# OPTIMAL MINIMUM CAPITAL REQUIREMENT: A PROFITABILITY AND RISK MANAGEMENT APPROACH

by

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## Abstract

World banking operations shook during the Subprime Mortgage Crisis of 2008. This financial turmoil demonstrated that the international banking regulation, Basel II, had serious shortcomings and was not powerful enough to prevent a banking crisis. Researchers agree that Basel II underestimated the minimum capital requirement (MCR), requiring banks to hold insufficient capital to prevent the last crisis. The present research proposes a risk management framework for MCR calculations. The methodology has three stages: first, the development of a bank profit function (formulated following accounting principles and Profit Function Theory); second, the application of the previous function to derive the bank risk function(following the guidance of Modern Portfolio Theory); finally, the insertion of the latter function as the objective function into a constrained optimization (return on equity constraint). This framework is tested with aggregate data from Canadian banks from the period 2000 to 2010. The optimized capital ($K_O$) is compared to the Basel II Standardized Approach MCR. The findings show that the profit function satisfactorily predicts bank profits, and it is useful development for future work. The risk function, however, does not properly represent the variance of profits. This together with the use of a linear constraint produced $K_O$ smaller than Basel II requirements, meaning that, as implemented, this framework cannot replace Basel II. Potential improvements to this framework are identified and proposed for further research. The most important contributions of this research are the development of the bank profit function and a preliminary exploration of this optimization framework to MCR determination. Although, in the current stage of its development, this framework cannot replace Basel II, it presents a new approach to MCR calculations that has the potential to strengthen international banking regulation.

## Preface

An initial prototype of a model was developed in the UBCO course Bank Risk (IGS 520K), in the fall of 2010. The model developed in that course, when tested with US bank data generated negative capital. Therefore, that model did not properly describe banking operations and was not useful for implementation in the real world.

With the support of Dr. Ross Hickey and Dr. Kenneth Carlaw, this model has been further developed by this author to include five major modifications on the bank profit function. In summary, the present bank profit function differs from the previous one by five additional variables: return on trading, noninterest income, noninterest expenses, other liabilities, and return on other liabilities, being the second and third in this list suggested by Dr. Ross Hickey. Additionally, the return on loans has also changed its random characteristic, being considered random variable in the present research by Dr. Kenneth Carlaw suggestion.

These major changes in the bank profit function transformed completely the derived bank risk function, increasing the number of terms of the latter function from six to twenty-eight. Furthermore, a different constraint was imposed, and the research was conducted using Canadian publicly banking data. All these distinct features make the present framework a completely different proposal.

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# 1 Chapter: Introduction

World banking operations shook during the last global financial crisis, which came to be known as the Subprime Mortgage Crisis of 2008. Researchers agree that this crisis could have been prevented if banks had, among other actions, held greater capital in their balance sheets (BCBS 2011, 1-69; Triana 2010, 45-48). The minimum capital requirement (MCR), a very common feature in banking regulations worldwide, aims to act as a reasonable minimum that will protect bank operations by serving as a buffer against unanticipated losses, as well as to provide an early warning to regulators of banks' troubles (Crouhy et al. 2000, 45; Estrella 1995, 1-12). The underestimation of MCR, which the majority of countries calculated following Basel II guidelines, created the conditions that contributed to the development of this crisis. The Basel II framework allowed banks to hold less capital than the amount that would have enabled them to resist this financial turmoil (BCBS 2011, 1-69). The purpose of this research is to develop an innovative risk management framework, which could be used to strengthen the minimum capital requirement calculation. This framework has been empirically tested using public domain aggregate data from Canadian banks.

The methodology used to develop this risk management framework involves three stages. In the preliminary stage, inputs of simplified balance sheet, statement of income, and the basic accounting equation are used to generate a bank profit equation, which is similar to the one proposed by the theory of profit function (McFadden 1978, 2-109). The second stage considers this bank profit equation and applies concepts of Modern Portfolio Theory to it, under the directions proposed by Markowitz (1952), to derive a bank risk function. For the calculation of this bank risk function, identified risk sources play the role of idiosyncratic variables. There are five major sources of risk considered in this framework: volatility of deposits, volatility of noninterest income, volatility of the return on loan portfolio, volatility of return on other assets, and the volatility of return on trading activities. The third stage of the methodology consists of the optimization of the bank risk function. This optimization minimizes the bank risk equation (objective function) subject to a return on equity (ROE) constraint, which acts as a benchmark condition and is used to bring the optimization into a real world application. This constrained optimization determines the optimized capital ($K_O$), which is then compared to the capital held by banks and the minimum capital requirements according to Basel II guidelines, for the period from 2000 to 2010. The proposed framework does not require credit ratings or developing internal models, risk metrics that failed to measure bank risk according to several authors (Triana 2010, 45-48; Preston 2010,20; King and Tarbert 2011, 1-18). The findings indicate that this framework, as implemented, cannot replace Basel II. However, important improvement opportunities are identified and proposed for future research, opening new horizons for a similar approach for strengthening minimum capital requirements. A detailed description of this framework is presented in Chapter 3 and results are discussed in Chapter 5.

A recurring theme in the banking literature is the relationship between bank inputs, outputs, and profits. Some researchers apply econometric techniques to regress bank profit on specific components of the profit function (developed by McFadden in 1978) to verify the influence of specific variables on bank profits. For instance, Mullineaux and Pyles (2010) explore the influence of marketing expenses on bank profits in the US. Orzechowski (2009) has developed a bank model using a profit function similar to the profit equation of the present research; however, he gives special treatment for each loan type, specifying a return for each (such as consumer loans, real estate, commercial and industrial loans) while in the present research all of them are combined into the return on loans. He also combines return on deposits with returns on other liabilities into a single return, while this research treats them as different returns. Furthermore, Orzechowski does not consider the return on trading activities and his main objective was to explain how bank capital and bank profits influence loan activity, whereas the present research aims to use this profit function to derive the bank risk function and further determine the optimized capital for bank operations. Freixas and Rochet (2008, 265-304) develop a profit function used to determine the optimal amount of reserves that a bank must hold, whereas the present research considers that banks convert all funding into loans and other assets, an assumption that ignores bank reserves.

Another important example of a recent contribution using the profit function is the work of Ganesan (2001), who analyzes bank profits in India using different versions of the profit function, two of which include some risk factors such as total credit-to-assets ratio. He concludes that interest cost, interest income, other income, deposits and the ratio of credit to total assets are the significant determinants of bank profitability. The present research corroborates some of Ganesan's conclusions, assuming that interest cost, interest income, other income, and deposits are determinants of bank profits; moreover, the present exercise adds to this list of variables influencing bank profits noninterest expenses, and also specifies two components of other income: noninterest income and net income or loss provided by trading activities. In another direction, Liang (1989) develops a bank profit function that includes a risk measurement to explain market concentration. The risk measurement in that paper is defined as the standard deviation of the net income-to-assets ratio over a ten year period; whereas we measure risk as the variance of bank returns (the square of the standard deviation ), as proposed by Markowitz (1952). Although these papers have made important contributions relating bank inputs and outputs to bank profits, and some of them have even included measurements of incurred risk in bank operation, none has derived the variance of profits from the profit function, to actually predict bank risk, which is an innovative feature of the present research.

The optimization of a risk function, similar to the one executed in the present research, was designed and presented in the article *The Optimization of a Quadratic Function Subject to Linear Constraints* (Markowitz 1955, 111-133). In this paper, Markowitz develops and discusses the minimization problems that involve quadratic functions of portfolio risk constrained by the expected value of the portfolio returns (portfolio yield). Whereas Markowitz developed his optimization in the context of an investment portfolio, the present research is concerned with the risk and profitability of bank operations, with a focus on determining the optimized capital. A second difference is that Markowitz presents the math theory and resolution of these optimization problems, whereas this research derives a bank risk function and empirically implements the optimization's results using Canadian aggregate data to check this framework's applicability.

The literature on banking regulation identifies several reasons for regulating banks: their important role in the payment system, their role in implementing central banks' monetary policies, and the risks assumed by governments as guarantors of the deposit insurance corporations as well as the lender of last resort.[1] All of these reasons make sound and safe banking practices a prime concern of governments since the opposite situation can

[1] Payment system is the system that transfers money between individuals and firms in an economy (Crouhy, Galai and Mark 2000, 45-96). Lender of last resort is an official institution, usually the country's central bank, which lends money to banks during crises, when no other financial institution is willing to do so (Grossman 2010, 98-104).

put tax payers' money at risk, as was the case during the crisis of 2008 (Crouhy et al. 2000, 45-96; Grossman 2010, 98-104; Booth, Alexander, and Britain 2009).

Regulators can take many different routes to control banking operations; yet, minimum capital requirements (MCR) have been among the most popular regulations in banking systems worldwide. MCR have also been the main pillar of international banking regulation as proposed by the Basel Accords. These Accords are a series of international banking agreements put together by the Basel Committee on Banking Supervision (BCBS) within the Bank for International Settlements. This committee has existed since 1974 (BCBS 2009, 1-8) and was initially composed of central bank governors from the G-10 (Balin 2008, 1-17). Basel I, II, and III are accords establishing supervisory standards - of which the most predominant feature has been the procedure for MCR calculation - and encouraging best practices in the global banking industry. The most recent accord, Basel III, was released in 2010, but its implementation will start only in 2013 and is expected to be completed by 2019 (King and Tarbert 2011, 1-18). Although this series of accords has helped to standardize bank regulation worldwide, its evolution is a continuing process, and there are shortcomings that have not yet been addressed.

Basel III was developed as a response to the Subprime Mortgage Crisis of 2008 (Cosimano and Hakura 2011, 1-34). In addition to the MCR, this accord will require a capital conservation buffer and a countercyclical buffer (a non-permanent buffer designed to be raised in good times and relaxed in bad times to make banks resist under extreme stress conditions of the bank industry's cycle), as protection against unpredictable losses (Rajan 2009, 397-402; BCBS 2011, 1-69; King and Tarbert 2011, 1-18). Despite the possible benefits of these additional buffers, Preston (2010) and Triana (2010) suggest that Basel III framework maintains the broadly criticized metrics for MCR calculation from Basel II (based on credit rating and the value-at-risk technique for risk measurement), therefore failing to improve the procedure for determining MCR. The present research proposes an alternative way to address this failing of the Accord that aims to strength international banking regulation with a focus on commercial bank operations.

The following chapters will present the motivations, theory, development and outcomes of this risk management framework for minimum capital requirement calculation. Chapter 2 discusses the importance of bank regulation and provides a brief review of the current international regulation, Basel II. Since the optimization framework will be tested with Canadian data, a contextualization of Canadian guidelines is presented. Chapter 3 describes the methodology for the optimization framework and the variables included. Chapter 4 offers data collection details and data description. Chapter 5discusses the results of the minimization of the bank risk function and compares the empirical optimized capital produced by the framework ($K_O$), the Basel II capital calculated using the standardized approach as applied in Canada ($K_{Basel\ II}$), and the actual capital held by banks in Canada ($K_{Market}$) for the period 2000 to 2010. Chapter 6 presents a summary of the main conclusions of this empirical test and explores potential future work.

# 2 Chapter: International Bank Regulation

## 2.1 Motivation

There are several reasons to regulate bank operations. From an economic point of view, it is valuable to do so due to the importance of these financial institutions in the payment system, which is the system that transfers money between individuals and firms in an economy (Crouhy, Galai and Mark 2000, 45-96). This key role grows in complexity when international financial transactions involving different currencies and time zones are taken into account. Therefore, banking regulation has been designed in part to guarantee that the payment system will perform properly.

Another important motivation for banking regulations is the fact that national governments are ordinarily the guarantors of national deposit insurance corporations, which guarantee a certain amount of the deposits in a bank, making safe bank operations a government's concern.[2] Furthermore, some governments can also play the role of lender of last resort as is the case in Canada (Cooke and Justice 2006, 14). A lender of last resort is an official institution, usually the country's central bank, which lends money to banks during crises, when no other financial institution is willing to do so. Often, it offers loans to banks that are experiencing financial difficulty or near collapse (Grossman 2010, 98-104). Therefore, there is a huge incentive for government to regulate bank operations, since it is their role to make sure banks operate without losses that ultimately will put taxpayer money at risk.

[2] For instance, the Canadian Deposit Insurance Corporation, CDIC, is a federal Crown corporation which insures deposits in all chartered banks in Canada up to $100,000 if the money is invested in covered savings accounts. For further details http://www.cdic.ca/e/coveredornot/coveredornot.html.

The spectrum of existing banking regulations includes several options such as restricting specific kinds of assets that banks can hold, defining how much of loans can be concentrated in one specific class of borrower, restricting bank borrowing, and imposing periodic publication of bank accounting statements. Ideally, bank regulation aims to foster stability and efficiency in an economy (Grossman 2010,128-134), however, depending on the regulation imposed and its limits, the goals of bank stability and efficiency can become competing instead of complementary. This is particularly true when the regulation stifles competition, thereby promoting stability but discouraging efficiency by preventing the economy from reaching its full potential. This situation could be generated for example by setting overly high capital requirements, or by limiting the interest paid on deposits, both of which are regulations designed to promote stability. Banking history worldwide has shown that the balance between efficiency and stability is not easy to achieve. Indeed, there are many cases in which countries have made a conscious choice to promote one over the other. For instance, Cook (2006) pointed out that

> Traditionally, Canadian regulators have favoured safety and soundness over competition in the financial sector. Domestic firms were protected from foreign competition and oligarchies of large national firms were encouraged (p. 15).

Cook's passage clearly emphasizes that Canada has so far opted for guaranteeing stability over efficiency in its banking system. Since some papers have shown that on average the output losses generated by a banking crisis can reach multiples of the country's GDP, this specific governments' tendency towards bank regulation is not to be faulted (BCBS 2010a,1-8).

On an international level, very strict and rigid banking regulations were observed amongst developed countries as a consequence of the Great Depression in the 1930s. In the 1960s and 1970s, however, these countries witnessed the deregulation of banking, which was a process of continuing increase in the flexibility towards banking regulation, allowing banks to engage in activities they were prevented to perform before (Grossman 2010, 284-289; Atindehou and Gueyie 1995, 285-295). The deregulation processes allowed for instance: branching, foreign entry into domestic banking, and permitting banks to engage in some financial innovations such as securitization and repurchase agreements.[3] This process, combined with the intensification of internationalization of banking in the 1980s, created an environment that allowed international banks to take advantage of the differences in banking regulation across countries, making national regulations insufficient to control bank risk-taking (Burns 1988, 11-24).

An interesting example of banks profiting from regulatory differences occurred in 1969. At that time, American banks established branches abroad, with the objective of increasing their liabilities, in order to be able to expand their loan portfolio and conduct banking free of American regulatory restraints. Specifically, banks collected deposits abroad, paying more than they would be allowed to under the American regulation (Regulation Q ceiling), and lent these funds to their head offices at home to extend loans to firms.[4] In spite of American regulators' pressures, some banks even executed transactions with American corporations using their foreign branches, rather than their offices in the US, just

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[3] Securitization involves the sale of interest in a package of loans originated by a bank. Repurchase agreement known as *repo*, is a transaction in which the bank sells securities in large amounts to a client under an agreement to repurchase the securities later for a higher price (increased by the *repo rate*) (Burns and Research 1988).

[4] In 1933 a new regulation was imposed on US national banks, and extended to all federally insured banks in1935.This new regulation made the payment of interest on demand deposits forbidden, and the Federal Reserve was empowered to set the ceiling interest rate on time and savings deposits, known as Regulation Q ceiling. The ceiling interest rate, or Regulation Q, had two main purposes: to reduce competitive pressures among banks for deposits, therefore increasing their profitability; and to encourage rural banks to loan locally instead of looking for better interest on their deposits with banks in large financial centers (Grossman 2010).

to avoid domestic regulations. This example demonstrates how cross country differences in banking regulations made national regulation ineffective in controlling bank risk-taking. The resulting uncertainty in bank profitability created by the extra expenses incurred abroad alarmed the American regulator, which replied using *moral suasion* (Burns and Research 1988, 15-20; Grossman 2010, 251-284).[5]

The development of international banking regulation, consequently, arose from a push from developed countries' governments towards an international harmonization of banking regulation. The objective was to prevent operations made internationally from threatening the safety of their internal banking system, and potentially causing losses that would need to be "covered" by governments. Another influence that pressed regulators towards international harmonization came from bankers in Europe and North America who complained of unfair competition from Japanese and Far East banks which were subject to less strict regulation and were, therefore, taking greater advantage of globalization's opportunities (Crouhy, Galai, and Mark 2006, 55-82).

In fact, the discussions about the harmonization of international banking regulation began in the aftermath of the famous Bankhaus Herstatt case in 1974. This German bank received a remittance from American banks on the day of its liquidation, June 26th, 1974, as part of scheduled foreign-exchange transactions. Due to the time zone difference between Germany and the US, Herstatt did not send the amount set by the agreements signed with the American banks, which therefore incurred large losses. This event prompted the formation of the Basel Committee on Banking Supervision (BCBS) in the

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[5] Moral suasion is an informal advisory for banks to not attempt avoiding domestic control by using their offshore branches.

same year. BCBS was composed by central banks' governors and bank regulatory agencies' representatives of the G-10 countries. This Committee's main tasks were to understand each G-10 banking regulation and to develop a feasible international regulation, which would promote international banking stability, by making all banks operate under similar requirements (Grossman 2010, 267-269).

Notwithstanding, a guideline for international banking regulation was not yet in place when bank losses, as a consequence of globalization, started to happen in a broader fashion. In the 1970s, banks from developed countries recycled the deposits of oil exporters (OPEC nations), loaning them at a floating interest rates to oil importers, concentrated in Latin America. Due to the collapse of markets for Latin American commodities and the increase in interest rates, these countries stopped paying back those loans, producing what is called the Third World Debt Crisis in the 1980s (Burns and Research 1988, 20-21). Rochlin (1993) observed that

> Of the six major Canadian banks, only the Toronto Dominion Bank recorded a profit in 1987; the provisions for Latin American debt, to a significant extent, were responsible for the poor results. During the late 1980s, share prices for Canadian bank stocks fell periodically as a result of a loss of domestic confidence - a loss arising from exposure to Third World loans (p. 161).

This expresses how deeply Canadian banks experienced the losses caused by the Third World Debt Crisis, which also generated major loan losses for American and European banks (Cooke2006, 1-24). The seriousness of this crisis accelerated the release of the first international banking framework: *International Convergence of Capital Measurement and Capital Standards*, known as Basel I, by BCBS in 1988. Considering that the only threat accounted for in Basel I framework was credit risk, it becomes clear how deeply developed countries' regulators felt the effects of the Third World Debt Crisis, which was mainly caused by counterparty default, the very definition of credit risk according to BCBS (1988).

## 2.2 Basel I

Basel I, as initially proposed by BCBS in 1988, focused on the specification of the minimum capital requirement, MCR, for banking operations. MCR should act as a buffer against unpredicted losses and provide an early warning of bank's insolvency (Estrella 1995, 1-12). According to the framework, the regulatory capital is equal to the sum of Tier 1 and Tier 2 capital, but Tier 1 must represent at least 50% of the total. These capital tiers differ in terms of the loss-bearing potential of their components (quality) as specified in Table 2.1.

**Table 2.1. Basel I Capital Tiers' Composition.**

| Capital | Components |
|---|---|
| Tier 1 | Paid-up capital: common shares and noncumulative perpetual preferred shares [1] |
| | Cash reserve (disclosed reserve) |
| Tier 2 | Undisclosed reserves |
| | General loan loss provision |
| | Hybrid debt instruments [2] |
| | Subordinated debt [3] |

*Source:* Information collected from *International Convergence of Capital Measurement and Capital Standards* by BCBS (1988), p.3-6.

*Note* [1]: Noncumulative preferred shares are considered preferred shares for which, if dividends are not paid in a specific year, they will not be paid in the future (dividends do not accumulate). The term perpetual prohibits the redemption of these shares within the first five years of issuance (OSFI 2007b).

[2]: Hybrid debt instruments are permanent and have some of the characteristics of both equity and debt. Some examples of such instruments are the long-term preferred shares in Canada and mandatory convertible debt instruments in the US.

[3]: a subordinated debt is a bond issued by a bank that it is going to be paid back by the bank only after all other creditors have been paid.

As emphasized by Larson (2011), the inclusion of the lower-quality Tier 2 capital as part of the regulatory capital in Basel I provides evidence of the existence of banks, even among G-10 economies, which did not possess enough owner's equity and had mainly relied on debts to finance banking operations (Larson 2011).

Basel I MCR was calculated according to the standard capital ratio, a ratio that must be at least eight percent, is given by equation 2.1a. The credit risk in 2.1a was calculated following the concept of *risk-weighted assets* (RWA), which contained on- and off-balance-sheet components and aimed to be a measure of bank risk exposure, based on credit risk. The uncertainty (risk) was measured according to counterparty's type and the specific financial instrument in the bank portfolio, concepts based on Risk Management Theory, which has evolved since the 1950s.

$$Standard\ Capital\ Ratio: \quad 0.08 = \frac{Regulatory\ Capital: Tier\ 1 + Tier\ 2}{Credit\ Risk} \tag{2.1a}$$

The Risk Management Theory has contributed to the identification, pricing, management, and control of financial risks. Its techniques have increasingly integrated management practice and reports in banking. The foundation of this theory relies on the principles of Modern Portfolio Theory, which recognize that different portfolios can be analysed based on the mean and variance of their rates of returns, as proposed by Markowitz (1952). In 1958, Franco Modigliani and Merton Miller showed that in the absence of corporate and income taxes, the capital structure of financial firms does not influence the value of the firm; therefore, increasing the leverage of the firm increases its risk, which must be compensated, under shareholders' perspective, by a higher expected rate of return.[6] Further contributions were made in 1964 and 1965 by Sharpe and Lintner respectively, who claimed that an asset should be priced according to its relative contribution to the overall risk of the portfolio it belongs to, a technique that came to be known as the *Capital Asset Pricing Model* (CAPM). In 1972, Robert Merton extended CAPM application by showing that it could be applied in a continuous time framework in which there is no jumps in stock prices. One year later, Fisher Black, Myron Scholes and Robert Merton developed the framework to price options. All of this theoretical progress was incorporated into banking practice swiftly, especially after the booming of financial innovations in the 1980s. However, the accounting approach to bank operation did not keep up with this evolution. The financial innovations created in the 1980s did not receive a place on banks' balance sheets: instead, they were accounted for on off-balance-sheet reports (Crouhy, Galai, and Mark 2001, 21-9; Dewatripont and Tirole 1994, 47-92). The growing importance of these instruments in bank portfolios made it clear that considering only on-balance-sheet instruments would not reflect the risk incurred by banking institutions anymore. Therefore, Basel I needed to address on- and off-balance-sheet risk exposures.

Since RWA were designed to measure credit risk, they were inserted as the denominator of equation 2.1a. It included measures of the credit risk of assets (on-balance-sheet items) together with credit risk of derivatives and non-derivative, both off-balance-sheet instruments that were treated differently in the framework.[7] A summary of the RWA

[6] Increasing leverage means that the firm takes on more borrowing relative to equity to fund its activity.
[7] A derivative is a financial contract of which the value is derived from an underlying asset, rate, or index. Some examples are future and options contracts (Tarbert 2000).

calculation for credit risk is presented in Figure 2.1 in which each risk source is formulated separately, and RWA is determined by their sum.

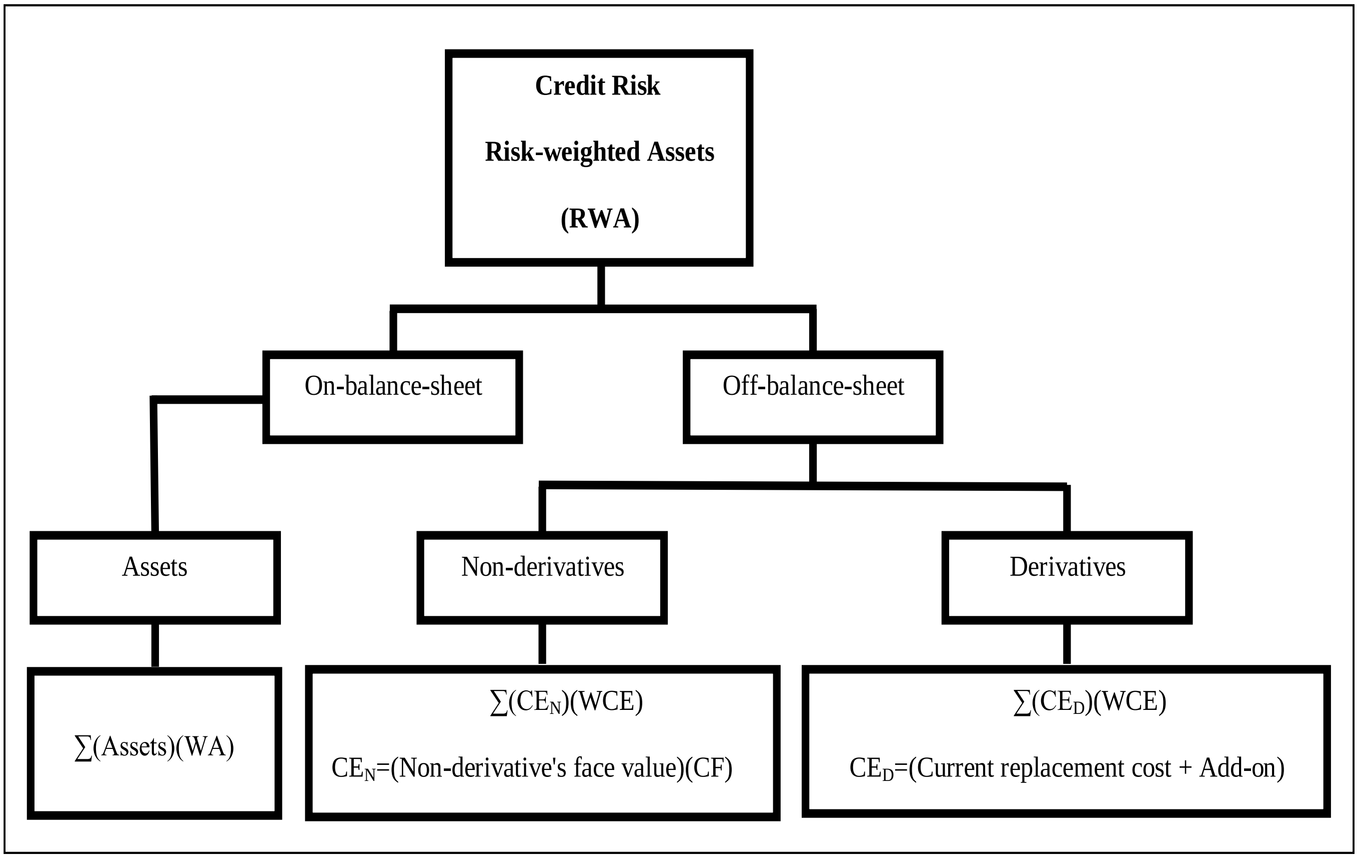


**Figure 2.1. Comprehensive Risk-weighted Assets (RWA) for Credit Risk.**

It is interesting to notice that the assets were directly multiplied by the risk weight for assets (WA), which varied according to the borrowers' type as given by Table 2.2. On the other hand, off-balance-sheet items needed to be first expressed as a credit equivalent (CE), which aimed to convert off-balance-sheet products into assets. CE has different definitions depending on whether it is converting a derivative or non-derivative instrument.[8] For non-derivative products, $CE_N$ is equal to the multiplication of the non-derivative's face value by a conversion factor, CF, which varies as given by Table 2.3. This credit equivalent is then multiplied by the risk weight for credit equivalent (WCE), which addresses the counterparty's credit risk, as illustrated in Table 2.4. Similarly, derivatives have to be first converted into credit equivalent and then multiplied by WCE; however, for derivatives $CE_D$ has a different definition: it is equal to the current replacement cost plus the add-on amount, as expressed in Figure 2.1. The current replacement cost is equal to the liquidation value of the derivative if positive, or otherwise zero. The add-on amount is given by the derivative's notional value[9] times the Basel add-on factor, which varies according to the maturity of the transaction and underlying asset involved in the derivative transaction, as given by Table 2.5 (Crouhy, Galai, and Mark 2006, 55-82). In summary, a simplified equation for RWA, after considering the different formulations of credit equivalent for derivative and non-derivative is expressed in equation 2.1b.

$$Risk-weighted\ Assets: \quad RWA = \sum(Assets)(WA) + \sum(CE_N)(WCE) + \sum(CE_D)(WCE) \tag{2.1b}$$

[8] Non-derivative examples are: letter of credit, banker's acceptance and sale and repurchase agreements.
[9] Notional value is the price reference stipulated in a derivative contract. It is not usually an accurate measure of economic value of the contract (Crouhy, Galai, and Mark 2001, 1-43).

**Table 2.2. Risk Weights for Assets, WA - On-balance-sheet Items.**

| Category | Description | Risk Weight |
|---|---|---|
| 1 | Claims on OECD governments[10] | 0% |
| 2 | Claims on other banks | 20% |
| 3 | Residential mortgages | 50% |
| 4 | Claims on the private sector and all other assets | 100% |
| 5 | Claims on domestic public sector entities | Regulators' discretion (0%, 20%, or 50%) |

*Source:* Information collected from *Basel I, Basel II, and Emerging Markets: A Nontechnical Analysis* by Bryan J. Balin (2008), p.3.

**Table 2.3. Conversion Factors for Non-derivatives – Off-balance-sheet Items.**

| Category | Off-balance-sheet Items | Risk Weight |
|---|---|---|
| 1 | Commitments with an original maturity of a year or less | 0% |
| 2 | Short-term self-liquidating trade-related contingencies such as letters of credit | 20% |
| 3 | Transaction-related contingencies such as performance bonds, revolving underwriting facilities (RUFs), and note issuance facilities (NIFs) | 50% |
| 4 | Direct credit substitutes, bankers' acceptances, standby letters of credit, sale and repurchase agreements, forward purchase of assets | 100% |

*Source: The Essentials of Risk Management* by Michel Crouhy, Dan Galai, Robert Mark(2006), Table 3-3, p. 61.©2006 McGraw-Hill Companies, Inc. Reproduced with permission.

**Table 2.4. Weights for Credit Equivalent, WCE - Off-balance-sheet Items.**

| Category | Type of Counterparty | Risk Weight |
|---|---|---|
| 1 | OECD governments | 0% |
| 2 | OECD banks and public-sector entities | 20% |
| 3 | Corporations and other counterparties | 50% |

*Source: The Essentials of Risk Management* by Michel Crouhy, Dan Galai, Robert Mark (2006), Table 3-2, p. 61. © 2006 McGraw-Hill Companies, Inc. Reproduced with permission.

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[10] OECD stands for the Organization for Economic Co-operation and Development, which currently has thirty four member countries. For more information: http:/www.oecd.org .

