable from a truss-analysis perspective, its basic assumptions limit its usefulness for the detailed analysis of MPC joints. As a result, our understanding of force distribution within connections is still incomplete.
2.2.2. Beam-foundation model

Crovella and Gebremedhin (1990) developed a 2D model representing concentrically-loaded tension-splice joints by assuming that an embedded tooth behaved as a cantilever beam supported by an elastic wood foundation (Figure 2.2).

![Figure 2.2 Beam-foundation model](image)

As shown in Figure 2.2, the tensile force is transferred to each tooth from the plate as a point load at the tooth base (where the tooth attaches to the plate). The reaction force is proportional to the corresponding deflection at every point along the beam. To obtain the deformation at the base of a tooth beam, the corresponding governing differential equation of bending needed to be solved with respect to appropriately chosen boundary conditions. The form of the differential equation for an elastic foundation is:

\[
EI \frac{d^4y}{dx^4} = -ky
\]

(2.2)

where \( y \) represents the lengthwise deflection of the beam, which is assumed to have the form \( y = e^{mx} \). \( E \) and \( I \) are the MOE and the moment of inertia of the beam, respectively, and \( k \) the foundation modulus of the wood surrounding the beam.
Two possible boundary conditions were considered. In each case, the end of the beam (tooth tip) away from the plate surface was considered to be free to translate and rotate. The other end of the beam, which was the base of the tooth, was assumed to have an unknown rotational stiffness. To determine the range of rotational stiffness, this base end was considered to be either 1) rigidly attached to the plate surface, thus allowing no rotation; or 2) pinned to the plate surface so that it is free to rotate. By applying these two conditions to the analysis, the rotational rigidity of the beam for the two cases could be determined respectively.

Modeling the beam-foundation required making further assumptions, such as a constant width along the beam length, and a constant foundation modulus as a fraction of the wood MOE (Wilkinson, 1971).

In contrast to Foshi’s model, the tensile force distribution among the tooth array was non-uniform over the connected area. The load value carried by the first tooth row (farthest from the centerline) was used as a benchmark to evaluate the loads carried by other columns of teeth in form of a ratio. Furthermore, the steel plate was not infinitely rigid (contrary to Foschi’s model). By summing up the deflection and load carried by the teeth, the elongation and force on the plate surface respectively, the total deformation \( \Delta \) and force \( P \) on the joint were determined. The stiffness of the joint \( K \) could then be calculated using the equation \( K = P/\Delta \).

A series of simple tests on tension-splice joints were conducted to verify the model. A good agreement was found between the predicted and experimental results when a semi-rigid connection at the tooth base was assumed. Sensitivity analyses were
performed to evaluate the effects of the above assumptions on stiffness. It was reported that the stiffness of joints was significantly dependent on the foundation modulus of wood.

Based on beam-elastic-foundation assumption, a number of studies (Crovella and Gebremedhin, 1990 and 1991, Gebremedhin et al., 1992) were carried out on the force distribution along the tooth array. MPC joints (mainly tension-splice joints) were studied experimentally and theoretically, using different plate types that differed with respect to plate geometry and tooth layout. It was concluded that the force distribution was not uniform among any of the plate types studied, but rather depended on the relative position of the teeth on the plate. In the case of a uniform tooth layout, for example, the row of teeth next to the centerline of the connector plate carried a greater load than the row of teeth farthest from the centerline. Thus it was recommended that the tooth layout be optimized so as to distribute the force uniformly, which could be done by changing the tooth number of selected section.

Groom and Polensek (1992) proposed another theoretical model based on beam-foundation theory; however, in this case the foundation modulus was attained from an embedment test in which a single tooth was driven into wood. The embedment test produced a nonlinear load-embedment curve; the foundation modulus was taken as the slope of the curve at any point. Inelastic interactions between the tooth and wood foundation, along with the changing moment of inertia accounting for changing dimensions along the tooth length were considered in the model. The study also considered the loading and plate direction with respect to grain orientation. The model predicted the load-displacement curves as well as the ultimate load of the MPC joints.
Riley and Gebremedhin (1999) developed 2D models that predicted axial and rotational stiffness values for both tension-splice joints and heel joints, based on the concept of beam-elastic-foundation. This study provided valuable insights into the performance of these two types of joints.

2.2.3. Equivalent spring or fictitious element

Sasaki and Takemura (1988) developed a model by replacing MPC joints with a set of three linear elastic springs, which represented axial, shear and rotational stiffness characteristics respectively. Replacing the semi-rigid MPC connection at the member end with springs allowed enabled a matrix analysis to be performed for the whole truss system. Full-scale load tests of MPC wood trusses were conducted, and the predicted values compared with the experimental results.

Cramer et al. (1990) developed a 2D finite-element model for testing the performance of tension-splice joints. The tooth-wood interface was simulated, by using a spring element to represent each tooth possessing two degrees of freedom with respect to the displacement occurring either in the plate’s primary axis or in the plate’s secondary axis. In 1993, Cramer et al. developed a more efficient method for computing the stiffness of MPC joints to account for in-plane bending due to joint eccentricity and nonlinear semi-rigid joint performance. This method also included an automated procedure to compute the geometrical characteristics of each plate-wood contact surface. The theoretical approach involved modeling each plate-wood connection as a single element with a set of three springs (two translational and one rotational) located at the center of gravity of each plate-wood contact area. The springs were connected to the
wood element, which was idealized as a frame member lying along the wood-member centerline, and to the plate element through rigid links. The model was semi-analytical because in that the analysis required the stiffness characteristics representing the contact area to be pre-computed from the individual tooth load-slip characteristics that were obtained from testing.

Gupta and Gebremedhin (1990) incorporated predetermined axial and rotational stiffness properties of truss joints by modifying the fixed-end forces and the element stiffness matrices of the truss members. In this approach, if any of the truss members were to become overstressed, it would not be obvious whether the problem had been initiated in the joint or in the member as the member stiffness values were modified by the stiffness of the joints.

Vatovec et al. (1995, 1996) developed a 3D model of the MPC tension-splice joints using the commercial software, ANSYS. In this research, the wood-to-tooth interaction, and gaps between wood members were taken into account. To simulate the wood-to-tooth interaction, each tooth of the metal plate was represented by three nonlinear spring elements at a single point. These three spring elements accounted for the stiffness in the three plate’s major directions respectively: parallel and perpendicular to the slots in the plane of the plate, and perpendicular to the plane of the plate. For each spring element, the nonlinear load-displacement relationship (stiffness) had been back-calculated based on tension-splice joint tests conducted on a per tooth basis. Contact elements were used to simulate the interaction of the two wood members while the joint was loaded in compression or bending. The model represented the axial load-deflection relationship relatively well. However, the rotational response of the model was poor due
to a lack of experimental data. It was also reported that the model was insensitive to changes in the modulus of elasticity of the wood and the steel, as well to whether or not the holes in the slots had been included.

Amanuel et al. (2000) developed a 2D finite element model of a tension-splice MPC joint by treating the wood-to-tooth interface as nonlinear contact elements. These contact elements resemble spring elements that transfer compressive and frictional forces between the wood and teeth as a joint is loaded. This model was able to predict the mechanics of tooth withdrawal and the axial stiffness of joints.

2.3. Conclusion

In this chapter, the literature regarding the modeling and testing of MPC joints was reviewed and some key issues were discussed. A review of both experimental and theoretical studies revealed that most of the early work had focused on the testing and modeling of tensile connections under axial loads to determine their strength and stiffness. The behavior of tensile connections was found to be relatively less complicated than that of other kinds of connections, such as heel joints, peak joints, web joints, etc. Another reason was that the investigation of the simpler tensile connection may serve to guide the study of more complex connections, thus enhancing our understanding of MPC connection as a whole.

As noted above, 2D modeling of MPC joints has received a great deal of attention. These joints were either simplified as spring elements acting at a single point on the plate, to represent the behaviour of a single tooth behavior; alternatively, a beam-on-foundation model was used with assumptions and simplifications such that the response of MPC
joints can’t be fully accurate. All these models somewhat overlooked or underestimated the tooth withdrawal behavior, which affects the strength and stiffness of MPC truss joints. Because of the complexity of the interaction between the tooth and wood medium in terms of the lateral wood bearing and interface friction, it is hard to effectively and computational efficiently realize this interaction in an existing commercial finite element programs, such as ANSYS, ABACUS etc. A sophisticated FE computer program implemented with carefully designed assumptions, which can simplify the tooth-to-wood interaction model, bring forth the possibility to integrate the tooth withdrawal feature in the MPC joint model to predict joint withdrawal failures.

Moreover, almost all past modeling efforts focused on the in-plane behavior of the MPC connection. The out-of-plane behavior has not been numerically modeled. The out-of-plane stiffness in particular may be critical for the stability analysis of compression web members and the estimation of associated bracing forces.
Chapter 3. Finite Element Method Based Model Formulation

Introduction

The purpose of this chapter is to discuss the development of a computer program to model and analyze MPC truss joints using the finite element method (FEM). This includes the formulation of the three FEM based elements, the idealization of a actual structure into a mathematical model, the Newton-Raphson iteration methods and the convergence criteria for problem solving, the tooth withdrawal failure criteria, and the coordination transformation. The resulting computer program written in FORTRAN is called Structural Analysis of Metal Plated Connection (SAMPC). Some generic FEM concepts and techniques utilized in the formulation in the study are presented for the completeness of the thesis.

3.1. Brick element

To simulate the wood member closely, the wood member is modeled as a solid to imitate the actual size and mechanical behaviors of the truss members. A typical 3D brick element with orthotropic material properties is used in this study. The element kinematic relationship, the displacement field and stress-strain constitutive relationship are described in sequence in this section as well as the next section.
3.1.1. Kinematics and displacement field

As illustrated in Figure 3.1, a brick element is defined by 8 nodes with its sides parallel to the three orthogonal directions X, Y, and Z in the local Cartesian coordinate system. Δx, Δy, and Δz are the side lengths of element. It’s important to note that the wood’s material axes, longitudinal, radial, and tangential (L, R, and T), are coincident with the coordinate axes X, Y, and Z.

The kinematic strain-displacement relationships of the brick element are defined in Equation (3.1), which includes the normal strains $\varepsilon_x$, $\varepsilon_y$, $\varepsilon_z$ and engineering shear strains $\gamma_{xy}$, $\gamma_{yz}$, $\gamma_{xz}$.

\[
\varepsilon_x = \frac{\partial u}{\partial x} \tag{3.1a}
\]

\[
\varepsilon_y = \frac{\partial v}{\partial y} \tag{3.1b}
\]

\[
\varepsilon_z = \frac{\partial w}{\partial z} \tag{3.1c}
\]

\[
\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{3.1d}
\]
\[
\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \tag{3.1e}
\]

\[
\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \tag{3.1f}
\]

The nodal displacement vector \( \mathbf{a} \) of the brick element consists of 3 translational degrees of freedom (DOF) \( u, v, \) and \( w \) defined in the X, Y and Z axes respectively for each of the 8 nodes as:

\[
\mathbf{a}^T = [u_1, v_1, w_1, \ldots, u_8, v_8, w_8] \tag{3.2}
\]

The displacement vector \([u \ v \ w]\) at an arbitrary location within the element can then be interpolated from the nodal displacements vector \( \mathbf{a} \) based on the shape functions as:

\[
u(x, y, z) = N_{0i}u_1 + \cdots + N_{0i}u_8 = \mathbf{N}_0^T \mathbf{a} \tag{3.3a}
\]

\[
v(x, y, z) = L_{0i}v_1 + \cdots + L_{0i}v_8 = \mathbf{L}_0^T \mathbf{a} \tag{3.3b}
\]

\[
w(x, y, z) = M_{0i}w_1 + \cdots + M_{0i}w_8 = \mathbf{M}_0^T \mathbf{a} \tag{3.3c}
\]

where

\[
N_{0i}, L_{0i}, M_{0i} = \text{shape functions}
\]

\[
N_0, L_0, M_0 = \text{shape function vectors}
\]

The shape functions in this study are all defined in the natural coordinate system. As illustrated in Figure 3.1, for the brick element in the study, the natural coordinate system in a 3D space has its axes \( \xi, \eta, \) and \( \zeta \) passing through the element geometrical center and parallel to Cartesian axes X, Y and Z respectively. And \( \xi = \pm 1, \eta = \pm 1, \) and
$\zeta = \pm 1$ at the edges of corresponding sides. Then the coordinate $x$ and $y$ within the element can be easily defined by:

$$
x = x_1 + \frac{(1+\xi)}{2} \Delta x \quad (3.4a)
$$

$$
y = y_1 + \frac{(1+\eta)}{2} \Delta y \quad (3.4b)
$$

$$
z = z_1 + \frac{(1+\zeta)}{2} \Delta z \quad (3.4c)
$$

where

$x_1, y_1, z_1 = \text{the coordinates of Node 1 in Figure 3.1}$

$\Delta x, \Delta y, \Delta z = \text{the side length of a brick element}$

Therefore, the derivatives $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ can be expressed in terms of $\xi, \eta, \zeta$ in order to calculate the strains in Equation (3.1).

$$
\begin{bmatrix}
\frac{\partial}{\partial x} \\
\frac{\partial}{\partial y} \\
\frac{\partial}{\partial z}
\end{bmatrix}
= \begin{bmatrix}
\frac{2}{\Delta x} & 0 & 0 \\
0 & \frac{2}{\Delta y} & 0 \\
0 & 0 & \frac{2}{\Delta z}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial}{\partial \xi} \\
\frac{\partial}{\partial \eta} \\
\frac{\partial}{\partial \zeta}
\end{bmatrix}
= \mathbf{J}^{-1}
\begin{bmatrix}
\frac{\partial}{\partial \xi} \\
\frac{\partial}{\partial \eta} \\
\frac{\partial}{\partial \zeta}
\end{bmatrix}
\quad (3.5)
$$

where $\mathbf{J}$ is the Jacobian matrix derived from the chain rule of differentiation.

The shape functions are then defined in the natural coordinate system as follows:

$$
N_{01} = V_{01} = M_{01} = \frac{1}{8}(1-\xi)(1-\eta)(1-\zeta) \quad (3.6a)
$$

$$
N_{02} = V_{02} = M_{02} = \frac{1}{8}(1+\xi)(1-\eta)(1-\zeta) \quad (3.6b)
$$
\[ N_{03} = V_{03} = M_{03} = \frac{1}{8}(1 + \xi)(1 + \eta)(1 - \zeta) \]  

(3.6c)

\[ N_{04} = V_{04} = M_{04} = \frac{1}{8}(1 - \xi)(1 + \eta)(1 - \zeta) \]  

(3.6d)

\[ N_{05} = V_{05} = M_{05} = \frac{1}{8}(1 - \xi)(1 - \eta)(1 + \zeta) \]  

(3.6e)

\[ N_{06} = V_{06} = M_{06} = \frac{1}{8}(1 + \xi)(1 - \eta)(1 + \zeta) \]  

(3.6f)

\[ N_{07} = V_{07} = M_{07} = \frac{1}{8}(1 + \xi)(1 + \eta)(1 + \zeta) \]  

(3.6g)

\[ N_{08} = V_{08} = M_{08} = \frac{1}{8}(1 - \xi)(1 + \eta)(1 + \zeta) \]  

(3.6h)

Since all deformation displayed in Equation (3.1) are expressed in the first derivatives of shape functions, from the Jacobian transform matrix in Equation (3.5), there is:

\[
\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \Delta x} \frac{\partial \Delta x}{\partial \xi} = \frac{\partial N_0^T}{\partial \xi} a = N_{1x}^T a
\]

(3.7a)

\[
\frac{\partial v}{\partial y} = \frac{\partial v}{\partial \Delta y} \frac{\partial \Delta y}{\partial \eta} = \frac{\partial L_0^T}{\partial \eta} a = L_{1y}^T a
\]

(3.7b)

\[
\frac{\partial w}{\partial z} = \frac{\partial M_0^T}{\partial \Delta z} \frac{\partial \Delta z}{\partial \zeta} = \frac{\partial M_0^T}{\partial \zeta} a = M_{1z}^T a
\]

(3.7c)

\[
\frac{\partial u}{\partial y} = \frac{\partial N_0^T}{\partial \eta} a = N_{1y}^T a
\]

(3.7d)

\[
\frac{\partial v}{\partial x} = \frac{\partial L_0^T}{\partial \xi} a = L_{1x}^T a
\]

(3.7e)

\[
\frac{\partial v}{\partial z} = \frac{\partial L_0^T}{\partial \zeta} a = L_{1z}^T a
\]

(3.7f)
\begin{equation}
\frac{\partial w}{\partial y} = \frac{\partial M_0^T}{\partial y} a = \frac{2}{\Delta y} \frac{\partial M_0^T}{\partial \eta} a = M_{1y}^T a \tag{3.7g}
\end{equation}

\begin{equation}
\frac{\partial w}{\partial x} = \frac{\partial M_0^T}{\partial x} a = \frac{2}{\Delta x} \frac{\partial M_0^T}{\partial \zeta} a = M_{1x}^T a \tag{3.7h}
\end{equation}

\begin{equation}
\frac{\partial u}{\partial z} = \frac{\partial N_0^T}{\partial z} a = \frac{2}{\Delta z} \frac{\partial N_0^T}{\partial \zeta} a = N_{1z}^T a \tag{3.7i}
\end{equation}

where the subscripts 1x, 1y and 1z denote the first derivatives with respect to coordinates x, y, and z respectively, and the same denotation for shape functions is used throughout the thesis.

Substitute Equation (3.7) into Equation (3.1), the strains of brick element can be rewritten as:

\begin{align*}
\varepsilon_x &= N_{1x}^T a \tag{3.8a} \\
\varepsilon_y &= L_{1y}^T a \tag{3.8b} \\
\varepsilon_z &= M_{1z}^T a \tag{3.8c} \\
\gamma_{xy} &= (N_{1y}^T + L_{1x}^T) a \tag{3.8d} \\
\gamma_{yz} &= (M_{1y}^T + L_{1z}^T) a \tag{3.8e} \\
\gamma_{xz} &= (M_{1x}^T + N_{1z}^T) a \tag{3.8f}
\end{align*}

### 3.1.2. Stress-strain constitutive relationship

The wood, by its nature, is an anisotropic material. In terms of engineering models, wood has usually been assumed as an orthotropic material, namely has
distinctively different properties in L (longitudinal), R (radial) and T (tangential) directions. Based on this assumption, a 3D orthotropic elastic stress-strain relationship is employed to formulate the wood element as:

\[ \sigma = D \varepsilon \]  \hspace{1cm} (3.9)

where

\[ \sigma^T = [\sigma_x \ \sigma_y \ \sigma_z \ \tau_{xy} \ \tau_{yz} \ \tau_{xz}] \]

\[ \varepsilon^T = [\varepsilon_x \ \varepsilon_y \ \varepsilon_z \ \gamma_{xy} \ \gamma_{yz} \ \gamma_{xz}] \]

\[ D^{-1} = \text{compliance matrix} = \begin{bmatrix}
\frac{1}{E_x} & -\frac{v_{xy}}{E_x} & -\frac{v_{xz}}{E_x} \\
-\frac{v_{yx}}{E_y} & \frac{1}{E_y} & -\frac{v_{yz}}{E_y} \\
-\frac{v_{zx}}{E_z} & -\frac{v_{zy}}{E_z} & \frac{1}{E_z} \\
\frac{1}{G_{xy}} & & \\
& \frac{1}{G_{yz}} & \\
& & \frac{1}{G_{xz}}
\end{bmatrix} \]

where

\[ E_x, E_y, E_z = \text{Young's moduli} \]

\[ G_{xy}, G_{yz}, G_{xz} = \text{shear moduli or moduli of rigidity} \]

\[ v_{xy}, v_{xz}, v_{yx}, v_{yz}, v_{zx}, v_{zy} = \text{Poisson's ratios} \]

And the \( D^{-1} \) matrix is presumed to be symmetric, so that:

\[ \frac{v_{yx}}{E_y} = \frac{v_{xy}}{E_x} \]  \hspace{1cm} (3.10a)
Equation (3.10) indicates that $v_{xy}, v_{xz}, v_{yx}, v_{yz}, v_{zx}$, and $v_{zy}$ are not independent quantities, and thus the number of Poisson’s ratio can be reduced to 3 major ones, namely $v_{xy}, v_{yz},$ and $v_{xz}$. The selection of $v_{xy}, v_{yz},$ and $v_{xz}$ over $v_{yx}, v_{zy},$ and $v_{zx}$ is based on the assumption that $E_x$ is larger than $E_y$, thus $v_{xy}$ is larger than $v_{yx}$ (ANSYS, 2010). Therefore in this study, special attention has to been paid in defining the local coordinate system of a brick element to ensure that the $E_x$ always coincides with $E_L$ in longitudinal direction. The stresses can then be calculated by Equation (3.11).

\[
\sigma_x = \frac{E_x}{H} \left[ 1 - (v_{yz})^2 \frac{E_z}{E_y} \right] \varepsilon_x + \frac{E_y}{H} \left[ v_{xy} + v_{xz} v_{yz} \frac{E_z}{E_y} \right] \varepsilon_x + \frac{E_z}{H} \left[ v_{xz} + v_{yz} v_{xy} \frac{E_y}{E_x} \right] \varepsilon_z
\]  
(3.11a)

\[
\sigma_y = \frac{E_y}{H} \left[ v_{xy} + v_{xz} v_{yz} \frac{E_z}{E_y} \right] \varepsilon_x + \frac{E_y}{H} \left[ 1 - (v_{xz})^2 \frac{E_z}{E_x} \right] \varepsilon_y + \frac{E_z}{H} \left[ v_{yz} + v_{xz} v_{xy} \frac{E_y}{E_x} \right] \varepsilon_z
\]  
(3.11b)

\[
\sigma_z = \frac{E_z}{H} \left[ v_{xz} + v_{yz} v_{xy} \right] \varepsilon_x + \frac{E_z}{H} \left[ v_{yz} + v_{xz} v_{xy} \frac{E_y}{E_x} \right] \varepsilon_y + \frac{E_z}{H} \left[ 1 - (v_{xy})^2 \frac{E_y}{E_x} \right] \varepsilon_z
\]  
(3.11c)

\[
\tau_{xy} = G_{xy} \gamma_{xy}
\]  
(3.11d)

\[
\tau_{yz} = G_{yz} \gamma_{yz}
\]  
(3.11e)

\[
\tau_{xz} = G_{xz} \gamma_{xz}
\]  
(3.11f)

where

\[
H = 1 - (v_{xy})^2 \frac{E_y}{E_x} - (v_{yz})^2 \frac{E_z}{E_y} - (v_{xz})^2 \frac{E_z}{E_x} - 2v_{xy} v_{yz} v_{xz} \frac{E_z}{E_x}
\]
As has been mentioned that the wood’s material axes L, R, and T are coincident with the axes X, Y, and Z, then

\[
E_x = E_L \quad (3.12a)
\]

\[
E_y = E_R \quad (3.12b)
\]

\[
E_z = E_T \quad (3.12c)
\]

\[
G_{xy} = G_{LR} \quad (3.12d)
\]

\[
G_{yz} = G_{RT} \quad (3.12e)
\]

\[
G_{xz} = G_{LT} \quad (3.12f)
\]

\[
v_{xy} = v_{LR} \quad (3.12g)
\]

\[
v_{yz} = v_{RT} \quad (3.12h)
\]

\[
v_{xz} = v_{LT} \quad (3.12i)
\]

where \(E_L, E_R\), and \(E_T\) are the elastic moduli along the longitudinal, radial and tangential axes of wood. \(G_{LR}, G_{RT}\) and \(G_{LT}\) are the shear moduli and also called the moduli of rigidity, and the subscripts, for example, LR denotes the shear strain in the LR plane and shear stresses in the LT and RT planes. And \(v_{LR}, v_{RT}\) and \(v_{LT}\) are the Poisson’s ratios. The first subscript of Poisson’s ratios refers to direction of applied stress and the second letter to direction of lateral deformation.

### 3.2. Thin plate element

Thin plate element is selected to model the steel truss plate due to the smallness of thickness dimension (0.91 mm or 0.036 in), which is about the \(1/100^{th}\) of the smallest in-
plane dimension (76.2 mm or 3 in). The formulation of the plate element is based on the following assumptions:

1. Straight lines perpendicular to the mid-surface (i.e. transverse normal) before deformation remain straight after deformation (Kirchoff hypothesis).

2. The transverse normals do not experience elongation (i.e. they are in-extensible) (Kirchoff hypothesis).

3. The transverse normals rotate such that they remain perpendicular to the mid-surface after deformation (Kirchoff hypothesis).

4. The change in the thickness of plate element is negligible, namely transverse strains $\gamma_{xz}$, $\gamma_{yz}$ and $\gamma_{zz}$ are assumed zero.

5. Bending deflection amplifications are taken into account due to the effect of axial compressive loads (P-Δ effect).

6. The plate element has a linear elastic material property and obeys isotropic stress-strain relationships.

These assumptions allow the plate structure undergoing small strains, small to moderate rotations, and relatively large transverse displacements (i.e. the transverse displacement is greater than the plate thickness). Also the stress and the strain can be measured respectively as force per unit un-deformed area and change in length to the original length.
3.2.1. Kinematics and displacement field

By considering the assumption [1] to [5], for the rotations of transverse normals are small or moderate (say 10°-15°, which is always the case in this study), the kinematic relationships take the form:

\[
\varepsilon_x = \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_0}{\partial x^2} + \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2
\]

\[
\varepsilon_y = \frac{\partial v_0}{\partial y} - z \frac{\partial^2 w_0}{\partial y^2} + \frac{1}{2} \left( \frac{\partial w_0}{\partial y} \right)^2
\]

\[
\gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y}
\]

where \( u_0, v_0, \) and \( w_0 \) denote the displacements of a point at mid-plane of the plate \( (x, y, 0) \) in \( (x, y, z) \) coordination directions. For normal strains in Equation (3.13a) and (3.13b), the first two terms account for the extension/compression and bending respectively, while for shear strain in Equation (3.13c), the first three terms account for the extension/compression and bending respectively, and the last terms for all cases account for deflection amplification due to P-\( \Delta \) effect.

