

Dynamic Resource Allocation for OFDM-based Cognitive Radio Systems

by

Gaurav Bansal

B. Tech., Indian Institute of Technology, Kanpur, 2005

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

Doctor of Philosophy

in

THE FACULTY OF GRADUATE STUDIES

(Electrical and Computer Engineering)

The University Of British Columbia

(Vancouver)

April 2011

© Gaurav Bansal, 2011

Abstract

Cognitive radio (CR) is an emerging technology that would improve spectrum utilization by exploiting unused spectrum in dynamically changing environments. We investigate the design of link adaptation algorithms (e.g., adaptive power and bit loading) for orthogonal frequency division multiplexing (OFDM)-based CR systems. Different power and bit loading schemes can be designed for CR users which exploits the time varying nature of fading gains across the OFDM subcarriers. However, one of the challenges here is to ensure that the interference caused to the primary users (PUs) remains below the target interference threshold. Therefore, not only do we need to consider the fading gains, but also the spectral distance of the subcarriers from the PU's band.

In this thesis, we propose an optimal power loading algorithm, assuming that the rate can be varied continuously, for an OFDM-based CR system. The down-link transmission capacity of the CR user is thereby maximized, while the interference introduced to the PU remains within a tolerable range. We investigate the case of discrete (or integer) rate adaptation. A sub-optimal scheme for integer bit loading is presented which approximates the optimal continuous rate value to a nearest integer. Next, we propose schemes that maximize the capacity of CR users when only imperfect channel state information (CSI) is available at the CR transmitter while guaranteeing the statistical interference constraint. Further, we propose resource allocation schemes for a multiuser scenario where power is loaded for CR users not only in the subcarriers where PU is not present (overlay fashion) but also in the subcarriers where PU is present (underlay fashion). Fi-

nally, for the scenarios where the link between CR source and destination might be weak and not reliable for communication, we employ relays and propose relay and power allocation schemes. Numerical results have been presented for all the proposed algorithms.

Preface

The publications that has resulted because of the research conducted in this thesis are as follows:

- Gaurav Bansal, Md. Jahangir Hossain, and Vijay K. Bhargava, “Adaptive Power Loading for OFDM-based Cognitive Radio Systems”, in *Proceedings of IEEE International Conference on Communications (ICC’07)*, pp. 5137-5142, Glasgow, Scotland, June 2007. (appears in chapter 2)
- Gaurav Bansal, Md. Jahangir Hossain, and Vijay K. Bhargava, “Optimal and suboptimal power allocation schemes for OFDM-based cognitive radio systems,” *IEEE Trans. on Wireless Commun.*, vol. 7, No. 11, pp. 4710-4718, Nov. 2008. (appears in chapter 2)
- Gaurav Bansal, Md. Jahangir Hossain, and Vijay K. Bhargava, “Discrete bit loading for OFDM-based cognitive radio systems”, in *Proceedings of National Conference on Communications (NCC’07)*, pp. 1104-1109, Jan. 2007, Kanpur, India. (appears in chapter 3)
- Gaurav Bansal, Md. Jahangir Hossain, and Vijay K. Bhargava, “Link Adaptation in OFDM-based Cognitive Radio Systems”, in *Cognitive Radio Communications Networks*, Springer-Verlag, pp. 189-212, 2007. (appears in chapter 3)
- Gaurav Bansal, Md. Jahangir Hossain, and Vijay K. Bhargava, “Adaptive power loading for OFDM-based cognitive radio systems with statistical in-

interference constraint,” accepted in *IEEE Trans. on Wireless Comm.*, Feb. 2011. (appears in chapter 4)

- Gaurav Bansal, Olivier Duval, and Francois Gagnon, “Joint Overlay and Underlay Power Allocation Scheme for OFDM-based Cognitive Radio Systems”, in *Proceedings of IEEE Vehicular Technology Conference (VTC’10)*, pp. 1-5, Taipei, Taiwan, May 2010. (appears in chapter 5)
- Gaurav Bansal, Md. Jahangir Hossain, and Vijay K. Bhargava, “Power allocation for multiuser OFDMA-based Cognitive Radio Systems with Joint Overlay and Underlay Architecture”, to be submitted as a journal paper. (appears in chapter 5)
- Dinesh Bharadia, Gaurav Bansal, Praveen Kaligineedi, and Vijay K. Bhargava, “Relay and Power Allocation Schemes for OFDM-based Cognitive Radio Systems”, under minor revision in a journal. (appears in chapter 6)

In the research contributions made in Chapters 2, 3, 4, and 5, I am the primary author and the researcher. I conducted literature review and identified the research problems. Formulation of the research problems, mathematical analysis and simulations were totally carried out by me. I also wrote the associated manuscripts for publication. I am second author in the contributions made in Chapter 6. For this particular work, I conducted the literature review, identified and formulated the research problem. I helped in the mathematical analysis and simulations. I wrote the manuscript for publication. Dr. Md. Jahangir Hossain is a co-author for contributions in Chapters 2, 3, 4, and 5. He provided some important technical feedback during the formulation of the research problems in those chapters. He also provided editorial corrections while writing the associated manuscripts for publication. Olivier Duval is a co-author in Chapter 5. He helped in conducting the mathematical analysis and writing the Introduction section of the associated manuscript. Prof. Francois Gagnon is a co-author for contributions in Chapter 5. He provided some editorial comments. Dinesh Bharadia is a co-author for the

contributions in Chapter 6. He conducted the mathematical analysis and wrote the computer programs for simulating performance of proposed schemes. Praveen Kaligineedi is a co-author in Chapter 6. He provided some feedback while formulating the research problem and provided important editorial feedback. My supervisor Prof. Vijay K. Bhargava is a co-author for contributions made in Chapters 2, 3, 4, 5, and 6 of this thesis. I consulted him during identification and formulation of the research problems. He also provided editorial feedback for the associated manuscripts.

Table of Contents

Abstract	ii
Preface	iv
Table of Contents	vii
List of Tables	xi
List of Figures	xii
Acknowledgments	xv
1 Introduction	1
1.1 Background	4
1.1.1 Opportunistic Spectrum Access Strategy: Overview	4
1.2 Motivation and Objective of the Thesis	7
1.2.1 Adaptive Power Loading Algorithms	8
1.2.2 Discrete Bit Loading Algorithms	9
1.2.3 Power Allocation with Imperfect CSI	10
1.2.4 Joint Overlay and Underlay Power Allocation Schemes	10
1.2.5 Relay and Power Allocation Schemes	10
1.3 Thesis Outline	11
2 Adaptive Power Loading for OFDM-based Cognitive Radio Systems	14
2.1 Introduction	14

2.2	System Model	17
2.2.1	Interference Introduced by CR User's Signal	20
2.2.2	Interference Introduced by PU's Signal	21
2.3	Optimal Scheme	21
2.4	Suboptimal Schemes	26
2.4.1	Scheme A	27
2.4.2	Scheme B	29
2.5	Comparison with Classical Schemes and Nulling Mechanism	30
2.5.1	Uniform Power Loading Scheme	30
2.5.2	Waterfilling Scheme	30
2.5.3	Nulling Mechanism	31
2.6	Numerical Results	33
2.7	Imperfect Channel-gain Information at the Transmitter	36
2.8	Conclusions	41
3	Discrete Bit Loading for OFDM-based Cognitive Radio Systems	43
3.1	Introduction	43
3.2	System Model	45
3.2.1	Interference Introduced by the Secondary User's Signal	46
3.2.2	Interference Introduced by the Primary User's Signal	46
3.3	Proposed Scheme A	47
3.4	Modifications in the Existing Schemes	50
3.4.1	Scheme B	50
3.4.2	Scheme C	51
3.5	Numerical Results	53
3.6	Effect of Subcarriers Nulling Mechanism	54
3.7	Conclusions	56
4	Adaptive Power Loading for OFDM-based Cognitive Radio Systems with Statistical Interference Constraint	60
4.1	Introduction	60

4.2	System Model	63
4.3	Problem Formulation and Optimal Power Allocation	66
4.4	Suboptimal and Classical Power Loading Schemes	69
4.4.1	Newly Proposed Suboptimal Scheme	70
4.4.2	Uniform Loading Scheme	71
4.4.3	Waterfilling Scheme	72
4.5	Numerical Results and Complexity of Algorithms	72
4.6	Conclusions	74
5	Power Allocation for Multiuser OFDMA-based Cognitive Radio Systems with Joint Overlay and Underlay Architecture	77
5.1	Introduction	77
5.2	System Model	79
5.3	Problem Formulation	82
5.4	Optimal Scheme	84
5.4.1	Subcarrier Allocation	84
5.4.2	Power Allocation	85
5.5	Suboptimal Scheme	86
5.6	Classical Schemes	89
5.6.1	Overlay Only Scheme	89
5.6.2	Underlay Only Scheme	90
5.7	Numerical Results	91
5.8	Conclusions	103
6	Relay and Power Allocation Schemes for OFDM-based Cognitive Radio Systems	104
6.1	Introduction	104
6.2	System Model	106
6.3	Problem Formulation and Proposed Schemes	108
6.3.1	Relay Allocation	110
6.3.2	Power Allocation	114

6.3.3	Optimal Solution	115
6.4	Numerical Results	116
6.5	Conclusions	118
7	Conclusions and Future Research Directions	119
7.1	Conclusions	119
7.2	Future Work and Work in Progress	121
7.2.1	Power Allocation Schemes for OFDM-based CR Systems with Imperfect Sensing	121
7.2.2	Cross Layer Design of OFDM-based CR Systems	122
7.2.3	Resource Allocation Schemes for Cooperative Relaying for CR Networks	122
	Bibliography	124

List of Tables

Table 4.1	Complexity of different schemes	75
-----------	---	----

List of Figures

Figure 1.1	Coexistence of primary and cognitive users according to spectrum pooling strategy.	5
Figure 1.2	Possible coexistence scenarios: Scenario 1.	6
Figure 1.3	Possible coexistence scenarios: Scenario 2.	7
Figure 2.1	Distribution of primary and CR users in the spatial domain. . .	18
Figure 2.2	Distribution of primary and CR users in the frequency domain.	19
Figure 2.3	Power profile for suboptimal schemes.	27
Figure 2.4	Maximum transmitted data rate of CR user versus interference introduced to the PU band by several schemes.	34
Figure 2.5	Transmit power of CR user versus interference introduced to the PU band by several schemes.	35
Figure 2.6	Maximum transmitted data rate of CR user versus interference introduced to the PU band by Scheme A for various nulling scenarios.	37
Figure 2.7	Maximum transmitted data rate of CR user versus interference introduced to the PU band by Scheme B for various nulling scenarios.	38
Figure 2.8	Maximum transmitted data rate of CR user versus interference introduced to the PU band by the uniform-loading scheme for various nulling scenarios.	39

Figure 2.9	Maximum transmitted data rate of CR user versus interference introduced to the PU band by the water-filling scheme for various nulling scenarios.	40
Figure 2.10	Maximum transmitted data rate of CR user versus average interference introduced to the PU band when the channel gain is not perfectly known at the CR transmitter.	42
Figure 3.1	Distribution of primary and secondary users	48
Figure 3.2	Interference introduced to the primary user band versus given data rate ($\text{BER} = 10^{-7}$)	55
Figure 3.3	Interference introduced to the primary user band versus given data rate ($\text{BER} = 10^{-7}$) for Scheme C under various nulling.	57
Figure 3.4	Interference introduced to the primary user band versus given data rate ($\text{BER} = 10^{-7}$) for Hughes-Hartogs scheme under various nulling.	58
Figure 3.5	Interference introduced to the primary user band versus given data rate ($\text{BER} = 10^{-7}$) for Chow et al. scheme under various nulling.	59
Figure 4.1	Co-existence of PU and CR users in the frequency domain.	64
Figure 4.2	Co-existence of PU and CR users in the spatial domain.	65
Figure 4.3	Maximum transmitted data rate versus power budget (P_T) for CR users.	74
Figure 4.4	Maximum transmitted data rate versus interference threshold for 2 nd PU band ($I_{th}^{(2)}$) for CR users.	75
Figure 4.5	Maximum transmitted data rate versus probability (a) with which instantaneous interference introduced to the PU band remains below interference threshold (I_{th}) for CR users.	76
Figure 5.1	Underlay and overlay power allocation	80
Figure 5.2	Power profile for suboptimal scheme	87

Figure 5.3	Total transmitted rate versus total power budget for various schemes	93
Figure 5.4	Total transmitted power versus total power budget for various schemes	94
Figure 5.5	Total transmitted rate versus interference threshold for various schemes	96
Figure 5.6	Total transmitted power versus interference threshold for various schemes	97
Figure 5.7	Power profile for various schemes for $I_{th}^{(l)} = 1 \times 10^{-6}$ Watts and total power budget = 10×10^{-3} Watts	98
Figure 5.8	Power profile for various schemes for $I_{th}^{(l)} = 1 \times 10^{-4}$ Watts and total power budget = 1×10^{-4} Watts	99
Figure 5.9	Power profile for various schemes for $I_{th}^{(l)} = 1 \times 10^{-6}$ Watts and total power budget = 10×10^{-3} Watts but for different values of λ_l	100
Figure 5.10	Total transmitted rate versus number of CR users for various schemes	101
Figure 5.11	Total transmitted rate versus probability (a) with which instantaneous interference introduced to the PU band remains below interference threshold (I_{th}) for various schemes	102
Figure 6.1	System model	107
Figure 6.2	PU and CR user distribution in frequency domain	116
Figure 6.3	Capacity of CR users versus interference threshold (I_{th}) for a fixed number (3) of relays.	117
Figure 6.4	Capacity of CR users versus relays for a fixed value (10^{-6} Watts) of interference threshold (I_{th}).	118

Acknowledgments

I would like to profoundly thank my supervisor Prof. Vijay K. Bhargava for his continuous guidance, inspiration and encouragement throughout my graduate studies. I have been very fortunate to work in a intellectual environment provided in the Information Theory and Systems (ITS) laboratory under Prof. Bhargava's mentorship. Special thanks to Dr. Md. Jahangir Hossain for all his invaluable suggestions and for always being there for research discussion. I am thankful to Prof. Lutz Lampe, Prof. Vikram Krishnamurthy, Prof. Robert Schober and Prof. Alireza Nojeh for serving on my departmental exam committee. I would like to acknowledge Natural Sciences and Engineering Research Council of Canada (NSERC) for supporting my graduate studies by awarding me Alexander Graham Bell Canada Graduate Scholarship. Last but not the least, I would like to thank all the members of ITS laboratory for their friendship and cooperation.

