Essays on Idiosyncratic Stock Return Volatility and Bank Lending Incentives

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Abstract

This dissertation studies two topics in finance. The first topic is on the firm-level stock return volatility. The second one is on lending incentives of banks.

Chapter 2 studies idiosyncratic stock return volatility in an investment model with convex capital adjustment costs. The model has two predictions: (1) in the cross-section, idiosyncratic return volatility has a V-shaped relationship with the asset growth rate; (2) in the time series, dispersion across firms in asset growth rates positively predicts average idiosyncratic volatility. Two sources of uncertainty drive the cross-sectional V-shape. Specifically, shocks to *expected* cash flows cause the high return volatility of *high* growth firms, while innovations to *unexpected* cash flows lead to the high return volatility of *low* (negative) growth firms. The time series prediction follows from the cross-sectional V-shape pattern. I document strong empirical support for the model's predictions of the relationship between the asset growth rate and idiosyncratic return volatility in both the cross-section and time series.

Chapter 3 provides explanations to two important questions that arise from the recent financial crisis: (i) Why don't banks keep enough cash to meet potential liquidity shocks and hence avoid fire sales of illiquid assets? (ii) Why do banks keep a large amount of cash on their balance sheet without lending after large capital injections from the government? This chapter examines both questions in a single framework by considering the lending incentives of banks when facing the risk of liquidity shocks. Banks make decisions based on the tradeoff between costs (fire sales of illiquid assets) and benefits (high returns from bank loans) of lending. This chapter shows that it may be optimal for banks to lend out cash and incur fire sales of assets under liquidity shocks, even if banks are endowed with enough cash to meet liquidity shocks. That is, fire sales of assets could be an *endogenous* outcome of banks' optimal decisions. At the same time, banks may still keep a large amount of cash after government capital injections to save the cost of fire sales, especially when banks are endowed with a large fraction of illiquid assets. Based on the results from the model, this study also provides policy implications for government intervention.

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Dedication

To my parents.

To my wife and our son.

Chapter 1

Introduction

This dissertation studies two topics in finance. In the first study, I investigate how frictions to real investment affect firm-level stock return volatility. This study contains both theory and empirical analyses. In the second study, I develop a theoretical model to investigate how liquidity risks affect the prices of illiquid bank assets as well as banks' lending incentives.

The motivation of the first study originally comes from a very interesting time series pattern in the data: the average firm-level return volatility of U.S. firms first increases over time and then drops down more recently. This pattern is documented by two papers. Campbell et al. (2001) document that over the period from 1962-1997, there was a noticeable increase in the idiosyncratic return volatility of U.S. firms relative to the reasonably stable market volatility. However, a more recent study by Brandt et al. (2010) shows that the upward trend in the average idiosyncratic volatility had been reversed by 2007.

This pattern has inspired extensive empirical studies on firm-level return volatility in both cross-section and time series. However, the existing explanations of firm-level return volatility show explanatory power only in the cross-section or the time series, but not in both. For example, proxies of growth options are useful in the time series but lack power in the cross-section. As another example, firm size and stock price have significant explanatory power in the cross-section, but not in the time series.

This motivates me to find a better explanation of firm-level return volatility that should have robust explanatory power in both the cross-section and time series. In addition, we know from prior literature that firms' real investment affects firms' expected stock returns. For example, Cooper et al. (2008) document that higher growth firms earn lower future returns. Therefore, it is natural to further investigate how real investment affects return volatility.

In the first essay of this dissertation, I develop an investment model with convex capital adjustment costs. Firms operate under a technology that is decreasing-return-to-capital. The first source of uncertainty that firms face is the demand uncertainty, which affects the firms' expected cash flows and investment opportunities. This is quite standard in the existing literature. What is novel in my model is that firms also face another source of uncertainty that is unrelated to the demand. This source of uncertainty generates unexpected cash flows. For example, operational risk or stochastic depreciation can generate unexpected cash flows that are insensitive to the firm's demand conditions. This cash flow volatility contributes directly to the return volatility.

The model has two predictions: (1) in the cross-section, idiosyncratic return volatility has a V-shaped relationship with the asset growth rate; (2) in the time series, dispersion across firms in asset growth rates positively predicts average idiosyncratic volatility.

The intuition is as follows. As a benchmark in the model, if there is no friction to investment, the return volatility is constant across firms. Therefore, investment frictions are the key factor to generate non-constant return volatility. More specifically, investment frictions affect return volatility through the following two mechanisms (or channels).

The first is a "growth channel," in which positive growth opportunities amplify shocks to expected cash flows. According to the real options literature, a firm value includes both the assets in place and growth options. Due to investment frictions, the relative importance of growth options in the firm value can vary across firms. In addition, since the growth options are levered claims on assets, they are more volatile than the assets in place. Therefore, more growth opportunities for a firm result in higher return volatility.

The second is a "market value channel," in which firms' low market values amplify shocks to unexpected cash flows. Again, due to investment frictions, the firm's market value can vary across firms with a given book capital. Since part of the stock return is the unexpected cash flows divided by the firm's market value, a low market value amplifies cash flow volatility. Therefore, for a given cash flow volatility, a low market value leads to high return volatility.

These two channels explain the model's cross-sectional prediction of a V-shaped relationship between idiosyncratic return volatility and asset growth rate. In the right segment of the V-shape, firms have high asset growth rates, and the growth options carry a large weight in the firm value. According to the growth channel, these firms should have high return volatility. In the left segment of the V-shape, firms have low/negative asset growth rates, and their market value is low relative to capital. According to the market value channel, these firms should also have high return volatility. In the middle of the V-shape, firms have moderate asset growth rates, and neither of the two channels is strong. So these firms have low return volatility.

The model's time series prediction follows from the cross-sectional V-shape pattern. If the heterogeneity in asset growth rates across firms is higher (i.e., more firms have either high or low growth rates), then more firms have higher idiosyncratic volatility, which means the average idiosyncratic volatility is also higher. This explains why a higher heterogeneity in asset growth rates predicts a higher average idiosyncratic volatility.

I document strong empirical support for the model's predictions. Specifically, in the cross-section, I sort firms according to their annual asset growth rate, which measures the firm's broad investment rate. I find that stocks with either high or low (negative) asset growth rates have higher idiosyncratic volatility than stocks with moderate asset growth rates. In other words, the cross-sectional idiosyncratic volatility shows a V-shape pattern in asset growth rate. In the time series, I find that the heterogeneity in asset growth rates among firms closely matches the up-and-down movement of the average idiosyncratic volatility. These two time series are highly positively correlated. In other words, higher heterogeneity in asset growth rates among firms predicts higher average idiosyncratic volatility.

The contribution of this study is twofold. First, I document new and interesting empirical relationships between idiosyncratic volatility and asset growth rates in both the cross-section and time series. Second, I show theoretically two important economic mechanisms that can help us understand the empirical findings in the data.

The second essay of this dissertation is inspired by the recent financial crisis. Specifically, I am intrigued by two important questions that arise from the crisis: (i) Why don't banks keep enough cash to meet potential liquidity shocks and hence avoid fire sales of illiquid assets? (ii) Why do banks keep a large amount of cash on their balance sheet without lending after large capital injections from the government?

I examine both questions in a single theoretical framework by considering the lending incentives of banks when facing the risk of liquidity shocks. The model has three dates and two types of agents: banks and non-bank institutions (such as hedge funds). At the initial date, banks are endowed with cash, non-tradable but recallable bank loans, and tradable illiquid assets, such as mortgage-backed-securities (MBS). The non-bank institutions are endowed with only cash. At the intermediate date, banks may face a liquidity shock, such as a sudden withdrawal from depositors or a failure in rolling-over short-term debt. At the final date, payoffs of assets are distributed to investors. Banks make decisions on asset sales and lending at the initial date to maximize their equity value. The non-bank institutions make investment decisions including purchasing banks' tradable assets to maximize their portfolio's expected value.

Banks make their decisions based on the tradeoff between the cost and benefit of lending. The cost of lending comes from the premature loan liquidation and the fire sales of illiquid assets to raise cash when banks face a liquidity shock. The benefit of lending is the high return from the newly issued bank loans. The banks make their optimal decisions such that the marginal cost equals the marginal benefit.

The first finding of the study is that fire sales of illiquid bank assets, meaning that bank assets are sold at prices lower than the fundamental value, can be an endogenous outcome of banks' optimal decisions. In other words, even if banks are endowed with enough cash to deal with the liquidity shock, they may still choose to lend out cash such that they have to sell their illiquid assets at a discount when hit by a liquidity shock. In other words, it may not be in the banks' interest to avoid fire sales of their illiquid assets.

The model also helps us understand why banks keep a large amount of cash on their balance sheet without lending it out after large capital injections from the government in the recent financial crisis. One direct explanation is that the liquidity shock is large, and the fire sales are very costly, so banks keep more cash to save the cost. This study provides another novel explanation. That is, with a fixed size of the liquidity shock and a fixed cash endowment, a higher endowment ratio of tradable-illiquid-assets to bank-loans makes fire sales more costly and therefore induces banks to keep an even larger amount of cash. This explanation is especially relevant during the recent financial crisis, in which many big banks are reported to be 'clogged' with a large amount of illiquid assets (such as MBS) on their balance sheet.

In addition, this study also generates policy implications for government intervention. For example, a direct capital injection to banks always dominates buying bank loans in terms of their effects on the real economy (i.e., total investment). However, a direct capital injection and buying tradable illiquid bank assets (such as MBS) can dominate each other, depending on the size of the government intervention fund.

The remainder of the dissertation is organized as follows. Chapter 2 investigates how frictions to real investment affect firm-level stock return volatility.

Chapter 3 develops a theoretical model to investigate how liquidity risks affect the prices of illiquid bank assets as well as banks' lending incentives. Chapter 4 concludes the dissertation.

Chapter 2

Asset Growth and Idiosyncratic Return Volatility [1]

2.1 Introduction

Traditionally, idiosyncratic return volatility, which measures idiosyncratic risk, has been important in understanding portfolio diversification, risk management, and valuation of stock options. Recent empirical studies emphasize the importance of idiosyncratic return volatility in stock returns.[2] Moreover, Campbell et al. (2001) document that over the period from 1962 to 1997, there was a noticeable increase in the idiosyncratic return volatility of U.S. firms relative to the reasonably stable market volatility. Accordingly, they also find that correlations among individual stocks and the explanatory power of the market model for a typical s-tock declined during the same period. Their finding has inspired extensive studies

[1]I would like to thank Jan Bena, Murray Carlson, Jason Chen, Adlai Fisher, Lorenzo Garlappi, Ron Giammarino, Alan Kraus, Kevin Wang (NFA discussant), Tan Wang, Ralph Winter, and seminar participants at Cheung Kong GSB, McGill University, Shanghai Jiaotong University, the University of British Columbia, the University of Calgary, the University of Texas at Austin, the University of Texas at Dallas, and the Northern Finance Association 2009 Meetings for their valuable comments and suggestions.

[2]See, for example, Ang et al. (2006, 2009) and Fu (2009) for the effect of idiosyncratic return volatility on stock returns.

that attempt to explain idiosyncratic return volatility in both the cross-section and time series.[3]

In this chapter, I study idiosyncratic return volatility in an investment model with capital adjustment costs. The model has two predictions. First, in the *cross-section*, idiosyncratic return volatility has a V-shaped relationship with the asset growth rate. Second, in the *time series*, dispersion across firms in asset growth rates positively predicts average idiosyncratic volatility. I further document strong empirical evidence that supports both predictions.

The cross-sectional V-shape of idiosyncratic return volatility is driven by two channels. The first is a "growth channel," in which positive growth opportunities amplify shocks to *expected* cash flows. This leads to high return volatility of high growth firms. The growth channel is closely related to the leverage effect of growth options on expected returns in the real options literature (see, for example, Carlson et al. (2004)). The second is a "market value channel," in which firms' low market values amplify shocks to *unexpected* cash flows. This results in high return volatility of negative growth firms. Moreover, the time series prediction follows from the cross-sectional V-shape pattern. That is, a higher degree of cross-sectional dispersion in the asset growth rates predicts a higher degree of volatility dispersion, or equivalently, a higher average of idiosyncratic return volatility.

To analyze the two channels noted above, I develop an investment model with convex capital adjustment costs. In the model, I assume that firms operate under a decreasing-return-to-capital technology and face two sources of uncertainty. One is demand uncertainty, which affects both the firm's market value and investment opportunities through *expected* cash flows. The other is the uncertainty of *unex-*

[3]Existing explanations of the upward trend in the average idiosyncratic return volatility include institutional ownership (Malkiel and Xu (2003), Bennett et al. (2003)), market composition of firms (Bennett and Sias (2006)), new listings (Brown and Kapadia (2007), Fink et al. (2010)), growth options (Cao et al. (2008)), product market competition (Gaspar and Massa (2006)), idiosyncratic volatility of fundamentals (Wei and Zhang (2006), Irvine and Pontiff (2009)), and financial reporting quality (Rajgopal and Venkatachalam (2006)). In addition, the cross-sectional explanations of idiosyncratic return volatility include long-term earnings growth (Malkiel and Xu (2003)), idiosyncratic volatility of profitability (Pastor and Veronesi (2003)), and retail trading (Brandt et al. (2010)).

pected cash flows, which is independent of the demand uncertainty. Since this second source of uncertainty does not affect expected cash flows, it has no effect on either firm value or investment. However, the volatility of unexpected cash flows contributes directly to the return volatility.

I first focus on the growth channel and consider a simple version of the model where firms only face demand uncertainty. To obtain intuition, I consider two special cases in which the return volatilities are constant. I analytically show that if the expected growth in demand is positive, the return volatility is lower with *infinite* adjustment costs than that with *no* adjustment cost. In other words, investment frictions can decrease return volatility.[4] The reason is that the flexibility to invest increases return volatility while the flexibility to disinvest decreases return volatility. When the expected growth is positive, investing becomes more important than disinvesting, and the overall effect of investment flexibility (i.e., less frictions) increases the return volatility in the frictionless case.

I then consider both the demand and cash flow uncertainty. In the general case where firms face *finite* adjustment costs, the return volatility is not constant. Within the growth channel, when firms are investing, the growth opportunities increase return volatility. On the other hand, when firms are disinvesting, the opportunities to disinvest reduce the return volatility. That is, the return volatility increases with the investment rate in the growth channel. In the market value channel, the low market values of negative growth firms amplify cash flow shocks. In other words, the return volatility decreases with the investment rate in the market value channel. Consider the combination of these two channels: the market value channel dominates for low (negative) growth firms; and the growth channel dominates for high growth firms. I confirm this intuition through numerical examples and demonstrate that there is a nonlinear, V-shaped relationship between the return volatility and asset growth rate. In addition, the model also generates time series implications by aggregating the cross-sectional observations. For given s-

[4]This contrasts with the result in Kogan (2004), where investment frictions always increase return volatility.

lopes and meeting point of the two segments of the V-shape, a higher degree of cross-sectional dispersion in asset growth rates predicts a higher cross-sectional average of idiosyncratic return volatility.

In support of the model's predictions, I document strong empirical evidence with U.S. common stocks from 1963 to 2009. In the *cross-section*, the idiosyncratic return volatility shows a V-shape with respect to firms' annual asset growth rate. That is, stocks with either high or low (i.e., negative) asset growth rates in the past year have higher idiosyncratic volatility in the following year than s-tocks with moderate growth. The stocks with the lowest idiosyncratic volatility are associated with firms that have an annual asset growth rate of approximately 5%. In the *time series*, the cross-sectional dispersion in annual asset growth rates positively predicts the cross-sectional average idiosyncratic return volatility in the next year.

I further document the robustness of the empirical findings. The V-shaped relationship in the cross-section is robust even after factors such as size and leverage are controlled for. In the time series, the asset growth measure is the most important predictor (among all well-known predictors of idiosyncratic volatility) of value-weighted, average idiosyncratic return volatility. In addition, the asset growth effect on idiosyncratic volatility dominates alternative explanations such as cash flow and its volatility, growth options, and forecasted long-term earnings growth.

The contribution of this study is twofold. First, this study analytically shows that investment frictions can *decrease* return volatility, which contradicts the intuition that investment frictions always increase return volatility due to the inflexible capital (see, for example, Kogan (2004)). Moreover, this study incorporates the market value channel, which involves unexpected cash flows, in explaining the *firm-level* return volatility. This extends the existing literature that emphasizes only the importance of the growth channel, which involves expected cash flows, in understanding the dynamics of asset returns. Second, this study is the first to empirically document: (1) the V-shaped relationship in the cross-section between the idiosyncratic return volatility and asset growth rates, and (2) the robust predictive power of asset growth rates on the average idiosyncratic return volatility in the time series.

This study is related to several steams of prior *theoretical* research. First, there is a large investment literature that studies the effect of frictions, such as irreversibility or convex adjustment costs, on firms' investment behavior (see, for example, Dixit and Pindyck (1994)). Another growing line of finance research, pioneered by Berk et al. (1999), examines the effect of investment on expected stock returns.[5] However, volatility of stock returns is typically not the focus of these studies.[6] In addition, existing theoretical research on return volatility mainly studies either *industry-level* or aggregate *market-level* volatility. For example, Kogan (2004) investigates the effect of investment frictions on industry-level return volatility. David and Veronesi (2009) study the aggregate market return volatility in an endowment economy. One notable exception is the work by Pastor and Veronesi (2003), who study *firm-level* return volatility in a learning framework. However, they do not consider firms' optimal investment and the effect of investment frictions on return volatility, which are the focus of my study.

My study is also related to a substantial *empirical* literature on idiosyncratic return volatility. Note that the existing explanations of idiosyncratic return volatility have two notable shortcomings.[7] First, most explanations of the observed time trend in average idiosyncratic volatility have difficulty in explaining why average idiosyncratic volatility falls during the period from 2000 to 2007. Brandt et al. (2010) show that the upward trend documented by Campbell et al. (2001) during the period from 1962 to 1997 has been reversed by 2007, falling below pre-1990s

[5]Other theoretical contributions include Gomes et al. (2003), Carlson et al. (2004, 2006, 2010), Kogan (2004), Zhang (2005), and Cooper (2006). There is also a growing empirical literature that tests investment-based models, see, for example, Cochrane (1991, 1996), Liu et al. (2009) and Li and Zhang (2010).

[6]Note that Carlson et al. (2010) investigate firm-level risk dynamics, which include both beta and total volatility, around seasoned equity offerings.

[7]See footnote 3 for a list of existing explanations of idiosyncratic return volatility in both the cross-section and the time series.

levels.[8] Second, none of the existing explanation shows robust explanatory power in *both* the time series *and* cross-section. For example, Rubin and Smith (2008) find that the market-to-book ratio, which is used by Cao et al. (2008) to proxy for the growth options, is useful in the time-series context but lacks power in the cross-section. They also find that most explanatory variables in the time series have no power after controlling for the lagged volatility. In contrast, my study shows that the asset growth rate has robust explanatory power on the idiosyncratic return volatility in both the cross-section and time series.

Finally, this study is also related to two empirical studies that involve asset growth and/or idiosyncratic return volatility. Cooper et al. (2008) find that firms with higher annual asset growth rates will experience lower future *returns*.[9] My findings show that the asset growth is also related to the return *volatility*. Lam and Wei (2009) use the idiosyncratic return volatility as a proxy for arbitrage risks to explain the asset growth anomaly documented by Cooper et al. (2008). They find that the asset growth anomaly is weaker for firms with lower idiosyncratic return volatility, which they attribute to the lower arbitrage risk of these firms.[10] My analysis suggests that one should be cautious when interpreting their finding since the asset growth and idiosyncratic return volatility are not independent of each other.

This chapter proceeds as follows. Section 3.2 presents the model and derives empirical implications. Section 2.3 describes the data. Section 2.4 provides main empirical evidence which supports the predictions of the model. Section 2.5 provides comprehensive robustness checks of the main empirical findings. Section 2.6 compares my findings to three closely related, alternative explanations of idiosyncratic return volatility in the literature. Finally, Section 3.5 concludes the chapter.

[8] Bekaert et al. (2010) also reject an upward trend in idiosyncratic volatility in 23 developed markets.

[9] Also see Titman et al. (2004) for the effect of capital investment on stock returns.

[10] Lipson et al. (2010) also find that the asset growth effect on return is weak for stocks with low idiosyncratic volatility.

2.2 Model

This section studies the effect of investment frictions, namely capital adjustment costs, on stock return volatility. Section 2.2.1 describes the model setup. In Section 2.2.2, I first study a simple version of the model and consider only the demand uncertainty, which directly affects firms' investment opportunities. To obtain intuition, I compare two special cases in which closed form solutions are available. I then consider the full version of the model in Section 2.2.3, where firms face both demand uncertainty and cash flow uncertainty. The model generates testable implications on the relationship between the stock return volatility and the asset growth rate.

2.2.1 Model Setup

Consider a firm which produces output using capital, denoted by K, and labor. The firm's production follows a constant return-to-scale Cobb-Douglas function. In addition, the firm faces an iso-elastic demand curve depending on a random variable X, which evolves over time according to the following geometric Brownian motion:

$$\frac{\mathrm{d}X_t}{X_t} = \mu_x \mathrm{d}t + \sigma_x \mathrm{d}Z_t^x, \tag{2.1}$$

where μ_x and σ_x are constants, and Z^x is a standard Brownian motion.

The firm incurs convex adjustment costs for new investment I. Following the neoclassical investment literature (Hayashi (1982)), I assume that the adjustment cost is homogeneous of degree one in I and K. Specifically, the adjustment cost, denoted by ϕ, is quadratic in the investment-to-capital ratio:

$$\phi(I,K) = \theta K \left(\frac{I}{K}\right)^2, \tag{2.2}$$

where the parameter $\theta \geq 0$ measures the degree of the adjustment cost. I choose the quadratic cost function for simplicity, but the implications of the model continue to hold for other convex cost function specifications.[11]

For simplicity, I assume that there is no capital depreciation, and the capital evolves according to:

$$\mathrm{d}K_t = I_t \mathrm{d}t. \tag{2.3}$$

The firm can costlessly and instantaneously adjust its labor input with a fixed wage. In other words, the firm always chooses the optimal amount of labor input without any adjustment cost. Then the firm's operating profit, i.e., revenue minus the labor costs, can be written compactly as follows:

$$\pi(X,K) = hX^{1-\alpha}K^{\alpha}, \tag{2.4}$$

where $h > 0$ scales the level of operating profit, and $0 < \alpha < 1$ captures the properties of both the firm's production technology and its demand curve.[12]

The firm's net cash flows have two components: the expected cash flow and the unexpected cash flow. The expected cash flow equals the operating profit minus the direct investment cost minus the adjustment cost. Conditional on the demand state variable (X), the expected cash flow is known. However, the realized cash flow can deviate from the expected cash flow due to the unexpected component.

There are many sources of uncertainty that may generate the unexpected component of the cash flow. Here I provide three important ones. First, the *operational risk* (e.g., software failures and product defects) or the *unexpected operating cost* generates uncertainty to the operating cash flows. Second, *stochastic depreciation* of capital, which may depend on exogenous factors such as weather conditions or technological obsolescence, also leads to uncertain net cash flows.[13] Third, the *stochastic non-operating income (expense)*, which is unrelated to a firm's normal operations, also contributes to the unexpected cash flow. Instead of explicitly modeling each single source of uncertainty, I capture their aggregate effect on the

[11] Note that the quadratic adjustment cost is widely used in the finance literature. See, for example, Cochrane (1996), Zhang (2005), Livdan et al. (2009), and Garlappi and Yan (2010).

[12] See Abel and Eberly (1996) for a similar setting.

[13] Ambler and Paquet (1994) consider stochastic depreciation in the real business cycle models.

cash flow volatility in a reduced form as follows.

Denote by C the cumulative net cash flow. The realized net cash flow in a short period of time $\mathrm{d}t$ is given by

$$\mathrm{d}C_t = [\pi(X_t, K_t) - I_t - \phi(I_t, K_t)]\,\mathrm{d}t + \sigma(X_t, K_t)\mathrm{d}Z_t^c. \tag{2.5}$$

The first term on the right hand side of Eq. (2.5) is the expected cash flow and the second term is the unexpected cash flow. Here the Brownian motion Z^c captures all uncertainty that affects the unexpected cash flow, and $\sigma(X,K)$ is the level of cash flow volatility that may depend on both the demand and capital levels. For simplicity, assume that the two Brownian motions Z^x and Z^c are independent. In other words, the uncertainty driving the unexpected cash flow is independent of demand uncertainty which drives the expected cash flow.

Assume that firms are all equity financed. In addition, investors are risk neutral and maximize the firms' expected cash flows discounted at a constant rate r. The firm value depends on both the demand and capital, denoted by $V(X,K)$, and it is determined by:

$$V(X_t, K_t) = \max_{\{I_s\}_{s\geq t}} E\left[\int_t^\infty e^{-r(s-t)}\mathrm{d}C_s \,\middle|\, X_t, K_t\right]. \tag{2.6}$$

To guarantee a finite firm value, assume $r > \mu_x$.

The firm's equity return has two components: one from the change in firm value and the other from the net cash flows. Specifically, the return in a short period of time $\mathrm{d}t$ is given by

$$\mathrm{d}R_t = \frac{\mathrm{d}V(X_t, K_t) + \mathrm{d}C_t}{V(X_t, K_t)}, \tag{2.7}$$

where R is the cumulative return.

2.2.2 Simple Version: Demand Uncertainty Only

In this section, I consider only the effect of demand uncertainty on return volatility, and do not consider cash flow uncertainty. Therefore, I set the cash flow volatility $\sigma(X,K) = 0$ in the model described in Section 2.2.1. Since demand uncertainty affects investment opportunities while cash flow uncertainty does not, this simplification highlights the effect of investment frictions on the return volatility directly through investment.

In order to obtain intuition, I consider two special cases. In the first case, firms can adjust their capital instantaneously without any adjustment cost (i.e., $\theta = 0$). In the second case, firms face an infinite adjustment cost (i.e., $\theta = \infty$), and therefore cannot change their capital. The following proposition characterizes firm values and compares the return volatility in the two cases.

Proposition 2.1 *The firm value and stock return volatility in the two special cases are given as follows.*

$$\text{Case 1 } (\theta = 0): \quad V_1(X, K(X)) = QK(X), \quad Var(R_1) = \left(1 - \frac{1}{Q}\right)^2 \sigma_x^2, \quad (2.8)$$

$$\text{Case 2 } (\theta = \infty): \quad V_2(X,K) = AX^{1-\alpha}K^{\alpha}, \quad Var(R_2) = (1-\alpha)^2 \sigma_x^2, \quad (2.9)$$

where both Q and A are constants, and $K(X)$ is proportional to X, which are all given in Appendix A.1. Here the subscripts '1' and '2' represent Case 1 and Case 2, respectively.

Furthermore, if $\mu_x > 0$, then firms with zero adjustment costs (Case 1) have a higher return volatility than firms with infinite adjustment costs (Case 2), i.e.,

$$Var(R_1) > Var(R_2). \quad (2.10)$$

If $\mu_x < 0$, the reverse is true.

The intuition is as follows. In the frictionless case, firms can adjust their capital instantaneously. The optimal investment policy for the firms is to maintain a constant capital-to-demand ratio (X/K). As a result, the firm value is linear in demand (X). This leads to constant return volatility. In other words, even if firms have different demand, frictionless investment always adjusts the capital such that firms are still identical after scaling by the capital (or the demand). Therefore, return volatility should be the same across firms. In the case with infinite adjustment cost, the capital level is fixed. The firm value is proportional to the operating profit, which follows a geometric Brownian motion. That is, infinite adjustment cost also leads to constant return volatility.

Compared with the case under infinite adjustment cost, the frictionless case has the extra flexibility of investing and disinvesting. Therefore, it must be the investment flexibility that drives the volatility difference between the two cases. The effect of investment flexibility on return volatility can be understood as follows. A demand shock affects directly the value of the existing assets as in the case with infinite adjustment cost. In the frictionless case, the flexibility of investing increases further the firm value after a positive demand shock because of the increased investment opportunities. Similarly, the firm value drops further after a negative demand shock due to the decline in growth opportunities. That is, flexibility of positive investment amplifies demand shocks. On the other hand, the flexibility of disinvesting cushions the decline of firm value after a negative demand shock and partially offsets the increase in firm value after a positive demand shock. That is, flexibility of negative investment decreases the return volatility. The overall effect of investment frictions on stock return volatility depends on the relative importance of investing flexibility and disinvesting flexibility. In the case where growth becomes important, i.e., firms have a positive expected growth in demand ($\mu_x > 0$), the effect of flexible investing dominates the effect of flexible disinvesting. As a result, investment flexibility increases the stock return volatility. In other words, investment frictions decrease the stock return volatility. However, if firms have a negative expected growth in demand ($\mu_x < 0$), the opposite is true.

