SHEAR BEHAVIOR OF REINFORCED CONCRETE DEEP BEAMS UNDER STATIC AND DYNAMIC LOADS

by

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Abstract

Reinforced concrete (RC) deep beams predominantly fail in shear which is brittle and sudden in nature that can lead to catastrophic consequences. Therefore, it is critical to determine the shear behavior of RC deep beams accurately under both static and dynamic loads.

In this study, a database of the existing experimental results of deep beams failing in shear under static loading was constructed. The database was used to propose two simplified shear equations using genetic algorithm (GA) to evaluate the shear strength of deep beams with and without web reinforcement under static loads. Reliability analysis was performed to calibrate the equations for design purposes. The resistance factors for the design equations were calculated for a target reliability index of 3.5 to achieve an acceptable level of structural safety.

A deep beam section designed following the building codes considering only static loads may behave differently under dynamic loading condition. Therefore, in this study, deep beams were analyzed under reversed cyclic loading to simulate the seismic effects. The ultimate load capacity, energy dissipation capacity, and ductility capacity were calculated in deep beams with different reinforcement ratios.

In RC structures, deep beams have interaction with other structural elements through connections. Therefore, to predict the shear behavior of deep beams in real structure under seismic loads, it is necessary to analyze a full structure with a deep beam. A seven storey RC office building with a deep transfer beam was designed following the CSA A23.3 standards. The structure was analyzed using non-linear pushover and non-linear dynamic time history analysis. The deep beam was evaluated for the shear deficiency under different earthquake records for the soil condition of the City of Vancouver. The analysis results showed a significant shear
deficiency of about 25% in the deep beam. The use of carbon fibre reinforced polymer (CFRP) resulted in increasing the shear capacity of a deep beam by up to 82%.

Keywords: Deep beam; shear strength; shear equation; genetic algorithm (GA); reversed cyclic load; seismic load.
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Dedication

To my parents, wife and the graceful gift from the almighty Allah- Nabeeha
Chapter 1: Introduction

1.1 Overview

Reinforced concrete deep beams have many applications in buildings, bridges, offshore structures and foundations. There are many structural elements which behave as a deep beam such as transfer beams, load bearing walls and coupling beams in buildings, pile caps in foundations, plate elements in the folded plates and bunker walls.

The existing research on deep beam behavior is reviewed in this section. The design and analysis of a deep beam needs more attention compared to a regular slender beam. Most of the previous researches on deep beam were focused on the shear behavior. This is because, the shear failure in deep beam was common and moreover, it is sudden and brittle. The objective of this literature review is to accumulate all the previous study done on the shear behavior in deep beam in order to identify the gaps in the past studies and set the objectives and areas for future research on deep beams.

The literature review has been divided into two sections. In the first part, the background on deep beams and the current ongoing research on the shear behavior of deep beams under static loads are discussed thoroughly. The methodology to calculate the shear capacity of deep beams available in the current building codes including American Concrete Institute (ACI 318) and Canadian Standards Associations (CSA A23.3) are discussed.

The next section of the literature review is focused on the shear behavior of deep beams under dynamic loads. Shear strengthening of deep beams using fibre reinforced polymer (FRP) are also
discussed in this section of the literature review. The basic material properties of the FRP and the behavior of the FRP systems as retrofitting and reinforcing elements are presented.

1.2 What is a Deep Beam?

A beam is generally regarded as a deep beam when its shear span, \( a \), to depth ratio, \( d \), is less than 2.5 \( (a/d<2.5) \) (ACI-ASCE Committee 426). The shear span is the distance from the loading point to the support. Fig. 1.1 shows a typical example of a slender and a deep beam. The main difference between a slender beam \( (a/d>2.5) \) and a deep beam is that in case of a slender beam the shear deformation is negligible and could be ignored while it must be considered in the analysis and design of a deep beam.

A predominant failure mode in deep beams is shear failure that can lead to catastrophic consequences. The Bernoulli-Navier's hypothesis for slender beams states that the distribution of strain through the depth of the beam is linearly proportional to the distance from the neutral axis. Their hypothesis was based on the assumption that shear deformation in a slender beam is negligible compared to flexural deformation. However, this is not applicable in case of deep beams. The strain distribution along the depth of a section in deep beam becomes non-linear with the decrease of the shear span to depth ratio (Collins and Mitchell 1991). Therefore, their theory cannot accurately predict the behavior of deep beams since the assumption of ‘plane sections remain plane’ does not apply and therefore, the theory will underestimate the shear strength which is unacceptable. This phenomenon in deep beam was investigated by many researchers and concluded that the sectional approach failed to predict the accurate behavior in case of a beam having \( a/d \) less than 2 (Fig. 1.2) (Rogowsky 1986, Collins and Mitchell 1991).
Figure 1.1 Slender beam and deep beam

Figure 1.2 Variation of shear stress with a/d ratio (Collins and Mitchell, 1991)
1.3 Deep Beam under Static Loads

The research on deep beams was first reported in the literature at the beginning of the twentieth century. The experimental investigation conducted mostly focused on the shear behaviour of deep beams with different parameters (Clark 1951, Kani 1967, Kong et al. 1970, Smith and Vantsiotis 1982, Oh and Shin 2001, Quintero-Febre et al. 2006, Tan et al. 1995). The experimental investigations were performed for four point loading condition. From the failure pattern of deep beams under static loading, the strut and tie model (STM) was developed (Schlaich 1987 and Marti 1985). The STM method analyze concrete members with a plastic truss analogy that transfers the forces from the loading point to the supports using concrete struts acting in compression and steel reinforcing ties acting in tension (Schlaich 1987 and Marti 1985). The struts and ties are interconnected at nodes.

The STM model is now gaining popularity in the research community and recently it has been incorporated in many design codes including ACI 318 and CSA A23.3. In North America, CSA A23.3-84 was the first design code to adopt the STM model as a design standard in concrete members considering the compression field theory (Collins 1978). Recently, ACI building code incorporated the STM method in Appendix A of the section “Building code requirements for structural concrete (ACI 318-05) and commentary (ACI 318R-05)”.

1.3.1 Strut and Tie Model Formulation

A strut and tie model is composed of a truss model with two components: a concrete strut acting in compression and a steel reinforcement acting in tension. The struts form in the diagonal direction along the line connecting the loading points to the supports and the ties form along the
longitudinal reinforcement. The strut and ties are interconnected by nodes. The forces in strut and tie must remain in equilibrium for the applied external loads.

1.3.1.1 Strut

The strut is the compression field in the deep beam. It can be bottle shaped or prismatic shaped (ACI 318-05). In bottle shaped strut, the width of the strut at its mid length is larger than the stress field width at the ends (Fig. 1.3). This is due to the transverse stresses at mid depth of the strut where it spread laterally. To simplify the STM design, bottle shaped struts are idealized as prismatic. The stress field remains parallel in prismatic strut along the axis of the strut and it has a uniform cross section over the length of the strut (Fig. 1.3).

1.3.1.2 Tie

A tie consists of the non-prestressing or prestressing reinforcement along with the portion of the surrounding concrete that is concentric with the axis of the tie (ACI 318-05). The surrounding concrete is included to define the zone in which the forces in the struts and ties are to be
anchored. However, for design purposes, the concrete in a tie is not considered to resist tensile axial force developed in the tie.

1.3.1.3 Nodes

Nodes are the intersection points on the beam in a strut-and-tie model between the axes of the struts, ties, and concentrated forces acting on the beam (ACI 318-05). At least three forces should act on a node in a strut and tie model to maintain equilibrium. The nodes are classified according to whether the forces applied on them are in compression (C) or in tension (T) (Fig. 1.4). If a node connects only compressive forces it is called a CCC node. A CCT node is a node that connects one tension force and two (or more) compression forces. Similarly, a CTT node connects one compression force and two (or more) tension forces. The fourth type of nodes, TTT, connects only tension forces (Fig. 1.4).

![Figure 1.4 Classification of nodes (a) CCC node, (b) CCT node, (c) CTT node and (d) TTT node](image)

1.3.1.4 Nodal Zones

The nodal zones are the volume of concrete around a node that is assumed to transfer strut and tie forces through the node (ACI 318-05). The nodal zones are classified as hydrostatic or non-hydrostatic nodal zones. In a hydrostatic nodal zone, the stresses on all the loaded faces of the
node are equal and the axis of the struts and/or ties are perpendicular to the loaded faces. For a non-hydrostatic nodal zone, the stresses taken on a surface perpendicular to the strut axis must be determined. ACI 318-05 defines the nodal zone as which is bounded by the intersection of the effective strut width, $w_s$, and the effective tie width, $w_t$ (Fig. 1.5).

![Figure 1.5 Hydrostatic nodal zones](image)

1.3.2 Code Provision for STM

Recently, many reinforced concrete design codes including CSA A23.3, ACI 318 and Eurocode 2 adopted the STM method after evaluating the model against experimental investigations. The design provisions of deep beams using the STM method are discussed in this section.

1.3.2.1 CSA A23.3-04

CSA A23.3-04 states that the flexural member with a clear span ($l_n$) to overall depth (h) ratio less than 2 must be designated as deep flexural member and the non-linear distribution of strains should be taken into account. According to CSA standard, the STM is an appropriate method to design deep flexural members. The code provision states that the strength of the strut is limited to the effective compressive stress of the concrete, $f_{ce}$, which is calculated using Equations 1.1
and 1.2 based on the modified compression field theory (Vecchio and Collins 1986).

\[
\frac{f_{ce}}{0.8 + 170\varepsilon_i} \leq 0.85 f'_c \quad (1.1)
\]

\[
\varepsilon_i = \varepsilon_s + (\varepsilon_s + 0.002) \cot^2 \theta_s \quad (1.2)
\]

where, \( \theta_s \) is the smallest angle between the compressive strut and the adjoining tensile ties, \( \varepsilon_s \) is the tensile strain in the tie inclined at \( \theta_s \) to the compressive strut and \( \varepsilon_i \) is the principal tensile strain.

Equation (1.2) assumes that the principal compressive strain in the direction of the strut is \(-0.002\) mm/mm. It can be observed from the above two equations that the concrete effective compressive stress, \( f_{ce} \), will increase with the increase of \( \theta_s \). CSA A23.3-04 does not specify any limitation on the value of \( \theta_s \). Rogowsky and MacGregor (1986) recommended a value for \( \theta_s \) from 25 degrees to 65 degrees.

In the nodal zone, the stress limit in concrete is as follows (CSA A23.3):

i. CCC node: \( 0.85 \phi_c f'_c \)

ii. CCT node: \( 0.75 \phi_c f'_c \)

iii. CTT or TTT node: \( 0.65 \phi_c f'_c \)

According to CSA A23.3-04, the minimum web reinforcement for deep flexural members is 0.2% of the gross concrete area in the horizontal and the vertical direction. An orthogonal grid of reinforcement should be located close to each face of the deep beam and the spacing of this reinforcement should not exceed 300 mm in each direction.
1.3.2.2 ACI 318-05

The definition of deep beam according to ACI 318-05 building code is “a member loaded on one face and supported on the opposite face so that compression struts can develop between the loads and the supports, and have either: (a) clear spans, $l_n$, equal to or less than four times the overall member depth; or (b) regions with concentrated loads within twice the member depth from the face of the support” (ACI Committee 318 2005). ACI 318-05 included the STM provisions for deep beam in its Appendix A. The design of a deep beam using the STM method is limited to members with an angle between the diagonal compression strut and the tension tie not less than 25 degrees.

According to ACI 318-05, the strength of a strut depends on the geometry of the strut, and the presence of reinforcement and how they are distributed across the strut. The effective compressive strength of the concrete in a strut is calculated using Equation (1.3).

$$f_{ce} = 0.85 \beta_s f_c$$  \hspace{1cm} (1.3)

where, $\beta_s$ =1.0 for struts with uniform cross section area over its length, 0.75 for bottle-shaped struts with distributed reinforcement crossing, 0.60 for bottle-shaped struts without distributed reinforcement crossing, and 0.40 for struts in tension members or the tension flanges of members.

The compression strength of nodal zones is calculated using Equation (1.4).

$$F_{nn} = f_{cen} A_{nz}$$  \hspace{1cm} (1.4)

where, $A_{nz}$ is the smaller of (a) the area of the face of the nodal zone perpendicular to the load acting on that face, and (b) the area of a section through the nodal zone perpendicular to the
line of action of the resultant force on the section,

\[ f_{cen} \] is the effective compressive strength of the concrete in the nodal zone \( = 0.85 \beta_n f'_c \),

\( \beta_n \) is a factor that depends on the node classification, 1.0 for CCC nodes, 0.80 for CCT nodes, and 0.60 for CTT nodes.

The yield strength for the non-prestressed tension ties in ACI 318-05 is limited to 550 MPa for longitudinal reinforcement and 410 MPa for shear reinforcement.

1.3.3 Drawback of STM

The basic drawback of the STM method is the complexity of the calculation required to predict the shear capacity. In the STM method, the designer is free to choose the truss dimensions (strut and tie) that carry the load through the disturbed region (D-region) to the supports. Therefore, many designers are not comfortable when applying the STM approach as there is no single solution for the STM model and it is recommended to assume more than one STM model to determine the capacity of a deep beam.

1.4 Deep Beams under Dynamic Loads

The behavior of deep beams under seismic loads is more complex compared to the behavior under static loads. Therefore, it is necessary to examine the behavior and failure pattern of a deep beam under seismic loads at ultimate loading conditions. Only experimental studies can provide accurate information, however, such studies involve testing a full scale deep beams which is quite expensive and it requires sensitive equipment to simulate the seismic loads. Moreover, the test methods are also time consuming. Therefore, the current investigation only focused on numerical analysis performed by finite element method. Two types of seismic analysis were
performed in this study and will be discussed in the following sections; the reversed cyclic analysis and the seismic analysis.

1.4.1 Reversed Cyclic Analysis

The experimental simulation of a real earthquake load in the laboratory requires sensitive equipment which is expensive. To overcome this limitation, researchers commonly performed reversed cyclic load tests on RC members. It has been observed that the problems arising in reinforced concrete structures can be traced by performing reversed cyclic tests. Therefore, it is necessary to predict the behavior under such conditions in order to develop safe and economical design of RC structures. The hysteresis model developed from the reversed cyclic analysis will provide information about ductility capacity, strength degradation, and energy dissipation of a deep beam during strong earthquake.

1.4.2 Seismic Analysis

It is possible to predict the behavior of single standalone deep beam from the reversed cyclic load analysis. However, in case of a real structure, the actual behavior of a deep beam will be affected by the integrated connection with other members such as columns or shear walls. Therefore, it is necessary to analyze the RC structure with a deep beam under seismic loads. In order to determine the exact performance among various seismic analysis, the non-linear static pushover analysis and non-linear dynamic time history analysis are the most common methods used to evaluate the behavior of RC structure under real earthquake loading. A detail description of these methods is given in Chapter 5.
1.5 Fibre Reinforced Polymer (FRP)

Fibre reinforced polymer (FRP) is a composite material which consists of fibres embedded in a resin matrix. Three kinds of fibres are available for structural applications: carbon, glass and aramid which are named as carbon fibre reinforced polymer (CFRP), glass fibre reinforced polymer (GFRP) and aramid fibre reinforced polymer (AFRP), respectively (Fig. 1.6). The term polymer is used to describe an array of extremely large molecules that consist of repeating units in which atoms are connected by covalent bonds. Resins are divided into two major groups: thermosetting and thermoplastic polymers. Polymer resins are good heat and electricity insulators. They are usually considered isotropic viscoelastic material. They creep under sustained load, and susceptible to ultraviolet degradation. The most common thermosetting resins used in civil engineering applications are: unsaturated polyesters, epoxies and vinylesters.

