SPOOFING PROOFING:

THE LOGICAL-NARRATOLOGICAL CONSTRUCTION OF CARROLL'S ALICE

BOOKS

by

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Abstract

This paper explores Charles Lutwidge Dodgson's (Lewis Carroll's) use of a *reductio ad absurdum* proof in both *Alice's Adventures in Wonderland* (1865) and *Through the Looking-Glass, and What Alice Found There* (1872). It seeks to show that Dodgson used this proof as a framework for the first novel in order to mock "new mathematics," including but not limited to *n*-dimensional theories of space, imaginary and negative numbers, and non-Euclidean geometries. It also problematizes this reading of *Wonderland* through its exploration of Dodgson's continued used of *reductio ad absurdum* as framework in *Looking-Glass*, in which he explores theoretical mathematics for which he held genuine interest. In doing so, the paper reviews Victorian developments in mathematics and the epistemological and theological shifts that these developments presaged. It also examines Dodgson's particular interests, and in particular, his contradictory views in mathematics. It therefore seeks to undermine the canonical view of Carroll as a simple, less-than-brilliant mathematician through its examination of his most famous books' fictional explorations of the worlds of mathematics.

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Dedication

A cliché, but a heartfelt one.

To my parents, for teaching me that my thoughts are worth something, for believing in me always, and for helping me communicate with the fairies in the flowerpots.

Introduction

In the decade that Charles Lutwidge Dodgson, a Student (equivalent to a Fellow) in mathematics at Christ Church, Oxford, and Deacon of the Church of England, published Alice's Adventures in Wonderland (1865), a book ostensibly written for children,¹ a revolution in English mathematics started—and it was one of which he did not approve. Non-Euclidean developments in Continental mathematics rudely awakened latenineteenth-century mathematicians in England, shaking the foundations of both their epistemological organizations of knowledge and their conceptions of truth,² in which Euclidean geometry, and the mathematics developed from the same, were considered perfectly, transcendentally truthful and occupied the highest place in the hierarchy of knowledge. In spite of his disapproval of the changes in his field, and although most of his publications focused on the studies of mathematics and logic, Dodgson followed Alice with a sequel, entitled Through the Looking-Glass, and What Alice Found There, in 1871. Ironically, of all Dodgson's works-fantasy tales and poems, mathematical theorems and problem-books, and political tracts—it is the *Alice* books that became, and remain to this day, the best loved and known.

A part of the charm of these books lies in their whimsy—in their cleverness of logic and language—but their continued popularity is most likely due to their growing "curiouser and curiouser" with each (re)reading (Carroll, *Alice's Adventures* 20). This

¹ Drafted at the request of his child-friend Alice Pleasance Liddell, the story was first delivered to Alice and two of her sisters orally in 1862, and the handwritten and -illustrated manuscript of *Alice's Adventures Under Ground* was given to Alice in 1864, at her repeated request.

² Both the Victorian system of knowledge and the Victorian idea (or ideal) of truth are discussed at length below.

means that their appeal and insightfulness grows as one grows older. Indeed, a number of late-twentieth- and early twenty-first-century commentators have suggested that the *Alice* books are more appealing to adults than children: Virginia Woolf, for example, famously stated, "The two Alices are not books for children; they are the only books in which we become children" (qtd. in Clark 158); Bernard M. Patten claims that *Alice's Adventure in Wonderland* "is a book for children all right, but it is also a book for scholars and logicians" (14); and Samuel M. Barton argued that "Mr. Dodgson ... is poking fun at fourth dimension students" of mathematics in *Through the Looking-Glass* (qtd. in Throesch 39).

The *Alice* books, then, are not simply books aimed to please readers of all ages, although they do delight and entertain readers of diverse ages. More precisely, Carroll's *Alice* books were a part of his mathematical-cum-epistemological project to staunchly defend what he saw as the transcendental truths of Euclidean geometry against the revolutionary claims of new mathematicians, and, in particular, against non-Euclideans. In doing so, I shall show that the *Alice* books use sophisticated understandings of logical and mathematical proofs as the narratological framework upon which Dodgson's stories—and satires—depend. In particular, I shall focus on Dodgson's use of the *reductio ad absurdum* proof (now more commonly known as "proof by contradiction") in order, by negative definition, to show that if the postulates of such a mathematics as non-Euclidean geometry were taken as true, many of the logical foundations of the Victorian epistemology would also have to change—and that these changes would create a world in which nobody behaved according to logical sense, a world, quite literally, of nonsense.

In writing the *Alice* novels, Dodgson uses the *reductio ad absurdum* proof as a narrative framework, opposing a Euclidean reality (or sense) to the non-Euclidean worlds of Wonderland and Looking-Glass World (nonsense). Within the story, in specific plot instances, as well as within his framework, Dodgson uses purposefully faulty logic and mathematics.³ Wonderland and Looking-Glass World, simply put, don't add up. The mathematics, when correct, uses a different base system or poses unsolvable, illogical (to Dodgson's Euclidean mind) questions.⁴ When the logical structures in the book are correct in their framing, their premises are incorrect (or, contrariwise, their conclusions are faulty when the premises are correct).

This method suggests that his narratological framework's proof is invalid,⁵ just as the proofs of the characters in the stories are invalid. That he uses *reductio ad absurdum* implies as much: it was the "proof" most commonly used by non-Euclidean mathematicians in their posits. But this creates a problem: if Dodgson's entire proof of absurdity is itself absurd, can he truly be said to be proving that the contents of that proof are absurd? What does the invalidity of the framing proof suggest, if it is intended to be invalid at all? And *why* would Dodgson frame his novels in this way if it (as it seemingly does) undermines the overall aim of the book?

³ As Edward Wakeling, Melanie Bayley, Martin Gardener, and Bernard M. Patten, amongst others, have documented at length.

⁴ See, as an example, Alice's multiplication table in *Alice's Adventures in Wonderland*, which Martin Gardener explains can be interpreted in several ways using different bases than base ten, the standard base for mathematics (Dodgson, *Annotated Alice* 23n4).

⁵ I use the term "invalid" in the mathematical sense, that is, as meaning that counterexamples can be found or that the proof does not come to a complete or logical conclusion.

Dodgson at times appears to be attempting to show that reality, as opposed to the irrealities of Wonderland/Looking-Glass World, is an invalid premise. By introducing his second inversion—Looking-Glass World—he suggests that his Wonderland proof of reality-as-false-premise is itself invalid, based on assumptions rather than on things that are, or can, be known. And what does he mean by suggesting that reality is invalid in the first place? This paper will examine whether Dodgson intended his proofs to be considered valid or invalid—and what he aimed for them to (dis)prove in the first place. In doing so, it seeks to identify the latent mathematical—and epistemological—meaning in the *Alice* books.

Based on evidence from earlier scholarly work regarding Dodgson's negative views on contemporary "advances" in mathematics and his own eccentric approaches, which favoured "old mathematics," or staunchly Euclidean mathematics, this paper will further argue that the *Alice* books attempt in some small way to satirize a specific reality: that of Victorian mathematicians and logicians. What they seek to prove, by spoofing proofing, or offering up purposefully faulty proofs, is that those who saw themselves as experts at the very edges of knowledge ("new" mathematicians) were in fact nothing more than Mad Hatters and March Hares—or, specifically in his own case but also in the case of other staunch Euclideans ("old" mathematicians), that they were nothing more than the already extinct Dodo.⁶

⁶ A number of critics have drawn attention to the biographical meaning of "Dodo": Gardener points out that the museum at Oxford had a stuffed dodo in it that was a favourite of Dodgson and the Liddell children and that Dodgson was nicknamed "Dodo" because his stammer often forced him to introduce himself as "Dodo-do-Dodgson" (27n10).

In order to prove this, I shall first give a brief outline of the struggle, in Victorian mathematics, between Euclideans and non-Euclideans, as well as discuss Carroll's position on these subjects and his understandings, based thereon, of what "true" and "logical" connote. I shall then discuss developments in children's literature in the age, with a particular focus on nonsense, after which I will knit together Carroll's position in relation to the epistemological struggle in mathematics and why he may have chosen to use children's nonsense literature as a tool of subtle satire and cultural subversion. In this section, I shall give particular focus to his enjoyment of puzzles and inversions, and of using the same (especially in logic) to both amuse and teach his child-friends. From here, I shall move into a direct analysis, through close-reading, of the two *Alice* books and a discussion of the repercussions of the publication of the second (Looking-Glass) upon possible meanings of the first (Wonderland) before I conclude that while Dodgson was certainly earnest in his attempts to undermine "new mathematics" in the first Alice book, by the time he wrote the second, he had likely begun to question whether or not his prior certainty that there could be no usefulness or merit in the developments he had previously mocked was waning.

Chapter 1: Fissures in Mathematical Truth: The Development of Non-Euclidean Geometries and Carroll's Conception of "Truth"

As has been detailed in works by historians such as Joan L. Richards and further explored by Carrollian Elizabeth Throesch, mathematics in England in the Victorian age depended entirely upon Euclid's *Elements*. Richards argues that "all English formulations ... of geometry were constructed from Euclidean geometry," and that these formulations defined the ways in which space and reality were conceived (61). Furthermore, it was upon this, Victorian intellectuals based their entire system of truth: Euclidean mathematics was seen to inscribe an area of absolute truth, and its study was seen to be a study of this absolute truth; the idea of a pure "mathematical truth—which had a primary ontological status independent of its practical applicability-... was widely accepted within the British mathematical community" (Richards 90) and, what is more, that the conception of knowledge within Victorian Britain depended upon the "transcendental truth mathematics was believed to describe" as the ideal of truth, "the perfect truth to which human intellect aspired" (Richards 90, 104). Importantly, this "perfect truth" was not simply the zenith of the British ideal of knowledge, however, but with theological conceptions of the hierarchical status of humans and with the defence of a belief in a Christian God in an increasingly secular world, for "knowledge of God was defended by being equated with the unquestionable status of geometrical truth" (Richards 104). This meant that to most Victorians, and especially to Victorian mathematicians, mathematics represented the closest humans could aspire to be to divine clarity and understanding; it was, to them, representative of a perfect nature, reflected in this world but not always manifest flawlessly in reality.

