A biologically-inspired eye model for testing oculomotor control theories

by

Mahkameh Lakzadeh

B.A.Sc., University of British Columbia, 2008

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Applied Science

in

THE FACULTY OF GRADUATE STUDIES
(Mechanical Engineering)

The University Of British Columbia
(Vancouver)

January 2012

© Mahkameh Lakzadeh, 2012
Abstract

This research presents a new biologically motivated robotic model of the human eye. The model incorporates aspects of the anatomy that are functionally important for understanding biological oculomotor systems. The 3DOF robotic eye is driven by 6 DC motors through low friction dyneema cables. The DC motors represent muscle actuation while dyneema cables represent the 6 extraocular muscles (EOM). The globe’s natural orbital support is emulated by a low-friction gimbal structure that supports the eye on the anteroposterior axis at the back of the globe, where there is no tendon interference. Moreover, we have used the Buckingham Π theorem dimensionless analysis to scale the geometric and dynamic properties of the biological eye according to the model’s specified dimensions and inertia. Lastly, to confirm the functionality of the eye and to verify that the initial design requirements have been satisfied, we have implemented a controller design to drive this redundant (6 actuators, 3 DOF) system. The presented robotic eye model is to be employed as a test bed for testing theories about oculomotor control. Furthermore, this system could also be used to assess proposed surgical corrections for various oculomotor diseases.
Table of Contents

Abstract . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ii

Table of Contents . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . iii

List of Tables . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . vi

List of Figures . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . vii

Glossary . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . x

Acknowledgments . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . xii

1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1
   1.1 Motivation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1
   1.2 Problem Statement . . . . . . . . . . . . . . . . . . . . . . . . 4
   1.3 Literature Review . . . . . . . . . . . . . . . . . . . . . . . . . 5
      1.3.1 Other Eye Models . . . . . . . . . . . . . . . . . . . . . . 7
   1.4 Research Objectives . . . . . . . . . . . . . . . . . . . . . . . 9
   1.5 Organization of Thesis . . . . . . . . . . . . . . . . . . . . . . 10
# Background

2.1 Anatomy

2.2 Eye Movement

## 2.2.1 Saccadic Eye Movement

## 2.2.2 Smooth Pursuit Eye Movement

# Design Requirements

3.1 Structural Requirements

3.2 Mechatronic Requirements

# Structural Design

4.1 Scaled Model Design

4.2 Musculotendon Representation

4.3 Globe Support Structure

### 4.3.1 Considered Solutions

### 4.3.2 The Gimbal Globe Support Structure

### 4.3.3 The Eye Robot - Structural Design

### 4.3.4 The Eye Robot - Manufacturing

### 4.3.5 The Eye Robot - Mechanical Characteristics

# Mechatronic Design

5.1 Architecture of the Controller

5.2 Sensors and Actuators

## 5.2.1 Actuators
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.2.2 Sensors</td>
<td>75</td>
</tr>
<tr>
<td>5.3 Kinematics and Control</td>
<td>77</td>
</tr>
<tr>
<td>5.3.1 Kinematics</td>
<td>78</td>
</tr>
<tr>
<td>5.3.2 Controller</td>
<td>84</td>
</tr>
<tr>
<td>5.4 Implementation</td>
<td>86</td>
</tr>
<tr>
<td>6 Assessment</td>
<td>88</td>
</tr>
<tr>
<td>7 Conclusion</td>
<td>106</td>
</tr>
<tr>
<td>7.1 Progress Assessment</td>
<td>106</td>
</tr>
<tr>
<td>7.2 Research Implications</td>
<td>108</td>
</tr>
<tr>
<td>7.3 Future Work</td>
<td>109</td>
</tr>
<tr>
<td>Bibliography</td>
<td>110</td>
</tr>
<tr>
<td>A Design Iteration</td>
<td>117</td>
</tr>
<tr>
<td>A.1 Considered Solutions</td>
<td>117</td>
</tr>
<tr>
<td>A.2 Gimbal Design</td>
<td>121</td>
</tr>
</tbody>
</table>
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>The selected dimensional variables</td>
<td>27</td>
</tr>
<tr>
<td>4.2</td>
<td>The biological and the scaled insertion points</td>
<td>36</td>
</tr>
<tr>
<td>4.3</td>
<td>Line of action of the extraocular muscles (EOMs)</td>
<td>36</td>
</tr>
<tr>
<td>4.4</td>
<td>The eye robot joint details</td>
<td>59</td>
</tr>
<tr>
<td>4.5</td>
<td>Evaluation of the gimbal elements under active conditions</td>
<td>64</td>
</tr>
<tr>
<td>4.6</td>
<td>Performance of the gimbal elements under activated motors</td>
<td>66</td>
</tr>
<tr>
<td>4.7</td>
<td>List of moment of inertia (MI) of the robot eye</td>
<td>68</td>
</tr>
<tr>
<td>5.1</td>
<td>Critical variables of the biological and robotic eye</td>
<td>73</td>
</tr>
<tr>
<td>6.1</td>
<td>Statistics of the performance of the robot eye given step inputs</td>
<td>91</td>
</tr>
<tr>
<td>6.2</td>
<td>Maximum error for the circular and linear trajectories</td>
<td>92</td>
</tr>
<tr>
<td>6.3</td>
<td>Rise times of robot’s and biological saccade responses</td>
<td>103</td>
</tr>
<tr>
<td>A.1</td>
<td>Joint details for the first rapid prototype of the eye robot</td>
<td>130</td>
</tr>
<tr>
<td>A.2</td>
<td>Performance evaluation of the eye robot prototype</td>
<td>133</td>
</tr>
</tbody>
</table>
List of Figures

Figure 1.1 Verifying known biological theories using physical models . . 4
Figure 2.1 3D model of the right eye . . . . . . . . . . . . . . . . . . . . . . 12
Figure 2.2 Action of the EOMs . . . . . . . . . . . . . . . . . . . . . . . . . . . 14
Figure 3.1 Position and velocity profile of a 10° saccade [31] . . . . . 22
Figure 4.1 Tendon attachment with steel cables . . . . . . . . . . . . . . . 33
Figure 4.2 Tendon attachment with dyneema cables . . . . . . . . . . . . . 33
Figure 4.3 Rectus lines of insertion, [60] . . . . . . . . . . . . . . . . . . . . . 35
Figure 4.4 Motor mountings and cable routings . . . . . . . . . . . . . . . 38
Figure 4.5 Ball terminated set screw support structure . . . . . . . . . . 40
Figure 4.6 Concept of the elastic stalk support . . . . . . . . . . . . . . . 41
Figure 4.7 Final design of the gimbal globe support structure . . . . . 43
Figure 4.8 C-bracket, the inner structure of the eye robot . . . . . . . . . 44
Figure 4.9 Solid model of the eye robot parts . . . . . . . . . . . . . . . . . . . 45
Figure 4.10 Exploded view of the globe’s inner structures . . . . . . . . . 48
Figure 4.11 Dimensioning of the eye robot part . . . . . . . . . . . . . . . . . . . 50
Figure 4.12 Acrylic support stand ........................................... 51
Figure 4.13 Conceptual design for incorporating the ocular muscle pulleys 53
Figure 4.14 Cross section of the roll joint ................................ 56
Figure 4.15 Pitch joints .......................................................... 57
Figure 4.16 Cross section of the globe-C-bracket assembly .......... 58
Figure 4.17 C-bracket assembly .............................................. 60
Figure 4.18 Globe assembly .................................................... 61
Figure 4.19 Final assembly ..................................................... 62
Figure 4.20 Beam bending diagram for the roll shaft ................. 63
Figure 4.21 FBD of the gimbal elements with activated motors ..... 65
Figure 4.22 Beam bending diagram for the pitch shafts .......... 67

Figure 5.1 General block diagram for position controlling the eye robot 70
Figure 5.2 Potentiometers ..................................................... 76
Figure 5.3 Voltage output of the potentiometer ......................... 77
Figure 5.4 General block diagram for position controlling the eye robot 78
Figure 5.5 Defined coordinate systems .................................... 79
Figure 5.6 Block diagram of the position controller .................. 84
Figure 5.7 Schematic of the controller implementation ............... 86

Figure 6.1 System’s response in yaw direction ......................... 89
Figure 6.2 System’s response in pitch direction ....................... 90
Figure 6.3 System’s response in roll direction ......................... 90
Figure 6.4 System’s response to rotations about axis on the coronal plane 92
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 6.5</td>
<td>System’s response to a circular path on the field of view</td>
<td>93</td>
</tr>
<tr>
<td>Figure 6.6</td>
<td>Error profile for a linear trajectory input</td>
<td>94</td>
</tr>
<tr>
<td>Figure 6.7</td>
<td>FBD of a tensioned cable with a wrap angle $\Phi$</td>
<td>96</td>
</tr>
<tr>
<td>Figure 6.8</td>
<td>Routing of the cables through nylon tubes</td>
<td>97</td>
</tr>
<tr>
<td>Figure 6.9</td>
<td>Bar graph representing the friction torques in the system</td>
<td>98</td>
</tr>
<tr>
<td>Figure 6.10</td>
<td>Comparison of model’s MI and the scaled MI of the real eye</td>
<td>101</td>
</tr>
<tr>
<td>Figure 6.11</td>
<td>Bar graph, evaluating the model’s MI</td>
<td>102</td>
</tr>
<tr>
<td>Figure 6.12</td>
<td>Comparison of robot’s and biological response</td>
<td>105</td>
</tr>
<tr>
<td>Figure A.1</td>
<td>Ball terminated set screws used to support the globe</td>
<td>118</td>
</tr>
<tr>
<td>Figure A.2</td>
<td>Ball terminated set screw support structure</td>
<td>118</td>
</tr>
<tr>
<td>Figure A.3</td>
<td>Elastic stalk support concept</td>
<td>120</td>
</tr>
<tr>
<td>Figure A.4</td>
<td>Conceptual gimbal support structure [22]</td>
<td>121</td>
</tr>
<tr>
<td>Figure A.5</td>
<td>Prototype1: the first design for the gimbal’s general shape</td>
<td>123</td>
</tr>
<tr>
<td>Figure A.6</td>
<td>Prototype2: the second design for the gimbal’s general shape</td>
<td>124</td>
</tr>
<tr>
<td>Figure A.7</td>
<td>Prototype3: the third design for the gimbal’s general shape</td>
<td>125</td>
</tr>
<tr>
<td>Figure A.8</td>
<td>The mechanism used for the lateral joint in Prototype3</td>
<td>126</td>
</tr>
<tr>
<td>Figure A.9</td>
<td>First rapid prototype of the eye robot</td>
<td>128</td>
</tr>
<tr>
<td>Figure A.10</td>
<td>Elements of first rapid prototype of the eye robot</td>
<td>129</td>
</tr>
<tr>
<td>Figure A.11</td>
<td>Globe assembly</td>
<td>130</td>
</tr>
<tr>
<td>Figure A.12</td>
<td>Free body diagram of E1D3 gimbal elements</td>
<td>132</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Definition</td>
<td></td>
</tr>
<tr>
<td>--------------</td>
<td>---------------------------------</td>
<td></td>
</tr>
<tr>
<td>VOR</td>
<td>vestibulo-ocular reflex</td>
<td></td>
</tr>
<tr>
<td>EOM</td>
<td>extraocular muscles</td>
<td></td>
</tr>
<tr>
<td>IP</td>
<td>insertion point</td>
<td></td>
</tr>
<tr>
<td>AZ</td>
<td>annulus of zinn</td>
<td></td>
</tr>
<tr>
<td>BTSC</td>
<td>ball terminated set screw</td>
<td></td>
</tr>
<tr>
<td>MI</td>
<td>moment of inertia</td>
<td></td>
</tr>
<tr>
<td>MR</td>
<td>medial rectus</td>
<td></td>
</tr>
<tr>
<td>LR</td>
<td>lateral rectus</td>
<td></td>
</tr>
<tr>
<td>SR</td>
<td>superior rectus</td>
<td></td>
</tr>
<tr>
<td>IR</td>
<td>inferior rectus</td>
<td></td>
</tr>
<tr>
<td>SO</td>
<td>superior oblique</td>
<td></td>
</tr>
</tbody>
</table>
IO  inferior oblique
AD  analog to digital convertor
DA  digital to analog convertor
Acknowledgments

I wish to extend my sincerest gratitude to my supervisors, Dr. Antony Hodgson and Dr. Dinesh Pai for their continuous support and inspiration. Under their supervision and guidance, I have grown intellectually and professionally and for that I will always be grateful. I also thank them for providing me with research fellowship and the equipment necessary for completing my thesis.

To my colleagues in the sensorimotor lab, Mr. Garrett Jones, Dr. Shinjiro Sueda, Mr. Sanghoon Yeo, Mr. Martin Lesmana, Dr. Timothy Edmunds and Mr. David Levin, you have become dear friends that I will cherish always. Thank you for your supporting hand, knowledge, and friendship over the past few years. I have learned many things from you and will always hold dear, memories of my time with you. To my dear friend Marius, whom I have had the pleasure of knowing since the start of my studies at UBC. You have been a constant encouragement, who has pushed me to become more passionate about my career. Thank you for helping me grow spiritually and professionally. To my dear friends Sama Ghnaim and Jeanie Chan, I extend my deepest appreciation for your continuos support and friendship.
I wish to thank my dear husband Hamed, whose unconditional support and love has kept me going throughout the years. Thank you for all the sleepless nights that you spent by my side. I have never met a more generous and loving person than you and I am truly grateful to have you in my life. Thank you for making me be a better version of myself.

Finally, I wish to extend my love and gratitude to my parents, who have selflessly devoted their lives to bettering mine. Thank you for making impossibles possible, thank you for allowing me to grow on my own while guiding me in the right direction, and most importantly thank you for teaching me the true meaning of unconditional love. For all this and more, I will always be indebted to you.

To my loving sister Pardis, you are my best friend and companion and I wouldn’t want to imagine life without you. Thank you for believing in me even when I don’t.

xiii
Chapter 1

Introduction

1.1 Motivation

Vision is a remarkable sensing capability which allows humans and other animals to perceive the world in great detail. It is clear that the use of this sense is essential for our ability to quickly assess our surrounding world and determine what actions to take to achieve our goals. Blindness, through retinal or optical nerve damage, is perhaps the most obvious failure of our visual system; nevertheless, malfunctions of our oculomotor system (i.e. the system that controls our eye movement), could prove to be equivalently impairing.

Although it may not be apparent, our ability to perceive the world is greatly dependant on the movement of our eyes; this sets biological visual systems apart from cameras. It is a curious fact that in order for us to see, we first need to recognize that there is something worth looking at [18, 29, 67]. Consequently,
our eyes are constantly in motion, both to stabilize the visual image on the retina and to move the fovea to objects of interest using fast saccadic eye movements. Saccades are necessary for our visual awareness because like a number of other animals humans have high clusters of visual receptors on an area 1mm in diameter, known as the fovea, at the centre of our field of view. Therefore to achieve maximum visual acuity the fovea needs to be directed onto the area of interest. Vestibulo-ocular reflex is another eye movement that facilitates a clear visual field by counteracting head motions by moving the eye in the opposite direction. The necessity of this movement becomes evident during activities such as walking or running during which our visual field remains unaffected despite the constant up and down movements of our head.

Much of the work for identifying the mechanisms of these eye movements are done in physiology labs, where neural connections are traced out and their responses to various stimuli are measured. However, to test their plausibility, theories about the neurology of eye movements must be tested by engineers on physical eye models. Such models have been developed for various body parts; for instance, Matsuoka et al. [36] has created a robotic index and thumb finger to study the motor control of human hand. In addition, to study the physical mechanics of the human jaw, Stavness et al. [55] have created an anatomically accurate, dynamic computer model of the human jaw.

Furthermore, understanding the tight integration between how we move our eyes and what we perceive is critical to understanding how certain diseases or injuries affect vision as well as finding potential solutions for treating these condi-
tions. One such condition is strabismus, where the eyes are not aligned with each other and thus their gazes are positioned on 2 different points in space which could prevent proper binocular vision and depth perception. Strabismus could either be the result of a brain malfunction or lack of coordination in the EOMs which could arise with age [9, 30]. Consequently, if our diagnoses lead to proposed surgical corrections, we will often want an accurate patient-specific model of the eye in order to perform simulated surgeries to determine the optimal outcome. While it is theoretically possible to build a purely virtual model for the vision system, it is wise to validate such models against real physical systems to ensure that no surprising un-modelled effects emerge, such as unaccounted dynamic properties or friction. Furthermore, in certain circumstances it maybe very difficult to simulate all the inputs which will be experienced in real life and in these situations it is necessary to have an accurate physical model of the vision system.

Finally, insights obtained from biology could aid the future development of robotic eyes and for such applications in robotics there must be tests performed on real physical systems. Vestibulo-ocular reflex (VOR) is one example of biological eye movement that would have practical applications in robotics. VOR is the eye movement in response to head motion whereby the eyes are moved in the opposite direction of the head motion thus, stabilizing the image on the retina. Understanding and implementing such control strategies would be very favourable in stabilizing camera images in a moving robot; a process which would require testing on a physical model.
1.2 Problem Statement

The steps required for transforming theories about various computational neuro-scientific models into facts can be represented by Figure 1.1. Specifically, known characteristics of a sensorimotor system, including its anatomy and control strategies are used to construct an anatomically accurate physical model, which consists of a geometrically accurate robot controlled by a biologically inspired control algorithm. Subsequently, known behaviors of the biological system will be implemented on the constructed model and its results will be compared to biological data to verify theories. Furthermore, a physical model could also help to uncover unknown behaviors of a sensorimotor system that could not be easily determined theoretically. Ultimately, the goal of such research is to reveal how our central nervous system controls our body movements.

