Essays in International Portfolio Choice and Monetary Policy

by

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Abstract

Empirical evidence shows that equity home bias is a prevailing fact for most countries, but standard international monetary business cycle models with nominal bonds hardly generate equity home bias for plausible preference parameter values. By incorporating inflation-indexed bonds, I show in chapter 2 that international monetary business cycle models can explain home bias in equities. Inflation-indexed bonds can hedge real exchange rate risk and domestic equities serve as a hedge against domestic labor income risk conditional on real exchange rates. Moreover, this chapter accounts for counter-cyclical movements of net foreign asset positions. These results are robust in environments either with complete markets or with incomplete markets.

How does international financial market integration alter international risk sharing? Chapter 3 develops a tractable center-periphery model with portfolio choice to investigate this issue. I compare three stages of financial integration. The first stage is financial autarky. The second stage is the central country becomes financially integrated with peripheral countries, but there is no financial integration between peripheral countries. The third stage is all financial markets are integrated into each other. From financial autarky to partial financial integration, volatility of consumption in all countries drops. From partial financial integration to full financial integration, volatility of consumption in peripheral countries decreases; however, consumption volatility rises in the central country when the central country is relatively large. When peripheral countries are large, the degree of international risk-sharing for all countries increases in the process of financial integration.

What's the optimal monetary policy in an economy with financial frictions? Chapter 4 investigates optimal monetary policies in a dynamic stochastic general equilibrium model with sticky prices, sticky wages and credit-market imperfections. Credit frictions distort allocations and prices, and generate large volatility of aggregate variables via the *financial accelerator*. Policy-makers can take advantage of a debt deflation channel to push down volatility of endogenous variables through the *financial decelerator*.

Abstract

In the optimized linear interest rate rule, interest rate decreases in asset prices but the response is quantitatively small. This optimized rule exhibits high persistence. Within a class of simple linear interest rate rules, a strict inflation-targeting rule has a larger welfare loss.

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Dedication

To my parents

Chapter 1

Introduction

Over the past few decades, capital flows across countries have reached a historical high and financial linkage across borders has become stronger and stronger. On the one hand, we observe a large increase in capital flows between developed countries and developing countries, also in conjunction with enlarged capital flows among developed countries. On the other hand, households in most countries still hold most of domestic equities in their country level equity portfolios. This thesis mainly tries to rationalize international portfolio choice in the context of financial globalization and explores how financial integration affects international risk-sharing. The Great Recession caused by $2007 - 2009$ financial crisis recalls an investigation on financial frictions and policy studies. I then study optimal monetary policies in an closed economy with financial frictions. In the following parts, I will illustrate these issues in three sections, which correspond to three following chapters in this thesis. In the first section, I discuss how to explain equity home bias in an open economy monetary model. In the second section, I explore how financial integration changes international risk-sharing. In the last section, I present how monetary and fiscal authorities implement policies in an economy with credit frictions.

1.1 Equity home bias in a monetary model

With the development of financial globalization in the past few decades, portfolio flows across countries have reached a historical high, and particularly, trade volume of assets across borders has surged dramatically since the early of 1990's. Nevertheless, countries still exhibit sizable home bias in equity portfolios. For instance, the share of country equity portfolios in domestic equities was 82% in the United States in 2005 (see Sercu and Vanpee (2007), summarized in table 1.1).[1] International portfolio choice

[1] Sercu and Vanpee (2007) has recently documented the share of country equity portfolios in domestic equities for a large number of countries, whereas Tesar (1993) provide early evidence on home bias in equities for the group of seven countries.

theories (e.g., Lucas 1982; Baxter and Jermann 1997) fail to explain this phenomenon often referred to the puzzle of home bias in equity portfolios (Obstfeld and Rogoff 2001).

A number of recent studies suggest potential resolutions to the equity home bias puzzle, relying on the notion that holding domestic equities provides a hedge for non-diversifiable income risk. Coeurdacier, Kollmann and Martin (2010) use an international real business cycle model with real bonds, while Engel and Matsumoto (2009a) and Engel and Matsumoto (2009b) employ an international monetary model with foreign currency forward contracts. Coeurdacier and Gourinchas (2011) offer empirical evidence on using bonds to hedge real exchange rate risk and domestic equities to hedge domestic labor income risk.[2] Nevertheless, in standard international monetary business cycle models with nominal bonds (e.g., Chari, Kehoe and McGrattan 2002), the equilibrium domestic equities held by domestic households are significantly sensitive to variations of preference parameter values. In contrast to existing theories, this chapter shows that international monetary business cycle models can be reconciled with equity home bias.

The contributions of this chapter are two-fold. First, I show that, by incorporating inflation-indexed bonds, international monetary business cycle models deliver home bias in equities for plausible preference parameter values. Inflation-indexed bonds can hedge real exchange rate risk and domestic equities can hedge against domestic labor income risk conditional on real exchange rates. Moreover, equity home bias is robust in environments either with complete markets, or incomplete markets. This explanation is motivated by the data. I present empirical evidence on bonds hedging real exchange rate risk and domestic equities hedging domestic labor income risk, conditional on real exchange rates, for a sub-sample of OECD countries (the group of seven countries, plus Denmark and Spain). Second, I explore in-

[2]These arguments are in line both with the hedging motive and the diversification motive promoted by Baxter and Jermann (1997). The hypotheses in Baxter and Jermann (1997) are based on the assumption of one-good world economy and therefore they focused on the unconditional correlation between labor income and capital income. Recent works study multiple tradable goods in a world economy and they focus on conditional correlation between labor income and capital income. Coeurdacier, Kollmann and Martin (2010) address home bias in equities in an international real business cycle model with real bonds and technology and investment efficiency shocks. In addition, Coeurdacier and Gourinchas (2011) exploit equity home bias in an endowment economy with real bonds and provided relevant empirical evidence. On the other hand, equity home bias studies by Engel and Matsumoto (2009a) and Engel and Matsumoto (2009b) explain the phenomenon as a result of nominal stickiness being highly strong, given that foreign currency forward contracts or one-period nominal bonds are equivalently to one-period real bonds when the degree of price stickiness tends to unity.

ternational portfolio dynamics. In the data sample, net foreign asset positions counter-cyclically move with output. I find that the model with either complete markets or incomplete markets can account for counter-cyclical movements of net foreign asset positions.[3]

Based on monetary business cycle models (see Chari, Kehoe and McGrattan 2002; Schmitt-Grohe and Uribe 2006), this chapter investigates international portfolio choice in a two-country dynamic stochastic general equilibrium (DSGE, hereafter) monetary model with an asset menu of equity and debt portfolios, investment, sticky prices and sticky wages.[4] I show that when asset markets are complete, the equilibrium equity portfolios can be decomposed into three components: market portfolio, portfolio for hedging labor income risk, and portfolio for hedging real exchange rate (or terms of trade) risk. Here, market portfolio is considered constant, one half, which reflects the traditional view on diversifying income risk across country borders (see, Lucas 1982; Baxter and Jermann 1997). It is argued that home bias in equity portfolios is present when: (a) domestic equities hedge domestic labor income risk conditional on real exchange rates, and (b) bonds hedge real exchange rate risk. Results indicate that condition (a) is satisfied in various environments, either with complete asset markets, or with incomplete asset markets, whereas condition (b) holds when inflation-indexed bonds are traded in international bond markets. These two conditions are motivated by the data. Evidence shows that 6 out of 9 country-pairs in the data sample support condition (a) and all of 9 country-pairs support condition (b).

In the benchmark model with producer currency pricing, monetary and technological shocks are considered, and hence, asset markets are complete

[3]The ratio of net foreign asset (NFA, hereafter, are the sum of net FDI outflows, net equity asset outflows, net debt asset outflows and reserves) to GDP in the United States declined steadily from 8% in 1970 to -23% in 2004. Moreover, net foreign assets counter-cyclically move with output for 8 out of 9 countries in the data sample, including the United States, Britain and Germany. The correlation between cyclical components of U.S. NFA-GDP ratio and U.S. GDP is -0.27 during $1970 - 2004$. From the text book definition of net foreign asset positions and current account, it is known that the change of net foreign asset positions is the sum of current account and wealth effect from changes of asset prices. The wealth channel here has a large contribution to the net foreign asset dynamics. The correlation between the cyclical components of wealth effects and of GDP is -0.11 in the United States. Coeurdacier, Kollmann and Martin (2010) report second moments for portfolios in an international real business cycle model with complete asset markets.

[4]There isn't investment in Engel and Matsumoto's work and the equilibrium equities are very sensitive to preference parameter values.

from a first-order approximation perspective.[5] Consequently, households across countries can achieve perfect risk sharing. However, international portfolio choice depends on the type of debt instruments available in asset markets. I find that in standard international monetary models with nominal bonds, the component of equity portfolios for hedging labor income risk is positive and extremely stable, whereas the component for hedging real exchange rate risk is sensitive to variations of preference parameter values. Accordingly, equilibrium equities vary significantly under different values of preference parameters.

I further find that a monetary model with real bonds can deliver robust home bias in equities, since returns on real bonds perfectly correlate with real exchange rates. Nonetheless, it is difficult to identify real bonds in the data. Conversely, inflation-indexed bonds seem to be a natural alternative to real bonds in hedging real exchange rate risk. A caveat here is that a significant difference between these two types of bonds remains because real bonds are state-contingent whereas inflation-indexed bonds are nominal, and therefore, non-state contingent. This chapter shows that inflation-indexed bonds in this model are quantitatively akin to real bonds in hedging real exchange rate risk. By incorporating inflation-indexed bonds, the model delivers robust equity home bias.

To this effect, an intuitive explanation for inflation-indexed bonds hedging real exchange rate risk is as follows. Monetary shocks have transitory effects on output, consumption and real exchange rates, but they have persistent effects on nominal prices. When nominal bonds, say, nominal consols or one-period nominal bonds, are traded internationally, relative interest payments on nominal bonds disconnect with real exchange rates because of high volatility of nominal prices. However, inflation-indexed bonds could bring this disconnection back, because the volatility of inflation rates is far smaller than that of nominal prices in a monetary model with monetary policy shocks. Thereby, inflation-indexed bonds quantitatively mimic real bonds in terms of hedging real exchange rate risk.

[5] Pricing-to-market is the other common pricing strategy for firms. Recent evidence from micro data (see Gopinath and Rigobon 2008 ; Gopinath, Itskhoki and Rigobon 2010) shows that 90% (97%) of U.S. imports (exports) are priced in dollars. When firms take advantage of pricing-to-market, real exchange rates disconnect with terms of trade, and consequently one debt instrument cannot hedge terms of trade and real exchange rate fluctuations simultaneously. However, it is shown that home equity holdings vary only slightly larger than those featuring producer currency pricing, responding to different values of preference parameters.

Table 1.1: Home bias in equity portfolios

Country	Share of portfolios in domestic equity (%, Year 2005)
Austria	58.5
Canada	76.6
Denmark	62.7
France	68.8
Germany	57.5
Italy	57.1
Japan	91.9
Spain	86.3
UK	65.0
US	82.2
Average	76.61

Source: Open access publications from Katholieke Universiteit Leuven, Sercu and Vanpee (2007), adapted by permission. Equity home bias in 2005, CPIS data.

Empirical data employed in this chapter takes into consideration inflation factors introduced by indexed bonds. The reason behind this rationale is that recent developments of bond markets have seen a burgeoning supply of inflation-index bonds in developed countries. For instance, Campbell, Shiller and Viceira (2009) document that the outstanding supply of Treasury Inflation-protected Securities (TIPS) accounted of 10% of marketable Treasury debt in the United States in 2008 and that inflation-indexed government bonds increased to 30% of British government bonds in 2008.[6] Besides the large trade of inflation-indexed bonds in the corresponding markets, the other consideration behind this rationale depends on bond returns observed in the data sample. Returns on bonds and bond return differentials across countries in the data sample show that bond returns are around one-half as volatile as output is. The model with inflation-indexed bonds could produce similar volatility of bond returns. Consequently, this model

[6]Inflation-indexed bonds are usually long-term bonds (term-to-maturity in excess of one year). Long-term bonds are as important as equities in international asset markets. Survey data from U.S. Treasury International Capital System showed that the share of long-term securities in foreign holdings of U.S. securities was 92% and that this share in U.S. holdings of foreign securities was 93% at the end of 2008. Here, long-term debt securities accounted for both 70% of U.S. external long-term liabilities and 30% of U.S. external long-term assets.

not only explains equity home bias but also matches the second moments of bond returns, thus providing a clear advantage over existing ones.

To address the issue of how different monetary policy specifications change the optimal portfolios, exogenous money supplies replace Taylor rules in the benchmark model. It reveals that equilibrium equity holdings remain fairly stable. An examination of whether preference specifications affect equilibrium equity portfolios follows next. The benchmark model uses a non-separable preference between consumption and real money balance, given that the latter affects marginal utility of consumption directly. To eliminate the direct effect of money balance on consumption, a separable preference between consumption and real money balance is employed. Results show that equity home bias is robust to preference parameter values.

When asset markets are complete, households in both countries achieve perfect risk sharing. However, when asset markets are incomplete, the degree of risk sharing is limited (see Corsetti, Dedola and Leduc 2008) and accordingly, portfolios are also adjusted correspondingly. This circumstance motivates further interest to understand how equity portfolios in incomplete markets are different from those in complete markets. To this end, market incompleteness is introduced by incorporating additional independent shocks into the benchmark model. According to the recent estimation of DSGE models (Justiniano, Primiceri and Tambalotti 2010), several other shocks are considered to largely contribute to business cycle fluctuations, including investment efficiency shocks, government expenditure shocks and preference shocks. Two versions of market incompleteness are considered here. In the first version, one extra shock is added to the benchmark model separately, whereas the second version includes all shocks. The results show that equilibrium equity portfolios seem to be invariant to the concurrence of these additional shocks.[7] To better understand how inflation matters for nominal bonds hedging real exchange rate risk, I then remove monetary shocks, keep only all real shocks, and use nominal bonds instead of inflation-indexed bonds in the model. The results show that nominal bonds can also hedge real exchange rate risk.

Pioneering researchers have proposed numerous ways to solve the puz-

[7] A question arises here as to equilibrium equity portfolios when only real shocks hit the economy. To answer this question, monetary shocks are disregarded and two versions of models with complete asset markets and real shocks are considered instead. When (1) only technology shocks and government expenditure shocks or (2) only technology shocks and preference shocks appear, the model generates super home bias in equities (home equity shares held by home households are larger than unity). However, incorporating monetary shocks can bring super home bias in equities down to home bias in equities.

zle of equity towards home.[8] In Cole and Obstfeld (1991), terms of trade themselves can facilitate risk sharing across borders under special parameterizations. Following this logic, Heathcote and Perri (2008) later argue that equity home bias is optimal in a two-country DSGE environment. However, their results are sensitive to preference parameter values. In contrast to these, efforts by a separate group of researchers took account of the influence of non-tradable goods. For instance, Baxter, Jermann and King (1998) show that, even when allowing for non-traded goods and non-traded factors, equities of domestic tradable goods never optimally exhibit home bias. Similar implications of a richer model environment were examined by Pesenti and van Wincoop (2002) and Collard, Dellas, Diba and Stockman (2009).[9] Hnatkovska (2010) study an economy with two-country, tradable goods, non-tradable goods sectors, and production. She shows that low diversification happens because variations in relative prices increase the riskiness of foreign assets and facilitate international risk-sharing. Yet another strand of research efforts emphasized hedging non-diversifiable human capital income (see Engel and Matsumoto 2009a; Engel and Matsumoto 2009b; Coeurdacier, Kollmann and Martin 2010; Coeurdacier and Gourinchas 2011).[10] Transaction cost is another important consideration in explaining equity home bias. Obstfeld (2007) Ohlin lecture provides an excellent summary on international asset trade and costs of trade.[11] Nonetheless, the latest literature on the subject reveals information asymmetry in explaining capital flows. For example, Tille and van Wincoop (2010) investigate the role of information dispersion in a general equilibrium environment.[12]

[8] An influential survey was originally conducted by Adler and Dumas (1983). Later, Lewis (1999) provide a summary of recent development of home bias in equities. The latest comprehensive survey work on open economy macro-finance can be found in Coeurdacier and Rey (2012).

[9] Tesar (1993) studies a two-period model with multiple non-tradable goods. In a partial equilibrium environment, Pavlova and Rigobon (2010) investigate portfolio choice in an endowment economy.

[10] Kollmann (2001) investigates the robustness of home bias in equities when these are the only international assets, whereas Collard, Dellas, Diba and Stockman (2009) illustrate that optimal equity shares are sensitive to trade elasticity.

[11] Coeurdacier (2009) shows that equities may not bias the home country for reasonable trade cost.

[12] Private information and the associated cost have been considered as important factors determining international equity flows (see Nieuwerburgh and Veldkamp 2009; Albuquerque, H. Bauer and Schneider 2009; Mondria and Wu 2010). However, from an empirical perspective, Didier, Rigobon and Schmukler (2011) present fund-level evidence showing that information cost might not be the key role in determining cross-country portfolios.

In the past few years, researchers have come up with various dynamic general equilibrium portfolio choice models. Devereux and Sutherland (2010a), Devereux and Sutherland (2011a) and Tille and van Wincoop (2010) independently and simultaneously develop a solution method advancing DSGE models with portfolio choice. Their proposed methods stay very close to the familiar first- and second-order approximation techniques.[13] In this chapter, the solution method for general equilibrium rational expectations models with higher order approximations are based on Schmitt-Grohe and Uribe (2004), and solutions to portfolio choice are based on the method introduced by Devereux and Sutherland (2010a) and Devereux and Sutherland (2011a).

1.2 International financial markets and international risk sharing

Financial integration starting from 1980's has significantly boosted international capital flows among developed economies and between developed countries and developing countries. This phenomenon has motivated a large expanding volume of literature that try to evaluate gains from financial integration (see recent studies, Bonfiglioli 2008; Kose, Prasad and Terrones 2009b; Nicolo and Juvenal 2012). Kose, Prasad, Rogoff and Wei (2009a) summarize that gains from financial integration are mixed in the literature.

Although they are large between developing countries and developed countries, capital flows among developing countries remain still limited. Figure 1.1-1.3 show that portfolio investments across developing countries are far lower than the corresponding commodity trade.[14] Developing countries hold a dominant part of portfolios either in the world center, such as the United States, or regional centers such as Britain and Singapore. For instance, the share of exports or imports of goods and services between Argentina and other developing countries such as Brazil, Chile and Mexico, in Argentina's total exports or imports is over 30% on average during $1997-2009$. However, in the same period, Argentina's portfolio investment assets in these developing countries only account for less than 5% of its to-

[13] Devereux and Sutherland (2007), Devereux and Sutherland (2008) and Devereux and Sutherland (2010b) apply their method to monetary policy and portfolio choices. Furthermore, Evans and Hnatkovska (2005) develop a similar solution method using perturbation and projection methods, whereas Stepanchuk and Tsyrennikov (2011) study international portfolios in a wealth-recursive equilibrium. Benigno and Nistico (2012) explore home bias under model uncertainty.

[14] Fratzscher and Imbs (2009) illustrate that portfolio investment exclusively increases risk-diversification across countries.

tal overseas portfolio investments.[15] Eichengreen and Park (2003) show that bilateral bank claims in Asia are much more smaller than Europe. Based on high-frequency indicators for equity market integration, Yu, Fung and Tam (2010) find that Asian countries have experienced financial integration for the past decade but this process is quite different from mature markets. In other words, financial integration among developing countries stays at a low level.

Naturally, two concerns are present about the observed low financial integration among developing economies. One is that developing countries have already achieved optimal allocation of assets across borders according to their income-output processes. The other is that some prevalent barriers prevent capital flows across developing economies. Table 1.2 and 1.3 show the pairwise correlation of growth rate of real GDP per capita among a sample of developing economies. The output growth rate correlations among Argentina, Brazil, Mexico and Chile are essentially zero. Asian developing economies except for China have a higher cross-correlation in the data sample, but it is almost the same as the correlation between these developing countries and regional core countries, such as Singapore and Japan. The growth rate of output in China seems uncorrelated with both advanced economies and developing economies. In terms of these income processes, the point of view that developing economies have already fully diversified their income risk across countries seems not consistent with the data. Alternatively, there might exist unexploited gains from financial integration for developing economies. These gains from diversification might be prevented by barriers such as capital account regulation, financial and economic underdevelopment (see quantitative evaluations in Mendoza, Quadrini and Rios-Rull 2009b; Bai and Zhang 2010; Bai and Zhang 2012). In Callen, Imbs and Mauro (2011), they show based on a sample of 74 countries that welfare gains remain untapped because of non-diversifiable enforcement costs.

An interesting question is whether further financial integration, say, financial integration among developing countries, could make all participating countries better off. Nevertheless, the literature doesn't say much about

[15] A country's external assets consists of portfolio investment assets, foreign exchange reserves and foreign direct investment. Portfolio investment assets have two sub-categories, equities and bonds. For many developing countries, foreign exchange reserves account for a dominant part of external assets. Nevertheless, portfolio investment assets are also an equally significant part of external assets in these countries. In practice, a geographical break of foreign exchange reserves is not available publicly, so the use of portfolio assets can basically capture the distribution of trade in assets for developing countries given the assumption that the geographical composition of portfolio assets held by monetary authorities is the same as other sectors within a country.

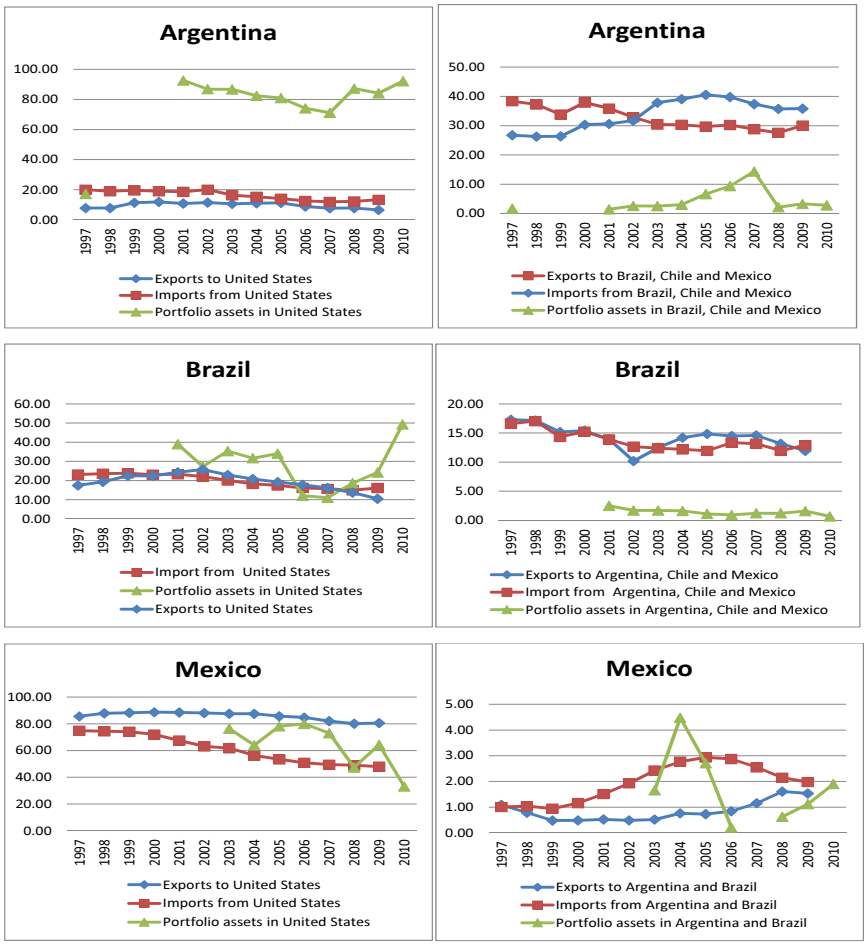

Figure 1.1: The graphs show the percentage share of imports to/exports from/portfolio assets in major foreign countries and regions when the home country is Argentina, Brazil and Mexico, respectively. The horizontal axis is year and the vertical axis is percentage. Data sources: Direction of Trade and CPIS datasets, IMF, 2011.

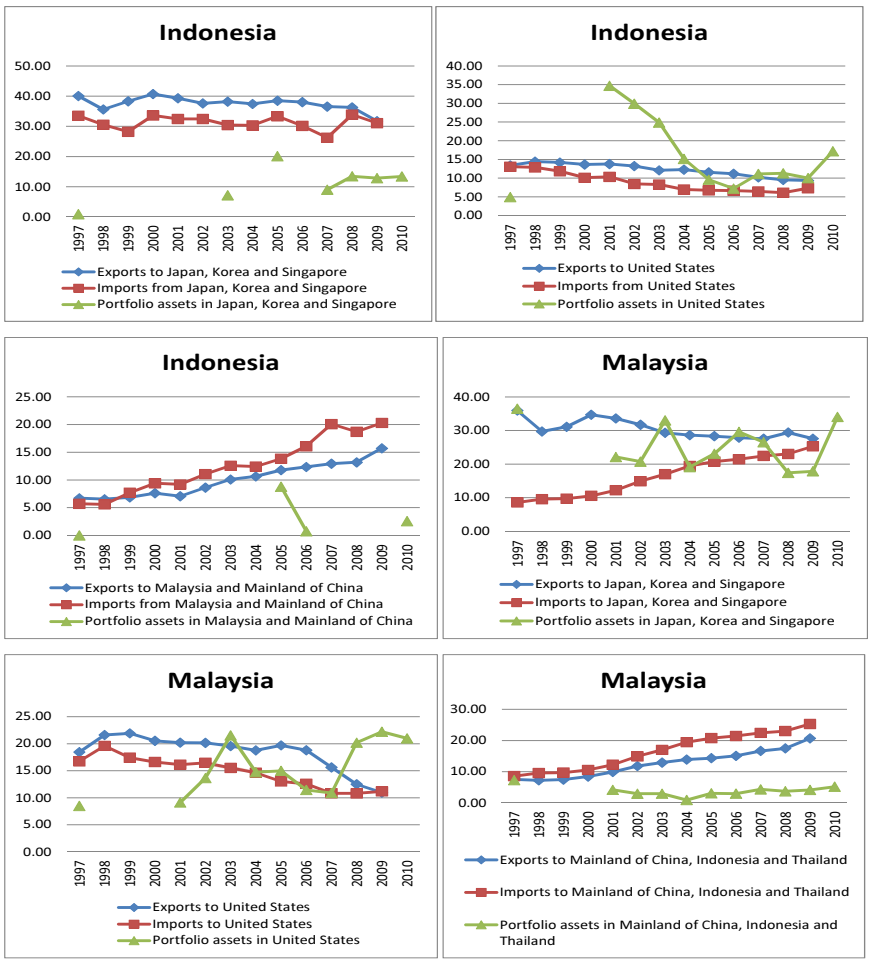

Figure 1.2: The graphs show the percentage share of imports to/exports from/portfolio assets in major foreign countries and regions when the home country is Indonesia and Malaysia, respectively. The horizontal axis is year and the vertical axis is percentage. Data sources: Direction of Trade and CPIS datasets, IMF, 2011.

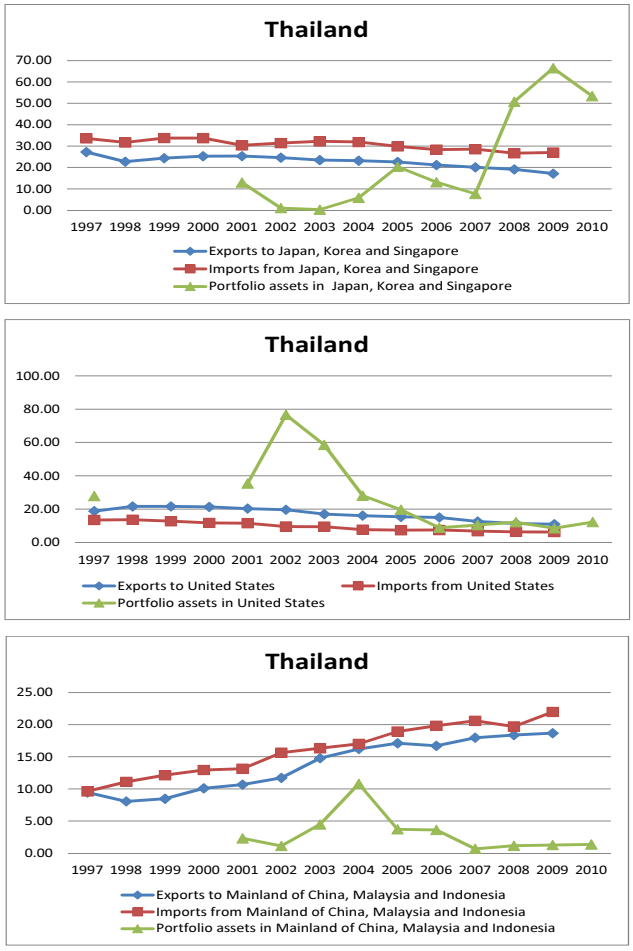

Figure 1.3: The graphs show the percentage share of imports to/exports from/portfolio assets in major foreign countries and regions when the home country is Thailand. The horizontal axis is year and the vertical axis is percentage. Data sources: Direction of Trade and CPIS data sets, IMF, 2011.

Table 1.2: Correlation of growth rate of real GDP per capita (Latin America)

	USA	Argentina	Brazil	Mexico	Chile
USA	1				
Argentina	0.0554	1			
Brazil	0.0877	0.3377	1		
Mexico	0.5294*	0.2636	-0.1491	1	
Chile	0.4512*	0.1922	0.0468	0.1736	1

Note: The table shows pairwise correlation of growth rate of real GDP per capita. A star denotes a significance level of 0.05. Yearly data during 1985 – 2009. Data source: Penn World Table 7.0.

Table 1.3: Correlation of growth rate of real GDP per capita (Asia)

	USA	Singapore	Japan	Indonesia	Malaysia	Thailand	China
USA	1						
Singapore	0.4376*	1					
Japan	0.4809*	0.6302*	1				
Indonesia	-0.2712	0.4384*	0.3509	1			
Malaysia	0.2739	0.8789*	0.6261*	0.6811*	1		
Thailand	-0.0239	0.5397*	0.6286*	0.7639*	0.7515*	1	
China	-0.0928	0.0893	-0.3075	0.0877	0.0622	0.0628	1

Note: The table shows pairwise correlation of growth rate of real GDP per capita. A star denotes a significance level of 0.05. Yearly data during 1985 – 2009. Data source: Penn World Table 7.0.

gains for developed countries when developing countries could fully diversify their portfolios worldly. I try to bridge this gap and investigate this issue in a center-periphery framework with international portfolio choice.

This chapter develops a tractable center-periphery framework with international portfolio choice. International financial architectures are divided into three stages according to financial development. The first stage is financial autarky, in which no financial assets are traded across borders. The second stage is partial financial integration, in which the central country becomes financially integrated with peripheral countries, but there is no financial integration between peripheral countries. This case is closed to the current financial architecture. The third stage is global financial integration, in which all financial markets are integrated. I find that (a) from financial autarky to partial financial integration, volatility of consumption in all

countries drops, and (b) from partial financial integration to global financial integration, volatility of consumption in peripheral countries decreases, but consumption volatility rises in the central country when the central country is large. In other words, peripheral countries monotonically increase international risk-sharing in the process of financial integration. Nevertheless, the core country achieves its highest risk-sharing in the medium stage of financial integration when peripheral countries are small. When peripheral countries are large, consumption volatility in all countries decreases in the process of financial integration.

From financial autarky to partial financial integration, each country has access to more financial assets and consequently achieves a better risk sharing than financial autarky. From partial financial integration to global financial integration, the mechanism lies in the demand effect induced by country size (see empirical evidence in Martin and Rey (2004)). Consider that the core country is far larger than other countries. From partial to global integration, the world demand for peripheral assets increases because of imperfect correlation of shocks, which implies peripheral asset prices go up and consequently their return rates decrease. However, the total demand for core assets reduces because of demand shifting towards peripheral assets, and then the asset price in the core country goes down and accordingly its return rate is higher. Both accounts imply a narrower cross-country premium between assets in the core and peripheral countries in partial financial integration than in global financial integration. The change of cross-country premium between different stages of financial integration can be treated as risk-sharing premium gained by peripheral countries. On the other hand, when the core country is large, income risk in this large country represents global income risk. In equilibrium, the response of consumption on impact can be decomposed into two parts. One is income effect, which comes from the change of output responding to shocks. The other is wealth effect, which represents the change of values of net foreign asset positions. When the core country is relatively large, it has a power over other assets in financial markets due to size effect, and the wealth effect favors the core country most in partial financial integration. When it is not large, the core's privilege disappears. Hence I obtain a monotonic reduction in consumption volatility for all countries with the proceeding of financial integration.

In the baseline model, I abstract the effect of changes of intertemporal prices on international risk-sharing. As emphasized by Cole and Obstfeld (1991), terms of trade can move around responding to shocks and thereby income risk can be automatically shared through relative price changes under particular parameterizations. I then extend the baseline model to a world

economy in which each country produces one tradable good and consumes all types of tradable goods. Labor supply is endogenous and exogenous shocks are productivity shocks. I obtain similar results.

A shortcoming of analyses in this chapter is that I don't compute welfare for each country in each stage of financial integration. In order to calculate welfare, I need to take third-order approximations to Euler equations for portfolios in all countries to obtain first-order approximations to portfolios, and to take second-order approximations to the rest of equations in the competitive equilibrium. This exercise will be left in my work in progress (Yu 2012).

This chapter contributes to two branches of literature. First, a strand of literature provides evaluation of gains from financial integration (Cole and Obstfeld 1991; Baxter and Crucini 1995; Brandt, Cochrane and Santa-Clara 2006; Martin 2010). Contrary to the international risk-sharing literature, I find that financial integration is not necessarily reduce consumption volatility for all participants. This view is related to the imperfect credit market literature (say, Matsuyama 2008) but I focus on a quite different environment in which capital inflow/outflow restrictions among developing economies are the source of imperfect credit markets. Recently, a growing literature focus on the effect of financial development on international risk sharing in environments with heterogeneous agents (for instance, Mendoza, Quadrini and Rios-Rull 2009a; Angeletos and Panousi 2011). They only consider household-specific shocks and there are no country-specific shocks in their models. I focus on instead country-specific shocks here. Second, this chapter also contributes to the stream of literature on international portfolio choice. While most of this literature focus on a two-country framework (for instance, Devereux and Sutherland 2011b), I develop a center-periphery framework with different stages of financial integration, rather than creating new assets in the process of financial integration (Devereux and Sutherland 2009).

In terms of framework, the center-periphery structure is similar to Pavlova and Rigobon (2008) and Goldberg and Tille (2009). Pavlova and Rigobon (2008) discuss portfolio choices and focus on international propagation of shocks when the central country is financially constrained. My work is quite different from theirs and I focus on financial integration and international risk-sharing. Goldberg and Tille (2009) explore monetary policy and invoice of international trade but without portfolio choice. The solution method used in this chapter is based on Devereux and Sutherland (2011a) who use higher order approximations to obtain optimal portfolios.

1.3 Credit friction, asset price and optimal monetary policy

The 2007 − 2009 financial turmoil revives a burgeoning interest in financial factors in the study of business cycles (i.e., Carlstrom and Fuerst 1997; Curdia and Woodford 2009; Curdia and Woodford 2010; Devereux and Yetman 2010; Gertler and Kiyotaki 2010; Kiyotaki and Moore 2012), a tradition starting from Bernanke and Gertler (1989).[16] The major point lies in that financial shocks have important effects on macroeconomy (see Jermann and Quadrini 2012) and/or that financial factors play an important role in amplifying the propagation of shocks in a closed economy (i.e., Kiyotaki and Moore 2012) or across boarders (see Devereux and Yetman 2010; Faia and Iliopulos 2010). Recent data show that the financial frictions are important factors generating large consumption and output volatility, which in turn incur some welfare losses. The question I am interested in in this chapter is whether high volatility of consumption and output could be avoided if monetary authorities have the proper instruments. If a financial accelerator can produce large responses of fundamentals to exogenous shocks, it is also true that policy makers could use a financial decelerator to stabilize the economy and improve welfare. Based on a medium scale closed economy model with sticky prices, sticky wages and financial frictions, this chapter shows that if the debt deflation channel is strong enough, monetary authorities should create deflation in booms and inflation in recessions in order to reduce the volatility of constrained borrowers'net worth.

In this chapter, I investigate optimal monetary policies in an economy with staggered nominal wage setting in addition to staggered price setting and credit-market imperfections. The specific questions I ask are as follows. How do monetary policies respond to asset prices when credit frictions exist in an economy? Does the response to financial factors make sense as a general policy? In other words, which kind of simple linear rules can implement the optimal monetary policy? As in recent contributions, volatility of price inflation prompts dispersion in prices across goods and henceforth inefficient output levels. Analogously, staggered wage contracts induce wage dispersion and hence inefficiencies in the distribution of employment across households (see Erceg, Henderson and Levin 2000). With credit market frictions, when nominal returns on debt are not state-contingent, varying the price level in response to shocks allows to transfer real wealth between

[16] Brunnermeier, Eisenbach and Sannikov (2012) provide an up-to-date survey on financial frictions in macroeconomics.

debtors and creditors, which in turn amplifies shocks through the financial accelerator (Bernanke, Gertler and Gilchrist 1999). Hence, achieving the optimal allocation would require to trade off costs of price dispersion and of wage dispersion, and the effect of debt deflation in the imperfect credit market as well.

I model credit frictions following the collateral channel of Kiyotaki and Moore (1997). Specifically, financial imperfections originate from a problem of debt contract enforcement between debtors and creditors. When debts require to be secured by collateral, which is also an input in production, the amount of debts positively co-moves with values of collateral. At the same time, the financial costs between external borrowing and internal funds negatively vary with borrowers' balance of sheet, which generate endogenous external finance premium and volatile asset price over the business cycle.

The model studied here has several key features besides credit frictions. First, it embeds monopolistic competition both in goods market and labor market, and Calvo-style nominal rigidities both in wage contracts and price resetting. Second, I consider intertemporal nominal debt contracts, in which debt contracts are signed at current period and will be repaid next period. Movements in inflation affect not only labor and goods market allocations but also the real repayment of debts. For instance, when an unanticipated positive technology shock happens, monetary authority should restore labor market efficiency by creating deflation such that the labor allocation is close to the efficient level in the absence of nominal inertia over the business cycle. On the other side, price stickiness requires stabilizing inflation to reduce the cost of price dispersion. Furthermore, at the same time, the associated deflation will transfer wealth from debtors to creditors, which creates an endogenous counter-cyclical net worth shock. When this induced net worth shock appears unexpectedly in a credit-frictionless economy, borrowers can smooth consumption and adjust their capital holdings through borrowing as much as they can until there is no arbitrage in asset markets. As a result, the adjustment of productive inputs are ignorable. Notwithstanding, the effect of net worth shocks on reallocation of productive factors in an economy with credit frictions is not negligible any more, since constrained borrowers can expand or shrink large amount of their holdings of productive factors via leverage, which could produce large reallocation of resources between debtors and creditors.

From the point of view of method, this chapter investigates monetary policies in terms of social welfare criterion. Unlike the output-inflation volatility frontier, a second-order approximation to an economy around its distorted steady state could provide correct rank of alternative monetary

policy rules (see Schmitt-Grohe and Uribe 2004). And thereby one can draw confident conclusion about the recommendable monetary policy.

The main finding is that monetary policy has high persistence in interest rate and it negatively reacts to asset prices, although the response is quite small conditional on controlling for inflation rate and output gap. The basic intuition is as follows. The financial accelerator could generate large responses of output to shocks in an economy absent of any nominal rigidity. However, the large volatility of output may be not desirable in terms of social welfare. With credit frictions, policy-makers could take advantage of the financial decelerator to drive down the volatility of output and thus enhance social welfare. In the model economy, asset price is mainly driven by borrowers' (also entrepreneurs') demand for capital. When a positive shock appears, a higher return to capital induces an increase in demand for capital and therefore a boom in asset price. However, credit constrained borrowers have to resort to lenders (also households) to obtain external funds. A lower interest rate responding to an increase in asset price would induce a lower supply of private credit and hence only part of demand for capital is satisfied in equilibrium, which consequently stabilizes the volatility of output. At the same time, the associated external finance premium increases under the optimal policy because positive technology shocks bring about a higher return to capital while the deflation in nominal prices induces a higher ex post real debt repayment and the latter effect dominates the former. Hence, borrowers' net worth decreases and thereby external finance premium goes up. However, the coefficient for asset price in the interest rate rule is low.

After comparing several simple interest rate rules widely discussed in the literature, I find that an interest rate rule with a large response to the lagged interest rate has a relatively low welfare loss. Strict inflation targeting and traditional Taylor rules have relatively large welfare losses.

Another finding of this chapter is that after incorporating fiscal policy into the model economy, at the long-run steady state, the optimal tax rate on capital is of -5.2%, and labor income tax rate is of 51%, for there is monopolistic power for final goods production, which in turn distorts the capital allocation in the entrepreneur sector. Subsidizing capital income could improve the efficiency of capital allocation. If all incomes are taxed at the same rate, then the income tax rate is 6.9%.

In monetary models, most of literature considering financial factors have a sole form of nominal rigidity - staggered price setting, and monetary policy rules that keep inflation rate constant also achieve optimal welfare level (see Faia and Monacelli 2007; Carlstrom, Fuerst and Paustian 2010; Curdia and Woodford 2009; Curdia and Woodford 2010). Nevertheless, staggered wage

contracts seem to play an important role in explaining cyclical patterns of the U.S. economy (see Christiano, Eichenbaum and Evans 2005; Schmitt-Grohe and Uribe 2006). So I take into account of wage stickiness in this chapter.

Taylor (2008) suggests that monetary policy should negatively respond to credit spread in the Taylor rules. However, after incorporating credit frictions in a dynamic stochastic general equilibrium model, Curdia and Woodford (2009) and Curdia and Woodford (2010) find that mere existence of spread matters little while spread adjustment can improve upon the Taylor rule but the size of response to credit is not robust to underlying nature of shocks. Carlstrom et al. (2010), De Fiore and Tristani (2009) and Benigno and Faia (2010) conclude that inflation stabilization is nearly optimal to technology shocks. The existing literature mainly focus on goods market frictions, i.e., price stickiness and credit market frictions (Monacelli 2008; Monacelli 2009). Either credit spread is produced in a reduced form, i.e., Curdia and Woodford (2009, 2010), or no role of credit spread and asset price is played in the model economy. Faia and Monacelli (2007) study an optimal linear interest rate rule in an economy of Carlstrom and Fuerst (1997); nevertheless, they only take into account of sticky prices and intra-period nominal debts, and therefore, stabilizing price inflation is apparently optimal. When inter-period nominal debts are introduced, ex post unanticipated variation in nominal prices associated with shocks could redistribute real wealth between debtors and lenders through the debt deflation channel, which has been discussed by Iacoviello (2002). However, monetary policy is not evaluated in terms of welfare criterion in his chapter. This chapter tries to bridge this gap and to investigate monetary policy based on welfare comparison in a richer environment.

Chapter 2

Equity Home Bias and International Portfolio Dynamics in a Monetary Open-Economy Model

Can we reconcile the observed equity home bias and the workhorse monetary business cycle model? This chapter will investigate this issue in deep detail. In the remainder of this chapter, section 2.1 describes the household's problem, the producer's problem, market clearings and the corresponding equilibrium. Section 2.2 derives a solution for the equilibrium equity portfolios and provides relevant evidence. Section 2.3 reports data set and estimates of exogenous processes. Section 2.4 introduces numerical results for optimal portfolios whose robustness is correspondingly verified. Section 2.5 extends the model with complete markets to models with incomplete markets, and section 2.6 discusses the dynamics of portfolios. Final remarks are in the last chapter.

2.1 The model

In this section, I describe the model economy and display the problems faced by households and all types of firms. In addition, I describe the monetary and fiscal policies. The sources of uncertainty in the benchmark model are a shock to technology and a shock to monetary policy.

The world economy consists of two equally large countries, denoted by H, the home country, and F, the foreign country. Goods and assets are internationally traded without cost. The population in each country has a continuum of households with unit measure who live infinite horizons. Households choose labor supply, consumption, money balance and portfolio in each period. Labor union sets wage contracts in a staggered way for each type of labor services. Wholesale firms competitively produce wholesale goods us-

ing labor services and physical capital and sell them to intermediate goods producers. Physical capital is accumulated by wholesale firms. Intermediate producers purchase homogeneous wholesale goods in the competitive goods market and differentiate them into imperfectly substitutable goods through different brands. They adjust price for each variety in a staggered way. The local bundlers combine intermediates into local goods composite and sell it to domestic households and export to the foreign country. Assume that firms use producer currency pricing and the law of one price holds at each individual good level because of no trade costs. Government maintains a balanced budget each period. International assets consist of equities and bonds. Dividends from all domestic firms, including wholesale firms and intermediate producers, are distributed to holders of domestic equities at the end of each period.

2.1.1 Consumption composite

Consumption composite in each country requires local intermediate goods and imported intermediate goods from the other country. It is aggregated via a CES form,

$$C_t = \left[a_C^{\frac{1}{\gamma}} C_{H,t}^{\frac{\gamma-1}{\gamma}} + (1-a_C)^{\frac{1}{\gamma}} C_{F,t}^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} \tag{2.1}$$

where $\gamma > 0$ is the elasticity of substitution between imports and exports. $C_{H,t}$ ($C_{F,t}$) is consumption demand for home (foreign) produced goods by home households. In the symmetric deterministic steady state, a_C is the share of consumption spending devoted to local goods. A preference bias towards local goods implies, $1/2 < a_C < 1$.

Expenditure minimization associated with the consumption composite (2.1) gives the consumer price index at the home country,

$$P_t = \left[a_C P_{H,t}^{1-\gamma} + (1-a_C) P_{F,t}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \tag{2.2}$$

where $P_{H,t}$ and $P_{F,t}$ respectively denote prices of home bundled goods and foreign bundled goods in terms of home currency.

We assume that the law of one price holds at individual good level and thereby, agents in both countries face the same prices of intermediate goods after adjusted by nominal exchange rates, i.e., $P_{F,t} = P_{F,t}^* \mathcal{E}_t$ and $P_{H,t}^* = P_{H,t}/\mathcal{E}_t$, with $\mathcal{E}_t$ the price of foreign currency in terms of domestic currency. Foreign variables are denoted by $*$ at their superscripts. Demand

for home goods by home consumers is given by $C_{H,t} = a_C(P_{H,t}/P_t)^{-\gamma}C_t$ and demand for foreign goods by home consumers has a form of $C_{F,t} = (1-a_C)(P_{F,t}/P_t)^{-\gamma}C_t$. Foreign country has a similar expression for consumer price index,

$$P_t^* = \left[a_C P_{F,t}^{*(1-\gamma)} + (1-a_C)P_{H,t}^{*(1-\gamma)}\right]^{\frac{1}{1-\gamma}}$$

where P_t^* is the price of foreign consumption composite in terms of foreign currency.

2.1.2 Households

Households in the home country have a life-time utility function of the form,

$$E_t\left[\sum_{\tau=t}^{\infty}\beta_\tau U(C_\tau, H_\tau, \frac{M_\tau}{P_\tau})\right] \tag{2.3}$$

where the operator E_t here represents the conditional expectation over all states of nature, C_τ is the home consumption, H_τ stands for labor supply and M_τ represents nominal money balance. Following the open economy literature, i.e., Schmitt-Grohe and Uribe (2003), I take use of endogenous discount factor,

$$\beta_{t+1} = \beta_t v(t), \text{ with } \beta_0 = 1,$$

where $v(t) \equiv \zeta_0 \bar{C}_t^{-\zeta_1}$, $0 < \zeta_1 < 1$, ζ_0 is a parameter which is chosen such that the discount factor at the steady state is equal to the inverse of risk-free rate. $\bar{C}_t$ is the aggregate home consumption. When they made decisions, households treat aggregate consumption as given. As in Chari, Kehoe and McGrattan (2002), the period utility function takes a form of,

$$\begin{aligned} U(C_t, H_t, \frac{M_t}{P_t}) =& \frac{1}{1-\sigma}\left[\left(\omega C_t^{\frac{\theta-1}{\theta}} + (1-\omega)\left(\frac{M_t}{P_t}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right]^{1-\sigma} \\ &+ \eta_0\frac{(1-H_t)^{1-\eta}}{1-\eta} \end{aligned} \tag{2.4}$$

where σ is the coefficient of relative risk aversion or the inverse of elasticity of intertemporal substitution, η characterizes Frisch elasticity of labor

supply.[17] ω describes the deterministic steady state share of consumption in the consumption and real money balance composite. θ characterizes the interest rate elasticity of money demand.

Before exploring households' budget constraints, let's look at first the labor market. Following Erceg et al.(2000), I assume that the home household is a monopoly supplier of a differentiated labor service, H_{jt}, $j \in [0, 1]$, as an imperfect substitute for other labor services. It's convenient to further assume that there is a labor union (see, Schmitt-Grohe and Uribe 2006) which combines home households' labor services and sells the service package to local firms. The aggregate labor input, H_t^d, has the Dixit and Stiglitz (1977) form,

$$H_t^d = \left(\int_0^1 H_{j,t}^{\frac{\theta_w - 1}{\theta_w}} dj \right)^{\frac{\theta_w}{\theta_w - 1}}, \ \theta_w > 0 \tag{2.5}$$

where θ_w is elasticity of substitution between labor services. The labor union minimizes the cost of providing a given amount of the aggregate labor services, H_t^d, given the wage rate $W_{j,t}$ set by the home households, and sells the service package to firms at the wage rate W_t,

$$W_t = \left(\int_0^1 W_{j,t}^{1-\theta_w} dj \right)^{\frac{1}{1-\theta_w}} \tag{2.6}$$

The demand for labor service j is given by,

$$H_{j,t} = \left(\frac{W_{j,t}}{W_t} \right)^{-\theta_w} H_t^d \tag{2.7}$$

Total labor services supplied by an individual household are then given by,

$$H_t = \int_0^1 H_{j,t} dj = H_t^d \int_0^1 \left(\frac{W_{j,t}}{W_t} \right)^{-\theta_w} dj = H_t^d s_{t+1}^w$$

with $s_{t+1}^w \equiv \int_0^1 \left(\frac{W_{j,t}}{W_t} \right)^{-\theta_w} dj$ describing wage dispersion in the labor market. The first equality above states that different labor services are indifferent to labor suppliers and therefore, the total labor supply is the sum of differentiated labor services. The second equality indicates that there is a wedge

[17] Christiano, Trabandt and Walentin (2010) propose another interpretation of labor supply H_t that it measures numbers of people working rather than hours of working. The value of η in the calibration is consistent with this interpretation. However, I still use Frisch labor supply elasticity for $1/\eta$ here for convenience.

between labor supply and labor demand. The higher the wage dispersion is in labor markets, the larger the wedge becomes. In the deterministic steady state, wage rates are the same cross labor services and thereby wage dispersion is one.

Notice that households face the same labor demand curve, choose identical consumption goods and hold the same financial assets. Therefore, the model is reduced to a representative agent model. Next I will use this exposition convenience instead in the following analysis.

The household's budget constraint at period t states that consumption expenditure plus asset positions for the next period must equal the total income,

$$C_t + q_t\psi_{H,t+1} + s_t q_t^* \psi_{F,t+1} + p_t^b B_{H,t+1} + s_t p_t^{b*} B_{F,t+1} + m_t = (q_t+d_t)\psi_{H,t}+s_t(q_t^*+d_t^*)\psi_{F,t}+(p_t^b+r_t^b)B_{H,t}+s_t(p_t^{b*}+r_t^{b*})B_{F,t}+w_t H_t^d+\frac{m_{t-1}}{\pi_t}+tr_t \tag{2.8}$$

For saving notions, a uppercase letter X_t denotes nominal price in local currency if applicable and the corresponding lower case x_t represents real price in terms of local real consumption, $x_t \equiv X_t/P_t$. The real exchange rate is defined as, $s_t \equiv \mathcal{E}_t P_t^*/P_t$.