Alternatively, the relative volatility in the two cases can be understood in the real options framework. In addition to assets-in-place as in the case with infinite adjustment cost, the frictionless case has the extra growth options. Since the call options (investing) are levered claims on assets, they are more volatile than assets-in-place. On the other hand, put options (disinvesting) are less volatile than assets-in-place. When firms have a positive expected growth in demand ($\mu_x > 0$), the call options become more important and the growth options increase the stock return volatility. This explains the higher return volatility in the frictionless case in Eq. (2.10). In contrast, if firms have a negative expected growth in demand ($\mu_x < 0$), the put options become more important and the growth options decrease the stock return volatility. This explains why the reverse of Eq. (2.10) is true when firms face a negative expected growth in demand.

It is worthwhile to discuss the difference between the result in Proposition 2.1 and that of Kogan (2004), who studies the industry-level return volatility and finds that investment frictions always increase return volatility. In that paper, firms are competitive and operate under a constant-return-to-capital technology. As a result, both the marginal value and the average value of capital always equal one in the frictionless world. In other words, there is no gain or loss from frictionless investment. Therefore, firms' return volatility is zero in the frictionless benchmark. Any inflexibility due to frictions (irreversibility or investment rate constraints) will lead to a deviation of the marginal value of capital from one, which in turn leads to high return volatility. However, as discussed above, the effect of growth options can be more important for monopoly firms assumed above, or more generally for firms under imperfect competition. As demonstrated through the two special cases in Proposition 2.1, frictions to investment can actually decrease stock return volatility if firms' expected growth is positive.

In the more general case with finite adjustment cost, the analytical solution is not available. However, based on the intuition from the two special cases, one can expect the following. When firms are investing, the call options to invest become more important in the firm value and therefore increase the return volatility. On the other hand, when firms are disinvesting, the put options to disinvest become more important in the firm value and therefore decrease return volatility. As a result, with finite adjustment costs, the stock return volatility will increase with the firms' willingness to invest. In other words, the return volatility will increase with the investment rate. In the next section, I use numerical examples to demonstrate this intuition.

2.2.3 Full Version: Demand and Cash Flow Uncertainty

Section 2.2.2 considers only demand uncertainty. More realistically, firms face both demand and cash flow uncertainty. In this section, I consider both sources of uncertainty and study the effect of capital adjustment costs on return volatility through both expected and unexpected cash flows.

The demand process is still given by Eq. (2.1). I further assume that the cash flow volatility is linear in the capital (K) but independent of the demand (X). That is,

$$\sigma(X,K) = \sigma_c K, \tag{2.11}$$

where σ_c is a constant.

The essence of this assumption is that the cash flow volatility does not decrease even as the demand becomes low. This is well supported by data. For example, empirical evidence indicates that the volatility of depreciation plus the non-operating expense as a percentage of total assets is non-decreasing as the average operating profit before depreciation becomes low (see the bottom panel of Figure A.1 in Appendix A.2). Moreover, the implications of the model carry over to other alternative functional forms for the cash flow volatility. For example, if the cash flow volatility is increasing in the capital to demand ratio (K/X) as the demand becomes low (which has direct support from the real data in Figure A.1), then the predictions of the model still hold.

However, I choose the functional form in Eq. (2.11) because it is intuitive and relatively easy to interpret. As discussed in Section 2.2.1, the cash flow volatility can be considered as an indirect way of capturing the uncertainty in operating costs, capital depreciation and non-operating income (expense). These factors grow with the size of the firm (i.e., the capital level) but are less sensitive to the demand conditions. For example, empirical evidence shows that the level of depreciation plus the non-operating expense is proportional to the capital level but independent of the operating income before depreciation (see the top panel of Figure A.1 in Appendix A.2). Therefore, it is natural to assume that their volatility is also proportional to the capital level.

Before showing the results for the general case with finite adjustment costs, I provide the following remark on the return volatility in the special frictionless case:

Remark 2.1 *Under the assumptions of the full model, the return volatility in the frictionless case is constant.*

This is due to the fact that both the firm value and investment policy are independent of the volatility of the demand process in the frictionless case. Recall that in the frictionless case, firms maintain a constant demand-to-capital ratio (X/K) and the firm value is proportional to capital. As a result, the cash flow volatility scales up the return volatility by a constant. Therefore, any return volatility differentiation across firms with different investment rates under finite adjustment costs is driven by the frictions to the investment, not directly by model assumptions.

Since analytical solutions for the model with finite capital adjustment costs are not available, I obtain numerical solutions using value function iterations (see Appendix A.3 for details of the numerical solution method). For numerical examples, I choose $\alpha = 0.8$. This value is used by Eberly et al. (2009) and consistent with the estimate of the average degree of returns to scale across industries by Burnside (1996). The adjustment cost parameter is set at $\theta = 0.15$, which is comparable with the estimation in Eberly et al. (2009). To generate reasonable dispersions in investment rate and return volatility, the other parameter values are chosen as follows: $h = 0.2, r = 0.1, \mu_x = 0.06, \sigma_x = 0.8, \sigma_c = 0.7$.

To demonstrate the effect of each source of uncertainty, I provide numerical examples of the relationship between stock return volatility and investment rate in the following two cases: (i) with demand uncertainty only (i.e., the simple

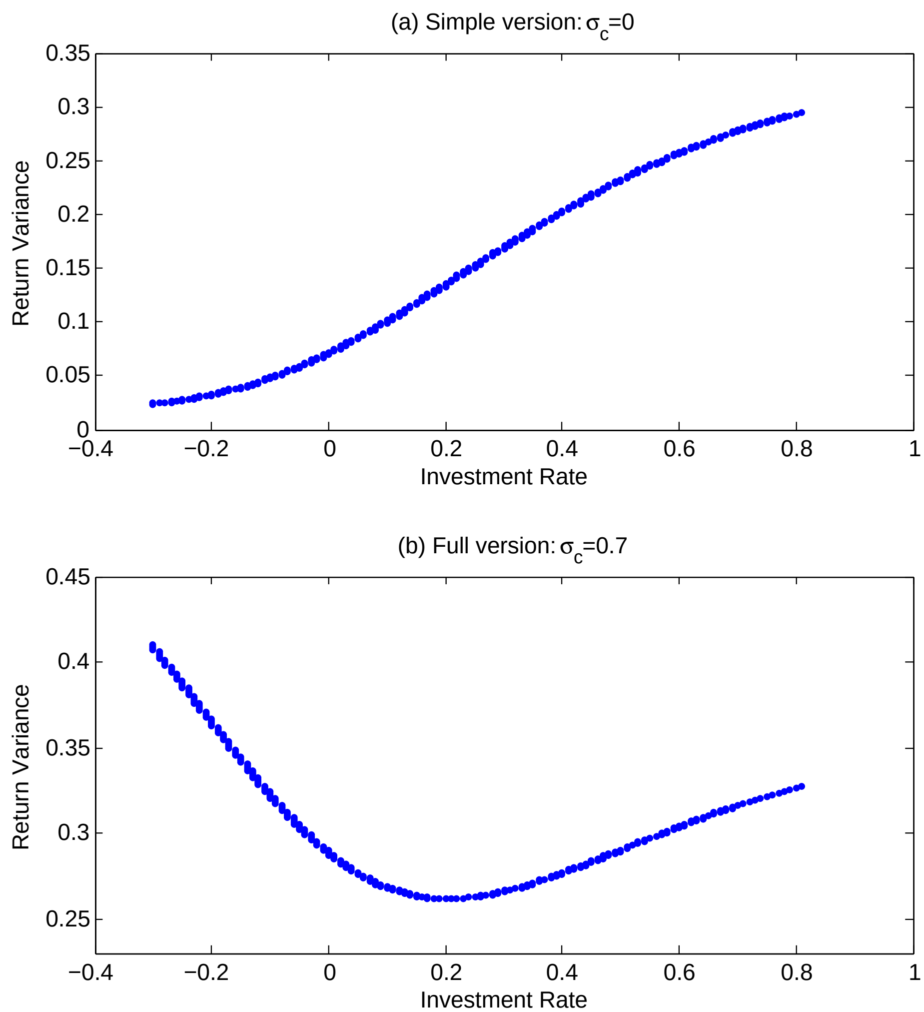

Figure 2.1: Numerical Examples: Return Volatility vs. Investment Rate
The figure reports results of the model using numerical methods. The common parameter values used are: $\alpha = 0.8, h = 0.2, r = 0.1, \mu_x = 0.06, \sigma_x = 0.8, \theta = 0.15$.

version of the model); (ii) with both demand and cash flow uncertainty (i.e., the full version of the model).

Panel (a) of Figure 2.1 shows that the return volatility in case (i) is monotonically increasing with the investment rate, as predicted by the simple version of the model. The intuition is that the options to invest increase the return volatility, while the options to disinvest decrease the return volatility. When firms are investing, the options of positive investment become more important, and therefore, these firms have higher return volatility. On the other hand, when firms are disinvesting, the options of negative investment become more important, leading to lower return volatility for these firms.

Panel (b) of Figure 2.1 shows that the return volatility in case (ii) has a V-shaped pattern with respect to the investment rate. Adding volatility ($\sigma_c K$) to the cash flow process enhances the return volatility dramatically in low (negative) investment regions, but has a smaller effect in high growth regions. This is because the contribution of the cash flow uncertainty to the return volatility is increasing in the capital-to-market-value ratio ($K/V(X,K)$). Since the firms's investment rate is decreasing in the capital-to-market-value ratio, the contribution of the cash flow uncertainty to the return volatility is decreasing in the investment rate. Therefore, the cash flow volatility has a strong effect on increasing the return volatility of disinvesting firms, but it has a smaller impact on the return volatility of investing firms.

Further numerical study shows that this nonlinear V-shaped relationship between return volatility and investment rate holds for a large range of parameter values. For example, with lower σ_c values, the nonlinear V-shaped relationship between return volatility and investment rate is similar to the case in panel (b) of Figure 2.1, with the volatility in the left segment being slightly lower.

From the above analysis, I derive two main implications of the model. The first states the relationship between cross-sectional stock return volatility and the asset growth rate.

Prediction 2.1 (Cross-section) *There is a nonlinear, V-shaped relationship between the stock return volatility and the asset growth rate in the cross-section.*

According to the model, firms with either low (negative) or high capital growth should have relatively high stock return volatility. On the other hand, firms with moderate growth rate (close to zero) should have relatively low stock return volatility. Therefore, one should expect a nonlinear relationship (a rough V-shape) between the stock return volatility and the asset growth rate, which measures the investment rate as a percentage of capital.

It is worth noting that an alternative model with only demand uncertainty plus operating leverage cannot quantitatively generate such a V-shape of return volatility in asset growth rate. When the firm's operating leverage is *exogenously* specified, I confirm in Appendix A.4 the intuition as in Carlson et al. (2004) that higher operating leverage results in higher return volatility. However, once firms can choose *endogenously* their operating leverage through optimal investment, the operating leverage effect is quantitatively dominated by the growth effect, which still results in a low return volatility for negative growth firms.

The second prediction concerns the time series of the average stock return volatility.

Prediction 2.2 (Time Series) *The cross-sectional dispersion in asset growth rates positively predicts the average stock return volatility.*

This time series prediction builds on the cross-sectional pattern. Consider a cross-section of firms that are ex ante identical. Under the model, stocks with extreme growth rates will have higher return volatility. Therefore, the more dispersed the firms' asset growth rates, the higher the average stock return volatility. That is, the dispersion in the cross-sectional asset growth rates should positively predict the average stock return volatility in the time series.

It is worth pointing out that these theoretical predictions hold for both total volatility and idiosyncratic volatility. This is because the growth effect related to the demand uncertainty as discussed in Section 2.2.2 is valid for both systematic and idiosyncratic demand shocks. The other source of uncertainty, the cash flow uncertainty, is purely firm specific and therefore affects only the idiosyncratic volatility. If the firm's demand subjects to only idiosyncratic shocks, then the idiosyncratic return volatility is also the total return volatility. So the predictions apply to both. In the more general scenarios, firms subject to both systematic and idiosyncratic demand shocks, then the predictions apply separately to both the idiosyncratic return volatility and the total return volatility. Despite this theoretical flexibility, I will test the model's predictions in the empirical analysis mainly using idiosyncratic volatility, which is the focus of this paper. As a robustness check, I will also present evidence in support of the model's predictions using total volatility.

2.3 Data

In this section, I first describe the data sources in Section 2.3.1. I then show the patterns of idiosyncratic return volatility with respect to the asset growth rate through sorting and graphing in Section 2.3.2.

2.3.1 Data Sources

I collect daily and monthly stock price and trading information from CRSP as well as the annual accounting data from COMPUSTAT. The main analysis covers all U.S. common stocks (CRSP share-code of 10 or 11) from 1963 to 2009. For the robustness analysis, I also collect the forecasted long-term EPS (Earnings Per Share) growth from IBES for the period from 1982 to 2009.

The main dependent variable is the annual idiosyncratic return volatility, which is calculated from daily return residues within each month over every twelve non-overlapping months. Specifically, for every year (from July to the next June) and for each stock, I run a time series regression of daily returns on the four daily factors, that is, the three Fama-French factors plus the momentum factor.[14] I then calculate within each month the variance of return residue from the factor regression. Finally, I average over the twelve non-overlapping months to calculate the annual frequency of idiosyncratic return volatility. Note that the volatility

[14] I require at least 60 daily returns within one year to prevent the factor model from over-fitting. In addition, at least 10 daily returns within one month are required to calculate the variance for each month.

measure is annualized, i.e., it is calculated by multiplying the daily variance by 252. In order to minimize the influence of outliers in this analysis, I winsorize the annual idiosyncratic volatility at the 99-percent level. To aggregate volatility, I use both equal-weighting and value-weighting schemes. For example, for value-weighting, I calculate the average idiosyncratic volatility by using market capitalization as the weight.[15] This generates a time series of annual average firm idiosyncratic volatility, which is the focus of the time series analysis.

The central explanatory variable in this study is the asset growth rate (gTA), which is calculated as the annual growth rate of total book assets. The main results are insensitive to different time horizons for calculating asset growth rate. For example, the asset growth rates calculated from the past two to three years all yield the same inferences. In the data, there are some extreme values of the annual growth rate of total assets; this mostly occurs in the case of small stocks. To deal with these extreme outliers, I winsorize the asset growth rate at the 99-percent level.[16]

To ensure that the asset growth effect on the idiosyncratic volatility is independent of other effects, I control for a battery of variables that have been shown in the literature to be related to idiosyncratic volatility. These variables include: size, market-to-book total assets (MABA), variance of MABA (VMABA), firm age, share price, turnover, leverage, a dividend dummy (DD), return on equity (ROE), and variance of ROE (VROE). Specifically, size is the market equity at the end of June. MABA is the fiscal year-end ratio of total book assets minus book equity plus market equity to total book assets. VMABA is the variance of MABA calculated using a past five-year rolling time series. Firm age is the number of years since a firm first appeared in CRSP.[17] Share price is the nominal price per share of the stock. Turnover is the ratio of trading volume per month to outstand-

[15]The interpretation is that the value-weighted average volatility is the average volatility if one randomly picks stocks with probabilities equal to their market equity shares. For equal-weighting, the probability of picking any stock is equal.

[16]Since asset growth rate is bounded above -1, I did not winsorize from below at one percentile.

[17]I start my CRSP sample from January 1926 and count the number of months since a firm first appears in CRSP.

ing shares. Leverage is the ratio of long-term debt to total assets. DD equals one for stocks that pay dividends and zero otherwise. ROE is the fiscal year-end ratio of earnings to book equity. VROE is the variance of ROE calculated using a past five-year rolling time series. Note that in regressions, I use the natural logarithms of both size and share price.

In the cross-sectional regressions, I assign accounting and price information to the explanatory variables that become available no later than the time at which the dependent variable becomes available. In particular, for regressions using average idiosyncratic volatility from July year t to June year $t+1$, I use the asset growth rate over the period $t-2$ to $t-1$, size at June t, MABA at $t-1$, VMABA calculated over the period $t-5$ to $t-1$, price at June t, average turnover during the past 12 months (July $t-1$ to June t),[18] leverage at $t-1$, DD at $t-1$, ROE at $t-1$, and VROE calculated over the period $t-5$ to $t-1$. Thus, I use past information to explain *and predict* future return volatility. This is very similar to the procedure first proposed by Fama and French (1992) to predict stock returns.

2.3.2 Patterns of Idiosyncratic Volatility

To show the relationship between idiosyncratic volatility and asset growth rate in the cross-section, I form deciles sorted by the asset growth rate for each year. For each decile and each year, I calculate the average idiosyncratic volatility and the average asset growth rate, as well as the other variables of interest. I then average these variables over time, which yields time series averages of the asset growth rate, idiosyncratic volatility, and the other variables for each decile. Table 2.1 shows the summary statistics for the asset growth sorted deciles. Note that the raw returns of the deciles sorted by the asset growth rate are decreasing in the asset growth rate, which is consistent with Cooper et al. (2008).

Using values from Table 2.1, I show in Fig.2.2 that the idiosyncratic volatility

[18] Rubin and Smith (2008) use the concurrent turnover and find significant coefficients in both time-series and cross-sectional regressions. However, it is unclear which is the driving force. Intuitively, high volatility will induce high trading as well. Using past trading volume mitigates such a concern.

Table 2.1: Summary Statistics for Asset Growth Sorted Deciles

The table reports summary statistics for portfolios sorted by asset growth rate. I first sort firms into deciles according to the annual asset growth rate (gTA) for each year from 1963 to 2009. I then calculate the average asset growth and the idiosyncratic volatility within each decile for each year. Finally, I average each decile over time to calculate the time series averages. All the other variables are calculated in the same manner. Idiosyncratic volatility (IVol) is the annualized variance that is calculated from daily return residues of the four-factor return regression; Systematic volatility (SVol) is the variance of the four-factor predicted returns. Return is the monthly stock raw return; Size is the market equity; Market assets to book assets (MABA) is the ratio of book assets minus book equity plus market equity to total assets; VMABA is the five-year rolling variance of MABA; Age is the number of years since a firm first appears in CRSP; Price is the nominal price per share; Turn is the turnover rate calculated as the monthly trading volume divided by outstanding shares; Lev is the leverage, calculated as long-term debts divided by total assets; DD is the dividend paying dummy; ROE is the return on equity; and VROE is the five-year rolling variance of ROE.

gTA Decile	1	2	3	4	5	6	7	8	9	10
(a) Equal-weighted										
gTA	-0.217	-0.051	0.004	0.040	0.073	0.110	0.156	0.231	0.391	1.271
IVol	0.807	0.494	0.338	0.270	0.247	0.258	0.275	0.313	0.373	0.500
SVol	0.043	0.035	0.029	0.028	0.027	0.029	0.032	0.035	0.041	0.051
Return	0.018	0.016	0.015	0.014	0.013	0.013	0.012	0.011	0.010	0.004
Size	298.5	661.4	1065	1348	1370	1566	1535	1241	989.6	690.2
MABA	1.665	1.303	1.272	1.318	1.370	1.482	1.635	1.821	2.066	2.462
VMABA	2.164	1.008	0.505	0.439	0.431	0.443	0.606	0.830	1.357	3.357
Age	11.88	15.39	17.48	17.95	17.66	16.79	14.86	12.64	10.35	7.373
Price	9.131	14.66	19.63	35.63	37.79	34.05	25.76	37.87	22.63	19.05
Turn	0.080	0.069	0.062	0.059	0.058	0.063	0.070	0.079	0.092	0.109
Lev	0.203	0.215	0.217	0.209	0.196	0.189	0.186	0.185	0.200	0.216
DD	0.258	0.457	0.721	0.772	0.740	0.747	0.673	0.637	0.459	0.433
ROE	-0.521	-0.086	0.021	0.061	0.075	0.087	0.092	0.090	0.069	0.013
VROE	0.604	0.208	0.104	0.078	0.074	0.065	0.085	0.120	0.180	0.413
(b) Value-weighted										
gTA	-0.191	-0.046	0.006	0.041	0.073	0.109	0.155	0.231	0.386	1.131
IVol	0.188	0.111	0.079	0.065	0.065	0.066	0.075	0.092	0.122	0.178
SVol	0.040	0.036	0.032	0.029	0.033	0.032	0.037	0.045	0.048	0.061
Return	0.013	0.012	0.011	0.010	0.010	0.010	0.010	0.009	0.008	0.006
MABA	1.701	1.563	1.471	1.588	1.750	1.986	2.175	2.537	2.805	2.876
VMABA	1.728	0.984	0.414	0.321	0.428	0.354	0.578	0.767	1.237	2.833
Age	28.03	34.55	36.66	36.71	36.92	36.95	32.70	27.19	21.95	17.81
Price	36.27	39.07	43.37	394.7	398.6	259.6	85.0	483.6	113.6	141.0
Turn	0.084	0.078	0.066	0.056	0.060	0.060	0.068	0.081	0.102	0.123
Lev	0.223	0.205	0.201	0.196	0.186	0.164	0.165	0.158	0.185	0.213
DD	0.703	0.839	1.222	1.255	1.023	1.138	1.074	1.423	0.829	0.653
ROE	-0.090	0.064	0.108	0.128	0.138	0.154	0.160	0.168	0.159	0.115
VROE	0.168	0.053	0.023	0.016	0.015	0.009	0.017	0.023	0.034	0.125

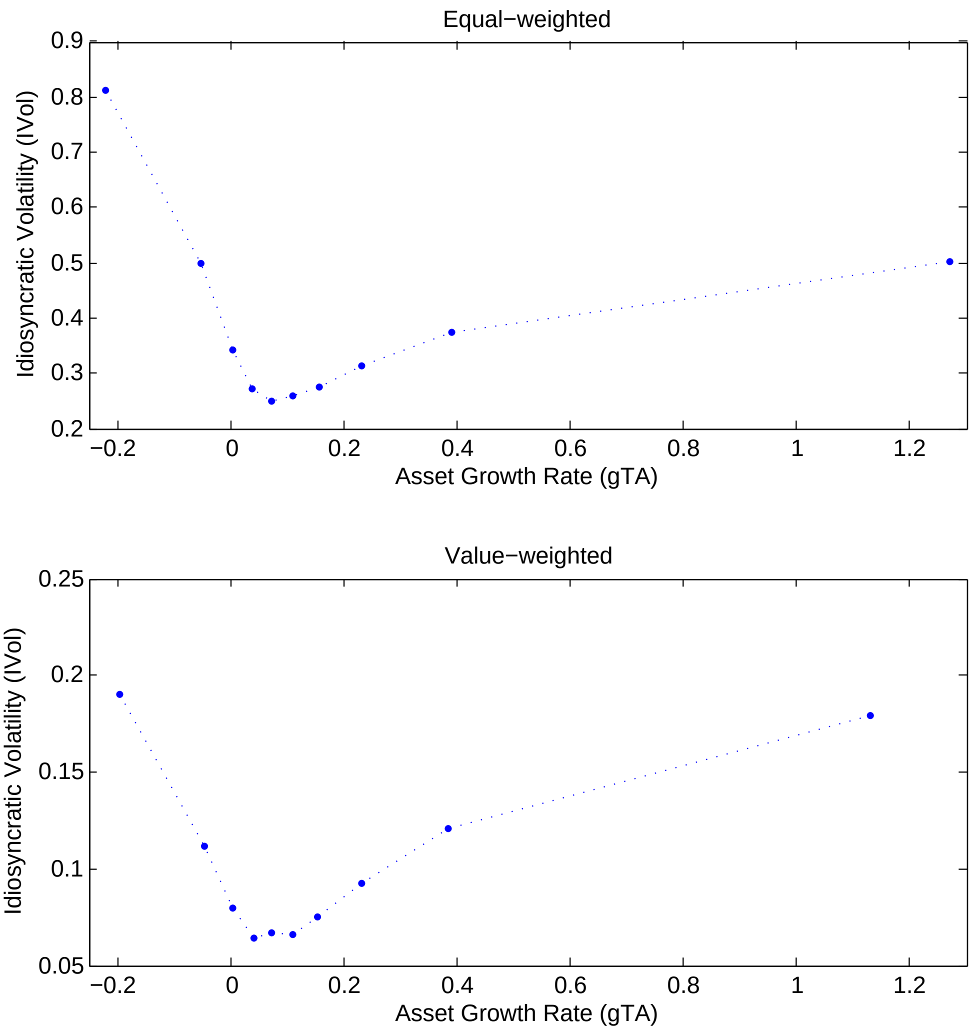

Figure 2.2: Idiosyncratic Volatility for Asset Growth Deciles

The figure plots the empirical relationship between idiosyncratic volatility and asset growth rate. First, I sort firms according to their asset growth rate into deciles for each year (1963 to 2009). I then calculate the average asset growth (gTA) and the idiosyncratic volatility (IVol) within each decile for each year. Finally, I average over time for each decile to calculate the time series averages.

has a V-shape with respect to the asset growth rate. For example, under equal-weighting, the idiosyncratic volatility decreases from 0.807 at decile 1 to its lowest value 0.247 at decile 5, and then increases to 0.500 at decile 10. For the asset growth rate, it increases from -21.7% at decile 1 to 7.3% at decile 5, and then increases dramatically to 127.1% at decile 10. The volatility at deciles 1 and 10 is more than two times higher than that at decile 5. The effect is similar under value-weighting. That is, the volatility decreases first and then increases as the asset growth rate increases. This is consistent with the model's first prediction and offers a guidance in choosing specifications of the regression in Section 2.4.1.

To visualize the relationship between idiosyncratic volatility and asset growth rate in the time series, I illustrate the time series of the average volatility and the aggregate asset growth measure in Fig.2.3. The upper panel compares the value-weighted average of the annual idiosyncratic volatility to the value-weighted annual aggregate asset growth measure from 1963 to 2009. The lower panel shows the equal-weighted time series. The aggregate asset growth measure (HMLgTA) is the difference in the average growth rate between high-growth and low-growth firms, i.e., the difference in the average asset growth rate between the two segments of the V-shape (see Section 2.4.2 for more details of constructing HMLgTA). Observe that the asset growth rate dispersion between high-growth and low-growth firms matches the average idiosyncratic volatility very well, especially the rising and then falling of the average idiosyncratic volatility around year 2000. One notable outlier is the high average idiosyncratic volatility in the year 2008 (more precisely, from July 2008 to June 2009), which involves dramatic price movements during the recent financial crisis. Note that although value-weighted measures are more relevant from an investment prospective, I present both equal-weighting and value-weighting calculations in the time series to make the analysis complete.

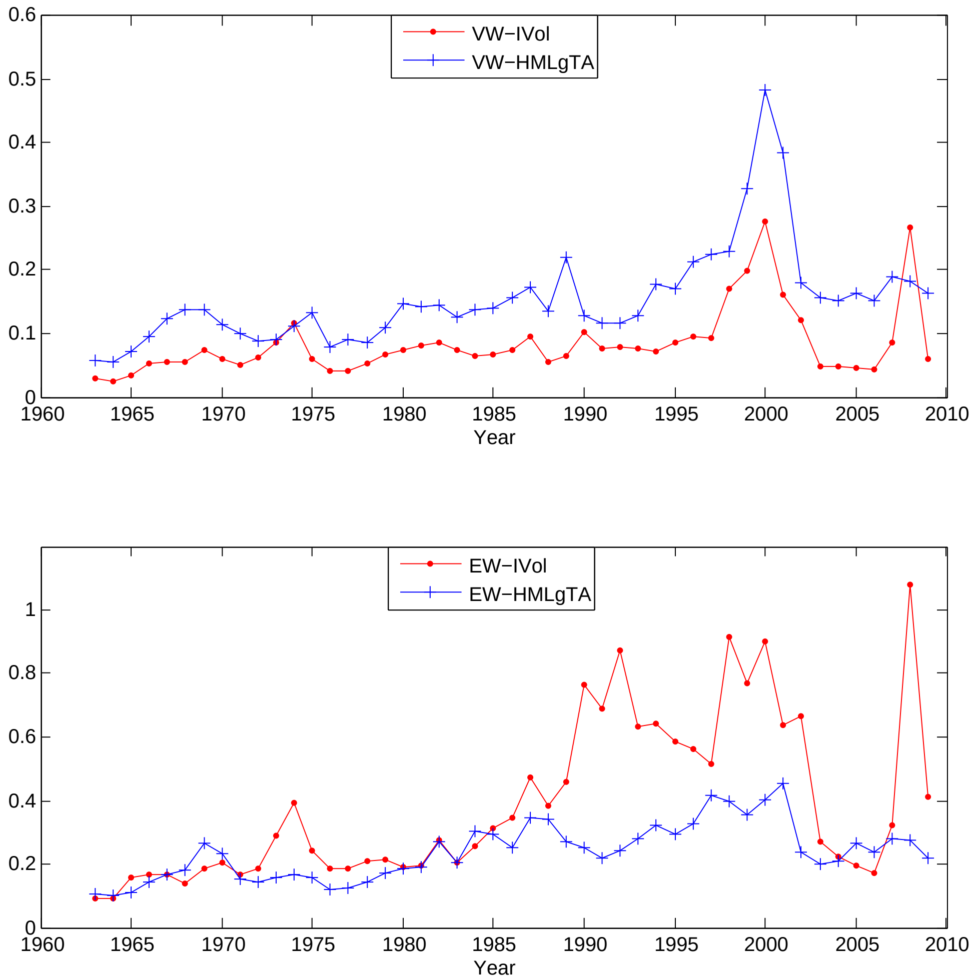

Figure 2.3: Time Series Idiosyncratic Volatility and Asset Growth: 1963–2009

The figure plots the value-weighted (VW) and equal-weighted (EW) time series of the average idiosyncratic volatility and the aggregate asset growth measure. The annual average idiosyncratic volatility (IVol) is the average monthly IVol over 12 non-overlapping months. The monthly IVol is calculated from daily residue returns from the four-factor return regression using a 12-month non-overlapping window. The aggregate asset growth measure, denoted by HMLgTA, is the difference in average asset growth rates between high-growth firms and low-growth firms, with the breaking annual asset growth rate of 5%.