This same method is applied to the study of the oculomotor system. Eye move-
ments are actuated by what appears to be a simple system of three agonist and antagonist muscle pairs. This implication of tractability has attracted many scientists and neurobiologists to the subject of eye movement over the years. As a result, there are more theories about the neurology of eye movements than of any other human movements. However, even though much is known about oculomotor physiology [32], many aspects of the mechanics of the eye, its musculature, and how it is controlled remain poorly understood (e.g., see [16] for a taste of the current controversies). Therefore, the study of this sensorimotor system would benefit from a physical model to unravel the unknown facts and confirm or dispute existing theories.

The research presented in this thesis is focused on creating a geometrically accurate robotic eye to be used as a testbed for studying the oculomotor system. As discussed above, in addition to a geometrically correct robot, an anatomically accurate model must also include a biologically-inspired control algorithm. However, this is beyond the scope of this thesis and thus development of a biological controller and implementing the various eye movements are left as future work.

1.3 Literature Review

Most current robotic visual systems are used purely as sensors that provide visual information, and so their designs are focused mostly on the mechatronic performance rather than biologically realistic movement [1–3, 7, 28, 42, 45, 68].

For instance, Pongas et al. [45], have designed a set of miniature eyes to be implemented in a humanoid doll. Two miniature CMOS cameras are placed in
spherical supports and a single motor-gear system controls their horizontal movement simultaneously. Two similar systems are responsible for the vertical movement of each individual eye. Mechanisms similar to the Pongas doll eyes have been created by other groups, including Asfour et al. [2] and Beira et al. [7]. Both Asfour and Beira have created 3DOF pair of eyes to be used in their humanoid robots. However, contrary to the model developed by Pongas et al. [45], these eyes have a common tilt controlled by a motor through a belt and separate pans, again driven by a motor through belt.

Yet another tilt/pan robotic eye unit has been developed by Miwa et al. [42] for their humanoid robot (WE-4). Once again, the tilt for both of the Miwa eyes are controlled by 1-DOF while their pan is controlled by 2-DOF. Tilt is driven by a belt mechanism through a DC motor and 2 other DC motors rotate the eyes horizontally using a tendon driven system and torsional springs.

There are also some visual systems that are inspired by the perceptual development used by humans, and specifically cognitive development in infants. For example, Zhang et al. [68] has produced a vision system that develops its vision gradually. This robot is capable of distinguishing objects from one another, tracking objects as well as understanding the relation between various objects. Such models are therefore more concerned with the visual processing component of vision rather than eye movements.

Thus to conclude, none of the vision units discussed above have focused on replicating the biological details that are responsible for human eye movements, features that are crucial if the model is to be used in scientific research or clinical
applications.

1.3.1 Other Eye Models

Although emulating the anatomy of human eyes is not typically included in the development of visual systems for robots, a few biologically inspired robotic eyes have been created for the purpose of scientific research. Most notable is the 2D physical model MAC-EYE, developed by Biamino et al. [8]. The MAC-EYE was designed with the intention of creating a tendon driven mechanism capable of implementing Listing’s Law\(^1\) and therefore, it incorporates human like kinematics and actuation properties. More specifically, this robotic eye is composed of a sphere, held in place by a low friction support, actuated by four DC motors through cables that represent the four recti muscles of the eye. However, since the MAC-EYE has been built for researching a specific ocular phenomenon, it lacks some essential features for testing a broader range of the existing oculomotor theories. In particular, it lacks the oblique muscles and even though the obliques may not be necessary for Listing’s Law-compatible movements, many eye movements such as ocular counter-roll and combined eye-head movements violate Listing’s Law. Furthermore, the MAC-EYE does not include an accurate placement of the tendons on the globe, a feature that is believed to have significant impact on oculomotor control [49].

Wang et al. [63] have recently constructed a biologically inspired robotic eye.

---

\(^1\)Listing’s Law states that all eye orientations that are achievable from a given initial gaze direction are reached by rotating the eye about a set of corresponding axes that lie on a specific plane
Unlike the MAC-EYE, this robotic eye has 3 Degrees of Freedom (DOF) and it is driven by 3 pairs of agonist/antagonist Pneumatic Artificial Muscles (PAMs). This Robotic eye is supported by a ball joint located in the centre of the globe. However, the chosen design and assembly process of this joint enforces a non-spherical globe shape, resulting in an inaccurate geometrical representation. In addition, the length-force properties of PAMs were tuned based on data from skeletal muscles which have been proven to be very different from EOMs [46]. Furthermore, similar to the MAC-EYE, this robotic eye disregards the significance of an exact EOM insertion point (IP) placement and sets the PAMs symmetrically about the globe’s axis.

Another pneumatically-actuated 2 DOF robot eye has been constructed by Lenz et al. [35]. A web camera mounted on a 2 DOF gimbal system represents the globe, which is driven by 4 pneumatic artificial muscles (PAM) emulating the EOMs. This robot is used to evaluate the performance of a vision stabilizing control algorithm that is inspired by the cerebellar function in vestibulo-ocular reflex (VOR) of humans. Once again in addition to the drawbacks of the PAM actuators, this model also does not possess other significant features of the ocular plant including 3 dimensionality, accurate tendon IP and an accurate globe geometry.

A number of simulated biologically inspired eye models have also been developed for the purpose of scientific research [38, 65]. For instance Wei [65] has created a bio-mechanical simulator of the oculomotor plant, which includes features such as “subject-specific orbit models” and accurate EOM strain properties.
Mehmood et al. [38] have also designed a simulated model of the human eye to assess the importance of tendon insertion points for mechanically implementing listing’s law. Nevertheless, as discussed previously, despite the evident advantages of a simulated model it is instructive to have a physical model.

1.4 Research Objectives

The objective of this research was to construct a biologically realistic robotic eye that can serve as a platform for understanding how the human oculomotor plant works. The motivation for this research is similar to that of the ACT hand developed by Matsuoka’s group [59], which was developed to study and discover mechanism and control strategies used to provide robust, adaptable and skillful human hand movements. Clearly, it is not possible to precisely replicate a biological eye because of the many important facts about the relevant anatomy that are still being discovered and due to the limitations of the technology available to implement an eye model. Rather, I hope to capture some significant features of biological eyes that are not present in typical robot eyes. Specifically: (1) 3D eye movement, which includes not only “pan and tilt” movements but also torsional movements about the visual axis, (2) actuation using six pairs of muscles/actuators (hence redundant, with an approximately antagonistic arrangement of actuators), (3) specific insertions and origins of muscles on the globe of the eye, which are believed to have a significant influence on oculomotor behavior and (4) proper dimensionless scaling of the geometric and dynamic properties of the ocular system, including but not limited to the globe diameter, and inertia; a process necessary for
a proper performance comparison between the model and the biological system.

1.5 Organization of Thesis

This thesis is organized as follows. Chapter 2 provides some background on the biology of the oculomotor system. Chapter 3 discusses the mechanical and electromechanical design requirements. Based on the laid out requirements I then present the mechanical design of the robotic eye in Chapter 4. Appendix A provides more details about the mechanical design process. The kinematics of the resulting system and its control are discussed in Chapter 5. Given the controller developed in Chapter 5 the performance of the robotic eye is assessed by implementing on the robot a series of movements resembling the biological eye movements the results of which are given in Chapter 6. Finally Chapter 7 concludes the thesis and discusses the future work.
Chapter 2

Background

The ocular plant is a complicated structure, and a full understanding of it requires an in-depth knowledge of both the anatomy and the physiology of early visual and cerebral processing. However, when we consider the eye positioning machinery, the subject of this thesis, only certain biological features and behaviours are necessary to be included in the system. Consequently, the scope of this review is limited to anatomical information that is of importance to the kinematics of the eye, such as the extraocular muscles and the neural function required for replicating biologically inspired eye movements. And it excludes details of acquiring the image including image mapping and the role of the retina and iris in early visual processing. For a more extensive discussion of these topics please refer to The neurology of eye movements by Leigh, R.J textbook [34].
2.1 Anatomy

The eye sits within a pyramid-shaped bony structure called the orbit. It is held within the middle of this space by a layer of fibrous connective tissue called Tenon’s capsule. The Tenon’s capsule provides the cavity for the eye to move within the periorbital fat, which maintains the globe in an anterior position. Three pairs of muscles known as the extraocular muscles (EOM) insert into the globe, and it is the innervation of these muscles that generate the force required to rotate the globe in three dimensions. Figure 2.1 illustrates the six EOM; they are: medial rectus (MR), lateral rectus (LR), superior rectus (SR), inferior rectus (IR), superior oblique (SO), inferior oblique (IO). This 3D model was developed by Wei et al. [65] from MRI data.

Figure 2.1: 3D model of the right eye, reconstructed from MRI [65]
The four recti muscles originate from the annulus of Zinn (AZ), a fibrous ring of tissue that encircles the optic nerve. The conical orbital walls point outward at approximately a $23^\circ$ angle. Thus, in primary position the superior and inferior rectus lines of action are approximately at a $23^\circ$ angle with respect to the anterioposterior axis of the globe [47]. The recti muscles have a fairly constant muscle length of about $37\text{mm} \pm 0.7$ while their tendon lengths vary between $3 - 7\text{mm}$ [61].

The SO is the longest EOM, with $30\text{mm}$ muscle length and $30\text{mm}$ tendon length. It also originates from the AZ but its path is redirected through the trochlea, a tissue pulley structure attached to the medial orbital wall, such that it is aligned with the ceiling of the orbital wall and makes an approximate $54^\circ$ angle with the anterioposterior axis of the globe [51, 61]. Unlike all the other EOMs, the IO arises from the medial orbital wall, anterioinferior to the globe. This $37\text{mm}$ long muscle continues from its origin in a path backward, upward and laterally between the orbital walls and the IR, and it inserts onto the globe through a $1 - 2\text{mm}$ tendon. This results in an approximate $51^\circ$ angle between the IO and globe’s vertical plane [62].

The EOMs are activated by the three cranial nerves III, VI and IV. The oculomotor or third cranial nerve supplies the SR, IR, MR, and IO while the abducens or the sixth cranial nerve is responsible for activating the LR. Finally, the fourth cranial nerve, also known as the trochlear nerve, supplies the SO muscle.
Abduction\(^1\) and adduction\(^2\) of the eye is primarily achieved by contracting the LR and MR respectively. These pairs of opposing muscles are called antagonist pairs. According to Sherrington’s law of reciprocal innervation, a muscle’s stimulation is accompanied by inhibiting its antagonist [21]. As a result, abduction is achieved by contracting the LR and relaxing the MR. Given the binocular vision of humans, synchronized movements of both eyes are required for directing the gaze in a given direction. Consequently, the neural control of the EOM of one eye is partnered with the control of the other eye. These partnering EOMs in each eye are known as yoke muscles. For example, in a right gaze the LR of the right eye is the yoke muscle for the MR of the left eye and vice versa. The activations of these yoke muscles are described by Hering’s law of equal innervation, which states that the muscles receive equal and simultaneous innervation, based on the position of the eye [23].

![Diagram of eye movements](Figure 2.2: The arrows indicate the pulling direction of the EOMs for the abducted and adducted eye)

The MR and LR muscles primarily act to abduct and adduct the eye regardless of the eye orientation. However, the action of the SR, IR, IO, SO varies depending

---

\(^1\)Abduction is the movement of the cornea away from the midline of the eye.  
\(^2\)Adduction is the movement of the cornea towards the midline of the eye.
on whether the eye is abducted or adducted. An abduction of 23°, aligns the SR and IR with the axis of the eye, which makes them primarily elevators and depressors. In this orientation, the SO intorts the eye (rotates the globe medially while the IO extorts the eye (rotates the globe temporally). Conversely, in an adducted eye, elevations and depressions are achieved primarily by the IO and SO, while intorsion and extortions become the duty of the SR and IR, respectively. Figure 2.2 illustrates the function of EOMs for the abducted and adducted eye.

2.2 Eye Movement

As mentioned previously, eye movements are an essential part of acquiring visual information. We move our eyes to either shift our gaze to direct the fovea onto the object of interest or to stabilize the image on the retina. Saccades, smooth pursuits, and vergence movements are gaze shifting mechanisms, while vestibular-ocular and optokinetic reflexes are the movements responsible for gaze stabilization during head movement. In addition, although not immediately apparent, maintaining visual gaze (or visual fixation) requires activating the EOMs as well as micro-saccades, which are necessary for proper stimulation of the photo-sensitive cells. This review is focused on the two most frequent eye movements: saccades and smooth pursuits. These are also the two main movements that are to be implemented on the eye model presented in this thesis. For more information about eye movements see [33, 66].
2.2.1 Saccadic Eye Movement

Saccades are rapid and abrupt simultaneous movements of both eyes, which essentially change the point of fixation in a short period of time. Saccades are the basic mechanisms for a number of eye movements including fixation and rapid eye movements. However, the main purpose of saccadic eye movements is to place small visually interesting parts of a scene onto the fovea, where they can be sensed with the highest resolution. This is because like many other animals with fovea, the human brain builds a mental 3D model of the scene based on the small parts of a scene sensed through the fovea. Saccades are the fastest rotational movements in the human body, reaching angular speeds of 500 Deg/sec in humans and up to 1000 Deg/sec in monkeys. Due to the extremely high angular velocities, no visual processing can be performed and therefore saccadic movements are controlled without visual feedback. Initiation of a saccade can take up to 200ms and it lasts between 20ms - 200ms, depending on the amplitude of the saccade [33, 52].

Saccade magnitudes could reach values as high as 90°. However, for magnitudes larger than 20° they are generally accompanied by a head movement. The angular velocity of the saccade is related to its amplitude. This relationship is linear for amplitudes of up to 60°, but as we approach the maximum possible eye velocity, this relationship becomes nonlinear and saturates. In fact, the proportionality of a saccade’s peak velocity to its magnitude is called the main sequence [5].

---

3The amplitude of a saccade is described as the angular distance traveled by the eye during the saccade.
and is what distinguishes saccades from other types of eye movement.

2.2.2 Smooth Pursuit Eye Movement

Smooth pursuit is the movement of our eyes as they track a moving target. Similar to saccades, the main goal of this movement is to keep the target close to the fovea. However, unlike saccades, pursuits are triggered by the movement of the target, and in fact in most cases subjects are not able to initiate a pursuit without a moving target. Consequently, the role of smooth pursuit movement is to match the eye velocity to that of the moving target. This velocity is normally relatively low (in the order of tens of degrees per second), which provides sufficient time for visual processing. This means that, unlike saccades, pursuits are controlled with constant visual feedback. However, despite this visual feedback, there are occasional large errors that are compensated for by corrective saccades. In fact, target velocities of larger than 30 Deg/sec are usually composed of alternating periods of pursuit and saccades.
Chapter 3

Design Requirements

This section gives an overview of the specified requirements for both the structural and the mechatronic design. These requirements were mainly selected to ensure that the final eye model would be an adequate testbed for studying the oculomotor system.

3.1 Structural Requirements

Our design decisions have primarily focused on creating an eye model that mimics the geometric and anatomic properties critical to the static and dynamic performance of the human eye. As discussed in the following paragraphs, such requirements are satisfied by a three dimensional, appropriately scaled model that represents the agonist/antagonist EOM accurately.

It is not trivial to extend controllers in 1D to 3D; as a result, a three dimensional model becomes crucial in understanding how human eye movements are
controlled. This is because, when rotations around a single axis are considered, angular velocity is simply the time derivative of the orientation, and artificial one dimensional oculomotor controllers rely on this fact. However, this does not hold in three dimensions since multidimensional rotations are non-commutative. Therefore, a significant part of unraveling the oculomotor controller is recognizing how this non-commutativity is dealt with. In fact, there is much debate over whether the neural controllers are accounting for this non-commutativity [25, 37] or are the orbital mechanics (orbital pulleys) compensating for this by making the ocular plant appear commutative to the neural controller [15, 16, 41, 48, 50]. Consequently, in addition to having three dimensions, the model must include, or at least have the potential of including, the orbital pulleys if it is to be used as a testbed for assessing the merits of the different sides of the debates.

As most movements in humans, ocular movements are accomplished by the pulling force of the agonist/antagonist muscle pairs. Therefore, an essential part of understanding the oculomotor controllers is recognizing the relationship between the direction of the torque produced by contracting the EOM and the resulting 3D orientation of the eye. Further [26] has proven analytically that it is incorrect to simplify muscle pairs by replacing them with “single bidirectional” muscles, as has been commonly done previously [17, 48, 50, 56]. Consequently, it is apparent that representing all 6 EOMs is a key feature that must be incorporated in the design of a geometrically accurate eye model. A full representation of the EOM entails the incorporation of:

1. Muscle actuation
2. Tendons (Force carriers between the globe and the actuation.)

3. Accurate tendon attachment locations

Biologically accurate tendon attachment locations on the globe are usually neglected in the modelling of the eye, mostly to simplify the kinematic analysis involved. Nevertheless, there is mathematical evidence that shows how the specific location of these attachments could in fact simplify the job of the neural controller [49]. Thus, to further examine such speculations, it is important to replicate this characteristic.

