Dimensionless cosmology

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Abstract

The variability of fundamental physical constants has been a topic of interest both theoretically and experimentally for many years. Although it is interesting to investigate the consequences of such a variation, it is important to realise that only the variation of dimensionless combination of constants can be meaningfully measured and discussed. In this thesis, I try to justify this way of thinking and apply it to two basic cosmological observables, Big Bang Nucleosynthesis and Cosmic Microwave Background anisotropies. I will mention some related studies that are either wrong or not complete because of being dimensionful.

Variation of constants could be considered on two different levels. On the first level one assumes that the constants are time invariant but they can assume different values in different Universes or patches of sky. A thought experiment describing a discussion with aliens having a different system of units with different coupling constants could be helpful, this idea will be returned to throughout the thesis. On the next level, the constants can be promoted to being smooth functions of time or space. It is good to have a firm understanding of what happens on the previous level before trying to consider genuinely variable constants. For variable constants we need to consider theories beyond the currently accepted ones, which are capable of consistently describing such a variation.

I briefly review the scalar-tensor theory of gravity as a possible way to describe the variation of the gravitational coupling. I give a brief historical review on the subject and consider the theory in the two so called 'frames', discussing about the benefits of each frame mathematically and the physical meaning of these frames. Such theories could form the frame work in which further study of variable constants could be carried out.

Preface

The introduction is partly based on a published work. Moss, A. Narimani, A. and Scott, D. Int. J. Mod. Phys. D, 19:22892294.

The main Body of the thesis, including chapters 2 to 5 are based on a submitted paper. Narimani, A. Moss, A. Scott, D. arXiv:1109.0492.

Prof. D. Scott proposed the main idea of these works. I was responsible for performing calculations and Dr. A. Moss helped me through Cosmic Microwave Background codes and calculations.

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Glossary

BAO Baryon Acoustic Oscillations

CMB Cosmic Microwave Background

ISW Integrated Sachs-Wolfe

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Chapter 1

Introduction

It doesn't matter how beautiful your theory is, it doesn't matter how smart you are or what your name is. If it doesn't agree with experiment, it's wrong. — Richard Feynman [*ch 7, Seeking New Laws*]

For a long while, the ultimate aim of physics has been to discover the underlying laws of nature. Through this process of discovery, physicists were expected to be able to explain the qualitative and quantitative aspects of some experiment in great detail. However, there are some practical difficulties associated with this way of thinking about physics which have been with us since the previous century.

A large body of work in theoretical physics involves areas about which we can have no direct empirical information, e.g. the interior of a blackhole or energies so high that they are beyond the capacity of any accelerator made on Earth. The goal for researchers in these fields seems to be to construct internally consistent theories in a way which might one day lead to some specific predictive power, without any contact with relevant measurement at least the near future. [7, 87]

Through the great improvements in theoretical physics over the past century, physicists are in a situation where they can try to find scientific answers to some old questions about the Universe. It is now possible to scientifically explore the Universe and see if it is infinite or finite in space, eternal or with a beginning, finely tuned or not. All of these issues date back to at least ancient Greece. Although these questions are seemed firmly in the realm of philosophers, modern cosmological observations now make them amenable to impirical investigation.

The possibility of variation in the laws of physics is an issue with some empirical problems. On the one hand, despite of its promise for extending our theoretical knowledge and understanding of Universe, the most likely energy scales for finding such a variation seems well beyond the reach of any accelerator on Earth. On the other hand, it needs needs to be extremely careful to make meaningful statements on this topic before interpreting experiments. Attempts to clarify how one might test for changing laws of physics end up requiring careful thought about what one means by a measurement, the importance of units in a measurement, and other basic considerations that have all been present since the very first physical experiments.

This thesis aims to try to find a consistent way of asking questions about the variation of fundamental constants in physics and, attempts to answer those questions with some phenomenological inspections in cosmology as a relevant place to seek for such a variation. There has been a wide and varied literature on this topic, from at least the time of the numerological ponderings of Kelvin and Dirac [32, 33]. Dirac argues in these papers that the ratio of electrical forces to gravitational forces between an electron and a proton is of the same order of magnitude as the age of the Universe on an atomic time scale. The age of the Universe is an increasing number while the ratio of the two forces is constant so in order to keep this order of magnitude equality he suggested that Newton's constant is changing with time. Dirac's 1937 paper on ' The Cosmological Constants' has been cited more than 500 times and there are numerous papers published since then claiming to constrain variations of the speed of light, c, electrical charge, e, Planck constant, h, or Newton's constant, G, a number of which we will mention in this thesis.

However, there is a long history of debate about whether one can measure the time-variation of a *dimensionful* constant. Although claims to the contrary continue to crop up, there is consensus among a long and illustrious line of physicists that only the variation of *dimensionless* combinations of constants can be meaningfully discussed. Among those pointing this out are Dicke [29], McCrea [82], Rees [105], Jeffreys [62], Duff et al. [36], Silk [116] and Wesson [133].

The basis for this view is that physical units are quite arbitrary, and that each individual constant can be removed through a suitable choice of units (see e.g. [35, 36, 108] for a comprehensive discussion). The simplest example is the speed of light, c, whose variation is now manifestly unmeasurable because of its designation as a fixed constant relating the definition of time and distance units. Dimensionless quantities, on the other hand, are independent of the choice of units and so should admit the possibility of genuinely observable changes.

From a different perspective, by considering how physics works empirically, one finds realises that any measurement can be reduced to dimensionless ratios of quantities – in other words, a measurement of some quantity is always made *relative* to some other quantity of the same dimensions. Therefore, the outcome of any actual measurement is a dimensionless number.

1.1 Dimensionless Gravity

Much has been written (see e.g. Uzan [129]) about variation in the fine structure constant,

$$\alpha_{\rm em} \equiv \frac{e^2}{4\pi\varepsilon_0\hbar c} \tag{1.1}$$

(where we have retained SI units in order not to obscure arguments about dimensions and units). A great deal of the activity in recent years has been inspired by claims of measurements or tight constraints on the variability of this constant with time [65, 85, 91, 103, 121].

There is a very close analogy between the Coulomb force and classical gravity, and hence one can define a 'gravitational fine structure constant' :

$$\alpha_{\rm g} \equiv \frac{Gm_{\rm p}^2}{\hbar c}. \tag{1.2}$$

This choice of notation goes back at least to Silk in 1977 [116]. It explicitly selects the proton mass as the 'gravitational charge', although any other particle mass can be used instead. Using current values for the fundamental constants one obtains $\alpha_{\rm g} = 5.9 \times 10^{-39}$. The fact that this is so small underscores the weakness of gravity compared with electromagnetism.

What the definition of $\alpha_{\rm g}$ means is that experimental observations should be sensitive only to G multiplied by the square of the gravitational charge (and normalized by Planck's constant and the speed of light). Just as in the case of variation of α_{em}, one is *not* allowed to ask *which* of the parts of α_{g} are varying with time.

Any discussion of the variation of the strength of gravity should start from this point. In other words, constraints which appear to have been placed on the variation of G only make sense if they can be interpreted as constraints on Gm_{p}^2.

It should also be noted that just because a quantity is dimensionless does *not* imply it is unit-independent. $\dot{G}/G$ (or $\Delta G/G$) suffers from exactly the same unit-dependence problems as $\dot{G}$. One could use a system of units in which $G = 1$ by definition, and hence there is no $\dot{G}$ in that system of units at all!

Having said all of this, it is perfectly reasonable to choose a system of units in which $\{c, h, m_{\text{p}}\}$ are fixed constants, and interpret any variation in α_{g} as a variation in G. The main point that one should always bear in mind is that there is only one degree of freedom in α_{g}. One can choose either G or m_{p} to represent that degree of freedom, but not both of them at the same time. And one needs to make sure to do this consistantly throughout a set of calculations.

1.2 Warm-Up Calculations

A well known example of the use of α_{g} in astrophysics is in the determination of the minimum mass of a star from first principles [15]:

$$M_* \sim \alpha_{\text{em}}^{3/2} \alpha_{\text{g}}^{-3/2} \left(\frac{m_{\text{e}}}{m_{\text{p}}}\right)^{-3/4} m_{\text{p}}, \tag{1.3}$$

or in other words the characteristic size of a star is $\sim 10^{57}$ protons. This is similar to the Chandrasekhar mass:

$$M_{\text{Ch}} \sim \alpha_{\text{g}}^{-3/2} m_{\text{p}} \sim 10^{57} m_{\text{p}}. \tag{1.4}$$

This means that if one were to compare such a mass measured today with the same mass measured billions of years ago, then one could determine that there had been a change because a *pure number* (in this case the number of nucleons in a star) was different.

An estimate developed later gives the mass of a galaxy from a cooling time argument (e.g.Rees and Ostriker [106], Silk [116]):

$$M_{\rm gal} \sim \alpha_{\rm em}^5 \alpha_{\rm g}^{-2} \left(\frac{m_{\rm p}}{m_{\rm e}}\right)^{1/2} m_{\rm p} \sim 10^{69} m_{\rm p}. \tag{1.5}$$

If we wish we can rewrite these in terms of the Planck mass by using $m_{\rm p} = \alpha_{\rm g}^{1/2} m_{\rm Pl}$.

We can also estimate (see e.g. Padmanabhan [100]) the characteristic lifetime of a Main Sequence star as:

$$t_* \sim \alpha_{\rm em}^{-1} \alpha_{\rm g}^{-1} \left(\frac{m_{\rm e}}{m_{\rm p}}\right)^{-1/2} \frac{\hbar}{m_{\rm e} c^2}. \tag{1.6}$$

We can rewrite this as a dimensionless number of Planck times

$$t_* \sim \alpha_{\rm em}^{-1} \alpha_{\rm g}^{-3/2} \left(\frac{m_{\rm e}}{m_{\rm p}}\right)^{-1} t_{\rm Pl}. \tag{1.7}$$

where

$$t_{\rm Pl} \equiv \left(\frac{\hbar G}{c^5}\right)^{1/2}. \tag{1.8}$$

1.3 Different Units

Comparing (1.6) with (1.7) we see that the same physical observable has different $\alpha_{\rm g}$ dependence based on the units used to measure that quantity. This might lead us think there should be a preferred system of units one can use, if we are going to make a prediction for the variation of fundamental constants.

