Measurements of Neutrino Interactions on Water using a Fine-Grained Scintillator Detector with Water Targets

by

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Abstract

Measurements of neutrino interaction cross-sections are important for the study of neutrino oscillations. For the T2K experiment, which has a near detector with a primarily carbon target and a far detector with an oxygen target, the cross-section difference is important for correctly comparing the neutrino fluxes at the two detectors. This dissertation presents a first measurement of the ratio between oxygen and carbon of the muon neutrino charge-current inclusive cross-section using the T2K near detector, ND280. The design and construction methods of the water target system in the ND280 Fine-Grained Detector (FGD) will be discussed, as will a new algorithm for reconstructing particle tracks contained within the FGD. The data analysis leading to the cross-section ratio measurement will be described. It is based on a statistical subtraction method which extracts the contribution of the FGD water mass to the total interaction rate for the FGD with water targets. We find an oxygen/carbon ratio of $1.129 \pm 0.114$ (stat) $\pm 0.044$ (syst) from the ND280 data, a $1.39\sigma$ difference from the Monte Carlo prediction of $0.954 \pm 0.029$. 
Preface

This dissertation is ultimately based on the experimental apparatus and data of the T2K experiment, the subject of a large international collaboration. None of the text of the dissertation is taken directly from previously published or collaborative articles.

The hardware design in Chapter 3 was done primarily by S. Yen and myself, except for the system in Section 3.5, which was designed by C. A. Miller. The construction and tests in Chapter 4 and Chapter 5 were performed by S. Yen and I, with assistance from many other collaborators for large-scale assembly work. The data analysis in Chapter 5 and elemental composition analysis in Chapter 6 are my original work.

The two-dimensional Hough and Radon transform reconstruction algorithms in Part III are of my own design and implementation, as are the reconstruction performance metrics of Chapter 9. The 3D matching algorithm described in Section 9.3 and used in conjunction with my 2D pattern recognition was written by T. Lindner and K. Ieki.

The event selection in Chapter 10 is a refinement and extension of a method developed by F. Sanchez and M. Ravonel, while I developed the subtraction method used to extract the cross-section ratio. Several of the systematic errors in Chapter 11 were estimated by other collaborators; these contributions are cited where appropriate. I adapted all of these systematics to my modified selection and analysis method, and developed the reduced FGD analysis of Section 11.6.1. The entire analysis was done within the context of the T2K ND280 $\nu_\mu$ group, with additional incidental contributions from many of that group’s members.
# Table of Contents

Abstract ....................................................... ii
Preface ......................................................... iii
Table of Contents ............................................ iv
List of Tables ................................................ xi
List of Figures ............................................... xiii
List of Acronyms ............................................. xviii
Acknowledgments ........................................... xx
Dedication ..................................................... xxii

I Introduction ................................................. 1

1 The Physics of Neutrinos ................................. 2
  1.1 Standard Model Neutrino Theory .................... 2
  1.2 Direct Observation of Neutrinos .................... 4
  1.3 Observation of Oscillation Phenomena ............. 6
    1.3.1 Natural Neutrinos ............................ 6
    1.3.2 Artificial Neutrinos ......................... 8
    1.3.3 Neutrino Beams ............................ 9
  1.4 Theory of Neutrino Mixing and Oscillation ....... 10
    1.4.1 Two-Flavour Mixing ....................... 10
1.4.2 Adding Another Flavour .................................. 13
1.4.3 Mixing Angles and Mass Differences ................. 14
1.4.4 Observables .............................................. 16
1.4.5 Matter Effects ............................................ 18
1.5 Neutrino Interaction Physics ................................. 19
1.5.1 Interactions on Free Nucleons ......................... 20
1.5.2 Relativistic Fermi Gas Model ......................... 23
1.5.3 Nuclear Shell Model .................................. 24
1.5.4 Nucleon-Nucleon Correlations ....................... 26
1.5.5 Resonant Pion Production ............................... 27
1.5.6 Deep Inelastic Scattering .............................. 28
1.5.7 Final State Interactions ................................. 29
1.6 Goal of Dissertation ....................................... 30

2 The T2K Experiment ........................................ 31
2.1 T2K Experimental Setup .................................. 31
2.1.1 T2K Production Beamline ............................... 31
2.1.2 Off-Axis Beam ........................................... 32
2.1.3 INGRID ................................................. 34
2.1.4 Super-Kamiokande ..................................... 34
2.2 Introduction to ND280 ..................................... 36
2.2.1 Interaction Modes in ND280 ......................... 37
2.2.2 The Tracker ............................................. 38
2.3 Fine-Grained Detectors .................................... 39
2.3.1 FGD Operation ......................................... 41
2.3.2 FGD Readout ........................................... 42
2.3.3 FGD Triggering ........................................ 43
2.3.4 FGD Calibration ....................................... 44

II FGD Passive Water Modules ............................. 46

3 Design and Materials ..................................... 47
3.1 The Need for Water Modules ............................ 47
3.2 Requirements .................................. 48
3.3 Raw Materials ................................. 49
  3.3.1 Polycarbonate Panels ..................... 49
  3.3.2 Polypropylene Skins ...................... 49
  3.3.3 Chemical Additives ....................... 50
3.4 Choice of Sealant and Adhesive .............. 51
  3.4.1 Requirements ............................. 51
  3.4.2 Isolated Testing ....................... 52
  3.4.3 Brief History of Usage .................. 53
  3.4.4 Glue for Skin Attachment ............... 54
3.5 Supply System ................................ 54

4 Construction of Modules ........................ 59
  4.1 Preparation of Panels ...................... 59
    4.1.1 Panel Cutting ......................... 59
    4.1.2 Inner Wall Drilling .................... 60
  4.2 Sealing Procedure .......................... 64
    4.2.1 Repair and Modification of Seal ........ 64
  4.3 Skin Attachment Procedure .................. 65
  4.4 Plumbing of Modules ....................... 66
  4.5 Mechanical Connection to the FGD ........... 68
  4.6 Prototype Module Construction ............ 69
    4.6.1 Scaled-Down Prototypes ............... 69
    4.6.2 Full-Size Prototypes ................. 70
  4.7 Production of Water Modules ............... 73
  4.8 Transport to Japan ......................... 75
  4.9 Assembly into FGD .......................... 75

5 Testing of Water Modules ....................... 77
  5.1 Module Filling for Tests .................... 77
  5.2 Testing at TRIUMF ......................... 77
  5.3 Initial Testing in Japan .................... 81
  5.4 Thickness and Volume Testing .............. 82
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.4.4</td>
<td>Calculating Pull Corrections</td>
<td>151</td>
</tr>
<tr>
<td>10.5</td>
<td>Results of Selection</td>
<td>152</td>
</tr>
<tr>
<td>11</td>
<td>Systematic Error Studies</td>
<td>158</td>
</tr>
<tr>
<td>11.1</td>
<td>Systematic Effects for Subtraction Analysis</td>
<td>158</td>
</tr>
<tr>
<td>11.2</td>
<td>General Methodology</td>
<td>159</td>
</tr>
<tr>
<td>11.3</td>
<td>FGD Masses</td>
<td>160</td>
</tr>
<tr>
<td>11.4</td>
<td>TPC Tracking Efficiency</td>
<td>161</td>
</tr>
<tr>
<td>11.5</td>
<td>TPC Charge Misidentification</td>
<td>162</td>
</tr>
<tr>
<td>11.6</td>
<td>Differences in FGD Granularity</td>
<td>166</td>
</tr>
<tr>
<td>11.6.1</td>
<td>The Reduced FGD1 Study</td>
<td>166</td>
</tr>
<tr>
<td>11.7</td>
<td>TPC Momentum Cut</td>
<td>168</td>
</tr>
<tr>
<td>11.8</td>
<td>TPC Muon PID</td>
<td>169</td>
</tr>
<tr>
<td>11.9</td>
<td>Neutral Current Interaction Background</td>
<td>170</td>
</tr>
<tr>
<td>11.10</td>
<td>Outside Fiducial Volume Backgrounds</td>
<td>171</td>
</tr>
<tr>
<td>11.10.1</td>
<td>Rate Uncertainty</td>
<td>172</td>
</tr>
<tr>
<td>11.10.2</td>
<td>FGD Matching $\chi^2$ Cut Uncertainty</td>
<td>172</td>
</tr>
<tr>
<td>11.10.3</td>
<td>Layer 23/37 Problem</td>
<td>173</td>
</tr>
<tr>
<td>11.10.4</td>
<td>Double-Skip Errors</td>
<td>175</td>
</tr>
<tr>
<td>11.10.5</td>
<td>High-Angle Tracks</td>
<td>175</td>
</tr>
<tr>
<td>11.10.6</td>
<td>Categorization of External Backgrounds</td>
<td>176</td>
</tr>
<tr>
<td>11.11</td>
<td>Other Systematic Effects</td>
<td>177</td>
</tr>
<tr>
<td>11.12</td>
<td>Summary of Systematics</td>
<td>178</td>
</tr>
<tr>
<td>12</td>
<td>Results of Analysis</td>
<td>180</td>
</tr>
<tr>
<td>12.1</td>
<td>Measurements of Ratio</td>
<td>180</td>
</tr>
<tr>
<td>12.2</td>
<td>Comparison with K2K Results</td>
<td>182</td>
</tr>
<tr>
<td>12.2.1</td>
<td>Reweighting</td>
<td>183</td>
</tr>
<tr>
<td>12.2.2</td>
<td>Reweighting of CC Inclusive Sample</td>
<td>183</td>
</tr>
<tr>
<td>12.3</td>
<td>Theoretical Prediction</td>
<td>184</td>
</tr>
<tr>
<td>12.4</td>
<td>Summary</td>
<td>187</td>
</tr>
<tr>
<td>12.4.1</td>
<td>Role in Oscillation Analysis</td>
<td>188</td>
</tr>
<tr>
<td>12.4.2</td>
<td>Future Directions</td>
<td>190</td>
</tr>
</tbody>
</table>
V Conclusion ............................. 192

13 Conclusion ............................. 193

Bibliography ............................. 196
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1.1</td>
<td>The twelve Standard Model fermions</td>
<td>3</td>
</tr>
<tr>
<td>Table 1.2</td>
<td>Nuclear model parameters for carbon and oxygen</td>
<td>23</td>
</tr>
<tr>
<td>Table 5.1</td>
<td>Variation in water level with pressure</td>
<td>84</td>
</tr>
<tr>
<td>Table 6.1</td>
<td>Elemental composition of the components of a typical XY layer</td>
<td>87</td>
</tr>
<tr>
<td>Table 6.2</td>
<td>Panel types and positions within the FGD of the nine water modules</td>
<td>88</td>
</tr>
<tr>
<td>Table 6.3</td>
<td>Elemental composition of empty old-style panels</td>
<td>93</td>
</tr>
<tr>
<td>Table 6.4</td>
<td>Correlation coefficients for empty old-style panels</td>
<td>93</td>
</tr>
<tr>
<td>Table 6.5</td>
<td>Elemental composition of empty new-style thick panels</td>
<td>94</td>
</tr>
<tr>
<td>Table 6.6</td>
<td>Correlation coefficients for empty new-style thick panels</td>
<td>94</td>
</tr>
<tr>
<td>Table 6.7</td>
<td>Elemental composition of empty new-style thin panels</td>
<td>95</td>
</tr>
<tr>
<td>Table 6.8</td>
<td>Correlation coefficients for empty new-style thin panels</td>
<td>95</td>
</tr>
<tr>
<td>Table 6.9</td>
<td>XY and virtual water densities of the empty panels</td>
<td>96</td>
</tr>
<tr>
<td>Table 6.10</td>
<td>Total areal densities of FGDs as built and filled with water</td>
<td>96</td>
</tr>
<tr>
<td>Table 6.11</td>
<td>FGD fiducial masses, including target water</td>
<td>96</td>
</tr>
<tr>
<td>Table 9.1</td>
<td>Cleanliness and completeness means for XZ projection</td>
<td>125</td>
</tr>
<tr>
<td>Table 9.2</td>
<td>Cleanliness and completeness means for YZ projection</td>
<td>126</td>
</tr>
<tr>
<td>Table 9.3</td>
<td>Cleanliness and completeness means for 3D tracks</td>
<td>129</td>
</tr>
</tbody>
</table>
Table 9.4 Number of good tracks in test sample for each isorecon algorithm. The original Radon transform has much fewer good tracks than the other two algorithms, but SBCAT still slightly outperforms the flexible Radon in 2D tracking, and more significantly outperforms it for 3D tracking.

Table 10.1 Nominal parameter values from Monte Carlo selection, as used in Equation 10.12

Table 10.2 Cumulative selection efficiencies at each analysis cut

Table 10.3 Relative efficiencies of each analysis cut (%)

Table 10.4 Selection purities at each analysis cut (%)

Table 10.5 Charge-current interaction classification by interaction channel for Monte Carlo

Table 11.1 Charge misidentification probabilities for the CC inclusive selection

Table 11.2 Event counts for full and reduced FGD1 selections in Monte Carlo

Table 11.3 Event counts for full and reduced FGD1 selections in data

Table 11.4 Particle identification efficiencies for muon control sample

Table 11.5 Classification of outside FGD fiducial volume background events

Table 11.6 Systematic errors on cross-section ratio

Table 12.1 Reduction table of Monte Carlo and data

Table 12.2 FGD1 Reduction table of Monte Carlo and data

Table 12.3 FGD2 Reduction table of Monte Carlo and data

Table 12.4 Nuclear model parameters for carbon and oxygen

Table 12.5 Comparison of oxygen/carbon ratios from this thesis and all other sources discussed in this chapter.
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1.1</td>
<td>Two-neutrino oscillation probability with maximal mixing and a set $\Delta m^2 L$</td>
<td>12</td>
</tr>
<tr>
<td>Figure 1.2</td>
<td>Illustration of normal and inverted neutrino mass hierarchies</td>
<td>15</td>
</tr>
<tr>
<td>Figure 1.3</td>
<td>Feynman diagrams of neutrino scattering in matter</td>
<td>18</td>
</tr>
<tr>
<td>Figure 1.4</td>
<td>Nuclear shell occupancy of carbon-12 (left) and oxygen-16 (right)</td>
<td>24</td>
</tr>
<tr>
<td>Figure 2.1</td>
<td>J-PARC neutrino beamline diagram</td>
<td>32</td>
</tr>
<tr>
<td>Figure 2.2</td>
<td>Diagram of secondary beam production target and neutrino production volume</td>
<td>33</td>
</tr>
<tr>
<td>Figure 2.3</td>
<td>Neutrino energy spectrum at a range of off-axis angles</td>
<td>33</td>
</tr>
<tr>
<td>Figure 2.4</td>
<td>Diagram of Super-Kamiokande detector</td>
<td>34</td>
</tr>
<tr>
<td>Figure 2.5</td>
<td>Exploded overview of ND280 detector, showing all sub-detectors and magnet elements</td>
<td>36</td>
</tr>
<tr>
<td>Figure 2.6</td>
<td>An example neutrino interaction in the ND280 Tracker</td>
<td>38</td>
</tr>
<tr>
<td>Figure 2.7</td>
<td>Cross-section photograph of typical FGD bar</td>
<td>39</td>
</tr>
<tr>
<td>Figure 2.8</td>
<td>Block diagram of FGD read-out electronics</td>
<td>42</td>
</tr>
<tr>
<td>Figure 3.1</td>
<td>Cross-section of polycarbonate panels</td>
<td>50</td>
</tr>
<tr>
<td>Figure 3.2</td>
<td>Freshly made ‘pucks’ of HE 1908 for testing water absorption. Both the mass and thickness of these samples were monitored over a multi-month immersion in water to test the effects of water absorption, prior to selection as the module sealant</td>
<td>52</td>
</tr>
</tbody>
</table>
Figure 3.3 Water absorption of HE 1908

Figure 3.4 Schematic of sub-atmospheric pressure water circulation system

Figure 3.5 Water system trial load and reservoir

Figure 3.6 Water supply pump, manifolds, and vacuum tank

Figure 3.7 Water module prototype with side valve

Figure 4.1 Saw blade used to cut polycarbonate panels

Figure 4.2 Pick used to slash thin walls of polycarbonate panel

Figure 4.3 Dremel attachments used to cut inner water panel walls

Figure 4.4 End of water module panel prior to sealing, showing horizontal slots and vertical slashes in the thin walls, and the round holes in the thick walls

Figure 4.5 Polypropylene channel for water module sealing

Figure 4.6 Approximate tiling of polypropylene skin pieces within water module outline

Figure 4.7 Pipe fitting on water module

Figure 4.8 Photograph of Water module strap brackets

Figure 4.9 Narrow prototype prior to filling

Figure 4.10 Crack in second prototype module

Figure 4.11 Third, 3/4 size prototype water module

Figure 4.12 Final production water module filled with water

Figure 4.13 Water modules being unpacked at J-PARC, showing the final water module design and the shipping crate used for their transport

Figure 4.14 The full production of FGD passive water modules stacked in the assembly area in J-PARC, showing the stainless steel straps attached to each module for hanging

Figure 5.1 Simple setup for filling water modules in the absence of the sub-atmospheric pressure circulation system

Figure 5.2 Buffer tank and valve assembly
Figure 5.3 Narrow water module wrapped in aluminized Mylar for tests

Figure 5.4 Apparatus for measuring water module internal pressure, showing the change in pressure inside a module overnight from the initial set point marked in red.

Figure 5.5 Variation of areal density of a single module with water depth at constant pressure, showing the increase in density due to bowing of the walls under the weight of the water.

Figure 5.6 Areal density of water in all modules

Figure 6.1 Areal densities of polypropylene sheet samples, showing the mean value of 72.4 mg/cm$^2$ used for the elemental composition and mass analysis.

Figure 7.1 Illustration of Hough transform parametrization

Figure 7.2 Illustration of line parameter clustering

Figure 7.3 Hough transform histogram of complex figure

Figure 7.4 Hit bars for particle track

Figure 7.5 Reconstructed tracks as favoured by Hough transform

Figure 8.1 Example event in FGD2, showing two tracks traversing the detector and a background hit

Figure 8.2 Example Radon transform histogram

Figure 8.3 Cluster labelling using a seed fill

Figure 8.4 Raw cluster labels in flexible Radon transform

Figure 8.5 Pruning of a flexible Radon track

Figure 8.6 Remaining Radon clusters after pruning and hit capture

Figure 9.1 Isorecon track cleanliness for 2D tracks in the XZ projection

Figure 9.2 Isorecon track completeness for 2D tracks in the XZ projection
| Figure 9.3 | Isorecon track cleanliness for 2D tracks in the YZ projection | 125 |
| Figure 9.4 | Isorecon track completeness for 2D tracks in the YZ projection | 126 |
| Figure 9.5 | Isorecon track finding efficiency in the XZ projection as a function of particle range | 127 |
| Figure 9.6 | Isorecon track finding efficiency in the XZ projection as a function of particle range | 127 |
| Figure 9.7 | Cleanliness of 3D isorecon tracks | 128 |
| Figure 9.8 | Completeness of 3D isorecon tracks | 129 |
| Figure 9.9 | Efficiency variation of 3D isorecon with track range | 130 |
| Figure 9.10 | Efficiency variation of 3D isorecon with true track angle | 130 |
| Figure 10.1 | Time distribution of ND280 beam spill tracks | 143 |
| Figure 10.2 | Number of hits in reconstructed TPC tracks | 144 |
| Figure 10.3 | Distribution of reconstructed vertices in XZ with FGD fiducial volume shown | 146 |
| Figure 10.4 | Inverse momentum times charge of Monte Carlo muon candidates | 147 |
| Figure 10.5 | Momentum distribution of Monte Carlo signal and background muon candidates | 148 |
| Figure 10.6 | Predicted TPC energy loss as a function of momentum for a variety of particles | 149 |
| Figure 10.7 | Measured energy loss and momentum for simulated particles in the TPC | 150 |
| Figure 10.8 | Muon pull of Monte Carlo control sample tracks | 152 |
| Figure 10.9 | Electron pull of muon control sample | 153 |
| Figure 10.10 | Neutrino interaction electron pull distribution fit with double Gaussian | 154 |
| Figure 10.11 | Total efficiency and purity | 156 |
| Figure 10.12 | FGD1 efficiency and purity | 156 |
| Figure 10.13 | FGD2 efficiency and purity | 157 |
Figure 11.1  Variation of rate ratio in mass Monte Carlo . . . . . . 162
Figure 11.2  Estimated TPC2 tracking efficiency . . . . . . . . . . 163
Figure 11.3  TPC charge misidentification probability as a function of
             momentum . . . . . . . . . . . . . . . . . . . . . . . 165
Figure 11.4  Example magnetic field distortion map . . . . . . . . 169
Figure 11.5  Matching $\chi^2$ of FGD hits to projected TPC tracks . . 173
Figure 11.6  Reconstructed end layer of throughgoing FGD tracks 174

Figure 12.1  Ratio of oxygen and carbon CCQE cross sections . . 185
List of Acronyms

APD  Avalanche Photo-Diode
ASUM  Analog Sum
CCQE  Charged-Current Quasi-Elastic
CMB  Crate Master Board
DAQ  Data Acquisition
DCC  Data Concentrator Card
DIS  Deep Inelastic Scattering
FEB  Front End Board
FGD  Fine-Grained Detector
FSI  Final State Interaction
J-PARC  Japan Proton Accelerator Research Complex
K2K  KEK to Kamioka experiment
LPB  Light Pulser Board
LSND  Liquid Scintillator Neutrino Detector
MCM  Master Clock Module
MPPC  Multi-Pixel Photon Counter
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ND280</td>
<td>Near Detector 280 meters</td>
</tr>
<tr>
<td>PID</td>
<td>Particle Identification</td>
</tr>
<tr>
<td>PMT</td>
<td>Photo-Multiplier Tube</td>
</tr>
<tr>
<td>P0D</td>
<td>Pi-Zero Detector</td>
</tr>
<tr>
<td>POT</td>
<td>Protons On Target</td>
</tr>
<tr>
<td>RFGM</td>
<td>Relativistic Fermi Gas Model</td>
</tr>
<tr>
<td>SBCAT</td>
<td>SciBar Cellular Automaton</td>
</tr>
<tr>
<td>SCA</td>
<td>Switched Capacitor Array</td>
</tr>
<tr>
<td>SMRD</td>
<td>Side Muon Range Detector</td>
</tr>
<tr>
<td>SNO</td>
<td>Sudbury Neutrino Observatory</td>
</tr>
<tr>
<td>T2K</td>
<td>Tokai to Kamioka experiment</td>
</tr>
<tr>
<td>TPC</td>
<td>Time-Projection Chamber</td>
</tr>
</tbody>
</table>
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For Leslie-Anne, who makes every day better.

In memory of Grant Victor Roberge (1956-2008)
Part I

Introduction
Chapter 1

The Physics of Neutrinos

The crucial discovery of the last two decades in neutrino physics is the discovery of nonzero neutrino mass through the mechanism of neutrino oscillation. The Tokai to Kamioka (T2K) experiment\[1\] is a long-baseline neutrino experiment currently operating in Japan which seeks to measure the parameters of this oscillation process. In particular, it aims to detect for the first time oscillation of muon neutrinos into electron neutrinos, and thus set a lower limit on the third neutrino mixing parameter $\theta_{13}$\[2\]. It also seeks to improve the precision of the measurement of muon neutrino disappearance and the mixing angle that governs that process, $\theta_{23}$\[3\]. Once a satisfactory value for $\theta_{13}$ is established, T2K can attempt to measure CP violation in neutrinos, with an aim towards a lepton-based explanation of the cosmic baryon asymmetry.

1.1 Standard Model Neutrino Theory

The Standard Model of particle physics contains twelve fermions divided into two families, quarks and leptons, and three generations, as shown in Table 1.1. Each generation contains an up-type quark, a down-type quark, a charged lepton, and a neutrino. The quarks carry colour charge, while the leptons are colourless and thus do not couple via the strong interaction. Neutrinos are also electrically neutral and so they only interact with the
Table 1.1: The twelve Standard Model fermions. The left table shows the six leptons, while the right table shows the six quarks.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Symbol</th>
<th>Charge</th>
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<tbody>
<tr>
<td>electron</td>
<td>$e$</td>
<td>-1</td>
</tr>
<tr>
<td>electron neutrino</td>
<td>$\nu_e$</td>
<td>0</td>
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<tr>
<td>muon</td>
<td>$\mu$</td>
<td>-1</td>
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<tr>
<td>muon neutrino</td>
<td>$\nu_\mu$</td>
<td>0</td>
</tr>
<tr>
<td>tau lepton</td>
<td>$\tau$</td>
<td>-1</td>
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<tr>
<td>tau neutrino</td>
<td>$\nu_\tau$</td>
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<thead>
<tr>
<th>Particle</th>
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<tbody>
<tr>
<td>up quark</td>
<td>$u$</td>
<td>2/3</td>
</tr>
<tr>
<td>down quark</td>
<td>$d$</td>
<td>-1/3</td>
</tr>
<tr>
<td>charm quark</td>
<td>$c$</td>
<td>2/3</td>
</tr>
<tr>
<td>strange quark</td>
<td>$s$</td>
<td>-1/3</td>
</tr>
<tr>
<td>top quark</td>
<td>$t$</td>
<td>2/3</td>
</tr>
<tr>
<td>bottom quark</td>
<td>$b$</td>
<td>-1/3</td>
</tr>
</tbody>
</table>

The existence of the neutrino was proposed in 1930 by Wolfgang Pauli, who used the hypothesis of a third, unseen decay product to explain the energy spectrum of beta decay. If the decaying nucleus were to emit only an electron, the fully constrained two-body kinematics would always produce the same electron energy; observations, however, showed a wide range of electron energies. While some were willing to accept this occurrence as evidence against the conservation of energy, momentum, and angular momentum, this was an unsatisfying conclusion given the fundamental nature of those conservation laws.

The posited neutrino was originally thought to be invisible and undetectable, a mere bookkeeping device for keeping the theory consistent. No evidence was seen for neutrino mass, so the neutrino was accepted as massless. Early theories of the weak interaction brought the neutrino into the heart of the theory.

The observation of pion and muon decays in the 1940s provided additional opportunities to balance the books. The decay of the pion produces a monoenergetic muon and no other visible particles, so the neutrino was pressed into service to serve as the other decay product. Muon decay needed two neutrinos to explain the energy spectrum of the emitted electron, generally the only visible decay product. While it was natural to assume that
all neutrinos are identical, later observational evidence showed that this was not true[5]. There were two ‘flavours’ of neutrino, the muon and electron neutrinos, and in addition the neutrino was shown to have distinct behaviour from the antineutrino.

The relationship between the neutrino and the antineutrino is complicated by parity violation in the weak interaction. The maximal violation of parity in the weak interaction[6] means that the weak interaction only couples to left-handed neutrinos and right-handed antineutrinos. It is then hypothesized that the neutrino and antineutrino are not distinct particles but merely the two different chiralities of a single particle. This is an attractive concept given that the ‘wrong-handed’ neutrinos would have no Standard Model interactions whatsoever and thus be fundamentally unobservable[7].

Furthermore, in the Standard Model Lagrangian, the mass term for an ordinary (Dirac) fermion has the form

$$\mathcal{L} = -m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

(1.1)

implying that for a massive particle both $\psi_L$ and $\psi_R$ must both exist. Since the Standard Model neutrino only has a visible $\psi_L$, this was treated as evidence for zero neutrino mass. However, if the neutrino and antineutrino were the same underlying particle with different chiralities, an additional mass mechanism called the Majorana mass is available. While neutrino mass is now a demonstrated fact, it remains an open question whether this mass is Dirac or Majorana in form.

### 1.2 Direct Observation of Neutrinos

The symmetries of Fermi’s original theory of beta decay pointed the way to the first direct observation of the neutrino. Starting with a standard $\beta^+$ decay,

$$p \Rightarrow e^+ + \nu_e + n$$

(1.2)
we can use crossing symmetry to show that $\beta^+$ decay should be able to be induced by an antineutrino:

$$\bar{\nu}_e + p \Rightarrow e^+ + n$$

(1.3)

In 1956 Cowan et al. [8] observed this reaction using an intense flux of antineutrinos produced by a nuclear reactor. The signal consisted of a coincidence between a positron annihilation and a neutron capture.

By analogy to the electron capture decays of some nuclei by the process

$$p + e^- \Rightarrow n + \nu$$

(1.4)

we expect that we can observe an induced “neutrino capture” decay by the process

$$n + \nu \Rightarrow p + e^-$$

(1.5)

If neutrinos are distinct from antineutrinos then an antineutrino cannot take part in this neutrino capture reaction. Pontecorvo [9] proposed to measure this using $^{37}$Cl, which has the advantage that the reaction product $^{37}$Ar is a noble gas which can be extracted practically on an atom-by-atom level. These atoms can then be counted by their radioactive decay back to $^{37}$Cl.

Davis [10] attempted to observe this reaction using antineutrinos and did not observe antineutrino capture, establishing the difference between neutrinos and antineutrinos.

Further direct observations of neutrinos were produced by Danby et al. [11] using neutrinos produced by pion decay. Interestingly, these neutrinos interacted only producing muons, showing that the neutrino produced in pion decay was distinct from the neutrino produced in beta decay. This and the discovery that neutrinos are different from antineutrinos led to the concept of lepton number conservation.

All leptons are given lepton number +1, while all anti-leptons are given lepton number -1. This quantity is conserved in all observed interactions. With the discovery of distinct neutrino flavours, the conservation of lepton flavour number was added, where muons and muon neutrinos conserve their
lepton numbers separately from those for electrons and electron neutrinos. With this framework, we can correctly predict the form of the muon decay, purely from conservation laws, as

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$  \hspace{1cm} (1.6)

The discovery of the tau lepton brought a third flavour of neutrino along with it, the tau neutrino. Due to the high tau mass, direct detection of the tau neutrino did not occur until 2001 by the DONUT experiment at Fermilab[12].

Strong evidence exists that there are only three active neutrino flavours. Measurement of the ‘invisible’ branching ratio of the $Z^0$ boson at LEP gives $N_\nu = 2.984\pm0.008[13]$. Nevertheless, the existence of further flavours of neutrino has not been ruled out, so long as they are ‘sterile’ and do not interact with the $Z^0$. Such particles could still have visible effects on the universe through neutrino oscillation (§1.4) or through gravitational interactions as dark matter.

1.3 Observation of Oscillation Phenomena

Neutrino oscillation phenomena have been observed, recognized and unrecognized, in a variety of different systems over the past 45 years. While it was proposed as a solution to several problems over the years, it was only conclusively observed in the late 1990s.

1.3.1 Natural Neutrinos

The first hints towards the eventual detection of neutrino oscillation were found in the solar neutrino problem, as discovered by Ray Davis in 1968[14]. Using a scaled-up version of the chlorine-argon radiochemical method of neutrino detection, placed deep in the Homestake mine in South Dakota, Davis detected only 1/3 as many neutrinos as predicted by standard theories of solar activity. Neutrino oscillation was proposed fairly early as a solution to this problem[15], but positive resolution of this would take decades.
While neutrino oscillations are the solution to the solar neutrino problem, the first evidence of their existence was found in a different system, that of atmospheric neutrinos. Atmospheric neutrinos are neutrinos produced in the upper atmosphere by the decay of particles in cosmic ray showers. Pions are by far the most common unstable particle produced in a cosmic ray shower, and so the flavour of atmospheric neutrinos should be described by the decay chain $\pi \rightarrow \mu \nu_\mu$, $\mu \rightarrow e\nu_\mu\nu_e$ (where I have neglected charges for clarity), giving a 2:1 ratio of muon neutrinos to electron neutrinos.

If we use a detector with 4$\pi$ acceptance, we can then probe a variety of oscillation distances by measuring the angles of incident neutrinos. The measurements described by the Super-Kamiokande experiment in 1998 show a substantial deficit of muon neutrinos increasing with the zenith angle and thus with propagation distance, while the electron neutrino flux was consistent with theory ignoring neutrino oscillations.[16] This result is consistent with muon neutrinos oscillating into tau neutrinos, and points towards one of the $\Delta m^2$s being much larger than the other.

The solar neutrino problem was eventually resolved by the Sudbury Neutrino Observatory (SNO) experiment[17]. SNO was a 1 kiloton water Čerenkov detector that used heavy water to allow a new measurement of the total neutrino flux. While the energy of solar neutrinos is too low to permit any charge-current interactions of $\nu_\mu$ and $\nu_\tau$, all three flavours participate in neutral-current interactions. The use of deuterium permits the disassociation reaction

$$\nu_\ell + d \Rightarrow \nu_\ell + p + n \quad (1.7)$$

Detection of the neutron from the disassociated deuterium allows measurement of the rate of this reaction, and thus a measurement of the total solar neutrino flux.

While SNO observed the same flux as earlier experiments through the charge-current interaction of $\nu_e$s, the neutral current measurement found a flux about three times larger, consistent with astrophysics and establishing neutrino oscillation as the solution to the solar neutrino problem[18]. Matter effects (Section 1.4.5) are strong in the Sun due to its high density, and result
in solar neutrinos being almost purely \( \nu_2 \).

### 1.3.2 Artificial Neutrinos

Measurements with natural neutrinos, while instrumental in establishing the existence of neutrino oscillations, are limited by the fixed conditions and variation in oscillation baseline. More precise measurements are possible with artificial neutrinos, which have a definite source position or positions, allowing the oscillation baseline to be tuned to a length where the oscillation effect should be near maximal.

Nuclear reactors produce a large flux of electron antineutrinos during operation. The highly neutron-rich fissile nuclei produce fission products which are themselves neutron-rich; these nuclei (and free neutrons) then restore the necessary balance through \( \beta^- \) decay, producing one \( \bar{\nu}_e \) for each decay that occurs. Approximately six beta decays occur for each fission event. These neutrinos are low-energy but plentiful, and allow for oscillation experiments with relatively short baselines. In particular, both solar and atmospheric scale baselines are available for reactor neutrinos, with the longer solar baseline still a practical 200 km.

Perhaps the best-known reactor neutrino experiment is KamLAND, which used a 1 kton liquid scintillator detector to observe the neutrino flux from a variety of nuclear reactors in central Japan\([19]\). This experiment was well-situated relative to the Japanese nuclear reactors to observe oscillations on the scale of the solar mass difference \( \Delta m^2_{12} \). KamLAND confirmed the hypothesis of neutrino oscillation by observing an energy-dependent difference in observed neutrino rate and thus the survival probability.

Shorter-baseline reactor neutrino experiments have been performed to attempt a measurement of \( \theta_{13} \). Until recently, the best experiment in this area was the CHOOZ experiment in France. As shown below in Equation 1.21 and Section 1.4.4, electron neutrino disappearance in this range is governed by \( \theta_{13} \) and the CHOOZ limit was the tightest pre-T2K limit available at \( \sin^2 2\theta_{13} < 0.15[20] \).