Chapter 1

Introduction

Radio spectrum is one of the most scarce and valuable resources for wireless communications. Given this fact, new insights into the use of spectrum have challenged the traditional static spectrum allocation approach to the spectrum management. Actual measurements have shown that most of the allocated spectrum is largely underutilized [1]. Spectrum-Policy Task Force appointed by Federal Communications Commissions (FCC) has also reported similar views about the underutilization of the allocated spectrum. Specifically, FCC has reported vast temporal and geographic variations in the usage of allocated spectrum with utilization ranging from fifteen to eighty five percent [2].

Spectral utilization can be improved significantly by giving opportunistic access of the frequency bands instead of static spectrum allocation. According to the opportunistic spectrum access policy, a group of potential users may use a frequency or spectrum band for wireless communications provided that the legacy users of this band are not deprived from the priority right to use the band. On the other hand, development of the software-defined radio (SDR) technology [3] has enabled a radio transceiver to perform its baseband processing functionalities e.g., modulation and demodulation using software and digital logic. The SDR, therefore, becomes the promising technology towards developing versatile wireless transceivers which will have the capability of accessing different radio networks

with different technologies. In order to facilitate opportunistic spectrum access, this versatile transceiver needs to be spectrally aware. This motivates the design of the cognitive radio (CR) technology [4]. The CR technology is an innovative radio design philosophy and involves smartly sensing the swaths of spectrum and then determining the transmission characteristics (e.g., symbol rate, power, bandwidth, latency) of a group of secondary users (SUs, also referred to as CR users)¹ based on the behavior of the users to whom the spectrum has been licensed [5], [6]. As such the CR has been proposed as a way to improve spectrum utilization by exploiting unused spectrum in a dynamically changing environment.

However, in order to fully exploit the CR paradigm, it requires to design adaptive access technologies for the CR systems. Therefore, the main focus of the wireless communication researchers community is to research and develop such enabling adaptive radio access technologies. Before becoming the CR systems a reality, it requires research in two major directions as outlined below.

- *Spectrum Sensing*: In order to identify and access a suitable portion of spectrum with a minimum interference to the legacy users i.e., the primary users (PUs), the first critical design challenge is to monitor the activity levels of the legacy users. This monitoring or sensing is very critical in the sense that it needs to process a very wide bandwidth and reliably detect presence of PUs. Therefore, spectrum sensing techniques should have a very high sensitivity, linearity, and dynamic range of the circuitry in the radio frequency front-end. In order to achieve these goals, various digital signal processing techniques, for example, matched filtering, energy detection, and cyclostationary feature detection have already been studied in the literature [7]. In order to come up with an effective spectrum sensing algorithm, it requires to consider the computational complexity, storage necessity, total search time and the knowledge the CR has regarding the PU signal characteristics. Burden on the signal processing techniques can be alleviated to a large extent by using cooperative diversity between CR spectrum sensors [8]. Few CR

¹Throughout this thesis, we use the terms SUs and CR users invariably.

spectrum sensors under independent fades can help in reducing individual sensitivity requirements.

- *Efficient Spectrum Utilization:* Based on the information of available spectrum as determined by the sensing algorithms, the next challenging task is to utilize the available spectrum in an efficient fashion by the SUs. As such the transmission capacity of the SUs is maximized while the interference introduced to the PUs remains within the tolerable range. Once an unused or suitable portion of the licensed spectrum is identified by the SUs, a number of challenging questions arises. One of these challenging questions is that what would be the physical layer transmission parameters e.g., transmission power and rate of SUs? As such the transmission capacity of the SUs is maximized while the interference introduced to the PUs is maintained within a satisfactory level. Due to the great flexibility in dynamically allocating the unused spectrum among the SUs as well as the easy analysis of the spectral activity of the PUs [9], orthogonal frequency division multiplexing (OFDM) has already been recognized as a potential transmission technology for the CR systems.

In this thesis we assume that spectrum sensing has been performed and the spectrum band which are available for CR transmission are known. This thesis focuses on exploring research challenges involved in the design of adaptive power and bit loading algorithms for an OFDM based CR system. In such a system, the SUs are considered to co-exist with the PUs by filling the unused portions of the frequency band and using OFDM modulation at the air interface. We provide some solutions to these challenging problems.

1.1 Background

1.1.1 Opportunistic Spectrum Access Strategy: Overview

One of the most challenging problems of opportunistic spectrum sharing is the successful co-existence of PUs and SUs in the same frequency band. Several strategies have been proposed in the literature for opportunistic spectrum access. Examples of these strategies have been surveyed in [10] and they include the spectrum pooling [9], the CR approach for usage of virtual unlicensed spectrum (CORVUS) [1], the DARPA's neXt Generation (XG) program [11, 12], the IEEE 802.22 [13], the dynamic intelligent management of spectrum for ubiquitous mobile network (DIMSUNet) [14], the OFDM-based cognitive radio (OCRA) network [15], the European dynamic radio for IP services in vehicular environments (DRiVE) [16]. In spectrum pooling architecture, CR system is highly flexible as in this manner, the spectrum bands, which are left idle by the licensed users, can be filled efficiently. The goal of this architecture is to overlay SUs on the existing licensed users without requiring any changes to the licensed system and hence, increase the spectrum utilization.

Spectrum Pooling

According to the spectrum pooling strategy of opportunistic spectrum sharing, the SUs access a licensed frequency bands by filling the unused portion of the spectrum without making any changes to the PUs' system. The notion of spectrum pooling was first introduced in [4]. Basically, the spectrum pooling represents the idea of merging spectral ranges from different licensed owners (GPRS, UMTS, military, emergency services, TV band, etc.) into a common pool. Then from this common pool, unused spectrum can be assigned to the cognitive users. For an example, the spectrum pooling strategy is shown in Fig. 1.1 where SUs co-exist in the same band by filling the unused or idle portion of the PUs' bands.

According to the spectrum pooling strategy, both SUs and PUs co-exist in the

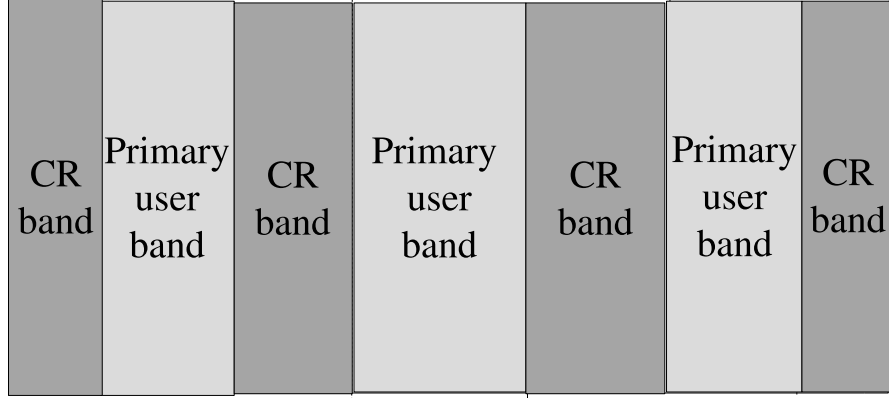


Figure 1.1: Coexistence of primary and cognitive users according to spectrum pooling strategy.

side by side band and their access technologies may be different. Therefore, the mutual interference is the limiting factor for the performance of both networks. Specifically, in [17] the authors have shown that using OFDM modulation causes mutual interference between the PU and the CR users due to the non-orthogonality of the transmitted signals. The amount of interference introduced to the PU's band by a CR user's subcarrier depends on the power allocated in that subcarrier as well as the spectral distance between the subcarrier and the PU's band. The authors have also studied the effect of subcarriers' nulling mechanism which reduces the interference in the PU's band.

The model presented in Fig. 1.1 is a generalized picture of coexistence of both types of users according to the spectrum pooling strategy. According to the interference model presented in [17], we will see that a SU's transmission in a given unused portion of the spectrum produce higher interference to the adjacent PU's band. In other words, the interference introduced into a PU band is dominated by the adjacent SUs' transmission. Interference from the far apart SUs decays as the distance increases. Therefore, if we consider only two dominant interferers, two

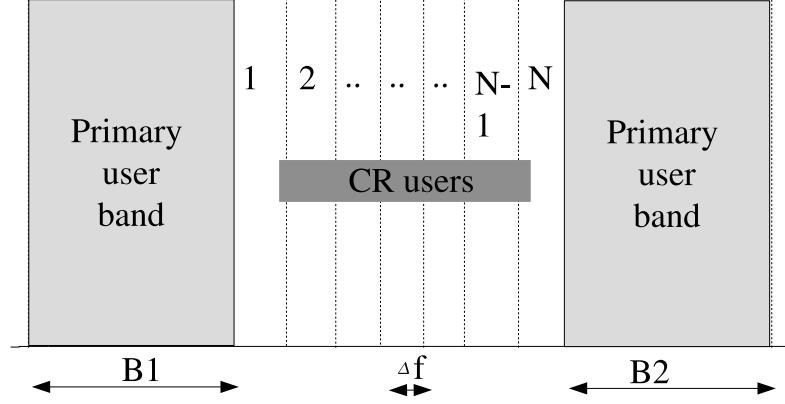


Figure 1.2: Possible coexistence scenarios: Scenario 1.

possible co-existence scenarios; namely scenario 1 and scenario 2, can be imagined as shown in Figs. 1.2, 1.3. Scenario 1 shows that the PUs may co-exist with the SU(s) that are located in the middle of the PUs. In scenario 2, SUs access the left and right side of the unused portion of the PUs band.

It is assumed that the frequency band ($B1$ and $B2$ in scenario 1, and B in scenario 2) which has been occupied by the PU(s) is known (see Figs. 1.2 and 1.3). The frequency band can be occupied by more than one PUs. The interference introduced to the PUs is the limiting factor for the successful coexistence. We assume that all the unused spectrum is used by a cognitive users which uses OFDM modulation format at the air interface. One of the main advantages of using OFDM is the flexibility that it provides in allocating the spectrum. The other advantage of using OFDM is that the FFT used in OFDM transmission can be used for the analysis of the spectral activity of the licensed users. The available bandwidth for CR transmission is divided into N subcarriers, $N/2$ on each side and each having a bandwidth of Δf .

Further it is assumed that the cognitive user does not have any knowledge of the PUs' access method whether it is also OFDM or not. If the PU also use OFDM modulation and the SU has knowledge of it, their transmission could be

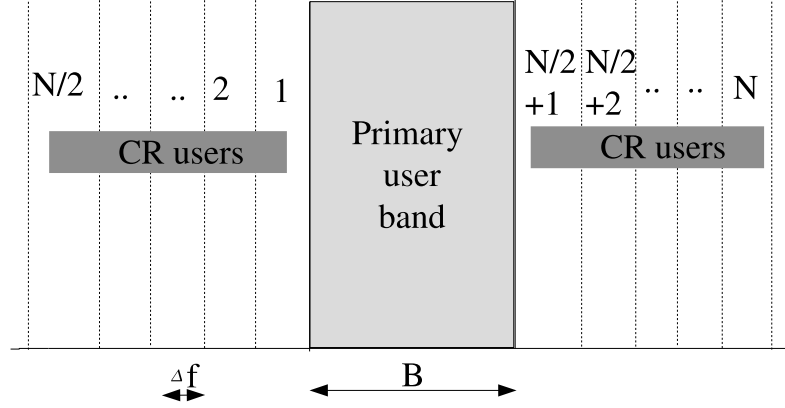


Figure 1.3: Possible coexistence scenarios: Scenario 2.

made orthogonal. However, in practice PU might not be using OFDM and even if it is using OFDM, it would be very difficult for CR user to know the required parameters of PU in order to maintain orthogonality. Due to the coexistence of primary and SUs in such fashion, there are two types of interference in the system [17]. One is introduced by the PU into the CR user band and the other is introduced by the cognitive user into the PU band as described below. The interference introduced by the PU to the CR band acts as a noise and limits the capacity of CR users. On the other hand, there would be limits imposed on the interference introduced by CR user into the PU band and will act as a limiting factor in the power allocation for CR users.

The literature review related to each of the research problems addressed in this thesis will be presented separately in the relevant chapters itself.

1.2 Motivation and Objective of the Thesis

The motivation for this research stems from the need to efficiently utilize the spectrum available for CR transmission. Since different subcarriers in an OFDM system may have different fading gains in a given channel access, use of the same

modulation order in all subcarriers leads to an inefficient utilization of overall spectrum [18]. Assuming that the channel state information (CSI) is available at the transmitter, different power, bit or both power and bit loading schemes have been proposed in literature. These loading schemes exploit the time varying nature of fading gains across the OFDM subcarriers in order to improve the overall system performance. Different loading algorithms have different end goals [19]. One broad class of bit loading algorithms minimizes the transmit power while attaining a fixed transmission rate as well as a given target bit error rate (BER) (see, for example, [20], [21]). In another version of bit loading algorithms, ergodic capacity is maximized at a fixed transmit power. All these algorithms have maximized the transmission capacity of OFDM-based systems and are useful for conventional wireless networks where there is only one group of users i.e., PUs. Since there is a mutual interference between the primary and the SUs when both type of users co-exist, use of the classical loading algorithms e.g., uniform power but variable rate algorithm and water-filling algorithm for SU's transmission may result in higher mutual interference to the PU's band. Hence, the design problem is that given an interference threshold prescribed by the PUs, how much power and how many bits should be loaded into each subcarrier such that the overall transmission capacity of the CR user is maximized.