In Mainland Europe, however, the idea of "truth" was different. Unlike in Britain, knowledge of God and knowledge in general were not hierarchically arranged or linked together as described above, and non-Euclidean mathematicians saw truth not as something transcendental and absolute but rather as something that could stand against all empirical claims to the contrary. In other words, truth was *experiential* (Richards 84–5); it was the sort of truth that we associate with the study of sciences today, empiric and evidence based. Where they were not based on experiment and proof, analyses were based on *analytic* truth, that which could be discussed and thought about—and appeared to make logical sense—but which could never be experienced (Richards 85); this is the sort of truth that we would most closely associate with mathematics today. Continental conceptions of truth, then, did not include what Richards calls "transcendental truth" (86), discussed above, that most British mathematicians believed their work achieved: a perfection of the rules that governed the imperfect world; a reflection of and a reaching towards divine truths and the rules of God; a mathematics that proved through its perfection the existence of a deity.

Because Continental mathematicians clearly delineated between experiential, or experimental truth (that which could be experienced in the real world) and analytic, theoretical truth (that which could be discussed but could not be experienced), they were able to discuss non-Euclidean geometries and *n*-dimensional spaces without hypostasizing them. "New math" British academics, however, began to argue that purely theoretical concepts to the real world (Throesch 37–9). "Old math" academics, however, lambasted them for this, and therefore often struggled to overcome newly formed prejudices against new mathematical ideas even when those ideas may have been useful.

Even beyond learned ways of thinking about truth and knowledge, giving up Euclidean ideas of space in such a short time would be as hard—if not harder—than giving up Christian ideas of time thanks to the discoveries of the natural sciences, including but not limited to Darwin's mid-century publications.⁷ As Richards reminds us, "the Euclidean view of geometrical space" had been central to Western epistemologies for "almost two and a half millennia" (62), longer than the Bible had been around.

Despite this, the fifth postulate—which states that if two lines in a plane, when intersected by a third line, produce on one side of the third line two angles whose sum is less than 180°, they will eventually meet on the side upon which the two angles whose sum is less than 180° occur⁸—had always been problematical, because it implies that lines drawn next to one another in an infinite plane with interior angles adding to exactly 180° will never meet (that is, that they are parallel). Even Euclid himself appears to have been troubled by it: he discussed it cursorily and did not base any other arguments upon it, unlike his other postulates; in fact, he was also unable to derive this postulate from the previous four postulates (Richards 62–3). It was because of continuing attempts to prove the fifth postulate directly, in fact, that it ended up undermining Euclidean geometry

⁷ Other discoveries and works of the pre- and early Victorian era that point to the same stretching of the concept of historical time to include millions rather than thousands of years include Charles Lyell and James Hutton, geologists and proponents of deep time; Étienne Geoffroy Saint-Hilaire, Jean-Baptiste Lamarck, and Robert Edmond Grant, early proponents of evolution; and Richard Owen, who pushed the creation of the Museum of Natural History in London. See Rudwick, Jones, Desmond, and Gruber.
⁸ This is sometimes known as the triangle postulate: if two lines angled towards one another have a third line drawn across them which produces two angles whose sum is less than 180°, they will eventually meet at a point (producing a triangle).

overall—Richards begins her history of non-Euclidean geometry with an Italian mathematician named Girolamo Saccheri, who in 1733 approached the postulate in a new way, by using *reduction ad absurdum*, or proof by contradiction (63).

In his book *Thinking from A to Z*, Nigel Warbuton defines *reductio ad absurdum* as "proving that a position is false, or at least untenable, by showing that if true it would lead to absurd consequences" (1).

A simple example of *reductio ad absurdum* is the following:

Problem: Prove that a triangle can only have *one* angle greater than 90°. Solution: Try to prove that a triangle can have *two* obtuse angles. We know that the angles of a triangle add up to 180° : that is, angles $A + B + C = 180^\circ$. But if A + B = x, where x is greater than 180° (as obtuse angles are, by definition, greater than 90°), then C must be negative to make $A + B + C = 180^\circ$. This would mean the angle would have to be less than nothing, an impossibility in geometric reality. Therefore, a triangle cannot have two obtuse angles.

We are most easily able to prove that "a triangle can have only one angle greater than 90° " by proving that it cannot have more than one angle greater than 90° —that is, we are able to prove the given statement by proving that any other possibility leads to absurd, or nonsensical, consequences, the definition of *reductio ad absurdum*.

Richards states that in his 1733 *reductio ad absurdum* proof, Girolamo Saccheri attempted to prove Euclid's fifth postulate with the "novel approach" which "opened up a vast new territory of mathematical innovation"—*reductio ad absurdum*: "he approached the problem … by an indirect route. Rather than attempting to prove the postulate true, he assumed that is was *not* true and tried to demonstrate the falsity of any alternative

postulate" (63). This was not a new way to approach a problem, however. Euclid used *reductio ad absurdum* in his own work: indeed, the central argument of one of his most famous proofs, the infinitude of primes, is *reductio ad absurdum*. Perhaps what Richards meant to suggest, then, was that the use of *reductio ad absurdum* in the field of mathematics during the eighteenth and nineteenth centuries began with the innovations of non-Euclidean geometry.

In and of itself, certainly, this logical move was not new; it was suggested as a rule by the founder of structured logic, Aristotle. In his *History of Quantification*, Daniel Bonevac suggests that *reductio ad absurdum* was "Aristotle's third inference rule," presented "together with rules about which categorical statement forms contradict which" (7). However, it is not until the advent of non-Euclidean geometries that it began to be used regularly in mathematics.

Saccheri was unable to prove the fifth postulate true using *reductio ad absurdum*, and he did not, through his ability to poke holes in the fifth postulate, reject Euclidean geometry. Rather, in his frustration at being unable to prove Euclid right, he chose instead to state that the Hypothesis of the Acute Angle (an "opposite" of Euclid's postulate which he was unable to prove false) had to be "absolutely false; because repugnant to the nature of the straight line" (qtd. in Richards 65). But despite Saccheri's determination that Euclid must be right, his work laid the foundations upon which non-Euclideans, nearly a century later, would build their theories. In fact, he quite literally built the foundations, for the non-Euclideans readily took up his model of proof, using his structure of negative definitions—to use a literary metaphor for indirect, contradictory

proofs—to express "doubt about the truth of a geometry constructed on such a conceptually flimsy postulate" (Richards 67).

Richards argues that it was not until the 1860s, when Carroll was writing *Alice's* Adventures in Wonderland, that the doubts of Continental non-Euclidean mathematicians and their proofs for the same made their way to England, claiming, "before late in the 1860s, there is little indication that any English mathematicians seriously considered the possibility that Euclid's fifth postulate could be doubted" (69). Here, I disagree with her. Even if there were not "serious doubts," there were certainly rumblings of discontent. And strangely, Richards herself points to some of these immediately after her above avowal: she relates in a footnote that William Whewell, one of the early Victorian proponents of a "category of necessary, mathematical truth" (90), felt it necessary to defend Euclidean geometry against attack after publication of Thomas Reid's work on retinal geometry (71); and she also draws attention to the admittedly non-mathematical, yet geometrically questioning, work of Bishop Berkeley, regarding vision (69). Furthermore, given the very nature of academic mathematical work in the Victorian era work, if published essays and public lectures began to be given in the early to mid-1860s, Carroll's contemporaries—I am sure, to his knowledge—would have been toying with the ideas foundational to their arguments years earlier.

A specific, British example that comes to mind of earlier examples of awareness of alternative spatial views is the work done in analytic algebra by Arthur Cayley in the 1840s. Cayley is mentioned in Richards's chapter on non-Euclidean geometry for his paper "Note on Lobachewsky's Imaginary Geometry" (1865), but she suggests that he "saw no connection between the analytical development and a need to radically

reconsider the nature of classically conceived space" (73). However, Cayley had been working even as a student on different conceptions of space, taking further William Rowan Hamilton's work on spheres and his development of quaternions to suggest a purely conceptual extension of algebra to consider spheres within *n*-dimensions (Crilly 81). While Cayley's early work applies his theories to three dimensions, in common with Euclid's *Elements* and our special reality, he was not opposed to positing the idea of a *"mathematical space* of '*n*-dimensions' represent[ing] an extension of three dimensions to an ideal realm, a step which entailed no physical reality for values of *n* greater than three" (Crilly 82). In fact, papers on *n*-dimensional space had been published in England since the 1820s, and *n*-dimensional space began to come into regular mathematical parlance in the 1840s and '50s (Crilly 83).

While I agree with Richards that perhaps a number of English mathematicians may not have had access to Continental developments (69), I do not agree with her that those at the important mathematical schools of the era—most notably, Cambridge, but also Oxford, where Dodgson studied and worked—would have been unaware, or "ignorant," in her terms, of the developments elsewhere in Europe. Tony Crilly points out that Cayley, for example, visited the Continent in the 1840s (81) and that Cambridge mathematics Fellows and students had enough of a knowledge of the two major Continental mathematics journals, one German and the other French, to have given them nicknames: *Crelle's Journal* and *Liouville's Journal* (84). What is more, most professional mathematicians, of whom Richards is speaking, being of the upper- or veryclose-to-upper classes, would have held at least a working knowledge of one of French or German, if not both. Her assumption that British mathematicians were unaware of the

work of those on the Continent seems to me not only far-fetched but founded on a presupposition that Anglophones know very little of other languages.