A new generation of short-baseline reactor experiments using near and
far detectors has recently established evidence for a non-zero value of $\theta_{13}$. The Daya Bay experiment uses a large complex of detectors around the eponymous high-power reactor in Southern China and sees strong indication for $\sin^2 2\theta_{13} \approx 0.1$\cite{21}. The RENO experiment in South Korea has reported similar results\cite{22}, while the Double CHOOZ experiment, using just a far detector, also finds evidence for non-zero $\theta_{13}$ at somewhat lower significance\cite{23}.

1.3.3 Neutrino Beams

Many recent experiments have used a neutrino beam produced by a particle accelerator as the neutrino source. Accelerator neutrinos are produced primarily through pion decay by

$$\pi^+ \Rightarrow \mu^+ + \nu_\mu$$  \hspace{1cm} (1.8)

The neutrinos are shaped into a beam through a Lorentz boost; the pions are produced at high energies and decay in flight imparting a large fraction of their forward momentum to the neutrino.

For a variety of reasons, the beam produced in this manner is not purely $\nu_\mu$s. Muon decay produces $\nu_e$s and $\bar{\nu}_\mu$s, and the high energy of the primary proton beam produces kaons, which decay through a variety of channels producing all available neutrino flavours. It is important to know the intrinsic $\nu_e$ fraction and the $\nu_\mu$ flux at the source, so neutrino beam experiments usually have two detectors, one near the source and one near the oscillation maximum.

One of the first long-baseline accelerator neutrino experiments was the KEK to Kamioka (K2K) experiment, which produced a neutrino beam at the KEK lab in Tsukuba, Japan and detected it at Super-Kamiokande, 280 km away. While K2K was a relatively low-statistics experiment, it did re-establish the existence of muon neutrino disappearance in the accelerator system\cite{24}. T2K is the successor to K2K, upgraded with a much higher neutrino flux and using the off-axis technique described in Section 2.1.2 for a narrower-band neutrino energy spectrum. The MINOS experiment,
using a $\nu_\mu$ produced at Fermilab and directed towards an iron-scintillator underground detector 720km away, has also observed neutrino oscillation of both neutrinos and antineutrinos with a few-GeV beam [25, 26].

The main result that is inconsistent with standard oscillation theory and parameters is also from an accelerator neutrino experiment, the short-baseline Liquid Scintillator Neutrino Detector (LSND) experiment at Los Alamos. The LSND group observed an excess of electron-like neutrino interactions in a beam of $\bar{\nu}_\mu$s, suggesting $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations at a much higher $\Delta m^2$ than the known mass-squared differences $\Delta m^2_{12}$ and $\Delta m^2_{23}$ from solar and atmospheric neutrino measurements[27]. More recent experiments, such as the MiniBOONE experiment at Fermilab have re-examined the LSND result; the current data rules out the LSND anomaly but introduces a new discrepancy at low neutrino energies[28].

1.4 Theory of Neutrino Mixing and Oscillation

The concept that neutrinos may oscillate between different identities has a long history. In 1957 Pontecorvo hypothesized oscillation between the neutrino and antineutrino analogous to that which occurs between the $K_0$ and $\bar{K}_0$ mesons to explain an ultimately-spurious appearance of neutrinos in a reactor antineutrino experiment[29]. While $\nu - \bar{\nu}$ oscillations faded into the background, a more fruitful concept appeared after the identification of the muon neutrino: neutrino flavour oscillation. In 1962 Maki, Nakagawa, and Sakata developed a theory of neutrino flavour mixing based on a simple but strange idea: the neutrino flavour eigenstates do not correspond to the neutrino mass eigenstates[30].

1.4.1 Two-Flavour Mixing

For two neutrinos, we can consider the two orthogonal mass eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$, and mix them into flavour eigenstates $|\nu_e\rangle$ and $|\nu_\mu\rangle$ by

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu
\end{pmatrix} = R(\theta) \begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix}
$$

(1.9)
where $R(\theta)$ is the 2D rotation matrix by the angle $\theta$. As neutrinos only interact through the weak interaction, which couples to flavour, only the flavour eigenstates will be ‘visible’ to the outside world with the structure of the mass eigenstates hidden to all Standard Model interactions. While the mass eigenstates will directly couple to the force of gravity, this is also experimentally inaccessible due to the weakness of gravity on the appropriate scales. Thus, only the behaviour of neutrinos as free particles can be sensitive to this structure, and it is here that the concept of oscillation re-enters the picture.

The key to this, as with any two-level quantum system in a mixed state, is with the time-dependent phase in the Schrodinger equation. Consider a muon neutrino, which we can write at the time of production as

$$|\nu_\mu\rangle = -\sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle$$ \hspace{1cm} (1.10)$$

Adding the time-dependent phases to each wavefunction and assuming the state as a whole has a definite momentum, $p$, we then get after a time $t$ and traversing a distance $L$ (using $c \equiv \hbar \equiv 1$):

$$|\nu_\mu\rangle (t) = -e^{i(pL-E_1t)} \sin \theta |\nu_1\rangle + e^{i(pL-E_2t)} \cos \theta |\nu_2\rangle$$ \hspace{1cm} (1.11)$$

It is at this point where we use the knowledge that the masses of the neutrino states are very small compared to the energy scale of any other physics that involves them, and as such any neutrino of interest will be highly relativistic. Thus $t \approx L$ and

$$E_i = \sqrt{p^2 + m^2} \approx (p + \frac{m^2}{2p})$$ \hspace{1cm} (1.12)$$

giving

$$|\nu_\mu\rangle (L) = -e^{-\frac{i m_1^2 L}{2p}} \sin \theta |\nu_1\rangle + e^{-\frac{i m_2^2 L}{2p}} \cos \theta |\nu_2\rangle$$ \hspace{1cm} (1.13)$$

or, equivalently,

$$|\nu_\mu\rangle (L) = -\sin \theta |\nu_1\rangle + e^{i \frac{i m_2^2 L}{2p}} \cos \theta |\nu_2\rangle$$ \hspace{1cm} (1.14)$$
At this point, we can calculate the probability that this \( \nu_{\mu} \) will interact as a \( \nu_e \) after travelling a distance \( L \), as

\[
P(\nu_{\mu} \rightarrow \nu_e) = |\langle \nu_e | \nu_{\mu}(L) \rangle|^2 = | - \cos \theta \sin \theta + e^{i\Delta m^2 L/2p} \sin \theta \cos \theta |^2
\]  

(1.15)

which, if you make the approximation \( p \approx E \), gives

\[
P(\nu_{\mu} \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left( \frac{k\Delta m^2 L}{E} \right)
\]  

(1.16)

where \( k = 1.27 GeV c^4 / (km \cdot eV^2) \), so that \( L \) is expressed in km, \( E \) in GeV, and \( \Delta m^2 \) in eV\(^2\). A graph of this function for \( \sin^2 2\theta = 1 \) can be found in Figure 1.1.

From Equation 1.16, we can see that the main experimental parameter of a neutrino oscillation experiment is the ratio of the baseline \( L \) to the neutrino energy \( E \). If we sample a range of values for this ratio, we can then fit for the values of the two physics parameters \( \theta \) and \( \Delta m^2 \). Specifically, as seen in Figure 1.1, \( \theta \) controls the amplitude of the probability function and \( \Delta m^2 \) controls its frequency.
Adding an additional neutrino flavour to the system makes the oscillation phenomena more complex. The two-by-two rotation matrix from Equation 1.9 is replaced by a three-by-three unitary matrix:

\[
U = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13}e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

(1.17)

where \(c_{ij}\) and \(s_{ij}\) are the cosines and sines of the three mixing angles: \(\theta_{12}\), \(\theta_{23}\), and \(\theta_{13}\), and \(\delta\) is a complex phase related to CP violation. Using the current state of neutrino experimental results, we find this matrix has the approximate value

\[
U \approx \begin{pmatrix}
0.9 & 0.5 & 0.15 \\
0.35 & 0.6 & 0.7 \\
0.35 & 0.6 & 0.7
\end{pmatrix}
\]

(1.18)

This matrix is generally called the PMNS matrix after Pontecorvo, Maki, Nakagawa, and Sakata, and in analogy to the CKM matrix of quark mixing. The content of the two mixing matrices is quite different; while the CKM matrix is approximately diagonal, the PMNS matrix has diagonal and non-diagonal elements of comparable magnitude. This, along with the inaccessibility of the neutrino mass eigenstates, makes the phenomena of neutrino mixing much different than quark mixing. Since the strong and electromagnetic interactions can act directly on the quark mass eigenstates, and the CKM matrix is approximately diagonal, quark physics can be conceptualized almost solely in terms of the mass eigenstates, with the flavour eigenstates only becoming relevant at a weak interaction vertex, while neutrino physics concerns itself deeply with the differences between the mass and flavour eigenstates.

The deep differences between the PMNS matrix for leptons and the CKM matrix for quarks make characterizing the former even more important. Prior to the discovery of neutrino oscillations at large mixing angles, the general assumptions was that the two matrices would be similar if not
identical, with mixing angles less than 15°. The difference in the mixing matrices is one of the largest asymmetries between quarks and leptons, and an understanding of the details of both matrices is thus an important goal for present-day particle physics. This difference, when combined with the equally dramatic disparity in mass between neutrinos and quarks, suggests the possibility that neutrino mass originates through a different mechanism than quark mass, such as the see-saw mechanism.

In addition to the neutrino mixing matrix, oscillation phenomena are also controlled by the differences of the three neutrino masses. Present experiments are generally sensitive to the two mass differences $|\Delta m_{12}^2|$ and $|\Delta m_{23}^2|$. We find that $|\Delta m_{23}^2| \gg |\Delta m_{12}^2|$, so we can treat $|\Delta m_{13}^2|$ as approximately equal to $|\Delta m_{23}^2|$.

1.4.3 Mixing Angles and Mass Differences

Currently there are several measurements of $\theta_{12}$ and $\theta_{23}$, often called the ‘solar’ and ‘atmospheric’ mixing angles, respectively, as well as some recent measurements of $\theta_{13}$. Now that we have a firm measurement of $\theta_{13}$, we can begin to look for CP violation in the neutrino sector. Neutrino CP violation potentially has deep cosmological implications, as the degree of CP violation seen in the quark sector is too small to explain the observed baryon asymmetry of the universe, i.e., the nonzero total baryon number of the observable universe.

There are also measurements of the mass squared differences $|\Delta m_{12}^2|$ and $|\Delta m_{23}^2|$. While the absolute value of these quantities is known, the sign of $\Delta m_{23}^2$ is currently unknown, which determines the mass hierarchy of the neutrinos. We can define $\Delta m_{12}^2$ to be positive; if $\Delta m_{23}^2$ is also positive then we have what is called the ‘normal hierarchy’, while if $\Delta m_{23}^2$ is negative we have the ‘inverted hierarchy’. The distinction between the mass hierarchies is shown graphically in Figure 1.2.

The mixing angle $\theta_{12}$ and its associated mass splitting $|\Delta m_{12}^2|$ are primarily studied by solar neutrino experiments and reactor neutrino experiments. The mixing angle is determined by solar neutrino observations, such
as done by SNO, and is known to be given by $\sin^2(2\theta_{12}) = 0.861^{+0.026}_{-0.022}$\cite{31}.

The mass splitting is better constrained by disappearance measurements of reactor $\bar{\nu}_e$s, with a best fit value of $|\Delta m^2_{12}| = (7.59^{+0.20}_{-0.21}) \times 10^{-5}$eV\textsuperscript{2}\cite{32,33}.

The mixing governed by $\theta_{23}$ and $|\Delta m^2_{23}|$ is studied with atmospheric neutrinos and long-baseline accelerator neutrinos. From the measurements of atmospheric neutrinos at Super-Kamiokande, we get $\sin^2(2\theta_{23}) > 0.95$\cite{34}, which is consistent with $\theta_{23} = \pi/4$ and has been corroborated by accelerator neutrino measurements done by MINOS, K2K, and T2K. The mass splitting is measured by MINOS to be $|\Delta m^2_{23}| = (2.32^{+0.12}_{-0.08}) \times 10^{-3}$eV\textsuperscript{2}\cite{35}.

The third mixing angle $\theta_{13}$ has been studied by long baseline accelerator experiments and by reactor neutrino experiments. In the summer of 2011, T2K found indications of nonzero $\theta_{13}$\cite{2}, with a value near $\sin^2(2\theta_{13}) = 0.10$. In March 2012, the Daya Bay reactor neutrino experiment measured $\sin^2 2\theta_{13} = 0.092 \pm 0.016$(stat) $\pm 0.005$(syst), a 5.2$\sigma$ deviation from the null hypothesis of $\theta_{13} = 0$\cite{36}. The RENO and Double CHOOZ experiments have reported comparable results \cite{22,23}.
Neutrino oscillation experiments measure oscillation probabilities, similar to those in Equation 1.16. Starting from a beam containing a single flavour, we can measure the fraction of the beam which is observed in each flavour after propagation. This is complicated by the threshold effects of the three charged lepton masses. Flavour identification is only possible in charge-current neutrino interactions, where the neutrino is transformed into the corresponding charged lepton, so it cannot occur when such a transformation is energetically impossible.

Since most oscillation experiments use energies where $\nu_\tau$s are undetectable, and all current electron neutrino experiments are below the threshold for $\nu_\mu$ as well, we have three observable quantities for neutrinos: electron neutrino survival probability ($P(\nu_e \to \nu_e)$), muon neutrino survival probability ($P(\nu_\mu \to \nu_\mu)$), and electron neutrino appearance probability ($P(\nu_\mu \to \nu_e)$), plus the corresponding probabilities for antineutrinos. Muon neutrino appearance ($P(\nu_e \to \nu_\mu)$) may be accessible in future if a source of electron neutrinos with energy above the muon production threshold becomes available.

Muon neutrino disappearance measurements are primarily sensitive to the mixing angle $\theta_{23}$ and the mass-squared difference $|\Delta m^2_{23}|$, i.e., the atmospheric mixing parameters. This occurs because $|\Delta m^2_{23}|$ is much larger than $|\Delta m^2_{12}|$, leading the two oscillations to occur on drastically different length scales. Ignoring the slow solar oscillation, we have the oscillation probability given by a simple two-flavour formula:

$$P(\nu_\mu \to \nu_\mu) \approx 1 - \sin^2 2\theta_{23} \sin^2 \left(1.27 \frac{L}{E} \Delta m^2_{23}\right)$$ (1.19)

Given the current non-observation of electron neutrino appearance, it can be assumed that the missing $\nu_\mu$s have oscillated into invisible $\nu_\tau$s.

Electron neutrino appearance is the only accessible process whose leading term depends on $\sin \theta_{13}$, meaning that it is our best handle on the value of $\theta_{13}$. At the oscillation maximum, the probability of electron neutrino
appearance is given, for sufficiently large $\theta_{13}$, by

$$P(\nu_\mu \rightarrow \nu_e) \approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \quad (1.20)$$

This probability has been constrained to below 5% by current experiments.

Electron antineutrino disappearance is a very useful measurement to make with reactor neutrinos as their energy ($\sim 3$ MeV) allows access to oscillations on both the scale of $\Delta m^2_{13} \approx \Delta m^2_{23}$ and $\Delta m^2_{12}$, as discussed in Section 1.3.2. The full three-flavour survival probability is

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 \left(1.27 \frac{L}{E} \Delta m^2_{13}\right)$$

$$- \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \left(1.27 \frac{L}{E} \Delta m^2_{12}\right) \quad (1.21)$$

While the third term of Equation 1.21 dominates the variation of $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ on long scales due to the larger size of the mixing angle $\theta_{12}$ as compared to $\theta_{13}$, on very short scales ($\sim 1$ km) it is suppressed and one can measure oscillation from the second term to determine $\theta_{13}$. The difficulty in this measurement is that the second term is still much smaller than 1, so a good normalization of the unoscillated flux is required. Current experiments in this area use a similar near/far comparison as accelerator neutrino experiments, complicated by the short baselines necessary for this measurement.

Only a subset of observables can show CP violation, manifesting as a difference in survival or appearance probability between neutrinos and antineutrinos. All neutrino survival probabilities must be CP invariant as the survival process is time-reversal invariant, and thus CP invariant by the CPT theorem. As such, the particular signature of CP violation is a difference in the fraction of neutrinos that oscillate away from their initial flavour into a given other flavour. In particular, this means that a two-neutrino system cannot have CP violation, there being only one possible flavour for a given neutrino flavour to oscillate into.

We see an analogous situation for CP violation in particle decays. CPT invariance requires that the decay rate for a particle and its antiparticle be equal, but CP violation can produce differences in its branching ratios.
Figure 1.3: Feynman diagrams of neutrino scattering in matter. The neutral current diagram on the left is valid for all neutrino flavours, while the charge current diagram on the right is only valid for electron neutrinos.

Then, given the restrictions on available observables discussed above, we only currently have access to neutrino CP violation through electron neutrino appearance in a muon neutrino beam. As a result, measurement of electron neutrino appearance and thus $\theta_{13}$ is a precondition for measurement of CP violation. We can define a CP asymmetry $A_{CP}$ for nonzero $\theta_{13}$ as

$$A_{CP} = \frac{P(\nu_{\mu} \rightarrow \nu_e) - P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e)}{P(\nu_{\mu} \rightarrow \nu_e) + P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e)}$$

(1.22)

which, ignoring matter effects, has the value

$$A_{CP} \approx \frac{\Delta m_{21}^2 L}{4E_{\nu}} \cdot \frac{\sin 2\theta_{12}}{\sin \theta_{13}} \cdot \sin \delta_{CP}$$

(1.23)

Interestingly, this asymmetry becomes larger as $\theta_{13}$ becomes smaller; the CP violating effect comes to dominate the electron neutrino appearance in that case.

1.4.5 Matter Effects

The above formalism describes neutrino oscillation when propagating in vacuum, but the presence of matter modifies the oscillation probabilities[37, 38]. In particular, matter as we know it is not a flavour-symmetric medium; the presence of electrons modifies the propagation Hamiltonian. Feynman dia-
grams for neutrino-electron scattering can be found in Figure 1.3.

All neutrinos can scatter off electrons through neutral current interactions. In the case where no momentum is transferred between the neutrino and electron (forward scattering), it produces an effective potential for propagating neutrinos, analogous to the effect of an index of refraction on the propagation of light through a medium. This potential does not affect oscillation probabilities as neutral current interactions are flavour-symmetric.

Forward scattering through charge-current interactions is also possible, but only for electron neutrinos, as shown on the right of Figure 1.3. Thus, for electron neutrinos, the potential due to matter has an additional term $\sqrt{2}G_F N_e$ due to charge-current interactions, where $G_F$ is the weak coupling constant and $N_e$ is the number density of electrons in the medium. This additional flavour-asymmetric potential can then modify the oscillation probabilities.

Ignoring the third neutrino flavour and the flavour-symmetric neutral current potential, the matter-effect modified Hamiltonian is

$$H = \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + \sqrt{2}G_F N_e \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix}$$

(1.24)

Matter effects are crucial for understanding the oscillation probabilities of solar neutrinos. With the dense, electron rich solar matter standing between us and the production of solar neutrinos in the core of the Sun, matter effects are the dominant contribution to the oscillation probabilities of solar neutrinos.

Additionally, matter effects produce an asymmetry between neutrinos and antineutrinos, as antineutrinos are unable to undergo the charge-current forward scattering from Figure 1.3. Matter effects are thus very important in understanding any possible CP violation in neutrino oscillation.

1.5 Neutrino Interaction Physics

To study neutrino oscillation, we must understand how neutrinos interact with the matter in our detectors. We relate the neutrino flux, which is mod-
ified by the oscillation physics, to the interaction rate with a quantity that depends on the target material called the interaction cross-section. Specifically, we can write down the relation

\[ R = \Phi N \sigma \]  

(1.25)

where \( R \) is the interaction rate, \( \Phi \) is the neutrino flux, \( N \) is the number of target particles, and \( \sigma \) is the interaction cross-section, which depends on the detailed physics of the interaction of the neutrino and the target particle.

Neutrino cross-section physics is relatively poorly understood, but is crucial to many long-baseline experiments including T2K. If identical detectors are used at both the near and far sites, cross-section uncertainties cancel, but often (such as for T2K) the near and far detectors are not identical. Thus, we must understand the interaction cross-sections to predict the event rate at the far detector from the data at the near detector.

There are a variety of ways a neutrino can interact with a nucleus, from simple neutral current scattering to interactions producing several secondary particles such as deep inelastic scattering (DIS). Neutrino flavour detection requires charge-current interactions, where the neutrino is transformed into a charged lepton. [Section 2.2.1] discusses the possible interaction modes in detail; here and for much of the thesis we are mainly interested in inclusive charge-current cross-sections, which include all processes where the neutrino becomes a charged lepton through a charge-current interaction with a nucleus. However, to model these interactions, we must treat each interaction channel separately, since each channel has different kinematics. The most significant charge-current interaction channels are quasi-elastic (CCQE), resonant pion production (CC1π), and deep inelastic scattering (CC-DIS).

1.5.1 Interactions on Free Nucleons

The simplest interaction case is for the interaction of a neutrino with a free nucleon. The free nucleon case is the foundation of neutrino-nucleus interaction models; we extend the physics of neutrino-nucleon interaction to the case of bound nuclei through the impulse approximation. Under the impulse
approximation, we treat the nucleus as a collection of free nucleons with some kinematic distribution, and then calculate the total neutrino-nucleus cross-section as the incoherent sum of the interaction cross-sections with the individual nucleons. Effectively, then, the neutrino interacts with a single nucleon inside the nucleus, which undergoes the entire neutrino interaction independent of any intranuclear interactions. The treatment of free-nucleon CCQE interactions in this section follows that of Gallagher, Garvey and Zeller[39], who provide a comprehensive review of neutrino CCQE interaction modelling.

We can write the interaction Hamiltonian for a lepton-nucleon interaction as the inner product of two 4-currents:

\[ H_{int} = \alpha J_{\text{lepton}}^\mu J_{\mu,\text{nucleon}} \]  

(1.26)

where \( \alpha \) is a constant of proportionality. It is instructive to consider electron-proton scattering first, which proceeds predominantly through the electromagnetic interaction.

In this case, we can write the electron current as

\[ J_{\mu,\text{lepton}}(k) = \bar{u}(k') \gamma^\mu u(k) \]  

(1.27)

where \( k \) and \( k' \) are the initial and final lepton momenta, and the nucleon current as[40]

\[ J_{\mu,\text{nucleon}}(p) = \bar{u}(p') \left[ F_1(q^2) \gamma_\mu + \frac{1}{2M} F_2(q^2) \sigma_{\mu\nu} p^\nu \right] u(p) \]  

(1.28)

where \( M \) is the nucleon mass, \( p \) and \( p' \) are the initial and final nucleon momenta, and \( F_1(q^2) \) and \( F_2(q^2) \) are two functions of the momentum transfer \( q^2 \) called form functions. The form functions encapsulate all of the details of the nucleon structure that affect the nucleon-electron interaction. These form factors are well-understood in the context of electron-nucleon scattering, and are also relevant in the context of neutrino scattering.

The weak interaction currents used in neutrino interactions are similar to those used for electromagnetic interactions, but have additional terms to
account for the axial-vector component of the weak interaction. The lepton current gains an additional factor, becoming

\[ J_{\mu,\text{lepton}}(k) = \bar{u}(k')(1 - \gamma^5)\gamma^\mu u(k) \] (1.29)

while the addition to the nucleon current is more complicated. We can write the nucleon weak current as

\[ J_{\mu,\text{nucleon}} = J_{\mu,V} + J_{\mu,A} \] (1.30)

where \( J_{\mu,V} \) is a vector current of the same form as Equation 1.28, and \( J_{\mu,A} \) is an axial vector current with the form

\[ J_{\mu,A}(p) = \bar{u}(p') \left[ G_A(q^2)\gamma^\mu + \frac{1}{2M} G_P(q^2)q^\mu \right] \gamma^5 u(p) \] (1.31)

where \( G_A \) and \( G_P \) are additional form factors that only appear in weak interactions.

The conserved vector current (CVC) and isospin-conserving properties of the electroweak interaction allow electron-nucleon scattering to determine the form of the weak nucleon vector current to a high degree of precision. Thus, it is only the parameters of the nucleon axial current that must be determined from neutrino scattering. Considerations of partially-conserved axial current (PCAC) determine the pseudoscalar form factor \( G_P \) in terms of the axial vector form factor \( G_A \) as \( G_P = 4m_N^2 G_A/(m_N^2 + Q^2) \), so only \( G_A \) must be determined directly from neutrino scattering experiments.

In all current neutrino models, \( G_A \) is parametrized with an empirical dipole formula,

\[ G_A(q^2) = \frac{g_A}{(1 + q^2/M_A^2)} \] (1.32)

which has two parameters, \( g_A \) and \( M_A \). \( g_A \) is the value of the form factor at zero momentum transfer and can be determined precisely through measurements of nuclear beta decay. These experiments\(^{41}\) give a value of \( g_A = 1.2671 \). The remaining parameter, \( M_A \) is the ‘axial mass’ parameter, and can be determined in neutrino-deuterium scattering. Since
Table 1.2: Nuclear model parameters for carbon and oxygen in the relativistic Fermi gas model (numbers taken from [47]).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$^{12}$C</th>
<th>$^{16}$O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fermi momentum ($p_F$)</td>
<td>221 MeV</td>
<td>225 MeV</td>
</tr>
<tr>
<td>Binding energy ($\epsilon_b$)</td>
<td>21 MeV</td>
<td>27 MeV</td>
</tr>
</tbody>
</table>

the nuclear effects in deuterium are small and relatively well-understood, the free-nucleon parameters are straightforward to extract. We find from these experiments and others that the best fit value for free nucleons is $M_A = (1.026 \pm 0.021) \text{GeV}[42]$.  

1.5.2 Relativistic Fermi Gas Model

While it is useful to consider neutrino interactions on free nucleons, the targets used in most experiments contain more complicated nuclei. For our purposes, the most significant nuclei are carbon-12 and oxygen-16. The simplest model of interactions with nucleons bound in a nucleus, and the one used by virtually all neutrino interaction simulations, is the Relativistic Fermi Gas Model (RFGM).

The RFGM models the nucleus as a zero-temperature Fermi gas with maximum momentum $k_F$. Nucleon binding is treated very simply, through the use of a fixed nucleon separation energy $\epsilon_B$. Neutrino interactions then proceed with the same form as discussed in Section 1.5.1 above, but with a neutrino momentum randomly drawn from the uniform Fermi distribution, and with the final state energy reduced by the nucleon binding energy. Thus, in the RFGM, the only contribution of the bound nuclear state is kinematic in nature.

Most implementations of the RFGM include an additional effect, namely Pauli blocking. All of the particle states in the Fermi gas are treated as filled, so the outgoing nucleon is forbidden from occupying any of them by the Pauli exclusion principle. In practice, this is implemented by forbidding any interaction that leaves the outgoing nucleon with a momentum less than $k_F$.  

23
The parameters of the RFGM are determined from electron scattering experiments, such as that of Moniz, et al., 1971[48]. The distribution of the electron-nucleus scattering cross-section as a function of energy transfer has a peak corresponding to quasi-elastic scattering; the width of this peak is directly related to the Fermi momentum, while its position is determined by the nucleon binding energy. Fits to this data for carbon-12 and oxygen-16 give the parameters found in Table 1.2.

The RFGM is the simplest model which reproduces the broad features of lepton-nucleus scattering, and as such neglects a number of important effects. Its effect on the scattering physics is purely kinematic; the interaction with a bound nucleon is assumed to be identical to that of a free nucleon with its particular momentum. No attempt is made to model the shape of the nuclear potential and the different states available within the nucleus, instead just treating the nuclear potential as an energy penalty for the outgoing nucleon. Despite this, it is used in virtually all simulations due to its simplicity and computational tractability.

1.5.3 Nuclear Shell Model

The next step in complexity after a Fermi gas model of the nucleus is the nuclear shell model. Unlike the RFGM, the shell model has a definite form for the nuclear potential, and does not treat all of the nucleons identically, instead assigning them different kinematic distributions due to their different states. The simplest shell model arises from a view of the nuclear potential as
a three-dimensional harmonic oscillator. The energy levels of this oscillator are then filled with nucleons to describe the nuclear ground state. There are separate sets of shells for neutrons and protons, as they do not exclude each other through the Pauli principle.

The number of nucleons required to have all occupied shells completely filled is called a ‘magic number’; nuclei with magic numbers of protons and/or neutrons are particularly stable and tightly-bound. The two lowest-energy shells are the 1s state with, with orbital angular momentum 0 and thus room for two nucleons, and the 1p state, with orbital angular momentum 1 and room for six nucleons, so the first two magic numbers are 2 and 8. Spin-orbit coupling splits the 1p state into two sub-states; the 1p_{3/2} and 1p_{1/2} states, where the subscript corresponds to the total angular momentum. These states are illustrated for carbon-12 and oxygen-16 in Figure 1.4. Oxygen-16 is ‘doubly-magic’, as it has a magic number of both neutrons and protons; thus the binding energy per nucleon for oxygen is higher than that for carbon-12.

While the shell model is simplest to derive from a spherical harmonic oscillator, it is not a realistic model of the nuclear potential. A hybrid between the harmonic oscillator and square-well potentials called the Woods-Saxon potential is often used[49]. Unlike the harmonic oscillator potential, which increases indefinitely travelling away from the origin, the Woods-Saxon potential is a finite well with smoothly-varying sides. The depth of this potential depends on the density of the nuclear matter and thus on the number of nucleons, and is slightly different for protons than neutrons due to the presence of Coulomb repulsion in the proton case.

The effect of Pauli blocking is more significant in a shell model than for the relativistic Fermi gas model. Since each nucleon has a specific state, the set of states forbidden for a transforming nucleon is known, and the Pauli blocking just prevents the nucleon from entering any of these states.

The shell model is a more realistic model of nuclear structure than the Fermi gas model, but it is not currently used in modelling of neutrino interactions. Its main advantage is that it correctly places the nucleons in discrete states, improving the kinematic distributions. This may also allow
the simulation of nuclear excited states produced in the neutrino interaction, which is not possible in the pure, continuous Fermi gas model. The main barrier to the usage of shell-model calculations in neutrino models is the computational complexity of shell-model systems, particularly for large nuclei. Additionally, the shell model still ignores the interactions between nucleons, except where they sum together to produce the averaged nuclear potential used to derive the nucleon shells. In particular, no pairwise nucleon-nucleon interaction is included, which has effects as described in the next section.

1.5.4 Nucleon-Nucleon Correlations

The impulse approximation, the Fermi gas model, and the shell model all treat the nucleons inside a nucleus as independent, uncorrelated particles contained within a fixed nuclear potential. While this model is usefully simple, it fails to account for the strength of interactions between nucleons in the nucleus. Short-range interactions between nucleons can cause two-nucleon fluctuations in the particle states different than any involving just one nucleon. During these fluctuations, the two nucleons have highly-correlated states and the impulse approximation is inadequate to describe the interaction physics.

Pairwise nucleon correlations affect neutrino interactions in two ways. First, the interactions between the correlated nucleons can cause them to be excited to momenta well in excess of the Fermi momentum[50]. These high-momentum nucleons are entirely ignored by Fermi gas models and modify the kinematic distribution of the neutrino interaction products. Second, the interaction may be considered to occur on the correlated pair, rather than on a single nucleon. Both of these nucleons can then be ejected from the nucleus, modifying the kinematics of the final state.

A recent controversy in neutrino interaction physics involves the value of the nucleon axial mass parameter in the axial-vector form factor, as discussed above in Section 1.5.1. While neutrino-deuterium interactions give $M_A = (1.026 \pm 0.021) \text{GeV}[51]$, recent measurements using the MiniBooNE detector suggest a value of $M_A = (1.35 \pm 0.17) \text{GeV}[52]$. The MiniBooNE result is
derived from an excess of CCQE events at reconstructed neutrino energies near 1 GeV, as compared to previous models and the higher-energy NOMAD experiment [53].

However, the addition of short-range nucleon-nucleon correlations to the interaction model may resolve this anomaly. In the MiniBooNE detector, the ejected nucleons are invisible, so ‘ordinary’ CCQE with a single nucleon is assumed for reconstruction of the neutrino energy. This energy will thus be inaccurate if multiple nucleons are involved in the neutrino interaction. Martini, et al. [54] calculated cross-sections using multi-nucleon correlations using the NOMAD $M_A$ value and found that they did indeed fit the MiniBooNE data, pointing to a possible resolution of this anomaly. However, calculations of multi-nucleon correlations are still crude at present, and do not yet allow calculation of the kinematics of all final-state particles. This currently prevents their use in neutrino event generators, and most quantitative tests against experimental data. Future development of models using multi-nucleon correlations will be important for resolving these issues.

1.5.5 Resonant Pion Production

The second simplest neutrino interaction process is resonant pion production, or resonant CC1$\pi$. In this case, the neutrino excites a nucleon into a $\Delta$ or $N^*$ resonance, which decays producing a pion and a nucleon which escape the nucleus. Since the resonant CC1$\pi$ channel involves the production of additional hadrons, it requires a more complicated interaction model than Charged-Current Quasi-Elastic (CCQE).

The most common model used to describe CC1$\pi$ reactions is the Rein-Sehgal (RS) model [55]. The RS model consists of a sum of all of the non-strange baryon resonances below 2 GeV, producing an amplitude for each of the neutrino-induced pion-production channels. The original model included 18 resonances and interference terms between neighbouring resonances, but more recent simulations using the RS model reduce the set of resonances to the 16 resonances listed as unambiguous by the PDG, and ignore the interference terms [47]. The nucleon current for CC1$\pi$ includes an axial vector
form factor modelled similarly to the one for CCQE in Equation 1.32, with a $M_A$ parameter estimated to be $M_A = (1.12 \pm 0.03) \text{GeV}[56]$.

The RS model only describes the single-nucleon interaction and does not include any nuclear structure effects. In all current simulations, the relativistic Fermi gas model described in Section 1.5.2 is used to modify the nucleon kinematics, with no other nuclear effects simulated. It is possible that resonance widths are modified in nuclear matter, but this is not modelled in any current neutrino interaction simulation. Experimental measurements are necessary to guide the theoretical understanding of resonant pion production.

### 1.5.6 Deep Inelastic Scattering

The third neutrino interaction channel I will discuss is Deep Inelastic Scattering (DIS). In a DIS interaction, the neutrino interacts directly with a single parton within a nucleon, producing a hadronic jet. DIS is primarily a high-energy interaction channel, as the neutrino wavelength must be small enough to resolve the inner structure of the nucleon. As such, DIS is a relatively small contribution to the interaction rate at the average T2K beam energies, but becomes more significant on the high-energy tail of the neutrino energy distribution.

