Investor Information and Asset Returns in Production Economies

by

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Abstract

In this thesis, I explore the implications of investor information for asset returns in general equilibrium economies with production.

In the first chapter, I study what determines the relationship between information quality and long-run risk in a Cox-Ingersoll-Ross type model with recursive preferences. Building on the recent work by Ai (2010), I separate the risk premium into the short-run and long-run components to highlight aspects of preferences that are important for this relationship. It is shown that the attitude towards temporal resolution of uncertainty determines the direction in which changes in information quality alter the compensation for long-run risk, while the elasticity of intertemporal substitution is important for the amplitude of this effect.

In the second chapter, I investigate how incomplete information affects asset returns in a real business cycle model with Epstein-Zin preferences. In the model economy, productivity is altered by both transitory and permanent shocks. The representative agent observes movements in productivity but cannot perfectly distinguish their sources. As a result he must solve a signal extraction problem. This incomplete information model is found to be quantitatively consistent with some common observations about asset prices and aggregate quantities, including, for example, the equity premium, the risk-free rate, the price-dividend ratio and the dynamics of consumption and output. Furthermore, the model generates a downward sloping term structure of equity risk as empirically observed − namely, assets with short-duration of cash flows have larger risk premium and return volatility than assets with long-duration of cash flows.

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Dedication

To my wife Florence and my son Ethan for their love and support.

Chapter 1

Introduction

A large literature has attached importance to incomplete information as a way of understanding aggregate behavior of asset prices.[1] This thesis contributes to this literature in both qualitative and quantitative manners. In Chapter 2, I study the relationship between incomplete information and long-run risk in a Cox-Ingersoll-Ross type model. I identify aspects of preferences that are important in determining this relationship. In Chapter 3, I study the quantitative implications of incomplete information in a real business cycle type model. I show that the incomplete-information real business cycle model is consistent with a variety of stylized facts including a downward-sloping term structure of equity.

1.1 Information Quality and Long-Run Risk: Revisiting the Role of Preferences

The long-run risk model of Bansal and Yaron (2004) has been a phenomenal success in the asset pricing literature. A rapidly growing research program finds the model useful for explaining the equity premium, size and book-to-market effects, momentum, long-term return reversals in stock prices, risk premium in bond markets, real exchange rate movements and more (see the review by Bansal (2007)). A crucial feature of Long-run risk models for modelling asset returns is a small but highly persistent component in consumption. This predictable component is usually assumed to be observable, but it is indeed hard to measure directly in consumption data. In light of this fact, some recent work on long-run risk dispense with the complete information assumption and assume that the predictable component

[1] Examples include Detemple (1986), Dothan and Feldman (1986), Merton (1987), Feldman (1989), Barsky and DeLong (1993), Timmermann (1993), Wang (1993), David (1997), Hansen et al. (1999), Veronesi (1999), Veronesi (2000), Brennan and Xia (2001), Brandt et al. (2004), Li (2005), Calvet and Fisher (2007), Bansal and Shaliastovich (2008a), Bansal and Shaliastovich (2008b), Croce et al. (2010), Brevik and d'Addona (2009), David and Veronesi (2009), Gollier and Schlee (2009), and Hansen and Sargent (2010).

of consumption is concealed from investors.[2]

To understand the consequence of such modification, it is important to grasp the conditions that govern the impact of incomplete information on asset returns. Ai (2010) derives such conditions in a general equilibrium model with production. He shows that the impact of incomplete information on asset returns is determined only by relative risk aversion, independent of the elasticity of intertemporal substitution (EIS). Specifically, incomplete information has the effect of increasing asset risk premia if relative risk aversion is larger than one. Ai then goes further to relate the intuition for this result to the linkage between hedging demand and long-run risk by separating the risk premium into myopic and hedging demand components. He explains that, in the case of risk aversion larger than one, incomplete information increases the premium for long-run risk and hence the hedging component of the risk premium. This explanation, however, implies that the relationship between incomplete information and long-run risk is solely dependent on the attitude towards static risk, regardless of whether investors prefer early or late resolution of uncertainty. Clearly this is not consistent with the common intuition.

The first essay of this thesis aims to provide a better understanding of how the impact of incomplete information is determined. To do so, I reexamine the model setup studied by Ai (2010), but use a different decomposition separating the risk premium into the contributions by short-run and long-run risk. I show that the attitude towards intertemporal risk controls the direction in which incomplete information impacts long-run risk, while the EIS determines the size of this impact. The overall effect on the risk premium, however, only depends on risk aversion, as emphasized by Ai (2010). The finding regarding the attitude towards intertemporal risk is consistent with the previous intuition based on endowment economy models. Complementary to Ai's result, this finding gives a clear interpretation of how incomplete information and the risk premium are related. The finding regarding the role of the EIS is due to the endogenous consumption smoothing channel emphasized by Kaltenbrunner and Lochstoer (2010) that will be elucidated in Chapter 2.

[2] A recent review by Pástor and Veronesi (2009) contains a survey on this strand of literature.

1.2 Incomplete Information and Asset Returns in Real Business Cycle Economies

A large body of research has sought to quantitatively explain aggregate asset prices with incomplete information. The bulk of this literature confines attention to endowment economies and thus abstracts from the fundamental linkage between financial markets and the real economy.[3] The recent work by Ai (2010) represents a first step towards understanding the quantitative implications of incomplete information in a production economy setting. However, the model he utilizes is based on the Cox-Ingersoll-Ross framework (CIR model) which is not the standard vehicle of analysis for exploring the real and financial linkage. In the second essay of this thesis, I show that the CIR model has difficulty replicating both macroeconomic and asset pricing moments at the same time. In light of this, I use a standard real business cycle model (RBC) to study the implications of incomplete information. The RBC models have proven a fruitful framework for understanding macroeconomic fluctuations and therefore have better potential to unite asset pricing and macroeconomic analysis.

I show that the standard real business cycle model augmented with incomplete information is consistent with a variety of empirical facts on asset prices such as the equity premium, the risk-free rate and the price-dividend ratio, while at the same time matching the volatility of consumption and output growth as well as their autocorrelation. Notably, the model also implies a negative slope for the term structure of equity and thus resolves an empirical challenge confronting several leading models of asset pricing. The mechanism generating the downward-sloping term structure is distinct from those proposed in the existing literature (Ai, Croce and Li (2010), Croce, Lettau and Ludvigson (2010)). To provide the intuition, I tie the term structure implication to the influence of incomplete information on endogenous consumption dynamics.

This essay contributes to the literature that aims to jointly explain macroeconomic quantities and asset prices. The starting point of this literature is the standard representative agent real business cycle model (e.g., King et al. (1988), Plosser (1989)). Rouwenhorst (1995) was the first to observe that, while this model is able to generate realistic processes for consumption and investment, it fails markedly at explaining asset prices. To provide a remedy, Jermann (1998) and Boldrin, Christiano and Fisher (2001)

[3]See, e.g., Brennan and Xia (2001), Calvet and Fisher (2007), Bansal and Shaliastovich (2008a), Bansal and Shaliastovich (2008b) and Croce et al. (2010).

introduce habit persistence and capital investment frictions. However, their models generate excess volatility in the risk free rate. Kaltenbrunner and Lochstoer (2010) and Croce (2010) resort to the idea of long-run risk. Their models do not inherit the problem with the risk free rate because of the assumption of high EIS. However, they did not make attempt to explain the term structure of equity. By contrast with these studies, my model is not only consistent with a set of stylized facts about asset prices and the macroeconomy but also provides an explanation why the term structure of equity is downward sloping.

The rest of the thesis proceeds as follows. In Chapter 2, I study the determinants of the relationship between information quality and long-run risk. In Chapter 3, I explore the implications of incomplete information in a standard real business cycle model, and provide a new mechanism for understanding the empirical relationship between equity risk premia and cash flow duration. Chapter 4 gives concluding remarks.

Chapter 2

Information Quality and Long-Run Risk: Revisiting the Role of Preferences

2.1 Introduction

In a seminal paper, Bansal and Yaron (2004) propose the long-run risk framework in which low frequency variations in consumption can provide a justification for the observed equity premium. Their study has catalyzed a burgeoning research program, which documents the success of long-run risk framework in explaining a wide array of asset pricing facts, including, for example, credit spreads, bond risk premia, cross-sectional stock returns, and real exchange rate movements (see Bansal et al. (2005), Bansal and Shaliastovich (2010), Bhamra et al. (2009), Chen (2009), Colacito and Croce (2008), Hansen et al. (2008), Kiku (2006), Piazzesi and Schneider (2006), and Rudebusch and Swanson (2008)).

In most of long-run risk models, investors are assumed to perfectly know the structure of the economy, including the stochastic process generating low frequency variations in consumption. Yet, in practice investors are not so well-informed about the stochastic properties of consumption, for the empirical evidence is indecisive about whether consumption comprises low frequency variations or is just purely i.i.d..[4] Even if investors understand that low frequency variations exist, they are still impelled to filter out high frequency shocks in consumption data in order to estimate the process of low frequency variations. Estimation errors will occur unless investors completely understand the temporal evolutions of the low-frequency and high-

[4]This raises the question how the low frequency shocks could be reflected in security prices if their existence cannot be ascertained by investors. An interesting explanation is provided by Hansen and Sargent (2010). In their work the representative agent assigns probabilities to two alternative models: one with predictable consumption and the other without. Concerns about model uncertainty induce the agent to put a very high probability to the model with predictable consumption.

frequency shocks, and are able to decompose each period's innovation into its component sources, for which the data provides no guide.

Recognizing that the complete information assumption is highly restrictive, the recent studies on long-run risk have set out to recast the models so that the stochastic process driving low frequency movements in consumption is concealed from investors.[5] These studies generate a set of results, showing that the incomplete information models are not only consistent with the observed equity premium and risk-free rate, but also capable of explaining some asset pricing issues that are otherwise difficult to address within the complete information context, such as large moves in asset prices, the relationship of the return properties of value and growth assets with their cash flow duration properties, and the statistical properties of wealth-consumption ratio (Bansal and Shaliastovich (2008a), Bansal and Shaliastovich (2008b), Croce et al. (2010), and Ai (2010)). These results are quite notable, and a precise understanding requires a good grasp of the conditions that allow incomplete information to figure prominently in asset prices. Focusing on the aggregate consumption claim, Ai (2010) derives such a condition analytically in a production economy setting. By decomposing the risk premium into the myopic and hedging demand components, he shows that risk aversion, as opposed to the EIS, determines the direction in which incomplete information affects the risk premium. In particular, the risk premium is increased if risk aversion is greater than one. To provide the intuition, Ai focuses on the connection between hedging demand and long-run risk, explaining that under the assumption of risk aversion greater than one, the required compensation for long-run risk and hence the hedging demand component of the risk premium is amplified by incomplete information. According to this intuition, the relationship between incomplete information and long-run risk depends on the attitude towards atemporal risk, not the attitude towards intertemporal risk. This stands in contrast to the understanding based on endowment economy models that the attitude towards intertemporal risk is a critical determinant of how incomplete information alters long-run risk (Bansal and Shaliastovich (2008a), Bansal and Shaliastovich (2008b), and Croce et al. (2010)). This is surprising, as it indicates a fundamental difference between endowment and production economies.

The analysis presented here derives its motivation from the above discrepancy. I revisit the economy Ai examines, and building on his results, I present an alternative decomposition, separating the risk premium of aggre-

[5] A recent review by Pástor and Veronesi (2009) contains a survey on this strand of literature.

gate consumption claim into the short-run and long-run components. This exercise highlights aspects of preferences that are important for the effect of information quality on long-run risk. It shows that the direction in which changes in information quality affect the required compensation for long-run risk depends on the attitude towards temporal resolution of uncertainty. If agents are eager to quickly resolve uncertainty about consumption, low information quality will result in high premium for long-run risk. The attitude towards intertemporal substitution, by contrast, is important in the determination of the amplitude of the effect that information quality has on the long-run risk exposure. If agents are more willing to shift consumption across time, changes in information quality will have larger impact on the long-run risk exposure.

To see the intuition for the first result, let us start by noting that the exposure of aggregate consumption claim to long-run risk is defined by the covariance of its incremental return with expected consumption growth. In the model economy, the incremental return and expected consumption growth are driven, respectively, by short-lived output shocks and long-lasting productivity innovations. In particular, they both increase when receiving a positive shock. Hence, the long-run risk exposure is larger when the two types of shocks has more tendency to move together. This happens when agents face incomplete information about future prospects so that they are forced to rely on changes in output to predict changes in productivity. Intuitively, the increased exposure to the risk associated with future consumption only gets compensated when agents value expedited resolution of uncertainty about the future. This production economy result is consistent with prior understanding based on endowment economy models (e.g., Bansal and Shaliastovich (2008a), Bansal and Shaliastovich (2008b) and Croce et al. (2010)). For the second result, the intuition can be understood from the association between intertemporal substitution of consumption and variability of expected consumption growth. From the definition of long-run risk exposure, if expected consumption growth becomes more responsive to productivity shocks, the long-run risk exposure will get a stronger impact from a decrease in information quality. The EIS matters for the response of expected consumption growth, and hence plays a significant role in determining the quantitative effect of information quality. To see this clearly, consider the situation where productivity is hit by a positive shock. In response to the shock, agents with high EIS are more willing to defer current consumption in exchange for high future consumption, leading to a larger increase in expected consumption growth. This in turn implies a larger covariance between the incremental return and expected consumption growth, and hence a larger exposure to long-run risk.

In this paper, the results are stated primarily for the aggregate consumption claim but this is just for expositional ease. I also perform the same analysis for an equity type asset modeled as a levered claim to aggregate consumption, and find the results to be qualitatively similar. The analysis of this paper is complementary to Ai's. While he identifies, among other results, aspect of preference that governs the effect of information quality on the *overall* risk premium, I determine aspects of preference that have an important influence on how information quality affects the *long-run* component of risk premium. This paper thus sheds additional light on the role of preferences in a long-run risk model with incomplete information.

The rest of the paper proceeds as follows. Section 1.2 provides an overview of the model Ai (2010) examines, including equilibrium properties of consumption and asset returns. Section 1.3 provides an intertemporal decomposition of the aggregate consumption risk premium, separating it into the short-run and long-run components, and then examines the roles played by various aspects of preferences in determining the asset pricing effect of information quality. Section 1.4 concludes.

2.2 The Model

The model is a continuous-time production economy of the Cox et al. (1985) type. A filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\}_{t \geqslant 0})$ is given, which satisfies the usual conditions and is endowed with a three-dimensional Brownian motion $\mathbf{B}_t \equiv (B_{K,t}, B_{\theta,t}, B_{e,t})'$ adapted to $\{\mathcal{F}_t\}_{t \geqslant 0}$.

2.2.1 Preferences

The economy is populated by a continuum of identical agents whose preferences are described by the stochastic differential utility of Duffie and Epstein (1992b). This class of utility is a continuous-time analogue of the recursive preferences proposed by Kreps and Porteus (1978), Epstein and Zin (1989) and Weil (1989). The utility index, U_t, is the solution to the integral equation

$$U_t = \mathbb{E}_t \int_t^\infty f(C_\tau, U_\tau)\mathrm{d}\tau, \tag{2.1}$$

where f is a normalized aggregator of current consumption and continuation utility that takes the form:

$$f(c,u) = \begin{cases} \frac{\beta}{\varphi}\left[c^{\varphi}(\alpha u)^{1-\varphi/\alpha} - \alpha u\right] & \text{if } 0 \neq \alpha \leqslant 1,\ 0 \neq \varphi \leqslant 1, \\ \beta u[\alpha \ln c - \ln(\alpha u)] & \text{if } 0 \neq \alpha \leqslant 1,\ \varphi = 0. \end{cases}$$

Here, β is the rate of time-preference, $\gamma \equiv 1 - \alpha$ is the coefficient of relative risk aversion (RRA), and $\psi \equiv (1-\varphi)^{-1}$ is the elasticity of intertemporal substitution (EIS). The recursive formulation thus provides a separate role for relative risk aversion and the elasticity of intertemporal substitution. In addition, it permits a representation of the attitude towards the timing of uncertainty resolution. Specifically, the agents prefer early (late) resolution of uncertainty if $\varphi > (<)\alpha$. In the special case where $\varphi = \alpha$, the recursive formulation collapses to the power specification of expected utility, under which the agents are neutral to the timing of uncertainty resolution.

2.2.2 Technology

There is a representative firm that produces a single physical good, the numeraire, which may be allocated to consumption or investment. The production technology exhibits stochastic constant returns to scale. Over a time interval of length $\mathrm{d}t$, it transforms y units of capital into $y + y \times (\theta_t \mathrm{d}t + \sigma_K \mathrm{d}B_{K,t})$ units of consumption good, where θ_t is the level of productivity and $\mathrm{d}B_{K,t}$ is an output shock with standard deviation σ_K. If between time t and $t + \mathrm{d}t$, $C_t \mathrm{d}t$ units of good are consumed, the total change in the capital stock K_t will be

$$\mathrm{d}K_t = \theta_t K_t \mathrm{d}t + \sigma_K K_t \mathrm{d}B_{K,t} - C_t \mathrm{d}t. \tag{2.2}$$

The probability distribution of current output depends on the current level of productivity, which is itself changing randomly over time. The development of θ will thus determine the production opportunities available to the economy in the future. It is postulated that θ evolves according to an Ornstein-Uhlenbeck process:

$$\mathrm{d}\theta_t = \kappa(\bar{\theta} - \theta_t)\mathrm{d}t + \sigma_\theta \mathrm{d}B_{\theta,t}, \tag{2.3}$$

where κ denotes the persistence of the productivity shock, $\bar{\theta}$ the long-run mean of θ_t, σ_θ the instantaneous volatility of productivity shocks. The short-run output shocks, $\mathrm{d}B_{K,t}$, are allowed to be correlated with the more long-lasting innovations in productivity, $\mathrm{d}B_{\theta,t}$, their correlation being denoted by ρ.

2.2.3 Information Structure and Bayesian Inference

Assume that all agents know the structure and parameters of the economy. However, they cannot observe the realizations of the productivity process $\{\theta_t\}_{t\geqslant 0}$, but continuously update their assessment using the information from endogenous quantities (wealth, capital stock, etc.) and an exogenous signal e_t. The stochastic differential equation describing the evolution of the exogenous signal takes the form $\mathrm{d}e_t = \theta_t \mathrm{d}t + \sigma_e \mathrm{d}B_{e,t}$, where the parameter σ_e controls the precision of the exogenous signal. Let $\mathcal{G}_t$ be the filtration that collects the information generated by the observations on endogenous quantities and the exogenous signal. Then, by Kalman-Bucy filter, the estimate of the state of technology, $m_t = \mathrm{E}[\theta_t|\mathcal{G}_t]$, evolves as

$$\mathrm{d}m_t = \kappa(\bar{\theta} - m_t)\mathrm{d}t + \sigma_m \mathrm{d}\tilde{B}_{m,t} \tag{2.4}$$

where

$$\sigma_m \equiv \sqrt{\left(\rho\sigma_\theta + \frac{Q}{\sigma_K}\right)^2 + \left(\frac{Q}{\sigma_e}\right)^2} \tag{2.5}$$

$$\mathrm{d}\tilde{B}_{m,t} \equiv \frac{1}{\sigma_m}\left[\left(\rho\sigma_\theta + \frac{Q}{\sigma_K}\right)\mathrm{d}\tilde{B}_{K,t} + \frac{Q}{\sigma_e}\mathrm{d}\tilde{B}_{e,t}\right], \tag{2.6}$$

Q is the steady-state estimation error (i.e., $Q = \lim_{t\to\infty} \mathrm{Var}[\theta_t|\mathcal{G}_t]$) given by

$$Q = \frac{(1-\rho^2)\sigma_\theta^2}{\left(\kappa + \rho\frac{\sigma_\theta}{\sigma_K}\right) + \sqrt{\left(\kappa + \rho\frac{\sigma_\theta}{\sigma_K}\right)^2 + (1-\rho)\sigma_\theta^2(\sigma_K^{-2} + \sigma_e^{-2})}} \tag{2.7}$$

The innovation processes, $\tilde{B}_{K,t}$ and $\tilde{B}_{e,t}$ are defined as

$$\mathrm{d}\tilde{B}_{K,t} \equiv \frac{1}{\sigma_K}\left[\frac{(\mathrm{d}K_t + C_t\mathrm{d}t)}{K_t} - m_t\mathrm{d}t\right],$$
$$\mathrm{d}\tilde{B}_{e,t} \equiv \frac{1}{\sigma_e}(\mathrm{d}e_t - m_t\mathrm{d}t).$$

Therefore, in the incomplete information economy, the dynamic of capital stock is perceived to be

$$\mathrm{d}K_t = m_t K_t \mathrm{d}t + \sigma_K K_t \mathrm{d}\tilde{B}_{K,t} - C_t\mathrm{d}t, \tag{2.8}$$

and the return to capital investment is given by

$$\mathrm{d}R_t^K = \frac{\mathrm{d}K_t + C_t\mathrm{d}t}{K_t} = m_t\mathrm{d}t + \sigma_K\mathrm{d}\tilde{B}_{K,t}. \tag{2.9}$$

2.2.4 Financial Markets

Agents have access to competitive financial markets where they trade continuously in three assets: a claim on the total stream of consumption good generated by the risky production technology, a riskless asset and a contingent claim. The supply of the aggregate consumption claim is strictly positive, while that of the riskless asset and the contingent claim is null. The riskless asset earns an instantaneous interest rate, r_t. Over any time interval $\mathrm{d}t$, the incremental value generated by the aggregate consumption claim includes both capital gains, $\mathrm{d}P_t$, and the stream of consumption good, $C_t\mathrm{d}t$, such that the incremental return on the aggregate consumption claim writes

$$\mathrm{d}R_t^C = \frac{\mathrm{d}P_t + C_t\mathrm{d}t}{P_t} = \mu_{R,t}^C\mathrm{d}t + \sigma_{R,t}^C\mathrm{d}\tilde{\mathbf{B}}_t, \tag{2.10}$$

where $\mu_{R,t}^C$ is a scalar, $\sigma_{R,t}^C$ a 1×2 vector, $\tilde{\mathbf{B}}_t \equiv (\tilde{B}_{K,t}, \tilde{B}_{e,t})'$. The contingent claim is issued and purchased by individuals. Its incremental return, $\mathrm{d}R_t^A$, is taken to be

$$\mathrm{d}R_t^A = \mu_{R,t}^A\mathrm{d}t + \sigma_{R,t}^A\mathrm{d}\tilde{\mathbf{B}}_t, \tag{2.11}$$

where $\mu_{R,t}^A$ is a scalar, $\sigma_{R,t}^A$ a 1×2 vector. Note that since the contingent claim is in zero net supply, its payoff specification is not important for the pricing of the aggregate consumption claim. It is introduced simply to span a complete market. As one can see, there are three investment vehicles in place and only two independent Brownian motions driving the economy, financial markets are therefore complete.