What is more, simply because a number of those working on non-Euclidean mathematics on the Continent did not *publish* their ideas until the early 1860s (according to her assertions, which we shall see below are not necessarily true) does not suggest that they—and their colleagues in Britain—did not correspond about their ideas. Indeed, if Karl Friedrich Gauss⁹ and Heinrich Christian Schumacher's "correspondence … on the subject was *published* between 1860 and 1863" (Richards 73, my emphasis), whisperings of the *ideas* regarding the same would likely have reached, at the very least, some mathematical professionals—even those living in Britain—prior to the publication of this correspondence.¹⁰ Sadly, Dodgson was generally quite vague when describing his mathematical work in his diaries, using phrases like "at work at the papers nearly all day, and all night" to describe it (185); however, some tantalizing hints suggest his desire to push back against non-Euclidean (and such theories as Dodgson would have seen as non-Euclidean, such as those of *n*-dimensional spaces) geometries in the years prior to the

⁹ Sometimes *Carl* Friedrich Gauss.

¹⁰ While I have attempted to verify that Dodgson may have been aware of this prior to the publication of *Alice's Adventures in Wonderland* in 1865, both historical fact and the scope of this project does not allow me to do so: not only were many of Dodgson's papers burnt following his death, and not only are many others are missing, but there are over 100,000 letters of his in various private and public collections all over the world (Cohen, Preface x–xi), only a very small fraction of which are published, and of those, few are from the period I wish to discuss. Further, several of Dodgson's diaries and his letter register have disappeared since they were last seen in 1898 (Dodgson, *Diaries* 143). This would certainly be an interesting arena for future archival research.

publication of *Wonderland*, for example, his statement, "I think an interesting collection might be made of axioms tacitly assumed by Euclid" (*Diaries* 189). Throesch, at the very least, appears convinced that Carroll was quite familiar with *n*-dimensional theories.

What is clear is that those whom most mathematical textbooks call the "founders" of non-Euclidean geometry died prior to the 1860s, so by necessity, most of their work was conceived and published prior to the time Richards outlines in her account.¹¹ What is more, the works of János Bólyai and Nicholai Lobachevskii, whom she calls the two founders of non-Euclidean geometry (others, such as Bonola, Crilly, and Trudeau, include also Gauss, Ferdinand Karl Schweikart, Arthur Cayley, and George Riemann, among others), were published in French in the late 1830s and in German in the early 1840s (Trudeau 158), after which time it was not received with the "deadening silence" Richards describes (69) but rather with relief: Gauss, in fact, gave up his own project in the area only because he saw a copy of Bólyai's treatise prior to its publication and felt "very glad that … the son of my old friend … takes the precedence of me in such a remarkable manner" (qtd. in Trudeau 158).

The point of this is not to discredit Richards's work, which is both thorough and useful. It is, however, intended to problematize some of the conclusions at which she arrives regarding the time at which knowledge of the non-Euclidean movement would have reached mathematical circles in England—and, most importantly, the time at which it would have reached Dodgson. I hope that the above has proven that it was at the very least likely that Dodgson was aware of and versed in the ideas of non-Euclideans at the

¹¹ See as examples Roberto Bonola's *Non-Euclidean Geometry*, Tony Crilly's *Arthur Cayley: Mathematician Laureate of the Victorian Age*, and Richard J. Trudeau's *The Non-Euclidean Revolution*.

time he was preparing *Alice's Adventures in Wonderland* for publication, between 1863 and 1865.

The discontent evident in the *Alice* books themselves suggests that Dodgson was aware of the ongoing challenges to mathematics as he knew it. I shall discuss this below, in chapters 3 and 4. Before I do so, however, I wish to comment on Dodgson himself, so that we may better understand why he takes the epistemological stance he does within the novels—and better place him within the discipline of Victorian mathematics.

Chapter 2: "Work for God": Dodgson, Mathematics, Religion, Epistemology, and Truth

As is discussed above, religion and mathematical truth were inextricably intertwined in the Victorian age, and Dodgson exemplified this perhaps more than most of his contemporaries. The son of a mathematician-clergyman, also named Charles Dodgson, the eldest son of the Dodgson family was captivated by mathematics from an early age: a commonly recited story in biographies of "Lewis Carroll" is of his going to his father when "only a very small boy" to insist that he "please explain" logarithms (Green 22). Dodgson enjoyed a sheltered childhood, during which time he took advantage of his Oxford-taught father's knowledge of advanced mathematics. Naturally precocious, Dodgson used to enjoy building mechanical gadgets and intricate puzzles for his family, including mazes in the garden. Eventually, he started a family magazine, for which he drew mazes, invented mathematical and logical puzzles, and wrote stories and verses.

At twelve, Dodgson was sent to Richmond school, where he excelled. He later went to Rugby, where he achieved numerous prizes. Headmaster of Richmond James Tate, in a letter to Dodgson's father, described the boy as follows:

He possesses, along with other and excellent natural endowments, a very uncommon share of genius.... He is capable of acquirements and knowledge far beyond his years, while his reason is so clear and jealous of error, that he will not rest satisfied without a most exact solution of whatever appears to him obscure. (qtd. in Green 22).¹²

¹² For a more complete picture of Carroll's childhood, see Green and Collingwood.

However, he also mentioned that the young Dodgson was "marvellously ingenious in replacing the ordinary inflexions of nouns and verbs ... by more exact analogies, or convenient forms of his own devising" (qtd. in Green 23). The tone of his letter suggests that some of these "convenient forms of his own devising" were not strictly within the curriculum, nor entirely desirable, in Tate's eyes.

Dodgson's determination to understand mathematics as deeply as possible was not quelled by his leaving school. Green suggests that "it is probable that Charles spent most of the year 1850 at [home], working alone (and probably with the aid of his father) in preparation for Oxford" (39). And it is clear from his letters home that he continued to work hard while he attended Oxford: within about two months of his beginning at Christ Church, he wrote home to his sister Mary to report that he had had "a sad incident, namely my missing morning chapel" because he had been up too late the night before, studying (Dodgson, *Letters* 8); and in June of 1851 (the year in which he began at Christ Church), he reported to his sister Louisa, "I amme uppe toe mine eyes yn worke" (11). His hard work and continual late nights appear to have paid off for Dodgson, though, as he achieved a Boulter Scholarship, a first in Mathematics, and eventually, a Studentship.¹³

However, Dodgson seemed to develop, as he grew older, a more fractious relationship with mathematics. While he seemed to champion mathematics as a transcendental truth, one that didn't necessarily have to hold in the real world, he also deplored many of the developments of his day that suggested new mathematical possibilities outside of what could be achieved in the world. Helena M. Pycior, for

¹³ As pointed out above, a Studentship at Oxford was the position equivalent to a Fellowship at Cambridge.

example, has seen in Dodgson's mathematical works and *Alice* books a disdain for symbolical algebra, "which stressed structure over meaning" (149): in fact, she suggests that even the language of Carroll's books is influenced by his interest in mathematics and logic, arguing that the similarity between Augustus de Morgan's introduction to symbolic algebra—"no word nor sign of arithmetic or algebra has one atom of meaning throughout this chapter"-and Alice's opinion that the poem in the Knave's trial doesn't have "an atom of meaning in it" was not only purposeful but suggests that "the Alices embodied ... Dodgson's misgivings about symbolical algebra" (149). Cambridge mathematician Arthur Cayley, with whom you are familiar due to the above discussion of his work on spheres, was one of the main proponents of the new algebra, which included such impossibilities (to Dodgson's eyes) as negative and imaginary numbers. As Pycior puts it, "these mathematicians invented a new approach to algebra in response to the problem of the negative and imaginary numbers, one which stressed structure and logical certainty rather than meaning and physical applicability[, but] ... opponents ... claimed that these numbers introduced absurdity into the paradigm of clear concepts and absolute truths" (151 - 52).

Pycior's convincing argument that Dodgson "probably became acquainted with the problem of negative and imaginary numbers very early in life" (160) suggests also that he was likely familiar with other work by those working in this area—which means that he would have been familiar with Cayley's non-Euclidean work, also. It was likely in 1854 that he began to read mathematical works related to, if not including, non-Euclidean developments on the Continent—a full decade before the publication of the first *Alice* books—when he studied in the summer with Bartholomew Price (161–62).

Many critics have attested to Dodgson's clinging to this idea of transcendent mathematical truth: Pycior suggests that "Dodgson ignored symbolical algebra because it was inimical to the traditional view of mathematics which he held to the end of his life: algebraic meaninglessness and arbitrariness went directly counter to his deep-seated belief in the absolute certainty or truth of mathematics" (162); Elizabeth Throesch argues that "he attacked contemporary attempts to update … Euclid's *Elements*" and that it is "highly unlikely that Carroll supported *n* dimensional and non-Euclidean geometries" (38). Certainly, truth was one of the things in which he was most interested, and to him, "the loss of meaningful mathematics is tantamount to the loss of human certainty, since as Dodgson indicated in his mathematical works, mathematics alone guaranteed absolute truth" (Pycior 168).

However, it seems that Dodgson was also insistent upon the arbitrariness of mathematics as he loved and understood it. Seth Lerer, in his seminal survey *Children's Literature: A Reader's History from Aesop to Harry Potter*, suggests that Dodgson was interested in whether or not languages are "conventional and arbitrary systems of signification" (193). He argues that Dodgson "set out to clarify the nature of his discipline by illustrating the confusions among academic logicians of his day," in which "he remarks that most authors of logic textbooks think of propositions as if they were real, almost living things" (193). Carroll himself stated that he believed that "no word has a meaning inseparably attached to it; a word means what the speaker intends by it, and what the hearer understand by it, and that is all" (qtd. in Lerer 193), almost exactly what Humpty Dumpty attests in his discussion with Alice in *Through the Looking-Glass*: "When I use a word, ... it means just what I choose it to mean—neither more nor less" (254).