Asset positions consist of money holdings, home and foreign equity holdings, and home and foreign debt portfolios. q_t (q_t^*) represents the price of home (foreign) equity and the corresponding term $\psi_{H,t+1}$ ($\psi_{F,t+1}$) represents the share of home (foreign) equities purchased by the home household at the end of period t. Holders of equities will receive the corresponding share of dividends, d_t (d_t^*), which are defined later. Similarly, p_t^b (p_t^{b*}) indicates the price of home (foreign) bonds. $B_{H,t+1}$ ($B_{F,t+1}$) denotes the quantity of home (foreign) bonds held by the home resident at the end of period t. Each unit of home (foreign) bonds gives r_t^b (r_t^{b*}) unit of real consumption composite to the claimant. Labor income is given by $w_t H_t^d$. Finally, the household receives a lump-sum government transfer tr_t. Foreign household has a similar exposition.

Before deriving the optimality conditions for the household, let me describe the bond market first. In the budget constraint (2.8) above, I present a generic form for bond returns. For instance, when nominal consols are traded across borders, they pay one unit of local currency per unit of bonds each period and then interest payments read $r_t^b = \frac{1}{P_t}$, $r_t^{b*} = \frac{1}{P_t^*}$. Other types of bond instruments will be introduced later.

In each period, the home household maximizes its life-time utility (2.3)

with respect to consumption, money balances, holdings of international assets, subject to the labor demand (2.7) and the budget constraint (2.8). The optimality conditions for consumption and holdings of financial assets are as follows,

$$U_{c,t} = E_t \left[v(t) U_{c,t+1} r_{i,t+1} \right], \text{ with } i = a_1, a_2, b_1, b_2 \tag{2.9}$$

where $U_{c,t}$ represents marginal utility of consumption at period t, and, $r_{i,t+1}$ expresses real gross returns to asset i in terms of consumption composite at the home country, which are defined later in equation (2.10). The first-order conditions derived above illustrate the usual "consumption Euler equation" which links the marginal cost of a foregone unit of current consumption to the expected marginal benefit from international asset returns in the following period. The real gross return to asset i, $r_{i,t+1}$, is defined as follows,

$$\begin{aligned} r_{a_1,t+1} &\equiv \frac{q_{t+1} + d_{t+1}}{q_t} \quad & r_{a_2,t+1} &\equiv \frac{s_{t+1}}{s_t} \frac{q^*_{t+1} + d^*_{t+1}}{q^*_t} \\ r_{b_1,t+1} &\equiv \frac{p^b_{t+1} + r^b_{t+1}}{p^b_t} \quad & r_{b_2,t+1} &\equiv \frac{s_{t+1}}{s_t} \frac{p^{b*}_{t+1} + r^{b*}_{t+1}}{p^{b*}_t} \end{aligned} \tag{2.10}$$

Analogously, the consumption Euler equations for the foreign household can be written as,

$$U_{c^*,t} = E_t \left[v^*(t) U_{c^*,t+1} \frac{s_t}{s_{t+1}} r_{i,t+1} \right], \text{ with } i = a_1, a_2, b_1, b_2$$

The optimal condition for real balance holdings is given by,

$$1 = \frac{U_{m,t}}{U_{c,t}} + \frac{1}{R_{t+1}} \tag{2.11}$$

where $U_{m,t}$ denotes marginal utility of real money balance. R_{t+1} is the risk-free nominal interest rate denominated in local currency, and it is determined by the following Euler equation,

$$\frac{1}{R_{t+1}} = E_t \left[v(t) \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right] \tag{2.12}$$

Let $X_{w,t}$ be the Lagrange multiplier for labor demand constraint (2.7), which is interpreted as wage markup. The household's optimal labor supply is set to equalize the marginal value of labor income divided by wage markup, $X_{w,t}$, and marginal disutility of labor supply, given nominal wages preset

by the household,

$$-\frac{U_{h,t}}{U_{c,t}} = \frac{w_t}{X_{w,t}} \tag{2.13}$$

The household sets nominal wages according to the staggered contracts as in Calvo (1983). In each period a fraction $1-\xi_w$ of wage contracts are renegotiated. When the household is able to reset wage contract j, it chooses $\tilde{W}_{j,t}$ to maximize its utility function, given the demand for labor service j and other wage contracts, while the remaining wage contracts, $W_{j,t}$, which cannot be re-optimized, are adjusted according to,

$$W_{j,t} = \pi_{t-1}^{\chi} W_{j,t-1}$$

where $\pi_t = P_t/P_{t-1}$ is home consumer price index inflation in period t and $\chi \in [0,1]$ represents indexation of wage contract to CPI inflation.

The first-order condition for optimal wage-setting $\tilde{w}$ can be written as,

$$\begin{aligned} 0 = E_t \Bigg\{ & \sum_{\tau=0}^{\infty} \beta_{t+\tau} \xi_w^{\tau} U_{c,t+\tau} H_{t+\tau}^d \left(\frac{\tilde{w}_t}{w_{t+\tau}} \right)^{-\theta_w} \prod_{k=1}^{\tau} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}} \right)^{\theta_w} \\ & \left[\frac{\theta_w - 1}{\theta_w} \frac{\tilde{w}_t}{\prod_{k=1}^{\tau} \frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}}} - \frac{w_{t+\tau}}{X_{w,t+\tau}} \right] \Bigg\} \end{aligned} \tag{2.14}$$

Here I have substituted the real wage rates, $w_{t+\tau} \equiv W_{t+\tau}/P_{t+\tau}$ and $\tilde{w}_{t+\tau} \equiv \tilde{W}_{t+\tau}/P_{t+\tau}$, for nominal wages, where the optimal nominal wage rate $\tilde{W}_t$ is reset by the household in period t. According to the optimal wage-setting, the household prices its labor service j so that the sum of expected discounted marginal utility of the income from an additional unit of labor supply equals its expected sum of discounted marginal disutility from labor services. When ξ_w tends to zero, the household can reset its wages every period.

2.1.3 Wholesale firms

At time t, the homogenous wholesale goods are produced by a perfectly competitive, representative firm. The firm produces goods by combining physical capital and labor aggregate according to the following technology,

$$Y_t = Z_{a,t} K_t^{\alpha} (H_t^d)^{1-\alpha} \tag{2.15}$$

where $0 < \alpha < 1$ and K_t denotes the time t home country's physical capital services. Total factor productivity, $Z_{a,t}$, is an exogenous random variable.

Physical capital stock, which is assumed to be owned by the wholesaler, and also finally owned by the household, evolves according to,

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) K_t + (1-\delta)K_t \tag{2.16}$$

Here, I_t denotes time t gross investment in the home country. The transformation function, Φ, characterizes the technology that transforms current investment into installed physical capital for use in the next period and it is continuously increasing and concave. $0 < \delta < 1$ is the physical rate of depreciation. In the following analysis, I use the specification as follows,

$$\Phi\left(\frac{I_t}{K_t}\right) = \frac{I_t}{K_t} - \frac{\phi}{2\delta}\left(\frac{I_t}{K_t} - \delta\right)^2$$

Parameter ϕ characterizes the elasticity of price of capital with respect to investment capital ratio.

The wholesaler's problem is to maximize the following expected discounted profits,

$$E_t \sum_{\tau=0}^{+\infty} Q_{t,t+\tau}\left(P_{w,t+\tau}Y_{t+\tau} - P_{I,t+\tau}I_{t+\tau} - W_{t+\tau}H^d_{t+\tau}\right)$$

subjective to technology (2.15) and capital evolvement (2.16). Where $Q_{t,t+\tau}$ is stochastic discount factor for nominal returns at period $t+\tau$, $Q_{t,t+\tau} \equiv \frac{\beta_{\tau+t}}{\beta_t}\frac{U_{c,t+\tau}P_t}{U_{c,t}P_{t+\tau}}$. $P_{w,t}$, $P_{I,t}$ stand for the price of wholesales and the price of investment goods, respectively.

The optimality condition for labor demand reads,

$$w_t = \frac{(1-\alpha)Y_t p_{H,t}}{H^d_t X_{p,t}}$$

where $X_{p,t} \equiv \frac{P_{H,t}}{P_{w,t}}$, the price mark-up for the wholesaler or the inverse of real marginal cost for intermediate producers.

The shadow price for physical capital $q_{k,t}$ relates to marginal product of investment via,

$$q_{k,t} = \left[\Phi'(I_t/K_t)\right]^{-1} p_{I,t}$$

Optimal investment decision is determined by the wholesaler's Euler

equation, which reads,

$$q_{k,t} = E_t \left\{ \Lambda_{t,t+1} \left[q_{k,t+1} \left((1-\delta) + \Phi\left(\frac{I_{t+1}}{K_{t+1}}\right) - \Phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}} \right) + \frac{\alpha Y_{t+1} p_{H,t+1}}{K_{t+1} X_{p,t+1}} \right] \right\}$$

where $\Lambda_{t,t+1}$ is stochastic discount factor for real returns at period $t+1$, $\Lambda_{t,t+1} \equiv \frac{\beta_{t+1}}{\beta_t}\frac{U_{c,t+1}}{U_{c,t}}$.

In each period, the dividends from the wholesaler are retained revenues, which equal the firm's total revenues minus wage bills and minus purchases of investment goods for use in the following period. Real dividends from the wholesale firm become,

$$d_{w,t} = p_{w,t} Y_t - p_{I,t} I_t - w_t H_t^d$$

Notice that the profits from the wholesale-good sector may not always be positive. For instance, when the current shock is highly adverse while the expectation of future shocks is quite promising, then it's optimal for the firm to issue debts or equities to finance its investment.[18] However, in the steady state of the model, the profit, $d_{w,t}$, is always positive and a small deviation from its steady state could rarely make profits negative when taking approximation around the steady state.

Similarly to consumption composite, gross investment is also aggregated using local goods and imported goods, which has a form of,

$$I_t = \left[a_I^{\frac{1}{\gamma_I}} I_{H,t}^{\frac{\gamma_I - 1}{\gamma_I}} + (1-a_I)^{\frac{1}{\gamma_I}} I_{F,t}^{\frac{\gamma_I - 1}{\gamma_I}} \right]^{\frac{\gamma_I}{\gamma_I - 1}}$$

where a_I represents the deterministic steady state share of investment expenditure on home goods, which has an analogous interpretation as a_C. Minimizing the cost of a unit of investment, given the price of goods, implies that investment expenditure on home goods is, $I_{H,t} = a_I (P_{H,t}/P_{I,t})^{-\gamma_I} I_t$, and on foreign goods is, $I_{F,t} = (1-a_I)(P_{F,t}/P_{I,t})^{-\gamma_I} I_t$.

Minimization investment cost implies the price of investment,

$$P_{I,t} = \left[a_I P_{H,t}^{1-\gamma_I} + (1-a_I) P_{F,t}^{1-\gamma_I} \right]^{\frac{1}{1-\gamma_I}} \tag{2.17}$$

[18] Modigliani-Miller theorem holds in the current environment.

2.1.4 Intermediate producers

There exists a continuum measure of intermediate producers in both countries, each of whom employs wholesale goods to produce one type of variety j by differentiating the wholesale products into various brands. Each variety is not a perfect substitute for other varieties, and thereby, intermediate producers have a monopoly power over consumers. The intermediate consumption bundle, $Y_{H,t}$, is aggregated from varieties, $Y_{H,j,t}$, $j \in [0,1]$, though the Dixit-Stiglitz technology,

$$Y_{H,t} = \left(\int_0^1 Y_{H,j,t}^{\frac{\theta_p - 1}{\theta_p}} dj \right)^{\frac{\theta_p}{\theta_p - 1}} \tag{2.18}$$

where $\theta_p > 1$ is the elasticity of substitution between varieties. The higher the market competitiveness is, the larger θ_p becomes.

Minimizing the cost of one unit of intermediate consumption bundle, the price reads,

$$P_{H,t} = \left(\int_0^1 P_{H,j,t}^{1-\theta_p} dj \right)^{\frac{1}{1-\theta_p}}$$

The demand for variety j becomes,

$$Y_{H,j,t} = \left(\frac{P_{H,j,t}}{P_{H,t}} \right)^{-\theta_p} Y_{H,t} \tag{2.19}$$

Intermediate producer j obtains nominal profits in period t,

$$D_{p,j,t} \equiv (P_{H,j,t} - P_{w,t}) \left(\frac{P_{H,j,t}}{P_{H,t}} \right)^{-\theta_p} Y_{H,t} \tag{2.20}$$

Intermediate producers choose prices to optimize their discounted profits subject to the market demand for their varieties (2.19). Prices of intermediates are set according to Calvo's style. In each period, there is a fraction ξ_p of intermediate producers who can re-optimize their prices while the remaining intermediate firms keep their prices unchanged. The intermediary j's objective function follows as,

$$\max_{P_{H,j,t}} E_t \sum_{\tau=0}^{+\infty} \left[\xi_p^\tau Q_{t,t+\tau} D_{p,j,t+\tau} \right] \tag{2.21}$$

The optimal price $\tilde{P}_{H,j,t}$ is set such that the sum of expected discounted

profits from selling intermediates equals the expected discounted cost from purchasing whole goods given the random duration of price readjustment,

$$\tilde{P}_{H,j,t} = \frac{\theta_p}{\theta_p - 1} \frac{E_t \left\{ \sum_{\tau=0}^{+\infty} \xi_p^\tau Q_{t,t+\tau} Y_{H,t+\tau} P_{H,t+\tau}^{1+\theta_p} / X_{p,t+\tau} \right\}}{E_t \left\{ \sum_{\tau=0}^{+\infty} \xi_p^\tau Q_{t,t+\tau} Y_{H,t+\tau} P_{H,t+\tau}^{\theta_p} \right\}} \tag{2.22}$$

Summing up real dividends over all intermediate producers yields,

$$d_{p,t} \equiv \int_0^1 d_{p,j,t} dj = p_{H,t} Y_{H,t} (1 - \frac{s_t^p}{X_{p,t}}) \tag{2.23}$$

with price dispersion in the intermediate variety market, $s_t^p \equiv \int_0^1 \left(\frac{P_{H,j,t}}{P_{H,t}} \right)^{-\theta_p}$.

2.1.5 Government

Following the literature and the practice in many central banks, monetary policy is represented by a Taylor rule, which is subject to stochastic financial shocks. Suppose that there are financial market shocks which affect the equilibrium nominal interest rate. According to the literature (McCallum (1999); Taylor and Williams (2010)), the Taylor rule takes a form of,

$$\log(R_{t+1}/R) = \rho_r \log(R_t/R) + (1-\rho_r) \left[\alpha_\pi \log(\pi_t/\pi) + \alpha_y \log(gdp_t/gdp) \right] + \epsilon_{r,t} \tag{2.24}$$

where $\epsilon_{r,t}$ denotes an innovation to the nominal interest rate R_{t+1}, which has an identical and independent normal distribution with mean zero. GDP in a country is defined as $gdp_t \equiv p_{H,t} Y_{H,t}$. ρ_r characterizes the persistence of nominal interest rate. α_π and α_y represent the degree of response of interest rate to inflation and output gap.

This simple monetary policy rule is designed to reflect the basic principle of monetary policy of leaning against the wind of inflation and output movements. I exclude nominal exchange rates from the simple interest rate rule since exchange rates have a complete pass-through in the current environment; in other words, the CPI inflation has included all of information contained in the changes of nominal exchange rates. On the other hand, researchers find that interest rate rules reacting to nominal exchange rates are inferior to those excluding them in terms of welfare(Taylor and Williams 2010).

When monetary authorities implement an interest rate rule, the quantity of money m_t circulated in the financial market is endogenously determined

by the intertemporal substitution of money balance. Fiscal authorities always maintain a balanced budget constraint. The seigniorage from issuing fiat money is redistributed in a lump-sum fashion to the domestic household in each period,

$$tr_t = m_t - \frac{m_{t-1}}{\pi_t} \tag{2.25}$$

2.1.6 Market clearings and competitive equilibrium

Market clearing conditions for home and foreign intermediate goods can be written as,

$$C_{H,t} + C^*_{H,t} + I_{H,t} + I^*_{H,t} = Y_{H,t} \tag{2.26}$$

$$C_{F,t} + C^*_{F,t} + I_{F,t} + I^*_{F,t} = Y^*_{F,t} \tag{2.27}$$

Market clearing conditions for international assets follow as,

$$\psi_{H,t+1} + \psi^*_{H,t+1} = 1 \qquad \psi_{F,t+1} + \psi^*_{F,t+1} = 1 \tag{2.28}$$

$$B_{H,t+1} + B^*_{H,t+1} = 0 \qquad B_{F,t+1} + B^*_{F,t+1} = 0 \tag{2.29}$$

According to wage aggregation equation (2.6), real wage evolves as

$$w_t^{1-\theta_w} = \xi_w w_{t-1}^{1-\theta_w} (\pi^{\chi}_{t-1}/\pi_t)^{1-\theta_w} + (1-\xi_w)\tilde{w}_t^{1-\theta_w} \tag{2.30}$$

Labor market clears,

$$H_t = \int_0^1 H_{j,t} dj = H_t^d \int_0^1 \left(\frac{w_{j,t}}{w_t}\right)^{-\theta_w} dj = H_t^d s_t^w \tag{2.31}$$

with wage dispersion s_t^w in the labor market, which evolves as,

$$s_t^w = (1-\xi_w)\left(\frac{\tilde{w}_t}{w_t}\right)^{-\theta_w} + \xi_w \left(\frac{w_{t-1}}{w_t}\right)^{-\theta_w} \left(\frac{\pi_t}{\pi^{\chi}_{t-1}}\right)^{\theta_w} s^w_{t-1} \tag{2.32}$$

The evolvement for the aggregate price has a form of,

$$1 = \xi_p \pi_{H,t}^{\theta_p - 1} + (1-\xi_p)\tilde{p}_{H,t}^{1-\theta_p} \tag{2.33}$$

with $\pi_{H,t}$ is PPI inflation and $\tilde{p}_{H,t} \equiv \frac{\tilde{P}_{H,j,t}}{P_{H,t}}$ when firm j re-optimizes its price $\tilde{P}_{H,j,t}$.

Price dispersion is measured by s_t^p, which evolves as,

$$s_t^p = (1-\xi_p)\tilde{p}_{H,t}^{-\theta_p} + \xi_p \pi_{H,t}^{\theta_p} s_{t-1}^p \tag{2.34}$$

Supply of wholesale goods is equal to demand for wholesale goods,

$$Y_t = \int_0^1 Y_{H,j,t} dj = s_t^p Y_{H,t} \tag{2.35}$$

Total dividends collected from all producers within a country read,

$$\begin{aligned} d_t &= p_{H,t} Y_{H,t} - (1-\alpha) p_{H,t} Y_{H,t} \frac{s_t^p}{X_{p,t}} - p_{I,t} I_t \\ &= p_{H,t} Y_{H,t} \left(1 - (1-\alpha) \frac{s_t^p}{X_{p,t}} - \frac{p_{I,t} I_t}{p_{H,t} Y_{H,t}} \right) \end{aligned} \tag{2.36}$$

Notice from the last equality in (2.36) that dividends are a time-varying fraction of GDP. When prices of goods are flexible, the labor income share, $(1-\alpha)\frac{s_t^p}{X_{p,t}}$, is constant. If an economy doesn't have capital accumulation, investment-GDP ratio, $\frac{p_{I,t} I_t}{p_{H,t} Y_{H,t}}$, is zero, and therefore, dividend-GDP ratio is constant. In the economy studied here, dividend-GDP ratio varies over time.

CPI inflation relates PPI inflation through the following way,

$$\pi_t = \frac{p_{H,t-1}}{p_{H,t}} \pi_{H,t} \tag{2.37}$$

Let $\kappa_t \equiv \{K_t,\ K_t^*,\ R_t,\ R_t^*,\ s_t^w,\ s_t^{w*},\ s_t^p,\ s_t^{p*},\ Z_t^A,\ Z_t^E,\ \}$ denote the state of the world at time t, where a set of exogenous variables $Z_t^E \equiv \left\{Z_{a,t}, Z_{a,t}^*, \epsilon_{r,t}, \epsilon_{r,t}^*\right\}$, and a set of portfolios,

$$Z_t^A \equiv \left\{\psi_{H,t}, \psi_{F,t}, \psi_{H,t}^*, \psi_{F,t}^*, B_{H,t}, B_{F,t}, B_{H,t}^*, B_{F,t}^*\right\}$$

A competitive equilibrium is a set of home household's decision rules, $C(\kappa_t)$, $C_H(\kappa_t)$, $C_F(\kappa_t)$, $H(\kappa_t)$, $m(\kappa_t)$, $\tilde{w}(\kappa_t)$, portfolio choices $\psi_H(\kappa_t)$, $\psi_F(\kappa_t)$, $B_H(\kappa_t)$, $B_F(\kappa_t)$; a set of foreign household's decision rules, $C^*(\kappa_t)$, $C_H^*(\kappa_t)$, $C_F^*(\kappa_t)$, $H^*(\kappa_t)$, $m^*(\kappa_t)$, $\tilde{w}^*(\kappa_t)$, portfolio choices $\psi_H^*(\kappa_t)$, $\psi_F^*(\kappa_t)$, $B_H^*(\kappa_t)$, $B_F^*(\kappa_t)$; a set of home firms' decision rules, $K(\kappa_t)$, $H^d(\kappa_t)$, $Y_H(\kappa_t)$, $I(\kappa_t)$, $I_H(\kappa_t)$, $I_F(\kappa_t)$, $d(\kappa_t)$, $\tilde{p}_H(\kappa_t)$; a set of foreign firms' decision rules, $K^*(\kappa_t)$, $H^{d*}(\kappa_t)$, $Y_F^*(\kappa_t)$, $I^*(\kappa_t)$, $I_H^*(\kappa_t)$, $I_F^*(\kappa_t)$, $d^*(\kappa_t)$, $\tilde{p}_H^*(\kappa_t)$; a set of price functions $q(\kappa_t)$, $q^*(\kappa_t)$, $p^b(\kappa_t)$, $p^{b*}(\kappa_t)$, $q_k(\kappa_t)$, $q_{k,t}^*(\kappa_t)$, $p_H(\kappa_t)$, $p_F^*(\kappa_t)$, $w(\kappa_t)$,

$w^*(\kappa_t)$, $R(\kappa_t)$, $R^*(\kappa_t)$, $s(\kappa_t)$, $r_i(\kappa_t)$, for $i = a_1, a_2, b_1, b_2$, $s^w(\kappa_t)$, $s^{w*}(\kappa_t)$, $s^p(\kappa_t)$, $s^{p*}(\kappa_t)$, $\pi(\kappa_t)$, $\pi^*(\kappa_t)$, $\pi_H(\kappa_t)$ and $\pi_F^*(\kappa_t)$; such that (1) households' decision rules solve the households' problems; (2) firms' decision rules solve the firms' problems; (3) the appropriate market clearing conditions hold.

2.2 Steady state equity portfolios

2.2.1 Steady states and log-linearizations

In the deterministic steady state, all variables are unchanged in both countries. Let a variable without time subscript denote its deterministic steady state, and a variable with a hat stands for its log deviation from its steady sate, i.e., $\hat{x}_t \equiv \log(x_t) - \log(x)$. Since there are two types of shocks, a technological shock and a monetary shock, and two types of assets, equities and bonds, in the world economy, asset market is complete up to a first-order approximation.[19] International risk sharing condition (Backus-Smith condition) can be written as,[20]

$$\sigma(\hat{C}_t - \hat{C}_t^*) = \hat{s}_t \tag{2.38}$$

The terms of trade, which is defined as $tot_t \equiv \frac{P_{F,t}}{P_{H,t}}$, relates the real exchange rate via $\hat{s}_t = (2a_C - 1)\widehat{tot}_t$ up to a first-order approximation.[21]

In the symmetric steady state, we have $\psi_H = \psi_F^*$, $\psi_F = \psi_H^* = 1 - \psi_H$, $B_H = B_F^*$, and $B_F = B_H^* = -B_H$. Now let's consider a static model in which the economy starts from its long-run steady state and households face uncertainty in the next period. Portfolio choice decisions are made in the current period. One can show that there exists a unique portfolio

[19] One can check that the spanning condition holds since the number of shocks is the same as the number of assets and that the rank condition holds as long as returns to assets are not perfectly correlated with each other. The assets traded in the international market here satisfy these two conditions.

[20] Notice that in the benchmark model with non-separate preference between consumption and real money balance, the log-linearized marginal utility of consumption contains real money balance and consumption. However, the share of consumption, ω, in the preference is close to one, 0.9977 in the calibration. Therefore, for exposition convenience but without loss of accuracy, I get rid of real money balance here. When preference is separable between consumption and real money balance, equation (2.38) holds exactly.

[21] Normalizing home (foreign) prices for imports and exports by the home (foreign) aggregate price in equation (2.2) and its foreign counterpart, applying the law of one price, substituting terms of trade and real exchange rates into these two price equations, we have $tot_t = \left[\frac{a_C s_t^{1-\gamma} - (1-a_C)}{a_C - (1-a_C)s_t^{1-\gamma}}\right]^{\frac{1}{1-\gamma}}$.

that satisfies the following static budget constraints for consumption and is consistent with the linearized risk sharing condition (2.38), [22]

$$C_t = w_t H_t^d + d_t \psi_H + d_t^*(1-\psi_H)s_t + r_t^b B_H - r_t^{b*} B_H s_t$$

$$C_t^* = w_t^* H_t^{d*} + \frac{d_t(1-\psi_H)}{s_t} + d_t^* \psi_H - \frac{r_t^b B_H}{s_t} + r_t^{b*} B_H$$

Taking a first-order log-linear approximation of static budget constraints around their steady state, then subtracting foreign consumption from home consumption, and finally replacing relative consumption with real exchange rate in the Backus-Smith condition (2.38), I obtain accordingly,

$$\frac{1}{\sigma}\hat{s}_t = \frac{d}{C}(2\psi_H - 1)\hat{d}_t^R + \frac{wH^d}{C}\widehat{wH}_t^R + 2\left[(1-\psi_H)\frac{d}{C} - \frac{B_H}{C}r^b\right]\hat{s}_t + 2r^b\frac{B_H}{C}\hat{r}_t^{bR} \tag{2.39}$$

with relative dividends $\hat{d}_t^R \equiv \hat{d}_t - \hat{d}_t^*$, relative labor income $\widehat{wH}_t^R \equiv (\hat{w}_t + \hat{H}_t^d) - (\hat{w}_t^* + \hat{H}_t^{d*})$, and relative interest payments $\hat{r}_t^{bR} \equiv \hat{r}_t^b - \hat{r}_t^{b*}$.

Let labor income share be $ls \equiv \frac{wH^d}{C}$ and equity income share $\frac{d}{C} = 1 - ls$. Projecting all relevant variables in the relative budget constraint (2.39) onto real exchange rates, and then subtracting the projection terms from the relative budget constraint, consequently, we have,

$$(1-ls)(2\psi_H - 1)(\hat{d}_t^R - \beta_{\hat{d}^R,\widehat{s}}\widehat{s}_t) + ls(\widehat{wH}^R - \beta_{\widehat{wH}^R,\widehat{s}}\widehat{s}_t) + 2\frac{B_H r^b}{C}(\hat{r}_t^{bR} - \beta_{\hat{r}^{bR},\widehat{s}}\widehat{s}_t) = 0 \tag{2.40}$$

where $\beta_{\hat{x},\widehat{s}} \equiv \frac{cov(\hat{x}_t,\widehat{s}_t)}{var(\widehat{s}_t)}$ denotes the coefficient in the projection of $\hat{x}_t$ onto $\widehat{s}_t$. Notations *cov* and *var* stand for covariance and variance, respectively. Multiplying both sides of equation (2.40) by $(\hat{d}_t^R - \beta_{\hat{d}^R,\widehat{s}}\widehat{s}_t)$ and taking expectations conditional on information up to period $t-1$, yields,

$$\psi_H = \underbrace{\frac{1}{2}}_{\text{Market portfolio}} + \underbrace{(-1)\frac{1}{2}\frac{ls}{1-ls}\beta_{w,d}^s}_{\text{Human capital hedge}} + \underbrace{(-1)\frac{r^b B_H}{d}\beta_{r,d}^s}_{\text{Exchange rate hedge}} \tag{2.41}$$

with

$$\beta_{w,d}^s \equiv \frac{cov(\widehat{wH}^R - \beta_{\widehat{wH}^R,\widehat{s}}\widehat{s}_t, \hat{d}_t^R - \beta_{\hat{d}^R,\widehat{s}}\widehat{s}_t)}{var(\hat{d}_t^R - \beta_{\hat{d}^R,\widehat{s}}\widehat{s}_t)}$$

[22] Notice that the portfolio choice is constant in a linearized dynamic system. One can also apply this result directly to obtain equation (2.39).

$$\beta^s_{r,d} \equiv \frac{cov(\hat{r}^{bR}_t - \beta_{\hat{r}^{bR},\widehat{s}}\widehat{s}_t, \hat{d}^R_t - \beta_{\hat{d}^R,\widehat{s}}\widehat{s}_t)}{var(\hat{d}^R_t - \beta_{\hat{d}^R,\widehat{s}}\widehat{s}_t)}$$

In the log-linearized model, optimal shares of home equities held by the home resident are determined by three terms. The first one is the *market portfolio*, in which one half comes from the home equities and the other half from foreign equities since both countries are symmetric. The market portfolio reflects the diversification motive emphasized by Baxter and Jermann (1997). The second term, *human capital hedge*, captures how well the home equity could hedge home human capital income risk. This term is expressed as a relative covariance between the component of human capital income orthogonal to real exchange rates and the component of dividends of equities orthogonal to real exchange rates. The last term, a relative covariance between the component of interest payments on bonds orthogonal to real exchange rates and the component of dividends orthogonal to real exchange rates, captures the relationship between bond assets and equity assets. Notice that bond position B_H shows up in home equity holdings, but this is not the trouble here.[23] The reason I use this expression is that we could see clearly the roles of equity assets in hedging human capital income risk and of bond assets in determining home equity bias. Following the convention in the literature (Engel and Matsumoto (2009a)), I label the last term in the home equity holding (2.41) as *exchange rate hedge* since this term disappears when relative bond returns are perfectly correlated with real exchange rates.

There are several points worth of making in equation (2.41). When there's no non-diversifiable income, i.e., labor income or income from non-tradable goods, and relative bond returns are uncorrelated with relative equity returns conditional on real exchange rates, households in both countries then perfectly diversify their income risk and hold only the *market portfolio* (see, Lucas (1982), which makes the share of home equities equal to one half.[24] When non-diversifiable income in an economy becomes unignorable, households in both countries have to take it into account and hedge

[23] We can obtain ψ_H and B_H by projecting both sides of equation (2.39) onto the relative dividends from equities $\hat{d}^R_t$ and onto the relative interest payments $\hat{r}^{bR}_t$ respectively and then use the same procedure in the main text to get expressions for ψ_H and B_H. See also, Coeurdacier and Gourinchas (2011).

[24] In a two Lucas trees world economy, in which both trees produce the same kind of fruit, households hold exactly the market portfolio since risk sharing implies households fully diversify idiosyncratic income shocks via choosing the market portfolio. This result is independent of preference parameters and of correlation between productivity shocks to trees.

this income risk using assets traded in the international market. If the relative returns on human capital is negatively correlated with relative equity returns conditional on real exchange rates, the motive for hedging human capital income risk leads the home household to hold more home equities since home equities pay more when labor income in the home country is low, which implies the term of *human capital hedge* in equation (2.41) is positive. When the correlation between relative returns on bonds and relative returns on equities is negligible, the motivation for hedging human capital income risk completely pins down how many home equities the home household holds. Home bias is present when the term *human capital hedge* is positive.

Nevertheless, even if *human capital hedge* term in equation (2.41) is positive, the types of bonds available in the international asset market are equally important in determining home equity holdings. Consider two simple model economies, in which both markets are complete, and the only difference comes from the types of bonds, for instance, one is with real consols whose payoffs are in local consumption goods and the other with nominal consols whose payoffs are denominated in national currency.

Obviously, the motives for holding market portfolio and for hedging labor income risk are the same in both countries since asset market is complete, but the motive for hedging exchange rate risk is different. In the country with real consols, relative bond returns are perfectly correlated with real exchange rate and thereby the *exchange rate hedge* term is zero under any type of shocks while this term in the economy with nominal consols might not be zero and it depends on types of shocks.

Table 2.1: Relative covariances of relative returns conditional on RER: Bonds

Variable	USA	UK	Japan	Germany	France	Canada	Italy	Denmark	Spain
$\beta^s_{r,d}$	0.00342	0.0471	0.0250	0.00561	0.0132	0.0344**	0.00417	0.0458***	-0.0152
	(0.0310)	(0.0288)	(0.0164)	(0.00717)	(0.00903)	(0.0153)	(0.00958)	(0.0105)	(0.0101)
Observations	61	61	61	61	61	61	61	61	61
Adj. R^2	0.0002	0.069	0.042	-0.009	0.012	0.095	-0.015	0.152	0.015

Note: Robust standard errors are in parentheses. *** stands for a significance $p < 0.01$, ** for $p < 0.05$ and * for $p < 0.10$. Foreign returns are constructed through a weighted average of home currency real returns on foreign assets. The weight is bilateral trade volume. The return differential is defined as home real return minus the weighted foreign returns. The effective real exchange rate is defined as a weighted average of bilateral real exchange rates using bilateral trade volume as a weight. Then for each type of assets, regressing the return differentials on changes of effective real exchange rates and obtain the residuals, which are orthogonal to the effective real exchange rates. Next regressing the residuals from the bond return differential regression on the residuals from the stock return differential regression yields the coefficient reported above. Returns on bond are the yields to 5-year government bonds. The appendix provides detailed information on data source and estimation method.

Table 2.2: Relative covariances of relative returns conditional on RER: Human capital

Variable	USA	UK	Japan	Germany	France	Canada	Italy	Denmark	Spain
$\beta^s_{w,d}$	-0.247***	0.00814	-0.0848	0.0979	-0.0540	0.123	-0.297*	-0.170	-0.331***
	(0.088)	(0.150)	(0.0995)	(0.0804)	(0.111)	(0.0778)	(0.160)	(0.123)	(0.104)
Observations	61	61	61	61	61	61	61	61	61
Adj. R^2	0.094	-0.017	-0.004	0.009	-0.011	0.022	0.044	0.019	0.130

Note: Robust standard errors are in parentheses. *** stands for a significance $p < 0.01$, ** for $p < 0.05$ and * for $p < 0.10$. Foreign returns are constructed through a weighted average of home currency real returns on foreign assets. The weight is bilateral trade volume. The return differential is defined as home real return minus the weighted foreign returns. The effective real exchange rate is defined as a weighted average of bilateral real exchange rates using bilateral trade volume as a weight. Then for each type of assets, regressing the return differentials on changes of effective real exchange rates and obtain the residuals, which are orthogonal to the effective real exchange rates. Next regressing the residuals from the bond return differential regression on the residuals from the stock return differential regression yields the coefficient reported above. Returns on equity are from stock index returns which include dividends and capital gains. Returns on human capital are estimated from data. The appendix provides detailed information on data source and estimation method.

2.2.2 Empirics on international asset markets

As emphasized by van Wincoop and Warnock (2006), "any explanation for portfolio home bias in the context of GE[general equilibrium] models only makes sense if the resulting covariance ratio is consistent with the data". Before going through the calibration of each term in equation (2.41), let's take a look at international asset markets first. Notice that the covariance ratios $\beta^s_{w,d}$ and $\beta^s_{r,d}$ don't depend on model specifications and can be obtained directly from the data. To obtain these covariance ratios, we need to know returns on human capital and returns on bonds and equities. The data used in this subsection is quarterly data. Data sources and the estimation method are described in the appendix. Since long-term bonds are the most widely traded bond assets in the bond asset category, I report the covariance ratio for bonds, $\beta^s_{r,d}$, for 5-year government bonds.[25] Table 2.1 shows that in a sub-sample of OECD countries, seven out of nine country pairs (*home v.s. foreign country* pair) have zero covariance ratio $\beta^s_{r,d}$. This covariance ratio in the remaining two country pairs, Canada and Denmark, is statistically significantly different from zero, but it is close to zero economically.[26] Now let's move to the covariance ratio for human capital, $\beta^s_{w,d}$. Table 2.2 presents the estimated results for $\beta^s_{w,d}$. For 6 out of 9 country pairs, the human capital covariance ratio, $\beta^s_{w,d}$, is negative. Particularly, this ratio is significantly negative for the USA, Italy and Spain, with -24.7%, -29.7% and -33.1%, respectively.

[25] The covariance ratio $\beta^s_{r,d}$ for other types of bonds, i.e., 3-month treasury bill, 2-year government bond, and 10-year government bond, is quite similar to 5-year government bond. The results are available upon request.

[26] This covariance ratio might not be accurately mapped from model to data since the volatilities of real exchange rates and of returns on stocks are far larger than the volatilities of returns on bonds in the data, while in general equilibrium models like the one in this chapter, volatilities of real exchange rates and of equity returns are similar to the volatility of bonds. Thereby, a close to zero covariance ratio $\beta^s_{r,d}$ in the data might be implied from extremely high volatility of real exchange rates that dominates the relative returns on bonds denominated in the same currency while relative returns on bonds denominated in national currency don't necessarily hedge real exchange rate fluctuations as long as their volatility is far lower than real exchange rates. However, following the literature, i.e., Coeurdacier (2009) and others, I adopt the interpretation of bonds hedging real exchange rate risk well when the covariance ratio $\beta^s_{r,d}$ is close to zero, ignoring the volatility wedge between models and data. However, I present second moments for bond return differentials (local return differentials) across borders in the data in the following section to get rid of volatile real exchange rates in the data.

2.3 Calibration

Since this chapter focuses not only on the steady state portfolios but also on first-order portfolios, while data on first-order portfolios is available at yearly frequency, thereby, I take use of annual data instead of quarterly. Variables and their sources are given in the appendix. The parameters of the model are chosen to be close to the standard choice for the values of parameters in literature. Parameters and their values are listed in table 2.3. Relative risk aversion parameter in the benchmark model is chosen to be $\sigma = 2$, which is adopted by Corsetti, Dedola and Leduc (2008) and Coeurdacier, Kollmann and Martin (2010). Following Obstfeld and Rogoff (2001) and Christiano, Eichenbaum and Evans (2005), the labor supply elasticity is set to be unity, which implies $\eta = 7/3$. The estimation of elasticity of substitution between imports and exports is ambiguous in literature. Corsetti, Dedola and Leduc (2008), who calibrate their model to the U.S. relative to a set of OECD countries on annual data, estimated that $\gamma = 0.85$, while Bernard et al. (2003) find that the elasticity equals 4 based on trade data. Here I assume that $\gamma = \gamma_I = 2$ in the benchmark model, as in Coeurdacier et al. (2010). I also consider a large range of trade elasticity and relative risk aversion to check the robustness of the results when necessary. The interest rate elasticity of money demand is chosen to be $\theta = 0.5$, which is taken from Hoffman, Rasche and Tieslau (1995), whose estimation is based on a cross-country study.

Empirical evidence suggests that the elasticity of substitution between differentiated labor services, θ_w, is between 2 and 6, which implies the wage markup rate is between 20% and 100%. Erceg, Henderson and Levin (2000) use the value of $\theta = 4$. I take $\theta = 6$ in the benchmark model. Elasticity of substitution between goods varieties is set at $\theta_p = 11$, implying a 10% price markup.

The wage contract duration parameter $\xi_w = 0.82^4$ implies an average contract duration of $1/(1 - 0.82^4) = 1.8$ years, around 7 quarters, which is slightly higher than the estimate in Christiano, Eichenbaum and Evans (2005).[27] Recently, Barattieri, Basu and Gottschalk (2010) find that the probability that an individual will experience a nominal wage change is at most 18% in the average quarter (a minimal duration of 5.6 quarters), which is based on Survey of Income and Program Participation for the period $1996 - 1999$ in the United States. Price duration parameter is set at $\xi_p = 0.75^4$, which implies average duration of price is around 1.5 years, which is

[27] Wage contract duration is 5 quarters when there is no habit formation in their model.

Table 2.3: Parameter values

Parameter	Description	Value
Parameters in utility function		
β	Discount factor	0.96
σ	Risk aversion	2
η	Frisch labor elasticity parameter	7/3
θ	Interest rate elasticity of money demand	0.5
$\gamma = \gamma_I$	Elasticity of substitution between home and foreign goods	2
$a_C = a_I$	Share of home-traded goods	0.85
θ_w	Elasticity of substitution between labor services	6
θ_p	Elasticity of substitution between intermediate goods	11
ξ_w	Wage contract duration parameter	0.82^4
ξ_p	Price duration parameter	0.75^4
ω	Share of consumption	0.9977
η_0	Coefficient for leisure	3.6405
χ	Wage indexation	0
Parameters in production		
δ	Depreciation rate	0.1
α	Capital share	0.30
ϕ	Curvature of adjustment cost function	0.1
Parameters in shock processes		
ρ_a	Persistence of TFP shocks	0.905
ρ_r	Persistence of money supply shocks	0.80
σ_{ϵ_a}	Standard deviation of TFP shocks	1.36%
σ_{ϵ_r}	Standard deviation of interest rate shocks	1.36%

in the reasonable range of price stickiness.

Both consumption and investment expenditure exhibit home bias in the U.S. and developed economies, and the share of investment expenditure on imports is higher than that of consumption. Following the international business cycle literature, i.e., Backus, Kehoe and Kydland (1995), I choose consumption and investment home bias to be $a_C = a_I = 0.85$, which implies imports-GDP ratio is 15% in the steady state. Corsetti, Dedola and Leduc (2008) assume that the share of home-traded goods in tradable consumption basket is 0.72 and the share of non-traded goods is 0.45, which implies imports-GDP ratio is $(1 - 0.72) \times (1 - 0.45) = 15.4\%$.

Since the data used in this chapter is yearly data, the discount factor, summarized by function β, is set at 0.96 in the steady state, which implies that an annualized risk-free rate of 4%.

The data in the sample shows that average fixed investment-GDP ratio is 20%. Then the share of capital in production function is calibrated as $\alpha = 0.3$. As is well known, without capital adjustment cost, the volatility of investment in the business cycle model is quite large compared to that of GDP. Thereby, the elasticity of price of capital with respect to investment-capital ratio is set such that the investment volatility relative to GDP volatility in the model matches that in the data. Finally, I set the depreciation rate of capital equal to 10% annually.

There are two other free parameters to be pinned down. One is η_0, which is chosen such that the labor supply at the steady state is $H = 0.3$, implying that the household will supply 30% of total hours to the labor market. The second is ω, which is set to match the yearly data on M1-GDP ratio, 15.2%, in the United States.

I collect a data set covering a group of 10 OECD countries, including U.S., Japan, U.K., Germany, France, Italy, Canada, Spain, Austria and Denmark. Once a home country is chosen, the foreign country is constructed through aggregating the rest of countries in the data sample using bilateral trade as weight. The appendix shows the data sources as well as the construction method for the foreign country. I estimate a **VAR** process of TFP using maximum likelihood for the U.S. and the constructed foreign country aggregates,

$$\begin{bmatrix} log(Z_{a,t}) \\ log(Z^*_{a,t}) \end{bmatrix} = \begin{bmatrix} 0.905 & 0 \\ 0 & 0.905 \end{bmatrix} \begin{bmatrix} log(Z_{a,t-1}) \\ log(Z^*_{a,t-1}) \end{bmatrix} + \begin{bmatrix} \epsilon_{a,t} \\ \epsilon^*_{a,t} \end{bmatrix}$$

with standard deviation of innovations $\sigma_{\epsilon_a} = \sigma_{\epsilon^*_a} = 1.36\%$, and cross correlation $corr(\epsilon_a, \epsilon^*_a) = 0.26$.

According to the Taylor principle in literature (say, Taylor and Williams, 2011), the coefficient α_π in the Taylor rule (2.24) should be larger than unity to avoid indeterminacy in equilibrium. I set $\alpha_\pi = 1.5$ and $\alpha_y = 0.1$, which are common used values in the Taylor rule. Assume that innovations in the Taylor for each country have a normal distribution with mean zero and constant standard deviation. Given these two parameter values, I estimate the persistence parameter, ρ_r, in the Taylor rule (2.24) for the Home and Foreign countries respectively based on the U.S.-foreign country pair and obtain $\rho_r = 0.80$ and the standard deviation $\sigma_{\epsilon_r} = 1.36\%$ with cross-country correlation $corr(\epsilon_r, \epsilon_r^*) = 0.5$.[28]

2.4 Numerical results

The usual solution method for rational expectations models can be applicable here for a linearized dynamic system. As is well-known, portfolio decisions in the linearized model are indeterminate and therefore, we need to take a second-order approximation to the Euler conditions for portfolios to pin down the zeroth order portfolios. Devereux and Sutherland (2011a) provide an algorithm to calculate the zeroth-order optimal portfolios.

Table 2.4: Optimal home equities held by home households (nominal consols)

	$\gamma = 0.50$	$\gamma = 0.85$	$\gamma = 1$	$\gamma = 1.1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$
$\sigma = 0.5$	-4.38	-12.64	42.53	8.68	0.96	-0.25	-0.89
$\sigma = 1$	-0.13	0.42	0.80	1.15	5.04	-10.16	-3.12
$\sigma = 1.5$	0.30	0.85	1.20	1.50	3.74	54.89	-4.71
$\sigma = 2$	0.46	1.00	1.33	1.60	3.47	17.02	-5.91
$\sigma = 5$	0.70	1.21	1.51	1.74	3.19	8.81	-9.49
$\sigma = 10$	0.76	1.27	1.55	1.78	3.12	7.76	-11.49

Note: This table reports the optimal share of home equities held by home households. σ is the coefficient of relative risk aversion and γ is the elasticity of substitution between imports and exports. Each unit of consol bond pays one unit of local currency each period. When $\gamma = 2$, $\sigma = 2$, home residents hold 1702% of foreign equities.

[28] The coefficients in the interest rate rule are consistent with the rule studied in Chari et al. (2002) and Christiano, Trabandt and Walentin (2010). The latter authors obtain the interest rate rule via a Bayesian estimation based on quarterly data in U.S., in which $\rho_r = 0.87$, $\alpha_\pi = 1.43$ and $\alpha_y = 0.07$.

Table 2.5: Optimal bond assets-GDP ratios (nominal consols)

	$\gamma = 0.50$	$\gamma = 0.85$	$\gamma = 1$	$\gamma = 1.1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$
$\sigma = 0.5$	1.76	4.35	13.07	2.40	0.01	0.37	0.53
$\sigma = 1$	0.40	0.19	0.06	0.06	1.31	3.45	1.19
$\sigma = 1.5$	0.25	0.04	0.08	0.18	0.92	17.06	1.68
$\sigma = 2$	0.19	0.02	0.13	0.23	0.84	5.14	2.05
$\sigma = 5$	0.10	0.10	0.20	0.29	0.77	2.56	3.16
$\sigma = 10$	0.07	0.12	0.23	0.30	0.76	2.24	3.78

Note: This table reports the ratio of bond assets to its GDP. σ is the coefficient of relative risk aversion and γ is the elasticity of substitution between imports and exports. Each unit of consol bonds pays one unit of local currency each period.

Table 2.6: Covariance ratios for hedging human capital income risk (nominal consols)

	$\gamma = 0.50$	$\gamma = 0.85$	$\gamma = 1$	$\gamma = 1.1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$
$\sigma = 0.5$	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.24
$\sigma = 1$	-0.22	-0.23	-0.23	-0.23	-0.23	-0.23	-0.24
$\sigma = 1.5$	-0.22	-0.22	-0.22	-0.23	-0.24	-0.23	-0.24
$\sigma = 2$	-0.22	-0.22	-0.22	-0.23	-0.27	-0.23	-0.23
$\sigma = 5$	-0.22	-0.22	-0.22	-0.22	-0.23	-0.23	-0.23
$\sigma = 10$	-0.22	-0.22	-0.22	-0.22	-0.22	-0.23	-0.23

Note: This table reports the ratio of covariance between human capital returns orthogonal to real exchange rates and equity returns orthogonal to real exchange rates to the variance of equity returns orthogonal to real exchange rates, $\beta^s_{w,d}$, in the main text. σ is the coefficient of relative risk aversion and γ is the elasticity of substitution between imports and exports. Nominal consols are traded in the international bond markets.

Table 2.7: Covariance ratios for hedging real exchange rate risk (nominal consols)

	$\gamma = 0.50$	$\gamma = 0.85$	$\gamma = 1$	$\gamma = 1.1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$
$\sigma = 0.5$	12.48	12.99	13.25	13.46	-14.87	13.23	14.22
$\sigma = 1$	10.92	10.74	7.90	16.61	13.06	13.36	14.11
$\sigma = 1.5$	10.19	5.81	14.65	13.29	12.72	13.17	13.98
$\sigma = 2$	9.69	24.67	13.33	12.79	12.28	13.05	13.87
$\sigma = 5$	8.38	13.11	12.29	12.16	12.33	12.80	13.73
$\sigma = 10$	7.53	12.51	12.04	11.98	12.22	12.72	13.67

Note: This table reports the ratio of covariance between human capital returns orthogonal to real exchange rates and equity returns orthogonal to real exchange rates to the variance of equity returns orthogonal to real exchange rates, $\beta^s_{r,d}$, in the main text. σ is the coefficient of relative risk aversion and γ is the elasticity of substitution between imports and exports.Nominal consols are traded in the international bond markets.

2.4.1 A model with nominal consols

Can a standard two-country monetary business cycle model with equities and nominal consols generate reasonable home bias in equity? The answer is it depends. Particularly, it depends critically on two preference parameters: the coefficient of relative risk aversion and trade elasticity.

In a model with nominal consols, interest payments on bond assets are denominated in national currency (i.e., one dollar interest payment by one unit of home bonds and one euro by one unit of foreign bonds). Express the interest payment in terms of national real consumption and obtain, $r^b_t = \frac{1}{P_t}$ and $r^{b*}_t = \frac{1}{P^*_t}$. Table 2.4 presents optimal home equities held by the home resident at steady state. The number in each cell in the table denotes the optimal shares of home equities for different combination of the coefficient of relative risk aversion and trade elasticity. Taking $\sigma = 2$ and $\gamma = 2$ for example, which is wildly used in the literature, the home resident holds 1702% of home equities, an extremely unreasonable value.

As many authors have pointed out, there are no consensus on the values for relative risk aversion and trade elasticity in the open-macroeconomics literature on the one hand (see, Corsetti, Dedola and Leduc 2008), and on the other hand, these two parameters seem to be important in determining asset holdings in the international macro-finance literature. For robustness check, table 2.4 lists home equity shares under a large range of relative risk

aversion, $\sigma \in [0.5, 10]$, and trade elasticity, $\gamma \in [0.5, 3]$. The numerical experiment illustrates that the share of home equities is extremely sensitive to relative risk aversion σ and trade elasticity γ. There's no clear increase or decrease pattern with the changes of these two parameters. On the side of bonds, table (2.5) illustrates the ratio of external bond assets to GDP. The bond assets-GDP ratio also swings a lot across different preference parameters.

A following question is how human capital hedge motive and exchange rate hedge motive contribute to the optimal shares of home equities. I then decompose the equity portfolio according to equation (2.41) and calculate each component. Combining steady state levels of labor income share, dividend-interest payment ratio and optimal bond holdings, with unconditional covariance ratios $\beta^s_{w,d}$ and $\beta^s_{r,d}$ delivers all terms in (2.41). Table 2.6 reports the covariance ratio $\beta^s_{w,d}$. For a wide range of parameter values, this ratio remains almost the same, around -0.22, which is consistent with the data, implying that the home household holds about 40% of home equity shares for hedging human capital income risk. Clearly, the large swings of optimal shares of home equities come from the exchange rate hedge motive. Table 2.7 shows the covariance ratio for hedging real exchange rate risk. We can see that nominal consols don't hedge real exchange rate risk well since $\beta^s_{r,d}$ has a very large value and vary largely with the preference parameters.

Do monetary or technological shocks change the responses of optimal equity shares hedging human capital income risk? The results (not reported here) show that either monetary shocks or technological shocks doesn't change the covariance ratio $\beta^s_{w,d}$ much.