Figure 1.6 Different types of FRP (ISIS 2007)
In the FRP matrix, the fibres carry the loads and are bonded together with the resin which allows the transfer of forces from fibre to fibre through shear stresses. The forms of FRP in structural engineering are reinforcing bars and tendons (applied internally in reinforced concrete or externally in prestressed concrete), and plates and sheets for external strengthening of members. FRP is a composite material; therefore its property completely depends on the formulation and constituents of the individual materials and their manufacturing processes. The properties of the FRP composite materials can be obtained following the experimental procedure described in CSA S806 and S807 and different ASTM test methods. The most common characteristics of FRP systems include high strength, light weight, corrosion resistance, fatigue resistance, insulation and small creep. All FRP systems have a linear elastic tensile behavior along the fibre direction (Fig. 1.7).

Fibre reinforced polymer (FRP) is becoming popular in concrete industry during the last two decades due to its high durability and high strength. The application of FRP includes repair, rehabilitation and strengthening of aged and new structures. The previous experimental investigations on FRP retrofitted RC beams using both FRP strip and wrapping showed a significant increase in the shear strength with respect to the unstrengthened beam (Tanarslan 2008, Altin 2010, Lim 2010, Tanarslan 2010, Colalillo and Sheikh (2011). Moreover, it was observed that the shear strength increase in beams depends on wrap length, anchorage and orientation of the fibre (Taljsten 2003, Sakar 2009, Lee 2011).
1.6 Research Significance

Many researchers proposed analytical equations to predict shear strength of deep beams from their experimental results. Therefore, the accuracy of the available equations is limited to the experimental tests they are derived from. In addition, the ACI and CSA strut and tie model (STM) is also too conservative and complicated to implement. Therefore, the proposed shear equations in the present research derived from the available experimental data in the literature will help engineers to predict the experimental shear capacity of a deep beam accurately. Moreover, the proposed shear equations are simple enough to be used in the design process compared to the complex STM method in the codes.
Previous researches were mostly focused on the shear behavior of RC deep beams under monotonic loads. The design equations by the STM method recommended in the ACI 314-10 and CSA A23.3 building codes are also based on the monotonic loading condition. Accordingly, the behavior and the shear capacity of a deep beam, under seismic load, designed using STM method is unknown. Therefore, the results from a finite element (FE) analysis of deep beam sections under reversed cyclic loading in the present research will help to predict the shear behavior in the loading and unloading conditions during an earthquake event.

Finally, the dynamic pushover and time history analysis performed on an RC deep transfer beam structure will help to capture the shear behavior of deep beams during a real earthquake. Moreover, the analysis will also help to capture the interaction of deep beam with other structural elements in building during an earthquake.

1.7 Scope and Objectives

The present research evaluates the shear behavior of RC deep beams under static and dynamic loads. The main objectives of the present research are as follows:

- To perform a parametric study in order to identify the effect of different parameters on the shear strength of deep beam and to compare the accuracy of the available shear equations including the ACI and the CSA STM methods for deep beam.
- To develop a simplified analytical shear equation to predict the shear strength of RC deep beams. In addition, a reliability analysis will be performed to propose resistance factors for design.
To perform a reversed cyclic loading analysis on a deep beam to evaluate the performance in terms of shear capacity, ductility capacity, energy dissipation capacity and deformation restoring capacity.

To analyze a RC building with a deep transfer beam under real earthquake records to observe the shear behavior along with the interactions with other structural elements in the building. The study will also investigate the use of FRP as a retrofitting technique in order to improve the shear capacity of a deep beam.

1.8 Thesis Organization

This thesis contains six chapters focused on the shear behavior of RC deep beams under static and dynamic loads.

Chapter 1 contains the background of RC deep beams. The procedure used in design codes to design deep beams with strut and tie model (STM) is also provided. An overview of seismic analysis and retrofitting techniques using FRP are also discussed.

Chapter 2 presents a thorough literature review on deep beams under static loading. The important parameters that affect the shear strength of RC deep beams are discussed. Previous shear equations including both the ACI and CSA STM models to predict the shear strength of RC deep beams with and without web reinforcement are compared with the experimental shear strength.

Based on the analysis presented in Chapter 2, an analytical model to predict the shear strength of deep beams with and without web reinforcement is presented in Chapter 3. The equations are calibrated for the resistance factors so that they can be used in the design of a deep beam.
A finite element analysis was used to determine the behavior of deep beams under reversed cyclic loading and it is discussed in Chapter 4. The behavior of a standalone deep beam with the variation of reinforcement ratios is presented.

Chapter 5 discusses the analysis of a deep beam under actual earthquake records. The shear deficiency of the deep beam under real earthquake is discussed and a retrofitted technique is recommended to improve the shear capacity. Chapter 6 reports the conclusions and summarizes the analysis results with some future recommendations.
Chapter 2: Shear Strength Prediction of Reinforced Concrete Deep Beam: A Review

2.1 Overview

In this chapter, the shear strength of deep beams predicted by models and equations proposed in the literature and the design codes were evaluated and compared to experimental results reported in the literature. The comparative study was performed to identify the accuracy of the available models and the design code. The performance of each equation was evaluated by determining the experimental shear strength to the predicted shear strength \( \frac{V_{\text{test}}}{V_{\text{calc}}} \), the inverse of the slope of a linear least squares regression of the calculated shear strength \( V_{\text{cal}} \) versus the experimental shear strength \( V_{\exp} \) plot (the \( \chi \) factor), the coefficient of variation (COV), the standard deviation (SD), the sample variance (VAR), the absolute error (E) and the correlation coefficient (COR). The performance index (PI) of each model was also calculated and compared based on their total penalty (\( p \)).

2.2 Experimental Database

A total of 381 experimental data points were gathered from 18 papers (Table 2.1). The experimental results were first compiled in order to establish relationships among various factors affecting the shear strength. The variables used in the experiments were beam width (b), height (h), effective depth (d), shear span to effective depth ratio (a/d), the compressive strength of concrete \( f_c' \), longitudinal reinforcement ratio (\( \rho \)), horizontal shear reinforcement ratio (\( \rho_h \)) and vertical shear reinforcement ratio (\( \rho_v \)). Table 2.1 shows the range (minimum and maximum) of
the variables used in the experiments. The experimental database has a wide range of \( a/d \) ratio from 0.13 to 2.5. The longitudinal, horizontal and vertical shear reinforcement ratios \( (\rho, \rho_h, \text{and} \rho_v) \) ranged from 0.1 to 4.08\%, 0 to 2.52\% and 0 to 2.65\%, respectively. The database was divided into two sections: a) deep beams with shear reinforcement and b) deep beams without any shear reinforcement.

### 2.3 Factors Affecting the Shear Strength of Deep Beams

The important factors that affected the shear capacity of the deep beams were shear span to depth ratio, compressive strength of concrete, longitudinal reinforcement, horizontal shear reinforcement and vertical shear reinforcement (Smith 1982). Fig. 2.1 shows the scattered plot of the normalized shear strength, \( \hat{V} \), versus each parameter. The normalized shear strength is calculated using Equation (2.1).

\[
\hat{V} = \frac{V_u}{\sqrt{f'_c bh}} \tag{2.1}
\]

where, \( \hat{V} \) is the normalized shear strength, \( V_u \) is the ultimate shear strength, \( f'_c \) is the concrete compressive strength, \( b \) is the width of the beam, and \( h \) is the height of the beam. The contribution of each factor on the shear capacity of deep beams is discussed below.

#### 2.3.1 Shear Span to Depth Ratio (a/d)

The shear strength of a deep beam largely depends on its span to depth ratio. This has been established after Kani’s investigations in the 1960s. Later other researchers (Rogowsky 1986 and Collins 1991) also investigated the size effect on deep beams and made the same conclusion. All experimental investigations on deep beams showed that the shear span to depth ratio is the main
parameter that affects their shear strength as it increases with the decrease of \( \frac{a}{d} \) ratio (Manuel 1971, Smith 1982, Mau 1989, Tan 1995, Ashour 2000, Londhe 2010). This is because as the \( \frac{a}{d} \) ratio decreases, the shear force transferred by the concrete strut directly to the supports. This mechanism is called the strut and tie action in deep beam. The normalized shear strength versus \( \frac{a}{d} \) ratio plot (Fig. 2.1a) shows that the shear strength of a deep beam is linearly proportional with \( \frac{a}{d} \) ratio.

### 2.3.2 Beam Span to Depth Ratio (\( \frac{l_n}{d} \))

Manuel et al. (1971) performed 12 experiments on deep beams with different span to depth ratio and commented that, similar to \( \frac{a}{d} \) ratio, \( \frac{l_n}{d} \) ratio has a significant influence on the shear strength of deep beam where the shear strength is inversely proportional to \( \frac{l_n}{d} \) ratio (Fig. 2.1b). This is because as the \( \frac{l_n}{d} \) ratio increased, a longer arch is required to transfer the load to the support and, at the same time, the mid span deflection increases which results in wider flexural crack and therefore, the shear strength decreases (Tan 1995).
Table 2.1  Beam database used to predict shear strength of RC deep beams

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<th>b (mm) Max</th>
<th>h (mm) Min</th>
<th>h (mm) Max</th>
<th>d (mm) Min</th>
<th>d (mm) Max</th>
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<th>a/d Max</th>
<th>f_c' (MPa) Min</th>
<th>f_c' (MPa) Max</th>
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Figure 2.1 Scatter plot for each factor contributing on the shear strength of deep beams.
2.3.3 Compressive Strength of the Concrete ($f^\prime_c$)

The shear strength is a function of the compressive strength, $f^\prime_c$. El-Sayed et al. (2006) showed that the shear strength increased by 10.7% when $f^\prime_c$ increased by 44.5% ($f^\prime_c$ from 43.6 MPa to 63 MPa). This increase is not proportional, however, because in the case of high strength concrete (> 60 MPa), the fractured aggregates at ultimate load will generate less friction compared to normal strength concrete (Fig. 2.1c). Similarly, Smith’s (1982) investigation on deep beam showed that $f^\prime_c$ has a great influence on the shear capacity. Their results showed that the capacity is higher in the case of a deep beam with high $f^\prime_c$ and low web reinforcement compared to a beam with low $f^\prime_c$ and high web reinforcement. However, their tests were limited to only normal strength concrete ($f^\prime_c = 16$ to 23 MPa). On the other hand, Londhe (2010) showed that the compressive strength of concrete ($f^\prime_c = 24$ to 37 MPa) has small effect on the shear increase of deep beams.

2.3.4 Longitudinal Reinforcement

Mau and Hsu (1989) conducted 64 experiments on deep beams and found that with the increase of longitudinal reinforcement, the shear strength of deep beam increased significantly. Similar studies by Ashour (2000) and Londhe (2010) found that the longitudinal reinforcement has linear correlation with the shear strength up to a certain limit for deep beams without shear reinforcement and beyond that it has no effect. Longitudinal reinforcement increases the shear strength of deep beams by reducing the crack width, by improving the interface shear transfer mechanism and by increasing the dowel action (Londhe 2010). Fig. 2.1d shows that the average shear strength increases linearly as the longitudinal reinforcement ratio increases up to 1.5% and beyond that it reaches a plateau.
2.3.5 Horizontal Shear Reinforcement

Although the reason to provide the horizontal shear reinforcement was to improve the shear capacity, some studies showed that it has no effect on the shear strength (Kong 1970). This can be observed in Fig. 2.1e, where the average shear strength remains almost constant with the change in horizontal shear reinforcement. Other researchers found that there will be a little increase in shear strength with the increase in horizontal shear reinforcement (Smith 1982). This is specially the case of low vertical shear reinforcement, where adding horizontal shear reinforcement ratio in deep beams will not have further contribution on its shear strength (Smith 1982). On the other hand, Ashour (2000) reported that horizontal shear reinforcement is more effective compared to vertical shear reinforcement in case of $\bar{a}/d < 0.75$.

2.3.6 Vertical Shear Reinforcement

Vertical web reinforcement is one of the major parameters that affect the shear strength of deep beams. The primary purpose of vertical web reinforcement is to provide confinement to the concrete which helps to improve the shear capacity of deep beams. In addition to this, it is more effective in improving the shear strength compared to horizontal shear reinforcement and in case of a shear failure it makes the beam fail in a more ductile manner. All studies showed that the shear strength of a deep beam increases linearly with the increase of the vertical shear reinforcement (Clark 1951, Kani 1967, Kong 1970, Smith 1982, Oh 2001, Quintero-Febre 2006, Tan 1995). However, Smith (1982) found that the contribution of the vertical shear reinforcement diminishes as the $\bar{a}/d$ decreases ($\bar{a}/d < 1$). Similar study by Ashour (2000) confirmed that the higher the $\bar{a}/d$ ratio ($\bar{a}/d > 0.75$), the higher the contribution of the vertical web reinforcement. On the contrary, Londhe (2010) reported that the shear strength increase was
observed up to a vertical shear reinforcement ratio of 1.25%. A similar result is observed in Fig. 2.1f, where it shows that the shear strength increases up to a vertical shear reinforcement ratio of 1.42% whereas, the average shear strength does not change much beyond a reinforcement ratio of 2%.

2.4 Shear Strength Prediction

The shear equation proposed by Ramakrishan et al. (1968), Kong (1972), Selvam (1976), Mau and Hsu (1989), Matamoros et al. (2003), Arabzadeh (2009), Londhe (2010) were used to predict the shear strength of deep beams. The shear strength of a deep beam was also calculated by both ACI 318 and CSA A23.3 STM equations. The shear equations are listed in Appendix A. This section will give a brief overview of each model.

The internal force system in the reinforced concrete deep beam is very complex. Moreover, concrete is a non-homogeneous material and its stress-strain distribution is highly non-linear. Therefore, it is difficult to predict a theoretical solution for the shear strength of deep beams. In 1968, Ramakrishnan (1968) derived an equation based on their experimental results to predict the shear strength of deep beam. The equation was developed on the basis of concrete splitting strength for the ultimate load causing shear failure in deep beams. Similar approach was adopted by Kong et al. (1972) considering concrete cylinder splitting tensile strength as the main variable to predict the shear strength. Their proposed equation was a function of clear shear span to depth ratio and the longitudinal and web reinforcement ratio. The constants in the equation were derived using 135 experiments on deep beams.
In 1976, Selvam (1976) proposed an equation from the equilibrium conditions considering the strength of concrete in compression and tension. The contribution of longitudinal reinforcement was also included. In addition to the diagonal tension failure in RC deep beams, a small compression zone was found at failure. Therefore, a compression ratio (\( m \)) was introduced in the governing equations which depend on the depth to span ratio (\( d / l_n \)) and the stress ratio (\( \xi \)).

Mau and Hsu (1989) derived a non-dimensional shear equation to predict the shear strength in deep beams considering the equilibrium condition of the effective shear element in the shear span. The equation is expressed in terms of four variables: shear span to depth ratio, compressive strength of concrete, horizontal shear reinforcement and vertical shear reinforcement. The model was calibrated using 64 experimental data from the literature.

Matamoros and Wong (2003) developed a design equation for the shear strength of deep beams based on the STM approach. The authors developed a simplified model considering three load transfer mechanisms in the strut and tie and proposed a correction factor which was calibrated using 175 experiments on deep beams. The loads were considered to be carried by the concrete and the horizontal and vertical shear reinforcements. Similarly, Arabzadeh (2009) developed an STM model for the ultimate shear strength of deep beams. He considered two load transfer mechanisms- diagonal concrete strut action by STM and resisting equivalent force perpendicular to the diagonal crack by shear reinforcement. The proposed equation was a function of concrete compressive strength and the horizontal and vertical shear reinforcements.

