DIRECTED SEARCH FOR DIFFERENTIATED GOODS

by

CE HUANG

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Abstract

In three directed search models with horizontal differentiation, this thesis characterizes the unique symmetric equilibrium for each model and studies the welfare property of equilibrium allocations.

In Chapter 2, horizontal differentiation is modeled as buyers' valuations being independent. In equilibrium, sellers use a mixed strategy with the support consisting of a countable number of prices. Equilibrium price dispersion exists and equilibrium allocation is constrained inefficient due to price dispersion.

Chapter 3 extends the model in Chapter 2 by allowing different degrees of horizontal differentiation. With large degrees of horizontal differentiation, sellers use a mixed strategy qualitatively similar to the equilibrium in Chapter 2. With small degrees of differentiation, sellers use a pure strategy.

Chapter 4 extends the model in Chapter 2 by allowing differentiation to be endogenous. Initially buyers are equally uncertain about the characteristics of sellers' goods and no differentiation exists. Then sellers choose prices together with the amounts of information disclosed to buyers about the characteristics of sellers' goods. Information disclosure leads to differentiation after buyers receive the information. It is shown that a seller's profit by disclosing full information is higher than that by disclosing partial information. In equilibrium both sellers disclose full information and use a pricing strategy that is identical to the equilibrium in Chapter 2.

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Chapter 1

Introduction

In a textbook version of a Walrasian competitive economy, the market is frictionless. There is a centralized price setting authority, and everyone else in the market observes the price, takes it as given, and makes buying or selling decisions optimally. Then the price adjusts so that the desired aggregate demand is equal to the desired aggregate supply. However, in real life markets, such a centralized price setting authority is often absent, and the transactions are conducted in a decentralized manner. Sellers decide on their own prices, and buyers choose from which seller to buy. The market is fraught with frictions, which prevent the market from bringing traders together and exhausting all desirable trades. Frictions include the difficulties in informing potential traders about the trading opportunities and in coordinating traders' decentralized decisions. Frictions can also generate important regularities in prices. A single price prevailing in a frictionless Walrasian market is at odds with the empirical observation of price dispersion or wage dispersion for similar goods or workers.

Early literature departs from the frictionless Walrasian analyses by assuming that agents randomly search among the other side of market, so the probability that an agent meets someone on the other side depends on the total numbers of agents on two sides of the market[1]. This approach ignores that the matching process is not completely random and different agents on the same side of the market can have different matching probabilities. Another strand of literature assumes that agents

[1] Examples include Diamond [7], Diamond [8], Mortensen [14] and Pissarides [19].

on one side first choose their respective terms of trade. Then, upon observing the distribution of the offered terms of trade, agents on the other side repeatedly sample a trading partner and choose how many samplings he or she does. Each sampling involves a cost for the agent and therefore, if the sampling cost outweighs the likelihood of finding the best trading partner, the agent may stop sampling before he or she finds the best trading partner. This literature integrates agents' rational choices into the matching model. An assumption made is that agents do not know the identities of trading partners who offer different terms of trade, and therefore can only randomly sample among all potential trading partners. This assumption contradicts with the emergence of information technologies that significantly reduce agents' cost of acquiring the information on identities of trading partners and the observation in two-sided market that agents can effectively direct their search toward specific potential partners.

The seminal work of Peters [15] develops another way of modelling decentralized matching. The matching process is modelled as a two stage game where in the first stage agents on one side of the market choose their respective terms of trade[2]. After observing all chosen terms of trade and identities of trading partners offering these terms of trade, agents on the other side select trading partners to visit, taking into account their payoffs from trading with all potential partners and the possibilities of trading. The friction is captured by the assumption that agents decide which partner to visit in an uncoordinated manner, and if too many agents visit the same partner, some of them will not trade successfully. This literature is called "directed search", because each agent on one side of the market can choose the term of trade to "direct" the visiting behaviours of the other side and through doing so, influence the probability he can trade with someone.

Recent directed search models begin to allow agents on either side of the market to be heterogeneous in terms of the values of trading with each other, and the information on this trading value can be dispersedly possessed by agents[3]. The typ-

[2]More generally, agents can choose a general mechanism, which determines the allocation and transfer price as functions of messages received from all visiting traders.

[3]In a goods market where sellers and buyers are on either side of the market, Eeckhout and Kircher [10] studies the case that sellers and buyers can be heterogeneous. In a labour market where two sides are vacancies and job seekers, Shimer [22] and Peters [18] study the case that vacancies and job seekers have different types.

ical assumption is that heterogeneity is valued in the same way by different agents. In other words, agents are differentiated by "types". Regardless of the identity of an agent on one side of the market, matching with high type agents on the other side leads to higher value than matching with low type agents.

However, there are cases where agents are differentiated horizontally. In other words, there are no high type agents that are more desirable to be matched nor low type agents that are less desirable to be matched with. Instead, agents' desirabilities for matching can vary for different agents on the other side, possibly due to agents' differing tastes or needs. In a market where the two sides are buyers and sellers[4], sellers' goods can be horizontally differentiated. For some buyers, some sellers' goods are more desirable than others, while for other buyers, the former sellers' goods are less desirable than the latter sellers'.

This thesis focuses on a simple environment with 2 buyers and 2 sellers. I consider three models with different degrees of horizontal differentiation and study the equilibrium allocation in each model. In Chapter 2, sellers' goods are horizontally differentiated, modelled with buyers' valuations of sellers' goods being independent. Depending on their valuations, each seller has some buyers who prefer her over the other seller regardless of sellers' prices. They are "loyal" buyers of the seller. The remaining buyers are indifferent between sellers' goods and choose a seller based on price comparison. They are "shoppers". A seller faces a trade-off between choosing a high price to only attract loyal buyers and a low price to attract both loyal buyers and shoppers. This trade-off leads to the nonexistence of pure-strategy equilibrium between sellers. A mixed-strategy equilibrium between sellers is constructed and is proved to be the only symmetric equilibrium between sellers. The mixed-strategy equilibrium features price-dispersion, in which sellers choose different prices for ex ante identical products.

In Chapter 3, sellers' goods are also horizontally differentiated but with different degrees of differentiation, modelled as different correlations between buyers' valuations. In this case, when the differentiation between sellers' goods is small, the unique symmetric equilibrium between sellers exhibits a pure-strategy equilibrium between sellers and when the differentiation is large, the unique sym-

[4] Throughout this thesis, buyers are male and sellers are female.

metric equilibrium exhibits a mixed-strategy equilibrium. When the differentiation reaches the threshold value, the mixed-strategy equilibrium converges to the pure-strategy equilibrium.

Chapter 4 considers a similar model as in Chapter 2, but now allowing the differentiation between sellers' goods to be endogenous. At the very beginning, buyers have no information about their valuations of sellers' goods and regard sellers' goods as perfect substitutes. Then sellers choose both prices and the amount of information disclosed to buyers by choosing the informativeness of the signal buyers receive about their true valuation of the seller's good. After observing sellers' prices and signal informativeness, buyers privately observe signals from both sellers and simultaneously decide which seller to select. Information provision by sellers leads to differentiation between sellers' goods. The more information sellers disclose, the more differentiated the sellers' goods are for the two buyers as they receive different signals from the sellers. The model finds in the unique symmetric subgame perfect Nash equilibrium, both sellers disclose full information to buyers and randomize over a countable number of different prices.

1.1 Related Literature

As stated above, this thesis is related to the literature on directed search. In this literature, sellers post prices or selling mechanisms first, then after observing all prices or mechanism postings, buyers choose which seller to visit. Buyers' searches among sellers are "directed" by the prices or mechanisms sellers post.

Burdett et al. [2] studies a directed search model where buyers think all sellers' goods are perfect substitutes and all buyers have the same valuation for sellers' goods. Sellers post prices first, then buyers decide which seller to visit. In order to model that buyers visit sellers in an uncoordinated way, buyers have to use a symmetric strategy for visiting sellers. Friction is captured by assuming sellers have limited capacities so that if too many buyers visit a given seller, some buyers will not get the seller's goods. A subgame perfect Nash equilibrium is identified where sellers post the same price and buyers randomly visit sellers.

This thesis considers a variant of the model in Burdett et al. [2] by introducing heterogeneous buyers and sellers. Sellers have different characteristics denoted by a one-dimensional type and similarly for buyers. Depending on the realizations of sellers' types and buyers' types, a buyer may prefer one seller over the other, or may like or dislike both sellers equally. Sellers in this model first post their prices, then buyers choose which seller to select. There are frictions due to buyers' uncoordinated visiting strategies and the limited capacities of sellers. I identify the unique symmetric pricing strategy of sellers. This pricing strategy is a mixed strategy with a countable support.

This thesis is also related recent directed search models that also introduce heterogeneous agents. Eeckhout and Kircher [10] considers a case where buyers and sellers have different types. The difference between their paper and the current thesis is that if all sellers charge the same price, in their paper, all types of buyers would agree on the relative attractiveness between any two types of sellers, while in the current thesis, some types of buyers would prefer one type of seller over the other, and other types of buyers have the reverse preference. In this sense, their model is about vertical differentiation while this thesis is about horizontal differentiation. Another difference is that, in Eeckhout and Kircher [10], each side of market is made up of a continuum of agents. When a single seller considers a deviation from the equilibrium posting strategy, he believes that in order to attract any buyer he has to provide buyers with some "market utility", which he takes as independent from the deviation. In a series of papers, Peters [16] and Peters [17] study the important question of whether the subgame perfect equilibrium of directed search with a finite number of agents and the rational expectation equilibrium where a market utility is assumed to be given, converge to the same limiting equilibrium as the number of agents converges to infinity. However, the current thesis conducts a fully strategic analysis by explicitly solving for the buyers' responses to all price postings that can possibly be equilibrium prices. Sellers' payoffs are determined by buyers' responses in the continuation equilibrium, without the market utility assumption. In the labour market setting, Shimer [22] considers a directed search model for a job market where firms have different productivities and job seekers have different abilities, and firms can post a wage schedule that depend on the job seekers' abilities. Peters [18] considers a directed search model with heterogeneous firms and job seekers, but the wage posted does not depend on the job seekers' abilities. Peters [18] also allows for an arbitrary distribution of job seekers' abilities and firms' productivities and investigates which kinds of firms match which kinds of job seekers in equilibrium. In both models the heterogeneity among agents is valuated in the same way across all agents on the other side of the market. For all firms, regardless of their productivities, the output of matching with a high ability job seeker is higher than the output of matching with a low ability job seeker. The difference between a goods market and a labour market is that in a labour market, firms (who post the terms of trade) care about which type of job seekers they hire as well as the wage, while job seekers (who observe the terms of trade posted by firms and decide which firm to approach) only care about the terms of trade and the probability of trading. However, in a goods market, sellers (who post the terms of trade) only care about the terms of trade and the probability of selling products, while buyers (who observe the terms of trade posted by sellers and decide which seller to approach) care about both the terms of trade and which seller they match with.

One major question in the directed search literature is how to explain price or wage dispersion, the phenomenon where identical goods have different prices or where identical workers have different wages. A directed search framework with homogeneous agents is in general not useful to get price dispersion as an equilibrium phenomenon. Burdett et al. [2] gets an equilibrium where a single price is charged[5]. Most other papers on directed search either have to rely on heterogeneity or have to use ex-post price dispersion. Ex-post price dispersion refers to the case where although sellers offer identical selling mechanism, because buyers visit sellers randomly and the final price of the selling mechanism depends on the realized number of visiting buyers, sellers offering the same mechanism end up charging different prices[6]. However, the current thesis suggests that equilibrium price dispersion might be generated by a simple interaction between horizontal

[5]An exception is Delacroix and Shi [6] which studies on-the-job search of workers and gets wage dispersion for homogeneous workers and jobs. The wage dispersion is due to firms using a pure strategy in wage schedule which depend on the wage of workers' current jobs. Another exception is Kircher [12] which allows workers to apply for more than one vacancies and vacancies can contact more than one applicant.

[6]See Shi [21], Shimer [22] and Dickens et al. [9] for models that generate equilibrium wage dispersion with heterogeneous workers and firms, and see Albrecht et al. [1] and Camera and Selcuk [3] for models that generate ex-post wage (price) dispersion in a homogeneous agents setting with symmetric equilibrium strategies.

differentiation and directed search.

This thesis is also related to the literature explaining price dispersion or wage dispersion without using directed search. Varian [24] considers a model with two groups of buyers having different amounts of price information. Buyers without pricing information randomly pick a seller and buy if the price is below the buyer's reservation price, while the informed buyers know all sellers' prices and choose the lowest price seller. Varian's model features a symmetric mixed-strategy equilibrium where sellers randomize among all the prices in an interval. As informed buyers always buy from the lowest price seller, there are no point masses in the equilibrium mixed strategy. Rosenthal [20] considers a model where each seller has "loyal" buyers who will buy as long as the price is lower than the buyers' reservation price. Remaining buyers are "shoppers", who regard all products as perfect substitutes and buy at the lowest price seller. The author finds in equilibrium sellers use a mixed strategy in setting prices.

In the equilibrium of this thesis, the market is segmented into loyal buyers and shoppers. Similarly as in Rosenthal [20], the presence of loyals and shoppers negates a pure strategy equilibrium. The difference between Rosenthal [20] and this thesis is that in Rosenthal [20] sellers have unlimited capacities so that shoppers always select the lowest price seller. This makes the profit function discontinuous. To be an equilibrium, sellers use a mixed strategy with an interval support. In this thesis sellers are capacity-constrained so that shoppers randomize in selection between sellers. As a result sellers' profits are continuous in prices. Nonetheless I show there is a mixed-strategy equilibrium, which differs considerably from Rosenthal [20] in that the support of the equilibrium strategy is a countable set.

From a different angle, this thesis is related to papers which study the mixed-strategy equilibrium in games with payoff discontinuities or continuities. Dasgupta and Maskin [5] shows the general condition for a game with discontinuous payoffs to have a mixed-strategy equilibrium and identifies properties of the mixed-strategy equilibrium. If players' payoffs are discontinuous when players take the same action, the mixed equilibrium strategy cannot take that action with strictly positive probability. On the reverse side, Sinitsyn [23] studies an environment where players' payoffs are continuous in the action profile of all players and finds that there exists a mixed-strategy equilibrium with finite number of actions. This thesis studies a case where players' payoffs are continuous at almost all common actions taken by players, but are discontinuous at a particular common action. The mixed-strategy equilibrium exhibits a countable number of actions approaching the action where payoffs are discontinuous.

Chapter 2

Directed Search with Horizontal Differentiation

2.1 Introduction

In the current chapter, the horizontal differentiation is modelled as a buyer's valuation of a seller's good depends on whether the buyer's taste matches with the characteristics of the seller's good. Both buyer's taste and seller's characteristics are binary variables. Depending on whether a buyer's taste matches with two sellers' characteristics, buyers are segmented into four groups: Two groups of buyers whose tastes match one seller but not the other, a group of buyers whose tastes match both sellers, and a group of buyers whose tastes do not match either seller. In equilibrium, the group of buyers whose tastes match one seller but not the other always selects the first seller as long as the seller's price is lower than buyers' reservation price. The group of buyers whose tastes match both sellers select sellers by comparing the prices of two sellers. When the two sellers' prices are close to each other, these buyers randomize between selecting two sellers with the probability of selecting one seller strictly decreasing in the seller's price. When one seller's price is sufficiently lower than the other seller's, these buyers select the lower price seller with probability 1.

From a seller's point of view, her potential buyers consist of two groups of buyers. One group of buyers have tastes that match the seller's characteristics but do not match the opponent seller's. These buyers are loyal buyers for the seller. Another group of buyers have tastes that match both sellers' characteristics. These buyers are called "shoppers" and they compare sellers' prices to decide which seller to select in the way described in the previous paragraph. A seller faces a choice of whether to compete for shoppers and if so, how many shoppers to attract. Attracting shoppers is good for the seller's probability to sell, but has a cost that a seller has to choose a lower price than opponent's. When the opponent seller's price is high, a price slightly lower than the opponent's can attract most shoppers because at a high price level these buyers are sensitive to the price difference between sellers. The seller's cost of attracting shoppers is low and therefore, her best response is to choose a price lower than the opponent and get all shoppers. When the opponent seller's price is low, to attract shoppers a seller has to choose a price much lower than the opponent's. This is because at a low price level shoppers are much less sensitive to price difference between sellers, and to attract these buyers a seller has to give them a sufficiently higher trading surplus than the opponent seller does. As the cost of attracting shoppers is very high, the seller's best response is to give up shoppers and to choose a price equal to the buyers' reservation price. This best response to the opponent's price leads to the nonexistence of a pure-strategy equilibrium in the sellers' game.

This chapter finds there exists a symmetric mixed-strategy equilibrium between sellers. The support of the equilibrium mixed strategy consists of a countable number of prices. For any two prices $p' > p$ in the support of the equilibrium strategy, if a shopper is offered the choice between p and p', he will select p for sure. At the same time, for any price p in the support, there is a $p < p''$, such that if a shopper has the choice between p and p'', he will select both with strictly positive probabilities[1]. The equilibrium strategy can be characterized by the first order condition for a seller to choose each price and the condition for the seller to get equal profit by choosing all prices.

In addition, this equilibrium is the only symmetric equilibrium in the sellers' game. To establish this, I show that all symmetric equilibria must have a countable support. All prices and the probabilities with which these prices are used are deter-

[1]This is different from Rosenthal [20] where shoppers always choose the lower price for sure.

mined by the fact that each price must satisfy a first order condition and the fact that each price must give the seller the same expceted profit. Given the characterized mixed-strategy equilibrium is the only mixed strategy satisfying these conditions, there do not exist symmetric equilibria, other than the characterized mixed-strategy equilibrium.

I show there cannot be a symmetric equilibrium with a finite support. If there was , the highest price would be either equal to the buyers' reservation price or lower. The first possibility cannot hold because if the opponent seller has a strictly positive probability of charging the buyers' reservation price, a seller has an incentive to charge a slightly lower price and get a discretely larger probability of selling by attracting all shoppers. The second possibility cannot hold either. If the highest possible price the opponent seller charges is strictly lower than the buyers' reservation price, a seller has either of two profitable deviations. If the opponent seller charges the highest price with a high probability, the seller wants to charge just slightly below that highest price. If the opponent charges the highest price with a low probability, the seller is better off by charging buyers' reservation price rather than the highest price of the opponent seller because at both prices the seller will not get shoppers. The seller's profit is higher by charging the higher price among these two prices, which is buyers' reservation price.

If there is an infinite number of prices converging to the buyers' (common) reservation price, as prices rise, the difference between any two adjacent prices becomes smaller. At the same time, shoppers' probability to visit sellers is more sensitive to price and the requirement on the distance between two prices to make these buyers visit the lower price is becoming smaller at a faster rate. Even at high prices, there is a price difference between any two prices such that shoppers would choose the lower price seller.

This chapter is organized as follows: Section 2.2 describes the model. Section 2.3 explicitly constructs a symmetric equilibrium where each seller randomizes over a countable number of prices. Section 2.4 proves the constructed equilibrium is the only symmetric equilibrium in the sellers' game. Section 2.5 discusses the welfare property of the characterized equilibrium and shows it is not constrained efficient. Section 2.6 gives some concluding remarks.

2.2 The Model

There are two sellers, A and B, and two buyers, 1 and 2. Each seller has one unit of indivisible good for which the seller's reservation price is 0. Each buyer wants to buy one unit of the good. A buyer's valuation of a seller's good can be either 0 or 1 with equal probabilities. Buyers' valuations are independent both across two sellers for a given buyer and across two buyers for a given seller. A buyer's valuations for two sellers' goods are known only to the buyer.

The model has three stages. At stage 1, two sellers simultaneously choose their prices, p_A and p_B. At stage 2, buyers observe the prices chosen by sellers, then, knowing their own valuations of two sellers' goods, simultaneously decide which seller to select. Each buyer can select at most one seller and can choose not to select any seller. At stage 3, the exchange happens. If only one buyer selects a seller, the seller sells the good to the buyer at the seller's price. If two buyers select the same seller, the seller randomly chooses one buyer and sells the good to the chosen buyer at the seller's price; the unchosen buyer will not get the good, nor will she be charged. If no buyer selects a seller, the seller keeps her good.

Sellers and buyers are risk-neutral. A seller's profit is her expected revenue, which is the seller's price multiplied by the probability of selling the good. The probability of selling is equal to the probability that at least one buyer selects the seller. A buyer's expected payoff from selecting a seller is the buyer's net valuation of the seller's good (the buyer's valuation of the seller's good subtracted by the seller's price) multiplied by the probability the buyer can get the seller's good, conditional on the buyer's valuations for the two sellers' goods. If the buyer does not select any seller, the buyer gets payoff 0.

The assumption that each buyer cannot select more than one seller implies if a buyer selects a seller but fails to match with the seller, the buyer fails to match with any seller. The setup that the seller randomly chooses one buyer to match with when the seller is selected by two buyers implies a strategic interaction between the two buyers in the sense that a buyer's probability of matching with a seller depends on the probability that the other buyer selects the same seller[2].

[2]This is different from Bertrand price competition models, where a buyer's payoff of buying from a seller is independent from whether or not the other buyers are buying from the same seller. This difference leads to buyers' different equilibrium response to sellers' prices, which in turn gives

This chapter focuses on symmetric subgame perfect Nash equilibrium of the game. Two buyers, if having the same valuations of two sellers' goods, use the same strategy in selecting sellers. Two sellers use the same strategy in choosing prices. The focus on symmetric strategy is due to the assumption that buyers cannot coordinate in selecting sellers and sellers cannot coordinate in choosing prices.

For any price profile by two sellers, a buyer's strategy is the probabilities of selecting sellers in each realization of the buyer's valuations of two sellers' goods,

$$\left\{ \begin{array}{c} \left(\theta_{00}^{A}(p_A,p_B),\theta_{00}^{B}(p_A,p_B)\right),\left(\theta_{01}^{A}(p_A,p_B),\theta_{01}^{B}(p_A,p_B)\right) \\ \left(\theta_{10}^{A}(p_A,p_B),\theta_{10}^{B}(p_A,p_B)\right),\left(\theta_{11}^{A}(p_A,p_B),\theta_{11}^{B}(p_A,p_B)\right) \end{array} \right\}$$

where $\theta_{st}^{j}(p_A,p_B)$ is, when seller A chooses price p_A and seller B chooses price p_B, the buyer's probability of selecting seller $j \in \{A,B\}$ when he has valuation s for seller A's good and valuation t for seller B's good. $1-\theta_{st}^{A}(p_A,p_B)-\theta_{st}^{B}(p_A,p_B)$ is the probability of choosing not to select any seller. $\theta_{st}^{A}(p_A,p_B)+\theta_{st}^{B}(p_A,p_B)\leq 1$.

Because the two buyers' valuations are independent, a buyer's knowledge of his own valuations does not give any information regarding the other buyer's valuations. Therefore regardless of what the buyer's valuations of two sellers' goods are, he thinks the other buyer's valuations of sellers' goods can be one of the four possible realizations, $\{(0,0),(0,1),(1,0),(1,1)\}$, with equal probabilities. Given this belief and the other buyer's strategy,

$$\left\{ \begin{array}{c} \left(\widetilde{\theta}_{00}^{A}(p_A,p_B),\widetilde{\theta}_{00}^{B}(p_A,p_B)\right),\left(\widetilde{\theta}_{01}^{A}(p_A,p_B),\widetilde{\theta}_{01}^{B}(p_A,p_B)\right) \\ \left(\widetilde{\theta}_{10}^{A}(p_A,p_B),\widetilde{\theta}_{10}^{B}(p_A,p_B)\right),\left(\widetilde{\theta}_{11}^{A}(p_A,p_B),\widetilde{\theta}_{11}^{B}(p_A,p_B)\right) \end{array} \right\},$$

the probability that the other buyer selects seller $j \in \{A,B\}$ is

$$\widetilde{\theta}^{j}(p_A,p_B)\equiv \tfrac{1}{4}\left(\widetilde{\theta}_{00}^{j}(p_A,p_B)+\widetilde{\theta}_{10}^{j}(p_A,p_B)+\widetilde{\theta}_{01}^{j}(p_A,p_B)+\widetilde{\theta}_{11}^{j}(p_A,p_B)\right),$$

as the probability that the other buyer selects seller j when he has valuation (s,t) is $\widetilde{\theta}_{st}^{j}(p_A,p_B)$ and the probability that the other buyer has valuation (s,t) is $\dfrac{1}{4}$.

Because seller j has only one good, if the other buyer selects seller j, a buyer's probability to match with seller j is $\frac{1}{2}$. If the other buyer does not select j, the buyer's probability to match with seller j is 1 . If the probability that the other buyer selects seller j is $\widetilde{\theta}^{j}(p_A,p_B)$, the buyer's probability to match with seller j is

different equilibrium pricing strategies of sellers.

$$\tfrac{1}{2}\widetilde{\theta}^j(p_A,p_B)+1-\widetilde{\theta}^j(p_A,p_B)=1-\tfrac{1}{2}\widetilde{\theta}^j(p_A,p_B)$$

If a buyer has valuation 1 for seller j's good, his net valuation for j's good is $1-p_j$. Therefore, given the other buyer's strategy, the buyer's expected payoff from selecting seller j is

$$(1-p_j)(1-\tfrac{1}{2}\widetilde{\theta}^j(p_A,p_B)).$$

If a buyer has valuation 0 for seller j's good, his net valuation for j's good is $-p_j$, and his expected payoff from selecting seller j is

$$-p_j(1-\tfrac{1}{2}\widetilde{\theta}^j(p_A,p_B)).$$

After characterizing a buyer's payoff as functions of the other buyer's strategy, we can find equilibria between buyers. Throughout the chapter, we focus on symmetric equilibria between buyers, based on the assumption that buyers cannot coordinate their strategies. This is a standard assumption in directed search literature.