**Table 2.5. Add-on Amount for Residual Maturity Time and Underlying Rates (Derivatives).**

| Category | Residual Maturity | Interest Rate Contract | Exchange rate Contract |
|---|---|---|---|
| 1 | Less than one year | 0% | 1.0% |
| 2 | One year and over | 0.5% | 5.0% |

*Source:* Adapted from *International Convergence of Capital Measurement and Capital Standards* by Basel Committee on Banking Supervision, BCBS (1988), p. 25.

### 2.2.1 Basel I Final Remarks

Overall, Basel I, as described above, was a hallmark in international banking regulation because it initiated the harmonization of MCR calculation. It was designed to be fully implemented only by internationally active banks in G-10 countries by 1992 – Canada implemented in 1988 (OSFI 2012a). Banks operating in G-10 countries had to comply with Basel I as this was a requirement imposed by their regulatory agencies. This regulatory framework made it easier for banks complying with it to engage in international banking opportunities. This fact might explains why Basel I guided regulators in a much broader spectrum than it was created for, including non-internationally active banks in developed jurisdictions, as well as several developing countries, reaching over a hundred countries worldwide (Balin 2008, 1-16; Holtorf, Muck, and Rudolf 2005, 79-80).

Notwithstanding its recognition as an important milestone, Basel I received strong criticism. There are five shortcomings that appeared consistently in the literature. The first four drawbacks were especially stressed and deeply explored by risk management authors. The first flaw identified in the framework was the inaccuracy of the risk-weights for measuring the risk incurred by banks. The Basel I *bucket approach*, as nicknamed by King and Tarbert (2011), included high-risk sovereign debts such as those from Mexico and

Greece together with Canadian low-risk debts in the same risk category, requiring banks to hold 0% of the claim amount as regulatory capital to offset potential losses. Because banks bore the same capital charge for different risks, they strategically chose assets with potential higher return, in order to produce greater financial results. Since higher return is often associated with greater risk, banks were therefore holding the riskiest assets in an attempt to boost their results. The fact that the risk categories for RWA calculation allowed, and according to King and Tarbert (2011) even encouraged, this riskier bank behavior, which is known in the risk management literature as regulatory arbitrage, the risk category approach was considered an imprudent inconsistency of Basel I (Crouhy et al. 2000, 45-96).The second fault involved risk inconsistencies across the *risk buckets*. For instance, any low-risk corporate loan was assumed to be a lot riskier (100% capital charge) than any claim to an OECD country, charged 0%. This made banks prefer high-risk sovereign debts to commercial loans as a regulatory arbitrage opportunity, reducing the quantity of commercial loans provided and producing what is called a *credit crunch* (Tarbert 2000, 1770-1810; Danielsson, Shin, and Zigrand 2011, 1-36).[11]

The third shortcoming was the fact that the framework for MCR calculation ignored the effects of diversification in the bank portfolio, in the RWA determination. By summing the risks of each instrument without considering the interaction or correlation between them, Basel I neglected the reduction of uncertainty produced by diversification, the keystone of Modern Portfolio Theory, which is the very foundation of modern risk management (Crouhy et al. 2000, 45-96; Markowitz 1991, 469-477). The fourth drawback is that, although the guideline included off-balance-sheet instruments in the calculation of RWA, it focused only

[11] *Credit crunch* occurs when banks curb new lending to businesses (Kashyap, Rajan, and Stein 2008, 14).

on credit risks when in fact, the main risk of these instruments, especially for derivatives, is market risk.[12] The fifth criticism of the Accord is that it failed to impose a strict definition of regulatory capital, which permitted countries' regulatory agencies to include into Tier 1 capital even preferred stocks with only ten years maturity, when they were supposed to allow only perpetual instruments into this Tier; there was a case in which subordinated debts were allowed to count as Tier 1.[13] These opportunities to inflate Tier 1 capital made undercapitalized banks look well capitalized, unbalancing the competition among banks in the international realm (Tarbert 2000, 1799-1810).

An intriguing prospect, important to add as a sixth weakness of Basel I, which has not been broadly discussed in the literature reviewed, is the missing justification for imposing a standard capital ratio equal to eight percent. The risk category approach is intuitive, although not straightforward. For instance, one can intuitively understand the creation of the risk-weighted asset categories by assuming that each risk category has historically presented a consistent rate of default, and that the bigger this rate the greater should be the capital charge. However, this assumption of consistency is quite a stretch, considering the diverse banking systems among G-10 countries in the late 1980s. What is not intuitive, however, is the reason why BCBS specified a capital ratio of eight percent. Freixas and Rochet (2008) suggest that regulators chose a target probability of failure, and made it equal to the probability that the capital ratio be smaller than the loss ratio.[14] Another attempt to explain the chosen capital ratio was presented by Estrella, Park and Peristiani (2000) who compared three different

[12] Market risk, as defined by Harper, Keller, and Pfeil (2005), is the uncertainty that the value of bank's portfolio changes as a response to the volatility of relevant parameters such as the price of underlying asset or interest rate.
[13] For other interesting examples see Tarbert (2000).
[14] Loss ratio is defined as total losses divided by the total of the credit exposure. For further information, see Freixas and Rochet (2008), 265-304.

ratios: leverage ratio, gross revenue ratio, and risk-weighted capital ratio from 1988 to 1992, in the US banking system.[15] The risk-weighted capital ratio was defined as tier 1 capital divided by risk-weighted assets. Their results ascertained that on average the mean and median for each year (of which the minimum during the period were 17.9% and 13.6%, respectively- both for 1990) were well above the four percent specified by Basel I. Their paper attests that Basel I MCR calculation would not result in changes in bank operations in the US and that the American scenario could not explain the BCBS percentage choice. In the literature reviewed, a unique perspective on the eight percent puzzle was offered by Tarbert (2000) who stated that

> The Basle Committee has left the world with little justification for its choice. Indeed, 'the 8% minimum is not grounded in any financial model of capital adequacy. Not only has the Committee failed to explain why 8% is the right minimum total capital ratio, it has never defined the question to which 8% is the answer' (p. 1805).[16]

The lack of justification for a standard capital ratio of eight percent as emphasized by the passage above may clarify the reason why some countries required standard capital ratio greater than the percentage set by BCBS, along with the inclusion of additional requirements not proposed in Basel I such as leverage ratios.[17] For instance, Canada had implemented a leverage ratio, the assets-to-capital multiple, since early 1980s, and maintained it after implementing the Accord (Crawford, Graham, and Bordeleau 2009, 45-50). The US introduced this additional ratio for capital adequacy in 1989 (Estrella 1995, 1-12). Therefore, an effective leveling of the international banking field was still unreached after Basel I

[15] Leverage ratio was Tier 1 capital divided by total assets; gross revenue ratio was tier 1 capital to total interest and noninterest income before taxes.
[16] Tarbert does spell Basel as Basle in his paper; this is also a common spelling in some German sources consulted.
[17] According to Estrella, Park and Peristiani (2000), in the US a well-capitalized bank would be those holding at least 6% Tier 1 risk-weighted capital ratio and 10% total risk-weighted capital ratio (Tier 1 plus Tier 2 capital).

implementation worldwide, meaning that the BCBS had failed on one of its main objectives (Rodriguez 2003, 115-126).

Basel I was amended in 1996 to incorporate market risk into the framework and to better reflect risk concerns that were extensively addressed by several authors. This amendment proposed two changes: the incorporation of several credit risk mitigations into the MCR calculation, and either the use of external credit rating agencies to determine the specific weight associated with each asset or credit equivalent in a standardized approach, or the use of internal bank credit ratings for more sophisticated institutions (Estrella, Park, and Peristiani 2000, 33-52). Exploring this topic more extensively is beyond the objectives of the present research, which aims to concentrate on Basel II, the international accord in place when the Subprime Mortgage Crisis of 2008 started discussed in a Canadian context in section 2.3, and to develop an innovative risk management approach for calculating the optimum capital for bank operations, which will be described in chapter 3.[18]

## 2.3 Basel II

The Basel II framework, like its predecessor Basel I, was used as a guide by bank regulators worldwide. This newer framework was first released in 2004 with the title: *Basel II: International Convergence of Capital Measurement and Capital Standards: a Revised Framework* (revised in 2006). Primarily, Basel II consists of three pillars to regulate banking operation: minimum capital requirement (MCR), supervisory oversight, and market discipline. The main focus of regulators has been on the first pillar, MCR, which will be

[18] Further explanation about Basel I and its shortcomings can be found in the works of Jones (2000),Tarbert (2000), Rodriguez (2003), Rudolf (2005), Balin (2008), Monroe (2010), Ojo (2010), King and Tarbert (2011), and Larson (2011).

discussed at length here. It is worth mentioning that pillar 2, supervisory oversight, motivates banks to bring forth their own methodology to determine MCR according to risks incurred (King and Tarbert 2011, 1-18). Furthermore, pillar 3, market discipline, ultimately idealizes that rigorous disclosure rules and transparency could contribute to lighten regulators' load of supervisory duties, transferring part of it to the market itself (Hartmann-Wendels, Grundke, and Spork 2005, 3-52). Since pillar 1 is the exploratory area of the present research, pillars 2 and 3 will be omitted from the following discussion.[19]

As aforementioned in the description of Basel I, the MCR specifies a reasonable amount of capital, which is intended to serve as a buffer to protect bank deposits and sustain banks' operation in case of unpredictable losses; it also aims to provide an early warning of banks' solvency and necessity of intervention to regulators and market participants.[20] MCR under Basel II was determined using measures of credit, market, and operational risk. Credit risk refers to risk of losses associated with borrowers' inability to pay back the loan. Differently from the first accord, Basel II required the use of credit rating agencies to determine the credit risk associated with each borrower, or this specific risk could be determined using bank internal models (for complex institutions). Market risk is the risk that fluctuations in market prices and interest rates could reduce banks' return and it was introduced in the Basel I amendment of 1996. Finally, operational risk is the risk of losses produced by inadequate computational systems, inappropriate controls, fraud, as well as human error (Crouhy, Galai, and Mark 2001, 31-9; Harper, Keller, and Pfeil 2005,765-784).

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[19] More information about pillars 2 and 3 are available in BCBS (2004), BCBS (2006), Ojo (2010), or King and Tarbert (2011).

[20] Solvency is related to the bank's ability to meet financial obligations, usually measured by how much its assets exceed liabilities (capital). Bankruptcy occurs whenever assets minus liabilities is negative (negative capital).

As emphasized by Crouhy, Galai, and Mark (2000) the classical and most challenging risk commercial banks are exposed to is credit risk. Famous cases that demonstrate this point are: Barings Bank and Credit Lyonnais. In the 1990s, the Barings Bank case was one that mixed market and operational risk, since the bank's employee Nick Leeson fraudulently traded - due to the lack of appropriate bank controls- and lost money in the Asian market. In this case, market risks due to the volatility of the Asian market assets traded, as well as fraud, one of the operational risks banks could be exposed to, combined to produce a few billion dollars in losses. On the other hand, the damage cause by imprudent lending (credit risk) at Credit Lyonnais in the 1980s produced losses greater than $20 billion. These cases paint a picture of how much more dangerous credit risk has historically been to commercial banks, when compared to operational and market risks (Crouhy et al. 2000, 72).

Similar to the Basel I, Basel II proposed MCR calculation by multiplying standard capital ratio (still equal to eight percent) by the risk-weighted assets (RWA) as represented in equation 2.2a. This equation shows a new feature of Basel II MCR: the inclusion of Tier 3 capital in the numerator. Although not clear in the same equation, Basel II also changed the RWA concept in relation to the previous framework. RWA is now a function of credit, market and operational risk as demonstrated by equation 2.2b (BCBS 2006,12).

$$Standard\ Capital\ Ratio\colon \quad 0.08 = \frac{Regulatory\ Capital\colon Tier\ 1 + Tier\ 2 + Tier\ 3}{RWA} \qquad (2.2a)$$

$$RWA = RWA\ for\ Credit\ Risk + 12.5(Market\ Risk + Operational\ Risk) \qquad (2.2b)$$

Basel II offered a range of alternative procedures for determining credit and operational risks, which were added to the market risk framework, introduced in the Basel I amendment of 1996. All options for MCR calculation are presented in Table 2.6, which also demonstrates the BCBS's ranking of the different approaches according to their complexity in terms of risk management requirements for development and implementation.

**Table 2.6. Different approaches for MCR Calculation According to Risk Type.**

| Risk Type | Simpler | Medium Complexity | High Complexity |
|---|---|---|---|
| Credit | Standardized Approach | Internal Ratings-Based (IRB) Foundation Approach | Internal Ratings-Based (IRB) Advanced Approach |
| Market | Standardized Approach | Internal Models Approach | |
| Operational | Basic Indicator Approach | Standardized Approach | Advanced Measurement Approach (AMA) |

*Source:* Based on information collected from *International Convergence of Capital Measurement and Capital Standards: A Revised Framework-Comprehensive Version* by the Basel Committee on Banking Supervision, BCBS (2006), p.12-203.

The different procedures presented by Basel II have the objective to match bank operations' sophistication with the risk management complexity required for MCR calculation. Therefore, more sophisticated operations would require greater risk management expertise, and a more advanced approach for MCR calculation. After receiving authorization from a designated regulator, some banks were permitted to produce their own risk management models to calculate MCR using either of the Internal Rating-Based (IRB) approaches for credit risk, the internal model approach for market risk, and the Advanced Measurement Approach for operational risk. The Accord suggested that for consistency, institutions applying the standardized approach for credit risk (banking book) should also employ the same framework for market risk (trading book), and those using the IRB approach in the banking book should implement the internal models approach for their trading book.[21] Once a bank started using the more sophisticated methodology for capital requirement calculation (IRB or internal models), they were expected to continue implementing, and returning to simpler approaches would be permitted only in extraordinary circumstances in Canada (OSFI 2007a, 261-292).

Overall the internal approaches were based on the value-at-risk (VaR) technique. The VaR concept defines the expected maximum loss of a portfolio given a confidence level and specific holding period. For Basel II compliance, a bank using any internal models must compute VaR on daily basis using a 99% one-tailed confidence level for a holding period of ten days of its portfolio value (Holtorf, Muck, and Rudolf 2005,89).[22] However, the Subprime Mortgage Crisis of 2008 attested that, while the VaR technique worked properly under normal market conditions, it underestimated losses in extreme circumstances (King and Tarbert 2011, 1-18; Triana 2010, 45-48; Nocera 2009, 24).

The choice of the approach used for assessing credit, market and operational risks was discretionary to the regulatory agency, which resulted in country-specific methodologies. In Germany there were only a few banks (fifteen in 2005) implementing internal models for market risk because of the higher cost involved in developing those models (Holtorf, Muck, and Rudolf 2005). The same was true in the US, where regulators went even further, requiring Basel II advanced procedures only for the largest banks, leaving the majority of banks with the ability to opt-in voluntarily, or maintain compliance with Basel I. The main

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[21] Trading book is composed by instruments held for trading or hedge purposes (for instance stocks and government securities). These instruments are tradable and can be sold in the market at their market value. On the other hand, the banking book consists of items that are not acquired for trading purposes and are meant to be held until maturity.

[22] For an example of market risk calculation using VaR see Holtorf, Muck and Rudolf (2005).

justification for not implementing Basel II in the entire American banking system was that the gain in risk sensitivity was outweighed by the cost involved for the banks to implement it (Monroe 2010, 33-5; Ojo 2010, 305-315).[23]

In contrast, in Canada, the majority of banks were using IRB approaches for the period from 2008 to 2011 (OSFI 2012b) as illustrated in Table 2.8. Therefore, it would be more realistic to express the Canadian scenario computing MCR according to the IRB approaches for credit risk. However, in order to implement IRB calculations, it would be necessary to have access to bank-specific information for each asset such as the probability of default, loss given default, exposure at default and maturity of each asset, and more importantly: the knowledge of a model approved by the regulatory agency (in this case, the Office of the Superintendent of Financial Institutions, OSFI); but, these information is not publically available. Similar limitations make internal modeling for market and operational risks also unfeasible. More details of Canadian banking regulation such as the regulatory regime and the number of institutions during the decade from 2000 to 2010 are also provided in Table 2.7.

[23] For further exploration of the US reasons of not applying Basel II to its entire banking system see Bulletin (2003).

**Table 2.7. Canadian Banking System Information.**

| Year | Regulatory Regime | Standardized Approach | IBR Approach for Credit Risk | Domestic Banks | Foreign Bank Subsidiary | Foreign Bank Branches | Bank Total |
|---|---|---|---|---|---|---|---|
| 2000 | Basel I[1] | | | 13 | 37 | 9 | 59 |
| 2001 | Basel I [1] | | | 14 | 33 | 17 | 64 |
| 2002 | Basel I [1] | | | 16 | 32 | 21 | 69 |
| 2003 | Basel I [1] | | | 18 | 29 | 22 | 69 |
| 2004 | Basel I [1] | | | 19 | 27 | 22 | 68 |
| 2005 | Basel I [1] | | | 21 | 26 | 24 | 71 |
| 2006 | Basel I [1] | | | 22 | 24 | 25 | 71 |
| 2007 | Basel I [1] | | | 20 | 24 | 29 | 73 |
| 2008 | Basel II | 8 | 68 | 21 | 25 | 30 | 76 |
| 2009 | Basel II | 9 | 68 | 22 | 25 | 30 | 77 |
| 2010 | Basel II | 10 | 67 | 22 | 26 | 29 | **77** |

*Source:* Information in columns two, three and four collected from emails from OSFI on March 30th 2012 and June 13th 2012 (OSFI 2012a; OSFI 2012b). Information of the last four columns acquired from OSFI Annual Reports from 1999-2000 to 2011-2012.
*Note [1]:* Basel I guidelines as amended in 1996 (OSFI 2012a).

At least for the standardized approach a generic procedure for capital adequacy calculation is publically available. Thus, the following description of Basel II as applied in Canada will cover the standardized approach for credit and market risk, and the basic indicator approach for operational risk. This description is provided to shed some light on regulatory banking practices, its complexity and how Basel II capital ($K_{Basel\ II}$) was estimated. $K_{Basel\ II}$ will be compared to the optimized capital ($K_O$), calculated according to the framework developed in this research, which is described in detail in Chapter 3. It is important to clarify that a faithful reproduction of the Basel II MCR calculation for Canadian banks will not be possible after 2008 because OSFI terminated the publication the report:

*Capital Adequacy-Components of Capital* that contained details of credit and market risk-weighted assets in January 2008.[24]

The next sections of this chapter will present a succinct description of the measures of credit, market and operational risk based on the Capital Adequacy Requirement Guideline (CAR). The CAR-A and CAR-A-1 were published in 2007 and are the Canadian guidelines based on *Basel II: International Convergence of Capital Measurement and Capital Standards: A Revised Framework – Comprehensive Version*, a publication released by BCBS in 2006. CAR guidelines are developed by the Office of the Superintendent of Financial Institutions, OSFI, which is the primary supervisory agency in Canada. The OSFI is responsible for the supervision of all federally regulated financial institutions: trust, loan, and insurance companies together with chartered banks since 1987 (OSFI 2012c).

### 2.3.1 Basel II in Canada

Banks following the CAR guidelines for Basel II should continuously meet capital multiple tests, which requires that the total assets divided by the sum of Tier 1 and Tier 2capital, be less or equal to twenty. Total assets include direct credit substitutes for specific off-balance-sheet items such as letters of credit, sale and repurchase agreements. Except with the Superintendent's prior approval, this multiple cannot exceed twenty.[25]All banks operating

[24] There are two interesting quarterly data reports available from 1996-2007 at OSFI website: *Capital Adequacy-Components of Capital* and *Summary Off-balance sheets Items*.
[25] For information about possible exceptions see pages 4-6 in OSFI (2007b).

in Canada should comply with the assets-to-capital multiple (aims to control for leverage), as illustrated in equation 2.3a.[26]

$$Assets - to - Capital\ Multiple: \ 20 \geq \frac{Total\ Assets}{(Tier\ 1 + Tier\ 2)} \tag{2.3a}$$

Banks are also required to continuously maintain a total capital ratio, defined as Risk Based Capital Ratio, equal to eight percent as presented in equation 2.3b,as well as a net Tier 1 capital ratio of four percent, according to equation 2.3c. Total capital is composed of three tiers: core capital (Tier 1), which comprises high quality capital; supplementary capital (Tier 2) which has two sub-categories: hybrid capital (Tier 2A) and limited life (Tier 2B); and Tier 3 capital, which is designed to meet only market risk requirements. Net Tier 1 capital for equation 2.3c is all the capital in this tier discounted by the goodwill and intangible assets such as mortgage servicing rights and purchased credit card relationships, if they exceed five percent of gross Tier 1 capital. Table 2.8 specifies the composition of each tier.

$$Risk\ Based\ Capital\ Ratio: 0.08 \leq \frac{Total\ Capital: Tier\ 1 + Tier\ 2 + Tier\ 3}{RWA\ Credit\ Risk + 12.5(Market\ Risk + Operational\ Risk)} \tag{2.3b}$$

$$Risk\ Based\ Capital\ Ratio: 0.04 \leq \frac{Net\ Tier\ 1\ Capital}{RWA\ Credit\ Risk + 12.5(Market\ Risk + Operational\ Risk)} \tag{2.3c}$$

[26] Assets-to-capital multiple is a key leverage control measure applied for Canada since the 1980s. It is not a Basel II requirement.

**Table 2.8. Capital Tier Composition According to Basel II Regulatory Framework as Applied in Canada.**

| Capital | Description | Components |
|---|---|---|
| Tier 1 | Core capital | Common shares, retained earnings, noncumulative perpetual preferred shares, qualifying innovative instruments[1], non-controlling interest from Tier 1 capital instruments, accumulated net after-tax foreign currency translation, and unrealized loss on available-for-sale equity securities |
| Tier 2A | Hybrid capital | Cumulative perpetual preferred shares, 99-year debentures, non-controlling interests from Tier 2A capital instruments, and general allowances[2] |
| Tier 2B | Limited life instruments | Limited life redeemable preferred shares, capital instruments issued in conjunction with a repackaging arrangement, other debentures and subordinated debt (which has to be amortized over its life), and non-controlling interests from Tier 2B. |
| Tier 3 | Market risk capital | Subordinated debt with minimum maturity of two years, not redeemable before maturity except with OSFI approval, that does not have to be amortized over its life, and for which the payment of interest or principal will be deferred if it could cause the institution to fall below the minimum capital requirement[3]. |

*Source:* Information collected from *Capital Adequacy Requirements (CAR)-A Simpler Approaches. No: A* by The Office of the Superintended of Financial Institutions Canada (OSFI) 2007.
*Note* [1]: Qualifying innovative instruments cannot exceed 15% of net Tier 1 and have specific restriction to be included in Tier 1. For more information, see appendix 2-I OSFI (2007b).
[2]: General allowances are limited to 1.25% of credit risk-weighted assets with OSFI approval needed.
[3]: For detailed description of the capital instruments and their characteristics see OSFI (2007b) chapter 2.
[4]: OSFI requires the declaration that this table is a reproduction of an official work, published by the Government of Canada, but it is not produced in affiliation with or endorsement of the Government of Canada.

#### 2.3.1.1 Credit Risk Standardized Approach

The credit risk Standardized Approach is calculated according to the same procedures illustrated before in Figure 2.1.[27] Nevertheless, to calculate the assets' credit risk, each asset is to be inserted in the calculation, net of specific allowances to account for risk mitigation for complying with the newer framework. Basel II also requires external credit rating, in

[27] See Figure 2.1 on page 16.

Canada, by one of the four authorized external credit assessment institutions: Dominion Bond Rating Service (DBRS), Moody's Investors Service, Standard and Poor's Rating Services (S&P), or Fitch Rating Services.[28] It is worthwhile mentioning that according to the Canadian credit rating standards, the capacity of the borrower to meet financial commitments varies from *AAA* (the strongest creditworthiness) to *D* (the weakest creditworthiness), and for Basel II guidelines, the lowest ratings specified is *Below B-* as illustrated by the scale in Figure 2.2. On-balance-sheet instruments are subject to a capital charge, which is determined by risk weight categories according to the borrower's creditworthiness as presented in Table 2.9.[29]

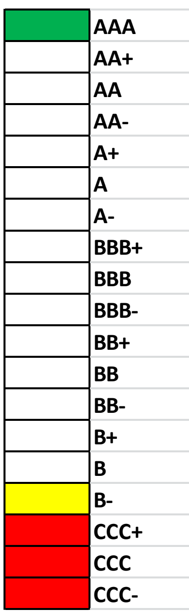


**Figure 2.2. Example of Canadian Credit Rating Scale (S&P 2012).**

[28] A credit rating expresses opinions about the ability of an issuer (corporation, state, or city government for example) to meet its financial obligations in accordance with the terms registered by the underlying contract (S&P 2012).
[29] For more information about difference in the denomination of risk categories among the credit rating agency authorized in Canada see page 52 of OSFI (2007b).

**Table 2.9. Risk-weighted Categories According to Borrower Type and Instrument Type.**

| Category | Credit Assessment | AAA to AA- | A+ to A- | BBB+ to BBB- | BB+ to BB- | B+ to B- | Below B- | Unrated |
|---|---|---|---|---|---|---|---|---|
| 1 | Claims on sovereign governments | 0% | 20% | 50% | 100% | 100% | 150% | 100% |
| 2 | Multilateral development banks | 20% | 50% | 50% | 100% | 100% | 150% | 50% |
| 3 | Deposit taking institutions, banks, and security firms | 20% | 50% | 100% | 100% | 100% | 150% | 100% |
| 4 | Residential mortgages | 35% | 35% | 35% | 35% | 35% | 35% | 35% |
| 5 | Commercial real estate | 100% | 100% | 100% | 100% | 100% | 100% | 100% |
| 6 | Retail portfolio (person or persons, non-corporate businesses) | 75% | 75% | 75% | 75% | 75% | 75% | 75% |
| 7 | Corporate claims | 20% | 50% | 100% | 100% | 150% | 150% | 100% |
| 8 | Public sector entities (PSE) | 20% | 50% | 100% | 100% | 100% | 150% | 100% |

*Source:* Information collected from *Capital Adequacy Requirement (CAR)-A Simpler Approaches. No:A* by OSFI (2007b), p.28-36.
*Note:* OSFI requires the declaration that this table is a reproduction of an official work, published by the Government of Canada, but it is not produced in affiliation with or endorsement of the Government of Canada.

Although not included in the previous table, mortgage-backed securities are an important instrument category, especially after receiving recognition as one of the factors contributing to the occurrence of the financial crisis of 2008(Booth, Alexander, and Britain 2009).[30] This instrument receives a capital charge of 0% when mortgages are guaranteed by the Canada Mortgage and Housing Corporation (CMHC), 35% when secured against residential mortgages, or 100% if under the National Housing Act mortgage-backed securities programs. It is worth adding that banks have an incentive to do business with

[30] A mortgage-backed security is a derivative whose value derives from unpaid mortgages. This instrument pays to the owner an interest rate usually related to the interest rates that the homeowners are paying on their mortgages.

reliable borrowers since overdue loans will require a much higher capital charge than those listed in Table 2.9. For instance, claims that are overdue by more than ninety days would be risk-weighted 100% or 150%, regardless of the capital charge applied before the claim became overdue.[31]

Some exceptions on the conditions expressed in Table 2.9 are allowed. For instance, claims on the Bank for International Settlements, the International Monetary Fund, the European Central Bank and the European Community were always charged 0%. Another exception is that some multilateral development banks were given 0% risk weight: International Bank for Reconstruction and Development, International Finance Corporation, Asian Development Bank, African Development Bank, European Bank for Reconstruction and Development, Inter-American Development Bank, European Investment Bank, European Investment Fund, Nordic investment Bank, Caribbean Development Bank, Islamic Development Bank, and Council of Europe Development Bank.[32] Furthermore, banks could use the risk scores assigned by the Export Credit Agencies (ECAs) instead of applying 100% for claims on unrated sovereigns as illustrated in Table 2.10.

**Table 2.10. Capital Charges According to the Export Credit Agencies Risk Scores.**

| Category | ECA 0-1 | ECA 2 | ECA 3 | ECA 4-6 | ECA 7 |
|---|---|---|---|---|---|
| Capital Charge | 0% | 20% | 50% | 100% | 150% |

*Source:* Adapted from *Capital Adequacy Requirement (CAR)-A Simpler Approaches. No: A* by OSFI (2007), p. 28.
*Note:* OSFI requires the declaration that this table is a reproduction of an official work, published by the Government of Canada, but it is not produced in affiliation with or endorsement of the Government of Canada.

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[31] A claim is charged 150%, if more than ninety days overdue and the provisions are less than 20% of the outstanding amount of the loan, or 100% when provisions are in the range of 20-100%.
[32] For detailed information on instruments or borrowers' type not included in this description see chapter 3 OSFI (2007b).

It is clear that if a sovereign had ECA seven, banks would prefer to consider it unrated as in the first line of Table 2.9 and be charged only 100% of the claim instead of 150% according to ECA; however, for any ECA score between zero and three, ECA criteria would require less capital charge.

In parallel, as in Basel I, non-derivative positions should be converted into a credit equivalent by the multiplication of its notional amount by conversion factors (CF).[33] There were no changes in the CF categories in Basel II (0%, 20%, 50% and 100%), therefore, they were maintained as previously presented in Table 2.2.[34] Credit risk on derivative instruments are calculated by the sum of the replacement cost at market value with the add-on amount; the latter is the multiplication of the notional amount of the instrument by the add-on factor that varies according to the instrument's residual maturity and underlying asset. However, the add-on amount categories have been extended for Basel II as presented in Table 2.11. Risk mitigation for derivatives was also recognized in the newer framework through the use of novations and other bilateral nettings.[35]

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[33] Notional amount is the nominal or face value of the instrument.

[34] Table 2.2 is on page 17. For detailed description of all non-derivatives that fit in each conversion factor's categories see pages 36-42 of OSFI (2007b).

[35] A novation is a written bilateral contract under which an obligation to deliver a given currency on a specific date is automatically amalgamated with all other obligations for the same currency and date. For more information on other bilateral nettings, see pages 44-48 of OSFI (2007b).

**Table 2.11. Add-on Amounts According to the Residual Maturity and Underlying Assets or Rates.**

| Category | Residual Maturity | Interest Rate Contract | Exchange-rate and Gold Contracts | Equity Contracts | Precious Metals Contracts[1] | Other Commodities Contracts |
|---|---|---|---|---|---|---|
| 1 | One year or less | 0.0% | 1.0% | 6.0% | 7.0% | 10.5% |
| 2 | Over one year to five years | 0.5% | 5.0% | 8.0% | 7.0% | 12.0% |
| 3 | Over five years | 1.5% | 7.5% | 10.0% | 8.0% | 15.0% |

*Source:* Adapted from *Capital Adequacy Requirement (CAR)-A Simpler Approaches. No: A* by OSFI (2007b), p.73.
*Note* [1]*:* Precious Metals Contracts excludes gold.
[2]*:* OSFI requires the declaration that this table is a reproduction of an official work, published by the Government of Canada, but it is not produced in affiliation with or endorsement of the Government of Canada.