Londhe (2010) proposed an analytical model based on 27 experimental results on deep beams. The ultimate shear strength was calculated by adding the contribution of concrete, longitudinal reinforcement, horizontal shear reinforcement and vertical shear reinforcement. The contribution
of concrete is predicted as a function of shear span to depth ratio and compressive strength of concrete.

2.5 Model and Codes Comparison: Results & Discussions

The performance of ACI and CSA building codes and the analytical models were compared with the experimental results compiled from the literature. The comparison was assessed with seven statistical parameters, which are commonly used for shear prediction models by researchers (Machial et al. 2012, Slater et al. 2012): a) the performance factor (PF) which is the ratio of the experimental shear strength to the calculated shear strength \( V_{\text{exp}}/V_{\text{cal}} \), b) the \( \chi \) factor which is the inverse of the slope of a linear least squares regression of the calculated shear strength \( V_{\text{cal}} \) versus the experimental shear strength \( V_{\text{exp}} \) plot, c) the standard deviation (SD), d) the sample variance (VAR), e) the coefficient of variation (COV), f) the coefficient of relationship (COR) and g) average absolute error (AAE).

In addition, a performance test was performed based on the performance factor (PF). A weighted penalty classification system was applied based on the demerit points classification model proposed by Collins (2001). The data points were categorized by their PF and a penalty (\( p \)) was applied to each of them. The penalties were chosen based on the structural safety. For example, high penalty value (\( p \)) is assigned on data points with PF less than one and those points are categorized as extremely dangerous (\( p = 5 \)) and dangerous (\( p = 3 \)). This is because data points with PF less than one is unacceptable in terms of safety. Similarly, high penalty value is also assigned for extremely conservative points (\( p = 4 \)). The penalty value (\( p \)) for different PF is shown in Tables 2.3 and 2.5. The performance of each model was determined in terms of its
performance index (PI). The PI is the summation of multiplying the number of data points in each category with their assigned penalty value. In order to better understand the shear strength, the database was divided into two groups, one with web reinforcement and the other without web reinforcement. The accuracy and performance of different models and code equations are reported in Tables 2.2 to 2.5.

2.5.1 Shear Strength with Web Reinforcement

2.5.1.1 Statistical Analysis

Table 2.2 shows the statistical comparison between the experimental and calculated shear strength for deep beams with web reinforcement. In the case of shear strength prediction with web reinforcement, both ACI and CSA codes were more conservative compared to the analytical models. Although the PF value was close for both of them, ACI had higher $\chi$ value (1.62) than CSA ($\chi =1.29$) which implies that the ACI code is more conservative. Moreover, both had very high SD and COV compared to other models. On the contrary, the COR value close to one indicated that both ACI and CSA code equations correlate better with the experimental results.

The average PF of Selvam (1976) model was only 1.02 which seemed to be a good prediction, but, the equation produced very low $\chi$ value (0.53) which indicates that the calculated shear strength is overpredicted. This resulted in a high AAE (61.2 %). Therefore, Selvam (1976) model was the least accurate model. A similar conclusion can be made for Ramakrishnan (1968) and Hsu (1989) models. Although these models had very low SD, VAR, COV and AAE, the $\chi$ value ($\chi <1$) made the model unsafe to be used in the design since it is overpredicted the experimental shear strength.
Among the analytical models, Londhe (2010) was the most conservative one since PF, $\chi$, SD, COV values were quite high. The equations proposed by Kong (1970), Matamoros (2003) and Arabzadeh (2009) predicted the shear strength quite accurately and had low SD, COV and AAE values compared to the code equations. Among these three models, the $\chi$ value of the model proposed by Matamoros (2003) seems to be unsafe ($\chi = 0.82$). Therefore, the remaining proposed models by Kong (1970) and Arabzadeh (2009) seem to be better and safe to be used in the design since they had low SD, COV and AAE values. Table 2.2 shows that the Kong (1970) model produced much lower PF, $\chi$, SD and COV values compared to the Arabzadeh (2009) model. On the contrary, Arabzadeh (2009) model produced lower AAE and higher COR values than the Kong (1970) model which makes the model more accurate.

From the above discussion, it is shown that it is very difficult to evaluate the accuracy of each model only by comparing the statistical parameters. Moreover, the evaluation is based only on descriptive statistical analysis, which may not provide enough information to assess the reliability of the model considering the structural safety. The performance test in the following section will help to better evaluate each model.
2.5.1.2 Performance Test

The performance index based on weighted penalty classification could give an indication of the safety of the predicted model. The weighted penalty on each model was applied based on “Demerits Point Classification” proposed by Collins (2001). The penalty ($p$) value was assigned based on the PF value from the statistical analysis where the classifications of penalty are from extremely dangerous ($p = 5$) to extremely conservative ($p = 4$). Table 2.3 shows performance test analysis of each model where a lower performance index (PI) value indicates a better model prediction. From statistical analysis, it was observed that both ACI and CSA codes are too conservative. But comparing the performance index with different equations, it was observed that the codes equations were better model since the PI was the lowest. Among the code equations, ACI produced low a PI (429) than the CSA (PI = 456) and it was the second best predicted model where only few points were in the dangerous zones. Most models such as, Ramakrishnan, Kong, Selvam, Mau and Londhe model had an average PF close to one and well below the code equations. Comparing the PI values, most of the models (Ramakrishnan, Kong, Selvam, Mau and Londhe) showed poor performance in predicting the shear strength since many predicted points remained below the safe zone. Only Matamoros (2003) and Arabzadeh (2009) proposed models showed very good prediction and had the lowest PI values (479 and 417 respectively) compared to other analytical models. However, observing the $\chi$ value, it can be concluded that Matamoros (2003) was not a safe model. Therefore, the model proposed by Arabzadeh (2009) was the most accurate and safe model that could predict the shear strength of deep beam with web reinforcement.
The predicted shear strength with web reinforcement were plotted against the experimental shear strength (Fig. 2.2a to Fig. 2.10a). Comparing the ACI and CSA models (Fig. 2.2a and Fig. 2.3a), CSA model has more points in the dangerous zone i.e., above the 45 degrees line. Therefore, although from the statistical analysis (Table 2.2) CSA model seems better, it is safer to use the ACI equation for predicting the shear strength of deep beams with web reinforcement which produced relatively low PI. The graph for the models produced by Selvam, Mau and Matamoros (Fig. 2.6a, 2.7a and 2.8a) showed that most points were in the dangerous zone and therefore these models were unacceptable for design purposes. Among the analytical models, Arabzadeh (Fig. 2.9a) showed good agreement with the experimental results and has fewer points in the dangerous zone. All the models are plotted together and compared in Fig. 2.11a where models with $\chi<1$ were excluded since they under-predicted the experimental shear strength.

### 2.5.2 Shear Strength without Web Reinforcement

#### 2.5.2.1 Statistical Analysis

Table 2.3 shows the statistical comparison between the experimental and calculated shear strength for deep beams without web reinforcement. The STM code equations for predicting
shear strength without web reinforcement were more conservative than the analytical models except Londhe (2010). Comparing ACI and CSA equations, ACI was more conservative which is reflected in the $\chi$ value (1.72) that was 24% higher than the CSA value. In addition, ACI had very high SD (0.97), VAR (0.95), COV (59%), AAE (41%) which were respectively 100%, 296%, 82% and 36% higher than the CSA values.

Table 2.4  Statistical analysis of shear strength prediction of RC deep beams without web reinforcement

<table>
<thead>
<tr>
<th>Model</th>
<th>Average PF</th>
<th>$\chi$</th>
<th>SD</th>
<th>VAR</th>
<th>COV (%)</th>
<th>COR</th>
<th>AAE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI 318-05</td>
<td>1.66</td>
<td>1.72</td>
<td>0.97</td>
<td>0.95</td>
<td>58.8</td>
<td>0.79</td>
<td>40.7</td>
</tr>
<tr>
<td>CSA 23.3</td>
<td>1.51</td>
<td>1.39</td>
<td>0.49</td>
<td>0.24</td>
<td>32.2</td>
<td>0.90</td>
<td>29.8</td>
</tr>
<tr>
<td>Ramakrishnan &amp; Ananthanarayana (1968)</td>
<td>0.95</td>
<td>1.29</td>
<td>0.59</td>
<td>0.35</td>
<td>61.9</td>
<td>0.56</td>
<td>76.9</td>
</tr>
<tr>
<td>Kong et al (1972)</td>
<td>1.11</td>
<td>1.37</td>
<td>0.69</td>
<td>0.48</td>
<td>62.6</td>
<td>0.56</td>
<td>59.7</td>
</tr>
<tr>
<td>Selvam (1976)</td>
<td>0.84</td>
<td>0.63</td>
<td>0.42</td>
<td>0.17</td>
<td>49.7</td>
<td>0.76</td>
<td>68.2</td>
</tr>
<tr>
<td>Mau &amp; Hsu (1989)</td>
<td>0.95</td>
<td>1.01</td>
<td>0.23</td>
<td>0.05</td>
<td>24.0</td>
<td>0.96</td>
<td>22.0</td>
</tr>
<tr>
<td>Matamoros (2003)</td>
<td>1.22</td>
<td>1.15</td>
<td>0.38</td>
<td>0.14</td>
<td>30.9</td>
<td>0.93</td>
<td>24.2</td>
</tr>
<tr>
<td>Arabzadeh et al. (2009)</td>
<td>0.96</td>
<td>1.11</td>
<td>0.30</td>
<td>0.09</td>
<td>31.7</td>
<td>0.93</td>
<td>29.6</td>
</tr>
<tr>
<td>Londhe (2010)</td>
<td>1.75</td>
<td>2.01</td>
<td>1.02</td>
<td>1.04</td>
<td>58.3</td>
<td>0.65</td>
<td>39.6</td>
</tr>
</tbody>
</table>

Selvam’s (1976) proposed model was the least accurate in its prediction since it had the lowest $\chi$ value (0.63) and a high AAE value (68.15%). On the contrary, Londhe (2010) model was the most conservative model where it’s average PF and $\chi$ values were 1.75 and 2.01, respectively. Moreover, the SD, VAR and COV values were found very high, therefore, it was observed that the calculated versus the experimental shear strength has too much scattered on the $V_{\text{cal}}$ vs $V_{\text{exp}}$ plot (Fig. 2.6 b). In case of Mau (1989) and Arabzadeh (2009) models, in spite of low SD, COV, VAR and AAE values, the models still overpredicted the shear strength since their average PF is just below one. Comparing all the models, it could be concluded that Matamoros (2003) analytical model predicted the shear strength more accurately than the other models since both PF (1.22) and $\chi$ value (1.15) are above one and the values were less conservative than the code
equations. Moreover, except for Mau’s (1989) model, the SD, COV, VAR and AAE values of the Matamoros (2003) proposed model were lower than the other models.

2.5.2.2 Performance Test

The performance index (PI) analysis for shear strength prediction of RC deep beams without web reinforcement is shown in Table 2.5. The PI analysis showed that the ACI STM model was highly conservative compared to the CSA code equation (PI of ACI = 161; PI of CSA = 114) which was already proven from the statistical analysis. The analytical models were primarily developed to predict the shear strength with web reinforcement. Therefore, from the performance index, it was clearly observed that these models (i.e., Ramakrishnan, Kong, Selvam, Mau and Arabzadeh) were poor in predicting the shear strength without web reinforcement. Consequently, they produced very high PI value (Table 2.5). As a result, more than 50% of the predicted points were below the safe zone. Only two models, Matamoros and Londhe, had low PI values (113 and 159, respectively). However, from the statistical analysis, it was clear that Londhe (2010) model is highly conservative where 33% of the points were in the conservative zone. In summary, it could be concluded that Matamoros (2003) proposed an analytical model that could predict the shear strength of a deep beam without web reinforcement accurately and safely. Along with the Matamoros (2003) model, CSA 23.3 STM model was another good method for accurately predicting the nominal strength of deep beams.

The predicted shear strength without web reinforcement were plotted against the experimental shear strength and presented in Fig. 2.2b to Fig. 2.11b. Unlike deep beams with web reinforcement, the CSA predicted shear strength showed better performance than the ACI where only 10% of the points are falling in the danger zone. Except Selvam (Fig. 2.6a), linear
regression lines for all the analytical models were below the 45 degrees line. Fig. 2.7b, 2.8b, 2.9b showed that Mau (1989), Matamoros (2003) and Arabzadeh (2009) were better models, since the predicted points were less scattered and within the narrow range of the 45 degrees line.

Table 2.5 Performance index for shear strength prediction of RC deep beams without web reinforcement

<table>
<thead>
<tr>
<th>PF</th>
<th>Classification</th>
<th>p</th>
<th>ACI 318</th>
<th>CSA 23.3</th>
<th>Ramakrishnan</th>
<th>Kong</th>
<th>Selvam</th>
<th>Mau &amp; Hsu</th>
<th>Matamoros</th>
<th>Arabzadeh</th>
<th>Londhe</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.75</td>
<td>Extremely dangerous</td>
<td>5</td>
<td>9</td>
<td>1</td>
<td>34</td>
<td>35</td>
<td>44</td>
<td>16</td>
<td>8</td>
<td>22</td>
<td>7</td>
</tr>
<tr>
<td>0.75-1.00</td>
<td>Dangerous</td>
<td>3</td>
<td>8</td>
<td>8</td>
<td>20</td>
<td>13</td>
<td>14</td>
<td>34</td>
<td>12</td>
<td>32</td>
<td>11</td>
</tr>
<tr>
<td>1.00-1.25</td>
<td>Reduced safety</td>
<td>0</td>
<td>20</td>
<td>18</td>
<td>12</td>
<td>6</td>
<td>12</td>
<td>23</td>
<td>34</td>
<td>14</td>
<td>17</td>
</tr>
<tr>
<td>1.25-1.75</td>
<td>Appropriate safety</td>
<td>1</td>
<td>18</td>
<td>29</td>
<td>9</td>
<td>14</td>
<td>12</td>
<td>11</td>
<td>23</td>
<td>16</td>
<td>21</td>
</tr>
<tr>
<td>1.75-3.00</td>
<td>Conservative</td>
<td>2</td>
<td>21</td>
<td>28</td>
<td>9</td>
<td>14</td>
<td>2</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>21</td>
</tr>
<tr>
<td>&gt;3.00</td>
<td>Extremely conservative</td>
<td>4</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7</td>
</tr>
</tbody>
</table>

*Performance index, PI = p × number of data points in each category*

Figure 2.2 Shear strength prediction by ACI code (a) with web reinforcement (b) without web reinforcement
Figure 2.3 Shear strength prediction by CSA code (a) with web reinforcement (b) without web reinforcement

Figure 2.4 Shear strength prediction by Ramakrishnan (a) with web reinforcement (b) without web reinforcement
Figure 2.5 Shear strength prediction by Kong (a) with web reinforcement (b) without web reinforcement

Figure 2.6 Shear strength prediction by Selvam (a) with web reinforcement (b) without web reinforcement
Figure 2.7 Shear strength prediction by Mau (a) with web reinforcement (b) without web reinforcement

Figure 2.8 Shear strength prediction by Matamoros (a) with web reinforcement (b) without web reinforcement
Figure 2.9  Shear strength prediction by Arabzadeh (a) with web reinforcement (b) without web reinforcement

Figure 2.10 Shear strength prediction by Londhe (a) with web reinforcement (b) without web reinforcement
2.6 Summary

This chapter presented a thorough literature review on the shear strength prediction of RC deep beams with and without web reinforcement. From the published literature a database of 381 beams was composed and used in the statistical analysis for comparing the prediction performance of current codes and analytical models. The following conclusions can be drawn from the study:

§ Based on the statistical analysis it was observed that both the ACI and CSA STM models underestimated the experimental shear strength. However, from the performance test, low performance index (PI) values confirmed that they were quite accurate and safe to be used in the design compared to many analytical solutions.

