PROBLEM POSING AS STORYLINE: COLLECTIVE AUTHORING OF MATHEMATICS BY SMALL GROUPS OF MIDDLE SCHOOL STUDENTS

by

ALAYNE CHERYL ARMSTRONG

B.A.H., Queen's University, 1988
M.A., University of Manitoba, 1995
B.Ed., University of British Columbia, 1997
M.A., University of British Columbia, 2006

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES

(Curriculum and Pedagogy)

THE UNIVERSITY OF BRITISH COLUMBIA
(Vancouver)

January 2013

Abstract

This dissertation investigates the problem posing patterns that emerge as small groups of students work collectively on a mathematics task, and describes the characteristics of problem posing that result.

This case study is a naturalistic inquiry about four small groups of Grade 8 students in the Lower Mainland of British Columbia who are working in a classroom setting, with the researcher acting as participant/observer and videographer.

The concept of author/ity is used to highlight human agency in mathematics. Small groups, as learning systems, are being considered to be "authors" of their discourse, and the improvisational nature of authoring is discussed. A parallel is drawn between the storyline of a literary work and the storyline that emerges as a group poses problems in order to work its way through a mathematical task.

The metaphor of a tapestry is used as a way of describing how the threads of group discourse weave together. To address the challenge of documenting collective behavior at the group level, a method of data analysis is introduced that "blurs" the data in order to capture patterns that emerge over time – transcripts are color-coded and then shrunk to create tapestries that provide visual evidence of collective problem posing patterns.

This dissertation finds that collective problem posing is an emergent process. Each group poses its own set of problems, and the number of problems posed and their frequency also vary, resulting in individual tapestries for each group. The tapestry patterns are then used to compare characteristics of the groups' discussions.

Problem posing appears to be an activity that these groups are able to do without receiving formal instruction or direction. The reposing of problems helps to structure each group's discussion, with the role that each problem plays in the conversation evolving as it reemerges. The concept of groups working as bricoleurs is also explored, with bricolage in mathematics being characterized as a creative and generative process.

The dissertation concludes with a discussion of expertise in school mathematics and what implications an "aesthetic of imperfection" might have in the mathematics classroom.

Preface

Some sections of Chapters 3, 8, and 9 are included in “Playing in liminal spaces: improvisation as a metaphor for prepared spontaneity in school mathematics,” an article I have been coauthoring with Dr. Susan Gerofsky, which is currently in review. My authorship of the article represents 75% of its completion.

This study was authorized by the Behavioral Research Ethics Board of the University of British Columbia, certificate # H10-02716

Table of Contents

List of Figures

Acknowledgements

This study was funded by the Social Sciences and Humanities Research Council of Canada (SSHRC) grant #452-2010-2389.

Thank you to School District #43 (Coquitlam) which funded a one year Educational Leave that allowed me time to conduct my research. My thanks also to the Coquitlam Teachers' Association, the Department of Curriculum and Pedagogy, and the Faculty of Graduate Studies which all provided funding that helped me attend conferences to present work related to this dissertation.

Thank you to my supervisor, Dr. Ann Anderson, and my committee members Dr. Susan Gerofsky and Dr. Anthony Clarke (and, briefly, Dr. Gaalen Erickson) for their wise guidance and encouragement and to Dr. Nathalie Sinclair, Dr. David Pimm, and Dr. Cynthia Nicol for their helpful input. A big thank-you to Glenn Davies for his support throughout this whole process, and also to my parents and family, friends, academic and teaching colleagues, and my students.

Chapter One: Prologue

> *Mathematical work does not proceed along the narrow logical path of truth to truth to truth, but bravely or gropingly follows deviations through the surrounding marshland of propositions which are neither simply and wholly true nor simply and wholly false.(Papert, 1980, p. 195)*

Mathematics weighs heavily on students in our schools. It is a subject with a reputation as being difficult and abstract, a solitary task meant only for those who have a natural capacity for it (Lafortune, Daniel, Pallascio, & Sykes, 1996; Sinclair, 2008). It is perceived by many students as a series of rules imposed by an outside source, be it textbook or teacher, with little recognition that student thinking can itself generate mathematics. If "[t]he only things mathematicians can be supposed to do with any certainty are scribble and think" (Rotman, 2006, p. 105), then in many classrooms most mathematics students are confined to the role of scribblers – writing, copying and calculating, rather than describing, explaining and proving.

Yet mathematics itself is a living and creative act (Boaler, 2008), and mathematicians themselves often collaborate in their work (Burton, 2004). So what is holding school mathematics back? Are we so conditioned to expect the act of mathematizing in school to proceed in a certain abstract formalized way that we are neglecting other ways in which mathematical learning may emerge? How else might we frame what it is that students are doing in mathematics?

Povey et al offer a reframing when they decenter the term *authority* – the traditional view of mathematical knowledge as external, fixed and absolute – to play with the concept of *author/ity*, splitting up the word to foreground the idea of there being an author (or authors) behind the scenes who negotiates this knowledge. Based on this shift in view, they write,

> teachers and learners… work implicitly (and, perhaps, explicitly) with an understanding that they are members of a knowledge-making community.... As such, meaning is understood as negotiated. External

> sources are consulted and respected, but they are also evaluated critically by the knowledge makers, those making meaning of mathematics in the classroom with whom *author/ity* rests. (Povey, Burton, Angier, & Boylan, 1999, p. 234)

In this dissertation, I extend these ideas beyond the individual. What if groups of students were considered to be authoring their mathematics, in the more creative sense, developing a storyline[1] as they go?

Education researchers have linked mathematics with the idea of student authoring in various ways, suggesting that written communication is an effective way for students to develop their mathematical thinking. Some have looked at pedagogical strategies such as high school level writing workshops (Fernsten, 2007; Wadlington & Hicks, 1994), while others have considered the use of pen pal letters (Crespo, 2003b), poetry (Altieri, 2005) and journal writing for students in elementary (Helton, 1995), and middle school (Albert & Antos, 2000; Baxter, Woodward, Olson, & Robyns, 2002). More recently, some researchers have been interested in how students display narrative aspects in their mathematical thinking when interacting with computer microworlds (Healy & Sinclair, 2007; Sinclair, Healy, & Sales, 2009).

In this dissertation, I work from a different perspective of student authoring. In reconsidering data from my previous work on the characteristics of group flow (Armstrong, 2008), I have noticed that those groups of students who appear to be most fully engaged in collaborating on solving mathematics tasks are the ones who are reformulating the task into their own problems. This seems to not only break the task down, so they are working with smaller, more manageable chunks, but to result in the development of a kind of a problem solving "storyline," one that differs from the storylines that are developed by other groups who are working on the same task. Perhaps these groups might be considered to be authoring their

[1] The concepts of story and storyline will be discussed in Chapter 4.

mathematics, and the emerging problem posing is propelling them along, both engaging their interest and motivating them to continue working towards solving the original task.

As such, I consider the process of authoring to be an improvisational one, and I explore aspects of improvisation that help to highlight characteristics of collective problem posing, such as the nature of expertise and the process of bricolage. In particular, I focus on the ideas that emerge in the collective discourse of small groups of students, in particular, the problems the students pose to one another as they work, and how the group creates a storyline as the discussion proceeds. In doing so, I investigate the following research questions:

- What problem posing patterns emerge as small groups of students work collectively on a mathematics task?
- What are the characteristics of problem posing as a collective process?

I believe that framing this situation as authoring helps to highlight doing school mathematics as a creative process, one that may help us to see students as gaining author/ity over school mathematics, making it something they do rather than something that is done to them.

Outline of dissertation

This investigation is a naturalistic case study of four small groups of Grade 8 students in a middle school in the Lower Mainland of British Columbia as they work in their mathematics classroom on an assigned task.

Chapter 2 begins with the discussion of what an author is, and how the concept of author/ity opens up the possibility of authorship to students. This concept is extended further in considering the group as author. The group as a learning system and a learning agent is discussed, as are the challenges of documenting this agent's thinking.

In Chapter 3, improvisation is proposed as a framework for considering the process of collective authoring. Aspects of improvisational theory are discussed, such as the level of attunement between group members, the relationship between structure and spontaneity, and the nature of bricolage. Examples from mathematics education are offered.

Chapter 4 opens with a discussion of the difference between story and storyline. The concept of problem posing is used to draw a parallel between the storyline of a play and the storyline that I argue emerges as a group of students pose problems while working on a mathematics task. The literature about mathematics problem posing is discussed.

Chapter 5 documents how the study was set up. Issues raised in documenting the collective authoring process are discussed. This chapter also introduces a method of data analysis that involves "blurring" the data in order to introduce the element of time into transcript data, and the metaphor of a tapestry is proposed.

Chapter 6 offers the findings of the study by first presenting a chart (and color key) of the problems posed by the groups and then the tapestries that resulted from their discussions. The stories of each of the four groups are described, and which is followed by charts of the problems posed by each group and the resulting storylines.

Chapter 7 examines the problem posing patterns that emerge through the use of the tapestries, and discusses the characteristics of the problem posing process, including how the role of a posed problem evolves throughout the session. I also consider how the groups work as bricoleurs as they consider the mathematics task.

Chapter 8 summarizes the findings of the case study, notes the problems that these findings themselves pose, and suggests avenues for future studies.

Finally, Chapter 9 discusses the implications of student author/ity for the role of expertise in school mathematics and argues for the development of an "aesthetic of imperfection."

Chapter Two: Who is the author?

What is an author?

"What is an author?" may at first seem to be a strange question to consider in regards to school mathematics, but it is, in fact, an important one. As Povey et al write,

> human meaning-making has been expunged from the accounts of mathematics that appear in standard texts; the contents are then portrayed in classrooms as authorless, as independent of time and place and as that which learners can only come to know by reference to external authority.... because the author(s) of the narrative remain hidden, mathematics becomes a cultural form suffused with mystery and power, a discourse that mystifies the basis for cultural domination. (1999, p. 235)

To work against this sense of mystery, and to consider the possibility of mathematics students themselves as authors, I begin by considering what an author is.

The word *author* comes from the Latin *auctor/auctoris*, meaning "one who increases, creates, fathers, founds or writes," and from *augere/auctum* meaning "to increase," and can be defined as "one who has created a document" or "the creator of something" (McArthur, 1992a, p. 98). The idea of "author" as creator has troubled some – can any author truly be considered the sole originator of a text? In his essay "The Death of the Author" (1968) Barthes argues no, noting that in early cultures narratives were delivered by shaman-like figures who acted more as conduit than creator, and that it was only the rise of positivism and capitalism that attached importance to the idea of an author being owner of a particular narrative. Ultimately, Barthes argues, there is no "Author." Even the person who is physically doing the writing is just a delivery mechanism for the language system that surrounds him: once the idea of "Author" is dead, "the writer no longer contains within himself passions, humors, sentiments, impressions, but that enormous dictionary, from which he derives a writing which can know no end or halt;

life can only imitate the book, and the book itself is only a tissue of signs, a lost, infinitely remote imitation" (1968, p. 5).

Others are uncomfortable with authors being seen as a means for "increasing" ideas. According to Foucault, to declare someone as an "author" is actually a way to delineate a particular set of ideas (such as identifying a piece of writing as belonging to the works of Freud) so that they can be more easily managed. For example, we may try to restrict the circulation of certain ideas by identifying who their "author" is and then punishing that person. For Foucault, the concept of authorship works as a "system of constraint" (1984, p. 119).

The concept of author, then, has been under fire. As mentioned in the introductory chapter, I am working from the ideas of Povey et al who also question the motion of author, although not as radically as Barthes or Foucault do. In their shifting of the word authority to author/ity, Povey et al unmask the authoritative, and seemingly authorless, mathematics text as the recorded interpretations of people over time[2]. Brown (1996) suggests that when the focus of mathematics educators turns more to mathematics activities rather than to the mathematics itself, interpretation plays far greater a role – for instance, the students' understanding of a mathematical situation, and how their interpretation changes as they notice new aspects of the situation and make new connections between them. This emphasis on interpretation, Brown argues, is similar to Gadamerian hermeneutics in that the meaning of the mathematics arises from the activity and the language used to frame it. And in that sense, it opens up the possibility of authorship to any of us who choose to engage in mathematics and communicate our interpretations to others. Mathematician Jonathan Borwein writes, "We respect *authority*, but value *authorship* deeply however much the two values are in conflict. For example, the more I recast someone else's ideas in my own words, the more I enhance my authorship while undermining the original authority of the notions" (2006, p. 3).

[2] In Chapter 3, I discuss how mathematics evolves over time through a process of bricolage.

If the role of author may be opened up to all individuals, I argue that it should be opened up to groups as well. For instance, Nicolas Bourbaki, who authored a number of mathematics texts, is actually the pseudonym used by a group of mathematicians in the early twentieth century (Mashaal, 2006). The kind of collective effort I refer to in this study, however, does not involve the passing around of a manuscript between mathematicians who are employed at different institutions, or a single person recording the ideas discussed by a group. Starting from the position that a group is a system, I suggest that the discourse a group produces cannot be parsed into individual contributions of its members, and therefore the idea of coauthors, although not technically incorrect, is inadequate. To do this, I first very briefly discuss how groups have been portrayed in educational literature, and then how a group can be considered to be a learner itself, and what to make of the discourse it generates.

What is a group?

Although it may seem that the identification of a group would be self-evident – the presence of two or more people in a defined location – I suggest that the characteristic of how "groupy" (Arrow, McGrath, & Berdahl, 2000, p. 34), or cohesive, a group is exists on a continuum of behavior. On one end of the spectrum might be a collection of people who happen to be in the same location. For instance, a line-up of strangers at a bus stop, seemingly ignoring each other, hardly seems to fit the definition of a group, but they have the potential to act as one. Should a disruption occur, such as the bus running over the curb and striking one of them, this event may prompt the rest of the strangers into interacting in order to help the injured party. At the other end of the spectrum are those groups whose behavior is so coordinated and features such a high level of interaction between the members that they seem to be behaving as a single unit: for instance, an improvising jazz band that has found its groove (Martin, Towers, & Pirie, 2006).

One way of distinguishing between group behaviors is in terms of how cohesively the members are behaving. When a group is acting *cooperatively*, everyone is working together to complete a task, but members of the group are focused on different parts of the task. When a group is working *collaboratively*, everyone in the group is working on the same task at the same time.[3] Finally, a group that is working *collectively* has such a high degree of coordinated interaction that it appears to be behaving as a single unit (Martin et al., 2006).

It is important to note, however, that for a group once a collective does not mean always a collective. Group behavior is more fluid than that. The level of cohesive activity in any group necessarily waxes and wanes according to the level of interest and other factors, and a peak state of cohesive effort is difficult to sustain for long. Consider a newly formed group. At first, if members do not know each other, interaction may be hesitant and limited to the more outgoing members. As the group members interact and find common ground, whether it is based on similar interests or a shared perspective on the task, the quantity and quality of interactions will grow. Assuming the group members are getting along, and are able to deal with any minor conflicts that arise, they might reach a state where they could be said to be collaborating. If they are really getting on well, communicating their ideas clearly, and building on each other's ideas to the point where there seems to be one central idea belonging to the group that is emerging, then one could argue that they are now a collective, perhaps even experiencing peak performance, or group flow (Sawyer, 2003). This group might continue to act as a collective, or as collaborators, or, once the task is completed, simply as a *collection*[4] of individuals who are familiar with each other and happen to be sitting around the same table, until another task is

[3] "Cooperative work is accomplished by the division of labor among participants, as an activity where each person is responsible for a portion of the problem solving…. Collaboration…[is] the mutual engagement of participants in a coordinated effort to solve the problem together" (Roschelle & Teasley, 1994, p. 70).
[4] *The Canadian Oxford Dictionary* (1998) defines *collection* as "Any group of things systematically assembled." There is little to no interaction between the things that have been assembled.

assigned, and the interaction begins again. Thus, a group's behavior exists on a continuum, and it is unrealistic to expect it to sustain a specific level of cohesion for long.

In defining groups, one must also consider that groups are not islands; they exist within groups, and in turn have groups within them. Imagine a group of four female students in a middle school mathematics classroom. The group members operate as individuals, part of the group, part of other small groups, and as a part of the larger class as a whole, all simultaneously (Davis & Sumara, 2006). Consider the relationships between these girls prior to the formation of this group[5]. Perhaps they only first met at the beginning of this school year, or perhaps they have long been friends. Perhaps they may have in fact attended the same "feeder" elementary school prior to attending this middle school (or not), their families may have lived in the same neighborhood, and their parents may have socialized together. The point is these girls are members of a number of systems, both historically and at the present time, a few of which I, as researcher, am aware but many more of which I am not.

There is more to consider. As any teacher knows, when a student enters a classroom she brings whole worlds with her. There are inner systems, the bodily systems, embedded within each girl. Has she had enough to eat? Has she had enough sleep? Is her immune system fighting off an infection? Is she emotionally upset about anything? Consider the other systems she herself is embedded within besides her class at school – her family, her sports team(s), her group of friends, her neighborhood, etc. All of these systems affect her make-up, who she is, and serve to inform her ideas and her actions within any group, and the group overall.

While these four girls are embedded in the small group, consider how the group itself is part of other systems – the class, the team, the grade, the school, the neighborhood, the school district, etc. Again, events that occur in one system will ultimately impact others that are

[5] This kind of group is defined by Arrow as being a "task force"(Arrow *et al*., 2000) assembled to perform a specific task and then disbanded afterwards.

connected to it. For example, if the school district decides that it will allocate more resources to literacy, individual students will eventually find that they are affected by that decision, in one way or another.

Finally, consider how that original small group itself is defined – by the teacher who controls who sits together, by the researcher who chooses to focus the camera on these particular girls instead of, perhaps, drawing the camera back to consider the behavior of other groups who happen to be sitting at that side of the classroom, or drawing the camera back even further and considering the behavior of all the classroom groups at once.

Working from systems theory, Arrow, McGrath and Berdahl summarize the embedded nature of any group nicely:

> Groups are open and complex systems that interact with the smaller systems (i.e. the members) embedded within them and the larger systems (e.g. organizations) within which they are embedded. Groups have fuzzy boundaries that both distinguish them from and connect them to their members and their embedding contexts.
> - Systems are open, complex, adaptive and dynamic
> - Systems entail recurrent patterns of interaction among elements at multiple levels
> - Systems have permeable boundaries that regulate the exchange of resources among levels. (2000, p. 34)

In short, part of what defines a group from a research perspective depends on how the observer sets the boundaries of the definition. While on the surface this dissertation will consider groups in a more traditional sense – as a certain number of people who are working together on a designated task for a designated amount of time – it also recognizes that these groups are embedded in larger systems and that the patterns of interaction that take place within the group are themselves interwoven in a fabric of discussion that stretches both backwards and forwards in time.

Research on effective group work

There has been much interest among researchers about how individuals might work more effectively together in groups[6]. A recent survey of naturalistic studies of classroom groups points out some of the findings about group work:

> children work more effectively in smaller than larger groups; the co-operative and collaborative approaches to group work are, generally more effective than individualistic and competitive approaches; there are modest academic gains; and pro-social and pro-school attitudes improve significantly in co-operative/collaborative groups. (Baines, Blatchford, & Kutnick, 2007, p. 57)

It has been argued that just because students have been grouped together for mathematics tasks, it does not mean they are working effectively together (Noddings, 1989) nor that all groups in a classroom will achieve the same results even when they are at the same level of ability (Barron, 2003). Yet, other researchers have found that students who have been taught effective ways to work and interact in groups (for example, E. G. Cohen, 1994; Mercer & Littleton, 2007; Webb *et al*., 2009) benefit from an improved quality of group experience, and different programs of group discussion such as Exploratory Talk (Mercer & Littleton, 2007) and Collaborative Reasoning (Reznitskaya et al., 2009) have been developed in order to facilitate this.

A survey by Webb and Palincsar points out that "the sense emerging from the literature is that the essence of collaboration is convergence – the construction of shared meanings for conversations, concepts and experiences" (1996, p. 848). For example, Cohen, in her review of studies of productive group work in elementary and secondary schools, notes that in group work "effective interaction should be more of a mutual exchange process, in which ideas, hypotheses, strategies and speculation are shared." (1994, p. 4). Other researchers see group work as a situation that is less about the sharing of individual ideas and more about the mutual development of shared ideas, for example the development of classroom sociomathematical

[6] Johnson and Johnson (2009) note that more than 1,200 studies about social interdependence have taken place in the past eleven decades.

norms (Yackel & Cobb, 1996). The concept of *taken-as-shared* helps to bridge the apparent gap between the individual and the group. Taken-as-shared meaning is developed between individuals through their social interactions, and evolves as students make adaptations "which [eliminate] perceived discrepancies between their own and others' mathematical activity while pursuing their goals" (Cobb, Yackel, & Wood, 1992, p. 118). Voigt writes that the concept of taken-as-shared goes beyond suggesting that individuals can come to agree that they have ascribed the same meaning to an idea: "from the observer's point of view, the meaning of taken-as-shared is not a partial match of the individual's constructions, nor is it a cognitive element. Instead, it exists in the process of interaction"(1996, p. 34), and not beyond it. He calls the relationship between mathematical meanings that are shared a *theme* and explains it using a simile: "the theme can be described as a river that finds[7] its own bed" (1995, p. 175). For Voigt, the theme lays both inside and outside the individual and the group – it is present neither in one, nor the other, but in the moments in which the individuals are negotiating and that the group itself is acting as one.

The overlooked learner

In the interest of increasing the effectiveness of small groups in classroom settings, researchers have tended to treat groups as collections of learners, and to analyze their interactions on an aggregate level. But perhaps there is "one often-overlooked learner: the classroom collective" (Davis & Sumara, 2005, p. 315) itself as an agent. Little documentation exists about the group itself as a learner, how its understanding unfolds (Martin et al., 2006), and how it thinks.

Although in casual conversation, a teacher might refer to what a certain group thinks or, for example, describe the personality of the class in period three (Bowers & Nickerson, 2001), it

[7] Here the word "finds" suggests that the river develops its own bed as it flows along.

can be difficult for researchers to conceptualize the group as a unit of analysis, even a small group. For instance, if one follows an acquisitionist view (Sfard, 1991) where the mind is seen to function as a container and learning is a matter of pieces of knowledge being transmitted from the teacher's mind, acquired by the student, and then stored in her mind, then the idea of group learning makes no sense. Once the group breaks up, as it inevitably must, and the members go their different ways, where does the group's learning go? There is no permanent structure – for instance, a group brain – to contain it. Thus, studies of small groups have often tended to focus on how working within the group affects the learning of the individuals within the group rather than on the group itself (Stahl, 2006).

Can a group think?

Even when considering learning as adapting to new circumstances, rather than storing chunks of knowledge, the concept of group learning is "a difficult, counter-intuitive way of thinking for many people"(Stahl, 2006, p. 16) due to the strong association of cognition with an individual psychological process. There is a benefit for the researcher who studies groups, however: the group's discourse may be considered to represent its thinking.

> Thinking embedded in collaborative practical activity must to a significant degree take the form of talk, gesture, use of artifacts, or some other publicly accessible mediational instrumentality; otherwise mutual formation of ideas would be rendered impossible. (Engeström, 1994, p. 45)

Stahl argues for this link between group discourse and group thinking.

> [W]hen we say that a group thinks, we are not postulating the group as a unitary physical object but are focusing on the unity of the group's discourse: the fact that effective collaborative discourse is best understood at the level of the group interaction rather than by focusing on the contributions of individual members. The group discourse has a coherence, and the references of the words within it are densely, inextricably interwoven. (2006, p. 399)

This is challenging to study: the discourse cannot "be analyzed by solely considering a sequence of statements that are made''(Yackel, 2002, p. 424). One might even argue that the individual pathways of growth of understanding within the collaboration do not exist at all (Martin et al., 2006). If we narrow our focus to the speech act itself, we can set boundaries to identify individual utterances,[8] but we must realize that it is only through the definitions we set out that the utterance is isolated; it cannot exist on its own. In his profile of anthropologist Ray Birdwhistell, McDermott quotes Birdwhistell's view of context:

> I like to think of it as a rope. The fibers that make up the rope are discontinuous; when you twist them together, you don't make *them* continuous, you make the *thread* continuous… the thread has no fibers in it, but, if you break up the thread, you can find the fibers again. So that, even though it may look in a thread as though each of those particles is going through it, that isn't the case. That's essentially the descriptive model. (McDermott, 1980, p. 4)

Adding to this, McDermott writes,

> It is not just that the fibers are analytically unavailable when one is focusing on the rope, it is that half the fibers do not exist except in contrast to other fibers and other parts of the background. All parts of the system define all the other parts of the system. Without the background, there are neither ropes nor fibers. (McDermott, 1996, p. 275)

Without the background or structure that boundaries provide, there are no utterances[9].

An utterance is linked to the past in that it is a response to another utterance, or utterances. This other utterance might be something that has just occurred in the group's ongoing conversation, or has taken place in the day or week or month or year – there are no time limits. Nor are there any limits to what it is that is recalled. It might be something spoken, a written text, a physical experience, a visual image, or it might be within an internal dialogue the subject has been having with herself. This adds to the researcher's challenge. Mercer and Littleton write,

---

[8] Bakhtin defined utterances as "not a conventional unit, but a real unit, clearly delimited by the change of speaking subjects"(Bakhtin, 1986, pp. 71-72). It is not clear what he would make of situations where the speech of different subjects overlaps.

[9] "[D]ialogic space is not a kind of 'thing' that one can identify with but more like a kind of relationship or a kind of 'difference', and not simply the easy kind of difference that one can see between two things but a 'constitutive difference' that helps bring the things apparently in relation into being in the first place" (Wegerif, 2010, p. 311).

> A profound problem for researchers wishing to understand how language is used to jointly construct knowledge (and, indeed, with understanding how conversational communication works at all) is inferring what knowledge resources speakers are using. Speakers may make explicit references to shared past experience or other types of common knowledge, but they often invoke such historical, temporal resources only implicitly. Observable features of interactions are likely to have unobservable determinants in the histories of the individuals, groups and institutional systems involved. (2007, p. 121)[10]

An utterance is a response to what has been, or what is currently, happening. But the utterance is also connected to the future, in that it is formed in anticipation of an impending utterance. Bakhtin writes,

> Every word is directed toward an answer and cannot escape the profound influence of the answering word that it anticipates…. Forming itself in an atmosphere of the already spoken, the word is at the same time determined by that which has not yet been said but which is needed and in fact anticipated by the answering word. Such is the situation in any living dialogue. (1981, p. 280)

Considering the dialogicality of a situation also means recognizing that an utterance does not belong to the one whom wrote/said/gestured it. Bakhtin writes, “The word in language is half someone else’s. It becomes one’s ‘own’ only when the speaker populates it with his own intentions” (1981, pp. 293-294). Thus, the “conversation” of a group “is crisscrossed by other places and temporalities, by absent third parties, who may express their voice through the participants’ discourse” (Grossen, 2009, p. 266) and also by the uptake and reuptake of individual threads of ideas. One might envision the utterance not as a link in a linear chain of threads but as a part of a fabric that comes from the past and stretches into the future. This fabric, in turn, might be considered to be a kind of tapestry, one with ripples spreading outwards from each little change that occurs as the multiple threads of linked discourses affect one another. I further develop the idea of the tapestry as a metaphor for analyzing the transcript data in Chapter 5.

[10] Barnes and Todd argue that it may even more challenging for those researchers who are observing a group that has a history of working together. “To take an extreme example, some long-standing groups generate catchphrases which for them carry implications which are closed to everyone else” (1995, p. 144).

Chapter 3: What is authoring?

Introduction

Povey et al (1999) define *authoring* as “the means through which a learner acquires facility in using community validated mathematical knowledge and skills” (p. 232), and in this chapter, I argue that authoring is an improvisational process. Improvisation, it might be said, is the way we live. It is an “act of creation that engages us all – the composition of our lives. Each of us has worked by improvisation, discovering the shape of our creation along the way, rather than pursuing a vision already defined” (Bateson, 2001, p. 1).

In this chapter, I work with, and build on, the definition of improvisation offered by mathematics education researchers Martin and Towers: “a collaborative practice of acting, interacting and reacting, of making and creating, in the moment, without script or prescription, and in response to the stimulus of one’s context and environment” (2009, p. 3). In doing so, I discuss aspects of improvisation theory – such as the relationship between structure and spontaneity, and the process of working with the resources that are on hand – which can inform our understanding of a how a group of mathematics students works collectively to author its own story.

Improvisation in education studies

In recent years, some educators have begun to explore (Sawyer, 2000b) the notion of improvisation as an important feature of teaching and learning mathematics. Researchers have described various kinds of improvisation performance (jazz, theatre, and other artistic disciplines) and drawn analogies to the practice of mathematics education. Much of this work has focused on teachers as improvisers. For example, King (2001) wrote about features of musical improvisation as a metaphor for features of mathematical pedagogy. Remillard (1999) contrasted the fixed nature of textbooks with teachers' in-action improvised decisions and responses to student needs in the classroom. Ribeiro, Monteiro, and Carrilo (2009) used improvisation as part of their model for teacher cognitive performance in classroom interactions. Neyland (2004a, 2004b, 2010) explored the political implications of the jazz metaphor itself for mathematics education.

Researchers have also employed the concept of improvisation as an analytic tool in regard to the development of new mathematic teachers. Borko and Livingston (1989) used improvisation as a framework for comparing the performances of secondary and elementary preservice teachers to those of their more experienced cooperating teachers. Sassi and Goldsmith (1995) employed a conceptual framework based on improvisation to discuss how elementary preservice teachers plan and prepare for teaching, structure their classroom activities, and respond in the moment, and to consider their level of improvisational understanding of the subject content itself. Maheux and Lajoie (2010) described how they used "informed improvisation" as a strategy in a series of role-plays with elementary preservice teachers as a way of preparing them for "acting in the moment" in the classroom. Towers and Martin (2009) analyzed how a mixed (early-childhood, elementary, and secondary) group of preservice teachers used improvisational actions to build a stronger idea of a mathematical concept as a teachable idea.

