New Approaches to Linear Graph Modeling of Distributed-Parameter Systems

by

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Abstract

Analytical modeling is an important fundamental step in the development of procedures such as simulation, design, control, and health monitoring of engineering systems. Typically, physical properties such as inertia, flexibility (or stiffness), capacitance, inductance, and energy dissipation (mechanical damping or electrical resistance) are spatially distributed in a physical dynamic system. Often in dynamic models, these characteristics are approximated by spatially “lumped” elements. For better accuracy, however, the true distributed nature of these parameters has to be incorporated into the model. Distributed parameter (DP) models are important in this context. This thesis concerns the representation of distributed parameter engineering systems using linear graphs (LG).

Among possible approaches for modeling of engineering systems, linear graphs are used in the present work due to its numerous advantages as discussed in the thesis. An engineering system may possess physical properties in many domains such as mechanical, electrical, thermal, and fluid. Mechatronic systems are multi-domain systems, which typically possess at least electro-mechanical characteristics. Linear graphs present a domain-independent unified approach for modeling multi-domain systems. Furthermore, linear graphs have beneficial features in the development of automatic procedures for modeling and designing engineering systems, which are long-term goals of the present work.

In this thesis, approaches are developed for the representation of distributed-parameter systems as LG models. Different approaches are presented for this purpose and compared. The LG modeling approach enables one to visualize the system structure before formulating the dynamic equations of the system. For example, for a DP system the structure of its LG model may possess a well-defined pattern. In this work, vector linear graphs are introduced to take advantage of these patterns. General notations and elements are defined for vector linear graphs. As a result of this development a new single element is generated for use in the modeling of distributed-parameter systems, particularly in the mechanical domain.

In this thesis, a software toolbox is enhanced and presented for LG modeling, which is able to automatically extract the state space equations of a mechatronic system. This software tool is provided free for academic use and is accessible through the Internet.

Throughout the thesis many comprehensive examples are provided to illustrate the developed concepts and procedures and their application.

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Abbreviations

LG	Linear Graphs
VLG	Vector Linear Graphs
DP	Distributed-Parameter
LP	Lumped-Parameter
PDE	Partial Differential Equation
ODE	Ordinary Differential Equation
SOV	Separation of Variables
No.	Number
Equ.	Equation

Nomenclature

Scalar	a
Vector	$[r]$
Matrix	$[M]$
Matrix with n rows, m columns	$[T]_{n\times m}$
Dirac delta function	$\delta(.)$

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1 Introduction

1.1 Modeling of Engineering Systems

A central theme in the study of engineering dynamic systems is modeling. A model is a sufficiently simplified representation of the actual system. It has several types of uses. A model may be used to analyze if certain design specifications are feasible before a physical prototype is built for experimental investigation. A model may be used in computer simulation of the system. Other uses of a model include model-based control, product qualification, and performance monitoring, and fault detection and diagnosis. A model should reflect the required features of the real system with sufficient accuracy. Thus, it should incorporate those aspects of the real system that are important in representing the needed characteristics and specifications, but should not be unnecessarily complex. Many engineering processes such as design, simulation, optimization, modification and so on particularly require a sufficient model. Thus, modeling is one of the fundamental steps in engineering and enough attention should be paid toward it.

There are several categories of models. It can be a scaled physical prototype of the actual system; for example, wind tunnel models for aircraft and breadboard models of electrical circuitry (a prototype model). A model may be represented in a numerical form using tables, curves, and so on (a numerical model). A model may be represented in a "mathematical" form where the values of the model parameters are determined using experimental data from the actual system and subsequently applying procedure of model identification (an experimental model). More basically, a mathematical representation of the systems may be formulated using the physics of the system phenomena (constitutive relations) and other applicable relations (e.g., continuity and compatibility). Such models are termed analytical models or mathematical models. This thesis concerns analytical modeling of engineering systems.

Because a model is a sufficiently simplified representation of the actual system, the fact that the model does not contain all the features of the real system is not a shortcoming. In general, a model does not have the capability of predicting all aspects of the real system. In fact, a model provides an estimation or approximation of the true behavior of the system. A variety of expertise including the domain expertise and expertise in modeling procedures, is needed in developing a model. An oversimplified model might omit some important features that affect the dynamics and could lead to inaccurate or unreliable result. On the other hand, an unduly complex and detailed model will increase the costs (including computational costs) which can hamper its applicability (e.g., in real-time control); may hide some important features while highlighting less important details; and may contain parameters that are impossible to estimate and make the analysis less accurate or even impossible.

Engineering systems may contain components or phenomena from different physical domains such as mechanical, electrical, thermal, and fluid. There exist some clear analogies among these domains [1]. In order to systematically integrate component models into the overall model of the system it is useful to use these analogies. They can result in unified and integrated approaches for modeling, design and analysis of engineering systems.

There are several modeling methodologies which can represent an engineering system using a unified and concurrent approach with respect to multi-domain physics. Linear graphs (LG), bond graphs (BG), virtual work principle, and block diagram modeling fall into this category. The present thesis primarily concerns LG modeling.

1.2 Research Objectives

System parameters such as inertia, flexibility, damping, fluid capacitance, electrical capacitance, inductance, electrical resistance, thermal conductivity, and thermal resistance are spatially distributed in physical components while in modeling approaches often they are modeled as lumped-parameter (LP) elements at specific spatial locations. Development of systematic and unified modeling methodology is a complex and challenging task, particularly when it comes to distributed-parameter (DP) models. LG, as a systematic modeling methodology, may be extended to DP systems.

The main goal of the present research is to improve or develop modeling methodologies of linear graph in the context of DP systems. Modeling approaches for DP systems in various physical domains will be investigated and finally a convenient representation is developed using LG. Some clear patterns that are generated in the LG structure are exploited for this purpose.

Also, the present thesis will take advantage of the systematic nature of LG to develop a computer program which is capable of automatically extracting the governing equations of a given system in the state-space form. This toolbox is provided through the Internet, particularly for academic users, at no charge.

1.3 Significance and Rationale

Based on the observations made in the present introduction, a good model should adequately reflect the required features of the real system to effectively characterize the dynamic behaviors that are under study. To develop modeling methodologies and extend their power to more general modeling approaches, it is necessary to investigate the modeling of a range of physical systems. In this context a significant challenge is presented by distributed-parameter (DP) systems. In an actual physical system of various physical domains, component parameters possess spatially distributed behavior. For example, in a mechanical system, flexibility, damping, and inertia properties are distributed to varying extents. Thus, in practical situations, DP systems are the common reality while it is much easier to model and analyze them using lumped-parameter models. *Albert Einstein* has said “Everything should be made as simple as possible, but not simpler.” This encapsulates the core philosophy of the present investigation. Specifically, in order to construct more accurate models that demonstrate the spatially distributed characteristics of the real systems, the system parameters should be spatially distributed as well in the mathematical model. However, the resulting model should not be considerably simpler or considerably more complex than what is needed for the particular purpose.

Approaches exist to model the dynamic behavior of DP systems. However, in the context of Mechatronics, which involves multi-domain systems, the present thesis particularly considers methodologies that can model the dynamics of engineering systems involving different physical domains in a unified and integrated manner. Because of the specific application we are pursuing in our laboratory which is the design evolution framework [2], it is important to adopt a methodology that brings about consistency and compatibility among physical domains. This is the main reason for selecting LG in this work. The rationale or adopting LG is discussed in detail in the next chapter.

1.4 Literature Review

The main goal of this research is the development of a unified and systematic modeling methodology in representation of DP systems in different physical domains. The past work that is available in the literature provides the foundation for the present research. As a particular application that is of significance in the activities of our laboratory, in the recent years there have been many efforts in the context of automated modeling and design, and design evolution [2-6]. In order to meet the practical requirements of these activities, a systematic modeling method that incorporates LG and the DP characteristics is desirable.

Distributed-parameter systems are analytically represented using partial differential equations in time and space [1,7,8]. There are various ways to analytically model DP systems [9]. However, for the purpose of the present research, two major approaches are presented to model DP systems: the micro-element approach where the model is formulated by discretizing the system into a very large number of sufficiently small elements; and the modal approach which uses separation of variables (SOV) into a time function reflecting the modal (natural) frequencies and a space function representing the ODE shapes of the dynamics. Both approaches will be further investigated in the present work, in the context of LG modeling.

In 1967 Brown presented an approach for modeling 1-dimnesional DP system using bond graphs (BG) [10]. There, many lumped elements with differential space in between were used. Barnard and Dransfield used the same approach in a study of hydraulic lines [11].

Karnopp *et al.* represented the equations of modal decomposition of a DP system by BG [12-14]. They used this procedure to represent the acoustics dynamics of a muffler system [15-16]. Later, Lebrun applied this method for modeling hydraulic lines [17] and hydraulic networks [18].

The BG modeling using the SOV procedure for DP systems has been widely used for different systems and applications. Karnopp and Allen used this procedure to incorporate bending modes of vehicle structures [19-21]. Margolis used the same technique to acquire mode shapes and natural frequencies of the Shiva-Nova laser space-frame at the Lawrence Livermore Laboratory [22]. This procedure has been used as well to model the aeroelastic response of aircraft wings [23] and the acoustic response of a respiratory system [24].

Margolis extended this procedure to handle some non-standard characteristics of a system. In particular, when non-causal forces act on a DP system, problems arise in the formulation [25]. In this case tedious algebraic loops must be solved in order to formulate the system equations. Specifically, Margolis presented a method to reduce or overcome these problems by introducing compliances for frequencies greater than those in the range of interest of the problem. This modification makes it possible to analyze the interaction of a DP system for inputs with different causality. He further developed this procedure by formulating an algorithm to model dynamic systems that incorporate DP systems [26].

1.5 Thesis Outline

This thesis is organized into six chapters and an appendix. Chapter 1 has provided an introduction to the modeling of engineering systems. It has been argued that an important area that needs to be further developed in the context of mechatronic modeling is DP system. In order to use the existing analogies among different physical domains, a unified modeling approach is preferred. Linear graphs are known to have clear advantages in this respect. Research objectives and the justification for them were indicated. A survey of background and pertinent literature was provided at the end of the chapter.

Chapter 2 highlights the importance of modeling and presents several useful modeling approaches. The rationale for adopting the LG modeling methodology in the present work is discussed in this chapter. A further discussion of the LG method and DP systems is provided at the end of the chapter.

Chapter 3 introduces vector linear graphs, which is an important contribution of the present thesis. The associated terminology and notation are described. An example in the area of multi-body systems is given to illustrate the application and advantages of this approach.

Chapter 4 investigates the LG representation of DP systems. DP systems from mechanical and hydraulic domains are modeled using the conventional LG method. Then, the procedure is extended to utilize vector linear graphs. A new LG element is introduced to represent a DP system using a straightforward procedure. This is an important contribution of the present work. Illustrative examples are given to further show the importance of the developed approaches.

Chapter 5 presents the LG2ss toolbox, a MATLB code which receives the LG representation of a system in the form of a matrix and generates the state space model of the system. The enhancement of the toolbox is also a contribution of the thesis. This code has been provided on the web for use at no cost through the Internet.

Chapter 6 concludes the thesis with a summary and a discussion of the work that has been carried out. The main contributions of the thesis are pointed out. Possible future work with further research possibilities is indicated.

Appendix A provides the PHP code that has been used to embed the LG2ss toolbox on the web.

2 Modeling Using Linear Graphs

2.1 Modeling of Mechatronic Systems

Dynamic modeling is a basic and fundamental step in the development and investigation of engineering processes. Models are used for a variety of purposes such as simulation, design, control, and health monitoring of engineering systems. For example, a model may be used to check whether the design specifications are feasible even before initiating the physical effort of prototyping. Many engineering systems fall within the realm of mechatronics. A mechatronic system involves more than one physical domains such as mechanical, electrical, fluid, and thermal, with significant dynamic interaction. In order to properly model, design and implement a mechatronic system, a comprehensive, integrated method should be used. When approaching mechatronic systems using conventional modeling methods, each physical domain is typically formulated and analyzed separately and differently. For example in case of an electromechanical system, mechanical and electrical domains are analyzed and designed separately and finally are combined together. However, for a system with multiple domains and dynamic interaction, as in the case of a mechatronic system, more compatible and optimal behavior may be obtained when they are analyzed and designed using a unified and integrated approach. Such a design may lead to greatly increased efficiency and reliability of the system and reduced overall cost.

A mechatronic system may be investigated in two different perspectives: power and information. From the perspective of power, energy levels that are needed to "drive" the system are substantially greater than what are needed to represent, transmit, and process information, which is needed, for example, in generating control actions. A shortcoming of conventional methods of modeling is that typically they are not able to integrate information channels and software control components into the model of the physics of the system. Utilizing a universal and unified approach to model complex and multi-domain engineering systems will directly benefit engineering activities such as simulation, design, health monitoring, control, and optimization.

As highlighted in the previous discussion, the purpose of this thesis is to develop a comprehensive integrated modeling methodology for mechatronic systems whose distributed parameters cannot be sufficiently approximated by conventional lumped models, so that the indicated shortcomings can be satisfactorily resolved. In this context, the two modeling languages that are widely used in the literature are bond graphs and linear graphs. These two modeling approaches represent the system components from different physical domains in the same manner in generating a global model. All dynamic equations derived from different domains are unified and represented in the same manner which can result in significant benefits. However, linear graphs have several advantages over bond graphs, as discussed in the sequel. Accordingly, the linear graph method has been chosen as the language for the developments presented in this thesis.

2.2 Modeling Approaches

A variety of modeling methodologies exist to represent different types of physical systems. Each of these methodologies has its own specific advantages and disadvantages. Although, one approach may be ideal to model a special type of problem it may not be applicable generally or for some other types. Modeling methods may be classified based on different aspects and criteria. For example, discrete versus continuous modeling, language-based versus graph-based modeling, numerical versus parametrical modeling, single domain versus multi-domain modeling, analytical versus experimental modeling, functional versus object-oriented modeling and so on. In literature there are surveys on these different classes of modeling [27].

Selection of a modeling method directly depends on the considered system, its application, and the needed information. Pertinent questions include the following: What is the system to be modeled (single domain, multi-domain, one dimensional, multi-dimensional, etc.); what is the modeling objective (system identification, health monitoring, optimization, inverse kinematics, control, etc.); what type of elements have to be modeled (lumped-parameter, distributed-parameter, linear, nonlinear, etc.)?

Mechatronic systems are interacting multi-domain in nature. To properly model them, a unified modeling approach should be adopted. Since a particular mechatronic system may contain elements, an object-oriented modeling method is more appropriate. Also the modeling language should be capable of describing the physical phenomenon in discrete time as well as continuous time.

Considering these requirements and based on available studies of various modeling methods, graphical modeling approaches are more applicable and convenient in modeling mechatronic systems and extracting their dynamic equations. Two particular methods of this class are widely considered in literature: linear graphs (LG) [1,28, 29] and bond graphs (BG) [14,30-32].

BG is a powerful integrated modeling tool which uses a graphical representation to describe elements from different physical domains. It has been widely used in the literature and several practical software tools have been developed based on that. At the Massachusetts Institute of Technology (MIT), Paynter proposed BG in 1959 [33]. BG considers energy exchanges among different parts of the system and between the system and the environment. In this method, energy is the product of an effort variable and a flow variable. The *0* and *1* junctions express the general form of loop and node rules of generalized Kirchhoff's laws.

A disadvantage of BG is that the structure of the BG model can be quite different from that of its physical system [27]. Also, this method is not properly capable of modeling physical systems in which discrete and continuous components are incorporated [27]. Conflicts in component causality may result as well in the bond graph representation.

In order to create a model using BG, energy directions and causality of the system must be established. In case of acausal systems for which input and output variables can be switched and also for cases where causality is variable through time (such as coulomb friction force around zero velocity), BG is not an appropriate modeling approach.

On the other hand, modeling with LG provides a more convenient methodology which does not have many shortcomings of BG as mentioned. LG is used in the present work particularly in the modeling of distributed parameter systems.