The FEM formulation of the thin plate element takes the same form as described in the thesis by He (2002) because of the same pattern of deformation. Originally, He applied the thin plate element to model the sheathing panel in shear wall application, to which the in-plane shear stress and strain is of great interest, and thus the additional shear strain \( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \) was included as degree of freedom. This may complicate the formulation of solution in terms of shape function, but provide a sophisticated numerical solution.
Figure 3.2 Plate element geometry

Shown in Figure 3.2, it’s a four-node plate element, and the DOF vector takes the form as:

\[
a^T = \begin{bmatrix}
    u_1, v_1, \frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial x}, w_1, \frac{\partial w_1}{\partial y}, \frac{\partial^2 w_1}{\partial x \partial y}, \ldots, u_4, v_4, \frac{\partial u_4}{\partial y} + \frac{\partial v_4}{\partial x}, \\
    w_4, \frac{\partial w_4}{\partial x}, \frac{\partial w_4}{\partial y}, \frac{\partial^2 w_4}{\partial x \partial y}
\end{bmatrix}
\]  

(3.14)

Then the displacement field can be represented by shape functions and degrees of freedoms as follows:

\[
u(x, y) = N_0 u_1 + N_2 \left( \frac{\partial u_1}{\partial y} \right) + \cdots + N_7 u_4 + N_8 \left( \frac{\partial u_4}{\partial y} \right) = N^T_0 a
\]

(3.15a)

\[
v(x, y) = L_0 v_1 + L_2 \left( \frac{\partial v_1}{\partial x} \right) + \cdots + L_7 v_4 + L_8 \left( \frac{\partial v_4}{\partial x} \right) = L^T_0 a
\]

(3.15b)

\[
w(x, y) = M_0 w_1 + M_2 \left( \frac{\partial w_1}{\partial x} \right) + M_3 \left( \frac{\partial w_1}{\partial y} \right) + M_4 \left( \frac{\partial^2 w_1}{\partial x \partial y} \right) + \cdots \\
+ M_13 w_4 + M_14 \left( \frac{\partial w_4}{\partial x} \right) + M_15 \left( \frac{\partial w_4}{\partial y} \right) + M_16 \left( \frac{\partial^2 w_4}{\partial x \partial y} \right) = M^T_0 a
\]

(3.15c)

where

\[N_0, L_0, M_0 = \text{shape functions}\]
$N_0, L_0, M_0 =$ shape function vectors

Follow the same procedure with the brick element, with the elemental natural coordinates illustrated in Figure 3.2, the local coordinates can be related to the natural coordinates by

$$x = x_1 + \frac{(1 + \xi)}{2} \Delta x$$  \hspace{1cm} (3.16a)

$$y = y_1 + \frac{(1 + \eta)}{2} \Delta y$$  \hspace{1cm} (3.16b)

where

$$x_1, y_1 = \text{coordinates of plate element Node 1 in Figure 3.2}$$

$$\Delta x, \Delta y = \text{side lengths of plate element}$$

Hence, the derivatives $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial \xi \eta}$ can be expressed in terms of $\xi, \eta$ and Jacobian matrix $J$,

$$\begin{bmatrix}
\frac{\partial}{\partial x} \\
\frac{\partial}{\partial y} \\
\frac{\partial}{\partial \xi \eta}
\end{bmatrix}
= \begin{bmatrix}
\frac{2}{\Delta x} & 0 & 0 \\
0 & \frac{2}{\Delta y} & 0 \\
0 & 0 & \frac{2}{\Delta x \Delta y}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial}{\partial \xi} \\
\frac{\partial}{\partial \eta} \\
\frac{\partial}{\partial \xi \eta}
\end{bmatrix}
= J^{-1}
\begin{bmatrix}
\frac{\partial}{\partial \xi} \\
\frac{\partial}{\partial \eta} \\
\frac{\partial}{\partial \xi \eta}
\end{bmatrix}$$  \hspace{1cm} (3.17)

Present the shape functions in natural coordinates in Equation (3.18), (3.19) and (3.20).

$$N_{01} = \frac{1}{4}(1 - \xi - \frac{3}{2} \eta + \frac{1}{2} \eta^3 + \frac{3}{2} \xi \eta - \frac{1}{2} \xi^3)$$  \hspace{1cm} (3.18a)

$$N_{02} = -\frac{1}{8}(1 - \xi - \eta - \eta^2 + \eta^3 + \xi \eta + \xi \eta^2 - \xi^3 \eta^3) \frac{\Delta y}{2}$$  \hspace{1cm} (3.18b)
\[ N_{03} = \frac{1}{4} (1 + \xi - \frac{3}{2} \eta + \frac{1}{2} \eta^3 - \frac{3}{2} \xi \eta + \frac{1}{2} \xi \eta^3) \] 

(3.18c)

\[ N_{04} = -\frac{1}{8} (1 + \xi - \eta - \eta^2 + \eta^3 - \xi \eta - \xi \eta^2 + \xi \eta^3) \frac{\Delta y}{2} \] 

(3.18d)

\[ N_{05} = \frac{1}{4} (1 + \xi + \frac{3}{2} \eta - \frac{1}{2} \eta^3 + \frac{3}{2} \xi \eta - \frac{1}{2} \xi \eta^3) \] 

(3.18e)

\[ N_{06} = -\frac{1}{8} (-1 - \xi + \eta - \eta^2 + \eta^3 - \xi \eta + \xi \eta^2 + \xi \eta^3) \frac{\Delta y}{2} \] 

(3.18f)

\[ N_{07} = \frac{1}{4} (1 - \xi + \frac{3}{2} \eta - \frac{1}{2} \eta^3 - \frac{3}{2} \xi \eta - \frac{1}{2} \xi \eta^3) \] 

(3.18g)

\[ N_{08} = -\frac{1}{8} (-1 - \xi - \eta + \eta^2 + \eta^3 + \xi \eta - \xi \eta^2 - \xi \eta^3) \frac{\Delta y}{2} \] 

(3.18h)

\[ L_{01} = \frac{1}{4} (1 - \eta - \frac{3}{2} \xi + \frac{1}{2} \xi^3 + \frac{3}{2} \xi \eta - \frac{1}{2} \xi^3 \eta) \] 

(3.19a)

\[ L_{02} = \frac{1}{8} (1 - \xi - \eta - \xi^2 + \xi^3 + \xi \eta + \xi^2 \eta - \xi^3 \eta) \frac{\Delta x}{2} \] 

(3.19b)

\[ L_{03} = \frac{1}{4} (1 - \eta + \frac{3}{2} \xi - \frac{1}{2} \xi^3 - \frac{3}{2} \xi \eta + \frac{1}{2} \xi^3 \eta) \] 

(3.19c)

\[ L_{04} = \frac{1}{8} (-1 - \xi + \eta + \xi^2 + \xi^3 + \xi \eta - \xi^2 \eta - \xi^3 \eta) \frac{\Delta x}{2} \] 

(3.19d)

\[ L_{05} = \frac{1}{4} (1 + \eta + \frac{3}{2} \xi - \frac{1}{2} \xi^3 + \frac{3}{2} \xi \eta - \frac{1}{2} \xi^3 \eta) \] 

(3.19e)

\[ L_{06} = \frac{1}{8} (-1 - \xi - \eta + \xi^2 + \xi^3 - \xi \eta + \xi^2 \eta + \xi^3 \eta) \frac{\Delta x}{2} \] 

(3.19f)

\[ L_{07} = \frac{1}{4} (1 + \eta - \frac{3}{2} \xi + \frac{1}{2} \xi^3 - \frac{3}{2} \xi \eta - \frac{1}{2} \xi^3 \eta) \] 

(3.19g)

\[ L_{08} = \frac{1}{8} (1 - \xi + \eta - \xi^2 + \xi^3 - \xi \eta - \xi^2 \eta + \xi^3 \eta) \frac{\Delta x}{2} \] 

(3.19h)
\[ M_{o1} = \frac{1}{4} \left( -\frac{3}{2} \xi + \frac{1}{2} \xi^3 - \frac{3}{2} \eta + \frac{1}{2} \eta^3 + \frac{9}{4} \xi \eta - \frac{3}{4} \xi^3 \eta - \frac{3}{4} \xi^2 \eta^3 + \frac{1}{4} \xi^3 \eta^3 \right) \]  
\[ (3.20a) \]

\[ M_{o2} = \frac{1}{8} \left( 1 - \xi - \xi^2 + \xi^3 - \frac{3}{2} \eta + \frac{1}{2} \eta^3 + \frac{3}{2} \xi \eta + \frac{3}{2} \xi^2 \eta - \frac{3}{2} \xi^3 \eta - \frac{1}{2} \xi \eta^3 \right) - \frac{1}{2} \xi^2 \eta^3 + \frac{1}{2} \xi^3 \eta^3 \frac{\Delta x}{2} \]  
\[ (3.20b) \]

\[ M_{o3} = \frac{1}{8} \left( 1 - \frac{3}{2} \xi + \frac{1}{2} \xi^3 - \eta - \eta^2 + \frac{3}{2} \xi \eta + \frac{3}{2} \xi^2 \eta - \frac{3}{2} \xi \eta^2 - \frac{3}{2} \xi \eta^3 \right) - \frac{1}{2} \xi^3 \eta^2 + \frac{1}{2} \xi^3 \eta^3 \frac{\Delta y}{2} \]  
\[ (3.20c) \]

\[ M_{o4} = \frac{1}{16} \left( 1 - \frac{3}{2} \xi + \frac{1}{2} \xi^3 - \frac{3}{2} \eta + \frac{1}{2} \eta^3 - \frac{9}{4} \xi \eta + \frac{3}{4} \xi^3 \eta + \frac{3}{4} \xi \eta^3 - \frac{1}{4} \xi^3 \eta^3 \right) \]  
\[ (3.20d) \]

\[ M_{o5} = \frac{1}{4} \left( 1 + \frac{3}{2} \xi - \frac{1}{2} \xi^3 - \frac{3}{2} \eta + \frac{1}{2} \eta^3 + \frac{9}{4} \xi \eta + \frac{3}{4} \xi^3 \eta + \frac{3}{4} \xi \eta^3 - \frac{1}{4} \xi^3 \eta^3 \right) \]  
\[ (3.20e) \]

\[ M_{o6} = \frac{1}{8} \left( -1 - \xi + \xi^2 + \xi^3 + \frac{3}{2} \eta - \frac{1}{2} \eta^3 + \frac{3}{2} \xi \eta - \frac{3}{2} \xi \eta^2 + \frac{3}{2} \xi \eta^3 - \frac{1}{2} \xi \eta^3 \right) + \frac{1}{2} \xi^2 \eta^3 + \frac{1}{2} \xi^3 \eta^3 \frac{\Delta x}{2} \]  
\[ (3.20f) \]

\[ M_{o7} = \frac{1}{8} \left( 1 - \frac{3}{2} \xi - \frac{1}{2} \xi^3 - \eta - \eta^2 + \xi^2 \eta - \xi^3 \eta - \xi \eta^2 - \xi \eta^3 \right) + \frac{1}{2} \xi^3 \eta^2 - \frac{1}{2} \xi^3 \eta^3 \frac{\Delta y}{2} \]  
\[ (3.20g) \]

\[ M_{o8} = \frac{1}{16} \left( -1 - \xi + \xi^2 + \xi^3 + \eta + \eta^2 - \eta^3 + \xi \eta - \xi^2 \eta + \xi^2 \eta^2 - \xi \eta^2 - \xi \eta^3 \right) - \xi \eta^3 + \xi^2 \eta^3 - \xi^3 \eta^2 + \xi^3 \eta^3 \frac{\Delta x \ \Delta y}{2} \]  
\[ (3.20i) \]

\[ M_{o9} = \frac{1}{4} \left( 1 + \frac{3}{2} \xi - \frac{1}{2} \xi^3 + \frac{3}{2} \eta - \frac{1}{2} \eta^3 + \frac{9}{4} \xi \eta - \frac{3}{4} \xi^3 \eta - \frac{3}{4} \xi \eta^3 + \frac{1}{4} \xi^3 \eta^3 \right) \]  
\[ (3.20j) \]
\[ M_{010} = \frac{1}{8} \left( \frac{-1 - \xi + \xi^2 + \xi^3 - \frac{3}{2} \eta + \frac{1}{2} \eta^3 - \frac{3}{2} \xi \eta + \frac{3}{2} \xi^2 \eta + \frac{3}{2} \xi^3 \eta + \frac{1}{2} \xi \eta^3}{2} \right) - \frac{1}{2} \left( \frac{\xi^3 \eta^3 - \xi^3 \eta^3}{2} \right) \Delta x \]

\[ M_{011} = \frac{1}{8} \left( \frac{-1 - \frac{3}{2} \xi + \frac{1}{2} \xi^3 - \eta + \eta^2 + \eta^3 - \frac{3}{2} \xi \eta + \frac{3}{2} \xi^2 \eta + \frac{3}{2} \xi^3 \eta + \frac{1}{2} \xi \eta^3}{2} \right) - \frac{1}{2} \left( \frac{\xi^3 \eta^3 - \xi^3 \eta^3}{2} \right) \Delta y \]

\[ M_{012} = \frac{1}{16} \left( 1 + \xi - \xi^2 - \xi^3 + \eta - \eta^2 - \eta^3 + \xi \eta - \xi^2 \eta - \xi^3 \eta - \xi^3 \eta^2 + \xi^2 \eta^2 \right) - \frac{1}{2} \left( \frac{\xi^3 \eta^3 + \xi^2 \eta^3 + \xi^3 \eta^2 + \xi^3 \eta^3}{2} \right) \Delta x \Delta y \]

Therefore, the derivatives needed in Equation (3.13) are:

\[ \frac{\partial u}{\partial x} = \frac{\partial N_0^T}{\partial x} a = \frac{2}{\Delta x} \frac{\partial N_0^T}{\partial \xi} a = N_{1x}^T a \]  

(3.21a)

\[ \frac{\partial u}{\partial y} = \frac{\partial N_0^T}{\partial y} a = \frac{2}{\Delta y} \frac{\partial N_0^T}{\partial \eta} a = N_{1y}^T a \]  

(3.21b)

\[ \frac{\partial v}{\partial x} = \frac{\partial L_0^T}{\partial x} a = \frac{2}{\Delta x} \frac{\partial L_0^T}{\partial \xi} a = L_{1x}^T a \]  

(3.21c)

\[ \frac{\partial v}{\partial y} = \frac{\partial L_0^T}{\partial y} a = \frac{2}{\Delta y} \frac{\partial L_0^T}{\partial \eta} a = L_{1y}^T a \]  

(3.21d)

\[ \frac{\partial w}{\partial x} = \frac{\partial M_0^T}{\partial x} a = \frac{2}{\Delta x} \frac{\partial M_0^T}{\partial \xi} a = M_{1x}^T a \]  

(3.21e)

\[ \frac{\partial w}{\partial y} = \frac{\partial M_0^T}{\partial y} a = \frac{2}{\Delta y} \frac{\partial M_0^T}{\partial \eta} a = M_{1y}^T a \]  

(3.21f)

\[ \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 M_0^T}{\partial x^2} a = \left( \frac{2}{\Delta x} \right)^2 \frac{\partial^2 M_0^T}{\partial \xi^2} a = M_{2x}^T a \]  

(3.21g)
Substitute Equation (3.21) into Equation (3.13), the strains of plate element can be rewritten as:

\[
\varepsilon_x = \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 - z \frac{\partial^2 w_0}{\partial x^2} = \left(N_{1x}^T - zM_{2x}^T \right)a + \frac{1}{2}a^T \left(M_{1x}M_{1x}^T\right)a
\]

\[
\varepsilon_y = \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - z \frac{\partial^2 w_0}{\partial y^2} = \left(L_{1y}^T - zM_{2y}^T \right)a + \frac{1}{2}a^T \left(M_{1y}M_{1y}^T\right)a
\]

\[
\gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} = 2z \frac{\partial^2 w_0}{\partial x \partial y} = \left(N_{1y}^T + L_{1x}^T - 2z M_{2xy}^T \right)a + a^T \left(M_{1x}M_{1y}^T\right)a
\]

### 3.2.2. Stress-strain constitutive relationship

The steel plate is assigned linear elastic material properties, due to the assumption that before the MPC joint fails, the steel plate is acting in its elastic range. Accordingly, the constitutive relationship is given by Hooke’s law for the plate element as:

\[
\sigma = D\varepsilon
\]

where

\[
\sigma^T = \begin{bmatrix} \sigma_x & \sigma_y & \tau_{xy} \end{bmatrix}
\]
\[ \varepsilon^T = \begin{bmatrix} \varepsilon_x & \varepsilon_y & \gamma_{xy} \end{bmatrix} \]
\[
[D] = \begin{bmatrix}
\frac{E}{1-v^2} & \frac{vE}{1-v^2} & 0 \\
\frac{vE}{1-v^2} & \frac{E}{1-v^2} & 0 \\
0 & 0 & G
\end{bmatrix}
\]

\[E = \text{Modulus of elasticity}\]
\[G = \text{shear modulus} = \frac{E}{2(1+v)}\]
\[v = \text{Poisson's ratio}\]

3.3. Tooth connection model

According to the literature reviewed, the interaction between the truss plate tooth and the wood, which is also regarded as the load-slip characteristics, plays a significant role in determining the strength and stiffness of truss plate joints, and thus partially affects the overall performance of the truss system. Therefore, these characteristics and the way they are incorporated in MPC truss joint model are the most important component of this study. Therefore, great efforts have been paid on understanding tooth-to-wood interaction, and modeling the actual behaviour of MPC joints in a more realistic means.

3.3.1. Modeling principles

As shown in Figure 3.3a and b, the single 3D tooth resembles a cantilever beam resting on the wood medium. While loading, the tooth may undergo bi-directional lateral shear forces \(P_y\) and \(P_z\). Torsion effects may occur but including this behaviour would
make this case very complicated. Since torsion may not contribute to the stiffness of joints, it was not considered in this study.

![Diagram](image)

Figure 3.3 The beam model (a and b) and the superimposing beam model (c) of a single tooth

As the tooth begins to deform, the lateral resistances along two orthogonal directions Y and Z are exerted by the wood medium along the tooth bearing interfaces. Due to the particular geometry of tooth in this study, the tooth is borne either on the face (with broad tooth-to-wood contact interface) or on the edge (with narrow tooth-to-wood contact interface). The lateral reaction forces of the wood on the tooth are not evenly distributed along the bearing interfaces, but gradually increase to a maximum at tooth base (where the tooth attaches to the plate), as illustrated in Figure 3.3a. The sum of the wood reaction along the tooth length isn’t in alignment with the external force $P_y$ or $P_z$. This results in the peeling moment $M_z$ or $M_y$, which causes the steel plate to rotate about the point where the tooth attaches to the plate, and thus in turn tends to pull the tooth out of wood. However, the friction forces $P_{xz}$ and $P_{xy}$ along the tooth-to-wood interfaces resist the withdrawal deformation.
It should be noted that, in general, as a fastener is driven into wood, the fastener damages a portion of the wood and creates a hole in the size of the fastener, and the driving process also causes the wood portion to be displaced and densified. As a result, an initial embedment pressure occurs at tooth-to-wood interfaces (Allotey, 2005). This embedment pressure is affected by a number of factors such as the viscoelastic and rheological properties of the wood, and the fastener driving method etc., such that it’s nearly impossible to measure it through tests. On the other hand, the initial embedment pressure vanishes as soon as the gap at the tooth-to-wood interface occurs, thus imposes no influence on fastener’s lateral maximum capacity. Hence, the initial embedment pressure is not considered in this model for simplification.

Furthermore, the superimposing beam model is used to simplify the model and promote program convergence. The single 3D beam of tooth (in Figure 3.3b) is simulated by two identical tooth beams I and II (in Figure 3.3c) of the same geometry as the original beam, and the two beams are coincident with each other at any point. Each beam performs a 2D behavior in the designated direction to account for lateral reaction of the beam independently. For example, as illustrated in Figure 3.3c, taking axis X along the tooth length while axis Y coincides with plate primary direction, which is parallel to plate slots, and Z perpendicular to Y as plate secondary direction. Beam I is in the X-Y plane subjected to lateral force $P_y$, and peeling moment $M_z$; while Beam II is in the X-Z plane subjected to $P_z$ and $M_y$. However, the original approach was to develop a single tooth beam model which can withstand the lateral reactions along two bearing surfaces. In doing so, the geometry and loading condition of the actual tooth can be preserved. The problem with this approach is that there were convergence difficulties during numerical
iteration. After several unsuccessful attempts to work with this approach, it was abandoned.

In FEM implementation, the model HYST developed by Foschi (2000) forms the basis of modeling the load-slip behavior of the individual tooth connection system. This original model has been extended and modified, and integrated into SAMPC as a subroutine, to calculate the load-slip relationship of the tooth connection element in two lateral directions and the peeling moments exerted by eccentric wood reaction forces. Meanwhile a tooth withdrawal model is also implemented to account for the withdrawal resistance to axial deformation.

The principle of HYST is that a single nail connector is modeled by an elasto-plastic 2D beam element with the specified yielding strength and modulus of elasticity (Figure 3.4a). And the surrounding embedment medium is modeled as a series of linear or nonlinear compression-only springs along the nail bearing surface to account for the lateral medium reaction forces. For the very resemblance between the nail connector and the truss plate tooth, HYST is assumed feasible to be employed in this study.

A single tooth is separated into two identical superimposing beams in this study, by applying a modified HYST model, each beam then has a set of lateral springs with distinct spring stiffness along the corresponding bearing interface, as shown in Figure 3.4b.
The load-displacement relationship of the two sets of lateral springs takes the same form of the 3-parameter exponential function as mentioned in Equation (2.1) and Figure 2.1.

The parameters $K$, $Q_1$ and $Q_0$ for the two sets of springs are dependent on the factors that affect the behavior of truss joints, such as wood and plate material properties as well as their orientation to the load, wood density and MC, etc. Therefore, it’s vital for this study to be able to find the proper spring parameters and then verify the accuracy of the model, which is elaborated in detail later in Chapter 5.

The tooth withdrawal behaviour is taken into account by applying withdrawal springs. Theoretically, when the withdrawal force invoked by peeling moment at the tooth-to-plate interface is greater than the shaft friction, the tooth starts to pull out. To simulate the friction resistance from wood medium, a set of withdrawal springs were set on the wood bearing interface along the tooth length originally. Some preliminary computations were conducted, and convergence difficulty was encountered again. Therefore, a single fictitious withdrawal spring is used at tooth tip instead, to account for
the withdrawal load-displacement relationship. This withdrawal spring takes the same form of Foschi’s 3-parameters function as in Equation (2.1), and the parameter calibration procedure is presented in Chapter 5.

Please note that unlike the friction force, the load-deformation relationship of the withdrawal spring is not directly determined by the contact surface and the pressure normal to the contact surface, but characterized by an exponential curve. In other words, strictly speaking, the withdrawal spring applied in the model is not a mathematical representation of tooth shaft friction, but a convenient analog to represent the tooth withdrawal procedure.

One of the important features of the thesis study is that the development of an analytical model capable of predicting the MPC joint failure in tooth withdrawal, which is one of the most common failure modes after all. The description of the mathematical schemes is presented later in Section 3.7.

The following assumptions are made in formulating the tooth beam element and spring element in HYST connection model and its modification:

(1) Each tooth is modeled as a number of beam-column elements, obeying an elastic-perfectly-plastic constitutive relation.

(2) The spring elements are set at the Gaussian points along the length of beam-column element. Therefore the total number of spring elements is dependent on the number of beam-column elements and the number of Gaussian points applied.
(3) The cross section of beam elements is assumed to remain plane and perpendicular to the deformed axis after deformation (Euler-Bernoulli hypothesis).

(4) The deflection amplification due to axial compressive loads (P-Δ effect) is considered.

### 3.3.2. Kinematics and displacement field

Despite the only difference between Beams I and II is in orientation, superimposing Beams I and II share exactly the same formulation procedures, such as strain-displacement relationship, displacement field, shape functions and their derivatives, etc. Therefore, to avoid redundancy, the formulation of a general beam-column element in Figure 3.5 representing both superimposing beams is presented here.