The broad objective of thesis is to develop thorough analytical modeling and numerical analysis, dynamic resource allocation schemes for OFDM-based cognitive radio systems such that the capacity of CR users can be maximized while interference introduced to the PU band is kept below a specified threshold. The specific objectives of the thesis can be described as:

1.2.1 Adaptive Power Loading Algorithms

According to the classical power and bit loading schemes e.g., water-filling, more power and bits should be loaded into the subcarrier which has higher channel gain. However, the amount of interference introduced by allowing transmission in a SU's subcarrier, depends on the location of the subcarrier with respect to

the PU's spectrum. From the interference point of view, more power should be loaded into a distant subcarrier. Therefore, it requires a judicious loading policy which not only consider the fading gains of the subcarriers but also the spectral distance of the subcarriers from the PU's band. The authors in [22] have proposed an unequal bit loading algorithm for a non-contiguous (NC)-OFDM-based CR system. However, in this scheme uniform power allocation among the OFDM subcarriers is used. In this thesis, we will see that use of uniform transmit power in each subcarrier can significantly reduce the transmission capacity of the SU. The power and bit loading algorithm for an OFDM-based CR system is formulated as a constrained optimization problem. This optimization problem maximizes the transmission capacity of the secondary subcarriers while keeping the interference introduced to the PU below a specified threshold.

1.2.2 Discrete Bit Loading Algorithms

The study in Section 1.2.1 is based on the operating assumption that the transmission rate can be varied continuously. Most of the coding and modulation schemes that are used in practice provide a discrete or integer transmission rate which has led the researchers to develop discrete bit loading algorithms for OFDM-based systems. In this context, a number of algorithms have been proposed in the literature. Examples of well-known algorithms include the Hughes-Hartogs [23] and the Chow et al. [24] algorithms. These algorithms are not directly applicable to an OFDM-based CR system as the interference they introduce to the adjacent PU's band may increase significantly. In this thesis, we propose a sub-optimal scheme based on Lagrange formulation such that it minimizes the interference introduced to PU band while transmitting at a fixed data rate and BER. We also propose schemes based on the modifications of Hughes-Hartogs [23] and Chow et al. [24] algorithms such that rather than minimizing the transmitting power (as required for the conventional scenario) they minimize the interference introduced to the PUs (as required for the CR scenario) while keeping a fixed data rate and BER.

1.2.3 Power Allocation with Imperfect CSI

The study in previous sections is based on the assumption that perfect CSI is available at the CR transmitter. Specifically, the channel gains between CR transmitter and CR receiver can be estimated at the CR transmitter through the feedback techniques. However, the instantaneous channel gains between CR transmitter and PU receiver is hard to estimate. Although by exploiting the pilot signals transmitted by the PU user it is possible to estimate the statistics (distribution and the mean value) of the channel fading gain. For such a scenario, the interference introduced to the PU band can be guaranteed only in a statistical manner. In this thesis, we propose schemes that maximize the capacity of CR users when only imperfect CSI is available at the CR transmitter while guaranteeing the statistical interference constraint.

1.2.4 Joint Overlay and Underlay Power Allocation Schemes

The study so far assumed that the CR power is only allocated in an overlay fashion to the subcarriers where PU is not present and only mutual interference between CR and PU is considered. However, power can also be loaded to subcarriers in an underlay fashion to the subcarriers where PU is present and the co-channel interference will also have to be taken into consideration. In this thesis, we propose resource allocation schemes for a multiuser scenario where power is loaded in a joint overlay and underlay fashion while making sure that the total interference introduced to the PU band remains below a threshold.

1.2.5 Relay and Power Allocation Schemes

CR systems may have a weak channel between CR source and CR destination and reliable communication might not be possible as CR source can not transmit at a very high power because of the limits on the interference that can be introduced to the PU band. In such a scenario, relays can be employed to provide an alternative path of communications as the data can be transmitted reliably at low transmission

power and hence, introducing less interference to the PU band. In this thesis, we propose relays and power allocation schemes for OFDM-based CR systems.

1.3 Thesis Outline

In chapter 2, we investigate an optimal power loading algorithm for an OFDM-based CR system. The downlink transmission capacity of the CR user is thereby maximized, while the interference introduced to the PU remains within a tolerable range. We also propose two suboptimal loading algorithms that are less complex. We also study the effect of a subcarrier nulling mechanism on the performance of the different algorithms under consideration. The performance of the optimal and suboptimal schemes is compared with the performance of the classical power loading algorithms, e.g., water-filling and uniform power but variable rate loading schemes that are used for conventional OFDM-based systems. Presented numerical results show that for a given interference threshold, the proposed optimal scheme allows CR base station (BS) to transmit more power in order to achieve a higher transmission rate than the classical loading algorithms. These results also show that although the proposed suboptimal schemes have certain degradation in performance compared to the optimal scheme, they outperform the classical loading algorithms. We also present numerical results for nulling mechanism. Finally, we investigate the effect of imperfect channel gain information at the transmitter.

In chapter 3, we explore the trade-offs between the data rate of an OFDM-based CR system and the interference introduced to the PU's band when both type of users co-exist in side by side band. In order to minimize the interference to the PU's band, a suboptimal algorithm is proposed. We also propose two schemes based on modifications in the existing discrete bit loading schemes namely the Hughes-Hartogs [23] and Chow et al. [24] schemes. Numerical results have been presented.

In chapter 4, we develop an optimal power allocation algorithm for the OFDM-based CR systems with statistical interference constraints imposed by different PUs. A suboptimal algorithm with reduced complexity is also investigated. Pre-

sented numerical results show that with our proposed optimal power allocation scheme CR user can achieve significantly higher transmission capacity for given power budget and interference constraints compared to the classical power allocation schemes namely, uniform and water-filling power allocation schemes. The sub-optimal scheme, which has same implementation complexity as the uniform power loading scheme, achieves higher transmission capacity than the conventional uniform power loading scheme. Water-filling scheme, which is optimal power allocation for a conventional OFDM-based system and has higher implementation complexity, outperforms the sub-optimal scheme.

In chapter 5, we present findings of our study of joint overlay and underlay power allocation schemes for orthogonal frequency division multiple access (OFDMA)-based multiuser CR system. In a departure from existing work in literature where power is allocated in either overlay only or underlay fashion, we propose schemes which perform a joint allocation. Specifically, the total capacity of CR is maximized while maintaining a total power budget and keeping the interference introduced to the PU band below a threshold. As the optimal schemes may offer prohibitively high complexity, we also propose a faster, suboptimal scheme. The results of simulations of the proposed joint overlay and underlay allocation schemes show a significant gain in transmission capacity over classical schemes which only transmit in either overlay or underlay fashion. Joint allocation maintains its advantage over either overlay or underlay only scheme even when implemented with a low-complexity suboptimal scheme as we empirically found out after implementing further simulations.

In chapter 6, we investigate the relay and power allocation problem for OFDM-based CR systems. We propose a method where the capacity of CR user employing relays is maximized while total transmission power is kept within a budget and the interference introduced to the PU band is kept within a prescribed threshold. The optimization problem is a mixed-integer problem, which is NP-hard. Hence in this chapter, we have proposed three sub-optimal schemes. The presented numerical results show that the performance of proposed suboptimal

schemes is close to the optimal solution.

In chapter 7, we conclude the thesis by highlighting our contributions. We also briefly suggest possible directions of future research.

Chapter 2

Adaptive Power Loading for OFDM-based Cognitive Radio Systems¹

2.1 Introduction

Cognitive radio (CR) has been proposed as a way to improve spectral efficiency by exploiting unused spectrum in dynamically changing environments. The CR design is an innovative radio design philosophy that involves smartly sensing the swaths of spectrum and then determining the transmission characteristics (e.g., symbol rate, power, bandwidth, latency) of a group of secondary users (SU) based on the behavior of the users to whom the spectrum has been licensed (referred to as primary users (PUs)). Although opportunistic spectrum access would allow

¹The papers based on the research work presented in this chapter have been published as: Gaurav Bansal, Md. Jahangir Hossain, and Vijay K. Bhargava, “Adaptive Power Loading for OFDM-based Cognitive Radio Systems”, in *Proceedings of IEEE International Conference on Communications (ICC’07)*, pp. 5137-5142, Glasgow, Scotland, and Gaurav Bansal, Md. Jahangir Hossain, and Vijay K. Bhargava, “Optimal and Suboptimal Power Allocation Schemes for OFDM-Based Cognitive Radio Systems”, in *IEEE Transactions on Wireless Communications*, vol. 7, no. 11, pp. 4710-4718, Nov. 2008.

CR users to identify and access available spectrum resources, one of the main concerns is to utilize these available spectrum resources in an efficient manner.

Due to its great flexibility in dynamically allocating unused spectrum among CR users, and the ease of analysis of the PU's spectral activity [9], orthogonal frequency division multiplexing (OFDM) has already been recognized in the literature as a potential transmission technology for CR systems. Since both CR user and PU may exist in side-by-side bands yet have different access technologies, mutual interference is the limiting factor for performance of both networks. Specifically, in [17] the authors have shown that using OFDM modulation causes mutual interference between the PU and CR users due to the non-orthogonality of the transmitted signals. The amount of interference introduced to the PU's band by a CR user's subcarrier depends on the power allocated in that subcarrier as well as the spectral distance between that particular subcarrier and the PU's band.

In order to exploit the time-varying nature of fading gains across the OFDM subcarriers, power loading algorithms have been proposed in the literature [18]. These algorithms maximize the transmission capacity of an OFDM-based conventional wireless network where there is only one group of users, i.e., PUs. Since there is mutual interference between CR user and PU when they co-exist in side-by-side bands, use of the classical loading algorithms e.g., uniform power but variable rate and water-filling algorithms, for CR system may result in higher mutual interference in the PUs' band. We consider a CR downlink transmission scenario where a CR transmitter/base station (BS) transmits information to a CR user using the radio spectrum that is unoccupied by the PU. For such downlink scenario interference introduced to the PUs is the limiting factor, rather than the transmit power of the CR user. In fact, such an interference-limited scenario limits the transmit power as well as the achievable transmission rate of CR user. Hence, the design problem is as follows. Given an interference threshold prescribed by PUs, how much power should be transmitted into each OFDM subcarrier for a given channel fading gain such that the total transmission rate of the CR user is maximized?

According to the classical power loading schemes, e.g., water-filling, more power should be loaded into the subcarrier with the higher channel gain. However, the amount of interference introduced by allowing transmission in a CR user's subcarrier depends on the location of the subcarrier with respect to the PU's spectrum. From the interference point of view, more power should be loaded into a subcarrier that is far away from PU's band. Therefore, a judicious loading policy is required that not only considers the fading gain of the subcarrier but also the spectral distance of the subcarrier from the PU's band.

Motivated by the aforementioned challenging task and the interference model studied in [17], in this chapter we propose an optimal power loading scheme using the Lagrange formulation. This loading scheme maximizes the downlink transmission capacity of the CR user while keeping the interference induced to the PUs below a specified threshold. As the complexity of the optimal scheme can be high for some applications, we also propose two suboptimal schemes. Our presented simulation results demonstrate the strength of our proposed schemes compared to the classical schemes for the CR scenario. Specifically, we show that our proposed schemes can load more power into CR user band in order to achieve higher transmission capacity for a given interference threshold as specified by the PUs' network. In a related work in [22], Wyglinski has proposed an unequal bit loading algorithm for a non-contiguous (NC)-OFDM-based CR system. In NC-OFDM-based transmission systems, subcarriers, which could potentially interfere with the transmission of other users have been proposed to deactivate. A combined spectrum pooling and adaptive bit loading for CR OFDM based system has been studied in [25] in order to improve the bit error rate (BER) of CR system. In order to minimize adjacent channel interference, a sidelobe suppression mechanism has been proposed in [26]. An efficient multiuser water filling algorithm under interference temperature constraints in the OFDMA-based CR networks has been proposed for an uplink scenario in [27]. The authors in [17] have studied the effect of the subcarrier's nulling mechanism, which reduces the interference in the PU's band. By nulling the subcarriers that are adjacent to the PU band, more power

can be loaded into far apart subcarriers for a given interference threshold. However, after nulling we cannot load any bits into the nearby subcarriers even when they have very good channel gain. Hence, nulling presents a trade-off, which we also study in this chapter. Specifically, we present the power profile of different schemes under various nulling scenarios. Numerical results show that for a given interference threshold, nulling of one adjacent subcarrier improves the achievable transmission rate of suboptimal and classical schemes, while nulling of two or more adjacent subcarriers may degrade their performance. Finally, we study the effect of imperfect channel gain information at the CR transmitter.

The organization of this chapter is as follows. While Section 2.2 describes the system model, Section 2.3 presents the problem formulation and the optimization solution for power adaptation. In Section 2.4, we propose two low-complexity suboptimal schemes. We compare performance of our proposed schemes with that of the classical schemes and nulling mechanism in Section 2.5. Selected numerical results are presented in Section 2.6. The effect of imperfect channel gain information at the transmitter is studied in Section 2.7. Finally, Section 2.8 concludes the chapter.

2.2 System Model

For an example, a possible co-existence scenario of PUs' radio and a CR user's radio in geographical location is shown in Fig. 2.1. In fact, there may be a number of co-existence scenarios. For example, one scenario can be co-located scenario where both primary receivers and CR receiver co-exist in same user's device. For example, in future generation laptops, multiple radios will exist in a given laptop and a medium access control (MAC) coordinator will co-ordinate their MAC functionalities. One of these radios can be based on CR technology. The other co-existence scenario can be closely-located scenario. In such scenario, different radio receivers may exist in different users' devices but these users can be closely located.