In "Jabberwocky," Carroll seems intent to show that words themselves are arbitrary, so long as the form is correct—but this is precisely what he argued *against* in mathematics. To complicate things further, Robin Wilson points out in *Lewis Carroll in Numberland*, that Carroll was interested in objects that could not be made in a threedimensional plane: for example, he illustrates, in *Sylvia and Bruno Concluded*, a Fortuna's Purse, with "no inside or outside ... [that] can be considered to contain the entire wealth of the world" (13–16). Wilson, in fact, argues that Dodgson was truly interested in the practicalities of theoretical mathematical discussion, such as what would occur if one were to fall through the centre of the earth (16–19). But if he is interested in the practical, worldly application of mathematics, why does he devote his time to Fortuna's Purse?

What is more, although Dodgson staunchly defended Euclid throughout his life and most especially in *Euclid and his Modern Rivals* (1879). The book closes with an attack on non-Euclideans, in which Euclid specifically defends the fifth postulate (regarding parallel lines):

Let me carry with me the hope that I have convinced you of the importance, if not the necessity, of retaining my order and numbering, and my method of treating straight Lines, angles, right angles, *and (most especially) Parallels*. Leave me these untouched, and I shall look on with great contentment while other changes are made—while my proofs are abridged and improved—while alternative proofs

are appended to mine—and while new Problems and Theorems are interpolated (225, my emphasis).

However, only nine years following the publication of *Euclid and His Modern Rivals*, in which Dodgson uses Euclid's ghost to chastise his contemporaries, Dodgson himself published *A New Theory of Parallels* (1888), which revised the classic axiom.

Indeed, Dodgson appears to have taken a singular, and at times contradictory, approach to mathematics. While he was, as Francine Abeles attests, certainly "aware of the work of his contemporaries in both mathematics in logic," given the contents of his published and unpublished writings, "he did not adopt the approaches of others, preferring instead to develop his own often idiosyncratic methods which sometimes involved reinventing what already existed" ("Trees" 26). This is perhaps why he was able to innovate as he did: Francine Abeles, for example, suggests that his devotion to religion and his shy nature (as regards his relations to his adult contemporaries) kept him on the outskirts of an increasingly secular mathematical and logical academic mainstream ("Trees" 33), thereby surpassing, in some ways, the pitfalls of Victorian mathematics and able also to view innovations with the gaze of a critical outsider.¹⁴

As I hope is evident by now, Dodgson not only had an eccentric approach to mathematics and logic but one that was often contradictory. He appears to have been insistent that the symbols used in mathematics and logic, like those used in languages, were simply a system of signs that could be undermined and were arbitrary—meaning could be assigned as desired. This does not, however, mean that he did not feel that

¹⁴ For more on the innovations Dodgson made, many of which were not appreciate until late in the twentieth century, see Abeles and Wakeling.

"mathematics alone guaranteed absolute truth" (Pycior 168); he did, perhaps because he was devoutly Christian and saw the truth of mathematics as linked to the truth of God, as discussed above. It simply means that he often worked outside of the existing system of signification while remaining devoutly attached to the signified.

Dodgson's faith was extremely important to him—and to his mathematical practice. Yet he continually suffered extreme feelings of both religious and academic unworthiness. Indeed, when he failed to win a scholarship in March of 1855, he wrote in his diary,

It is tantalising to think how easily I might have got it, if only I had worked properly during this term, which I fear I must consider wasted. However, I have now got a year before me.... I record this resolution to shame myself with, in case March / 56 finds me still unprepared, knowing how many similar failures there have been in my life already. (*Lewis Carroll's Diaries* 1:78)

And he wrote in 1867, likely because of his propensity for fun, games, and theatre, that his having a place in the Church was "almost a desecration with my undisciplined and worldly affections" (qtd. in Hinde 28). That said, he appears to have oscillated between viewing his deficits negatively and viewing them with humour, however, stating in a letter to his sister, in a clearly ironic and humorous tone, that "if ever impudence and importunity deserved to succeed, I did" (Dodgson, *Looking-Glass Letters* 35).

A large part of Dodgson's drive to succeed in life both mathematically and religiously was his father. As mentioned above, the elder Charles Dodgson was also a mathematician. He, like his son, had been awarded a Studentship at Oxford, but he gave it up when he met his soon-to-be-wife, as Students were, at the time, not permitted to

marry. His father held various positions in the Church of England throughout his life.¹⁵ Years after his father's death, ¹⁶ Dodgson continued to call it "the greatest sorrow of my life" (qtd. in Cohen, Preface xii). Almost all introductions to works on Dodgson underscore his desire to please his father and to live up to his example.

Dodgson was himself a deacon of the Church of England, and, according to Cohen, was throughout his life "genuinely devout, relying on inner instinct perhaps more than external teachings as a basis for divine truth" ("Dodgson"). While Dodgson was to grow to differ from his father in his practice of faith, growing to disdain the Church of England due to his feeling that many "believe the Bible is true, because our Holy Mother, the Church, tells us it is" (qtd. in Hinde 26), faith remained central to his life and his work, shaping his views on everything from himself to his academic writings. This means that not only were the mathematical concepts upon which he built his career, and which he believed to his core, threatened by non-Euclidean mathematicians but his epistemology, based on Kantian and Christian ideas of "truth," was threatened as well.

It is evident that Dodgson saw his work in mathematics as fulfilling his Godgiven talents and as something he did in service of God. He wrote to one of his sisters that his last book, *Symbolic Logic*, was his "work for God" (qtd. in Patten 11), and Cohen attests that "Dodgson's writing meant a great deal to him; writing was the main course by which he could do something for others, to fulfill[1] a deep religious desire to contribute something to humanity—it was his offering to God" ("Dodgson").

¹⁵ The elder Charles Dodgson was at the time of Dodgson's birth a curate, but he later became a rector, a chaplain, an archdeacon, and, finally, a canon.

¹⁶ Dodgson's father died in 1868, three years after Wonderland and three years before Looking-Glass.

Chapter 3: "It occurred to her that she ought to have wondered at this, but at the time it all seemed quite natural": Dodgson, Inversions, Pretending, Games, and Nonsense

Dodgson famously claimed that "words mean more than we mean to express when we use them; so a whole book ought to mean a great deal more than the writer means" (qtd. in Wheat 114), a quip that signals his interest in the slipperiness of the meaning of words and his belief in the arbitrariness of symbols. Importantly, despite his continued insistence that there is no *moral* to the *Alice* books, we must not assume that there is no *meaning* to them; that is, while they may not carry any intended philosophical or ethical lesson, the books clearly intersect with Dodgson's other loves, mathematics and logic, and have something to say about them.¹⁷ Indeed, Dodgson, in a letter to his publisher written in June 1864, stated, "In spite of your 'morality,' I want something sensational" for the title of the book (Dodgson, *Looking-Glass Letters* 64). Why else would he ask in an 1871 letter to his child friend Amy Hughes, "How are you getting on, I wonder, with guessing those puzzles from 'Wonderland'? If you think you've found out any of the answers, you may send them to me; and if they're wrong, I won't tell you they're right!" (Dodgson, "Selected Letters" 50).

A great lover of puzzles, Dodgson had been inventing mathematical and logical games for his family and, later, friends since his childhood. I briefly mentioned above

¹⁷ In these definitions of "moral" and "meaning," I define "moral" as relating to good or bad conduct within a given society's ethically or philosophically constructed rules of conduct, whether spoken or unspoken, and "meaning" as an underlying message that is meant to be conveyed but is not necessarily linked with certain behaviours categorized as right or wrong nor necessarily bounded by any societal rules.

that he had started a magazine for his family that included puzzles and problems, as well as riddles and poems. This was only the beginning of a love affair that lasted his entire life.

As an adult, Dodgson often amused himself by inventing puzzles and problems. In fact, several of his books were intended as games or fun brain teasers: *The Game of Logic* (1887), *Pillow Problems thought out during Sleepless Nights* (1893), and *Symbolic Logic* (1896) were all published with the intention that they would be fun ways for children to learn logic and basic mathematical arguments in schools (or on their own). But it was not only in his published works that Dodgson was devoted to the fun in logical and mathematical puzzles: he was well-known for being playful in person, but he was most especially remembered by his child friends for his letters. As Cohen suggests, in his letters, Dodgson

rises above the ordinary, the basic, and whisks himself and his reader off to a world of nonsense. He creates puzzles, puns, pranks; he teases, he feigns, he fantasizes. He sends letters in verse, sometimes in verse set down in prose to see if his reader will detect the hidden metres and rhymes; letters written backwards so that one has to hold them up to a looking-glass to read them; letters with hoaxes and acrostics; rebus letters with other visual effects. (Preface xi)

In fact, there are several books dedicated solely to his whimsical letters.¹⁸

¹⁸ See, as examples, Thomas Hinde's selection of letters, *Looking-Glass Letters*; G.D. Schott's "Mirror Writing: Allen's Self Observations, Lewis Carroll's 'Looking-glass' Letters, and Leonardo da Vinci's Maps"; and Morton N. Cohen's *The Selected Letters of Lewis Carroll*.

Of particular interest as regards his mirror-written letters is that they involve reversals or inversions of the usual way of seeing. Dodgson was fascinated by inversions in both his mathematical and logical work. Abeles points out that Carroll's Method of Trees, which is almost identical to logical methods that are used in modern-day logical work and can solve extremely complicated, multi-premised problems, depended upon the use of *reductio ad absurdum*, "a form of inverse reasoning that he enjoyed using in his mathematical writings" (Abeles, "Trees" 30–31). Bartley calls the method "a mechanical test of validity through a *reductio ad absurdum* argument for a large part of the logic of terms" (qtd. in Abeles, "Trees" 26).