To clarify the issue, let us think of a physical observable with units of time, T, which based on a theoretical prediction has some $\alpha_{\rm g}$ dependence like $T \propto \alpha_{\rm g}^p$, when expressed in seconds. The same quantity has a different $\alpha_{\rm g}$ dependence in terms of the Planck time: $T \propto \alpha_{\rm g}^{p-1/2}$. On the observational side, T is measured with some uncertainty δT which could be used to constrain a possible time variation of $\alpha_{\rm g}$. Now, the question is: are these two different theoretical predictions the same when compared with observational data?

The answer is: *Yes*. There is some G dependence in $t_{\rm Pl}$ that should be taken into account. Considering this dependency, one finds the same results in both cases.

Let us temporarily choose G to represent α_{g}. The observed value of T in seconds, including measurement error, is $T + \delta T$. One can infer the following constraint on G based on this observation:

$$\frac{T+\delta T}{T} = \left(\frac{G+\delta G}{G}\right)^{p}. \tag{1.9}$$

The observed value of T in Planck times is $(T + \delta T)/t_{\mathrm{Pl}}$. If one assumes a variation in the strength of gravity, then t_{Pl} varies as well. Therefore, the inferred constraint on G would be:

$$\left(\frac{T+\delta T}{T}\right)\cdot\left(\frac{t_{\mathrm{Pl}}}{t'_{\mathrm{Pl}}}\right) = \left(\frac{T+\delta T}{T}\right)\cdot\left(\frac{G}{G+\Delta G}\right)^{\frac{1}{2}} = \left(\frac{G+\Delta G}{G}\right)^{p-\frac{1}{2}}, \tag{1.10}$$

and

$$\frac{t_{\mathrm{Pl}}}{t'_{\mathrm{Pl}}} = \left(\frac{G}{G+\Delta G}\right)^{\frac{1}{2}} \tag{1.11}$$

therefore (1.10) is the same as (1.9).

1.4 Most General Case

It is reasonable to see if one can express the results of any general physical observation in terms of a few dimensionless parameters. In most of cosmology the relevant physical quantities for specific problems are formed from the set $P = \{c, h, \varepsilon_0, e, G, m_{\mathrm{p}}, m_{\mathrm{e}}, k, T\}$, plus extensive variables, such as distance, mass, rate, redshift, etc. Then, from dimensional analysis, the only dimensionless quantities which can be constructed are

$$\alpha_{\mathrm{em}} \equiv \frac{e^2}{4\pi\varepsilon_0 \hbar c}, \quad \alpha_{\mathrm{g}} \equiv \frac{G m_{\mathrm{p}}^2}{\hbar c}, \quad \mu \equiv \frac{m_{\mathrm{e}}}{m_{\mathrm{p}}}, \quad \theta \equiv \frac{kT}{m_{\mathrm{p}} c^2}, \tag{1.12}$$

and combinations of them. The importance of the dynamical variable temperature, T, in this set, as opposed to the other dimensional constants, will be highlighted in the following section.

This analysis shows that if a specific quantity is dimensionless (which should be the case if one is talking about a physical measurement), and contains one of the members of P, then the other elements have to appear in a way that leads to at least one of the dimensionless ratios introduced in (1.12). In this paper we will explore consequences of adopting this dimensionless way of thinking for measuring observables in physical cosmology.[1]

There are many papers in the literature (e.g. [9, 25, 52, 53, 57, 76, 77, 83, 96, 101, 102, 112, 113, 117, 122]) which try to answer a question such as: "What would happen to observable X, if the speed of light or the gravitational constant or Planck's constant had a different value". Such questions are not well defined, since one can tune the other parameters of the dimensional set P, such that the dimensionless ratios in (1.12) remain the same. Since any observable is essentially dimensionless, it can only depend on these dimensionless ratios, and one can deduce that nothing happens to the observable X if, for example, G changes. On the other hand one can meaningfully consider how X might change if $\alpha_{\rm g}$ varies.

The necessity of using dimensionless ratios is commonly illustrated through an experiment which tries to measure a variation in the speed of light (e.g. [58]). The basic idea is that any measurement of the speed of light will in the end reduce to clocks and rulers. Since the separation of the two ends of any metre stick is a function of the spacing between its atoms, which is a function of the speed of light, the length of the metre stick depends on the speed of of light, and this makes any putative variation of *c unmeasurable*. What we say is a bit different. We believe that any apparent variation in the speed of light is *irrelevant* in any measurable quantity, regardless of special relativity. A variation in the speed of light is spurious because it is dimensional, not because of its fundamental role in modern physics. Once one understands this connection to a choice of units, it becomes clear that the same principle applies to all dimensional constants – one can talk about the variation of $\alpha_{\rm em}$, for example, but it is meaningless to ask whether this is due to a variation of one of e, ε_0, $\hbar$ or c. In this paper we will explore consequences of adopting this dimensionless thinking to measuring observables in physical cosmology.

As a prelude to the rest of the paper, let us consider (BBN) and the final abundance ratio of helium atoms relative to baryons – a measurement which is clearly dimensionless. We will discuss BBN in a dimensionless manner in more detail

[1] In a somewhat different context the importance of using dimensionless numbers in cosmology was stressed earlier by Wesson [134].

later on, but it may be helpful first to explain the logic and sketch the main idea. When we say "nothing happens to observable X if G changes", one might naively think that changing G would, for example, change the expansion rate of the Universe and hence lead to observable consequences. The point is that there is no observable which depends solely on the expansion rate. There also has to be some other rate (like the recombination rate, neutron decay rate, etc.) that one is considering in the problem. The ratio of these two rates (the expansion rate and the other relevant rate) is dimensionless and, therefore, has to depend on the dimensionless ratios and not just G. Based on this, the general dependence of the primordial helium abundance $Y_\mathrm{p} = Y_\mathrm{p}\left(c, h, \varepsilon_0, e, G, m_\mathrm{p}, m_\mathrm{e}, m_\mathrm{n}\right)$ can always be reduced to $Y_\mathrm{p} = Y_\mathrm{p}\left(\alpha_\mathrm{g}, \alpha_\mathrm{em}, \mu_\mathrm{e}, \mu_\mathrm{n}\right)$. μ_e and μ_n are respectively the electron and neutron mass ratios relative to the proton mass. Therefore, the dimensional constants should not be regarded as independent, and any variation in these constants should always be understood as a variation in the dimensionless ratios.

In this thesis I will consider different aspects of cosmology in relation to fundamental constants. The next section deals with some difficulties which arise when one tries to track a variation in the fundamental constants within the standard framework of cosmology. In chapter 2 I try to show explicitly and in a completely dimensionless manner how a change in α_g will affect two different aspects of the early Universe, namely BBN and recombination. I keep everything explicit in that chapter, but also simplify some of the equations. Therefore they lack the full accuracy required for comparing with data, but nevertheless demonstrate the main physics. In chapter 3 I work through the publicly available codes for computing Cosmic Microwave Background (CMB) anisotropies, make them manifestly dimensionless, and present the results of a variation of α_g or α_em on the CMB power spectra. In chapter 4 I briefly discuss how this work could be extended to other areas of physical cosmology.

In cosmology, a particularly important constant is Newton's constant, G. This is because it enters the dynamics of the background, the growth of perturbations and large-scale geometric effects. There have been many studies of how one might constrain $\dot{G}/G$ using cosmological or astrophysical data (e.g. [44]), as we will discuss further below. It seems odd that studies of α_em and μ are usually careful to point out the importance of only considering dimensionless constants, while papers on G (or even sections in review articles) tend to ignore this entirely.

1.5 Some Challanging Issues

We can imagine two different scenarios when discussing variable constants in cosmology, such as $\alpha_{\rm g}$ or $\alpha_{\rm em}$. In the first, we assume our Universe is described by a fixed set of physical laws and we attempt to measure the values of the (dimensionless) free parameters in this model. In the second (and perhaps more conventional) approach we consider a possible time variation of the constants. These ideas are of course related, but it is simplest to first concentrate on the notion of constants which are *constant* in time, but could vary among different universes (or patches within the same universe).

In Bridgman's classic book on dimensional analysis [13], he uses the idea of a different country, with a widely different set of units, in order to help explain the relationship between units and measurements. Since cosmology involves parameters which apply to the whole Universe, one can effectively extend Bridgman's idea and consider some alien civilization in a separate universe to our own, or at least a volume in which the dimensionless parameters may be different from ours. An imaginary conversation with observers in the other universe is a useful thought experiment for understanding what is really measurable. These observers would likely have a completely different set of units (and dimensional constants), so any such dialogue should only focus on the dimensionless aspects. The goal of the discussion would be to communicate the value of the dimensionless constants in our Universe, and learn the aliens' values. To do this we "simply" have to sort out concepts like "rest mass" and "charge" and so on, in order to be able to meaningfully say things like "the mass ratio of the two most common particles is 1836 for us, what about you?" Then we would describe the fundamental forces, making sure that we were talking about the same thing as the aliens, before progressing to defining the dimensionless couplings.

Moving on to the second scenario, where the constants might change, one would attempt to constrain the space-time variation of the dimensionless constants in our Universe using cosmological observables. The standard models of cosmology and particle physics cannot accommodate a genuinely variable $\alpha_{\rm g}$ or $\alpha_{\rm em}$. In this case one has to promote these parameters to dynamical fields, such as in scalar-tensor theories of gravity. The detailed study of such models is beyond the scope of this thesis but I will have a brief review of the theory in the appendix. For full consistency one would then have to check that any additional terms in relevant equations (the Friedmann equation for example) are negligible at the times under consideration. Here I will be assuming that the variation of the constants is small enough that these conditions are likely to be satisfied. However, it is worth noting that this may not be true in every case. To do better one would need an explicit theory for the time variation, while I will try to keep everything general here.

In cosmology one requires an additional set of parameters (as well as the dimensionless physical constants) to specify the model. This is a little different from setting the values of (dimensionless) physical constants, since the cosmological parameters are (at least statistically speaking) chosen from among a set of possible universes, all with the same physics. Some parameters (or ratios) may be fully deterministic, but others may have stochastic values. Someday we might have a theory which tells us exactly the value for some of the parameters, but in the current state of cosmology, we consider them as free parameters, which are unknown a priori. The usual set consists of (at least): $\{\rho_{\rm B}, \rho_{\rm M}, \rho_{\rm R}, \rho_{\Lambda}, H_0, T_0, A, n, \ldots\}$. These are the energy densities in various components (baryons, matter, radiation and vacuum), the expansion rate, CMB temperature, and amplitude and slope of the initial conditions. Usually the densities are expressed in terms of the critical density (which involves G) to give the set $\{\Omega_{\rm B}, \Omega_{\rm M}, \Omega_{\rm R}, \Omega_{\Lambda}\}$. Parameters such as the amplitude A and tilt n of the scalar power spectrum probably depend on fundamental constants, but in a way which is as yet unknown.