In spectral domain, we consider the same side-by-side CR radio access model

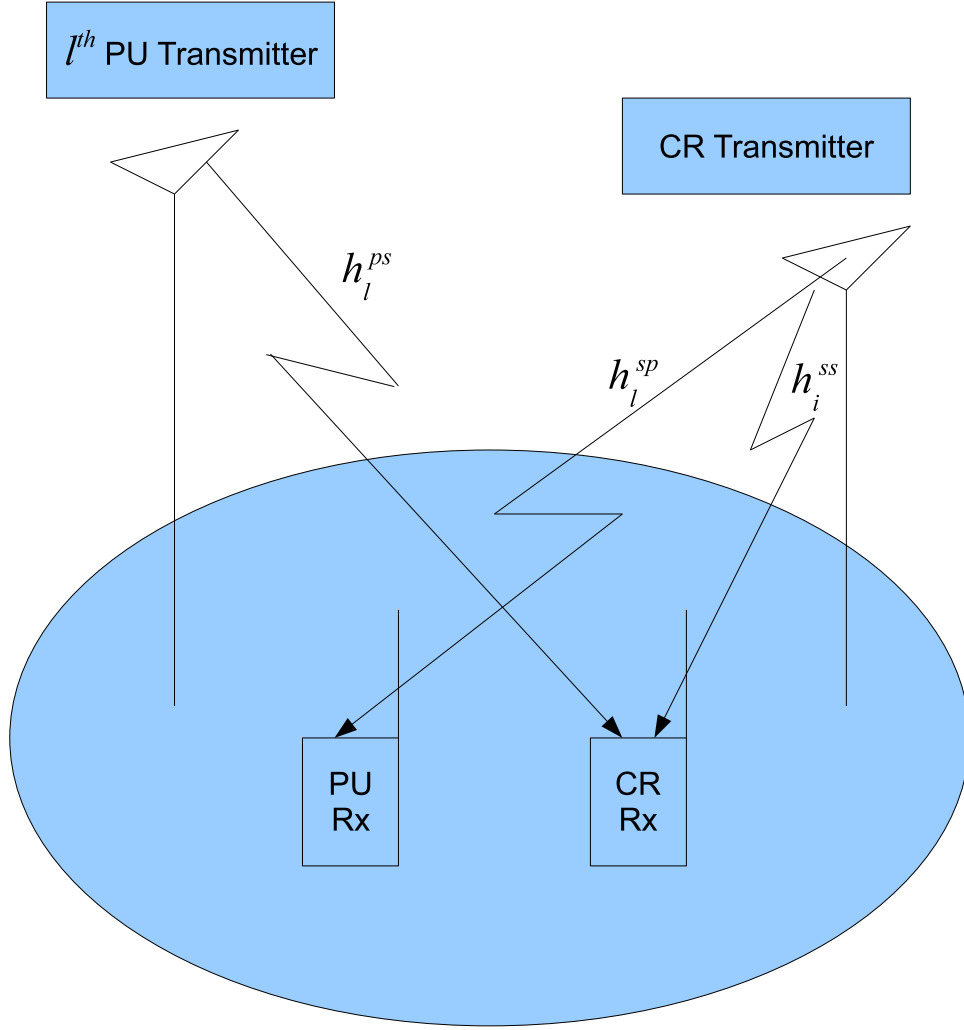


Figure 2.1: Distribution of primary and CR users in the spatial domain.

as is assumed in [17]. Basically, it is assumed that the frequency bands of bandwidth B_1, B_2, \dots, B_L , in Hz which have been occupied by the PU(s) $1, 2, \dots, L$, are sensed by the CR system and known (see Fig. 2.2) to it. The unoccupied band sensed by the CR system for possible transmission is located on each side of the PU bands $1, 2, \dots, L$, as shown in Fig. 2.2. The available bandwidth for CR transmission is divided into N subcarrier based OFDM system. It is assumed that the

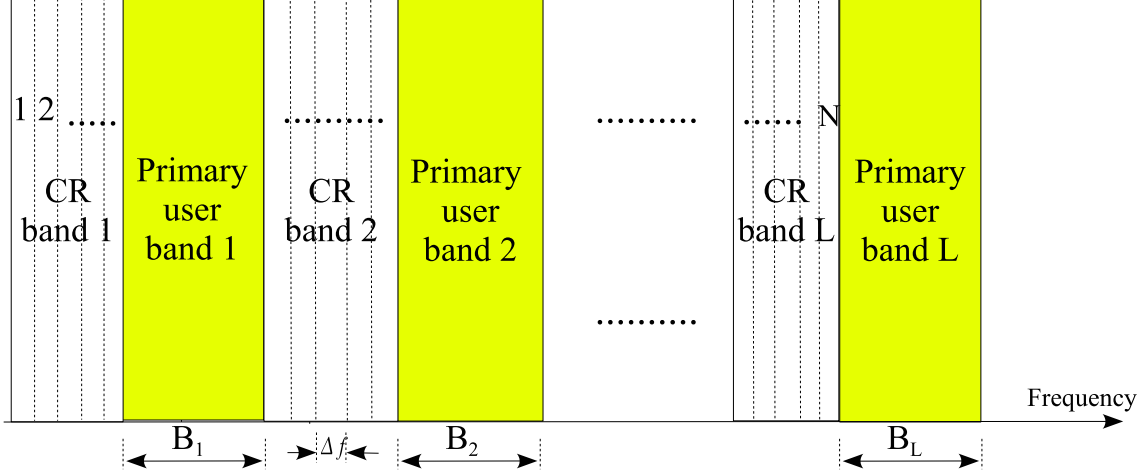


Figure 2.2: Distribution of primary and CR users in the frequency domain.

bandwidth for each CR subcarrier is Δf Hz. It is assumed that the access mechanism/modulation format in PUs' band is not known to the CR system.

In downlink transmission scenario shown in Fig. 2.1, in general, there are three instantaneous fading gains: (1) between the CR user's transmitter and CR user's receiver for the i^{th} subcarrier denoted as h_i^{ss} (it is assumed that h_i^{ss} 's are independent and identically distributed (i.i.d.)); (2) between the CR user's transmitter and l^{th} PU receiver, denoted as h_l^{sp} (assumed to be i.i.d.); and (3) between the l^{th} PU's transmitter and the CR user's receiver, denoted as h_l^{ps} (assumed to be i.i.d.). We assume these instantaneous fading gains are perfectly known at the CR user's transmitter. Specifically, we assume that the CR user's receiver can estimate channel gains h_i^{ss} and h_l^{ps} and report to the CR transmitter. With co-located scenario discussed earlier i.e., both primary and CR receivers are located in same device and primary receiver can estimate the channel h_l^{sp} which is reported to the CR transmitter. On the other hand, if PU and CR receivers are located in different devices, we assume that the CR transmitter can estimate the channel gain h_l^{sp} from the emitted signal from the PU's receiver.

Further, we note that for scenarios where perfect knowledge of instantaneous

channel gains is not available, the throughput obtained through proposed resource allocation algorithm in this chapter will serve as an upper bound on the maximum achievable throughput. In Section 2.8, we will study the case where the instantaneous fading gains are not perfectly known at the CR transmitter rather the average fading gains of PUs are known at the CR transmitter.

The transmit power is adjusted in each CR user's subcarrier. With an ideal coding scheme, the transmission rate at the i^{th} carrier, R_i , for the transmit power, P_i and channel fading gain, h_i^{ss} , is connected via the Shannon capacity formula, and is given by

$$R_i(P_i, h_i^{ss}) = \Delta f \log_2 \left(1 + \frac{|h_i^{ss}|^2 P_i}{\sigma^2 + \sum_{l=1}^L J_i^{(l)}} \right), \quad (2.1)$$

where σ^2 denotes the additive white Gaussian noise (AWGN) variance, and $J_i^{(l)}$ denotes the interference introduced by the l th PU band into the i^{th} OFDM subcarrier. It is assumed that the interference introduced by PUs into the SUs can be modeled as AWGN which may not be valid for low number of PUs. However, for larger number of PUs, AWGN assumption is a good approximation.

As it is mentioned in [17] that due to the coexistence of PU and CR users in side by side bands, there are two types of interference in the system. One is introduced by the PUs into the CR user's band, and the other is introduced by the CR user into the PUs' band. In what follows, we provide brief description and mathematical models for interference between CR user and PUs.

2.2.1 Interference Introduced by CR User's Signal

Assuming an ideal Nyquist pulse, the power density spectrum of the i^{th} subcarrier in the CR user band can be written as [17]

$$\phi_i(f) = P_i T_s \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2, \quad (2.2)$$

where P_i is the total transmit power in the i^{th} subcarrier and T_s is the symbol duration. Let us denote the interference introduced by the i^{th} subcarrier of CR to the l th PU's band as $I_i^{(l)}(d_{il}, P_i)$. This interference is the integration of the power density spectrum of the i^{th} subcarrier across the l th PU band, and can be written as

$$I_i^{(l)}(d_{il}, P_i) = |h_l^{sp}|^2 P_i T_s \int_{d_{il}-B_l/2}^{d_{il}+B_l/2} \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 df, \quad (2.3)$$

where d_{il} represents the distance in frequency between the i^{th} subcarrier of the CR user band and the l th PU band, and B_l represents occupied bandwidth by l th PU.

2.2.2 Interference Introduced by PU's Signal

The power density spectrum of the PU signal after M-fast Fourier transform (FFT) processing can be expressed by the following expected value of the periodogram [17] :

$$E\{I_N(w)\} = \frac{1}{2\pi M} \int_{-\pi}^{\pi} \phi_{PU}(e^{jw}) \left(\frac{\sin(w-\psi)M/2}{\sin(w-\psi)/2} \right)^2 d\psi, \quad (2.4)$$

where w represents the frequency normalized to the sampling frequency and $\phi_{PU}(e^{jw})$ is the power density spectrum of the PU signal. The PU signal has been taken to be an elliptically filtered white noise process with an amplitude P_{PU} [17]. The interference introduced by the l th PU signal to the i^{th} subcarrier, denoted as $J_i^{(l)}(d_{il}, P_{PU})$, will be the integration of the power density spectrum of the PU signal across the i^{th} subcarrier, and can be written as

$$J_i^{(l)}(d_{il}, P_{PU}) = |h_l^{ps}|^2 \int_{d_{il}-\Delta f/2}^{d_{il}+\Delta f/2} E\{I_N(w)\} dw. \quad (2.5)$$

2.3 Optimal Scheme

Our objective is to maximize the total transmission rate of CR user while keeping the instantaneous interference introduced to the PUs below a certain threshold,

expressing mathematically as,

$$C = \max_{P_i} \sum_{i=1}^N \Delta f \log_2 \left(1 + \frac{|h_i^{ss}|^2 P_i}{\sigma^2 + \sum_{l=1}^L J_i^{(l)}} \right), \quad (2.6)$$

subject to,

$$\sum_{l=1}^L \sum_{i=1}^N I_i^{(l)}(d_{il}, P_i) \leq I_{th}, \quad (2.7)$$

and

$$P_i \geq 0, \quad \forall i = 1, 2, \dots, N, \quad (2.8)$$

where C denotes the transmission capacity of the CR user, N denotes the total number of OFDM subcarriers, I_{th} denotes the total interference threshold prescribed by the L PU bands, $K_i^{(l)} = |h_l^{sp}|^2 T_s \int_{d_{il}-B_l/2}^{d_{il}+B_l/2} \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 df$, and λ is a finite deterministic value. It should be noted that we are solving the problem for a single CR user.

Theorem: The total transmission capacity is maximized by

$$P_i^* = \max \left\{ 0, \frac{1}{\lambda \sum_{l=1}^L K_i^{(l)}} - \frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \right\} \quad (2.9)$$

Proof: Considering the fact that maximization of a concave function is equivalent to minimization of its negative value and introducing the Lagrange multiplier λ for the inequality constraint in Eq. (2.7), and Lagrange multipliers μ_i for the inequality constraint in Eq. (2.8), we can write the Karush-Kuhn-Tucker (KKT)

conditions as [28]

$$\begin{aligned}
P_i^* &\geq 0, \\
&\forall \quad i = 1, 2, \dots, N \\
\sum_{l=1}^L \sum_{i=1}^N I_i^{(l)}(d_{il}, P_i^*) &\leq I_{th}, \\
\mu_i &\geq 0, \\
&\forall \quad i = 1, 2, \dots, N \\
\mu_i P_i^* &= 0, \\
&\forall \quad i = 1, 2, \dots, N \\
-\frac{1}{\left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} + P_i^*\right)} - \mu_i + \lambda \sum_{l=1}^L \underbrace{\frac{\partial I_i^{(l)}}{\partial P_i^*}}_{K_i^{(l)}} &= 0, \\
&\forall \quad i = 1, 2, \dots, N
\end{aligned} \tag{2.10}$$

Now, we can eliminate μ_i from Eq. (2.10) and write the equation as follows,

$$\begin{aligned}
P_i^* &\geq 0 \\
\forall \quad i &= 1, 2, \dots, N, \\
\sum_{l=1}^L \sum_{i=1}^N I_i^{(l)}(d_{il}, P_i^*) &\leq I_{th}, \\
-\frac{P_i^*}{\left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} + P_i^*\right)} + \lambda P_i^* \sum_{l=1}^L K_i^{(l)} &= 0, \\
\forall \quad i &= 1, 2, \dots, N, \\
\frac{1}{\left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} + P_i^*\right) \sum_{l=1}^L K_i^{(l)}} &\leq \lambda, \\
\forall \quad i &= 1, 2, \dots, N.
\end{aligned} \tag{2.11}$$

If $\lambda < 1 / \left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \sum_{l=1}^L K_i^{(l)} \right)$, this last condition in Eq. (2.11) can only hold if $P_i^* > 0$ as $\sum_{l=1}^L K_i^{(l)} > 0$, which by the third condition in Eq. (2.11) implies that $\lambda = 1 / \left(\left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} + P_i^* \right) \sum_{l=1}^L K_i^{(l)} \right)$. Solving for P_i^* , we can write $P_i^* = 1 / \left(\lambda \sum_{l=1}^L K_i^{(l)} \right) - \left(\sigma^2 + \sum_{l=1}^L J_i^{(l)} \right) / |h_i^{ss}|^2$. Now, if $\lambda \geq 1 / \left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \sum_{l=1}^L K_i^{(l)} \right)$, then $P_i^* > 0$, is impossible, because it would imply $\lambda \geq 1 / \left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \sum_{l=1}^L K_i^{(l)} \right) > 1 / \left(\left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} + P_i^* \right) \sum_{l=1}^L K_i^{(l)} \right)$, which violates the third condition in Eq. (2.11) as the condition can be written as,

$$P_i^* \sum_{l=1}^L K_i^{(l)} \left(\lambda - \frac{1}{\left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} + P_i^* \right) \sum_{l=1}^L K_i^{(l)}} \right) = 0, \quad \forall i = 1, 2, \dots, N. \tag{2.12}$$