DIS interactions require significantly different modelling methods than other interaction channels involving an interaction with a whole nucleon. The kinematic distributions of partons within a nucleon require a fuller QCD description than the nucleons within a nucleus. The GENIE neutrino interaction simulation uses a cross-section model for neutrino-parton scattering developed by Bodek and Yang[57] for DIS interactions. The Bodek-Yang model treats vector and axial-vector form factors identically, necessitating an empirical correction that is applied as an overall multiplier to the calculated cross-section[47] to reproduce the observed neutrino cross-section at 100 GeV.

In principle, the intra-nucleon nature of DIS interactions makes them relatively insensitive to nuclear structure effects. The neutrino-nucleus DIS
cross-section would then scale linearly with the mass number $A$. We still expect some kinematic effects from the motion of the nucleon, as the momentum of the partons in a nucleon are comparable in scale to the Fermi momentum of the nucleons in a nucleus. Current neutrino simulations include a kinematic contribution from the relativistic Fermi gas model for DIS, similar to that for all other neutrino interaction channels.

1.5.7 Final State Interactions

A nuclear effect that is common to all neutrino interaction channels is the Final State Interaction (FSI). In the impulse approximation used to model neutrino interactions, the neutrino interacts instantaneously with a nucleon, transforming the initial state particles to the final state particles without any immediate reference to the remainder of the system. Once the final state particles are produced, they must then exit the nucleus. Since many of these particles are strongly-interacting, they can interact with the nuclear matter in many different ways before leaving the nucleus.

Consider, for example, the case of pion production. A pion produced inside the nucleus has many opportunities to interact with nucleons before exiting. These interactions, such as absorption and charge exchange, can change both the multiplicity and charge of the visible exiting particles. Secondary interactions with nearby nuclei can also produce similar effects to FSIs, and are generally simulated with a particle propagation framework such as GEANT.

In current neutrino interaction generators, the FSIs are treated separately from the primary neutrino interactions. After simulating a primary neutrino interaction and determining the final state particles and their kinematics, the propagation of each strongly-interacting particle through the nucleus is simulated based on our understanding of hadron interaction physics.

We model only hadronic FSIs, as the cross-section for additional lepton interactions within the nucleus is relatively small. As such, the charge-current inclusive analysis presented in this dissertation is relatively insensitive to FSIs, by depending only on the details of the outgoing lepton. For
more channel-sensitive experiments, FSIs can affect the results by changing the apparent neutrino interaction channel of the final state. Improved modelling of these interactions is thus necessary for a comprehensive understanding of a channel-sensitive analysis.

1.6 Goal of Dissertation

The combination of a plastic scintillator-based near detector and a water Čerenkov far detector makes the T2K oscillation measurement sensitive to differences in neutrino cross-section between carbon and oxygen. This thesis will outline a measurement of the ratio of the neutrino charge-current inclusive cross-section for oxygen to that for carbon. As many of the nuclear effects are poorly understood, a measurement could help refine neutrino interaction modelling efforts, which are a non-negligible source of systematic errors in the T2K oscillation measurement.

This thesis is divided into five parts, each describing a separate area of effort:

- **Part I** of this thesis describes the basics of neutrino oscillation physics and the design of the T2K experiment.

- **Part II** describes the hardware component of the thesis work, which involved building a water target system for the FGD component of the near detector, for the purpose of measuring neutrino interactions on oxygen.

- **Part III** describes the development of a new track-finding algorithm for tracks contained within the FGD.

- **Part IV** uses an analysis of the neutrino interactions in the FGD along with a statistical subtraction method to calculate the oxygen/carbon cross-section ratio.

- **Part V** contains a summary of the full thesis project.
Chapter 2

The T2K Experiment

2.1 T2K Experimental Setup

T2K is a long-baseline accelerator neutrino experiment in the pattern described in Section 1.3.3. The beam for T2K is generated at the Japan Proton Accelerator Research Complex (J-PARC) lab in Tokai, Japan, and has a 295 km baseline to the far detector at Super-Kamiokande. In addition to the beam and Super-K, T2K also includes two near detectors, INGRID and Near Detector 280 meters (ND280). As the T2K beam is off-axis (see Section 2.1.2 below), the two detectors are used to measure the beam both on- and off-axis. ND280 is an off-axis magnetic spectrometer described in detail in Section 2.2, while INGRID is an on-axis detector for monitoring the beam shape and direction.

2.1.1 T2K Production Beamline

The neutrino beam for T2K is produced using the 30 GeV proton beam from the J-PARC Main Ring accelerator. A diagram of the neutrino beamline can be found in Figure 2.1. The proton beam is fully extracted in a single cycle and directed onto a 91.4 cm long graphite target to produce the secondary meson beam. This production target is contained in the first of three magnetic focussing horns. The horns serve as a selective focussing mechanism that is tuned to focus pions of the desired charge while de-focussing those
The secondary beam of primarily pions is then allowed to propagate through a long decay volume to produce the tertiary neutrino beam. This 93 m long tunnel is filled with helium gas to minimize scattering of mesons, and ends in a beam dump to remove all non-neutrino particles from the beam.

### 2.1.2 Off-Axis Beam

A key difference between T2K and earlier accelerator neutrino experiments is the use of an off-axis beam. The kinematics of pion decay give a much narrower neutrino energy distribution as seen in Figure 2.3. This off-axis angle can then be tuned to align the beam maximum with the oscillation maximum at the far detector. For T2K, the off-axis angle was chosen as 2.5°, which maximizes the flux in the region of the first oscillation maximum at Super-K. The higher intensity of the T2K beam also aids in the use of the off-axis technique, as the off-axis intensity is lower than the central intensity.
Figure 2.2: Diagram of secondary beam production target and neutrino production volume

Figure 2.3: Neutrino energy spectrum at a range of off-axis angles. Main body of 2.5 degree spectrum highlighted on T2K oscillation probability graph, illustrating the optimal choice of off-axis angle.
2.1.3 INGRID

In addition to ND280 (Section 2.2), the near detector complex at J-PARC contains the on-axis INGRID detector. INGRID consists of a cross-shaped arrangement of small scintillator/iron neutrino detectors for characterization of the beam intensity and cross-sections. The data from INGRID is used to monitor the stability of the beam flux and direction with time. The beam direction information is also important for determining the precise off-axis angle to Super-K and ND280 for prediction of neutrino fluxes and spectra.

2.1.4 Super-Kamiokande

The Super-Kamiokande (Super-K) detector is a large water Čerenkov detector located in western Japan, originally commissioned in 1996. It is situated in the Mozumi mine 1km underneath Mt. Ikenoyama, and consists of a cylindrical tank holding 50,000 m$^3$ of water, as shown in Figure 2.4. Inside the water, particles travelling faster than the speed of light in water produce cones of Čerenkov light, which are recorded in an array of 11,129 Photo-Multiplier Tubes (PMTs) which cover 40% of the outer wall of the tank.
inner detector. This provides sufficient spatial resolution to reconstruct the full Čerenkov ring. A volume beginning 2m from the wall of the inner detector is used as the fiducial volume, containing 22.7 kton of water. The outer detector is used as a veto for events originating outside the detector and for particles that leave the detector.

Super-K has been used for a variety of measurements in the past, most prominently to measure atmospheric neutrinos (see Section 1.3.1). It is the successor to the Kamiokande detector built in the 1980s to search for proton decay, and continues the proton decay search alongside its program of neutrino physics. As the T2K beam is directed on Super-K for only a small fraction of the time, these other measurements can continue with minimal interference from T2K operation.

Super-K is a full ring imaging Čerenkov detector which exploits the convenient Čerenkov threshold in water of $\beta \approx 0.75$. Most atmospheric and T2K $\nu_\mu$s and $\nu_e$s have sufficiently high energy to produce leptons above threshold in CC interactions, which emit Čerenkov radiation for a while before falling below threshold. The Čerenkov cone produces a ring of light on the instrumented wall of the inner detector, whose width measures the distance travelled before falling below threshold, and thus the energy of the producing particle.

An important characteristic of Super-K for its use in T2K is its ability to perform particle identification. Muons travel through the detector with no or few scatterings, producing a sharp-edged ring, while electrons shower and undergo multiple scattering, making the electron ring much more ‘fuzzy’. This PID capability is crucial to the electron neutrino appearance measurement.

An inherent limitation of Super-K as a Čerenkov detector is that it can only observe the lighter, higher energy particles produced in a neutrino interaction. In particular, the high Čerenkov threshold for protons ($\approx 1.5$ GeV/c) means that the recoil proton from the neutrino interaction is invisible, along with any low-energy pions that may have been produced. An important task for ND280 is to characterize the interaction modes that can mimic the crucial CCQE mode in Super-K, as discussed below in Section 2.2.1.
2.2 Introduction to ND280

The ND280 detector is the primary apparatus for measuring the properties of the T2K neutrino beam at the source, unoscillated. It is a magnetized spectrometer with a variety of subdetectors. A diagram of ND280 can be found in Figure 2.5.

The heart of ND280 consists of two tracking detectors, the Pi-Zero Detector (P0D), and the Tracker, which in turn consists of three Time-Projection Chambers (TPCs) and two Fine-Grained Detectors (FGDs). The P0D is designed mainly to measure neutral current neutrino interactions, while the tracker is designed to characterize the high-energy leptons produced in charge-current neutrino interactions. Surrounding these detectors is an electromagnetic calorimeter (ECAL), which is built to capture electromagnetic showers of electrons and photons that escape the basket. In addition, the magnet yoke is interleaved with scintillator bars making up the Side Muon Range Detector (SMRD), which tracks penetrating muons as they escape the detector, and also allows measurement of cosmic rays as they traverse the entire ND280.

Figure 2.5: Exploded overview of ND280 detector, showing all subdetectors and magnet elements.
2.2.1 Interaction Modes in ND280

The various ND280 subdetectors are designed to measure a variety of neutrino interaction modes. The ideal mode for observing neutrino interactions is the charge-current quasi-elastic (CCQE) mode,

\[ \nu_\ell + n \rightarrow \ell^- + p \]  

(2.1)

The simple kinematics of CCQE allow the incident neutrino energy to be reconstructed purely from the momentum of the resulting charged lepton. As such, it is the signal mode for measurements of energy-dependent oscillation probabilities, and is the selection goal in the analysis of Super-K data.

At T2K energies, the most common non-QE charge-current interaction has pion emission through the excitation of a \( \Delta \) resonance, known as resonant CC1\( \pi \). There are multiple CC1\( \pi \) combinations available:

\[ \nu_\ell + p \rightarrow \ell^- + \Delta^{++} \rightarrow \ell^- + p + \pi^+ \]  

(2.2)

\[ \nu_\ell + n \rightarrow \ell^- + \Delta^+ \rightarrow \ell^- + p + \pi^0 \]  

(2.3)

\[ \nu_\ell + n \rightarrow \ell^- + \Delta^+ \rightarrow \ell^- + n + \pi^+ \]  

(2.4)

Distinguishing CCQE interactions from CC1\( \pi \) interactions is an important design goal of ND280. There is also a contribution to the CC1\( \pi \) rate from coherent CC1\( \pi \) interactions, where the neutrino scatters off the nucleus as a whole rather than a single nucleon. Multiple pions and nucleons can be produced in DIS events, where the neutrino scatters off a component quark producing hadron jets. Roughly, the CCQE interaction dominates at low neutrino energy (< 1GeV), with the CC1\( \pi \) peaking just above 1 GeV and DIS becoming the dominant interaction in the tens of GeV range.

The most prominent neutral current interaction in ND280 is the NC1\( \pi^0 \) interaction,

\[ \nu_\ell + N \rightarrow \nu_\ell + N + \pi^0 \rightarrow \nu_\ell + N + 2\gamma \]  

(2.5)

NC1\( \pi^0 \) is an important background to electron neutrino appearance, as the photons produced in the \( \pi^0 \) decay mimic electrons in Super-K. Usually,
NC1π^0 can be distinguished by the presence of two rings from the two decay photons, but there are a variety of ways in which one of the rings can be missed producing an electron-like background event. The ND280 P0D is designed to measure the rate of NCπ^0 events to refine the estimate of this background rate at Super-K.

2.2.2 The Tracker

The charge-current analyses in this dissertation are centred on the Tracker component of ND280. The Tracker combines the precision tracking of the three large TPCs with the instrumented target mass of the FGDs to measure both the long-range leptons and short-range hadrons produced in neutrino charge-current interactions. A sample CCQE event in the tracker is shown in Figure 2.6.

The 3D tracking of the TPCs in the magnetic field combines with its precise measurements of \( dE/dX \) permit the Tracker to identify muons and electrons, and to determine the momentum and charge of these particles. These tracks are matched to hits in the FGDs to trace the tracks between...
two TPCs or back to the interaction vertex, if it is located within the FGD. The FGD is also used to reconstruct short tracks originating at the vertex and stopping within the FGD. (See Section 2.3 and Part III for more details on the FGD and FGD-only tracking)

### 2.3 Fine-Grained Detectors

The ND280 Tracker relies on the three TPCs for most of its tracking performance, but they do not make up a suitable neutrino detector by themselves. As the density of the TPC active volume is low, very few neutrino interactions will occur within the TPC. As a result, the TPCs are interleaved with solid scintillator detectors called Fine Grained Detectors, or FGDs\[58\]. Each FGD contains 1.1 tonnes of target material, but the composition differs between the two FGDs; the first FGD is composed entirely of plastic scintillator, while the second has scintillator layers interleaved with passive water modules (described in Part II).

The active elements of the FGDs are made of 9.61mm square, 1864.3 mm long bars of extruded plastic scintillator. These bars include a high-opacity...
TiO$_2$ coating to isolate the individual scintillator bars, and are threaded with a wavelength-shifting fibre to collect the scintillation light. The cross-section of a typical FGD bar is shown in Figure 2.7. The fibres in turn are connected to solid-state photosensors called Multi-Pixel Photon Counters (MPPCs), one for each bar. The scintillator bars are assembled into alternating layers of 192 vertical or horizontal bars, allowing tracking of particles along both the X and Y axes. Each pair of horizontal and vertical layers is called an XY module.

The two FGDs have identical mechanical structures and electronics, and identical active areas, but contain different mixes of modules. The first FGD (FGD1) contains fifteen XY modules, while FGD2 contains seven XY modules and six water modules. The total thickness in each case is approximately 30 cm. The modules are then sealed inside an aluminum dark box frame with thin covers to reduce their effect on exiting particles. The modules are hung from the top of the dark box by steel straps and are sandwiched together by the pressure of the cover plates.

The thinness of the FGDs is designed to complement the precision tracking of the TPCs. Most of the penetrating charged particles will exit through the large upstream and downstream faces of the FGD into the neighbouring TPC which can track the particles with high precision in three dimensions. The magnetic field of the ND280 detector allows the TPC to measure the momentum and charge of these particles, which can then be tracked back to their origin in the FGD.

An important purpose of the FGD is to detect short-range particles produced at the neutrino interaction vertex. These tracks allow us to differentiate between different interaction modes, and aid in the determination of the vertex position. FGD-only tracking is key to distinguishing CCQE interactions from various other charge-current interactions, both through direct detection of pions or other secondaries, and through the additional constraint on the kinematics through observation of the proton. Methods for reconstructing tracks contained within the FGD are described in Part III.

The FGD must also be able to tag delayed activity in the form of Michel electrons. These electrons are produced by the decay of stopped muons,
meaning that they are delayed from the primary event by the muon lifetime of \( \sim 2.2 \, \mu s \). As a result, the FGD must continue recording hits over a relatively long time period, and a long readout buffer of about 10 \( \mu s \) is thus used with the FGD.

### 2.3.1 FGD Operation

The FGD operates by collecting the scintillation light produced as charged particles traverse the plastic scintillator bars that comprise the FGD active volume. This light is well-contained within each scintillator bar by the titanium dioxide coating. The hole in the middle of each bar contains a 1mm diameter wavelength-shifting optical fibre. One end of each fibre is mirrored for improved light collection, while the other is mechanically coupled to an MPPC. To provide room for the mounting of the MPPCs, the orientation of the fibre alternates direction along the length of a module.

The active area of an MPPC consists of an array of small Avalanche Photo-Diode (APD) pixels operated in Geiger mode. When a photon strikes a pixel the diode breaks down and conducts briefly. The total current is summed for all pixels giving an output current proportional to the number of pixels struck, and thus to the number of incident photons if that number is small compared to the number of pixels (667 for the devices used in the FGD).

The output signal from each MPPC is then fed to a continuous waveform sampler based on a Switched Capacitor Array (SCA). The capacitors of the SCA are set up in a loop and they sample the input voltage at 50 MHz. There are 511 cells in the SCA leading to a total readout time of 10.1 \( \mu s \). Prior to the SCA, the MPPC signal passes through a combination preamplifier and shaper that lengthens the pulse by a factor of 25, allowing the short (\( \sim 20 \, \text{ns} \)) pulse to be traced in detail by the SCA. When the detector is triggered, the samples stored in each SCA can then be digitized using a 12-bit flash ADC. The SCAs and the preamp are contained in a custom ASIC shared with the TPC readout called AFTER. The AFTER ASICs provide two preamp/SCA combinations per MPPC, one low-gain channel and one
high-gain channel, to better match the dynamic range of the MPPCs to the range of the digitizer.

2.3.2 FGD Readout

The FGD electronics are built such that the majority of the readout is outside the dark box, in a series of 24 minicrates around the sides of each FGD. There are a number of connections between the MPPCs on one end and the Data Acquisition (DAQ) computers on the other, as outlined in Figure 2.8.

Inside the dark box, each MPPC is mounted on a small ‘daughtercard’, and connected to one of the six ‘busboards’ mounted along each side of each XY module. These busboards are simple circuit boards that aggregate the signals from the MPPCs to a single ribbon cable connector, contain temperature and humidity sensors for monitoring the internal conditions of the detector, and contain LEDs positioned by the mirrored ends of fibres for testing of channels.

The minicrates are centred on the ‘backplanes’, which form part of the dark box wall and pass electrical signals between the inside and outside of the dark box. Each backplane has fifteen busboards (seven in FGD2) connected to it on the inside, one for each XY module, plus four (two) Front
End Boards (FEBs), a Crate Master Board (CMB), and a Light Pulser Board (LPB), attached on the outside.

The FEBs house two AFTER ASICs (Section 2.3.1) which sample the MPPC waveforms and the 12-bit flash ADC used for rapidly digitizing these waveform samples. They also contain a charge pump for generating the $\sim 70$ V bias required to run the MPPCs, and fine controls to set the precise voltage separately for each MPPC. Each FEB manages 64 MPPCs, and interfaces with the CMB for control and data acquisition.

The CMBs control the data acquisition process using an FPGA and transfer the data from the FEBs to the rest of the DAQ. The four or eight ASICs in the CMB’s minicrate are read out in parallel by the FPGA, and the data transmitted to the Data Concentrator Card (DCC) for further processing. This process takes approximately 2 ms, occupying the FGD electronics for only a small period after triggering.

The DCCs are the interface from the FGD electronics to the rest of the ND280 system. Each DCC controls four CMBs, triggering their readout and formatting the data so it can be transmitted to the main computer systems via Ethernet.

### 2.3.3 FGD Triggering

ND280 triggering is generally controlled by the Master Clock Module (MCM), so named because it produces the 100 MHz reference clock signal for all sub-detectors. Three types of MCM triggering are important for the FGD: beam spill triggering, Trip-T cosmic triggering, and FGD cosmic triggering.

Beam spill triggers occur whenever a beam spill occurs in the accelerator, causing protons to impact on our production target. These triggers are ultimately produced outside ND280 and are used to acquire neutrino interaction data.

The Trip-T cosmic trigger is produced by the subdetectors based on the Trip-T readout electronics, namely the ECAL, P0D, and SMRD. These detectors can produce a cosmic trigger from a coincidence of two modules, such as the top upstream and bottom downstream segments of the SMRD.
Trip-T cosmics sample the entire detector, but the rate of cosmics passing through the Tracker is quite low, necessitating the FGD cosmic trigger. While the FGD data acquisition is generally triggered externally by the arrival of a beam spill, the FGD also has the capacity to trigger itself for the measurement of cosmic rays. The signals for each MPPC are added together in groups of eight to produce the Analog Sum (ASUM) signal. Each of these ASUM groups has a threshold value at which it ‘fires’. If a designated number (usually 2) of the ASUM groups in a minicrate simultaneously, the crate itself is said to have fired. We trigger if we have coincidental firing of two crates in FGD1 with two crates in FGD2, so the FGD-triggered cosmics all pass through both FGDs and the entirety of the middle TPC.

2.3.4 FGD Calibration
The purpose of the FGD calibration is to translate from the raw MPPC waveforms to a set of discrete energy deposit events, or hits. Specifically, we must translate the raw height of the MPPC pulse to a normalized measure of light yield that is uniform between bars and between times. This process has several steps, based on the various characteristics of the scintillator/fibre/photodetector system.

The output voltage of an MPPC is discretized, with each hit pixel providing a roughly equal contribution to the output pulse. Thus, the first normalization is to normalize the pulse height by the height of a pulse where only a single pixel fires. This can be measured using the dark noise of the MPPCs, where pixels spontaneously fire due to thermal effects. From the pulse height distribution of dark noise, we can extract a single photon pulse height for normalizing physics pulses. This value varies with temperature and operating voltage and is corrected for those effects through an empirical formula, determined through the use of an electron beam at TRIUMF and FGD cosmic rays, which should have a constant energy deposit between channels. It is also corrected for intrinsic variations between FGD bars, and the effects of MPPC saturation, to obtain a value for the charge deposited
in the FGD.

Each separate pulse from the MPPC is then converted into a hit, storing the channel number (and implicitly the hit coordinates), the observed charge, and the fit time for the pulse. These hits then form the input for the track reconstruction, as discussed in Part III.
Part II

FGD Passive Water Modules
Chapter 3

Design and Materials

3.1 The Need for Water Modules

During the design of ND280[58], one thing that was seen as crucial was the ability to measure interaction rates on water. As most of the target mass of ND280 is composed of plastic scintillator and metal, and the difference in interaction cross-sections between carbon and oxygen is poorly understood, ways of including water inside the ND280 detectors are necessary.

The concept chosen for the FGD is to include passive water targets in one of the two FGDs, and then extract the water component by comparing the two detectors. The general plan for the water measurement is to scale the interaction rate in FGD1 to the mass of the plastic in FGD2, and then subtract from the total FGD2 rate to find the rate purely on water. The details of the first such FGD analysis can be found in [Part IV].

For this statistical subtraction to work well, the water modules must be designed so that their chemical composition is, as closely as possible, the sum of the compositions of some quantity of scintillator material and some quantity of water. This ensures that the component remaining following the subtraction accurately reproduces the properties of water.
3.2 Requirements

As the FGDs are made of a stack of vertical modules and kept rigid in part by light compression of the stack along the beam axis, the water modules must be of rigid construction, rather than in the form of a flexible bag. The active areas of the FGDs are only 30 cm thick, compared to their almost 186 cm square cross-section. As a result, the water modules must be thin (2.54 cm) and share the 186 cm square cross-section. The volume permits the use of 70–90 L of water per module.

Assembly of the FGDs is done by stacking the modules on a table, assembling the frame around them, and then tilting the assembly until the modules are vertical. To include water modules, they must be able to bear the weight placed upon them in this process while empty, and then be filled when vertical.

Additionally, the relatively small size of the FGD rules out separate containment within the dark box for scintillator and water modules, nor is it practical to keep the FGD electronics away from the water modules. Thus, the water modules must be designed to be leak-proof to protect the remainder of the detector from water damage. Due to the 0.2 T magnetic field inside the ND280 detector, no ferromagnetic materials can be used in the modules themselves.

To maintain the integrity of the module seal, no level sensors were installed within the water modules. External water level sensors must be extremely thin to fit; none that fit were sufficiently sensitive. Instead, flow through the supply system is taken as proof that the water module is full.

The design chosen in response to these needs consists of hollow polycarbonate panels, sealed at the top and bottom, and connected to a sub-atmospheric pressure water supply system. The sub-atmospheric pressure inside the panels ensures that a leak in a water module will draw air into the panel, rather than leak water into the surrounding FGD. The supply system must circulate the water to maintain the internal pressure and water level and flush out air bubbles. The water must also be kept clean and free of bacterial growth, necessitating the use of a chemical additive in the target
water.

3.3 Raw Materials
A variety of materials were used in constructing the FGD water modules. The main constituents are polycarbonate panels, polypropylene skins, and glue.

3.3.1 Polycarbonate Panels
Polycarbonate is chosen for the main water vessel for several useful properties. It is rigid, relatively impermeable to water, and requires relatively thin walls to maintain its structure. The oxygen in polycarbonate also increases the amount of ‘water’ present within the module, helping to balance the elemental composition of the module.

In addition, hollow polycarbonate panels with strong internal structure are commercially available for use as greenhouse walls. The availability off the shelf of these raw materials was an important factor in the choice of polycarbonate.

The structure of these polycarbonate panels is shown in Figure 3.1. There are three different ‘types’ of polycarbonate panel that were used. The first six modules were made with the US-sourced Suntuf Sunlite Structured 1” panels acquired in 2006, having an internal structure with five inner walls in each 1” square cell. Further panels were acquired in 2008 from Palram in the UK, which have a different structure, with narrower cells, having only three inner walls each. It was discovered after building modules from these panels that one of the ones we used was manufactured substantially thinner than the others, and that the remaining new panels were slightly thinner than the old panels.

3.3.2 Polypropylene Skins
The ultimate analysis goals for the FGD water system require a clean cancellation of the plastic parts of both detectors, leaving only water behind. This puts requirements on the elemental ratios of the water modules, as
Figure 3.1: Cross-section of polycarbonate panels used as the basis for the passive water modules. The left illustration shows the original (old-style) panels used to construct all prototypes and the first five production modules, while the right illustration shows the new cross-section of the second lot of panels used for the last three production modules. The vertical and outside walls shown are thick, while the other walls are substantially thinner.

It was found that the water module was deficient in hydrogen and carbon relative to what was needed. As a result, a 1/32” polypropylene skin was added on each side of the polycarbonate panel.

In addition to modifying the elemental composition, the polypropylene skins serve a useful purpose in containing the water in the module. Water can slowly diffuse through the walls of the polycarbonate panels, increasing the ambient humidity of the FGD. High humidity inside the FGD dark box is undesirable due to its effect on the electronics. The polypropylene skins inhibit the diffusion to a level where it is manageable with a flow of dry air through the dark box.

3.3.3 Chemical Additives

The water is treated with anti-microbial and anti-corrosive agents to prevent damage to the modules and circulation system. Biological growth in the water is inhibited by a 0.25% concentration of Germall Plus biocide,
manufactured by Sutton Laboratories. Biological growth could damage the seal of the module or block the flow of water, so it is important to avoid it as much as possible.

Corrosion of fittings into the water in the system would also cause difficulty, even if it is very slow. Due to the presence of both brass and stainless steel components in the water system, electrochemical effects could increase corrosion rates. To combat the corrosive effects, 10 ppm of tolyltriazole is also added to the water. Tolyltriazole bonds to the surface of copper and copper-alloy parts, creating a protective film of non-metallic material.

3.4 Choice of Sealant and Adhesive

A crucial part of the FGD water module design process was the selection of appropriate adhesives and sealants. The difficulty of gluing polycarbonate and polypropylene required careful investigation of the available options.

3.4.1 Requirements

There were a number of requirements on the sealant used in the FGD water modules. They fall broadly into three categories: interactions with polycarbonate, interactions with water, and the properties of the uncured sealant.

Foremost among these requirements was that it must bond sufficiently to polycarbonate to maintain a seal, and that it should bond as well as possible. In addition, the sealant must not damage the polycarbonate. This is particularly relevant during the curing of the glue, as this is when the most extreme changes occur. The curing must not be sufficiently exothermic to damage the polycarbonate, nor may the sealant contract enough during curing to strain the structure of the panel.

The sealant will then be immersed in water for most of the operational lifetime of the module, and must maintain its properties while so immersed. If the sealant were to swell from water absorption, and then shrink when the water is drained, leaks in the module could develop. The pressure of the water should not cause the bonds to delaminate, or to chemically alter the glue surface causing failure.
Finally, it must be possible to manipulate the sealant so as to produce a full, even seal of the panel. Its pot life must be sufficient to allow us to fill a channel the width of the module and immerse the module before it solidifies. It must be fluid enough to fill the full channel and viscous enough to make it practical to contain.

### 3.4.2 Isolated Testing

A variety of tests were conducted on the various glue candidates. Tests were done to measure the strength of the adhesion and their properties when immersed in water.

We measured water absorption by casting small ‘pucks’ of the sealants under study, as shown in Figure 3.2, and then immersing some of them in water for a long period of time. Both the thickness and mass of these pucks was measured periodically to determine the effects of water absorption on a mass of the glue. Results for the final sealant HE 1908 can be found in Figure 3.3.

The strength of bond achieved with each glue was tested using pairs of small, consistently sized polycarbonate bars, staggered lengthwise with an
Figure 3.3: Water absorption of HE 1908, illustrated by the change in mass of a set of samples immersed in water for 75 days. The dashed lines at the bottom show the control samples that were left in air rather than water. Clear saturation is observed at an acceptable level of increase.

overlapping glue joint at one end. These glue bonds were tested with two different tests: a ‘peel’ test where the bars were held horizontal and one end weighed down with calibrated weights, and a ‘shear’ test where the bars were instead held vertically. These tests were done for dry samples, and for samples that have soaked in water for long intervals. These tests were important for validating the choice of sealant. Tough-Seal 21 was found to lose almost 90% of its adhesion in the peel test after 2.5 weeks in 31°C water, while the HE 1908 lost almost none of its adhesion, which was already 45% more than for Tough-Seal 21. These results strongly favoured our decision to use HE 1908.

3.4.3 Brief History of Usage

The earliest prototypes were sealed with Stycast 1266, the best option we had found at that point. However, water absorption tests suggested that GSP 1339 was a better option. The first two full-size prototypes were built with this sealant, but we experienced failures as described in Section 4.6.2. After further tests, the epoxy elastomer Tough-Seal 21 was used for the next prototype, but the viscosity of the uncured epoxy, its strong tendency
to adhere to the sides of the gluing mould, and its high degree of water absorption disfavoured its continued use. Tests of the polyurethane sealant HE 1908 found it to be favourable, and it was used for all sealing beginning with the fourth and final prototype.

3.4.4 Glue for Skin Attachment

The adhesive for the water module skin was chosen through a similar process as the module sealant, outside of the requirement that the adhesive have favourable properties for water absorption. It was determined early that the only glue available with sufficient bonding strength of polypropylene to polycarbonate was CLR 1390/CLH 6025. We then needed to determine whether we could improve the adhesion through surface treatments. We studied sanding of the polycarbonate, sanding of the polypropylene, and priming with Mitsui Unistole P primer. Peel and shear tests akin to those done for polycarbonate/polycarbonate gluing (Section 3.4.2) were done and all three treatments were found to have a beneficial effect.

3.5 Supply System

As the FGD water modules are installed in close proximity to sensitive electronics, there must be some protection against water leaks. The small size of the FGD rules out physical isolation, so instead we chose to use sub-atmospheric pressure inside the water modules to prevent the water from escaping. Sub-atmospheric pressure will draw in air through a leak rather than allowing water to flow out. To maintain this sub-atmospheric pressure even in the presence of a leak, a circulating water system is necessary.

The sub-atmospheric pressure supply system was designed by C.A. Miller using the same principles as his design for the ND280 detector cooling system. The design uses two reservoirs and two pumps to provide both vacuum and circulation. A schematic of the system can be found in Figure 3.4.

The main principle of the system is to draw water through the water modules from a large open reservoir to a small vacuum tank held at a controlled, sub-atmospheric pressure by an air pump. The water level in the
Figure 3.4: Schematic of sub-atmospheric pressure water circulation system. Water is drawn through the modules from an open reservoir by the low pressure in a vacuum vessel; the water is then pumped back to the reservoir to maintain a moderate water level in the vacuum vessel.

vacuum vessel is maintained by pumping water back into the reservoir. The open reservoir is kept on a platform at 2.4 m above the floor, while the base of FGD2 is 3.8 m above the floor and the vacuum vessel is 0.8 m above the floor. The water level in the reservoir is at approximately 3 m from the floor during normal operation, while the water level in the vacuum vessel is approximately 20 cm.

Starting from the open reservoir at atmospheric pressure (approximated here as 1 bar), we see that the pressure at the base of FGD2, 80 cm above the water level in the reservoir, is 920 mbar. The pressure at the top of the FGD module, 2.3 m higher, is 687 mbar. The vacuum vessel is maintained at 300
mbar absolute pressure, which draws water down from the tops of the water modules to generate a circulation flow, which is adjusted to be approximately 1 L/min. As the modules are higher than the reservoir, subatmospheric pressure is maintained within all FGD2 water modules and hoses.

The water supply system was initially built at TRIUMF in early 2009, before shipping and reassembly in Tokai. Photographs of the system at TRIUMF can be found in Figure 3.5 and Figure 3.6.

The supply system was originally tested with the narrow prototype described in Section 4.6.1. This prototype had two 1/4” valves attached to the side for venting tests, one near the bottom, and one two meters higher near the top. The valve attachment is shown in Figure 3.7. While connected to the system at approximately 0.3 bar pressure, we tested opening the top valve. We were able to open the valve fully without any water escaping. With the top valve fully open, we then opened the bottom valve. In this
Figure 3.6: Water circulation system test installation showing the water circulation pump in centre, vacuum tank on left, and manifolds on right. These components were placed at floor level as they would be installed on the floor of the ND280 pit.

In this case, water flowed freely out of the bottom valve, as the inflow of air at the top raised the internal pressure.

The system was later tested with a full-size water module (as seen in Figure 3.5) without significant incident. It was then tested with the fully-assembled FGD2 for several weeks prior to the test of FGD2 in the M11 beamline at TRIUMF.

While the circulation system is necessary for maintaining the pressure in the presence of significant leaks, the vertical alignment of the system components is capable of producing a static sub-atmospheric pressure in the water modules in the case of power failure. This was tested following the March 2011 Tohoku earthquake, when J-PARC was without electricity for almost two months. When the system was restarted on May 10, 2011, the modules appeared to have remained below atmospheric pressure. Over an extended period of time, air leaks into the system will slowly displace water back into the reservoir, lowering the water level in the modules. The
Figure 3.7: Water module prototype showing side valve. The side valve was opened while the prototype was connected to the test water circulation system to ensure the system was able to control a sizable leak in the body of the module. No water leaked out, even with the valve fully open.

water level in the modules was unknown when restarting the system after the earthquake.
Chapter 4

Construction of Modules

4.1 Preparation of Panels

The polycarbonate panels required a significant amount of processing before they were ready for sealing. In particular, they must be cut to length, and the internal cells connected to form a single contiguous volume to be filled with water as a unit.