An essential part of creating a geometrically accurate model is replicating the human eye geometry. However, physical constraints including available part sizes and accessibility of the testbed create limitations on how feasible this duplication can be. Nevertheless, the model’s size and inertia effects the required forces, natural frequencies and velocities. As a result it is crucial to have a procedure whereby all the essential properties are scaled in relation to the model’s specified dimensions and inertia. Here, we have used the Buckingham $\pi$ theorem to meet this requirement. By ensuring that key $\Pi$ variables are equivalent in the real eye and our model, we can scale the model up at our convenience without losing any information.

The globe support structure is another important component of this design. A suitable structure must allow for the three dimensional movement of the globe within the same range as the biological eye. In addition, the design must allow room for a clear routing of the tendons and allow accessibility to various parts
of the model as it is to be used as a testbed. Actually, this part proved to be the most involved in terms of design work since simply reproducing the biological equivalent of connective tissues is not trivial. Section 2.1.4 includes explaining the detailed design of this structure, including the additional requirements that had to be taken into account.

It must be noted that a number of reasonable assumptions were made early on. Firstly, the eyeball is taken to be a perfect sphere, which is a very close approximation aside from the slight anterior bulging where the cornea is located. It is also assumed that the centre of rotation of the eye is fixed and located at the centre of the sphere. This is a minor assumption as, in actuality, the centre of the eye is only 1.3 mm posterior to this assumed location [19]. Furthermore, the centre of the real eye has non-zero velocity, however these velocities are very small and are generally ignored [19]. Finally, mechanical constraints such as material strength, weight, friction and sizing played a significant role in the final decisions made throughout the design process. Consequently, as the design process progressed, a number of other assumptions were made, which will be explained in context in the forthcoming chapter.

3.2 Mechatronic Requirements

Before discussing the established requirements for an appropriate controller, I would like to reiterate that the primary purpose of this thesis is to introduce an anatomically accurate eye model. Consequently, the main goal of implementing a control algorithm is to evaluate the performance of this robot eye. This means
the presented control algorithm is not inspired by existing theories about how the brain controls the oculomotor plant. Nonetheless, the desired performance of this controller was chosen to be comparable with eye behaviour during a saccadic eye movement. This was to ensure a reasonable equivalence between the performance of the eye model and the documented behaviour of human eyes. Saccades are one of the most studied and well known movements of the human body. Over the years, scientists such as DA. Robinson and many others have characterized the

---

**Figure 3.1: Position and velocity profile of a 10° saccade [31]**
behaviour of saccadic eye movements [6, 20, 31, 33, 52]. Figure 3.1 illustrates the position and velocity profile of a 10° saccade. This saccade is completed in 40ms with the velocity reaching values as high as 300 Deg/sec. These values increase with increasing saccade magnitude, a relationship described by the term main sequence of saccades [5]. Furthermore, the steady state error of a saccade is on the orders of 10th of a degree and saccadic eye movements are characterized by their overdamped response1 [33]. Ultimately, these elements became the basis of the requirements set out for the control algorithm presented in this thesis. Specifically the controller must enforce:

1. An overdamped response
2. A steady state error of $\leq 0.1°$

---

1A response where there is no oscillation about the steady state value, as illustrated in Figure 3.1 A
Chapter 4

Structural Design

This chapter will present the guidelines used to design the tendon-driven eye model presented in this thesis. The chapter is organized based on the Structure Requirements outlined in Chapter 3. Scaled Model Design is discussed in Section 4.1. Section 4.2 will explain how Musculotendons are represented. Finally in Section 4.3, I will discuss the design and manufacturing process of the Globe Support Structure.

4.1 Scaled Model Design

The human eye is approximately 24 mm in diameter with an inertia\(^1\) of approximately \(4 \times 10^{-7} \text{Kgm}^2\) [69]. In addition, the orbital connective and fatty tissues that support the globe exert passive forces depending on the globe’s position [53]. An important part of creating an anatomically accurate model is reproducing these

\(^1\)This includes the inertia of the optic-nerve head, Tenon’s capsule, and the EOMs.
geometric and dynamic properties. However, mechanical constraints make it difficult to reproduce these essential properties at such a small scale. This creates implications for how comparable our model will be to the biological eye. For instance, if the eye model does not share the same diameter, inertia, stiffness and damping properties, then the step and velocity responses of our model may be irrelevant to the biological responses. Consequently, it is crucial to devise a technique whereby common methods such as step and velocity responses can be used to assess the model when exact duplication of geometric and dynamic properties are not feasible.

One such method is the Buckingham Π theorem. Buckingham states that laws of physics can be expressed in a form independent of the units used [10, 12]. In other words, let’s consider a problem that can be expressed by a set of dimensionally homogenous equations with \( n \) “dimensional variables”, \( q_1, q_2, \ldots, q_n \), as illustrated in Equation 4.1.

\[
F(q_1, q_2, \ldots, q_n) = 0 \quad \text{or} \quad q_1 = f(q_2, \ldots, q_n)
\]  

(4.1)

Now if we let \( k \) denote the number of fundamental dimensions that are required to describe the \( n \) dimensional parameters (In the case of the ocular system these dimensions are, mass \( (M) \), length \( (L) \), time \( (T) \)) , then we can express Equation 4.1 by \( j = n - k \) dimensionless and independent quantities known as Π groups, \( \Pi_1, \Pi_2, \ldots, \Pi_j \), as seen in Equation 4.2. These “Π groups” are not unique; nevertheless they are independent of each other, in the sense that each one may vary
while the other stay constant.

\[ G(\pi_1, \pi_2, \ldots, \pi_j) = 0 \quad \text{or} \quad \pi_1 = g(\pi_2, \ldots, \pi_j) \quad (4.2) \]

The Π theorem has 2 favourable attributes that could decrease the complexity of the problem in hand considerably:

1. When analytical equations are difficult or impossible to set up and solve for a given problem, the Pi theorem (more generally, dimensional analysis) can be used to infer the existing relations between some of the variables of the problem.

2. The size of a given problem is reduced by the number of fundamental parameters present in the problem.

Physical modelling, where a given system is scaled up or down, is an example of when Π analysis can be used to establish the relationship amongst the parameters of a given physical system. This relationship stays unchanged across all scales of the model.

Consequently, to create a physical model of the ocular plant, I used Π analysis to obtain a set of non-dimensional variables that must stay homogeneous across all sizes. This provides a platform for a valid conversion of physical quantities across the eye model and the biological eye. The following lists the steps that I took to arrive at the non-dimensional parameters for the ocular plant.

1. I determined the important variables governing the eyeball, while considering the physical significance of each chosen parameter. Most important in
choosing these governing quantities is their independence from one another, in other words modifying one parameter must not affect the others.

2. I then expressed each parameter in terms of the basic units of mass (M), length (L), and time (T). Table 4.1 gives a list of these parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Fundamental Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>τ</td>
<td>[Nm]</td>
<td>$[ML^2T^{-2}]$</td>
<td>Applied torques</td>
</tr>
<tr>
<td>B</td>
<td>[Nm/rad/s]</td>
<td>$[ML^2T^{-3}]$</td>
<td>Ocular damping constant</td>
</tr>
<tr>
<td>K</td>
<td>[Nm/rad]</td>
<td>$[ML^2T^{-2}]$</td>
<td>Ocular stiffness constant</td>
</tr>
<tr>
<td>I</td>
<td>[Kgm$^2$]</td>
<td>$[ML^2]$</td>
<td>Inertia</td>
</tr>
<tr>
<td>D</td>
<td>[m]</td>
<td>[L]</td>
<td>Globe diameter</td>
</tr>
<tr>
<td>ω</td>
<td>[rad/s]</td>
<td>$[T^{-1}]$</td>
<td>Globe Velocity</td>
</tr>
<tr>
<td>α</td>
<td>[rad/s$^2$]</td>
<td>$[T^{-2}]$</td>
<td>Globe acceleration</td>
</tr>
</tbody>
</table>

3. Given the 7 quantities concerned in expressing the dynamic behavior of the eye and the 3 basic dimensions required to express them, I arrived at $7-3 = 4$ non-dimensional quantities or $\pi$ groups, which were formed as a combination of the 7 dimensional quantities. These $\pi$ groups take the general form of:

$$\Pi = I^a D^b \tau^c B^d K^e \omega^f \alpha^g$$

Then the dimensional equivalent is expressed as:

$$\Pi = M^a L^{2d} L^b M^{e} L^{2c} T^{-2e} M^{d} L^{-2d} T^{-3d} M^{e} L^{2e} T^{-2e} T^{-f} T^{-g}$$

To have $\Pi$ be a non-dimensional number, the powers of all the separate
dimensions must add up to zero. Hence we arrive at the equations in 4.3.

\[ a + c + d + e = 0 \]
\[ 2a + b + 2c + 2d + 2e = 0 \] \hspace{1cm} (4.3)
\[ -2c - 3d - 2e - f - 2g = 0 \]

4. There are 7 unknowns and 3 equations therefore a solution may be found by arbitrary choosing values for 4 and solving the equations in 4.3 to find the other 3 unknowns. By selecting \( c = 1 \), \( e = 0 \), \( f = 0 \), and \( g = 0 \) we arrive at \( \Pi_\tau \):

\[ \Pi_\tau = \frac{\tau^3}{IB^2} \]

\( \Pi_K \) is found by selecting \( d = 0 \), \( e = 1 \), \( f = 0 \), and \( g = 0 \):

\[ \Pi_K = \frac{K}{\tau} \]

Setting \( c = 0 \), \( d = 0 \), \( f = 1 \), and \( g = 0 \) we find \( \Pi_\omega \) as:

\[ \Pi_\omega = \omega \sqrt{\frac{I}{K}} \]

Finally when we set \( c = 0 \), \( d = 0 \), \( f = 0 \), and \( g = 1 \) we find \( \Pi_\alpha \) as:

\[ \Pi_\alpha = \alpha \frac{I}{K} \]

Subsequently, the ocular plant may be expressed as a function of the resulting
Π variables:

$$G\left(\tau^3, \frac{K}{\tau}, \omega\sqrt{\frac{I}{K}}, \omega I K \right) = 0$$

This could be solved for any of the Π groups, for instance solving it for Πτ

$$\frac{\tau^3}{IB^2} = g\left(\frac{K}{\tau}, \omega\sqrt{\frac{I}{K}}, \omega I K \right)$$

(4.4)

It is important to realize that the Π groups are not just numbers; rather, they represent important quantities that give insight to the physics of the model. For instance, let’s consider the situation where we wish to find the required applied torque to move the eye model to a desired position, given data from the biological eye. In this case we can apply 4.4 to the eye model and the biological eye, with the primes representing the eye model.

$$\frac{\tau'^3}{IB'^2} = g\left(\frac{K'}{\tau'}, \omega'\sqrt{\frac{I'}{K'}}, \omega' I K' \right)$$

$$\frac{\tau^3}{IB^2} = \frac{\tau'^3}{IB'^2}$$

(4.5)

Then, as the Pi theorem states, the arguments of function g are the same for both the model and the biological eye. Therefore, the value of the function would be the same in both cases and, as demonstrated in 4.6, the model’s torque can be found in terms of the given biological torques:

$$\frac{\tau^3}{IB^2} = \frac{\tau'^3}{IB'^2}$$

$$\tau'^3 = \frac{IB'^2}{IB^2} \tau^3$$

(4.6)
An identical process would be carried out for finding other characteristics for the eye model, such as stiffness, given the known eye behaviours. Moreover, applying this method in reverse would be valuable in the validation process. Namely, by appropriately scaling model quantities such as the velocity profile, we can directly compare the performance of the model eye and the biological eye.

4.2 Musculotendon Representation

As discussed previously, creating an anatomically accurate model of the human eye also includes developing an actuation system that mimics the passive and active properties of the human EOM. Here I have chosen to use a tendon-driven structure where muscle active contractions, simulated by electromagnetic actuators, are transferred to the eye globe through cables that represent the ocular tendons. The details of these design decisions are explained in section A and B, respectively.

Furthermore, I have chosen to model and provide the passive properties of the EOMs in software rather than using a physical representation. This is because the perceived passive force of the ocular plant is associated with more than just the EOMs; it also includes other biological features such as connective orbital tissues and the optic nerve. A software representing the passive forces allows us to lump or separate these different parts, which becomes important when deciding on an appropriate ocular control algorithm. For instance, the authors of [49] have argued that to reach an accurate understanding of the ocular control algorithms used by the brain, it is essential to separate the muscular and orbital passive proper-
ties, whereas typically the ocular plant dynamics are lumped together in the most
commonly used ocular models [48, 54, 58]. Consequently, this method of repre-
sentation is preferred despite the complexity that it adds to the control algorithm.

A. Muscle Actuation

There have been a number of attempts to design an actuator that mimics the prop-
erties of human muscles. Amongst these are shape memory alloys [44], and pneu-
matic actuators [13] and specifically the most commonly used pneumatic actuator,
the MacKibben actuator, or air muscle [14, 57]. All these actuators have
been designed to mimic properties of skeletal muscles, which are known to be
very different from the EOMS [46]. These actuators essentially have reproduced
the quasi-static force-length relationship of the skeletal muscles and for the most
part do not match the compliance characteristics of the real muscles. It is possible
to construct a custom-made air muscle with properties of the EOMs, however a
further shortcoming of pneumatic muscles is that their activation representation is
not easily adjustable and thus it becomes difficult to capture the unique features
of the EOMs. As a result, we have decided to use DC motors as our actuators.
This allows incorporating the biological EOM and orbital properties in the control
algorithm. In addition, unlike the fixed activation and force-displacement relation-
ship produced by the pneumatic actuators, controlled DC motors allow for EOM
activation representation that is adjustable for each muscle.
B. Tendon Representation

As is commonly assumed [17, 48, 50, 54, 56], we have chosen to model the tendons as thin cables rather than as bands, which would more accurately model the physical shape of real EOM [26]. This is a reasonable assumption since the cable’s width would only affect the area over which existing tendon forces are applied and would not affect the force balance on the globe. Furthermore, regardless of this assumption, the cables provide a simple yet appropriate replacement for the biological tendons, since just like the tendons they can only exert a pulling force. This is an important resemblance, which in theory provides a set of control challenges for the tendon-driven eye model that are similar to those that our brain must overcome.

I considered a number of different cables before deciding on a material that met the preceding specifications. Most importantly the cable must be lightweight with very high strength, and should be stiff relative to the compliance that is to be implemented in the controller. High stiffness and low weight would allow us to ignore the dynamic properties of the cables, thus making them ideal force carriers between the motors and the globe. This is also enables us to replicate the existing stiffness in the biological tendons purely in software without being concerned with uncertain material properties. Moreover, the cables must also have a smooth finish and a low coefficient of friction in order to minimize the friction with contacting surfaces.

Low stretch ($< 0.04\% \text{ inch/ft}$ for 20 million cycles), nylon coated, stainless steel cables were the initial selection and they were attached to small screws
threaded through the thickness of the globe, as shown in Figure 4.1. Furthermore, the presence of the nylon coating creates a smooth, low friction sliding contact between the cables and the globe. However, these steel cables have a large bending stiffness; namely they can not easily bend around sharp corners, which makes the routing and fastening of the cables rather difficult.

For this reason, the steel cables were replaced with dyneema cables, a High
Modulus Polyethylene (HMPE) fibre. With a tensile strength of 3200 MPa, dyneema offers an extremely high strength-to-weight ratio. Dyneema also offers low elongation (E-modulus of 110 GPa, which is about 4 orders of magnitude larger than the 70 kPa [64] E-modulus of EOM tendons). This allows us to ignore the elastic and dynamic properties of the dyneema cables and view them as an ideal force transporters, which has great implications on simplifying the calculations required to find the actual torque transferred from the motors to the globe. Furthermore, compared to the steel cables the dyneema cables are small in diameter (0.8mm vs. 0.3mm) which makes it easier to handle them. They also benefit from very small bending stiffness, thus considerably simplifying the fastening and routing of the cables. Figure 4.2 illustrates the cable-globe attachments with the dyneema cables; here the cable is simply run through small holes in the globe and is secured in place by knotting it at the other end. This process greatly simplified the assembly process and it also reduced interference between fasteners of the cable and various parts of the model.

As mentioned in Chapter 3, it has been shown that biologically accurate tendon insertion locations could have great implications on how the brain controls eye movements [49], and thus it is important to include this feature in the design of our eye robot. Therefore, since the eye robot presented here is 2.5\(^2\) times larger than the biological eye, all the biological attachment data from literature [60] must also be scaled by this factor.

\(^2\) The eye robot has a 60mm diameter and the biological eye has a diameter of 24mm. This dimension for the model was limited by the size of the available camera.
EOM-globe attachments span a curved line, as shown in Figure 4.3. However, to simplify the modelling and analyzing process we have chosen to approximate this curve as a single point called the insertion point (IP), which is illustrated in Figures 4.1 and 4.2. This is a common assumption made by many researchers working on building an eye model [8, 49, 63] and no concerns have been reported about the potential implications of this assumption on how the oculomotor system is controlled. The IPs are taken to be at the mid point of the insertion curve, and it is the location of these selected points that are then scaled by the diameter ratio of the eye model to the human eye. Table 4.2 shows biological IPs, taken from the literature [49], and the corresponding scaled versions used in the model.