Given the return processes, the dynamics of wealth is readily obtained. Let a_t and b_t denote, respectively, the proportion of wealth invested in the aggregate consumption claim and the contingent claim. Since the agent's actions must be self-financing, the remaining proportion $1-a_t-b_t$ is invested in the risk-free asset. The agent's wealth dynamics may be written as

$$\begin{aligned}\mathrm{d}W_t &= \left[a_tW_t(\mu_{R,t}^C - r_t) + b_tW_t(\mu_{R,t}^A - r_t) + r_tW_t - C_t\right]\mathrm{d}t \\ &\quad + a_tW_t\sigma_{R,t}^C\mathrm{d}\tilde{\mathbf{B}}_t + b_tW_t\sigma_{R,t}^A\mathrm{d}\tilde{\mathbf{B}}_t \\ &\equiv W_t\mu_W\mathrm{d}t + W_t\sigma_W\mathrm{d}\tilde{\mathbf{B}}_t\end{aligned} \tag{2.12}$$

where μ_W is a scalar, σ_W a 1×2 vector.

2.2.5 Equilibrium Characterization

Finally, I describe some characteristics of the equilibrium that are relevant for the ensuing analysis.

In this economy, the optimization problem of the representative agent is to select consumption and investment plans to maximize expected lifetime utility as measured by (2.1), subject to (2.4) and (2.12). In performing the optimization the representative agent takes the risk-free rate, r_t, and the price of the aggregate consumption claim, P_t, as given. An equilibrium is reached if r_t and P_t are such that the representative agent optimally invests all of his wealth into the aggregate consumption claim and nothing into the risk-free asset and the contingent claim. Comparing (2.9) and (2.10) reveals that, in equilibrium, P_t must equal the amount of physical capital, K_t. Otherwise, P_t will differ from K_t by the value of a continual payout stream of $\mu_{R,t}^C - m_t$ which is the difference between expected return on physical investment and expected return on aggregate consumption claim. The agent will thus have incentive to expand or contract their capital investments until $\mu_{R,t}^C = m_t$. Therefore, the equality $P_t = K_t$ must hold in equilibrium. This line of reasoning is found in Cox et al. (1985). Since $P_t = K_t$, we can use (2.8) to rewrite (2.10) as

$$\mathrm{d}R_t^C = \frac{\mathrm{d}K_t + C_t\mathrm{d}t}{K_t} = m_t\mathrm{d}t + \sigma_K \mathrm{d}\tilde{B}_{K,t}. \tag{2.13}$$

The optimal decision of the representative agent leads to an equilibrium consumption rule represented by

$$C(W_t, m_t) = x^{-1}(m_t)W_t, \tag{2.14}$$

where $x(m_t)$ is the wealth-consumption ratio given by

$$x(m_t) = \beta^{-\psi} H(m_t)^{-\frac{1-\psi}{1-\gamma}}, \tag{2.15}$$

where $H(m_t) : (-\infty, +\infty) \to \mathbb{R}^+$ satisfies an ordinary differential equation specified in the appendix. It is useful to note that $H(m_t)$ is a positive function, and is strictly decreasing (increasing) if $\gamma > 1$ ($\gamma < 1$). By implication, $x(m_t)$ is strictly increasing (decreasing) in m_t if $\psi > 1$ ($\psi < 1$) (see Ai (2010)).

2.3 An Intertemporal Decomposition for the Risk Effect of Information Quality

In this section, I provide an alternative decomposition to Ai (2010), separating the risk premium into the short-run and long-run components. I use this decomposition to study the determinants for the direction and amplitude of the effect of information quality on long-run risk.

2.3.1 Intertemporal Structure of Risk

I begin by illustrating the intertemporal structure of risk encoded in the state price density. The state price density, π_t, can be written as

$$\pi_t = \left(\beta e^{-\beta t}\right)^{\frac{\alpha}{\varphi}} C_t^{-\gamma} \left(x_t e^{\int_0^t x_s^{-1} \mathrm{d}s}\right)^{-\frac{\varphi-\alpha}{\varphi}}. \tag{2.16}$$

The derivation for the this expression is found in the appendix. Unlike the one implied by power utility, this state price density depends on the wealth-consumption ratio x_t, i.e., the value of the claim to aggregate consumption per unit consumption. This feature reflects the concern of recursive utility agents about future consumption growth.

Using Ito's lemma, I differentiate π_t to obtain

$$\frac{\mathrm{d}\pi_t}{\pi_t} = \mathbb{E}_t\left(\frac{\mathrm{d}\pi_t}{\pi_t}\right)\mathrm{d}t - \gamma\left[\frac{\mathrm{d}C_t}{C_t} - \mathbb{E}_t\left(\frac{\mathrm{d}C_t}{C_t}\right)\right] - (\varphi-\alpha)\left[\varphi^{-1}\frac{\mathrm{d}x_t}{x_t} - \mathbb{E}_t\left(\varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right)\right]. \tag{2.17}$$

The expression in the first square bracket represents shocks to contemporaneous consumption growth. By Ito's lemma, the expression in the second square bracket is written as

$$\varphi^{-1}\frac{\mathrm{d}x_t}{x_t} - \mathbb{E}_t\left(\varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right) = \frac{\psi}{\alpha}\frac{H'(m_t)}{H(m_t)}\sigma_m \mathrm{d}\tilde{B}_{m,t}. \tag{2.18}$$

By the properties of $H(\cdot)$, $\frac{1}{\alpha}\frac{H'(m_t)}{H(m_t)} > 0$. As a result, (2.18) has the same sign as the expected growth shock $\mathrm{d}\tilde{B}_{m,t}$. Thus, equation (2.17) shows how agents feel about the short-run and long-run uncertainty in consumption. While they always feel bad when contemporaneous consumption growth is low, their reaction to the uncertainty in expected consumption growth depends on their attitude towards the timing of uncertainty resolution. If agents prefer early resolution of uncertainty (i.e., $\varphi > \alpha$), the pricing kernel will rise in response to unfavorable expected growth shocks (i.e., $\mathrm{d}\tilde{B}_{m,t} < 0$), reflecting the fear of agents for such events.

For any risky asset with incremental return $\mathrm{d}R_t$, this pricing kernel imposes a restriction on the instantaneous risk premium of the form

$$\mu_{R,t} - r_t = \gamma \mathrm{Cov}_t\left(\mathrm{d}R_t, \frac{\mathrm{d}C_t}{C_t}\right) + (\varphi-\alpha)\,\mathrm{Cov}_t\left(\mathrm{d}R_t, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right). \tag{2.19}$$

In this expression, the consumption risk premium is separated into its frequency components. The first term captures the compensation for the exposure to contemporaneous growth shocks, i.e., the short-run component of risk premium; the second term captures the compensation for the exposure to expected growth shocks, i.e., the long-run component of risk premium. Note that the price of long-run risk is given by $\varphi - \alpha$. Intuitively, this price is determined by the strength of preferences for temporal resolution of uncertainty. It gets larger, the more eager agents are to resolve uncertainty quickly.

2.3.2 Determinant of the Relationship between Information Quality and Long-Run Risk

This subsection studies the determinant for the relationship between information quality and long-run risk. To develop intuition in the simplest case possible, I being with the claim on aggregate consumption stream. Since the price of risk is constant, information quality can only impinge on long-run risk through the risk exposure (see (2.19)). In the benchmark case of fully observable economy, the long-run risk exposure is measured by

$$\mathrm{Cov}\left(\mathrm{d}R_t^C, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\middle|\mathcal{F}_t\right) = \frac{\psi}{\alpha}\frac{G'(\theta_t)}{G(\theta_t)}\rho\sigma_\theta\sigma_K. \tag{2.20}$$

Note that $\mathcal{F}_t$ is the information set of the complete information economy and $G(\cdot)$ satisfies an ODE specified in the appendix. Introducing noisy information changes the long-run risk exposure to

$$\mathrm{Cov}\left(\mathrm{d}R_t^C, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\middle|\mathcal{G}_t\right) = \frac{\psi}{\alpha}\frac{H'(m_t)}{H(m_t)}(\rho\sigma_\theta\sigma_K + Q), \tag{2.21}$$

where $\mathcal{G}_t$ is the information set of the partially observable economy.[6] The resultant change is particularly easy to assess if the two types of production shocks are uncorrelated ($\rho = 0$).

Proposition 1. *In a Cox-Ingersoll-Ross production economy with $\rho = 0$, noisy information increases long-run risk for the claim on aggregate consumption if agents prefer early resolution of uncertainty (i.e., $\varphi > \alpha$).*

This result follows immediately by setting $\rho = 0$ in (2.20) and (2.21), which shows that the full information economy is immune from long-run risk, while the incomplete information economy is positively exposed. The implication of this differential exposure for the long-run risk compensation depends on the attitude towards temporal resolution of uncertainty. The long-run risk compensation will increase if early resolution of uncertainty

[6] The risk exposure is derived from (2.10) and (2.18).

is preferred (i.e., $\varphi > \alpha$), and conversely if late resolution of uncertainty is desired (i.e., $\varphi < \alpha$). In relation to Ai (2010), this result clarifies the intuition that the attitude towards *intertemporal* risk, rather than the attitude towards *atemporal* risk, determines the direction in which information quality affects long-run risk. It also confirms that, in Ai (2010)'s quantitative investigation, noisy information indeed operates through the long-run risk channel to magnify the risk premium, as the preference parameters were restricted to imply an inclination towards early resolution of uncertainty.

Consider now the more general case where $\rho \neq 0$. Since an analytical solution is not available in this case, I make parametric assumptions to proceed. Table 2.1 summarizes the parameter values. These values are taken from Ai (2010)'s benchmark calibration, except that a range of values is allowed for the correlation coefficient (ρ) and the EIS (ψ). Table 2.2 reports the results of simulating the model. These are consistent with the intuition developed for the case of $\rho = 0$. Specifically, when the EIS (ψ) takes such values that imply preferences for early resolution of uncertainty (i.e., $\psi = 0.6, 0.8, 1.5, 2.0$), long-run risk is inversely related with information quality; the relationship is reversed when the EIS is sufficiently small such that agents desire late resolution of uncertainty (i.e., $\psi = 0.1, 0.3$). Furthermore, this pattern stands out for all levels of shock correlation under consideration.

2.3.3 Determinant of the Magnitude of the Effect of Information Quality on Long-Run Risk

I now explore the determinant factors influencing the quantitative effect of information quality on long-run risk. Evidently the EIS has an important role to play in this regard. For a given level of risk aversion, the desire of agents for a quick resolution of uncertainty and thus the price of long-run risk rises with the EIS. As a result, changes in information quality may have large impact on long-run risk when the EIS is large. Apart from this, the EIS can also affect the extent to which changing information quality alters the long-run risk exposure. Table 2.4 demonstrates the latter effect, showing that, when information quality gets low, a larger EIS results in a more pronounced increase in the long-run risk exposure.

To understand the intuition for the second effect, I begin by recalling that the exposure to long-run risk is measured by the covariance of incremental return with expected consumption growth. From (2.13) and (2.18), the incremental return and expected consumption growth are, respectively, driven by the perceived shocks to output and productivity (i.e., $\mathrm{d}\tilde{B}_{K,t}$ and $\mathrm{d}\tilde{B}_{m,t}$). In the presence of incomplete information, optimal updating creates a tendency for the two types of shocks, and thus the incremental return and expected consumption growth, to covary positively. The tendency is stronger the larger is the EIS, because high EIS leads expected consumption growth to respond more aggressively to the productivity shock, $\mathrm{d}\tilde{B}_{m,t}$. This will become clear after inspecting the dynamics of consumption.

By Ito's lemma, the dynamics of consumption write

$$\frac{\mathrm{d}C_t}{C_t} = \mu_C(m_t)\mathrm{d}t + \sigma_K \mathrm{d}\tilde{B}_{K,t} - \frac{\psi - 1}{1-\gamma}\frac{H'(m_t)}{H(m_t)}\sigma_m \mathrm{d}\tilde{B}_{m,t}, \tag{2.22}$$

where

$$\begin{aligned}\mu_C(m_t) = \psi\left(m_t - \beta - \frac{\varphi\gamma\sigma_K^2}{2}\right) - \frac{\varphi\psi(\gamma-\psi)\sigma_m^2}{2\alpha^2}\left(\frac{H'(m_t)}{H(m_t)}\right)^2 \\ - \frac{\varphi\psi\gamma}{\alpha}\frac{H'(m_t)}{H(m_t)}(\rho\sigma_\theta\sigma_K + Q).\end{aligned}$$

Equation (2.22) shows that following a favorable productivity shock (i.e., $\mathrm{d}\tilde{B}_{m,t} > 0$), consumption drops initially if $\psi > 1$, but rises if $\psi < 1$. What is behind this behavior is a tension between income and substitution effects that are engendered by the change of productivity. When productivity increases, the agents on the one hand want to accelerate consumption because they feel wealthier, on the other hand want to defer consumption in exchange for the prospect of higher future consumption growth. If $\psi > 1$, the substitution effect dominates the income effect, leading to a negative response in consumption. If $\psi < 1$, the opposite occurs. The less is consumed at the present, the more will be invested and consequently the more expected consumption growth will be boosted. Therefore, the extent of response in expected consumption growth increases with the EIS. Figure 1 illustrates this graphically, plotting the dynamic response of consumption growth for different values of the EIS.

2.3.4 The Overall Effect of Information Quality on the Risk Premium: An Alternative Interpretation

On the basis of the preceding analysis, I provide an alternative interpretation to Ai (2010) for the effect of information quality on the overall risk premium. Using the loglinear method of Campbell et al. (2004), Ai (2010) shows that noisy information causes the risk premium to increase if risk aversion is greater than one. To provide the intuition, Ai focuses on the association between information quality and hedging demand. Building on his results, I first confirm in Table 2.3 the prediction of his loglinear solutions by accurate numerical solutions. Then, I show the relationship between information quality and the risk premium can be understood from the alterations in intertemporal structure of risk.

First, let us review the intuition behind Ai's result. In giving the intuition, Ai writes the consumption risk premium as

$$\mu^C_{R,t} - r_t = \gamma\sigma^2_K - \frac{H'(m_t)}{H(m_t)}(\rho\sigma_\theta\sigma_K + Q), \tag{2.23}$$

where the first component results from myopic demand for asset, and the second component (including the minus sign) results from demand for hedging. As we can see, only the hedging demand component would be affected by changes in information quality. Ai shows that, if $\gamma > 1$, the hedging demand component is positive, and gets larger as information quality gets low. Intuitively, in the case of $\gamma > 1$, noisy information creates a negative demand for hedging, and the demand is larger, the lower the quality of information. Hence, there must be a corresponding rise in the risk premium so that the market for aggregate consumption claim clears.

The intuition that risk aversion rather than the EIS determines the sign of hedging demand was provided by Bhamra and Uppal (2006) in a dynamic consumption-portfolio choice model with recursive preferences. In addition, these authors showed that the EIS as opposed to risk aversion controls the size of hedging demand. This intuition also presents itself here, and will become clear if we approximate (2.23) using the log-linear method of Campbell et al. (2004):

$$\mu^C_{R,t} - r_t \approx \gamma\sigma^2_K - \frac{1-\gamma}{\bar{x}^{-1}+\kappa}(\rho\sigma_\theta\sigma_K + Q), \tag{2.24}$$

where $\bar{x}$ is the unconditional wealth-consumption ratio. Since $\bar{x}$ depends on the EIS (see (2.15) for the definition of x), (2.24) makes clear that the EIS affects the size of the hedging demand component.

The hedging demand component, when seen through the lens of intertemporal structure of risk, is just an outcome of the interaction between the short-run and long-run risk factors. With this in mind, it is not difficult to realize that we should look at how exactly such interaction has occurred in order to understand the relation between information quality and the risk premium. To do so, consider the following equality

$$\mathrm{Cov}_t\left(\mathrm{d}R^C_t, \frac{\mathrm{d}W_t}{W_t}\right) = \mathrm{Var}_t\left(\frac{\mathrm{d}W_t}{W_t}\right). \tag{2.25}$$

The left hand side is the covariance between return on aggregate consumption claim and return on wealth. The right hand side is the instantaneous volatility of return on wealth. This equality holds since the equilibrium dictates that all the wealth must be invested in the aggregate consumption claim so that the return process of this asset is identical to that of wealth.

Next, I substitute out $\mathrm{d}W_t/W_t$ from the left hand side of (2.25) using the identity

$$\frac{\mathrm{d}W_t}{W_t} = \frac{\mathrm{d}C_t}{C_t} + \frac{\mathrm{d}x_t}{x_t} + \frac{\mathrm{Cov}_t(\mathrm{d}C_t, \mathrm{d}x_t)}{W_t}. \tag{2.26}$$

With a little algebraic manipulation, I obtain

$$\underbrace{\mathrm{Cov}_t\left(\mathrm{d}R_t^C, \frac{\mathrm{d}C_t}{C_t}\right)}_{\text{Short-run risk exposure}} = \mathrm{Var}_t\left(\frac{\mathrm{d}W_t}{W_t}\right) - \varphi\,\underbrace{\mathrm{Cov}_t\left(\mathrm{d}R_t^C, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right)}_{\text{Long-run risk exposure}}, \tag{2.27}$$

where the left hand side represents the short-run risk exposure; the right hand side depends on the instantaneous volatility of return on wealth and the long-run risk exposure. Note that the third term on the right hand side of (2.26) is deterministic and therefore drops out from (2.27). As can be noticed from the derivation, the general form of this equation does not rely on whether the model is an endowment or production economy. However, the fact that the model under study is a Cox-Ingersoll-Ross production economy does make a difference: it simplifies (2.27) to the extent that the instantaneous volatility of return on wealth is now a constant. Furthermore, this constant equals the instantaneous volatility of return on physical investment, i.e.,

$$\mathrm{Var}_t\left(\frac{\mathrm{d}W_t}{W_t}\right) = \sigma_K^2, \tag{2.28}$$

simply because the worth of financial wealth and the amount of physical capital are equal to each other in equilibrium.

Equation (2.27) allows us to see how the economy's short-run and long-run exposures to consumption risk change relative to each other when information quality varies. From the preceding subsection, it has been observed that the economy tends to be more positively exposed to long-run risk when information quality gets low. When this happens, the exposure to short-run risk and therefore the required compensation will be reduced if $\varphi > 0$, or equivalently, $\psi > 1$. This observation is in line with what Ai (2010) finds regarding the relation between information quality and the realized volatility of consumption growth. Based on loglinear approximate solutions, he finds low information quality to decrease the realized volatility of consumption growth when $\psi > 1$.