2.3 Linear Graphs

Linear graphs present a powerful tool for modeling of physical systems [7, 8]. Particularly a unique state space model for a dynamic system can be systematically obtained using linear graphs. LG uses branches (interconnecting lines) to represent system elements and nodes to represent interconnection points (junctions) of elements. Two types of variables, through variables and across variables, which are analogous among different physical domains, are used as element variables. The product of an across variable and the corresponding through variable is power. An important feature of LG models is that there exists a one to one correspondence between the model and the physical system, from component and structure points of view. In LG once the graphic model is built, a unique state space model can be generated [1,28, 29]. To do so three classes of equations should be formulated:

1- Continuity equations which are a generalization of Kirchhoff's node equations (current balance; equilibrium of forces, etc.) among the through variables at each node.
2- Compatibility equations which are the generalization of Kirchhoff's loop equations (for voltages; velocities, etc.) of the across variables for each closed path of the graph.
3- Constitutive equations which show the physical relation among the across and through variables of an element.

LG was first formalized by Trent [34], and rapidly developed in the 1960s as a general methodology to model multi-domain engineering systems. In the beginning the system components were not categorized and the constitutive equation for each element was obtained analytically or experimentally [35-37]. In [38] Chandrashekar and Kesavan used LG to formulate complex electrical networks. In 1973, Andrews and Kesavan introduced a vector-network model which utilized LG to model unconstrained 3D systems [39]. Later, Li and Andrews extended this work to model constrained 2D systems [40]. Chou *et al.* used LG formulation along with joint coordinates to model open-loop multi-body systems [41] and Baciu and Andrews extended this study for systems with kinematic constraints [42].

McPhee continued these studies and introduced branch coordinates in LG for modeling multi-body systems [43,44]. In 1997 Huang and Whitehouse extracted dynamics equations of rigid multi-body systems using LG and compared them with equations obtained using the Newton-Euler and Lagrange-Hamilton methods [45].

In recent years, McPhee *et al.* have done a great deal of research on modeling of spatial flexible multi-body systems and symbolic computing using the vector formation of LG [46-49]. Diaz developed this area for different physical domains (electrical, hydraulic and mechanical) [50].

In all the mentioned studies, when a system contains both one dimensional non-mechanical parts and 3D mechanical bodies, different formulations have been used to obtain the dynamic equations. However, [51] has attempted to provide a unified approach for LG modeling of these complex systems.

Further to the mentioned advantages, LG modeling has the following benefits over BG modeling. In LG, the model topology bears direct resemblance to the corresponding physical system. In fact, LG possesses topological analogies among different physical domains. For example, in the LG representation, a series connection in the mechanical domain is analogous to a series connection in the electrical domain, which is not the case for BG modeling.

As a result of these advantages LG facilitates the identification similarities and generalizations across domains. For example, Smith introduced a new element, *inerter* in the mechanical domain which is the properly analogous component for capacitors in the electrical domain. His finding is based on observations made using LG modeling [52].

In specific applications such as the design evolution framework that is being developed in our laboratory, multi-domain problems are encountered. Then it is necessary to utilize a consistent modeling approach that preserves the analogies among different domains in a consistent and compatible manner. LG is well suited as a modeling technique, in this context.

2.4 Distributed-Parameter Systems

Distributed parameter (DP) systems are those whose physical parameters are spatially distributed at least in part and their approximation by lumped elements leads to a model that is not valid. Analytically they are represented by PDEs in time and space [14]. All physical components demonstrate distributed properties to some extent. Typically, inertia is distributed over a segment in space and may exhibit stiffness and resistance characteristics. Similarly springs and dampers are spatially distributed and exhibit inertial characteristics. However, in many applications components are assumed to be lumped at a single spatial location at a given time.

There are engineering systems for which lumping of the real components to a few discrete spatial locations is far from obvious or may lead to significant model errors. For these systems compliance and inertia properties should be considered distributed over the geographic space of the component and should be modeled accordingly. The wings of an aircraft, solar panels of a spacecraft and long hydraulic pipes are examples of such systems. Thus, modeling approaches must be developed to be able to accurately represent distributed properties for different components in different physical domains.

In DP systems, the properties are considered to be distributed throughout the component as interconnected infinitesimally small elements. When the system moves, each of these infinitesimal elements acts relative to each other in a continuous fashion.

Another way of representing the behavior of a DP system is to describe the time and the spatial behavior. The time response of a DP system may be represented using appropriate natural frequencies (or eigenvalues). Its shape may be described in space using a continuous function of space variables (mode shapes or eigenfunctions). From the vibration point of view, DP systems are represented by infinite number of natural frequencies and mode shapes. A mode shape is a pattern of motion along the space domain of a system in which all parts oscillate with the same frequency and with a fixed phase relation.

In this thesis DP systems are represented by using LG modeling methodologies. Different approaches will be developed for LG to properly represent a DP system in a unified and systematic framework. As a result, modeling of the systems in which DP components interact with lumped parameter (LP) components, is made consistent and straightforward.

3 Vector Linear Graphs (VLG)

In applications where multi-dimensional system components are present, expressing the physics of the system by scalar linear graphs is a tedious and somewhat complex task. In such circumstances, multi-dimensional notations would be an asset. Moreover, when specific similarities and patterns in the scalar linear graph model of a system are recognized, they can be represented in a compact notation which simplifies the model representation. This multi-dimensional and compact notation is called vector linear graphs (VLG) which uses vector representations, and vector calculations in obtaining the governing equations of the system.

Vector calculation has been used in bond graphs. Bondorson [53] first introduced vector bond graphs and used them to model one-dimensional distributed-parameter systems using lumped micro-elements. Later, vector bond graphs were used in multi-body systems and dynamics of mechanisms [54-56]. Other applications of vector bond graphs, such as introducing mechanical joint constraints, are found in [57-59]. In these applications, the methods used to obtain system equations require the vector bond graphs to be resolved into several scalar bonds. Thus it results in decompression of the graphical representation of the system [60].

3.1 Notation and Elements

To extend the linear graph notation to the VLG notation, some minor changes are made in representation of the element variables. In VLG the through and across variables and also the elemental parameters are denoted by vectors and/or matrices, thereby representing the multi-dimensional nature of the system. In this new representation, unlike conventional linear graphs, each element is a multi-port element. In other words, they are interacting with the neighboring elements through several ports of energy. Thus, each single branch in a VLG represents a bunch of branches in the corresponding LG. The general form of a VLG element is shown in Figure 3-1. Both through and across variables are n-vectors defined as:

$$[f] = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix} \qquad [v] = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

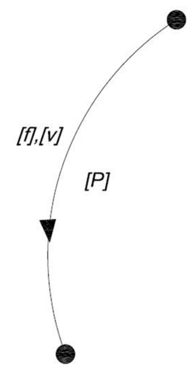

Figure 3-1 General form of a vector linear graph element.

Power associated with each element is a scalar and is the algebraic summation of the product of the corresponding through and across variables (or the dot product of the two vectors):

$$P = \sum_{i=1}^{n} f_i \cdot v_i = [f]^T [v] \qquad 3\text{-}1$$

This will apply to all conventional elements of linear graphs. The elemental parameter and constitutive equation for each conventional element would be represented in the same manner as before but in a matrix form. For example, in the case of mechanical domain the elements are defined as in Table 3-1.

Table 3-1 Elemental equations of vector linear graphs for mechanical domain.

Element	**Constitutive equation**
Inertia	$[f_m]_{n\times 1} = [M]_{n\times n}[\dot{v}_m]_{n\times 1}$
Spring	$[\dot{f}_k]_{n\times 1} = [K]_{n\times n}[v_k]_{n\times 1}$
Damper	$[f_d]_{n\times 1} = [B]_{n\times n}[v_d]_{n\times 1}$
Transformer	$[v_o]_{n\times 1} = [T]_{n\times m}[v_i]_{m\times 1}$ $[f_i]_{m\times 1} = -[T]_{m\times n}^T[f_o]_{n\times 1}$
Gyrator	$[v_o]_{n\times 1} = [G]_{n\times m}[f_i]_{m\times 1}$ $[v_i]_{m\times 1} = -[G]_{m\times n}^T[f_o]_{n\times 1}$

We can easily verify the power conservation in transformers and gyrators. For a transformer we have:

$$[v_o]_{n\times 1} = [T]_{n\times m}[v_i]_{m\times 1} \rightarrow [v_o]^T{}_{1\times n} = [v_i]^T{}_{1\times m}[T]^T{}_{m\times n}$$

$$\rightarrow [v_o]^T{}_{1\times n}[f_o]_{n\times 1} = [v_i]^T{}_{1\times m}[T]^T{}_{m\times n}[f_o]_{n\times 1}$$

$$\Rightarrow [v_o]^T{}_{1\times n}[f_o]_{n\times 1} + [v_i]^T{}_{1\times m}[f_i]_{m\times 1} = 0$$

This states that the summation of input power and output power is zero. The same procedure can be followed for the gyrator elements. Since vector linear graphs use vector notations, each single equation formulation in a VLG represents a group of equation formulations in the corresponding conventional LG. Thus, it reduces the amount of formulation manipulations and computational cost. Another advantage of VLG is that the graphical representation of the model is condensed since each element in VLG represents a group of elements in the corresponding conventional LG. Advantages of vector linear graphs will be more evident in application and practice, as indicated in the illustrative example of this thesis. A popular application of vector modeling is in multi-body systems. In the following section, illustrative examples are provided to demonstrate the possible application of VLG.

3.2 Illustrative Example

The two-link manipulator is a popular example, which may represent many industrial and academic applications. There are many procedures of dynamic modeling for robot manipulators. Dynamic modeling of robots has been done with conventional linear graphs [61] and bond graphs [14, 62] as well. These graphical modeling methods have several advantages over others. One advantage is that the model can be easily constructed only using the kinematic relations of the manipulator without deriving the complicated governing dynamic equations. Moreover, due to one-to-one correspondence of the linear graph model with the physical system, changes to the system can be easily applied on the model. Only minor changes to the model structure and the parameters of the elements are required, without having to reformulating the dynamic equations. In this illustrative example, VLG is used to generate a model of a robot manipulator, with more ease than with conventional linear graphs.

A schematic representation of a two-link manipulator and its parameters are illustrated in Figure 3-2. To model a multi-body system, velocities of the points of interest should be related to the generalized coordinates of the system [63]. Here these generalized coordinates are defined as absolute angular rotations of the links:

$$q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

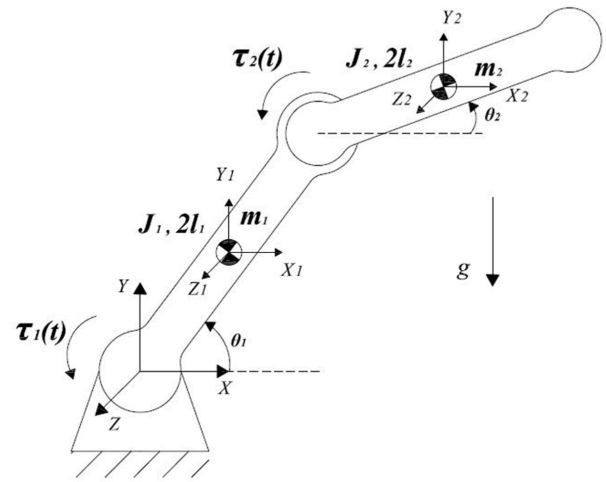

Figure 3-2 Two-link robot manipulator.

The absolute velocity of the mass center of each link can be determined by:

$$\begin{bmatrix}\dot{X}_1\\ \dot{Y}_1\\ \dot{Z}_1\end{bmatrix} = \begin{bmatrix}-l_1 sin(\theta_1) & 0\\ l_1 cos(\theta_1) & 0\\ 0 & 0\end{bmatrix}\dot{q}$$

$$\begin{bmatrix}\dot{X}_2\\ \dot{Y}_2\\ \dot{Z}_2\end{bmatrix} = \begin{bmatrix}-2l_1 sin(\theta_1) - l_2 sin(\theta_2) & -l_2 sin(\theta_2)\\ 2\ l_1 cos(\theta_1) + l_2 cos(\theta_2) & l_2 cos(\theta_2)\\ 0 & 0\end{bmatrix}\dot{q}$$

After combining these equations we come up with the following transformation equation:

$$\begin{bmatrix}\dot{X}_1\\ \dot{Y}_1\\ \dot{Z}_1\\ \dot{X}_2\\ \dot{Y}_2\\ \dot{Z}_2\end{bmatrix} = \begin{bmatrix}-l_1 sin(\theta_1) & 0\\ l_1 cos(\theta_1) & 0\\ 0 & 0\\ -2l_1 sin(\theta_1) - l_2 sin(\theta_2) & -l_2 sin(\theta_2)\\ 2l_1 cos(\theta_1) + l_2 cos(\theta_2) & l_2 cos(\theta_2)\\ 0 & 0\end{bmatrix}\dot{q} = [T].\dot{q}$$

It is this transformation that helps us to build the linear graph model shown in Figure 3-3. Since the transformer parameter $[T]$ has time-varying variables, it is termed a modulated transformer. In this linear graph model vectors and matrices are defined as in Table 3-2.

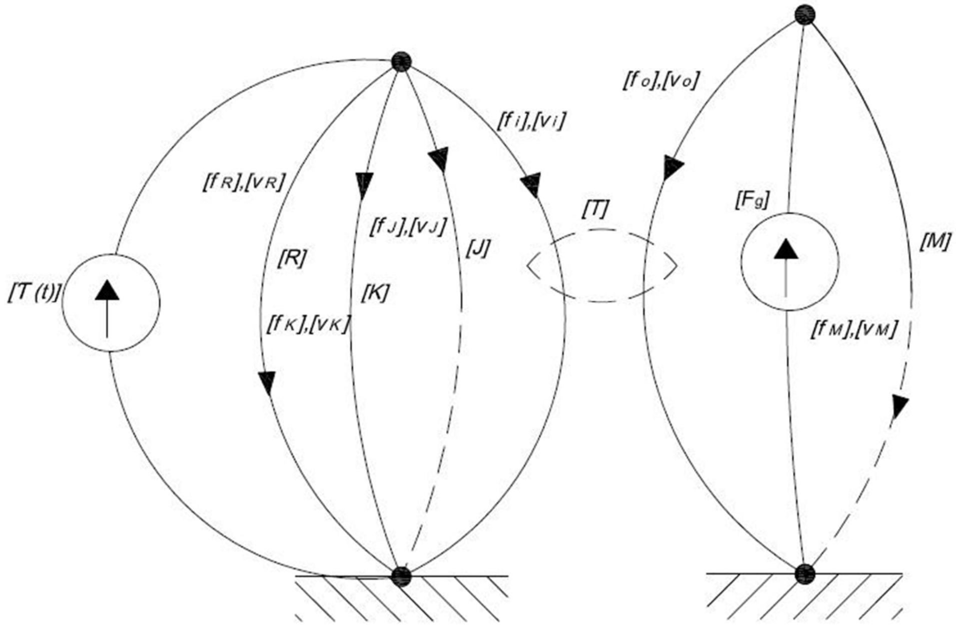

Figure 3-3 Linear graph of two-link robot manipulator using vector linear graphs.

Table 3-2 Vectors and Matrices used in model of two-link robot manipulator.

