HANDLING UNCERTAINTY IN HYDROLOGIC ANALYSIS AND DROUGHT RISK ASSESSMENT
USING DEMPSTER-SHAFER THEORY

by

Amin H. Zargar Yaghoobi

M.Sc., Memorial University of Newfoundland, 2009
M.Tech., Indian Institute of Technology Delhi, 2005
B.Sc., University of Mazandaran, 2003

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

The College of Graduate Studies

(Civil Engineering)

THE UNIVERSITY OF BRITISH COLUMBIA

(Okanagan)

December 2012

© Amin Zargar, 2012
ABSTRACT

The aim of this thesis is to enhance some of the hydrologic analyses involved in drought risk assessment (DRA) to uncertainty-driven analyses therefore improving the accuracy and informativeness of DRA. In DRA, risk, or the expected loss from drought hazard is estimated by integrating the magnitude of hazard (i.e., drought severity) with vulnerability (i.e., susceptibility to losses from drought). Most hydrologic analyses including DRA are traditionally performed in a deterministic setting, ignoring data quality and uncertainty issues. Uncertainty can affect the accuracy of modeling results and undermine subsequent decision making. In order to handle uncertainty in DRA, this thesis uses the Dempster-Shafer theory (DST) which provides a unified platform for modeling and propagating uncertainty in the forms of variability, conflict and incompleteness. First, DST is used to model and propagate uncertainty arisen from a high degree of conflict between two datasets of a drought hazard indicator, the snow water equivalent. Four DST combination rules are used for conflict-resolution and results unanimously indicate a high possibility of drought. Second, the Standardized Precipitation Index (SPI) is used as a generic measure of hazard and is linked directly with wildfire risk in current and future climate scenarios. Using DST, modifications are introduced into SPI, enabling the integration of uncertainty analysis with SPI processes. The resulting enhanced SPI can model the effects of long-term shifts in climate normals on drought hazard while simultaneously evaluating the significance of these shifts within the range of surrounding uncertainty. Later, vulnerability to wildfire is simulated using enhanced SPI and two additional variables: evaporation and firefighting capacity. The estimated risk indicates that forests in Okanagan Basin are vulnerable to wildfires during periods of 2040-2069 and 2070-2099 unless the firefighting capacity is enhanced with a presumed rate. Through the successful implementation of DST into DRA processes, this research demonstrates the capability of DST in improving hydrologic analyses and enhancing informativeness in the water resources arena in general.
PREFACE

Five journal papers have been published or submitted (under review) from this thesis. Complete references are provided below:


A version of Chapter 2 has been published as paper (1). I wrote this paper, while Naser and Khan provided review and feedback, and Sadiq finalized the manuscript. Versions of Chapter 3 have been published as: (a) paper (2); the section that appears in this thesis was written by me, while Sadiq made corrections and finalized the manuscript, and (b) paper (3); I wrote this paper, while Naser and Khan provided review and Neumann assisted with data collection. Sadiq finalized the manuscript. A version of Chapter 4 has been published as paper (3). A version of Chapter 5 has been submitted as paper (1). A version of Chapter 6 has been submitted as paper (4). I wrote this paper, while Khan provided review and Sadiq finalized the manuscript. A version of Chapter 7 has been submitted as paper (5). Similarly, I wrote this paper, while Khan provided review and Sadiq finalized the manuscript.
TABLE OF CONTENTS

ABSTRACT .......................................................................................................................... ii
PREFACE ............................................................................................................................. iii
TABLE OF CONTENTS ....................................................................................................... iv
LIST OF TABLES ................................................................................................................ viii
LIST OF FIGURES ............................................................................................................. ix
LIST OF ABBREVIATIONS ................................................................................................ xi
LIST OF SYMBOLS .......................................................................................................... xiv
ACKNOWLEDGEMENTS .................................................................................................. xv

CHAPTER 1 INTRODUCTION ......................................................................................... 1
  1.1 Background .................................................................................................................. 1
  1.2 Research objectives .................................................................................................... 3
  1.3 Thesis structure .......................................................................................................... 4

CHAPTER 2 BACKGROUND ........................................................................................... 5
  2.1 Drought ...................................................................................................................... 5
  2.2 Drought characterization concepts ............................................................................ 6
      2.2.1 Drought definitions ............................................................................................. 6
      2.2.2 Drought types and characteristics ..................................................................... 6
      2.2.3 Drought indicators ............................................................................................ 8
      2.2.4 Drought indices .................................................................................................. 8
  2.3 Drought management and drought risk assessment ..................................................... 9
      2.3.1 Drought hazard .................................................................................................. 11
        2.3.1.1 Severity and frequency ................................................................................ 11
        2.3.1.2 Climate change and droughts ...................................................................... 11
      2.3.2 Drought vulnerability ......................................................................................... 11
        2.3.2.1 Causal and impact assessment ..................................................................... 12
        2.3.2.2 Temporal trends ........................................................................................... 13
      2.3.3 Drought risk characterization ............................................................................. 13

CHAPTER 3 HYDROLOGICAL UNCERTAINTY ANALYSIS .............................................. 15
  3.1 Uncertainty modeling ................................................................................................. 15
  3.2 Dempster–Shafer theory (DST) ................................................................................ 16
      3.2.1 Basic concepts .................................................................................................... 17
3.2.1.1 Basic probability assignment (bpa) ................................................................. 17
3.2.1.2 Belief function (bel) ....................................................................................... 18
3.2.1.3 Plausibility function (pl) ............................................................................... 19
3.3 Conflict-handling using DST data fusion .................................................................. 20

CHAPTER 4 HANDLING CONFLICT IN DROUGHT HAZARD: A CASE OF SNOW WATER

4.1 Overview .................................................................................................................. 26
4.2 SWE data sources and DST ..................................................................................... 26
4.2.1 Conflict in SWE data .......................................................................................... 26
4.2.2 Study area ............................................................................................................ 27
4.2.3 Data sources ......................................................................................................... 28
4.2.3.1 Source A .......................................................................................................... 28
4.2.3.2 Source B .......................................................................................................... 30
4.2.4 Representation of bpa using probability box (p-box) ............................................. 30
4.2.5 Formulation of bpa .............................................................................................. 31
4.2.6 Dependency in data sources ................................................................................ 32
4.3 High-conflict data fusion through DST .................................................................... 33
4.3.1 Dempster–Shafer (D-S) rule of combination ......................................................... 34
4.3.2 Yager rule of combination ................................................................................... 36
4.3.3 Mixture ................................................................................................................ 37
4.3.4 Proportional conflict redistribution rule no. 6 (PCR6) .......................................... 39
4.3.5 Decision making in DST ..................................................................................... 41
4.4 Discussion ................................................................................................................ 42
4.4.1 Combination results ............................................................................................. 42
4.4.2 Propagating data uncertainty using DST ............................................................. 43
4.5 Summary .................................................................................................................. 44
CHAPTER 5 CRITICAL REVIEW AND SELECTION OF A DROUGHT INDEX ................. 46
5.1 Overview ........................................................................................................... 46
5.2 A review of drought indices ............................................................................ 46
  5.2.1 Taxonomy of drought indices ...................................................................... 46
  5.2.2 Major drought indices ................................................................................ 47
    5.2.2.1 Operational drought indices ................................................................. 47
    5.2.2.2 Other notable drought indices ............................................................... 52
  5.2.3 Recent developments in drought indices ................................................... 54
    5.2.3.1 Recent advances in drought indices by type ......................................... 54
    5.2.3.2 Aggregation of drought indices ............................................................ 56
    5.2.3.3 Climate change effects ......................................................................... 57
  5.3 Application of drought indices for drought risk assessment ................................ 57
    5.3.1 The role of vulnerability .......................................................................... 58
    5.3.2 Drought risk assessment using SPI ........................................................ 58
  5.4 Summary ........................................................................................................ 59

CHAPTER 6 UNCERTAINTY-DRIVEN CHARACTERIZATION OF DROUGHT HAZARD USING ENHANCED STANDARDIZED PRECIPITATION INDEX ................................................. 60
6.1 Overview ........................................................................................................ 60
6.2 Climate change and its effects on the Okanagan Basin precipitation .............. 61
6.3 SPI modifications to accommodate climate change ....................................... 62
6.4 Study area ...................................................................................................... 63
6.5 Uncertainty in SPI ........................................................................................ 64
  6.5.1 Standardized Precipitation Index ............................................................... 65
  6.5.2 Sources of uncertainty in SPI ..................................................................... 67
6.6 Enhanced SPI ................................................................................................ 69
  6.6.1 Modeling uncertainty using p-box .............................................................. 69
  6.6.2 Generalizing deterministic interpolation to uncertainty-driven interpolation ................................................... 72
    6.6.2.1 Deterministic spatial interpolation ......................................................... 72
  6.7 Uncertainty-driven interpolation .................................................................. 74
6.8 Incorporating climate change effects .............................................................. 76
  6.8.1 Change in precipitation means ................................................................. 76
  6.8.2 Change in the distribution of precipitation and extreme drought ............ 78
  6.8.3 Enclosing uncertainty .............................................................................. 81
LIST OF TABLES

Table 2-1 – Examples of drought impacts category and subcategory by NDMC (2012) .................................................. 12
Table 3-1 – An example frame of discernment .................................................................................................................. 19
Table 3-2 – Belief and plausibility functions for the example interval .................................................................................. 20
Table 3-3 – Some data fusion applications in hydrological modelling .................................................................................... 23
Table 3-4 – Common DST fusion rules ............................................................................................................................... 25
Table 4-1 – The bpa for three intervals derived from the distribution of each source .............................................................. 32
Table 4-2 – Results of D-S rule of combination .................................................................................................................... 35
Table 4-3 – D-S rule of intersection results (Ø: null set) .......................................................................................................... 37
Table 4-4 – Results of Yager combination ............................................................................................................................. 37
Table 4-5 – Masses calculated for each interval according to the assigned weight in the mixture rule .................. 38
Table 4-6 – Results of mixture combination ....................................................................................................................... 38
Table 4-7 – Results of the conjunctive rule applied on the two sources (PCR6) ............................................................... 40
Table 4-8 – Results of PCR6 combination ........................................................................................................................... 41
Table 5-1 – Advantages and disadvantages of popular drought indices .............................................................................. 48
Table 5-2 – Phenomena reflected by specific-duration SPIs and their applications .......................................................... 50
Table 5-3 – Additional notable drought indices ..................................................................................................................... 53
Table 6-1 – Thresholds for SPI categories ............................................................................................................................ 67
Table 6-2 – Statistics for change in extreme drought occurrences for modeled and grid datasets ......................... 83
Table 7-1 – Model and validation R² for five samples (TBA in ha, Eref in mm) ................................................................. 99
Table 7-2 – Dependent variable is TBA .............................................................................................................................. 100
Table 7-3 – Interannual summer precipitation distribution parameters for the Okanagan Basin ........................................ 101
Table 7-4 – Climate change induced increase in wildfire due to extreme drought (M=Model) (FSC=100%,
i.e., similar to 2009) .................................................................................................................................................. 102
Table 7-5 – Percentage increase in wildfire impact area, i.e., risk, due to extreme drought ........................................... 102
LIST OF FIGURES
Figure 1-1 – Thesis structure .............................................................................................................................................. 4
Figure 2-1 – The sequence for the occurrence of different drought types. Modified after NDMC (2006b) .............. 7
Figure 2-2 – Components of drought risk assessment (DRA) (Hayes et al. 2004) ......................................................... 11
Figure 2-3 – The presence of hazard, exposure and vulnerability results in risk ............................................................. 14
Figure 3-1 – The three distributions represent a case of (a) consonant, (b) consistent, (c) disjoint, and (d) arbitrary evidence ........................................................................................................................................ 21
Figure 4-1 – Red Deer River watershed (right map Copyright Government of Alberta, Used with Permission) ........................................................................................................................................ 28
Figure 4-2 – SWE grids for zone 1 of the Red Deer River watershed from (a) source A and (b) source B (1- April 2009, values in mm) .................................................................................................................................. 29
Figure 4-3 – The frequency histogram of (a) source A and (b) source B of SWE data within zone 1 of the Red Deer River watershed (note the difference in y-axis scales) .................................................. 29
Figure 4-4 – A p-box example (the X-axis is a generic scalar example and not representative of a specific unit) ........................................................................................................................................ 31
Figure 4-5 – The p-box for source A (a) and source B (b) ................................................................................................. 32
Figure 4-6 – The p-box of the results obtained from D-S rule of combination ................................................................. 35
Figure 4-7 – The p-box of Yager combination .................................................................................................................. 37
Figure 4-8 – The p-box of mixture combination ............................................................................................................. 39
Figure 4-9 – The resultant p-box from PCR6 .................................................................................................................. 41
Figure 4-10 – Hypothetical hydrographs calculated based on the resultant bpa from PCR6 data fusion .................. 44
Figure 6-1 – The location of Okanagan Basin .............................................................................................................. 62
Figure 6-2 – DEM for the Okanagan Basin and 25 km vicinity (X-axis and Y-axis labels represent longitude and latitude respectively) ......................................................................................................... 64
Figure 6-3 – SPI process ................................................................................................................................................ 65
Figure 6-4 – Histogram for the number of years with data available ........................................................................... 68
Figure 6-5 – Stations labeled by year count and magnitude of standard error for fitted values of k and \( \theta \) ............ 69
Figure 6-6 – P-box (a) with 68% (1\( \sigma \)) of \( \pm \)SE of k and \( \theta \) (station 1125381), and (b) with superimposed empirical data, best fit, 50% (dashed) and 95% of SE (station 1124980) ......................................................... 72
Figure 6-7 – Elevation-precipitation scatter plot (the area of the bubbles is proportional to the product of fit parameters standard error and normalized elevation difference) .............................................. 74
Figure 6-8 – Cokriging interpolation of stations summer mean daily precipitation ......................................................... 74
Figure 6-9 – Discretization of gamma distribution using 10 equi-percentiles ............................................................. 76
Figure 6-10 – Predicted change in precipitation average for the Okanagan Basin for the CGCM3 A2 scenario run #5 for (a) 2050s (2040-2069) and (b) 2080s (2070-2099) based on 1961-1990 normals (PCIC 2011) ........................................................................................................................................ 78
Figure 6-11 – Variations in yearly mean precipitation 1900-2006 and change during 2040-2069 and 2070-2099 (PCIC 2011) ........................................................................................................... 78
Figure 6-12 – Log(Mean) vs. log(Variance) for station data (modeled) ................................................................. 80
Figure 6-13 – Log(Mean) vs. log(Variance) for grid data ...................................................................................... 81
Figure 6-14 – Change in extreme drought occurrence due to climate change .................................................. 81
Figure 6-15 – Finding standard errors multiplier .............................................................................................. 82
Figure 6-16 – Change in summer extreme drought modeled using station-based data .................................. 84
Figure 6-17 – Change in summer extreme drought modeled using grid data ................................................ 84
Figure 6-18 – Histograms of percent increase in summer extreme drought for the Okanagan Basin (x-axis labels represent bin mid-points; 2050s are shown in darker color) .................................................. 86
Figure 6-19 – Probability of enclosing for modeled data for (a) 2050s and (b) 2080s ................................. 87
Figure 6-20 – Probability of enclosing for grid data for (a) 2050s and (b) 2080s ................................................ 87
Figure 6-21 – SE_k × SE_θ for (a) modeled and (b) grid data .............................................................................. 88
Figure 7-1 – Location of wildfires in the Okanagan Basin during the summer months 1919-2009 ............ 97
Figure 7-2 – Temporal variations 1919-2009 of summer log(TBA) and SPI for the Okanagan Basin, 1919-2009 (dashed line represents log(TBA)) .................................................................................. 97
Figure 7-3 - Temporal variations of summer log(TBA) and E_{ref} for the Okanagan Basin, 1919-2009 (dashed line represents log(TBA)) ..................................................................................... 98
Figure 7-4 – Different models for change in wildfire fighting capacity (FSC-1, FSC-2 and FSC-3) ............... 98
Figure 7-5 – Scatterplots for SPI - log(TBA) regression ...................................................................................... 99
Figure 7-6 – The variation between logs of summer precipitation variance and mean for the Okanagan Basin based on grid data ........................................................................................................ 100
Figure 7-7 – The left tail of the gamma distribution for summer precipitation for three periods: 1961-1990, 2050s and 2080s ................................................................................................................. 102
### LIST OF ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANN</td>
<td>Artificial Neural Networks</td>
</tr>
<tr>
<td>CCIN</td>
<td>Canadian Cryospheric Information Network</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>CFS</td>
<td>Canadian Forest Service</td>
</tr>
<tr>
<td>CGCM3</td>
<td>Canadian Coupled Global Climate Model version 3</td>
</tr>
<tr>
<td>CMC</td>
<td>Canadian Meteorological Centre</td>
</tr>
<tr>
<td>CMI</td>
<td>Crop Moisture Index</td>
</tr>
<tr>
<td>CSDI</td>
<td>Crop-Specific Drought Index</td>
</tr>
<tr>
<td>CWFIS</td>
<td>Canadian Wildland Fire Information System</td>
</tr>
<tr>
<td>DHA</td>
<td>Drought Hazard Analysis</td>
</tr>
<tr>
<td>DRA</td>
<td>Drought Risk Assessment</td>
</tr>
<tr>
<td>DSmT</td>
<td>Dezert-Smarandache Theory</td>
</tr>
<tr>
<td>DST</td>
<td>Dempster-Shafer Theory</td>
</tr>
<tr>
<td>DT</td>
<td>Daily Transpiration deficit</td>
</tr>
<tr>
<td>DTx</td>
<td>DT for x days</td>
</tr>
<tr>
<td>D-S</td>
<td>Dempster-Shafer rule</td>
</tr>
<tr>
<td>enKF</td>
<td>ensemble Kalman Filter</td>
</tr>
<tr>
<td>ENSO</td>
<td>El Niño/La Niña-Southern Oscillation</td>
</tr>
<tr>
<td>FAO</td>
<td>Food and Agriculture Organization</td>
</tr>
<tr>
<td>FDS</td>
<td>Fuzzy Dempster-Shafer</td>
</tr>
<tr>
<td>FSC</td>
<td>Fire suppression capacity</td>
</tr>
<tr>
<td>GCM</td>
<td>General Circulation Model</td>
</tr>
<tr>
<td>GHG</td>
<td>Greenhouse Gases</td>
</tr>
<tr>
<td>HDI</td>
<td>Hybrid Drought Index</td>
</tr>
<tr>
<td>IDNDR</td>
<td>International Decade for Natural Disaster Reduction</td>
</tr>
<tr>
<td>IDW</td>
<td>Inverse Distance Weighted</td>
</tr>
<tr>
<td>IPCC</td>
<td>Intergovernmental Panel on Climate Change</td>
</tr>
<tr>
<td>IWRM</td>
<td>Integrated Water Resources Management</td>
</tr>
<tr>
<td>Acronym</td>
<td>Definition</td>
</tr>
<tr>
<td>---------</td>
<td>------------</td>
</tr>
<tr>
<td>LB</td>
<td>Lower-bound</td>
</tr>
<tr>
<td>MLR</td>
<td>Multiple Linear Regression</td>
</tr>
<tr>
<td>MoFLNRO</td>
<td>Ministry of Forests, Lands and Natural Resource Operations</td>
</tr>
<tr>
<td>NDMC</td>
<td>National Drought Mitigation Center</td>
</tr>
<tr>
<td>NDVI</td>
<td>Normalized Difference Vegetation Index</td>
</tr>
<tr>
<td>NIR</td>
<td>Near-Infrared</td>
</tr>
<tr>
<td>PASG</td>
<td>Percent Average Seasonal Greenness</td>
</tr>
<tr>
<td>PCA</td>
<td>Principal Component Analysis</td>
</tr>
<tr>
<td>PCIC</td>
<td>Pacific Climate Impacts Consortium</td>
</tr>
<tr>
<td>PCR5</td>
<td>Proportional Conflict Redistribution Rule no. 5</td>
</tr>
<tr>
<td>PCR6</td>
<td>Proportional Conflict Redistribution Rule no. 6</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Distribution Functions</td>
</tr>
<tr>
<td>PDO</td>
<td>Pacific Decadal Oscillation</td>
</tr>
<tr>
<td>PDSI</td>
<td>Palmer Drought Severity Index</td>
</tr>
<tr>
<td>PRISM</td>
<td>Pliocene Research Interpretation and Synoptic Mapping</td>
</tr>
<tr>
<td>RDI</td>
<td>Reconnaissance Drought Index</td>
</tr>
<tr>
<td>RSM</td>
<td>Relative Soil Moisture</td>
</tr>
<tr>
<td>rSPI</td>
<td>relative SPI</td>
</tr>
<tr>
<td>SWE</td>
<td>Snow Water Equivalent</td>
</tr>
<tr>
<td>SWIR</td>
<td>Short Wave Infrared</td>
</tr>
<tr>
<td>SOSA</td>
<td>Start of Season Anomaly</td>
</tr>
<tr>
<td>SPEI</td>
<td>Standardized Precipitation Evapotranspiration Index</td>
</tr>
<tr>
<td>SPI</td>
<td>Standardized Precipitation Index</td>
</tr>
<tr>
<td>SRES</td>
<td>Special Report on Emissions Scenarios</td>
</tr>
<tr>
<td>SSM</td>
<td>Special Sensor Microwave Imager</td>
</tr>
<tr>
<td>SWSI</td>
<td>Surface Water Supply Index</td>
</tr>
<tr>
<td>TBA</td>
<td>Total Burned Area</td>
</tr>
<tr>
<td>TBL</td>
<td>Triple Bottom Line</td>
</tr>
<tr>
<td>TPSS</td>
<td>Thin-Plate Smoothing Splines</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Full Form</td>
</tr>
<tr>
<td>--------------</td>
<td>-----------</td>
</tr>
<tr>
<td>UB</td>
<td>Upper-bound</td>
</tr>
<tr>
<td>UNFCCC</td>
<td>United Nations Framework Convention on Climate Change</td>
</tr>
<tr>
<td>USDM</td>
<td>U.S. Drought Monitor</td>
</tr>
<tr>
<td>WESTPO</td>
<td>Western Governors' Policy Office</td>
</tr>
<tr>
<td>WMB</td>
<td>Wildfire Management Branch</td>
</tr>
<tr>
<td>WMO</td>
<td>World Meteorological Organization</td>
</tr>
<tr>
<td>WWII</td>
<td>World War II</td>
</tr>
</tbody>
</table>
### LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bel</td>
<td>Belief function</td>
</tr>
<tr>
<td>$E_{ref}$</td>
<td>Reference evaporation</td>
</tr>
<tr>
<td>$K$</td>
<td>Conflict</td>
</tr>
<tr>
<td>$k$</td>
<td>Gamma distribution shape parameter</td>
</tr>
<tr>
<td>$m, bpa$</td>
<td>Basic probability assignment</td>
</tr>
<tr>
<td>$\emptyset$</td>
<td>Null set</td>
</tr>
<tr>
<td>$P$</td>
<td>Precipitation</td>
</tr>
<tr>
<td>$pl$</td>
<td>Plausibility function</td>
</tr>
<tr>
<td>$Q$</td>
<td>Discharge</td>
</tr>
<tr>
<td>$q$</td>
<td>Yager’s basic probability assignment</td>
</tr>
<tr>
<td>$SE$</td>
<td>Standard error</td>
</tr>
<tr>
<td>$V$</td>
<td>Variance</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Significance level</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Gamma distribution</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Lower incomplete gamma function</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Gamma distribution scale parameter</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>Ignorance</td>
</tr>
</tbody>
</table>
ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to some of the faculty and staff at UBC Okanagan’s School of Engineering. Firstly, I would like to recognize Dr. Rehan Sadiq, who in a very professional and supportive manner helped streamline my workflow for achieving the outcome I envisioned from my PhD. This outcome would not have been as successful under any other undertaking. I would also like to thank my committee members and other involved faculty. The guidance from committee member Prof. Faisal I. Khan at the Faculty of Engineering and Applied Science, Memorial University of Newfoundland was instrumental and is highly acknowledged. Dr. Bahman Naser is also highly acknowledged for his guidance and insightful feedback.

The financial support from Natural Sciences and Engineering Research Council of Canada (NSERC) under Discovery Grant and PGS –D2 grant programs is acknowledged. The following people are individually gratefully acknowledged:

- Dr. Scott Ferson of Applied Biomathematics is highly acknowledged for his valuable inputs for Chapters 4 and 6.
- Dr. Donald Wilhite and Dr. Michael Hayes of the NDMC are acknowledged for their support regarding the materials used in the review in Chapter 2.
- Dr. Florentin Smarandache of the University of New Mexico and Dr. Jean Dezert of the French Aerospace Lab are also acknowledged for their valuable inputs for Chapter 4. Dr. Arnaud Martin of ENSIETA (France) is acknowledged for providing the code for PCR6 used in Chapter 4.
- Donald Gayton of FORREX and Kevin Skrepnek of Kamloops Fire Centre (Wildfire Management Branch) are greatly acknowledged for providing data and insight for Chapter 7.
- The anonymous reviewers for the submitted papers based on which this thesis has been written are greatly acknowledged.

Foremost, I would like to acknowledge my exceptional parents who first sowed seeds of knowledge in me and for the support of my entire family, including my siblings, who encouraged me throughout the pursuit of my academic endeavours.
CHAPTER 1 INTRODUCTION

This thesis will address uncertainty, part of the intricacies involved in drought risk assessment enabling better understanding of drought risk, hence, improving decision making and drought management. This chapter provides a general introduction to this research. It first provides brief background information, then highlights the need for this research and thereby sets the objectives. These sections are intended to lay the conceptual groundwork, helping the reader understand the significance of the research problem pursued. Finally, a section describes the structure of the thesis which will serve to guide the reader through the rest of the thesis.

1.1 Background

Drought is a natural hazard that is instigated by severe shortage in precipitation. As a meteorological phenomenon, drought can occur to any region. Given the pervasive nature of water dependence in different aspects of human society and nature, drought inevitably has profound and pervasive consequences on society and the environment. Drought consequences can reach far beyond the drought-stricken area and the loss can be in the order of billions of dollars (Wilhite 2000).

Given the high stakes involved, decision makers require reliable information to manage drought impacts. A primary tool for drought management is the risk assessment framework which was adopted in mid-1990s enabling proactive planning for drought impacts and consequences (Andreu et al. 2007; Hayes et al. 2004; Wilhite et al. 2000; Wu and Wilhite 2004). This framework measures, risk, or the possible undesired consequences of a hazard by integrating hazard and vulnerability (Hayes et al. 2004).

It has long been understood that data and models used in drought risk assessment (DRA) (as in other hydrologic analyses) are subject to stochastic and subjective uncertainties that undermine the reliability of hydrologic information. Uncertainty can include (but is not limited to) the natural spatio-temporal variability (also, stochasticity) in hydrologic variables but also human-caused lack of knowledge (epistemic uncertainty) in the form of incompleteness in hydrologic records, and disagreement (conflict) between multiple data sources of any feature. For the former, climate change is predicted to shift the long-term characteristics of variability (“climate normals” e.g., mean precipitation) which further undermines the accuracy of results produced from models that assume stationary normals.

Traditional deterministic hydrologic models do not appreciate such uncertainties which can propagate through subsequent modeling without scrutiny. As a result, stakeholders and decision makers are left
without appropriate awareness and are prone to make poorly-informed decisions. On the other hand, uncertainty-driven analysis can characterize the uncertainty associated with modeling, placing the traditional deterministic results within the frame of surrounding uncertainty.

In order to handle the problems associated with uncertainty, this work first acknowledges that methods for modeling uncertainty in hydrological analysis have evolved beyond classical and Bayesian theories, and deterministic settings need to be replaced by such generalized and uncertainty-driven settings (Hall 2003). Furthermore, uncertainty should be classified as variability (aleatory) and epistemic uncertainty (Ferson et al. 2003; Oberkampf et al. 2000; Sentz and Ferson 2002). Methods that are capable of handling both types of uncertainty, separately but within a single framework, will be very helpful. Soft-computing and generalized models of information such as fuzzy sets theory (Zadeh 1965) or Dempster-Shafer Theory of evidence (DST) (Dempster 1967; Shafer 1976) generalize the mapping of data from a crisp (deterministic) mapping to a mapping of a set of possible and probable values and are hence capable of modeling epistemic uncertainty such as conflict, non-specificity, vagueness and incompleteness.

This thesis envisions incorporating soft-computing methods in DRA: methods that have the capability to handle uncertainty, partial truth, and approximation. These methods can integrate uncertainty modeling with DRA, characterize resulting uncertainty and improve subsequent decision making. Previous works have shown that the DST method can improve drought characterization and classification (e.g., Eierdanz et al. 2008; Ghosh and Mujumdar 2007; Raje and Mujumdar 2010). This thesis further pursues this vision by generalizing some DRA data and models by uncertainty-driven methods, and further investigates DST’s capacity in handling additional uncertainties in DRA. This application will concern the two components of DRA: (a) hazard analysis and (b) vulnerability analysis, described in more detail in following.