Overall, comparing the rules just described it is clear that the basic structure of the Basel I methodology for credit risk calculation was maintained in Basel II. However, the latter had more risk categories for assets, which varied not only with the borrowers' and instruments' type, but also with the specific credit rating released by an external agency. Additionally, Basel II presented broader and more complex add-on amount categories for derivative positions. The newer framework also recognized the influence of risk mitigation by deducting provisions, novations, and other nettings from instruments' face value. Therefore, Basel II credit risk standardized approach presented strides in relation to Basel I.

#### 2.3.1.2 Market Risk Standardized Approach

Market risk matters for MCR calculations only when the value of trading book assets or trading book liabilities (when the financial institution is the issuer of trading instruments) is at least 10% of total assets and exceeds one billion dollars; otherwise banks are exempt of any capital charge for market risk. Basel II considers market risk the uncertainty of losses in *on*- and off-balance-sheet instruments caused by movements of market prices. In the standardized approach, each market risk category is calculated individually and summed to produce the total market risk capital charge.

There are six market risk categories: interest rate, equity, commodities, foreign exchange, options and credit derivatives. From this list, interest rate and equity positions bear capital charges for specific risk and general market risk; commodities and foreign exchange are subject only to general market risk charge. Options capital charge could be determined by the simplified method or scenario method, which account for the peculiar underlying instrument's capital charge (interest rate, equity, commodity or foreign exchange) incorporated into the option risk charge. Credit derivatives capital charge could include general market risk, specific risk and counterparty credit risk.[36] Specific risk is the uncertainty of loss caused by adverse price fluctuations that take place due to factors related to the issuer. In its turn, general market risk is the uncertainty of loss produced by adverse fluctuations in market prices not related specifically to the issuer.

It is interesting that banks have the opportunity to choose between the standardized approach and the internal model approach for market risk capital charge calculation, for each market risk category. However, when the bank holds significant positions in a specific market risk category, the regulatory agency would require internal models implementation.[37] The next subsections will briefly describe each market risk category and their respective capital requirements, the sum of which is the total market risk capital charge in the Basel II framework as illustrated in Figure 2.3.

[36] Some examples of credit derivatives are total return swap, credit default swap and credit-linked note.
[37] The regulatory agency in Canada is the Office of the Superintendent of Financial Institutions Canada (OSFI).

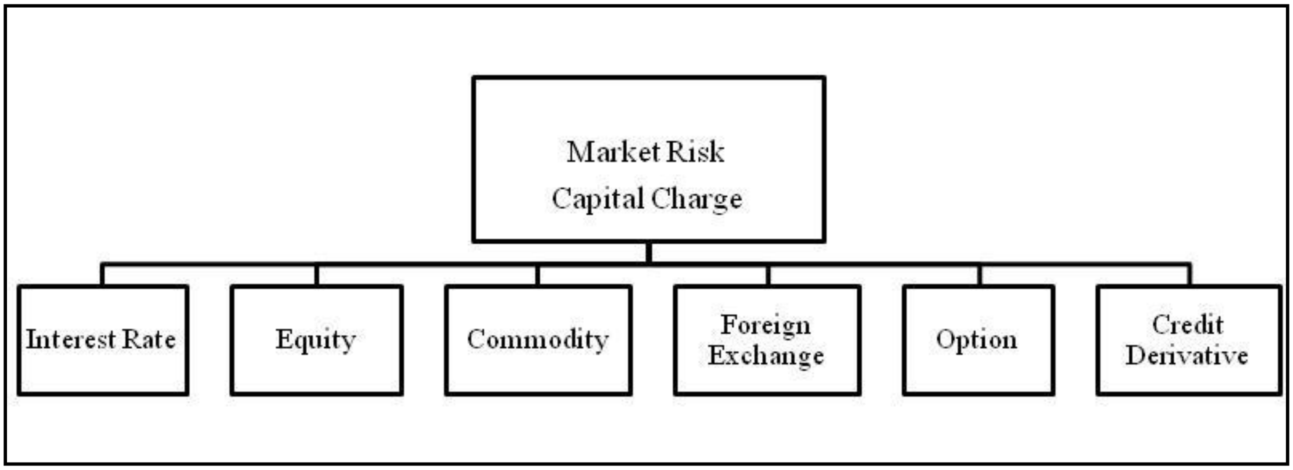


**Figure 2.3. Market Risk Capital Charge Components.**

### 2.3.1.2.1 Interest Rate Risk

The interest rate risk requirement is the sum of the capital charge for specific risk and general market risk for each different currency in the trading book. The specific risk is given by equation 2.3d, for which the risk factor is determined according to the issuer, credit risk rating and residual maturity as presented in Table 2.12. Unlike the rules for credit risk, for Market risk purposes, credit rating agencies are not restricted to DBRS, Moody's Investors Services, Standard & Poor's, Fitch Rating Services. The *Government* category in Table 2.12 includes any instrument issued or guaranteed by the Government of Canada, or the government of a Canadian territory or province.[38] On the other hand, the *Qualifying* category incorporates securities issued or guaranteed by a public sector entity, a multilateral development bank, a bank (since the securities were not counted as capital for this bank), or an authorized security firm in Belgium, Canada, France, Germany, Italy, Japan, Luxembourg, Netherlands, Sweden, Switzerland, United Kingdom or the United States. Finally, the O*ther*

[38] For more details about the government category, see OSFI (2007a), p. 306.

category is composed of securities that do not fit into the *Government* or *Qualifying* categories.

$$Specific\ Risk = (Absolute\ Value\ in\ the\ Trading\ Book)(Risk\ Factor) \quad (2.3d)$$

**Table 2.12. Interest Rate Specific Risk Categories and Respective Risk Factors.**

| Category | External Credit Assessment | Residual Term to Final Maturity | Risk Factor |
|---|---|---|---|
| Government | AAA to AA- | All | 0.00% |
| | A+ to BBB- | Six months or less | 0.25% |
| | | Between 6-24 months | 1.00% |
| | | Greater than 24 months | 1.60% |
| | BB+ to B- | All | 8.00% |
| | Below B- | All | 12.00% |
| | Unrated | All | 8.00% |
| Qualifying | All | Six months or less | 0.25% |
| | | Between 6-24 months | 1.00% |
| | | Greater than 24 months | 1.60% |
| Other | BB+ to BB- | All | 8.00% |
| | Below BB- | All | 12.00% |
| | Unrated | All | 8.00% |

*Source:* Adapted from *Capital Adequacy Requirement (CAR). No: A-1* by OSFI (2007a), p.305.
*Note:* OSFI requires the declaration that this table is a reproduction of an official work, published by the Government of Canada, but it is not produced in affiliation with or endorsement of the Government of Canada.

The general market risk capital charge component uses the maturity method, which consists of the sum of three components: basis risk charge, net position charge, and yield curve risk charge. The following explanation of this element's calculation is complex and would be best understood using a concrete example such as the one presented later on in the text along with its representation in Figure 2.4.[39]

The basis risk capital charge starts the process by inserting the long and short notional principal of each debt instrument, with opposite signs, in the time-bands (columns W and X in Figure 2.4). Positions with similar sign are summed in the same time-band line and multiplied by risk-weight, producing weighted long and short positions in each time-band (Y and Z columns).[40] Basis risk is then calculated by summing all weighted matched positions and multiplying the result by 10% (the absolute value, AV, of each term in the green column A1 is summed).[41] The basis risk charge is an absolute value (does not carry any sign), and the same is true for the second component of general market risk: net position charge. However, for the latter the sum is computed accounting for signs carried by weighted unmatched positions and the absolute value is taken in the end, being the result multiplied by 100% (every term in the pink column A2 is summed, and the AV of this result is multiplied by 1) .

Finally, the yield curve charge is calculated by ignoring the time-bands internal limits of the weighted unmatched (the pink area A2) and considering only the zone limits around it. Then, considering the entire weighted unmatched (A2) positions in zones one, two, and three total intra-zone matched (column B1) and unmatched (column B2) are defined. Intra-zone weighted matched totals of zones 2 (the AV of the yellow box in B1) and 3 (the blue box in

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[39] Basis risk is the uncertainty that arises when the instrument used to hedge specific positions does not fluctuate exactly in the way to offset the losses of the positions taken. Whenever banks have more earning assets than paying liabilities, interest rate risk arises when the market interest rates declines. Nevertheless, banks with negative net positions will experience an increase in income as interest rate arises. The yield curve risk is the uncertainty caused by having assets and liabilities based on different benchmark rates. Thus, banks should evaluate the movement in yield curves and the impact of that to their portfolio value (Hull 2007).

[40] The time-bands and respective risk weights demonstrated in Figure 2.4 are for coupon 3% or more. For information about coupons smaller than 3% see OSFI (2007a), p.310.

[41] A matched is the smaller value between the long and short position. It does not have a sign is the absolute value of the smaller position. The unmatched amount is the difference between the long and short positions, and it maintains the sign of the greatest between positions.

column B1) are summed and multiplied by 30%; this result is added to 40% of the intra-zone weighted matched total of zone 1 (the red box in column B1). Then, the weighted unmatched totals of zones are compared (in column B2): if the zones one and two match, this amount (the orange box in C1), is multiplied by 40% and added to the intra-zones previous summation. The same procedure is repeated for the inter-zones 2-3 and 1-3, the former if matched total being multiplied by 40% (the green box in C1) and the latter by 100% (the black box in C1). The yield curve risk charge is, therefore, the sum of intra- and inter-zones matches as illustrated in Figure 2.4.

An interesting example considers a fictitious bank that holds four different interest rate instruments. Suppose this portfolio is composed of: an interest swap of two hundred million, for which the institution receives floating rate interest and pays fixed, with next interest rate reset after twelve months, remaining life of swap of five years; a long position in interest rate future with market value of eighty million dollars, delivery date after six months, life of underlying government security of 5 years;[42] a government bond at market value of fifty-five million, remaining maturity of two months, and coupon of eight percent; a qualifying bond, worth twenty million, remaining maturity of three years, and coupon of nine percent. The time-bands are fulfilled according to their specification and following definition of the signs and counter entries as specified by the "Position Reporting for the Maturity Method" (OSFI 2007a, 314) as represented in Figure 2.4, which also illustrates the calculations involved to produce the general market risk capital charge for interest risk in this example.

[42] The current interest rate is assumed to be identical to the swap and future instruments are based on.

| Zone | Time-bands | Positions ($ millions) Long (+) W | Short (-) Z | Risk Weights (%) | Risk-weighted positions Long (+) | Short (-) | Positions ($ millions) Matched weighted (A) | Unmatched (A) (- or +) | Zone | Intra-zone ($ millions) Matched weighted (B) | Unmatched (B) (- or +) | Inter-zone ($ millions) Matched weighted (C) | Unmatched (C) (- or +) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | up to 1 month | | | 0,00% | | | | | | | | | |
| 1 | 1-3 months | 55 | | 0,20% | 0,11 | | | 0,11 | 1 | | | | |
| | 3-6 months | | -80 | 0,40% | | -0,32 | | -0,32 | | 0,32 | 1,19 | | |
| | 6-12 months | 200 | | 0,70% | 1,4 | | | 1,4 | | | | | |
| | 1-2 years | | | 1,25% | | | | | | | | | 1,54 |
| 2 | 2-3 years | 20 | | 1,75% | 0,35 | | | 0,35 | 2 | | 0,35 | | |
| | 3-4 years | | | 2,25% | | | | | | | | | |
| | 4-5 years | 80 | -200 | 2,75% | 2,2 | -5,5 | 2,2 | -3,3 | | | | | |
| | 5-7 years | | | 3,25% | | | | | | | -3,3 | 0,35 | -2,95 |
| 3 | 7-10 years | | | 3,75% | | | | | 3 | | | | |
| | 10-15 years | | | 4,50% | | | | | | | | | |
| | 15-20 years | | | 5,25% | | | | | | | | 1,19 | -2,11 |
| | Over 20 years | | | 6,00% | | | | | | | | | |

**Basis Risk Charge**=10%{Σ|(matched time-bands)|}=0.1(2.2)=0.22 $ milions

**Net Position Charge**=100%{| Σ(unmatched time-bands) |} =|0.11-0.32+1.4+0.35-3.3|=|-2.11|=2.11 $ millions

**Yield Curve Risk Charge**= 40%(matched weighted intra-zone 1) + 30%(matched weighted intra-zone 2 + matched weighted intra-zone 3) +
40%[matched weighted inter-zone (1-2) + matched weighted inter-zone (2-3)] + 100%[matched weighted inter-zone (1-3)]
= 0.4(0.32)+0.3(0+0)+0.4(0+0.35)+1(1.19)=1.458 $ millions

**Interest Rate General Market Risk Capital Charge**= Basic Risk Charge+Net Position Charge+Yield Curve Risk Charge = 0.22+2.11+1.458=**3.788 $ millions**

**Figure 2.4. Interest Rate General Market Risk Capital Charge Example.**

#### 2.3.1.2.2 Equity Risk

Equity risk requirement, similarly to interest rate risk, aims to protect the banks against the risk of diverse price movement of the equity positions (whether short or long) relative to the market (specific risk) and against the equity market price fluctuation as a whole (general market risk). Therefore, the equity risk requirement is also calculated by the sum of the capital charge for specific risk and general market risk. Banks must present the calculations for equities of countries other than Canada separately for each country, and consider also exchange rate risk exposure for these foreign positions.

Specific equity risk is calculated by the gross position, which sums the absolute value of all long and short equity positions, considering, however, the net basis whenever the institution holds long and short positions of the same equity. This calculation must also include derivatives, inserted at market value. The capital requirement is 8% of the gross position. If the portfolio could be defined as liquid and well-diversified, the capital charge for equity risk could be reduced to only 4%.[43]

General market risk calculation follows a similar procedure: instruments are valued at market value, and the institution global net position is generated (long minus short positions) for each country in which the institution holds equity instruments. Additionally, options are excluded from this calculation and there are specific treatments for some equity derivatives to include them in the global net position calculation.[44] Then, the capital requirement for general market risk is calculated for each country as 8% of the country's net position.

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[43] For details of the definition of liquid and well-diversified portfolio, see OSFI (2007a), p. 323.
[44] For information on the equity derivatives calculations, see OSFI (2007a), p. 324-6.

#### 2.3.1.2.3 Foreign Exchange Position Risk

The calculation of the minimum capital requirement for holding or taking foreign currency positions and gold is executed by the *shorthand* method. Although gold is in fact a commodity, it is treated as a foreign exchange position due to the fact that its volatility is more similar to foreign currencies than commodities (OSFI 2007a, 327). The capital requirement is applied to the trading and non-trading book. First, the exposures in a single currency are measured. Then, the capital charge for the portfolio in different currencies is calculated, which is defined as the maximum between the net open long positions and the net open short positions in each currency, which is converted at spot exchange rate into Canadian dollars.[45] This converted maximum is summed with the absolute value of the net open position in gold. This sum is then multiplied by 8% to generate the foreign exchange capital charge.

#### 2.3.1.2.4 Commodity Risk

The standardized approach for commodity market risk measurement uses the *simplified measurement* method, which determines a specific capital charge on the net and another charge on the gross position in each commodity category. The net open positions (long minus short) of spot and forward commodities are expressed in standard units of measurement (for instance barrels, kilos), and different commodities may not be offset in the calculation. Each commodity is converted at market spot rates into Canadian dollars. After

[45] Spot exchange rate is the instantaneous exchange rate for immediate delivery.

this conversion all converted net open positions are summed and multiplied by 15%, generating the net open commodity position capital charge.

The gross position requirement aims to protect against basis risk, interest rate risk, and forward gap risk.[46] To account for those risks, institutions' gross positions are summed for a particular commodity (long plus short positions), converted to Canadian dollars, summed with converted gross positions of all commodities held, and finally multiplied by 3% to generate the gross position capital charge. The commodity risk capital charge is the sum of net position and gross position capital charges.

#### 2.3.1.2.5 Option Risk

Options contracts incorporate the risk of the underlying assets involved such as commodities, exchange rate and stocks. There are two procedures in the standardized approach to calculate the capital requirement: the *simplified* method, which is designed for institutions that purchase options but do not write them, and the *scenario* method, which is appropriate for banks that not only buy but also write options. In this section the *simplified* method will be explored.[47]

Overall, the calculation of the option capital charge depends on the strategy implemented by the institution, which are illustrated in Table 2.13. The option capital charge has two components: one intrinsic to the option contract and other related to the underlying asset. Whenever the option is *in the money* the estimated profit, if the institution exercises

[46] Basis risk is the uncertainty in a hedge strategy if the price of the instrument used to hedge does not fluctuate identically, or does not perfectly offset the price volatility of the asset being hedge - see Hull (2007), p.45-71. Forward gap risk is the risk that the forward price may change for reasons other than a change in the interest rates. See Hull (2007), p. 97-124.
[47] For a description of the scenario method see OSFI (2007a), p. 333-6.

the option, is the option risk; when the option is *out of the money*, this risk is considered zero. The capital charge regarding the underlying asset is calculated following guidelines of the four previous subsections.[48]

**Table 2.13. Option Market Risk Capital Charge – Simplified Method.**

| Position | Calculation |
|---|---|
| Long the underlying instrument and long the put option | Capital Charge= (Underlying asset price)(quantity)(specific risk% + general market risk%)- (amount option is in the money, or zero)(quantity) |
| Short the underlying instrument and long the call | See immediately above procedure. |
| Long call, or long put option | Capital Charge=Min[(Underlying asset price)(quantity)(specific risk% + general market risk %), (market value of the option)(quantity)] |

*Source:* Adapted from *Capital Adequacy Requirement (CAR). No:A-1* by OSFI (2007a), p.332.
*Note:* OSFI requires the declaration that this table is a reproduction of an official work, published by the Government of Canada, but it is not produced in affiliation with or endorsement of the Government of Canada.

#### 2.3.1.2.6 Credit Derivative Risk

Credit risk derivative capital requirement involves different entries depending on whether the institution is the guarantor or the beneficiary of the instrument. Its calculation will internally apply the interest rate risk capital charges, holding also a potential counterparty credit risk charge as specified in Table 2.14.

[48] *In the money* option means that it is worth exercising the option, it may produce gains after accounting for the option cost. While an option *out of the money* is one that is not worth exercising. For an example of option market risk charge see OSFI (2007a), p. 332.

**Table 2.14. Credit Derivative Capital Charges.**

| Instrument | Risk Type | Guarantor | Beneficiary |
|---|---|---|---|
| Total Return Swap | Specific Risk | Long positions in the reference assets | Short positions in the reference assets |
| | General Market Risk | Long or short position in the reference asset and a short or long position in the notional bond | Short or long position in the reference asset and long or short position in the notional bond |
| | Counterparty Credit Risk | Add-on factor | Add-on factor |
| Credit Default Swap | Specific Risk | Long positions in the reference assets | Short positions in the reference assets |
| | General Market Risk | Usually no risk charges | Usually no risk charges |
| | Counterparty Credit Risk | Add-on factor required for some transactions | Add-on factor |
| Credit-linked Note | Specific Risk | Long position in the reference assets + long position on the note issuer | Short position in the reference assets |
| | General Market Risk | Long position in the note | No risk from market movements |
| | Counterparty Credit Risk | No counterparty risk | No counterparty risk |

*Source:* Adapted from *Capital Adequacy Requirement (CAR).No: A-1* by OSFI (2007a), p.303.
*Note:* OSFI requires the declaration that this table is a reproduction of an official work, published by the Government of Canada, but it is not produced in affiliation with or endorsement of the Government of Canada.

The specific risk capital charge is the result of the multiplication of the absolute values of the derivative position (*mark-to-market*) by respective risk factors as described before in Table2.12 taking a residual maturity of 6-24 months, and the appropriated category (*Government*, *Qualifying*, or *Other*).[49] Netting of positions of the same category is allowed

[49] The *mark-to-market* value is the most current market valuation of an instrument.

under special conditions.[50] The general market risk capital charge is calculated following the *maturity* method of interest rate risk explained in section 2.3.1.2.1. The total credit derivative risk charge is the sum of the specific risk, general market risk, and counterparty credit risk if applicable.

#### 2.3.1.3 Operational Risk Basic Indicator Approach

Every bank applying the Basic Indicator Approach must hold a fixed percentage (alpha, equal to 15%) of its average gross income, GI, of the last three years to offset unpredicted losses caused by operational risk. GI is determined by the sum of net interest income and net non-interest income, which are defined according to the Canadian General Accepted Accounting Principles (CGAAP). The gross income should not count any realized profits or losses from the sale of securities in the bank book (usually specified in the accounting reports as *held to maturity* and *available for sale*), extraordinary or irregular items, or any income derived from insurance; it should also be gross of provisions (unpaid interest, for instance) and operating expenses. Additionally, any negative or zero annual gross income is excluded from the calculation as presented by equation 2.3e.

$$\textit{Basic Indicator Approach}: \qquad Capital = \frac{\sum_{i=1}^{n}[\alpha(GI_n)]}{n} \tag{2.3e}$$

[50] For conditions are specified, see OSFI (2007a), p. 338-9.

Every bank implementing this approach for operational risk is also encouraged by OSFI to comply with the BCBS's guidelines released in February 2003 entitled: *Sound Practices for the Management and Supervision of Operational Risk*.

### 2.3.2 Basel I and II Brief Comparison

There are several similarities between Basel II (revised in 2006) and its precursor, Basel I (first released in 1988). Both use the same standard capital ratio for bank minimum capital adequacy, which is required to be at least eight percent. If we consider the amended version of Basel I (released in 1996), its definition of regulatory capital was maintained in Basel II (the numerator of the ratio was unaltered and equal to the sum of Tier 1, 2 and 3) (Larson 2011).

Despite these commonalities, Basel II creates a new three-pillar structure to reinforce minimum capital requirements (MCR) already present in Basel I, and implements supervisory oversight and market discipline. Considering that market risk was incorporated into Basel I in the amendment of 1996 and restricting the following analysis only to the standardized approach, Basel II is more risk sensitive and complex than the first Accord since it incorporated operational risk into the framework. The Basel II MCR calculation also extended the risk mitigation deductions in bank exposures, improving this aspect of the Accord. Another innovation was the incorporation of the market risk for derivatives, targeted to solve one of the flaws of Basel I pointed out in the literature as mentioned in subsection 2.2.1.

As previously described, the newer framework also implements more risk categories for assets and derivatives; the former varying not only with the borrowers' and instruments' type but also with the specific credit rating released by an external agency; the latter incorporating different commodities and extended residual maturities for credit risk measurements. These developments enriched the quality of risk measurement of each instrument held, addressing part of the *intra-* and *inter*-risk-category inconsistencies, shortcomings highlighted in the discussions of Basel I in section 2.2.1. Nevertheless, this progress was not enough to prevent regulatory arbitrage, especially considering the existence of an unrated risk category, which requires less capital charge than some instruments rated *below B- or BB-*, motivating riskier firms to stay unrated and banks to incur higher risks without being penalized by a greater capital charge (Crouhy, Galai, and Mark 2006, 55-82).[51]

Although it accounted for diversification effects in the trading book, the newer accord still fails to recognize diversification and correlations in the banking book and between trading and banking books.[52] Therefore, albeit to a lesser degree than Basel I, Basel II still maintains its precursor's shortcoming, regarding the diversification effects. Basel II also failed to impose a stricter definition of, or reform, the regulatory capital, allowing unbalanced competition among countries and therefore not completely solved another criticism emphasized by the reviewed literature on Basel I (Section 2.2.1).

Basel II was implemented less broadly than its predecessor, and an important example of that is the US. This lower popularity is credited to its high complexity, the

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[51] This unrated versus low credit rating inconsistencies are clearly illustrated in Table 2.9 and 2.12.

[52] Trading book is composed of instruments or commodities held for trading or hedge purposes, which are tradable and can be disposed of in the market at their market value such as stocks, commodities and government securities – all these instruments are subject to market risk. Basel II framework openly accounts for diversification while measuring Market Equity Risk, as described in section 2.3.1.2. Banking book consists of items that are not acquired for trading purposes. Banking book components are presumed to be held until maturity mainly because they are less liquid than trading book items, and usually bear credit risk (Allen & Overy 2009).

requirement for more advanced risk management expertise for banks and regulators alike, and the high costs associated with developing internal models or obtaining credit ratings.

Analyzing all shortcomings presented in section 2.2.1, Basel II was successful at targeting the market risk of derivatives. It improved detection and accounted for diversification in the trading book, but there is still room to account for it in the banking book. The framework partially addressed risk inconsistencies within and between the risk categories, yet it still allows regulatory arbitrage by requiring less capital for unrated entities than the percentage charged of those presenting low credit ratings. Basel II still fails to impose a stricter definition of regulatory capital and to explain the reasons for maintaining a standard capital ratio of eight percent.

### 2.3.3 Basel II Final Remarks and Consequences

Besides the shortcomings inherited from its precursor, Basel II also received its own specific criticism, for instance including operational risk into pillar one (minimum capital requirement) instead of under pillar two (supervisory oversight) was claimed as an inconsistency by some authors (Bulletin 2003, 1-9; Rodriguez 2003, 115-126). Another flaw emphasized in the literature is the reliance on credit ratings (for standardized approach) and value-at-risk metrics, VaR, (for internal models) as accurate measures of credit and market risks. Some critics have claimed that these are not accurate risk measures for two main reasons: the quality and rigor of the assessment, and the conflict of interest inherent to the process. It is worth adding that the conflict of interest arises in the process of obtaining a credit rating in two situations: the rating agencies are hired by those they produce the ratings for, and the internal models are designed by the banks themselves, which have a clear motivation to make the capital charges as small as possible (Larson 2011, 1-31; Preston 2010, 20; Triana 2010, 45-48; Magazine 2008, 12-20). An additional drawback is that since risk weights are grounded on performance expectations (whether using rating reports or VaR), the framework generates what Benink, Danielsson and Jonsson (2008) defined as endogenous risk – the risk that emerges when the action of some market participants creates similar expectations and influence others, exerting impact on market prices. The main consequence is a pro-cyclical behavior, which induces banks to withdraw credit just before and during recession periods, and to overly extend it in good times. This pro-cyclical behavior amplifies the fluctuation of the economic cycle, provoking credit crunches in recessions and over inflating credit in recovery times (Gordy and Howells 2006 ,395-417; Balin 2008, 1-17; Benink, Daníelsson, and Jónsson 2008, 85-96; Ojo 2010, 305-315; Danielsson, Shin, and Zigrand 2011, 1-36). Moreover, since the risk-weighted assets (RWA) metric is already smaller than total assets and the regulatory capital is eight percent of RWA, the MCR calculated by Basel II is very small, permitting high *leverage*. This small requirement allows banks to rely on borrowing to fund their operation and hold little capital against unpredictable losses (Ojo 2010, 305-315; Triana 2010, 45-48).[53]

Additionally, there are issues related to the adequacy of the skills and incentives of regulators to monitor bank operations, especially regarding the approval of the internal models. This aspect coupled with the high regulatory discretion conceded to the regulatory agencies suggests that Basel II could have contributed to regulatory forbearance and even corruption (Rodriguez 2003, 115-126; Balin 2008, 1-17). A topic that remained untouched in the reviewed literature, but one that deserves criticism is that, although Basel II has required

[53] *Leverage* is a measure of the amount of debt that the bank uses to finance assets. A high leverage means that a bank is using significantly more debts compared to the capital held to finance its activity.

banks to publicly provide more information through the income statement and balance sheet enforced by pillar three (market discipline), it made no requirements regarding off-balance-sheet reports, which opens opportunities for hiding risks from the public, making it unfeasible for shareholders, depositors and firms to effectively measure the real risks incurred by banks.

Considering that minimum capital requirement is also designed to provide an early warning to regulators and market participants of banks' solvency, the failure of many banks in the financial crisis of 2008 revealed that Basel II, as implemented, did not work properly in this regard. A clear example of inadequate risk-sensitivity in the Basel II framework was the Northern Rock Bank case in the UK. This institution held high-risk instruments which caused it to fail in early 2008. On the occasion of its default, this bank held capital exceeding Basel II requirements. This highlights the fact that the framework in place measured (weighted) risks improperly. As a result, capital requirements were overly low to provide sufficient buffer to avoid the crisis, and the desired warning signal in the case of a bank in trouble (Ojo 2010, 305-315).

Overall, criticism on all the approaches imposed for Basel II to calculate MCR has increased since the crisis of 2008, putting in question their credibility as risk management techniques (Grossman 2010, 251-289). This crisis corroborated the claim that capital levels have eroded, allowed by a regulatory framework that underestimated regulatory capital, which made banks unable to absorb losses in stress conditions (King and Tarbert 2011, 1-18).

To respond to this crisis, the third Accord has been released in two publications: *Basel III: A Global Regulatory Framework for more Resilient Banks and Banking Systems*, and *Basel III: International Framework for Liquidity Risk Measurement, Standards and*

*Monitoring* both prepared in 2010, by the Basel Committee on Banking Supervision (BCBS).[54] It is noteworthy that even the BCBS (2011) recognized Basel II's failure in determining an effective MCR by stating that:

> The global banking system entered the crisis with an insufficient level of high quality capital. The crisis also revealed the inconsistency in the definition of capital across jurisdictions and the lack of disclosure that would have enabled the market to fully assess and compare the quality of capital across jurisdictions. A key element of the new definition of capital is the greater focus on common equity, the highest quality component of a bank's capital (p. 12).

Basel III implementation is scheduled to commence in 2013. The document concerning capital requirements finally reforms the definition of regulatory capital, even eliminating the Tier 3 component.[55] The framework also requires a capital conservation buffer and a countercyclical buffer, as well as the regulatory capital that maintains the definition of eight percent standard capital ratio, against asset losses. However, the newer accord has already received strong criticism as discussed by King and Tarbert (2011):

> There are those who believe aspects of the reforms outlined above will hamper economic recovery. […] Others have suggested more radical reform and a doing away with the risk-weighted approach altogether (p. 3).

This passage highlights that BCBS has not convinced some authors of the effectiveness of reforms proposed for capital adequacy, or that this proposal will make recovery from the crisis any less difficult. Despite the possible benefits of these additional buffers, and the reform in the regulatory capital definition, the Basel III framework still fails to fix the procedure for determining MCR. Specifically it has not changed the risk metrics used, which still rely on value-at-risk (VaR) or credit rating reports, which proved to be unreliable during the crisis (Preston 2010; Triana 2010, 45-48). An alternative solution to

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[54] *Basel III: A Global Regulatory Framework for more Resilient Banks and Banking Systems* was reviewed in June 2011.
[55] A shortcoming highlighted since the release of the first version of the Accord (1988).

improve MCR calculation would be one that eliminates the risk-weighted asset concept and reforms the entire framework rather than simply adding extra buffers. The present research proposes an innovative risk management framework to strengthen the minimum capital requirement. This optimized capital aims to be less complicated and more risk sensitive than the Basel II framework. The development of the proposed framework is described in Chapter 3.