In addition to an accurate representation of the IP, routing of the tendons is
Table 4.2: The biological and the scaled insertion points (The coordinate is taken to be at the centre of the eye globe)

<table>
<thead>
<tr>
<th>Extraocular muscle</th>
<th>Biological IP [mm]</th>
<th>Scaled IP [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MR</td>
<td>[8.4, -9.2, 0]</td>
<td>[20.2, -22.0]</td>
</tr>
<tr>
<td>LR</td>
<td>[6.7, 10.5, 0]</td>
<td>[16.2, 25.0]</td>
</tr>
<tr>
<td>SR</td>
<td>[7.3, 0, 10]</td>
<td>[17.6, 0, 24.1]</td>
</tr>
<tr>
<td>IR</td>
<td>[7.7, 0, -9.8]</td>
<td>[18.4, 0, -23.5]</td>
</tr>
<tr>
<td>SO</td>
<td>[-4.5, 3, 11.2]</td>
<td>[-10.8, 7.1, 26.9]</td>
</tr>
<tr>
<td>IO</td>
<td>[-7.9, 9.6, 0]</td>
<td>[-19, 23, 0]</td>
</tr>
</tbody>
</table>

another key factor in reproducing the lines of action of the EOMs. To achieve this, the anchoring points, sometimes called pulleys, of each tendon were placed to force the cable to follow the same geometry as the EOM. This geometry was specified based on the anterioposterior axis of the globe and the angle that each EOM makes with it. These angles are listed in Table 4.3.

Table 4.3: Angles between the anterioposterior axis of the globe and the EOMs [60]

<table>
<thead>
<tr>
<th>Extraocular muscle</th>
<th>Angle [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR</td>
<td>23</td>
</tr>
<tr>
<td>IR</td>
<td>23</td>
</tr>
<tr>
<td>SO</td>
<td>54</td>
</tr>
<tr>
<td>IO</td>
<td>51</td>
</tr>
</tbody>
</table>

Two types of anchoring were used, one for the recti muscles and one for the obliques. In the biological ocular plant, the four recti muscles and the superior oblique muscle originate from a common conical structure, called the AZ. In the eye robot this structure is reproduced using a low friction teflon cube, Figure 4.4. The teflon block has a small circular opening for each converging tendon to pass
through. There is a minor modification in our model, however: only the four recti tendons are routed through the modelled AZ structure. This decision was made in order to develop the most efficient and convenient motor arrangement, with the four recti motors located behind the globe and the oblique motors positioned to the right of the eye model.

For the obliques, once again attention was directed towards reproducing the tendon’s precise lines of action, rather than exactly replicating the biological anchoring location. In this case the tendons were routed through very low friction plastic tubes glued into hooks as seen in Figure 4.4. This provided a low friction solution which was easy to incorporate into the design. Figure 4.4 also illustrates the motor mounting arrangements as well as the motor-cable attachment mechanism. Small aluminum static pulleys (OD=15m) were screwed to the motor shafts with setscrews and used to attach the cables to the motors; the cables were then passed through openings in the pulleys and were knotted to keep them in place.

4.3 Globe Support Structure

The biological eyeball is suspended inside the conical walls of the orbit in an elegant manner. It is supported and protected by a layer of fibrous connective tissue called “Tenon’s Capsule”. The Tenon’s Capsule provides a cavity for the eye to move within the periorbital fat which maintains the globe in an anterior position. Furthermore, there are ligaments connecting the EOM to the orbital walls and it is the action of these ligaments that is responsible for the smooth and dampened eye movements.
However, while it is important to capture characteristics of this orbital support, exactly replicating all of its features is not necessary to achieve a similar functionality. For instance directly representing all the features such as compliance of the globe, periorbital fat, and lubrication inside the orbit is not critical. Rather it is important to reproduce the purpose of these elements. Consequently, the modelled support mechanism must keep the globe in an anterior position, provide a smooth and interference free path for the cables, and also allow for the 3D rotation of the globe. Moreover, as mentioned in Section 4.2 the elastic and damping forces of the connecting tendons will be accounted for in software.
To satisfy these requirements, I have custom designed a globe support structure that provides a practical and functional solution for the problem of supporting a spherical globe given the mechanical and material constraints. The corresponding design decisions were greatly influenced by the need for a flexible and open structure. This is essential for easy access to all parts of the model, which would facilitate operation and future modifications. The following design criteria were used throughout the various stages of the design process:

1. Provide a low friction mechanism that allows for the 3 rotational DOF of the globe, with the centre of rotation fixed at the centre of the globe

2. Ensure clearance of the cables throughout the entire range of operation

3. Present an optimal location for the camera and its connecting cables

### 4.3.1 Considered Solutions

Before deciding on the final design of the globe support structure, a number of methods were considered; models of some of these were built and tested in order to identify the design limitations. The initial concept was based on supporting the globe on ball-terminated screws[43], as illustrated in Figure 4.5. This was a simple design in terms of construction; however, the task of optimally locating the screws to avoid interference with the cables and their IP's proved to be complex. In addition the ball-terminated screws increased the dynamic friction of the system. I then considered supporting the globe using elastic stalks connected to the ground, Figure 4.6. However this design was also rejected since the elastic stalks
would not provide a purely rotational support and would allow the globe to translate. Furthermore, similar to the screw supports preventing interference with the cables requires optimally locating the elastic stalk attachment to the globe which is challenging to find. I finally decided on a gimbal support structure, which is what the final support structure is based on. The detailed design of these solutions are provided in Appendix A.

4.3.2 The Gimbal Globe Support Structure

It was apparent from early on that an off-the-shelf gimbal will not be sufficient to meet the specific characteristics required from a globe support structure. Consequently, gimbal support structure was custom designed to include the following desired features:

1. Have a single attachment point between the gimbal and the globe, to ensure
Figure 4.6: Concept of the elastic stalk support

clear routing of the cables.

2. Present an optimal location for the camera and its connecting cable.

3. Incorporate angle sensors for the 3 DOF.

4. Have yaw, the most frequent eye movement, as the inner most gimbal rotation to minimize the device’s moment of inertia (MI) in this direction.

Ideally, the MI would be equal around all axes of the model. However, in practice this is difficult to achieve with a gimbal system since the globe’s MI around all of the gimbal axes is increased by that of the moving gimbal elements except for the inner most rotation where only the globe is in motion. This results in 3 different MIs for the globe depending on which direction the globe is moving. Therefore, it is extremely important to not only reduce the individual added MIs due to the

\[3\text{See Section 4.3.3 for definition}\]
gimbal parts, but to also minimize the difference between them. The two factors that determine MI are the mass of the part and the moment arm\(^4\). Consequently, the structural strength of the gimbal system needs to be optimized to minimize the amount of mass needed in each stage.

With these specifications in mind, the general shape of the gimbal was decided based on an iterative design process. Various structural designs were prototyped, examined and adjusted in order to arrive at a suitable final design. In the following section, I will explain the detailed design of the final eye robot. For the design details of the earlier versions of this gimbal system please refer to Appendix A.

### 4.3.3 The Eye Robot - Structural Design

Figure 4.7 illustrates the final design of the eye robot with the gimbal support structure. Considering the right hand coordinate system illustrated in this figure, yaw is rotation about the Z axis, pitch is rotation about the Y axis, and finally roll defines rotations about the X axis.

An important aspect of gimbal design is the sequence of rotations it implements. As discussed previously, since a gimbal forms a serial kinematic chain, the proximal axes of rotations have larger inertia than the distal ones. The standard Helmholtz and Fick gimbals used in the eye movement literature [22] have torsion (“roll”) as the final axis, but the most common eye movement is a horizontal rotation (“yaw”). Therefore our gimbal implements pitch, followed by roll, with yaw as the final rotation. The key to achieving this sequence lies in the unique de-

\(^4\)Moment arm is defined as the radius of gyration of the gimbal part
Figure 4.7: The eye robot, final design of the gimbal globe support structure and placement of the gimbal elements, which are discussed in the following paragraphs.

Figure 4.8 illustrates the inner gimbal element labeled C-bracket. This element along with the globe’s interior were custom-designed to bear the shaft and the rotational joints of the yaw and roll movements. The C-bracket and the globe, respectively, accommodate the rotational joints and shafts required for the globe’s yaw rotation. The globe is divided into two sections, the top hemisphere and the bottom hemisphere. Each hemisphere has 8\textit{mm} long bosses on the bottom of its inner surface. These bosses are drilled down 10\textit{mm} into the thickness of the globe to hold the stainless steel precision shafts that support the yaw rotation,
Figure 4.8: C-bracket, the inner structure of the eye robot

The corresponding rotational joints for these shafts are the 2 openings that are drilled across the length of the C-bracket, as labelled in Figure 4.9c. Given this arrangement, during yaw rotations the globe is the only element in motion. Thus, the globe must include a posterior slot, labeled “trace slot” in Figure 4.9a, to accommodate the roll shaft that is attached to the stationary C-bracket inside the globe.

The roll rotations of the eye robot are supported by a single joint, located at the centre of the C-bracket, Figure 4.9c. The globe and its content cantilever off of the stainless steel precision shaft responsible for this rotation and this shaft is fastened to the back-ring (Figure 4.9e) and rotates with respect to the rest of the
Figure 4.9: Solid model of the eye robot parts
gimbal. Considering such attachments, it is apparent that during intorsion\textsuperscript{5} and extorsion\textsuperscript{6} the globe’s MI is increased by the MI of the C-bracket, which moves with the globe.

Finally, pitch rotations are supported by the two joints inside the support stands, labelled in Figure 4.7 and the two shafts fastened to either sides of the outer-ring along its centre line, Figure 4.9d. This makes pitch the outermost rotation, which means that during elevation and depression of the globe all the gimbal elements (C-bracket, back-ring, and outer-ring) are also in motion, thus increasing the globe’s MI. To reduce the inevitable increase that the gimbal imposes on the globe’s MI during pitch and roll rotations, I have decreased the moment arm of each element as much as possible given the geometrical and structural constraints. Specifically, by placing the C-bracket inside the globe and making its height as short as possible, the element is brought close to the centre of rotation of the globe, which significantly reduces its effective MI. Furthermore, I have carefully chosen the diameter of the outer-ring and back-ring to minimize the gap between the ring and the surface of the globe, without compromising a clear routing for the cables.

\textsuperscript{5}Rotation towards the centre of the body
\textsuperscript{6}Rotation away from the centre of the body.
4.3.4 The Eye Robot - Manufacturing

The Eye Robot - Sizing and Material Selection

The majority of the dimensions in the eye robot were constrained by the measurements of a suitable camera. The camera is to be used in the implementation of a biologically inspired controller and thus its incorporation in the design is of great importance. The selection process for the camera was mainly driven by its size and its capability to provide live image information with a minimum speed of 30 frames per second. Based on these requirements the Point Grey, Dragonfly2 camera was selected. Dragonfly2 is a remote head camera, meaning that the camera can be detached from its processing board thus, making it relatively compact, 20 $mm \times 30 \ mm \times 23 \ mm$. Furthermore, this camera has a speed of 60 frames per second, which is sufficiently above the required speed of 30 frames per second.

Figure 4.10 illustrates an exploded view of the globe’s inner structures. The C-bracket holds the roll and yaw joints and their subsequent angle sensors. Thus, I began the sizing process by first selecting the dimensions of the C-bracket based on the smallest available angle sensor, a 11 $mm \times 11 \ mm \times 2.2 \ mm$ potentiometer labelled in Figure 4.8. To get an accurate measurement from the potentiometers, they must be fixed to the C-bracket. Therefore, lengths $a$ and $c$, as labelled in Figure 4.11a, must be at least as long as the 11$mm$ width of the potentiometer. Given the values chosen for the lengths of $a$ and $c$, I calculated the smallest possible inner diameter that would leave sufficient space for the camera inside the
globe without any interference between it and the C-bracket, as illustrated in Figure 4.10. Thus, by choosing a 2\textit{mm} wall thickness, the globe’s final dimensions are \textit{ID} = 56 \textit{mm} and \textit{OD} = 60 \textit{mm}. This 2 \textit{mm} globe wall thickness does not provide sufficient support for the 17 \textit{mm} yaw shafts; this is the reason for adding the two 6 \textit{mm} bosses inside the globe, labelled in Figure 4.9b. The thickness of the back wall( \( t_b \)) and top and bottom walls ( \( t_t \)) of the C-bracket, labelled in Figure 4.11a, were determined last. \( t_b \) was selected to accommodate the two 4 \textit{mm} thick ball bearings required to support the roll joint, while \( t_t \) was restricted by the 6 \textit{mm} high polymer bushings used to reduce friction in the yaw joints. It should be noted that the size of these bearings were limited by the allowable 4 \textit{mm} diameter hollow potentiometer shaft.

The back-ring and the outer-ring inner radii were chosen to bring them as close as possible to the circumference of the globe thus, reducing their effective
MI. Furthermore, the 8 mm thickness of these elements is a direct consequence of having to accommodate the 4 mm roll and yaw shafts, which were imposed by the hollow shaft potentiometers.

I selected the outer radii of the outer-ring and back-ring to reduce their mass to a minimum without jeopardizing their structural strength. As a result, I began by choosing a material that would provide the optimum mass-strength ratio considering the manufacturing methods available to me. Given the somewhat complex shape of these elements I deduced that a 2D or 3D prototyping process would be the most suitable option. Therefore, I chose 3D rapid prototyping to fabricate the earlier version of the eye robot out of polycarbonate, the strongest materials available for this method (Tensile strength, 68 MPa). However, when this earlier version of the eye robot was tested under maximal loading it was apparent that the 4.8% tensile elongation of polycarbonate is too large to meet the maximum desired deformation of 0.1 mm. Consequently, the high stiffness and strength of aluminium was utilized to achieve the deformation requirements and keep the mass relatively low. This material choice narrowed the manufacturing process to “Water Jet Cutting”, a 2D prototyping method. Thus the final dimensions of the outer-ring and back-ring were calculated considering the maximum applied forces as well as the precision capabilities of the Water Jet Cutter being used. The force analysis and dimensional optimization of these elements were carried out using Cosmos, the analysis simulator in SolidWorks.

The optimized thicknesses of the back-ring and the outer-ring achieved by the process that was just explained are not suitable for supporting the 4 mm diameter
Figure 4.11: Dimensioning of the eye robot part
shafts of roll and pitch rotations and would therefore induce undesirable stresses in the outer gimbal elements. This is because the optimization process used disregarded the requirement that the shafts for the pitch and roll rotations must respectively be supported by the 2 mm thick outer-ring (Figure 4.11c) and < 1 mm thick back-ring (Figure 4.11b). In addition, the globe and everything inside of it cantilevers off of the roll shaft held by the radial thickness of the back-ring (Figure 4.11b) and subsequently the entire weight of the gimbal structure is supported by the pitch shafts held by the radial thickness of the outer-ring (Figure 4.11c). Consequently, the outer-ring and back-ring were protruded 8 mm (respectively labelled as pitch and roll shaft boss in Figure 4.11b and Figure 4.11c) at their shaft connection areas. This provided the required support for the load bearing shafts without adding excessive material and weight.

![Support Stand](image)

**Figure 4.12:** Acrylic support stand
An acrylic structure, labelled “support stand” in Figure 4.12, acts as the support that holds the outer gimbal element as well as the anchoring mechanisms of the oblique muscles. The 48 mm depth of this structure extends back to the furthest point of the back-ring, a design decision intentionally made to facilitate future modifications for including the orbital pulleys in the design. The orbital pulleys attach the globe to the inner walls of the socket and as discussed in Section 3.1, their function is a controversial subject amongst the experts and requires further investigation. Thus it is important that the design would include the potential of adding the pulleys. Much like the trochlea of the SO, the orbital pulleys are thought to act as flexible anterior anchoring points for the Recti muscles [27, 39]. The conceptual design for mechanically representing these connective elements is to fasten panels with rows of holes (labelled “pulley routing plate” in Figure 4.13) to the back walls of the extended acrylic support thus reproducing a posterior anchoring for the cables as they are routed through the appropriate holes inside the panels. A sketch of this conceptual design is illustrated in Figure 4.13.

**Manufacturing Process**

Rapid prototyping methods provide a fast and relatively cheap alternative for manufacturing prototypes, which greatly facilitates iterative testings of various designs. Consequently, 2D water jet cutting and 3D fused deposition modelling were utilized for fabricating the eye robot, and, as discussed briefly in the previous section, the most suitable processes were chosen based on the materials selected.

As suggested by its name, a water jet cutter uses high velocity and pressure jet
of water to cut through metals and other materials such as granite. Here, I utilized the 2D “Abrasive water-jet cutting machine MAXIEM 1515” available at UBC’s mechanical machine shop to fabricate the aluminum and acrylic pieces of the eye robot. This cutter is connected to a 20hp direct drive pump and has a position accuracy of $\pm 0.005''$ and a Kerf of $0.035''(\pm 0.01'')$ for 1/8” mild steel\(^7\). Using a water jet cutter provides a quick and efficient prototyping method, capable of cutting intricate shapes, directly from computerized engineering drawings, into the

\(^7\)Specifications are provided by the MECH Machine Shop website
material of choice. Furthermore, the pressurized water jet used to cut through the material also acts as a cooling agent thus, preserving the material’s intrinsic properties during the cutting process. This feature is especially beneficial when the material involved is easily deformable by heat, such as various types of plastics.