2.4.2 The benchmark model with inflation-indexed bonds

Inflation-indexed bonds

Until now, I have analyzed different components of optimal shares of home equities held by the home household when nominal consols are traded in the international bond markets, and find that the incentive for hedging human capital income risk is quite stable and is also consistent with what we have observed in the data. Bonds seem to be a key to obtain a robust home equity shares across different preference parameters and across diverse shocks. Recently, inflation-indexed government bonds have become available in many countries, including Canada, France, Japan, U.K. and U.S., and the stock of these bonds grows rapidly in bond markets. Britain first issued inflation-indexed government bonds in the 1980s and the U.S.

government first introduced Treasury Inflation-protected Securities (TIPS) in 1997. Up to 2008, the outstanding supply of inflation-indexed bond in the U.K. grew to around 30% of British public debts and TIPS accounted for 10% of marketable U.S. Treasury debt (Campbell, Shiller and Viceira 2009). Inflation-indexed bonds in these countries usually have long maturities and naturally, they are corresponding to the inflation-indexed consols in the model studied here.

Beyond the evidence above, a natural concern appears. It seems that equity home bias has existed for a long time, even before the first issuance of inflation-linked bonds. However, notice that, in the early years, international capital flows were significantly restricted and consequently we observed equity home bias in decades ago because of lack of financial integration. Equity home bias becomes a puzzle only recently when international financial markets are integrated. The time of issuance of inflation linked bonds was quite consistent with the start of massive financial market integration, which implies that inflation linked bonds could be an endogenous response to financial market integration.[29] Accordingly, Inflation linked bond seem a good and reasonable candidate hedging against real exchange rate risk.

Before going through how well inflation-indexed bonds hedge real exchange rate risk, let's begin with real consol bonds, which pay a fixed amount of consumption composite to claimants each period. The key insight for hedging real exchange rate risk comes from the relative interest payments. The relative interest payment on the real bond is $r_t^{rR} \equiv \frac{1}{s_t}$ and its log-linearization version reads,

$$\hat{r}_t^{rR} \equiv -\hat{s}_t \tag{2.42}$$

which implies that the relative interest payment $\hat{r}_t^{rR}$ perfectly moves with real exchange rate $\hat{s}_t$; in another word, real consols could perfectly hedge real exchange rate risk. This point has confirmed by Coeurdacier et al. (2010). Table 2.8 reports gross returns and relative interest payments on bonds in detail.

When nominal consols are traded in the international asset markets, the relative interest payment r_t^{cR} by nominal consols in period t is $r_t^{cR} \equiv \frac{P_t^*}{P_t}\frac{1}{s_t}$. Log-linearizing this relative interest payment yields,

$$\hat{r}_t^{cR} \equiv (\hat{P}_t^* - \hat{P}_t) - \hat{s}_t \tag{2.43}$$

Notice that relative prices show up in the relative interest payment above

[29] I thank the external examiner, Mathias Hoffmann, brought up this concern.

besides real exchange rates. We know that relative prices varies a lot with monetary shocks (non-stationary nominal prices) and relative interest payment $\hat{r}_t^{cR}$ disconnects with real exchange rate $\hat{s}_t$. Thereby nominal consols are not good for hedging real exchange rate risk, which is the case in the model with nominal consols. Nevertheless, when monetary shocks are not important, or there are not monetary shocks, relative prices change mildly and nominal consols could be helpful to hedge real exchange rate risk. This point will be confirmed in the next section of incomplete asset markets when I shut down nominal rigidities or monetary shocks.

Now let me describe inflation-indexed consols. Suppose that the economy starts from its steady state, say, period 0. Assume that the face value for one unit of consols is set at P^b and the preset interest rate is $R-1$ per period. Without inflation indexation, interest payment is $P^b(R-1)$ dollars per period. Now suppose that interest rate is fully indexed to past inflation. Then one unit of such consols delivers, $P^b(R-1)\Pi_{s=0}^{t-1}\pi_s$, with $\pi_0 = 1$, dollars to claimants in period t. This payment can be simplified as $P^b(R-1)\frac{P_{t-1}}{P_0} = (R-1)P_{t-1}$, where I have used the assumption of $P^b = P_0 = 1$ for simplicity. In many cases, bond issuers negotiate interest rates on long-term bonds with bond holders. Thereby instead of paying fixed interest rate $R-1$ per period, bond issuers promise to pay time-varying interest rates. Assume that the interest rate on consols varies with short-term interest rate, say $R_t - 1$, in period t, determined in the previous period $t-1$. Thereby, each unit of this type of inflation-indexed bond delivers $(R_t-1)P_{t-1}$ dollars in period t. The relative real interest payment on inflation-indexed bond can then be written as,

$$r_t^{dR} \equiv \frac{R_t - 1}{R_t^* - 1}\frac{\pi_t^*}{\pi_t}\frac{1}{s_t} \tag{2.44}$$

Take a log-linearization of r_t^{dR} in the equation above and read,

$$\hat{r}_t^{dR} \equiv \left(\hat{\pi}_t^* - \frac{1}{1-\beta}\hat{R}_t^*\right) - \left(\hat{\pi}_t - \frac{1}{1-\beta}\hat{R}_t\right) - \hat{s}_t \tag{2.45}$$

Notice that, besides real exchange rates, there are two extra terms, inflation rate adjusted by short-term interest rate in each country, show up in the equation above. It is this relative inflation rates that reduce the volatility of relative interest payment $\hat{r}_t^{dR}$ when the elasticity of intertemporal substitution is low. With low EIS (or high risk aversion), households have higher incentive to smooth consumption, which implies nominal in-

Table 2.8: Types of bonds

Bond types	Gross real return (Home)	Relative interest payment ($\frac{\text{Home}}{\text{Foreign}}$)
Real consol	$\frac{P^b(R-1)P_t+P_t^b}{P_{t-1}^b}\frac{1}{\pi_t}$	$\frac{P^b(R-1)}{P^b(R-1)}\frac{1}{s_t}=\frac{1}{s_t}$
One-period nominal bond	$\frac{R_t}{\pi_t}$	$\frac{R_t-1}{R_t^*-1}\frac{1}{\mathcal{E}_t}$
Nominal consol	$\frac{P^b(R-1)+P_t^b}{P_{t-1}^b}\frac{1}{\pi_t}$	$\frac{P^b(R-1)}{P^b(R-1)}\frac{P_t^*}{P_t}\frac{1}{s_t}=\frac{P_t^*}{P_t}\frac{1}{s_t}$
Inflation-indexed bond	$\frac{P^b(R_t-1)P_{t-1}+P_t^b}{P_{t-1}^b}\frac{1}{\pi_t}$	$\frac{P^b(R_t-1)}{P^b(R_t^*-1)}\frac{\pi_t^*}{\pi_t}\frac{1}{s_t}=\frac{R_t-1}{R_t^*-1}\frac{\pi_t^*}{\pi_t}\frac{1}{s_t}$

terest rates respond positively with expected inflation rates, which in turn reduces the volatility of the terms in the brackets in equation (2.45). Consequently, inflation-indexed bonds could hedge real exchange rate risk pretty well. In the following analysis of portfolio dynamics, I make use of this type of inflation-indexed bond as the benchmark instead of nominal consols to study international portfolio choices.

In order to get a sense of how bonds are different in terms of relative interest payments, I plot a simulation path of 125 periods based on the benchmark calibration. Figure 2.1 shows relative interest payments to different bonds, which are expressed in units of local consumption goods $\hat{r}_t^{iR}+\hat{s}_t$ with $i=r,c,d$. Relative interest payments on inflation-indexed bonds move very tightly around those on real bonds. Nevertheless, relative interest payments on nominal bonds, including nominal consols and one-period nominal bonds, have a large volatility compared with inflation-indexed bonds since monetary shocks have persistent effects on nominal prices.

Given the benchmark calibration, I calculate the steady state home equity holdings by home households. Table 2.9 show that with inflation-indexed consols traded internationally, households in either country optimally hold around 90% of their domestic equity shares. Compared with the model with nominal bonds, the equilibrium equity shares in the model with inflation-indexed bonds are quite robust across different values of trade elasticity and elasticity of intertemporal substitution.

How well does domestic equity hedge domestic labor income risk, conditional on real exchange rates? Does the inflation-indexed bond hedge real exchange rate risk? Following the decomposition of the steady state equity portfolio in equation (2.41), it shows that the covariance ratio for hedging la-

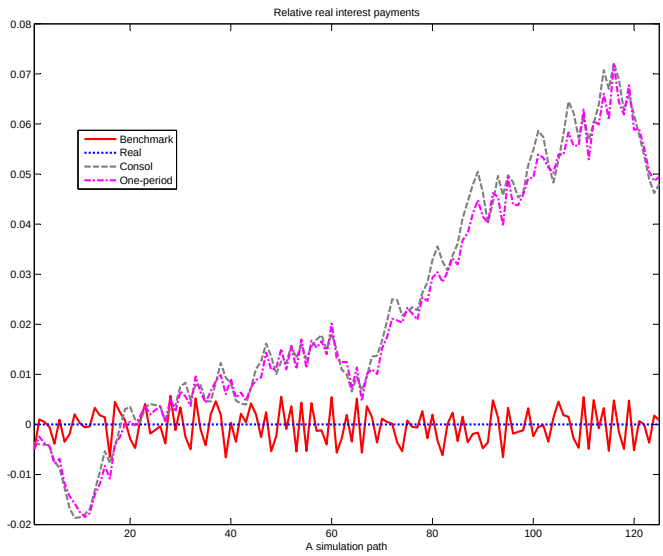

Figure 2.1: A simulation path for relative interest payments on different types of bonds based on the benchmark calibration. The red solid line is for the benchmark model with inflation-indexed bonds. The blue dotted line represents a model with real bonds indexed to local produced goods. The pink dashed line denotes a model with one-period nominal bonds. The grey dashed line is for a model with nominal consols.

Table 2.9: Optimal home equities held by home residents (inflation-indexed bonds)

	$\gamma = 0.50$	$\gamma = 0.85$	$\gamma = 1$	$\gamma = 1.1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$
$\sigma = 0.5$	1.11	1.04	1.03	1.02	1.00	1.00	1.05
$\sigma = 1$	1.12	1.01	0.98	0.97	0.94	0.93	0.99
$\sigma = 1.5$	1.11	0.97	0.94	0.93	0.89	0.90	0.96
$\sigma = 2$	1.10	0.95	0.92	0.90	0.87	0.87	0.94
$\sigma = 5$	1.06	0.89	0.85	0.84	0.81	0.82	0.91
$\sigma = 10$	1.04	0.86	0.82	0.81	0.78	0.80	0.89

Note: This table reports the optimal share of home equities held by home households in a model with TFP shocks and monetary shocks. Inflation-indexed consol derivatives are used. The model specification is same as the benchmark model. σ is the coefficient of relative risk aversion and γ is the elasticity of substitution between imports and exports.

Table 2.10: Covariance ratios for hedging real exchange rate risk (inflation-indexed bonds)

	$\gamma = 0.50$	$\gamma = 0.85$	$\gamma = 1$	$\gamma = 1.1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$
$\sigma = 0.5$	0.37	0.32	0.30	0.29	0.24	0.19	0.09
$\sigma = 1$	0.51	0.44	0.42	0.40	0.34	0.28	0.17
$\sigma = 1.5$	0.55	0.49	0.46	0.44	0.38	0.31	0.20
$\sigma = 2$	0.58	0.51	0.48	0.47	0.40	0.33	0.21
$\sigma = 5$	0.61	0.55	0.52	0.51	0.44	0.37	0.24
$\sigma = 10$	0.64	0.57	0.54	0.52	0.45	0.38	0.25

Note: This table reports the ratio of covariance between human capital returns orthogonal to real exchange rates and equity returns orthogonal to real exchange rates to the variance of equity returns orthogonal to real exchange rates, $\beta^s_{r,d}$, in the main text. σ is the coefficient of relative risk aversion and γ is the elasticity of substitution between imports and exports. Inflation-indexed bonds are traded in the international bond markets.

bor income risk, $\beta^s_{w,d}$, is around -0.23 (similar to table 2.6 and not reported here) and remains quite stable across preference parameters, which matches the data pretty well. The covariance ratio for hedging real exchange rate risk, $\beta^s_{r,d}$, shown in table 2.10, is highly small and close to zero. For instance, in the benchmark calibration, $\sigma = \gamma = 2$, $\beta^s_{r,d} = 0.33$ for inflation-indexed bonds, while $\beta^s_{r,d} = 13.05$ for nominal bonds.[30] To wrap up, a model with inflation-indexed bonds can deliver quite robust equity home bias, and furthermore, the covariance ratios for hedging real exchange rate risk and labor income risk are also consistent with the data.

Data and model moments for asset returns

Inflation-indexed bonds could effectively hedge real exchange rate risk and hence a model with these bonds could deliver robust home bias in equity portfolios. A following question is then how well the second moments for bonds generated by the model are consistent with those in the data. I find that the model studied here not only explains home bias in equity portfolios, but also produces second moments for bond returns which are consistent with the data. Table 2.11 presents second moments for CPI inflation rate, equity return rate and bond return rate both in the model and in the data. Because of data availability on equity returns and bond returns, I only report the second moments for the quarterly data. The first column labeled data shows that inflation rate, bond real return rate and bond real return rate differential across borders (home real return rate net of foreign real return rate) are one half as volatile as output when the U.S. is the home country.[31] Inflation rate is pro-cyclical, bond return rate is weakly counter-cyclical and bond return rate differential is acyclical in the data. The column labeled model (1) shows the corresponding model moments in the benchmark model with inflation-indexed bonds. The volatilities and cyclicality for inflation rate, bond return rate and bond return rate differential in the model are quite closed to the data. As in the literature, the model studied here hardly captures equity returns in the stock markets. For comparison convenience, I also report second moments in a model with nominal bonds. The column labeled model (2) shows that the volatilities of bond return rate and bond return rate differential are almost two times as large as those in the data.

[30] When I use exogenous money supplies (in the following section) rather than Taylor rules , $\beta^s_{r,d}$ is close to zero in terms of magnitude, say, around 0.1.

[31] In calculating bond return rate differentials, both return rates are denominated in local real consumption composite, rather than in the same consumption composite.

Table 2.11: Data and model moments for asset returns

	Data	Model (1)	Model (2)	Model (3)	Model (4)
Standard deviation relative to GDP					
Inflation rate	0.57	0.66	0.66	0.70	0.70
Bond real return rate	0.60	0.54	0.89	0.42	0.71
Bond real return rate differential	0.48	0.48	0.92	0.41	1.01
Equity real return rate	5.55	0.41	0.41	0.35	0.35
Cross correlation with GDP					
Inflation rate	0.36	0.47	0.47	0.81	0.81
Bond real return rate	-0.15	-0.17	-0.14	0.49	-0.80
Bond real return rate differential	-0.04	-0.09	-0.13	0.28	-0.50
Equity real return rate	0.06	0.39	0.39	0.34	0.34

Note: Model specifications are model (1) with Taylor rules and inflation-indexed bonds, model (2) with Taylor rules and nominal consols, model (3) with exogenous money supplies and inflation-indexed bonds, and model (4) with exogenous money supplies and nominal consols. Bond return rate refers to the return rate on 5-year government bonds. Bond return rate differential in the data is the average of 5-year government bond return rate differentials between the U.S. and other countries in the data sample. Because of data availability, the moments in the data column are calculated based on quarterly data starting from 1980s and data periods vary with countries. The numbers in the model columns show second moments generated by various models. The model period is one-year.

Alternative monetary policy: exogenous money supply

An alternative way of conducting monetary policy for monetary authorities is to control the money supply circulated in the economy. Suppose that monetary policy is represented as a growth rate schedule, $g_{m,t}$, which is subject to stochastic financial shocks. Assume that there are financial market shocks which affect the quantity of money circulated in the financial market. The quantity of real money balance m_t in the economy is assumed to evolve as,

$$m_t = \frac{m_{t-1}}{\pi_t} g_{m,t} \tag{2.46}$$

Based on the U.S. and foreign country pair, I estimate a first-order **VAR** process for monetary growth, which reads,

$$\begin{bmatrix} g_{m,t} \\ g^*_{m,t} \end{bmatrix} = \begin{bmatrix} 0.357 & 0 \\ 0 & 0.357 \end{bmatrix} \begin{bmatrix} g_{m,t-1} \\ g^*_{m,t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{m,t} \\ \epsilon^*_{m,t} \end{bmatrix}$$

with standard deviation of innovations $\sigma_{\epsilon_m} = \sigma_{\epsilon^*_m} = 4.15\%$, and cross correlation $corr(\epsilon_m, \epsilon^*_m) = 0.20$, which is slight larger than 0.1 in Kollmann (2001) but lower than 0.5 in Chari, Kehoe and McGrattan (2002).

I redo the experiments above and find that a model with inflation-indexed consols could deliver quite robust home bias in equity portfolios. The decomposition of steady state equity holdings are similar to the case with Taylor rules. Particularly, the covariance ratio $\beta^s_{r,d}$ for hedging real exchange rate risk is highly close to zero, around 0.1. The second moments for bond return rates are reported in table 2.11. The column labeled model (3) shows that bond return rates and bond return rate differentials are less volatile than GDP, as in the data. The last column labeled model (4) shows that a model with nominal bonds generates quite high volatilities for bond return rates and return rate differentials.

2.4.3 Alternative preference specification

When preference is nonseparable in consumption and real money balance demand, marginal utility of consumption depends on real money balance and thus portfolio Euler equations also in turn contain real money balance. In order to compare the results in the new Keynesian model to those in real business cycle model without money balance, it's better to make the

preference separable in consumption and money balance,

$$U(C_t, H_t, \frac{M_t}{P_t}) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \eta_0 \frac{H_t^{1+\eta}}{1+\eta} + \frac{\nu_0}{1-\nu} \left(\frac{M_t}{P_t}\right)^{1-\nu} \tag{2.47}$$

Based on this alternative utility function, I recalibrate the model given the targets described in section 2.3 and resolve for steady state portfolios. Similarly to the model with nominal bonds and non-separable preference, the numerical result shows that optimal shares of home equities held by the home household remain sizably sensitive to trade elasticity and relative risk aversion. Contrastingly, the motive for hedging human capital income risk remains stable and consistent with data. Inflation-indexed bonds could bring robust equity home bias back as in the benchmark model.

2.4.4 Nominal rigidities in a model with nominal bonds

How does optimal shares of home equities held by home households change with nominal rigidities in a model with nominal bonds? Engel and Matsumoto (2009a) firstly investigate the effect of nominal rigidities on portfolio choices and find that home equity shares are increasing with price stickiness given the trade elasticity. However, this monotonic relationship doesn't sustain in a model with investment. Table 2.12 displays optimal shares of home equities under different combination of nominal rigidities $\xi \equiv \xi_w = \xi_p$ (price stickiness is set as the same as wage stickiness) and trade elasticity γ when money supply shocks and TFP shocks are present.[32] The smaller ξ is, the more flexible prices and wages are. When nominal rigidities are remarkably high, the household holds most of home equities. For instance, the price and wage duration must be longer than $\frac{1}{1-0.7} = 3.3$ years in order to make a household with log-preference bias toward home equities. Nevertheless, the motive for hedging human capital income risk seems not changing much with nominal rigidities. The bottom graphs in figure 2.2 show the covariance ratio $\beta^s_{w,d}$ and the implied equity shares for hedging human capital income risk in an economy with log-preference and unitary trade elasticity. Even deviating from log preference and unit trade elasticity, say $\sigma = 2$ and $\gamma = 2$, the optimal equity shares for hedging human capital income risk remains quite similar to the case with log-preference. The bottom graphs in figure 2.3 illustrate this point. Based on these analyses, that a model with high nominal rigidities can deliver robust home bias in equity lies in a fact that

[32] Engel and Matsumoto (2009a) treat money supplies as exogenous shocks. To facilitate comparison, I also use money supply shocks in this and the following subsection.

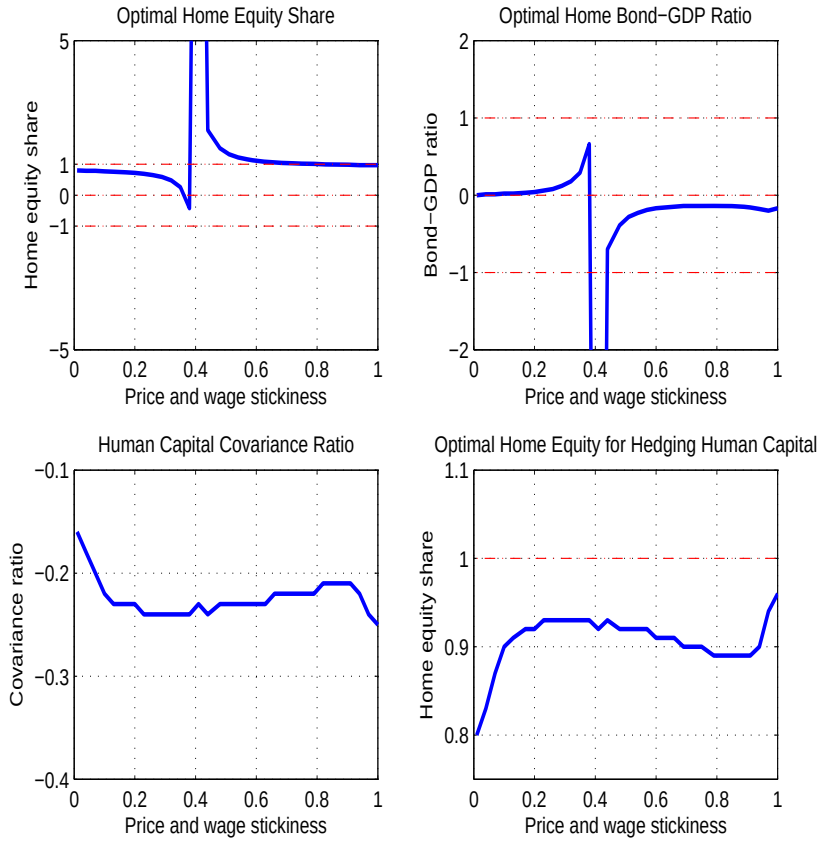

Figure 2.2: Optimal portfolios and nominal rigidities. $\sigma = \gamma = 1$. The last graph reports the sum of equity portfolios for hedging human capital income risk and the market portfolio.

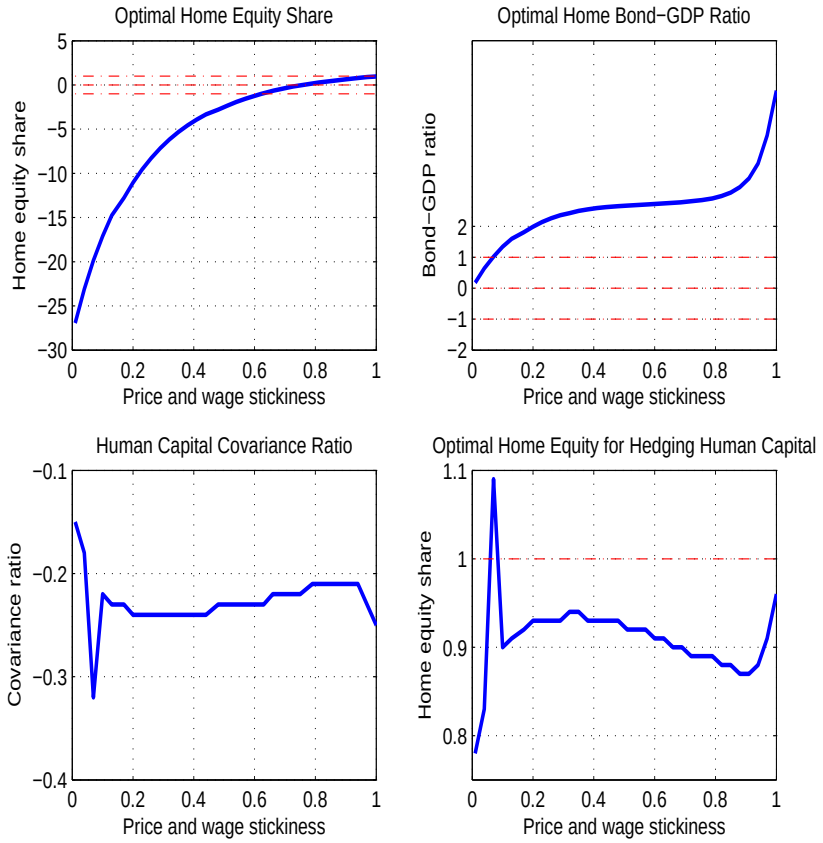

Figure 2.3: Optimal portfolios and nominal rigidities. $\sigma = \gamma = 2$. The last graph reports the sum of equity portfolios for hedging human capital income risk and the market portfolio.

nominal bond becomes actually equivalent to real bond when the degree of price stickiness tends to unit.

Table 2.12: Optimal home equity shares of home equities: nominal consols and various nominal rigidities

	$\gamma = 0.50$	$\gamma = 0.85$	$\gamma = 1$	$\gamma = 1.1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$
$\xi = 0.05$	0.45	1.01	1.40	1.74	4.86	-22.12	-3.46
$\xi = 0.10$	0.43	1.02	1.41	1.77	5.20	-17.37	-3.35
$\xi = 0.15$	0.42	1.02	1.44	1.81	5.69	-13.71	-3.22
$\xi = 0.20$	0.39	1.02	1.46	1.87	6.46	-10.85	-3.08
$\xi = 0.25$	0.36	1.02	1.50	1.95	7.81	-8.59	-2.91
$\xi = 0.30$	0.33	1.03	1.55	2.07	10.70	-6.81	-2.73
$\xi = 0.35$	0.27	1.03	1.64	2.25	20.82	-5.35	-2.53
$\xi = 0.40$	0.20	1.04	1.77	2.59	-74.45	-4.17	-2.30
$\xi = 0.45$	0.09	1.05	2.04	3.34	-9.90	-3.21	-2.06
$\xi = 0.50$	-0.10	1.09	2.80	6.55	-4.32	-2.42	-1.81
$\xi = 0.55$	-0.49	1.22	17.68	-7.36	-2.24	-1.77	-1.54
$\xi = 0.60$	-1.70	0.34	-0.82	-0.97	-1.16	-1.22	-1.26
$\xi = 0.65$	31.94	0.86	0.23	-0.01	-0.51	-0.76	-0.97
$\xi = 0.70$	2.68	0.92	0.56	0.38	-0.07	-0.38	-0.68
$\xi = 0.75$	1.71	0.94	0.72	0.60	0.24	-0.05	-0.38
$\xi = 0.80$	1.36	0.95	0.81	0.73	0.46	0.22	-0.10
$\xi = 0.85$	1.19	0.95	0.87	0.82	0.64	0.46	0.20
$\xi = 0.90$	1.08	0.96	0.91	0.88	0.77	0.65	0.47
$\xi = 0.95$	1.01	0.96	0.94	0.93	0.88	0.82	0.72
$\xi = 1.00$	0.97	0.96	0.96	0.96	0.95	0.94	0.92

Note: This table reports the optimal home equities held by home households. ξ is the probability of price and wage non-adjustment each period as of Calvo's style and γ is the elasticity of substitution between imports and exports. Each unit of consol bond pays one unit of local currency each period.

2.4.5 Home bias in consumption in a model with nominal bonds

Is home bias in equity portfolios related to home bias in consumption? Based on the assumption of unitary trade elasticity and of log preference in consumption, Heathcote and Perri (2008) show that home bias in equity is positively correlated with home bias in consumption in a two-country real business cycle model. Figure 2.4-2.5 confirms their results in a model with

Table 2.13: Optimal home equity shares held by home households: nominal consols and no home bias in consumption

	$\gamma = 0.50$	$\gamma = 0.85$	$\gamma = 1$	$\gamma = 1.1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$
ψ_H	-3.30	1.32	1.00	0.33	-0.53	-0.84	-1.05
Bond-GDP ratio	1.32	-0.14	-0.01	0.18	0.42	0.49	0.52
$\psi_{H,w}$	0.84	0.92	0.94	0.96	1.00	1.03	1.04
$\psi_{H,r}$	4.14	-0.40	-0.06	0.62	1.53	1.87	2.09
$\beta^s_{w,d}$	-0.19	-0.23	-0.24	-0.25	-0.28	-0.29	-0.30

Note: This table reports optimal portfolios held by home households. γ is the elasticity of substitution between imports and exports. The first row labeled ψ_H shows the optimal portfolio share of home equities by home households. The following row is the bond-GDP ratio. The third row labeled $\psi_{H,w}$ is the optimal equity share used to hedge human capital risk and $\psi_{H,r}$ is the optimal share of home equities for the exchange rate hedge motive. The covariance ratio of human capital and equity returns $\beta^s_{w,d}$ is shown in bottom line of the table. The model specification is as the benchmark except that there's no home bias in consumption, $a_C = 0.5$.

nominal bonds, TFP shocks and money supply shocks. The top graph of figure 2.4 shows that with log preference and unitary trade elasticity, optimal shares of home equities increase with the share of home consumption and investment expenditure on home goods when this share is relatively large, say, 70%. Nevertheless, this relationship collapses when we deviate from the unitary trade elasticity. The top graph of figure 2.5 shows that with a coefficient of relative risk aversion $\sigma = 2$ and trade elasticity $\gamma = 2$, we obtain a reverse relationship between the share of home equities and the share of home consumption expenditure on home produced goods. Home equity shares exhibit either supper home bias (with a share of home equities higher than one) when home bias in consumption is large or super anti-home bias (with a share of home equities lower than zero) when the share of consumption expenditure on local goods is relatively low. Next we turn to the component of optimal shares of home equities for hedging human capital income risk. The bottom two rows of graphs in figure 2.4-2.5 show that the covariance ratio $\beta^s_{w,d}$ of human capital income with dividends from equities conditional on real exchange rates doesn't vary much with the consumption expenditure share on local goods, with a range of $-0.2 - -0.3$, which is consistent with the data and the implied home equity shares for hedging human capital income risk lie within $0.35 - 0.53$.

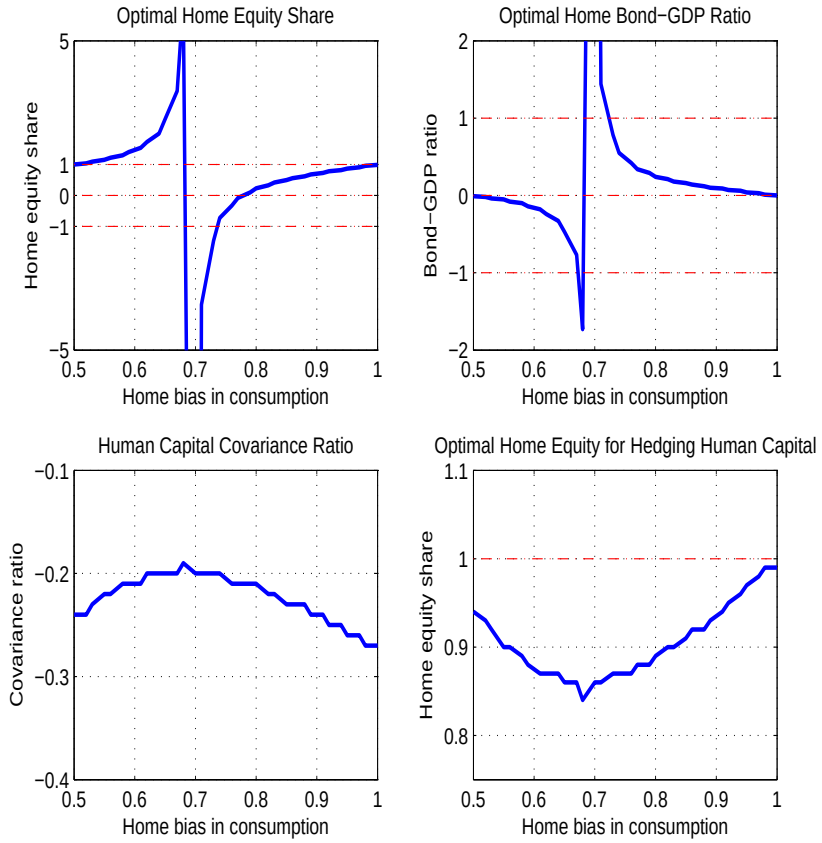

Figure 2.4: Optimal portfolios and home bias in consumption. $\sigma = \gamma = 1$. The last graph reports the sum equity portfolios for hedging human capital income risk and the market portfolio.

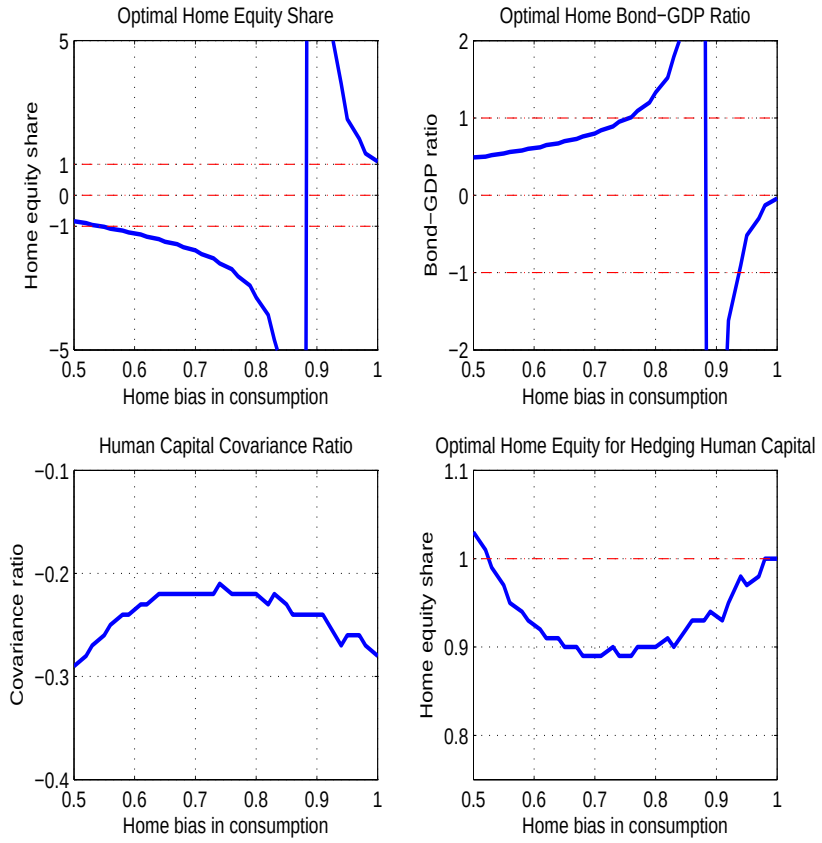

Figure 2.5: Optimal portfolios and home bias in consumption. $\sigma = \gamma = 2$. The last graph reports the sum equity portfolios for hedging human capital income risk and the market portfolio.

In the literature, unit trade elasticity plays an important role in determining optimal portfolios. Cole and Obstfeld (1991) firstly investigate the effect of portfolio diversification on welfare and find that complete home bias in equity is optimal when trade elasticity is unitary. Engel and Matsumoto (2009a) (EM, hereafter) and Devereux and Sutherland (2008) also confirm this finding.[33] In their models, they both assume that there isn't home bias in consumption, that is, $a_I = a_C = 0.5$.[34] In order to compare with their results, I produce optimal portfolios when there isn't home bias in consumption in table 2.13. The first row labeled ψ_H is the optimal shares of home equities, following which bond assets-GDP ratio is reported. The next two rows show the decomposition of home equity holdings. $\psi_{H,w}$ is the sum of market portfolio 1/2 and the optimal portfolio for hedging human capital income risk. $\psi_{H,r}$ is the remaining hedge motive for equities conditional on real exchange rates. When trade elasticity is unity, the home household completely hold home equities and zero bond assets, financial autarky is optimal even when international financial markets are available to all households. However, once deviating from unitary trade elasticity, I get either super home bias or super foreign bias in equities. The large shift of optimal equity shares across preference parameters is mainly induced by bond holdings since the motive for hedging human capital income risk is quite stable across trade elasticity.

To facilitate further comparison with EM' model, I develop a model with pricing-to-market. The appendix describes the model structure in detail. In the model with pricing-to-market, I use nominal consols. I further assume that there isn't physical capital and home bias in consumption. Other pa-

[33] Engel and Matsumoto (2009b) develop further a model including home bias in consumption based on EM (2009a). They solve for the foreign equity holdings by home residents and compare how the equity holdings vary with home bias in consumption and price and wage stickiness.

[34] They share many features in their models. First, they assume no home bias in consumption. Second, labor is the only input in production (no capital). Third, they use price stickiness as nominal rigidities. Forth, technological and monetary shocks are the exogenous driving forces. In terms of the asset menu, Devereux and Sutherland (2008) exploit different types of asset menu, including real bond indexed to locally produced goods, nominal bonds denominated in each currency and equities, while Engel and Matsumoto (2009a) assume that equities and forward currency forward contracts are traded. Their results for home equity holdings are based on a key assumption in their models: no home bias in consumption. One can show that in a first-order approximated version of this type of model (even with investment and wage stickiness), asset holdings don't depend on (a) relative risk aversion, (b) variance and persistence of shocks, (c) monetary policy, either monetary growth rates or coefficients in the Taylor rule. Home bias in equity then mainly depends on the trade elasticity and the extent of price stickiness.

rameters are the same as the benchmark model. The appendix also provides a version of model with foreign currency forward contracts. Since both of models generate similar results, I report the results of the model with nominal consols. Table A.1 presents the optimal home equities, which are very similar to table 1 in Engel and Matsumoto (2009a), since both of models have similar structures.[35] When deviating from no home bias in consumption, I get a similar portfolio shares of home equities as in table 2.4. In another experiment, I introduce investment into the model, and find that, even though there is no home bias in consumption, the model could still generate relatively robust home bias in equities. The home equity holdings are similar to an economy with $a_c = 0.68$ and without investment. However, this investment channel becomes weaker when home bias in consumption is higher or the degree of nominal rigidity is larger.

To sum up, when nominal consols are the mainly type of bond assets in the international financial market, home bias in consumption and investment seems to play a quite limited role in solving the puzzle of home bias in equity portfolios while price stickiness seems to be a promising candidate of solving this puzzle but we need incredibly high price stickiness to deliver robust home bias in equities! The reason for the latter case is that the exchange rate hedge term (the last term) in the optimal equity portfolio (2.41) tends to disappears as prices become more stickier because nominal consols are asymptotically equivalent to real consols.

2.5 Incomplete asset markets

This section extends the benchmark model with complete asset markets to one with incomplete markets. The way I introduce market incompleteness is to incorporate additional shocks into the baseline model. In the following subsections, three types of extra shocks will be considered separately. The first subsection takes into account of investment specific shocks (see Fisher 2006; Greenwood et al. 1988; Greenwood et al. 1997; Greenwood et al. 2000), since empirical evidence shows that investment specific shocks have a large contribution to business cycle fluctuations (Justiniano, Primiceri and Tambalotti 2010). In the second subsection I add government expenditure shocks to the benchmark model because government spending and taxes have important effects on consumption and output (see Blanchard and Perotti 2002; Mertens and Ravn 2010; Fisher and Peters 2010; Lez-Salido and Vall 2007;

[35] The parameters used here are different from EM's but the patterns for shares of home equities are exactly the same.

Ramey 2011; Romer and Romer 2010). In the third subsection, I will investigate optimal shares of home equities in a model with preference shocks since many researchers find that preference shifts play a non-ignorable role in explaining business cycle movements (see Ireland 2001; Justiniano, Primiceri and Tambalotti 2010). At the end of this section, I include all shocks in the model in order to compare quantitative effects of shocks on optimal equity portfolios. Monetary policy is conducted through exogenous money supplies in this section.

2.5.1 Investment efficiency shocks

Let the investment efficiency shock be $Z_{i,t}$, which effects the transforming efficiency between investment and capital goods. Physical capital accumulation now becomes,

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) K_t Z_{i,t} + (1-\delta)K_t \tag{2.48}$$

Shadow price for physical capital yields,

$$q_{k,t} = \left[\Phi'(I_t/K_t)\right]^{-1} p_{I,t}/Z_{i,t} \tag{2.49}$$

Optimal investment decision then follows,

$$\begin{aligned} q_{k,t} =& E_t \Lambda_{t,t+1} \left\{ q_{k,t+1} \left[(1-\delta) + Z_{i,t+1} \left(\Phi(\frac{I_{t+1}}{K_{t+1}}) - \Phi'\left(\frac{I_{t+1}}{K_{t+1}}\right) \frac{I_{t+1}}{K_{t+1}} \right) \right] \right. \\ & \left. + \frac{\alpha Y_{t+1} p_{H,t+1}}{K_{t+1} X_{p,t+1}} \right\} \end{aligned} \tag{2.50}$$

Other optimality conditions are the same as in the benchmark model. As in Coeurdacier et al. (2010), investment efficiency shocks are an AR(1) process,

$$\begin{bmatrix} log(Z_{i,t}) \\ log(Z^*_{i,t}) \end{bmatrix} = \begin{bmatrix} 0.79 & 0 \\ 0 & 0.79 \end{bmatrix} \begin{bmatrix} log(Z_{i,t-1}) \\ log(Z^*_{i,t-1}) \end{bmatrix} + \begin{bmatrix} \epsilon_{i,t} \\ \epsilon^*_{i,t} \end{bmatrix}$$

where $\sigma_{\epsilon_i} = \sigma_{\epsilon^*_i} = 1.73\%$, $corr(\epsilon_i, \epsilon^*_i) = 0.19$.

Coeurdacier et al. (2010) explore a real model with technological and investment efficiency shocks and show that optimal equity shares don't respond to trade elasticity and relative risk aversion. After shutting down monetary shocks and removing nominal rigidities, and keeping inflation-

indexed consols rather than nominal consols, the model investigated here degenerates to a real model analogous to Coeurdacier et al. (2010) and the numerical results deliver quite similar robust results as theirs. I then add nominal rigidities and monetary shocks into the real model. Asset markets now become incomplete because there are six types of shocks but only four types of assets. The results show that the marginal contribution of nominal rigidities and the additional shocks to optimal home equity shares are moderate. However, when nominal consols, instead of inflation-indexed consols, are traded in international financial markets, optimal home equity shares become quite sensitive to preference parameters again. Table 2.14 displays the optimal equity shares. Analogously to the benchmark model, equity shares are varying much across different preference parameter values. In another experiment, which isn't reported here, I get rid of monetary shocks and keep only real shocks in the monetary model with nominal consols. Quite contrasting to the results in the previous model with monetary shocks, the optimal equity shares bias the home countries and are robust to preference parameters.

Table 2.14: Optimal home equity shares held by home households: nominal consols and investment shocks

	$\gamma = 0.50$	$\gamma = 0.85$	$\gamma = 1$	$\gamma = 1.1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$
$\sigma = 0.50$	-4.29	-0.37	0.48	0.76	0.85	0.49	-0.05
$\sigma = 1.00$	-0.21	0.56	0.80	0.87	0.57	-0.04	-0.60
$\sigma = 1.50$	0.25	0.82	0.98	1.02	0.65	-0.09	-0.71
$\sigma = 2.00$	0.42	0.93	1.08	1.12	0.79	0.02	-0.68
$\sigma = 5.00$	0.67	1.11	1.25	1.32	1.30	0.80	-0.10
$\sigma = 10.00$	0.74	1.16	1.31	1.39	1.55	1.31	0.47

Note: This table reports the optimal share of home equities held by home households in a model with TFP shocks, investment efficiency shocks and monetary shocks. σ is the coefficient of relative risk aversion and γ is the trade elasticity. Nominal consols are traded. Each unit of bond asset pays one unit of local currency each period.

2.5.2 Government expenditure shocks

A model with government expenditure shocks

Assume that government consumption composite has a form of,

$$G_t = \left[a_G^{\frac{1}{\gamma_G}} G_{H,t}^{\frac{\gamma_G - 1}{\gamma_G}} + (1 - a_G)^{\frac{1}{\gamma_G}} G_{F,t}^{\frac{\gamma_G - 1}{\gamma_G}} \right]^{\frac{\gamma_G}{\gamma_G - 1}} \tag{2.51}$$

Minimizing the cost of unit government expenditure on goods implies that the price of government consumption composite is as follows,

$$P_{G,t} = \left[a_G P_{H,t}^{1-\gamma_G} + (1 - a_G) P_{F,t}^{1-\gamma_G} \right]^{\frac{1}{1-\gamma_G}} \tag{2.52}$$

The government expenditure on home goods can be written as $G_{H,t} = a_G (P_{H,t}/P_{G,t})^{-\gamma_G} G_t$ and on foreign goods is, $G_{F,t} = (1-a_G)(P_{F,t}/P_{G,t})^{-\gamma_G} G_t$. Government revenues consist of two parts. The first part comes from seigniorage revenues via printing fiat money and the second from tax revenues. Assume that labor income tax rate and profit tax rate to be constant at τ_h and τ_f, respectively. Assume further that no tax occurs on financial assets. Tax revenues through levying on labor services are,

$$\int_0^1 W_{j,t} H_{j,t} \tau_h dj = W_t H_t^d \tau_h \tag{2.53}$$

and profit taxes are $D_t \tau_f$. Assume government balances its budget every period,

$$D_t \tau_f + W_t H_t^d \tau_h + M_t - M_{t-1} = G_t P_{G,t} + Tr_t \tag{2.54}$$

Notice that government expenditures are exogenous, tax revenues are endogenously determined by households and firms. The sum of seigniorage income and tax revenues may not completely cover government expenditure each period. For simplicity, I introduce lump-sum transfers between government and households to balance government budget constraint each period.

The household's budget constraint will now take into account of taxes,

$$\begin{aligned} C_t + q_t \psi_{H,t+1} + s_t q_t^* \psi_{F,t+1} + p_t^b B_{H,t+1} + s_t p_t^{b*} B_{F,t+1} + m_t = \\ (q_t + d_t(1 - \tau_f)) \psi_{H,t} + s_t (q_t^* + d_t^*(1 - \tau_f)) \psi_{F,t} + (p_t^b + r_t^b) B_{H,t} + \\ s_t (p_t^{b*} + r_t^{b*}) B_{F,t} + w_t H_t^d (1 - \tau_h) + \frac{m_{t-1}}{\pi_t} + tr_t \end{aligned} \tag{2.55}$$

After-tax gross returns can then be written as follows,

$$r_{a_1,t+1} \equiv \frac{q_{t+1} + d_{t+1}(1-\tau_f)}{q_t} \qquad r_{a_2,t+1} \equiv \frac{s_{t+1}}{s_t}\frac{q^*_{t+1} + d^*_{t+1}(1-\tau_f)}{q^*_t}$$

$$r_{b_1,t+1} \equiv \frac{p^b_{t+1} + r^b_{t+1}}{p^b_t} \qquad r_{b_2,t+1} \equiv \frac{s_{t+1}}{s_t}\frac{p^{b*}_{t+1} + r^{b*}_{t+1}}{p^{b*}_t}$$

The optimal intratemporal substitution between consumption and leisure yields,

$$-\frac{U_{h,t}}{U_{c,t}} = \frac{w_t(1-\tau_h)}{X_{w,t}} \tag{2.56}$$

which states that marginal rate of substitution between labor supply and consumption equals after-tax wage.

Goods market clearing conditions now state that private consumption, government consumption and investment demands for each good are equal to the supply of each good,

$$C_{H,t} + C^*_{H,t} + I_{H,t} + I^*_{H,t} + G_{H,t} + G^*_{H,t} = Y_{H,t} \tag{2.57}$$

$$C_{F,t} + C^*_{F,t} + I_{F,t} + I^*_{F,t} + G_{F,t} + G^*_{F,t} = Y^*_{F,t} \tag{2.58}$$

Other optimal conditions are the same as in the benchmark model.

Calibration and equity portfolios

The new unknown parameters are γ_G, τ_h, and τ_f. Assume that tax rates on labor income and on profits are the same, $\tau_h = \tau_f$, and they are chosen such that the ratio of government consumption to GDP is 16.4%, which is taken from the data sample. In the calibration, tax rates are set at $\tau_h = \tau_f = 20.1\%$, which lie in the range of tax rates discussed in the literature. Assume further that $\gamma_G = \gamma$ in the calibration.

Government expenditure shocks are estimated from the data sample and evolve as,

$$\begin{bmatrix} log(G_t/G) \\ log(G^*_t/G) \end{bmatrix} = \begin{bmatrix} 0.61 & 0 \\ 0 & 0.61 \end{bmatrix} \begin{bmatrix} log(G_{t-1}/G) \\ log(G^*_{t-1}/G) \end{bmatrix} + \begin{bmatrix} \epsilon_{g,t} \\ \epsilon^*_{g,t} \end{bmatrix}$$

where $\sigma_{\epsilon_g} = \sigma_{\epsilon^*_g} = 2.32\%$, $corr(\epsilon_g, \epsilon^*_g) = 0.60$.

Now let's turn to the numerical experiments. First, consider a model with technological shocks, monetary shocks and government expenditure shocks, in which nominal consols are traded internationally. Table 2.15

shows that introducing government expenditure shocks doesn't produce robust optimal shares of home equities. For instance, at $\sigma = 2$ and $\gamma = 2$, the home household has an extremely large short position in home equities.

How do the optimal shares of home equities hedging human capital income risk vary with shocks happening in an economy? In another experiment, I investigate optimal equity shares in a model with inflation-indexed consols and real shocks (technological and government expenditure shocks). The results (not reported here) show that equity portfolios exhibit super home bias for all combinations of trade elasticity and relative risk aversion. For instance, when $\sigma = \gamma = 2$, home residents hold 184% of home equity and short sell 84% of foreign equities. It seems that even with real bond to hedge real exchange rate risk, home bias in equity still depends on how the components of human capital income orthogonal to real exchange rates (or terms of trade) and returns to equities orthogonal to real exchange rates (or terms of trade) respond to shocks and how large the responses are. Although returns to local equities can still be used to hedge local labor income risk, it is quite hard to conclude that the shares of home equities lie exact between one half and one to all main types of shocks. Put it another way, the puzzle of home bias in equity remains in a real model with technological shocks and government expenditure shocks! Nevertheless, this trouble seems to disappear when I incorporate nominal rigidities and monetary shocks into the model with inflation-indexed bonds.

Table 2.15: Optimal home equity shares held by home households: nominal consols and government expenditure shocks

	$\gamma = 0.50$	$\gamma = 0.85$	$\gamma = 1$	$\gamma = 1.1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$
$\sigma = 0.50$	-12.14	8.53	3.90	2.23	0.26	-0.57	-1.15
$\sigma = 1.00$	-0.54	0.39	1.34	2.37	-8.99	-3.64	-2.60
$\sigma = 1.50$	0.07	0.99	1.86	2.40	28.26	-6.86	-3.39
$\sigma = 2.00$	0.29	1.17	1.81	2.39	12.67	-10.20	-3.89
$\sigma = 5.00$	0.59	1.40	1.93	2.38	4.64	-26.75	-4.99
$\sigma = 10.00$	0.67	1.45	1.95	2.37	5.37	-17.14	-5.36

Note: This table reports the optimal share of home equities held by home households in a model with TFP shocks, investment efficiency shocks and monetary shocks. σ is the coefficient of relative risk aversion and γ is the trade elasticity. Nominal consols are traded. Each unit of bond asset pays one unit of local currency each period.

2.5.3 Preference shocks

Let a preference shift for the home household be $Z_{b,t}$, which enters the household's preference through subjective discount factor β_t (as in Justiniano et al. 2010). The preference of the home representative household now becomes,

$$E_t \sum_{\tau=0}^{+\infty} \left\{ \beta_{t+\tau} Z_{b,t+\tau} U(C_{t+\tau}, H_{t+\tau}, \frac{M_{t+\tau}}{P_{t+\tau}}) \right\}$$

Optimality conditions for portfolios yield,

$$U_{c,t} = E_t \left[v(t) \frac{Z_{b,t+1}}{Z_{b,t}} U_{c,t+1} r_{i,t+1} \right], \text{ with } i = a_1, a_2, b_1, b_2$$

Intertemporal preference shifts alter the household's optimal consumption path. When $Z_{b,t}$ is higher than its expected value at period t, the household is willing to consume more today and consume less in future periods. Observe first that preference shocks (or taste shifts) change the stochastic discount factor and act as demand shocks, which die out in the long run. Second, only temporary shocks have real effects on consumption and output. Justiniano et al. (2010) provide an estimation of preference shocks based on U.S. macroeconomic data and the following process for preference shocks is borrowed from theirs,

$$\begin{bmatrix} log(Z_{b,t}) \\ log(Z^*_{b,t}) \end{bmatrix} = \begin{bmatrix} 0.67^4 & 0 \\ 0 & 0.67^4 \end{bmatrix} \begin{bmatrix} log(Z_{b,t-1}) \\ log(Z^*_{b,t-1}) \end{bmatrix} + \begin{bmatrix} \epsilon_{b,t} \\ \epsilon^*_{b,t} \end{bmatrix}$$

where $\sigma_{\epsilon_b} = \sigma_{\epsilon^*_b} = 0.16\%$, $corr(\epsilon_b, \epsilon^*_b) = 0.20$.