Suppose in a symmetric continuation equilibrium in the buyers' game, both buyers select seller j with probability $\theta^j(p_A,p_B)$. Seller j's probability of selling her good is

$$1-\left(1-\theta^j(p_A,p_B)\right)^2$$

And seller j's expected profit is

$$\left[1-\left(1-\theta^j(p_A,p_B)\right)^2\right]p_j$$

It is straightforward to show that sellers will not charge prices strictly less than 0, the lowest possible valuation of buyers, or prices strictly greater than 1, the highest possible valuation of buyers. I focus on the subgames where prices of both sellers are between 0 and 1.

2.3 Construction of Equilibrium

The main result of this chapter is stated here.

Proposition 1. *There exists a symmetric subgame perfect Nash equilibrium where a seller's strategy is to randomize over a countably infinite number of prices.*

To prove the proposition, we first characterize buyers' symmetric continuation equilibrium for every profile of two sellers' prices, as summarized in Lemma 1. Taking the characterized buyers' continuation equilibrium as given, we get each seller's profit associated with two sellers' price choices. Then by solving a set of equations, we find a mixed strategy in choosing prices, denoted by $\left\{\left(p_k^*, x_k^*\right)\right\}_{k=1}^{\infty}$, with the probability of choosing p_k^* being x_k^* for all k. At last, we show that $\left\{\left(p_k^*, x_k^*\right)\right\}_{k=1}^{\infty}$ is a symmetric equilibrium in the sellers' game, which completes the proof of the proposition.

We first explain buyers' equilibrium strategy with the following definition.

Definition 1. *Denote* $\underline{p}(p) \equiv \frac{7}{6}p - \frac{1}{6}$ *and* $\overline{p}(p) \equiv \frac{6}{7}p + \frac{1}{7}$. *Say* p' *locally competes with* p *when* p' *satisfies* $\underline{p}(p) \leq p' \leq \overline{p}(p)$.

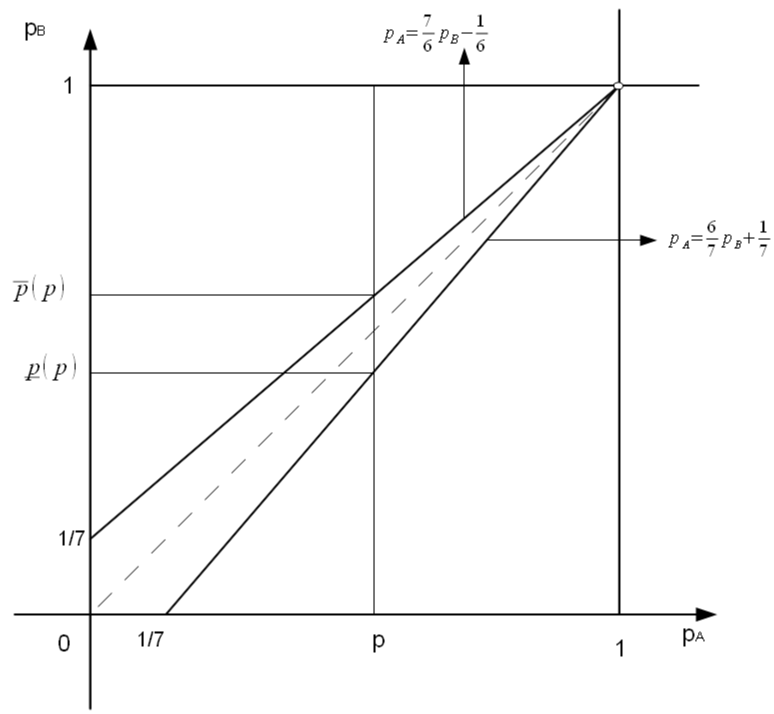
The reason we say two sellers' prices locally compete with each other when two sellers' prices satisfy $\underline{p}(p_B) \leq p_A \leq \overline{p}(p_B)$ is, in buyers' continuation equilibrium in the subgame following (p_A, p_B), a buyer will randomize between selecting two sellers when he has valuation 1 for both sellers, with the probability of selecting a seller strictly decreasing in the seller's price and strictly increasing in the opponent seller's price.

The range of prices that locally compete with a price $0 < p < 1$ is shown in Figure 2.1. It shrinks as p increases. As p converges to 1, the measure of locally competing prices converges to 0 with only price 1 locally competing with price 1 itself.

The following lemma shows the buyers' symmetric continuation equilibrium.

Lemma 1. *In the subgames where sellers choose* $0 \leq p_A \leq 1$ *and* $0 \leq p_B \leq 1$*, there exists a symmetric equilibrium in the buyers' game such that*

Figure 2.1: Prices that Locally Compete with $0 < p < 1$



$$\left\{\begin{array}{l} \theta_{00}^{A*}(p_A,p_B)=0,\theta_{00}^{B*}(p_A,p_B)=0 \\ \theta_{01}^{A*}(p_A,p_B)=0,\theta_{01}^{B*}(p_A,p_B)=1 \\ \theta_{10}^{A*}(p_A,p_B)=1,\theta_{10}^{B*}(p_A,p_B)=0 \end{array}\right\},$$

$$\theta_{11}^{A*}(p_A,p_B)=\begin{cases} 1 & if \quad p_A < \underline{p}(p_B) \\ \dfrac{7(1-p_A)-6(1-p_B)}{(1-p_A)+(1-p_B)} & if \quad \begin{array}{c} \underline{p}(p_B) \le p_A \le \overline{p}(p_B) \\ and\ p_A \neq 1\ or\ p_B \neq 1 \end{array} \\ 0 & if \quad p_B < \underline{p}(p_A) \\ \dfrac{1}{2} & if \quad p_A = p_B = 1 \end{cases}$$

$and\ \theta_{11}^{B*}(p_A,p_B)=1-\theta_{11}^{A*}(p_A,p_B).$

If two buyers both use the symmetric equilibrium strategy as stated in Lemma

1, seller A's probability to sell is

$$\Pi(p_A, p_B) \equiv 1 - \left[1 - \frac{1}{4}\left(1 + \theta_{11}^{A*}(p_A, p_B)\right)\right]^2,$$

which is strictly increasing in $\theta_{11}^{A*}(p_A, p_B)$.

Therefore, when $p_A < \underline{p}(p_B)$, $\Pi(p_A, p_B) = \frac{3}{4}$, which is the maximum level of $\Pi(p_A, p_B)$, denoted as $\overline{\Pi}$.

When $\underline{p}(p_B) \leq p_A \leq \overline{p}(p_B)$ and $p_A \neq 1$ *or* $p_B \neq 1$,

$$\Pi(p_A, p_B) = \frac{13}{4}(1 - p_B)\frac{2(1 - p_A) - \frac{5}{4}(1 - p_B)}{((1 - p_A) + (1 - p_B))^2};$$

When $p_B < \underline{p}(p_A)$, $\Pi(p_A, p_B) = \frac{7}{16}$, which is the minimum level of $\Pi(p_A, p_B)$, denoted as $\underline{\Pi}$.

When $p_A = p_B = 1$, $\Pi(p_A, p_B) = \dfrac{39}{64}$.

It is straightforward to show that $\Pi(p_A, p_B)$ is continuous at all (p_A, p_B) such that $0 < p_A \leq 1$ and $0 < p_B \leq 1$ except $(1, 1)$.

Consider a mixed strategy of a seller, denoted as $\{(p_k, x_k)\}_{k=1}^{\infty}$ where p_k is chosen with probability x_k, such that for all k, $0 < p_k < p_{k+1} < 1$ and p_k does not locally compete with p_{k+1}. Because $\{p_k\}_{k=1}^{\infty}$ has an infinite number of prices and $0 < p_k < p_{k+1} < 1$ for all k, $\{p_k\}_{k=1}^{\infty}$ must converge to some price less than or equal to 1. This means for a sufficiently large k, p_k is very close to p_{k+1}. To satisfy that p_k does not locally compete with p_{k+1} despite p_k being very close to p_{k+1} for sufficiently large k, p_k has to be very close to 1 to make the measure of p_k's locally competing region close to 0. In other words, $\{p_k\}_{k=1}^{\infty}$ must converge to 1. Note that price 1 is chosen with probability 0.

Based on these properties of $\{(p_k, x_k)\}_{k=1}^{\infty}$, consider two equations. The first equation requires that when the opponent seller adopts the mixed strategy $\{(p_k, x_k)\}_{k=1}^{\infty}$, a seller's profit by choosing price p_k is equal to her profit by choosing price 1. As price 1 is chosen with probability 0 by the opponent seller, the seller's profit by choosing price 1 is $\underline{\Pi}$. The second equation requires that the seller has no profitable local deviation from choosing p_k.

$$\left[\underline{\Pi}\sum_{i=1}^{k-1}x_i+\Pi(p_k,p_k)x_k+\overline{\Pi}\left(1-\sum_{i=1}^{k}x_i\right)\right]p_k=\underline{\Pi} \tag{2.1}$$

$$\underline{\Pi}\sum_{i=1}^{k-1}x_i+\left[\Pi(p_k,p_k)x_k+\frac{\partial\Pi\left(p_k,p_k\right)}{\partial p_A}p_kx_k\right]+\overline{\Pi}\left(1-\sum_{i=1}^{k}x_i\right)=0 \tag{2.2}$$

As there is no local competition between p_l and p_{l+1} for all $l\in N$, when a seller chooses price p_k and the opponent seller chooses any price among $p_1,p_2,...,p_{k-1}$, the probability that the seller can sell is $\underline{\Pi}$. If the opponent seller chooses p_k, the probability that the seller can sell is $\Pi(p_k,p_k)$. When the opponent seller chooses any price among $p_{k+1},p_{k+2},...$, the probability that the seller can sell is $\overline{\Pi}$. Therefore the expected probability that the seller can sell by choosing p_k is the term in the square bracket on the left hand side of (2.1).

As the seller slightly changes her price away from p_k, if the opponent seller chooses any price among $p_1,p_2,...,p_{k-1}$, the probability that the seller can sell is not affected by the seller's price change and the marginal profit change associated with the seller's price change is equal to the probability of selling $\underline{\Pi}$. If the opponent seller chooses price p_k, the probability that the seller can sell is affected with the marginal change being $\dfrac{\partial\Pi\left(p_k,p_k\right)}{\partial p_A}$ therefore the marginal profit change is the probability of selling $\Pi(p_k,p_k)$ plus $\dfrac{\partial\Pi\left(p_k,p_k\right)}{\partial p_A}p_k$, which is the profit change associated with the change in probability of selling. If the opponent seller chooses any price among $p_{k+1},p_{k+2},...$, the probability that the seller can sell is not affected and the marginal profit change is again equal to the probability of selling $\overline{\Pi}$. Therefore the marginal change in the seller's expected profit is the left hand side of (2.2).

The following lemma shows that the above two equations can fully characterize a mixed strategy. In addition, the characterized mixed strategy satisfies the properties that there is no local competition between any two adjacent prices and price 1 is chosen with probability 0.

Lemma 2. *(2.1) and (2.2) have a unique solution, denoted as* $\left\{\left(p_k^*,x_k^*\right)\right\}_{k=1}^{\infty}$, *such* *that for all k,* $0 < p_k^* < p_{k+1}^* < 1$ *and* $0 < x_k^*$*. Also* $lim_{k\to\infty} p_k^* = 1$ *and* $\sum_{k=1}^{\infty} x_k^* = 1$*. In addition, there is no local competition between* p_k^* *and* p_{k+1}^* *for all k.*

Note that $\sum_{k=1}^{\infty} x_k^* = 1$ implies $lim_{k\to\infty} x_k = 0$, therefore price 1 is chosen with probability 0.

We can prove a stronger result, which says there is no overlapping between the locally competing regions of p_k^* and that of p_{k+1}^* for all k.

Lemma 3. *The solution to equations (2.1) and (2.2) satisfies* $\overline{p}\left(p_k^*\right) < \underline{p}\left(p_{k+1}^*\right)$ *for all k.*

The next lemma shows that the characterized mixed strategy is a symmetric equilibrium, which complete the proof of Proposition 1.

Lemma 4. $\left\{\left(p_k^*, x_k^*\right)\right\}_{k=1}^{\infty}$ *is a symmetric equilibrium in the sellers' game.*

When the opponent seller uses mixed strategy $\left\{\left(p_k^*, x_k^*\right)\right\}_{k=1}^{\infty}$, a seller's profit by choosing p such that p is in local competing region of p_k^* for some k can be shown to be strictly concave function in p. Therefore the first order condition for the seller to choose p_k^* implies that the seller's profit by choosing p_k^* is greater than the seller's profit by choosing any other price in the local competing region of p_k^*. When the seller's price p is in the range $\left[\overline{p}\left(p_{k-1}^*\right), \underline{p}\left(p_k^*\right)\right]$, the seller's profit can be shown to be a constant multiplied by p, and therefore strictly increasing in p. Because the seller's profit is continuous at $\underline{p}\left(p_k^*\right)$, we can show the seller's profit by choosing p is less than her profit by choosing p_k^*. Because any $0 < p < 1$ must either fall into the local competing region of p_k^* or the gap between the local competing regions of p_{k-1}^* and p_k^* for some k , and the seller receives the same profit by choosing p_k^* for all k as the profit by choosing price 1, we have the seller's profit by choosing any p such that $0 < p \leq 1$ is less than or equal to the seller's profit by choosing p_k^*. Therefore $\left\{\left(p_k^*, x_k^*\right)\right\}_{k=1}^{\infty}$ is a symmetric equilibrium in the sellers' game.

2.4 Uniqueness of Equilibrium

We establish a stronger claim that the characterized mixed strategy in the last section is the only symmetric equilibrium in the sellers' game, given buyers' symmetric continuation equilibrium as in Lemma 1.

Proposition 2. *The equilibrium established in Proposition 1 is the only symmetric equilibrium between sellers.*

The proposition establishes that price dispersion is a necessary outcome of horizontal differentiation in directed search introduced in this chapter.

The proof of Proposition 2 follows a sequence of lemmas.

Lemma 5. *Any symmetric equilibrium between sellers is in a mixed strategy.*

Intuitively, buyers with valuation 1 for seller A and valuation 0 for B are the loyal buyers of seller A because they will always select seller A. Similarly, buyers with valuation 0 for seller A and valuation 1 for B are the loyal buyers of seller B. Buyers with valuation 1 for both sellers decide which seller to select based on comparison between sellers' prices. Call them shoppers. Sellers face a trade-off in choosing prices. A high price can extract more surplus from loyal buyers, but will turn away shoppers and lead to a low probability of selling. A low price can attract shoppers and lead to a high probability of selling. When the opponent seller's price is high, choosing a price lower than the opponent to attract shoppers is more profitable than choosing price 1 and attracting only loyal buyers. When the opponent seller's price is low, choosing a lower price to attract shoppers is less profitable than choosing price 1 and attracting the loyal buyers only. Therefore, as the opponent seller's price decreases, the seller's best response will change from choosing a price lower than the opponent's to choosing price 1. The discrete jump in the seller's best response leads to the nonexistence of a pure-strategy pricing equilibrium.

Lemma 6. *In any symmetric equilibrium in the sellers' game, price 1 is chosen with probability 0.*

To see why in any symmetric equilibrium strategy a seller cannot choose 1 with strictly positive probability, consider a seller who chooses price 1 with strictly positive probability. If the other seller chooses price 1, since a shopper gets payoff 0 from selecting either seller, he selects either seller with equal probabilities. If the other seller chooses a price slightly less than 1, since the shopper gets strictly positive payoff from selecting the other seller and gets payoff 0 from selecting the seller, he selects the other seller with probability 1. So when the seller chooses price 1 with strictly positive probability, by choosing a price slightly less than 1, the other seller can attract shoppers and get a discretely larger probability of selling and discretely larger profit than by choosing price 1. Therefore, for the other seller, choosing price 1 with strictly positive probability cannot be a best response. This implies in any symmetric equilibrium in the sellers' game, price 1 cannot be chosen with strictly positive probability.

Lemma 7. *In any symmetric equilibrium between sellers, there is no local competition between any two prices in the support of equilibrium strategy.*

To see why Lemma 7 is true, suppose there are two prices in a symmetric equilibrium's support, and there is local competition between these two prices. Because the derivative of a seller's profit with respect to the seller's price can be easily shown to be strictly less than $\overline{\Pi}$ and strictly decreasing in the seller's price, we can show the first order condition cannot hold at both prices. In particular, when the first order condition holds at the lower price, a seller has an incentive to slightly decrease her price below the higher price.

Lemma 8. *In any symmetric equilibrium in the sellers' game, the supremum of equilibrium strategy support is equal to 1.*

To see why Lemma 8 is true, suppose the support of a symmetric equilibrium has a supreme strictly less than 1. From Lemma 7, the support must consist of a finite number of prices, because if the support consists of an infinite number of prices and the price sequence has a supreme strictly less than 1, for sufficiently large k, p_k is in local competition with p_{k+1}, which violates Lemma 7. For a finite number of prices, there must be a highest price. For the mixed strategy to be a symmetric equilibrium, a seller has no incentive to deviate from choosing the highest price to choosing price 1, and also the first order condition must be satisfied at the highest price. However, the appendix shows these two conditions cannot be both satisfied. In particular, if a seller has no incentive to deviate to choosing price 1, the seller has an incentive to slightly decrease her price from the highest price in the support.

Proposition 2 follows from the above four lemmas. These lemmas imply any symmetric equilibrium in the sellers' game takes the form $\{(p_k, x_k)\}_{k=1}^{\infty}$ with the probability of charging price p_k being x_k and $lim_{k\to\infty} p_k = 1$. To be a symmetric equilibrium, $\{(p_k, x_k)\}_{k=1}^{\infty}$ has to satisfy necessary equilibrium conditions, i.e., when the opponent seller takes strategy $\{(p_k, x_k)\}_{k=1}^{\infty}$, a seller's profit by choosing p_k is the same for all $k \in N$, and the seller has no profitable local deviation away from choosing p_k. Given p_k does not locally compete with p_{k+1} for all $k \in N$, these two necessary conditions are just equations (2.1) and (2.2). Therefore, $\{(p_k, x_k)\}_{k=1}^{\infty}$ must also be a solution to these equations. Because the equations have a unique solution $\left\{\left(p_k^*, x_k^*\right)\right\}_{k=1}^{\infty}$, $\{(p_k, x_k)\}_{k=1}^{\infty}$ must be identical to $\left\{\left(p_k^*, x_k^*\right)\right\}_{k=1}^{\infty}$.

Therefore, there do not exist symmetric equilibria other than $\left\{\left(p_k^*, x_k^*\right)\right\}_{k=1}^{\infty}$.

2.5 Welfare Analysis

Now that we have characterized the unique symmetric equilibrium of the game in which sellers use the mixed strategy established in Proposition 1 and buyers use the strategy for selecting sellers as stated in Lemma 1, we can characterize the expected total surplus generated by the equilibrium allocation. In particular, we are interested in the following question: whether the equilibrium allocation maximizes the total surplus.

Consider a social planner, who can observe buyers' valuations and can also decide which seller the buyers select, but is constrained to decide a buyer's selection of sellers only based on the buyer own valuations (not based on the valuation profile of two buyers). In addition, the social planner must assign the two buyers the same selection action if they have the same valuations. In other words, the selecting actions of buyers are constrained to be symmetric and the social planner cannot coordinate selection actions of buyers. We consider the social planner's problem of the optimal selection action of buyers that maximizes the total surplus under the above constraints, and get the maximum value of the total surplus.

By comparing the total surplus under equilibrium allocation with that under social planner, we get the following result.

Proposition 3. *The symmetric subgame perfect Nash equilibrium generates a total surplus strictly less than the total surplus by a social planner who cannot coordinate buyers' selection actions.*

It is well known that in directed search models, when the social planner can not coordinate buyers' selection actions among sellers, letting buyers independently randomize among sellers with equal probability of selecting each seller maximizes total welfare. To see why the total surplus of the equilibrium allocation is less than the total surplus of the social planner, note that in the equilibrium sellers use a mixed strategy in setting prices, so there is a positive probability that one seller charges a lower price than the other seller. If both buyers have valuation 1 for two sellers, they both select the lower price seller and only one of them can get the seller's good. In other words, because in equilibrium buyers' selections are based on sellers' prices that are observed by both buyers, the buyers' selections become positively correlated with each other. Buyers are therefore more likely to select the same seller than they are when their selections are completely independent, which is the case in the social planner's problem.

2.6 Concluding Remarks

This chapter investigates a directed search model in which sellers simultaneously choose prices then buyers simultaneously select sellers without coordination. Sellers' goods are horizontally differentiated and buyers have independent valuations for sellers' goods. In the unique symmetric equilibrium, sellers use a mixed strategy where a countable number of prices can be charged. There is equilibrium price dispersion. The price dispersion makes buyers more likely to select the same seller and the equilibrium allocation becomes constrained inefficient.

We can consider an extension of the current model to a directed search model with more than two sellers and two buyers. Also, the buyers' valuation space can be more general. A conjecture is that there still exists a similar mixed-strategy equilibrium because the result of the current chapter seems to hinge on the directed search of buyers, but not on the particular details of the number of buyers and sellers and the valuation space of buyers. Explicitly verifying this conjecture is the direction for future work.

Another direction for future work is to relax the restriction that sellers choose take-it-or-leave-it prices to more general selling mechanisms, such as auctions.

Chapter 3

Pricing Differentiated Products in Directed Search

3.1 Introduction

Similarly with Chapter 2, this chapter considers sellers' symmetric equilibrium pricing strategy in a directed search model where sellers' goods are differentiated. When a buyer has high valuation for one seller's good and low valuation for the other's, the buyer's decision in selecting which seller to contact is independent from two sellers' prices and the buyer will contact the seller for whom the buyer has a high valuation as long as the seller's price is lower than the buyer's reservation price. This buyer is the loyal buyer of the seller. When a buyer has high valuation for both sellers' goods, the buyer decides which seller to buy from based on sellers' prices and probability of getting each seller's product. This buyer is called "shopper". Sellers face a trade-off between choosing a high price with a low selling probability by attracting only loyal buyers and choosing a low price with a high selling probability by attracting both loyal buyers and shoppers.

In a directed search framework under various compositions of buyers with different valuations, this chapter's main finding is the condition under which different kinds of symmetric equilibrium in the sellers' game can appear. The model finds that ceteris paribus, the less differentiated sellers' products are, the more shoppers there are and the more likely the model will have a symmetric pure-strategy equilibrium in the sellers' game. In the equilibrium, each seller competes for these shoppers by choosing a low price. On the other hand, the more differentiated sellers' products are, the more loyal buyers there are and the more likely the model will have a symmetric mixed-strategy equilibrium.

Comparative static analysis shows when the model has a pure-strategy equilibrium in the sellers' game, as sellers' products become slightly more differentiated, the symmetric equilibrium price increases. When products are more differentiated, there are more loyal buyers. With more loyal buyers, the incentive of a seller to extract surplus from loyal buyers through raising price is stronger. So seller's price increases. However, the price will not increase without bound. As price increases, the incentive to marginally decrease the price to compete for shoppers becomes stronger. This is because a one-unit price decrease at a higher price level by a seller leads to a proportionally bigger surplus increase for shoppers, and therefore, a larger switch by shoppers from the other seller to the seller, giving the seller stronger incentive to decrease price to attract shoppers.

As sellers' products become sufficiently differentiated, the proportion of loyal buyers is so large that competing for shoppers is less profitable than choosing buyers' reservation price and attracting only loyal buyers. However, if both sellers choose this price, a seller has an incentive to slightly lower her price. To see this, note that when both sellers choose buyers' reservation price, shoppers get zero payoff from contacting either seller therefore they randomize between contacting two sellers with equal probabilities. As a seller slightly decreases her price below the buyers' reservation price, all shoppers will switch to the seller because they get strictly positive surplus from trading with the seller. Therefore, each seller has an incentive to lower her price to attract shoppers. As sellers' prices drop to a level slightly lower than buyers' reservation price, shoppers' surplus from trading with each seller is still very low, therefore a price cut by a seller will make matching with the seller more desirable than matching with the other and attract shoppers to the seller. Therefore each seller still wants to cut her price to get more shoppers. As the price level drops to a sufficiently low level where sellers have no incentive to decrease their prices even further to attract shoppers, they find the current price is so low that choosing buyers' reservation price is more profitable. When two sellers both choose buyers' reservation price, each of them wants to decrease her price slightly. Another cycle of price cutting begins. The model therefore does not have a symmetric pure-strategy equilibrium.

The symmetric equilibrium exhibits a mixed-strategy equilibrium with the support consisting of a countable number of prices. When the degree of differentiation between sellers' products is just big enough for the model to have a mixed-strategy equilibrium, the lowest price of the equilibrium price support (which is made of a countable number of prices) is charged with probability close to 1. In other words, the mixed-strategy equilibrium is "close" to a pure-strategy equilibrium. As sellers' products become even more differentiated, the probability to choose the lowest price decreases and the probability to choose other prices in the support increases. Therefore, the product differentiation introduced in the current chapter theoretically bridges the gap between mixed-strategy equilibrium and pure-strategy equilibrium.