4.1.1 Panel Cutting

The first step in constructing a water module was to cut a polycarbonate panel to design length. The module cutting served a dual purpose; in addition to matching the module length to the requirements of the design, we produced a new, more carefully cut end on each side of the module. The first production run (see Section 4.7) had the panels cut to a nominal length of 1873 mm, while the protrusion of sealant on the finished modules led us to cut 1 cm shorter (1863 mm) in the second production run. The second run panels came with difficult-to-remove plastic and foil covers on each end, further requiring a fresh cut on both ends.

It is important to have a clean cut on the end of the panel. In general, cutting one of these panels with a saw can introduce microscopic cracks, leaving the finished module more susceptible to developing leaks. The factory cutting of the panels is of unknown quality, so we need to re-cut each
end more carefully. To this end, we purchased a high-quality, low-vibration titanium carbide circular saw blade, as seen in Figure 4.1. Using this blade and a large table saw, we were able to get clean, even cuts on both ends of the panels.

4.1.2 Inner Wall Drilling

Initially, the polycarbonate panels consisted of isolated cells which would be sealed separately if the ends were immediately sealed. As such, each of the inner walls must be pierced at each end to allow water and air to flow between them and to make the water module a single connected volume. This procedure was different for the thick and thin inner walls.

Many of the holes necessary were drilled out using a Dremel high-speed rotary tool. As the holes must be made with some depth into the panel, to allow room for the sealant to fill each cell, the Dremel was necessary to cut at right angles to the cell direction. For the thin walls, a small circular cutter attachment, as shown in the left Figure 4.3, was used to cut a slot
Figure 4.2: Photograph of the pick used to slash thin walls of polycarbonate panel. The walls were slashed from the full insertion depth of the pick to a Dremel-cut slot 2 cm from the end of the panel.

Figure 4.3: Dremel attachments used to cut inner water panel walls. The attachment on the left was used to cut a horizontal slot in the thin walls 2 cm from the end, while the attachment on the right was used to cut two round holes in each internal thick wall 1.5–2 cm from the end.
parallel to the outside edge along the length of each inner wall, at 1.5-2 cm into the body of the panel. A right-angled pick, as shown in Figure 4.2, was then used to pierce the thin wall about 10 cm from the end and then it was drawn back out, slashing a tear in the inner wall and joining to the Dremel-cut slot, providing a T-shaped aperture between each sub-cell. The result can be seen in Figure 4.4.

A different procedure was needed for the thick walls, as they are too thick to be pierced by the picks. Instead, two round holes were cut through each thick wall with a different Dremel tool, pictured on the right of Figure 4.3, consisting of a 6 mm spherical drill at the end of the tool shaft. This bit was then pressed laterally against the thick wall to cut a round hole \( \sim 2 \) cm into the body of the panel. The heat generated by the rapidly-moving tool was sufficient to melt into the polycarbonate and produce a hole, but the resulting molten plastic often cooled into a partial covering of the hole. Fortunately, the physical properties of this plastic were altered sufficiently that these remnants could be removed by de-burring with a flat-
Figure 4.5: Polypropylene channel for water module sealing, showing the channel base and extended walls, along with the Teflon block used to seal the end of the channel. To seal the panel ends, glue was poured into the bottom of the channel, and then the polycarbonate panel was firmly inserted.

Due to structural deficiencies in the molded side cells, one cell on each side of the polycarbonate panel was left disconnected from the rest of the module. This cell remains air-filled throughout the operation of the FGD. After this, the water-filled width was 1760 mm for the original modules with 1” cells, and 1780 mm for the newer modules with narrower cells. The nominal width of the polycarbonate panel, including the air-filled cells, is 1809 mm.

Following the drilling, the cells were cleared of any debris using compressed air, and then the area intended for sealing was cleaned by swabbing the inside of the cells with isopropanol. Once a panel was cut, drilled, and cleaned, it was ready to be sealed. After one end was sealed, a hole was drilled in the glue to allow the interior of the panel to vent during the second sealing.
4.2 Sealing Procedure

The key tool used for sealing the modules was a polypropylene channel of appropriate (~ 1”) width to fit snugly on the end of a polycarbonate panel. The sides of the channel were extended using sheets of polypropylene, and the entire channel assembly was lined with two layers of polyethylene sheeting for each sealing. Two Teflon blocks of appropriate width were set up at appropriate points along the length of the channel to provide a glue reservoir the length of a polycarbonate panel. The assembled channel can be seen in Figure 4.5. This reservoir was marked into quarters to ensure uniformity of glue filling.

Each set of HE 1908 containers contained approximately 1 L of two-part adhesive. This is slightly more than was necessary for sealing one end of a water module. 800 mL of sealant was measured out and mixed in a large plastic beaker with a wooden paint stir-stick. This mixed adhesive was poured into the channel, taking care to pour approximately equal portions into each quarter. The glue was then left to flow through the channel and equalize in level for a short period. After five minutes of levelling, the module was carefully and uniformly lowered into the channel.

Once the module was securely inserted into the sealant-filled channel, it was then secured in the vertical position while the sealant cured. The panel was positioned against the edge of a large table, and bungee cords were used to tie the panel to the table. The tilt of the panel was adjusted until it was fully vertical, and a second table was placed in front of the panel to further prevent tipping. The assembly was then left to cure overnight. Once the glue had cured, we were then able to peel the channel and lining off the end of the module.

4.2.1 Repair and Modification of Seal

While we took precautions to maintain a uniform glue thickness of 1 cm or more through the entire module, the process was still imprecise and imperfect. This was particularly evident when sealing the second end, leaving an airtight vessel that could occasionally push back against the sealant. Thus,
we developed a procedure for adding additional glue to deficient cells.

First, we drill through the existing glue to produce a 5/64” channel into
the panel for each insufficiently-sealed cell. The panel was then hung with
the end under repair facing down, and a small batch of HE 1908 mixed.
The liquid polyurethane was then loaded into a syringe through the plunger
end, being too viscous to be drawn up in the usual way. This syringe was
attached to a large (16 gauge) needle with a blunted end for user safety and
reduced risk of puncturing the sealant.

Having prepared the glue syringe, we then inject sealant into the panel
to fill each cell to a safe depth. The sealant was injected through the full
length of the drill hole to fill it, and then the hole was covered with masking
tape to prevent leakage. The glue was sufficiently viscous to flow slowly
through the hole and be stopped by the tape.

Since humidity or liquid water could interfere with the normal curing
process of the sealant, we must ensure that the interior of the module is
dry. If the module to be repaired had previously held water, the water was
removed by flushing it with dry nitrogen overnight.

### 4.3 Skin Attachment Procedure

The primary difficulties in attaching the skins to the surface of the water
module were to make sure that the glue layer was of uniform thickness and
that no large air bubbles are trapped underneath the skin. An attachment
procedure was devised to control both of these factors, using a granite table
to make sure that the module is flat during the gluing.

The polypropylene skins were processed extensively prior to the attach-
ment procedure. An array of small holes was drilled into the sheet to allow
air to escape during the gluing. One surface of each sheet was sanded and
treated with Mitsui Chemicals Unistole P primer for better adhesion. The
surface of the polycarbonate was also sanded, but it was not treated.

The polypropylene sheets were too small to cover the full size of the
water module, so three pieces were tiled together to make the skin. There
was a 180×122 cm piece, a 63.5×122 cm piece, and a 61cm square piece,
Figure 4.6: Approximate tiling of polypropylene skin pieces within water module outline. Since the polypropylene sheets were too small to cover the full area of the water module surface, they were cut appropriately to cover the surface as well as possible.

The chosen epoxy was very viscous in its uncured form. To spread it evenly over the module surface, grooved rollers were used, similar to the assembly of the XY modules. The pot life of the glue was sufficient to spread it over an entire side at once. Having spread the glue over the surface, the polypropylene sheets were then lowered as smoothly as possible over the surface by hand, and then smoothed additionally by hand.

Once the skins were attached, the entire module was covered with a plastic sheet and aluminum pressure plates placed over the full area of the module. The epoxy was cured for 12-18 hours in this configuration before the module was ready for the next step.

4.4 Plumbing of Modules

For connection to the supply system, the water modules must present standard fittings at both the top and bottom. These fittings are installed on opposite corners of the module such that water must fill the panel both horizontally and vertically before full circulation is established. The method chosen for the final modules after some early prototyping rigidly connects a
Figure 4.7: Pipe fitting on water module, showing the 1/4” brass Swagelok tee embedded in a block of HE 1908 sealant and held in place against sideways pressures by two brass screws. This provides a single 1/4” pipe fitting for attaching the water module to the circulation system.

First, a 1/4” hole is drilled through the sealant and NPT pipe threads cut into its sides. A tee-fitting with one NPT connection and two 1/4” Swagelok connections is then connected to the module, its threads coated in five-minute epoxy for a quick bond between the tee and the module. The two Swagelok connectors are then parallel with the top/bottom surface of the module. The connector facing the inside of the module is plugged with a Swagelok cap, while the outside-facing connector is the outside-accessible fitting of the module.

The Swagelok tee is protected from twisting by the addition of two long screws, one on each side of the neck of the plugged end. Once the tee and screws are in place, the entire assembly except the open fitting is permanently embedded in a block of HE 1908 sealant. The resulting fitting is thoroughly sealed to the body of the water module, mechanically bonded with it, and resistant to torsional strain. On the bottom of each module, a solid block of HE 1908 was cast opposite the fitting so that the module may
Figure 4.8: Photograph of water module strap brackets. Five of these aluminum brackets were glued to the bottom of each water module to hold the five stainless steel straps by which the module is suspended within the FGD and distribute the load on the bottom of the fragile module body.

rest evenly on the floor.

4.5 Mechanical Connection to the FGD

Modules are held in place within the FGD three ways. They are suspended from the top of the dark box by thin stainless steel straps, they are held in place horizontally by pressure from the cover plates, and they are held at the corners by brackets, some padded and some unpadded.

Compatibility in these cases between water modules and XY modules is somewhat difficult due to their different mechanical characteristics. The steel straps are held in place with aluminum brackets on both types of module, but the different shapes require different kinds of bracket. The XY modules have small brackets that bolt into the body of the module between busboards, but it is not possible to bolt a bracket into a water module. The water module brackets are wide to spread the load more evenly, as shown in Figure 4.8, and are glued to the bottom of the module using HE 1908.

The water modules are also narrower and taller than the square XY modules so we need additional parts to fit within the corner brackets. Acrylic bars cut to the correct size are attached to the edges of each water module.
These pieces of acrylic are attached using one of our failed candidate sealants, Tough-Seal 21, which firmly bonds acrylic to polycarbonate.

Each FGD module has spacers made of G10 fibreglass attached to its surface to hold it apart from its neighbours and provide a standard width to the FGD stack. The design of the spacers for the water modules is slightly different to accommodate the greater fragility of the water modules.

While the fully-rigid scintillator modules have square spacers, the spacers chosen for the water modules are instead 2.58x5.16 cm rectangles, allowing them to span multiple cells. This configuration spreads the compression load between multiple walls and reduces the pressure on the hollow module surface. The spacers were attached with double-sided masking tape, multiple layers being used in some circumstances to ensure the module thickness matched the specification. There are thirty of these spacers on each side of the FGD, roughly evenly distributed over the surface.

### 4.6 Prototype Module Construction

A variety of prototype modules were built during the design stage of the water system. These included both partial modules for testing of material properties and full-size modules to test the module construction process.

#### 4.6.1 Scaled-Down Prototypes

In the fall of 2006, two small-scale water module prototypes were built as a proof of concept. These two modules were the full length of a panel, 210 cm, but were only 30 cm wide. These prototypes were sealed using the original first choice sealant, Stycast 1266, and the second was fitted with two side valves for leak testing. A photograph of a narrow prototype can be found in Figure 4.9.

The first narrow prototype was successfully filled with water in September 2006. It was subsequently damaged when it fell over, dragging the vacuum pump behind it off the 2m high shelf and onto the back of the panel. While this severely dented the plastic it does not appear to have breached the outside wall of the polycarbonate panel, making us more optimistic about
Figure 4.9: Narrow water module prototype prior to filling. Prior to the construction of full-sized water modules, these narrow prototypes were built as a proof-of-concept for the module sealing and filling method.

the strength of the final modules.

A replacement was built and successfully held water over a long period of time, including a continuous seven-month stretch from January-July 2007.

4.6.2 Full-Size Prototypes

The first full-size water panel was built in the summer of 2007, after the initial selection of GSP 1339 as the module sealant. One end of the module was sealed with GSP, and then we attempted to fill the end cell with Stycast 1266. A number of difficulties were found with this approach. Initially, the glue was poured into the panel through a funnel, but its viscosity is such that it would not pour sufficiently quickly to fill the cell while it remained liquid. The cell was partially filled with this method and then later finished
filling using a syringe. Some of the adhesive leaked out into adjacent cells through an erroneously-drilled hole in the thick walls.

After this prototype was sealed at the other end with GSP it was filled with water and left filled as a leak test. After approximately ten days of being filled, the module developed a leak. Several more leaks developed over the subsequent days, and while we were able to seal many of them by covering them with a thin G10 sheet, more developed. The majority of these leaks were near the side cell that was filled with Stycast.

Concluding that the Stycast-filled cell was stressing the polycarbonate, we began construction on a second prototype using the GSP sealant. We also adopted the more careful module cutting procedure described in Section 4.1. Unfortunately, the sealant and panel cracked in the mold, developing a crack approximately 6 mm wide and 200 mm long, as seen in Figure 4.10, thus rendering the second prototype unusable. As the panels
are expensive, the construction of full-size prototypes was halted until a solution could be found.

Initial investigation of the GSP curing process suggested that the rigid epoxy contracted while it cured, producing a mechanical stress on the end of the module. This effect was small but it was observed for both the GSP and Stycast epoxies. The cracked module thus appears to have relieved the lateral stress by rupturing at a weak point.

Further investigation of the curing process found that high temperatures were produced as the sealant cured in place. Measurements of GSP and Stycast curing in a cup observed temperatures over 150 °C, above the point where polycarbonate can be damaged by heat. Thus, low contraction and curing temperatures became further criteria for the selection of the water module sealant.

The requirement of low contraction led us to abandon rigid epoxies. The next sealant used on a prototype water module was the epoxy elastomer Tough-Seal 21. We cut the damaged end from the second prototype and sealed it with Tough-Seal for a roughly 3/4-height prototype module, as seen in Figure 4.11.
This third prototype successfully held water for several months, but the Tough-Seal 21 proved difficult to work with, and susceptible to large amounts of water absorption. Thus for the final prototype we switched to a third sealant, HE 1908 polyurethane. This first HE 1908 module succeeded in all tests, and we moved to produce the remaining water modules using the same procedure, beginning in spring 2008. The prototype was included in our final set of nine modules as module 0.

4.7 Production of Water Modules

Once the fourth full-size prototype had been proven sound, full-scale production of water modules was begun. Three water module production runs were done; two of full-size FGD modules and an additional production of smaller modules for the HARPSICHORD detector, which is a scaled-down FGD for use in pion scattering measurements at TRIUMF.

Following the prototyping process, we had five polycarbonate panels on hand. Including the final prototype, this was sufficient to fill the FGD but allowed no spares. These panels were the basis of the summer 2008 production of water modules. One of these modules had a damaged cell on one end that was cut off concurrently with the end cutting. This non-uniformity in width caused some trouble with the production process and measurements of module capacity. The modules produced in this run were given numbers 1-5. A picture of a production water module can be found in Figure 4.12.

At this time, we attempted to acquire an additional stock of panels for the production of spare modules. Unfortunately, our original supplier was no longer available, and the panels were no longer marketed in North America. Eventually, we were able to find a UK supplier, and acquired six additional panels through the generous assistance of the University of Sheffield.

Three further modules were built through the first half of 2009. These modules used the new UK panels, which had a somewhat different cell structure, as seen above in Figure 3.1. This difference and the slight difference in the panel thicknesses required only minor, superficial changes in the pro-
Figure 4.12: Final production water module, sealed with HE 1908 polyurethane. The module is shown here filled with green water to test its watertightness, and has the full set of polypropylene skins attached to demonstrate their effectiveness as a vapour barrier.

During our initial tests of the first production run in the water supply system, small air bubbles were observed in some of the outlet lines. These modules were first checked for any large leaks. Any invisible leaks were then sealed by painting each end of the faulty panels with more sealant, and then pumped down to a -19”Hg vacuum to draw the sealant into any leaks. The repair process was fully successful.

The HARPSICHORD detector is a scaled-down version of the FGD for use in pion scattering experiments at TRIUMF. In 2010 a set of six small (29x32 cm) water modules were produced for use in HARPSICHORD using one of the remaining three polycarbonate panels. These were identical to the larger modules except that the polypropylene skins were bolted into the acrylic bars at the edges rather than glued on.
4.8 Transport to Japan

The water modules were shipped to J-PARC separately from the rest of the FGD. As they were shipped by air, they were well-cushioned to withstand possible turbulence. The crate was cushioned around the edges and layers of foam were inserted between each module and the one beneath it. A picture of the water modules in their crate can be found in Figure 4.13, and a picture of the full set of modules stacked at J-PARC can be found in Figure 4.14. Prior to shipping, the modules were rinsed with a 0.25% Germall Plus disinfectant solution, to inhibit growth in any residual water.

All nine completed water modules were shipped together to Tokai, and arrived on June 25, 2009. They were tested as described in Chapter 5 prior to their use in the FGD.

4.9 Assembly into FGD

After testing the nine modules at J-PARC (Chapter 5), modules 0-4 and 6 were selected as the final modules for FGD2. Modules 5 and 7 were rejected
due to their physical irregularities; module 5 was narrower than all others and module 7 thinner. Module 8 developed a flow problem that we were unable to repair, and was thus also rejected.

An FGD is assembled by stacking modules on a rigid 1” thick steel plate, positioned together with an alignment jig. The dark box frame is then assembled around the stack and attached to the modules, and the whole assembly tilted to vertical. Then the plate is removed and the covers attached to the dark box. The FGD water modules are connected with 1/4” tubing to passthroughs in the corners of the dark box. For simplicity, the modules were stacked such that the panel number increases uniformly following the beam direction.
Chapter 5

Testing of Water Modules

5.1 Module Filling for Tests

For most of these tests, the supply system described in Section 3.5 was unavailable or inappropriate. Instead, a simplified supply system, as seen in Figure 5.1, was used to fill the water modules for various tests.

First, the water module is attached to a vertical support structure. A buffer tank is connected to the outlet (top), and then, through a valve and pressure gauge, connected to a vacuum pump, as shown in Figure 5.2 for a narrow panel. At the inlet (bottom), a hose was connected to a bucket of water, which is kept full through the filling process. The water is then drawn into the module by the vacuum pump. This process takes approximately one hour for a full-size module. To drain the module, the valve at the top is opened, allowing the water to drain naturally into the bucket.

5.2 Testing at TRIUMF

When each module was constructed, we tested the watertightness of each seal by filling the module and monitoring the water level over 1–2 weeks. This test was done twice, once with the module in the usual orientation, and once with it inverted. All production modules passed this test.

Longer-term tests were conducted on the prototype modules. It was
**Figure 5.1:** Simple setup for filling water modules in the absence of the sub-atmospheric pressure circulation system. The pressure in the buffer tank was monitored to protect the module from excessive vacuum during filling.

**Figure 5.2:** Buffer tank and valve assembly, showing a half-filled buffer tank, pressure gauge, vent valve, and pump valve, in order from right to left.
important to test the ability of the prototypes to hold water over a long period of time. The narrow prototype and the final full-size prototype both held water for over six months in our test setup. Some additional long-term tests were done on these modules.

The primary long-term test was the thermal cycling test. For this test, the module was wrapped in an electric blanket and then insulated with additional blankets. The electric blanket was connected to a timer that powered it for twelve out of every twenty-four hours. Temperature and humidity sensors were put within the insulated enclosure and monitored. This setup allowed the module to reach thermal equilibrium with the lab by the end of the blanket-off cycle.

The temperature within the enclosure varied from approximately 22°C to 30°C over the course of a cycle. No significant increase in humidity relative to ambient occurred over the three months of testing, and neither the narrow prototype or the full-size prototype developed leaks or experienced excessive water diffusion.

The seal on the modules is not perfect; both the sealant and the module walls can have water molecules diffuse through them and escape to the environment. The narrow prototype was used primarily for studies of this effect, being observed over the course of more than one year. Initially, it was tested with no skins and we observed a diffusion rate of 1.43 mL/day, by measuring the change in the water level over the course of several months.

We then used the narrow module to test diffusion mitigation methods. The polypropylene skins lower the water diffusion rate, and their effect on the narrow prototype was to decrease the diffusion to (1.0 ± 0.1) mL/day. This, when scaled to the size of the full FGD, was (68 ± 6.8) mL/day, which was judged to be unacceptable, requiring a dry air flow rate of 5 L/min to prevent increases in humidity.

The other mitigation method we tested was to wrap the module in aluminized Mylar. The narrow water module was wrapped in Mylar with a small area near the top left open to view the water level. A photograph of the wrapped prototype can be found in Figure 5.3. This was then left for several months and a diffusion rate of (0.31 ± 0.02) mL/day was observed,
Figure 5.3: Narrow water module wrapped in aluminized Mylar for tests of a possible vapour control method. A small area of the module was left exposed for water level measurements, which were carried out over the course of several months. While the Mylar decreased the water diffusion through the surface of the module, tests with a full-size module showed acceptable diffusion rates without the difficult-to-scale Mylar wrap.

which is $(3.5 \pm 0.2) \text{ mL/day}$ scaled to a full-size module. This was a an improvement of a factor of 3 over the polypropylene-only rate and we briefly considered wrapping the full-size water modules in aluminized Mylar.

The estimates of the water diffusion rate for the full FGD were based on scaling the narrow module to a full module by the ratio of their widths. There are several possible issues with this, including the contributions of the sides, the different length of the narrow prototype, and a different choice of sealant between the early prototype and the final module. Once a full-size production module was available with polypropylene skins, it was left filled for approximately 1 month at the end of 2008. Over this time, no drop in water level was observable, giving an upper limit on the water diffusion for a single module of 1.3 mL/day.

This was substantially less than our estimate based on the narrow panel,
even with the aluminized Mylar wrap. Scaling to an FGD of six water modules, this requires less than 1 L/min of dry air to clear out, which is quite practical to supply, and does not require the invasive Mylar wrap.

5.3 Initial Testing in Japan

After the water modules arrived at J-PARC on June 25 2009, as described in Section 4.8, a number of tests were conducted to ensure their continued integrity. The most basic test done was the vacuum test. Each module was sealed at one end with a Swagelok plug, and the other end was connected to a combination pressure gauge/valve assembly, as pictured in Figure 5.4. We then pumped down the inside of the module to approximately -20” Hg (-0.668 atm) and marked the position of the pressure indicator on the gauge. The module was then left at vacuum overnight (or longer) and the increase in pressure measured. An increase of less than 0.5” Hg over 24 hours was considered acceptable.

The initial tests of the modules immediately following their unpacking was successful for all modules. The test was repeated following the assembly
of FGD2 and successfully found some loose connections between the FGD modules and the frame of the dark box.

5.4 Thickness and Volume Testing

In the second stage of the water module testing in Tokai, we tested their ability to hold water without leaking over the course of a few days. These tests provided us the opportunity to make detailed measurements of the capacity of the water modules and assess whether the water module thickness varies significantly over the height of the panel.

When we filled the modules for these tests, we measured the mass of water we introduced to each module using a bathroom scale. The mass was measured for the module as a whole and in steps. Each step introduced approximately 18 kg of water into the module. After each step, the water within the module was allowed to level out, and the depth of the water recorded at three points across its width. During this process, the pressure within the module was monitored to maintain a consistent pressure for all level measurements.
Total Areal Density of Water in Modules

Figure 5.6: Total areal densities of water in all nine filled modules (dashed lines are ±1σ on the average). The modules all have relatively similar density except module 7, which was rejected due to the clear difference in water capacity.

From this, we can measure the variation of the areal density of the water as a function of water depth. We expect that, as more water is added at a constant top pressure, the pressure at the bottom of the module should increase due to the weight of the water, causing the sides of the panel to bow outwards slightly. This variation of areal density with depth is shown in Figure 5.5. From the fit shown, we see that the average areal density increases with depths at $1.1 \times 10^{-4} \text{g/cm}^2$ per cm of additional depth.

Using the measurement of the total water volume for each module, we can calculate the average areal density of water in the module. Results are shown in Figure 5.6. Module 7, which was made with the unusually thin polycarbonate panel, is a clear outlier, while the remaining eight modules fall within a relatively narrow band.

5.4.1 Pressure Dependence

For some of the modules, we also measured the dependence of the water level on the pressure. As the pressure decreases, the walls contract, raising the water level and lowering the areal density. After filling the module, we
<table>
<thead>
<tr>
<th>Panel</th>
<th>Slope (%/atm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>no data</td>
</tr>
<tr>
<td>1</td>
<td>1.03 ± 0.10</td>
</tr>
<tr>
<td>2</td>
<td>1.16 ± 0.10</td>
</tr>
<tr>
<td>3</td>
<td>no data</td>
</tr>
<tr>
<td>4</td>
<td>1.10 ± 0.10</td>
</tr>
<tr>
<td>5</td>
<td>0.60 ± 0.10</td>
</tr>
<tr>
<td>6</td>
<td>0.91 ± 0.10</td>
</tr>
<tr>
<td>7</td>
<td>0.45 ± 0.09</td>
</tr>
<tr>
<td>8</td>
<td>1.40 ± 0.09</td>
</tr>
<tr>
<td>Mean</td>
<td>0.96 ± 0.31</td>
</tr>
</tbody>
</table>

Table 5.1: Variation in water level with pressure (fit from data between -20” Hg and atmosphere). Two modules were tested with an earlier process that omitted this step, the mean of the other seven slopes was used for their correction.

made level measurements at a variety of pressures, with no connection to the water source. This was done over a short period of time so differences in evaporation are negligible. Treating the variation of level with pressure as linear, we fit the rate of change of level with pressure for each module, as shown in Table 5.1. The mean slope was used for pressure correction of the modules with no data.

The values from Figure 5.6 were all corrected to a nominal pressure of -20” Hg using these slopes.
Chapter 6

Elemental Composition

6.1 Introduction

A detailed accounting of the elemental composition of the FGD is important for a variety of reasons. For example, knowing the composition of the modules based on their design and on measurements of their individual components allows us to accurately simulate them in our Monte Carlo.

An important use of the elemental composition comes when considering the water subtraction analysis (Part IV). We must know the composition of the mass in each fiducial volume, so that we know what scaling to use to subtract the FGD1 rate from the FGD2 rate.

This chapter will concentrate on the elemental composition of the empty water modules. The contribution of the water is known, and can be measured separately as described in Section 5.4. This analysis only considers the elements that occur in the main body of the water module, i.e., the sealant and FGD attachment components are neglected.

Provided a sufficiently stringent fiducial volume cut is used, this elemental composition analysis is sufficient for treatment of the detector in physics analyses. Given the nominal seal thickness of 2 cm and a small air gap at the top of the module, this requires that the fiducial volume cut be placed no more than 905 mm from the centre of the FGD along the vertical direction. The water volume is narrower than it is high, with the narrowest in-use
modules having a water width of 1760 mm, or 880 mm from the centre to the edge.

The elemental composition of the FGD XY modules was calculated by Scott Oser[59]. They are summarized in Table 6.1. Like the water module analysis, the XY module analysis only considers components in the centre of the module, and ignores the stainless steel straps.
### Table 6.1: Elemental composition of the components of a typical XY layer, in g/cm² of each element. (From Oser, 2010[59])

<table>
<thead>
<tr>
<th>Component</th>
<th>C</th>
<th>O</th>
<th>H</th>
<th>Ti</th>
<th>Si</th>
<th>N</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scintillator bars</td>
<td>1.7651 ± 0.0067</td>
<td>0.0248 ± 0.0039</td>
<td>0.1468 ± 0.0006</td>
<td>0.0355 ± 0.0059</td>
<td>0.0010 ± 0.00004</td>
<td>1.973 ± 0.0104</td>
<td></td>
</tr>
<tr>
<td>G10</td>
<td>0.0196 ± 0.0015</td>
<td>0.0331 ± 0.0001</td>
<td>0.0034 ± 0.0018</td>
<td>0.0218 ± 0.0043</td>
<td>0.0013 ± 0.0013</td>
<td>0.079</td>
<td></td>
</tr>
<tr>
<td>Plexus MA590</td>
<td>0.0484 ± 0.0060</td>
<td>0.0215 ± 0.0027</td>
<td>0.0065 ± 0.0008</td>
<td>0.0009 ± 0.0001</td>
<td>0.000002</td>
<td>0.0774 ± 0.0096</td>
<td></td>
</tr>
<tr>
<td>fiber</td>
<td>0.0155</td>
<td>0.000002</td>
<td>0.0013</td>
<td>0.000002</td>
<td>0.000002</td>
<td>0.0169</td>
<td></td>
</tr>
<tr>
<td>XY module</td>
<td>1.849 ± 0.0092</td>
<td>0.0794 ± 0.0048</td>
<td>0.1579 ± 0.0021</td>
<td>0.0355 ± 0.0059</td>
<td>0.0218 ± 0.0043</td>
<td>0.0031 ± 0.0012</td>
<td>2.147 ± 0.0144</td>
</tr>
</tbody>
</table>
Table 6.2: Panel types and positions within the FGD of built water modules. The panel positions inside the FGD are numbered from most upstream to most downstream, and are in the same order as the module serial numbers for simplicity.

<table>
<thead>
<tr>
<th>Module #</th>
<th>Position</th>
<th>Panel</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>Old-style</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>Old-style</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>Old-style</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>Old-style</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>Old-style</td>
</tr>
<tr>
<td>5</td>
<td>Spare</td>
<td>Old-style</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>New-style thick</td>
</tr>
<tr>
<td>7</td>
<td>Spare</td>
<td>New-style thin</td>
</tr>
<tr>
<td>8</td>
<td>Unusable (defective drain)</td>
<td>New-style thick</td>
</tr>
</tbody>
</table>

6.2 Materials

The elemental composition analysis is based on separate analyses of the various components of the water modules.

6.2.1 Polycarbonate

The polycarbonate panels used to build the passive water modules have a complicated internal structure, as shown in Figure 3.1. The panels are described in detail in Section 3.3.1. Table 6.2 describes which modules were built with which type of panel, and where they sit in the FGD.

Common polycarbonate (e.g. Lexan) has the chemical formula C\(_{16}\)H\(_{14}\)O\(_3\). This corresponds to these percentages by mass: 75.7% C, 18.8% O, 5.5% H. A sample of each of the three types of panel was cut to measure the average areal density of the material. The mass was measured using a high-precision Mettler scale and thus the mass measurements have a small (< 0.1%) error.

Since the panels have a periodic internal structure, it is important that the samples measured to determine the areal density have an integer number of cells. The two samples of the new Palram panels were measured to have a length consistent with an integer multiple of the cell width, so the
uncertainty on this width measurement was directly translated into an uncertainty on the density, giving $333 \pm 6$ mg/cm$^2$ for the thick panel and $314 \pm 5$ mg/cm$^2$ for the thin panel. The Sunlite sample, however, was measured to have $n = 7.87 \pm 0.04$ cells, and so the density needed to be corrected.

The correction was based on a parametrization of the mass of the sample as a sum of eight vertical thick walls, plus two thick walls and three thin walls per cell. Since the cells are square, the thick outer walls were assumed to have the same mass as the thick vertical walls, so the mass of the sample can be written as

$$m_{\text{total}} = (\lceil n \rceil + 2n)m_{\text{thick}} + 3nm_{\text{thin}} = (23.74 \pm 0.08)m_{\text{thick}} + (23.62 \pm 0.08)m_{\text{thin}}$$

To determine the values of $m_{\text{thick}}$ and $m_{\text{thin}}$, I measured the thicknesses of the thin and thick walls, finding that the thin walls are $0.18 \pm 0.01$ mm thick and the thick walls were $0.92 \pm 0.01$ mm thick, giving a thickness ratio of $0.20 \pm 0.01$. Assuming that the volume density of the two parts of the panel are the same, I calculated that $m_{\text{thick}} = (4.15 \pm 0.04)$g and thus the missing $0.13 \pm 0.04$ cells should weigh $1.39 \pm 0.34$ g. I then calculated the corrected density as $342 \pm 3$ mg/cm$^2$. As the sample of old-style panel was approximately twice the size of the other samples, the errors from the measurement process are reduced compared to those from the other samples.

### 6.2.2 Polypropylene

Polypropylene has the monomer formula C$_3$H$_6$. Each module has two skins of 1/32” nominal thickness polypropylene, as described in Section 3.3.2. A variety of samples of the polypropylene sheeting were cut from spare pieces to determine the areal density of the polypropylene. The densities of these samples are plotted in Figure 6.1. The scatter in these points is substantially larger than the uncertainty on each point, so the scatter was used as the error in the PP mass for the elemental abundance calculations. Thus, the measured areal density of polypropylene skin material is $72.4 \pm 2.3$ mg/cm$^2$. 

89
Figure 6.1: Areal densities of polypropylene sheet samples, showing the mean value of 72.4 mg/cm$^2$ used for the elemental composition and mass analysis.

6.2.3 Skin Epoxy (CLR 1390/CLH 6025)

The elemental composition of the skin epoxy was given in an email from the manufacturer as 43% C, 30% O, 5% H, 2% N, 8% Mg, 1% Ca, and 11% Si. The difficulty in calculating the contribution in this component is instead based on the difficulty in determining how much was actually used. The first skin gluing was measured to consume 1317 g of epoxy over a 181x181 cm area, but this test was never repeated. The total amount of epoxy mixed for later gluings was $\sim$1800 g, but not all of this was usable. The value used for this analysis was $1400 \pm 200$ g, giving a density of $43 \pm 6$ mg/cm$^2$.

6.2.4 G10 Spacers

There are thirty spacers on each side of a water module, measuring 2.58x5.16 cm in area. The total area covered by these spacers is 399 cm$^2$, which is 1.15% of the nominal 34745 cm$^2$ panel area. The contribution of these spacers will be averaged over the total area of the panel.

These spacers were attached to the panels with varying amounts of
double-sided masking tape, and, in the case of the new-style thin panel, posterboard. The old-style panels had two layers of masking tape on one side and one layer on the other. The thick new-style panels had two layers of masking tape on one side and three on the other, and the thin new-style panels had four layers of tape on one side and two layers of tape and one layer of posterboard on the other side.

The composition of G10 was taken from a CMS memo\cite{60} as 26.8% C, 41.7% O, 6.6% H, and 22.0% Si. The tape and posterboard were assumed to be composed of cellulose (C$_{6}$H$_{10}$O$_{5}$).