2.4 Results

This section provides empirical evidence that supports the model's predictions. Sections 2.4.1 and 2.4.2 present results in the cross-section and time series, respectively. Section 2.4.3 provides results from multiple regressions which control for some other variables related to idiosyncratic volatility that have been proposed in the prior literature.

2.4.1 Cross-sectional Analysis

To analyze the asset growth effect on stock return volatility, I adopt Fama and MacBeth's (1973) method for cross-sectional regressions. Specifically, for each year (which is averaged over the months from July of year t to June of year $t+1$), I run cross-sectional regressions of idiosyncratic stock return volatility on the previous year's asset growth rate. To maintain robustness, all t-statistics in the Fama-MacBeth regressions are adjusted for the first order autocorrelation of the estimates from cross-sectional regressions.[19]

Table 2.2 presents the Fama-MacBeth regression results of five different regression specifications in which the asset growth rate is the only independent variable. Regression (1) runs a regression of idiosyncratic volatility on the asset growth rate, ignoring the V-shaped relationship, which is predicted by the theoretical model and observed by the simple sorting in Section 2.3.2. The coefficient is insignificant, and the R-square is extremely low (0.85%). Regression (2) uses the absolute value of the asset growth rate as a naive approach to capture the V-shaped relationship between the idiosyncratic volatility and asset growth rate. Although the coefficient is significantly positive, the R-square (1.80%) is still very low. Regression (3) incorporates the asymmetric effect of the asset growth rate with a two-segment piecewise linear model; and the R-square increases to 8.80%. If I further allow a free breaking point between low-growth and high-growth rates

[19] That is, the simple t-statistic of a Fama-MacBeth regression is multiplied by a factor of $\sqrt{(1-\rho)/(1+\rho)}$, where ρ is the first order autocorrelation of time series coefficient estimates in the cross-sectional regressions.

Table 2.2: Asset Growth and Idiosyncratic Volatility in Cross-section

The table reports estimates from Fama-MacBeth regressions of the idiosyncratic volatility on the asset growth rate for the sample period from 1963 to 2009. Denote by *c* the asset growth rate breaking point in the two-segment piecewise linear model. LgTA equals gTA - c for asset growth rate lower than c, and zero otherwise. HgTA equals gTA - c for asset growth rate higher than c, and zero otherwise. The t-statistics are adjusted for the first order autocorrelation of coefficient estimates from cross-sectional regressions.

	gTA	Abs(gTA)	LgTA	HgTA	Breaking point: c	Adj. R^2 (%)
(1)	-0.019 (-0.66)					0.85
(2)		0.139 (10.5)				1.80
(3)			-1.97 (-5.99)	0.127 (8.58)	Fixed at 0	8.80
(4)			-1.723 (-5.25)	0.170 (9.92)	0.053 (6.25)	9.72
(5)			-1.74 (-5.91)	0.161 (10.37)	Fixed at 0.05	9.57

(regression (4)), the R-square increases to 9.72%. Regression (5) fixes the breaking point at 5%, which is approximately the breaking point fitted by regression (4). The R-square (9.57%) is almost the same as in regression (4).

Overall, the coefficient of the low-growth segment is significantly negative, and the coefficient of the high-growth segment is significantly positive, with the breaking point around 5%. Moreover, the slope of the low-growth segment is about 10 times steeper than that of the high-growth segment. The slopes of both segments of the V-shape are highly significant in regression (5) with t-statistics of -5.91 and 10.37, which indicates that the idiosyncratic volatility is an asymmetric V-shape with respect to the asset growth rate. This provides formal evidence that supports the cross-sectional prediction of my theoretical model.

Since the V-shape is piecewise linear, I can transform the nonlinear regression models to linear ones. Such a transformation makes it much easier to implement regressions and saves computing time, especially in the estimation of robust s-tandard errors. As shown in Table 2.2, the adjusted R-squares in the nonlinear regressions (3) to (5) are very similar. This implies that the goodness-of-fit in the cross-section is insensitive to the choice of the breaking point in the asset growth rate. To transform the regressions into linear ones, I fix the breaking point at 5% and split the asset growth rate (gTA) into two variables, LgTA and HgTA. If the asset growth rate is *lower* than 5%, then LgTA equals gTA minus 5%; otherwise, LgTA equals zero. If the asset growth rate is *higher* than 5%, then HgTA equals gTA minus 5%; otherwise, HgTA equals zero. (Therefore, the sum of LgTA and HgTA always equals the asset growth rate minus 5%.) These two variables, LgTA and HgTA, capture the V-shape of volatility with respect to the asset growth rate in standard linear models without using nonlinear estimations.

2.4.2 Time Series Analysis

To assess the time series effect of the asset growth rate on return volatility, I form time series based on both the aggregate asset growth measure and the aggregate idiosyncratic volatility. Specifically, I calculate the cross-sectional average idiosyncratic volatility for each month. I then average over 12 months, from July to the next June, to generate the annual measure of the aggregate idiosyncratic volatility. Inspired by the cross-sectional result that the volatility has a V-shaped relationship in the asset growth rate, I adopt the dispersion of the cross-sectional asset growth rate between high-growth and low-growth firms as the annual aggregate asset growth measure. The dispersion, denoted by HMLgTA, is calculated as the average of HgTA minus the average of LgTA.[20]

To assess the level of residual autocorrelations, I calculate the Durbin-Watson (DW) statistic in all time series regressions.[21] Note that the t-statistics for all

[20]An alternative to the HMLgTA measure is the cross-sectional standard deviation of asset growth rate, which gives the same inferences in the time series regressions. These two measures have a correlation of 0.97.

[21]The deviation of the Durbin-Watson statistic from 2 shows the autocorrelation of regression residues. Specifically, DW smaller than 2 shows positive autocorrelation, while DW larger than 2

Table 2.3: Asset Growth and Idiosyncratic Volatility in Time Series

The table reports estimates from the time series regression of the average idiosyncratic volatility on the aggregate asset growth measure for the sample period from 1963 to 2009. The time series of average idiosyncratic volatility ($\overline{\text{IVol}}$) is calculated as the cross-sectional average idiosyncratic volatility for each year. The aggregate asset growth measure, denoted by HMLgTA, is the difference in average asset growth rates between high-growth firms and low-growth firms, with the breaking annual asset growth rate of 5%. I use both value-weighting (panel a) and equal-weighting (panel b) to calculate these averages. DW is the Durbin Watson statistic. The t-statistics are calculated using Newey-West adjusted standard errors with a lag length of 2.

	HMLgTA	DW	Adj. R^2 (%)
(a) Value-weighted	0.516 (10.4)	2.06	57.77
(b) Equal-weighted	1.94 (8.61)	1.21	43.03

annual time series regressions are adjusted for the residual autocorrelation through Newey-West adjusted standard errors with a lag length of 2 ($T^{1/3}/2 \approx 2$ with the total observation T=47).

Consider first the case in which all aggregate variables are value-weighted. Panel (a) of Table 2.3 shows that the aggregate asset growth measure is significantly positive in the time series (with the t-statistic equal to 10.4). Moreover, the aggregate asset growth measure demonstrates strong explanatory power for the average stock idiosyncratic volatility in terms of R-square (57.77%).

Panel (b) of Table 2.3 shows the regression result for equal-weighted aggregate measures. Again, the aggregate asset growth measure has very strong explanatory power in the time series.

From the above results, the asset growth measure has strong explanatory power for the idiosyncratic volatility in both equal-weighted and value-weighted time series. These results provide formal support of the time series prediction of my

shows negative autocorrelation.

theoretical model.

2.4.3 Multiple Regression Analysis

In this section, I further control for other variables that are related to return volatility, such as size, age and price. Note that some variables considered in the following analyses are highly correlated. For example, the correlation coefficient is 0.72 between price and size, -0.5 between ROE and VROE, and 0.43 between age and size. Therefore, it is not appropriate to include all variables in one regression. Instead, I control for the lagged volatility as well as *one* other variable at a time. My goal here is to demonstrate the significance of the asset growth rate in cross-sectional idiosyncratic volatility, and show that the influence of asset growth on idiosyncratic volatility cannot be explained by any control variables or the lagged volatility.[22]

Table 2.4 presents the univariate and multiple cross-sectional regressions that use the Fama-MacBeth approach. In the univariate regressions (panel (a) of Table 2.4), all variables except for MABA, Turnover, and Leverage are significant in terms of the t-statistic at the 5-percent level. The results show that (1) volatility is positively autocorrelated; (2) larger firms have lower volatility; (3) firms with highly volatile MABA have high volatility; (4) older firms have lower volatility; (5) higher price-per-share stocks have lower volatility; (6) dividend paying firms have lower volatility; and (7) stocks with higher return-on-equity and lower volatility of ROE have lower volatility. In terms of R-square, the most significant explanatory variable is lagged volatility, followed by price, size, DD and ROE.

Panel (b) of Table 2.4 first reports the pure asset growth effect and then controls for the lagged idiosyncratic volatility and one of the remaining variables. There are two major findings. First, the asset growth effect is still significant after controlling for the lagged volatility. Second, and more important, the asset growth

[22] In the cross-sectional regression including all explanatory variables, HgTA is still highly significant, while LgTA is insignificant. This still shows the explanatory power of asset growth for at least high growth firms.

Table 2.4: Multiple Regressions: Cross-section

The table reports Fama-MacBeth regressions in the cross-section for both univariate regressions (panel a) and multiple regressions (panel b). The dependent variable is the idiosyncratic volatility (IVol); and the sample covers the period from 1963 to 2009, with 47 cross-sections. LagVol is the lagged idiosyncratic volatility. LgTA equals gTA - 5% for asset growth rate values lower than 5%, and zero otherwise. HgTA equals gTA - 5% for asset growth rate values higher than 5%, and zero otherwise. See the description of all the other explanatory variables in Table 2.1. The t-statistics are adjusted for the first order autocorrelation of coefficient estimates from cross-sectional regressions. Note that the natural logarithms of both size and price are used.

	LgTA	HgTA	LagVol	Size	MABA	VMABA	Age	Price	Turn	Lev	DD	ROE	VROE	
(a) Univariate Regressions														
Coeff.			0.814	-0.151	0.018	0.032	-0.009	-0.335	0.20	0.018	-0.215	-0.39	0.603	
t-stat			(19.95)	(-3.90)	(1.55)	(6.01)	(-4.05)	(-3.97)	(0.79)	(0.40)	(-5.03)	(-9.06)	(2.20)	
Adj.R^2(%)			47.16	21.95	1.66	2.79	4.42	39.29	2.24	0.24	11.12	9.00	4.21	
(b) Multiple Regressions														Adj.R^2(%)
(1)	-1.74	0.161												9.57
	(-5.91)	(10.4)												
(2)	-0.42	0.064	0.797											49.27
	(-3.95)	(5.39)	(18.6)											
(3)	-0.26	0.057	0.723	-0.06										52.80
	(-2.88)	(6.07)	(19.0)	(-3.44)										
(4)	-0.41	0.060	0.802		0.001									49.37
	(-3.78)	(5.39)	(18.6)		(0.29)									
(5)	-0.41	0.053	0.801			0.006								49.86
	(-3.74)	(7.07)	(18.8)			(4.12)								
(6)	-0.42	0.050	0.781				-0.003							49.63
	(-3.93)	(5.40)	(18.5)				(-2.99)							
(7)	-0.062	0.049	0.61					-0.166						55.73
	(-0.92)	(5.65)	(16.9)					(-3.70)						
(8)	-0.40	0.073	0.817						-0.33					51.46
	(-3.60)	(4.95)	(18.1)						(-3.34)					
(9)	-0.42	0.065	0.821							0.053				49.66
	(-4.17)	(4.17)	(16.9)							(2.14)				
(10)	-0.37	0.044	0.760								-0.097			50.05
	(-3.56)	(4.93)	(18.2)								(-4.04)			
(11)	-0.22	0.063	0.770									-0.121		50.04
	(-3.50)	(5.84)	(18.3)									(-7.94)		
(12)	-0.387	0.059	0.789										0.133	49.50
	(-3.81)	(5.41)	(18.5)										(2.01)	

is a significant predictor of stock return volatility in the cross-section even after the lagged volatility and other volatility related variables are controlled for. The low-growth segment has a highly significant negative coefficient, and the high-growth segment has a highly significant positive coefficient.[23]

The result in Table 2.4 shows that the asset growth rate is still significant in

[23]The only exception is the seventh regression in panel (b) of Table 2.4 where the price is added: while HgTA is still highly significant, LgTA is insignificant (but still has a negative coefficient).

explaining the idiosyncratic volatility in the multiple cross-sectional regressions. Other variables such as price-per-share and the dividend dummy also have significant effects in the cross-section. However, if these variables are valid explanations, they should be significant in the time series as well. Therefore, a supplementary time series test is crucial, as it will distinguish the asset growth effect from other alternative explanations.

In the time series, I construct the other aggregate variables in a similar manner as I did for the aggregate volatility. For example, the aggregate market-to-book assets measure is calculated as the cross-sectional average using either equal-weighting or value-weighting of market-to-book assets. Data show that the correlations among these explanatory variables are very high. For example, in the value-weighted time series, the correlation coefficient is 0.89 between size and VROE, 0.84 between size and turnover, and 0.80 between VMABA and VROE. Similarly, in the equal-weighted time series, the correlation coefficient is 0.95 between size and turnover, 0.74 between VMABA and VROE, and -0.89 between ROE and VROE. Given such high correlations among these time series, it is not appropriate to control for several variables at the same time due to potential multicollinearity.[24] This is especially true for relatively short time series, as I have only 47 time series observations. Therefore, I only control for the lagged volatility and assess the explanatory power of one remaining variable at a time. My goal is to find those variables that are still significant after the lagged volatility is controlled for.

Table 2.5 shows the results of the univariate and multiple regressions using value-weighted aggregate measures. There are four important results. First, the asset growth measure is the most important explanatory variable in terms of the adjusted R-square in both the univariate (57.77%) and bivariate (55.73%) regressions. Second, the lagged volatility is not significant once the asset growth is controlled for. This shows the superior explanatory power of the asset growth

[24] Even in the full regression including all the control variables, the asset growth measure is still highly significant in the value-weighted time series, although it is insignificant in equal-weighting.

Table 2.5: Multiple Regressions: Value-weighted Time Series

The table reports results of the time series regressions using value-weighted data from 1963 to 2009, with 47 observations. The dependent variable is the average idiosyncratic volatility ($\overline{\text{IVol}}$). DW is the Durbin Watson statistic. The t-statistics are calculated using Newey-West adjusted standard errors with a lag length of 2.

	HMLgTA	LagVol	Size	MABA	VMABA	Age	Price	Turn	Lev	DD	ROE	VROE	DW	Adj. R^2(%)
(a) Univariate Regressions														
Coeff.	0.516	0.513	0.026	0.046	0.018	0.004	0.032	0.355	-2.24	-0.007	1.03	0.814		
t-stat	(10.4)	(2.25)	(2.32)	(4.34)	(1.61)	(0.94)	(0.55)	(2.32)	(-2.01)	(-1.00)	(2.36)	(2.48)		
Adj.R^2(%)	57.77	25.23	27.92	39.98	17.36	0.65	-0.72	15.93	15.65	-1.60	7.32	28.43		
DW	2.06	1.87	1.27	1.15	1.12	1.02	0.93	1.26	1.19	0.96	1.13	1.38		
(b) Multiple Regressions													DW	Adj. R^2(%)
(1)	0.514	0.001											2.06	55.73
	(5.63)	(0.01)												
(2)		0.322	0.017										1.79	33.13
		(1.20)	(1.46)											
(3)		0.261		0.037									1.68	44.79
		(1.50)		(3.00)										
(4)		0.398			0.008								1.74	26.35
		(1.60)			(0.74)									
(5)		0.512				0.0001							1.87	23.49
		(1.99)				(0.03)								
(6)		0.551					0.068						1.94	30.01
		(2.67)					(1.47)							
(7)		0.417						0.129					1.83	24.94
		(1.25)						(0.62)						
(8)		0.427							-1.31				1.90	28.60
		(2.01)							(-1.59)					
(9)		0.510								-0.005			1.87	23.79
		(2.21)								(-0.90)				
(10)		0.474									0.674		2.01	27.59
		(1.91)									(1.41)			
(11)		0.295										0.523	1.80	31.44
		(1.17)										(1.34)		

even compared to the lagged volatility.[25] Third, all the other competing variables except MABA are not significant once the lagged volatility is added to the regression. Lastly, even though MABA is still significant in the bivariate regression, its adjusted R-square (44.79%) is much lower than that of the asset growth

[25] Rubin and Smith (2008) demonstrate in a simulation that inferences in regressions with both highly persistent dependent variable (i.e., the time series of average volatility) and independent variables (e.g., average price or average market-to-book) suffer from a spurious regression problem, as discussed in Ferson et al. (2003). They also find that the Newey and West (1987) approach is inadequate to correct such a problem. Instead, simply controlling for the lagged dependent variable (i.e., the average volatility) gives correct inferences without eroding the power of the tests. Notice also the large difference in the Durbin-Watson statistics between the univariate and bivariate regressions in Table 2.5, with bivariate regressions much closer to 2.

Table 2.6: Multiple Regressions: Equal-weighted Time Series

The table reports results of the time series regressions using equal-weighted data from 1963 to 2009, with 47 observations. The dependent variable is the average idiosyncratic volatility ($\overline{\text{IVol}}$). DW is the Durbin Watson statistic. The t-statistics are calculated using Newey-West adjusted standard errors with a lag length of 2.

	HMLgTA	LagVol	Size	MABA	VMABA	Age	Price	Turn	Lev	DD	ROE	VROE	DW	Adj. R^2(%)
(a) Univariate Regressions														
Coeff.	1.94	0.689	0.121	0.392	0.055	-0.006	-0.364	2.47	-7.76	-0.835	-1.68	0.728		
t-stat	(8.61)	(4.98)	(1.75)	(3.30)	(2.14)	(-0.19)	(-5.76)	(2.40)	(-2.99)	(-5.06)	(-4.97)	(2.36)		
Adj.R^2(%)	43.03	47.78	11.93	24.49	7.08	-2.1	26.38	21.37	18.51	35.28	44.04	26.60		
DW	1.21	2.18	0.73	0.73	0.71	0.59	0.84	0.77	0.81	1.08	1.32	0.94		
(b) Multiple Regressions													DW	Adj. R^2(%)
(1)	1.11	0.478											2.18	56.58
	(2.73)	(2.85)												
(2)		0.645	0.037										2.14	47.74
		(3.69)	(0.73)											
(3)		0.590		0.233									2.01	55.32
		(4.00)		(2.25)										
(4)		0.712			-0.009								2.22	46.76
		(4.06)			(-0.46)									
(5)		0.691				-0.005							2.18	46.68
		(5.02)				(-0.25)								
(6)		0.589					-0.135						2.10	49.34
		(3.54)					(-2.30)							
(7)		0.604						0.878					2.07	48.85
		(3.12)						(0.97)						
(8)		0.611							-3.12				2.13	49.45
		(3.40)							(-1.24)					
(9)		0.531								-0.367			2.02	51.20
		(2.38)								(-1.44)				
(10)		0.458									-0.752		2.04	50.31
		(2.24)									(-1.80)			
(11)		0.608										0.179	2.10	47.66
		(3.09)										(0.71)		

(55.73%). This shows that the aggregate asset growth is the most significant explanatory variable for the aggregate idiosyncratic volatility in the value-weighted time series.

Table 2.6 shows both the univariate and multiple regression results for equal-weighted time series. Three results are worth noting. First, the aggregate asset growth rate retains its explanatory power even after the lagged volatility is controlled for. Furthermore, in the bivariate regressions, the aggregate asset growth measure gives the highest adjusted R-square (56.58%). Finally, MABA and price are also significant after the lagged volatility is controlled for.

In sum, the results in Tables 2.4 to 2.6 show that the asset growth measure has significant explanatory power in both the cross-section and the time series. MABA is the only other variable which is also significant in both the equal-weighted and value-weighted time series. However, MABA lacks power in the cross-section. Even in the time series, the asset growth effect is more important than MABA in terms of R-square in explaining the average idiosyncratic volatility. This implies that the asset growth measure is the *only* explanation which is significant in both the cross-section and time series.

2.5 Robustness Checks

In this section, I provide additional checks to show that the effect of the asset growth on firm-specific volatility is robust. Specifically, I demonstrate that the main result in Section 2.4 still holds (1) for alternative regression methods (pooled cross-section regressions and panel regressions with fixed effects), (2) for the pre-1995 time series, (3) using alternative measures of volatility, (4) using the monthly frequency of volatility in regressions, and (5) when past stock returns are controlled for. I then document that using alternative measures such as (6) the lagged investment-to-assets ratio and (7) the contemporaneous asset growth rate yield the same inferences as the lagged asset growth rate.

2.5.1 Alternative Regression Methods

In the main analysis in Section 2.4, I adopt Fama-MacBeth regressions in the cross-section. For robustness checks, I use two alternative regression methods: (i) pooled cross-section regressions and (ii) fixed effect panel regressions. In both cases, the standard errors are calculated by double clustering by firm and year.[26]

Panel (a) of Table 2.7 presents the results from the pooled cross-section regressions. Similar to the results of the main analysis, the slope is significantly negative for low (negative) growth firms and significantly positive for high growth firms.

[26] See Petersen (2009) for a comprehensive study that compares different approaches to panel data regressions in the finance literature. See Cameron et al. (2006) and Thompson (2011) for clustered standard errors in multiple dimensions.

Table 2.7: Alternative Regression Methods

The table reports the results using alternative regression methods. The dependent variable is the idiosyncratic volatility (IVol). Pooled cross-section regression runs a single cross-section regression. The panel regression includes firm and year fixed effects. Let the time series average of a firm's growth rate be $\overline{gTA}$. Then $\widehat{LgTA}$ equals gTA - $\overline{gTA}$ for asset growth rate values lower than $\overline{gTA}$, and zero otherwise. $\widehat{HgTA}$ equals gTA - $\overline{gTA}$ for asset growth rate values higher than $\overline{gTA}$, and zero otherwise. All the t-statistics are calculated using standard errors clustered by both firm and year.

(a) Pooled Cross-section Regression

		LgTA	HgTA	LagVol	R^2(%)
	(1)	-2.14	0.169		6.87
		(-11.3)	(12.2)		
	(2)	-0.586	0.072	0.714	36.68
		(-5.05)	(5.14)	(14.3)	

(b) Fixed Effect Panel Regression

		$\widehat{LgTA}$	$\widehat{HgTA}$	LagVol	R^2(%)
Fixed Firm Effect:	(3)	-0.491	0.042		43.39
		(-9.27)	(3.63)		
	(4)	-0.239	0.020	0.422	49.95
		(-4.74)	(2.12)	(9.98)	
Fixed Firm and Year Effect:	(5)	-0.467	0.036		47.04
		(-11.5)	(4.46)		
	(6)	-0.236	0.020	0.420	53.08
		(-6.29)	(2.96)	(10.7)	

This is still true after the lagged volatility is controlled for. The R-squares are comparable to, though lower than, those of the Fama-Macbeth regressions.

To capture the potential heterogeneity among firms, i.e., firms may have different levels of fundamental risks (e.g., demand risks) and/or growth potentials (e.g., the average growth rate), I run additional panel regressions with fixed firm effects. For firms with different average growth rates, the return volatility should be high when the growth rate is far away from each firm's average growth rate, and low when the growth rate is close to the firm's average growth rate. Therefore, I break the asset growth measure of a firm into two measures $\widehat{\text{LgTA}}$ and $\widehat{\text{HgTA}}$, using the firm's time series average growth rate as the breaking point. Panel (b) of Table 2.7 presents the results. Again, there is a V-shaped relationship between the idiosyncratic volatility and the asset growth rate. With firm fixed effects (regressions (3) and (4) in Table 2.7), firms that grow at a rate far away from their normal average have higher idiosyncratic return volatility. The results are similar if I control for both the fixed firm effect and the fixed year effect (regressions (5) and (6) in Table 2.7).

In summary, the two alternative methods, i.e., the pooled cross-section regression and the panel regression with fixed effects, yield the same inferences as the Fama-MacBeth regressions. After the firm heterogeneity is controlled for, firms that grow at rates which are either higher or lower than their normal average growth rate tend to have higher return volatility.

2.5.2 Pre-1995 Time Series

Rubin and Smith (2008) find that the existing explanations of the average idiosyncratic volatility lack power in the pre-1995 period, which excludes the episode of rising and then falling idiosyncratic volatility around year 2000. Here I conduct time series regressions and show that the asset growth has a robust explanatory power for the pre-1995 sub-sample. As shown in Section 2.4.3, MABA is the only variable that is significant in the value-weighted time series regression in addition to the asset growth measure and the lagged volatility. Table 2.8 re-

Table 2.8: 1963-1995 Time Series

The table reports the time series regression results using data from 1963 to 1995, with 33 observations. The dependent variable is the average idiosyncratic volatility ($\overline{\text{IVol}}$). All variables are value-weighted. DW is the Durbin Watson statistic. The t-statistics are calculated using Newey-West adjusted standard errors with a lag length of 2.

	LagVol	HMLgTA	MABA	DW	Adj.R^2(%)
(1)	0.385	0.182		1.93	33.37
	(2.97)	(2.06)			
(2)	0.548		0.004	1.88	25.37
	(3.48)		(0.57)		

ports the time series regression result and shows that MABA is insignificant in the pre-1995 period, which is consistent with Rubin and Smith (2008). This implies that the explanatory power of MABA comes mostly from the short period of time around year 2000. In contrast, the aggregate asset growth measure is still significant in this sub-sample, even when the episode around year 2000 is excluded.

2.5.3 Alternative Measures of Volatility

So far I have defined the idiosyncratic volatility relative to the four-factor model. To show that the results are not driven by this specific choice of model, I use alternative methods to calculate the idiosyncratic volatility. Specifically, I consider alternative idiosyncratic volatility measures which are adjusted by (i) the Fama-French three-factor model; (ii) the CAPM; and (iii) the market return without beta estimations. Finally, I also look at the asset growth effect on the total volatility, which is model-free.

It is worth noting that the systematic volatility, which equals the total volatility minus the idiosyncratic volatility, is also reported in Table 2.1 for asset growth rate sorted deciles. Under equal-weighting, the systematic volatility accounts for only 10% of the total volatility. Under value-weighting, the systematic volatility ranges from 17% to 34% of the total volatility across the asset growth deciles.[27] Similar to the idiosyncratic volatility, the systematic volatility shows a V-shape with respect to the asset growth rate, indicating that the total volatility is also a V-shape.

Table 2.9 shows the results of both the cross-section and time series using different definitions of the idiosyncratic volatility as well as the total volatility. These yield the same results as the main analysis, i.e., the four-factor adjusted idiosyncratic volatility (compare Table 2.9 with Tables 2.2 and 2.3).

2.5.4 Monthly Frequency of Idiosyncratic Volatility

In the main analysis in Section 2.4, I use the annual frequency of idiosyncratic volatility in the regressions, since most explanatory variables are measured annually. Alternatively, I can also run monthly frequency regressions with monthly volatility and annual explanatory variables. That is, I can simply assign annual measures to each of the 12 months in the following year. This approach has been widely used in monthly expected return regressions (see, e.g., Fama and French (1992)).