(a) Drought hazard analysis: Uncertainty is handled in two applications. First, to deal with high conflict among multiple sources of data for a drought hazard indicator namely the snow water equivalent (SWE). SWE is an important indicator for the snow accumulated in spring and the resulting volume of runoff in summer. The multitude of sources of SWE data presents a decision making challenge for water resource managers, especially when sources are highly conflicting. Using DST’s framework, four applicable DST data fusion are identified and the high conflict between the data sources is resolved. The resulting uncertainty-driven fused data are argued to be more informative and contain a measure of uncertainty that can be incorporated in subsequent processes and analyses including decision making. For this, a method to propagate the uncertainty along with data in subsequent modeling process is
thereby introduced. Second, DST is used to enhance a drought hazard index: the Standardized Precipitation Index (SPI) by handling (i) incompleteness and (ii) variability. (i) Hydrologic records often include missing data which cause *incompleteness* in data which results in inaccuracy in analysis since the records used are of variable length and non-uniform quality. This is more evident in remote and mountainous regions such as the Okanagan Basin, BC. Using DST, the uncertainty from hydrologic records of various lengths is characterized. This measure reflects the reliability of an SPI value regardless of the length of precipitation record used. Additionally, a novel method for propagating uncertainty along with data in spatial cokriging interpolation is introduced. (ii) In addition, a method to model shift in variability due to climate in SPI is introduced. Current characterization of SPI assumes stationary normals for precipitation and ignores long term trends in precipitation. Since summer precipitation has a long term trend (for the Okanagan Basin it is towards lower precipitation), this work investigates the effects of this trend on the frequency of drought occurrence in a future climate scenario.

(b) Vulnerability analysis: The enhanced SPI-based hazard analysis is complemented with vulnerability analysis for drought-induced wildfire risk assessment in the Okanagan Basin. Using two additional parameters, namely, evapotranspiration and firefighting capacity, the loss from wildfires (total burned area) has been linked to drought. Wildfire drought risks are characterized for current and future scenarios.

1.2 Research objectives

The overall objective of this research is to improve the accuracy and informativeness of drought risk assessment (DRA) by replacing the traditional deterministic approach with enhanced uncertainty-driven methods.

The following specific objectives are envisioned:

1. Incorporating epistemic uncertainty in the form of data conflict and incompleteness in DRA hence improving its accuracy and informativeness.

2. Incorporating the effects of shift in precipitation normals on precipitation variability in SPI in order to model the change in drought hazard, and comparing the significance of this change with respect to surrounding uncertainty.

3. Performing enhanced vulnerability analysis and risk assessment for drought induced wildfire in the Okanagan Basin, BC.
1.3 Thesis structure

The structure of this work is illustrated in Figure 1-1. The concepts in each chapter usually follow the ideas put forward in the preceding chapter. Both chapters 4 and 6 address issues related to uncertainty-driven hazard analysis. However, a separate intermediate chapter (Chapter 5) is dedicated to a critical review of drought indices in the context of their application to DRA.

Following this general introduction, Chapter 2 provides a literature review of general concepts related to drought and DRA. In Chapter 3, uncertainty analysis frameworks including Dempster-Shafer theory and data fusion techniques are described. Chapter 4 handles conflict in SWE, an indicator of drought hazard, using DST data fusion techniques. In Chapter 5, drought indices are reviewed critically and SPI is identified as an appropriate drought index for drought hazard analysis. In Chapter 6, uncertainty-driven drought hazard analysis using SPI is formulated. In this chapter, incompleteness in precipitation records and long-term shifts in climate normals are addressed. Chapter 7 performs vulnerability analysis and risk assessment for wildfires using enhanced SPI. Finally, conclusions are drawn and recommendations for future research have been provided in Chapter 8.

Figure 1-1 - Thesis structure
CHAPTER 2 BACKGROUND

The conceptual background in this work scopes over the topics of drought risk assessment and uncertainty analysis formulations. This chapter explains basic concepts in drought characterization and risk assessment. Drought indices as the primary tools for DRA are critically reviewed separately in detail in Chapter 5. The concepts behind uncertainty analysis in hydrologic modeling have been discussed in Chapter 3.

2.1 Drought

Drought is a natural hazard that occurs due to substantial deficiency in precipitation. Drought causes substantial loss and disruption to human life. For instance, drought accounts for 22% of the total damage from disasters, and along with earthquakes, floods and windstorms cause the highest global economic loss (Guha-Sapir et al. 2004; IDNDR 1995; Wilhite 1991). Droughts also account for ~33% of the people affected by all natural hazards which is the highest impact among natural hazards (IDNDR 1995). Some regions in Canada such as the Prairies and the interior of British Columbia are particularly susceptible to drought. The 2001/02 drought in the Canadian Prairies resulted in a gross domestic product drop of ~$5.8 billion and a loss of more than 41,000 jobs (Wheaton et al. 2005).

The threat from drought has further gained importance due to additional factors including:

- increasing drought frequency and severity due to climate change effects (Bates et al. 2008),
- decreasing safe water resources due to increasing contamination of water resources by urbanization and industrialization, and finally,
- increasing inhabitation of climatically marginal regions i.e., regions with more uncertain water availability (Wilhite 2005).

From the viewpoint of water resource management, such drivers undermine the balance between water supply and demand, and challenge its long-term sustainability. A disruption in water supply affects water-dependent activities which in turn undermines the natural flow of dependent goods and services. Such vital dependence of society’s functioning on water results in a complex network of impacts once drought occurs (Wilhite et al. 2007).

The pervasive nature of drought impacts is best addressed by the Integrated Water Resources Management (IWRM) paradigm (Kenabatho and Montshiwa 2006). IWRM promotes a “co-ordinated development and management of water, land and related resources in order to maximize the resultant
economic and social welfare in an equitable manner without compromising the sustainability of vital ecosystems” (Jønch-Clausen and Fugl 2001).

2.2 Drought characterization concepts

2.2.1 Drought definitions

The definition of drought has by itself proven complex and challenging; although the majority of people may consider some extreme precipitation shortage as drought, how to objectively characterize it for planning and management is a challenging issue. Drought can generally be defined as the extreme persistence of precipitation deficit (González and Valdés 2006) over a specific region for a specific period of time (Beran and Rodier 1985; Correia et al. 1994). In addition to the elements of ‘persistence’ of ‘precipitation deficit’ and ‘bounded by time and space’, definitions have expanded to include impacts on the environment and on society (Tsakiris and Vangelis 2004). In this viewpoint, drought impacts are part of the drought definition and drought definition is a function of both the magnitude of the water shortage as well as local susceptibility and ground conditions. In this regard, Wilhite (2004) emphasizes the human demand placed on water supply and regards drought susceptibility as a context-dependent matter. As such, part of the complexity in drought definitions stems from such subjectivity with respect to type and extent of drought impacts (Eierdanz et al. 2008). This challenge is reflected in the conceptual development of non-meteorological drought indices; although more than 91 drought impacts can be identified (NDMC 2011a), drought indices make use of only a handful of impact-indicators including vegetation health, evapotranspiration or water resources levels.

It is also important to differentiate between conceptual and operational definitions of drought (Wilhite and Glantz 1985). Conceptual definitions are formulated in general terms for overall understanding and establishing drought policy (NDMC 2006a). Operational definitions of drought (e.g., agricultural or hydrological) objectively define criteria for the start, the end and the severity of drought for a specific application. These characteristics are described in the following section.

2.2.2 Drought types and characteristics

By implementing an operational definition of drought, three main physical drought types are established: meteorological, agricultural and hydrological droughts. In a broad definition, these droughts occur in the following order: precipitation deficiency instigates meteorological drought which subsequently impacts soil moisture content (i.e., agricultural drought). Low recharge from the soil to
water features (e.g., streams and lakes) causes a delayed hydrological drought. Figure 2-1 provides a general schematic of this sequence.

In addition to their type, droughts are fundamentally characterized in three main dimensions: severity, duration and spatial distribution. Some authors have included additional characteristics such as: frequency, magnitude (or cumulated deficit), predictability, rate of onset (i.e., a slow or fast onset) and timing. Unfortunately, usage of the terms severity, intensity, and magnitude is not universal, and sometimes their meanings are even switched. For example, Yevjevich (1967) uses the vocabulary of run-sum, run-length, and run-intensity for the associated terms of severity, duration, and magnitude used by Dracup et al. (1980). This thesis uses the widely adopted terminology of Salas (1993):

**Duration:** Depending on the region and impacts under study, drought duration can vary between a week and a few years. Because of drought’s dynamic nature, a region can experience wet and dry spells
simultaneously when considering various timescales. As such, in shorter durations the region experiences dryness or wetness, while in longer-term, it experiences the opposite (NCDC 2010).

**Magnitude:** The accumulated deficit of water (e.g., precipitation, soil moisture, or runoff) below some threshold during a drought period.

**Intensity:** The ratio of drought magnitude to its duration.

**Severity:** Two usages are provided for drought severity:

1. the degree of the precipitation deficit (i.e., magnitude), or
2. the degree of impacts resultant from the deficit (Wilhite 2004).

**Geographic extent:** The areal coverage of the drought. This area expands and contracts during the drought. This area can span several hydrological or administrative regions (e.g., watersheds or provinces, respectively).

**Frequency (return period):** The frequency or return period of drought is defined as the average time between drought events that have a severity that is equal to- or greater than a threshold.

### 2.2.3 Drought indicators

Along with precipitation, additional variables such as evapotranspiration and streamflow are also used to more comprehensively characterize drought. Using water balance and hydrological models, such variables or indicators are used in combination to derive a drought index. Such indicators can be meteorological, hydrological or water supply-and-demand in nature. Meteorological indicators include precipitation and cloud cover; hydrological indicators include streamflow and groundwater level; water supply and demand indicators include reservoir storage. In practice, however, some indicators such as precipitation, potential evapotranspiration, and soil cover and vegetation cover characteristics have had wider applications and influence (Tsakiris and Vangelis 2005).

### 2.2.4 Drought indices

Drought indices are quantitative variables used to characterize the severity of drought. They provide a generalized picture of drought conditions that is used for reporting and comparing drought levels. A drought index is obtained by integrating several indicators into a single numerical value. Compared to raw indicator data, drought indices are more readily useable for decision making (Hayes 2006). In this regard, drought indices provide processed information for operational hazard and consequences.
quantification. Some drought indices such as the Normalized Difference Vegetation Index (NDVI) only reflect consequences while others such as the Standardized Precipitation Index (SPI) are hazard-only indices.

Using this relatively simple methodology, drought indices have developed into the primary tool for communicating drought levels between researchers and organizations. Some prominent indices are currently operationally used for the publication of weekly grid-based drought condition maps which are publicly accessible.

Operationally, using an index for drought characterization serves the following purposes:

- drought detection and real-time monitoring (Niemeyer 2008)
- declaring the beginning or end of a drought period (Tsakiris et al. 2007)
- allowing drought managers to declare drought levels and instigate drought responses measures;
- drought evaluation (Niemeyer 2008)
- representing the concept of drought in a region (Tsakiris et al. 2007)
- correlating with quantitative drought impacts over variable scales of geography and time; and
- facilitating the communication of drought conditions among various interested entities.

For the purpose of risk assessment, some drought indices are widely used as a surrogate measure of hazard (Shahid and Behrawan 2008; Wilhelmi and Wilhite 2002; Wu et al. 2004) and are reviewed in detail in Chapter 5.

2.3 Drought management and drought risk assessment

Drought management aims to minimize the subsequent impacts of a considerable interruption in water supply. For this generic objective, drought management specifically addresses the disruption in the sustainable functioning of the economic, environmental and social aspects of the society.

Traditionally, this objective was pursued by a crisis (disaster) management approach until mid-1990s when a risk-based approach gradually emerged (Wilhite et al. 2000). Risk analysis has been established as a proactive approach for preparedness and contingency planning instead of the “reactive” crisis management approach. A risk measure relates the enormity of hazard to the losses sustained by the population and provides water resources planners and stakeholders with an improved tool in decision making. In addition, the characterization of risk enables comparing and ranking of risk resulting in
improved preparedness by improved allocation of resources, for example, by early mobilization of adaptive capacity.

In the context of drought, risk analysis has hence emerged as a standard framework for the evaluation of the loss associated with drought (Hayes et al. 2004; Laughlin and Clark 2000). Drought risk analysis is constituted by two tasks: first, drought risk assessment (DRA) that characterizes and quantifies risk, and second, drought risk management that identifies best management strategies to control or minimize adverse effects. According to Hayes et al. (2004), DRA comprises hazard analysis and vulnerability analysis that jointly quantify drought risk.

A complete assessment of the potential consequences of drought (i.e., risk) through a comprehensive risk assessment is a difficult process. Drought initiates a complex network of impacts which makes the quantification of drought losses through vulnerability analysis difficult (Wilhite et al. 2007). In addition, knowledge gaps and uncertainties undermine the accuracy of variables used in hazard- and vulnerability analyses. Part of this difficulty is due to the unique characteristics of drought which even includes its definition. Drought definition is non-uniformly treated; although the majority may characterize drought as an “extreme water shortage event”, however different population groups and economic sectors disproportionately and non-uniformly sustain subsequent losses. Drought vulnerability analysis is hence subjective to the characteristics of the affected region, whereas the hazard assessment considers a strictly stochastic variable.

Hayes et al. (2004) provided a DRA framework that adopts two main components of risk assessment (Figure 2-2). First, hazard analysis, that provides a probabilistic characterization of drought using frequency (return period) and drought severity. Temporal trends in hazard analysis concerns the multi-temporal analysis of the hazard behaviour, e.g., long-term climatic patterns resulting from climate change. Second, vulnerability analysis, which is based on three components: causal assessment, impact assessment and temporal trends of droughts. These analyses study factors such as the composition of the population, development, land conditions and economic sectors. DRA’s components are described in detail in the following sections.
2.3.1 Drought hazard

2.3.1.1 Severity and frequency

Hazard is a condition that poses a threat to an asset. In the context of DRA, hazard is regarded as a stochastic and independent process that affects human and environmental assets. Drought hazard is closely tied with, and is regularly quantified using drought indices. Drought hazard is characterized using drought dimensions: severity, frequency, magnitude, geographic extent and duration. Temporal trends are drivers that alter the behavior of drought hazard in the long-term, e.g., climate change.

2.3.1.2 Climate change and droughts

Currently, analysis and planning for drought hazard rely on historical climate data, for example, 30 years of a certain reference period. The long-term trends in such data are assumed to be stationary. Temporal trends in hazard can however arise from predicted non-stationarity in long-term climate patterns (Bates et al. 2008). In general, two dynamics can affect future drought hazard (Ghosh and Mujumdar 2007):

1. The dynamic of future climate scenarios that depend on greenhouse gas emission levels.
2. The dynamics involved in atmospheric circulations and their effects on local climates.

2.3.2 Drought vulnerability

Vulnerability is the term linking hazard to consequences and risk. It is defined as the degree of susceptibility to the drought hazard (Wilhelmi 1999; Wilhelmi and Wilhite 2002). Vulnerability is a variable construct and will change based on the nature of the hazard, e.g., by drought severity. Vulnerability also varies depending on ground conditions and coping capacity. For example, compared to developed countries, less developed countries are more vulnerable due to limited financial resources and human capacity (Wilhite 2005).
According to Hayes et al. (2004), the assessment of drought vulnerability in an area consists of the following three tasks:

2.3.2.1 Causal and impact assessment

Following the onset of drought, a network of interconnected impacts occurs to an area beyond the drought-stricken region. According to Wilhite et al. (2007), the dependence of numerous sectors of the economy on water for the production of goods and providing services causes intricate connections among drought impacts. Drought hence variably impacts different economic sectors and population groups.

Impact assessment: Impact assessment comprises of the identification of the relevant impacts. Hayes et al. (2004) suggested inventoried susceptible social, economic, and environmental impacts to sectors in a particular region. As such, the list of possible impacts is reduced to applicable impacts. For example, NDMC (2012) compiled a generic list of 91 drought impacts under three main sustainability categories: economic, environmental and social categories (this is also referred to as triple bottom line or TBL). Impacts in each TBL category are further sub-categorized. Examples of such sub-categories are presented in Table 2-1. Similar lists appeared earlier in WESTPO (1977); Wilhite and Glantz (1985). Country-specific lists using the similar TBL categories can be found in, e.g., FAO (2004); Nguyen et al. (2009).

<table>
<thead>
<tr>
<th>Category</th>
<th>Example subcategory</th>
<th>Example impacts</th>
</tr>
</thead>
</table>
| Economic     | Costs and losses to agricultural producers | • Annual and perennial crop losses  
                          • Damage to crop quality                                                     |
| Environmental| Damage to plant communities           | • Increased number and severity of fires  
                          • Wind and water erosion of soils, reduced soil quality  
                          • Loss of biodiversity                                                      |
| Social       | Health                               | • Mental and physical stress (e.g., anxiety, depression, loss of security, domestic violence)  
                          • Reductions in nutrition (e.g., high-cost food limitations, stress-related dietary deficiencies) |

Causal assessment: A causal analysis traces outward from each impact the multiple environmental, social, and economic underlying factors that contribute to a resulting impacts (Ribot 1996). In such a
causal chain, precipitation deficiency instigates other impacts and is placed top among additional variables that accentuate the negative consequences of drought. For example, the impact of ‘water shortage’ can be accentuated by not having alternative water reserves or not having water conservation plans in place. Methods such as impact tree diagrams and scenario building have been used to model the network of impacts from drought (Hayes et al. 2004). Impact tree diagrams show the causal relationships among impacts (NDMC 2006b). Scenario building explores a range of possible future scenarios to better understand potential causal relationships (NDMC 2006b). Scenarios consider how dynamic vulnerability factors such as population increase and degrading water quality could influence drought impacts.

Scenarios analysis can be developed by two approaches (Warwick et al. 2003):

a) Prospect planning: studying the current situation and drivers (e.g., improving technology) that propel the present situation into the future or

b) Retrospect planning: designing a snapshot of the future situation and studying backward how that state can be achieved.

2.3.2.2 Temporal trends

Temporal trends concern the dynamics in demographics and policies. Important factors include resilience and adaptation which affect long-term planning for droughts and need to be included in risk analysis. These include issues such as technology, population demographics (e.g., in terms of human capacity or economic sector), behaviour (e.g., willingness to adapt), and enacted policies (Hayes et al. 2004). Resilience refers to the capability of society and supporting ecological systems to tolerate disturbances caused by drought and to restore to a stable state. This includes, for instance, boosting the firefighting capacity. Adaptation refers to activities undertaken in the system in order to adjust to drought disturbances. As a result of these activities, vulnerability is reduced and resilience is increased (IPCC 2007). Adaptation capacity differs among societies, for example, developing countries are restricted by human capacity and financial resources (UNFCCC 2007).

2.3.3 Drought risk characterization

Following hazard and vulnerability analyses, hazard and vulnerability are integrated to estimate risk. Multiple formulations have been proposed for the derivation of risk. Some researchers include exposure, i.e., the presence of an asset (e.g., human population) to risk characterization and define risk as the product of these three terms (Schneiderbauer and Ehrlich 2006).
\[ \{R\} = \{H\} \times \{V\} \times \{E\} \quad (2-1) \]

where:
- \(R\) = risk
- \(H\) = hazard
- \(E\) = exposure
- \(V\) = vulnerability (dimensionless)

Figure 2-3 - The presence of hazard, exposure and vulnerability results in risk

However, in most drought risk analysis literature, the exposure is already assumed 1, i.e., an asset is present where and when the hazard occurs. A product operator ensures that risk is only realized when both hazard and vulnerability are present. Eq. (2-1) can hence be simplified as (Iglesias 2008):

\[ \{R\} = \{H\} \times \{V\} \quad (2-2) \]

In this formulation, the risk represents the probability of harmful consequences or the expected damages resulting from interactions of hazard and vulnerable conditions.
CHAPTER 3 HYDROLOGICAL UNCERTAINTY ANALYSIS

3.1 Uncertainty modeling

Uncertainty arises from current and future unknowns and includes unknowns in current physical measurements or the occurrence of future events. In philosophical terms, uncertainty has been subject of extensive research and is hence non-uniformly regarded among different disciplines. This is reflected in the variety of proposed epistemological categorizations (see Ayyub and Klir (2006) for a list). This thesis adopts the definition by Ayyub and Klir (2006) for the fields of engineering and sciences: uncertainty is one type of ignorance (all unknown) and is a conscious ignorance, “a recognized self-ignorance through reflection”; it arises when knowledge is incomplete due to inherent deficiencies with acquired knowledge.

Uncertainty in the environment may also be represented from the viewpoint of the procedure that causes it: In the process of modeling the environment, uncertainty inevitably arises due to humans’ inability to capture the true complexity of real-world systems. This inherent inability requires applying simplifications in the creation of abstract models of the environmental features and mechanisms. This deliberate departure from comprehensive modeling is one source of uncertainty in environmental data and models. The resulting uncertainty undermines the accuracy and reliability of the outputs obtained from environmental models.

By analyzing the uncertainty associated with environmental modeling, it is possible to reduce the consequences from uncertain data and models. This can be done through managing uncertainty, e.g., by reducing uncertainty, or by communicated uncertainty, e.g., by providing uncertainty-driven results enabling more informed decision making. Uncertainty analysis in hydrology has recently become more important as hydrologic modeling has “matured”, moving beyond the traditional deterministic approaches (Hall 2003).

As our understanding about the nature and types of uncertainty has improved, methods to handle uncertainty have expanded. Uncertainty has been categorized into two major types: aleatory and epistemic uncertainty (Fishburn 1994; Oberkampf et al. 2000; Winkler 1996). Aleatory uncertainty is also termed variability, stochastic uncertainty or simply stochasticity. This uncertainty is irreducible as it arises from the natural variations within a system. For example, monthly precipitation is treated stochastically since precise knowledge that would otherwise determine its causing mechanism is
unavailable. Variability is regularly parameterized and represented by either probability density functions (PDF) or cumulative distribution functions (CDF).

Epistemic uncertainty (also termed subjective uncertainty) arises from limited knowledge about the system. It can be reduced by improving the means by which systems are observed and modeled. Some types of epistemic uncertainty include incompleteness, vagueness, ambiguity and conflict. Ayyub and Klir (2006) provided a taxonomy of various epistemic uncertainty and methods to handle them. Among epistemic uncertainties, incompleteness and conflict frequently occur in hydrologic analysis. Incompleteness arises from missing data and can be handled by Dempster-Shafer Theory (DST) (Dempster 1967; Shafer 1976). Conflict arises from disagreement between multiple data available for a given phenomenon, such as measurements from different sensors, methods or models. Conflict can similarly be handled by DST and is discussed in detail in Sections 3.3 and 3.3.1.

Traditionally, in handling uncertainty, probabilistic methods have predominantly been used. Such applications usually focus on aleatory uncertainty. As such, some researchers still emphasize the capability of probabilistic methods for handling different uncertainties (Laviolette and Seaman 1994). On the other hand, probabilistic methods have been challenged by others for handling data that are subject to epistemic uncertainties that result from lack of knowledge about the system (Dubois and Prade 1994). These include additional types of epistemic uncertainty such as vagueness and ambiguity (Hoffman and Hammonds 1994; Limbourg 2008). For instance, information may be expressed in linguistic terms (e.g., ‘low’, ‘medium’ and ‘high’) which are inherently vague, and not probabilistic. Expert opinion is one case of such information and is efficiently modelled by methods such as fuzzy logic (Sandri et al. 1995). Ambiguity arises when information is missing that would otherwise specify the choice between alternatives (Klir and Folger 1988).

Ayyub and Klir (2006) provide a taxonomy of different types of uncertainty and methods to handle them. From this taxonomy those uncertainties appearing in hydrological data and handling methods have been addressed in works such as Bardossy et al. (1990) and Hall (2003).

### 3.2 Dempster–Shafer theory (DST)

The Dempster–Shafer theory of evidence (DST) (Dempster 1967; Shafer 1976) is based on Bayesian probability theory. DST generalizes Bayesian inference by formally allowing probability to be assigned to “non-disjoint” sets. For example, if hypotheses include sets of: \{low\}, \{medium\} and \{high\}, DST would enable probability to be assigned to an additional \{low, medium\} set. Similarly, if a cumulative distribution function is thought to contain a set of scalar hypotheses for a variable, DST can additionally
assign probability to interval hypotheses. As such, by formally allowing a more precise allocation of evidence to both disjoint and non-disjoint sets, DST enables a finer representation of uncertainty information compared to Bayesian theory. If data consist of disjoint hypotheses, DST’s frame of discernment reduces to that of a Bayesian characterization.

One of the advantages of DST is its capacity to handle conflict and incompleteness simultaneously in a formal unified framework. This capacity renders this framework well for modelling uncertainties specific to hydrological data. DST can also model additional types of epistemic uncertainties such as vagueness by using its extensions. Vagueness can be handled by incorporating fuzzy membership functions within the framework of fuzzy Dempster-Shafer (FDS) (Yen 1990).

3.2.1 Basic concepts

The frame of discernment (Θ) is the fundamental set in DST and consists of an exhaustive set of mutually exclusive hypotheses or propositions. For example, for ‘water quality’ the set of propositions can be defined to include ‘low’ (L), ‘medium’ (M) or ‘high’ (H). No other sets exist in the frame of discernment (the property of being exhaustive), and the intersection between pairs of sets is a null set (e.g., L ∩ M = ∅), i.e., they are mutually exclusive. The power set, 2^Θ is defined as the set of all possible subsets of Θ (including the empty set ∅). For example, if the frame of discernment is comprised of three sets, Θ = {L, M, H}, its power set will consist of 8 subsets as following:

\[2^\Theta = \{\emptyset, \{L\}, \{M\}, \{H\}, \{L, M\}, \{M, H\}, \{L, H\}, \{L, M, H\}\}\]

Among the subsets, the last subset (\{L, M, H\} = Θ) denotes complete ignorance as it fails to provide any specific information. Each subset in the power set of Θ is called a focal element. Subsets can also be intervals, such as [[2.5 4]] or [[4 17] U [22 35]].

3.2.1.1 Basic probability assignment (bpa)

Based on the evidence provided, each focal element may be assigned a degree of belief \(\in [0, 1]\), where 0 represents no belief and 1 represents complete belief. The degree of belief for each proposition is termed a basic probability assignment (bpa), or mass function (m), e.g., \(m(\{L, M\}) = 0.5\).

DST uses a generalized notion of probability termed a basic probability assignment bpa, or mass function, \(m. bpa\) is the proportion of all relevant and available evidence (such as empirical evidence or expert knowledge), that support a particular focal element. The bpa ranges between 0 and 1.
It should be noted that \( bpa \) is not analogous to the classical definition of probability, rather, it is a mapping of the power set to the interval between 0 and 1, where the \( bpa \) of the null set is \( m(\emptyset) = 0 \), and the summation of the \( bpas \) of all subsets (i.e., all possibilities) of the power set is 1 (Sentz and Ferson 2002). The proposition \( m(A) \) has the following properties:

\[
\sum_{A \subseteq \Theta} m(A) = 1 \tag{3-1}
\]

\[
\forall A \subseteq \emptyset \ 0 \leq m(A) \leq 1 \tag{3-2}
\]
i.e., according to (3-2), the probability of an event lies between 0 and 1. Suppose that the evidence is \( m(M) = 0.7 \) on a frame of discernment \( \Theta = \{L, M, H\} \). As required by (3-1), the total \( bpa \) should sum to 1, therefore 0.3 is assigned to ignorance, i.e., \( m(\emptyset) = m(L, M, H) = 0.3 \). All the remaining subsets have zero probability mass. In comparison to Bayesian theory, DST requires all missing evidence to be assigned to ignorance while Bayesian theory equally distributes missing evidence to the remainder disjoint subsets (Laplace Principle of Insufficient Reason).

Equation (3-1) corresponds to a closed world (exhaustive) assumption, meaning that no other state than the universal set elements can possibly be achieved. If no evidence relevant to any focal element is available, the remainder \( bpa \) is assigned to ignorance (\( \emptyset \)). Equation (3-2) requires the summation of \( bpa \)'s of focal elements to equal to 1.

### 3.2.1.2 Belief function (\( bel \))

The lower bound for probability in DST (as well as in other frameworks) is belief. For a proposition of interest \( A_i \), the belief function is defined as the sum of all the \( bpa \)'s of the proper subsets \( A_k \) of the proposition of interest \( A_i \), i.e., \( A_k \subseteq A_i \) for proposition \( A_i \). The general relation between \( bpa \) and belief is expressed as:

\[
bel(A_i) = \sum_{A_k \subseteq A_i} m(A_k) \tag{3-3}
\]

The belief function has two other properties:

\[
\begin{bmatrix}
bel(\emptyset) = 0 \\
bel(\Theta) = 1
\end{bmatrix} \tag{3-4}
\]
Consider the frame of discernment given in Table 3-1; for intervals in the first row bpa are given in second row.