From the above noted assumptions (3) and (4), the normal strain of the beam element is expressed as:

\[
\varepsilon = \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_0}{\partial x^2} + \frac{1}{2} \left( \frac{\partial u_0}{\partial x} \right)^2
\]

where \( u_0 \) and \( w_0 \) (can be \( v_0 \)) denote the axial and transverse displacements of a point at the neutral axis. The first term of strain accounts for the extension or compression, the second for the bending, and the last term for deflection amplification due to P-Δ effect.
The length of the beam is discretized into beam-column elements with 2 nodes and 3 DOF at each node. The nodal DOF vector of beam element is:

\[ \mathbf{a}^T = \left[ w_1, \frac{\partial w_1}{\partial x}, u_1, w_2, \frac{\partial w_2}{\partial x}, u_2 \right] \]  

(3.25)

The displacements at arbitrary location within the beam-column element can be expressed as:

\[ u(x) = N_{01}u_1 + N_{02}u_2 = \mathbf{N}_0^T \mathbf{a} \]  

(3.26a)

\[ w(x) = M_{01}w_1 + M_{02} \left( \frac{\partial w_1}{\partial x} \right) + M_{03}w_2 + M_{04} \left( \frac{\partial w_2}{\partial x} \right) = \mathbf{M}_0^T \mathbf{a} \]  

(3.26b)

The shape functions in natural coordinate are:

\[ N_{01} = \frac{1}{2} (1 - \xi) \]  

(3.27a)

\[ N_{02} = \frac{1}{2} (1 + \xi) \]  

(3.27b)

\[ M_{01} = \frac{1}{2} \left( 1 - \frac{3}{2} \xi + \frac{1}{2} \xi^3 \right) \]  

(3.27c)

\[ M_{02} = \frac{1}{4} \left( 1 - \xi - \xi^2 + \xi^3 \right) \frac{\Delta x}{2} \]  

(3.27d)

\[ M_{03} = \frac{1}{2} \left( 1 + \frac{3}{2} \xi - \frac{1}{2} \xi^3 \right) \]  

(3.27e)

\[ M_{04} = \frac{1}{4} \left( -1 - \xi + \xi^2 + \xi^3 \right) \frac{\Delta x}{2} \]  

(3.27f)

From Equation (3.26), there is:

\[ \frac{\partial u}{\partial x} = \frac{\partial \mathbf{N}_0^T}{\partial x} \mathbf{a} = \frac{2}{\Delta x} \frac{\partial \mathbf{N}_0^T}{\partial \xi} \mathbf{a} = \mathbf{N}_1^T \mathbf{a} \]  

(3.28a)

\[ \frac{\partial w}{\partial x} = \frac{\partial \mathbf{M}_0^T}{\partial x} \mathbf{a} = \frac{2}{\Delta x} \frac{\partial \mathbf{M}_0^T}{\partial \xi} \mathbf{a} = \mathbf{M}_1^T \mathbf{a} \]  

(3.28b)
\[
\frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 M_0^T}{\partial x^2} a = \left( \frac{2}{\Delta x} \right)^2 \frac{\partial^2 M_0^T}{\partial \xi^2} a = M_2^T a
\]  

(3.28c)

Now Equation (3.24) can be rewritten as:

\[
\varepsilon = \left( N_1^T - z M_2^T \right) a + \frac{1}{2} a^T M_1 M_1^T a
\]  

(3.29)

### 3.3.3. Stress-strain constitutive equations

The stress-strain behavior of the beam-column element is defined by an elastic-perfectly-plastic model as:

\[
\sigma = \begin{cases} 
E \varepsilon & \text{if } \varepsilon \leq \varepsilon_{\text{yield}} \\
\sigma_{\text{yield}} & \text{if } \varepsilon_{\text{yield}} \leq \varepsilon 
\end{cases}
\]  

(3.30)

Where

\[
E = \text{Modulus of elasticity}
\]

\[
\sigma_{\text{yield}}, \varepsilon_{\text{yield}} = \text{yielding stress and strain of the truss plate}
\]

### 3.3.4. Tooth connection load-displacement relationship

In 3D space, the deformations of the tooth connector are determined from the relative movements between the plate and underlying wood member, as illustrated in Figure 3.6. The dashed line depicts an un-deformed plate and wood member contour, and the white circle represents the original tooth position at the plate-wood contact interface. When loading occurs, the plate moves relatively from the wood member in three coordinate directions, and rotates about two principle plate axes. The two black circles indicate respectively the tooth top (tooth-plate attachment point) and where the tooth
embeds into the wood. The relative distances between the two black circles in three translational directions ($\Delta x$, $\Delta y$ and $\Delta z$) as well as the plate rotations ($\Delta \theta_x$ and $\Delta \theta_y$) altogether define the tooth deformations as expressed in Equation (3.31).

\[
\Delta x = u_T - u_W \quad \text{(3.31a)}
\]
\[
\Delta y = v_T - v_W \quad \text{(3.31b)}
\]
\[
\Delta z = w_T - w_W \quad \text{(3.31c)}
\]
\[
\Delta \theta_x = \theta_{xT} \quad \text{(3.31d)}
\]
\[
\Delta \theta_y = \theta_{yT} \quad \text{(3.31e)}
\]

Here, $u_W, v_W$ and $w_W$ are the wood displacements at the tooth-wood bearing point, while $u_T, v_T$ and $w_T$ are the plate displacements of the tooth top (tooth-plate attachment point). $\theta_{xT}$ and $\theta_{yT}$ are the plate rotations. And $u_T$ and $v_T$ are given by

\[
u_T = u_{p0} - \frac{t}{2} \frac{\partial w_{p0}}{\partial x} \quad \text{(3.32a)}
\]
\[
v_T = v_{p0} - \frac{t}{2} \frac{\partial w_{p0}}{\partial y} \quad \text{(3.32b)}
\]
where, \( t \) is the thickness of plate, and \( u_{p0}, v_{p0}, w_{p0} \) are the plate displacements at the neutral-plane.

Conventionally, in the element meshing, extra nodes may be set on the tooth-plate attachment and the tooth-wood interface to obtain the deformations of the tooth connection. This would increase the model’s DOF significantly and complicate the formulation of shape functions, and then unfavorably increase overall computation time to a big extent. Herein, the tooth deformations are calculated as the interpolations of those of associated plate element nodes and wood element nodes. Therefore, the corresponding DOF vector can be defined by:

\[
a^T = \left[ u_{1r}, v_{1r}, \frac{\partial u_{1r}}{\partial y} + \frac{\partial v_{1r}}{\partial x}, w_{1r}, \frac{\partial w_{1r}}{\partial y}, \frac{\partial^2 w_{1r}}{\partial x \partial y}, \ldots, u_{4r}, v_{4r}, \frac{\partial u_{4r}}{\partial y} + \frac{\partial v_{4r}}{\partial x} \right]_{P(28)}^{(52)}
\]

(3.33)

The elemental DOF vector \( a \) in Equation (3.33) consists of 52 components, and the first 28 components are degrees of freedom of the associated plate element, and the other 24 components are degrees of freedom of the associated wood brick element.

Given the tooth associated brick element and plate element, from Equations (3.3), (3.15) and (3.21), the displacement components in Equation (3.32) can be rewritten as

\[
\Delta x = \left[ \left( \mathbf{N}_0 - \frac{t}{2} \mathbf{M}_{1x} \right)_p^T - \left( \mathbf{N}_0 \right)_w^T \right] \cdot a = \mathbf{Q}_x^T \cdot a
\]

(3.34a)

\[
\Delta y = \left[ \left( \mathbf{L}_0 - \frac{t}{2} \mathbf{M}_{1y} \right)_p^T - \left( \mathbf{L}_0 \right)_w^T \right] \cdot a = \mathbf{Q}_y^T \cdot a
\]

(3.34b)
\[ \Delta z = \left[ (M_0)_p^T - (M_0)_w^T \right] \cdot \mathbf{a} = Q_z^T \cdot \mathbf{a} \quad (3.34c) \]

\[ \Delta \theta_x = (M_{1x})_p^T \cdot \mathbf{a} = Q \theta_x^T \cdot \mathbf{a} \quad (3.34d) \]

\[ \Delta \theta_y = (M_{1y})_p^T \cdot \mathbf{a} = Q \theta_y^T \cdot \mathbf{a} \quad (3.34e) \]

where \( Q_x, Q_y, Q_z, QQ_x \) and \( QQ_y \) are the combination vector of the shape functions.

Once the displacements are calculated by Equation (3.34), the lateral displacements \( \Delta x \) and \( \Delta y \) can be input into the modified HYST subroutine to calculate the reaction forces \( F(\Delta x) \) and \( F(\Delta y) \), as well as the peeling moments \( M(\Delta x) \) and \( M(\Delta y) \) as output.

Please note that the reaction forces \( F(\Delta x) \) and the peeling moments \( M(\Delta x) \) are calculated only if \( \Delta x > 0 \), namely tooth acts against the bearing wood in loading direction, otherwise if \( \Delta x < 0 \), there would be a gap forming between tooth and wood, and no reaction forces are invoked to resist the deformation of tooth: \( F(\Delta x) \) and \( M(\Delta x) \) are zero.

In the case \( \Delta x > 0 \), the force of each lateral spring at Gaussian point can be calculated during the HYST analysis, say \( F_x^i \) or \( F_y^i \). The lateral reaction forces \( F(\Delta x) \) or \( F(\Delta y) \) at the tooth-plate attachment can then be calculated as the sum of these spring forces along the corresponding direction. Additionally, the modification has been added in HYST to calculate the peeling moments \( M(\Delta x) \) or \( M(\Delta y) \) as the sum of the product of the spring force \( F_x^i \) or \( F_y^i \) and the distance of the spring to tooth head \( L_x^i \) or \( L_y^i \).
\[ F(\Delta x) = \sum_{n} F_{x}^{i} \]  
\[ F(\Delta y) = \sum_{n} F_{y}^{i} \]  
\[ M_{x}(\Delta x) = \sum_{n} F_{x}^{i} \cdot L_{x}^{i} \]  
\[ M_{y}(\Delta y) = \sum_{n} F_{y}^{i} \cdot L_{y}^{i} \]

where \( n \) is the amount of springs engaged.

A withdrawal spring model is integrated in this study as a subroutine to calculate the withdrawal force \( F(\Delta z) \). When \( \Delta z \) is greater than zero, that is the plate separates from the wood, the withdrawal force \( F(\Delta z) \) is exerted and calculated by Foschi’s 3-parameter model in Equation (2.1); while \( \Delta z \) is negative, namely the plate moves against the wood, the stiffness of the spring is considered infinitely big assuming the plate can’t penetrate the wood surface.

3.4. Formulation of MPC truss joint model

3.4.1. The principle of virtual work

The principle of virtual work is applied throughout the FEM formulation of MPC joint. The principle states that if a body is in equilibrium, the total virtual work done by actual internal as well as external forces in moving through their respective virtual displacement is zero (Reddy, 1993). The analytical form of the principle over a typical element is given by:

\[ \delta W = \delta W_{I} - \delta W_{E} = 0 \]  

The internal virtual strain energy is:
\[ \delta W_I = \int_V \delta \varepsilon^T \sigma dV \]  

(3.37)

where \( V \) is the volume of the element, and \( \delta \varepsilon \) is the virtual strain vector, \( \sigma \) is the corresponding stress vector.

The work done by external applied forces is:

\[ \delta W_E = \int_V \delta a^T F_B dV + \int_S \delta a^T F_S dS + \sum \delta a^T R_C \]  

(3.38)

In which, \( \delta a \) is an arbitrary virtual global displacement vector, \( F_B \) and \( F_S \) denote the body force and surface force vector respectively, and \( R_C \) denotes the consistent nodal force vector.

Before forming the virtual work for the MPC truss joint system, the virtual work at element level is required for the solution.

### 3.4.2. Internal virtual work in brick element

From Equation (3.38), the internal virtual strain energy of brick element is:

\[
\left( \delta W^e_I \right)_W = \int_V \delta \varepsilon_x^T \sigma_x dV + \int_V \delta \varepsilon_y^T \sigma_y dV + \int_V \delta \varepsilon_z^T \sigma_z dV + \\
\int_V \delta \gamma_{xy}^T \tau_{xy} dV + \int_V \delta \gamma_{yz}^T \tau_{yz} dV + \int_V \delta \gamma_{xz}^T \tau_{xz} dV
\]  

(3.39)

Using the kinematic relationships developed in Equation (3.8) and the stress-strain constitutive relationships in Equation (3.11), Equation (3.39) can be expressed in terms of shape functions and natural coordinates.
\[
(\delta W_f)^T_w = \left( \frac{\Delta x}{2} \frac{\Delta y}{2} \frac{\Delta z}{2} \right) \delta a^T \left[ \iint \iint \left( DXW \cdot N_{1x} \cdot N_{1x}^T + DXYW \cdot L_{1x} \cdot N_{1x}^T \right) \right] d\xi d\eta d\zeta + \\
\int \int \int \left( DXYW \cdot N_{1x} \cdot L_{1y}^T + DYW \cdot L_{4y} \cdot L_{4y}^T + DYZW \cdot M_{1z} \cdot M_{1z}^T \right) d\xi d\eta d\zeta + \\
\int \int \int \left( DXZW \cdot N_{1x} \cdot M_{1z}^T + DYZW \cdot L_{1y} \cdot M_{1z}^T + DZW \cdot M_{1z} \cdot M_{1z}^T \right) d\xi d\eta d\zeta + \\
\int \int \int GXYW \cdot \left( N_{1y} + L_{4x} \right) \left( N_{1y} + L_{4x} \right)^T d\xi d\eta d\zeta + \\
\int \int \int GYZW \cdot \left( M_{1y} + L_{4z} \right) \left( M_{1y} + L_{4z} \right)^T d\xi d\eta d\zeta + \\
\int \int \int GXZW \cdot \left( M_{1x} + N_{1z} \right) \left( M_{1x} + N_{1z} \right)^T d\xi d\eta d\zeta \}
\]

= \delta a^T \Theta^T_w \delta a

(3.40)

Where

\[DXW = \frac{E_x}{h} \left[ 1 - \left( v_{yx} \right)^2 \frac{E_z}{E_y} \right]\]

\[DYW = \frac{E_y}{h} \left[ 1 - \left( v_{xz} \right)^2 \frac{E_z}{E_x} \right]\]

\[DZW = \frac{E_z}{h} \left[ 1 - \left( v_{xy} \right)^2 \frac{E_y}{E_x} \right]\]

\[DXYW = \frac{E_y}{h} \left[ v_{xy} + v_{xz} v_{yz} \frac{E_z}{E_y} \right]\]

\[DXZW = \frac{E_z}{h} \left[ v_{xz} + v_{yz} v_{xy} \right]\]

\[DYZW = \frac{E_z}{h} \left[ v_{yz} + v_{xz} v_{xy} \frac{E_y}{E_x} \right]\]

\[GXYW = G_{xy}\]
\[ GYZW = G_{yz} \]
\[ GXZW = G_{xz} \]

All of the definitions of shape functions and material properties can be found in Section 3.1.

### 3.4.3. Internal virtual work in plate element

The internal virtual work of plate element is:

\[
\left( \delta W_i \right)_p = \int_V \delta \varepsilon_x^T \sigma_x dV + \int_V \delta \varepsilon_y^T \sigma_y dV + \int_V \delta \gamma_{xy}^T \tau_{xy} dV
\]

(3.41)

From Equations (3.22) and (3.23), the internal virtual work is:
\[
(\partial W_p) = \frac{\Delta x \Delta y}{2} \delta a^T \left\{ \iint_{\xi \eta} \left( DP \cdot N_{1x}N_{1x}^T + KP \cdot M_{2x}M_{2x}^T \right) d\xi d\eta + \right.
\]
\[
\iint_{\xi \eta} DP \cdot \left[ \frac{1}{2} \left( N_{1x} + M_{1x}M_{1x}^T \cdot a \right) \cdot a^T \cdot M_{1x}M_{1x}^T + M_{1x}M_{1x}^T \cdot a \cdot N_{1x}^T \right] d\xi d\eta + \right.
\]
\[
\iint_{\xi \eta} \left( DGP \cdot L_{1y}N_{1x}^T + KGP \cdot M_{2x}M_{2x}^T \right) d\xi d\eta + \right.
\]
\[
\iint_{\xi \eta} DGP \cdot \left[ \frac{1}{2} \left( L_{1y} + M_{1y}M_{1y}^T \cdot a \right) \cdot a^T \cdot M_{1y}M_{1y}^T + M_{1y}M_{1y}^T \cdot a \cdot L_{1y}^T \right] d\xi d\eta + \right.
\]
\[
\iint_{\xi \eta} DNP \cdot \left( N_{1x}L_{1y}^T + L_{1y}N_{1x}^T \right) + KNP \cdot \left( M_{2x}M_{2x}^T + M_{2y}M_{2y}^T \right) d\xi d\eta + \right.
\]
\[
\iint_{\xi \eta} DNP \cdot \left[ \frac{1}{2} \left( L_{1y} + M_{1y}M_{1y}^T \cdot a \right) \cdot a^T \cdot M_{1y}M_{1y}^T + M_{1y}M_{1y}^T \cdot a \cdot N_{1x}^T \right] d\xi d\eta + \right.
\]
\[
\iint_{\xi \eta} DNP \cdot \left( N_{1y}L_{1x} + L_{1x}N_{1y}^T \right) \left( N_{1y} + L_{1x} \right) + KGP \cdot M_{xy}M_{xy}^T \right] d\xi d\eta + \right.
\]
\[
\iint_{\xi \eta} DGP \cdot \left( M_{1x}M_{1y}^T \cdot a + M_{1y}M_{1x}^T \cdot a \right) \cdot \left( a^T \cdot M_{1x}M_{1y}^T + N_{1y} + L_{1x} \right) d\xi d\eta + \right.
\]
\[
\iint_{\xi \eta} DGP \cdot \left( N_{1y} + L_{1x} \right) \cdot a^T \cdot M_{1x}M_{1y}^T \right] a
\]

(3.42)

where

\[ t = \text{the thickness of plate element} \]

\[ DP = \frac{Et}{1-v^2} \]

\[ DNP = \frac{Evt}{1-v^2} \]

\[ DGP = Gt = \frac{Et}{2(1+v)} \]
\[ KP = \frac{Et^3}{12(1-v^2)} \]
\[ KNP = \frac{Et^3v}{12(1-v^2)} \]
\[ KGP = \frac{t^3G}{3} \]

The above virtual work expression includes two parts, namely the linear part to account for the work done by the elongation and bending of the neutral plane of the steel plate, and the nonlinear part to account for the work done by the large deflection due to P-\(\Delta\) effects. The second nonlinear part is path-dependent, which needs to be updated correspondingly throughout the nonlinear iteration implemented in this study. Therefore, it is convenient to present these two parts of virtual work separately as in Equation (3.43) and (3.44).

\[
\left( \delta W_f^e \right)_{LP} = \frac{\Delta x \Delta y}{2} \delta \mathbf{a}^T \left\{ \iint_{\xi \eta} \left( DP \cdot N_{1x}N_{1x}^T + KP \cdot M_{2x}M_{2x}^T \right) d\xi d\eta + \right.
\]
\[
\left. \iint_{\xi \eta} \left( DGP \cdot L_{1y}L_{1y}^T + KGP \cdot M_{2x}M_{2x}^T \right) d\xi d\eta + \right.
\]
\[
\left. \iint_{\xi \eta} \left[ DNP \cdot (N_{1x}L_{1x}^T + L_{1y}N_{1x}^T) + KNP \cdot (M_{2x}M_{2y}^T + M_{2y}M_{2x}^T) \right] d\xi d\eta + \right.
\]
\[
\left. \iint_{\xi \eta} \left[ DGP \cdot (N_{1y} + L_{1x}) \left( N_{1y} + L_{1y} \right)^T + KGP \cdot M_{xy}M_{xy}^T \right] d\xi d\eta \right\} \mathbf{a}
\]
\[
= \delta \mathbf{a}^T \cdot \Theta_{LP}^e \cdot \mathbf{a}
\]

(3.43)
\[
\left( \delta W_i^{e} \right)_{NP} = \frac{\Delta x \Delta y}{2} \delta a^T \left\{ \int \int \frac{1}{2} \left( N_{1x} + M_{1x} M_{1x}^T \cdot a \right) \cdot a^T \cdot M_{1x} M_{1x}^T + M_{1x} M_{1x}^T \cdot a \cdot N_{1x}^T \right} d \xi d \eta + \\
\int \int DGP \cdot \left[ \frac{1}{2} \left( L_{1y} + M_{1y} M_{1y}^T \cdot a \right) \cdot a^T \cdot M_{1y} M_{1y}^T + M_{1y} M_{1y}^T \cdot a \cdot L_{1y}^T \right] d \xi d \eta + \\
\int \int DNP \cdot \left[ \frac{1}{2} \left( N_{1x} + M_{1x} M_{1x}^T \cdot a \right) \cdot a^T \cdot M_{1x} M_{1x}^T + M_{1x} M_{1x}^T \cdot a \cdot N_{1x}^T \right] d \xi d \eta + \\
\int \int DNP \cdot \left[ \frac{1}{2} \left( L_{1y} + M_{1y} M_{1y}^T \cdot a \right) \cdot a^T \cdot M_{1y} M_{1y}^T + M_{1y} M_{1y}^T \cdot a \cdot N_{1y}^T \right] d \xi d \eta + \\
\int \int DGP \cdot \left( M_{1x} M_{1x}^T \cdot a + M_{1y} M_{1y}^T \cdot a \right) \cdot \left( a^T \cdot M_{1y} M_{1y}^T + N_{1y}^T + L_{1y}^T \right) d \xi d \eta + \\
\int \int DGP \cdot \left( N_{1y} + L_{1x} \right) \cdot a^T \cdot M_{1x} M_{1x}^T \right) a
= \delta a \cdot \Theta_{NP}^e \cdot a
\]

(3.44)

### 3.4.4. Internal virtual work in a tooth connection

The internal virtual work done by a single tooth connection is:

\[
\left( \delta W_i \right)_T = \left[ F_x (\Delta x) \delta \Delta x + F_y (\Delta y) \delta \Delta y + F_z (\Delta z) \delta \Delta z + \\
M_x (\Delta x) \delta \Delta \theta_x + M_y (\Delta y) \delta \Delta \theta_y \right]
\]

From Equation (3.34), the virtual deformations of a tooth connection are:

\[
\delta \Delta x = Q_x^T \cdot \delta a
\]

(3.46a)

\[
\delta \Delta y = Q_y^T \cdot \delta a
\]

(3.46b)

\[
\delta \Delta z = Q_z^T \cdot \delta a
\]

(3.46c)

\[
\delta \Delta \theta_x = QQ_x^T \cdot \delta a
\]

(3.46d)

\[
\delta \Delta \theta_y = QQ_y^T \cdot \delta a
\]

(3.46d)
Therefore, substitute Equation (3.46) into Equation (3.45), the internal virtual work of a tooth connection element can be rewritten as

\[
(\delta W_i)_T = \left[ F_x (\Delta x) \cdot Q_x^T \cdot \delta a + F_y (\Delta y) \cdot Q_y^T \cdot \delta a + F_z (\Delta z) \cdot Q_z^T \cdot \delta a + M_x (\Delta x) \cdot Q_{x}^T \cdot \delta a + M_y (\Delta y) \cdot Q_{y}^T \cdot \delta a \right] (3.47)
\]

### 3.4.5. Formulation of system equations

Given the internal virtual work for the wood brick element, plate element and tooth connection element, adapt the Equation (3.36) for the formulation of MPC truss joint system yields

\[
\delta W = \sum (\delta W_i^e)_p + \sum (\delta W_i^e)_w + \sum (\delta W_i)_T - \sum \delta a^T R_c = 0 \tag{3.48}
\]

Differentiate Equation (3.48) with respect to \( \delta a \), the out-of-balance load vector \( \Psi \) can be obtained:

\[
\Psi = \sum \Psi_w + \sum \Psi_p + \sum \Psi_T - R_c \tag{3.49}
\]

From Equation (3.40), (3.43), (3.44) and (3.47), the out-of-balance force components at element level are:

\[
\Psi_w = \Theta_w a \tag{3.50a}
\]

\[
\Psi_p = (\Theta_{LP}^e + \Theta_{NP}^e) \cdot a \tag{3.50b}
\]

\[
\Psi_T = \left[ F_x (\Delta x) Q_x^T + F_y (\Delta y) Q_y^T + F_z (\Delta z) Q_z^T + M_x (\Delta x) Q_x^T + M_y (\Delta y) Q_y^T \right] \tag{3.50c}
\]

By differentiating Equation (3.50) with respect to \( \delta a \), the tangent stiffness matrix can be obtained to conduct Newton-Raphson iteration, which is briefly described in Section 3.6.
\[ \frac{\partial \psi}{\partial \mathbf{a}} = \sum \Theta^e_w + \sum \Theta^e_{LP} + \sum \Theta^e_{NP} + \]
\[ \sum \left[ \frac{\partial F_x(\Delta x)}{\partial \Delta x} \cdot Q_x \cdot Q^T_x + \frac{\partial F_y(\Delta y)}{\partial \Delta y} \cdot Q_y \cdot Q^T_y + \frac{\partial F_z(\Delta z)}{\partial \Delta z} \cdot Q_z \cdot Q^T_z \right] \]
\[ + \sum \left[ \frac{\partial M_x(\Delta x)}{\partial \Delta x} \cdot QQ_x \cdot Q^T_x + \frac{\partial M_y(\Delta y)}{\partial \Delta y} \cdot QQ_y \cdot Q^T_y \right] \]

In Equation (3.51), the terms of \( \Theta^e_w \) and \( \Theta^e_{LP} \) are linear, and the rest components are nonlinear, namely path dependent.

It’s important to note that the out-of-balance force vector and tangent stiffness matrix for all FEM element types are constructed in local element coordinate system; however, following the FEM procedures, the structural out-of-balance force vector and tangent stiffness matrix need to be in global coordinate system. Therefore, special attention has to be paid to the coordinate transformation as described in Section 3.7.

### 3.5. Compatibility and completeness of shape function

While constructing the shape functions as described in the previous sections, some specific requirements (compatibility and completeness) have been considered to ensure that approximate solutions converge to reasonable accurate results.

The specification of compatibility depends on how strains are defined in terms of derivatives of the displacement fields. For example, for the brick element, the strains are defined by first derivative of the displacement fields, and then the continuity of the displacement fields across element edges has to be satisfied. This is called \( C^0 \) continuity. However, for the thin plate element, the strains are based on second derivatives of
displacement fields. In this case, continuity of first derivatives of displacement fields across element edge is demanded. This is called $C^1$ continuity.

The completeness requires that the shape functions of the element must be able to represent the rigid body displacements and constant strain states. This conditions can be simply checked by $\sum N_i = 1$, which is fulfilled for all elements used in this thesis.

### 3.6. Newton-Raphson iteration and convergence criteria

As has been noted in the previous sections, the model developed for MPC joints is nonlinear. The nonlinearity has been introduced by the large deflection of the plate element enforced by P-$\Delta$ effect, the nonlinear tooth-to-wood interaction, and the tooth withdrawal process. The nonlinear finite element equations in this study are solved through the Newton-Raphson iteration procedure.

Suppose the initial guess of the structural displacement vector is $a_0$ (Figure 3.7), initially, the out-of-balance force can be expanded about $a_0$ using Taylor series expansion as follows:
\[ \Psi(\mathbf{a}) = \Psi(\mathbf{a}_0) + \left. \frac{\partial \Psi}{\partial \mathbf{a}} \right|_{\mathbf{a} = \mathbf{a}_0} \cdot \Delta \mathbf{a}_0 = 0 \]  

(3.52)

Let \( K_T \) denote the tangent stiffness matrix, then

\[ K_T = \frac{\partial \Psi}{\partial \mathbf{a}} \quad (3.53) \]

The Equation (3.52) then becomes:

\[ \Psi(\mathbf{a}_0) = -K_{T0}(\mathbf{a}_0) \cdot \Delta \mathbf{a}_0 \]  

(3.54)

And the correction of the initial \( \mathbf{a}_0 \) can be calculated by:

\[ \Delta \mathbf{a}_0 = \frac{-K_{T0}(\mathbf{a}_0)}{\Psi(\mathbf{a}_0)} \]  

(3.55)

A new estimated displacement vector can be obtained by:

\[ \mathbf{a}_1 = \mathbf{a}_0 + \Delta \mathbf{a}_0 \]  

(3.56)

A repeating procedure continues until prescribed convergence criteria are satisfied and the solution is achieved within current load or displacement step.