\section*{6.3 Analysis of Composition}

The elemental composition of the panels was investigated based on the densities and elemental compositions of the four components. Six elements were considered separately: carbon, oxygen, hydrogen, magnesium, silicon, and titanium. All other mass was considered together as ‘Other’, which includes nitrogen, calcium, and chlorine. The overall composition of the empty panel was calculated, then broken down into three segments: XY module-like, water-like, and remnant mass. The water density is considered with the module both full and empty.

First, the water module density and elemental composition was calculated from the separate measurements for each component. Care was taken to correctly evaluate the correlations between the elements. Having the final composition and covariance matrix, we can then calculate our three segments.

The total XY module-like density was chosen to use all of the carbon in the water module. We scale the density of the XY modules by the ratio of the carbon densities of the XY and water modules to find the water module ‘plastic’ density $\rho_p$, following Equation \ref{equation:6.2}.

$$\rho_s = \frac{\rho_{C_{w}}}{\rho_{C_{xy}}} \cdot \rho_{XY}$$ (6.2)

Any remaining oxygen is then paired with an appropriate mass of hy-
drogen to form a ‘virtual’ water density $\rho_w$, as in Equation 6.3,

$$\rho_w = f_O \left( \rho_{Ow} - \rho_{Oxy} \frac{\rho_{Cw}}{\rho_{Cxy}} \right)$$  \hspace{1cm} (6.3)

where $f_O$ is the fraction of the density of water due to oxygen. In this way, we extract an ‘XY density’ and a ‘virtual water density’ from the composition of each water module. If we subtract these densities from the total water module density, we find the ‘remnant’ density $\rho_r$. The water module design was planned so as to minimize this remnant density.

6.4 Results
<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>O</th>
<th>H</th>
<th>Mg</th>
<th>Si</th>
<th>Ti</th>
<th>Other</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polycarbonate panel</td>
<td>259.0 ± 2.3</td>
<td>64.1 ± 0.6</td>
<td>18.9 ± 0.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>342.0 ± 3.0</td>
</tr>
<tr>
<td>Polypropylene skins</td>
<td>124.1 ± 3.9</td>
<td>0</td>
<td>20.7 ± 0.7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>144.8 ± 4.6</td>
</tr>
<tr>
<td>Skin Epoxy</td>
<td>36.8 ± 5.1</td>
<td>25.5 ± 3.6</td>
<td>4.5 ± 0.6</td>
<td>6.8 ± 0.9</td>
<td>9.7 ± 1.4</td>
<td>0</td>
<td>2.8 ± 0.4</td>
<td>86.1 ± 12.0</td>
</tr>
<tr>
<td>Spacers</td>
<td>2.1 ± 0.0</td>
<td>3.1 ± 0.0</td>
<td>0.5 ± 0.0</td>
<td>0</td>
<td>1.5 ± 0.0</td>
<td>0</td>
<td>0.2 ± 0.0</td>
<td>7.4 ± 0.0</td>
</tr>
<tr>
<td>Total (without water)</td>
<td>422.0 ± 6.9</td>
<td>92.7 ± 3.6</td>
<td>44.5 ± 0.9</td>
<td>6.8 ± 0.9</td>
<td>11.2 ± 1.4</td>
<td>0</td>
<td>3.0 ± 0.4</td>
<td>580.2 ± 13.2</td>
</tr>
<tr>
<td>Remnant</td>
<td>0.0 ± 7.2</td>
<td>0.0 ± 5.2</td>
<td>-0.9 ± 1.1</td>
<td>6.8 ± 0.9</td>
<td>6.2 ± 1.7</td>
<td>-8.1 ± 1.3</td>
<td>2.3 ± 0.5</td>
<td>6.3 ± 14.2</td>
</tr>
</tbody>
</table>

**Table 6.3:** Elemental composition of empty old-style panels (mg/cm²) (errors in totals column include correlation terms)

<table>
<thead>
<tr>
<th>C</th>
<th>O</th>
<th>H</th>
<th>Mg</th>
<th>Si</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>1.000</td>
<td>0.791</td>
<td>0.976</td>
<td>0.748</td>
<td>0.748</td>
</tr>
<tr>
<td>O</td>
<td>1.000</td>
<td>0.697</td>
<td>0.988</td>
<td>0.988</td>
<td>0.748</td>
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<tr>
<td>H</td>
<td>1.000</td>
<td>0.678</td>
<td>0.678</td>
<td>0.748</td>
<td>0.748</td>
</tr>
<tr>
<td>Mg</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Si</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
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<tr>
<td>Other</td>
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<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

**Table 6.4:** Correlation coefficients for empty old-style panels (symmetrical bottom half omitted)
<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>O</th>
<th>H</th>
<th>Mg</th>
<th>Si</th>
<th>Ti</th>
<th>Other</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polycarbonate panel</td>
<td>252.2 ± 4.5</td>
<td>62.5 ± 1.1</td>
<td>18.4 ± 0.3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>333.0 ± 6.0</td>
</tr>
<tr>
<td>Polypropylene skins</td>
<td>124.1 ± 3.9</td>
<td>0</td>
<td>20.7 ± 0.7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>144.8 ± 4.6</td>
</tr>
<tr>
<td>Skin Epoxy</td>
<td>36.8 ± 5.1</td>
<td>25.5 ± 3.6</td>
<td>4.5 ± 0.6</td>
<td>6.8 ± 0.9</td>
<td>9.7 ± 1.4</td>
<td>0</td>
<td>2.8 ± 0.4</td>
<td>86.1 ± 12.0</td>
</tr>
<tr>
<td>Spacers</td>
<td>2.2 ± 0.0</td>
<td>3.3 ± 0.0</td>
<td>0.5 ± 0.0</td>
<td>0</td>
<td>1.5 ± 0.0</td>
<td>0</td>
<td>0.2 ± 0.0</td>
<td>7.7 ± 0.0</td>
</tr>
<tr>
<td>Total (without water)</td>
<td>415.3 ± 7.9</td>
<td>91.2 ± 3.7</td>
<td>44.0 ± 1.0</td>
<td>6.8 ± 0.9</td>
<td>11.2 ± 1.4</td>
<td>0</td>
<td>3.0 ± 0.4</td>
<td>571.6 ± 14.2</td>
</tr>
<tr>
<td>Remnant</td>
<td>0.0 ± 8.2</td>
<td>0.0 ± 5.3</td>
<td>-0.7 ± 1.2</td>
<td>6.8 ± 0.9</td>
<td>6.3 ± 1.7</td>
<td>-8.0 ± 1.3</td>
<td>2.3 ± 0.5</td>
<td>6.8 ± 15.1</td>
</tr>
</tbody>
</table>

**Table 6.5:** Elemental composition of empty new-style thick panels (mg/cm²) (errors in totals column include correlation terms)

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>O</th>
<th>H</th>
<th>Mg</th>
<th>Si</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>1.000</td>
<td>0.792</td>
<td>0.957</td>
<td>0.649</td>
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<td>0.649</td>
</tr>
<tr>
<td>O</td>
<td>1.000</td>
<td>0.720</td>
<td>0.953</td>
<td>0.953</td>
<td>0.953</td>
<td></td>
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<tr>
<td>H</td>
<td>1.000</td>
<td>0.647</td>
<td>0.647</td>
<td>0.647</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mg</td>
<td></td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
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<td>Si</td>
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<tr>
<td>Other</td>
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<td></td>
<td></td>
<td></td>
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<td>1.000</td>
</tr>
</tbody>
</table>

**Table 6.6:** Correlation coefficients for empty new-style thick panels (symmetrical bottom half omitted)
<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>O</th>
<th>H</th>
<th>Mg</th>
<th>Si</th>
<th>Ti</th>
<th>Other</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polycarbonate</td>
<td>237.8 ± 3.8</td>
<td>58.9 ± 0.9</td>
<td>17.3 ± 0.3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>314.0 ± 5.0</td>
</tr>
<tr>
<td>Polypropylene skins</td>
<td>124.1 ± 3.9</td>
<td>0</td>
<td>20.7 ± 0.7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>144.8 ± 4.6</td>
</tr>
<tr>
<td>Skin Epoxy</td>
<td>36.8 ± 5.1</td>
<td>25.5 ± 3.6</td>
<td>4.5 ± 0.6</td>
<td>6.8 ± 0.9</td>
<td>9.7 ± 1.4</td>
<td>0</td>
<td>2.8 ± 0.4</td>
<td>86.1 ± 12.0</td>
</tr>
<tr>
<td>Spacers</td>
<td>2.5 ± 0.0</td>
<td>3.6 ± 0.0</td>
<td>0.5 ± 0.0</td>
<td>0</td>
<td>1.5 ± 0.0</td>
<td>0</td>
<td>0.2 ± 0.0</td>
<td>8.4 ± 0.0</td>
</tr>
<tr>
<td>Total (without water)</td>
<td>401.2 ± 7.5</td>
<td>88.0 ± 3.7</td>
<td>43.0 ± 0.9</td>
<td>6.8 ± 0.9</td>
<td>11.2 ± 1.4</td>
<td>0</td>
<td>3.0 ± 0.4</td>
<td>553.3 ± 13.8</td>
</tr>
<tr>
<td>Remnant</td>
<td>0.0 ± 7.8</td>
<td>0.0 ± 5.2</td>
<td>-0.2 ± 1.1</td>
<td>6.8 ± 0.9</td>
<td>6.5 ± 1.6</td>
<td>-7.7 ± 1.3</td>
<td>2.4 ± 0.5</td>
<td>7.8 ± 14.7</td>
</tr>
</tbody>
</table>

**Table 6.7:** Elemental composition of empty new-style thin panels (mg/cm²) (errors in totals column include correlation terms)

<table>
<thead>
<tr>
<th>C</th>
<th>O</th>
<th>H</th>
<th>Mg</th>
<th>Si</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>1.000</td>
<td>0.791</td>
<td>0.963</td>
<td>0.685</td>
<td>0.685</td>
</tr>
<tr>
<td>O</td>
<td>1.000</td>
<td>0.712</td>
<td>0.967</td>
<td>0.967</td>
<td>0.967</td>
</tr>
<tr>
<td>H</td>
<td>1.000</td>
<td>0.659</td>
<td>0.659</td>
<td>0.659</td>
<td>0.659</td>
</tr>
<tr>
<td>Mg</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Si</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Other</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 6.8:** Correlation coefficients for empty new-style thin panels (symmetrical bottom half omitted)
<table>
<thead>
<tr>
<th>Panel Type</th>
<th>XY Module</th>
<th>Virtual Water</th>
<th>Correlation</th>
<th>Remnant</th>
</tr>
</thead>
<tbody>
<tr>
<td>old-style</td>
<td>490.0 ± 9.0</td>
<td>84.0 ± 4.0</td>
<td>0.652</td>
<td>6.3 ± 14.2</td>
</tr>
<tr>
<td>new-style thick</td>
<td>482.3 ± 10.0</td>
<td>82.6 ± 4.1</td>
<td>0.669</td>
<td>6.8 ± 15.1</td>
</tr>
<tr>
<td>new-style thin</td>
<td>465.9 ± 9.5</td>
<td>79.7 ± 4.0</td>
<td>0.668</td>
<td>7.8 ± 14.7</td>
</tr>
</tbody>
</table>

**Table 6.9:** XY and virtual water areal densities of the empty panels (mg/cm²), with the remaining density of matter that does not have either an XY module-like or water-like elemental composition.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>FGD1 Areal Density</td>
<td>30.0 ± 0.2</td>
</tr>
<tr>
<td>FGD2 Plastic Areal Density</td>
<td>15.8 ± 0.1</td>
</tr>
<tr>
<td>FGD2 Water Areal Density</td>
<td>13.8 ± 0.0</td>
</tr>
</tbody>
</table>

**Table 6.10:** Total areal densities of the interior regions of the FGDs as built and filled with water, splitting FGD2 into FGD1-like and water-like components. (g/cm²)

The elemental composition of the three types of empty water modules can be found in Tables 6.3, 6.5, and 6.7. The correlations for the three types can be found in Tables 6.4, 6.6, and 6.8. In each of these cases, the remnant density is consistent with zero and has a central value between 1.0–1.5% of the total density central value.

A summary of the components of each of the three types of water module can be found in Table 6.9 and Table 6.10 describes the areal densities of the FGD fiducial volumes, and Table 6.11 contains the total fiducial masses, broken down between plastic and water in FGD2.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>FGD1 Mass</td>
<td>831.0 ± 5.6</td>
</tr>
<tr>
<td>FGD2 Plastic Mass</td>
<td>438.1 ± 2.7</td>
</tr>
<tr>
<td>FGD2 Water Mass</td>
<td>383.3 ± 0.6</td>
</tr>
<tr>
<td>FGD Plastic Ratio</td>
<td>0.527 ± 0.005</td>
</tr>
</tbody>
</table>

**Table 6.11:** FGD fiducial volume masses, including target water (kg)
Part III

FGD-Only Track
Reconstruction
Chapter 7

Principles of FGD Reconstruction

7.1 Overview of FGD Reconstruction

The goal of the FGD reconstruction is to find the tracks left by particles as they traverse the FGD. This process is largely determined by the structure of the FGD and its place within the larger ND280 detector.

The input to the FGD reconstruction is a set of FGD hits, each of which has a time, a deposited charge, and a hit bar number. The bar number provides a 2D position for the hit, in either the XZ or YZ plane. The third coordinate corresponds to the extended dimension of the bar and is thus indeterminate. To remove any hits due to noise in the detector, only hits with a measured charge greater than 5 photoelectrons are used for the reconstruction. In addition, we have the set of reconstructed tracks in the TPCs, which are reconstructed prior to the FGD reconstruction.

The first step in the FGD reconstruction uses the added information from the TPC to reconstruct tracks that pass through both an FGD and a TPC. The TPC tracks are extrapolated into the FGD and matched to FGD hits that fall along the extrapolated path. Further details on the TPC reconstruction and TPC/FGD matching can be found below in Section 10.2.

Once the relevant hits have been matched to TPC tracks, a second re-
construction stage works on the remaining hits to reconstruct FGD-only tracks. This stage is referred to as the FGD isolated reconstruction, or “FGD isorecon”. The isorecon consists in turn of two stages: a pattern recognition stage run separately on the two projections, and a “3D matching” stage which pairs up tracks in the two projections to make tracks with full three-dimensional path information.

The FGD acquires data over a relatively long interval (10 μs) to capture delayed activity from muon decays. We would like to treat delayed activity separately from the initial beam-interaction activity, and also to separate interactions from different beam bunches, so the FGD reconstruction is performed separately on one or more time bins. These bins are selected through the simple method of sorting all FGD hits by time and placing bin divisions any time the time interval between consecutive hits exceeds 100 ns.

7.2 The Role of FGD Isorecon

There are several goals for the use of FGD isorecon information. For the most part, FGD-only tracks are short tracks; most long tracks, such as muon tracks, will leave the FGD and enter the TPC. As such, the main role for the FGD-only tracks is in distinguishing neutrino interaction topologies. Seeing one or more pions in the FGD that originate at the same vertex as the muon generally rules out the possibility of a CCQE interaction, and we can with sufficient statistics distinguish between different types of non-QE interactions by examining the pion tracks.

Additionally, in the long term we plan to find the tracks of recoil protons in the FGD using the FGD isorecon. Seeing the proton improves our vertex finding and improves our knowledge of the neutrino interaction kinematics. However, the range of protons in the FGD is quite short; many of them are too short to hit the three X-bars and three Y-bars currently necessary to reconstruct a 3D isorecon track. The current reconstruction does not attempt to solve this problem, preferring to improve the reconstruction efficiency for longer tracks.

Another use for the FGD isorecon is for trajectories at a high angle to
the beam axis. These trajectories will exit through the side of the FGD and will not enter the TPC, and thus will not be found by the TPC/FGD matching despite being long. While it is possible to match these tracks to activity in the barrel ECAL that surrounds the FGDs on the outside, this activity gives far less information than the 3D tracking of the TPC, making it impractical to use for seeding. As such, the FGD isorecon is used to find these ‘sideways’ tracks.

### 7.3 Preexisting FGD Pattern Recognition

The original pattern recognition for the FGD isorecon was based on a cellular automaton called SciBar Cellular Automaton (SBCAT)[61]. The SBCAT algorithm builds from local connections between hit clusters in adjacent layers to find the longest available continuous tracks. First, the FGD hits are clustered together in groups of up to three hits in a single layer. These clusters are then used together as the ‘hits’ for the remainder of the pattern recognition.

Next, the algorithm constructs the set of all available ‘segments’ joining together clusters in successive FGD layers. Segments are built from clusters in adjacent layers, or next-to-adjacent layers if no intervening hits exist in the adjacent layer, if the distance between the two clusters is within a given threshold. All valid pairs of clusters are added to the set of segments; a cluster can be part of arbitrarily many segments.

Once the set of segments is constructed, the algorithm begins to join together segments into tracks. As each segment contains hit clusters from two different layers, the starting point is with pairs of segments that share one of their clusters. These are joined together if the result is sufficiently collinear. A graph of joined segments is built, and then the algorithm selects the longest connected paths through this graph as the track.

The layer-orientation of the SBCAT algorithm can cause problems for particle tracks oriented more than 45 degrees from the z-axis, as these tracks will traverse too few layers in comparison to their path length within the layer. As such, the FGD reconstruction runs SBCAT twice, once with the
layer orientation matching with the physical XY modules of the FGD, and again rotated 90 degrees so that the layers are now the 192 X or Y bars and the bars in all the modules at a given transverse coordinate are taken to be in the same layer.

7.4 **Hough and Radon Transforms**

Both the Hough and Radon transforms are mathematical transformations to the data which reduce the track finding problem in the detector space to a cluster finding problem in a transform space. They both do this by considering a set of straight lines in the detector and assigning a value to each, related to the number of FGD hits along the line.

A major expected benefit for a Hough or Radon transform reconstruction over SBCAT is in the angular dependence of its efficiency. SBCAT’s layer-based structure gives its output a preferred direction, while the Hough and Radon transforms are rotationally symmetric and should have similar efficiency in all directions.

Additionally, the Hough and Radon transform reconstructions have the possibility of using vertex information from the TPC/FGD matching to ‘seed’ the reconstruction of short tracks. While this has not yet been implemented successfully, it remains an idea that can be integrated smoothly with the concepts of the Hough and Radon transforms.

7.5 **The Hough Transform**

The Hough transform is a mathematical construct that relates points and lines in the plane to lines and points, respectively, in a separate Hough plane. The coordinates in the Hough plane correspond to the parameters of a straight line in the original plane. For pattern recognition, we consider the source as a set of \((x, y)\) points, and use the transform to find lines, or more generally, paths built of these points.
Figure 7.1: Illustration of Hough transform parametrization, where the red line is described by the two polar coordinates \((r, \theta)\) of the normal vector to the line from the origin.

### 7.5.1 Conventional Parametrization

There are practical difficulties with using the traditional slope/intercept parametrization of a 2D line in a Hough transform. We have a finite area of interest and would like to only consider lines that pass through this area in a practical implementation. However, lines passing through a finite area can still have arbitrarily large slopes and intercepts, necessitating an unbounded Hough space.

An alternative, polar parametrization has the virtue of representing all lines traversing a finite area of the plane in a bounded Hough space. We express the equation of the line as

\[
r = x \cos \theta + y \sin \theta
\]  

(7.1)

where \(r\) and \(\theta\) are fixed and correspond to the polar coordinates of the line’s closest approach to the origin. This parameterization is illustrated in Figure 7.1. The resulting parameter space is bounded on three sides, as all lines can be expressed with \(r \in [0, \infty)\) and \(\theta \in [-\pi, \pi)\). For a bounded area, we can calculate a maximum \(r\), giving us a bounded Hough space.

If we have a fixed pair of line parameters \((r, \theta)\), we can write the coordi-
nates of the line described in Equation 7.1 parametrically in \( s \) as

\[
x(s) = s \sin \theta + r \cos \theta \quad \text{(7.2)}
\]

\[
y(s) = -s \cos \theta + r \sin \theta \quad \text{(7.3)}
\]

### 7.5.2 Lines in Hough Space

We can see from Equation 7.1 that the set of points in the Hough space corresponding to all lines passing through a given point \((x_0, y_0)\) in the original plane is a sinusoidal curve given by

\[
r(\theta) = x_0 \cos \theta + y_0 \sin \theta \quad \text{(7.4)}
\]

The intersection of two such curves in Hough space gives the parameters of the line that joins the two underlying points in real space. If we denote these two points as \((x_0, y_0)\) and \((x_1, y_1)\), Equation 7.4 gives the intersection at

\[
\theta = \arctan \frac{x_0 - x_1}{y_1 - y_0} \quad \text{(7.5)}
\]

and

\[
r = \frac{x_0 y_1 - y_0 x_1}{\sqrt{(x_0 - x_1)^2 + (y_1 - y_0)^2}} \quad \text{(7.6)}
\]

If we have multiple points that are roughly collinear, the intersection points of their respective Hough curves will form a cluster around the best-fit line passing through those points. Thus, the Hough transform turns the track-finding problem into a cluster-finding problem, amenable to a number of simple solutions. An illustration of this phenomenon can be found in Figure 7.2. The three roughly collinear red points produce similar red lines which cluster in Hough space, while the black lines connecting other points are substantially different and will be distant in Hough space.

### 7.6 Implementation of the Hough Transform

The Hough transform implemented for the FGD reconstruction used a histogram binning to find these clusters in Hough space. A structure of bins
in \((r, \theta)\) space was defined, and then the intersection point corresponding to each pair of hits was calculated and added to the corresponding bin. Once the transform is complete, the hits corresponding to the points in each bin can be considered a track. An illustration of this process can be found in Figure 7.3. The three lines in the coordinate space figure are reflected by three clear peaks in the Hough transform space, while accidental coincidences between lines and with noise points provide a diffuse background.

As implemented, the Hough transform extracts multiple tracks from the Hough histogram through an iterative approach. First, the most likely track is selected from the transform, as defined by the bin with the most hits. A track object is created using these hits, and the hits are removed from consideration by the Hough transform. Then, the transform is re-run on the remaining hits and the process repeats until either there are no hits remaining, or the remaining hits do not produce any bins with a size above a given threshold (usually three hits).
7.7 Deficiencies of the Hough Transform

The Hough transform as described and implemented above treats the FGD hits as pointlike, existing only at the geometric centre of each bar. This causes the algorithm to favour track hypotheses that pass through the centre of bars. The bar granularity of the FGD is such that this is a significant problem.

A particle traversing the FGD will hit bars as shown in Figure 7.4. We see that the track passes through the corner of several bars and does not favour the centre of any bars. Figure 7.5 shows the track solution favoured...
Figure 7.4: Hit bars in the FGD for a given particle, following the trajectory marked in red. Ideally, the reconstruction should reconstruct this pattern as a single particle track.

Figure 7.5: Tracks, marked in blue, reconstructed by the Hough transform for the set of FGD hits shown in black. Due to its preference for tracks that pass through the centres of bars, the large-scale continuity of these hits is lost by the Hough transform in favour of a local collinearity.

by the Hough transform, which considers each hit to exist at its precise geometrical centre. This leads to a preference for broken, horizontal tracks over a single diagonal track. As such, we must replace the Hough transform with an algorithm that accommodates the significant extent of the FGD bars.

A limit to the sensitivity of the Hough transform as implemented comes from the tradeoff inherent in choosing bin boundaries. Making the granu-
larity of the binning finer gives the reconstruction more possible track trajectories, but if the intersection points are binned too finely, the intersection points will be split up and the cluster corresponding to a track will disintegrate. Additionally, it is impossible to set bin boundaries \textit{a priori} in such a way that they will not bisect clusters and thus divide tracks.

7.8 The Radon Transform

The replacement for the Hough transform that was investigated was based on the Radon transform. Like the Hough transform, the Radon transform is a transform from an \((x,y)\) coordinate space to a line space \((r,\theta)\). While the Hough transform is a discrete transform transforming points in the coordinate space to lines in the transform space, the Radon transform is a continuous transform from a function in coordinate space to a function in transform space.

Consider a function \(f(x,y)\) in coordinate space. The Radon transform of this function is defined by the integral

\[
g(r,\theta) = \int f(x(r,\theta,s),y(r,\theta,s))ds
\]

where the functions \(x(r,\theta,s)\) and \(y(r,\theta,s)\) are described by \textit{Equation 7.2} and \textit{Equation 7.3}, respectively.

The Radon transform is an important mathematical element of a number of real-world sensing processes. For instance, the raw output of a CT scan is the Radon transform of the density of the subject material.

7.9 Discretizing the Radon Transform

If we treat the set of hits as a function \(f(x,y)\) given by

\[
f(x,y) = \begin{cases} 
1 & \text{if } (x,y) \text{ is inside a hit bar} \\
0 & \text{otherwise}
\end{cases}
\]

the continuous Radon transform of this function will have peaks corresponding to track hypotheses that pass through the most bars and are thus the
most likely tracks. However, like the pointlike Hough transform, this favours tracks that pass through bar centres, as the likelihood associated with passing through each bar is proportional to how much bar it passes through. Thus, we must use a different method to apply this to the discrete hit data.

Consider the figure in the \((x, y)\) plane as a set of regions \(R_i\), each with an associated weight value \(w_i\). To discretize the Radon transform, we replace the integral in Equation 7.7 with a sum over these regions:

\[
g(r, \theta) = \sum_i w_i \Psi_i(r, \theta) \tag{7.9}
\]

where \(\Psi_i(r, \theta)\) is 1 when the line described by \(r\) and \(\theta\) intersects the region \(R_i\) and is 0 otherwise. For the FGD reconstruction, the regions \(R_i\) are squares corresponding to the cross-sections of hit bars, each given unit weight. Effectively, this discretized Radon transform counts the number of hit areas traversed by each line, using that as the transform value.

This transform is still continuous in the \((r, \theta)\) plane, so for computational purposes we need to sample the transform function at an array of discrete points. Unlike the method for binning the Hough transform, there are no sensitivity tradeoffs for increasing the density of this array. Where finer Hough binning can ultimately split up tracks, finer Radon sampling is merely more computationally expensive.

### 7.10 Drawbacks of Hough and Radon Transforms for Reconstruction

The prime drawbacks of the Hough and Radon transforms as track reconstruction algorithms stem from being purely a collinearity criterion, with no requirement of connectedness. Since all points along the hypothesis line are equivalent for calculating the transform value, there can be large gaps between the hits in a reconstructed track which are unrealistic in the track of a continuously-moving real particle.

Additionally, the collinearity requirement forces the hits to line up on a single straight line, irrespective of any curvature the particle track may
have in reality. We expect that tracks will curve in the YZ plane due to the magnetic field, and also that tracks in the FGD will curve due to multiple scattering in the dense material. A successful reconstruction program using these transforms must mitigate both of these problems to allow realistic reconstruction of FGD-only tracks. The various mitigation methods used in the FGD Radon transform implementations will be described in Chapter 8.
Chapter 8

Implementation of Radon Transform for FGD

8.1 Initial Implementation

Initially, the Radon transform was implemented in the same framework as was used for the Hough transform. The transform was run over the set of FGD hits not matched to TPC tracks, the most likely track was extracted, the hits in the track removed from consideration, and the transform re-run on the remaining hits until all tracks are extracted.

An important part of calculating the Radon transform is evaluating whether a given line in the \((r, \theta)\) parametrization intersects a hit in the FGD. We model the hit as a square centred at \((x_0, y_0)\) with side width \(2\delta\), so the corners of the hit are given by

\[
(x_0 \pm \delta, y_0 \pm \delta), (x_0 \pm \delta, y_0 \mp \delta)
\]

(8.1)

First, we rotate by \(-\theta\) into a new coordinate system \((x', y')\). Since in this case we have

\[
x' = x \cos \theta + y \sin \theta
\]

(8.2)
Equation 7.1 gives that

\[ r = x' \quad (8.3) \]

Since the line in question is now vertical, only the \( x' \) coordinates of the corners are pertinent. These coordinates, from Equation 8.1 and Equation 8.2, are

\[ x' = x'_0 \pm \delta (\cos \theta + \sin \theta), \quad x' = x'_0 \pm \delta (\cos \theta - \sin \theta) \quad (8.4) \]

where \( x'_0 = x_0 \cos \theta + y_0 \sin \theta \). We can then transform the system by \(-x'_0\) to \( x'' = x' - x'_0 \), centering the hit on the y-axis and placing the line at

\[ x'' = r - x'_0 \quad (8.5) \]

The corners of the hit are then

\[ x'' = \pm \delta (\cos \theta + \sin \theta), \quad x'' = \pm \delta (\cos \theta - \sin \theta) \quad (8.6) \]

so the line intersects the square if \( r - x'_0 \) is between the largest and smallest of these \( x'' \)-coordinates. By inspection, we see that the largest must be

\[ x'' = \delta (|\cos \theta| + |\sin \theta|) \quad (8.7) \]

and that the largest and smallest are symmetric about \( x'' = 0 \). As such, the final criterion for intersection is

\[ |r - x'_0| \leq \delta (|\cos \theta| + |\sin \theta|) \quad (8.8) \]

Another consideration for implementing the Radon transform is the placement of the coordinate origin. The line parametrization becomes degenerate at \( r = 0 \), where the lines \((r, \theta)\) and \((r, \theta + \pi)\) become coincident. As such, the \( r \) values used in the sampling grid, separated by \( \delta r \), begin at \( r = \delta r / 2 \) rather than at \( r = 0 \), and also we avoid any issues of hits intersecting the origin by placing it outside the FGD. Conveniently, the origin of the ND280 coordinate system is just upstream of FGD1, and it was used as
the coordinate origin for the Radon transform.

### 8.1.1 Oversampling

To allow for some leeway in the collinearity criterion of the Radon transform reconstruction, a method from computer graphics was adapted to the reconstruction algorithm, namely oversampling. Oversampling is a method used for anti-aliasing computer generated images, where the image is rendered at a higher resolution, and then multiple pixels are averaged together to produce the colour value for each pixel in the final image.

For the Radon transform reconstruction, this means finding the set of hits for each hypothesis, and then combining hypotheses in such a way that the set of hits for the combination is the union of the sets of hits for each constituent hypothesis. For the standard Radon transform, a 4x4 grid of hypotheses was combined in this way for each final hypothesis. Each final hypothesis had its own distinct set of sub-hypotheses, producing a grid structure in Radon space.

### 8.1.2 Pruning

The pruning stage is an attempt to fix one of the deficiencies described in Section 7.10, namely the lack of connectedness criteria. The aim is to remove any ‘stray’ hits picked up from distant regions of the detector and leave a track with a reasonable degree of continuity.

The pruning algorithm used in the original Radon transform implementation was a direct re-application of the pruning process developed for the Hough transform reconstruction. In this algorithm, the collinearity of the Hough tracks was used as the basis for a simple connectedness criterion. If we sort the hits along the length of the track, large gaps between adjacent hits will appear where there are discontinuities. These gaps are then used to divide the track into connected pieces.

Since all of the hits in the track are roughly collinear, the distance from the endmost hit is a good proxy for the distance along the line, and so the hits were sorted by this distance. It is important to choose the endmost hit
or hits from both sides will be interleaved in the sorted list, in which case the track will be destroyed by the increasingly large gaps between ‘adjacent’ hits. Unfortunately, the endmost hit is difficult to define without already having the sorted list.

Due to this deficiency, better sorting schemes were investigated, but none were found to have improved performance. The failure of these sorting schemes to provide improved pruning performance was a major impetus for the development of a new Radon transform reconstruction (Section 8.2) and an entirely different pruning algorithm (Section 8.5.1).

8.2 The Flexible Radon Transform

After extensive tests on the original Radon transform reconstruction, it became clear that a new approach was necessary. The main difference is in the combination of different Radon lines to form a track hypothesis. The fixed oversampling clusters introduce similar problems as the bin boundaries of the Hough transform; a rigid structure is imposed \textit{a priori} without reference to the shape of the data.

The new approach was named the flexible Radon transform reconstruction, as the core difference is in replacing the rigid oversampling boundaries with a flexible clustering of Radon lines based on the shape of the Radon histogram. The new algorithm divided into three stages:

- The transform stage, which applies the Radon transform to the hits and produces an image in Radon space corresponding to the input hits
- The clustering stage, where groups of Radon lines are selected and grouped together into track hypotheses
- The post-processing stage, where the set of track hypotheses is analysed and altered to remove redundant tracks and remove spurious connections

The post-processing stage is necessary to address the deficiencies described in Section 7.10 and to ensure that the algorithm produces few duplicated tracks.
8.3 The Transform Stage

The Radon transform component of the flexible Radon reconstruction is largely identical to that used for the earlier implementation. The hypotheses are spaced 2.5 mm apart along the $r$ axis and 25 mrad apart along the $\theta$ axis, which gives a 808 by 251 grid for 202,808 total hypotheses. This is three times the 65,920 hypotheses (bundled into 4120 groups) used in the initial implementation. Since the flexible Radon runs the transform only once rather than iteratively, the larger space is not infeasible.

We translate the Z-position of hits in FGD2 such that FGD2 occupies the same volume in coordinate space as FGD1, thus simplifying the implementation of the Radon transform. An example event in FGD2 can be seen in Figure 8.1. Its Radon transform can be seen in Figure 8.2.

Later versions of the flexible Radon transform added a hit preclustering stage prior to the Radon transform stage. Preclustering merges together adjacent hits in the same layer into an object that is treated as a unit for the purpose of the Radon transform; it is a single region with a weight equal to the number of hits included in the cluster. The preclustering used for the final version of the flexible Radon transform had a maximum cluster size of three hits; any grouping larger than three hits was separated into clusters of three hits or fewer.
Figure 8.2: Example Radon transform histogram, showing the Radon weights for the track hypotheses with hits as shown in Figure 8.1. Two clusters corresponding to the two tracks are seen where the blue curves converge.

8.4 The Clustering Stage

The heart of the flexible Radon transform is a clustering stage based on a seed fill algorithm, similar to that used in image processing. For image processing and editing, the seed fill is used to find a contiguous region of a single colour in an image, such as for the ‘paint bucket’ tool in an image editing program.

For use in the flexible Radon reconstruction, the seed fill was used to label each of the separate clusters of Radon hypotheses above threshold, by ‘filling’ each cluster in turn with a distinct numeric label. The simplest way to define the seed fill is as a recursive process:

1. If the current hypothesis is already labelled or its weight is below threshold, stop.

2. Label the current hypothesis with the current label

3. Repeat all three steps for each of the eight adjacent hypotheses in Radon space (orthogonal and diagonal)
Figure 8.3: Cluster labelling using a seed fill. Each set of points with the same colour and number in the diagram on the right is treated as a separate cluster for the purpose of finding track clusters in Radon space.