# 3 Chapter: Optimization Framework

The proposed risk management framework consists of an optimization problem, which involves the minimization of the bank risk function subject to a return on equity (ROE) constraint. This constraint is imposed to give the developed framework a real world application that accounts for important bank profitability metrics (ROE), which is publicly available in Canada.[56] The methodology used to develop this framework involves three stages. In the preliminary stage, inputs of simplified balance sheets and income statements of banks are used to generate a bank profit equation, which is then expanded using an accounting equation that states that total assets are equal to liabilities plus the owner's equity (Jensen 2007, 25-73).

The second stage derives a bank risk function that considers the sources of risk to bank operations: risks from the loan portfolio (risks of the banking book); risks related to the trading book (risks incurred in trading assets); volatility of deposits that might cause an early sale of an asset at an adverse price; and volatility of noninterest income that can vary according to the volume of new contracts generated (such as investment management, chequing account fees). These risk sources are not controllable by banks and have the potential to produce losses. This stage applies the concepts of Modern Portfolio Theory to derive the bank risk function, defined as the variance of bank returns (or bank profit). The procedure employed to calculate this variance acknowledges all possible linear dependence among these risk sources, making no assumptions about the linear dependence (covariance) between different risks.

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[56] ROE aggregate of the banking sector is recorded in OSFI Annual Reports, which are available at: http://www.osfi-bsif.gc.ca/osfi/index_e.aspx?DetailID=647.

The third stage consists of a constrained optimization (minimization) of the bank risk equation (object function) subject to a ROE constraint. The constrained optimization applies the Lagrange Multiplier technique (Yang 2008, 79-89), and is solved using Maple 15 (math software). The solution is programmed using Stata 11 (statistical software), producing the main outcome of this development: the constrained optimized capital ($K_O$).

After executing this framework to generate $K_O$, a second objective of the present research is to compare this estimate to the real capital held by banks ($K_{Market}$), and the capital that would be required by Basel II ($K_{Basel\ II}$), using historic data from 2000 to 2010. It is important to mention that from 1997 to 2007 Canada followed Basel I, implementing Basel II only in 2008 (OSFI 2007a, 1-360; OSFI 2007b, 1-141; OSFI 2001, 1-4). Therefore, this study considers a hypothetical application of Basel II in Canada from 2000 to 2007, by calculating $K_{Basel\ II}$ for those years.

The following sections provide a detailed description of each stage of the proposed framework. Section 3.1 describes the formulation of the bank profit equation based on current accounting procedures. Section 3.2 derives a bank risk function following Modern Portfolio Theory as proposed by Markowitz (1952). Section 3.3 focuses on the minimization of the bank risk function subject to the ROE constraint.

## 3.1 The Bank Profit Function

The profit equation of any business is calculated based on total income minus total expenses. The income statement of a bank demonstrates its profit, or gross income ($\pi$) as a function of five main factors: interest income, interest expenses, noninterest income, noninterest expenses, and other income. An expression for bank profits, $\pi$, and its expansion is represented in Figure 3.1.

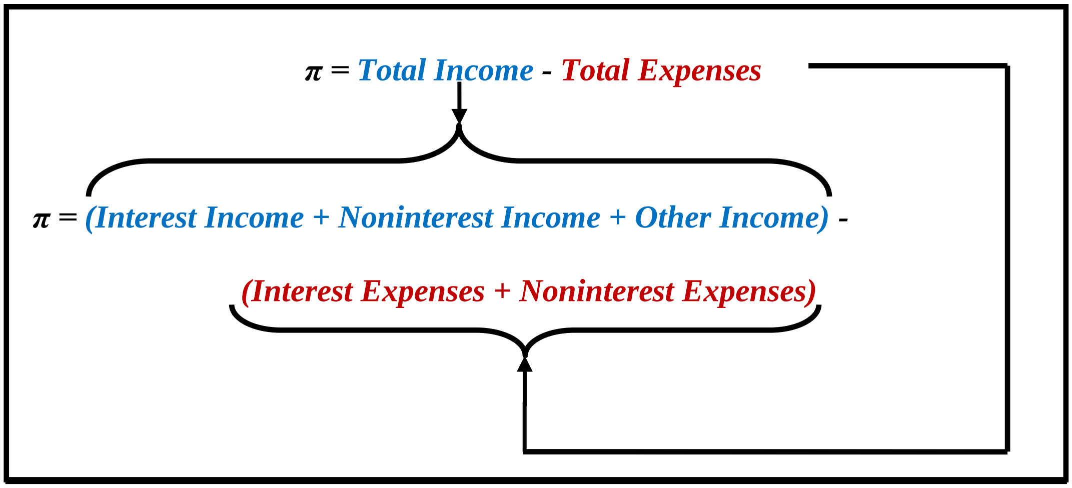

**Figure 3.1. Accounting Bank Profit Equation and Its Expansion.**

A simplified bank balance sheet is composed of total assets, which is the sum of total loans ($L$) plus other assets ($O_A$); liabilities, which consist of deposits ($D$) and other liabilities ($O_L$); and owners' equity, or capital ($K$). Based on the accounting equation, total assets must be equal to liabilities plus owners' equity (capital) as represented in equation 3.1a.

$$L + O_A = D + O_L + K \quad (3.1a)$$

Equation 3.1a does not completely represent bank operations, since off-balance-sheet instruments are not represented. An extensive body of literature shows that accounting is lagging in relation to banking practices, and that the objective of computing capital adequacy would be better accomplished if it included off-balance-sheet instruments together with the other inputs and outputs of banking operations in equation 3.1a (Larson 2011, 1-31; Crouhy, Galai, and Mark 2001, 1-43). This issue is also discussed by the Basel Committee on Banking supervision (BCBS 1986), which claims that:

> individual types of risk associated with most off-balance-sheet business are in principle no different from those associated with on-balance-sheet business. It therefore suggests that off-balance-sheet risks cannot and should not be analysed separately from the risks arising from on-balance-sheet business, but should be regarded as an integral part of banks' overall risk profiles (p. 1-2).

In order to implement BCBS's suggestion above, it would be desirable to include off-balance-sheets, $O_{BS}$, in the accounting equation 3.1a. However, aggregate off-balance-sheet figures are available only from 1996-2007 in the *Summary Off-balance Sheet Items* report on the OSFI website, and even this available material is difficult to consistently translate into inputs and outputs of bank operation because no sufficient specification of the instruments is provided. As a result, even acknowledging the importance of including off-balance-sheet items, this goal is not feasible with the data available, and the present research does not include $O_{BS}$ figures in the framework developed. Nevertheless, further exploration to include $O_{BS}$ items would be an interesting topic for further research.

Combining risk management and economic concepts, a gross profit function considering banks' inputs ($D$, $O_L$, and $K$), outputs ($L$ and $O_A$), and their returns was developed. This profit function is based on the following assumptions:

I. Accounting equation is valid, and allows the isolation of $O_A$ by rearranging terms as illustrated in equation 3.1b.

$$O_A = D + O_L + K - L \tag{3.1b}$$

II. All inputs ($D$, $O_L$, and $K$) and outputs ($L$ and $O_A$) are assumed to be greater than zero.

III. Banks convert all funding inputs into loan, $L$, or other assets, $O_A$. In other words, reserves are not contemplated into this profit function.

IV. Capital in this framework is composed of capital stock (common shares plus preferred shares), contributed surplus, retained earnings, non-controlling interest in subsidiaries, and accumulated other comprehensive income or loss.

V. Everything that banks have control over is considered deterministic. $D$ and $O_A$ is the only non-deterministic component of equation 3.1b. Considering the inputs, banks can define the amount they borrow $O_L$, and how much to rise as capital, $K$; therefore, both are considered constant. The exception among inputs in the framework is deposits, $D$. Since banks do not have real control of how much and when a client will withdraw from savings or chequing accounts, $D$ is the only input considered uncontrollable (a risk source). Among the outputs, $L$ is assumed to be constants during the year because banks can control the amount to loan out; nevertheless, since $O_A$ is given by equation 3.1b, it is idiosyncratic as a result of $D$ randomness.

VI. Banks cannot control noninterest income, $N_I$. They cannot accurately predict the flow of clients in and out of their portfolio (generating chequing accounts' fees for example); therefore, this variable is also considered uncontrollable, or a risk source.

VII. The rates of return on deposits, $r_D$, and the rate of return on other liabilities, $r_{OL}$, are assumed constant in a year period. This is because it is assumed to be a competitive environment, and in order to stay operating; all banks need to practice the same remuneration to their depositors and debtors.

VIII. All the other returns, $r_L$, $r_{OA}$, and $r_T$, represent potential sources of loss (risk) and are uncontrollable by banks. Even considering that for competitiveness banks must all practice the same loan rate, the real rate of return on loans will be different, since banks cannot accurately control when and how much borrowers' defaults will be, such that each bank's loan portfolio will have a different quality and, therefore, distinct return on loans, $r_L$. Since each bank will have a specific other assets and trading instruments portfolio, both $r_T$ and $r_{OA}$ are also considered risk sources.

IX. Interest income is produced by loans, $L$, and other assets, $O_A$.

X. The charge for impairment is totally caused by loans' defaults. So, charge for impairment is deducted from the total income on loans.

XI. Other income (represented by $r_T O_A$) is composed of trading income plus gains or losses on instruments held for other than trading purposes. This type of income is produced usually by off-balance-sheet instruments, gold and silver (the last two are common components of $O_A$). However, considering the lack of data about off-balance-sheet figures, $O_A$ is used as a proxy for [($O_A$)+(*off-balance-sheet*)]. In other words, it is assumed that other income (or trading income) is produced uniquely by $O_A$.

XII. The description of the variables employed in the profit equation is shown in Table 3.1, which also illustrates the controllable nature of each variable, as defined by the assumptions made above.

**Table 3.1. Variables' Description and Their Controllable Nature for Bank Operation.**

| Variable | Description | Controllable by Bank Operation | Uncontrollable by Bank Operation (source of risk) |
|---|---|---|---|
| $D$ | Deposits | | YES |
| $K$ | Capital (owners' equity) | YES | |
| $L$ | Total loans | YES | |
| $N_E$ | Noninterest expenses | YES | |
| $N_I$ | Noninterest income | | YES |
| $O_A$ | Other assets | | YES |
| $O_L$ | Other liabilities | YES | |
| $r_D$ | Return on deposits | YES | |
| $r_L$ | Net return on loans | | YES |
| $r_{OA}$ | Return on other assets | | YES |
| $r_{OL}$ | Return on other liabilities | YES | |
| $r_T$ | Return on trading instruments | | YES |
| $W$ | Random Product 1 ($r_{OA}D$) | | YES |
| $Z$ | Random Product 2 ($r_T D$) | | YES |

The bank profit function is determined by applying these assumptions to the main incomes and expenses illustrated in Figure 3.1, for which some elements are specified by equations 3.1c, 3.1d, and 3.1e, as represented in equation 3.1f. Applying the $O_A$ definition from equation 3.1b into 3.1.f, the bank profit equation is finally expressed in equation 3.1g (after rearrangements). Considering that two terms in equation 3.1g are the product of two sources of risk, they will be substituted as follows: $W$ for the product $r_{OA}D$ and $Z$ for $r_T D$, into equation 3.1h, which is the bank profit function that is carried further in this innovative framework. It is noteworthy to mention that equation 3.1h is similar to the profit function as proposed by McFadden (1978), but inputs' and outputs' prices are substituted in the present framework by inputs' and outputs' returns.

$$Interest\ Income = r_L L + r_{OA} O_A \tag{3.1c}$$

$$Other\ Income = r_T O_A \tag{3.1d}$$

$$Interest\ Expenses = r_D D + r_{OL} O_L \tag{3.1e}$$

$$\pi = \{[r_L L + r_{OA} O_A] + N_I + r_T O_A\} - \{[r_D D + r_{OL} O_L] + N_E\} \tag{3.1f}$$

$$\pi = \{[r_L L + r_{OA}(O_L + K - L)] + N_I + r_T(O_L + K - L) + r_{OA} D + r_T D\} - \{[r_D D + r_{OL} O_L] + N_E\} \tag{3.1g}$$

$$\pi = r_L L + r_{OA}(O_L + K - L) + N_I + r_T(O_L + K - L) + W + Z - r_D D - r_{OL} O_L - N_E \tag{3.1h}$$

### 3.2 The Bank Risk Function

The second stage in developing the framework starts with the recognition of the risk sources that were introduced while describing the assumptions in the previous section (uncontrollable variables in Table 3.1). The sources of risk identified in equation 3.1h are: return on loans ($r_L$), return on other assets ($r_{OA}$), return on trading instruments ($r_T$), noninterest income ($N_I$), deposits ($D$), as well as the products of $r_{OA}$ and $r_T$ with $D$, which are represented by $W$ and $Z$, respectively. In other words, the gross profit equation has seven dimensions: $r_L$, $r_{OA}$, $r_T$, $N_I$, $D$, $W$ and $Z$.

Modern Portfolio Theory defines profit as the expected return, and risk as the variance of returns, commonly characterized in the risk management literature by its standard deviation.[57] Specifically, in a seminal paper, Markowitz (1952) states:

> The concepts "yield" and "risk" appear frequently in financial writings. Usually if the term "yield" were replaced by "expected yield" or "expected return", and "risk" by "variance of return", little change of apparent meaning would result (p. 89).

Working with the Markowitz approach (translating the *expected return* definitions into the present exercise) the bank gross profit is given by expected profit, $E[\pi]$, represented in equation 3.2a, in which the expected value of the risk sources is represented by the variables with subscript $_*$.

$$E[\pi] = r_{L*}L + r_{OA*}(O_L + K - L) + N_{I*} + r_{T*}(O_L + K - L) + W_* + Z_* - r_D D_* - r_{OL}O_L - N_E \quad (3.2a)$$

[57] Standard deviation is the square root of the variance of a random variable.

Additionally, the variance of $\pi$ (*variance of return*), calculated considering the aforementioned risk sources, represents the bank risk function. Under the variance interpretation of bank risk, the risk sources play the role of random variables for calculation of the variance of profit. In this exercise, no assumption of independence of the risk sources is made, to allow for all possible interactions, which is a fair picture of interdependence of financial risks. Nevertheless, considering all possible correlations among identified risks makes deriving the variance of bank profit ($Var[\pi]$) non-trivial. According to Wooldridge (2009), when there is no assumption of independence among idiosyncratic variables, the equation for the variance is given by equation 3.2b. In this equation, instances of $X_i$ are the idiosyncratic variables or the risk sources in the present exercise, represented by: $r_L$, $r_{OA}$, $r_T$, $N_I$, $D$, $W$ and $Z$. $Var(X_i)$ stands for the variance of the risk source $X_i$, and $Cov(X_i,X_j)$ represents the covariance between $X_i$ and $X_j$.

$$Var[\sum_{i=1}^{n}(a_i X_i)] = \sum_{i=1\ j=1}^{n} \left(a_i{}^2 Var(X_i) + 2a_i a_j Cov(X_i, X_j)\right) \qquad (3.2b)$$

The direction provided by equation 3.2b is used to calculate the bank risk function, (the variance of profits), applying the classification of the variables according to their controllable nature, as illustrated in Table 3.1. The controllable variables are assumed to be constant, and the uncontrollable variables treated as idiosyncratic variables for determining the variance of profit equation. This bank risk function, which is illustrated by equation 3.2c, will be taken further in this innovative framework. It is important to mention that the risk management literature usually represents the variance of a risk variable, for instance $r_L$, by $\sigma_{r_{L^2}}$, and the covariance of this variable with $r_T$, by $\sigma_{rTrL}$.

$$Var[\pi] = L^2 var(r_L) + (O_L + K - L)^2 var(r_{OA}) + var(N_I) + (O_L + K - L)^2 var(r_T) + r_D{}^2 var(D) + var(W) + var(Z) + 2L(O_L + K - L)cov(r_L, r_{OA}) + 2Lcov(N_I, r_L) + 2L(O_L + K - L)cov(r_L, r_T) - 2Lr_D cov(D, r_L) + 2Lcov(r_L, W) + 2Lcov(r_L, Z) + 2(O_L + K - L)cov(N_I, r_{OA}) + 2(O_L + K - L)^2 cov(r_{OA}, r_T) - 2r_D cov(D, Z) + 2cov(W, Z) - 2r_D(O_L + K - L)cov(D, r_{OA}) + 2(O_L + K - L)cov(r_{OA}, W) - 2r_D cov(D, W) + 2(O_L + K - L)cov(r_{OA}, Z) + 2(O_L + K - L)cov(N_I, r_T) - 2r_D cov(D, N_I) + 2cov(N_I, W) + 2cov(N_I, Z) - 2r_D(O_L + K - L)cov(D, r_T) + 2(O_L + K - L)cov(r_T, W) + 2(O_L + K - L)cov(r_T, Z) \qquad (3.2c)$$

It is important to clarify that assuming controllable variables as constant is a simplification justified by the modeling process to keep equation 3.2c simpler than it would be otherwise. In reality, $K$, $O_L$, and $L$ do vary. The application of this type of simplification has been endorsed by Crouhy, Galai and Mark (2006) in their proposition:

> Real life is complicated and includes many details that models cannot and maybe should not, accommodate. The role of models is to highlight the most important factors and the relationships among these factors. A good financial model is one that helps separate […] the major explanatory variables from a noisy background (p. 109).

The framework developed in this research focuses on the identified risk sources ("*the major explanatory variables*"), separating these variables from those controllable by bank ("*the noisy background*"), to allow for a risk management interpretation of banking operations that reflects only the relevant components of banking risk taking to further relate them to the regulatory capital.

## 3.3 Constrained Optimization of the Bank Risk Function

The third and last stage of this framework is the optimization itself. The bank risk equation can be considered a univariate quadratic function of *K*, which is assumed to be continuous (differentiable at every point). Its first derivative is also assumed to present this property. Applying this univariate assumption, it is adequate to define equation 3.2c, the bank risk function, as $Var[\pi(K)]$.

For this function, the convexity of the curve is defined by the second-order condition (second derivative of the function) as summarized in Table 3.2.

**Table 3.2. Definition of the Convexity of the Objective Function.**

| Condition | Convex | Concave |
|---|---|---|
| 2nd-order insufficient | $Var[\pi(K)]'' \geq 0$ | $Var[\pi(K)]'' \leq 0$ |
| 2nd-order sufficient | $Var[\pi(K)]'' > 0$ | $Var[\pi(K)]'' < 0$ |
| [1] Function shape | U-Shape | ∩-shape |

*Sources:* Adapted from *Fundamental Methods of Mathematical Economics* by Alpha C. Chiang and Kevin Wainwright, 4th Edition (p. 235), and *OPMT 5701 Lecture Notes One variable Optimization* by Kevin Wainwright (2006).
*Note [1]*: when the 2nd-order sufficient holds for all *x*.

The analysis of the first and second-order conditions of the bank risk equation, $Var[\pi(K)]$, are for a descriptive exercise, recommended. The first-order condition is given by $Var[\pi(K)]'$, which is presented by equation 3.3a.

$$Var[\pi(K)]' = \{2Lcov(r_L, r_T) + 2Lcov(r_L, r_{OA}) - 2r_D cov(D, r_{OA}) - 2r_D cov(D, r_T) + 2(K + O_L - L)var(r_{OA}) + 4(K + O_L - L)cov(r_{OA}, r_T) + 2(K + O_L - L)var(r_T) + 2cov(N_I, r_{OA}) + 2cov(r_T, W) + 2cov(r_T, Z) + 2cov(r_{OA}, W) + 2cov(r_{OA}, Z) + 2cov(N_I, r_T)\} = 0 \quad (3.3a)$$

Solving equation 3.3afor $K$ will define the stationary point, $(K_s, Var[\pi(K_s)])$, as determined in equation 3.3b.

$$K_s = \frac{-1}{var(r_{OA}) + var(r_T) + 2cov(r_{OA}, r_T)}\{Lcov(r_L, r_T) + Lcov(r_L, r_{OA}) - r_D cov(D, r_{OA}) - r_D cov(D, r_T) + O_L var(r_{OA}) - Lvar(r_{OA}) + 2O_L cov(r_{OA}, r_T) + cov(r_{OA}, W) - 2Lcov(r_{OA}, r_T) + cov(r_T, W) + O_L var(r_T) - Lvar(r_T) + cov(N_I, r_{OA}) + cov(N_I, r_T) + cov(r_T, Z) + cov(r_{OA}, Z)\} \quad (3.3b)$$

Solution 3.3b is interesting because it relates bank capital to bank risks (variance and the linear dependence among sources of risks, the latter given by the covariance terms), characteristics that would make it ideal for a minimum capital requirement. However, since $K_S$ levels vary between 1.27 to 30.81 times the capitals actually held by banks ($K_{Market}$), it does not have real world application, therefore, will not be further explored in this research.[58]

The second-order condition is given by $Var[\pi(K)]''$, as illustrated in equation 3.3c.

$$Var[\pi(K)]'' = 2var(r_{OA}) + 2var(r_T) + 4cov(r_{OA}, r_T) \quad (3.3c)$$

Applying the conditions specified in Table 3.2, the bank risk function will have a minimum extreme whenever equation 3.3d below holds and a maximum if 3.3e is verified. The characteristics of the bank risk function are explored in Section 5.1.2.

$$Minimum: 2var(r_{OA}) + 2var(r_T) + 4cov(r_{OA}, r_T) > 0 \quad (3.3d)$$

$$Maximum: 2var(r_{OA}) + 2var(r_T) + 4cov(r_{OA}, r_T) < 0 \quad (3.3e)$$

[58] $K_S$ results are presented in Appendix B from 2000 to 2010.

Conditions are simulated to determine the optimized capital ($K_O$) that would minimize the bank risk function, while still considering bank profitability levels as measured by the benchmark return on equity, ROE. Equation 3.3f illustrates the constraint, in which the numerator can be formulated as the expected profit, $E[\pi]$, minus taxes and other deductions, $T_x$, as suggested by equation 3.3g (the subscript $*$ means the expected value of the risk sources).

$$ROE: \qquad \alpha = \frac{Net\ Income_{Period}}{Average\ Capital_{Period}} \tag{3.3f}$$

$$\text{Net Income}_{\text{Year}} = r_{L*}L + r_{OA*}(O_L + K - L) + r_{T*}(O_L + K - L) + N_{I*} + W_* + Z_* - r_D D_* - r_{OL}O_L - N_E - T_X \tag{3.3g}$$

Therefore, the constraint has the formulation expressed in equation 3.3h.

$$ROE: \alpha = \frac{r_{L*}L + r_{OA*}(O_L + K - L) + r_{T*}(O_L + K - L) + N_{I*} + W_* + Z_* - r_D D_* - r_{OL}O_L - N_E - T_X}{K} \tag{3.3h}$$

In equation 3.3h, the value of the input $\alpha$ is the ROE reported by the Office of the Superintendent of Financial Institutions Canada (OSFI). This estimate is annually reported in the OSFI Annual Report, which is represented for the years 2000 to 2010 in Figure 3.2 (OSFI 2005; OSFI 2007c; OSFI 2008; OSFI 2012d). Since the ROE is published as a fiscal year's statistic, for consistency the values of all equations' inputs are introduced in the framework considering a year period from April 1$^{st}$ to March 31$^{st}$.

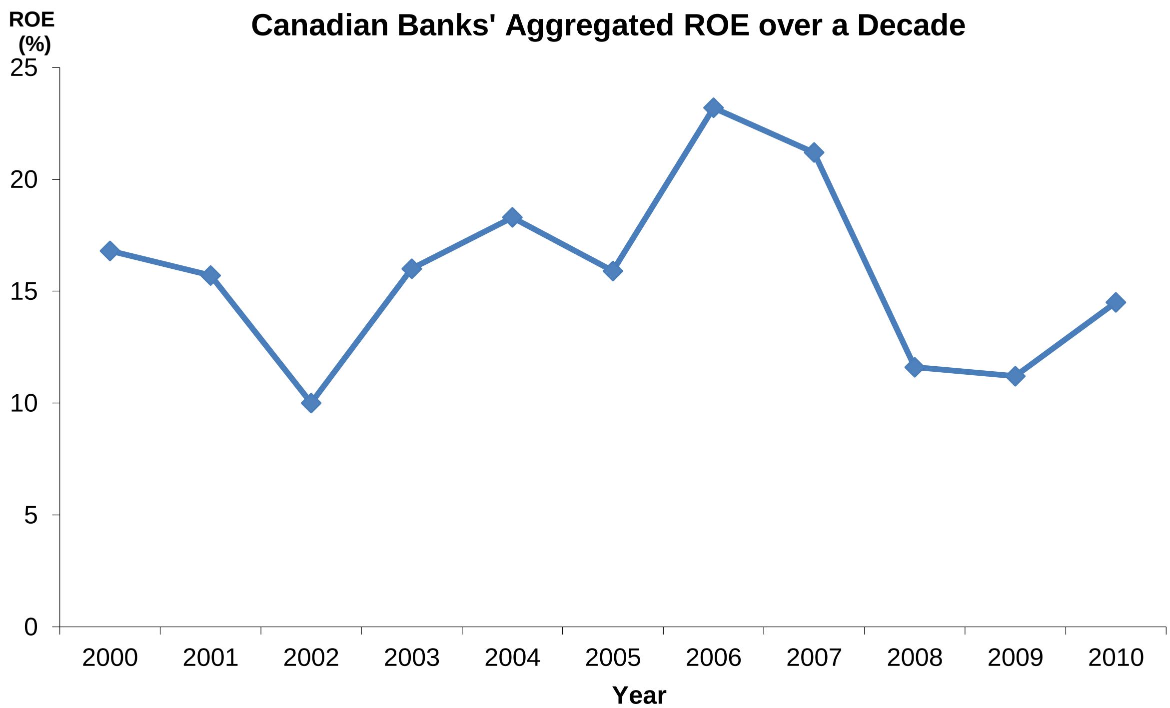


**Figure 3.2. Canadian ROE at Aggregate level for Banks from 2000 to 2010.**

The optimization was solved applying the Lagrange Multiplier Algorithm. A copy of all stages of this math solution is provided in Appendix A. In summary, the solution of this optimization determines $K_O$ as illustrated in equation 3.3i.[59]

$$K_O = \frac{(N_{I*} + r_{L*}L + r_{T*}O_L + r_{OA*}O_L - r_{OA*}L + W_* + Z_* - r_{T*}L - r_D D_* - N_E - T_x - r_{OL}O_L)}{(\alpha - r_{OA*} - r_{T*})} \tag{3.3i}$$

It is interesting to mention that the linear constraint, ROE, governs this optimization result, since $K_O$ is exactly the formulation of solving for $K$ in equation 3.3h. Therefore, $K_O$ as defined in 3.3i has no relationship with any risk measures (any variance or covariance of the

[59] Again the subscript $*$ represents the mean of the risk source for that specific year.

risk sources). This result is intrinsic to the nature of the proposed exercise: a limitation of the optimization of a univariate quadratic function subject to a linear constraint. Important additional information is that the Lagrange Multiplier term gathers all risk measures in a very complex equation, as presented in Appendix A. The Lagrange Multiplier indicates the change in the objective function ($Var[\pi(K)]$) that would occur per unit of change in the left-hand side of the equality constraint ($\alpha$), or in other words by what amount the bank risk function can be reduced if the constraint ROE is relaxed (Mohan and Deep 2009, 6-30).

The $K_O$ results for each year from 2000 to 2010 are presented in Chapter 5, specifically in Section 5.1. This Section also discusses the comparison of $K_O$ with $K_{Market}$ and $K_{Basel\ II}$. The estimates of every input, output, variance, and covariance included in the bank risk function are provided in Appendix B.

# 4 Chapter: Data Collection

As previously mentioned, this entire framework is based on inputs from balance sheets, income statements, and benchmark return on equity (ROE) at the aggregate level of the bank sector in Canada. Monthly balance sheet data and quarterly income statement figures were collected from the online reports *Consolidated Balance Sheet* and *Summary Income Statement* provided by the Office of the Superintendent of Financial Institutions Canada (OSFI) website.[60] These data were selected in the category *Total All Banks*, which includes domestic, foreign and foreign bank subsidiaries operating in Canada. The data were separated into eleven subsets, one for each year from 2000 to 2010. Since ROE is measured for a fiscal year, which in Canada starts on April 1st and ends on March 31st, all annual average values are calculated considering the same limits.

The balance sheet data required for the framework was calculated as follows: other liability ($O_L$) was determined by total liabilities (which in the OSFI balance sheet report include capital) minus capital ($K_{Market}$) minus deposits ($D$). Following the basic accounting equation 3.1a, other assets ($O_A$) was determined by $K_{Market}$ plus $O_L$ plus $D$ minus loans ($L$).

A quarterly income statement report includes figures of *interest income* (which includes income provided by loans and other assets), *interest expenses* (which specifies the amount paid for deposits and for other liabilities), *noninterest expenses*, *charge for impairment*, *deductions*, and *other income* – the latter composed of *trading income*, *gains or losses on instruments held for other than trading purposes*, and *noninterest income*. Income statement data are accumulative in the reports during the year, which means that the fourth

[60] For access to the data collected see http://www.osfi-bsif.gc.ca/osfi/index_e.aspx?ArticleID=554.

quarter's interest income is the sum of all four quarters' interest income in the specific year (from January to December). In order to properly measure the returns, income statement data were transformed into the unitary contribution of each quarter by subtracting previous quarters from the amount registered in the reports. These data were then related to the balance sheet group to produce the inputs for the framework as specified in Table 4.1.

**Table 4.1. Description of Variables' Calculation.**

| Variable | Controllable Nature | Calculation |
|---|---|---|
| $D_*$ | Uncontrollable | Annual average($D$) |
| $K_{Market}$ | Controllable | Annual average($K_{Market}$) |
| $L$ | Controllable | Annual average($L$) |
| $N_E$ | Controllable | Annual average(Total noninterest expenses) |
| $N_I$ | Uncontrollable | Annual average[(Total other income)-(Trading income + Gains or losses on instruments held for other than trading purposes)] |
| $O_A$ | Controllable | Annual average($D$+$K$+$O_L$-$L$) |
| $O_L$ | Controllable | Annual average(Total Liabilities including capital-$K_{Market}$-$D$) |
| $r_D$ | Controllable | Annual average[(Interest expenses on demand and fixed term deposits)/$D$] |
| $r_{L*}$ | Uncontrollable | Annual average[(Interest income on loans - Charge for impairment)/$L$] |
| $r_{OA*}$ | Uncontrollable | Annual average[(Interest income - Interest income on loans)/$O_A$] |
| $r_{OL}$ | Controllable | Annual average[(Interest expenses – Interest expenses on demand and fixed term deposits)/$O_L$] |
| $r_{T*}$ | Uncontrollable | Annual average[(Trading income + Gains or losses on instruments held for other than trading purposes)/$O_A$] |
| $T_x$ | External Parameter | Annual average(Provisions for income taxes + Non-controlling interests in subsidiaries + Extraordinary items and discontinued operations + Net income attributable to non-controlling interests) |

*Note*: The subscript $_*$ stands for the annual average of uncontrollable variables (risk sources), and the lack of the subscript means that the inputs was collected in the same month as the others in the same equation.