The obvious question that arises here is "how do we define the cosmological model in a dimensionless way?" We can return to our alien-conversation thought experiment to help sort this out. To communicate information on the background cosmology, we would need to discuss what we mean by "baryons", "photons", "neutrinos", etc., and then give their densities in some way. However, there is an additional issue within the realm of cosmology, which arises because many of the usual parameters (the Ωs, H, etc.) depend on time. Hence, even if we have established that we live in the same Friedmann model as our alien friends, we still have to establish whether we are observing that model at the same epoch or not.

The best way to do this would be to agree on a fiducial period in the evolution of the Universe and then discuss where we are relative to that. An obvious choice is the epoch of matter-radiation equality (discussed in a related context in [125]), but there are plenty of other possibilities: when the Hubble rate is equal to a particular reaction rate; when the Thomson optical depth is unity; when matter has the same energy density as the vacuum, etc. Assuming that the Universe has flat geometry (a clearly dimensionless statement), then we can give the values of the Ωs at the agreed fiducial epoch, together with one number to fix that epoch, say the value of θ at equality. Then we only need to give the value of *one* parameter today in order to fix the epoch at which we live (making sure this is a dimensionless parameter of course). This could be any one of the Ωs today, or the value of $z_{\rm eq}$, or the value of H_0t_0, or θ_0 ($\equiv kT_0/m_{\rm p}c^2$). Anything which is essentially monotonic (and changing in time) and dimensionless will do (so H_0t_0 is fine now, but useless billions of years ago, when it was hardly changing). In addition, we need a way to describe the amplitude of density perturbations, which is dimensionless – but this is relatively simple, by converting from A to the dimensionless power at some fiducial scale (e.g. $\Delta^2_{\mathscr{R}}(k_0)$ used by *WMAP* [69], σ_8 derived from galaxy clustering or Martin Rees' Q^2 [124]). The slope n is already dimensionless and so offers no difficulty. The real problems come from the fact that the *other* main cosmological parameters are epoch-dependent quantities.

This gives rise to two complications when describing the observable Universe, which are not there when one is discussing models of laboratary physics. Firstly, if one is not careful, then it is possible to effectively fix *more than one* of the parameters today, leading to inconsistent results. The second part is that different choices of the "what-time-is-it?" parameter are not entirely equivalent, because some contain a dependency on other physics parameters.

Take for example the choice of either the CMB temperature T_0, or Hubble constant, H_0 (made suitably dimensionless). These two are related to each other via $\Omega_{\rm R}$ and $\alpha_{\rm g}$ using the Friedmann equation

$$H_0^2 = \frac{8\pi^3}{45}\,\frac{\alpha_{\rm g}}{h^2\Omega_{\rm R}}\,\frac{k_{\rm B}^4\,T_0^4}{m_{\rm p}^2\,c^4}\,. \tag{1.13}$$

Therefore, either H_0 or T_0 is enough to determine today's epoch, and it is not consistent to use the Friedmann equation and treat *both* of these as free parameters. However, it turns out that it matters whether we choose T_0 or H_0, since the pair (T_0, H_0) in *our* Universe do not satisfy the above equation in a universe with a different $\alpha_{\rm g}$.

The thermal energy to rest mass dimensionless ratio, θ, needs some explanation. Unlike the other dimensionless ratios, this one is not built through pure physical constants but contains a "variable", the temperature. Temperature is a property of many body systems, which describes the phase space occupation of the particles. The role of Boltzman constant in the set P is to make temperature dimensionless. Boltzman constant is a Universal constant which can not be replaced with other physical constants [88]. In an alternative approach, one can give reason for the presence of temperature among fundamental constants because he actually needs a variable to describe thermodynamical systems which are evolving with time.

Chapter 2

Analytic Calculations

In order to illustrate how one ensures that calculation are dimensionless, we focus on two cosmological examples.

2.1 Dimensionless BBN

BBN is an area of astrophysics which has been thoroughly investigated for signs of variation in the fundamental constants (see e.g. [56, 72, 97] for non-gravitational couplings and BBN). Among the constants which are effective during BBN, the gravitational constant plays a key role, and hence there have been many published studies of the effects of a varying G on the primordial abundance of the light elements (see e.g. [3, 5, 21, 23, 24, 28, 59, 61, 74, 136]). I explicitly calculate the abundance of helium synthesized during BBN in this section, focusing on the dominant parts of the physics and neglecting some of the finer details. The calculation, though crude, shows the role of α_{g} in primordial nucleosynthesis and explicitly reveals the dimensionality in the relevant physics.

The key parameter in BBN is the ratio of number densities of neutrons to protons, which can be defined as:

$$R \equiv \frac{n_{\mathrm{n}}}{n_{\mathrm{p}}} = e^{-u}. \tag{2.1}$$

The quantity u is the ratio of mass difference to freeze-out temperature:

$$u \equiv \frac{\left(m_{\rm n} - m_{\rm p}\right) c^2}{k\,T_{\rm f}}. \tag{2.2}$$

$T_{\rm f}$ is explicitly the temperature at which the following reactions freeze out:

$$\mathrm{e}^- + \mathrm{p} \to \nu_{\rm e} + \mathrm{n}; \qquad \mathrm{p} + \bar{\nu}_{\rm e} \to \mathrm{e}^+ + \mathrm{n}. \tag{2.3}$$

According to Bernstein Bernstein [8], the rate for these reactions is:

$$\lambda = \frac{255}{\tau_{\rm n}\, u^5} \left(12 + 6u + u^2\right), \tag{2.4}$$

where $\tau_{\rm n}$ is the lifetime of the neutron. In order to identify the effects of the gravitational coupling constant, we can write this lifetime as

$$\tau_{\rm n} = \frac{f_1(\alpha_{\rm w})h}{m_{\rm p}\, c^2}, \tag{2.5}$$

where f_1 is a dimensionless function of the weak coupling constant. The freeze-out temperature is set by the equality of this rate and the expansion rate of the Universe:

$$\lambda = H \quad \Rightarrow \quad w\,u^3 - u^2 - 6u - 12 = 0, \tag{2.6}$$

where we have defined w through

$$w \equiv \frac{f_1}{255}\sqrt{\frac{8\pi g_*}{3}}\left(1 - \frac{m_{\rm p}}{m_{\rm n}}\right)^2 \alpha_{\rm g}^{1/2},$$

and g_* takes care of the number of relativistic species. This cubic equation can be solved to obtain the freeze-out temperature as a function of gravitational coupling:

$$u = \frac{1}{3}\,\frac{(18w+1)^{2/3} + (18w+1)^{-1/3} + 1}{w}. \tag{2.7}$$

After this time the most important remaining reaction is the β-decay of neutrons, which continues until the formation of deuterium. Deuterium formation is delayed due to the large photon-to-baryon ratio. Its abundance is fixed at a temperature $T_{\rm D}$, which is roughly given by the equality of the number density of photons with the number density of the deuterium which has been formed:

$$\exp\left(\frac{B_{\rm D}}{kT_{\rm D}}\right)\eta = 1. \tag{2.8}$$

Here η is the ratio of baryons to photons and $B_{\rm D}$ is the binding energy of deuterium, which can be written as a dimensionless function of the strong and electromagnetic fine structure constants times the proton mass, $B_{\rm D} = f_2(\alpha_{\rm s}, \alpha_{\rm em})\, m_{\rm p}\, c^2$. The temperature $T_{\rm D}$ could also be converted to an age through the Friedmann equation. By the time of deuterium formation neutrinos have already frozen out and electron-positron pairs have annihilated. Thus the ratio of the age of the Universe to the neutron lifetime becomes

$$v \equiv \frac{t}{\tau_{\rm n}} = \frac{1}{f_1 f_2^2}\sqrt{\frac{-3\ln(\eta)}{54\pi}}\,\alpha_{\rm g}^{-1/2}, \tag{2.9}$$

and the primordial helium fraction (by mass) is

$$Y_{\rm p} = 2\,\frac{R}{1+R}e^{-v}. \tag{2.10}$$

We have aimed at an expression for the helium abundance which is manifestly dimensionless, i.e. it only depends on dimensionless ratios of physical quantities, including $\alpha_{\rm g}$. This simple analysis leads to a primordial helium fraction of about $Y_{\rm p} \sim 0.22$, which is within 10 percent of the value coming from more complicated numerical BBN codes. Now we can put all this together to track the effects of a possible variation of $\alpha_{\rm g}$ on $Y_{\rm p}$:

$$\begin{aligned}\delta Y_{\rm p} \quad = \quad & \frac{e^{-(u+v)}}{(1+R)^2}\,\alpha_{\rm G}^{1/2} \\ & \times\left[\frac{u^3}{3wu^2-2u-6}\frac{f_1}{255}\sqrt{\frac{8\pi g_*}{3}}\left(1-\frac{m_{\rm p}}{m_{\rm n}}\right)^2 + \frac{1+R}{f_1 f_2^2}\sqrt{\frac{-3\ln(\eta)}{45\pi}}\alpha_{\rm G}^{-1}\right] \\ & \times\frac{\delta\alpha_{\rm G}}{\alpha_{\rm G}}. \end{aligned} \tag{2.11}$$

The first term in the square brackets comes from the change in the neutron's freeze-out fraction and the second is due to a change in the age of the Universe. Both terms have the same order of magnitude $\left(\sim 10^{-2}\right)$ and have the same sign. A higher α_{g} leads to more neutrons at the freeze-out time and a lower age for the Universe, both of which enhance the primordial helium fraction.

If one takes the measurement error on Y_{p} to be ~ 0.005, and assumes a power-law variation for $\alpha_{\mathrm{g}} \propto t^{-x}$, this simple analysis shows that x should be less than 0.005, which is consistent with the results of other studies (see e.g. Yang et al. [136]).