The model uses two relative convergence criteria:

\[ \frac{|\Psi_i|}{|\Psi_f|} \leq \varepsilon_f \quad \text{or} \quad \frac{|\Delta \mathbf{a}_i|}{|\mathbf{a}_j|} \leq \varepsilon_d \]  

(3.57)

where \( |\Psi_i| \) and \( |\mathbf{a}_i| \) are the norms of the out-of-balance force vector and the incremental displacement at the end of the \( i^{th} \) iteration within one loading step, while \( |\Psi_f| \) and \( |\mathbf{a}_j| \) are the norms of the initial out-of-balance force vector and the initial incremental displacement vector at the first iteration within the loading step.

The selection of the tolerance is really a matter of the balance between the computational cost and the precision of the solution. A tolerance level of 1.0E-3 is carefully chosen for both the out-of-balance force and the incremental displacement at the
early stage of iteration. As the tooth starts to pull out of the wood according to the prescribed tooth withdrawal criterion, the iteration goes into nonlinear stage and the tolerance is loosened by certain amount accordingly, such that the program can be computationally efficient with acceptable accuracy.

3.7. Tooth withdrawal and failure criteria

According to the reviewed publications, researchers have conducted a great deal of work on modeling the truss joint behavior to evaluate the strength and stiffness of MPC truss joints, which has provided favorable results compared with corresponding joint tests. However, because of the complexity of the composite structure of MPC truss joints and many affecting factors, scarce literature has addressed computational modeling of MPC connection failure in tooth withdrawal. The implementation of this failure criterion in MPC truss joint model would tend to be a great exploration, and also be one of major distinctive features of this thesis.

In general, four types of failure modes are possible while MPC connection under different loading conditions: tooth withdrawal, wood splitting, plate buckling and plate yielding in tension or shear. Sometimes these four types of failure would not occur independently, but intertwine with one another depending on the joint configuration and loading condition. For example, tooth withdrawal concurs with wood splitting or plate yielding, etc. For MPC truss joints subjected to tension and out-of-plane bending, which are of most interest in this thesis, the major failure mode is tooth withdrawal referring to the literature reviewed and the joint tests conducted later in Chapters 4 and 6. Thus the tooth withdrawal is considered as the only failure mode in this study.
Since the wood medium underlying the tooth has elastic-plastic property, when loading occurs, the medium experiences lateral plastic deformations. And these deformations are not uniform along the tooth bearing surfaces, but gradually increase to the maximum at the tooth base. That is the closer the location is to the tooth base, the bigger the lateral deformation is. When a tooth start to withdraw, the lower portion of the tooth moves upwards and tends to fit into the bigger gap left by the adjacent upper portion; meanwhile a void is formed at the tooth tip. As the tooth is gradually pulled out by a prescribed amount (say half of the total length as an example), the tooth is considered failed and no longer contribute to the strength and stiffness of the joint. When a critical amount of the teeth fail in this way, the MPC joint can be considered to be failed, due to it can’t provide the adequate strength and/or stiffness to withstand the load.

To closely simulate the withdrawal behavior in FEM implementation, a specially designed analytical scheme is applied. As illustrated in Figure 3.8, a single tooth has been discretized into \( n \) beam-column elements (or segments), and each element has the length of \( L/n \). \( L \) is the total length of the tooth. For simple description, one lateral spring element is assumed set on each segment to account for the lateral resistance from the wood medium, thus there are \( n \) lateral spring elements along the tooth length, denoted as \( s_1 \) to \( s_n \). \( u_i \) denotes the lateral displacement of spring element \( i \). Superscripts I and II denote the withdrawal steps.
When the withdrawal displacement of a tooth, which is calculated from the relative deformation between the plate and the wood in Equation (3.31), is greater than a segment length, Step I in Figure 3.8a takes place: the top beam element is assumed pulled out that leaves a void in size of a tooth segment at the tip (illustrated in dashed line). Then the lateral displacements of the springs have to be updated to reflect the change by assigning the displacement $u_{2I}$ of spring $s_2$ to spring $s_1$, $u_{3I}$ to spring $s_2$, and so on and so forth. The withdrew portion of the tooth $s_n$ is set free by not integrating its stiffness components into HYST structural stiffness matrix. When two tooth segments are pulled out from the wood as shown in Figure 3.8b, the further displacement updates occur accordingly. The repetitive procedure continues until tooth failure criteria are satisfied.

As is found out, the tooth withdrawal failure criterion may vary with the joint configuration and the loading condition. Thus the specific failure criteria will be discussed later with the corresponding MPC joint configuration and loading cases.
3.8. Coordinate transformation of elements in 3D space

In the process of forming the FEM formulation of MPC truss joints, the coordinate transformations are important due to the need of transforming displacements, forces as well as stiffness matrices from local (or elemental) coordinate system into global (or structural) coordinate system, or vice versa.

3.8.1. Vector transformation

Consider a point $A$ in a 3D space as in Figure 3.9, which may as well be considered as a vector, whose components in the $x$, $y$ and $z$ direction are $x_1$, $y_1$ and $z_1$, and $x'_1$, $y'_1$ and $z'_1$ in $x'$, $y'$ and $z'$ coordinate system. Thus,

$$
\begin{bmatrix}
    x'_1 \\
    y'_1 \\
    z'_1
\end{bmatrix} = T_{3x3} \begin{bmatrix}
    x_1 \\
    y_1 \\
    z_1
\end{bmatrix}
$$

where $T_{3x3}$ is the transformation matrix taking the form:

$$
T_{3x3} = \begin{pmatrix}
    l_1 & m_1 & n_1 \\
    l_2 & m_2 & n_2 \\
    l_3 & m_3 & n_3
\end{pmatrix}
$$
And the components $l_i$, $m_i$, and $n_i$ are defined as the cosine of the angle between axes indicated in Table 3.1

Table 3.1 The angle reference for transformation matrix

<table>
<thead>
<tr>
<th></th>
<th>$x'$</th>
<th>$y'$</th>
<th>$z'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$l_1$</td>
<td>$m_1$</td>
<td>$n_1$</td>
</tr>
<tr>
<td>$y$</td>
<td>$l_2$</td>
<td>$m_2$</td>
<td>$n_2$</td>
</tr>
<tr>
<td>$z$</td>
<td>$l_3$</td>
<td>$m_3$</td>
<td>$n_3$</td>
</tr>
</tbody>
</table>

The transformation matrix $T_{3x3}$ is orthogonal, which means its inverse is equal to its transpose. Therefore the inverse of the transformation in Equation (3.58) is

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = T_{3x3}^{-1} \begin{bmatrix} x_1' \\ y_1' \\ z_1' \end{bmatrix}$$

(3.60)

By its definition, vector $A$ can also represent a displacement vector or a force vector.

### 3.8.2. Stiffness matrix transformation

Following the standard procedure in FEM, the elemental stiffness matrices are first constructed in local coordinate system, and then transformed into the global coordinate system before being assembled into a global stiffness matrix. The plate element and the brick element are developed separately in their own local coordinate systems. However, given the global coordinate coincides with the plate local coordinate
system, only the stiffness of the wood element is needed to be transformed from the local coordinate system into the global coordinate system.

According to Equation (3.58), given the global displacements \( u, v \) and \( w \) at any point in a brick element, the local displacements \( u', v' \) and \( w' \) can be achieved by

\[
\begin{align*}
\begin{bmatrix}
u' \\
v' \\
w'
\end{bmatrix} &= T_{3\times3} \begin{bmatrix}
u \\
v \\
w
\end{bmatrix} \\
&= T_{3\times3} \begin{bmatrix}
u' \\
v' \\
w'
\end{bmatrix}
\end{align*}
\]

(3.61)

And thus the stiffness matrix transformation for the brick element is given by

\[
K_{wg} = T_w^T K_{we} T_w
\]

(3.62)

Where \( K \) is the stiffness matrix of a brick element, \( T \) is the stiffness transformation matrix, and the subscription ‘w’ denotes wood member, ‘wg’ and ‘we’ denote that the stiffness matrices are defined in the global or local coordinate system, respectively. \( T_w \) is defined in Equation (3.63).

\[
T_w = \begin{bmatrix}
T_{3\times3} & 0 & 0 \\
0 & T_{3\times3} & 0 \\
0 & 0 & T_{24\times24}
\end{bmatrix}
\]

(3.63)

As noted in Section 3.3.4, the deformations of a tooth were calculated from the DOFs of associated plate element and wood element. The calculation can only be carried out when the displacement vectors of the plate element and wood element are in the same
coordinate system, namely the global coordinate system. Then the stiffness coefficients of
the associated wood element need to be transformed from local to global, while the ones
relating the plate element remain unchanged. Therefore the connection stiffness
transformation is:

$$K_{ng} = T_n^T K_{ne} T_n$$  \hspace{1cm} (3.64)

where the subscription ‘n’ denotes the n\textsuperscript{th} tooth connection element. $K_{ng}$ and $K_{ne}$ are the
stiffness matrices of the n\textsuperscript{th} tooth connection element, which are defined in global and
elemental coordinate system respectively. $T_n$ is the stiffness transformation matrix, and
defined in Equation (3.65).

$$T_c = \begin{bmatrix}
I_{7 \times 7} & I_{7 \times 7} & I_{7 \times 7} & 0 \\
I_{7 \times 7} & I_{7 \times 7} & T_{3 \times 3} & T_{3 \times 3} \\
I_{7 \times 7} & T_{3 \times 3} & T_{3 \times 3} & T_{3 \times 3} \\
0 & T_{3 \times 3} & T_{3 \times 3} & T_{3 \times 3} \\
& T_{3 \times 3} & T_{3 \times 3} & T_{3 \times 3} \\
& & T_{3 \times 3} & 52 \times 52
\end{bmatrix}$$  \hspace{1cm} (3.65)

where, I is an identity matrix of the size 7x7.
3.9. Summary

In this chapter the formulation of FEM based elements (wood brick element, thin plate element and tooth connection element) has been described, and a numerical model SAMPC has been developed based on FEM principles. Due to the fact that the model is nonlinear, the Newton-Raphson iteration is employed, and specified convergence criteria are set to achieve the solution. The tooth withdrawal failure mode is identified as the only failure mode in this thesis to explore the possibility of predicting the MPC truss joint failure in particular loading conditions. The coordinate transformation has been briefly described here to introduce how force vector, displacement vector and stiffness matrix in local coordinate system can be transformed into global coordinate system, and vice versa.
Chapter 4. Experimental Study

Introduction

Six types of MPC truss joints were tested to calibrate the modeling parameters and verify the numerical model SAMPC. These six joint types include four standard types and two non-standard ones. The material specification, specimen fabrication, testing procedures, and data analyses are described in this chapter.

4.1. Material

All lumber used in this study was machine-stress-rated (1650f-1.5E) nominal two-by-four (38x89mm) Spruce-Pine-Fir (SPF) lumber purchased from a local lumber company. The lumber was cut into 2.1-meter (7 ft) long sections due to the size limitation of the conditioning chamber. The lumber sections were conditioned in a controlled chamber of 20°C and 65% relative humidity for at least one month until constant weight was achieved, and then the MOE of the lumber was determined using an E-Computer on the basis of vibration frequency prior to specimen fabrication.

20-gauge (0.9 mm or 0.036 in in thickness) metal truss plates were commercial products supplied by plate manufacturer Mitek Canada, Inc. These plates are commonly used in light-frame truss construction. The plates had slot teeth layout (shown in Figure 4.1), and were 75 mm x 125 mm (3 by 5 in) in area, with a tooth density of 8 teeth per 25.4 mm x 25.4 mm (square inch). Average tooth length, width and thickness were 9.5 mm (3/8 in), 3.18 mm (or 0.125 in), and 0.9mm (or 0.036 in), respectively.
4.2. Specimen fabrication

Six joint types were assembled and tested. Joint type AA, AE, EA and EE (Figure 4.1) were four standard joint configurations discussed in the CSA standard S347 (2004), and tested to obtain model input parameters. These four cases represented four simple force-plate-grain orientations: AA — plate major direction (along the slot length direction) parallel to load, wood grain parallel to load; AE — plate perpendicular to load, wood grain parallel to load; EA — plate perpendicular to load, wood grain parallel to load; EE — plate perpendicular to load, wood grain perpendicular to load. The two non-standard configurations (Figure 4.2) included the joints with plate to grain angles of 45 and 60 degrees, which were tested on the purpose for model verification.
During the specimen fabrication, CSA S347-99 (2004) – Method of Test for Evaluation of Truss Plates Used in Lumber Joints – was followed. Thirty joint specimens were fabricated for each of the four standard joint types, and 25 joint specimens for each of the two non-standard types, which met the minimum number requirement of 10 specimens of CSA standard. No knots were located in the region of truss plate
embedment. The teeth within the end distance of 12 mm and the edge distance of 6 mm were removed to meet the end and edge distance requirement. Since the tested plates were relatively small in size compared with the commercial fabrication, no keeper nails needed and installed.

Furthermore, additional considerations were given on plate allocation. Plates were overplated to induce failure in the teeth to occur on only one half of the tested joints, which had the designated plate-to-grain orientation. For instance of four standard joint types, as shown in Figure 4.1, about 30% of the plate area was placed on one wood member, which had the designated plate-to-grain orientation, while 70% of the plate area on the other wood member. Meanwhile the teeth within the dashed line box near the wood-to-wood contact were removed before the plates were installed. This not only met CSA S347 requirement of the end distance of 12 mm, but also produced relatively large end distance, about 19.6mm (or 0.773 in) for case AA and EA and 14.3mm (or 0.5625 in) for AE and EE, to eliminate possible wood-block-shear-through failure, which is a brittle failure as the end distance is insufficient in the tensile case. In doing so, for all four standard cases, the joint half of interest (or tested joint half) contained 24 teeth in each of the two side plates while 72 teeth on the opposing wood member.

This procedure assumed that the joint half with more teeth produces none or small enough plate-to-wood slip to make its contribution to the total displacement insignificant. To assess this assumption, the perimeter of the plate on that half of joint was highlighted with thin reference lines before test. During and after testing, the plate was visually inspected by being compared with the initial reference mark. There was no pronounced
plate-to-wood slip and plate distortions observed on that half of joint. Therefore, in the following, only the behavior of tested joint half was investigated.

The similar unbalanced plating arrangement and other adjustments were implemented to two non-standard joint types. For instance, during the pre-test of 45-degree joint to obtain the testing rate, it’s found that the tooth withdrawal failure occurred on the half of the joint where the load was applied, which implied the lateral restraint on tested joint half was insufficient. A strip of metal plate (about 1 in, as marked in dashed line in Figure 4.3), which contained 24 teeth, was then trimmed to force failure in the tested half.

A specially designed pressing jig and a universal testing machine Sintech with 245 kN capacity were used to press the plate onto the wood (Figure 4.3). The pressing jig was designed to restrict movement of the wood member and the truss plate, and to permit accurate alignment of the truss plate onto the joint. Wood members were butted tightly against one another before the plate were installed. Only one plate was pressed at a time. The plate was pressed into the wood so that the teeth were fully embedded in the lumber and no gaps remained between the plate surface and the wood, which was in accordance with CSA standard. Meanwhile with the facilitation of the jig, the plates were ensured no embedment in the lumber.
4.3. Specimen tests

After fabrication, the joints were again stored in the condition chamber for at least 7 days prior to testing for allowing for relaxation of stresses induced by pressing.

The test setups regarding four standard and two non-standard joint tests were shown in Figure 4.4. Steel grips were designed to hold the joints in place and apply loads with steel rods. The loading or holding grips were at least 125 mm (5 in) away from the truss plates or the displacement monitor devices, which ensured these grips didn’t interfere with the joint tests, so that the forces or constraints can be assumed evenly distributed over the cross section near the connections. In order to minimize the bending moment effects on the plate, the load was applied in alignment with the centroid of the plate.
a. the test setup of joint AA and EA

b. the test setup of joint AE and EE

c. the test setup of 60-degree joint

d. the test setup of 45-degree joint

Figure 4.4 The joint test setups

One or two prototype specimens from each joint type were tested to assess the test procedure, estimate the load capacity, and thus set the speed for the following tests
accordingly. The displacement-controlled test speed of 1mm/min was selected to attain maximum load for about 5 minutes, which was consistent with the CSA S347 recommendation that failure should occur in not less than 5 minutes or more than 10 minutes. Almost all joints were tested beyond maximum load except for a few joints that failed in a brittle wood failure. Two displacement transducers (linear variable differential transducers, or LVDTs) were used to measure relative displacements between the two wood members. Force and displacement were continually collected by a computer data acquisition system.

Depending on joint configuration, the transducers recorded the displacement at different locations. For instance, marked in Figure 4.4, transducers were placed 50 mm (or 2 in) from wood-to-wood contact on the tested joint half in case AA and EA, right on the thickness surface in case AE and EE, 32 mm (1-1/4 in) in 60-degree case, and 16 mm (5/8 in) on one side and 32 mm (1-1/4 in) on the other in 45-degree case.

Right after testing at the end of each day, a 25 mm (1 in) wide wood sample was cut from the location near the plate on the tested half of the joint to test for specific gravity (SG) and moisture content (MC). To avoid the time variation, the samples were marked, sealed in individual zipper bags and stored in an air tight container until they were tested as a group.
4.4. Test results and analysis method

4.4.1. Test results for MOE, SG and MC

The modulus of elasticity (MOE) of the lumber, as measured with E-computer, ranged from 7.41 GPa to 12.56 GPa with a mean value of 9.40 GPa and a coefficient of variation of 11%. The MOE values for all tested lumber are listed in Appendix A.

ASTM D 2395 Method A and D 4442 Method A (2008) were referred to determine SG and MC, respectively. The average SG and MC regarding to specific joint type are listed in Table 4.1, and the values for each tested specimen are listed in Appendix A.

4.4.2. Failure mode

Generally, the six joint configurations exhibited quite similar major failure pattern, namely teeth withdrawal associated with plate peeling, but with some variations.

Almost all specimens of cases AA and EE, in which the tensile force was applied parallel to wood grain orientation, failed in teeth withdrawal associated with plate peeling (Figure 4.5). As the loading proceeded, the total load was transferred to the bearing wood medium through the tooth-wood interface. When the rows of teeth started to crush the wood, the gaps at the tooth-wood interfaces started to form. The gaps at the first row of teeth (furthest to the wood-to-wood contact) always occurred sooner and became wider over time than the teeth of the adjacent row. Due to the peeling moment caused by eccentric lateral resistance from the medium, the plate started to peel and the first row of teeth which had wider gap started to pull out; and meanwhile the load redistribution
occurred spontaneously throughout the process. When the first row of teeth withdrew by a certain amount, about 1/2 of the tooth length from visual observation, the tested specimen reached its maximum load, and the teeth on the other rows had also withdrawn by a small amount. It was assumed that at this moment, the first row had little or no engagement in load bearing, and the remainder of the teeth carried the load.

![Figure 4.5 Tooth withdrawal failure](image)

Besides the tooth withdrawal and plate peeling failure, which was most frequent, wood splitting was also observed in AE and EE cases (Figure 4.6 and Figure 4.7), where the force was perpendicular to grain direction. Due to the fact that wood is weak in tension perpendicular to grain, with the widening of the tooth-wood interface gaps, small wood cracks occurred at some teeth. And these cracks developed over time and connected with the cracks at adjacent teeth to form a line at either first row (Figure 4.6) or second row of teeth (Figure 4.7). The specimens, which developed the splitting line at first row of teeth, tended to have brittle wood failure with relatively high maximum load; otherwise for the splitting lines occurred at the second row as the first row of teeth had already withdrawn from the wood, the maximum load occurred before wood splitting.
Likewise, most of two non-standard joint specimens failed in tooth withdrawal associated with plate peeling, and the tooth withdrawal tended to start at the plate corner. Wood splitting occasionally occurred along grain (Figure 4.8 and Figure 4.9).
Figure 4.8 The failure of 60-degree joint

Figure 4.9 The failure for 45-degree joint

It is important to note that in most cases, the joint failure was non-symmetric. That is during the loading, the teeth on one side plate tended to pull out from the wood sooner than the teeth on the other side plate (Figure 4.10). This may have been caused by the natural variability in localized wood properties (such as the presence of undetected defects within the plated area) and/or joint fabrication. Even though caution was exercised during joint fabrication not to under- or over-press the plates, one of the plates may be under-pressed or over-pressed causing the non-symmetrical behaviour.
4.4.3. Results of joint tests

The data from a few joint specimens were discarded due to several reasons, such as pilot joints tested to acquire proper test speed, a few 45-degree joint specimens tested before configuration adjustment, fixture failure and failure at locations other than near the plates.

Test data for all joints were plotted and fitted into curves. The average of the two transducer readings was used to avoid any errors due to asymmetric specimen deflections in the four standard joint cases. In two non-standard cases, due to the asymmetric configurations, separate data curves were plotted and fitted based on these two transducers’ recordings to represent the load-displacement relationships of the joint.

Foschi’s (2000) five-parameter \( (Q_0, Q_1, Q_2, K \text{ and } D_{\text{max}}) \) exponential model in Equation (4.1) and Figure 4.11 was employed to fit the load-displacement curves of all joints. This model gave a complete load-displacement curve fitting from zero load to failure.
\[
P(\Delta) = \begin{cases} 
(Q_0 + Q_1 \Delta)(1.0 - \exp(-K\Delta/Q_0)) & \text{if } \Delta \leq D_{\text{max}} \\ 
\exp(Q_3 (\Delta - D_{\text{max}})^2) & \text{if } \Delta > D_{\text{max}}
\end{cases}
\]

where \(P_{\text{max}} = (Q_0 + Q_1 D_{\text{max}})(1.0 - \exp(-K D_{\text{max}}/Q_0))\)

\[
Q_3 = \frac{\log 0.8}{\left((Q_2 - 1.0) D_{\text{max}}\right)^2}
\]

Figure 4.11 Foschi’s 5-parameter exponential equation

The following method was used to define the appropriate curve-fitting parameters for each joint data. First, a computer-based least square regression technique was used to determine the five parameters for each joint data. Then the parameters defining the average load-displacement relationship for each joint type were obtained by averaging the parameter values from the individual tests.

The derived average load-displacement parameters for six joint configurations are listed in Table 4.1. And the curve fitting parameters for each recorded joint specimen along with specimen’s SG and MC, peak load as well as R square value, are listed in Appendix A. Figure 4.12, Figure 4.13, Figure 4.14 and Figure 4.15 show the fitted load-
displacement curves for individual tests and average curves for four standard joint tests. It should be noted that for two non-standard cases, the load-displacement data were plotted and fitted for each of the two transducers and are denoted as 60-1, 60-2, 45-1 and 45-2 in Figure 4.16, Figure 4.17, Figure 4.18 and Figure 4.19.

It was found that Foschi’s model fitted with the test data quite well that the coefficient of determination (R square value) was around 0.99 and the calculated parameters were stated in Appendix A. And the coefficient of variation for parameter K in Table 4.1 varied between 0.063 and 0.143, and the other parameters also have quite small variation ranges. This might indicate that the quality of fabricated joint specimens was quite uniform, given the coefficients of variation of MOE and SG were small too, and the sample sizes were sufficient for this type of test. Therefore, it could be concluded that the average curves were adequate to represent the load-displacement relationship of all tested joint types in later model calibration and verification analyses, which are described in Chapter 5.

The average load-displacement curves of four standard joint types were compared and are shown in Figure 4.20. Containing the same 24 teeth on the tested joint half, types AA and EA, which were loaded in tension parallel to wood grain, had higher load capacity and initial stiffness than those of types AE and EE, which were loaded in tension perpendicular to grain. This is consistent with the fact that wood is weak in tension perpendicular to grain. And the curve of AA or AE showed a quite resemblance to that of EA or EE.
Despite the random material variability in wood, there are three distinctive variations between joint type AA and EA, or AE and EA: the tooth-wood bearing area (AA or AE has wider tooth-wood bearing area than EA or EE), the moment of inertia of the tooth cross section (the tooth beam is subjected to the transverse load along its strong axis of the cross section in case EA and EE), and the tooth layout (AA or AE has 2 rows and 12 teeth on each row, while EA or EE has 4 rows and 6 teeth on each row). Now the question is how these three aspects work in determining the behavior, and in turn the strength and stiffness of MPC joints in tension. An elaborate mathematical simulation would be employed to gain an insight of this question in the next chapter.