Further, syllogisms, a form of deductive reasoning in which a conclusion is reached from two premises, were fascinating to Dodgson—in fact, he was particularly interested in sorites, or series of syllogisms (Abeles, "Trees" 26). What fascinated Dodgson in particular was that "the *validity* of a Syllogism is quite independent of the *truth* of its Premisses" (qtd. in Abeles, "Trees" 25)—the very thing for which he had, earlier in his life, criticized in "new mathematics," that the form mattered more than the applicability of the math to real-world problems.

As you can see, what is most striking about the games and puzzles in his letters is the ways in which they are tied to his mathematical and logical work. Aside from the mirror-written letters' being tied to his interested in inversions, as outlined above, many of his letters, for example, required readers to use a magnifying glass to read the minute script (Schott 2159), clearly linked to his geometrical interest in proportions, which is also explored in *Alice's Adventures in Wonderland*, as discussed below. And as Abeles points out, he frequently used anagrams, acrostics, and ciphers in his letters to his child

friends ("Formal Logic"); what is more, some of these "games" were in fact quite mathematically complex. For example, his matrix cipher "is the first cipher that uses a nonstandard arithmetic, and it is the first cipher system based on a mathematical group. Moreover, it incorporates encipherment instructions within the ciphertext itself, ... foreshadowing the notion of a stored program that would come almost a century later" (Abeles, "Formal Logic" 701).

It is indeed interesting that some of Dodgson's most intriguing mathematical work remains unpublished, and were often used only for the amusement of friends in letters (Abeles, "Formal Logic" 701): in fact, Abeles argues that his "ciphers, anagrams, acrostics and poems[, found, for the most part, in his letters and unpublished work,] represent one of the closest associations of his mathematical-literary interests" ("Formal Logic" 708). She also poignantly asks, "Why did Dodgson write his symbolic logic books under his pseudonym?" ("Formal Logic" 42) I would answer that Dodgson published his symbolic logic books under his pseudonym not simply because he felt that they would sell better under the famous name of "Lewis Carroll" but because his mathematical, logical, and literary works all dealt with the same topics.

Chapter 4: "'No room! No room!'": Mathematical and Logical Criticisms of Alice Many critics of Dodgson's *Alice* mathematics and logic have focused on specific incidents within the books to determine what Dodgson is trying to say, and have fallen on the side of his satirizing "new mathematics." Pycior, for example, argues that Dodgson discusses negatives and their creation three times in Alice, the most clear instance of which is during the Mad Hatter's tea party: "the Mad Hatter proclaims the impossibility of subtracting something from nothing" and comments that while Alice cannot take less than nothing, "it's very easy to take more than nothing"; when Alice later asks the Mock Turtle what happened on the twelfth day of study at school, if the previous day had included zero hours of study and the hours of study constantly decreased, the Turtle and Gryphon change the subject; and in *Through the Looking-Glass*, Alice tells the Red Queen that she cannot subtract nine from eight (for it would leave a negative) (164). She points out that in the first two instances-the ones that occur in Wonderland-Dodgson takes "the concept [of negatives] literally, and force[s] his readers to consider less tea than that contained in an empty cup and fewer hours of study than none[,] ... present[ing] physical situations in which 'quantity less than nothing' was nonsensical' (164).

Melanie Bayley suggests that Dodgson uses *reductio ad* absurdum to deconstruct "the 'semi-logic' of the new abstract mathematics, mocking its weakness by taking these premises to their logical conclusions, with mad results," and she examines Alice's changes in size as commentaries upon correct, realistic proportions (as opposed to non-Euclidean and symbolical examinations of geometry and algebra).¹⁹ She suggests that the

¹⁹ Sadly, Bayley never discusses what she means when she says that Dodgson uses *reductio ad absurdum* in *Alice*. I infer that she means that Wonderland is an absurd place—but this is more the logical than the

main targets are De Morgan's symbolical algebra, Poncelet's projective geometry, and Hamilton's quaternions and theories regarding the relationship of mathematics to the fourth dimension of time. While it is unlikely that Dodgson would target De Morgan, as they were on good terms, and as "the mathematical (and logical) interests, work, and styles of the two men overlapped, with De Morgan clearly inspiring the younger mathematical don" (Pycior 159), his disdain for the other two mathematicians she mentions—at least at the time he wrote and published *Wonderland*—was clear, if contradictory, as discussed above.

Patten suggests that Dodgson is showing, by way of demonstration through the character of Alice, faulty ways of thinking—faulty logic—to imply what may be right (or, at the very least, to show what is wrong). This is precisely what Dodgson is doing through the structure of the books as well—showing what is wrong as a humorous way to imply what may be right (or, at the very least, to limit the number of times people use logic illogically or faultily). Yet some of the "types of defective thinking" that Patton lists are those that Dodgson uses. One example is the "partial selection of evidence," for one cannot represent in a fictional story the entirety of society or even of a part of society, and Dodgson only present us with two possible opposites (Wonderland and Looking-Glass World), when there may be many more (43). Another example is the "neglect of opposing or contradictory evidence," by way of assuming that readers would know just what and whom he was mocking in Victorian Oxford (43).

mathematical use of the term *reductio ad absurdum*, and I wish here to explore the formulation of the narrative as a proof by contradiction.

I wish to examine what he was doing around the time that he published the *Alice* books to have a better understanding of what may have been on his mind when he was composing them. I have already given apt attention to the influences on Dodgson throughout his life that may have shaped the way in which he viewed the world—and, most specifically and importantly in this instance, the world of mathematical truth. It is clear then, when we become aware that "from 1855 to 1871, Dodgson published five books and pamphlets on Euclid's geometry, a pamphlet on trigonometry, a book on analytic geometry and one on linear algebra" (Abeles, "Formal Logic" 697–98), that geometric concepts and debates are likely what shaped the *Alice* books. While it is true that Dodgson was intrigued by logic at this time—and certainly more so towards the publication of *Through the Looking-Glass*—he didn't start working seriously within logic until the 1880s, after the publication of both books. It is therefore much more likely that any focus within the books—and certainly, in *Wonderland*—is geometric, or rather, that it is in the logic of geometric proofs.²⁰

²⁰ This is not to say that the only valid interpretation of the *Alice* books is one that is based on geometry (or logic, or mathematics). This is certainly not the case. There are many other interpretations that are intriguing and well sourced, such as Laura Mooneyham White's "Domestic Queen, Queenly Domestic," which examines the influence of Ruskin's *Sesame and Lilies* (1865) on Dodgson's Looking-Glass gardens and queens. There are, too, the many discussions of the use of the English language within the books, although I side with Martin Gardener and believe that most psychoanalytic and allegorical interpretations of *Alice* are far off the mark: as he says, "the hypothesis must not be ruled out that it is only by accident that a pencil ... is shaped the way it is" (xv). However, that Dodgson was working in the field of geometry at the same time as he was working on the *Alice* books does suggest that one extremely likely influence, and therefore an interpretation that is valid and based on a knowledge of the author's influences and epistemologies, is a mathematical one.

Chapter 5: Contrary to Contrariwise: *Wonderland*, *Reductio ad Absurdum*, and Mathematical Critique

As has been shown above, Dodgson was particularly interested in inversions, reversals, syllogisms, and mirror-images—opposites, and proofs from the same. This can clearly be seen in both his mathematical work and in the *Alice* books themselves, one of which takes place upside down and one of which takes place in reflection.²¹ As was stated above, such logical and pseudo-logical moves shall be my focus here, and in particular, I wish to examine how Dodgson used the *reductio ad absurdum* proof—otherwise known as proof by contradiction—as a narratological framework of, and within, *Alice's Adventures in Wonderland*.²² To do so, I shall first have to examine how nonsense and science came together in the Victorian age and then how *Wonderland* uses tropes of Victorian scientific writing as well as children's literature to inform its narrative mode. From there, I shall trace Dodgson's "proof" through the book, before discussing what it may, or may not, mean.

Seth Lerer suggests that there are many links between the rise of nonsense in Victorian children's books and the writings of Darwin. It is not so much the content of Darwin's writings that were influential, in Lerer's eyes, but the some of the constructions which were later adopted in children's books—and the sense of *wonder* that Darwin's writings carried: various forms of the word "wonder" appear in *Origin of the Species* forty-one times (Lerer 175). Yet another aspect of writing that Lerer sees children's

²¹ For more on Dodgson's mirror-writing and other mirror-related obsessions, see as examples G.D. Schott, Francine Abeles, and Bernard M. Patten.

²² And, later, in *Through the Looking-Glass, and What Alice Found There*.

literature as having adopted from Darwin is the voice of the narrator, a voice of "wonder and assurance," often using direct addresses or humorous asides to the reader (176). Lerer suggests that mid-nineteenth-century science taught Victorians "that the world is changing; that discoveries challenge our understanding of reality; and that what once seemed fantasy could all be real" (178)—yet there was certainly a push back against this idea that anything at all "could be real": "Darwin's impact on Victorian fantasy lay in this tension between fact and fiction: between the scientific observation and the need to make a metaphor; between exact measurements and sense impressions" (Lerer 179).