This chapter is organized as follows: Section 3.2 describes the setup of the model. Section 3.3 discusses the conditions under which the unique symmetric equilibrium exhibits a pure-strategy equilibrium and a mixed-strategy equilibrium in the sellers' game respectively. Section 3.4 considers how the equilibrium strategy changes when the degree of differentiation changes. Section 3.5 concludes and discusses future works.

3.2 The Model

There are two sellers, A and B, and two buyers, 1 and 2. Each seller has one unit of indivisible good for which the seller's reservation price is 0. A buyer's valuation of a seller's good can be either 1 or 0, which is private information to the buyer.

This chapter considers a general distribution of the profile of two buyers' valuations for two sellers' goods in order to capture situations with various correlations in buyers' valuations of sellers. The correlations can be both across buyers and across sellers. The joint distribution of two buyers' valuations of two sellers' goods (four random variables) is shown in Table 3.1.

The model has three stages. At stage 1, two sellers simultaneously choose their prices p_A and p_B. At stage 2, buyers observe prices chosen by sellers, then, knowing their own valuations of two sellers' goods, simultaneously decide which

Table 3.1: Joint Distribution of Buyers' Valuations of Sellers' Goods

Valuation realization	Prob.
One buyer has valuation 0 for both sellers The other buyer has valuation 0 for both sellers	η_0
One buyer has valuation 0 for both sellers The other buyer has valuation 0 for one seller and valuation 1 for the other	η_1
One buyer has valuation 0 for both sellers The other buyer has valuation 1 for both sellers	η_2^β
One buyer has valuation 0 for one seller and valuation 1 for the other seller The other buyer has valuation 1 for the first seller, valuation 0 for the second	η_2
One buyer has valuation 0 for one seller and valuation 1 for the other seller The other buyer has valuation 0 for the first seller, and valuation 1 for the second	η_2^σ
One buyer has valuation 1 for both sellers The other buyer has valuation 0 for one seller and valuation 1 for the other	η_3
One buyer has valuation 1 for both sellers The other buyer has valuation 1 for both sellers	η_4

seller to select. Each buyer can select at most one seller. At stage 3, the exchange happens. If only one buyer selects a seller, the seller sells the good to that buyer at the seller's price. If two buyers select the same seller, the seller randomly chooses one buyer and sells the good to the chosen buyer at the seller's price; the unchosen buyer will not get the good, nor will he be charged. If no buyer selects a seller, the seller keeps his good.

Sellers and buyers are all risk-neutral. A seller's profit is the seller's probability of selling the good multiplied by the seller's price. The probability of selling is equal to the probability that at least one buyer selects the seller. A buyer's expected payoff from selecting a seller is the buyer's net valuation of the seller's good (the buyer's valuation of the seller's good less the seller's price) multiplied by the probability the buyer believe he can get the seller's good. The buyer's payoff from not selecting any seller is 0.

The assumption that each buyer cannot select more than one seller implies if a buyer selects a seller but fails to trade with the seller, the buyer fails to match with any seller. The setup that when both buyers select the same seller, the seller can accommodate at most one and randomly choose to trade implies a strategic interaction between two buyers, in the sense that a buyer's probability of trading with a seller depends on the probability that the other buyer selects the same seller[1].

A buyer's strategy is, when two sellers choose p_A and p_B, the probabilities of selecting one of two sellers for each of the buyer's possible valuations of two sellers' goods,

$$\left\{ \begin{array}{c} \left(\theta_{00}^A(p_A,p_B),\theta_{00}^B(p_A,p_B)\right),\left(\theta_{01}^A(p_A,p_B),\theta_{01}^B(p_A,p_B)\right) \\ \left(\theta_{10}^A(p_A,p_B),\theta_{10}^B(p_A,p_B)\right),\left(\theta_{11}^A(p_A,p_B),\theta_{11}^B(p_A,p_B)\right) \end{array} \right\}$$

where $\theta_{st}^j(p_A,p_B)$ is, when seller A announces price p_A and seller B announces price p_B, the buyer's probability of selecting seller $j \in \{A,B\}$ if the buyer's valuation for A's good is s and for B's good is t. $1-\theta_{st}^A(p_A,p_B)-\theta_{st}^B(p_A,p_B)$ is the probability of choosing not to select any seller. $\theta_{st}^A(p_A,p_B)+\theta_{st}^B(p_A,p_B) \leq 1$.

Because seller j has only one good, if the other buyer selects seller j, a buyer's probability to match with seller j is $\frac{1}{2}$. If the other buyer does not select j, the buyer's probability to match with seller j is 1. If the probability conditional on the buyer's own valuation that the other buyer selects seller j is $\widetilde{\theta}^j(p_A,p_B)$, the buyer's probability to match with seller j is

$$\tfrac{1}{2}\widetilde{\theta}^j(p_A,p_B)+1-\widetilde{\theta}^j(p_A,p_B)=1-\tfrac{1}{2}\widetilde{\theta}^j(p_A,p_B)$$

If a buyer has valuation 1 for seller j's good, his net valuation for j's good is $1-p_j$. Therefore, given the other buyer's strategy, the buyer's expected payoff from selecting seller j is

$$(1-p_j)(1-\tfrac{1}{2}\widetilde{\theta}^j(p_A,p_B)).$$

If a buyer has valuation 0 for seller j's good, his net valuation for j's good is $-p_j$, and his expected payoff from selecting seller j is

$$-p_j(1-\tfrac{1}{2}\widetilde{\theta}^j(p_A,p_B)).$$

[1]This is different from Bertrand price competition models, where a buyer's payoff of buying from a seller is independent from whether or not the other buyers are buying from the same seller. This difference leads to buyers' different equilibrium response to sellers' prices, which in turn gives different equilibrium pricing strategies of sellers.

Suppose in a symmetric continuation equilibrium in the buyers' game, both buyers select seller j with probability $\theta^j(p_A, p_B)$. Seller j's probability of selling her good is

$$1 - \left(1 - \theta^j(p_A, p_B)\right)^2$$

And seller j's expected profit is

$$\left[1 - \left(1 - \theta^j(p_A, p_B)\right)^2\right] p_j.$$

The chapter focuses on symmetric (possibly mixed-strategy) equilibria between buyers in selecting sellers, and symmetric equilibria between sellers in choosing prices. The focus on symmetric strategy is due to the assumption that buyers cannot coordinate on selecting sellers, and sellers cannot coordinate on choosing prices. In other words, the chapter focuses on the symmetric subgame perfect Nash equilibria of the game.

It can be shown that choosing negative prices cannot be equilibrium strategies for sellers. Also, choosing prices strictly greater than 1 or price 0 cannot be equilibrium strategies either. This implies equilibrium prices of sellers are strictly positive and less than or equal to 1. Given prices are in this range, in any equilibrium between buyers, buyers with valuation 1 for one seller's good and valuation 0 for the other seller select the first seller for sure because selecting the first seller gives non-negative payoff while selecting the second gives strictly negative payoff. For buyers with valuation 0 for both sellers, they do not select any seller because selecting either seller gives strictly negative payoff.

When $\eta_4 = 0$ and at least one of η_2^β and η_3 is not equal to 0, it is possible for one buyer to have valuation 1 for both sellers but not possible for two buyers to both have valuation 1 for both sellers. For a buyer with valuation 1 for both sellers, the other buyer either has valuation 1 for one seller and valuation 0 for the other, or has valuation 0 for both sellers. Therefore, the other buyer's strategy is determined. For the buyer, the probability of getting seller A's good can be shown to be the same as the probability of getting seller B's good. Therefore the buyer's selection between two sellers is to select the lower price seller with probability 1. From a seller's point of view, choosing a price slightly lower than the opponent gives the seller discretely larger probability of selling by attracting those buyers with valuation 1 for both sellers. It can be shown there exists a symmetric equilibrium in the sellers' game where each seller randomizes over an interval of prices [2].

When $\eta_4 = 0$, $\eta_2^{\beta} = 0$ and $\eta_3 = 0$, it is not possible for any buyer to have valuation 1 for both sellers. Therefore all buyers' strategies are independent from sellers' prices. The equilibrium between sellers is to both choose price 1.

The remainder of this chapter will focus on situation with $\eta_4 > 0$.

For buyers with valuation 1 for both sellers, there are three cases to consider. When seller A's price is sufficiently lower than seller B's, their selection strategy is to select seller A with probability 1 and seller A's probability of selling is a constant, denoted as $\overline{\Pi}$. When seller A's price is sufficiently higher than B's, these buyers select seller B with probability 1 and seller A's probability of selling is a constant with regards to the particular level of seller A's price, denoted as $\underline{\Pi}$. When A's price is close to B's, these buyers randomize between selecting A and B and seller A's probability of selling depends on two sellers prices, denoted as $\Pi(p_A, p_B)$ where

$$\Pi(p_A, p_B) \equiv$$
$$\tfrac{1}{2}\eta_1 + \eta_2 + \tfrac{1}{2}\eta_2^{\sigma} + \tfrac{1}{2}\eta_3 + (\eta_2^{\beta} + \tfrac{1}{2}\eta_3) f(p_A, p_B) + \eta_4 f(p_A, p_B)(2 - f(p_A, p_B))$$

$$\text{and } f(p_A, p_B) \equiv \frac{(\eta_2^{\beta} + \frac{3}{4}\eta_3 + 2\eta_4)(1 - p_A) - (\eta_2^{\beta} + \frac{3}{4}\eta_3 + \eta_4)(1 - p_B)}{\eta_4[(1 - p_A) + (1 - p_B)]}.$$

In the last case, we say there is local competition between two prices while in the first two cases, there is no local competition between two prices.

3.3 Symmetric Equilibrium between Sellers

This section investigates the symmetric equilibrium in the sellers' game.

Define p^* to be the solution of equation

$$\frac{\partial \Pi(p, p)}{\partial p_A} p + \Pi(p, p) = 0 \tag{3.1}$$

[2] The analysis is analogous to Varian (1980) so is omitted here.

The main result of this chapter can be stated here.

Proposition 4. *When $\underline{\Pi} \leq \Pi(p^*, p^*)p^*$, the unique symmetric subgame perfect Nash equilibrium is such that each seller chooses p^*. When $\Pi(p^*, p^*)p^* < \underline{\Pi}$, each seller randomizes over a countable number of prices in the unique symmetric subgame perfect Nash equilibrium.*

To prove, it is first established that when seller B chooses p^*, seller A's profit as a function of A's price has two local maximum points, p^* and 1. Therefore, only p^* or 1 can be global maximum points of A's profit.

If $\underline{\Pi} \leq \Pi(p^*, p^*)p^*$, seller A's profit by choosing p^* is greater than or equal to her profit by choosing 1 and therefore, p^* is the global maximum point of A's profit. So p^* is a symmetric equilibrium between sellers.

To prove p^* is the only symmetric equilibrium in the sellers' game, suppose by contradiction that there exists another symmetric equilibrium in the sellers' game.

One possibility is that the symmetric equilibrium is in pure strategy where sellers choose price p'. A necessary condition for p' to be a symmetric equilibrium is when opponent seller chooses p', p' is a local maximizing point for a seller's profit. Therefore p' satisfies equation 3.1. As equation 3.1 has a unique solution p^*, p' must coincide with p^*, which establishes that there can be only one symmetric pure-strategy equilibrium in the sellers' game where sellers choose p^*.

Another possibility is that the symmetric equilibrium is in mixed strategy. We can similarly show that, as in Chapter 2, any two prices in the support of any symmetric equilibrium strategy do not locally compete with each other. Based on this result, there are two possibilities regarding the support of the symmetric equilibrium mixed strategy. One possibility is that the support of the equilibrium mixed strategy is made of finite number of prices. Another possibility is that the support of the equilibrium mixed strategy is made of infinite number of prices. We can show that in either case, necessary equilibrium conditions cannot simultaneously hold at all prices. Therefore there do not exist symmetric mixed-strategy equilibria between sellers.

If $\Pi(p^*, p^*)p^* < \underline{\Pi}$, choosing price p^* with probability 1 is not a symmetric equilibrium, because the seller's profit by choosing p^* is low and choosing price 1 is more profitable. As shown before, the only candidate for a symmetric pure-strategy equilibrium is p^*. As p^* is not a symmetric equilibrium and therefore, there do not exist symmetric pure-strategy equilibria between sellers.

By following similar steps as in Chapter 2, we can show that when $\Pi(p^*, p^*)p^* < \underline{\Pi}$, these exists only one symmetric mixed-strategy equilibrium in the sellers' game where sellers randomize over a countable number of prices.

3.4 Comparative Statics

η_2^β represents a degree of no differentiation between sellers' products since it is the probability that one buyer has valuation 1 for both sellers while the other buyer has valuation 0 for both sellers, and so the two sellers are perfect substitutes from the perspective of each buyer. Similarly, η_4 represents a degree of no differentiation between sellers' products. η_3 represents a degree of horizontal differentiation between sellers' products since it is the probability that one buyer has valuation 1 for both sellers while the other buyer has valuation 1 for one seller and 0 for the other seller. Therefore one buyer regards two sellers as perfect substitutes while the other buyer thinks two sellers are selling differentiated goods. Similarly, η_1, η_2 and η_2^σ are also measures of differentiation.

When η_4 increases while holding probabilities η_1, η_2, η_2^σ, η_2^β and η_3 to be constants, condition $\underline{\Pi} \leq \Pi(p^*, p^*)p^*$ is more likely to hold. In addition, when η_4 approaches 1, the other probabilities approach 0. Sellers' goods are almost the same goods. We can show that condition $\underline{\Pi} \leq \Pi(p^*, p^*)p^*$ will hold for sufficiently large η_4 and the unique symmetric equilibrium in the sellers' game exhibits sellers choose p^*.

When η_3 increases while holding probabilities η_1, η_2, η_2^σ, η_2^β and η_4 to be constants, condition $\Pi(p^*, p^*)p^* < \underline{\Pi}$ is more likely to hold. In addition, when η_3 approaches 1, which means the sellers' goods are almost horizontally differentiated, we can show that condition $\Pi(p^*, p^*)p^* < \underline{\Pi}$ will hold. The unique symmetric equilibrium in the sellers' game exhibits each seller randomizes over a countable number of different prices.

When any of η_1, η_2 and η_2^σ increases while holding probabilities η_2^β, η_3 and η_4 to be constants, condition $\Pi(p^*, p^*)p^* < \underline{\Pi}$ is more likely to hold. Because η_1, η_2 and η_2^σ are all measure of differentiation, as the measure of differentiation increases, the unique symmetric equilibrium in the sellers' game exhibits each seller randomizes over a countable number of prices.

Corollary 1. *With sufficiently small degree of differentiation between sellers' goods, the symmetric equilibrium exhibits sellers use a pure strategy in choosing prices. With a sufficiently large degree of differentiation, the symmetric equilibrium exhibits sellers randomize among a countable number of different prices.*

The appendix shows in addition that when any of η_1, η_2 and η_2^σ increase, p^* increases and the lowest price of the price support of mixed-strategy equilibrium, denoted as p_1^*, also increases. It can also be shown that the probability of choosing p_1^* decreases.

3.5 Conclusion

This chapter considers a directed search model where two sellers, each with one unit of horizontally differentiated goods, try to match with two buyers, each of whom wants one unit of good. Sellers simultaneously choose prices, then after observing both sellers' price choices, buyers simultaneously select sellers without coordination. When two buyers select the same seller, each of them trades with the seller with equal probabilities.

The model finds whenever the correlation structure between buyers' valuations satisfies a certain condition, there exists a symmetric pure-strategy equilibrium in the sellers' game, and whenever the condition is violated, there exists a symmetric mixed-strategy equilibrium between sellers where each seller randomizes over a countable number of prices.

Future work includes extending the binary space of buyers' valuations into a continuous space. The $\{0,1\}$ case is special in the sense that sellers cannot profitably extract surplus from buyers who have valuation 0 for the seller's good. Therefore, in the current model the competition between sellers is for buyers who have valuation 1 for both sellers.

Although the current model is a very special in the buyers' valuation space, a general insight of the model is that the selection among sellers by buyers, who are the target of sellers' competition, changes continuously as the profile of the utilities sellers offer changes. As one seller provides sufficiently better utility than other sellers do, the buyer selects the seller with probability 1. This feature holds in other directed search models. It is of interest to investigate whether the mixed-strategy equilibrium among sellers with a support consisting of a countable number of prices can still appear in other directed search models.

Chapter 4

Information Provision in Directed Search

4.1 Introduction

In numerous economic environments, sellers not only decide the prices of their products, but also provide information about the products to potential buyers, who initially know little about the products. This is true particularly for new or sophisticated products. Consider the examples of free samples, movie trailers, and free trial computer games. By controlling how much information the free samples contain about the real products, sellers control how much information buyers know about sellers' products. However, buyers' valuation after receiving the new information is typically known to buyers, but not to sellers. For example, providing free samples to potential buyers gives buyers more information about the product, but the effect of that information on the buyers' valuation is typically known only to the buyers, given their personal tastes and needs. Instead, sellers only know the distribution of valuations among buyers conditional on the new information. Disclosing information on the characteristics of the sellers' goods changes buyers' valuations in a non-monotonic way. Releasing information increases some buyers' valuations, whose tastes match with the disclosed characteristics, and decreases others', whose tastes it does not match. In other words, more information leads to a mean-preserving-spread in the distribution of buyers' valuations conditional on the new information.

Sellers face a trade-off in deciding how much information to release to potential buyers. Disclosing more information raises the valuation of buyers whose tastes match with the characteristics of seller's good, meaning the seller can charge a higher price to extract part of this extra surplus. However, disclosing information lowers some other buyers' valuations, which leads to decreased market share for the seller. This paper studies this trade-off in a decentralized matching model where two sellers try to match with two buyers by posting prices and disclosing product information to the buyers.

The matching process is modelled as a two stage game in which the sellers first simultaneously post their prices and product information. Then after the buyers privately draw signals, they simultaneously select sellers from whom to buy. There is no coordination between the buyers in choosing sellers, so in equilibrium, two buyers may select the same seller and only one buyer will be able to trade with the seller. There is, therefore, strategic interaction between buyers in the sense that a buyer has to consider the other buyer's strategy when deciding which seller to select. A buyer does not necessarily buy from the seller who offers him the highest valuation less the price. Rather, because there is some uncertainty about whether a buyer can get the seller's good, a buyer must also consider the probability of successfully trading with the seller. The main result is that providing full information dominates providing only partial information.

The argument is that regardless of the opponent seller's information provision and price, increasing the provided information to be fully informative while increasing the price accordingly always leads to higher profits for a seller. In an environment with frictional search for sellers, buyers have to take into account the probability of successfully trading with sellers. Providing full information and increasing price accordingly maintains the valuations (less the seller's price) of buyers who receive high signal realizations, but strictly increases the probability these buyers can trade with the seller. This is because buyers who receive low signal realizations no longer select that seller. Therefore these high signal buyers select the seller with equal or higher probability after the seller deviates to full information. A seller's profit is the probability of a sale multiplied by the seller's price. Because each seller has only one unit of goods, as long as there remain buyers seeking to buy, the seller has a good probability of making a sale. The fact that high signal buyers select the seller with equal or higher probability limits the loss in the seller's probability of making a sale after the seller deviates to full information and a higher price. It turns out it is more profitable to sell only to these high signal buyers at full information and a higher price than attracting both high signal and low signal buyers at a lower price.

This paper is related to Lewis and Sappington [13] which studies how much information a monopolistic seller should let potential buyers know about buyers' private tastes about the seller's good. Lewis and Sappington [13] identify conditions under which a monopolistic seller should either disclose no information or disclose all information. This paper is also related to recent models which consider information provision by sellers in a competitive environment. Ivanov [11] considers an information provision model where multiple sellers non-cooperatively decide how much information and how high a price to charge. There is a representative buyer in Ivanov's model for whom the sellers' information affects his valuation for each seller separately. After receiving information from all sellers, the buyer buys from the seller who gives him the highest valuation less the price. Providing full information becomes the unique equilibrium among sellers only when the number of sellers reaches some threshold value. If there are only two sellers, providing full information is not an equilibrium. In a two-stage setup, Damiano and Li [4] consider a model where two sellers compete to sell to a representative buyer. In the first stage, sellers choose how much information to provide. In the second stage, after knowing each other's information provision, sellers choose their prices. Although there is an equilibrium in the sellers' game in the first stage where full information provision is an equilibrium, there is also an asymmetric equilibrium where a seller provides full information and the other seller provides no information. The difference between the current paper and the above representative buyer models is in the current paper there is competition between buyers. Therefore the cost of providing all information in terms of reduced market share is smaller in the current paper because high signal buyers will select sellers with higher probability.

In terms of the approach to modelling the matching process, this paper is also related to the directed search literature. In a series of papers, Peters [15], Peters [16] and Peters [17] study the equilibrium outcome of the matching game between a group of sellers and a group of buyers, in which sellers first publicly post their respective prices, then after observing all sellers' prices, buyers decide which seller to select without coordination. The focus of Peters' series of papers is to investigate, when the number of players is very large, whether the symmetric subgame perfect Nash equilibrium of the matching game approaches the competitive equilibrium of the game. Burdett et al. [2] studies a similar model and their aim is to derive the number of matches as a function of the numbers of buyers and sellers. Unlike the above papers, this paper considers information provision by sellers in an otherwise standard directed search model.

This chapter is organized as follows: Section 4.2 describes the setup of the model. Section 4.3 discusses the dominance of full information provision. Section 4.4 analyzes the welfare property of the equilibrium. Section 4.5 concludes and discusses future works.

4.2 The Model

Consider a model where there are two sellers, A and B, each having one unit of good, and two buyers, 1 and 2, each wanting one unit of good. A buyer's valuation of a seller's good can be either 0 or 1, with equal probability. A buyer's valuation of one seller's good is independent of his valuation of the other seller's good, and it is also independent of the other buyer's valuations. Initially, buyers do not know their valuations for sellers' goods. Each seller can provide the two buyers with some information about their valuations of the seller's good. This paper considers a class of symmetric binary information structures for each good. The signal of each information structure takes realization either 0 or 1. Each information structure can be represented by a probability α, which is both the probability of the signal being 1 conditional on the buyer's valuation of the good being 1, and the probability of the signal being 0 conditional on the buyer's valuation of the good being 0. For a higher α, buyers are more sure about their valuations of a seller's good from the signal realization. α is therefore called the precision of the signal and a higher α stands for more information about buyers' valuations disclosed to buyers. Without loss of generality, we can restrict attention to the case where $\alpha \in [\frac{1}{2}, 1]$.

Although the seller can choose the precision of the signal, she does not observe the realization of the signal. Only the buyer can observe the realization of the signal.

The timing of the model is as follows. First, sellers simultaneously post their prices and signal precisions. Second, upon observing the prices and signal precisions of the two sellers, each buyer independently and privately draws a signal from each seller and observes the signal realization. Then buyers simultaneously decide which seller to select (or decide not to select either seller). A buyer's payoff of not selecting any seller is 0. If a buyer is the only buyer selecting the seller, the buyer trades with the seller at the seller's posted price. If two buyers select the same seller, the seller randomly picks one buyer between them with equal probabilities and trades with the buyer at the seller's posted price. The other buyer is turned away, does not trade with any seller and receives payoff 0. If no buyer selects a seller, the seller keeps the good and receives profit 0.

A buyer can select seller A, seller B, or neither. Because the buyer receives 0 payoff when a buyer selects a seller but fails to get the seller's good, the buyer's expected payoff of selecting a seller is the probability the buyer can get the seller's good, multiplied by the difference between the seller's price and the buyer's expected valuation of the seller's good conditional on the signal realization.

Given a seller's signal precision is α_j, after a buyer receives a realization 1 of the signal from the seller, the buyer's expected valuation of the seller, conditional on the signal realization being 1, is

$$E(buyer\ i's\ valuation\ of\ seller\ j|realization\ of\ signal$$

$$received\ by\ buyer\ i\ from\ seller\ j = 1)$$

$$= 1 \cdot Prob(buyer\ i's\ valuation\ of\ seller\ j's\ good =$$
$$1|realization\ of\ signal\ received\ by\ buyer\ i\ from\ seller\ j = 1)$$

$$+0 \cdot Prob(buyer\ i's\ valuation\ of\ seller\ j's\ good =$$
$$0|realization\ of\ signal\ received\ by\ buyer\ i\ from\ seller\ j = 1)$$

$$= \alpha_j$$

When a buyer receives a realization 0 of the signal from the seller, the buyer's expected valuation of the seller, conditional on the signal realization being 0, is

$$E(buyer\ i's\ valuation\ of\ seller\ j|realization\ of\ signal$$

$$received\ by\ buyer\ i\ from\ seller\ j = 0)$$

$$= 1 \cdot Prob(buyer\ i's\ valuation\ of\ seller\ j's\ good =$$
$$1|realization\ of\ signal\ received\ by\ buyer\ i\ from\ seller\ j = 0)$$

$$+0 \cdot Prob(buyer\ i's\ valuation\ of\ seller\ j's\ good =$$
$$0|realization\ of\ signal\ received\ by\ buyer\ i\ from\ seller\ j = 0)$$

$$= 1 - \alpha_j$$

The more precise a seller's signal is, the larger the difference between the expected valuation of buyers who receive signal 0 and expected valuation of buyers who receive signal 1 is.