The results so far point us to two aspects of preferences. The attitude towards intertemporal substitution (i.e., whether $\psi > 1$ or $\psi < 1$) determines the direction in which information quality affects the short-run component of risk premium, while the attitude towards temporal resolution of uncertainty (i.e., whether $\varphi > \alpha$ or $\varphi < \alpha$) plays a similar role for the long-run component of risk premium. The ensuing discussion will distinguish scenarios along these two dimensions to understand how the sign of the hedging demand component of risk premium is determined by the interaction of the short-run and long-run risks.

Consider first (i) $\psi < 1$, $\varphi > \alpha$ and (ii) $\psi > 1$, $\varphi < \alpha$. In these cases, information quality affects the short-run and long-run components of risk premium in the same direction: low information quality increases both components in scenario (i), and decreases both components in scenario (ii). The parameter restriction of case (i) is equivalent to $\gamma > 1$, while that of case (ii) is equivalent to $\gamma < 1$. Therefore, in these cases, $\gamma > 1$ is a sufficient condition for noisy information to increase the risk premium.

To examine the remaining cases, I find it useful to express the change of consumption risk premium in terms of its short-run and long-run component sources. Define $\Delta\mathrm{Cov}_t\left(\mathrm{d}R_t^C, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right) \equiv \mathrm{Cov}\left(\mathrm{d}R_t^C, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}|\mathcal{G}_t\right) - \mathrm{Cov}\left(\mathrm{d}R_t^C, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}|\mathcal{F}_t\right)$ as the difference in long-run risk exposure between the economies with full and partial observability. From (2.27), it follows that the difference in short-run risk exposure satisfies

$$\Delta\mathrm{Cov}_t\left(\mathrm{d}R_t^C, \frac{\mathrm{d}C_t}{C_t}\right) = (-\varphi)\Delta\mathrm{Cov}_t\left(\mathrm{d}R_t^C, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right). \tag{2.29}$$

Then, using (2.19), the change of consumption risk premium is represented by

$$\begin{aligned}\Delta(\mu_{R,t}^C - r_t) = & \underbrace{\gamma}_{\text{Price of short-run risk}} \underbrace{(-\varphi)\Delta\mathrm{Cov}_t\left(\mathrm{d}R_t^C, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right)}_{\text{The effect of noisy information on short-run risk exposure}} \\ & + \underbrace{(\varphi - \alpha)}_{\text{Price of long-run risk}} \underbrace{\Delta\mathrm{Cov}_t\left(\mathrm{d}R_t^C, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right)}_{\text{The effect of noisy information on long-run risk exposure}},\end{aligned} \tag{2.30}$$

where the upper and bottom lines of (2.30) capture, respectively, the effect of changing information quality on the short-run and long-run components of consumption risk premium. Collecting terms gives

$$\Delta(\mu^C_{R,t} - r_t) = (\varphi - \alpha - \gamma\varphi)\Delta\mathrm{Cov}_t\left(\mathrm{d}R^C_t, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right). \tag{2.31}$$

Now I use (2.30) and (2.31) to examine the remaining cases: (*iii*) $\psi > 1$, $\varphi > \alpha$ and (*iv*) $\psi < 1$, $\varphi < \alpha$. For these cases, the following result is obtained under the assumption of $\rho = 0$.

Proposition 2. *Suppose that $\rho = 0$. For case (iii) and (iv), lowering information quality affects the short-run and long-run components of risk premium in opposite directions. Furthermore, if $\gamma > 1$, the risk positively affected has sufficiently high price relative to the risk negatively affected, and the risk premium is therefore increased by noisy information.*

To obtain this result, let us start with case (*iii*). Note that, under the assumption of $\rho = 0$, $\Delta\mathrm{Cov}_t\left(\mathrm{d}R^C_t, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right) = \frac{\psi}{\alpha}\frac{H'(m_t)}{H(m_t)}Q > 0$. As can be seen from (2.30), this implies that introducing noisy information affects the short-run and long-run risk exposures in opposite directions. A higher overall risk premium will result only if the effect associated with long-run risk dominates. For this to happen, (2.31) suggests that the critical condition is $\varphi - \alpha > \gamma\varphi$, where $\varphi > 0$. This condition is intuitive: the price of long-run risk, which is the left hand side is the price of long-run risk, must be sufficiently high relative to that of short-run risk, which is positively related to the right hand side. Furthermore, this condition is equivalent to $\gamma > 1$.

Consider next the case (*iv*) $\psi < 1$, $\varphi < \alpha$. In this case, since $\varphi < 0$, introducing noisy information amplifies both the short-run and long-run exposures. The increased exposure to long-run risk, however, has a negative effect on the risk premium, since agents prefer late resolution of uncertainty. In this situation, for the risk premium to increase, the effect associated with short-run risk must be sufficiently large. (2.31) says that the required condition is the same as in the previous case, $\varphi - \alpha - \gamma\varphi$, which can be rewritten as $\gamma > \frac{\alpha - \varphi}{-\varphi}$. This condition now has a different interpretation: it says that, for the positive effect of short-run risk to dominate the negative effect of long-run risk, the short-run risk must have sufficiently high price. It is easy to see that this condition is equivalent to $\gamma > 1$.

For the more general case $\rho \neq 0$, the conclusion of Proposition 2 may still hold, provided that $\Delta\mathrm{Cov}_t\left(\mathrm{d}R^C_t, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right) > 0$. This is indeed the case, as is seen from Table 2.4. For a range of values of ψ and ρ, the long-run risk exposure is shown to be positively affected by noisy information.

Essentially, this subsection reinterprets the relation between information quality and the risk premium from the perspective of intertemporal structure of risk. This allows us to see clearly how the risk effect of information quality stems from relative changes in the short-run and long-run risk factors.

2.3.5 Extending the Analysis to a Dividend Claim

So far the analysis has been focused on the aggregate consumption claim. Now I assess the extent to which the implications obtained for this asset carry over to a dividend claim. As in Abel (1999), Bansal and Yaron (2004) and Ai (2010), the dividend claim is modeled as a levered claim on aggregate consumption. Specifically, dividend growth is assumed to evolve as

$$\frac{\mathrm{d}D_t}{D_t} = \phi \frac{\mathrm{d}C_t}{C_t} - A\mathrm{d}t + \sigma_D \mathrm{d}B_{D,t}, \tag{2.32}$$

where the parameter ϕ captures the idea of leverage; the Brownian motion $B_{D,t}$ is assumed to be independent from other shocks in the economy.[7].

In the appendix, I characterize the price-dividend ratio, $\Gamma(m_t;\sigma_e) \equiv S_t/D_t$, as the solution to a second-order ODE. Given the price-dividend ratio, the equity risk premium is derived as

$$\mu^S_{R,t} - r_t = \gamma \mathrm{Cov}_t\left(\mathrm{d}R^S_t, \frac{\mathrm{d}C_t}{C_t}\right) + (\varphi - \alpha)\mathrm{Cov}_t\left(\mathrm{d}R^S_t, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right), \tag{2.33}$$

where the first and second covariance represent, respectively, the short-run and long-run exposure to risk given by

$$\mathrm{Cov}_t\left(\mathrm{d}R^S_t, \frac{\mathrm{d}C_t}{C_t}\right) = \phi\sigma^2_K + (\rho\sigma_\theta\sigma_K + Q)v(m_t) - \varphi\mathrm{Cov}_t\left(\mathrm{d}R^S_t, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right), \tag{2.34}$$

$$\mathrm{Cov}_t\left(\mathrm{d}R^S_t, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right) = \frac{\psi}{\alpha}\frac{H'(m_t)}{H(m_t)}\left[\phi(\rho\sigma_\theta\sigma_K + Q) + v(m_t)\sigma^2_m\right]. \tag{2.35}$$

Note that $v(m_t)$ is given by $\frac{\Gamma'(m_t)}{\Gamma(m_t)} - \frac{\phi\varphi\psi}{\alpha}\frac{H'(m_t)}{H(m_t)}$, where $\Gamma(m_t)$ satisfies a second-order ODE given in the appendix.

[7] This assumption is also maintained by Ai (2010).

From equation (2.34), the short-run risk exposure depends on the long-run risk exposure as well as the conditional volatility of equity return, where the latter is given by

$$\mathrm{Var}_t\left(\frac{\mathrm{d}S_t}{S_t}\right) = \phi\sigma_K^2 + (\rho\sigma_\theta\sigma_K + Q)v(m_t). \tag{2.36}$$

Note that, unlike its counterpart for the aggregate consumption claim (see (2.28)), this quantity is time-varying and dependent on information quality, so it is difficult to analytically determine how the short-run and long-run exposures change relative to each other when information quality varies. Also, note that even in the case of $\rho = 0$, equation (2.35) does not yield a closed form representation for the effect of information quality on the long-run risk exposure. For these reasons, I will rely entirely on numerical analysis to understand the role of EIS for the equity claim.

The parameters governing the dynamic of dividend growth are specified as in Ai (2010). They are chosen as $\phi = 2.05$, $A = 0.034$, $\sigma_D = 0.11$. Results from the analysis are reported in Tables 2.5, 2.6 and 2.7. These results are consistent with those obtained for the aggregate consumption claim. Table 2.5 shows that the long-run component of equity risk premium and changes in information quality are positively related when agents prefer late resolution of uncertainty (i.e., $\psi < \frac{1}{\gamma}$), and negatively related when agents prefer early resolution of uncertainty (i.e., $\psi > \frac{1}{\gamma}$). In contrast, Table 2.6 shows that the overall equity risk premium is consistently increased by incomplete information, independent of the EIS and the attitude towards temporal resolution of uncertainty. Noting that risk aversion is set to $\gamma = 2$, the results in Table 2.6 accord well with those concerning the relation between information quality and the risk premium of aggregate consumption claim. Finally, Table 2.7 shows that the EIS matters quantitatively for how information quality affects the long-run risk exposure. In particular, the size of the effect of information quality on the long-run component of equity risk premium tends to increase with the EIS, in line with the results documented for the aggregate consumption claim. According to this analysis, whether the asset under study is a levered or unlevered claim on aggregate consumption has no crucial effects on how the EIS and the attitude towards temporal resolution of uncertainty determine the effect of information quality on long-run risk.

2.4 Conclusion

I have revisited the role of preferences in an incomplete-information long-run risk model that Ai (2010) examines. By separating the short-run and long-run components of risk premium, I highlight two aspects of preferences that have an important influence on how information quality affects long-run risk. First, the attitude towards temporal resolution of uncertainty controls the direction of the effect of information quality on the long-run component of risk premium. Low information quality tends to be associated with large premium for long-run risk if agents prefer early resolution of uncertainty. Second, the EIS tends to be important in the determination of the amplitude of the effect of information quality on the long-run risk exposures. The larger the EIS, the more the economy is (positively) exposed to long-run risk when information quality gets low. These results complement those of Ai and help to further understand the relationship between information quality and long-run risk.

Table 2.1: Parameter Values

Parameter	Description	Value
β	Subjective discount rate	0.014
γ	Risk aversion	2
$\bar{\theta}$	Long-run mean of productivity	0.035
κ	Persistence of productivity shocks	0.027
σ_θ	Volatility of productivity shocks	0.005
σ_K	Volatility of output shocks	0.099
σ_e	Information quality	$\{0,\infty\}$
ρ	Correlation of shocks	{-0.7, -0.4, -0.1, 0.1, 0.4, 0.7}
ψ	Elasticity of intertemporal substitution	{0.1, 0.3, 0.6, 0.8, 1.5, 2}

Table 2.2: Information Quality and the Long-Run Component of Consumption Risk Premium

	$\rho=-0.7$		$\rho=-0.4$		$\rho=-0.1$		$\rho=0.1$		$\rho=0.4$		$\rho=0.7$	
EIS	$\sigma_e=0$	$\sigma_e=\infty$	$\sigma_e=0$	$\sigma_e=\infty$	$\sigma_e=0$	$\sigma_e=\infty$	$\sigma_e=0$	$\sigma_e=\infty$	$\sigma_e=0$	$\sigma_e=\infty$	$\sigma_e=0$	$\sigma_e=\infty$
$\psi=0.1$	0.59	-0.18	0.37	-0.38	0.11	-0.61	-0.12	-0.79	-0.57	-1.10	-1.00	-1.26
$\psi=0.3$	0.30	-0.09	0.19	-0.19	0.05	-0.29	-0.06	-0.37	-0.26	-0.51	-0.49	-0.62
$\psi=0.6$	-0.16	0.05	-0.09	0.10	-0.02	0.14	0.03	0.17	0.11	0.21	0.21	0.26
$\psi=0.8$	-0.49	0.14	-0.29	0.29	-0.07	0.41	0.07	0.49	0.31	0.60	0.55	0.70
$\psi=1.5$	-1.84	0.50	-1.00	0.94	-0.24	1.28	0.23	1.47	0.90	1.71	1.52	1.91
$\psi=2.0$	-3.16	0.81	-1.60	1.44	-0.37	1.90	0.35	2.14	1.31	2.41	2.14	2.68

Notes - This table displays the effect of incomplete information on the long-run component of consumption risk premium (i.e., $(\varphi-\alpha)\mathrm{Cov}_t(\mathrm{d}R_t^C,\varphi^{-1}\frac{\mathrm{d}x_t}{x_t})$) for general correlation structure of shocks $\rho\neq 0$. The results are obtained by simulating the continuous-time model and then time-aggregating to an annual frequency. All entries are in percentage. The reported values are the average of 5000 simulations.

Table 2.3: Information Quality and the Consumption Risk Premium under General Correlation Structure of Shocks

	$\rho = -0.7$		$\rho = -0.4$		$\rho = -0.1$		$\rho = 0.1$		$\rho = 0.4$		$\rho = 0.7$	
EIS	$\sigma_e = 0$	$\sigma_e = \infty$	$\sigma_e = 0$	$\sigma_e = \infty$	$\sigma_e = 0$	$\sigma_e = \infty$	$\sigma_e = 0$	$\sigma_e = \infty$	$\sigma_e = 0$	$\sigma_e = \infty$	$\sigma_e = 0$	$\sigma_e = \infty$
$\psi = 0.1$	1.23	2.18	1.50	2.44	1.83	2.72	2.11	2.95	2.68	3.34	3.20	3.54
$\psi = 0.3$	1.21	2.19	1.50	2.44	1.83	2.69	2.10	2.88	2.61	3.24	3.18	3.51
$\psi = 0.6$	1.17	2.19	1.49	2.44	1.84	2.66	2.09	2.81	2.51	3.03	2.99	3.28
$\psi = 0.8$	1.14	2.20	1.48	2.44	1.84	2.64	2.08	2.77	2.47	2.95	2.88	3.13
$\psi = 1.5$	1.04	2.21	1.46	2.43	1.84	2.60	2.08	2.69	2.41	2.81	2.72	2.92
$\psi = 2.0$	0.91	2.23	1.27	2.41	1.85	2.59	2.08	2.67	2.40	2.76	2.68	2.85

Notes - This table displays the effect of information quality on the consumption risk premium for general correlation structure of shocks $\rho \neq 0$. The results are obtained by simulating the continuous-time model and then time-aggregating to an annual frequency. All entries are in percentage. The reported values are the average of 5000 simulations.

Table 2.4: EIS and the Informational Effect on the Long-Run Risk Exposure of the Aggregate Consumption Claim

	$\rho = -0.7$	$\rho = -0.4$	$\rho = -0.1$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.7$
$\psi = 0.1$	0.10	0.09	0.09	0.08	0.07	0.03
$\psi = 0.3$	0.29	0.29	0.26	0.23	0.19	0.10
$\psi = 0.6$	0.63	0.57	0.48	0.42	0.30	0.15
$\psi = 0.8$	0.84	0.77	0.64	0.56	0.39	0.20
$\psi = 1.5$	1.76	1.46	1.14	0.93	0.61	0.29
$\psi = 2.0$	2.65	2.03	1.51	1.19	0.73	0.41

Notes - This table displays the incremental change in the long-run risk exposure generated by the change in information quality. The long-run risk exposure is defined by $\mathrm{Cov}_t\left(\mathrm{d}R_t^C, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right)$. The results are obtained by simulating the continuous-time model and then time-aggregating to an annual frequency. The reported values are the average of 5000 simulations.

Table 2.5: Information Quality and the Equity Risk Premium

	$\rho = -0.7$		$\rho = -0.4$		$\rho = -0.1$		$\rho = 0.1$		$\rho = 0.4$		$\rho = 0.7$	
EIS	$\sigma_e = 0$	$\sigma_e = \infty$	$\sigma_e = 0$	$\sigma_e = \infty$	$\sigma_e = 0$	$\sigma_e = \infty$	$\sigma_e = 0$	$\sigma_e = \infty$	$\sigma_e = 0$	$\sigma_e = \infty$	$\sigma_e = 0$	$\sigma_e = \infty$
$\psi = 0.1$	2.06	5.17	3.68	6.67	5.86	8.55	7.97	10.35	12.38	13.77	14.99	15.69
$\psi = 0.3$	2.09	5.10	3.62	6.45	5.49	7.99	7.07	9.25	10.39	11.92	13.58	14.29
$\psi = 0.6$	2.13	4.99	3.51	6.12	5.02	7.25	6.12	8.03	7.95	9.31	10.05	10.77
$\psi = 0.8$	2.14	4.92	3.43	5.90	4.76	6.81	5.66	7.39	7.04	8.25	8.48	9.10
$\psi = 1.5$	2.12	4.65	3.21	5.18	4.12	5.56	4.64	5.77	5.34	6.04	5.93	6.26
$\psi = 2.0$	1.98	4.48	3.13	4.76	3.89	4.93	4.30	5.02	4.79	5.14	5.13	5.29

Notes - This table displays the effect of information quality on the equity risk premium for general correlation structure of shocks $\rho \neq 0$. The results are obtained by simulating the continuous-time model and then time-aggregating to an annual frequency. All entries are in percentage. The reported values are the average of 5000 simulations.

Table 2.6: Information Quality and the Long-Run Component of Equity Risk Premium

	$\rho = -0.7$		$\rho = -0.4$		$\rho = -0.1$		$\rho = 0.1$		$\rho = 0.4$		$\rho = 0.7$	
EIS	$\sigma_e = 0$	$\sigma_e = \infty$	$\sigma_e = 0$	$\sigma_e = \infty$	$\sigma_e = 0$	$\sigma_e = \infty$	$\sigma_e = 0$	$\sigma_e = \infty$	$\sigma_e = 0$	$\sigma_e = \infty$	$\sigma_e = 0$	$\sigma_e = \infty$
$\psi = 0.1$	-0.03	-0.42	-0.75	-1.05	-1.77	-1.91	-2.79	-2.83	-4.55	-4.71	-5.60	-5.84
$\psi = 0.3$	0.05	-0.21	-0.29	-0.51	-0.71	-0.87	-1.08	-1.18	-1.88	-1.96	-2.53	-2.58
$\psi = 0.6$	-0.07	0.11	0.08	0.24	0.25	0.38	0.37	0.48	0.58	0.66	0.83	0.87
$\psi = 0.8$	-0.33	0.32	0.10	0.69	0.56	1.06	0.87	1.30	1.36	1.67	1.88	2.04
$\psi = 1.5$	-2.25	1.05	-0.94	2.00	0.32	2.73	1.14	3.14	2.32	3.67	3.44	4.12
$\psi = 2.0$	-3.46	1.62	-1.97	2.82	-0.33	3.62	0.83	4.03	2.51	4.51	4.00	5.01

Notes - This table displays the effect of information quality on the long-run component of equity risk premium for general correlation structure of shocks $\rho \neq 0$. The results are obtained by simulating the continuous-time model and then time-aggregating to an annual frequency. All entries are in percentage. The reported values are the average of 5000 simulations.

Table 2.7: EIS and the Informational Effect on the Long-Run Risk Exposure of the Equity Claim

	$\rho = -0.7$	$\rho = -0.4$	$\rho = -0.1$	$\rho = 0.1$	$\rho = 0.4$	$\rho = 0.7$
$\psi = 0.1$	0.05	0.04	0.02	0.01	0.02	0.03
$\psi = 0.3$	0.20	0.17	0.12	0.08	0.06	0.04
$\psi = 0.6$	0.54	0.48	0.39	0.33	0.24	0.12
$\psi = 0.8$	0.87	0.79	0.67	0.57	0.41	0.21
$\psi = 1.5$	2.48	2.21	1.81	1.50	1.01	0.52
$\psi = 2.0$	3.39	3.19	2.63	2.13	1.33	0.67

Notes - This table displays the incremental change in the long-run risk exposure generated by the change in information quality. The long-run risk exposure is defined by $\mathrm{Cov}_t\left(\mathrm{d}R_t^C, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right)$. The results are obtained by simulating the continuous-time model and then time-aggregating to an annual frequency. The reported values are the average of 5000 simulations.