However, when 3D shapes such as the globe of the eye robot are required, water jet cutting becomes inadequate. As a result, I used the rapid prototyping services of the company “RED EYE” to fabricate the globe. RED EYE uses “Fused Deposition Modelling (FDM)”, an additive manufacturing method that transforms 3D CAD models into physical prototypes. The CAD file is sliced and is used to define a tool path for the heated extrusion head. The extrusion head then deposits 0.125mm layers of liquified thermoplastics, which hardens immediately after release, to build the part one later at a time, from bottom top. As a result, materials practical for this process are limited by their heat resistance properties and include various types of plastics. I chose to use ABS M30 for its high impact, tensile and flexural strength\(^8\). Moreover, ABS M30 posses a strong layer bonding, which results in high quality, durable and strong parts, capable of including detailed features. In essence FDM is an excellent method for producing durable prototypes with great dimensional accuracy\(^9\).

\(^8\) Tensile Strength = 36MPa, Tensile Elongation = 4\%, Flexural Stress = 61MPa
\(^9\) Depending on the size of the part, achievable accuracies vary between ±0.127mm or ±0.0015mm/mm, the greater of the two.
Joint Details

Each joint must provide a low friction and rigid support for the gimbal elements, while applying sufficient constraints to eliminate joint play by restraining its movement to a specific rotational direction. Thus, a suitable design of the gimbal joints is essential for obtaining a high quality performance from the eye robot.

The roll joint in particular requires an intricate design because it is a single cantilevered joint that must support the entire weight of the globe and its content, while firmly holding them in place under the applied tendon loads. I utilized radial ball bearings to satisfy the load bearing requirements of this joint. The bearing balls, which are held in place by the rotating inner race and the fixed outer race, greatly reduce the rotational friction while transmitting the radial loads attributed to the cantilevering masses. To ensure that the bearings provide sufficient support I have chosen to use two bearings housed in the 10\text{mm} diameter counter bore hole at the centre point of the C-bracket’s back wall, labelled in Figure 4.14. Subsequently, one end of the roll shaft fixes to and rotates with the inner-race of the bearings, while the other end attaches to the back-ring using a setscrew, Figure 4.14. To increase the contact area between the shaft and the setscrew and thus, ensuring a rigid attachment, I have cut a D profile on the shaft and used a flat end setscrew. Next, I examined the loads on the roll joint in order to apply the appropriate joint constraints. Pulling forces on the globe, exerted by the activated cables, are transferred to the C-bracket, the resulting force path of these loads are illustrated in Figure 4.14. To prevent the C-bracket from moving under these axial loads, a force path must be provided to transfer the loads to the back-ring (This
path is illustrated in Figure 4.14). Thus, I have used a stainless steel spacer to provide a medium for this force transfer. Moreover, placing a snap ring (labelled in Figure 4.14) against the inner race of the front bearing provides the necessary forward joint constraint to prevent unintentional disassembly.

The entire gimbal system rests on the two pitch joints, Figure 4.15. Thus, similar to the roll joint, these joints must sustain a large radial load while allowing a low frictional rotation. Consequently, again I used two radial ball bearings to achieve these goals and housed them inside a 10mm diameter counter-bore hole on the side walls of the supporting stands. Same as the roll shaft, each pitch shaft fits to and rotates with the inner race of the bearings on one end and attaches to holes inside the outer-ring using a setscrew. Once again, I utilized a D profile on the shaft and a flat-ended setscrew to produce a rigid attachment. Considering the relatively low sideway forces, horizontal slack is not of much concern. Nonethe-
less, to ensure a strong bond between the bearing and its shaft, I glued the outer surface of the yaw shaft to the bearing’s inner race.

Because of their placement, yaw joints do not sustain much of the weight of the assembly and thus, these joints are the simplest in design. Therefore, given the absence of large radial loads, reducing rotational friction becomes the top demand from a bearing. Consequently, I have chosen to use flanged polymer bushings for reducing the friction in yaw rotation. These bushings are friction fitted into the holes on the top and bottom walls of the C-bracket, where the yaw shafts, attached to the globe, slide through them with a loose fit. The effective vertical slack is ultimately dependant on the tolerances of the manufactured parts; however, slack is substantially reduced by sandwiching the C-bracket between the globe’s top
and bottom hemisphere, Figure 4.16 illustrates the cross-section of this globe, C-bracket assembly. However, in doing so, as the globe rotates in the yaw direction, the smooth surface of the bushing’s flange will move against the uneven surface of the globe’s boss. Therefore, in order to avoid the sliding friction due to this movement I have placed a smooth stainless steel washer where the bushing flange meets the globe’s boss. Table 4.4 summarizes the details of all the joints explained in this section.
Assembly Process

The assembly process begins by the assemblage of the roll joint. I start by snapping a zinc plated steel retaining ring onto the roll shaft. I then tight fit the two bearings onto the shaft, making sure that the bearings and the snap ring all press against each other tightly. Subsequently, this combination is attached to the C-bracket by using lock-tight to fix the outer race of the bearings to the back wall of the bracket. Next, the steel retaining ring is pressed against the rear bearing, sandwiching the bearings between itself and the snap ring. At this point, I pass the roll potentiometer through the end of the roll shaft and glue the sensor onto the bracket’s back wall using epoxy. Figure 4.17 illustrates the exploded view of this assembly.

To continue the assembly process, I friction fit the polymer bushings and yaw shafts inside their housing in the C-bracket and the top and bottom globe hemisphere, respectively. The stainless steel washers are then glued onto the surface of the globe’s bosses to reduce their effective sliding friction. Finally, before fitting the two globe pieces together, I placed the C-bracket’s assembly through the yaw shaft in the bottom hemisphere and glued the yaw potentiometer onto the floor of

<table>
<thead>
<tr>
<th>Rotational Direction</th>
<th># of Joints</th>
<th>Fixed to</th>
<th>Rotates wrt</th>
<th>Fixed by</th>
<th>Rotates in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yaw</td>
<td>2</td>
<td>Globe</td>
<td>C-bracket</td>
<td>Friction Fit</td>
<td>Polymer Bushings</td>
</tr>
<tr>
<td>Roll</td>
<td>1</td>
<td>Back-ring</td>
<td>C-bracket</td>
<td>Setscrew</td>
<td>Radial Ball Bearings</td>
</tr>
<tr>
<td>Pitch</td>
<td>2</td>
<td>Outer-ring</td>
<td>Support Stand</td>
<td>Setscrew</td>
<td>Radial Ball Bearings</td>
</tr>
</tbody>
</table>

**Table 4.4:** The eye robot joint details
Figure 4.17: C-bracket assembly, the numbers represent the assembly sequence

The C-bracket. Figure 4.18 illustrates the exploded view of this assembly.

The final step is attaching the outer gimbal element to the support stands so that the globe/C-bracket combination can be fastened to it. Thus, before assembling the three pieces of the support stand, I fixed the bearings to the housings inside the two sidewalls of the support stand and glue the pitch shafts to the inner race of these bearings using cyanoacrylate (super glue). As discussed, this is done to enhance the connection bond of the shaft and the bearings thus eliminating joint play under the forces of the oblique cables. I then sandwich the outer gimbal element between the two sidewalls and fasten it to the pitch shafts as illustrated in the exploded view shown in Figure 4.19. Finally, the pitch potentiometer is glued
Figure 4.18: Globe assembly, the numbers represent the assembly sequence to the outer surface of the support stand wall and the globe is attached to the outer gimbal element.

4.3.5 The Eye Robot - Mechanical Characteristics

I have assessed the overall structural strength of the system by evaluating the strength of its building blocks under the two existing conditions of deactivated and activated motors. However, different elements and various parts of the same element are loaded depending on which condition is being considered. When
Figure 4.19: Final assembly, the numbers represent the assembly sequence considering the deactivated situation, the yaw and roll shafts and the elements that these shafts are attached to, namely the back and outer rings, are loaded under the weight of the structural elements. On the other hand, when the motors are activated, the tensioned cables pull on the globe, loading all the gimbal elements. The structural strength of a part under the applied loads is expressed in terms of its factor of safety. However, it is also important for the components to have small deflections since large deflections would affect the positioning accuracy of the
globe. Implementing a definite upper boundary for these deflections is difficult. However, considering the size of the structure I have imposed a rough maximum allowable deflection of 0.1 mm.

Under static conditions, when the motors are not running, the weight of the globe and the gimbal elements are the only forces on the structure and thus the shafts are under bending loads. The total weight on the roll shaft is distributed over the length of the two bearings. Thus, by taking the equivalent concentrated load $P$ to be at the midpoint of this length we arrive at the beam bending diagram shown in Figure 4.20 with a maximum deflection given by:

$$\delta_{\text{max}} = \frac{Pa^2}{6EI}(3l - 1) \quad (4.7)$$

![Beam bending diagram for the roll shaft](image)

**Figure 4.20**: Beam bending diagram for the roll shaft

Thus, considering the 69 g weight on the 4 mm stainless steel$^{10}$ shaft, the max-

$^{10}E_{SS316} = 1.93 \times 10^{11} \text{ Pa}$

63
imum deflection of the roll shaft is $8.5 \times 10^{-7}$ mm, which is considerably below the 0.1 mm. The reaction forces of this mass loading is applied to the holes in back-ring and the inner-bracket, where these elements attach to the roll shaft. The effect of these reaction forces are evaluated using Cosmos, the analysis simulator in Solidworks and quantified as the deflection of the elements and their factors of safety\textsuperscript{11}; these results are summarized in Table 4.5.

<table>
<thead>
<tr>
<th>Gimbal Element</th>
<th>Maximum Deflection [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner-bracket</td>
<td>$2.15 \times 10^{-6}$</td>
</tr>
<tr>
<td>Back-ring</td>
<td>$1.47 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

\textbf{Table 4.5:} Performance of the gimbal elements when the motors are not activated

When all the motors are activated, the recti and oblique cables pull the globe back and to the left and thus, loading all the gimbal elements as shown in Figure 4.21. The applied forces that were used to carry out these analysis were taken to be the tension in the cables when the DC motors were activated by the maximum allowable current. Letting $F_{max}, I_{max}, K_i, \text{ and } R_{pulley}$ respectively represent, maximum cable tension [N], maximum output current [A] from the amplifier, motor torque constant [Nm/A], and motor pulley radius [m], the maximum tension in each cable is described by the following:

$$ T_{max} = I_{max} K_i / R_{pulley} = 7N $$

\textsuperscript{11} This factor of safety is calculated based on the 27.6 MPa yield strength of the aluminum alloy 1060 used
Figure 4.21: Free body diagram of the gimbal elements when the motors are activated (the reaction forces are represented by the blue arrows)
Consequently, $F_{\text{max}}\text{−back−ring}$, the maximum applied load to the back ring, is calculated by summing the maximum tension ($T_{\text{max}}$) of the four recti cables. While $F_{\text{maxouter−ring}}$, the maximum applied load to the outer-ring, is the sum of the maximum tension in the two oblique cables.

<table>
<thead>
<tr>
<th>Gimbal Element</th>
<th>Maximum Deflection [mm]</th>
<th>F.O.S</th>
<th>Deflection F.O.S</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-bracket</td>
<td>$1.1 \times 10^{-3}$</td>
<td>7</td>
<td>88</td>
</tr>
<tr>
<td>Back-ring</td>
<td>$1.9 \times 10^{-2}$</td>
<td>2</td>
<td>53</td>
</tr>
<tr>
<td>Outer-ring</td>
<td>$1.3 \times 10^{-2}$</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

**Table 4.6:** Performance of the gimbal elements under activated motors

(*Deflection F.O.S is calculated as $De_{\text{req}}/De_{\text{actual}}$, where the required deflection is set as 0.1mm*)

The performance of the gimbal elements under these loadings was once again simulated using Cosmos by evaluating the factor of safety and maximum deflection of each element. Table 4.6 summarizes these results. The maximum deflection of all three majorly loaded elements are significantly smaller than 0.1mm and the *Deflection F.O.S* in Table 4.6, described as $\frac{\text{Allowable Deflection}}{\text{Maximum Part Deflection}}$, quantifies these results. The structural strength of each part is quantified by the *F.O.S* values in Table 4.6. These values are based on the yield strength of the aluminum, alloy 1060, used in the eye robot. Once again, it is apparent that the support structure is capable of supporting the maximum possible loads on the device.

Two other components that are considerably loaded when the motors are activated are the two shafts responsible for pitch rotation. The maximum applied loads on these shafts are when the four recti cables are pulling on the globe; this
load is divided across the two steel shafts. Upon viewing the robot structure su-
periorly, Figure 4.22 illustrates the deflection of a pitch shaft under such loadings.
Here \( P \), the applied load, is half of the maximum applied loads from the recti
motors. The maximum deflection for this beam bending condition is given by

\[
\delta_{\text{max}} = \frac{P l^3}{12EI}
\]  

As it was thoroughly discussed, the design of the eye robot was greatly influ-
enced by the need to minimize the \( M I \) of the gimbal elements, considering the
mechanical limitations, and specifically to minimize this effect on yaw rotations.
Table 4.7 lists the \( M I \) for each of the rotational directions (Yaw, Pitch, roll). The
second column of this table describes the directional \( M I \), while the third column
illustrates the effect of the gimbal elements on the globe’s \( M I \), by expressing the
ratio between the actual directional \( M I \), including the gimbal elements, and the
globe’s directional \( M I \) without the gimbal elements. Consequently, it is appar-

Figure 4.22: Beam bending diagram for the pitch shafts

Equation 4.8, using which the maximum deflection of the pitch shafts are calcu-
lated to be \( 5 \times 10^{-6} \text{ mm} \), a value that is substantially smaller than 0.1mm.
ent that the main goal of the design process, namely minimizing the effect of the gimbal elements on the globe’s yaw MI, is met given the 1.07 ratio in the yaw direction. Furthermore, as expected, globe’s roll MI is also not very much influenced by the gimbal elements, while its pitch MI, the outer most gimbal rotation, is the most effected.

<table>
<thead>
<tr>
<th>Rotational Direction</th>
<th>MI With Gimbal Parts [Kgm²]</th>
<th>Actual directional MI</th>
<th>Globe’s directional MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yaw</td>
<td>$2 \times 10^{-5}$</td>
<td></td>
<td>1.07</td>
</tr>
<tr>
<td>Pitch</td>
<td>$6 \times 10^{-5}$</td>
<td></td>
<td>2.6</td>
</tr>
<tr>
<td>Roll</td>
<td>$2.5 \times 10^{-5}$</td>
<td></td>
<td>1.14</td>
</tr>
</tbody>
</table>

Table 4.7: MI of the three rotational directions of the globe, including the gimbal
Chapter 5

Mechatronic Design

The purpose of this chapter is to present the mechatronics design of the eye robot. The first section gives an overview of the controller architecture including the controlled variables and the corresponding inputs and measured outputs. Next, based on this architecture the selection process of the required sensors and actuators will be discussed. The eye robot kinematics and the control algorithm design that results from it are discussed and finally the implementation of this control algorithm is described at the end of the chapter.

5.1 Architecture of the Controller

As mentioned in Section 3.2, the motivation for the design and implementation of a control algorithm for the eye robot is to evaluate the kinematic and dynamic performance of this robotic eye and as such the algorithm is not meant to mimic the existing oculomotor control theories, this will be left for a future project. Conse-
sequently, for the intended purposes of the controller presented here, I have regarded the eye robot as a redundant tendon driven robot that is to be driven to a desired position. More specifically, controlling the globe’s 3D orientation is dependent on the ability to project this 3D quantity to the 6D space of the motor angles. The general architecture of such controller is illustrated by the block diagram in Figure 5.4, where:

- $\theta_d = \text{Desired } [3 \times 1] \text{ globe orientation}$
- $\hat{\theta}_g = \text{Measured } [3 \times 1] \text{ globe orientation}$
- $\Delta \theta_g = \text{Error between the desired globe orientation and the current orientation}$
- $J(\theta_g) = \text{Jacobian mapping}$
- $\Delta L = [6 \times 1] \text{ Change in tendon length vector}$
- $F = [6 \times 1] \text{ Force input vector to the 6 DC – motors}$

**Figure 5.1:** General block diagram for position controlling the eye robot
As described by this block diagram, the orientation of the globe of the eye robot can be controlled with a feedback controller. Using the kinematics of the eye robot structure we arrive at the Jacobian, which maps the 3D globe orientation error, calculated as the difference between the desired and the measured values, to the corresponding 6D change in the tendon lengths. Then we position control the 6 motors to the appropriate angle such that the desired tendon length ($\Delta L$) is wrapped around the motor. The output of this controller is the resulting force input to the eye structure.

5.2 Sensors and Actuators

In this section I will present the process of selecting the position sensors and DC motor actuators required to implement the controller described in Section 5.4.

5.2.1 Actuators

As mentioned previously, it was decided to implement the contraction of the 6 EOMs using DC motors. To perform as suitable actuators, the motors must meet the force and dynamic requirements of the eye robot structure. Thus, I began by considering the maximum power that the motors need to supply, which can be described by the following power equation.

$$P_{max req} = \tau_{max req} \cdot \omega_{max R} \quad (5.1)$$
Here, $\tau_{maxR}$ is the highest required torque and $\omega_{maxR}$, the maximum angular velocity that the globe would have. $\omega_{maxR}$ was scaled from the oculomotor system data reported in the literature using the dimensionless $\Pi$ groups derived in Chapter 4.