The results of the monthly cross-sectional regression (panel (a) of Table 2.10) are similar to those of the annual regressions (Table 2.4). Also, the results of the time series regressions with monthly data using value-weighted and equal-weighted time series (panel (b) of Table 2.10) are similar to the results using annual data (Tables 2.5 and 2.6). The only noticeable difference is an increase in the R-squares after controlling for the lagged volatility, which is natural due to the higher frequency of monthly volatility.

[27] Note that the systematic volatility as a fraction of the total volatility (i.e., the factor model R-square) is an inverted V-shape with respect to the asset growth rate. That is, the stocks with extreme growth have lower R-squares. In other words, the factor model explains fewer fractions of return variations of extreme growth stocks, which also have extreme return volatility.

Table 2.9: Alternative Measures of Volatility

The table shows the cross-secitonal and value-weighted time series regressions with each of the following alternative measures of volatility as the dependent variable: (1) Fama-French three-factor adjusted idiosyncratic volatility, (2) CAPM adjusted idiosyncratic volatility, (3) market adjusted idiosyncratic volatility, and (4) total volatility.

(a) Cross-section

	LgTA	HgTA	LagVol	Adj. R^2(%)
(1) Fama-French 3-factor adjusted IVol	-1.747	0.163		9.57
	(-5.92)	(10.4)		
	-0.427	0.065	0.797	49.20
	(-3.95)	(5.39)	(18.7)	
(2) CAPM adjusted IVol	-1.771	0.166		9.62
	(-5.92)	(10.5)		
	-0.434	0.065	0.797	49.24
	(-3.92)	(5.49)	(18.7)	
(3) Market adjusted IVol	-1.786	0.169		9.69
	(-6.01)	(10.88)		
	-0.440	0.065	0.797	49.27
	(-3.97)	(5.50)	(18.7)	
(4) Total Volatility	-1.781	0.177		9.73
	(-5.84)	(11.33)		
	-0.445	0.067	0.791	48.97
	(-3.96)	(6.00)	(19.2)	

(b) Time Series

	HMLgTA	LagVol	DW	Adj.R^2(%)
(1) Fama-French 3-factor adjusted IVol	0.534		2.04	58.26
	(10.7)			
	0.525	0.014	2.06	56.26
	(5.74)	(0.11)		
(2) CAPM adjusted IVol	0.578		2.07	54.68
	(10.4)			
	0.581	-0.008	2.06	52.58
	(6.20)	(-0.07)		
(3) Market adjusted IVol	0.666		2.09	53.93
	(9.83)			
	0.680	-0.024	2.06	52.00
	(6.39)	(-0.24)		
(4) Total Volatility	0.827		2.06	38.13
	(10.2)			
	0.819	0.011	2.08	35.75
	(8.10)	(0.17)		

Table 2.10: Monthly Frequency of Idiosyncratic Return Volatility

The table reports the regression results using monthly frequency of idiosyncratic volatility as the dependent variable, with a total of 558 months. The cross-sectional regressions correspond to the annual frequency results reported in Table 2.4. The time series regressions correspond to the annual frequency results reported in Tables 2.5 and 2.6.

(a) Cross-section

	LgTA	HgTA	LagVol	Adj. R^2(%)
	-1.365	0.139		6.10
	(-9.85)	(16.94)		
	-0.301	0.031	0.770	61.89
	(-16.0)	(30.5)	(141.9)	

(b) Tim Series

	HMLgTA	LagVol	DW	Adj.R^2(%)
Value-weighted	0.405		0.543	43.41
	(6.67)			
	0.095	0.756	2.24	75.16
	(3.47)	(16.1)		
Equal-weighted	1.548		0.231	37.15
	(10.8)			
	0.184	0.889	1.91	87.49
	(3.15)	(28.8)		

2.5.5 Controlling for Past Stock Returns

Arguably, the asset growth effect on idiosyncratic volatility can be driven by stocks with extreme past returns, i.e., past "losers" and "winners." The reason is as follows. Extreme return stocks tend to have extreme concurrent asset growth and high concurrent return volatility. Since the return volatility is highly persistent, stocks with extreme past returns, and hence extreme past asset growth, will have higher future return volatility. In other words, the asset growth effect documented in this study might simply be a mechanical result of volatility persistency.

Note first that I have taken into account the persistence of idiosyncratic volatility in the multiple regression analysis in Section 2.4.3. Although I do find that the idiosyncratic volatility is highly persistent with a first order autocorrelation of 0.814 (see Table 2.4 for a univariate regression of the idiosyncratic volatility on its lag), the asset growth measure is still significant when the lagged volatility is controlled for.

To further address this concern, I double sort stocks independently on the asset growth and past returns. Specifically, I form portfolios for July of year t to June of year $t+1$ by using returns concurrent to the asset growth measure in year $t-1$.[28] For each past return quintile, I then calculate the average idiosyncratic volatility for each asset growth decile, similar to Fig.2.2. Fig.2.4 plots the average idiosyncratic volatility for each asset growth decile and past return quintile. Although the extreme losers have much higher idiosyncratic volatility, for each past return quintile there is a V-shaped relationship between the idiosyncratic volatility and the asset growth rate.[29] This implies that the asset growth effect on the cross-sectional idiosyncratic volatility still exists even after the past returns are controlled for.

Since there is a similar V-shaped relationship, I follow the same procedure to process the data as I did for the asset growth rate. In particular, I separate the stocks into low and high past return groups, with the breaking point of 0.9% in the monthly return. I then assign the two past return measures to each stock accordingly. Similarly, in the time series regressions, I use the cross-sectional dispersion in past return as the aggregate past return measure. This dispersion is calculated as the average of the high past return measure minus the average of the low past return measure.[30]

[28]Using returns from July $t-1$ to June t yields the same inference.

[29]Note that the extreme losers account for only 20% of the sample when using equal-weighting, and even less when using value-weighting. Note also that the double sorting is independent. Therefore, the number of stocks in each pair of past return and asset growth rate should not be the same in general.

[30]The time series results are qualitatively the same if the cross-sectional standard deviation of past returns is used instead.

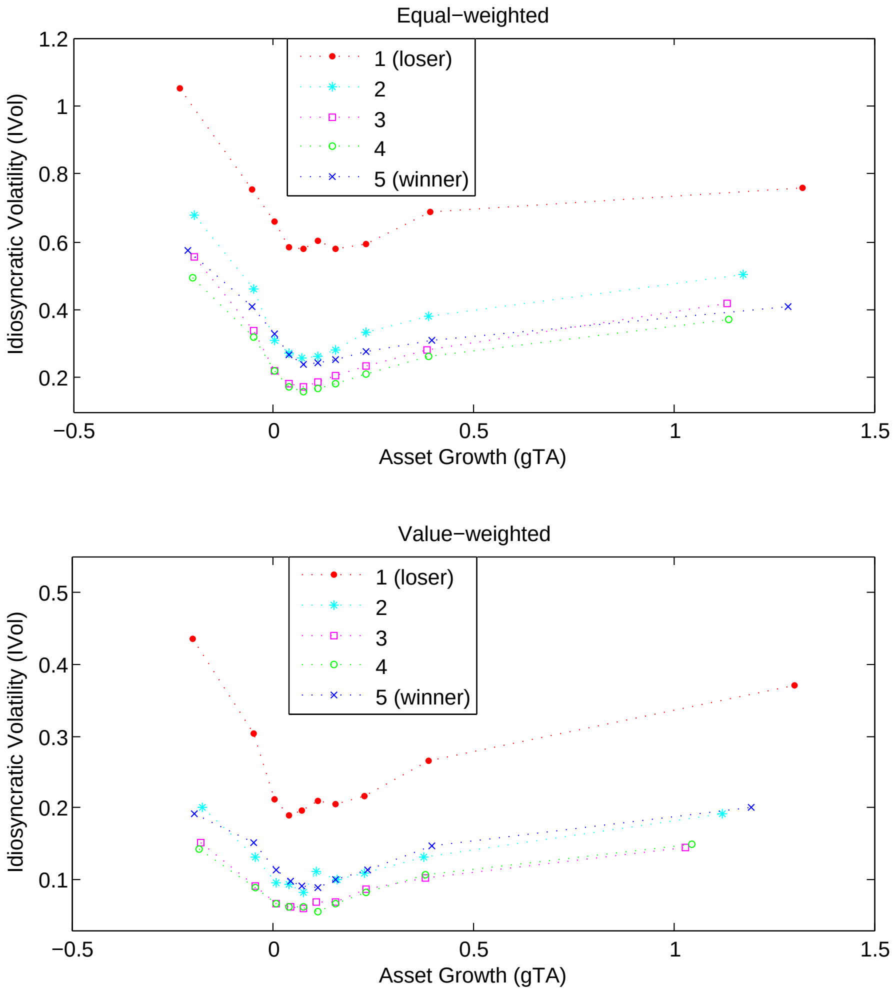

Figure 2.4: Idiosyncratic Volatility for Momentum and Asset Growth Portfolios

The figure plots the average idiosyncratic volatility of portfolios double sorted by past return and asset growth. The sorting method and data sample are the same as those of Figure 2.2, except that past return is also used. I double sort independently over the past 12 months log returns (quintiles) and last year's asset growth (deciles). Specifically, I sort stocks for year t (July t to June $t+1$) according to the average log returns of the past 12 months, over which the asset growth rate for year $t-1$ is calculated. I then sort the stocks according to the last year's asset growth rate.

Table 2.11: Past Return and Idiosyncratic Return Volatility

The table reports regression results when past stock returns are controlled for. For cross-sectional regressions, the procedure is the same as that of Table 2.4. The definition of LReturn and HReturn is similar to that of LgTA and HgTA, except that the breaking point for past return is set at 0.9%. Note that the past return is calculated as the monthly average log returns for the same period in which the asset growth rate is calculated. In the time series, HMLReturn is calculated in a way similar to the calculation of HMLgTA.

(a) Cross-section

	LgTA	HgTA	LagVol	LReturn	HReturn	Adj. R^2(%)
(1)				-7.14 (-4.78)	2.06 (10.3)	12.82
(2)			0.829 (19.2)	-1.91 (-3.67)	-0.12 (-0.66)	51.19
(3)	-1.25 (-7.08)	0.115 (8.93)		-5.93 (-4.42)	1.62 (10.2)	17.94
(4)	-0.28 (-4.1)	0.063 (6.04)	0.812 (19.1)	-1.67 (-3.59)	-0.35 (-2.27)	51.69

(b) Time Series

		HMLgTA	LagVol	HMLReturn	DW	Adj.R^2(%)
	(1)			3.66 (1.64)	1.47	17.78
Value-weighted	(2)		0.456 (2.60)	0.596 (0.27)	1.85	23.69
	(3)	0.515 (5.79)	-0.07 (-0.46)	0.693 (0.69)	2.06	54.95
	(4)			11.49 (2.78)	1.04	15.46
Equal-weighted	(5)		0.820 (8.34)	-5.12 (-1.38)	2.25	48.38
	(6)	1.10 (2.99)	0.602 (3.81)	-4.75 (-1.40)	2.25	57.14

Panel (a) of Table 2.11 shows that although the lagged returns have explanatory power in the cross-section, the asset growth measures are still highly significant when both the lagged return and lagged volatility are controlled for. However, the effect of past winners is not robust, as the coefficient changes from negative (regressions (1) and (3)) to positive (regressions (2) and (4)). Panel (b) of Table 2.11 shows the time series results. For both value-weighted and equal-weighted time series, the dispersion in the past return turns out to be insignificant once either the lagged volatility or the asset growth is added into the regression. Note that, however, the asset growth measure is still highly significant in both cases. These results indicate that the asset growth effect on idiosyncratic volatility in the cross-section as well as in the time series is robust and not driven by extreme past return stocks.

2.5.6 Investment-to-assets Ratio

In the main analysis, I use the asset growth rate as the broad measure of a firm's investment rate. Alternatively, one can use the investment-to-assets ratio as the measure of a firm's investment rate. I follow Lyandres et al. (2008) and measure investment-to-assets ratio as the annual change in net property, plant, and equipment (PPE) plus the annual change in inventories divided by the lagged book value of assets. I exclude financial firms (with siccd between 6000 and 6999) for this industrial specific measure.

The results are reported in Table 2.12. The investment-to-assets ratio has a similar relationship with the idiosyncratic volatility as the broader asset growth measure in both the cross-section and the time series. Specifically, in the cross-section, there is a V-shaped relationship between the idiosyncratic volatility and the investment-to-assets ratio. In the time series, the cross-sectional standard deviation of investment-to-assets ratio positively explains/predicts the average idiosyncratic volatility.

Table 2.12: Investment-to-assets Ratio and Idiosyncratic Volatility

The table reports the regression results using the investment-to-assets ratio as an alternative measure of investment rate. The definition of LIA and HIA is similar to that of LgTA and HgTA in Table 2.4, except that the breaking point for investment-to-assets ratio (IA) is set at 4%. In the time series, the cross-sectional standard deviation of IA ratio (sdIA) is used as an alternative measure of dispersion in firms' investment.

(a) Cross-section

	LIA	HIA	LagVol	Adj. R^2(%)
	-0.413 (-2.87)	0.203 (3.63)	0.834 (18.7)	48.70

(b) Time Series

	sdIA	LagVol	DW	Adj.R^2(%)
Equal-weighted	1.593 (1.42)		0.59	4.23
	1.164 (2.13)	0.698 (5.95)	2.27	53.21
Value-weighted	0.681 (2.58)		0.81	15.51
	0.424 (2.24)	0.575 (3.10)	1.81	44.81

2.5.7 Contemporaneous Asset Growth

In the main analysis, I use the *lagged* asset growth to explain and predict the current idiosyncratic volatility. It is also interesting to explore the relationship between the *contemporaneous* growth rate and idiosyncratic volatility.

In the following regressions, I use the asset growth measure which is concurrent with the idiosyncratic volatility measure. Specifically, I regress the average idiosyncratic volatility from January to December of year t on the asset growth between year $t-1$ and t. Table 2.13 reports a V-shaped relationship between the idiosyncratic volatility and the contemporaneous asset growth in the cross-section.

It also shows a positive relationship in the time series between the average idiosyncratic volatility and the cross-sectional dispersion in the contemporaneous asset growth rate. Both results are similar to the findings in the main analysis. Furthermore, the *contemporaneous* asset growth has a stronger relationship with the idiosyncratic volatility than the *lagged* asset growth in terms of goodness-of-fit in the cross-section. In particular, the adjusted R-square is 13.55% with contemporaneous asset growth (see Table 2.13), while it is 9.57% with the lagged asset growth measure (see Table 2.2).

2.6 Asset Growth versus Alternative Explanations

In this section, I compare the explanatory power of the asset growth with three alternative explanations in the prior literature: (1) cash flow (profitability) and its time series variance (Pastor and Veronesi (2003) and Wei and Zhang (2006)), (2) growth options proxied by market-to-book assets (Cao et al. (2008)), and (3) forecasted long-term earnings growth (Malkiel and Xu (2003)). The following results show that the asset growth effect on the idiosyncratic volatility dominates all these alternative explanations.

2.6.1 Asset Growth versus Cash Flow Volatility

Pastor and Veronesi (2003) examine the idiosyncratic risk in a valuation model where investors learn about profitability. (They define profitability as the cash flow per dollar book value of equity.) They find that younger stocks, stocks that pay no dividends, and high market-to-book equity stocks have more volatile returns. Wei and Zhang (2006) use return-on-equity (ROE) and its time series variance (VROE) as the proxies of profitability and its uncertainty.

Irvine and Pontiff (2009) also study the effect of unexpected cash flow shocks on the average idiosyncratic volatility in the time series. They interpret the increasing cash flow volatility as a result of intensified competition among firms over time. However, they do *not* provide *cross-sectional* relationship between the

Table 2.13: Contemporaneous Asset Growth and Idiosyncratic Volatility

The table reports the regression results when the contemporaneous asset growth rate is used as the explanatory variable instead of the lagged growth rate. The asset growth measures are concurrent with the idiosyncratic volatility measure. Specifically, in the cross-section, the average idiosyncratic volatility of January t to December t is regressed on the asset growth rate between years $t-1$ and t. In the time series, the cross-sectional average volatility is regressed on the concurrent cross-sectional dispersion of asset growth (HMLgTA).

(a) Cross-section

	LgTA	HgTA	LagVol	Adj. R^2(%)
	-1.813	0.137		13.55
	(-3.35)	(8.47)		
	-0.944	0.035	0.705	53.01
	(-2.92)	(4.09)	(15.2)	

(b) Time Series

	HMLgTA	LagVol	DW	Adj.R^2(%)
Equal-weighted	1.56		0.59	38.08
	(5.39)			
	0.419	0.796	1.44	79.27
	(2.14)	(10.26)		
Value-weighted	0.644		1.26	61.95
	(8.79)			
	0.517	0.379	2.02	72.74
	(6.44)	(8.47)		

cash flow volatility and idiosyncratic volatility. Since Wei and Zhang (2006) provides *both* cross-section and time series results, I follow their measures of cash flow (i.e., ROE) and volatility of cash flow (i.e., VROE).

In the robustness analysis, I control for ROE and VROE one at a time and find that they are both significant in the cross-sectional regressions but insignificant in the bivariate time series regressions. To further show the significance of the asset growth measure, I control for ROE and VROE simultaneously. Results in Table

Table 2.14: Asset Growth vs. Cash Flow Volatility

The table reports the comparison between the asset growth effect and the cash flow effect on the idiosyncratic volatility. The cross-sectional regression is conducted similar to that of Table 2.4. The value-weighted time series regressions are conducted similar to those in Table 2.5.

(a) Cross-section

	LgTA	HgTA	LagVol	ROE	VROE	Adj. R^2(%)
	-0.21	0.063	0.767	-0.13	0.123	50.19
	(-3.34)	(6.01)	(18.3)	(-7.92)	(0.99)	

(b) Time Series

	HMLgTA	LagVol	ROE	VROE	DW	Adj.R^2(%)
		0.274	0.588	0.491	1.90	32.97
		(0.99)	(1.28)	(1.16)		
	0.630	0.031	0.279	-0.385	2.26	56.78
	(4.99)	(0.25)	(0.61)	(-1.72)		

2.14 show that the asset growth measure is still significant in the cross-sectional regressions. Table 2.14 also reports the result of the value-weighted time series regressions after controlling for ROE and VROE. It turns out that the aggregate asset growth measure is the only significant variable among the four variables (the asset growth measure, ROE, VROE, and the lagged volatility) in the time series regression. These results indicate that the asset growth has a stronger effect than the cash flow and its volatility measures documented in the prior literature.

2.6.2 Asset Growth versus Growth Options

Cao et al. (2008) argue that firms with more growth options, proxied by the market-to-book assets ratio (MABA), or with higher variations in growth options, proxied by the time series variance of MABA (VMABA), should have higher idiosyncratic volatility. However, they only test their explanation in a time series without showing cross-sectional evidences. I conduct such a cross-sectional test with measures in Cao et al. (2008) and do not find any support of their explanation. As shown in Table 2.15, MABA is insignificant in the cross-sectional regression. Note, however, that the asset growth measures are still significant in the cross-sectional regressions after both MABA and VMABA are controlled for. Table 2.15 also shows that the aggregate asset growth measure retains its power after controlling for both MABA and VMABA in the value-weighted time series.

It is worthwhile to point out that there also exists a V-shaped relationship between the idiosyncratic volatility and the market-to-book assets ratio.[31] Note that the V-shape of idiosyncratic volatility versus MABA is quite different from the V-shape of idiosyncratic volatility versus asset growth rate in terms of the number of firms in the two segments of the V-shape. In the case of the asset growth rate, the two segments of the V-shape have almost the same number of stocks, with the breaking point in decile 5 (see Table 2.1). However, in the case of MABA, the left segment has much fewer stocks and the breaking point is in decile 3.

Similar to the asset growth, the V-shape with respect to MABA can be captured by a two-segment piecewise linear model. In particular, I split the MABA into two measures, LMABA and HMABA (similar to splitting gTA into LgTA and HgTA).[32] The results of the cross-sectional regressions are given in Table 2.15. The pure MABA effect on the idiosyncratic volatility only delivers a relatively low R-square of 3.02%. Recall from Table 2.4 that the pure asset growth effect has an R-square of 9.57%. Moreover, breaking MABA into two segments does not affect the significance of the asset growth measure. Similar to the case of asset growth, I use the difference between the average of HMABA and the average of LMABA in the time series regressions instead of the simple average of MABA

[31] I find a similar V-shape for deciles sorted by book-to-market equity ratio. This is consistent with the findings by Carlson et al. (2004) and Chen (2009), but raises concerns on the finding of Pastor and Veronesi (2003) that market-to-book equity has a significant and positive effect on idiosyncratic volatility in the cross-section. Recall from the results using annual cross-sectional regressions in Table 2.4 that MABA is not significant in the univariate cross-sectional regression and the R-square is very low (1.66%).

[32] An alternative way to capture the V-shape is to add a square term of MABA in the cross-sectional regressions; and the results are qualitatively the same.

Table 2.15: Asset Growth vs. Growth Options

The table reports the comparison between the asset growth effect and the growth options effect on the idiosyncratic volatility. The definition of LMABA and HMABA is similar to that of LgTA and HgTA, except that the breaking point for market-assets-to-book-assets (MABA) is set at 0.95. The dispersion of MABA (HMLMABA) is constructed in the same way as HMLgTA. The cross-sectional regression is conducted similar to that of Table 2.4. The value-weighted time series regressions are conducted similar to those in Table 2.5.

(a) Cross-section–I

LgTA	HgTA	LagVol	MABA	VMABA	Adj. R^2(%)
-0.41 (-3.66)	0.053 (6.59)	0.800 (18.7)	-0.004 (-0.89)	0.007 (4.60)	50.00

(b) Cross-section–II

LgTA	HgTA	LagVol	LMABA	HMABA	Adj. R^2(%)
			-1.13 (-2.51)	0.033 (2.76)	3.02
-0.398 (-3.76)	0.061 (5.38)	0.799 (18.6)	-0.236 (-2.38)	0.004 (1.02)	49.49

(c) Time Series–I

HMLgTA	LagVol	MABA	VMABA	DW	Adj.R^2(%)
	0.309 (1.62)	0.043 (2.92)	-0.006 (-1.15)	1.76	44.65
0.456 (4.33)	0.061 (0.48)	0.022 (2.52)	-0.010 (-3.79)	2.16	58.92

(d) Time Series–II

	LagVol	HMLMABA	DW	Adj.R^2(%)
1963–2009 Period		0.046 (5.33)	1.18	42.20
	0.245 (1.47)	0.037 (3.45)	1.67	46.37
1963–1995 Period		-0.005 (-0.49)	0.90	-2.08
	0.545 (3.46)	0.004 (0.54)	1.88	25.26

used in the robustness analysis.[33] I choose the breaking point of 0.95 for MABA, which is obtained by fitting the breaking point in a two-segment, piecewise linear model. Table 2.15 shows that although the cross-sectional dispersion in MABA is significant in the full sample, it has no explanatory power in the value-weighted, pre-1995 time series.

The above results show that the two measures used by Cao et al. (2008) (MABA and VMABA) in the time series lack explanatory power in the cross-section. In contrast, after both measures are controlled for, the asset growth maintains its explanatory power in both the cross-section and the time series. Moreover, after I take into account the V-shape of idiosyncratic volatility in MABA, MABA still lacks power in the cross-section and the pre-1995 time series. This indicates that the asset growth measure is a better explanation for idiosyncratic volatility than MABA, the proxy of growth options.

2.6.3 Asset Growth versus Long-term Earnings' Growth

In this section, I distinguish between the asset growth measure and the expected long-term growth, which Malkiel and Xu (2003) proxy with the expected growth of long-term earnings, i.e., the earnings per share (EPS) growth as forecasted by IBES over the next 3 to 5 years. Although both the asset growth rate and EPS growth are growth measures, they have obvious differences. First, the asset growth rate measures the total asset growth, while the EPS growth is the growth of earnings *per share*. So if a firm grows by issuing new shares without changing its EPS, the asset growth measure still captures this growth while EPS does not. Second, the asset growth is the realized past asset growth, while EPS growth is the forecasted future growth. While the realized asset growth has no noise, the forecasted EPS growth can have substantial noises. Third, the asset growth measure is available for almost all public firms since 1963, while EPS long-term growth is only available from early 1980s and for relatively large stocks.

[33] I also use the cross-sectional standard deviations of MABA in the time series regressions. The results are qualitatively the same.

Nevertheless, I still separate the two types of growth in the cross-sections where both measures are available. I collect long-term EPS growth from IBES for the period from 1982 to 2009. I use the median of monthly forecasts to calculate the annual average of the forecasted EPS long-term growth. I then match the long-term growth rate calculated from July of year $t-1$ to June of year t based on IBES forecasts with the average monthly volatility from July of year t to June of year $t+1$. The pooled cross-sectional correlation between the asset growth rate and long-term EPS growth is only 0.38, indicating that the two are not highly correlated.

It is worth noting that Malkiel and Xu (2003) also find an asymmetric V-shaped relationship between the idiosyncratic volatility and long-term EPS growth.[34] I set up a two-segment, piecewise linear model of idiosyncratic volatility with respect to the forecasted long-term EPS growth. I confirm that there indeed exists an asymmetric V-shaped relationship between the idiosyncratic volatility and the long-term EPS growth. However, comparing regressions (1) and (2) in Table 2.16, I also find that introducing the V-shape generates a very small gain in R-square relative to a simple linear model.[35] This contrasts with my results, which show that both the low-growth and high-growth segments are very important in the cross-sectional idiosyncratic volatility regressions. Moreover, the long-term EPS growth is not significant once lagged volatility is added to the regression (regressions (3) and (4) in Table 2.16). Note, however, that the asset growth is highly significant in this smaller sample (regressions (5) and (6) in Table 2.16).

More important, when the long-term EPS growth is controlled for in regressions (7) to (10) in Table 2.16, the asset growth effect is still significant and has the same asymmetric V-shape as in the main analysis. This shows that the past realized asset growth has explanatory power for idiosyncratic volatility in additional

[34]Note that the idiosyncratic volatility in Malkiel and Xu (2003) is calculated using returns in three years (one year before and two years after the IBES statistic period). My idiosyncratic volatility measure is calculated using returns of 12 months after the asset growth rate is known.

[35]Note that Malkiel and Xu (2003) do not report their R-squares in the two-segment, piecewise linear model (see Table 9 in Malkiel and Xu (2003)).

Table 2.16: Asset Growth vs. Long-term EPS Growth

The table reports the comparison between the asset growth effect and the Long-term EPS Growth effect on the idiosyncratic volatility. The long-term EPS growth (EPSLTG) is obtained from IBES. The cross-sectional regressions use the data from 1982 to 2009. The asset growth measures, LgTA and HgTA, are the same as in Table 2.4. The definitions of LEPSLTG and HEPSLTG are similar to those of LgTA and HgTA. The breaking points for gTA and EPSLTG are set at 2% and 2.5% respectively, which are obtained from the fitted breaking points in the two-segment, piecewise linear models. The t-statistics are adjusted for the first order autocorrelation of estimates in the cross-sectional regressions.

	LgTA	HgTA	LagVol	EPSLTG	LEPSLTG	HEPSLTG	Adj. R^2 (%)
(1)				1.054 (3.41)			8.13
(2)					-0.092 (-0.51)	1.063 (3.48)	8.21
(3)			1.124 (10.3)	0.046 (0.37)			40.96
(4)			1.124 (10.4)		0.086 (0.39)	0.046 (0.37)	40.89
(5)	-1.14 (-5.31)	0.114 (7.11)					6.24
(6)	-0.161 (-2.90)	0.022 (3.26)	1.119 (11.2)				40.56
(7)	-1.07 (-5.65)	0.046 (3.40)		p.983 (3.54)			11.74
(8)	-1.07 (-5.64)	0.044 (3.44)			0.310 (3.24)	0.987 (3.56)	11.81
(9)	-0.157 (-2.84)	0.021 (2.79)	1.117 (10.3)	-0.001 (-0.02)			40.87
(10)	-0.160 (-2.90)	0.020 (2.67)	1.118 (10.3)		0.145 (0.77)	-0.003 (-0.03)	40.94

to the forecasted long-term EPS growth.