Input torques on each link	$[\tau(t)] = \begin{bmatrix} \tau_1(t) \\ \tau_2(t) \end{bmatrix}$
Rotational inertia of each link	$[J] = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}$
Joint compliance	$[K] = \begin{bmatrix} K_1 & 0 \\ -K_2 & K_2 \end{bmatrix}$
Resistance to model friction in each joint	$[R] = \begin{bmatrix} R_1 & 0 \\ -R_2 & R_2 \end{bmatrix}$
Gravitational force on mass center of each link	$[f_g] = [0 \quad -m_1 g \quad 0 \quad 0 \quad -m_2 g \quad 0]^T$
Mass matrix	$[M] = \begin{bmatrix} [m_1] & 0 \\ 0 & [m_2] \end{bmatrix}$ and $[m_1] = m_1[I_3]$, $[m_2] = m_2[I_3]$

The joint compliance has been added to represent the elastic deformation of the joint components [64, 65]. Note that the link mass is dependent on the link rotational inertia. Thus, the state variables are defined as the derivatives of the generalized coordinates and the compliance forces of the robot joints:

$$\text{State variables: } X = \begin{bmatrix} [f_K]_{2\times1} \\ [v_J]_{2\times1} \end{bmatrix}$$

The node equations:

$$[\tau(t)] - [f_J] - [f_K] - [f_R] - [f_i] = 0$$

$$[f_g] - [f_o] - [f_M] = 0$$

The loop equations:

$$[v_J] - [v_K] = 0$$

$$[v_J] - [v_R] = 0$$

$$[v_J] - [v_i] = 0$$

$$[v_J] - [v_K] = 0$$

$$[v_o] - [v_M] = 0$$

Eliminate the auxiliary variables:

$$[\dot{f_K}] = [K][v_J]$$

$$[\dot{v_J}] = [J]^{-1}([\tau(t)] - [f_K] - [R][v_J] - [f_i])$$

$$[\dot{v_J}] = [J]^{-1}\{\,[\tau(t)] - [f_K] - [R][v_J] + [T]^T([f_g] - [M][\dot{T}][v_J] + [M][T][\dot{v_J}])\}$$

$$[\dot{v_J}] = \{[I] - [J]^{-1}[T]^T[M][T]\}^{-1}\{[J]^{-1}\{\,[\tau(t)] - [f_K] - [R][v_J] + [T]^T([f_g] - [M][\dot{T}][v_J])\}\}$$

Governing equations:

$$[\dot{f_K}] = [K][v_J]$$

$$[\dot{v_J}] = \{[I] - [J]^{-1}[T]^T[M][T]\}^{-1}\{[J]^{-1}\{\,[\tau(t)] - [f_K] - [R][v_J] + [T]^T([f_g] - [M][\dot{T}][v_J])\}\}$$

Due to the nonlinearities that arise from the geometric constraints in the transformer element, the latter governing equation should be linearized in order to obtain a linear state space model.

This procedure can be extended and generalized to higher order robot manipulators and robots with other types of joints, where the modulated transformers relate mass centers of links to the absolute velocities of the joints. Damping and compliance matrices should be obtained carefully to correctly relate the corresponding forces to the adjoining joint absolute velocities.

3.3 Summary

In modeling of some types of systems the model graph follows a uniform pattern. In this chapter it was shown that such uniform patterns may be exploited to simplify the modeling process in generation of the model graph and in the equation formulation. VLG was introduced for this purpose. VLG uses matrix and vector algebra instead of scalar algebra and combines several equation lines into a single equation line, providing a compact representation. The notations and elements for VLG were defined. An illustrative example of a multi-body system was presented to demonstrate the application of VLG.

4 Modeling of Distributed-Parameter (DP) Systems

4.1 Modeling DP Systems Using Linear Graphs

In developing linear graph models and extending their power toward a universal modeling approach, it is necessary to investigate the modeling of a range of physical systems. A challenging type of systems in this context is the distributed-parameter (DP) systems. The physical parameters of a real component are spatially distributed to some extent. In a realistic and accurate model it should be treated as a distributed-parameter system and not a lumped-parameter one. Distributed-parameter systems are analytically represented by partial differential equations in time and space.

Two main methods are available to model distributed-parameter systems: microelement method (model consists of a very large number of very small interconnected elements) and modal analysis (where time and space behavior are separated). In the following sections these approaches are presented and extended to linear graph (LG) modeling.

4.1.1 Microelement Approach

This approach for modeling one-dimensional systems uses a continuous chain of interconnected lumped components. In the mechanical domain the applicable lumped component is a simple oscillator consisting of ideal, lumped micro-elements of mass, spring and damper. In particular, the springs account for the elastic energy storage properties and dampers account for energy dissipation properties of the system. Each such micro-component (simple oscillator) corresponds to a very small length segment of the system. In theory, to have an exact model of a distributed system one should have an infinite number of oscillators; thus the length of each component should be infinitesimal, tending to zero in the limit. However modeling of an infinite number of components is not practically possible.

Each simple oscillator has two state variables. Hence, increasing the number of elements results in an excessively large state space and correspondingly excessive resources for analyzing, computation and simulation of the system.

As an illustrative example consider a simple bar with specific boundary conditions and a general distributed external force, as shown in Figure 4-1.

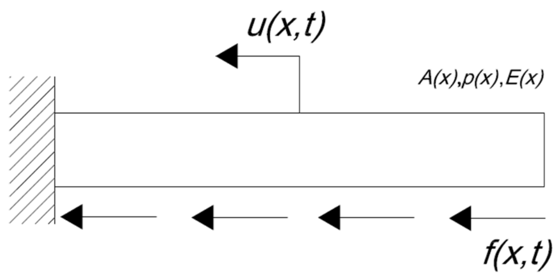

Figure 4-1 A simple bar subjected to external force.

In Figure 4-2 the representation of this continuous bar using lumped elements, as previously discussed, is shown. For simplicity and since mass and spring properties are more critical, the damping property is not included in this example. It should be noted, however, that damping can be incorporated through similar steps, and the overall procedure remains the same.

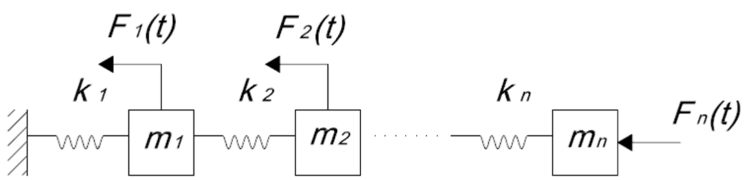

Figure 4-2 Lumped-parameter representation of the bar.

In order to apply the present approach, the required number of lumped elements should be specified. To this end, let *Δx* be the length of each element. The length of the bar (*L*) is divided by this elemental length to give the number of lumped elements. From [14] the corresponding mass and stiffness of the element is given by:

$$m_i = \int_{\Delta x} \rho_i(x) A_i(x) dx \qquad 4\text{-}1$$

$$k_i = \frac{\int_{\Delta x} E_i(x) A_i(x) dx}{\Delta x} \qquad 4\text{-}2$$

In order to incorporate the external force, it should be noted that in continuous elements, the external force is considered to be distributed over the spatial domain of the element. The force exerted on each lumped element is given by:

$$F_i(t) = \int_{\Delta x} f(x,t) dx \qquad 4\text{-}3$$

in which *f(x,t)* is the force exerted per unit length. The linear graph of the corresponding system is shown in Figure 4-3.

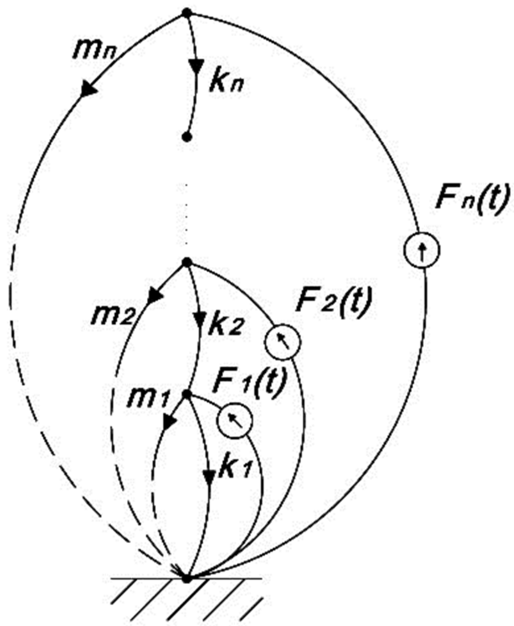

Figure 4-3 Linear graph representation of the lumped model of a bar.

4.1.2 Separation of Variables (Modal) Approach

Consider the structural bar discussed in the previous section. The continuous model that is obtained by letting Δx to zero corresponds to the partial differential equation (PDE) [1466]:

$$F(x,t) + \frac{E\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2} \tag{4-4}$$

This PDE governs the dynamics of the bar. Modal analysis may be used to solve this PDE in time (t) and space (x) variables [66]. The separation approach separates the displacement function as the product of a time function and a space function. As a result the PDE is transformed into a set of ODEs. Specifically, the response function is expressed as:

$$u(x,t) = Y(x).\,g(t) \tag{4-5}$$

In modal analysis, first the external force exerted on the system is ignored and the corresponding homogenous differential equation (which represents the "free" response) is solved for the space variable. In fact, it is this free, natural response that is given by the modal response. The detailed procedure is given in [66].

The response is expressed as the modal expansion:

$$u(x,t) = \sum_{n=1}^{\infty} Y_n(x).\,g_n(t) \qquad 4\text{-}6$$

Here $Y_n(x)$ is the *n*th mode shape, which is given by one of the solutions of the ordinary differential equation in the space variable. An important characteristic of the mode shapes is its orthogonality [66] the mode shapes are orthogonal to one another. This reflects the linear independence of the separate mode shapes, which can be combined to generate any general shape. The system response, forced or unforced, may be represented as a linear combination of the mode shapes; only the time functions (generalized coordinates) differ in the two cases. It should be noted that the mode shapes should satisfy the boundary conditions of the system.

After substitution of $u(x,t) = \sum_{n=1}^{\infty} Y_n(x).\,g_n(t)$ 4-6 in the main non-homogenous equation and application of orthogonality, we obtain:

$$m_n \ddot{g}_n + k_n g_n = \int_L f(x,t) Y_n(x) dx \qquad 4\text{-}7$$

in which:

$$m_n = \int_L \rho(x) A(x) Y_n^2(x) \qquad 4\text{-}8$$

$$k_n = m_n \omega_n^2 \qquad 4\text{-}9$$

Also, ω_n is the natural frequency corresponding to the *n*th mode shape and is obtained from the ordinary differential equation in the time variable. In the present example one has:

$$m_n = m = \frac{\rho A L}{2}$$

$$k_n = m \frac{E\pi^2 (2n-1)^2}{4\rho L^2}$$

It is $m_n \ddot{g}_n + k_n g_n = \int_L f(x,t) Y_n(x) dx$ 4-7 that is used to represent the DP system in linear graphs. This equation is similar to that of a simple mass-spring system. Hence, the linear graphs representation can be obtained conveniently. In order to better understand the approach, the distributed force on the bar is substituted with a point force *F(t)* on the right end of the bar. We do this without loss of generality of the problem. In this case .Equ4-7 may be rewritten as:

$$m_n \ddot{g}_n + k_n g_n = \int_L F(t).\,\delta(x-L).\,Y_n(x) dx \Rightarrow m_n \ddot{g}_n + k_n g_n = F(t) Y_n(L) \qquad 4\text{-}10$$

where δ(.) is the Dirac delta function. The LG representation of $m_n \ddot{g}_n + k_n g_n = \int_L F(t).\,\delta(x-L).\,Y_n(x) dx \Rightarrow m_n \ddot{g}_n + k_n g_n = F(t) Y_n(L)$ 4-10 is shown in Figure 4-4. The force exerted on the system is transmitted to all modes through appropriate transformers. In fact the transformer elements are used to excite the modes by the external force.

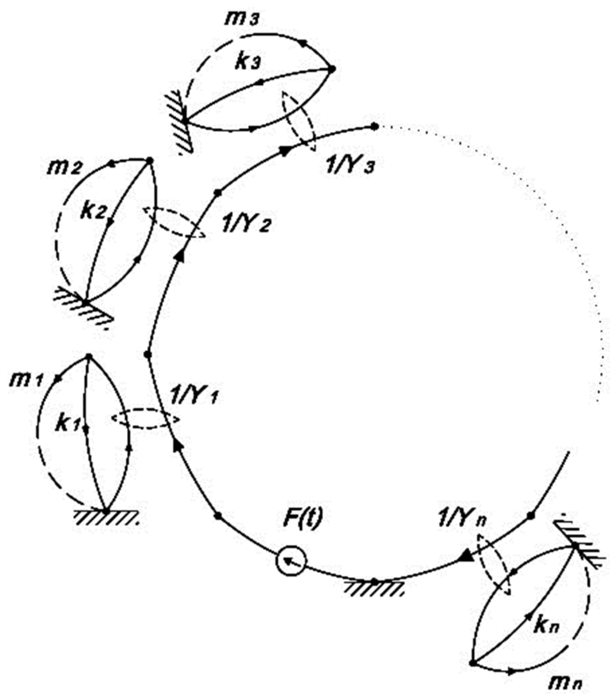

Figure 4-4 Linear graph presentation of the bar using modal separation.

This graph can be used to obtain the state space model of the system which may be used in simulation, control, and so on. Using the linear graph for each mode, one has:

$$v_n = \dot{g}_n = X_{2n-1}$$

$$f_n = k_n g_n = X_{2n}$$

Here v is the across variable, f is the through variable, and X is the state variable. The state space equation is obtained as:

$$\frac{d}{dt}\begin{bmatrix} v_1 \\ f_1 \\ \vdots \\ v_n \\ f_n \end{bmatrix} = A\begin{bmatrix} v_1 \\ f_1 \\ \vdots \\ v_n \\ f_n \end{bmatrix} + B[F(t)]$$

in which:

$$A = \begin{pmatrix} 0 & -\frac{1}{m_1} & \cdots & 0 & 0 \\ k_1 & 0 & & 0 & 0 \\ \vdots & & \ddots & \vdots & \\ 0 & 0 & \cdots & 0 & -\frac{1}{m_n} \\ 0 & 0 & & k_n & 0 \end{pmatrix} \quad B = \begin{pmatrix} \frac{Y_1(L)}{m_1} \\ 0 \\ \vdots \\ \frac{Y_n(L)}{m_n} \\ 0 \end{pmatrix}$$

If we consider the velocities of all positions at all times as the desired output we have:

$$\frac{du}{dt}(x,t) = [Y_1(x) \quad 0 \quad \dots \quad Y_n(x) \quad 0] \begin{bmatrix} v_1 \\ f_1 \\ \vdots \\ v_n \\ f_n \end{bmatrix} \qquad \text{4-11}$$

Since these system equations may be derived using the direct solution of the partial differential equation of the system, it is important to indicate the relevance of linear graphs in this context. In particular, the linear graphs can represent the multi-physics of a more general multi-domain system in a unified manner. For example, if the force exerted on the rod is created from another system and if the rod interacts with other domains (electrical, thermal, fluid), the same unified approach is applicable. Furthermore, the procedure is very systematic and the resulting state space model is unique.

As an advantage of linear graphs, it is easy to distinguish that subsystems corresponding to each mode are connected in parallel, unlike the subsystems in the micro-element approach. As a result higher-frequency subsystems (or unexcited modes) may be removed without seriously affecting the model accuracy.

System simulation using this model is very straightforward. Figure 4-5 shows the displacement of the example bar at the point where the force is exerted (at $x = L$). It is assumed that the bar is made of regular steel of length 1m, square cross-section of side 10cm. A sinusoidal external force of amplitude 1N and frequency of 1Hz is applied.