Notice that the volatility of preference shocks is quite small compared to technological and monetary shocks. It's highly possible that in a model with technological, monetary and preference shocks, the first two shocks dominate the last one. Table 2.16 shows the numerical experiments. As in the previous subsection, I consider first a model with inflation-indexed consols and real shocks (technological and preference shocks). The numerical results display that home equity exhibits super home bias, as shown in the case with government spending shocks. Home residents hold 165% of home equities in the benchmark calibration! However, after adding monetary shocks and nominal rigidities into the model, I can get a reasonable and robust home bias in equities.

Table 2.16: Optimal home equity shares held by home households: nominal consols and preference shocks

	$\gamma = 0.50$	$\gamma = 0.85$	$\gamma = 1$	$\gamma = 1.1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$
$\sigma = 0.50$	16.51	3.84	2.64	2.10	0.90	0.22	-0.37
$\sigma = 1.00$	-0.78	-0.27	0.46	2.01	-2.57	-1.78	-1.52
$\sigma = 1.50$	0.05	0.79	1.42	2.12	-139.45	-3.93	-2.20
$\sigma = 2.00$	0.31	1.03	1.57	2.11	12.39	-6.27	-2.65
$\sigma = 5.00$	0.64	1.29	1.72	2.10	5.59	-25.90	-3.74
$\sigma = 10.00$	0.73	1.35	1.75	2.10	4.94	-129.96	-4.23

Note: This table reports the optimal share of home equities held by home households in a model with TFP shocks, preference shocks and monetary shocks. σ is the coefficient of relative risk aversion and γ is the trade elasticity. Nominal consols are traded. Each unit of bond asset pays one unit of local currency each period.

2.5.4 All shocks

In this subsection, I examine how incomplete asset markets affect equity portfolios when all of shocks considered above are included in the model. This is a horse race among different shocks. When nominal bonds are used in the international market, as in other cases discussed above, home equity holdings are quite sensitive to trade elasticity and relative risk aversion. However, the component of equity holdings for hedging labor income risk remains quite stable, varying between -0.27 and -0.23. When inflation-indexed bonds are traded in the international market, robust equity home bias is restored (see table 2.17).

Until now, we might conclude that monetary shocks and inflation matters for nominal bonds hedging real exchange rate risk. As a final remark in this section, I shut down monetary shocks and keep the remaining four types of real shocks and nominal rigidities in the model. The asset markets are still incomplete since there are four assets and eight shocks in the model. The numerical experiment shows that, even with nominal bonds, the model could produce quite robust home bias in equities (table 2.18).

Table 2.17: Optimal home equity shares held by home households: inflation-indexed bonds and all shocks

	$\gamma = 0.50$	$\gamma = 0.85$	$\gamma = 1$	$\gamma = 1.1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$
$\sigma = 0.50$	-0.74	-0.09	0.21	0.38	-0.69	-19.77	-16.00
$\sigma = 1.00$	0.94	0.93	0.93	0.93	0.91	0.89	0.83
$\sigma = 1.50$	0.95	0.92	0.91	0.90	0.87	0.84	0.78
$\sigma = 2.00$	0.95	0.91	0.89	0.88	0.85	0.81	0.75
$\sigma = 5.00$	0.93	0.88	0.86	0.85	0.81	0.77	0.71
$\sigma = 10.00$	0.92	0.87	0.85	0.84	0.80	0.75	0.69

Note: This table reports the optimal share of home equities held by home households in a model with TFP shocks, monetary shocks, investment efficiency shocks, government expenditure shocks and preference shocks. Inflation-indexed consol derivatives are used. The model include all shocks consider in the main text. σ is the coefficient of relative risk aversion and γ is the trade elasticity.

Table 2.18: Optimal home equity shares held by home households: nominal consols and real shocks

	$\gamma = 0.50$	$\gamma = 0.85$	$\gamma = 1$	$\gamma = 1.1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$
$\sigma = 0.50$	1.09	0.94	0.91	0.80	0.72	0.60	0.42
$\sigma = 1.00$	-0.25	0.63	0.86	0.80	-0.51	-1.33	-1.57
$\sigma = 1.50$	0.82	0.97	0.70	0.73	0.73	0.71	0.68
$\sigma = 2.00$	0.86	0.17	0.71	0.71	0.69	0.65	0.59
$\sigma = 5.00$	0.91	0.66	0.69	0.68	0.64	0.60	0.53
$\sigma = 10.00$	0.95	0.67	0.68	0.67	0.63	0.58	0.52

Note: This table reports the optimal share of home equities by home households in a model with TFP shocks, investment efficiency shocks, government expenditure shocks and preference shocks. Nominal consols are used here. The model include all real shocks consider in the main text. Parameters are the same as cases with incomplete markets. σ is the coefficient of relative risk aversion and γ is the trade elasticity.

2.6 International portfolio dynamics

2.6.1 Numerical solution method

Building on Devereux and Sutherland (2010a), a model has to be taken a third-order approximation around its steady state in order to find the first-order accurate equilibrium portfolios. The policy functions for equilibrium portfolios thus can be written as a function of state variables as usual, $\psi_{j,t} = \psi_j + \varrho_j \cdot \hat{\kappa}_t$, $B_{j,t} = B_j + \varrho_j \cdot \hat{\kappa}_t$, with $j = H$, or F. $\hat{\kappa}_t$ denotes a vector for the percentage deviation of state variables, which are defined in section 2.1.6, and ϱ_j is the corresponding coefficient vector in the policy function.[36] The coefficient vector ϱ_j can be obtained using a third-order approximation of the household's consumption Euler equations and a second-order approximation of the rest of equilibrium equations in the model. Here, I use Schmitt-Grohe and Uribe (2004) algorithm to calculate the second-order approximation and rewrite the formula for equilibrium portfolios as in Devereux and Sutherland (2010a) such that Schmitt-Grohe and Uribe's method can be applied here directly. The Technical Appendix provides steps in detail on calculating first-order components of foreign asset dynamics.

2.6.2 Measuring international portfolios

Before turning to the quantitative properties of the empirical and theoretical economy, I start with summarizing international financial variables used in this chapter. Net foreign equity assets at the end of period t are measured as $(\psi_{H,t+1} - 1)q_t + \psi_{F,t+1} q_t^* s_t$ and net foreign bond assets can be written as $B_{H,t+1} p_t^b + B_{F,t+1} p_t^{b*} s_t$. The sum of net foreign equity assets and net foreign bond assets are net foreign assets. Substituting net foreign assets, dividends, consumption and investment demands into the home household's budget constraint, yields,

$$\begin{aligned} NFA_{t+1} &= NX_t + NFI_t + \text{Value-of-Asset}_t \\ &= CA_t + NFA_t + VAL_t \end{aligned} \tag{2.59}$$

where NFA_{t+1} denotes net foreign asset positions at the end of period t. Net export (trade balance), NX_t, is exports net imports, and net factor income from abroad, NFI_t, is the factor income paid by foreigners (dividend and interest payment inflows, here) minus factor income paid to foreigners (dividend and interest payment outflows). From the textbook definition of

[36] Including newly defined variables besides the original variables in the state vector: net foreign asset positions at the end of period in both countries.

current account, we know current account is given by $CA_t = NX_t + NFI_t$. Value-of-Asset$_t$ represents the period t value of external assets held at the end of period $t-1$, which includes the net foreign asset positions at the end of period $t-1$, NFA_t, and wealth effect in period t, VAL_t.

Net equity purchases are the changes of equity portfolios, $(\psi_{H,t+1} - \psi_{H,t})q_t + (\psi_{F,t+1} - \psi_{F,t})q_t^* s_t$, which are equity asset outflows minus equity asset inflows. Net bond purchases are analogously defined as $(B_{H,t+1} - B_{H,t})p_t^b + (B_{F,t+1} - B_{F,t})p_t^{b*} s_t$, which are bond asset outflows minus bond asset inflows. Changes of net foreign asset positions are given by $\Delta NFA_{t+1} \equiv NFA_{t+1} - NFA_t$. Up to a first-order approximation, current account is zero.[37] Taking a first-order approximation around the steady state, changes of net foreign asset positions can be written as,

$$\begin{aligned} \Delta NFA_{t+1} &= (\psi_H - 1)q\Delta\hat{q}_t + \psi_F q^*(\Delta\hat{q}_t^* + \Delta\hat{s}_t) + B_H p^b \Delta\hat{p}_t^b \\ &\quad + B_F p^{b*}(\Delta\hat{p}_t^{b*} + \Delta\hat{s}_t) + \mathcal{O}(\| \epsilon \|) \\ &\equiv VAL_t + \mathcal{O}(\| \epsilon \|) \end{aligned} \tag{2.60}$$

where $\Delta x_{t+1} \equiv x_{t+1} - x_t$. Changes of net foreign assets mainly come from wealth effects, VAL_t. Changes of net equity assets are the sum of net equity purchases and wealth effects from equity assets. Similarly, changes of net bond assets are equal to net bond purchase plus wealth effects from bond assets.

[37] See also the technical appendix in Coeurdacier et al. (2010).

Table 2.19: Second moments (1)

			NSP	NSP	NSP	SP
Variables	Data (US)	Data (Mean)	Money and TFP	TFP	Money	Money and TFP
Home equity share (%)	82	77(0.17)	84	84	84	87
Std. relative to GDP						
Investment	2.85	2.98(0.25)	2.80	1.87	3.41	2.75
Consumption	0.78	0.87(0.31)	0.50	0.65	0.36	0.54
Employment	0.68	0.70(0.46)	1.13	0.30	1.52	1.12
Real exchange rate	3.58	3.49(0.54)	0.66	0.78	0.55	0.61
Net foreign assets	0.96	1.34(0.30)	1.55	2.67	0.54	1.28
Δ(Net foreign assets)	0.98	1.24(0.32)	0.73	1.15	0.41	0.64
Δ(Net foreign equity assets)	0.90	1.10(0.39)	1.47	1.03	2.09	1.74
Δ(Net foreign bond assets)	0.45	0.83(0.60)	1.16	1.07	1.83	1.32
Series corr.						
GDP	0.54	0.56(0.24)	0.55	0.90	0.26	0.56
Investment	0.71	0.64(0.17)	0.30	0.86	0.16	0.30
Consumption	0.62	0.63(0.16)	0.81	0.93	0.54	0.84
Employment	0.55	0.63(0.18)	0.19	0.79	0.17	0.20
Real exchange rate	0.73	0.64(0.16)	0.71	0.93	0.39	0.67
Net foreign assets	0.44	0.41(0.49)	0.87	0.89	0.70	0.86
Δ(Net foreign assets)	-0.03	-0.07(1.70)	-0.02	0.02	-0.12	-0.02
Δ(Net foreign equity assets)	-0.03	-0.04(3.41)	-0.06	-0.07	-0.30	-0.10
Δ(Net foreign bond assets)	0.19	0.06(1.98)	0.13	0.45	-0.22	-0.04

Table 2.20: Second moments (1) (continued)

			NSP	NSP	NSP	SP
Variables	Data (US)	Data (Mean)	Money and TFP	TFP	Money	Money and TFP
Cross corr. with home GDP						
Investment	0.85	0.88(0.08)	0.93	0.95	0.97	0.92
Consumption	0.89	0.80(0.13)	0.87	0.90	0.90	0.87
Employment	0.89	0.73(0.36)	0.68	-0.32	0.97	0.63
Real exchange rate	0.07	0.04(8.05)	0.60	0.58	0.59	0.57
Net foreign assets	-0.20	-0.06(4.42)	-0.29	-0.61	-0.47	-0.44
Δ(Net foreign assets)	-0.27	-0.20(0.62)	-0.29	-0.17	-0.53	-0.31
Δ(Net foreign equity assets)	-0.21	-0.02(11.27)	0.16	0.02	-0.13	0.05
Δ(Net foreign bond assets)	-0.17	-0.26(0.58)	-0.39	-0.19	0.03	-0.21
Cross country corr.						
GDP	0.58	0.28(1.21)	0.23	0.19	0.28	0.30
Investment	0.20	0.19(1.57)	0.40	0.30	0.43	0.41
Consumption	0.46	0.11(3.43)	0.76	0.80	0.69	0.82
Employment	0.53	0.24(1.07)	0.13	0.57	0.11	0.13
Return on equity assets	0.53	0.44(0.30)	0.53	0.46	0.54	0.52
Return on bond assets	0.53	0.56(0.14)	0.53	0.73	0.52	0.52

Note: The column labelled *Data (US)* reports the data moments for the US and *Data (Mean)* reports the weighted average of data moments in the data sample. Numbers in parentheses in column *Data (Mean)* denote coefficient of variation. *NSP* stands for a model without TFP spillover across borders and *SP* represents a model with TFP spillover. The column labelled *Money and TFP* displays simulated moments in the model with TFP shocks and money supply shocks. The column labelled *TFP* reports model moments for the model with TFP shocks only and *money* for the model with money supply shocks only.

Table 2.21: Second moments (2)

			NSP	SP	SP
Variables	Data (US)	Data (Mean)	Taylor rule	Investment	Gov. expense
Home equity share (%)	82	77(0.17)	87	88	83
Std. relative to GDP					
Investment	2.85	2.98(0.25)	2.44	3.05	2.89
Consumption	0.78	0.87(0.31)	0.58	0.54	0.63
Employment	0.68	0.70(0.46)	0.93	1.13	1.17
Real exchange rate	3.58	3.49(0.54)	0.70	0.62	0.65
Net foreign assets	0.96	1.34(0.30)	2.19	1.38	1.39
Δ(Net foreign assets)	0.98	1.24(0.32)	0.84	0.67	0.69
Δ(Net foreign equity assets)	0.90	1.10(0.39)	0.73	1.52	0.86
Δ(Net foreign bond assets)	0.45	0.83(0.60)	1.36	1.29	0.62
Series corr.					
GDP	0.54	0.56(0.24)	0.69	0.56	0.55
Investment	0.71	0.64(0.17)	0.38	0.35	0.30
Consumption	0.62	0.63(0.16)	0.90	0.84	0.85
Employment	0.55	0.63(0.18)	0.19	0.20	0.22
Real exchange rate	0.73	0.64(0.16)	0.80	0.69	0.68
Net foreign assets	0.44	0.41(0.49)	0.92	0.87	0.86
Δ(Net foreign assets)	-0.03	-0.07(-1.70)	-0.01	0.01	-0.02
Δ(Net foreign equity assets)	-0.03	-0.04(-3.41)	-0.04	-0.16	-0.14
Δ(Net foreign bond assets)	0.19	0.06(1.98)	-0.04	-0.12	0.48

Table 2.22: Second moments (2) (continued)

			NSP	SP	SP
Variables	Data (US)	Data (Mean)	Taylor rule	Investment	Gov. expense
Cross corr. with home GDP					
Investment	0.85	0.88(0.08)	0.91	0.88	0.92
Consumption	0.89	0.80(0.13)	0.88	0.83	0.84
Employment	0.89	0.73(0.36)	0.46	0.63	0.59
Real exchange rate	0.07	0.04(8.05)	0.61	0.54	-0.20
Net foreign assets	-0.20	-0.06(-4.42)	-0.52	-0.44	-0.44
Δ(Net foreign assets)	-0.27	-0.20(-0.62)	-0.15	-0.34	-0.32
Δ(Net foreign equity assets)	-0.21	-0.02(-11.27)	-0.10	-0.12	-0.07
Δ(Net foreign bond assets)	-0.17	-0.26(-0.58)	0.11	0.00	-0.26
Cross country corr.					
GDP	0.58	0.28(1.21)	0.37	0.30	0.27
Investment	0.20	0.19(1.57)	0.61	0.29	0.41
Consumption	0.46	0.11(3.43)	0.84	0.81	0.84
Employment	0.53	0.24(1.07)	0.49	0.16	0.14
Return on equity assets	0.53	0.44(0.30)	0.69	0.53	0.53
Return on bond assets	0.53	0.56(0.14)	0.59	0.54	0.54

Note: The column labelled *Data (US)* reports the data moments for the US and *Data (Mean)* reports the weighted average of data moments in the data sample. Numbers in parentheses in column *Data (Mean)* denote coefficient of variation. *NSP* stands for a model without TFP spillover across borders and *SP* represents a model with TFP spillover. The column labelled *Taylor rule* displays simulated moments in the model with TFP shocks and interest rate shocks. The column labelled *Investment* reports model moments for the model with TFP shocks, investment efficiency shocks and money supply shocks and *Gov. expense* for the model with TFP shocks, government expenditure shocks and money supply shocks.

2.6.3 Data moments v.s. model moments

First of all, I review some properties of international business cycles in major developed economies in my data set. These properties refer to the second moments of yearly time series detrended with Hodrik-Prescott filter and to cross correlations between these series. The first data columns labelled "Data(US)" in table 2.19-2.20 and table 2.21-2.22 show the (relative) second moments and cross correlations for U.S. economy (the home country) and for the *world* economy which is aggregated from 9 other countries in the data using bilateral trade volume as weights. The next data columns labeled "Data(Mean)" in these two tables report the output-weighted average of second moments and the average of cross correlations when each country in the sample is treated as the home country and the rest of 9 countries as the foreign country. Net foreign assets, changes of net foreign assets and changes of net foreign equity/bond assets are normalized by national GDP. GDP, investment, consumption are in real terms and logged. Real exchange rates are logged as well. Finally, all variables are detrended by Hodrick-Prescott filter (the smoothing parameter is 100). Instead of reporting the actual standard deviations, I list the standard deviations of variables relative to that of GDP and I also use the relative moments in numerical experiments. The data shows that volatilities of net foreign assets, net foreign equity assets and net foreign bond assets are of the same order as output both in levels and first differences. Net foreign assets counter-cyclically move with output.

Now we turn to the simulated second moments of the model. The column labeled "Money and TFP" displays the results for a model with inflation-indexed bonds and exogenous money supply shocks. The following three columns present results for the model with TFP shocks only, monetary supply shocks only and a model with technology spill-over, respectively.[38] All of these columns show that net foreign assets generate the same volatilities as output and that they have high persistence and are counter-cyclical.

How does the model generate counter-cyclical movement of net foreign asset positions? Recall that the change of NFA is equal to the wealth effect (see equation (2.60)). In the steady state of a symmetric equilibrium, all real variables are the same across countries, $q = q^*$, $p^b = p^{b*}$, and the share of home equity held by home consumer is identical to the share of foreign equity held by foreigners, that is, $\psi_H + \psi_F = 1$ and analogously, $B_H + B_F = 0$.

[38] When the number of shocks is less than the number of assets, at least some of assets are indeterminate in equilibrium. In the computation, I force the variance of one type shock close to zero such that I can calculate the steady state portfolios.

Changes of net foreign assets can be rewritten as,

$$\begin{aligned}\Delta NFA_{t+1} =& \psi_F q^* \left(\Delta \hat{q_t^*} + \Delta \hat{s}_t - \Delta \hat{q}_t\right) + B_H p^b \left(\Delta \hat{p_t^b} - \Delta \hat{p_t^{b*}} - \Delta \hat{s}_t\right) \\ &+ \mathcal{O}\left(\| \epsilon \|\right)\end{aligned} \tag{2.61}$$

Consider now a positive TFP shock happening at the home country. Outputs on impact and in future periods arise because of higher technological level on impact and higher investment in the following periods, which causes an increase of real dividends in the following periods and a rise of real interest payments because of expected deflation in the near future at home. Higher expected dividends and interest payments push up prices of home equities and home bonds.[39] What happens in the foreign county is quite different from the home country. Investment at the foreign country doesn't change much, and consequently, the price of foreign equities only goes up a little bit. However, the foreign real interest rate rises because of deflation spilled over from the home country, which implies foreign bond price has a similar increment as home bond price. On the other hand, home real exchange rates depreciate in response to the positive TFP shock at home. Thereby, the relative price of foreign bonds $\frac{p_t^{b*} s_t}{p_t^b}$ increases. Relative price of foreign equities $\frac{q_t^* s_t}{q_t}$ depends on the relative increments in real exchange rates and in home equity price. In the model studied here, the latter has a larger increment than the former, and then the relative price of home equities drops on impact. In the baseline calibration, home residents have a long position in home bond ($B_H > 0$) and $0 < \psi_F < 1$, which implies the total effect of a technological shock on net foreign assets is negative. Similar intuition holds for other types of shocks. Table 2.21 shows that either in a model with complete asset markets or with incomplete asset markets, net foreign asset positions counter-cyclically move with output.

Besides portfolio dynamics, I also produce the second moments for other aggregate variables. The benchmark model with complete asset markets delivers similar second moments as in Chari et al. (2002). However, both of theirs and mine suffer common weaknesses. That is, cross country consumption correlation is two high compared to the cross correlation between output. Volatility of real exchange rates is much lower than that in data. Nevertheless, cross country correlations for investment and employment in

[39] Since households smooth their consumption paths, stochastic discount factors in both countries don't move around much. Thereby, asset prices are mainly pinned down by dividend flows or interest payment flows in future periods.

the current chapter are consistent with data both in a model with complete asset markets or incomplete asset markets.

Chapter 3

International Financial Markets and International Risk Sharing

How does financial integration increase international risk sharing? Are all participating countries better off in the process of financial globalization? I'll investigate this issue based on a center-periphery framework in this chapter. This chapter is organized as follows. Section 3.1 describes a simple one-good world economy and characterizes the optimal portfolios for each country in each stage of financial integration. Section 3.2 extends our results to the case of many goods. Final remarks are in chapter 5.

3.1 A one-good economy

The world economy consists of three countries, country A, B, and C. Country A is the central country, which has a continuum of population with $n_A = \frac{1}{2}$ measure, and the index for population lies in $[0, \frac{1}{2})$. Country B and C are peripheral countries and they have equal population size of $n_B = n_C = \frac{1}{4}$. The index of population in country B has a range of $[\frac{1}{2}, \frac{3}{4})$ and the population in country B lies in $[\frac{3}{4}, 1]$. There is an infinitely lived representative agent in each country. In order to illustrate how portfolio choices affect international risk sharing, I first consider a world economy with only one tradable good. In other words, this section focuses on the aspect of common discount factor in international asset pricing and doesn't take into account of the effect of terms of trade on portfolios and international risk-sharing. I'll leave this part to the third section. Each country can issue equities which entitle the claimant a corresponding share of profits in that country. Whether equities can be traded across borders depends on the prevailing international financial architecture. I'll describe it later. The household in each country receives an endowment and chooses consumption and a portfolio. There are no transaction cost for international trade

in goods and assets. The uncertainty in this world economy comes from endowment shocks in these three countries.

There are three types of financial architecture in the world economy, which correspond to three stages of international financial integration. The first stage is financial autarky, in which only goods and services can be traded across borders. Trade balance is maintained period by period and there is no international capital flows. With the development of international financial markets, assets can be gradually traded across countries, but not for all countries. In the current financial architecture, capital flows between developing economies and developed economies are far larger than those among developing economies. Brazil, Argentina, Indonesia, Malaysia and Thailand are good examples of peripheral countries in this type of financial architecture. The United States and other regional countries, say, the Britain and Singapore are examples of the core country. I call this case as partial financial integration, in which capital can flow freely between the central and peripheral countries, but capital flows are restricted between peripheral countries. The last case is full financial integration, in which all countries can have access to domestic and all foreign financial markets. In this model economy, there are three types shocks, one endowment shock in each country. Full financial integration implies that asset markets are complete up to a first-order approximation.

3.1.1 Households

Households in country i have a life-time utility function of the form,

$$E_t \left[\sum_{\tau=t}^{\infty} \beta_{i,\tau} U(C_{i,\tau}) \right] \tag{3.1}$$

where the operator E_t here represents the conditional expectation over all states of nature, $C_{i,\tau}$ is the consumption in country $i = A, B, C$ in period τ. Following the open macroeconomic literature, i.e. Schmitt-Grohe and Uribe (2003), I take use of an endogenous discount factor to make sure that the rational expectations equilibrium exists,

$$\beta_{i,t+1} = \beta_{i,t} v(i,t), \text{ with } \beta_{i,0} = 1,$$

where $v(i,t) \equiv \zeta_0 \bar{C}_{i,t}^{-\zeta}$, $0 < \zeta < 1$. ζ_0 is a parameter which is chosen such that the discount factor at the steady state is equal to the inverse of risk-free rate. $\bar{C}_{i,t}$ is the per capita consumption. When they made decisions, households

treat per-capita consumption as given. The household's preference has a form of power utility,

$$U(C_{i,t}) = \frac{(C_{i,t})^{1-\sigma} - 1}{1-\sigma} \tag{3.2}$$

where σ is the coefficient of relative risk aversion or the inverse of elasticity of intertemporal substitution.

3.1.2 International financial markets

With development of financial markets, asset markets in different countries gradually integrate into one another. Without loss of generality, I consider three stages of international financial market development. The first one is financial autarky, in which there are no trade in assets across countries. the second stage is that financial markets are partially integrated across borders, in which central country A has access to all financial markets in the world while peripheral countries can only hold domestic assets and assets in the core country (see figure 3.1). The last stage is the fully financially integrated world, in which each country has access to all financial markets in the world. Suppose that each country provides one type of asset, equity. Each equity share in country i gives the claimant a corresponding share of profits in that country. For tractability, assume that equity assets have a duration of one-period. Let the price of equity be $Q_{i,t}$ in country i, and the profits are endowments $Y_{i,t}$ in country i. The gross return on equities in country i can be written as,

$$R_{i,t} = \frac{Y_{i,t}}{Q_{i,t}} \tag{3.3}$$

Let $X^j_{i,t}$, $i, j = A, B, C$, be the share of equities issued by country j and held by country i at the end of period $t-1$. I normalize the total share of equities in each country to be unity. The net foreign asset positions, $NW_{i,t+1}$, then can be written as the sum of holdings of foreign equities by the home household net home equities held by foreigners,

$$NW_{A,t+1} \equiv \left(X^A_{A,t+1} - 1\right) Q_{A,t} + X^B_{A,t+1} Q_{B,t} + X^C_{A,t+1} Q_{C,t} \tag{3.4}$$

$$NW_{i,t+1} \equiv \left(X^i_{i,t+1} - 1\right) Q_{i,t} + X^A_{i,t+1} Q_{A,t} + X^j_{i,t+1} Q_{j,t} \tag{3.5}$$

with $i, j = B, C$ and $i \neq j$.

The aggregate resource constraint faced by the central country has the

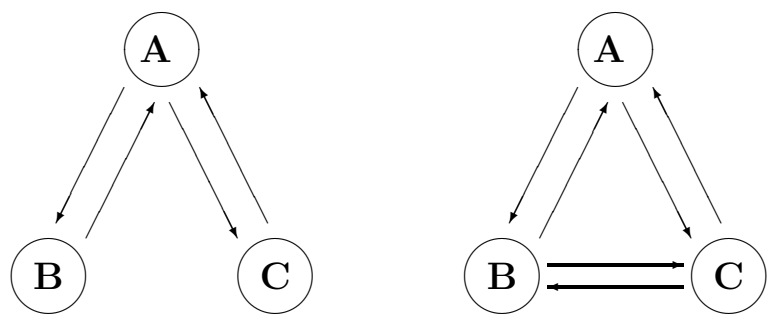

Figure 3.1: Financial architecture. The left panel is for partial financial integration. The right panel is for global financial integration. A denotes for the central country. B and C stand for peripheral countries.

following form,

$$C_{A,t} + NW_{A,t+1} = Y_{A,t} + NW_{A,t}R_{A,t} + \alpha'_{A,t}R_{x,t} \tag{3.6}$$

The market value of equities in peripheral country B and C held by the central country is, $\alpha_{A,t} \equiv \left[X^B_{A,t}Q_{B,t}; X^C_{A,t}Q_{C,t}\right]$ and excess returns $R_{x,t} \equiv [R_{B,t} - R_{A,t}; R_{C,t} - R_{A,t}]$. Total expenditures on the left hand side consist of consumption expenditure and a portfolio taken to the next period. The right hand side contains resources of total income, which include profits, $Y_{A,t}$, and gross returns on net foreign assets characterized by the last two terms in the equation above. Notice that the household's budget constraint in country A can be obtained through dividing the population size on both sides of equation (3.6). I focus on aggregate resource constraint instead.

Peripheral countries face similar resource constraints. Total income from output and returns on equity assets equals total expenses,

$$C_{i,t} + NW_{i,t+1} = Y_{i,t} + NW_{i,t}R_{A,t} + \alpha'_{i,t}R_{x,t} \tag{3.7}$$

with $i = B, C$. The market value of equity holdings in peripheral country B is $\alpha_{B,t} \equiv \left[(X^B_{B,t} - 1)Q_{B,t}; X^C_{B,t}Q_{C,t}\right]$ and it is $\alpha_{C,t} \equiv \left[X^B_{C,t}Q_{B,t}; (X^C_{C,t} - 1)Q_{C,t}\right]$ in country C.

I will focus on three stages of financial market development. In the first stage, financial markets are autarky, and the household in each country only hold his own equities, $X^i_i = 1$, and $X^j_i = 0$, with $i, j = A, B, C$ and $i \neq j$. In the second stage, the household in the central country has

access to global financial markets and can hold any types of assets in the world. The financial market in the central country is also available to households in peripheral countries. Nevertheless, the household in one peripheral country is completely excluded from the other peripheral country because the under-development of financial markets in peripheral countries. For instance, capital account regulation in developing economies prevents foreigners from investing in the domestic market, or contract enforcement problems are so prevailing, or information cost is so high that no foreigners would like to invest in these countries. An extreme case considered here is that a peripheral country is completely excluded from the other peripheral country, $X_i^j = 0$, $i, j = B, C$ and $i \neq j$. In the third case, financial markets are globally integrated, households in any country can participate trading assets in any financial market.

In the financial autarky case, consumption equals output in each period and each country, $C_{i,t} = Y_{i,t}$ and $X_i^i = 1$, $i = A, B, C$. When households can choose foreign equities, the optimality conditions for portfolios are characterized by the consumption Euler equations in each country. I list these conditions in two separate sets. When the financial architecture is in stage two, only the household in the core country can choose three types of equities and peripheral countries can have access to domestic and the core country's equities. The optimality conditions for holding equities follow as,

$$U_{C,t}^A = E_t \left[v(A,t) U_{C,t+1}^A R_{j,t+1} \right] \text{ , with } j = A, B, C \tag{3.8}$$

$$U_{C,t}^B = E_t \left[v(B,t) U_{C,t+1}^B R_{j,t+1} \right] \text{ , with } j = A, B \tag{3.9}$$

$$U_{C,t}^C = E_t \left[v(C,t) U_{C,t+1}^C R_{j,t+1} \right] \text{ , with } j = A, C \tag{3.10}$$

where $U_{C,t}^i$ denotes marginal utility of consumption in country i in period t. These conditions state that the forgone marginal utility of one dollar today equals the expected discounted marginal utility tomorrow from gross returns to such one foregone dollar.

When the world economy is in global financial integration of stage three, households in each country can trade assets with the other two countries. The optimality conditions for portfolios read,

$$U_{C,t}^i = E_t \left[v(i,t) U_{C,t+1}^i R_{j,t+1} \right] \text{ , with } i, j = A, B, C \tag{3.11}$$

Endowments in each country follow an auto-regressive process of order one,

$$\log Y_{i,t+1} = (1-\rho) \log Y_i + \rho \log Y_{i,t} + \epsilon_{i,t+1} \tag{3.12}$$

where Y_i stands for the steady state endowments in country i and $0 < \rho < 1$ is the persistence of endowment shocks. The innovation $\epsilon_{i,t+1}$ has an $i.i.d$ normal distribution with mean zero and standard deviation σ_ϵ. For tractability of the solution, I assume that innovations across borders are uncorrelated.

3.1.3 Competitive equilibrium

The asset market clearing condition requires that the sum of equity shares issued by each country is unity,

$$X^j_{A,t} + X^j_{B,t} + X^j_{C,t} = 1 \text{ , with } j = A, B, C \tag{3.13}$$

Goods market clearing condition implies that total absorption of goods is equal to total endowments,

$$C_{A,t} + C_{B,t} + C_{C,t} = Y_{A,t} + Y_{B,t} + Y_{C,t} \tag{3.14}$$

Notice also that the global net foreign asset positions should be zero, $NW_{A,t} + NW_{B,t} + NW_{C,t} = 0$ and $\alpha_{A,t} + \alpha_{B,t} + \alpha_{C,t} = 0$.

Now I have described the world economy. The rational expectations competitive equilibrium is the defined as follows,

Definition 1. *A competitive equilibrium consists of a sequence of asset prices $Q_{i,t}$, $i = A, B, C$, $t = 0, 1, 2, \cdots$ and a sequence of allocations $C_{i,t}$ and of portfolios $X^j_{i,t}$, $i, j = A, B, C$, $t = 0, 1, 2, \cdots$, such that (a) given prices, consumption and portfolio decisions by the household in each country solve the corresponding household's problem, and (b) prices clear asset markets in each country.*

In the deterministic steady state, endowments in each country are constant. Let the steady state subjective discount factor be $0 < \beta \equiv v(i, \cdot) < 1$, $i = A, B, C$. Gross returns to assets equal $R = \frac{1}{\beta}$. I consider a symmetric steady state in which consumption per capita are the same in all countries. Therefore, net foreign asset positions are equal to zero in the steady state and consumption equals output.

3.1.4 Solving the model

As shown in Devereux and Sutherland (2011a), I need to take a second-order approximation to Euler equations (3.8)-(3.10) and a first-order approximation to other equilibrium conditions to obtain a solution for the steady

state portfolio since certainty equivalence implies that portfolio is indeterminate in a linearized model. In order to pin down the steady state portfolio, I need first to solve a linearized version of the rational expectations equilibrium and then substitute the corresponding terms into the second-order approximation to the Euler equations to obtain the steady state portfolios. After obtaining the portfolio, I substitute them back into the solution to the rational expectations equilibrium to get the final solution to the linearized dynamic system. Notice that up to a first-order approximation, expected excess returns on equities (say, the return of equity A relative to the return of equity B) are zero. Thereby, the linearized version of the dynamic system in partial financial integration is exactly the same as global financial integration. The only difference between these two financial regimes lies in the second-order approximation to consumption Euler equations.

The linearized system

Let a variable with a hat denote the log deviation from its deterministic steady state. Since population in each country is fixed, therefore, the log deviation for an aggregate variable is the same as a per capita variable. First, taking a log-linearization to resource constraints (3.6)-(3.7), the relative resource constraints read,

$$\begin{aligned}\frac{3}{4}\widetilde{NW}_{B,t+1}+\frac{1}{4}\widetilde{NW}_{C,t+1}=&\frac{3}{4\beta}\widetilde{NW}_{B,t}+\frac{1}{4\beta}\widetilde{NW}_{C,t}+\frac{3}{4}\widetilde{\alpha}'_B\widehat{R}_{x,t}+\frac{1}{4}\widetilde{\alpha}'_C\widehat{R}_{x,t}\\&-\frac{1}{2}\Delta\widehat{Y}_{B,t}+\frac{1}{2}\Delta\widehat{C}_{B,t}\end{aligned}\tag{3.15}$$

$$\begin{aligned}\frac{1}{4}\widetilde{NW}_{B,t+1}+\frac{3}{4}\widetilde{NW}_{C,t+1}=&\frac{1}{4\beta}\widetilde{NW}_{B,t}+\frac{3}{4\beta}\widetilde{NW}_{C,t}+\frac{1}{4}\widetilde{\alpha}'_B\widehat{R}_{x,t}+\frac{3}{4}\widetilde{\alpha}'_C\widehat{R}_{x,t}\\&-\frac{1}{2}\Delta\widehat{Y}_{C,t}+\frac{1}{2}\Delta\widehat{C}_{C,t}\end{aligned}\tag{3.16}$$

where $\widetilde{NW}_{B,t}\equiv\frac{4NW_{B,t}}{\widetilde{C}}$ and $\widetilde{NW}_{C,t}\equiv\frac{4NW_{C,t}}{\widetilde{C}}$ are the ratios of net foreign assets to total output in country B and C, respectively. Notice that net foreign asset positions are zero at the steady state, and therefore, $\widetilde{NW}_{i,t}$, $i=B,C$ is a first-order term. $\widetilde{\alpha}_i\equiv\frac{4\bar{\alpha}_i}{\beta\widetilde{C}}$ with $i=B,C$ denotes the ratio of the steady state net equity holdings to total output in peripheral country i. $\Delta\widehat{C}_{i,t}\equiv\widehat{C}_{A,t}-\widehat{C}_{i,t}$ expresses the consumption of central country A relative to the consumption in peripheral country i, and similarly, the relative

endowment is given by $\Delta\widehat{Y}_{i,t} \equiv \widehat{Y}_{A,t} - \widehat{Y}_{i,t}$.

Taking a first-order approximation to the Euler equations (3.8)-(3.10), subtracting (3.9) from (3.8) and subtracting (3.10) from (3.8) respectively, yields,

$$(\sigma - \zeta)\Delta\widehat{C}_{B,t} = E_t\left[\sigma\Delta\widehat{C}_{B,t+1}\right] \tag{3.17}$$

$$(\sigma - \zeta)\Delta\widehat{C}_{C,t} = E_t\left[\sigma\Delta\widehat{C}_{C,t+1}\right] \tag{3.18}$$

Four equations (3.15)-(3.18) form a dynamic system with two endogenous state variables $\widetilde{NW}_{B,t+1}$, $\widetilde{NW}_{C,t+1}$, and two endogenous control variables, $\Delta\widehat{C}_{B,t}$, $\Delta\widehat{C}_{C,t}$. Following Blanchard and Kahn (1980) and Devereux and Sutherland (2011a), the appendix provides steps for the solution in detail. I then substitute the solution to consumption, asset prices and excess returns into the second-order approximation to consumption Euler equations and obtain the steady state portfolios. In the second stage with partial financial integration, the second-order approximations to Euler equations (3.8)-(3.10) read,

$$E_t\left\{\begin{bmatrix} \sigma\Delta\widehat{C}_{B,t+1} & 0 \\ 0 & \sigma\Delta\widehat{C}_{C,t+1} \end{bmatrix}\widehat{R}_{x,t+1}\right\} = 0$$

In the third stage with global financial integration, the second-order approximations to Euler equations (3.11) yield,

$$E_t\left\{\sigma\Delta\widehat{C}_{B,t+1}\widehat{R}_{x,t+1}\right\} = 0$$

$$E_t\left\{\sigma\Delta\widehat{C}_{C,t+1}\widehat{R}_{x,t+1}\right\} = 0$$

Observe that there are two restrictions on portfolio choices in partial financial integration while there are four restrictions on portfolio choices in global financial integration. Combining with three asset market clearing conditions (3.13) and two net foreign asset positions, the portfolio in each country can be completely pinned down.

Results

Table 3.1 and table 3.2 reports the steady state equity holdings by different countries in partial financial integration and global financial integration, respectively. In global financial integration, households achieve effective risk-sharing through fully diversifying portfolios in international financial markets. For instance, the core country A holds $\frac{1}{2}\frac{1}{1-\beta\rho}$ shares of equities

issued by country B and C, respectively, and it holds $1-\frac{1}{2}\frac{1}{1-\beta\rho}$ shares of domestic equities. Peripheral country B holds $\frac{1}{4}\frac{1}{1-\beta\rho}$ shares of equities of country A, $1-\frac{3}{4}\frac{1}{1-\beta\rho}$ shares of domestic equities and $\frac{1}{4}\frac{1}{1-\beta\rho}$ shares of equities of country C. When international financial markets are partially integrated, peripheral countries don't have the chance to fully diversify their income risk. The only channel of improving risk sharing is to hold equities issued by the core country. Table 3.1 shows that peripheral country B or C chooses to purchase $\frac{2}{7}\frac{1}{1-\beta\rho}$ shares of equities in the core country, which are higher than those in full financial integration. The core country would hold more equities in the peripheral countries in partial financial globalization than those in global financial integration. I summarize the solution for portfolios in the following proposition.

Proposition 1. *In a one-good endowment world economy with population size $n_A=\frac{1}{2}$, $n_B=\frac{1}{4}$ and $n_C=\frac{1}{4}$, the steady state portfolios don't depend on the coefficient of relative risk aversion. Moreover, peripheral countries hold more equities issued by the core country in partial financial integration than those in global financial integration.*

Table 3.1: Optimal equity shares held by each country in partial financial integration ($n_A=\frac{1}{2}$, $n_B=n_C=\frac{1}{4}$)

	Equity A	Equity B	Equity C
Country A	$1-\frac{4}{7}\frac{1}{1-\beta\rho}$	$\frac{4}{7}\frac{1}{1-\beta\rho}$	$\frac{4}{7}\frac{1}{1-\beta\rho}$
Country B	$\frac{2}{7}\frac{1}{1-\beta\rho}$	$1-\frac{4}{7}\frac{1}{1-\beta\rho}$	0
Country C	$\frac{2}{7}\frac{1}{1-\beta\rho}$	0	$1-\frac{4}{7}\frac{1}{1-\beta\rho}$

Table 3.2: Optimal equity shares held by each country in global financial integration ($n_A=\frac{1}{2}$, $n_B=n_C=\frac{1}{4}$)

	Equity A	Equity B	Equity C
Country A	$1-\frac{1}{2}\frac{1}{1-\beta\rho}$	$\frac{1}{2}\frac{1}{1-\beta\rho}$	$\frac{1}{2}\frac{1}{1-\beta\rho}$
Country B	$\frac{1}{4}\frac{1}{1-\beta\rho}$	$1-\frac{3}{4}\frac{1}{1-\beta\rho}$	$\frac{1}{4}\frac{1}{1-\beta\rho}$
Country C	$\frac{1}{4}\frac{1}{1-\beta\rho}$	$\frac{1}{4}\frac{1}{1-\beta\rho}$	$1-\frac{3}{4}\frac{1}{1-\beta\rho}$

Once obtaining the steady state portfolios, I substitute these portfolios into the solution to the linearized dynamic system and get the final solution.

Based on these solutions, I compare the volatility of consumption across financial architectures, which is reported in proposition 2.

Proposition 2. *In a one-good endowment world economy with population size $n_A = \frac{1}{2}$ and $n_B = n_C = \frac{1}{4}$, the conditional variance of consumption in all countries are the same in financial autarky and they are also the same in global financial integration. If $0 < \frac{1-\beta+\frac{\beta\zeta}{\sigma}}{1-\beta\rho} < \frac{7}{3}$, the conditional variance of consumption in each country in each financial regime is ranked as,*

$$var_{t-1}(\widehat{C^p_{A,t}}) < var_{t-1}(\widehat{C^g_{A,t}}) < var_{t-1}(\widehat{C^a_{A,t}})$$

$$var_{t-1}(\widehat{C^g_{B,t}}) < var_{t-1}(\widehat{C^p_{B,t}}) < var_{t-1}(\widehat{C^a_{B,t}})$$

$$var_{t-1}(\widehat{C^g_{C,t}}) < var_{t-1}(\widehat{C^p_{C,t}}) < var_{t-1}(\widehat{C^a_{C,t}})$$

where var_{t-1} denotes variance conditional on information up to period $t-1$ and superscript p, g and a stand for a variable in partial financial integration, global financial integration and financial autarky, respectively.

Proposition 2 states the main result in this chapter. The volatility of consumption in peripheral countries continually decreases in financial integration while it is not necessary for the core country to experience monotonically reduction in consumption volatility. As we know, in the process of international financial integration, peripheral countries have access to more and more international financial markets and their income risk can be gradually diversified. Thereby, consumption in these countries becomes much smoother. Nevertheless, the core country in the second stage of financial integration has already got the opportunity to diversify its income risk in global financial markets. It can do better in this medium stage than in full financial integration since peripheral countries are mutually excluded from each other in the medium stage.

The proof of proposition 2 can be directly obtained from the solution to the linearized system. In financial autarky, consumption equals output in each country and each period, $var_{t-1}(\widehat{C^a_{i,t}}) = \sigma^2_\epsilon$ with $i = A, B, C$. In global financial integration, individual income risk is fully diversified through international financial markets, and the volatility of consumption is $var_{t-1}(\widehat{C^g_{i,t}}) = \frac{3}{8}\sigma^2_\epsilon$ with $i = A, B, C$. In partial financial integration, the core country can take advantage of holding global assets to achieve a better risk-sharing than the global financial integration. For the core country, its consumption variance is $\sigma^2_\epsilon \left(\frac{1}{2} - \frac{1}{14}\frac{1-\beta+\frac{\beta\zeta}{\sigma}}{1-\beta\rho}\right)^2 + 2\sigma^2_\epsilon \left(\frac{1}{4} + \frac{1}{28}\frac{1-\beta+\frac{\beta\zeta}{\sigma}}{1-\beta\rho}\right)^2$. For

the peripheral countries, the variance of consumption in either country becomes $\sigma_\epsilon^2\left(\frac{1}{4}-\frac{1}{4}\frac{1-\beta+\frac{\beta\zeta}{\sigma}}{1-\beta\rho}\right)^2+\sigma_\epsilon^2\left(\frac{1}{2}+\frac{1}{14}\frac{1-\beta+\frac{\beta\zeta}{\sigma}}{1-\beta\rho}\right)^2+\sigma_\epsilon^2\left(\frac{1}{4}+\frac{5}{28}\frac{1-\beta+\frac{\beta\zeta}{\sigma}}{1-\beta\rho}\right)^2$. Comparing terms listed above, I can get the results in proposition 2.

Notice that $0<\beta<1$, $0<\rho<1$, $\sigma>0$ and $0<\zeta<1$. As long as ζ is not too large and σ is not too small, condition $0<\frac{1-\beta+\frac{\beta\zeta}{\sigma}}{1-\beta\rho}<\frac{7}{3}$ holds. For reasonable parameter values used in the literature, say, ζ is close to zero and $\sigma>0.5$, this condition definitely holds.

To better understand how portfolios alter the volatility of consumption across international financial regimes, let's turn to the impact response of consumption to exogenous endowment shocks. Let $\xi\equiv\frac{1}{14}\frac{1-\beta+\frac{\beta\zeta}{\sigma}}{1-\beta\rho}>0$. The response of consumption in country A to endowment shocks on impact reads,

$$\frac{\partial\widehat{C^p_{A,t}}}{\partial\boldsymbol{\epsilon}_t}=\underbrace{[1,0,0]^\intercal}_{\text{Income effect}}+\underbrace{\left[-\frac{1}{2}-\xi,\frac{1}{4}+\frac{1}{2}\xi,\frac{1}{4}+\frac{1}{2}\xi\right]^\intercal}_{\text{Wealth effect}}$$

$$\frac{\partial\widehat{C^g_{A,t}}}{\partial\boldsymbol{\epsilon}_t}=\underbrace{[1,0,0]^\intercal}_{\text{Income effect}}+\underbrace{\left[-\frac{1}{2},\frac{1}{4},\frac{1}{4}\right]^\intercal}_{\text{Wealth effect}}$$

$$\frac{\partial\widehat{C^a_{A,t}}}{\partial\boldsymbol{\epsilon}_t}=\underbrace{[1,0,0]^\intercal}_{\text{Income effect}}$$

where superscript $\intercal$ denotes the transpose of a vector. The vector of endowment shocks is $\boldsymbol{\epsilon}_t=[\epsilon_{A,t},\epsilon_{B,t},\epsilon_{C,t}]^\intercal$. When international financial markets are integrated, there are two channels affecting households' consumption. The first channel is income effect, which is generated by endowment shocks. The household in country A increases his consumption one for one responding to the domestic endowment shock, given returns on his portfolio unchanged, while the consumption doesn't change with foreign endowment shocks through this income channel. The income effect is the same across financial regimes from the equations above. The second channel is wealth effect, which works through the changes of returns on portfolios. Since the household in country A has a negative net position in country A's equities, in other words, foreigners have a long position in equities issued by country A, the household needs to pay dividends to foreigners. In partial integration, country A's household is required to pay $\frac{1}{2}+\xi$ dollars to households in other countries, while he only needs to pay $\frac{1}{2}$ dollars to foreigners in global finan-

cial integration. On the other hand, the household receives dollar payments from other countries. The results show that country A receives more dividends from abroad in partial financial integration than in global financial integration.

The sum of wealth effect and income effect determines a country's aggregate exposure to income risk. In financial autarky, the domestic household faces individual income risk. In global financial integration, the income risk is fully diversified through international financial markets. Since the core country accounts for one half population of the world, it can only diversify one half of domestic income risk in global financial integration. Nevertheless, the core country could diversify more domestic income risk by holding more foreign equities and less domestic equities in partial global financial integration. Thereby, the degree of international risk-sharing is higher in partial financial integration when the core country is relatively large.

For peripheral country B (similarly for country C, omitted here because of symmetry), the response of consumption to endowment shocks on impact is as follows,

$$\frac{\partial \widehat{C^p_{B,t}}}{\partial \boldsymbol{\epsilon}_t} = \underbrace{[0,1,0]^\intercal}_{\text{Income effect}} + \underbrace{\left[\frac{1}{2}+\xi, -\frac{3}{4}+\frac{5}{2}\xi, \frac{1}{4}-\frac{7}{2}\xi\right]^\intercal}_{\text{Wealth effect}}$$

$$\frac{\partial \widehat{C^g_{B,t}}}{\partial \boldsymbol{\epsilon}_t} = \underbrace{[0,1,0]^\intercal}_{\text{Income effect}} + \underbrace{\left[\frac{1}{2}, -\frac{3}{4}, \frac{1}{4}\right]^\intercal}_{\text{Wealth effect}}$$

$$\frac{\partial \widehat{C^a_{B,t}}}{\partial \boldsymbol{\epsilon}_t} = \underbrace{[0,1,0]^\intercal}_{\text{Income effect}}$$

The income effect is the same across financial regimes and also the same across countries responding to a domestic endowment shock. Compared to global financial integration, households in the peripheral countries in partial financial integration hold more domestic equities and equities issued by the core country. Therefore, risk exposure to domestic and the core country's income risk is higher in the medium stage of financial integration for peripheral countries. Observe that when $\frac{1-\beta+\frac{\beta\zeta}{\sigma}}{1-\beta\rho} < 1$, or equivalently, $(1-\rho)\sigma > \zeta$, a rise in country C's endowment can spill over to the other peripheral country and consumption in country B increases, even though these two countries don't have a direct financial and trade linkage between each

other. This kind of spill-over is propagated through the core country, which holds equity shares issued by country B. When the endowment in country C increases, wealth in both the core country and country C increases. Higher consumption induced by larger wealth leads to households' more savings in order to smooth consumption overtime, which in turn drive up asset prices in all countries. Households then re-balance their portfolios responding to shocks. The net foreign asset position in country B drops and consumption goes up accordingly.

A following question is if the core country achieves a higher risk-sharing in partial integration, why doesn't the core country in global integration choose the same portfolio as in partial integration? The reason is that households face different asset prices in these two regimes and thereby the optimal portfolio for the core country in partial integration is no longer feasible in global integration.

Country size and risk-sharing

In the baseline model, I consider that the core country has a dominant population in the world. In order to explore whether the results in the baseline model are robust to country size, I investigate the optimal portfolios and consumption volatility under different country sizes. Peripheral countries are still assumed to be symmetric, $n_B = n_C = n$, while the size of the central country becomes $n_A = 1 - 2n$. Let the core country be the larger country, $0 < n \leq \frac{1}{3}$.

Following the same procedure of solving the rational expectations equilibrium in the previous section, the optimal portfolios for each country are listed in table 3.3 and table 3.4. Similar to the benchmark case, peripheral countries would hold more shares of equities issued by the core country in partial financial integration than in global financial integration. Proposition 3 shows that global financial integration lowers consumption volatility for all countries compared to financial autarky, regardless of country sizes. Moreover, when the size of the core country is relatively small (it is still the largest country), for instance, $n_A = \frac{1}{3}$, consumption volatility in the core country monotonically decreases in the process of financial integration. Nevertheless, when it is large enough, the core country achieves the best risk-sharing in partial financial integration, as stated in proposition 2.