A given $\Pi$ variable stays constant across all dimensions, for instance the dimensionless variable, $\Pi_\omega = \omega \sqrt{I_K}$, would be equal for both the biological eye and the robot. Thus, the maximum angular velocity of the robot can be described, in terms of the biological value, by Equation 5.2.

$$\omega_R(\sqrt{\frac{I_B}{K_R}}) = \omega_B(\sqrt{\frac{I_B}{K_B}})$$

$$\omega_R = \omega_B \sqrt{\frac{I_B K_R}{K_B I_R}}$$

Equation 5.2

In a similar manner $K_R$ in Equation 5.2 can be found by using $\Pi_K = \frac{K}{\tau}$ and $\Pi_\tau = \frac{\tau^3}{I_B^2}$, which respectively correspond to the system’s stiffness and input torque. If we assume that the damping of the robot $B_R$ is twice the damping in the biological eye, $B_B^1$, then the input robot torque can be expressed in terms of the reported maximum $12[mNm]$ [69] active torque of the EOMs as:

$$\tau_{aR} = \tau_{aB}(\frac{I_R(2B_R)}{I_B B_B})^{1/3}$$

$$\tau_{aR} = \tau_{aB}(\frac{4 I_R}{I_B})^{1/3} = 103[mNm]$$

Equation 5.3

$^1$This is a reasonable assumption since the damping of the eye robot will be implemented in software and thus it can be tuned to a desired value.
Thus, $K_R$ is now defined by:

$$K_R = \tau_{a_B}(\frac{L_{k_R}}{L_{J_B}})^{1/3} \tau_{a_B}K_B \quad (5.4)$$

Substituting Equation 5.4 in 5.2 $\omega_R = 3.5 [rad/s]$. Table 5.1 lists all the biological values and their corresponding robot equivalent\textsuperscript{2}. The maximum required power can now be calculated:

$$P_{maxreq} = \tau_{maxreq} \cdot \omega_{maxreq} = 0.4W$$

Keeping these requirements in mind, I compared them to the the specifications of the available Maxon Amax DC motors in the lab (AMax-25, Maxon Motor AG). These permanent magnet DC motors have a nominal torque rating (i.e. Maximum allowable continuous torque given a 12V input voltage) of 0.042 [Nm] and a corresponding nominal angular velocity of 325 [rad/s]. Therefore, the nom-

\textsuperscript{2}In order to assess the worst possible scenario, in these calculations I have used the highest MI of the eye robot, which is in the pitch direction.
inal output power of these motors is:

\[ P_{\text{nominal}} = \tau_{\text{nominal}} \cdot \omega_{\text{nominal}} \]

\[ = 14W \gg P_{\text{max req}} \]

Considering that the nominal output power of the available motors were much larger than the calculated maximum required torques, I decided to use the Maxon Amax-25 PM DC motors. However, upon making this decision there are other factors that must be taken into consideration such as the effect of the motor friction. Specifically, the minimum required torque from these motors must be larger than the no load torque of the maxon motors, considering the pulley diameters available for the given motor shafts. For these motors I have selected a pulley with a groove diameter of 13mm. The minimum torque of the EOMs is the tonic torque that exist when the eye is stationary and it is reported to be 4.9[mNm] [40, 52], which scales to 42[mNm] given the eye robot dimensions. Thus, to provide this minimum torque the motor must supply,

\[ \tau_{\text{Mmin}} = \tau_R \frac{R_{\text{pulley}}}{R_{\text{globe}}} \]

\[ \tau_{\text{Mmin}} = 9 \text{ [mNm]} \]

The reported no load friction for the Maxon motor is 1.5[mNm] which is more than 8 times smaller than the minimum requirements and thus it would not create any limitations. Furthermore, although power requirements are a good initial tool for selecting the appropriate DC motor, it is not sufficient for the motor to only be
capable of producing the required power, rather it must be capable of providing the appropriate maximum torque and speed without exceeding its power ratings. Thus, considering the pulley ratios, the maximum required torque from the motor is $\tau_M = 103[mNm] \frac{13nm}{60nm} = 22[mNm]$, which is about half of the nominal torque of the Maxon Amax-32. Similarly the maximum required speed from the motor is $\omega_M = 3.5[rad] \frac{60mm}{13mm} = 16[rad/sec]$, compared to the 325[rad/sec] nominal speed of the selected motors. It is therefore apparent that the selected motors exceed the requirements by a large margin and thus they would be able to manage factors, such as system friction, that are not accounted for in the calculations presented here.

5.2.2 Sensors

As discussed in Section 5.1 the orientation of the globe will be controlled using the feedback system. Therefore, we need a position sensor to measure the yaw, pitch and roll of the eye robot considering the biological and mechanical design requirements.

Foveola, the portion of the fovea with the highest visual acuity has a 0.35 mm diameter, which translates to 1deg of visual angle. Therefore, the globe orientation should be controlled to a fraction of this value, which we have selected to be $1/10^{th}$, and this requirement should be reflected in the resolution of the position sensor. Consequently, the selected sensor must measure angles with an accuracy of $< 0.1^\circ$. Furthermore, as illustrated in the controller block diagram, illustrated in Figure 5.4, the controlled quantity is a change in the globe orientation and thus,
it is important to know where we are starting from. This means that we need an absolute measurement of the yaw, pitch and roll. A factor that played an important roll in the selection process of the angle sensor was the size and the configuration of the sensor. As briefly discussed in Chapter 4, two of the three sensors must fit on the C-bracket inside of the globe, a requirement that imposed a big restriction on the selection process. In view of the requirements discussed in the above paragraph, I have chosen to use potentiometers (RDC506, Alps Electric, Tokyo) for measuring the yaw, pitch and roll of the eye model. The primary reason for selecting these sensors is their small dimensions, Figure 5.2. A further advantage of potentiometers is that they produce absolute angle measurement. When driven at its rated voltage of 5 V, RDC506 has a linear output with a slope of 0.016 V/Deg, Figure 5.3. The potentiometers are connected to a 16-bit analog to digital convertor (AD) (PCI-6620, National Instruments, USA). Therefore, considering the 320° angular range of the potentiometer, the effective resolution is

\( \text{Figure 5.2: Potentiometers used for measuring the angular position of the globe} \)
given by \( \frac{320^\circ}{216 - 1} = 0.005^\circ \), which is 2 orders of magnitude smaller than the desired maximum angular error of \( 0.1^\circ \).

![Figure 5.3: Voltage output of the potentiometer, given the angular movement input. This plot is taken from RDC506 spec sheet.](image)

A draw back of using a potentiometer is the friction that it introduces to the system. It is important that this friction is small relative to the operating torques. RDC506 has a maximum friction torque of \( 2[mNm] \), which is much smaller than the \( 42[mNm] \) tonic eye robot torques.

5.3 Kinematics and Control

This section includes the derivation of the control algorithm discussed in Section 5.1, the block diagram of the controller is repeated here in Figure 5.4. This control algorithm requires analyzing the kinematics of the eye robot structure and implementing a position controller.
5.3.1 Kinematics

In this section I have used the kinematics of the eye robot structure to arrive at the Jacobian mapping of the controller in Figure 5.4. The presented Jacobian would map 3D displacements of the globe to the corresponding 6D changes in the tendon lengths.

The rotational kinematics of the eye can be expressed with the aid of a rotation matrix. To define this rotational kinematics, let us consider two right handed coordinate systems, a *globe* frame, denoted by \( \mathbf{g} \) in Figure 5.5, which is attached to the centre of the globe and moves with the globe, and a *world* frame, denoted by \( \mathbf{w} \) in Figure 5.5, which is fixed to the centre of the globe and stays stationary as the globe rotates. Furthermore, we define the reference or primary position of the globe as the instance in space where the head and globe coordinates coincide with each other. Then the globe-fixed coordinate frame’s transformation matrix is...
Figure 5.5: We define two right hand coordinate systems both of which have origins at the centre of the globe. The globe coordinate rotates with the globe, while the world coordinate stays fixed.

given by matrix multiplication of the yaw, pitch and roll rotation matrices given
in Equations 5.5 - 5.7. 

\[ R_{\text{yaw}}(\alpha) = \begin{vmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{vmatrix} \] (5.5)

\[ R_{\text{pitch}}(\beta) = \begin{vmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{vmatrix} \] (5.6)

\[ R_{\text{roll}}(\gamma) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos(\gamma) & -\sin(\gamma) \\ 0 & \sin(\gamma) & \cos(\gamma) \end{vmatrix} \] (5.7)

The order of this matrix multiplication is determined by the kinematics of the gimbal, which has a serial kinematic chain with intersecting joint axes. Therefore, the gimbal imposes rotations about a moving coordinate frame (known as passive rotation) with a pitch, roll, yaw rotation sequence and thus the corresponding transformation matrix for the globe-fixed coordinate is given by

\[ R_g = R_{\text{pitch}}R_{\text{roll}}R_{\text{yaw}} \]

This can be used for estimating the orientation of the globe from the gimbal joint rotation sensors.

---

\(^3\)Vector quantities are distinguished by bold font lower case variables and matrices are differentiated by bold font upper case variables.
Subsequently, in order to drive the globe to a desired orientation expressed by \( \mathbf{R}_g \), we must transform this 3D orientation to the equivalent motor angles. This can be done by relating a small displacement \( \Delta \mathbf{R}_g \) of the globe to a small displacement of the motors (\( \Delta \theta \)).

A small (right) displacement \( \Delta \mathbf{R}_g \) of the globe is most conveniently parameterized in exponential coordinates. Namely, I parametrize \( \Delta \mathbf{R}_g \) by a unit vector representing the equivalent rotation axis and the angle describing the rotation about this axis. There are a number of methods for computing the exponential coordinate of a rotation matrix [11]. Here I have used the matrix-to-quaternion and quaternion-to-exponential formula to convert \( \mathbf{R}_g \) to its exponential form. A quaternion is defined to be \((q_o, \mathbf{q})\), where \( q_o \) is a scaler and \( \mathbf{q} \) is a vector. Thus, I have formulated a conversion from SO(3) to the angle-axis form as follows:

**Matrix to Quaternion**

\[
\begin{align*}
\mathbf{d} &= \mathbf{R}_g - \mathbf{R}_g^T, \\
\mathbf{r} &= \begin{pmatrix}
-d(2,3) \\
d(1,3) \\
-d(1,2)
\end{pmatrix} \\
\theta_{gq} &= \tan^{-1}\left(\frac{\|\mathbf{r}\|}{\text{trace}(\mathbf{R}_g) - 1}\right) \\
\mathbf{q}_{g} &= \left[\begin{array}{c}
\cos\left(\frac{\theta_{gq}}{2}\right) \\
\frac{\mathbf{r}}{\|\mathbf{r}\|}\sin\left(\frac{\theta_{gq}}{2}\right)
\end{array}\right]
\end{align*}
\]

\(4\)The \( g \) subscripts refer to the globe quantitates.
**Quaternion to Exponential**

\[ \phi = 2 \cos^{-1}(q_{og}) \]

\[ v = \begin{cases} 
\frac{q_{gn}}{\sin(\phi/2)} & \text{if } \Phi \neq 0 \\
0 & \text{if } \Phi = 0 
\end{cases} \]  \hspace{1cm} (5.9)

This parameterization can then be described by:

\[ \Delta R_g = \exp[\theta_g] \]

where the direction of the vector \( \theta_g \) describes the direction of the unit vector \( v \) and its magnitude is the angle of rotation \( \theta_g \). The bracket ([ ]) notation is used to represent the matrix of the cross-product \( \theta_g \times \).

For control we need to relate the globe’s displacement \( (\theta_g) \) and torque \( (\tau_g) \) to the cable motors’ displacements \( (\theta) \) and torque \( (\tau) \). If we assume, as in [8], that a cable \( i \) takes the shortest path between its insertion point \( p_i \) and the anchor or pulley point \( a_i \), then

\[ N_i = \frac{R_g p_i \times a_i}{\| R_g p_i \times a_i \|} \]  \hspace{1cm} (5.10)

Here \( N_i \) is the vector normal to the plane of the cable at the insertion, expressed in world coordinates. Therefore a tension \( f_i \) in the cable, wrapped on a globe of radius \( r_g \), will contribute a torque \( \tau_{g,i} = r_g f_i N_i \) on the globe. The tension is
produced by the DC motor, wrapping around a motor pulley of radius $r_m$. Thus

$$\tau_{g,i} = r_g \frac{\tau_m}{r_m} N_i.$$  \hfill (5.11)

From this we can extract and assemble the columns of the Jacobian $J$, which is defined by the relation $\tau_g = J^T \tau$.

$$J^T = r_g \left( \begin{array}{cccc} \frac{1}{r_1} & \cdots & \frac{1}{r_6} \end{array} \right).$$  \hfill (5.12)

Then principle of virtual work can be used to relate $\Delta \theta_g$ to $\Delta \theta$. Namely, the work done by the motors must be equivalent to the work done by the globe. Therefore,

$$\tau_g \cdot \Delta \theta_g = \tau \cdot \Delta \theta$$

This inner product can also be expressed by

$$\begin{array}{c} \tau_g^T \Delta \theta_g = \tau^T \Delta \theta \\ \tau^T J \end{array}$$

Therefore the displacements are now related by

$$\Delta \theta = J \Delta \theta_g$$
5.3.2 Controller

The block diagram in Figure 5.6 expands on the more general controller architecture discussed earlier in the chapter. Here the "Position Controller" block is expanded to illustrate the detail of this part of the controller. In this section the final control architecture used in the eye robot will be described.

Figure 5.6: Block diagram for the position controller used to drive the eye robot. \( \hat{\theta}_g \) represents the current globe angle measured by the potentiometers.

The \( J(\hat{\theta}_g) \) block in Figure 5.6 uses the kinematics mapping developed in the previous section to relate globe orientation to motor angles. \( J(\hat{\theta}) \) is updated every time step based on the measured globe angles. For a desired globe orientation, \( \theta_d, J(\hat{\theta}_g) \) maps the globe position error, \( \Delta \theta_g \), to tendon displacements, \( \Delta L \), which are scaled by the motor pulley diameters to correspond to motor angle commands,
$\Delta \theta$. This relationship is described by the following equations

$$\Delta \theta = r_s \left( \frac{1}{r_1} N_1 \cdots \frac{1}{r_6} N_6 \right)^T \Delta \theta_s$$

Finally, given the motor angle commands, the motors are current controlled to the desired displacements using a PID controller.

The eye robot is a 2N configuration tendon driven system, using the classification of tendon driven robots in the literature [24]. In a 2N configuration, one DOF of the system is controlled by 2 actuators pulling on antagonist tendons. Given the structure of this antagonist system, the torque from each motor can be transformed to the joint only by pulling (i.e., you can not push on a cable). A control challenge that emerges from this characteristic of a 2N tendon driven robot is ensuring that tendons are tensioned under both passive (antagonist) and active (agonist) conditions. Therefore, we have implemented a simple logic similar to [24] to ensure that the cables are tensioned at all times. This is represented by the "Tension Logic" block in Figure 5.6. The block takes the motor voltage command, $V_m$, and outputs the rectified motor voltage command, $V_{mrec}$ using the following logic:

\[
\begin{align*}
& \text{if } V_m > 0 \\
& \text{output} = V_m \\
& \text{else} \\
& \text{output} = 1
\end{align*}
\]
5.4 Implementation

I programmed and designed the control algorithm using Simulink, a visual programming language developed by MathWorks. Simulink is an interactive environment whose “customizable block libraries” greatly simplifies the design of the control system. Furthermore, the “Real-Time Workshop” in Simulink provides a convenient procedure for testing and implementing the designed controller on the eye robot. The real time workshop consists of the Host Computer, the Target Computer, and the plant which includes the sensors and the actuators as well as the mechanical structure, Figure 5.7 maps the relationship between these three components for the setup used to control the eye robot.

![Diagram of control system](image)

**Figure 5.7:** Communication map of the controlling computer and the eye robot

The host computer runs Simulink, Simulink Coder, and a C compiler and communicates with the target via an IP communications link. Through this connection
the Simulink model code, generated by Simulink Coder, is downloaded onto the target computer, which ultimately sends motor commands and receives sensor data to and from the eye robot through the digital to analog convertor (DA) and AD boards. The sensor data saved on the target computer, can either be viewed by the target or acquired and saved by the host to further examine the performance of the controller.

The PID controller outputs a current command corresponding to desired motor torque. Thus, we require a motor control amplifier capable of current control mode. Furthermore, position control of a motor requires the motor control amplifier to be capable of controlling acceleration and deceleration in both directions. Finally, input and output of the amplifier must have a linear relationship to eliminate the effect of the motor amplifier on the dynamics of the eye robot. To satisfy the above requirements, I have chosen a Maxon servomotor control amplifier (LSC 30/2, Maxon Motor AG). LSC 30/2 is a linear 4 quadrature servo controller capable of accelerating and decelerating Amax 32 in both directions. This servo amplifier has a number of operational modes, including a current control mode. In the current control mode LSC 30/2 takes as input voltage commands (between 0 – 10[V]) from the PID controller and outputs current commands (between 0 – 2A) proportional to the commanded voltage, thus keeping the motor current at the value predetermined by the PID controller.
Chapter 6

Assessment

In this chapter, the mechanical and mechatronics capabilities of the eye robot and its ability to function as a testbed for scientific research is assessed. First, I evaluate the tracking accuracy of the eye robot for the principal directions of rotation and 2D trajectories. I then verify that the robot’s MIs are comparable with the scaled inertia of the real eye. Finally, I assess the capability of the eye robot to achieve mechanical responses similar to the real eye. In particular, the rise time and peak velocity of the robot’s response to commanded steps, which resemble saccadic eye movements, are compared to saccade recordings from human subjects.