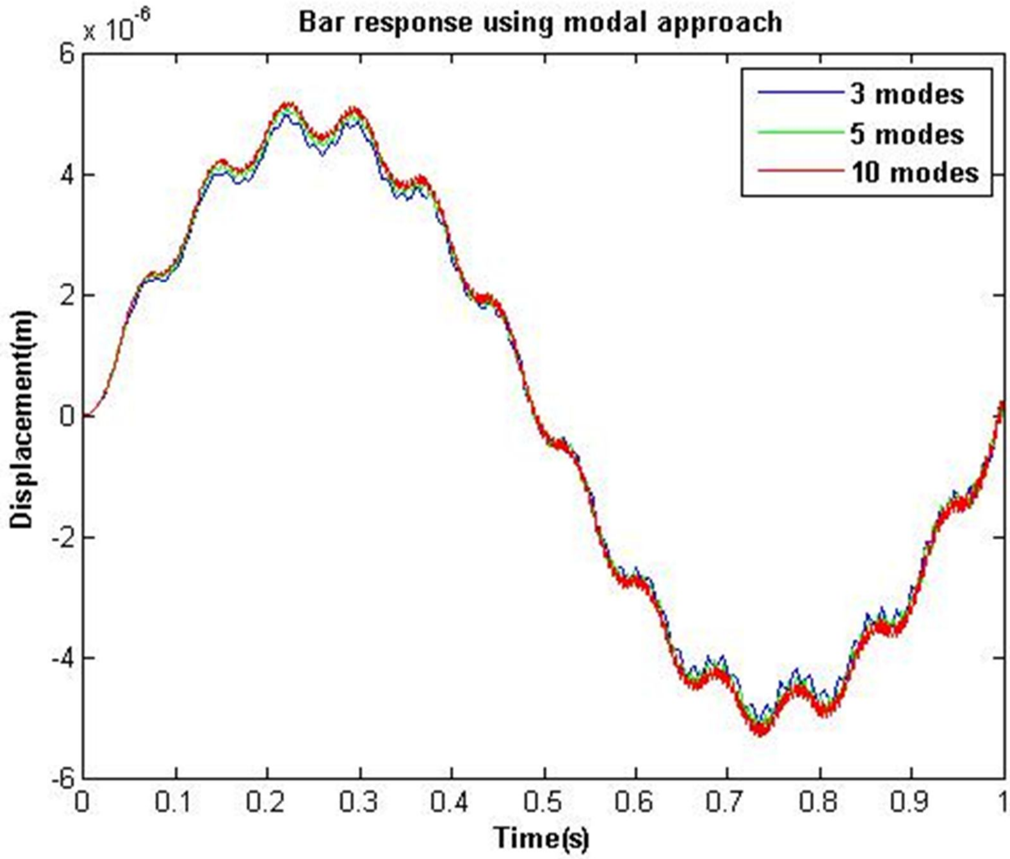

Figure 4-5 Time response of the bar using first several modes.

As it is seen from Fig 4-5, as the included number of modes is increased, the response becomes smoother and more accurate. Beyond the first five modes, the improvement is not significant. Also it is evident that these results are exactly the same as the results obtained from the conventional modal analysis. Using this approach the properties of any desirable point on bar may be evaluated.

It should be noted that in this method, the mode shapes are known a priori for the specific boundary conditions. Typically they are determined prior to LG modeling. Mode shapes may be obtained using the well-known experimental or theoretical methods.

This approach of separation of variables (SOV) may be generalized for any type of external forces exerted in different points on the system. However, as the number of points at which the forces are exerted is increased, the LG representation becomes more complex. In the following sections, based on the SOV approach described in this section, other methods are introduced to reduce this complexity.

4.1.3 Comparison and Discussion

The approaches provided in this section are useful for unified modeling of systems in which DP segments interact with lumped-parameter parts. Either approach (micro-element or SOV) may be used depending on the needs of the problem.

The micro-element approach is easy to understand uses straightforward physical discretization of the DP system. However, as the number of micro elements increases, more computational resources are needed for solution. Identification of the system parameters (e.g., stiffness and damping) at the interfaces of the discrete elements is not straightforward. As demonstrated, the LG representation of a DP system using this method shows an "overlapped-growth" pattern. This pattern means that the micro elements are coupled together and the removal of one element destroys the validity of the entire model. Discontinuity of displacement field is another drawback for this approach.

On the other hand, the SOV approach retains the distributed characteristic of the system. A parallel pattern with a looped link is recognizable in the LG representation. This pattern indicates that each mode is independent and uncoupled from the other modes (a higher-frequency LG element or an element corresponding to node may be removed without seriously sacrificing the model accuracy). Also, better accuracy is possible with a given number of modes, compared to the micro-element approach with the same number of discrete elements.

From the above discussion it is evident that the SOV approach provides more convenience and accuracy within the integrated system. This approach has been adopted as the basis of the presentations of the foregoing sections. This approach may be generalized to include various DP elements and in multiple domains. In order to demonstrate the generalization of the approach, the procedure is applied for a 2D structural beam and a hydraulic pipeline, in the following section.

4.2 LG Representation of 2D Structural Beam

The approach of SOV may be applied using the same procedure as before, for a 2-dimensional mechanical system such as a structural beam. First the governing continuous dynamic equation is obtained and then through the modal decomposition, the equations for time response and the mode shapes in the spatial domain, for the specified boundary conditions, for various modes are developed. The forced partial differential equation of a Bernoulli-Euler beam is [66]:

$$F(x,t) = \rho A \frac{\partial^2 v}{\partial t^2} + \frac{EI\partial^4 v}{\partial x^4} \qquad 4\text{-}12$$

After modal decomposition and following the procedure as in the previous section, the equation for generalized coordinates is obtained as:

$$m_n \ddot{g}_n + k_n g_n = \int_L f(x,t) Y_n(x) dx \tag{4-13}$$

in which:

$$m_n = \int_L \rho(x) A(x) Y_n^2(x) \tag{4-14}$$

$$k_n = m_n \omega_n^2 \tag{4-15}$$

The mode shapes are given by

$$Y_n(x) = A' cosh(\lambda_n x) + B' sinh(\lambda_n x) + C' cos(\lambda_n x) + D' sin(\lambda_n x) \tag{4-16}$$

$$\lambda_n = \frac{\rho A}{EI} \omega_n^2 \tag{4-17}$$

Here $Y_n(x)$ is the nth mode shape and ω_n is the natural frequency corresponding to the nth mode shape which is determined from the ordinary differential equation in the time variable. The coefficients *A', B', C'* and *D'* are determined up to one unknown, using the four boundary conditions introduced by the problem. The remaining unknown may be determined by the initial conditions. Finally the transverse response in space and time is given by:

$$v(x,t) = \sum_{n=1}^{\infty} Y_n(x) . g_n(t) \tag{4-18}$$

The LG representation is inspired by $m_n \ddot{g}_n + k_n g_n = \int_L f(x,t) Y_n(x) dx$ 4-13 for each mode. It is evident that this representation has exactly the same form as that for the structural bar in the previous section. The only difference is in the parameters; specifically, the modal elements and the formula to calculate them. As an illustrative example, the LG model of the beam in Figure 4-6 is presented in Figure 4-7.

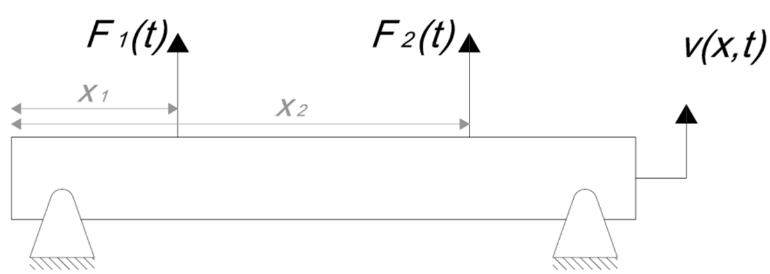

Figure 4-6 Structural 2D beam subjected to two external point forces.

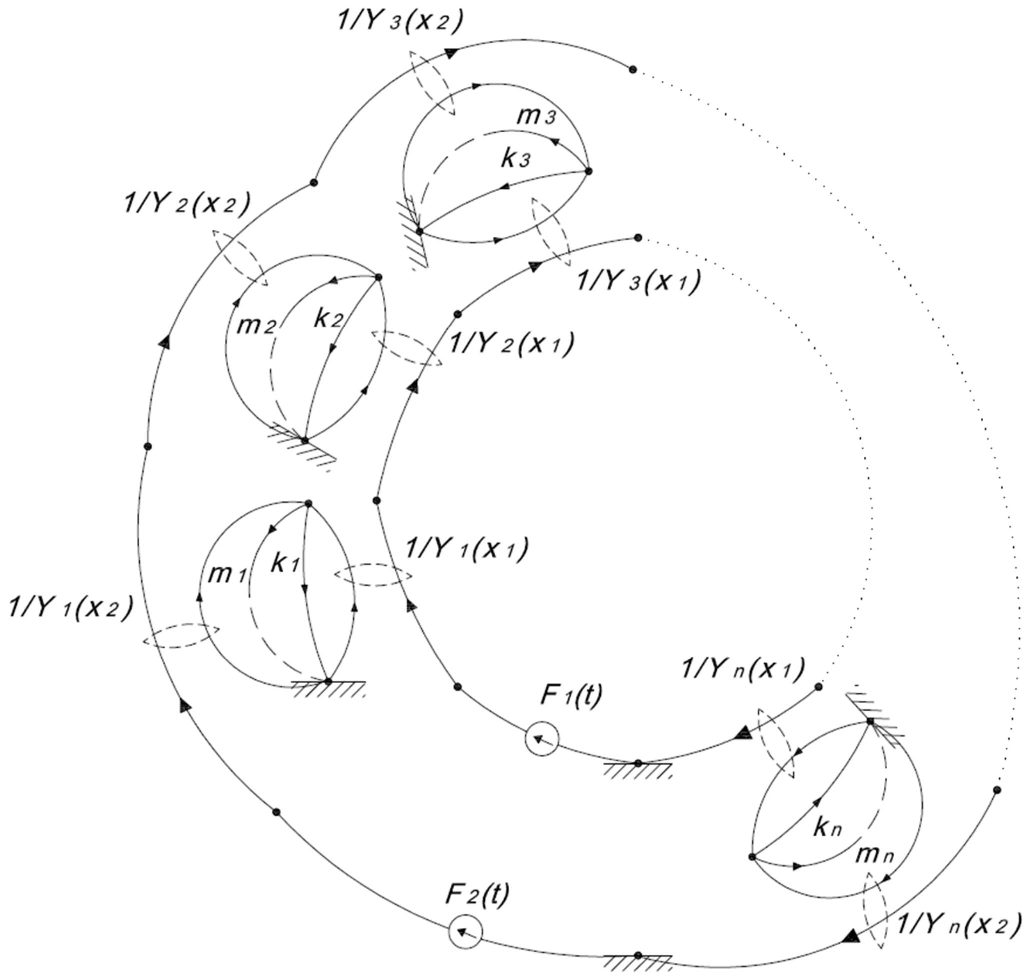

Figure 4-7 LG representation of a 2D beam.

4.3 LG Representation of Pipelines

A critical part of hydraulic and electro-hydraulic systems is the pipeline which facilitates the fluid power transmission. A pipeline is more accurately interpreted as a DP subsystem for which the mathematical representation is provided by a set of partial differential equations.

In pipelines, especially long pipelines, the pressure propagates with a time delay and pressure pulsation. Due to the compressibility and inertia of the fluid, the fluid pressure waves propagate at the speed of sound, which is approximately 1000 m/s for oil. For a pipeline of length L_{pipe}, the time for the pressure waves to propagate through the pipeline is $t_w = Lpipe/a$. Consequently, if the pipeline is short, e.g., L_{pipe} = 1 m, the corresponding propagation time is 1 ms. Thus, the pressure differences in the pipeline will disappear after 1 ms. In this case, it is reasonable to assume that the pipeline is a static device.

However, for long pipelines, the propagation time introduces a time delay that may be significant if the exciting input frequency approaches the resonant frequency of the pipelines [67]

The flow of a compressible fluid in an elastic pipe line is described by the continuity and momentum relations [16]:

$$\frac{\partial P(x,t)}{\partial t} + \frac{\rho a^2}{A}\frac{\partial Q(x,t)}{\partial x} = \frac{\rho a^2}{A} q(x,t) \qquad \text{4-19}$$

$$\frac{\partial Q(x,t)}{\partial t} + \frac{A}{\rho}\frac{\partial P(x,t)}{\partial x} = \frac{F(x,t)}{\rho} \qquad \text{4-20}$$

where *A* is the cross-section area of the pipe, ρ is the fluid mass density, *F(x,t)* is the externally applied force per unit length, and *q(x,t)* represents volume flow rate into the pipe per unit length. Finite wall stiffness can be taken into account by correcting the fluid bulk modulus β and corresponding acoustic velocity *a*, as shown by Wylie et al. [68]. With the assumption of laminar flow, the fluid friction losses can be modeled using the Hagen-Poiseuille equation [69]:

$$F_f(x,t) = \rho B Q(x,t) \qquad \text{4-21}$$

The friction coefficient *B* depends on the fluid kinematic viscosity v_0 and the effective radius of the pipeline R_{pipe} and can be expressed as:

$$B = {8 v_0}/{R_{pipe}}^2 \qquad \text{4-22}$$

The partial differential equations $\frac{\partial P(x,t)}{\partial t} + \frac{\rho a^2}{A}\frac{\partial Q(x,t)}{\partial x} = \frac{\rho a^2}{A} q(x,t)$ 4-19 and $\frac{\partial Q(x,t)}{\partial t} + \frac{A}{\rho}\frac{\partial P(x,t)}{\partial x} = \frac{F(x,t)}{\rho}$ 4-20 can be combined in two ways to form a forced wave equation. With suitable second derivatives for time *t* and space *x*, the pressure or flow may be eliminated as:

$$\rho\frac{\partial^2 Q}{\partial t^2} - \rho a^2\frac{\partial^2 Q}{\partial x^2} + \rho a^2\frac{\partial q}{\partial x} - \frac{\partial F}{\partial t} = 0 \qquad \text{4-23}$$

$$\frac{A}{\rho a^2}\frac{\partial^2 P}{\partial t^2} - \frac{A}{\rho}\frac{\partial^2 P}{\partial x^2} + \frac{1}{\rho}\frac{\partial F}{\partial x} - \frac{\partial q}{\partial t} = 0 \qquad \text{4-24}$$

$\rho\frac{\partial^2 Q}{\partial t^2} - \rho a^2\frac{\partial^2 Q}{\partial x^2} + \rho a^2\frac{\partial q}{\partial x} - \frac{\partial F}{\partial t} = 0$ 4-23 determines the flow from an external force; e.g., a pressure acts on the pipe, and works in the flow from pressure causality. $\frac{A}{\rho a^2}\frac{\partial^2 P}{\partial t^2} - \frac{A}{\rho}\frac{\partial^2 P}{\partial x^2} + \frac{1}{\rho}\frac{\partial F}{\partial x} - \frac{\partial q}{\partial t} = 0$ 4-24 is suitable for determining the pressure from an injected flow and states the flow causality.

4.3.1 The Pressure Causality

For the first equation we assume that the injected flow along the pipe is zero (*q(x,t)=0*). The forcing in this case consists of friction losses along the pipe and the pressure force at the ends of the pipe, as given by:

$$F(x,t) = F_f(x,t) + A(P_0\delta(x) + P_l\delta(x-l)) \qquad 4\text{-}25$$

Hence $\rho\frac{\partial^2 Q}{\partial t^2} - \rho a^2\frac{\partial^2 Q}{\partial x^2} + \rho a^2\frac{\partial q}{\partial x} - \frac{\partial F}{\partial t} = 0$ 4-23 may be rewritten as:

$$\rho\frac{\partial^2 Q}{\partial t^2} - \rho a^2\frac{\partial^2 Q}{\partial x^2} - \rho B\frac{\partial Q}{\partial t} = A(\dot{P}_0\delta(x) + \dot{P}_l\delta(x-l)) \qquad 4\text{-}26$$

The normal modes for this equation are found by considering the corresponding boundary conditions:

$$\frac{\partial Q}{\partial x}(0,t) = \frac{\partial Q}{\partial x}(l,t) = 0 \qquad [P_0, P_l] \text{ as inputs} \qquad 4\text{-}27$$

The Separation of Variables (SOV) is applied as before. The inertia, fluid capacitance and fluid resistance parameters for the i^{th} mode are obtained and are presented in Table 4-1. The procedure is described in detail in [16, 17].

Table 4-1 Normal mode parameters in pressure causality.