Figure 2.1: EIS and the Response of Expected Consumption Growth

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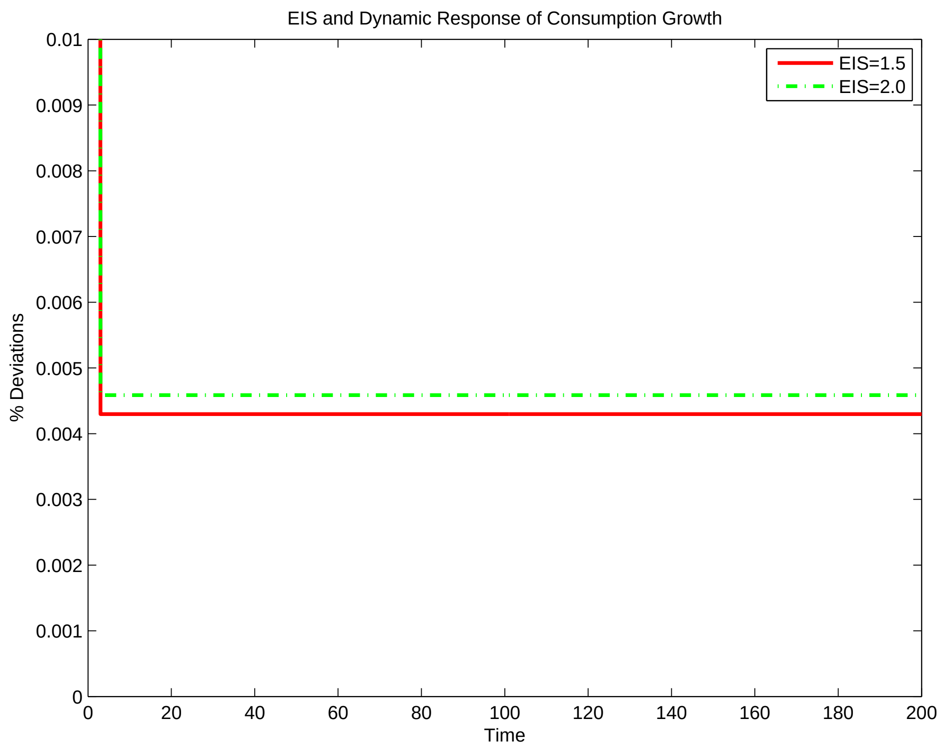


Notes: For different values of EIS, this figure depicts the response of expected consumption growth to a positive one standard deviation shock to productivity. The other parameter values used to obtain the responses are $\beta = 0.014$, $\gamma = 2$, $\bar{\theta} = 0.035$, $\kappa = 0.027$, $\sigma_\theta = 0.005$, $\sigma_K = 0.099$, $\sigma_e = \infty$, $\rho = 0$.

Chapter 3

Incomplete Information and Asset Returns in Real Business Cycle Economies

3.1 Introduction

Financial markets are riddled with incomplete information. Investors face uncertainty about many of the parameters and variables governing economic fundamentals, and they learn about these hidden quantities by observing data. This learning process is fettered by the large amount of randomness permeating financial markets. Estimation errors thus occur inevitably. An extensive literature has sought to understand whether these estimation errors might help to explain aggregate stock market behavior. Most of this literature has focused on endowment economy, including, for example, Brennan and Xia (2001), who adopt the standard expected utility framework, and Calvet and Fisher (2007), Bansal and Shaliastovich (2008a), Bansal and Shaliastovich (2008b) and Croce et al. (2010), who develop models with Epstein-Zin utility. Remarkably, these studies find incomplete information to be important in explaining a wide variety of asset market phenomena, including the equity premium puzzle, the risk-free rate puzzle, volatility and higher moments of stock returns, asset price jumps, the effect of idiosyncratic volatility on returns and cross-sectional stock return properties.[8]

These results are encouraging, but relying on exogenous, reduced-form specifications of aggregate quantities is somewhat unsatisfying. Indeed, aggregate quantities are inextricably linked to asset prices, for the latter indicate investment value and thereby guide the allocation of capital across firms and the arrangement of consumption and investment across time and states

[8] Contributions to the study of asset prices in partially observable endowment economies also include, but are not limited to, Wang (1993), Veronesi (1999, 2000), Brandt et al. (2004), Li (2005), Brevik and d'Addona (2009), Hansen and Sargent (2010) and Ju and Miao (2010). Some partial equilibrium analysis are found in Merton (1987), Barsky and DeLong (1993), Timmermann (1993) and David and Veronesi (2009, 2011).

of nature. By implication, the effects of incomplete information impinging on asset prices necessarily have their impacts on aggregate quantities. Therefore, the ultimate coherence and success of incomplete information based solutions hinge not only on their ability to explain the behavior of asset prices, but also on their consistency with the regularities in real economic activity. The inseparable linkage between asset prices and the macroeconomy thus provides a compelling motivation to extend the investigation from endowment economy setting to general equilibrium model with production. Ai (2010) takes a step in this direction.[9] Working in the framework of Cox et al. (1985) (CIR) augmented with recursive preferences, he shows that incomplete information (about productivity) can help to produce an adequate fit not only for the moments of aggregate asset prices, such as the equity premium, the risk-free rate and the price-dividend ratio, but also for the moments of aggregate consumption, such as the volatility and autocorrelation of consumption growth.

While this is a notable success, the CIR model Ai employs is not the standard framework for quantitatively uniting general equilibrium macroeconomics and asset pricing. In this endeavor the real business cycle (RBC) model is most commonly used, because it has enjoyed a measure of success in replicating important features of macroeconomic time series.[10] The CIR model, on the other hand, falls short along this dimension. An example is provided in Table 3.2, showing that the CIR model, when made consistent with salient features of consumption and asset prices as in Ai (2010), predicts too large an investment-capital ratio and too much variability in the growth rates of output and investment. Compared with the actual U.S. economy between 1929 and 1998, the investment-capital ratio is more than 50 times larger, and the growth rates of output and investment are, respectively, 5 times and 20 times more volatile.

In this paper, I use a standard real business cycle framework to study the implications of incomplete information. The model economy is subject to stochastic disturbances in productivity that have independent transitory and permanent components. Agents observe productivity levels but cannot distinguish between transitory and permanent changes as they occur. Consequently, they must use all available information to form optimal forecasts.

[9] Early analysis of production economies under incomplete information include Detemple (1986), Dothan and Feldman (1986), Feldman (1989) and David (1997). However, their attention was not placed on the real and financial linkage.

[10] The RBC model has been extensively used in the asset pricing literature. Important examples include, among others, Rouwenhorst (1995), Jermann (1998), Tallarini (2000), Boldrin et al. (2001) and Guvenen (2009).

Preferences are identical across agents, which have the recursive formulation proposed by Epstein and Zin (1989). Now commonly used in asset pricing, such preferences not only give a convenient way to separate risk aversion and the elasticity of intertemporal substitution, but also accommodate the idea that individuals may be concerned with temporal resolution of uncertainty. As a result of the second feature, the temporal distribution of risk matters to the decision maker. In particular, if agents prefer early resolution of uncertainty, as assumed in this paper, they will exhibit aversion to long-run risk.

This model setup takes minimal necessary deviations from the standard paradigm of real business cycles, thereby providing a clear benchmark for the question of interest. However, there is a cost of such simplicity − in the model endogenous dividends are less procyclical than consumption and may even be countercyclical. This is a problem endemic to other standard models as well, such as Kaltenbrunner and Lochstoer (2010) and Gourio (2010). In this paper, instead of searching for a remedy, I simply choose to work with a dividend stream that is calibrated to the historical moments of aggregate dividends. Such compromise in modeling cash flows is certainly not satisfactory. But still, important insights into the implications of incomplete information can be gathered from the stochastic discount factor channel.

Using a parametric version of this model, with parameters chosen to match several features of aggregate quantities, I study the extent to which it is consistent with the empirical facts about asset prices. For key moments of asset prices, such as the equity premium, the risk-free rate and the price-dividend ratio, this model matches the data closely. But interestingly, this outcome comes predominantly from permanent shocks instead of from incomplete information. This observation is in the same spirit as those made by Kaltenbrunner and Lochstoer (2010) and Croce (2010), suggesting the importance of permanent shocks for asset prices. However, despite being less of a factor for quantitative implications, incomplete information has a crucial role to play in a qualitative aspect. Absent this element, the model predicts that short-term dividend strips have smaller risk premia than long-term dividend strips, contrary to the empirical evidence that the term structure of equity risk is downward sloping.[11] This problem is resolved when incomplete information is added. This result is worth noting for two reasons. First, the term structure evidence of equity risk has posed

[11]For the empirical evidence on the term structure of equity risk, see, e.g., Cornell (1999, 2000), Dechow et al. (2004), Da (2009), and Binsbergen, Brandt and Koijen (2010).

a challenge to several leading models of asset pricing, such as the external habit formation model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), and the variable rare disasters model of Gabaix (2010).[12] Understanding what model structures can account for this evidence is an important step in furthering our knowledge about how aggregate asset prices are determined. Second, the explanation presented in this paper is coherent, in the sense that it conforms to some key features of the macroeconomy. Of course, this paper is not the first attempt to get at this issue. The novelty here rests in the particular mechanism that I will elucidate.

The mechanism can be understood in terms of the consumption dynamics. For Epstein-Zin preferences, the temporal distribution of risk matters, and as a result the equilibrium asset prices reflect not only consumption fluctuations at short horizons but also those at long horizons. In this economy, because agents are assumed to desire early resolution of uncertainty, the equity risk premium depends positively on both types of fluctuations. The term structure of equity risk is thus also tied to the consumption dynamics − if short-term fluctuations in consumption play a much larger role than long-term fluctuations, the term structure of equity risk can be downward sloping. This linkage underlies the implication of incomplete information for the term structure of equity risk. The following discussion will delve into the intuition.

The natural point to start is the benchmark case of complete information, where the transitory and permanent shocks to productivity are separately observable. As is well known, consumption adjusts to these shocks in very different ways. With a temporary rise in productivity, consumption increases in the short run and then declines back to the steady state as the productivity gain diminishes over time. By contrast, when the productivity rise is permanent, consumption not only increases in the initial period but continues to ramp up over time. Since agents in this economy desire early resolution of uncertainty, the long-run impact of permanent shocks is especially onerous (e.g., Kaltenbrunner and Lochstoer (2010); Croce (2010)). Accordingly they require higher risk premia for assets with larger exposure to such impacts − precisely those assets whose cash flows are weighted more towards the far future. This leads to an upward slope for the term structure of equity risk.

Against this benchmark, the intuition for the effect of incomplete infor-

[12] See Binsbergen, Brandt and Koijen (2010) for comparison of these models to the term structure evidence.

mation becomes clear. Deviating from complete information means that, whenever the agents observe an increase (decrease) in productivity, they revise upward (downward) their beliefs for both the transitory and permanent shocks. In the short run, these two types of shocks work in the same direction for consumption, and as a result they reinforce each other. Over time, as the transitory and permanent shocks drive consumption in opposite directions, their positive correlation makes them offset each other's impact on long-term consumption. These effects on short-term and long-term consumption combine to make consumption significantly more volatile in the short term than in the long term, thereby skewing the temporal distribution of risk towards the short end. In turn this causes a downward slope for the term structure of equity risk.

This mechanism is based on the fact that transitory and permanent shocks to productivity have opposing long-run impacts on consumption. The implication of this fact for asset prices has been considered by Kaltenbrunner and Lochstoer (2010) and others in models featuring recursive preferences and complete information. A lesson learned there is that permanent shocks are more important than transitory shocks as a source of asset market fluctuations. This seems to warrant asset pricing studies to focus exclusively on disturbances with permanent effects. However, by analyzing an economy with incomplete information, this paper highlights that transitory shocks have a role to play in shaping the term structure of equity risk. I believe this observation merits special attention.

There are other attempts to match the term structure of equity risk. Lettau and Wachter (2007, 2011) propose reduced-form models that generate higher risk premia for short-term dividend strips than for long-term dividend strips. Croce et al. (2010) also provide an information-based general equilibrium explanation but their model is an endowment economy. Moreover, they rely on a different mechanism which critically depends on the obscuring effect of incomplete information. In their economy of incomplete information, small long-run shocks to the growth rates of consumption and dividends are substantially overshadowed by large short-run shocks. This effect is strong enough that the process for dividend growth appears close to i.i.d. under signal extraction. For this reason short-term dividend strips are deemed more risky than their long-term counterparts.

This paper is part of an expanding research effort to construct models that jointly match the stylized facts of macroeconomic variables and aggregate asset prices. Based upon the real business cycle framework, considerable progress has been made in this area by way of imparting deviations from rational expectations (e.g., Cagetti et al. (2002)) or enriching the standard model with sophisticated preferences, real and financial frictions, as well as sources of risk that are potentially of large concerns for investors (e.g., Jermann (1998); Boldrin et al. (2001); Tallarini (2000); Kaltenbrunner and Lochstoer (2010); Kuehn (2008); Gourio (2010); Guvenen (2009)). Complementary to these prior studies, this paper undertakes an investigation into a situation with incomplete information. This informational friction combined with Epstein-Zin preferences makes some progress towards understanding the term structure of equity risk, while also maintaining consistency with some key moments of asset prices and the macroeconomy. A related work by Ai et al. (2010) is able to match a similar set of asset pricing facts also in a model of production economy. Their mechanism is nevertheless entirely different, based on complete information and the distinction between physical and intangible capital.

The remainder of the paper is organized as follows. The next section contains a formal description of the model and details the filtering and control problem faced by the representative agent. Section 3 describes the choice of parameter values and examines the implications for various moments of aggregate asset prices and aggregate quantities. Section 4 explores the implications for the term structure of equity risk and illustrates the mechanism through which these implications arise. Section 5 concludes.

3.2 The Model

This section outlines a standard real business cycle model with both transitory and permanent shocks to productivity. These shocks are not separately observable, and as a result agents must make inferences about the productivity state based on historical observations. I use this model to study the implications of incomplete information. The key elements of the model are described below.

3.2.1 Preferences and the Stochastic Discount Factor

The representative agent in this economy has preferences of the kind proposed by Epstein and Zin (1989)

$$U_t = \left[(1-\beta)C_t^{\varphi} + \beta(\mathbb{E}_t[U_{t+1}^{\alpha}])^{\varphi/\alpha}\right]^{1/\varphi}, \tag{3.1}$$

where C_t is the level of consumption in period t, β is the time discount factor, the random variable U_{t+1} is the continuation value of a consumption plan from period $t+1$ onwards, and $\mathbb{E}_t[\cdot]$ is the conditional expectation operator. These preferences generalize power utility so that the elasticity of intertemporal substitution (EIS) is separated from the coefficient of relative risk aversion (RRA). In the recursion (3.1), $\gamma \equiv 1-\alpha$ is the coefficient of relative risk aversion and $\psi \equiv (1-\varphi)^{-1}$ is the EIS.[13] For these preferences, the timing of uncertainty resolution generally matters to the decision maker. Specifically, if $\varphi > \alpha$, the decision maker prefers uncertainty about the future to be resolved sooner rather than later; the opposite is true if $\varphi < \alpha$ (Epstein and Zin (1989)).

For the utility function (3.1), Epstein and Zin (1989) show that the stochastic discount factor is given by

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{\varphi-1} \left(\frac{U_{t+1}^{\alpha}}{\mathbb{E}_t[U_{t+1}^{\alpha}]}\right)^{1-\frac{\varphi}{\alpha}}. \tag{3.2}$$

To understand the intuition for this stochastic discount factor, one can rewrite it as[14]

$$m_{t+1} = \frac{\alpha}{\varphi}\log\beta - \gamma\Delta c_{t+1} - (\varphi-\alpha)\,\varphi^{-1}\tilde{r}_{w,t+1}, \tag{3.3}$$

where m_{t+1} is the log stochastic discount factor, and $\tilde{r}_{w,t+1}$ is the "adjusted" return on wealth defined as

$$\tilde{r}_{w,t+1} = \log\frac{W_{t+1}}{C_{t+1}} - \log\left(\frac{W_t}{C_t} - 1\right). \tag{3.4}$$

The formula (3.3) shows that there are two contributions to the stochastic discount factor. One is the contribution of realized consumption growth familiar from Rubinstein (1976), Lucas (1978) and Breeden (1979) model of asset pricing. The other contribution is from the "adjusted" return on wealth $\tilde{r}_{w,t+1}$, which is present only when $\varphi = \alpha$. From the definition of $\tilde{r}_{w,t+1}$, we know that it reflects changes in expected consumption growth, because changes in the log wealth-consumption ratio can be represented as

$$\log\frac{W_{t+1}}{C_{t+1}} - \log\frac{W_t}{C_t} \approx \varphi\left\{\mathbb{E}_{t+1}\left[\sum_{j=1}^{\infty}\varrho^j\Delta c_{t+1+j}\right] - \mathbb{E}_t\left[\sum_{j=1}^{\infty}\varrho^j\Delta c_{t+j}\right]\right\}, \tag{3.5}$$

[13] The class of recursive preference structure is originally suggested by Kreps and Porteus (1978). Epstein and Zin (1989) extend their work to stationary infinite-horizon settings and propose a more general class of risk preferences. Weil (1989) also suggests and applies a constant elasticity version to asset pricing.

[14] This is obtained by first substituting out future utility from (3.2) with the return on wealth r_w (see Epstein and Zin (1991)), then replacing r_w with the "adjusted" return on wealth $\tilde{r}_w$, where $\tilde{r}_w = r_w - \Delta c$.

where ϱ is a constant given by $\varrho = 1 - \exp(\overline{c-w})$, with $\overline{c-w}$ denoting the unconditional log consumption-wealth ratio (see, e.g., Campbell (1999)). Then it follows that $\varphi^{-1}\tilde{r}_{w,t+1}$ moves in the same direction as expected consumption growth. Therefore, the second contribution to the stochastic discount factor comes from the impact of expected consumption growth. When there is a negative shock to expected consumption growth, the stochastic discount factor will rise if $\varphi > \alpha$, reflecting the fear of information-loving agents for such events. The implication is reversed if $\varphi < \alpha$. Thus, the attitude towards the timing of uncertainty resolution translates into concerns about the temporal distribution of risk.

3.2.2 Production and Capital Accumulation

There is a single final good in this economy and it is produced according to a constant-returns-to-scale neoclassical production technology. In each period, the output produced with K_t units of capital and N_t hours of labor is given by

$$Y_t = Z_t(X_t N_t)^{\nu} K_t^{1-\nu}, \tag{3.6}$$

where $\nu \in (0,1)$ is the labor's share of output, and Z_t and X_t are random variables concerning productivity which will be described in more detail below. In this economy, because utility is derived only from consumption, it is optimal for agents to allocate their entire endowment of time to productive work. The endowment of time is normalized to one so that $N_t = 1$ for all t.

To aid in exposition, define the total factor productivity A_t as[15]

$$A_t \equiv Z_t X_t^{\nu}. \tag{3.7}$$

I permit temporary changes in total factor productivity through Z_t. In particular, $z_t \equiv \ln Z_t$ is assumed to follow a first-order autoregressive process

$$z_t = \rho_z z_{t-1} + \varepsilon_t, \tag{3.8}$$

where $|\rho_z| \in (0,1)$ and ε_t is i.i.d. normally distributed with mean zero and variance σ_ε^2. Permanent technological shifts are restricted to be in labor productivity X_t, which ensures consistency with balanced growth (see King et al. (1988)). Assume that X_t has the dynamics

$$X_t = e^{\theta_t} X_{t-1}, \tag{3.9}$$

[15]This follows from rewriting the production function as $Y_t = Z_t X_t^{\nu} N_t^{\nu} K_t^{1-\nu}$.

where θ_t is the stochastic growth rate of X. The law of motion for θ_t obeys

$$\theta_t = (1 - \rho_\theta)\mu_\theta + \rho_\theta \theta_{t-1} + \xi_t, \tag{3.10}$$

where $|\rho_\theta| \in (0, 1)$, μ_θ is the long-run mean of θ and ξ_t is i.i.d. normally distributed with mean zero and variance σ_ξ^2. In addition, ξ_t is assumed to be uncorrelated with ε_t. Given (3.7) and (3.9), the growth rate of total factor productivity is

$$g_t \equiv \ln\left(\frac{A_t}{A_{t-1}}\right) = \nu\theta_t + z_t - z_{t-1}. \tag{3.11}$$

Compared to those considered in the existing asset pricing literature, the production function (3.6) is more general, allowing for both temporary and permanent shocks to productivity.[16] The motivation for this choice is to accommodate the possibility that output may be affected by more than one type of disturbance.