Although most of the papers on this subject could be considered to be valid when one interprets them in a dimensionless manner (i.e. translating a variation in G into a variation in α_{g}), making the whole argument dimensionless reveals some assumptions which are obscured by dimensions. For example, sometimes statements are made like "whenever you see G, interpret it as G times some quark mass m_{x}, or a combination of G and Λ_{QCD}" (see e.g. Iocco et al. [58]). But actual measurements of G have always been made using normal atoms, and therefore when numbers are plugged into equations in order to derive some proposed time variation for G (or more properly, α_{g}), then α_{g} is effectively the one introduced in equation (1.12). In this sense, those papers which are assuming a simultaneous time variation for G and m_{p}, are not valid (see e.g. Dent and Stern [28], Iocco et al. [58]).

2.2 Redshift of Recombination

Cosmological recombination of hydrogen is mainly controlled by the population of the first excited state Zel'Dovich et al. [138]. This is because of the high optical depth for photons coming from transitions direct to the ground state. The rate of recombination to this first excited state is given as [34, 119]

$$\Gamma = n_{\mathrm{B}}\, \alpha^{(2)} = 9.78\, n_{\mathrm{B}} \left(\frac{E_0}{kT}\right)^{1/2} \ln\left(\frac{E_0}{kT}\right) \frac{\hbar^2 \alpha_{\mathrm{em}}^2}{m_{\mathrm{e}}^2\, c}, \tag{2.12}$$

here 'B' stands for baryons (i.e. protons and neutrons), 'e' for electrons, $\alpha^{(2)}$ is the recombination rate to the second energy level of hydrogen and E_0 is the binding energy of hydrogen, which is equal to $\alpha_{\mathrm{em}}^2 m_{\mathrm{e}} c^2/2$. We have assumed that all of the atoms are ionized. It also simplifies things considerably (without qualitatively changing the physics) if we ignore the mass difference of protons and neutrons, i.e. set $m_{\mathrm{B}} = m_{\mathrm{p}}$, then

$$n_{\mathrm{B}} = \frac{\rho_{\mathrm{B}}}{m_{\mathrm{B}}} = \frac{\rho_{0\mathrm{B}}}{m_{\mathrm{p}}} \left(\frac{T}{T_0}\right)^3 . \tag{2.13}$$

This gives a rate

$$\Gamma = 7.0 \left(\frac{m_{\mathrm{e}}\, c^2}{k\, T_0}\right)^{1/2} \frac{\hbar^2 \alpha_{\mathrm{e}}^3}{m_{\mathrm{e}}^2\, c} \frac{\rho_{0\mathrm{B}}}{m_{\mathrm{p}}} \ln\left(\frac{E_0}{k\,T}\right) \left(\frac{T}{T_0}\right)^{5/2} . \tag{2.14}$$

Assuming for further simplicity that $\Omega_{\mathrm{M}} = 1$ (this assumption could easily be relaxed later), the Hubble constant is

$$H = \left(\frac{8\pi G \rho_0}{3}\right)^{1/2} \left(\frac{T}{T_0}\right)^{3/2} , \tag{2.15}$$

and one can also use the equalities

$$\rho_{0\mathrm{B}} = \rho_{0\mathrm{R}} \frac{\Omega_{\mathrm{B}}}{\Omega_{\mathrm{R}}}, \quad \rho_0 = \rho_{0\mathrm{R}} \frac{\Omega_{\mathrm{M}}}{\Omega_{\mathrm{R}}}, \tag{2.16}$$

together with

$$\rho_{0\mathrm{R}} = a T_0^4 = \frac{8\pi^5 k^4}{15 c^3 h^3} T_0^4 . \tag{2.17}$$

Putting all of these together, one can find an expression for the redshift of recombination:

$$1 + z_{\mathrm{r}} \propto \frac{T_{\mathrm{r}}}{T_0} = 68\, \alpha_{\mathrm{g}}^{1/2}\, \alpha_{\mathrm{em}}^{-3}\, \mu^{3/2}\, \theta_0^{-3/2} \left(\frac{\Omega_{\mathrm{M}}}{\Omega_{\mathrm{R}}}\right)^{1/2} \left(\frac{\Omega_{\mathrm{B}}}{\Omega_{\mathrm{R}}}\right)^{-1} . \tag{2.18}$$

Again we can see that an observable, which is the redshift of recombination, depends only on the dimensionless ratios of physical quantities, together with some other dimensionless parameters. Although this expression contains much of the essential physics, unfortunately the numerical factor (which is neglected to be written down) is far from correct. That is because I have not taken account of the partial hydrogen ionization. It would be possible to take this further by using an approximation to the Saha equation to correct for the ionized fraction, changing the scalings with the dimensionless parameters. However, this rapidly gets complicated, and accuracy really needs the numerical solution to the relevant differential equations (which is done in the next section).

Nevertheless the main point is that one can see an important dependence on the dimensionless quantities of interest. The change in the redshift of recombination is one of the main cosmological effects of varying α_{g}. There are also other effects on CMB anisotropies, as we will see in the next chapter.

Chapter 3

Numerical Calculations

3.1 Varying αs and CMB Anisotropies

In the previous sections I wrote down expressions which were explicitly dimensionless, but at the expense of accuracy. Let us now numerically explore the effects of a varying α_{em} or α_{g} on the CMB anisotropies. I use RECFAST v1.5 [114] for the recombination history of the Universe and CMBFAST [115] for the calculation of CMB angular power spectra. It turns out that one would face some difficulties if one were to blindly dive into the codes and try to change "G" or add some factors of α_{em} in the relevant parts. There are several issues which should be considered before one can promote these constants into dynamical parameters in the code. It turns out that this choice not only affects the feasibility of the problem, but also the outcome of the numerical calculation. As an example, CMBFAST is written so that it uses the total density contributions, $\{\Omega\}$, both for the strength of inertia in acoustic modes, and as gravitational charge. Therefore, if one chooses these ratios as the free parameters, in place of densities, it is much easier to trace the effects of a variation in the gravitational constant, because there is an additional factor of "G" wherever the code needs the Ωs as seeds for gravitational collapse.

3.1.1 Recombination through RECFAST

One can trace the effects of a varying fine structure constant or gravitational constant on the process of recombination through RECFAST (see e.g. [51, 66, 111]). In order to convert the code to a form where it can run with a different $\alpha_{\rm em}$, one has to track all of the relevant dependencies [89]. Most of the dependence shows up in the energy levels, since $E \propto \alpha_{\rm em}^2$. The 'Case-B' recombination rate (see Rybiki and Lightman p. 282 [110]) has an $\alpha_{\rm em}^5$ dependence and the complete analytic form of the 2s–1s two photon rate ([20, 120]) varies as $\alpha_{\rm em}^8$. Triplet transitions of helium (discussed in Lach and Pachucki [71]), case-B recombination and two photon transition rates for helium have the same scaling as for hydrogen. With this information in hand one can trace the effects of a different $\alpha_{\rm em}$ (or a time-dependent $\alpha_{\rm em}$) on recombination. The G dependence for recombination lies entirely in the Hubble constant. Putting the rates together and defining the dimensionless ratios,

$$A \equiv \frac{n_{\rm p}\,\alpha}{H}, \qquad B \equiv K\,\Lambda n_{\rm p}, \qquad C \equiv K\,\beta\,n_{\rm p}, \qquad D \equiv \frac{\beta}{H}, \tag{3.1}$$

the Saha equation [114] will take the form:

$$\frac{dx_{\rm p}}{dz} = \left[x_{\rm e}\,x_{\rm p}A - \left(1-x_{\rm p}\right)D\right]\frac{\left[1+B(1-x_{\rm p})\right]}{(1+z)\left[1+(B+C)(1-x_{\rm p})\right]}. \tag{3.2}$$

This is explicitly for hydrogen, but there is a similar form for helium too. These dimensionless ratios have the following dependencies on the coupling constants:

$$A\left(\alpha_{\rm em},\alpha_{\rm g}\right) \propto \alpha_{\rm em}^5\,\alpha_{\rm g}^{-0.5}; \tag{3.3}$$

$$B\left(\alpha_{\rm em},\alpha_{\rm g}\right) \propto \alpha_{\rm em}^2\,\alpha_{\rm g}^{-0.5}; \tag{3.4}$$

$$C\left(\alpha_{\rm em},\alpha_{\rm g}\right) \propto \alpha_{\rm em}^{-1}\,\alpha_{\rm g}^{-0.5}; \tag{3.5}$$

$$D\left(\alpha_{\rm em},\alpha_{\rm g}\right) \propto \alpha_{\rm em}^5\,\alpha_{\rm g}^{-0.5}. \tag{3.6}$$

Figure 3.1 shows the results of a 1% increase in the fine structure constant and separately of the gravitational constant through the history of recombination. This is consistent in sign with equation Equation : recom – an increase in $\alpha_{\rm em}$ leads to a higher ionization fraction, which leads to a lower redshift for recombination. It is also noticeable that even a 1% increase in $\alpha_{\rm em}$ can lead to more than 1% variation in the recombination history, while such a variation in $\alpha_{\rm g}$ does not leave any significant trace, due to the weaker power-law scalings above and the relatively thin last scattering surface.

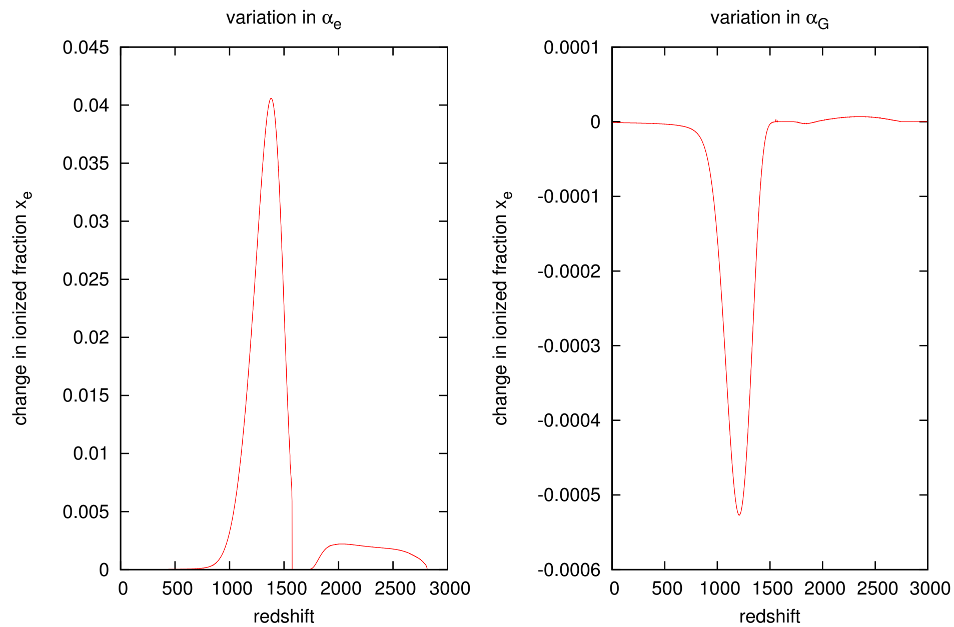

Figure 3.1: Effects on the ionization history of 1% increases in $\alpha_{\rm em}$ (left) and $\alpha_{\rm g}$ (right).