This paper focuses on the case where two buyers, if they get the same signals from two sellers, use the same strategy in selecting sellers[1]. A buyer's (common) strategy is defined as the probabilities of selecting seller A and seller B for possible signal realizations from two sellers, following the prices and signal precisions of two sellers.

$$\{\left(\pi_{00}^A(\alpha_A,p_A,\alpha_B,p_B),\pi_{00}^B(\alpha_A,p_A,\alpha_B,p_B)\right),\left(\pi_{01}^A(\alpha_A,p_A,\alpha_B,p_B),\pi_{01}^B(\alpha_A,p_A,\alpha_B,p_B)\right)$$
$$\left(\pi_{10}^A(\alpha_A,p_A,\alpha_B,p_B),\pi_{10}^B(\alpha_A,p_A,\alpha_B,p_B)\right),\left(\pi_{11}^A(\alpha_A,p_A,\alpha_B,p_B),\pi_{11}^B(\alpha_A,p_A,\alpha_B,p_B)\right)\}$$

where $\pi_{st}^j(\alpha_A,p_A,\alpha_B,p_B)$ is the probability of selecting seller $j \in \{A,B\}$ when the buyer receives signal s from seller A and signal t from seller B if seller A's signal precision is α_A and price is p_A and seller B's signal precision is α_B and price is p_B. $1-\pi_{st}^A(\alpha_A,p_A,\alpha_B,p_B)-\pi_{st}^B(\alpha_A,p_A,\alpha_B,p_B)$ is the probability of not selecting any sellers following α_A, p_A, α_B and p_B. $\pi_{st}^A(\alpha_A,p_A,\alpha_B,p_B)+\pi_{st}^B(\alpha_A,p_A,\alpha_B,p_B) \leq 1$.

From a buyer's point of view, given the other buyer's strategy as stated above, the probability that the other buyer selects seller A is

[1]The focus on a symmetric strategy is due to the assumption that no coordination between buyers exists in selecting sellers.

$$\pi^A(\alpha_A, p_A, \alpha_B, p_B) \equiv \begin{array}{l} \frac{1}{4}[\pi^A_{00}(\alpha_A, p_A, \alpha_B, p_B) + \pi^A_{01}(\alpha_A, p_A, \alpha_B, p_B) \\ +\pi^A_{10}(\alpha_A, p_A, \alpha_B, p_B) + \pi^A_{11}(\alpha_A, p_A, \alpha_B, p_B)] \end{array}$$

With probability $\pi^A(\alpha_A, p_A, \alpha_B, p_B)$ the other buyer selects seller A, and the buyer trades with seller A with probability $\frac{1}{2}$. With complementary probability the other buyer does not select seller A, and the buyer trades with seller A with probability 1. The probability that the buyer can trade with seller A upon selecting seller A is therefore

$$1 - \frac{1}{2}\pi^A(\alpha_A, p_A, \alpha_B, p_B)$$

The buyer's expected payoff from selecting seller A is therefore

$$\{E[buyer's\ valuation\ of\ seller\ A's\ good|realization\ of\ signal$$

$$received\ by\ the\ buyer\ from\ seller\ A] - p_A\} \times \left[1 - \frac{1}{2}\pi^A(\alpha_A, p_A, \alpha_B, p_B)\right]$$

The buyer's expected payoff from selecting seller B is

$$\{E[buyer's\ valuation\ of\ seller\ B's\ good|realization\ of\ signal$$

$$received\ by\ the\ buyer\ from\ seller\ B] - p_B\} \times \left[1 - \frac{1}{2}\pi^B(\alpha_A, p_A, \alpha_B, p_B)\right]$$

where

$$\pi^B(\alpha_A, p_A, \alpha_B, p_B) \equiv \begin{array}{l} \frac{1}{4}[\pi^B_{00}(\alpha_A, p_A, \alpha_B, p_B) + \pi^B_{01}(\alpha_A, p_A, \alpha_B, p_B) \\ +\pi^B_{10}(\alpha_A, p_A, \alpha_B, p_B) + \pi^B_{11}(\alpha_A, p_A, \alpha_B, p_B)] \end{array}$$

A seller's profit is defined as the probability of selling the good multiplied by the price. Seller A's probability of selling when buyer 1 and 2 use the same strategy in choosing sellers, as stated above, is

$$1 - \left[1 - \pi^A(\alpha_A, p_A, \alpha_B, p_B)\right]^2$$

where $\left[1-\pi^A(\alpha_A, p_A, \alpha_B, p_B)\right]^2$ is the probability that neither buyer selects the seller A. The complimentary probability of this is the probability that at least one buyer selects seller A, which is the probability for seller A to sell.

From above, the probability that a buyer can trade with seller A is a decreasing function in $\pi^A(\alpha_A, p_A, \alpha_B, p_B)$ and seller A's probability of selling is an increasing function in $\pi^A(\alpha_A, p_A, \alpha_B, p_B)$. Therefore the probability that a buyer can trade with seller A decreases as the seller's probability of selling increases. The higher the probability seller A can sell means the lower the probability a buyer can trade with seller A.

The solution concept is a subgame perfect Nash equilibrium in which buyers use symmetric strategy in selecting sellers. That is, for any profile of sellers' prices and information provisions, given the other buyer uses a pure or mixed strategy, a buyer wants to use the same strategy. This is due to the assumption that there is no coordination between buyers. Also, this chapter focuses on symmetric equilibria between sellers.

4.3 Full Information Provision

Proposition 5. *The unique symmetric subgame perfect Nash equilibrium features that sellers provide full information and randomize over a countable number of prices.*

To prove the proposition, we first show that any symmetric subgame perfect Nash equilibrium must feature full information provision by sellers. This follows from the result that a seller always has a profitable deviation to the perfectly precise signal, regardless of the other seller's price and signal precision. In particular, if seller A is providing less than full information, i.e., $\alpha_A < 1$, and if seller A gets strictly positive probability of selling under $(\alpha_A, p_A, \alpha_B, p_B)$, seller A's profit is greater under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ than her profit under $(\alpha_A, p_A, \alpha_B, p_B)$. If $\alpha_A < 1$ and seller A's probability of selling under $(\alpha_A, p_A, \alpha_B, p_B)$ is zero, seller A can get more profit under $(1, \varepsilon, \alpha_B, p_B)$ where ε is positive and very close to 0.

Lemma 9. *If $\alpha_A < 1$ and seller A's probability of selling under $(\alpha_A, p_A, \alpha_B, p_B)$ is strictly positive, seller A's profit under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ is strictly greater* *than seller A's profit under* $(\alpha_A, p_A, \alpha_B, p_B)$*, regardless of seller B's information provision* α_B *and price* p_B*.*

Lemma 9 claims that seller A has a profitable deviation from partial information provision to full information provision and a higher price such that high signal buyer's net surplus of A is unaffected.

This deviation to full information and higher price will immediately make seller A lose the low signal buyers, if A initially has some of the low signal buyers. High signal buyers are more likely to select A because while they receive the same net surplus from A, their probability of getting A's good is higher as there is no longer competition for A's product from low signal buyers.

This implies if seller A initially does not have low signal buyers, seller A's deviation to full information is at no cost for A's probability of selling and is strictly profitable due to the higher price.

Now consider the situation where A initially has low signal buyers, a situation that requires A's initial price not to be too high. In particular, it has to be lower than the low signal buyer's initial valuation.

To establish the deviation is still profitable, we consider the change in A's prices and probabilities of selling associated with the deviation.

The increase in price is exactly equal to the increase in the high signal buyer's valuation associated with better information. Due to the symmetry between high and low signals, the increase in the high signal buyer's valuation is exactly equal to the decrease in the low signal buyer's valuation, which is equal to the low signal buyer's initial valuation because the buyer's valuation decreases to 0 under full information. Therefore the price increase is equal to the low signal buyer's initial valuation. Given the initial price is lower than the low signal buyers' initial valuation, the price increase is higher than the initial price, which implies the new price is at least twice as high as the initial price.

The decrease in probability of selling cannot be very high. The argument is high signal buyers are more likely to select A than low signal buyers. Given low signal buyers constitutes one half of the total population of buyers, among buyers who select A initially at most one half can be low signal buyers. When seller A deviates to full information, losing low signal buyers can lead to at most one half drop in seller A's probability of selling.

Therefore, given the price is at least twice as high while probability of selling is at most one half as low, the profit of A must increase.

Lemma 10. *If $\alpha_A < 1$ and seller A's probability of selling under $(\alpha_A, p_A, \alpha_B, p_B)$ is zero, seller A's profit under $(1, \varepsilon, \alpha_B, p_B)$ is strictly greater than seller A's profit under $(\alpha_A, p_A, \alpha_B, p_B)$ for a sufficiently small ε, regardless of seller B's information provision α_B and price p_B.*

Seller A's probability of selling under $(\alpha_A, p_A, \alpha_B, p_B)$ is zero only when (α_B, p_B) satisfies $p_B \leq 1 - \alpha_B$, because otherwise seller A can at least get the buyer who gets signal 1 from A and signal 0 from B and sell at strictly positive probability. It is claimed that seller A's profit is strictly positive under $(1, \varepsilon, \alpha_B, p_B)$ where $\varepsilon > 0$ but very small. Under $(1, \varepsilon, \alpha_B, p_B)$, for buyers who get signal 1 from seller A and get signal 0 from seller B, their net valuation of A's good is close to 1, and their net valuation of B's good is $1 - \alpha_B - p_B$, which is strictly less than $\frac{1}{2}$, because $1 - \alpha_B \leq \frac{1}{2}$ and p_B is a positive number. For ε sufficiently close to 0, the buyer's net surplus from seller A will be larger than twice the net surplus from seller B. The buyer will visit seller A for sure. This implies the seller can sell at a strictly positive probability under $(1, \varepsilon, \alpha_B, p_B)$. Therefore, seller A's profit is strictly positive under $(1, \varepsilon, \alpha_B, p_B)$.

Given the result of Lemma 9, a seller can always increase her profit by deviating to full information regardless of the opponent seller's price and information provision. Therefore as long as with strictly positive probability the opponent seller is taking information provision and pricing policies that give the seller strictly positive probability of selling, the seller can strictly increase her profit by deviating to full information and a higher price. If all information provision and pricing policies taken by the opponent seller with strictly positive probabilities gives the seller zero probability of selling, the seller can strictly increase her profit by deviating to full information and a small but positive price. This establishes that as long as buyers are playing according to their symmetric continuation equilibrium, any subgame perfect Nash equilibrium features that sellers provide full information. Given Chapter 1 shows that, conditional on both sellers providing full information, the unique symmetric pricing equilibrium in the sellers' game is that sellers randomize over a countable number of prices, the unique symmetric subgame perfect Nash equilibrium of the game must feature that sellers provide full information and randomize over a countable number of prices.

4.4 Welfare Analysis

Given the model analyzed in this chapter has a unique symmetric subgame perfect Nash equilibrium where two sellers both disclose full information and randomize over a countable number of prices, the equilibrium allocation is identical to the equilibrium allocation in Chapter 1. Therefore the total surplus of the equilibrium allocation of this chapter is equal to that of Chapter 1.

Now consider the case where sellers are constrained to disclose no information to buyers. Therefore, each buyer has valuation $\frac{1}{2}$ for both sellers' goods. This is the case analyzed in Burdett et al. [2] except that each buyer's valuation is $\frac{1}{2}$ rather than 1. It can be easily verified that in the symmetric subgame perfect Nash equilibrium, both sellers choose price $\frac{1}{4}$ and both buyers randomize between selecting either seller with equal probabilities. The equilibrium total surplus can be shown to be $\frac{3}{4}$, which is strictly lower than the total surplus of the equilibrium allocation in this chapter. Therefore, endowing sellers with the ability to disclose information improves the efficiency of the allocation, compared to the case where sellers cannot disclose information.

4.5 Conclusion

This paper investigates the information provision by sellers when they can also simultaneously decide the prices of their goods in a model where buyers use directed search to decide which seller to select. The main result is that providing full information is always more profitable than partial information provision for sellers.

As can be seen in the proof, this strong result is associated with strong assumptions made in the model. One is that a buyer has valuation 0 and 1 with equal probabilities. Another is the symmetric nature of signal structure of each seller. Investigating whether full information is also always more profitable than partial information in a more general setup is a question for future work to address.

Another possible future extension of the current chapter is to ask whether the dominance of full information provision still holds in a more general environment where buyers' valuations are drawn from a continuous distribution. Still another possible extension is more general search friction of buyers. For example, instead of assuming each visiting buyer trades with the seller with equal probability, we can consider a more general selling mechanism such as ex-post negotiation.

Chapter 5

Conclusion

This thesis studies a decentralized way to match sellers with buyers by allowing sellers to choose their respective prices and buyers to select sellers without coordination after observing all of the prices chosen by sellers. Sellers can only sell goods to buyers who select them. When only one buyer selects a seller, the seller sells her good to the buyer at the seller's chosen price. When multiple buyers select the same seller, the seller randomly picks one buyer and sells the goods to him. Other buyers are turned away and do not trade with any other seller.

The innovation of this thesis is to introduce horizontal differentiation between sellers' goods in a model of directed search. The general result is that under enough horizontal differentiation, there is a mixed-strategy equilibrium that is considerably different from the mixed-strategy equilibrium in previous research.

This thesis also shows how reducing the level of horizontal differentiation can make the mixed-strategy equilibrium to disappear and make a pure-strategy equilibrium to emerge. The pure-strategy equilibrium is exactly the same equilibrium as in a standard directed search model.

When sellers can choose information disclosure policy at the time that they choose prices, there is a unique symmetric equilibrium in the sellers' game to disclose full information and randomize over a countable number of prices.

Future work includes extending the binary space of buyers' valuations into a continuous space. The $\{0,1\}$ case is special in the sense that sellers cannot profitably extract surplus from buyers who have valuation 0 for the seller's good.

Therefore in the current thesis, the competition between sellers is for buyers who have valuation 1 for both sellers. We can consider another extension of the current model to a directed search model with more than two sellers and two buyers.

A conjecture is that there still exists a similar mixed strategy equilibrium under a more general setup because the result of the current thesis seems to hinge on the directed search of buyers, but not on the particular details of the number of buyers and sellers, or the valuation space of buyers. Explicitly verifying this conjecture is a direction for future work.

As can be seen in the proof in Chapter 4, the result that sellers disclose full information is associated with strong assumptions made in the model. One is that a buyer can have valuation 0 and 1 with equal probabilities. Another is the symmetric nature of the signal structure of each seller. Investigating whether in a more general setup providing full information is also always more profitable than providing partial information is a question for future work to address.

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Appendix A

Appendix for Chapter 2

A.1 Proof of Lemma 1

In the subgames in which seller A chooses p_A and seller B chooses p_B, suppose the other buyer uses the strategy in selecting sellers as stated in Lemma 1. This implies the other buyer selects seller A with probability $\frac{1+\theta(p_A,p_B)}{4}$, and selects seller B with probability $\frac{2-\theta(p_A,p_B)}{4}$. Therefore a buyer's payoff by selecting seller A, when the buyer's valuation of A's good is 1, is

$$(1-p_A)\left(1-\frac{1}{2}\frac{1+\theta(p_A,p_B)}{4}\right)$$

and the buyer's payoff by selecting seller B, when the buyer's valuation of B's good is 1, is

$$(1-p_B)\left(1-\frac{1}{2}\frac{2-\theta(p_A,p_B)}{4}\right).$$

In subgames in which $p_A < \frac{7}{6}p_B - \frac{1}{6}$, the other buyer's strategy as stated in Lemma 1 is to select seller A with probability $\theta(p_A,p_B) = 1$. The buyer's payoff by selecting seller A is therefore

$$(1-p_A)\left(1-\frac{1}{2}\frac{1+\theta(p_A,p_B)}{4}\right) = \frac{3}{4}(1-p_A)$$

and payoff by selecting seller B is

$$(1-p_B)\left(1-\frac{1}{2}\frac{2-\theta(p_A,p_B)}{4}\right)=\frac{7}{8}(1-p_B).$$

Because $p_A<\frac{7}{6}p_B-\frac{1}{6}$ implies $\frac{3}{4}(1-p_A)>\frac{7}{8}(1-p_B)$, selecting seller A with probability 1 is optimal for the buyer.

In subgames in which $\frac{7}{6}p_B-\frac{1}{6}\leq p_A\leq\frac{6}{7}p_B+\frac{1}{7}$ and $p_A\neq 1$ or $p_B\neq 1$, the other buyer's strategy as stated in Lemma 1 is to select seller A with probability $\theta(p_A,p_B)=\frac{7(1-p_A)-6(1-p_B)}{(1-p_A)+(1-p_B)}$. The buyer's payoff by selecting seller A is

$$(1-p_A)\left(1-\frac{1}{2}\frac{1+\theta(p_A,p_B)}{4}\right)=\frac{\frac{13}{8}(1-p_B)}{(1-p_A)+(1-p_B)}(1-p_A)$$

and payoff by selecting seller B is

$$(1-p_B)\left(1-\frac{1}{2}\frac{2-\theta(p_A,p_B)}{4}\right)=\frac{\frac{13}{8}(1-p_A)}{(1-p_A)+(1-p_B)}(1-p_B).$$

Therefore the buyer's expected payoff by selecting A is equal to the payoff by selecting B.

So selecting seller A with probability $\frac{7(1-p_A)-6(1-p_B)}{(1-p_A)+(1-p_B)}$ is optimal for the buyer.

In subgames in which $\frac{6}{7}p_B+\frac{1}{7}<p_A$, the other buyer's strategy as stated in Lemma 1 is to select seller A with probability $\theta(p_A,p_B)=0$. The buyer's payoff by selecting seller A is

$$(1-p_A)\left(1-\frac{1}{2}\frac{1+\theta(p_A,p_B)}{4}\right)=\frac{7}{8}(1-p_A)$$

and payoff by selecting seller B is

$$(1-p_B)\left(1-\frac{1}{2}\frac{2-\theta(p_A,p_B)}{4}\right)=\frac{3}{4}(1-p_B).$$

Because $\frac{6}{7}p_B+\frac{1}{7}<p_A$ implies $\frac{7}{8}(1-p_A)<\frac{3}{4}(1-p_B)$, selecting seller A with probability 0 is optimal for the buyer.

In the subgame in which $p_A=p_B=1$, the other buyer's strategy as stated in Lemma 1 is to select seller A with probability $\theta(p_A,p_B)=\frac{1}{2}$. The buyer's payoff by selecting seller A is

$$(1-p_A)\left(1-\frac{1}{2}\frac{1+\theta(p_A,p_B)}{4}\right)=0$$

and payoff by selecting seller B is

$$(1-p_B)\left(1-\frac{1}{2}\frac{2-\theta(p_A,p_B)}{4}\right)=0.$$

Because the payoff by selecting A is equal to the payoff by selecting B, selecting seller A with probability $\frac{1}{2}$ is optimal for the buyer.

In conclusion, when the other buyer use the strategy in selecting sellers as stated in Lemma 1, the same strategy in selecting sellers is also optimal for the buyer. Therefore, the strategy in selecting sellers as stated in Lemma 1 is a symmetric equilibrium between buyers.

When two prices p and p' satisfy $\frac{7}{6}p'-\frac{1}{6}\leq p\leq\frac{6}{7}p'+\frac{1}{7}$, we say they locally compete with each other. Otherwise we say they do not locally compete with each other.

A.2 Proof of Lemma 2

First, (2.1) and (2.2) can be rewritten as the following first order difference equation.

$$p_{k+1}=\frac{56+\sqrt{56^2+36\frac{76p_k-11}{p_k^2}}}{2\frac{76p_k-11}{p_k^2}}\ for\ k\in N \tag{A.1}$$

with initial condition

$$p_1=\frac{98+\sqrt{21889}}{390} \tag{A.2}$$

And the probability of choosing p_k is x_k, with x_k being given by

$$x_k = \frac{28}{65}\frac{1-p_k}{p_k^2} \; for \; k \in N \tag{A.3}$$

(A.2) implies that $p_1^* \approx 0.63064$, therefore p_1^* satisfies $0 < p_1^* < 1$, and it is straightforward to show $x_1^* = \dfrac{28}{65}\dfrac{1-p_1^*}{p_1^{*2}}$ satisfies $0 < x_1^* < 1$.

Second, we can show that price sequence $\{p_k^*\}_{k=1}^{\infty}$ as defined by difference equation (A.1) and initial condition (A.2) satisfies $0 < p_k^* < 1$, $\dfrac{7}{6}(1-p_{k+1}^*) \leq 1-p_k^*$ for all $k \in N$.

Because of (A.1), it can be shown that when $\dfrac{11}{48} < p_k^*$, comparing p_{k+1}^* with p_k^* is equivalent with comparing 1 with p_k^*, and comparing p_{k+1}^* with 1 is equivalent with comparing p_k^* with 1.

Therefore, when $\dfrac{11}{48} < p_k^* < 1$, $p_k^* < p_{k+1}^* < 1$.

Because p_1^* satisfies $\dfrac{11}{48} < p_1^* < 1$, we have $p_1^* < p_2^* < 1$.

Therefore p_2^* also satisfies $\dfrac{11}{48} < p_2^* < 1$, which in turns implies $p_2^* < p_3^* < 1$.

By repeating this argument, for all $k \in N$, $p_k^* < p_{k+1}^* < 1$.

Given $\left\{p_k^*\right\}_{k=1}^{\infty}$ is strictly increasing and has an upper bound of 1, $lim_{k\to\infty} p_k^*$ exists and is less than or equal to 1.

By taking limit on the two sides of (A.1) as $k \to \infty$, we can solve for $lim_{k\to\infty} p_k^* = 1$.

When $\dfrac{11}{48} < p_k^*$, comparing $\frac{7}{6}(1-p_{k+1}^*)$ with $1-p_k^*$ is equivalent with comparing p_k^* with 1. Because $\dfrac{11}{48} < p_k^* < 1$, for all $k \in N$, $\frac{7}{6}(1-p_{k+1}^*) < 1-p_k^*$, which is $p_k^* < \dfrac{7}{6}p_{k+1}^* - \dfrac{1}{6}$. In addition, comparing $\dfrac{6}{7}p_k^* + \dfrac{1}{7}$ with $\dfrac{7}{6}p_{k+1}^* - \dfrac{1}{6}$ is equivalent with comparing p_k^* with 1. Because $p_k^* < 1$, $\dfrac{6}{7}p_k^* + \dfrac{1}{7} < \dfrac{7}{6}p_{k+1}^* - \dfrac{1}{6}$.

Now we show that $\{x_k^*\}_{k=1}^{\infty}$ as defined by using the price sequence $\{p_k^*\}_{k=1}^{\infty}$ and (A.3) satisfies $0 < x_k^* < 1$ for all $k \in N$ and $\sum_{k=1}^{\infty} x_k^* = 1$.

From $p_k^* < 1$ for all $k \in N$ and (A.3), we know $0 < x_k^*$ for all $k \in N$.

Rearrangement of (A.1) leads to the following equation:

$$\frac{76p_k^* - 11}{p_k^{*2}} = \frac{56p_{k+1}^* + 9}{p_{k+1}^{*2}} \tag{A.4}$$

Therefore

$$
\begin{aligned}
\sum_{k=1}^{\infty} x_k^* &= lim_{N\to\infty}\left(\sum_{k=1}^{N} x_k^*\right)\\
&= lim_{N\to\infty}\left[\sum_{k=1}^{N}\left(\frac{28}{65}\frac{1-p_k^*}{p_k^{*2}}\right)\right]\\
&= lim_{N\to\infty}\left[\frac{7}{325}\sum_{k=1}^{N}\left(\frac{56p_k^*+9}{p_k^{*2}}-\frac{76p_k^*-11}{p_k^{*2}}\right)\right]\\
&= lim_{N\to\infty}\left[\frac{7}{325}\left(\frac{56p_1^*+9}{p_1^{*2}}-\frac{76p_N^*-11}{p_N^{*2}}\right)\right]\\
&= \frac{7}{325}\left(\frac{56p_1^*+9}{p_1^{*2}}-65\right)\\
&= 1
\end{aligned}
$$

where the third from the last equality is got by using difference equation (A.4) to cancel out terms, the second from the last equality is due to $lim_{N\to\infty}p_N^* = 1$, and the last equality is due to (A.2).

Because $0 < x_k^*$ for all $k \in N$, from $\sum_{k=1}^{\infty} x_k^* = 1$, we know $x_k^* < 1$ for $k \in N$.

A.3 Proof of Lemma 4

In the text, it is already shown that $\Gamma_A\left(p_k^*\right) = \underline{\Pi}$ and $\dfrac{\partial\Gamma_A(p)}{\partial p}|_{p=p_k^*} = 0$ for all k.

It is straightforward to show that $\dfrac{\partial^2\Gamma_A(p)}{\partial p^2} < 0$ for all p such that $\underline{p}\left(p_k^*\right) < p < \overline{p}\left(p_k^*\right)$. Therefore, $\Gamma_A(p) \le \Gamma_A\left(p_k^*\right)$ for all p such that $\underline{p}\left(p_k^*\right) \le p \le \overline{p}\left(p_k^*\right)$.

It can be shown that $\Gamma_A(p) \le \Gamma_A\left(\underline{p}\left(p_k^*\right)\right)$ for all p such that $\overline{p}\left(p_{k-1}^*\right) < p < \underline{p}\left(p_k^*\right)$. Therefore, $\Gamma_A(p) \le \Gamma_A\left(p_k^*\right)$ for all p such that $\overline{p}\left(p_{k-1}^*\right) \le p \le \overline{p}\left(p_k^*\right)$.