The clusters are extracted from the Radon space figure by running this process repeatedly, each time choosing the highest-weighted remaining unlabelled hypothesis. The label is incremented between each iteration of the process, so the highest-weighted overall cluster is labelled 1, the next highest labelled 2, and so on. Once the most significant unlabelled hypothesis is below threshold, all clusters of hypotheses above threshold will have been labelled. Each labelled cluster is then used to produce a track candidate, containing the set of all hits touched by at least one of the Radon hypotheses in the cluster. The labelling process for a simple figure is shown in Figure 8.3.

For the Radon transform, we add some additional constraints and considerations. The Radon space does not have hard boundaries; rather, three of the four edges have periodicity conditions. The space is periodic in $\theta$, so the seed fill simply wraps around the sides of the histogram along that axis. The $r$ axis is more complicated. At the high end, we have a hard boundary. The maximum $r$ is chosen such that any points beyond it correspond to lines that do not intersect the FGD, so we can safely assume that these points would have a transform value of zero and cut off the seed fill at that edge.

The situation is more complicated at the low end on the $r$-axis, that is, at $r = 0$. The seed fill algorithm would like to move across the boundary at the $\theta$-axis to negative values of $r$, but $r$ in the sample is defined to be positive. However, examining Equation 7.1, we can see that the line defined
by \((-r, \theta)\) is equivalent to the line defined by \((r, \theta + \pi)\). Thus, a step to lower \(r\) from \((r, \theta)\), where \(r\) is its minimum value, is taken to be a step to \((r, \theta + \pi)\), subtracting \(2\pi\) as necessary to remain in the range \([-\pi, \pi]\).

Another constraint that was added to the seed fill was a requirement that the fill process only move to neighbours whose value is less than or equal to that of the current point. We found that the islands corresponding to different tracks could be connected in Radon space by a ‘bridge’ of points above threshold. Requiring that the fill only move ‘downhill’ prevents these clusters from being joined up by the seed fill.

For the flexible Radon transform as implemented, the threshold was chosen as three hits, as this is the fewest number of points needed to define a line on the plane as more than a mathematical coincidence. Higher thresholds were not studied as the sensitivity of the reconstruction to short tracks is very important, and a higher threshold would increase the minimum length of a reconstructible track.

The set of clusters for the Radon histogram in Figure 8.2 is shown in Figure 8.4. As can be seen in the figure, there are more than 50 clusters at this stage of the reconstruction, requiring post-processing to find the two tracks of Figure 8.1.

### 8.5 The Post-Processing Stage

The post-processing step is necessary to reduce the large number of track hypotheses that are produced in the clustering stage to a set of tracks that better reflect the number of particles passing through the FGD. A significant drawback of the Radon transform reconstruction is its tendency to find spurious ‘alias’ and ‘ghost’ tracks. Alias tracks arise when portions of multiple actual tracks are collinear and thus are lumped together by the Radon transform. This is particularly noticeable for vertical tracks; all the hits in a given FGD layer line up cleanly along a single line. Ghost tracks, on the other hand, are tracks that are subsets of other larger tracks, which arise from slightly disconnected clusters in Radon space. While alias tracks can have high weights, ghost tracks will have smaller weights than the large
tracks that they ghost.

As the properties of the two types of spurious tracks are quite different, the post-processing has two stages which deal with them separately. The pruning stage deals with high-weight alias tracks, breaking them apart into connected pieces, while the hit capture stage deals with low-weight ghost tracks, eliminating tracks which are subsets of other, larger tracks.

8.5.1 Flexible Pruning

An entirely different pruning mechanism was used in the flexible Radon transform than was used for the original implementation. Not only does the flexibility of the track make the assumption of linearity suspect, no sorting algorithm was found that would work in enough cases to be worthwhile. Instead, the pruning was performed on a layer-by-layer basis.

The hits in each layer were divided into segments, where the segments are separated by two or more bars which were not hit. The hits in each segment are thus roughly continuous and are treated as a unit for the remainder of the
Figure 8.5: Pruning of a flexible Radon track; a track with all shown hits is separated into the differently-coloured pieces. The pruning separates each layer into nearly-continuous pieces and then selects one of these per layer for use in each final track. Skipped layers cause more stringent vertical continuity conditions in selecting pieces.

The pruning algorithm. This stage implements the ‘vertical’ pruning criterion as only one segment per layer is included in a final track.

Once all of the layers are broken into segments, we must re-integrate the track by connecting together segments in the z-direction. Beginning from a seed segment, we proceed downstream and then upstream by layer and select a segment from each layer for inclusion in the track. Generally, the closest or most-overlapping segment in each successive layer is chosen.

The criteria for segment selection differ depending on whether layers are skipped. If more than one layer passes without an eligible segment, the track rebuilding is ended in the current direction. If a single layer is skipped, only segments that are within six bars of the previous segment are considered. For adjacent layers, the gap is allowed to be as much as four times the total number of bars in both segments.

To avoid ambiguities, it is ideal to seed the selection algorithm from a layer with only one segment. If such a layer is available, it is used as the
initial seed, otherwise the seed is picked arbitrarily from the most upstream layer.

The track selection algorithm is run iteratively until all segments are used. All tracks found in the pruning are added to the set of candidates, and if the largest of these tracks is still the largest candidate overall, it proceeds to the hit capture stage. Figure 8.5 illustrates the pruning process, with an initial track consisting of all visible hits split by the pruner into the separately-coloured pieces.

8.5.2 Hit Capture

Once a track has passed through the pruning stage, we want to finalize it as an FGD-only track. While it is desirable to allow tracks to share hits, the prevalence of duplicate or ‘ghost’ tracks made it infeasible for the time being. Thus, the hits in each final track candidate are removed from all other tracks before further tracks are extracted from the set of candidates. This process has been given the name ‘hit capture’.

A track captures its hits and is returned as an actual FGD iso-track only if it is the candidate with the most hits remaining after the pruning step. Once this has been established, the hits are removed from all other candidates, and any candidates dropping below two hits are discarded entirely. If there are any candidates remaining after hit capture, the process of pruning and hit capture continues with the largest remaining candidate.

After the postprocessing steps, the example event has two remaining clusters as shown in Figure 8.6. There are two remaining clusters which correspond to the two tracks shown in Figure 8.1.
Figure 8.6: Remaining Radon clusters from Figure 8.4 after pruning and hit capture. All but the two physically-significant clusters are removed in post-processing, leaving the two tracks we see in Figure 8.1
Chapter 9

Analysis of Reconstruction Performance

9.1 Definitions

To evaluate the performance of the FGD isorecon in reproducing the paths of the underlying true particles, we must understand the structure of the Monte Carlo truth information available for the simulated FGD. Each simulated particle is associated with a truth trajectory, which describes the true path of the particle, and has a unique ID number.

Any number of particles can pass through a bar simultaneously, so there is no one-to-one mapping from hits to truth trajectories. When associating reconstructed tracks to true trajectories, then, we must consider all of the trajectories contributing to each of the track hits. For this analysis, tracks are associated to trajectories by counting how many hits each trajectory contributes to, and picking the trajectory associated with the most hits.

It is important to have a clear definition of the quantities used to evaluate the performance of the reconstruction. Having associated a reconstructed track with a true trajectory, there are three quantities available: the number of hits in the reconstructed track $N_R$, the number of hits contributed to by the true trajectory $N_T$, and the number of hits in the reconstructed track.
contributed to by the associated true trajectory $N_C$.

From these, we can define two figures of merit for the track: completeness and cleanliness. The completeness of a track is the fraction of hits from the true trajectory found by the reconstruction, i.e.,

$$\text{Completeness} = \frac{N_C}{N_T} \quad (9.1)$$

while the cleanliness is the fraction of hits in the reconstructed track with contributions from the true trajectory,

$$\text{Cleanliness} = \frac{N_C}{N_R} \quad (9.2)$$

In the analysis below, a track is marked as ‘good’ if both its cleanliness and completeness are above 90%.

We also define a reconstruction efficiency for each isorecon as the fraction of truth trajectories that are associated with a reconstructed track. Only ‘reconstructible’ tracks are used for determining the efficiency, where a track is reconstructible in a projection if it contributes to at least three hits, and if its total track length is at least 5 cm.

For these tests, the TPC/FGD matching is disabled, allowing the isorecon to operate on all of the FGD hits and attempt to find tracks even for trajectories that leave the FGD for the TPC. Also, only FGD1 was considered; only the truth trajectories with at least one end in FGD1 are used for this study, and only reconstructed tracks in FGD1 are considered.

In this chapter, we compare the baseline SBCAT algorithm to both the original Radon transform reconstruction and the flexible Radon transform. The Monte Carlo sample used for these tests contains only interactions generated within the central ‘basket’ area of the ND280 detector, so that more of the simulated tracks are pertinent to this analysis.

### 9.2 Performance for 2D Tracking

First, we analyze the performance of the different isorecon algorithms for 2D tracking. It is useful to study both projections as they are not symmetrical;
as the magnetic field of the ND280 detector is along the X-axis, we expect tracks in the YZ projection to curve while being straight in the XZ projection. The curvature of YZ tracks is of particular concern for line-based reconstruction algorithms such as the Radon transform, so we must ensure that the algorithm has acceptable performance in both cases.

We find the cleanliness and completeness of the XZ tracks from the three algorithms in Figures 9.1 and 9.2. We can see that the track cleanliness for...
<table>
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<th>Isorecon</th>
<th>Mean Cleanliness (%)</th>
<th>Mean Completeness (%)</th>
</tr>
</thead>
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<tr>
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<td>63.0</td>
<td>52.5</td>
</tr>
<tr>
<td>Radon</td>
<td>59.1</td>
<td>53.8</td>
</tr>
<tr>
<td>Flexible Radon</td>
<td>66.5</td>
<td>55.1</td>
</tr>
</tbody>
</table>

Table 9.1: Cleanliness and completeness means for XZ projection

Figure 9.3: Isorecon track cleanliness for 2D tracks in the YZ projection. Qualitatively, the original Radon appears the worst, and the flexible Radon the best.

the original Radon transform is poorer than for either SBCAT or the flexible Radon transform, while the completeness distribution is quite similar in all cases. The distribution of cleanliness has two fairly clear features; there are markedly fewer tracks with cleanliness less than 0.3 from the flexible Radon, and markedly fewer tracks with cleanliness greater than 0.8 from the original Radon transform.

This comparison can be made more quantitative by considering the mean values of the cleanliness and completeness distributions. Table 9.1 shows these means. Comparable information for YZ tracks is found in Figures 9.3 and 9.4, and Table 9.2. There are only minor differences between the projections, confirming that the linearity of the Radon transform reconstruction is not a problem in that plane.

We can also examine the track-finding efficiency for the three algorithms
Figure 9.4: Isorecon track completeness for 2D tracks in the YZ projection, mostly similar for all algorithms

<table>
<thead>
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<th>Isorecon</th>
<th>Mean Cleanliness (%)</th>
<th>Mean Completeness (%)</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>Radon</td>
<td>59.1</td>
<td>53.2</td>
</tr>
<tr>
<td>Flexible Radon</td>
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<td>54.9</td>
</tr>
</tbody>
</table>

Table 9.2: Cleanliness and completeness means for YZ projection as a function of true track range. Figures 9.5 and 9.6 show this for the XZ and YZ projections, respectively. We can see that the efficiencies are similar and approximately 70% at ranges above about 250 mm, while below that there is a fairly clear hierarchy, where the SBCAT reconstruction has better efficiency than the flexible Radon, which in turn performs better than the original Radon transform.

9.3 Performance for 3D Tracking

For use in physics analysis, it is important to have 3D tracks in the FGD. 2D tracks are poorly defined, in the sense that there can be any form of unknown movement in the transverse direction, modifying the path length and thus the energy loss characteristics of the particle. The same 3D matching code is used for all isorecon algorithms, so testing 3D tracking efficiency deals both with the performance of the isorecon algorithm on its own and its interaction...
Figure 9.5: Isorecon track finding efficiency in the XZ projection as a function of particle range. SBCAT shows a slight advantage for short tracks which diminishes for longer tracks.

Figure 9.6: Isorecon track finding efficiency in the YZ projection as a function of total particle range. SBCAT is the best for short tracks but the comparison becomes more equal for long tracks.

with the requirements of the 3D matcher.

The 3D matching is based on two criteria, a position criterion and a charge criterion. Two tracks are matched together by position if their start and end Z-planes are both within a certain tolerance of each other, and if the total number of planes encompassed by the tracks is above a given threshold. If multiple YZ tracks match an XZ track through the position criterion, the pair with the least difference in average charge per hit between the
projections is chosen and matched together into a 3D track. The parameters of the position criteria are different depending on the complexity of the reconstructed event; if there are more than four total 2D tracks and fewer than 90% of the FGD hits are used by the reconstructed tracks, then the ends must be within 1 plane and the total track must span at least four planes. Otherwise, the total track can span any number of planes and the layer-matching criterion is gradually loosened from 1 layer difference to 3 until a viable match is found. The tracks that are matched together are then refit to provide a track with 3D information.

Figure 9.7: Cleanliness of 3D isorecon tracks, showing the flexible Radon producing fewer tracks with very low cleanliness.

Cleanliness and completeness of the 3D tracks can be found in Figure 9.7 and Figure 9.8, respectively. We see similar results as for the 2D tracks, though the low ends of both the cleanliness and completeness distributions are suppressed. This is because poorly-reconstructed tracks are less likely to be matchable with a corresponding track in the other projection. Nevertheless, the features identified above, showing few flexible Radon tracks with cleanliness less than 0.3 and fewer original Radon tracks with cleanliness greater than 0.8, remain in the 3D matched tracks. The cleanliness and completeness means are shown in Table 9.3.

The efficiency of the isorecon algorithms is shown in Figure 9.9 as a function of true track range. Once again, we see a plateau above 250 mm,
Figure 9.8: Completeness of 3D isorecon tracks. This is very similar for all algorithms, suggesting that this is largely a property of the 2D-3D matching algorithm which is the same for all track-finding algorithms.

<table>
<thead>
<tr>
<th>Isorecon</th>
<th>Mean Cleanliness (%)</th>
<th>Mean Completeness (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SBCAT</td>
<td>66.3</td>
<td>56.5</td>
</tr>
<tr>
<td>Radon</td>
<td>61.7</td>
<td>56.4</td>
</tr>
<tr>
<td>Flexible Radon</td>
<td>68.0</td>
<td>57.6</td>
</tr>
</tbody>
</table>

Table 9.3: Cleanliness and completeness means for 3D tracks

though the overall efficiency is lower (between 50% and 60%) due to the inefficiencies of the 3D matching algorithm. Additionally, we still see a clearly superior efficiency at low range from the SBCAT algorithm as compared with the two Radon transform reconstructions.

With 3D tracks, we can study the angular efficiency of the isorecon algorithm, as shown in Figure 9.10. We can see that the angular behaviour of the Radon transform is quite different from SBCAT; the Radon transforms see a drop in efficiency at very forward angles, while the efficiency of SBCAT drops at angles perpendicular to the beam direction.
Figure 9.9: Efficiency variation of 3D isorecon tracks with track range. A clear advantage is seen for SBCAT with short tracks.

Figure 9.10: Efficiency variation of 3D isorecon with true track angle from the Z-axis (horizontal in the beam direction). SBCAT has a clear advantage for tracks close to the forward direction and a clear disadvantage for tracks close to 90 degrees from the Z-axis.
Table 9.4: Number of good tracks in test sample for each isorecon algorithm. The original Radon transform has much fewer good tracks than the other two algorithms, but SBCAT still slightly outperforms the flexible Radon in 2D tracking, and more significantly outperforms it for 3D tracking.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>XZ</th>
<th>YZ</th>
<th>3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>SBCAT</td>
<td>12747</td>
<td>12736</td>
<td>9719</td>
</tr>
<tr>
<td>Radon</td>
<td>10817</td>
<td>10663</td>
<td>8304</td>
</tr>
<tr>
<td>Flexible Radon</td>
<td>12357</td>
<td>12340</td>
<td>9136</td>
</tr>
</tbody>
</table>

9.4 Summary and Outlook

A summary of the number of good tracks found by each reconstruction, where a good track has both cleanliness and completeness greater than 90%, can be found in Table 9.4. SBCAT shows slightly better performance, but both it and the flexible Radon are clearly better than the original Radon.

Overall, we see that the flexible Radon transform makes a modest improvement in some of our measures over the SBCAT algorithm, while falling short in others. It is clear, however, that the original Radon algorithm has some significant deficiencies. Deficiencies are seen in the track-finding efficiency of the flexible Radon transform at very forward angles and at short ranges as compared with SBCAT. The full causes of this effect are as yet unknown, but it is likely that the hit capture step, as described in Section 8.5.2, is at least partially responsible.

Since the flexible Radon as currently constituted allows each hit to belong to only a single track, tracks considered earlier in the post-processing can remove hits critical to the reconstruction of shorter tracks. As the tracks are considered in order of their hit count, which tracks roughly with the track length, longer tracks are favoured over shorter tracks as the ‘owner’ of FGD hits.

The short-range deficiencies of the Radon transform reconstructions are related to the problem at forward track angles, as the geometry of the FGD ensures that forward tracks are also short; a perfectly horizontal track having
a maximum length in the FGD of 30 cm. Thus, a large fraction of the horizontal and near-horizontal tracks will be in the falling edge of Figure 9.9.

Along with these inefficiencies, the flexible Radon transform reconstruction has been found to be too flexible in what final tracks it considers. For example, it has been observed to reconstruct tracks that pass through clear but oblique vertices. The available tests of isorecon performance do a poor job of catching this type of failure, so no quantitative information on this problem is available.

As a result of these difficulties and the relatively modest improvement provided in the cases where the flexible Radon transform was superior, the flexible Radon transform as described in this thesis did not replace SBCAT as the default algorithm for FGD-only track reconstruction. However, the Radon transform concept was developed further by others, and a derivative of this work is now used as the default reconstruction algorithm for FGD-only tracks.
Part IV

Subtraction Analysis for Measurement of Water Cross-Sections
Chapter 10

Method of Analysis

10.1 Principle of Subtraction Analysis

A key goal of the FGD is to observe the interaction rates and cross-sections of neutrinos on water. The passive water modules described in Part II and included in FGD2 provide water for this measurement, but interactions on water must be separated from interactions in the carbon-based plastic scintillator material of the FGD active area.

Ideally, the neutrino interaction vertex could be measured with sufficient accuracy to determine whether a given interaction occurred in the water or in the plastic. However, this accuracy is not available in current analyses, and will never be available for some classes of events, particularly events with a single visible track. Present methods rely on finding an FGD bar that appears to be the vertex, and thus will always find vertices within the FGD plastic.

An alternative to detailed vertex reconstruction is to run a statistical subtraction, not identifying the water events individually but determining their numbers en masse. Conceptually, the simplest form of subtraction analysis involves taking data with and without water in the detector. This method is used in ND280 by the P0D, but it is less suitable for the FGD. The water in the FGD makes up a much larger fraction of the total detector mass than the water in the P0D, so the difference in energy loss between a
full and empty FGD is much larger in comparison to the actual energy loss. This systematic difference makes it much more difficult to compare the full and empty FGDs. The momentum distribution of particles exiting the FGD would differ, as they would lose less energy in the empty FGD than in the full FGD. Track multiplicities would differ, with fewer tracks stopping in the FGD and fewer tracks disappearing into the FGD dead material. This is less of an impediment to a CC inclusive measurement but would complicate a channel-specific measurement significantly.

Instead, the FGD statistical subtraction is based on a comparison of the two separate FGDs. FGD1, being made up entirely of plastic scintillator, accurately models the non-water target mass of FGD2. Thus, it should be possible to scale the interaction rate in FGD1 by the ratio of plastic masses of the two FGDs to estimate the plastic contribution to the FGD2 interaction rate. As a result, we can subtract the scaled FGD1 rate from the FGD2 rate to extract an estimate of the water interaction rate, i.e. we extract the rate of interactions in water $N_w$ as

$$N_w = N_2 - \frac{m_p}{m_1}N_1$$  \hspace{1cm} (10.1)

where $m_p$ is the total mass of plastic in the FGD2 fiducial volume, including water module walls, and $m_1$ is the mass of the FGD1 fiducial volume. In this analysis, we define the interaction cross-section per unit mass, rather than per target nucleon; these two definitions are close if the cross-section scales with the atomic mass number $A$. To isolate this underlying cross-section physics, we express our result as a mass-scaled ratio of the water and plastic rates,

$$R = \frac{N_w m_1}{N_1 m_w}$$  \hspace{1cm} (10.2)

where $m_w$ is the mass of water in the FGD2 fiducial volume. This ratio should be equal to the ratio of the cross-sections for water and plastic, with the beam flux cancelling in the ratio, provided that the dependence of the cross-section on the neutrino energy is similar for the two materials. We can
see this if we expand the event rates into an integral over energy, as

\[ N_w = m_w \int \Phi(E)\sigma_w(E) dE \]  \hspace{1cm} (10.3)

and

\[ N_1 = m_1 \int \Phi(E)\sigma_p(E) dE \]  \hspace{1cm} (10.4)

where \( \Phi(E) \) is the beam flux, \( \sigma_w(E) \) is the water cross-section, and \( \sigma_p(E) \) is the plastic cross-section. If the cross-sections are approximately proportional (\( \sigma_w(E) \approx R\sigma_p(E) \)), we can reproduce the constant of proportionality with Equation 10.2.

This cancellation is aided by the characteristics of the T2K beam. As shown in Figure 2.3, the off-axis beam used for T2K has a relatively narrow energy distribution, with the majority of the neutrinos below 1 GeV. Thus, for the approximation to be valid, we need the cross-section ratio to be approximately flat over the energy region of the T2K beam, rather than over the entire spectrum of possible neutrino energies. This assumption is borne out by current models, which observe a variation over our energy range of less than 10%.

Now Equations 10.2 and 10.1 use the true rate of interactions in each detector, uncorrected for the efficiency and purity of the selection. With an estimate of these efficiencies and purities, derived from the Monte Carlo truth information, we can correct the measured event rates to retrieve the necessary true rates. This correction is complicated by the observation of different efficiencies in FGD2 for interactions in different types of module; while FGD1 is uniform and has efficiency \( \epsilon_1 \), the FGD2 water modules have efficiency \( \epsilon_w \) and the FGD2 XY modules have efficiency \( \epsilon_p \).

To accommodate these different efficiencies, we must alter Equation 10.1. First, we break down the total FGD2 rate as

\[ N_2 = N_p + N_{pw} + N_w \]  \hspace{1cm} (10.5)

where \( N_p \) is the true rate in the XY modules, \( N_{pw} \) is the true rate in the plastic component of the water modules, and \( N_w \) is the true rate in the
water component of the water modules.

To re-state this in terms of the measured FGD2 rate $n_2$, we need to add
the efficiencies and the FGD2 purity $P_2$. Of the $n_2$ selected events in FGD2,
$P_2 n_2$ of them are signal events, which can be broken down by Equation 10.5
as

$$P_2 n_2 = \epsilon_p N_p + \epsilon_w (N_{pw} + N_w) \quad (10.6)$$

Now both $N_p$ and $N_{pw}$ are on an XY-module-like elemental composition, so
we estimate them based on the FGD1 rate by scaling by masses

$$N_p \approx \frac{m_{xy}}{m_1} N_1, N_{pw} \approx \frac{m_{pw}}{m_1} N_1 \quad (10.7)$$

where $m_{xy}$ is the mass of the XY modules in the FGD2 fiducial volume, and
$m_{pw}$ is the mass of the plastic component of the water modules in the FGD2
fiducial volume. Equation 10.6 then becomes

$$P_2 n_2 = \left( \epsilon_p \frac{m_{xy}}{m_1} N_1 + \epsilon_w \frac{m_{pw}}{m_1} \right) N_1 + \epsilon_w N_w \quad (10.8)$$

The ratio $m_{xy}/m_1$ is simply the ratio of the number of XY modules in
the two FGD fiducial volumes, i.e., 6/14. This allows us to express the ratio
$m_{pw}/m_1$ in terms of the known total FGD2 plastic mass $m_p = m_{xy} + m_{pw}$.
We can then solve Equation 10.8 for $N_w$ to substitute in Equation 10.2:

$$N_w = \frac{P_2 n_2}{\epsilon_w} - \left[ \frac{m_{xy}}{m_1} \epsilon_p + \left( \frac{m_p}{m_1} - \frac{m_{xy}}{m_1} \right) \right] N_1 \quad (10.9)$$

$$= \frac{P_2 n_2}{\epsilon_w} - \left( \frac{6}{14} \left( \frac{\epsilon_p}{\epsilon_w} - 1 \right) + \frac{m_p}{m_1} \right) N_1 \quad (10.10)$$

Combining Equation 10.2 and 10.10 gives

$$R = \frac{m_1}{m_w} \left[ \frac{P_2 n_2}{\epsilon_w N_1} + \frac{6}{14} \left( 1 - \frac{\epsilon_p}{\epsilon_w} \right) - \frac{m_p}{m_1} \right] \quad (10.11)$$

Equation 10.11 still depends on the true FGD1 rate $N_1$, but since we
can treat FGD1 with a single efficiency, we have $N_1 = P_1 n_1/\epsilon_1$ and can
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1/m_w$</td>
<td>2.17</td>
</tr>
<tr>
<td>$m_p/m_1$</td>
<td>0.526</td>
</tr>
<tr>
<td>$P_1$</td>
<td>0.917</td>
</tr>
<tr>
<td>$P_2$</td>
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</tr>
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<td>0.408</td>
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<tr>
<td>$\epsilon_w$</td>
<td>0.414</td>
</tr>
<tr>
<td>$\epsilon_p$</td>
<td>0.403</td>
</tr>
</tbody>
</table>

Table 10.1: Nominal parameter values from Monte Carlo selection, as used in Equation 10.12

substitute in Equation 10.11 to get the final rate calculation:

$$R = \frac{m_1}{m_w} \left[ \frac{P_2 \epsilon_1}{P_1 \epsilon_w} \cdot \frac{N_2}{N_1} + \frac{6}{14} \left( 1 - \frac{\epsilon_p}{\epsilon_w} \right) - \frac{m_p}{m_1} \right]$$  \hspace{1cm} (10.12)

Table 10.1 shows the values of the parameters in Equation 10.12 as derived from the standard Monte Carlo selection.

10.2 Overview of Event Reconstruction

The event reconstruction used for this analysis has three relevant stages: reconstruction of tracks in the TPC, matching of TPC tracks to FGD hits, and global reconstruction of tracks in ND280. The FGD-only track reconstruction, as described in Part III] was not used in this analysis; as an inclusive charge-current selection, the details of interaction topology given by the FGD-only tracks are irrelevant to the final sample.

10.2.1 TPC Track Finding

As a particle passes through a TPC, it deposits a trail of ionization in the TPC gas. The TPC permits us to reconstruct this track with full three-dimensional information. The interior of the TPC has a strong electric field that causes the ionized electrons to drift through the gas to a grid detector pads at each end. This drift is slow enough at approximately 8 cm/µs that
the arrival time of the charge from different distances from the anode plane can be distinguished, providing a three-dimensional image of the ionization, one dimension of which is the arrival time.

Converting the drift coordinate from a time to an actual distance requires two pieces of information. The drift speed of the charge in the detector can be characterized and is generally well-regulated by the electric field strength and the gas mixture, but the conversion also requires a time offset to place it within the detector. The ND280 TPCs are all surrounded by scintillator detectors, and can use their timing to supply this offset $T_0$. If available, the TPC reconstruction uses hits in the FGDs to supply the offset.

Once the TPC hits are translated into full 3D coordinates, tracks are reconstructed using a cellular automaton pattern recognition package. These reconstructed tracks are then the seed for the next step in the reconstruction process.

10.2.2 TPC/FGD Matching

After the tracks are reconstructed in each TPC, they are matched to hits in the FGD. Each track is projected into any adjacent FGDs, so TPC1 tracks are projected forward into FGD1, TPC2 tracks are projected backward into FGD1 and forward into FGD2, and TPC3 tracks are projected backward into FGD2. The tracks are matched to FGD hits incrementally using a Kalman filter[61]. Checking through the FGD hits, beginning in the closest layer to the TPC, a “matching $\chi^2$” is calculated for each hit by projecting the track to the Z position of the hit and using the difference in their positions:

$$\chi^2 = (v_1 - v_2)(C_1 + C_2)^{-1}(v_1 - v_2)^T$$ (10.13)

where $v_1$ is the extrapolated vector position, $v_2$ is the hit position, and $C_1$ and $C_2$ their respective covariance matrices. The error used for the hit is just the extent of the bar. The hit is included in the track if the $\chi^2$ is under a given value, and the track state updated to include the new information from the added hit.

This process continues until the far side of the FGD is reached, no more
FGD hits are available, or if multiple layers in succession have no successfully matched hits.

10.2.3 Global Reconstruction and Vertex Finding

Following the reconstruction in each subdetector, the ND280 reconstruction performs a global reconstruction, where component tracks from the different subdetectors are stitched together into a complete particle track. For tracks in the TPC and FGD there are actually two stages to this. First, in the Tracker reconstruction, tracks from different TPCs are stitched together. As described above, the TPCs are matched to the FGDs separately, so a track passing through all three TPCs would still be three separate tracks following the TPC/FGD matching. Overlaps in the FGD hits between different TPC tracks allow them to be stitched together into one, coherent Tracker track.

After the Tracker reconstruction stitches together TPC/FGD tracks that can traverse multiple TPCs, the global reconstruction matches together tracks in the various subdetectors using a Kalman filter. These tracks are disjoint, requiring a different matching method than the FGD overlaps used in the Tracker. After matching together subdetector tracks, the global reconstruction refits the complete track to provide a continuous track fit from end to end.

The reconstructed vertex is taken from the end of the global track. If the global fit indicates that the particle is travelling downstream, the upstream end is used, while if it indicates that the particle is travelling upstream, the downstream end is used. If the end is in an FGD, the last hit has only two coordinates, so the fit is extrapolated to the Z-position of that hit to define the third coordinate.

10.2.4 Momentum and Charge Determination

Having matched together tracks in all of the subdetectors, we need to determine the momentum and charge of the particle underlying each track. The foundation of the momentum measurement is found in the TPC reconstruction, where each track is fit to a helix to find the track curvature in
the magnetic field. The sign of this curvature provides the charge of the particle, while its magnitude allows the momentum to be calculated from the relation

\[ p = Bqr \]  

(10.14)

where \( B \) is the value of the magnetic field, \( q \) is the electron charge, and \( r \) is the radius of curvature.

These calculations of charge and momentum are done separately for each TPC and also for the fitted global track, but the momentum determination for global tracks in the version of the reconstruction that was used contained a bug preventing its use in analysis. Thus, we must derive the track momentum directly from a measurement in a TPC.

It is clearest to derive the momentum information from a single TPC, but this requires us to choose which TPC to use. First, the upstream end of the track is found by examining the TPC and FGD constituents and choosing the one with the smallest initial Z coordinate. Then the TPC constituent with an end closest to the upstream end is selected and its curvature used to find the momentum and charge.

While the trajectory of the particle in the TPC is used to find its momentum, most often it will have originated in an FGD. Since the FGD is much more dense than the TPC, there can be a significant energy loss as the particle leaves the FGD. Thus, an energy loss correction is applied, integrating the Bethe-Block formula over the FGD material from the selected vertex to the edge of the detector.

### 10.3 FGD Neutrino Selection

The key prerequisite to the subtraction analysis is a measurement of the interaction rates in each FGD. This section and the next will describe how \( \nu_\mu \) CC interactions are selected in the FGD. We characterize these interactions as producing a muon-like track that originates in an FGD and proceeds into a TPC for momentum and \( \frac{dE}{dx} \) measurement. The selection proceeds in two broad stages: selection of neutrino interactions in the FGD, and separation of CC \( \nu_\mu \) events from other types of interactions. This section will cover the
former, while Section 10.4 describes the latter.

The FGD neutrino selection I used contains the following cuts:

1. Disregard any tracks assigned an invalid status by the reconstruction
2. Disregard tracks falling outside beam bunch times
3. Disregard tracks with no TPC component
4. Disregard tracks with no FGD component
5. Veto on P0D track
6. Require minimum number of TPC track points
7. Select highest momentum remaining track as muon candidate
8. Require track origin to be within FGD fiducial volume

This selection will be elaborated in the following subsections.

10.3.1 Beam Timing Cut

The T2K beam came in six distinct bunches per spill during the run period used for this analysis, with a consistent timing relative to the beam spill trigger. For later running, the number of bunches was increased to eight. The timing distribution of these tracks can be found in Figure 10.1. Interactions in each of the bunches can be treated as separate events. The contents of each event are divided into bunches by their timing, each bunch being defined by the mean bunch time ±86 ns. If no reconstructed objects fall within a beam bunch window, the event is discarded, otherwise the objects that fall outside all bunch windows are discarded.

10.3.2 TPC and FGD Component Cuts

We only consider tracks that have both an FGD and a TPC constituent. Only tracks with an FGD component can originate in the FGD fiducial volume, so only these tracks are considered. The TPC is necessary for momentum and charge determination, and for Particle Identification (PID), so we can only detect muons that leave TPC tracks.
10.3.3 P0D Veto

Since we wish to isolate activity due to neutrino interactions in the FGD, we must veto interactions that occur upstream and then enter the FGD. The fiducial volume cut alone does not do this, as it only constrains the track with the highest measured momentum in the TPC, and is vulnerable to reconstruction failures that break the track in the middle of the FGD. Many analyses require there be no tracks in the upstream TPC, but this can be overly broad in the case of backwards-going secondary tracks, which can reach the TPC from an interaction in the upstream part of the FGD. Instead, this selection uses information from the P0D to veto incoming events.

All of the reconstructed tracks in a bunch are checked for components in the P0D. If any of the tracks has a P0D component that ends within 6 cm of the downstream face of the P0D, the event is vetoed. In the ND280 coordinate system, we veto events with any P0D segment ending or beginning at $z > -1000$ mm. (The downstream face of the P0D is at $z = -939$ mm.) As the P0D is very large, coincident contained interactions can occur,
so only the downstream end is used as a veto.

10.3.4 TPC Track Quality Cut

The TPC track curvature fit is a crucial reconstruction step for this analysis; it measures the particle momentum and charge, both of which are used in the selection. Thus, we must ensure that the fit is of sufficient quality that the values we obtain from it are reliable. Unfortunately, in the version of the reconstruction used for this analysis, the variance provided for the momentum is not accurate, and as such a proxy measurement to the fit quality must be used.

The number of TPC hits included in the track, or, equivalently, the number of degrees of freedom of the curvature fit, was the proxy used for quality of fit, as we expect longer tracks to have better-defined fits. We can see the distribution of track hit counts in the TPC in Figure 10.2. A threshold of 18 hits was used for this analysis. In the MC, this cut is seen to accept approximately 96% of the signal reaching it.
10.3.5 FGD Fiducial Volume Cut

The FGD fiducial volume includes cuts along all side of the FGD active volume except the downstream face. On the upstream side, we cut away 20 mm to use the first two scintillator layers as veto planes. Since we cannot distinguish an interaction actually occurring in the upstream scintillator layer from one occurring in the dead material upstream of the FGD, removing it from the fiducial volume increases our fiducial volume purity.