	I	***C***	***R***	***Y***
i=0	$I_0 = \frac{\rho L}{A}$	$C_0 = \infty$	$R_0 = B\frac{\rho L}{A}$	$Y_0 = 1$
i=1,2,3,…	$I_i = \frac{\rho L}{2A}$	$C_i = \frac{2AL}{\pi^2\rho a^2 i^2}$	$R_i = B\frac{\rho L}{2A}$	$Y_i = \cos(i\pi)$

The mode shapes are cosines with arbitrary amplitudes, usually set to unity. Considering the boundary conditions in $\frac{\partial Q}{\partial x}(0,t) = \frac{\partial Q}{\partial x}(l,t) = 0$ $[P_0, P_l]$ as inputs 4-27 we obtain the mode shapes as:

$$Y_i(x) = \cos\left(\frac{i\pi x}{l}\right) \qquad i = 0,1,2,\ldots \qquad 4\text{-}28$$

The corresponding resonant frequencies are $\omega_i = \frac{i\pi a}{l}$. For the model with fluid pressure as the input and the flow rate as output, the Linear Graph model is shown in Figure 4-8.

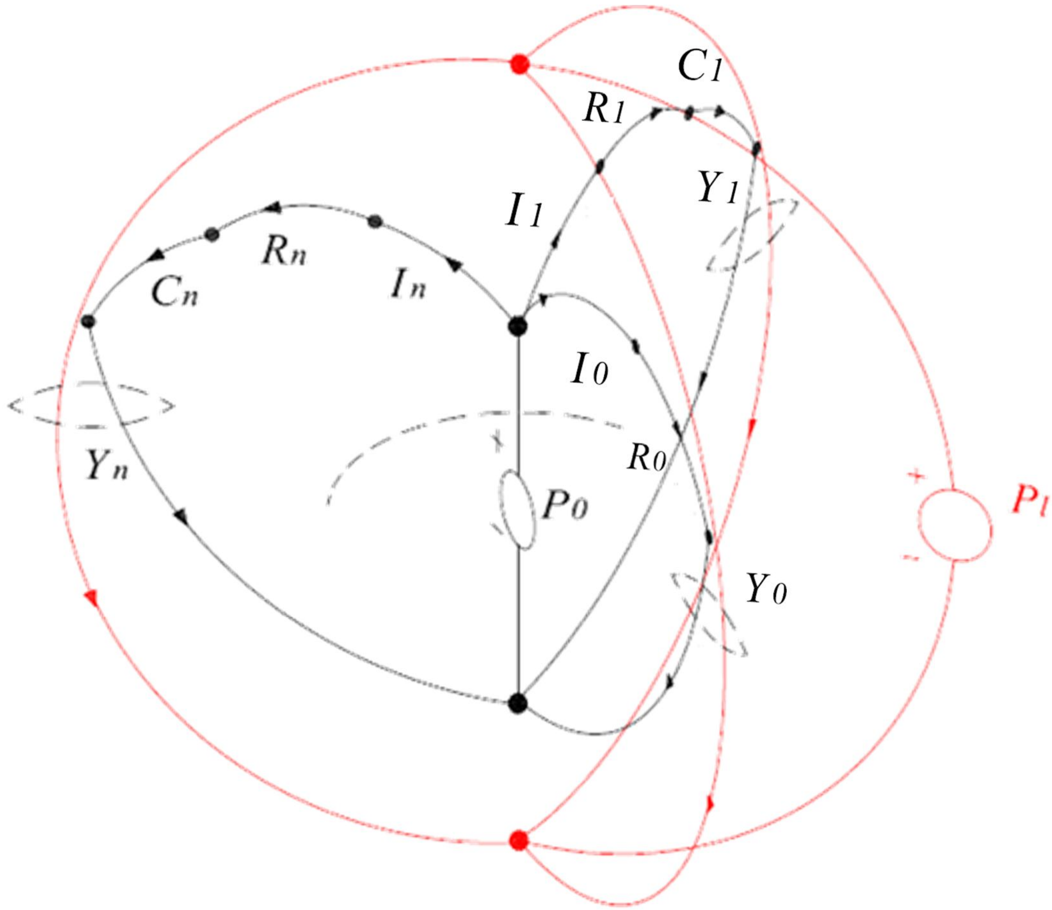

Figure 4-8 3D LG model of a hydraulic line in pressure causality.

The linear graph model has been drawn in 3D space

4.3.2 The Flow Causality

$A\rho a2\partial 2P\partial t2-A\rho\partial 2P\partial x2+1\rho\partial F\partial x-\partial q\partial t=0$ 4-24. The forcing consists only of friction losses along the tube:

$$F(x,t) = F_f(x,t) \qquad 4\text{-}29$$

The injected flow is present only at the ends of the tube:

$$q(x,t) = Q_0\delta(x) + Q_l\delta(x-l)) \qquad 4\text{-}30$$

The boundary conditions compatible with this input causality are:

$$\frac{\partial P}{\partial x}(0,t) = \frac{\partial P}{\partial x}(l,t) = 0 \qquad [Q_0, Q_l] \text{ as inputs} \qquad 4\text{-}31$$

Hence $\frac{A}{\rho a^2}\frac{\partial^2 P}{\partial t^2} - \frac{A}{\rho}\frac{\partial^2 P}{\partial x^2} + \frac{1}{\rho}\frac{\partial F}{\partial x} - \frac{\partial q}{\partial t} = 0$ 4-24 may be rewritten as:

$$\frac{A}{\rho a^2}\frac{\partial^2 P}{\partial t^2} - \frac{A}{\rho}\frac{\partial^2 P}{\partial x^2} - \frac{AB}{\rho a^2}\frac{\partial Q}{\partial t} = \dot{Q}_0\delta(x) + \dot{Q}_l\delta(x - l)) \qquad 4\text{-}32$$

Using the SOV method, the inertia, fluid capacitance and fluid resistance parameters for the i^{th} mode are obtained as given in Table 4-2.

Table 4-2 Normal mode parameters in flow causality.

	I	***C***	***R***	***Y***
i=0	$I_0 = \infty$	$C_0 = \frac{AL}{\rho a^2}$	$R_0 = B\frac{AL}{\rho a^2}$	$Y_0 = 1$
i=1,2,3,…	$I_i = \frac{2\rho L}{i^2 \pi A}$	$C_i = \frac{AL}{2\rho a^2}$	$R_i = B\frac{AL}{2\rho a^2}$	$Y_i = \cos(i\pi)$

The mode shapes are cosines with arbitrary amplitudes which are usually set to unity. Considering the boundary conditions in $\partial P \partial x(0,t) = \frac{\partial P}{\partial x}(l,t) = 0$ $[Q_0, Q_l]$ as inputs 4-31 we obtain the mode shapes as:

$$Y_i(x) = \cos\left(\frac{i\pi x}{l}\right) \qquad i = 0,1,2,\ldots \qquad 4\text{-}33$$

The corresponding resonant frequencies are given by $\omega_i = \frac{i\pi a}{l}$. For the model with flow rate as the input and fluid pressure as the output, the Linear Graph model is shown in Figure 4-9.

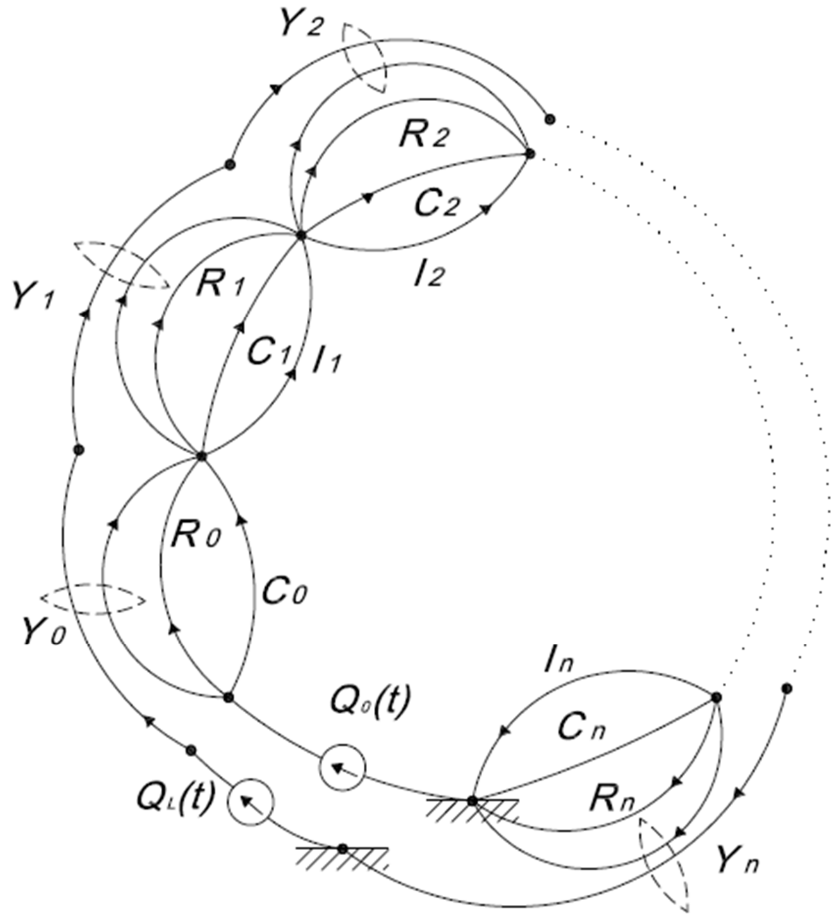

Figure 4-9 LG model of hydraulic line in flow causality.

This graph is exactly the same as the graph resulting from a mechanical DP system, and the parallel pattern is present at the same plane as the modes.

4.4 Modeling DP Systems Using Vector Linear Graphs

In the previous section, approximate models of mechanical DP systems were obtained using linear graph modeling and modal decomposition. By studying the linear graph representation of a mechanical DP system we note that it consists of a parallel pattern that grows uniformly according to the number of modes that is used to approximate the system. Specifically the model of each DP system creates a uniform topological pattern. Because of this uniform pattern it contains uniform equations that can be represented in a compact form by vector mathematics. Thus, we need to use VLG which consists of elements with matrix or vector constitutive equations.

The use of VLG will considerably reduce the equation manipulations and computations since it will assemble the corresponding mode shape equations as vector-matrix equations. Furthermore it can lead to the introduction of a new element in LG in representing distributed systems.

Two illustrative examples are provided now to demonstrate the advantages of vector linear graphs in modeling mechanical DP systems. These examples consist of DP systems which may be parts of an overall dynamic system. In the following examples a general number of modes are solved for each DP system. The considered number of modes depends on the nature of the system and should be determined by the engineering knowledge.

4.4.1 One-Dimensional Structural Bar

Consider a typical structural bar with specific boundary conditions and an external force, as shown in Figure 4-10. Mathematical modeling procedure for this bar has been presented in the previous sections. As before, the damping properties are not taken into account for simplicity. However, including the damping would not change the procedure.

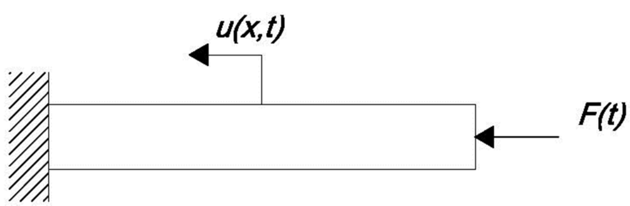

Figure 4-10 Structural bar with a point external load.

Using the developments of the previous section, for convenience, we define the elemental parameters and equations given in Table 4-3.

Table 4-3 Elemental equations for mechanical DP systems in vector linear graphs.

Element	Constitutive equation	Parameter
Inertia	$[f_m]_{n\times 1} = [M]_{n\times n}[\dot{v}_m]_{n\times 1}$	$[M] = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & m_n \end{bmatrix}$

Spring	$[\dot{f}_k]_{n\times 1} = [K]_{n\times n}[v_k]_{n\times 1}$	$[K] = \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & k_n \end{bmatrix}$
Transformer	$[f_o]_{n\times 1} = -[r]_{n\times 1}[f_i]_{1\times 1}$ $[v_i]_{1\times 1} = [r]^T_{1\times n}[v_o]_{n\times 1}$	$[r] = \begin{bmatrix} Y_1(L) \\ Y_2(L) \\ \vdots \\ Y_n(L) \end{bmatrix}$

In which:

$$m_i = m = \frac{\rho A L}{2} \quad , \quad i = 1,2,\dots,n \qquad \text{4-34}$$

$$k_i = k = m\frac{E\pi^2(2i-1)^2}{4\rho L^2} \quad , \quad i = 1,2,\dots,n \qquad \text{4-35}$$

Here ρ, A, and L are density of the material, cross-sectional area, and length, respectively, of the bar. Also, $Y_i(L)$ is the value of the ith mode shape at the forcing point.

The order of the system equations depend on the number of modes included in the model, which is based on the specific application. Since we have a mass element and a spring element for each mode, the order of the system equation for each mechanical DP system would be twice the number of included modes.

The linear graph model for this bar is shown in Figure 4-11, using both conventional LG and VLG.

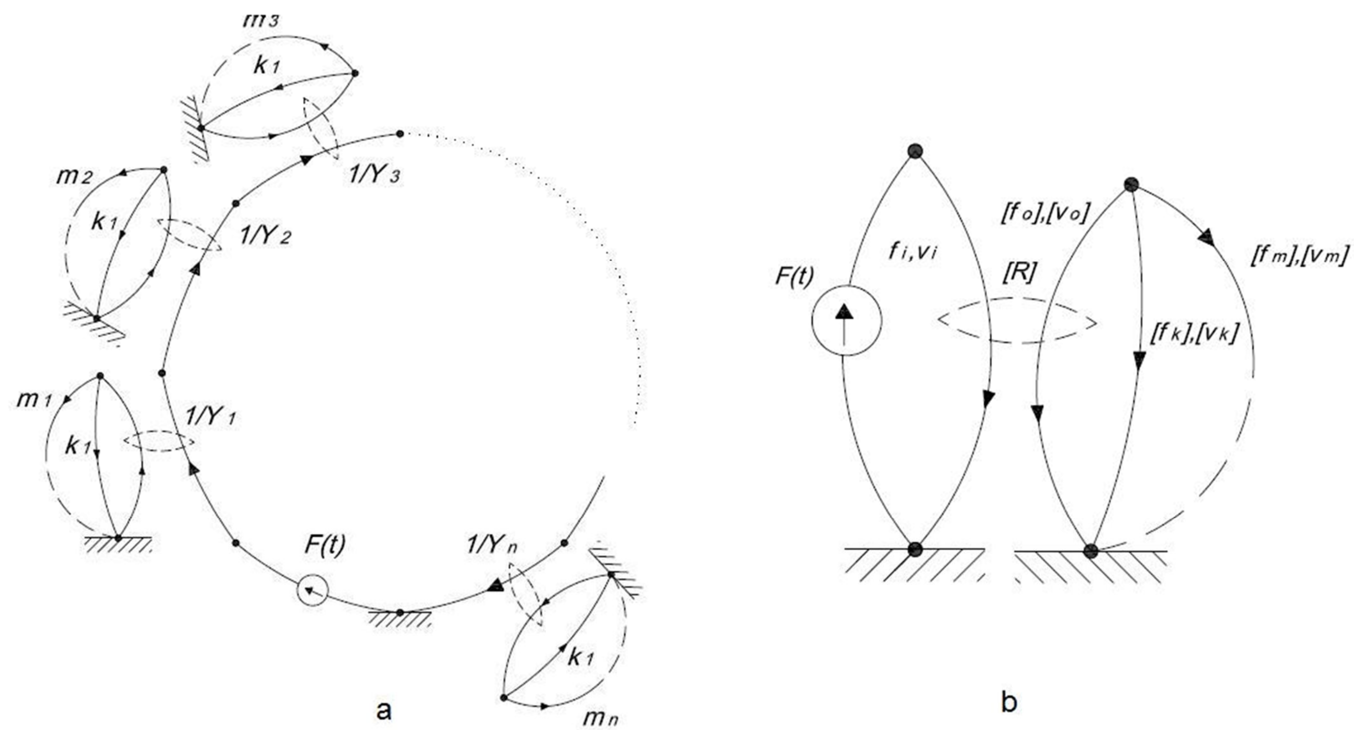

Figure 4-11 Model of 2D structural bar: (a) conventional linear graph; (b) Vector linear graph.