These are the same problems with which Dodgson struggles in *Wonderland*, but Dodgson seems specifically to do so within the field of mathematics. On the one hand, he appears to be genuinely invested in "old mathematics," in the belief that Euclid's *Elements* points to the sort of truth I discussed in the above sections—an absolute truth tied to an epistemological system that is, in turn, tied to a faith in God. But, on the other hand, he struggled with where to place mathematics: in reality or in some transcendent area outside of reality. Dodgson firmly pushed aside "new mathematics" because it was impossible, yet he also appears, as outlined above, to have dismissed those who took mathematical and logical "truths" as real things: they were, to him, concepts only. Why, then, was he so determined that concepts that could not occur in reality, like negative numbers, should not be grouped mathematically with concepts that could apply to the real world? And where did he draw the line between what was and what was not acceptable?

If *Alice's Adventures in Wonderland* is to be interpreted as an inverse or opposite of reality (reality turned on its head), then, I argue, it is in fact a *reductio ad absurdum*. It falsely seeks to "prove" that its premises are correct; or, rather, it seeks to disprove them

through an attempt to prove them, to show that the conclusions to which one would come are impossible.²³ However, there is a problem with using this type of proof, especially for complex problems that have many opposites, as Warbuton points out: "there is usually no touchstone for absurdity; one person's absurdity is another's common sense. Unless a view implies a contradiction there is no easy way of demonstrating its absurdity" (2). How are we to interpret the steps within the proof of *Wonderland* if we are unsure of both what Dodgson is trying to prove and how many opposites are possible?

What complicates a reading of *Wonderland* as a pure critique is that while Dodgson, as discussed above, regularly used *reductio ad absurdum* in his mathematical practice, so too did those whom he appears to be criticizing. As was shown in "Fissures in Mathematical Truth," it in fact appears to have been one of the first (unintentional) non-Euclideans who introduced this logical move into the late-eighteenth- and nineteenth-century mathematical mainstream, or at the very least repopularized it: Girolamo Saccheri did so when he attempted to solve Euclid's fifth postulate by showing that it couldn't *not* be true. (But he failed to prove this—and in failing to prove it, he opened up the space in which the anti-Euclideans operated.) However, as discussed above, Euclid *also* used it—and so, too, did Dodgson. Many of his most intriguing works are, in fact, dependent upon this sort of proof.

Is, then, *Wonderland* meant to be precisely the sloppy sort of proof that Dodgson railed against—one that did not have clearly stated premises and failed to follow through to a clear conclusion? I think not. I believe that Dodgson felt that anyone well-versed

²³ As a simple, practical example, please refer to my attempt to show that triangles can have two obtuse angles (angles greater than 90°) in "Fissures in Mathematical Truth," above.

enough in mathematics, as it stood at the time of his writing, would be able to decipher what he was attempting to say. However, I wish to leave the possibility open to discussion that *Wonderland* was either disingenuous in trying to reach its conclusions (that is, tongue-in-cheek) or that Dodgson was mocking sloppy mathematical work by mimicking those he was satirizing.

That said, I interpret the framing proof of *Wonderland* as genuine. I believe that Dodgson used *Wonderland* to show that the sorts of mathematics discussed therein were impossible in reality (the opposite of *Wonderland*) by virtue of showing what would occur should the premises of non-Euclidean geometricians, symbolic algebraists, and those pursuing studies of *n*-dimensional spaces be true: complete chaos; a shifting, unpredictable reality in which all logic is really illogic and nothing much makes sense. This is probably part of the reason that he chose the name "Wonderland" for his book,²⁴ rather than the other titles he considered: for the Victorians, "wonder," or imagination, was the opposite of "reason," or clear, logical thinking; it was aligned with passion and a lack of control instead of with reserve and judgement. And, more, it is probably what is

adventures

Of all these, I at present prefer 'Alice's Adventure in Wonderland.' (Dodgson, *Looking-Glass Letters* 64)

²⁴ In a June 1864 letter to Tom Taylor, Dodgson stated,

I first thought of 'Alice's Adventures Under Ground,' but that was pronounced too much like a lesson book, in which instruction about mines would be administered in the form of a grill; then I took 'Alice's Golden Hour,' but that I gave up, having a dark suspicion that there is already a book called 'Lily's Golden Hours.' Here are the other names that I have thought of: Alice among the elves Alice's hour in elf-land goblins doings wonderland.

behind his desire to hide that the entire Wonderland experience is a dream until the very end of the book,²⁵ so that it seems, as it appeared to Dodgson to seem to many of his contemporaries, like a possible reality, or like something that could be possible given certain circumstances.

In consideration of this, it is important to note that while dream narratives are now almost a cliché in alternative-reality and fantasy fiction, at the time of Dodgson's writing, they were not only uncommon but *Wonderland* is one of the first in which almost the entire narrative occurs within a dream space; one well-known precursor is Shakespeare's *A Midsummer Night's Dream*, but in that instance, we are guided to believe that while the human characters believe their supernatural experiences to be a shared dream, the fairies are, in fact, real beings within the world. This is in fact the case in most fantastic literature written prior to *Wonderland*: either the fantastic beings within the tale have a rational explanation or the fantastic beings were always a part of reality, but simply an undiscovered part.²⁶ Indeed, as discussed above, most Victorians, because they were

²⁵ In the same letter as was mentioned in the last note, Dodgson wrote, "The whole thing is a dream, but *that* I don't want revealed till the end" (Dodgson, *Looking-Glass Letters* 64).

²⁶ I think here of examples in children's literature, such as Charles Kingsley's *The Water-Babies* (serialized 1862–1863), in which the author asserts that "There must be fairies; for this is a fairy tale: and how can one have a fairy tale if there are no fairies?" And in sensation fiction, such as Wilkie Collins's *The Moonstone* (serialized 1859–1860), in which seemingly fantastic circumstances are rationally explained.

bombarded by seemingly impossible discoveries, like platypi, which were originally "dismissed as a hoax" (Lerer 178), were quite willing to believe in fairies.²⁷ Having established the importance of the dream to the framing of the novel-and most especially, the importance of *hiding* that Wonderland is a dream space until the end of the novel—let us now return to examining the proof, in which the opposite of what we are trying to prove as valid is itself taken as though it were valid. One of the most important chapters in attempting to read the framework of the novel is the first, in which Alice falls down the rabbit-hole. If we wish to take the rabbit-hole as the liminal area between a space of mathematical truth and a space of mathematical fallacy, we must consider Alice's own reactions to the rabbit-hole to prove that this is so. And in doing so, we must read Alice as a practitioner or student of mathematics.²⁸ Dodgson writes, "down went Alice after it [the White Rabbit], never once considering how to get out [of the rabbithole] again" (12). Alice, the naïve but well-intending mathematician, who is following the Rabbit out of curiosity, doesn't think about the practical outcome of her experiment or whether the Rabbit's human behaviour makes sense within the reality in which she exists. (The narrator states, "When she thought it over afterwards, it occurred to her that she ought to have wondered at this [speaking rabbit], but at the time it all seemed quite natural" [12].) Instead, she thinks only of satisfying her curiosity, much as those

²⁷ I think here of the famous pictures of Elsie Wright and Frances Griffiths (1917), which the girls admitted, in the 1980s, were taken with the aid of paper fairies—but which at the time were believed by many (including Arthur Conan Doyle) to be real.

 $^{^{28}}$ I feel it important to reiterate here my belief that this is one of many multivalent readings of Alice and of the *Alice* books. I in no way seek to argue that this is the only, or the best, way to read Alice; it is simply one of many valid readings.

experimenting in imaginary and negative numbers, or with non-Euclidean geometry and *n*-dimensions, did: they knew it wasn't possible in this reality, but they were curious about what would occur *were* it possible, and operated in a purely hypothetical space—a space of curiosity.

What is more, when Alice begins to fall, the narrator tells us that "she tried to look down and make out what she was coming to, but it was too dark to see anything" (13). In falling, she is following the Rabbit to unforeseeable conclusions and consequences. But rather than worrying about this, Alice distracts herself with tangents she finds along the way: "she looked at the sides of the well, and noticed that they were filled with cupboards and pictures hung upon pegs" (13). Her first distraction is "a jar from one of the shelves[,] ... labelled 'orange marmalade'" that is, "to her great disappointment[,] ... empty" (13). If we read the rabbit-hole as the way to Wonderland, the land of nonsensical mathematics, we can read this jar of marmalade as one of the many "empty" distractions that the mathematicians whose notions define Wonderland's operations found, and discarded, along the way.

Given the way in which Dodgson frames Alice's fall—her experience of the liminal space between the possible and impossible realms of mathematics—he makes his views regarding those who travel to Wonderland (like Alice) clear: they are not certain where they are going; they have no idea why they are going to these unknown mental spaces (other than for curiosity's sake); and they are easily distracted by tangential interests on the way, but usually find such tangents "empty."

More, Dodgson gives us a clue to his narrative framework (i.e., that this is a proof by contradiction, or a proof by disproof of opposites) in one of Alice's linguistic slips during her fall:

'I wonder if I shall fall right *through* the earth! How funny it'll seem to come out among the people that walk with their heads downwards! The Antipathies, I think—' (she was rather glad there was no one listening, this time, as it didn't sound at all the right word) '—but I shall have to ask them what the name of the country is, you know. Please, Ma'am, is this New Zealand or Australia?' (14)

Of course, what Alice *wishes* to say is the "Antipodes"; however, in using the word "antipathies," she signals not only opposites and contraries but ones that are marked with aversion or dislike.²⁹ What is more, "antipathies" are markedly things that are *opposed to nature*, or not of the natural, real world, which is precisely what Dodgson wishes to make the mathematics of Wonderland out to be. He signals this further by Alice's dreamy reversal of "Do cats eat bats?" into "Do bats eat cats?" as she falls (14). In this space, he suggests, "she couldn't answer either question, so it didn't much matter which way she put it" (14). Of course, it does matter the way one puts things, especially in mathematics. This is, in fact, what Dodgson is working with: if we reverse things to make them opposite, then we have a *reductio ad absurdum* proof!