Hence, $P_i^* = 0$ if $\lambda \geq 1 / \left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \sum_{l=1}^L K_i^{(l)} \right)$. Therefore, we can write the

optimal power profile as

$$\begin{aligned}
P_i^* &= \frac{1}{\lambda \sum_{l=1}^L K_i^{(l)}} - \frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \\
&\quad \text{if } \lambda < \frac{1}{\left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \right) \sum_{l=1}^L K_i^{(l)}} \\
&= 0 \quad \text{if } \lambda \geq \frac{1}{\left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \right) \sum_{l=1}^L K_i^{(l)}}
\end{aligned} \tag{2.13}$$

or, more simply, can be written as following,

$$P_i^* = \max \left\{ 0, \frac{1}{\lambda \sum_{l=1}^L K_i^{(l)}} - \frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \right\}. \tag{2.14}$$

Substituting Eq. (2.14) into Eq. (2.7) we can write

$$\sum_{l=1}^L \sum_{i=1}^N K_i^{(l)} \times \max \left\{ 0, \frac{1}{\lambda \sum_{l=1}^L K_i^{(l)}} - \frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \right\} \leq I_{th}. \tag{2.15}$$

Now as both left-hand side and right-hand side of Eq. (2.15) are positive values, the maximum of left-hand side would be achieved when the right-hand side is maximum. Hence, to maximize the capacity, i.e. to maximize the power, we solve Eq. (2.15) at the boundary and it is written as,

$$\sum_{l=1}^L \sum_{i=1}^N K_i^{(l)} \times \max \left\{ 0, \frac{1}{\lambda \sum_{l=1}^L K_i^{(l)}} - \frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \right\} = I_{th}. \tag{2.16}$$

The lefthand side of Eq. (2.16) is a piecewise-linear increasing function of $\frac{1}{\lambda \sum_{l=1}^L K_i^{(l)}}$, with breakpoints at $\left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \right)$, and hence the equation has a unique solution that is readily determined. Hence Proved.

It is important to mention that the power allocation policy in Eq. 2.9 is indeed an waterfilling policy. However, the cutoff value for the channel gain or the threshold for this waterfilling policy is weighted by the inverse of the interference term $\sum_{l=1}^L K_i^{(l)}$. Specifically, the policy suggests that more power should be allocated to the subcarrier which has relatively better channel quality and is relatively far away from the PU's band.

2.4 Suboptimal Schemes

By using the above scheme, we can calculate the optimal power allocation policy that maximizes the transmission capacity of the CR user while keeping the interference introduced to the PUs below the specified threshold. However, several iterations may be required in finding the value of λ from Eq. (2.16) and the complexity of the optimal scheme is $O(N \log N)$. It should be noted that the complexity of the optimal scheme is not very high, but however there might be some systems which have strict requirements on the complexity and hence, in the following section, we propose sub-optimal schemes based on heuristics that have a complexity of $O(1)$.

Heuristic schemes proposed in this section are based on the fact that the interference introduced to a PU band by the CR user subcarrier increases as the spectral distance between them decreases. If we ignore the second term $(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2})$ from the power profile of Eq. (2.13), the power is inversely proportional to the parameter $\sum_{l=1}^L K_i^{(l)}$, which depends on the spectral distance of the i^{th} subcarrier from the PU band. As such, in order to reduce the interference to a particular PU band, less power should be assigned to the subcarriers that are near to that PU band, and more power should be assigned to the subcarriers that are far from the PU band.

There are two types of CR user bands as illustrated in Fig. 2.2. One is adjacent only to one PU band (CR user band 1) and the other is surrounded by PU bands on both sides (CR user band 2, 3, ..., L). The allocation of less power to the nearby subcarriers suggests that the power can be distributed like a *single ladder* profile

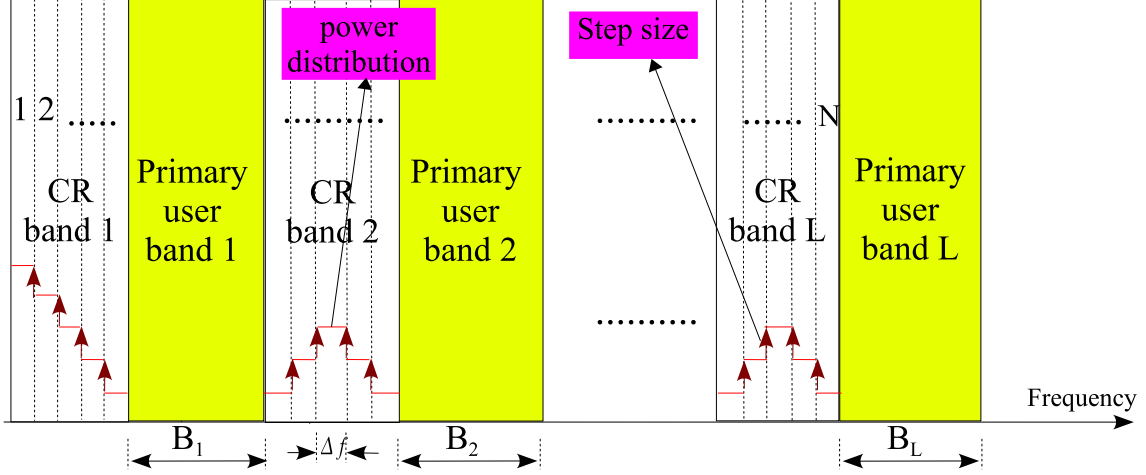


Figure 2.3: Power profile for suboptimal schemes.

in CR user band 1 and like a *step ladder* profile in other CR user subcarriers (2, 3, ..., L), as shown in Fig. 2.3. The total transmit power is determined such that the total interference introduced by all the subcarriers is equal to the interference threshold. Based on the step size of the power profile, we propose two suboptimal schemes as follows.

2.4.1 Scheme A

While allocating power using Scheme A for a particular CR user subcarrier, we consider the effect only on the PU band where it causes the most amount of interference, i.e., the PU band to which it is closest. Power is distributed such that the subcarriers that are adjacent to the PU bands are given power P . Then we increase the power by P as we move away from the PU bands. Hence, to subcarriers that are adjacent to the PU bands we allocate power P , to those that are right next to them we allocate $2P$, and so on. For proposing this scheme, without loss of generality we assume that each CR user band occupies $\frac{N}{L} \geq 1$ subcarriers. Hence, we

can write the power profile as follows:

$$\begin{aligned}
P_i^A &= \left(\frac{N}{L} + 1 - i \right) P \\
&\quad \forall i \in \left\{ 1, 2, \dots, \frac{N}{L} \right\} \\
&= \left(i - \frac{N}{L} \right) P \\
&\quad \forall i \in \left\{ \frac{N}{L} + 1, \dots, \left\lceil \frac{3N}{2L} \right\rceil \right\} \\
&= \left(\frac{2N}{L} + 1 - i \right) P \\
&\quad \forall i \in \left\{ \left\lceil \frac{3N}{2L} \right\rceil + 1, \dots, \frac{2N}{L} \right\} \\
&\vdots \\
&= \left(i - \frac{(L-1)N}{L} \right) P \\
&\quad \forall i \in \left\{ \frac{(L-1)N}{L} + 1, \dots, \left\lceil \frac{(2L-1)N}{2L} \right\rceil \right\} \\
&= (N+1-i) P \\
&\quad \forall i \in \left\{ \left\lceil \frac{(2L-1)N}{2L} \right\rceil + 1, \dots, N \right\}
\end{aligned} \tag{2.17}$$

We now need to find P . We can write,

$$\sum_{i=1}^N \sum_{l=1}^L K_i^{(l)} P_i = I_{th}. \tag{2.18}$$

From eq. (2.17) and eq. (2.18), we can write:

$$P = \frac{I_{th}}{x} \tag{2.19}$$

where

$$\begin{aligned}
x &= \sum_{i=1}^{\frac{N}{L}} \sum_{l=1}^L K_i^{(l)} \left(\frac{N}{L} + 1 - i \right) \\
&+ \left(\sum_{i=\frac{N}{L}+1}^{\lceil \frac{3N}{2L} \rceil} \sum_{l=1}^L K_i^{(l)} \left(i - \frac{N}{L} \right) \right. \\
&\quad \left. + \sum_{i=\lceil \frac{3N}{2L} \rceil+1}^{\frac{2N}{L}} \sum_{l=1}^L K_i^{(l)} \left(\frac{2N}{L} + 1 - i \right) \right) \\
&+ \dots \\
&+ \left(\sum_{i=\frac{(L-1)N}{L}+1}^{\lceil \frac{(2L-1)N}{2L} \rceil} \sum_{l=1}^L K_i^{(l)} \left(i - \frac{(L-1)N}{L} \right) \right. \\
&\quad \left. + \sum_{i=\lceil \frac{(2L-1)N}{2L} \rceil+1}^N \sum_{l=1}^L K_i^{(l)} (N + 1 - i) \right) \tag{2.20}
\end{aligned}$$

2.4.2 Scheme B

In this scheme, the step size of the ladder is taken to be inversely proportional to $\sum_{l=1}^L K_i^{(l)}$. Hence, the power in the i^{th} subcarrier can be written as

$$P_i^B = P / \sum_{l=1}^L K_i^{(l)}, \tag{2.21}$$

where P will be determined by the value of I_{th} , as follows. Eq. (2.18) will hold in this case as well and by substituting Eq. (2.21) into Eq. (2.18) the power allocation policy for scheme B, P_i^B , can be expressed as

$$P_i^B = \frac{I_{th}}{N \sum_{l=1}^L K_i^{(l)}}. \tag{2.22}$$

In next section we describe classical schemes namely, uniform power loading and waterfilling scheme, and nulling mechanism which we will use as benchmarks to measure the performance of the proposed schemes.

2.5 Comparison with Classical Schemes and Nulling Mechanism

The classical OFDM loading algorithms, the uniform power loading and waterfilling schemes, are suboptimal for such an interference limited scenario, as they do not have constraints on interference.

2.5.1 Uniform Power Loading Scheme

In the uniform power loading scheme, uniform power P is loaded into each sub-carrier. In fact, the uniform power loading scheme is a special case of sub-optimal scheme A, where P_i^A is assumed as P . Therefore, using Eq. (2.18) for a given interference threshold I_{th} , the power allocated to the i^{th} subcarrier with uniform power loading P_i^U can be easily expressed as

$$P_i^U = \frac{I_{th}}{\sum_{i=1}^N \sum_{l=1}^L K_i^{(l)}}. \quad (2.23)$$

2.5.2 Waterfilling Scheme

For distributing power according to the waterfilling scheme, we first determine the total power used by the uniform scheme (P_{Total}^U) in transmitting at the given interference threshold. Using this total power as the power constraint, the power

profile for the waterfilling scheme can be written as [29]

$$\begin{aligned}
P_i^{\text{WF}} &= \frac{1}{\lambda} - \frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \quad \text{if } \lambda < \frac{1}{\left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2}\right)} \\
&= 0 \quad \text{if } \lambda \geq \frac{1}{\left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2}\right)}
\end{aligned} \tag{2.24}$$

or, more simply, as $P_i^{\text{WF}} = \max \left\{ 0, \frac{1}{\lambda} - \frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \right\}$. Now, the constant λ can be calculated from the following:

$$\sum_{i=1}^N \max \left\{ 0, \frac{1}{\lambda} - \frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \right\} = P_{\text{Total}}^{\text{u}} \tag{2.25}$$

It should be noted that the interference introduced by the water-filling scheme will not be equal to I_{th} and can be written as

$$I^{\text{WF}} = \sum_{l=1}^L \sum_{i=1}^N K_i^{(l)} P_i^{\text{WF}}. \tag{2.26}$$

2.5.3 Nulling Mechanism

In [17], the authors studied the effect of nulling the subcarriers and showed that the interference introduced to the PU band can be reduced by nulling those subcarriers (i.e., allocating zero power) that are adjacent to the PU band, since the adjacent subcarriers produce the maximum amount of interference. This procedure implies that for a given interference threshold, more power can be allocated to the far apart subcarriers than to the neighboring subcarriers. Thus, one would expect that a higher transmission rate can be achieved using more power. However, nulling the adjacent subcarriers loses frequency diversity, as the adjacent subcarrier is assigned with zero power even when it has a very good channel gain.

Therefore, nulling creates a trade-off. Hence, here we study our proposed suboptimal schemes namely Scheme A, and Scheme B, and classical uniform-loading and water-filling scheme with the effect of nulling. It should be noted that we do not study effect of nulling on the optimal scheme, as it is already optimal, and automatically nulls the subcarriers that need to be nulled.