§ In the case of deep beams with web reinforcement, the ACI STM produced low PI than the CSA STM and only 6% of the data points among the test results were in the dangerous zone.
compared to 13% for CSA STM model. On the other hand, in the case of deep beam without web reinforcement, the CSA STM model produced 29% less PI compared to the ACI STM model. Moreover, it was the best model for predicting the shear strength than all the analytical equations except Matamoros (2003).

Among the 7 analytical models, it was clear from the statistical analysis that Londhe’s (2010) proposed model was the most conservative for predicting the shear strength of deep beams both with and without web reinforcement.

Both descriptive statistical analysis and performance test conformed that Arabzadeh (2009) proposed model was better in predicting the shear strength for deep beams with web reinforcement and Matamoros (2003) for deep beams without web reinforcement compared to other models.
Chapter 3: Shear Strength Prediction of RC Deep Beams and Calibration for the Resistance Factors

3.1 Overview

The strut and tie model (STM) in the building codes (ACI 318 and CSA A23.3) do not have any simplified equation to calculate the shear capacity of deep beams. Thus, engineers are free to choose the truss dimensions (strut and tie) that carry the load through the D-regions (disturbed regions) to its supports. Each STM truss model may give different shear capacity for the same deep beam section. Therefore, it is more appropriate to assume more than one STM truss model for a particular deep beam which is completely ambiguous.

From the previous analysis presented in Chapter 2, it was observed that the available equations in the literature for the shear prediction model either under predicted or over predicted the experimental shear strength. The comparative study of nine different shear prediction models including ACI 318 and CSA A23.3 STM models were presented based on the descriptive statistics and the performance test. Although the performance of the ACI and CSA STM model was better compared to the other equations, both STM models were conservative and their COV was found high.

In this Chapter, an analytical model to predict the nominal shear strength of deep beams by genetic algorithm (GA) will be developed. Moreover, for design purposes, the equations will be calibrated and a resistance factors will be recommended for each shear equation.
3.2 Experimental Database

The same 381 test results on deep beams described in Chapter 2 were used for the analysis in this chapter (Table 2.1). The database was divided into two parts: a) deep beam with shear reinforcement and b) deep beam with no shear reinforcement. Here, two shear equations are proposed for the deep beams with and without web reinforcement. From the analysis presented in Chapter 2, it was observed that the variables mostly affecting the shear strength of deep beams were \(\frac{a}{d}\) ratio, longitudinal reinforcement ratio \((\rho)\), horizontal and vertical shear reinforcement ratio \((\rho_h, \text{ and } \rho_v, \text{ respectively})\). Therefore, these were included as the primary variables when developing the shear equations.

3.3 \(2^k\) Factorial Design

Factorial designs are widely used to identify the important parameters affecting the experimental results (Montgomery 2008). Moreover, it examines the interaction effect among the variables. The present study only considered the \(2^k\) factorial design in which there are two levels of each factor that affects the experimental response. These levels were quantitative values and each represented a high and a low side of the data range. For the factorial design, the response of the output was considered linear over the ranges of the variables with a 5% risk considered in the factorial design to reject the null hypothesis i.e., the probability of ‘Type I’ error, \(\alpha = 0.05\).

In this study, the response was the shear strength of a deep beam, while from the experimental results reported in the literature, there was 8 variables that influenced this response, namely, \(b, h, f'_c, f_y, a/d, \rho, \rho_h, \rho_v\) (Table 2.1). In order to reduce the number of variables, the response and the main variables were normalized following equation (3.1). The reinforcement ratios were
normalized by multiplying them by the ratio of the steel yield strength to the concrete compressive strength \( (f_y / f'_c) \) in order to include the variation of both \( f_y \) and \( f'_c \) in the proposed equation, while the shear strength was normalized by \( f'_c \) and the cross-section of the tested beams \( (bh) \). With the reduced number of variables, the general shear equation for a deep beam with and without web reinforcement is presented in Equations (3.2) and (3.3).

\[
\begin{align*}
\hat{V}_{\text{test}} &= \frac{V_{\text{test}}}{f'_c bh} \\
\hat{\rho} &= \rho \left( \frac{f_y}{f'_c} \right) \\
\hat{\rho}_h &= \rho_h \left( \frac{f_y}{f'_c} \right) \\
\hat{\rho}_v &= \rho_v \left( \frac{f_y}{f'_c} \right)
\end{align*}
\]  

\( (3.1) \)

\[
\hat{V} = f \left( \frac{a}{d}, \hat{\rho}, \hat{\rho}_h, \hat{\rho}_v \right)
\]  

\( (3.2) \)

\[
\hat{V} = f \left( \frac{a}{d}, \hat{\rho} \right)
\]  

\( (3.3) \)

The percent contribution and the \( P \)-value for each factor were calculated to identify the most significant factors as well as the interactions among the factors. The \( P \)-value is the probability of obtaining a test statistics at least as extreme as the observed value when the null hypothesis is true (Montgomery 2008). Two-way, three-way and four-way interactions were considered. The interactions helped include the higher order non-linear terms in the prediction of shear equations which eventually improved the accuracy of the equations.
3.4 Results from the Factorial Design

The $2^4$ factorial design was performed and the sum of squares, $F$-value, $P$-values and the percent contribution of each factor and their interactions were calculated using the commercial software Minitab.

3.4.1 Deep Beam with Web Reinforcement

Table 3.1 presents the results from the $2^4$ factorial design for a deep beam with web reinforcement. The $P$-value for the main parameters $a/d$, and were found to be 0.0, 0.029 and 0.018, respectively. Comparing with $\alpha = 0.05$, it can be concluded that the risk to reject the null hypothesis was less than 5%. The results also indicated that among the four variables, $a/d$, and are the most significant parameters that affect the shear strength of the RC deep beams. Beside the $P$-value, the percent contribution of $a/d$ ratio was around 56%, which indicates that $a/d$ has the highest influence on the shear strength of deep beams with web reinforcement.

Moreover, from the factorial design, it was observed that the parameters were highly interacting with each other which eventually contributed non-linearly to the shear strength. Among the two-way interactions, and produced the lowest $P$-value of 0.027 ($\alpha = 0.05$). The parameters and were also interacting with each other and produced a $P$-value of 0.07, but this interaction was ignored since it exceeded the value for $\alpha = 0.05$. Three-way and four-way interactions were also observed in the analysis. Among the three-way interactions, , and produced a $P$-value of 0.001 and their total percent contribution was 8.15% which was the second highest after the contribution of $a/d$ ratio. The parameters $a/d$, and were also highly interacting with
each other and therefore, a low $P$-value of 0.009 was obtained. All the parameters were found to interact four-way (four-way interaction) and produced a $P$-value of 0.001.

Table 3.1 Factorial design shear strength with web reinforcement

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Sum of Squares</th>
<th>% Contribution</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main effects</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(a/d)$</td>
<td>0.056089</td>
<td>55.52</td>
<td>73.98</td>
<td>0</td>
</tr>
<tr>
<td>$\hat{\rho}$</td>
<td>0.003634</td>
<td>3.60</td>
<td>4.79</td>
<td>0.029</td>
</tr>
<tr>
<td>$\hat{\rho}_h$</td>
<td>0.001296</td>
<td>1.28</td>
<td>1.71</td>
<td>0.192</td>
</tr>
<tr>
<td>$\hat{\rho}_v$</td>
<td>0.004327</td>
<td>4.28</td>
<td>5.71</td>
<td>0.018</td>
</tr>
<tr>
<td><strong>2-way Interactions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(a/d)*\hat{\rho}$</td>
<td>0.001896</td>
<td>1.88</td>
<td>2.5</td>
<td>0.115</td>
</tr>
<tr>
<td>$(a/d)*\hat{\rho}_h$</td>
<td>0.000013</td>
<td>0.01</td>
<td>0.02</td>
<td>0.897</td>
</tr>
<tr>
<td>$(a/d)*\hat{\rho}_v$</td>
<td>0.000031</td>
<td>0.03</td>
<td>0.04</td>
<td>0.839</td>
</tr>
<tr>
<td>$\hat{\rho}*\hat{\rho}_h$</td>
<td>0.002508</td>
<td>2.48</td>
<td>3.31</td>
<td>0.07</td>
</tr>
<tr>
<td>$\hat{\rho}*\hat{\rho}_v$</td>
<td>0.001998</td>
<td>1.98</td>
<td>2.63</td>
<td>0.106</td>
</tr>
<tr>
<td>$\hat{\rho}_h*\hat{\rho}_v$</td>
<td>0.003736</td>
<td>3.70</td>
<td>4.93</td>
<td>0.027</td>
</tr>
<tr>
<td><strong>3-way Interactions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(a/d)<em>\hat{\rho}^2</em>\hat{\rho}_h$</td>
<td>0.002556</td>
<td>2.53</td>
<td>3.37</td>
<td>0.068</td>
</tr>
<tr>
<td>$(a/d)<em>\hat{\rho}_h^2</em>\hat{\rho}_v$</td>
<td>0.001281</td>
<td>1.27</td>
<td>1.69</td>
<td>0.195</td>
</tr>
<tr>
<td>$(a/d)<em>\hat{\rho}_h</em>\hat{\rho}_v^2$</td>
<td>0.005202</td>
<td>5.15</td>
<td>6.86</td>
<td>0.009</td>
</tr>
<tr>
<td>$\hat{\rho}_h^2*\hat{\rho}_v^2$</td>
<td>0.008228</td>
<td>8.15</td>
<td>10.85</td>
<td>0.001</td>
</tr>
<tr>
<td><strong>4-way Interactions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(a/d)<em>\hat{\rho}^2</em>\hat{\rho}_h^2*\hat{\rho}_v$</td>
<td>0.008222</td>
<td>8.14</td>
<td>10.84</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Fig. 3.1 shows the main effects of each parameter contributing to the shear strength. It was observed that the shear strength increase in a deep beam with the increase of longitudinal reinforcement ratio $(\rho)$ and the horizontal and vertical shear reinforcements ratios $(\hat{\rho}_h\text{ and }\hat{\rho}_v)$. On the contrary, the shear span to depth ratio $(a/d)$ inversely affected the shear strength in a deep beam. The two-way interaction was plotted in Fig. 3.2. The interaction plot clearly indicated that there were substantial interactions between $(a/d)$ and $(\hat{\rho}_h, \hat{\rho}_v)$ which was also
observed from the P-value and the percent contribution results. Moreover, the significant and non-significant parameters and their interactions were also identified from the normal plot (Fig. 3.3).

Figure 3.1 Main effects plot for the parameters \(a/d, \hat{\rho}, \hat{\rho}_h\), and \(\hat{\rho}_v\) over the shear strength

Figure 3.2 Interactions between \(\hat{\rho}\) and \(\hat{\rho}_h\) and between \(\hat{\rho}_h\) and \(\hat{\rho}_v\)
3.4.2 Deep Beam without Web Reinforcement

Similar to shear strength of deep beam with web reinforcement, the statistical analysis results from the factorial design for a deep beam without web reinforcement are presented in Table 3.2. The $P$-values for the main parameters $a/d$ and $\beta$ are 0.001 and 0.006, respectively. Therefore, comparing with $\alpha = 0.05$, it could be concluded that both of them are important parameters which was also confirmed from the normal plot of significant and non-significant parameters (Fig. 3.4). The percent contribution of $a/d$ ratio is approximately 55%, which was found similar to the previous analysis for a deep beam with web reinforcement. The other important parameter was the main longitudinal reinforcement, $\beta$, which has a percent contribution of 38%. The interaction between $a/d$ and $\beta$ was found not to be significant ($P$-value of 0.25).
Table 3.2  Factorial design chart for main effects & interactions: without web reinforcement

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Sum of Squares</th>
<th>% Contribution</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a / d)</td>
<td>0.018314</td>
<td>55.20</td>
<td>11.42</td>
<td>0.001</td>
</tr>
<tr>
<td>ˆρ</td>
<td>0.012707</td>
<td>38.30</td>
<td>7.93</td>
<td>0.006</td>
</tr>
<tr>
<td>(a / d)* ˆρ</td>
<td>0.002159</td>
<td>6.51</td>
<td>1.35</td>
<td>0.249</td>
</tr>
</tbody>
</table>

Figure 3.4  Normal plot for significant and non-significant factors (without web reinforcement)

3.5 Proposed Shear Strength Equation using Genetic Algorithm

Genetic Algorithms (GAs) first developed by John Holland (1970). The basic mechanism of the GAs was based on the biological evolution. GA has a wide area of applications which is used to understand the processes in natural systems. A detailed description of GA is found in Ashour (2003).

The present study used the GA technique to develop an equation to predict the shear strength of deep beams. The same two groups from the experimental database were used to develop the
shear equations for deep beam with and without web reinforcement. The analysis from the factorial design was incorporated in the proposed equations by GA. The primary population size used was 1000. The mean square error was used as a fitness function to search for a solution. The mathematical operators \(\{+, -, \ast, /, \sqrt{\cdot}, \}_{\text{set}}\) were considered for the program runs.

### 3.5.1 Shear Equation for Deep Beam with Web Reinforcement

From the factorial design, it was found that \(a/d\), \(\rho\), and \(\rho_v\) were the most significant parameters that contribute to the shear strength. In addition, 2-way, 3-way and 4-way interactions among the parameters were observed between \(a/d\), \(\rho\), \(\rho_v\) and \(\rho\). Therefore, in addition to the main parameters, these high order non-linear interaction terms were included in the proposed equation in order to achieve the highest accuracy. The shear equation developed by the GA technique with all the important parameters and their interactions is presented in Equation (3.4).

\[
V = 0.1976 - 0.5897 (a/d)^{0.1625} + 0.6215 \rho^{0.09661} + 0.4631 \rho_v^{1.185} - 320.3 (\hat{\rho}_h \hat{\rho}_v)^{2.391} \\
+ 20.63 (\hat{\rho}_h \hat{\rho}_v)^{1.884} + 20.14 ((a/d) \hat{\rho}_h \hat{\rho}_v)^{1.073} - 21.04 ((a/d) \hat{\rho}_h \hat{\rho}_v)^{0.9088} 
\]

\[(3.4)\]

For design purposes, Equation (3.4) was simplified without losing accuracy. For the simplified equation, only the parameters and interactions which have the percent contribution higher than 5% were considered (Table 3.1). The GA was performed again and the simplified equation was proposed (Equation 3.5). By ignoring some parameters and their interactions, a small amount of accuracy was sacrificed, but eventually the equation became quite simple.

\[
V = \frac{2}{5} - \frac{1}{4} (a/d)^{0.23} + 0.85 (\hat{\rho}_h \hat{\rho}_v)^{1/10} - \frac{3}{5} ((a/d) \hat{\rho}_h \hat{\rho}_v)^{1/16} - 200 ((a/d) \hat{\rho}_h \hat{\rho}_v)^{2.65} 
\]

\[(3.5)\]
3.5.2 Shear Equation for Deep Beam without Web Reinforcement

Similarly, from the factorial design for deep beam without web reinforcement, it was observed that \(\frac{a}{d}\) and \(\rho\) were the most significant parameters that affected the shear strength and there was very little interaction between them. Therefore, only these two terms were included in the GA solution for the shear equation. The equation from the GA analysis for deep beam without web reinforcement is presented in Equation (3.6).

\[
V = 1.74 - 2(\frac{a}{d})^{0.044} + \frac{1}{2}\rho^{0.14}
\]  

(3.6)

3.6 Comparative Study among the Shear Equations

The performance of the proposed shear equations were compared with the ACI and CSA codes against six statistical parameters: 1) performance factor (PF): ratio of experimental shear strength to calculated shear strength \((V_{\text{exp}}/V_{\text{cal}})\), 2) \(\chi\) factor: inverse of the slope of a linear least squares regression of the calculated shear strength \((V_{\text{cal}})\) versus the experimental shear strength \((V_{\text{exp}})\) plot, 3) standard deviation (SD), 4) sample variance (VAR), 5) coefficient of variation (COV), and 6) average absolute error (AAE).