<table>
<thead>
<tr>
<th>$A_i$</th>
<th>$\emptyset$</th>
<th>[2.5 6]</th>
<th>[6 9]</th>
<th>[9 11.2]</th>
<th>[2.5 9]</th>
<th>[6 11.2]</th>
<th>[2.5 6] $\cup$ [9 11.2]</th>
<th>[2.5 11.2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m(A_i)$</td>
<td>0</td>
<td>0.5</td>
<td>0.3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The calculation of belief functions for two focal elements are shown below. See Table 3-2 for belief functions of the entire interval.

$$bel([2.5 \ 6] \cup [9 \ 11.2]) = m([2.5 \ 6]) + m([9 \ 11.2]) = 0.5$$

$$bel([2.5 \ 11.2]) = m([2.5 \ 6]) + m([6 \ 9]) + m([9 \ 11.2]) + m([2.5 \ 9]) + m([6 \ 11.2])$$

$$+ m([2.5 \ 6] \cup [9 \ 11.2]) + m([2.5 \ 11.2]) = 1$$

### 3.2.1.3 Plausibility function ($pl$)

The upper bound for probability is plausibility, which is the summation of bpa’s of all sets, $A_k$ that intersect with the set of interest, $A_i$, i.e., $A_k \cap A_i \neq \emptyset$. Plausibility is defined as:

$$pl(A_i) = \sum_{A_k \cap A_i \neq \emptyset} m(A_k) \quad (3-5)$$

Belief and plausibility functions are linked to each other through the doubt function, defined as the complement of belief:

$$pl(A_i) = 1 - bel(\neg A_i) \quad (3-6)$$

where $\neg A_i$ is the complement of $A_i$. It is also possible to derive the following relationships for belief and plausibility:

$$pl(A_i) \geq bel(A_i) ; pl(\emptyset) = 0 ; pl(\emptyset) = 1 ; pl(\neg A_i) = 1 - bel(A_i)$$

For the data provided in Table 3-1, the plausibility function for [2.5 6] can be derived as:

$$pl([2.5 \ 6]) = m([2.5 \ 6]) + m([2.5 \ 9]) + m([2.5 \ 6] \cup [9 \ 11.2]) + m([2.5 \ 11.2]) = 0.7$$

In similar fashion the calculated plausibility functions for all intervals is given in Table 3-2.
Table 3.2 - Belief and plausibility functions for the example interval

<table>
<thead>
<tr>
<th>$A_i$</th>
<th>$\emptyset$</th>
<th>$[2.5\ 6]$</th>
<th>$[6\ 9]$</th>
<th>$[9\ 11.2]$</th>
<th>$[2.5\ 9]$</th>
<th>$[6\ 11.2]$</th>
<th>$[2.5\ 6] \cup [9\ 11.2]$</th>
<th>$[2.5\ 11.2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m(A_i)$</td>
<td>0</td>
<td>0.5</td>
<td>0.3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>bel($A_i$)</td>
<td>0</td>
<td>0.5</td>
<td>0.3</td>
<td>0</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>pl($A_i$)</td>
<td>0</td>
<td>0.7</td>
<td>0.5</td>
<td>0.2</td>
<td>1</td>
<td>0.5</td>
<td>0.7</td>
<td>1</td>
</tr>
</tbody>
</table>

3.3 Conflict-handling using DST data fusion

In a multisource data environment, strategies are sought to make maximal use of data. Consider situations where multiple sources of data are available for a particular feature and a decision is to be made based on the unified data. For that feature, all available sources represent potential candidates as input for the analysis. There is hence a need to complementarily utilize multiple data sources in analyses and decision making. Issues such as the pertinence of the sources of data and their reliabilities need to be considered.

3.3.1 Data conflict

Conflict arises from disagreement between multiple datasets available for a given phenomenon. In hydrology, data may be acquired from ground measurements, remote sensing, or model estimation. It is therefore possible to have conflicting values for various hydrological features. These include, for example, multi-source permeability measurements (Mathon et al. 2010) and multiple general circulation models (GCM) for the prediction of long-term climate normals (Raje and Mujumdar 2010).

Conflict can be of different types and magnitudes. The magnitude of conflict (or simply “conflict” hereafter) refers to the degree of disagreement among the data. Multisource data or multisource evidence can be in different arrangements and variably agree (or disagree). The arrangement of evidence can be: consonant-, consistent-, disjoint- and arbitrary (Ferson et al. 2003). Consider the case of three bounded distributions that are obtained from multiple independent sources and which are illustrated in Figure 3.1. The four possible arrangements of evidence related to these data are described below. If these distributions were identical, conflict would not exist, i.e., conflict would be equal to zero.

- **Consonant evidence**: In consonant evidence, evidences are nested within another (Figure 3.1a). Therefore, at least one evidence is completely shared among all sources.
- **Consistent evidence**: All sources partly agree on at least some evidence (Figure 3.1b).
- **Disjoint evidence**: Evidences do not share any data or subset of data (Figure 3-1c).

- **Arbitrary evidence**: Evidences arbitrarily share subsets of data (Figure 3-1d).

In a probabilistic setting, conflict may be quantified using the Dempster-Shafer theory. For the variable \( X \), frequency distributions can be used for deriving probabilities and different hypotheses can be developed based on the probability (mass) assigned to intervals within the range of the distributions. Conflict \( K \) is calculated by first intersecting the probability of hypotheses (i.e., intervals) from a source to those of other sources and summing the resulting probabilities. \( K \) is calculated by subtracting the summed probabilities from 1.

![Figure 3-1 - The three distributions represent a case of (a) consonant, (b) consistent, (c) disjoint, and (d) arbitrary evidence](image)

### 3.3.2 Data fusion

Combining or fusing information is commonly used to reduce or treat conflicts. A fusion process combines data from multiple sources into a single estimate that is expected to be more accurate and/or more informative than the individual sources (Dasarathy 1997). Nevertheless, care must be taken to understand the involved intricacies such as the impacts of uncertainty on the resultant (fused) dataset. In this regard, data fusion may not be advisable for a number of scenarios (Bloch *et al.* 2001). For example, if data are recorded at different times, they cannot be directly combined. Also, given prior knowledge, it may be possible to select subsets from the sources that are consistent and reliable and then combine only this information. It may also be possible to correct erroneous information to reduce data conflict and then perform the fusion. Sometimes data fusion can be delayed pending additional information which may help resolve the conflict.
Data fusion is hereby interpreted as *mathematical techniques that use multiple - possibly disagreeing - sources of information for a feature to produce a more accurate and/or informative value for it*. By reviewing several data fusion techniques, this section examines the application of such techniques in handling multisource data in hydrologic applications.

A comprehensive literature review indicates that there are numerous methods available for data fusion. However, such methods may not explicitly state conflict-handling as their purpose. A large number of data fusion techniques use the *updating* capacity within Bayes inference to constantly improve a data-driven model with new data. Data assimilation is one of such methods in which observed data are merged with modelled outputs to improve the estimation of the state of some hydrological variable (Alavi et al. 2009). Ensemble Kalman Filter (enKF) is a Monte-Carlo implementation of the Bayesian update problem (Evensen 1994; Mandel 2007). Given a probability density function (PDF) of the state of the modelled system (a prior) and likelihood, enKF uses the Bayes theorem to obtain PDF of the posterior (Mandel 2007). Other data assimilation techniques include direct insertion (Rodell et al. 2004) and optimal interpolation (Brasnett 1999).

The following section reviews some data fusion techniques used for hydrological analyses. The justification for this review stands on the intrinsic merits of any data fusion method for application to the characteristics of data and mechanisms in hydrological modeling. For example, as data fusion techniques have major applications in military image processing applications, their performance in terms of computational cost is regarded as a merit however this requirement is loosely sought in hydrological applications. From another perspective, in hydrological applications, the information at the tails of a distribution (i.e., drought vs. heavy rainfall) or any uncertain number contains important characteristics which need to be preserved as hydrological applications often rely on such marginal information for risk characterization. Issues of high variability and human-induced (epistemic) uncertainty need also to be comprehensively considered for hydrological applications.

**3.3.3 Data fusion techniques**

Depending on a number of general criteria, such as the method used to model data, different formulae for conflict resolution are available. For example if data are expressed in a possibilistic (fuzzy logic) setting, fuzzy fusion is naturally selected. Other criteria include the nature of the problem comprising the presence of additional uncertainties in the data (e.g., incompleteness), the degree of the conflict, and the reliability of the data sources. Table 3-3 summarizes a few data fusion algorithms used for hydrological modelling.
Table 3-3 - Some data fusion applications in hydrological modelling

<table>
<thead>
<tr>
<th>Source</th>
<th>Methods</th>
<th>Application</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abrahart and See (2002)</td>
<td>ANN and fuzzy logic</td>
<td>rainfall-runoff models</td>
<td>Evaluated six data fusion strategies; identified ANN as the best method</td>
</tr>
<tr>
<td>Solomatine and Dulal (2003)</td>
<td>M5 model trees</td>
<td>rainfall-runoff models</td>
<td>Compared the performance of ANN against M5 model trees; M5 model trees showed minor improvement</td>
</tr>
<tr>
<td>Hégarat-Mascle and Seltz (2004)</td>
<td>DST</td>
<td>forest cover and land cover change detection</td>
<td>Presented a DST-based method that combines different change detection indices</td>
</tr>
<tr>
<td>See (2008)</td>
<td>Simple averaging, ANN, fuzzy logic, M5 model trees and instance-based learning</td>
<td>flow forecasting models</td>
<td>All investigated data fusion algorithms were found to perform well; M5 was argued to be superior for reasons of transparency and ease of development</td>
</tr>
<tr>
<td>Mathon et al. (2010)</td>
<td>DST</td>
<td>permeability measurements</td>
<td>Three DST combination rules were studied; no method was found clearly superior</td>
</tr>
<tr>
<td>Raje and Mujumdar (2010)</td>
<td>DST</td>
<td>drought prediction</td>
<td>Five DST combination rules were applied for handling conflict in three general circulation models (GCMs)</td>
</tr>
</tbody>
</table>

3.3.3.1 Statistical data fusion (classical inference)

Statistical data fusion techniques use an observed sample of data to derive conclusions about involved mechanisms or distribution (Hall and McMullen 2004). Such techniques use empirical probabilities to determine if a proposed hypothesis or its alternative/contrary is valid. In general, this method considers two hypotheses at a time (null $H_0$, and alternative hypotheses, $H_1$). The joint probability is then computed having an assumed hypothesis. A test of significance is then performed to determine the probability/likelihood that data would actually be observed if indeed the assumed hypothesis is true.

3.3.3.2 Bayesian fusion

Bayesian fusion uses Bayes probability theory in Equation (3-7) to fuse multisource data. As a statistical inference method, it represents all types of uncertainty (e.g., variability and epistemic uncertainties) in a unified measure of probability. For a single source, the Bayes formula calculates the probability of a hypothesis being true given some event/observation ($a posteriori$) from a combination of the prior probability ($a priori$) of the hypothesis and conditional probability ($likelihood$) of the event/observation. If $H_1, H_2, \ldots, H_t$ are mutually exclusive and exhaustive hypotheses (e.g., ‘illness’ or ‘not illness’).
explaining an event (observation) \( E \) (e.g., ‘water contamination’), Bayesian inference is mathematically expressed as (3-7):

\[
P(H_i|E) = \frac{P(E|H_i)P(H_i)}{\sum_i P(E|H_i)P(H_i)} = \frac{P(E|H_i)P(H_i)}{P(E)}
\]

\[
\sum_i P(H_i) = 1
\]

where \( P(H_i|E) = a \text{ posteriori} \) probability that hypothesis \( H_i \) is true given the evidence \( E \)

\( P(H_i) = a \text{ priori} \) probability of hypothesis \( H_i \) being true

\( P(E) = a \text{ priori} \) probability of evidence \( E \)

\( P(E|H_i) = \text{conditional probability of observing } E, \text{ given } H_i \text{ is true.} \)

When having multiple evidence \( E_i \), e.g., knowledge elicited from expert \( i = 1 \) to \( n \), for a set of hypotheses \( H_j \), e.g., chemical X is toxic \( (H_1) \) or not \( (H_2) \), the probability of \( H_1 \) being true is written as (Hall and McMullen 2004):

\[
P(H_1|E_1, E_2, ..., E_n) = \frac{P(H_1)P(E_1|H_1) ... P(E_n|H_1)}{\sum_j P(H_j)[P(E_1|H_j)P(E_2|H_j) ... P(E_n|H_j)]}
\]

Bayesian inference is well suited for expressing subjective probabilities (e.g., expressed by experts), in addition to empirical probabilities used in classical inference.

### 3.3.3.3 DST fusion

In DST fusion, first, sources are expressed as DST data then data are fused using DST combination rules. The appropriate DST rule of combination is selected using data characteristics such as the degree of conflict and the reliability of data sources (Bloch 1996) and variability and dependence among data sources (Ferson et al. 2003). Ferson et al. (2003) identified in excess of 10 rules with newer rules being continuously proposed, e.g., Florea et al. (2009), Haenni (2005), Huynh (2009), Schubert (2011) and Smarandache and Dezert (2006). There is, however, no clear formal guideline as to which method to apply for a given problem. Thus, the choice for the method of fusion needs to be justified by the user. Ferson et al. (2003) provide a guideline in the form of questions that can aid in this process.
In general, research has identified disjunctive (union) types of operators (rules) appropriate for high conflict and conjunctive (intersection) rules for low conflict situations. In particular, high conflict has been subject to extended research and new schemes have been proposed. Some schemes such as the robust combination rules (Florea et al. 2009) perform according the degree of conflict; they act as conjunctive rule for low conflict and as disjunctive rule for high conflict data. Other dynamic rules proposed for high conflict data include proportional conflict redistribution rules that dynamically redistribute conflict among evidence based on initial bpa (Smarandache and Dezert 2006). Some common DST combination rules along with their advantages and disadvantages are shown in Table 3-4.

<table>
<thead>
<tr>
<th>Combination rules</th>
<th>Ref.</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dempster’s Rule</td>
<td>Dempster (1967)</td>
<td>Simple calculations for low-conflict situations</td>
<td>Ignores conflicting evidence that may contain substantial mass</td>
</tr>
<tr>
<td>Yager’s Rule</td>
<td>Yager (1987)</td>
<td>Takes the conflicting mass into account</td>
<td>The conflicting mass is assigned to entire power set, resulting in large ignorance in the result</td>
</tr>
<tr>
<td>Mixture (averaging)</td>
<td>Ferson et al. (2003)</td>
<td>Inputs user’s weight assignment to each source and proportionally incorporates all the evidence</td>
<td>Neglects the existence of conflict</td>
</tr>
</tbody>
</table>

### 3.3.3.4 Other fusion techniques

Other fusion techniques include artificial neural networks, fuzzy fusion and rough sets, which may be applied after considering the nature of the data and the degree of conflict. For example, for data prone to vagueness and a high degree of conflict, fuzzy fusion with disjunctive aggregation operator may be used. Data fusion techniques with application in environmental risk assessment are reviewed in Zargar et al. (2013).
CHAPTER 4 HANDLING CONFLICT IN DROUGHT HAZARD: A CASE OF SNOW WATER EQUIVALENT

4.1 Overview

This chapter focuses on the problem of modelling and propagating uncertainty arisen from two highly conflicting datasets for a drought hazard indicator: the snow water equivalent (SWE). The objective is to investigate DST’s formalism for handling high-conflict data and to develop a framework for propagating the resulting uncertainty. In the first analytical step, DST data fusion is used to handle conflict and produce a single estimation of the data that can be used in analysis and subsequent uncertainty propagation. This estimation is expected to be more accurate or more informative than either of the sources alone (Dasarathy 1997). Four DST combination rules are applied to fuse the two sources into a single source with associated uncertainty. For each combination rule, a probability box (p-box) is produced, allowing an efficient comparison of the results. At the second part, the resultant p-box is used as input into a snowmelt runoff model and demonstrates an approach to propagate uncertainty along with data. This enables the assessment of the effect of uncertainty on predicted discharge and the probability of the resulting hydrologic drought.

4.2 SWE data sources and DST

Snowmelt runoff models are used to calculate runoff from snow-driven watersheds and predict drought or water surplus for dry periods of summer. These models use variables such as daily temperature and precipitation, SWE as well as parameters such as degree-day factor and watershed characteristics (e.g., area-elevation curve) to calculate runoff. SWE is the water content of a snow column and is usually expressed in millimetres. Snowmelt runoff models may be used in grid based or lumped model configuration. In this work, the case is based on the lumped version of the Snowmelt Runoff Model (SRM) (Martinec et al. 1998). In a lumped approach, the watershed is first divided into several zones of similar elevation called elevation bands. For SRM, 500m elevation bands have been recommended (Martinec et al. 1998). For each zone, an SWE value is calculated by simple arithmetical averaging of cell values within each zone. The mean SWE is subsequently inputted into the model.

4.2.1 Conflict in SWE data

SWE data can be captured or modeled using a variety of methods including remote sensing and ground-based surveying. For remote sensing approaches, the microwave electromagnetic spectrum has been found to capture characteristics such as snow extent, snow depth and snow water equivalent (DeWalle
and Rango 2008). Currently, SWE maps are extracted from passive microwave sensors using various algorithms discussed in, for instance Chang et al. (1997); Kunzi et al. (1982); Rango et al. (1979). Platforms for snow cover mapping include Advanced Very High Resolution Radiometer (AVHRR) from the National Oceanic and Atmospheric Administration (NOAA) with a spatial resolution of 1.1km and NASA’s Terra Moderate Resolution Imaging Spectroradiometer (MODIS) with a spatial resolution of 250m.

Ground-based approaches include periodic snow-course surveying and continuous snow pillow measurements. Ground-based SWE observations are interpolated over a given watershed to produce a continuous surface of SWE (DeWalle and Rango 2008).

The availability of SWE data from different sources leads to multiple values for consideration as potential candidates for input into a snowmelt runoff model. This, however also results in dissimilarity and disagreement among data. The comparison and analysis of the available datasets, the representative value from each dataset, the mean SWE, is an abstract and narrowly informative notion. Compared to this single-valued and deterministic measure, the distribution curve (frequency-based probabilities) of each source can be used instead to provide more information. Using these distributions, it is possible to analyze local conflict (and agreement) between the two sources.

4.2.2 Study area

The Red Deer River watershed in Alberta (Canada) is a part of the South Saskatchewan River basin (Figure 4-1). The watershed is 49,000 km² in area and its elevation ranges from 568m to 3231m. For use in SRM, the watershed is divided into elevation bands of 533m resulting in a total of five zones (cf. Section 4.2.3.1 for map of the zones). Data for 1-April-2009 was considered as this day is commonly used to represent the annual maximum snow accumulation (Bohr and Aguado 2001). This study focuses on zone 1 with elevation varying between 568m and 1101m. Zone 1 is used because it comprises the largest portion of the watershed and thus includes the largest number of grid cells. This zone is also associated with minimum error in SWE estimation from passive microwave data because of its low topographical variation (Derksen et al. 2003).
4.2.3 Data sources

This work considers two available datasets from remotely-sensed and ground-based SWE observations as potential inputs for the SRM model.

4.2.3.1 Source A

“Source A” data was obtained from the repository of passive microwave remote sensing data available at the Canadian Cryospheric Information Network (CCIN, website: http://www.ccin.ca/cms/en/socc/snow/currentSnow.aspx). The CCIN weekly updates the SWE map for the three Canadian Prairie provinces (Alberta, Saskatchewan, and Manitoba) using data acquired from the Special Sensor Microwave Imager (SSM/I) satellite.
An algorithm is used to calculate SWE for each pixel (cell) based on brightness temperature and the composition of land cover types (CCIN 2011). The algorithm considers four types of land cover applicable to the Canadian Prairie: open Prairie environments (O) and three types of forest: coniferous (C), deciduous (D), and sparse (S). For each pixel, SWE is calculated by spatially-weighted-averaging of four different SWE values proportional to the land cover. This equation is given in (4-1) (Derksen et al. 2003):

\[
SWE = F_O(SWE_O) + F_C(SWE_C) + F_D(SWE_D) + F_S(SWE_S)
\]  

(4-1)

where \(F_i\) and \(SWE_i\) are respectively, fraction of land cover and SWE. \(SWE_i\) are calculated using four different algorithms where SWE is a function of the brightness temperature difference between the vertically polarized 37 and 19 GHz channels (CCIN 2011; Derksen et al. 2003).

The data have a general nominal resolution of 25km with an accuracy of ±15mm SWE relative to land measurements in open vegetation and areas of low relief (see Derksen et al. 2003, for a thorough discussion of errors associated with this dataset). The map and frequency distribution of the passive microwave SWE data are shown in Figure 4-2a and Figure 4-3a.

![Figure 4-2](image1.png)

Figure 4-2 – SWE grids for zone 1 of the Red Deer River watershed from (a) source A and (b) source B (1-April-2009, values in mm)

![Figure 4-3](image2.png)

Figure 4-3 – The frequency histogram of (a) source A and (b) source B of SWE data within zone 1 of the Red Deer River watershed (note the difference in y-axis scales)
4.2.3.2 Source B

Source B data were derived from gridded snow depth data produced by the Canadian Meteorological Centre (CMC, website: http://www.asc-csa.gc.ca/eng/sciences/committees-saeac.asp). The CMC uses snow depth measurements from climate stations, meteorological aviation reports and special aviation reports to produce daily snow depth data for part of the Northern Hemisphere (Brasnett 1999). According to the documentation information, for the watershed, five stations reported any snow depth observation during the March-April months of 2009.

An SWE map was obtained by multiplying snow depth by a density value appropriate to its ecoregion (Sturm et al. 1995) and the measurement date (1-April-2009) (Brown and Mote 2009) (Figure 4-2b). According to Sturm et al. (1995), the Red Deer River basin spans both alpine and prairie ecoregions. In order to obtain SWE values snow depth had to be multiplied by snow density. Snow densities for the two ecoregions are available as mean monthly values. By averaging the March and April snow density for each region: 263.5 kg/m$^3$ and 312.0 kg/m$^3$ for alpine and 261.0 kg/m$^3$ and 308.0 kg/m$^3$ for prairie, the snow density for 1-April was calculated: for alpine 287.75 kg/m$^3$ and for prairie 284.5 kg/m$^3$.

This dataset has a variable grid resolution of 1/3° Gaussian which is approximately 23km at 51-52°N where the watershed is located. Information about the quality and accuracy of this dataset was not documented with the data (the interested reader is referred to Brown and Mote (2009) for a general discussion on the methodology and possible sources of error). The dataset’s frequency distribution is shown in Figure 4-3b.

4.2.4 Representation of bpa using probability box (p-box)

The bpas can be represented using two simple tools: probability-box (p-box) and D-S structures (Ferson et al. 2003). D-S structures are constructed by using bel. and pl. functions as lower and upper bounds in the plot. D-S structures can be interpreted as a series of cumulative distribution functions (CDF). The true CDF for the variable is expected to be positioned between the bel. and the pl. functions.

The p-box is constructed by stacking probability boxes. The X-axis of boxes represents a hypothesis, i.e., an element of the universal set (e.g., a single value or an interval range for scalar variable) and Y-axis is the bpa (evidence for the element) (Figure 4-4). The capacity of DST to separately and simultaneously model variability and epistemic uncertainty can conveniently be shown using a probability-box (p-box) (Ferson et al. 2003). P-box is a generalized CDF that uses a band instead of line in traditional CDF. This band models epistemic uncertainty. Generally, the wider this band, the larger epistemic uncertainty.
Similar to the D-S structures, the true CDF for the variable is expected to lie between the lower and upper functions in p-box. The p-box corresponding to the bpa described in Table 4-1 is plotted in Figure 4-4. The dashed line represents the upper bound and the solid line represents the lower bound.

![Figure 4-4 – A p-box example (the X-axis is a generic scalar example and not representative of a specific unit)](image)

Both p-box and D-S structures can represent a variety of continuous and discrete probability distributions. These plots can be converted to each other. Discretization is the process applied to continuous monotonic CDF to express them as interval probability (Tucker and Ferson 2003). In a p-box, finer intervals (on the X-axis) and a narrower gap between the lower and upper bands (in the Y direction) indicate less uncertainty. Two measures of uncertainty can hence be defined (Limbourg 2008), first, randomness, which arises from the way masses are distributed in the p-box. Randomness is similar to uncertainty in probabilistic models. Second, the non-specificity, which arises from the width of the intervals. A bpa consisting of wider focal elements is less specific.

The general term discretization error, (Tucker and Ferson 2003) refers to loss of information due to a low number of categories when cutting (discretizing) a continuous CDF into boxes. Discretization error can be managed by choosing discretization levels that would allow rigorous inference to still be made on p-box.

### 4.2.5 Formulation of bpa

The bpa for each source was derived using the frequency distributions as evidence (Figure 4-3). For simplicity, each distribution was discretized into three intervals. As earlier mentioned, a higher number of intervals is desirable (Moloney 1986) as it results in less non-specificity or discretization error (Limbourg 2008; Tucker and Ferson 2003). Masses can be assigned by segmenting the CDF by a fixed percentile, (e.g., 1%) resulting in probability boxes (e.g., 100 boxes for 1% percentiles) (Tucker and Ferson 2003). For source B, since the distribution is skewed by a large number of zero values and source A contains no information in this range, a small interval [0 to 0.05] was chosen to contain the data.
points occurring in this range and the rest of the distribution was divided into identical 16.5%
percentiles resulting in two intervals: [0.05 40.01] and [40.01 102.4]. For source A, equal intervals
were used ([19.35 30.49], [30.49 41.63] and [41.63 52.76]) as using equal percentiles will result in a
larger first box, i.e., [19.35 >30.49]. Since source B contains large evidence for the SWE range close to
zero, a smaller interval will be more appropriate to analyze the local conflict between the two sources
in the [0 >19.35] range.

The bpa and p-box resulting from the PDF of each source are shown in Table 4-1 and Figure 4-5
respectively.

<table>
<thead>
<tr>
<th>m_A</th>
<th>0.29</th>
<th>0.57</th>
<th>0.14</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>[19.35 30.49]</td>
<td>[30.49 41.63]</td>
<td>[41.63 52.76]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>m_B</th>
<th>0.67</th>
<th>0.17</th>
<th>0.16</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>[0.00 0.05]</td>
<td>[0.05 40.01]</td>
<td>[40.01 102.4]</td>
</tr>
</tbody>
</table>

P-box of SWE data for source A

P-box of SWE data for source B

4.2.6 Dependency in data sources

In DST, the relationship among data sources needs to be considered because interdependency among
data sources can bias the solution. The classical Dempster-Shafer rule assumes independent data
sources. Some attempts have been made to modify the DST rule to fuse data in cases where data
sources do not satisfy independency criteria (e.g., the evidential neural network) (Basir et al. 2005).
Different combination rules have been found to have different sensitivities to dependency (Ferson et al.
2004). For this reason, before any fusion rule is performed, dependency among data sources must be
investigated and the method for combination must be selected accordingly. In this work, the sources of
information rely on distinct systems for recording SWE data and are hence assumed independent.
4.3 High-conflict data fusion through DST

The appropriate DST data fusion rule can be selected considering data characteristics such as the degree of conflict among sources and the reliability of each data source (Bloch 1996). The variability and the dependence among data sources should also be considered (Ferson et al. 2003; 2004). Ferson et al. (2003) earlier identified more than 10 rules with newer rules proposed (Florea et al. 2009; Haenni 2005; Huynh 2009; Schubert 2011; Smarandache and Dezert 2006). There is, however, no clear formal guideline as to which method to apply for a given problem. Thus, the choice for the method of fusion needs to be justified by the user. Some literature, for example a guideline in the form of series of questions provided by Ferson et al. (2003) can aid in this process.

The data considered in this work can be described as to being subject to high conflict, being low in number of sources (two sources) and being accompanied with limited knowledge about accuracy and hence reliability. For this, equal reliability (or unreliability) can be assigned to such datasets. For selecting data fusion rules, high conflict can thus be regarded as the major indicator, which has been subject to extended research (e.g., Florea et al. 2009; Huynh 2009; Schubert 2011). In general, research has identified disjunctive (union) types of operators (rules) appropriate for high conflict and conjunctive (intersection) rules for low conflict situations. In this regard, based on the D-S measure of conflict ($K$), schemes such as the robust combination rules (Florea et al. 2009) would variably perform; they act as conjunctive rules for low conflict and like disjunctive rules for high conflict data. Other dynamic rules proposed for high conflict data include proportional conflict redistribution rules that dynamically distribute conflict among evidence based on initial masses (refer to Section 4.3.4).

Mixture is an example of a disjunctive rule followed by discounting; it sums evidence ($bpa$) based on their weighted averages (refer to Section 4.3.3). Assigning weights is essential based on an argument that in high conflict at least one of the sources is to some extent unreliable (Haenni 2002; 2005). Weights are assigned by analysts capable of distinguishing between the reliability of sources and also expressing mathematically such distinctions (Dubois and Prade 1992). Discount and combine rule is another type of averaging rules. Similar to mixture, this rule, first discounts the sources and then combines the resulting functions with a rule (e.g., D-S rule) using a discounting function. The discounting function similarly makes use of the reliability information. Discount and combine differs from mixture in that it uses $bel$ function for calculation while mixture uses $bpa$ ($m$). The difficult task of determining the extent of the unreliability of the sources and the discount rate complicate discounting schemes. Some solutions to this problem are discussed in Huynh (2009), Schubert (2011) and Smets (2007).
This study applied four rules to fuse data sources A and B: D-S initial rule of combination (to demonstrate the behaviour of a conjunctive rule), Yager which universally attributes conflict as ignorance, mixture and proportional conflict redistribution rule number 6. The choice of these rules is further justified in each section. The results from each rule are then compared and discussed.