Similarly, we also cut 100 mm on each of the outside edges. This vetoes interactions in the dead materials in the FGD frames, but has additional purposes in FGD2. The water modules (Part II) are not square like the scintillator modules, but are longer in the vertical direction and narrower in the horizontal. Along the horizontal axis the 100 mm cut keeps the fiducial volume fully inside the uniform, water-filled part of the water modules. Additionally, the vertical ends of the water modules, though they protrude past the scintillator, are filled with sealant. Cutting 100 mm inside the extent of the scintillator, we exclude this portion.

The resulting FGD fiducial volume is an 1664 mm wide square perpendicular to the beam direction, and is 311 mm thick along the beam direction, for a total fiducial volume of $0.862 \text{ m}^3$. In the ND280 coordinate system, this is:

- $-832.17 \text{ mm} < x < 832.17 \text{ mm}$
- $-777.17 \text{ mm} < y < 887.17 \text{ mm}$
- $135.95 \text{ mm} < z < 447.05 \text{ mm}$ (FGD1), or
- $1493.95 \text{ mm} < z < 1807.05 \text{ mm}$ (FGD2)

This volume is illustrated in Figure 10.3.

The fiducial volume is defined identically for the two FGDs despite the differences in structure. This means that the most upstream part of the FGD2 fiducial volume is in a passive water module, possibly increasing the chance of event migration out of the fiducial volume. A Monte Carlo study found that this was not a significant effect.
Figure 10.3: Distribution of reconstructed vertices in XZ with FGD fiducial volume shown outlined in red. The vertices are chosen as the beginning of the muon candidate (highest momentum) track, and this graph shows the distribution immediately prior to the fiducial volume cut. The layer structure of FGD2 is visible in this plot, as the vertex locations are limited to be within the active area of the detector with this scheme.

10.4 Inclusive Charge-Current Interaction Selection

Once we have determined that the event contains a neutrino interaction in an FGD, we must determine whether this event is the result of a charge-current $\nu_\mu$ interaction. In practice, this means that we must establish whether the highest-momentum particle passing the cuts in Section 10.3 (the “muon candidate”) is a $\mu^-$. Three cuts are used for this selection:

1. The track must have a reconstructed charge that is negative
2. The track momentum must be greater than 200 MeV/c
3. The TPC PID ‘pull’ for the muon hypothesis must be less than 2.5, and that for the electron hypothesis must be greater than 2.5.

As this is an inclusive CC measurement, there is no constraint on other activity in the detector beyond that provided by the upstream veto.
Figure 10.4: Inverse momentum times charge of Monte Carlo muon candidates. Two peaks are seen, one positive and the other negative. The negative peak is primarily signal, while the positive peak is primarily background. Thus, when we require a negative charge we improve the signal/background ratio.

10.4.1 Charge Cut

The reconstructed charge of the particle, as determined by the sign of the track curvature in the magnetic field, must be negative. We use the charge fit in the immediate downstream TPC if available, otherwise the immediate upstream TPC is used. The global fit to the track using all available subdetector information is not used, as it has been found to be less accurate than the local TPC fit in the current reconstruction. The inverse momentum times charge of the muon candidates prior to this cut is shown in Figure 10.4, and we can see that the background is primarily positive, while the signal is primarily negative. Signal events may have positive muon candidate charge in the case where the candidate is incorrectly selected and is not the outgoing muon.

10.4.2 Momentum Cut

A momentum cut is used to cut down on the outside-FGD backgrounds. One way an event outside the FGD can be mistaken for one in the FGD is if
Figure 10.5: Momentum distribution of Monte Carlo signal and background muon candidates. We see that the backgrounds peak at low momentum and cross the signal at about 200 MeV/c. Lower-momentum particles in ND280 are more likely to go upstream or transverse to the beam, and are more likely to stop in the FGD if they encounter it. Thus, they are more likely to be erroneously reconstructed beginning in the wrong detector.

A π⁺ or other positive interaction product travels upstream and stops in a FGD. Current software cannot distinguish this, in general, from a negative track travelling downstream.

The kinematics of the neutrino interaction disfavour the production of high-momentum upstream-going particles, so a cut that removes low-momentum muon candidates should reduce this background with a relatively small effect on the signal. 200 MeV/c was chosen as the cut and was seen in the MC truth information to substantially improve the selection purity. The momentum distribution of the muon candidates for signal and background events is shown in Figure 10.5. The cut has an efficiency of approximately 96%, while increasing the purity from 69% to 81.
Figure 10.6: Predicted TPC energy loss as a function of momentum for a variety of particles. The differences are due to the difference in mass and therefore velocity at a given momentum, as the Bethe-Bloch formula predicts the energy loss of a particle is primarily a function of its velocity. This relation allows us to separate different particle types in the reconstruction.

10.4.3 PID Cut

The current reconstruction does not yet have an overall, global PID subsystem. Instead, this analysis used a PID based on a single TPC. This TPC is chosen to be the immediate downstream TPC, unless there is no component there, in which case the immediate upstream TPC is used instead. The TPC is able to measure the energy lost by a traversing particle and this information is used for particle identification.

The energy loss of a charged particle in matter is given by the Bethe-Bloch formula:

$$- \frac{dE}{dx} \propto \frac{1}{\beta^2} \left[ \frac{1}{2} \ln(K\beta^2\gamma^2) - \beta^2 \right]$$  \hspace{1cm} (10.15)

where $\beta$ and $\gamma$ are the relativistic kinematic variables of the moving particle, and $K$ is a constant depending on the properties of the matter. If we measure the momentum of the particle from its curvature, different masses will give us different values of $\beta$ and $\gamma$ and thus different values for the energy loss. Thus, measuring the energy loss allows us to distinguish between particles of different masses, and thus determine the identity of the observed particle.
Figure 10.7: Measured energy loss and momentum for simulated particles in the TPC, showing the expectation for three hypotheses as above (Figure 10.6). We can see that there are clear bands in the histogram corresponding to the muon and proton expectations, with a fainter band corresponding to electrons and very few kaons.

The predicted energy loss in the TPC as a function of momentum for muons, pions, electrons, and protons can be found in Figure 10.6. The distribution of reconstructed energy loss and momentum in the TPC for a Monte Carlo sample is shown in Figure 10.7, and can be seen to roughly match the structure of the predictions.

If we measure the energy loss and momentum of a particle traversing the TPC, we can calculate a set of ‘pulls’ for each possible particle. Since we have a theoretical prediction of what the relationship between energy loss and momentum for each particle is, we can quantify how well the data point fits each hypothesis by the pull

\[
pull = \frac{\text{data} - \text{prediction}}{\text{error}}
\]  

(10.16)

To select muons using these pulls, we require that the muon pull of the candidate track have a deviation from the mean of less than 2.5, and
the electron pull have a deviation that is greater than 2.5. Other particle hypotheses are not considered as they do not increase the sensitivity of the PID cut.

10.4.4 Calculating Pull Corrections

Since the TPC pull distributions are not centred on zero, we must subtract the mean if we are to have a symmetric cut on the peak of the distribution. This section details the process where these means were calculated. Each mean is calculated separately for each of the three TPCs, so there are six quantities that must be measured.

For the muon pull, a control sample of throughgoing particles was used to find the pull mean. The criteria for tracks to be included in this sample were:

1. Component in all three TPCs
2. More than 18 hits in each TPC segment
3. Track starts far upstream ($Z < -2000$ mm)

We then assume that these tracks are preponderantly muons, since we expect that any electrons would shower in the lead and brass sheets in the P0D, preventing them from producing long clean tracks. Thus, we use the fitted mean of their muon pull distribution as the correction in the main analysis PID cut. The muon pull distribution for the MC control sample can be found in Figure 10.8. The means of the pull distributions are corrected to zero, but their widths are not corrected to unity.

The process for correcting the electron pull is more complicated, as we do not have a control sample of electrons as we do for muons. Thus, we must instead extract a correction from the main sample of FGD neutrino interactions, which contains both muons and electrons. The electron pull distribution of this sample will then have two peaks: one near zero corresponding to the electrons, and another away from zero corresponding to the muons.
Figure 10.8: Muon hypothesis pull of Monte Carlo control sample tracks. This quantity measures the deviation of the observed energy loss from the energy loss expected for its momentum if we assume it is a muon. The mean of this distribution is used to correct the muon pull values in the analysis to have a mean of zero.

We can characterize the muon peak in the electron pull distribution using the control sample. A fit to this sample, as shown in Figure 10.9, gives us the mean and width of the muon pull for use in fitting the distribution from the neutrino sample. That distribution is fit with a double Gaussian, one for each peak, as shown in Figure 10.10. The fit for the muon peak is seeded with the mean from the control sample, and its width is fixed to that from the control sample fit. The mean found for the other Gaussian is then used as the correction to the electron pulls for the PID cut.

10.5 Results of Selection

The Monte Carlo sample used for this study used the GENIE neutrino interaction simulation\cite{47} to generate neutrino interactions throughout the body of the ND280 detector, including both the magnet and the detector mass
Figure 10.9: Electron pull of muon control sample. This allows us to characterize the peak in the total electron pull distribution corresponding to the muons, and thus to fit for the peak in the final distribution (Figure 10.10) corresponding to the electrons.

Table 10.2: Cumulative selection efficiencies at each analysis cut, split out per FGD. The columns of absolute event numbers show the number of true signal events selected at each stage of the analysis.
Figure 10.10: FGD Neutrino interaction electron pull distribution fit with double Gaussian. The left-hand peak contains muons and is similar to that shown in Figure 10.9, while the right peak contains electrons. It is the right peak we want to centre at zero and thus the right peak mean is used to correct the electron pull in the final analysis.

<table>
<thead>
<tr>
<th>Cut</th>
<th>FGD1</th>
<th>FGD2</th>
<th>Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spill Timing</td>
<td>96.9</td>
<td>93.7</td>
<td>3.25</td>
</tr>
<tr>
<td>Global PID Validity</td>
<td>100.0</td>
<td>100.0</td>
<td>0.03</td>
</tr>
<tr>
<td>TPC Track</td>
<td>86.0</td>
<td>88.7</td>
<td>-2.72</td>
</tr>
<tr>
<td>FGD Segment</td>
<td>98.6</td>
<td>97.3</td>
<td>1.32</td>
</tr>
<tr>
<td>P0D Veto</td>
<td>97.1</td>
<td>99.1</td>
<td>-1.98</td>
</tr>
<tr>
<td>TPC Track Length</td>
<td>96.6</td>
<td>96.6</td>
<td>0.01</td>
</tr>
<tr>
<td>FGD Fiducial</td>
<td>90.2</td>
<td>90.6</td>
<td>-0.41</td>
</tr>
<tr>
<td>TPC Negative</td>
<td>67.9</td>
<td>68.3</td>
<td>-0.37</td>
</tr>
<tr>
<td>Momentum</td>
<td>96.6</td>
<td>95.6</td>
<td>1.02</td>
</tr>
<tr>
<td>Muon PID</td>
<td>89.6</td>
<td>89.4</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 10.3: Efficiencies of each analysis cut (%), relative to the efficiency of the previous cut, with the difference of the two FGDs for each cut shown.
Table 10.4: Selection purities at each analysis cut (%). No purities are available for the separate FGDs prior to the FGD fiducial volume cut, as it is there that the two FGDs are distinguished in the analysis.

<table>
<thead>
<tr>
<th>Purity</th>
<th>FGD1</th>
<th>FGD2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Cuts</td>
<td>-</td>
<td>-</td>
<td>0.4</td>
</tr>
<tr>
<td>Spill Timing</td>
<td>-</td>
<td>-</td>
<td>0.8</td>
</tr>
<tr>
<td>Global PID Validity</td>
<td>-</td>
<td>-</td>
<td>0.8</td>
</tr>
<tr>
<td>TPC Track</td>
<td>-</td>
<td>-</td>
<td>5.9</td>
</tr>
<tr>
<td>FGD Segment</td>
<td>-</td>
<td>-</td>
<td>8.1</td>
</tr>
<tr>
<td>P0D Veto</td>
<td>-</td>
<td>-</td>
<td>12.2</td>
</tr>
<tr>
<td>TPC Track Length</td>
<td>-</td>
<td>-</td>
<td>12.6</td>
</tr>
<tr>
<td>FGD Fiducal</td>
<td>58.3</td>
<td>51.0</td>
<td>54.4</td>
</tr>
<tr>
<td>TPC Negative</td>
<td>73.2</td>
<td>65.9</td>
<td>69.4</td>
</tr>
<tr>
<td>Momentum</td>
<td>84.3</td>
<td>78.5</td>
<td>81.3</td>
</tr>
<tr>
<td>Muon PID</td>
<td>91.8</td>
<td>89.6</td>
<td>90.7</td>
</tr>
</tbody>
</table>

inside the magnet. This is equivalent to approximately $3.9 \times 10^{20}$ Protons On Target (POT) of neutrino beam. The configuration of ND280 in the simulation mirrors the state of the detector as of February 2010.

The selection was run over the full ND280 data set from T2K Run 1 (February-June 2010). This represents $3.0 \times 10^{19}$ POT of well-captured beam exposure.

The efficiency and purity of the selection through the analysis cuts is described in Table 10.2 and Table 10.4, respectively. Separate purities are not available for the two FGDs prior to the fiducial volume cut as that is the step where the selection is split between the FGDs. The purities were constructed as the fraction of the selected events that have a true $\nu_\mu$ charge-current interaction vertex within the FGD fiducial volume. Only events in the correct fiducial volume were considered for the separate FGD1 and FGD2 purities. The efficiency and purity of the selection are plotted in Figure 10.11 for all FGD interactions, in Figure 10.12 for FGD1, and in Figure 10.13 for FGD2.

Of the 10% background in the selected sample, approximately 6% of it is
Figure 10.11: Efficiency and purity of entire selection through the series of analysis cuts. The muon selection cuts (after TPC Negative) are seen to give substantial purity gains with minimal efficiency penalty.

Figure 10.12: FGD1 efficiency and purity, showing the same overall shape as Figure 10.11.
Figure 10.13: FGD2 efficiency and purity, showing the same overall shape as Figure 10.11 and Figure 10.12.

<table>
<thead>
<tr>
<th>Interaction Channel</th>
<th>FGD1 %</th>
<th>FGD2 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCQE</td>
<td>5353</td>
<td>5186</td>
</tr>
<tr>
<td>CC Resonant</td>
<td>2371</td>
<td>2352</td>
</tr>
<tr>
<td>CC Coherent</td>
<td>88</td>
<td>87</td>
</tr>
<tr>
<td>CC DIS</td>
<td>1865</td>
<td>1918</td>
</tr>
<tr>
<td>Neutral Current</td>
<td>321</td>
<td>340</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>9998</td>
<td>9883</td>
</tr>
</tbody>
</table>

Table 10.5: Charge-current interaction classification by interaction channel for Monte Carlo events. Includes backgrounds from outside the FGD fiducial volume, and shows the primacy of CCQE interactions at the T2K beam energies.

charge-current $\nu_\mu$ interactions outside the fiducial volume erroneously reconstructed within the fiducial volume (discussed in Section 11.10), and approximately 3% are $\nu_\mu$ neutral current interactions (discussed in Section 11.9). The remaining 1% of the selected events are interactions of other neutrino types, mainly $\bar{\nu}_\mu$ and $\nu_e$. Table 10.5 shows the division of the selected CC events between the different interaction channels.
Chapter 11

Systematic Error Studies

11.1 Systematic Effects for Subtraction Analysis

The constraints of the subtraction analysis make the set of relevant systematic errors different than for a basic counting analysis. Systematic effects that are correlated between the two FGDs are expected to cancel, while the systematic differences between the two FGDs and their respective downstream TPCs add additional systematic errors. We consider the following detector systematics:

- Differences in tracking efficiency between TPCs (Section 11.4)
- Differences in charge misidentification probabilities between TPCs (Section 11.5)
- Differences in granularity and active volume due to the different structure of the two FGDs (Section 11.6)
- Differences in momentum resolution between TPCs (Section 11.7)
- Differences in PID efficiencies between TPCs (Section 11.8)
- Differences in efficiency due to the different solid angle covered by the upstream veto volume (Section 11.11)
• Differences in TPC time determination efficiency (Section 11.11)

• Differences in migration between the two FGDs due to the assumption of downstream motion (Section 11.11)

Each of these systematics are considered to affect the efficiency of the selection in the two FGDs. Another detector systematic with a different mechanism for action is the determination of the FGD masses and the FGD2 mass fractions between water and plastic; the errors in this measurement process affect the masses used for scaling the rate and is described in Section 11.3.

Along with detector systematics, we also must consider the modelling of our backgrounds in the Monte Carlo. The backgrounds we must consider are:

• Uncertainties in the difference of the neutral current interaction rate between water and plastic (Section 11.9)

• Interactions outside the FGD reconstructed inside the FGD (Section 11.10)

• Migration of events between FGDs (Section 11.11)

• Contamination of sample by cosmic rays (Section 11.11)

These systematics are considered to affect the purity of the selection in the FGDs.

11.2 General Methodology

Each systematic effect was evaluated using a toy Monte Carlo. We calculate the efficiency-corrected ratio of event rates in FGD2 and FGD1 using Equation 10.12, and vary the efficiency, density, and purity terms by the underlying distribution of each systematic effect. The resulting values of $R$ are then histogrammed and fit with Gaussians to determine the systematic error on the ratio from each effect. The Monte Carlo values of $N_1$ and $N_2$ were used in all cases.

Detector systematics were treated as modifying the efficiency; we treat the overall efficiency as calculated with the Monte Carlo truth information.
as the product of the efficiencies of each step in the selection. Thus if we have an efficiency in a given step \( \epsilon_i \) from another source, with an error or \( \Delta \epsilon_i \), we can decompose the known efficiency as

\[
\epsilon = \epsilon_r \cdot \epsilon_i
\]  

(11.1)

where \( \epsilon_r \) is a ‘remnant’ efficiency representing the rest of the operations. In general, we have separate values for \( \epsilon, \epsilon_i, \) and \( \Delta \epsilon_i \) for each FGD.

To find the systematic error on the ratio, we then draw changes in efficiency \( \delta \) from a unit Gaussian and use it to modify the efficiencies of the two FGDs as

\[
\epsilon'_1 = \epsilon_{r1} \cdot (\epsilon_{i1} + \Delta \epsilon_{i1} \delta)
\]  

(11.2)

and

\[
\epsilon'_2 = \epsilon_{r2} \cdot (\epsilon_{i2} \pm \Delta \epsilon_{i2} \delta)
\]  

(11.3)

where the sign of \( \Delta \epsilon_{i2} \delta \) in Equation 11.3 is positive if the effect is correlated between the two FGDs and negative if the effect is anticorrelated between the two FGDs. We then calculate the ratio as above with the new efficiencies and fit the resulting distributions with Gaussians to find the systematic error on the ratio from this effect.

Backgrounds were treated as modifying the purity; if we throw a change in the background of \( \delta N_b \) the new purity is given by

\[
P' = 1 - \frac{N_b + \delta N_b}{N + \delta N_b}
\]  

(11.4)

where \( N \) is the total event count in the detector and \( N_b \) is the number of background events calculated as \( N_b = N(1 - P) \). As with the efficiencies discussed above, we can have correlated or anticorrelated \( \delta N_b \) values for the two FGDs.

### 11.3 FGD Masses

The calculation of the water and plastic densities of FGD2, as described in Chapter 6 is based on a variety of measurements, each with their own
variance. The error on these measurements is propagated to an error on the final rate ratio using a toy MC over 18 input variables:

- Seven correlated elemental densities of the XY modules
- Seven correlated elemental densities of the common parts of the water modules
- The total areal density of water in FGD2
- The areal densities of the two types of polycarbonate panels
- The areal density of the tape used to attach the G10 spacers

The SciPy random number generator is used to calculate the correlated random numbers. For each trial, the corrected rate ratio was calculated using Equation 10.12 and the Monte Carlo event counts. The resulting distribution of rate ratios can be found in Figure 11.1. A Gaussian was fit to this distribution, whose mean is the central value of the cross-section ratio and whose width is the value of the systematic error. The final value of the systematic error was found to be 0.93%.

### 11.4 TPC Tracking Efficiency

This systematic error was evaluated based on a study done by Anthony Hillairet[62]. In this study, clean throughgoing muon tracks were identified in a subset of the available detectors, and the presence of a track in an additional detector is checked. The track criteria are chosen such that this additional track segment is almost certainly present in reality. TPC1 and TPC3 tracks were used to check the TPC2 efficiency, while TPC2 and P0D tracks were used to check the TPC1 efficiency. The unavailability of tracking information in the downstream ECAL made the measurement of TPC3 efficiency somewhat more difficult; TPC1 and TPC2 were used as the reference while the output angle from TPC2 was constrained so that a track would be expected in TPC3. The efficiency was calculated as

$$\epsilon_{\text{TPC}} = \frac{N_t}{N_r} \quad (11.5)$$
where $N_r$ is the number of reference tracks, and $N_t$ is the number of reference tracks with tracks in the TPC under study.

The efficiency was measured separately for each Main Ring run and for the Monte Carlo (shown as Run 40). The MC efficiency was taken as the central value of this efficiency, and the variance was found by choosing the error bar limit that was furthest from that value and symmetrizing it around the central value. To be conservative and maximize any possible effect, the errors in the two FGDs were considered to be completely anticorrelated. We find the efficiency for TPC2 to be $(99.5 \pm 0.6)\%$ and that for TPC3 to be $(99.2 \pm 0.5)\%$. The final value of the systematic error on the cross-section ratio was found to be $2.39\%$.

### 11.5 TPC Charge Misidentification

The charge misidentification in the TPCs was studied by Javier Caravaca and described in T2K technical note 48[63]. The control sample used for this test was made by selecting tracks that begin within the fiducial volume.
of the P0D, with times consistent with beam bunches, and components in all three TPCs with at least 40 degrees of freedom each.

If we measure the fraction of these tracks with consistent charge in all three TPCs, $P_{\text{same}}$, and assuming that each TPC has the same probability $P_{\text{wrong}}$ of reconstructing the wrong charge, we can relate the measurement to the charge misidentification probability as

$$P_{\text{same}} = (1 - P_{\text{wrong}})^3 + P_{\text{wrong}}^3 \quad (11.6)$$

or, solving for $P_{\text{wrong}}$,

$$P_{\text{wrong}} = \frac{1}{2} \left( 1 - \sqrt[3]{\frac{4P_{\text{same}} - 1}{3}} \right) \quad (11.7)$$

However, we would like to study the difference in charge misidentification
properties between TPC2 and TPC3, so a more complicated formalism than Equation 11.7 is necessary.

For simplicity, we assume that TPC1 and TPC2 have the same probability of charge misidentification. The largest difference between the three TPCs is that TPC3 is nearer the edge of the magnet, giving it a much less uniform magnetic field than the other two. As such, we can treat TPC1 and TPC2 as identical, with a charge misidentification probability which can be calculated from the fraction of tracks with the same charge in the two detectors analogously to Equation 11.7:

\[ P_{\text{same}12} = (1 - P_{\text{wrong}12})^2 + P_{\text{wrong}12}^2 \Rightarrow P_{\text{wrong}12} = \frac{1}{2} \left( 1 - \sqrt{1 - 2(1 - P_{\text{same}12})} \right) \]  

(11.8)

Then, we can do the same two-TPC comparison with TPC2 and TPC3, i.e.,

\[ P_{\text{same}23} = (1 - P_{\text{wrong}12})(1 - P_{\text{wrong}3}) + P_{\text{wrong}12}P_{\text{wrong}3} \]  

(11.9)

which if we know \( P_{\text{wrong}12} \) from above gives us \( P_{\text{wrong}3} \) as

\[ P_{\text{wrong}3} = \frac{P_{\text{same}23} + P_{\text{wrong}12} - 1}{2P_{\text{wrong}12} - 1} \]  

(11.10)

The study of charge misidentification determined the misidentification probability as a function of momentum, as shown in Figure 11.3. As a result, we must convolve the momentum distribution of the selected tracks with this probability to find the overall charge misidentification probability. This was done using the Monte Carlo momentum distribution for both the data and Monte Carlo misidentification probabilities. The resulting misidentification probabilities can be found in Table 11.1.

The difference in the misidentification probabilities was treated as the central value for the inefficiency due to this effect, which was varied by its error in the toy Monte Carlo. To be conservative, this error was taken as anticorrelated between the two FGDs. The resulting systematic error was found to be 1.43%.
Figure 11.3: TPC charge misidentification probability as a function of momentum, shown for both TPC2 and TPC3. These probabilities were convolved with the observed momentum distribution to find a single value of the charge misidentification probability for each case (shown in Table 11.1).

Table 11.1: Charge misidentification probabilities for the CC inclusive selection, as extracted from the momentum-dependent probabilities in Figure 11.3 through convolution with the momentum distribution of the selected muons. The data-MC difference was used as the variation in the efficiency for the systematic error propagation.
11.6 Differences in FGD Granularity

One significant source of systematic differences between the two FGDs is their different fraction of active versus dead material, and resulting difference in granularity in the beam direction. Active layers in FGD1 are adjacent, with very little dead material between them in proportion to the thickness of the active material, while the active modules in FGD2 are separated by more than 2.5 cm of passive water module. As such, tracks in FGD2 have fewer hits and may be lost entirely if enough of the particle’s path passes through a passive water module. The matching of TPC tracks to FGD hits must project farther to each successive layer, possibly modifying its effectiveness.

The difference in the structure of the FGDs could also affect the fiducial volume cut. Interactions in the FGD2 dead material outside the fiducial volume could reconstruct inside the FV if the interaction products enter it while travelling through the dead material.

11.6.1 The Reduced FGD1 Study

It is possible to alter the FGD1 data such that its granularity becomes much more closely comparable to FGD2. If we discard all hits in every other module, we produce an FGD with eight active modules separated by seven dead modules. While this is not an exact facsimile of FGD2 due to the greater width of the water modules, we can directly compare the FGD1 results in this case with the standard FGD1 results in both Monte Carlo and data to constrain the systematic effects of the difference in the two FGDs. In addition, we expect most of the differences between the reduced FGD1 and FGD2 to be modelled well by the Monte Carlo; since we expect the cross-section difference to be small, the most significant difference is in the amount of dead material between layers. A water layer in FGD2 is 2.8 g/cm$^2$ of dead material, while a deactivated XY module is 2.1 g/cm$^2$; the simulated geometry reflects these numbers well and thus should not contribute to the systematic error.

For the reconstruction to function with the altered FGD1 data identically to data taken with an eight-module FGD1, the numbering of the layers must
Table 11.2: Selected event counts for full and reduced FGD1 selections in Monte Carlo. The differences seen between the two cases are small, suggesting that the granularity of the FGDs is not a large effect for an inclusive CC measurement.

<table>
<thead>
<tr>
<th></th>
<th>FGD1</th>
<th>FGD2</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>3446</td>
<td>3357</td>
<td>0.9741</td>
</tr>
<tr>
<td>Reduced</td>
<td>3428</td>
<td>3341</td>
<td>0.9745</td>
</tr>
</tbody>
</table>

be modified such that the remaining layers are contiguously numbered, as the layer numbers are used in the reconstruction to establish the continuity of a track. With added software to effect this renumbering, the reconstruction and analysis were re-run on data and Monte Carlo samples and compared to the results with the standard reconstruction. We expect the efficiency for FGD1 to go down while that for FGD2 should stay the same, but small changes to the FGD2 rate are possible. For an example, an event could migrate from FGD1 to FGD2 if all of its FGD1 hits are removed by the reduction.

The results for the GENIE Monte Carlo can be found in Table 11.2. Due to time constraints, the reduced reconstruction was only run on approximately 30% of the Monte Carlo sample; the full FGD1 counts in Table 11.2 are from running over the same events. We see that the event rates are reduced by approximately 0.5% but that the FGD1/FGD2 ratio is consistent between the two cases. This suggests that the difference in structure between FGD1 and FGD2 had little effect on the final results for the two detectors, and corroborates what we see in Table 10.3, where the fiducial volume cut has very similar efficiencies in the two cases. The data, as shown in Table 11.3, shows a similarly small effect to the Monte Carlo. While the ratio differs by approximately 0.5%, the low number of data events makes this difference statistically insignificant. Thus, systematic effects due to the difference in FGD structure were considered to be negligible.
<table>
<thead>
<tr>
<th></th>
<th>FGD1</th>
<th>FGD2</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>705</td>
<td>757</td>
<td>1.074</td>
</tr>
<tr>
<td>Reduced</td>
<td>701</td>
<td>758</td>
<td>1.081</td>
</tr>
</tbody>
</table>

**Table 11.3:** Event counts for full and reduced FGD1 selections in data. The fractional difference in event counts is larger than for the Monte Carlo (Table 11.2), but the difference is still statistically insignificant.

### 11.7 TPC Momentum Cut

The difference in momentum resolution between the TPCs can affect the efficiency of the TPC momentum cut. Since TPC3 is nearer to the end of the magnet, the magnetic field strength varies more through the body of TPC3 than in the other two TPCs. We would expect that TPC3 would produce a less precise measurement of the momentum as a result. This becomes a systematic data/MC difference since the simulation used for this analysis was run with a perfect, constant magnetic field in the TPC. Casey Bojchek has studied the systematic migration of events between momentum bins due to these effects, and he was able to apply his work to the case of a 200 MeV/c momentum cutoff. This work is based on T2K Technical Note 71\[64\].

Before the installation of the ND280 subdetectors in the fall of 2009, the magnetic field within the UA1 magnet was mapped. We can refine this map within the TPC using the calibration pattern printed on the central cathode. This pattern is scanned with a photoionization laser to produce a charge image which drifts through the TPC and is detected. An empirical distortion correction was calculated by comparing the observed pattern in data with the simulated calibration pattern. An example distortion map for one TPC can be found in Figure 11.4; the circles represent the nominal positions of each calibration target, and the lines show the deviation of the observed image from nominal, magnified by a factor of 20.

The dataset was reprocessed three times with three different magnetic field states used in the reconstruction and calibration. First, it was run
with the same perfect field used in the simulation. Then, it was re-run with both the field map and the empirical distortion correction, and, finally, with just the empirical correction. In each of these cases the fraction of tracks below the 200 MeV/c momentum cut was calculated. The difference between the perfect field and the mapped field was used as a correction to the MC efficiency, while the difference between the mapped field and empirical correction was used as the error on that correction. The errors in the two FGDs were treated as anticorrelated to set a conservative upper limit on the effect. The final systematic error was found to be 0.39%.

11.8 TPC Muon PID

The systematic errors due to the muon PID process were studied by Claudio Giganti[65]. While it was applied to a newer version of the PID code, the
Table 11.4: Particle identification efficiencies for muon control sample, as supplied by Claudio Giganti[65]. The data-MC difference was used as the error on the efficiency for systematic error propagation.

<table>
<thead>
<tr>
<th>TPC</th>
<th>Data</th>
<th>MC</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>98.8%</td>
<td>99.1%</td>
<td>0.3%</td>
</tr>
<tr>
<td>3</td>
<td>98.5%</td>
<td>99.5%</td>
<td>1.0%</td>
</tr>
</tbody>
</table>

method is the same that was used in the 2010 ND280 analysis described in T2K Technical Note 15[66]. This method used a control sample of clean throughgoing tracks that were assumed to be muons, and calculated the fraction of these tracks that pass the PID criteria for both data and Monte Carlo and both TPC2 and TPC3. These events were required to have exactly one track in each TPC with at least 60 TPC hits each and the reconstructed track momentum was required to be above 1 GeV/c.

In the end, the PID selection efficiencies for both TPC2 and TPC3 were estimated for both data and Monte Carlo, and can be found in Table 11.4. The difference between the two was taken to be the systematic error on the Monte Carlo selection efficiency.

Although the PID cut used in the selection used the upstream TPC when the downstream TPC was unavailable, this was a small enough fraction of the events that the TPC2 number was used for FGD1 and the TPC3 number for FGD2 for determining the systematic error. To be conservative, the two FGDs were taken to be anticorrelated during the toy error Monte Carlo. We find a 2.83% systematic error in the final ratio due to this effect.

11.9 Neutral Current Interaction Background

Upon a suggestion from the T2K Neutrino Interaction Working Group conveners[67], the uncertainty on the neutral current background in water was taken to be 10%. In this analysis the difference in neutral current cross-section between water and plastic was taken to be independent of the charge-current cross-section difference, to provide an upper bound on the effect. The neutral
current rate in FGD2 was estimated using the Monte Carlo truth information, and the rate in water was estimated by multiplying this by the fraction of the FGD2 mass that is made of water (47%). This rate was then varied by 10% in the toy Monte Carlo and the resulting variance in the ratio was found from a Gaussian fit. This systematic applies only to FGD2, so the FGD1 purity is not changed by this toy Monte Carlo.

11.10 Outside Fiducial Volume Backgrounds

There are a variety of ways an interaction outside the FGD fiducial volume can be reconstructed as one inside the FGD fiducial volume. Each of these components of the out-of-FGD background can be affected by different systematic effects. There are three broad categories of these backgrounds: neutral particles entering the FGD, charged particles originating outside the tracker, and charged particles originating inside the tracker. Neutral particles (mainly neutrons and photons) do not represent a reconstruction failure as the neutral particle is invisible to the detector, and thus we do not apply reconstruction systematics to them. The Tracker mainly consists of light elements, which we believe are well-modelled by our neutrino interaction generators, so we only apply reconstruction systematics, not interaction rate systematics, to background events originating in the Tracker. Finally, interactions outside the Tracker can be on heavy nuclei such as iron or lead, where the nuclear interaction models are more uncertain, so we apply a rate uncertainty as well as the reconstruction systematics to these backgrounds.

Four reconstruction systematics were identified that were relevant to the out-of-FGD background in the subtraction analysis:

1. Differences in the TPC/FGD matching $\chi^2$ distributions
2. The “Layer 23” track breaking failure mode
3. Track breaking for very horizontal tracks due to multiple skipped layers
4. Reconstruction problems for high-angle tracks in the FGD
These four effects and the overall rate uncertainty will be discussed further below. The majority of this work uses the results in T2K Technical Note 098[68].