Agents in this economy may either consume the final good or invest it to accumulate capital. Following the adjustment costs literature, I assume a nonlinear evolution for how investment is converted into capital:

$$K_{t+1} = (1 - \delta)K_t + G\left(\frac{I_t}{K_t}\right)K_t, \tag{3.12}$$

where $\delta \in [0, 1)$ denotes the depreciation rate of capital, I_t is investment in period t and $G(\cdot)$ is a positive, concave function. The function $G(\cdot)$ accounts for the presence of adjustment costs in capital accumulation. As in Jermann (1998), $G(\cdot)$ takes the form

$$G\left(\frac{I_t}{K_t}\right) = \frac{a_1}{1 - 1/\eta}\left(\frac{I_t}{K_t}\right)^{1-1/\eta} + a_2, \tag{3.13}$$

where a_1, a_2 are constants, and the curvature parameter η determines the severity of adjustment costs. The constants a_1 and a_2 are chosen so that the steady state of the model does not depend on the parameter η.

Finally, agents face resource constraint in each period which requires the total use of the final good not to exceed output:

$$C_t + I_t \leqslant Y_t. \tag{3.14}$$

[16] For example, Jermann (1998), Boldrin et al. (2001), Guvenen (2009), Kaltenbrunner and Lochstoer (2010) restrict productivity shocks to be either temporary or permanent.

3.2.3 Information Structure and the Decision Problem

To stipulate an information structure, I assume that the economy has been in progress long enough for the agent to learn the parameters and structure of the economy. However, the agent cannot distinguish between the transitory and permanent shocks. Information about the shocks is communicated to him through the observations on total factor productivity. The decision problem thus contains an element of learning. Furthermore, because of the separation between estimation and control in this economy, the decision problem can be decomposed into two pieces: an estimation problem in which the agent infers the composition of productivity shocks using all information available; and a control problem in which he chooses optimal consumption and investment plans based on his current estimate.[17] I begin with the estimation problem.

At time zero the agent is assumed to have a joint normal prior distribution over z and θ. As time goes by, z and θ evolve according to (3.8) and (3.10), respectively, and the agent updates his beliefs in a fully Bayesian fashion using the historical observations on total factor productivity (i.e., $\mathcal{F}_t \equiv \{A_{t-s}\}_{s=0}^{\infty}$). To derive the agent's beliefs, it is convenient to work with a state space representation of the information structure. This representation consists of measurement and transition equations. The measurement equation describes the relationship between the observed growth rate of total factor productivity g and the unobserved variables θ and z. Let π_t be a vector containing θ_t, z_t and z_{t-1}, i.e., $\pi_t \equiv [\theta_t, z_t, z_{t-1}]^\top$. Then the measurement equation is just a vector reformulation of equation (3.11):

$$g_t = B\pi_t, \tag{3.15}$$

where $B \equiv [\nu, 1, -1]$. The transition equation summarizes the evolution of the unobserved variables, which is a vector autoregressive process with independent multivariate normal innovations:

$$\pi_{t+1} = \Phi\pi_t + \Gamma + Hu_{t+1}, \tag{3.16}$$

where

$$\Phi \equiv \begin{bmatrix} \rho_\theta & 0 & 0 \\ 0 & \rho_z & 0 \\ 0 & 1 & 0 \end{bmatrix}, \qquad H \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$\Gamma \equiv \left[(1-\rho_\theta)\mu_\theta, 0, 0\right]^\top, \quad u_{t+1} \equiv [\xi_{t+1}, \varepsilon_{t+1}]^\top.$$

[17] See Detemple (1986) and Gennotte (1986) for the discussion of the separation principle in dynamic equilibrium asset pricing models.

Then, deriving the optimal estimates for productivity shocks is a straightforward application of the Kalman filter. Let m_t be the expected value of π_t, i.e., $m_t = \left[\tilde{\theta}_t, \tilde{z}_t, \tilde{z}_{t-1}\right]^\top$, and Σ_t be the prediction error covariance of π_{t+1} conditional upon the information set $\mathcal{F}_t$. In this setting, the covariance matrix Σ_t converges to a steady state Σ which satisfies the following algebraic Riccati equation (see, e.g., Harvey (1989)):

$$\Sigma = \Phi\Sigma\Phi^\top - \Phi\Sigma B^\top (B\Sigma B^\top)^{-1} B\Sigma\Phi^\top + HQH^\top, \tag{3.17}$$

where

$$Q \equiv \begin{bmatrix} \sigma_\xi^2 & 0 \\ 0 & \sigma_\varepsilon^2 \end{bmatrix}.$$

Then by the Kalman filter, the estimate m_t evolves over time according to

$$m_{t+1} = \Phi m_t + \left[\Omega - \Sigma B^\top (B\Sigma B^\top)^{-1} B\right]\Gamma + \Sigma B^\top (B\Sigma B^\top)^{-1} \tilde{u}_{t+1}, \tag{3.18}$$

where Ω is a 3×3 identity matrix, and $\tilde{u}_{t+1} \equiv g_{t+1} - B\Phi m_t$ is i.i.d. normally distributed with mean zero and variance $B\Sigma B^\top$.

Coming to the control problem, we see that the state of the economy is characterized by the capital and productivity levels, K_t and A_t, and the agent's assessment of the transitory and permanent shocks, as measured by $\tilde{z}_t$ and $\tilde{\theta}_t$. To determine the equilibrium consumption, I operate directly on the planner's problem which is expressed as

$$V(K_t, A_t, \tilde{\theta}_t, \tilde{z}_t) = \max_{C_t} \left\{ (1-\beta) C_t^\varphi + \beta \mathbb{E}\left[V(K_{t+1}, A_{t+1}, \tilde{\theta}_{t+1}, \tilde{z}_{t+1})^\alpha \middle| \mathcal{F}_t \right]^{\varphi/\alpha} \right\}^{1/\varphi} \tag{3.19}$$

subject to (3.12), (3.14) and (3.18). This problem is non-stationary because other variables in the economy inherit the trend of productivity process. To make it amenable to analysis by dynamic programming methods, in the appendix I transform the problem into one that is stationary in scaled variables.

3.2.4 Financial Assets

To close the model, it only remains to introduce financial assets. There are two assets in this economy: a risk-free asset and an equity. The risk-free asset is a one-period lived asset that delivers one unit of consumption in all states of nature. Its gross return is given by

$$R_{f,t+1} = \frac{1}{\mathbb{E}_t[M_{t+1}]}. \tag{3.20}$$

The equity represents a claim to aggregate stock market dividends. Similar to Kaltenbrunner and Lochstoer (2010), log dividend growth, $\Delta d_{t+1} \equiv \log \frac{D_{t+1}}{D_t}$, is specified as

$$\Delta d_{t+1} = \mu_d + \lambda_1(ac_t - \overline{ac}) + \lambda_2 \tilde{u}_{t+1} + \sigma_d \zeta_{t+1}, \tag{3.21}$$

where $\zeta_{t+1} \sim \mathcal{N}(0,1)$ is i.i.d. and independent of the productivity shocks; ac_t is log productivity-consumption ratio, i.e., $ac_t \equiv \log \frac{A_t}{C_t}$, with mean $\overline{ac}$. Since ac_t predicts future consumption growth, λ_1 captures common predictability of consumption and dividends;[18] λ_2 captures systematic shocks to dividends; σ_d is the idiosyncratic dividend volatility. In the next section these parameters will be chosen in accordance with the historical behavior of stock market dividends.

Given the dividend process, the equity price P_t is computed from the representative agent's stochastic discount factor, M_{t+1}, at equilibrium quantities. That is,

$$P_t = \mathbb{E}_t \left[M_{t+1}(P_{t+1} + D_{t+1}) \right]. \tag{3.22}$$

This works because in the model the equilibrium quantities chosen by the social planner are identical to the competitive outcome.

3.3 Implications for Asset Prices and Aggregate Quantities

In this section I use a simple benchmark calibration to examine the model's implications for asset prices and aggregate quantities. To gain more intuition about the asset pricing implications, I also compare the results with those obtained under complete information.

3.3.1 Calibration

In the asset pricing literature, a common strategy for calibrating the real business cycle model is a two-step procedure (e.g., Jermann (1998), Boldrin et al. (2001)). The first step, which follows the real business cycle tradition, is to restrict some parameters based on evidence from growth observations and micro studies. The second step is to choose the remaining parameter values so that the model comes as close as possible to a set of business

[18] The working paper version of Kaltenbrunner and Lochstoer (2010) contains evidence on the predictive power of productivity-consumption ratio both in the data and in their simulated RBC model (with complete information).

cycle and asset pricing moments. This strategy will be followed here. Assuming that a model period is one month, I calibrate the model to match some moments of annual U.S. data over the period 1929 to 1998. Since the model abstracts from inflation and population growth, the calibration is conducted with respect to real, per capita empirical counterparts. The parameter choices are summarized in Table 3.3.

The model is calibrated at a quarterly frequency. Following Bansal and Yaron (2004), I set the relative risk aversion, γ, at 10, and the elasticity of intertemporal substitution, ψ, at 1.5. But it should be noted that so far there is a fair amount of uncertainty regarding its magnitude. Campbell and Mankiw (1989), Campbell (2003), and Yogo (2004) find the EIS to be small and in many cases statistically indistinguishable from zero, while Attanasio and Weber (1993), Vissing-Jørgensen (2002), and Bansal and Yaron (2004) report estimates of larger than one. The labor's share of output, ν, is set to 0.64 as in Jermann (1998) and Kaltenbrunner and Lochstoer (2010). δ is assumed to be 0.025 to imply a 10% annual capital depreciation rate. The persistence parameters, ρ_z and ρ_θ, and the relative size of shocks, $\sigma_\xi/\sigma_\varepsilon$, are important for the incomplete information economy, because they determine the beliefs attached to temporary and permanent stochastic variations in productivity. I calibrate their values to match reasonably closely the serial correlation structure of U.S. total factor productivity, including the first order autocorrelation of level and growth rate in quarterly data and the first order autocorrelation of growth rate in annual data. The subjective discount factor β, the mean parameter μ, the adjustment cost parameter η, and the standard deviation of temporary shocks σ_ε, are restricted using four moments: the average risk-free rate, the standard deviation of output growth, and the mean and standard deviation of consumption growth. Finally, I select the remaining parameters to produce an empirically-relevant process for dividends.

3.3.2 Macroeconomic and Asset Pricing Moments under the Benchmark Calibration

Table 3.4 reports the simulation results for the benchmark calibration. The table also includes the corresponding empirical moments as a guide to the model fit. For comparison purpose, predictions from Ai (2010)'s incomplete-information CIR model are also presented.

The model has reasonable performance for asset prices. The implied average equity risk premium is 4.79% and equity return volatility is 17.07%, both of which are reasonably close to their empirical counterparts, 6.33%

and 19.42%. This comes as no surprise – just like in Kaltenbrunner and Lochstoer (2010) and Croce (2010), long-run risk arises here as a result of endogenous consumption adjustment to permanent shocks. Finally, because of the assumption of high EIS, my model is able to produce a low risk-free rate volatility of 0.90%, which closely matches the empirical target of 0.97%.

The model also matches macroeconomic variables reasonably well. In the data output growth has a first-order autocorrelation of 0.42 and investment-capital ratio is on average 0.17. The model in this paper predicts 0.35 and 0.11, respectively, for these moments. By contrast, the CIR model in Ai (2010) produces a weaker output growth persistence of 0.21 and a much larger investment-capital ratio which is about 50 times the size of the empirical target. The predictions of my model are quantitatively reasonable not only in that they come close to the data, but also in that they are compatible with the empirical volatility of consumption and output growth. As noted previously, the parameter values are restricted in such a way that consumption and output growth are as volatile in the model as they are in the data. Such a parametric restriction is not easy to impose on the CIR model considered by Ai (2010). Using his calibration, I observe that consumption growth volatility is closely matched but output growth volatility ends up being about 5 times as volatile as the data suggest. Simultaneously matching consumption and output growth volatility is also important from an asset pricing point of view, because it helps to ensure that the implications derived for financial markets are consistent with a realistic extent of consumption smoothing. One dimension along which my model significantly deviates from empirical evidence is investment growth – the implied investment growth fluctuates only about half as much as in the data.[19] Along this dimension Ai's model is also misaligned with evidence. The times series of investment growth it generates is about 20 times more volatile than is actually the case. Overall, it appears that my model has a comparative advantage in matching aggregate quantities, which is only to be expected as the RBC type models are designed to mimic macroeconomic reality.

A reasonable match with the macroeconomic side is important in ensuring more coherent predictions about asset prices. Thus, for a careful examination of the role that incomplete information plays in shaping asset prices, the evidence presented here leans toward the RBC model being the more suitable choice. This by no means downplays the importance of the CIR model which has its own virtues such as analytical tractability and

[19] This problem may be mitigated by using the adjustment costs function introduced by Croce (2010), but such modification is not pursued in this paper.

transparency.

3.3.3 Comparison to Economies with Complete Information

To better understand the role of incomplete information in affecting financial market outcome, I go on to compare the economies with and without incomplete information. Using the benchmark parameter values, Table 3.5 shows that the complete information counterpart is less risky, exhibiting a lower equity premium and a higher risk-free rate. In other words, incomplete information has the effect of increasing asset market risk, consistent with findings from previous studies (e.g., Brennan and Xia (2001), Ai (2010)).

Besides incomplete information, permanent shocks are another driver of asset prices in the model economy, which operate by endogenously inducing long-run consumption risk (e.g., Kaltenbrunner and Lochstoer (2010), Croce (2010)). The more persistent are the impact of permanent shocks, the larger is the resulting long-run risk. To understand the quantitative effect of incomplete information relative to permanent shocks, I consider an experiment in the context of complete information economy, obtained through a perturbation in the persistence of permanent shocks. More specifically, the persistence of permanent shocks in the complete information economy (ρ_θ) is increased from 0.965 to 0.975, while the parameters of the incomplete information economy are left unchanged. Table 3.6 shows that, as a result of this parameter change, the complete information economy now has quantitative implications quite close to those of the incomplete information economy, on both the macroeconomic and asset pricing side.

This similarity in asset pricing predictions is due to a familiar property of asset pricing models featuring Epstein-Zin preferences. For Epstein-Zin preferences, the temporal distribution of risk matters so that the persistence of consumption growth is reflected in equilibrium asset prices. In particular, if Epstein-Zin agents prefer early resolution of uncertainty, as assumed in this paper, they dislike persistence in consumption growth (e.g., Piazzesi and Schneider (2006)). Introducing incomplete information has the effect of increasing consumption persistence and hence contributes to risk. More specifically, observing changes in productivity under incomplete information leads agents to update beliefs about temporary and permanent shocks in the same direction, so that current consumption growth is made more positively correlated with expected consumption growth.

The comparison above does not seem to favor the incomplete information model, as the complete information model can fit the data about equally well. However, this outcome comes from an incomplete picture where only standard asset pricing moments are taken into account. An important feature of the data that has so far been left out of consideration is the term structure of equity risk premia. Authors such as Lettau and Wachter (2007) and Binsbergen et al. (2010) emphasize that the term structure of equity risk premia contains valuable information for assessing equilibrium asset pricing models. The next section therefore evaluates the model along this dimension.

3.4 Implications for the Term Structure of Equity Risk Premia

The term structure of equity risk premia refers to the relationship between the amount of risk compensation required for equity cash flows and the amount of time before the cash flows materialize. Binsbergen et al. (2010) propose to measure this relationship by recovering the prices of dividend strips, which are claims to dividends paid over future time intervals, from derivative market data. They find under the no-arbitrage condition that short-term dividend strips are significantly more risky than long-term dividend strips. In other words, the term structure of equity risk premia is downward-sloping. Furthermore, they show that this data feature is hard to reconcile with several important equilibrium asset pricing models, such as Campbell and Cochrane (1999), Bansal and Yaron (2004) and Gabaix (2010). Below I consider the implications of incomplete information for the term structure of equity risk premia.

3.4.1 The Role of Information Structure

Following Lettau and Wachter (2007) and Binsbergen et al. (2010), the equity is regarded as a portfolio of zero-coupon dividend strips with different maturities. Letting $P_t^{(n)}$ be the price of a dividend strip at time t which pays dividend n periods in the future, and $R_{t+1}^{(n)}$ be the one-period return on this dividend strip, i.e., $R_{t+1}^{(n)} = P_{t+1}^{(n-1)}/P_t^{(n)}$, the zero-coupon dividend strips are valued under no-arbitrage, so that

$$P_t^{(n)} = \mathbb{E}_t\Big[M_{t+1}P_{t+1}^{(n-1)}\Big], \tag{3.23}$$

where $P_t^{(1)} = \mathbb{E}_t[M_{t+1}D_{t+1}]$. Then the term structure of equity risk premia in the model economy is defined by the relationship of $\mathbb{E}_t\big[r_{t+1}^{(n)} - r_{f,t+1}\big]$ with n.

To study this relationship, I simulate the model 5000 times to compute the average annualized risk premium and return volatility for each dividend strip. The results are plotted in Figure 3.5, where the parameters are set to the benchmark values. As illustrated, the dividend strip risk premia under complete information increase with maturity. The risk premium is 4.4% per annum for a strip paying dividends one quarter from now and 5.2% per annum for a strip paying dividend 30 years from now. Similar pattern is also observed in the annualized return volatility. Thus, contrary to the evidence, the term structure of equity risk premia is upward-sloping. On the other hand, the dividend strip risk premia under incomplete information decrease with maturity. The risk premium is 5.4% per annum for a strip paying dividends one quarter from now and 4.5% for a strip paying dividends 30 years from now. This pattern is repeated in the annualized return volatility of dividend strips. The term structure of equity risk premia is thus downward-sloping as the data suggests. Consistent with the pattern of the term structure of equity risk premia, Figure 3.2 shows that the price-dividend ratio is downward sloping under complete information but upward sloping under incomplete information. Taken together, these results provide evidence for incomplete information as a potentially important factor in generating an empirically relevant equity term structure.

The idea that information structure might help to understand the term structure of equity risk premia is also entertained by Croce et al. (2010). They consider a discrete-time endowment economy similar to the long-run risk specification of Bansal and Yaron (2004) but augment it with incomplete information. While endowment economy model is a useful starting point, the fact that it abstracts from production precludes consideration of whether the explanation is consistent with macroeconomic outcomes. In contrast, this paper develops the analysis within the class of neoclassical production economies that rest at the core of modern macroeconomic theory. The discipline imposed by the macroeconomic implications of this particular class of model is a key point of departure from the aforementioned work.

Before getting into the details, it is important to note that the model's prediction for the term structure of equity risk premia is not a manifestation of the long-run insurance channel. As noted in Croce et al. (2010), with an Epstein-Zin representative agent favoring early resolution of uncertainty, the term structure of equity risk premia will slope down if the long-run insurance channel is at work, namely that the innovations in expected consumption growth shift the stochastic discount factor and equity returns in the same direction (holding other shocks fixed). The reason is that in this type of situation long-horizon variations of consumption growth have a negative effect on the equity risk premium. Croce, Lettau and Ludvigson point out that this strategy of matching the term structure evidence of equity risk premia is unappealing because the associated equity premium is too small and can even be negative. My result does not rely on this strategy. Under my calibration long-run swings in consumption growth contribute to higher stock market risk.

3.4.2 Impulse Response of Consumption

To understand the intuition, I start with an impulse response analysis for the dynamics of aggregate consumption. As noted before, the temporal distribution of risk matters for Epstein-Zin preferences, so that the equilibrium asset prices reflect not only shocks to realized consumption growth, as in the case of power utility, but also reflect shocks to expected consumption growth. The linkage between asset prices and temporal variations of consumption growth implies that the term structure of equity risk premia is tied to the relative variability of consumption growth over short and long horizons. Knowing how this relative variability would be affected by incomplete information helps understand the model prediction for the temporal composition of equity premium.

First, consider the complete information case. Figure 3.5 reports the impulse responses of consumption for this case. These are the responses to temporary and permanent increases in productivity, obtained using the calibrated parameter values. As shown, consumption has very different adjustment to temporary and permanent shocks. With a permanent shock consumption grows forever after, but with a temporary shock consumption eventually reverts to the steady state. These are well-known dynamics in the real business cycle literature. In terms of asset pricing, recent studies such as Kaltenbrunner and Lochstoer (2010) note the importance of permanent shocks relative to temporary shocks. Unlike temporary shocks, permanent shocks induce highly persistent fluctuations in consumption growth. This type of risk places more burden than those occurring over short horizons when agents wish to resolve uncertainty sooner. Consequently, long-horizon dividend strips are more risky and the term structure of equity risk premia is upward-sloping.