While students may be a part of the improvisation going on in the classrooms in these studies, the role of teacher is obviously the center of the research. In one of the few studies to focus on student behavior, Martin, Towers and Pirie (2006) used an improvisational framework to characterize the collective mathematical understanding of elementary students as creative and emergent.

A framework for collective behavior

When framing intragroup communications, improvisation "provides a means to characterize this process and offers a way of pointing to the conditions under which the collective might grow and develop" (Martin et al., 2006, p. 158). For instance, when a group is newly formed, how do the members come to learn to work together? A study by Bastien and Hostager (1988) looked at the level of improvisation performance by a group of musicians who had never worked with each other before. Although there was no sheet music, and no time to get to know each other or rehearse, as the session continued, the group began to improvise more complicated songs, until at the end they performed a piece that was on the upper limit of the improvisational continuum[11]. The authors suggested that a "centering strategy" had taken place:

> the jazz musicians began with a center that consisted of shared information regarding jazz music theory, song structures, behavior norms, and communicative codes. This center of shared information specified potential paths of musical invention for the musicians, who then selectively invented ideas along some of these paths. The group, in turn, then selectively adopted some of these ideas/paths and implemented them into organizational practice as shared bases for further musical invention…. The center of shared knowledge was extended outward by incorporating all the ideas/paths implemented in the previous songs and the group became capable of inventing and coordinating more complex musical variations. (Bastien & Hostager, 1988, p. 596)

As is evident in the example cited above, the more group members develop a mutual pool of ideas and techniques, the more they can attend to each other's actions and intentions. In jazz, in theatre, in any kind of collective work, the idea of attention, both aural and visual, is crucial. In one improvised activity, jazz musicians "trade fours" or "trade eights" – where "fours" or "eights" refer to the number of measures that the soloist plays before the next soloist begins. It is a fast-paced exchange and, rather than start with completely new ideas, each soloist continues playing in the way the previous soloist did, but tweaks it slightly in order to transform it. "These

[11] The concept of improvisation as a continuum of behavior will be discussed later in this chapter.

performances seem to work because the performers are closely attuned to each other; monitoring the other performer's actions at the same time that they continue their own performance, they are able to quickly hear or see what the other performers are doing, and then to respond by altering their own unfolding, ongoing activity" (Sawyer, 2003, p. 37). As a result, the behavior of the group becomes more convergent and coherent.

In improvisational theatre, the situation is much the same. Sawyer cites an interview with actor Pete Gardner about the benefits of rehearsals for establishing trust and habits of interactions between actors. Rehearsals result in "a sensitivity that comes with knowing each other," Gardner says. "[Y]ou wouldn't be as attuned [with a stranger] and you wouldn't be hearing the differences in their voices as they're changing and as they're saying things" (Sawyer, 2003, pp. 64-65).

To work collectively, group members need to be available to other's ideas when they arise, able to recognize the best chances to take, or the most interesting direction to pursue. And they need to be able to recognize how an offered idea builds on another idea, and then on how other ideas in turn, can build on that. Returning to the example of a jazz group, the players are not necessarily "equal" in terms of their instrumental prowess, their experience, their understanding of the music, yet their contributions are equally valued. There is an "etiquette" (Becker, 2000) involved which encourages everyone to participate and, perhaps more importantly, for all members to attend to one another very carefully so they can pick up on the "better idea" (Becker, 2000) and build on that. This blending of multiple contributions to form a single one brings to mind the image of a tapestry: group members weaving together threads of ideas to build on a pattern they recognize as emerging, with the possibility of "collectively chang[ing] their notion of what is good as the work progresses" (Becker, 2000, p. 175).

Improvisation as a spectrum of behavior

As mentioned previously, the coherence and level of interaction within a group varies as its work proceeds, and as that changes, so does the group's reliance on the known structure of what it is performing. Jazz groups provide a model of how this works.

> When musicians improvise, it is usually based on the repetition of the song structure. These guiding structures are nonnegotiable, impersonal limitations: musicians do not have to stop to create agreements along the way…. These moderate constraints serve as benchmarks that occur regularly and predictably throughout the tune, signaling the shifting context to everyone. (Barrett, 2002, p. 145)

How much the musicians rely on these structural benchmarks depends on the genre of music. In an interview with Berliner (1994, pp. 66-71), jazz musician Lee Konitz suggests that there is actually a spectrum of improvisational behaviors that can occur, depending on the proportion of structure to spontaneity. This idea of "full spectrum improvisation" (Weick, 2002) is very suitable for use in considering what takes place in a mathematics classroom, as it offers a variety of ways a combination of constraints and freedoms can be proportioned. To explore the concept of different levels of improvisation, I have chosen to focus on a chart (see Figure 2.1) developed by Michael H. Zack (2000, p. 232) a former jazz musician currently working as an organization scientist, which employs different metaphors to expand on the stages of improvisation proposed by Konitz (third column of chart). I will discuss these stages, first in terms of settings more traditionally associated with improvisation, such as jazz and theatre, then in terms of what each stage might mean in terms of a group's performance in a mathematics classroom.

Figure 2.1 Genres of Improvisation

Music Genre	Extent of Improvisation	Konitz's Stages	Organizing Metaphor	Communication Metaphor	Dynamics
Classical	Minimal to none	Interpretation	Functional hierarchy	Formal; structured; predefined; linear	Rigid
Traditional jazz/swing	Constrained with strong structure	Embellishment	Job shop/process platform	Predictable but flexible scripts; adjacency pairs	Flexible
Bebop (for example, music in the style of Charlie Parker or Dizzy Gillespie)	Extensive; harmony and basic tune structure can be modified	Variation	Network	Complex but structured conversation	Organic
Postbop (for example, music in the style of John Coltrane or Miles Davis)	Maximal; content and structure emerge	Improvisation	Functional anarchy	Emergent, spontaneous, mutually constructed conversation	Chaotic[12]

Interpretation[13] – In jazz, this occurs when a song is performed, and the musicians play all the notes, in the original order, but vary musical features such as attack, stress, tempo, and dynamics in order to create their own interpretation. In theatre, the performers follow the script, using all of the words, in the correct order, but use their voices, facial expressions and gestures to provide their own interpretation of what the written text means. In a mathematics classroom, an interpretation might occur when students are given a task that involves finding the perimeter of a rectangle. One student might add all of the individual sides together, while another might

[12] The term "chaotic" is used in the sense of chaos: "when a system's output varies so erratically it seems random"(N. Johnson, 2007, p. 40) .

[13] Some might question if "interpretation," found at the most rigid end of the improvisation spectrum proposed here, is even improvisation at all. Weick notes that improvisation is more than just paraphrasing – there is transformation (2002). I would argue that, in the case of interpretation, there is a very limited amount of transformation, akin to putting one's toe in the waters of improvisation, and the existence of this level is important to illustrate the presence of improvisation even in the most structured behaviors.

calculate 2 x length + 2 x width, and yet another might use distributive theory and offer 2 x (length + width). Ultimately they are using the same concept and (assuming the calculations are accurate) will arrive at the same correct answer. It is like following a path through the woods – you can walk, you can skip, you can run, you can do a series of somersaults, but the path defines the boundary conditions by which you navigate the forest.

Embellishment – In this case, when jazz musicians play the song it is recognizable, but they rephrase the melody by moving parts around to anticipate or delay them, or they may add ornamentation. In theatre, performers might change the wording in their script, adding to or subtracting from their speeches. The plot and the characters are still the same, but there is more flexibility as to what words are delivered to the audience. An embellishment in a mathematics class might occur when students are working with example spaces (Watson & Mason, 2005). For instance, they might be provided with a particular number (e.g. 3) and then asked to generate examples of the same concept on their own (e.g. the three bears of the fairy tale, a drawing of three flowers, three sides of a triangle, 0.5 x 6, 17 + 5 – 19, square root of 9, $2x + x$, etc.). In embellishment, the variety of paths available may be predictable yet there is more than one, and there is flexibility as to which one(s) the student chooses to take.

Variation – Here, the song is played, but the jazz musicians insert completely new passages into the song, although it is clear that these passages are related to the song. In theatre, performers might be presenting the story of Cinderella. There may be digressions in the scenes that are unrelated to the original story, or new minor characters might be introduced, but they are all related to the telling of the story and, in the end, the basic plot is followed. In the mathematics classroom, students might be presented with something like the Bill Nye task that the students in my study were assigned.

The Bill Nye Fan Club Party

> The Bill Nye Fan Club is having a year-end party, which features wearing lab coats and safety glasses, watching videos and singing loudly, and making things explode. As well, members of the club bring presents to give to the other members of the club. Every club member brings the same number of gifts to the party.
>
> If the presents are opened in 5 minute intervals, starting at 1:00 pm, the last gift will be opened starting at 5:35 pm. How many club members are there?

In this kind of task, students are given certain parameters for the task, and are aware that there is certain correct answer (or range of acceptable answers), but do not know what that answer is. Here, as each student works, he/she becomes "a *bricoleur* who assembles new things from old. In the process, an idea for an entirely new object may arise, leading to new mathematical questions, new exploration, and possibly revision of previous ideas" (Watson & Mason, 2005, p. 157)[14]. Thus, even in a fairly structured task there is the potential for creativity.

[Pure] Improvisation – At this end of the spectrum, there is the potential for complete transformation. The song, played through once, provides a starting point but as the musicians continue parts of the melody are changed, or completely replaced, to such an extent that the resulting performance no longer resembles the original song. In theatre, the actors may start with Cinderella, but then surround her with other characters (Superman, Jay Leno, etc.), strand them on a desert island, note that it is Thanksgiving, and take it from there, throwing in other ideas and characters, until the resulting story is completely removed from the original fairy tale. In mathematics class, pure improvisation might occur during an open-ended task that has no set answer, where students might be playing around with one problem, when something about it triggers a series of thoughts until the ideas they are exploring and inventing are nowhere near their starting point and they are now in new territory. What is occurring may at first seem to be

[14] I further discuss the concept of *bricolage* later in this chapter.

random, and it is only as the performers/students look back that they would be able to discern a pattern to their actions and ideas. "The improvisational process is one of laying a path down in walking" (Montuori, 2003, p. 246).

The concept of improvisation comprising a spectrum of moves suggests different properties than does "simple stand-alone"[15] improvisation, as it foregrounds the role played by memory, past experience, diverse contributions, and reliance on structure (Weick, 2002, p. 52). Weick argues that when considering improvisation as a spectrum "any one activity may contain all four gradations, as sometimes happens in jazz" (2002, p. 52) or, I would argue, when a group works on mathematics. In this dissertation, I will focus on how groups behave when presented with a fairly structured mathematics task, the kind that is often assigned in mathematics classrooms and the kind that one would not normally associate with creativity. The idea of improvisation as being comprised of varying levels of behavior offers a window for considering what room there is for play even within structure.[16]

Expertise, improvisation and school mathematics

Although spontaneity is the characteristic most commonly associated with improvisation, another important aspect of the improvising process is the expertise that arises from specialized experience: "the discipline and experience on which improvisers depend, and … the actual practices and processes that engage them. Improvisation depends, in fact, on thinkers having absorbed a broad base of… knowledge" (Weick, 2002, p. 51).

Let us consider the importance of expertise and conventions to the improvisational process, first in jazz and theatre, and then in school mathematics. In jazz, a performer must have

[15] The popular conception of improvisation is as a spontaneous form of creativity, one that is either turned "on" or "off."

[16] As well, considering improvisation to be a spectrum of behaviors is helpful for the researcher as it removes the pressure of trying to determine if a group is acting collectively enough for long enough to be identified as improvising.

some expertise before she is able to improvise. This comes in at least two forms – being able to physically perform the maneuvers required (i.e. use of the instrument itself must be mastered); and having knowledge of different melodies, chord progressions, and riffs. Frank Barrett, a former jazz musician and a current associate professor in business and public policy, writes, "Learning to play jazz is a matter of learning the theory and rules that govern musical progressions. Once integrated, these rules become tacit and amenable to complex variation and transformation, much like learning the rules of grammar and syntax as one learns to speak (Barrett, 2002, p. 137). In theatre improvisation, again, one must be able to use the instrument (voice, body posture, and gesturing), as well as elements of knowing the craft. Their "standards" include the basic improvisation games actors learn as part of their training and techniques such as how to indicate that one is ready to take over another performer's part during a skit (i.e. tap him on the shoulder). Conventions, both in jazz and improvised theatre, provide a kind of structure that's hidden to the neophyte audience member, and these are rules that "emerge informally in a community of practice, over the years with continual experimentation with what works and what doesn't work" (Sawyer, 2003, p. 52). In mathematics, the equivalent situation might take the form of knowing relationships between numbers, and how (and why) to use certain algorithms and formulas. In all cases, the performer must be able to use these conventions fluently to develop a high level of automaticity. If a mathematics student is constantly struggling to work out multiplying two numbers together, perhaps falling back on repeated addition instead, it may be difficult for him to move beyond that to make connections with related concepts such as ratio and proportion, and therefore limit his, and the group's, ability to improvise.

It is not enough, however, to be able to play a song, recite a script, or solve an equation quickly and proficiently. To work entirely from a script, striving for accuracy to the original, only shows an expertise in reproduction. Improvisation requires playing with the structure, using some creativity, and this can occur to varying degrees, depending on how free and loose[17] the performers are being.

Reaching a stage of peak experience is difficult, and improvisation provides a collapsed binary to describe this state through the idea of "prepared spontaneity." Jazz musicians need a certain expertise in order to improvise, and that takes planning – they might be more accurately called "practicers" (Berliner, 1994, p. 494) than practitioners. The goal is what Barrett calls "aimless aiming": to

> integrate ideas, freeing attention so that players can think strategically about their choice of notes and the overall direction of their solos. Hargreaves et al (Hargreaves, Cork, & Setton, 1991, p. 53) hypothesize that when improvisers employ automatic thinking to execute patterns they are free to plan the overall strategy of the piece (2002, p. 138).

The audience is often unaware of the planning and practice that lies behind this.

In school mathematics, this kind of automatic thinking is known as mathematical fluency, and traditionally mathematics classes have focused on students attaining this. But in the case of school mathematics, the practice is often the only thing of which the audience (students and general public) is aware. Hewitt argues that

> [p]ractice is clearly required for something new to become something which is known so well that it can be used when little or no conscious attention is given to it. However, there are many times when the carrying out of repetitive tasks through a series of questions in a traditional exercise does not succeed in helping that skill be retained beyond a relatively short period of time. (1996, p. 29)

Unlike in jazz, the practice in mathematics class, unfortunately, usually does not lead to anything further, other than perhaps more practice. However, as Hewitt notes, it is "[t]he desire to practice your art [that] subordinates the tools to carry out that art"(1996, p. 32). Perhaps in a situation where a group is exploring problems of their own choosing (or invention), through student desire

[17] In terms of the willingness to move away from a prescribed structure.

the possibility exists in pursuing these problems where they can subordinate the facts and the drills to the task at hand and focus on the ideas themselves.[18]

Improvisation as bricolage

Paul Berliner, a professor of ethnomusicology, writes: “[i]mprovisation involves reworking pre-composed material and designs in relation to unanticipated ideas conceived, shaped, and transformed under the special conditions of performance, thereby adding unique features to every creation” (1994, p. 241). This act of subordinating one’s resources to the task at hand has been characterized as a process of *bricolage*. Originating from the French verb *bricoler*, bricolage means to tinker, to fiddle with, or, in contemporary French, to “do it yourself.” The term bricolage first appears in Claude Lévi-Strauss’s anthropological work *The Savage Mind* (1966), in which he contrasts what he considers to be the analytic method of scientific theorizing of the Western world with the more concrete theorizing, or bricolage, employed in other more “primitive” cultures. He writes:

> The “bricoleur” is adept at performing a large number of diverse tasks; but, unlike the engineer, he does not subordinate each of them to the availability of raw materials and tools conceived and procured for the purpose of the project. His universe of instruments is closed and the rules of his game are always to make do with “whatever is at hand,” that is to say with a set of tools and materials which is always finite and is also heterogeneous because what it contains bears no relation to the current project, or indeed to any particular project, but is the contingent result of all the occasions there have been to renew or enrich the stock or to maintain it with the remains of previous constructions or destructions. (1966, pp. 17-18)

[18] Neyland (2010) proposes the term *effortless mastery*, a concept informed by improvisation, to function as an opposite to *maths anxiety*.

Materials and tools are not designed in advance for a specific project. Instead, the bricoleur does what he can with what is immediately available, even if at first it does not seem suitable for the task.

The idea of tinkering and fiddling suggests both a sense of spontaneity as well as a lack of professionalism, or a lack of seriousness, the feeling of just messing around with whatever is at hand. In education literature about teaching practices, some academics have picked up on this idea of being ill-prepared and working in the moment to criticize teachers' work. For instance, it is argued that the bricoleur teacher borrows and uses old teaching methods and resources without improving them, resulting in pedagogical inadequacies (Hatton, 1989). Undesirable, bricolage is depicted as a method teachers are forced to fall back on due to working conditions – without enough time or resources to engage in proper planning, the bricoleur teacher ends up gathering lesson and unit plans from here and there in order to get by (Scribner, 2005).

These interpretations of bricolage as merely wholesale "borrowing" and as a stop-gap measure are too simplistic. A teacher's reliance on another's lesson plan may involve using what is on hand, but in doing so he/she is "reinventing based on evolving intentions" (Reilly, 2009, p. 383) and the results may be far different from the purpose of the original source. Returning to what Lévi-Strauss writes, when the bricoleur is following the "rules of his game" he seeks to challenge himself. As well, the heterogeneous tools and materials with which he works are not blank but are marked by "all the occasions" they have been previously used. Original meaning is sedimented in the artifact being used, but as artifacts are recombined with other ones through bricolage, the resulting connections develop into new codes of meaning (Barker, 2004). One can see this, for instance, with fashion (Harajuku, a Japanese street style that mixes, for instance, traditional Japanese attire with modern Western wear), or with writing (punk writer Kathy Acker's *Great Expectations* which "plagiarizes" heavily from Dickens's novel of the same title but to a much different effect). Bakhtin writes of how one can take a word into a new context:

“attach it to new material, put it in a new situation in order to wrest new answers from it, new insights into its meaning, and even wrest from it new words of its *own* (since another’s discourse, if productive, gives birth to a new word from us in response)” (1981, p. 346). Bricolage, then, has the potential to be creative and subversive, disruptive and generative.

Turkle and Papert compare the style of the bricoleur with that of Lévi-Strauss’s engineer, whom they rename “the planner.” In a study of both grade school and college age computer programming students, Turkle and Papert define bricolage as “a style of organizing work that invites descriptions such as negotiational rather than planned in advance”(1990, p. 144). Working from Piaget’s theory of intellectual development, although rejecting the hierarchy of his proposed stages, the authors characterize the “planner,” as having a “formal” method as compared to the bricoleur who uses a “concrete” method (1990, p. 136). In comparing these learners, they write:

> The bricoleur resembles the painter who stands back between brushstrokes, looks at the canvas, and only after this contemplation, decides what to do next. For planners, mistakes are missteps; for bricoleurs they are the essence of a navigation by mid-course corrections. For planners, a program is an instrument for premeditated control; bricoleurs have goals, but set out to realize them in the spirit of a collaborative venture with the machine. For planners, getting a program to work is like “saying one’s piece”; for bricoleurs it is more like a conversation than a monologue. In cooking, this would be the style of those who do not follow recipes and instead make a series of decisions according to taste. While hierarchy and abstraction are valued by the structured programmers’ planner’s aesthetic, bricoleur programmers prefer negotiation and rearrangement of their materials. (1990, p. 136)

I quote this passage at length because it highlights some characteristics of improvisation that are also important to group work in mathematics, namely: the importance of mistakes as “mid-course corrections,” the contemplation that takes place during the work process, and the characterization of working being “more like a conversation than a monologue” involving a negotiation and rearrangement of ideas. This describes, in part, how a small group works. Interestingly, Turkle and Papert characterize the method of the planner, the “standard, canonical” style of computer programming, as being “mathematical” (1990). In doing so, they raise an important point: what is often privileged in school mathematics is the kind of formal thinking that underlies the planner’s process, namely identifying the problem as being a particular type and then using the appropriate formula, rather than exploring the concepts themselves[19]. But what if school mathematics itself were performed through bricolage?

Mathematics as bricolage

There is a long tradition in Western thought stretching back to Plato and his ideal forms, of mathematics as an eternal absolute, and that it is only through thinking and theorizing by an elite group (i.e. mathematicians) that its laws and axioms can be uncovered. The rest of us attempt to learn the rules and then apply them. Lakoff and Nuñez call this “standard folk theory of what mathematics is for our culture” (2000, p. 340) the “Romance of Mathematics,” and they argue that its influence has had a number of negative effects:

> It intimidates people, alienates them from math, maintains an elite and justifies it. It rewards incomprehensibility, and this inaccessibility perpetuates the romance. The alienation and inaccessibility contributes to the division in our society of people who can function in an increasingly technical economy and those that can’t – social and economic stratification of society. (2000, p. 341)

Mathematics itself is an evolving invention, a human process developed and refined by various societies throughout its history. Lakatos, the philosopher who first set this idea out clearly (Ernest, 1998), argued for what he called “quasi-empiricism” in his *Proofs and Refutations* (1976). Here mathematics is not portrayed as a static Platonic form that is discovered, but as a process, an evolving aspect of culture. The conversation between teacher and students as they discuss the Euler characteristic at first seems to be a Socratic dialogue where the teacher is apprenticing his students into traditional conventions of proper mathematical

[19] This may raise the issue of what if a problem does not call for an exploration of the concepts. I wonder if it matters what the problem calls for – the text (or writer) has no control of the reader’s form of interaction. For instance, the Bill Nye task does not explicitly ask its readers to explore the concept of factors, yet the four groups in this study do just that.

arguments. However, the alternative narrative provided by the footnotes undermines this interpretation, showing how “acceptable” mathematical strategies have varied during different eras of history, and pointing to an analogy between political ideologies and scientific theories (Lakatos, 1976, p. 49). Returning to the main storyline of his book, it becomes clear from the characters’ arguments that the process of refining a mathematical proof is never-ending. There is always something else to consider. Through his characters’ working, and reworking, of Euler’s axioms, Lakatos illustrates how the field of mathematics evolves through a process of bricolage.

This idea has been picked up in mathematics education literature as well. Although one might expect mathematicians to do their work through rational rules-based deduction, Bauersfeld uses terms such as tinkering and bricolage to characterize the messy “pragmatic adaptations” (1994, p. 144) that are actually involved in mathematical thinking. In describing the work of students in mathematics, Hershkowitz et al define the term *building-with* as the process of combining familiar objects as components to resolve a problem (Hershkowitz, Schwarz, & Dreyfus, 2001, p. 214), arguing that this kind of reconstruction is central to the process of abstraction. This process is not a linear, unidirectional one. Herbel-Eisenmann and Wagner characterize “mathematisation as the moves between the personal and impersonal, between context and abstraction. Mathematics lives in this tension” (2007, p. 13). The process of this movement, I suggest, embodies the elements of bricolage.

Approached using the principles of bricolage, mathematics is not a rule-bound, predictable school subject. Of having students working with example spaces (an activity described earlier in this chapter) Watson and Mason write, “In our experience, the bricolage of example construction can yield surprising results, because the knowledge and resources being brought to the task are different for different learners” (Watson & Mason, 2005, p. 80). With the possibility of the interplay of structure and spontaneity bringing “surprising results,” it is worth considering bricolage and its inherent improvisational quality as an approach to teaching and learning mathematics.

Chapter 4: What is the story?

Introduction

In this chapter I suggest that a story is a narrative driven by conflicts, or problems. I begin by distinguishing between the terms "story" and "storyline," arguing that a story provides the whole narrative, including plot, character and setting, while the storyline is the thread of conflicts within the story. After connecting the idea of conflict with the problems, I suggest that the act of students posing their own problems is parallel to the development of a storyline. Then I discuss the literature around mathematical problem posing. As many of these studies have largely focused on posed problems as products generated by individuals in test-like settings, I consider possibilities that have been neglected, such as problem posing on a collective level, and problem posing as a process rather than as an end product.

What is a story?

A *story* is a broad term in that it is not tied to a specific format, level of truth or purpose:

> A narrative, spoken or written, in prose or in verse, true or fictitious, related so as to inform, entertain, or instruct the listener or reader. A story has a structure that may be more or less formal, unfolds as a sequence of events and descriptions (even when devices like flashbacks alter the flow of time), and concerns one or more characters in one or more settings (McArthur, 1992b, p. 987)

That it can be spoken and that it is the result of a series of events make the story suitable for describing what a group creates in the course of its conversational work together. To tie this in more with mathematical discussion, it helps to reduce the story to a more basic form – its storyline. A *storyline* may be defined as:

> The sequence or flow of events in a story: the unelaborated routine of the *plot*, as opposed to the *theme* that the plot treats. A common story line is *Boy meets girl – boy loses girl – boy finds girl*, and a twist in such a story

> line might be *girl meets boy – girl loses boy – girl finds another boy*.
> (Nash, 1992, p. 987)

More simply put, a storyline may be regarded as the linear sequence of "what happens next."

To consider what it is that "happens next," it is helpful to note what German critic Gustave Freytag proposes in his work *Technique of the Drama* (1900). Freytag pictures the structure of a five-act play as a kind of pyramid, which includes the following parts: introduction, complication, climax, resolution, and catastrophe. The "complication," or what has come to be called "rising action," is of particular interest here as it is something that is spurred on by a series of events, or conflicts, with each one triggering the next, in much the same way that a storyline works. In this chapter, I argue that a storyline emerges when a group of students work on a mathematics task, and that the problems the group poses function as the conflicts, or events, that make up the storyline[20].

What drives a story?

What usually drives a story is a sense that something needs to be resolved. It may be a disagreement, a disconnect, an uncomfortable gap in understanding, or a conflict, but it is this something that provides an impetus to further action. William Shakespeare's play, *Romeo and Juliet*, provides a good example of how the central conflict of a story line can generate a number of other conflicts, which help to drive the story to its conclusion. A boy (Romeo Montague) and a girl (Juliet Capulet) meet at a feast hosted by the Capulets and fall in love; each belongs to opposite sides of a longtime feud between the Montagues and the Capulets and thus their friends and families will not approve of the match. How can they be together? They secretly marry and

[20] In order to highlight the way that problems operate in stories and make the connection with what occurs in group discourse during a mathematical task, I have chosen to work with a simplified conception of story and storyline. However, it should be noted that not all stories contain conflicts that are resolved (or are resolvable), and some more experimental stories may not contain conflict at all.

decide to wait for an opportune time to reveal the news to the world. However, this soon precipitates other conflicts, including the following:

- Mercutio versus Tybalt regarding Romeo's disguised and unauthorized presence at the Capulet feast;
- Romeo versus Tybalt regarding Tybalt's slaying of Mercutio;
- Romeo versus the kingdom in terms of a suitable punishment for his slaying of Tybalt;
- Juliet versus Lord Capulet regarding his wish to marry Juliet to Count Paris;
- Romeo's misinterpretation of a message about Juliet's "death;"
- Romeo versus Paris when they unexpectedly meet up at the Capulet family vault where the unconscious but seemingly dead Juliet lies;
- Romeo's decision to drink a poison in order to join Juliet in death;
- Juliet's decision to use Romeo's dagger to stab herself when she awakens and discovers the scene around her.

It is one thing to author a literary story, generating a storyline based on conflicts, but is it another to author the solution to a mathematics task? Just how original can you be in solving, for example, "the Locker Problem," a task that thousands and thousands of students have been assigned over the years and one that has a single, correct answer? Again, there is a parallel to this situation in one of the "classic" storylines that recur time and time again in literature. Shakespeare's central problem of "star-cross'd lovers" in *Romeo and Juliet* is echoed in our contemporary *West Side Story* and even in the more recent *High School Musical* – and Shakespeare's play itself is a descendant of Arthur Brookes' 1562 poem *The Tragicall Historye of Romeus and Juliet,* which is itself a translated interpretation of one of Bandello's Italian short stories *Novelle* (Drabble, 1985). Yet each author has made the story his/her own by varying the storyline. While the overarching conflict is the same (young couple from opposite sides of warring worlds comes to a tragic end), it is how the smaller conflicts, or problems, are settled that makes each text unique. If we focus our attention on the paths that students take in solving a mathematics task rather than on the final answer, then we may be better attuned to the unique storylines that develop.

Problem posing

In considering how painters work, Sawyer notes that some artists have an improvisational style that creativity researchers call *problem-finding* which involves "constantly searching for her or his visual problem while painting" (2000a, p. 153). In their discussion of improvisational theatre, Vera and Crossan further elaborate: "As part of the creative process, actors find a problem for themselves, spend some time solving the problem, and find a new problem during the solving of the last one" (2004, p. 737). The term "problem finding" suggests that the problem exists independently of the people who find it which belies what I believe to be the emergent nature of the process. Instead, I use the term *problem posing* which is grounded in mathematics education literature and has been defined as "the creation of questions in a mathematical context and… the reformulation, for solution, of ill structured existing problems" (Pirie, 2002, p. 929).