To summarize the results from the comparisons in Sections 2.6.1 to 2.6.3, the asset growth measure retains its explanatory power in both the cross-section and the time series, even after several closely related alternative explanations are controlled for. Moreover, the asset growth measure shows stronger explanatory power than these alternatives. Specifically, although cash flows and volatility of cash flows (proxied by ROE and VROE respectively) show some explanatory power in the cross-section, they do not show explanatory power in the value-weighted time series. Similarly, although MABA has explanatory power in the full time series (though not in the pre-1995 period), it lacks power in the cross-section. Finally, the expected growth in long-term earnings, proxied by forecasted long-term EPS growth, becomes insignificant once lagged volatility is added to the cross-sectional regressions.

2.7 Conclusion

This chapter studies the effect of frictions to real investment on return volatility. The model predicts a nonlinear V-shaped relationship between the idiosyncratic return volatility and the asset growth rate. The high return volatility of low (negative) growth firms is due to the amplification of shocks to unexpected cash flows by firms' low market values. In contrast, the high return volatility of high growth firms is driven by the amplification of shocks to expected cash flows through growth opportunities. Aggregating the V-shape in each cross-section, the model also predicts a positive correlation between the cross-sectional dispersion in the asset growth rates and the average idiosyncratic return volatility.

This study also documents strong empirical evidence that supports the model's predictions. Empirical analysis confirms the V-shaped relationship between the idiosyncratic return volatility and the asset growth rate in the cross-section. That is, in the cross-section, stocks with either high positive or negative asset growth rates have high idiosyncratic return volatility in the next year. In the time series, a higher cross-sectional dispersion of the firm-level asset growth rate empirically predicts a higher average idiosyncratic return volatility in the next year.

This study further documents the robustness of the empirical findings. The V-shaped relationship in the cross-section is robust even after controlling for factors such as size and leverage. The asset growth measure turns out to be the most important predictor of the average idiosyncratic return volatility in the value-weighted time series. Moreover, the asset growth effect on idiosyncratic return volatility, which is driven by investment frictions, empirically dominates alternative explanations of idiosyncratic return volatility such as cash flow and its volatility, growth options, and forecasted long-term earnings growth.

Chapter 3

Endogenous Asset Fire Sales and Bank Lending Incentives [1]

3.1 Introduction

The financial crisis of 2007-2009, starting with the U.S. subprime mortgage crisis, has been the worst since the Great Depression. Its global effects include the failure and bailout of key financial institutions, the decline in consumer wealth estimated in trillions of U.S. dollars, substantial financial commitments incurred by many governments, and a significant decline in economic activities.[2]

Many illiquid assets, such as mortgage-backed securities, have been sold at fire-sale prices due to the poor market condition during the crisis. Huge losses from these securities have impacted the lending capacity of financial institutions such as banks, which in turn slows economic activities. Concerns regarding the stability of key financial institutions have driven governments around the world to

[1] I would like to thank Jan Bena, Adlai Fisher, Lorenzo Garlappi, Ron Giammarino, Hernan Ortiz-Molina, Tan Wang, Ralph Winter, and seminar participants at the University of British Columbia and the Northern Finance Association 2010 Meetings for their valuable comments and suggestions.

[2] For reviews of the recent crisis, see Gorton (2008), Brunnermeier (2009), Diamond and Rajan (2009a), Blanchard (2009), and Krishnamurthy (2009).

provide funds to encourage lending and restore confidence in the financial system. For example, the Troubled Asset Relief Program ("TARP") allows the United States Treasury to purchase or insure up to $700 billion of "troubled" assets, which are illiquid and difficult-to-value, from banks and other financial institutions.[3] One of the important goals of "TARP" is to encourage banks to resume lending at the levels before the crisis, both to each other and to consumers and businesses.

Despite the large U.S. government injection of capital to the banking sector after September 2008, bank lending shrank dramatically. Ivashina and Scharfstein (2009) document that new loans to large borrowers (through syndicated lending) falls by 47% during the last quarter of 2008. Such a decline in lending is also manifested in the aggregate flow of funds of bank loans in the U.S. commercial banking sector. As shown in Fig. 3.1, the flow of bank loans decreases dramatically from $1660B in the 3rd quarter of 2008 to -$514B in the 4th quarter, and to -$526B and -$351B in the first two quarters of 2009. Related to the drop of bank loans, the spread between the bank loan rate and the federal funds rate increases during the same period. According to the Federal Reserve's Survey of Terms of Business Lending,[4] the spread between the C&I loan rate and the federal funds rate at U.S. domestic banks increases from 2.52% in the third quarter of 2008 to 3.13%, 3.06%, 3.06%, and 3.27% in the next four quarters (see Fig. 3.2).

On the other hand, banks seem willing to keep a large holding of cash on their balance sheet during the same period. According to the Federal Reserve's H.8 report (Assets and Liabilities of Commercial Banks in the U.S.), the aggregate cash to total assets ratio of commercial banks has increased dramatically since

[3] As reported by SIGTARP (2009), "TARP" is just one part of a much broader U.S. Federal Government effort to stabilize and support the financial system. Since the onset of the financial crisis in 2007, the Federal Government, through many agencies, has implemented dozens of programs. The maximum total amount of support that Federal Government and agencies has specified that they could provide is estimated to be as large as $20 trillion U.S. dollars.

[4] The Survey of Terms of Business Lending collects data on gross loan extensions made during the first full business week in the middle month of each quarter. The authorized size for the survey is 348 domestically chartered commercial banks and 50 U.S. branches and agencies of foreign banks. The sample data are used to estimate the terms of loans extended during that week at all domestic commercial banks and at all U.S. branches and agencies of foreign banks.

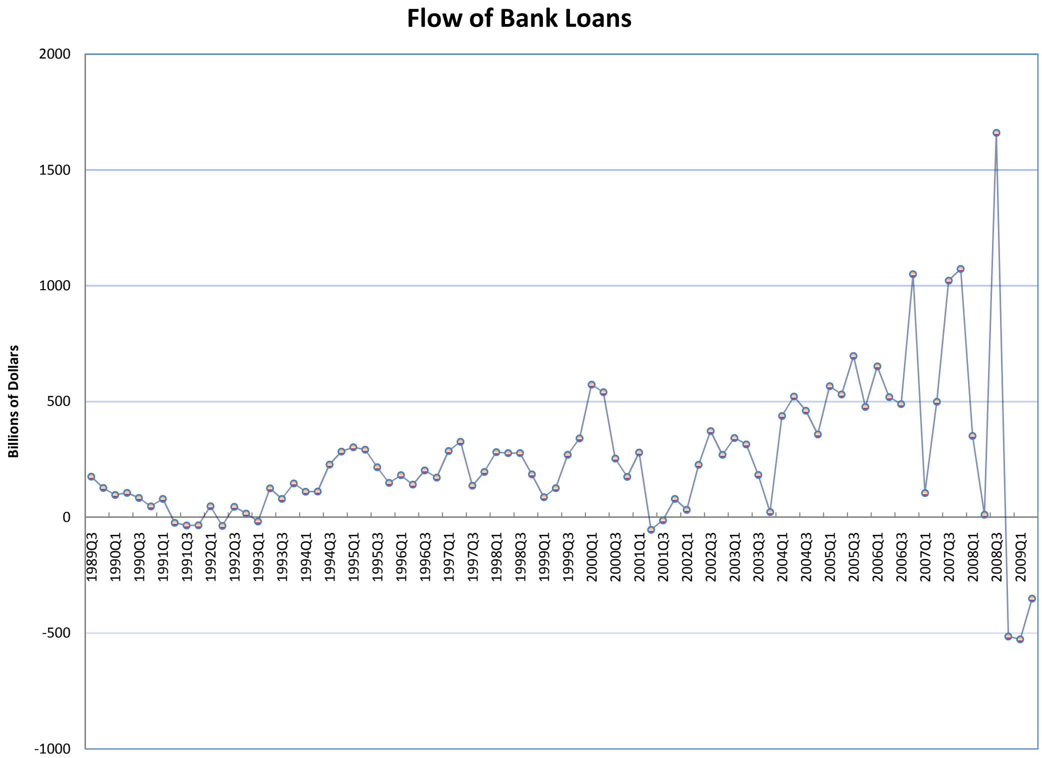

Figure 3.1: Quarterly Flow of Bank Loans (annualized) of the U.S. Commercial Banking Sector: 1989:Q3–2009:Q2
(Source: Flow of Funds, Federal Reserve.)

September 2008. As shown in Fig. 3.3, the cash ratio is only 2.87% in August 2008. It then quickly increases to 3.44% (September), 5.08% (October), and 7.19% (November), and then stays roughly at 8% through December 2008 to August 2009. As shown in Fig. 3.3, such a high cash ratio (8%) has not been seen for the recent 20 years.

Two important questions arise from the above observations: (i) Why don't banks keep enough cash to meet potential liquidity shocks and hence avoid fire sales of illiquid assets? (ii) Why do banks keep a large amount of cash on their balance sheet without lending it out after large capital injections from the gov-

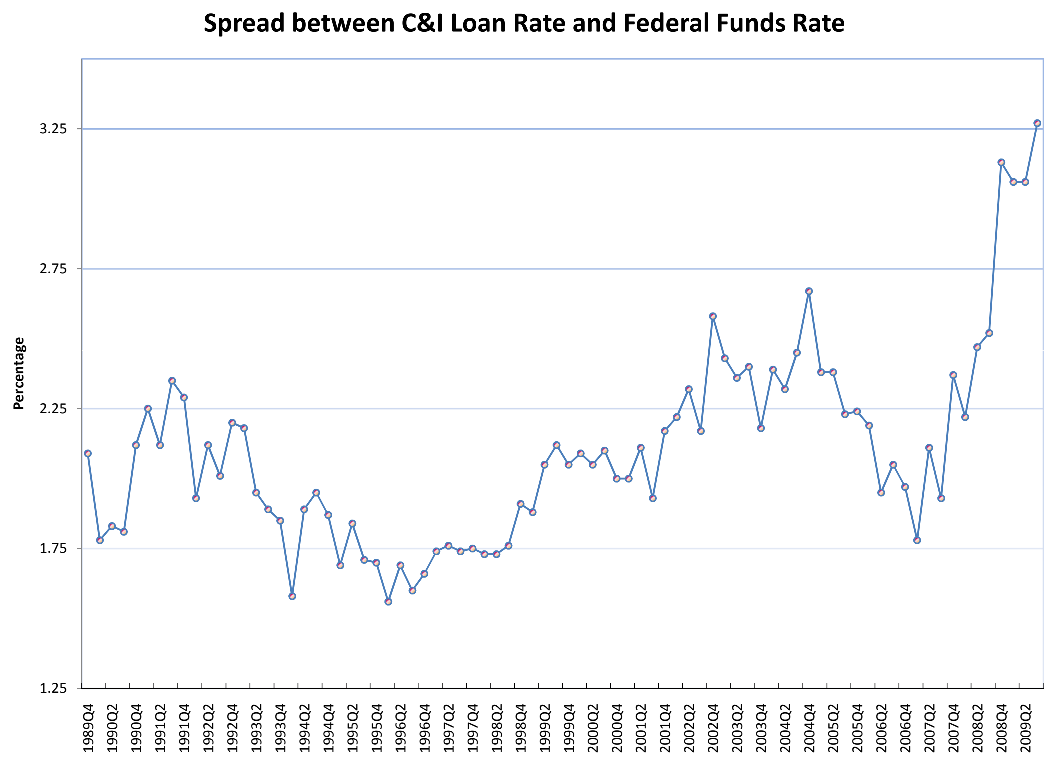

Figure 3.2: Commercial and Industrial Loan Rates at the U.S. Domestic Banks: 1989:Q4–2009:Q3
(Source: Survey of Terms of Business Lending, Federal Reserve.)

ernment? In this study, I use a single framework to answer both questions by examining lending incentives of banks when they face liquidity risks.

The key ingredient of the model is the fire sales of illiquid assets when banks face the risk of liquidity shocks. Banks are endowed with cash, non-tradable bank loans and tradable illiquid assets such as mortgage-backed securities. The non-tradable bank loans can be liquidated prematurely at a discounted value. The illiquid assets can be sold to private buyers who have a limited amount of cash available. If banks have less cash (plus outside buyers' excess cash) than the liquidity demand from banks' short-term creditors, they have to liquidate some of their loans and sell illiquid assets at a fire-sale price (a price lower than the

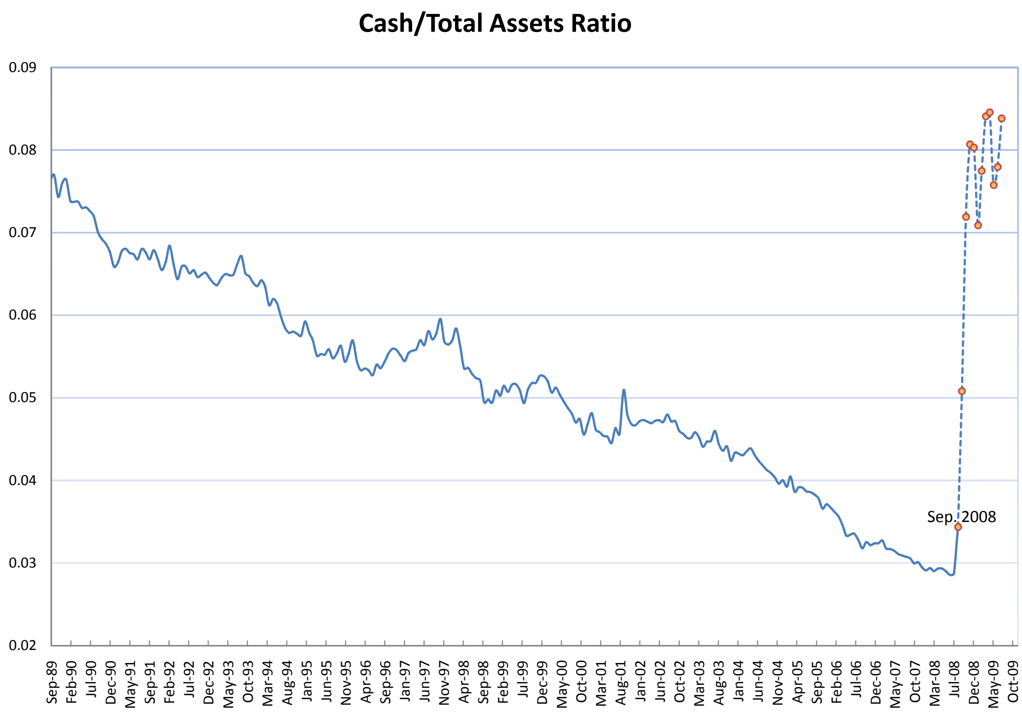

Figure 3.3: Monthly Cash Holdings to Total Assets Ratio of U.S. Commercial Banks: Sep. 1989 – Aug. 2009
(Source: Assets and Liabilities of Commercial Banks in the U.S. (H.8), Federal Reserve.)

fundamental value).

Banks make decisions based on the tradeoff between the cost and benefit of lending. The cost of lending comes from the premature loan liquidation and the fire sales of illiquid assets to raise cash when banks face a liquidity shock. The benefit of lending is the high return from the potential bank loans. This study shows that it can be optimal for banks to lend out cash and therefore incur asset fire sales under a liquidity shock, even if banks are endowed with enough cash to meet the liquidity shock. That is, fire sales of assets could be an endogenous outcome of banks' optimal decisions. This provides an answer to the first question raised above.

At the same time, although the forgone return from the potential bank loans might be high, the cost of potential fire sales could be even higher. As a result, banks might want to keep a large amount of cash to save the cost of potential fire sales. Since the cost of potential fire sales is directly linked to the severity of the liquidity shock (e.g., the size of the shock), more severe liquidity shock will induce a larger cash holding of banks. In addition to this direct explanation, my model provides another important dimension to understand the cash holding of banks. Specifically, for given severity of a liquidity shock, a higher bank illiquid assets to loans ratio makes fire sales more costly and therefore induces banks to keep an even larger amount of cash on their balance sheet rather than lending out. This provides an answer to the second question raised above. This novel explanation is especially relevant during the recent financial crisis, in which many big banks are reported to be 'clogged' with a large amount of illiquid assets (such as mortgage-backed securities) on their balance sheet.

Moreover, this study also provides implications for government policy to alleviate the credit crunch in the recent financial crisis. I compare three government intervention policies, and rank them according to their effects on the real economy and investment. Specifically, this study shows that when the government intervention fund size is relatively small, buying illiquid bank assets at face values and a direct capital injection to banks are equivalent; and both dominate buying bank loans at the face value. As the intervention fund increases such that there is no illiquid asset left in the case of buying assets, depending on parameter values, buying illiquid assets can dominate or be dominated by both a direct capital injection and buying bank loans. I also show that a direct capital injection (weakly) dominates buying bank loans.

Diamond and Rajan (2009b) is the closest work to this study in terms of modeling framework. They assume that banks have no cash and choose the timing of the illiquid assets trading to meet potential liquidity shocks. One of banks' choices is to sell illiquid assets conditional on a liquidity shock, in which assets can be sold only at a very low fire-sale price due to the limited amount of outside liquidity. Alternatively, banks can sell illiquid assets before the liquidity shock, where the price of the illiquid assets has already incorporated the expected fire sales in the future. Their main result is that if banks are insolvent in the future conditional on the liquidity shock, banks might refuse to sell their illiquid assets before shocks arrive, even if they could remain solvent by doing so. The intuition is similar to the "debt overhang" of Myers (1977) and the "risk shifting" of Jensen and Meckling (1976). The model of Diamond and Rajan (2009b) provides an explanation to the freeze of the illiquid asset markets, such as the market of mortgage-backed securities.

The model in this study extends Diamond and Rajan (2009b) with four key differences. First, the focus of this study is bank lending incentives and their effect on new bank lending and prices of illiquid asset fire sales. In contrast, Diamond and Rajan (2009b) focus on the freeze of illiquid assets trading. Second, while in Diamond and Rajan (2009b) the insolvency of banks under a liquidity shock is a necessary *and* sufficient condition for the asset market freezing, it is only a necessary but *not* sufficient condition in my study. Third, my study endogenizes fire sales of illiquid assets under a liquidity shock, while Diamond and Rajan (2009b) assume fire sales exogenously. Fourth, this study analyzes the effect of fire sales on the lending of banks, which is the major source of funds for many businesses and the focus of government intervention. However, Diamond and Rajan (2009b) only discuss the effect of fire sales on lending through a non-banking channel.

Another closely related paper is Acharya et al. (2009). They also look at banks' endogenous choice of liquidity which involves fire sales of illiquid assets. However, there are three major differences between my model and theirs. First, while they only look at the choice between cash and illiquid assets, my study breaks the illiquid assets into non-tradable bank loans and other tradable illiquid assets. Therefore, my study yields new results such as the effect of the loan to illiquid asset ratio on cash holding, new bank lending, and asset fire-sale prices. Given that banks do hold both bank loans and other illiquid assets, this extension is important in studying the bank's lending incentives. Second, in their model, the benefit of holding cash is from participating in the potential fire sales of illiquid assets of other troubled banks. In my model, however, different players have different incentives for holding cash. For private buyers, it is the same as that in Acharya et al. (2009). For banks, holding cash is to alleviate the potential loss from their own fire sales of illiquid assets. Both of these motives to hold cash should be relevant in reality. However, it seems that the precautionary motive is more relevant for banks to hold cash during financial crises. Third, although both my study and Acharya et al. (2009) use the "cash-in-the-market" pricing approach of Allen and Gale (1998), the cause of fires sales is different. In Acharya et al. (2009), fire sales of illiquid assets are linked to bank failures due to shocks to asset values. In my study, however, fire sales are a result of liquidity shortage and bank failures do not necessarily occur.

This chapter proceeds as follows. Section 3.2 sets up the model. Section 3.3 carries out the model analysis and derives implications on bank loans, bank cash holdings, and illiquid asset fire-sale prices. Section 3.4 discusses the model's implications for government policy. Finally, Section 3.5 concludes the chapter. The Appendix B contains all the proofs.

3.2 Model

This section describes the model. In the model, there are three dates: 0 (initial date), 1 (intermediate date), and 2 (final date).

3.2.1 Agents

There are two types of agents: banks and non-bank institutions. Each type has a continuum of identical agents with measure one.

Banks are financed with equity and demand deposits (or by rolling over short-term debt). For simplicity, banks are assumed to have monopoly power in financing and pay an interest rate of zero to their depositors.[5] The objective of banks is to maximize the expected bank equity value.

The non-bank institutions (e.g., private equity, hedge funds) are the only potential buyers of banks' tradable assets. For this reason, I refer to them as the *private buyers* thereafter. These private buyers are all-equity financed. The objective for them is to maximize the expected value of their holdings.

Both banks and private buyers are risk neutral; and the universal discount rate is zero.

3.2.2 Assets and Investment Opportunities

Banks are endowed with three types of assets: tradable but illiquid assets (such as mortgage-backed securities), non-tradable bank loans, and cash (or cash-equivalents). The private buyers are endowed with cash only. Both the tradable bank assets and bank loans are assumed to be riskless in terms of their date-2 values. That is, all assets are risk-free in terms of their fundamental values.

The illiquidity of banks' tradable assets (such as mortgage-backed securities) is due to the fact that private buyers, who are the only potential buyers of these assets, might have a limited amount of cash available. As a result, these assets may trade at a price that could be much lower than the fundamental value (i.e., at a fire-sale price).

The endowment bank loans are ex-ante identical but can be recalled prematurely at date 1 for values uniformly distributed between zero and the full face value.[6] However, bank loans cannot be traded directly to private buyers.[7]

In addition, both banks and private buyers can always choose to hold cash with a raw return of one. This can be alternatively interpreted as granting both of them

[5]Therefore, depositors can claim the same amount at either date 1 or date 2.

[6]Existing loans cannot be recalled at date 0 due to the lack of borrower's cash at date 0. Similarly, new loans issued at date 0 cannot be recalled at date 1 due to the lack of borrower's cash at date 1.

[7]We can think of banks having a special monitoring technology which is crucial to maintain the loan value. But private buyers lack such a skill and therefore are unwilling to buy bank loans directly.

the access to a risk-free storage technology.

In addition to endowments described above, both banks and buyers have investment opportunities at date 0. Banks can make new loans that mature at date 2, with a maximum size of L. These potential loans have different but sure rates of return ranging uniformly between $[1, \bar{R}]$. Similarly, the private buyers can invest in other long-term projects that generate cash only at date 2, with a maximum size of I. The different but sure gross returns of these projects are also uniformly distributed between $[1, \bar{R}]$. For simplicity, new bank loans and buyers' investment opportunities are assumed to be independent (so they won't compete with each other).

3.2.3 Endowments

At date 0, banks are endowed with assets that will be worth Z in total at date 2. Of these assets, an amount of D is financed with demand deposits and the rest by equity. Let α, β, and γ be the fractions of tradable assets, bank loans, and cash, respectively. That is, banks are endowed with tradable assets of αZ, bank loans of βZ, and cash of γZ.[8] Private buyers are endowed with θ amount of cash at date 0.

3.2.4 Timing of Events

The timeline of the model is shown in Fig. 3.4. At the initial date, both banks and private buyers will decide on their own investments. The tradable bank assets may be traded on this date.

The key event happens at the intermediate date. At date 1, banks face a liquidity shock with a probability q, upon which banks have to raise fD of cash to meet the liquidity demand. Such a liquidity demand can be interpreted as sudden

[8] If the total size of the bank portfolio is fixed at Z, then $\alpha + \beta + \gamma = 1$. However, I would like to keep all three variables rather than eliminating one of them. This provides extra flexibility when I analyze the government intervention, which could potentially change the size of the banks' portfolio (e.g., government direct capital injection to banks). In that case, $\alpha + \beta + \gamma > 1$ with a fixed Z. Moreover, it also facilitates comparative static analyses later in the study.

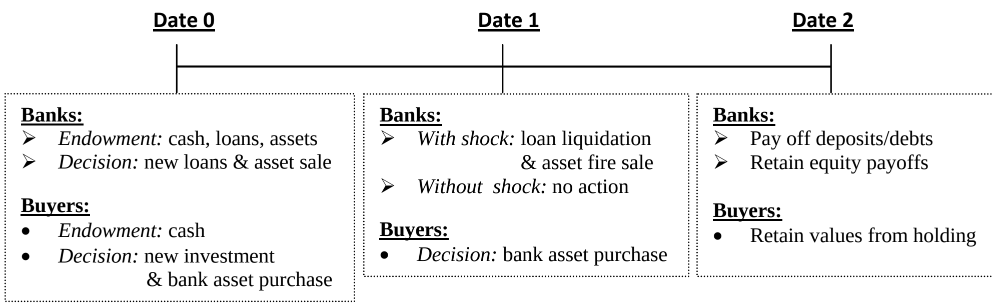

Figure 3.4: Model Timeline

withdrawal from a fraction of depositors due to their personal liquidity needs,[9] or alternatively, as a partial failure of short-term debt roll-over.[10] If banks do not have enough cash of their own, they have to prematurely liquidate some of their loans, or sell a part of their tradable assets, or more likely do both.

At the final date 2, all bank loans and securities pay off. Banks's equity-holders pay off the deposits/debts and keep the equity value. Private buyers retain all payoffs from their holdings. It is the *expected* payoff at the final date that matters for both banks and private buyers.

3.3 Analysis

I start this section by considering the new investments made at date 0 by both the banks and private buyers for a given cut-off rate of return. In the following, I first formulate the agents' optimization problems in Section 3.3.1, I then analyze the illiquid assets trading, focusing on the timing and condition to trade in Section 3.3.2. I establish results for endogenous fire sales of illiquid assets in Section

[9]If deposits are insured by FDIC, then the depositors' motive of bank run as in Diamond and Dybvig (1983) is absent.

[10]Bear Stearns failed in March 2008 because it was unable to renew the short-term debt that it used to finance long-term investments in mortgage-backed securities. Also, according to Ivashina and Scharfstein (2009), after the failure of Lehman Brothers in September 2008 there was a run by short-term bank creditors, making it difficult for banks to roll over their short-term debt.

3.3.3, specify the equilibrium conditions in Section 3.3.4, and finally carry out comparative static analyses in Section 3.3.5.

Since the sure return of available new loans is uniformly distributed between $[1,\bar{R}]$, the banks would like to make loans with higher returns first, and move downward along the return distribution as more loans are made. To arrive at a minimum loan rate of R, where $1 \leq R \leq \bar{R}$, the banks are required to lend out cash with an amount of

$$L(R) = \frac{\bar{R}-R}{\bar{R}-1}L, \tag{3.1}$$

which generates new loans with a corresponding face value of

$$F(R) = \frac{L}{\bar{R}-1}\int_{R}^{\bar{R}} r \cdot \mathrm{d}r = \frac{L}{2}\frac{\bar{R}^2-R^2}{\bar{R}-1}. \tag{3.2}$$

For example, if all the new loans are employed by the banks (i.e., $R=1$), then $L(1) = L$ and $F(1) = L(\bar{R}+1)/2$.

By the same argument, the new investment that the private buyers make at date 0 for a given minimum required return of R is

$$I(R) = \frac{\bar{R}-R}{\bar{R}-1}I. \tag{3.3}$$

This new investment generates a value at date 2 which equals

$$V(R) = \frac{I}{2}\frac{\bar{R}^2-R^2}{\bar{R}-1}. \tag{3.4}$$

As a result, the private buyers will have $[\theta - I(R)]^+$ of cash left available for asset trading.

3.3.1 Agents' Optimization Problem

The banks can either sell their tradable assets when hit by the liquidity shock at date 1 or sell earlier at date 0, anticipating the liquidity shock at date 1. If the price of the illiquid assets is low under a liquidity shock at date 1, then the date 0 price will incorporate such a discount in expectation as well.

The private buyers are indifferent between buying illiquid bank assets at date 0 and date 1 if they get the same expected return at date 2. Let the price of per unit of date-2 face value of illiquid assets be P_0 at date 0, and P_1 at date 1 under a liquidity shock. (Note that the price at date 1 is 1 without any liquidity shock.) Then P_0 and P_1 satisfy:

$$\frac{1}{P_0} = \frac{q}{P_1} + (1-q), \tag{3.5}$$

where the left hand side is the return for the buyers if illiquid assets are bought at date 0; the right hand side is the expected return of buying at date 1 (with or without a liquidity shock). For a given date-1 price under a liquidity shock (P_1), we can find from Equation (3.5) the maximum date-0 price that the buyers are willing to pay: $P_0 = 1/[q/P_1 + (1-q)]$.