These observations help indicate how the consumption dynamics are likely to change if agents are partially informed about the sources of uncertainty. In this case, belief updating leads to positively correlated estimates for temporary and permanent components of productivity growth. For consumption growth, this may generate additional volatility over short horizons since initial consumption adjustments to temporary and permanent shocks are made positively correlated. Over longer horizons it causes another effect that lessens consumption growth fluctuations. The latter effect arises because learning about productivity shocks gives rise to the perception that temporary and permanent shocks work in exactly opposite directions on future consumption. More concretely, an observed rise in productivity makes agents believe that there has been an upward swing in both the temporary and permanent components of growth. In turn, they anticipate that as the temporary growth effect gradually evaporates there will a downward force exerted on consumption that partially offsets the impact of permanent shocks over longer horizons. This effect can reduce the variability of future consumption growth. Consistent with this intuition, the impulse response reported in Figure 3.4 shows that consumption under incomplete information displays a rather smooth profile over longer horizons. Such a consumption profile tends to imply higher risk of short-term dividend strips over long-term dividend strips. Therefore, the term structure of equity risk premia can be downward-sloping under incomplete information.

3.4.3 Relation to Croce, Lettau and Ludvigson (2010)

Related to this paper, Croce et al. (2010) contemplate another intuition regarding the relation between incomplete information and the term structure of equity risk premia. In their endowment economy, the representative agent is confronted with a situation in which i.i.d. and persistent shocks to the growth rates of consumption and dividends are not separately observable and must be estimated from historical observations. Because persistent shocks are assumed to be much smaller than i.i.d. shocks, they are assigned far smaller probability, so that changes in consumption and dividend growth are perceived to mostly reflect the impact of i.i.d. shocks. This effect is strong enough in their model to make the term structure of equity risk premia downward-sloping.

It is useful to know if such obscuring effect is also present in my model. A simple numerical exercise can help clarify it. Before we go any further, recall that learning about productivity shocks in the model economy is done by observing increments of the productivity process. More specifically, agents compare productivity levels in consecutive periods, A_{t-1} and A_t, to find out the growth rate (i.e., $g_t = \nu(\mu + \theta_t) + z_t - z_{t-1}$), and then estimate its temporary and permanent components by Bayesian updating. To quantify the estimates, I conduct the following exercise for the calibrated economy. Consider an example where the positions of temporary and permanent growth components at time $t-1$ are set to their long-run average values and the beliefs held by agents coincide with the truth, i.e., $\tilde{\theta}_{t-1} = \theta_{t-1} = 0$, $\tilde{z}_{t-1} = z_{t-1} = 0$. Suppose that θ_t and z_t both increase by one standard deviation, i.e., $\theta_t = 0.21\%$ and $z_t = 4.1\%$. Then the growth of productivity rises by 4.35%. Applying Kalman filter on the observation, the posterior beliefs are $\tilde{\theta}_t = 0.09\%$, $\tilde{z}_t = 1.96\%$ and $\tilde{z}_{t-1} = -1.60\%$. This result is listed in the first row of Table 3.7.

Several effects are observed. First, z_t has a larger estimated magnitude than θ_t. Second, there is a downward revision in the belief assigned to the lagged value of temporary shock z_{t-1}. Third, it appears that signal extraction does not strongly reduce the perceived importance of permanent shocks. The underlying permanent shock is 5% the size of temporary shock, and it is estimated to be 4.6% as large.

The first effect is simply because temporary shocks are assumed to have a larger standard deviation. To understand the second effect, consider the responses of total factor productivity and beliefs given in Figure 3.5. As shown, the growth of total factor productivity increases following a positive temporary shock. However, from the subsequent period onwards, the growth of total factor productivity turns negative as the shock dies out gradually. In the case of negative shocks, the response of total factor productivity is the mirror image of the current one. The agent understands these. Therefore, when observing a rise in total factor productivity, he recognizes not only the possibility that the change is caused by a contemporaneous positive shift in the temporary or permanent component of productivity, but also the possibility that the rise in total factor productivity just reflects the evaporation of a negative temporary shock that has occurred in the previous period. Hence, the agent decreases his belief about z_{t-1} while increasing his belief about z_t and θ_t. This effect is not present in Croce et al. (2010), where the randomness concealing persistent growth shocks is i.i.d.. The third effect has to do with the second one. The downward belief adjustment for the past temporary shock has the effect of dampening the assessment of contemporaneous temporary shock, which helps to keep permanent shocks from being significantly overshadowed in posterior beliefs. This is not the case in Croce et al. (2010) because their model does not feature the second effect described above.

An important factor that affects the posterior beliefs about temporary and permanent shocks is their relative size. If permanent shocks are considerably small relative to temporary shocks, it is possible that they will be assigned negligible weight in posterior beliefs. Table 3.7 illustrates this by examining the effect of altering $\sigma_\xi/\sigma_\varepsilon$ on posterior beliefs. It shows that as

$\sigma_\xi/\sigma_\varepsilon$ decreases – that is, permanent shocks decrease in importance relative to temporary shocks – the estimate is lowered for the event that a given change of productivity growth reflects a permanent shift, and raised for the opposite event that the change is temporary. Furthermore, when $\sigma_\xi/\sigma_\varepsilon$ is small enough agents are led to consider the change in productivity growth as largely ephemeral. For example, in the case of $\sigma_\xi/\sigma_\varepsilon = 0.5\%$, the permanent shock appears rather negligible, with an estimated magnitude of only 0.04% the size of temporary shock. For less extreme values of $\sigma_\xi/\sigma_\varepsilon$ the obscuring effect seems fairly week and can even be reversed. For example, when $\sigma_\xi/\sigma_\varepsilon =$ the permanent shock is estimated to be the size of temporary shocks though it is actually only as large.

Thus, depending upon the relative size of permanent and temporary shocks, the obscuring effect may or may not be present in my model. While this effect is present in the calibrated economy, it is rather weak and unlikely to be the main force driving the term structure of equity risk premia. This gives an indication that the implication my model has for the term structure of equity risk premia is largely driven by the aforementioned offsetting effect. The intuition for the offsetting effect is distinct from, yet complementary to, that of Croce, Lettau and Ludvigson. In addition, this intuition derives from a standard production economy model as opposed to the endowment economy considered by Croce, Lettau and Ludvigson. Importantly, this suggests that the idea that information structure might help understand the term structure of equity risk premia can be extended to a more general class of economies.

Another factor influencing beliefs is the persistence of productivity shocks. Table 3.8 shows that decreased persistence of permanent shocks lowers beliefs about permanent shocks but raises beliefs about temporary shocks. As a result, the perceived importance of permanent shocks is reduced relative to temporary shocks. Similarly, it is observed in Table 3.9 that decreased persistence of temporary shocks also reduces the relative importance of permanent shocks, though to a lesser extent. This may seem surprising. On intuitive grounds one would expect less persistent shocks to always receive less weight in belief updating. This is not the case here due to the effect associated with the lagged temporary shock. Table 3.9 shows that, as temporary shocks become less persistent, the assessment of their lagged values receive less downward adjustment. This results in increased beliefs about the contemporaneous temporary shock. Although beliefs about permanent shocks are also increased as a result of decreased persistence of temporary shocks, the effect is overwhelmed by the one aforementioned.

The effects discussed above also have implications for the term structure of equity risk premia. Figure 3.6 shows the consequence of varying the persistence and relative variability of productivity shocks on the slope of the term structure of equity risk premia. The top panel shows that increasing the persistence of permanent shocks, ρ_θ, can make the term structure of equity risk premia. The bottom panel shows that increasing the size of permanent shocks relative to temporary shocks has a similar effect. But the magnitude is larger compared to increasing ρ_θ. This is consistent with the results documented in Table 3.73.9 that beliefs appear more sensitive to variations in the relative size of permanent and temporary shocks than to variations in the shock persistence.

3.5 Conclusion

In this paper, I have analyzed a real business cycle model in which a representative agent with Epstein-Zin preferences cannot separately identify transitory and permanent shocks to productivity. For aggregate asset pricing, this model economy gives a glimpse of what is possible to learn and accomplish by introducing incomplete information into the neoclassical stochastic growth framework.

For calibrated parameter values, the model is found to be consistent with some salient features of aggregate asset prices and macroeconomic quantities, including the equity premium, the risk-free rate, the price-dividend ratio, and the variation and autocorrelation of consumption and output growth. Most notably, it provides an explanation for why the term structure of aggregate equity risk is downward sloping − an empirical challenge that confronts several leading models of asset pricing. The key to this particular success is the assumed inability of agents to discriminate perfectly between the transitory and permanent movements in productivity. In this regard, this paper shares the perspective of Croce et al. (2010) that places an emphasis on incomplete information as a way of reconciling the cash flow duration evidence. But the mechanism at work here is quite different, based on the fact that transitory and permanent shocks to productivity have opposing effects on long-term consumption. Simple as it is, this mechanism brings a new insight into understanding the empirical relationship between equity risk premia and cash flow duration.

To help convey the insight in its sharp form, the model is deliberately kept simple. Only the most basic association between Bayesian learning and neoclassical stochastic general equilibrium production is studied. The fact that the model is not able to endogenously generate realistic behavior for dividends tells us there are important margins, along which decisions are made, that have not been captured in this simple world. That being the case, adventuring into a more full-fledged incomplete information model of production economy would be a necessary next step. In addition, it would be necessary to estimate the model's parameters. Although the calibration exercise goes some way to show the model's potential, it cannot be a substitute for rigorous estimation work. I leave these tasks to future research.

Table 3.1: Parameter Values for Ai (2010)'s Model

Parameter	Symbol	Value
Subjective discount rate	β	0.014
Risk aversion	γ	2
Long-run mean of productivity	$\bar{\theta}$	0.035
Autocorrelation of persistence shocks	κ	0.027
Conditional volatility of persistent shocks	σ_θ	0.005
Volatility of i.i.d. shocks	σ_K	0.099
Elasticity of intertemporal substitution	ψ	2
Dividend growth rate parameter	A	0.034
Leverage parameter	ϕ	2.05
Idiosyncratic volatility of dividend growth	σ_D	0.11

Notes: This table presents the parameter values that Ai (2010) uses in calibrating a Cox, Ingersoll and Ross (1985) type production economy model.

Table 3.2: Quantitative Implications of Ai (2010)'s Model

Statistic	Data	Ai (2010)
Panel A - Macroeconomic Moments		
$\mathrm{E}[\Delta c]$ (%)	1.80	1.79
$\sigma[\Delta c]$ (%)	2.75	2.80
$AC1[\Delta c]$	0.49	0.48
$\sigma[\Delta y]$ (%)	4.26	(21.49)
$AC1[\Delta y]$	0.42	(0.21)
$\sigma[\Delta i]$ (%)	17.24	(378.28)
$AC1[\Delta i]$	0.43	(0.11)
$\mathrm{E}[I/K]$	0.17	(8.79)
$\mathrm{E}[I/Y]$	0.15	(0.50)
Panel B - Asset Pricing Moments		
$\sigma[\Delta d]$ (%)	11.49	10.65
$AC1[\Delta d]$	0.21	0.30
$\mathrm{Corr}[\Delta c, \Delta d]$	0.55	0.53
$\mathrm{E}[r_f]$ (%)	0.86	0.86
$\sigma[r_f]$ (%)	0.97	0.77
$\mathrm{E}[r_e - r_f]$ (%)	6.33	5.06
$\sigma[r_e]$ (%)	19.42	23.20
$\mathrm{E}[\exp(p-d)]$	26.56	27.22
$\sigma[p-d]$	0.29	0.34

Notes: This table expands Table II and V in Ai (2010) by reporting the simulation results of his model for output and investment. The "Data" column contains estimates based on annual U.S. data between 1929 and 1998. Asset pricing moments are taken from Bansal and Yaron (2004). Macroeconomic moments are based on data from the Bureau of Economic Analysis. I use Ai's calibrated parameters and simulation procedure to compute the moments for output and investment shown in parentheses. The notations have the following meaning: Δc, Δd, Δi and Δy denote, respectively, the log growth rate of consumption, dividend, investment and output; I/K is investment-capital ratio; I/Y is investment-output ratio; $p-d$ is log price-dividend ratio; r_f, r_e and $r_e - r_f$ are respectively the risk-free rate, the log return on equity and the equity risk premium; $\mathbb{E}$, σ, Corr, $AC1$ denote respectively the mean, the standard deviation, the correlation and the first-order autocorrelation.

Table 3.3: Parameter Values for the RBC Model

Parameter	Symbol	Value
Subjective discount factor	β	0.998
Relative risk aversion	γ	10
Elasticity of intertemporal substitution	ψ	1.5
Labor's share of output	ν	0.64
Average productivity growth rate	μ	0.51%
Capital depreciation rate	δ	0.025
Conditional volatility of the transitory shock	σ_ε	4.1%
Conditional volatility of the trend growth shock	σ_ξ	0.05*σ_ε
Autocorrelation of the trend growth shock	ρ_θ	0.965
Autocorrelation of the transitory shock	ρ_z	0.95
Capital adjustment costs parameter	η	8
Dividend growth rate parameter	μ_d	1.02%
Leverage parameters	$\{\lambda_1, \lambda_2\}$	{0.15, 1.63}
Idiosyncratic volatility of dividend growth	σ_d	3.14%

Notes: This table presents calibrated parameter values for the incomplete-information RBC model at a quarterly frequency.

Table 3.4: Quantitative Implications of the RBC Model

Statistic	Data	Ai (2010)	RBC Model
Panel A - Macroeconomic Moments			
$\mathrm{E}[\Delta c]$ (%)	1.80	1.79	1.81
$\sigma[\Delta c]$ (%)	2.75	2.80	2.76
$AC1[\Delta c]$	0.49	0.48	0.49
$\sigma[\Delta y]$ (%)	4.26	(21.49)	4.26
$AC1[\Delta y]$	0.42	(0.21)	0.31
$\sigma[\Delta i]$ (%)	17.24	(378.28)	8.01
$AC1[\Delta i]$	0.43	(0.11)	0.21
$\mathrm{E}[I/K]$	0.17	(8.79)	0.11
$\mathrm{E}[I/Y]$	0.15	(0.50)	0.23
Panel B - Asset Pricing Moments			
$\sigma[\Delta d]$ (%)	11.49	10.59	10.77
$AC1[\Delta d]$	0.21	0.29	0.23
$\mathrm{Corr}[\Delta c, \Delta d]$	0.55	0.53	0.54
$\mathrm{E}[r_f]$ (%)	0.86	0.86	0.86
$\sigma[r_f]$ (%)	0.97	0.77	0.90
$\mathrm{E}[r_e - r_f]$ (%)	6.33	5.06	4.79
$\sigma[r_e]$ (%)	19.42	19.56	17.07
$\mathrm{E}[\exp(p-d)]$	26.56	25.71	22.39
$\sigma[p-d]$	0.29	0.34	0.24

Notes: This table reports various macroeconomic and asset pricing moments from the data and from simulating my incomplete information model. The empirical asset pricing moments are from Bansal and Yaron (2004), which are based on annual data from 1929 to 1998. The empirical macroeconomic moments are calculated for the same sample period using annual data from the Bureau of Economic Analysis. The notations have the following meaning: Δc, Δd, Δi and Δy denote, respectively, the log growth rate of consumption, dividend, investment and output; I/K is investment-capital ratio; I/Y is investment-output ratio; $p - d$ is log price-dividend ratio; r_f, r_e and $r_e - r_f$ are respectively the risk-free rate, the log return on equity and the equity risk premium; $\mathbb{E}$, σ, Corr, $AC1$ denote respectively the mean, the standard deviation, the correlation and the first-order autocorrelation. The statistics of the CIR model are based on my replication using the benchmark calibration of Ai (2010).

Table 3.5: Comparison between RBC Economies with and without Incomplete Information: (1)

Statistic	Data	Incomplete Info	Complete Info
Panel A - Macroeconomic Moments			
E$[\Delta c]$ (%)	1.80	1.81	1.82
$\sigma[\Delta c]$ (%)	2.75	2.76	2.45
$AC1[\Delta c]$	0.49	0.49	0.46
$\sigma[\Delta y]$ (%)	4.26	4.26	3.94
$AC1[\Delta y]$	0.42	0.31	0.25
$\sigma[\Delta i]$ (%)	17.24	8.01	7.64
$AC1[\Delta i]$	0.43	0.21	0.17
E$[I/K]$	0.17	0.11	0.10
E$[I/Y]$	0.15	0.23	0.21
Panel B - Asset Pricing Moments			
$\sigma[\Delta d]$ (%)	11.49	10.77	10.23
$AC1[\Delta d]$	0.21	0.23	0.20
Corr$[\Delta c, \Delta d]$	0.55	0.54	0.50
E$[r_f]$ (%)	0.86	0.86	0.97
$\sigma[r_f]$ (%)	0.97	0.90	1.04
E$[r_e - r_f]$ (%)	6.33	4.79	4.33
$\sigma[r_e]$ (%)	19.42	17.07	16.72
E$[\exp(p-d)]$	26.56	22.39	23.45
$\sigma[p-d]$	0.29	0.24	0.23

Notes: This table compares results from simulating the RBC model with and without incomplete information. The empirical asset pricing moments are from Bansal and Yaron (2004), which are based on annual data from 1929 to 1998. The empirical macroeconomic moments are calculated for the same sample period using annual data from the Bureau of Economic Analysis. The notations have the following meaning: Δc, Δd, Δi and Δy denote, respectively, the log growth rate of consumption, dividend, investment and output; I/K is investment-capital ratio; I/Y is investment-output ratio; $p-d$ is log price-dividend ratio; r_f, r_e and $r_e - r_f$ are respectively the risk-free rate, the log return on equity and the equity risk premium; $\mathbb{E}$, σ, Corr, $AC1$ denote respectively the mean, the standard deviation, the correlation and the first-order autocorrelation.

Table 3.6: Comparison between RBC Economies with and without Incomplete Information: (2)

Statistic	Data	Incomplete Info $\rho_\theta = 0.965$	Complete Info $\rho_\theta = 0.975$
Panel A - Macroeconomic Moments			
$\mathrm{E}[\Delta c]$ (%)	1.80	1.81	1.84
$\sigma[\Delta c]$ (%)	2.75	2.76	2.78
$AC1[\Delta c]$	0.49	0.49	0.49
$\sigma[\Delta y]$ (%)	4.26	4.26	4.25
$AC1[\Delta y]$	0.42	0.31	0.30
$\sigma[\Delta i]$ (%)	17.24	8.01	7.97
$AC1[\Delta i]$	0.43	0.21	0.21
$\mathrm{E}[I/K]$	0.17	0.11	0.12
$\mathrm{E}[I/Y]$ (%)	0.15	0.23	0.24
Panel B - Asset Pricing Moments			
$\sigma[\Delta d]$ (%)	11.49	10.77	10.89
$AC1[\Delta d]$	0.21	0.23	0.24
$\mathrm{Corr}[\Delta c, \Delta d]$	0.55	0.54	0.53
$\mathrm{E}[r_f]$ (%)	0.86	0.86	0.88
$\sigma[r_f]$ (%)	0.97	0.90	0.92
$\mathrm{E}[r_e - r_f]$ (%)	6.33	4.79	4.83
$\sigma[r_e]$ (%)	19.42	17.07	16.72
$\mathrm{E}[\exp(p-d)]$	26.56	22.39	22.65
$\sigma[p-d]$	0.29	0.24	0.26

Notes: This table compares various macroeconomic and asset pricing moments for the model economies with complete and incomplete information. The empirical asset pricing moments are from Bansal and Yaron (2004), which are based on annual data from 1929 to 1998. The empirical macroeconomic moments are calculated for the same sample period using annual data from the Bureau of Economic Analysis. The notations have the following meaning: Δc, Δd, Δi and Δy denote, respectively, the log growth rate of consumption, dividend, investment and output; I/K is investment-capital ratio; I/Y is investment-output ratio; $p-d$ is log price-dividend ratio; r_f, r_e and $r_e - r_f$ are respectively the risk-free rate, the log return on equity and the equity risk premium; $\mathbb{E}$, σ, Corr, $AC1$ denote respectively the mean, the standard deviation, the correlation and the first-order autocorrelation.

Table 3.7: The Relation of Posterior Beliefs with the Relative Size of Temporary and Permanent Shocks

$z_t = 2.2\%,\ z_{t-1} = 0.$						
$\sigma_\xi/\sigma_\varepsilon$	θ_t	θ_t/z_t	$\tilde{\theta}_t$	$\tilde{z}_t$	$\tilde{z}_{t-1}$	$\tilde{\theta}_t/\tilde{z}_t$
8.0%	0.18%	8.0%	0.10%	1.00%	-1.24%	10.24%
7.0%	0.15%	7.0%	0.08%	1.07%	-1.17%	7.84%
6.0%	0.13%	6.0%	0.07%	1.15%	-1.09%	5.76%
5.0%	0.11%	5.0%	0.05%	1.25%	-0.99%	4.00%
4.0%	0.09%	4.0%	0.04%	1.37%	-0.86%	2.56%
1.0%	0.02%	1.0%	0.00%	1.91%	-0.30%	0.16%
0.5%	0.01%	0.5%	0.00%	2.04%	-0.16%	0.04%

Notes: This table shows how the posterior beliefs about temporary and permanent shocks vary with their relative size $\sigma_\xi/\sigma_\varepsilon$. σ_ξ and σ_ε denote, respectively, conditional standard deviations of permanent and temporary shocks. θ_t, z_t and z_{t-1} denote the true values of temporary and permanent shocks, $\tilde{\theta}_t$, $\tilde{z}_t$ and $\tilde{z}_{t-1}$ denote the respective posterior estimates. When changing $\sigma_\xi/\sigma_\varepsilon$, I let σ_ξ vary but keep σ_ε and the other parameters fixed at the benchmark values. The results reported for θ_t and z_t are obtained as a result of one standard deviation shift from the long-run average.