Return on equity values were acquired from OSFI Annual Reports from 2000 to 2010 (OSFI 2005; OSFI 2007c; OSFI 2008; OSFI 2012d). Data of risk-weighted assets (RWA) for market and credit risks were collected from the OSFI website from a report entitled *Capital Adequacy-Components of Capital*. This report was terminated in December 2007, what coupled with the fact that there is no publicly available off-balance-sheet report after 2007 makes computing an estimate of Basel II regulatory capital from 2008 to 2010 unfeasible. Therefore, Basel II required capital will be presented only from 2000 to 2007 in Chapter 5. The RWA for operational risk was calculated following the Risk Basic Indicator Approach as described in section 2.3.1.3 (which uses income statement data), and added to eight percent of RWA (based on the OSFI reports previously mentioned), generating MCR Basel II capital, $K_B$.

However, $K_B$ includes other items that do not compose $K_{Market}$ or $K_O$, therefore $K_B$ was transformed into an estimate that has a similar definition to the others to allow fair comparison.[61] This transformation was implemented by multiplying $K_B$ by the ratio of the framework capital definition divided by the Basel II definition of total capital (the sum of Tier 1, 2 and 3 minus required deductions), this operation transformed $K_B$ into the $K_{Basel\ II}$, which then could be compared to $K_{Market}$ and $K_O$ in the discussion of results provided in Chapter 5 - equation 4.1a demonstrate $K_B$ transformation procedure.[62]

$$K_{Basel\ II} = \frac{K_{Framework\ Definition}}{(Tier\ 1 + Tier\ 2 + Tier\ 3)_{Basel\ II\ Definition}} . K_B \qquad (4.1a)$$

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**61** $K_{Market}$ and $K_O$ are both composed of common shares, preferred shares, contributed surplus, retained earnings, accumulated other comprehensive income or loss, and non-controlling interest in subsidiaries.

**62** Table 2.8 on page 31 presents each Tier composition according to Basel II framework.

# 5 Chapter: Results and Discussions

## 5.1 Constrained Minimization of The Bank Risk Function

The inputs described in Chapters 3 and 4 were implemented using Stata 11 (statistical software) to produce the outputs illustrated in Table 5.1. This table shows the results for the constrained optimized capital ($K_O$), the Lagrange Multipliers, the value of the bank risk function at $K_O$ ($Var[\pi(K_O)]$), the capital actually held by banks ($K_{Market}$), the $K_O$-to-$K_{Market}$ ratio, the Basel II requirement ($K_{Basel\ II}$) and the $K_O$-to-$K_{Basel\ II}$ ratios from 2000 to 2010.

**Table 5.1. Results of the Constrained Minimization of the Bank Risk Function.**

| Year | Lagrange Multiplier | $Var[\pi(K_O)]$ | $K_O$ (\$Billions)[1] | $K_{Market}$ (\$Billions) | $K_O/K_{Market}$ Ratio | $K_{Basel\ II}$[2] (\$Billions) | $K_O/K_{Basel\ II}$ Ratio |
|---|---|---|---|---|---|---|---|
| 2000 | $-2.94\text{x}10^{27}$ | $1.00\text{x}10^{18}$ | $1.13\text{x}10^{1}$ | $7.38\text{x}10^{1}$ | $1.53\text{x}10^{-1}$ | $5.43\text{x}10^{1}$ | $2.08\text{x}10^{-1}$ |
| 2001 | $-4.12\text{x}10^{27}$ | $-1.14\text{x}10^{18}$ | $1.18\text{x}10^{1}$ | $8.04\text{x}10^{1}$ | $1.47\text{x}10^{-1}$ | $5.72\text{x}10^{1}$ | $2.06\text{x}10^{-1}$ |
| 2002 | $-3.75\text{x}10^{27}$ | $6.14\text{x}10^{17}$ | $1.68\text{x}10^{1}$ | $8.23\text{x}10^{1}$ | $2.04\text{x}10^{-1}$ | $5.92\text{x}10^{1}$ | $2.84\text{x}10^{-1}$ |
| 2003 | $-7.61\text{x}10^{26}$ | $4.63\text{x}10^{17}$ | $1.78\text{x}10^{1}$ | $8.36\text{x}10^{1}$ | $2.13\text{x}10^{-1}$ | $5.83\text{x}10^{1}$ | $3.05\text{x}10^{-1}$ |
| 2004 | $-1.39\text{x}10^{26}$ | $-2.25\text{x}10^{17}$ | $1.82\text{x}10^{1}$ | $8.72\text{x}10^{1}$ | $2.09\text{x}10^{-1}$ | $5.89\text{x}10^{1}$ | $3.09\text{x}10^{-1}$ |
| 2005 | $-1.76\text{x}10^{27}$ | $-1.55\text{x}10^{18}$ | $2.10\text{x}10^{1}$ | $9.25\text{x}10^{1}$ | $2.27\text{x}10^{-1}$ | $6.49\text{x}10^{1}$ | $3.24\text{x}10^{-1}$ |
| 2006 | $+2.21\text{x}10^{27}$ | $-1.08\text{x}10^{17}$ | $1.84\text{x}10^{1}$ | $1.03\text{x}10^{2}$ | $1.79\text{x}10^{-1}$ | $7.19\text{x}10^{1}$ | $2.56\text{x}10^{-1}$ |
| 2007 | $-7.21\text{x}10^{27}$ | $-1.94\text{x}10^{18}$ | $1.75\text{x}10^{1}$ | $1.13\text{x}10^{2}$ | $1.55\text{x}10^{-1}$ | $7.98\text{x}10^{1}$ | $2.19\text{x}10^{-1}$ |
| 2008 | $-2.50\text{x}10^{27}$ | $3.42\text{x}10^{17}$ | $2.70\text{x}10^{1}$ | $1.36\text{x}10^{2}$ | $1.99\text{x}10^{-1}$ | | |
| 2009 | $+2.24\text{x}10^{28}$ | $2.51\text{x}10^{17}$ | $3.47\text{x}10^{1}$ | $1.56\ \text{x}10^{2}$ | $2.22\text{x}10^{-1}$ | | |
| 2010 | $-1.09\text{x}10^{28}$ | $6.00\text{x}10^{18}$ | $3.83\text{x}10^{1}$ | $1.68\ \text{x}10^{2}$ | $2.28\text{x}10^{-1}$ | | |

*Note* [1]: The data collected as well as the results illustrated in Appendix B are all in thousands of dollars (1000\$). In order to make the numbers easier to understand, capital results are converted in \$Billions in Table 5.1.

[2]: It is important to recall that the information needed to calculate Basel II values (Total RWA for credit and market) is not available after 2007 because OSFI terminated the report *Capital Adequacy – Components of Capital* in the first quarter of 2008.

The Lagrange Multipliers represented in Table 5.1 have dimensions varying from minus $-10^{28}$ to $10^{28}$. This range expresses the magnitude of change in the objective function (the value of $Var[\pi(K)]$) that would result for every unit of change in the right-hand side of the equality constraint ($\alpha$). This characteristic demonstrates how sensitive the bank risk function is in relation to a change in the value of the chosen constraint (the ROE value reported by OSFI, in this case). Another important characteristic of the result is that $K_O$ figures are smaller than $K_{Market}$ ($K_O$-to-$K_{Market}$ ratios vary from 0.147 to 0.228). $K_O$ are also less than 35% the size of $K_{Basel\ II}$ values ($K_O$-to-$K_{Basel\ II}$ ratios varying from 0.206 to 0.324). Figure 5.1 visually illustrates these comparisons.

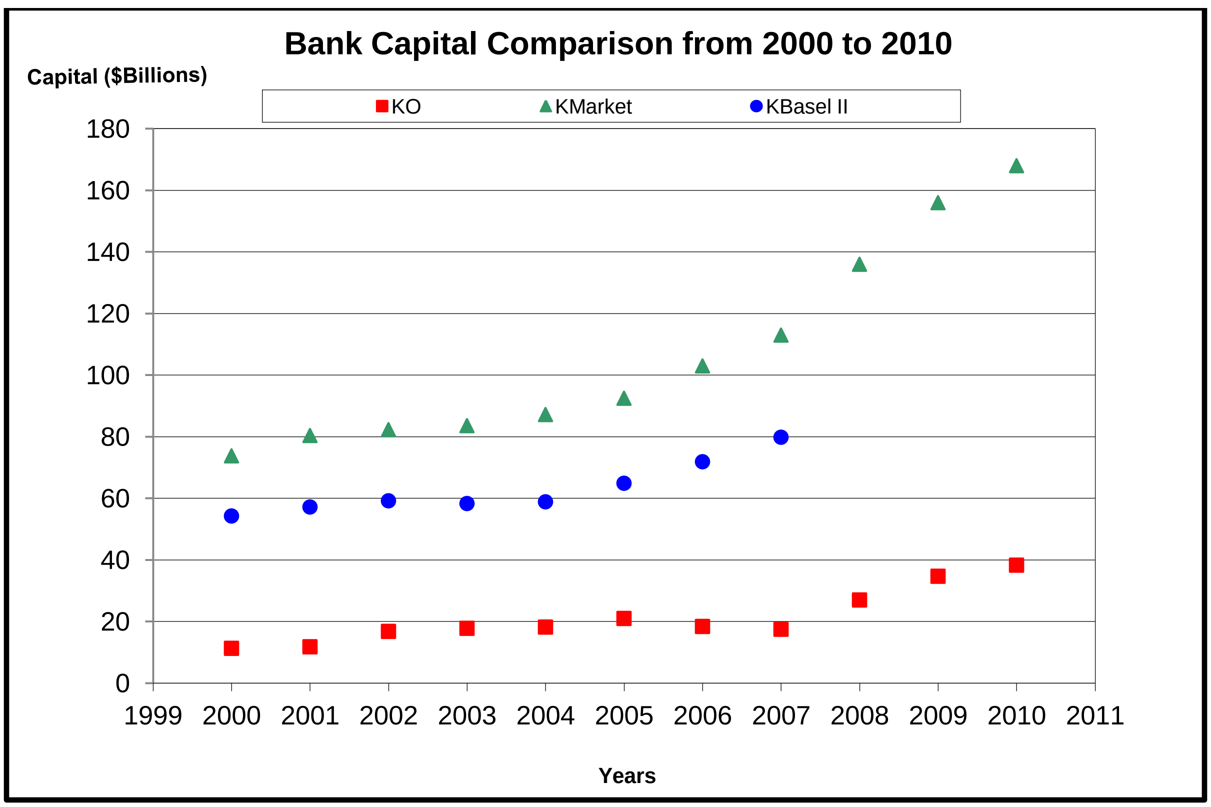


**Figure 5.1. $K_O$, $K_{Market}$ and $K_{Basel\ II}$ Comparison.**

This figure makes it clear that the $K_{Basel\ II}$ follows $K_{Market}$ variation at an almost constant difference. This observation suggests that there was a targeted difference between the minimum regulatory capital requirement and the actual capital held by banks in Canada throughout 2000 to 20120.

A question that might arise from this exercise is: what are the risk-weighted assets (RWA) ratios for each of the compared capitals? This question is especially important because the $K_{Basel\ II}$ figures presented account only for the capital as defined by this research framework, which has a stricter definition than the sum of Tier 1, Tier 2 and Tier 3 as in Basel II definition. Figure 5.2 presents the comparison for $K_O$, $K_{Basel\ II}$ and $K_{Market}$ RWA ratios.

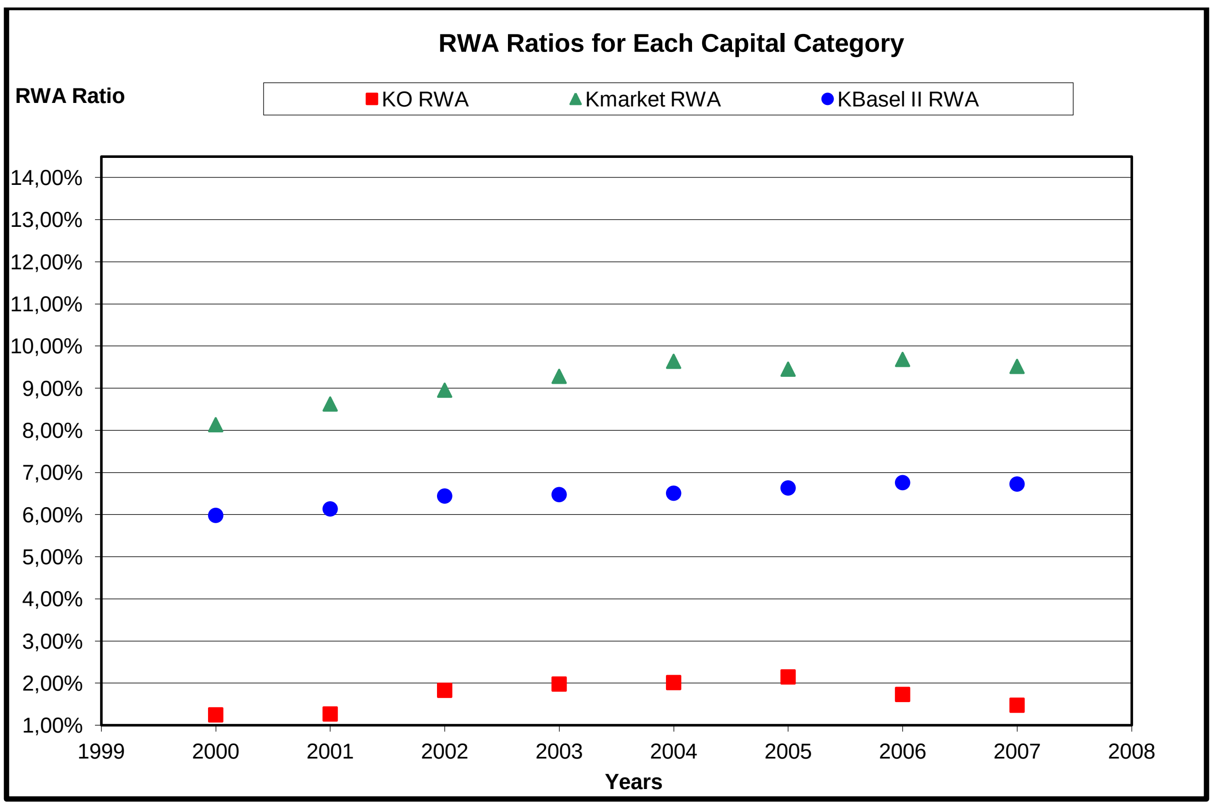


**Figure 5.2. Risk-weighted Assets (RWA) Capital Ratios.**

The previous graph illustrates that $K_{Basel\ II}$ RWA ratio is on average 6.46%, while the $K_{Market}$ RWA ratio is consistently approximately 3percent points higher, with average 9.16%. It reinforces the statement about a targeted difference resulting from banks strategically controlling their operation to maintain a RWA ratio of 3% above the minimum capital required by Basel II. $K_O$ RWA ratio figures range from 1.25% to 2.15% (average 1.71%).

It is important to add that Basel III has reformed capital definition, and banks' capital will be similar to this framework capital's definition. Therefore, considering that Canadian banks have already been operating around 9.16% RWA under this new definition, Basel III will not be binding to the banks until the extra buffers are implemented (conservation and countercyclical).

The main purpose of the present research is to propose a risk management framework for strengthening minimum capital requirement (MCR), which according to the literature implies requiring capital greater than the Basel II requirements. The $K_O$ results suggest that the proposed framework does not perform the expected outcome. Therefore, in the current stage of its development, it cannot replace Basel II. However, it is necessary to understand possible causes of these results. In order to assess the contributions of this framework to banking regulation reform, deeper exploration of each step of this framework is conducted in the following sections.

### 5.1.1 Analyzing the Profit Function

The gross profit function represented by equation 5.1a below is a reproduction of equation 3.1g, after rearranging it to include the random products $r_{OA}.D$ ($W$) and $r_T.D$ ($Z$).

This rearrangement helps to identify each income and expense category reported by banks in their income statement. Table 5.2 specifies the definitions of the terms in equation 5.1a.

$$\pi = \{[r_L L + r_{OA}(D + O_L + K - L)] + r_T(D + O_L + K - L) + N_I\} - \{[r_D D + r_{OL} O_L] + N_E\} \tag{5.1a}$$

**Table 5.2. Specification of Each Term of the Profit Function.**

| Term | Definition in the Income Statement |
|---|---|
| $r_L L$ | Liquid Income on Loans |
| $r_{OA}(D + O_L + K - L)$ | Other Assets Income |
| $r_L L + r_{OA}(D + O_L + K - L)$ | Total Interest Income |
| $r_T(D + O_L + K - L)$ | Trading Income |
| $N_I$ | Noninterest Income |
| $r_D D$ | Deposit Expenses |
| $r_{OL} O_L$ | Other Liability Expenses |
| $N_E$ | Noninterest Expenses |

The following figures illustrate the average composition of total income and total expenses as specified in Table 5.3 in the period from 2000 to 2010. Figure 5.3 demonstrates the importance of each income component, while Figure 5.4 the expense components.

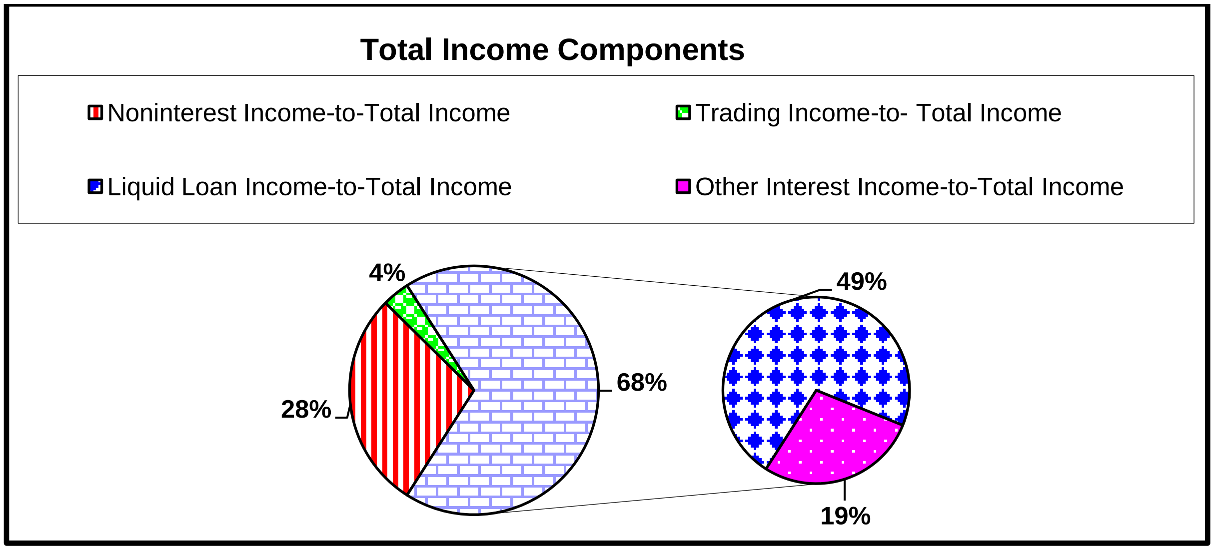


**Figure 5.3. Bank Total Income Complements.**

Figure 5.3 illustrates the importance of interest income for banking operations, which accounts for 68% of the total income produced, being 49% of this total generated by loan activities. On the other hand, Figure 5.4 demonstrates the large the impact of noninterest expenses on the total expenses of the bank operations, accounting for 51% of total expenses.

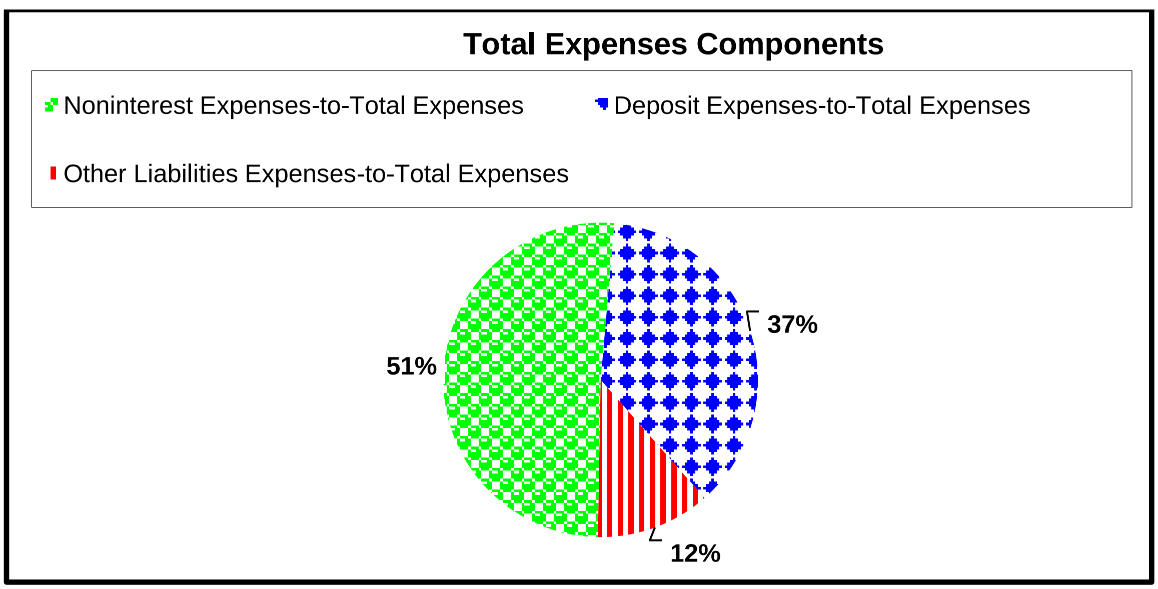


**Figure 5.4. Bank Total Expense Components.**

The graph presented in Figure 5.5 demonstrates how accurately the profit function developed (equation 5.1a) predicts the real gross profit generated by bank operations (the average gross income annually reported by OSFI). This graph indicates that although the bank profit function did not include off-balance-sheet items, it satisfactorily predicts the profit produced by banks. The lowest deviation from the expected gross profit represents 99.05% the actual profit generated (year 2008), and the highest is only 5.51% greater than the reference line (year 2010), which is an acceptable difference (-0.95% to +5.51%distant from the reference line). These results assert that the bank profit function developed properly represents bank operations. Therefore, this development is a significant contribution of the present research, and it has the potential to be applied for different purposes in further work.

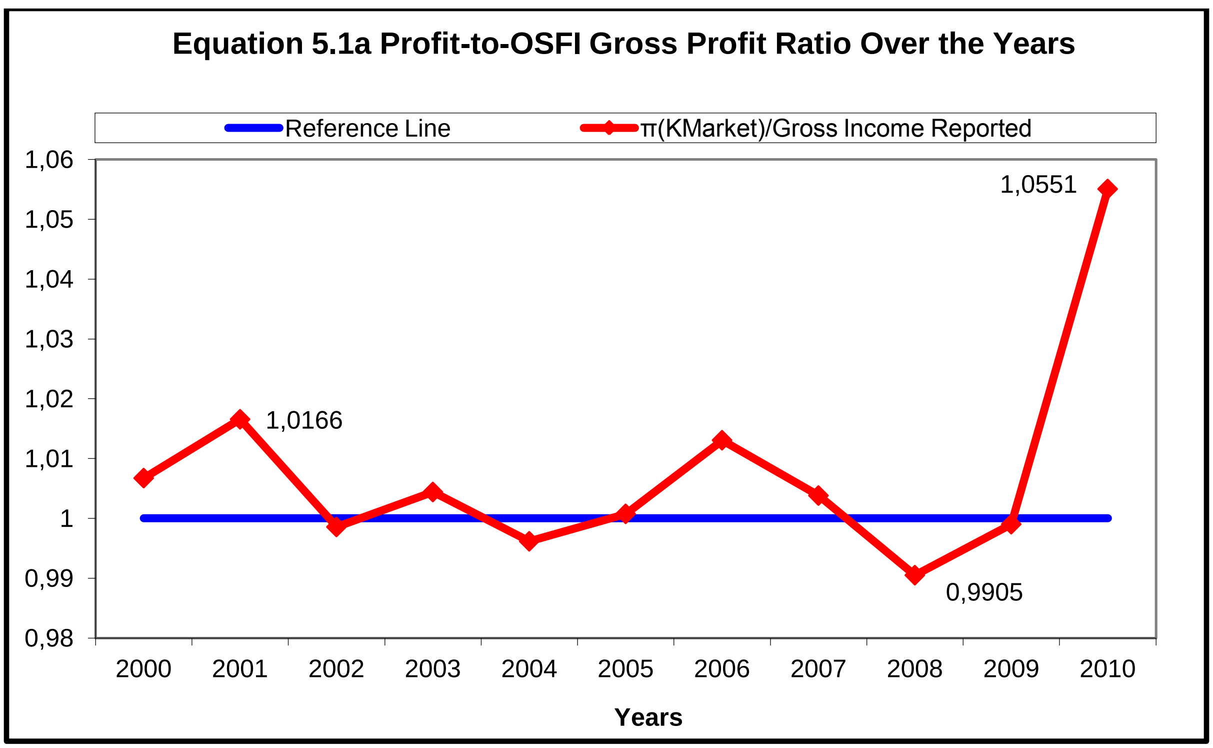


**Figure 5.5. Deviations of the Profit Function Compared to OSFI Gross Profit for the Studies Period.**

### 5.1.2 Analyzing the Bank Risk Function

The study of the bank risk function starts by the analysis of the curvature of the function. Table 5.3 shows the results for the second-order condition (given by: $Var[\pi(K)]'' = 2var(r_{OA}) + 2var(r_T) + 4cov(r_{OA}, r_T)$), which determines the convexity of the bank risk function derived ($Var[\pi(K)]$). This Table also provides the capital held by banks ($K_{Market}$) and the values of bank risk function at market levels, for years from 2000 to 2010.

**Table 5.3. The Convexity of the Bank Risk Function and Its Value at Market Levels for 2000 to 2010.**

| Year | $var(r_{OA})$ | $var(r_T)$ | $cov(r_{OA}, r_T)$ | $Var[\pi(K)]''$ | Convexity of $Var[\pi(K)]$ | $K_{Market}$ (1000$)[1] | $Var[\pi(K_{Market})]$ |
|---|---|---|---|---|---|---|---|
| 2000 | $1.82x10^{-7}$ | $1.32x10^{-7}$ | $6.71x10^{-1}$ | $2.68x10^{0}$ | Convex | $7.38x10^{7}$ | $8.31x10^{17}$ |
| 2001 | $3.75x10^{-7}$ | $4.07x10^{-7}$ | $7.72x10^{-1}$ | $3.09x10^{0}$ | Convex | $8.04x10^{7}$ | $-1.01x10^{18}$ |
| 2002 | $5.55x10^{-8}$ | $3.73x10^{-7}$ | $2.38x10^{-1}$ | $9.51x10^{-1}$ | Convex | $8.23x10^{7}$ | $5.14x10^{17}$ |
| 2003 | $3.95x10^{-8}$ | $2.95x10^{-8}$ | $2.06x10^{-1}$ | $8.23x10^{-1}$ | Convex | $8.36x10^{7}$ | $3.80x10^{17}$ |
| 2004 | $6.77x10^{-8}$ | $1.93x10^{-7}$ | $5.48x10^{-2}$ | $2.19x10^{-1}$ | Convex | $8.72x10^{7}$ | $-1.93x10^{17}$ |
| 2005 | $1.16x10^{-7}$ | $1.60x10^{-7}$ | $2.59x10^{-1}$ | $1.04x10^{0}$ | Convex | $9.25x10^{7}$ | $-1.33x10^{18}$ |
| **2006** | **$1.39x10^{-7}$** | **$8.57x10^{-8}$** | **$-5.55x10^{-1}$** | **$-2.22x10^{0}$** | **Concave** | **$1.03x10^{8}$** | **$-4.85x10^{16}$** |
| 2007 | $2.02x10^{-7}$ | $3.92x10^{-6}$ | $8.70x10^{-1}$ | $3.48x10^{0}$ | Convex | $1.13x10^{8}$ | $-1.70x10^{18}$ |
| 2008 | $5.77x10^{-7}$ | $5.45x10^{-7}$ | $4.39x10^{-2}$ | $1.76x10^{-1}$ | Convex | $1.36x10^{8}$ | $2.76x10^{17}$ |
| **2009** | **$2.29x10^{-6}$** | **$5.30x10^{-7}$** | **$-7.05x10^{-1}$** | **$-2.82x10^{0}$** | **Concave** | **$1.56x10^{8}$** | **$2.86x10^{17}$** |
| 2010 | $1.06x10^{-6}$ | $4.18x10^{-7}$ | $7.56x10^{-1}$ | $3.02 x10^{0}$ | Convex | $1.68x10^{8}$ | $4.92x10^{18}$ |

*Note* [1]: $K_{Market}$ is presented in thousands of dollars ($1000) to enable the reader to detect the riskiness level in which banks were operating in the following figure.

Since the covariance term of the second-order condition (*Var[π(K)]''*) is greater than the others, its sign governs whether the second derivative is positive or negative. Moreover, $cov(r_{OA},r_T)$ would be negative when return on other assets (such as government bonds), $r_{OA}$, increases at the same time that return on trading instruments (for instance stocks), $r_T$, decreases, or vice-versa. Although usually $r_{OA}$ and $r_T$ vary in the same direction (covariance is customarily positive), a reverse situation did occur in 2006 and 2009, as illustrated by the red data in Table 5.3. The negative covariance for the year 2006 could be justified by the analysis of data presented by MacGee (2009). According to that source, in 2006 a drop in $r_{OA}$ could have been provoked by the increase of central bank target interest rates, while $r_T$ might have been pushed up by the rise in housing prices, making $cov(r_{OA},r_T)$ smaller than zero. The same paper also reports that in 2009 the effects were exactly the opposite: central bank target interest rates decreased (as a result, $r_{OA}$ went up) and housing prices decreased ($r_T$ went down), producing again a negative covariance term ($cov(r_{OA},r_T)<0$). It is worth mentioning that the inverse movement of these rates of returns ($r_{OA}$ going up and $r_T$ going down) were reported during the Subprime Mortgage Crisis of 2008 in the US (Mackenzie 2011; Manda 2010), which suggests that this crisis reached the Canadian banking system a year later, in 2009.

The bank risk function (*Var[π(K)]*) for the years 2000 to 2010 is illustrated in Figure 5.6. It is interesting to note the four well-defined patterns that stand out in the graphs: one concave (2006 and 2009) and three convex. The first convex group is composed of years 2004 and 2008; the second 2002, 2003, and 2005; the third consists of the bank risk function for the years 2000, 2001, 2007, and 2010. These four patterns describe banking risk-taking behavior throughout the decade, according to the bank risk function developed.