Dodgson underscores all of this when Alice comes to the bottom of her fall. She looks up and finds that "it was all dark overhead" (15). Alice, now in the space of

²⁹ The *Oxford English Dictionary* defines "antipathies" as both "contrariety of feeling, disposition, or nature (between persons or things); natural contrariety or incompatibility" and "feeling against, hostile feeling towards; constitutional or settled aversion or dislike" ("antipathies").

nonsensical mathematics, cannot see, or find her way back to, the world of sense. Her only option is to continue to follow the White Rabbit down "another long passage" to a hall with "doors all round" that were "all locked" (15). As we see in the second chapter, her only recourse to get out of this hall is to start to play with the rules of reality as we know them, shifting proportions and relative sizes until she is small enough to pass through the "little door about fifteen inches high" that opens with "a little golden key" and the passage beyond (15–26). But whenever she is small enough to fit through the door, she doesn't have the key to open it, and in the end, she instead falls into a sea of her own tears (cried "when she was nine feet high" during her size shifts to get the key and get through the door [27]) and ends up swimming to a different place all together.

While many may argue that this makes sense, given that the setting is a dream, it is important to note that Dodgson has purposefully set this scene so that we are unaware that Alice is in a dream space at this point in time, as outlined above. These shifts in size and space, then, are meant to be taken as possibilities in a world of opposites, a possible extension of our own world. If you recall the reactions to non-Euclidean and *n*-space mathematics, outlined above, these shifts become a clear commentary upon what occurs to people and to spaces when proportions and spatial relations are played with, when the mathematical certainties—transcendental truths—of human knowledge are thrown out or questioned. As Throesch outlines in her discussion of Wonderland and hyperspace philosophy, "the Victorian conception of the fourth dimension of space ... was often conflated with the new non-Euclidean geometries that were becoming increasingly popular in the second half of the nineteenth century" (37). Here, Dodgson is clearly

coupling the two areas of discussion in order to show their absurdity.³⁰ And as has been pointed out by various critics before me, he does so again later in the novel.

The most notable scenes in which he deals with size and proportion are the abovediscussed scene, in which Alice attempts to reach the right size to go through the door at the bottom of the rabbit-hole and, at one point, nearly goes "out altogether, like a candle," and wonders "what I should be like then" (18), a clear commentary on negative numbers and impossible spaces; Alice's entrapment in the White Rabbit's house, during which time, the creatures of Wonderland seem to see no problem in their idea of pulling someone the size of a house out the chimney (47-49); the famous Caterpillar scene, which Bayley assures us is linguistically tied to proportion (" the word 'temper' [at this time] retained its original sense of 'the proportion in which qualities are mingled', ... [s]o the Caterpillar could well be telling Alice to keep her body in proportion—no matter what her size"); the subsequent scene, in which she first eliminates her torso and legs, so that "her chin was pressed so closely against her foot, that there was hardly room to open her mouth" (62), and she then grows and meets a pigeon, who accuses Alice of being a serpent because her proportions are all mixed up, and her neck, extremely long (63); the scene with the baby that becomes a pig (74–76); and, lastly, the final scene in

³⁰ I hope, however, that I have sufficiently complicated Throesch's assertion that Dodgson was exploring, in the *Alice* books, "what he perceived to be the dangers of separating symbol from meaning in mathematics" (*Wonderland* 38), given his fraught and contradictory relationship with the problem. Dodgson's foremost belief seems to me to have been that while symbols are arbitrary (and therefore do not matter too much), meaning is still there and must never be forgotten. We see this demonstrated, for example, in "Jabberwocky," in *Through the Looking-Glass*.

Wonderland, in which the King and Queen of Hearts attempt to try Alice in a court of law for being too tall and she scatters their deck of cards (*Wonderland* 141–48).

In addition to this, Dodgson plays with the idea of *n*-dimensional spaces as they were linked to spiritualism. K.G. Valente has discussed this in detail, so I will summarize here: as he suggests, "a confluence of various mathematical trends, including but not limited to the development of non-Euclidean geometries and the continued ascendance of algebraic methodologies, precipitated numerous exchanges on the subject" which, manifest in the popular imagination, "offered Spiritualists an alternative account of phenomena then widely believed to involve the passage of matter through matter" (129, 131); the fourth dimension "would allow for manipulations not possible in the confines of three-dimensional space" (131). While Valente focusses his discussion on articles published in the last quarter of the nineteenth century, he does reference the 1846 essay of Gustav Fechner "Space has Four Dimensions," which argued for "extending the scope of geometrical investigations to include such a possibility" (129–30). Thus, it seems likely to me, given what other critics have already discussed within Dodgson's work and what I see there, that some of his manipulations of space are commentaries upon the impossibility of spiritualist notions within real space.

Most effectively, we can see this in the figure of the Cheshire Cat. His most famous trick (likely because it is repeated throughout the Disney film *Alice in Wonderland*) is, in the book, shown in the scene in which he gives directions to Alice immediately prior to the Mad Hatter's tea party: the Cat appears and vanishes several times, to ask what became of the Duchess's baby (who has turned into a pig), and Alice says, "I wish you wouldn't keep appearing and vanishing so suddenly: you make one

quite giddy," so the Cat vanishes "quite slowly, beginning with the end of the tail and ending with the grin, which remained some time after the rest had gone" (*Wonderland* 79). Of course, perhaps the best scene which shows the dilemma of having a fourth dimension which we cannot see due to a Plato's Cave-like blindness, to use the original analogy (Valente 129), is that in which the Cheshire Cat's head (only) shows up at Alice's croquet match with the Queen. The Queen, naturally, calls for its head to be cut off, but

the executioner's argument was, that you couldn't cut off a head unless there was a body to cut it off from.... The King's argument was, that anything that has a head could be beheaded.... The Queen's argument was, that is something wasn't done about it in less than no time, she's have everybody executed, all round. (*Wonderland* 105)

The Cat, being in several dimensions at once, cannot be dealt with entirely in the third dimension. In addition, this scene once again refers to the idea of less than nothing. If the Queen were to cut off everyone's heads "in less than no time," she would have to have done so already.

This brings us to the concept of time, which was closely linked to the fourth dimension. In fact, it was posited as *being* the fourth dimension. We can see the beginnings of this when Hamilton, in an 1853 lecture on his concept of quaternions, defined the fourth dimension thusly: "It seemed (and still seems) to me natural to connect this extra-spatial unit with the conception of time" (qtd. in Bayley). We see Alice beginning to lose her temporal bearings almost as soon as the story begins, when she falls down the rabbit-hole—lead by the White Rabbit, who is constantly "too late" and

obsessively checks his pocket-watch (*Wonderland* 12)—for she is unsure whether "the well was very deep or she fell very slowly, for she had plenty of time as she went down to look about her, and to wonder what was going to happen next" (*Wonderland* 12).

Time continues to be problematic in Wonderland. For example, as Bayley has pointed out, the tea party is all about time: the Hatter's clock measures the day rather than the minute and hour, and it, like Alice's, doesn't "tell you what year it is" (*Wonderland* 83–84); the Hatter hypostasizes time after Alice says that it can be "wasted," as though it is a material thing, claiming that "If you knew Time as well as I do, … you wouldn't talk about wasting *it*. It's *him*" (85); and when Alice asks what happens when they come to the beginning again, after being told that they circulate the table to void washing dishes, they change the subject (87). These are clearly references to *n*-dimensional theory; when Hamilton claimed that it was "natural to connect this extra-spatial unit with the conception of time," he implied that time was naturally and necessarily linked to materiality; and, what is more, the fourth element (time) is needed, in Hamilton's quaternion system, to allow "rotations to be calculated algebraically" (Bayley), which is likely why the Hatter, Hare, and Dormouse begin to rotate around the table when a fourth person joins them (Alice) and stop when she leaves.³¹

³¹ Bayley claims that the dysfunction of the tea party is due to Time's being absent, because he and the Hatter "quarreled last March" (*Wonderland* 86), but there is no hint in the book that he would otherwise have been present at the table. What is more likely is that Dodgson is pointing out the arbitrariness of calling the fourth quaternion required for rotational calculations "time," by having a little girl sit at the table to become the fourth element instead, because there was no proof other than Hamilton felt it "natural" that it was time.

More, Dodgson plays with the concept of less time than nothing towards the end of the book. For example, the Queen makes constant references to having things done in "less than no time," which is, as mentioned above, impossible; or "in about half no time," which would be no time, for half of zero is still zero (*Wonderland* 105, 111); the Mock Turtle and Gryphon claim that "lessons" are "lessons" because they lessen every day, but when Alice asks what they do on the twelfth day (one less than zero), they change the subject (118); and during the Knave's trial, the King tries to have the jury give their verdict before the evidence is presented (132) and the Queen demands, "Sentence first verdict afterwards" (146), reversing a logical, sequential order of events.

Similarly to her loss of her temporal bearings, when Alice falls down the rabbithole, she also loses her spatial bearings: "'I wonder what Latitude or Longitude I've got to?' (Alice had no idea what Latitude was, or Longitude either, but thought they were very grand words to say)" (*Wonderland* 13). In the world she is entering, our basic truths—and especially ones related to mathematics—don't "signify" (*Wonderland* 25), to use her own words; as Gardener points out, even the base for multiplication shifts from ten (*Annotated Alice* 23n4); and, similarly, space doesn't "signify," as she shows through her "try" at geography. As mentioned above in my discussion of proportion, the creatures of Wonderland, including Alice, once she falls down the rabbit-hole, seem to have no clear idea of space: when Alice falls into the sea in the first chapter, it is unclear where she is falling from, and she suddenly shifts from being in a hall to being in a natural landscape without explanation—as the narrator puts it, "everything seemed to have changed since her swim in the pool, and the great hall … had vanished completely" (*Wonderland* 26–31, 40); Bill, the lizard, is sent down the White Rabbit's chimney to pull

Alice out of the house—but she is almost as large as the house, and would therefore never fit up a chimney (*Wonderland* 47–49); the Cheshire Cat plays with space through his ability to pop in and out of it; the Hare and the Hatter tell Alice that there is "No room! No room!" at the tea table, even though, as she points out, "There's *plenty* of room" (*Wonderland* 81); Alice is able to find her way back to the great hall at the bottom of the rabbit-hole through "a door leading right into" a tree (*Wonderland* 92); and the Hatter, when he is told that he may "stand down" in the court of law, replies, "I can go no lower.... I'm on the floor as it is" (*Wonderland* 136).