Each vector modal mass and spring carries *n* state variables.

$$\text{State variables: } X = \begin{bmatrix} [f_k] \\ [v_m] \end{bmatrix}_{2n\times 1}$$

The node equations:

$$F(t) - f_i = 0$$

$$-[f_o] - [f_m] - [f_k] = 0$$

The loop equations:

$$[v_o] - [v_m] = 0$$

$$[v_k] - [v_m] = 0$$

$$V - v_i = 0$$

Eliminate auxiliary variables:

$$[\dot{v_m}] = [M]^{-1}([r]F(t) - [f_k])$$

$$[\dot{v_m}] = -[M]^{-1}[f_k] + [M]^{-1}[r]F(t)$$

$$[\dot{f}_k] = [K][v_m]$$

State space equation:

$$\dot{X} = \begin{bmatrix} 0 & [K] \\ -[M]^{-1} & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ [M]^{-1} \times [r] \end{bmatrix} F(t)$$

4.4.2 Milling Machine Structure

In this example the state space equations of a milling machine structure are derived. The system consists of two beams with specified boundary conditions, which are interacting with each other, as shown in Figure 4-12(b). External dynamic force from the table is exerted on the knee and a set of spring-damper combination is used between the beams to represent the interaction constraints. Each beam can be approximated using different number of modes. In this example, one beam uses n modes and the other uses m modes. The vector linear graph model of this system is given in Figure 4-13. The general procedure of development is the same as in the previous example.

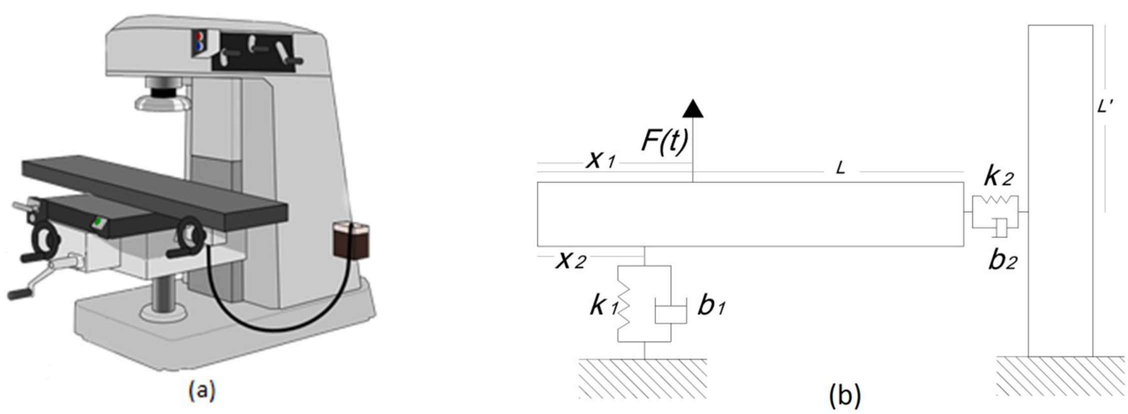

Figure 4-12 Milling machine structure.

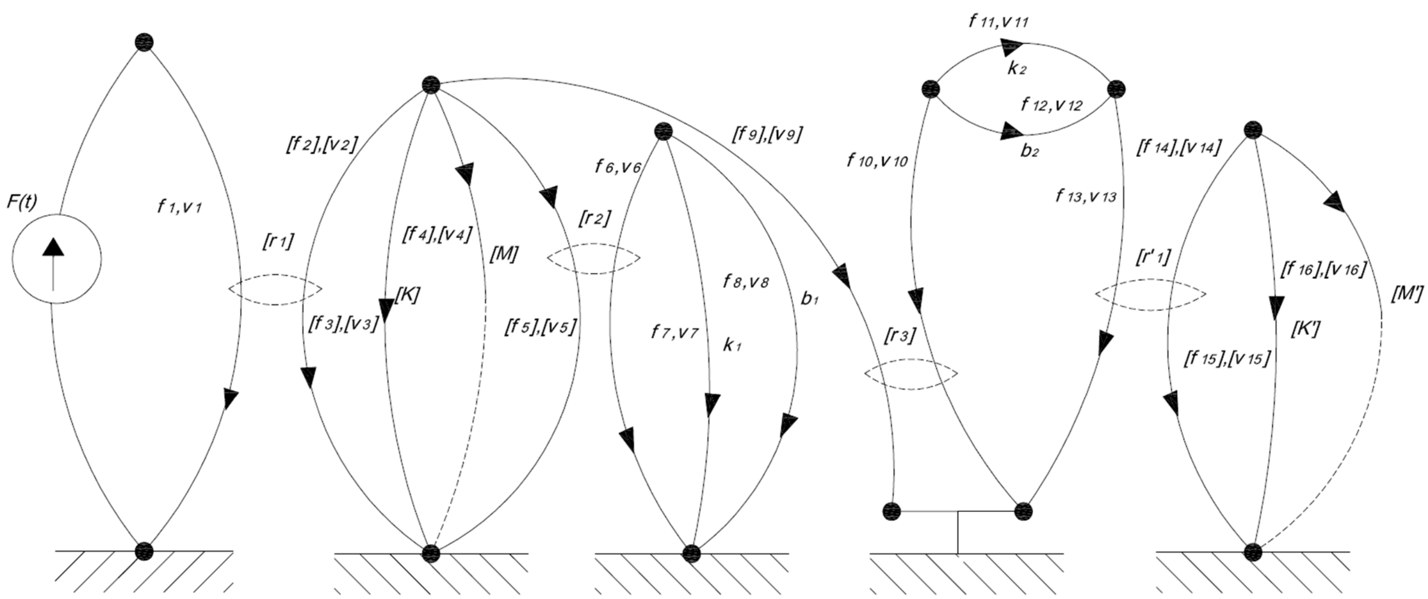

Figure 4-13 LG of milling machine structure.

$$\text{State variables } X = \begin{bmatrix} \begin{bmatrix} [f_3] \\ [v_4] \end{bmatrix}_{2n\times 1} \\ f_7 \\ f_{11} \\ \begin{bmatrix} [f_{15}] \\ [v_{16}] \end{bmatrix}_{2m\times 1} \end{bmatrix}$$

$$[r_1] = \begin{bmatrix} Y_1(x_1) \\ Y_2(x_1) \\ \vdots \\ Y_n(x_1) \end{bmatrix} \quad [r_2] = \begin{bmatrix} Y_1(x_2) \\ Y_2(x_2) \\ \vdots \\ Y_n(x_2) \end{bmatrix} \quad [r_3] = \begin{bmatrix} Y_1(L) \\ Y_2(L) \\ \vdots \\ Y_n(L) \end{bmatrix} \quad [r'_1] = \begin{bmatrix} Y'_1(L') \\ Y'_2(L') \\ \vdots \\ Y'_n(L') \end{bmatrix}$$

Here $Y_i(x)$ is the shape function of the vertical beam and $Y_i(x)$ is the shape function of horizontal beam. It should be noted that boundary conditions are considered while obtaining the shape functions.

The node equations:

$$F(t) - f_1 = 0$$

$$-[f_2] - [f_3] - [f_4] - [f_5] - [f_9] = 0$$

$$-f_6 - f_7 - f_8 = 0$$

$$-f_{10} - f_{11} - f_{12} = 0$$

$$f_{11} + f_{12} - f_{13} = 0$$

$$-[f_{14}] - [f_{15}] - [f_{16}] = 0$$

The loop equations:

$$[v_3] - [v_4] = 0$$

$$[v_4] - [v_5] = 0$$

$$[v_4] - [v_9] = 0$$

$$v_6 - v_7 = 0$$

$$v_8 - v_7 = 0$$

$$v_{11} - v_{12} = 0$$

$$v_{10} - v_{12} - v_{13} = 0$$

$$[v_{14}] - [v_{16}] = 0$$

$$[v_{15}] - [v_{16}] = 0$$

Eliminate auxiliary variables:

$$[\dot{f_3}] = [K_1][v_4]$$

$$[\dot{v_4}] = [M]^{-1}(-[f_2] - [f_3] - [f_5] - [f_9]) = [M]^{-1}([r_1]F(t) - [f_3] - [f_5] - [f_9])$$

$$[\dot{v_4}] = [M]^{-1}\{[r_1]F(t) - [f_3] + [r_2](-f_7 - b_1[r_2]^T[v_4]) + [r_3](-f_{11} - b_2[r_3]^T[v_4] + b_2[r'_1]^T[v_{16}])\}$$

$$\dot{f_7} = k_1 \cdot v_7 = k_1 \cdot v_6 = k_1([r_2]^T[v_4])$$

$$\dot{f_{11}} = k_2 \cdot v_{11} = k_2(v_{10} - v_{13}) = k_2([r_3]^T[v_4] - [r'_1]^T[v_{16}])$$

$$[\dot{f_{15}}] = [K'][v_{16}]$$

$$[\dot{v_{16}}] = [M']^{-1}(-[f_{14}] - [f_{15}]) = [M']^{-1}([r'_1] \cdot f_{13} - [f_{15}])$$

$$[\dot{v_{12}}] = [M']^{-1}\{[r'_1](f_{11} + b_2[r_3]^T[v_4] - b_2[r'_1]^T[v_{16}]) - [f_{15}]\}$$

State space equations:

$$\dot{X} = \begin{bmatrix} 0 & [K] & 0 & 0 & 0 & 0 \\ -[M]^{-1} & -[M]^{-1}(b_1[r_2][r_2]^T + b_2[r_3][r_3]^T) & -[M]^{-1}[r_2] & -[M]^{-1}[r_3] & 0 & b_2[M]^{-1}[r_3][r'_1]^T \\ 0 & k_1[r_2]^T & 0 & 0 & 0 & 0 \\ 0 & k_2[r_3]^T & 0 & 0 & 0 & -k_2[r'_1]^T \\ 0 & 0 & 0 & 0 & 0 & [K'] \\ 0 & b_2[M']^{-1}[r'_1][r_3]^T & 0 & [M']^{-1}[r'_1] & -[M']^{-1} & -b_2[M']^{-1}[r'_1][r'_1]^T \end{bmatrix} X + \begin{bmatrix} 0 \\ [M]^{-1}[r_1] \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F(t)$$

This procedure may be compared with the previous procedure of LG modeling. A reduction in equation formulations and simplicity of the graphic model are observed.

4.5 A New Element in Linear Graphs

In this section, we continue to extend the linear graph representation of a mechanical DP system. A new single LG element is introduced for this purpose. In the previous section we reduced the model by assembling all the modal equations into two modal mass and modal spring matrix elements. The amount of equation manipulations can be reduced even further by merging all modal elements and transformers of a DP system into a single element. An element with constitutive equations that encapsulate all the modal equations is obtained in this manner.

Assume that p forces are acting on a mechanical DP element. We wish to approximate the element to the nth order. This element represents two sets of n state variables (i.e., n modal masses and n modal springs) and p number of transformers. Thus, it encapsulates $2n$ constitutive equations and p transformer equations. Considering the previous method of representing DP elements, one set of constitutive equations is for the A-type elements and the other set is for the T-type elements.

Since in distributed elements, unlike lumped elements, power can be transferred through several points, this new element should be a multiport element. The number of ports depends on the number of external forces that interact with the element, which is equal to p. If the distributed system is in integral causality, the constitutive equations for this element would be straightforward and are given by

$$\frac{d}{dx}\begin{bmatrix}[f]\\ [v]\end{bmatrix} = \begin{bmatrix} 0 & [K] \\ -[M]^{-1} & 0 \end{bmatrix}\begin{bmatrix}[f]\\ [v]\end{bmatrix} + \begin{bmatrix} 0 \\ [M]^{-1} \times [R]_{n\times p}[f_{ext}]_{p\times 1}\end{bmatrix} \qquad 4\text{-}36$$

Here $[f]$ and $[v]$ are state variable vectors, $[M]$ and $[K]$ are modal mass and modal stiffness matrices, $[R]$ is the mode shape matrix which excites the external forces, and $[f_{ext}]$ is external force vector.

$$[R] = [[r_1] \quad [r_2] \quad \cdots \quad [r_p]] \tag{4-37}$$

$$[f_{ext}] = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_p \end{bmatrix} \tag{4-38}$$

The transformers in the element build up the following relation for the across variables:

$$[v_{ext}]_{p\times 1} = [R]^T_{p\times n} \times [v_i]_{n\times 1} \tag{4-39}$$

and

$$[v_{ext}] = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_p \end{bmatrix} \tag{4-40}$$

This element is shown in Figure 4-14. This element can be used in systems that also contain LP systems. In this case, transformers of the DP element act like gates to switch between one-dimensional and multi-dimensional LGs. In the following section, the same examples as in the previous section are solved again to further demonstrate the advantages of the new linear graph element.

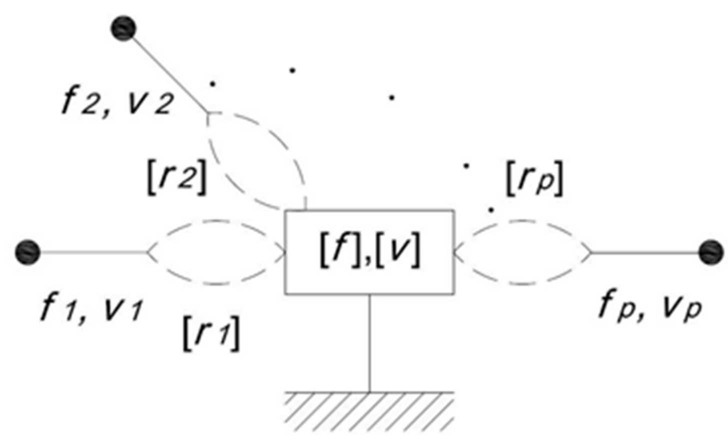

Figure 4-14 The mechanical DP element in linear graphs.

4.5.1 One-Dimensional Structural Bar

In this section the structural bar shown in Figure 4-10 is solved again with the use of the new mechanical DP element. The LG of this system is illustrated in Figure 4-15.

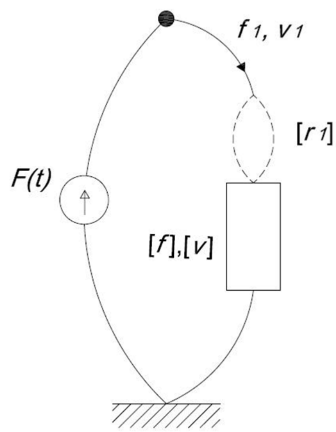

Figure 4-15 LG model of the structural bar using the new element.

$$\text{State variables: } X = \begin{bmatrix} [f] \\ [v] \end{bmatrix}_{2n\times 1}$$

The constitutive equation:

$$\frac{d}{dx}\begin{bmatrix} [f] \\ [v] \end{bmatrix} = \begin{bmatrix} 0 & [K] \\ -[M]^{-1} & 0 \end{bmatrix}\begin{bmatrix} [f] \\ [v] \end{bmatrix} + \begin{bmatrix} 0 \\ [M]^{-1}[r_1]f_1 \end{bmatrix}$$

and

$$[r_1] = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}$$

The node equations:

$$F(t) - f_1 = 0$$

Eliminate auxiliary variables:

$$\frac{d}{dx}\begin{bmatrix} [f] \\ [v] \end{bmatrix} = \begin{bmatrix} 0 & [K] \\ -[M]^{-1} & 0 \end{bmatrix}\begin{bmatrix} [f] \\ [v] \end{bmatrix} + \begin{bmatrix} 0 \\ [M]^{-1}[r_1]F(t) \end{bmatrix}$$

State space equation:

$$\dot{X} = \begin{bmatrix} 0 & [K] \\ -[M]^{-1} & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ [M]^{-1}[r_1] \end{bmatrix} F(t)$$

Note how the extraction of the state space model was made easier and faster, where fewer steps were needed. In this example it was not necessary to use any loop equations.