Here, we restrict ourselves to two cases of nulling, namely, the one-nulling and two-nulling cases. By a “one-nulling” we mean that for each side of the PU band, we null one subcarrier that is adjacent to it. Without loss of generality, we assume that each CR user band occupies $\frac{N}{L}$ subcarriers. Hence, in Fig. 2.2 we null $2L - 1$ subcarriers, namely,

$$O \in \left\{ \left(\frac{N}{L} \right), \left(\frac{N}{L} + 1 \right), \left(\frac{2N}{L} \right), \left(\frac{2N}{L} + 1 \right), \dots, (N) \right\}. \quad (2.27)$$

Similarly, in two-nulling, for each side of the PU band we null the two subcarriers adjacent to it. Hence, for two-nulling in Fig. 2.2 we null $2(2L - 1)$ subcarriers, namely,

$$T \in \left\{ \left(\frac{N}{L} - 1 \right), \left(\frac{N}{L} \right), \left(\frac{N}{L} + 1 \right), \left(\frac{N}{L} + 2 \right), \right. \\ \left. \left(\frac{2N}{L} - 1 \right), \left(\frac{2N}{L} \right), \left(\frac{2N}{L} + 1 \right), \dots, (N) \right\}. \quad (2.28)$$

Now for studying the nulling mechanism, we load zero power in the subcarriers in Eq. (2.27) or Eq. (2.28) for one and two nulling respectively. For the remaining sub-carriers, we load powers according to the scheme under study, such that the total interference introduced to the primary user band is equal to I_{th} , and the power profile for the schemes can be derived accordingly. In the following section, we present simulation results that compare the performances of the various proposed schemes with the classical schemes and nulling mechanism for the CR scenario.

2.6 Numerical Results

In the numerical results presented in this section, we assume the values of L and N to be 4 and 20, respectively.² We assume the value of T_s to be 4μ seconds. Here we assume the values of Δf to be 0.3125 MHz, which is same as subcarrier frequency spacing in wireless local area network (LAN) standards [30], [31]. The values of B_1 , B_2 , B_3 , and B_4 has been taken to be 1 MHz, 2 MHz, 5 MHz, and 10 MHz respectively. The value of σ^2 is assumed to be 10^{-3} W. The value of amplitude P_{PU} is assumed to be 10. The channel gains h_i^{ss} , h_l^{sp} , and h_l^{ps} is assumed to be Rayleigh fading with an average channel power gain of 10dB. Since the channel fading gains for different realizations of channel gain can be different, an average transmission capacity of 100,000 independent simulation runs is considered.

In Fig. 2.4, we plot the achievable transmission rate of the CR user versus interference introduced to the PU band for different schemes under consideration. From this figure, we observe that for a given interference threshold, the optimal scheme achieves the highest transmission rate for CR users. We can also see that the transmission rate that can be achieved using the sub-optimal schemes is higher than that achieved by the classical uniform-power loading and water-filling algorithms. It can also be observed that scheme B performs better than scheme A.

In Fig. 2.5, we present the transmit power of the CR user versus the interference introduced to the PU band for various schemes under consideration. We can observe from Fig. 2.5 that the optimal scheme allows transmission of higher power than the other schemes for a given interference threshold. The classical uniform power loading and water-filling schemes are able to load the least amount of power and achieve lower transmission capacity for CR users, as they do not judiciously take interference into account in their loading policy for a given interference threshold.

The transmission capacity of the CR user versus interference introduced to

²In practice the values of L and N would be high, but for simplicity in the simulation analysis we have assumed the values to be 4 and 20, respectively. Also, it should be noted that the trends in the results presented in this chapter would still hold for other values of L and N .

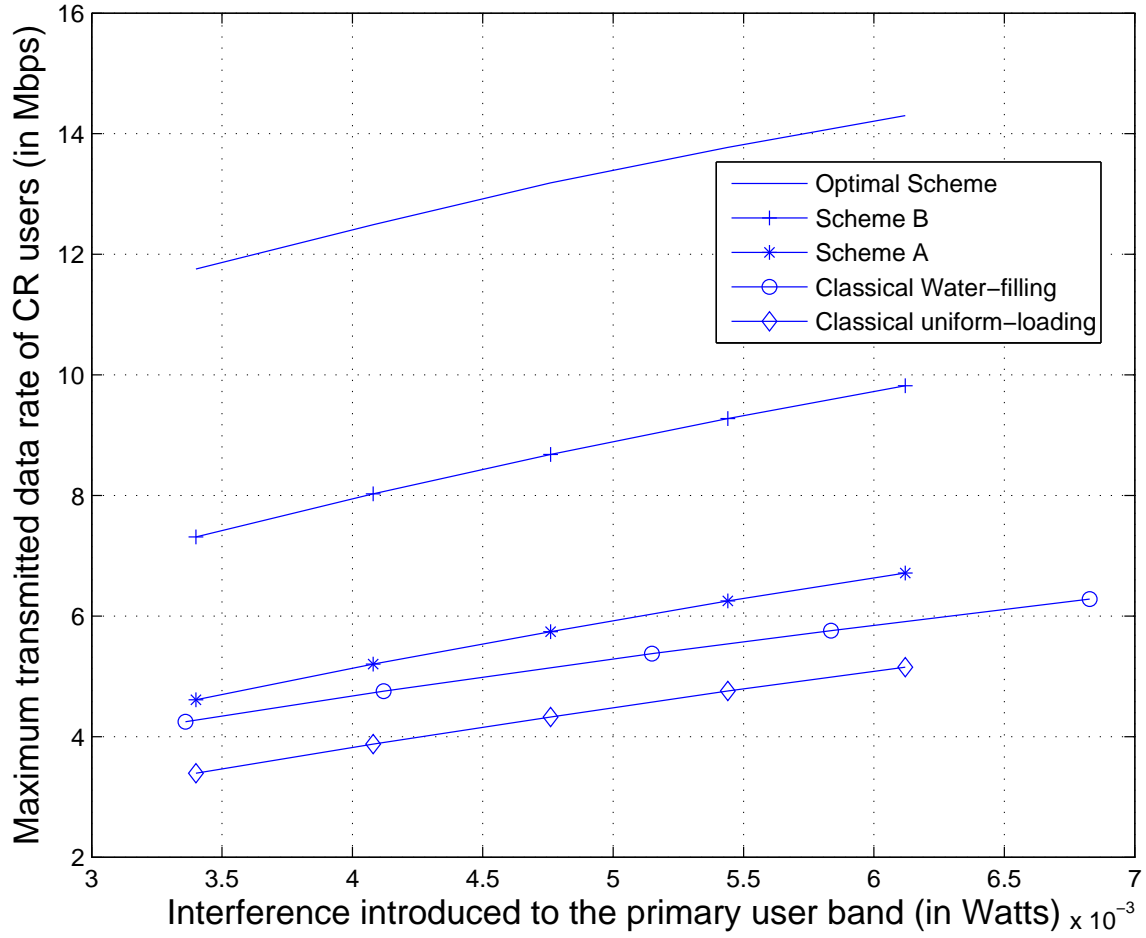


Figure 2.4: Maximum transmitted data rate of CR user versus interference introduced to the PU band by several schemes.

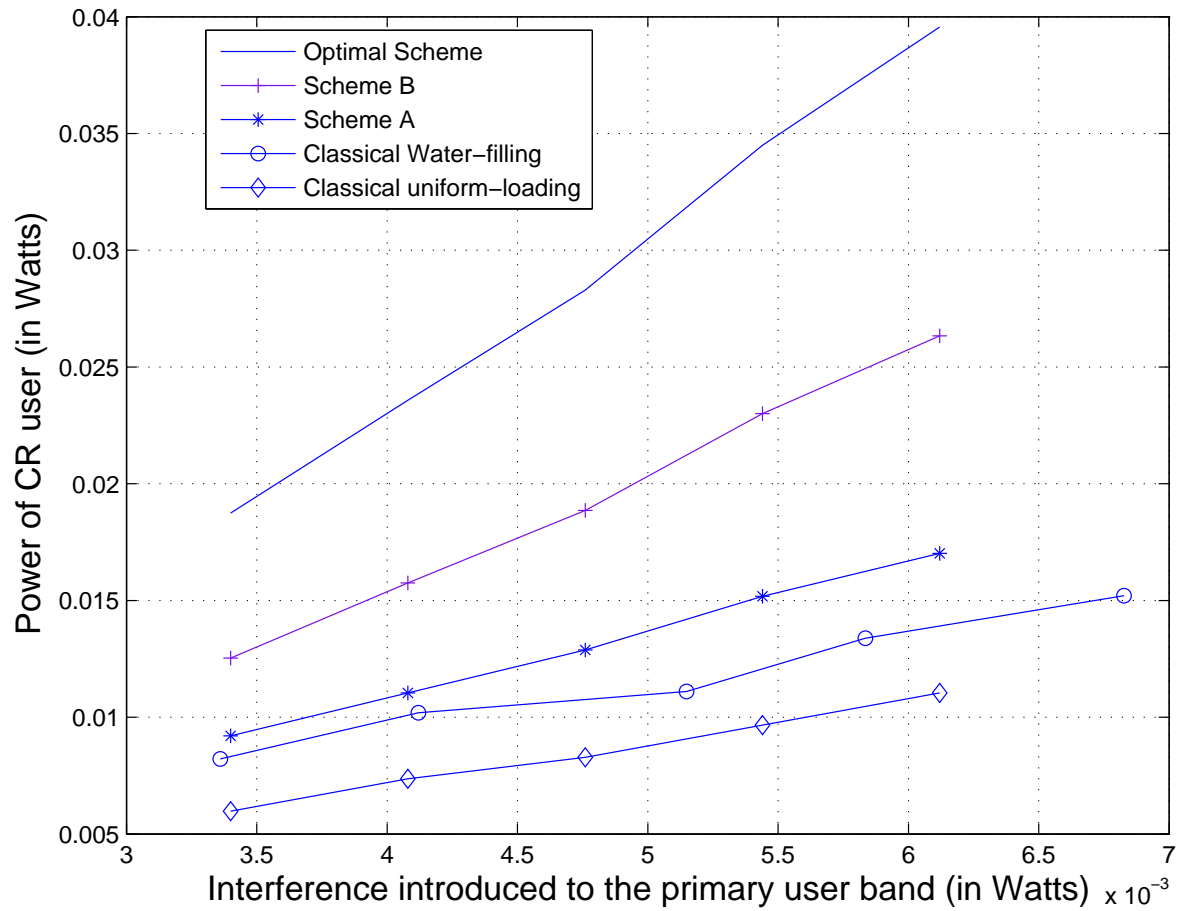


Figure 2.5: Transmit power of CR user versus interference introduced to the PU band by several schemes.

the PU band is plotted for scheme A, scheme B, the classical uniform-loading scheme and the classical water-filling scheme for various nulling scenarios in Fig. 2.6, Fig. 2.7, Fig. 2.8, and Fig. 2.9, respectively. For the sake of comparison, we have also plotted the transmission capacity of the optimal scheme in Figs. 2.6, 2.7, 2.8, and 2.9. It can be observed from the figures that after nulling, the performance of the suboptimal and classical schemes improves compared with the no-nulling case. However, the optimal scheme still achieves the highest capacity for a given interference threshold. Interestingly, it can be observed from Figs. 2.6, 2.7, 2.8, and 2.9 that for all the schemes, performance degrades for the two-nulling case compared with the one-nulling case. This result is because of the tradeoff between the interference that can be reduced by nulling additional subcarriers and the capacity that can be achieved by keeping them active. From these selected numerical results, it can be concluded that nulling additional subcarriers does not always help to improve overall system performance. We did not consider further nulling, as the associated performance degradation was checked via simulation.

2.7 Imperfect Channel-gain Information at the Transmitter

In the numerical results presented in this section, we use the values of L , N , T_s , Δf , B_1 , B_2 , B_3 , B_4 , σ^2 and P_{PU} supplied in Section 2.6. Further, the channel gains h_i^{ss} , h_i^{sp} , and h_i^{ps} are assumed to be Rayleigh fading with an average channel power gain of 10dB. Now, we assume that the CR transmitter can know the h_i^{ss} exactly as it can receive the feedback from CR receiver. But for some scenarios, e.g., closely-located scenario, it may not be possible for the CR transmitter to know the instantaneous values of h_i^{sp} and h_i^{ps} . However, the average channel gains of h_i^{sp} and h_i^{ps} can be predicted as they are closely located. Therefore, in this section, we study the performance of various algorithms assuming that only average channel gains of h_i^{sp} and h_i^{ps} are known at the CR user's BS. It is important to mention that in such case, interference constraint imposed to PUs is met in an average sense. We run our simulations for the scenario when CR transmitter only knows

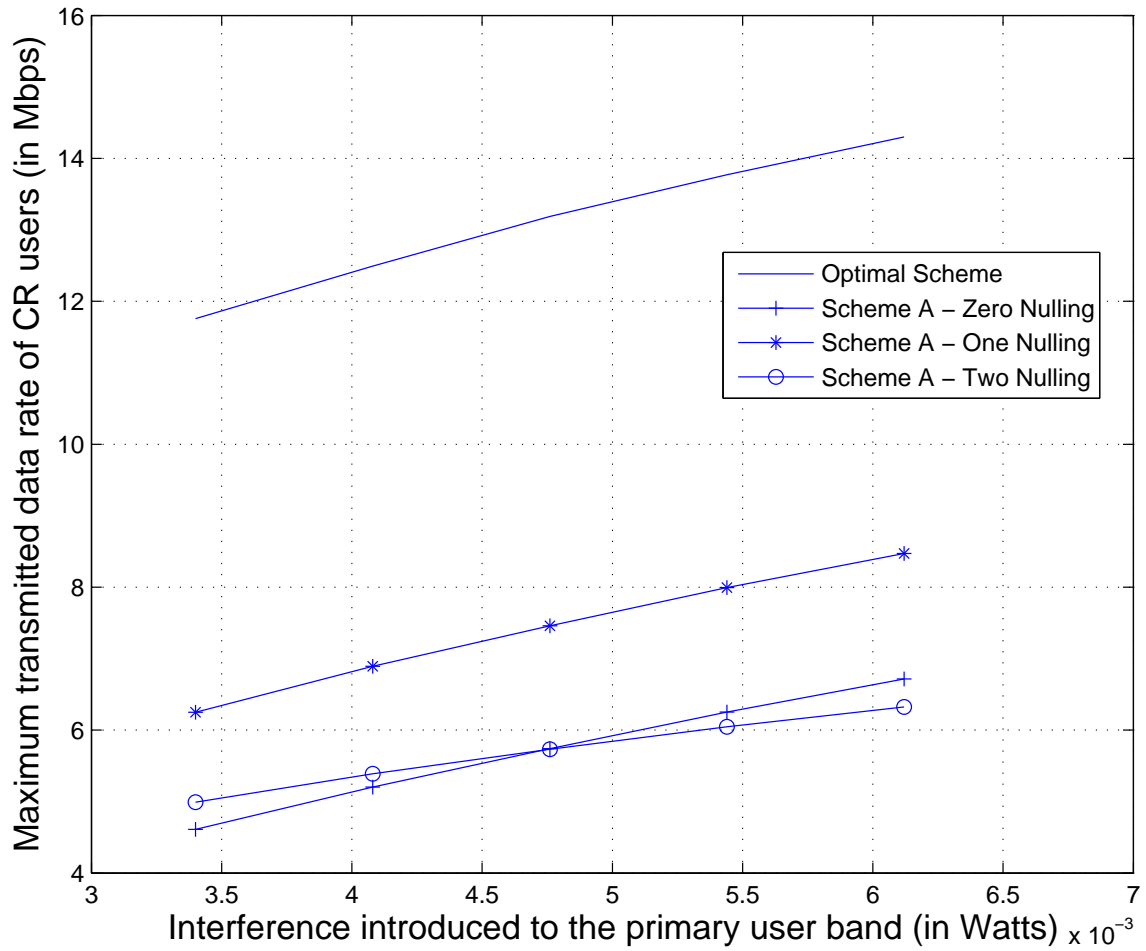


Figure 2.6: Maximum transmitted data rate of CR user versus interference introduced to the PU band by Scheme A for various nulling scenarios.