![Load-displacement curves (AA)](image)

Figure 4.12 Load-displacement curves for AA truss joint
Figure 4.13 Load-displacement curves for AE truss joint

Figure 4.14 Load-displacement curves for EA truss joint

Figure 4.15 Load-displacement curves for EE truss joint
Figure 4.16 Load-displacement curves for 60-1 truss joint

Figure 4.17 Load-displacement curves for 60-2 truss joint

Figure 4.18 Load-displacement curves for 45-1 truss joint
Figure 4.19 Load-displacement curves for 45-2 truss joint

Figure 4.20 Comparison of the load-displacement relationships for four standard joint tests
Table 4.1 Joint test results and curve fitting parameters

<table>
<thead>
<tr>
<th>Joint Type</th>
<th>Number of joints recorded</th>
<th>Maximum load (kN)</th>
<th>Q0 (kN)</th>
<th>Q1 (kN/mm)</th>
<th>K (kN/mm)</th>
<th>DMAX (mm)</th>
<th>Q3</th>
<th>SG (g/cm³)</th>
<th>MC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>28</td>
<td>Average c.o.v.</td>
<td>5.839</td>
<td>4.067</td>
<td>3.185</td>
<td>53.841</td>
<td>0.598</td>
<td>2.000</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.014</td>
<td>0.027</td>
<td>0.064</td>
<td>0.039</td>
<td>0.035</td>
<td>0.051</td>
<td>0.08</td>
</tr>
<tr>
<td>AE</td>
<td>30</td>
<td>Average c.o.v.</td>
<td>4.038</td>
<td>4.484</td>
<td>0.078</td>
<td>14.596</td>
<td>0.775</td>
<td>1.910</td>
<td>0.47</td>
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<td></td>
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<td>0.024</td>
<td>0.071</td>
<td>4.514</td>
<td>0.029</td>
<td>0.056</td>
<td>0.098</td>
<td>0.08</td>
</tr>
<tr>
<td>EA</td>
<td>30</td>
<td>Average c.o.v.</td>
<td>5.223</td>
<td>4.150</td>
<td>2.043</td>
<td>51.314</td>
<td>0.565</td>
<td>2.587</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.012</td>
<td>0.037</td>
<td>0.118</td>
<td>0.037</td>
<td>0.022</td>
<td>0.026</td>
<td>0.06</td>
</tr>
<tr>
<td>EE</td>
<td>28</td>
<td>Average c.o.v.</td>
<td>3.886</td>
<td>4.820</td>
<td>-0.389</td>
<td>15.925</td>
<td>0.618</td>
<td>2.149</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.028</td>
<td>0.062</td>
<td>0.841</td>
<td>0.024</td>
<td>0.040</td>
<td>0.052</td>
<td>0.09</td>
</tr>
<tr>
<td>60-1</td>
<td>22</td>
<td>Average c.o.v.</td>
<td>7.703</td>
<td>10.669</td>
<td>0.312</td>
<td>18.846</td>
<td>0.829</td>
<td>1.569</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.030</td>
<td>0.178</td>
<td>4.323</td>
<td>0.061</td>
<td>0.052</td>
<td>0.039</td>
<td>0.07</td>
</tr>
<tr>
<td>60-2</td>
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<td>5.671</td>
<td>2.919</td>
<td>29.926</td>
<td>0.870</td>
<td>1.660</td>
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<td></td>
<td></td>
<td></td>
<td>0.030</td>
<td>0.060</td>
<td>0.151</td>
<td>0.063</td>
<td>0.069</td>
<td>0.056</td>
<td>0.07</td>
</tr>
<tr>
<td>45-1</td>
<td>18</td>
<td>Average c.o.v.</td>
<td>6.769</td>
<td>3.914</td>
<td>5.015</td>
<td>24.233</td>
<td>0.717</td>
<td>1.628</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.038</td>
<td>0.126</td>
<td>0.123</td>
<td>0.077</td>
<td>0.082</td>
<td>0.139</td>
<td>0.08</td>
</tr>
<tr>
<td>45-2</td>
<td>18</td>
<td>Average c.o.v.</td>
<td>6.769</td>
<td>7.586</td>
<td>1.496</td>
<td>16.264</td>
<td>1.175</td>
<td>1.657</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.038</td>
<td>0.112</td>
<td>0.681</td>
<td>0.143</td>
<td>0.100</td>
<td>0.064</td>
<td>0.08</td>
</tr>
</tbody>
</table>
4.5. Summary

This chapter described the experimental study on the MPC truss joints tested under tension. The tested joints include the four standard joint types and two non-standard joint types. The joint failure modes were discussed, and the potential reasons for these failure patterns were explored. The test data processing techniques were applied to obtain the tested truss joint’s load-displacement relationships, which will be used as a reference for the model parameter calibration and model verification discussed in the next chapter.
Chapter 5. MPC Truss Joint Parameter Calibration and Model Verification

Introduction

This chapter describes the input and output parameters of FEM-based program SAMPC developed in Chapter 3. By using the results of four standard joint tests described in Chapter 4, SAMPC was calibrated with the modeling parameters for spring elements. By comparing the model predictions with the test results from two non-standard joints, the model was evaluated for its accuracy and robustness.

5.1. Assumptions

By applying the numerical formulation described in Chapter 3, SAMPC was constructed to simulate the structural behavior of six types of MPC joints tested in tension, which has been described experimentally in Chapter 4. Herein, the inputs for the model geometrical configuration, corresponding boundary conditions and loading conditions are discussed.

During the development of model inputs, great efforts had been devoted to try to simulate the geometry, boundary conditions and loading conditions of the actual truss plate joints. However, due to the intrinsic complexity of the problem on hand, several carefully designed idealizations and assumptions were implemented, as the MPC joint model was constructed mathematically. By doing so, the modeling procedure was
simplified and the computational cost was reduced, but more importantly the essential components or features of the problem at hand were kept:

(1) Only one-fourth of the joint is modeled. Noting the utilization of over-plating scheme in joint fabrication, the load-displacement relationship of the joint is assumed only attributed to the displacement of the tested joint half (with desired plate-grain orientation), and the relative displacement of truss plate and the wood member on the other half is assumed zero. Thus the tested joint half, which consists of one wood member, corresponding metal plate area and tooth-wood connection, is considered in this model. Additionally, all modeled truss plate joints are geometrically symmetric with respect to the neutral plane parallel to the truss plates. These altogether make it possible to use the one-fourth of joint with one side plate to simulate the behavior of the truss joint.

(2) It’s possible to simulate the whole wood piece of the tested joint half by remaining its full length, but this would demand a substantial increase of

Figure 5.1 Modeling schematics
computational time. Thus the wood member is truncated at the point where the pinned gripping connection would not interfere with the plate connection, so that the restraining force from the gripping connection can be considered evenly distributed over the wood member cross section at the truncated surfaces, as shown in Figure 5.1. The appropriate boundary conditions are applied accordingly and addressed in Assumption (12).

(3) The truss plate is manufactured with staggered rows of slots with 2.54 mm (0.1 in) from one another row (Figure 5.2a). To include this feature in the program would complicate the analysis significantly, and may not contribute to the accuracy of the evaluation of joint strength and stiffness much. Instead, through shifting the slots towards the slot center by 1.27 mm (0.05 in), the slots are arranged in alignment so as to facilitate plate element meshing (Figure 5.2b). As a result, the overall shape of the plate, the number of teeth, and the teeth density are kept the same.
(4) To address the slot openings on the plate in the procedure of constructing the stiffness matrix of the structure, these slot elements are considered zero contribution to the overall structure, or in other words, zero stiffness. By doing so, the unfavorable complexity of force concentration around the opening is ignored.

(5) To reduce the computational time, a special meshing scheme is employed to the wood member. The fine mesh is applied at the vicinity of the plated area and loading location, while the relatively coarse mesh is applied elsewhere. And a sensitivity study showed that compared with uniformly fine mesh, this meshing scheme really accelerated the overall computation, while no appreciable changes to the overall response were observed.

(6) The cross section of the tooth remains the same along its length, even though the actual configuration of the tooth is twisted and tapered toward the tip. Assigning the uniform tooth configuration may not affect the overall stiffness of joint (Groom, 1992). From the measurement of the truss plates provided by Mitek...
Canada, Inc, the tapered segment takes up approximately 1/5th of the total tooth length (about 1.82 mm out of 9.1mm). It’s deemed proper to neglect the difference by assuming the tooth length is the sum of half of tapered segment plus the non-tapered length, thus the length of 8.19mm is obtained and used.

(7) The deformation of tooth connection at the tooth top, where the tooth attaches to the plate, is calculated from the displacements of the associated plate element and the underlying wood element. The identification number of the associated plate element and wood element are important inputs to the program. Since the tooth is always allocated at the common side of two adjacent plate elements, the selection of associated plate element needed attention. Take the example in Figure 5.3a, the tooth layout resembles that in cases AA and EA that the load direction is parallel to tooth slot direction (plate primary direction). The little black cross marks represent the teeth A and B at the middle points of corresponding sides. Since the element on the right side of tooth A is an opening, the tooth A’s displacements then are obtained by interpolating over its associated element I via nodal displacements and shape functions. Accordingly, the associated plate element for tooth B is element II. Similarly in Figure 5.3b, the tooth layout resembles the case AE and EE that the load direction is perpendicular to plate primary direction and the associated element for teeth A and B is elements I and II respectively.
(8) The wood material properties in the model include modulus of elasticity in the three principal orientations (longitudinal, radial, and tangential). The longitudinal MOE is assigned based on the average test result (stated in Section 4.4.1), and the MOE properties for the other two orientations are derived from the longitudinal MOE values with the guidelines given in the Wood Handbook (Forest Products Laboratory, 2010).

When the purchased lumber arrived in the lab, a great deal of blue stain caused by Mountain Pine Beetle and fungi was observed in the lumber. This indicates that the Lodgepole Pine made up a large portion of the SPF lumber batch, also given Canadian spruce, pine and fir share the similarity in mechanical properties, thus the Elastic ratios and Poisson’s ratios are taken from the those of Lodgepole Pine in Wood Handbook: $MOE_R = 0.068MOE_L$, $MOE_T = 0.102MOE_L$, $G_{LR} = 0.046MOE_L$, $G_{LT} = 0.005MOE_L$, and $G_{RT} = 0.049MOE_L$. As has been
addressed in Section 3.1.2, instead of 6 Poisson’s ratios of the combination of three orthotropic directions, three of them are used: \( v_{LR} \), \( v_{RT} \), and \( v_{LT} \). And all these values used in the model are listed in Table 5.1.

(9) The steel plate elements are assigned elastic-plastic bilinear material properties, and the modulus of elasticity, yield stress and Poisson’s ratio are provided by the truss plate manufacture, Mitek, Canada, Inc, and are listed in Table 5.1.

(10) The teeth that have the same plate-to-grain orientation are assumed to have the same tooth-to-wood interaction properties in terms of strength capacity and stiffness. The tooth-to-wood interaction is defined by a series of lateral springs along two lateral directions and a withdrawal spring. The properties of these springs are characterized by an exponential relationship defined in Equation (2.1) and Figure 2.1. Therefore, the same spring parameters \( K \), \( Q_1 \) and \( Q_0 \) are assigned to the corresponding springs of the same plate-to-grain orientation.

However, the localized properties of wood supporting the teeth are not uniform, due to the natural variation of wood. It is not practical to represent the localized properties of wood; therefore, the assumption of uniform tooth-to-wood properties is a convenient strategy to simplify the problem.

(11) Either displacement control or load control can be applied as input history in the program. In this study, displacement control is selected as input history over load control for two reasons: the actual joint tests are conducted with displacement control; and the displacement control method will allow the analysis to go beyond the maximum capacity of the system (load control method can’t fulfill) to
include the post-peak behavior of the structure. The post-peak performance of MPC joints is not the key interest of this study, but leads to the potential of the model improvement with the facility of advanced technique, like arc length method. Illustrated in Figure 5.1, the stepping displacement is applied sidewise at each plate node uniformly, which is consistent with the assumption that MPC connection on the loading half is so stiff that there is no relative movement between the plate and the wood member.

Since the maximum loads were achieved at different iteration steps for six types of joint tests, the total 100 steps of Newton-Raphson iteration with the stepping increment of 0.01 mm per step are chosen to calculate the joint deformation up to 1 mm for all six cases.

(12) A tooth withdrawal failure criterion is proposed herein to address the joint failure mode. Although a number of researchers have recognized that the tooth withdrawal governs the load carrying capacity of a joint and addressed the necessity of withdrawal failure modeling, no published research regarding MPC tooth withdrawal failure criterion has been found. Therefore a tentative scheme is explored that the tooth is considered to be failed if a gap of 40% tooth length has formed between the plate and wood at the tooth location. And the feasibility and robustness of this scheme will be investigated later in this chapter.

(13) Two boundary conditions are imposed to properly address the symmetric conditions of the model: vertical roller boundary conditions are applied along the right end of the steel plate, as well as along the bottom edge of the wood member,
to restrain the translation in the direction perpendicular to the planes of symmetry, and rotation about the axes defining the planes of symmetry. Moreover, the restraints from translating along two major plate directions are applied at the truncated wood ends to reflect the reactions invoked at the pinned gripping connection.

(14) A Visual Basic (VB) program has been developed to generate the geometrical input information for SAMPC, which includes geometrical size and mesh size of the plate, the wood member and the teeth, and the layout patterns of tooth arrays. The same VB program also served to visualize the original and deflection diagrams of the MPC joints with available model solutions. The diagrams depicted by the program aid in assessing the accuracy of model input information and monitoring the deformation of loaded joints. To demonstrate the program, the original geometry and deflection diagrams of modeled MPC joints that are generated by the program, which are presented in Appendix B.

<table>
<thead>
<tr>
<th>Table 5.1 Material properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
</tr>
<tr>
<td>Wood</td>
</tr>
<tr>
<td>Wood</td>
</tr>
<tr>
<td>Wood</td>
</tr>
<tr>
<td>Wood</td>
</tr>
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<td>Wood</td>
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<tr>
<td>Wood</td>
</tr>
<tr>
<td>Wood</td>
</tr>
<tr>
<td>Steel Plate</td>
</tr>
<tr>
<td>Steel Plate</td>
</tr>
<tr>
<td>Steel Plate</td>
</tr>
</tbody>
</table>
5.2. Model parameter calibration

As addressed in Chapter 3, there are three sets of spring elements involved in the MPC joint model: two sets of lateral springs accounting for the wood reaction as a nonlinear elastic-plastic foundation along two orthogonal plate directions, and the other set of withdrawal springs which accounts for the withdrawal resistance to axial deformation. Each set of springs is defined by the 3-paramater exponential model in Equation (3.24). The experiments of four standard joints in tension were conducted in Chapter 4 in order to calibrate these spring parameters along different force-plate-grain orientations.

In all four cases, the teeth are bearing laterally on both face and edge. In Figure 5.4, the shaded rectangles represent the tooth cross sections, the parallel lines represent the wood grain orientation, and the force P represents the wood reaction. The notation is applied as: the first letter refers to the orientation of the bearing surface, such as F is face bearing and T is edge bearing; and the second letter refers to the orientation of the wood grain to the load, such as A is parallel and E is perpendicular. And the combinations of two tooth-to-grain orientations constitute the sets of lateral springs to be calibrated in four standard joints tested in tension. That is, FA and TE with tension loaded along the plate primary axis (normal to tooth face) for AA joint type, FE and TA with tension loaded along the plate primary axis for AE, TA and FE with tension loaded along the plate secondary axis (normal to tooth edge) for EA, and TE and FA with tension loaded along the plate secondary axis for EE.
5.2.1. Calibration procedure

Based on the theoretical formulations described in Chapter 3 and the assumptions addressed early in this chapter, a computer model was constructed, and the model inputs for four standard joint tested were established accordingly. A nonlinear function optimization procedure was followed to estimate the spring parameters. The procedure is summarized as follows:

(1) Preliminarily approximate values for all spring parameters are selected through trial and error to provide a visual best-fit solution against the average load-displacement test data collected for four standard joint tests. The reason for this step is to narrow down the range of parameters considered in the optimization scope and thus speed up the whole optimization process. It should be noted that to investigate the accuracy of the modeling parameters, the theoretical displacements of selected points were calculated and compared with the experimental
displacements measured at the same points, namely the transducer measuring locations.

In this step, a sensitivity investigation was conducted on all four cases by varying the spring parameters by an arbitrary amount. It was observed that the changes of the spring parameter perpendicular to loading direction had a minimal influence on the overall solution. Then, to speed up the optimization, the three sets of springs to be calibrated for each joint type were reduced to the lateral spring along the loading direction (namely, FA for AA joint, FE for AE joint, TA for EA, and TE for EE), and the withdrawal spring along the tooth length direction.

(2) The predicted reaction force at each loading step until failure is compared to the experimental data by computing an objective function $\Phi$ as:

$$\Phi = \sum_{N} \left( F_{i,\text{test}} - F_{i,\text{prediction}} \right)^2$$  \hspace{1cm} (5.1)

where $N$ is the total loading step until failure, $F_{i,\text{test}}$ is the average force from test data at the $i^{th}$ loading step, and $F_{i,\text{prediction}}$ is the model prediction with current sets of spring parameters. Since the tested four standard joints did not fail at the same load step, $N$ is not a constant for all cases, but depended on the loading condition and failure criteria. It should be noted that the joint is considered to be failed when the maximum load is reached, and the maximum load herein is also regarded as failure load or critical load.
(3) The explicit expression of the objective function $\Phi$ is not available. Running the program to search for the optimal parameters is computationally costly. Thus an alternative using the response surface method is proposed. This method allows a relatively simple polynomial function of the model parameters of interest, as given in Equation (5.2) to replace actual structural response. And due to the theoretical solution is not sensitive to springs perpendicular to loading direction, the Equation (5.2) contains six variables, namely three characteristic variables for spring parallel to loading $(m_0, m_1, k)$ and the withdrawal spring $(m_{w_0}, m_{w_1}, k_{w})$.

$$\Phi = \Phi(m_0, m_1, k, m_{w_0}, m_{w_1}, k_{w})$$  \hspace{1cm} (5.2)

The preliminary spring parameters are taken as the initial points. Each parameter is perturbed in turn for 5 times with an arbitrary increment while keeping the other variables unchanged, then step (2) is repeated to obtain $\Phi$ value accordingly. A nonlinear regression procedure is then conducted to find out the coefficients in Equation (5.2). In doing so, a response surface is formed in a multi-variable space, and the objective of the procedure is to find out the optimal model parameters on the response surface to minimize the $\Phi$ value.

(4) A nonlinear function minimization procedure using the quasi-Newton method is employed to estimate the optimal model parameters to be calibrated. Then through an iteration procedure, the initial choices of spring parameters are modified. An optimal solution is considered achieved when the objective
function value does not change by more than 1% of the previous value, or $\Phi$ is smaller than a number corresponding to a maximum difference of about the 1N between the experimental results and the theoretical solutions. When either of these criteria are reached, the optimization process is terminated.

5.2.2. Calibration results

5.2.2.1. Spring parameters

Following the procedure described previously, the optimal spring parameters were attained for four standard MPC joint types respectively and are listed in Table 5.2. Six parameters were attained with respect to the lateral spring along the loading direction as well as the withdrawal spring. The load-displacement relationships of joints were also derived from SAMPC based on these optimal spring parameters. To demonstrate the goodness of model prediction, the corresponding R-square values were calculated from zero to the maximum load, and are listed in Table 5.2 as well. The maximum load carrying capacity is of most concern in this study. Figure 5.5, Figure 5.6, Figure 5.7, and Figure 5.8 show the comparisons between the load-displacement curves obtained from the average test results and theoretical solutions for four standard truss joint tests respectively.
<table>
<thead>
<tr>
<th>Spring Parameter</th>
<th>Unit</th>
<th>Joint Type</th>
<th>AA(FA)*</th>
<th>AE(FE)*</th>
<th>EA(TA)*</th>
<th>EE(TE)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_0$</td>
<td>(kN)</td>
<td></td>
<td>0.06</td>
<td>0.04</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>$m_1$</td>
<td>(kN/mm)</td>
<td></td>
<td>0.03</td>
<td>0.025</td>
<td>0.01</td>
<td>0.001</td>
</tr>
<tr>
<td>$k$</td>
<td>(kN/mm)</td>
<td></td>
<td>0.3</td>
<td>0.1</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>$mw_0$</td>
<td>(kN)</td>
<td></td>
<td>0.005</td>
<td>0.00265</td>
<td>0.02</td>
<td>0.008</td>
</tr>
<tr>
<td>$mw_1$</td>
<td>(kN/mm)</td>
<td></td>
<td>1e-5</td>
<td>1e-4</td>
<td>1e-4</td>
<td>1e-4</td>
</tr>
<tr>
<td>$kw$</td>
<td>(kN/mm)</td>
<td></td>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.2</td>
</tr>
</tbody>
</table>

$R^2$ - 0.987 0.974 0.989 0.978

Note: $m_0$, $m_1$, and $k$ denote the parameters of lateral springs parallel to the loading direction; $mw_0$, $mw_1$, and $kw$ denote the withdrawal spring parameters.
*FA, FE, TA and TE in the parentheses denote the tooth-to-wood orientation which was calibrated with corresponding joint type.

Figure 5.5 Test result and model prediction for AA truss joint
Figure 5.6 Test result and model prediction for AE truss joint

Figure 5.7 Test result and model prediction for EA truss joint

Figure 5.8 Test result and model prediction for EE truss joint
The force and deformation of each tooth, as the output of SAMPC, were calculated at each loading step. Thus the vertical deformation (or withdrawal deformation) was updated spontaneously according to the tooth withdrawal displacement update scheme elaborated in Section 3.7, until tooth withdrawal failure criterion was met.

It was found that in the model solutions, the teeth on the same row exhibited almost the same performance in terms of deformation and force. The lateral deformations and resulting lateral resistances calculated from the modified HYST for the teeth in the same row showed little variations while loading. Similarly, the withdrawal deformation was also quite consistent for the teeth in the same row, and thus the withdrawal displacement update tended to occur at nearly the same loading step for the teeth in the same row. This uniform performance of the teeth in the same row from the model solutions wasn’t quite consistent with the joint tests. From the visual inspection of the tested joints, the teeth in the same row didn’t seem to have uniform withdrawal as some teeth tend to withdraw sooner than other teeth. The uniform performance of the teeth in the same row resulted from the model assumption [10] addressed in Section 5.1.

As a result, when the withdrawal displacement updated according to the withdrawal displacement update scheme, a pronounced disturbance was observed, and this led to the staggering force-displacement curves (shown in Figures 5.5 to 5.8). In particular, this was more pronounced when the withdrawal update occurred to the first tooth row (the row of teeth located furthest from the wood-to-wood gap).
Despite the disruptions due to the withdrawal displacement update, in all four cases, the model solutions agreed fairly well up to the maximum load with the experimental data, and the corresponding R-square values ranged from 0.974 to 0.989. As the loading proceeded beyond the maximum load, the model predicted load-displacement curves experienced abrupt decreases in strength, whereas the experimental results showed the smooth strength degrading curves.

The inconsistency between the model solutions and test results may come from two aspects: (1) the uniform tooth-to-wood properties defined in the model assumption [10] discussed in Section 5.1; (2) the withdrawal failure criterion discussed in the model assumption [12] that the tooth is assumed to be failed as 40% of its length has pulled out of the wood. Since the teeth in the same row have similar model predicted deformation, when the prescribed withdrawal failure criterion was first met at a tooth row, all the teeth in the row failed simultaneously, and then the load portion carried by this row couldn’t be redistributed into the other tooth rows (the load sharing among the tooth rows will be investigated later in Section 5.2.2.2). This resulted in a sudden drop of the strength and stiffness of the joint.

The sudden degradation of joint strength indicated two things: (1) it testified that the tooth withdrawal plays an important role in predicting the strength and stiffness of MPC joints, and (2) it implied that the uniform tooth-to-wood properties assumption and tooth withdrawal failure criteria can’t capture the post-peak property of the MPC joints, and isn’t applicable to post-peak evaluation.
Originally, two approaches were investigated to allow the strength and stiffness to degrade gradually by either adjusting the spring parameters (or tooth-to-wood properties) or altering the withdrawal failure criterion. The first approach was to reduce the stiffness of tooth springs by a certain percentage, say 20%, after the tooth withdraws by 40% of the length. The other one is to assume that the spring stiffness varies with respect to the relative tooth location to the wood edge. Both approaches encountered computational instability and difficulty in convergence (e.g. singular stiffness matrix), when a relatively large deformation occurred, say 40% tooth withdrawal. This may indicate that the relatively large deformation induced a nonlinear response in the system, which is highly likely coming from the nonlinearity of tooth springs under large deformation. Due to these issues, these two approaches were abandoned.

Since the maximum load carrying capacity is of most interest in this study, the inconsistency between the theoretical solutions and test results beyond the maximum load is considered acceptable for this thesis. But this inconsistency implies that the model requires further study to predict the post-peak properties of MPC truss joints.

Moreover, the comparisons between the force-displacement spring curves, which were calibrated for four tooth-to-grain orientations in Figure 5.4, are shown in Figure 5.9 and Figure 5.10. These spring curves were depicted by applying the spring parameters in Table 5.2 into the exponential model in Equation (2.1) and Figure 2.1. Both figures indicated that the load-plate-grain (or load-tooth-grain)
orientation had distinctive influence on determining the lateral spring characteristic parameters as well as the withdrawal spring characteristic parameters.

Theoretically, the lateral tooth-to-wood deformation can be attributed to two parts: the deformation of local underlying wood by crushing or splitting (when loading parallel to the wood fiber and tooth bearing on edge), and the deformation of teeth as a beam is subjected to a transverse load. Although the edge orientation has a smaller tooth-bearing area, as the strong axis of the cross section, the initial stiffness of the lateral spring is not less than that of face orientation. For example, FA and TA have quite similar initial stiffness (0.3 for FA, and 0.4 for TA); likewise, FE and TE have the same initial stiffness of 0.1. However, the stiffness of lateral spring TA tends to degrade much more than other cases when subjected to high loads. This is because that in the case of TA (with the edge bearing along the wood grain), as the loading progresses along the grain direction, the tooth edge starts to cut through the wood fiber because wood is weak in tension perpendicular to grain, and thus the stiffness of TA softens significantly. In case TE (with the edge bearing perpendicular to the wood grain), similarly with the tooth bearing on the edge, but the underlying wood experiences compression perpendicular to the grain, which has greater strength capacity than tension perpendicular to the grain. Thus the stiffness degradation of TE is more moderate than that of TA.

During the parameter calibration for the withdrawal springs, it was found that not only the load-plate-grain orientation had an influence on the parameter values, but the tooth withdrawal criterion and the tooth layout also played important roles. Based on the tooth withdrawal failure criterion proposed in this study, the maximum load carrying capacity of joint system is directly associated with tooth withdrawal failure at the first
row in all four cases. Therefore, to withstand the same level of maximum strength, cases EA and EE, which have half the number of teeth in the first row than cases AA and AE, tended to have more vertical resistance from each tooth. This explained why the withdrawal springs in EA and EE had greater initial stiffness than those of AA and AE.

It was also observed that the post-peak stiffness of withdrawal springs tended to be zero. This may suggest the possibility of eliminating spring parameter $m_w$ in model parameter calibrations by simply assuming its value to be zero.