For $p < \underline{p}(p_1^*)$, $\Gamma_A(p) \le \Gamma_A\left(\underline{p}(p_1^*)\right)$.

As for all $0 < p < 1$, $\exists k \in N$, such that $\overline{p}\left(p_{k-1}^*\right) \le p \le \overline{p}\left(p_k^*\right)$, so $\Gamma_A(p) \le \Gamma_A\left(p_k^*\right)$ for all $0 < p < 1$. Because $\Gamma_A\left(p_k^*\right) = \underline{\Pi}$ for all $k \in N$, a seller has no profitable deviation from choosing p_k^* for all k. Therefore $\left\{\left(p_k^*, x_k^*\right)\right\}_{k=1}^{\infty}$ is a symmetric equilibrium between sellers.

A.4 Proof of Lemma 5

Seller A's best response in p_A for seller B's price $0 < p_B < 1$ is

$$p_A(p_B) = \begin{cases} 1 & if \quad 0 \leq p_B < \frac{6}{7}\frac{\underline{\Pi}}{\overline{\Pi}} + \frac{1}{7} \\ 1\, or\, \frac{\underline{\Pi}}{\overline{\Pi}} & if \quad p_B = \frac{6}{7}\frac{\underline{\Pi}}{\overline{\Pi}} + \frac{1}{7} \\ \frac{7}{6} \times p_B - \frac{1}{6} & if \quad \frac{6}{7}\frac{\underline{\Pi}}{\overline{\Pi}} + \frac{1}{7} < p_B < 1 \end{cases}$$

Seller B's best response in p_B for seller A's price $0 < p_A < 1$ can be symmetrically defined. The two best response functions have no intersection point in the set of (p_A, p_B) such that $0 \leq p_A < 1$ and $0 \leq p_B < 1$. This implies any (p_A, p_B) such that $0 \leq p_A < 1$ and $0 \leq p_B < 1$ cannot be a pure strategy pricing equilibrium between two sellers.

A's best response to $p_B = 1$ is not well defined: At $p_B = 1$, A's profit is $\overline{\Pi} p_A$ for $p_A < 1$, and is $\Pi(1,1)$ for $p_A = 1$. Because $\overline{\Pi} > \Pi(1,1)$, A wants to set p_A arbitrarily close to 1 but different from 1. This implies $(p_A = 1, p_B = 1)$ is not a pure strategy equilibrium between two buyers because given the opponent is choosing 1, a seller has incentive to choose slightly below 1.

Similarly, any (p_A, p_B) such that $p_A = 1$ and $p_B < 1$ cannot be a pure strategy equilibrium either. Given A chooses price 1, B wants to choose slightly below (and strictly below) 1, and given B chooses a price slightly below 1, A wants to choose $\frac{7}{6}p_B - \frac{1}{6}$, which is strictly less than 1 for a p_B strictly below 1.

A.5 Proof of Lemma 6

Proof. Suppose a seller chooses price 1 with probability $\gamma > 0$ and chooses prices less than 1 with probability $1-\gamma$. The other seller's profit by choosing price 1 is $\gamma\Pi(1,1) + (1-\gamma)\underline{\Pi}$, because with probability γ the seller chooses price 1 so the other seller sells her good with probability $\Pi(1,1)$ by choosing price 1, and with probability $1-\gamma$ the seller chooses prices less than 1 so the other seller sells her good with probability $\underline{\Pi}$ by choosing price 1.

The other seller's profit by choosing a price $p < 1$ is greater than or equal to $\gamma\overline{\Pi}p + (1-\gamma)\underline{\Pi}p$, because with probability γ the seller chooses price 1 so the other seller sells her good with probability $\overline{\Pi}$ by choosing price p, and with probability $1-\gamma$ the seller chooses prices strictly less than 1 so the other seller's probability of selling her good by choosing price p is no less than $\underline{\Pi}$ since $\underline{\Pi}$ is the lowest possible probability for the other seller to sell her good.

Compare $\gamma\Pi(1,1)+(1-\gamma)\underline{\Pi}$ with $\gamma\overline{\Pi}p+(1-\gamma)\underline{\Pi}p$.

Because $\Pi(1,1)<\overline{\Pi}$ and $0<\gamma$, $\dfrac{\gamma\Pi(1,1)+(1-\gamma)\underline{\Pi}}{\gamma\overline{\Pi}+(1-\gamma)\underline{\Pi}}<1$.

When $\dfrac{\gamma\Pi(1,1)+(1-\gamma)\underline{\Pi}}{\gamma\overline{\Pi}+(1-\gamma)\underline{\Pi}}<p$, $\gamma\Pi(1,1)+(1-\gamma)\underline{\Pi}<\gamma\overline{\Pi}p+(1-\gamma)\underline{\Pi}p$.

So given the seller has a positive probability of choosing price 1, the other seller has a profitable deviation from choosing price of 1 to some price strictly less than but very close to 1. This establishes that any symmetric equilibrium between two sellers cannot have a strictly positive probability of choosing price of 1. □

Because in any symmetric equilibrium between sellers, price 1 is chosen with probability 0, given opponent seller uses a symmetric equilibrium strategy, a seller's profit by choosing price 1 is $\underline{\Pi}$, because opponent seller chooses prices strictly less than 1. This implies to be a symmetric equilibrium, equilibrium profit of a seller has to be at least $\underline{\Pi}$ to deter a seller's deviation of choosing price 1.

Suppose a symmetric equilibrium strategy has a support with a lower bound $\underline{p}$. Given opponent seller takes an equilibrium strategy (which has a lower bound $\underline{p}$), a seller's profit by choosing $\underline{p}$ is less than or equal to $\overline{\Pi}\underline{p}$. Because equilibrium profit (which is also the profit a seller gets by choosing $\underline{p}$) is greater than or equal to $\underline{\Pi}$, this implies $\overline{\Pi}\underline{p}\geq\underline{\Pi}$, i.e., $\underline{p}\geq\dfrac{\underline{\Pi}}{\overline{\Pi}}$.

Suppose a symmetric equilibrium strategy has a support with a least upper bound $\overline{p}$. A seller's profit by choosing $\overline{p}$ is less than or equal to $\Pi(\overline{p},\overline{p})\overline{p}$. From $\Pi(\overline{p},\overline{p})\overline{p}\geq\underline{\Pi}$, we have $\overline{p}\geq\dfrac{\underline{\Pi}}{\Pi(\overline{p},\overline{p})}$. This implies that in any symmetric equilibrium, prices in the support of the equilibrium strategy must be greater than or equal to $\dfrac{\underline{\Pi}}{\Pi(\overline{p},\overline{p})}$.

A.6 Proof of Lemma 7

Denote $M(p_A,p_B)\equiv\dfrac{\partial\Pi(p_A,p_B)}{\partial p_A}$.

It is straightforward to show that $\Pi(p_A,p_B)$ is strictly decreasing and concave in p_A.

Seller A's expected profit is $\Gamma_A(p_A,p_B)\equiv\Pi(p_A,p_B)\times p_A$, so the first order derivative of seller A's expected profit with respect to p_A is

$$\frac{\partial \Gamma_A(p_A, p_B)}{\partial p_A} = \Pi(p_A, p_B) + M(p_A, p_B) \times p_A.$$

When $\frac{7}{6}p_B - \frac{1}{6} \leq p_A \leq \frac{6}{7}p_B + \frac{1}{7}$, $\Pi(p_A, p_B) \leq \overline{\Pi}$, and $M(p_A, p_B) < 0$, so the first order derivative of seller A's expected profit with respect to seller A's price is strictly less than $\overline{\Pi}$. Observe

$$\frac{\partial^2 \Gamma_A(p_A, p_B)}{\partial p_A^2} = 2M(p_A, p_B) + \frac{\partial^2 \Pi(p_A, p_B)}{\partial p_A^2} \times p_A.$$

Because $M(p_A, p_B) < 0$ and $\frac{\partial^2 \Pi(p_A, p_B)}{\partial p_A^2} < 0$ due to strict concavity of $\Pi(p_A, p_B)$ in p_A, therefore $\frac{\partial^2 \Gamma_A(p_A, p_B)}{\partial p_A^2} < 0$, it follows that the derivative of seller A's expected profit with respect to p_A is strictly decreasing in p_A.

Consider a symmetric mixed strategy equilibrium with the distribution denoted as $F(.)$. Denote the lowest price in the price support of $F(.)$ as p_1. Suppose there is another price p' in the price support that locally competes with p_1.

When opponent seller using mixed pricing strategy $F(.)$, the derivative of a seller's expected profit with respect to the seller's price when the seller chooses price p_1 is

$$\begin{gathered}\int_{p_1}^{\frac{6}{7}p_1+\frac{1}{7}} \left[\Pi(p, \widetilde{p}) + M(p, \tilde{p}) \cdot p\right]|_{p=p_1} dF(\widetilde{p}) + \\ \overline{\Pi}\left(F(\tfrac{6}{7}p' + \tfrac{1}{7}) - F(\tfrac{6}{7}p_1 + \tfrac{1}{7})\right) + \overline{\Pi}\left(1 - F(\tfrac{6}{7}p' + \tfrac{1}{7})\right)\end{gathered} \tag{A.5}$$

The derivative of a seller's expected profit with respect to the seller's price when the seller chooses price p' is

$$\begin{gathered}\int_{p_1}^{\frac{6}{7}p_1+\frac{1}{7}} \left[\Pi(p, \widetilde{p}) + M(p, \tilde{p}) \cdot p\right]|_{p=p'} dF(\widetilde{p}) + \\ \int_{\frac{6}{7}p_1+\frac{1}{7}}^{\frac{6}{7}p'+\frac{1}{7}} \left[\Pi(p, \widetilde{p}) + M(p, \tilde{p}) \cdot p\right]|_{p=p'} dF(\widetilde{p}) + \overline{\Pi}\left(1 - F(\tfrac{6}{7}p' + \tfrac{1}{7})\right)\end{gathered} \tag{A.6}$$

Compare the first terms of (A.5) and (A.6). Note that opponent seller's price

$\widetilde{p}$ between $[p_1, \frac{6}{7}p_1 + \frac{1}{7}]$ is now locally competing with both p_1 and p', and because the derivative of a seller's expected profit with respect to the seller's price, $\Pi(p,\widetilde{p}) + M(p,\tilde{p}) \cdot p$, is strictly decreasing in its own price conditional on the seller's price is locally competing with opponent seller's price, the first term of (A.5) is strictly greater than the first term of (A.6) as long as opponent seller has strictly positive probability of charging a price in $[p_1, \frac{6}{7}p_1 + \frac{1}{7}]$.

Compare the second terms of (A.5) and (A.6). It can be shown that second term of (A.5) is strictly greater than the second term of (A.6).

Therefore (A.5) is strictly greater than (A.6), which implies first order conditions could not hold simultaneously at price p_1 and p'. It is therefore established that there is no prices in a symmetric equilibrium price support that locally compete with the lowest price of the support. A symmetric mixed strategy pricing equilibrium $F(.)$ must have a discrete mass at price p_1, denote it as x_1. And there exists p_2 in the support of $F(.)$ such that in the support of $F(.)$ there is no prices between p_1 and p_2, and there is no local competition between p_1 and p_2.

Suppose there is a price $p'' > p_2$ in the support of $F(.)$ that locally competes with p_2.

When opponent seller is using mixed pricing strategy $F(.)$, the derivative of a seller's expected profit with respect to the seller's price when the seller's price is p_2 is

$$\begin{aligned} &\underline{\Pi} x_1 + \int_{p_2}^{\frac{6}{7}p_2+\frac{1}{7}} [\Pi(p,\widetilde{p}) + M(p,\tilde{p}) \cdot p]\,|_{p=p_2} dF(\widetilde{p}) + \\ &\overline{\Pi}\left(F(\tfrac{6}{7}p'' + \tfrac{1}{7}) - F(\tfrac{6}{7}p_2 + \tfrac{1}{7})\right) + \overline{\Pi}\left(1 - F(\tfrac{6}{7}p'' + \tfrac{1}{7})\right) \end{aligned} \tag{A.7}$$

and the derivative of a seller's expected profit with respect to the seller's price when the seller's price is p'' is

$$\begin{aligned} &\underline{\Pi} x_1 + \int_{p_2}^{\frac{6}{7}p_2+\frac{1}{7}} [\Pi(p,\widetilde{p}) + M(p,\tilde{p}) \cdot p]\,|_{p=p''} dF(\widetilde{p}) + \\ &\int_{\frac{6}{7}p_2+\frac{1}{7}}^{\frac{6}{7}p''+\frac{1}{7}} [\Pi(p,\widetilde{p}) + M(p,\tilde{p}) \cdot p]\,|_{p=p''} dF(\widetilde{p}) + \overline{\Pi}\left(1 - F(\tfrac{6}{7}p'' + \tfrac{1}{7})\right) \end{aligned} \tag{A.8}$$

Similarly as the comparison before, it can be shown (A.7) is strictly greater than (A.8), which implies there cannot be prices in the support of a symmetric equilibrium mixed pricing strategy $F(.)$ that locally compete with p_2. $F(.)$ must have a discrete mass at price p_2, denote it as x_2, and there exists p_3 in the support of $F(.)$ such that in the support of $F(.)$ there is no other prices between p_2 and p_3, and there is no local competition between p_2 and p_3.

Then by using the same argument for the third lowest price and all other prices, it is proved that the equilibrium price support has to be discrete and there cannot be two prices in the price support that locally compete with each other.

A.7 Proof of Lemma 8

Suppose there exists a symmetric equilibrium pricing strategy such that the support has a supreme strictly less than 1. From Lemma 7, the symmetric equilibrium pricing strategy must have finite number of prices.

When opponent seller uses a pricing strategy made of finite number of prices, where the largest price $\overline{p}$ is strictly less than 1 and $\overline{p}$ is charged with probability $\overline{x}$ and prices below $\frac{7}{6}\overline{p} - \frac{1}{6}$ are charged with probability $1-\overline{x}$, a seller's expected profits by charging $\overline{p}$ is

$$(1-\overline{x})\underline{\Pi}\overline{p} + \overline{x}\Pi(\overline{p},\overline{p})\overline{p}$$

Because a seller's expected profit by charging price 1 is $\underline{\Pi}$, the condition that the seller does not want to deviate to price 1 is

$$(1-\overline{x})\underline{\Pi}\overline{p} + \overline{x}\Pi(\overline{p},\overline{p})\overline{p} \geq \underline{\Pi}$$

which implies

$$\overline{x} \geq \frac{\underline{\Pi}}{\Pi(\overline{p},\overline{p}) - \underline{\Pi}} \frac{1-\overline{p}}{\overline{p}} \tag{A.9}$$

On the other hand, a seller's first order condition at $\overline{p}$ is

$$(1-\overline{x})\underline{\Pi}+\overline{x}\left[\Pi(\overline{p},\overline{p})+M(\overline{p},\overline{p})\overline{p}\right]=0,$$

which implies

$$\overline{x}=\frac{\underline{\Pi}}{\underline{\Pi}-\Pi(\overline{p},\overline{p})-M(\overline{p},\overline{p})\overline{p}} \tag{A.10}$$

From the functional form

$$\Pi(p_j,p_{j'})\equiv\frac{13}{4}(1-p_{j'})\frac{2(1-p_j)-\frac{5}{4}(1-p_{j'})}{\big((1-p_j)+(1-p_{j'})\big)^2}$$

we can get

$$M(p_j,p_{j'})=\frac{13}{4}(1-p_{j'})\frac{2(1-p_j)-\frac{9}{2}(1-p_{j'})}{\big((1-p_j)+(1-p_{j'})\big)^3}$$

Therefore

$$M(p_{j'},p_{j'})=-\frac{65}{64}\frac{1}{(1-p_{j'})}$$

Substitute into (A.10), we have

$$\overline{x}=\frac{\underline{\Pi}}{\underline{\Pi}-\Pi(\overline{p},\overline{p})+\frac{65}{64}\frac{\overline{p}}{1-\overline{p}}}$$

Because the lower bound of a symmetric pricing equilibrium support is no less than $\dfrac{\underline{\Pi}}{\Pi(\overline{p},\overline{p})}=\dfrac{28}{39}$, $\overline{p}$ satisfies $\dfrac{11}{65}<\overline{p}<1$.

It can be shown that $\dfrac{11}{65}<\overline{p}<1$ implies

$$\frac{\underline{\Pi}}{\Pi(\overline{p},\overline{p})-\underline{\Pi}}\frac{1-\overline{p}}{\overline{p}}>\frac{\underline{\Pi}}{\underline{\Pi}-\Pi(\overline{p},\overline{p})+\frac{65}{64}\frac{\overline{p}}{1-\overline{p}}}.$$

Therefore condition that if a seller has no profitable deviation from $\overline{p}$ to 1, i.e., (A.9) holds, then (A.10) would be violated, in particular, the seller will has an incentive to slightly decrease her price below $\overline{p}$.

A.8 Proof of Proposition 3

Now we turn to the discussion of expected total surplus in the equilibrium characterized in Proposition 1. Denote a buyer's action of not selecting any seller as selecting ϕ.

As shown in Table A.1, the expected total surplus under the characterized equilibrium is the sum of expected total surplus under every profile of valuations of two buyers multiplied by the probability of the respective profile of valuations. The calculation shows the expected total surplus is

$$\frac{1}{16}\{8Pr(p_A > p_B) + \frac{17}{2}Pr(p_A = p_B) + 8Pr(p_A < p_B) + 11\}$$

which is, using $Pr(p_A = p_B) = 1 - Pr(p_A > p_B) - Pr(p_A < p_B)$, and because seller A and seller B use the symmetric mixed pricing strategy, $Pr(p_A > p_B) = Pr(p_A < p_B)$, the expected total surplus is

$$\frac{39}{32} - \frac{1}{16}Pr(p_A < p_B)$$

Table A.1: Expected Total Surplus under Equilibrium Allocation

Valuation	Equilibrium Allocation	Surplus	Expected surplus
(1,1) (1,1)	$\begin{cases} \text{Both buyers select B} & if\ p_A > p_B \\ \text{Buyer mixes between selecting A and B} & if\ p_A = p_B \\ \text{Both buyers select A} & if\ p_A < p_B \end{cases}$	$\begin{cases} 1 & if\ p_A > p_B \\ \frac{3}{2} & if\ p_A = p_B \\ 1 & if\ p_A < p_B \end{cases}$	$Pr(p_A > p_B)$ $+\frac{3}{2}Pr(p_A = p_B)$ $+Pr(p_A < p_B)$
(1,1) (0,1)	$\begin{cases} \text{Both buyers select B} & if\ p_A > p_B \\ \text{1 mixes between A and B, 2 selects B} & if\ p_A = p_B \\ \text{1 selects A, 2 selects B} & if\ p_A < p_B \end{cases}$	$\begin{cases} 1 & if\ p_A > p_B \\ \frac{3}{2} & if\ p_A = p_B \\ 2 & if\ p_A < p_B \end{cases}$	$Pr(p_A > p_B)$ $+\frac{3}{2}Pr(p_A = p_B)$ $+2Pr(p_A < p_B)$
(1,1) (1,0)	$\begin{cases} \text{1 selects B, 2 selects A} & if\ p_A > p_B \\ \text{1 mixes between A and B, 2 selects A} & if\ p_A = p_B \\ \text{Both buyers select A} & if\ p_A < p_B \end{cases}$	$\begin{cases} 2 & if\ p_A > p_B \\ \frac{3}{2} & if\ p_A = p_B \\ 1 & if\ p_A < p_B \end{cases}$	$2Pr(p_A > p_B)$ $+\frac{3}{2}Pr(p_A = p_B)$ $+Pr(p_A < p_B)$
(1,1) (0,0)	$\begin{cases} \text{1 selects B, 2 selects } \phi & if\ p_A > p_B \\ \text{1 mixes between A and B, 2 selects } \phi & if\ p_A = p_B \\ \text{1 selects A, 2 selects } \phi & if\ p_A < p_B \end{cases}$	$\begin{cases} 1 & if\ p_A > p_B \\ 1 & if\ p_A = p_B \\ 1 & if\ p_A < p_B \end{cases}$	1
(0,1) (1,1)	$\begin{cases} \text{Both buyers select B} & if\ p_A > p_B \\ \text{1 selects B, 2 mixes between A and B} & if\ p_A = p_B \\ \text{1 selects B, 2 selects A} & if\ p_A < p_B \end{cases}$	$\begin{cases} 1 & if\ p_A > p_B \\ \frac{3}{2} & if\ p_A = p_B \\ 2 & if\ p_A < p_B \end{cases}$	$Pr(p_A > p_B)$ $+\frac{3}{2}Pr(p_A = p_B)$ $+2Pr(p_A < p_B)$
(0,1) (0,1)	Both buyers select B	1	1
(0,1) (1,0)	1 selects B, 2 selects A	2	2
(0,1) (0,0)	1 selects B, 2 selects ϕ	1	1
(1,0) (1,1)	$\begin{cases} \text{1 selects A, 2 selects B} & if\ p_A > p_B \\ \text{1 selects A, 2 mixes between A and B} & if\ p_A = p_B \\ \text{Both buyers select A} & if\ p_A < p_B \end{cases}$	$\begin{cases} 2 & if\ p_A > p_B \\ \frac{3}{2} & if\ p_A = p_B \\ 1 & if\ p_A < p_B \end{cases}$	$2Pr(p_A > p_B)$ $+\frac{3}{2}Pr(p_A = p_B)$ $+Pr(p_A < p_B)$
(1,0) (0,1)	1 selects A, 2 selects B	2	2
(1,0) (1,0)	Both buyers select A	1	1
(1,0) (0,0)	1 selects A, 2 selects ϕ	1	1
(0,0) (1,1)	$\begin{cases} \text{1 selects } \phi \text{, 2 selects B} & if\ p_A > p_B \\ \text{1 selects } \phi \text{, 2 mixes between A and B} & if\ p_A = p_B \\ \text{1 selects } \phi \text{ 2 selects A} & if\ p_A < p_B \end{cases}$	$\begin{cases} 1 & if\ p_A > p_B \\ 1 & if\ p_A = p_B \\ 1 & if\ p_A < p_B \end{cases}$	1
(0,0) (0,1)	1 selects ϕ, 2 selects B	1	1
(0,0) (1,0)	1 selects ϕ, 2 selects A	1	1
(0,0) (0,0)	1 selects ϕ, 2 selects ϕ	0	0
Total Surplus			$\frac{39}{32} - \frac{1}{16}Pr(p_A < p_B)$

Table A.2: Optimal Buyer's Selection Action (Social Planner Cannot Coordinate)

Buyer's valuation	Buyer's optimal action
(1,1)	Select A and B with equal probabilities
(1,0)	Select A
(0,1)	Select B
(0,0)	ϕ

Table A.3: Optimal Expected Total Surplus (Social Planner Cannot Coordinate)

Buyers' valuation profile	Allocation to achieve maximum surplus	Surplus
(1,1) (1,1)	Each buyer selects A and B with equal probabilities	$\frac{3}{2}$
(1,1) (0,1)	1 selects A and B with equal probabilities, 2 selects B	$\frac{3}{2}$
(1,1) (1,0)	1 selects A and B with equal probabilities, 2 selects A	$\frac{3}{2}$
(1,1) (0,0)	1 selects A and B with equal probabilities,2 selects ϕ	1
(0,1) (1,1)	1 selects B, 2 selects A and B with equal probabilities	$\frac{3}{2}$
(0,1) (0,1)	1 selects B, 2 selects B	1
(0,1) (1,0)	1 selects B, 2 selects A	2
(0,1) (0,0)	1 selects B, 2 selects ϕ	1
(1,0) (1,1)	1 selects A, 2 selects A and B with equal probabilities	$\frac{3}{2}$
(1,0) (0,1)	1 selects A, 2 selects B	2
(1,0) (1,0)	1 selects A, 2 selects A	1
(1,0) (0,0)	1 selects A, 2 selects ϕ	1
(0,0) (1,1)	1 selects ϕ, 2 selects A and B with equal probabilities	1
(0,0) (0,1)	1 selects ϕ, 2 selects B	1
(0,0) (1,0)	1 selects ϕ, 2 selects A	1
(0,0) (0,0)	1 selects ϕ, 2 selects ϕ	0
Total expected surplus		$\frac{39}{32}$

Appendix B

Appendix for Chapter 3

B.1 Proof that Sellers will not Choose Negative Prices or Prices strictly greater than 1

Suppose, by contradiction, a seller chooses a negative price. To show this cannot be equilibrium strategy, there are two cases to consider.

One case is the seller has a strictly positive probability of selling by choosing the negative price. This implies the seller gets strictly negative profit. The seller will get higher profit by choosing price 0, which guarantees profit 0.

Another case is that the seller has probability 0 of selling by choosing the negative price. Zero probability of selling means no buyer selects the seller. This can happen only when the other seller also chooses a negative price, because otherwise a buyer who has valuation 0 for both sellers will select the first seller and therefore the first seller, being selected by at least one buyer, has a strictly positive probability of selling. The other seller attracts all buyers and sells with probability 1. Therefore, as argued in the first case, the other seller has a profitable deviation to price 0. After the other seller deviates to price 0, the seller now can sell with strictly positive probability, and as argued in the first case, the seller has a profitable deviate to price 0.