3.6.1 Shear Equation for Deep Beam with Web Reinforcement

The statistical analysis and comparison between the proposed equations and the ACI and CSA STM models were presented in Table 3.3. The statistical analysis showed that both the ACI and CSA STM model were too conservative. The average PF for ACI and CSA equations was 1.78 and 1.74. On the other hand, the proposed equations (general and simplified) have an average PF close to 1 which is also reflected in the \(\chi\) value. The statistical parameters e.g. standard deviation (SD), variance (VAR) and the coefficient of variation (COV) were reduced significantly in the
case of the proposed equation compared to the code equations. The COV for the original ACI and CSA STM equation was more than 40% whereas it was reduced to less than one-half (19%) in the case of the proposed shear equation. The AAE of the proposed equation was only one-third of that of the original ACI and CSA STM model. Fig. 3.5 portrays the comparison among between the proposed shear equations with the ACI and CSA STM model. The linear least square regression line along with the $\chi$ value indicated the improvement in accuracy achieved using the proposed shear equations compared to the code equations.

As expected, comparing the simplified equation to the general proposed equation, the accuracy was reduced. The standard deviation of the simplified equation increased from 0.19 to 0.26. Similarly the COV increased from 19.19% to 25.74% and the average absolute error increased from 14% to 22%. Despite this, the statistical indicators (Average PF, $\chi$ value, SD, VAR, COV and AAE) for the simplified equation were lower than the code equations. Finally, it is worth mentioning that the proposed simplified equation was much simpler to implement and use compared to the code equations.

<table>
<thead>
<tr>
<th>Model</th>
<th>Average PF</th>
<th>$\chi$</th>
<th>SD</th>
<th>VAR</th>
<th>COV (%)</th>
<th>AAE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI 318</td>
<td>1.78</td>
<td>1.84</td>
<td>0.71</td>
<td>0.51</td>
<td>40.08</td>
<td>40.73</td>
</tr>
<tr>
<td>CSA 23.3</td>
<td>1.74</td>
<td>1.56</td>
<td>0.75</td>
<td>0.57</td>
<td>43.38</td>
<td>38.89</td>
</tr>
<tr>
<td>Proposed equation (general)</td>
<td>0.99</td>
<td>1.02</td>
<td>0.19</td>
<td>0.04</td>
<td>19.19</td>
<td>13.83</td>
</tr>
<tr>
<td>Proposed equation (Simplified)</td>
<td>1.01</td>
<td>1.07</td>
<td>0.26</td>
<td>0.07</td>
<td>25.74</td>
<td>22.20</td>
</tr>
</tbody>
</table>
3.6.2 Shear Equation for Deep Beam without Web Reinforcement

Table 3.4 shows the comparison between the proposed equation and the ACI and CSA STM models. Similar to the equation with web reinforcement, both the ACI and CSA STM models were highly conservative. The average PF for ACI and CSA equation was 1.66 and 1.51, respectively. On the other hand, the average PF for the proposed equation was only 0.99. Moreover, the χ value of the proposed equation was only 1.07 lower than the ACI and the CSA STM models (1.72 and 1.39, respectively). Beside this, the descriptive statistical parameters i.e., standard deviation (SD), variance (VAR) and the coefficient of variation (COV) were reduced significantly in the case of the proposed equation compared to the ACI and the CSA STM models. The COV for the ACI and CSA STM was quite high and among them CSA STM model produced the lower COV compared to ACI. The COV for the proposed predicted equation was
reduced by 60% and 28% compared to the ACI and CSA model, respectively. The average error of the proposed equation was reduced to 53% compared to the ACI model and 36% compared to the CSA model. The linear least square regression line is plotted in Fig. 3.6 which shows the improvement made by the proposed shear equation in terms of accuracy.

<table>
<thead>
<tr>
<th>Model</th>
<th>Average PF</th>
<th>χ</th>
<th>SD</th>
<th>VAR</th>
<th>COV (%)</th>
<th>AAE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI 318</td>
<td>1.66</td>
<td>1.72</td>
<td>0.97</td>
<td>0.95</td>
<td>58.77</td>
<td>40.68</td>
</tr>
<tr>
<td>CSA 23.3</td>
<td>1.51</td>
<td>1.39</td>
<td>0.49</td>
<td>0.24</td>
<td>32.23</td>
<td>29.84</td>
</tr>
<tr>
<td>Proposed equation</td>
<td>0.99</td>
<td>1.07</td>
<td>0.23</td>
<td>0.05</td>
<td>23.23</td>
<td>19.23</td>
</tr>
</tbody>
</table>

Figure 3.6 Comparison of shear equations: predicted vs experimental (without web reinforcement)

3.7 Reliability Analysis

Reliability is a measure of the likelihood of a failure. In structural engineering, it is the margin of safety of a structural system at the ultimate limit state (i.e., just before collapse). In a reliability
model, $Q$ represents the load effect on the structural system and $R$ represents the resistance of the structural system. Both $Q$ and $R$ are the random variables which can be determined from probability density functions. Nowak and Collins (2000) described various methods to calculate the reliability. In general, reliability can be expressed by a ‘limit state function’ or ‘performance function’ $g$ such as

$$g(R, Q) = R - Q$$  \hspace{1cm} (3.7)

where, $g$ is the safety margin. If $g \geq 0$, the structure is safe (desired performance); if $g < 0$, the structure is unsafe (undesired performance). The probability of failure is equal to the probability that the undesired performance will occur. The probability of failure, $P_f$ can be expressed as,

$$P_f = P(R - Q < 0) = P(g < 0)$$  \hspace{1cm} (3.8)

If $R$ and $Q$ are normally distributed, then the reliability index ($\beta$) can be expressed as,

$$\beta = \frac{m_R - m_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}}$$  \hspace{1cm} (3.9)

where, $m_R$, $m_Q$ are the mean values of resistance and total load effect, respectively and $\sigma_R$, $\sigma_Q$ are the standard deviation of resistance and total load, respectively. The reliability index can be considered as a function of failure $P_f$,

$$\beta = -\Phi^{-1}(P_f)$$  \hspace{1cm} (3.10)

where, $\Phi^{-1}$ is the inverse standard normal distribution function.

### 3.7.1 Resistance Models

The resistance of a structure can be expressed as,

$$R = R_n \times M \times F \times P_f$$  \hspace{1cm} (3.11)
where, \( R_n \) is the nominal resistance, \( M \) is the material factor, \( F \) is the fabrication factor, and \( P_r \) is the professional factor. The bias factor and the coefficient of variation of the resistance, \( R \), is given by,

\[
m_R = R_n \times \lambda_M \times \lambda_F \times \lambda_P
\]

\[
V_R = (V_M^2 + V_F^2 + V_P^2)^{1/2}
\]

where, \( \lambda_M, \lambda_F, \lambda_P \) are the bias factors of \( M, F, P \) and \( V_M, V_F, V_P \) are the coefficients of variation of \( M, F, P \), respectively.

For the present study, the bias factor and the coefficient of variation for material and fabrication were taken from the previous research performed by Nowak and Szerszen (2001). The professional factor, \( P_r \), is the ratio of the experimental shear strength to the shear strength predicted by the optimized shear equations. Statistics of the professional factors such as the bias factor, standard deviation and coefficient of variation for each optimized shear equations were computed separately.

3.7.2 Load Models

The present study only considered gravity loads, namely the dead and live load combinations in the load model. The statistical parameters for the dead and live loads were obtained from the previous research by Nowak (1999). For example, the bias factor for cast-in-place concrete is 1.05 with a coefficient of variation of 0.10. The bias factor for a 50 year live load is 1.0 with a coefficient of variation of 0.18. Both the ultimate limit state load cases specified by ACI 318 and CSA A23.3 building codes were considered. Therefore, the shear equations were calibrated for ACI 318 and CSA A23.3 ultimate limit states and two sets of resistance factors were calculated.
The ACI 318 and CSA A23.3 ultimate limit state are described in Equation (3.14) and (3.15), respectively:

\begin{align*}
1.4D < \phi R \\
1.2D + 1.6L < \phi R \\
1.4D < \phi R \\
1.25D + 1.5L < \phi R
\end{align*}

(3.14) (3.15)

where, \(D\) is the dead load, \(L\) is the live load and \(\phi\) is the resistance factor

Reliability indexes for the shear equations were calculated for the full range of \(D/(D + L)\) ratio. The resistance factor for each shear equation was recommended based on the target reliability index \(\beta_{T}\) of 3.5. In case of CSA A23.3, the resistance factors were calculated by keeping the original resistance for concrete \((\phi_c = 0.65)\) and steel \((\phi_s = 0.85)\) recommended in the code. For design purposes, an additional resistance factor, \(\phi\) was recommended in addition to \(\phi_c\) and \(\phi_s\).

3.7.3 Results from the Reliability Analysis

The reliability index values for the proposed shear equations with and without web reinforcement are presented in Figs. 3.7, 3.8 and 3.9. The resistance factors were reported to the nearest of 0.05. The analysis showed that the reliability index skewed at \(D/(D + L)\) ratio of 0.9 which was considered as a critical point. This is because, at \(D/(D + L)\) ratio of 0.9, the mean values of the resistance \((m_R)\) was found to be minimum and therefore, the difference between the mean value of the resistance \((m_R)\) and the total load effect \((m_Q)\) was lower than at other load ratios. The recommended resistance factors \((\phi)\) for the ACI and CSA load factors were 0.75 and 0.85, respectively for the proposed general equation with web reinforcement. On the other hand, the recommended \(\phi\) values for the ACI and CSA load factors were 0.8 and 0.75, respectively for
the simplified shear equation. Similarly, the resistance factors ($\phi$) for the proposed shear equation without web reinforcement were found to be 0.75 and 0.8 for the ACI and CSA load factors, respectively. The reliability index and the recommended resistance factors are presented in Table 3.5. The design shear equations with and without web reinforcement are presented in Equations (3.16) and (3.17) where, Equation (3.16) is the simplified shear equation for deep beam with web reinforcement.

$V_u = \frac{2}{5} - \frac{1}{4}(a / d)^{0.23} + 0.85(\hat{\rho}_h \hat{\rho}_v)^{1/10} - \frac{3}{5} ((a / d) \hat{\rho}_h \hat{\rho}_v)^{1/16} - 200((a / d) \hat{\rho}_h \hat{\rho}_v)^{2.65}$

ACI: $\phi = 0.8$, CSA: $\phi = 0.75$ (in addition to $\phi_c = 0.65$ and $\phi_s = 0.85$)

$V = 1.74 - 2(a / d)^{0.044} + \frac{1}{2} \rho^{0.14}$

ACI: $\phi = 0.75$, CSA: $\phi = 0.8$ (in addition to $\phi_c = 0.65$ and $\phi_s = 0.85$)

Figure 3.7 Reliability index for the proposed general shear equation with web reinforcement
Figure 3.8  Reliability index for the proposed simplified shear equation with web reinforcement

Figure 3.9  Reliability index for the proposed shear equation without web reinforcement
Table 3.5  Reliability index and resistance factors for load combination of \( D+L \)

<table>
<thead>
<tr>
<th>Proposed equation</th>
<th>Code used</th>
<th>Reliability Index, ( \beta )</th>
<th>Recommended ( \varphi )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \varphi=0.9 )</td>
<td>( \varphi=0.85 )</td>
</tr>
<tr>
<td>general with web reinforcement</td>
<td>ACI 318</td>
<td>2.50</td>
<td>2.83</td>
</tr>
<tr>
<td></td>
<td>CSA A23.3</td>
<td>3.47</td>
<td>3.68</td>
</tr>
<tr>
<td>simplified with web reinforcement</td>
<td>ACI 318</td>
<td>3.01</td>
<td>3.27</td>
</tr>
<tr>
<td></td>
<td>CSA A23.3</td>
<td>3.19</td>
<td>3.31</td>
</tr>
<tr>
<td>without web reinforcement</td>
<td>ACI 318</td>
<td>2.52</td>
<td>2.84</td>
</tr>
<tr>
<td></td>
<td>CSA A23.3</td>
<td>3.20</td>
<td>3.37</td>
</tr>
</tbody>
</table>

3.8 Summary

This chapter presented the statistical analysis performed using \( 2^k \) factorial design and the analytical equations developed using GA to predict the shear strength of RC deep beams with and without web reinforcement. From the literature, a database of 381 beams and their test results was composed and used in the prediction. The following conclusions can be drawn from the study:

1. The most significant parameters affecting the shear strength of deep beams were identified by the factorial design. The results (\( p \)-value) from \( 2^k \) full factorial showed that \( a/d \) ratio, and the horizontal and vertical shear reinforcement ratio were the most contributing parameters to the shear strength where the percent contribution of \( a/d \) ratio was the highest (56%). Moreover, two-way, three-way and four-way interactions were also observed among the parameters and their combined percent contribution was more than 25%. Therefore, inclusion of these higher order non-linear terms in the shear equation improved its accuracy.

2. A comparative study among the proposed shear equations with the ACI and CSA STM models was also presented. The performance factor and the \( \chi \) value of the STM models were quite high, however, they were reduced to one in the case of proposed equations. Moreover,
from the descriptive statistical analysis, the value of SD, VAR, COV and AAE improved the accuracy of the proposed shear equation over the ACI and CSA STM model.

3. The analysis presented in Chapter 2 showed that the models used for shear prediction with web reinforcement, performed poorly when used to predict shear strength without web reinforcement, and vice versa. Therefore, two separate equations were developed in the present study.

4. The objective of the proposed shear equations development was to predict the nominal shear strength accurately. Therefore, the equations were calibrated to be used for design purposes. The resistance factors were recommended from a reliability index analysis. In the case of CSA load factors, the resistance factor ($\phi$) for the ultimate shear strength was calculated in addition to the current material resistance factors for concrete ($\phi_c = 0.65$) and steel ($\phi_s = 0.85$). Therefore, in order to calculate the ultimate shear strength according to ACI code, the proposed model (Equations 3.16 and 3.17) need to be multiplied with only the resistance factor ($\phi$). On the other hand, in the case of CSA code, the proposed model (Equations 3.16 and 3.17) need to multiply with the resistance factor ($\phi$) along with the material resistance factors recommended in the code.
Chapter 4: Shear Behavior of Reinforced Concrete Deep Beam under Reversed Cyclic Loading

4.1 Overview

The codes design philosophy for reinforced concrete (RC) structures is to provide safety against collapse at ultimate loads and functionality at working loads. Therefore, to ensure the structural safety and serviceability it is necessary to predict the structure’s ultimate and deformational capacity and its cracking pattern. Experimental studies can provide accurate information in this regard, however, they are expensive and time consuming, especially to conduct experiment on a full scale structure like deep beam or shear wall under earthquake or reversed cyclic loading. Moreover, it is difficult to predict the exact failure behavior under dynamic loads of a real structure from their scaled test results. Therefore, analytical models could be an economic, reliable and fast solution to predict the behavior of a full scale structure under static or dynamic load instead of experimental testing. It has been observed that the problems arising in the reinforced concrete structures due to seismic loads can be traced by performing reversed cyclic tests. Therefore, in this chapter, the behavior of a deep beam under reversed cyclic loading will be investigated.