Consider first the private buyers' optimization problem. The private buyers will choose the cut-off rate for making new long-term investment by solving the following maximization problem:

$$\text{Private Buyers' Optimization Problem: } \max_R \left[V(R) + \frac{\theta - I(R)}{P_0} \right], \; s.t. \;\; \theta \geq I(R). \tag{3.6}$$

The solution of this problem can be easily obtained. The private buyers would like to maximize their profit by investing in those long-term projects that generate returns at least as high as what they can get from buying the banks' assets. That is, the private buyers will set $1/P_0$ as their minimum required rate of return for investing in the long-term project. As a result, the private buyers will have $[\theta - I(1/P_0)]$ of cash left for asset trading.[11]

Next, consider the banks' optimization problem. At date 0, the banks have to decide how many new loans to make (or equivalently, how much cash to retain and how many assets to sell). Since the banks are endowed with cash themselves,

[11] In a general case, it could happen that $\theta < I(1/P_0)$. In this case, the private buyers will invest all their money in the long-term projects, which generate higher returns than buying bank assets. I rule out this uninteresting case since there is no interaction between the private buyers and banks.

they can always choose to use their own cash endowment first to make new loans. However, there are potentially two cases in which the banks would like to sell assets to the private buyers at date 0. In the first case, banks exhaust all their own cash but still want to make additional loans. Then they need to sell illiquid assets to the private buyers to raise additional cash for further lending. In the second case, if it is in the banks' own interest to hold more cash at date 0 (for example, the banks may want to prevent the potential insolvency at date 1), the banks then would like to sell their assets to the private buyers for cash at date 0.[12]

Suppose that the banks choose to make new loans at date 0 with rates of return no lower than R (the optimal choice will be determined in the equilibrium). Then the banks will have cash of $\gamma Z - L(R)$ (a negative amount of cash simply means that the banks have to sell assets at date 0 to the private buyers to raise cash for lending). At date 1, under the liquidity shock, the banks have to raise cash of fD through loan liquidations and illiquid asset fire sales. To determine how many loans should be liquidated, note that the banks would be *willing* to recall loans with liquidation values equal to or greater than the illiquid asset fire-sale price.[13] For every date-2 dollar of endowment loans, a fraction of $1 - P_1$ will be liquidated (since the liquidation value is uniformly distributed) which generates cash with an amount of

$$\int_{P_1}^{1} l \cdot \mathrm{d}l = \frac{1}{2}(1 - P_1^2).$$

Therefore, the total cash from loan liquidations is $\beta Z(1 - P_1^2)/2$.

From the above analysis, the cash constraint under a liquidity shock at date 1

[12] Although both the new loan making and the illiquid asset trading can happen at the same date, we can think of banks making new loans using their own cash and then trading with the private buyers if more cash is needed. Such an interpretation simplifies the analysis without affecting the results.

[13] It is possible that the minimum liquidation value of loans is below the fire-sale price of illiquid assets. That is, the banks might *have to* liquidate loans with liquidation value less than the asset price. Such a scenario happens when there is no benefit from lowering the asset price (i.e., $\alpha Z P_1 = \theta - I(1/P_0)$) and the cash raised is still below required. For the ease of presentation, I ignore these special cases in the main analysis. I will return to these cases when I discuss the effect of government intervention.

can be written as follows:

$$[\gamma Z - L(R)] + \frac{1}{2}\beta Z(1 - P_1^2) + \left[\theta - I(\frac{1}{P_0})\right] = fD, \tag{3.7}$$

where the first term is the bank cash left; the second is the bank loan liquidation cash from endowment loans; and the third term is the proceeds from bank asset sales (at either date 0 or date 1).[14]

Generally, the bank equity value will depend on when the bank assets are sold. First consider the case where the bank assets are sold only at date 1. If there is no liquidity shock at date 1, then the date-2 bank equity value will be $Z - D + F(R) - L(R)$. Note that $Z - D$ is the equity's endowment value and $F(R) - L(R)$ is the NPV from the newly made loans. If there is a liquidity shock at date 1, then the equity value at date 2 will be

$$\left\{\beta Z P_1 + F(R) + \left[\alpha Z - \frac{\left[\theta - I(\frac{1}{P_0})\right]}{P_1}\right] - (1-f)D\right\}^+, \tag{3.8}$$

where the first term is the endowment loans left after liquidation; the second term is the face value of the new loans; the third term is the illiquid assets left after fire sales at P_1 at date 1;[15] and the last term is the banks' liability after the liquidity withdrawal at date 1.

As a result, the expected payoff for the bank equity if assets are traded only at

[14]The left hand side of (3.7) should be no less than the right hand side. For the ease of presentation, in the main analysis I consider only the interior solutions (i.e., $R > 1$ and $P_1 < 1$), which satisfy equation (3.7) .

[15]For simplicity, I assume $\alpha Z P_1 > \theta - I(1/P_0)$ such that there are still some illiquid assets left in the banks' portfolio after fire sales. Otherwise, scenarios described in footnote 13 might occur.

date 1 is

$$E_0^1(R) = q\left\{\beta Z P_1 + F(R) + \left[\alpha Z - \frac{\left[\theta - I(\frac{1}{P_0})\right]}{P_1}\right] - (1-f)D\right\}^+$$
$$+ (1-q)\left[Z - D + F(R) - L(R)\right], \tag{3.9}$$

where the subscript '0' means that the equity expectation value is formed at date 0, and the superscript '1' indicates that asset selling only happens at date 1.

If the assets are only traded at date 0, then the cash constraint under the liquidity shock will still be the same as equation (3.7). However, the expected payoff for the bank equity should be modified as follows:

$$E_0^0(R) = q\left\{\beta Z P_1 + F(R) + \left[\alpha Z - \frac{\left[\theta - I(\frac{1}{P_0})\right]}{P_0}\right] - (1-f)D\right\}^+$$
$$+ (1-q)\left\{Z - D + F(R) - L(R) + \left[\theta - I(\frac{1}{P_0})\right]\left(1 - \frac{1}{P_0}\right)\right\}. \tag{3.10}$$

Therefore, the banks' optimization problem can be written compactly as follows:

$$\text{Banks' Optimization Problem:} \quad \max_{R,t=\{0,1\}} E_0^t(R), \tag{3.11}$$
$$\text{s.t. liquidity constraint (3.7),}$$
$$\gamma Z + [\theta - I(\frac{1}{P_0})] \geq L(R), \quad \text{if } t = 0,$$
$$\gamma Z \geq L(R), \quad \text{if } t = 1.$$

That is, the banks will choose how much to lend at date 0 and whether to trade at date 0 or date 1, subject to the liquidity constraint at date 1 and the cash constraint at date 0.[16]

[16]For simplicity, I assume that asset trading occurs at either date 0 or date 1 (but not both). However, the results will be similar if the banks are allowed to choose a combination of trading at

3.3.2 Asset Trading

In this section, I analyze the banks' illiquid asset trading based on the agents' optimization problems developed in Section 3.3.1. I establish two results, one regarding the timing of trading and the other on the condition of trading freeze at date 0.

The following lemma presents results regarding the timing of the asset trading.

Lemma 3.1 *Assume that the banks are always solvent and have some illiquid assets left after selling.*

(a) If the banks have enough cash to make the equilibrium amount of new loans by using only their cash endowment, then the banks are indifferent between trading their illiquid assets at date 0 and date 1;

(b) Otherwise, banks will prefer selling assets at date 0 and use the proceeds to make additional new loans.

The result in (a) is intuitive. When there is no solvency issue and the banks still have their own cash left, there is no need to trade at date 0. In addition, the date 0 price will in expectation incorporate the date 1 fire-sale price under a liquidity shock. At the equilibrium, both the banks and buyers will be indifferent between the two trading dates. For most analysis in the following, I assume that the condition in (a) holds. However, the result in (b) is important to the following discussion on asset freeze. The intuition is as follows. If banks exhaust all their cash to extend new loans, then it may be optimal for them to sell some of their illiquid assets to the private buyers and then use the cash proceeds to make more loans. This will happen if the benefit from new loans is greater than the cost of fire sales.

Furthermore, this analysis modifies the main result of Diamond and Rajan (2009b), who provide an explanation of the freeze of illiquid bank assets. They find that if the banks are insolvent at date 1 under a liquidity shock, then they will

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both dates.

not sell assets at a fire-sale price at date 0, even they could be solvent by doing so. That is, in their model the insolvency of banks under a liquidity shock at date 1 is a necessary and sufficient condition for the illiquid asset market to freeze. However, my model extends Diamond and Rajan (2009b) and incorporate new investment opportunities for banks. Therefore, results in Diamond and Rajan (2009b) should be modified as follows.

Proposition 3.1 *The insolvency of the banks at date 1 under a liquidity shock is only a necessary but not a sufficient condition for the no-trading of illiquid bank assets at date 0. Under some other conditions (e.g., the benefit from new loans is greater than the cost of fire sales), the banks would like to sell assets at a fire-sale price at date 0 and use the cash proceeds to make new high yield bank loans.*

Suppose that the banks become just solvent under a liquidity shock at date 1. According to Lemma 3.1, the banks may prefer selling assets at date 0 and use the proceeds to extend more loans. Such results will naturally continue to hold even the banks are insolvent under a liquidity shock at date 1. Such a modification comes from the fact that banks are granted with additional opportunities of making new loans, while in Diamond and Rajan (2009b) the banks can only hold cash after asset fire sales at date 0. Proposition 3.1 is illustrated through numerical examples shown in Fig. 3.5. The figure shows that the banks may still prefer date-0 trading even they are insolvent at date 1 under a liquidity shock.

3.3.3 Endogenous Fire Sales

In this section, I analyze endogenous fire sales of banks' illiquid assets. Note that the following results can be obtained without considering the equilibrium.

Intuitively, the banks would like to make new loans with high returns if the cost of doing so is low. In the absence of tradable illiquid assets, the banks consider the tradeoff between the return from new loans and the cost of loan liquidations. Since the return of available new loans is much greater than 1, the banks would always make more loans and induce endogenous loan liquidations. Now consider

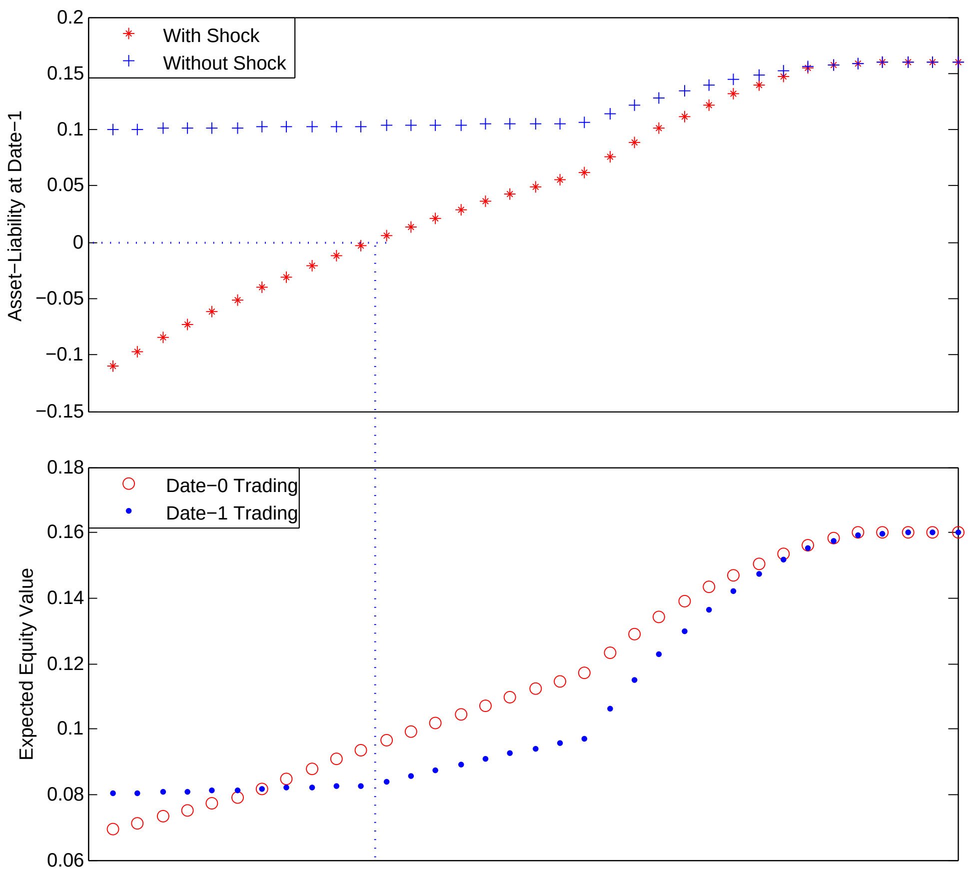

Figure 3.5: Examples of Timing of Trading

The parameter values used in these examples are as follows: $q = 0.2$, $\bar{R} = 1.3$, $I = 0.4$, $L = 0.4$, $Z = 2$, $D = 1.9$, $\beta = 0.6$. From the left to the right, f decreases from 0.3 to 0.125, θ increases from 0.4 to 0.75, γ increases from 0.0005 to 0.235, and $\alpha = 1 - \gamma - \beta$.

the case where the banks' portfolio contains illiquid assets. Are the banks still willing to make more new loans such that illiquid assets will be sold at a fire-sale price?

Under some conditions, it is optimal for the banks to make new loans and retain less cash than required under a liquidity shock even they are endowed with enough cash for such a shock. Such an optimal choice leads to fire sales (assets sold at a price lower than the fundamental value) endogenously. While under some other conditions, the opposite is true: the banks would like to hold cash instead of lending out to prevent costly fire sales of illiquid assets. The main results are summarized in the following proposition.[17]

Proposition 3.2 *Let the banks' cash endowment (γZ) plus the private buyers' cash under no fire-sale ($(\theta - I)^+$) be no less than the cash requirement under a liquidity shock (fD). But the banks' cash after making all new loans ($\gamma Z - L$) plus the private buyers' cash under no fire-sale ($(\theta - I)^+$) is smaller than the liquidity shock (fD). In addition, the banks are always solvent and have some of their illiquid assets left after trading.*

(i) If the private buyers are unwilling to provide cash without fire sales (i.e., $\theta \leq I$), then it is always optimal for the banks to make new loans such that illiquid bank assets will be traded at a price lower than the fundamental value (endogenous fire sales);

(ii) If the private buyers are willing to provide cash even without fire sales (i.e., $\theta > I$) and the banks are not relying heavily on the private liquidity under a shock (i.e., $\theta - I$ is small), then result in (i) still holds;

(iii) Otherwise, it is optimal for the banks to keep enough cash and make less loans (even the forgone returns from new loans are much higher than 1),

[17] I assume that the private buyers are always relevant in the equilibrium, such that there is always interaction between the banks and the buyers. Otherwise, it is uninteresting because the illiquid assets will never be traded and the private buyers will be out of the model.

such that assets will be traded at their fare prices in stead of a fire-sale price (no fire-sale).

The assumptions specified in the above proposition makes sure that any resulting fire sales of assets are endogenous (if the banks choose not to lend, then there is no asset fire sale). It also rules out uninteresting cases where fire sales of assets are not a concern (e.g., the banks and private buyers have enough cash to cover both new investment opportunities and potential liquidity demand).

The intuition of the above proposition is easy to obtain. To understand result (i), note first that it is always optimal for the banks to make loans with returns greater than the implied cost from loan liquidations (i.e., $1/P_0$), if there is no asset trading involved (i.e., the private buyers are still unwilling to provide cash). As more loans are made, eventually the implied return from the loan liquidation makes the private buyers marginally indifferent between investing in their own long-term projects and buying banks' assets. Now, if banks make an additional infinitesimal amount of new loans, then the cost from fire sales is small enough (as banks only need to raise additionally that infinitesimal amount of cash under a liquidity shock), compared to the high returns from the lending. Therefore, it is optimal for banks to lend out more and endogenously incur fire sales of their illiquid assets.

The intuition of results (ii) and (iii) are very similar to that for (i) but with one key difference. That is, in cases (ii) and (iii), banks depend on private buyers' cash even without fire sales (as $\theta > I$). Then any additional lending will result in fire sales of illiquid assets. Although the fire-sale price could be very close to the fundamental value, the cost of such fire sales could be large compared to the gain from new loans. This is because a small drop in the price will affect not only the incremental asset selling but also all existing ones, which could be large. On the other hand, the benefit from the additional loans is always proportional to the added size of new loans, which is small at the margin. Therefore, depending on the size of the existing asset trading (or equivalently, the private buyers' willingness to provide cash) there can be endogenous fire sales (result (ii)) or no fire-sale (result

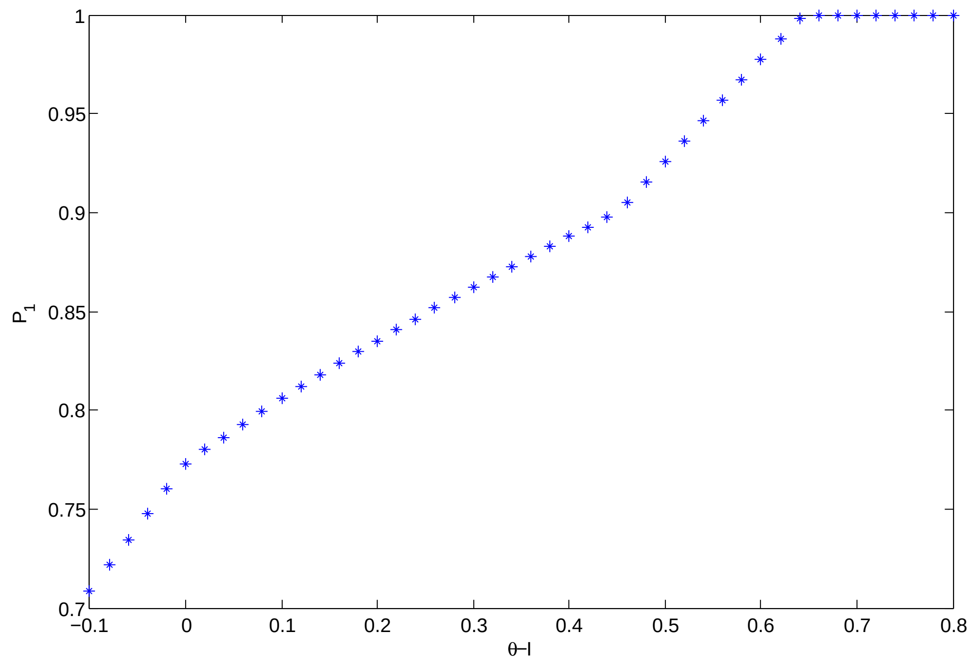

Figure 3.6: Examples of Endogenous Fire Sales

The parameter values used in these examples are as follows: $q = 0.4$, $\bar{R} = 1.3$, $I = 0.2$, $L = 0.6$, $Z = 2$, $D = 1.8$, $f = 0.25$, $\beta = 0.5$. The sum of bank cash endowment and private buyer's excess cash, $\gamma Z + (\theta - I)^+$, is no less than the size of the shock ($fD = 0.45$). I set the γ to be zero if the private buyer's excess cash $(\theta - I)$ is larger than the size of the shock. As usual, $\alpha = 1 - \gamma - \beta$.

(iii)).

I show numerical examples of endogenous fire sales (or no fire sales) in Fig. 3.6. These examples confirm that there are always asset fire sales if $\theta \leq I$. As banks depend more and more on private buyers' cash to meet the liquidity shock, i.e., as $\theta - I$ becomes larger and larger, banks will switch from endogenous fire sales to no fire-sale at all.

The above results of endogenous fire sales of banks' illiquid assets support the assumption in Diamond and Rajan (2009b). In their model, they assume that banks have no cash at date 0. As a result, the fire sales of assets are inevitable. In other words, the fire sales of assets are assumed exogenously in Diamond and Rajan (2009b). Proposition 3.2 shows that such fire sales can arise endogenously, generalizing Diamond and Rajan's assumption of exogenous asset fire sales.

3.3.4 Equilibrium

In this section, I derive equilibrium conditions which will be used in Section 3.3.5. In the following, I assume banks are always solvent and have some of their own cash left at the equilibrium. As a result, the result (a) in Lemma 3.1 applies and banks are indifferent between trading at date 0 and date 1. Therefore, without loss of generality, I assume trading happens only at date 1. In addition, I assume fire sales exist in the equilibrium (i.e., results (i) and (ii) of Proposition 3.2 hold).[18]

To find the equilibrium condition, compare the marginal benefit of lending and the marginal cost of liquidation and fire sales, subject to the cash constraint in (3.7). Alternatively, we can compare the marginal benefit and cost by decreasing or increasing one additional dollar of cash at date 1 under a liquidity shock. Specifically, I look at the marginal effect of one dollar change in the left hand side of equation (3.7), which I refer to as "cash-under-shock", on the expected date-2 bank equity value.

In the case of decreasing one additional dollar of cash at date 1, banks decrease the cut-off rate of return for new loans (R) and fix the liquidation price (P_1). The marginal benefit of lending to the bank equity (for every dollar decrease in "cash-under-shock") is

$$MB_L = -\left.\frac{\partial E_0^1}{\partial(\text{cash-under-shock})}\right|_{\text{fix } P_1} = qR + (1-q)(R-1). \tag{3.12}$$

The marginal benefit of lending is simply the expected marginal benefit in

[18]The same analysis can be extended to cover more general cases only with more complications. In all the numerical examples shown in this study, I take into account the possible no fire-sale solution (i.e., $P_1 = 1$).

the two states. If the cash constraint is ignored, the benefit of lending out one additional dollar is R in the shock state and $R-1$ in the non-shock state.

I can find the marginal cost of liquidation/fire-sale to the bank equity similarly. For every dollar increase in "cash-under-shock," the banks decrease the fire-sale price (P_1) and fix the cut-off rate for new loans (R). This yields

$$MC_S = -\left.\frac{\partial E_0^1}{\partial(\text{cash-under-shock})}\right|_{\text{fix } R} = q\left[\frac{1}{P_1} + \frac{\theta - I(\frac{1}{P_0})}{\beta Z P_1^3 + \frac{Iq}{R-1}}\right]. \tag{3.13}$$

Since fire sales happen only in the shock state, the marginal cost is proportional to the probability of the shock (q). When the cash constraint is ignored, the first term in the brackets on the right hand side of (3.13) is the marginal cost of loan liquidation and fire sales, the second term in the brackets can be interpreted as the additional cost of the existing fire sales of assets.

Intuitively, the marginal benefit of lending and the marginal cost of liquidation and fire sales should be the same at the equilibrium. Otherwise, banks can increase their equity value by adjusting their lending subject to the cash constraint at date 1 under a liquidity shock. The equilibrium with fire sales is characterized in the following proposition.[19]

Proposition 3.3 *Assume that banks are always solvent and have some of their own cash left at the equilibrium. Then the equilibrium asset prices (P_1^* and P_0^*) and the minimum required rate of loans (R^*) are determined by the following system of equations:*

$$q\left[\frac{1}{P_1} + \frac{\theta - I(\frac{1}{P_0})}{\beta Z P_1^3 + \frac{Iq}{R-1}}\right] = R - (1-q); \tag{3.14}$$

$$[\gamma Z - L(R)] + \frac{1}{2}\beta Z(1-P_1^2) + \left[\theta - I(\frac{1}{P_0})\right] = fD. \tag{3.15}$$

[19]It is equivalent to take the first order derivative of E_0^1 in Eqn. (3.9) with respect to R subject to the cash constraint in Eqn. (3.7) (see the proof in Appendix B). The approach taken in the main text has its advantage of easy interpretation.

The other quantities in equilibrium are determined as follows: banks' cash holding at date 0 ($\gamma Z - L(R^*)$), new loan face value ($F(R^*)$), private buyers' cash holding ($\theta - I(1/P_0^*)$), and private buyers' minimum required return for new investment ($1/P_0^*$).

Note that the left hand side of Eqn. (3.14) is the marginal cost (in terms of the equity value) of raising an additional dollar through decreasing the date-1 asset price P_1. Similarly, the right hand side is the corresponding benefit from decreasing the minimum loan rate R (i.e., making more loans). Equation (3.14) means that in equilibrium the marginal cost of loan liquidation and fire sales of illiquid assets equals the marginal benefit of making additional loans. In other words, equation (3.14) gives the optimal lending condition. The second equation is the cash constraint at date 1 under a liquidity shock. These two conditions determine the equilibrium.

3.3.5 Comparative Statics

The comparative statics in this section provide implications on banks' lending incentives. The following analysis also shows the bank lending effect on illiquid asset fire-sale prices, the minimum required returns for new bank loans and private buyers' new investments.

First, consider the effect of the illiquid assets to bank loan ratio in banks' portfolio for a given cash endowment on banks' lending incentives and other e-quilibrium quantities.

Proposition 3.4 *For a given fraction of cash endowment (γ) and all else being equal, a higher illiquid assets to loan ratio (or equivalently, a higher $\alpha = 1 - \gamma - \beta$) in the banks' endowment portfolio will lead to the following:*

(i) Banks will lend less and require a higher minimum rate for new bank loans at date 0;

(ii) Banks will hold more cash at hands; and the total amount of cash from banks and buyers is larger;

(iii) Under some further conditions (e.g., the banks do not rely heavily on the buyers' cash to meet the liquidity demand), the illiquid bank assets will sell at lower fire-sale prices (at both date 0 and 1). As a result, the buyers will also hold more cash.

The intuition of (i) and (ii) is straightforward. Fixing a cash endowment fraction, a higher illiquid assets fraction leads to a less loan endowment. This in turn leads to less cash from loan liquidation at date 1 conditional on a liquidity shock. This makes banks acting more conservatively. The result in (iii) is due to the complicated structure of marginal cost of lending on the existing asset trading (see Eqn. (3.14)).

This result helps to explain the decrease in bank lending during the recent financial crisis given that many big banks are reported to be 'clogged' with a large amount of illiquid assets, such as mortgage-backed securities, on their balance sheet. Banks in the recent crisis are endowed with a high fraction of illiquid assets (i.e., a high α). Therefore, holding all other factors fixed (e.g., the exogenous liquidity shock), the high fraction of illiquid assets in the banks' portfolio contributes to the observed decline in lending and the sharp increase in the bank cash holding. This is a novel explanation comparing to other traditional explanations that are based on the severity of the liquidity shock itself. This explanation is illustrated through numerical examples as shown in Fig. 3.7. In the two cases with different parameter values in Fig. 3.7, banks lend less and require a higher minimum rate for new bank loans with a larger fraction of illiquid assets. As a result, the cash to asset ratio is higher for a higher illiquid asset fraction. It also shows that the asset price does not necessarily decrease with the illiquid asset fraction.

The following proposition summarizes the effect of cash endowment (i.e., the fraction γ) on lending and asset prices:

Proposition 3.5 *For a given fraction of loan endowment (β) and all else being equal, a higher fraction of cash (γ) in the banks' endowment portfolio will lead to the following:*

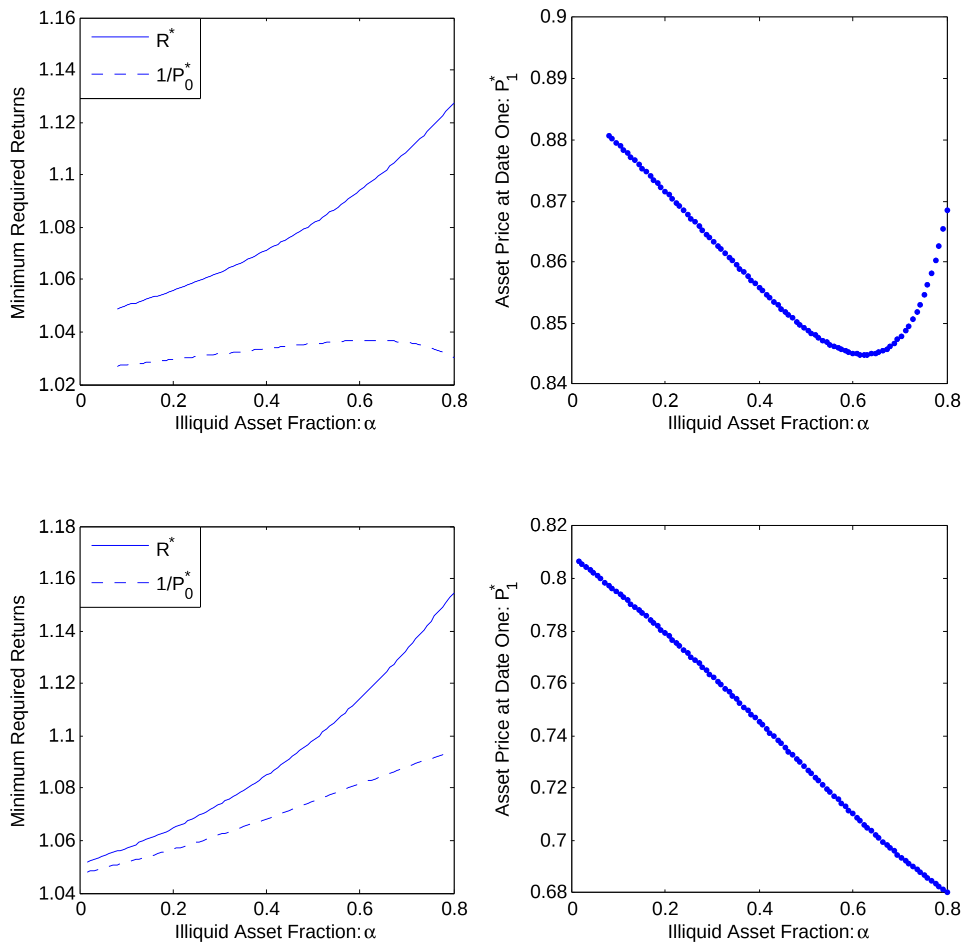

Figure 3.7: Asset Fire-sale Prices and Minimum Required Returns under Different Illiquid Asset Fraction

In the left panels, the solid line represents minimum required returns for banks to make new loans at date 0; the dotted line represents buyers' required returns corresponding to buying the assets at price P_0^* at date 0 (or equivalently, buying at date 1 with price P_1^*). The parameter values used in these examples are as follows: $q = 0.2$, $\bar{R} = 1.2$, $I = 0.3$, $L = 0.4$, $Z = 2$, $D = 1.6$, $f = 0.25$, $\gamma = 0.2$, and $\beta = 1 - \gamma - \alpha$. $\theta = 0.4$ in the upper two panels and $\theta = 0.25$ in the lower two panels.