Suppose, by contradiction again, a seller chooses a price strictly greater than 1. As buyers have the outside option of not selecting any seller, which gives the buyer 0 payoff, choosing a price strictly greater than 1 by a seller (the highest possible valuation of buyer) leads to no buyer selecting the seller and therefore 0 selling probability and 0 profit for the seller. Choosing price 0 also leads to 0 expected profit for the seller. I claim that a seller can get strictly positive expected profit by choosing a positive price very close to 0, as long as the other seller's price is non-negative. Consider a buyer who has valuation 1 for the seller and 0 for the other seller. Regardless of which seller the other buyer selects, as long as the seller's price is strictly positive and very close to 0 and the other seller's price is non-negative, the buyer gets strictly positive payoff from selecting the seller and non-positive payoff from selecting the other seller. Therefore in equilibrium the buyer selects the seller with probability 1, which means the seller can get a strictly positive selling probability and a strictly positive profit by choosing a positive price very close to 0.

B.2 Buyers' Symmetric Continuation Equilibrium

To determine the symmetric continuation equilibrium value of $\pi_{11}^{A*}(p_A, p_B), \pi_{11}^{B*}(p_A, p_B)$, for $0 < p_A \leq 1, 0 < p_B \leq 1$ and p_A,p_B not simultaneously 1, suppose the other buyer uses strategy

$$\left(\pi_{00}^A = 0, \pi_{00}^B = 0\right), \left(\pi_{01}^A = 0, \pi_{01}^B = 1\right), \left(\pi_{10}^A = 1, \pi_{10}^B = 0\right), \left(\pi_{11}^A = a, \pi_{11}^B = 1 - a\right)$$

Then a buyer's expected payoff from selecting seller A, conditional on the buyer's valuations being 1 for both sellers, is

$$(1 - p_A)\left(1 - \frac{1}{2}\frac{\frac{1}{4}\eta_3 + \eta_4 a}{\frac{1}{2}\eta_2^\beta + \frac{1}{2}\eta_3 + \eta_4}\right)$$

where the second term is the expected probability of trading with seller A successfully conditional on the buyer's valuation being 1 for both sellers. This is because from a buyer's point of view, with probability $\frac{1}{2}\eta_2^\beta$, the buyer has valuation 1 for both sellers and the other buyer has valuation 0 for both sellers. The reason for the probability to be $\frac{1}{2}\eta_2^\beta$ is that η_2^β is the probability that a buyer has valuation 1 for both sellers and the other buyer has valuation 0 for both sellers. Due to symmetry between two buyers, the probability for a particular buyer to have valuation 1 for both sellers and the other buyer to have valuation 0 for both sellers is $\frac{1}{2}\eta_2^\beta$. Similarly, with probability $\frac{1}{4}\eta_3$, the buyer has valuation 1 for both sellers and the other buyer has valuation 1 for seller A and valuation 0 for seller B; With probability $\frac{1}{4}\eta_3$, the buyer has valuation 1 for both sellers and the other buyer has valuation 0 for A and valuation 1 for B; With probability η_4, the buyer has valuation 1 for both sellers and the other buyer has valuation 1 for both sellers. Therefore conditional on the buyer having valuation 1 for both sellers, the probability distribution of the other buyer's valuations is: With probability $\dfrac{\frac{1}{2}\eta_2^\beta}{\frac{1}{2}\eta_2^\beta+\frac{1}{2}\eta_3+\eta_4}$, the other buyer has valuation 0 for both sellers; With probability $\dfrac{\frac{1}{4}\eta_3}{\frac{1}{2}\eta_2^\beta+\frac{1}{2}\eta_3+\eta_4}$, the other buyer has valuation 1 for A and 0 for B; With the same probability, the other buyer has valuation 0 for A and 1 for B; With probability $\dfrac{\eta_4}{\frac{1}{2}\eta_2^\beta+\frac{1}{2}\eta_3+\eta_4}$, the other buyer has valuation 1 for both sellers. Because the other buyer selects A with probability 1 when he has valuation 1 for A and 0 for B, and selects A with probability a when he has valuation 1 for both sellers, conditional on the buyer has valuation 1 for both sellers, the probability that the other buyer selects A is $\dfrac{\frac{1}{4}\eta_3+\eta_4 a}{\frac{1}{2}\eta_2^\beta+\frac{1}{2}\eta_3+\eta_4}$. When the other buyer selects A, the buyer gets the seller's good with probability $\frac{1}{2}$, and when the other buyer does not select A, the buyer gets the seller's good with probability 1. Therefore the expected probability that the buyer gets A's good conditional on the buyer's valuation being 1 for both sellers is

$$\frac{1}{2}\cdot\frac{\frac{1}{4}\eta_3+\eta_4 a}{\frac{1}{2}\eta_2^\beta+\frac{1}{2}\eta_3+\eta_4}+1\cdot\left(1-\frac{\frac{1}{4}\eta_3+\eta_4 a}{\frac{1}{2}\eta_2^\beta+\frac{1}{2}\eta_3+\eta_4}\right)=1-\frac{1}{2}\frac{\frac{1}{4}\eta_3+\eta_4 a}{\frac{1}{2}\eta_2^\beta+\frac{1}{2}\eta_3+\eta_4}.$$

Similarly the buyer's expected payoff from selecting B, conditional on the buyer's valuations being 1 for both sellers, is

$$(1-p_B)\left(1-\frac{1}{2}\frac{\frac{1}{4}\eta_3+\eta_4-\eta_4 a}{\frac{1}{2}\eta_2^\beta+\frac{1}{2}\eta_3+\eta_4}\right)$$

By comparing the expected payoffs from selecting A and selecting B, the buyer's best response to the other buyer's selecting strategy is

$$\left(\theta_{00}^A(a)=0,\theta_{00}^B(a)=0\right),\left(\theta_{01}^A(a)=0,\theta_{01}^B(a)=1\right),\left(\theta_{10}^A(a)=1,\theta_{10}^B(a)=0\right)$$

$$\theta_{11}^{A}(a)=\begin{cases}1 & ,if\ f(p_A,p_B)>a\\ \in[0,1] & ,if\ f(p_A,p_B)=a\\ 0 & ,if\ f(p_A,p_B)<a\end{cases}$$

where $f(p_A,p_B)\equiv\dfrac{(\eta_2^{\beta}+\frac{3}{4}\eta_3+2\eta_4)(1-p_A)-(\eta_2^{\beta}+\frac{3}{4}\eta_3+\eta_4)(1-p_B)}{\eta_4[(1-p_A)+(1-p_B)]}$.

$f(p_A,p_B)$ is strictly decreasing in p_A and strictly increasing in p_B for $0<p_A\leq 1$, $0<p_B\leq 1$ and p_A,p_B not simultaneously equal to 1.

The unique symmetric continuation equilibrium value of $\theta_{11}^{A*}(p_A,p_B),\theta_{11}^{B*}(p_A,p_B)$, for $0<p_A\leq 1$, $0<p_B\leq 1$ and p_A,p_B not simultaneously equal to 1, can be characterized as

$$\theta_{11}^{A*}(p_A,p_B)=\begin{cases}1 & ,\ f(p_A,p_B)\geq 1\\ f(p_A,p_B) & ,\ 0<f(p_A,p_B)<1\\ 0 & ,\ f(p_A,p_B)\leq 0\end{cases}$$

and $\theta_{11}^{B*}(p_A,p_B)=1-\theta_{11}^{A*}(p_A,p_B)$.

Define the notion that p_A is sufficiently lower than p_B when $f(p_A,p_B)\geq 1$, p_A is sufficiently higher than p_B when $f(p_A,p_B)\leq 0$, and p_A is close to p_B when $0<f(p_A,p_B)<1$. In the first two cases, there is no local competition between p_A and p_B, while in the last case, there is local competition between two prices.

Since sellers do not know the particular realization of buyers' valuations, but know the joint distribution of their valuations, given buyers' equilibrium strategy in the subgame with p_A and p_B such that $0<p_A\leq 1$ and $0<p_B\leq 1$, seller A's expected probability of selling is

$$\tfrac{1}{2}\eta_1+\eta_2+\tfrac{1}{2}\eta_2^{\sigma}+\tfrac{1}{2}\eta_3+(\eta_2^{\beta}+\tfrac{1}{2}\eta_3)\theta_{11}^{A*}(p_A,p_B)+\\ \eta_4\theta_{11}^{A*}(p_A,p_B)\left(2-\theta_{11}^{A*}(p_A,p_B)\right)$$

This is because with probability $\frac{1}{2}\eta_1+\eta_2+\frac{1}{2}\eta_2^{\sigma}+\frac{1}{2}\eta_3$, at least one buyer chooses seller A with probability 1. This is because among the scenarios where a buyer has valuation 1 for one seller and valuation 0 for the other while the other buyer has valuation 0 for both sellers, half of them involve a buyer has valuation 1 for seller A. Therefore half of η_1, which is the total probability of these scenarios, one buyer has valuation 1 for seller A so seller A can sell with probability 1. Similarly we can get other probabilities. With probability $\eta_2^\beta + \frac{1}{2}\eta_3$, only one buyer chooses seller A with probability $\pi_{11}^{A*}(p_A, p_B)$ and the other buyer does not choose the seller, so seller A can sell with probability $\pi_{11}^{A*}(p_A, p_B)$. With probability η_4, both buyers choose seller A with probability $\pi_{11}^{A*}(p_A, p_B)$, so seller A can sell with probability $1 - \left(1 - \pi_{11}^{A*}(p_A, p_B)\right)^2 = \pi_{11}^{A*}(p_A, p_B)\left(2 - \pi_{11}^{A*}(p_A, p_B)\right)$.

And seller B's expected probability of selling is

$$\frac{1}{2}\eta_1 + \eta_2 + \frac{1}{2}\eta_2^\sigma + \frac{1}{2}\eta_3 + (\eta_2^\beta + \frac{1}{2}\eta_3)\left(1 - \theta_{11}^{A*}(p_A, p_B)\right) + \\ \eta_4\left(1 - \theta_{11}^{A*}(p_A, p_B)\right)\left(1 + \theta_{11}^{A*}(p_A, p_B)\right).$$

Seller A's expected probability of selling is strictly increasing in $\theta_{11}^{A*}(p_A, p_B)$.

When $1 < f(p_A, p_B)$, we have$\theta_{11}^{A*}(p_A, p_B) = 1$ and seller A's expected probability of selling reaches its maximum level, denote it as $\overline{\Pi}$. When $f(p_A, p_B) < 0$, we have $\theta_{11}^{A*}(p_A, p_B) = 0$, and seller A's expected probability of selling reaches its minimum level, denote it as $\underline{\Pi}$. When $0 \le f(p_A, p_B) \le 1$, denote seller A's expected probability of selling as $\Pi(p_A, p_B)$.

B.3 Proof of Proposition 4

First, the following results will be useful regarding the properties of symmetric mixed-strategy equilibrium between sellers.

Lemma 11. *In any symmetric mixed-strategy equilibrium between sellers, any two prices in the support do not locally compete with each other.*

The proof of this lemma follows exactly as the proof of Lemma 7, so is omitted here.

Lemma 12. *Any symmetric equilibrium between sellers cannot have a support made of more than two and finite number of prices.*

Proof. Consider by contradiction, there is a symmetric mixed-strategy equilibrium between sellers with a support made of a finite number of prices $\{p_1, p_2, ..., p_K\}$, where $2 \le K$. From Lemma 11, p_k does not locally compete with p_{k+1} for all $k = 1, ..., K-1$. Denote the probabilities of choosing p_k as x_k. The necessary conditions for this mixed strategy to be a symmetric equilibrium include the first order condition at p_K, which can be shown to be

$$[\underline{\Pi}(1-x_K)+\Pi(p_K,p_K)x_K]+\frac{\partial\Pi(p_K,p_K)}{\partial p_A}x_K p_K=0 \tag{B.1}$$

from which we can solve for x_K,

$$x_K=\frac{\underline{\Pi}}{\underline{\Pi}-\Pi(p_K,p_K)}-\frac{\partial\Pi(p_K,p_K)}{\partial p_A}p_K \tag{B.2}$$

Another necessary equilibrium condition is that a seller's profit by choosing p_K is no less than her profit by choosing 1.

$$\underline{\Pi}\leq[\underline{\Pi}(1-x_K)+\Pi(p_K,p_K)x_K]\,p_K \tag{B.3}$$

Substitute equation B.2 into the above equation and rearrange, we have

$$p_K\leq\frac{\Pi(p_K,p_K)-\underline{\Pi}}{-\frac{\partial\Pi(p_K,p_K)}{\partial p_A}(1-p_K)} \tag{B.4}$$

From the first order condition for a seller to choose p_{K-1},

$$\underline{\Pi}(1-x_{K-1}-x_K)+\Pi(p_{K-1},p_{K-1})x_{K-1}+\frac{\partial\Pi(p_{K-1},p_{K-1})}{\partial p_A}x_{K-1}p_{K-1}+\overline{\Pi}x_K=0 \tag{B.5}$$

Subtract B.1 by B.5, we have

$$\left[\overline{\Pi}-\Pi(p_K,p_K)-\frac{\partial\Pi(p_K,p_K)}{\partial p_A}p_K\right]x_K=\left[\underline{\Pi}-\Pi(p_{K-1},p_{K-1})-\frac{\partial\Pi(p_{K-1},p_{K-1})}{\partial p_A}p_{K-1}\right]x_{K-1} \tag{B.6}$$

A seller's profit by choosing p_{K-1} is

$$\left[\underline{\Pi}(1-x_{K-1}-x_K)+\Pi(p_{K-1},p_{K-1})x_{K-1}+\overline{\Pi}x_K\right]p_{K-1} \tag{B.7}$$

From B.5, we know

$$\underline{\Pi}(1-x_{K-1}-x_K)+\Pi(p_{K-1},p_{K-1})x_{K-1}+\overline{\Pi}x_K=-\frac{\partial\Pi(p_{K-1},p_{K-1})}{\partial p_A}x_{K-1}p_{K-1} \tag{B.8}$$

which implies a seller's profit by choosing p_{K-1} is

$$-\frac{\partial\Pi(p_{K-1},p_{K-1})}{\partial p_A}x_{K-1}p_{K-1}^2 \tag{B.9}$$

Similarly we can show that a seller's profit by choosing p_K is

$$-\frac{\partial\Pi(p_K,p_K)}{\partial p_A}x_K p_K^2 \tag{B.10}$$

From the necessary equilibrium condition that a seller gets the same profit by choosing p_K and p_{K-1}, we have

$$-\frac{\partial\Pi(p_K,p_K)}{\partial p_A}x_K p_K^2=-\frac{\partial\Pi(p_{K-1},p_{K-1})}{\partial p_A}x_{K-1}p_{K-1}^2 \tag{B.11}$$

Divide B.6 by B.11, we have

$$\frac{\overline{\Pi}-\Pi(p_K,p_K)-\frac{\partial\Pi(p_K,p_K)}{\partial p_A}p_K}{-\frac{\partial\Pi(p_K,p_K)}{\partial p_A}p_K^2}=\frac{\underline{\Pi}-\Pi(p_K,p_K)-\frac{\partial\Pi(p_{K-1},p_{K-1})}{\partial p_A}p_{K-1}}{-\frac{\partial\Pi(p_{K-1},p_{K-1})}{\partial p_A}p_{K-1}^2} \tag{B.12}$$

The left hand side of B.12 can be shown to be a strictly decreasing function in p_K and because $p_K<1$, we know the left hand side is strictly greater than 1. The right hand side of B.12 can be shown to be less than or equal to 1. To see this, note p_{K-1} is strictly less than p_K and because of B.4, we know that

$$p_{K-1}\leq\frac{\Pi(p_{K-1},p_{K-1})-\underline{\Pi}}{-\frac{\partial\Pi(p_{K-1},p_{K-1})}{\partial p_A}(1-p_{K-1})} \tag{B.13}$$

which can be shown is equivalent to the right hand side of B.12 is less than or equal to 1. Given the left hand side of B.12 is strictly greater than 1 and the right hand side of B.12 is less than or equal to 1, we know that B.12 cannot hold. This implies that necessary equilibrium conditions cannot hold at both two prices p_K

and p_{K-1}. Therefore, the support of any symmetric mixed-strategy equilibrium cannot be made of more than two and finite number of prices. $\square$

B.3.1 Mixed-strategy Symmetric Equilibrium

When $\Pi(p^*,p^*)p^* < \underline{\Pi}$, similarly with Chapter 1, we can construct a candidate for symmetric equilibrium between sellers.

Consider an increasing, infinite sequence of prices $\{p_k\}_{k=1}^{\infty}$ with a sequence of probabilities $\{x_k\}_{k=1}^{\infty}$ corresponding to each price, such that for all k, p_k does not locally compete with p_{k+1} for all k.

This implies that $lim_{k\to\infty} p_k = 1$.

Necessary conditions for the mixed strategy $\{p_k\}_{k=1}^{\infty}$ and $\{x_k\}_{k=1}^{\infty}$ to be a symmetric equilibrium between sellers are that when opponent seller uses the mixed strategy, a seller gets the same profit by choosing p_k as the profit by choosing 1 and the seller has no profitable local deviation from p_k for all k.

These conditions can be written as the following equations.

For all $k \in N$,

$$[\underline{\Pi} \cdot \sum_{j=1}^{k-1} x_j + \Pi(p_k,p_k) \cdot x_k + \overline{\Pi} \cdot (1 - \sum_{j=1}^{k} x_j)]p_k = \underline{\Pi} \tag{B.14}$$

$$\underline{\Pi} \cdot \sum_{j=1}^{k-1} x_j + \Pi(p_k,p_k) \cdot x_k + \overline{\Pi} \cdot (1 - \sum_{j=1}^{k} x_j) + \frac{\partial \Pi(p_k,p_k)}{\partial p_A} \cdot x_k \cdot p_k = 0 \tag{B.15}$$

We show that when $\Pi(p^*,p^*)p^* < \underline{\Pi}$, the above two equations characterize $\{p_k\}_{k=1}^{\infty}$ and $\{x_k\}_{k=1}^{\infty}$, and the characterized mixed strategy is a symmetric equilibrium between sellers.

First, we can rewrite equations (B.14), and (B.15) into a first order difference equation in prices,

$$\frac{\Pi(p_{k+1},p_{k+1}) - \overline{\Pi} + \frac{\partial \Pi(p_{k+1},p_{k+1})}{\partial p_A} p_{k+1}}{\frac{\partial \Pi(p_{k+1},p_{k+1})}{\partial p_A} p_{k+1}^2} = \frac{\Pi(p_k,p_k) - \underline{\Pi} + \frac{\partial \Pi(p_k,p_k)}{\partial p_A} p_k}{\frac{\partial \Pi(p_k,p_k)}{\partial p_A} p_k^2} \tag{B.16}$$

with initial condition

$$\frac{\Pi(p_1,p_1)-\overline{\Pi}+\frac{\partial\Pi(p_1,p_1)}{\partial p_A}p_1}{\frac{\partial\Pi(p_1,p_1)}{\partial p_A}p_1^2}=\frac{\overline{\Pi}}{\underline{\Pi}} \tag{B.17}$$

with x_k being determined by

$$x_k=\frac{\underline{\Pi}}{-\frac{\partial\Pi(p_k,p_k)}{\partial p_A}p_k^2}. \tag{B.18}$$

Because $\dfrac{\partial\Pi(p,p)}{\partial p_A}=-\dfrac{(\eta_2^{\beta}+\frac{1}{2}\eta_3+\eta_4)(2\eta_2^{\beta}+\frac{3}{2}\eta_3+3\eta_4)}{4\eta_4}\dfrac{1}{1-p}$ is a strictly decreasing and continuous function in p for all $p\in(0,1)$, the left hand side of (B.17), $\dfrac{\Pi(p_1,p_1)-\overline{\Pi}+\frac{\partial h(p_1,p_1)}{\partial p_A}p_1}{\frac{\partial\Pi(p_1,p_1)}{\partial p_A}p_1^2}$, is a strictly decreasing and continuous function for all $p_1\in(0,1)$.

Furthermore, $\dfrac{\Pi(p_1,p_1)-\overline{\Pi}+\frac{\partial\Pi(p_1,p_1)}{\partial p_A}p_1}{\frac{\partial\Pi(p_1,p_1)}{\partial p_A}p_1^2}|_{p_1=\frac{\underline{\Pi}}{\overline{\Pi}}}=\dfrac{\Pi(p_1,p_1)-\overline{\Pi}}{\frac{\partial\Pi(p_1,p_1)}{\partial p_A}p_1^2}|_{p_1=\frac{\underline{\Pi}}{\overline{\Pi}}}+\dfrac{\overline{\Pi}}{\underline{\Pi}}>\dfrac{\overline{\Pi}}{\underline{\Pi}}$.

And $lim_{p_1\to 1^-}\dfrac{\Pi(p_1,p_1)-\overline{\Pi}+\frac{\partial\Pi(p_1,p_1)}{\partial p_A}p_1}{\frac{\partial\Pi(p_1,p_1)}{\partial p_A}p_1^2}=lim_{p_1\to 1^-}\dfrac{\Pi(p_1,p_1)-\overline{\Pi}}{\frac{\partial\Pi(p_1,p_1)}{\partial p_A}p_1^2}+lim_{p_1\to 1^-}\dfrac{1}{p_1}=1<\dfrac{\overline{\Pi}}{\underline{\Pi}}$.

Therefore, initial condition (B.17) has a unique solution, denoted as p_1^*, such that $0<p_1^*<1$. Furthermore, p_1^* is strictly greater than $\dfrac{\underline{\Pi}}{\overline{\Pi}}$.

In addition, when $\Pi(p^*,p^*)p^*<\underline{\Pi}$, we have

$$\left\{\frac{\partial\Pi(p,p)}{\partial p_A}p+\Pi(p,p)\right\}|_{p=\frac{\underline{\Pi}}{\Pi(p,p)}}<0$$

which implies that

$$\frac{\Pi(p,p)-\overline{\Pi}+\frac{\partial\Pi(p,p)}{\partial p_A}p}{\frac{\partial\Pi(p,p)}{\partial p_A}p^2}|_{p=\frac{\underline{\Pi}}{\Pi(p,p)}}<\frac{\overline{\Pi}}{\underline{\Pi}}.$$

Because $\dfrac{\Pi(p_1^*,p_1^*)-\overline{\Pi}+\frac{\partial\Pi(p_1^*,p_1^*)}{\partial p_A}p_1^*}{\frac{\partial\Pi(p_1^*,p_1^*)}{\partial p_A}p_1^{*2}}=\dfrac{\overline{\Pi}}{\underline{\Pi}}$ and $\dfrac{\Pi(p_1,p_1)-\overline{\Pi}+\frac{\partial\Pi(p_1,p_1)}{\partial p_A}p_1}{\frac{\partial\Pi(p_1,p_1)}{\partial p_A}p_1^2}$ is strictly decreasing in p_1, the above inequality implies $p_1^* < \dfrac{\underline{\Pi}}{\Pi(p_1^*, p_1^*)}$.

Second, we show that p_2^* satisfies $p_1^* < p_2^* < 1$. From difference equation (B.16), we have

$$\frac{\Pi(p_1, p_1) - \underline{\Pi} + \frac{\partial \Pi(p_1, p_1)}{\partial p_A} p_1}{\frac{\partial \Pi(p_1, p_1)}{\partial p_A} p_1^2} = \frac{\Pi(p_2, p_2) - \overline{\Pi} + \frac{\partial \Pi(p_2, p_2)}{\partial p_A} p_2}{\frac{\partial \Pi(p_2, p_2)}{\partial p_A} p_2^2} \tag{B.19}$$

From equation (B.19), p_2^* can be solved by substituting the value of p_1^*. Because $\dfrac{\Pi(p_1^*, p_1^*) - \overline{\Pi} + \frac{\partial \Pi(p_1^*, p_1^*)}{\partial p_A} p_1^*}{\frac{\partial \Pi(p_1^*, p_1^*)}{\partial p_A} p_1^{*2}} = \dfrac{\overline{\Pi}}{\underline{\Pi}}$, it can be shown that

$$\frac{\Pi(p_1^*, p_1^*) - \underline{\Pi} + \frac{\partial \Pi(p_1^*, p_1^*)}{\partial p_A} p_1^*}{\frac{\partial \Pi(p_1^*, p_1^*)}{\partial p_A} p_1^{*2}} = \frac{\overline{\Pi}}{\underline{\Pi}} + \frac{\overline{\Pi} - \underline{\Pi}}{\underline{\Pi}\left(\Pi(p_1^*, p_1^*) - \overline{\Pi}\right)} \left(\overline{\Pi} - \frac{\underline{\Pi}}{p_1^*}\right)$$

And because $p_1^* < \dfrac{\underline{\Pi}}{\Pi(p_1^*, p_1^*)}$, we have $\dfrac{\overline{\Pi}}{\underline{\Pi}} + \dfrac{\overline{\Pi} - \underline{\Pi}}{\underline{\Pi}\left(\Pi(p_1^*, p_1^*) - \overline{\Pi}\right)} \left(\overline{\Pi} - \dfrac{\underline{\Pi}}{p_1^*}\right) > 1$, which implies

$$\frac{\Pi(p_1^*, p_1^*) - \underline{\Pi} + \frac{\partial \Pi(p_1^*, p_1^*)}{\partial p_A} p_1^*}{\frac{\partial \Pi(p_1^*, p_1^*)}{\partial p_A} p_1^{*2}} > 1$$

The right hand side of (B.19) is a strictly decreasing and continuous function in p. Its value at $p = p_1^*$ is strictly greater than the left hand side of (B.19), and its limit value as $p \to 1^-$ is equal to 1, which is strictly less than the left hand side of (B.19), as shown in the above inequality. Therefore, equation (B.19) as an equation in p_2 has a unique solution between p_1^* and 1.