4.3.1 Dempster–Shafer (D-S) rule of combination

The D-S rule of combination (note that D-S rule of combination is distinguished from the Dempster-Shafer theory, DST) emphasizes solely the agreement between evidence and ignores all conflicting evidence. As such, a strict conjunctive logic (through an AND-type operator or product) is employed for the combination of evidence. D-S rule is popularly used in DST fusion literature as a baseline approach and a benchmark for capturing the magnitude of conflict. The aggregation of two sources \( m_A \) and \( m_B \) using D-S rule of combination is determined by (e.g., Deng et al. 2011):

\[
m_{AB}(A_i) = \frac{\sum_{A_p \cap A_q = A_i} m_A(A_p)m_B(A_q)}{1-K} \quad \text{when } A_i \neq \emptyset
\]

(4-2)

\[
K = \sum_{A_p \cap A_q = \emptyset} m_A(A_p)m_B(A_q)
\]

(4-3)

where \( K \) is the degree of conflict in two sources of evidence and \( m_A(A_p) \) and \( m_B(A_q) \) represent the corresponding masses for propositions \( p \) and \( q \). The denominator \( (1 - K) \) in the above equation is a normalization factor that counterbalances the effect of conflicting evidence on the aggregated output. Note that (4-2) and (4-3) can also be written as:

\[
m_{AB}(A_i) = \frac{\sum_{A_p \cap A_q = A_i} m_A(A_p)m_B(A_q)}{1 - \sum_{A_p \cap A_q = \emptyset} m_A(A_p)m_B(A_q)}
\]

(4-4)

In D-S rule of combination, weights can be assigned to each source. Weights assignment or discounting is done by multiplying the \( bpa \) of each source with the weight that ranges from 0 to 1 and their sum equals to 1. In this study, as no information indicating greater credibility for any of the sources was available, equal weights were assigned to each source.

D-S rule of combination is performed in two stages. First, intersect all possible combinations of the two sources. Each interval is intersected with all sets/intervals from the other source. For completely conflicting combinations, the associated masses of intersection are assigned to conflict. For example,
for non-intersecting combinations [0 0.05] and [41.63 52.76] (cf. Table 4-1), the product of their \( bpa \) 
\( 0.67 \times 0.14 = 0.0938 \) is aggregated with other masses to calculate \( K \).

The degree of conflict \( K \) for all non-intersecting intervals is:
\[
K = 0.1943 + 0.3819 + 0.0938 + 0.0238 + 0.0464 = 0.7402
\]

which indicates high conflict among data sources.

Second, the resultant \( bpa \) is divided by the normalization factor \((1 - K) = 0.2598\).

\[
m_{AB}([19.35 \ 30.49]) = \frac{0.0493}{0.2598} = 0.1897 \approx 0.19
\]

\[
m_{AB}([40.01 \ 41.63]) = \frac{0.0912}{0.2598} = 0.3510 \approx 0.35
\]

\[
m_{AB}([30.49 \ 40.01]) = \frac{0.0969}{0.2598} = 0.3729 \approx 0.37
\]

\[
m_{AB}([41.63 \ 52.76]) = \frac{0.0224}{0.2598} = 0.0862 \approx 0.09
\]

The results from the D-S rule of combination indicate that the probability of SWE occurring in the 
[30.49 40.01] range is highest followed by [40.01 41.63], [19.35 30.49] and [41.63 52.76]. These 
results are summarized in Table 4-2 and the resultant p-box is shown in Figure 4-6.

<table>
<thead>
<tr>
<th>( A_p )</th>
<th>[19.35 30.49]</th>
<th>[30.49 40.01]</th>
<th>[40.01 41.63]</th>
<th>[41.63 52.76]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m(\cdot) )</td>
<td>0.19</td>
<td>0.37</td>
<td>0.35</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Table 4-2 - Results of D-S rule of combination

Figure 4-6 – The p-box of the results obtained from D-S rule of combination
4.3.2 Yager rule of combination

One deficiency in the D-S rule fusion is that it neglects the conflicting evidence. Zadeh (1984) presented an example of a patient who is diagnosed by two physicians A and B. Physician A diagnosed the patient with disease x with a probability of 99% and disease y with a probability of 1%. Physician B, however believed that the patient has disease z with probability = 99% but had disease y with probability = 1%

The frame of discernment for the disease can hence be expressed as $\Theta = \{x, y, z\}$. The DS rule of combination implies that:

$$\text{Degree of conflict} = K = 0.9999$$
$$\text{Normalization factor} = 1 - K = 0.0001.$$  

$m_{AB}(x) = 0, m_{AB}(y) = 1$ and $m_{AB}(z) = 0$.

Although this is an exaggerated example, these results show how D-S rule of combination is counterintuitive in high conflict cases (as it only relies on non-conflicting evidence).

Yager (1987) proposed an alternative combination rule to D-S to address the above problem, as well as deficiencies such as the need for a non-associative operator. In Yager rule, the basic probability mass assignment is known as the ground probability assignments ($q$). Yager rule differs from D-S in two main aspects: first, unlike $m(A)$, avoids using a normalization factor on $q(A)$; and second, conflict is attributed to the universal set ($\Theta$) which inflates ignorance.

In Yager rule of combination, the ground probability assignment associated with $A_i$ is obtained from the intersection of subsets $A_p$ and $A_q$ (Ferson et al. 2003):

$$q_{AB}(A_i) = \sum_{A_p \cap A_q = A_i} m_A(A_p)m_B(A_q)$$  \hspace{2cm} (4-5)

which is equal to the non-normalized values of D-S rule. Conflicting mass is added as ignorance ($\Theta$) to ground probability assignments to calculate masses:

$$m(A_i) = K + \sum_{A_p \cap A_q \neq \emptyset} m_A(A_p)m_B(A_q)$$  \hspace{2cm} (4-6)

As evident, the conflict $\sum_{A_p \cap A_q \neq \emptyset} m_A(A_p)m_B(A_q)$ is assigned to the ignorance (universal set) $[0, 102.4]$. For this, all hypotheses (intervals) are inflated, e.g., $m([19.35, 30.49]) = 0.0493 + 0.74 \equiv$
0.79. Since the probability assignments for ignorance in both sources (i.e., \(m_A(\emptyset)\) and \(m_B(\emptyset)\)) is zero, \(m(\emptyset)\) will be equal to conflict: 0.74.

**Table 4-3 - D-S rule of intersection results (Ø: null set)**

<table>
<thead>
<tr>
<th>(m_A)</th>
<th>(m_B)</th>
<th>(m_A)</th>
<th>(m_B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[19.35 30.49]</td>
<td>0.67 [0 0.05]</td>
<td>0.17 [0.05 40.01]</td>
<td>0.16 [40.01 102.4]</td>
</tr>
<tr>
<td>0.29 [19.35 30.49]</td>
<td>0.1943 Ø</td>
<td>0.0493 [19.35 30.49]</td>
<td>0.0464 Ø</td>
</tr>
<tr>
<td>0.57 [30.49 41.63]</td>
<td>0.3819 Ø</td>
<td>0.0969 [30.49 40.01]</td>
<td>0.0912 [40.01 41.63]</td>
</tr>
<tr>
<td>0.14 [41.63 52.76]</td>
<td>0.0938 Ø</td>
<td>0.0238 Ø</td>
<td>0.0224 [41.63 52.76]</td>
</tr>
</tbody>
</table>

The order of the probabilities of intervals after applying the Yager rule of combination is similar to D-S’s however inflated by conflict. These results are summarized in Table 4-4. The corresponding p-box is shown in Figure 4-7.

**Table 4-4 - Results of Yager combination**

<table>
<thead>
<tr>
<th>(A_p)</th>
<th>[19.35 30.49]</th>
<th>[30.49 40.01]</th>
<th>[40.01 41.63]</th>
<th>[41.63 52.76]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q(.))</td>
<td>0.05</td>
<td>0.10</td>
<td>0.09</td>
<td>0.02</td>
</tr>
<tr>
<td>(m(.))</td>
<td>0.79</td>
<td>0.84</td>
<td>0.83</td>
<td>0.76</td>
</tr>
</tbody>
</table>

**Figure 4-7- The p-box of Yager combination**

### 4.3.3 Mixture

Mixture involves two operations: a disjunctive rule that sums all \(bpa\) followed by discount based on assigned weights. Mixture was proposed for handling high levels of conflict (Murphy 2000) and according to Ferson et al. (2003) it is suitable data that are subject to variability. Since variability (probability distributions) was the basis for deriving \(bpa\) for the sources (cf. Section 4.2.5) and conflict was high \((K = 0.74)\), mixture was considered as another combination rule. Mixture retains all
information of evidence in its outcome and hence is suitable for uncertainty propagation. However, mixture fails to converge toward certainty (i.e., reduce uncertainty) as it simply “adds” evidence, leaving the initial configuration of bpa’s intact and does not locally strengthen any of the hypotheses (intervals).

The equation for mixture combination is simple (Sentz and Ferson 2002, Raje and Mujumdar 2010):

\[ m_{1...n}(A) = \frac{1}{n} \sum_{i=1}^{n} w_i m_i(A) \]  

(4-7)

Depending on the weight \( (w_i) \) assigned by the user, the evidence is first weighted and then all intervals from each source of evidence are pooled. Here, an equal weight \( w_i \) of 1 for all evidence are assumed and these two datasets \( (n = 2) \) are averaged (shown in Table 4-5). For example, \( m([0.05 19.35]) = \frac{1}{2} (1 \times 0 + 1 \times 0.17) \approx 0.09 \).

<table>
<thead>
<tr>
<th>Source</th>
<th>Interval</th>
<th>Initial m</th>
<th>Mixture m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source A</td>
<td>[19.35 30.49]</td>
<td>0.29</td>
<td>0.145</td>
</tr>
<tr>
<td></td>
<td>[30.49 41.63]</td>
<td>0.57</td>
<td>0.285</td>
</tr>
<tr>
<td></td>
<td>[41.63 52.76]</td>
<td>0.14</td>
<td>0.07</td>
</tr>
<tr>
<td>Source B</td>
<td>[0.00 0.05]</td>
<td>0.67</td>
<td>0.335</td>
</tr>
<tr>
<td></td>
<td>[0.05 40.01]</td>
<td>0.17</td>
<td>0.085</td>
</tr>
<tr>
<td></td>
<td>[40.01 102.4]</td>
<td>0.16</td>
<td>0.08</td>
</tr>
</tbody>
</table>

The results from mixture indicate that the [30.49 40.01] range is still most probable followed by [40.01 41.63] however the [0.00 0.05] range ranks third. These results are provided in Table 4-6 and the corresponding p-box is shown in Figure 4-8.

<table>
<thead>
<tr>
<th>( A_p )</th>
<th>[0.00 0.05]</th>
<th>[0.05 19.35]</th>
<th>[19.35 30.49]</th>
<th>[30.49 40.01]</th>
<th>[40.01 41.63]</th>
<th>[41.63 52.76]</th>
<th>[52.76 102.4]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m(.) )</td>
<td>0.33</td>
<td>0.09</td>
<td>0.24</td>
<td>0.37</td>
<td>0.36</td>
<td>0.15</td>
<td>0.08</td>
</tr>
</tbody>
</table>
4.3.4 Proportional conflict redistribution rule no. 6 (PCR6)

Martin and Osswald (2006) developed the Proportional Conflict Redistribution Rule no. 6 (PCR6) within the Dezert-Smarandache theory (DSmT). DSmT generalizes DST by allowing the intersection of focal elements in the outcome (relaxes the requirement of mutual exclusivity). For example, if the cardinality is 2: $\Theta = \{L, H\}$, the power set of DSmT will include an extra set, $\{L \cap H\}$ in addition to the four sets that are already in the DST power set ($\emptyset$, $\{L\}$, $\{H\}$, $\{L, H\}$). In DSmT the power set is called the hyper-power set denoted by $S^\emptyset$. The number of outcomes from a hyper-power set is considerably higher than a power set with the same cardinality. For example, a cardinality of 3 will have 19 focal elements (compared to 8 in DST); in similar fashion, a cardinality of 4 will have 167 (compared to 16) and so on (Smarandache and Dezert 2006).

Similar to DST, various fusion rules have been developed within DSmT, e.g., the classic and the hybrid DSm rule (Smarandache and Dezert 2009). PCR6 belongs to a family of conflict redistribution rules which redistributes evidence based on the initial mass of the focal elements (Smarandache and Dezert 2009). Prior to PCR6, five conflict redistribution rules were proposed, among which PCR5 is considered to be a mathematically robust method to redistribute the conflicting mass judicially (Smarandache and Dezert 2006). Since in DSmT focal element intersection is allowed, cardinality can be problematic. PCR6 compared to PCR5 was found to perform faster and produce more intuitive results for hyper-power set cardinalities of more than 2 (Martin and Osswald 2006). (For more information about the PCR-family, the reader is referred to Smarandache and Dezert (2006)).

Here, PCR6 is applied on a DST power set. For this purpose, the intervals are configured to make the intersection of all focal elements null. For this similar values were set for the intervals of the two belief structures and weightings assigned based on the distribution of each source. The value of 45.0 mm SWE
was selected instead of 41.63 and 40.01 mm SWE (Table 4-7) to more evenly distribute the evidence among intervals for source B.

PCR6 is applied in three steps:

1) Apply the conjunctive rule (Smarandache and Dezert 2006):

\[
m_{AB}(C) = \sum_{X_1, X_2 \in S^\emptyset, X_1 \cap X_2 = C} m_A(X_1)m_B(X_2)
\]

(4-8)

The results from this step are shown in Table 4-7.

2) Calculate the conflict using:

\[
K_{AB} = \sum_{X_1, X_2 \in S^\emptyset, X_1 \cap X_2 = \emptyset} m_A(X_1)m_B(X_2)
\]

(4-9)

\[
K_{AB} = 0.81(0.29 + 0.65 + 0.06) + 0.00(0.01 + 0.06 + 0.12) + 0.01(0.65 + 0.06) + 0.29(0.06 + 0.12) + 0.06(0.06) + 0.65(0.12) = 0.95
\]

3) Redistribute the conflict based on initial mass of focal elements:

\[
m_{PCR6}(A) = m_{12...s}(A)
\]

\[
+ \sum_{i=1}^{s} m_i(A_i)^2 \sum_{\bigcap_{k=1}^{s-1} Y_{\sigma_i(k)} \cap A = \emptyset} \left( \frac{\prod_{j=1}^{s-1} m_{\sigma_i(j)}(Y_{\sigma_i(j)})}{m_i(A) + \sum_{j=1}^{s-1} m_{\sigma_i(j)}(Y_{\sigma_i(j)})} \right)
\]

(4-10)

---

**Table 4-7 - Results of the conjunctive rule applied on the two sources (PCR6)**

<table>
<thead>
<tr>
<th></th>
<th>[0.00 19.35]</th>
<th>[19.35 30.49]</th>
<th>[30.49 45.0]</th>
<th>[45.0 102.4]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_A)</td>
<td>0.00</td>
<td>0.29</td>
<td>0.65</td>
<td>0.06</td>
</tr>
<tr>
<td>(m_B)</td>
<td>0.81</td>
<td>0.01</td>
<td>0.06</td>
<td>0.12</td>
</tr>
<tr>
<td>(m_{AB}(\cdot))</td>
<td>0.00</td>
<td>0.00</td>
<td>0.04</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Equation (4-10) is discussed in detail by Martin and Osswald (2006). The PCR6 MATLAB® code from Martin (2008) was used to solve (4-10) (Table 4-8):
The results indicate that in contrary to previous methods, the probability of the [00.00 19.35] range ranks highest, followed by the [30.49 45.0] range. The resulting p-box is shown in Figure 4-9.

### Figure 4-9 - The resultant p-box from PCR6

#### 4.3.5 Decision making in DST

Following the combination of evidence, a decision can be made on the hypotheses (intervals) in the resulting p-box. In decision, representative hypotheses from the resultant p-box are selected and the rest are neglected. Theoretically, this decision will result in the reduction of uncertainty. Following the decision, remaining calculations on the data can then be performed using selected hypotheses. Most studies use the maximum \( bpa \) criterion (Martin 2008). For example, for the mixture rule the decision interval would be \([30.49 40.01]\) (refer to Table 4-7). As our objective initially stated, handling uncertainty will be limited to uncertainty modelling and propagation, and as such, reducing uncertainty by decision will not be considered. In a situation where data are at lower conflict, this decision can also be performed, reducing data uncertainty. Regardless of the decision step, the uncertainty information contained in a fused p-box is nevertheless informative and may be used for uncertainty propagation and consideration by subsequent users (e.g., decision makers). Consequently, the capacity to retain maximal information for uncertainty propagation will be used as an additional indicator for discussing the suitability of combination rules.

<table>
<thead>
<tr>
<th>( A_p )</th>
<th>[00.00 19.35]</th>
<th>[19.35 30.49]</th>
<th>[30.49 45.0]</th>
<th>[45.0 102.4]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_{PCR6}() )</td>
<td>0.51</td>
<td>0.10</td>
<td>0.35</td>
<td>0.04</td>
</tr>
</tbody>
</table>
4.4 Discussion

4.4.1 Combination results

In the D-S rule of combination, conflicting combinations i.e., [0 19.35] and [52.76 102.4] were eliminated (Figure 4-6) and their conflicting mass was used to strengthen intersecting evidence during the normalization process. The outcome intervals were hence confined to a small interval [19.35 52.76] containing less variability and less epistemic uncertainty (especially compared to Yager’s). Yager’s rule assigned the conflicting mass to the entire interval (ignorance), resulting in a wide band indicating large epistemic uncertainty in the output p-box (Figure 4-7). The range of outcomes for Yager and remainder two combination rules encompassed the entire possible range ([0 102.4]) indicating higher variability.

Mixture produced a slimmer band containing less epistemic uncertainty. Since the evidence were assigned equal weights, all hypotheses equally contributed to the output p-box. In PCR6, since this rule proportionally re-distributed the conflicting evidence based to the initial masses, in the outcome p-box, intervals [0 19.35] and [30.49 45.0] can be seen to include a larger mass (and are hence more probable). However, the discretization error and large non-specificity resulted in higher ignorance in the PCR6 outcome compared to mixture’s.

The application of the results in water resources management can further be discussed in terms of drought management and additionally flood forecasting. For D-S, a non-intersecting interval [0.00 0.05] containing a relative large mass of 0.67 was ignored and its mass was incorporated in the normalization factor that is used to adjust other intersecting masses. In real-world applications, the large zero class can potentially indicate severe drought hazard conditions. The large ignorance in Yager’s result was largely inconclusive and prevented users to extract information from the resulting p-box. Using other methods, local conflicts and agreements are more distinctively identified, and inferences for decision making purposes can achieved in an uncomplicated manner. The main weakness in PCR6 (and DSmt in general) is the large non-specificity caused by the size of the interval containing the zero values ([0.00 19.35]). The user is limited in number of intervals in PCR6. If a finer discretization were to be used, the higher number of elements would drastically increase in the required calculations. For the case of this work, the mixture rule was identified as the most appropriate among the four applied rules. As a disjunctive rule, mixture represents cautious behaviour. For highly-conflicting data, it can be viewed as an approach that allows maximal information to be retained in the outcome and is hence
useful for uncertainty propagation. The results can be indicative of the likelihood for both drought as well as water abundance.

4.4.2 Propagating data uncertainty using DST

DST can be used as a framework for propagating data uncertainty within hydrologic models. Using the p-box method, instead of a deterministic value, data along with their uncertainty can be inputted into any hydrologic process to obtain uncertainty-driven outputs. Processes are granular operations which in combination form a hydrologic model. They can range in complexity from a simple arithmetic operation to more complex relationships. Using this sequence (or chain) of processes, data uncertainty is propagated to obtain final results along with their uncertainty information.

Formally, a system model (here, a hydrologic process) is defined that inputs uncertain variables $x$ where $x := (x_1, ..., x_n)^T$ and fixed parameters $d$ to output variable $z$ where $z = \Phi(x, d)$ (Limbourg 2008). Uncertain variables $x$ are subject to an uncertainty model (e.g., a PDF or a p-box) while others remain fixed (similar to $d$). Depending on the method used for uncertainty propagation, an uncertain output for $z$ is obtained. Here, the p-box resulting from the fusion process is used as input into the process. Instead of a single deterministic value, the endpoints of each of the intervals in the p-box are used as input, and for each a separate output is obtained.

For example, consider the p-box that resulted from the PCR6 fusion in Figure 4-9. (PCR6’s p-box is used for simplicity, as it contains less endpoints compared to mixture.). This p-box can be used in a linear snowmelt model to obtain discharge $Q$ based on an uncertain variable, $SWE$ (Figure 4-10). In a similar fashion as the Monte Carlo simulation, however with considerably fewer inputs, the endpoints of all intervals (i.e., 0, 19.35, 30.49, 45.0 and 102.4) are used as inputs to the model. For each value, a distinct output is obtained (five in total). These five values represent the lower and upper bounds (endpoints) of $Q$ and along with the associated bpos (0.51, 0.10, 0.35 and 0.04) they collectively constitute the p-box for $Q$. This p-box replaces the deterministic variables that normally pass through each granular process. As shown in Figure 4-10, at the final step, in total five outputs (i.e., hydrographs) are produced that represent uncertainty in the form of a p-box. These five hydrographs are hypothetically obtained for April 1, 2009 and each pair corresponds to an interval from the PCR6 data fusion (Figure 4-9). For instance, the band that is bounded by hydrographs $Q(SWE = 0)$ and $Q(SWE = 19.35)$ has the highest mass: (0.51), while other bands contain a combined mass of 0.49.
4.5 Summary

Generally, there is evidence that there is greater value in combining datasets than in using single sources alone (e.g., Raje and Mujumdar 2010; See 2008). This chapter demonstrated the utility of DST for modelling and propagating uncertainty arisen from data conflict in hydrological modelling of a drought hazard indicator. Uncertainty-preserving models such as DST enable modelling of various types of aleatory and epistemic uncertainties thus enabling hydrological applications to move beyond deterministic approaches. The approach that was presented by evidence theory adds to the methods already established for uncertainty handling in drought hazard analysis. In this work, the application of four DST fusion rules to high conflicting data was investigated. Among these rules, mixture and PCR6 produced more intuitively plausible results compared to the D-S rule which only considered the narrow intersecting intervals. Both mixture and PCR6 produced less ignorance in comparison to Yager’s rule. Mixture retained all the mass from different sources in its outcome. This characteristic is helpful in highly conflicting situations that frequently arise in hydrological modelling. In addition to preserving all information, mixture enables users to study conflict locally within the p-box. PCR6 suffers from non-specificity resulting from the wider intervals. PCR6 (and DSmT in general) is constrained by the limited number of intervals that can be established since larger cardinalities considerably increase the computational cost. This problem defined as non-specificity (Limbourg 2008) can be attributed to a larger deficiency arisen from discrete computing methods used within hydrological applications (where variables are commonly continuous in nature). Regardless of such shortcomings, DST fusion produced uncertainty-driven and hence more informative SWE data for subsequent analyses including DRA.

A simple method was also illustrated for propagating uncertainty of a single variable with hydrologic processes. The resultant p-box from any combination rule can be used for propagating uncertainty in
the chain of processes within a hydrologic model. Instead of a single value, lower and upper bound values from the intervals can be used for input into any hydrologic process enabling the representation of uncertainty (in the form of \textit{bpa}) along with the calculated results.
CHAPTER 5 CRITICAL REVIEW AND SELECTION OF A DROUGHT INDEX

5.1 Overview

The objective of this chapter is to review available drought indices for application in DRA. This chapter is divided into two parts; the first part will review drought indices based on their inherent benefits (Section 5.2 and Appendices I to III). Second, advantageous drought indices are reviewed specifically from a DRA application point of view (Section 5.3).

5.2 A review of drought indices

Drought indices are quantitative measures that characterize drought levels by assimilating data from one or several variables (indicators) such as precipitation and SWE into a single numerical value. An index is thus more readily useable than raw indicator data. The nature of drought indices reflects different events and conditions; they can reflect climate dryness anomalies (mainly based on precipitation) or correspond to delayed agricultural and hydrological impacts such as soil moisture loss or lowered reservoir levels. In addition, categorization of drought indices can be based on the data and technology used. For example, a considerable number of indices use remote-sensing imagery to detect vegetation health as an indicator of drought impacts.

This section provides a comprehensive review of 74 indices out of the nearly 150 available. Using nine primary references (Hayes 2006; Hayes et al. 2000; Heim 2002; Kallis 2008; Keyantash and Dracup 2002; Niemeyer 2008; Quiring 2009; Steinemann 2003; Steinemann et al. 2005; Tsakiris et al. 2007) a preliminary list of drought indices was compiled from which prominent drought indices were selected. This list includes various operational, research and proposed drought indices. The trend within the development of each index category is further described.

5.2.1 Taxonomy of drought indices

Since the development of a drought index can conceptually be based on multiple factors, e.g., drought’s nature/characteristics and the impacts considered, multiple drought indices have been developed. This is in addition to continuing technological developments (especially in the field of remote-sensing), the need to customize indices to specific climatic and hydrologic regimes (e.g., Vicente-Serrano et al. 2010), and the recent trend in aggregating existing indices to new ones to cover more impacts and applications (e.g., Brown et al. 2008). More than 150 drought indices have been developed (Niemeyer 2008) and additional indices have recently been proposed (Cai et al. 2011; Karamouz et al. 2009; Rhee et al. 2010; Vasiliades et al. 2011; Vicente-Serrano et al. 2010).
The variety of proposed drought indices also reflects the variation in perceptions about drought. This includes the basic definition of drought which varies among different applications. For example, agricultural drought primarily focuses on absent soil moisture content while hydrological drought examines the lagged effects of precipitation deficiency on various water features. This section provides the fundamental concepts based on which drought indices have been developed.

Commonly, drought indices are categorized based on the type of impacts they relate to. The taxonomy can also be based on the variables they incorporate (Steinemann et al. 2005) or on the use of disciplinary data (Niemeyer 2008). Three popular categories are meteorological, agricultural and hydrological drought indices. Niemeyer (2008) adds three categories to this list: comprehensive, combined and remote-sensing based drought indices. Comprehensive drought indices use a variety of meteorological, agricultural and hydrological variables to draw a comprehensive picture of drought. The Palmer Drought Severity Index (PDSI) is an example of this approach. Remote sensing-based drought indices use information from remote-sensing sensors to map the condition of the land (e.g., the Normalized Difference Vegetation Index, NDVI, Tucker 1979). Combined (also termed hybrid and aggregate) drought indices are derived by incorporating existing drought indicators and indices into a single measure. The US Drought Monitor (NDMC 2010) is an example.

This review is based on the categorization by Niemeyer (2008) omitting the “comprehensive” category.

5.2.2 Major drought indices

5.2.2.1 Operational drought indices

This section describes six drought indices that are frequently used in forecasting, monitoring and planning operations. Because of their prevalence, they warranted a longer description. The advantages and disadvantages of these indices are summarized in Table 5-1. Some drought indices specifically reflect one type of impacts or application, while others can be configured to correspond to varying impacts and thus drought type. For example, SPI, which is a meteorological drought, can be deployed for longer timescales to reflect agricultural and hydrological impacts.

**Percent of normal**: The percent of normal precipitation is a meteorological drought index that describes the drought as the precipitation deviation from the normal (average). The normal usually corresponds to the mean of the past 30 years. Percent of normal is calculated by dividing a given precipitation by the normal. The time scale of the analysis can vary from a single month to a year. The main advantage of this index is its simplicity and transparency which makes it useful for communicating
drought levels to the public (Keyantash and Dracup 2002). The percent of normal enables analysis for a single region and a specific period within a year. The statistical construct of this index has been criticized for inconsistency in two aspects (Hayes 2006). First, since no statistical transformation is used for the distribution of the precipitation record, the difference between the median and the mean value can undermine its accuracy. Second, since the distributions for seasons and regions are different, this index cannot be used to compare cross-season and cross-region droughts. As such this method lacks robustness required for operational use in planning and management.

**Deciles:** The method of deciles or 10 percentiles is based on dividing the distribution of monthly record precipitation into 10% parts (Gibbs and Maher 1967). Extended lengths of precipitation data record are required for accurate estimation. Deciles may be computed for any chosen period or window. Different categories of drought exist in the Australian Drought Watch Service. Generally, deciles method considers only the lowest 10% and two categories are used for characterizing rainfall deficiency: **severe** and **serious** rainfall deficiency. The former indicates the lowest five percent of recorded rainfall and the latter, the second lowest five percent.

### Table 5-1 - Advantages and disadvantages of popular drought indices

<table>
<thead>
<tr>
<th>DI, source and inputs</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
</table>
| **SPI (McKee *et al.* 1993)**  
Precipitation | - Simplicity; SPI relies only on precipitation data.  
- As SPI is adaptable for the analysis of drought at variable time scales; it can be used for monitoring agricultural and hydrological drought.  
- Comparing precipitation departure from normal for various regions with highly different climates is possible.  
- Equally represents both wet and dry climates and hence can be used for monitoring wet periods. | - Uses only precipitation, loosely connected to ground conditions. Potential evapotranspiration is a valuable additional indicator (Hu and Willson 2000; Tsakiris and Vangelis 2005; Vicente-Serrano *et al.* 2010).  
- Limitations of the precipitation data including accuracy of measurements, the number of gauging stations and length of the record.  
- Lacks the ability to identify regions with greater tendency to droughts. Requires knowledge of the local climatology. |
| **PDSI (Palmer 1965)**  
Precipitation, temperature | - More comprehensive than precipitation-only indices; evapotranspiration and soil moisture are also considered  
- Can use basic data for | - Arbitrary selection of beginning and end intensity values and algorithms  
- Less transparency because of more sophisticated computation  
- Calibrated for U.S. Great Plains’ conditions; limited |
calculation: precipitation and air temperature for which records for a long time back exist
- Most effective where impacts sensitive to soil moisture

Factors in antecedent conditions
- Applicability in locations with climatic extremes, mountainous terrain, or snow-pack unless calibrated
- Variable performance across regions and time periods
- Applicability to regions with extreme climate, e.g., highly variable rainfall or runoff, mountainous areas
- Handling of snow and soil freeze
- Neglecting the lag between rainfall and runoff lag
- PDSI uses the Thornthwaite method to estimate potential evapotranspiration. This technique has wide acceptance, but it is still only an approximation.
- All Palmer Indices are hardly appropriate for droughts within water management systems as they exclude water storage, snowfall, and other supplies. They also do not take human water balance impacts such as irrigation into account (Steinemann et al. 2005).