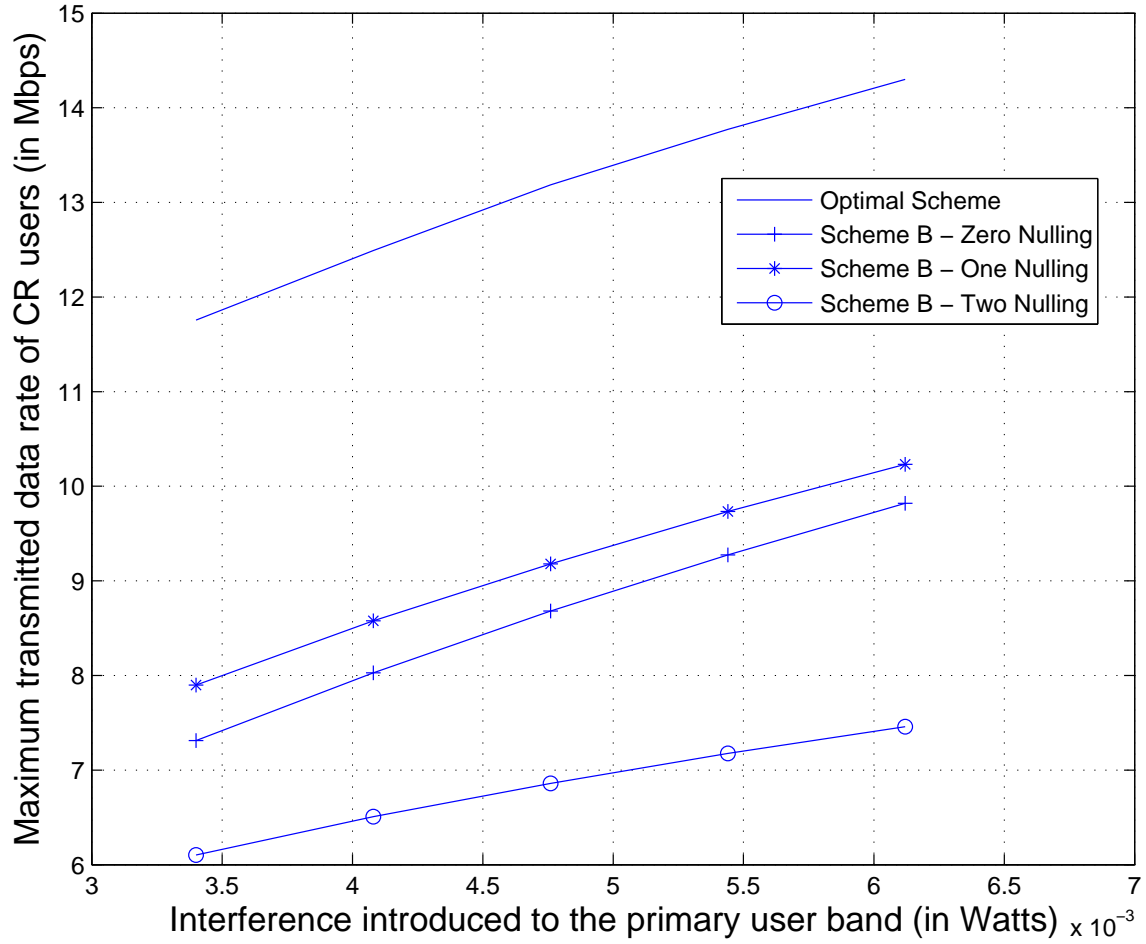


Figure 2.7: Maximum transmitted data rate of CR user versus interference introduced to the PU band by Scheme B for various nulling scenarios.

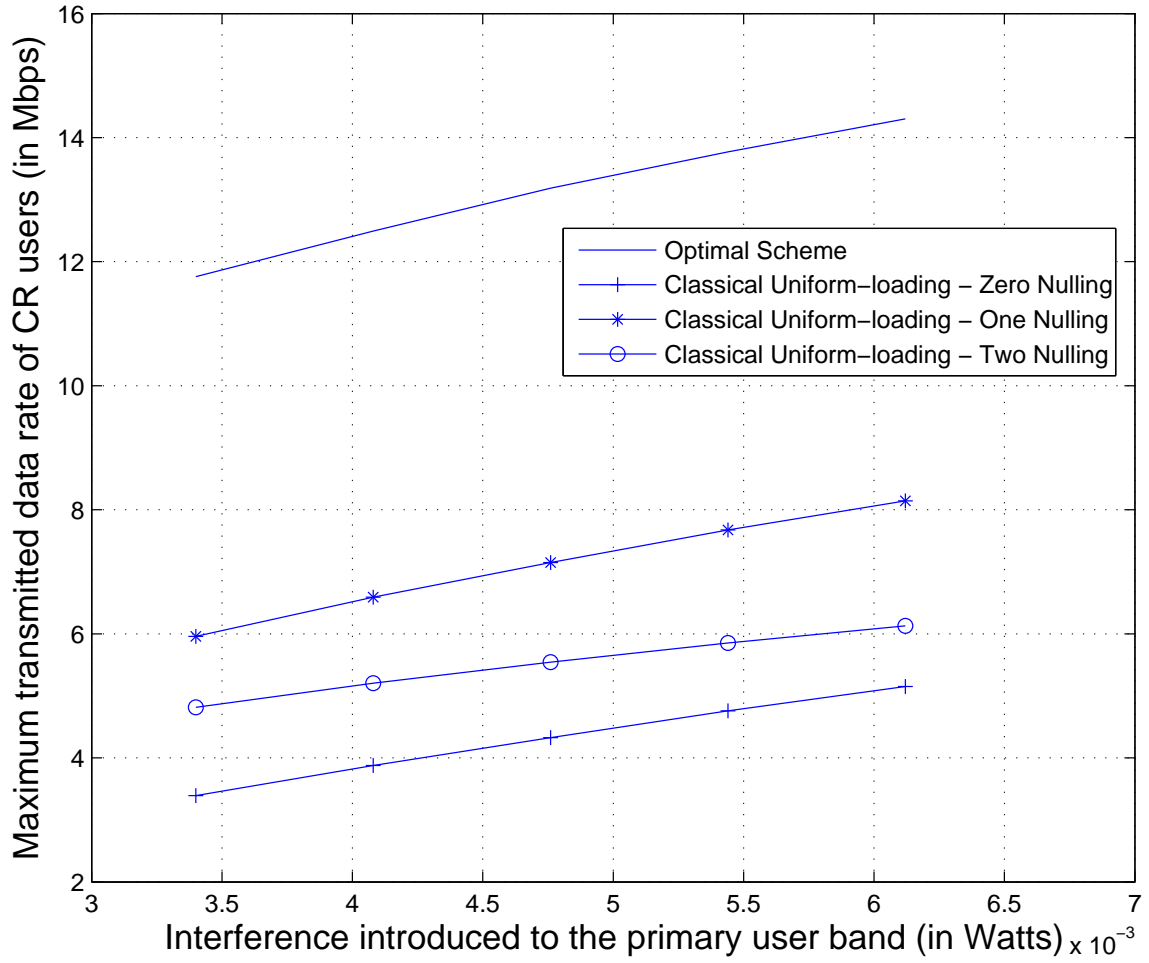


Figure 2.8: Maximum transmitted data rate of CR user versus interference introduced to the PU band by the uniform-loading scheme for various nulling scenarios.

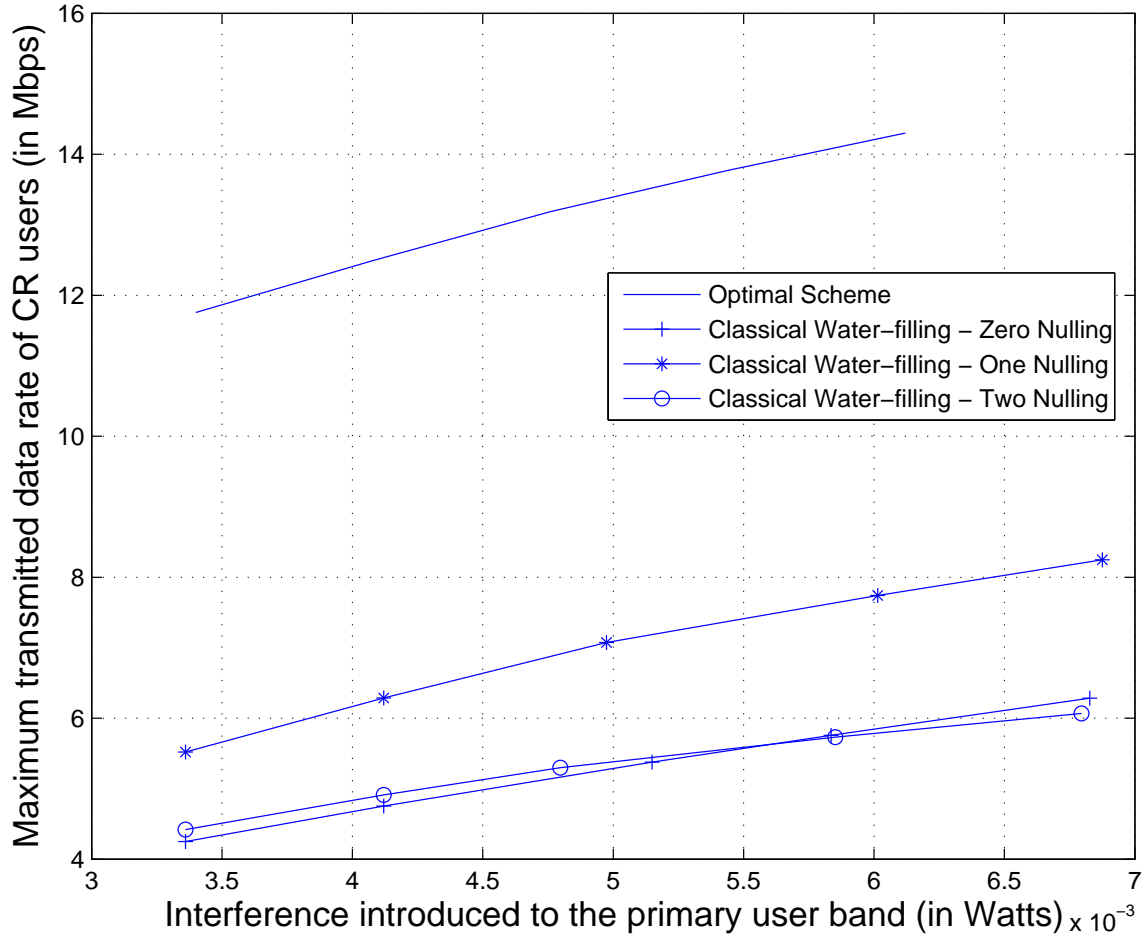


Figure 2.9: Maximum transmitted data rate of CR user versus interference introduced to the PU band by the water-filling scheme for various nulling scenarios.

the average channel power gain of h_l^{SP} and h_l^{PS} . In Fig. 2.10, we have plotted the transmission capacity of the CR user versus interference introduced to the PU band for various schemes for the scenarios of perfect and imperfect channel information at the transmitter. Again, we have presented the curve of average transmission capacity for 100,000 simulation iterations. It can be observed from Fig. 2.10, that performance degrades for the imperfect scenario. However, it should be noted that even with the imperfect channel gain information at the CR transmitter, proposed optimal and suboptimal schemes perform better than the classical schemes.

2.8 Conclusions

In this chapter, we have developed an optimal power loading algorithm that maximizes the downlink transmission data rate of the CR user while the interference introduced to the PU remains within a given limit. We have also proposed two suboptimal power loading algorithms that have less complexity but can achieve a performance close to the optimal one. Presented numerical results show that the classical loading algorithms used for conventional wireless networks perform the worst for the CR scenario among the schemes considered. We have also studied the effect of nulling mechanisms on the performances of various schemes. Selected numerical results have demonstrated that the optimal scheme performs the best, and that the one-nulling case achieves better data rate performance than cases with a greater number of nullings, as well as zero-nulling cases. Finally, we have studied the case where the channel gain information is not perfectly known at the CR transmitter and found that even in this case proposed schemes perform better than the classical schemes.