Proposition 3. *In a one-good endowment world economy with population size $n_A = 1 - 2n$ and $n_B = n_C = n$, $0 < n \leq \frac{1}{3}$, the conditional variance of consumption in all countries are the same in financial autarky and they*

are also the same in global financial integration; volatility of consumption is lower in global financial integration, that is, $var_{t-1}(\widehat{C^g_{i,t}}) < var_{t-1}(\widehat{C^a_{i,t}})$, *with* $i = A, B, C$. *If* $n(1+\chi) > \frac{2}{3} > n\chi$, *with* $\chi \equiv \frac{2}{2-n} - \frac{\beta n\left(1-\rho-\frac{\zeta}{\sigma}\right)}{(2-n)(1-\beta\rho)}$, *the ranking of conditional variance of consumption for the core country reads,*

$$var_{t-1}(\widehat{C^g_{A,t}}) < var_{t-1}(\widehat{C^p_{A,t}}) < var_{t-1}(\widehat{C^a_{A,t}})$$

If $n(1+\chi) < \frac{2}{3}$, *the conditional variance of consumption in the core country is ranked as follows,*

$$var_{t-1}(\widehat{C^p_{A,t}}) < var_{t-1}(\widehat{C^g_{A,t}}) < var_{t-1}(\widehat{C^a_{A,t}})$$

The conditional variance of consumption in peripheral countries is ranked as follows,

$$var_{t-1}(\widehat{C^g_{j,t}}) < var_{t-1}(\widehat{C^p_{j,t}}) < var_{t-1}(\widehat{C^a_{j,t}}) \ ,j = B, C$$

Risk-sharing premium

Another way to explore the cost and benefit of international risk-sharing is to examine the cost of insurance in different stages of financial development. When households can pool individual income risk together, the cost of insurance would fall. Lower cost of insurance implies higher expected returns on insurance given the ex post payoffs by assets. As we know, expected returns on assets are the same across asset types up to a first-order approximation. I need to use higher order approximations to pin down expected asset returns. Based on the Euler conditions for country A's household (other Euler equations deliver the same results), the expected excess returns on the equity issued by country j, $j = B, C$, can be expressed as,

$$\begin{aligned} E_t\left(R_{j,t+1} - R_{A,t+1}\right) &= R\sigma E_t\left(\widehat{C}_{A,t+1}(\widehat{R}_{j,t+1} - \widehat{R}_{A,t+1})\right) \\ &= R\sigma cov_t\left(\widehat{C}_{A,t+1}, \widehat{R}_{j,t+1} - \widehat{R}_{A,t+1}\right) \end{aligned}$$

where cov_t denotes covariance conditional on information up to date t. This condition indicates that equity j will entitle its holders an excess return if it can not hedge against consumption risk.

Once solving for the dynamic system, I obtain excess returns on equities in different stages of financial integration. In partial financial integration,

the excess returns read,

$$E_t\left(R^p_{j,t+1} - R^p_{A,t+1}\right) = R\sigma_A^2\sigma(3n\chi - 1) \text{ with, } j = A, B \tag{3.19}$$

When the core country is large, that is, a small n such that $3n\chi - 1 < 0$, the expected return on equity A is higher than that on equity j since equity j can effectively hedge against the consumption income risk for country A's household. Thereby, equity A should compensate its share holders more in order to make them hold this type of asset. When $3n\chi - 1 > 0$, equity j needs to pay its holders excess returns. Notice that $\frac{\partial\chi}{\partial n} > 0$, when peripheral countries are large enough, the latter condition holds.

In global financial integration, asset markets are complete up to a first-order condition. The excess returns on equities are as follows,

$$E_t\left(R^g_{j,t+1} - R^g_{A,t+1}\right) = R\sigma_A^2\sigma(3n - 1) \text{ with, } j = A, B \tag{3.20}$$

When three countries are equally large, $n = \frac{1}{3}$, returns on all equities are the same up to a second-order approximation. Nevertheless, equity A has an excess return over other equities in all other cases with $0 < n < \frac{1}{3}$. When the core country is large, the equity shares in this country account for a dominant part of assets in international financial markets. In other words, equity A is backed by the major source of uncertainty in the world economy. The household in country A wants to get rid of his income risk by selling his own equity shares to foreigners. Nevertheless, on the demand side, households in peripheral countries can't completely absorb the amount of equity shares sold by the core country because of their smaller country sizes. In equilibrium, the price of equity A must be driven down and the expected return on equity A must be pushed up to attract foreigners to hold this type of equity.

With the liberalization of international financial markets, households in peripheral countries gradually have access to more financial assets in other countries. The expansion of the number of assets available to households could reduce the cost of insurance against country-specific shocks. In partial financial integration, the demand for equity A by peripheral households is higher than that in global integration, which in turn drives up the price of equity A and consequently lowers its expected excess return rate. Peripheral households gain a premium in asset markets from partial integration to global integration. Let the *international risk-sharing premium* be measured

by,

$$\begin{aligned} RSP_j &\equiv E_t\left(R^g_{A,t+1} - R^g_{j,t+1}\right) - E_t\left(R^p_{A,t+1} - R^p_{j,t+1}\right) \\ &= 3nR\sigma_A^2\sigma(\chi - 1) > 0 \text{ with, } j = A, B \end{aligned} \tag{3.21}$$

which states that households in peripheral countries obtain a positive risk-sharing premium in the process of financial integration. This premium increases in the size of peripheral countries and in the coefficient of relative risk aversion.

Some special shock processes

In the previous section, I assume that shocks across countries are not correlated. The motivation for this simplified assumption is to introduce the need for risk-sharing across borders. If shocks in peripheral countries are perfectly positively correlated, of course, there is no need for the peripheral countries to share individual income risk in global financial integration because of symmetry. The volatility of consumption will be the same both in partial integration and global integration. On the other hand, if these shocks are perfectly negatively correlated, peripheral countries can arrive at perfect risk-sharing by holding their equities only. The core country doesn't have other asset markets to share its income risk. The volatility of consumption for the core country in global financial integration will be the same as the case of financial autarky. For general shocks processes, whether financial integration improves international risk-sharing depends on country sizes, as outlined in proposition 3.

Table 3.3: Optimal equity shares held by each country in partial financial integration ($n_A = 1 - 2n$, $n_B = n_C = n$)

	Equity A	Equity B	Equity C
Country A	$1 - \frac{4n}{2-n}\frac{1}{1-\beta\rho}$	$\frac{2(1-2n)}{2-n}\frac{1}{1-\beta\rho}$	$\frac{2(1-2n)}{2-n}\frac{1}{1-\beta\rho}$
Country B	$\frac{2n}{2-n}\frac{1}{1-\beta\rho}$	$1 - \frac{2(1-2n)}{2-n}\frac{1}{1-\beta\rho}$	0
Country C	$\frac{2n}{2-n}\frac{1}{1-\beta\rho}$	0	$1 - \frac{2(1-2n)}{2-n}\frac{1}{1-\beta\rho}$

Table 3.4: Optimal equity shares held by each country in global financial integration ($n_A = 1 - 2n$, $n_B = n_C = n$)

	Equity A	Equity B	Equity C
Country A	$1 - \frac{2n}{1-\beta\rho}$	$\frac{1-2n}{1-\beta\rho}$	$\frac{1-2n}{1-\beta\rho}$
Country B	$\frac{n}{1-\beta\rho}$	$1 - \frac{1-n}{1-\beta\rho}$	$\frac{n}{1-\beta\rho}$
Country C	$\frac{n}{1-\beta\rho}$	$\frac{n}{1-\beta\rho}$	$1 - \frac{1-n}{1-\beta\rho}$

3.2 A model with multiple tradable goods

In the previous section, I focus on the common discount factor and abstract from an important aspect of international risk-sharing, terms of trade. As emphasized by Cole and Obstfeld (1991), terms of trade could play an important role in international risk sharing. The basic logic is that terms of trade will appreciate responding to an adverse productivity shock in a country. Even though international financial markets may not be available to households in some countries, as long as goods and services can freely flow across borders, terms of trade could still buffer the adverse effect of a negative productivity shock. A follow-up question is how international financial integration improves international risk-sharing when I take into account of both terms of trade and the common discount factor. Another new element in this section is that I consider a production economy in which labor services are the only input. With endogenous labor supply, households have an extra channel to smooth consumption overtime.

The model structure is similar to the previous section. I present only the new model elements here. Country size is the same as baseline case, $n_A = \frac{1}{2}$ and $n_B = n_C = \frac{1}{4}$. Each country has two types of agents, households and firms. Households in each country choose a consumption composite, labor supply and an international portfolio in each period. Firms produce a unique type of tradable good. Therefore, firms in different countries produce different kinds of tradable goods. The productivity suffers a random shock in each country. Final goods producers in each country combine three tradable goods into domestic consumption composites, which is consumed by domestic households only. I will investigate three stages of financial development: financial autarky, partial financial integration and global financial integration.

3.2.1 Consumption composite

Consumption composites in each country require locally produced goods and imported goods from the other two countries. The consumption composite in country A, $C_{A,t}$, has a form of,

$$C_{A,t} = \left[\alpha_1^{\frac{1}{\gamma}}(C_{A,t}^A)^{\frac{\gamma-1}{\gamma}} + \left(\frac{1}{2}\right)^{\frac{1}{\gamma}}(1-\alpha_1)^{\frac{1}{\gamma}}(C_{A,t}^B)^{\frac{\gamma-1}{\gamma}} + \left(\frac{1}{2}\right)^{\frac{1}{\gamma}}(1-\alpha_1)^{\frac{1}{\gamma}}(C_{A,t}^C)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}} \tag{3.22}$$

where $\gamma > 0$ is the elasticity of substitution between imports and exports. $C_{A,t}^j$, $i = A, B, C$, is consumption demand for goods produced at country j by the household in country A. In the symmetric deterministic steady state, α_1 is the share of consumption spending devoted to goods produced by country A. A preference bias towards local goods implies, $1/2 < \alpha_1 < 1$.

Expenditure minimization associated with the consumption composite (3.22) gives the consumer price index at country A as,

$$P_{A,t} = \left[\alpha_1(P_{A,t}^A)^{1-\gamma} + \frac{1-\alpha_1}{2}(P_{A,t}^B)^{1-\gamma} + \frac{1-\alpha_1}{2}(P_{A,t}^C)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \tag{3.23}$$

where $P_{A,t}^j$, $j = A, B, C$, denotes the price of good j in terms of currency in country A. Demand for goods produced at country A by country A consumers is given by $C_{A,t}^A = \alpha_1(P_{A,t}^A/P_{A,t})^{-\gamma}C_{A,t}$. Demand for country j, $j = B, C$, goods by country A consumers has a form of $C_{A,t}^j = \frac{1-\alpha_1}{2}(P_{A,t}^j/P_{A,t})^{-\gamma}C_{A,t}$.

Analogously, in the peripheral world, country B and C have a similar consumption composite defined over three types of goods,

$$C_{i,t} = \left[(1-\alpha_1)^{\frac{1}{\gamma}}(C_{i,t}^A)^{\frac{\gamma-1}{\gamma}} + (\alpha_1\alpha_2)^{\frac{1}{\gamma}}(C_{i,t}^i)^{\frac{\gamma-1}{\gamma}} + (\alpha_1(1-\alpha_2))^{\frac{1}{\gamma}}(C_{i,t}^j)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}} \tag{3.24}$$

where $C_{i,t}$, $i = B, C$, denotes consumption composite in country i. $C_{i,t}^j$, $j = B, C$ and $j \neq i$, represents the consumption demand for goods produced at country j (source country) by agents in country i (destination country). α_1 is the share of consumption expenditure on goods produced by peripheral countries. α_2 is the share of local goods in the peripheral consumption composite at country B and C. Thereby, agents in country B or C spend a

share of $\alpha_1\alpha_2$ on local products. Expenditure minimization implies that,

$$P_{i,t} = \left[(1-\alpha_1)(P^A_{i,t})^{1-\gamma} + \alpha_1\alpha_2(P^i_{i,t})^{1-\gamma} + \alpha_1(1-\alpha_1)(P^j_{i,t})^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \quad (3.25)$$

with $i = B, C$. $P_{i,t}$ denotes the consumer price index in country i and $P^j_{i,t}$ is the price of goods produced at country j in terms of currency at country i. Demand for goods produced in country j, $j = B, C$, by agents at country i, $i = B, C$ can be written as, $C^A_{i,t} = (1-\alpha_1)\left(P^A_{i,t}/P_{i,t}\right)^{-\gamma} C_{i,t}$, $C^i_i = \alpha_1\alpha_2\left(P^i_{i,t}/P_{i,t}\right)^{-\gamma} C_{i,t}$ and $C^j_i = \alpha_1(1-\alpha_2)\left(P^j_{i,t}/P_{i,t}\right)^{-\gamma} C_{i,t}$ with $i \neq j$.

Since goods can be freely traded across borders without any cost, the law of one price holds for each good variety. Furthermore, suppose that all countries take use of international currency, say, dollar, as their account unit, thereby, $P^A_{A,t} = P^A_{B,t} = P^A_{B,t}$, $P^B_{A,t} = P^B_{B,t} = P^B_{C,t}$ and $P^C_{A,t} = P^C_{B,t} = P^C_{C,t}$. For simplicity, let the goods produced in country A be the numeraire and set $P^A_{A,t} = 1$.

3.2.2 Households

Households in country i have a life-time utility function of the form,

$$E_t\left[\sum_{\tau=t}^{\infty} \beta_{i,\tau} U(C_{i,\tau}, N_{i,\tau})\right] \quad (3.26)$$

where the operator E_t here represents the conditional expectation over all states of nature, $C_{i,\tau}$ is the consumption, and $N_{i,\tau}$ stands for labor supply in country $i = A, B, C$. $\beta_{i,\tau}$ is the endogenous subjective discount factor. The household's preference has a form of,

$$U(C_{i,t}, N_{i,t}) = \frac{(C_{i,t})^{1-\sigma} - 1}{1-\sigma} - \eta_0 \frac{(N_{i,t})^{1+\eta}}{1+\eta} \quad (3.27)$$

where σ is the coefficient of relative risk aversion or the inverse of elasticity of intertemporal substitution, η characterizes Frisch elasticity of labor supply.[40]

Labor market is competitive, and therefore labor supply is determined

[40] Christiano, Trabandt and Walentin (2010) propose another interpretation of labor supply $N_{i,t}$ that it measures numbers of people working rather than hours of working. The value of η in the calibration is consistent with this interpretation. However, I still use Frisch labor supply elasticity for $1/\eta$ here for convenience.

by real wage and marginal rate of substitution between consumption and leisure,

$$-\frac{U^i_{N,t}}{U^i_{C,t}} = \frac{W_{i,t}}{P_{i,t}} \tag{3.28}$$

where $U^i_{C,t}$ and $U^i_{N,t}$ stand for marginal utility of consumption and marginal disutility of labor supply, respectively in country i at time t. $W_{i,t}$ is wage rate.

3.2.3 Financial markets

There are three types of equities in financial markets. The claimants of equities obtain a corresponding share of profits in the issuance country. Let the profits from production be $\Pi_{i,t}$ at country i, which is defined later. The gross return on equities at country i can be written as,

$$R_{i,t} = \frac{\Pi_{i,t}}{Q_{i,t}}$$

The resource constraint faced by agents in the central country has the following form,

$$P_{A,t}C_{A,t} + NW_{A,t+1} = N_{A,t}W_{A,t} + \Pi_{A,t} + NW_{A,t}R_{A,t} + \alpha'_{A,t}R_{x,t} \tag{3.29}$$

Expenditures on the left hand side consist of consumption expenditure and a portfolio taken to the next period. The right hand side contains the resources of total income, which include labor income, $N_{A,t}W_{A,t}$, profits, $\Pi_{A,t}$, and gross returns on net foreign assets in the last two terms.

Peripheral countries face similar resource constraints. The total income from sales of output and returns on equity assets equals total expenses,

$$P_{i,t}C_{i,t} + NW_{i,t+1} = N_{i,t}W_{i,t} + \Pi_{i,t} + NW_{i,t}R_{A,t} + \alpha'_{i,t}R_{x,t} \tag{3.30}$$

with $i = B, C$. Net equities in peripheral country B is $\alpha_{B,t} \equiv [(X^B_{B,t} - 1)Q_{B,t}; X^C_{B,t}Q_{C,t}]$ and in country C is $\alpha_{C,t} \equiv [X^B_{C,t}Q_{B,t}; (X^C_{C,t} - 1)Q_{C,t}]$.

In partial financial integration, the core country A has access to global financial assets, the optimality condition for its portfolio choice is,

$$\frac{U^A_{C,t}}{P_{A,t}} = E_t\left[v(A,t)\frac{U^A_{C,t+1}}{P_{A,t+1}}R_{j,t+1}\right] \text{ , with } j = A, B, C \tag{3.31}$$

This condition is the usual consumption Euler equation for equities issued by country j, which states that the forgone marginal utility of one dollar today equals the expected discounted marginal utility tomorrow from gross returns to such one dollar asset j purchased in period t.

In the peripheral sphere, households in country B (or C) have access to equities in the central country A and their own asset market. Their corresponding optimal conditions for portfolios yield,

$$\frac{U^B_{C,t}}{P_{B,t}} = E_t\left[v(B,t)\frac{U^B_{C,t+1}}{P_{B,t+1}}R_{j,t+1}\right] \text{ , with } j = A, B \tag{3.32}$$

$$\frac{U^C_{C,t}}{P_{C,t}} = E_t\left[v(C,t)\frac{U^C_{C,t+1}}{P_{C,t+1}}R_{j,t+1}\right] \text{ , with } j = A, C \tag{3.33}$$

In global financial integration, households in any country can trade equities with households in other countries. The optimality conditions are as follows,

$$\frac{U^i_{C,t}}{P_{i,t}} = E_t\left[v(i,t)\frac{U^i_{C,t+1}}{P_{i,t+1}}R_{j,t+1}\right] \text{ , with } i, j = A, B, C \tag{3.34}$$

3.2.4 Firms

At time t, the homogenous goods in each country are produced by a competitive, representative firm. The firm produces the good using labor services according to the following technology,

$$Y_{i,t} = Z_{i,t}N_{i,t}^{1-\theta} \tag{3.35}$$

with $0 < \theta < 1$. $1-\theta$ is labor income share and $N_{i,t}$ denotes the time t labor demand. Total factor productivity, $Z_{i,t}$, is an exogenous random variable, which has the following form of,

$$\log Z_{i,t+1} = (1-\rho)\log Z_i + \rho \log Z_{i,t} + \epsilon_{i,t+1}$$

where Z_i stands for the steady state technology in country i and ρ characterizes the persistence of technology shocks. Technology innovation $\epsilon_{i,t}$ has an $i.i.d.$ normal distribution with mean zero and standard deviation $\sigma_{\epsilon,i}$.

In the competitive labor market, labor demand is determined by the

marginal product of labor and wages,

$$W_{i,t} = \frac{P^i_{i,t} Y_{i,t}(1-\theta)}{N_{i,t}}$$

Subtracting payrolls from the firm's total revenues, I obtain profits in country i,

$$\Pi_{i,t} = P^i_{i,t} Y_{i,t} - N_{i,t} W_{i,t}$$

3.2.5 Competitive equilibrium

The asset market clearing condition requires that the sum of equity shares for each type of equity is unity,

$$X^j_{A,t} + X^j_{B,t} + X^j_{C,t} = 1 \text{ , with } j = A, B, C \tag{3.36}$$

Goods market clearing condition implies that total absorption of good j is equal to its total production in country j ,

$$C^j_{A,t} + C^j_{B,t} + C^j_{C,t} = Y_{j,t} \text{ , with } j = A, B, C \tag{3.37}$$

Notice also that the global net foreign asset positions should be zero, $NW_{A,t} + NW_{B,t} + NW_{C,t} = 0$ and $\alpha_{A,t} + \alpha_{B,t} + \alpha_{C,t} = 0$.

The rational expectations competitive equilibrium is the defined as follows,

Definition 2. *The competitive equilibrium consists of a sequence of goods prices $P^i_{i,t}$, wages $W_{i,t}$ and asset prices $Q_{i,t}$, $i = A, B, C$, $t = 0, 1, 2, \cdots$ and a sequence of allocations $C_{i,t}$, $N_{i,t}$ and of portfolios $X^j_{i,t}$, $i, j = A, B, C$, $t = 0, 1, 2, \cdots$, such that (a) given prices, consumption, labor and portfolio decisions by the household in each country solve the household's and firms's problems, and (b) prices clear labor markets, goods markets and asset markets in each country.*

3.2.6 Steady state

In the deterministic steady state, the technological level keeps constant in each country. I consider a symmetric steady state in which consumption per capita, output per capita and labor service per capita are the same in all countries. Thereby, net foreign asset positions are equal to zero in the steady state and consumption per capital equals output per capita. Relative prices in the steady state are equal to one. Let a variable without time subscript

denote its deterministic level and a variable with a tilde on the head denote per capita variable.

After some algebraic manipulation, I have the steady state technology $Z_B = Z_C = 2^{-\theta} Z_A$, and labor supply,

$$\widetilde{N} = \left[\frac{1-\theta}{\eta_0} \left(2^{\theta} Z_A \right)^{1-\sigma} \right]^{\frac{1}{1+\eta-(1-\theta)(1-\sigma)}}$$

and output thereby can be written as,

$$\widetilde{Y} = 2^{\theta} Z_A \left[\frac{1-\theta}{\eta_0} \left(2^{\theta} Z_A \right)^{1-\sigma} \right]^{\frac{1-\theta}{1+\eta-(1-\theta)(1-\sigma)}}$$

Other variables in the steady state are as follows. $Y_A = \frac{1}{2}\widetilde{Y}$, $Y_B = Y_C = \frac{1}{4}\widetilde{Y}$, $\Pi_i = \theta Y_i$, $Q_i = \theta\beta Y_i$ with the inverse of gross returns to assets, β, $C_i = Y_i$.

3.2.7 Solving portfolios and the dynamic system

First, wee need to work out the log-linearized version of the model. Let a variable with a hat denote the log deviation from its deterministic steady state. Since population in each country is fixed and thereby log derivation for an aggregate variable is the same as a per capita variable. Let the goods produced by the central country be numeraire, $P^A_{A,t} = 1$. Log-linearized consumption demand for each good in each country yields,

$$\begin{aligned}
&\widehat{C^A_{A,t}} = \gamma\widehat{P_{A,t}} + \widehat{C_{A,t}}, \quad \widehat{C^B_{A,t}} = \gamma(\widehat{P_{A,t}} - \widehat{P^B_{B,t}}) + \widehat{C_{A,t}}, \quad \widehat{C^C_{A,t}} = \gamma(\widehat{P_{A,t}} - \widehat{P^C_{C,t}}) + \widehat{C_{A,t}} \\
&\widehat{C^A_{B,t}} = \gamma\widehat{P_{B,t}} + \widehat{C_{B,t}}, \quad \widehat{C^B_{B,t}} = \gamma(\widehat{P_{B,t}} - \widehat{P^B_{B,t}}) + \widehat{C_{B,t}}, \quad \widehat{C^C_{B,t}} = \gamma(\widehat{P_{B,t}} - \widehat{P^C_{C,t}}) + \widehat{C_{B,t}} \\
&\widehat{C^A_{C,t}} = \gamma\widehat{P_{C,t}} + \widehat{C_{C,t}}, \quad \widehat{C^B_{C,t}} = \gamma(\widehat{P_{C,t}} - \widehat{P^B_{B,t}}) + \widehat{C_{C,t}}, \quad \widehat{C^C_{C,t}} = \gamma(\widehat{P_{C,t}} - \widehat{P^C_{C,t}}) + \widehat{C_{C,t}}
\end{aligned}$$

The log-linearized versions of price indices (3.23) and (3.25) can be written as,

$$\begin{aligned}
&\widehat{P_{A,t}} = \frac{1-\alpha_1}{2}(\widehat{P^B_{B,t}} + \widehat{P^C_{C,t}}) \qquad \widehat{P_{B,t}} = \alpha_1\alpha_2\widehat{P^B_{B,t}} + \alpha_1(1-\alpha_2)\widehat{P^C_{C,t}} \\
&\widehat{P_{C,t}} = \alpha_1(1-\alpha_2)\widehat{P^B_{B,t}} + \alpha_1\alpha_2\widehat{P^C_{C,t}}
\end{aligned} \tag{3.38}$$

Combining optimal labor supply (3.28) and production (3.35), yields

labor demand in country $i = A, B, C$, and the linearized equations read,

$$\widehat{N_{i,t}} = \frac{1}{\theta+\eta}\left(\widehat{P^i_{i,t}} - \widehat{P_{i,t}} - \sigma\widehat{C_{i,t}} + \widehat{Z_{i,t}}\right) \tag{3.39}$$

$$\widehat{Y_{i,t}} = \frac{1}{\theta+\eta}\widehat{Z_{i,t}} + \frac{1-\theta}{\theta+\eta}\left(\widehat{P^i_{i,t}} - \widehat{P_{i,t}} - \sigma\widehat{C_{i,t}}\right) \tag{3.40}$$

Linearizing goods market clearing conditions (3.37) and get,

$$2\widehat{Y_{A,t}} = 2\alpha_1\widehat{C^A_{A,t}} + (1-\alpha_1)\widehat{C^A_{B,t}} + (1-\alpha_1)\widehat{C^A_{C,t}} \tag{3.41}$$

$$\widehat{Y_{B,t}} = (1-\alpha_1)\widehat{C^B_{A,t}} + \alpha_1\alpha_2\widehat{C^B_{B,t}} + \alpha_1(1-\alpha_2)\widehat{C^B_{C,t}} \tag{3.42}$$

$$\widehat{Y_{C,t}} = (1-\alpha_1)\widehat{C^C_{A,t}} + \alpha_1(1-\alpha_2)\widehat{C^C_{B,t}} + \alpha_1\alpha_2\widehat{C^C_{C,t}} \tag{3.43}$$

I then express output, labor, and prices as functions of consumption and exogenous shocks. Substitute these variables into the linearized version of budget constraints (3.29)-(3.30) and obtain,

$$\begin{aligned}\frac{3}{4}\widetilde{NW}_{B,t+1} + \frac{1}{4}\widetilde{NW}_{C,t+1} =& \frac{3}{4\beta}\widetilde{NW}_{B,t} + \frac{1}{4\beta}\widetilde{NW}_{C,t} + \frac{3}{4}\widetilde{\alpha}'_B\widehat{R}_{x,t} + \frac{1}{4}\widetilde{\alpha}'_C\widehat{R}_{x,t} \\ &+ \Gamma_{A1}\Delta\widehat{Z}_{B,t} + \Gamma_{A2}\Delta\widehat{Z}_{C,t} + \Gamma_{A3}\Delta\widehat{C}_{B,t} + \Gamma_{A4}\Delta\widehat{C}_{C,t}\end{aligned} \tag{3.44}$$

$$\begin{aligned}\frac{1}{4}\widetilde{NW}_{B,t+1} + \frac{3}{4}\widetilde{NW}_{C,t+1} =& \frac{1}{4\beta}\widetilde{NW}_{B,t} + \frac{3}{4\beta}\widetilde{NW}_{C,t} + \frac{1}{4}\widetilde{\alpha}'_B\widehat{R}_{x,t} + \frac{3}{4}\widetilde{\alpha}'_C\widehat{R}_{x,t} \\ &+ \Gamma_{A2}\Delta\widehat{Z}_{B,t} + \Gamma_{A1}\Delta\widehat{Z}_{C,t} + \Gamma_{A4}\Delta\widehat{C}_{B,t} + \Gamma_{A3}\Delta\widehat{C}_{C,t}\end{aligned} \tag{3.45}$$

where $\widetilde{NW}_{B,t} \equiv \frac{4NW_{B,t}}{\widetilde{C}}$ and $\widetilde{NW}_{C,t} \equiv \frac{4NW_{C,t}}{\widetilde{C}}$ are the ratios of net foreign assets to GDP in country B and C, respectively. Notice that net foreign asset positions are zero at the steady state, and therefore, $\widetilde{NW}_{i,t}$, $i = B, C$ is a first-order term. $\widetilde{\alpha}_i \equiv \frac{4\bar{\alpha}_i}{\beta\widetilde{C}}$ with $i = B, C$ denotes the ratio of the zeroth order net equity holdings to GDP in the peripheral country i. $\Delta\widehat{C}_{i,t} \equiv \widehat{C}_{A,t} - \widehat{C}_{i,t}$ expresses consumption of central country A relative to consumption at peripheral country i, and similarly, relative technological level is given by $\Delta\widehat{Z}_{i,t} \equiv \widehat{Z}_{A,t} - \widehat{Z}_{i,t}$. Γ_{As} with $s = 1, 2, 3, 4$ are functions of structural parameters in the model, which are given in the appendix.

Taking a first-order approximation to the Euler equations (3.31)-(3.33), subtracting (3.32) from (3.31) and subtracting (3.33) from (3.31) respec-

tively, yields,

$$(\sigma-\zeta)\Delta\widehat{C}_{B,t}+\Delta\widehat{P}_{B,t}=E_t\left[\sigma\Delta\widehat{C}_{B,t+1}+\Delta\widehat{P}_{B,t+1}\right]$$
$$(\sigma-\zeta)\Delta\widehat{C}_{C,t}+\Delta\widehat{P}_{C,t}=E_t\left[\sigma\Delta\widehat{C}_{C,t+1}+\Delta\widehat{P}_{C,t+1}\right]$$

with $\Delta\widehat{P}_{i,t}\equiv\widehat{P}_{A,t}-\widehat{P}_{i,t},\ i=B,C$. Substituting relative prices out in equations above using relative consumption and technology, I obtain,

$$\begin{aligned}E_t\left[(\sigma-\Gamma_{B3})\Delta\widehat{C}_{B,t+1}-\Gamma_{B4}\Delta\widehat{C}_{C,t+1}\right]&=-E_t\left[\Gamma_{B1}\Delta\widehat{Z}_{B,t+1}+\Gamma_{B2}\Delta\widehat{Z}_{C,t+1}\right]\\&+(\sigma-\zeta-\Gamma_{B3})\Delta\widehat{C}_{B,t}-\Gamma_{B4}\Delta\widehat{C}_{C,t}+\Gamma_{B1}\Delta\widehat{Z}_{B,t}+\Gamma_{B2}\Delta\widehat{Z}_{C,t}\end{aligned}\tag{3.46}$$

$$\begin{aligned}E_t\left[(\sigma-\Gamma_{B3})\Delta\widehat{C}_{C,t+1}-\Gamma_{B4}\Delta\widehat{C}_{B,t+1}\right]&=-E_t\left[\Gamma_{B1}\Delta\widehat{Z}_{C,t+1}+\Gamma_{B2}\Delta\widehat{Z}_{B,t+1}\right]\\&+(\sigma-\zeta-\Gamma_{B3})\Delta\widehat{C}_{C,t}-\Gamma_{B4}\Delta\widehat{C}_{B,t}+\Gamma_{B1}\Delta\widehat{Z}_{C,t}+\Gamma_{B2}\Delta\widehat{Z}_{B,t}\end{aligned}\tag{3.47}$$

where Γ_{Bs}, $s=1,2,3,4$ are functions of parameters in the model, which are defined in the appendix.

Up to a first-order approximation, portfolio choices are indeterminate because of certainty equivalence, I need a second-order approximation to obtain the zeroth-order portfolio. Following Devereux and Sutherland (2011a), I take a second-order approximation to the Euler equations (3.31)-(3.33) in partial financial integration, and obtain,

$$E_t\left\{\begin{bmatrix}-\sigma\Delta\widehat{C}_{B,t+1}-\Delta\widehat{P}_{B,t+1} & 0\\ 0 & -\sigma\Delta\widehat{C}_{C,t+1}-\Delta\widehat{P}_{C,t+1}\end{bmatrix}\widehat{R}_{x,t+1}\right\}=0\tag{3.48}$$

Notice that when financial markets are not fully accessible to all countries, the asset market is incomplete. There are two constraints on portfolio choices imposed by equations (3.48). Combining with three asset market clearing conditions (3.36) and another two symmetric asset holdings, say country B v.s C and central country v.s. peripheral countries, I have seven equations and seven unknowns for portfolio X_i^j. Once I solve the linearized rational expectations equilibrium and I can obtain the zeroth portfolio.

When financial markets are globally integrated together, the second-

order approximations to Euler equations (3.34) read,

$$E_t\left[\widehat{R}_{x,t+1}(-\sigma\Delta\widehat{C}_{B,t+1}-\Delta\widehat{P}_{B,t+1})\right]=0$$
$$E_t\left[\widehat{R}_{x,t+1}(-\sigma\Delta\widehat{C}_{C,t+1}-\Delta\widehat{P}_{C,t+1})\right]=0$$

There are four constraints on portfolio choices imposed by Euler equations. Combining with three asset market clearing conditions and two symmetric conditions for asset holdings, say country B v.s C and central country v.s. peripheral countries, I have nine equations and nine unknowns for the portfolio X_i^j.

Until now, I have reduced the linearized dynamic system to a system of four equations, that is, two resource constraints (3.44)-(3.45) and two Euler equations (3.46)-(3.47). There are four unknowns $\Delta\widehat{C}_{B,t}$, $\Delta\widehat{C}_{C,t}$, $\widetilde{NW}_{B,t+1}$ and $\widetilde{NW}_{C,t+1}$. $\Delta\widehat{Z}_{B,t}$ and $\Delta\widehat{Z}_{C,t}$ are exogenous variables. Since excess return vector $\widehat{R}_{x,t+1}$ is unpredictable conditional on information up to date t and thereby is uncorrelated with relative consumption $\Delta\widehat{C}_{i,t}$ and relative technology $\Delta\widehat{Z}_{i,t}$, $i=B,C$. Let $\xi_{i,t+1}\equiv\widetilde{\alpha}_i'\widehat{R}_{x,t+1}$, which is uncorrelated with variables at date t. Let a vector $x_t\equiv[\widetilde{NW}_{B,t};\widetilde{NW}_{C,t}]$, which contains the predetermined variables, and a vector $y_t\equiv[\Delta\widetilde{C}_{B,t};\Delta\widetilde{C}_{C,t}]$, which includes control variables. The four-equation system can be rewritten as follows,

$$\begin{bmatrix} x_{t+1} \\ E_t(y_{t+1}) \end{bmatrix}=\widetilde{\mathbf{A}}\begin{bmatrix} x_t \\ y_t \end{bmatrix}+\widetilde{\mathbf{f}}_t \tag{3.49}$$

Coefficient matrix $\widetilde{\mathbf{A}}$ and exogenous process $\widetilde{\mathbf{f}}_t$ are defined in the appendix. There are four eigenvalues for system (3.49),

$$\lambda_1=1-\frac{\zeta}{\sigma-\frac{\left[\alpha_1(2\alpha_2-1)-\frac{\sigma(\theta-1)}{\eta+\theta}\right](\alpha_1-2\alpha_1\alpha_2)}{\gamma-\alpha_1^2\gamma(2\alpha_2-1)^2+\frac{(\alpha_1(2\alpha_2-1)-1)(\theta-1)}{\eta+\theta}}} \qquad \lambda_3=\frac{1}{\beta}$$
$$\lambda_2=1-\frac{\zeta}{\sigma+\frac{(2\alpha_1-1)\left[\frac{\sigma(\theta-1)}{\eta+\theta}-2\alpha_1+1\right]}{2(\alpha_1-1)\left[2\alpha_1\gamma-\frac{\theta-1}{\eta+\theta}\right]}} \qquad \lambda_4=\frac{1}{\beta}$$

Observe that $\lambda_2<1$ when $\frac{1}{2}\leq\alpha_1\leq 1$ and $\lambda_3>1$, $\lambda_4>1$. $\lambda_1<1$ over a wide range of reasonable parameter values. Thereby, the dynamic system in this model has a unique solution according to Blanchard and Kahn (1980).

The appendix provides details about how to find the solution. For the control vector, I have,

$$y_{t+1} = D_1\xi_{t+1} + D_2\varepsilon_{t+1} + D_3 \begin{bmatrix} x_{t+1} \\ \Delta\widehat{Z}_t \end{bmatrix} \tag{3.50}$$

where coefficient vectors (matrices) D_s, $s = 1, 2, 3$, are defined in the appendix. Relative technology vector is $\Delta\widehat{Z}_t \equiv [\Delta\widehat{Z}_{B,t}; \Delta\widehat{Z}_{C,t}]$, exogenous shock vector is given by $\varepsilon_t \equiv [\epsilon_{A,t}; \epsilon_{B,t}; \epsilon_{C,t}]$ and unexpected excess returns to assets $\xi_{t+1} \equiv [\xi_{A,t+1}; \xi_{B,t+1}]$. The evolution of state vector x_{t+1} is determined by the upper line of equation (3.49).

Ex post excess returns on assets can be expressed as a function of exogenous shocks,

$$\widehat{R}_{B,t+1} - \widehat{R}_{A,t+1} = R_{11}\xi_{t+1} + R_{21}\varepsilon_{t+1} \tag{3.51}$$

$$\widehat{R}_{C,t+1} - \widehat{R}_{A,t+1} = R_{12}\xi_{t+1} + R_{22}\varepsilon_{t+1} \tag{3.52}$$

where R_{sk}, $s, k = 1, 2$, are constant and defined in the appendix. Recall that the expression for unexpected excess returns to assets $\xi_{t+1} = \widetilde{\alpha}\widehat{R}_{x,t}$ with $\widetilde{\alpha} \equiv [\widetilde{\alpha}'_B; \widetilde{\alpha}'_C]$. In the benchmark model, international financial markets are partially integrated and there is no financial asset flows cross peripheral countries. Thereby $\widetilde{\alpha} = \begin{bmatrix} \widetilde{\alpha}_{11} & 0 \\ 0 & \widetilde{\alpha}_{22} \end{bmatrix}$ with some constant $\widetilde{\alpha}_{ii}$, $i = 1, 2$.

Next I write the marginal utility of peripheral countries relative to the central country as,

$$-\sigma\Delta\widehat{C}_{B,t+1} - \Delta\widehat{P}_{B,t+1} = D_{11}\xi_{t+1} + D_{21}\varepsilon_{t+1} + others_{t+1} \tag{3.53}$$

$$-\sigma\Delta\widehat{C}_{C,t+1} - \Delta\widehat{P}_{C,t+1} = D_{12}\xi_{t+1} + D_{22}\varepsilon_{t+1} + others_{t+1} \tag{3.54}$$

where $others_{t+1}$ stands for terms which are predetermined at data $t + 1$. Similarly to R_{sk}, D_{sk}, $s, k = 1, 2$, are also constant and are defined in the appendix.

I then substitute relative excess returns in equation (3.51)-(3.52) and relative consumption governed by equation (3.50) into the second-order approximation to Euler equations (3.48) and achieve the following results.

Result 1. *When international financial markets are partially integrated, say, there isn't asset trade within peripheral countries, optimal equities* $\widetilde{\alpha}$

held by each country are determined by the following two equations,

$$(D_{11}\widetilde{H} + D_{21})\Sigma\widetilde{\widetilde{R}}_1 = 0 \tag{3.55}$$

$$(D_{12}\widetilde{H} + D_{22})\Sigma\widetilde{\widetilde{R}}_2 = 0 \tag{3.56}$$

where $\widetilde{H} = \widetilde{\alpha}\widetilde{\widetilde{R}}$, $\widetilde{\widetilde{R}} = \widetilde{R}^{-1}\begin{bmatrix} R_{21} \\ R_{22} \end{bmatrix}$, *and* $\widetilde{R} = \mathbf{I}_2 - \begin{bmatrix} R_{21} \\ R_{22} \end{bmatrix}\widetilde{\alpha}$. $\widetilde{\widetilde{R}}_i$ *is the transpose of the i-th row of matrix* $\widetilde{\widetilde{R}}$. Σ *is the variance-covariance of exogeneous shock vector* ε_t.

Result 1 shows that the optimal portfolio choices are determined by a nonlinear equation systems. When the financial market in a peripheral country is separated from the other peripheral country, the asset market is incomplete.

Analogously, complete financial integration makes asset market complete up to a first-order approximation, in which agents in each country could achieve perfect risk sharing up to a first-order approximation. The optimal equities held by each country can be obtained by the following result.

Result 2. *When international financial markets are fully integrated, the optimal equities* $\widetilde{\alpha}$ *held by each country are determined by the following four equations,*

$$(D_{11}\widetilde{H} + D_{21})\Sigma\widetilde{\widetilde{R}}' = 0 \tag{3.57}$$

$$(D_{12}\widetilde{H} + D_{22})\Sigma\widetilde{\widetilde{R}}' = 0 \tag{3.58}$$

where $\widetilde{H} = \widetilde{\alpha}\widetilde{\widetilde{R}}$, $\widetilde{\widetilde{R}} = \widetilde{R}^{-1}\begin{bmatrix} R_{21} \\ R_{22} \end{bmatrix}$, *and* $\widetilde{R} = \mathbf{I}_2 - \begin{bmatrix} R_{21} \\ R_{22} \end{bmatrix}\widetilde{\alpha}$. Σ *is the variance-covariance of exogenous shock vector* ε_t.

Once obtain the optimal zeroth-order portfolio $\widetilde{\alpha}$, I have the final solution to the linearized rational expectations equilibrium. The results are summarized in the following proposition.

Proposition 4. *The solution to the linearized rational expectations equilibrium is as follows,*

$$y_t = D_3\begin{bmatrix} x_t \\ \Delta\widehat{Z}_{t-1} \end{bmatrix} + (D_1\widetilde{H} + D_2)\varepsilon_t \tag{3.59}$$

$$x_{t+1} = \frac{1}{\beta} x_t + S_1 \rho \Delta \widehat{Z}_{t-1} + S_2 y_t + (\widetilde{H} + S_1 \Phi)\varepsilon_t \tag{3.60}$$

$$\Delta \widehat{Z}_t = \rho \Delta \widehat{Z}_{t-1} + \Phi \varepsilon_t \tag{3.61}$$

where $S_1 = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} \Gamma_{A1} & \Gamma_{A2} \\ \Gamma_{A2} & \Gamma_{A1} \end{bmatrix}$, $S_2 = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} \Gamma_{A3} & \Gamma_{A4} \\ \Gamma_{A4} & \Gamma_{A3} \end{bmatrix}$ *and* $\Phi = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$. *The control variables are stacked in vector* $y_t = [\Delta \widehat{C}_{B,t}; \Delta \widehat{C}_{C,t}]$ *and endogenous state variables are stacked in* $x_t = [\widetilde{NW}_{B,t}; \widetilde{NW}_{C,t}]$. *Exogenous variables in (3.61) are included in* $\Delta \widehat{Z}_t = [\Delta \widehat{Z}_{B,t}; \Delta \widehat{Z}_{C,t}]$.

3.2.8 Impulse-responses

Based on the proposition 4, consumption in each country can be explicitly written as,

$$\begin{aligned} \widehat{C}_{A,t} =& \frac{1}{2} \frac{\theta + \eta}{\theta + \eta + (1-\theta)\sigma} [1,1] \left[S_1 \Phi + S_2 (D_1 \widetilde{H} + D_2) \right] \varepsilon_t \\ & - \frac{1-\alpha_1}{2} \frac{1+\eta}{\theta + \eta + (1-\theta)\sigma} [1,1] \left[\Gamma_{P1} \Phi - \Gamma_{P2} (D_1 \widetilde{H} + D_2) \right] \varepsilon_t \\ & + \frac{1+\eta}{\theta + \eta + (1-\theta)\sigma} [1,0,0] \varepsilon_t + others_t \\ \equiv & f_{CA} \varepsilon_t + others_t \end{aligned} \tag{3.62}$$

There are three innovation terms affecting unexpected change in consumption conditional on predetermined variables at date t, $others_t$. The top line on the right hand of equation (3.62) shows the *wealth effect*, which indicates the effect of unanticipated excess returns to external wealth on consumption due to unanticipated shocks. The second line displays the *terms of trade effect*, which states that consumption moves with the change of relative prices due to innovations. The third line reports the *income effect*, which comes from the change of output due to technological changes given all else constant. The income effect appears only when domestic innovation is present.

Consumption in the peripheral countries has a form of,

$$\widehat{C}_{B,t} = f_{CA} \varepsilon_t - \left[(D_1 \widetilde{H} + D_2) \right]_U \varepsilon_t + others_t \tag{3.63}$$

$$\widehat{C}_{C,t} = f_{CA} \varepsilon_t - \left[(D_1 \widetilde{H} + D_2) \right]_L \varepsilon_t + others_t \tag{3.64}$$

where $[\cdot]_U$ and $[\cdot]_L$ denote the upper row and lower row of matrix $[\cdot]$. These two consumption response functions come directly from the policy function for relative consumption in proposition 4.

Analogously, labor service in the central country has a similar exposition,

$$\begin{aligned}\widehat{N}_{A,t} = &-\frac{1}{2}\frac{\sigma}{\theta+\eta+(1-\theta)\sigma}[1,1]\left[S_1\Phi + S_2(D_1\widetilde{H}+D_2)\right]\varepsilon_t \\ &-\frac{1-\alpha_1}{2}\frac{1-\sigma}{\theta+\eta+(1-\theta)\sigma}[1,1]\left[\Gamma_{P1}\Phi - \Gamma_{P2}(D_1\widetilde{H}+D_2)\right]\varepsilon_t \\ &+\frac{1-\sigma}{\theta+\eta+(1-\theta)\sigma}[1,0,0]\varepsilon_t + others_t \\ \equiv &f_{NA}\varepsilon_t + others_t\end{aligned} \tag{3.65}$$

Notice that labor supply negatively responds to external wealth since higher external wealth implies higher consumption and then make leisure more desirable, which induces deduction of labor services in the labor market. The response of labor supply to price and technology changes depend on the elasticity of intertemporal substitution. When agents have log-preference $\sigma = 1$, income effect and terms of trade effect play no roles in determining labor services. Labor demands in the peripheral countries are characterized as follows,

$$\widehat{N}_{B,t} = f_{NA}\varepsilon_t - f_{NBR}\varepsilon_t + others_t \tag{3.66}$$

$$\widehat{N}_{C,t} = f_{NA}\varepsilon_t - f_{NCR}\varepsilon_t + others_t \tag{3.67}$$

where f_{NBR} and f_{NCR} are constant vectors given in the appendix.

3.2.9 Calibration and numerical results

As I have shown that the optimal portfolio held by each country is determined by nonlinear equations in result (1) and (2). In order to calculate the business cycle properties as well as to compare volatility of consumption and output across different financial architectures, I resort to numerical results instead. The parameters in table 3.5 used here are very standard and taken from the literature. The model period is assumed to be yearly. Thereby, the discount rate in the steady state is set as $\beta = 0.96$, with a 4% annualized interest rate. There are two parameters in the endogenous discount factor, ζ and ζ_0. ζ is set to be a small positive number, say, 0.001, and ζ_0 is chosen such that the steady state discount rate is $\beta = 0.96$. The elasticity of intertemporal substitution is chosen at $\sigma = 2$. The Frisch labor elastic-

Table 3.5: Parameter values

Parameter	Description	Value
β	Steady state discount rate	0.96
σ	Risk aversion	2
η	Frisch labor elasticity parameter	1
η_0	Coefficient for leisure	1
γ	Elasticity of substitution between home and foreign goods	2
$\alpha_1 = \alpha_2$	Share of home-traded goods	0.85
ζ	Parameter in the endogenous discount factor	0.001
ζ_0	Parameter in the endogenous discount factor	varying
$1-\theta$	Labor income share	$\frac{2}{3}$
ρ	Persistence of TFP shocks	0.90
σ_ϵ	Standard deviation of TFP shocks	1.36%

ity is $\eta = 1$. Elasticity of substitution between imports and local goods is assumed to be $\gamma = 2$ in the benchmark case. There is no consensus about this trade elasticity in the literature (see a discussion in chapter 2) and I take use of a wide range of values of trade elasticity in the following section as a sensitivity check. The expenditure share of home goods in the home consumption basket is set at $\alpha_1 = \alpha_2 = 0.85$, which implies that households in country A spend 85% of income on goods produced by country A and that household in country B and C spend $85\% \times 85\% = 72.25\%$ of income on locally produced goods. Labor income share is $1-\theta = 2/3$, which implies that dividend share is 1/3. Technologies are assumed to follow an AR(1) process with persistence $\rho = 0.90$ and a standard deviation of $\sigma_\epsilon = 1.36\%$.

First, let's look at the zeroth order portfolio held by each country under the benchmark calibration. Table 3.6 reports the cross-country portfolios under the partially financial integration and the global integration. When financial markets are partially integrated, households in country A sell short their own equities and have a large long position in foreign equities. Country B and Country C also sell short their own equities and have a large long position in equities of Country A. These optimal shares of equities seem unreasonable compared to those in the data. Like other researches (see chapter 2 for details), equity holdings in each country exhibit super foreign bias and are quite sensitive to structural parameters. When all financial markets are integrated into a united world financial market, households in

Table 3.6: Optimal equity shares held by each country under different financial architectures

	Partial integration			Global integration		
	Equity A	Equity B	Equity C	Equity A	Equity B	Equity C
Country A	-28.22	29.22	29.22	-23.75	24.75	24.75
Country B	14.61	-28.22	0.00	12.38	-29.99	6.24
Country C	14.61	0.00	-28.22	12.38	6.24	-29.99

each country alter their portfolios to obtain extra gains from risk sharing across borders. The right columns in table 3.6 illustrates this point. As in proposition 1, households in peripheral countries would hold more equities in the core country in partial financial integration than those in global financial integration.

Does financial integration reduce the volatilities of consumption and output? The answer is it depends. Table 3.7 shows the conditional volatility of consumption, output and real exchange rates. We can see that volatility of consumption in all countries significantly drop when world economies parade from financial autarky to partial financial integration. The extra gains from partial financial integration to global financial integration are mixed. In peripheral country B and C, volatility of consumption decreases; nevertheless, in country A, the volatility of consumption increases a little bit. We know that country A has access to global financial markets both in partial integration and global integration. The effect of financial integration in the third stage (global financial integration) on country A seems unclear since the market is incomplete in the second stage (partial financial integration). Nevertheless, in the peripheral countries, household could have access to more financial markets from financial autarky to partial integration, and then to global integration, which implies that households in these two countries can share their risk with the rest of world and thereby reduce the volatility of consumption. Labor supply has a similar pattern as consumption. Partial integration reduces labor volatility by a large extent. Global financial integration seems moderately increase the volatilities of labor in all countries.

Table 3.7: Conditional volatility (%): the benchmark

	Autarky	Partial integration	Global integration
Consumption in A	0.897	0.739	0.745
Consumption in B	0.675	0.637	0.629
Labor in A	0.449	0.294	0.295
Labor in B	0.587	0.389	0.394
GDP in A	1.063	1.257	1.251
GDP in B	0.851	1.014	0.998
RER in A	0.969	1.010	0.991
RER in B	0.774	0.823	0.832
TOT in A	1.384	1.443	1.416
TOT in B	1.215	1.296	1.315

Note: This table reports the conditional standard deviation for variables of interest in different financial regimes. Parameter values are the benchmark calibration. RER is effective real exchange rate and TOT is effective terms of trade.

Table 3.8: Conditional volatility (%): an alternative calibration

	Autarky	Partial integration	Global integration
Consumption in A	0.833	0.833	0.833
Consumption in B	0.833	0.833	0.833
Labor in A	0	0	0
Labor in B	0	0	0
GDP in A	1.36	1.36	1.36
GDP in B	1.36	1.36	1.36
RER in A	0	0	0
RER in B	0	0	0
TOT in A	1.666	1.666	1.666
TOT in B	1.696	1.696	1.696

Note: This table reports the conditional standard deviation for variables of interest in different financial regimes. Parameter values are the benchmark calibration except for $\sigma = \gamma = 1$ and $\alpha_1 = \alpha_2 = 0.5$. RER is effective real exchange rate and TOT is effective terms of trade.