Thus, in Wonderland, at least, Dodgson appears to be maligning "new" mathematics through his use of *reductio ad absurdum*. In the end, he dismisses the space of non-Euclidean, *n*-dimensional, symbolic algebraic mathematics as only a "curious dream," the dream of a little girl, and nothing more (148).

Chapter 6: "*Here*, you see, it takes all the running *you* can do to keep in the same place": Dodgson, Concepts, and a Crumbling "Truth" in *Through the Looking-Glass* The second *Alice* book, published six years after the first, has less of a mathematical focus than the first; instead, its focus is holding a looking-glass to society. However, it does pick up on some of the mathematical themes of the first, and it complicates them. In it, Alice, curious at what the backs of things look like in Looking-Glass World, climbs up through the parlor mirror to explore it, and she once again finds herself in a world in which things seem the opposite of what they are in her usual world.

This second book is also an inverse proof, but rather than looking at things inversed (upside-down), it examines them in reverse—in mirror image. Even the framing of the novel is reversed, compared to *Wonderland*: the real world part takes place indoors during the winter instead of outdoors during the summer; Alice is told where she is going and helped by the populace of Looking-Glass World rather than confused by them; and the hopeful opening poem of *Wonderland* is instead a melancholic closing poem in *Looking-Glass*. Despite that it is meant to be reversed, however, the narrative of Alice's Looking-Glass adventure shares some similarities with that of her adventure in Wonderland: for example, she comes across the White King's messengers, Hatta and Haigha (pronounced Hatta and Hey-ah: the Hatter and the Hare) half way through her quest, drinking tea and eating bread-and-butter (*Wonderland* 276–271); she spends time with the Red Queen in a garden, and she is able to get to the Red Queen only by taking a roundabout path (190); and Alice ends the dream by throwing everything into chaos and telling off the royalty (316–17).

However, because Dodgson is this time reflecting society rather than inverting it, he does not mock "new" mathematics the way that he did in the previous *Alice* book. Instead, his focus in this book is logic and puzzles. He uses as a second frame for the book a chess game, which is explained in full in the front of the book, and he examines things as they would be if they were reversed, as in a mirror, reflecting his love of puzzles and mirror-writing.

In this second book, Dodgson limits his examinations and mathematical plays to things that, in some ways, make sense in the real world; he invokes the seeming paradoxes of existence, such as that if you want to get to the place in the mirror that looks as though it is "ahead" of you, you must go in the opposite direction (for things that are behind you are reflected ahead of you). Perhaps the clearest example of this is Alice's walking within Looking-Glass World; in order for Alice to get where she wants to go, she has to try to go in the opposite direction:

Alice ... set out at once towards the Red Queen. To her surprise, she lost sight of her in a moment, and found herself walking in [the opposite direction].... A little provoked, she drew back, and after looking everywhere for the Queen (whom she spied out at last, a long way off), she thought she would try the plan, this time, of walking in the opposite direction.

It succeeded most beautifully. She had not been walking a minute before she found herself face to face with the Red Queen. (190)

His examinations of space and time, too, look at real-world problems. For example, to stay in the same spot, geographically speaking, one must stand still, as Alice points out: "In *our* country, … you'd generally get to somewhere else—if you ran very

fast for a long time, as we've been doing" (*Wonderland* 195). However, if one wishes to stay in the same place in the *universe* on a rotating globe, and in the same place temporally, with the date line where it is, one must move with the earth, as the Red Queen mentions: "*here*, you see, it takes all the running *you* can do, to keep in the same place" (195–96). Dodgson here points out the paradox of space and time: in a way, to stay "still" in time, as we have set up our time system, you would have to run at the rotational speed of Earth, and to stay in the same spot in space (not geographic space but universal space), you would also have to keep moving.

He also examines what it would be like to be in a mirror world temporally in a different way: taken literally, in a "reversed" world, the world would turn in the opposite direction, and time would run backwards. This is why the White Queen comments that "living backwards ... always makes one a little giddy at first" (233); she must put a plaster on her finger while she is explaining time and memory to Alice, because she knows that at the end of their conversation, she will prick her finger on a brooch—and immediately after she puts on the plaster, she begins screaming in pain because her finger is bleeding, and then she pricks it (235–36).³²

However, his framing of the book as a *reductio ad absurdum*, like the first book, suggests that these readings are in fact intended to make fun of theoretical mathematics: just as in Wonderland, things in Looking-Glass World are not correct; they take place in a dream space and are a part of a disproof—the framing suggests, in short, that these things should be impossible.

³² If this seems a little confusing, think of film: this is the same idea that the 1978 *Superman* movie explores, when Superman flies around the earth to reverse the death of Lois Lane.

It is likely, therefore, that Dodgson is poking fun once again. He seems to be particularly interested in this second book in the idea of perception. In this instance, I believe that he is taking to task the work on retinal geometry that began with Berkeley and Reid in the late-eighteenth century (Richards 69–70).

That said, Dodgson was himself fascinated by mirrors and inversions. If this is simply an exploration of the possibilities of a theoretical mirror space, what does that say about his *reductio ad absurdum* framing of not only this second *Alice* book but the first? Is this meant to show something he is willing to examine theoretically, is it once again meant to mock those who take impossibilities seriously and therefore examine them, or is it meant to undermine the original reading I suggested of *Wonderland* and to suggest that perhaps he was wrong in his youth—that there is, after all, some merit in examining physically impossible occurrences mathematically?

As we have seen above, Dodgson seemed to genuinely believe in some of the other tropes and troubles he examined in *Through the Looking-Glass*. A large preoccupation of the book is language, especially whether or not names mean something. As shown above, Dodgson felt that symbols and signs were, for the most part, arbitrary, as is shown in the scene with Humpty Dumpty, when the egg claims he is the master of words, and in the poem "Jabberwocky," which shows that the meaning of a piece of writing can still be understood by most readers even if almost half of the words are complete nonsense. ³³ This suggests that perhaps Dodgson's reversals in the second *Alice* book were not meant as a *reductio ad absurdum* but rather as a genuine exploration of the

³³ For a detailed analysis of language in *Through the Looking-Glass*, see Beatrice Turner's "Which is to be master?": Language as Power in *Alice in Wonderland* and *Through the Looking-Glass*."

possibilities of a world in which symbols and signs are lost, confused, and shifting and in which everything is reversed. This means that the theoretical mathematics explored in the second book are those that are, in fact, of interest to Dodgson—that he was genuine in his exploration of them rather than mocking. If this is in fact the case, it is possible that he is purposefully undermining his earlier work, *Alice's Adventures in Wonderland*. By presenting us with theoretical, explorative mathematics in which he was genuinely interested within another world of opposites, Dodgson destabilizes his earlier mockery of "new mathematics."

Conclusion

Many critics have written Charles Dodgson off as someone who, in the words of Martin Gardener, "made no significant contributions to mathematics," gave "lectures [that] were humorless and boring," and "was a fuzzy, prim, fastidious, cranky, kind, gentle bachelor whose life was sexless, uneventful, and happy" (xvi); and who, according to Morton N. Cohen, was a "shy, stammering, sheltered academic" (Preface xiii–xiv). But as Eugene Senta, Francine Abeles, and Helena Pycior argue, Dodgson was not as simple and insignificant as many paint him. Indeed, all three suggest that much of his mathematical work "was far ahead of its time" and "remarkable" (Senta 435), as well as "introduce[ing] important methods that foreshadowed modern concepts and techniques in automated reasoning" (Abeles, "Formal Logic" 33).

Dodgson, as I have shown, was not as clear-cut and conservative as many critics—even fans—suggest he was; he is not so easily put in a box as a "boring" man who somehow managed to produce two of the best-loved books in the English language. He was a man whose opinions at times oscillated, who felt torn between his intellectual curiosities and the concept of "truth" he had inherited from his dogmatic, intellectual father, his teachers, and his culture. As Andrew Wheat says,

Such a temperament—bent on certainty and exactness—would, it seems, if it did not opt for dogmatism, be likely to move in one of two directions as the limitations of the human intellect became increasingly apparent: astonished wonder at a world where "very few things indeed were really impossible" (as Alice senses) or bewildered disillusionment, perhaps bordering on downright despair. In Dodson/Carroll's case we find suggestions of both. (108)

I suggest that while the first *Alice* book was genuine in its *reductio ad absurdum* attempt to undermine the efforts of "new" mathematics, it is clear from Dodgson's later works that he was beginning to shift his ideals by the time he wrote the second *Alice* book. The second book is, therefore, not only genuine in its own claims but is meant to undercut the mathematical commentary of the first, to make readers doubt the mockery there presented. While, as Pycior proclaims, Dodgson was "concerned … with the breakdown of older mathematical certainty under the impact of the new" (169), he perhaps came, by his death, to see within the new some intellectual promise. I argue that this shift—and crisis—is shown in the change in tone towards mathematical content between the first and the second *Alice*, but it is shown also by his publication of *A New Theory of Parallels*, ten years before his death.

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