The wording of this definition raises an issue worth exploring: the use of the word *question* and what it means in relation to the word *problem*. The two are often used interchangeably in everyday discussion – they frequently show up in each other's definitions – but they are not the same thing. In short: all questions contain problems, but not all problems are phrased as questions.

In everyday life, problems have a bad reputation. *Roget's Superthesaurus* lists synonyms such as difficulty, complication, knot, trouble, dilemma, quandary, mess, pickle, predicament, can of worms, headache, pain in the neck, and hassle (McCutcheon, 1995, p. 403), all of them negative. According to *The Canadian Oxford Dictionary*, a problem is defined as "a doubtful or difficult matter requiring a solution" yet, in a mathematics context, a problem is "an inquiry starting from given conditions to investigate or demonstrate a fact, result or law" (1998, p. 1153).

Depending on your viewpoint then, a problem in itself is not a negative thing. Still there is an element of discomfort about it, a sense that something needs to be resolved or fixed. To recognize a problem is to be aware of a gap, a disparity, a limitation, an unknown, a dissonance, a variance, a conflict, or a disconnection**.**

On the other hand, a question refers to the grammatical structure of an utterance, namely the interrogative form. This kind of utterance points to the existence of problem but is not the problem itself. Other language structures, not to mention physical gestures and facial expressions, can also point to problems and this makes equating problems with questions troublesome for researchers. For this reason, in this dissertation I use a revised version of Pirie's definition of problem posing (2002): "the creation of *problems* in a mathematical context and… the reformulation, for solution, of ill structured existing problems."

Types of problem posing

Working from this revised definition of problem posing, one might argue that there are two kinds of problem posing, depending on the purpose of the problem being posed (Silver, 1994), and where it occurs in relation to the problem solving process. In the first half of the definition, a new problem is generated from a situation. The problem posing has the purpose of *problem formulation*, or "What new problems are suggested by this situation, problem or experience?" (Silver, 1994). Here the problem is generated from the situation itself, perhaps using techniques such as "what-if-not"[21]. Silver (1994) suggests that it also occurs after one has finished solving a problem, similar to Pólya's "look back and reflect" stage, to consider any new situations or problems that have arisen. Others disagree, arguing that "working from

[21] This is a technique that can be used to generate posed problems, by taking a problem and changing one of its parameters by asking "what if?" (S. I. Brown & Walter, 2005).

situations[22]… is not the same as working from problems. Part of the activity is, in fact, the formulation of [local] problems that may arise out of definitions and rules that are developed in the discussion of the situation" (Banwell, Saunders, & Tahta, 1972).

In the second half of the definition – the "How can I (re)formulate this problem so that it can be solved?" type (Silver, 1994) – a related problem is generated in response to the original problem, as a way of making that original problem more accessible (Pólya, 1957). This could take the form of modifications (Whitin, 2004), perhaps rewording the original problem, or setting it in a more accessible context, so that it can be directly solved or putting boundary conditions on it. It could also take the form of purposely extending the original problem (Whitin, 2004) to see what else there is to investigate in it. Problem reformulation might also occur through the creation of new problems that are produced as one pursues the original, as a way of breaking the original problem up into more manageable pieces, or of dealing with situations that need to be resolved before the original problem itself can be tackled. This description echoes Bakhtin, who writes about how an idea "is questioned, […] is put in a new situation in order to expose its weak sides, to get a feel for its boundaries " (Bakhtin, 1981, p. 348).

Despite what on the surface may seem a very pragmatic way to go about solving a problem, this reformulation is also a creative process as the evolving problem is generative in that it establishes new links, relationships, and even variables (Kilpatrick, 1987). The following description of that process is helpful:

> Duncker (1945) offered the insight that each stage of the solution of a problem constitutes the problem's reformulation. Thus the mediating phases provide opportunity for problem-posing along the way. These re-formulations are products of creative thought. Finding and posing the problem is the critical outer layer of the problem solving process. Once that layer is peeled away, it reveals further layers within which new problems reside, problems that must be addressed as steps in the finding of a grand solution (Lewis, Petrina, & Hill, 1998).

[22] A situation is a current set of circumstances from which a mathematical exploration might proceed. For instance, having noted that 1/2= 2/4, students might use this situation as a springboard for investigating other equivalencies.

Benefits of problem posing

The current National Council of Teachers of Mathematics' Standards document (2000) notes that problem posing is an important component of problem solving, recognizing it as an indication of a "mathematical disposition." For teachers, this is probably at the heart of its appeal in a classroom situation – students believing they can do mathematics (Baxter, 2005). In an article discussing problem posing as a teaching strategy, Whitin argues that problem posing can improve the atmosphere of a mathematics classroom, as "a strategy that builds a spirit of intellectual excitement and adventure by legitimizing asking questions and freeing learners from the one-answer syndrome" (2004, p. 129). Sensing less pressure to get the right answer, some students may feel more confident about their mathematical abilities, and consequently may find participating in class activities to be less stressful (Baxter, 2005; Buerk, 1982). Problem posing builds on the sense of surprise, and sometimes dissatisfaction, that children inherently have about situations (Whitin, 2004). It gives students a license to "not to know something" yet still be able to notice and comment on it. As a result, students of any age may feel encouraged to further develop their mathematical curiosity, which may in turn motivate them to investigate (and learn) further. Students are more likely to be intrigued by the problems they pose themselves (Banwell et al., 1972; Crespo & Sinclair, 2008) since these questions tend to reflect their own interests. This too helps to lessen student anxiety and promote confidence about dealing with mathematical situations (Buerk, 1982). In an interview, Marion Walter[23] argues that because a wider variety of interests are being addressed, problem posing potentially provides a more diverse group of students with opportunities for success (Baxter, 2005) than would normally be the case with more traditional mathematics tasks. This benefits not only these particular students, but the

[23] Marion Walter coauthored, with Stephen Brown, *The Art of Problem Posing (*2005), which explores the relationship between problem solving and problem posing and develops problem posing as a educational activity.

community as well. As more people become engaged with mathematics, “we invite a much wider range of mathematical ideas into the conversation” (Baxter, 2005, p. 126).

Silver argues that “to understand what mathematics is, one needs to understand the activities or practice of persons who are makers of mathematics” (1994, p. 22), namely mathematicians. Pólya believed that self-directed problem posing was one of the most important parts of a mathematician’s work (1990 (1954)), and some researchers and educators[24] perceive problem posing as an excellent way to introduce students to mathematical habits of mind. Walter suggests that through problem posing students learn to be more careful observers of mathematical attributes, which in turn may allow them to begin to modify them and extend them, investigating them further (Whitin, 2004, p. 135). “[I]n coming to ask questions on mathematical concepts, students might come to understand those concepts in a more generalized, less context-dependent way” (Pirie, 2002, p. 929). These generalizations may help lead students to rules, an important labor-saving device of mathematicians (Whitin, 2004, p. 134). Of course, most students are not aiming to become mathematicians; still, problem posing offers them an opportunity to develop thinking strategies that will help them cope in the real world (English & Lesh, 2003) where they will encounter and deal with ill-structured problems. In reviewing five studies involving a total of 800 middle school students, Silver and Shapiro conclude that problem posing in the classroom encourages students to mathematize situations, which may help them to connect mathematics sensibly to real-life situations (1992). And finally, because questions always seem to trigger more questions, students become aware that “there is always unfinished business” (Whitin, 2004) both in mathematics class, and in life. Given these possibilities, it is not

[24] A couple of years after he wrote an article lamenting about how researchers had neglected problem posing as a topic of study (Silver, 1994), Silver noted the number of articles about problem posing geared towards practitioners that were appearing in popular mathematics teacher journals, and wrote “it appears that practitioner interest is running far ahead of the development of credible techniques for assessing mathematical problem posing and the accumulation of solid research evidence regarding its nature” (Silver, Mamona-Downs, Leung, & Kenney, 1996, p. 521).

surprising then that problem posing has not only been recommended as “a *goal* of instruction but also as a *means* of instruction” (Kilpatrick, 1987, p. 123).

How does one become a problem poser?

Mathematician Peter Hilton suggests that “Computation involves going from a question to an answer. Mathematics involves going from an answer to a question” (Flannery, 2002). One of the goals of problem posing research has been to determine how students can become better problem posers. Are there specific traits that predispose them to be better at generating questions? Are some instructional conditions that are more beneficial than others? How does problem solving relate to problem posing?

Perhaps a fundamental place to begin is with the relationship between problem posing and a student’s level of mathematical ability. Is problem posing for everyone? An early article (Ellerton, 1986) which discussed a subsample of a larger study, compared eight “more able” children aged six to seven years with eight “less able” ones and found that the more able students were able to generate more difficult mathematical problems. Ellerton was neither very clear about what “more able” meant, nor about how the quality of questions produced was judged. Other studies have tried to be more specific. As tests for creativity often prompt participants to generate as many questions as they can (Silver, 1994), it seems possible that creative ability and problem posing are linked. However, some studies of preservice teachers (S.S. Leung, 1993; Shukkwan S. Leung & Silver, 1997) have been unable to establish a connection. The case for a link between problem posing ability and the students’ knowledge of mathematics is not much clearer. Some studies have argued that students and preservice teachers with a strong knowledge of mathematics are also strong problem posers (English, 1997; S.S. Leung, 1993; Shukkwan S. Leung & Silver, 1997), while another was unable to find a connection between the two for elementary preservice teachers (Crespo, 2003a). In a survey study of more than 500 middle school students who had not had any instruction in problem posing, Silver and Cai (1996) found that all of the subjects – regardless of ability, mathematical, creative or otherwise – were able to pose at least one problem, and that more than half of the students were able to generate sets of related problems. This suggests that problem posing is something that is available to all students, and that it is a complex process, requiring more than a good number sense or a set of creative skills.

When asked to problem pose, what do students who have not been formally taught "problem posing" tend to do? Cai & Hwang (2002) studied Chinese and American Grade 6 students, and found that the American students tended to produce rule-based extensions of the problem relying on concrete (i.e. working with a number-based pattern, such as adding 2 each time) strategies, while the Chinese students tended to use abstract (i.e. algebraic formulae) strategies and symbolic representation. There was no difference between the groups, however, in terms of the variety of problems produced. In their study of middle school teachers and prospective secondary school teachers, Silver et al (1996) found that their subjects used both affirming[25] and negating[26] processes to create problems, and that their production of clusters of related problems suggested that the subjects were posing problems in a systematic manner. Silver and Cai (2005) asked middle school students to pose three questions based on a story problem they were given. The researchers noted that the problems generated tended to be solvable (i.e. within the students' mathematical capabilities), chained (that is, produced using an associative process, in that the first problem provided a cue for the next two) and increasing in mathematical complexity (based on semantic structural relations). In their initial case study of two college-aged students each individually working on a problem posing task, Ciferelli and Cai (2005) first

[25] "Accepting the givens" – when one accepts and works with the parameters of a situation (Brown & Walter, 2005) as the starting point of an investigation.
[26] "Challenging the givens" – when one alters one (or more of the parameters) in order to create a new starting point for an investigation (Brown & Walter, 2005).

suggested that the problems posed were produced in an associative manner. However, after following up with these particular students by having them work on an additional task (Cifarelli & Cai, 2006), the researchers concluded that a recursive model – where the ideas generated by the solving of one posed problem influences what problem is posed next, and so on – would be more appropriate.

Researchers have also investigated what conditions help to promote problem posing. With Grade 3 students, English (1998) found that a familiar context helped the students generate more questions. Formal symbolism, which was fairly new and thus more unfamiliar to them, they found difficult to interpret, and so they had difficulty coming up with possible problems for cards which display number sentences like "12 – 8 = 4." When presented with an informal context which lacks symbolic representation, such as a photograph of children playing with some objects, or a sentence such as "Sarah has five dolls on one shelf in her room and four toy cars on another shelf" (1998, p. 88) students were much more successful. With preservice teachers, Crespo and Sinclair (2008) found that having time to explore the problem situation and become familiar with it and its constraints helped their subjects pose a greater number of problems and mathematically richer ones. Roth (1995) noted the benefits of exploration time in his case study of three Grade 4 science students working on an engineering inquiry project. He argued that "the longer students experiment in a given domain, the more they structure their interactions with the environment" (1995, p. 371) because of their increased familiarity with the context.

Student interest in the problem posing situation itself has also been found to improve their performance. Roth (1995) described his Grade 4 and 5 science students as being completely engaged in their design task, which involved both problem posing and problem solving, in part because it was based on their ideas and work. Crespo (2003a) found that providing preservice teachers with a genuine audience of Grade 4 pen pals added meaning to their problem posing task and increased their willingness to engage in and discuss the problem posing process.

Judging a situation to be problematic, and thus being worth posing problems about, has also been shown to increase student interest (Crespo & Sinclair, 2008).

"Problem formulating appears to require facility in identifying the important features of a problem, abstracting from previous problems encountered, and seeing problems as organized into related classes" (Kilpatrick, 1987, p. 142), a facility that distinguishes the experts from the novices. In a review of problem posing literature, Silver and Marshall (1989) noted that experts, spend more time than novices engaging in problem formulating and reformulating. Pirie (2002) writes, "To pose mathematical questions at any level… involves more than being able to do the mathematics. It requires some understanding of the mathematical concepts involved – at the very least a feel for when a concept can be appropriately invoked" (p. 929). Experts notice the way mathematical situations are structured and which ones are relevant, while novices become bogged down in details, or do not have the repertoire to bring into play.

Students can be supported as they move from a novice level to an expert level through various forms of instructor intervention ranging from introductory activities to specific problem posing strategies to participating in problem posing (and solving) programs (S. I. Brown & Walter, 2005; Crespo, 2003a; Crespo & Sinclair, 2008; English, 1997, 1998; S.S. Leung, 1993; Pirie, 2002). Students were able to solidify their problem posing strategies simply by having time to explore and practice. Crespo and Sinclair (2008) determined that by the end of a mathematics methods course, when preservice teachers were given ample time to explore problem situations, they were able to generate richer problems than they were before the instructional interventions. In another methods course, preservice teachers who posed weekly problems for their assigned Grade 4 pen pals to solve, started off by tending to pose computational-based questions with a narrowed mathematical scope, apparently to help their pupils avoid making errors. After 11 weeks of seeing their pupils' responses and collaborating with their own classmates, the preservice teachers in general were posing more open-ended, complex questions and, in their design of these questions, showing a greater acceptance of pupil error as part of the learning process (Crespo, 2003a). Grade 3 students who participated in a problem posing program in small groups developed a better ability to pose problems with good structural complexity than a control group who had not participated in the program (English, 1998). Although they did not use instructional interventions in their study, Cai and Hwang (2002) found that their Chinese and American subjects' problem posing techniques reflected the type of mathematical training the schools in their respective countries offer.

Collective problem posing

In problem posing, what occurs at an individual level in many ways parallels what occurs at the level of a collective. Kilpatrick writes,

> Group work seems to provide a natural context for problem formulating. When students work together, they often identify problems that would be missed if they were working alone. A poorly formulated idea brought up by one student can be tossed around the group and reformulated to yield a fruitful problem. Students participate in a dialogue with others that mirrors the kind of internal dialogue that good problem formulators appear to have with themselves. (1987, pp. 141-142)

Some argue that group work has the potential to provide a safe structure for building problem posing competence (Silver & Marshall, 1989), and offers the opportunity for students to work together less competitively and more productively (S. I. Brown & Walter, 2005). Yet, despite these and similar recommendations (English, 1997; Lester, 1994; Silver, 1994; Silver et al., 1996), there is little in the literature about how problem posing works on a collective level. Instead, studies largely focus on the quantity and type of problems that individuals pose. However, what little research there is suggests that further study of collective problem posing could be promising. A study of preservice teachers in a mathematics methods class (Crespo, 2003a) noted that the subjects found engaging in collaborative problem posing to be a particularly helpful way to generate questions to send to their Grade 4 pen pals, and one that encouraged them to continue to work together for the remainder of their course. My study further explores the nature of collective problem posing by looking at the problem posing process and examining its characteristics.

Problem posing and problem solving

Given the intertwined relationship of problem posing and problem solving, it has perhaps been inevitable that researchers would have explored the relationship between the two. Although Walter insists that not all good problem solvers are good problem posers (Baxter, 2005), other researchers (English, 1997; Silver et al., 1996) have argued that there is a connection. Silver and Cai (1996) noted that good problem solvers were able to generate a greater number of problems and more complex problems than poor problem solvers were. However, they added a caveat, noticing that poor problem solvers tended to misrepresent the problems they were solving, leading the researchers to wonder if some kind of information processing deficit also might have been in play in student performance on problem posing tasks as well. Cai and Hwang (2002) suggested that the types of problems posed by Chinese Grade 6 students were related to the type of critical thinking they used to approach problem solving. In their investigation of how problem posing related to different stages of the problem solving process, Silver et al (1996) found that students tended to pose more problems prior to solving a problem, in comparison to the amount they posed during or after the problem solving. This result may have been due to a natural tendency to ask more questions about a new task, or a reflection of a study situation where the students were given more time to pose questions before the problem solving task than afterwards.

In a self-described “growl” contained in his letter to the editors of *For the Learning of Mathematics*, Tahta (1984) wonders what the value of problem posing is when one can just toss out a series of questions automatically. As Crespo and Sinclair point out, “one can vary the givens to produce a new problem without establishing any personal relationship with, assessment of, or feeling for the problem" (2008, p. 397). Walter argues when you generate a list of problems, certain ones will reoccur (Baxter, 2005), yet does that mean they indicate important themes, or are they just more likely to be posed because they're more strongly associated with the original situation? One also needs to consider the significance of the questions created. If problem posing can help students think more like mathematicians do, does it actually help them push the boundaries of their knowledge as it does for mathematicians? The literature is inconclusive. Both Silver et al (1996) and Crespo and Sinclair (2008) found that teachers and preservice teachers posed problems they could not necessarily solve and that did not have "nice" solutions, but Silver and Cai (1996) were unable to determine if the middle school students in their study could or could not solve the questions they generate. Other studies have not raised the issue at all.

A product or a process?

The majority of the studies described here rely on their subjects' written work, a static product, as the focus of analysis. While this has the advantage of allowing researchers the ability to draw on a large pool of subjects, it also has the effect of (appropriately enough) triggering yet more questions about the research itself. In an excellent discussion of the results of one such study (Silver & Cai, 1996), the researchers wondered if middle school students only recorded problems they knew they could solve; perhaps they were able to generate more complex questions, but hesitated to write them down because they were not able to solve them. Concerns were also raised about how students were interpreting the task. As mentioned earlier, the weak responses of some of the students in the problem solving part of the task may have been due in part to an information processing deficit. Ultimately, Silver and Cai wondered if their study was actually measuring the students' ability to problem solve, or their inability to read and interpret the task itself, or perhaps even their inability to record and report? The researchers also questioned the trend of simpler questions being posed before the more complex ones were. Perhaps the subjects originally had the more complex question in mind first but decided to record the simpler questions, which may have been developed later, at the beginning of their written responses. Some subjects in this study appeared to treat the task as a comprehension exercise, creating questions based on what they inferred from the task, while others asked questions of a more philosophical and non-mathematical nature about the characters described in the task (for instance, “Why did Julio drive farther?”). And what were researchers to do with subject responses that were not really problems, but had the potential to be rich sources of mathematics? Was it just that the students had not framed their ideas in a more appropriate problem format?

All of these issues point to problem posing being difficult, and perhaps simply inappropriate, to capture with a written end product. Hence, my study examines the problem posing process, considering how the problems posed help to structure the problems that appear later and describing the different patterns of posed problems that emerge.

Chapter 5: Documenting the authoring process

Purpose of Study

This dissertation documents the emergent collective problem posing of small groups in two Grade 8 middle school classes while solving a task[27] entitled "The Bill Nye Fan Club Party."

The Bill Nye Fan Club Party

> The Bill Nye Fan Club is having a year-end party, which features wearing lab coats and safety glasses, watching videos and singing loudly, and making things explode. As well, members of the club bring presents to give to the other members of the club. Every club member brings the same number of gifts to the party.
>
> If the presents are opened in 5 minute intervals, starting at 1:00 pm, the last gift will be opened starting at 5:35 pm. How many club members are there?

My research questions are:

- What problem posing patterns emerge as small groups of students work collectively on a mathematics task?
- What are the characteristics of problem posing as a collective process?

In this dissertation, I consider the behavior of groups actively engaged in mathematics on a collective level, and the participants in this study were selected through purposeful sampling, a kind of biased sampling that is "an intentional consequence of [the] research design" (D. Edwards & Mercer, 1987, p. 26). The case itself is "a complex entity located in a milieu or

---

[27] Although "The Bill Nye Fan Club Party" is itself a problem, and on the worksheet distributed to the students it is titled as "Problem of the Day," throughout this dissertation I will refer to it as a "task." I do this for two reasons. The first is to try to reduce the confusion that would otherwise result from referring both to it and to what the students pose as "problems." To call the students' posed problems "sub-problems" (as I do in the discussion of problem posing in Chapter 4) suggests that they are somehow inferior to the original Bill Nye problem which the students are discussing, or that there is a hierarchy of problems that exists. The word "task" also points to the Bill Nye problem as being initiated by the teacher (and, in this case, the researcher), which is true of many activities that take place in mathematics classrooms.

situation embedded in a number of contexts or backgrounds"(Stake, 2005, p. 449), and as the researcher, I set the boundaries of the case by choosing what level to focus on and how close or far away to focus the video camera lens. In this case, although acknowledging the reflexive relationship between individuals and their group, I largely focus on the group as a collective learning agent.

This case study is a naturalistic inquiry where I was acting as a participant/observer and videographer in the classroom, someone who was visible but who avoided interactions and discussions with students during class. I hoped to minimize my presence as researcher in the classroom in two ways. As I had been a teacher at this particular school for several years (although not during the year when the data was collected), most students were familiar with my presence in the building and recognized that I knew their teachers, and this helped to minimize my status as an intruder. To further reduce any disruption, I made brief visits to each class before data collection began to help accustom students to my presence in their classroom. At the same time, this also allowed me to become more familiar with how the dynamics in each class were developing and to consider what type of tasks might be least disruptive to the two classes' routines.

Discussion of Methodology

Data Collection

The study did not begin until March so that the social norms, values, and routines of each class had time to be established. There was a pilot taping with the teacher's (Mrs. Shug[28]) homeroom class in early March which enabled Mrs. Shug and me to work out the locations of groups, cameras, and audio recorders, and to develop routines for introducing each task to the classes. This pilot taping was followed by an extended break (for a one week school project

[28] The names of all participants in the study, including the teacher, have been changed to preserve their anonymity.

unrelated to the study and for the school's two week spring break holidays). Session tapings took place in April and May, roughly every two weeks depending on the school schedule, for a total of five sessions for each class, with each session lasting approximately 40 minutes.

In this study, two stationary video cameras were each focused on a group that Mrs. Shug and I had identified as having a strong potential to work collectively (this identification is discussed later in this chapter). Also visible in the background were other groups participating in the study, meaning that each "video-taped" group was in fact being recorded by two cameras, each with a different angle. The cameras also recorded each group participating in the study whenever it happened to be presenting its ideas to the class.

There are challenges in audio-recording in a middle school classroom. Middle school classroom activities are generally noisy, particularly when there are 30 students in the room who are actively participating. As well, the video-camera's built-in microphone is often physically located too far away from the group it is recording to pick up the group's discussion consistently. To get around this, I placed an audio-recorder with each of the video groups to ensure that the group's discussion was adequately captured. I used both the video and audio recordings in transcribing the sessions.

In addition, I audio-recorded two additional student groups [29] – as the workings of any group cannot be predicted, these groups served as a back-up in case they had active on-task discussions but the two videotaped groups did not.

I took field notes throughout the sessions from a location at the back of the classroom, and compared these notes to the video and audio recordings to clarify events captured in the tapings. Other data sources included the task sheets where group members recorded their work

[29] These audiotaped groups were also visible in the background of the videos of the main groups. This allowed me to view where each audiotaped group member was positioned – and if anyone arrived or left during the session – and their gross physical movements

and solutions, and the class whiteboard where some groups chose to write their ideas while presenting their solutions to the rest of the class.

There are several advantages to using video data. It allows the researcher to view and review the data in a way that is not possible in real-time observation. The data can be viewed on multiple occasions, allowing the viewer to pick up details that might have been missed during initial viewings (for example, subject discourse that was not obviously audible the first time), and to focus his/her attention on various aspects of the situation. It also enables the researcher to develop conjectures that he/she can reflect on during later viewings and compare them to other viewed data. Later in this chapter, I further discuss how the treatment of video data might be used to expand and enhance the researcher's view of small group work in other ways.

Although videotaping may be considered "the least intrusive, yet most inclusive, way of studying the phenomenon"(Pirie, 1996, p. 554), it is impossible for the presence of the video cameras not to have an impact on the classroom environment. Even the most technologically confident subject cannot help but be aware of a camera lens being trained on her, of being watched, no matter how inconspicuously the video recorder may be positioned. It is also important to be aware of the limitations of videotaped data. There is much going on in the classroom that the camera misses, not only what is taking place during the session itself, but also past events that help to shape what occurs in the present. As well, where the camera is placed in the room, and what is framed by its lens, even the type of equipment used, affects the kind of data that is gathered (Pirie, 1996). A comparison of the video data with the audio data, my field notes, and student artifacts helped to work against these limitations.

Research Participants

The research took place at a Grade 6-8 middle school in a large suburban school district in British Columbia. The school has a multicultural population with wide-ranging economic backgrounds and family situations. In order to further create a “fair” distribution of student types within a class, there is a class-building process where there is an effort to distribute gender, academic ability, behavior issues, and other characteristics evenly amongst the classes in an attempt to ensure that no class would be deemed to be preferable to another. The tentative class lists are then reviewed by the school counselor, school administrators, the student services teachers, and the classroom teachers.

The middle school age group is known for its high energy and for its enthusiasm for socializing, making its members ideally suited for working in groups while tackling mathematics tasks. By the Grade 8 level, most students have an understanding of mathematical operations as well as of basic mathematical concepts. As well, at middle school there is a freedom from the kind of formal examinations found at the secondary level and this allows for more flexibility in curriculum content, as well as in the daily schedule, making it less disruptive to integrate research tasks in with the students’ regular classroom activities.

Mrs. Shug is a very experienced classroom teacher, considered to be a master teacher by her peers, able to handle the presence of video and audio equipment in her classroom, and flexible and collaborative by nature. I conceived of my relationship with her as a kind of partnership whereby I trusted her to act as a kind of a liaison between my needs as a researcher and the needs of her students. For that reason, when I offered her a selection of tasks to choose from, I trusted her to choose the ones that would best suit the needs and interests of her students. She seemed confident in doing so, and on one occasion requested a few more tasks from which to choose.

Studying two mathematics classes taught by the same teacher made things easier for me logistically in terms of setting up the one classroom for videotaping. It also made things easier for Mrs. Shug in that both of her classes were participating in the same tasks, just as they were in their regular curricular activities. In an attempt to reduce the likelihood of the members of one class telling members of the other class what the solution to an upcoming task was, the classes worked on different problems during different weeks in hopes that students would either forget to tell their friends or, if told, these friends would forget what they had been told.[30]

Sixteen students from each class of 30 students (so, just over half) participated in the study for a total of 32 students. Although student level of mathematical ability may be an issue for some mathematics education studies, in a project for one of my mathematics education graduate level courses[31], I found that although some students might be considered to have a lower level of mathematics knowledge than their peers, their ability to ask questions and draw out the ideas of their peers may mean they are sought out as task partners. As Bowers and Nickerson note, "In fact, those students who do not follow the developmental trajectory of the mode[32] often add the spice and initiative needed to propel the evolution of new practices" (2001, p. 3). It was for this reason that Mrs. Shug and I selected students for the videotaped groups based not on their mathematical grades but on whom Mrs. Shug thought would feel most comfortable in front of a video camera, and would be willing to actively and cooperatively discuss the mathematical tasks with their peers. Thus, the groups were composed of students who were all working at grade level but who had mixed levels of ability (and confidence in their abilities) in mathematics. Some of the groups were composed of one gender while other groups

[30] There was only verbal evidence in one group of one member of that group knowing what "the answer" was in advance – with Michael announcing very early in a session to the members of NIJM that he already knew the answer to the Snow Day task (see Appendix A). However, the other members of his group did not appear to be interested in his information, wanting to solve the problem for themselves, and he did not raise the matter again.

[31] Another graduate student and I observed one of my own Grade 8 mathematics classes during problem solving tasks. We were investigating behavioral patterns of students when they were allowed to choose where to work in the classroom and with whom they wanted to work (if anyone).

[32] In this quote, the word "mode" refers to the majority of the group.

were mixed, depending on the friendship groups in that particular class. We made adjustments to group composition during the study when certain students were absent, and in certain cases where the group dynamics were not working out[33].

Ethics

All data gathered through video and audio technology and from participant documents and artifacts was collected with explicit permission from the participants and in full compliance with BREB guidelines. Permission for the project was granted by both the school's administration and the school district office, and Mrs. Shug also granted consent. Students in the class were informed about the research and were provided with assent forms for themselves and consent forms for their parents/guardians. Mrs. Shug managed the distribution and collection of these forms. Those students who chose not to participate in the study still took part in the mathematical tasks, but the groups they worked in were not videotaped or audiotaped, and I did not formally observe them for my research.

Confidentiality has been maintained through altering identifiable details in recorded data. Notes, student's written solutions, transcripts, audiotapes, videotapes and CD-ROMS are secured in a locked filing cabinet in my home office, and computer files are on the hard drive of my home computer and password protected.