Unitary trade elasticity

As discussed in the literature, when the elasticity of substitution between imports and exports, γ, is unity, terms of trade may effectively hedge against income risk across countries (Cole and Obstfeld 1991). Table 3.8 reports the second moments for variables of interest when $\gamma = \sigma = 1$. Not surprisingly, terms of trade can effectively hedge idiosyncratic income risk. Moreover, the effective real exchange rates are constant (when $\gamma = 1$) and labor supply is also unchanged (when $\gamma = \sigma = 1$). I can show that equities are perfect substitutes up to a first order approximation, which implies that international portfolios are indeterminate. In other words, any feasible portfolio choice is optimal for households in all countries.

Alternative shock processes

In the baseline calibration, I assume that the correlation between productivity innovations is zero. This assumption is suitable for developing countries in Latin America (see, table 1.2). Obverse that output among Asian economies (see table 1.3) seems more synchronized than other developing economies. In order to see whether the ranking of consumption volatility change with shock processes, I set the cross correlation to be $corr_t(\epsilon_{A,t+1}, \epsilon_{B,t+1}) = corr_t(\epsilon_{A,t+1}, \epsilon_{C,t+1}) = 0.4$ and $corr_t(\epsilon_{B,t+1}, \epsilon_{C,t+1}) = 0.7$. Numerical results show that the ranking of consumption volatility is

unchanged.

Sensitivity analysis

As in the literature, there is no consensus on the values of the trade elasticity and the elasticity of intertemporal substitution. I then vary these two values around their benchmark values. The results show that the rank of consumption volatility is the same as the benchmark case.

Notice that the first-order approximation of the model can't be used to analyze welfare changes across financial architectures, a complete welfare analysis is required to take a third order approximation to Euler equations for portfolios and a second approximation to other equations in the dynamic system. It is out of the scope of this chapter (see a work in progress, Yu 2012). However, volatility of consumption is still an important indicator for welfare analysis.

Chapter 4

Credit Friction, Asset Price and Optimal Monetary Policy

Financial frictions play an important role in propagating shocks across sectors and countries, particularly at the time of financial crises, in which households and firms face binding collateral constraints. The constrained borrowers are then forced to cut their borrowing and consequently to reduce their demand for goods and assets. This in turn generates quite volatile output and consumption. In terms of the perspective of consumption smoothing, large consumption volatility isn't a good thing. So there is room for policy makers to pursue optimal policies in order to stabilize the economy. This chapter will study how monetary and fiscal authorities quantitatively conduct optimal policies in the case of binding collateral constraints. I first outline the model in Section 2. After defining the equilibrium in Section 3, I specify functions used in the model and calibrate them in Section 4. In Section 5, I present the Ramsey steady state and discuss the optimal monetary and fiscal policy as well as second moments of several key variables. Section 6 displays the Ramsey optimal monetary policy and the optimized linear monetary policy rule. In Section 7, I investigate the welfare of alternative implementable interest rate rules. Conclusion is made in the final chapter.

4.1 The Model

There are two types of agents in the economy, households, and entrepreneurs, with the same population size of unit measure. Both of them are long-lived and risk averse.

4.1.1 Households

The household supplies l_t hours of labor, makes consumption decision c_t, purchases risk-free bonds b_{t+1} and chooses holdings of capital good k^s_{t+1} for

the next period, which consists of, for instance, housing and other durable goods. The preference for a household is given by,

$$E_0 \left\{ \sum_{t=0}^{+\infty} \beta^t U(c_t, k_t^s, l_t) \right\} \tag{4.1}$$

Consumption composite c_t is aggregated via a CES form,

$$c_t = \left[\int_0^1 c_{it}^{1-\frac{1}{\phi}} di \right]^{\frac{1}{1-\frac{1}{\phi}}} \tag{4.2}$$

where c_{it} is a consumption variety in the consumption composite. ϕ is the elasticity of substitution between consumption varieties.

Nominal price for consumption is P_t, which is aggregated by individual prices P_{it},

$$P_t = \left[\int_0^1 P_{it}^{1-\phi} di \right]^{\frac{1}{1-\phi}} \tag{4.3}$$

There are continuous labor markets with unit measure. Following Schmitt-Grohe and Uribe (2006), nominal wage W_t^j in labor market j is set by households or labor union. Households face the same labor demand curve and choose identical consumer goods and capital goods. Given the wages prevailing in labor markets, households allocate hours across different labor markets. The household supplies l_t^j hours to labor market j to satisfy demand for that sort of labor service,

$$l_t^j = \left(\frac{w_t^j}{w_t} \right)^{-\tilde{\phi}} l_t^d$$

where $w_t^j = \frac{W_t^j}{P_t}$, $w_t = \frac{W_t}{P_t}$. $\tilde{\phi}$ is the elasticity of substitution between labor services. l_t^d is total labor demanded by entrepreneurs. Total hours supplied by household i are,

$$l_t = \int_0^1 l_t^j dj = l_t^d \int_0^1 \left(\frac{w_t^j}{w_t} \right)^{-\tilde{\phi}} dj \tag{4.4}$$

The household's budget constraint reads,

$$c_t + q_t\left[k^s_{t+1} - (1-\delta)k^s_t\right] + b_{t+1} = \frac{b_t R_t}{\pi_t} + (1-\tau^f_t)(F_t + D_t) + (1-\tau^h_t) l^d_t \int_0^1 w^j_t \left(\frac{w^j_t}{w_t}\right)^{-\tilde{\phi}} dj \tag{4.5}$$

where q_t is the price of capital goods, R_t nominal gross interest rate, F_t profits from retailers, D_t profits from capital goods producers, $\pi_t = \frac{P_t}{P_{t-1}}$ gross price inflation, δ depreciation rate for capital goods, τ^f_t profit tax and τ^l_t labor income tax. The left hand side of equation (4.5) denotes a household's expenditure, which consists of consumption of final goods, change of capital goods plus replacement cost, purchase of bonds. The income for the household contains debt repayments from entrepreneurs, after-tax profits and after-tax labor income. Profits come from labor union and retailers, which are postponed to later.

Wages are set by labor union in terms of Calvo style. At each period a fraction of $\tilde{\alpha}$ markets could reset their nominal wages, while the remaining markets adjust wages according to $W^j_t = W^j_{t-1}\pi^{\tilde{\chi}}_{t-1}$ or $w^j_t = w^j_{t-1}\pi^{\tilde{\chi}}_{t-1}/\pi_t$, $0 \leq \tilde{\chi} \leq 1$ denotes the degree of indexation to inflation.

The household's objective is to choose allocations { c_t, l_t, b_{t+1}, w^j_t, k^s_{t+1} } to maximize preference (4.1), facing constraints (4.4) and (4.5). Let λ_t and $\frac{\lambda_t(1-\tau^l_t)w_t}{\mu_t}$ be multipliers for constraints (4.4), (4.5), respectively. The Lagrangian functions is,

$$\begin{aligned}\mathcal{L} =& E_0\{\sum_{t=0}^{+\infty}\beta^t\{U(c_t,k^s_t,l_t) + \lambda_t[\frac{b_t R_t}{\pi_t} + (1-\tau^f_t)F_t \\ &+ (1-\tau^h_t)l^d_t\int_0^1 w^j_t\left(\frac{w^j_t}{w_t}\right)^{-\tilde{\phi}} dj - c_t - q_t\left(k^s_{t+1} - (1-\delta)k^s_t\right) - \tau^{ls}_t - b_{t+1}] \\ &+ \frac{\lambda_t(1-\tau^l_t)w_t}{\mu_t}\left[l_t - l^d_t\int_0^1\left(\frac{w^j_t}{w_t}\right)^{-\tilde{\phi}} dj\right]\}\}\end{aligned} \tag{4.6}$$

F.O.N.C.s are as follows,

$$U_c(t) = \lambda_t \tag{4.7}$$

$$\lambda_t = \beta R_{t+1} E_t \left\{ \frac{\lambda_{t+1}}{\pi_{t+1}} \right\} \tag{4.8}$$

$$-U_l(t) = \frac{\lambda_t (1-\tau_t^l) w_t}{\mu_t} \tag{4.9}$$

$$\lambda_t q_t = \beta E_t \left\{ \lambda_{t+1} q_{t+1} (1-\delta) + U_k(t+1) \right\} \tag{4.10}$$

Equation (4.7) states that marginal utility of consumption equals the shadow price of real income. Equation (4.8) shows that a marginal increment of savings in period t increases the quantity of available goods at time $t+1$ by an amount of R_{t+1}/π_{t+1}, which has a discounted marginal value $\beta R_{t+1} E_t \left\{ \frac{\lambda_{t+1}}{\pi_{t+1}} \right\}$. The marginal benefit is equal to the marginal cost of the initial purchase of bond in period t, λ_t. The optimal condition for capital in equation (4.10) has a similar interpretation. For labor supply (4.9), marginal dis-utility of labor supply is equal to the marginal value of after-tax labor income divided by wage mark-up μ_t.

Notice that in a symmetric equilibrium, all labor markets are the same. Consider a particular labor market j in which wage could be reset to $\tilde{w}_t$ at period t. The Lagrangian function is written as,

$$\begin{aligned}
\mathcal{L}^w =& E_t \{ \sum_{s=0}^{+\infty} (\beta\tilde{\alpha})^s [\lambda_{t+s} \tilde{w}_{t+s} l_{t+s}^d \left(\frac{\tilde{w}_{t+s}}{w_{t+s}} \right)^{-\tilde{\phi}} (1-\tau_{t+s}^l) \\
& - \lambda_{t+s} \frac{(1-\tau_{t+s}^l) w_{t+s}}{\mu_{t+s}} l_{t+s}^d \left(\frac{\tilde{w}_{t+s}}{w_{t+s}} \right)^{-\tilde{\phi}}] \} \\
=& E_t \{ \sum_{s=0}^{+\infty} (\beta\tilde{\alpha})^s [\lambda_{t+s} \tilde{w}_t^{1-\tilde{\phi}} l_{t+s}^d \left(\frac{\prod_{k=1}^s \pi_{t+k-1}^{\tilde{\chi}}}{\prod_{k=1}^s \pi_{t+k}} \right)^{1-\tilde{\phi}} w_{t+s}^{\tilde{\phi}} (1-\tau_{t+s}^l) \\
& - \lambda_{t+s} \frac{(1-\tau_{t+s}^l) w_{t+s}}{\mu_{t+s}} l_{t+s}^d \tilde{w}_t^{-\tilde{\phi}} \left(\frac{\prod_{k=1}^s \pi_{t+k-1}^{\tilde{\chi}}}{\prod_{k=1}^s \pi_{t+k}} \right)^{-\tilde{\phi}} w_{t+s}^{\tilde{\phi}}] \}
\end{aligned} \tag{4.11}$$

Given prices $\{w_{t+s}, l_{t+s}^d, \tau_{t+s}^l, \lambda_{t+s}\}$ for all $s \geq 0$, the F.O.N.C.s can be

simplified as,

$$
\begin{aligned}
0 =& E_t \left\{ \sum_{t=0}^{+\infty} (\beta\tilde{\alpha})^t \lambda_{t+s} l_{t+s}^d \left(\frac{\tilde{w}_t}{w_{t+s}} \right)^{-\tilde{\phi}} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}} \right)^{\tilde{\phi}} \right. \\
& \left. \left[\frac{\tilde{\phi}-1}{\tilde{\phi}} \frac{(1-\tau_{t+s}^l)\tilde{w}_t}{\prod_{k=1}^{s} \frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}} - \frac{w_{t+s}(1-\tau_{t+s}^l)}{\mu_{t+s}} \right] \right\}
\end{aligned}
\tag{4.12}
$$

The first term in the bracket stands for the marginal utility from labor income by increasing one unit of labor service at period t. The second term in the bracket represents the marginal cost from labor supply. The optimal wage resetting rule requires that, after taking account of the probability of not being able to reset wages in all future periods, the expected discounted value of labor income equals the expected discounted cost of labor service.

Write the discounted expected utility from labor income as,

$$
f_t^1 \equiv \frac{\tilde{\phi}-1}{\tilde{\phi}} E_t \left\{ \sum_{t=0}^{+\infty} (\beta\tilde{\alpha})^t \lambda_{t+s} l_{t+s}^d \left(\frac{\tilde{w}_t}{w_{t+s}} \right)^{-\tilde{\phi}} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}} \right)^{\tilde{\phi}} \frac{(1-\tau_{t+s}^l)\tilde{w}_t}{\prod_{k=1}^{s} \frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}} \right\}
$$

and the discounted expected dis-utility from labor supply reads,

$$
f_t^2 \equiv E_t \left\{ \sum_{t=0}^{+\infty} (\beta\tilde{\alpha})^t \lambda_{t+s} l_{t+s}^d \left(\frac{\tilde{w}_t}{w_{t+s}} \right)^{-\tilde{\phi}} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}} \right)^{\tilde{\phi}} \frac{w_{t+s}(1-\tau_{t+s}^l)}{\mu_{t+s}} \right\}
$$

Write f_t^1 and f_t^2 in recursive forms,

$$
f_t^1 = \frac{\tilde{\phi}-1}{\tilde{\phi}} \tilde{w}_t \lambda_t (1-\tau_t^l) \left(\frac{w_t}{\tilde{w}_t} \right)^{\tilde{\phi}} l_t^d + \tilde{\alpha}\beta E_t \left[\left(\frac{\pi_{t+1}}{\pi_t^{\tilde{\chi}}} \right)^{\tilde{\phi}-1} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t} \right)^{\tilde{\phi}-1} f_{t+1}^1 \right] \tag{4.13}
$$

$$
f_t^2 = -U_l t \left(\frac{w_t}{\tilde{w}_t} \right)^{\tilde{\phi}} l_t^d + \tilde{\alpha}\beta E_t \left[\left(\frac{\pi_{t+1}}{\pi_t^{\tilde{\chi}}} \right)^{\tilde{\phi}} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t} \right)^{\tilde{\phi}} f_{t+1}^2 \right] \tag{4.14}
$$

$$
f_t^1 = f_t^2 \tag{4.15}
$$

4.1.2 Entrepreneurs

Entrepreneurs own firms. They hire labor services l_t^d from labor markets and purchase capital goods k_{t+1}^e as productive inputs in the next period from capital market. Output produced by entrepreneurs is the whole-sale good, which cannot be directly consumed. Production is of constant return to scale for a given level of technology level a_t,

$$Y_t = F(k_t^e, l_t^d; a_t) \tag{4.16}$$

The log of technology evolves as an AR(1) process with persistence of ρ_a,

$$ln(a_t/\bar{a}) = \rho_a ln(a_{t-1}/\bar{a}) + \epsilon_t^a \tag{4.17}$$

where ϵ_t^a follows as $i.i.d.$ normal distribution with mean zero and a standard deviation of σ_a. $\bar{a}$ is the value of a_t at its long-run steady state, which is normalized to be one.

Let the nominal price of the whole-sale good be P_t^w. The price markup for the whole-sale good then can be written as $X_t = \frac{P_t}{P_t^w}$, a similarly expression in Bernanke, Gertler and Gilchrist (1999).

Entrepreneurs lack of sufficient internal funds to finance purchases of capital goods. An alternative way of obtaining external funds is to issue corporate bonds. Nevertheless, because of contract enforcement problems, entrepreneurs can only issue an amount of corporate bonds up to a fraction of their holdings of capital goods. In the spirit of Kiyotaki and Moore (1997), entrepreneurs face a following borrowing constraint in nominal terms,

$$B_{t+1}^e \leq \kappa E_t \left(\frac{Q_{t+1} k_{t+1}^e}{R_{t+1}} \right) \tag{4.18}$$

where B_{t+1}^e is the nominal purchase of bonds at the end of period t, whose payoff in the following period is $B_{t+1}^e R_{t+1}$. Q_{t+1} stands for nominal price of capital at period $t+1$. Write it in real terms,

$$b_{t+1}^e \leq \kappa E_t \left(\frac{q_{t+1} k_{t+1}^e \pi_{t+1}}{R_{t+1}} \right) \tag{4.19}$$

where $b_{t+1}^e \equiv B_{t+1}^e / P_t$, $q_{t+1} \equiv Q_{t+1}/P_{t+1}$.

Entrepreneurs' preference is defined,

$$E_0 \left\{ \sum_{t=0}^{+\infty} (\beta\gamma)^t U(C_t^e) \right\} \tag{4.20}$$

with $< 0\gamma < 1$. They are less patient than households.

In a symmetric equilibrium, all entrepreneurs are identical. A representative entrepreneur faces a following budget constraint,

$$c_t^e + q_t \left[k_{t+1}^e - (1-\delta)k_t^e\right] + \frac{b_t^e R_t}{\pi_t} = (1-\tau_t^k)\left(\frac{Y_t}{X_t} - w_t l_t^d\right) + b_{t+1}^e \tag{4.21}$$

The left-hand side is an entrepreneur's expenditure in period t, which includes consumption, installment of new capital plus replacement cost, and debt payments. The income side contains returns to physical capital (the first term on the right-hand side) and newly issued corporate bonds.

The entrepreneur's objective is to choose $\{ c_t^e, l_t^d, b_{t+1}^e, k_{t+1}^e \}$ to maximize (4.20) subject to (4.19) and (4.21). Let $\tilde{\lambda}_t$ and $\tilde{\lambda}_t\varphi_t$ be multipliers for these two constraints respectively. The entrepreneurial Lagrangian function reads,

$$\begin{aligned}\mathcal{L}^e =& E_0\{\sum_{t=0}^{+\infty}(\beta\gamma)^t\{U(C_t^e) + \tilde{\lambda}_t[(1-\tau_t^k)\left(\frac{Y_t}{X_t} - w_t l_t^d\right) + b_{t+1} \\ & - c_t^e - q_t\left[k_{t+1}^e - (1-\delta)k_t^e\right] \\ & - \tau_t^{le} - \frac{b_t R_t}{\pi_t}] + \tilde{\lambda}_t\varphi_t\left(\kappa\frac{q_{t+1}k_{t+1}^e\pi_{t+1}}{R_{t+1}} - b_{t+1}^e\right)\}\}\end{aligned} \tag{4.22}$$

The optimality conditions include,

$$U_e(t) = \tilde{\lambda}_t \tag{4.23}$$

$$q_t\tilde{\lambda}_t = \tilde{\lambda}_t\varphi_t\kappa E_t\left(\frac{q_{t+1}\pi_{t+1}}{R_{t+1}}\right) + E_t\left\{\gamma\beta\tilde{\lambda}_{t+1}\left[q_{t+1}(1-\delta) + (1-\tau_{t+1}^k)\frac{F_{kt+1}}{X_{t+1}}\right]\right\} \tag{4.24}$$

$$\tilde{\lambda}_t = \tilde{\lambda}_t\varphi_t + E_t\left(\gamma\beta\frac{\tilde{\lambda}_{t+1}R_{t+1}}{\pi_{t+1}}\right) \tag{4.25}$$

$$\frac{F_{lt}}{X_t} = w_t \tag{4.26}$$

These equations have straightforward interpretations. In equation (4.23), marginal benefit of consumption is equal to the shadow price of real income, $\tilde{\lambda}_t$. The right-hand side of equation (4.24) states that a marginal increase of net worth in capital investment in period t enlarges both the quantity of available returns to capital at time $t+1$ by an amount of $q_{t+1}(1-\delta) + (1-\tau_{t+1}^k)\frac{F_{kt+1}}{X_{t+1}}$, which has a marginal value of $\tilde{\lambda}_{t+1}$, and the value of levered

investment opportunity at time t by the amount of $\varphi_t \kappa E_t \left(\frac{q_{t+1}\pi_{t+1}}{R_{t+1}}\right)$. The sum of these two effects equals the marginal value of wealth worth of one unit of capital in period t. Optimal debt issuance has a similar interpretation.

Combining equation (4.24) and (4.25), yields an expression for external finance premium φ_t,

$$\varphi_t = E_t \left\{ \gamma\beta \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \frac{\left[\frac{q_{t+1}(1-\delta)}{q_t} + (1-\tau^k_{t+1})\frac{F_{kt+1}}{q_t X_{t+1}}\right] - \frac{R_{t+1}}{\pi_{t+1}}}{1-\kappa E_t\left(\frac{q_{t+1}\pi_{t+1}}{q_t R_{t+1}}\right)} \right\} \tag{4.27}$$

which measures the value of relaxing the borrowing constraint or the value of levered investment opportunity. Notice that $\gamma\beta\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}$ is a representative entrepreneur's pricing kernel. The numerator in the last fraction denotes the excess return on holding capital goods relative to bond repayment at time $t+1$, multiplied by leverage ratio $1/\left(1-\kappa E_t\left(\frac{q_{t+1}\pi_{t+1}}{q_t R_{t+1}}\right)\right)$, which is the marginal value of relaxing borrowing constraint.

Capital goods are produced via using final consumption goods. The law of motion for aggregate capital is,

$$i_t = k_{t+1} - (1-\delta)k_t \tag{4.28}$$

Investment in capital goods incurs adjustment costs, $c(i_t)$. No arbitrage condition requires that price of capital goods should be the same as marginal cost of investment,

$$q_t = c'(i_t) + 1 \tag{4.29}$$

and profits from capital goods production are redistributed to households, which are equal to,

$$D_t = i_t q_t - (c(i_t) + i_t) \tag{4.30}$$

Labor services used by entrepreneurs are aggregated via a constant elasticity of substitution (CES) technology,

$$l^d_t = \left[\int_0^1 (l^{dj}_t)^{1-\frac{1}{\tilde{\phi}}} dj\right]^{\frac{1}{1-\frac{1}{\tilde{\phi}}}} \tag{4.31}$$

and nominal wage is accordingly given by,

$$W_t = \left[\int_0^1 (W^j_t)^{1-\tilde{\phi}} dj\right]^{\frac{1}{1-\tilde{\phi}}} \tag{4.32}$$

4.1.3 Retailers

Retailers purchase whole-sale goods from entrepreneurs in the competitive whole-sale goods market, differentiate them by color or brands without additional costs, and then sell them in the final good market. Market demand for brand i is $y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\phi} y_t$, where y_t is total absorption, which is defined as,

$$y_t = c_t + c_t^e + i_t + c(i_t) + g_t \tag{4.33}$$

where g_t is government expenditure.

Retailer of brand i obtains a profit of F_{it},

$$F_{it} = \left(\frac{P_{it}}{P_t} - \frac{1}{X_t}\right)\left(\frac{P_{it}}{P_t}\right)^{-\phi} y_t$$

When retailers get the chance to reset their prices, they choose $\tilde{P}_t$; otherwise, price will be adjusted with past inflation, $P_t = P_{t-1}\pi_{t-1}^{\chi}$. Retails' problem is to maximize the sum of expected discounted profits from period t onward and the Lagrangian function follows as

$$\begin{aligned}\mathcal{L}^r =& E_t \left\{ \sum_{s=0}^{+\infty} \alpha^s \Lambda_{t,t+s} P_{t+s} \left[\left(\frac{\tilde{P}_t}{P_t}\right)^{1-\phi} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right)^{1-\phi} y_{t+s} \right.\right. \\ & \left.\left. - \frac{1}{X_{t+s}} \left(\frac{\tilde{P}_t}{P_t}\right)^{-\phi} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right)^{-\phi} y_{t+s} \right] \right\}\end{aligned} \tag{4.34}$$

where $\Lambda_{t,t+s} \equiv \beta^s \frac{U_c(t+s)}{U_c(t)} \frac{P_t}{P_{t+s}}$

The F.O.N.C.s follow as,

$$\begin{aligned}0 =& E_t \left\{ \sum_{s=0}^{+\infty} \alpha^s \Lambda_{t,t+s} P_{t+s} \left[\left(\frac{\tilde{P}_t}{P_t}\right)^{1-\phi} \frac{1}{\tilde{P}_t} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right)^{1-\phi} y_{t+s} \right.\right. \\ & \left.\left. - \frac{\phi}{\phi-1} \frac{1}{X_{t+s}} \frac{1}{\tilde{P}_t} \left(\frac{\tilde{P}_t}{P_t}\right)^{-\phi} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right)^{-\phi} y_{t+s} \right] \right\}\end{aligned} \tag{4.35}$$

This equation has an analogous explanation as wage resetting. The expected discounted value of sale proceeds is equal to the expected discount cost of providing these final goods.

Rewriting the equation above into two parts, x_t^1 and x_t^2, yields,

$$\begin{aligned} x_t^1 &= \frac{\phi}{\phi-1} E_t \left[\sum_{s=0}^{+\infty} \alpha^s \Lambda_{t,t+s} \frac{P_{t+s}}{\tilde{P}_t} \frac{1}{X_{t+s}} \left(\frac{\tilde{P}_t}{P_t} \right)^{-\phi} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}} \right)^{-\phi} y_{t+s} \right] \\ &= \frac{\phi}{\phi-1} \frac{1}{X_t} y_t \tilde{p}_t^{-\phi-1} + \beta\alpha E_t \left\{ \frac{U_c(t+1)}{U_c(t)} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-\phi-1} \left(\frac{\pi_t^{\chi}}{\pi_{t+1}} \right)^{-\phi} x_{t+1}^1 \right\} \end{aligned} \tag{4.36}$$

where $\tilde{p}_t = \frac{\tilde{P}_t}{P_t}$.

$$\begin{aligned} x_t^2 &= E_t \left[\sum_{s=0}^{+\infty} \alpha^s \Lambda_{t,t+s} \frac{P_{t+s}}{\tilde{P}_t} \left(\frac{\tilde{P}_t}{P_t} \right)^{1-\phi} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}} \right)^{1-\phi} y_{t+s} \right] \\ &= y_t \tilde{p}_t^{-\phi} + \beta\alpha E_t \left\{ \frac{U_c(t+1)}{U_c(t)} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-\phi} \left(\frac{\pi_t^{\chi}}{\pi_{t+1}} \right)^{1-\phi} x_{t+1}^2 \right\} \end{aligned} \tag{4.37}$$

$$x_t^1 = x_t^2 \tag{4.38}$$

Total profits distributed to households are,

$$\begin{aligned} F_t &= \int_0^1 F_{it} di = \int_0^1 \left(\frac{P_{it}}{P_t} - \frac{1}{X_t} \right) \left(\frac{P_{it}}{P_t} \right)^{-\phi} y_t di \\ &= y_t \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{1-\phi} di - \frac{y_t}{X_t} \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\phi} di \\ &= y_t - \frac{y_t}{X_t} s_t^p \end{aligned} \tag{4.39}$$

where s_t^p is price dispersion and is defined as,

$$
\begin{aligned}
s_t^p &\equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\phi} di \\
&= (1-\alpha)\left(\frac{\tilde{P}_t}{P_t}\right)^{-\phi} + \alpha(1-\alpha)\left(\frac{\tilde{P}_{t-1}\pi_{t-1}^{\chi}}{P_t}\right)^{-\phi} + \alpha^2(1-\alpha)\left(\frac{\tilde{P}_{t-2}\pi_{t-1}^{\chi}\pi_{t-2}^{\chi}}{P_t}\right)^{-\phi} \\
&\quad + \cdots \\
&= (1-\alpha)\tilde{p}_t^{-\phi} + \alpha\left(\frac{\pi_t}{\pi_{t-1}^{\chi}}\right)^{\phi} s_{t-1}^p
\end{aligned}
\tag{4.40}
$$

4.1.4 Government

Government collects tax revenues τ_t and issues government bond b_{t+1}^g to finance government expenditure g_t. Tax revenues consist of labor income tax, income tax from the entrepreneurial sector and profit tax,

$$
\begin{aligned}
\tau_t &= \tau_t^h l_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\phi}} dj + \tau_t^f (F_t + D_t) + \tau_t^k \left(\frac{Y_t}{X_t} - w_t l_t^d\right) + \tau_t^{lh} + \tau_t^{le} \\
&= \tau_t^h l_t^d w_t + \tau_t^f F_t + \tau_t^k \left(\frac{Y_t}{X_t} - w_t l_t^d\right)
\end{aligned}
\tag{4.41}
$$

Flow of funds for the government in a cashless economy is given by,

$$
b_{t+1}^g - \frac{b_t^g R_t}{\pi_t} = g_t - \tau_t \tag{4.42}
$$

Let $n_{t+1} \equiv R_{t+1} b_{t+1}^g$ be government's liability at the end of period t. Government's flow of funds can be written as,

$$
\frac{n_{t+1}}{R_{t+1}} + \tau_t = g_t + \frac{n_t}{\pi_t} \tag{4.43}
$$

Government expenditure is driven by an exogenous process,

$$
ln(g_t/g) = \rho_g ln(g_{t-1}/g) + \epsilon_t^g \tag{4.44}
$$

where g denotes government's steady state expenditure.

4.1.5 Aggregation and market clearing conditions

Law of large number implies that nominal price evolution follows as,

$$P_t^{1-\phi} = \alpha(P_{t-1}\pi_{t-1}^{\chi})^{1-\phi} + (1-\alpha)\tilde{P}_t^{1-\phi} \tag{4.45}$$

Writing it in real terms, yields,

$$1 = \alpha(\pi_{t-1}^{\chi}/\pi_t)^{1-\phi} + (1-\alpha)\tilde{p}_t^{1-\phi} \tag{4.46}$$

Whole-sale good is transformed to final goods via a one-for-one technology. Demand for whole-sale good is,

$$Y_t = y_t \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\phi} di = s_t^p y_t \tag{4.47}$$

Similarly, nominal wage evolves as,

$$w_t^{1-\tilde{\phi}} = \tilde{\alpha} w_{t-1}^{1-\tilde{\phi}} (\pi_{t-1}^{\tilde{\chi}}/\pi_t)^{1-\tilde{\phi}} + (1-\tilde{\alpha})\tilde{w}_t^{1-\tilde{\phi}} \tag{4.48}$$

Labor market clears,

$$l_t = \int_0^1 l_t^j dj = l_t^d \int_0^1 \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\phi}} dj = l_t^d s_t^w \tag{4.49}$$

where wage dispersion is evolving as,

$$s_t^w = (1-\tilde{\alpha})\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\phi}} + \tilde{\alpha}\left(\frac{w_{t-1}}{w_t}\right)^{-\tilde{\phi}} \left(\frac{\pi_t}{\pi_{t-1}^{\tilde{\chi}}}\right)^{\tilde{\phi}} s_{t-1}^w \tag{4.50}$$

Capital market clearing condition reads,

$$k_{t+1} = k_{t+1}^e + k_{t+1}^s \tag{4.51}$$

Bond market clearing condition is,

$$b_{t+1} = b_{t+1}^e + b_{t+1}^g \tag{4.52}$$

In the case with nominal rigidities, an interest rate rule is applied here,

$$ln\left(\frac{R_{t+1}}{R}\right) = \alpha_\pi ln\left(\frac{\pi_t}{\pi}\right) + \alpha_{\pi_w} ln\left(\frac{\pi_{Wt}}{\pi_W}\right) + \alpha_y ln\left(\frac{y_t}{y}\right) + \alpha_q ln\left(\frac{q_t}{q}\right) + \rho_r ln\left(\frac{R_t}{R}\right) \tag{4.53}$$

where α_π, α_{π_w}, α_y, α_q and ρ_r are coefficients for the corresponding target variables.

4.2 Characterizing the equilibrium

4.2.1 Competitive equilibrium

A symmetric equilibrium is defined in the usual way. Given the household's, entrepreneur's and government's initial debts, { b_0, b_0^e, b_0^g }, initial capital holdings { k_0^s, k_0^e }, initial price and wage dispersion { s_0^p, s_0^w } and the stochastic processes, $\{g_t, a_t\}$, a symmetric equilibrium is an allocation, { c_t, k_{t+1}^s, b_{t+1}, l_t, μ_t, λ_t, $\tilde{w}_t$, f_t^1, f_t^2, $\tilde{p}_t$, x_t^1, x_t^2, F_t, D_t, k_{t+1}^e, l_t^d, Y_t, b_{t+1}^e, φ_t, c_t^e, $\tilde{\lambda}_t$, i_t, n_{t+1}, τ_t, k_{t+1}, y_t }, price system, { π_t, w_t, q_t, X_t }, evolvement of price and wage dispersion, { s_t^w, s_t^p }, and government policy, { τ_t^l, τ_t^k, τ_t^f, R_{t+1} }, such that:

- { c_t, k_{t+1}^s, b_{t+1}, l_t, μ_t, λ_t, $\tilde{w}_t$, f_t^1, f_t^2 } solve the household's problem subjective to the sequence of household budget constraints and staggered wage setting;
- { k_{t+1}^e, l_t^d, Y_t, b_{t+1}^e, φ_t, c_t^e, $\tilde{\lambda}_t$, i_t } solve the entrepreneur's problem subjective to the sequence of entrepreneur's flow of funds and borrowing constraints;
- { $\tilde{p}_t$, x_t^1, x_t^2, F_t } solve retailer's problem constrained by staggered price setting;
- { n_{t+1}, τ_t } satisfy government budget constraints, given government policy;
- the whole-sale good market clears
- the final goods markets clear;
- the labor markets clear;
- the capital market clears;

- the bond market clear;
- and $R_t \geq 1$, and $\varphi_t \geq 0$, for all $t \geq 0$

The competitive equilibrium satisfies equations (4.7)-(4.10), (4.13)-(4.15), (4.16), (4.19), (4.21), (4.23)-(4.26), (4.28), (4.29), (4.30), (4.33), (4.36)-(4.40), (4.41), (4.43), (4.46), (4.47), (4.48), (4.49), (4.50), (4.51), (4.52), exogenous processes, (4.17), (4.44), and government policy.

4.2.2 The Ramsey equilibrium

The Ramsey planner's problem is to find the fiscal and monetary policies such that in the competitive equilibrium, the weighted sum of the household's and entrepreneur's expected lifetime utility reaches the highest level. In choosing optimal policies, the government commits to honor its chosen policies in the past. In all periods, private agents maximize their objective functions taking the whole policy plan as given.

Formally, the Ramsey equilibrium is a set of allocations $\{ c_t, k^s_{t+1}, b_{t+1}, l_t, \mu_t, \lambda_t, \tilde{w}_t, f^1_t, f^2_t, \tilde{p}_t, x^1_t, x^2_t, F_t, D_t, k^e_{t+1}, l^d_t, Y_t, b^e_{t+1}, \varphi_t, c^e_t, \tilde{\lambda}_t, i_t, n_{t+1}, \tau_t, k_{t+1}, y_t \}$, price system, $\{ \pi_t, w_t, q_t, X_t \}$, evolvement of price dispersion, $\{ s^w_t, s^p_t \}$ and government policy, $\{ \tau^l_t, \tau^k_t, \tau^f_t, R_{t+1} \}$, for all $t \geq 0$ that maximize,

$$W_0 \equiv E_0 \left\{ \sum_{t=0}^{+\infty} \left[\omega \beta^t U(c_t, k^s_t, l_t) + (1-\omega)(\beta\gamma)^t U(C^e_t) \right] \right\} \tag{4.54}$$

where ω is the weight on households in the planer's objective function. The complete set of equilibrium conditions are listed in the appendix.

4.3 Specification and calibration

Preferences for households and entrepreneurs have forms of,

$$U(c_t, k^s_t, l_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + jln(k^s_t) - \frac{l_t^{1+\theta}}{1+\theta}$$

$$U(C^e_t) = \frac{(c^e_t)^{1-\sigma} - 1}{1-\sigma}$$

Production follows as,

$$F(k^e_t, l^d_t; a_t) = a_t (k^e_t)^{\alpha_0} (l^d_t)^{1-\alpha_0}$$

Investment adjustment cost is set as,

$$c(i_t) = \frac{\psi}{2}\left(\frac{i_t}{k_t}\right)^2 k_t$$

Parameters in this system are { σ, β, γ, j, θ, δ, ψ, α_0, ϕ, $\tilde{\phi}$, κ, α, $\tilde{\alpha}$, χ, $\tilde{\chi}$, $\bar{a}$, g, ρ_a, σ_a, ρ_g, σ_g }.

We need extra constraints to figure out the parameters. Parameters are divided into two groups. The first group of parameters are chosen from the literature. Preferences are set as log-preferences for both households and entrepreneurs, $\sigma = 1$. $\beta = 1.04^{-1/4}$ such that the annual real interest rate is 4% when inflation is zero. Risk-spread is set to be 2% which induces $\gamma = 0.98$. Frisch labor elasticity is set at $\theta = 0.47$. Depreciation rate is $\delta = 0.025$. Share of capital in production is chosen to be $\alpha_0 = 0.3$. As in Christiano, Eichenbaum and Evans (2005), elasticity of substitution between final goods is $\phi = 6$, and elasticity of substitution across labor services is set as $\tilde{\phi} = 21$. The probability of resetting price is $\alpha = 0.6$, and $\tilde{\alpha} = 0.64$ for wage readjustment. Indexation to inflation for price $\chi = 0$, and $\tilde{\chi} = 0$, in the benchmark model. Technology level is normalized to be $\bar{a} = 1$. Technology shocks and government expenditure shocks are taken from Schmitt-Grohe and Uribe (2006), $\rho_a = 0.8556$, $\sigma_a = 0.0064$, $\rho_g = 0.87$, $\sigma_g = 0.016$. Loan-to-value $\kappa = 0.80$, which is consistent with literature, i.e., Iacoviello (2002). The weight of households in the social welfare function is set be to 1/2 in the benchmark model.

Following Mendoza, Razin and Tesar (1994), income tax rates are set as $\tau^k = 0.407$ and $\tau^h = 0.258$. I set $\tau^f = \tau^k$, and lump-sump taxes are not applicable.

The second group of parameters consists of the elasticity of utility with respect to capital goods consumption j, investment adjustment cost ψ, and government expenditure at the deterministic steady state g. I choose j and ψ such that investment-GDP ratio is $s_i = 16.03\%$, which is the quarterly average value of fixed investment-GDP ratio in USA over $1951 - 2009$, and holdings of capital by households to holdings of capital by entrepreneurs ratio s_e is 1.32 in USA, which is defined as $\frac{\text{(residential fixed assets + consumer durables)}}{\text{nonresidential fixed assets}}$. Government expenditure-GDP ratio is set to be $s_t = 17\%$, based on which government expenditure is solved. Table 4.1 lists structural parameters used by this chapter. The appendix provides details of deterministic steady state.

Table 4.1: Structural parameters

Parameter	Value	Description
β	$(1.04)^{-1/4}$	Households' subjective discount factor (quarterly)
σ	1	Elasticity of intertemporal substitution
γ	0.98	Entrepreneur's relative subjective discount factor (quarterly)
θ	0.47	Frisch labor elasticity
δ	0.025	Depreciation rate (quarterly)
ϕ	6	Price-elasticity of final goods varieties
$\tilde{\phi}$	21	Wage-elasticity of labor varieties
κ	0.8	Loan-to-Value ratio
α	0.6	Fraction of firms not setting prices optimally
$\tilde{\alpha}$	0.64	Fraction of labor markets not setting wages optimally
χ	0	Degree of price indexation
$\tilde{\chi}$	0	Degree of wage indexation
α_0	0.3	Share of capital in production
J	0.2652	Preference parameter
ψ	40.5305	Capital adjustment cost parameter
σ_a	0.0064	Std. dev. of technology innovation
ρ_a	0.8556	Serial correlation of technology innovation
σ_g	0.016	Std. dev. of government expenditure
ρ_g	0.87	Serial correlation of government expenditure
ω	0.5	Social welfare parameter

4.4 Ramsey steady state and dynamics

4.4.1 Ramsey steady state

In the deterministic long-run steady state of the Ramsey equilibrium, all endogenous variables keep constant. Table 4.2 displays the Ramsey steady-state values of inflation, nominal interest rate, labor and capital income tax rates under a few environments of interests.

Consider first a case in which profit tax rate is the same as capital income tax in an economy with credit frictions, shown in the first line of table 4.2. The steady-state inflation is zero because nominal rigidities such as price stickiness, wage stickiness and nominal interest rate are completely forecastable in the deterministic steady state. Capital income tax rate is negative, an order of -5.2% rather than zero. The intuition is that monopolistic competition in the final goods markets provides also monopolistic power with entrepreneurs who use capital goods in their production. A Subsidy to capital-goods could improve the efficiency of capital goods market. This finding is consistent with Schmitt-Grohe and Uribe (2006) who get a subsidy of -6.3%. Tax rate on labor income is 52%, which is higher than that of Schmitt-Grohe and Uribe (2006).

Furthermore, I investigate whether credit frictions could affect the optimal tax rate in the Ramsey steady state. The second line in table 4.2 reports that when there are no credit frictions, i.e., $\varphi = 0$, inflation is still zero, and subsidy to capital income becomes -5.8%, similar to the case with credit frictions, while labor income tax rate drops dramatically to 26%. The reason is that capital is efficiently allocated (in the sense of absence of credit constraints) in an economy without credit frictions, thereby labor supply may be higher in that economy and so do output and consumption. Government can still finance its expenditure via a lower labor income tax rate.

Consider an income tax regime, in which all of incomes are taxed by the same rates, $\tau_f = \tau_k = \tau_l$. The last line of table 4.2 illustrates that income tax rate is around 7% in an economy with credit frictions. In the following analysis, I will focus on a regime without fiscal policy.

Table 4.2: Ramsey steady state

Economy				Steady State (%)			
τ_f	χ	$\tilde{\chi}$	φ	π	R	τ_h	τ_k
τ_k	0	0	$1-\gamma$	0.0	4.00	50.89	-5.16
τ_k	0	0	0	0.0	4.00	26.32	-5.78
$\tau_k = \tau_l$	0	0	$1-\gamma$	0.0	4.00	6.90	6.90

4.4.2 Ramsey dynamics

In this section, I will study the cyclical implications under the Ramsey monetary policy. I shut down active fiscal policies in this section. Table 4.3 shows the steady state, standard deviation, serial correlation, and correlation with output for a number of variables of interest. The second moments are computed using Monte Carlo simulations. I perform 1000 simulations of 200 periods for each variable. The second moments are averages over the 1000 simulations. Note that the standard deviation of inflation and interest rate are at moderate levels.

In the real business cycle literature with financial frictions, output usually negatively correlates with external finance premium, in which positive (negative) technology shocks make entrepreneurs wealthier (poorer) and their associated external finance premia decrease (increase) with the rise (decrease) of net worth, which in turn enhances (drives down) output in the economy. When nominal debt is introduced into the financial market, another concern is the debt deflation. When there are time-varying nominal prices, real wealth will be redistributed between debtors and creditors. Accordingly, the credit constrained debtors have to accommodate asset holdings, i.e., capital here, which in turn propagates shocks through this collateral channel.

Table 4.3: Cyclical implications of optimal policy

Variable	Steady State	Standard Deviation	Serial Correlation	Correlation with Output
R_t	4	0.49	-0.2	-0.17
π_t	0	0.93	0.37	-0.28
y_t	1.31	1.18	0.92	1
c_t	0.97	0.96	0.92	1
c_t^e	0.05	0.91	0.93	0.98
φ_t	0.02	2.14	0.5	0.46
q_t	2.18	0.75	0.91	0.95
i_t	0.18	1.46	0.92	1
k_t^e	3.85	0.47	0.99	0.29
l_t	0.82	0.39	0.75	0.54
w_t	0.93	0.91	0.95	0.98

Note: R_t and π_t are expressed in percentage per year. The steady state values of other variables are expressed in levels. The second moments in the table corresponding to percent deviations from their steady state values.

In a model with sticky price, nominal debt and borrowing constraint, policy-makers trade off price dispersion cost and debt inflation. Iacoviello (2002) analyzes an economy in this kind of environment. He finds that nominal debt could reduce the volatilities of output and inflation for a number of shocks. However, his results are not based on social welfare maximization. From the point of view of social welfare, volatilities of consumption and labor supply are not good things because welfare function is concave in consumption and labor supply. In the current environment, monetary policy affects the economy through three channels. The first is price rigidity, which requires low volatility of inflation. The second is wage rigidity, which requires changes of inflation to make real wages reach their efficient levels. The last one works via the collateral channel, through which inflation volatility could accelerate or decelerate propagation of shocks. When the endogenous variables are stationary processes, large volatility maybe reduce ex ante welfare. If this happens, the Ramsey planner could use monetary policy to offset this large volatility through the collateral channel. Two variables from table 4.3, external finance premium and price of capital, illustrate this point. In the benchmark model, volatility of external finance premium is 2.1%, which is nearly twice as output while asset price is less volatile than output. Moreover, asset price is highly serially correlated but the serial correlation of external risk premium is moderate. The most surprising result is the positive correlation between output and external finance premium, which implies that entrepreneurs' net worth decreases when there are positive technology shocks. The intuition is that, with the Ramsey policy, inflation drops a lot when there are positive technology shocks hitting the economy, and hence debt deflation makes entrepreneurs pay more to creditors, although returns to capital increase associated with positive technology shocks. The debt deflation channel dominates the higher returns to capital and thus entrepreneurs' net worth is lower than their steady state level; hence it follows an rise in external finance premium.

Consider then how nominal rigidities affect asset prices and external finance premium. Panel A in table 4.4 shows the second moments of variables of interest under different price and wage rigidities when debts are issued in nominal terms. Under flexible price and wage ($\alpha = 0$, $\tilde{\alpha} = 0$), inflation volatility is highly large while volatility of nominal interest rate is pretty small, which is consistent with the findings in literature, i.e., Schmitt-Grohe and Uribe (2006). However, nominal debts still play a role in buffering volatility of output and consumption. The standard deviation of external finance premium is $\varphi = 2\%$ and highly positively correlates with output.

When there is only price stickiness in the economy, ($\alpha = 0.6$, $\tilde{\alpha} = 0$),

Table 4.4: Degree of nominal rigidity and optimal policy

α	$\tilde{\alpha}$	Statistics	R_t	π_t	y_t	φ_t	q_t	i_t
Panel A: Nominal Debt								
0.64	0.6	Mean	4	0	1.31	0.02	2.18	0.18
		STD	0.49	0.93	1.18	2.14	0.75	1.46
		Ser. Corr.	-0.2	0.37	0.92	0.5	0.91	0.92
		Corr. with y_t	-0.17	-0.28	1	0.46	0.95	1
0	0.64	STD	0.56	2.29	1.38	2.33	0.89	1.73
		Ser. Corr.	-0.2	0.01	0.81	0.72	0.78	0.8
		Corr. with y_t	-0.26	-0.36	1	0.93	0.95	1
0.6	0	STD	1.42	0.62	1.3	0.98	0.84	1.63
		Ser. Corr.	-0.1	-0.24	0.91	0.07	0.89	0.9
		Corr. with y_t	0.17	-0.02	1	-0.4	0.95	1
0	0	STD	0.12	1.99	1.36	1.75	0.88	1.71
		Ser. Corr.	0.83	-0.07	0.85	0.73	0.83	0.85
		Corr. with y_t	-0.85	-0.3	1	0.93	0.95	1
Panel B: Indexed Debt								
0.64	0.6	Mean	4	0	1.31	0.02	2.18	0.18
		STD	1.84	1	1.22	2.74	0.79	1.54
		Ser. Corr.	-0.18	0.3	0.86	0.34	0.83	0.84
		Corr. with y_t	0.25	-0.05	1	-0.05	0.95	1
0	0.64	STD	2.61	4.21	1.48	5.97	0.98	1.89
		Ser. Corr.	-0.23	-0.33	0.81	0.68	0.77	0.79
		Corr. with y_t	0.3	0.26	1	-0.69	0.95	1
0.6	0	STD	2.15	0.8	1.39	2.82	0.92	1.77
		Ser. Corr.	-0.17	-0.36	0.85	0.34	0.82	0.84
		Corr. withy_t	0.26	0.33	1	-0.75	0.95	1
0	0	STD	2.55	-	2.34	50.92	1.63	3.12
		Ser. Corr.	-0.1	-	0.88	0.65	0.86	0.87
		Corr. with y_t	0.14	-	1	-0.84	0.96	1

Note: R_t and π_t are expressed in percentage per year. The steady state values of other variables are expressed in levels. The second moments in the table corresponding to percent deviations from their steady state values.

monetary policy should remedy the distortion of price dispersion in goods market. Inflation volatility drops dramatically and nominal interest rate is more volatile than the flexible case. On the other hand, lower inflation volatility makes debt inflation channel silent and external finance premium then negatively co-moves with output.

When wage stickiness appears only in the economy, ($\alpha = 0$, $\tilde{\alpha} = 0.64$), monetary policy needs to improve labor market efficiency through moving real wage close to the efficient level. Inflation volatility increases and so does external finance premium.

In the benchmark model, $\alpha = 0.6$, $\tilde{\alpha} = 0.64$, volatilities of inflation and nominal interest rate are between the case with flexible wages and the case with flexible prices. External finance premium also lie in that range but with a positive correlation with output.

Across different cases, volatilities of asset price, output and net investment don't change too much and their correlation with output are quite similar across different environments. The reason is that asset price plays a role in allocating capital between households and entrepreneurs, and so does investment. A rise in asset price will decrease the demand for capital and a fall in asset price has an opposite effect. Collateral constraints make asset price more volatile in real business cycles (see the bottom line in Panel B of table 4.4). Nevertheless, debt deflation could dump this kind of volatility. That is why the correlation between inflation and output is negative and significant, and external finance premium positively moves with output.

In panel B of table 4.4, I shut down debt deflation channel, that is, nominal debt is fully indexed to inflation and hence nominal interest rate is thereby the real interest rate. In the benchmark case, $\alpha = 0.6$, $\tilde{\alpha} = 0.64$, the volatility of variables in the table increases because without debt deflation entrepreneurs needn't repay as much as the case with debt deflation. The financial accelerator via an increase in entrepreneurs' net worth drives the economy much volatile, which holds across regimes with different nominal rigidities. Notice that when there is only price stickiness in an economy, the volatility of inflation is still positive, 0.8%, rather than zero. The reason is that we take an approximation to the dynamic system around its distorted steady state, where monopolistic power exists both in retail sector and labor union, beyond credit frictions. Consequently, there is a tradeoff between inflation volatility and output volatility.[41] When both prices and wages

[41] I also consider a case in which subsidies to monopolistic price-setters and wage-setters are applicable in the deterministic steady state. The results which are not reported here display that distortion of credit frictions dominates the distortion of monopolistic power both in goods and labor markets. The relative large positive volatility of inflation reflects

are flexible, the economy with fully inflation-indexed debts exhibits exactly the same second moments as in a real business cycle model. Volatilities of output, external finance premium, asset price and net investment are much higher than those with nominal rigidities.

Therefore, the basic results in this section are that Ramsey planner is willing to bear some cost of price dispersion and/or wage dispersion to reduce the volatility of endogenous variables since large volatility of variables is more undesirable comparing to the inefficiency of output from nominal rigidities. When nominal debt is introduced into the economy, debt deflation provides Ramsey policy-maker with another channel to push down the aggregate volatility through the financial decelerator.

4.5 Ramsey and optimized impulse responses

In this section, I will find the optimized linear interest rate rule which contains only observable macroeconomic variables. I define the distance between the competitive equilibrium under a linear rule and the Ramsey equilibrium as follows. Let IR_T^R denotes the impulse-responses of variables associated with the Ramsey equilibrium of length T quarters. Analogously, let IR_T^{CE} denotes the impulse-responses associated with the competitive equilibrium of length T quarters under a particular linear rule. Let the vectorized difference be $x \equiv vec(IR_T^R - IR_T^{CE})$. The linear rule that minimizes the distance $x^T x$ is called the optimized linear rule.

The optimized linear rule is given by,[42]

$$\begin{aligned}\log\left(\frac{R_{t+1}}{R}\right) =& 0.154\log\left(\frac{\pi_t}{\pi}\right) + 0.0874\log\left(\frac{\pi_{Wt}}{\pi_W}\right) + 0.0048\log\left(\frac{y_t}{y}\right) \\ &- 0.0145\log\left(\frac{q_t}{q}\right) + 0.797\log\left(\frac{R_t}{R}\right)\end{aligned} \tag{4.55}$$

which exhibits a passive response to inflation. Notice that nominal interest rate negatively responds to the change of asset prices. The logic is similar to Faia and Monacelli (2007) but on an opposite argument. Asset price volatility in an economy with credit frictions is large because of collateral

a large gap between the distorted steady state and the efficient steady state.

[42]Credit spread between external financial premium and risk free rate is quite small. Including the spread in the linearized interest rule wound't change the coefficients for other variables.

constraints, which in turn increase the volatilities of capital allocations between households and entrepreneurs, and so does investment. The increase in investment volatility in turn induces an increase in consumption volatility and therefore reduces ex ante social welfare. The Ramsey policy aims to reduce the volatility of output and consumption. Associated with a decrease of interest rate responding to ascending in asset price, households are less willing to lend entrepreneurs some funds, and therefore, demand for capital in intermediate goods production is restrained. The volatility of consumption is brought down. Notice that asset prices positively move with output. Consequently, the negative response of interest rate to asset price is not large, conditional on output gap. In other words, once we control for inflation rate and output gap, the optimal interest rate doesn't necessarily respond to asset prices.