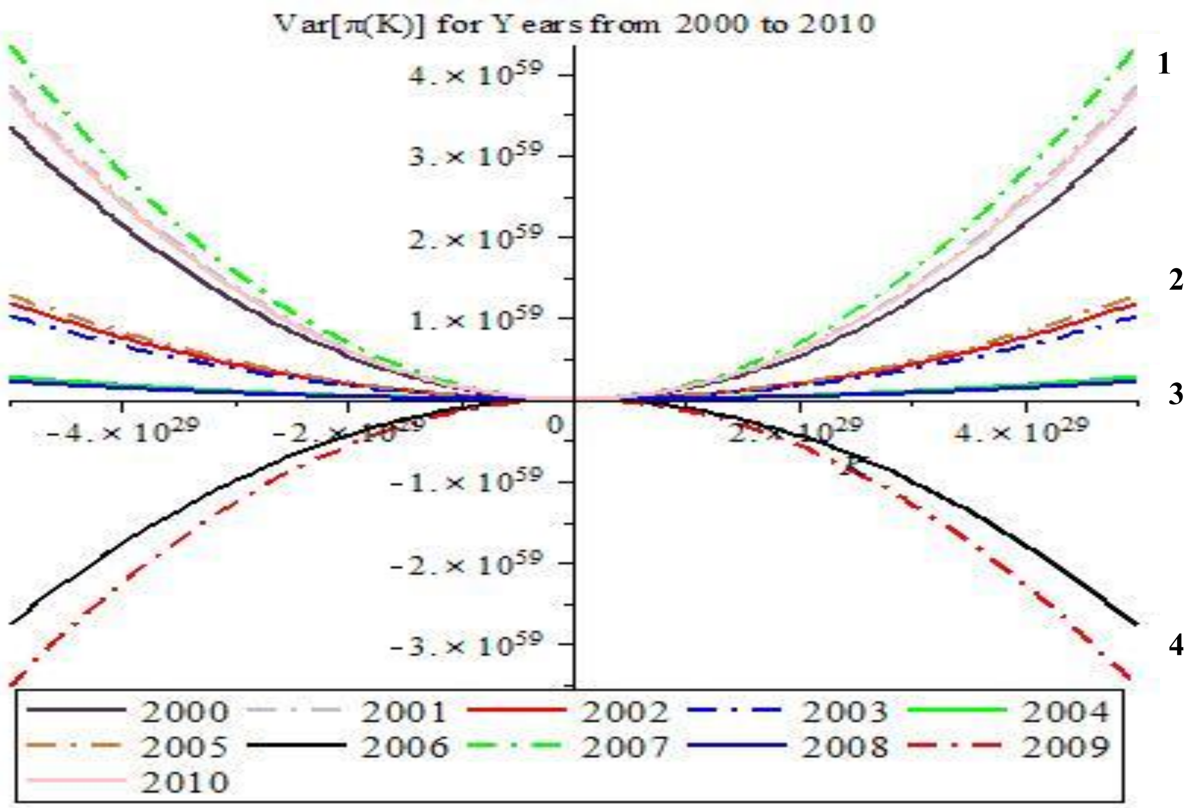


**Figure 5.6. The Bank Risk Function for the Period 2000-2010.**

Examining patterns 1 to 4 in Figure 5.5, it looks as if banking operation cycle has a stage of high risk-taking behavior (pattern 1, at higher risk levels), two intermediary risk-taking stages (lighter curvature in pattern 2 and a very low curvature parabola in pattern 3), followed by periods of risk aversion, represented by negative levels of the bank risk function (pattern 4). However, it is worth adding that the negative values of the bank risk function at market level (as reported in Table 5.3 specifically for years 2004, 2005, 2006 and 2007) mean that banks operations were risk averse during these years. This risk-averse characteristic, however, contradicts the literature about the Subprime Mortgage Crisis, which claims that banks have taken excessive risks in the years just before the crisis (Kashyap, Rajan, and Stein 2008, 431-471; Booth, Alexander, and Institute of Economic Affairs of the

Great Britain 2009, 1-199; Diamond and Rajan 2009, 606-610). This conceptual contradiction raises the question of the fitness of the bank risk function developed to properly represent bank risk-taking. Figure 5.7 illustrates the fitness of the bank risk function derived, comparing it to the real variance of the gross profit as reported by OSFI.

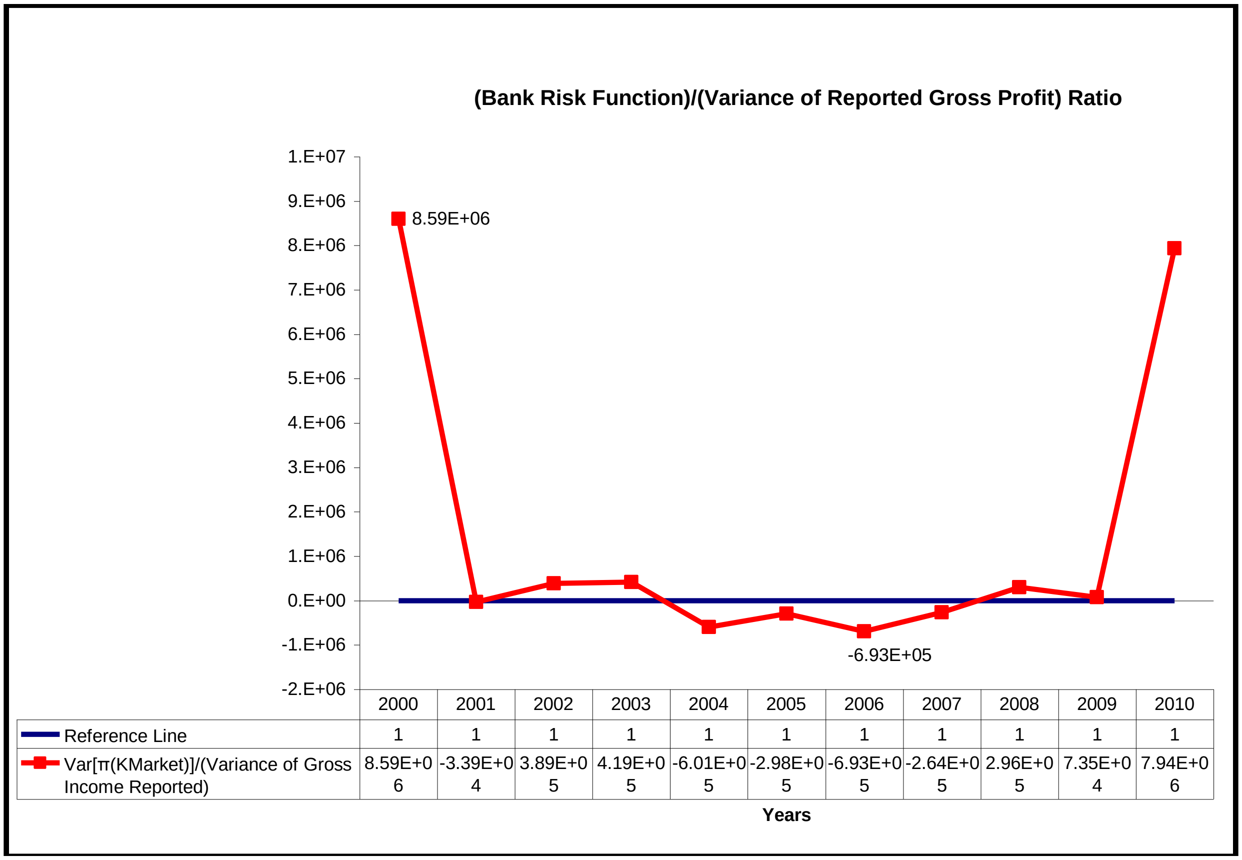


**Figure 5.7. Deviations of the Bank Risk Function Compared to the Variance of the OSFI Gross Profit.**

The analysis of the values represented in Figure 5.7 indicates that the bank risk function derived does not properly represent the observed variance of profits, since the deviations from the real variance of gross profits are significant (varying from $-10^6$ to $+10^6$). Although the derivation of the bank risk function was based on a reliable statistical procedure, its expression might have included terms that are not significant to the variance of profits, and/or omitted important ones. This conclusion indicates that some of the assumptions made at the second stage of the framework development are inappropriate, which means that some risk sources may not be significant, and/or a few are still to be considered by future work. The efforts made so far are, nevertheless, valuable since the terms of the profit function (equation 5.1a) and some of the current bank risk function (equation 3.2c) may be tested as explanatory variables in a regression of the observed variance of profits, in order to generate a more appropriate bank risk function. The development of this new bank risk is suggested as a topic for further research.[63]

#### 5.1.3 Analyzing the Optimization

As discussed in Section 3.3, the linear constraint governs the optimization result, since $K_O$ is exactly the formulation of solving for $K$ in the ROE (equation 3.3h). This observation suggests that future work may avoid a linear constraint. Additionally, a careful analysis of equation 3.3f sheds light on the fact that ROE is inversely proportional to the capital held by banks.[64] As a result, assuming that the underestimation of capital by Basel II occurred implies that ROE levels were actually overestimated in this period. This conclusion indicates that ROE as released by OSFI is potentially an inappropriate measure of banking profitability. In order to compensate for this inappropriateness, more research into the calibration of the ROE reported by OSFI, or the exploration of another nonlinear profitability constraint is useful as future work.

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[63] Equation 3.2c is on page 67, and equation 5.1a on page 80.
[64] Equation 3.3f is on page 70.

# 6 Chapter: Conclusion

This research implements an innovative approach for minimum capital requirement (MCR) calculation. The framework tested is based on the development of a bank profit function (stage 1). This bank profit function is used to derive the bank risk function (stage 2), which in its turn is inserted as the objective function into a constrained optimization (stage 3). The main outcome of this framework is the determination of the constrained optimized capital ($K_O$).

Although the profit function developed in this research does not include off-balance-sheet items, the findings show that it satisfactorily predicts bank profits, and it has the potential to be applied in future research for a variety of purposes in addition to MCR calculation. The bank risk function, however, does not properly represent the variance of profits. Nevertheless, this unfitness has not influenced the optimization result, since $K_O$ formula is determined by the isolation of capital into the return on equity equation, a limitation that is imposed by an optimization of a univariate function subject to a linear constraint. $K_O$ results generated by the present framework are smaller than Basel II requirements, for the years 2000 to 2010. This outcome ($K_O < K_{Basel\ II}$) means that, as implemented, the framework cannot replace Basel II, since the underestimation of regulatory capital, as calculated by Basel II, is claimed as a factor contributing to the occurrence of the Subprime Mortgage Crisis of 2008 by several authors. The analysis of the risk-weighted assets (RWA) capital ratios indicates that Canadian banks have held its RWA ratio (9.16%) consistently above the minimum specified by Basel II (6.46%), under the capital definition proposed by the present framework from 2000 to 2010. The same analysis also suggests that the transition to Basel III requirement, which has capital description similar to this framework capital definition, might not be difficult for Canadian banks until the implementation of the extra capital buffers: conservation and countercyclical.

The analysis of this framework stages suggest some opportunities for improvements. In order to determine more realistic bank risk function, potential improvements to this framework may be achieved by regressing the observed variance of profits on explanatory variables such as the components of the profit function. Additional improvement may also be granted by the minimization of this new bank risk function subject to a nonlinear profitability constraint.

Overall, the most important contributions of the present research are the development of the bank profit function and a preliminary exploration of an optimization framework to MCR determination. Although in the current stage of its development, this framework cannot replace Basel II, it presents a new approach to MCR calculations that might have the potential to strengthen banking regulation.

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# Appendices

### Appendix A  Optimization Stages

All stages of the optimization of the bank risk function (*Var[π(K)]*) are presented in the following eight pages. Maple 15, the math software used to solve this optimization, has a syntax slightly different from the one applied throughout the text. For instance, the covariance terms presented previously as $cov(r_T, r_{OA})$ will be represented by $cov_{rTrOA}$; similarly the variance of $r_T$ (previously presented as $var(r_T)$) corresponds in the following figure to $var_{rT}$; the expected value of random variables, before represented by the variable with subscript * ($r_{T^*}$), is symbolized by the subscript $_E$ ($r_{TE}$). As discussed in Chapters 3 and 5, it is noteworthy how the solutions for the Optimized Capital ($K_O$) and the Lagrange Multiplier ($\lambda$) have distinct characteristics, being all risk measures (variance and covariance of risk sources) incorporated in a very complex definition of $\lambda$, and a simplistic equation defining $K_O$ (basically the isolation of $K$ in the ROE equation).

> $\mathrm{pi} := r_L \cdot L + r_{OA} \cdot (K + O_L - L) + r_T \cdot (K + O_L - L) - r_D \cdot D + W + Z + N_I - r_{OL} \cdot O_L - N_E$

$$\pi := r_L\, L + r_{OA}\,(K + O_L - L) + r_T\,(K + O_L - L) - r_D\, D + W + Z + N_I - r_{OL}\, O_L - N_E \quad (1)$$

> $var\pi := L^2 \cdot varr_L + (K + O_L - L)^2 \cdot varr_{OA} + var_{NI} + (K + O_L - L)^2 \cdot varr_T + (r_D)^2 \cdot var_D + var_W + var_Z + 2 \cdot L \cdot (K + O_L - L) \cdot cov_{rLrOA} + 2 \cdot L \cdot cov_{NIrL} + 2 \cdot L \cdot (K + O_L - L) \cdot cov_{rLrT} - 2 \cdot L \cdot r_D \cdot cov_{DrL} + 2 \cdot L \cdot cov_{rLW} + 2 \cdot L \cdot cov_{rLZ} + 2 \cdot (K + O_L - L) \cdot cov_{NIrOA} + 2 \cdot (K + O_L - L)^2 \cdot cov_{rOArT} - 2 \cdot r_D \cdot (K + O_L - L) \cdot cov_{DrOA} + 2 \cdot (K + O_L - L) \cdot cov_{rOAW} + 2 \cdot (K + O_L - L) \cdot cov_{rOAZ} + 2 \cdot (K + O_L - L) \cdot cov_{NIrT} - 2 \cdot r_D \cdot cov_{DNI} + 2 \cdot cov_{NIW} + 2 \cdot cov_{NIZ} - 2 \cdot r_D \cdot (K + O_L - L) \cdot cov_{DrT} + 2 \cdot (K + O_L - L) \cdot cov_{rTW} + 2 \cdot (K + O_L - L) \cdot cov_{rTZ} - 2 \cdot r_D \cdot cov_{DW} - 2 \cdot r_D \cdot cov_{DZ} + 2 \cdot cov_{WZ}$

$$var\pi := -2\, L\, r_D\, cov_{DrL} + 2\, L\,(K + O_L - L)\, cov_{rLrOA} + 2\, L\,(K + O_L - L)\, cov_{rLrT} - 2\, r_D\,(K + O_L - L)\, cov_{DrOA} - 2\, r_D\,(K + O_L - L)\, cov_{DrT} + L^2\, varr_L + (K + O_L - L)^2\, varr_{OA} + var_{NI} + var_Z + 2\, L\, cov_{NIrL} + 2\, L\, cov_{rLW} + 2\, L\, cov_{rLZ} + 2\,(K + O_L - L)^2\, cov_{rOArT} + (K + O_L - L)^2\, varr_T + r_D^2\, var_D + var_W - 2\, r_D\, cov_{DZ} - 2\, r_D\, cov_{DW} - 2\, r_D\, cov_{DNI} + 2\, cov_{NIW} + 2\, cov_{NIZ} + 2\, cov_{WZ} + 2\,(K + O_L - L)\, cov_{NIrOA} + 2\,(K + O_L - L)\, cov_{rTW} + 2\,(K + O_L - L)\, cov_{rTZ} + 2\,(K + O_L - L)\, cov_{rOAW} + 2\,(K + O_L - L)\, cov_{rOAZ} + 2\,(K + O_L - L)\, cov_{NIrT} \quad (2)$$

> $FOC := diff(var\pi, K)$

$$FOC := 2\, L\, cov_{rLrOA} + 2\, L\, cov_{rLrT} - 2\, r_D\, cov_{DrOA} - 2\, r_D\, cov_{DrT} + 2\,(K + O_L - L)\, varr_{OA} + 4\,(K + O_L - L)\, cov_{rOArT} + 2\,(K + O_L - L)\, varr_T + 2\, cov_{NIrOA} + 2\, cov_{rTW} + 2\, cov_{rTZ} + 2\, cov_{rOAW} + 2\, cov_{rOAZ} + 2\, cov_{NIrT} \quad (3)$$

> $SOC := diff(FOC, K)$

$$SOC := 2\, varr_{OA} + 4\, cov_{rOArT} + 2\, varr_T \quad (4)$$

> $K_S := solve(FOC = 0, K)$

$$K_O := -\frac{1}{varr_{OA} + 2\, cov_{rOArT} + varr_T}\left(L\, cov_{rLrOA} + L\, cov_{rLrT} - r_D\, cov_{DrOA} - r_D\, cov_{DrT} + varr_{OA}\, O_L - varr_{OA}\, L + cov_{rOAW} + 2\, cov_{rOArT}\, O_L - 2\, cov_{rOArT}\, L + cov_{rTW} + varr_T\, O_L - varr_T\, L + cov_{NIrOA} + cov_{NIrT} + cov_{rTZ} + cov_{rOAZ}\right) \quad (5)$$

> *# Now lets consider the ROE constraint for the minimization of varπ.*

> *# The constraint ROE* $= \alpha - \dfrac{(Net\ Income)}{\text{Average Capital}} = 0$

> $constraintROE := \alpha - \frac{1}{K}\left(r_{LE} \cdot L + r_{OAE} \cdot (K + O_L - L) + r_{TE} \cdot (K + O_L - L) - r_D \cdot D_E + W_E + Z_E + N_{IE} - r_{OL} \cdot O_L - N_E - Tx\right)$

$$constraintROE := \alpha - \frac{1}{K}\left(r_{LE} L + r_{OAE}\left(K + O_L - L\right) + r_{TE}\left(K + O_L - L\right) - r_D D_E + W_E + Z_E + N_{IE} - r_{OL} O_L - N_E - Tx\right) \quad (6)$$

> $Lagrange := var\pi + \lambda \cdot constraintROE$

$$Lagrange := -2 L r_D cov_{DrL} + 2 L \left(K + O_L - L\right) cov_{rLrOA} + 2 L \left(K + O_L - L\right) cov_{rLrT} - 2 r_D \left(K + O_L - L\right) cov_{DrOA} - 2 r_D \left(K + O_L - L\right) cov_{DrT} + L^2 varr_L + \left(K + O_L - L\right)^2 varr_{OA} + var_{NI} + var_Z + 2 L cov_{NIrL} + 2 L cov_{rLW} + 2 L cov_{rLZ} + 2 \left(K + O_L - L\right)^2 cov_{rOArT} + \left(K + O_L - L\right)^2 varr_T + r_D^2 var_D + var_W - 2 r_D cov_{DZ} - 2 r_D cov_{DW} - 2 r_D cov_{DNI} + 2 cov_{NIW} + 2 cov_{NIZ} + 2 cov_{WZ} + 2 \left(K + O_L - L\right) cov_{NIrOA} + 2 \left(K + O_L - L\right) cov_{rTW} + 2 \left(K + O_L - L\right) cov_{rTZ} + 2 \left(K + O_L - L\right) cov_{rOAW} + 2 \left(K + O_L - L\right) cov_{rOAZ} + 2 \left(K + O_L - L\right) cov_{NIrT} + \lambda \left(\alpha - \frac{1}{K}\left(r_{LE} L + r_{OAE}\left(K + O_L - L\right) + r_{TE}\left(K + O_L - L\right) - r_D D_E + W_E + Z_E + N_{IE} - r_{OL} O_L - N_E - Tx\right)\right) \quad (7)$$

> $Lagrange\lambda := diff\left(Lagrange, \lambda\right)$

$$Lagrange\lambda := \alpha - \frac{1}{K}\left(r_{LE} L + r_{OAE}\left(K + O_L - L\right) + r_{TE}\left(K + O_L - L\right) - r_D D_E + W_E + Z_E + N_{IE} - r_{OL} O_L - N_E - Tx\right) \quad (8)$$

> $LagrangeK := diff\left(Lagrange, K\right)$

$$LagrangeK := 2 L cov_{rLrOA} + 2 L cov_{rLrT} - 2 r_D cov_{DrOA} - 2 r_D cov_{DrT} + 2 \left(K + O_L - L\right) varr_{OA} + 4 \left(K + O_L - L\right) cov_{rOArT} + 2 \left(K + O_L - L\right) varr_T + 2 cov_{NIrOA} + 2 cov_{rTW} + 2 cov_{rTZ} + 2 cov_{rOAW} + 2 cov_{rOAZ} + 2 cov_{NIrT} + \lambda \left(-\frac{r_{OAE} + r_{TE}}{K} + \frac{1}{K^2}\left(r_{LE} L + r_{OAE}\left(K + O_L - L\right) + r_{TE}\left(K + O_L - L\right) - r_D D_E + W_E + Z_E + N_{IE} - r_{OL} O_L - N_E - Tx\right)\right) \quad (9)$$

> $solution := solve\left(\{Lagrange\lambda, LagrangeK\}, \{\lambda, K\}\right)$

$$solution := \Big\{K = \frac{1}{\alpha - r_{OAE} - r_{TE}}\left(r_{LE} L + r_{OAE} O_L - r_{OAE} L + r_{TE} O_L - r_{TE} L - r_D D_E + W_E + Z_E + N_{IE} - r_{OL} O_L - N_E - Tx\right), \lambda = -\big(2 \big(\alpha Z_E cov_{rTZ} + \alpha Z_E cov_{NIrOA} + \alpha Z_E cov_{NIrT} + \alpha Z_E cov_{rOAW} + \alpha Z_E cov_{rOAZ} - 4 N_E N_{IE} cov_{rOArT} - 2 N_E N_{IE} varr_{OA} - 2 N_E N_{IE} varr_T - 4 N_E W_E cov_{rOArT} - 2 N_E W_E varr_{OA} - 2 N_E W_E varr_T - 4 N_E Z_E cov_{rOArT} - 2 N_E Z_E varr_{OA} - 2 N_E Z_E varr_T + N_E cov_{rTW} r_{OAE} + N_E cov_{rTW} r_{TE} + N_E cov_{rTZ} r_{OAE} + N_E cov_{rTZ} r_{TE} \quad (10)$$

$$
\begin{aligned}
&+ N_E\, \mathit{cov}_{NIrOA}\, r_{OAE} + N_E\, \mathit{cov}_{NIrOA}\, r_{TE} + N_E\, \mathit{cov}_{NIrT}\, r_{OAE} + N_E\, \mathit{cov}_{NIrT}\, r_{TE} + N_E\, \mathit{cov}_{rOAW}\, r_{OAE} \\
&+ N_E\, \mathit{cov}_{rOAW}\, r_{TE} - 2\, Tx\, Z_E\, \mathit{varr}_{OA} - 2\, Tx\, Z_E\, \mathit{varr}_{T} + Tx\, \mathit{cov}_{rTW}\, r_{OAE} + Tx\, \mathit{cov}_{rTW}\, r_{TE} \\
&+ Tx\, \mathit{cov}_{rTZ}\, r_{OAE} + Tx\, \mathit{cov}_{rTZ}\, r_{TE} + Tx\, \mathit{cov}_{NIrOA}\, r_{OAE} + Tx\, \mathit{cov}_{NIrOA}\, r_{TE} + Tx\, \mathit{cov}_{NIrT}\, r_{OAE} \\
&+ Tx\, \mathit{cov}_{NIrT}\, r_{TE} + Tx\, \mathit{cov}_{rOAW}\, r_{OAE} + Tx\, \mathit{cov}_{rOAW}\, r_{TE} + Tx\, \mathit{cov}_{rOAZ}\, r_{OAE} + Tx\, \mathit{cov}_{rOAZ}\, r_{TE} \\
&- \alpha\, N_E\, \mathit{cov}_{rTW} - \alpha\, N_E\, \mathit{cov}_{rTZ} - \alpha\, N_E\, \mathit{cov}_{NIrOA} - \alpha\, N_E\, \mathit{cov}_{NIrT} - \alpha\, N_E\, \mathit{cov}_{rOAW} \\
&- \alpha\, N_E\, \mathit{cov}_{rOAZ} + \alpha\, N_{IE}\, \mathit{cov}_{rTW} + \alpha\, N_{IE}\, \mathit{cov}_{rTZ} + \alpha\, N_{IE}\, \mathit{cov}_{NIrOA} + \alpha\, N_{IE}\, \mathit{cov}_{NIrT} \\
&+ \alpha\, N_{IE}\, \mathit{cov}_{rOAW} + \alpha\, N_{IE}\, \mathit{cov}_{rOAZ} + \alpha\, W_E\, \mathit{cov}_{rTW} + \alpha\, W_E\, \mathit{cov}_{rTZ} + \alpha\, W_E\, \mathit{cov}_{NIrOA} \\
&+ \alpha\, W_E\, \mathit{cov}_{NIrT} + \alpha\, W_E\, \mathit{cov}_{rOAW} + \alpha\, W_E\, \mathit{cov}_{rOAZ} + \alpha\, Z_E\, \mathit{cov}_{rTW} - W_E\, \mathit{cov}_{rOAZ}\, r_{OAE} \\
&- W_E\, \mathit{cov}_{rOAZ}\, r_{TE} - Z_E\, \mathit{cov}_{rTW}\, r_{OAE} - Z_E\, \mathit{cov}_{rTW}\, r_{TE} - Z_E\, \mathit{cov}_{rTZ}\, r_{OAE} - Z_E\, \mathit{cov}_{rTZ}\, r_{TE} \\
&- Z_E\, \mathit{cov}_{NIrOA}\, r_{OAE} - Z_E\, \mathit{cov}_{NIrOA}\, r_{TE} - Z_E\, \mathit{cov}_{NIrT}\, r_{OAE} - Z_E\, \mathit{cov}_{NIrT}\, r_{TE} - Z_E\, \mathit{cov}_{rOAW}\, r_{OAE} \\
&- Z_E\, \mathit{cov}_{rOAW}\, r_{TE} - Z_E\, \mathit{cov}_{rOAZ}\, r_{OAE} - Z_E\, \mathit{cov}_{rOAZ}\, r_{TE} + N_E\, \mathit{cov}_{rOAZ}\, r_{OAE} + N_E\, \mathit{cov}_{rOAZ}\, r_{TE} \\
&+ 4\, N_{IE}\, W_E\, \mathit{cov}_{rOArT} + 2\, N_{IE}\, W_E\, \mathit{varr}_{OA} + 2\, N_{IE}\, W_E\, \mathit{varr}_{T} + 4\, N_{IE}\, Z_E\, \mathit{cov}_{rOArT} \\
&+ 2\, N_{IE}\, Z_E\, \mathit{varr}_{OA} + 2\, N_{IE}\, Z_E\, \mathit{varr}_{T} - N_{IE}\, \mathit{cov}_{rTW}\, r_{OAE} - N_{IE}\, \mathit{cov}_{rTW}\, r_{TE} - N_{IE}\, \mathit{cov}_{rTZ}\, r_{OAE} \\
&- N_{IE}\, \mathit{cov}_{rTZ}\, r_{TE} - N_{IE}\, \mathit{cov}_{NIrOA}\, r_{OAE} - N_{IE}\, \mathit{cov}_{NIrOA}\, r_{TE} - N_{IE}\, \mathit{cov}_{NIrT}\, r_{OAE} - N_{IE}\, \mathit{cov}_{NIrT}\, r_{TE} \\
&- N_{IE}\, \mathit{cov}_{rOAW}\, r_{OAE} + L^2\, \mathit{cov}_{rLrOA}\, r_{OAE}^2 + L^2\, \mathit{cov}_{rLrOA}\, r_{TE}^2 + L^2\, \mathit{cov}_{rLrT}\, r_{OAE}^2 + L^2\, \mathit{cov}_{rLrT}\, r_{TE}^2 \\
&+ 2\, L^2\, \mathit{cov}_{rOArT}\, r_{LE}^2 + L^2\, r_{LE}^2\, \mathit{varr}_{OA} + L^2\, r_{LE}^2\, \mathit{varr}_{T} + 2\, \mathrm{D}_E^2\, \mathit{cov}_{rOArT}\, r_{\mathrm{D}}^2 + \mathrm{D}_E^2\, r_{\mathrm{D}}^2\, \mathit{varr}_{OA} + \mathrm{D}_E^2 \\
&r_{\mathrm{D}}^2\, \mathit{varr}_{T} + 2\, \mathrm{O}_L^2\, \mathit{cov}_{rOArT}\, r_{OL}^2 + \mathrm{O}_L^2\, r_{OL}^2\, \mathit{varr}_{OA} + \mathrm{O}_L^2\, r_{OL}^2\, \mathit{varr}_{T} + L\, \mathit{cov}_{rTW}\, r_{OAE}^2 + L\, \mathit{cov}_{rTW}\, r_{TE}^2 \\
&+ L\, \mathit{cov}_{rTZ}\, r_{OAE}^2 + L\, \mathit{cov}_{rTZ}\, r_{TE}^2 + L\, \mathit{cov}_{NIrOA}\, r_{OAE}^2 + L\, \mathit{cov}_{NIrOA}\, r_{TE}^2 + L\, \mathit{cov}_{NIrT}\, r_{OAE}^2 \\
&+ L\, \mathit{cov}_{NIrT}\, r_{TE}^2 + L\, \mathit{cov}_{rOAW}\, r_{OAE}^2 + L\, \mathit{cov}_{rOAW}\, r_{TE}^2 + L\, \mathit{cov}_{rOAZ}\, r_{OAE}^2 + L\, \mathit{cov}_{rOAZ}\, r_{TE}^2 \\
&- \mathrm{O}_L\, \mathit{cov}_{rTW}\, r_{OAE}^2 - \mathrm{O}_L\, \mathit{cov}_{rTW}\, r_{TE}^2 - \mathrm{O}_L\, \mathit{cov}_{rTZ}\, r_{OAE}^2 - \mathrm{O}_L\, \mathit{cov}_{rTZ}\, r_{TE}^2 - \mathrm{O}_L\, \mathit{cov}_{NIrOA}\, r_{OAE}^2 \\
&- \mathrm{O}_L\, \mathit{cov}_{NIrOA}\, r_{TE}^2 - \mathrm{O}_L\, \mathit{cov}_{NIrT}\, r_{OAE}^2 - \mathrm{O}_L\, \mathit{cov}_{NIrT}\, r_{TE}^2 - \mathrm{O}_L\, \mathit{cov}_{rOAW}\, r_{OAE}^2 - \mathrm{O}_L\, \mathit{cov}_{rOAW}\, r_{TE}^2 \\
&- \mathrm{O}_L\, \mathit{cov}_{rOAZ}\, r_{OAE}^2 - \mathrm{O}_L\, \mathit{cov}_{rOAZ}\, r_{TE}^2 - Tx\, \alpha\, \mathit{cov}_{rTW} - Tx\, \alpha\, \mathit{cov}_{rTZ} - Tx\, \alpha\, \mathit{cov}_{NIrOA} \\
&- Tx\, \alpha\, \mathit{cov}_{NIrT} - Tx\, \alpha\, \mathit{cov}_{rOAW} - Tx\, \alpha\, \mathit{cov}_{rOAZ} + 4\, Tx\, N_E\, \mathit{cov}_{rOArT} + 2\, Tx\, N_E\, \mathit{varr}_{OA} \\
&+ 2\, Tx\, N_E\, \mathit{varr}_{T} - 4\, Tx\, N_{IE}\, \mathit{cov}_{rOArT} - 2\, Tx\, N_{IE}\, \mathit{varr}_{OA} - 2\, Tx\, N_{IE}\, \mathit{varr}_{T} - 4\, Tx\, W_E\, \mathit{cov}_{rOArT} \\
&- 2\, Tx\, W_E\, \mathit{varr}_{OA} - 2\, Tx\, W_E\, \mathit{varr}_{T} - 4\, Tx\, Z_E\, \mathit{cov}_{rOArT} - N_{IE}\, \mathit{cov}_{rOAW}\, r_{TE} - N_{IE}\, \mathit{cov}_{rOAZ}\, r_{OAE} \\
&- N_{IE}\, \mathit{cov}_{rOAZ}\, r_{TE} + 4\, W_E\, Z_E\, \mathit{cov}_{rOArT} + 2\, W_E\, Z_E\, \mathit{varr}_{OA} + 2\, W_E\, Z_E\, \mathit{varr}_{T} - W_E\, \mathit{cov}_{rTW}\, r_{OAE} \\
&- W_E\, \mathit{cov}_{rTW}\, r_{TE} - W_E\, \mathit{cov}_{rTZ}\, r_{OAE} - W_E\, \mathit{cov}_{rTZ}\, r_{TE} - W_E\, \mathit{cov}_{NIrOA}\, r_{OAE} - W_E\, \mathit{cov}_{NIrOA}\, r_{TE}
\end{aligned}
$$