(i) The illiquid assets will trade at a higher price at both date 0 and date 1;

(ii) Banks will lend more and require a lower minimum required loan rate;

(iii) Private buyers will invest more in their own long-term projects.

Note that the above results hold in two cases: (i) β is fixed and $\gamma = 1 - \alpha - \beta$ is higher; (ii) both α and β are fixed and γ is higher. A higher cash fraction can be a result of government capital injection to the banks. Proposition 3.5 shows that it is beneficial for the government to intervene from the social point of view. This is not surprising given that the inefficiency in this model is driven purely by liquidity problems. Fig. 3.8 shows numerical examples that illustrate Proposition 3.5. In figure 3.8, the banks' minimum required rate for loans is decreasing with the cash fraction (γ). On the other hand, the asset price is increasing with the cash fraction.

I also conduct other comparative statics results which yield similar results. For example, more new loans opportunities for banks (i.e., a higher L), less endowment of private buyers' cash (i.e., a lower value of θ), and a larger liquidity shock (i.e., a larger fD) lead to lower fire-sale price and higher the minimum required return for new investment. Higher private buyers' own investment opportunities (i.e., higher I) lead to lower fire-sale prices and smaller total cash holdings. On the other hand, a higher probability of the liquidity shock (i.e., a higher q) will lead to higher fire sale prices and larger total cash holdings.

3.4 Policy Implication and Discussion

This section considers the model's implications for the government intervention policy. Assume that the government has a fixed amount of cash (gZ) that can be used to intervene at date 0. Consider three alternative intervention policies: (i) direct capital injection (DCI) to banks (assuming the government will take an equivalent worth of equity or debt from the banks); (ii) buying the existing bank loans (BL) at their face value; and (iii) buying bank illiquid assets (BA) at their

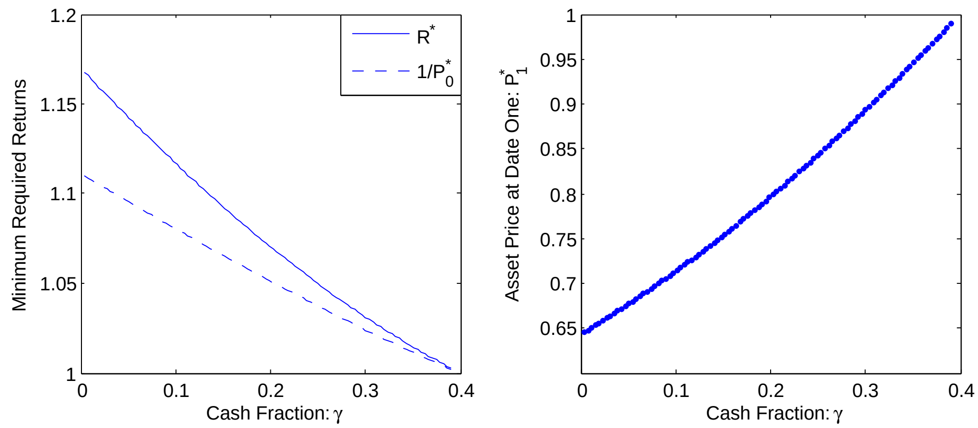

Figure 3.8: Asset Fire-sale Prices and Minimum Required Returns under Different Cash Endowment
In the left panels, the solid line represents minimum required returns for banks to make new loans at date 0; the dotted line represents buyers' required returns corresponding to buying the assets at price P_0^* at date 0 (or equivalently, buying at date 1 with price P_1^*). The parameter values used in these examples are as follows: $q = 0.2$, $\bar{R} = 1.2$, $I = 0.3$, $L = 0.4$, $Z = 2$, $D = 1.6$, $f = 0.25$, $\theta = 0.3$, and $\beta = 0.5$. Note that $\alpha = 1 - \gamma - \beta$ in the examples. The results are the same if I fix $\alpha = 0.5$.

face values.[20] These alternative policies will be ranked based on their effects on the real economy, i.e., the total investment by both the banks and private buyers.[21]

[20]The results will be the same if the government intervenes using the post-intervention equilibrium prices rather than the face values. The only difference is that there might be a transfer of value from banks to the government if equilibrium prices are used. Such a transfer of value does not distort banks' incentives due to the fact that incremental government intervention always makes them no worse off. However, the assumption that government intervene using face values greatly simplifies the analysis.

[21]The results are qualitatively the same if the government uses the social values (i.e., the sum of expected values for both banks and private buyers) as the ranking criterion. However, from a social welfare or political point of view, government might still prefer policies that can generate more job opportunities, which presumably depend more on the size (rather than value generated) of the total investment in the economy.

The government's optimization problem is

$$\text{Government's Optimization Problem:} \quad \max_{DCI,BL,BA} \left[L(R^*) + I(1/P_0^*)\right], \quad \text{s.t. a fixed intervention fund of } gZ. \tag{3.16}$$

where the equilibrium bank lending ($L(R^*)$) and the private buyers' long-term investment ($I(1/P_0^*)$) will depend on which policy the government chooses.

To rule out cases in which each individual policy cannot be carried out, assume that the government intervention fund is no greater than the face value of either the illiquid bank assets or the bank loans.[22] However, depending on the size of the government intervention, I consider cases in which no illiquid assets are left after trading with the private buyers at date 0.

It is worth noting that selling bank assets to private buyers is equivalent to issuing bank equities (or debts) to the private buyers. However, in the policy analysis, these two approaches may be different depending on the size of the government intervention. If banks sell equities to private buyers, then the government policies of direct capital injection and buying bank assets are equivalent, and both dominate buying bank loans. On the other hand, if banks only sell assets but not equities to private buyers, it will lead to more interesting results. The analysis in this section under the assumption of only asset sales (but not equities) to the private buyers is still a relevant one if I assume the government can coordinate (or force) banks' selling decisions, given that selling assets to private buyers can dominate selling equities to them.[23]

[22] The analysis can be extended to cases in which banks assets or the bank loans are smaller than the government fund by combining more than one policy. For example, if the government fund is larger than the illiquid bank assets, the extra fund besides buying the assets can be injected to banks directly. However, the mixed policy makes it less clear which policy is better than the others. Moreover, such a complication brings no new insight to the main analysis.

[23] An alternative argument for favoring selling banks assets to private buyers over selling bank equities involves information asymmetry over bank loans. If we modify the model such that private buyers have less information regarding the value of bank loans, then they may refuse to buy bank equities. However, they do not have problem to buy the illiquid bank assets. A similar argument applies to issuing bank debt to private buyers.

The following corollary compares buying bank loans and buying the illiquid bank assets.

Corollary 3.1

(i) If there are still some illiquid bank assets left at date 0 after trading, then buying illiquid bank assets leads to the following: (1) banks will lend more at date 0 and require a lower minimum loan rate; (2) the total new investments from banks and private buyers are larger.

(ii) Otherwise, depending on parameter values, buying the bank loans and buying the illiquid bank assets can dominate each other.

Result (i) is a direct result of Proposition 3.4. Result (ii) is due to the disentanglement of illiquid asset price and the marginal loan liquidation value. Specifically, the asset price is higher than the marginal loan liquidation value in (ii).

It is also worth pointing out that in (i) not only the combined cash invested is higher but also the combined value from investing is higher. This is a result of the fact that banks make loans with returns much higher than the required return for the private buyers.[24] This implies that in (i) buying bank assets is better than buying bank loans from a social point of view, since more valuable real investments are made.

Similarly, the following corollary compares the direct capital injection and buying illiquid bank assets.

Corollary 3.2

(i) If there are still some illiquid bank assets left at date 0 after trading, the direct cash injection and buying illiquid assets are equivalent;

(ii) Otherwise, depending on parameter values, buying illiquid assets and the direct injection of capital to banks can dominate each other.

[24] Recall the equilibrium condition satisfies $R^* > 1/P_0^*$. Therefore, for every dollar drop in the private buyers' investment, the banks will make more than one additional dollar of loans with returns greater than the forgone returns of the private buyers' investment. As a result, the total value from new investment is also higher.

Result (i) is straightforward. If banks have extra assets left after trading with private buyers, taking away assets at their face values is identical to taking an equivalent equity stake in banks. Result (ii) is again due to the fact that the illiquid asset price is different from the marginal loan liquidation value.

Finally, the following corollary compares buying bank loans and the direct capital injection.

Corollary 3.3

(i) If there are still asset fire sales (i.e., $P_1 < 1$), the direct cash injection strictly dominates buying bank loans;

(ii) Otherwise, the direct cash injection and buying bank loans are equivalent.

This results shows that the direct injection of capital to banks always weakly dominates buying bank loans. Result (i) is due to the fact that buying bank loans decreases the cash amount from loan liquidation. This leads to more conservative bank actions and inefficiency occurs. If there are no asset fire sales, banks need not liquidate loans at date 1. Therefore, buying loans does not have any negative impact on banks compared to the direct capital injection (result (ii)).

Fig. 3.9 shows numerical examples that illustrate Corollaries 3.1, 3.2, 3.3. In the figure, for a relative small amount of government intervention fund, buying assets and the direct capital injection are equivalent, and they both dominate buying bank loans in terms of total real investment. As the fund size increases such that there is no asset left under the buying asset policy, buying assets first dominates the other two policies. As the fund size increases, both buying loans and the direct capital injection dominate buying assets. In addition, the figure also shows that the direct capital injection weakly dominates buying bank loans for any size of intervention fund.

The policy implications discussed above are closely related to the finding of Philippon and Schnabl (2009), who compare the efficiency of three alternative intervention policies: buying equity, purchasing existing assets, and providing

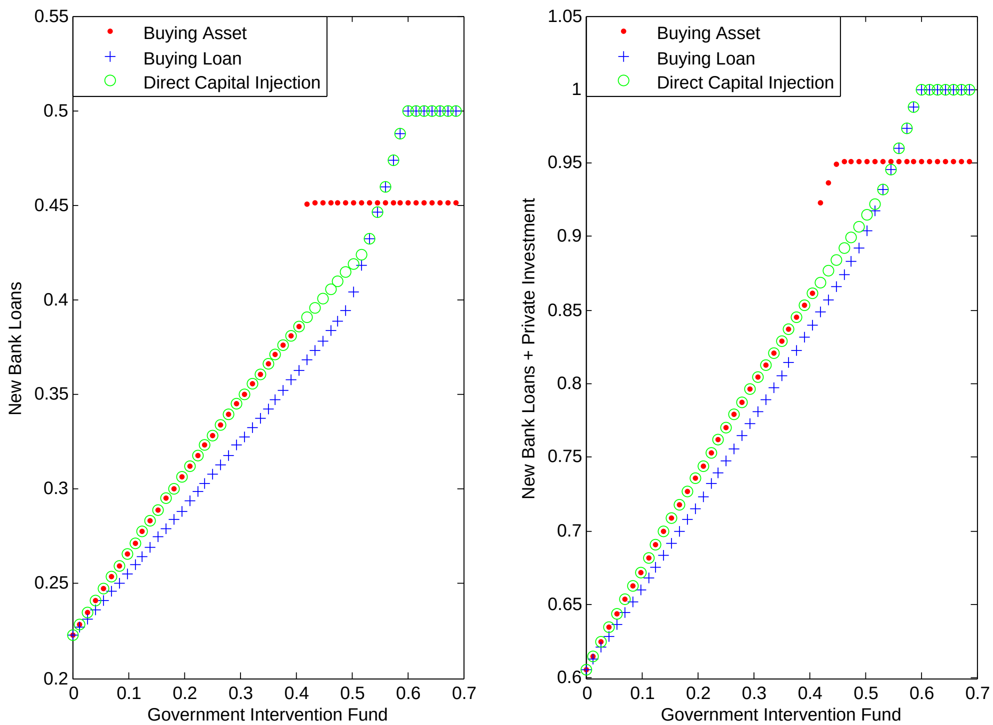

Figure 3.9: New Bank Loans and Total Investment under Government Intervention

The parameters used in these examples are as follows: $q = 0.2$, $\bar{R} = 1.2$, $I = 0.5$, $L = 0.5$, $Z = 2$, $D = 1.8$, $f = 0.25$, and $\theta = I + 0.25 = 0.75$. The bank initial endowments are: $\alpha = 0.35$, $\beta = 0.6$, and $\gamma = 0.05$.

debt guarantees. They find that all three policies are equivalent under symmetric information. However, with asymmetric information between firms and the government, buying equity dominates the other two intervention policies. Similar to Philippon and Schnabl (2009), my study also compares the efficiency of three government intervention policies: a direct capital injection (i.e., buying equity), buying bank loans, and buying bank illiquid assets. However, I find that buying equity and buying illiquid assets can dominate each other, depending on parameter values as well as the intervention size. More important, this ranking of intervention efficiency does not require information asymmetry in my study contrasting to Philippon and Schnabl (2009).

Moreover, the results in my study are also very useful to understand some seemingly surprising developments during the recent financial crisis. For example, Ivashina and Scharfstein (2009) find that banks cut their lending less if they have better access to deposit financing and thus they were not as reliant on short-term debts. I can simply explain this finding using the comparative statics presented above. In the terms of my current model, banks with better access to insured deposits and relying less on short-term debt will face a smaller liquidity shock. As shown in the last section, a smaller liquidity shock implies that banks will make more new loans, which is consistent to Ivashina and Scharfstein's findings.

Another feature of the recent crisis is that many banks are "clogged" with large holdings of illiquid assets, such as mortgage-backed securities. My model implies that a higher fraction of illiquid assets on banks' balance sheet will induce them to hold more cash and lend less when facing a liquidity shock. This matches the high cash to asset ratio of commercial banks during the past 10 months (see Fig. 3.3) as well as the sharp decline in the flow of bank loans recently (see Fig. 3.1).

3.5 Conclusion

This chapter studies banks' lending incentives under the risk of a liquidity shock. Banks make their lending decisions based on the tradeoff between the cost and benefit of lending. The cost of lending comes from the fire sales of illiquid assets to raise cash when the bank is hit by a liquidity shock. The benefit of lending is the high returns from the new bank loans. This study shows that it may be optimal for banks to lend out cash and therefore incur asset fire sales under a liquidity shock, even banks are endowed with enough cash to meet the liquidity shock. That is, fire sales of assets could be an endogenous outcome of banks' optimal decisions.

By the same tradeoff argument, banks may be willing to keep a large amount of cash to save the cost of potential asset fire sales. Although the forgone returns from the bank loans might be high, the costs of potential asset fire sales could be even higher. In addition, a higher ratio of bank illiquid assets to bank loans can induce banks to keep an even larger amount of cash on their balance sheet rather than lending out. As many banks are 'clogged' with large holdings of illiquid assets in the recent financial crisis, this study provides a novel explanation as to why banks hold a large amount of cash after government capital injections.

Moreover, this study also provides implications for the government intervention policy. According to the model, for a given amount of public fund for intervention, a direct capital injection dominates buying bank loans. However, buying illiquid assets and a direct injection of capital to banks can dominate each other, depending on parameter values as well as the size of the government intervention. This result can help the government to choose the most efficient intervention policy.

Chapter 4

Conclusions

To conclude, this dissertation covers two essays in finance, one on idiosyncratic stock return volatility and the other on bank lending incentives.

Chapter 2 investigates how frictions to real investment affect firm-level stock return volatility. The theoretical model predicts a nonlinear V-shaped relationship between the idiosyncratic return volatility and the asset growth rate. The high return volatility of low (negative) growth firms is due to the amplification of shocks to unexpected cash flows by firms' low market values. In contrast, the high return volatility of high growth firms is driven by the amplification of shocks to expected cash flows through growth opportunities. Aggregating the V-shape in each cross-section, the model also predicts a positive correlation between the cross-sectional dispersion in the asset growth rates and the average idiosyncratic return volatility. Formal empirical analysis provides strong evidence that supports these predictions in both the cross-section and the time series.

This chapter further documents the robustness of the empirical findings. The V-shaped relationship in the cross-section is robust even after controlling for factors such as size and leverage. The asset growth measure turns out to be the most important predictor of the average idiosyncratic return volatility in the value-weighted time series. Moreover, the asset growth effect on idiosyncratic return volatility, which is driven by investment frictions, empirically dominates alternative explanations of idiosyncratic return volatility such as cash flow and its volatility, growth options, and forecasted long-term earnings growth.

Chapter 3 develops a theoretical model to investigate how liquidity risks affect the prices of illiquid bank assets as well as banks' lending incentives. The first result of this study is that fire sales of assets could be an endogenous outcome of banks' optimal decisions. The intuition is as follows. Banks make their lending decisions based on the tradeoff between the cost and benefit of lending. The cost of lending comes from the fire sales of illiquid assets to raise cash when the bank is hit by a liquidity shock. The benefit of lending is the high returns from the new bank loans. This chapter shows that it may be optimal for banks to lend out cash and therefore incur asset fire sales under a liquidity shock, even banks are endowed with enough cash to meet the liquidity shock. In other words, it may not be in the banks' interest to avoid fire sales of their illiquid assets.

The second result of this study is that banks may be willing to keep a large amount of cash to save the cost of potential asset fire sales. Although the forgone returns from the bank loans might be high, the costs of potential asset fire sales could be even higher. In addition to a large liquidity shock, a higher ratio of bank illiquid assets to bank loans can induce banks to keep an even larger amount of cash on their balance sheet rather than lending out. As many banks are 'clogged' with large holdings of illiquid assets in the recent financial crisis, this study provides a novel explanation as to why banks hold a large amount of cash after government capital injections.

Finally, this study also generates policy implications for the government intervention. For example, for a given amount of public fund for intervention, a direct capital injection dominates buying bank loans in terms of their effect on the total real investment. However, buying illiquid assets and a direct injection of capital to banks can dominate each other, depending on parameter values as well as the size of the government intervention.

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Appendix A

Appendix for Chapter 2

A.1 Proof of Proposition 2.1

Consider first the frictionless case. For any given value of X, the firm instantaneously chooses the optimal capital $K(X)$. Let $y \equiv X/K(X)$. Since the firm value $V(X,K(X))$ is homogeneous of degree one in X and K, I can rewrite it as $V(X,K(X)) = v(1,K(X)/X)X$. Then the marginal value of capital, $\partial v(1,1/y)/\partial(1/y)$, equals one and is independent of X. Therefore, the ratio y has to be a constant for any value of X. In other words, the optimal investment policy of the firm is to keep a constant ratio of X/K. Given this optimal policy, the firm value will depend only on the value of X. The HJB equation is

$$\begin{aligned} rV(X) &= \pi(X) + \mu_x X V_x(X) + \frac{1}{2}\sigma_x^2 X^2 V_{xx}(X) - \frac{\mu_x X}{y} \\ &= hy^{-\alpha}X - \frac{\mu_x X}{y} + \mu_x X V_x(X) + \frac{1}{2}\sigma_x^2 X^2 V_{xx}(X). \end{aligned} \tag{A.1}$$

The solution of (A.1) after eliminating the bubble terms is

$$V(X) = \frac{hy^{-\alpha} - \mu_x y^{-1}}{r - \mu_x} X \equiv Q_x X. \tag{A.2}$$

Using the marginal value of capital $\partial Q_x/\partial(1/y) = 1$, the optimal ratio is determined as

$$y = \left(\frac{r}{\alpha h}\right)^{\frac{1}{1-\alpha}}. \tag{A.3}$$

I can then rewrite $V(X, K(X)) = QK(X)$, where $K(X) = X/y$ and $Q = Q_x y = (r/\alpha - \mu_x)/(r - \mu_x)$.

In the case with infinite adjustment cost, firms do not invest and the capital is constant. The HJB equation is

$$\begin{aligned} rV(X,K) &= \pi(X,K) + \mu_x X V_x(X,K) + \frac{1}{2}\sigma_x^2 X^2 V_{xx}(X,K) \\ &= hX^{1-\alpha}K^{\alpha} + \mu_x X V_x(X,K) + \frac{1}{2}\sigma_x^2 X^2 V_{xx}(X,K). \end{aligned} \tag{A.4}$$

The solution of (A.4) after eliminating the bubble terms is

$$V(X,K) = AX^{1-\alpha}K^{\alpha}, \quad \text{where } A = \frac{h}{r - [(1-\alpha)\mu_x - \alpha(1-\alpha)\sigma_x^2/2]}. \tag{A.5}$$

Recognize that this is the Gordon growth formula.

It is easy to obtain the volatility from firm values according to Eq. (2.7). In the frictionless case, denote the return by R_1, which follows

$$\mathrm{d}R_1 = \frac{\mathrm{d}V_t + \mathrm{d}C_t}{V_t} = \mathscr{O}(\mathrm{d}t) + \frac{Q_x \mathrm{d}X_t - \mathrm{d}X_t/y}{Q_x X_t} = \mathscr{O}(\mathrm{d}t) + (1 - 1/Q)\frac{\mathrm{d}X_t}{X_t}, \tag{A.6}$$

where $\mathscr{O}(\mathrm{d}t)$ represents terms of the same order of magnitude as $\mathrm{d}t$. Then the volatility in Eq. (2.8) follows.

Similarly, in the infinite adjustment cost case, denote the return by R_2, which follows

$$\mathrm{d}R_2 = \frac{\mathrm{d}V_t + \mathrm{d}C_t}{V_t} = \mathscr{O}(\mathrm{d}t) + \frac{\mathrm{d}X_t^{1-\alpha}}{X_t^{1-\alpha}} = \mathscr{O}(\mathrm{d}t) + (1-\alpha)\frac{\mathrm{d}X_t}{X_t}. \tag{A.7}$$

Then the volatility in Eq. (2.9) follows.

To compare the volatilities, note that $Q = 1/\alpha + \mu_x(1/\alpha - 1)/(r - \mu_x)$, which is greater (less) than $1/\alpha$ if $\mu_x > 0$ ($\mu_x < 0$). The result in Eq. (2.10) follows. ■

A.2 Motivating Empirical Observation on Cash Flow Volatility

Figure A.1 plots the average and standard deviation of the non-operating cost with respect to the earnings before interest, taxes, depreciation and amortization (EBITDA). The non-operating cost proxies for the unexpected cash flows, and the EBITDA proxies for the expected cash flows. The figure is generated with the following steps:

1. Extract quarterly income statement items (OIBDPQ, PIQ) from COMPUSTAT (1973Q1–2009Q4).

2. Define COST=(OIBDPQ-PIQ), which includes depreciation, interest expenses, nonoperating expenses, and special items. All measures are scaled by the lagged total book assets. Winsorize the income measures at 1% and 99% for each quarter.

3. Calculate the average and standard deviation of COST and the average of OIBDPQ within a rolling window of 20 quarters (requires at least 8 observations within the window).

4. Form deciles according to the average OIBDPQ for each quarter, and calculate the equal-weighted averages of the average and standard deviation of COST and the average of OIBDPQ for each decile and each quarter. Calculate the time series averages of these measures for each decile.

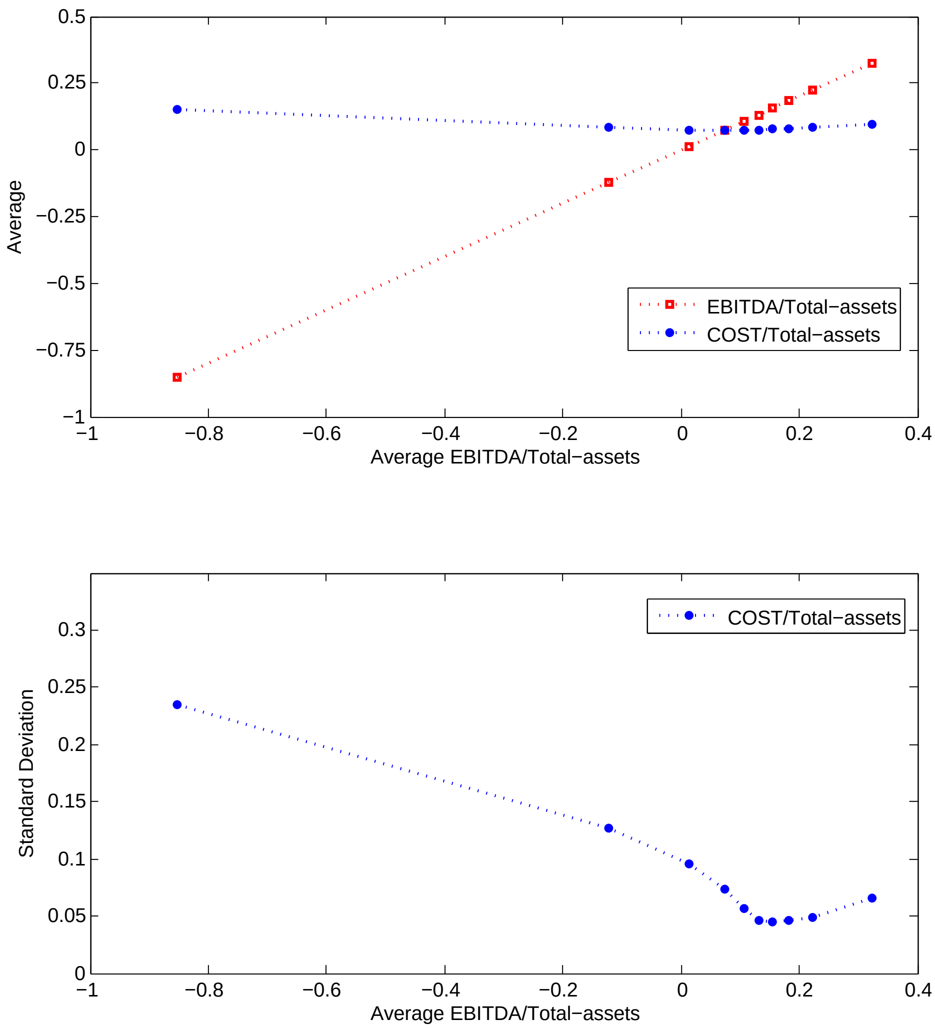

Figure A.1: Quarterly Nonoperating Cost vs. EBITDA

The non-operating cost (COST) includes depreciation, interest expenses, nonoperating expenses, and special items. The Earnings Before Interest, Taxes, Depreciation and Amortization (EBITDA) are the quarterly Operating Income Before Depreciation (OIBDPQ) in Compustat. Both the COST and EBITDA are scaled by book total assets.

A.3 Numerical Solution Method

In the full model, discretize the time into small interval Δt. The firm value is determined by

$$V(X,K) = \max_{I}\left\{\left[hX^{1-\alpha}K^{\alpha} - I - \phi(I,K)\right]\Delta t + e^{-r\Delta t}E[V(X',K')|X,K]\right\}, \tag{A.8}$$

where $X' = X + \Delta X$ and $K' = K + I\Delta t$.

Define $y = X/K$, $i = I/K$, $v(y) = V(X,K)/K$. Then the above problem reduces to

$$v(y) = \max_{i}\left\{\left[hy^{1-\alpha} - i - \theta i^2\right]\Delta t + (1+i\Delta t)e^{-r\Delta t}E[v(y')|y]\right\}, \tag{A.9}$$

$$\text{where} \quad y' = y\frac{1+\mu_x\Delta t + \sigma_x\Delta Z_x}{1+i\Delta t}, \text{and } \Delta Z_x \sim \mathcal{N}(0,\Delta t). \tag{A.10}$$

To solve the problem numerically, discretize y between $[\underline{y},\ \bar{y}]$. Note, however, that y' may be off these specified grids of y for general discretization of ΔZ_x (e.g., discretize ΔZ_x into 1000 equally spaced grids within $\pm 3\sqrt{\Delta t}$). Therefore, I need interpolate and extrapolate the value function $v(y)$ when calculating the expectation $E[v(y')|y]$. This can be done, for example, using the `interp1` function in Matlab.