Fenwick and Fong (1979) first performed the cyclic load tests on five RC slender beams to examine the hinge formation. They observed a shear strength degradation of beams under cyclic loading in addition to the formation and growth of significant cracks along the length of the beam. Stevens et al. (1991) performed experiments on RC panel under reversed cyclic shear. They found that the cyclic shear causes the RC member to yield in the weaker reinforcement and
ultimately to fail in concrete crushing. Similarly, Fang et al. (1993) performed cyclic loading tests on moderately deep RC beams with high strength concrete (HSC). They observed that HSC under cyclic loading exhibited less spalling in the plastic hinges and slower strength degradation compared to normal strength concrete. Moreover, they observed that as the $a/d$ ratio decreases the pinching effect in the hysteresis loops became more prominent.

Several studies showed that the shear capacity of RC beams was reduced significantly under cyclic loading which causes low inelastic shear deformation in RC beams compared to monotonic loads (Aschheim and Moehle 1992, Wong et al. 1993, and Choi and Park 2010). This is due to the widening of the flexure-shear cracks in the plastic hinge zones that weakens the aggregate interlock at the cracks (Priestley et al. 1994 and Martín-Pérez and Pantazopoulou 1998). Therefore, in the case of earthquake design of special moment frames, ACI 318-05 building code neglects the shear contribution of concrete, $V_c$, to the total shear resistance if the seismic design shear load, $V_e$ (plastic hinge region) is equal to or more than one-half of the maximum required shear strength and the factored axial compression is less than $A_g f'_c / 20$.

The previous studies on cyclic behavior were limited to only RC slender beams. Researchers only studied deep beam behavior under static load and none of the studies analyzed the behavior of RC deep beams under reversed cyclic loading. Therefore, it is necessary to observe and predict the shear behavior of RC deep beams in order to design them accurately in seismic zones. The present Chapter aims to predict the behavior and the ultimate capacity of RC deep beams under reversed cyclic loading using finite element method (FEM).
4.2 Methodology

In order to predict the behavior of RC deep beams under seismic loads, the beam was analyzed under reversed cyclic loading using FEM. At first, the FEM model simulated the experimental study performed by Islam et al. (2005) on a deep beam under static load. The FEM model was verified with the test results from that study. Then the FEM model was used to simulate the behavior of deep beam under reversed cyclic load. The reversed cyclic load was applied in downward and upward direction with displacement increment. The analysis was performed with the variation of longitudinal reinforcement, horizontal and vertical reinforcement ratios. The FEM package ABAQUS/Standard was used for the FEM modeling and analysis of RC deep beams.

4.3 Model Description

4.3.1 Geometry

The experimental work on deep beam was adopted from previous work by Islam et al. (2005). The dimensions of the tested deep beam were 2000 mm long, 150 mm wide and 800 mm high. The beam was tested under four point loading where, the distance between the loading points were 400 mm. The shear span to depth ratio of the deep beam was 0.8 (Fig. 4.1). Two 20 mm and two 25 mm deformed bar were used as longitudinal reinforcement. The beam was reinforced horizontally and vertically with shear reinforcements consisting of 4 mm wire at 100 mm spacing. The yield strength of the 20 mm and 25 mm bars were 543 MPa and 500 MPa, respectively and the yield strength of the 4 mm wire was 553 MPa. The concrete compressive
strength, \( f'_c \), and splitting tensile strength, \( f_t \), were 37.8 MPa and 3.83 MPa, respectively. The geometry and reinforcement detail of the deep beams is shown in Fig. 4.1.

4.3.2 Material Constitutive Models

4.3.2.1 Concrete

The concrete is modeled in ABAQUS using the damage plasticity model. The inelastic concrete behavior is represented in the damage plasticity model by using the concepts of isotropic damaged elasticity in combination with isotropic tensile and compressive plasticity. The model is useful to simulate the RC member subjected to monotonic and cyclic loading with the steel confinement. Moreover, the model captures the irreversible damage of fractured concrete with the combination of multi hardening plasticity and isotropic damaged elasticity. It also allows the stiffness recovery effects during cyclic load reversals. In the damage plasticity model, the
concrete is considered to fail in tensile cracking or compressive crushing (Hibbitt, Karlsson, & Sorensen Inc. 2009).

The present study considered the model proposed by Saenz (1964) to represent the uniaxial stress-strain relationship in compression (Fig. 4.2). The relationship between stress-strain is as follows:

\[
\sigma_c = \frac{E_c \varepsilon_c}{1 + (R + R_\varepsilon - 2) \left(\frac{\varepsilon_c}{\varepsilon_0}\right) - (2R - 1) \left(\frac{\varepsilon_c}{\varepsilon_0}\right)^2 + R \left(\frac{\varepsilon_c}{\varepsilon_0}\right)^3}
\]

(4.1)

where,

\[
R = \frac{R_\varepsilon (R_\sigma - 1)}{(R_\varepsilon - 1)^2}, \quad R_\varepsilon = \frac{E_c}{E_0}, \quad E_0 = \frac{f_t}{\varepsilon_0}
\]

(4.2)

In Equation (4.2), the values for \( \varepsilon_0 \), \( R_\varepsilon \) and \( R_\sigma \) are taken as 0.0025, 4.0 and 4.0, respectively, as reported by Hu and Schnobrich (1989).

The uniaxial tension behavior is presented in Fig. 4.3. The first part of the graph (Fig. 4.3a) represents the elastic curve which is established from the elastic modulus, \( E_c \), and the tensile strength, \( f_t \). The elastic modulus, \( E_c \) is calculated using the following formula,

\[
E_c = 4700 \sqrt{f_c}
\]

(4.3)

The post-peak tension failure of concrete in damage plasticity model was specified using the fracture energy method (Hillerborg et al. 1976). Fig. 4.3b displays the post-peak tension failure where the fracture energy, \( G_f \), is the area under the softening curve and was assumed equal to 120 J/m\(^2\) (Hibbitt, Karlsson, & Sorensen Inc. 2009).
The strength degradation mechanism in concrete is more complex under uniaxial cyclic loading, this is due to the opening and closing of previously formed micro-cracks, as well as their interaction. It is observed that there is some recovery of the elastic stiffness as the load changes direction during a uniaxial cyclic test (Hibbitt, Karlsson, & Sorensen Inc. 2009). The stiffness
recovery effect is known as the “unilateral effect”. The effect is usually more pronounced as the load changes from tension to compression, causing tensile cracks to close, which results in the recovery of the compressive stiffness. The damaged plasticity model assumes that the reduction of the elastic modulus is given in terms of a scalar degradation.

The concrete damage plasticity model in ABAQUS requires the values of elastic modulus, Poisson’s ratio, compressive and tensile behaviour of concrete and plastic damage parameters. The five plastic damage parameters are the dilation angle \( \psi \), the flow potential eccentricity \( \varepsilon \), the ratio of initial equibiaxial compressive yield stress to initial uniaxial compressive yield stress \( \frac{\sigma_{b0}}{\sigma_{c0}} \), the ratio of the second stress invariant \( K_c \) and the viscosity parameter \( \mu \) that defines viscoplastic regularization of the concrete constitutive equations in ABAQUS/Standard analysis. The values of the plastic damage parameters were taken as the recommended value by ABAQUS and were set to 26 degrees, 0.1, 1.16, 0.66, and 0.01, respectively. The Poisson’s ratio for concrete was considered as 0.2.

4.3.2.2 Steel Reinforcement

The reinforcement was considered as elastic–plastic material and identical in tension and compression (Fig. 4.4). The elastic modulus, \( E_s \), was taken as 210 GPa (Islam et al. 2005). A Poisson’s ratio of 0.3 was used for the steel. In the FEM modelling, the steel reinforcement was considered as an embedded element in the concrete with a perfect bond between the two materials.
To simulate the behavior of the steel reinforcement under cyclic loading, a nonlinear isotropic/kinematic hardening model was used (Hibbitt, Karlsson, & Sorensen Inc. 2009). The model captures the inelastic behavior of reinforcement subjected to cyclic loading.

4.3.3 FE Model

The FE model for the concrete beam and the steel rebar are shown in Fig. 4.5. Four-node linear tetrahedral elements (C3D4) were used for the concrete and 20-node quadratic brick with reduced integration elements (C3D20R) were used for the steel reinforcement (Fig. 4.6). The quadratic brick element allowed to model a smooth surface for the circular cross-section of the reinforcement bar. A displacement based four point bending load was applied on the beam.
4.3.4 Validation of FEM model

The experimental and the numerical load-deflection curve for the mid span deflection of the deep beam is shown in Fig. 4.7. Fig. 4.7 shows that the FEM model predicts the load-deflection curve
for deep beam to be slightly stiffer compared to the experimental results. The failure of the deep beam was assumed by concrete crushing at an ultimate strain of 0.0035 for the FEM model. The load-deflection curve shows that the deep beam was failed at ultimate load of 1050 kN which was only 1% higher compared to the experimental ultimate failure load (1040 kN). However, the maximum deflection at failure was found 12% higher in the FEM model (5.36 mm) compared to the experimental results (4.78 mm). The difference in the FEM and the experimental results could be due to the perfect bond assumption between concrete and steel in addition to the uncertainty involved in the actual material strength. In general, the load-deflection curve from the experiment and the FEM analysis were in good agreement. The good agreement indicates that the constitutive models used for concrete and steel able to capture the fracture behavior of deep beam accurately.

Figure 4.7 Load-displacement for the deep beam under monotonic loading
4.4 Deep Beam under Reversed Cyclic Loading

To predict the behavior of RC deep beams under seismic loads, the beam was analyzed under reversed cyclic loading. The load was applied by displacement control. Fig. 4.8 shows the intensity of applied displacement in each cycle where 1mm downward displacement was applied in the first loading cycle. All the beams were loaded two cycles for each level of displacement. The displacement on a subsequent set of cycle was increased by 1 mm.

The same validated model for the deep beam under monotonic loading was analyzed for the reversed cyclic loading. Ten different deep beam sections were analyzed. The beams had different longitudinal reinforcement, horizontal and vertical shear reinforcements (Table 4.1). The longitudinal reinforcement varied from 0.7% to 3.27% and the horizontal and vertical shear reinforcements were both varied from 0 to 0.4%.

![Figure 4.8 Reversed cyclic load history in FEM analysis](image)
### Table 4.1 Reinforcement details of the deep beam specimens

<table>
<thead>
<tr>
<th>Beam Specimens</th>
<th>a/d ratio</th>
<th>Longitudinal Reinforcement</th>
<th>Vertical Shear Reinforcement, 4mm wire</th>
<th>Horizontal Shear Reinforcement, 4mm wire</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0</td>
<td>0.8</td>
<td>2-20 mm + 2-25 mm dia</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B1</td>
<td>0.8</td>
<td>2-20 mm + 2-25 mm dia</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>B2</td>
<td>0.8</td>
<td>2-20 mm + 2-25 mm dia</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>B3</td>
<td>0.8</td>
<td>2-20 mm + 2-25 mm dia</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>B4</td>
<td>0.8</td>
<td>2-20 mm + 2-25 mm dia</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>B5</td>
<td>0.8</td>
<td>2-20 mm + 2-25 mm dia</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>B6</td>
<td>0.8</td>
<td>2-20 mm + 2-25 mm dia</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>B7</td>
<td>0.8</td>
<td>2-20 mm + 2-25 mm dia</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>B8</td>
<td>0.8</td>
<td>2-20 mm dia</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>B9</td>
<td>0.8</td>
<td>6-25 mm dia</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

#### 4.5 Results and Discussions

#### 4.5.1 Hysteresis Curves

The hysteresis curves represent the load-displacement ($P - \Delta$) relationship of RC structures under reversed cyclic loading. Fig. 4.9 shows the $P - \Delta$ curves for the 10 different deep beams.

The following observations are made from the hysteresis curves:

- In the first cycle of loading the relationship between the vertical load, $P$, and the mid span deflection, $\Delta$, is linear before cracking of concrete occurs.
- Within the elastic range, the residual deformation is negligible.
- After the cracking of concrete, the stiffness of the beam decreased and the hysteresis loops changed direction from an elastic straight line to an inelastic curvature. As the displacement increased, the stiffness (slope of the curve) experienced further decrease (became flatter).
• The stiffness increased with the increase of the vertical shear reinforcements compared to the horizontal shear reinforcement. This phenomenon indicates that the load carrying capacity of the beam section increased more with the increase of the vertical shear reinforcements.

• The beam stiffness increased with the increase of the longitudinal reinforcement ratio where the shear reinforcement ratio remains constant.

The shear strength of each beam is calculated from the load bearing capacity of the specimens at failure. The variation of the shear strength with the change in the reinforcement ratio is plotted in Fig. 4.10. The shear capacity of the deep beam was increased by 66% with the increase of vertical shear reinforcement ratio from 0.0% to 0.4% (Fig 4.10a). On the other hand, as the horizontal shear reinforcement ratio increased from 0.0% to 0.4%, the shear capacity increased by 89% (Fig 4.10b). Therefore, it can be concluded that both vertical and horizontal shear reinforcement are equally important to improve the shear capacity of deep beam. The longitudinal reinforcement also improved the shear capacity by 29% with the increase of its reinforcement ratio from 0.7% to 3.4%.
Figure 4.9  Load deflection curve at mid span of the deep beams
Figure 4.10 Effect of different parameters on Shear Strength of deep beams

4.5.2 Skeleton Curves

The envelopes of the hysteresis curves can be represented by the skeleton curves, which are shown in Fig. 4.11 where, points A and A' represent the yield load-displacement point and points B and B’ represent the ultimate load-displacement point. The following observations can be made from the load-displacement envelope curves.
• Based on the behavior, each curve can be divided into two parts: a) elastic (pre-yield) and b) plastic (post-yield).

• In the elastic phase, the load-deflection relationship is linear until the cracking of concrete. The non-linear portion of the curve appears after the concrete cracks.

• After cracking of the concrete, the slope of the skeleton curves decreased. The change in the slope indicates that the beam section is subjected to inelastic deformation. At the ultimate phase, the beam experience large displacement with relatively constant loading.

• The yield load and yield displacement of Beams B1, B2, B3 and B4 increased with the increase of both vertical and horizontal shear reinforcements.

• The skeleton curves for beam B0 and B1 were similar. This concludes that without vertical shear reinforcement, horizontal shear reinforcements alone cannot improve the load carrying capacity of a deep beam.
Figure 4.11 Skeleton curves for the reversed cyclic loading
4.5.3 Deformation Restoring Capacity

During a strong earthquake, a RC structure experience plastic deformation. Therefore, the residual deformation during an earthquake need to be measured as it gives an indication of the deformation restoring capacity of a structure. The deformation restoring capacity affects the serviceability of a structure after being subjected to an earthquake. The deformation restoring capacity can be measured in terms of the residual deformation ratio, $\Delta_r / \Delta_u$, where, $\Delta_r$ is the residual displacement after the full reversed cyclic loading and $\Delta_u$ is the maximum displacement of the corresponding skeleton curves.

Table 4.2 shows the displacement restoring capacity of different deep beam specimens. The variation of deformation restoring capacity with the change in the reinforcement ratio is plotted in Fig. 4.12. The following observations are made from the table and figures.

- The displacement restoring capacity for the downward loading cycles were higher compared to the upward loading cycles. This is due to the high residual displacement observed after the first cycle for each level of applied displacement.
- Deep beam B1 (without any vertical shear reinforcement) shows the least deformation restoring capacity among all the beams studied.
- The capacity increased by 140% after providing only 0.1% of vertical shear reinforcement; although beyond that, it shows a negative trend with the increase of the vertical shear reinforcements. The capacity decreased by 26% with an increase of vertical shear reinforcement ratio from 0.1% to 0.4% (Fig. 4.12a).
• The improvement of deformation restoring capacity of deep beam specimens, B3, B5, B6 and B7, with the increase of horizontal shear reinforcement is very small which indicates that the horizontal shear reinforcement have little impact on the deformation restoring capacity. However, the capacity decreased after the increase of the horizontal shear reinforcement ratio from 0.2% to 0.4%.