Because $-\dfrac{\partial \Pi(p, p)}{\partial p_A} p(1-p)$ is a strictly increasing function in p and $p_2^* > p_1^*$, we have

$$-\frac{\partial \Pi(p_2^*, p_2^*)}{\partial p_A} p_2^*(1-p_2^*) > -\frac{\partial \Pi(p_1^*, p_1^*)}{\partial p_A} p_1^*(1-p_1^*)$$

Because $\dfrac{\Pi(p_1^*, p_1^*) - \underline{\Pi} + \frac{\partial \Pi(p_1^*, p_1^*)}{\partial p_A} p_1^*}{\frac{\partial \Pi(p_1^*, p_1^*)}{\partial p_A} p_1^{*2}} > 1$, we have $-\frac{\partial \Pi(p_1^*, p_1^*)}{\partial p_A} p_1^*(1-p_1^*) > \Pi(p_1^*, p_1^*) - \underline{\Pi}$, therefore

$$-\frac{\partial \Pi(p_2^*, p_2^*)}{\partial p_A} p_2^*(1 - p_2^*) > \Pi(p_2^*, p_2^*) - \underline{\Pi}$$

which is equivalent to

$$\frac{\Pi(p_2^*, p_2^*) - \underline{\Pi} + \frac{\partial \Pi(p_2^*, p_2^*)}{\partial p_A} p_2^*}{\frac{\partial \Pi(p_2^*, p_2^*)}{\partial p_A} p_2^{*2}} > 1.$$

Third, we can show that p_3^* satisfies $p_2^* < p_3^* < 1$.

$$\frac{\Pi(p_2, p_2) - \underline{\Pi} + \frac{\partial \Pi(p_2, p_2)}{\partial p_A} p_2}{\frac{\partial \Pi(p_2, p_2)}{\partial p_A} p_2^2} = \frac{\Pi(p_3, p_3) - \overline{\Pi} + \frac{\partial \Pi(p_3, p_3)}{\partial p_A} p_3}{\frac{\partial \Pi(p_3, p_3)}{\partial p_A} p_3^2} \tag{B.20}$$

Given $\frac{\Pi(p_2^*, p_2^*) - \underline{\Pi} + \frac{\partial \Pi(p_2^*, p_2^*)}{\partial p_A} p_2^*}{\frac{\partial \Pi(p_2^*, p_2^*)}{\partial p_A} p_2^{*2}} > 1$ as shown in step 2, we can similarly prove that equation (B.20) has a unique solution p_3^* between p_2^* and 1.

These steps can be carried out for any $k \in N$, therefore, for all $k \in N$, $p_k^* < p_{k+1}^* < 1$.

Because $\left\{p_k^*\right\}_{k=1}^{\infty}$ is strictly increasing sequence, and has an upper bound 1, $lim_{k\to\infty} p_k^*$ exists and $lim_{k\to\infty} p_k^* \leq 1$.

Take limits of the two sides of difference equation (B.16) as $k \to \infty$, we can solve that $lim_{k\to\infty} p_k^* = 1$.

$$
\begin{aligned}
\sum_{k=1}^{N} x_k^* &= \sum_{k=1}^{N} \frac{\underline{\Pi}}{-\frac{\partial \Pi\left(p_k^*, p_k^*\right)}{\partial p_A} p_k^{*2}} \\
&= \frac{\underline{\Pi}}{\overline{\Pi}-\underline{\Pi}} \sum_{k=1}^{N} \frac{\overline{\Pi}-\underline{\Pi}}{-\frac{\partial \Pi\left(p_k^*, p_k^*\right)}{\partial p_A} p_k^{*2}} \\
&= \frac{\underline{\Pi}}{\overline{\Pi}-\underline{\Pi}} \sum_{k=1}^{N} \left[\frac{\Pi(p_k^*, p_k^*) - \underline{\Pi} + \frac{\partial \Pi\left(p_k^*, p_k^*\right)}{\partial p_A} p_k^*}{-\frac{\partial \Pi\left(p_k^*, p_k^*\right)}{\partial p_A} p_k^{*2}} - \frac{\Pi(p_k^*, p_k^*) - \overline{\Pi} + \frac{\partial \Pi\left(p_k^*, p_k^*\right)}{\partial p_A} p_k^*}{-\frac{\partial \Pi\left(p_k^*, p_k^*\right)}{\partial p_A} p_k^{*2}} \right] \\
&= \frac{\underline{\Pi}}{\overline{\Pi}-\underline{\Pi}} \left[-\frac{\Pi(p_1^*, p_1^*) - \overline{\Pi} + \frac{\partial \Pi\left(p_1^*, p_1^*\right)}{\partial p_A} p_1^*}{-\frac{\partial \Pi\left(p_1^*, p_1^*\right)}{\partial p_A} p_1^{*2}} + \frac{\Pi(p_N^*, p_N^*) - \underline{\Pi} + \frac{\partial \Pi\left(p_N^*, p_N^*\right)}{\partial p_A} p_N^*}{-\frac{\partial \Pi\left(p_N^*, p_N^*\right)}{\partial p_A} p_N^{*2}} \right] \\
&= \frac{\underline{\Pi}}{\overline{\Pi}-\underline{\Pi}} \left[\frac{\overline{\Pi}}{\underline{\Pi}} + \frac{\Pi(p_N^*, p_N^*) - \underline{\Pi} + \frac{\partial \Pi\left(p_N^*, p_N^*\right)}{\partial p_A} p_N^*}{-\frac{\partial \Pi\left(p_N^*, p_N^*\right)}{\partial p_A} p_N^{*2}} \right] \\
&= \frac{\overline{\Pi}}{\overline{\Pi}-\underline{\Pi}} + \frac{\underline{\Pi}}{\overline{\Pi}-\underline{\Pi}} \frac{\Pi(p_N^*, p_N^*) - \underline{\Pi} + \frac{\partial \Pi\left(p_N^*, p_N^*\right)}{\partial p_A} p_N^*}{-\frac{\partial \Pi\left(p_N^*, p_N^*\right)}{\partial p_A} p_N^{*2}}
\end{aligned}
$$

where the fourth equality is to use (B.16) to cancel out terms, and the fifth equality is to use (B.17).

$$
\begin{aligned}
\textstyle\sum_{k=1}^{\infty} x_k^* = lim_{N\to\infty} \sum_{k=1}^{N} x_k^* &= lim_{N\to\infty} \left[\frac{\overline{\Pi}}{\overline{\Pi}-\underline{\Pi}} + \frac{\underline{\Pi}}{\overline{\Pi}-\underline{\Pi}} \frac{\Pi(p_N^*, p_N^*) - \underline{\Pi} + \frac{\partial \Pi\left(p_N^*, p_N^*\right)}{\partial p_A} p_N^*}{-\frac{\partial \Pi\left(p_N^*, p_N^*\right)}{\partial p_A} p_N^{*2}} \right] \\
&= \frac{\overline{\Pi}}{\overline{\Pi}-\underline{\Pi}} + \frac{\underline{\Pi}}{\overline{\Pi}-\underline{\Pi}} lim_{N\to\infty} \frac{\Pi(p_N^*, p_N^*) - \underline{\Pi} + \frac{\partial \Pi\left(p_N^*, p_N^*\right)}{\partial p_A} p_N^*}{-\frac{\partial \Pi\left(p_N^*, p_N^*\right)}{\partial p_A} p_N^{*2}} \\
&= \frac{\overline{\Pi}}{\overline{\Pi}-\underline{\Pi}} + \frac{\underline{\Pi}}{\overline{\Pi}-\underline{\Pi}} (-1) \\
&= 1
\end{aligned}
$$

where the third equality is due to $lim_{N\to\infty} p_N^* = 1$.

In conclusion, we prove that for all $k \in N$, $p_k^* < p_{k+1}^* < 1$, $0 < x_k^*$, $lim_{k\to\infty} p_k^* = 1$, and $\sum_{k=1}^{\infty} x_k^* = 1$.

Therefore, $\left\{\left(p_k^*, x_k^*\right)\right\}_{k=1}^{\infty}$ is a valid mixed strategy in choosing prices.

At last, we show that for all $k \in N$, p_k^* maximizes seller A's profit when seller B uses mixed strategy $\left\{\left(p_k^*, x_k^*\right)\right\}_{k=1}^{\infty}$.

p_1^* is determined by the solution to equation (B.17).

It is easy to show that when $\Pi(p^*,p^*)p^* < \underline{\Pi}$,

$$p_1^* > \frac{\Pi(p_1^*,p_1^*) - \underline{\Pi}}{\Pi(p_1^*,p_1^*) - \underline{\Pi} - \frac{\partial \Pi(p_1^*,p_1^*)}{\partial p_A}(1-p_1^*)}$$

Because for all $k \in N$, $p_{k+1}^* > p_1^*$, we have for all $k \in N$,

$$p_k^* > \frac{\Pi(p_k^*,p_k^*) - \underline{\Pi}}{\Pi(p_k^*,p_k^*) - \underline{\Pi} - \frac{\partial \Pi(p_1^*,p_1^*)}{\partial p_A}(1-p_1^*)}.$$

The difference equation (B.16) can be solved for p_{k+1}^* as a function of p_k^*.

Denote the highest price p such that $f\left(p,p_k^*\right) > 0$ as $\overline{p}\left(p_k^*\right)$, the lowest price such that $f\left(p,p_k^*\right) < 1$ as $\underline{p}\left(p_k^*\right)$.

Compare $\overline{p}\left(p_k^*\right)$ with $\underline{p}\left(p_{k+1}^*\right)$, by substituting the expression for p_{k+1}^*, and rearrange, we have the above comparison is equivalent with comparing

$\left(1-p_k^*\right)\left[\left(6\eta_4^2+4\eta_3\eta_4+7\eta_2^\beta\eta_4+\frac{3}{4}\eta_3^2+\frac{5}{2}\eta_2^\beta\eta_3+2\eta_2^{\beta 2}\right)p_k^*-\left(3\eta_4^2+\eta_3\eta_4+2\eta_2^\beta\eta_4\right)\right]$ with 0.

Because p_k^* satisfies $p_k^* < 1$ and $p_k^* > \dfrac{\Pi(p_1^*,p_1^*) - \underline{\Pi}}{\Pi(p_1^*,p_1^*) - \underline{\Pi} - \frac{\partial \Pi(p_1^*,p_1^*)}{\partial p_A}(1-p_1^*)}$, we have

$\left(1-p_k^*\right)\left[\left(6\eta_4^2+4\eta_3\eta_4+7\eta_2^\beta\eta_4+\frac{3}{4}\eta_3^2+\frac{5}{2}\eta_2^\beta\eta_3+2\eta_2^{\beta 2}\right)p_k^*-\left(3\eta_4^2+\eta_3\eta_4+2\eta_2^\beta\eta_4\right)\right] < 0$

Therefore $\overline{p}\left(p_k^*\right) < \underline{p}\left(p_{k+1}^*\right)$.

Given opponent seller is using $\{(p_k^*,x_k^*)\}_{k=1}^\infty$, consider whether a seller has a profitable deviation to prices other than $\{p_1^*,p_2^*,...\}$.

Consider some price $\widetilde{p} \in \left[\underline{p}\left(p_k^*\right),\overline{p}\left(p_k^*\right)\right]$ but different from p_k^*. By charging $\widetilde{p}$, the seller is locally competing with opponent seller when opponent seller is charging p_k^*, and is not locally competing with opponent when opponent is charging price other than p_k^*. If the prices opponent seller is locally competing with the seller's price, the first order derivative of seller's profit with respect to the seller's price is decreasing in seller's price. If the opponent's price is not locally competing with the seller's price, the first order derivative of seller's profit with respect to the seller's price is a constant. This implies when opponent is using strategy $\{(p_k^*,x_k^*)\}_{k=1}^\infty$, the derivative of the seller's expected profit with respect to the seller's price is also decreasing in seller's own price. From the main text of the paper, the derivative of the seller's expected profit with respect to the seller's price when the seller's price is p_k^* is zero. Because the derivative of the seller's expected profit with respect to the seller's price is decreasing in seller's own price, the derivative of the seller's expected profit with respect to the seller's price is positive when the seller's price is in the range of $[\underline{p}(p_k^*),\ p_k^*)$ while the derivative is negative when the seller's price is in the range of $(p_k^*, \overline{p}(p_k^*)]$, so the seller's expected profit by charging price p_k^* is greater than expected profit by charging any other price $\widetilde{p} \in \left[\underline{p}(p_k^*), \overline{p}(p_k^*)\right]$.

Now consider prices in the range $\left(\overline{p}\left(p_{k-1}^*\right), \underline{p}(p_k^*)\right)$. When the seller's price is in this range, the seller is not locally competing with any of opponent's prices. So seller's expected probability of selling is a positive constant, which means expected profit at $\underline{p}(p_k^*)$ is strictly greater than profit at any other price in the range. Because the seller's expected profit is continuous at $\underline{p}(p_k^*)$, and expected profit at p_k^* is greater than profit at $\underline{p}(p_k^*)$, expected profit at p_k^* is greater than expected profit at any price in the range $\left(\overline{p}\left(p_{k-1}^*\right), \underline{p}(p_k^*)\right)$.

Therefore, the seller's expected profit at p_k^* is strictly greater than expected profit at any other price in $\left[\overline{p}\left(p_{k-1}^*\right), \overline{p}(p_k^*)\right]$. This holds for all $k = 1, 2, ...$ so we have seller has no profitable deviation to prices other than $\{p_1^*, p_2^*, ...\}$.

Therefore, when $\Pi(p^*, p^*)p^* < \underline{\Pi}$, the mixed strategy $\{p_1^*, p_2^*, ...\}$ with corresponding probabilities $\{x_1^*, x_2^*, ...\}$ is a symmetric equilibrium between sellers.

According to Lemma 11 and Lemma 12, we know any symmetric mixed-strategy equilibrium must have a support consisting of infinite number of prices such that any two prices in the support do not locally compete with each other. Therefore any symmetric mixed-strategy equilibrium takes the form $\{p_1, p_2, ...\}$ with corresponding probabilities $\{x_1, x_2, ...\}$ such that p_k does not locally compete with p_{k+1} for all $k \in N$. This implies that necessary conditions for $\{p_1, p_2, ...\}$ and $\{x_1, x_2, ...\}$ to be a symmetric equilibrium include equations B.14 and B.15. Given these two equations have a unique solution $\{p_1^*, p_2^*, ...\}$ and $\{x_1^*, x_2^*, ...\}$, $\{p_1, p_2, ...\}$ and $\{x_1, x_2, ...\}$ must coincide with $\{p_1^*, p_2^*, ...\}$ and $\{x_1^*, x_2^*, ...\}$ in order for $\{p_1, p_2, ...\}$ and $\{x_1, x_2, ...\}$ to be a symmetric equilibrium.

This establishes the characterized mixed strategy $\{p_1^*, p_2^*, ...\}$ and $\{x_1^*, x_2^*, ...\}$ is the unique symmetric mixed-strategy equilibrium between sellers.

Given there is no symmetric pure-strategy equilibrium between sellers when

$\Pi(p^*, p^*)p^* < \underline{\Pi}$, $\{p_1^*, p_2^*, ...\}$ and $\{x_1^*, x_2^*, ...\}$ is the unique symmetric equilibrium between sellers.

B.3.2 Pure-strategy Symmetric Equilibrium

As argued in the main text, if we can prove that when the opponent seller chooses price p^*, only p^* and 1 can be local maximum points for a seller's profit, then we can show condition $\underline{\Pi} \leq \Pi(p^*, p*)p^*$ implies there is a pure-strategy symmetric equilibrium where sellers choose p^*.

We consider three cases for seller A's price when B's price is p^*.

When seller A's prices are sufficiently higher than p^*, seller A's probability of selling is $\underline{\Pi}$ regardless of the particular level of A's price, and price 1 leads to the highest profit among all prices that are sufficiently higher than p^*. As prices strictly higher than 1 lead to 0 probability of selling and strictly lower profit than price 1, 1 is a local maximum point for seller A's profit.

When A's price is in local competition with p^*, seller A's profit is given by $\Pi(p_A, p^*)\, p_A$. Since by definition of p^*, first order derivative of seller A's profit with respect to p_A is zero at p^* and $\Pi(p_A, p^*)\, p_A$ is strictly concave in p_A, seller A's profit at p^* is higher than profit by choosing prices nearby and is therefore a local maximum point.

When seller A's price is sufficiently lower than p^*, seller A's probability of selling is $\overline{\Pi}$ regardless of particular level of A's price, and the highest price among all prices sufficiently lower than p^* leads to the highest profit. Whether or not this price is local maximum point for seller A depends on whether seller A's profit at this price is higher than her profits at prices slightly higher. Price slightly higher mean the price is in local competition with p^*, and seller A's probability of selling is a strictly concave function of the particular level of price. As the first order condition is satisfied at p^*, and p^* is strictly higher than the price in question, the derivative is strictly positive at the price in question. This is implies the highest price among all prices sufficiently lower than p^* is not a local maximum point for seller A.

Therefore, seller A's profit has only two local maximum points, 1 and p^*. As long as seller A's profit by choosing p^* is greater than or equal to profit by choosing

1, which is condition $\underline{\Pi} \leq \Pi(p^*, p^*)p^*$, p^* is a symmetric equilibrium between sellers.

Then we can show there do not exist mixed-strategy symmetric equilibria between sellers when $\underline{\Pi} \leq \Pi(p^*, p^*)p^*$. Suppose by contradiction that there exists a symmetric mixed-strategy equilibrium. Based on Lemma 11, the support must be discrete. Based on Lemma 12, the support must be made of infinite number of prices.

Given the support is made of an infinite number of price such that any two prices in the support do not locally compete with each other, we know the infinite number of prices must converge to 1.

Denote these infinite number of prices as $\{p_k\}_{k=1}^{\infty}$ and the corresponding probabilities of choosing p_k as x_k, similarly as in Chapter 2, we can show necessary equilibrium conditions are equations B.14 and B.15.

When $\underline{\Pi} < \Pi(p^*, p^*)p^*$, we can show that p_2^* is strictly greater than 1. Therefore the characterized p_2^* cannot be in the support of a mixed strategy.

When $\underline{\Pi} = \Pi(p^*, p^*)p^*$, we can show that $p_1^* = p^*$, $x_1^* = 1$, $p_k^* = 1$ and $x_k^* = 0$ for all $k \geq 2$. Therefore the characterized mixed strategy coincides with the pure strategy p^*.

In both cases, there does not exist a symmetric mixed-strategy equilibrium.

Therefore, when $\underline{\Pi} \leq \Pi(p^*, p^*)p^*$, p^* is the only symmetric equilibrium between sellers.

B.4 Comparative Statics

Because $-\dfrac{\partial \Pi(p,p)}{\partial p} p$ is a strictly increasing function in p, the condition that $\underline{\Pi} \leq \Pi(p^*, p^*)p^*$ is equivalent to the following condition:

$$-\frac{\partial \Pi(p,p)}{\partial p} p\Big|_{p=\frac{\underline{\Pi}}{\Pi(p,p)}} \leq \Pi(p,p)$$

which can be shown is

$$\frac{\left(\eta_2^{\beta} + \frac{1}{2}\eta_3 + \eta_4\right)\left(2\eta_2^{\beta} + \frac{3}{2}\eta_3 + 3\eta_4\right)}{4\eta_4} - \frac{\Pi(p,p)\left(\Pi(p,p) - \underline{\Pi}\right)}{\underline{\Pi}} \leq 0.$$

As η_1, η_2 and η_2^σ increases, $\underline{\Pi}$ and $\Pi(p,p)$ both increases with $\Pi(p,p)-\underline{\Pi}$ unchanged therefore, the left hand side of the above inequality increases. Therefore the above inequality is less likely to hold.

As η_3 increases, it can be shown that the left hand side of the above inequality increases. Therefore the above inequality is less likely to hold, too.

As η_4 increases, it can be shown that the left hand side of the above inequality decreases. Therefore the above inequality is more likely to hold.

In addition, as η_1, η_2 and η_2^σ increases, it can also be shown that p^* increases as $\Pi(p^*,p^*)$ increases and p_1, which is the lowest price among the countable number of prices in the support of mixed-strategy equilibrium, increases and the probability of choosing p_1 decreases.

Appendix C

Appendix for Chapter 4

C.1 Buyers' Symmetric Continuation Equilibrium

C.1.1 $\frac{1}{2} < \alpha_A \leq 1$, $\frac{1}{2} < \alpha_B \leq 1$, $1-\alpha_A < p_A < \alpha_A$ and $1-\alpha_B < p_B < \alpha_B$

Case 1. $(\alpha_A - p_A) \leq \dfrac{6}{7}(\alpha_B - p_B)$

There exists a symmetric Nash equilibrium between two buyers:

$$\left\{ \begin{array}{c} \left(\pi_{00}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 0, \pi_{00}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 0\right), \left(\pi_{01}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 0, \pi_{01}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 1\right) \\ \left(\pi_{10}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 1, \pi_{10}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 0\right), \left(\pi_{11}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 0, \pi_{11}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 1\right) \end{array} \right\}$$

Case 2. $\dfrac{6}{7}(\alpha_B - p_B) < (\alpha_A - p_A) < \dfrac{7}{6}(\alpha_B - p_B)$

There exists a symmetric Nash equilibrium between two buyers:

$$\left\{ \begin{array}{c} \left(\pi_{00}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 0, \pi_{00}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 0\right), \left(\pi_{01}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 0, \pi_{01}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 1\right) \\ \left(\pi_{10}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 1, \pi_{10}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 0\right), \\ \left(\pi_{11}^{A*} = \dfrac{7(\alpha_A - p_A) - 6(\alpha_B - p_B)}{(\alpha_A - p_A) + (\alpha_B - p_B)}, \pi_{11}^{B*} = \dfrac{7(\alpha_B - p_B) - 6(\alpha_A - p_A)}{(\alpha_A - p_A) + (\alpha_B - p_B)}\right) \end{array} \right\}$$

Case 3. $\dfrac{7}{6}(\alpha_B - p_B) \leq (\alpha_A - p_A)$

There is a symmetric Nash equilibrium between two buyers:

$$\left\{\begin{array}{c} \left(\pi_{00}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{00}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\left(\pi_{01}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{01}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right) \\ \left(\pi_{10}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{10}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\left(\pi_{11}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{11}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right) \end{array}\right\}$$

C.1.2 $\frac{1}{2}<\alpha_A\leq 1, 0<p_A\leq 1-\alpha_A$ and $\frac{1}{2}<\alpha_B\leq 1$, $1-\alpha_B<p_B\leq\alpha_B$

Case 1. $\alpha_A-p_A\leq\alpha_B-p_B$

There exists a symmetric Nash equilibrium between buyers:

$$\left\{\begin{array}{c} \left(\pi_{00}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{00}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\left(\pi_{01}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{01}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right) \\ \left(\pi_{10}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{10}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\left(\pi_{11}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{11}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right) \end{array}\right\}$$

Case 2. $\alpha_B-p_B<\alpha_A-p_A<\frac{7}{5}(\alpha_B-p_B)$

There is a symmetric Nash Equilibrium between buyers:

$$\left\{\begin{array}{c} \left(\pi_{00}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{00}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\left(\pi_{01}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{01}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right) \\ \left(\pi_{10}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{10}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right), \\ \left(\pi_{11}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=\dfrac{6(\alpha_A-p_A)-6(\alpha_B-p_B)}{(\alpha_A-p_A)+(\alpha_B-p_B)},\pi_{11}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=\dfrac{7(\alpha_B-p_B)-5(\alpha_A-p_A)}{(\alpha_A-p_A)+(\alpha_B-p_B)}\right) \end{array}\right\}$$

Case 3. $\frac{7}{5}(\alpha_B-p_B)\leq\alpha_A-p_A$, $1-\alpha_A-p_A<\frac{7}{5}(\alpha_B-p_B)$

There exists a symmetric Nash equilibrium between two buyers:

$$\left\{\begin{array}{c} \left(\pi_{00}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{00}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\left(\pi_{01}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{01}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right) \\ \left(\pi_{10}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{10}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\left(\pi_{11}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{11}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right) \end{array}\right\}$$

Case 4. $\frac{7}{5}(\alpha_B-p_B)\leq 1-\alpha_A-p_A<2(\alpha_B-p_B)$

There exists a symmetric Nash Equilibrium between two buyers:

$$\left\{\begin{array}{c} \left(\pi_{00}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{00}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right), \\ \left(\pi_{01}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=\dfrac{5(1-\alpha_A-p_A)-7(\alpha_B-p_B)}{(1-\alpha_A-p_A)+(\alpha_B-p_B)},\pi_{01}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=\dfrac{8(\alpha_B-p_B)-4(1-\alpha_A-p_A)}{(1-\alpha_A-p_A)+(\alpha_B-p_B)}\right) \\ \left(\pi_{10}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{10}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right)\left(\pi_{10}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{10}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right) \end{array}\right\}$$