Due to the short time interval ($\Delta t = 1/252$ for daily frequency), the discount factor $e^{-r\Delta t}$ is close to one. This makes the convergence of the value function iteration very slow. Therefore, I use the modified policy function iteration to speed up the convergence with the following steps:

1. Discretize y such that $\ln(y)$ is equally distributed along $[\ln(\underline{y}),\ \ln(\bar{y})]$. The range is chosen such that it includes the optimal value of y in the frictionless benchmark as given by Eq. (A.3).

2. Initialize the value function $v^0(y)$ along specified grids (e.g., 1000 grids).

3. For a given value function $v^n(y)$, find the optimal policy $i^n(y)$ according to

Eq. (A.9). Fix this policy function and iterate the value function for a given number of times (e.g., 500). Record the resulting value function as $v^{n+1}(y)$.

4. Repeat step 3 until the value function converges (i.e., for a small value of ε, $\sup|(v^{n+1}(y)-v^n(y))/v^n(y)|<\varepsilon$), and the policy function stays the same for a given number of policy iterations (e.g., 50). The resulting value function and the policy function are the solutions of the model.

Once the value function $v(y)$ and the corresponding policy function $i(y)$ have been found, the stock return with y changing to y' within Δt is given by

$$\Delta R(y,y') = \frac{\left[hy^{1-\alpha}-i-\theta i^2\right]\Delta t+(1+i\Delta t)v(y')+\sigma_c\Delta Z_c}{v(y)}-1, \tag{A.11}$$

where y' is random and given by Eq. (A.10).

The variance of this return (annualized) is straightforward to obtain:

$$\sigma^2(R(y)) = \frac{Var(\Delta R(y,y')|y)}{\Delta t}. \tag{A.12}$$

A.4 Operating Leverage and Idiosyncratic Return Volatility

Carlson et al. (2004) emphasize the importance of operating leverage in explaining the book-to-market effects. The argument goes as follows. When demand for a firm's product decreases, the firm's book-to-market ratio increases. Assuming that fixed operating costs are proportional to capital, the riskiness of returns increases due to higher operating leverage. That is, higher book-to-market stocks are riskier as a result of higher operating leverage.

Even though the above argument focuses on systematic risks, it also applies to idiosyncratic return volatility. By a similar argument, when demand for a firm's product decreases, the operating leverage increases and the firm would like to disinvest. This in turn will result in higher idiosyncratic return volatility for negative growth firms. However, as discussed in the main text, negative growth firms also have less growth options. This in turn results in lower return volatility. Therefore, it is important to investigate whether an alternative model with operating leverage can quantitatively generate similar predictions as produced by the model presented in the main text.

To incorporate operating leverage, I follow Carlson et al. (2004), and introduce a fixed operating cost, which are proportional to capital (fK). To emphasize the importance of operating leverage, in the following I consider modification to the simple version of the model in 2.2.2, where firms face only demand uncertainty. After adding the fixed cost, the net cash flow is rewritten as

$$\mathrm{d}C_t = [\pi(X_t, K_t) - I_t - \phi(I_t, K_t) - fK]\,\mathrm{d}t. \tag{A.13}$$

Following the same steps as in Section A.1, it is easy to obtain the return volatility for the two special cases:

$$\text{Case 1 } (\theta = 0): \qquad Var_{R_1} = \left(1 - \frac{r - \mu_x}{(r+f)/\alpha - (f + \mu_x)}\right)^2 \sigma_x^2, \tag{A.14}$$

$$\text{Case 2 } (\theta = \infty): \qquad Var_{R_2}(X/K) = \left[\frac{(1-\alpha)A(X/K)^{1-\alpha}}{A(X/K)^{1-\alpha} - f/r}\right]^2 \sigma_x^2, \tag{A.15}$$

where A is given by Eq. (A.5).

Panel (a) of Fig.A.2 plots the return volatility with respect to the demand-to-capital ratio (X/K). In the frictionless case, the demand-to-capital ratio is still constant. This leads to constant operating leverage and return volatility. In the special case with infinite adjustment cost, the firm's capital is exogenously given. The operating leverage is therefore also exogenously given and driven by the demand condition. Confirming the intuition as in Carlson et al. (2004), higher operating leverage (i.e., lower demand-to-capital ratio) results in higher return volatility.

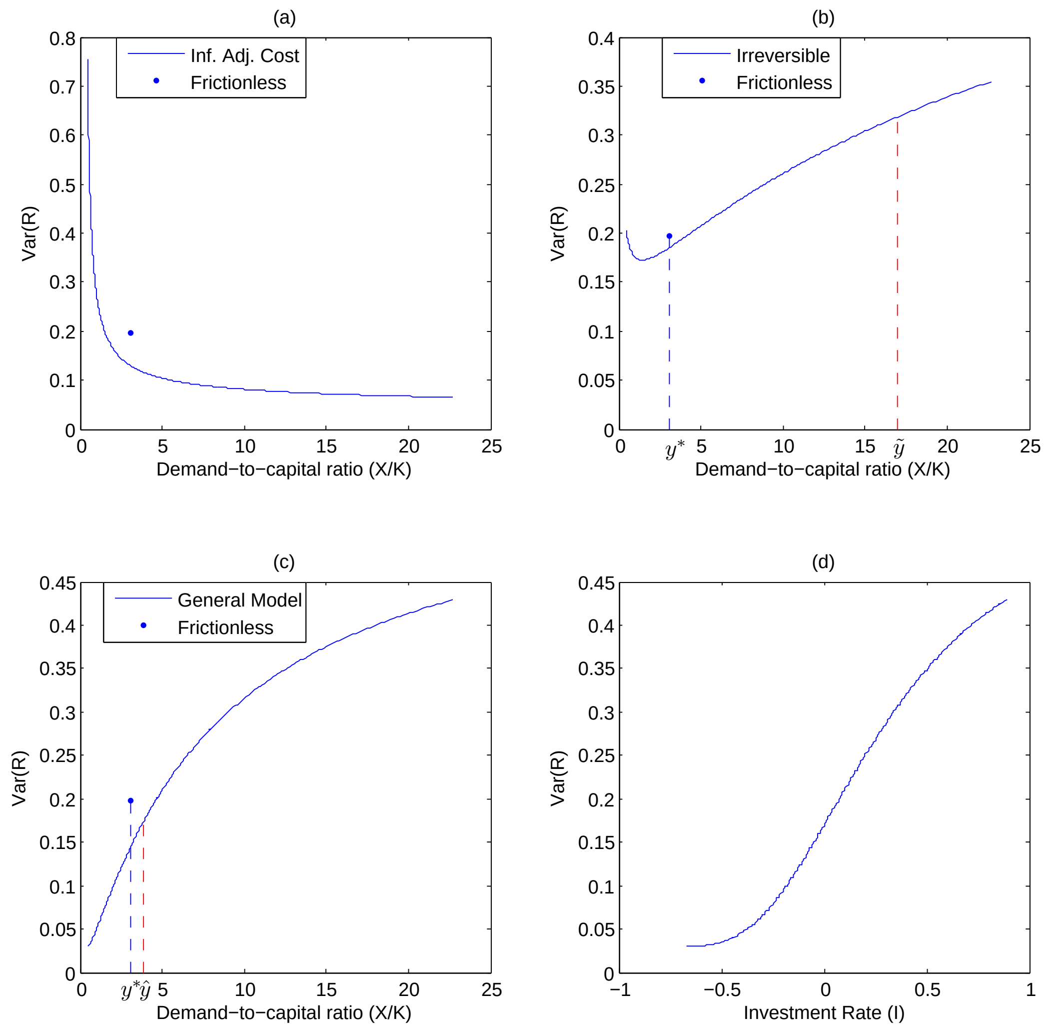

Figure A.2: Operating Leverage Effect on Return Volatility
The figure reports results of an alternative model with operating leverage. The common parameter values used are: $\alpha = 0.8$, $h = 0.2$, $r = 0.1$, $\mu_x = 0.06$, $\sigma_x = 0.8$, $f = 0.1$. In panel (b), $\theta^- = \infty$ and $\theta^+ = 0.15$ for the irreversible case. In panels (c) and (d), $\theta^- = \theta^+ = 0.15$ for the general case.

To demonstrate the importance of endogenous choice of operating leverage, I investigate one slight deviation of the case with infinite adjustment costs. Specifically, assuming investment is irreversible but subject to a finite adjustment cost for positive investment. In terms of the cost parameter,

$$\theta = \begin{cases} \theta^- = \infty & \text{if } I < 0; \\ \theta^+ & \text{if } I \geq 0. \end{cases} \tag{A.16}$$

Panel (b) of Fig.A.2 demonstrates that once the firm has the opportunity to invest, which means the operating leverage is partially endogenously determined, the threshold of demand-to-capital ratio for investment increases dramatically from y^* in the frictionless case to $\tilde{y}$. In this case, the effect of operating leverage is mostly dominated by the growth effect that I discussed in the main text. Only in the region with extremely low demand-to-capital ratio (comparing to the investment threshold $\tilde{y}$) the return volatility is slightly higher, but still much lower than that in the high demand-to-capital regions.

Finally, if the investment is costly reversible, panel (c) of Fig.A.2 shows that the threshold of demand-to-capital ratio for investment ($\hat{y}$) is only slightly higher than that of the frictionless case (y^*). Note that the parameter values except the fixed cost (f) used in panel (c) of Fig.A.2 are the same as that in Fig.2.1. In this case, the operating leverage effect is totally submerged by the growth effect as firms have more and more put options when the demand becomes low. This results in a low return volatility for low demand-to-capital firms. Panel (d) of Fig.A.2 plots the same return volatility as in panel (c) but with respect to investment rate. The numerical example shows that the return volatility is increasing with the investment rate.

Overall, the analysis presented in this section shows that the operating leverage effect is important only when it is exogenously specified. Once the firms can choose endogenously their operating leverage (through investment), then the operating leverage effect on the return volatility is quantitatively dominated by the growth effect for firms with negative growth. In other words, operating leverage cannot quantitatively explain the high return volatility of negative growth firms.

Appendix B

Appendix for Chapter 3

B.1 Proofs

Proof of Lemma 3.1: Since date-0 trading and date-1 trading share the same cash constraint (3.7), we only need compare the equity values directly for the two trading schemes in Eqns. (3.9, 3.10).

$$E_0^1 - E_0^0 = \left[\theta - I(\frac{1}{P_0})\right]\left[\frac{1}{P_0} - \left(\frac{q}{P_1} + (1-q)\right)\right] = 0, \tag{B.1}$$

where Eqn. (3.5) is used in the last step. Therefore, banks are indifferent between date-0 and date-1 trading. This proves the first bullet.

For the second bullet, let's first look at one special example that makes the claim obvious. Imagine that banks have no cash, and private buyers have excess cash (after their own investment) which is larger than the liquidity shock (fD). At the beginning, the asset should trade at the fare value ($P_0 = P_1 = 1$). If banks trade at date 1, then there is no gain or loss directly from trading. However, if banks trade at date 0, they can use the cash proceeds to make loans with a high return of $\bar{R}$ without incurring any cost (i.e., still no loan liquidation or asset fire sales). It is obvious that banks would prefer to trade earlier. Such a special example can be extended to more general cases, but the intuitions are still the same.

Let's assume that banks are required to keep the cash from the date-0 asset trade. Then the result in Eqn. (B.1) holds. That is, the equity value would be the same as that of the date-1 trading. Now, let's relax the cash holding requirement. Once the banks get cash in hands they would decide how much to lend according to the sign of

$$\frac{\mathrm{d}E_0^0}{\mathrm{d}R} = \frac{-L}{\bar{R}-1}\left[R - \frac{1}{P_0} - \frac{q\left[\theta - I(\frac{1}{P_0})\right]}{\beta Z P_1^3 + \frac{Iq}{\bar{R}-1}}\right]. \tag{B.2}$$

Therefore, if $R > \frac{1}{P_0} + \frac{q\left[\theta - I(\frac{1}{P_0})\right]}{\beta Z P_1^3 + \frac{Iq}{\bar{R}-1}}$, then banks would prefer to lend out the cash (i.e., R will be lower). If the banks do not have enough cash endowment for lending at date 0, they would have to sell assets at date 0 to get the cash for further lending, which is beneficial for them. ■

Proof of Proposition 3.1: From Lemma 3.1, if the banks are solvent at date 1 under a liquidity shock, then either they are indifferent between the two trading dates or they prefer date-0 trading. Therefore, the insolvency of banks at date 1 under liquidity shock is a necessary condition for the asset freeze at date 0.

Imagine that banks are just solvent under a liquidity shock at date 1. Then according to Lemma 3.1, banks might prefer date-0 trading if the available loan rate is high relative to the cost of fire sales. Now, if banks are just insolvent, they would also prefer date-0 trading by the same reason. Therefore, the insolvency of banks at date 1 under a liquidity shock is not a sufficient condition to freeze the asset market at date 0. ■

Proof of Proposition 3.2: There are two cases. For case I (corresponding to case (i) in the proposition), $\theta \leq I$. I would like to show that it's always optimal for banks to lend out cash and incur liquidation of loans and fire sales of illiquid assets.

Using Eqns. (3.7, 3.9), the equity value changes with the amount of new loans extended by banks (or equivalently, R, the cutoff rate of new loans) as follows.

$$\mathrm{d}E_0^1 = q\left[\mathrm{d}F(R) - \frac{\mathrm{d}L(R)}{P_1}\right] + (1-q)[\mathrm{d}F(R) - \mathrm{d}L(R)] + q\frac{\left[\theta - I(\frac{1}{P_0})\right]^+}{P_1^2}\mathrm{d}P_1. \tag{B.3}$$

At the start of the liquidation of loans (or further at the start of the fire sales), $\theta \leq I(1/P_0)$. Combining the above equation with Eqn. (3.5), we have,

$$\frac{\mathrm{d}E_0^1}{\mathrm{d}R} = q\left[\frac{-RL}{\bar{R}-1} - \frac{-L}{(\bar{R}-1)P_1}\right] + (1-q)\left[\frac{-RL}{\bar{R}-1} + \frac{L}{\bar{R}-1}\right] = \frac{-L}{\bar{R}-1}(R - \frac{1}{P_0}). \tag{B.4}$$

At the start of liquidation (without fire sales), $P_0 = P_1 = 1$ and $R > 1$ (since it is assumed that cash remaining after all investment opportunities are taken is smaller than fD, so, when cash equals fD, $R > 1$). Therefore the derivative in the above equation is negative, meaning that banks would prefer to make more loans (i.e., lower R). As more cash is lent out, eventually the assets will start to trade. Assume $\tilde{R} = 1/\tilde{P}_0$ solves $\beta Z(1-\tilde{P}_1^2)/2 + [\gamma Z - L(\tilde{R})] = fD$. If $\tilde{R} > I^{-1}(\theta)$ (see footnote 17), then $R > 1/P_0$ at the start of fire sales (since $R > \tilde{R}$, and $1/P_0 < 1/\tilde{P}_0$). As a result, the above derivative is again negative. Therefore, it is optimal for banks to lend out more cash and incur fire sales as well.

Case II corresponds to cases (ii) and (iii) in the proposition. In this case, $\theta > I(1) \geq I(1/P_0)$. Note first that banks would like to lend out cash until the combined cash from banks and the private buyers equal the liquidity shock fD, as there is only benefit (high loan rate) but no cost (no loan liquidation and asset fire sales) up to that point. We next look at the banks' lending decision at that point. Combining Eqns. (3.7, 3.9), we have,

$$\frac{\mathrm{d}E_0^1}{\mathrm{d}R} = \frac{-L}{\bar{R}-1}\left[R - \frac{1}{P_0} - \frac{q\left[\theta - I(\frac{1}{P_0})\right]}{\beta Z P_1^3 + \frac{Iq}{\bar{R}-1}}\right]. \tag{B.5}$$

Therefore, banks' lending decisions will depend on the sign of the above derivative. If $\theta - I$ is small, then the derivative is negative for a large R and high prices. That is, banks would prefer to lend more and incur both the loan liquidation and asset fire sales (case (ii)). On the other hand, if $\theta - I$ is large enough such that the above derivative is positive, then banks would not lend out cash any more. As a result, there is no loan liquidation and asset fire sales (case (iii)). ■

Proof of Proposition 3.3: The equilibrium conditions are derived in the main text. Here I provide an alternative approach to derive the optimal bank lending condition. The equilibrium must be that Eqn. (B.5) is zero. That is

$$R = \frac{1}{P_0} + \frac{q\left[\theta - I(\frac{1}{P_0})\right]}{\beta Z P_1^3 + \frac{Iq}{\bar{R}-1}}. \tag{B.6}$$

This is equivalent to Eqn. (3.14) as specified in the proposition. ■

Proof of Proposition 3.4: The system of two equations:

$$A_1Y_1 + B_1Y_2 + C_1 = 0, \tag{B.7}$$

$$A_2Y_1 + B_2Y_2 + C_2 = 0, \tag{B.8}$$

has solutions given by

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{pmatrix} -C_1 \\ -C_2 \end{pmatrix} = \frac{-1}{A_1B_2 - A_2B_1} \begin{pmatrix} B_2C_1 - B_1C_2 \\ A_1C_2 - A_2C_1 \end{pmatrix}. \tag{B.9}$$

Let x be the variable of interest. Take first order derivatives of equilibrium conditions in Eqns. (3.14, 3.15) with respect to x:

$$\overbrace{\left[\frac{\partial MC_S}{\partial P_1}-\frac{\partial MB_L}{\partial P_1}\right]}^{A_1<0}\frac{\partial P_1^*}{\partial x}+\overbrace{\left[\frac{\partial MC_S}{\partial R}-\frac{\partial MB_L}{\partial R}\right]}^{B_1<0}\frac{\partial R^*}{\partial x}+\overbrace{\left[\frac{\partial MC_S}{\partial x}-\frac{\partial MB_L}{\partial x}\right]}^{C_1}=0; \tag{B.10}$$

$$\underbrace{-\left[\beta ZP_1^*+\frac{Iq}{(\bar{R}-1)P_1^{*2}}\right]}_{A_2<0}\frac{\partial P_1^*}{\partial x}+\underbrace{\frac{L}{\bar{R}-1}}_{B_2>0}\frac{\partial R^*}{\partial x}+\underbrace{\frac{\partial(\text{cash-under-shock})}{\partial x}}_{C_2}=0. \tag{B.11}$$

Since $A_1B_2-A_2B_1<0$, using the above three equations together, we have,

$$\text{Sign}\begin{pmatrix}\frac{\partial P_1^*}{\partial x}\\ \frac{\partial R^*}{\partial x}\end{pmatrix}=\text{Sign}\begin{pmatrix}B_2C_1-B_1C_2\\ A_1C_2-A_2C_1\end{pmatrix}. \tag{B.12}$$

Now, if $x=\alpha$ and $\beta=1-\alpha-\gamma$, then $C_1>0$ and $C_2<0$. Therefore, we have,

$$\text{Sign}\begin{pmatrix}\frac{\partial P_1^*}{\partial \alpha}\\ \frac{\partial R^*}{\partial \alpha}\end{pmatrix}=\text{Sign}\begin{pmatrix}B_2C_1-B_1C_2\\ A_1C_2-A_2C_1\end{pmatrix}=\begin{pmatrix}\pm\\ +\end{pmatrix}. \tag{B.13}$$

Therefore, the equilibrium cutoff rate for new bank loans is higher the higher the α (i.e., the higher the asset to loan ratio). That is, the banks will keep a larger amount of cash in hands.

If $\frac{\partial P_1^*}{\partial \alpha}<0$, then private buyers's cash $[\theta-I(\frac{1}{P_0^*})]$ will be larger for a larger α. So the total cash is larger. For $\frac{\partial P_1^*}{\partial \alpha}\geq 0$, note that the total cash holding of banks and buyers is

$$[\gamma Z-L(R)]+\left[\theta-I(\frac{1}{P_0^*})\right]=fD-\frac{1}{2}(1-\alpha-\gamma)Z(1-P_1^{*2}). \tag{B.14}$$

Note that the RHS of the above equation increases with α. So do the total cash holding on the LHS.

From Eqn. (B.13), $\frac{\partial P_1^*}{\partial \alpha} < 0$ if and only if $B_2C_1 - B_1C_2 < 0$. That is, the necessary and sufficient condition for the asset price to decrease with α is given by

$$\overbrace{\frac{L}{\bar{R}-1}}^{B_2} \frac{\overbrace{q\left[\theta - I(\frac{1}{P_0^*})\right] ZP_1^{*3}}^{C_1}}{\left[\beta ZP_1^{*3} + \frac{Iq}{\bar{R}-1}\right]^2} - \overbrace{(-1)}^{B_1} \overbrace{\left[-\frac{1}{2}Z(1-P_1^{*2})\right]}^{C_2} < 0. \tag{B.15}$$

This condition is more likely to be satisfied if the banks do not rely heavily on the buyer's cash to meet the liquidity demand (i.e., a small value of $[\theta - I(\frac{1}{P_0^*})]$). ■

Proof of Proposition 3.5: Follow the proof above for Proposition 3.4, but set $x = \gamma$ and fix α and β (or alternatively, set $x = \gamma$, $\alpha = 1 - \beta - \gamma$, and fix β). In both cases, $C_1 = 0$ and $C_2 > 0$. Therefore,

$$\text{Sign}\begin{pmatrix} \frac{\partial P_1^*}{\partial \gamma} \\ \frac{\partial R^*}{\partial \gamma} \end{pmatrix} = \text{Sign}\begin{pmatrix} B_2C_1 - B_1C_2 \\ A_1C_2 - A_2C_1 \end{pmatrix} = \begin{pmatrix} + \\ - \end{pmatrix}. \tag{B.16}$$

That is, the illiquid assets will trade at a higher price, and banks will lend more and require a lower minimum required rate for new loans with more cash endowment. Similarly, the private buyers will invest more cash in their own long-term investment opportunity. ■

Proof of Corollary 3.1: The first bullet is a direct result of Propositions 3.4. For the second bullet, we simply show the results through numerical examples in Fig. 3.9. See also the proof of Corollary 3.2. ■

Proof of Corollary 3.2: Let's first look at the direct government capital injection (DCI) of gZ to banks as the benchmark. After the cash injection, $\alpha + \beta + \gamma = 1 + g$. Further assuming that government takes a stake of equity which worths gZ in banks.

According to Lemma 3.1, banks are indifferent between trading assets at date

0 and date 1. Without loss of generality, assume banks trade assets only at date 0. Then the equity value for banks after subtracting government's share is,

$$E_0^0(DCI) = q\left\{\beta Z P_1 + F(R) + \left[\alpha Z - \frac{\theta - I(\frac{1}{P_0})}{P_0}\right] - (1-f)D - gZ\right\} \quad \text{(B.17)}$$
$$+ (1-q)\left\{Z - D + F(R) - L(R) + \left[\theta - I(\frac{1}{P_0})\right]\left(1 - \frac{1}{P_0}\right)\right\},$$

subject to

$$[\gamma Z - L(R)] + \frac{1}{2}\beta Z(1 - P_1^2) + \left[\theta - I(\frac{1}{P_0})\right] = fD. \quad \text{(B.18)}$$

The resulting equilibrium condition is the same as before,

$$R(DCI) = \frac{q}{P_1} + (1-q) + q\frac{\theta - I(\frac{1}{P_0})}{\beta Z P_1^3 + \frac{Iq}{\bar{R}-1}}. \quad \text{(B.19)}$$

Now, if the government buys illiquid assets (BA) at the face value at date 0 for the same amount of cash, then

$$E_0^0(BA) = q\left\{\beta Z K_1 + F(R) + \left[(\alpha - g)Z - \frac{\theta - I(\frac{1}{P_0})}{P_0}\right] - (1-f)D\right\} \quad \text{(B.20)}$$
$$+ (1-q)\left\{Z - D + F(R) - L(R) + \left[\theta - I(\frac{1}{P_0})\right]\left(1 - \frac{1}{P_0}\right)\right\},$$

and

$$[\gamma Z - L(R)] + \frac{1}{2}\beta Z(1 - K_1^2) + \left[\theta - I(\frac{1}{P_0})\right] = fD, \quad \text{(B.21)}$$

where I have differentiated the loan liquidation price K_1 and the asset price P_1. Let's define similarly

$$\frac{1}{K_0} \equiv \frac{q}{K_1} + (1-q).$$

Notice that if $(\alpha - g)Z > \frac{\theta - I(\frac{1}{K_0})}{K_0}$, then $E_0^0(DCI) = E_0^0(BA)$ and $K_1 = P_1$. That is, the two methods, the direct injection of capital and buying the bank illiquid assets, give the same equity values. More important, these two methods will have the same effect on the bank lending and the private buyer's own investment activities. That is, they will have the same effect on the real economy.

On the other hand, if $(\alpha - g)Z < \frac{\theta - I(\frac{1}{K_0})}{K_0}$, then it must be that $(\alpha - g)Z = \frac{\theta - I(\frac{1}{P_0})}{P_0}$. (We label this case as BA2.) Therefore, the minimum loan liquidation value is smaller than the asset price at date 1 ($K_1(BA2) < P_1(BA2)$).

Since P_0 is fixed for a given g, the new equilibrium condition becomes

$$R(BA2) = \frac{q}{K_1} + (1-q). \tag{B.22}$$

Combining Eqns. (B.18, B.19, B.21, B.22), it is easy to show that $K_1(BA2) < P_1(DCI)$. At the start of BA2 ($P_1(BA2) < 1$), the total investment under BA2 is higher than that under DCI by a similar argument that we did for (B.14). However, as the government intervention fund increases such that $P_1(BA2) = 1$, the above argument might not hold any more. This is because both the banks and the private buyers' investment will be fixed under BA2 with the government fund crowding out the private buyers' cash dollar by dollar.

Therefore, depending on parameter values as well as the size of the intervention, buying assets could induce banks to lend the same, more, or less new loans, comparing to the direct capital injection to banks. This is also true for the private buyers' investment. Fig. 3.9 provides supporting numerical examples. ■

Proof of Corollary 3.3: The equity value under buying loans (BL) is given by

$$\begin{aligned} E_0^0(BL) = & q\left\{(\beta - g)ZP_1 + F(R) + \left[\alpha Z - \frac{\theta - I(\frac{1}{P_0})}{P_0}\right] - (1-f)D\right\} \\ & + (1-q)\left\{Z - D + F(R) - L(R) + \left[\theta - I(\frac{1}{P_0})\right]\left(1 - \frac{1}{P_0}\right)\right\}, \end{aligned} \tag{B.23}$$

subject to

$$[\gamma Z - L(R)] + \frac{1}{2}(\beta - g)Z(1 - P_1^2) + \left[\theta - I(\frac{1}{P_0})\right] = fD. \tag{B.24}$$

If $P_1 = 1$, then Eqns. (B.17, B.18) are equivalent to Eqns. (B.23, B.24). Therefore, the two policies are equivalent.

If $P_1 < 1$, then the following optimal condition holds.

$$R(BL) = \frac{1}{P_0} + \frac{q\left[\theta - I(\frac{1}{P_0})\right]}{(\beta - g)ZP_1^3 + \frac{Iq}{\bar{R}-1}}. \tag{B.25}$$

If $R(BL) \leq R(DCI)$, then $P_1(BL) < P_1(DCI)$ (comparing (B.18) and (B.24)). Now, if (B.19) holds, then it must be that $R(BL) > R(DCI)$ (comparing (B.19) and (B.25)). This leads to a contradiction. Therefore, banks will lend less under the policy of buying bank loans.

Next, let's look at the total investment.

$$\text{DCI:} \quad L(R) + I(\frac{1}{P_0}) = \gamma Z + \theta - fD + \frac{1}{2}\beta Z(1 - P_1^2). \tag{B.26}$$

$$\text{BL:} \quad L(R) + I(\frac{1}{P_0}) = \gamma Z + \theta - fD + \frac{1}{2}(\beta - g)Z(1 - P_1^2). \tag{B.27}$$

If $P_1(BL) < P_1(DCI)$, then private buyers will also invest less under buying loans. As a result, the total investment is also lower under buying loans. If $P_1(BL) \geq P_1(DCI)$, then the RHS of (B.27) is smaller than the RHS of (B.26). Therefore, the total investment is again lower under buying loans. ■