4.5.2 Milling Machine Structure

Here, the system as shown in Figure 4-12 is solved by using of the new LG element. The linear graph model is illustrated in Figure 4-16.

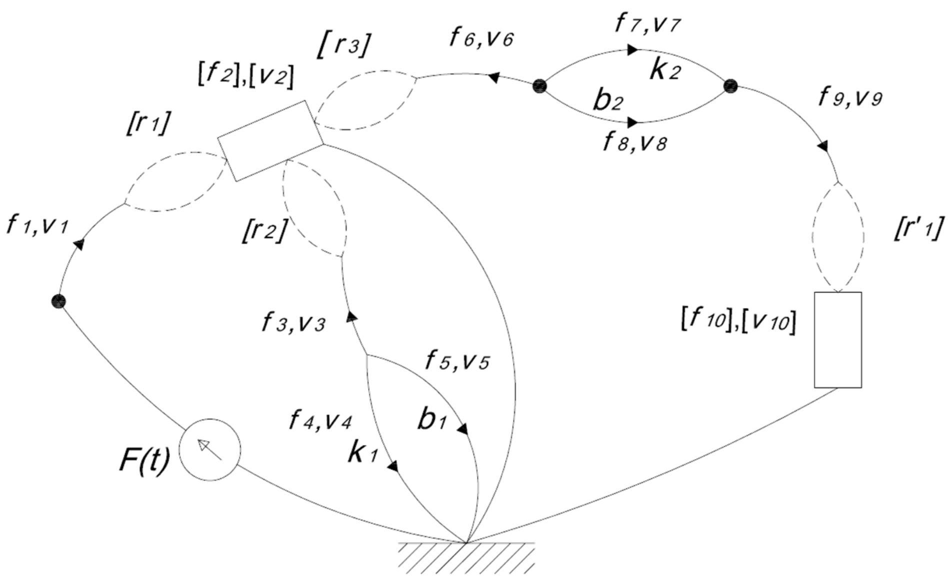

Figure 4-16 LG model of the milling machine structure using the new LG element.

$$\text{State variables: } X = \begin{bmatrix} \begin{bmatrix} [f_2] \\ [v_2] \end{bmatrix}_{2n\times 1} \\ f_4 \\ f_7 \\ \begin{bmatrix} [f_{10}] \\ [v_{10}] \end{bmatrix}_{2m\times 1} \end{bmatrix}$$

The constitutive equations:

$$\frac{d}{dx}\begin{bmatrix} [f_2] \\ [v_2] \end{bmatrix} = \begin{bmatrix} 0 & [K] \\ -[M]^{-1} & 0 \end{bmatrix}\begin{bmatrix} [f_2] \\ [v_2] \end{bmatrix} + \begin{bmatrix} 0 \\ [M]^{-1}[R_1]\begin{bmatrix} f_1 \\ f_3 \\ f_6 \end{bmatrix} \end{bmatrix}$$

and $[R_1] = [[r_1] \quad [r_2] \quad [r_3]] = \begin{bmatrix} Y_1(x_1) & Y_1(x_2) & Y_1(L) \\ Y_2(x_1) & Y_2(x_2) & Y_2(L) \\ \vdots & \vdots & \vdots \\ Y_n(x_1) & Y_n(x_2) & Y_n(L) \end{bmatrix}$

$$\dot{f}_4 = k_1 \cdot v_4$$

$$\dot{f}_7 = k_2 \cdot v_7$$

$$\frac{d}{dx}\begin{bmatrix}[f_{10}]\\ [v_{10}]\end{bmatrix} = \begin{bmatrix}0 & [K']\\ -[M']^{-1} & 0\end{bmatrix}\begin{bmatrix}[f_{10}]\\ [v_{10}]\end{bmatrix} + \begin{bmatrix}0\\ [M']^{-1}[r'_1]f_9\end{bmatrix}$$

and
$$[r'_1] = \begin{bmatrix}Y'_1(L')\\ Y'_2(L')\\ \vdots\\ Y'_n(L')\end{bmatrix}$$

The node equations:

$$F(t) - f_1 = 0$$

$$-f_3 - f_4 - f_5 = 0$$

$$-f_6 - f_7 - f_8 = 0$$

$$f_7 + f_8 - f_9 = 0$$

The loop equations:

$$v_4 - v_5 = 0$$

$$v_7 - v_8 = 0$$

$$v_5 - v_3 = 0$$

$$v_6 - v_7 - v_9 = 0$$

Eliminate auxiliary variables:

$$\frac{d}{dx}\begin{bmatrix}[f_2]\\ [v_2]\end{bmatrix} = \begin{bmatrix}0 & [K]\\ -[M]^{-1} & 0\end{bmatrix}\begin{bmatrix}[f_2]\\ [v_2]\end{bmatrix} + \begin{bmatrix}0\\ [M]^{-1}[R_1]\begin{bmatrix}F(t)\\ f_3\\ f_6\end{bmatrix}\end{bmatrix}$$

$$\frac{d}{dx}\begin{bmatrix}[f_2]\\ [v_2]\end{bmatrix} = \begin{bmatrix}0 & [K]\\ -[M]^{-1} & 0\end{bmatrix}\begin{bmatrix}[f_2]\\ [v_2]\end{bmatrix} + \begin{bmatrix}0\\ [M]^{-1}[R_1]\begin{bmatrix}F(t)\\ -f_4 - b_1[r_2]^T[v_2]\\ -f_7 - b_2([r_3]^T[v_2] - [r'_1]^T[v_{10}])\end{bmatrix}\end{bmatrix}$$

$$\dot{f}_4 = k_1 \cdot v_4 = k_1 \cdot v_3 = k_1[r_2]^T[v_2]$$

$$\dot{f}_7 = k_2 \cdot v_7 = k_2(v_6 - v_9) = k_2([r_3]^T[v_2] - [r'_1]^T[v_{10}])$$

$$\frac{d}{dx}\begin{bmatrix}[f_{10}]\\ [v_{10}]\end{bmatrix} = \begin{bmatrix}0 & [K']\\ -[M']^{-1} & 0\end{bmatrix}\begin{bmatrix}[f_{10}]\\ [v_{10}]\end{bmatrix} + \begin{bmatrix}0\\ [M']^{-1}[r'_1]f_9\end{bmatrix}$$

$$\frac{d}{dx}\begin{bmatrix}[f_{10}]\\ [v_{10}]\end{bmatrix}=\begin{bmatrix}0 & [K']\\ -[M']^{-1} & 0\end{bmatrix}\begin{bmatrix}[f_{10}]\\ [v_{10}]\end{bmatrix}+\begin{bmatrix}0\\ [M']^{-1}[r'_1](f_7+b_2([r_3]^T[v_2]-[r'_1]^T[v_{10}])\end{bmatrix}$$

State equations:

$$\dot{X} = \begin{bmatrix} 0 & [K] & 0 & 0 & 0 & 0 \\ -[M]^{-1} & -[M]^{-1}(b_1[r_2][r_2]^T + b_2[r_3][r_3]^T) & -[M]^{-1}[r_2] & -[M]^{-1}[r_3] & 0 & b_2[M]^{-1}[r_3][r'_1]^T \\ 0 & k_1[r_2]^T & 0 & 0 & 0 & 0 \\ 0 & k_2[r_3]^T & 0 & 0 & 0 & -k_2[r'_1]^T \\ 0 & 0 & 0 & 0 & 0 & [K'] \\ 0 & b_2[M']^{-1}[r'_1][r_3]^T & 0 & [M']^{-1}[r'_1] & -[M']^{-1} & -b_2[M']^{-1}[r'_1][r'_1]^T \end{bmatrix} X + \begin{bmatrix} 0 \\ [M]^{-1}[r_1] \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F(t)$$

4.6 Summary

Modeling of distributed systems using an integrated tool such as linear graphs opens up a wide domain of problems, which then may be handled easier and in a more systematic manner. To illustrate this possibility, the present chapter presented examples of DP systems subject to specific external forces. Dynamics of the system was investigated using the linear graphs approach. This approach may be generalized to consider various distributed elements and in multiple domains. For example, beams, rotating shafts, pipelines and 2-dimensional and 3-dimensional structures may be analyzed in a unified manner. Mechanical subsystems may be subjected to heat transfer and may interact with fluid flow. The subsystem may form a part of an electrical subsystem (e.g., armature of a motor). The linear graph approach will provide a unified method for modeling such multi-domain problems.

Two approaches were provided to mathematically model DP systems in a unified manner: micro-element approach and separation of variables (SOV) approach. Either of these approaches could be used depending on the problem type and the needs. It was discussed that the SOV approach provided increased convenience and accuracy within the integrated system. This approach was specifically developed for 1D and 2D mechanical structural elements and hydraulic pipelines.

LG is a powerful tool which helps us visualize the system before formulating the system equations. Sometimes sub-graphs of the linear graph model of a specific system may be similar. In this chapter we took advantage of these similarities to simplify the dynamic model of dynamic systems. The representation of DP systems using the SOV approach was presented and simplified in the linear graph modeling context by assembling the modal properties. In order to simplify the model, vector linear graphs which use vector algebra, were introduced. To illustrate the application of vector linear graphs, examples of mechanical DP system and multi-body system were discussed. An example of two-link robot manipulator showed how vector linear graphs can be used to simplify modeling of multi-body systems. Once a linear graph model is established, a unique state-space model may be derived, which may be used in computer simulation, control, design, fault diagnosis, and so on.

The application of vector linear graphs for solving DP systems resulted in a new linear graph element. Although this new element models mechanical DP systems, it can be generalized to the hydraulic domain and further to other physical domains. This DP element can be used in mixed systems containing lumped-parameter elements from any physical domain. In this chapter, application of vector linear graphs was devoted to the mechanical domain. The development of vector linear graphs and DP elements for other physical domains will follow the same general approach.

A comparison of the computational effort of the developed methodologies demonstrates the computational efficiency of these methods. In Table 4-4, the complexity of the DP bar model and associated computation is summarized in a comparative manner for the three cases: LG modeling with and without the new DP element, and VLG modeling. It is assumed that 4 modes are used in the system model.

Table 4-4 comparison on computation efficiency and complexity of DP bar models.

Method	No. of Branches	No. of Nodes	No. of Elements	No. of Constitutive Equations	No. of Loop Equations	No. of Node Equations	Total No. of Equations
LG using conventional elements	17	9	13	16	9	8	33
VLG	5	3	4	4	3	2	9
LG using the new DP element	2	2	2	2	1	1	4

The newly introduced element for LG modeling provides a convenient tool to model DP systems. It reduces the computational effort as it requires fewer equation formulations. As evident from Table 4-4, the methodologies developed in this thesis result in improved computational efficiency and reduced model complexity. This type of comparison is not applicable for BG modeling since a single element to model a DP system is not available in BG modeling.

5 LG2ss Toolbox

LG2ss is a MATLAB function which generates symbolic dynamic equations of a system directly from the structure of the system and uses the linear graph modeling method [70]. This function is available online at lg2ss.mech.ubc.ca . The LG matrix and the output matrix (which are defined later in this chapter) are the inputs to the function. This facility may be used on most web browsers. MATLAB need not be installed on the computer of the user in order to use this LG facility.

The LG2ss produces the following set of linear state equations:

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu} + \boldsymbol{E}\dot{\boldsymbol{u}} \qquad 5\text{-}1$$

$$\mathrm{y} = \boldsymbol{Cx} + \boldsymbol{Du} + \boldsymbol{F}\dot{\boldsymbol{u}} \qquad 5\text{-}2$$

where $\boldsymbol{x}$ is the state vector, $\boldsymbol{y}$ is the output vector, and $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$, $\boldsymbol{E}$, and $\boldsymbol{F}$ are the matrices of the state-space model. The symbolic state space model of the system represented by LG2ss toolbox consists of nine fields: states, inputs, outputs – vectors of symbolic variable names for the state variables, system inputs, and system outputs.

A, B, C, D, E, F – the symbolic state-space $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$, $\boldsymbol{E}$, $\boldsymbol{F}$ matrices as defined in $\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu} + \boldsymbol{E}\dot{\boldsymbol{u}}$ 5-1 and $\mathrm{y} = \boldsymbol{Cx} + \boldsymbol{Du} + \boldsymbol{F}\dot{\boldsymbol{u}}$ 5-2.

The matrices should be constructed in the same way as in standard MATLAB. To create a matrix use the matrix constructor operator []. Create a row in the matrix by entering elements within the brackets. Separate each element with a comma or space. To start a new row, terminate the current row with a semicolon. For example, to input the matrix $\begin{bmatrix} 1 & 6 & 1 \\ 2 & 5 & 2 \end{bmatrix}$, enter into the box as:

[1 6 1; 2 5 2] or [1,6,1; 2,5,2]

5.1 Specifying LG Model in LG2ss

The linear graph of a system is specified in LG2ss as two numerical matrices:

• A *b*-by-*4* matrix, called the LG matrix, which represents the linear graph structure of the system, where *b* is number of branches.

• A *q*-by-*3* matrix, called the outputs matrix, which represents the system outputs, where *q* is the number of desired outputs.

5.1.1 LG Matrix

Each row of the LG matrix specifies a branch of the linear graph model. For single-port elements the specification row is of the form: [*branch_number*, *tail_node*, *head_node*, *element_type*],

where the *tail_node* and the *head_node* are the numbers assigned to the tail and head nodes of the branch specified by *branch_number*.

For each two-port element, the *consecutive* specification rows are of the form:

[*branch_a number*, *tail_a_node*, *head_a_node*, *element_type*]

[*branch_b number*, *tail_b_node*, *head_b_node*, *element_type*]

where *tail_a*, *head_a*, *tail_b* and *head_b* are the numbers assigned to the tail and head nodes of the directed graph edges representing the two sides of a two-port element (*branch_a* and *branch_b*). The branch numbering must be done in such a way that *branch_b_number* is equal to *branch_a_number* plus one.

The *element_type* is an integer number between 1 and 7, which indicates the type of element, as defined in Table 5-1

Table 5-1 Number definition for element types.

Element type	**Element type number**
A_type	1
T_type	2
D_type	3
A_type source	4
T_type source	5
Transformer	6
Gyrator	7

5.1.2 Output Matrix

Each row of the output matrix specifies a desired output variable of the model. The specification row is of the form:

[*output_number, element_number(branch_number), output_type*]

Output_type indicates an integer number between 1 and 4, according to Table 5-2, which shows the type of the variable of the element specified by *element_number*. For example, the row [3 5 2] means the third desired output is *f5* (the through variable associated with branch 5).

Table 5-2 Number definition for output variable types.

Variable type	Variable type number
Across variable	1
Through variable	2
Integrated across variable	3
Integrated through variable	4

5.2 LG2ss Internal Parameters and Elemental Equations

The state space equations generated by LG2ss use pre-defined symbolic parameters as specified in Table 5-3.

Table 5-3 Pre-defined symbolic parameters.

Element type	Symbolic parameters	Elemental equation
1 (generalized A-type element)	C, f, v, dv	$f = C\ dv$
2 (generalized T-type element)	L, f, v, df	$v = L\ df$
3 (generalized D-type element)	R, f, v	$v = R\ f$
4 (generalized A-type source)	f, v, df	v: *input*
5 (generalized T-type source)	f, v, dv	f: *input*
6 (transformer element)	tf, f_a, v_a, f_b, v_b	$v_b = tf\ v_a$ $f_b = (-1/tf)\ f_a$
7 (gyrator element)	gy, f_a, v_a, f_b, v_b	$v_b = gy\ f_a$ $f_b = (-1/gy)\ v_a$

5.3 Illustrative Examples

5.3.1 Electrical Circuit

A simple electrical circuit is shown in Figure 5-1 for which the desired outputs are V_C, I_{L2}. The *LGm* and *Om* matrices are input to the dialogue box provided on the website.