Tasks

The tasks were all "Problems of the Day" that I had been using for the past decade with my own middle school level mathematics classes. For this study, I revised the wording of the

[33] In one case, a group of four boys did not get along well, although they were not fighting, and ten minutes into the taping they had all left their group area to work with friends in other groups! In another case, two girls were actively discussing the mathematics but in Cantonese. After trying to pair them with English-speaking friends within the group, only to have the Cantonese girls drift together again, Mrs. Shug and I felt that perhaps their level of English was not strong enough to expect them to be working on the tasks in English. We moved the girls to an audiotape group with another pair of students who preferred to work quietly on their own, where the two girls could continue to investigate the problems in Cantonese.

problems to include the names of the subjects' current teachers[34], as students in my own classes seemed to enjoy this aspect of the problems. The situations in the problems were more silly than realistic and students appeared to like that as well.

Prior to each of the regular sessions I offered Mrs. Shug two or three problems from which to choose to use as the task. Her choices were motivated by a desire not to intimidate students whom she knew to be anxious about mathematics, and to offer problems that would be of high interest to the students in general. The tasks were selected to be independent of the topics being taught in the students' regular mathematics class, so students would feel they would be able to fully participate in the study even if they were not "getting" the mathematics lessons in the regular class, and so that Mrs. Shug did not feel obligated to try to teach to the tasks to try to prepare her students. In past years, Mrs. Shug had taught problem solving to her mathematics classes as a separate unit one, day per week, later in the school year (she had not done so yet for this year's classes in anticipation of my study filling that role[35]), thus she was comfortable with problem solving tasks being separate from her regular curriculum. As I hoped to document the students within the group interacting with each other rather than with the teacher, I asked Mrs. Shug to interact with the groups as little as possible during the sessions and to avoid offering problem solving strategies (i.e. giving hints as to how to solve the task) when students did approach her with questions. This was difficult for her at first, especially as one of her classes was very dependent on teacher feedback[36] but as the new routine became established, she enjoyed having her students work more independently.

[34] I have revised the names again for the appendices of this dissertation in order to preserve the teachers' anonymity.
[35] In early planning for the study, Mrs. Shug mentioned that she was looking forward to this opportunity to try problem solving tasks that were new to her with her students. To maintain consistency with the kind of work Mrs. Shug normally assigned during her problem solving units of study, all the tasks I offered her were fairly structured, with one correct answer but flexibility in how to arrive at that answer.
[36] At one point during this particular class's second taping, there was actually a little line of four students following Mrs. Shug around the room like ducklings. By the fourth taping, this type of behavior had largely disappeared.

When introducing the tasks to the students, Mrs. Shug's instructions included the following: all the information students needed was contained within the problem itself (this instruction was included after the pilot taping featured a question where several students attempted to look up the answer using their laptop computers); students were to talk to their neighbors in their group about the problem; they were to look for patterns in their work; they were to write down all of their work on the task worksheet provided (Appendix A); and they needed to be ready to present their ideas to the rest of the class towards the end of the session.

Students were asked to work on these tasks in class (i.e. it was not a homework assignment), and were not offered any direction for written solutions other than to show their work on the task sheet itself (attaching extra paper as necessary). These written solutions were not marked nor returned to the students. I felt that if I had marked them that would have given me an extra authority in the class that would have been inappropriate, and I did not want to add to the Mrs. Shug's work load by assigning the marking to her. Instead, to provide feedback, encourage accountability and to stress the importance of being able to communicate one's ideas clearly, we implemented a system where, towards the end of the class, two or three groups were randomly selected (Mrs. Shug drew names written on popsicle sticks from a container) to present their solutions. Students who volunteered to explain their work were also offered a chance to speak after the main presenters had finished. Almost all students in the class appeared to be quite motivated to try to solve the problems and eager to share their ideas with their peers, and we found we did not have to implement any other pedagogical structures to keep students on task. Mrs. Shug facilitated during the presentations, guiding her students in a discussion of different problem solving strategies, whether or not the solutions being presented "made sense," and what kind of patterns students may have noticed.

Analysis

Choosing groups

I was in Mrs. Shug's classroom for videotaping on eleven occasions, which resulted in 41 usable group recordings in total[37]. As I was using a grounded theory approach (Glaser & Strauss, 1967), I selected groups "for their ability to contribute to the developing/emergent theory" (Miles & Huberman, 1994, p. p. 28) – namely those who were working collectively on the tasks. In order to identify these groups, I began by watching and listening to the class activities to determine if all members in each group were actively participating in the whole group's activities. This did not preclude short periods of time where students in the group were working individually, or talking in pairs, as long as most of the discussion about the task occurred as a whole group. These observations helped me determine which recordings to listen to and/or view further.

Once all of the data was collected, I watched the video recordings closely looking for body gestures and postures that suggested attentiveness to the task and openness to the ideas of other members. I listened to the audio recordings to ensure that students were getting along, offering ideas, accepting and building on ideas offered by others, and that all students had opportunities to contribute to the conversation.

In the recordings, some students appear to be much quieter than others, but I did not rule them out; it is important not to confuse quiet behavior with a lack of interest or participation. Sometimes these students may have been much less talkative than their peers, but they still offered contributions that helped to develop the group discussion. Other times, utterances of

[37] In almost all of these eleven occasions, four groups were recorded. During one session, due to a high number of study participants being absent, only three groups were recorded. During two other sessions, certain group recordings were quickly ruled out. On one occasion, described in a previous footnote, although four groups were recorded, one of the groups was composed of four boys who decided within ten minutes to split up to work with other groups, leaving the video camera recording an empty desk area for the remainder of the session. On another occasion, four groups were recorded but one group of girls remained largely silent as the girls worked individually, with little group conversation about the task at all.

certain students were sometimes missed by the recording equipment due to their distance from the video camera and audio recorder, the low volume of their voices, or louder/closer students were speaking at the same time resulting in an overlapping conversation that drowned out other discussion.

Groups whose members were all actively and positively engaged in the task, and where the majority of the discussion was taking place as a whole group, were identified as working collectively: of the 41 recordings of group work, I deemed that ten met this criteria. As I was interested in comparing groups who were working on the same task, I chose the Bill Nye task because four of the eight groups who completed it worked collectively in doing so[38]. The four groups[39] this study focuses on are:

- NIJM (Nitara, Isaiah, Jacob and Michael);
- DATM (Derek, Amaya, Timothy and Meredith);
- REGL (Rebekkah, Eliana, Geri and Lucy);
- JJKK (Jessica, Julianna, Kady and Katia).

In this dissertation, I usually refer to these groups by their acronymic names (i.e. NIJM) as a way of characterizing each group as an entity in itself.

Transcription process

After choosing which groups to transcribe, I watched the videos again to get an idea of the events that occurred during the session. I listened to the audiotapes to prepare an initial transcript of what was said, and then used the video and audio tapes to complete the transcribing. Viewing the videotapes allowed me to see postures, gestures, and facial expressions that helped

[38] The Bill Nye task was the last one on which the two classes worked. I wonder if more groups were able to work collectively solving it because of the experience they had gained working on the previous tasks.
[39] NIJM, DATM and REGL are in one of Mrs. Shug's mathematics classes, while JJKK is in the other. NIJM, REGL and JJKK are all videotaped with supplemental audiotaping. DATM is mainly audiotaped, but appears in the background of NIJM's video so it is possible to view the body postures, gestures, and some facial expressions of the group members.

me to interpret what was being said, although I did not include descriptions of these as part of the transcripts[40]. I viewed and listened to the tapes several times, a layered and iterative process which enabled me to complete the transcriptions.

Analysis process

As this study involves elaborating upon and building theory about problem posing as an improvisational process, I analyzed the data using a constant comparison method (Glaser & Strauss, 1967). I began by reading one of the group transcripts in order to identify the problems posed by that group and to sort them into categories. I then read another transcript and identified the problems posed by a second group. This new data caused me to revise my original categories. I then read the transcripts of the third and fourth groups, again adjusting categories to fit the new data. Then I returned to each of the transcripts in turn, re-reading them and making further category adjustments until saturation[41] had occurred.

Identifying where problems were being posed was not a straightforward process. Since people are not necessarily accurate in articulating what they mean, and perhaps middle school students are less eloquent than older students or adults may be, this had some implications for my analysis. As mentioned in Chapter 4, the words “problem” and “question” are similar in meaning but are not identical, so my analysis was not a matter of looking for all the places in the transcript where someone happened to be asking a question. A question might point to a problem that was unrelated to the mathematical task (for instance, a student asking to drink from a peer’s bottle of water), while a statement might point to a problem. In their study of peer group discussions in elementary school classroom situations, Barnes and Todd found their initial attempts to code the discussion by identifying questions to be frustrating: “we found we could

[40] Although I made note of side-discussions between the students when they occurred, unless they were particularly short or appeared to trigger ideas related to the Bill Nye task, I did not transcribe them.
[41] At the point of saturation, the categories “incorporate and accommodate data in a good fit, with no discrepant cases” (L. Cohen, Manion, & Morrison, 2007, p. 493).

not make sense of the purposes to which questions were being put if we looked at isolated cases out of context. We had to look back at what had gone before and forward to what followed" (1995, p. 148). For instance, yes/no questions are not necessarily any more open than "wh" questions (who, what, where, when, why) – it all depends on the context in which they are posed. Ultimately, Barnes and Todd concluded that "inquiry might progress in utterances posed in any form" (1995, p. 154) whether they be questions or statements, individually or jointly constructed.[42]

Thus, the process of determining whether or not a group had posed a problem was necessarily an interpretative one. I was looking more at the conversational fabric around the utterance, both before the utterance occurred (what did the intent of the utterance seem to be?) and afterwards (namely, how did the group respond to the utterance?). For example, an utterance which initially appeared to be pointing out a piece of information could be treated by the group as a "What if this were true?" or a "What would happen if we try this?" type of posed problem. Sometimes there needed to be some conversational give-and-take between group members before the problem was articulated clearly enough that the group could proceed to explore it.

Another aspect of problem posing that I had to account for when identifying problems was the evolution of each problem as the conversation proceeded. Sometimes there was a quick spurt of activity within the group to publicly frame the problem (the type of give-and-take mentioned above) but more frequently the evolution occurred more subtly over the course of the discussion. Sometimes the same problem was being posed with different wording each time it appeared in a discussion. For instance, how a student phrases an utterance when speaking to a peer may be different from how she phrases it in addressing an authority figure (or different peer), even when it is the same idea that she is seeking to convey, because she may anticipate

[42] After much thought, I decided to phrase the problem categories that emerged in the form of questions because the question is the grammatical form in the English language that is most commonly associated with problems.

different responses[43]. At other times the same wording for the problem was used but, because the context had changed, the meaning of the problem being posed had shifted with the context. For instance, sometimes the meaning evolved as a group discussed a problem; sometimes the meaning shifted when the problem reemerged later in the session because the context had changed. I provide evidence of how posed problems evolve during group discussion in Chapter 7.

In considering her data, a researcher faces a dilemma similar to one that may challenge an artist – how can she see her subject (the data) with fresh eyes? Betty Edwards, an art educator best known for her strategies for learning based on the perceptual skills of drawing,[44] writes, "We tend to see what we expect to see or what we decide we have seen. This expectation or decision, however, often is not a conscious process" (1999, p. xxv). To get beyond these preconceived ideas, artists need to perceive their subjects differently. In the same way, a researcher needs to make her data strange in hopes of revealing new patterns and insights.

An example that may help to explicate this situation also comes from the field of art. The Impressionist painters were working in the nineteenth century when photography was becoming a more accessible and popular pastime. In the face of a device valued for its ability to capture a detailed representation of reality, the Impressionists sought to offer something the camera could not. The camera captures a snapshot, an instant of time, a "dot" of light. "[T]he impressionists realized that light was both a dot and a blur. If the camera captured the dot, the impressionist represented the blur. They want to capture *time* in the paintings, showing how a bale of hay changes in the afternoon shadows" (Lehrer, 2007, p. 100).

---

[43] This anticipated response may be immediate, it may be farther in the future, or it might never occur. Even in talking to oneself, aloud or internally, one is anticipating some kind of self-reaction to what one has uttered.

[44] Edwards argues that the global skill of drawing is made up of five perceptual skills: the perception of edges, spaces, relationships, lights and shades and, ultimately, the whole (gestalt). As an art instructor, to help her students learn to really see their subjects she would have them take on unusual (at the time) tasks, like doing a drawing of an upside-down Picasso piece. Because the students did not recognize what it was they were supposed to be drawing, this forced them to focus on the values of the sketch (the edges, etc.) rather than trying to capture the piece as a whole. Ultimately, this enabled them to "see" the Picasso work in a new way.

How might one “blur” what is physically seen? Lehrer describes the painter Cezanne as staring “at his subject until it melted”(2007, p. 102). Similarly, many artists use a technique called “squinting” (Graves, 1994) which is very much what it sounds like: the artist un-focuses his eyes, and closes the lids slightly, altering the perception of what he sees by reducing both the amount of detail and color he takes in so that the subject is simplified to a series of tones – i.e. contrasts between dark and light. In doing so, he is blurring the visual data to reduce his subject to its visual essence.

For the researcher, the video camera records the events that unfold in front of it, instant by instant, dot by dot. The researcher can stop the video flow at any moment, effectively freezing time to create a single snapshot of a scene that can be examined to reveal details that might otherwise pass by unnoticed. There are also other ways she can play with time in the video in order to blur the data. She can replay a scene repeatedly, which gives her an opportunity to look past the most obvious events of the scene in order to notice background details that may not have been evident at first. She can also change the speed of the video by increasing it or slowing it down, again, to highlight aspects that might otherwise remain obscure. For example, in my previous work on collective flow (Armstrong, 2008), I found that playing the video at an increased speed served to emphasize the students’ gestures, and this in turn helped me to determine if there were episodes when the students’ gestures became synchronized with one another.

For this study, in order to capture some of the characteristics of collective problem posing, I needed to blur the data in a different manner, in this case to better see the group’s conversation as a whole. While some have employed the metaphor of a crystal to describe the potential for multiple interpretations that qualitative research admits (Janesick, 2003), the metaphor that I am using to document the patterns of collective problem posing is that of the “tapestry.” Composed of strands of fabric and color, a tapestry reveals different faces depending on its physical distance from the observer. From afar, which would be the equivalent of summarizing a group conversation and then considering it from both a temporal and contextual distance, the tapestry shows a panoramic scene – a whole composed of a number of parts. Closer, the landscape of the tapestry might still be evident, but now the individual strands are more visible. Move closer still, and now the individual strands are the focus and the overall scene is no longer clear – much in the same way in which it may be easy to follow the individual turns of a conversation but difficult to summarize the gist of the discussion as a whole while it is taking place. At this level, an overall pattern is invisible, but individual contributions and ideas stand out. These strands of individual utterances are ones that weave together into a tapestry as the conversation proceeds.

The production of the transcript tapestry involves a data blurring process, which starts with the transcript itself. Once I identify the posed problem categories (for this study, there were 31 categories in total), I color code the utterances in the transcripts according to the problem posing category they best fit. The color-coded transcripts are then shrunk in size, using computer screenshots, to the point where the words of the transcript are no longer visible and the lines of color coding appear as a visual pattern. The resulting tapestry provides an overall image of the problems posed in the group's session – a blurring of data that provides a visual storyline.

Chapter 6: The groups and the stories they tell

Introduction

In this chapter I describe findings about the individual groups, in terms of their stories and their storylines, based on the problems that are posed during their sessions. I first provide a brief reminder of the task itself and the general setting in which the groups were working. I then introduce some of the data that emerged – the problems that were posed, a key of the color coding, and the tapestries for each of the groups. Then I offer the following for each of the groups: a story that emerges from its session; two data charts of the problems posed (the first showing the chronology and the second showing the frequency); a discussion of the problems posed and the resulting storyline.

The task

The four groups (NIJM, REGL, JJKK and DATM) are presented with the Bill Nye task. All of these groups are working in their own regular Grade 8 mathematics classroom settings. The task is given to them on sheets of paper, distributed one for every two people. Their teacher, Mrs. Shug, reads the task aloud to each class, issues some general directions and then offers students an opportunity to ask her questions. Then the groups are given approximately 25 minutes to work on the task. They are encouraged to work independently of Mrs. Shug, but she is willing to answer questions if a group calls her over. By the end of each of their sessions, all of these four groups have found the correct answer: 8 club members attend the party, each bringing 7 gifts. Looking only at their final answers, one might assume that each of the groups "gets" the question and leave it at that. Looking at the tapestries in Figure 6.1 it is apparent that each group takes a different path, creating a different storyline, in developing its solution to the task. Figure 6.2 provides a color-key of the problems posed.

Figure 6.1: Tapestries

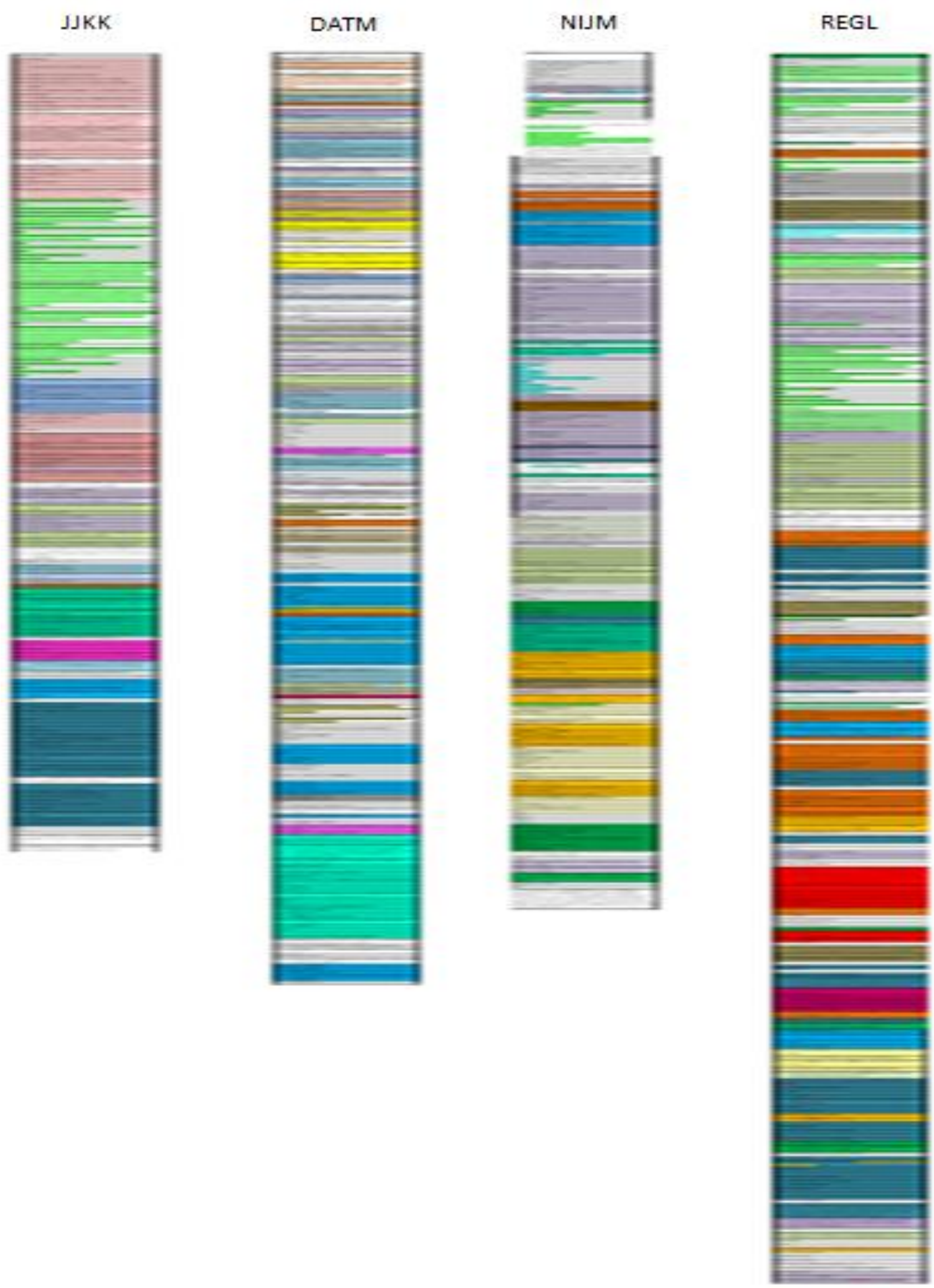


Figure 6.2: Key to color codes

Color name	Problem posed
Lavender	Do we use time and divide by 5 [number of intervals]?
Bright blue	What about if everyone brings x gifts each?
Wheat	Is there an extra 5 minutes? (because last gift is opened starting at 5:35)
Light blue	How many people are there?
Medium blue	What are the factors of x?
Lime green	What is meant by an interval?
Taupe	Do all members give to everyone?
Goldenrod	Do they also bring gifts for themselves?
Orange	Does everyone bring the same amount of gifts?
Sky blue	How many gifts are there?
Pale yellow	What if there are x people?
Pine green	How do we think outside the box?
Teal	Is it a square root?
Fuchsia	Why did we get x?
Coral	How long does it take to open all the gifts?
Periwinkle	Can they take breaks in between opening gifts?
Ivory	Does it start at one o'clock?
Gray	What is a tournament?
Red	What if it's an exchange?
Light green	How long does it take to open one gift?
Forest green	Can't we just count how many people?
Light purple	How many gifts does each person bring?
Pink	How many gifts are opened in an hour?
Brown	Is another group's answer right?
Purple	Can they bring partial gifts?
Light pink	What if someone doesn't get a gift?
Dark gray	How do we know if we're right?
Khaki	What if there are x people and gifts?
Tan	Does it take 5 minutes to open one gift or 5 minutes to open all the gifts that one person brings?
Aquamarine	How can we use the 24 hour clock?
Yellow	Can they open gifts at the same time?

NIJM

Story

NIJM is one of the first groups in the class to receive the Bill Nye task worksheet, and the group starts posing problems even before Mrs. Shug formally introduces the task to the class and leads a brief class discussion. The first problem the group poses ("Do we use time and divide by 5?") leads to a strategy of counting out every five minutes of the time period given in order to determine the number of five minute intervals in the gift giving. The next two problems ("How many people are there?" and "How many gifts are there?") are ones related to information provided by the task description, but neither is picked up by the group which returns to considering "Do we use time and divide by 5?"

NIJM then ponders "What is an interval?" By coincidence, this problem is then addressed by Mrs. Shug when she introduces the task to the class, and three members of NIJM participate in the class discussion that follows, offering possible definitions for the term "interval." Mrs. Shug then asks the class to begin working on the task.

Early in its session, NIJM has pondered possible strategies for tackling the task – the counting out strategy mentioned above, as well as a guess and check strategy ("What if everyone brings x gifts each?") that could work to determine the number of club members and gifts. The group drops this second problem due to a lack of information ("But we don't know how many people were there."), but will return to it later when the group has calculated what it needs to know in order to start a guess and check process.

As NIJM performs the counting and recounting, it also deals with related issues. "Is it a square root?" emerges as a kind of prediction about what possible answers might be ("Since we're learning about square roots, I'm pretty sure it's about square roots"). They try it and discover that the answer is a decimal which would not be appropriate.

I: We can’t have seven and a half people going to the party though!
[Laughter]
M: It could be like the show *Two and a Half Men*.

NIJM wonders whether or not an answer called out by another group in the class is correct (NIJM appears to conclude that it is not), and there is a bit of competition within the group when one member hazards a prediction as to the number of intervals, prompting much discussion about whether or not his guess will be correct. All of these other issues, which are discussed and quickly dealt with, refer the group back to “Do we use time and divide by 5?”

Once the group has a couple of numbers to work with (55 and 56 as the possible number of intervals), they start checking details: Does the gift opening start at 1 o’clock? Is the number of intervals 55 or 56? When the members of the group agree that the number is 56, they look for a strategy to find factors of 56. Considerations about details of the Bill Nye task emerge such as “Do the partygoers bring presents for themselves?” After being posed a couple of times, this particular problem finally appears to be dismissed:

N: It’s obvious. Who would buy themselves a present?
J: Yeah.
N: Hey, they have other members to give them a present.

“What if someone doesn’t get a gift?” is dealt with more bluntly: “Everyone gets a gift.” The group then returns to the guess and check strategy that emerged earlier in the session. It is noteworthy that NIJM is able to consider problems as needed, and discuss ideas without anyone in the group becoming confused. Having determined that 8 people bring 7 gifts, there is concern that this solution may be too easy – has NIJM thought outside the box enough as Mrs. Shug suggested to the class in her introduction to the task[45]?

J: Yeah, eight people bring seven gifts.
I: Yeah.
J: We got it.
N: Did we get it?
I: I think so.

[45] See Appendix B for transcript of Mrs. Shug’s introduction of the POD task to each of the two classes.

N: That was easy.
N: No, it shouldn’t be this easy. She just said that.
J: Because we *did* think outside the box.
I: We *did*, we [*sic*] this calculation.
M: How much time is that? Like, how much time did we spend on this?
I: Five minutes.
N: Five, ten minutes. Maybe seven.
I: Try something else.

This leads to the suggestion: “If I draw it, maybe we can, maybe we can try something else, more outside the box.”

NIJM begins a recount of the number of intervals to see if there might be another answer. In the nine minutes left before Mrs. Shug warns the class that presentations will begin soon, the group chats about various topics as it completes its count.

Posed problems

Figure 6.3a: Chronology of problems posed by NIJM

Problems posed
Do we use time and divide by five?
How many people are there?
How many gifts are there?
Do we use time and divide by five?
What is meant by an interval?
What is meant by an interval?
Do we use time and divide by five?
Does everyone bring the same amount of gifts?
Do we use time and divide by five?
Does everyone bring the same amount of gifts?
What if everyone brings x gifts each?
How many people are there?
What if everyone brings x gifts each?
Do we use time and divide by five?
Is it a square root?
How many gifts are there?
Do we use time and divide by five?
How many gifts are there?
Do we use time and divide by five?
Is another group's answer right?
Do we use time and divide by five?
Can they bring partial gifts?
Do we use time and divide by five?
What are the factors of x?
Is it a square root?
Do we use time and divide by five?
Does it start at one o'clock?
How many gifts are there?
Is there an extra 5 minutes (because the last gift starts at 5:35?
How do we think outside the box?
What are the factors of x?
Is it a square root?
Do they also bring gifts for themselves?
Do all members give to everyone?
What if someone doesn't get a gift?
Do they also bring gifts for themselves?
What is meant by an interval?
What if there are x people?
Do they bring gifts for themselves?
What if there are x people?
Do they bring gifts for themselves?
What if there are x people?
How do we think outside the box?
Do we use time and divide by five?
How do we think outside the box?

Figure 6.3b: Frequency of problems posed by NIJM

Problem posed	**# of times emerges or re-emerges**
Do we use time and divide by 5?	11
How many gifts are there?	4
Do they bring gifts for themselves?	4
What is meant by an interval?	3
What if there are x people?	3
Is it a square root?	3
How do we think outside the box?	3
What if everyone brings x gifts each?	2
What are the factors of x?	2
How many people are there?	2
Does everyone bring the same amount of gifts?	2
What if someone doesn't get a gift?	1
Is there an extra 5 minutes (because the last present starts at 5:35)	1
Is another group's answer right?	1
Does it start at one o'clock?	1
Do all members give to everyone?	1
Can they bring partial gifts?	1
17 different problems posed	**45 problems total**

Storyline

Indulging in only a few conversational digressions during the solving process, NIJM is focused and systematic in its approach to this task. It takes the group roughly 13 minutes to solve the problem, and the members spend the remainder of the session briefly looking for an alternate solution and then just chatting. Perhaps the efficiency of NIJM in solving the task is due in part to having posed a problem so early in the session which serves to structure the group's subsequent discussion.

As Figure 6.3b shows, there is one problem that dominates NIJM's discussion, and that is "Do we use time and divide by 5?" which is posed a total of 11 times. It is the first problem posed by the group, even before the class discussion occurs, and it reemerges ten more times in the session, acting as a kind of central thread[46] for roughly the first half of the group's discussion. The other problems posed in this session are not only posed far fewer times (four or less), but many of them appear to be offshoots of "Do we use time and divide by 5?" considering side-issues related to this central problem. Some of these emerge only once – such as "Does it start at 1 o'clock?" which serves to clarify the parameters of the task, and "What if someone doesn't get a gift?" They are discussed, and apparently resolved, before the group carries on. Those that re-emerge, show an evolution in their purpose [47]. For instance, "Does everyone bring the same amount of gifts?" when first posed, seems to be pointing out an aspect of the task which the group has just read aloud. When it is reposed, the group agrees that the resulting number of gifts will be an "even" (by which they appear to mean "whole") number. In another instance, "Is it a square root?" first emerges as a kind of prediction. When it next occurs, the group appears to agree that the answer cannot be a square root because the square root of 56 would be a decimal. In its final appearance, the group actually performs the square root calculation and reaffirms that it is not an appropriate strategy.

When the group determines the answer to the task there are a few problems that appear to be posed as a kind of a check. Sometimes, the posed problems help to juxtapose the situation in the task with what would likely occur in the real world. The problem of "Do they bring gifts for themselves?" is brought up when the group is trying to determine the factors of 56. After being posed a couple of times, this problem does not appear again.

In another type of check, "How do we think outside the box?" re-emerges as a consideration of whether or not the proposed solution is too easy, and then again as a prompt to try solving the task another way.

[46] I will further discuss the role of the central thread in Chapter 7.
[47] The evolution of posed problems will be further discussed in Chapter 7.

REGL

Story

The very first discussion REGL has is in response to Mrs. Shug's direction to the class about thinking outside the box. In a way, "How do we think outside the box?" acts as a frame for REGL's session, not only because it appears at the beginning and near the end of the session, but because it foregrounds how the group will deal with getting stuck.