Table 3.8: The Relation of Posterior Beliefs with the Persistence of Permanent Shocks

$\theta_t = 0.11\%,\ z_t = 2.2\%,\ z_{t-1} = 0,\ \theta_t/z_t = 5\%.$				
ρ_θ	$\tilde{\theta}_t$	$\tilde{z}_t$	$\tilde{z}_{t-1}$	$\tilde{\theta}_t/\tilde{z}_t$
0.98	0.07%	1.00%	-1.22%	7.29%
0.95	0.04%	1.35%	-0.89%	3.14%
0.92	0.03%	1.58%	-0.67%	1.78%
0.89	0.02%	1.72%	-0.54%	1.19%
0.86	0.02%	1.82%	-0.44%	0.89%
0.83	0.01%	1.89%	-0.38%	0.70%

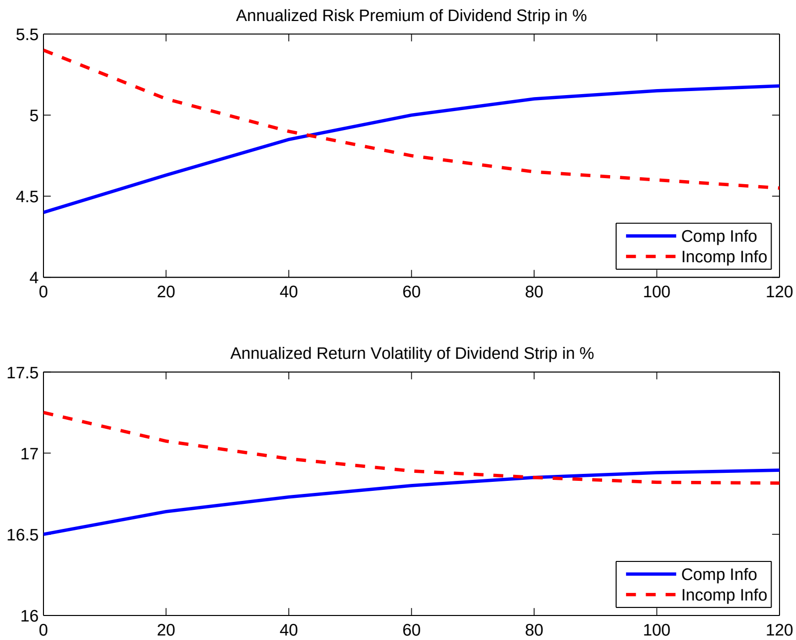
Notes: This table shows how the posterior estimates of temporary and permanent shocks vary with respect to the persistence of permanent shocks ρ_θ. θ_t, z_t and z_{t-1} denote the true values of temporary and permanent shocks, $\tilde{\theta}_t$, $\tilde{z}_t$ and $\tilde{z}_{t-1}$ denote the respective posterior estimates. All parameters other than ρ_θ are given in the benchmark calibration. θ_t and z_t are obtained as a result of one standard deviation shift from the long-run average.

Table 3.9: The Relation of Posterior Beliefs with the Persistence of Temporary Shocks

$\theta_t = 0.11\%,\ z_t = 2.2\%,\ z_{t-1} = 0,\ \theta_t/z_t = 5\%.$				
ρ_z	$\tilde{\theta}_t$	$\tilde{z}_t$	$\tilde{z}_{t-1}$	$\tilde{\theta}_t/\tilde{z}_t$
0.98	0.05%	1.18%	-1.06%	3.90%
0.95	0.05%	1.28%	-0.96%	4.04%
0.92	0.06%	1.35%	-0.88%	4.12%
0.89	0.06%	1.41%	-0.83%	4.17%
0.86	0.06%	1.45%	-0.78%	4.20%
0.83	0.06%	1.49%	-0.74%	4.23%

Notes: This table shows how the posterior estimates of temporary and permanent shocks vary with respect to the persistence of temporary shocks ρ_z. θ_t, z_t and z_{t-1} denote the true values of temporary and permanent shocks, $\tilde{\theta}_t$, $\tilde{z}_t$ and $\tilde{z}_{t-1}$ denote the respective posterior estimates. All parameters except ρ_z are given in the benchmark calibration. θ_t and z_t are obtained as a result of one standard deviation shift from the long-run average.

Figure 3.1: Information Structure and the Term Structure of Equity Risk Premia



Notes: The top panel shows log equity risk premium as a function of maturity; the bottom panel shows the standard deviation of excess returns on zero-coupon equity as a function of maturity. Parameters are fixed at the benchmark values.

Figure 3.2: Information Structure and the Term Structure of Price-Dividend Ratio

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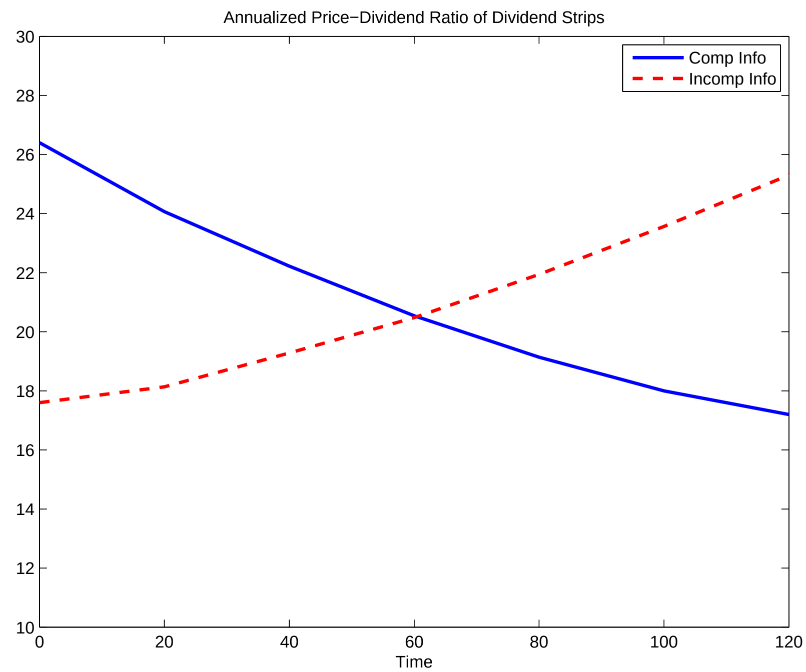

Notes: This figure shows (annualized) price-dividend ratio as a function of maturity. Parameters are fixed at the benchmark values.

Figure 3.3: Consumption Response under Complete Information

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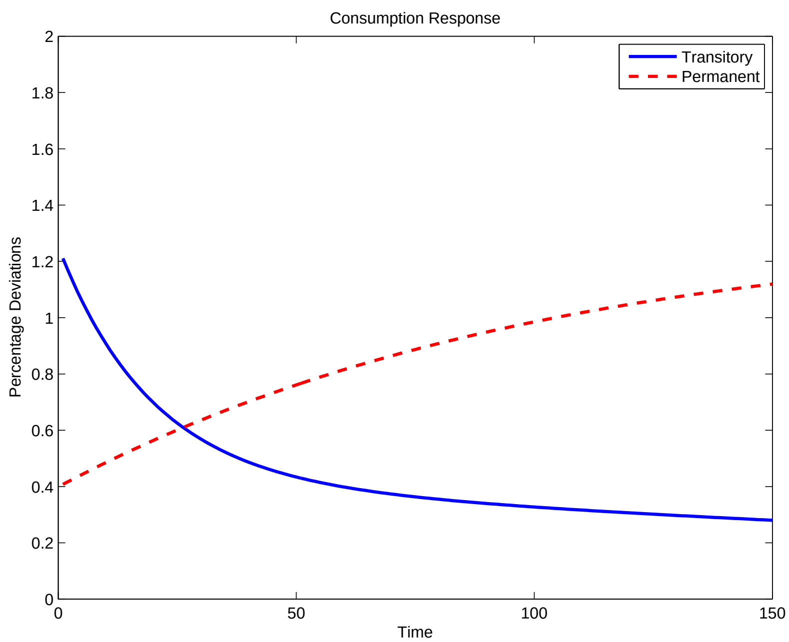

Notes: This figure plots the response of consumption under complete information using the benchmark parameter values. The results reported are the average outcome of simulating the model for 5000 times.

Figure 3.4: Consumption Response under Incomplete Information

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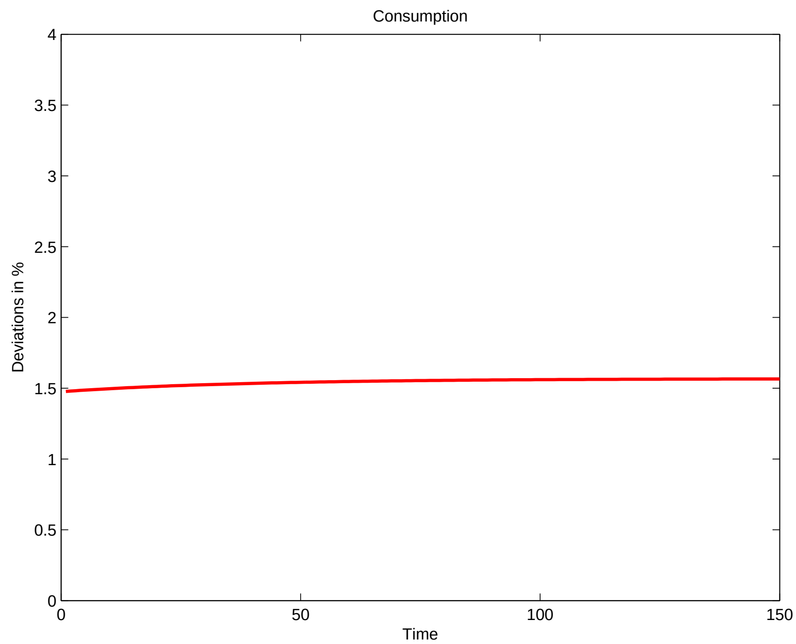

Notes: This figure plots the response of consumption under incomplete information using the benchmark parameter values. The results reported are the average outcome of simulating the model for 5000 times.

Figure 3.5: Belief Response to Productivity Shocks

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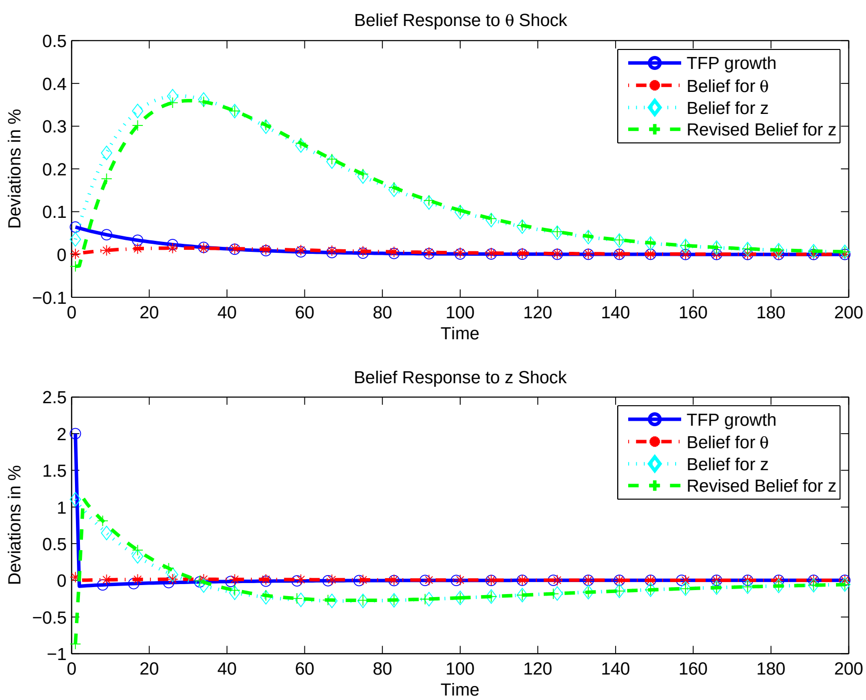

Notes: This figure plots the response of beliefs to temporary and permanent shocks for the calibrated model. The upper panel shows the response to one standard deviation increase in the permanent component of productivity, while the bottom panel shows the response to the temporary component.

Figure 3.6: Comparative Statics for the Term Structure of Equity Risk Premia

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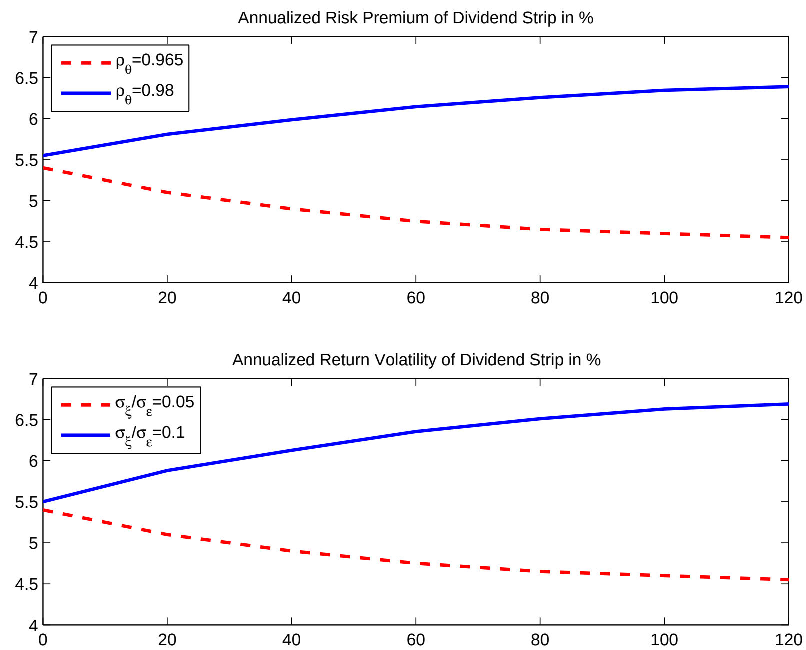

Notes: This figure shows the effects of varying the persistence and relative variability of productivity shocks on the term structure of equity risk premia.

Chapter 4

Conclusion

In this thesis, I have investigated the impact of incomplete information on asset returns in general equilibrium economies with production. Both essays examine models with Epstein-Zin type preferences. In these models, the manner in which uncertainty resolves over time matters to the decision maker, and as a consequence the long-run risk, in terms of low frequency variations in consumption, is reflected in the equilibrium asset prices. Incomplete information affects the temporal resolution of uncertainty and therefore can potentially plays a larger role in these models than in those with the standard expected utility.

In the first essay, I use a Cox-Ingersoll-Ross model to analytically identify aspects of preferences that are important for the impact of incomplete information on the compensation for long-run risk. Two results are obtained. First, the direction in which incomplete information affects the long-run component of risk premium is determined by the attitude towards the temporal resolution of uncertainty. Agents with desire to resolve uncertainty sooner require higher risk compensation for the long run when information about the future is incomplete. Second, the extent to which incomplete information affects the long-run exposure to risk is determined by the attitude towards intertemporal substitution. The larger the elasticity of intertemporal substitution, the stronger the impact that incomplete information has on the long-run risk exposure. These results are obtained through separating the risk premium into the short-run and long-run components. They stand in contrast to those suggested by prior work based on myopic demand versus hedging demand decomposition (Ai (2010)).

In the second essay, I use a standard real business cycle model to quantitatively examine the implications of incomplete information for equilibrium asset returns. I show that with incomplete information the model is quantitatively consistent with some salient features of asset prices and the macroeconomy. More specifically, the model matches key moments of asset prices such as the equity premium, the risk free rate and the price-dividend ratio, while at the same time being consistent with the some key macroeconomic moments such as the volatility of consumption and output growth as well as their autocorrelation. Furthermore, the model is able to generate a negative slope for the term structure of equity. That is, the model predicts that assets with cash flow fluctuations weighted more towards the near future have larger risk premia and return volatility, consistent with the empirical evidence. This result is notable because accounting for the term structure evidence of equity is an empirical challenge for several leading asset pricing models such as the external habit formation model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004) and the variable rare disasters model of Gaibaix (2010). The mechanism that generates this result differs from those in the existing literature and provides new insights into understanding why equity risk premia are negatively related to cash flow duration.

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Appendix A

Appendix to Chapter 2

A.1 The ODEs Characterizing the Functions $H(m)$ and $G(\theta)$

The function $H(m_t; \sigma_e)$ satisfies

$$0 = \frac{\alpha}{\varphi\psi}\beta^{\psi} H(m_t)^{-\frac{\varphi\psi}{\alpha}} + [\kappa(\bar{\theta} - m_t) + \alpha(\rho\sigma_\theta\sigma_K + Q)]\frac{H'(m_t)}{H(m_t)} + \frac{\sigma_m^2}{2}\frac{H''(m_t)}{H(m_t)} + \alpha\left(m_t - \frac{\beta}{\varphi} - \frac{\gamma\sigma_K^2}{2}\right). \tag{A.1}$$

Then, the full information counterpart $G(\theta_t)$ satisfies

$$0 = \frac{\alpha}{\varphi\psi}\beta^{\psi} G(\theta_t)^{-\frac{\varphi\psi}{\alpha}} + [\kappa(\bar{\theta} - \theta_t) + \alpha\rho\sigma_\theta\sigma_K]\frac{G'(\theta_t)}{G(\theta_t)} + \frac{\sigma_\theta^2}{2}\frac{G''(\theta_t)}{G(\theta_t)} + \alpha\left(\theta_t - \frac{\beta}{\varphi} - \frac{\gamma\sigma_K^2}{2}\right). \tag{A.2}$$

A.2 Derivation of the State Price Density

I build on the results of Duffie and Epstein (1992a) and Ai (2010) to derive the pricing kernel. Duffie and Epstein show that the state price density for stochastic differential utility is given by

$$\pi_t = e^{\int_0^t f_v(C_s, J_s)\mathrm{d}s} f_c(C_t, J_t), \tag{A.3}$$

where f_c and f_v are the partial derivatives of f with respect to its first and second arguments, respectively, and J is the value function. Furthermore, we have

$$f_v(C_t, J_t) = \frac{\beta}{\rho}\left(1 - \frac{\rho}{\alpha}\right) C_t^{\rho}(\alpha J_t)^{-\rho/\alpha} - \frac{\beta}{\rho}\alpha \tag{A.4}$$

$$f_c(C_t, J_t) = \beta C_t^{\rho-1}(\alpha J_t)^{1-\rho/\alpha} \tag{A.5}$$

Ai shows that the value function J is given by

$$J(W_t, m_t) = H(m_t)\frac{W_t^\alpha}{\alpha}. \tag{A.6}$$

Substituting (A.4), (A.5) and (A.6) into (A.3), and noting that $x_t = \beta^{-\psi} H(m_t)^{-\frac{1-\psi}{1-\gamma}}$, I rearrange terms to obtain the expression of state price density given by (2.16).