• The average displacement restoring capacity for Beam B3, B8 and B9 with the variation of longitudinal reinforcement remain almost constant (Fig. 4.12c).

<table>
<thead>
<tr>
<th>Specimens</th>
<th>$\Delta_r$, mm</th>
<th>$\Delta_u$, mm</th>
<th>$\Delta_r/\Delta_u$</th>
<th>Average</th>
</tr>
</thead>
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<tr>
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<td>Downward cycle</td>
<td>Upward cycle</td>
<td>Downward cycle</td>
<td>Upward cycle</td>
</tr>
<tr>
<td>B0</td>
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<td>0.24</td>
<td>2.90</td>
<td>2.63</td>
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<tr>
<td>B1</td>
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<td>0.20</td>
<td>2.91</td>
<td>2.71</td>
</tr>
<tr>
<td>B2</td>
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<td>3.06</td>
<td>4.85</td>
<td>4.70</td>
</tr>
<tr>
<td>B3</td>
<td>2.98</td>
<td>2.79</td>
<td>4.91</td>
<td>4.72</td>
</tr>
<tr>
<td>B4</td>
<td>2.66</td>
<td>1.84</td>
<td>4.97</td>
<td>4.1</td>
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<tr>
<td>B5</td>
<td>2.47</td>
<td>1.12</td>
<td>3.83</td>
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<tr>
<td>B6</td>
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<td>3.86</td>
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<td>B7</td>
<td>3.18</td>
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<tr>
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<td>1.21</td>
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</tr>
<tr>
<td>B9</td>
<td>2.74</td>
<td>2.10</td>
<td>4.97</td>
<td>3.77</td>
</tr>
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</table>
Figure 4.12 Effect of different parameters on deformation restoring capacity of deep beams

4.5.4 Ductility Capacity

Ductility is an important index for seismic evaluation of RC structures which is measured in terms of displacement capacity. The ductility, $\mu$, is the ratio of the ultimate displacement, $\Delta_u$ (at ultimate load), to the yield displacement, $\Delta_y$ (at the yield load). The ductility capacity of all the
specimens is listed in Table 4.3. The variation of ductility capacity with the change in the reinforcement ratio is plotted in Fig. 4.13. The important observations are listed below.

- Deep beams, B0 and B1, without any vertical shear reinforcement have the minimum ductility capacity of 3.48 and 3.56, respectively.
- The ductility capacity increased by up to 71% with the increase of vertical shear reinforcement ratio.
- The ductility capacity was also increased with the increase of horizontal shear reinforcement ratio. Fig. 4.13b shows that the ductility capacity increases up to 37% with the increase of horizontal shear reinforcement ratio from 0 to 0.4%.
- The ductility capacity of a deep beam was increased by 22% with the increase of the longitudinal reinforcement ratio from 0.7% to 1.8%. However, the capacity decreased in a small amount by 7% when the longitudinal reinforcement ratio was increased from 1.8% to 3.4% (Fig. 4.13c).

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Loading direction</th>
<th>$\Delta_y$, mm</th>
<th>$\Delta_u$, mm</th>
<th>$\Delta_u/\Delta_y$</th>
<th>Average</th>
</tr>
</thead>
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<td>Upward cycle</td>
<td>Downward cycle</td>
<td>Upward cycle</td>
<td></td>
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</tr>
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<td>B2</td>
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<td>4.91</td>
<td>4.72</td>
<td>5.46</td>
</tr>
<tr>
<td>B4</td>
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<td>0.67</td>
<td>4.81</td>
<td>3.73</td>
<td>6.06</td>
</tr>
<tr>
<td>B5</td>
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<td>0.65</td>
<td>3.83</td>
<td>3.70</td>
<td>5.25</td>
</tr>
<tr>
<td>B6</td>
<td>0.90</td>
<td>0.65</td>
<td>3.86</td>
<td>3.72</td>
<td>4.29</td>
</tr>
<tr>
<td>B7</td>
<td>0.95</td>
<td>0.62</td>
<td>5.70</td>
<td>4.63</td>
<td>6.00</td>
</tr>
<tr>
<td>B8</td>
<td>0.87</td>
<td>0.63</td>
<td>3.87</td>
<td>2.74</td>
<td>4.45</td>
</tr>
<tr>
<td>B9</td>
<td>0.98</td>
<td>0.71</td>
<td>4.97</td>
<td>3.77</td>
<td>5.07</td>
</tr>
</tbody>
</table>
4.5.5 Energy Dissipation Capacity

The performance of the RC structures could be evaluated in terms of energy dissipation capacity of the member in cyclic loading. The energy dissipation capacity in each loading cycle was calculated from the area under the hysteresis loop (Fig. 4.9). The cumulative energy dissipated in
each beam under the reversed cyclic loading is shown in Fig. 4.14. The following observations are made from the figure.

- The amount of dissipated energy increased with increase of displacement in the plastic zone.

- The deep beam specimens were subjected to the same amount of downward or upward displacement twice in two cycles and the results showed that the amount of dissipation energy in second cycle is higher compared to first cycle. This is due to the residual displacement after the first cycle of the loading.

- The results from all the beam specimens show that the greater reinforcement ratio results in higher amount of energy dissipation.

- The variation of energy dissipation capacity with the change in the reinforcement ratio is plotted in Fig. 4.15. The capacity was found the least with no vertical shear reinforcement (4808 kN-mm) and it increased by 500% after providing only 0.1% vertical shear reinforcement ratio (Fig. 4.15a). However, the capacity improvement was not significant (10%) after increasing the vertical shear reinforcement from 0.1% to 0.4%.

- The energy dissipation capacity was also increased with the increase of horizontal shear reinforcement ratio by 216% from 0 to 0.4% (Fig. 4.15b). Similarly, the energy dissipation capacity of a deep beam was increased by 134% with the increase of the longitudinal reinforcement ratio from 0.7% to 1.8% (Fig. 4.15c).
Figure 4.14 Energy dissipation of the deep beam specimens
Figure 4.15 Effect of different parameters on energy dissipation capacity of deep beams

4.6 Summary

This chapter presented the behavior of deep beams under reversed cyclic loading. Ten reinforced concrete deep beams with different reinforcement ratios were analyzed using FEM to predict their behavior. The following conclusions can be drawn from the study:
It has been observed from the analysis results that the deep beam without any vertical shear reinforcement showed the least capacity in terms of load carrying capacity, deformation restoring capacity, ductility capacity and the energy dissipation capacity.

The load-deflection curve for a deep beam was found in the elastic range until the crack appeared in the concrete and after that inelastic branch of the hysteresis loop appears with the increase in deflection. The residual displacement in the first cycle was very small and it increases with the increase of the inelastic deformation.

It was observed that only horizontal shear reinforcement could not improve the load carrying capacity of the deep beams. In presence of vertical shear reinforcement, the increase in horizontal shear reinforcement could improve the shear capacity of the deep beams.

The residual displacement for the downward loading cycles was found higher compared to upward loading cycles. Without any vertical shear reinforcement, the deep beams exhibited the minimum deformation restoring capacity. The deformation restoring capacity increases up to 140% and 28% with the increase of vertical and horizontal shear reinforcement respectively.

The ductility of the deep beams increases with the increase of shear reinforcement. However, without vertical shear reinforcement, the increase of horizontal shear reinforcement could not improve the ductility capacity. Moreover, in presence of high longitudinal reinforcement ratio, the ductility of the deep beams decreased significantly.
The increase in the amount of reinforcement ratio caused the deep beams to absorb more energy during reversed cyclic loading. The total energy in the second cycles of loading is higher compared to the first cycle for the same displacement level. This is because the deep beam specimens subjected to some residual displacement after completion of first cycle of loading.
Chapter 5: Strengthening of Shear Deficit RC Deep Beams by FRP

5.1 Overview

The strut and tie model (STM) recommended in the building codes (ACI 318 and CSA A23.3) considers only the design due to static loads. Therefore, it is important to investigate the behavior of deep beams designed using STM under seismic loads.

Fibre reinforced polymer (FRP) is becoming a popular strengthening material in concrete industry during the last two decades due to its light weight, high durability and high strength. The application of FRP includes the repair and the strengthening of aged and new structures. Numerous studies reported in the literature investigated the shear strengthening of RC slender beams using FRP, where either FRP strips or sheets were applied (Colotti et al. 2001, Adhikary et al. 2004, Tanarslan 2010, Lim 2010, Altin et al. 2010). All experimental investigations using FRP showed a significant increase in the shear strength with respect to the unstrengthened beams (Tanarslan et al. 2008, Altin et al. 2010, Lim 2010, Tanarslan 2010). This increase in shear strength was dependent on wrap length, anchorage and orientation of the fibres (Taljsten 2003, Sakar et al. 2009, Lee et al. 2011).

None of the studies reported in the literature considered FRP strengthening of deep beam for seismic loads. Even for slender beam, only few studies focused on the shear strengthening of RC beams under cyclic loads (Anil 2006, Anil 2008, Sakar et al. 2009, Colalillo 2011). The experimental results of these few studies showed a substantial improvement in shear capacity, ductility, stiffness and energy dissipation capacity of the CFRP wrapped beams. For example, the shear strength using externally CFRP wrapped beam was found 56% higher under cyclic loads than a control beam (Anil 2008). However, the improvement observed mostly depends on the

The objective of this chapter is to analyze a typical RC building with a deep beam at its first floor. The deep beam was designed using STM method according to CSA A23.3 building code. The analysis was performed using non-linear pushover and non-linear time history analysis to evaluate shear strength under seismic loads. Moreover, the capacity of the structure was also checked in terms of lateral displacement, inter-storey drift ratio and base shear. A CFRP retrofitting technique was recommended as a viable solution to improve the capacity in the members which were found deficient in seismic capacity.

5.2 Structure for the Case Study

A two-dimensional office building that has seven floors with a deep beam at the first floor was adopted from the literature (Li et al. 2003). The total height and width of the building is 28.9 m and 22.8 m, respectively (Fig. 5.1). The building has three types of columns, exterior, interior and base columns. The two interior columns were discontinued at the first storey to facilitate the parking at the ground floor. The loads carried by these interior columns were, therefore, transferred to the base columns through a deep beam which worked as a transfer beam that has a span length of 19.6 m. The base columns were fixed at the supports and the beam column joints were considered rigid. The RC members were designed following the CSA A23.3-04 standard whereas the deep beam was designed following the STM procedure recommended in the CSA code (Fig. 5.2). The shear span to depth ratio of the deep beam was 2.27 (a/d<2.5). The compressive strength of the concrete was considered 30 MPa and the yield strength of the steel was 400 MPa for both flexure and shear reinforcement. The Mander’s confined concrete model
(Mander et al. 1988) was used in SAP2000 to model the stress-strain behavior of the concrete and the steel. The hysteresis behavior of the materials was captured by following the Takeda model (Takeda et al. 1970).

Figure 5.1 Elevation of the modeled building
5.3 Non-Linear Pushover Analysis

The nonlinear static pushover analysis is an approximate method, where the structure is being analyzed applying an incremental lateral load. The distribution of the load may remain constant or vary with respect to the height of the structure. The load is increased until the structure reaches a target displacement or a complete failure. From there, the capacity of the structure can be determined. The pushover analysis can be performed using both force-based (Satyarno 1998, Elnashai 2001, Antoniou 2002) or displacement-based (Antoniou 2004, Pinho 2005) approach. The load pattern for the pushover can be chosen in several ways such as (a) an incremental force (b) a constant force or (c) an incremental acceleration applied along the height of the structure.

In the present study, the deep beam structure was modeled in SAP2000 where the nonlinear pushover analysis was performed considering the P-delta effects. The pushover force was applied as a constant incremental acceleration at the base of the structure along the horizontal direction until a complete failure was achieved. The structure is pushed to a target displacement
of 1000 mm at the corner of the top floor. The damping of the structure was considered 5\%. The shear capacity of the beams (including deep beam) and columns at each floor were calculated from the pushover analysis.

5.4 Time History Analysis

The non-linear dynamic time history analysis was used to evaluate the inelastic behavior of the structure during a real earthquake. The analysis captures the demand on the structure during an earthquake. Although the method proved that it can capture the most comprehensive and accurate behavior of a structure under an earthquake load, it requires more computational effort than the static pushover analysis. Moreover, earthquakes are random and it is unlikely that the same earthquake will be repeated in the future. So, it will be unwise and insufficient to design a building considering only one earthquake record. Consequently, the structure must be analyzed with more than one earthquake record. While some researchers recommended to consider at least three records for the design (Bhatti 1981), the present study analyzed the structure for twenty four earthquake records. The detail information of the records is listed in Table 5.1. The earthquakes occurred in different years where the magnitude on a Richter scale varied from 4.9 to 7.68 with a wide range of peak ground acceleration (PGA). The time history records are selected from the Pacific Earthquake Engineering Research Center (PEER) ground motion database website (PEER 2011).

In order to perform the time history analysis, the earthquake records must be scaled or matched to the response spectrum of the area where the structure is located. The response spectral matching improves the matching of the natural records to the target spectrum over some desired period range (McGuire 2002, Hancock 2006). All twenty four time history records considered
for the time history analysis were matched with the design spectral response acceleration for the City of Vancouver, BC (Fig. 5.3) using the SeismoMatch software. The matching of the earthquake records were performed considering a damping of 5% with a scale factor of 1.0. A typical un-scaled and scaled time history records (New Zealand earthquake) is shown in Fig. 5.4. Next, the nonlinear dynamic time history analysis for each earthquake record was performed in SAP2000, where the ground acceleration was applied in horizontal direction with a scale factor of $g$ (9.8 m/s) and a constant damping of 5%.

<table>
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<tr>
<th>Record</th>
<th>Event</th>
<th>Year</th>
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<th>Event</th>
<th>Year</th>
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5.5 Analysis Results and Discussions

5.5.1 Capacity of the Structure

The capacity of the structure was determined from the nonlinear pushover analysis where the structure was pushed until it failed. From the pushover analysis, the capacity of the structure in terms of lateral displacement, inter-storey drift ratio, total base shear and beam shear in each floor were calculated.
5.5.1.1 Storey Drift and Inter-Storey Drift Ratio

The ductility of the structure was measured in terms of the displacement and the inter-storey drift capacity. The displacements and the inter-storey drifts from the pushover analysis are showed in Fig. 5.5. The structure was pushed to a target displacement of 1000 mm at the top floor, but it was fully collapsed at a displacement of 541 mm at the top floor. Therefore, the maximum storey drift capacity was 541 mm at the top storey, whereas the storey drift was 87 mm at the deep beam level which is only 16% of the maximum drift.

The inter-storey drift ratio was calculated from the displacements of each storey as a percentage of the storey height (Fig. 5.5b). The second floor exhibited the maximum inter-story drift ratio of 2.4% and the ratio gradually decreased at the upper storeys. At the deep beam level, the inter-storey drift ratio was only 1.17% which indicated that the first floor at the deep beam level was not ductile enough compared to other floors. The FEMA 356 guideline describes three damage levels for a building structure depending on the inter-storey drift ratios: immediate occupancy (drift ratio <1%), life safety (drift ratio 1~2%) and collapse prevention (drift ratio 2~4%). From the analysis, it can be concluded that the damage level of the structure was between life safety and collapse prevention as the maximum inter-storey drift ratio at the second floor was found to be 2.4% (Fig. 5.5b).
5.5.1.2 Total Base Shear

The base shear capacity of the structure was calculated from the pushover analysis to be 2893 kN.