Figure 4.1 displays the responses of variables of interest to an unexpected one-percent positive technology shock a_t at date t. The solid lines denote responses in the Ramsey equilibrium and the dashed line stands for responses in the competitive equilibrium associated with the optimized linear rule (4.55). The equilibrium dynamics of non-policy variables in the optimized linear rule mimic those associated with the Ramsey equilibrium quite well.

Labor supply drops nearly -0.6% on impact in the Ramsey policy. The reason is that because of deflation, households obtain much real wealth transfer from their nominal debt payment by entrepreneurs. This kind of income effect dominates the substitution effect which comes from higher real wages. Accordingly, labor supply drops while consumption increases. The response of output jumps up to 0.6% on impact because of reduction of labor supply. The Ramsey planner responds to the positive technology shock by tightening monetary policy, while the optimized linear rule calls for an easing of monetary policy. A deflation immediately follows by the tightening monetary policy. With the easing of monetary conditions, inflation goes gradually back to its steady state level. At the same time, the associated external finance premium increases on impact because entrepreneurs' net worth declines for the sake of debt deflation. In aggregate, entrepreneurs can still borrow 0.4% of their initial debts to finance their purchases of capital.

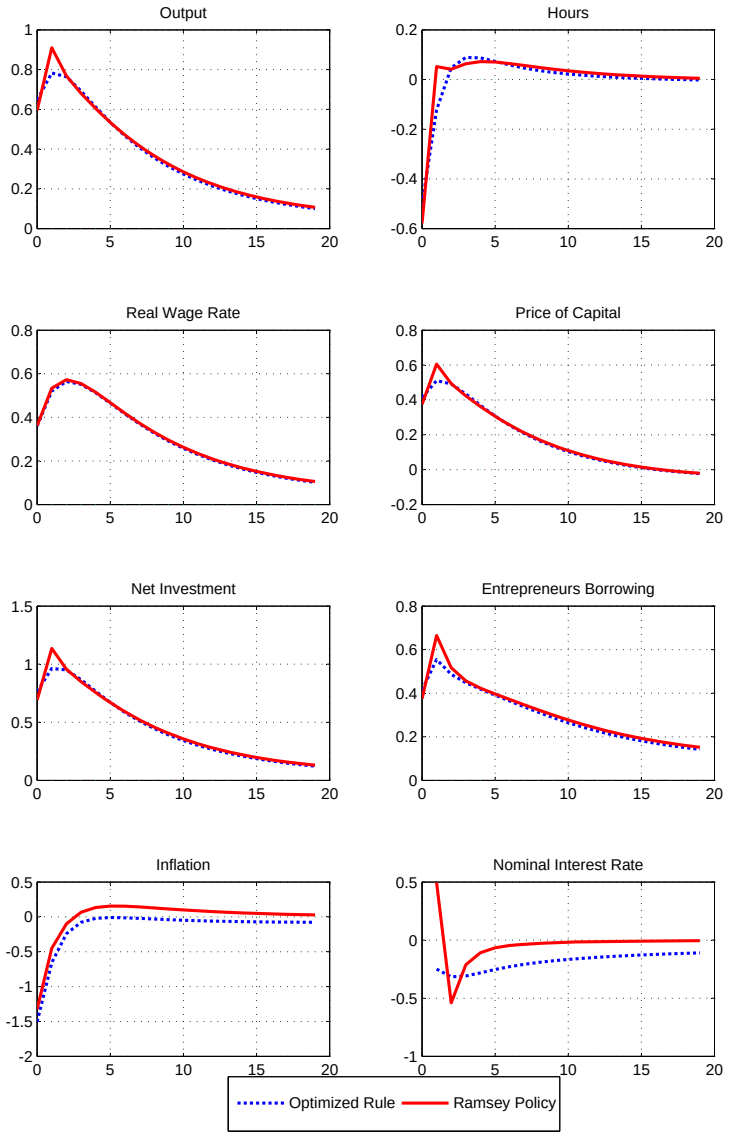

Figure 4.1: Responses of variables to one percent positive technology shock a_t in an economy with credit frictions. Red solid line denotes the Ramsey policy and blue dashed line is the optimized linear interest rule.

4.6 Welfare evaluation: optimized v.s. linear interest rate rules

This section I will assess alternative interest rate rules based on social welfare evaluation. I take second order approximations to the competitive equilibrium induced by a specified interest rule around its deterministic steady state. In particular, this analysis focuses on the conditional and unconditional expected discounted utility of agents in the economy. The conditional welfare is calculated conditional on the steady state levels of variables in the economy. Social welfare function is given by formula (4.54) and it can also be written in recursive forms.

Households' welfare can be written in a recursive form,

$$\begin{aligned} W_{h,t} &= U(c_t, k_t^s, l_t) + E_t \left\{ \sum_{s=1}^{+\infty} \beta^s U(c_{t+s}, k_{t+s}^s, l_{t+s}) \right\} \\ &= U(c_t, k_t^s, l_t) + \beta E_t \left\{ W_{h,t+1} \right\} \end{aligned} \tag{4.56}$$

Entrepreneurs' welfare becomes,

$$\begin{aligned} W_{e,t} &= U(C_t^e) + E_t \left\{ \sum_{s=1}^{+\infty} \beta^s U(c_{t+s}^e) \right\} \\ &= U(c_t^e) + \beta\gamma E_t \left\{ W_{e,t+1} \right\} \end{aligned} \tag{4.57}$$

Social welfare is of the form,

$$W_t = \omega W_{h,t} + (1-\omega) W_{e,t} \tag{4.58}$$

4.6.1 Comparing simple interest rate rules

Simple interest rules considered in this analysis consist of the following specifications: (a) a strict inflation targeting rule (IR, hereafter), (b) a simple Taylor rule (TRI), (c) a Taylor rule (TR), (d) a simple Taylor rule with a positive response to asset price (TRI $+\alpha_q > 0$), (e) a simple Taylor rule with a negative response to asset price (TRI $+\alpha_q < 0$), (f) a Taylor rule with a positive response to asset price (TR $+\alpha_q > 0$), (g) a Taylor rule with a negative response to asset price (TR $+\alpha_q < 0$), (h) an alternative Taylor rule with a smoothing term (TR + inertia 1), (i) an alternative Taylor rule with a smoothing term (TR + inertia 2), (j) an alternative Taylor rule with a high response to output gap (TR + high output response), (k) an alter-

native Taylor rule with a low response to output but with high persistence (TR + low output response), (l) a Taylor rule with a positive response to wage inflation (TR + π_w), (m) the optimized linear rule. Table 4.5 displays the coefficients for each specific simple interest rate rule.

Table 4.5: Simple interest rate rules

Rule	Description				
	α_π	α_{π_w}	α_y	α_q	ρ_r
(a) IT	$\pi_t = 0$ for all t				
(b) TRI	1.5	0	0	0	0
(c) TR	1.5	0	0.5	0	0
(d) TRI + $\alpha_q > 0$	1.5	0	0	0.2	0
(e) TRI +$\alpha_q < 0$	1.5	0	0	-0.2	0
(f) TR + $\alpha_q > 0$	1.5	0	0.5	0.2	0
(g) TR + $\alpha_q < 0$	1.5	0	0.5	-0.2	0
(h) TR + inertia 1	3	0	0.8	0	1
(i) TR + inertia 2	1.2	0	1	0	1
(j) TR + high output response	1.5	0	1	0	0
(k) TR + low output response	1.2	0	0.06	0	1.3
(l) TR + π_w	1.5	1	0.5	0	0
(m) Optimized linear rule	0.154	0.0874	0.0048	-0.0145	0.797

Some remarks are made here. The coefficient of asset price in the linear rules (d)-(g) are taken from Faia and Monacelli (2007). Rules (h) and (i) react to the lagged interest rate with a coefficient of one. Rule (h) has a higher response to inflation. Thereby, these two rules have considerable inertia. Rule (j) has a high response to output gap, which is proposed by researchers in the literature. Rule (k) is suggested by Rotemberg and Woodford (1998).

First, I explore the second moments of nominal interest rate, inflation, output, external finance premium, asset price and net investment in the competitive equilibrium induced by the specified linear interest rules (see table 4.6). An noticeable observation is that volatility of external finance premium varies a lot across linear interest rate rules. One way to measure a stabilizing policy rule is to investigate its policy frontier, the pair of standard deviation of inflation and standard deviation of output gap. Figure 4.2 plots this policy frontier under different policy rules. The Ramsey policy (point R) yields the smallest inflation-output-gap standard deviation combination (the closest point to the origin) and the optimized linear rule (point m) generates a larger but near optimal pair. The simple Taylor rule (Rule b) and Taylor rule with wage inflation (Rule l) have the largest output

Table 4.6: Cyclical implications of simple interest rate rules

Rule	Statistics	R_t	π_t	y_t	φ_t	q_t	i_t
IT	Mean	4	0	1.31	0.02	2.18	0.18
	STD	6.54	0	3.55	63.96	2.53	4.78
	Ser. Corr.	-0.24	-0.24	0.77	0.67	0.75	0.77
	Corr. with y_t	0.3	-0.31	1	-0.84	0.97	1
TRI	STD	8.05	5.38	6.12	237.86	4.56	8.56
	Ser. Corr.	0.76	0.76	0.75	0.61	0.73	0.74
	Corr. with y_t	-0.67	-0.96	1	-0.97	0.97	1
TR	STD	2.74	2.77	0.7	15.53	0.42	0.81
	Ser. Corr.	0.78	0.78	0.77	0.68	0.74	0.76
	Corr. with y_t	-0.77	-1	1	0.93	0.96	1
$TRI + \alpha_q > 0$	STD	2.95	2.6	1.73	36.94	1.19	2.27
	Ser. Corr.	0.77	0.77	0.77	0.59	0.74	0.76
	Corr. with y_t	-0.75	-0.98	1	-0.93	0.96	1
$TRI + \alpha_q < 0$	STD	3.93	1.9	2.79	156.47	2.24	4.14
	Ser. Corr.	0.54	0.25	0.71	0.63	0.7	0.71
	Corr. with y_t	-0.37	-0.48	1	-0.95	0.99	1
$TR + \alpha_q > 0$	STD	2.69	2.77	0.6	20.67	0.34	0.66
	Ser. Corr.	0.78	0.77	0.76	0.65	0.72	0.75
	Corr. with y_t	-0.76	-1	1	0.91	0.95	1
$TR + \alpha_q < 0$	STD	2.86	2.81	0.88	6.91	0.55	1.06
	Ser. Corr.	0.78	0.78	0.77	0.76	0.74	0.76
	Corr. with y_t	-0.77	-1	1	0.97	0.96	1
TR + inertia 1	STD	2.24	2.08	1.79	33.59	1.23	2.35
	Ser. Corr.	0.87	0.74	0.83	0.57	0.81	0.82
	Corr. with y_t	-0.8	-0.99	1	-0.89	0.96	1
TR + inertia 2	STD	1.9	2.16	0.58	26.13	0.32	0.63
	Ser. Corr.	0.89	0.71	0.92	0.64	0.93	0.94
	Corr. with y_t	-0.92	-0.94	1	0.77	0.95	1
TR + high output response	STD	2.94	3.06	0.41	31.35	0.21	0.41
	Ser. Corr.	0.82	0.79	0.74	0.64	0.68	0.72
	Corr. with y_t	-0.76	-1	1	0.86	0.96	0.99
TR + low output response	STD	0.51	1.06	1.14	2.59	0.73	1.42
	Ser. Corr.	0.94	0.41	0.92	0.4	0.92	0.93
	Corr. with y_t	-0.85	-0.56	1	0.44	0.95	1
TR $+\pi_w$	STD	8.05	5.26	6.63	258.73	4.93	9.28
	Ser. Corr.	0.75	0.74	0.76	0.61	0.74	0.75
	Corr. with y_t	-0.67	-0.96	1	-0.97	0.97	1
Optimized linear rule	STD	0.51	1.06	1.14	2.59	0.72	1.41
	Ser. Corr.	0.94	0.41	0.92	0.39	0.92	0.93
	Corr. with y_t	-0.85	-0.56	1	0.44	0.95	1

and inflation volatility. Strictly inflation targeting seems far away from the efficient policy rule. Other rules with a lagged interest rate also could reduce the volatility of output and inflation.

4.6.2 Welfare evaluation

The comparison of inflation-output volatility pairs is an ad hoc measure of efficiency of a policy rule. Table 4.7 provides the conditional and unconditional expected discounted utility of private agents defined by equation (4.58). Policy rules with high coefficient of lagged interest rate (Rules h, i, k, m) have both higher conditional and unconditional social welfare. In most of cases, including asset price in Taylor rules could enhance social welfare. And the traditional Taylor rule and strict inflation targeting rule lie in the range of the second preferable choices.

Table 4.7: Welfare results

Rule	Conditional Welfare		Unconditional Welfare	
	Welfare	Lost^a	Welfare	Lost^b
IT	-61.3968	22.82	-61.4608	28.95
TRI	-63.0322	186.36	-63.2317	206.04
TR	-61.336	16.74	-61.3977	22.64
$TRI + \alpha_q > 0$	-61.3016	13.3	-61.3358	16.45
$TRI + \alpha_q < 0$	-61.7567	58.81	-61.9513	78
$TR + \alpha_q > 0$	-61.3369	16.83	-61.4008	22.95
$TR + \alpha_q < 0$	-61.3456	17.7	-61.4103	23.9
TR + inertia 1	-61.2562	8.76	-61.2846	11.33
TR + inertia 2	-61.2718	10.32	-61.314	14.27
TR + high output response	-61.3864	21.78	-61.4723	30.1
TR + low output response	-61.2676	9.9	-61.3232	15.19
TR + π_w	-63.1706	200.2	-63.4479	227.66
Optimized linear rule	-61.1686	0	-61.1713	0

Notes: Lost^{a,b}is defined as the difference between the optimized linear interest rule and the corresponding simple interest rate rule, with a multiplier of 100.

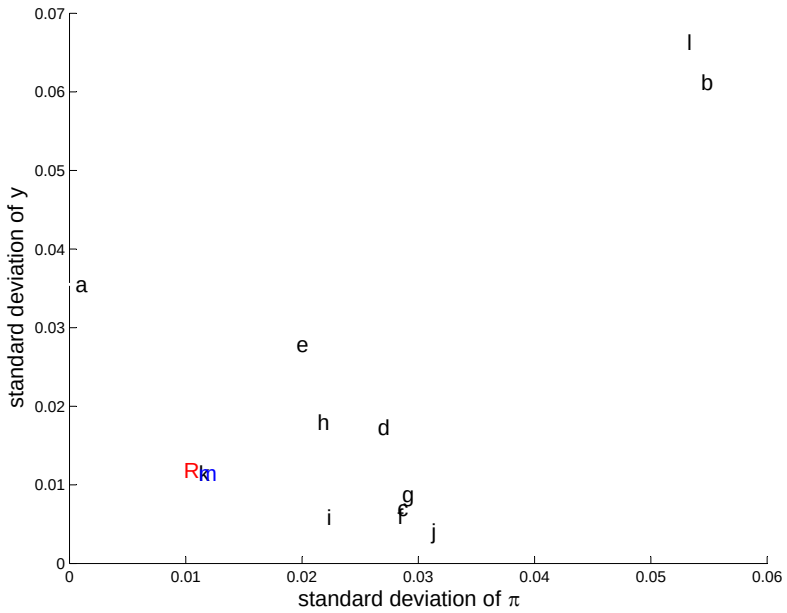

Figure 4.2: Policy frontiers: simply linear interest rate rules versus Ramsey policy. The letters on the graph are the corresponding policy rules described in table 4.5. "R" stands for the Ramsey policy.

Chapter 5

Conclusions

Home bias in equity portfolios has been a puzzle for decades. The main questions addressed in chapter 2 are: (1) Can standard international monetary business cycle models with nominal bonds deliver home bias in equity?; (2) Can optimal home equities be obtained while ensuring that the equilibrium equities are insensitive to preference parameter values?; (3) Are the relevant covariance ratios in the proposed model consistent with those in the data sample? It has been shown that the answers to these questions depend on the kind of bonds available in the international financial markets and whether monetary shocks have real effects on fundamentals.

This chapter quantitatively investigates portfolio choice in a medium-scale, two-country monetary business cycle model with sticky prices and sticky wages. The main mechanism for home bias in equity portfolio lies in two key covariance ratios. One is the covariance ratio between relative labor income that is orthogonal to real exchange rates (or terms of trade), and relative returns to national equities that is orthogonal to real exchange rates. The other is the covariance ratio between relative returns on bonds that is orthogonal to real exchange rates, and relative returns to national equities that is orthogonal to real exchange rates. The model shows that either in a model with complete asset markets or with incomplete asset markets, major shocks considered in the literature, such as, technological shocks, monetary shocks, investment efficiency shocks, government expenditure shocks and preference shifts, either separately or jointly, contribute to generating a negative moderate covariance ratio for hedging human capital income risk. This covariance ratio generated by the model is also consistent with the data. Nevertheless, the latter covariance ratio for hedging real exchange rate risk depends on the type of bonds traded internationally. Numerical experiments show that a model with nominal bonds and monetary shocks hardly deliver a robust and close-to-zero covariance ratio for hedging real exchange rate risk , whereas a model with inflation-indexed bonds or without monetary shocks can present such a close-to-zero covariance ratio that is consistent with the data. The motivation for adapting inflation-indexed bonds stems from the recent development in bond markets. This is the main

contribution of this work.

Table 5.1: Optimal bond assets-GDP ratio: inflation-indexed bonds

	$\gamma = 0.50$	$\gamma = 0.85$	$\gamma = 1$	$\gamma = 1.1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$
$\sigma = 0.5$	1.16	0.67	0.50	0.39	0.01	0.41	1.06
$\sigma = 1$	0.90	0.28	0.07	0.06	0.49	0.91	1.54
$\sigma = 1.5$	0.78	0.08	0.14	0.27	0.71	1.12	1.73
$\sigma = 2$	0.70	0.04	0.27	0.40	0.84	1.24	1.83
$\sigma = 5$	0.50	0.32	0.55	0.68	1.11	1.49	2.04
$\sigma = 10$	0.41	0.43	0.67	0.80	1.21	1.58	2.11

Note: This table reports the ratio of bond assets to GDP in the benchmark model. σ is the coefficient of relative risk aversion and γ is the trade elasticity.

Table 5.2: Bond-GDP ratio

Country	Debt assets - GDP ratio (%)	Debt liabilities - GDP ratio (%)	External debts - GDP ratio (%)
Austria	51.6	68.8	17.1
Canada	20.2	51.2	31.0
Denmark	36.6	75.6	39.0
France	43.9	48.4	4.5
Germany	43.7	44.3	0.6
Italy	29.9	41.7	11.8
Japan	30.2	22.9	-7.4
Spain	23.2	36.3	13.1
UK	126.3	139.6	13.3
US	18.3	26.9	8.5
Average	34.3	41.1	6.8

Data source: *Journal of International Economics*, Lane and Milesi-Ferretti (2007), adapted by permission.

The other contribution of this chapter is exploring the second moments both for portfolios and aggregate variables. Net foreign asset positions exhibit counter-cyclical movement with output both in the data and in all models studied here, either with complete markets or with incomplete asset markets. The model also generates sizable international co-movements for aggregate variables.

Besides the findings mentioned above, there are other factors worth addressing. First, a model with technological shocks and government expen-

diture shocks generates robust super home-bias in equities. A similar super home-bias in equities is obtained when incorporating only technology shocks and preference shocks into a model. It is shown that super-home bias disappears when additional shocks, say, monetary shocks, are present in the model. Second, the bond assets-GDP ratio produced by the model varies significantly across different values of preference parameters, although the model could generate robust and reasonable home bias in equities. Table 5.1 shows this bond assets-GDP ratio in the benchmark model, while the data in table 5.2 displays reasonably-stable bond assets-GDP ratios for many countries. This remains an unsolved puzzle in this work and will likely draw further attention from other researchers. Suggestions for future work include: (1) taking into account the phenomena of home bias in equities and of reasonable debt-GDP ratios in a unified framework; (2) investigating portfolio choice in a multiple-country environment that incorporate international propagation and portfolio adjustments responding to shocks in other countries with different country sizes; (3) studying optimal portfolio choice under different stages of financial integration and financial development; (4) exploring cross-sectional implications between the volume of inflation-indexed bonds and equity home bias and validating the mechanism developed in this chapter in a further step.

How does international risk sharing change with financial integration? Chapter 3 investigates this issue in a center-peripheral framework. I find that financial integration indeed enhances international risk sharing for peripheral countries, but it not necessarily the case for the central country. The main mechanism lies in demand effect induced by country size. When the central country is large, the external demand for assets is low and partial financial integration could provide a better risk sharing for the core country.

There are several important aspects untouched in this chapter. First, I focus on consumption volatility. For a complete welfare analysis, I need to resort to higher order approximations to obtain first-order dynamics of assets. This will be left in my ongoing work (Yu 2012). Second, I investigate equity assets only and ignore other type of assets such as debt assets. Future work should include this type of assets and explore asset structures and risk sharing.

When both wages and prices are determined by staggered nominal contracts, and some credit-constrained agents can only issue non-state contingent nominal debt, chapter 4 shows how to conduct optimal monetary policies. In the benchmark model, interest rate should decrease when asset price rises but the response is very small. In terms of welfare maximization, credit frictions provide policy-makers with another channel to drive

down the volatility of consumption and labor supply, which is omitted in a credit-frictionless economy. In the Ramsey policy, output and asset price volatilities are relatively small because external finance premium is positively co-moves with output, which is opposite, however, in real models (flexible price and wage setting and real debt).

I also compare several linear interest rate rules according to their inflation-output-gap volatility frontiers and social welfare induced by each rule. Rules with a higher response to lagged interest rate perform better than rules without responding to the lagged interest rate. The Taylor rule and a strict inflation targeting rule produce a similar welfare loss, both of which are worse than the rule including an inertia term. One caveat is that the external finance premium co-moves positively with output in the optimal policy because the deflationary effect of a technological shock dominates the effect of loosening collateral constraints. This seems counterfactual with the data. However, the bottom line in this exercise is that if aggregate variables, such as consumption and output, in an economy with credit frictions, are quite volatile, monetary authorities could use the debt deflation channel and the financial decelerator to stabilize the economy and consequently to improve welfare.

Bibliography

Adler, Michael, and Bernard Dumas (1983) 'International portfolio choice and corporation finance: A synthesis.' *Journal of Finance* 38(3), 925–84

Albuquerque, Rui, Gregory H. Bauer, and Martin Schneider (2009) 'Global private information in international equity markets.' *Journal of Financial Economics* 94(1), 18–46

Angeletos, George-Marios, and Vasia Panousi (2011) 'Financial integration, entrepreneurial risk and global dynamics.' *Journal of Economic Theory* 146(3), 863–896

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland (1995) 'International business cycles: Theory and evidence.' In *Frontiers of Business Cycle Research,* ed. Thomas F. Cooley (Princeton University Press) pp. 331–356

Bai, Yan, and Jing Zhang (2010) 'Solving the feldstein-horioka puzzle with financial frictions.' *Econometrica* 78(2), 603–632

— (2012) 'Financial integration and international risk sharing.' *Journal of International Economics* 86, 17–32

Barattieri, Alessandro, Susanto Basu, and Peter Gottschalk (2010) 'Some evidence on the importance of sticky wages.' Working Papers 10-11, Federal Reserve Bank of Boston

Baxter, Marianne, and Mario J Crucini (1995) 'Business cycles and the asset structure of foreign trade.' *International Economic Review* 36(4), 821–54

Baxter, Marianne, and Urban J Jermann (1997) 'The international diversification puzzle is worse than you think.' *American Economic Review* 87(1), 170–80

Baxter, Marianne, Urban J. Jermann, and Robert G. King (1998) 'Nontraded goods, nontraded factors, and international non-diversification.' *Journal of International Economics* 44(2), 211–229

Benigno, Pierpaolo, and Ester Faia (2010) 'Globalization, pass-through and inflation dynamics.' Kiel Working Papers 1604, Kiel Institute for the World Economy, March

Benigno, Pierpaolo, and Salvatore Nistico (2012) 'International portfolio allocation under model uncertainty.' *American Economic Journal: Macroeconomics* 4(1), 144–89

Bernanke, Ben, and Mark Gertler (1989) 'Agency costs, net worth, and business fluctuations.' *American Economic Review* 79(1), 14–31

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist (1999) 'The financial accelerator in a quantitative business cycle framework.' In *Handbook of Macroeconomics,* ed. J. B. Taylor and M. Woodford, vol. 1 of *Handbook of Macroeconomics* (Elsevier) chapter 21, pp. 1341–1393

Blanchard, Olivier, and Roberto Perotti (2002) 'An empirical characterization of the dynamic effects of changes in government spending and taxes on output.' *The Quarterly Journal of Economics* 117(4), 1329–1368

Blanchard, Olivier Jean, and Charles M Kahn (1980) 'The solution of linear difference models under rational expectations.' *Econometrica* 48(5), 1305–11

Bonfiglioli, Alessandra (2008) 'Financial integration, productivity and capital accumulation.' *Journal of International Economics* 76(2), 337–355

Brandt, Michael W., John H. Cochrane, and Pedro Santa-Clara (2006) 'International risk sharing is better than you think, or exchange rates are too smooth.' *Journal of Monetary Economics* 53(4), 671–698

Brunnermeier, Markus K., Thomas M. Eisenbach, and Yuliy Sannikov (2012) 'Macroeconomics with financial frictions: A survey.' Levine's Working Paper Archive 786969000000000384, David K. Levine, February

Callen, Michael, Jean Imbs, and Paolo Mauro (2011) 'Pooling risk among countries.' Working paper in Paris School of Economics

Calvo, Guillermo A. (1983) 'Staggered prices in a utility-maximizing framework.' *Journal of Monetary Economics* 12(3), 383–398

Campbell, John Y (1996) 'Understanding risk and return.' *Journal of Political Economy* 104(2), 298–345

Campbell, John Y., Robert J. Shiller, and Luis M. Viceira (2009) 'Understanding inflation-indexed bond markets.' *Brookings Papers on Economic Activity* 40(Spring), 79–138

Carlstrom, Charles T, and Timothy S Fuerst (1997) 'Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis.' *American Economic Review* 87(5), 893–910

Carlstrom, Charles T., Timothy S. Fuerst, and Matthias Paustian (2010) 'Optimal monetary policy in a model with agency costs.' *Journal of Money, Credit and Banking* 42(s1), 37–70

Chari, V V, Patrick J Kehoe, and Ellen R McGrattan (2002) 'Can sticky price models generate volatile and persistent real exchange rates?' *Review of Economic Studies* 69(3), 533–63

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans (2005) 'Nominal rigidities and the dynamic effects of a shock to monetary policy.' *Journal of Political Economy* 113(1), 1–45

Christiano, Lawrence J., Mathias Trabandt, and Karl Walentin (2010) 'Dsge models for monetary policy analysis.' In *Handbook of Monetary Economics,* ed. Benjamin M. Friedman and Michael Woodford, vol. 3 of *Handbook of Monetary Economics* (Amsterdam, the Netherlands: Elsevier) chapter 7, pp. 285–367

Coeurdacier, Nicolas (2009) 'Do trade costs in goods market lead to home bias in equities?' *Journal of International Economics* 77(1), 86–100

Coeurdacier, Nicolas, and He Rey (2012) 'Home bias in open economy financial macroeconomics.' CEPR Discussion Papers 8746, C.E.P.R. Discussion Papers, January

Coeurdacier, Nicolas, and Pierre-Olivier Gourinchas (2011) 'When bonds matter: Home bias in goods and assets.' NBER Working Papers 17560, National Bureau of Economic Research, Inc, November

Coeurdacier, Nicolas, Robert Kollmann, and Philippe Martin (2010) 'International portfolios, capital accumulation and foreign assets dynamics.' *Journal of International Economics* 80(1), 100–112

Cole, Harold L., and Maurice Obstfeld (1991) 'Commodity trade and international risk sharing : How much do financial markets matter?' *Journal of Monetary Economics* 28(1), 3–24

Collard, Fabrice, Harris Dellas, Behzad Diba, and Alan Stockman (2009) 'Goods trade and international equity portfolios.' School of Economics Working Papers 2009-14, University of Adelaide, School of Economics

Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc (2008) 'International risk sharing and the transmission of productivity shocks.' *Review of Economic Studies* 75(2), 443–473

Curdia, Vasco, and Michael Woodford (2009) 'Credit frictions and optimal monetary policy.' BIS Working Papers 278, Bank for International Settlements, March

_ (2010) 'Credit spreads and monetary policy.' *Journal of Money, Credit and Banking* 42(s1), 3–35

De Fiore, Fiorella, and Oreste Tristani (2009) 'Optimal monetary policy in a model of the credit channel.' Working Paper Series 1043, European Central Bank, April

Devereux, Michael B., and Alan Sutherland (2007) 'Monetary policy and portfolio choice choice in an open economy macro model.' *Journal of the European Economic Association* 5(2-3), 491–499

Devereux, Michael B., and Alan Sutherland (2008) 'Financial globalization and monetary policy.' *Journal of Monetary Economics* 55(8), 1363–1375

_ (2009) 'A portfolio model of capital flows to emerging markets.' *Journal of Development Economics* 89(2), 181–193

_ (2010a) 'Country portfolio dynamics.' *Journal of Economic Dynamics and Control* 34(7), 1325–1342

_ (2010b) 'Valuation effects and the dynamics of net external assets.' *Journal of International Economics* 80(1), 129–143

Devereux, Michael B., and Alan Sutherland (2011a) 'Country portfolios in open economy macro?models.' *Journal of the European Economic Association* 9(2), 337–369

Devereux, Michael B., and Alan Sutherland (2011b) 'Evaluating international financial integration under leverage constraints.' *European Economic Review* 55(3), 427–442

Devereux, Michael B., and James Yetman (2010) 'Leverage constraints and the international transmission of shocks.' *Journal of Money, Credit and Banking* 42(s1), 71–105

Didier, Tatiana, Roberto Rigobon, and Sergio L. Schmukler (2011) 'Unexploited gains from international diversification : patterns of portfolio holdings around the world.' Policy Research Working Paper Series 5524, The World Bank, January

Dixit, Avinash K, and Joseph E Stiglitz (1977) 'Monopolistic competition and optimum product diversity.' *American Economic Review* 67(3), 297–308

Eichengreen, Barry, and Yung Chul Park (2003) 'Why has there been less financial integration in asia than in europe?' Institute of European Studies, Working Paper Series qt6823v94w, Institute of European Studies, UC Berkeley, January

Engel, Charles, and Akito Matsumoto (2009a) 'The international diversification puzzle when goods prices are sticky: It's really about exchange-rate hedging, not equity portfolios.' *American Economic Journal: Macroeconomics* 1(2), 155–88

— (2009b) 'International risk sharing: Through equity diversification or exchange rate hedging?' IMF Working Papers 09/138, International Monetary Fund, July

Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin (2000) 'Optimal monetary policy with staggered wage and price contracts.' *Journal of Monetary Economics* 46(2), 281–313

Evans, Martin D. D., and Viktoria Hnatkovska (2005) 'Solving general equilibrium models with incomplete markets and many assets.' NBER Technical Working Papers 0318, National Bureau of Economic Research, Inc, October

Faia, Ester, and Eleni Iliopulos (2010) 'Financial globalization, financial frictions and optimal monetary policy.' Globalization and Monetary Policy Institute Working Paper 52, Federal Reserve Bank of Dallas

Faia, Ester, and Tommaso Monacelli (2007) 'Optimal interest rate rules, asset prices, and credit frictions.' *Journal of Economic Dynamics and Control* 31(10), 3228–3254

Fisher, Jonas D. M. (2006) 'The dynamic effects of neutral and investment-specific technology shocks.' *Journal of Political Economy* 114(3), 413–451

Fisher, JonasD.M., and Ryan Peters (2010) 'Using stock returns to identify government spending shocks.' *Economic Journal* 120(544), 414–436

Fratzscher, Marcel, and Jean Imbs (2009) 'Risk sharing, finance, and institutions in international portfolios.' *Journal of Financial Economics* 94(3), 428–447

Gertler, Mark, and Nobuhiro Kiyotaki (2010) 'Financial intermediation and credit policy in business cycle analysis.' In *Handbook of Monetary Economics,* ed. Benjamin M. Friedman and Michael Woodford, vol. 3 of *Handbook of Monetary Economics* (Elsevier) chapter 11, pp. 547–599

Goldberg, Linda, and Cedric Tille (2009) 'Macroeconomic interdependence and the international role of the dollar.' *Journal of Monetary Economics* 56(7), 990–1003

Gopinath, Gita, and Roberto Rigobon (2008) 'Sticky borders.' *The Quarterly Journal of Economics* 123(2), 531–575

Gopinath, Gita, Oleg Itskhoki, and Roberto Rigobon (2010) 'Currency choice and exchange rate pass-through.' *American Economic Review* 100(1), 304–36

Gourinchas, Pierre-Olivier, and He Rey (2007) 'International financial adjustment.' *Journal of Political Economy* 115(4), 665–703

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W Huffman (1988) 'Investment, capacity utilization, and the real business cycle.' *American Economic Review* 78(3), 402–17

Greenwood, Jeremy, Zvi Hercowitz, and Per Krusell (1997) 'Long-run implications of investment-specific technological change.' *American Economic Review* 87(3), 342–62

— (2000) 'The role of investment-specific technological change in the business cycle.' *European Economic Review* 44(1), 91–115

Heathcote, Jonathan, and Fabrizio Perri (2008) 'The international diversification puzzle is not as bad as you think.' CEPR Discussion Papers 6982, C.E.P.R. Discussion Papers, October

Hnatkovska, Viktoria (2010) 'Home bias and high turnover: Dynamic portfolio choice with incomplete markets.' *Journal of International Economics* 80(1), 113–128

Hoffman, Dennis L., Robert H. Rasche, and Margie A. Tieslau (1995) 'The stability of long-run money demand in five industrial countries.' *Journal of Monetary Economics* 35(2), 317–339

Iacoviello, Matteo (2002) 'House prices, borrowing constraints and monetary policy in the business cycle.' Boston College Working Papers in Economics 542, Boston College Department of Economics, October

Ireland, Peter N. (2001) 'Sticky-price models of the business cycle: Specification and stability.' *Journal of Monetary Economics* 47(1), 3–18

Jermann, Urban, and Vincenzo Quadrini (2012) 'Macroeconomic effects of financial shocks.' *American Economic Review* 102(1), 238–71

Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti (2010) 'Investment shocks and business cycles.' *Journal of Monetary Economics* 57(2), 132–145

Kiyotaki, Nobuhiro, and John Moore (1997) 'Credit cycles.' *Journal of Political Economy* 105(2), 211–48

Kiyotaki, Nobuhiro, and John Moore (2012) 'Liquidity, business cycles, and monetary policy.' NBER Working Papers 17934, National Bureau of Economic Research, Inc, March

Kollmann, Robert (2001) 'Explaining international comovements of output and asset returns: The role of money and nominal rigidities.' *Journal of Economic Dynamics and Control* 25(10), 1547–1583

Kose, M Ayhan, Eswar Prasad, Kenneth Rogoff, and Shang-Jin Wei (2009a) 'Financial globalization: A reappraisal.' *IMF Staff Papers* 56(1), 8–62

Kose, M. Ayhan, Eswar S. Prasad, and Marco E. Terrones (2009b) 'Does financial globalization promote risk sharing?' *Journal of Development Economics* 89(2), 258–270

Lane, Philip R., and Gian Maria Milesi-Ferretti (2001) 'The external wealth of nations: measures of foreign assets and liabilities for industrial and developing countries.' *Journal of International Economics* 55(2), 263–294

_ (2007) 'The external wealth of nations mark ii: Revised and extended estimates of foreign assets and liabilities, 1970-2004.' *Journal of International Economics* 73(2), 223–250

Lewis, Karen K. (1999) 'Trying to explain home bias in equities and consumption.' *Journal of Economic Literature* 37(2), 571–608

Lez-Salido, Jordi Galnd J. David, and Javier Vall (2007) 'Understanding the effects of government spending on consumption.' *Journal of the European Economic Association* 5(1), 227–270

Lucas, Robert Jr. (1982) 'Interest rates and currency prices in a two-country world.' *Journal of Monetary Economics* 10(3), 335–359

Martin, Ian (2010) 'how much do financial markets matter? Cole-Obstfeld revisited.' Manuscript, Graduate School of Business, Standford University

Martin, Philippe, and Helene Rey (2004) 'Financial super-markets: size matters for asset trade.' *Journal of International Economics* 64(2), 335–361

Matsuyama, Kiminori (2008) 'Aggregate implications of credit market imperfections.' In 'NBER Macroeconomics Annual 2007, Volume 22' NBER Chapters (National Bureau of Economic Research, Inc) pp. 1–60

McCallum, Bennett T. (1999) 'Issues in the design of monetary policy rules.' In *Handbook of Macroeconomics,* ed. J. B. Taylor and M. Woodford, vol. 1 of *Handbook of Macroeconomics* (Elsevier) chapter 23, pp. 1483–1530

Mendoza, Enrique G., Assaf Razin, and Linda L. Tesar (1994) 'Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption.' *Journal of Monetary Economics* 34(3), 297–323

Mendoza, Enrique G., Vincenzo Quadrini, and Jose-Victor Rios-Rull (2009a) 'Financial integration, financial development, and global imbalances.' *Journal of Political Economy* 117(3), 371–416

_ (2009b) 'On the welfare implications of financial globalization without financial development.' In 'NBER International Seminar on Macroeconomics 2007' NBER Chapters (National Bureau of Economic Research, Inc) pp. 283–312

Mertens, Karel, and MortenO. Ravn (2010) 'Measuring the impact of fiscal policy in the face of anticipation: A structural var approach.' *Economic Journal* 120(544), 393–413

Monacelli, Tommaso (2008) 'Optimal monetary policy with collateralized household debt and borrowing constraints.' In 'Asset Prices and Monetary Policy' NBER Chapters (National Bureau of Economic Research, Inc) pp. 103–146

Monacelli, Tommaso (2009) 'New keynesian models, durable goods, and collateral constraints.' *Journal of Monetary Economics* 56(2), 242–254

Mondria, Jordi, and Thomas Wu (2010) 'The puzzling evolution of the home bias, information processing and financial openness.' *Journal of Economic Dynamics and Control* 34(5), 875–896

Nicolo, Gianni De, and Luciana Juvenal (2012) 'Financial integration, globalization, and real activity.' CESifo Working Paper Series 3737, CESifo Group Munich, March

Nieuwerburgh, Stijn Van, and Laura Veldkamp (2009) 'Information immobility and the home bias puzzle.' *Journal of Finance* 64(3), 1187–1215

Obstfeld, Maurice (2007) *International Risk Sharing and the Costs of Trade* (Stockholm School of Economics: The Ohlin Lectures, May 23-24)

Obstfeld, Maurice, and Kenneth Rogoff (2001) 'The six major puzzles in international macroeconomics: Is there a common cause?' In 'NBER Macroeconomics Annual 2000, Volume 15' NBER Chapters (National Bureau of Economic Research, Inc) pp. 339–412

Pavlova, Anna, and Roberto Rigobon (2008) 'The role of portfolio constraints in the international propagation of shocks.' *Review of Economic Studies* 75(4), 1215–1256

Pavlova, Anna, and Roberto Rigobon (2010) 'An asset-pricing view of external adjustment.' *Journal of International Economics* 80(1), 144–156

Pesenti, Paolo, and Eric van Wincoop (2002) 'Can nontradables generate substantial home bias?' *Journal of Money, Credit and Banking* 34(1), 25–50

Ramey, Valerie A. (2011) 'Identifying government spending shocks: It's all in the timing.' *The Quarterly Journal of Economics* 126(1), 1–50

Romer, Christina D., and David H. Romer (2010) 'The macroeconomic effects of tax changes: Estimates based on a new measure of fiscal shocks.' *American Economic Review* 100(3), 763–801

Rotemberg, Julio J., and Michael Woodford (1998) 'An optimization-based econometric framework for the evaluation of monetary policy: Expanded version.' NBER Technical Working Papers 0233, National Bureau of Economic Research, Inc, May

Schmitt-Grohe, Stephanie, and Martin Uribe (2003) 'Closing small open economy models.' *Journal of International Economics* 61(1), 163–185

— (2004) 'Solving dynamic general equilibrium models using a second-order approximation to the policy function.' *Journal of Economic Dynamics and Control* 28(4), 755–775

Schmitt-Grohe, Stephanie, and Martin Uribe (2006) 'Optimal fiscal and monetary policy in a medium-scale macroeconomic model.' In 'NBER Macroeconomics Annual 2005, Volume 20' NBER Chapters (National Bureau of Economic Research, Inc) pp. 383–462

Sercu, Piet, and R Vanpee (2007) 'Home bias in international equity portfolios: a review.' Open access publications from Katholieke Universiteit Leuven urn:hdl:123456789/175483, Katholieke Universiteit Leuven

Stepanchuk, Sergiy, and Viktor Tsyrennikov (2011) 'International portfolios: an incomplete markets general equilibrium approach.' Technical Report. Cornell University

Taylor, John B. (2008) 'Monetary policy and the state of the economy.' Testimony before the Committee on Financial Services, U.S. House of Representatives. February 26.

Taylor, John B., and John C. Williams (2010) 'Simple and robust rules for monetary policy.' In *Handbook of Monetary Economics,* ed. Benjamin M. Friedman and Michael Woodford, vol. 3 (Amsterdam, the Netherlands: Elsevier) pp. 829–859

Tesar, Linda L. (1993) 'International risk-sharing and non-traded goods.' *Journal of International Economics* 35(1-2), 69–89

Tille, Cric, and Eric van Wincoop (2010) 'International capital flows.' *Journal of International Economics* 80(2), 157–175

van Wincoop, Eric, and Francis E. Warnock (2006) 'Is home bias in assets related to home bias in goods?' NBER Working Papers 12728, National Bureau of Economic Research, Inc, December

Yu, Changhua (2012) 'Evaluating international financial integration in a center-periphery economy.' Work in progress, University of British Columbia, June

Yu, Ip-Wing, Kang-Por Fung, and Chi-Sang Tam (2010) 'Assessing financial market integration in asia - equity markets.' *Journal of Banking & Finance* 34(12), 2874–2885

Appendix A

Appendix to Chapter 2

A.1 Quarterly data on asset returns

Total return indices are from Datastream: Canada (S&P/TSX composite index), Denmark (Copenhagen KFX DS-calculated), Germany (DAX 30 performance), France (France-DS market), Italy (Italy-DS market), Japan (Topix), Spain (Spain-DS market), UK (FTSE all share), USA (S&P 500 composite).

Bond yields are from Thomson Reuters bond yields in Datastream. Yields are government bond bid yields with maturity of 3-month, 2-year, 5-year and 10-year.

Labor income is the compensation of employees from the OECD total labor cost on *OECD.stat* database. Notice that the share labor income in total income from non-corporate firms and gross mixed income is relatively small, less than 10% in many countries, even less than 5% in some countries. Because of data unavailability of mixed income in quarterly frequency, I take only use of compensation of employees.

Bilateral trade volume is constructed as follows. Let $ex_{i,j,t}$ be the current value of export (in U.S. dollars) from country i to country j at period t. When country i is the home country, the weight of country j in the *foreign country* counterpart is $\frac{ex_{i,j,t}+ex_{j,i,t}}{\sum_j(ex_{i,j,t}+ex_{j,i,t})}$. Data on $ex_{i,j,t}$ is from the *Direction of Trade* of IMF database.

Nominal exchange rate is from taken from the *International Financial Statistics* of IMF database.

A.2 Annual data on national accounts and net foreign assets

Annual data series on real GDP, the GDP deflator, export and import prices, real private consumption, real fixed investment, consumer price index, current account, nominal exchange rate, population, discount rate and treasure-bill rate, which were taken from IMF's *International Financial Statistics* (IFS for short) database, for the period $1970 - 2004$. Employment data is taken from OECD's national accounts (*OECD.stat* database) for the same period. Money supply for U.S., Japan and Canada, using M1 definition of money, were taken from IMF's IFS database. U.K. doesn't have M1 definition of money, and I take the closest measure of narrow money, M0, instead, which was drawn from the online database in Bank of England (table code is *LPMAVAE*). The rest of countries in the data

sample are all members of the Euro-Zone, and, instead, I use the growth of narrow money (M1) in EU-16 countries, which is drawn from OECD's national accounts, as a proxy measure of money growth in these countries.

Home and Foreign counterpart are constructed as follows. When U.S. is the Home country, I then constructed trade-weighted aggregates for Japan, U.K., Germany, France, Italy, Canada, Spain, Austria and Denmark. The trade weights were constructed by adding up exports from Home country to Foreign country i and exports from Foreign country i to Home country and dividing by the total home trade with the rest of world in the data sample. Of course, the sum of trade weight is equal to one. Export data, in current U.S. dollars, were taken from IMF's Direction of Trade (1980-2004, CD-ROM) and historical Direction of Trade ($1970-1978$, Computer file. Sources: STUDYNO = 07628 International Monetary Fund, DIRECTION OF TRADE (Computer file), 2nd release, Washington, DC, International Monetary Fund (producer), 1979, Ann Arbor, MI: Inter-University Consortium for Political and Social Research (distributor), 1979.). The bilateral trade data in year 1979 was filled by using interpolation method. The *foreign country* is constructed by aggregating the rest of economies in the sample using trade weights described above. Real GDP, real private consumption, real fixed investment and real money balance series are purchasing power parity exchange rates adjusted in 2000 U.S. dollars. Consumption based real exchange rate dada is defined using consumer price index weighted by trade weight. Real imports and exports series are deflated by import and export prices, respectively. Data on foreign equity asset positions and foreign bond asset positions were taken from Lane and Lane and Milesi-Ferretti (2001) and Lane and Milesi-Ferretti (2007).

A.3 Estimating returns on human capital

This section displays the method of estimating returns on human capital. From the asset pricing perspective, labor income in an economy can be considered as dividends from human capital. As in the main text, wh_t denotes the real labor income in the home country. Let v_t stands for the real price of human capital in the home country. Following Campbell (1996), I assume that dividend-price ratio is stationary and its unconditional expectation is constant, say, at its long-run deterministic steady state. Let $\widetilde{x}$ represent the log of x_t. Then the realized log of gross return on human capital has a form of,

$$\begin{aligned}\widetilde{r}_{n,t+1} &\equiv \log(wh_{t+1} + v_{t+1}) - \log(v_t) \\ &= \log\left(1 + \frac{v_{t+1}}{wh_{t+1}}\right) + \widetilde{wh}_{t+1} - \widetilde{v}_t \\ &= \kappa + \rho\widetilde{v}_{t+t} + (1-\rho)\widetilde{wh}_{t+1} - \widetilde{v}_t + \mathcal{O}\left(\| \epsilon \|\right)\end{aligned} \tag{A.1}$$

with $\kappa \equiv \log(1-\rho)$. ρ characterizes the steady state inverse of gross return to human capital, $\frac{1}{\rho} \equiv \frac{v+wh}{v}$. Then up to a first-order approximation, the log of realized gross return to human capital can be expressed as the sum of a constant

and a weighted average of prices and dividends. Let the log of dividend-price ratio be $\widetilde{\zeta}_t \equiv \widetilde{wh}_t - \widetilde{v}_t$. Then the log of return to human capital (A.1) can be written as,

$$\widetilde{r}_{n,t+1} = \kappa + \widetilde{\zeta}_t - \rho\widetilde{\zeta}_{t+1} + \Delta\widetilde{wh}_{t+1} + \mathcal{O}\left(\| \epsilon \|\right) \tag{A.2}$$

where $\Delta\widetilde{wh}_{t+1} \equiv \widetilde{wh}_{t+1} - \widetilde{wh}_t$. Since the dividend-price ratio is stationary, $\widetilde{\zeta}_{t+j}$ doesn't explode as j increases. Writing equation (A.2) forward recursively yields that the current price-dividend ratio is a weighted average of future log of returns to human capital adjusted by labor income growth,

$$\widetilde{\zeta}_t = \sum_{j=0}^{\infty} \rho^j \left(\widetilde{r}_{n,t+1+j} - \Delta\widetilde{wh}_{t+1+j}\right) - \frac{\kappa}{1-\rho} + \mathcal{O}\left(\| \epsilon \|\right) \tag{A.3}$$

As in Campbell (1996), I assume further that the conditional expected return to financial wealth equals the conditional expected return to human capital *within* a country, that is, $E_t\left\{\widetilde{r}_{n,t+j}\right\} = E_t\left\{\widetilde{r}_{a_1,t+j}\right\}$ for all $j > 0$. Taking expectations on both sides of price-dividend ratio (A.3) conditional on the information up to period t and substituting $E_t\left\{\widetilde{r}_{a_1,t+j}\right\}$ for $E_t\left\{\widetilde{r}_{n,t+j}\right\}$, obtains,

$$\widetilde{\zeta}_t = E_t\left\{\sum_{j=0}^{\infty} \rho^j \left(\widetilde{r}_{a_1,t+1+j} - \Delta\widetilde{wh}_{t+1+j}\right)\right\} - \frac{\kappa}{1-\rho} + \mathcal{O}\left(\| \epsilon \|\right) \tag{A.4}$$

Next, substituting $\widetilde{\zeta}_t$ in equation (A.4) into equation (A.2) yields an expression for the log of return to human capital $\widetilde{r}_{n,t+1}$. Thereby, the unexpected return to human capital can be written as a linear combination of labor income growth and returns to financial wealth,

$$\begin{aligned}
\widetilde{r}_{n,t+1} - E_t\widetilde{r}_{n,t+1} &= -\rho\left(\widetilde{\zeta}_{t+1} - E_t\widetilde{\zeta}_{t+1}\right) + \Delta\widetilde{wh}_{t+1} - E_t\Delta\widetilde{wh}_{t+1} + \mathcal{O}\left(\| \epsilon \|\right) \\
&= -\rho(E_{t+1} - E_t)\sum_{j=0}^{\infty} \rho^j \left(\widetilde{r}_{a_1,t+2+j} - \Delta\widetilde{wh}_{t+2+j}\right) \\
&\quad + (E_{t+1} - E_t)\Delta\widetilde{wh}_{t+1} + \mathcal{O}\left(\| \epsilon \|\right) \\
&= (E_{t+1} - E_t)\sum_{j=0}^{\infty} \rho^j \Delta\widetilde{wh}_{t+1+j} - (E_{t+1} - E_t)\sum_{j=1}^{\infty} \rho^j \widetilde{r}_{a_1,t+1+j} \\
&\quad + \mathcal{O}\left(\| \epsilon \|\right)
\end{aligned} \tag{A.5}$$

The equation above shows that increases in expected future labor income growth (the first term on the right hand side at the bottom line) cause a positive return on human capital, while rises in expected future financial wealth returns (the second term on the right hand side at the bottom line) have a negative effect on returns to human capital since dividend flows from human capital are discounted at a higher

rate.

To estimate the right hand side of the last line of equation (A.5) and to obtain the innovations to human capital returns, I adapt a vector autoregressive approach of Campbell. I put the real stock index return $\widetilde{r}_{a_1,t}$ as the first element of a vector z_t and real labor income growth $\Delta\widetilde{wh}_t$ as the second element of the vector z_t. Other variables observed at the end of period t, denoted by x_t, which contain useful information to forecast future stock returns and labor income growths, are stacked following these two variables. Thereby, $z_t \equiv [\widetilde{r}_{a_1,t}, \Delta\widetilde{wh}_t, x_t^T]^T$. Suppose that z_t follows a first-order VAR process,

$$z_{t+1} = AZ_t + \varepsilon_{t+1} \tag{A.6}$$

where A is the companion matrix of the VAR and innovations ε_{t+j} for $j > 1$ are orthogonal to z_t.