Tracking Accuracy

As explained in section 3.2, the aim is for the eye robot to have a fast, overdamped response and to reach the target position to within $\frac{1}{10}^{th}$ of a degree. An
overdamped response is characterized by the percent overshoot (P. OS) \(^1\) of the step response. Specifically, a completely overdamped response would have zero P. OS.

I use step inputs in yaw, pitch and roll to assess the quality of tracking in the principal directions of rotation. Figures 6.1 - 6.3 illustrate the position and velocity profile of the eye robot’s response to step inputs in the principal directions. The dashed line represents the target position while the solid line illustrates the eye robot’s response to these inputs.

![Figure 6.1: System’s response to 5°, 15°, and 24° inputs in yaw](image)

Similar step inputs were repeated 50 times in all three rotational directions and the resulting mean steady state error (\(E_{ss}\)), and \(P. OS\) are presented in Table 6.1. The \(E_{ss}\) in all directions are on the orders of \(\frac{1}{100}\) of degrees, demonstrating that the eye robot is in fact capable of achieving the desired tracking accuracy.

\[^1P. OS = 100 \times \frac{\text{Overshoot}}{\text{Steady state value}}\]
Figure 6.2: System’s response to $5^\circ$, $15^\circ$, and $20^\circ$ inputs in pitch

Figure 6.3: System’s response to $5^\circ$, $10^\circ$, and $17^\circ$ inputs in roll
in the three principal directions of rotation. Furthermore, examining the step responses reveals that for the most part they are close to overdamped with very small overshoots. These small overshoots are attributed to the tradeoffs involved in designing the controller. Achieving a fast overdamped response using a PID controller introduces two potentially opposing goals: short rise-time and low overshoot, which enforces optimizing the PID parameter choices to achieve the best tradeoff between these requirements. Namely, an overdamped response requires decreasing the proportional gain and increasing the derivative gain, both of which increase the rise-time of the response. This effect can therefore be minimized if we allowed for a slower rise-time. However, the most important characteristic of a saccadic movement is the speed of the saccade and, in fact, the saccade responses can be accompanied by small overshoots. As such, here I have chosen achieving a small rise-time as the primary goal and have accepted small overshoots.

Table 6.1 lists the statistics of these values for 50 step inputs with the step magnitudes varying between $5^\circ - 30^\circ$, in the three directions.

**Table 6.1:** Statistics for steady state error ($E_{ss}$) and percent overshoot ($P.OS$) for the eye robot’s step input response based on 50 data points

<table>
<thead>
<tr>
<th>Rotational Direction</th>
<th>$E_{ss}$ [Deg]</th>
<th>POS [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yaw</td>
<td>0.03 ± 0.04</td>
<td>1 ± 0.25</td>
</tr>
<tr>
<td>Pitch</td>
<td>0.06 ± 0.07</td>
<td>2.2 ± 0.95</td>
</tr>
<tr>
<td>Roll</td>
<td>0.05 ± 0.06</td>
<td>1.6 ± 0.50</td>
</tr>
</tbody>
</table>

To evaluate the system’s ability in tracking 2D trajectories I commanded rotations about axes on the coronal plane$^2$ at a rate of 50 $Deg/s$. The rotations mapped

$^2$The vertical plane that divides the body into ventral and dorsal planes
to a linear and a circular path on the field of view. Figures 6.4 and 6.5 illustrate responses of the eye robot to these commanded trajectories. Once again, the dotted line is the commanded path and the solid line represents the system’s response. The trajectories were each repeated 10 times and the resulting maximum errors are reported in Table 6.2.

**Table 6.2:** Maximum error for the circular and linear trajectories

<table>
<thead>
<tr>
<th>Trajectory path</th>
<th>Max Error [Deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>$2 \pm 0.5$</td>
</tr>
<tr>
<td>Circular</td>
<td>$3 \pm 0.1$</td>
</tr>
</tbody>
</table>
Figure 6.5: System’s response to a circular path on the field of view

These results indicate that the tracking accuracy of the eye robot is decreased considerably, for commanded 2D trajectories. Such large tracking errors are likely due to the static friction of the motors. This is apparent when considering the circular trajectory, because the maximum deviation of the robot from its commanded path occurs at the 4 corners of the circle, where the velocity becomes zero instantaneously and thus, the motors must stop and start moving. Furthermore, when considering a linear trajectory the error must decrease over time. Figure 6.6 illustrates that the error profile of a given linear path increases until it reaches a maximum before starting to decrease and remaining at a constant value for the rest of the path.
The assumption that motors static friction is responsible for the large tracking errors found can be validated further by comparing the measured maximum errors and the minimum required error that would put the motors in motion, considering their static friction. The reported static friction of the Maxon Amx motors is quantified by a no-load current of 58 mA. Thus, under no-load conditions, a minimum of 58 mA is required to rotate the motor shaft. Furthermore, these motors are being controlled through amplifiers that provide a linear mapping between commanded voltages (from the PID output), varying between 0 V – 10 V, and motor current inputs, varying between 0 A – 1.5 A. This means that a minimum of

$$\frac{0.058 \, A}{\frac{1.5 \, A}{10 \, V}} = 0.4 \, V$$
is required to rotate the motor shaft. Considering that the PID controller has a proportional gain of 8, this voltage value corresponds to an angle input of,

\[
\frac{0.4 \, V}{8} = 0.05 \, \text{rad} = 2.9 \, \text{Deg}
\]

Consequently, a minimum of about 3 Deg error is required before the motors are able to rotate. This value is comparable with the maximum reported errors in Table 6.2. Consequently, in order for the eye robot to be able to perform trajectories with the acceptable tracking accuracy of 0.1 Deg (0.002 rad) the motors must be replaced with ones with a lower static friction value or we should explicitly compensate for the static friction in the control algorithm. One such algorithm has been developed by Astma [4].

Substitution of the motors would be further justified if it is found that the motor static torques are in fact the dominating source of friction in the system. To examine this possibility I have calculated and compared the motor stiction to the friction forces, and their subsequent effect on the required globe torque, that are present in the following system components:

- Dyneema tendons and globe surface
- Dyneema tendons routing through plastic tubes
- Bearing and bushing frictions

The Maxon motors have a reported static friction torque of 1.5 mNm. This is
converted to its equivalent torque on the globe, $\tau_{mg}$, by the following equation.

$$\tau_{mg} = 1.5 \text{ mNm} \frac{R_g}{R_m} = 7 \text{ mNm}$$

Here $R_g$ and $R_m$ are respectively the globe and motor pulley radius. I have quantified the effect of the friction between the cables and the globe surface by calculating the maximum torque that the dyneema cable could apply to the globe before it slips. This is given by the product of the normal force exerted on to the globe due to the tension in the cable, and the coefficient of static friction of dyneema. During static conditions the cables are tensioned by the motors with 0.84 N. The normal force due to this tension can be calculated by examining the FBD of the cable, illustrated in Figure 6.7. Considering the force balance of this FBD, $N$, can

![Figure 6.7: FBD of a tensioned cable with a wrap angle $\Phi$](image)

be expressed as a function of the cable tension, $F$, and the wrap angle of the cable, $\Phi$ by:

$$N = 2F \sin(\Phi/2)$$

The maximum possible wrap angle is $60^\circ$, when the globe is fully rotated to
the right or left, and Dyneema’s coefficient of static friction, \( \mu_d \), ranges between 0.08 – 0.12. Thus, the maximum torque, \( \tau_{gc} \), that is applied to the globe due to the cable-globe friction is:

\[
\tau_{gc} = 2\mu_d F \sin(\phi/2) R_g = 3 \text{ mNm}
\]

Similarly, routing of the cables through nylon tubes introduces a friction torque, \( \tau_{cr} \), on the globe that can be described as a function of the cable entering angle, \( \theta_i \), and exiting angle, \( \theta_o \), as illustrated in Figure 6.8. In this figure \( \Delta F \) represents the force dissipated due to friction which is approximated by,

\[
\Delta F = F(e^{\mu(\theta_i+\theta_o)} - 1) = 0.072 \text{ N}
\]

Therefore, the torque on the globe due to this friction force is, 2 mNm. Finally, the bearing friction torque, \( \tau_{bb} \) is calculated to be

\[
\tau_{bb} = F_r \mu_{bb} r_{ib} = 0.01 \text{ mNm}
\]
Here, $F_r$ is the maximum radial force, distributed between the two pitch ball bearings, and it is applied by the tension in the four recti cables as they pull the globe posteriorly. $\mu_{bb}$ is the coefficient of static friction, which is reported as 0.003 for the single row ball bearing used here, and finally $r_{ib}$ is the bearing’s bore radius. The friction in the bushings of the yaw joints are maximum when the globe is being pulled back by the 4 recti muscles, with a force $F$. Therefore, considering that the coefficient of friction, $\mu_p$, of the bushing polymer (G300), ranges between 0.1 and 0.15, the maximum friction torque, $\tau_b$, on the globe due to the polymer

![Comparison of Friction Torques in the System](image)

**Figure 6.9:** Bar graph comparing the friction torques in the system. $T_{mg}$ is the motor static friction torque, $T_{gc}$ is the dyneema cable and globe surface friction torque, $T_{rc}$ is the friction torque due to the routing of the cables, $T_b$ is the bushing friction torque, and $T_{bb}$ is the ball bearing friction torque.
bushings is calculated to be

$$\tau_b = F \mu p r_s = 1 \text{ mNm}$$

where $r_s$ is the radius of the shaft. These results are illustrated by the bar graph in Figure 6.9. From this graph it is easy to realize that the motor stiction force is indeed the dominant source of friction in the system, by more than 2 times, and thus reducing this friction source would have a significant effect on the performance on the robot.

**Assessment of MI**

Another significant factor, for assessing the capability of this eye system to emulate biologically plausible motions, is the MI of the system. It has been shown that the inertial forces in the biological eye are negligible in comparison to the existing passive forces [52]. Thus, a primary design goal is to maintain this relationship amongst the model’s forces and many of the design decisions are driven by the requirement of reducing the MI especially in yaw, which is the most frequent eye movement direction. To determine whether the model MIs are sufficiently small, we need to compare the model’s MIs with the scaled MI of the real eye. The design requirement is met if the model’s MIs, in all three principal directions, are smaller than or in the same order magnitude of the scaled MI.

An exact scaling of the real eye’s MI requires detailed geometric and material properties, which are difficult to find. Nonetheless, we could estimate the MI scale
factor to the right order of magnitude, by viewing the real eye and the model as spheres. In doing so, the moment of inertia of the model and the real eye can be expressed by,

\[ I \propto M r^2 \Rightarrow I \propto (\rho r^3)r^2 \]

\[ I \propto r^5 \]

Then the scale factor between the MI of the real eye, \( I_E \), and the model eye, \( I_M \), is defined by the ratio of their radii as follows,

\[ \frac{I_M}{I_E} = \left( \frac{R_M}{R_E} \right)^5 \]

Therefore, considering that the eye robot’s globe diameter is 2.5 times the real eye, the \( 4E - 7 \ Kgm^2 \) inertia of the real eye is scaled, by a factor of about 100, to \( 4E - 5 \ Kgm^2 \).

Figure 6.10 compares the model’s inertias, in its principal directions \( (I_y, I_p, I_r) \), with this scaled MI, \( I_s \). It is evident that the model’s MI in all 3 directions are within the same order of magnitude as the scaled MI. These results imply that the anisotropy of the MIs of yaw, roll and pitch, imposed by the gimbal structure, will not have a significant impact on the dynamic response of the system since all inertial forces are negligible. MI, \( I_s \).

The MIs used in Figure 6.10 only include the gimbal and the globe and ignore the motor rotor’s inertia. Thus, for a more accurate comparison with the scaled biological MI, which includes the inertia of the EOMs, we must include inertia of the motors. Considering that the motors rotate about their own axis and are connected
Comparing the Scaled MI with the Model MIs

Figure 6.10: Comparison of model’s MI and the scaled MI of the real eye to the globe through the dyneema cables, their effective inertia at the centre of the globe can be expressed by Equation 6.1. Here, \( I_{re} \) is the effective motor inertia at the globe, \( I_r \) is the actual motor inertia and \( R_g \) and \( R_p \) are respectively the globe and the motor pulley radii.

\[
I_{re} = I_r \left( \frac{R_g}{R_p} \right)^2
\]  

(6.1)

Therefore, the \( 4 \times 10^{-6} \ Kgm^2 \) inertia of the motor rotor\(^3 \) is scaled to its equivalent \( 8 \times 10^{-5} \ Kgm^2 \) inertia at the globe centre.

The results of including the motor inertias are presented by the bar graph in Figure 6.11. The addition of the motor inertias imposes a 1 order of magnitude difference between the scaled biological inertia and the robot’s inertias. Nevertheless, Robinson [52] has shown that inertial forces in the biological eye remain negligible even when they are increased by 2 orders of magnitude. Thus, the 1 or-

\(^3\)Value taken from the motor data sheet
der of magnitude difference between the MI of the robot and the scaled biological value is still sufficient for considering the inertial forces as negligible compared to the passive and active forces. Furthermore, upon examining Equation 6.1, it

![Comparing the Scaled MI with the Model MIs](image)

**Figure 6.11:** Comparison of model’s MI and the scaled MI of the real eye, including the motor’s inertias

is apparent that the effective MIs of the motors can be considerably reduced by increasing the radius of the motor pulleys. However, this must be done while accounting for the maximum torque required from the motors and ensuring that changing the pulley radius will not compromise this requirement.

### Speed Evaluation

Being able to achieve speeds comparable to the real eye is another important quality that the eye robot must possess if it is to function as a testbed for implementing real eye movements; this is particularly important for saccades, the fastest eye
movement. When considering a saccade, we can quantify the speed of the response with its rise time\(^4\) and peak velocity. The eye robot must be capable of reaching peak velocities achieved by real saccadic movements. However, these peak velocities must be appropriately scaled using \(\Pi \omega = \omega \sqrt{\frac{I}{K}}\), the angular speed \(\Pi\) group developed in Section 4.1. Therefore, given a biological peak velocity, \(\omega_{\text{max}B}\), the equivalent robot peak velocity, \(\omega_{\text{max}R}\) is defined by:

\[
\omega_{\text{max}R} = \omega_{\text{max}B} \sqrt{\frac{K_{RI_B}}{K_{BI_R}}}
\]

which scales down the model’s peak velocity, as a function of the ratio of the biological inertia to the robot’s inertia.

In Figure 6.12, responses of the eye robot to 5 Deg and 15 Deg step commands in yaw and pitch are compared with 5 Deg and 15 Deg human saccade recordings in yaw and pitch.

Table 6.3: Rise times of robot’s and biological saccade responses

<table>
<thead>
<tr>
<th></th>
<th>Yaw5 Deg</th>
<th>Yaw15 Deg</th>
<th>Pitch5 Deg</th>
<th>Pitch15 Deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Eye [ms]</td>
<td>13</td>
<td>23</td>
<td>18</td>
<td>23</td>
</tr>
<tr>
<td>Robot Eye [ms]</td>
<td>23</td>
<td>38</td>
<td>31</td>
<td>35</td>
</tr>
</tbody>
</table>

The biological data, shown as the dashed line, are recorded using a 2D eye tracking system, which uses an infrared camera to track the pupil. The robot data plotted here, are not scaled by the \(\Pi\) variables; nonetheless, the robot reaches peak velocities as high as the biological values, indicating that it is certainly ca-

\(^4\)Here rise time is defined as the time required to reach 63% of the steady state value.
pable of achieving comparable scaled speeds. Furthermore, the rise times of these responses are reported in Table 6.3. Once again, despite the fact that the robot values have not been scaled appropriately, they are very close to the saccadic rise times. Thus, considering speed requirements, the presented eye robot provides a suitable testbed for examining oculomotor behaviors.

Although the eye robot is capable of reaching biologically comparable peak velocities, its velocity profile starts to increase at a much lower rate in comparison with the biological response. This steep slope in the real eye’s velocity profile, suggests that the biological controller turns on to a maximum rate before cutting out once the target position is reached. This is a known characteristic of the saccadic Pulse-Step controller. Another indication of existence of such saccadic controller is the system’s response to a force step input. When a step force is applied to the biological globe, it rotates with a time constant of 250 ms [52]. However, as illustrated in Figure 6.12, typical saccades have markedly shorter durations. This indicates that the eye is not normally driven with a step input force, rather it is driven to its desired orientation, with a pulse of maximum torque and a step input is only applied to keep the globe at this desired orientation. Consequently, it appears that the motor size is sufficient to allow the eye robot to achieve peak velocities comparable to the real eye, and that the limitations in achieving comparable biological tracking rates are limitations of the current controller rather than the intrinsic capabilities of the hardware.
Figure 6.12: Comparison of the eye robot responses, to step inputs, with human saccade recordings
Chapter 7

Conclusion

7.1 Progress Assessment

The objective of this research is to create a novel physical model of the human eye to be used as a platform for testing theories regarding oculomotor control as well as providing insight on how to control tendon driven robots.

The design of this model was divided into two parts: the structural (Chapter 4) and the mechatronics (Chapter 5) design. In Chapter 3, it was shown that to create an eye model that mimics the geometric and anatomic properties critical in the static and dynamic performance of the human eye, we must have an appropriately scaled three dimensional model of the biological eye that represents the agonist/antagonist EOMs accurately.