$$
\begin{aligned}
&- W_E\, cov_{NIrT}\, r_{OAE} - W_E\, cov_{NIrT}\, r_{TE} - W_E\, cov_{rOAW}\, r_{OAE} - W_E\, cov_{rOAW}\, r_{TE} \\
&- \mathrm{D}_E\, \mathrm{O}_L\, r_{\mathrm{D}}\, r_{TE}\, varr_T - \mathrm{O}_L\, cov_{DrT}\, r_{\mathrm{D}}\, r_{OAE}\, r_{OL} + 2\, \mathrm{O}_L\, cov_{DrT}\, r_{\mathrm{D}}\, r_{OAE}\, r_{TE} \\
&- \mathrm{O}_L\, cov_{DrT}\, r_{\mathrm{D}}\, r_{OL}\, r_{TE} - \mathrm{O}_L\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{OAE}\, r_{OL} + 2\, \mathrm{O}_L\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{OAE}\, r_{TE} \\
&- \mathrm{O}_L\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{OL}\, r_{TE} + L\, \mathrm{D}_E\, cov_{rLrT}\, r_{\mathrm{D}}\, r_{OAE} + L\, \mathrm{D}_E\, cov_{rLrT}\, r_{\mathrm{D}}\, r_{TE} \\
&- 4\, L\, \mathrm{D}_E\, cov_{rOArT}\, r_{\mathrm{D}}\, r_{LE} + 2\, L\, \mathrm{D}_E\, cov_{rOArT}\, r_{\mathrm{D}}\, r_{OAE} + 2\, L\, \mathrm{D}_E\, cov_{rOArT}\, r_{\mathrm{D}}\, r_{TE} \\
&- 2\, L\, \mathrm{D}_E\, r_{\mathrm{D}}\, r_{LE}\, varr_{OA} - 2\, L\, \mathrm{D}_E\, r_{\mathrm{D}}\, r_{LE}\, varr_T + L\, \mathrm{D}_E\, r_{\mathrm{D}}\, r_{OAE}\, varr_{OA} + L\, \mathrm{D}_E\, r_{\mathrm{D}}\, r_{OAE}\, varr_T \\
&+ L\, \mathrm{D}_E\, r_{\mathrm{D}}\, r_{TE}\, varr_{OA} + L\, \mathrm{D}_E\, r_{\mathrm{D}}\, r_{TE}\, varr_T + L\, \mathrm{O}_L\, cov_{rLrOA}\, r_{OAE}\, r_{OL} \\
&- 2\, L\, \mathrm{O}_L\, cov_{rLrOA}\, r_{OAE}\, r_{TE} + L\, \mathrm{O}_L\, cov_{rLrOA}\, r_{OL}\, r_{TE} + L\, \mathrm{O}_L\, cov_{rLrT}\, r_{OAE}\, r_{OL} \\
&- 2\, L\, \mathrm{O}_L\, cov_{rLrT}\, r_{OAE}\, r_{TE} + L\, \mathrm{O}_L\, cov_{rLrT}\, r_{OL}\, r_{TE} + 2\, L\, \mathrm{O}_L\, cov_{rOArT}\, r_{LE}\, r_{OAE} \\
&- 4\, L\, \mathrm{O}_L\, cov_{rOArT}\, r_{LE}\, r_{OL} + 2\, L\, \mathrm{O}_L\, cov_{rOArT}\, r_{LE}\, r_{TE} + 2\, L\, \mathrm{O}_L\, cov_{rOArT}\, r_{OAE}\, r_{OL} \\
&+ 2\, L\, \mathrm{O}_L\, cov_{rOArT}\, r_{OL}\, r_{TE} + L\, \mathrm{O}_L\, r_{LE}\, r_{OAE}\, varr_{OA} + L\, \mathrm{O}_L\, r_{LE}\, r_{OAE}\, varr_T \\
&- 2\, L\, \mathrm{O}_L\, r_{LE}\, r_{OL}\, varr_{OA} - 2\, L\, \mathrm{O}_L\, r_{LE}\, r_{OL}\, varr_T + L\, \mathrm{O}_L\, r_{LE}\, r_{TE}\, varr_{OA} + L\, \mathrm{O}_L\, r_{LE}\, r_{TE}\, varr_T \\
&+ L\, \mathrm{O}_L\, r_{OAE}\, r_{OL}\, varr_{OA} + L\, \mathrm{O}_L\, r_{OAE}\, r_{OL}\, varr_T + L\, \mathrm{O}_L\, r_{OL}\, r_{TE}\, varr_{OA} - L\, \alpha\, \mathrm{D}_E\, cov_{rLrOA}\, r_{\mathrm{D}} \\
&- L\, \alpha\, \mathrm{D}_E\, cov_{rLrT}\, r_{\mathrm{D}} + 2\, L\, \alpha\, \mathrm{D}_E\, cov_{rOArT}\, r_{\mathrm{D}} + L\, \alpha\, \mathrm{D}_E\, r_{\mathrm{D}}\, varr_{OA} + L\, \alpha\, \mathrm{D}_E\, r_{\mathrm{D}}\, varr_T \\
&+ L\, \alpha\, \mathrm{O}_L\, cov_{rLrOA}\, r_{OAE} - L\, \alpha\, \mathrm{O}_L\, cov_{rLrOA}\, r_{OL} + L\, \alpha\, \mathrm{O}_L\, cov_{rLrOA}\, r_{TE} + L\, \alpha\, \mathrm{O}_L\, cov_{rLrT}\, r_{OAE} \\
&- L\, \alpha\, \mathrm{O}_L\, cov_{rLrT}\, r_{OL} + L\, \alpha\, \mathrm{O}_L\, cov_{rLrT}\, r_{TE} + 2\, L\, \alpha\, \mathrm{O}_L\, cov_{rOArT}\, r_{LE} - 4\, L\, \alpha\, \mathrm{O}_L\, cov_{rOArT}\, r_{OAE} \\
&+ 2\, L\, \alpha\, \mathrm{O}_L\, cov_{rOArT}\, r_{OL} - 4\, L\, \alpha\, \mathrm{O}_L\, cov_{rOArT}\, r_{TE} + L\, \alpha\, \mathrm{O}_L\, r_{LE}\, varr_{OA} + L\, \alpha\, \mathrm{O}_L\, r_{LE}\, varr_T \\
&- 2\, L\, \alpha\, \mathrm{O}_L\, r_{OAE}\, varr_{OA} - 2\, L\, \alpha\, \mathrm{O}_L\, r_{OAE}\, varr_T + L\, \alpha\, \mathrm{O}_L\, r_{OL}\, varr_{OA} + L\, \alpha\, \mathrm{O}_L\, r_{OL}\, varr_T \\
&- 2\, L\, \alpha\, \mathrm{O}_L\, r_{TE}\, varr_{OA} - 2\, L\, \alpha\, \mathrm{O}_L\, r_{TE}\, varr_T - L\, \alpha\, cov_{DrT}\, r_{\mathrm{D}}\, r_{LE} + L\, \alpha\, cov_{DrT}\, r_{\mathrm{D}}\, r_{OAE} \\
&+ L\, \alpha\, cov_{DrT}\, r_{\mathrm{D}}\, r_{TE} - L\, \alpha\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{LE} + L\, \alpha\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{OAE} + L\, \alpha\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{TE} \\
&+ L\, \mathrm{D}_E\, cov_{rLrOA}\, r_{\mathrm{D}}\, r_{OAE} + L\, \mathrm{D}_E\, cov_{rLrOA}\, r_{\mathrm{D}}\, r_{TE} + L\, \mathrm{O}_L\, r_{OL}\, r_{TE}\, varr_T + L\, cov_{DrT}\, r_{\mathrm{D}}\, r_{LE}\, r_{OAE} \\
&+ L\, cov_{DrT}\, r_{\mathrm{D}}\, r_{LE}\, r_{TE} - 2\, L\, cov_{DrT}\, r_{\mathrm{D}}\, r_{OAE}\, r_{TE} + L\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{LE}\, r_{OAE} + L\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{LE}\, r_{TE} \\
&- 2\, L\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{OAE}\, r_{TE} - 2\, \alpha\, \mathrm{D}_E\, \mathrm{O}_L\, cov_{rOArT}\, r_{\mathrm{D}} - \alpha\, \mathrm{D}_E\, \mathrm{O}_L\, r_{\mathrm{D}}\, varr_{OA} - \alpha\, \mathrm{D}_E\, \mathrm{O}_L\, r_{\mathrm{D}}\, varr_T \\
&- \alpha\, \mathrm{O}_L\, cov_{DrT}\, r_{\mathrm{D}}\, r_{OAE} + \alpha\, \mathrm{O}_L\, cov_{DrT}\, r_{\mathrm{D}}\, r_{OL} - \alpha\, \mathrm{O}_L\, cov_{DrT}\, r_{\mathrm{D}}\, r_{TE} - \alpha\, \mathrm{O}_L\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{OAE} \\
&+ \alpha\, \mathrm{O}_L\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{OL} - \alpha\, \mathrm{O}_L\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{TE} - 2\, \mathrm{D}_E\, \mathrm{O}_L\, cov_{rOArT}\, r_{\mathrm{D}}\, r_{OAE} \\
&+ 4\, \mathrm{D}_E\, \mathrm{O}_L\, cov_{rOArT}\, r_{\mathrm{D}}\, r_{OL} - 2\, \mathrm{D}_E\, \mathrm{O}_L\, cov_{rOArT}\, r_{\mathrm{D}}\, r_{TE} - \mathrm{D}_E\, \mathrm{O}_L\, r_{\mathrm{D}}\, r_{OAE}\, varr_{OA} \\
&- \mathrm{D}_E\, \mathrm{O}_L\, r_{\mathrm{D}}\, r_{OAE}\, varr_T + 2\, \mathrm{D}_E\, \mathrm{O}_L\, r_{\mathrm{D}}\, r_{OL}\, varr_{OA} + 2\, \mathrm{D}_E\, \mathrm{O}_L\, r_{\mathrm{D}}\, r_{OL}\, varr_T
\end{aligned}
$$

$$- \mathrm{D}_E \mathrm{O}_L r_{\mathrm{D}} r_{TE} varr_{OA} - L cov_{NIrOA} r_{LE} r_{TE} + 2 L cov_{NIrOA} r_{OAE} r_{TE} - L cov_{rTZ} r_{LE} r_{TE}$$

$$+ 2 L cov_{rTZ} r_{OAE} r_{TE} - L cov_{NIrOA} r_{LE} r_{OAE} - L Z_E r_{OAE} varr_{OA} - L Z_E r_{OAE} varr_T$$

$$- L Z_E r_{TE} varr_{OA} - L Z_E r_{TE} varr_T - L W_E r_{TE} varr_{OA} - L W_E r_{TE} varr_T - L Z_E cov_{rLrOA} r_{OAE}$$

$$- L Z_E cov_{rLrOA} r_{TE} - L Z_E cov_{rLrT} r_{OAE} - L Z_E cov_{rLrT} r_{TE} + 2 L N_{IE} r_{LE} varr_T$$

$$- L N_{IE} r_{OAE} varr_{OA} - L N_{IE} r_{OAE} varr_T - L N_{IE} r_{TE} varr_{OA} - L N_{IE} r_{TE} varr_T$$

$$- L W_E cov_{rLrOA} r_{OAE} - L W_E cov_{rLrOA} r_{TE} - L W_E cov_{rLrT} r_{OAE} - L W_E cov_{rLrT} r_{TE}$$

$$- L cov_{rTW} r_{LE} r_{OAE} - L cov_{rTW} r_{LE} r_{TE} + 2 L cov_{rTW} r_{OAE} r_{TE} - L cov_{rTZ} r_{LE} r_{OAE}$$

$$- L \alpha cov_{rOAZ} r_{OAE} - L \alpha cov_{rOAZ} r_{TE} + L N_E cov_{rLrOA} r_{OAE} + L N_E cov_{rLrOA} r_{TE}$$

$$+ L N_E cov_{rLrT} r_{OAE} + L N_E cov_{rLrT} r_{TE} - 4 L N_E cov_{rOArT} r_{LE} + 2 L N_E cov_{rOArT} r_{OAE}$$

$$+ 2 L N_E cov_{rOArT} r_{TE} - 2 L N_E r_{LE} varr_{OA} - 2 L N_E r_{LE} varr_T + L N_E r_{OAE} varr_{OA}$$

$$+ L N_E r_{OAE} varr_T + 4 L Z_E cov_{rOArT} r_{LE} - 2 L Z_E cov_{rOArT} r_{OAE} - 2 L Z_E cov_{rOArT} r_{TE}$$

$$+ 2 L Z_E r_{LE} varr_{OA} + 2 L Z_E r_{LE} varr_T + 4 L W_E cov_{rOArT} r_{LE} - 2 L W_E cov_{rOArT} r_{OAE}$$

$$- 2 L W_E cov_{rOArT} r_{TE} + 2 L W_E r_{LE} varr_{OA} + 2 L W_E r_{LE} varr_T - L W_E r_{OAE} varr_{OA}$$

$$- L W_E r_{OAE} varr_T - \mathrm{O}_L^2 r_{OAE} r_{OL} varr_T - \mathrm{O}_L^2 r_{OL} r_{TE} varr_{OA} - \mathrm{O}_L^2 r_{OL} r_{TE} varr_T$$

$$+ \mathrm{O}_L cov_{DrT} r_{\mathrm{D}} r_{OAE}^2 + \mathrm{O}_L cov_{DrT} r_{\mathrm{D}} r_{TE}^2 + \mathrm{O}_L cov_{DrOA} r_{\mathrm{D}} r_{OAE}^2 + \mathrm{O}_L cov_{DrOA} r_{\mathrm{D}} r_{TE}^2$$

$$- L Tx \alpha cov_{rLrOA} - L Tx \alpha cov_{rLrT} + 2 L Tx \alpha cov_{rOArT} + L Tx \alpha varr_{OA} + L Tx \alpha varr_T$$

$$+ L Tx cov_{rLrOA} r_{OAE} + L Tx cov_{rLrOA} r_{TE} + L Tx cov_{rLrT} r_{OAE} + L Tx cov_{rLrT} r_{TE}$$

$$- 4 L Tx cov_{rOArT} r_{LE} + 2 L Tx cov_{rOArT} r_{OAE} + 2 L Tx cov_{rOArT} r_{TE} - 2 L Tx r_{LE} varr_{OA}$$

$$- 2 L Tx r_{LE} varr_T + L Tx r_{OAE} varr_{OA} + L Tx r_{OAE} varr_T + L Tx r_{TE} varr_{OA} + L Tx r_{TE} varr_T$$

$$- L \alpha N_E cov_{rLrOA} - L \alpha N_E cov_{rLrT} + L N_E r_{TE} varr_{OA} + L N_E r_{TE} varr_T$$

$$- L N_{IE} cov_{rLrOA} r_{OAE} - L N_{IE} cov_{rLrOA} r_{TE} - L N_{IE} cov_{rLrT} r_{OAE} - L N_{IE} cov_{rLrT} r_{TE}$$

$$+ 4 L N_{IE} cov_{rOArT} r_{LE} - 2 L N_{IE} cov_{rOArT} r_{OAE} - 2 L N_{IE} cov_{rOArT} r_{TE} + 2 L N_{IE} r_{LE} varr_{OA}$$

$$+ L^2 \alpha cov_{rLrOA} r_{LE} - L^2 \alpha cov_{rLrOA} r_{OAE} - L^2 \alpha cov_{rLrOA} r_{TE} + L^2 \alpha cov_{rLrT} r_{LE}$$

$$- L^2 \alpha cov_{rLrT} r_{OAE} - L^2 \alpha cov_{rLrT} r_{TE} - 2 L^2 \alpha cov_{rOArT} r_{LE} + 2 L^2 \alpha cov_{rOArT} r_{OAE}$$

$$+ 2 L^2 \alpha cov_{rOArT} r_{TE} - L^2 \alpha r_{LE} varr_{OA} - L^2 \alpha r_{LE} varr_T + L^2 \alpha r_{OAE} varr_{OA}$$

$$+ L^2 \alpha r_{OAE} varr_T + L^2 \alpha r_{TE} varr_{OA} + L^2 \alpha r_{TE} varr_T - L^2 cov_{rLrOA} r_{LE} r_{OAE}$$

$$- L^2 cov_{rLrOA} r_{LE} r_{TE} + 2 L^2 cov_{rLrOA} r_{OAE} r_{TE} - L^2 cov_{rLrT} r_{LE} r_{OAE} - L^2 cov_{rLrT} r_{LE} r_{TE}$$

$$+ 2 L^2 cov_{rLrT} r_{OAE} r_{TE} - 2 L^2 cov_{rOArT} r_{LE} r_{OAE} - 2 L^2 cov_{rOArT} r_{LE} r_{TE} - L^2 r_{LE} r_{OAE} varr_{OA}$$

$$- L \alpha cov_{rTW} r_{OAE} - L \alpha cov_{rTW} r_{TE} + L \alpha cov_{rTZ} r_{LE} - L \alpha cov_{rTZ} r_{OAE} - L \alpha cov_{rTZ} r_{TE}$$

$$+ L \alpha cov_{NIrOA} r_{LE} - L \alpha cov_{NIrOA} r_{OAE} - L \alpha cov_{NIrOA} r_{TE} + L \alpha cov_{NIrT} r_{LE}$$

$$- L \alpha cov_{NIrT} r_{OAE} - L \alpha cov_{NIrT} r_{TE} + L \alpha cov_{rOAW} r_{LE} - L \alpha cov_{rOAW} r_{OAE}$$

$$- L \alpha cov_{rOAW} r_{TE} + L \alpha cov_{rOAZ} r_{LE} + 2 L \alpha N_E cov_{rOArT} + L \alpha N_E varr_{OA} + L \alpha N_E varr_T$$

$$+ L \alpha N_{IE} cov_{rLrOA} + L \alpha N_{IE} cov_{rLrT} - 2 L \alpha N_{IE} cov_{rOArT} - L \alpha N_{IE} varr_{OA} - L \alpha N_{IE} varr_T$$

$$+ L \alpha W_E cov_{rLrOA} + L \alpha W_E cov_{rLrT} - 2 L \alpha W_E cov_{rOArT} - L \alpha W_E varr_{OA} - L \alpha W_E varr_T$$

$$+ L \alpha Z_E cov_{rLrOA} + L \alpha Z_E cov_{rLrT} - 2 L \alpha Z_E cov_{rOArT} - L \alpha Z_E varr_{OA} - L \alpha Z_E varr_T$$

$$+ L \alpha cov_{rTW} r_{LE} - L cov_{NIrT} r_{LE} r_{OAE} - L cov_{NIrT} r_{LE} r_{TE} - L^2 r_{LE} r_{OAE} varr_T$$

$$- L^2 r_{LE} r_{TE} varr_{OA} - L^2 r_{LE} r_{TE} varr_T - L \mathrm{O}_L cov_{rLrOA} r_{OAE}^2 - L \mathrm{O}_L cov_{rLrOA} r_{TE}^2$$

$$- L \mathrm{O}_L cov_{rLrT} r_{OAE}^2 - L \mathrm{O}_L cov_{rLrT} r_{TE}^2 - L cov_{DrT} r_{\mathrm{D}} r_{OAE}^2 - L cov_{DrT} r_{\mathrm{D}} r_{TE}^2 - L cov_{DrOA} r_{\mathrm{D}}$$

$$r_{OAE}^2 - L cov_{DrOA} r_{\mathrm{D}} r_{TE}^2 + \alpha \mathrm{D}_E cov_{DrT} r_{\mathrm{D}}^2 + \alpha \mathrm{D}_E cov_{DrOA} r_{\mathrm{D}}^2 + 2 \alpha \mathrm{O}_L^2 cov_{rOArT} r_{OAE} - 2 \alpha$$

$$\mathrm{O}_L^2 cov_{rOArT} r_{OL} + 2 \alpha \mathrm{O}_L^2 cov_{rOArT} r_{TE} + \alpha \mathrm{O}_L^2 r_{OAE} varr_{OA} + \alpha \mathrm{O}_L^2 r_{OAE} varr_T - \alpha$$

$$\mathrm{O}_L^2 r_{OL} varr_{OA} - \alpha \mathrm{O}_L^2 r_{OL} varr_T + \alpha \mathrm{O}_L^2 r_{TE} varr_{OA} + \alpha \mathrm{O}_L^2 r_{TE} varr_T - \mathrm{D}_E cov_{DrT} r_{\mathrm{D}}^2 r_{OAE}$$

$$- \mathrm{D}_E cov_{DrT} r_{\mathrm{D}}^2 r_{TE} - \mathrm{D}_E cov_{DrOA} r_{\mathrm{D}}^2 r_{OAE} - \mathrm{D}_E cov_{DrOA} r_{\mathrm{D}}^2 r_{TE} - 2 \mathrm{O}_L^2 cov_{rOArT} r_{OAE} r_{OL} - 2$$

$$\mathrm{O}_L^2 cov_{rOArT} r_{OL} r_{TE} - \mathrm{O}_L^2 r_{OAE} r_{OL} varr_{OA} + 2 Tx^2 cov_{rOArT} + Tx^2 varr_{OA} + Tx^2 varr_T + 2$$

$$N_E^2 cov_{rOArT} + N_E^2 varr_{OA} + N_E^2 varr_T + 2 N_{IE}^2 cov_{rOArT} + N_{IE}^2 varr_{OA} + N_{IE}^2 varr_T + 2$$

$$W_E^2 cov_{rOArT} + W_E^2 varr_{OA} + W_E^2 varr_T + 2 Z_E^2 cov_{rOArT} + Z_E^2 varr_{OA} + Z_E^2 varr_T$$

$$+ 2 \mathrm{O}_L W_E cov_{rOArT} r_{OAE} - 4 \mathrm{O}_L W_E cov_{rOArT} r_{OL} + 2 \mathrm{O}_L W_E cov_{rOArT} r_{TE}$$

$$+ \mathrm{O}_L W_E r_{OAE} varr_{OA} + \mathrm{O}_L W_E r_{OAE} varr_T - 2 \mathrm{O}_L W_E r_{OL} varr_{OA} - 2 \mathrm{O}_L W_E r_{OL} varr_T$$

$$+ \mathrm{O}_L W_E r_{TE} varr_{OA} + \mathrm{O}_L W_E r_{TE} varr_T + 2 \mathrm{O}_L Z_E cov_{rOArT} r_{OAE} - 4 \mathrm{O}_L Z_E cov_{rOArT} r_{OL}$$

$$+ 2 \mathrm{O}_L Z_E cov_{rOArT} r_{TE} + \mathrm{O}_L Z_E r_{OAE} varr_{OA} + \mathrm{O}_L Z_E r_{OAE} varr_T - 2 \mathrm{O}_L Z_E r_{OL} varr_{OA}$$

$$- 2 \mathrm{O}_L Z_E r_{OL} varr_T + \mathrm{O}_L Z_E r_{TE} varr_{OA} + \mathrm{O}_L Z_E r_{TE} varr_T + \mathrm{O}_L cov_{rTW} r_{OAE} r_{OL}$$

$$- 2 \mathrm{O}_L cov_{rTW} r_{OAE} r_{TE} + \mathrm{O}_L cov_{rTW} r_{OL} r_{TE} + \mathrm{O}_L cov_{rTZ} r_{OAE} r_{OL} - 2 \mathrm{O}_L cov_{rTZ} r_{OAE} r_{TE}$$

$$+ \mathrm{O}_L cov_{rTZ} r_{OL} r_{TE} + \mathrm{O}_L cov_{NIrOA} r_{OAE} r_{OL} - 2 \mathrm{O}_L cov_{NIrOA} r_{OAE} r_{TE} + \mathrm{O}_L cov_{NIrOA} r_{OL} r_{TE}$$

$$+ \mathrm{O}_L cov_{NIrT} r_{OAE} r_{OL} - 2 \mathrm{O}_L cov_{NIrT} r_{OAE} r_{TE} + \mathrm{O}_L cov_{NIrT} r_{OL} r_{TE} + \mathrm{O}_L cov_{rOAW} r_{OAE} r_{OL}$$

$$- 2 \mathrm{O}_L cov_{rOAW} r_{OAE} r_{TE} + \mathrm{O}_L cov_{rOAW} r_{OL} r_{TE} + \mathrm{O}_L cov_{rOAZ} r_{OAE} r_{OL}$$

$$- 2\,\mathrm{O}_L\, cov_{rOAZ}\, r_{OAE}\, r_{TE} + \mathrm{O}_L\, cov_{rOAZ}\, r_{OL}\, r_{TE} + W_E\, cov_{DrT}\, r_{\mathrm{D}}\, r_{OAE} + W_E\, cov_{DrT}\, r_{\mathrm{D}}\, r_{TE}$$

$$+ W_E\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{OAE} + W_E\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{TE} + Z_E\, cov_{DrT}\, r_{\mathrm{D}}\, r_{OAE} + Z_E\, cov_{DrT}\, r_{\mathrm{D}}\, r_{TE}$$

$$+ Z_E\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{OAE} + Z_E\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{TE} + 2\, L\, cov_{NIrT}\, r_{OAE}\, r_{TE} - L\, cov_{rOAW}\, r_{LE}\, r_{OAE}$$

$$- L\, cov_{rOAW}\, r_{LE}\, r_{TE} + 2\, L\, cov_{rOAW}\, r_{OAE}\, r_{TE} - L\, cov_{rOAZ}\, r_{LE}\, r_{OAE} - L\, cov_{rOAZ}\, r_{LE}\, r_{TE}$$

$$+ 2\, L\, cov_{rOAZ}\, r_{OAE}\, r_{TE} - 2\, Tx\, \alpha\, \mathrm{O}_L\, cov_{rOArT} - Tx\, \alpha\, \mathrm{O}_L\, varr_{OA} - Tx\, \alpha\, \mathrm{O}_L\, varr_T$$

$$+ Tx\, \alpha\, cov_{DrT}\, r_{\mathrm{D}} + Tx\, \alpha\, cov_{DrOA}\, r_{\mathrm{D}} + 4\, Tx\, \mathrm{D}_E\, cov_{rOArT}\, r_{\mathrm{D}} + 2\, Tx\, \mathrm{D}_E\, r_{\mathrm{D}}\, varr_{OA}$$

$$+ 2\, Tx\, \mathrm{D}_E\, r_{\mathrm{D}}\, varr_T - 2\, Tx\, \mathrm{O}_L\, cov_{rOArT}\, r_{OAE} + 4\, Tx\, \mathrm{O}_L\, cov_{rOArT}\, r_{OL} - 2\, Tx\, \mathrm{O}_L\, cov_{rOArT}\, r_{TE}$$

$$- Tx\, \mathrm{O}_L\, r_{OAE}\, varr_{OA} - Tx\, \mathrm{O}_L\, r_{OAE}\, varr_T + 2\, Tx\, \mathrm{O}_L\, r_{OL}\, varr_{OA} + 2\, Tx\, \mathrm{O}_L\, r_{OL}\, varr_T$$

$$- Tx\, \mathrm{O}_L\, r_{TE}\, varr_{OA} - Tx\, \mathrm{O}_L\, r_{TE}\, varr_T - Tx\, cov_{DrT}\, r_{\mathrm{D}}\, r_{OAE} - Tx\, cov_{DrT}\, r_{\mathrm{D}}\, r_{TE}$$

$$- Tx\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{OAE} - Tx\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{TE} - \alpha\, \mathrm{D}_E\, cov_{rTW}\, r_{\mathrm{D}} - \alpha\, \mathrm{D}_E\, cov_{rTZ}\, r_{\mathrm{D}}$$

$$- \alpha\, \mathrm{D}_E\, cov_{NIrOA}\, r_{\mathrm{D}} - \alpha\, \mathrm{D}_E\, cov_{NIrT}\, r_{\mathrm{D}} - \alpha\, \mathrm{D}_E\, cov_{rOAW}\, r_{\mathrm{D}} - \alpha\, \mathrm{D}_E\, cov_{rOAZ}\, r_{\mathrm{D}}$$

$$- 2\, \alpha\, N_E\, \mathrm{O}_L\, cov_{rOArT} - \alpha\, N_E\, \mathrm{O}_L\, varr_{OA} - \alpha\, N_E\, \mathrm{O}_L\, varr_T + \alpha\, N_E\, cov_{DrT}\, r_{\mathrm{D}}$$

$$+ \alpha\, N_E\, cov_{DrOA}\, r_{\mathrm{D}} + 2\, \alpha\, N_{IE}\, \mathrm{O}_L\, cov_{rOArT} + \alpha\, N_{IE}\, \mathrm{O}_L\, varr_{OA} + \alpha\, N_{IE}\, \mathrm{O}_L\, varr_T$$

$$- \alpha\, N_{IE}\, cov_{DrT}\, r_{\mathrm{D}} - \alpha\, N_{IE}\, cov_{DrOA}\, r_{\mathrm{D}} + 2\, \alpha\, \mathrm{O}_L\, W_E\, cov_{rOArT} + \alpha\, \mathrm{O}_L\, W_E\, varr_{OA}$$

$$+ \alpha\, \mathrm{O}_L\, W_E\, varr_T + 2\, \alpha\, \mathrm{O}_L\, Z_E\, cov_{rOArT} + \alpha\, \mathrm{O}_L\, Z_E\, varr_{OA} + \alpha\, \mathrm{O}_L\, Z_E\, varr_T$$