As in Campbell (1996), x_t contains a relative bill rate (the difference between 3-month Treasury bill rate and its one-year backward moving average) and the yield spread between long (5-year) and short (3-month) term government bonds. These two variables are considered to be informative about future stock returns and labor income growth in the literature. Let e_1 be a vector with the same dimension as z_t, whose first element is one and whose other elements are all zero, or in another word, $\widetilde{r}_{a_1,t} = e_1^T z_t$. Similarly, let e_2 be a vector which picks up the second element in z_t, that is, $\Delta\widetilde{wh}_t = e_2^T z_t$. The data used here are quarterly data which is described in the data appendix. The *foreign* country variables are constructed as follows. First, labor income and stock index returns are deflated by CPI and then I remove seasonal components of each series using X-12. Second, express all domestic variables in the *home* currency using bilateral real exchange rates. Third, using the bilateral trade as weights, take a weighted average of the corresponding variable in other countries to obtain an artificial *foreign* variable. Each series is demeaned and therefore no constant term in the VAR above. I estimate the VAR processes in the home country and its foreign counterpart separately since stock returns and labor income growth are mainly determined by its own characteristics within a country. Coeurdacier and Gourinchas (2011) estimate a similar process like equation (A.6) using internationally differenced variables and other variables, such as log consumption expenditure, relative bond returns and cyclical external imbalances nxa in Gourinchas and Rey (2007).

Before estimating the VAR process in equation (A.6), we need to decide the optimal lags. The optimal lags for most of countries in the sample (i.e., USA, Japan, Germany, France, Italy, Denmark and Australia and their foreign counterparts, UK, Canada, and the foreign counterpart of Spain) is one. I then chose the first-order VAR specification for all countries in the sample. As in the literature, the steady state return parameter is set to $\rho = 0.98$ in the quarterly data in order to reflect the risk-premium observed in the data on asset prices. After obtaining the coefficient A, then the conditional expectations of vector z_t becomes $E_t z_{t+1+j} = A^{j+1} z_t$ with

$j \geq 0$. The expected changes in returns on stocks can be written as,

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \widetilde{r}_{a_1,t+1+j} = e_1^T \rho A (I - \rho A)^{-1} \varepsilon_{t+1} \tag{A.7}$$

with an identity matrix I. The expected change in labor income growth then becomes,

$$(E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta \widetilde{wh}_{t+1+j} = e_2^T (I - \rho A)^{-1} \varepsilon_{t+1} \tag{A.8}$$

Thereby, the innovations in the return on human capital can be expressed as,

$$\widetilde{r}_{n,t+1} - E_t \widetilde{r}_{n,t+1} = (e_2^T - e_1^T \rho A)(I - \rho A)^{-1} \varepsilon_{t+1} \tag{A.9}$$

Analogous to Coeurdacier and Gourinchas (2011), the estimation show that innovations to the returns on human capital are quite volatile, with a similar order as returns on stocks.

A.4 Alternative currency pricing: pricing-to-market

Producer currency pricing assumes a complete exchange rate pass-through. Recent evidence from micro data (see, Gopinath and Rigobon 2008; Gopinath, Itskhoki and Rigobon 2010) shows that 90% (97%) of U.S. imports (exports) are priced in dollars.[43] Therefore, there is producer currency pricing in exports and local currency pricing in imports for U.S.. In this section, we consider another extreme case in which firms take use of local currency pricing instead of producer currency pricing, and subsequently the exchange rate pass-through is zero.

Assume that goods markets are segmented across borders. Firms in both countries can set their prices in terms of local currency, which implies that the law of one price doesn't hold even for an individual variety. The main difference between these two pricing strategies is the intermediate producers' maximization problems, market clearing conditions for goods and the evolvement of prices. In the following analysis, except for the currency in pricing, the environment considered in this section is the exactly same as in section 2.1, and therefore we just list the equations which are different from those in the producer currency pricing.

[43] Gopinath, Itskhoki and Rigobon (2010) report the fraction of nondollar priced imports from most of counties is small. For instance, Germany is 40%, Japan 21%, UK 20% and Canada 4%.

A.4.1 Intermediate producers

Intermediate producer j obtains nominal profits from both markets in period t,

$$D_{p,j,t} \equiv (P_{H,j,t} - P_{w,t}) \left(\frac{P_{H,j,t}}{P_{H,t}}\right)^{-\theta_p} Y_{H,t} + (\mathcal{E}_t P^*_{H,j,t} - P_{w,t}) \left(\frac{P^*_{H,j,t}}{P^*_{H,t}}\right)^{-\theta_p} Y^*_{H,t} \tag{A.10}$$

where the first term on the right hand side denotes the revenues from sales of intermediate goods at Home country minus cost of wholesale goods and the second term shows revenues from abroad net of cost of wholesales, all of which are in terms of Home currency.

Producer j chooses $\tilde{P}_{H,j,t}$ and $\tilde{P}^*_{H,j,t}$ to maximize its expected discounted profits (2.21). The optimal prices are set such that the sum of expected discounted profits from selling intermediates at Home (Foreign) equals the expected discounted cost from purchasing whole goods at Home (Foreign) given the random duration of price readjustment. It follows that the optimal prices are given by,

$$\tilde{P}_{H,j,t} = \frac{\theta_p}{\theta_p - 1} \frac{E_t \left\{ \sum_{\tau=0}^{+\infty} \xi_p^\tau Q_{t,t+\tau} Y_{H,t+\tau} P_{H,t+\tau}^{1+\theta_p} / X_{p,t+\tau} \right\}}{E_t \left\{ \sum_{\tau=0}^{+\infty} \xi_p^\tau Q_{t,t+\tau} Y_{H,t+\tau} P_{H,t+\tau}^{\theta_p} \right\}}$$

$$\tilde{P}^*_{H,j,t} = \frac{\theta_p}{\theta_p - 1} \frac{E_t \left\{ \sum_{\tau=0}^{+\infty} \xi_p^\tau Q_{t,t+\tau} Y^*_{H,t+\tau} P^{*\theta_p}_{H,t+\tau} P_{H,t+\tau} / X_{p,t+\tau} \right\}}{E_t \left\{ \sum_{\tau=0}^{+\infty} \xi_p^\tau Q_{t,t+\tau} \mathcal{E}_{t+\tau} Y^*_{H,t+\tau} P^{*\theta_p}_{H,t+\tau} \right\}}$$

Foreign intermediate producers have similar rules for optimal prices $\tilde{P}_{F,j,t}$ and $\tilde{P}^*_{F,j,t}$.

It then follows that total real dividends accumulated by Home intermediate firms are given by,

$$\begin{aligned} d_{p,t} &\equiv \int_0^1 d_{p,j,t} dj \\ &= Y_{H,t} p_{H,t} + Y^*_{H,t} p^*_{H,t} s_t - \left(\frac{Y_{H,t} s^p_{H,t} p_{H,t}}{X_{p,t}} + \frac{Y^*_{H,t} s^{p*}_{H,t} p_{H,t}}{X_{p,t}} \right) \end{aligned}$$

Dividends to intermediaries consist of total revenues from sales of intermediates in Home and Foreign goods markets net of total cost of purchasing wholesale goods. $s^p_{H,t}$ represents the price dispersion in the home market and $s^{p*}_{H,t}$ is its foreign counterpart.

A.4.2 Market clearings and the competitive equilibrium

The market clearing conditions for each good at each country are as follows,

$$C_{H,t} + I_{H,t} = Y_{H,t} \qquad C^*_{H,t} + I^*_{H,t} = Y^*_{H,t}$$
$$C_{F,t} + I_{F,t} = Y_{F,t} \qquad C^*_{F,t} + I^*_{F,t} = Y^*_{F,t}$$

The supply of wholesale goods links to the demand for it via the following equation,

$$Y_t = \int_0^1 Y_{H,j,t} dj + \int_0^1 Y^*_{H,j,t} dj = s^p_{H,t} Y_{H,t} + s^{p*}_{H,t} Y^*_{H,t}$$

The Home goods price dispersion in Home market is measured by $s^{p*}_{H,t}$, which evolves as,

$$s^{p*}_{H,t} = (1-\xi_p)\tilde{p}^{-\theta_p}_{H,t} + \xi_p \pi^{\theta_p}_{H,t} s^{p*}_{H,t-1}$$

Home goods in the Foreign market and Foreign goods in the Home and Foreign markets have similar price dispersion evolvements.

The aggregate dividends at Home country distributed to equity holders have a form of,

$$d_t = \frac{\alpha p_{H,t} Y_{H,t}}{X_{p,t}} - p_{I,t} I_t + d_{p,t}$$

Labor markets and financial markets are exactly the same as those in the benchmark model with producer currency pricing. The competitive equilibrium consists of a sequence of prices and of allocation rules for households, wholesalers and intermediate producers such that (1) households' decision rules solve their own utility maximization problems given the prevailing asset prices, goods prices and wages; (2) The decision rules for firms solve their profit maximization problems; (3) Goods markets, labor markets and asset markets clear each period.

A.4.3 Steady state portfolios

Notice that under pricing to market, exchange rate pass-through is zero and consequently real exchange rate doesn't perfectly move along with terms of trade even up to a first order approximation. Consequently, even though a bond asset whose interest payment is indexed to locally produced goods does completely hedge terms of trade fluctuations, it doesn't yet hedge real exchange rate risk completely, and vice versa. We then evaluate the different components of optimal shares of home equities. Analogous to the benchmark model, home equity holdings are extremely sensitive to the preference parameters when nominal consols are traded in the international financial market. The home household holds 412% of foreign equities when trade elasticity is 2 and the coefficient of relative risk aversion is $\sigma = 2$. However, the component of optimal equity shares hedging human capital income risk is similar to the benchmark model.

A.5 Foreign currency forward contract

In Engel and Matsumoto (2009a), they show that foreign exchange forward contracts can hedge exchange rate risk. This section aims to examine whether their claims hold in the model studied in this chapter. The asset menu now available to households in both countries becomes equity assets and foreign exchange forward contracts. First, let's write the household's budget constraint in local currency as,

$$P_tC_t + M_t + Q_t\psi_{H,t+1} + \mathcal{E}_tQ_t^*\psi_{F,t+1} = \\ \psi_{H,t}(Q_t + D_t) + \psi_{F,t}\mathcal{E}_t(Q_t^* + D_t^*) + (\mathcal{E}_t - \mathcal{F}_t)\tilde{\psi}_t + W_tH_t^d + M_{t-1} + Tr_t \tag{A.11}$$

where $\mathcal{F}_t$ is the delivery price of a forward contract and $\mathcal{E}_t$ is the spot exchange rate at maturity of the contract. $\tilde{\psi}_t$ stands for the units of forward contracts purchased by the home household. When $\tilde{\psi}_t \geq 0$, the home household has a long position in a forward contract. The optimal condition for a foreign currency forward contract gives,

$$0 = E_t\left[v(t)\frac{U_{c,t+1}}{P_{t+1}}\left(\mathcal{E}_{t+1} - \mathcal{F}_{t+1}\right)\right] \tag{A.12}$$

where $\mathcal{E}_t - \mathcal{F}_t$ denotes the payoff from a long position in a forward contract on one unit of foreign currency.

Table A.1: Optimal home equity shares held by home households: nominal consols and local currency pricing

	$\gamma = 0.50$	$\gamma = 0.85$	$\gamma = 1$	$\gamma = 1.1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$
$\sigma = 0.50$	3.54	1.53	1.00	0.71	-0.16	-0.84	-1.65
$\sigma = 1.00$	3.54	1.53	1.00	0.71	-0.16	-0.84	-1.65
$\sigma = 1.50$	3.54	1.53	1.00	0.71	-0.16	-0.84	-1.65
$\sigma = 2.00$	3.54	1.53	1.00	0.71	-0.16	-0.84	-1.65
$\sigma = 5.00$	3.54	1.53	1.00	0.71	-0.16	-0.84	-1.65
$\sigma = 10.00$	3.54	1.53	1.00	0.71	-0.16	-0.84	-1.65

Note: The number shown above is the optimal share of home equities held by home residents. Each unit of consol bond pays one unit of local currency each period. Producers use local currency pricing. There's home bias in consumption and no physical capital. Shock process parameters are $\sigma_m^2 = 2\sigma_a^2$, $\rho_a = 0.905^4$ and $\rho_m = 0.357$. Other parameters are the same as in the benchmark model.

Let the real returns on one unit of foreign currency from a long position in a forward contract be,

$$r_{c,t} \equiv \frac{s_t}{\pi_t^*} - \frac{f_t}{\pi_t} \tag{A.13}$$

where $f_t \equiv \frac{\mathcal{F}_t P^*_{t-1}}{P_{t-1}}$, which has a similar interpretation as real exchange rates.

The delivery price of a forward contract in equation (A.12) then is determined by the following condition,

$$E_t \left[\Lambda_{t,t+1} r_{c,t+1}\right] = 0 \tag{A.14}$$

Rewrite Household's budget constraint (A.11) in terms of real consumption,

$$\begin{aligned} &C_t + q_t \psi_{H,t+1} + s_t q^*_t \psi_{F,t+1} + m_t = \\ &\quad w_t H^d_t + (q_t + d_t)\psi_{H,t} + s_t(q^*_t + d^*_t)\psi_{F,t} + r_{c,t}\psi_t + \frac{m_{t-1}}{\pi_t} + tr_t \end{aligned} \tag{A.15}$$

with $\psi_t \equiv \frac{\tilde{\psi}_t}{P^*_{t-1}}$, the number of forward contracts on foreign currency in terms of real foreign consumption.

As usual, net foreign asset position,nfa_{t+1}, here is defined as,

$$nfa_{t+1} = q_t \left(\psi_{H,t+1} - 1\right) + s_t q^*_t \psi_{F,t+1}$$

Budget constraint (A.15) can be written as,

$$\begin{aligned} nfa_{t+1} &= w_t H^d_t + d_t - C_t + q_{t-1}(\psi_{H,t} - 1) r_{1,t} + s_{t-1} q^*_{t-1} \psi_{F,t} r_{2,t} + r_{c,t}\psi_t \\ &= w_t H^d_t + d_t - C_t + r_{2,t} nfa_t + \alpha^T_t r_{x,t} \end{aligned}$$

where $\alpha_t \equiv \left[q_{t-1}(\psi_{H,t} - 1), \psi_t\right]^T$ and excess returns on home equity and forward contract, $r_{x,t} \equiv \left[r_{1,t} - r_{2,t}, r_{c,t}\right]^T$.

I redo the experiment and obtain the optimal equity shares. The experiment shows that a model with foreign currency forward contracts coincides with a model with one-period nominal bonds, both of which are almost the same as a model with nominal consols. Optimal shares of home equities are quite sensitive to trade elasticity and relative risk aversion.

Appendix B

Appendix to Chapter 3

B.1 Log-linearization

B.1.1 Budget constraints

The linearized budget constraint can be written as,

$$\frac{3}{4}\widetilde{NW}_{B,t+1} + \frac{1}{4}\widetilde{NW}_{C,t+1} = \frac{3}{4\beta}\widetilde{NW}_{B,t} + \frac{1}{4\beta}\widetilde{NW}_{C,t} + \frac{3}{4}\widetilde{\alpha}'_B\widehat{R}_{x,t} + \frac{1}{4}\widetilde{\alpha}'_C\widehat{R}_{x,t} + \Gamma_{A1}\Delta\widehat{Z}_{B,t} + \Gamma_{A2}\Delta\widehat{Z}_{C,t} + \Gamma_{A3}\Delta\widehat{C}_{B,t} + \Gamma_{A4}\Delta\widehat{C}_{C,t} \tag{B.1}$$

$$\frac{1}{4}\widetilde{NW}_{B,t+1} + \frac{3}{4}\widetilde{NW}_{C,t+1} = \frac{1}{4\beta}\widetilde{NW}_{B,t} + \frac{3}{4\beta}\widetilde{NW}_{C,t} + \frac{1}{4\beta}\widetilde{\alpha}'_B\widehat{R}_{x,t} + \frac{3}{4\beta}\widetilde{\alpha}'_C\widehat{R}_{x,t} + \Gamma_{A2}\Delta\widehat{Z}_{B,t} + \Gamma_{A1}\Delta\widehat{Z}_{C,t} + \Gamma_{A4}\Delta\widehat{C}_{B,t} + \Gamma_{A3}\Delta\widehat{C}_{C,t} \tag{B.2}$$

where Γ_{As} with $s = 1, 2, 3, 4$ are functions of structural parameters in the model, which are given as,

$$\Gamma_{A1} = \frac{1}{2}\left[-\frac{\Gamma_{Y1} + \Gamma_{Y3}}{2} + \Gamma_{P1}(1-\alpha_1) - \frac{\Gamma_{P3}}{2}(1 + \alpha_1 - 2\alpha_1\alpha_2)\right]$$
$$\Gamma_{A2} = \frac{1}{2}\left[-\frac{\Gamma_{Y1} - \Gamma_{Y3}}{2} + \Gamma_{P1}(1-\alpha_1) + \frac{\Gamma_{P3}}{2}(1 + \alpha_1 - 2\alpha_1\alpha_2)\right]$$
$$\Gamma_{A3} = \frac{1}{2}\left[1 + \frac{\Gamma_{Y2} + \Gamma_{Y4}}{2} - \Gamma_{P2}(1-\alpha_1) + \frac{\Gamma_{P4}}{2}(1 + \alpha_1 - 2\alpha_1\alpha_2)\right]$$
$$\Gamma_{A4} = \frac{1}{2}\left[\frac{\Gamma_{Y2} - \Gamma_{Y2}}{2} - \Gamma_{P2}(1-\alpha_1) - \frac{\Gamma_{P4}}{2}(1 + \alpha_1 - 2\alpha_1\alpha_2)\right]$$

where Γ_{Ys} and Γ_{Ps} with $s = 1, 2, 3, 4$ are defined as follows,

$$\Gamma_{Y1} = \frac{2\alpha_1\gamma(1+\eta)}{2\alpha_1\gamma(\theta+\eta) + 1 - \theta} \qquad \Gamma_{Y2} = \frac{(1-\theta)(2\alpha_1\gamma\sigma - 2\alpha_1 + 1)}{2\alpha_1\gamma(\theta+\eta) + 1 - \theta}$$
$$\Gamma_{Y3} = \frac{\gamma(1+\eta)\left[\alpha_1(2\alpha_2 - 1) + 1\right]}{\gamma(\theta+\eta)\left[\alpha_1(2\alpha_2 - 1) + 1\right] + 1 - \theta} \qquad \Gamma_{Y4} = \frac{(1-\theta)\left[\sigma\gamma + (\sigma\gamma - 1)\alpha_1(2\alpha_2 - 1)\right]}{\gamma(\theta+\eta)\left[\alpha_1(2\alpha_2 - 1) + 1\right] + 1 - \theta}$$

$$\Gamma_{P1} = \frac{1+\eta}{\eta+\theta}\frac{1}{\Gamma_1} \qquad \Gamma_{P2} = \left[\frac{(1-\theta)\sigma}{\eta+\theta} + 2\alpha_1 - 1\right]\frac{1}{\Gamma_1}$$
$$\Gamma_{P3} = \frac{1+\eta}{\eta+\theta}\frac{1}{\Gamma_2} \qquad \Gamma_{P4} = \left[\frac{(1-\theta)\sigma}{\eta+\theta} + \alpha_1(2\alpha_2 - 1)\right]\frac{1}{\Gamma_2}$$

Γ_1 and Γ_2 are defined as,

$$\Gamma_1 = 2(1-\alpha_1)\left[2\alpha_1\gamma + \frac{1-\theta}{\eta+\theta}\right]$$
$$\Gamma_2 = [\alpha_1(2\alpha_2 - 1) - 1]\left\{\gamma\left[\alpha_1(2\alpha_2 - 1) + 1\right] + \frac{1-\theta}{\eta+\theta}\right\}$$

Notice that $\Gamma_1 > 0$ and $\Gamma_1 < 0$ and thereby, $\Gamma_{P1} > 0$, $\Gamma_{P2} > 0$, $\Gamma_{P3} < 0$. $\Gamma_{Y1} > 0$, $\Gamma_{Y3} > 0$.

A special case. When trade elasticity is equal to unity $\gamma = 1$ and there is no home bias in consuming peripheral goods for peripheral countries $\alpha_2 = \frac{1}{2}$, terms of trade can automatically adjust the value of production in the peripheral countries, which implies that when labor supply is inelastic (or the economy is an endowment economy), equities in the peripheral countries are perfect substitutes.

B.1.2 First-order approximation to Euler equations

Euler equations for portfolio choices can be approximated by the following equations,

$$(\sigma - \zeta)\Delta\widehat{C}_{B,t} + \Delta\widehat{P}_{B,t} = E_t\left[\sigma\Delta\widehat{C}_{B,t+1} + \Delta\widehat{P}_{B,t+1}\right] \tag{B.3}$$
$$(\sigma - \zeta)\Delta\widehat{C}_{C,t} + \Delta\widehat{P}_{C,t} = E_t\left[\sigma\Delta\widehat{C}_{C,t+1} + \Delta\widehat{P}_{C,t+1}\right] \tag{B.4}$$

where relative consumer prices $\Delta\widehat{P}_{B,t} \equiv \widehat{P}_{A,t} - \widehat{P}_{B,t}$ and $\Delta\widehat{P}_{C,t} \equiv \widehat{P}_{A,t} - \widehat{P}_{C,t}$ are defined as,

$$\Delta\widehat{P}_{B,t} = \Gamma_{B1}\Delta\widehat{Z}_{B,t} + \Gamma_{B2}\Delta\widehat{Z}_{C,t} - \Gamma_{B3}\Delta\widehat{C}_{B,t} - \Gamma_{B4}\Delta\widehat{C}_{C,t}$$
$$\Delta\widehat{P}_{C,t} = \Gamma_{B2}\Delta\widehat{Z}_{B,t} + \Gamma_{B1}\Delta\widehat{Z}_{C,t} - \Gamma_{B4}\Delta\widehat{C}_{B,t} - \Gamma_{B3}\Delta\widehat{C}_{C,t}$$

where Γ_{Bs}, $s = 1, 2, 3, 4$ are defined as,

$$\Gamma_{B1} = \frac{1-2\alpha_1}{2}\Gamma_{P1} - \frac{\Gamma_{P3}}{2}(\alpha_1 - 2\alpha_1\alpha_2)$$
$$\Gamma_{B2} = \frac{1-2\alpha_1}{2}\Gamma_{P1} + \frac{\Gamma_{P3}}{2}(\alpha_1 - 2\alpha_1\alpha_2)$$
$$\Gamma_{B3} = \frac{1-2\alpha_1}{2}\Gamma_{P2} - \frac{\Gamma_{P4}}{2}(\alpha_1 - 2\alpha_1\alpha_2)$$
$$\Gamma_{B4} = \frac{1-2\alpha_1}{2}\Gamma_{P2} + \frac{\Gamma_{P4}}{2}(\alpha_1 - 2\alpha_1\alpha_2)$$

In deriving relative prices $\Delta\widehat{P}_{B,t}$ and $\Delta\widehat{P}_{C,t}$, I have used the fact that prices of products in each country can be expressed as functions of relative consumption and relative technology,

$$\widehat{P}^{B}_{B,t} = \left[\frac{\Gamma_{P1}-\Gamma_{P3}}{2}, \frac{\Gamma_{P1}+\Gamma_{P3}}{2}\right]\begin{bmatrix}\Delta\widehat{Z}_{B,t}\\ \Delta\widehat{Z}_{C,t}\end{bmatrix} - \left[\frac{\Gamma_{P2}-\Gamma_{P4}}{2}, \frac{\Gamma_{P2}+\Gamma_{P4}}{2}\right]\begin{bmatrix}\Delta\widehat{C}_{B,t}\\ \Delta\widehat{C}_{C,t}\end{bmatrix} \tag{B.5}$$

$$\widehat{P}^{C}_{C,t} = \left[\frac{\Gamma_{P1}+\Gamma_{P3}}{2}, \frac{\Gamma_{P1}-\Gamma_{P3}}{2}\right]\begin{bmatrix}\Delta\widehat{Z}_{B,t}\\ \Delta\widehat{Z}_{C,t}\end{bmatrix} - \left[\frac{\Gamma_{P2}+\Gamma_{P4}}{2}, \frac{\Gamma_{P2}-\Gamma_{P4}}{2}\right]\begin{bmatrix}\Delta\widehat{C}_{B,t}\\ \Delta\widehat{C}_{C,t}\end{bmatrix} \tag{B.6}$$

Substituting relative prices $\Delta\widehat{P}_{B,t}$ and $\Delta\widehat{P}_{C,t}$ into the linearized Euler equations and I obtain,

$$\begin{aligned} E_t\left[(\sigma-\Gamma_{B3})\Delta\widehat{C}_{B,t+1} - \Gamma_{B4}\Delta\widehat{C}_{C,t+1}\right] &= -E_t\left[\Gamma_{B1}\Delta\widehat{Z}_{B,t+1} + \Gamma_{B2}\Delta\widehat{Z}_{C,t+1}\right] \\ &+ (\sigma-\zeta-\Gamma_{B3})\Delta\widehat{C}_{B,t} - \Gamma_{B4}\Delta\widehat{C}_{C,t} + \Gamma_{B1}\Delta\widehat{Z}_{B,t} + \Gamma_{B2}\Delta\widehat{Z}_{C,t} \end{aligned} \tag{B.7}$$

$$\begin{aligned} E_t\left[(\sigma-\Gamma_{B3})\Delta\widehat{C}_{C,t+1} - \Gamma_{B4}\Delta\widehat{C}_{B,t+1}\right] &= -E_t\left[\Gamma_{B1}\Delta\widehat{Z}_{C,t+1} + \Gamma_{B2}\Delta\widehat{Z}_{B,t+1}\right] \\ &+ (\sigma-\zeta-\Gamma_{B3})\Delta\widehat{C}_{C,t} - \Gamma_{B4}\Delta\widehat{C}_{B,t} + \Gamma_{B1}\Delta\widehat{Z}_{C,t} + \Gamma_{B2}\Delta\widehat{Z}_{B,t} \end{aligned} \tag{B.8}$$

B.1.3 Solving the dynamic system

Let $x_t \equiv [\widetilde{NW}_{B,t}; \widetilde{NW}_{C,t}]$ and $y_t \equiv [\widetilde{C}_{B,t}; \widetilde{C}_{C,t}]$. A four-equation system can be rewritten as follows,

$$\mathbf{A}\begin{bmatrix} x_{t+1} \\ E_t(y_{t+1})\end{bmatrix} = \mathbf{B}\begin{bmatrix} x_t \\ y_t\end{bmatrix} + \mathbf{f}_t \tag{B.9}$$

with

$$\mathbf{A} \equiv \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ 0 & 0 & \sigma-\Gamma_{B3} & -\Gamma_{B4} \\ 0 & 0 & -\Gamma_{B4} & \sigma-\Gamma_{B3} \end{bmatrix}, \quad \mathbf{B} \equiv \begin{bmatrix} \frac{3}{4\beta} & \frac{1}{4\beta} & \Gamma_{A3} & \Gamma_{A4} \\ \frac{1}{4\beta} & \frac{3}{4\beta} & \Gamma_{A4} & \Gamma_{A3} \\ 0 & 0 & \sigma-\zeta-\Gamma_{B3} & -\Gamma_{B4} \\ 0 & 0 & -\Gamma_{B4} & \sigma-\zeta-\Gamma_{B3} \end{bmatrix}$$

$$\mathbf{f}_t \equiv \begin{bmatrix} \Gamma_{A1} & \Gamma_{A2} \\ \Gamma_{A2} & \Gamma_{A1} \\ \Gamma_{B1} & \Gamma_{B2} \\ \Gamma_{B2} & \Gamma_{B1} \end{bmatrix}\begin{bmatrix}\Delta\widehat{Z}_{B,t}\\ \Delta\widehat{Z}_{C,t}\end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -\Gamma_{B1} & -\Gamma_{B2} \\ -\Gamma_{B2} & -\Gamma_{B1} \end{bmatrix}\begin{bmatrix} E_t\left[\Delta\widehat{Z}_{B,t+1}\right] \\ E_t\left[\Delta\widehat{Z}_{C,t+1}\right]\end{bmatrix} + \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix}\xi_{A,t}\\ \xi_{B,t}\end{bmatrix}$$

Let $\widetilde{\mathbf{A}} \equiv \mathbf{A^{-1}B}$ and $\widetilde{\mathbf{f}}_t \equiv \mathbf{A^{-1}f}_t$. I can rewrite the dynamic system (B.9) as,

$$\begin{bmatrix} x_{t+1} \\ E_t(y_{t+1}) \end{bmatrix} = \widetilde{\mathbf{A}} \begin{bmatrix} x_t \\ y_t \end{bmatrix} + \widetilde{\mathbf{f}}_t \tag{B.10}$$

Following Blanchard and Quah (1980), applying Jordan decomposition to $\widetilde{\mathbf{A}}$ and obtain

$$\widetilde{\mathbf{A}} = \mathbf{V\Lambda V^{-1}}$$

where $\mathbf{\Lambda}$ is a diagonal matrix with eigenvalues of $\widetilde{\mathbf{A}}$, λ_i, $i = 1, 2, 3, 4$, on its diagonal in a descending order. $\mathbf{V}$ is a matrix whose columns are the corresponding orthogonal eigenvector to eigenvalue λ_i. These four eigenvalues can be written explicitly as,

$$\lambda_1 = 1 - \frac{\zeta}{\sigma - \frac{\left[\alpha_1(2\alpha_2-1) - \frac{\sigma(\theta-1)}{\eta+\theta}\right](\alpha_1 - 2\alpha_1\alpha_2)}{\gamma - \alpha_1^2\gamma(2\alpha_2-1)^2 + \frac{(\alpha_1(2\alpha_2-1)-1)(\theta-1)}{\eta+\theta}}} \qquad \lambda_3 = \frac{1}{\beta}$$

$$\lambda_2 = 1 - \frac{\zeta}{\sigma + \frac{(2\alpha_1-1)\left[\frac{\sigma(\theta-1)}{\eta+\theta} - 2\alpha_1 + 1\right]}{2(\alpha_1-1)\left[2\alpha_1\gamma - \frac{\theta-1}{\eta+\theta}\right]}} \qquad \lambda_4 = \frac{1}{\beta}$$

Observe that $\lambda_2 < 1$ when $\frac{1}{2} \leq \alpha_1 \leq 1$ and $\lambda_3 > 1$, $\lambda_4 > 1$. $\lambda_1 < 1$ holds for a large range of parameter values.

Let $\begin{bmatrix} x_{2,t+1} \\ E_t(y_{2,t+1}) \end{bmatrix} \equiv \mathbf{V}^{-1} \begin{bmatrix} x_{t+1} \\ E_t(y_{t+1}) \end{bmatrix}$. The system (B.10) can be written as,

$$\begin{bmatrix} x_{2,t+1} \\ E_t(y_{2,t+1}) \end{bmatrix} = \begin{bmatrix} \Lambda_{11} & 0 \\ 0 & \Lambda_{22} \end{bmatrix} \begin{bmatrix} x_{2,t} \\ y_{2,t} \end{bmatrix} + \tilde{\tilde{\mathbf{f}}}_t \tag{B.11}$$

with $\tilde{\tilde{\mathbf{f}}}_t \equiv \mathbf{V}^{-1}\mathbf{A}^{-1}\mathbf{f}_t$. Λ_{11} is a diagonal matrix with the first two less-than-one eigenvalues on its main diagonal and Λ_{22} is a diagonal matrix with the last two larger-than-one eigenvalues on the diagonal. The lower partition of system (B.11) then becomes,

$$y_{2,t} = -M\Delta\widehat{Z}_t - N\xi_t \tag{B.12}$$

where $M \equiv \Lambda_{22}^{-1}(\mathbf{I}_2 - \Lambda_{22}^{-1}\rho)^{-1}[\mathbf{V}^{-1}\mathbf{A}^{-1}F]_L$ and $N \equiv \Lambda_{22}^{-1}[\mathbf{V}^{-1}\mathbf{A}^{-1}G]_L$. $[\cdot]_L$ denotes the bottom row of a matrix. F and G are two constant matrices, given by,

$$F \equiv \begin{bmatrix} \Gamma_{A1} & \Gamma_{A2} \\ \Gamma_{A2} & \Gamma_{A1} \\ \Gamma_{B1}(1-\rho) & \Gamma_{B2}(1-\rho) \\ \Gamma_{B2}(1-\rho) & \Gamma_{B1}(1-\rho) \end{bmatrix}, \quad G \equiv \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Substituting $y_{2,t+1}$ back into x_{t+1} and y_{t+1}, I obtain the solution for controls y_{t+1},

$$y_{t+1} = D_1\xi_{t+1} + D_2\varepsilon_{t+1} + D_3 \begin{bmatrix} x_{t+1} \\ \Delta\widehat{Z}_t \end{bmatrix} \tag{B.13}$$

D_s, $s = 1, 2, 3$ are defined as,

$$\begin{aligned} D_1 &\equiv -\left[\left[\mathbf{V}^{-1}\right]_{22}\right]^{-1} N \\ D_2 &\equiv -\left[\left[\mathbf{V}^{-1}\right]_{22}\right]^{-1} M\Phi \\ D_3 &\equiv -\left[\left[\left[\mathbf{V}^{-1}\right]_{22}\right]^{-1}\left[\mathbf{V}^{-1}\right]_{21}, \left[\left[\mathbf{V}^{-1}\right]_{22}\right]^{-1} M\rho\right] \end{aligned}$$

and $\Phi \equiv \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$.

Unexpected excess returns follow as,

$$\widehat{R}_{B,t+1} - \widehat{R}_{A,t+1} = R_{11}\xi_{t+1} + R_{21}\varepsilon_{t+1} \tag{B.14}$$

$$\widehat{R}_{C,t+1} - \widehat{R}_{A,t+1} = R_{12}\xi_{t+1} + R_{22}\varepsilon_{t+1} \tag{B.15}$$

where R_{11}, R_{21}, R_{12} and R_{22} are constant and are given by,

$$\begin{aligned} R_{11} &= -\Gamma'_{RB2}D_1 & R_{21} &= \Gamma'_{RB1}\Phi - \Gamma'_{RB2}D_2 \\ R_{12} &= -\Gamma'_{RC2}D_1 & R_{22} &= \Gamma'_{RC1}\Phi - \Gamma'_{RC2}D_2 \end{aligned}$$

and

$$\begin{aligned} \Gamma_{RB1} &= \begin{bmatrix} \frac{\Gamma_{P1}-\Gamma_{P3}}{2} - \frac{\Gamma_{Y1}+\Gamma_{Y3}}{2} \\ \frac{\Gamma_{P1}+\Gamma_{P3}}{2} - \frac{\Gamma_{Y1}-\Gamma_{Y3}}{2} \end{bmatrix} & \Gamma_{RB2} &= \begin{bmatrix} \frac{\Gamma_{P2}-\Gamma_{P4}}{2} - \frac{\Gamma_{Y2}+\Gamma_{Y4}}{2} \\ \frac{\Gamma_{P2}+\Gamma_{P4}}{2} - \frac{\Gamma_{Y2}-\Gamma_{Y4}}{2} \end{bmatrix} \\ \Gamma_{RC1} &= \begin{bmatrix} \frac{\Gamma_{P1}+\Gamma_{P3}}{2} - \frac{\Gamma_{Y1}-\Gamma_{Y3}}{2} \\ \frac{\Gamma_{P1}-\Gamma_{P3}}{2} - \frac{\Gamma_{Y1}+\Gamma_{Y3}}{2} \end{bmatrix} & \Gamma_{RC2} &= \begin{bmatrix} \frac{\Gamma_{P2}+\Gamma_{P4}}{2} - \frac{\Gamma_{Y2}-\Gamma_{Y4}}{2} \\ \frac{\Gamma_{P2}-\Gamma_{P4}}{2} - \frac{\Gamma_{Y2}+\Gamma_{Y4}}{2} \end{bmatrix} \end{aligned}$$

Now I write relative marginal utility as,

$$-\sigma\Delta\widehat{C}_{B,t+1} - \Delta\widehat{P}_{B,t+1} = D_{11}\xi_{t+1} + D_{21}\varepsilon_{t+1} + others_{t+1} \tag{B.16}$$

$$-\sigma\Delta\widehat{C}_{C,t+1} - \Delta\widehat{P}_{C,t+1} = D_{12}\xi_{t+1} + D_{22}\varepsilon_{t+1} + others_{t+1} \tag{B.17}$$

where

$$\begin{aligned} D_{11} &= \Gamma'_{UB1}D_1 & D_{21} &= \Gamma'_{UB1}D_2 - \Gamma'_{UB2}\Phi \\ D_{12} &= \Gamma'_{UC1}D_1 & D_{22} &= \Gamma'_{UC1}D_2 - \Gamma'_{UC2}\Phi \end{aligned}$$

and

$$\begin{aligned} \Gamma_{UB1} &\equiv \begin{bmatrix} -\sigma + \Gamma_{B3} \\ \Gamma_{B4} \end{bmatrix} & \Gamma_{UB2} &\equiv \begin{bmatrix} \Gamma_{B1} \\ \Gamma_{B2} \end{bmatrix} \\ \Gamma_{UC1} &\equiv \begin{bmatrix} \Gamma_{B4} \\ -\sigma + \Gamma_{B3} \end{bmatrix} & \Gamma_{UC2} &\equiv \begin{bmatrix} \Gamma_{B2} \\ \Gamma_{B1} \end{bmatrix} \end{aligned}$$

The last term $others_{t+1}$ represents a collection of terms which are predetermined

at date $t+1$.

Substituting excess returns in equation (B.14)-(B.15) and relative marginal utilities in equation (B.16)-(B.17) into the second-order approximation to Euler equations, yields,

$$E_t\left\{\begin{bmatrix} D_{11}\xi_{t+1}+D_{21}\varepsilon_{t+1} & 0 \\ 0 & D_{12}\xi_{t+1}+D_{22}\varepsilon_{t+1}\end{bmatrix}\begin{bmatrix} R_{11}\xi_{t+1}+R_{21}\varepsilon_{t+1} \\ R_{12}\xi_{t+1}+R_{22}\varepsilon_{t+1}\end{bmatrix}\right\}=0 \tag{B.18}$$

Now use $\xi_{i,t+1}\equiv\widetilde{\alpha}_i'\widehat{R}_{x,t+1}$, $i=B,C$ to get rid of ξ_{t+1} in the equations above, rearrange related terms and then arrive at a solution for portfolio $\widetilde{\alpha}$, which is summarized in result 1 and 2. Once obtaining the zeroth-order portfolio $\widetilde{\alpha}$, I substitute it into the solution to control variables in equation (B.13) and obtain a solution to the rational expectations equilibrium. Proposition 4 reports this solution in the main text.

B.2 Impulse responses

Relative labor supply $\Delta\widehat{N}_{i,t}\equiv\widehat{N}_{A,t}-\widehat{N}_{i,t}$ can be written as,

$$\Delta\widehat{N}_{B,t}=\frac{1}{\theta+\eta}\left\{\left(\Gamma'_{NCB}-\sigma\begin{bmatrix}1\\0\end{bmatrix}^T\right)(D_1\widetilde{H}+D_2)-\left(\Gamma'_{NZB}-\sigma\begin{bmatrix}1\\0\end{bmatrix}^T\right)\Phi\right\}\varepsilon_t$$

$$\Delta\widehat{N}_{C,t}=\frac{1}{\theta+\eta}\left\{\left(\Gamma'_{NCC}-\sigma\begin{bmatrix}0\\1\end{bmatrix}^T\right)(D_1\widetilde{H}+D_2)-\left(\Gamma'_{NZC}-\sigma\begin{bmatrix}0\\1\end{bmatrix}^T\right)\Phi\right\}\varepsilon_t$$

with

$$\Gamma_{NZB}\equiv\begin{bmatrix}(1-\alpha_1)\Gamma_{P1}-\frac{\Gamma_{P3}}{2}(1+\alpha_1-2\alpha_1\alpha_2)\\(1-\alpha_1)\Gamma_{P1}+\frac{\Gamma_{P3}}{2}(1+\alpha_1-2\alpha_1\alpha_2)\end{bmatrix}$$

$$\Gamma_{NCB}\equiv\begin{bmatrix}(1-\alpha_1)\Gamma_{P2}-\frac{\Gamma_{P4}}{2}(1+\alpha_1-2\alpha_1\alpha_2)\\(1-\alpha_1)\Gamma_{P2}+\frac{\Gamma_{P4}}{2}(1+\alpha_1-2\alpha_1\alpha_2)\end{bmatrix}$$

$$\Gamma_{NZC}\equiv\begin{bmatrix}(1-\alpha_1)\Gamma_{P1}+\frac{\Gamma_{P3}}{2}(1+\alpha_1-2\alpha_1\alpha_2)\\(1-\alpha_1)\Gamma_{P1}-\frac{\Gamma_{P3}}{2}(1+\alpha_1-2\alpha_1\alpha_2)\end{bmatrix}$$

$$\Gamma_{NCC}\equiv\begin{bmatrix}(1-\alpha_1)\Gamma_{P2}+\frac{\Gamma_{P4}}{2}(1+\alpha_1-2\alpha_1\alpha_2)\\(1-\alpha_1)\Gamma_{P2}-\frac{\Gamma_{P4}}{2}(1+\alpha_1-2\alpha_1\alpha_2)\end{bmatrix}$$

B.3 Financial autarky

When financial markets in each country are segmented from each other, trade in goods must balance in each period. The dynamic system then reads ,

$$\begin{bmatrix}\Delta\widehat{C}_{B,t}\\\Delta\widehat{C}_{C,t}\end{bmatrix}=-\begin{bmatrix}\Gamma_{A3}&\Gamma_{A4}\\\Gamma_{A4}&\Gamma_{A3}\end{bmatrix}^{-1}\begin{bmatrix}\Gamma_{A1}&\Gamma_{A2}\\\Gamma_{A2}&\Gamma_{A1}\end{bmatrix}\begin{bmatrix}\Delta\widehat{Z}_{B,t}\\\Delta\widehat{Z}_{C,t}\end{bmatrix}$$

Substituting this solution into price equation (B.5) and (B.6), I obtain a solution to prices. I then substitute prices into the log-linearized production functions and labor market clearing conditions to get a solution to consumption and output in each country.

Appendix C

Appendix to Chapter 4

C.1 A complete set of equilibrium conditions

$$c_t^{-\sigma} = \beta R_{t+1} E_t \left\{ \frac{c_{t+1}^{-\sigma}}{\pi_{t+1}} \right\} \tag{C.1}$$

$$q_t c_t^{-\sigma} = \beta E_t \left\{ c_{t+1}^{-\sigma} q_{t+1}(1-\delta) + \frac{j}{k_{t+1}^s} \right\} \tag{C.2}$$

$$f_t^1 = \frac{\tilde{\phi}-1}{\tilde{\phi}} \tilde{w}_t c_t^{-\sigma}(1-\tau_t^l) \left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\phi}} l_t^d + \tilde{\alpha}\beta E_t \left[\left(\frac{\pi_{t+1}}{\pi_t^{\tilde{\chi}}}\right)^{\tilde{\phi}-1} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\phi}-1} f_{t+1}^1 \right] \tag{C.3}$$

$$f_t^2 = l_t^{\theta} \left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\phi}} l_t^d + \tilde{\alpha}\beta E_t \left[\left(\frac{\pi_{t+1}}{\pi_t^{\tilde{\chi}}}\right)^{\tilde{\phi}} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\phi}} f_{t+1}^2 \right] \tag{C.4}$$

$$f_t^1 = f_t^2 \tag{C.5}$$

$$Y_t = a_t (k_t^e)^{\alpha_0} (l_t^d)^{1-\alpha_0} \tag{C.6}$$

$$b_{t+1}^e = \kappa E_t \left(\frac{q_{t+1} k_{t+1}^e \pi_{t+1}}{R_{t+1}} \right) \tag{C.7}$$

$$c_t^e + q_t \left[k_{t+1}^e - (1-\delta) k_t^e \right] + \tau_t^{le} + \frac{b_t^e R_t}{\pi_t} = (1-\tau_t^k) \left(\frac{Y_t}{X_t} - w_t l_t^d \right) + b_{t+1}^e \tag{C.8}$$

$$\begin{aligned} q_t (c_t^e)^{-\sigma} =& (c_t^e)^{-\sigma} \varphi_t \kappa E_t \left(\frac{q_{t+1}\pi_{t+1}}{R_{t+1}} \right) + E_t \left\{ \gamma\beta (c_{t+1}^e)^{-\sigma} \left[q_{t+1}(1-\delta) \right.\right. \\ & \left.\left. + (1-\tau_{t+1}^k) \frac{\alpha_0 Y_{t+1}}{X_{t+1} k_{t+1}^e} \right] \right\} \end{aligned} \tag{C.9}$$

$$(c_t^e)^{-\sigma} = (c_t^e)^{-\sigma} \varphi_t + E_t \left(\gamma\beta \frac{(c_{t+1}^e)^{-\sigma} R_{t+1}}{\pi_{t+1}} \right) \tag{C.10}$$

$$\frac{(1-\alpha_0) Y_t}{X_t l_t^d} = w_t \tag{C.11}$$

$$i_t = k_{t+1} - (1-\delta) k_t \tag{C.12}$$

$$q_t = \psi\left(\frac{i_t}{k_t}\right) + 1 \tag{C.13}$$

$$y_t = c_t + c_t^e + i_t + c(i_t) + g_t \tag{C.14}$$

$$x_t^1 = \frac{\phi}{\phi-1}\frac{1}{X_t}y_t\tilde{p}_t^{-\phi-1} + \beta\alpha E_t\left\{\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\phi-1}\left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{-\phi}x_{t+1}^1\right\} \tag{C.15}$$

$$x_t^2 = y_t\tilde{p}_t^{-\phi} + \beta\alpha E_t\left\{\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\phi}\left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{1-\phi}x_{t+1}^2\right\} \tag{C.16}$$

$$x_t^1 = x_t^2 \tag{C.17}$$

$$F_t = y_t - \frac{y_t}{X_t}s_t^p \tag{C.18}$$

$$D_t = i_t q_t - \left(\frac{\psi}{2}\left(\frac{i_t}{k_t}\right)^2 k_t + i_t\right) \tag{C.19}$$

$$s_t^p = (1-\alpha)\tilde{p}_t^{-\phi} + \alpha\left(\frac{\pi_t}{\pi_{t-1}^{\chi}}\right)^{\phi}s_{t-1}^p \tag{C.20}$$

$$\tau_t = \tau_t^h l_t^d w_t + \tau_t^f(F_t + D_t) + \tau_t^k\left(\frac{Y_t}{X_t} - w_t l_t^d\right) + \tau_t^{lh} + \tau_t^{le} \tag{C.21}$$

$$\frac{n_{t+1}}{R_{t+1}} + \tau_t = g_t + \frac{n_t}{\pi_t} \tag{C.22}$$

$$1 = \alpha(\pi_{t-1}^{\chi}/\pi_t)^{1-\phi} + (1-\alpha)\tilde{p}_t^{1-\phi} \tag{C.23}$$

$$Y_t = s_t^p y_t \tag{C.24}$$

$$w_t^{1-\tilde{\phi}} = \tilde{\alpha}w_{t-1}^{1-\tilde{\phi}}(\pi_{t-1}^{\tilde{\chi}}/\pi_t)^{1-\tilde{\phi}} + (1-\tilde{\alpha})\tilde{w}_t^{1-\tilde{\phi}} \tag{C.25}$$

$$l_t = l_t^d s_t^w \tag{C.26}$$

$$s_t^w = (1-\tilde{\alpha})\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\phi}} + \tilde{\alpha}\left(\frac{w_{t-1}}{w_t}\right)^{-\tilde{\phi}}\left(\frac{\pi_t}{\pi_{t-1}^{\tilde{\chi}}}\right)^{\tilde{\phi}}s_{t-1}^w \tag{C.27}$$

$$k_{t+1} = k_{t+1}^e + k_{t+1}^s \tag{C.28}$$

$$\text{Fiscal Policy} \tag{C.29}$$

$$\text{Monetary Policy} \tag{C.30}$$

$$ln(a_t/\bar{a}) = \rho_a ln(a_{t-1}/\bar{a}) + \epsilon_t^a \tag{C.31}$$

$$ln(g_t/g) = \rho_g ln(g_{t-1}/g) + \epsilon_t^g \tag{C.32}$$

C.2 The deterministic steady state

The deterministic steady state in this system tries to replicate the long-run average values for variables of interest in the U.S.. The long-run average annualized GDP deflator over $1951 - 2009$ is 1.0344, then $\pi = 1.0344^{1/4}$. Following Mendoza, Razin and Tesar (1994), income tax rates are set as $\tau^k = 0.407$ and $\tau^h = 0.258$. I set $\tau^f = \tau^k$, and lump-sump taxes are not applicable, $\tau^{le} = \tau^{ls} = 0$.

$$R = \frac{\pi}{\beta}$$

$$\tilde{p} = \left(\frac{1 - \alpha\pi^{(1-\chi)(\phi-1)}}{1-\alpha}\right)^{\frac{1}{1-\phi}}$$

$$X = \frac{\phi}{\phi - 1}\tilde{p}^{-1}\frac{1 - \alpha\beta\pi^{(1-\chi)(\phi-1)}}{1 - \alpha\beta\pi^{(1-\chi)\phi}}$$

$$s^p = \frac{(1-\alpha)\tilde{p}^{-\phi}}{1 - \alpha\pi^{(1-\chi)\phi}}$$

$$\frac{w}{\tilde{w}} = \left(\frac{1 - \tilde{\alpha}\pi^{(1-\tilde{\chi})(\tilde{\phi}-1)}}{1-\tilde{\alpha}}\right)^{\frac{1}{\tilde{\phi}-1}}$$

$$s^w = \frac{(1-\tilde{\alpha})(w/\tilde{w})^{\tilde{\phi}}}{1 - \tilde{\alpha}\pi^{(1-\tilde{\chi})\phi}}$$

$$\psi = \frac{-1 + \varphi\kappa\beta + \beta\gamma\left[1 - \delta + (1-\tau^k)\frac{\alpha_0(1+s_e)s^p\delta}{s_i X}\right]}{\delta - \varphi\kappa\delta\beta - \beta\gamma\left[(1-\delta)\delta + (1-\tau^k)\frac{\alpha_0(1+s_e)s^p\delta^2}{2s_i X}\right]}$$

$$q = \psi\delta + 1$$

$$s_k = \frac{s_i}{\delta + \psi\delta^2/2}$$

$$\frac{c^e}{y} = (1-\tau^k)\frac{\alpha_0 s^p}{X} - (\delta + \kappa - \kappa\beta)q\frac{s_k}{1+s_e}$$

$$\frac{c}{y} = 1 - s_i - s_g - c^e/y$$

$$j = \frac{q(1/\beta - 1 + \delta)s_e s_k}{1+s_e}\frac{1}{c/y}$$

$$l = \left(\frac{\tilde{\phi}-1}{\tilde{\phi}}\frac{(1-\tau^l)(1-\tilde{\alpha}\beta\pi^{(1-\tilde{\chi})\tilde{\phi}})}{1-\tilde{\alpha}\beta\pi^{(1-\tilde{\chi})(\tilde{\phi}-1)}}\frac{s^w}{X}\frac{(1-\alpha_0)s^p}{w/\tilde{w}}\frac{1}{c/y}\right)^{\frac{1}{1+\theta}}$$

$$l^d = l/s^w$$

$$y = \left((l^d)^{1-\alpha_0} \left(\frac{s_k}{1+s_e} \right)^{\alpha_0} \frac{1}{s^p} \right)^{\frac{1}{1-\alpha_0}}$$

$$c^e = \frac{c^e}{y} y$$

$$c = \frac{c}{y} y$$

$$k = s_k y$$

$$k^e = \frac{s_k}{1+s_e} y$$

$$k^s = \frac{s_e s_k}{1+s_e} y$$

$$g = s_g y$$

$$b^e = \kappa q k^e \beta$$

$$Y = y s^p$$

$$F = (1 - s^p/X) y$$

$$i = \delta k$$

$$D = iq - (i + \psi \delta^2 k/2)$$

$$\tilde{w} = \frac{(1-\alpha_0) y s^p s^w}{X l(w/\tilde{w})}$$

$$\tau = \tau^l w l^d + \tau^f (F + D) + \tau^k (Y/X - w l^d)$$

$$n = g \left(1 - \frac{\tau}{y s_g} \right) \frac{1}{1/\pi - 1/R}$$