The beginning of the session is also marked by a flurry of different ideas being offered. All of these are acknowledged by the group, but although none are taken up immediately, neither are any of them criticized or dismissed, suggesting that the group is surfacing a number of ideas with which it might work.

REGL decides that the number of gifts needs to be determined first by "Can we use time and divide by 5?" and seeks to describe the situation described in the task on a metaphorical level – for instance, wondering if an interval is the same as an intermission:

> R: When I think of intervals I think of like plays and there's all these like intervals and people stop for twenty minutes to have a snack or something.
> L?: I thought it was intermission.
> R: Oh right. Well they both sound the same; they are the same – intervals, intermission. Whatever.

A conversation with Mrs. Shug ultimately resolves the interval definition. Then the problem of "Is there an extra 5 minutes?" alternates with the "Can we use time and divide by 5?" problem until 56 is calculated to be the number of intervals.

REGL appears to be aware right away that it needs to seek out the factors of 56 – at one point the group ponders if there is a general rule to follow in order to figure out all of the factors of any numbers – and there is some discussion about whether this strategy is too simple, if it is "outside the box" enough. Initially, all of the factors except 7 and 8 are considered, and since none of these other factors are deemed to work, the group starts to explore the parameters of the task further (Do the club members bring gifts for themselves? Do they bring the same number of gifts? Do they give to everyone?).

Thinking outside the box, a new idea is introduced, based on the metaphor that the gift giving at the party is an exchange.

> G: Yeah it's fifty-five right? [stops writing]. And then see if there's ten people? So it's different than these people then this person so they give out one, two, three, four, five, six, seven, eight, nine, ten presents. Ten. And the second person gives out one, two, three, four, five, six, seven, eight, nine, because you've already given.
> E: But everyone has to give. But everyone has to give the same amount of presents.
> G: Yeah.
> R: Yeah.
> G: But like isn't it like exchange?
> R: No, it's like giving out.
> E: Because, okay, every club member brings the same number of gifts to the party.
> R: /So it's always/
> G: But it doesn't make sense/
> ?: It doesn't make sense.
> R: I'm sure there's more to it.
> E: Yeah I, that was a good theory. Omigod I wish we could have done something cool like that so we could show it.
> R: Yeah, I know. We could do lines everywhere and what [*sic*].

Although the metaphor of an exchange does not work, it is praised by the group as being "a good theory" and a "cool" way to show a solution: "We could do lines everywhere."

REGL continues to review the problems it has posed so far, and eventually comes upon the factors of 8 times 7 equaling 56. Recognizing this as the likely answer, there is some discussion about whether this strategy is too simple, if it is "outside the box" enough. REGL even discusses whether the Bill Nye task itself is worded clearly enough. The group then discusses how it might present its solution to the class and then double-checks its calculations.

Posed problems

Figure 6.4a: Chronology of problems posed by REGL

Problem(s) posed
How do we think outside the box? [as statement]
How long does it take to open one gift?
How many people are there?
What is meant by an interval?
How long does it take to open one gift?
Do we use time and divide by 5?
How do we think outside the box?
[followed by more reading aloud of question]
Does everyone bring the same amount of gifts?
What is meant by an interval?
What's a tournament?
Do all members give to everyone?
How many people are there?
How many gifts are there?
Do we use time and divide by 5?
How long does it take to open one gift?
What is meant by an interval?
Is there an extra 5 minutes? (because last gift is opened starting at 5:35)
Do we use time and divide by 5?
What is meant by an interval?
Do we use time and divide by 5?
What is meant by an interval?
How long does it take to open one gift?
Do we use time and divide by 5?
Is there an extra 5 minutes? (because last gift is opened starting at 5:35)
Do we use time and divide by 5?
Is there an extra 5 minutes? (because last gift is opened starting at 5:35)
Does everyone bring the same amount of gifts?
What are the factors of x?
Do all members give to everyone?
Can't we just count how many people?
Does everyone bring the same amount of gifts?
What if everyone brings x gifts each?
What are the factors of x?
How do we think outside the box?
Do we use time and divide by 5?
What are the factors of x?
How do we think outside the box?
Does everyone bring the same amount of gifts?
What if everyone brings x gifts each?
Does everyone bring the same amount of gifts?
What are the factors of x?
Does everyone bring the same amount of gifts?
Do they also bring gifts for themselves?
What are the factors of x?
Do we use time and divide by 5?

Problem(s) posed
What if it’s an exchange?
Does everyone bring the same amount of gifts?
How do we think outside the box?
What if it’s an exchange?
Do all members give to everyone?
What are the factors of x?
What if everyone brings x gifts each?
Does everyone bring the same amount of gifts?
What are the factors of x?
How do we think outside the box?
What are the factors of x?
What if everyone brings x gifts each?
What if there are x people?
Is there an extra 5 minutes? (because last gift is opened starting at 5:35)
What are the factors of x?
Do they also bring gifts for themselves?
What are the factors of x?
How do we think outside the box?
What are the factors of x?
Do we use time and divide by 5?
Is there an extra 5 minutes? (because last gift is opened starting at 5:35)
Do they also bring gifts for themselves?
Do we use time and divide by 5?

Figure 6.4b: Frequency of problems posed by REGL

Problem posed	# of times emerges or re-emerges
What are the factors of x?	11
Do we use time and divide by 5?	10
Does everyone bring the same amount of gifts?	8
How do we think outside the box?	7
What is meant by an interval?	5
Is there an extra 5 minutes? (because last gift is opened starting at 5:35)	5
How long does it take to open one gift?	4
Do all members give to everyone?	3
Do they also bring gifts for themselves?	3
How many people are there?	2
What if everyone brings x gifts each?	2
What if it's an exchange?	2
What's a tournament?	1
How many gifts are there?	1
Can't we just count how many people?	1
What if there are x people?	1
16 different problems posed	**66 problems total**

Storyline

REGL poses its first four problems almost one after another, without any of them being taken up by the group. "Can we use time and divide by 5?" is then posed and it becomes the group's first central thread, appearing 10 times during the session. This problem first alternates with the problem "What is an interval?" with the two problems gradually starting to meld together, until a conversation with Mrs. Shug resolves the interval problem. Then "Can we use time?" alternates with "Is there an extra 5 minutes?" until REGL calculates 56 to be the number of intervals.

The second central thread begins almost immediately. REGL appears to be already aware that it needs to seek out the factors of 56, posing "What are the factors of x?" 11 times during the remainder of the session. Although the group discusses the factors 4, 14, 2 and 28, for whatever reason 7 and 8 are missed completely, and the group becomes stuck. To get around this obstacle, while continuing to consider “What are the factors of x?” REGL reposes 26 of the problems that it had discussed earlier in the session, and poses five new problems as well. The alternation of different problems being posed results in the continuation of the thready[48] pattern physically evident earlier in the tapestry (Figure 6.1). Eventually, the group finds that 7 and 8 are the factors that solve the task, and it then reposes a few more problems to check its work.

JJKK

Story

JJKK’s discussion begins with the problem “How many gifts are opened in an hour?” Although there is agreement in the group that this is a good problem to pursue, and that 12 is the number of gifts, a thorough discussion follows, including support from the teacher, to ensure that whole group understands how and why this could be calculated. A discussion/debate about “What is meant by an interval?” ensues, which also requires Mrs. Shug’s support, this time in the form of an explanation of the term interval. This leads to a related problem (“Can they take breaks between opening gifts?”) and Mrs. Shug helps here as well by repeating the explanation of what an interval is. Once this is resolved, “How many gifts are opened in an hour?” re-emerges. There is some debate about whether the 12 calculated earlier refers to the number of presents or the number of minutes, and Mrs. Shug helps again by encouraging a shyer student to explain her ideas in order to resolve the issue. “How long does it take to open all the gifts?” emerges, and while there is confusion about some misspoken wording, it is cleared up quickly. Mrs. Shug offers a suggestion about how to keep track of the calculations using a clock diagram (which is not taken up by the group). She leaves and the group continues with its own calculations, pursuing the problem “Do we use time and divide by 5?”

[48] Slim bands of color that alternate with slim bands of other colors. Pattern types will be discussed in Chapter 7.

The group also considers “Is there an extra 5 minutes?” as the calculations proceed. Two problems reemerge, one which (“How many people are there?”) is pointed back to the group by Mrs. Shug (“That’s what I’m asking you.”), and one (“How many gifts does each person bring?”) which is not immediately taken up by the group. A problem that Mrs. Shug herself poses (“Does everyone bring the same amount?”) is not addressed by the group either. She leaves again.

“Is it a square root?” is now introduced. Although the problem of “Is there an extra 5 minutes?” is raised, it is not taken up for discussion, and as a result the group continues working under the assumption that there are 55 intervals. The square root of 55 turns out to be a decimal which the group rounds down to 7 and then agrees on as being a reasonable answer. Then JJKK divides 7 back into 55 and gets 7.8. At first this seems to lead to the conclusion that 7 people brought 7 gifts, a statement which is repeated within the group a couple of times. Then a statement is made that 8 people brought 7 gifts. The idea of rounding up 7.8 is then discussed. Although the group still seems to be hanging on to the idea of 55 intervals (“Yeah, but you have fifty-five as the total”), the group notes that 8 times 7 equals 56. Although not explicitly stated, it appears that this answer of 56 is close enough to 55 to satisfy the group. When Mrs. Shug warns the class that there are only five minutes left before presentations begin, JJKK does not engage in any further group discussion. Instead, each member begins writing on her own sheet of paper.

Posed problems

Figure 6.5a: Chronology of problems posed by JJKK

Problems posed
How many gifts are opened in an hour?
How many people are there?
How many gifts does each person bring?
How many gifts are opened in an hour?
What is meant by an interval?
Can they take breaks in between opening gifts?
How many gifts are opened in an hour?
How long does it take to open all the gifts?
Do we use time and divide by 5?
How long does it take to open all the gifts?
Do we use time and divide by 5?
Is there an extra 5 minutes? (because last gift starts at 5:35)
Do we use time and divide by 5?
Is there an extra 5 minutes? (because last gift starts at 5:35)
How many people are there?
How many gifts does each person bring?
Does everyone bring the same amount?
Is it a square root?
Why did we get x?
How many gifts does each person bring?
How many people are there?
What if everyone brings x gifts?
What are the factors of x?
What are the factors of x?

Figure 6.5b: Frequency of problems posed by JJKK

Problems Posed	**# of times emerges or re-emerges**
How many gifts are opened in an hour?	3
How many people are there?	3
How many gifts does each person bring?	3
Do we use time and divide by 5?	3
How long does it take to open all the gifts?	2
Is there an extra 5 minutes? (because last gift is opened starting at 5:35)	2
What is meant by an interval?	1
Can they take breaks in between opening gifts?	1
Did everyone bring the same amount?	1
Is it a square root?	1
Why did we get x?	1
What if everyone brings x gifts each?	1
What are the factors of x?	1
13 different problems posed	**23 problems total**

Storyline

The chunky[49] pattern displayed in the first third of JJKK's tapestry (Figure 6.1) – large blocks of color that tend not to reemerge in the pattern – is quite distinctive from the tapestries of the other three groups. The chunkiness reflects how a problem is posed, discussed at some length until some kind of agreement is reached, and then disappears, presumably either having been resolved or dropped completely. In comparison to the other groups, JJKK rarely reposes problems.

Take, for instance, the problem "How many gifts are opened in an hour?" which is posed twice. In both instances the problem appears to be posed in order to clarify the idea within the group that 12 gifts would be opened in an hour. The first time it emerges, the group is discussing where the "12" comes from, with one member proposing this calculation as a way to begin, and gradually the other members of the group coming on board. The second time the problem occurs, there is a discussion to clarify whether 12 means the number of minutes or the number of intervals. Mrs. Shug is called in by the group to take part in both discussions, and she acts as a kind of interpreter, helping to make meaning clearer for individual members.

For approximately the first half of the session, whenever JJKK poses a problem, it discusses it immediately and, at times, at length. Perhaps the group needs more discussion time for each problem in the beginning in order to build cohesiveness within the group in terms of how to work together and how to interpret each other's statements. Given how much discussion appears to be required to establish common meanings, posing a lot of problems to consider at the same time would be to risk confusion within the group. However, in the second half of the session, the tapestry pattern becomes less chunky. Perhaps this suggests that the group members are now communicating well enough that they can assume mutual understanding without a thorough discussion taking place.

[49] Thick bands of color in the tapestry. For more discussion about this kind of pattern, see Chapter 7.

DATM

Story

DATM[50] does not start working until well after the class discussion has taken place, and even at this point only three group members are present. The session begins with an off-task conversation, followed by several problems that are posed to interpret the meaning of the Bill Nye task but that are not taken up by the group. Another problem is posed ("Do we use time and divide by 5?") but this is almost immediately challenged as not actually addressing what the task is asking about the number of club members. A debate ensues between Amaya and Derek while Timothy reads the task worksheet over on his own.

> D: Seriously, we're trying to figure out how many people were there.
> A: [laughing] No, I know, it takes steps.
> D: how are you going to do that? Are you going to divide that by what?
> A: Gah. [starts counting]
> …
> D: I'm trying to figure out what you're trying to do.
> A: I'm trying to figure out what I'm trying to do too.
> D: 'Cause you're like trying to figure out to figure time. We don't want time, we want people.
> A: Yes, it takes time to figure out people.

This issue does not appear to be resolved.

The arrival of the fourth student, Meredith, interrupts the debate but this seems to offer the group a fresh start. An earlier problem re-emerges ("How long it takes to open gifts?") along with another problem about whether club members can open gifts at the same time. There is a delay while the group attempts to attract Mrs. Shug's attention in order to get her help with these problems. After she provides some clarification, the group continues working.

Now there is some movement within the group. Meredith leaves; the boys start working together as a pair; and Amaya asks a student from the REGL group to go over some calculations

[50] Because of this particular group's dynamics, I will be referring to individual group members.

with her. The conversations overlap at this point, but both pairs appear to be dealing with the same two problems: the re-emerging "Do we use time and divide by 5"? and the new "Is there an extra 5 minutes?" After a few minutes, Meredith returns and DATM reforms, with the student from REGL returning to her group. After a brief discussion, 56 seems to be accepted as the number of intervals, but then the problem of "Why did we get [56]?" is posed. It is not taken up, and then 55 is asserted to be the number of gifts. "Is there an extra 5 minutes?" re-emerges twice and, although not explicitly addressed, its appearance seems to correct the number of intervals from 55 to 56 because 28 is now introduced as a possible factor. DATM asks Mrs. Shug "Do all members give to everyone?" and in her reply, Mrs. Shug poses the problem of "Does everyone bring the same amount of gifts?" The group poses its problem again, Mrs. Shug says that the club members do give to everyone, but DATM still seems to be confused about the issue after she leaves.

DATM now works with a guess-and-test method, and proposes that there are 8 people and 7 gifts. It is not clear from where these numbers come.[51] In the discussion that follows, DATM still seems to be trying to confirm what the number 56 actually represents, people or gifts, and some insults are traded at this point. The attempts to bring in other examples in order to prove or disprove what 56 represents leads to confusion within the group, as does a reintroduction of the factors 7 and 8. The problem "How do we know if we're right?" is posed. Meredith leaves again, and the boys have a fairly long discussion about being thirsty and playing basketball, while Amaya continues her calculations.

The three students then start trying to use the 24 hour clock, something that was required to solve the Power Outage task from a previous session (see Appendix A) to establish what 56 represents, which leads to a debate about which is the better way to convert time, counting

[51] REGL, the nearest other group, is not working with factors yet at this time so that group is not the source of the factors, even though Amaya had been working with a member of REGL previously.

around a physical analogue clock (“Wow, that’s a waste. You didn’t have to do all that. You could have just looked at the clock”) or using a mathematical method. When the fourth student returns, Mrs. Shug announces to the class that presentations will begin in five minutes. There is another discussion about water and then about whom in the group will do the actual speaking if DATM is called up to present a solution to the class. The group hurriedly agrees that 8 people brought 7 gifts, but there still seems to be a bit of confusion.

> M: We got fifty-six people?
> A: No, we got eight people.
> D: Eight people and seven presents.
> A: There’s eight people at the party and they each have seven presents.
> D: That’s a lame party.

Mrs. Shug then addresses the class as a whole, and the presentation of solutions begins.

Problems posed

Figure 6.6a: Chronology of problems posed by DATM

Problems posed
Does it take 5 minutes to open one gift or 5 minutes to open all the gifts that one person brings?
Do all members give to everyone?
Does it take 5 minutes to open one gift or 5 minutes to open all the gifts that one person brings?
Do all members give to everyone?
How many people are there?
Does everyone bring the same amount of gifts?
Do we use time and divide by 5?
How many people are there?
Does it start at one o'clock?
Do we use time and divide by 5?
How many people are there?
Do we use time and divide by 5?
How many people are there?
How long does it take to open all the gifts at the party?
How many people are there?
Does it take 5 minutes to open one gift or 5 minutes to open all the gifts that one person brings?
Do we use time and divide by 5?
Can they open gifts at the same time?
Do we use time and divide by 5?
Can they open gifts at the same time?
Does it take 5 minutes to open one gift or 5 minutes to open all the gifts that one person brings?
Can they take breaks in between opening gifts?
Do we use time and divide by 5?
How many people are there?
Do we use time and divide by 5?
Is there an extra 5 minutes? (because last gift starts at 5:35)
Is there an extra 5 minutes? (because last gift starts at 5:35)
Do we use time and divide by 5?
Is there an extra 5 minutes? (because last gift starts at 5:35)
Do we use time and divide by 5?
How many people are there?
Do we use time and divide by 5?
How many people are there?
Is there an extra 5 minutes? (because last gift starts at 5:35)
Why did we get x?
How many people are there?
Do we use time and divide by 5?
Is there an extra 5 minutes? (because last gift starts at 5:35)
Do we use time and divide by 5?
Is there an extra 5 minutes? (because last gift starts at 5:35)
What if there are x people and gifts?
Do all members give to everyone?

Problems posed
Does everyone bring the same amount of gifts?
Do all members give to everyone?
What if everyone brings x gifts each?
Do they also bring gifts for themselves?
Does everyone bring the same amount of gifts?
What if everyone brings x gifts each?
Is there an extra 5 minutes? (because last gift starts at 5:35)
What if everyone brings x gifts each?
How many people are there?
Does everyone bring the same number of gifts?
Do all members give to everyone?
What if everyone brings x gifts each?
What if there are x people and gifts?
What if everyone brings x gifts each?
How do we know if we're right?
What if everyone brings x gifts each?
Why did we get x?
How can we use the 24 hour clock?
What if everyone brings x gifts each?

Figure 6.6b: Frequency of problems posed by DATM

Problems posed	**# of times emerges or re-emerges**
Do we use time and divide by 5?	12
How many people are there?	10
Is there an extra 5 minutes? (because last gift starts at 5:35)	7
What if everyone brings x gifts each?	7
Do all members give to everyone?	5
Does it take 5 minutes to open one gift or 5 minutes to open all the gifts that one person brings?	4
Does everyone bring the same amount of gifts?	4
Can they open gifts at the same time?	2
Why did we get x?	2
Does it start at one o'clock?	1
How long does it take to open all the gifts?	1
Can they take breaks in between opening gifts?	1
What if there are x people and gifts?	1
Do they also bring gifts for themselves?	1
How do we know if we're right?	1
How can we use the 24 hour clock?	1
16 different problems posed	**61 problems total**

Storyline

The beginning of DATM's discussion is dominated by two problems – "Do we use time and divide by 5?" and "How many people are there?" – which alternate with each other while the group debates about which to tackle first in solving the Bill Nye task.

The arrival of Meredith appears to spur the group to explore the parameters of the task. Some details seem to be settled quite quickly ("Does it start at one o'clock?") and problems like these are not reposed. Others require more discussion and, in one case ("Does it take 5 minutes to open one gift or 5 minutes to open all the gifts that one person brings?"), some clarification from Mrs. Shug.

As the membership of the group is quite fluid, with pairs forming and dissolving within the group, and Meredith coming and going, reposed problems like "Is there an extra 5 minutes? (because last gift starts at 5:35)" seem to remind group members of the parameters of the task. Other problems like "Why did we get *x*?" and "How do we know we're right?" are less philosophical questions than they are requests for clarification about what the group has calculated.

When the group first determines that the factors are 7 and 8, there are a few problems reposed as a kind of check on the answer. In discussing these calculations, DATM ends up discussing a problem that is unique to this group, "How can we use the 24 hour clock?" There is a rush to agree on the final answer as the group has run out of time.

Creating a path

At first, the structure of the Bill Nye task would not appear to allow for many creative possibilities. To solve it, one must understand what the range of time is for opening the gifts, determine the number of time intervals that exist within that time frame, and then find the pair of factors of the number such that one factor is one greater than the other (i.e. 8 and 7). Yet, in working through this apparently straightforward task, these four groups take very different paths to arrive at the same correct solution.

NIJM works like a well-oiled machine, posing problems as soon as it receives the task sheet and quickly honing in on a problem that offers a mathematical strategy. This problem acts as a central thread that seems to serve in structuring the group's discussion (this role is discussed further in the next chapter), while other problems help to clarify the parameters of the task and apparently assure the group that it is on track. For NIJM, the problems posed efficiently plot out its storyline, reaching an answer, and then quickly double-checking that its solution is correct. In terms of the levels of improvisation discussed in Chapter 3, NIJM falls more into the Embellishment stage than the Variation stage because of the efficient nature of its process – there are few digressions during its journey.

If NIJM's posed problems help it speed along on its path from Point A to Point B, REGL's posed problems appear to take it on a longer, more winding journey. The group often juggles a few problems at once, making connections between them that seem to prompt discussions that are somewhat deeper than those the other groups have: REGL discusses metaphors that might help them understand the problem; it wonders if there is a general rule for factoring rather than mechanically cranking each of the factors out; it wonders about the wording of the task itself. The presence of these kinds of digressions suggests that the group is working in the Variation stage of the improvisation spectrum. When the group becomes stuck, REGL not only reposes previous problems to check the accuracy of its previous discussion, posing a few new ones in the process, but it continues to consider the problems a few at a time.

JJKK's story is one of a group slowly getting itself up to speed as it makes its journey. Like NIJM, it seems to be following a fairly straightforward path, but here JJKK seems to be using a centering strategy (Bastien & Hostager, 1988), as discussed in Chapter 3, in order to work more cohesively. At first the problems are posed and dealt with one at a time, discussed at length at one sitting, although not necessarily in depth. As the session proceeds, and JJKK appears to be communicating more effectively, the discussions of each problem tend to be less lengthy, although there is still a sense of the group carefully putting one foot in front of the other as it continue to deals with a single problem at a time.

Finally, DATM's storyline seems somewhat erratic at first, as Meredith and the girl from REGL come and go, internal off-task discussions take place, and some group members seem unable to communicate their thinking clearly enough to the rest of the group. The problems that are posed, and reposed, help to bring the group together, even allowing them to pursue digressions, like the discussion of the 24 hour clock, and thus reach the Variation stage of improvisation. By the end of the session, although it is rushed, DATM has determined an answer to the task.

That the groups come up with different paths is not surprising. They are comprised of students with different experiences and interests, and the working dynamics of each group certainly varies. Perhaps what should be surprising is how school mathematics often seems to prize efficiency over depth of thought and creativity. Yet even in this straightforward task there are lots of paths to follow, each one of them bringing up the potential for valuable mathematical discussion: When do we round up and when do we round down? How do we know if we're right? Is there only one way to calculate this, or can we find a better way? Even fairly structured tasks contain within them room for improvisation, and thus for the development of unique storylines.

Chapter 7: Overall trends

Introduction

In this chapter I discuss problem posing patterns across the group sessions and the characteristics of the problems that are posed. As mentioned in the previous chapter, the storyline created by each group working on the Bill Nye task is quite different, and it is no surprise that this is reflected in what problems are posed (Figure 7.1). For instance, it is interesting that there are relatively few common problems posed by all four of the groups (four in total) compared to the number of problems that are unique to certain groups (14 in total).

First, I discuss what is revealed by the colors in the tapestry: how problems weave in and out of the discussion; how early problems seem to reemerge later in the session; how certain colors seem to dominate certain tapestries, and how a few colors are unique to certain tapestries. I then argue how the role of a posed problem depends on the context of the discussion when it happens to be posed, and how even the same problem can take on different roles depending on when it reemerges in the conversation. Finally, I suggest how the groups work as bricoleurs as they consider the Bill Nye task.

Figure 7.1: Color coding chart of posed problems by frequency

Problem posed (generalized)	JJKK	DATM	NIJM	REGL	#
Do we use time and divide by 5 [number of intervals]?	X	X	X	X	4
What about if everyone brings x gifts each?	X	X	X	X	4
Is there an extra 5 minutes? (because last gift is opened starting at 5:35)	X	X	X	X	4
How many people are there?	X	X	X	X	4
What are the factors of x?	X		X	X	3
What is meant by an interval?	X		X	X	3
Do all members give to everyone?		X	X	X	3
Do they also bring gifts for themselves?		X	X	X	3
Does everyone bring the same amount of gifts?	[X]	X	X	X	3
How many gifts are there?			X	X	2
What if there are x people?			X	X	2
How do we think outside the box?			X	X	2
Is it a square root?	X		X		2
Why did we get x?	X	X			2
How long does it take to open all the gifts?	X	X			2
Can they take breaks in between opening gifts?	X	X			2
Does it start at one o'clock?		X	X		2
What is a tournament?				X	1
What if it's an exchange?				X	1
How long does it take to open one gift?				X	1
Can't we just count how many people?				X	1
How many gifts does each person bring?	X				1
How many gifts are opened in an hour?	X				1
Is another group's answer right?			X		1
Can they bring partial gifts?			X		1
What if someone doesn't get a gift?			X		1
How do we know if we're right?		X			1
What if there are x people and gifts?		X			1
Does it take 5 minutes to open one gift or 5 minutes to open all the gifts that one person brings?		X			1
How can we use the 24 hour clock?		X			1
Can they open gifts at the same time?		X			1

Tapestry color patterns

A tapestry offers a visual representation of problem posing during a conversation, and a comparison of the four tapestries shows how the same task can be the basis of different problem posing patterns, even when each group concludes with the same correct answer.

The width of the color bands indicates the approximate length of time a problem is being discussed, and how many, if any, other problems are able to "bump against" it (Davis & Simmt, 2003). For example, in the first half of JJKK's tapestry, it is evident that each posed problem is discussed for a considerable length of time, as shown by thick bands, or "chunks," of color. This produces a "chunky" pattern that is quite unique amongst the four groups, and reflects the fact that JJKK poses and reposes far fewer problems than the other groups do (23 in total, compared to REGL's 66, DATM's 61 and NIJM's 45). In comparison, a group like REGL, who tends to consider a few problems at the same time, generally has a more "thready" pattern, where slim bands, or "threads," of color alternate with other bands of color. DATM's tapestry has a thready pattern of lavender ("Can we take time and divide by 5?) and light blue ("How many people?") when the group is debating which of these two problems to pursue first. For NIJM, DATM and REGL there tends to be a thready pattern of different colors at the beginning of their tapestries, suggesting that a number of different problems are posed and put "on the table," so to speak. This thready pattern also tends to occur later in the sessions when groups have come up with a tentative answer, showing the kind of problem reposing that occurs to check a possible answer.

Another striking aspect of group work that a tapestry helps to illustrate is how posed problems weave in and out of conversations. A color may appear briefly early in a session – for instance, medium blue in NIJM ("What are the factors of x?") – and not appear again until over halfway through when it begins to occur quite frequently. This occurs in all the sessions, when a problem is posed, seemingly ignored only to be reposed later in the discussion. Other problems that seem to have been discussed and resolved also come up later for more discussion. This suggests that the mention of a posed problem early on in a session may help to seed a later discussion. It also seems to highlight the idea of all ideas being part of the tapestry, visible or not – no utterance truly disappears.

The physical amount of color that appears is also something to consider. For example, a lavender color ("Do we use time and divide by 5?") dominates NIJM's tapestry. Although this color appears in all of the tapestries, it does not occur in the same locations in each of the tapestries, nor does it cover the same area. For instance, JJKK has little lavender in comparison to the pink ("How many gifts are opened in an hour?") that dominates the top of its tapestry and a medium shade of blue ("What are the factors of x?") that anchors the bottom. For NIJM, however, not only is there a lot of lavender, but it appears regularly and alternates with other colors, particularly in the first half of that tapestry. The lavender seems to act as a kind of central thread for the discussion. In a later section I consider how certain problems like "Do we use time and divide by 5?" may help to structure discussions for some groups, providing a central thread from which other problems may spring.

While lavender and a few other colors appear in all of the tapestries, there are many other colors which do not. For instance, there is a shade of teal ("Is it a square root?") that only appears in NIJM and JJKK. And still other colors are unique to certain groups, like the light green ("How can we use the 24 hour clock?") that occurs at the end of DATM's tapestry. This suggests that while a few problems are posed by all groups, most others are not. This is an issue regarding bricolage that I return to in the next section.

In summary, the tapestries illustrate some of the patterns found in group conversations, helping to highlight when certain problems are being discussed, how often, and their duration. While the tapestries serve to make these trends more visible, they also have limitations. They do not show the viewer what is actually being discussed – the visual distance provided by shrinking the color-coded transcripts also obscures the words being said, making the identities of the posed problems themselves rather anonymous unless one is also referring to the color-coding key.