3.1.2 Perturbations through CMBFAST

The effects of an alteration in the recombination history could in principle be measured through the CMB power spectra. This section complements sec : BBN in terms of comparing a hypothetical variation in the fundamental constants with known physics at a particular epoch – $z \sim 1000$ in this case (rather than $z \sim 10^9$ for BBN). As is discussed in Moss et al. [89], a different fine structure constant leaves its main imprint on the recombination history, and this shows up in CMBFAST through two main effects: (1) the derivative of the opacity, which is proportional to Thomson scattering, with an $\alpha_{\rm em}^2$ dependence; and (2) the number density of free electrons, which is basically the "freeze-out" value at the end of recombination. Therefore, one can almost ignore the effects of a different fine structure constant after recombination. The case is different for $\alpha_{\rm g}$. As is discussed in Riazuelo and Uzan [107], a different G will lead to a different sound horizon at the last scattering surface and a different distance from this surface to us, the combina-

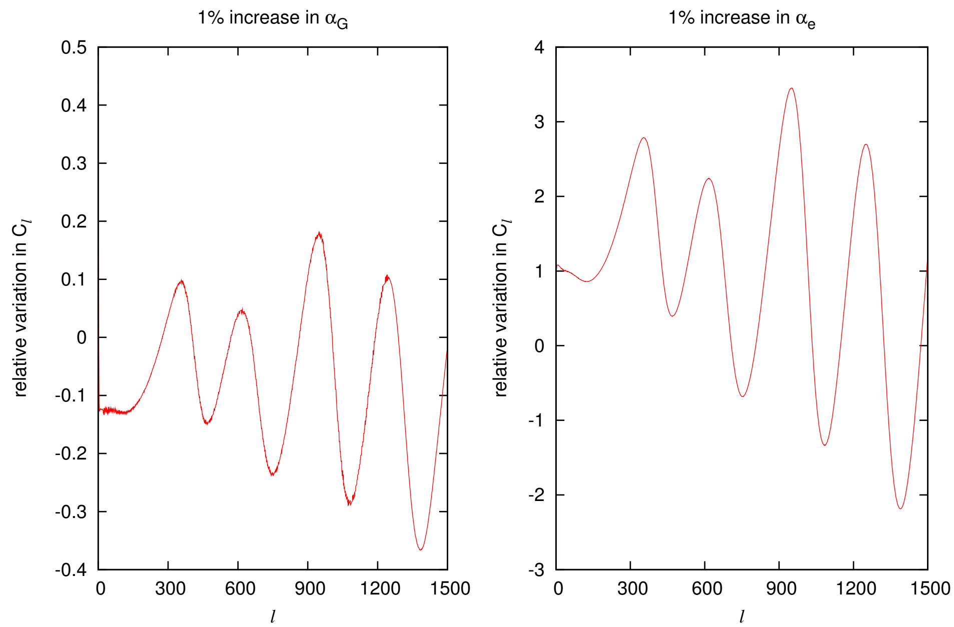

Figure 3.2: Effects on the CMB anisotropies of a 1% increase in α_g (left) and α_{em} (right). The vertical axis is percentage.

tion of which will change the position of the peaks of the power spectrum. There is also an Integrated Sachs-Wolfe (ISW) effect which can enhance the amplitude of perturbations. This is examined in Zahn and Zaldarriaga [137] and, as explained in Chan and Chu [18] for the case of a constant but different G, the effects are much less important than they are for a time-varying G. The results of a 1% increase in α_{em} and α_g are shown in figure 3.2.

In [137], the authors propose a scaling in G, ($G \to \lambda^2 G$), which is equivalent to the scaling of α_g considered here. However, I cannot confirm their results, since they have assumed that $\{T_0, H_0, \Omega_R\}$ have the *same* values as in the standard model. This is incorrect, because these three are not independent and a change in α_g will result in a different value for at least one of them (depending on which one we have chosen as our dependent variable). The time varying α_g or α_{em} results are shown in figure 3.3.

Our study is certainly not the first to explore the effects of variable fundamental

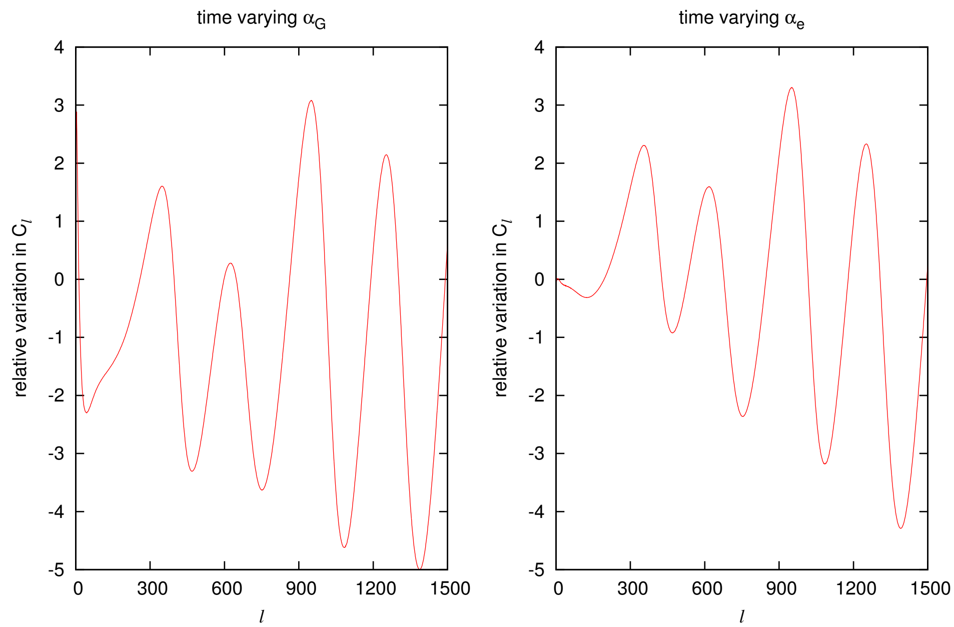

Figure 3.3: Effects on the CMB anisotropies of time-varying constants. We have specifically used $\alpha_{\rm g} \propto t^{-0.002}$ (left) and $\alpha_{\rm em} \propto t^{-0.002}$ (right). The vertical axis is percentage.

constants on the CMB (see e.g. [18, 86, 137] for G and [4, 50, 80, 81, 109] for $\alpha_{\rm em}$). The main point here is that we have insisted on doing everything in a dimensionless way to avoid finding any apparent effects which are simply a result of the choice of units. As an example, Boucher and et al. [11] suggest that one can search for a violation of the Strong Equivalence Principle using the CMB power spectra. The method, though at first sight quite elegant, is totally dimensional. When one starts to make it dimensionless, it becomes evident that the whole argument has to be reconsidered. A basic assumption is that a variation in G can make a difference in gravitational mass relative to the inertial mass of baryons. However, this is in general wrong, since one can *always* use the freedom of setting the gravitational charge and inertial mass of a given body (and only for one object) equal to each other.

Several particular ideas for exploring the variation of G within cosmology have been described elsewhere. We are suspicious of any of these studies in which the dimensionful quantity G is discussed along with other dimensionful constants, such as H_0 (e.g. [42, 73, 75, 90, 128]). Specific ideas for time-varying constants may lead to specific forms for the function of time, e.g. $\alpha_{\rm em}(t)$. Following such detailed predictions are beyond the scope of the present paper. However, the methodology set out here should be useful in testing any explicit model that is proposed.

Chapter 4

Discussion

I have already considered the effects of a variable α_{g} or α_{em} on BBN and CMB anisotropies, with the main emphasis on making things dimensionless. One can extend this approach to other observables in cosmology such as weak lensing, Baryon Acoustic Oscillations (BAO), large-scale structure, ISW effect and the use of supernovae as standard candles.

BAO follows the same basic physics as CMB anisotropies (see e.g. Eisenstein and Hu [38]), so there should in principle be the same kind of α_{g} dependence. The important timescales for BAO are the equality epoch, z_{eq} and the drag epoch, z_{d}. The first, z_{eq}, is purely set by the initial conditions of the Universe, and the time today, setting the epoch when $\Omega_{\mathrm{M}} = \Omega_{\mathrm{R}}$. The drag epoch is defined as the time when baryons are freed from the Compton scattering of photons, and therefore z_{d} has the same α_{g} or α_{em} dependence as z_{r}. These special redshifts set the scaling conditions for BAO. After the drag epoch, one should solve for the matter transfer function, where this function is basically the same as the CMB transfer function in terms of α_{g} or α_{em} dependence. It seems clear therefore, that if one wanted to use BAO to constrain the variation of α_{g}, it would be straightforward to follow the same procedure discussed in chapter 3.

In gravitational lensing (see [70] for an earlier study) there is a danger of becoming confused by dimensions, since both the estimate of the lensing mass and the curvature depend on G, and they are mixed together in α_{g}. The simple case of an Einstein ring is illuminating. Here the lens and the source are colinear and in Euclidean space the radius of the Einstein ring is the geometric mean of the Schwarschild radius of the lens and the distance to the lens. This means that for a relatively nearby lens at distance d the lensing angle is $\Theta_{\rm E} \sim \sqrt{GM/c^2 d}$, and if we estimate the mass through measuring a velocity dispersion v^2 over a radius r, then $\Theta_{\rm E} \sim \sqrt{\Theta v^2/c^2}$, where $\Theta \equiv r/d$ is the apparent angular size of the object. Viewed this way we see that it is hard to use lensing to measure the strength of gravity, because the obvious dimensionless observables leave no G dependence!