A.3 Derivation of Equations (2.34) and (2.35)

By Ito's lemma, the process for consumption growth is obtained as

$$\frac{\mathrm{d}C_t}{C_t} = \mu_C(m_t)\mathrm{d}t + \sigma_K \mathrm{d}\tilde{B}_{K,t} - \frac{\varphi\psi}{\alpha}\frac{H'(m_t)}{H(m_t)}\sigma_m \mathrm{d}\tilde{B}_{m,t}, \tag{A.7}$$

where

$$\begin{aligned}\mu_C(m_t) \equiv m_t - x_t^{-1} - \frac{\varphi\psi}{\alpha}\frac{H'(m_t)}{H(m_t)}\left[\kappa(\bar{\theta} - m_t) + \rho\sigma_\theta\sigma_K + Q\right] \\ - \frac{\sigma_m^2}{2}\frac{\varphi\psi}{\alpha}\left[\frac{H''(m_t)}{H(m_t)} + \frac{\gamma-\psi}{\alpha}\left(\frac{H'(m_t)}{H(m_t)}\right)^2\right].\end{aligned} \tag{A.8}$$

Notice that the second order derivative of $H(m_t)$ can be substituted out using (A.1). The resulting expression of $\mu_C(m_t)$ is

$$\begin{aligned}\mu_C(m_t) = \psi\left(m_t - \beta - \frac{\varphi\gamma\sigma_K^2}{2}\right) - \frac{\varphi\psi(\gamma-\psi)\sigma_m^2}{2\alpha^2}\left(\frac{H'(m_t)}{H(m_t)}\right)^2 \\ - \frac{\varphi\psi\gamma}{\alpha}\frac{H'(m_t)}{H(m_t)}(\rho\sigma_\theta\sigma_K + Q).\end{aligned} \tag{A.9}$$

From Ito's lemma, the process for dividend growth is

$$\frac{\mathrm{d}D_t}{D_t} = \mu_D(m_t)\mathrm{d}t + \phi\sigma_K \mathrm{d}\tilde{B}_{K,t} - \frac{\phi\varphi\psi}{\alpha}\frac{H'(m_t)}{H(m_t)}\sigma_m \mathrm{d}\tilde{B}_{m,t} + \sigma_D \mathrm{d}B_{D,t}, \tag{A.10}$$

where

$$\mu_D(m_t) = \phi\mu_C(m_t) - A. \tag{A.11}$$

I conjecture that the equity price is separable in dividend, i.e., $S_t = D_t\Gamma(m_t)$. By Ito's lemma

$$\frac{\mathrm{d}S_t}{S_t} = \left(\mu_R^S(m_t) - \Gamma^{-1}(m_t)\right)\mathrm{d}t + \sigma_{S,K}\mathrm{d}\tilde{B}_{K,t} + \sigma_{S,m}\mathrm{d}\tilde{B}_{m,t} + \sigma_D \mathrm{d}B_{D,t}, \tag{A.12}$$

where

$$\sigma_{S,K} \equiv \phi\sigma_K, \tag{A.13}$$

$$\sigma_{S,m} \equiv \left(\frac{\Gamma'(m_t)}{\Gamma(m_t)} - \frac{\phi\varphi\psi}{\alpha}\frac{H'(m_t)}{H(m_t)}\right)\sigma_m \tag{A.14}$$

$$\begin{aligned}\mu_R^S(m_t) \equiv \mu_D(m_t) + \Gamma^{-1}(m_t) + \frac{\Gamma'(m_t)}{\Gamma(m_t)}\Bigg[&\kappa(\bar{\theta} - m_t) + \phi(\rho\sigma_\theta\sigma_K + Q) \\ &- \frac{\phi\varphi\psi}{\alpha}\frac{H'(m_t)}{H(m_t)}\sigma_m^2\Bigg] + \frac{\sigma_m^2}{2}\frac{\Gamma''(m_t)}{\Gamma(m_t)}.\end{aligned} \tag{A.15}$$

By equation (2.19), the equity risk premium satisfies

$$\mu_R^S(m_t) - r_t = \gamma \mathrm{Cov}_t\left(\frac{\mathrm{d}S_t}{S_t}, \frac{\mathrm{d}C_t}{C_t}\right) + (\varphi - \alpha)\mathrm{Cov}_t\left(\frac{\mathrm{d}S_t}{S_t}, \varphi^{-1}\frac{\mathrm{d}x_t}{x_t}\right). \tag{A.16}$$

Equation (A.16) characterizes the price-dividend ratio $\Gamma(m_t)$. It is an ODE of the form

$$0 = 1 + \lambda_0(m_t)\Gamma + \lambda_1(m_t)\Gamma' + \lambda_2\Gamma'', \tag{A.17}$$

where

$$\lambda_0(m_t) \equiv \mu_D(m_t) - r_t + \phi\Bigg[(\rho\sigma_\theta\sigma_K + Q)\left(1 + \frac{\varphi\psi\gamma}{\alpha}\right)\frac{H'(m_t)}{H(m_t)} \tag{A.18}$$

$$- \gamma\sigma_K^2 - \frac{\varphi\psi}{\alpha}\left(\frac{H'(m_t)}{H(m_t)}\right)^2\sigma_m^2\Bigg] \tag{A.19}$$

$$\lambda_1(m_t) \equiv \kappa(\bar{\theta} - m_t) + (\phi - \gamma)(\rho\sigma_\theta\sigma_K + Q) + \left(1 - \frac{\phi\varphi\psi}{\alpha}\right)\frac{H'(m_t)}{H(m_t)}\sigma_m^2 \tag{A.20}$$

$$\lambda_2 \equiv \frac{\sigma_m^2}{2}. \tag{A.21}$$

Appendix B

Appendix to Chapter 3

B.1 Measurement of Aggregate Quantities

The data used to measure aggregate quantities are from the National Income and Product Accounts compiled by the Bureau of Economic Analysis. I use annual observations over the period 1929-1998.
1. **Consumption** (C_t). Consumption is constructed as per capita nondurable consumption expenditure deflated by the NIPA nondurable consumption price index plus services deflated by the NIPA services price index.
2. **Investment** (I_t). Physical investment is measured by per capita private fixed investment deflated by the NIPA private fixed investment price index.
3. **Capital** (K_t). The capital stock series is chosen to match the investment series. Accordingly, I construct total capital stock as current cost of producer structures and equipment plus current cost private residential capital. Per capita capital is obtained by dividing the total capital stock by the size of population.
4. **Measured Output** (Y_t). Output is measured as the sum of consumption and investment. The other components of the actual output are excluded because they are not explicitly modeled in my theoretical economy.

B.2 Numerical Solution of the CIR Model

This appendix describes the procedure used in this paper for replicating the quantitative results of Ai (2010). The procedure involves exactly discretizing his model which is of the Cox et al. (1985) type. To provide details, I begin with a brief review of his model setup.

On the consumer side of the economy, there is a representative agent who is endowed with the continuous-time recursive preferences of Duffie and Epstein (1992b):

$$U_t = \mathbb{E}_t \int_t^\infty \frac{\beta}{\varphi}\big[C_\tau^\varphi (\alpha U_\tau)^{1-\varphi/\alpha} - \alpha U_\tau\big]\mathrm{d}\tau, \quad 0 \neq \alpha \leqslant 1,\ 0 \neq \varphi \leqslant 1, \quad \text{(B.1)}$$

where β is the rate of time-preference, $\gamma \equiv 1-\alpha$ is the coefficient of relative risk aversion, and $\psi \equiv (1-\varphi)^{-1}$ is the elasticity of intertemporal substitution.

On the production side, the capital stock K_t changes stochastically over time according to

$$\mathrm{d}K_t = \theta_t K_t \mathrm{d}t + \sigma_K K_t \mathrm{d}B_{K,t} - C_t \mathrm{d}t, \tag{B.2}$$

where $\mathrm{d}B_{K,t}$ is an i.i.d. shock, and the level of technology, θ_t, follows an Ornstein-Uhlenbeck process:

$$\mathrm{d}\theta_t = \kappa(\bar{\theta} - \theta_t)\mathrm{d}t + \sigma_\theta \mathrm{d}B_{\theta,t}, \tag{B.3}$$

with $\mathrm{d}B_{\theta,t}$ being independent of $\mathrm{d}B_{K,t}$.[20]

The equity is taken to be a levered claim on aggregate consumption, with cash flow stream evolving as

$$\frac{\mathrm{d}D_t}{D_t} = \phi \frac{\mathrm{d}C_t}{C_t} - A\mathrm{d}t + \sigma_D \mathrm{d}B_{D,t}. \tag{B.4}$$

Agents in this economy cannot observe the realizations of the productivity process $\{\theta_t\}_{t\geqslant 0}$, and they continuously update their assessment using the observation on capital stock derived from equation (B.2). Their estimate for θ_t evolves as

$$\mathrm{d}\hat{\theta}_t = \kappa(\bar{\theta} - \hat{\theta}_t)\mathrm{d}t + \frac{Q}{\sigma_K}\mathrm{d}\tilde{B}_{K,t}, \tag{B.5}$$

where $\mathrm{d}\tilde{B}_{K,t}$ and Q are, respectively, the innovation and the steady-state estimation error, given by

$$\mathrm{d}\tilde{B}_{K,t} = \frac{1}{\sigma_K}\left[\frac{(\mathrm{d}K_t + C_t\mathrm{d}t)}{K_t} - \hat{\theta}_t \mathrm{d}t\right], \tag{B.6}$$

$$Q = \frac{\sigma_\theta^2}{\kappa + \sqrt{\kappa^2 + \sigma_\theta^2 \sigma_K^{-2}}}. \tag{B.7}$$

The optimization problem of the representative agent is to choose consumption-investment plan so as to maximize expected lifetime utility. The corresponding HJB equation is

$$\max_{C_t}\left\{\frac{\beta}{\varphi}\left[C_t^{\varphi}(\alpha J(K_t,\hat{\theta}_t))^{1-\varphi/\alpha} - \alpha J(K_t,\hat{\theta}_t)\right] + \mathcal{L}J(K_t,\hat{\theta}_t)\right\} \tag{B.8}$$

[20] While Ai (2010) allows for a nontrivial correlation structure between $\mathrm{d}B_{\theta,t}$ and $\mathrm{d}B_{K,t}$, his quantitative analysis focuses on the case of zero correlation.

$$\text{s.t.} \qquad \mathrm{d}K_t = \hat{\theta}_t K_t \mathrm{d}t + \sigma_K K_t \mathrm{d}\tilde{B}_{K,t} - C_t \mathrm{d}t, \tag{B.9}$$

$$\mathrm{d}\hat{\theta}_t = \kappa(\bar{\theta} - \hat{\theta}_t)\mathrm{d}t + \frac{Q}{\sigma_K}\mathrm{d}\tilde{B}_{K,t}, \tag{B.10}$$

where $\mathcal{L}$ is a differential operator with respect to K_t and $\hat{\theta}_t$. Converting this problem into discrete time gives

$$J(K_t, \hat{\theta}_t) = \max_{C_t} \left\{(1-e^{-\beta\Delta})C_t^{\varphi} + e^{-\beta\Delta}\mathbb{E}_t\left[J(K_{t+\Delta}, \hat{\theta}_{t+\Delta})^{\alpha}\right]^{\varphi/\alpha}\right\}^{1/\varphi} \tag{B.11}$$

$$\text{s.t.} \qquad K_{t+\Delta} = \exp\left[\left(\hat{\theta}_t - f(\hat{\theta}_t) - \frac{\sigma_K^2}{2}\right)\Delta + \sigma_K \tilde{\epsilon}_{K,t+\Delta}\sqrt{\Delta}\right]K_t,$$

$$\hat{\theta}_{t+\Delta} = \bar{\theta}(1 - e^{-\kappa\Delta}) + e^{-\kappa\Delta}\hat{\theta}_t + \sigma_{\hat{\theta}}\tilde{\epsilon}_{K,t+\Delta},$$

where $f(\hat{\theta}_t) \equiv \frac{C_t}{K_t}$, and $\sigma_{\hat{\theta}} \equiv \frac{Q}{\sigma_K}\sqrt{(1 - e^{-2\kappa\Delta})/(2\kappa)}$.

It is straightforward to verify that the discrete-time solution converges to its continuous-time counterpart as time interval Δ gets small. In the computation, I choose $\Delta = 1/400$ under which my solutions are sufficiently close to Ai's. Solving the Bellman equation involves only the state variable $\hat{\theta}$. The other state variable K drops out because the Bellman equation is proportional to K. The Bellman equation is solved by modified policy iteration on a grid of $\hat{\theta}_t$ with 100 points. The algorithm iterates until the percentage change in $f(\hat{\theta}_t)$ is less than 10^{-5}.

Because output and investment are not explicitly defined in Ai (2010), I must define these variables before computing their moments. The definition used here follows Leland (1974) and Eaton (1981), who consider linear technology settings similar to the CIR model. Specifically, output is taken to be $Y_t = \hat{\theta}_t K_t$, which implies investment to be $I_t = \hat{\theta}_t K_t - C_t$. I also follow these authors to interpret the i.i.d. change in capital stock as stochastic depreciation. This interpretation is consistent with Ai (2010).

B.3 Decentralization

To explicitly define the assets the model is pricing, consider a decentralized formulation of the planner's problem (3.19). The decentralization scheme is in the style of Jermann (1998) and Donaldson and Danthine (2002). Assume that there is a representative investor who stands for a continuum of investors and a representative firm who behaves competitively. The investor and the firm are subject to the same informational constraint as the planner in the centralized setup. There is a risk-free asset and an equity claim to the firm's dividend stream. The equity claim is in positive supply, while the risk-free asset is in zero net supply.

The investor chooses to consume C_t out of his wealth W_t subject to the budget constraint

$$P_t S_t + H_t \leqslant W_t - C_t, \tag{B.12}$$

where P_t is the ex-dividend equity price, S_t is the number of shares of equity, H_t is the value of human capital. Suppose that human capital is completely tradeable.

To rewrite the budget constraint in wealth-return form, I must define wealth:

$$W_t \equiv (P_t + D_t) S_{t-1} + H_t + \varpi_t N_t, \tag{B.13}$$

where D_t denotes dividends, $\varpi_t N_t$ denotes labor income. In words, wealth at the beginning of period t is the sum of equities including dividends, and human capital including current labor income.

Multiplying both sides of the budget constraint by W_{t+1} and rearranging yields

$$W_{t+1} = \frac{(P_{t+1} + D_{t+1}) S_t + H_{t+1} + \varpi_{t+1} N_{t+1}}{P_t S_t + H_t} (W_t - C_t), \tag{B.14}$$

where the first term on the right hand side is the return on wealth from date t to date $t+1$, $R_{w,t+1}$. One can decompose the return on wealth into contributions from equities and human capital:

$$R_{w,t+1} = \pi_{e,t} R_{e,t+1} + \pi_{h,t} R_{h,t+1}, \tag{B.15}$$

where the returns and their respective weights are defined as $\pi_{e,t} = \frac{P_t S_t}{P_t S_t + H_t}$, $R_{e,t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}$, $\pi_{h,t} = \frac{H_t}{P_t S_t + H_t}$, and $R_{h,t+1} = \frac{H_{t+1} + \varpi_{t+1} N_{t+1}}{H_t}$. Then, the dynamic of wealth reads

$$W_{t+1} = (W_t - C_t) R_{w,t+1}. \tag{B.16}$$

The behavior of the investor is characterized by the solution to the following problem subject to the budget constraint (B.16):

$$U_t = \max_{C_t, \pi_{e,t}} \left\{ C_t^{\varphi} + \beta \mathbb{E}\left[U_{t+1}^{\alpha} | \mathcal{F}_t \right]^{\varphi/\alpha} \right\}^{1/\varphi}.$$

Then the first-order condition with respect to $\pi_{e,t}$ gives the Euler equation for the equity

$$1 = \mathbb{E}[M_{t+1} R_{e,t+1} | \mathcal{F}_t], \tag{B.17}$$

where M_{t+1} is the stochastic discount factor. The return on the risk-free asset is determined by

$$R_{f,t+1} = \frac{1}{\mathbb{E}[M_{t+1}|\mathcal{F}_t]}. \tag{B.18}$$

Euler equations (B.17) and (B.18) define the returns on the equity claim and the risk-free asset, with the intertemporal marginal rate of substitution M_{t+1} computed from the planner's equilibrium allocation.

Now let us turn to the firm's side. The firm begins period t with the stock of capital K_t carried over from the previous period. In each period, the firm has to decide how much to invest. Assume that the firm does not issue new shares and finances its investment exclusively through retained earnings. The dividends to investors are then equal to

$$D_t = Y_t - I_t - \varpi_t N_t.$$

In this setting of effectively complete markets, the firm's objective is clear: maximize at each point of time the cum-dividend stock market value of the firm $Q_t \equiv P_t + D_t$ which is equal to the present discounted value of all current and future expected cash flows. The representative firm's decision problem can be written as

$$Q(K_t, A_t, \tilde{\theta}_t, \tilde{z}_t) \equiv \max_{\{I_{t+s}\}_{s=0}^{\infty}} \mathbb{E}\left[\sum_{s=0}^{\infty} M_{t+s} D_{t+s} \middle| \mathcal{F}_t\right] \tag{B.19}$$

subject to the evolution of capital stock, total factor productivity and investors' beliefs. The corresponding optimality condition is

$$1 = \mathbb{E}\big[M_{t+1} R_{I,t+1}|\mathcal{F}_t\big], \tag{B.20}$$

where $R_{I,t+1}$ is the return to the firm's investment.

Market clearing in goods market requires that all produced goods are either consumed or invested such that $Y_t = C_t + I_t$. Thus, goods market clearing condition is identical to the planner's resource constraint. Financial markets also clear. In each period, the representative investor holds all firm shares and the claim on human capital.

B.4 Derivation of Equation (3.5)

This appendix uses the loglinear method of Campbell (1993) to derive equation (3.5) in Section 3.2.1. Note that the Euler equation for the consumption claim is

$$1 = \mathbb{E}_t \left[\beta^{\vartheta} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\vartheta}{\psi}} R_{w,t+1}^{\vartheta} \right], \tag{B.21}$$

where I use the definition $\vartheta \equiv \frac{\alpha}{\varphi}$ to lighten the notation.

The Euler equation (B.21) can be rewritten in log form by taking a second-order Taylor approximation. Its log version takes the form

$$0 \approx \vartheta \log \beta - \frac{\vartheta}{\psi} \mathrm{E}_t \Delta c_{t+1} + \vartheta \mathrm{E}_t r_{w,t+1} + \frac{1}{2} \left[\left(\frac{\vartheta}{\psi} \right)^2 \sigma_{c,t}^2 + \vartheta^2 \sigma_{w,t}^2 - \frac{2\vartheta^2}{\psi} \sigma_{cw,t} \right]. \tag{B.22}$$

Here, lower case letters indicate logs. $\sigma_{c,t}^2$ denotes $\mathrm{Var}_t(\Delta c_{t+1})$ and other expressions of the form σ_{xy} are defined in analogous fashion (with subscript w representing r_w).

Rewrite (B.22) as a linear relationship between expected consumption growth and expected return on wealth portfolio with a slope equal to the EIS ψ. This relationship is

$$\mathrm{E}_t \Delta c_{t+1} \approx \omega + \psi \mathrm{E}_t r_{w,t+1}, \tag{B.23}$$

where

$$\omega = \psi \log \beta + \frac{1}{2} \left[\left(\frac{\vartheta}{\psi} \right) \sigma_{c,t}^2 + \vartheta \psi \sigma_{w,t}^2 - 2\vartheta \sigma_{cw,t} \right].$$

Next, loglinearize the investor's budget constraint (Eq. (B.16)), and then solve it forward to get an expression for the log wealth-consumption ratio

$$w_t - c_t \approx \mathrm{E}_t \sum_{j=1}^{\infty} \varrho^j (\Delta c_{t+j} - r_{w,t+j}) + \frac{\varrho \kappa}{\varrho - 1}, \tag{B.24}$$

where ϱ and κ are constants given by $\varrho = 1 - \exp(\overline{c - w})$, $\kappa = \log \varrho + (1 - 1/\varrho) \log(1 - \varrho)$. Here lower case letters are used for logs. Substituting (B.23) into (B.24), the desired result follows.

B.5 Numerical Solution of the RBC Model

To make the planner's problem stationary, I divide all time-t variables by X_{t-1}. This works because the homogeneity of preferences, production function and capital accumulation equation implies that the value function is homogeneous of degree one in (K_t, X_{t-1}). The Bellman equation for the stationary problem is

$$\widehat{V}(\widehat{K}_t, \tilde{\theta}_t, \tilde{z}_t) = \max_{\widehat{C}_t} \left\{ (1-\beta)\widehat{C}_t^{\varphi} + \beta \mathbb{E}\Big[e^{\alpha \tilde{\theta}_t} \widehat{V}(\widehat{K}_{t+1}, \tilde{\theta}_{t+1}, \tilde{z}_{t+1})^{\alpha} \Big| \mathcal{F}_t \Big]^{\varphi/\alpha} \right\}^{1/\varphi}$$

subject to

$$\begin{aligned} \widehat{I}_t &= e^{\tilde{z}_t + \nu \tilde{\theta}_t} \widehat{K}_t^{1-\nu} - \widehat{C}_t, \\ \widehat{K}_{t+1} &= e^{\tilde{\theta}_t} \left[(1-\delta)\widehat{K}_t + G\left(\frac{\widehat{I}_t}{\widehat{K}_t} \right) \widehat{K}_t \right], \end{aligned}$$

and the informational constraint described by (3.18).

I solve the model by the method of modified policy iteration on a 3-dimensional discrete grid. The grid for K, $\tilde{\theta}$ and $\tilde{z}$ has 80, 15 and 15 elements, respectively. I find the consumption policy by iterating over the Bellman equation until percentage change in the policy function is less than 10^{-5}. Given the optimal consumption policy, the stationary price functional is solved numerically as a fixed point problem on the same grid until the percentage change in price function is less than 10^{-5}.