![Figure 5.9](image-url)  
**Figure 5.9** Force-displacement relationship of lateral spring element for joint AA, AE, EA, and EE

![Figure 5.10](image-url)  
**Figure 5.10** Force-displacement relationship of withdrawal spring element for joint AA, AE, EA, and EE
5.2.2.2. Load distribution among tooth rows

In this study, the force and the deformation of each tooth were calculated at each loading step as an output. To get the insight on how the tooth reacted during the loading, the load distribution along with the percentage difference among the rows of teeth at three different loading steps were compared and reported in Table 5.3. To demonstrate the relationship between the load distribution and tooth withdrawal among tooth rows, the tooth withdrawal conditions associated with these loading steps are also addressed in Table 5.3. The three specific loading steps consist of one-third of the maximum load (suggested design value by TPI design specifications), two-thirds of the maximum load, and the maximum load. However, on the theoretical load-displacement path of the tested joints, usually there are no data points available at exactly one-third and two-thirds of the maximum load. To avoid introducing extra errors by interpolation, the points which are closest to the exact points were used.
<table>
<thead>
<tr>
<th>Joint Type</th>
<th>Load level with respect to the ultimate load</th>
<th>Tooth Row Reaction Force (kN)</th>
<th>Withdrawal Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>row #1</td>
<td>row #2</td>
</tr>
<tr>
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<td>~1/3</td>
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<tr>
<td></td>
<td></td>
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<td>~2/3</td>
<td>1.83</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(49.6%)</td>
<td>(50.4%)</td>
</tr>
<tr>
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<tr>
<td></td>
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<td>(56.1%)</td>
</tr>
<tr>
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</tr>
<tr>
<td></td>
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<td>(56.1%)</td>
<td>(43.9%)</td>
</tr>
<tr>
<td></td>
<td>~2/3</td>
<td>1.285</td>
<td>1.386</td>
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<tr>
<td></td>
<td></td>
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<td>2.149</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(46.7%)</td>
<td>(53.3%)</td>
</tr>
<tr>
<td>EA</td>
<td>~1/3</td>
<td>0.562</td>
<td>0.486</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(29.7%)</td>
<td>(25.7%)</td>
</tr>
<tr>
<td></td>
<td>~2/3</td>
<td>0.896</td>
<td>0.922</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(25.1%)</td>
<td>(25.9%)</td>
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<td></td>
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<td>(20.2%)</td>
<td>(22.8%)</td>
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<tr>
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<td>0.269</td>
<td>0.256</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(26.8%)</td>
<td>(25.4%)</td>
</tr>
</tbody>
</table>
According to the data in Table 5.3, it is recognized here again that the assumption of even load distribution among all teeth is incorrect. Before the tooth withdrawal update occurs in all four joint types, the first tooth row (the furthest to the wood-to-wood gap) takes up a bigger portion of the load than the other rows. This is more pronounced for joint type AA and AE, which have only two tooth rows, and the first row takes up nearly 60% of the total load. As loading progresses, the tooth withdrawal update starts to occur in tooth rows, first in the first row, then in the next row, and so on and so forth. This results in the changes of the lateral stiffness of tooth rows, and consequently the load distribution occurs. And right before the first tooth row fails in withdrawal at the maximum load, the tooth row which has less or no tooth withdrawal carries most of the load compared to the other rows. Right before the withdrawal failure occurs, the first tooth row still carries rather substantial portions of the load, about 40% for AA and AE, and 20% for EA and EE. Thus when failure takes place, the first tooth row is eliminated from the tooth array in accordance with the tooth withdrawal failure scheme. Unlike in
reality, the tooth withdrawal progresses gradually and the load redistributes in continuity, the sudden elimination of the tooth row leads to an abrupt drop of joint load-carrying capacity by around 40% or 20%.

5.3. Model verification

The above procedure outlined the calibration of the modeling spring parameters, which were used to approximate semi-rigid behavior of truss plate joints. The calibration was based on test data of the actual four standard joints in tension. While the model performed well in four standard joint configurations as indicated in the comparison of theory with experiments, the more important question is how well it can predict the strength and stiffness of the joint with arbitrary force-plate-grain angles. Tests of two joints with 60 degree and 45 degree plate-to-grain angles (Figure 4.2 and Figure 4.4) can be used to address this question.

5.3.1. Hankinson’s formula

Due to its distinct fibre structure, wood is an orientation-sensitive material in nature. Thus the lateral load-slip relationship of tooth-to-wood connection elements is highly dependent on the force-plate-grain orientation, which has been justified in a great deal of publications. Conventionally, the determination of the spring parameters for an arbitrary force-plate-grain orientation demands interpolating between the parameters of the four standard orientations. A series of interpolation techniques are available, such as linear interpolation (Vatover 1997), the step function method (ICBO, 1979) and Hankinson’s formula (1921). Hankinson’s formula was used in this thesis to obtain the
corresponding spring characteristic parameters for intermediate force-plate-grain angles. The feasibility and the accuracy of Hankinson’s formula have been tested and verified in McCarthy’s studies (1986). It should be noted that in both 60 degree and 45 degree plate-to-grain cases, the loads were applied along the plate’s primary axis (slot direction), thus the force-plate-grain orientation issue was reduced to a plate-to-grain orientation issue. And for cases where the loads do not coincide with the plate two major orientations, the more general procedure of obtaining spring characteristic parameters could be followed as described by Foschi (1977).

![Diagram](image)

Figure 5.11 Truss plate tooth-to-grain orientation

As shown in Figure 5.11, \( \theta \) is the angle between the plate primary axis and the grain direction (in the following, the angle between the plate primary axis and the grain direction will be called plate-to-grain orientation, and \( \theta \) is measured counter-clockwise), meanwhile the force direction is parallel to the plate primary axis. By applying Hankinson’s formula, the spring initial stiffness parameters \( k_f \) and \( k_t \) can be defined as
\[ k_f = \frac{k_{fa} k_{fe}}{k_{fa} \sin^2 \theta + k_{fe} \cos^2 \theta} \]  

(5.3a)

\[ k_t = \frac{k_{ta} k_{te}}{k_{ta} \sin^2 \theta + k_{te} \cos^2 \theta} \]  

(5.3b)

\[ k_{fa}, k_{fe}, k_{ta} \text{ and } k_{te} \] are the spring stiffness parameters for the four tooth-to-grain orientations in Figure 5.11. The same procedure was also applied to spring stiffness parameters \( m_0 \) and \( m_1 \). By taking \( \theta \) in Equation 5.3 as 60 degree and 45 degree respectively, the calculated values for the two non-standard joint configurations are listed in Table 5.4.

As calibrated and stated in Section 5.2, the behavior of the tooth withdrawal spring element is also substantially dependent on the plate-to-grain orientation, thus the same Hankinson’s method was followed to obtain the characteristic parameters for the withdrawal springs. Then the interpolation of the withdrawal spring parameters was taken between EA and EE orientations, since the resulting spring parameters (listed in Table 5.4) gave the best predictions in terms of the maximum strength.

<table>
<thead>
<tr>
<th>Table 5.4 Spring parameters</th>
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</tr>
<tr>
<td>( m_{f0} )</td>
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<tr>
<td>( m_{f1} )</td>
</tr>
<tr>
<td>( k_f )</td>
</tr>
<tr>
<td>( m_{t0} )</td>
</tr>
<tr>
<td>( m_{t1} )</td>
</tr>
<tr>
<td>( k_t )</td>
</tr>
<tr>
<td>Spring Parameter</td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>$m_w_0$</td>
</tr>
<tr>
<td>$m_w_1$</td>
</tr>
<tr>
<td>$k_w$</td>
</tr>
</tbody>
</table>

$R^2$ — 0.996 0.972

5.3.2. 60-degree and 45-degree

This section compares the theoretical predictions to experimental observations of joints with plate-to-grain angles of 60 degree and 45 degree, and also demonstrates that only a limited amount of general information is needed as model inputs.

In forming the model inputs, the assumption (1) through (11) described in Section 5.1 are also applied. And additional assumptions are applied as:

1. As has been described previously, the Hankinson’s interpolation method is employed to determine the modeling parameters of the three sets of spring elements at intermediate plate-to-grain angles. Thus, except for the particular geometrical information, the other information needed as model input is directly derived from the parameter calibration on four standard MPC joints.

2. Teeth near the gaps at the wood-to-wood end are eliminated from the calculation, due to wood failure during fabrication or limited end distance resulting in less engagement in the joint action. Accordingly, the total of 36 teeth was taken into account for the 60-degree case, and 33 teeth for the 45-degree case.
(3) The boundary conditions are applied to reflect the potential torsion deformation due to the asymmetric loading. The pinned restraint and roller restraint were applied at the center of the two truncated planes respectively.

Figure 5.12 and Figure 5.13 compare theoretical solutions with the experimental data. Again, the corresponding R-square values were calculated up to the maximum load capacity to indicate the goodness of the theoretical predictions against the joint tests, and are listed in Table 5.4.

![Model verification (60)](image1)

Figure 5.12 Test result and model prediction for 60-degree truss joint

![Model verification (45)](image2)

Figure 5.13 Test result and model prediction for 45-degree truss joint
In Figure 5.12 and Figure 5.13, the two experimental average curves indicate that the displacements measured at two different reference points during tests, as indicated in Figure 4.4. Although the displacements at the two different reference points of joints show pronounced variations, particularly in the 45-degree case, the theoretical solutions show no significant difference, because the theoretical load-displacement curves calculated at the corresponding reference points tend to coincide with each other.

Meanwhile, in both cases, the theoretical model gives fairly good predictions of the maximum load, with ratios of the predicted maximum force to the average test data of 99.1% and 97.9% for 60-degree and 45-degree cases respectively. However, the displacements at the maximum load differ from the ones of experiments.

The main reason for these differences and inconsistencies may be attributed to two aspects: the wood density variation (early/late wood), the presence of knots, internal checks and other defects in wood have been ignored in the numerical model; and the neglect of the material variability in wood by assuming the identical springs’ characteristics to all teeth engaged. Since the predicted results are within the conservative zone compared with the test data, and thus are considered acceptable.

Overall, the predicted results agreed well with the tests results up to the maximum load, which indicated that the feasibility of SAMPC for predicting the MPC joints while subjected to tension has been justified.

Unlike the four standard joints that the teeth on the same row tended to perform the same way in terms of forces and deformations, due to the asymmetric geometry and loading condition in 60-degree and 45-degree cases, the teeth at the plate corner
withstood more per-tooth force than the other teeth, and thus tended to withdraw sooner. This was consistent with test observations. Meanwhile, instead of the whole row of teeth failing at the same time, this resulted in the smoother load-displacement curves as shown in Figure 5.12 and Figure 5.13.

5.4. Some general remarks

Throughout the processes of model parameter calibration and model verification, SAMPC’s solutions are found to be quite sensitive to the modeling parameters; in particular, the lateral spring parameters along the loading direction as well as the withdrawal spring parameters. Inappropriate parameter assignment would not only give the incorrect answer, but may also result in unexpected non-convergence.

The withdrawal criterion and withdrawal failure criterion are schematic solutions, which may not reflect the real behavior of tooth withdrawal behavior (friction involved), but bring up the opportunities for future study.

Due to the inconsistency of between the model solutions with the test results after the maximum load, further research is needed to study the post-peak properties of the MPC truss joints.

5.5. Summary

This chapter presented the results of model spring parameter calibration against the four standard joint test data, and computer program verification against the two non-standard joint tests. The modeling assumptions were discussed. The theoretical results
were compared with the test data, and within the loading zone of most concern (from zero load to maximum load), fairly good agreement was achieved.
Chapter 6. Modeling of MPC Truss Joint in out-of-plane

Bending and Reliability Analyses of Lateral Bracing Forces in

MPC Wood Trusses

Introduction

In this chapter, due to industry interest, a truss design-prompted question is explored: limit state design of lateral bracing of MPC truss system. In order to determine the lateral bracing forces required for trusses, the out-of-plane bending behavior of MPC truss joints is studied experimentally and analytically. Due to the fact that the tooth withdrawal performance governs the stiffness of MPC joints while subjected to out-of-plane bending, the possibility of the application of SAMPC in simulating MPC joints subjected to out-of-plane bending is investigated and discussed. Based on the experimental studies on typical web-to-chord truss joints, reliability studies of MPC truss web bracing systems are carried out by using a series of techniques, such as: structural modeling, nonlinear curve regression, and response surface method. Sensitivity analyses are also conducted. Finally an approach to evaluate the lateral bracing force of the brace members in MPC truss systems is proposed to facilitate the bracing design.

6.1. Background of lateral bracing system in MPC truss

MPC wood trusses are designed to resist bending, compression, and tensile loads in the plane of the truss which usually implies forces acting in the strong direction of the two-by-member structure. Lateral bracing is required to resist the lateral or out-of-plane load and provide system stability to carry the full design load. Without it, some truss
members may buckle at loads far less than what they are designed to support. The length-to-width ratio in the weak-axis direction of truss compressive web member is assumed to be reduced by lateral bracing to ensure system stability. The number of braces and the bracing points are determined to provide sufficient support to prevent web buckling.

In metal plated wood trusses, the 2% rule is the most common and accepted design practice to estimate the required bracing forces in compression members. The 2% rule is a strength-based model which assumes the lateral bracing force to be 2% of the compression force of the web (Throop 1947). Along with many assumptions, the 2% rule model assumes web members are pin connected with the chord members and a bracing is connected to the web by a zero-stiffness hinge (Throop 1947). Waltz et al. (2000) pointed out that the 2% rule may not be appropriate without considering the bracing stiffness, which is not in compliance with the theoretical assumption (Timoshenko and Gere 1961) that bracing stiffness is present to reinforce slender compression members.

Winter (1958) developed a simplified model to derive the bracing force and stiffness for bracing design with the consideration of the strength and stiffness requirements of a lateral braced beam-column system. Plaut (1993a, b) modified Winter’s model for more conservative consideration. Underwood et al. (2001) investigated the lateral bracing of multiple-web or multiple-chord systems and proposed a method to calculate the bracing force for design purposes. Song and Lam (2009) developed a numerical model to study the structural behavior of 3D beam-columns subjected to compression loads and biaxial bending moments. Most of these lateral braced-web system models were based on simply-supported beam-column assumptions.
However, the MPC joints at the ends of the compression webs have some rotational restraints. The rotational stiffness of the MPC joints in the buckling plane can increase the critical buckling load which can influence the estimation of lateral bracing forces in the members.

Moreover, the critical buckling load of braced webs can be affected by the intrinsic randomness in the material’s properties, structural behavior and construction. Therefore reliability analysis is a rational choice to evaluate the lateral bracing force in a quantitative manner. Up until now there has been no publication that considers out-of-plane rotational stiffness of MPC connections or the reliability analysis of lateral braced-web truss system.

This chapter investigated the influence of the wood-to-wood contact gap on the out-of-plane rotational stiffness of MPC connections and evaluated the rotational stiffness experimentally; testified the applicability of program SAMPC for evaluating the stiffness of MPC joint while subjected to out-of-plane bending; modified Song and Lam’s model (2009) with the consideration of semi-rigid MPC web-to-chord connections; and applied the response surface method to perform reliability-based structural analyses of the single- and double-braced web systems and sensitivity analyses. Uncertainties involving the modulus of elasticity (MOE) of lumber, the out-of-plane rotational stiffness of the MPC connection, the translational stiffness of the brace-to-web nail connection, the initial mid-span deflection of web members due to the construction bias, and the load eccentricity, were considered when formulating the performance function for reliability analyses. Sensitivity analyses were conducted to investigate the relative contributions of different uncertainties to the performance of the single-braced web systems under axial...
compression, including the translational stiffness of the nail connection and the out-of-plane rotational stiffness of the MPC connection.

6.2. Experimental study

Since the out-of-plane rotational stiffness of MPC connections plays an important role in estimating the bracing forces of truss members, the analytical and experimental studies were conducted to investigate the influence of the wood-to-wood contact gap on the out-of-plane rotational stiffness of the MPC connections, and to quantify the rotational stiffness of the MPC connections.

6.2.1. Material

The 38 x 89mm (nominal 2 by 4 in) 1650f-1.5E Spruce Pine Fir (SPF), machine-stress-rated (MSR) lumber was used for both webs and bracings. The lumber used in this study was purchased from the same supplier as the one used in joint tensile tests in Chapter 4, but from different batches. Thus the material properties were measured independently for these two batches of lumber. The lumber was stored in a conditioning chamber at 20°C and 65% relative humidity for one month prior to specimen fabrication. Each 2.1 m (7 ft) long piece was numbered and tested for MOE using an E-Computer that determines MOE on the basis of vibration frequency.

20-gauge (0.9 mm or 0.0356 in in thickness) and 18-gauge (1.18 mm or 0.0466 in in thickness) metal truss plates were supplied by Mitek Canada, Inc. The 20-gauge metal truss plate came from the same batch as had been used to fabricate the specimens for
tensile truss tests in Chapter 4. The 18-gauge truss plate shared the same configuration with the 20-gauge ones, and the only variation was the thickness of the plate.

6.2.2. Sample fabrication

Two-member MPC joints (as shown in Figure 6.1a) were first tested in bending to investigate the influence of a wood-to-wood contact gap on the out-of-plane rotational stiffness of the MPC connections. Three-member MPC joint (as shown in Figure 6.1b) bending tests were then conducted to derive the out-of-plane rotational stiffness of typical connections.

Figure 6.1 Specimen configuration, a. two-member joint specimen; b. three-member joint specimen (1” = 25.4 mm)

In the testing of the two-member joint, five specimens were fabricated with either no gap or with a 2 mm gap between two members. The metal truss plates used were all 20-gauge. The three-member joint was fabricated with either 20- or 18-gauge metal truss plates. Ten joints were fabricated for each type of truss plate.

After fabrication, the specimens were again placed in the conditioning chamber for at least 7 days prior to testing.
In the two-member MPC joint test, the specimens were mounted on a test apparatus by clamping the main member to a steel frame (Figure 6.2). The displacement-controlled load was applied at a distance of 229 mm (9 in) from the center line of the main member of the joint using a steel bar.

![Figure 6.2 The two-member joint bending test setup](image)

In the three-member joint test (Figure 6.3), the main chord member and the "vertical" web member (illustrated in Figure 6.1b) were both clamped to the steel frames in such a way that the deflection only occurred in the second web member. The displacement-controlled load was applied at the distance of 241 mm (9.5 in) from the center line of the main member of the joint.
In both cases, to eliminate deflection due to the vertical shear force, a steel plate was placed underneath the specimen along the edge of the wood-to-wood contact (Figure 6.4). With this support, the deflection of the second wood member was only caused by bending about the long axis of the main member.

A prototype specimen was used to estimate the capacity and set the speed for the following tests. The test speeds were selected to attain the maximum load in not less than 5 minutes or more than 10 minutes. The applied load and the absolute deflection of the joint at the loading point were continuously collected by a computer data acquisition
system. The rotation was calculated as the deflection value divided by the loading arm, namely the distance of the loading point to the center line of the main member; and the bending moment was calculated as the applied load times the loading arm.

The specific gravity and moisture content of the wood were determined in accordance with ASTM standards D4442 and D2395 (2009). After the bending tests, a 25.4 mm (1 in) square specimen was taken from the piece of lumber where teeth withdrawal occurred, and taken from the area between the clamp and the truss plate. Each specimen was stored in an individual impermeable plastic bag to prevent changes in moisture content before weighing.

6.2.3. Test results and discussion

Moisture content of the specimens ranged from 11.77% to 12.83%, with a mean value of 12.22%. Specific gravity ranged from 0.36 to 0.47, with a mean value of 0.43; MOE, as measured with the E-computer, ranged from 9.02 GPa to 15.19 GPa, with the mean of 11.37 GPa.

For both cases, all the bending tests failed due to teeth withdrawal from the main member and associated wood splitting, and no pronounced plate buckling on the compression sides was observed. This may be because that plate slides relative to the wood, due to the strong stiffness of plate and tooth-wood connection.

It is important to consider a deformation limit of the joint in order to prevent excessive deformation, which is not allowed in service. Herein the linear portion of rotational stiffness of the joint was of most interest. The moment-rotation relationships of
specimens were taken up to the radian of 0.01. The average response was calculated by averaging moments of all five specimens with respect to rotation. Then one-parameter linear polynomials were applied to fit the average curves. This parameter indicates the rotational stiffness of the MPC joint in terms of moment and rotation.

Figure 6.5 and Figure 6.6 depict the experimentally derived moment and rotation relationships of the two-member joints without a gap or with a 2 mm gap respectively.

![Moment and rotation curve for no gap joint](image1)

Figure 6.5 The moment and rotation relationships of no gap joint

![Moment and rotation curve for 2mm gap joint](image2)

Figure 6.6 The moment and rotation relationships of 2mm gap joint
The average moment at 0.01 radian for no-gap joints was $1.497 \times 10^5$ N.mm, and $1.485 \times 10^5$ N.mm for 2 mm gap joints. The rotational stiffness was $1.497 \times 10^7$ N.mm/rad for no gap joints and $1.485 \times 10^7$ N.mm/rad for 2 mm gap joints, which was 0.8% lower than the former one. Therefore, it is safe to conclude that the influence on rotational stiffness for the wood-to-wood contact gap smaller than 2 mm, which is always the case in real metal plate truss construction, is very small and can be considered to be negligible in truss design.

Figure 6.7 and Figure 6.8 are the moment and rotation relationships for 20-gauge and 18-gauge three-member MPC joints respectively.

![Moment and rotation curve of 20-gauge plate](image)

Figure 6.7 The moment and rotation relationships of 20-gauge MPC joint
The average moment at 0.01 radian for 20-gauge truss plate joints was $1.464 \times 10^5$ N.mm, and $1.444 \times 10^5$ N.mm for 18-gauge truss plate joints. The calculated rotational stiffness was $1.464 \times 10^7$ N.mm/rad for 20-gauge joints and $1.444 \times 10^7$ N.mm/rad for 18-gauge joints. Since there was only a small variation in rotational stiffness between the two commercial gauged metal truss plates, the same out-of-plane rotational stiffness of metal plate connections at the webs-to-chords joints, $1.464 \times 10^7$ N.mm/rad, would later be used as an input parameter for the reliability-based structural analyses of the lateral-braced web system under axial compression.

6.3. Bending joint modeling

In the previous section, two-member and three-member MPC joints were tested in out-of-plane bending. In the three-member joint test, the main and the third wood member were tightly clamped to the setup fixture. As the vertical loading progressed, the second member tended to rotate about the longitudinal axis of the main and the third member simultaneously. This resulted in a bi-axial out-of-plane rotation scenario. Due to
the intrinsic complexity in behavior, the attempt to numerically simulate the three-member joint was abandoned. Therefore, a numerical model was constructed using the program SAMPC to simulate the two-member MPC joint subjected to out-of-plane bending. The model solutions were compared and justified with the test results.

6.3.1. MPC joint model

During the numerical model construction, the following assumptions were applied with respect to the joint geometry, boundary conditions and loading conditions:

(1) In the two-member joint test, nearly the same rotational stiffness of up to 0.01 radians for the joint with or without a gap, suggests that for the wood-to-wood contact gap smaller than 2 mm, there is no significant influence of the gap on the out-of-plane rotational stiffness of MPC connections within the zone of interest. Thus the FEM model was constructed so that wood members closely touched without any gap, such that gap closure and plate buckling and/or sliding through were not accounted for in the model for simplicity. In doing so, spring elements with constant large stiffness were used to connect the coinciding nodes, belonging to the touching wood members. These coinciding nodes were located in the bottom lines of the wood-to-wood contact surface.

(2) Since no wood-to-wood contact gap was considered, it was possible to assume that the compression force due to the bending was solely carried by wood-to-wood contact, which had a much larger resistance than the steel plate. Thus the steel plate in the compression zone was neglected in the model. As a result, the
FEM model was composed of two wood members, and one steel plate on the joint’s tension zone.

(3) The main wood member in Figure 6.1a, which was restrained to the steel frame, was truncated to reduce the problem size. To reflect the restraints of the steel frame, the vertical rollers were applied across the bottom plane of the main wood member, and the pin-and-roller boundary conditions along two orthogonal directions in the plane were applied on the end sides of the member.

(4) In the test, to eliminate deflection due to the vertical shear force, a steel plate was placed underneath the wood-to-wood contact edge as shown in Figure 6.4. Accordingly, the vertical roller boundary conditions were applied along the edge lines to account for this vertical restraint.

(5) Since the model solution was calculated within the linear zone, instead of nonlinear springs, the linear springs were used for lateral direction and withdrawal direction.

(6) Displacement control was applied normal to the joint plane at 0.1 mm per loading step, and up to 2.3 mm, which was at the radian of 0.01 with respect to the centre axis of the main member. Given the bend arm was 228.6 mm (9 in) in the two-member joint case, measured from the loading point to the centre axis of the main member, the displacement control was applied.

(7) The 20-gauge metal truss plates came from the same batch as was used in tensile tests in Chapter 4, so the material properties given in Table 5.2 were also used here. As for wood material properties, the longitudinal MOE was assigned based
on the average test result, and the MOE properties for the other two orientations were derived from the longitudinal MOE values with the guidelines given in the Wood Handbook (Forest Products Laboratory, 2010). The same elastic ratios and Poisson’s ratios were used for bending joints here as those for tensile joints. All these material property values for model input are listed in Table 6.1.

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<th>Material</th>
<th>Material Properties</th>
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6.3.2. Model results

Following the FEM procedure and the assumptions stated previously, the model was constructed.

According to the assumption [2], this model consisted of two wood members, and one truss side plate. The loading wood member had the plate-grain orientation resembling that of joint type AA, thus was denoted as BA; while the other wood member had the plate-grain orientation resembling that of joint type AE, and thus was denoted as BE. Due
to the similarity of the plate-grain orientation, the spring parameters of cases AA and AE were employed as model inputs for the two wood members respectively. Due to the assumption [5] that linear springs were used to represent the lateral resistance and withdrawal resistance, the spring stiffness parameter $k$ of 0.3 kN/mm and $k_w$ of 0.1 kN/mm of AA were then used for BA, and $k$ of 0.1 kN/mm and $k_w$ of 0.1 kN/mm of AE were then used for BE.

However, the bending model with the parameters of AA and AE did not yield desirable results. This may be attributed to the following reasons:

1) Contrary to the assumption [1] that wood-to-wood contact had infinite stiffness to resist the crushing of the wood members, the wood can become deformed due to the wood fiber densification at the initial stage of loading. Therefore, assuming the constant infinite stiffness to the wood-to-wood contact throughout the entire loading process may not reflect the real situation.

2) In assumption [2], compression force due to the bending was solely carried by wood-to-wood contact, in other words, the compression was carried only by the contact spring elements located at the bottom lines of the wood-to-wood contact surface. In fact, before large deformation is induced by the bending moment, compression is borne by the wood-to-wood contact area, which varies with time.

3) Also according to assumption [2], the plate buckling and plate slippage on the joint compression zone were not taken into account.