The evolving role of individual posed problems

A notable trend across the sessions is how the role a posed problem plays in a discussion changes each time it is posed even if, on the surface, the wording of the problem appears to be much the same. The reason a problem is posed, or the role it plays when it is posed, is not a static thing. For instance, a problem may be re-posed as a reminder of what the group has already agreed upon, or a way of making sure that the group is on the same page, rather than raising an issue of contention.

In this section, I discuss how the problem of "Do we use time and divide by 5?" is posed and then evolves in each of the groups' sessions. This analysis is based on the charts found in Appendix C which summarize each group's response to each problem posed or reposed during its session. I identify the function of the posed problem in terms of how the group appears to respond to it each time the problem is posed, as well as how the speaker poses it (since I cannot know the intention, I can only interpret the speaker's expression of the idea and the expression of the reaction to it, or lack of reaction, by the group)

NIJM

"Do we use time and divide by 5?" is the very first problem posed by this group, even before the class discussion has occurred, where it is offered as a counting method. This is followed in short order by two other posed problems which seem to function as a kind of response to the original problem. "Do we use time and divide by 5?" is raised a second time as the proposed counting method, and this time is explored by the group in a little more detail. The class discussion follows, and shortly afterwards the group poses the problem again, this time as a consideration of the idea that there might be something easier the group could do than counting out the intervals in order to determine how many there are. Nothing else is suggested, and when "Do we use time and divide by 5?" is posed a fourth time, almost immediately, it prompts the counting method to begin. The fifth time the problem is posed, it is suggested that this problem will lead NIJM to determine the number of gifts each party-goer will bring. The group agrees to continue with the counting method and that if the number of intervals for one hour can be determined the group can "keep doing it" from there. The next two times the problem is raised it refers to ongoing calculations. When "Do we use time and divide by 5?" emerges for the eighth time, it is in reference to predictions the group is making as to what the final answer will be. When it occurs again, the counting is continuing. The tenth time the problem is posed, the counting has been completed and the group is considering a recount. This is followed by much discussion of other posed problems. The problem reemerges for the eleventh, and final, time at the very end of the session, when the group is checking its solution, and assigning different members of the group to perform a recount. This leads to a discussion of whether or not there is another way to determine a solution.

In summary, in NIJM's session "Do we use time and divide by 5?" is posed in order to:

- propose a method of entry into the task;
- discuss what method would be easiest;
- discuss how it might eventually lead to solving the entire task;
- estimate/predict possible answers;
- narrate ongoing calculations;
- check possible answers.

REGL

"Do we use time and divide by 5?" is first offered in the REGL's session as a proposal about how to tackle the Bill Nye task, but it is not taken up for discussion by the group. After some conversation about the task, "Do we use time and divide by 5?" is posed a second time and this time REGL discusses what method might be involved in pursuing this particular problem.

The problem emerges very shortly thereafter again as a possible first step in solving the task and the group accepts this and begins to work on it. When it is posed for a fourth time, very quickly afterwards, the discussion begins to merge with that of another posed problem “What is meant by an interval?” and the group talks about the calculations so far. When “What is meant by an interval?” appears to be resolved, the group returns to “Do we use time and divide by 5?” a fifth time and continues the calculations. A connection is then made with the problem “Is there an extra 5 minutes?” and the calculations are completed with this second problem in mind. A discussion of several other posed problems occurs. The seventh time “Do we use time and divide by 5?” is posed, it functions as a brief recap of what the group has determined so far, as if to ensure that all members are on the same page. The discussion then returns to other problems. “Do we use time and divide by 5?” appears for an eighth time as a check to see if a counting method would also give a total of 55 intervals. There is further discussion of other problems. The final two times “Do we use time and divide by 5?” appears are at the very end of the session as a means of checking the group’s final solution.

In summary, during REGL’s session “Do we use time and divide by 5?” functions to:

- propose a method of entry into the task;
- focus the group on something in particular to calculate;
- connect with other problems the group poses in discussing the task;
- establish group cohesiveness (and a common purpose);
- double-check calculations and solutions.

DATM

“Do we use time and divide by 5?” is first raised as a possible method of approaching the task, and is connected with a task completed by the class on a previous occasion. It is then posed as a counting method, spurring calculations to begin, and then continues as a debate about what the group should be doing: finding the number of time intervals, or finding the number of people. The problem seems to be adopted as the group’s central thread. It is discussed by two pairs of students when the group briefly splits up, and the next several times this problem is raised for the pairs, and then for the entire group, it plays the same role: it prompts a discussion of the accuracy of the calculations. “Do we use time and divide by 5?” is then raised an eleventh time in order to confirm that the number of gifts is 55. The twelfth, and last, time the problem is posed, it is as a warning not to confuse the number of minutes in an hour with the number of gifts (an error one group member had made previously).

In summary, during DATM’s session “Do we use time and divide by 5?” functions to:

- propose a method of entry into the task;
- spur a debate about which problem the group should pursue;
- focus the group on something in particular to calculate;
- discuss calculations;
- discuss the accuracy of calculations;
- warn about confusing what various numbers represent.

JJKK

This group’s situation is an interesting one to consider. In order for the posed problems to evolve in terms of function they need to reoccur, and this tends not to happen with JJKK, particularly in the first half of its session. After fairly long discussion (at least compared to the other groups) the problems tend to disappear from the conversation. In fact, only three of the 11 questions the group poses occur again, with “Do we use time and divide by 5?” being among the three. The first time it occurs, the problem prompts a discussion of how to do the calculations. The second (and last) time it is posed, the group is checking its answer.

In summary, during JJKK’s session “Do we use time and divide by 5?” functions to:

- discuss how to perform calculations;
- check the group’s answer.

Summary

Overall, it seems that a posed problem can play a number of roles. In general terms, a posed problem may function to:

- generate discussion;
- estimate;
- discuss strategy;
- spur debate;
- focus attention;
- confirm agreement;
- make an objection;
- prompt action;
- narrate/check ongoing calculations;
- remind group about problem parameters;
- check an answer.

The role a posed problem plays depends on the boundaries set by the observer. For instance, consider the boundary of time (i.e. from what point in time the problem is being regarded). In the immediate present, a problem might seem to be posed as a suggestion, but in hindsight it might appear to be posed as a warning about a parameter of the task that the group has failed to consider. Another example of a boundary is that of perspective. From the perspective of the student who first poses the problem, the purpose might be to refer to what happened in a previous session. However, from the perspective of a group member who was absent from that session, the posed problem may offer an idea that is completely new. In short, the evolving role of the posed problem confirms the dialogic nature of the situation.

Bricolage as a mathematical process

A traditional way to define bricolage is as “working with what you have,” and here again the boundaries set by the perspective of the observer come into play. For researchers, this ability to recognize what it is that groups "have" will depend on how familiar they are with the groups they are observing. In the case of this study, I have access to video and audio recordings from the study of certain groups working on tasks prior to the Bill Nye one (see Appendix A) and so I have some idea of the recent history of the groups; through informal discussions with Mrs. Shug I have an idea of what topics the students were currently working on in their regular mathematics classes; I could hear any references that students made to these, or other, past experiences as they worked, and I could see the tools students were using during the Bill Nye task[52].

From my point of view, the groups all have the following with which to work. All four groups have been taking mathematics classes from Mrs. Shug all year. They are all present for a session in a previous week where they worked on the Power Outage[53] task. In their current regular mathematics lessons, they are all studying square numbers. The members of NIJM, REGL and DATM (with the exception of Meredith of DATM) are all present for Mrs. Shug's introduction to the Bill Nye task which advises the class to think "outside the box" (It should be noted that Mrs. Shug does not include this instruction in the introduction to the task that she gives to JJKK's class, which is the first of the two classes to be assigned the task). All of these might be considered to be experiences these groups "have." It is interesting though that not all of the groups use these experiences in the same way or, in some cases, at all.

During their sessions, both REGL and NIJM refer to Mrs. Shug's instructions to the class. REGL appears to take her advice to heart, posing and reposing "How do we think outside the box?" seven times. This makes this problem the one REGL poses the fourth most often (Figure 6.4b) not far off from its most frequent problem, "What are the factors of x?" which is posed 11 times. As mentioned in Chapter 6, "How do we think outside the box?" provides a kind

[52] The students mainly work with pencil and paper. There is a wall multiplication chart that at times some students from REGL and DATM appear to be looking at (JJKK and NJIM are seated on the other side of the classroom), but no direct references are made to this chart in any of the sessions.
[53] See Appendix A.

of framework for REGL's discussion, seeming to guide them to discuss topics at a deeper level. NIJM poses the problem "How do we think outside the box?" three times, once when the group is just starting to discuss "What are the factors of x?" and twice at the very end of the session when it has an answer and is wondering if there might be another way to solve the task. The third of the groups who heard Mrs. Shug's instructions, DATM, does not pose "How do we think outside the box?" at all[54].

DATM is the only group to pose the "How can we use the 24 hour clock?" problem, one that appears to be based on the solution to the Power Outage task that the class worked on during a previous session. Amaya makes it a part of her calculations from the beginning of the session, and towards the end of the session, when she is trying to explain to Derek how she calculated the number of intervals, the group (minus the absent Meredith) ends up discussing how one can physically count around a clock to determine the same thing.

Finally, NIJM is one of two groups (JJKK is the other) to refer to the square roots topic the class is currently studying in its regular mathematics lessons. The group first bases a prediction about the answer on it:

> I: This is square roots. It's square roots guys.
> M: Oh… I see, I see what they're doing.
> I: Since we're learning about square roots, I'm pretty sure it's about square roots.

The problem is dropped temporarily until the group determines the number of five minute intervals. Then it re-emerges.

[54] It is interesting that all four groups converge on the same solution (i.e. 8 people and 7 gifts each) although the wording of the Bill Nye task does not delimit other factors of 56 from being an answer (e.g. 14 people with 4 gifts each). Despite much discussion about the nature of the gift-giving and various problems, such as "Does everyone get a gift?" "Do all members give to everyone?" "Do they also bring gifts for themselves?" and "What if someone doesn't get a gift?" being posed, there is minimal divergence in the groups' imagining of what kind of party this might be. Social conventions appear to be an influence – as NIJM notes at one point, "Who would buy themselves a present?" – and perhaps the convention of 'normally one correct answer to a school mathematics task' is in play as well.

> M: Who has a calculator? It's a decimal We need a calcu, yeah, wait, you can't. That's not a square root, then. You need a decimal.
> N: It's not a square root. We already knew that.

Although the problem seems to be dismissed, it comes up again as the group double-checks its ideas.

> N: Okay. Let's divide fifty-six. (?)
> I: Can I have the calculator? [N takes calculator from her pencil case but uses it herself]
> N: I just want to find the square root of fifty-six. It will be point something.
> I: Yeah, it's going to be.
> N: Fifty-six. Seven point four eight three three.
> I: Seven point five.
> N: Yeah [N puts her calculator back in the pencil case].
> I: We can't have seven and a half people going to the party though!
> [Laughter]
> M: It could be like the show *Two and a Half Men.*

This problem, as a method of determining factors, helps NIJM to put some parameters on possible answers (i.e. they need to be whole numbers). In contrast, JJKK does not mention the classroom connection when discussing the square roots. ("Oh! I got it! I got it, I got it, I got it. We're supposed to find the square root. Yeah, we're supposed to find the square root."). JJKK is not concerned about the decimal that it calculates as a result, and never explicitly rejects the square root idea, even when it determines the answer to be 8 club members each bringing 7 gifts.

It may be sorely tempting for mathematics teachers to try to frontload facts and algorithms to their students before assigning them problems to solve, hoping that this will ensure that the students are fully prepared with everything they will need to be successful in their mathematics tasks. Yet, as teachers inevitably discover, and as we can see from the groups described above, just because students have been exposed to mathematical facts and ideas, does not mean they will draw on them when it might be useful or appropriate. For every group like NIJM and REGL who worries whether they have considered Mrs. Shug's instruction to "think outside the box" enough, there's another group like DATM who appears to ignore the idea completely. Or consider JJKK, a group who works quite closely with Mrs. Shug through approximately half of its session. Even when she poses a problem for the group to consider, one that seems likely be worthwhile to pursue given that it has been suggested by the teacher, JJKK does not take it up and continues posing its own problems instead. These examples point past bricolage being considered to be a process of "working with what you have," to being a process of "triggering what you have." Consider NIJM and REGL. Although the two groups pose a number of the same problems (Figure 7.1) their tapestry patterns turn out to be quite different. For instance, NIJM decides on a direction right away, whereas REGL offers a number of ideas in a short period of time before going deeper into discussions. Although REGL poses "Do we use time and divide by 5?" almost as many times as NIJM does, it spends less time actually discussing the calculations, perhaps because its method (dividing the total number of minutes by five) takes less time than does the counting method that dominates NIJM's work, or perhaps because REGL finds other problems more engaging[55]. As well, REGL poses other problems at the same time it considers "Do we use time and divide by 5?" which helps to maintain a thready pattern of different colors. As a result, the lavender color appears less prominently in REGL's tapestry than it does in NIJM's. Another difference stems from REGL missing the factors of 7 and 8 in its initial calculations, generating a round of reposing problems as the group tries to figure out what it might be missing.

The different tapestry patterns that develop for each of the groups suggest the emergent nature of bricolage. The starting task may be the same, but as each group works, the context evolves, the needs change, and different patterns of problems develop resulting in tapestries as unique as an individual person's fingerprints.

[55] According to Figure 6.3b, "Do we use time and divide time by 5?" emerges for NIJM 11 times, far more often than other problems they pose (4 is the next highest number). In comparison, according to Figure 6.4b, REGL poses "Do we use time and divide by 5?" 10 times, but also poses "What are the factors of x?" 11 times, "Does everyone bring the same amount of gifts?" eight times, and "How do we think outside the box?" seven times. This suggests that REGL may find a wider number of problems to be of interest.

Chapter 8: Conclusions

This dissertation examines the behavior of small groups of students engaged in collective problem posing in their grade 8 mathematics classrooms. At the beginning of this dissertation, the roles of authority and author/ity in mathematics classrooms are compared. In a traditional mathematics classroom, there is much authority vested in the textbook and the teacher as transmitters of knowledge. The concept of author/ity, however, in highlighting human agency in the creation of mathematics helps to position the student as a knowledge-maker, a potential author. I briefly explore who can be an author, and argue that a group as a collective learning system can be considered to be an author through the discourse it creates. I then characterize the process of authoring as a form of improvisation. The use of an improvisational framework highlights the relationship between structure and spontaneity in performing school mathematics, as well as the potential of students to work as bricoleurs, generating their own pathways as they tackle even structured tasks. I briefly explore the framework of story, suggesting that the problems posed by groups as they work parallel the development of a storyline. Noting the dialogic nature of collective discussion, I propose the tapestry as a metaphor for framing group discourse and describe a way of blurring the data in order to create tapestries based on transcriptions that visually depict the storylines created over time by the groups.

Summary of findings

In this section, I discuss my findings for each of the research questions: What problem posing patterns emerge as small groups of students work collectively on a mathematics task, and what are the characteristics of problem posing as a collective process? As is perhaps appropriate for a dissertation about this topic, the process of pursuing these particular research questions means the emergence of new problems for consideration. I discuss some of these and suggest possible directions for further research.

What problem posing patterns emerge as small groups of students work collectively on a mathematics task?

In tackling this question, there are two challenges: problem posing needs to be documented at the level of the collective, and as a process rather than a product (i.e. a list of posed problems) so that the emerging patterns can be traced. To do so, I employ a method of blurring the data by introducing the element of time. The result is the creation of a colored tapestry of each group's session, which makes the patterns visible and shows the problem posing of each group, as an agent, over time.

As the tapestries show at a glance, each group follows its own path in working on the task. For instance, a different set of problems emerges for each of the four groups. For those of us who are used to how textbooks often set out rich problems with a set of related problems that are meant to guide the reader, it may come as a bit of a surprise that a fairly structured task like Bill Nye can generate a variety of problems, and that this variety can ultimately lead the groups to the same correct answer.

However, a few problems are posed by all four groups (Do we use time and divide by 5? What about if everyone brings x gifts each? Is there an extra 5 minutes? How many people are there?) from both of Mrs. Shug's mathematics classes. Does this common set of problems indicate that there are certain problems that must be posed in order to solve a specific task? Or is a commonly posed set of problems more dependent on a context shared by the groups? It would be interesting to see if a common set of posed problems would still emerge in a study with groups from a wider variety of settings. If it did, this could lend support to the textbook practice of providing specific guiding questions. If it did not, one might wonder if the textbook's guiding questions might actually interfere with the readers' own thinking about the task.

Another pattern that emerges from the data is how each group poses a different number of problems. The difference between the number of individual problems posed is not large, ranging from 12 (JJKK) to 17 (NIJM), and this raises some questions. How might changing the structure of the task affect this range? Would a more open task offer groups room to pose a greater variety of problems? Or is the number of individual problems posed more indicative of how well a group works together? Perhaps a more cohesive group would be more willing to take risks in posing problems, and thus pose more of them.

Perhaps more striking than the number of individual problems posed is the total number of problems posed overall, including reposed problems. Here the range between the groups widens considerably, with JJKK posing 23, NIJM posing 45, DATM posing 61, and REGL posing 66. One might posit that the difference is due to each group's "personality." For example, REGL, who tends to explore concepts more deeply and connect ideas more frequently than the other groups, poses more problems than JJKK, who tends to argue about one problem at a time until a consensus appears to be reached. The difference between the groups also may be due to their varying levels of confidence in their mathematical abilities. At the beginning of its session, when JJKK poses a problem, the purpose seems to be to clarify what mathematics is being performed. Once the group reaches an agreement about how to proceed, there appears to be no reason to repose the problem. For REGL, problems are posed in a more interwoven manner, as the group tends to discuss less of the "how to" and more of the "why" of the task at hand. This raises issues about what is most significant about problem posing – which particular problems are raised, how often they are raised, when they are raised, or in what combination? While some problem posing studies in the literature have focused on the number of problems posed, or the quality of problems posed, my findings suggest that the pattern in which problems are both posed and reposed may ultimately tell us more about students' mathematical behavior and understanding.

Another noteworthy finding is that problems do not emerge in the same order for each of the groups. For instance, three groups begin with operational problems[56] (NIJM, JJKK and REGL) whereas one (DATM) begins with a problem that is more interpretive[57] of the task. The varied ways in which groups in this study approach the Bill Nye task may suggest that educators need to be careful of presenting problem solving heuristics as lock-step procedures to be followed in a specific order. It would be interesting to explore if the order is a matter of chance (whatever problem happened to be uttered first), an indication of the group's level of understanding of the task's parameters, or a routine that particular group has developed for addressing tasks (for example, a group might always read a new task aloud to see if everyone understands what it means). It would also be valuable to explore the patterns of investigative and operational problems that emerge as the groups work. If it becomes evident that mathematically stronger groups do follow certain types of problem posing pattern, then it might be important for educators to guide weaker groups to develop similar routines.

Problems reemerge, and they do so with different frequencies for different groups, and for different lengths of discussion. This also contributes to the uniqueness of the tapestry patterns that emerge for each group. It would be interesting to know if a group has, or develops over time, its own style, visually represented by a type of tapestry pattern. Does each group truly have a unique fingerprint? If so, this might help to promote the view of school mathematics as a creative subject.

The tapestry patterns offer an initial indication of the type of discussion taking place for each group. A chunky pattern suggests that each posed problem is being dealt with at length. A

[56] This would be a problem that is about how to solve the task.
[57] This would be a problem having to do with understanding the wording of the task.

thready pattern suggests that the group is considering more than one problem at a time – either that problems are being thrown out for the consideration of the group (for instance, as a way of generating a starting point at the beginning of a session, or as a way of checking an answer later on), or that problems are being connected or compared in some way. However, it might also indicate that group members are arguing about what to do next (as in the case of DATM, when Amaya and Derek cannot seem to agree about where to start) or perhaps trying to push for a particular problem without listening to each other's opinion. It is important, then, when working with tapestries, to keep checking back with the original transcript so that the content of the discussion is kept in mind during the analysis.

What are the characteristics of problem posing as a collective process?

One of the most basic things for which this case study provides evidence is the ability of groups to problem pose collectively without having been directed to do so, and without having received any formal instructions on how to do so. In a sense, this addresses Tahta's concern about problem posing potentially becoming an exercise in compiling a meaningless list of problems (see Chapter 5). In this study, the groups are problem posing with a purpose.

This study also suggests that problem posing is a generative process. Even for JJKK, with its tendency to deal with one problem at a time, problems trigger the posing of other problems, and the group remains motivated and busy until close to the end of the session when the members each drift towards writing up her own solution.

The reposing of problems proves to be a very important aspect of the problem posing process. It is particularly interesting to note how the purpose of each problem evolves as the context of the discussion and the needs of the group change. For instance, a problem may be posed to suggest a method early in the session, and later it may be reposed to check a possible solution.

Even though the four groups have some common experiences with which to work, the fact that certain groups do not necessarily draw on these experiences, or if they do, do not do so in the same way as other groups, suggests that the process of bricolage is more than simply sitting down and “working with what you have.” It is more a matter of interplay between structure and spontaneity, a process that is non-linear, multidirectional and unpredictable.

Summary of significance

This dissertation contributes to the literature on group work by offering a description of collective behavior that works on the level of the group as an agent. It also describes an analysis technique for considering collective behavior that introduces time as an element, blurring the data in order to provide visual evidence of emergent problem posing patterns. The tapestries that are developed from the transcripts provide images of the storylines that develop, providing visual evidence of the different paths groups develop while working on the same mathematical task. These images might be of use to educators and preservice teachers in considering the ways in which their own students may approach activities.

As well, this dissertation addresses a gap in the problem posing literature by providing a description of the collective problem posing process, noting the patterns that may occur, how problems are reposed and how the role of these reposed problems evolves as the session continues. It suggests that perhaps the strength of problem posing is not the generation of a list of problems at the end of the task, but the emerging patterns of problems as the group’s discussion continues and how they help to structure pathways to a solution.

This dissertation adds to the growing literature about improvisation in education by considering how students, rather than teachers, work as improvisers and bricoleurs, and it develops the idea of improvisation as a continuum of behavior that occurs during mathematics activities. Future studies might build on these initial findings to better understand the worth of the improvisational framework in mathematics education.

Finally, as mentioned in Chapter 3, an issue for mathematics educators to consider is how we might best balance structure versus spontaneity in the mathematics classroom. My overall findings about collective problem posing patterns highlight the presence of spontaneity even in what appears to be a structured task. Although its limited scope necessarily precludes generalizations about the behavior of mathematics students, this study suggests the creative processes that may be occurring in our mathematics classes right under our very noses. What advantage might we take of them?

Chapter 9: Epilogue

> *All the mathematical methods and relationships that are now known and taught to schoolchildren started as questions, yet students do not see the questions. Instead, they are taught content that often appears as a long list of answers to questions that nobody has ever asked. (Boaler, 2008, p. 27)*

This dissertation begins with a discussion of the concept of author/ity. In a knowledge-making community students are aware of the fact that someone does author the mathematics in the books around them, and that in their work as bricoleurs they too are part of this authoring tradition. This may help to demystify math and to alleviate students' fear of the subject. As well, not only does the notion of author/ity foster a more equal relationship between learners and teacher, but it also gives responsibility to the students to break away from being passive empty vessels waiting to be filled with facts, and instead take on the role of making meaning of mathematics for themselves. What might happen if educators embraced author/ity?

When reading the stories of the four groups in this study, one might be tempted to judge them. Which of the groups does the task well? Are JJKK's ideas as valid as the efficient NIJM's? Which of the four groups is the best? Yet another way to frame this is to go back to the comparison with literary stories that started this dissertation: Would we treat these groups the same way if they were developing literary stories rather than mathematical solutions? I believe there is a degree of judgment in mathematics education, a subject that is conceived as being more "black and white" (or, you're either right or wrong) than other subjects, that is not found in the arts or the humanities. Perhaps one of the reasons for this has to do with the role of expertise in school mathematics.

Expertise is a valued commodity. An expert is someone who has a deep understanding of their field, someone from whom one can seek useful advice. To become an expert takes time: you need to know your craft, how to use your instrument, and you must be able to create a high quality of performance. An improvisational framework can help us to reconsider the nature of expertise and what purpose there may be to being an expert.

Mathematics suffocates from the idea of the expert. In coming across as a big wall of structured ideas, in a way that few other school subjects do (see, for example Buerk, 1982), mathematics makes it clear that you can never know everything, that only a very select few can achieve expert status. For many learners, mathematics is a subject that turns the popular saying "it's the journey, not the destination" on its head. When the final product is privileged over the process of getting to that final product, there comes an expectation that mathematical understanding will be instantaneous. As a result, even students whom others deem to be "mathematical" may be haunted by the sense that they do not really understand what they are doing if they do not get an answer immediately.

No one is an expert at everything. Expertise is defined by a certain set of boundaries around the subject matter. If you are an expert, you are an expert in a certain area. If you stay within those boundaries, you remain an expert. Those boundaries are usually shifting in one way or another over time. For instance, if you are a musician, your body is changing as you get older, so you need to keep practicing to stay in shape. Techniques change, and you may learn those. Other artists are trying new things; and you push the boundaries too. The true expert is in fact an innovator, pursuing what is known in Zen practice as "beginner's (or novice) mind."

The "expert" mind can become constricted with ideas of how things "should be" and, as composer Debussy once wrote, these experts may have "neither the will nor the courage to break with their successes, failing to seek new paths and give birth to new ideas…. They have neither the courage nor the temerity to leave what is certain for what is uncertain" (Romesburg, 2001, pp. 239-240). This can result in repeating old ideas rather than embracing the flux and possibilities of life. The beginner's mind, on the other hand, is fresh and accepting: "it is always ready for anything; it is open to everything. In the beginner's mind there are many possibilities; in the expert's mind there are few" (Suzuki, 1985, p. 21).

Where an expert might have the confidence and the ability to set up a mathematically problematic situation to solve where there are recognizable boundaries as well as new areas to explore, students in school mathematics classes are rarely able to do that, and may instead crave predictability and known patterns. In school, the teacher or the textbook sets the problem, and the student is expected to find the correct answer – the student is seldom assigned the role of expert, and rarely seeks out the role of innovator, not taking advantage of the possibilities of his/her own beginner's mind.

However, if the students are given space within a task that lies in their own area of expertise, no matter how limited this area may seem to an observer, this potentially sets them up in the role of the expert. They then have a chance to become an innovator, posing problems to explore this new territory. In describing a group of contact dancers, Debra Cash writes that they were "inventing new problems on the spot and pushing themselves to answer their own questions in ways that stressed and valorized the unexpected" (Cash, 2000, p. 179). Problem posing has the potential.

The improvisational nature of problem posing has other potential as well. In discussing how organizations might benefit from a more improvisational style of working, Weick suggests an "aesthetic of imperfection" whereby mistakes are embraced:

> Errors now become viewed as experiments from which people can learn,
> as oddities to be incorporated or made normal, as items to be isolated from

> ongoing processes so their effects will be localized, as inevitable when personal activity rather than an impersonal product is being assessed, as potentially the right notes for some other song, as an excuse to say "let it pass," as evidence that involvement is high, as transient flaws that will make sense as events unfold. (1995, p. 191)

As has been discussed earlier in this dissertation, problems may be posed when a conflict or a gap of understanding develops. When REGL is unable to find factors of 56 that solve the task, the group considers a different model of the task situation. To take a risk like this, a standard of assessment needs to be in place where the "rightness" of an answer is not the only criteria of success, and where mistakes are not an inhibition but an inspiration. In jazz, there is something called the "save" (Barrett, 2002), which occurs when a player plays a wrong note, but then they, or someone else in the collective swoops in and uses it as a springboard into something else. What is accidental is treated as intentional; "wrong" becomes "right," and the expert has made a discovery.

Treating mathematics as more a fluid subject, rather than a series of isolated lessons and units may help as well. In the case of REGL's gift exchange model, what I as the researcher/observer have designated as the start and the stop times of the group's discussion is what defines this particular session of group work. In this session, we see that there is an attempt at a "save" made by REGL in declaring the gift exchange idea to be a "good theory," but there is no evidence of the model acting as a springboard for the group to another idea. However, were we able to extend the time boundaries into the future and include different locations of discussion, the situation might be different. Perhaps later on REGL will remember the gift exchange model, and realize that it might fit a new mathematical task that is being investigated. Perhaps, the group might figure out what it was about the gift exchange model that did not fit the task parameters, and that in turn might trigger an idea about how to solve another problem.

Expanding the time and situational parameters, then, is another way that what is "wrong" ultimately may turn out to be "right."

In the opening of this dissertation, I described how students may feel alienated from school mathematics because of its reputation. It is my hope that what I have written here might spark the reader to start to problematize that reputation. School mathematics has more in common with the humanities and the arts than many realize, it just takes a different perspective to appreciate it. There is a story here that needs to be told, and my hope is that this dissertation helps to trigger a new storyline.

Bibliography

Altieri, J. (2005). Creating poetry: Reinforcing mathematical concepts. *Teaching Children Mathematics, 12*(1), 18-25.

Armstrong, A. (2008). The fragility of group flow: The experiences of two small groups in a middle school mathematics classroom. *The Journal of Mathematical Behavior, 27*(2), 101-115.

Arrow, H. M., McGrath, J., & Berdahl, J. (2000). *Small groups as complex systems: Formation, coordination, development, and adaptation*. Thousand Oaks, CA: Sage.

Baines, E., Blatchford, P., & Kutnick, P. (2007). Pupil grouping for learning: Developing a social pedagogy of the classroom. In R. M. Gillies, A. F. Ashman & J. Terwel (Eds.), *The teacher's role in implementing cooperative learning in the classroom* (Vol. 7, pp. 56-72): Springer.