<table>
<thead>
<tr>
<th>NDVI (multiple sources)</th>
<th>Visible red band, near infrared bands</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Simple algorithms</td>
<td>- While resolution is high (1km) (compared to weather stations) Advanced Very High Resolution Radiometer (AVHRR) covers a large land area (Ji and Peters 2003)</td>
</tr>
<tr>
<td>- Current NDVI algorithms can reduce noise from atmospheric conditions (e.g., clouds) and effects of the sun-surface geometry with respect to the sensor. It hence broadly distinguishes vegetated areas from other surfaces.</td>
<td></td>
</tr>
<tr>
<td>- NDVI actually measures dryness (rather than interpolation or extrapolation).</td>
<td></td>
</tr>
<tr>
<td>- Resolution: The resolution of NDVI datasets extracted from MODIS sensor is 250m and lacks accuracy for some applications. These include monitoring change in riparian buffer zones and urban areas (Nagler et al. 2005).</td>
<td>- Soil conditions effects: NDVI is sensitive to darker and wet soil background (Huete et al. 1985). In wet conditions, the reflectance may not be equal in two bands and as such, the NDVI may vary with soil moisture variations.</td>
</tr>
<tr>
<td></td>
<td>- Saturation: In dense vegetation and/or multilayered canopy, where large biomass is present NDVI imagery tends to saturate.</td>
</tr>
<tr>
<td></td>
<td>- Non-linearity: Similar to other ratio-based Standardized Vegetation Indices (SVI), NDVI suffers from scaling and non-linearity.</td>
</tr>
<tr>
<td></td>
<td>- Atmospheric interference (Atmospheric path radiance): Atmospheric interference can contaminate pixels. This contamination can be due to cloud, seasonal smoke, aerosols, haze, etc. Currently available algorithms are capable of partially removing the contaminated pixels.</td>
</tr>
<tr>
<td></td>
<td>- Anisotropy: Surfaces, especially vegetation variably reflect light in different directions. The effects of variable geometry of illumination and the position of the vegetation relative to the swath of the sensor need to be considered.</td>
</tr>
<tr>
<td></td>
<td>- Vegetation stress and moisture correlation: Vegetation stress is influenced by more factors than moisture conditions alone. These include regional rainfall patterns and soil type as well events such as floods, insect infestation, wildfire, etc. (Ji and Peters 2003).</td>
</tr>
</tbody>
</table>
Standardized Precipitation Index (SPI): SPI (McKee et al. 1993) is a popular meteorological drought index that is also solely based on precipitation data. Similar to the percent of normal, SPI compares precipitation at any point with its multi-year average. SPI overcomes the discrepancies resulting from using a non-standardized distribution by transforming the distribution of the precipitation record to a standard normal distribution. For this, the precipitation record is first fitted to a gamma distribution which is then transformed into a normal distribution using an equal-probability transformation. The mean is then set to zero and as such, values above zero indicate wet periods and values below zero indicate dry periods. For any given drought, its SPI score represents how many standard deviations its cumulative precipitation deficit deviates from the normalized average (Drought Watch 2010). If a value of less than zero is consistently observed and it reaches a value of -1 or less, a drought is said to have occurred (McKee et al. 1993). The detailed procedure for calculating SPI is provided in Section 6.5.1.

An important aspect is the development of the SPI is its ability to calculate drought levels for flexible time scales. SPI can theoretically be computed for any time period; however it is typically applied for meaningful periods which are usually 3 months up to a few years and correspond to certain drought impacts including change soil moisture content and different water features. Since, over time the precipitation deficit gradually and variably affects different water resources (e.g., streamflow, groundwater and snowpack), the multitude of SPI durations can be used to reflect changes in different water features. Table 5-2 provides different timescales of SPI with related effects (NDMC 2006c). Some drought impacts associated with certain SPI lengths are listed below (NDMC 2011b).

**Table 5-2 – Phenomena reflected by specific-duration SPIs and their applications**

<table>
<thead>
<tr>
<th>SPI duration</th>
<th>Phenomena reflected</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-month SPI</td>
<td>Short-term conditions</td>
<td>Short-term soil moisture and crop stress (especially during the growing season)</td>
</tr>
<tr>
<td>3-month SPI</td>
<td>Short- and medium-term moisture conditions</td>
<td>A seasonal estimation of precipitation</td>
</tr>
<tr>
<td>6-month SPI</td>
<td>Medium-term trends in precipitation</td>
<td>Potential for effectively showing the precipitation over distinct seasons. E.g., for California, the 6-month SPI can effectively indicate of the amount of precipitation from Oct. to Mar.</td>
</tr>
<tr>
<td>9-month SPI</td>
<td>Precipitation patterns over a medium time scale</td>
<td>If SPI&lt;sub&gt;9&lt;/sub&gt; &lt; -1.5 then it is a good indication that substantial impacts can occur in agriculture (and possibly other sectors)</td>
</tr>
<tr>
<td>12-month SPI</td>
<td>Long-term precipitation patterns</td>
<td>Possibly tied to streamflows, reservoir levels, and also groundwater levels</td>
</tr>
</tbody>
</table>
In December 2009, the Inter-Regional Workshop on Indices and Early Warning Systems for Drought was held (“Lincoln Declaration on Drought Indices”, WMO, 2009). One of the goals of the workshop involving representatives from 22 countries was to help determine the best “meteorological” index and then recommend that all national meteorological services use this index. This would make comparisons in drought severity between countries in the same region, and also between regions possible. The SPI was chosen by participants as the one to use (Hayes et al. 2011).

The global acceptance and adaptation of SPI can be attributed to a number of advantages including low input data requirements and retaining statistical consistency (Agnew 2000; Cancelliere et al. 2007). The latter is beneficial as it enables calculating drought severity independent of the magnitude of mean precipitation and therefore allows comparison among different climates (Agnew 2000). In application, since the score in SPI represents how many standard deviations the precipitation deficit deviates from the normalized average therefore it is possible to set statistical thresholds to universally define classes of drought. The statistical nature of SPI makes probabilistic risk assessment possible. Operational regional and global maps for SPI are currently available at e.g., NCDC (2012) and EDO (2012). Details of SPI calculation procedure including involved equations are provided in Section 6.5.1.

**Palmer Drought Severity Index (PDSI):** PDSI (Palmer 1965) is a popular meteorological drought index, especially in the USA. The PDSI bases its definition of drought on water supply-and-demand instead of precipitation anomaly. Emphasis is on abnormalities in moisture deficiency rather than weather anomalies (Guttman 1999). PDSI uses precipitation, temperature and the local Available Water Content (AWC) data for soil. Using these inputs, PDSI computes four terms in the water balance equation: evapotranspiration, runoff, soil recharge and moisture and gives a more complete picture of the water balance (Niemeyer 2008 categorizes PDSI as a "comprehensive" drought index) and has remained popular despite criticism. Improvements include self-calibration capacity (Wells et al. 2004) and modifications to the evapotranspiration estimation methods replacing the original Thornthwaite method (Thornthwaite 1948) with other formulations.

**U.S. Drought Monitor (USDM):** The USDM (Svoboda et al. 2002) is a composite drought index. The USDM integrates multiple indices such as SPI and PDSI as well as indicators such as vegetation and hydrologic conditions into a weekly map of drought. This information is later subject to expert interpretation for refinement. Because of its composite nature, USDM can respond to the needs of various water users including water planners and the agriculture industry. USDM is currently widely used at the organizational level, research and by the media. The index is increasingly considered outside the USA.
**Normalized Difference Vegetation Index (NDVI):** NDVI is a remote sensing-based index that measures vegetation conditions (Rouse *et al.* 1974). NDVI uses the Advanced Very High Resolution Radiometer (AVHRR) reflected red and near-infrared channels to calculate if the vegetation is healthy, or unhealthy and sparse (e.g., suffering from drought or insect infestation). The formula for NDVI is given in (5-1):

\[
NDVI = \frac{NIR - R}{NIR + R}
\]

where \(NIR\) is near-infrared spectral reflectance and \(R\) is the visible red spectral reflectance. Under healthy conditions chlorophyll (the green substance that produces carbohydrates in plants) absorbs light, reflecting less \(R\). Lower \(R\) values result in higher NDVI value. Unhealthy plants reflect higher \(R\) resulting in lower NDVI. NDVI has extensively been used as a base index for a number of remote sensing indices that measure vegetation conditions, e.g., Vegetation Condition Index, VCI (Kogan 1990).

**5.2.2.2 Other notable drought indices**

Other notable drought indices are summarized in Table 5-3 (introduced prior to year 2000) and Appendix I (introduced after year 2000). Additional drought indices and indicators are comprehensively summarized in Appendix II. In the following section, the trend in the development of drought indices has been discussed under each drought type category.
### Table 5-3 – Additional notable drought indices


<table>
<thead>
<tr>
<th>Drought index and reference(s)</th>
<th>Type</th>
<th>Inputs</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z-index - Palmer (1965)</td>
<td>M</td>
<td>SM, ET</td>
<td>Monthly standardized anomaly of available moisture; intermediate term within PDSI (cf. Section 5.2.2.1); used for monitoring short-term droughts</td>
</tr>
<tr>
<td>Palmer Modified Drought Index (PMDI) - Palmer (1965)</td>
<td>M</td>
<td>SM, ET</td>
<td>Modified PDSI; main difference is in the calculated beginning and ending time of drought/wet periods; compared to Palmer Hydrological Drought Index (PHDI) responds more quickly and can be used for real-time monitoring</td>
</tr>
<tr>
<td>Keetch-Byram Drought Index (KBDI) - Keetch and Byram (1968)</td>
<td>M</td>
<td></td>
<td>Analyzes P and SM in the water budget model; used by fire control managers to monitor forest fires</td>
</tr>
</tbody>
</table>
| Effective Drought Index (EDI) - Byun and Wilhite (1999) | M    |     | Developed in response to weaknesses in then-available drought indices, weaknesses include imprecision in the drought beginning, ending and accumulated stress; ignoring the aggravating effects of runoff and ET; and incapability for real-time monitoring because of being monthly based  

\[
EP = \sum_{n=1}^{i} \frac{P_m}{n}, \quad EP = \text{effective precipitation, } i = \text{duration of summation (number of dry days + 365), } P_m = \text{precipitation of } m \text{ days before} \]

\[
SEP = \frac{DEP}{Se(EP)}; \quad SEP = \text{standardized } DEP, St(EP) = \text{standard deviation of daily } EP
\]

| Palmer Hydrological Drought Index (PHDI) - Palmer (1965) | H    | SM, ET | Analyzes precipitation and temperature in the PDSI water balance model; compares meteorological and hydrological drought across space and time (Heim 2002) |
| Surface Water Supply Index (SWSI) - Shafer and Dezman (1982) | H    | SP, ReS | Developed in response to PDSI’s limitations for mountain snow hydrology; calculates the weighted average of the standardized anomalies for P, ReS, SP and runoff, the four primary features in the surface water budget; used for river basins in western USA |
| Reclamation Drought Index (RDI) - Weghorst (1996) | H    | SP, ReS | Similar to SWSI, however incorporates temperature-variable demand and duration into the index; calculated basin-wide. |
| Crop Moisture Index (CMI) - Palmer (1968) | A    |     | Analyzes precipitation and temperature in a water balance model |
| Crop Specific Drought Index (CSDI) – Meyer et al. (1993) | A    | P, T, ET | Requires soil and crop phenology information in addition to climatological data; estimates soil water availability for different zones and soil layers by daily intervals. CSDI-based indices include: Corn Drought Index (CDI) (Meyer and Pulliam 1992) and Soybean Drought Index (SDI) (Meyer and Hubbard 1995) |
Crop Water Stress Index (CWSI) - Idso et al. (1981); Jackson et al. (1981)

CWSI = 1 - \frac{AET}{PET} \text{ where } AET = \text{actual ET and PET = potential ET (Jackson 1981). The terms are replaced by the difference in canopy and air temperature. Applied for irrigation scheduling.}

Normalized Difference Infrared Index (NDII) - Hardisky et al. (1983)

NDII is highly correlated with canopy and leaf water content (Equivalent Water Thickness, EWT); EWT is related to VWC; NDII is used for monitoring VWC.

NDII = \frac{R_{850} - R_{1650}}{R_{850} + R_{1650}} ; R_{850} = \text{the land-surface reflectance of the NIR channel; } R_{1650} = \text{the land-surface reflectance at 1650nm}

Vegetation Condition Index (VCI) - Kogan (1990)

Determines the departure of current NDVI from the minimum NDVI with respect to long-term NDVI; measures the health of vegetation; (used in USDM); VCI for week/month j is calculated from:

VCI_j = \frac{NDVI_j - NDVI_{min}}{NDVI_{max} - NDVI_{min}} \times 100 ; NDVI_{max} \text{ and } NDVI_{min} = \text{the maximum and minimum NDVIs, respectively, in the record for the specific month/week; } NDVI_j \text{ is the NDVI for the month under study}

Temperature Condition Index (TCI) - Kogan (1995)

VCI was modified to use the brightness temperature instead of NDVI as input; determines the deviation the month from recorded maximum; the idea is the higher the temperature, the higher the drought

TCI_j = \frac{BT_{j} - BT_{min}}{BT_{max} - BT_{min}} \times 100 ; BT_{max} \text{ and } BT_{min} = \text{the maximum and minimum BTs, respectively, in the record for the specific month/week; } BT_j = \text{month j's BT}

Vegetation Health Index (VHI) - Kogan (1995)

Combines VCI and TCI; uses a weight factor \( \alpha \) for the contributions of VCI and TCI's; \( \alpha \) is set to 0.5 lacking information; found more effective than other vegetative drought indices (Kogan 1990, 2001)

VHI = \alpha VCI + (1 - \alpha)TCI

Normalized Difference Water Index (NDWI) - Gao (1996)

Complementary to NDVI; determines VWC based on physical principles.

NDWI = \frac{NIR - SWIR}{NIR + SWIR} ; NIR = \text{reflectance (radiance) for NIR; SWIR = reflectance (radiance) for SWIR}

5.2.3 Recent developments in drought indices

The specific recent developments for meteorological, agricultural and hydrological drought indices are detailed below. Other directions are described in following sections.

5.2.3.1 Recent advances in drought indices by type

Meteorological drought indices: The development and implementation of a drought index heavily depends on data availability (Steinemann et al. 2005). Earlier drought indices used meteorological data readily available from synoptic meteorological stations (Niemeyer 2008). These include precipitation-only indices such as RAI (Van-Rooy 1965), BMDI (Bhalme and Mooley 1980), DSI (Bryant et al. 1992), NRI (Gommes and Petrassi 1994), EDI (Byun and Wilhite 1999) and DFI (González and Valdés 2006). For reasons such as better correlation with drought impacts and accounting for temporal trends in temperature, additional meteorological variables have been considered. These include modifications to
SPI (McKee et al. 1993), to develop the more comprehensive RDI (Tsakiris and Vangelis 2005) that incorporates evapotranspiration resulting in better association with impacts from agricultural and hydrological droughts. Vicente-Serrano et al. (2010) developed SPEI that is sensitive to long-term trends in temperature change. If such trends are absent, SPEI performs similarly to SPI. KBDI (Keetch and Byram 1968) also considered temperature and has had wide application to wildfire monitoring. PAI (Pálfai 1991) considered ground water in addition to these two indicators and have mainly been applied to basins within Hungary.

**Agricultural drought indices:** Approaches to characterize agricultural drought mainly evolve around monitoring soil water balance and the subsequent deficit in the event of a drought. This applies to the seven non-remote-sensing agricultural drought indices considered in this work: RSM (e.g., Thornthwaite and Mather 1955), CMI (Palmer 1968) which is similar to PDSI however models short-term agricultural by considering moisture deficit only in the top five feet of soil column (Byun and Wilhite 1999; Narasimhan and Srinivasan 2005), and CSDI (Meyer et al. 1993) originally designed for corn and its variant for soybean (Meyer and Hubbard 1995). DTx (Matera et al. 2007) calculates the daily transpiration deficit (DT) for x days. DTx uses the CRITeRIA soil moisture balance model (Zinoni and Marletto 2003) for with inputs including soil, crop and weather conditions in addition to temperature anomalies which affect evapotranspiration.

Increased spatial and temporal resolutions were sought in developing SMDI and ETDI (Narasimhan and Srinivasan 2005). This approach considers the soil component of the SWAT hydrologic model which has a resolution of 16km² (compared to then 7000km² to 160,000km² resolutions of SPI and PDSI). Within the top 2m of the soil component, “soil profile”, SMDI characterizes soil moisture deficit at varying depths: top 2ft (SMDI₂), 4ft (SMDI₄) and 6ft (SMDI₆). SMDI₂ and ETDI (which considers evapotranspiration deficit) were suggested for short-term drought conditions monitoring and SMDI₆ for long-term monitoring.

Remote sensing based vegetation indices such as NDVI (Tucker 1979), EVI (Liu and Huete 1995), VegDRI (Brown et al. 2008), TCI (Kogan 1995) and NDWI (Gao 1996) are also used to monitor general vegetation state and health (Sivakumar et al. 2011).

**Hydrological drought indices:** This group of indices aims at providing a comprehensive characterization of delayed hydrologic impacts of drought. Earlier, the sophisticated PHDI (Palmer 1965) model considered precipitation, evapotranspiration, runoff, recharge and soil moisture. The PDSI family of indices however lacked the snow component accumulation which led to the development of SWSI
(Shafer and Dezman 1982), probably the most popular of this group. Later, RDI (Weghorst 1996) improved SWSI by incorporating temperature and hence calculated a variable water demand as input.

RSDI (Stahl 2001) bases its model on homogeneous drought-stricken regions which are composed of several neighbouring low-flow gauging stations. RSDI first calculates the deficiency in streamflow compared to historic values and then uses cluster analysis to delineate the drought-stricken regions. Two later indices consider a water balance model: GRI (Mendicino et al. 2008) and Water Balance Derived Drought Index (Vasiliades et al. 2011), the former focuses on groundwater resources and uses geo-lithological conditions information in a distributed water balance model, while the latter uses a model that artificially simulates runoff for ungauged and low-data watersheds.

**Expanding the remote-sensing capacity:** New sensors and algorithms have constantly enabled the incorporation of improved remotely-sensed information in drought characterization. New sensors have higher spatial resolution, a current shortcoming in drought indices products (Niemeyer 2008). Novel noise reduction algorithms and other atmosphere correction algorithms improve the thematic accuracy of remote-sensing datasets.

Remote sensing indices are diverse and new indices are frequently proposed. While NDVI has remained popular, other indices such as VegDRI, VCI (Kogan 1990), TCI and VHI (Kogan 1995) are currently operationally used (NDMC 2011c; NOAA 2011). Traditionally used bands include near-infrared (NIR), red and short-wavelength infrared (SWIR). The Land Surface Temperature (LST) has been used as additional source along with NDVI to improve drought characterization accuracy (Cai et al. 2011; Lambin and Ehrlich 1995; Prihodko and Goward 1997; Rhee et al. 2010; Wan et al. 2004; Wang et al. 2001).

5.2.3.2 **Aggregation of drought indices**

Non-hybrid indices are mainly useful for particular places and specific objectives (applications) and do not provide a comprehensive characterization of drought events. Combining drought indices has been increasingly discussed as a means to incorporate and more effectively exploit information that is readily available and proven to be useful in field-specific drought indices (Kallis 2008; Niemeyer 2008; Sivakumar et al. 2011). In a follow-up to the Lincoln Declaration (WMO 2009), Sivakumar et al. (2011) recommended the creation of a new composite hydrologic drought index that would cover streamflow, precipitation, reservoir levels, snowpack, and groundwater levels. In general, hybrid drought indices can provide a stronger correlation with actual impacts sustained in the ground.
Most hybrid drought indices are comparatively recent, including the USDM (Svoboda et al. 2002) and VegDRI (Brown et al. 2008). VegDRI combines SPI and PDSI in addition to two NDVI-based indicators: Percent Average Seasonal Greenness (PASG) and Start of Season Anomaly (SOSA). Karamouz et al. (2009) combined the SPI, SWSI and PDSI to develop the integrated HDI.

5.2.3.3 Climate change effects

The predicted non-stationarity in future climates (IPCC 2007) has instigated research for the inclusion of future temporal patterns in drought characterization. The SPEI (Vicente-Serrano et al. 2010) accounts for the increase in the duration and magnitude of droughts resulting from higher temperatures. Additional research has been conducted for specific regions including Mpelasoka et al. (2008) for Australia and Dubrovsky et al. (2009) for the Czech Republic.

5.3 Application of drought indices for drought risk assessment

The review of drought indices can further be examined from the viewpoint of their relevance to various constituents of DRA: hazard, vulnerability and consequences/risk. According to this viewpoint, the prominent drought indices reviewed correspond - to various degrees - to hazard and consequences. While some solely reflect a meteorological hazard, e.g., SPI, deciles and percent of normal, others such as PDSI, PDHI, SWSI and RDI incorporate variables such as streamflow and SWE reflecting already established consequences. On the other extreme, vegetation indices such as NDVI only reflect the already established consequences of drought (vegetation health).

As such, given that drought indices correspond to a certain part of the hazard–consequences spectrum, a more appropriate and better representative incorporation of drought indices in drought management and risk assessment has been an ongoing theme. This effort is reflected in the aggregation of drought indices (previously discussed in Section 5.2.3.2). By incorporating additional indicators of hazard/consequences, this aggregation promises to complement the picture provided by a hazard-only or consequences-only drought index. This incorporation has however been heavier on the consequences side and commonly a dilemma in drought characterization can be identified: a discontinuity as to what extent prominent drought indices such as SPI correspond to consequences sustained on the ground. In other words such drought indices correspond to limited, mainly climatic variables of hazard such as precipitation. Tsakiris and Vangelis (2005) discussed a “balance” between “input” variables (e.g., precipitation) and “output” variables (e.g., evapotranspiration) and use this argument in the development of the evapotranspiration-driven Reconnaissance Drought Index (RDI). Similarly, Tsakiris et al. (2006) used RDI for drought assessment of a basin in Greece. RDI is found to
better reflect ground conditions compared to the simpler SPI that is only based on precipitation. For this, Tsakiris et al. (2007) considered two categories of drought indices: general indices and specific indices. General drought indices are used to provide an overview of the drought severity and occurrence (e.g., SPI) while specific drought indices mostly correlate droughts to the anticipated impacts on various sectors within the economy, the environment and the society (e.g., NDVI for agriculture) (Tsakiris et al. 2007).

5.3.1 The role of vulnerability

While hazard and consequences are measured using relevant indices in a rather straightforward fashion, the definition of vulnerability has not been as simple. Vulnerability’s definition has been dependent on the characteristics of the case study. For this, vulnerability has usually been treated variable and as the term linking a specific hazard to specific consequences. For example, Wilhelmi and Wilhite (2002) produced agricultural drought vulnerability maps for Nebraska using four factors: probability of seasonal crop moisture deficiency, soil root zone available water holding capacity, land use types, and irrigated cropland. They used a numerical weighting scheme to assess the drought vulnerability potential of each factor which were then combined using GIS. Shahid and Behrawan (2008) considered four socio-economic and three physical/structural factors in the derivation of a vulnerability index. Socio-economic factors included: population density, female to male ratio, poverty level, and percentage of people depending on agriculture, and physical/structural factors such as percentage of irrigated land, soil moisture holding capacity and food production per unit area. Fontaine et al. (2009) used an impacts-based approach to develop a vulnerability index. They primarily based their vulnerability model on three predefined variables:

- exposure: a combination of frequency and severity,
- sensitivity: susceptibility of water consumers to drought effects, and
- adaptive capacity: ability to manage adverse effects of drought.

5.3.2 Drought risk assessment using SPI

The two preceding sections viewed drought indices as occurring somewhere between the hazard and consequences continuum and vulnerability was defined as the term linking consequences to hazard. Although SPI was earlier identified as an advantageous index, it only reflects hazard. A case for embracing SPI for DRA is provided in following:
1) Considering that risk corresponds to specific economic, social and environmental impacts on the ground (NDMC 2011a), vulnerability is studied in application- or impact-specific settings. Hazard, however, needs to be regarded as a generic, uniform and independent phenomenon which can be characterized as a standalone process. (This approach is commonly embraced for characterizing drought hazard in the context of meteorological drought indices such as the SPI (e.g., Cancelliere et al. 2007; Shahid and Behrawan 2008; Wu and Wilhite 2004). The use of multiple, non-standardized hazard index results in disparate and non-uniform characterization for hazard. SPI hence can be regarded simplified and representative of the needs of the generic water user (Brown et al. 2008; Karamouz et al. 2009).

2) A risk measure however needs to link the hazard to the potential losses sustained by different sectors within the population. Indices of consequence including NDVI or those which include both hazard and consequence (such as PDHI), model specific drought impacts (vegetation health and streamflow, respectively), while indices such as SPI provide generic measures of hazard. Using such a generic measure, vulnerability for a wide range of impacts can be subsequently modeled.

5.4 Summary

Using drought indices is a pragmatic way to assimilate large amounts of data into quantitative and operational measures. This review initially presented descriptions for 74 drought indices. These indices were discussed in terms of application for DRA and were found to variably reflect hazard and consequences. In addition to its already established advantages, SPI was identified as an outstanding as it reflects the fundamental phenomenon that triggers drought. As a generic measure of hazard, SPI has a robust statistical basis and can be linked to impacts by relevant auxiliary variables and vulnerability measures, whereas, other drought indices non-uniformly characterize drought and are defined for specific applications and impacts.
CHAPTER 6 UNCERTAINTY-DRIVEN CHARACTERIZATION OF DROUGHT HAZARD USING ENHANCED STANDARDIZED PRECIPITATION INDEX

6.1 Overview

Regardless of its robustness, any probabilistic hazard analysis method such as SPI is prone to stochastic and epistemic uncertainty which can limit the usefulness of its results. Current SPI methods are undermined by inaccuracy that stems from climate change effects on variability (i.e., non-stationarity in climate normals) (Katz and Brown 1992) and incomplete climatic records (<30 years) which result in SPI values of non-uniform quality and reliability in a study area. Often records shorter than the recommended 30 year period are used. Consequently, two shortcomings can be identified in SPI: a) lack of a mechanism to model shifts in long-term climate normals (Dubrovsky et al. 2009; Vicente-Serrano et al. 2004) and b) inability to adapt to areas where data are scarce and records are inconsistent, especially for mountainous and remote regions.

The objective of this chapter is to address the following shortcomings: a) modeling shift in precipitation variability and the consequent change in drought frequency, and b) modeling uncertainty and ranking the significance of the changing frequency with regard to the associated uncertainty. The conceptual argument of this work is divided into three main components:

1) This work first adapts the generalized framework by DST to enhance SPI (Section 6.6). Among uncertainty-handling formulations, DST can handle variability and epistemic uncertainties including conflict and incompleteness (Ayyub and Klir 2006b; Ferson et al. 2003; Sentz and Ferson 2002). The p-box method is used to efficiently model data uncertainty alongside regular procedures (Ferson et al. 2003).

2) One of the limits of any uncertainty-driven analysis is the added computation due to discretization of continuous uncertainty bounds, especially if this uncertainty is to be propagated through subsequent processes. Here, a method is introduced to retain continuous bounds using only two discretized segments of the gamma-distributed uncertainty band (Section 6.6.2).

3) In the final phase, using the enhanced framework the effects of the shifts in precipitation normals on precipitation distribution and drought frequency are modeled (Section 6.8). The change in drought frequency is then compared to the surrounding uncertainty (i.e., the uncertainty that surrounds any deterministic data). This method is discussed through an application to extreme summer drought in Okanagan Basin, BC, Canada as a case study.
6.2 Climate change and its effects on the Okanagan Basin precipitation

The increase in atmospheric greenhouse gases (GHG) levels is understood to result in an increase in the average Earth temperature. Higher temperatures increase evaporation thus elevating atmospheric moisture. Elevated atmospheric moisture will alter atmospheric circulation and hence change precipitation regimes mainly as an increase in mean annual precipitation (IPCC 2007). This “non-stationarity” in climate behaviour has prompted researchers to include long-term trends in precipitation normals. Change in precipitation mean is estimated using coupled ocean-atmosphere general circulation models (GCM) which simulate shifts in future temperature normals based on various emission scenarios. Emission scenarios are driven by factors such as economic growth and development, environmental sustainability and population growth.

GCMs provide inputs for researchers to objectively project precipitation changes. While shift in the precipitation average has most commonly been used, precipitation variability (i.e., distribution of interannual precipitation) has been equally emphasized (Katz and Brown 1992). Drought is a phenomenon that is instigated by precipitation events found at the left tail of such distributions (low extremes).