Therefore, adjusted coefficients were proposed to account for these model shortcomings. Through a calibration procedure, the adjusting coefficient was determined
as 0.1 for case BA \((k_{BA} = 0.1 \times k_{AA} \text{ and } k_{wBA} = 0.1 \times k_{wAA})\) and 10 for case BE \((k_{BE} = 10 \times k_{AE} \text{ and } k_{wBE} = 10 \times k_{wAE})\). The resulting tooth spring stiffness parameters are listed in Table 6.2. The test results and calibrated model prediction of the moment-rotation relationship are shown in Figure 6.9.

**Table 6.2 Tooth spring parameters for out-of-plane bending**

<table>
<thead>
<tr>
<th>Spring Parameter</th>
<th>Unit</th>
<th>Joint Type</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(k)</td>
<td>(kN/mm)</td>
<td>BA</td>
<td>0.03</td>
<td>1</td>
</tr>
<tr>
<td>(k_w)</td>
<td>(kN/mm)</td>
<td>BE</td>
<td>0.01</td>
<td>1</td>
</tr>
<tr>
<td>(R^2)</td>
<td></td>
<td></td>
<td>—</td>
<td>0.973</td>
</tr>
</tbody>
</table>

![Moment and rotation curve](image)

*Figure 6.9 The comparison of the moment and rotation relationship for two-member none-gap bending*

The model solution was only taken up to 0.01 radians, due to the main interest of this study it is the only linear part of the rotational stiffness. A close agreement was achieved with an R-square value of 0.973.

Fairly good model prediction with the calibrated adjusting coefficients justifies the possibility of using SAMPC to evaluate the out-of-plane rotational stiffness of MPC
connections. It brings the potential of refining the model by including the gap closure and plate buckling into SAMPC to eliminate the use of adjusting coefficients.

6.4. Web-bracing structural modeling

For the purpose of reliability analysis on the braced truss web structure, a 3D FEM-based structural analysis computer program SATA (Structural Analysis of Truss Assemblies), which was developed by Song (2009), was used as the platform for the numerical modeling studies.

Two FEM-based models, illustrated in Figure 6.10 and Figure 6.11 respectively, were built to perform the eccentric compression of braced beam-column systems, which resembles single- and double-braced web systems under axial compression.

Figure 6.10 The FEM analog of single-braced truss web system

Figure 6.11 The FEM analog of double-braced truss web system
Figure 6.11 The FEM analog of double-braced truss web system

To create the structural model, the following assumptions were considered:

1) The web and the bracing members were 38 x 89 mm (nominal 2 by 4 in) Spruce Pine Fir (SPF) MSR 1650f-1.5E dimension lumber, and were modeled by nonlinear beam-column elements. The wood beam-columns were 3048 mm (10 ft) in length and braced by 610 mm (2 ft) long brace members at either mid-span (for single bracing case) or one-third span (for double bracing case). The geometrical sizes of the web and the bracing members were suggested by a local truss manufacturer. Basic material properties including MOE, parallel-to-wood-strain compression and tension strengths were taken from material property tests of Song’s study (Song and Lam, 2009 and 2010), and were adjusted with size effect consideration. MOE of the dimension lumber was considered as one of the random variables in the reliability analysis, and was assumed 3-parameter Weibull distributed. The characteristic distribution parameters from Song and Lam (2009 and 2010) and Song et al. (2010) are listed in Table 6.4.

2) 2-10d Common nails, 76.2 mm (3 in) in length and 3.76 mm (0.148 in) in diameter, were considered as a whole for the brace-to-web nail connection, which was modeled with a nonlinear spring element with six degrees of freedom (Figure 6.12).
Song and Lam (2009 and 2010) and Song et al. (2010) experimentally investigated the force-to-displacement relationship of each degree of freedom (DOF). The spring force and displacement relationship of each degree of freedom was defined by Foschi’s 3-parameter exponential model (1974), as described in Equation (2.1).

In Song’s study, the model was simplified by assuming that the load displacement curve leveled out at the large nail displacement, namely, $m_1$ was set to zero. The results of $m_0$, $m_1$ and $k$ in the function for each degree of freedom from Song (Song and Lam, 2009 and 2010 and Song et al., 2010) were used in this research (quoted in Table 6.3). The translational stiffness $K_x$ of the nail connections along the length direction (X axis in Figure 6.12) of the bracing member was considered as another random variable, and was assumed to be of normal distribution with the mean and the standard deviation are listed in Table 6.4.

![Figure 6.12 Nail connection](image)

Table 6.3 The modeling parameters for the nail connection from Song (2009)

<table>
<thead>
<tr>
<th>Type</th>
<th>Maximum capacity $m_0$</th>
<th>Slope of asymptote $m_1$</th>
<th>Initial slope $k$</th>
</tr>
</thead>
</table>

138
<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St.d.</th>
<th>Mean</th>
<th>St.d.</th>
<th>Mean</th>
<th>St.d.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
<td>(N)</td>
<td>(N)</td>
<td>(N/mm)</td>
<td>(N/mm)</td>
<td>(N/mm)</td>
<td>(N/mm)</td>
</tr>
<tr>
<td>X</td>
<td>3219.67</td>
<td>168.68</td>
<td>0</td>
<td>0</td>
<td>1370.99</td>
<td>297.09</td>
</tr>
<tr>
<td>Y</td>
<td>862.42</td>
<td>130.79</td>
<td>0</td>
<td>0</td>
<td>2006.74</td>
<td>1166.62</td>
</tr>
<tr>
<td>Z</td>
<td>3141.03</td>
<td>557.6</td>
<td>0</td>
<td>0</td>
<td>824.71</td>
<td>122.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Unit</th>
<th>(kN.m)</th>
<th>(kN.m)</th>
<th>(kN.m)</th>
<th>(kN.m)</th>
<th>(kN.m)</th>
<th>(kN.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rx</td>
<td>0.018</td>
<td>0.005</td>
<td>0</td>
<td>0</td>
<td>0.441</td>
<td>0.203</td>
</tr>
<tr>
<td>Ry</td>
<td>0.025</td>
<td>0.009</td>
<td>0</td>
<td>0</td>
<td>3.33</td>
<td>0.939</td>
</tr>
<tr>
<td>Rz</td>
<td>0.039</td>
<td>0.005</td>
<td>0</td>
<td>0</td>
<td>1.813</td>
<td>0.484</td>
</tr>
</tbody>
</table>

3) The MPC connection at the end of the web was simulated as a linear spring element with the semi-rigid out-of-plane rotational restraint. The derivation of the out-of-plane rotational stiffness of the MPC connection was addressed in Section 6.2.3. For the purpose of reliability analysis, the derived out-of-plane rotational stiffness $K_r$ of the MPC connection was fitted into 3-parameter Weibull distribution, and the characteristic parameters are listed in Table 6.4. Except for the out-of-plane rotational degree of freedom, the other degrees of freedom of spring elements in the model were fully restrained.

4) The braced beam-column analog was loaded in eccentric compression in the weak axis Y of the specimen’s cross section (as illustrated in Figure 6.10 and Figure 6.11). In practice, the load eccentricity may occur in both the weak axis Y and the strong axis X, but since MPC connections are assembled in a symmetric manner along strong axis X, the eccentricity in the strong axis can be very small due to the relatively small thickness. Therefore, for the modeling purpose, the eccentricity in
X axis was ignored for simplicity. The eccentricity of compression (EC) in the weak axis Y was assumed to be normally distributed with a mean of 4 mm (5% of the width of lumber) and a standard deviation of 2 mm.

5) From truss manufacturing and construction practice, some initial mid-span out-of-plane lateral deflections (ΔI) can be expected in the web members. According to Song’s study (Song and Lam, 2009 and 2010 and Song et al., 2010), based on the measurements taken from 30 web members in trusses, the mean value and standard deviation of the initial mid-span lateral deflections were -0.93 mm and 2.42 mm, respectively. From this a normal distribution was adopted.

Table 6.4 The FEM modeling parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Unit</th>
<th>3-parameter weibull distribution</th>
<th>Mean</th>
<th>St.d.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>location</td>
<td>scale</td>
<td>shape</td>
</tr>
<tr>
<td>x1 (x1)</td>
<td>MOE(38x89)</td>
<td>(Gpa)</td>
<td>8.834*</td>
<td>1.666*</td>
<td>1.56*</td>
</tr>
<tr>
<td>x2 (x2)</td>
<td>Kr1^</td>
<td>(kN-m/rad)</td>
<td>8.545</td>
<td>6.652</td>
<td>4.15</td>
</tr>
<tr>
<td>x3 (x3)</td>
<td>Kr2^</td>
<td>(kN-m/rad)</td>
<td>8.545</td>
<td>6.652</td>
<td>4.15</td>
</tr>
<tr>
<td>x4 (x4)</td>
<td>Kx1^</td>
<td>(N-m)</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>(x5)</td>
<td>Kx2^</td>
<td>(N-m)</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>x5 (x6)</td>
<td>EC</td>
<td>(mm)</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>x6 (x7)</td>
<td>ΔI</td>
<td>(mm)</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Note: x1 “x6 are the notations of modeling parameters for the single bracing application, while (x1)“(x7) are the notations of modeling parameters for the double bracing application.

* indicates the quotations from Song (Song and Lam, 2009 and 2010 and Song et al., 2010).

^ Kr1 and Kr2 represent the out-of-plane rotational stiffness of MPC joints, Spring1 and Spring2 (see Figure 6.10 and Figure 6.11) respectively; Kx1 and Kx2 represent the translational stiffness of the nail connections along the length direction, Spring3 and Spring4 respectively.
6.5. Reliability study

6.5.1. Response surface function

6.5.1.1. Single bracing

Since the explicit expression of the structural solution for the lateral bracing force was not available for reliability analysis, a response surface method was then employed to estimate the structural reliability of the single- and the double-braced web systems under axial compression. This method replaced the actual structural response with the approximated response function of random variables of interest, which provided a simple and explicit means for reliability study.

A simple quadratic polynomial function was adopted for the single-braced web system:

\[
F(x_1, \ldots, x_6) = A_0 + \sum_{i=1}^{6} (A_i \cdot x_i) + \sum_{i=1}^{6} (B_i \cdot x_i^2) + \sum_{i=1}^{6} \sum_{j=1}^{6} C_{ij} \cdot x_i \cdot x_j
\]  

(6.1)

In which, the \( F \) is the ratio of the lateral bracing force divided by the axial buckling force in webs, as defined in Equation (6.2). \( x_1 \) to \( x_6 \) are the 6 modeling random variables as stated in Table 6.1. \( A_0, A_i, B_i, \) and \( C_{ij} \) are unknown coefficients to define the response surface.

\[
F = \frac{\text{lateral bracing force}}{\text{buckling force in web}} \times 100\%
\]  

(6.2)

To form the response function, random variable sampling was conducted using Microsoft Excel. By applying the built-in random number generation function of Excel, 500 combinations of random variables were generated based on the probability
distribution information stated in Table 6.4. These sampling values were used as inputs for the SATA web-bracing model to calculate the beam-column buckling force and the corresponding bracing force, and then the system response values \( F \) were calculated by Equation (6.2) accordingly.

The sample size of 500 was chosen for both single- and double-braced truss web systems, because 1) it would limit the computational expense and 2) from the preliminary study, it was found that the system responses based on the randomly generated samples usually clustered around a certain range, and the increase of sample size would not necessarily reduce the confidence interval. Therefore, the sample size of 500 was assumed to be appropriate and adequate for this study.

By applying the least square method, the coefficients in Equation (6.1) were evaluated based on the responses of the samples of random variables. Meanwhile, a statistical optimization procedure was applied to remove statistically insignificant second-order and cross-product terms in Equation (6.1).

\[
F(x_1, \ldots, x_6) = A_0 + \sum_{i=1}^{6} (A_i \cdot x_i) + B_1 \cdot x_1^2 + B_2 \cdot x_3^2 + B_3 \cdot x_4^2 + B_4 \cdot x_5^2 \\
+ C_1 \cdot x_1 \cdot x_3 + C_2 \cdot x_1 \cdot x_6 + C_3 \cdot x_2 \cdot x_3 + C_4 \cdot x_3 \cdot x_6
\]  

(6.3)

The coefficients of the response surface function in Equation (6.3) are listed in Table 6.5.

Table 6.5 The coefficients of the response surface function for single bracing case
<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A0</td>
<td>-1.19</td>
<td>B2</td>
<td>-0.00202</td>
</tr>
<tr>
<td>A1</td>
<td>0.108</td>
<td>B3</td>
<td>0.0432</td>
</tr>
<tr>
<td>A2</td>
<td>-0.0582</td>
<td>B4</td>
<td>0.00121</td>
</tr>
<tr>
<td>A3</td>
<td>0.166</td>
<td>C1</td>
<td>-0.00712</td>
</tr>
<tr>
<td>A4</td>
<td>-0.175</td>
<td>C2</td>
<td>0.00153</td>
</tr>
<tr>
<td>A5</td>
<td>-0.00553</td>
<td>C3</td>
<td>-0.00218</td>
</tr>
<tr>
<td>A6</td>
<td>0.187</td>
<td>C4</td>
<td>-0.00536</td>
</tr>
<tr>
<td>B1</td>
<td>0.00298</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: all parameters are dimensionless.

With the calibrated coefficients and the same sampling values of random variables, the response surface function gave predictions that were compared with the sample responses from SATA model in Figure 6.13.

![Graph](image)

Figure 6.13 The comparison between the sampling dots and the response surface prediction for single bracing case

It can be seen that the response surface method prediction of bracing force over axial buckling force ratios agreed well with the sampling responses, and the coefficient of determination $R^2$ was calculated as 0.947. All the ratios fell into -1.5~1.5 range. The minus sign indicated that the bracing member was in tension. This happened in the cases
where the initial lateral mid-span deflection of webs bowed opposite to the bracing side, and then when a compression load was applied, the web tended to bow along, resulting in tension in the bracing member. It is justified that in the continuous lateral braced (CLB) truss system, when all webs bow to one direction, the bracing force is the maximum. However, when webs bow in opposite directions, the compression force in bracing can be offset by a certain amount. It also indicated that the bracing force from the single member analog analysis was a conservative estimate for CLB bracing design purposes.

### 6.5.1.2. Double bracing

Similar to the single bracing case, a quadratic polynomial function with respect to 7 random variables (as stated in Table 6.4) served as the platform for the response surface function of the double bracing case.

\[
F(x_1, ..., x_7) = A_0 + \sum_{i=1}^{7} (A_i \cdot x_i) + \sum_{i=1}^{7} (B_i \cdot x_i^2) + \sum_{i=1}^{7} \sum_{j=1}^{7} C_{ij} \cdot x_i \cdot x_j
\]  

(6.4)

Likewise, 500 times of sampling with respect to each random variable were conducted, and the structural responses in the form of \(F\) value were calculated by Equation (6.4).

Based on the sampling results, it was observed that the calculated \(F\) values for Brace 1 and Brace 2 can differ as large as 678% \((= \frac{(F \text{ of Brace1} - F \text{ of Brace 2})}{(F \text{ of Brace1})} \times 100\%\) ), therefore separate response surface functions were taken into account for Brace 1 and Brace 2, as expressed in Equation (6.5) and Equation (6.6). The coefficients of the response surface functions are listed in
Table 6.6 and Table 6.7 respectively.

\[ F(x_1, ..., x_7) = A_0 + \sum_{i=1}^{7} (A_i \cdot x_i) + \sum_{i=1}^{7} (B_i \cdot x_i^2) + C_1 \cdot x_1 \cdot x_4 + C_2 \cdot x_1 \cdot x_5 + C_3 \cdot x_2 \cdot x_3 + C_4 \cdot x_4 \cdot x_7 + C_5 \cdot x_5 \cdot x_6 + C_6 \cdot x_6 \cdot x_7 \]  
\[ (6.5) \]

\[ F(x_1, ..., x_7) = A_0 + \sum_{i=1}^{7} (A_i \cdot x_i) + \sum_{i=1}^{7} (B_i \cdot x_i^2) + C_1 \cdot x_1 \cdot x_2 + C_2 \cdot x_1 \cdot x_3 + C_3 \cdot x_1 \cdot x_5 + C_4 \cdot x_2 \cdot x_5 + C_5 \cdot x_3 \cdot x_6 + C_6 \cdot x_6 \cdot x_7 \]  
\[ (6.6) \]

Table 6.6 The coefficients of the response surface function for Brace 1 in double bracing case

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A0</td>
<td>13.1</td>
<td>B4</td>
<td>-0.331</td>
</tr>
<tr>
<td>A1</td>
<td>0.097</td>
<td>B5</td>
<td>-0.329</td>
</tr>
<tr>
<td>A2</td>
<td>-1.07</td>
<td>B6</td>
<td>-0.00442</td>
</tr>
<tr>
<td>A3</td>
<td>-0.791</td>
<td>B7</td>
<td>-0.00449</td>
</tr>
<tr>
<td>A4</td>
<td>-2.4</td>
<td>C1</td>
<td>0.326</td>
</tr>
<tr>
<td>A5</td>
<td>1.89</td>
<td>C2</td>
<td>-0.119</td>
</tr>
<tr>
<td>A6</td>
<td>-0.0277</td>
<td>C3</td>
<td>0.0261</td>
</tr>
<tr>
<td>A7</td>
<td>0.123</td>
<td>C4</td>
<td>-0.0405</td>
</tr>
<tr>
<td>B1</td>
<td>-0.0195</td>
<td>C5</td>
<td>0.059</td>
</tr>
<tr>
<td>B2</td>
<td>0.0256</td>
<td>C6</td>
<td>0.014</td>
</tr>
<tr>
<td>B3</td>
<td>0.0148</td>
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<td></td>
</tr>
</tbody>
</table>

Note: all parameters are dimensionless.

Table 6.7 The coefficients of the response surface function for Brace 2 in double bracing case

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A0</td>
<td>10.6</td>
<td>B4</td>
<td>-0.146</td>
</tr>
<tr>
<td>A1</td>
<td>-0.071</td>
<td>B5</td>
<td>0.116</td>
</tr>
<tr>
<td>A2</td>
<td>-0.951</td>
<td>B6</td>
<td>-0.0065</td>
</tr>
<tr>
<td>A3</td>
<td>-0.414</td>
<td>B7</td>
<td>-0.00033</td>
</tr>
<tr>
<td>A4</td>
<td>0.333</td>
<td>C1</td>
<td>0.0196</td>
</tr>
<tr>
<td>A5</td>
<td>0.03</td>
<td>C2</td>
<td>0.0315</td>
</tr>
<tr>
<td>A6</td>
<td>-0.0255</td>
<td>C3</td>
<td>-0.146</td>
</tr>
<tr>
<td>A7</td>
<td>0.128</td>
<td>C4</td>
<td>0.0749</td>
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</table>

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>-0.027</td>
<td>C5</td>
<td>0.00662</td>
</tr>
</tbody>
</table>
With the calibrated coefficients and the sampling values of random variables, the response surface function gave the predictions which were compared with the sample responses from SATA model in Figure 6.14 and Figure 6.15.

![Graph showing predicted vs sampled lateral bracing force to web buckling force ratio.](image)

Figure 6.14 The comparison between the sampling points and the response surface method prediction for Brace 1 in double bracing case
It can be seen that the sampling dots tended to cluster around the range of -1 to +1, while the dots around the upper bound and lower bound of the -2 to 1.5 range were relatively sparse. This phenomenon also occurred in the single bracing case, but was more pronounced in the double bracing case. This may indicate that when the critical buckling occurred to the double-braced truss web, there was high probability that both braces withstood less than about 1% of the critical buckling load (compression only) in web members. This was reasonable due to load sharing between two bracing members.

Meanwhile, by visually comparing Figure 6.13 with Figure 6.14 and Figure 6.15, it can be seen that by introducing one more variable, the variation of the response surface prediction with respect to sampling points increased. The calculated coefficient of determination $R^2$ was 0.872 for Bracing 1 and 0.877 for Bracing 2. It might indicate that in the double bracing case, the response surface function might not yield a good evaluation of bracing force over axial buckling force ratio outside the range of -1 to +1.
6.5.2. Performance function

In this study, the performance function for reliability analysis is given below:

\[ G = S - F(x) \]  

(6.7)

For the purpose of discussion and comparison to the 2% rule, \( F \) is defined (in Equation (6.2)) as a ratio of the lateral bracing force divided by the axial compression load in the web. \( S \) is a value in percentage and represents the critical point with respect to having a certain reliability level to be found out. In this study \( S = 1\%, 1.5\%, 2\% \) were studied.

The reliability analysis software RELAN (Foschi et al. 2007) was used to estimate the failure probability of the single- and double-braced truss web systems, namely the system response \( F \) exceeds the safety level \( S \). RELAN was used to calculate the probability of failure in two ways: (1) by using the first order reliability method (FORM) with importance sampling and; (2) using the Monte Carlo simulation. Table 6.8, Table 6.9, and 6.10 give the results of reliability analyses for single and double bracing cases respectively, in terms of the failure probabilities and the associated reliability indices \( \beta \) for three \( S \) levels.
### Table 6.8 Reliability indices and failure probability for single bracing case

<table>
<thead>
<tr>
<th>S level</th>
<th>Method</th>
<th>Probability of failure (%)</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FORM/Important sampling</td>
<td>0.640(0.728)</td>
<td>2.489(2.443)</td>
</tr>
<tr>
<td>S=1%</td>
<td>Monte Carlo (1e5)*</td>
<td>0.705</td>
<td>2.455</td>
</tr>
<tr>
<td></td>
<td>FORM /Important sampling</td>
<td>0.0198(0.0241)</td>
<td>3.543(3.490)</td>
</tr>
<tr>
<td>S=1.5%</td>
<td>Monte Carlo (1e6)*</td>
<td>0.0194</td>
<td>3.548</td>
</tr>
<tr>
<td></td>
<td>FORM /Important sampling</td>
<td>2.17e-4 (2.877e-4)</td>
<td>4.594(4.535)</td>
</tr>
<tr>
<td>S=2%</td>
<td>Monte Carlo (1e9)*</td>
<td>9.26e-5</td>
<td>4.811</td>
</tr>
</tbody>
</table>

Note: FORM is the abbreviation for first order reliability method. The numbers in the parentheses are the important sampling results. * indicates the sample size of the Monte Carlo simulation method.

### Table 6.9 Reliability indices and failure probability for Brace 1 in double bracing case

<table>
<thead>
<tr>
<th>S level</th>
<th>Method</th>
<th>Probability of failure (%)</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FORM /Important sampling</td>
<td>0.788(0.612)</td>
<td>2.414(2.505)</td>
</tr>
<tr>
<td>S=1%</td>
<td>Monte Carlo (1e5)*</td>
<td>0.675</td>
<td>2.470</td>
</tr>
<tr>
<td></td>
<td>FORM /Important sampling</td>
<td>0.043(0.027)</td>
<td>3.335(3.457)</td>
</tr>
<tr>
<td>S=1.5%</td>
<td>Monte Carlo (1e5)*</td>
<td>0.030</td>
<td>3.432</td>
</tr>
<tr>
<td></td>
<td>FORM /Important sampling</td>
<td>1.66e-3 (8.81e-4)</td>
<td>4.150(4.293)</td>
</tr>
<tr>
<td>S=2%</td>
<td>Monte Carlo (1e6)*</td>
<td>1.100e-3</td>
<td>4.244</td>
</tr>
</tbody>
</table>

Note: FORM is the abbreviation for first order reliability method. The numbers in the parentheses are the important sampling results. * indicates the sample size of the Monte Carlo simulation method.
Table 6.10 Reliability indices and failure probability for Brace 2 in double bracing case

<table>
<thead>
<tr>
<th>S level</th>
<th>Method</th>
<th>Probability of failure (%)</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>S=1%</td>
<td>RSM/Important sampling</td>
<td>0.131(0.129)</td>
<td>3.009(3.013)</td>
</tr>
<tr>
<td></td>
<td>Monte Carlo (1e5)*</td>
<td></td>
<td>0.190</td>
</tr>
<tr>
<td>S=1.5%</td>
<td>RSM/Important sampling</td>
<td>2.17e-3(1.35e-3)</td>
<td>4.089(4.198)</td>
</tr>
<tr>
<td></td>
<td>Monte Carlo (1e6)*</td>
<td></td>
<td>1.3e-3</td>
</tr>
<tr>
<td>S=2%</td>
<td>RSM /Important sampling</td>
<td>3.57e-5(8.05e-6)</td>
<td>4.958(5.240)</td>
</tr>
<tr>
<td></td>
<td>Monte Carlo (1e8)*</td>
<td></td>
<td>7.0e-6</td>
</tr>
</tbody>
</table>

Note: FORM is the abbreviation for first order reliability method.
The numbers in the parentheses are the important sampling results.
* indicates the sample size of the Monte Carlo simulation method.

6.5.3. Discussion

The reliability research on beam-columns under eccentric axial loads and lateral loads was reported by Foschi et al. (1989). By using the RELAN program and a non-linear finite element analysis of the beam-column, reliability indices β for limit state design purposes were calibrated for two types of lateral load (uniformly distributed and center point load), three different values of slenderness ratios and four combinations of dead loads with live loads, and with respect to the variation of the ratio of the total nominal lateral load to the nominal axial compression load.

For the reliability index β in our study to be valid over a range of slenderness and dead-to-live-load ratio, the resulting β of the 38 mm x 184 mm (nominal 2” by 8”) SS SPF beam-column subjected to the center-point lateral load from Foschi et al. (1989) was
averaged over the different ratios of the lateral load to the axial compression load and is listed in Table 6.11.

<table>
<thead>
<tr>
<th>Results</th>
<th>Lateral center point load to axial load ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Mean</td>
<td>2.67</td>
</tr>
<tr>
<td>St.d.</td>
<td>0.23</td>
</tr>
<tr>
<td>c.o.v.</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Figure 6.16 The comparison of reliability index $\beta$ for single bracing case

Figure 6.17 The comparison of reliability index $\beta$ for double bracing case
The comparison of the results of the reliability study (FORM) of single- and double-braced web systems to Foschi’s evaluation was depicted in Figure 6.16 and Figure 6.17 respectively. The reliability indices from FORM were used to plot the Figure 6.16 and Figure 6.17, because they would be relatively more conservative rather than those from importance sampling and the Monte Carlo simulation. The line of Foschi’s evaluation had been truncated at 3% of lateral force to axial load ratio to emphasize the range of interest. In Figure 6.16, the lines from reliability analysis for brace intersected with Foschi’s evaluation at the point of 1.1%. In Figure 6.17, the line for Brace 1 intersected with Foschi’s evaluation at the point of 1.16%, while the line for Bracing 2 was extended and intersected with Foschi’s evaluation at the point of 0.85%. Therefore if one is considering Foschi’s values as reference reliability indices, when the critical $S$ value is greater than 1.1% for single bracing case, or 1.16% for Brace 1 and 0.85% for Bracing 2 for double bracing case, the system would reserve higher reliability.