Let us look at this in a little more detail. In cosmology one can work out the lensing angle to be (e.g. [16])

$$\Theta_{\rm E} = \sqrt{\frac{4GMd_{\rm LS}}{c^2\, d_{\rm L}\, d_{\rm S}}}, \tag{4.1}$$

where $d_{\rm LS}$ is the distance from the source to the lens, and $d_{\rm S}$ and $d_{\rm L}$ are, respectively, the distances from us to the source and to the lens. However, this seemingly innocent dimensionless equation is not useful for any experiment designed to measure a change in the gravitational constant. This equation, or any equation for lensing, should be turned into an equation which only has $\alpha_{\rm g}$ or $\alpha_{\rm em}$ dependence before it can be used as a test of fundamental constant variation. One can work out all of the distances in the above equation and end up with the following relation:

$$\begin{aligned} \Theta_{\rm E} &= A\, \alpha_{\rm G}^{3/4}\, \theta\, \frac{M}{m_{\rm p}}, & (4.2) \\ \text{with } A &\equiv \sqrt{\frac{8\,\pi^3}{45\Omega_{\rm R}}} \sqrt{\frac{1+z_{\rm L}}{N_{\rm L}} - \frac{1+z_{\rm L}}{N_{\rm L}}}. & (4.3) \end{aligned}$$

Here $z_{\rm L}$ and $z_{\rm S}$ are, respectively, the redshift of the lens and the source and $N_{\rm L}$ and $N_{\rm S}$ are defined as:

$$N_{\rm x} \equiv \int_{\frac{1}{1+z_{\rm x}}}^{1} \frac{dy}{y^2\,\sqrt{\Omega_\Lambda + \Omega_{\rm M} y^3\, \Omega_{\rm R} y^4}}, \tag{4.4}$$

where x can be either S or L.

Speaking in more general terms (and again thinking about communicating with another civilization), we can see two possibilities. The first is that one could imagine choosing a lens which consists of a fixed number of particles, so that one knows the value of the pure number M/m_{p}. That may be interesting from a philosophical point of view, but is hardly related to how we carry out observations in cosmology. A second idea is that perhaps one could carry out a statistical survey over many galaxy lenses, measuring statistics which depend on a characteristic galaxy mass M_{gal}. If that mass depends on fundamental constants through a cooling argument (e.g. [106]) then it may be that in a lensing survey the observables depend on the number $\alpha_{\mathrm{em}}^5 \alpha_{\mathrm{g}}^{-2} \mu^{-1/2}$. Since this would be different in universes with different values of the constants, then this is potentially measurable, and hence it *might* be possible to constrain a redshift dependence of lensing observables.

These ideas do not seem particularly practical, and so we are left unclear about whether gravitational lensing could ever be used to constrain the time variation of α_{g}. Things will presumably be unambiguous in explicit self-consistent models which contain a variable strength of gravity, and we leave this topic for future studies of different models.

There are many other astrophysical phenomena which can in principle be used to test the variation of fundamental constants (see e.g. García-Berro et al. [44], Uzan [129]). Large-scale structure and various measures of the power spectrum of galaxy and matter clustering (see e.g. Nesseris et al. [95]), will also have dependence on α_{g}. It will again be important to ensure that this dependence is dimensionless, and that cosmological parameters are not over-constrained when doing so. In other words, one needs to realise that changing α_{g} can effectively change the epoch at which we live.

Supernovae have also been very useful as approximately standard candles in cosmology, and such studies can also be adapted to constrain a combination of fundamental constant variations [37, 43, 46, 94]. Like other stellar sources, there is a dependence on $\alpha_{\mathrm{g}}^{-1/2}$ in the evolutionary timescale (see e.g. [27, 45, 49, 79, 126]), and hence the use of supernovae, combining the standard candle property with luminosity distance, will involve a different combination of fundamental constant variations than for BAO and lensing studies. A combination of cosmological probes will therefore enable variations among different parameters to be distinguished.

Finally, it worths mentioning that among the set of different astrophysical phenomena, cosmological studies have the potential of probing not only the time dependency of $\alpha_{\rm g}$, but also any scale dependence. It is important to work out an appropriately consistent theory to describe these phenomena with a spatially variable fundamental constant. Although I have not considered such models here, I still suppose that it will be important to avoid dimensional quantities.

Chapter 5

Conclusions

I am not advocating that all of cosmology henceforth should be represented in dimensionless forms. However, I would say that care has to be taken when dealing with variation of physical constants in cosmology. This is particularly because of the cosmic time ambiguity, which could lead to *over*-constraining the cosmological parameters.

In terms of the standard cosmological picture, one could define dimensionless parameters at some fiducial epoch. If we choose matter-radiation equality as this epoch, then we can define the background cosmology by giving the values of the Ωs *then*, as shown in Tabel 5.1. (I have assumed massless neutrinos in the table, contributing to the "radiation" by the usual factor of 0.68 times the photon energy density. I have also assumed a flat background, and adopted values from the *WMAP* 6-parameter fits Komatsu et al. [69].) We then need to give one quantity (out of a wide range of possibilities) to define the time *today*. I have pointed out how it is possible to reach invalid conclusions about the variation of fundamental constants (particularly G) by fixing too many parameters in the cosmological model.

Explicit physical mechanisms for the variation of fundamental constants will lead to specific forms for $\alpha_{\mathrm{g}}(t)$ etc. I believe that the proper place to start investigating such theories is to make sure that there is a robust basis for comparing cosmologies with different *constant* values of the constants. Only then can one effectively deal with time-variable quantities.

A time-dependent (or space-dependent) α_{g} is not consistent with General Rel-

Table 5.1: Dimensionless cosmological model parameters. Calculated values and uncertainties are from the Markov chains from the *WMAP* 6-parameter fits [69].

Parameter	Value	Uncertainty
Coupling constants		
$\alpha_{\rm em}$	7.297×10^{-3}	...
$\alpha_{\rm g}$	5.906×10^{-39}	...
Quantities at equality		
Ω_{γ}	0.296	...
Ω_{ν}	0.204	...
$\Omega_{\rm CDM}$	0.415	0.004
$\Omega_{\rm B}$	0.085	0.004
Ω_{Λ}	4.4×10^{-11}	1.2×10^{-11}
θ	8.0×10^{-10}	0.3×10^{-10}
Definitions of "now"		
$z_{\rm eq}$	3200	130
θ_0	2.503×10^{-13}	...

ativity. One of the only explicit theories for accommodating such a variation is a scalar-tensor theory of gravity. Some studies have already investigated CMB anisotropies and other cosmological constraints in the simplest form of scalar-tensor model (e.g. [1, 19, 135]), the so-called Brans-Dicke theory [12]. While this version of a scalar-tensor theory seems to be already ruled out by solar system experiments [104], there are more general scalar-tensor theories which pass the solar system tests [123], and might be promising avenues of exploration (e.g. [92, 127]). These may provide analogues for investigating the variation in physical constants proposed in some brane-world scenarios [26, 84, 93]. In studying the empirical tests of such models I recommend keeping everything dimensionless in order not to be misled by apparent variations that may be unmeasurable.

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Appendix A

Scalar-Tensor Theory of Gravity in a Nutshell

A.1 Introduction

The Scalar-Tensor theory of gravity starting with Jordan [63], came into existence based on Dirac's pondering about the variation of the gravitational constant [32]. Later on, Hoyle [54] used a scalar field to describe a Universe with constant creation of matter, compatible with the ideas of Bondi and Gold [10] about a steady state Universe. Hoyle's thoery is basically a scalar-tensor theory with the scalar field as the matter creator operator.

Scalar-tensor theory was not taken serious until about 1961 when Brans and Dicke [12] discussed General Relativity(GR) and its incompatibility with Mach's principle. However, Brans-Dicke theory lost its initial attraction for a while due to the following works of Dicke himself. He worked out BBN in their theory and suggested some tests using solar observations. All of these tests turned out to be compatible with General Relativity, without any need for further modification. It is hard to say that these tests ruled out the idea, since GR is just a special case in Brans-Dicke, but GR is simpler and has less free parameters, therefore, being compatible with GR is usually taken as disfavouring Brans-Dicke.

In 1970, Wagoner [130] wrote the most general form of a scalar-tensor theory. Wagoner's theory is dynamic in the sence that the theory can evolve with time;

gravity can start from a Brans-Dicke like theory in the early epochs of the Universe and evolve into GR at later times. It is also possible to look for scale-dependent gravity models which are the same as GR on small scales but are deviate from it on larger scales.

There was a revival of interest in scalar-tensor theories after the type-Ia supernova observations showed acceleration in 1998 [2], which brought in the first observational support for a cosmological constant. Many theorists tried to look for theories of gravity that could lead to cosmological acceleration at late times or on large scales without the mysterious ad hoc cosmological constant fluid [17, 22, 39, 47, 60, 68]. Multidimensional inspired theories, f(R) theories and of course, scalar-tensor theories, gained lots of attention since then and there has been a large body of work discussing the internal relations among these seemingly different theories (see e.g. [41] and references therein).

In this appendix I will go through the basics of Brans-Dicke theory and the more general forms of a scalar-tensor theory while trying to make conceptual, theoretical, and observational arguments for why one should care about scalar-tensor theories in general.

A.2 Scalar-Tensor Theory on Conceptual Grounds

As is described in [12], one can think of two different approaches regarding space-time structure. In the first approach, space-time is an absolute physical structure with its own physical and geometrical properties. One is allowed to define inertial frames and therefore absolute motions in this space-time structure, without any reference to the rest of matter present in space. In the second approach, space-time does not have any physical or geometrical structure of its own. Space-time gains its physical meaning from the matter therein, and one can define a motion in space-time only *relative* to other physical objects. This second approach is called Mach's principle [78].

While the second approach seems more attractive for physicists nowadays, there is not any accepted physical theory which is in complete accordance with Mach's principle. Even in GR, that space-time geometry is affected by the matter distribution, it is not uniquely specified by the distribution of objects in space [12].

There are two main thought experiments which led Brans and Dicke to their theory of gravity. The idea behind both of these is that if one assumes Mach's principle, then any fictitious force on a test particle should be reinterpreted as the effect of the motion of other particles in the Universe relative to the test particle. In the first experiment, they imagine an observer with a gyroscope and a gun in a laboratory in an empty space-time. The observer can lean out of the window of his laboratory and shot a small bullet tangent to the circular walls of his lab which will make his laboratory start rotating. The gyroscope in the lab should point towards a nearly fixed direction relative to the direction of motion of the rapidly receding bullet. In the Mach's principle approach to space-time, this means that the tiny bullet is more important in determining the motion of gyroscope than the massive walls of the laboratory.