For the province of British Columbia (BC), Canada, where the Okanagan Basin is located (Figure 6-1) (see Section 6.4 for further details), it is predicted that warming and subsequent change in precipitation will be greater than the global average (Pike et al. 2008). In general, four types of hydrologic regimes can be found in the province (Pike et al. 2008): rain-dominated, snowmelt-dominated, mixed/hybrid, and glacier-augmented. The snowmelt-dominated regimes such as the Okanagan Basin are dominated by snow accumulation and melt processes and characterized by spring peak flows, late summer and winter low flows. The downscaling of GCMs has predicted the following changes for such watersheds (Cohen et al. 2004; Cohen and Kulkarni 2001; Van der Gulik and Neilsen 2008; Pike et al. 2008; Rauscher et al. 2008; Shrestha et al. 2011; Stewart 2009; Summit et al. 2009):

1. Winter, summer, and overall annual temperatures will increase over the next 80 years,

2. precipitation will shift seasonally, with more falling in winter and less in summer,

3. the snowmelt-dominated system will have a shorter snow accumulation season due to warmer weather and the rain-on-snow effect, all resulting in a smaller snowpack volume,

4. the start of the spring freshet will occur earlier,
5. the earlier onset of spring melt will lengthen the period between late summer and early autumn low flows, and

6. hotter summers and longer growing seasons will further increase agricultural and other human water demands, while threatening aquatic ecosystems and species, resulting an overall increased pressure on water resources.

![Figure 6-1 - The location of Okanagan Basin](image)

6.3 SPI modifications to accommodate climate change

The statistical foundation of SPI assumes that certain characteristics of the precipitation distribution are stationary over time. Accordingly, if a large precipitation sample/record is used, the precipitation distribution for any location will approach the true distribution (here, assumed gamma). Given the predicted non-stationarity in future climate normals, the temporal trends in SPI have been studied and some researchers have introduced SPI modification to take into account shifts in precipitation normals.

Moreira et al. (2006) calculated SPI for the period of 1932 to 1999, for the region of Alentejo, Portugal to find possible trends in temporal evolution. They divided this period into three periods of 22 or 23 years. They used a loglinear model to study the drought class transitions among these three periods. They found similar behaviors for the first and last periods, both worse than drought events in the second period. The transition from the second to the third period was considered as evidence of aggravating drought conditions attributed to climate change. Li et al. (2008) considered the 30-year period of 1970-1999 of the Amazon region, Brazil and discovered a negative (drought inclining) trend of -0.32 change in SPI per decade. Dubrovsky et al. (2009) introduced the relative SPI (rSPI) to study drought for the Czech Republic. Compared to the traditional “self-calibrated” SPI, rSPI is calibrated using a reference weather series which is then applied to a tested series. Present conditions (1961–2000) were compared to future conditions (2060–2099) obtained from five GCMs. Results indicate a decrease in summer precipitation and increase in both winter and spring precipitation. Vicente-Serrano
et al. (2010) emphasized the role of temperature increase and increase in evapotranspiration processes on drought severity. The proposed method is multi-scalar, i.e., applicable to different hydrological systems, and is capable of differentiating drought types in the context of global warming. Absent temporal trends in temperature (e.g., by climate change), the proposed method performs similar to SPI.

6.4 Study area

The study area hereby introduced to help with the explanation of the problem and model. The Okanagan Basin is a snow-dominated semi-arid watershed located in south central British Columbia (Figure 6-1). The Okanagan Valley is located at the middle of the Basin. The Basin has an approximate area of 8,000 km² with a length and width of just above 180 km (N-S) and 100 km (E-W), respectively. The Basin has 30 major tributaries that eventually flow into the main water feature in the Basin, the Okanagan river-lake system. The Okanagan Lake itself extends over 135 km long. At its southern end, the Okanagan River flows into the larger Columbia River Basin which drains westward into the Pacific Ocean.

The Okanagan Basin has a complex terrain with elevation ranging from 350 m at the Okanagan Lake at valley bottom to 1,600 to 2,000 m in the surrounding mountain ranges (Duke et al. 2008). The Basin has a dry continental climate with a snowmelt-dominated hydrology (Merritt and Alila 2004). Being in an semi-arid climate and a high proportion of agricultural activities, the long term sustainability of water resources especially during summer and late-summer is an issue of concern. IPCC (2007) predicted reduced rainfall over continental interiors during summer as a result of increase in evaporation. Regional studies have predicted reduced snowfall from the proportion of total annual precipitation as a result of increase in temperature, peaking the pressure on water use in the Basin during late summer low-flows (Cohen et al. 2004; Cohen and Kulkarni 2001; Van der Gulik and Neilsen 2008; Pike et al. 2008; Rauscher et al. 2008; Shrestha et al. 2011; Stewart 2009; Summit et al. 2009).

Elevation has a dominant influence on Okanagan Basin hydrology. The digital elevation model (DEM) was obtained from the Geobase website (www.geobase.ca). According to the dataset, the elevation in the Okanagan Basin ranges between 270 m and 2312 m. Geobase provides DEM at two scales: 1:250,000 and 1:50,000 (hereby used). The 1:50,000-scale data has a resolution of 0.75 × 0.75 arc seconds. These measurements correspond to uniform 23 m spacing in S-N direction, and 15.2 m to 14.7 m in E-W spacing. Considering the spatial extents of the Basin (~180 km in N-S and ~100 km in E-W), for its scale, the Basin has a diverse terrain: for an elevation mean of 1092 m, it has a standard deviation of 449 m. The elevation dataset was clipped with a 25 km buffer surrounding the Okanagan
Basin (Figure 6-2) to minimize “boundary effect”. Boundary effect partly refers to the non-presence of features in the region outside the analysis area which introduces inaccuracy in the boundary region when spatial analyses such as interpolation are performed. The 25 km buffer was used in subsequent analyses.

Station-wide precipitation data were obtained from the Daily Climatological Data (DLY04) of the Canadian National Climate Data and Information Archive (climate.weatheroffice.gc.ca). Since a 25 km buffer zone was used, analyses spanned to a small area within the US state of Washington (WA). Data for US stations were obtained from the National Climatic Data Center (www.ncdc.noaa.gov). Records for a total of 131 stations, 122 in BC, Canada and 9 in WA, USA for the period of 1900-2006 were obtained. Station-wide temporal completeness varies in all monthly, seasonal and annual windows.

![Figure 6-2 – DEM for the Okanagan Basin and 25 km vicinity (X-axis and Y-axis labels represent longitude and latitude respectively)](image)

6.5 Uncertainty in SPI

In water resources studies, DST has successfully been used to enhance hydrological features to uncertainty-driven features. These features include permeability (Mathon et al. 2009), GCM (Raje and Mujumdar 2010) and snow water equivalent (Zargar et al. 2012b).
6.5.1 Standardized Precipitation Index

SPI is a statistical index with a relatively simple and straightforward procedure. For some specifications, e.g., distribution to fit, or minimum monthly completeness, the applied criteria have varied. The procedure used for each point (a station or a grid cell) is described. The process is illustrated in steps in Figure 6-3.

1- Prepare data

   a. Select a time window: The length of this window corresponds to different impacts. The period considered (e.g., a specific month or season) should reflect specific phenomena. In this work summer SPI is considered, spanning three months from June 1 to August 31 (92 days in total) (step ①).
b. Length of record: SPI requires at least 30 years of preferably continuous data. 50 years or more are recommended (Guttman 1999). Regardless, studies often incorporate non-continuous records of non-uniform lengths (step ①).

c. Completeness: Monthly, and as such, seasonal daily precipitation data often include missing days. Criteria for using data subject to incompleteness vary. The minimum 90% rule is hereby used: for year X of the record, each month should contain records for at least 90% of the days (>27 days for June and >28 days for July and August). If less than 90% of days’ records exist, year X is excluded (step ①).

d. Average days to seasonal mean and arrange data: Average daily values to seasonal means and arrange values in a continuous data series (step ②).

2- Fit a distribution

a. Fit a gamma distribution to the series and obtain distribution parameters: McKee et al. (1993) originally fitted the gamma distribution to the data series. Gamma distribution is considered suitable for interannual precipitation, e.g., Lloyd-Hughes and Saunders (2002) while Pearson Type III distribution, a 3-parameter gamma distribution, was alternatively suggested by Guttman (1999). By fitting the gamma distribution, gamma distribution parameters shape (k) and scale (θ) are obtained (step ③).

b. Obtain scores: Using obtained k and θ parameters on the cumulative gamma distribution formula: \( F(P; k, \theta) = x \), where:

\[ F: \text{Cumulative gamma distribution} \]
\[ P: \text{Precipitation for a year} \]

obtain a year’s score, x on the fitted gamma distribution (step ④, cf. Section 6.6.1 for more details).

3- Transform to standardized normal distribution

a. Transform gamma scores and obtain SPI: Using an equi-percentile transformation, transform the gamma distribution to a standardized normal distribution and use this transformation to obtain standard normal score for the gamma scores obtained at Step
2.b. The transformation is regularly done using a relationship by Abramowitz and Stegun (1965) (see Bordi and Sutera 2001) (step ⑤).

b. Step ⑥ is required for spatially interpolating station-wide SPI values to cover the study area.

Using the SPI score, drought can further be categorized using Table 6-1 (McKee et al. 1993). For example, if a year scores less than -2, it is categorized as extreme drought and occurs with a cumulative probability between 0 and 2.3%. Alternative classification schemes are suggested by Agnew (2000) and NDMC (2008). For example, in NDMC (2008) the classification is finer and extreme drought has been termed “exceptional drought” (category D4). Here the classification by McKee et al. (1993) is employed. This classification is global and is applicable to SPI of any duration.

<table>
<thead>
<tr>
<th>Drought category</th>
<th>Extremely dry</th>
<th>Severely dry</th>
<th>Moderately dry</th>
<th>Near normal</th>
<th>Wet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold</td>
<td>SPI &lt; -2</td>
<td>-2 &lt; SPI &lt; -1.5</td>
<td>-1.5 &lt; SPI &lt; -1</td>
<td>-1 &lt; SPI &lt; 1</td>
<td>+1 &lt; SPI</td>
</tr>
<tr>
<td>Corresponding probability density</td>
<td>-47.7%</td>
<td>-47.7% to -43.3%</td>
<td>-43.3% to -34.1%</td>
<td>-34.1% to 34.1%</td>
<td>34.1% to 50%</td>
</tr>
</tbody>
</table>

6.5.2 Sources of uncertainty in SPI

Similar to any hydrologic analysis, SPI’s model is prone to various sources of uncertainty. Sources of uncertainty can be numerous and an exhaustive screening of all sources of uncertainty is impossible and beyond the scope of this work.

Theoretically, uncertainty could be separated from ignorance (i.e., all unknown) as a type of known unknown (Ayyub and Klir 2006a). Models of the environment are abstractions and are inherently uncertain: they deliberately omit some complexity resulting in some degree of uncertainty. Accordingly, in practice uncertainty can be regarded as relative to an ideal definition or baseline for the data and model. Such a “nominal ground” can serve as a basis from which any aberration would be termed uncertainty (Aalders 1996).

Uncertainty hereby will refer to variability and incompleteness. Variability is modeled by assuming precipitation as a stochastic variable that follows a certain (gamma) distribution. For completeness, the nominal ground is defined as following:
where (1) (ideally), for each station records for summers of all 107 years are available (1900-2006), and (2) for each month, the records for at least 90% of the days are available.

For the former requirement, this imperfection (<107) is formalized as incompleteness uncertainty, for the latter, months with less than 90% days of available data are excluded. By adopting these constraints, it is also assumed that in ideal conditions the empirical distribution perfectly matches the fitted gamma distribution, i.e., all error in distribution fitting arises from incompleteness. Higher orders of error shall then be traced and attributed to other sources of uncertainty (e.g., inaccuracy in precipitation values), all of which are otherwise assumed perfect. Additional sources of uncertainty that have been ignored include accuracy of precipitation values, positional accuracy of stations, variability due to cycles such as the Pacific Decadal Oscillations (PDO), etc. Additional work is required for these terms, especially for the latter, where the added effects of cyclical patterns with climate change anomalies can be substantial (IPCC 2007). It should be noted that by requiring the minimum 30 years of data, SPI implicitly includes such cycles.

The SPI procedure requires a certain degree of completeness of monthly data (cf. Section 6.5.1). For example, for a minimum 90% completeness in summer: availability of >27 days for June, and >28 days for July and August. The distribution of completeness for the months of summer (June-July-August) reveals that 78% of the 41730 possible months (107 years × 3 months × 130 stations) contain no data at all. 21% contain equal or more than 90% of days, from which, 14% contain data for all days of the month and the remaining 1% is ranging 0% to 90% complete. Among the 107 years, the temporal coverage of stations varies for data available for a single year to 101 years (for station 1123390) (Figure 6-4).

![Figure 6-4 – Histogram for the number of years with data available](image-url)
Gamma distributions were subsequently fitted to all available stations with 7 or more years of data. The standard errors for the fits are shown in Figure 6-5. According to the earlier assumption, this error is hereby primarily attributed to the incompleteness of record in different stations. This is confirmed by Figure 6-5 where the standard errors from $k$ and $\theta$ (represented by the diameter of the circles) is conversely proportional to the number of years of data available for the station.

6.6 Enhanced SPI

Enhancing SPI’s model hereby refers to generalizing the deterministic definition of variables in SPI to a definition that enables modeling data imperfections including variability and epistemic uncertainty. This generalization is modeled using the Dempster–Shafer theory that is described in the following. In the two subsequent sections, uncertainty modeling and propagation is described.

6.6.1 Modeling uncertainty using p-box

The main concept in the development of an enhanced SPI or any other index is to replace the deterministic definition of the involved variable(s) with a generalized uncertainty-driven definition.
When calculating SPI, the use for the precipitation distribution for parameters \((k \text{ and } \theta)\) is limited to the calculation of distributions for interannual precipitation. (These scores are then transformed to a normalized standard distribution and SPI is calculated). \(k\) and \(\theta\) however contain valuable information about precipitation variability at each station or cell (for grid data). This information can be propagated along with normal calculations and retained for additional operations such as spatial interpolation. This also resonates with Rhee et al. (2008) that argued interpolation prior to calculating drought index values provides the most consistent results. Variability-driven precipitation may hence be interpolated instead of SPI.

Moreover, due to data incompleteness, \(k\) and \(\theta\) were imperfectly fitted to data and resulted in epistemic uncertainty (represented by parameters standard error, \(SE\)). Epistemic uncertainty may also simultaneously be propagated with variability.

a) Modeling variability

The granular feature for which uncertainty needs to be defined is first set to: precipitation at each point. A point refers to a station for station-based data, or a cell, for grid data. The traditional deterministic mapping of data to a variable is expanded by a set of probabilistic hypotheses for the variable: precipitation at each point \(j\), \(V_j\), is defined as a mapping of a set of possible values \(x_i, i = 1, ..., n\) (universal set) \(V_j \xrightarrow{bpa} x_i\). The probability of belonging of the set of possible values \(x_i\) to the variable \(V_j\) is defined as \(bpa\). \(bpa\) for variability corresponds to the gamma distribution that is fitted to data at point \(j\) given in Eq. (6-1).

\[
\Gamma(k, \theta) = f(x; k, \theta) = \frac{1}{\theta^k \Gamma(k)} x^{k-1} e^{-\frac{x}{\theta}}
\]

(6-1)

for \(x \geq 0\) and \(k, \theta > 0\)

Accordingly, instead of a single deterministic value \(x\) for the variable, a range of possible values \(x_i\) with gamma-defined probability are mapped to \(V_j\). Subsequent operations may be performed on the variability-driven \(V_j\). If only variability is considered, the gamma distribution is shown using the CDF as a single line in p-box.
b) Modeling epistemic uncertainty

The epistemic uncertainty results from the degree of (in)completeness at each station. As it was earlier assumed, theoretically, if “complete” data were available, the gamma distribution would perfectly fit the empirical data.

The deficiency in fitting a gamma distribution to data is modeled using the standard error ($SE$) of fit parameters $k$ and $\theta$ (i.e., $SE_k$ and $SE_\theta$). $SE_k$ and $SE_\theta$ were estimated using the MASS library of the R statistical software (Ripley et al. 2012). The MASS library fits the gamma distribution using a direct optimization of the log-likelihood and calculates $SE$ from a numerical approximation of the information matrix (Ripley et al. 2012).

$SE_k$ and $SE_\theta$ refer to a normally distributed probability function that surround best fit $k$ and $\theta$. The introduction of parameters $SE_k$ and $SE_\theta$ representing epistemic uncertainty into $k$ and $\theta$ can be done using confidence bands. Using $SE$, confidence bands $\hat{k}(x) \pm SE_k$ and $\hat{\theta}(x) \pm SE_\theta$ with coverage probability $1 - \alpha$ are defined simultaneously satisfying the following conditions:

$$\Pr(\hat{k} - SE_k \leq k \leq \hat{k} + SE_k) = 1 - \alpha \quad (6-2)$$

$$\Pr(\hat{\theta} - SE_\theta \leq \theta \leq \hat{\theta} + SE_\theta) = 1 - \alpha \quad (6-3)$$

where: $\alpha = $ significance level

The lower- and upper-bound of the p-box are the percentages by which the confidence band encapsulates the best fit distribution. In Figure 6-6a, the p-box for station 1125381 with coverage probability of $1 \times SE$ corresponding to 68% is shown. At 68% coverage probability, the “true” fit is expected to rest in the confidence band defined by $\Gamma(k + SE_k , \theta - SE_\theta)$ (lower bound) and $\Gamma(k - SE_k , \theta + SE_\theta)$ (upper bound). While in Figure 6-6b, for station 1124980, the coverage probability at 50% corresponding to $0.67 \times SE$ and 95% corresponding to $1.96 \times SE$ are shown. In both p-box, the curve in the middle represents the best fit line.
6.6.2 Generalizing deterministic interpolation to uncertainty-driven interpolation

6.6.2.1 Deterministic spatial interpolation

To cover the entire study area, station-wide values for mean need to be spatially interpolated (step 6 in Figure 6-3). Current schemes for SPI interpolation include inverse distance weighted (IDW) (Patel et al. 2007), multiple linear regression (MLR) model (Loukas and Vasiliades 2004; Naoum and Tsanis 2003), cubic spline (Bonsal and Regier 2007) and bilinear interpolation (Lloyd-Hughes and Saunders 2002). Rhee et al. (2008) compared four methods (Thiessen polygons, IDW, and thin-plate smoothing splines (TPSS) and ordinary kriging) at different spatial units. Thiessen polygons and TPSS showed relatively large errors. IDW and kriging were found superior due to their lowest cross-validation error however IDW was chosen because of a simpler procedure. Rhee et al. (2008) also found that interpolation of indicators (e.g., precipitation) prior to calculating drought index values provides the most consistent drought categorization between various spatial units; however additional error may be introduced. Akhtari et al. (2009) compared four interpolation methods: Thiessen polygons, weighted moving average, TPSS, kriging and co-kriging for SPI and Effective Drought Index (EDI). They found kriging-based methods to be superior with the exception of their higher computational cost.

For the interpolation of precipitation over complex terrain, kriging and cokriging have been found superior, especially when data are sparse, using elevation as an auxiliary variable can considerably improve accuracy (Azimi-Zonooz et al. 1989; Diodato 2005). Studies indicate modifications for improved cokriging performance: Azimi-Zonooz et al. (1989) found disjunctive cokriging to outperform ordinary
cokriging, however at higher computational cost. Diodato (2005) proposed using a topographical index as an additional auxiliary variable. Earlier, Knotters et al. (1995) found kriging combined with regression to outperform co-kriging when the target variable (precipitation) and the auxiliary variable (elevation) insufficiently correlate.

For the cokriging process, the relationship between elevation and precipitation and spatial autocorrelation need to be considered. The elevation-precipitation relationship can be established using a variety of linear, curvilinear and exponential relationships. The scatter plot and relationship for the Okanagan Basin stations elevation and their summer mean daily precipitation is shown in Figure 6-7. Spatial autocorrelation (Moran’s I) was calculated for mean at 0.31, with a Z-score of 9.81. While spatial autocorrelation was sufficiently established, elevation-precipitation relationship was found limited. The area of the bubbles in Figure 6-7 is proportional to the product of fit parameters standard error and normalized elevation error. Elevation error is due to the difference in elevation in stations record and the elevation as extracted from the DEM. This is mainly due to the imprecise location coordinates provided in older weather stations. The lack of performance in Figure 6-7 could not however be clearly attributed to the product of three error factors: \( k \) and \( \theta \) standard errors and normalized elevation difference.

Here, a cokriging approach with elevation as the auxiliary variable is applied. Considering that elevation variations largely affect the hydrologic regime in the Okanagan Basin and data are sparse locally at higher elevations (cf. Figure 6-2 and Figure 6-5), the use of elevation as auxiliary variable is essential. A specific reason for using the cokriging approach over methods such as kriging combined with regression despite the limited performance of the elevation-precipitation relationship was the prevalent use of this method in spatial interpolation of precipitation. This work hence implements enhancements on this popular method. The result of deterministic cokriging interpolation is shown in Figure 6-8.
Figure 6-7 – Elevation-precipitation scatter plot (the area of the bubbles is proportional to the product of fit parameters standard error and normalized elevation difference)

Figure 6-8 – Cokriging interpolation of stations summer mean daily precipitation

6.7 Uncertainty-driven interpolation

Uncertainty-driven spatial interpolation using fuzzy $\alpha$-cuts (Bardossy et al. 1989; Phil and Diamond 1989) and with the addition of Bayesian prior knowledge (Bandemer and Gebhardt 2000) were explored. A p-box based interpolation scheme is presented in following where a continuous “p-box surface” will cover the study area. For this, arithmetic operations need to be applied on p-box. These operations are discussed in Ferson et al. (1999) and Williamson and Downs (1990).
For continuous bounds in p-box, the p-box is first discretized into boxes (Figure 6-9). Arithmetic operations can be performed on boxes where $x$ is represented by interval endpoints and the height of the box represents probability. Arithmetic operations are performed on pairwise combinations of boxes from two p-boxes.

Although discretization is essential for p-box operations on continuous bounds, nevertheless this process causes additional uncertainty due to following issues:

1. Any discretization causes loss of information. The uncertainty arisen from discretization in p-box is termed non-specificity. The coarser the segments, the more non-specific the resultant information,

2. moving from parametric representation of data by $k$ and $\theta$ to discrete data, p-box will become a generic p-box and parametric information will be lost,

3. since all possible pairwise combinations are considered, calculations can be voluminous, especially for finer discretization boxes/segments, and

4. in general in hazard analysis and risk assessment, information at the tails of the distribution are essential. 1% percentiles for discretization have been used in Sentz and Ferson (2002).

However, no matter how fine the discretization, the analysis may not maintain the original accuracy.

To overcome the shortcomings in discretization, a combination of discretization and parametric approaches is hereby introduced. In this work, instead of discretizing the entire range in p-box by percentiles, the p-box bounds LB and UB, is discretized by two percentiles ($40^{th}$ and $60^{th}$ percentiles). Interpolated maps for these percentiles were obtained. Using these two percentiles at each point, the gamma distribution can be reconstructed. The following system of two equations and two unknowns, $k$ and $\theta$ was then solved:

\[ \Gamma_{CDF}(k, \theta) = \frac{1}{\Gamma(k)} \gamma(k, \frac{x_{CDF=0.40}}{\theta}) = 40\% \]  \hspace{1cm} (6-4)

\[ \Gamma_{CDF}(k, \theta) = \frac{1}{\Gamma(k)} \gamma(k, \frac{x_{CDF=0.60}}{\theta}) = 60\% \]  \hspace{1cm} (6-5)

where: $\gamma = \text{is the lower incomplete gamma function}$
\[ \Gamma(k) = \text{gamma function} \]

This follows an assumption that interannual precipitation distribution at any given point in the study area follows the gamma distribution. The results from this method were tested for best fit. The mean from interpolating means: \( M_{cokriging} \) was compared to calculated from fit parameters \( k \) and \( \theta: M_{k \times \theta} \) and it was confirmed by a coefficient of determination of \( R^2 = 0.9999 \).

\[ \text{Figure 6-9 – Discretization of gamma distribution using 10 equi-percentiles} \]

### 6.8 Incorporating climate change effects

The effects of climate change on precipitation in the Okanagan Basin are characterized by a general increase in annual precipitation, a shift from snowfall to rainfall and reduced summer precipitation. These changes are regularly reported as shift in mean. Change in distribution is equally important as it enables modeling extreme events including drought (e.g., Katz and Brown 1992). This section describes an earlier developed model (Waggoner 1989) for calculating the effects of shift in precipitation means on precipitation variability and extreme drought occurrence. The change in extreme drought occurrence is then studied within the surrounding uncertainty which represents the range of possible values due to epistemic uncertainty.

#### 6.8.1 Change in precipitation means

Data for projected precipitation means were obtained from the Pacific Climate Impacts Consortium (PCIC) ClimateWNA tool (PCIC 2011; Wang et al. 2011). ClimateWNA downscales PRISM (Daly et al. 2002) to calculate monthly (2.5 x 2.5 arcmin for the reference period: 1961-1990), seasonal and annual climate variables for western North America (CFCG 2011; Hamann and Wang 2005; Wang and Hamann 2006).
The dataset consists of absolute values for climate change based on 1961-1990 normals with following attributes:

**Projected years:** 2050s (2040-2069) and 2080s (2070-2099)

**GCM:** Canadian Coupled Global Climate Model version 3 (CGCM3)

**Scenario:** The A2 scenario from the IPCC Special Report on Emissions Scenarios (SRES) (IPCC 2000). The A2 family scenario describes a heterogeneous world characterized with interest in local and regional economic growth and technological dynamics (CCCma 2010). High-population growth (15 billion by 2100) and high emission level is associated with this scenario.

**Run:** The dataset is the fifth run of this scenario.

It is worth noting that while the resulting projections from GCMs have often proven reliable at regional scale (IPCC 2007), they nevertheless needed to be downscaled for finer spatial scales by ClimateWNA. The uncertainty arisen from the process of downscaling GCM projections to local scales has not been considered here.

The maps for projected change in precipitation means for 2050s (2040-2069) and 2080s (2070-2099) are shown in Figure 6-10. Since projected precipitation is based on 1961-1990 normals (PCIC 2011), an assumption was made that these normals are representative of the 1900-2006 normals (based on which station-based SPI and climate change maps were produced). The variations in yearly mean precipitation within 1900-2006 and change during 2040-2069 ($-0.127 \text{ mm/d} $) and 2070-2099 ($-1.911 \text{ mm/d} $) (PCIC 2011) are shown in Figure 6-10. This assumption was verified by analyzing the trend in average precipitation for the 1900-2006 period. The mean for the 1961-1990 period ($\mu_{1961-1990} = 14.96 \text{ mm/d} $) (Figure 6-11) was tested against the mean for the 1900-2006 period ($\mu_{1900-2006} = 14.38 $) using a two sample analysis. The result for the $t$-test was a $p$-value of 0.56 corresponding to the $t$-test statistic of -0.57.
6.8.2 Change in the distribution of precipitation and extreme drought

Modeling variability is imperative since it reveals the change in drought by comparing the distributions for current and future precipitation distributions. Waggoner (1989) proposed a method for modeling the change in gamma distribution parameters using shift in mean. The underlying assumption is that the correlation between current variance and mean can be used as a basis for calculating future gamma distribution parameters. Here, the procedure for modeling change in distribution is explained by using both station-based (DLY04, 1900-2006) and grid datasets (1961-1990) (PCIC 2011) as base datasets. The
station-based dataset is ‘raw’ station data and provides significant low $SE_k$ and $SE_\theta$ for certain stations. The grid based dataset provides more uniform $SE$ for the entire study area.

**Station-based data (“modeled”):** Waggoner (1989) first establishes an empirical log-log relationship between the variance and mean in the data. The scatter plot for $\log(V)$ against $\log(M)$ is shown in Figure 6-12.