It can be seen that the rule of thumb of 2% for MPC truss bracing design would be conservative for the metal plated wood truss system given that pinned web end assumptions may be invalid. The analytical result itself is based on a single web system also gave a conservative estimation when applied to the truss system, considering other system stability applications which can stiffen the whole truss system, and thus result in reduced compression force that can be passed on to bracing. Here components such as roof sheathing, diagonal braces, etc. would add to the system stiffness. Therefore, $S$ value can take the value of 1.16% for both single bracing and double bracing cases. However, for bracing design consideration, a relatively conservative $S$ of 1.25% is recommended here, namely the bracing force is 1.25% times the axial compression force in webs. The
interpolated $\beta$ is 3.0 for a single bracing case, and 2.9 for Brace 1 and 3.5 for Brace 2 in double bracing case accordingly.

It should be noted that the reliability and the sensitive study (will be discussed in the next section) were carried out on MSR lumber, which is most common in MPC wood truss construction practices in Canada. This suggests that the results of this study, in terms of the structural reliability index and lateral force to axial load ratio, may not be applicable to visually graded lumber, because of the greater variability of MOE in visually graded lumber. But the same analytical procedures can be followed to assess the reliability of lateral-braced truss web systems fabricated with visually graded lumber.

6.6. Sensitivity analyses

Sensitivity analyses were conducted to investigate the relative contributions of different random variables to the performance of the single-braced truss web system under axial compression. Two random variables were studied here: (1) the translational stiffness of the nail connection along the longitudinal (length) direction $K_x$ (X axis in Figure 6.10) and (2) the out-of-plane rotational stiffness of the MPC web-to-chord connection $K_r$.

The procedure of the sensitivity study involved varying the characteristic distribution parameters of each of these random variables over a specified range and then repeating the above reliability analyses to determine the failure probability and the reliability indices $\beta$ with respect to the same three $S$ levels. For simplicity each random variable was varied one at a time in the analyses.
6.6.1. Kx—the translational stiffness of the nail connection along the longitudinal direction

According to Song and Lam (2009 and 2010), the translational stiffness of the nail connection along the longitudinal direction Kx followed the normal distribution with the mean and the standard deviation of 1.372 N·m and 0.297 N·m respectively. In this sensitivity analysis, the coefficient of variation was kept intact, but the mean values were changed to 0.686 N.m and 2.058 N·m (0.5 and 1.5 times the original value) alternatively, and then the standard deviation became 0.1485 N·m and 0.4455 N·m respectively. While the Kx was changing, the other five random variables were kept the same as characterized in Table 6.4, Table 6.12 and Table 6.13 showed the results of reliability analyses based on the changes.

<table>
<thead>
<tr>
<th>S level</th>
<th>Method</th>
<th>Probability of failure (%)</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>S = 1%</td>
<td>FORM</td>
<td>0.671</td>
<td>2.471</td>
</tr>
<tr>
<td></td>
<td>Monte Carlo (1e5)*</td>
<td>0.739</td>
<td>2.436</td>
</tr>
<tr>
<td>S = 1.5%</td>
<td>FORM</td>
<td>0.023</td>
<td>3.503</td>
</tr>
<tr>
<td></td>
<td>Monte Carlo (1e5)*</td>
<td>0.024</td>
<td>3.477</td>
</tr>
<tr>
<td>S = 2%</td>
<td>FORM</td>
<td>2.05e-05</td>
<td>4.610</td>
</tr>
<tr>
<td></td>
<td>Monte Carlo (1e9)*</td>
<td>1.03e-07</td>
<td>4.785</td>
</tr>
</tbody>
</table>

Note: * indicates the sample size of Monte Carlo simulation method.
Table 6.13 Reliability indices and failure probability at Kx’s mean = 2.058 N.m

<table>
<thead>
<tr>
<th>S level</th>
<th>Method</th>
<th>Probability of failure (%)</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>S = 1%</td>
<td>FORM</td>
<td>0.612</td>
<td>2.507</td>
</tr>
<tr>
<td></td>
<td>Monte Carlo (1e5)*</td>
<td>0.620</td>
<td>2.504</td>
</tr>
<tr>
<td>S = 1.5%</td>
<td>FORM</td>
<td>0.018</td>
<td>3.580</td>
</tr>
<tr>
<td></td>
<td>Monte Carlo (1e5)*</td>
<td>0.013</td>
<td>3.667</td>
</tr>
<tr>
<td>S = 2%</td>
<td>FORM</td>
<td>1.18e-5</td>
<td>4.750</td>
</tr>
<tr>
<td></td>
<td>Monte Carlo (1e10)*</td>
<td>2.06e-06</td>
<td>5.173</td>
</tr>
</tbody>
</table>

Note: * indicates the sample size of Monte Carlo simulation method.

Figure 6.18 The Kx and reliability index β relationship

It can be seen from Table 6.12 and Table 6.13 that the change of reliability index β ranged from 1.5% (= (2.507 - 2.471) / 2.471 for case S=1%) to 8% (= (5.173 - 4.785) / 4.785, for case S=2%) with the changes of Kx. It is also illustrated in Figure 6.18 that in all three S levels the reliability indices β linearly increased by a very small amount with the increase of Kx, and the increase is almost negligible. Therefore, it is safe to say that the failure probabilities and the reliability indices β are not significantly influenced by
variations in Kx. It is probably because the nail connection along the longitudinal
direction was too stiff that \( \beta \) was not affected much by the change of Kx.

6.6.2. \( Kr \)—the out-of-plane rotational stiffness of the MPC web-to-chord
connection

The out-of-plane rotational stiffness of the MPC web-to-chord connection \( Kr \) was
considered as another random variable in the sensitivity study. Similar to random variable
Kx, the mean of \( Kr \) was varied to 7.3 kN.m/rad and 21.9 kN.m/rad (0.5 and 1.5 times the
origin), but the coefficient of variation didn’t change, and the three characteristic Weibull
parameters changed correspondingly. The results of the reliability analyses are listed in
Tables 8 and 9.

<table>
<thead>
<tr>
<th>S level</th>
<th>Method</th>
<th>Probability of failure (%)</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>S = 1%</td>
<td>FORM</td>
<td>2.305</td>
<td>1.940</td>
</tr>
<tr>
<td></td>
<td>Monte Carlo (1e5)*</td>
<td>2.693</td>
<td>1.890</td>
</tr>
<tr>
<td>S = 1.5%</td>
<td>FORM</td>
<td>0.348</td>
<td>2.635</td>
</tr>
<tr>
<td></td>
<td>Monte Carlo (1e5)*</td>
<td>0.260</td>
<td>2.639</td>
</tr>
<tr>
<td>S = 2%</td>
<td>FORM</td>
<td>1.25e-3</td>
<td>3.403</td>
</tr>
<tr>
<td></td>
<td>Monte Carlo (1e6)*</td>
<td>1.65e-3</td>
<td>3.306</td>
</tr>
</tbody>
</table>

Note: * indicates the sample size of Monte Carlo simulation method.
Table 6.15 Reliability indices and Failure Probability at Kr’s mean = 21.9 kN.m/rad

<table>
<thead>
<tr>
<th>S level</th>
<th>Method</th>
<th>Probability of failure (%)</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>S = 1%</td>
<td>FORM</td>
<td>0.242</td>
<td>2.849</td>
</tr>
<tr>
<td></td>
<td>Monte Carlo (1e5)*</td>
<td>0.265</td>
<td>2.808</td>
</tr>
<tr>
<td>S = 1.5%</td>
<td>FORM</td>
<td>2.37e-3</td>
<td>4.146</td>
</tr>
<tr>
<td></td>
<td>Monte Carlo (1e5)*</td>
<td>2.30e-3</td>
<td>4.165</td>
</tr>
<tr>
<td>S = 2%</td>
<td>FORM</td>
<td>3.54e-6</td>
<td>5.570</td>
</tr>
<tr>
<td></td>
<td>Monte Carlo (1e10)*</td>
<td>1.03E-07</td>
<td>6.296</td>
</tr>
</tbody>
</table>

Note: * indicates the sample size of Monte Carlo simulation method.

It is illustrated in Figure 6.19 that in all S levels reliability indices increase linearly with Kr, and the increase ranges from 47% to 64%. When the S value goes up, β is likely to increase more with the increase of Kr. It is clear, from the analysis, that the reliability of the single-braced web system is very sensitive to the changes of Kr, and thus determining the Kr value properly is essential for bracing design.
6.7. Conclusion

This chapter presented a study to evaluate the critical buckling load and lateral bracing force of single- and double-braced truss web systems. The experimental and analytical studies were conducted to investigate the gap influence over rotational stiffness of MPC joints and quantify the out-of-plane rotational stiffness. The results showed that the variation was small and negligible. Out-of-plane bending joint modeling was conducted using the program, SAMPC, which verified the applicability of the program to evaluate the out-of-plane stiffness of MPC joints. A structural FEM analog was built with the consideration of semi-rigid web-end fixity provided by MPC truss joints. A response surface method was employed to estimate the reliability of single- and double-braced truss web systems under buckling load. The randomness of the MOE of lumber, the out-of-plane rotational stiffness of the MPC web-to-chord connection, the translational stiffness of the nail connection along the longitudinal direction, the initial lateral mid-span deflection of web members, and the load eccentricity, were considered in the formulation of the performance function. The sensitivity analyses were performed to investigate the influence of the translational stiffness of the nail connection and the reliability of the out-of-plane rotational stiffness of the MPC web-to-chord connection. Based on the results of the reliability analyses, it indicated that the 2% rule based on pined web end assumption is quite conservative for metal plated wood truss bracing design practice, and a rational value of 1.25% was recommended to replace the 2% rule when the rotational stiffness of the chord member is considered. From the sensitivity analyses, the reliability of the system was not significantly affected by the translational stiffness of the nail connection along the longitudinal direction; nevertheless, the
reliability index $\beta$ shows a linear relation with the out-of-plane rotational stiffness of the MPC web-to-chord connection. Therefore, determining the $K_r$ value properly is essential for truss bracing design.

The results of the reliability study can be applied to machine-stress-rated lumber that is most common in the MPC wood truss construction practices in Canada. The same analytical procedures can be applied to assess the reliability of lateral-braced truss web systems fabricated with visually graded lumber.
Chapter 7. Conclusions

7.1. Conclusions

A FEM based program (SAMPC) was developed to model the nonlinear behavior of MPC joints in wood trusses. The model is capable of accounting for a large number of factors influencing MPC joint performance:

1. To retain the real geometry of MPC joints as much as possible, 3D brick elements with orthotropic material properties were used to model the wood members, a thin plate element to model the truss plate, and an elaborate tooth connection element to model individual teeth.

2. The slot opening of the truss plate, made by die-punched teeth, was considered while forming the mathematical solution.

3. The tooth connection model, which considered the tooth shank as a beam element with elastic-plastic behaviour, employed three sets of springs to account for underlying wood resistance along the two lateral directions of each tooth and withdrawal resistance along the longitudinal axis. All the springs had featured nonlinear load-displacement curves. Implementing these springs allowed the model to evaluate adequately the load-displacement relationship of the tooth connection with various plate-to-grain orientations by using Hankinson’s interpolation.
4. The model can predict the plate peeling and the tooth withdrawal by monitoring the relative gaps between the plate and wood under stepwise loading. A withdrawal displacement updating scheme was used to simulate the tooth withdrawal process. For the first time in any MPC joint study, a tentative tooth withdrawal failure criterion based on the percentile of tooth withdrawal, was proposed to predict joint failure during tooth withdrawal.

5. Nonlinearity was incorporated in the model by considering the large deformation of the plate induced by the P-Δ effect, by the nonlinear stiffness of the tooth springs, and by the tooth withdrawal process. The nonlinear finite element equations in this study were solved using the integrated Newton-Raphson iteration procedure.

Two series of experiments were conducted to calibrate the model parameters and verify the model: material property tests aimed at generating the model inputs, the structural tests of MPC joints in tension aimed at developing a database for model parameter calibration and model verification.

By comparing the model solutions against the joint test data, an optimization procedure was followed to derive the model parameters. The calibrated model predictions showed a fairly good agreement with the experimental results up to the critical load (maximum load). Based on the results from the parameter calibration, the joint model of angled plate-to-grain orientations predicted the behavior of joint adequately well up to the critical load, thus demonstrating the applicability of this model to more complex joint configurations.
For the first time in an MPC study, the modeling of the out-of-plane rotational stiffness of MPC joints was investigated. The bending joint model was again built by using the SAMPC program. An adjusting factor was used to compromise the model’s limitation due to model assumptions. The adjusting factor can be calibrated by means of MPC joint bending tests. The calibrated model predictions agreed well with the test results.

A reliability analysis was conducted to evaluate the critical buckling load and lateral bracing force of a single-/double-braced wood truss web system. A number of variables affecting the performance of braced truss web systems were investigated. As a result, a design ratio of adequate reliability was recommended for amending bracing design.

Sensitivity analyses showed that the reliability of the single-braced system is not significantly affected by the translational stiffness of the nail connection along the longitudinal direction axes. The reliability index, however, shows a linear relation with the out-of-plane rotational stiffness of the MPC web-to-chord connection.

7.2. Significance

In the case of the investigated truss joints, the SAMPC program shows its ability to simulate MPC joints in elaborate detail, including material properties, 3D model geometry, the plate slots, the location and orientation of the teeth, the wood grain angle, boundary conditions, loading conditions, etc. This detailed model greatly facilitates a better understanding of joint behavior under realistic joint configurations and loading conditions. The ability of the model to predict the behavior of MPC joints, really brings
up the potential to model joints fabricated from different wood species and truss plate types, featuring more complex joint configurations, and capable of being subjected to more complex loading conditions. The information derived from the model results can be used not only to evaluate the adequacy of a given structural design, but also to facilitate truss plate, truss joint and overall truss design.

The tooth withdrawal process was carefully considered in the model, because it is a major reason of MPC joint failure and one of substantial sources of nonlinearity of joint behavior. The tooth withdrawal displacement updating scheme successfully simulated the tooth withdrawal process. As a tentative exploration, a tooth withdrawal failure criterion has been proposed, for the first time in MPC joint study, to predict the ultimate load of joints failed in tooth withdrawal. The model results were adequately satisfactory. Due to the similarity between the truss plate tooth and the dowel type connectors, in which a deformable beam bears on flexible foundations (such as wood), it is assumed that the methodology of the withdrawal displacement updating can also be applied to improve the modeling of dowel type connectors.

Also for the first time in an MPC study, the modeling of the out-of-plane rotational stiffness of MPC joints has been investigated. The satisfactory model prediction again verifies the versatility of the computer program in modeling various joint configurations.

The reliability analyses presented a study to evaluate the critical buckling load and lateral bracing force of single-/double-braced wood truss web systems. The probability characteristics of a wide range of the model variables were investigated. The
output of results from this reliability study can be used as a basis for truss compression web/brace design.

### 7.3. Limitations and recommendations

Several limitations due to model simplifications and assumptions, and recommendations for the future work regarding the computer program SAMPC could be made based on the results yielded from this study.

In this thesis, only relatively small truss plates were used, which along with the special plate allocation design, provoked joints to fail in tooth withdrawal. However, MPC joints may also fail in plate rupture in tension or shear, particularly when larger truss plates are engaged. Considering steel plate properties in the program, the model can be extended to account for post-yielding nonlinear performance of steel plate.

Likewise, the model, by including wood failure stress in tension perpendicular to grain, is possible to identify joint failure in wood rupture (tension perpendicular to grain).

Although the effort have been made to develop a model to predict the tooth withdrawal failure of MPC truss joints, the model solutions show a close agreement with the joint test results before the maximum load, but aren’t consistent with test results past the maximum load. This inconsistency may come from the assumption of the uniform tooth-to-wood properties, the tooth withdrawal failure criteria, or the numerical nonlinearity caused by the large deformation after the maximum strength. This brings forth the necessity of the further research on model’s post-peak properties.
The major objective of this thesis is to numerically model the behaviour of the MPC truss joints loaded in a static state (uni-directional tension or out-of-plane bending). Therefore, the program isn’t able to consider the dynamic demands, which is also beyond the scope of this thesis. The program HYST was originally developed by Foschi (2000) to simulate the static and dynamic behavior of nail connectors. Unlike its static applications, in which only the uni-directional lateral springs are engaged, both the lateral springs on the two opposite lateral directions and the gap closure between the nail and wood medium are engaged. Therefore, the possible modifications for the SAMPC program can be made to investigate the dynamic performance of MPC joints, such as the loading history and the peeling moment updating.

As the mathematical representation of the longitudinal tooth-to-wood interaction, the simplified withdrawal spring model, incorporated with withdrawal displacement updating and withdrawal failure criterion, provided relative solutions for MPC joint analyses. However, more sophisticated models should take into account the interaction of tooth-to-wood longitudinal interaction with respect to other possible factors, such as the lateral wood medium pressure along tooth length.

In reality, the load-slip relationship of single tooth is determined not only by the steel material properties, geometrical size, but also by the tooth bearing face against the wood and the wood properties. During the visual inspection of truss plate, the teeth were die-punch protruded from the plate with random bearing faces, which is not consistent with the assumption of uniform characteristics for all tooth. Meanwhile since wood is a highly variable material, the wood medium may not provide the uniform lateral resistance on the tooth bearing area. Therefore, the procedure of treating each tooth-to-wood...
interaction with the uniform load-slip relationship (elasticity) may bring the intrinsic limitation, such as the concurrent withdrawal failure of a tooth row.

In the model calibration procedure described in Chapter 5, the characteristic model parameters were derived by fitting the model against the average test data. As a result, the fitting parameters are mean values. However, based on the results from the experiments, a database was developed consisting of tested truss joint configurations. The statistical quantities of values for each model parameter, such as the mean, standard deviation, and probability distribution, can be readily obtained through statistical analysis. Generating this statistical information raises the possibility of conducting reliability studies on the strength and stiffness of MPC truss joints.

In the out-of-plane bending joint model, an adjusting factor was used to account for the possible gap closure at the wood-to-wood end, as well as plate buckling. A more advanced model is required to take into account of these aspects. Such a model, which would include a large-deformation nonlinear analytical procedure, could be used to evaluate MPC joints under in-plane bending.

The results from the detailed models provide important insights regarding how MPC joints perform under designed loading, thus facilitating our ability to determine their mechanical properties of MPC joints. However, these model may be too costly to run and too complex to use for everyday truss design. Thus, SAMPC could be used by truss manufacturers as a means of evaluating and improving determinate analysis methods.
Bibliography


Appendix A: Test Results of MOE, SG and MC Tests and Curve Fitting Results of MPC Truss Joint Tensile Tests

1. Test results of MOE, SG and MC

The 7-foot long SPF lumber was stacked into a pile with the properly placed wood blocks to separate the lumber so that air can move across the wood surfaces. Then the lumber was stored in a conditioning chamber at 20°C and 65% relative humidity until the specimens reached constant weight. The MOE of lumber was determined using an E-Computer on the basis of vibration frequency prior to specimen fabrication. Table A.1 lists the results of the 88 lumber pieces measured, and the average was used in the FEM model formulation.

<table>
<thead>
<tr>
<th>No.</th>
<th>Vibration MOE (GPa)</th>
<th>No.</th>
<th>Vibration MOE (GPa)</th>
<th>No.</th>
<th>Vibration MOE (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.04</td>
<td>31</td>
<td>7.40</td>
<td>61</td>
<td>9.37</td>
</tr>
<tr>
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The SG and MC were tested in accordance with ASTM D2395, Method A and ASTM D 4442, Method A (ASTM 2008). Before these tests, the specimens were sealed in an airtight plastic bag after being cut from the joint specimens, and then put away in an airtight jar to avoid the time variation. The test results of SG and MC are listed in Tables A.2 to A.9 for each individual specimen.

2. Curve fitting results of MPC truss joint tests

Six types of MPC truss joint were tested in tension in the purpose of calibrating model parameters and verifying the model. Originally, there were 30 joint specimens fabricated for each of four standard joint types, and 25 joint specimens for each of two non-standard joint types. However, the data from a few joint specimens were discarded due to several reasons, such as pilot joints tested to acquire proper test speed, a few 45-degree joint specimens tested before configuration adjustment, fixture failure and failure at locations other than near the plates.

The curve fitting procedure was carried out on each individual joint test. The resultant fitting parameters, maximum loads and associated R square values are presented in Tables A.2 to A.9. It should be noted that for the four standard joint types, the fitting parameters were obtained based on the average of the two transducer’s readings and were denoted as AA, AE, EA, and EE; while as for two non-standard cases, due to the asymmetric deformation recording location, the fitting parameters based on the two transducers’ recordings are listed separately and denoted as 60-1, 60-2, 45-1 and 45-2.
Table A.2 Curve fitting parameters of joint type AA

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c.o.v. 0.08 0.04 0.027 0.064 0.039 0.035 0.051 0.014

Note: Q3 is dimensionless and corresponds to a load level equal to 80% of the maximum.
Table A.3 Curve fitting parameters of joint type AE

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Note: Q3 is dimensionless and corresponds to a load level equal to 80% of the maximum.
Table A.4 Curve fitting parameters of joint type EA

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c.o.v. 0.06 0.03 0.037 0.118 0.037 0.022 0.026 0.012

Note: Q3 is dimensionless and corresponds to a load level equal to 80% of the maximum.
Table A.5 Curve fitting parameters of joint type EE

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STDV 0.04 0.01 0.296 0.327 0.385 0.025 0.112 0.107
c.o.v. 0.09 0.05 0.062 -0.841 0.024 0.040 0.052 0.028

Note: Q3 is dimensionless and corresponds to a load level equal to 80% of the maximum.
Table A.6 Curve fitting parameters of joint type 60-1

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<th>Q1 (kN/mm)</th>
<th>K (kN/mm)</th>
<th>DMAX (mm)</th>
<th>Q3 PEAKLOAD (kN)</th>
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Note: Q3 is dimensionless and corresponds to a load level equal to 80% of the maximum.
Table A.7 Curve fitting parameters of joint type 60-2

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Average 0.50 15.35 5.671 2.919 29.926 0.870 1.660 7.703
STDV 0.03 0.47 0.338 0.440 1.887 0.060 0.093 0.227
c.o.v. 0.07 0.03 0.060 0.151 0.063 0.069 0.056 0.030

Note: Q3 is dimensionless and corresponds to a load level equal to 80% of the maximum.
Table A.8 Curve fitting parameters of joint type 45-1

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Note: Q3 is dimensionless and corresponds to a load level equal to 80% of the maximum.
Table A.9 Curve fitting parameters of joint type 45-2

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<th>Spec. #</th>
<th>SG (g/cm$^3$)</th>
<th>MC (%)</th>
<th>Q0 (kN)</th>
<th>Q1 (kN/mm)</th>
<th>K (kN/mm)</th>
<th>DMAX (mm)</th>
<th>Q3</th>
<th>PEAKLOAD (kN)</th>
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Note: Q3 is dimensionless and corresponds to a load level equal to 80% of the maximum.
Appendix B: The Deformation Plot of MPC Model

One Visual Basic (VB) program has been developed to generate the geometrical input information for SAMPC, which includes the geometrical size and mesh size of the plate, the wood member and the teeth, and the layout patterns of tooth arrays. The same VB program also serves to visualize the original and deflection diagrams of the MPC truss joints with available model solutions. To demonstrate the program, the examples of modeled MPC joints are given here.

The geometry of joints and the element meshing procedure were constructed based on the assumptions discussed in Section 5.1. The black squares in the diagrams illustrate the plate slots; the red dots illustrate the tooth location relative to the metal plate and the wood member. The axes X, Y and Z follow the rule of Cartesian coordination system. The axes X and Y are oriented along two plate directions: either primary (tooth slot direction) or secondary direction (perpendicular to the plate primary direction), and the axis Z is perpendicular to the plane XY.

Only the top view of the original joint diagram and the side view of the deformed diagram are shown here, which are believed to give a clear demonstration of how the geometry of joints and the element meshing were constructed and what the deformation is relative to the original diagram. The deformation diagram is depicted based on the model solutions right before the withdrawal failure of the first tooth takes place.
Due to the deformations are often relatively small and less pronounced when compared to the geometrical joint size, a scale index of 15 is used to magnify the deflection by 15 times, to achieve a better demonstration of the withdrawal deformation.

Figure B.1 The original plot of Joint AA (top view)

Figure B.2 The model-calculated deformation plot of Joint AA (side view)
Figure B.3 The original plot of Joint AE (top view)

Figure B.4 The model-calculated deformation plot of Joint AE (side view)
Figure B.5 The original plot of Joint EA (top view)

Figure B.6 The model-calculated deformation plot of Joint EA (side view)
Unlike the four standard joints in Figure B.1 to Figure B.8, in which the teeth on
the same row tended to perform the same in terms of forces and deformations, the
theoretical solutions show that in intermediate plate-to-grain angle joints (60 and 45
degrees), the teeth at the plate corner (in the circles of Figure B.9 and Figure B.12),
withstood greater per-tooth force than the other teeth, and thus tended to withdraw sooner,
which is due to the asymmetric geometry and loading condition in 60-degree and 45-degree cases. This was consistent with test observation.

In addition to the top view of original MPC joints diagram, two side views in the planes XZ and YZ are presented here to demonstrate the geometry and the element meshing of joints, as well as the deformation relative to the original diagram. The deformation of the top sides of metal plates (withstand the largest vertical deformation) are depicted in Figure B.10 and Figure B.13, to avoid the visual confusion caused by image overlapping.
Figure B.9 The original plot of Joint 60 (top view)

Figure B.10 The model-calculated deformation plot of Joint 60 (in the XZ plane)

Figure B.11 The model-calculated deformation plot of Joint 60 (in the YZ plane)
Figure B.12 The original plot of Joint 45 (top view)

Figure B.13 The model-calculated deformation plot of Joint 45 (in the XZ plane)

Figure B.14 The model-calculated deformation plot of Joint 60 (in the YZ plane)