Case 5. $2(\alpha_B - p_B) \leq 1 - \alpha_A - p_A$

There exists a symmetric Nash Equilibrium between buyers:

$$\left\{ \begin{array}{l} \left(\pi_{00}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 1, \pi_{00}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 0\right), \left(\pi_{01}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 1, \pi_{01}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 0\right) \\ \left(\pi_{10}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 1, \pi_{10}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 0\right), \left(\pi_{11}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 1, \pi_{11}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 0\right) \end{array} \right\}$$

C.1.3 $0 < p_A \leq 1 - \alpha_A$ and $0 < p_B \leq 1 - \alpha_B$

Case 1. $\alpha_A - p_A < \frac{1}{2}(1 - \alpha_B - p_B)$

For buyers receiving all signal combinations, valuation of A's goods is low comparing to the valuation of B's goods. Buyers always visit B.

$$\left\{ \begin{array}{l} \left(\pi_{00}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 0, \pi_{00}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 1\right), \left(\pi_{01}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 0, \pi_{01}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 1\right) \\ \left(\pi_{10}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 0, \pi_{10}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 1\right), \left(\pi_{11}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 0, \pi_{11}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 1\right) \end{array} \right\}$$

Case 2. $\frac{1}{2}(1 - \alpha_B - p_B) \leq \alpha_A - p_A \leq \frac{5}{7}(1 - \alpha_B - p_B)$

For buyers receiving $(1,0)$, valuation of A's goods is comparable to valuation of B's goods. These buyers mix between A and B. For buyers receiving signal $(1,1)$,$(0,0)$,and $(0,1)$, valuation of A's goods is low compared to valuation of B's goods, they visit *B*.

$$\left\{ \begin{array}{c} \left(\pi_{00}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 0, \pi_{00}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 1\right), \left(\pi_{01}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 0, \pi_{01}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 1\right) \\ \left(\pi_{10}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = \dfrac{8(\alpha_A - p_A) - 4(1 - \alpha_B - p_B)}{(\alpha_A - p_A) + (1 - \alpha_B - p_B)}, \pi_{10}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = \dfrac{5(1 - \alpha_B - p_B) - 7(\alpha_A - p_A)}{(\alpha_A - p_A) + (1 - \alpha_B - p_B)}\right) \\ \left(\pi_{11}^{A*}(\alpha_A, p_A, \alpha_B, p_B) = 0, \pi_{11}^{B*}(\alpha_A, p_A, \alpha_B, p_B) = 1\right) \end{array} \right\}$$

Case 3. $\dfrac{5}{7}(1 - \alpha_B - p_B) \leq (\alpha_A - p_A) \leq \dfrac{5}{7}(\alpha_B - p_B),\ (1 - \alpha_A - p_A) \leq \dfrac{5}{7}(1 - \alpha_B - p_B)$

For buyers receiving signal $(1,0)$, valuation of A's goods is high compared to valuation of B's goods, these buyers visit A. For buyers receiving $(0,0)$, $(0,1)$ and $(1,1)$ valuation of A's goods is low compared to valuation of B's goods, they visit *B*.

$$\left\{\begin{array}{c} \left(\pi_{00}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{00}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right),\left(\pi_{01}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{01}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right) \\ \left(\pi_{10}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{10}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\left(\pi_{11}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{11}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right) \end{array}\right\}$$

Case 4. $\frac{5}{7}(\alpha_B-p_B)\leq(\alpha_A-p_A)\leq\alpha_B-p_B,\ \frac{\alpha_B-\alpha_A}{\alpha_B-\frac{1}{2}}\leq\frac{p_A-p_B}{\frac{1}{2}-p_B}$

For buyers receiving $(1,0)$, valuation of A's goods is high compared to valuation of B's goods, so they visit A. For buyers receiving $(1,1)$, valuation of A's goods is now comparable to valuation of B's goods so they mix between A and B. For buyers receiving $(0,0)$ and $(0,1)$, valuation of A's goods is low compared to valuation of B's goods, they visit *B*.

$$\left\{\begin{array}{c} \left(\pi_{00}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{00}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right),\left(\pi_{01}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{01}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right) \\ \left(\pi_{10}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{10}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right), \\ \left(\pi_{11}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=\frac{7(\alpha_A-p_A)-5(\alpha_B-p_B)}{(\alpha_A-p_A)+(\alpha_B-p_B)},\pi_{11}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=\frac{6(\alpha_B-p_B)-6(\alpha_A-p_A)}{(\alpha_A-p_A)+(\alpha_B-p_B)}\right) \end{array}\right\}$$

Case 5. $\frac{5}{7}(1-\alpha_B-p_B)\leq(1-\alpha_A-p_A)\leq1-\alpha_B-p_B,\ \frac{\alpha_B-\alpha_A}{\alpha_B-\frac{1}{2}}>\frac{p_A-p_B}{\frac{1}{2}-p_B}$

For buyers receiving $(1,0)$, valuation of A's goods is high compared to valuation of B's goods so they visit A. For buyers receiving $(0,0)$, valuation of A's goods is comparable to valuation of B's goods so they mix between A and B. For buyers receiving $(0,1)$ and $(1,1)$, valuation of A's goods is low compared to valuation of B's goods,they visit *B*.

$$\left\{\begin{array}{c} \left(\pi_{00}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=\frac{7(1-\alpha_A-p_A)-5(1-\alpha_B-p_B)}{(1-\alpha_A-p_A)+(1-\alpha_B-p_B)},\pi_{00}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=\frac{6(1-\alpha_B-p_B)-6(1-\alpha_A-p_A)}{(1-\alpha_A-p_A)+(1-\alpha_B-p_B)}\right) \\ \left(\pi_{01}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{01}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right), \\ \left(\pi_{10}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{10}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right)\left(\pi_{11}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{11}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right) \end{array}\right\}$$

Case 6. $(\alpha_B-p_B)\leq(\alpha_A-p_A),\ (1-\alpha_A-p_A)\leq(1-\alpha_B-p_B)$

For buyers receiving $(1,0)$ and $(1,1)$, valuation of A's goods is high compared to valuations of B's goods so they visit A. For buyers receiving $(0,0)$ and $(0,1)$, valuation of A's goods is low compared to valuations of B's goods, so they visit *B* for sure.

$$\left\{\begin{array}{c}\left(\pi_{00}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{00}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right),\left(\pi_{01}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{01}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right)\\ \left(\pi_{10}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{10}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\left(\pi_{11}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{11}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right)\end{array}\right\}$$

Case 7. $(\alpha_A - p_A) \leq (\alpha_B - p_B)$, $(1-\alpha_B - p_B) \leq (1-\alpha_A - p_A)$

For buyers receiving $(1,0)$ and $(0,0)$, valuation of A's goods is high compared to valuation of B's goods, so they visit A. For buyers receiving $(1,1)$ and $(0,1)$, valuations of A's goods are low compared to valuation of B's goods, so they visit B.

$$\left\{\begin{array}{c}\left(\pi_{00}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{00}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\left(\pi_{01}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{01}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right)\\ \left(\pi_{10}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{10}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\left(\pi_{11}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{11}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right)\end{array}\right\}$$

Case 8. $(1-\alpha_B - p_B) \leq (1-\alpha_A - p_A) \leq \frac{7}{5}(1-\alpha_B - p_B)$, $\dfrac{\alpha_B-\alpha_A}{\alpha_B-\frac{1}{2}} \leq \dfrac{p_A-p_B}{\frac{1}{2}-p_B}$

For buyers receiving $(1,0)$ and $(1,1)$, valuation of A's goods is high compared to valuations of B's goods so they visit A. For buyers receiving $(0,0)$ valuation of A's goods is comparable to valuation of B's goods, so they mix between A and B. For buyers receiving $(0,1)$ valuation of A's goods is low compared to valuation of B's goods, so they visit B.

$$\left\{\begin{array}{c}\left(\pi_{00}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=\dfrac{6(1-\alpha_A-p_A)-6(1-\alpha_B-p_B)}{(1-\alpha_A-p_A)+(1-\alpha_B-p_B)},\pi_{00}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=\dfrac{7(1-\alpha_B-p_B)-5(1-\alpha_A-p_A)}{(1-\alpha_A-p_A)+(1-\alpha_B-p_B)}\right)\\ \left(\pi_{01}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{01}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right),\\ \left(\pi_{10}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{10}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right)\left(\pi_{11}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{11}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right)\end{array}\right\}$$

Case 9. $(\alpha_B - p_B) \leq (\alpha_A - p_A) \leq \frac{7}{5}(\alpha_B - p_B)$, $\dfrac{\alpha_B-\alpha_A}{\alpha_B-\frac{1}{2}} > \dfrac{p_A-p_B}{\frac{1}{2}-p_B}$

For buyers receiving $(1,0)$ and $(0,0)$, valuation of A's goods is high compared to valuation of B's goods, so they visit A. For buyers receiving $(1,1)$, valuation of A's goods is comparable to valuation of B's goods, so they mix between A and B. For buyers receiving $(0,1)$, valuation of A's goods is low compared to valuation of B's goods, so they visit B.

$$\left\{\begin{array}{c}\left(\pi_{00}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{00}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\left(\pi_{01}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{01}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right)\\ \left(\pi_{10}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{10}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\\ \left(\pi_{11}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=\dfrac{6(\alpha_A-p_A)-6(\alpha_B-p_B)}{(\alpha_A-p_A)+(\alpha_B-p_B)},\pi_{11}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=\dfrac{7(\alpha_B-p_B)-5(\alpha_A-p_A)}{(\alpha_A-p_A)+(\alpha_B-p_B)}\right)\end{array}\right\}$$

Case 10. $1-\alpha_A-p_A\leq\frac{7}{5}(\alpha_B-p_B),\ \frac{7}{5}(1-\alpha_B-p_B)\leq 1-\alpha_A-p_A,\ \frac{7}{5}(\alpha_B-p_B)\leq\alpha_A-p_A$

For buyers receiving $(1,0)$ and $(1,1)$, valuation of A's goods is high compared to valuations of B's goods so they visit A. For buyers receiving $(0,0)$ valuation of A's goods is high compared to valuation of B's goods, so they visit *A*. For buyers receiving $(0,1)$, valuation of A's goods is low compared to valuation of B's goods, they visit *B*.

$$\left\{\begin{array}{c}\left(\pi_{00}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{00}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\left(\pi_{01}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=0,\pi_{01}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=1\right)\\ \left(\pi_{10}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{10}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\left(\pi_{11}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{11}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right)\end{array}\right\}$$

Case 11. $\dfrac{7}{5}(\alpha_B-p_B)\leq(1-\alpha_A-p_A)\leq 2(\alpha_B-p_B)$

For buyers receiving $(1,0)$,$(1,1)$ and $(0,0)$, valuation of A's goods is high compared to valuations of B's goods so they visit A. For buyers receiving $(0,1)$, valuation of A's goods is comparable to valuation of B's goods, they mix between A and B.

$$\left\{\begin{array}{c}\left(\pi_{00}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{00}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right)\\ \left(\pi_{01}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=\dfrac{5(1-\alpha_A-p_A)-7(\alpha_B-p_B)}{(1-\alpha_A-p_A)+(\alpha_B-p_B)},\pi_{01}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=\dfrac{8(\alpha_B-p_B)-4(1-\alpha_A-p_A)}{(1-\alpha_A-p_A)+(\alpha_B-p_B)}\right)\\ \left(\pi_{10}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{10}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right)\left(\pi_{11}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{11}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right)\end{array}\right\}$$

Case 12. $2(\alpha_B-p_B)<(1-\alpha_A-p_A)$

For buyers receiving all signals, valuation of A's goods is high compared to valuation of B's goods. Buyers always visit A.

$$\left\{\begin{array}{c}\left(\pi_{00}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{00}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\left(\pi_{01}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{01}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right)\\ \left(\pi_{10}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{10}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right),\left(\pi_{11}^{A*}(\alpha_A,p_A,\alpha_B,p_B)=1,\pi_{11}^{B*}(\alpha_A,p_A,\alpha_B,p_B)=0\right)\end{array}\right\}$$

C.2 Proof of Lemma 9

For ease of notation, denote $\pi_{st}^{j} \equiv \pi_{st}^{j}(\alpha_A, p_A, \alpha_B, p_B)$, $\pi_{st}^{j'} \equiv \pi_{st}^{j}(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ for $(s,t) \in \{(0,0),(0,1),(1,0),(1,1)\}$, $j \in \{A,B\}$, and $\pi^{j} \equiv \pi^{j}(\alpha_A, p_A, \alpha_B, p_B)$, $\pi^{j'} \equiv \pi^{j}(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ for $j \in \{A,B\}$. We need the following result to proceed.

Lemma 13. *When $p_B \leq 1-\alpha_B$ or $p_A > 1-\alpha_A$ and $p_B > 1-\alpha_B$, if seller A's probability of selling under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ is strictly less than that under $(\alpha_A, p_A, \alpha_B, p_B)$, seller B's probability of selling under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ is greater than or equal to that under $(\alpha_A, p_A, \alpha_B, p_B)$.*

Proof. When $p_B \leq 1-\alpha_B$, a buyer gets nonnegative payoff from selecting seller B, because the buyer's net valuation for seller B's good, conditional on getting B's good, is nonnegative and the probability of getting B's good is strictly positive. Given a buyer can always get a nonnegative payoff from selecting at least one seller, in equilibrium the buyer either selects seller A or seller B. This means, for all $(s,t) \in \{(0,0),(0,1),(1,0),(1,1)\}$, $\pi_{st}^{A} + \pi_{st}^{B} = 1$, which implies $\pi^{A} + \pi^{B} = 1$. Note that after seller A deviates to $(1, p_A + (1-\alpha_A))$, because seller B's information provision and price do not change, by the same argument, we have $\pi^{A'} + \pi^{B'} = 1$. If A's probability of selling is strictly less under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ than that under $(\alpha_A, p_A, \alpha_B, p_B)$, which implies $\pi^{A'} < \pi^{A}$, then we have $\pi^{B'} > \pi^{B}$. This implies B's probability of selling is greater under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ than that under $(\alpha_A, p_A, \alpha_B, p_B)$.

When $p_A > 1-\alpha_A$, $p_B > 1-\alpha_B$, a buyer gets negative payoff from selecting a seller when getting signal 0 from the seller. This implies the buyer will choose to not select any seller when getting signal 0 from both sellers, i.e.,$\pi_{00}^{A} + \pi_{00}^{B} = 0$. If the buyer gets signal 1 from at least one seller, the buyer's probabilities of selecting two sellers will add up to one because the buyer gets positive payoff from selecting at least one seller. Therefore, $\pi_{10}^{A} + \pi_{10}^{B} = 1, \pi_{01}^{A} + \pi_{01}^{B} = 1, \pi_{11}^{A} + \pi_{11}^{B} = 1$. This implies $\pi^{A} + \pi^{B} = \frac{3}{4}$. Note that after seller A deviates to $(1, p_A + (1-\alpha_A))$, because seller B's information provision and price do not change, by the same argument as before, we have $\pi^{A'} + \pi^{B'} = \frac{3}{4}$. If A's probability of selling is strictly less under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ than that under $(\alpha_A, p_A, \alpha_B, p_B)$, which implies $\pi^{A'} < \pi^{A}$, because $\pi^{A} + \pi^{B} = \frac{3}{4}$ and $\pi^{A'} + \pi^{B'} = \frac{3}{4}$, we have $\pi^{B'} > \pi^{B}$, which in turn implies seller B's probability of selling under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ is greater than that under $(\alpha_A, p_A, \alpha_B, p_B)$. $\square$

As argued in the main text, if seller A's probability of selling under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ is strictly greater than or equal to that under $(\alpha_A, p_A, \alpha_B, p_B)$, seller A's profit under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ is strictly greater than that under $(\alpha_A, p_A, \alpha_B, p_B)$.

Now we focus on the situation where seller A's probability of selling under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ is strictly less than that under $(\alpha_A, p_A, \alpha_B, p_B)$. In other words, $\pi^{A'} < \pi^A$. To prove that seller A's profit under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ is strictly greater than that under $(\alpha_A, p_A, \alpha_B, p_B)$, there are several cases to consider.

Case 1. $p_A \leq 1-\alpha_A$, $p_B \leq 1-\alpha_B$

Sub-case 1.1 $\pi^A_{11} > 0$

This implies $\pi^A_{10} > 0$.

The argument is the following. A buyer's payoff from selecting seller A under signal (1,1) is equal to the payoff of selecting A under signal (1,0). This is because first the buyer's net valuations of seller A's good conditional on getting A's good successfully is the same across two signal realizations since the buyer gets signal 1 from seller A in both cases. Second the probabilities of getting A's good are the same across these two signal realizations. This is because two buyers' signals are statistical independent, so getting a different pair of signals from two sellers does not change the buyer's belief on the other buyer's signals. The buyer's payoff from selecting B under signal (1,1) is higher than or equal to the payoff from selecting B under signal (1,0) because the buyer's net valuation of B's good conditional on successfully getting B's good under signal (1,1) is higher than or equal to that under signal (1,0)[1]. Given $\pi^A_{11} > 0$, i.e., in equilibrium the buyer selects A with positive probability under signal (1,1), the buyer must also select A with positive probability under signal (1,0) because selecting A is equally attractive while selecting B is less attractive.

[1] A buyer's payoff from selecting B under signal (1,1) is equal to that under signal (1,0) only when B is providing no information at all, i.e., $\alpha_B = \dfrac{1}{2}$. In this case, signal (1,1) and signal (1,0) should be seen as the same signal because B's signals are completely uninformative. Therefore $\pi^A_{11} = \pi^A_{10}$.

From $\pi_{11}^A > 0$ and $\pi_{10}^A > 0$, combined with $\pi^A > \pi^{A'}$ and, by Lemma 13, $\pi^B \leq \pi^{B'}$, it can be proved that $\pi_{11}^{A'} = 1$ and $\pi_{10}^{A'} = 1$ as shown in the following argument.

Under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$, upon getting signal (1,1), a buyer's net valuation of seller A's good conditional on successfully getting A's good is the same as that of the buyer upon getting signal (1,1) under $(\alpha_A, p_A, \alpha_B, p_B)$. In both cases, the buyer's net valuation of seller A's good is $\alpha_A - p_A$. From $\pi^{A'} < \pi^A$, the probability for a buyer to get seller A's good under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$, $1 - \dfrac{1}{2}\pi^{A'}$, is strictly greater than the probability for a buyer to get A's good under $(\alpha_A, p_A, \alpha_B, p_B)$, $1 - \dfrac{1}{2}\pi^A$. Therefore under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$, a buyer's payoff from selecting A upon getting signal (1,1) is strictly greater than the buyer's payoff from selecting A upon getting signal (1,1) under $(\alpha_A, p_A, \alpha_B, p_B)$. Similarly, from $\pi^B \leq \pi^{B'}$, a buyer's payoff from selecting B upon getting signal (1,1) under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ is strictly less than that of the buyer upon getting signal (1,1) under $(\alpha_A, p_A, \alpha_B, p_B)$. Therefore, given the buyer selects A with strictly positive probability upon getting signal (1,1) under $(\alpha_A, p_A, \alpha_B, p_B)$, the buyer will select seller A with probability 1 upon getting signal (1,1) under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$, because under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$, selecting A is strictly more attractive and selecting B is strictly less attractive.

Therefore, $\pi^{A'} \equiv \dfrac{\pi_{11}^{A'} + \pi_{10}^{A'}}{4} = \dfrac{1}{2}$, and A's probability of selling under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ is $1 - (1-\pi^{A'})^2 = \frac{3}{4}$. Even if A's probability of selling under $(\alpha_A, p_A, \alpha_B, p_B)$ is 1, which is the highest probability seller A can sell under $(\alpha_A, p_A, \alpha_B, p_B)$, seller A's probability of selling under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ is still larger than one half of the probability of selling under $(\alpha_A, p_A, \alpha_B, p_B)$. This means A's profit under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ is strictly larger than A's profit by under $(\alpha_A, p_A, \alpha_B, p_B)$ since $p_A + (1-\alpha_A) \geq 2p_A$ from the condition that $p_A \leq 1 - \alpha_A$.

Sub-case 1.2 $\pi_{11}^A = 0$

This implies $\pi_{01}^A = 0$, which in turn implies $\pi^A \leq \frac{1}{2}$. This implies seller A's probability of selling under $(\alpha_A, p_A, \alpha_B, p_B)$ is less than or equal to $\frac{3}{4}$. There are two possibilities to consider.

One possibility is $\pi_{10}^{A} > 0$. From $\pi_{10}^{A} > 0$ combined with $\pi^{A} > \pi^{A'}$ and by Lemma 13, $\pi^{B} \leq \pi^{B'}$, we have $\pi_{10}^{A'} = 1$. This implies $\pi^{A'} \geq \frac{1}{4}$ and seller A's probability of selling under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ is greater than or equal to $\frac{7}{16}$. Because $\frac{7}{16}$ is greater than half of $\frac{3}{4}$, which is the upper bound of seller A's probability of selling under $(\alpha_A, p_A, \alpha_B, p_B)$, A's profit under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ is strictly larger than A's profit under $(\alpha_A, p_A, \alpha_B, p_B)$.

Another possibility is $\pi_{10}^{A} = 0$. This implies $\pi_{00}^{A} = 0$. Therefore, $\pi^{A} = 0$. This is the case ruled out because it is assumed that seller A can sell with a strictly positive probability under $(\alpha_A, p_A, \alpha_B, p_B)$.

Case 2. $p_A \leq 1-\alpha_A$, $p_B > 1-\alpha_B$

In this case, we know $\pi_{10}^{A} = 1$,$\pi_{10}^{B} = 0$ and $\pi_{00}^{A} = 1$, $\pi_{00}^{B} = 0$. $\pi_{01}^{A} = 1 - \pi_{01}^{B}$, and $\pi_{11}^{A} = 1 - \pi_{11}^{B}$. Therefore, $\pi^{A} = \dfrac{2 + \pi_{01}^{A} + \pi_{11}^{A}}{4}$ and $\pi^{B} = \dfrac{2 - \pi_{01}^{A} - \pi_{11}^{A}}{4}$.

Also we know that $\pi_{10}^{A'} = 1$, $\pi_{10}^{B'} = 0$ and $\pi_{00}^{A'} = 0$, $\pi_{00}^{B'} = 0$. $\pi_{01}^{A'} = 0$, $\pi_{01}^{B'} = 1$, and $\pi_{11}^{A'} = 1 - \pi_{11}^{B'}$. Therefore, $\pi^{A'} = \dfrac{1 + \pi_{11}^{A'}}{4}$ and $\pi^{B'} = \dfrac{2 - \pi_{11}^{A'}}{4}$.

There are two sub-cases to consider.

Sub-case 2.1 $\pi_{01}^{A} > 0$.

This implies $\pi_{11}^{A} = 1$ and $\pi_{11}^{B} = 1 - \pi_{11}^{A} = 0$.

Therefore, $\pi^{A} = \dfrac{3 + \pi_{01}^{A}}{4}$ and $\pi^{B} = \dfrac{1 - \pi_{01}^{A}}{4}$.

From $\pi^{A'} = \dfrac{1 + \pi_{11}^{A'}}{4}$, we have $\pi^{A} > \pi^{A'}$, and because $\pi^{B'} = \dfrac{2 - \pi_{11}^{A'}}{4}$, we have $\pi^{B} \leq \pi^{B'}$.

From $\pi^{A} > \pi^{A'}$ and $\pi^{B} \leq \pi^{B'}$, we have $\pi_{11}^{A'} \geq \pi_{11}^{A}$, which implies $\pi_{11}^{A'} = 1$. Therefore, $\pi^{A'} = \dfrac{1}{2}$. Seller A's expected probability of selling is $\frac{3}{4}$. Similarly with Sub-case 1.1, seller A's expected profit under $(1, p_A + (1-\alpha_A), \alpha_B, p_B)$ is strictly larger than A's profit under $(\alpha_A, p_A, \alpha_B, p_B)$.

Sub-case 2.2 $\pi_{01}^{A} = 0$.

In this sub-case, $\pi^{A} = \dfrac{2 + \pi_{11}^{A}}{4}$, $\pi^{A'} = \dfrac{1 + \pi_{11}^{A'}}{4}$,$\pi^{B} = \dfrac{2 - \pi_{11}^{A}}{4}$ and $\pi^{B'} = \dfrac{2 - \pi_{11}^{A'}}{4}$.

Suppose $\pi_{11}^{A} > \pi_{11}^{A'}$. Because $\pi_{11}^{A} + \pi_{11}^{B} = 1$, $\pi_{11}^{A'} + \pi_{11}^{B'} = 1$, and $\pi_{11}^{A} > \pi_{11}^{A'}$, we have $\pi_{11}^{B} < \pi_{11}^{B'}$. This implies $\pi^{B} < \pi^{B'}$.

Because $\pi^A > \pi^{A'}$ and $\pi^B < \pi^{B'}$, we have $\pi_{11}^A \leq \pi_{11}^{A'}$, which contradicts with $\pi_{11}^A > \pi_{11}^{A'}$.

Therefore, it must be $\pi_{11}^A \leq \pi_{11}^{A'}$.

Because $\pi^A \equiv \dfrac{2+\pi_{11}^A}{4}$,and $\pi^{A'} \equiv \dfrac{1+\pi_{11}^{A'}}{4} \geq \dfrac{1+\pi_{11}^A}{4} = \pi^A - \dfrac{1}{4}$.

It can be shown that seller A's probability of selling under $\pi^{A'}$, $1-\left(1-\pi^{A'}\right)^2$, is greater than or equal to half of the probability of selling for seller A under π^A, $1-\left(1-\pi^A\right)^2$.