The relationship from fitting a linear line to the scatter plot is deduced as:

$$ V = 0.56M^{1.52} \tag{6-6} $$

Where $V$ represents the variance of precipitation and $M$ represent the mean of precipitation. For the gamma distribution, the following relationships hold true:

$$ M = k\theta \tag{6-7} $$

$$ V = k\theta^2 \tag{6-8} $$

From (6-6) and (6-8) Waggoner (1989) calculated $k$:

$$ k = 1.80M^{0.48} \tag{6-9} $$

Using (6-9), the relationship between change in $k$ and change in $M$ can be written as

$$ k + \Delta k = 1.80(M + \Delta M)^{0.48} \tag{6-10} $$

Where $\Delta k$ and $\Delta M$ are change in and $k$ and $M$, respectively. $k$ for climate change conditions ($k_{cc}$) is equal to $k + \Delta k$. $\Delta k$ can be calculated using current $M$ and $k$ as initial conditions.

$$ \Delta k = 1.80(M_{cc})^{0.48} - k \tag{6-11} $$

Where $M_{cc} = M$ under climate change conditions. $\theta_{cc}$: $\theta$ for climate change conditions, can be calculated from (6-12):
Grid data ("grid"): For grid based data, the empirical relationship between the variance and mean is shown in Figure 6-13. This high correlation can be attributed as an artifact of the grid data generation procedure. Accordingly, $\Delta k$, $k_{cc}$ and $\theta_{cc}$ can be calculated from:

$$\theta_{cc} = \frac{M_{cc}}{k_{cc}}$$

(6-12)

Accordingly,

$$\Delta k = 7.276(M_{cc})^{0.0615} - k$$

(6-13)

McKee et al. (1993) defined extreme drought as precipitation occurring with amount lower than those of 2.3% of lowest precipitation occurrences ($SPI < -2$). Using the distributions from current and projected conditions, it is possible to calculate departure in extreme drought occurrence due to change in mean and distribution. This amount is shown as the shaded area in Figure 6-14. This approach may equally be applied to other classes of drought.

\[ y = 1.5209x - 0.2546 \]

\[ R^2 = 0.6205 \]

Figure 6-12 – Log(Mean) vs. log(Variance) for station data (modeled)
6.8.3 Enclosing uncertainty

As earlier discussed, incompleteness resulted in lack of fit and was modeled by the standard errors of fit parameters $k$ and $\theta$. $SE_k$ and $SE_\theta$ are defined for the standard deviation ($\sigma$) of a standard normal distribution that surrounds each parameter $k$ and $\theta$ for the best fit line $\Gamma(k, \theta)$. P-box was accordingly constructed using $1 \times \sigma$ of standard errors of $k$ and $\tau$ ($SE = \sigma$). On the constructed p-box, at $\Gamma_{CDF}(k, \theta) = 2.3\%$ (i.e., $SPI = -2$), it is possible to establish the normal distribution by knowing two values at the lower- and upper bounds $1 \times \sigma$, 68.1\%, i.e., $k \pm 1 \times SE_k$ and $\theta \pm 1 \times SE_\theta$. The normal
distribution is shown in Figure 6-15. This normal distribution may be used to compare and rank the magnitude of extreme drought change with regard to uncertainty.

![Diagram](figure6_15.png)

**Figure 6-15 – Finding standard errors multiplier**

Here, this comparison is performed by finding the corresponding “α” multiplier to standard error. Theoretically, a p-box can be viewed as a series of CDFs that correspond to k and θ for a certain CDF that is located within lower bound and upper bound of a p-box. In this p-box, the band encloses 68.3% (1 × σ for standard errors of k and θ) of the probability (probability that true value lies within these bounds) (Figure 6-14 and Figure 6-15). In Figure 6-15, lines m and m’ represent ±1σ that together enclose 68.1% of probability at $\Gamma_{CDF} = 2.3\%$ (extreme drought). Lines p and p’ enclose a probability corresponding to $±a.SE$ at $\Gamma_{CDF} = 2.3\%$.

Theoretically, the change in extreme drought would correspond to a certain probability and a certain “α” multiplier of $SE$ on this normal distribution. α is hence the probability of exceeding either of the bounds. The multiplier α of standard errors $SE_k$ and $SE_θ$ (α), is defined such that for decrease in precipitation ($M - ΔM$) one has:

$$M - ΔM = (k - a.SE_k)(θ - a.SE_θ) \quad (6-14)$$

The corresponding CDF to α will occur closer to the p-box’s upper-bound (UB). For increase in precipitation one has:

$$M + ΔM = (k + a.SE_k)(θ + a.SE_θ) \quad (6-15)$$
which will occur closer to the p-box’s lower-bound (LB). Accordingly to Equations (6-14) and (6-15), small changes in precipitation and wide distributions result in the small the significance (multiplier) of the drought occurrence when there is uncertainty.

\( a \) in Equations (6-14) and (6-15) may be solved using (6-16) and (6-17) respectively:

\[
a^2(SE_E SE_\theta) - a(SE_E \theta + SE_\theta k) + \Delta M = 0 \quad (6-16)
\]

\[
a^2(SE_E SE_\theta) + a(SE_E \theta + SE_\theta k) - \Delta M = 0 \quad (6-17)
\]

### 6.9 Results and discussion

#### 6.9.1 Station-based and grid-based data

Changes in summer extreme drought occurrence during 2050s and 2080s (shaded area in Figure 6-14) are shown in Figure 6-16 and Figure 6-17. The histograms for these maps are shown in Figure 6-18. The results are first discussed in general perspective followed by cross-dataset and cross-period perspectives:

1- **General outlook**: Except for a small area in grid 2050s, all models indicate an increase in extreme drought frequency. The statistics for these maps are given in Table 6-2.

<table>
<thead>
<tr>
<th></th>
<th>Modeled 2050s</th>
<th>Grid 2050s</th>
<th>Modeled 2080s</th>
<th>Grid 2080s</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.28%</td>
<td>0.12%</td>
<td>3.45%</td>
<td>2.80%</td>
</tr>
<tr>
<td><strong>Variance</strong></td>
<td>3.1E-05</td>
<td>1.7E-04</td>
<td>4.1E-03</td>
<td>1.4E-02</td>
</tr>
<tr>
<td><strong>Std. Dev.</strong></td>
<td>0.06%</td>
<td>0.13%</td>
<td>0.64%</td>
<td>1.18%</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>0.28%</td>
<td>0.13%</td>
<td>3.36%</td>
<td>2.49%</td>
</tr>
</tbody>
</table>

In a general glance, the distribution of change in summer drought occurrence during 2050s and 2080s is mostly on par with the distribution of summer mean precipitation change in Figure 6-10. This increase in occurrence is correlated with elevation; more extreme droughts are predicted for lower elevations compared to higher elevations. Over 30 years, the transition from 2050s to 2080s will be characterized by increased variability. Using calculated extreme drought probability, values for return period of extreme drought can be updated from their current ~43 years (100/2.3%).
Figure 6-16 – Change in summer extreme drought modeled using station-based data

Figure 6-17 – Change in summer extreme drought modeled using grid data

2- Cross-dataset: The cross-dataset examination reveals the following points:

- A narrower range, standard deviation and variance for modeled for both 2050s and 2080s compared to grid (cf. Table 6-2). This can be attributed to the specifications of the modeling or interpolation processes used for obtaining the dataset. The modeled dataset exhibits a “flattening” effect from co-kriging, varying less at the local scales (more noticeable in rough parts of the terrain).
- The modeled results are more conservative because of the inability of the interpolation methods to insufficiently vary with the terrain. This can be attributed low point density and the mediocre correlation between mean precipitation at stations with elevation.

- The center masses of the two datasets almost agree. For modeled 2050s, a clear peak at 0.3% can be seen. For grid 2080s, a clearer peak compared to modeled 2080s can be observed at 1.75%.

3- Cross-period: The cross-period examination reveals the following points:

- In average, there is more than 10 times increase in extreme drought occurrences in 2080s compared to 2050s, which is a significant value over three decades.

- There is a narrower range and standard deviation in 2050s compared to 2080s. There is a low skew for 2050s for both datasets: -0.10 and 0.05; for 2080s the values are higher and positive: 0.42 and 1.20.

- Both maps for 2080s indicate a high proportion of change to occur approximately within the 2.5% to 3.5% range.

6.9.2 Probability of enclosing results

The results from scoring change in extreme drought (Figure 6-10) with respect to uncertainty (‘α’), modeled by p-box of each cell in Figure 6-16 and Figure 6-17 are shown in Figure 6-19 and Figure 6-20. As expected, maps for 2080s indicate higher probability of enclosing due to higher magnitudes of change in 2080s. Theoretically, since the uncertainty bounds are infinite (due to the definition of standard error), the highest probability of enclosing was achieved below 100% and at grid at 2080s at 36% (α = 0.46σ). While probability of enclosing for all points was limited to the 0 and 36% range, 2080s map indicates higher probability of enclosing.

According to (6-16), the factors in probability of enclosing are $SE_k$, $SE_\theta$ (Figure 6-21) and $\Delta M$ (Figure 6-10), in addition to $k$ and $\theta$. Probability of enclosing is a measure for reliability. Both maps vary in reliability. Grid produced a smaller range in accordance with Figure 6-21 map which was more homogenous and conservative $SE_k \times SE_\theta$. 

85
The lack of continuity in Figure 6-20 can be attributed to artifacts of the grid model in which $SE_k$ and $SE_\theta$ are discontinuous (Figure 6-21). The same patterns can be seen for both periods. However, these patterns becomes more visible in 2080s as the magnitude of change ($\Delta M$) is larger.

Overall, the scoring reveals overwhelming magnitudes of epistemic uncertainty compared to magnitude of change in extreme drought occurrence.

Figure 6-18 – Histograms of percent increase in summer extreme drought for the Okanagan Basin (x-axis labels represent bin mid-points; 2050s are shown in darker color)
Figure 6-19 – Probability of enclosing for modeled data for (a) 2050s and (b) 2080s

Figure 6-20 – Probability of enclosing for grid data for (a) 2050s and (b) 2080s
6.10 Summary

By replacing the deterministic definition of data and processes in a drought hazard analysis model with the enhanced p-box method, this work satisfied several objectives. Using an enhanced interpolation method, variability was propagated in the entire study area and precipitation threshold for each point corresponding to extreme drought ($P < 2.3\%$) were obtained. Using these thresholds and shift in precipitation means the decrease or increase in extreme drought was modeled. The changes in occurrence of extreme drought were calculated by modeling the effects of change in mean on $k$ parameter, and then $\theta$ parameter (Waggoner 1989). Results indicated that except for a small area in the grid dataset, summer extreme drought will rise. This mean increase is between 0.12% and 0.28% for 2050s and 2.80% and 3.45% for 2080s. The range and variability of change in summer extreme drought rose from 2050s to 2080s.

An important concern in modeling the effects of climate change on watershed hydrology is the reliability of results due to different sources of uncertainty. In analyses where data are subject to uncertainty, a common obstacle to processing the uncertainty along with data (uncertainty propagation) is the increase in computational requirements. This includes the $n$-fold increase in computation due to $n$-fold discretization of continuous measures of uncertainty, e.g., lower and upper-bounds of a gamma function. This work introduced a novel method for preserving continuity in uncertainty bounds in hydrologic processes. By using two points along each bounds and solving a system of equations, continuous lower and upper-bounds for uncertainty at interpolated points were obtained. For risk analysis - which often consider extreme tails of bounds for hazard characterization -
this enhancement can more accurately represent those probabilities. By propagating uncertainty in interannual precipitation, this work compared the modeled change in extreme drought with the surrounding uncertainty arisen from incomplete records. The highest score for significance was 36% (Figure 6-19) warranting the highest attention.

Uncertainty analysis benefits modeling in different aspects, including placing the traditional deterministic results within the framework of surrounding uncertainty thus aiding subsequent process and analysis (including decision making) with more informative results.
CHAPTER 7 DROUGHT RISK ASSESSMENT IN THE CONTEXT OF WILDFIRE VULNERABILITY

7.1 Overview

Droughts can cause substantial losses by triggering a complex network of impacts that pervasively affect society and the environment. The potential occurrence of such losses, i.e., the drought risk, depends on both the magnitude of the hazard and the vulnerability on the ground. While hazard was effectively modeled in a stochastic and standalone manner using enhanced SPI in Chapter 6, the assessment of drought impacts has proven more complex. Underlying issues include: difficulty in temporally isolating impacts, the subjective nature of impacts which varies by region, application and economic sectors, and complex cause-and-effect relationship among impacts (e.g., drought effects on the cycle of land erosion and flood frequency) (Wilhite and Glantz 1985; Wilhite 2000; Wilhite et al. 2000, 2007).

As discussed in Section 5.3, vulnerability is the linking term between a predefined drought hazard with its impacts. In addition, since drought hazard is defined in dimensions including severity, geographic extent and duration; vulnerability needs to relate a specific definition of hazard to possible consequences (risk). Risk may itself be ultimately quantified in environmental, social and economic dimensions using relevant measures, e.g., dollar value for social or emergy (Odum 1996) for environmental consequences.

In representing hazard and consequences, drought indices are variably placed within the spectrum of the process that starts from the occurrence hazard and ends with adverse consequences.

Meteorological drought indices such as the widely-adapted SPI is a hazard-only index that can be used for generic (top level) risk assessment. Drought indices may additionally incorporate indicators such as vegetation health and streamflow to reflect risk for agricultural and hydrological applications.

Earlier studies have demonstrated the advantages of SPI for hazard characterization (Guttman 1998, 1999; Hayes et al. 2000; Tsakiris and Vangelis 2004). However, the linking of SPI with drought consequences, i.e., vulnerability analysis has been limited. SPI has so far been tied to few impacts and with limited success (Tsakiris and Vangelis 2005; Tsakiris et al. 2006). This relationship can be improved by incorporating additional variables such as evapotranspiration (Tsakiris et al. 2006) and indices such as the Crop-Specific Drought Index (CSDI) (Wu and Wilhite 2004). Moreover, original SPI is insensitive to shifts in climate normals and modifications have been recommended (Dubrovsky et al. 2009; Vicente-Serrano et al. 2010).
Given the above shortcomings, two objectives have been pursued in this chapter:

First, in order to expand SPI’s connection with drought impacts, it analyzes the vulnerability of forests as one asset susceptible to extreme drought. In this exercise, since wildfire is a 1st level impact, the wildfires can be studied in a standalone manner as they do not result from higher-level impacts. (Wildfires rather cause downstream impacts such as water quality degradation and respiratory ailments.) This work establishes a relationship between historical drought hazard and an associated loss (Total Burned Area) and hence integrates SPI-based summer drought hazard with wildfire vulnerability for a case study of Okanagan Basin (BC, Canada). Models incorporating evapotranspiration and firefighting capacity have also been developed. These models are then validated. Using these models, current risk to extreme drought ($SPI \leq -2$) is characterized.

Second, using the enhanced SPI model introduced in Chapter 6, shifts in climate normals are modeled and future wildfires due to change in drought occurrence behavior are characterized. This goal is realized by using shift in precipitation and evapotranspiration normals, and drought occurrence probability for the CGCM3 A2 scenario. The change in wildfires from climate change induced increase in extreme drought occurrence is finally derived.

### 7.2 SPI for wildfire risk assessment

As a generic drought index, SPI can be adapted for DRA in generic applications (Patel et al. 2007; Shahid and Behrawan 2008; Tsakiris et al. 2006) or specific applications, e.g., in the study of agricultural drought impacts (Patel et al. 2007; Tsakiris et al. 2006; Wu and Wilhite 2004). For the latter, additional indicators such as evapotranspiration have been used with precipitation in order to improve SPI correlation with drought impacts (Tsakiris and Vangelis 2005; Tsakiris et al. 2006; Wu and Wilhite 2004). These studies confirm evapotranspiration as a primary auxiliary variable with precipitation in agricultural drought risk assessment. Similarly, for studying drought impacts on forests, additional factors such as climatic indicators and indicators of ground conditions are used to determine wildfire risk. These factors are explained in the following section.

### 7.3 Weather and wildfire ground vulnerability factors and mitigation

According to Agee (1993), wildfire behavior is dependent on a triangle comprised of weather and two ground vulnerability factors: fuel and topography. Among these factors Cary et al. (2006) place greater importance on climate and weather rather than topography and fuel. Mitigation through suppression
has played a prominent role since its introduction in the beginning of the 20\textsuperscript{th} century. These factors influencing wildfire behavior are described in following:

7.3.1 Weather

Moisture, temperature and wind are three important hazard factors influencing wildfires behavior (Agee 1993; WMB 2012a): Moisture in the form of water vapor and precipitation can slow wildfire or extinguish it altogether. Wet fuel decreases the fire spread rate by preventing ignition and slowing propagation. Strong winds increase fire spread in the direction of the wind. Winds additionally increase oxygen supply, flatten flames which pre-heat adjacent areas ahead and blow sparks to areas ahead causing spot fires. Higher temperatures pre-heat fuel making them burn more rapidly and also effect the ground movement of air currents.

7.3.2 Ground factors

Ground factors including fuel and topography determine the susceptibility of forests to wildfire. Fuel spacing and quantity in addition to type: large and slow burning compared to light and fast burning determine fire behavior.

Topographical factors such as slope, aspect and terrain features can also determine wildfire behavior. High slopes can aid fire spread rate by preheating adjacent fuel. Aspect determines if sunlight is received in that direction. Sunlight results in higher temperature and drier fuel. Terrain features such as mountains can decrease wind flow, while valleys can pass winds at high speeds increasing the rate of spread.

7.3.3 Mitigation through suppression

Wildfire fighting plays an important role in the determination of the total area burned (Cumming 2005). In the Canadian context, fire suppression was successfully applied during the first 75 years of the 20\textsuperscript{th} century. However, in the mid-1970s, there was realization that total fire exclusion is economically unfeasible, ecologically undesirable, and physically impossible (CFS 2010). Based on province or territory priorities, fire suppression is mainly applied for areas where certain assets are at risk (e.g., wildland-urban interfaces, high-value industrial forests and recreational sites), while fires in remote areas may be left to burn as part of the natural cycle that maintains the health and diversity of the forest (CFS 2012). Currently, around 90\% of wildfires are fought in Canada (CFS 2010).
7.4 Drought risk assessment

7.4.1 Drought induced vulnerability analysis of wildfires

The first step in drought vulnerability analysis framework (cf. Section 2.2) (Hayes et al. 2004) starts with impact assessment. A relevant starting checklist for any impact assessment is the 91 impacts provided in a rather comprehensive fashion by NDMC (2012). This work focuses on a single impact, wildfire that can occur in forested regions. NDMC (2012) categorized wildfire by the following hierarchy: All impacts > Environmental > Damage to plant communities > Increased number and severity of fires.

In the next step, the increased number and severity of fires (termed hereon wildfire) needs to be studied within the causal network of other drought impacts.

The causal assessment of drought impacts nevertheless remains a complex task (Iglesias et al. 2009; Wilhite et al. 2007). Using literature (Wilhite and Glantz 1985; Wilhite 2000; Wilhite et al. 2000, 2007) and NDMC (2012) the following factors contribute to this complexity:

Interdependencies: Some impacts do not follow a straightforward cause-and-effect mechanism and the network is complicated by direct or indirect feedback mechanisms among impacts. For example, “wind and water erosion of soils, reduced soil quality” and “impaired productivity of forest land” can have mutual effects. Another example includes “reduced productivity of cropland (wind erosion, long-term loss of organic matter, etc.)” and “damage to crop quality”

Spatial and temporal variations: Impacts can vary widely in spatial and temporal dimensions. The “secondary” drought impacts (also “indirect” and “induced” impacts) concern effects after the initial pulse (Kulshreshtha and Klein 1989; Wilhite and Glantz 1985). Such macro-level impacts (e.g., reduction in national income) can occur to a large region and can be captured using an input-output model for the region (Kulshreshtha and Klein 1989). Similarly, in the temporal dimension, some impacts are almost immediately realized while some take years to be sensed.

Sequence of impacts: Sometimes a general sequence of impacts can be identified from immediate impacts (level 1, e.g., “insect infestation”) to some intermediate impacts, level i, e.g., “annual and perennial crop losses” to some monitored final impact, level n, e.g., “income loss for farmers due to reduced crop yields”. Drought impacts can nevertheless occur out of sequence due to specific ground conditions. Noting that “insect infestation” and “annual and perennial crop losses” can have a feedback mechanism, level i can be affected by level i + m (any subsequent level).
**Subjectivity of links:** The strength or even the existence of a link between two impacts (nodes) varies between cases. Regarding the previous example, during crop losses, insects may or may not be present to infest the damaged crop.

As evident, drought impacts are sometimes interconnected, occur in a wide temporal and/or spatial window, and some have subjective causal relationships. Wildfire, a level 1 impact, is on the other hand to a high degree independent of such factors. Although, ground conditions such as insect infestation and accumulated fuel can exacerbate wildfires, nevertheless, from the initiation of drought to the start of wildfire, other drought impacts have little effect on wildfire behavior. Wildfires themselves are however parent impacts for dependent impacts such as increase in respiratory ailments, erosion, water quality degradation due to erosion and firefighting agents, and loss of recreational land.

Following causal assessment, the study of temporal trends concerns the dynamics in wildfire occurrence frequency. Being a level 1 impact, wildfires vulnerability will change as a function of future drought behavior (e.g., change in climate normals) in addition to ground forest conditions (e.g., increase in insect infestations). For the province of British Columbia, Canada, recently the largest pine beetle outbreak in North America has been recorded. Changes in (a) species and ecosystem distribution, due to global warming and (b) the range and impact of forest pests and diseases is predicted (MoFLNRO 2012).

### 7.4.2 3-month SPI as generic drought hazard measure

SPI is a precipitation-only drought index (detailed SPI calculation procedure is found in Chapter 6). The correlation of precipitation with wildfires, especially during the days leading up to wildfires is well studied and currently implemented in wildfire rating systems (e.g., CWFIS 2012). The correlation of precipitation in a standardized format and for prolonged periods (three months and above) remains to be studied. The switch to standardized precipitation has already been proven successful in agricultural applications (Wu et al. 2004).

Although SPI can theoretically be derived for any time window (including as short as daily or weekly), it will only be useful if the time window reflects a meaningful phenomenon from a DRA perspective. For agricultural regions, 3-month SPI reflects short- and medium-term soil moisture conditions (NDMC 2012b). The estimation of summer seasonal precipitation departures is especially important due to water use conflicts, low-flows in snow driven watersheds and high evapotranspiration potentials.
SPI has nevertheless shortcomings that need to be considered. These include: insensitivity to climate cycles such as the Pacific Decadal Oscillation (PDO) and El Niño/La Niña-Southern Oscillation (ENSO), insensitivity to long-term shifts in climate normals (climate non-stationarity) (Dubrovsky et al. 2009; Vicente-Serrano et al. 2010; Zargar et al. 2012c) and weak correlation with vegetation impacts (Tsakiris and Vangelis 2005; Tsakiris et al. 2006).

The first shortcoming can be overcome to some extent by using a precipitation record longer than the cycle’s frequency. In this manner, the cycle’s abnormalities will be reflected in the calculation of SPI (nevertheless, in an aggregated manner with other years). For climate non-stationarity, a method that take into account projected shift in precipitation mean was developed in Chapter 6. The last shortcoming can be tackled by considering auxiliary variables such as temperature, evaporation and evapotranspiration.

7.5 Case study: Wildfire in the Okanagan Basin

7.5.1 Summer drought and wildfires

Drought is a recurrent event in the Okanagan Basin. Summer drought has gained importance due to current and predicted end-of-summer low-flows. Among the drought impacts listed in NDMC (2012), wildfire is an important, level 1 impact of drought in the Okanagan Basin, BC. The most recent of large wildfires occurred in 2003, starting in Aug. 16 and continuing onto September. Over 28,000 ha of forest and park were lost, more than 27,000 people were evacuated and 239 homes were destroyed (City of Kelowna 2009).

Historically, between 1919 to 2009, wildfires have occurred with following monthly distribution: 0.06% in March, 0.36% in April, 0.41% in May, 1.23% in June, 47.07% in July, 46.39% in August, 3.55% in September and 0.94% in October (Wildfire Management Branch 2009, see Section 7.5.2).

In response to growing concerns about wildfire, the British Columbia Forest Service was created in 1912. Detection capability was enhanced with the subsequent establishment of lookout towers and communication systems. Firefighting capability has further been enhanced since WWII with the widespread use of airplanes (Gayton 2012).

7.5.2 Variables and data

The variables considered and their data sources are described in following:
**Total burned area (TBA):** TBA (ha) is the total area of wild land burned during the summer months (June, July and August: JJA). The data were obtained from DataBC (WMB 2009). The data start at 1919 and end at 2011. Older data have been acquired mostly through digitization and more recent data are manually acquired using GPS. TBA is often represented by a log transformation. The perimeters for fires in the Okanagan Basin during JJA months from 1919 to 2009 are shown in Figure 7-1.

**Precipitation (SPI):** Precipitation (mm) data were obtained from the ClimateWNA tool (Wang et al. 2011) which downscales PRISM (Daly et al. 2002) to calculate monthly (2.5 x 2.5 arcmin for the reference period: 1961-1990), seasonal and annual climate variables for western North America. Using this precipitation dataset, SPI values were calculated for the 1919-2009 period (version 4.62 of ClimateWNA models for years up to 2009). The yearly variation of TBA with SPI is shown in Figure 7-2.

**Reference evaporation (E_{ref}):** The ClimateWNA tool provides reference evaporation data for 1901-2009 and future periods (2020s, 2050s and 2080s). The method is based on Hargreaves evapotranspiration equation. The yearly variations of summer E_{ref} with TBA are shown in Figure 7-3.

**Fire suppression capacity (FSC):** Information about fire suppression capacity (%) was obtained through communication with Gayton (2012), who suggests that the era of modern fire suppression activity in the Okanagan Basin began in the 1930’s, was interrupted by WWII, and came into full force in the years after the War. Currently the Kamloops Fire Centre is responsible for wildfire fighting in the Okanagan Basin and provides information such as number of crews and personnel (WMB 2012b). However in the event of large fires, capacity from adjacent regions is also incorporated and subsequently these figures do not reflect the complete wildfire fighting capacity. Moreover, since records for the historic capacity (1947-2009) were not available, in this work yearly FSC values were defined as a nominal variable that is a function of the 2009 capacity (set to 100%). The starting capacity (1947) was arbitrarily assumed 50% linearly increasing to 100% in 2009 (see Figure 7-4.3). For completeness and verification purposes, two other models, a constant capacity and a linearly increasingly capacity from zero were also considered Figure 7-4.1 and Figure 7-4.2.

In using FSC as a single mitigating variable on overall risk, this work acknowledges simplifications involved in the variable’s parameterization including a linear 50% to 100% increase in FSC from 1947 to 2009. This work hence cautions against trivializing the rather large complexity inherent to wildfires suppression. FSC by itself is a surrogate parameter for many factors and does not consider technological changes since 1947 such as communication, GIS tools and decision-support systems – which can have significant and non-linear effects on mitigation capacity.
Figure 7-1 - Location of wildfires in the Okanagan Basin during the summer months 1919-2009

Figure 7-2 – Temporal variations 1919-2009 of summer $\log(TBA)$ and SPI for the Okanagan Basin, 1919-2009 (dashed line represents $\log(TBA)$)
7.5.3 Model and validation

Since log-scale was used and SPI is already a standardized variable, simple (Model 1) and multiple linear regression models (2 and 3) were used. The models are shown in Table 7-1.

**Sampling:** The linear model was developed by splitting the 91 years of data to two subsets (45-46). This random sampling was repeated 5 times using MS Excel software. One subset was used for deriving the...
model and the other for validating. The formulae and $R^2$ for these five samples are given in Table 7-1. The sampling confirms the initial sample $R^2$. The scatterplot for models comprising of SPI as only independent variable are shown in Figure 7-5.

<table>
<thead>
<tr>
<th>Model</th>
<th>Sample set</th>
<th>Formula</th>
<th>Model $R^2$</th>
<th>Validation $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>$\log(TBA) = -0.83 \times SPI + 2.207$</td>
<td>0.340</td>
<td>0.374</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$\log(TBA) = -0.788 \times SPI + 2.077$</td>
<td>0.342</td>
<td>0.369</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$\log(TBA) = -0.961 \times SPI + 2.278$</td>
<td>0.496</td>
<td>0.230</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$\log(TBA) = -0.868 \times SPI + 2.020$</td>
<td>0.336</td>
<td>0.396</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$\log(TBA) = -0.793 \times SPI + 2.242$</td>
<td>0.332</td>
<td>0.372</td>
</tr>
<tr>
<td>II</td>
<td>1</td>
<td>$\log(TBA) = +0.008 \times E_{ref} - 0.714 \times SPI - 2.626$</td>
<td>0.363</td>
<td>0.423</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$\log(TBA) = +0.005 \times E_{ref} - 0.717 \times SPI - 0.919$</td>
<td>0.353</td>
<td>0.445</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$\log(TBA) = +0.017 \times E_{ref} - 0.744 \times SPI - 8.068$</td>
<td>0.580</td>
<td>0.246</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$\log(TBA) = +0.011 \times E_{ref} - 0.784 \times SPI - 5.035$</td>
<td>0.382</td>
<td>0.442</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$\log(TBA) = +0.007 \times E_{ref} - 0.645 \times SPI - 1.874$</td>
<td>0.349</td>
<td>0.454</td>
</tr>
<tr>
<td>III</td>
<td>1</td>
<td>$\log(TBA) = -1.736 \times FSC + 0.001 \times E_{ref} - 0.673 \times SPI + 2.498$</td>
<td>0.582</td>
<td>0.551</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$\log(TBA) = -2.237 \times FSC - 0.005 \times E_{ref} - 0.622 \times SPI + 6.506$</td>
<td>0.705</td>
<td>0.517</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$\log(TBA) = -1.392 \times FSC + 0.010 \times E_{ref} - 0.729 \times SPI - 3.385$</td>
<td>0.679</td>
<td>0.486</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$\log(TBA) = -2.184 \times FSC + 0.000 \times E_{ref} - 0.664 \times SPI + 3.353$</td>
<td>0.663</td>
<td>0.540</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>$\log(TBA) = -1.402 \times FSC + 0.004 \times E_{ref} - 0.552 \times SPI + 0.735$</td>
<td>0.486</td>
<td>0.647</td>
</tr>
</tbody>
</table>

Figure 7-5 – Scatterplots for SPI - log(TBA) regression

Variable wildfire fighting capacity: The multivariate regression models were repeated additionally incorporating variable FSC and are shown in Table 7-2. As predicted, FSC-3 (linear line increases with initial firefighting capacity) performed the best.
### Table 7-2 – Dependent variable is TBA

<table>
<thead>
<tr>
<th>Independent</th>
<th>Model</th>
<th>Model $R^2$</th>
<th>Validation $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPI, $E_{ref}$, FSC-1</td>
<td>$TBA = -1.736 \times FSC + 0.004 \times E_{ref} - 0.648 \times SPI + 0.211$</td>
<td>0.530</td>
<td>0.513</td>
</tr>
<tr>
<td>SPI, $E_{ref}$, FSC-2</td>
<td>$TBA = -1.325 \times FSC + 0.000 \times E_{ref} - 0.702 \times SPI + 3.035$</td>
<td>0.570</td>
<td>0.537</td>
</tr>
<tr>
<td>SPI, $E_{ref}$, FSC-3</td>
<td>$TBA = -1.736 \times FSC + 0.001 \times E_{ref} - 0.673 \times SPI + 2.498$</td>
<td>0.582</td>
<td>0.551</td>
</tr>
</tbody>
</table>

### 7.6 Climate change impacts

In order to model the effects of climate change on precipitation distribution, the method by Waggoner (1989) was adopted. The involved steps are described in following:

1) Using either grid or station points for the basin, establish an relationship between $log(Var)$ and $log(Mean)$. The scatter for $log(Var)$ vs. $log(Mean)$ is shown in Figure 7-6.