The second thought experiment deals with a particle falling in the gravitational potential of the Sun. In a coordinate system so chosen that the particle is at rest, the gravitational pull of the Sun should be balanced by the gravitational pull due to the rest of the mass in the Universe. The acceleration of the falling particle due to Sun's gravity can be approximated through Newton's law as $a \sim GM_{\odot}/r^2$. Based on dimensional analysis and the relevant parameters in this problem, one can approximate the acceleration due to the matter distribution in the Universe as $a \sim M_{\odot} R c^2 / M r^2$, where R is the radius of the Universe and M is the total mass in the causally related Universe. By equating these two accelerations one gets:

$$G \sim \frac{R c^2}{M}, \tag{A.1}$$

which means that the gravitational constant should vary with time.

A.3 Brans-Dicke Theory

From the preceding general arguments, Brans and Dicke deduced that a Mach-compatible theory would presumably have a time-varying gravitational constant in an expanding Universe. Einstein's theory is clearly inconsistent with such a hypothesis and should therefore be extended to a theory which admits a time varying gravitational coupling. They started with the Einstein-Hilbert Lagrangian:

$$L_{\mathrm{EH}} = \sqrt{g}\left(R + \frac{16\pi G}{c^4} S_{\mathrm{m}}\right). \tag{A.2}$$

Here R is the Ricci scalar and S_{m} is the Lagrangian of matter, including all non-gravitational interactions. The quantity g is the absolute value of the determinant of the space-time metric. They argued that it is not wise to replace G with a field in this Lagrangian, since that would lead to a non-conserved energy momentum tensor for matter defined via

$$T_{\mu\nu} \equiv \frac{2}{\sqrt{g}} \frac{\delta S_{\mathrm{m}}}{\delta g_{\mu\nu}}, \tag{A.3}$$

with $g_{\mu\nu}$ representing the metric of the space-time. Furthermore, the particles will not move along the geodesics of the metric any more if S_{m} is coupled to a space-time dependent field. Based on these considerations, they divided the Einstein-Hilbert Lagrangian by G and replaced it with a scalar field ($\phi = 1/G$):

$$L'_{\mathrm{EH}} = \sqrt{g}\left(\frac{R}{G} + \frac{16\pi}{c^4} S_{\mathrm{m}}\right) \Longrightarrow L_{\mathrm{BD}} = \sqrt{g}\left(\phi R + \frac{16\pi}{c^4} S_{\mathrm{m}} - \frac{\omega}{\phi}\, \phi_{,\mu}\phi^{,\mu}\right). \tag{A.4}$$

The last term in the Brans-Dicke Lagrangian, L_{BD}, is the usual kinetic term of a scalar field and the role of the field in the denominator is to make ω a dimensionless constant. The interesting point is that right after writing this Lagrangian the authors say: "In any sensible theory ω must be of the general order of magnitude of unity".

A.4 Brans-Dicke Theory on Observational Grounds

Solar System tests.— Brans and Dicke immediately worked out the weak field limit of their theory and found the correction to general relativity due to adding a scalar field. They found that the amount of precession of Mercury in their theory is

$$\frac{4+3\omega}{6+3\omega} \times (\text{value of general relativity}), \tag{A.5}$$

which is always less than the value predicted by GR. The Brans-Dicke prediction approached GR assymptotically as $\omega \to \infty$ and this limit is often taken as the GR limit. Dicke argued that the oblateness of the Sun can make a Newtonian $1/r^3$ potential [31]. This would resolve some of the famous $43''$ discrepancy between the observed prehelion of the orbit of Mercury and the prediction of Newtonian gravity. Therefore, the post-Newtonian corrections should be smaller than $43''$, in favour of Brans-Dicke's theory. Following studies [48] showed that the oblateness of the Sun is smaller than what Dicke had proposed by about an order of magnitude.

In 1970 Nordtvedt [98] parametrised the most general theory-independent form of a metric in the weak field limit, up to terms of order $(1/c^4)$ for g_{00} and $(1/c^3$ and $1/c^2)$, respectively, for g_{0k} and g_{kk}. The results of comparing observations with Brans-Dicke's theory showed that the maximum fractional contribution of scalars to the Newtonian and post-Newtonian potential is less than one part in a thousand [99].

Big Bang Nucleosynthesis.— Dicke [30] worked out BBN in his own theory. He assigned an energy momentum tensor to the scalar field which would increase the expansion rate of the Universe. His conclusion was that his theory allows a Universe without any primordial helium forming.

Cosmic Microwave Background.— Chen and Kamionkowski worked out the perturbation theory in detail for Brans-Dicke theory [19]. They also carried out the numerical calculations and checked for possible degeneracies of ω with other cosmological parameters. Their conclusion is that *PLANK* should be able to identify values as large as $\omega \simeq 3000$ with a standard deviation of about 0.00062.

A.5 More General Scalar-Tensor Theories

Tensor-MonoScalar Theories.— In 1970, Wagoner wrote down the most general tensor-mono scalar theory [130]. This is important, since adding a scalar seems the most simple and natural extension of general relativity in the field theory language. The list of his assumptions are:

a) The theory should be convarient, leading to tensorial equations.

b) The equations of motion are derived from a Lagrangian of the form $L = \sqrt{g}(L_{\mathrm{g}} + L_{\mathrm{I}})/16\pi G$, with L_{g} the Lagrangian for gravitational fields alone and L_{I} the Lagrangian for the coupling of gravity with all other forms of interaction. These fields vanish at infinity.

c) Gravity is mediated with a tensor and a scalar.
d) The field equations are at most second order differential equations, leading to the following form for L_{g}:

$$L_{\mathrm{g}} = h(\phi)\,R + l(\phi)\,\phi_{,\mu}\phi^{,\mu} + 2\,\lambda(\phi). \tag{A.6}$$

e) Lastly, mutual coupling of gravity and matter of the following form is assumed:

$$L_{\mathrm{I}} = L_{\mathrm{I}}(A^2(\phi)g_{\mu\nu},...). \tag{A.7}$$

Here A is an arbitrary function of the scalar field. This form of coupling guarantees that a local inertial system can always be made in which the laws of special relativity and electromagnetism hold. It is also claimed by Nordtvedt [99] that the only consistent field theories without causality problems, negative energy modes, etc., are those which respect this form of coupling.

By employing the following redefinitions

$$\begin{aligned} \tilde{g}_{\mu\nu} &= h\,g_{\mu\nu}, \\ \frac{d\phi}{d\Phi} &= h\left\{h\,l - \frac{3}{2}\left(\frac{dh}{d\phi}\right)^2\right\}^{-1/2} \end{aligned} \tag{A.8}$$

the Lagrangian is written in its final form (suppressing tilde signs):

$$L = \frac{1}{16\pi G}\sqrt{g}\left(R - \Phi_{,\mu}\Phi^{,\mu} + 2\,\lambda(\Phi) + L_I(A^2(\Phi)g_{\mu\nu},...)\right). \tag{A.9}$$

This form of Lagrangian, which does not have any coupling of the scalar field to the Ricci scalar, but couples the scalar field with matter fields, has its own benefits which I will describe later. It is refered to as a Lagrangian written in the Einstein conformal frame. If one performs a conformal transformation on the metric of this Lagrangian, such that $\bar{g}_{\mu\nu} = A^2\,g_{\mu\nu}$, then the new matter Lagrangian takes the form that we want, $\bar{L}_{\mathrm{I}} = \bar{L}_{\mathrm{I}}(\bar{g}_{\mu\nu},...)$. Under such a transformation, the determinant of the metric transforms with a factor of A^8, i.e. $\bar{g} = A^8\,g$, and the Ricci scalar transforms as [131]:

$$R = A^2\left\{\bar{R} + 6\Box\ln A - 6\,\bar{g}^{\mu\nu}(\nabla_\mu \ln A)(\nabla_\nu \ln A)\right\}. \tag{A.10}$$

The second term on the right hand side of this equation is a surface term which is zero by assumption.

Putting all of these together one gets the Lagrangian written in the so called 'Jordan-Fierz' [63] frame (or simply Jordan frame),

$$L = \frac{1}{16\pi}\sqrt{\bar{g}}\left\{\phi\bar{R} - \frac{\omega(\phi)}{\phi}\phi_{,\mu}\phi^{,\mu} + 2\bar{\lambda}(\phi) + L_I(\bar{g}_{\mu\nu},...)\right\}, \tag{A.11}$$

with the following substitutions:

$$GA^2(\Phi) = \phi^{-1}; \tag{A.12}$$

$$\alpha^2 = (2\omega(\phi)+3)^{-1}; \tag{A.13}$$

$$\frac{GA^8(\Phi)}{\lambda(\Phi)} = \bar{\lambda}^{-1}. \tag{A.14}$$

Here α is defined as

$$\alpha \equiv \frac{d\ln A(\Phi)}{d\Phi}. \tag{A.15}$$

As we can see, the Brans-Dicke theory is a special case of (A.11), with $\bar{\lambda} = 0$ and a constant value for ω.

Tensor-MultiScalar Theories.— As I mentioned before, writing the Lagrangian in the Einstein frame has some benefits. One of them is that it can straightforwardly be generalised to the tensor-multi scalar case, which seems to be the most general plausible theory for gravity mediate via a tensor and a number of scalar fields. Nordtvedt [99] writes the Lagrangian for the tensor-multi scalar case as

$$L = \frac{1}{16\pi G}\sqrt{g}\left\{R - g^{\mu\nu}\langle\Phi_{,\mu}\Phi_{,\nu}\rangle + 2\lambda(\Phi) + L_I(A^2(\Phi)g_{\mu\nu},...)\right\}, \tag{A.16}$$

where the angular brackets define an inner product in the space of scalar fields,

$$\langle\Phi_{,\mu}\Phi_{,\nu}\rangle = \gamma_{ab}\Phi^a_{,\mu}\Phi^b_{,\nu}, \tag{A.17}$$

and γ_{ab} is the metric in the space of these fields.