5.5.1.3 Beam Shear Capacity

The shear capacity at the beams at each storey was calculated from the pushover analysis. The capacity of the beams is displayed in Fig. 5.6. It was observed that the average capacity of the regular slender beam from storey 2 to 7 ranged from 223 to 234 kN. On the other hand, the maximum capacity for the deep beam was 1921 kN.

5.5.2 Seismic Demand versus Capacity of the Structure

A total of 24 time history records of real earthquakes were selected to capture all the possible vibration the structure may face under a future earthquake. From the nonlinear time history analysis, the seismic demand for these earthquakes in terms of displacements, inter-storey drift ratio, total base shear and beam shear were calculated. The total seismic demand was compared with the capacity of the structure which is calculated from the pushover analysis.
5.5.2.1 Storey Drift and Inter-Storey Drift Ratio

The displacements demand for different records were ranging from 71.65 mm to 106.37 mm at the top floor (Fig. 5.7a). The maximum displacement was recorded for the Coyote Lake earthquake. Comparing the demand with the capacity found from the pushover analysis (541 mm) at the top floor, a factor of safety of 5.08 for the structure was determined. At the deep beam level, the maximum drift demand was 15.39 mm where the capacity was 87 mm i.e., the capacity/demand ratio was 5.65. Comparing the demand versus capacity of the structure, it can be concluded that the structure has sufficient strength to resist the lateral drift (Fig. 5.7a).
Figure 5.7  Demand vs capacity (a) displacement (b) inter-storey drift
The maximum inter-storey drift ratio was 0.544% (Fig. 5.7b) at the fourth floor for the same Coyote Lake earthquake. Comparing the capacity from the pushover analysis (inter-storey drift ratio of 2.12%), the beam has a safety factor of 3.9. Moreover, based on the drift demand from all earthquake records, it can be concluded that the structure will not reach its immediate occupancy damage level (drift ratio <1%) and therefore, will remain safe for any severe earthquake (FEMA 356). However, it will be unwise to make such a conclusion without checking the shear capacity of the structure, i.e., global shear capacity (base shear) and local shear capacity (beam shear).

5.5.2.2 Total Base Shear

The base shear demand and capacity ratio for the earthquake records are displayed in Fig. 5.8. Twenty two earthquake records (92%) showed that the demand exceeded the maximum capacity of the structure. The base shear from the pushover analysis was found to be 2893 kN where the maximum demand from the time history analysis was 4931 kN for the Parkfield earthquake; i.e., 70% higher than the capacity. On average, the base shear demand was 25% higher than the original capacity of the structure.

5.5.2.3 Beam Shear

The shear demand of the beams on each floor from the time history analysis is presented in Fig. 5.9. The shear capacity in the top six floors was found to be satisfactory. However, the deep beam at the first floor was found to have a significant shear deficiency. The maximum shear demand was 2921 kN for the Parkfield earthquake which is 52% higher than the deep beam’s capacity (1921 kN) from the pushover analysis. The shear demand to capacity ratio at the deep
beam is displayed in Fig. 5.10. The plot shows that the deep beam has on average a shear deficiency of 22% when considering all the earthquake records.
Retrofitting with FRP

The time history analysis with the real earthquake records showed a significant shear strength deficiency in the deep beam. If an earthquake similar to the ones considered hit in the future the deep beam will fail. The deep beam at the first storey not only carries the floor loads but also transfers the loads from the two discontinued interior columns to the foundations; therefore, failure of the deep beam will eventually lead to the complete collapse of the structure. Therefore, shear strengthening is necessary to prevent the structure from the catastrophic consequence in the future. The present study focused on using carbon fibre reinforced polymer (CFRP) retrofitting technique.

In order to model the application of CFRP sheets, concrete was considered fully confined with FRP. Numerous empirical and semi-empirical models were proposed in the literature to describe the behavior of the FRP confined concrete. Most of these models including the ACI 440 were developed following the Mander et al. (1988) model which is based on confinement with steel (Fig. 5.11b) (Spoelstra et al. 1999, Fam et al. 2001). Some other models were developed from experimental studies using circular FRP confined sections (Samaan et al. 1998 and Campione et
al. 2003) which were later modified into a model for rectangular sections (Fig. 11c). The present study adopted the FRP confined concrete model developed by Rousakis et al. (2007). The unconfined area in the concrete section was confined by FRP U-wrap (Fig. 5.10c). A unidirectional CFRP sheet of 0.117 mm thick was used for the concrete confinement. The tensile modulus of CFRP sheet was 240 GPa and the failure strain was 15.5%. One to five layers of wrapping was considered to increase the shear capacity as well as the base shear of the total structure. The stress-strain curves for unconfined and FRP-confined concrete are showed in Fig. 5.12. The capacity of the retrofitted structure was calculated after performing the nonlinear pushover analysis in SAP2000. Similarly, the seismic demand of the structure was calculated from the nonlinear dynamic time history analysis using the same 24 earthquake records (Table 5.1).

In addition to the shear deficiency at the deep beam, the analysis results showed that the base shear of the structure was found to be insufficient by 25% on average for all the earthquake records. Therefore, the base columns were retrofitted with the same number of layers of CFRP wrapping system as the deep beam.

Figure 5.11 Unconfined and confined concrete sections
5.6.1 Results for the Retrofitted Structure

The CFRP wrapping showed a substantial shear capacity improvement for the deep beam (Fig. 5.13). The increase in the shear capacity of the deep beam ranged from 49.5% for one layer of FRP wrapping up to 82% for five layers of FRP wrapping (Fig. 5.13). After applying one layer of FRP wrap, the average capacity to demand ratio increased to 1.12 (Fig. 5.15). However, the deep beam still exhibited weakness for some earthquakes where the demand exceeded the capacity (Parkfield earthquake, Fig. 5.14). Therefore, the deep beam required to be retrofitted with more than one layer of CFRP. Deep beam retrofitted by three and five layers of FRP provided an increase in shear strength of 62% and 82% respectively. Moreover, it increased the average capacity to demand ratio to 1.22 and 1.37 for three and five layers of CFRP respectively (Fig. 5.15).

Similar to the deep beam, the FRP wrapping on the columns improved the capacity improvement ranged from 60% to 117% for different number of CFRP layers (Fig. 5.16). While the capacity to
demand ratio increased to 1.25 for one layer of FRP, the base shear demand still remain insufficient for two earthquake records (Nortia and Parkfield). Therefore, more FRP layers were applied on the columns and analyzed. Three and five layers of FRP provided a base shear increase of 83% and 117%, respectively, corresponding to capacity to average demand ratio was improved to 1.42 and 1.69, respectively.

Figure 5.13 Shear capacity of unretrofitted and retrofitted beams
Figure 5.14 Shear demand and capacity after strengthening

Figure 5.15 Shear capacity improvement after CFRP wrapping around beam
5.7 Summary

This chapter presented the behavior of a deep beam integrated in a structure under seismic loads and recommended a retrofitting technique for the shear deficit members. The following conclusions can be drawn from the study:

- The ductility of the structure was sufficient in terms of horizontal drift and inter-storey drift ratio considering the FEMA guideline for structural serviceability in earthquakes at different damage level. However, the ductility at the deep beam level was about half compared to the upper floors.

- Time history analysis using 24 earthquake records showed that the seismic demand of the structure was exceeded for 92% of the considered records. In addition, the base shear capacity of the structure was 25% below the average seismic demand.
The deep beam showed a significant shear deficiency for the entire earthquake records used in the time history analysis where on average the deficiency was 22%. The shear strength increased in the retrofitted deep beam by 49% for one CFRP layer and up to 82% for five CFRP layers.

The study also recommended retrofitting the base columns to increase the base shear capacity. The average capacity to demand ratio increased by 25% using one layer of CFRP in the columns. However, two of the earthquake records showed that the seismic demand of the total base shear exceeded the maximum capacity of the structure. Therefore, the column was analyzed with three and five layers of CFRP retrofitting. In both cases, the FRP system provided a sufficient capacity for all earthquake records and also allowed a substantial factor of safety against collapse (42% and 69% respectively).
Chapter 6: Conclusions and Future Recommendations

6.1 Summary and Conclusions

The shear behavior of RC deep beams was analyzed under static and dynamic loading. The adequacy of the design guidelines in the building codes (CSA A23.3 and ACI 318) was evaluated. The following conclusions can be drawn from the present research:

- A database of 381 test results on RC deep beams was composed from the literature to perform a statistical analysis and to compare the shear strength prediction equations proposed by different researchers in addition to the CSA A23.3 and ACI 318 code equations. Most of the previous shear equations were either over predicting or under predicting the experimental shear capacity. This is because, they were derived from a limited number of test results. However, although ACI and CSA strut and tie model (STM) for deep beam design was found conservative, the performance of the STM equations was found to be better in terms of weighted penalty \( p \) value and therefore was safe to use in the design compared to other analytical models.

- A parametric study was performed using the same databases to identify the most important parameters that affect the shear strength of RC deep beams. The information was used to develop simplified shear equations for deep beam with and without web reinforcements.

- A factorial analysis was performed to identify the percent contribution of each parameter to the shear capacity of deep beam. It was observed that the shear span to depth ratio \( \alpha/d \) ratio) is the most significant factor on shear strength. Moreover, the factorial analysis showed that the parameters were highly interacting among each other and the total
percent contribution of the interaction terms was one-fourth of the deep beam shear capacity.

Two simplified shear equations were proposed for deep beam with and without web reinforcement. The equations were calibrated using reliability analysis. The resistance factors were recommended from a target reliability index of 3.5 which offered more safety in the design.

The shear behavior of a single standalone deep beam was analyzed under the reversed cyclic loading. The deep beam was modeled using the finite element method and was analyzed with the variation of longitudinal reinforcement ratio, and horizontal and vertical web reinforcement ratios. The load deflection curves were plotted to understand the failure pattern. It was observed that by increasing the vertical shear reinforcement, the shear capacity, energy dissipation capacity, ductility and deformation resorting capacity was increased by 66%, 500%, 71% and 140%, respectively. On the other hand, by increasing the horizontal shear reinforcement, the shear capacity, energy dissipation capacity, ductility and deformation resorting capacity was increased by 89%, 216%, 37% and 27%, respectively. Therefore, it can be concluded that the effect of vertical shear reinforcement was more pronounced than the horizontal shear reinforcement.

The performance of deep beams in a RC building was analyzed by performing a non-linear pushover and non-linear time history analysis. The analysis captured the behavior of the deep beam in a real earthquake where the deep beam was also interacting with other structural elements. It was observed that the deep beam designed following CSA A23.3 STM model was found to have a significant shear deficiency for different
earthquake records. Therefore, a CFRP retrofitting technique was recommended to improve the capacity of the deep beams.

6.2 Recommendations for Future Research

The proposed design shear equations were based on previous experimental results where most of the tests were performed on small scaled deep beam. Therefore, the proposed shear equations need to be verified with full scaled test results. Moreover, the variation of the shear behavior between a small scaled and full scaled deep beam could be investigated as a future research. The parametric study showed that some parameters have no clear trend on the shear strength of RC deep beam due to lack of experimental data. Therefore, either more experiment could be performed or the gap among the data points could be filled using the FEM.

The analysis and results found from the dynamic analysis require the experimental validation. The reversed cyclic loading can be applied on deep beam in Laboratory to find the exact crack formation and failure pattern till the ultimate load. This will give more understanding in the exact shear behavior in single standalone deep beams under seismic loads.
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## Appendices

### Appendix A: Shear Equations

<table>
<thead>
<tr>
<th>Model</th>
<th>Shear Equation</th>
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| ACI 318-08  | $F_{ns} = f_{ce} A_{cs}$  
$F_{nt} = A_{ts} f_y$  
$f_{ce} = 0.85 \beta_s f_c$  
$f_{cs} = 0.85 \beta_n f_c$  
$V_u = \min \left\{ \frac{F_{ns} \sin \theta}{F_{nt}} \right\}$                                      |
| CSA 23.3-04 | $F_{ns} = \phi f_{cu} A_{ts}$  
$f_{cu} = \frac{f'_c}{0.8 + 170 \varepsilon_i} \leq 0.85 f'_c$  
$\varepsilon_i = \varepsilon_s + (\varepsilon_s + 0.002) \cot^2 \theta_s$  
$F_{nt} = \phi f_y A_{st}$  
$V_u = F_{ns} \sin \theta$                                        |
| Ramakrishnan & Ananthanarayana (1968) | $V_n = \beta K f_{bi} bh$  
$\beta = 2$ for one point and two point load  
$K = \pi / 2$ for cylinder split test  
$f_i = 7.2 \sqrt{f_c}$; $f'_c$ in psi |
| Kong et al. (1972) | $V_n = C_1 \left( 1 - 0.35 \frac{X}{h} \right) f_{sp} bh + C_2 \sum A_{yi} \frac{Y}{h} \sin^2 \alpha$  
$C_1 = 1.4$ and $C_2 = 130 \text{MPa}$ for plain bar and $300 \text{MPa}$ for deformed bar |
Selvam (1976)  
\[ P_u = \psi_s f'_{sp} bh \]
\[ \psi_s = \left[ \xi m^2 - (1 - m)^2 + \alpha^2 + 2\xi m(1 - m) \right] \]
\[ m = \frac{m_u \exp(1 - \beta)}{\pi \alpha \beta}; \quad m_u = \frac{1}{1 + \xi}; \quad \xi = \frac{0.85 f_c}{f_{sp}}. \]

Mau and Hsu (1989)  
\[ \frac{V}{f_c} = \frac{1}{2} \left[ K (\omega_h + C) + \sqrt{K^2 (\omega_h + C)^2 + 4(\omega_h + C)(\omega_y + C)} \right] \]
\[ K = \frac{2d_v}{h}; \quad 0 < a / h \leq 0.5 \]
\[ = \frac{d_v}{h} \left[ h \left( \frac{4}{a} - \frac{2a}{3h} \right) \right]; \quad 0.5 < a / h \leq 2 \]
\[ = 0; \quad a / h > 2 \]
\[ \omega_h = \frac{\rho_{fy}}{f_c}; \quad \omega_y = \frac{\rho_{fy}}{f_c}; \quad C = \frac{\sigma_l}{f_c} \]
\[ V_n = vbd_f. \]

Matamoros and Wong (2003)  
\[ V = 0.3 f_{bw} \frac{a}{d} + \frac{\rho_{wv} \beta_f}{f_c} + (1 - a / d) \rho_{wh} \frac{bd}{f_{cy}} \]
\[ w_{st} = l_s \sin \theta + h_a \cos \theta \]

Arabzadeh et al. (2009)  
\[ V_u = \left( \frac{f_c}{a} \right)^{0.7} A_{st} \sin \theta + 0.09 (\rho_y)^{0.35} A_{wp} \cos \theta \]
\[ A_{wp} = A_v \cos \theta + A_h \sin \theta \]

Londhe (2010)  
\[ V = V_c + V_{ms} + V_{wh} + V_{wv} \]
\[ V_c = \alpha_1 \left[ (1 - 0.3a / d) \sqrt{0.8 f_{ck} bd} \right]; \quad V_{ms} = \alpha_2 \left( \frac{100 A_{d} \sin^2 \theta_i}{D} \right) \]
\[ V_{wh} = \alpha_2 \left[ \sum_{i=1}^n \frac{100 A_{iwh} y_i \sin^2 \theta_i}{D} \right]; \quad V_{wv} = \alpha_2 \left[ \sum_{i=1}^n \frac{100 Y_{iwh} y_i \sin^2 \theta_i}{D} \right] \]
\[ \alpha_1 = \left( \frac{0.375 C_1}{\gamma_{mc}} \right); \quad \alpha_2 = \left( \frac{0.75 C_2}{100 \gamma_{mc}} \right) \]