$$+ \alpha\, \mathrm{O}_L\, cov_{rTW}\, r_{OAE} - \alpha\, \mathrm{O}_L\, cov_{rTW}\, r_{OL} + \alpha\, \mathrm{O}_L\, cov_{rTW}\, r_{TE} + \alpha\, \mathrm{O}_L\, cov_{rTZ}\, r_{OAE}$$

$$- \alpha\, \mathrm{O}_L\, cov_{rTZ}\, r_{OL} + \alpha\, \mathrm{O}_L\, cov_{rTZ}\, r_{TE} + \alpha\, \mathrm{O}_L\, cov_{NIrOA}\, r_{OAE} - \alpha\, \mathrm{O}_L\, cov_{NIrOA}\, r_{OL}$$

$$+ \alpha\, \mathrm{O}_L\, cov_{NIrOA}\, r_{TE} + \alpha\, \mathrm{O}_L\, cov_{NIrT}\, r_{OAE} - \alpha\, \mathrm{O}_L\, cov_{NIrT}\, r_{OL} + \alpha\, \mathrm{O}_L\, cov_{NIrT}\, r_{TE}$$

$$+ \alpha\, \mathrm{O}_L\, cov_{rOAW}\, r_{OAE} - \alpha\, \mathrm{O}_L\, cov_{rOAW}\, r_{OL} + \alpha\, \mathrm{O}_L\, cov_{rOAW}\, r_{TE} + \alpha\, \mathrm{O}_L\, cov_{rOAZ}\, r_{OAE}$$

$$- \alpha\, \mathrm{O}_L\, cov_{rOAZ}\, r_{OL} + \alpha\, \mathrm{O}_L\, cov_{rOAZ}\, r_{TE} - \alpha\, W_E\, cov_{DrT}\, r_{\mathrm{D}} - \alpha\, W_E\, cov_{DrOA}\, r_{\mathrm{D}}$$

$$- \alpha\, Z_E\, cov_{DrT}\, r_{\mathrm{D}} - \alpha\, Z_E\, cov_{DrOA}\, r_{\mathrm{D}} + 4\, \mathrm{D}_E\, N_E\, cov_{rOArT}\, r_{\mathrm{D}} + 2\, \mathrm{D}_E\, N_E\, r_{\mathrm{D}}\, varr_{OA}$$

$$+ 2\, \mathrm{D}_E\, N_E\, r_{\mathrm{D}}\, varr_T - 4\, \mathrm{D}_E\, N_{IE}\, cov_{rOArT}\, r_{\mathrm{D}} - 2\, \mathrm{D}_E\, N_{IE}\, r_{\mathrm{D}}\, varr_{OA} - 2\, \mathrm{D}_E\, N_{IE}\, r_{\mathrm{D}}\, varr_T$$

$$- 4\, \mathrm{D}_E\, W_E\, cov_{rOArT}\, r_{\mathrm{D}} - 2\, \mathrm{D}_E\, W_E\, r_{\mathrm{D}}\, varr_{OA} - 2\, \mathrm{D}_E\, W_E\, r_{\mathrm{D}}\, varr_T - 4\, \mathrm{D}_E\, Z_E\, cov_{rOArT}\, r_{\mathrm{D}}$$

$$- 2\, \mathrm{D}_E\, Z_E\, r_{\mathrm{D}}\, varr_{OA} - 2\, \mathrm{D}_E\, Z_E\, r_{\mathrm{D}}\, varr_T + \mathrm{D}_E\, cov_{rTW}\, r_{\mathrm{D}}\, r_{OAE} + \mathrm{D}_E\, cov_{rTW}\, r_{\mathrm{D}}\, r_{TE}$$

$$+ \mathrm{D}_E\, cov_{rTZ}\, r_{\mathrm{D}}\, r_{OAE} + \mathrm{D}_E\, cov_{rTZ}\, r_{\mathrm{D}}\, r_{TE} + \mathrm{D}_E\, cov_{NIrOA}\, r_{\mathrm{D}}\, r_{OAE} + \mathrm{D}_E\, cov_{NIrOA}\, r_{\mathrm{D}}\, r_{TE}$$

$$+ \mathrm{D}_E\, cov_{NIrT}\, r_{\mathrm{D}}\, r_{OAE} + \mathrm{D}_E\, cov_{NIrT}\, r_{\mathrm{D}}\, r_{TE} + \mathrm{D}_E\, cov_{rOAW}\, r_{\mathrm{D}}\, r_{OAE} + \mathrm{D}_E\, cov_{rOAW}\, r_{\mathrm{D}}\, r_{TE}$$

$$+ \mathrm{D}_E\, cov_{rOAZ}\, r_{\mathrm{D}}\, r_{OAE} + \mathrm{D}_E\, cov_{rOAZ}\, r_{\mathrm{D}}\, r_{TE} - 2\, N_E\, \mathrm{O}_L\, cov_{rOArT}\, r_{OAE} + 4\, N_E\, \mathrm{O}_L\, cov_{rOArT}\, r_{OL}$$

$$
\begin{aligned}
&- 2\, N_E\, \mathrm{O}_L\, cov_{rOArT}\, r_{TE} - N_E\, \mathrm{O}_L\, r_{OAE}\, varr_{OA} - N_E\, \mathrm{O}_L\, r_{OAE}\, varr_T + 2\, N_E\, \mathrm{O}_L\, r_{OL}\, varr_{OA} \\
&+ 2\, N_E\, \mathrm{O}_L\, r_{OL}\, varr_T - N_E\, \mathrm{O}_L\, r_{TE}\, varr_{OA} - N_E\, \mathrm{O}_L\, r_{TE}\, varr_T - N_E\, cov_{DrT}\, r_{\mathrm{D}}\, r_{OAE} \\
&- N_E\, cov_{DrT}\, r_{\mathrm{D}}\, r_{TE} - N_E\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{OAE} - N_E\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{TE} + 2\, N_{IE}\, \mathrm{O}_L\, cov_{rOArT}\, r_{OAE} \\
&- 4\, N_{IE}\, \mathrm{O}_L\, cov_{rOArT}\, r_{OL} + 2\, N_{IE}\, \mathrm{O}_L\, cov_{rOArT}\, r_{TE} + N_{IE}\, \mathrm{O}_L\, r_{OAE}\, varr_{OA} + N_{IE}\, \mathrm{O}_L\, r_{OAE}\, varr_T \\
&- 2\, N_{IE}\, \mathrm{O}_L\, r_{OL}\, varr_{OA} - 2\, N_{IE}\, \mathrm{O}_L\, r_{OL}\, varr_T + N_{IE}\, \mathrm{O}_L\, r_{TE}\, varr_{OA} + N_{IE}\, \mathrm{O}_L\, r_{TE}\, varr_T \\
&+ N_{IE}\, cov_{DrT}\, r_{\mathrm{D}}\, r_{OAE} + N_{IE}\, cov_{DrT}\, r_{\mathrm{D}}\, r_{TE} + N_{IE}\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{OAE} + N_{IE}\, cov_{DrOA}\, r_{\mathrm{D}}\, r_{TE})) \Big/ \\
&\big(\alpha^3 - 3\, \alpha^2\, r_{OAE} - 3\, \alpha^2\, r_{TE} + 3\, \alpha\, r_{OAE}^2 + 6\, \alpha\, r_{OAE}\, r_{TE} + 3\, \alpha\, r_{TE}^2 - r_{OAE}^3 - 3\, r_{OAE}^2\, r_{TE} \\
&- 3\, r_{OAE}\, r_{TE}^2 - r_{TE}^3\big)\big\}
\end{aligned}
$$

**Figure 6.1. All Stages of the Minimization of the Bank Risk Function (Var[$\pi$(K)]).**

## Appendix B Framework Empirical Inputs and Outputs

The following pages contain figures that specifythe mean of all framework empirical inputs and outputs.

| Year | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | Average |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| L | 8.81E+08 | 9.37E+08 | 9.63E+08 | 9.52E+08 | 1.00E+09 | 1.10E+09 | 1.23E+09 | 1.38E+09 | 1.53E+09 | 1.51E+09 | 1.76E+09 | 1.20E+09 |
| $O_L$ | 4.11E+08 | 4.68E+08 | 4.92E+08 | 5.21E+08 | 5.38E+08 | 5.98E+08 | 6.80E+08 | 7.77E+08 | 8.92E+08 | 8.59E+08 | 8.96E+08 | 6.48E+08 |
| Total Assets | 1.53E+09 | 1.66E+09 | 1.73E+09 | 1.77E+09 | 1.86E+09 | 2.05E+09 | 2.29E+09 | 2.57E+09 | 2.92E+09 | 2.89E+09 | 3.06E+09 | 2.21E+09 |
| D | 1.04E+09 | 1.11E+09 | 1.16E+09 | 1.17E+09 | 1.23E+09 | 1.36E+09 | 1.51E+09 | 1.68E+09 | 1.89E+09 | 1.87E+09 | 1.99E+09 | 1.46E+09 |
| $r_D$ | 1.20E-02 | 8.82E-03 | 5.73E-03 | 5.29E-03 | 4.87E-03 | 6.09E-03 | 8.03E-03 | 8.82E-03 | 6.59E-03 | 3.53E-03 | 3.09E-03 | 6.62E-03 |
| $r_L$ | 1.76E-02 | 1.42E-02 | 1.10E-02 | 1.23E-02 | 1.16E-02 | 1.25E-02 | 1.42E-02 | 1.44E-02 | 1.15E-02 | 8.56E-03 | 8.17E-03 | 1.24E-02 |
| $N_I$ | 6.78E+06 | 6.80E+06 | 7.04E+06 | 7.02E+06 | 7.63E+06 | 8.48E+06 | 8.67E+06 | 9.60E+06 | 9.90E+06 | 9.68E+06 | 1.05E+07 | 8.37E+06 |
| $O_A$ | 6.48E+08 | 7.21E+08 | 7.67E+08 | 8.19E+08 | 8.56E+08 | 9.49E+08 | 1.06E+09 | 1.19E+09 | 1.39E+09 | 1.38E+09 | 1.30E+09 | 1.01E+09 |
| $r_{OA}$ | 8.20E-03 | 7.24E-03 | 5.73E-03 | 5.08E-03 | 5.21E-03 | 5.79E-03 | 6.40E-03 | 6.64E-03 | 5.37E-03 | 3.72E-03 | 4.03E-03 | 5.76E-03 |
| $r_{OL}$ | 7.12E-03 | 5.90E-03 | 4.36E-03 | 4.08E-03 | 3.97E-03 | 5.29E-03 | 6.36E-03 | 6.01E-03 | 3.93E-03 | 2.47E-03 | 2.38E-03 | 4.71E-03 |
| $r_T$ | 2.90E-03 | 2.19E-03 | 9.77E-04 | 1.32E-03 | 1.33E-03 | 1.64E-03 | 1.79E-03 | -3.89E-06 | -1.08E-03 | 1.18E-03 | 8.37E-04 | 1.19E-03 |
| W | 8.61E+06 | 8.11E+06 | 6.64E+06 | 5.92E+06 | 6.43E+06 | 7.92E+06 | 9.79E+06 | 1.13E+07 | 1.02E+07 | 6.99E+06 | 8.17E+06 | 8.19E+06 |
| Z | 3.04E+06 | 2.44E+06 | 1.13E+06 | 1.54E+06 | 1.64E+06 | 2.25E+06 | 2.72E+06 | -1.40E+05 | -2.06E+06 | 2.22E+06 | 1.71E+06 | 1.50E+06 |
| $N_E$ | 1.02E+07 | 1.10E+07 | 1.11E+07 | 1.11E+07 | 1.14E+07 | 1.25E+07 | 1.20E+07 | 1.25E+07 | 1.33E+07 | 1.45E+07 | 1.48E+07 | 1.22E+07 |
| $T_X$ | 1.27E+06 | 8.97E+05 | 8.33E+05 | 1.32E+06 | 1.54E+06 | 1.47E+06 | 1.56E+06 | 1.01E+06 | 6.61E+05 | 1.73E+06 | 2.17E+06 | 1.32E+06 |
| $K_{Market}$ | 7.38E+07 | 8.04E+07 | 8.23E+07 | 8.36E+07 | 8.72E+07 | 9.25E+07 | 1.03E+08 | 1.13E+08 | 1.36E+08 | 1.56E+08 | 1.68E+08 | 1.07E+08 |
| varrl | 2.57E-07 | 5.87E-06 | 7.98E-07 | 7.17E-08 | 9.21E-08 | 4.48E-07 | 3.60E-07 | 2.39E-07 | 1.10E-06 | 6.96E-07 | 5.91E-07 | 9.57E-07 |
| varroa | 1.82E-07 | 3.75E-07 | 5.55E-08 | 3.95E-08 | 6.77E-08 | 1.16E-07 | 1.39E-07 | 2.02E-07 | 5.77E-07 | 2.29E-06 | 1.06E-06 | 4.64E-07 |
| varrt | 1.32E-07 | 4.07E-07 | 3.73E-07 | 2.95E-08 | 1.93E-07 | 1.60E-07 | 8.57E-08 | 3.92E-06 | 5.45E-07 | 5.30E-07 | 4.18E-07 | 6.18E-07 |
| vard | 5.50E+14 | 1.03E+15 | 1.88E+14 | 2.60E+14 | 5.43E+14 | 9.42E+14 | 4.92E+15 | 4.35E+15 | 4.54E+15 | 2.50E+14 | 2.58E+15 | 1.83E+15 |
| varni | 8.32E+10 | 2.34E+13 | 1.26E+10 | 9.84E+10 | 3.56E+10 | 6.44E+11 | 1.39E+11 | 3.95E+11 | 3.52E+11 | 1.18E+11 | 3.26E+11 | 2.33E+12 |
| varw | 1.68E+11 | 1.96E+11 | 4.31E+10 | 4.36E+10 | 1.59E+11 | 4.59E+11 | 9.25E+11 | 6.77E+10 | 1.17E+12 | 8.11E+12 | 5.33E+12 | 1.52E+12 |
| varz | 1.59E+11 | 4.07E+11 | 4.90E+11 | 4.81E+10 | 3.10E+11 | 3.38E+11 | 1.21E+11 | 1.21E+13 | 1.95E+12 | 1.87E+12 | 1.82E+12 | 1.78E+12 |

**Figure 6.2. Framework Empirical Inputs and Outputs - Part I.**

| Year | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | Average |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| covrlroa | -1.61E-01 | 9.96E-01 | -7.46E-01 | -6.32E-01 | 7.47E-01 | 8.05E-01 | 7.78E-01 | 6.58E-01 | 5.63E-01 | -9.81E-01 | -8.28E-01 | 1.09E-01 |
| covnirl | -7.32E-01 | 3.42E-01 | 1.46E-01 | 9.91E-01 | -1.80E-01 | 8.08E-01 | 4.49E-01 | 2.70E-01 | -4.22E-01 | 1.59E-01 | -5.52E-01 | 1.16E-01 |
| covrlrt | -7.28E-01 | 7.17E-01 | 1.56E-01 | 1.33E-01 | -4.72E-01 | 7.76E-01 | -9.34E-01 | 9.01E-01 | -7.27E-01 | 5.58E-01 | -8.75E-01 | -4.50E-02 |
| covdrl | 7.31E-02 | -9.27E-01 | 4.88E-01 | 4.66E-01 | 4.46E-01 | 8.42E-01 | 6.80E-01 | -7.67E-01 | -6.18E-01 | 3.23E-01 | -6.71E-01 | 3.05E-02 |
| covrlw | -1.34E-01 | 9.91E-01 | -7.83E-01 | -5.20E-01 | 7.57E-01 | 8.43E-01 | 7.94E-01 | 2.35E-01 | 5.33E-01 | -9.77E-01 | -8.22E-01 | 8.33E-02 |
| covrlz | -6.83E-01 | 6.75E-01 | 1.72E-01 | 1.64E-01 | -4.21E-01 | 8.26E-01 | -8.98E-01 | 9.08E-01 | -7.01E-01 | 5.65E-01 | -8.81E-01 | -2.49E-02 |
| covniroa | -2.52E-01 | 3.63E-01 | 5.03E-01 | -6.28E-01 | -2.64E-01 | 8.49E-01 | 4.62E-01 | 5.18E-01 | -5.38E-01 | -3.10E-01 | 8.81E-01 | 1.44E-01 |
| covroart | 6.71E-01 | 7.72E-01 | 2.38E-01 | 2.06E-01 | 5.48E-02 | 2.59E-01 | -5.55E-01 | 8.70E-01 | 4.39E-02 | -7.05E-01 | 7.56E-01 | 2.37E-01 |
| covdroa | -4.05E-01 | -9.54E-01 | -8.43E-01 | -4.40E-01 | 3.97E-01 | 8.72E-01 | 6.38E-01 | -9.62E-01 | -9.87E-01 | -1.35E-01 | 9.50E-01 | -1.70E-01 |
| covroaw | 8.69E-01 | 9.78E-01 | 9.78E-01 | 9.38E-01 | 9.44E-01 | 9.84E-01 | 9.14E-01 | 7.44E-01 | 9.98E-01 | 1.00E+00 | 1.00E+00 | 9.41E-01 |
| covroaz | 5.57E-01 | 7.32E-01 | 2.19E-01 | 1.47E-01 | 8.86E-02 | 3.51E-01 | -4.22E-01 | 8.74E-01 | 1.04E-01 | -7.11E-01 | 7.78E-01 | 2.47E-01 |
| covnirt | 9.78E-02 | 5.63E-01 | 8.17E-01 | 2.56E-01 | 5.77E-01 | 4.84E-01 | -5.71E-01 | 2.14E-01 | 4.52E-01 | 8.27E-01 | 6.94E-01 | 4.01E-01 |
| covdni | -4.25E-01 | -3.28E-01 | -7.71E-01 | 5.81E-01 | 7.59E-01 | 9.98E-01 | 9.60E-01 | -3.05E-01 | 6.62E-01 | -6.47E-01 | 9.85E-01 | 2.24E-01 |
| covniw | -5.05E-01 | 3.73E-01 | 3.57E-01 | -4.71E-01 | 6.07E-02 | 9.29E-01 | 7.78E-01 | 9.07E-01 | -4.86E-01 | -3.22E-01 | 8.91E-01 | 2.28E-01 |
| covniz | 3.95E-03 | 6.01E-01 | 8.11E-01 | 2.89E-01 | 6.08E-01 | 5.82E-01 | -3.42E-01 | 2.41E-01 | 3.92E-01 | 8.25E-01 | 7.14E-01 | 4.30E-01 |
| covdrt | 8.36E-02 | -8.98E-01 | -7.18E-01 | 7.86E-01 | 4.14E-01 | 5.07E-01 | -7.51E-01 | -9.62E-01 | 8.59E-02 | -5.75E-01 | 7.39E-01 | -1.17E-01 |
| covrtw | 7.74E-01 | 6.44E-01 | 2.97E-02 | 5.30E-01 | 1.94E-01 | 3.56E-01 | -7.01E-01 | 3.67E-01 | 9.72E-02 | -7.18E-01 | 7.59E-01 | 2.12E-01 |
| covrtz | 9.77E-01 | 9.97E-01 | 1.00E+00 | 9.98E-01 | 9.98E-01 | 9.93E-01 | 9.64E-01 | 1.00E+00 | 9.98E-01 | 1.00E+00 | 9.99E-01 | 9.93E-01 |
| covdw | 1.00E-01 | -8.70E-01 | -7.10E-01 | -1.02E-01 | 6.77E-01 | 9.45E-01 | 8.95E-01 | -5.36E-01 | -9.74E-01 | -1.17E-01 | 9.56E-01 | 2.41E-02 |
| covdz | 2.92E-01 | -8.64E-01 | -7.05E-01 | 8.22E-01 | 4.72E-01 | 6.03E-01 | -5.49E-01 | -9.61E-01 | 2.19E-02 | -5.67E-01 | 7.61E-01 | -6.14E-02 |
| covwz | 7.64E-01 | 6.02E-01 | 1.07E-02 | 4.79E-01 | 2.42E-01 | 4.52E-01 | -5.15E-01 | 3.86E-01 | 1.56E-01 | -7.24E-01 | 7.82E-01 | 2.40E-01 |

**Figure 6.3. Framework Empirical Inputs and Outputs - Part II.**

| Year | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | Average |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Impairment/Total Income | 4.00E-02 | 7.10E-02 | 8.52E-02 | 3.32E-02 | 1.71E-02 | 1.90E-02 | 1.87E-02 | 2.70E-02 | 6.19E-02 | 1.00E-01 | 5.98E-02 | 4.85E-02 |
| Loan Income/Total Income | 5.45E-01 | 5.40E-01 | 5.10E-01 | 5.03E-01 | 4.78E-01 | 4.79E-01 | 5.12E-01 | 5.49E-01 | 5.59E-01 | 4.98E-01 | 4.95E-01 | 5.15E-01 |
| Other Interest Income/Total Income | 1.74E-01 | 1.87E-01 | 1.78E-01 | 1.70E-01 | 1.76E-01 | 1.86E-01 | 1.92E-01 | 2.08E-01 | 2.08E-01 | 1.57E-01 | 1.56E-01 | 1.81E-01 |
| Noninterest Income/Total Income | 2.20E-01 | 2.18E-01 | 2.82E-01 | 2.83E-01 | 3.01E-01 | 2.83E-01 | 2.43E-01 | 2.49E-01 | 2.76E-01 | 2.96E-01 | 3.19E-01 | 2.70E-01 |
| Trading Income/Total Income | 6.14E-02 | 5.46E-02 | 3.00E-02 | 4.42E-02 | 4.47E-02 | 5.24E-02 | 5.37E-02 | -5.55E-03 | -4.30E-02 | 4.87E-02 | 3.03E-02 | 3.38E-02 |
| Deposit Expenses-to-Total Expenses Ratio | 4.88E-01 | 4.14E-01 | 3.34E-01 | 3.17E-01 | 3.07E-01 | 3.46E-01 | 4.28E-01 | 4.67E-01 | 4.28E-01 | 2.85E-01 | 2.69E-01 | 3.71E-01 |
| Other Liabilities Expenses/Total Expenses | 1.15E-01 | 1.16E-01 | 1.09E-01 | 1.11E-01 | 1.10E-01 | 1.34E-01 | 1.52E-01 | 1.46E-01 | 1.19E-01 | 9.05E-02 | 9.27E-02 | 1.18E-01 |
| Noninterest Expenses/Total Expenses | 3.97E-01 | 4.70E-01 | 5.56E-01 | 5.72E-01 | 5.83E-01 | 5.21E-01 | 4.20E-01 | 3.87E-01 | 4.54E-01 | 6.24E-01 | 6.38E-01 | 5.11E-01 |
| $K_{Market}$ | 7.38E+07 | 8.04E+07 | 8.23E+07 | 8.36E+07 | 8.72E+07 | 9.25E+07 | 1.03E+08 | 1.13E+08 | 1.36E+08 | 1.56E+08 | 1.68E+08 | 1.07E+08 |
| $K_S$ | 1.05E+09 | -5.72E+08 | 1.67E+09 | 1.59E+09 | -2.05E+09 | -2.85E+09 | 3.75E+08 | -6.33E+08 | 3.51E+09 | 1.98E+08 | 2.85E+09 | 4.67E+08 |
| $K_O$ | 1.13E+07 | 1.18E+07 | 1.68E+07 | 1.78E+07 | 1.82E+07 | 2.10E+07 | 1.84E+07 | 1.75E+07 | 2.70E+07 | 3.47E+07 | 3.83E+07 | 2.12E+07 |
| $\alpha$=(OSFI ROE) | 1.68E-01 | 1.57E-01 | 1.00E-01 | 1.60E-01 | 1.83E-01 | 1.59E-01 | 2.32E-01 | 2.12E-01 | 1.16E-01 | 1.12E-01 | 1.45E-01 | 1.59E-01 |
| SOC | 2.68E+00 | 3.09E+00 | 9.51E-01 | 8.23E-01 | 2.19E-01 | 1.04E+00 | -2.22E+00 | 3.48E+00 | 1.76E-01 | -2.82E+00 | 3.02E+00 | 9.49E-01 |
| Lagrange Multiplier ($K_O$) | -2.94E+27 | -4.12E+27 | -3.75E+27 | -7.61E+26 | -1.39E+26 | -1.76E+27 | 2.21E+27 | -7.21E+27 | -2.50E+27 | 2.24E+28 | -1.09E+28 | -8.61E+26 |
| Gross Income Reported | 3.84E+06 | 3.34E+06 | 2.96E+06 | 4.57E+06 | 5.35E+06 | 5.33E+06 | 6.44E+06 | 5.33E+06 | 4.31E+06 | 6.22E+06 | 7.92E+06 | 5.05E+06 |
| $\pi(K_{Market})$ | 3.87E+06 | 3.40E+06 | 2.95E+06 | 4.59E+06 | 5.32E+06 | 5.34E+06 | 6.52E+06 | 5.35E+06 | 4.27E+06 | 6.21E+06 | 8.35E+06 | 5.11E+06 |
| $\pi(K_O)$ | 3.18E+06 | 2.75E+06 | 2.51E+06 | 4.17E+06 | 4.87E+06 | 4.81E+06 | 5.82E+06 | 4.72E+06 | 3.80E+06 | 5.62E+06 | 7.72E+06 | 4.54E+06 |

**Figure 6.4. Framework Empirical Inputs and Outputs - Part III.**

| Year | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | Average |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Variance of Gross Income Reported | 9.67E+10 | 2.98E+13 | 1.32E+12 | 9.07E+11 | 3.21E+11 | 4.46E+12 | 7.00E+10 | 6.43E+12 | 9.33E+11 | 3.89E+12 | 6.20E+11 | 4.44E+12 |
| $Var[\pi(K_{Market})]$ | 8.31E+17 | -1.01E+18 | 5.14E+17 | 3.80E+17 | -1.93E+17 | -1.33E+18 | -4.85E+16 | -1.70E+18 | 2.76E+17 | 2.86E+17 | 4.92E+18 | 2.66E+17 |
| $Var[\pi(K_S)]$ | -4.57E+17 | -1.67E+18 | -6.79E+17 | -5.50E+17 | -6.94E+17 | -5.82E+18 | 3.34E+16 | -2.67E+18 | -7.23E+17 | 2.89E+17 | -5.95E+18 | -1.72E+18 |
| $Var[\pi(K_O)]$ | 1.00E+18 | -1.14E+18 | 6.14E+17 | 4.63E+17 | -2.25E+17 | -1.55E+18 | -1.08E+17 | -1.94E+18 | 3.42E+17 | 2.51E+17 | 6.00E+18 | 3.37E+17 |
| Credit and Market RWA (1000$) | 8.25E+08 | 8.42E+08 | 8.22E+08 | 7.97E+08 | 7.97E+08 | 8.67E+08 | 9.48E+08 | 1.07E+09 | | | | |
| Operational risk (1000$) | 6.56E+06 | 7.24E+06 | 7.81E+06 | 8.32E+06 | 8.63E+06 | 8.98E+06 | 9.24E+06 | 9.61E+06 | | | | |
| $K_B$ (1000$) or $K_{BaselII}$ without considering the ratio | 7.26E+07 | 7.46E+07 | 7.36E+07 | 7.21E+07 | 7.24E+07 | 7.83E+07 | 8.51E+07 | 9.50E+07 | | | | |
| $K_{Framework\ Definition}$/ (Total capital Basel II) | 7.47E-01 | 7.67E-01 | 8.05E-01 | 8.09E-01 | 8.13E-01 | 8.29E-01 | 8.45E-01 | 8.40E-01 | | | | |
| Total RWA | 9.08E+08 | 9.32E+08 | 9.19E+08 | 9.01E+08 | 9.05E+08 | 9.79E+08 | 1.06E+09 | 1.19E+09 | | | | |
| $K_{\textbf{Basel II}}$ | 5.43E+07 | 5.72E+07 | 5.92E+07 | 5.83E+07 | 5.89E+07 | 6.49E+07 | 7.19E+07 | 7.98E+07 | | | | |
| $K_{BaselII}$ RWA Ratio | 5.98E-02 | 6.13E-02 | 6.44E-02 | 6.47E-02 | 6.51E-02 | 6.63E-02 | 6.76E-02 | 6.72E-02 | | | | |
| $K_S$ RWA Ratio | 1.16E+00 | -6.14E-01 | 1.82E+00 | 1.77E+00 | -2.27E+00 | -2.91E+00 | 3.53E-01 | -5.33E-01 | | | | |
| $K_O$ RWA Ratio | 1.25E-02 | 1.27E-02 | 1.83E-02 | 1.98E-02 | 2.01E-02 | 2.15E-02 | 1.73E-02 | 1.47E-02 | | | | |
| $K_{Market}$ RWA Ratio | 8.13E-02 | 8.62E-02 | 8.95E-02 | 9.28E-02 | 9.64E-02 | 9.45E-02 | 9.68E-02 | 9.52E-02 | | | | |
| $K_O/K_{BaselII}$ | 2.08E-01 | 2.06E-01 | 2.84E-01 | 3.05E-01 | 3.09E-01 | 3.24E-01 | 2.56E-01 | 2.19E-01 | | | | |
| $K_{market}/K_{BaselII}$ | 1.36E+00 | 1.41E+00 | 1.39E+00 | 1.43E+00 | 1.48E+00 | 1.43E+00 | 1.43E+00 | 1.42E+00 | | | | |
| $K_O/K_{Market}$ | 1.53E-01 | 1.47E-01 | 2.04E-01 | 2.13E-01 | 2.09E-01 | 2.27E-01 | 1.79E-01 | 1.55E-01 | 1.99E-01 | 2.22E-01 | 2.28E-01 | 1.94E-01 |
| $K_O$ for $\alpha_{4.855}$=(OSFI ROE)/4.855 | 7.56E+07 | 7.60E+07 | 1.13E+08 | 1.03E+08 | 1.03E+08 | 1.26E+08 | 1.04E+08 | 9.69E+07 | 1.54E+08 | 2.05E+08 | 2.15E+08 | 1.25E+08 |
| ($K_O$ for $\alpha_{4.855}$)/$K_{Market}$ | 1.02E+00 | 9.45E-01 | 1.37E+00 | 1.23E+00 | 1.18E+00 | 1.36E+00 | 1.01E+00 | 8.58E-01 | 1.13E+00 | 1.31E+00 | 1.28E+00 | 1.17E+00 |
| $K_O$ for $\alpha_{3.26}$=(OSFI ROE)/3.26 | 4.40E+07 | 4.50E+07 | 6.53E+07 | 6.42E+07 | 6.47E+07 | 7.69E+07 | 6.53E+07 | 6.15E+07 | 9.66E+07 | 1.26E+08 | 1.35E+08 | 7.68E+07 |
| ($K_O$ for $\alpha_{3.26}$)/$K_{market}$ | 5.96E-01 | 5.60E-01 | 7.93E-01 | 7.68E-01 | 7.42E-01 | 8.31E-01 | 6.34E-01 | 5.44E-01 | 7.10E-01 | 8.08E-01 | 8.04E-01 | 7.18E-01 |

**Figure 6.5. Framework Empirical Inputs and Outputs - Part IV.**