Field Equations.— The field equations of the Lagrangian in the Jordan-Fierz frame, equation 11, are:

$$
\begin{aligned}
\bar{R}_{\mu\nu} - \frac{1}{2}\bar{R}\bar{g}_{\mu\nu} - \bar{\lambda}\,\bar{g}_{\mu\nu} &= 8\pi\phi^{-1}\bar{T}_{\mu\nu} + \phi^{-2}\omega(\phi)\{\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\phi_{,\rho}\phi^{,\rho}\} \\
&\quad + \phi^{-1}\{\phi_{,\nu;\mu} - \bar{g}_{\mu\nu}\Box\phi\} \\
\Box\bar{\phi} + \frac{d\bar{\lambda}}{d\bar{\phi}} &= \frac{1}{2\omega(\phi)+3}\left(8\pi\bar{T} - \frac{d\omega}{d\phi}\bar{g}^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right) \\
\Box &\equiv \partial_\mu\partial^\mu. \qquad \text{(A.18)}
\end{aligned}
$$

As can be seen from the above equation, the second derivatives of the scalar and tensor fields are related to each other in the Jordan-Fierz frame, and therefore this frame mixes the scalar and tensorial modes with each other. In fact the scalar field is acting like a source for the tensor field. One of the other benefits of the Einstein frame is that these modes are decoupled from each other. The field equations in the Einstein frame are:

$$
\begin{aligned}
R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} - \lambda\,g_{\mu\nu} &= \Phi_{,\mu}\Phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\Phi_{,\rho}\Phi^{,\rho} + 8\pi G T_{\mu\nu}; \\
\Box\Phi + \frac{d\lambda}{d\Phi} &= -4\pi G\alpha(\Phi)T. \qquad \text{(A.19)}
\end{aligned}
$$

A.6 More General Theories in Practice

As was mentioned before, the Brans-Dicke theory is essnetially ruled out by solar system observations. However, the situation is different for more general scalar-tensor theories. Keeping in mind that general relativity is the limit of Brans-Dicke theory with $\omega \to \infty$, there is a natural mechanism in the tensor-mono scalar theory of Wagoner or general tensor-multi scalar theories which drives the coupling ω to infinity at late cosmological times. In this section, I will discribe this so called "attractor mechanism" which runs the scalar-tensor theories towards general relativity at recent cosmological times. I will follow Nordtvedt [99], who has shown it in the Einstein frame. Barrow [6] has worked out the cosmological equations in the Jordan-Fierz frame for vacuum-and radiation-dominated cases. I will check the equations for a special situation the ω starts from a finite value in the the early Universe and tends to infinity at late cosmological times. In the following, I will work in the Einstein frame, using equation (A.19), with $\lambda = 0$.

As is beautifully derived in Weinberg [132], one obtains the following metric as soon as one assumes isotropy and homogeneity in space:

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right). \tag{A.20}$$

Here k is the spatial curvature indicator, which takes the values of $\{0, +1, -1\}$, respectively, for flat, spherical or hyperbolic spaces. I will assume $k = 0$ in the following. It is also argued in [132] that the perfect fluid energy momentum tensor, is the only choice under the assumptions of isotropy and homogeneity. Plugging this metric and energy momentum tensor into the field equations of (A.19), one obtains the following equations for the scale factor, a, and the scalar field:

$$\begin{aligned} -3\frac{\ddot{a}}{a} &= 4\pi G(\rho + 3P) + 2(\dot{\Phi})^2; \\ 3\left(\frac{\dot{a}}{a}\right)^2 &= 8\pi G\rho + 2(\dot{\Phi})^2; \\ \ddot{\Phi} + 3\frac{\dot{a}}{a}\dot{\Phi} &= -4\pi G\alpha(\rho - 3P). \end{aligned} \tag{A.21}$$

Here p and ρ are the pressure and the density of matter in the Einstein frame. The energy momentum tensor is not conserved in this frame, but one obtains the following counterpart equation:

$$\frac{d(\rho a^3)}{dt} + p\frac{d(a^3)}{dt} = (\rho - 3P)a^3 \frac{d(A(\Phi))}{dt}. \tag{A.22}$$

which considers the scalar field contribution to the total energy.

One can decouple the scalar field, Φ, and the scale factor, $a(t)$, by introducing the following variable,

$$\tau \equiv \ln(a) + \text{constant}, \tag{A.23}$$

such that

$$d\tau = \frac{\dot{a}}{a} dt, \tag{A.24}$$

and writing all of the above differential equations in "τ-time". The resultant equation for the scalar field is:

$$\frac{2}{3-(\Phi')^2}\Phi'' + (1-\Gamma)\Phi' = -(1-3\Gamma)\alpha(\Phi), \tag{A.25}$$

where $\Gamma \equiv p/\rho$ and a prime denotes $d/d\tau$. The above equation looks like a damped harmonic oscillator with $2/(3-(\Phi')^2)$ as its mass, $(1-\Gamma)$ its damping factor and $\alpha(\Phi)$ its driving force. Looking at (A.13), one finds that $\alpha \sim 1/\omega$, and therefore, if ($\alpha = d\ln A/d\Phi$) tends towards zero, the theory is being driven towards general relativity. (A.25) opens up such a possibility and is the "attractor mechanism".

In this equation, and in the whole scalar-tensor theory, α is a free God given function without any constraints on it! I took two possibilities for this function and solved (A.25) during the radiation and matter dominated eras for both of these cases. In one of them I took α to be the solution of an underdamped oscillator, and in the other, I took α to be an exponentially decreasing function of time. During radiation domination, $\gamma = 1/3$, and the term containing α is negligible. $\Phi =$ constant is a plausible solution for this era. Using this solution as an initial condition, I solved the equation numerically for the matter domination era. The results are shown in Fig. A.1.

A.7 Scalar-Tensor Theories on Theoretical Grounds

Unifying Interactions.— Attempts to unify gravity with other interactions mostly end up with prediction of massless scalar fields [99]. This can be seen from the early Kaluza-Klein [64, 67] attempts of unifying general relativity with electromagnetism and the compactification of the fifth dimension arising in that theory. This may indeed be a two way street, since in the standard model of elementary particles, particles gain mass through a scalar field, and this could be thought of as a manifestation of gravity.

Looking at the current state of theoretical physics, the are three scalar fields in the two standard models for elementary particles and cosmology. A scalar field exists in elementary particle physics, namely the Higgs particle. And in cosmology, one scalar field is needed for inflation and probably another for dark energy. It seems that it would be nice to unify these three scalar fields or find a relation

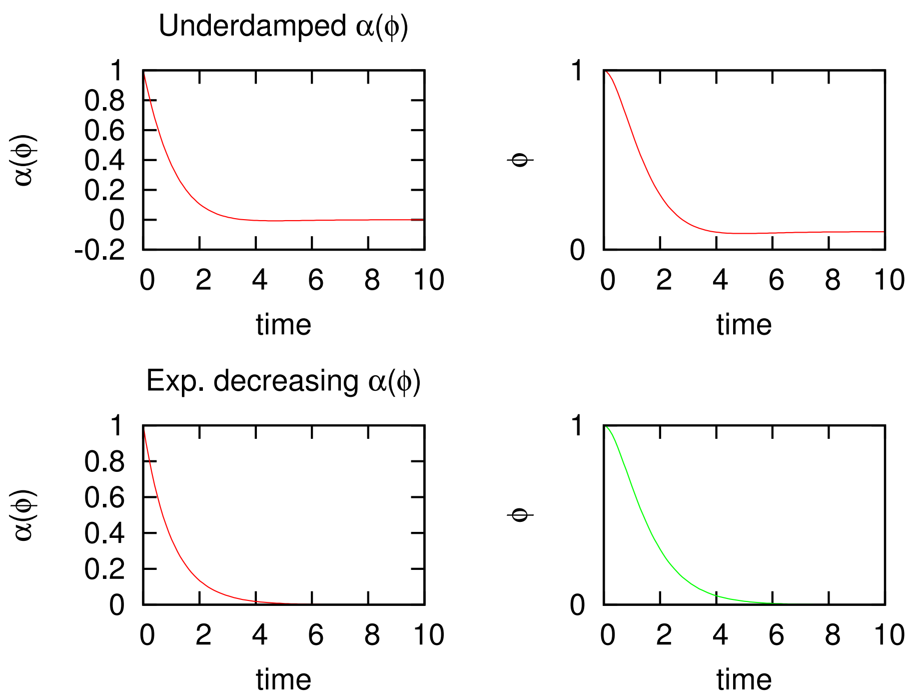

Figure A.1: Inverse of the scalar field coupling, α, and the scalar field, Φ, in artbirary units.

between them.

Different Theories of Gravity.— It has become clear over time that scalar-tensor theories are conformally equivalent to a wide range of gravitational theories, including any f(R) theory [118], or some multidimensionally insipred theories of gravity [41]. Despite this mathematical equivalence among different theories, there has been disagreement about which theory is physical. Mathematical equivalency means that the space of the solutions of these theories is the same, but lack of physical equivalency means that these theories predict different values for the same physical observables.

Physical Frame.— As was shown before, there are two so called *frames* in the context of scalar-tensor theories, the Einstein frame and the Jordan frame. While these two frames are related to each other through a conformal transformation, which is basically a redifinition of the fields, they are not nevertheless physically equivalent. As is discussed in [14, 40, 41], each frame has its own benefits and drawbacks, both mathematically and physically.

The main drawback of the Jordan frame is that the scalar field energy is either negative definite or non-definite. This leads to stability problems at the classical level and the lack of any ground state at the quantum level. The positive feature of the Jordan frame is that the scalar field is minimally coupled to matter fields, and one can therefore use the usual standard model calculations in this frame.

In the Einstein frame the scalar field has a positive definite energy, but it couples to the matter fields. This coupling calls into question the weak equivalence principle and the soundness of using standard model predictions in this frame.

Equivalence Principle.— There are two versions of the equivalence principle, the *weak* equivalence principle and the *strong* equivalence principle [12]. According to the first one, two test bodies will fall at the same rate in a gravitational field if they are released at the same time and from the same place. In this sense, the coupling strength of gravity to particles might vary with time or even from place to place, but this strength is the same for all particles in every single place at a certain moment of time. On the other hand, the strong version of the equivalence principle says that gravity has a constant coupling to matter,at every time and every place! It is obvious that scalar-tensor theories do not respect the equivalence principle in its strong sense, but they might respect the weak form of it, as long as they have the same coupling for every particle. It has been generally believed that the weak equivalence principle does not hold in the Einstein frame, since the matter particles do not move on geodesics of the metric anymore, but Hui and Nicolis [55] proved that the Einstein frame respects the weak equivalence principle as long as the scalar field self energy can be ignored.

A.8 Future Developements

Scalar-tensor theories form a set of consistent theories in which the strength of gravity can vary with time. By carrying out explicit candidates within these theories, we should be able to find self-consistent predictions for how effectively α_g can affect various cosmological observables. That will be the focus of future work.