Bakhtin, M. (1981). *The dialogic imagination* (C. Emerson & M. Holquist, Trans.). Austin, Texas: University of Texas Press.

Bakhtin, M. (1986). *Speech genres and other late essays* (V. W. McGee, Trans.). Austin, Texas: University of Texas Press.

Banwell, C., Saunders, K., & Tahta, D. (1972). *Starting points: For teaching mathematics in middle and secondary schools*. London, UK: Oxford University Press.

Barker, C. (Ed.) (2004) The SAGE Dictionary of Cultural Studies. London: SAGE.

Barnes, D. R., & Todd, F. (1995). *Communication and learning revisited: Making meaning through talk*. Portsmouth, NH: Boynton/Cook Publishers.

Barrett, F. J. (2002). Creativity and improvisation in jazz and organizations. Implications for organizational learning. In K. N. Kamoche, M. P. E. Cunha & J. V. D. Cunha (Eds.), *Organizational improvisation* (pp. 135-162). London: Routledge.

Barron, B. (2003). When smart groups fail. *The Journal of the Learning Sciences, 12*(3), 307-359.

Barthes, R. (1968). *The death of the author*. Aspen, (5&6).

Bastien, D. T., & Hostager, T. J. (1988). Jazz as a process of organizational innovation. *Communication Research*, *15*(6), 582-602.

Bateson, M. C. (2001). *Composing a life*. New York: Grove Press.

Bauersfeld, H. (1994). Theoretical perspectives on interaction in the mathematics classroom. In R. Biehler, R. W. Scholz, R. Staber & B. Winkelmann (Eds.), *Didactics of mathematics as a scientific discipline*. Dordrecht, The Netherlands: Kluwer Academic Publisher.

Baxter, J. A. (2005). Some reflections on problem posing: A conversation with Marion Walter. *Teaching Children Mathematics, 12*(3), 122-128.

Baxter, J. A., Woodward, J., Olson, D., & Robyns, J. (2002). Blueprint for writing in middle school mathematics. *Mathematics Teaching in the Middle School, 8*(1), 52-56.

Becker, H. S. (2000). The etiquette of improvisation. *Mind, Culture and Activity, 7*(3), 171-176.

Berliner, P. (1994). *Thinking in jazz*. Chicago, IL: University of Chicago Press.

Boaler, J. (2008). *What's math got to do with it? Helping children learn to love their least favorite subject - and why it's important for america*. New York, New York: Viking.

Borko, H., & Livingston, C. (1989). Cognition and improvisation: Differences in mathematical instruction by expert and novice teachers. *American Educational Research Journal*, *26*(4), 473-498.

Borwein, J. (2006). Implications of experimental mathematics for the philosophy of mathematics. In B. Gold & R. Simons (Eds.), *Current issues in the philosophy of mathematics from the viewpoint of mathematicians and teachers of mathematics*: Mathematical Association of America.

Bowers, J. S., & Nickerson, S. (2001). Identifying cyclic patterns of interaction to study individual and collective learning. *Mathematical Thinking and Learning, 3*(1), 1-28.

Brown, S. I., & Walter, M. I. (2005). *The art of problem posing*. Mahwah, NJ: Lawrence Erlbaum Associates.

Brown, T. (1996). Intention and significance in the teaching and learning of mathematics. *Journal for Research in Mathematics Education, 27*(1), 52-66.
Buerk, D. (1982). An experience with some able women who avoid mathematics. *For the Learning of Mathematics, 3*(2), 19-24.
Burton, L. (2004). *Mathematicians as enquirers. Learning about learning mathematics*. Norwell, Massachusetts: Kluwer Academic Publishers.
Cai, J., & Hwang, S. (2002). Generalized and generative thinking in US and Chinese students' mathematical problem solving and problem posing. *Journal of Mathematical Behavior, 21*, 401-421.
Cash, D. (2000). Response to Becker's "The etiquette of improvisation". *Mind, Culture and Activity, 7*(3), 177-179.
Cifarelli, V. V., & Cai, J. (2005). The evolution of mathematical explorations in open-ended problem-solving situations. *The Journal of Mathematical Behavior, 24*(3-4), 302-324.
Cifarelli, V. V., & Cai, J. (2006). *The role of self-generated problem posing in mathematics exploration.* Paper presented at the 30th Conference of the International Group for the Psychology of Mathematics Education, Prague.
Cobb, P., Yackel, E., & Wood, T. (1992). Interaction and learning in mathematics classroom situations. *Educational Studies in Mathematics, 23*(1), 99-122.
Cohen, E. G. (1994). Restructuring the classroom: Conditions for productive small groups. *Review of Educational Research, 64*(1), 1-35.
Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education* (6th ed.). New York, NY: Routledge.
Crespo, S. (2003a). Learning to pose mathematical problems: Exploring changes in preservice teachers' practices. *Educational Studies in Mathematics, 52*, 243-270.
Crespo, S. (2003b). Using math pen-pal letters to promote mathematical communication. *Teaching Children Mathematics, 10*, 34-39.
Crespo, S., & Sinclair, N. (2008). What makes a problem mathematically interesting? Inviting prospective teachers to pose better problems. *Journal of Mathematics Teacher Education, 11*(5), 395-415.
Davis, B., & Simmt, E. (2003). Understanding learning systems: Mathematics education and complexity science. *Journal for Research in Mathematics Education, 34*(2), 137-167.
Davis, B., & Sumara, D. J. (2005). Complexity science and educational action research: Toward a pragmatics of transformation. *Educational Action Research, 13*(3), 453-464.
Davis, B., & Sumara, D. J. (2006). *Complexity and education. Inquiries into learning, teaching, and research*. New York: Lawrence Erlbaum Associates.
Drabble, M. (1985). Romeo and juliet. In M. Drabble (Ed.), *The Oxford Companion to English Literature* (5th ed., pp. 1155). Oxford: Oxford University Press.
Duncker, K. (1945). On problem-solving. *Psychological Monographs, 58*(5).
Edwards, B. (1999). *The new drawing on the right side of the brain*. New York, NY: Jeremy P. Tarcher/Putnam.
Edwards, D., & Mercer, N. (1987). *Common knowledge. Development of understanding in the classroom*. London: Metheun & Co., Ltd.
Ellerton, N. (1986). Children's made-up mathematics problems—a new perspective on talented mathematicians. *Educational Studies in Mathematics, 17*(3), 261-271.
Engeström, Y. (1994). Teachers as collaborative thinkers: Activity-theoretical study of an innovative teacher team. In I. Cargren, G. Handal & S. Vaage (Eds.), *Teachers' minds and actions: Research on teachers' thinking and practice* (pp. 43-61). Bristol, PA: Farmer.
English, L. D. (1997). Promoting a problem posing classroom. *Teaching Children Mathematics*.
English, L. D. (1998). Children's problem posing within formal and informal contexts. *Journal for Research in Mathematics Education, 29*(1), 83-106.
English, L. D., & Lesh, R. A. (2003). Ends-in-view problems. In R. A. Lesh & H. M. Doerr (Eds.), *Beyond constructivism. Models and modeling perspectives on mathematics problem solving, learning and teaching* (pp. 297-316). Mahwah, NJ: Lawrence Erlbaum Associates, Inc., Publishers.

Ernest, P. (1998). *Social constructivism as a philosophy of mathematics*. Albany, NY: State University of New York Press.
Fernsten, L. A. (2007). A writing workshop in mathematics: Community practice of content discourse. *Mathematics Teacher, 101*(4), 273-278.
Flannery, S. (2002). *In code: A mathematical journey*. Chapel Hill, N.C.: Algonquin Books.
Foucault, M., & Rabinow, P. (1984). What is an author? In P. Rabinow (Ed.), *The Foucault reader* (pp. 101-120). New York, NY: Pantheon.
Freytag, G. (1900). *Technique of the drama. An exposition of dramatic composition and art*. Chicago, IL: Scott, Foresman and Company.
Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research* (1st ed.). Chicago: Aldine Publishing Company.
Graves, D. R. (1994). *Life drawing in charcoal*. Mineola, NY: Dover Publications.
Grossen, M. (2009). Social interaction, discourse and learning. In K. Kumpulainen, C. E. Hmelo-Silver & M. César (Eds.), *Investigating classroom interaction: Methodologies in action* (pp. 263-275). Rotterdam: Sense Publishers.
Hargreaves, D. J., Cork, A. C., & Setton, T. (1991). Cognitive strategies in jazz improvisation: An exploratory study. *Canadian Journal of Research in Music Education, 33*, 47-54.
Hatton, E. (1989). Lévi-Strauss's "bricolage" and theorizing teachers' work. *Anthropology and Education Quarterly, 20*(2), 74-96.
Healy, L., & Sinclair, N. (2007). If this is our mathematics, what are our stories. *International Journal of Computers for Mathematical Learning, 12*(1), 3-21.
Helton, S. M. (1995). I thik the citanre will hoder lase: Journal keeping in mathematics class. *Teaching Children Mathematics, 1*(6), 336-340.
Herbel-Eisenmann, B., & Wagner, D. (2007). A framework for uncovering the way a textbook may position the mathematics learner. *For the Learning of Mathematics, 27*(2), 8-14.
Hershkowitz, R., Schwarz, B., & Dreyfus, T. (2001). Abstraction in context: Epistemic actions. *Journal for Research in Mathematics Education, 32*(2), 195-222.
Hewitt, D. (1996). Mathematical fluency: The nature of practice and the role of subordination. *For the Learning of Mathematics, 16*(2), 28-35.
Janesick, V. J. (2003). The choreography of qualitative research design. Minuets, improvisations, and crystallization. In N. K. Denzin & Y. S. Lincoln (Eds.), *Strategies of qualitative inquiry* (pp. 46-79). Thousand Oaks, CA: SAGE.
Johnson, D. W., & Johnson, R. T. (2009). An educational psychology success story: Social interdependence theory and cooperative learning. *Educational Researcher, 38*(5), 365-379.
Johnson, N. (2007). *Simply complexity. A clear guide to complexity theory*. Oxford, UK: Oneworld Publications.
Kilpatrick, J. (1987). Problem formulating: Where do good problems come from? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 123-148). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc., Publishers.
King, K. D. (2001). Conceptually-orientated mathematics teacher development: Improvisation as a metaphor. *For the Learning of Mathematics, 21*(3), 9-15.
Lafortune, L., Daniel, M.-F., Pallascio, R., & Sykes, P. (1996). Community of inquiry in mathematics for higher education. *Analytic Teaching, 16*(2), 82-89.
Lakatos, I. (1976). *Proofs and refutations. The logic of mathematical discovery*. Cambridge: Cambridge University Press.
Lakoff, G., & Núnez, R. E. (2000). *Where mathematics come from: How the embodied mind brings mathematics into being*. New York: Basic Books.
Lehrer, J. (2007). *Proust was a neuroscientist*. New York, NY: Houghton Mifflin Harcourt.
Lester, F. K. (1994). Musings about mathematical problem-solving research: 1970-1994. *Journal for Research in Mathematics Education, 25*(6), 660-675.
Leung, S. S. (1993). *The influence of task formats, mathematics knowledge, and creative thinking.* Paper presented at the The 17th International Conference of the International Group for the Psychology of Mathematics Education, Tsukuba, Japan.

Leung, S. S., & Silver, E. A. (1997). The role of task format, mathematics knowledge, and creative thinking on the arithmetic problem posing of prospective elementary school teachers. *Mathematics Education Research Journal*, *9*(1), 5-24.

Levi-Strauss, C. (1966). *The savage mind*. Chicago: University of Chicago Press.

Lewis, T., Petrina, S., & Hill, A. M. (1998). Problem posing - adding a creative increment to technological problem solving. *Journal of Industrial Teacher Education, 36*(1). Retrieved from http://scholar.lib.vt.edu/ejournals/JITE/v36n1/lewis.html

Maheux, J.-F., & Lajoie, C. (2010). On improvisation in teaching and teacher education. *Complicity: An International Journal of Complexity and Education, 8*(2), 86-92.

Martin, L., Towers, J., & Pirie, S. E. B. (2006). Collective mathematical understanding as improvisation. *Mathematical Thinking and Learning*, *8*(2), 149-183.

Mashaal, M. (2006). *Bourbaki: A secret society of mathematicians* (A. Pierrehumbert, Trans.). Providence, RI: American Mathematical Soceity.

McArthur, T. (1992a). Author. In T. McArthur (Ed.), *The Oxford comparion to the English language* (pp. 98). Oxford, UK: Oxford University Press.

McArthur, T. (1992b). Story. In T. McArthur (Ed.), *The Oxford companion to the English language* (pp. 987). Oxford, UK: Oxford University Press.

McCutcheon, M. (1995). *Roget's super thesaurus*. Cincinnati, OH: Writer's Digest Books.

McDermott, R. P. (1980). Profile: Ray. L. Birdwhistell. *Kinesis Report, 2*(3), 1-4; 14-16.

McDermott, R. P. (1996). Acquisition of a child by a learning disability. In S. Chaiklin & J. Lave (Eds.), *Understanding practice: Perspectives on activity and content* (pp. 269-305). Cambridge, UK: Cambridge University Press.

Mercer, N., & Littleton, K. (2007). *Dialogue and the development of children's thinking. A sociocultural approach*. Routledge: New York, NY.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Thousand Oaks, CA: SAGE Publication.

Montuori, A. (2003). The complexity of improvisation and the improvisation of complexity: Social science, art and creativity. *Human Relations, 56*(2), 237-255.

Nash, W. (1992). Story line. In T. McArthur (Ed.), *The Oxford companion to the English language* (pp. 987). Oxford, UK: Oxford University Press.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Albert, L. R., & Antos, J. (2000). Daily journals. *Mathematics Teaching in the Middle School, 5*(8), 526-531

Neyland, J. (2004a). Playing outside: An introduction to the jazz metaphor in mathematics education. *Australian Senior Mathematics Journal*, *18*(2), 8-16.

Neyland, J. (2004b). *Social justice and the jazz metaphor*. Paper presented at the MERGA 27, Townsville, Australia.

Neyland, J. (2010). *Effortless mastery and the jazz metaphor*. Paper presented at the MERGA 33, Fremantle, WA, Australia.

Noddings, N. (1989). Theoretical and practical concerns about small groups in mathematics. *The Elementary School Journal*, *89*(5), 606-623.

Papert, S. (-1980). *Mindstorms* (1st ed.). New York: Basic Books Inc.

Pirie, S. E. B. (1996). *Classroom video-recording: When, why and how does it offer a valuable data sourse for qualitative research?* Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Panama City, FL.

Pirie, S. E. B. (2002, October 26-29, 2002). *Problem posing: What can it tell us about students' mathematical understanding?* Paper presented at the Annual Meeting of the North American Cahpter of the International Group for the Psychology of Mathematics Education, Athens, GA.

Pólya, G. (1957). *How to solve it. A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.

Pólya, G. (1990 (1954)). *Mathematics and plausible reasoning* (Vol. 1: Induction and Analogy in Mathematics). Princeton, N.J.: Princeton University press.

Povey, H., Burton, L., Angier, C., & Boylan, M. (1999). Learners as authors in the mathematics classroom. In L. Burton (Ed.), *Learning mathematics: From hierarchies to networks* (pp. 232-245). London: Falmer Press.

Reilly, M. A. (2009). Opening spaces of possibility: The teacher as bricoleur. *Journal of Adolescent and Adult Literacy, 52*(5), 376-384.

Remillard, J. T. (1999). Curriculum materials in mathematics education reform: A framework for examining teachers' curriculum development. *Curriculum Inquiry, 29*, 315-342.

Reznitskaya, A., Luo, L., Clark, A.-M., Miller, B., Jadallah, M., Anderson, R. C., . . . Jahiel, K. (2009). Collaborative reasoning: A dialogic approach to group discussions. *Cambridge Journal of Education, 39*(1), 29-48.

Ribeiro, C. M., Monteiro, R., & Carrilo, J. (2009). *Professional knowledge in an improvisation episode: The importance of a cognitive model*. Paper presented at the Proceedings of Sixth Congress of the European Society for Research in Mathematics Education, Lyon, France.

Romesburg, H. (2001). *The life of the creative spirit*. Philadelphia, PA: Xlibris Corporation.

Roschelle, J., & Teasley, S. D. (1994). The construction of shared knowledge in collaborative problem solving. *NATO ASI Series F computer and systems sciences, 128*, 69-97.

Roth, W.-M. (1995). From "wiggly structures" to "unshaky towers": Problem framing, solution finiding and negotation of courses of actions during a civil engineering unit for elementary students. *Research in Science Education, 25*(4), 365-381.

Rotman, B. (2006). The semiotics of mathematics. In R. Hersh (Ed.), *18 unconventional essays on the nature of mathematics* (pp. 97-127). New York: Spring Science+Business Media, Inc.

Sassi, A. M., & Goldsmith, L. T. (1995). *Beyond recipes and behind the magic: Mathematics teaching as improvisation*. Paper presented at the North American Chapter of the International Group for the Psychology of Mathematics Education, Columbus, OH.

Sawyer, R. K. (2000a). Improvisation and the creative process: Dewey, collingwood, and the aesthetics of spontaneity. *The Journal of Aesthetics and Art Criticism, 58*(2), 149-161.

Sawyer, R. K. (2000b). Improvisational cultures: Collaborative emergence and creativity in improvisation. *Mind, Culture and Activity, 7*(3), 180-185.

Sawyer, R. K. (2003). *Group creativity. Music, theater, collaboration*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Scribner, J. P. (2005). The problems of practice: Bricolage as a metaphor for teachers' work and learning. *The Alberta Journal of Educational Research, 51*(4), 295-310.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics, 22*, 1-36.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19-28.

Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27*(5), 521-539.

Silver, E. A., & Cai, J. (2005). Assessing students' mathematical problem posing. *Teaching Children Mathematics, 12*(3), 129-135.

Silver, E. A., Mamona-Downs, J., Leung, S. S., & Kenney, P. A. (1996). Posing mathematical problems: An exploratory study. *Journal for Research in Mathematics Education, 27*(3), 293-309.

Silver, E. A., & Marshall, S. P. (1989). Mathematical and scientific problem solving: Findings, issues, and instructional implications. In B. F. Jones & L. Idol (Eds.), *Dimensions of thinking and cognitive instruction* (pp. 265-290). Hillsdale, N.J.: Lawrence Erlbaum Associates, Inc., Publishers.

Silver, E. A., & Shapiro, L. J. (1992). Examinations of situation-based reasoning and sense-making in students' interpretations of solutions to a mathematics story problem. In J. P. Ponte, J. F. Matos, J. M. Matos & D. Fernandes (Eds.), *Mathematical problem solving and new information technologies* (Vol. 89, pp. 113-123). Berlin: Springer-Verlag.

Sinclair, N. (2008). Attending to the aesthetic in the mathematics classroom. *For the Learning of Mathematics, 28*(1), 29-35.

Sinclair, N., Healy, L., & Sales, C. O. R. (2009). Time for telling stories: Narrative thinking with dynamic geometry. *ZDM Mathematics Education, 41*, 441-452.
Stahl, G. G. (2006). *Group cognition: Computer support for building collaborative knowledge*. Cambridge: MIT Press.
Stake, R. (2005). Qualitative case studies. In N. K. Denzin & Y. S. Lincoln (Eds.), *The handbook of qualitative research* (pp. 443-465). Thousand Oaks, CA: SAGE.
Suzuki, S. (1985). *Zen mind, beginner's mind*. New York: John Weatherhill, Inc.
Tahta, D. (1984). Comment. [letter to the editors]. *For the Learning of Mathematics, 4*(2), 32.
Towers, J., & Martin, L. (2009). The emergence of a "better" idea: Preservice teachers' growing understanding of mathematics for teaching. *For the Learning of Mathematics, 29*(3), 44-48.
Turkle, S., & Papert, S. (1990). Epistemological pluralism: Styles and voices within computer culture. *Signs: Journal of Women in Culture and Study, 16*(1), 128-157.
Vera, D., & Crossan, M. M. (2004). Theatrical improvisation: Lessons for organizations. *Organization Studies, 25*(5), 727-749.
Voigt, J. (1995). Thematic patterns of interaction and sociomathematical norms. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 163-201). Hillsdale, NJ: Lawrence Erlbaum Associates.
Voigt, J. (1996). Negotiation of mathematical meaning in classroom processes: Social interaction and learning mathematics. In L. Steffe, P. Nesher, P. Cobb, G. Goldin & B. Greer (Eds.), *Theories of mathematical learning* (pp. 21-50). Mahmah, New Jersey: Lawrence Erlbaum.
Wadlington, E., & Hicks, K. (1994). *Using process writing to teach problem solving to middle school and junior high students*. Paper presented at the Conference of the Association for Childhood Education International Study, New Orleans, LA.
Watson, A., & Mason, J. (2005). *Mathematics as a constructive activity. Learners generating examples*. Mahwah, NJ: Lawrence Erlbaum Associates.
Webb, N. M., Franke, M. L., De, T., Chan, A. G., Freund, D., Shien, P., & Melkonian, D. K. (2009). "Explain to your partner": Teachers' instructional practices and students' dialogue in small groups. *Cambridge Journal of Education, 39*(1), 49-70.
Webb, N. M., & Palincsar, A. S. (1996). Group processes in the classroom. In D. C. Berliner & R. C. Calfee (Eds.), *Handbook of educational psychology* (pp. 841-873). New York, NY: Macmillan Library Reference USA.
Wegerif, R. (2010). Dialogue and teaching thinking with technology. Opening, expanding and deepening the "interface". In K. Littleton & C. Howe (Eds.), *Educational dialogues. Understanding and promoting productive interaction* (pp. 304-322). London, UK: Routledge.
Weick, K. E. (1995). Creativity and the aesthetics of imperfection. In C. M. Ford & D. A. Gioia (Eds.), *Creative action in organizations. Ivory tower visions and real world voices* (pp. 187-194). Thousand Oaks, CA: Sage Publications, Inc.
Weick, K. E. (2002). Improvisation as a mindset for organizational analysis. In K. N. Kamoche, M. P. Cunha & J. V. Cunha (Eds.), *Organizational improvisation* (pp. 49-70). London, UK: Routledge.
Whitin, D. J. (2004). Building a mathematical community through problem posing. In R. N. P. Rubenstein & G. W. Bright (Eds.), *Perspectives on the teaching of mathematics: Sixty-sixth yearbook* (pp. 129-140). Reston, VA: National Council of Teachers of Mathematics.
Yackel, E. (2002). What can we learn from analyzing the teacher's role in collective argumentation. *Journal of Mathematical Behavior, 21*, 423-440.
Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.
Zack, M. H. (2000). Jazz improvisation and organizing: Once more from the top. *Organization Science, 11*(2), 227-234.

Appendix A: Tasks used in the study

World's Largest Pizza[58]

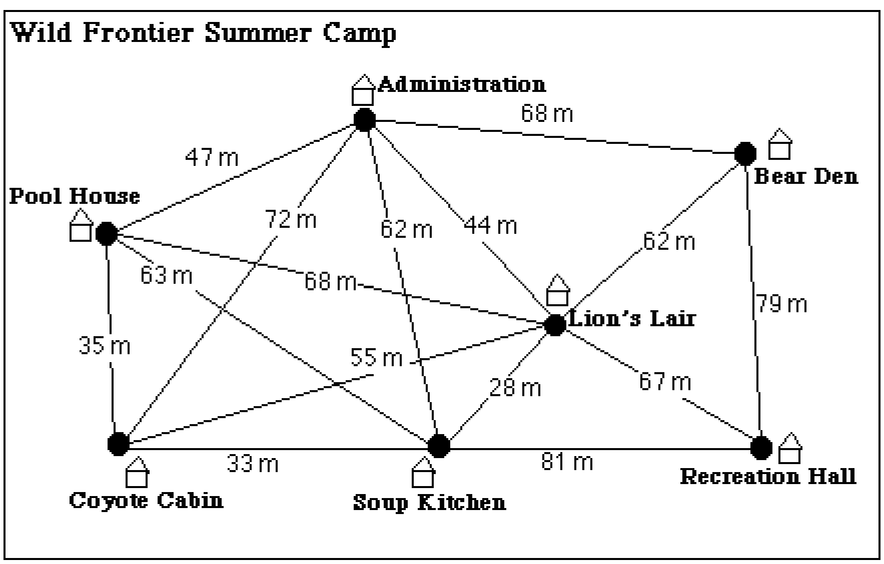
It was Mrs. Chow's birthday, and Mrs. Shug decided to throw her a surprise party. But not just any surprise party – Mrs. Shaw wanted it to be a huge party, and she wanted to invite every single person that Mrs. Chow had ever known in her entire life. Since there were going to be so many people at this party, Mrs. Shug decided to order a gigantic pizza, the largest pizza in the world.

This pizza was so big that it was going to cost Mrs. Shug a small fortune just to cut it. The World's Largest Pizza Cutting Company charged Mrs. Shug $500 per straight cut. Obviously, Mrs. Shug was going to have to slice up the pizza in as few cuts as possible in order to save money.

What are the minimum and maximum numbers of people Mrs. Shug can invite to the surprise party if she uses 3 cuts? 4 cuts? 5 cuts? 6 cuts? *n* cuts? Assume that each person at the party gets one piece of pizza.

Mr. Rollee's Summer Camp[59]

Mr. Rollee, the new owner of Wild Frontier Summer Camp, has asked the electrician to create an internal telephone system to connect his office in the administration building to all the other camp buildings. How should his telephone network be designed, so that all these buildings are connected, directly or indirectly, to him using the minimum amount of wire? The map shows the distance between the phone connections in the seven buildings.



[58] Based on a problem in Tsuruda (1994).
[59] Based on a problem found on the Math Forum website (www.mathforum.org).

Power Outage[60]

One day while Mrs. Shug was at school, the electricity went out at home. When she had left for school that morning, all the clocks were working and agreed that the time was 6:30. When she got home they all displayed different times.

The wind-up clock, which was unaffected by the electricity, read 5:21. The analog electric clock stops running when you unplug it from the wall, and it starts up where it left off when you plug it back in. That clock said it was 3:50. Her digital electric clock, which resets itself to midnight when the electricity goes out, flashes until you correct the time. It was flashing 6:03 a.m.

Assuming the electricity went out just once, what time did it go out, and how long was it off?

Snow Day[61]

It hasn't happened in a long time, particularly at this time of year, but if it ever snowed hard enough for long enough, school would close for the day. Yes, you'd be sad but you'd get over the pain in time. Anyway… School closings are announced on local TV and radio stations, but sometimes you have to watch or listen for a long time before they announce your school. Mr. Bill would get advance notice of any closings. He then calls Mrs. Jones and Mrs. Washington. Mrs. Jones calls Mr. Johns and Mrs. Grundy. Mrs. Washington calls Mrs. Shug and Mr. Gold. The phone tree continues with Mrs. Shug calling Mrs. Chow and Ms. Anderson, and Mr. Gold calling Mrs. Vance and Mr. Smith, and so on.

We are going to assume that all the teachers want to get back to sleep as soon as possible, so they are not going to chat, and each call will take only one minute.
One morning Mr. Bill finds out that there is no school and makes his first call at 6:00 a.m. Ten minutes later, at 6:10 a.m., everyone on the phone chain is back asleep. How many phone calls have been made?

What time will the last calls be made if the phone chain is extended to include 1,000 teachers in the school district?

The Bill Nye Fan Club Party[62]

The Bill Nye Fan Club is having a yearend party, which features wearing lab coats and safety glasses, watching videos and singing loudly, and making things explode. As well, members of the club bring presents to give to the other members of the club. Every club member brings the same number of gifts to the party.

If the presents are opened in 5 minute intervals, starting at 1:00 pm, the last gift will be opened starting at 5:35 pm. How many club members are there?

---

[60] Based on a problem found on the Math Forum website (www.mathforum.org).
[61] Based on a problem found on the Math Forum website (www.mathforum.org).
[62] Based on a problem in Tsuruda (1994).

Appendix B: Transcripts of class discussions prior to group work

Mrs. Shug's homeroom (containing JJKK)

Mrs. Shug:	Alright, ladies and gentlemen, so Problem of the Day… [reads problem aloud to class]. Again, everything is right here. So you are looking for how many club members are there. Questions?
Unidentified:	Can every club member bring only one gift?
Mrs. Shug:	No, well, they certainly could bring but every club member brings the same number of gifts to the party. So if everyone decided to bring one, yeah, but you've got to think about when they started opening and when they started closing.
Jessica:	So are you trying to find out how many presents were opened every hour? Or just…
Mrs. Shug:	You're trying to find out how many club members are there.
Julianna:	How many presents are they supposed to bring?
Mrs. Shug:	That's a good question. That's part of it.

Mrs. Shug's other mathematics class (containing NIJM, REGL and DATM)

Mrs. Shug:	So a reminder you're going to be working in groups. Usually it starts out with two and then you can work with four and you can work larger than that. But everything that is important is going to be on the paper in front of you. Another thing to remember is that this is the same one they [the other class] did yesterday and they kind of got tricked in a sense – "oh, this is too easy." So look outside the box, (?), think outside the box of the birthday present today. So, you are going to have an opportunity to chat with neighbors to try to figure it out and then again your names will be pulled out of, the straws, and you will have to come up and explain yourself. So a big part of this is can you clearly communicate what you are thinking? How you are getting to that point? Just getting the answer, great, but how did you get there and were you to repeat this and want to explain it to someone you have to be able to communicate this to someone else. So when you get it, read it over and then we will read it through to make sure that you understand.

Mrs. Shug hands out the problem sheets, one for every two students. She gives the class a couple of minutes to read the problem over. She then reads the question aloud to the class but stops at the term "five minute intervals."