11.10.1 Rate Uncertainty

While the ND280 Tracker consists mainly of light elements, the remainder of the detector includes a number of components made of iron, brass, or lead. Since the neutrino interaction cross-section is better understood for light elements, the Monte Carlo prediction for the interaction rate outside the Tracker is more uncertain. We have two different neutrino interaction simulations, NEUT and GENIE, as well as the neutrino interaction data, so we can estimate this rate uncertainty by comparing the data and MC rates for each non-Tracker subdetector. For each of these, the difference in data/MC rate ratio between the two different MC generators was calculated and the largest of these differences used as the systematic rate uncertainty. We found the largest difference between NEUT and GENIE for the magnet/SMRD, which was about 20%. This effect is the same in both FGDs and is treated as perfectly correlated.

11.10.2 FGD Matching $\chi^2$ Cut Uncertainty

When matching TPC tracks to FGD hits, we calculate a $\chi^2$ for each hit describing how well it matches with the TPC track and whatever prior FGD hits were selected. The acceptance cut on this $\chi^2$ is the same for both Monte Carlo and data, but we discovered that the distribution of this variable is different in the two situations, as can be seen in Figure 11.5. While they have roughly the same shape, the MC $\chi^2$ distribution is narrower by a factor of 1.3. Thus, we are accepting hits further onto the tail of the distribution for data than for MC.

The size of this systematic effect was estimated by re-running the reconstruction over the MC with a $\chi^2$ cut lower by a factor of 1.3, and calculating the difference in the total number of out-of-FGD backgrounds after the CC inclusive selection. The uncertainty thus assigned was 10%. As with the
Figure 11.5: Matching $\chi^2$ of FGD hits to projected TPC tracks (data in black, MC in red). A discrepancy is seen between data and Monte Carlo with the MC having noticeably lower $\chi^2$ values than the data. This result is consistent with the MC hits having a $\chi^2$ distribution that is narrower by a factor of 1.3 than the data hits. Reconstruction of the MC was re-run with a corresponding lower $\chi^2$ cut for estimation of the background due to this effect.

rate uncertainty, this is correlated between the two FGDs.

11.10.3 Layer 23/37 Problem

The version of the reconstruction used for this analysis has a bug that is generally called the ‘Layer 23 problem’. When matching hits in the FGD to tracks in the TPC, an incremental method is used that evaluates hits in both projections in layer order, and then uses those hits to refine the matching criteria for deeper hits. If a certain number of hits in one projection are matched without any intervening hits in the other projection, the matching ends. We can see this in Figure 11.6 as a spike in the number of throughgoing tracks reconstructed as ending in layer 23.
Figure 11.6: Reconstructed end layer of throughgoing FGD tracks (data in black, MC in red). We expect these tracks to all reconstruct to layer 0, so any other value generally indicates a reconstruction failure. Two problems are visible here: the spike at layer 23, described in Section 11.10.3, and the overall excess of data over Monte Carlo at intermediate layer numbers, attributed to the double-skip errors of Section 11.10.4.

The method of operation of the TPC means that the hit position along the drift direction (the X-direction) is found in a more complicated fashion than the position along the other two directions. Thus, there can be mistakes in the X position of TPC activity which has correct positions along the other two axes. When such activity is matched to the FGD, the YZ extrapolation of the TPC track is correctly aligned with the associated FGD hits but the XZ extrapolation is mispositioned and does not find any FGD hits.

In this situation, the FGD/TPC matcher will match YZ FGD hits until the condition described above is triggered, at which point the matching will end. In FGD1, the last layer matched will be layer 23. (The corresponding layer in FGD2 is layer 37.) Thus, we have a large excess of tracks that end in layer 23, many of which are throughgoing or otherwise originating outside the FGD fiducial volume.

The systematic error on the background due to these events was estimated at 150%, as a large excess of data tracks over Monte Carlo events was
observed to be ending in layer 23, as shown in Figure 11.6. We assume that this effect is the same in both FGDs as the matching conditions and TPC distortions are the same for both.

### 11.10.4 Double-Skip Errors

This is a second systematic due to the ending condition of the TPC/FGD matching software. If no hits are found in more than one consecutive layer, the matching stops, creating a track beginning in the FGD fiducial volume. In most situations, this is not problematic, but for particle trajectories that are close to horizontal, such a situation can arise from the positioning of the FGD dead material. A near-horizontal track can pass through the dead material of bars in multiple layers, producing a long interruption in the set of hits produced by the particle.

We see in the graph in Figure 11.6 that the number of tracks ending in each layer between the front two and layer 23 is roughly uniform, and differs significantly between data and Monte Carlo. We attribute these failures to the double-skip effect and observe that the difference between the data and Monte Carlo is approximately a factor of 2. As a result, the number of these events is assigned a 100% systematic error. Since we do not know how correlated this effect is between the two FGDs, we treat it conservatively as anticorrelated in the error Monte Carlo.

### 11.10.5 High-Angle Tracks

The TPC/FGD matching used in this analysis had poor performance for steep tracks, which often caused them to break partway through the FGD creating an apparent event in the FGD fiducial volume. Particles affected by this problem would have been generated in the ECAL or magnet surrounding the FGD and thus are not vetoed by the P0D upstream veto.

The systematic error in this case was estimated using throughgoing cosmic rays. After selecting only cosmic rays that traverse the FGD at a high angle, the rate at which the track was reconstructed beginning in the fiducial volume was measured. The data/Monte Carlo ratio of this rate was found
Table 11.5: Classification of outside FGD fiducial volume background events, following the scheme in Section 11.10.6. The different background types have different systematic errors associated with them, as described throughout Section 11.10.

<table>
<thead>
<tr>
<th>Classification</th>
<th>FGD1 %</th>
<th>FGD2 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Tracker</td>
<td>175</td>
<td>242</td>
</tr>
<tr>
<td>Parent Neutron or Gamma</td>
<td>40</td>
<td>43</td>
</tr>
<tr>
<td>Backward</td>
<td>128</td>
<td>152</td>
</tr>
<tr>
<td>High Angle (FGD sides)</td>
<td>34</td>
<td>37</td>
</tr>
<tr>
<td>Very Forward (2 missed layer)</td>
<td>39</td>
<td>46</td>
</tr>
<tr>
<td>Layer 23/37</td>
<td>36</td>
<td>37</td>
</tr>
<tr>
<td>Everything Else</td>
<td>96</td>
<td>153</td>
</tr>
<tr>
<td>Total</td>
<td>548</td>
<td>710</td>
</tr>
</tbody>
</table>

Table 11.5: Classification of outside FGD fiducial volume background events, following the scheme in Section 11.10.6. The different background types have different systematic errors associated with them, as described throughout Section 11.10.

to be $1.15 \pm 0.30$. The systematic error was taken to be the linear sum of the error on the ratio and its deviation from unity, i.e. it was measured to be 45%. As with the double-skip error, the lack of knowledge of the parallels between the FGDs leads to the conservative assumption of anticorrelated errors.

11.10.6 Categorization of External Backgrounds

To determine the size of each background in the event selection, a simplified and extended version of a categorization scheme developed by Fanny Dufour[68] was used on the Monte Carlo events with interaction vertices outside the FGD fiducial volume. The following criteria were applied to the selected events in order, making 7 categories:

1. True vertex inside the tracker volume
2. Parent or grandparent particle neutral
3. Backwards-going (true track direction pointing upstream)
4. High-angle track (true track direction $\cos \theta$ positive but $> 0.5$)
5. Very forward track (reconstructed track slope in XZ or YZ plane with magnitude less than 0.07)

6. Reconstructed vertex in layer 23/37

7. All other events

The result of this categorization on the background Monte Carlo events can be found in [Table 11.5].

The matching $\chi^2$ uncertainty (Section 11.10.2) was applied to events in all categories, while the rate uncertainty (Section 11.10.1) was applied to the sum of all categories besides category 1. The other uncertainties each applied to specific categories: the layer 23/37 error (Section 11.10.3) to category 6, the double-skip error (Section 11.10.4) to category 5, and the high-angle error (Section 11.10.5) to category 4. The systematics are varied simultaneously in a toy Monte Carlo, and the resulting systematic error was found to be 2.06%.

11.11 Other Systematic Effects

There are several systematic effects that were considered that were found to be negligible. As the P0D veto criterion is the same for both FGDs, it has a smaller effect on FGD2 due to the smaller solid angle covered by the veto volume. While this in principle affects the numbers of both signal and background events, the background should be treated correctly by the analysis in Section 11.10 above, leaving only the signal for consideration. In the Monte Carlo, we found that no signal events surviving the other selection cuts were cut by the P0D veto, making the efficiency difference a negligible systematic.

In principle, there could be differences in the frequency of failure of the TPC $T_0$ determination between the two TPCs. In particular, TPC2 has both FGDs available in which to find the $T_0$, while TPC3 only has FGD2. The $T_0$ sets the X-offset of the TPC, meaning that without a $T_0$ we are unable to reconstruct a 3D track in the TPC. It was found that these $T_0$ failures
were very rare in all cases (<0.25%) and as such this was not considered a significant systematic.

Relative timing of hits in the two FGDs determines the direction of a track, but this was not considered in the reconstruction. We found that this had no effect on the neutrino event selection, as none of the tracks with timing contradicting their reconstructed directions passed the fiducial volume cut when considered in either direction. As such, any systematic differences between the two FGDs are irrelevant.

We expect some background from events occurring in one FGD fiducial volume and being reconstructed in the other. If this migration happens preferentially in one direction, this can lead to errors in the final ratio. In the Monte Carlo, we do see more FGD1 events migrated to FGD2 than the converse, but the total size of this background was found to be very small (0.1–0.2%). As a result, this systematic is considered to be negligible.

In principle, cosmic rays could contaminate the beam spill data. These are not modelled in the Monte Carlo and would be a systematic difference between the data and simulation. However, the beam spill cut has a total time window of about 1 µs, which for the few Hz cosmic ray rate is unlikely to contain a significant number of cosmic rays.

11.12 Summary of Systematics

The final table of systematic errors can be found in [Table 11.6](#). We find that the error budget of this analysis is dominated by statistical errors; the limited amount of data gives us a 12% statistical error. The most significant detector systematics are the systematic on the muon PID and the systematic on the TPC tracking efficiency, while the out-of-FGD fiducial volume background provides the next largest systematic.
### Table 11.6: Systematic errors on cross-section ratio, shown as a percentage of the MC ratio. We see that the measurement is statistics-limited, with virtually all of the final error coming from the statistical error.

<table>
<thead>
<tr>
<th>Systematic</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical</td>
<td>11.91</td>
</tr>
<tr>
<td>FGD Mass Breakdown</td>
<td>0.93</td>
</tr>
<tr>
<td>TPC Tracking Efficiency</td>
<td>2.39</td>
</tr>
<tr>
<td>Charge Confusion</td>
<td>1.43</td>
</tr>
<tr>
<td>Momentum Cut</td>
<td>0.39</td>
</tr>
<tr>
<td>Muon PID</td>
<td>2.83</td>
</tr>
<tr>
<td>FGD Layer Differences</td>
<td>neg.</td>
</tr>
<tr>
<td>P0D Veto Efficiency</td>
<td>neg.</td>
</tr>
<tr>
<td>TPC T0 Determination</td>
<td>neg.</td>
</tr>
<tr>
<td>Track Direction</td>
<td>neg.</td>
</tr>
<tr>
<td>Water NC Interactions</td>
<td>0.35</td>
</tr>
<tr>
<td>Out of FGD FV</td>
<td>2.06</td>
</tr>
<tr>
<td>Wrong FGD</td>
<td>neg.</td>
</tr>
<tr>
<td>Cosmic Ray Contamination</td>
<td>neg.</td>
</tr>
<tr>
<td>Total (systematic only)</td>
<td>4.60</td>
</tr>
<tr>
<td>Total</td>
<td>12.76</td>
</tr>
</tbody>
</table>
Chapter 12

Results of Analysis

Having done the FGD neutrino selection, the oxygen/carbon cross-section ratio can be calculated. In this chapter, we compare the ratio found for the FGD data with the ratio from the GENIE Monte Carlo, and compare the result of this dissertation with previous comparisons of neutrino cross-sections on oxygen and carbon. Experimental measurements of CCQE cross-sections from the K2K experiment will be considered, as will a pair of theoretical models with different treatments of nuclear structure.

12.1 Measurements of Ratio

We select 19881 Monte Carlo events, 9998 in FGD1 and 9883 in FGD2, and 1569 data events, 757 in FGD1 and 812 in FGD2, as inclusive charge-current interactions in the FGD fiducial volume, using the selection described in Chapter 10. The reduction of both the Monte Carlo and data samples through the series of cuts can be found in Tables 12.1, 12.2, and 12.3. Using the efficiencies, purities, and masses described above, and Equation 10.12, we calculate the oxygen/carbon cross-section ratio $R$ as

$$R_{MC} = 0.954 \pm 0.029$$  \hspace{1cm} (12.1)
Table 12.1: Reduction table of Monte Carlo and data, showing the event counts at each stage of the analysis. Early differences in the event fraction can be due to the higher beam power in the MC as compared to the data for the period considered in this analysis.

<table>
<thead>
<tr>
<th>Cut</th>
<th>MC</th>
<th>%</th>
<th>Data</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Cuts</td>
<td>11546512</td>
<td>100.0</td>
<td>1107202</td>
<td>100.0</td>
</tr>
<tr>
<td>Spill Timing</td>
<td>5212718</td>
<td>45.1</td>
<td>608974</td>
<td>55.0</td>
</tr>
<tr>
<td>Global PID Validity</td>
<td>5112442</td>
<td>44.3</td>
<td>598915</td>
<td>54.1</td>
</tr>
<tr>
<td>TPC Track</td>
<td>625070</td>
<td>5.4</td>
<td>94788</td>
<td>8.6</td>
</tr>
<tr>
<td>FGD Segment</td>
<td>441939</td>
<td>3.8</td>
<td>70166</td>
<td>6.3</td>
</tr>
<tr>
<td>P0D Veto</td>
<td>288181</td>
<td>2.5</td>
<td>32181</td>
<td>2.9</td>
</tr>
<tr>
<td>TPC Track Length</td>
<td>271257</td>
<td>2.3</td>
<td>30412</td>
<td>2.7</td>
</tr>
<tr>
<td>FGD Fiducial</td>
<td>56528</td>
<td>0.5</td>
<td>5169</td>
<td>0.5</td>
</tr>
<tr>
<td>TPC Negative</td>
<td>30194</td>
<td>0.3</td>
<td>2605</td>
<td>0.2</td>
</tr>
<tr>
<td>Momentum</td>
<td>24759</td>
<td>0.2</td>
<td>2097</td>
<td>0.2</td>
</tr>
<tr>
<td>Muon PID</td>
<td>19881</td>
<td>0.2</td>
<td>1569</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 12.2: FGD1 Reduction table of Monte Carlo and data, beginning at the FGD1/FGD2 classification prior to the FGD fiducial volume cut.

<table>
<thead>
<tr>
<th>Cut</th>
<th>MC</th>
<th>%</th>
<th>Data</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>FGD Fiducial</td>
<td>26779</td>
<td>100.0</td>
<td>2392</td>
<td>100.0</td>
</tr>
<tr>
<td>TPC Negative</td>
<td>14475</td>
<td>54.1</td>
<td>1214</td>
<td>50.8</td>
</tr>
<tr>
<td>Momentum</td>
<td>12152</td>
<td>45.4</td>
<td>1000</td>
<td>41.8</td>
</tr>
<tr>
<td>Muon PID</td>
<td>9998</td>
<td>37.3</td>
<td>757</td>
<td>31.6</td>
</tr>
</tbody>
</table>

Table 12.3: FGD2 Reduction table of Monte Carlo and data.
for MC, with the error coming from finite MC statistics, and

\[ R_{\text{data}} = 1.129 \pm 0.114(\text{stat}) \pm 0.044(\text{syst}) \quad (12.2) \]

for data, giving a deviation of 1.39\( \sigma \).

We can verify that the efficiency and purity corrections are functioning correctly by setting all efficiencies and purities in Equation 10.12 to 1, and using the signal counts from the MC before any cuts. We get that

\[ R = \frac{m_1}{m_w} \left[ \frac{N_2}{N_1} - \frac{m_p}{m_1} \right] \quad (12.3) \]

where now the rates \( N_1 \) and \( N_2 \) are the true signal rates. With the observed values of 22415 and 21641 for \( N_1 \) and \( N_2 \), respectively, we get a ratio of 0.952, consistent with the corrected value 0.954.

In searching for comparisons with other measurements and predictions, we find no other results for inclusive charge-current \( \nu_\mu \) interactions on both carbon and oxygen. The other results we have found were specific to CCQE interactions. As CCQE is the dominant interaction process through much of the T2K energy range, this should still be a useful comparison for the FGD subtraction result; Table 10.5 shows that the selected sample is about 53% CCQE.

### 12.2 Comparison with K2K Results

Prior to T2K, the only experiment to measure cross-sections on both carbon and oxygen in the same beam was K2K. It is thus useful to compare the K2K results with the new result in this dissertation.

The K2K results are presented as measurements of the nucleon ‘axial mass’ \( M_A \), which is a parameter in the standard formulation of neutrino CCQE cross-sections, described in more detail in Section 1.5.1. Specifically, \( M_A \) occurs in a dipole expansion of the nuclear axial vector form factor

\[ F_A(Q^2) = \frac{g_A}{(1 + Q^2/M_A^2)^2} \quad (12.4) \]
which is particularly important in neutrino scattering since it is only involved in weak interactions. As such, these results cannot be compared directly with the ND280 result in Section 12.1. Instead, we plan to regenerate the cross-section ratio from our Monte Carlo, with the $M_A$ values tuned to match those measured by K2K. The method used for this calculation is called reweighting.

### 12.2.1 Reweighting

The ND280 Monte Carlo simulation is a large, time-consuming process. Neutrino interactions, once generated from a cross-section model, are propagated through the detector, simulating the form of the actual ND280 data, and then passed through the full, complex calibration and reconstruction suite. A production of the size used for this analysis is the product of over two CPU-years of computer time. As such, it is impractical to regenerate the full sample for small tweaks to the initial cross-section model.

The solution to this is the use of event reweighting. Fully-generated events are run through the interaction generator again, and the interaction probability from the original generation compared with the probability of an identical interaction in the tweaked model. A weight value is then generated from these probabilities; events that become less likely are assigned weights less than 1, while events that become more likely are assigned weights greater than 1. We then estimate the event count resulting from the changed model by summing the weights of the selected events.

### 12.2.2 Reweighting of CC Inclusive Sample

The K2K SciFi detector was used to measure the value of $M_A$ for CCQE interactions in water. They found a value of $M_A = (1.2 \pm 0.12)\text{GeV}/c^2$[69] for an oxygen target. With the SciBar detector, the corresponding value was measured for a carbon target as $M_A = (1.144 \pm 0.077(\text{fit})^{+0.078}_{-0.072}(\text{syst}))\text{GeV}/c^2$[70]. We would like to use reweighting to apply these values to our simulation of the ND280 data.

We apply the reweighting to the FGD inclusive charge-current $\nu_\mu$ sample.
described in Chapter 10. The interaction vertex associated with the muon candidate track is identified by finding the one closest in position to the true origin of the track. For each event, this vertex is reweighted at six values of $M_A$: the K2K oxygen value, the K2K carbon value, and at $\pm 1\sigma$ from each of those values.

Having calculated the weights for all six scenarios, we then calculate totals for each FGD. The weights for the K2K carbon $M_A$ are used for FGD1, and the weights for the oxygen $M_A$ are used for FGD2. While this is not the ideal granularity, as FGD2 contains both plastic and water, it was not practical to apply this at a more specific level. Since $M_A$ only affects CCQE interactions, all non-QE interactions are left with unit weight, implicitly assuming the correctness of the standard GENIE model in comparing carbon and oxygen. While this assumption is probably incorrect in detail, the unavailability of inclusive or non-QE CC measurements on carbon and oxygen requires us to make it.

The weight totals were then used in Equation 10.12 to calculate a central value and error limits for the “K2K ratio”. To estimate the error value conservatively, the errors on the two $M_A$ values were treated as anticorrelated.

The resulting central ratio was found to be 0.963, with a lower error bound of 0.915 and an upper error bound of 1.012. The errors are very close to being symmetric, with a difference of only 0.001 in the two error bars. Taking the larger of the errors and symmetrizing, we get the “K2K ratio” as

$$R_{K2K} = 0.963 \pm 0.049$$

(12.5)

This is consistent with the MC prediction of the cross-section ratio in ND280.

### 12.3 Theoretical Prediction

A theoretical comparison of the CCQE cross-sections for carbon and oxygen has been done by Butkevich, 2009[71]. The ratio of carbon and oxygen cross-sections was calculated for two different models at a variety of neutrino energies, as shown in Figure 12.1. The relativistic Fermi gas model (RFGM) is the standard baseline model for neutrino-nucleus interactions, where the
Figure 12.1: Ratio of oxygen and carbon CCQE cross sections, from [71]. RFGM is the basic relativistic Fermi gas model, while RDWIA is the relativistic distorted wave impulse approximation model developed in the paper, which is run with and without two-nucleon short range correlations (SRC). We see a large effect from the inclusion of short-range correlations, suggesting that a measurement of this cross-section ratio might be useful to determine whether SRCs are a significant effect in neutrino-nucleus scattering. (@American Physical Society, 2009, used with permission)

Table 12.4: Relativistic Fermi gas nuclear model parameters for carbon and oxygen (numbers taken from [47])

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$^{12}$C</th>
<th>$^{16}$O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fermi momentum ($p_F$)</td>
<td>221 MeV</td>
<td>225 MeV</td>
</tr>
<tr>
<td>Binding energy ($\epsilon_b$)</td>
<td>21 MeV</td>
<td>27 MeV</td>
</tr>
</tbody>
</table>

nucleus is treated as a system of independent, quasi-free nucleons with a flat momentum distribution.

There are two parameters in the RFGM that vary between different nuclei: the Fermi momentum $p_F$ and the nuclear binding energy $\epsilon_b$. Table 12.4 shows that the difference in these parameters between carbon-12 and oxygen-16 is quite small. As can be seen in Figure 12.1, these nuclear
effects are small, with the O/C ratio for the RFGM being very close to unity throughout the energy range. In addition, the RFGM includes the effect of Pauli blocking. An interaction is Pauli blocked if the target nucleon, being transformed from a neutron to a proton or vice versa, finds the state to be occupied in its new identity to be full, preventing the interaction from occurring.

The RFGM ignores the various nuclear structures that can influence the interaction cross-section, particularly at low momentum transfer ($Q^2$). The nuclear shell structure is ignored, as are any short-range correlations (SRC) between nucleons and final state interactions of the ejected nucleon. The paper introduces as an alternative model based on a relativistic distorted-wave impulse approximation (RDWIA) which accounts for all of these effects.

Short-range correlations are effects that cause two nucleons to, at least temporarily, act collectively as a single quantum object. A neutrino can interact with such a correlated pair when it encounters a nucleus, rather than with a single quasi-free nucleon. In these cases, the neutrino interaction may eject both of the correlated nucleons, producing a more complicated final state than the usual CCQE interactions. In particular, this invalidates the usual neutrino energy calculation for CCQE interactions, using the over-constrained two-body kinematics to determine the incident neutrino energy without observing the ejected proton. Multi-nucleon CCQE interactions may often be experimentally indistinguishable from the single-nucleon case, so their simulation is an important component in understanding CCQE interaction data.

These short-range correlations may help to resolve the ‘axial mass anomaly’, where recent measurements of $M_A$ (Section 12.2) on nuclear targets show a value ($\sim 1.2\text{GeV}$) that is inconsistent with the $(1.03 \pm 0.02)\text{GeV}$ measured in neutrino-deuteron scattering prior to 1990.

We convert these cross-section ratios as a function of energy to a value we can compare with our FGD subtraction result by doing a flux-weighted average. In this case, we use the true neutrino energy distribution from our selected GENIE MC sample. For each event, we find the value of the oxygen/carbon from the theoretical curve corresponding to the true neutrino
Table 12.5: Comparison of oxygen/carbon ratios from this thesis and all other sources discussed in this chapter.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ND280 Data Subtraction</td>
<td>$1.129 \pm 0.114\text{(stat)} \pm 0.044\text{(syst)}$</td>
<td></td>
</tr>
<tr>
<td>GENIE MC Subtraction</td>
<td>$0.954 \pm 0.029$</td>
<td></td>
</tr>
<tr>
<td>GENIE MC Truth Subtraction</td>
<td>$0.952$</td>
<td></td>
</tr>
<tr>
<td>K2K $M_A$ Measurements (CCQE)</td>
<td>$0.963 \pm 0.049$</td>
<td></td>
</tr>
<tr>
<td>Theoretical prediction (RDWIA)</td>
<td>$0.912$</td>
<td></td>
</tr>
<tr>
<td>Theoretical prediction (RFGM)</td>
<td>$1.0001$</td>
<td></td>
</tr>
</tbody>
</table>

energy. The average of these values for the full neutrino selection is then taken as the theoretical value of the ratio from that model. Using the curves in Figure 12.1, we find an oxygen/carbon ratio for the CCQE $\nu_{\mu}$ cross-section of 0.912 from the RDWIA and 1.0001 from the RFGM. As with the experimental comparison in Section 12.2, the comparison of this ratio with the FGD subtraction result is imperfect due to the latter’s inclusion of non-QE interactions.

### 12.4 Summary

Table 12.5 summarizes the various values of the oxygen/carbon ratio calculated in this chapter. The FGD subtraction analysis provides the first measurement of an inclusive charge-current cross-section ratio between oxygen and carbon in a single neutrino beam. There is a 1.39σ difference between the measured ratio and the GENIE Monte Carlo prediction, which may indicate that current models underestimate the effect of nuclear structure on neutrino interactions. We cross-check the GENIE prediction with a simpler one based on the true event counts in each FGD and find that they are consistent.

Both the K2K measurement and the RDWIA model with short-range correlations predict low oxygen/carbon ratios for CCQE interactions, which make up 53% of the neutrino sample used for the inclusive measurement. Much of the reduction in ratio between the RDWIA model and the simpler RFGM model is due to the inclusion of short-range nucleon correlations; this
reduction is supported by the K2K data but does not appear in the FGD subtraction result. This suggests that there may be counteracting effects from the non-QE interactions that would raise the total cross-section ratio.

The work in this dissertation assumes that the two cross-sections have approximately the same energy dependence, allowing the flux to cancel in the ratio of interaction rates. If this assumption were to break down, giving us a cross-section ratio that varies with energy, this could modify the comparison between expected and measured neutrino interaction rates at the far detector. While the expectation has the same energy distribution as the near detector flux used in calculating the ratio, the oscillated flux will have a different energy distribution leading to a different relationship between the measured rate and the flux. The models used in this work predict that the energy dependence of the cross-section ratio is relatively small, but this could introduce an additional systematic error on the oscillation observable without an energy-dependent cross-section ratio measurement.

12.4.1 Role in Oscillation Analysis

The measurement of the carbon-oxygen ratio is important to the T2K oscillation analysis, since the near detector is primarily carbon-based while the far detector is primarily oxygen-based. Current oscillation analyses include a near detector component that is purely FGD1-based, using the interaction rate in FGD1 to normalize the beam flux for extrapolation from J-PARC to Super-Kamiokande, and does not include any carbon/oxygen cross-section differences outside those simulated by the neutrino interaction model, thus resulting in an error if the carbon/oxygen ratio in the Monte Carlo is not correct.

There are several cross-section uncertainties included in the oscillation analysis that are treated as uncorrelated between the carbon-based near detector and the oxygen-based far detector. These include a large uncertainty in the spectral function, which is an alternative formulation of the nuclear kinematic distribution to the relativistic Fermi gas model of Section 1.5.2. The next most significant of these uncertainties are the uncertainty in the
cross-section difference between electron and muon neutrinos, and uncertainties in the pion-less decay rate of the $\Delta$ particles produced in resonant CC1\(\pi\) interactions. Overall, the total C/O cross-section uncertainty contributes a 7.5% error on the near/far ratio\[72\].

This 7.5% uncertainty is an ‘external’ uncertainty (based on constraints from other experiments) in the context of the T2K measurement, as it results from the application of other experimental results to the T2K cross-section model. Effectively, it acts as an uncertainty on the carbon-oxygen ratio in the neutrino interaction simulation. To use the measurement in this dissertation in this framework, we can renormalize the MC cross-section ratio to the measured value, allowing us to replace the 7.5% uncertainty with the uncertainty of the cross-section ratio measurement. For the current measurement, the resulting error would be larger than provided by external constraints, though it would remove a significant model-dependence from the result.

The current public result from T2K for electron neutrino appearance has 11 events at Super-K with a predicted background of 3.2 ± 0.4 events\[73\]. Both signal and background would shift with the cross-section ratio, with a shift on the order of 17% for the current result. Given the small number of events currently available, this shift would have a small effect on the significance of the result, and the change in systematic error would be negligible compared to the large statistical error.

However, the picture changes with the addition of more data. Eventually, there will be many more events at Super-Kamiokande and the 7.5% systematic error from the cross-section model (and its possible model-dependence) will become a significant problem. At the same time as the improved statistics at Super-K make the cross-section error a bigger problem, improved statistics at ND280 will make the error on the cross-section ratio measurement smaller, such that it will become a net improvement to replace the 7.5% current error with that from the cross-section ratio measurement.
12.4.2 Future Directions

There are a number of important ways this work can be extended. The current result does not have sufficiently high significance or sufficient detail to contribute to the overall understanding of neutrino-nucleus interactions, but there is a clear path towards a measurement that can improve the theoretical understanding and reduce the systematic errors on the T2K oscillation measurement.

The first improvement is to increase the statistical sensitivity of the measurement by including more data. Currently, the T2K experiment has roughly four times as much data recorded as was used for the measurement in this dissertation. Including this data and correcting for the systematic differences between the different T2K run periods will cut the statistical error in half, dramatically reducing the total error on the measurement. Greater statistical significance could allow the observation of significant differences of data from the Monte Carlo or could allow the measurement to serve as a cross-check on the neutrino modelling used for our prediction.

The next improvement would be to separate the CC inclusive sample into quasi-elastic and non-quasi-elastic fractions. Since the interaction modelling is channel-specific, separating the sample in this way will allow greater insight into the validity of the different models used for the different interaction processes. As seen in this chapter, all of the previous measurements and calculations comparable to this result are CCQE only, so a CCQE ratio measurement could be used to distinguish between the validity of the different models in Figure 12.1. Since the non-QE fraction is more poorly understood, a measurement of the CC non-QE ratio could help guide the models to a better understanding of the physics of non-QE interactions.

However, separating the sample by interaction channel will also introduce additional systematic effects that must be studied. For example, Michel electron tagging in the FGD is an important component of CCQE event selection, as the presence of a Michel electron suggests that a pion was produced in the initial neutrino interaction. This is complicated by the irreducible difference in Michel tagging efficiency between the two FGDs due to
the large amount of dead material in FGD2. While a Michel electron produced in FGD1 has a large probability of interacting with active detector material, one produced in FGD2 can be fully contained within a passive water layer, thus being entirely invisible to the detector and thus the analysis. The reduced FGD1 reconstruction of [Section 11.6.1] will be a useful tool in evaluating this systematic effect, as it allows us to observe the fraction of Michel electrons seen in FGD1 that disappear when half of the layers are made into dead material for both MC and data.

The third major improvement that can be made to this analysis is to separate the measurement by muon momentum and angle. The FGD1 analysis used to normalize the near-far ratio is currently divided into momentum and angle bins, and dividing the FGD2 and subtraction analyses in the same way allows us to measure the spectral dependence of the cross-section ratio, which will become more important as the T2K oscillation measurement evolves from a pure counting experiment into a more detailed spectral fit which may be distorted by differences in the cross-section ratio for different spectral regions. This analysis will require substantially larger amounts of data than is currently available for good statistical significance, so the continuing increase in the T2K beam power and thus the absolute interaction rate will be very useful.
Part V

Conclusion
Chapter 13

Conclusion

We have presented a first measurement of the inclusive charge-current neutrino cross-section ratio between oxygen and carbon. This measurement is based on the combination of water- and plastic-based components in the T2K ND280 Fine-Grained Detector.

The FGD contains a passive target water system consisting of six water vessels interleaved with the active scintillator layers of one of the two installed FGDs. These passive water modules were built at TRIUMF from hollow, 2.54 cm-thick polycarbonate panels designed as greenhouse windows, and designed to provide optimal separation of the component elements into polystyrene-like and water-like fractions with minimal remainder. In operation, they are attached to a circulating supply system which maintains the interior of the modules at sub-atmospheric pressure for leak control, protecting the other components of the FGD installed at close proximity to the water modules.

A new reconstruction algorithm for FGD-only particle tracks was designed and implemented, providing a modest improvement in the efficiency of the FGD-only reconstruction. This algorithm was based on a discretized Radon transform, which converts the track-finding problem in the detector coordinate system to a cluster-finding problem in an abstract transform space. While this implementation of a Radon transform reconstruction was not used to replace the original cellular-automaton algorithm for FGD-only
reconstruction, it was the basis for additional work that has now become the default method for the reconstruction of FGD-only tracks.

The data analysis for the cross-section ratio was based on the concept of a subtraction analysis, where the contribution of interactions in the FGD2 plastic is estimated by scaling the FGD1 rate by the respective masses of plastic, and then the FGD2 water rate is found by subtracting the estimated FGD2 plastic rate.

The rates in the two detectors were determined using an FGD $\nu_\mu$ interaction selection in two parts. First, the selection of tracks in the detector is narrowed to include only valid tracks which span at least one FGD and one TPC, and an upstream veto is applied to reject outside interactions that enter the Tracker. Then, the highest momentum remaining track is selected as a muon candidate, and tested for an origin in the FGD fiducial volume, negative charge, sufficient momentum, and muon-like energy loss. If all of these tests pass, the event is selected as an FGD neutrino interaction in its origin FGD.

The analysis finds that the mass-weighted charge-current inclusive cross-section ratio of oxygen to carbon is $1.129 \pm 0.114\text{(stat)}\pm 0.044\text{(syst)}$. This is a $1.39 \sigma$ deviation from the GENIE Monte Carlo prediction of $0.954 \pm 0.029$, which is not significant enough to clearly show these values are different. No other experimental measurements or theoretical predictions of this value are available, but they are available for a similar ratio restricted to charge-current quasi-elastic interactions.

Measurements of the CCQE cross-section from K2K predict, when reweighting the GENIE prediction to align with the K2K cross-section parameters, a ratio of $0.963 \pm 0.049$, which is consistent with the prediction from the stock GENIE model. A theoretical prediction using a relativistic distorted wave impulse approximation gives a ratio under the T2K energy distribution of 0.912. The difference of this value from the measurement in this thesis may indicate nuclear effects of opposite sense in non-QE neutrino interactions. As the measurement is statistics-limited, more data would help to resolve this issue, as would a complementary measurement of the exclusive CCQE cross-section ratio. Once the overall measurement is no longer limited by
statistics, an energy-dependent measurement would be a useful addition, to ensure that the ratio varies as slowly as suggested by the interaction models.
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201


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