![Figure 7-6 – The variation between logs of summer precipitation variance and mean for the Okanagan Basin based on grid data](image)

\[
y = 1.803x - 0.566 \\
R^2 = 0.9517
\]

\[
log(V) = 1.803 \times log(M) - 0.566 \quad (7-1)
\]

\[
V = 0.272M^{1.803} \quad (7-2)
\]

2) Using the mean and variance relationships in gamma distribution (7-3) and (6-8), calculate $k$ and $\Delta k$ can be as a function of $M$ ((7-5), (7-6) and (7-7)).
Using the new $k$, the altered distributions were obtained (Table 7-3). These distributions are shown along with the original distribution in Figure 7-7. The change in burned area resulting from climate change induced increase in extreme drought occurrence can be modeled (Table 7-4) using the parameters of the altered distributions (Table 7-3). The baseline for $E_{ref}$ (1961-1990 average) in Table 7-4 was 610.2 and FSC was set to 100%, or the 2009 capacity. The same calculations were also applied with a variable FSC from the trapezoidal model (Table 7-5). Using the trapezoidal model, if the growth in firefighting capacity is continued, in 2050s, FSC will increase to 137% and 161% by 2080s (Table 7-5).

Table 7-3 – Interannual summer precipitation distribution parameters for the Okanagan Basin

<table>
<thead>
<tr>
<th></th>
<th>$P$ (mm)</th>
<th>$k$</th>
<th>$\theta$</th>
<th>$V$</th>
<th>$P$ at $\Gamma_{CDF} = 2.3%$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1961-1990s</td>
<td>46.49</td>
<td>8.82</td>
<td>5.27</td>
<td>245.04</td>
<td>20.70</td>
</tr>
<tr>
<td>2050s</td>
<td>45.27</td>
<td>7.80</td>
<td>5.80</td>
<td>262.68</td>
<td>18.94</td>
</tr>
<tr>
<td>2080s</td>
<td>41.13</td>
<td>7.66</td>
<td>5.37</td>
<td>221.02</td>
<td>17.04</td>
</tr>
</tbody>
</table>
Figure 7-7 – The left tail of the gamma distribution for summer precipitation for three periods: 1961-1990, 2050s and 2080s

Table 7-4 - Climate change induced increase in wildfire due to extreme drought (M=Model) (FSC=100%, i.e., similar to 2009)

<table>
<thead>
<tr>
<th>Corresponding SPI for $P$ at $F_{CDF} = 2.3%$ on 1961-1990</th>
<th>$\Delta E_{ref}$</th>
<th>Corresponding burned area (ha)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>M-1</td>
</tr>
<tr>
<td>1961-1990</td>
<td>-2.00</td>
<td>0</td>
</tr>
<tr>
<td>2050s</td>
<td>-2.20</td>
<td>67.41</td>
</tr>
<tr>
<td>2080s</td>
<td>-2.44</td>
<td>117.57</td>
</tr>
</tbody>
</table>

Table 7-5 - Percentage increase in wildfire impact area, i.e., risk, due to extreme drought

<table>
<thead>
<tr>
<th>Period</th>
<th>% increase in TBA, FSC=100%</th>
<th>% decrease in TBA, increasing FSC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1947-2009</td>
<td>M-1 47%</td>
<td>M-3 -65%</td>
</tr>
<tr>
<td></td>
<td>M-2 363%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M-3 54%</td>
<td></td>
</tr>
<tr>
<td>2050s</td>
<td>M-1 130%</td>
<td>M-3 -79%</td>
</tr>
<tr>
<td></td>
<td>M-2 1563%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M-3 143%</td>
<td></td>
</tr>
</tbody>
</table>

* M-1 to M-3 represent the three assumed models for change in wildfire fighting capacity

7.7 Discussion and summary

Using a linear regression model, a relationship between the summer SPI and wildfire - a top level impact from drought - was established. The model was strengthened by using two additional variables:
evaporation and firefighting capacity with $R^2 = 0.58$. Since information about firefighting capacity was scant, a unitless variable was used (percent of 2009 capacity) and it was assumed this capacity has increased from 50% to 100% in 2009. This model was used to calculate the risk of wildfire under current summer extreme drought conditions ($SPI \leq -2$). The risk under current conditions with average 1919-2009 evaporation rate was estimated at 386.4ha of total burned wilderness/forest area (TBA) for the entire summer.

Climate change is predicted to alter the distribution of precipitation and hence drought; consequently the effects of climate change during 2050s and 2080s on precipitation and drought occurrence were also modeled. Using altered precipitation and reference evaporation based on the CGCM3 A2 climate change scenario, if current (2009) baseline wildfire fighting capacity is kept, for 2050s, there will 54% increase in wildfires and for 2080s, 143%. If wildfire fighting capacity increases by the assumed 1947 to 2009 rate, TBA will drop by 65% and 79% for 2050s and 2080s respectively.

This chapter presented the second part of DRA using enhanced SPI. As a measure of hazard, SPI previously garnered widespread acclaim; however, this work confirmed its suitability for DRA as a whole. Through the successful assessment of vulnerability of wilderness/forest to current and future drought hazard, this work demonstrated the capacity of SPI to correlate with wildfire, a top-level drought impact. This correlation was strengthened by incorporating reference evaporation and further reinforced using firefighting capacity.

As discussed earlier, drought impacts constitute a network of impacts that are interconnected and pervasive to various human activities and the environment. Considering that wildfire is a parent node to several subsequent impacts such as soil erosion or respiratory ailments, by developing vulnerability and risk estimates, loss from wildfires can be used as input to subsequent impacts. Vulnerability and risk assessment may accordingly be performed. Together this sequence of impacts will uncover a fuller picture of vulnerability and risk in part of the network of impacts.
8.1 Summary and conclusions

The individual achievements of chapters 4 through 7 have been summarized at the end of each chapter. This chapter presents general concluding remarks, and further remarks on the cumulative contribution of preceding chapters.

Overall, this work successfully enhanced the data representation for two hydrologic features (Snow Water Equivalent (SWE) and interannual precipitation) from a deterministic to an uncertainty-driven data representation, enabling more informative decision making under uncertainty. Using the enhanced models, highly conflicting SWE datasets for Red Deer Basin, AB, were treated and an uncertainty-driven SWE was produced allowing the incorporation of uncertainty in subsequent analyses and decision making. For the Okanagan Basin (BC), drought hazard and the risk of wildfires for current conditions and a future climate scenario (CGCM3 A2) were successfully estimated. Hazard conditions were found high, especially for 2070-2099. Meanwhile, through the introduction of the “probability of enclosing” measure, medium to high uncertainty was detected (probability of enclosing ≤ 36%) in some areas of the hazard map. This confirms the need for such advanced methods to comprehensively handle uncertainty especially for hydrologic impacts and implications of climate change.

More specifically, the following conclusions can be drawn from the objectives described earlier in Chapter 1:

1. The Dempster-Shafer theory (DST) (Dempster 1967; Shafer 1976) was confirmed as a rigorous framework for handling epistemic uncertainties in the form of incompleteness and conflict in hydrologic analysis. In the first step, using DST, the deterministic definition of data was replaced with an uncertainty-driven definition, one that is capable of incorporating and dealing with uncertainty along with original data. Subsequently, uncertainty-driven drought hazard analysis was performed. This outcome was achieved in two parts:
   a. Using data fusion, a high degree of conflict was successfully handled in a drought hazard indicator (SWE). Four data fusion rules were implemented and the mixture rule was identified as most appropriate. The outcome was a unified uncertainty-driven, but more reliable data.
   b. DST was used to handle incompleteness in precipitation records used in SPI, a prominent drought hazard index. By treating incompleteness using DST, heterogeneity in station-wide
data was resolved. The results from these models enhanced the informativeness of the traditional deterministic approach, for example, by drawing attention to other possible drought scenarios.

In addition, this work introduced methods for propagation of uncertainty. For conflict, a methodology to propagate uncertainty in snowmelt run-off models was demonstrated. For the case of incompleteness, uncertainty was successfully and efficiently propagated in the spatial interpolation of SPI using a novel method.

2. By enhancing SPI, DRA can now model variability along with epistemic uncertainty - and hence model shifts in climate normals. Using the CGCM3 A2 scenario, the hazard of extreme drought for summer in the Okanagan Basin, is predicted to increase between 0.12% and 0.28% for 2040-2069 and 2.8% and 3.45% for 2070-2099. The model helps to rank the estimated SPI within the range of surrounding uncertainty. The “probability of enclosing” helps in identifying the reliability of climate change by comparing the magnitude of shift in precipitation normals against surrounding uncertainty. These enhanced measures will help decision makers to make more informed decisions by providing knowledge of where data are more reliable.

3. In the final phase, using the enhanced SPI model and an assumed increase in firefighting capacity from 1947 to 2009, wildfire risk was characterized. DRA of wildfires revealed a high increase in the risk if evaporation was also considered and firefighting capacity remained constant (as in 2009) for the Okanagan Basin forests. The deficiency in firefighting capacity will be met if the firefighting capacity is enhanced with the presumed rate that started in 1947 and continued until 2009.

8.2 Contributions

The contributions of this work included a fundamental and conceptual enhancement to hydrologic analyses where the deterministic vision of hydrologic features was expanded to a broader uncertainty-driven vision. This approach advocates replacing the fundamental deterministic mapping of a single value to an attribute (i.e., deterministic data) with a set of values and further integrating this enhanced framework with hydrologic modeling. As this work demonstrated, this can be achieved through providing mechanisms for integrating uncertainty modeling and propagation with regular processes in hydrologic analyses. As a result of this integration, the accuracy and informativeness of data resulting from hydrologic analyses and DRA were improved. This work also introduced a novel and much needed method for more efficiently integrating data uncertainty modeling along with regular modeling. This included using minimal calculations in establishing and solving a system of equations that represent the continuous uncertainty bounds that demark uncertainty around the “true” value for a feature. This
enhancement was essential in preserving continuous bounds and avoiding potential large non-specificity. Throughout the application of the framework to the Okanagan Basin, enhanced and more reliable data were provided for the first time to the end user community including decision makers. Safer and more informed decision making was hence made possible.

8.3 Limitations

The following are the main limitations identified in this work:

1. This work addressed limited sources of uncertainty. Additional possible sources of uncertainty include accuracy of precipitation values, positional accuracy of stations and variability due to cycles such as the pacific decadal oscillations (PDO).

2. Not all types of epistemic uncertainty were covered in this work (e.g., vagueness and ambiguity). In addition, (in-)completeness was only studied in a single parameter (i.e., precipitation) and similarly conflict was only considered in SWE. The application of DST to handle additional types of epistemic uncertainty for additional parameters needs to be further investigated.

3. A single, although, extreme climate scenario was used. The complete picture of future scenarios that can range between stationarity (the continuation of current conditions) and more extreme climate change is missing.

4. Drought was considered only as a meteorological phenomenon with a single drought impact. The hazard from hydrologic and agricultural droughts remains to be explored. The application of SPI to subsequent lower-level impacts can also be a topic of future research.

8.4 Recommendations

Improvements to this work can be recommended in following directions:

1. Including additional uncertainties in the enhanced SPI model, for example, positional accuracy of stations and the effects of climatic cycles that can result in improved estimates and more informative results.

2. Evaluating the performance of additional mathematical theories of uncertainty such as fuzzy sets theory (Zadeh 1965) and rough sets theory (Pawlak 1982) when applied to a similar problem.

3. Repeating the analysis using additional climate change scenarios, e.g., CGCM3 A1B and B1 that can account for other possible future scenarios.
4. Expanding DRA to incorporate additional impacts using triple bottom line sustainability dimensions, i.e., social, environmental and economic impacts.

5. Performing a comprehensive analysis of all the factors that affect wildfire behavior including the effect of multiyear drought and insect infestation. Also, future work can consider additional scenarios for change in firefighting capacity.

6. Developing an integrated decision support tool based on drought risk assessment components in order to aid in the management of current and future drought. This can be accompanied by superimposing water demand-and-supply maps with those developed in this work. This analysis can help in identifying areas hardest hit by climate-change-induced drought and help decision making by identifying, e.g., supply deficiency.
REFERENCES


American Research Press, Rehoboth, NM.

Smarandache, F., and Dezert, J. (2009). *Advances and applications of DSmT for information fusion III.*
American Research Press, Rehoboth, NM.


## APPENDICES

### Appendix A: Recent Drought Indices

Drought indices by category and date introduced after year 2000 (MHA: combination; Agre: Aggregate)

<table>
<thead>
<tr>
<th>Drought index and reference(s)</th>
<th>Type</th>
<th>Motive/Requirements</th>
<th>Novelty and notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Regional Streamflow Deficiency Index (RSDI)</strong>&lt;br&gt;Stahl (2001)</td>
<td>H</td>
<td>Characterizing drought within each homogeneous region.</td>
<td>Uses flow duration curves and flows that exceed 90% of the time (Q90). Using the time series of streamflow, a deficiency index is computed which is used to identify homogenous regions using cluster analysis. RSDI is computed for each homogeneous region.</td>
</tr>
<tr>
<td><strong>Aggregate Drought Index (ADI)</strong>&lt;br&gt;Keyantash and Dracup (2004)</td>
<td>MHA</td>
<td>PDSI limitations including geographic biases, not sufficiently considering snowfall processes, and complex, empirical formulations based on the climate of US Midwestern states. SWSI does not consider evaporation and soil moisture.</td>
<td>ADI is a multivariate, aggregate index that inputs six hydrologic variables of precipitation, streamflow, reservoir storage, evapotranspiration, soil moisture and snow water content. Uses five to six variables. The first principle component (PC1) is normalized by its standard deviation.</td>
</tr>
</tbody>
</table>
| **Soil Moisture Deficit Index (SMDI)** and **Evapotranspiration Deficit Index (ETDI)**<br>Narasimhan and Srinivasan (2005) | A | By considering the spatial variability of hydrological parameters of soil type and land cover as well as meteorological parameters, it is possible to improve previous indices such as SPI, PDSI, CMI and SWSI; the hydrologic system is better modeled and soil moisture deficit monitoring is possible at a finer resolution. | SMDI and ETDI use a high-resolution comprehensive hydrologic model that incorporates a crop growth model. Weekly values are calculated for different soil layers and depths. The difference is that SMDI considers soil moisture in its calculations while ETDI considers the water stress ratio: \[
\frac{PET - AET}{PET}
\]. Indices increase spatial (16km²) and temporal (weekly) resolution. Weekly values reflect short-term dry conditions which is very helpful during plant growth phases. |
<p>| <strong>Reconnaissance Drought Index (RDI)</strong>&lt;br&gt;Tsakiiris and Vangelis (2005) | M | Precipitation alone is inadequate and less realistic estimate of moisture deficit; the severity of drought is underestimated without PET. In addition, it is more difficult to correlate the damages from drought when PET is omitted from the equation. Achieve a balance between two major meteorological parameters of | RDI is more comprehensive than SPI. Advantages include: - being physically based; RDI calculates the aggregated deficit between the evaporative demand of the atmosphere and precipitation. - being flexible for different periods of time. - better association with hydrological... |</p>
<table>
<thead>
<tr>
<th>Index Name</th>
<th>Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standardized Precipitation Evapotranspiration Index (SPEI)</strong></td>
<td>In an illustrative experiment, SPI could not identify the pattern of increase in the duration and magnitude of droughts resultant from higher temperatures. SPEI was required to overcome the shortcomings of SPI in addressing the consequences of climate change on drought behaviour.</td>
<td>Vicente-Serrano <em>et al.</em> (2010)</td>
</tr>
<tr>
<td><strong>Modified Perpendicular Drought Index (MPDI)</strong></td>
<td>The earlier developed PDI (Ghulam <em>et al.</em>, 2007b) was found to lack accuracy on surfaces that are variable between bare soils and densely vegetated agricultural fields. For bare soils, both indices performed equally.</td>
<td>Ghulam <em>et al.</em> (2007a)</td>
</tr>
<tr>
<td><strong>Normalized Multi-Band Drought Index (NMDI)</strong></td>
<td>Enhancing the sensitivity of Normalized Difference Water Index (NDWI) and Normalized Difference Infrared Index (NDII) to drought severity.</td>
<td>Wang and Qu (2007)</td>
</tr>
<tr>
<td><strong>Vegetation Drought Response Index (VegDRI)</strong></td>
<td>To characterize specific droughts; combines indices: NDVI, SPI and PDSI.</td>
<td>Brown <em>et al.</em> (2008)</td>
</tr>
<tr>
<td><strong>Hybrid Drought Index (HDI)</strong></td>
<td>Combined the SPI, SWSI and PDSI.</td>
<td>Karamouz <em>et al.</em> (2009)</td>
</tr>
</tbody>
</table>
### Appendix B: Additional Drought Indices


<table>
<thead>
<tr>
<th>Index/Indicator</th>
<th>Type</th>
<th>Source</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Soil Moisture (RSM)</td>
<td>A</td>
<td>E.g., Thornthwaite and Mather (1955)</td>
<td>RSM is calculated the water balance from various methods. Takes climate, soil and crop variables including potential ET and precipitation; soil physical properties; and crop characteristics and crop management practices (Sivakumar et al. 2011). Reported in percentage.</td>
</tr>
<tr>
<td>Agricultural Drought Index (DTx)</td>
<td>A</td>
<td>Matera et al. (2007)</td>
<td>Uses a water balance model and crop transpiration to calculate an integrated transpiration deficit over a period of time.</td>
</tr>
<tr>
<td>Rainfall Anomaly Index (RAI)</td>
<td>M</td>
<td>Van-Rooy (1965)</td>
<td>Uses the average precipitation over weekly, monthly, or annual time periods to characterize relative drought. Relative drought is then ranked with respect to the 10 most severe droughts in the long-term record, based on which the drought is then assigned a magnitude (Wanders et al. 2010).</td>
</tr>
<tr>
<td>Bhalme and Mooly Drought Index (BMDI)</td>
<td>M</td>
<td>Bhalme and Mooley (1980)</td>
<td>Considers the percent departure of monthly or annual precipitation from its long-term means (Byun and Wilhite 1999).</td>
</tr>
<tr>
<td>Drought Severity Index (DSI)</td>
<td>M</td>
<td>Bryant et al. (1992)</td>
<td>Uses the accumulated monthly deficit of precipitation in preceding months in a window of time, e.g., 3- or 6-month to characterize drought.</td>
</tr>
<tr>
<td>Drought Frequency Index (DFI)</td>
<td>M</td>
<td>González and Valdés (2006)</td>
<td>Uses the mean frequency of recurrence as the scale for evaluating drought significance.</td>
</tr>
<tr>
<td>Weighted PDSI</td>
<td>H,M</td>
<td>Palmer (1965)</td>
<td>Uses PDSI of the current and the preceding week; efficient indicator of surface runoff drought (Vasiliades et al. 2011).</td>
</tr>
<tr>
<td>Groundwater Resource Index (GRI)</td>
<td>H</td>
<td>Mendicino et al. (2008)</td>
<td>Uses a simple distributed water balance model. Considers geo-lithological conditions that affect the summer hydrologic response to winter precipitation.</td>
</tr>
<tr>
<td>Water Balance Derived Drought Index</td>
<td>H</td>
<td>Vasiliades et al. (2011)</td>
<td>Uses the UTHBAL water balance model (Loukas et al. 2007) to simulate runoff. The index is then derived by normalizing and standardizing the synthetic runoff to the mean runoff.</td>
</tr>
<tr>
<td>Sperling Drought Index (SDI)</td>
<td>H</td>
<td>Droughtscore.com (2007)</td>
<td>Easily understandable measure (dry&lt;100&lt;wet); uses long-term precipitation patterns, groundwater- and reservoir levels, and the Palmer drought indices for drought characterization in longer time windows.</td>
</tr>
<tr>
<td>Vegetation Outlook (VegOut)</td>
<td>Agre Tadesse and Wardlow (2007)</td>
<td>Combines climate information and RS observations of current vegetation conditions with oceanic index data and environmental biophysical information such as land cover type, irrigation status, soils, and ecological setting to provide a future outlook of general vegetation conditions.</td>
<td></td>
</tr>
<tr>
<td>-----------------------------</td>
<td>-----------------------------------</td>
<td>-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Ratio Vegetation Index (RVI)</td>
<td>RS Pearson and Miller (1972)</td>
<td>$RVI = \frac{NIR}{R}$</td>
<td></td>
</tr>
</tbody>
</table>
| Weighted Difference Vegetation Index (WDVI) | RS Clevers (1988); Richardson and Wiegand (1977) | $WDVI = NIR - \gamma R$  
$\gamma$ = the slope of the soil line (Qi et al. 1994). |
| Perpendicular Vegetation Index (PVI) | RS Richardson and Wiegand (1977) | $PVI = \sin(a)NIR - \cos(a)red$  
$a$ = the angle between the soil line and the NIR (Ray 1994). |
| Difference Vegetation Index (DVI); Vegetation Index (VI) | RS Lillesand and Kiefer (1987); Richardson and Everitt (1992) | $DVI = NIR - red$  
(Ray 1994). |
| Leaf Water Content Index (LWCI) | RS Hunt et al. (1987) | Uses remotely-sensed leaf water content to classify plant health. |
| Soil Adjusted Vegetation Index (SAVI) | RS Huete (1988) | $SAVI = \frac{NIR - R}{NIR + R + L} (1 + L)$  
$L$ = soil adjustment factor to account for first-order soil background variations (Qi et al. 1994) |
| Transformed SAVI (TSAVI1) | RS Baret et al. (1989) | $TSAVI1 = \frac{\gamma (NIR - \gamma)(R - i)}{R + \gamma NIR - \gamma i}$  
$\gamma$ = soil line slope  
i = intercept (Qi et al. 1994). |
| Infrared Percentage Vegetation Index (IPVI) | RS Crippen (1990) | Argued that the red subtraction in NDVI was unnecessary (Ray 1994).  
$IPVI = \frac{NIR}{NIR + R} = \frac{NDVI + 1}{2}$ |
| Second TSAVI (TSAVI2) | RS Baret and Guyot (1991) | $TSAVI2 = \frac{\gamma(NIR - \gamma R - i)}{\gamma NIR + R - \gamma i + X(1 + \gamma^2)}$  
$X$ = a factor to minimize the soil effects ($X = 0.08$); rest similar to TSAVI1. |
| Atmospherically Resistant Vegetation Index (ARVI) | RS Kaufman and Tanre (1992) | First in the series of indices that have built-in atmospheric correction (Ray 1994). Replaced $R$ in NDVI with $RB$, where:  
$RB = R - \gamma(B - R)$  
and $\gamma$ is a correction parameter which was found to be efficiently applicable to all surfaces at $\gamma = 1$ and ARVI is thus:  
$ARVI = \frac{NIR - RB}{NIR + RB}$ |
| Modified Soil Adjusted Vegetation Index (MSAVI) | RS Qi et al. (1994) | $MSAVI1 = SAVI = \frac{NIR - R}{NIR + R + L} (1 + L)$  
$L = 1 - 2\gamma NDVI \times WDVI$  
$\gamma$ = the primary soil line parameter. |
<table>
<thead>
<tr>
<th>Vegetation Index</th>
<th>Author(s)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second MSAVI (MSAVI2)</td>
<td>Qi et al. (1994)</td>
<td><em>MSAVI2</em> = ( \frac{2NIR + 1 - \sqrt{(2NIR + 1)^2 - 8(NIR - R)}}{2} ) Ui uses annual NDVI to study annual land surface dryness and vegetation dynamics.</td>
</tr>
<tr>
<td>Anomaly Vegetation Index (AVI)</td>
<td>Chen et al. (1994)</td>
<td>Combines vegetation information with remotely-sensed temperature data to detect drought.</td>
</tr>
<tr>
<td>Water Supplying Vegetation Index (WSVI)</td>
<td>Chen et al. (1994)</td>
<td>Combines vegetation information with remotely-sensed temperature data to detect drought.</td>
</tr>
<tr>
<td>Cubed Ratio Vegetation Index (CRVI)/(CVI)</td>
<td>Thenkabail et al. (1994)</td>
<td>TVX combines NDVI and LST as evidence for drought. The relationship of NDVI and LST has been subject of extended research (Qin et al. 2008). Lambin and Ehrlich (1995) express TVX as:</td>
</tr>
<tr>
<td>Temperature-Vegetation Index (TVX)</td>
<td>Lambin and Ehrlich (1995); Prihodko and Goward (1997)</td>
<td>TVX combines NDVI and LST as evidence for drought. The relationship of NDVI and LST has been subject of extended research (Qin et al. 2008). Lambin and Ehrlich (1995) express TVX as:</td>
</tr>
<tr>
<td>Modified NDVI (MNDVI); Enhanced Vegetation Index (EVI)</td>
<td>Liu and Huete (1995)</td>
<td>NDVI applied to MODIS. Uses feedback loops to minimize both atmospheric and soil bias that is present in NDVI and other VIs.</td>
</tr>
<tr>
<td>Vegetation Temperature Condition Index (VTCl)</td>
<td>Wang et al. (2001)</td>
<td>Uses NDVI and LST. Ranges between 0 and 1. For a specific NDVI value: NDVIi, pixel VTCl is:</td>
</tr>
<tr>
<td>Global Vegetation Moisture Index (GVMI)</td>
<td>Ceccato et al. (2002)</td>
<td>Retrieves the vegetation water content.</td>
</tr>
<tr>
<td>Standardized Vegetation Index (SVI)</td>
<td>Peters et al. (2002)</td>
<td>Uses weekly NDVI to calculate the probability of the deviation of vegetation conditions from normal.</td>
</tr>
</tbody>
</table>
**Dryness Index (TVDI)**

\[
TVDI = \frac{T_s - T_{s_{\text{min}}}}{a + bNVI - T_{s_{\text{min}}}}
\]

- \(T_{s_{\text{min}}}:\) the minimum surface temperature in the concept of \(T_s/NVI\) triangle space
- \(T_s:}\) pixel's observed surface temperature
- \(a\) and \(b\) are parameters defining the dry edge, obtained from a linear fit for \(T_{s_{\text{max}}} (\text{maximum surface temperature}):\)
  \[
  T_{s_{\text{max}}} = a + bNVI
  \]

**Cumulative Water Balance Index (CWBI)**

- **Authors:** Dennison et al. (2003)
- **Description:** Measures regional drought stress by cumulatively summing the difference between precipitation and reference ET over a window of time.

**Soil Water Index (SWI)**

- **Authors:** Wagner et al. (2003)
- **Description:** Used the microwave C-band scatterometer data to derive a global soil moisture data set for years 1992-2000.

**Land Surface Temperature (LST)**

- **Authors:** Wan et al. (2004)
- **Description:** Since LST is sensitive to the drought, it was used as additional indicator along with NDVI.

**Vegetation Condition Albedo Drought Index (VCADI)**

- **Authors:** Ghulam et al. (2007a)
- **Description:** First in the series that was followed by PDI and MPDI (Ghulam et al. 2007b,c).

**Perpendicular Drought Index (PDI)**

- **Authors:** Ghulam et al. (2007b)
- **Description:** Second in the VCADI, PDI and MPDI series (Ghulam et al. 2007a,b,c).

**Remote Sensing Drought Risk Index (RDRI)**

- **Authors:** Liu et al. (2008)
- **Description:** Used cloud indexes over China as an indicator for drought.

**Total Storage Deficit Index (TSDI)**

- **Authors:** Yirdaw et al. (2008)
- **Description:** Calculates average terrestrial water storage from gravity measurements obtained from gravity recovery and climate experiment (GRACE) satellite mission.

**Vegetation Water Stress Index (VWSI)**

- **Authors:** Ghulam et al. (2008)
- **Description:** Uses NIR and shortwave infrared (SWIR) wavelengths. VWSI has a strong correlation with fuel moisture content which is indicative of wheat drought.

**Scaled Drought Condition Index (SDCI)**

- **Authors:** Rhee et al. (2010)
- **Description:** Current RS drought indices are for use mainly in arid regions. SDCI is designed for agricultural drought monitoring in both arid and humid regions. Combines NDVI and LST with precipitation data.

**Vegetation Water Supply Index (VWSI)**

- **Authors:** Cai et al. (2011)
- **Description:** Combines LST and NDVI. Performs more efficiently on agriculture fields with densely covered vegetation areas.