Mrs. Shug:	Can someone tell me what an interval is?
Nitara:	It's like (?)
Mrs. Shug:	No, not quite the (?). What's an interval?
Michael:	Like five, and then you ten, I don't know.

Isaiah: So one minute, so you'd open the present then five minutes later you open another one.

Mrs. Shug: Okay? So it's a time that's a baseline. Okay? So every five minutes, so every interval of five minutes [She continues to read question aloud to the class]. So there's a couple of key ways it's being worded in there so watch it carefully. So five minute intervals, starting at one. The last gift will be opened starting at five thirty-five pm. How many fan club members are there? Questions? All right. Think outside the box. Go.

Appendix C: Group problem transcripts

Chart 1 – Group NIJM

Problem posed	Group response	Occurrence	Who discussed
Do we use time and divide by five?	Brief discussion of counting method followed by objections (in next two posed problems]	1st occurrence, comes up again	Michael, Isaiah
How many people are there?	Very brief, in response to problem posed above	1st occurrence, comes up again	Nitara
How many gifts are there?	Very brief, also in response to above posed problem	1st occurrence, comes up again	Nitara
Do we use time and divide by five?	Statement of counting method with a little more detail.	Reoccurrence, comes up again	Michael
What is meant by an interval?	Discussion of how timing of intervals might work	1st occurrence, comes up again	Isaiah Michael
What is meant by an interval?	Part of class discussion of POD	Reoccurrence, comes up again	Mrs. Shug, Nitara, Michael, Isaiah
	Discussion of whether to work as a whole group or as two pairs		
Do we use time and divide by five?	Is there an easier way than counting? Not taken up.	Reoccurrence, comes up again	Isaiah
Does everyone bring the same amount of gifts?	Seems to be recalling the POD just read aloud	1st occurrence, comes up again	Nitara
Do we use time and divide by five?	Counting begins	Reoccurrence, comes up again	Michael
Does everyone bring the same amount of gifts?	Agreement that this means the result will be a whole number – originally termed as 'even' but then corrected	Reoccurrence, resolved	Nitara, Isaiah
What if everyone brings x gifts each?	5 is proposed with a total of 25 gifts.	1st occurrence, comes up again	Michael, Isaiah, Jacob
How many people are there?	Not taken up	Reoccurrence,	Nitara
What if everyone brings x gifts each?	1 is proposed, perhaps as an extreme test case, with a total of 1 person at the party bringing a gift for himself.	Reoccurrence,	Michael, Isaiah, Jacob
Do we use time and divide by five?	Suggested that by doing this they'll then get the number of gifts each person brought. Group agrees to try it, to use a counting method and that finding the amount for one hour would work (can "keep doing it" from there). This keeps group busy for awhile	Reoccurrence, comes up again	Nitara, Isaiah, Michael, Jacob
Is it a square root?	A prediction. Then noted that this is the topic they're currently studying in their regular mathematics class.	1st occurrence, comes up again	Isaiah
How many gifts are there?	A prediction of 55.	1st occurrence,	Michael (do I

Problem posed	Group response	Occurrence	Who discussed
		comes up again	have the right speaker?)
Do we use time and divide by five?	Counting	Reoccurrence, comes up again	Nitara
How many gifts are there?	55 is discussed as a prediction.	Reoccurrence, comes up again	Jacob, Isaiah, Nitara, Michael
Do we use time and divide by five?	Counting continues	Reoccurrence, comes up again	Jacob, Nitara
Is another group's answer right?	Another group has been overheard saying that 87 is the answer.	Only occurrence	Michael, Isaiah
Do we use time and divide by five?	Counting continues as do the estimates as to what the answer might be]	Reoccurrence, comes up again	Jacob, Isaiah, Michael, Nitara
Can they bring partial gifts?	Not taken up.	Only occurrence	Isaiah
Do we use time and divide by five?	Counting continues	Reoccurrence, comes up again	Nitara, Michael, Isaiah
What are the factors of x?	What are the factors of 56? Not taken up.	Reoccurrence, comes up again	Nitara
Is it a square root?	Looking for a calculator but then realization that it can't be a square root because it isn't a whole number	Reoccurrence, comes up again	Michael, Nitara
Do we use time and divide by five?	A discussion of recounting.	Reoccurrence, comes up again	Nitara, Michael, Isaiah
Does it start at one o'clock?	A discussion of when the gift opening would begin.	Only occurrence, resolve	Jacob, Isaiah
How many gifts are there?	Prediction of 55 repeated.	Reoccurrence, comes up again	Jacob
Is there an extra 5 minutes (because the last gift starts at 5:35)	A debate occurs as to whether there would be 55 or 56 intervals and why. Not resolved.	1st occurrence, comes up again	Nitara, Jacob, Isaiah, Michael
How do we think outside the box?	Group discussion about how there must be more to the problem than there seems to be	1st occurrence, comes up again	Michael, Jacob, Nitara
What are the factors of x?	Using calculator to determine what goes into 56.	Reoccurrence, comes up again	Nitara, Isaiah
Is it a square root?	They find that the square root of 56 is a decimal, and discuss how they can't have half people going to a party	Reoccurrence, resolved	Nitara, Isaiah, Michael
Do they also bring gifts for themselves?	Discussed the idea of the partygoers not bringing gifts for themselves.	1st occurrence, comes up again	Isaiah, Nitara
Do all members give to everyone?	No agreement is reached about this	Only occurrence	Michael, Nitara
What if someone doesn't get a gift?	Accepted that everyone gets a gift.	Only occurrence	Michael, Nitara
Do they also bring gifts for themselves?	Seems to be asserted that they don't.	Reoccurrence, comes up again	Isaiah
What is meant by an interval?	Seems to be a reminder.	Last occurrence	Nitara
What if there are x people?	8 people are proposed and a discussion about why this would probably work	1st occurrence, comes up again	Isaiah, Nitara, Jacob

Problem posed	Group response	Occurrence	Who discussed
Do they bring gifts for themselves?	Now it's settled - "it's obvious" "who would bring a gift for themselves?" – using real-life example	Reoccurrence, comes up again	Isaiah, Nitara, Jacob
What if there are x people?	They start doing calculations based on 8 people	Reoccurrence, comes up again	Jacob, Isaiah, Nitara
Do they bring gifts for themselves?	Testing idea to be sure they'd don't bring gifts for selves. Now confirmed.	Reoccurrence, resolved	Isaiah, Nitara
What if there are x people?	Discussion of how 8 people, 7 gifts gives them a total of 56 gifts and how that makes sense as the answer	Reoccurrence, resolved	Isaiah, Jacob
How do we think outside the box?	A discussion about whether this has been too easy, or maybe they did think outside the box	Reoccurrence, comes up again	Nitara, Jacob, Isaiah
Do we use time and divide by five?	Rechecking the calculations leading to their answer, different people doing the counting this time	Reoccurrence, resolved	Isaiah, Michael
How do we think outside the box?	Stemming from problem above, is there another way to get the answer as a way of thinking outside the box.	Reoccurrence. unresolved	Isaiah

Chart 2: Group DATM

Problem(s) posed (and how posed)	Group response	Occurrence	Who discussed
Does it take 5 minutes to open one gift or 5 minutes to open all the gifts that one person brings?	In response another student asks that the question be reread and there's a brief discussion about that.	1st occurrence, comes up again	Amaya
Do all members give to everyone?	Not taken up	1st occurrence, comes up again	Amaya
Does it take 5 minutes to open one gift or 5 minutes to open all the gifts that one person brings?	Not taken up	Reoccurrence, comes up again	Amaya
Do all members give to everyone?	Not taken up.	Reoccurrence, comes up again	Derek
How many people are there?	No one knows.	1st occurrence, comes up again	Amaya, Derek
Does everyone bring the same amount of gifts?	The immediate response to this is unintelligible	1st occurrence, comes up again	Amaya
Do we use time and divide by 5?	24 hour clock is briefly (was used in a previous POD)	1st occurrence, comes up again	Amaya
How many people are there?	Not taken up.	Reoccurrence, comes up again	Derek
Does it start at one o'clock?	Agreement.	Only occurrence	Amaya
Do we use time and divide by 5?	Use of a counting method	Reoccurrence, comes up again	Amaya, Derek
How many people are there?	Debate about how to approach this question – directly (D) or by finding out other info first (A). This debate continues through much of the session.	Reoccurrence, comes up again	Derek, Amaya
Do we use time and divide by	Calculations continues	Reoccurrence,	Amaya

Problem(s) posed (and how posed)	Group response	Occurrence	Who discussed
5?		comes up again	
How many people are there?	Debate continues.	Reoccurrence, comes up again	Derek, Amaya
How long does it take to open all the gifts at the party?	M has recently arrived in group and is trying to determine what's going on.	Only occurrence	Meredith
How many people are there?	Debate continues	Reoccurrence, comes up again	Derek
Does it take 5 minutes to open one gift or 5 minutes to open all the gifts that one person brings?	No decision made	Only occurrence	Meredith, Derek
Do we use time and divide by 5?	Calculations continue	Reoccurrence, comes up again	Amaya
Can they open gifts at the same time?	They consider asking the teacher	1st occurrence, comes up again	Meredith, Derek
Do we use time and divide by 5?	Calculations continue	Reoccurrence, comes up again	Amaya
Can they open gifts at the same time?	They decide to get teacher. They ask researcher in meantime but she refers them to teacher. Takes a while but Mrs. Shug comes over, tells them one gift is opened at a time. While she's there…	Reoccurrence, resolved	Meredith, Amaya, Mrs. Shug
Does it take 5 minutes to open one gift or 5 minutes to open all the gifts that one person brings?	Not taken up because immediately followed by…	Reoccurrence, resolved	Amaya
Can they take breaks in between opening gifts?	Another student piggy-backs this problem in there while Mrs. Shug is there. Teacher suggests that partygoers open gift in 1 minute and then have a four minute break.	Only occurrence	Meredith, Mrs. Shug
Do we use time and divide by 5?	Calculations continue	Reoccurrence, comes up again	Timothy
How many people are there?	Overlapping speech so can't tell if this is taken up. The group splits into D&T, and A and E from REGL. Meredith leaves briefly.	Reoccurrence, comes up again	Derek
Do we use time and divide by 5?	Ideas presented to E.	Reoccurrence, comes up again	Amaya
Is there an extra 5 minutes? (because last gift starts at 5:35)	Can't tell if taken up – overlapping discussion	1st occurrence, comes up again	Timothy
Is there an extra 5 minutes? (because last gift starts at 5:35)	Taken up in form of calculations	Reoccurrence, comes up again	Eliana from REGL
Do we use time and divide by 5?	Some confusion in calculations when 60 thrown in as number of gifts rather than number of minutes {A & E]; T doing his own calculations.	Reoccurrence, comes up again	Amaya, Timothy, Eliana
Is there an extra 5 minutes? (because last gift starts at 5:35)	Accepted	Reoccurrence, comes up again	Timothy, Derek
Do we use time and divide by 5?	Mistake noted in calculations	Reoccurrence, comes up again	Timothy Derek
How many people are there?	Mistake discussed.	Reoccurrence, comes up again	Timothy, Derek

Problem(s) posed (and how posed)	Group response	Occurrence	Who discussed
Do we use time and divide by 5?	M returns and group reforms.	Reoccurrence, comes up again	Meredith
How many people are there?	M comments on T's mistake.	Reoccurrence, comes up again	Meredith, Derek
Is there an extra 5 minutes? (because last gift starts at 5:35)	Not taken up.	Reoccurrence, comes up again	Meredith
Why did we get x?	Question about method but not taken up	1st occurrence, comes up again	Derek, Amaya
How many people are there?	Confirmation that it's 56 gifts, but still don't know number of people	Reoccurrence, comes up again	Derek, Amaya
Do we use time and divide by 5?	55 introduced as number of gifts	Reoccurrence, comes up again	Meredith
Is there an extra 5 minutes? (because last gift starts at 5:35)	Not taken up	Reoccurrence, comes up again	Timothy
Do we use time and divide by 5?	Meant as a warning not to get mixed up with 60.	Reoccurrence, comes up again	Amaya
Is there an extra 5 minutes? (because last gift starts at 5:35)	Not taken up	Reoccurrence, comes up again	Derek
What if there are x people and gifts?	28 is proposed	1st occurrence	Amaya, Derek
Do all members give to everyone?	Posed to Mrs. Shug	Reoccurrence, comes up again	Amaya
Does everyone bring the same amount of gifts?	Part of POD read aloud	Reoccurrence, comes up again	Mrs. Shug
Do all members give to everyone?	In reposing this to Mrs. Shug, an example of there being 20 people is brought in. Mrs. Shug confirms they bring gifts for everyone. Still some confusion	Reoccurrence, comes up again	Meredith, Mrs. Shug, Derek
What if everyone brings x gifts each?	Group now working with a guess and test method, suggesting there are 8 people and 7 gifts. (not sure where these numbers suddenly came from)	1st occurrence, will reoccur	Amaya, Meredith, Derek
Do they also bring gifts for themselves?	Confirmed that they don't.	Only occurrence	Amaya, Derek.
Does everyone bring the same amount of gifts?	Seems to be a reminder.	Reoccurrence, comes up again	Meredith
What if everyone brings x gifts each?	Discussed but not resolved. Example used of 4 travelers.	Reoccurrence, comes up again	Amaya, Timothy, Meredith, Derek
Is there an extra 5 minutes? (because last gift starts at 5:35)	Seems to be a reminder.	Reoccurrence, comes up again	Meredith
What if everyone brings x gifts each?	Discussion continues. 56 gifts proposed.	Reoccurrence, comes up again	Derek, Meredith
How many people are there?	Discussion about how 56 gifts doesn't mean 56 people	Reoccurrence, comes up again	Derek, Meredith
Does everyone bring the same number of gifts?	Seems to be a reminder	Reoccurrence, comes up again	Derek
Do all members give to everyone?	Brief discussion	Reoccurrence, comes up again	Meredith, Derek
What if everyone brings x gifts each?	Taken up with some insults exchanged	Only occurrence, not resolved	Meredith

Problem(s) posed (and how posed)	Group response	Occurrence	Who discussed
What if there are x people and gifts?	Introduced to disprove last problem but student ends up getting confused	Only occurrence, not resolved	Derek
What if everyone brings x gifts each?	8 people and 7 gifts proposed again. Time thrown in and more confusion. 56 confirmed as total number of gifts.	Reoccurrence, comes up again	Derek, Amaya, Meredith, Timothy
How do we know if we're right?	Echoed but not taken up.	Only occurrence, not resolved	Derek, Meredith
What if everyone brings x gifts each?	A guess of 56 gifts offered. Not taken up.	Reoccurrence, comes up again	Derek
Why did we get x?	Wondering about method used to calculate 56 but no explanation offered.	Reoccurrence, not resolved	Derek, Amaya
How can we use the 24 hour clock?	This has been mentioned in passing as part of A's method but now is being addressed as a problem. As there is no mention of this kind of clock in the problem, this seems to refer to a previous POD that did use it. Different methods are compared.	Only occurrence, resolved	Amaya, Derek, Timothy
What if everyone brings x gifts each?	Mrs. Shug has announced that presentations will begin in 5 minutes. Suddenly eight people and seven gifts is the agreed-upon answer with no discussion about how or why it would work. Conclusion is that it's a lame party]	Reoccurrence, resolved	Meredith, Amaya, Derek

Chart 3: Group JJKK

Problem(s) posed	Group response	Occurrence	Who discussed
How many gifts are opened in an hour?	During class discussion of POD.	1st occurrence, comes up again	Jessica
How many people are there?	During class discussion of POD. Mrs. Shug says that they are supposed to find out how many club members there are.	1st occurrence, comes up again	Mrs. Shug
How many gifts does each person bring?	During class discussion of POD. Mrs. Shug notes that this is part of what is being asked by the POD.	1st occurrence, comes up again	Julianna
How many gifts are opened in an hour?	12 is proposed as an answer and there is a lengthy discussion about this was determined. The main 2 debaters have difficulty understanding each other (one student in particular has difficulty expressing what she means) and Mrs. Shug ends up being called over as a kind of a translator. Eventually 12 is agreed upon.	Reoccurrence, comes up again	Kady, Jessica, Katia, Julianna, Mrs. Shug
What is meant by an interval?	Another big discussion. Again, one student is having difficulty expressing herself – she seems consistent on the idea, for a while, that it means	Only occurrence	Jessica, Kady, Katia, Mrs. Shug

Problem(s) posed	Group response	Occurrence	Who discussed
	unwrapping gifts for 5 minutes then a 5 minute break. When Mrs. Shug, again translating between the two debaters, says that the opening of the next gift starts right away, student says that's kind of what she meant.		
Can they take breaks in between opening gifts?	Student apparently still hooked into idea that club members are opening gifts for a full hour. When Mrs. Shug says it's more than an hour, the student says she knows that – so again, is language failing her or is she covering for herself? She tries to introduce her idea of intervals again but Mrs. Shug says there are no breaks	Only occurrence	Jessica, Mrs. Shug
How many gifts are opened in an hour?	12 is reintroduced, but student with difficulty now seems to think it refers to minutes. Takes some discussion and help from Mrs. Shug to sort this out.	Reoccurrence, resolved	Kady, Jessica, Mrs. Shug
How long does it take to open all the gifts?	Calculations begin with Mrs. Shug standing by. A little more confusion about minutes versus hours but cleared up quickly. Mrs. Shug makes more suggestion about how to keep on track using a clock diagram]	1st occurrence, comes up again	Kady, Mrs. Shug, Julianna, Jessica
Do we use time and divide by 5?	More calculations	1st occurrence, comes up again	Katia
How long does it take to open all the gifts?	A final statement on the matter from Mrs. Shug.	Reoccurrence, resolved	Mrs. Shug
Do we use time and divide by 5?	Discussion of calculations	Reoccurrence, comes up again	Katia, Julianna
Is there an extra 5 minutes? (because last gift starts at 5:35)	Discussion and agreement.	1st occurrence, comes up again	Jessica
Do we use time and divide by 5?	Rest of group checks calculations.	Reoccurrence, group doesn't discuss it again	Jessica, Kady
Is there an extra 5 minutes? (because last gift starts at 5:35)	Although group agrees on this, 55 is the total number they calculate.	Reoccurrence, group doesn't raise it again	Jessica, Kady
	[interruption while Mr. M talks to Jessica]		
How many people are there?	Mrs. Shug replies that this is part of the POD	Reoccurrence, comes up again	Katia, Mrs. Shug
How many gifts does each person bring?	Not taken up.	Reoccurrence	Katia
Does everyone bring the same amount?	Not taken up.	Only occurrence	Mrs. Shug
Is it a square root?	Group does not directly say that this is what they're currently studying in the regular mathematics classes. Discussion about what square root means – calculated as 7.4.	Only occurrence, resolved	Katia, Jessica, Kady
Why did we get x?	Student asking where 7 came from. No explanation that they rounded down.	Only occurrence	Kady

Problem(s) posed	Group response	Occurrence	Who discussed
How many gifts does each person bring?	Group agrees that that's what they've figured out so far.	1st occurrence, comes up again	Jessica
How many people are there?	Not taken up.	Reoccurrence, resolved	Jessica
What if everyone brings x gifts?	A general acceptance that everyone brought 7 gifts once they've decided to round down.	Only occurrence	Jessica, Katia
What are the factors of x?	Group divides 7 into 55. Some discussion about an estimate. Answer is 7.8. Then, an echo-ey discussion about how 7 people brought 7 gifts. Reading aloud from POD sheet. When Mrs. Shug is called over, student explaining to her says there are 8 people.	Only occurrence, resolved	Jessica, Kady, Katia
What are the factors of x?	A discussion about whether it's 7 or 8 people. Group still has 55 as their total, but recognize that 7 x 8 is 56 which is apparently close enough. They decide 7.8 and round up.	Reoccurrence	Kady, Katia
	Time warning given. No further group discussion.		

Chart 4: Group REGL

Problem(s) posed [and how posed]	Group response	Occurrence	Who discussed
How do we think outside the box? [as statement]	No real response	1st occurrence, comes up again	Eliana
How long does it take to open one gift?	Suggestion offered	1st occurrence, comes up again	Rebekkah, Lucy
How many people are there?	Not taken up	1st occurrence, comes up again	Lucy
What is meant by an interval?	Suggestion offered	1st occurrence, comes up again	Lucy, Rebekkah
How long does it take to open one gift?	Group doesn't dispute this being a problem but doesn't take it up either	1st occurrence, comes up again	Unidentified voice
	Group reads aloud from POD		
Do we use time and divide by 5?	Student offers reason for posing. Not taken up.	1st occurrence, comes up again	Rebekkah
How do we think outside the box?	Followed directly by reading aloud from POD	Reoccurrence, comes up again	Eliana
[followed by more reading aloud of question]			
Does everyone bring the same amount of gifts?	Agreement but no discussion	1st occurrence, comes up again	Unidentified voice
What is meant by an interval?	Previous suggestion reoffered	Reoccurrence, comes up again	Eliana, Rebekkah
What's a tournament?	A satisfactory definition is worked out	Only occurrence, resolved	Geri, Eliana
Do all members give to everyone?	Agreement that partygoers each bring gifts for everyone	1st occurrence, comes up again	Geri, Eliana, Rebekkah
How many people are there?	Seem to function as reminder. Not taken up.	Reoccurrence, does not occur again	Rebekkah

Problem(s) posed [and how posed]	**Group response**	**Occurrence**	**Who discussed**
How many gifts are there?	Discussion of why need to answer this first.	Only occurrence	Rebekkah, Eliana
Do we use time and divide by 5?	Discussion of method involved.	Reoccurrence, comes up again	Eliana, Rebekkah
How long does it take to open one gift?	Discussion leads to next posed problem	Reoccurrence, comes up again	Eliana
What is meant by an interval?	Problem posed and then suggestion offered by the same student	Reoccurrence, comes up again	Rebekkah
Is there an extra 5 minutes? (because last gift is opened starting at 5:35)	Agreement	1st occurrence, comes up again	Eliana, Rebekkah
Do we use time and divide by 5?	Problem posed and then suggestion offered by the same student. Two group members start writing.	Reoccurrence, comes up again	Rebekkah, Eliana
What is meant by an interval?	Intent to ask teacher. Problem not taken up by group.	Reoccurrence, comes up again	Eliana
Do we use time and divide by 5?	This problem and the previous one now seem to be coming together. Group discusses calculations.	Reoccurrence, comes up again	Geri, Eliana, Rebekkah
What is meant by an interval?	Problem is picked up and discussed by group, where metaphors of an intermission and of taking a break are raised. Teacher is called over and this question is repeated	Reoccurrence, resolved	Eliana, Rebekkah, Lucy?
How long does it take to open one gift?	Rephrasing of previous question by another student in group. Teacher answers and provides more information to clarify. This appears to resolve this problem and the previous one]	Reoccurrence, resolved	Rebekkah, Eliana, Mrs. Shug
Do we use time and divide by 5?	Group continues with calculations	Reoccurrence, comes up again	Rebekkah, Eliana
Is there an extra 5 minutes? (because last gift is opened starting at 5:35)	Discussion	Reoccurrence, comes up again	Eliana, Rebekkah
Do we use time and divide by 5?	This posing is related to previous problem so not taken up directly	Reoccurrence, comes up again	Eliana
Is there an extra 5 minutes? (because last gift is opened starting at 5:35)	More group discussion. Calculations completed.	Reoccurrence, comes up again	Geri, Eliana
	[Interruption from A of DATM to discuss A's calculations so far]		Eliana
Does everyone bring the same amount of gifts?	The implications of this are discussed. The group decides that they do.	Reoccurrence, comes up again	Rebekkah, Eliana
What are the factors of x?	Discussion of different factors of 56 – 2, 4, 14, 28	1st occurrence, comes up again	Rebekkah, Eliana, Geri
Do all members give to everyone?	Not taken up.	Reoccurrence, comes up again	Rebekkah
Can't we just count how many people?	This problem isn't clear, either from the student who poses it or the group's response.	Only occurrence?	Eliana, Geri
Does everyone bring the same amount of gifts?	Seems to be a statement rather than a point of discussion. Not taken up.	Reoccurrence, comes up again	Eliana

Problem(s) posed [and how posed]	Group response	Occurrence	Who discussed
What if everyone brings x gifts each?	A discussion where "4" is echoed by different members of the group.	1st occurrence, comes up again	Rebekkah, Geri, Eliana
What are the factors of x?	This has actually changed slightly to the more general "how do we know what the factors of 56 are?" as if there might be a rule for them to follow to determine the factors.	Reoccurrence, comes up again	Eliana, Rebekkah
How do we think outside the box?	Not taken up.	Reoccurrence, comes up again	Rebekkah
Do we use time and divide by 5?	This seems to be a recap of what the group has determined so far. Not taken up.	Reoccurrence, comes up again	Eliana
What are the factors of x?	Group looks up at multiplication wall chart	Reoccurrence, comes up again	Rebekkah, Eliana
How do we think outside the box?	Expressed as a worry that the way they are proceeding is too simple. Not taken up.	Reoccurrence, comes up again	Rebekkah, Eliana
Does everyone bring the same amount of gifts?	Although this seemed to be agreed upon before, the group takes this up.	Reoccurrence, comes up again	Geri, Rebekkah
What if everyone brings x gifts each?	This seems to be a combo of the factors problem and how many each person brings problem. 14 is discussed as possible number of gifts.	Reoccurrence does not occur again.	Eliana, Rebekkah
Does everyone bring the same amount of gifts?	Group's discussion notes that there are only 56 gifts and how that limits the possibilities of the POD situation	Reoccurrence, comes up again	Eliana, Rebekkah
What are the factors of x?	More discussion of 4 as a possible factor.	Reoccurrence, comes up again	Eliana, Rebekkah
Does everyone bring the same amount of gifts?	Different factors are considered – the discussion is a combination with how many does each person bring	Reoccurrence, comes up again	Eliana, Rebekkah
Do they also bring gifts for themselves?	A brief discussion of why they don't	1st occurrence, comes up again	Rebekkah, ?
What are the factors of x?	[14 doesn't work but they're on the right track – not taken up by group]	Reoccurrence, comes up again	Eliana
Do we use time and divide by 5?	Discussion of whether a counting method would give a total of 55.	Reoccurrence, comes up again	Geri, Eliana
What if it's an exchange?	One student proposes a combinations-based algorithm. The group points out that this doesn't fit the POD situation because each partygoer has to give out the same number of gifts	1st occurrence, comes up again	Geri, Eliana, Rebekkah
Does everyone bring the same amount of gifts?	Seems to be in support of why the problem above doesn't fit.	Reoccurrence, comes up again	Eliana
How do we think outside the box?	Seems to be to mollify the student above	Reoccurrence, comes up again	Rebekkah
What if it's an exchange?	Group further comforts student by saying that solution would	Reoccurrence, resolved	Eliana, Rebekkah

Problem(s) posed [and how posed]	Group response	Occurrence	Who discussed
	have been good had it worked because of the potential for being able to draw the connections		
Do all members give to everyone?	Discussion about if the POD is worded clearly enough about this	Reoccurrence, does not occur again	Eliana, Rebekkah
What are the factors of x?	Discussion of 28 and 14 as factors of 56	Reoccurrence, comes up again	Rebekkah
What if everyone brings x gifts each?	Discussion about what the POD is actually saying. Group agrees that all partygoers bring the same number of gifts and have to give out all the gifts that they bring. Kind of a combo but leads to….	Reoccurrence	Eliana, Rebekkah
Does everyone bring the same amount of gifts?	Discussion about whether they should ask Mrs. Shug to clarify.	Reoccurrence, comes up again	Eliana
What are the factors of x?	Establishing that there are 56 gifts.	Reoccurrence, comes up again	Eliana
How do we think outside the box?	This is blended with the last problem - they are looking for a simple solution, but suppose it's too simple?	Reoccurrence, comes up again	Rebekkah
What are the factors of x?	Establishing 2 is a factor (leads from last problem)	Reoccurrence, comes up again	Rebekkah
What if everyone brings x gifts each?	Discussion of how 28 gifts doesn't work	Reoccurrence resolved	Eliana, Rebekkah
What if there are x people?	Discussion of how 28 people doesn't work	1st occurrence. resolved	Eliana, Rebekkah
Is there an extra 5 minutes? (because last gift is opened starting at 5:35)	Discussion to confirm that this situation makes sense.	Reoccurrence, comes up again	Eliana
What are the factors of x?	Excited discussion about the factors 7 and 8.	Reoccurrence, comes up again	Rebekkah, Eliana, Geri
Do they also bring gifts for themselves?	Agreed that partygoers wouldn't give gifts to themselves.	Reoccurrence, comes up again	Rebekkah, Eliana
What are the factors of x?	Confirmed that there would be 8 people.	Reoccurrence, resolved. POD is solved.	Eliana, Rebekkah
How do we think outside the box?	Discussion about how the answer seems too simple, yet it did take time to figure it out	Reoccurrence, comes up again	Eliana, Rebekkah
What are the factors of x?	Group shares answer with NIJM, and then discusses as a group about how it would work.	Reoccurrence, communication with another group	Eliana, Rebekkah, Geri
Do we use time and divide by 5?	Seems to be a test of their answer.	Reoccurrence, double-check	Eliana
Is there an extra 5 minutes? (because last gift is opened starting at 5:35)	Seems to be a test of their answer	Reoccurrence, double-check	Eliana
Do they also bring gifts for themselves?	Seems to be a test of their answer	Reoccurrence, double-check	Rebekkah
Do we use time and divide by 5?	A couple of members start doing a counting method to check the number of intervals.	Reoccurrence, double-check	Rebekkah, Eliana