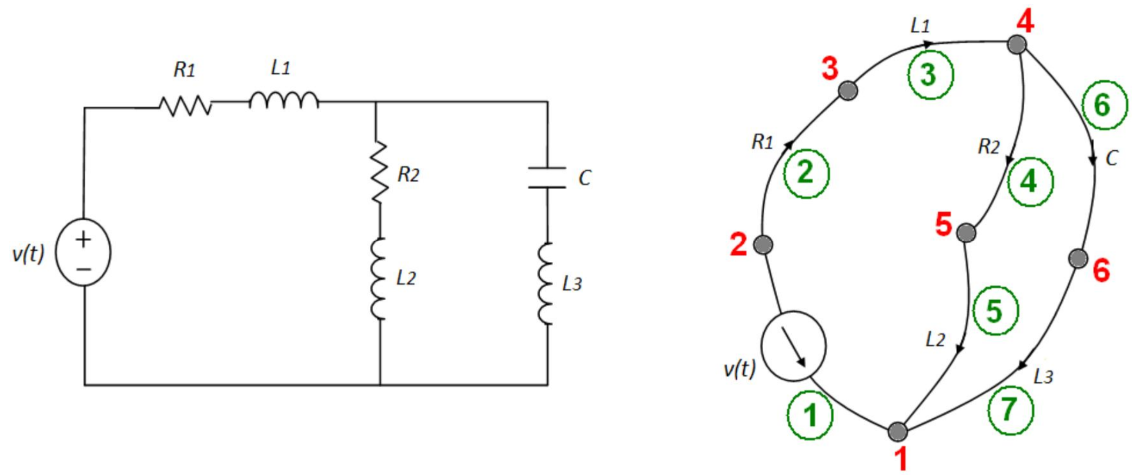

Figure 5-1 Electrical circuit.

Given below is what is shown on the page after the inputs are submitted.

$$LGm = \begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & 2 & 3 & 3 \\ 3 & 3 & 4 & 2 \\ 4 & 4 & 5 & 3 \\ 5 & 5 & 1 & 2 \\ 6 & 4 & 6 & 1 \\ 7 & 6 & 1 & 2 \end{bmatrix} \quad , \quad Om = \begin{bmatrix} 1 & 6 & 1 \\ 2 & 5 & 2 \end{bmatrix}$$

LG matrix = [1 2 1 4;2 2 3 3;3 3 4 2;4 4 5 3;5 5 1 2;6 4 6 1;7 6 1 2]

Output matrix = [1 6 1;2 5 2]

The matrices ***A, B, C, D, E*** and ***F*** for this system are obtained.

A = [-(L3*R4 + L7*R2 + L7*R4/(L3*L5 + L3*L7 + L5*L7), L3/(L3*L5 + L3*L7 + L5*L7), -(L7*R2)/(L3*L5 + L3*L7 + L5*L7)],
[0, 0, 1/C6],
[(L3*R4 - L5*R2)/(L3*L5 + L3*L7 + L5*L7), -(L3 + L5)/(L3*L5 + L3*L7 + L5*L7), -(L5*R2)/(L3*L5 + L3*L7 + L5*L7)]

B = [L7/(L3*L5 + L3*L7 + L5*L7],
[0],
[L5/(L3*L5 + L3*L7 + L5*L7)]

C = [0, 1, 0],
[1, 0, 0]

D = [0],
[0]

E = [0],
[0],
[0]

F = [0],
[0]

Note that *v(t) ≡ v1, R1 ≡ R2, L1 ≡ L3, R2 ≡ R4, L2 ≡ L5, C ≡ C6, L3 ≡ L7.*

5.3.2 Rotary-Motion System with Gearbox

A Rotary-motion system with gearbox is shown in Figure 5-2 for which the desired outputs are v_7 [1].

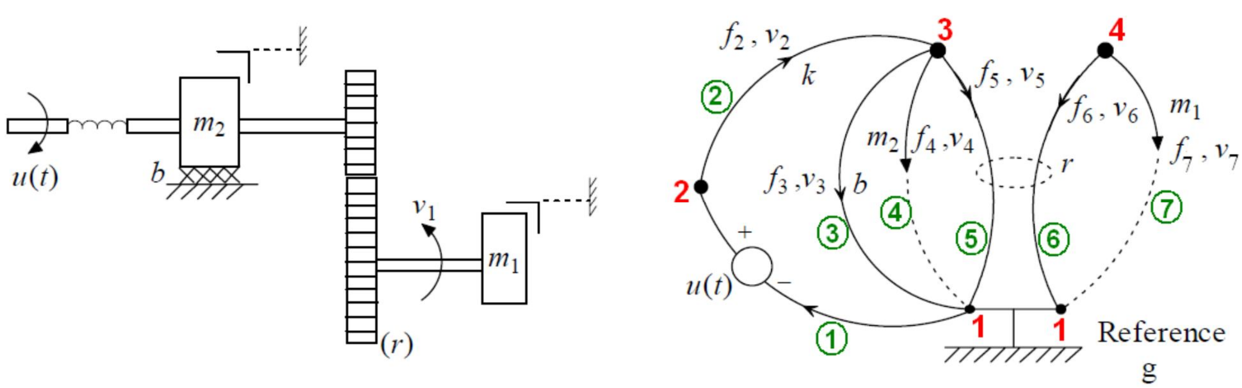

Figure 5-2 Rotary motion system with a gearbox.

The *LGm* and *Om* matrices are the inputs to the dialogue box provided on the website. Given below is what is shown on the page after the inputs are submitted. Desired output:

$$LGm = \begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & 2 & 3 & 2 \\ 3 & 3 & 1 & 3 \\ 4 & 3 & 1 & 1 \\ 5 & 3 & 1 & 6 \\ 6 & 4 & 1 & 6 \\ 7 & 4 & 1 & 1 \end{bmatrix} \quad , \quad Om = [1 \quad 7 \quad 1]$$

LG matrix = [1 2 1 4;2 2 3 2;3 3 1 3;4 3 1 1;5 3 1 6;6 4 1 6;7 4 1 1]

Output matrix = [1 7 1]

A = [0, -1/(L2*TF5],
[TF5/(C7*TF5^2 + C4), -1/(R3*(C7*TF5^2 + C4))]

B = [1/L2],
[0]

C = [0, 1]

D = 0

E = [0],
[0]

F = 0

Note that: $u(t) \equiv v_1$, $k \equiv L_2/2$, $L_1 \equiv L_3$, $b \equiv 1/R_3$, $m_2 \equiv C_4$, $r \equiv TF_5$, $m_1 \equiv C_7$.

5.3.3 Robotic Sewing System

A robotic sewing system is shown in Figure 5-3 for which the desired outputs are f_4, x_7 [1].

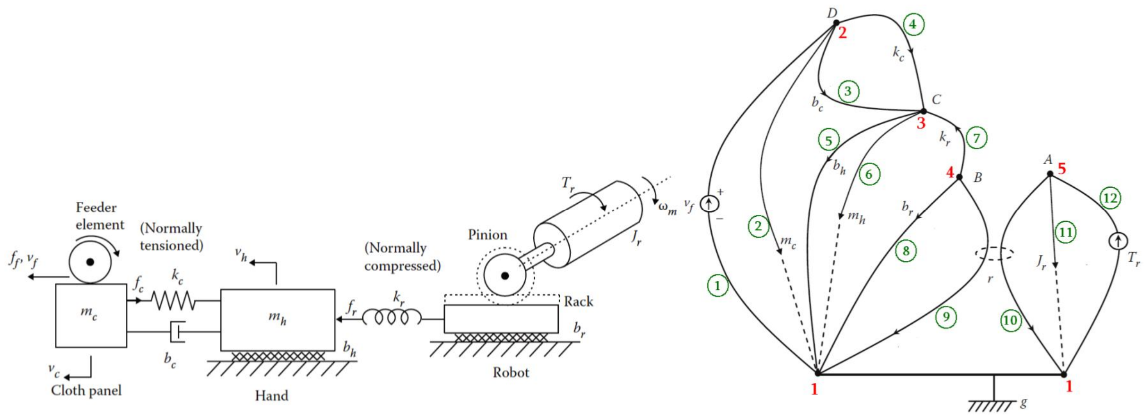

Figure 5-3 Robotic sewing system.

The *LGm* and *Om* matrices are the inputs to the dialogue box provided on the website. Given below is what is shown on the page after the inputs are submitted.

$$LGm = \begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & 2 & 1 & 1 \\ 3 & 2 & 3 & 3 \\ 4 & 2 & 3 & 2 \\ 5 & 3 & 1 & 3 \\ 6 & 3 & 1 & 1 \\ 7 & 4 & 3 & 2 \\ 8 & 4 & 1 & 3 \\ 9 & 4 & 1 & 6 \\ 10 & 5 & 1 & 6 \\ 11 & 5 & 1 & 1 \\ 12 & 5 & 1 & 5 \end{bmatrix} \quad , \quad Om = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 7 & 3 \end{bmatrix}$$

LG matrix = [1 2 1 4;2 2 1 1;3 2 3 3 ;4 2 3 2;5 3 1 3;6 3 1 1;7 4 3 2;8 4 1 3;9 4 1 6;10 5 1 6;11 5 1 1;12 5 1 5]

Output matrix = [1 4 2;2 7 3]

A = [0, -1/L4, 0, 0, 0],
[1/C6, -(R3 + R5/(C6*R3*R5), 1/C6, 0, 0],
[0, -1/L7, 0, 1/(L7*TF9), 0],
[0, 0, -1/(C11*TF9), -1/(C11*R8*TF9^2), 0],
[0, -1, 0, 1/TF9, 0])

```
B = [1/L4, 0],
    [1/(C6*R3, 0],
    [0, 0],
    [0, -1/C11],
    [0, 0]

C = [1, 0, 0, 0, 0],
    [0, 0, 0, 0, 1]

D = [0, 0],
    [0, 0]

E = [0, 0],
    [0, 0],
    [0, 0],
  [0, 0],
  [0, 0]

F = [0, 0],
    [0, 0]
```

Note that: $v_f \equiv v_1$, $T_r \equiv f_{12}$, $m_c \equiv C_2$, $b_c \equiv 1/R_3$, $k_c \equiv 1/L_4$, $b_h \equiv 1/R_5$, $m_h \equiv C_6$, $k_r \equiv 1/L_7$, $b_r \equiv 1/R_8$, $r \equiv TF_9$, $J_r \equiv C_{11}$.

5.4 Acknowledgment

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6 Conclusion

6.1 Summary

Modeling of engineering systems is a fundamental step that is necessary for many engineering activities such as simulation, design, control, optimization monitoring and fault diagnosis, and so on. Thus, sufficient attention should be given to dynamic modeling. An important area in engineering systems is distributed parameter (DP) systems. A suitable modeling method is needed in this area to provide unified and accurate representations, particularly for multi-domain systems, which will enable finer representation and determination of the system dynamic behavior.

In multi-domain applications, as in the design evolution framework that is being developed in our laboratory, it is advantageous to use analogies among different physical domain and use consistent, unified and integrated modeling methods. In this context linear graph (LG) modeling has proven to have many advantages over other available methods. In particular:

- It is an integrated approach that is applicable for multi-domain systems.

- It uses a systematic procedure to extract system dynamic equations and thus can be automated. As a result it can be incorporated with other systematic algorithms.

- It provides a pictorial representation of the physical system which enables visualization of the system characteristics (such as topology) before model formulation.

- LG model provides a one to one correspondence with its physical system.

- It provides a unique state space model once the graph is built.

- It is consistent with structural analogies among different physical systems.

- It helps identify similarities across physical domains.

Vector linear graphs (VLG) provide a tool to simplify the graphic model and the equation formulation of complex systems. This tool utilizes the possible patterns and similarities in the topology of an LG model.

In this thesis two main approaches to model DP systems using LG modeling was investigated: the micro-element method and the separation of variables (SOV) method. The results of applying these approaches were discussed and compared. It was shown that the SOV approach provides increased accuracy while retaining the distributed characteristics of the system, for a given level of model complexity. The LG representations for 1-D and 2-D mechanical structures and hydraulic lines were developed using SOV approach and the results were studied.

A new element was introduced in this thesis to effectively represent mechanical DP systems that have causal inputs. This LG element is a new tool for incorporating DP systems into an overall dynamic system model.

LG2ss is a MATLAB function which utilizes the systematic nature of the LG modeling method. In the present thesis this toolbox was developed as an online application so that it could be available through the Internet for academic use.

Throughout this thesis many illustrative examples were provided to effectively convey the concepts that were explored, developed, or applied. Furthermore, these examples helped to better identify the advantages, shortcomings, and differences among the introduced procedures and approaches.

6.2 Main Contributions

In the analytical realm the most important contribution of this thesis was the development of an LG modeling method for DP systems and investigation of the graphical patterns that were generated in these models. The VLG method was introduced to take advantage of these graphical patterns and simplify the model with respect to both graphical representation and equation formulation.

Based on VLG and observations made in the modeling of DP systems, a new LG element was defined. This element provided a straightforward capability for modeling dynamic systems where DP and LP segments would exist with dynamic interaction.

These studies demonstrated the advantages and potential capabilities of the LG modeling method over other graphical, unified modeling methods.

As another contribution the LG2ss MATLAB function was facilitated into a webpage application to be used at no cost by academic users through the Internet.

6.3 Possible Future Work

In this thesis the LG representation of DP systems was developed particularly for mechanical and hydraulic dynamic systems. However, the SOV approach in LG modeling may be generalized to cover other physical domains as well. This development is left as a possible future work.

The simplification of the LG models obtained using the SOV approach was done by VLG and the introduction of a new element in LG for the mechanical domain. There is room to develop these procedures for other physical domains. This effort may result in a common global element in LG to represent DP systems of all physical domains.

In this thesis the introduced element is applicable for the cases where only causal forces are exerted on the DP system. As a possible future work, the effects of input causality on equation formulation of the DP system may be investigated. Inputs with mixed causalities should be studied as well. This will facilitate automatic modeling of dynamic systems where DP system segments are present in the overall system and accept mixed input causalities.

Furthermore, there is room to further develop and enhance the LG2ss toolbox. It can be improved by incorporating the VLG modeling procedure to reduce the amount of computation and equation formulation. The newly introduced element for modeling DP systems may be programmed into the toolbox so that LG2ss would be able to model dynamic systems that incorporate DP system segments.

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Appendix A PHP Code for LG2SS Web Deployment

This appendix presents the PHP code that was written for the web deployment of the LD2SS toolbox.

```
<html>
<div align="center">
<img src="LG2ss.jpg"/>
 </div>
 <header>
 <div align="center">
 <p>
<font size="3" face="arial" color="black">
[Text]
 <br/><br/></div><div align="center">
 For more details on this applicatin please download the
 <a href="manual.pdf"> manual</a>  PDF file
</font>
</p></div> <hr/>
 </header>
       <body>
       <div align="center">
       <form action="" method="post">
           LG matrix: <input name="m1" />
                           <br/>
                           Output matrix: <input name="m2" /><br/></div>
                           <div align="center">
           <input type="submit" /><br /></div>
       </form>
                  </div>
    </body>
</html>

<?php
if(isset($_POST['m1'])) {
       $f= rand (1e5,1e6);
       exec($c='matlab -nosplash -nodesktop -sd '."C:\\output".' -r "try;s=LG2ss('.preg_replace('/[^0-9; +\-\.\[\]]/', '', $_POST['m1']).','.preg_replace('/[^0-9; +\-\.\[\]]/', '', $_POST['m2']).','".$f."'); catch err; msg= err.message; quit force; end"', $r);
```

```
	echo '<p>LG matrix = '.$_POST['m1'].'</p><p>Output matrix = '.$_POST['m2'].'</p><hr/>';

	for (; !file_exists("C:\output\\".$f.".txt"); sleep(1));
	$file = fopen("C:\output\\".$f.".txt", "r") or exit("Unable to open file!");
	//Output a line of the file until the end is reached
	while(!feof($file))
	{
	echo fgets($file). "<br />";
	}
fclose($file);
	unlink("C:\output\\".$f.".txt");
	// echo '<img src="data:image/jpeg;base64,'.file_get_contents('C:\output\image.jpeg').'" />';
}
```