Subsequently, Chapter 4 provided solutions for incorporating these requirements in the design of the eye robot. Using Π dimensional analysis, the ocu-
lar plant was expressed as a function of four non-dimensional variables called \( \Pi \) groups. These groups were used to scale biological quantities such as torque, stiffness, velocity and acceleration to appropriate values for the scaled up eye model. The activation of the EOMs are represented by 6 DC motors, which drive the globe through cables that represent the ocular tendons. Furthermore, the insertion and anchoring locations of the tendons are chosen to ensure that the cables are routed in the correct angular position. Finally, a custom made gimbal structure provides a low friction support, which allows the eye robot’s globe to rotate in three directions. Considering that yaw is the most frequent eye rotation direction, it is important to ensure that the velocity of the globe is maximized in this direction, which means reducing the globe’s inertia in yaw. Furthermore, given that a gimbal forms a serial kinematic chain, distal axes of rotations have smaller inertia than the proximal ones. Therefore, an important aspect of gimbal design is to implement a sequence of rotation where yaw would be the the inner most gimbal rotation. The presented gimbal implements pitch, followed by roll, with yaw as the final rotation and the unique design and placement of the gimbal elements is key to achieving this sequence.

In Chapter 5, a control algorithm was designed to evaluate the kinematic and dynamic performance of the eye robot. To develop the presented controller, the eye robot was regarded as a redundant tendon driven robot, to be driven to a desired position. The globe’s 3D orientation was subsequently controlled by projecting this 3D quantity to the 6D space of the tendon lengths using the jacobian of the eye system, which was developed based on the system’s kinematics.
Finally, in Chapter 6, we assessed the acceptability of the eye system for emulating oculomotor behaviors by verifying 3 main characteristics of the eye robot. Specifically, it was shown that the eye robot is capable of achieving satisfactory tracking accuracy in the 3 principle directions of rotation. However, due to high motor static frictions, the model does not have the same performance for 2D trajectories. By comparing the robot’s responses, to commanded step inputs, with human saccade responses, it is evident that the robot is capable of reaching biologically comparable speeds. Furthermore, I demonstrated that the MI$s of the robot are sufficiently small in comparison with the scaled biological inertia and thus, similar to the oculomotor system, the inertial forces in our eye model are negligible.

7.2 Research Implications

The main contribution of this thesis is a novel 3 dimensional tendon driven model of the human eye and a corresponding controller for driving this redundant system. This study has focused on designing a geometrically accurate model, which can be used for several purposes. Most notably, it incorporates the aspects of the anatomy that are functionally crucial for using this model as testbed for implementing oculomotor control theories. This physical model can also be used in conjunction with simulated models of the eye to understand diseases or injuries affecting vision and find treatments for these conditions. In addition, using this testbed to explore biological visual systems could give insight in how a biologically inspired vision system could be effective in real world applications.
7.3 Future Work

A major limitation of the current design is the frictional forces that are present in the system. Most notable are the friction forces introduced between the cables and the globe’s surface due to the rough finish on the globe. This drawback may be resolved by applying a smoother globe finish, which is left as future work for following versions of the model.

Another significant source of friction is the static friction in the motors. These frictional losses are attributable to the rotational bearings as well as the graphite commutator; their overall value is determined by the input current required to run the motor under no-load conditions. For the Maxon motor being used here, this value is reported as 58 mA or 1.4 mNm. This could potentially lead to complications when implementing a biological controller, especially when slow tracking motions are involved. A brushless DC motor could be used to reduce the frictional losses.

Furthermore, as discussed previously, there are two major steps for transforming theories regarding various computational neuroscientific models into facts:

1. Building a geometrically accurate physical model

2. Developing a biologically inspired controller

The main focus of this research is the creation of the physical model and thus the development of a biologically inspired controller is left as future work.
Bibliography


114


Appendix A

Design Iteration

A.1 Considered Solutions

The initial approach for the design of an optimum globe support structure was to rest the globe on ball terminated set screw (BTSC) A.1. Eight of these supporting set screws are threaded into four arc shaped holders which support the globe along two of its diameters making a $45^\circ$ angle with the vertical line through the centre of the globe. Figure A.2 illustrates a prototype that was built based on this concept by a previous colleague [43].

Initially this appeared as an appealing solution primarily because it was a reasonably simple design and it presented the accessible and open concept structure necessary for facilitating experimentation and modifications. Nevertheless, despite this open structure, the clearance of the cables proved to be a difficult problem to solve. Specifically, it was very challenging to find an optimal location for
the BTSCs, as to avoid interference with the IPs throughout the entire range of motion. Moreover, the ball bearing arrangement had an undesirable high friction, which would add uncertainties to the system and complicate the control process.

To improve on the ball bearing support structure we began exploring alternative conceptual designs inspired by the biological orbital support. In an attempt to
mimic the ligament connections of the ocular plant the elastic stalk support design was considered. Essentially as illustrated in A.3, the globe is grounded through spring elements that are fixed to the ground at one end and to the globe at the other end. In addition to having a close resemblance to the orbital support, the elastic stalks also serve as built in passive stiffness. Therefore by carefully selecting the properties of the the spring elements we could have a physical representation of the ocular passive elements and reduce the complexity of the control algorithm.

However despite their advantages, the elastic stalks are not sufficient to keep the globe from translating. This is because the design is only a partial representation of the biological orbital support and consequently it lacks certain characteristic elements that account for this translating problem in the biological version. Specifically, the elastic stalk design lacks the equivalent of the periorbital fat and tenon’s capsule which are responsible for limiting the globe’s motion to a purely rotational one. Furthermore, this design shares a similar drawback with the ball bearing structure. Namely once again an optimal location of the elastic stalk attachments is challenging to find without creating interference with the cables. Given this common difficulty it was concluded that an effective design must minimize its contact points with the surface of the globe.

Considering the limitations of the mentioned design concepts, we ultimately decided on supporting the globe in a gimbal system. A gimbal is any support structure that can pivot about an axis, but usually it is defined as a set of three pivoting supports whose axis of rotations are orthogonal to one another. As illustrated in Figure A.4, the gimbal parts are linked to each other and so as one part
rotates its rotation is passed on to the other two parts. Consequently any object, such as the globe, that is fastened to the most inner rotating element could be oriented in any direction by rotating the gimbal parts. Additionally, this orientation is characterized by a moving reference frame whose order of hierarchy is determined by the linking sequence of the gimbal parts. For instance suppose that the globe is mounted onto the shaft of the gimbal in Figure A.4, then its location is described by first a vertical rotation, followed by a horizontal rotation and finally a torsional rotation about the line of sight. Nevertheless this sequence of rotation is not explicit and may be chosen according to the preferred order of eye movement.

Although the design of the gimbal structure is much more involved, it has a number of advantages compared to the ball terminated set screw as well as the
Figure A.4: Conceptual gimbal support structure [22]

elastic stalk design. The gimbal design could be optimized to minimize the contact points of the supporting structure with the globe’s surface. This would considerably reduce the complications regarding cables interference. Moreover, given that each gimbal part rotates about a single axis, it becomes much more trivial to find a low friction solution for the motion of each part (example, bearings), ultimately leading to a low friction support structure. Finally using a gimbal system enables the potential of measuring the three individual rotations that make up the final orientation of the globe. This data would be valuable for developing a control algorithm.

A.2 Gimbal Design

The following sections explain the design iterations that were carried out in order to arrive at a suitable shape for the gimbal support structure.
A. General Gimbal Shape

It is extremely challenging to use a theoretical approach for finding a suitable gimbal shape that would provide cable clearance throughout the globe’s entire range of motion. Consequently, the general shape of the gimbal was chosen using an iterative design process, where physical prototypes were built based on conceptual designs. These elementary prototypes had the unique purpose of evaluating the shape and kinematic characteristics of a given gimbal design. Therefore, they were not concerned with perfecting the details of the mechanical design, including the final size of the globe being used.

Most of the initial prototypes were built from readily available and easily manufacturable materials. For instance, various shapes of the gimbal elements were cut from plywood, while styrofoam and plastic spheres were used to represent the globe. Screws fastened on one end, and rotating freely in a through hole at the other end, represented rotational joints. Furthermore, in order to get an accurate understanding of whether a given gimbal shape was suitable for routing the tendons, the cable’s insertion points were scaled based on the diameter of the globe being used and tendons were routed to represent the same configuration as the biological tendons.

Figure A.5 shows the prototype of the first gimbal design. This gimbal’s sequence of rotation is given by pitch, yaw and roll. Much of the design decisions of this initial prototype were motivated by the aim of creating a system that minimized the gimbal attachments to the globe while ensuring tendon clearance. Consequently, the globe cantilevers off of the innermost element of the gimbal,
therefore reducing the globe-gimbal attachments to 1 point. This point is located at the equivalent location of the optic nerve in the real eye where there are no tendons present and the irregular shape of the second gimbal element and the relatively large ring, provide an excellent platform for routing the cables without any interference with the gimbal parts. However, these beneficially large and irregular shaped elements lead to an unacceptable increase in the MI of the gimbal elements. Therefore, to account for this significant drawback the gimbal parts must be reduced in size and weight while still ensuring cables clearance. Most importantly, the gimbal in Figure A.5 does not have yaw as its inner most rotation. This was a challenging requirement that took a few design iterations to achieve.

**Figure A.5:** Prototype1: the first design for the gimbal’s general shape

The second iteration of the design, illustrated in Figure A.6, focused on solving the simpler drawback of prototype1, reducing the MI of the gimbal parts while
keeping the routing path of the cables clear. Consequently, this second iteration has the same sequence of rotation as prototype1 but with much smaller MI. This is achieved through reducing the mass and moment arm of the gimbal elements by having lightweight circular shaped elements and bringing them closer to the outer circumference of the sphere. The semi and 3/4 circular shapes of this gimbal’s elements were chosen to reduce weight while ensuring a clear routing path for the oblique cables. In fact, although prototype2 is about half the size of prototype1 ($D_{max1} = 150 \text{ mm} \ vs. \ D_{max2} = 70 \text{ mm}$), it provides an interference free path for routing all of the cables.

![Prototype2: the second design for the gimbal’s general shape](image)

**Figure A.6:** Prototype2: the second design for the gimbal’s general shape

Despite the improvements of prototype2, a third iteration of the gimbal’s general shape design had to be carried out in order to set yaw as the inner most rotation of the gimbal support structure. As a result, prototype3 was entirely driven by this
inclination, while keeping the previously determined requirements in mind. Figure A.7 shows the wooden version of prototype3. This gimbal implements a pitch, roll, yaw rotation sequence, which is achieved by the unique shape and placement of the gimbal elements. Most notably is the placement of the C-bracket, as labeled in A.8, that bears the rotational joints of yaw and roll. The globe is custom designed to include the yaw shafts, Figure A.8. These shafts are fastened to the globe and rotate freely in the yaw joints located on the top and bottom part of the C-bracket. Therefore, during yaw rotations the C-bracket stays still and the globe must have a posterior slot (labeled “trace slot” in A.8) to accommodate the C-bracket’s extension through it’s back. As in the previous prototypes, roll is handled by a single joint, located at the centre of the C-bracket. The shaft for this rotation is fastened to the back plate of the outer ring. Therefore, during roll
the globe’s $MI$ is increased by that of the C-bracket that moves with it. Finally, pitch in this gimbal is achieved by the rotation of the two shafts, fixed along the centre line of the outer ring, through the joints in the support stands, Figure A.8. Given this arrangement, the globe has the largest added $MI$ during pitch since all the gimbal elements (the outer ring, its back plate and the C-bracket) are moving with the globe. The added inertias to roll and pitch are minimized by placing the C-bracket inside the globe, which reduces the moment arm of this element immensely. Furthermore, we have brought the outer ring as close as possible to the surface of the globe, which once again reduces the added inertia by reducing the moment arm. In addition, the gimbal element’s mass were optimized by careful material selection and shape design to further reduce the added $MI$. 

**Figure A.8:** The mechanism used for the lateral joint in Prototype3
B. Rapid Prototype Model of Prototype3

The general gimbal shape achieved by prototype3 satisfies the major design requirements set originally, namely:

- Placing the most frequent eye movement, yaw, as the inner most rotation
- Minimizing the gimbal element’s $M_1$ by reducing the mass and moment arm
- Decreasing the gimbal/globe contact to a single point
- Providing a clear routing path for all the cables

Consequently, to further investigate the strengths and weaknesses of this design a more precise version of prototype3 was built. The main motivation for building this model was to implement and examine the detailed mechanical design that had been neglected in prototype3. These include joint friction, adequate joint constraints, size and material properties.

Considering the complicated nature of the gimbal parts I used rapid prototyping to construct them. This “additive layer manufacturing” provides a simple process for fabricating complex shaped parts that would otherwise be very difficult to produce using traditional methods such as machining. However, choosing rapid prototyping as the manufacturing technique limits the material selection to the list of plastics suitable for this process. Amongst these I chose polycarbonate, one of the strongest available materials with a tensile strength of 57 MPa. Figure A.9 illustrates the final results, various parts of the model are labeled.
The overall dimension of the model was selected based on the lowest possible globe size. The globe’s 46 mm inner diameter was restrained by the size of the available camera, 20 mm × 30 mm × 23 mm. Furthermore, to accommodate the C-bracket the globe is parted longitudinally into two components that are friction fitted together, illustrated in Figure A.10. In this model, the yaw shafts are combined into one continuous 2 mm stainless steel precision shaft which is fixed in the through hole down the length of the C-bracket, Figure A.10. This arrangement of the yaw rotation shaft acts as a reinforcement in the horizontal direction when the globe is pulled back by the activated cables. Table A.1 summarizes the details of
Figure A.10: Elements of first rapid prototype of the eye robot
E1D3 joints. Subsequent to selecting an appropriate globe size the C-bracket was
designed to fit inside the empty space of the posterior globe and the outer ring
diameter was optimized to minimize its moment arm without jeopardizing a clear
routing of the cables.

Figure A.11: Globe assembly

Table A.1: Joint details for the first rapid prototype of the eye robot

<table>
<thead>
<tr>
<th>Rotational Direction</th>
<th>Fixed wrt</th>
<th>Rotates wrt</th>
<th>Fixed by</th>
<th>Rotational joint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yaw</td>
<td>C bracket</td>
<td>Globe</td>
<td>Glued</td>
<td>Through Hole</td>
</tr>
<tr>
<td>Roll</td>
<td>Outer Ring</td>
<td>C Bracket</td>
<td>Threaded</td>
<td>Through Hole</td>
</tr>
<tr>
<td>Pitch</td>
<td>Outer Ring</td>
<td>Support Stand</td>
<td>Threaded</td>
<td>Through Hole</td>
</tr>
</tbody>
</table>

An important part of the detailed mechanical design includes studying the
structural strength of the system, which is determined by the strength of the indi-
vidual elements. Therefore, it is crucial that the entire gimbal structure withstands
the applied forces from the activated motors (transferred through the cables) without failing. Here, failure of a part is defined as a deflection of more than 0.1 \text{mm}, rather than the typically used yield strength of the material. This requirement is essential for providing a rigid and reliable support structure as well as ensuring a precise and purely rotational globe movement. For instance, if the outer ring was to be deflected to a point that the globe’s centre would be shifted from that of the outer ring’s, the rotational axes would no longer coincide and would therefore add an undesired translational portion to the globe’s motion. To evaluate the structural strength of the this prototype the individual elements were examined under the maximum applied forces. This maximum was taken to be the force transferred to the specific element, when all the 6 motors are fully activated. If $F_{\text{max}}$, $I_{\text{max}}$, $K_i$, and $R_{\text{pulley}}$ respectively represent, maximum motor force produced, maximum output current of the amplifier, motor torque constant, and motor pulley radius, then the maximum motor force can be described by the following relationship.

$$F_{\text{max}} = I_{\text{max}}K_i/R_{\text{pulley}} = 7 \text{ N}$$

The maximum applied force to the gimbal system is when all 6 cables are being pulled with the maximum calculated motor force of 7N. Considering this situation, the tension in the 4 recti are predominantly taken by the C-bracket, while those of the obliques are transferred through the outer ring, Figure A.12. Subsequently, the structural strength of the C-bracket and the outer ring were examined using Cosmos in Solid Works. The corresponding maximum deflection and factor of
safety of the gimbal elements are summarized in Table A.2\textsuperscript{1}. Considering, the unacceptably low ”deflection F.O.S.” of this prototype imposed the requirement of a new design. Furthermore, insufficient joint constraint caused undesired globe translation under activated motors and lack of bearing elements in the joints lead to high rotational friction. Consequently, the final eye robot, presented in Section 4.3, was designed to incorporate these requirements.

\textsuperscript{1} The F.O.S of each element is calculated based on the 57 MPa yield strength of polycarbonate.
Table A.2: Performance of the eye robot prototype under the applied motor loads

<table>
<thead>
<tr>
<th>Component</th>
<th>Maximum Deflection [mm]</th>
<th>F.O.S</th>
<th>Deflection F.O.S</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-bracket</td>
<td>1.74</td>
<td>1.85</td>
<td>0.057</td>
</tr>
<tr>
<td>Outer Gimbal Structure</td>
<td>4.9</td>
<td>1.81</td>
<td>0.02</td>
</tr>
</tbody>
</table>