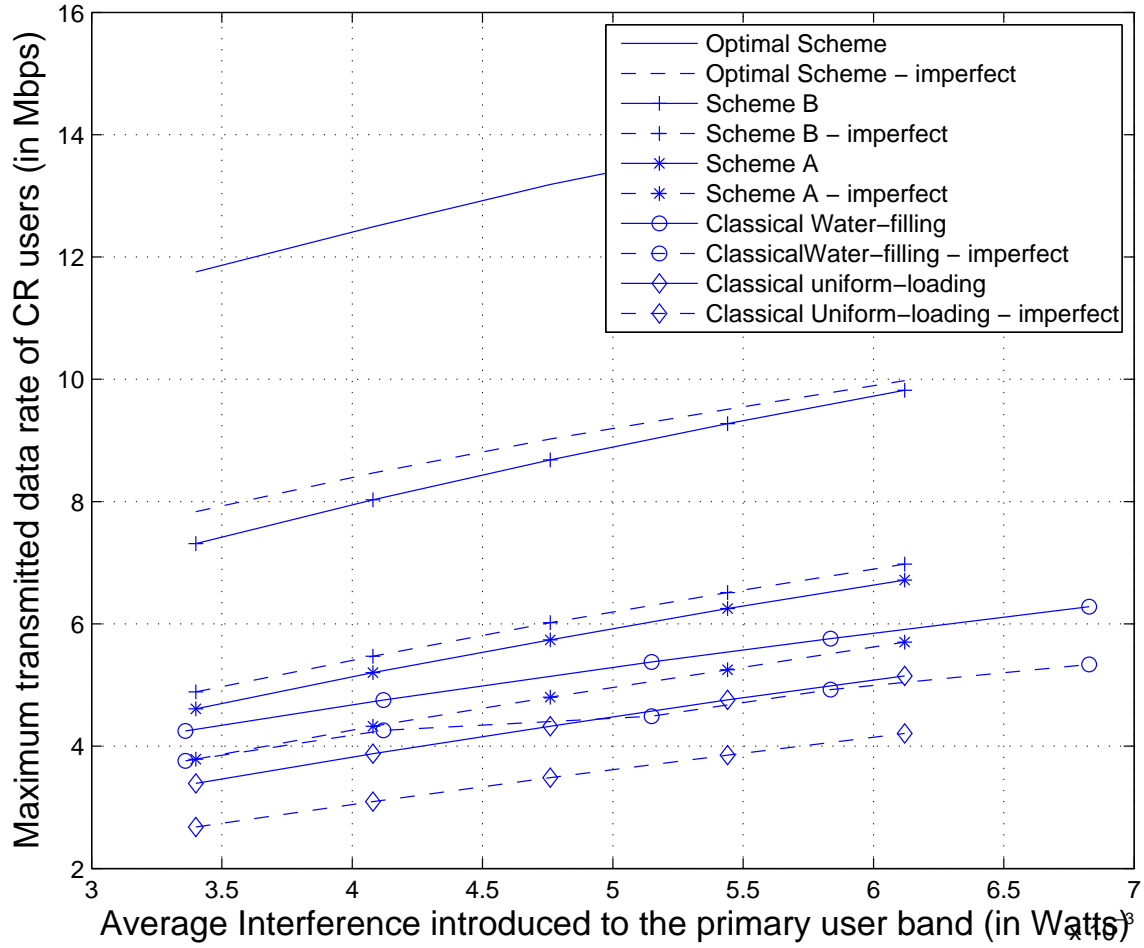


Figure 2.10: Maximum transmitted data rate of CR user versus average interference introduced to the PU band when the channel gain is not perfectly known at the CR transmitter.

Chapter 3

Discrete Bit Loading for OFDM-based Cognitive Radio Systems¹

3.1 Introduction

Cognitive radio (CR) has been proposed as a way to improve spectrum efficiency by giving opportunistic access of the frequency bands to a group of CR users for whom the band has not been licensed. Although opportunistic spectrum access would allow CR users to identify and access available spectrum resources, one of the main concerns is to utilize the available spectrum resources in an efficient manner. As such the interference introduced by the CR users to the primary users' band is kept below a prescribed threshold determined by interference temperature [6].

¹The papers based on the research work presented in this chapter have been published as: Gaurav Bansal, Md. Jahangir Hossain, and Vijay K. Bhargava, "Link Adaptation in OFDM-based Cognitive Radio Systems", in *Cognitive Radio Communications Networks*, Springer-Verlag, pp. 189-212, 2007, and Gaurav Bansal, Md. Jahangir Hossain, and Vijay K. Bhargava, "Discrete bit loading for OFDM-based cognitive radio systems", in *Proceedings of National Conference on Communications (NCC'07)*, pp. 1104-1109, Jan. 2007, Kanpur, India.

Due to great flexibility in dynamically allocating the unused spectrum among the CR users as well as the easy analysis of the spectral activity of the primary users [9], orthogonal frequency division multiplexing (OFDM) has already been recognized in the literature as a potential transmission technology for the CR systems. Since both the secondary and primary users may exist in side by side band and their access technologies may be different, the mutual interference is the limiting factor for performance of both networks. Specifically, in [17] the authors have shown that using OFDM modulation causes mutual interference between the primary and the secondary users due to the non-orthogonality of the transmitted signals. The amount of interference introduced to the primary user's band by a given subcarrier depends on the power allocated in that subcarrier as well as the distance between the subcarrier and the primary user's band.

In order to exploit the time varying nature of fading gains across the OFDM subcarriers discrete bit loading algorithms have been proposed in literature [23],[24]. These algorithms have minimized the transmitted power of an OFDM system while transmitting at a fixed data rate and at a fixed bit error rate (BER) and are useful for conventional wireless networks where there is only one group of users i.e., primary users. Since there is a mutual interference between CR and primary users when both group of users co-exist in side by side band, the discrete bit loading algorithms proposed for conventional systems e.g., [23],[24] results in higher mutual interference to the primary users. Hence, the design goal in CR system is quite different problem as the interference to the primary users should be below the given threshold. According to the classical discrete bit loading schemes e.g., [23] and [24], more power and bits should be loaded into the subcarrier which has higher channel gain. However the amount of interference, introduced by allowing secondary user's transmission, in a given subcarrier depends on the location of the subcarrier with respect to the primary user's spectrum and from this point of view, more power and hence, more bits should be loaded into a distant subcarrier. Therefore, it requires a judicious loading policy which not only considers the fading gain on the subcarriers but also the distances of the subcarriers from the

primary user band in order to minimize interference.

In this chapter, we propose a sub-optimal scheme namely scheme A, based on Lagrange formulation such that it minimizes the interference introduced to the primary users while transmitting at a fixed data rate and BER. Further, we propose an optimal scheme namely scheme B by modifying the existing Hughes-Hartogs algorithm [23] and a sub-optimal scheme namely scheme C by modifying Chow et al. algorithm [24] such that rather than minimizing the transmitting power (as required for the conventional scenario), they minimize the interference introduced to the primary users (as required for the CR scenario) while keeping a fixed data rate and BER. In all cases we assume that there is no constraint on transmit power. This assumption is practical for CR radio system specially for downlink scenario as the interference is the limiting factor. Simulation results show that the proposed schemes A, B and C introduce less interference to the primary user band as compared to existing Hughes-Hartogs and Chow et al. schemes for a specified data rate and BER of CR users. Also, scheme A is of very low complexity as compared to scheme B.

The organization of this chapter is as follows. In Section 3.2, we give the system model. The optimization problem has been formulated and solved in a sub-optimal fashion in Section 3.3. Further in Section 3.4, we present the modification in the existing optimal and suboptimal schemes. In Section 3.5, we give selected numerical results. Finally, Section 3.6 concludes the chapter.

3.2 System Model

We consider the same side by side CR access model as assumed in [17]. Basically, it is assumed that the frequency band which has been occupied by the primary users is known and the primary user band of bandwidth B is located in the middle and the bandwidth available for secondary user transmission is located on each side of primary user band as shown in Fig. 3.1. There can be more than one primary user occupying the primary user band of bandwidth B . OFDM modulation scheme is employed for the secondary users and the available bandwidth for

secondary users is divided into N subcarriers, $N/2$ on each side and each having a bandwidth of Δf .

We assume that each subcarrier has a bandwidth that is much smaller than the coherence bandwidth of the channel and that the instantaneous channel gains are perfectly known at the transmitter. The transmit bits are adaptively loaded in each subcarrier. Due to the coexistence of primary and secondary users in such fashion, there are two types of interference in the system [17]. One is introduced by the primary user into the CR user band and the other is introduced by the cognitive user into the primary user band as described below.

3.2.1 Interference Introduced by the Secondary User's Signal

The power density spectrum of the i^{th} subcarrier in CR user band can be written as [17]

$$\phi_i(f) = P_i T_s \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2, \quad (3.1)$$

where P_i is the total transmit power emitted by the i^{th} subcarrier in CR user's band and T_s is the symbol duration. The interference introduced by the i^{th} subcarrier to the primary user band is the integration of the power density spectrum of the i^{th} subcarrier across the primary user band and can be written as

$$I_i(d_i, P_i) = P_i T_s \int_{d_i - B/2}^{d_i + B/2} \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 df, \quad (3.2)$$

where d_i represents the spectral distance between the i^{th} subcarrier of CR user band and the primary user band. $I_i(d_i, P_i)$ represents the interference introduced by the i^{th} subcarrier for a transmit power, P_i into the primary user's band.

3.2.2 Interference Introduced by the Primary User's Signal

The power density spectrum of the primary user signal after the M-fast Fourier transform (FFT) processing can be expressed by the following expected value of

the periodogram [17]

$$E\{I_N(w)\} = \frac{1}{2\pi M} \int_{-\pi}^{\pi} \phi_{PU}(e^{jw}) \left(\frac{\sin(w - \psi)M/2}{\sin(w - \psi)/2} \right)^2 d\psi, \quad (3.3)$$

where w represents the frequency normalized to the sampling frequency and $\phi_{PU}(e^{jw})$ is the power density spectrum of the primary user signal. Primary user signal has been taken as an elliptically filtered white noise process with an amplitude P_{PU} [17]. The interference introduced by the primary user signal to the i^{th} subcarrier will be the integration of the power density spectrum of the primary user signal across the i^{th} subcarrier and can be written as

$$J_i(d_i, P_{PU}) = \int_{d_i - \Delta f/2}^{d_i + \Delta f/2} E\{I_N(w)\} dw, \quad (3.4)$$

where $J_i(d_i, P_{PU})$ represents the interference introduced by the primary user signal into the i^{th} subcarrier of CR user's band. For a large number of PUs this interference introduced into a CR subcarrier can be assumed to be additive white Gaussian noise (AWGN) from central limit theorem. Here, we assume a single PU but in a practical system there would be lot of PUs interfering with the CR subcarrier and hence, we assume the interference J_i to be AWGN.

3.3 Proposed Scheme A

In this section, we propose a sub-optimal scheme which minimizes the interference introduced to the primary user band while transmitting at a fixed data rate and BER. We use Lagrange formulation for solving the optimization problem, but as it is difficult to solve the combinatorial optimization we assume the rates to be continuous and after the optimization we round them to integer numbers and

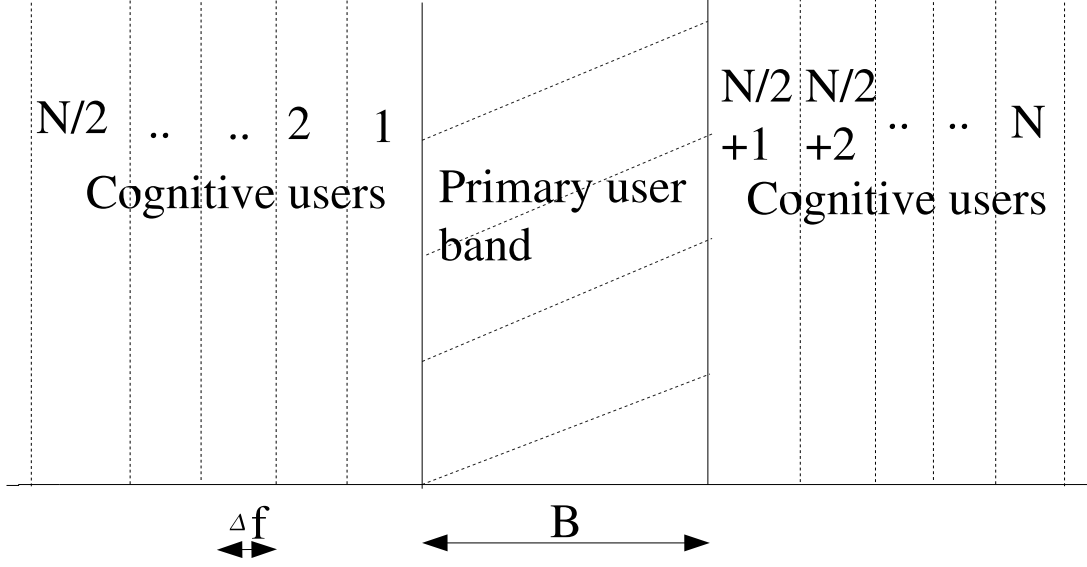


Figure 3.1: Distribution of primary and secondary users

hence, the scheme is sub-optimal. It can be written mathematically as follows:

$$\min_{R_i \in \mathcal{R}} \sum_{i=1}^N I_i(d_i, P_i(R_i)), \quad (3.5)$$

$$s.t. \quad : \quad \sum_{i=1}^N R_i = R_{spec}, \quad (3.6)$$

$$BER_i(P_i, R_i, h_i) \leq BER_{spec}, \quad (3.7)$$

where $\mathcal{R} = \{0, 1, 2, \dots\}$ represents the integer transmission rate, N denotes the total number of secondary users, R_i denotes the transmitted rate for the i^{th} subcarrier, BER_i denotes the BER for the i^{th} subcarrier. From the assumption in Section 3.2.2. that the interference J_i is AWGN, BER can be expressed as [32]

$$BER_i = 0.2 \exp \left[\frac{-1.6 P_i h_i^2}{(\sigma^2 + J_i)(2^{R_i} - 1)} \right] \quad (3.8)$$

and R_{spec} and BER_{spec} respectively denotes the specified data rate and BER.

Now, from Eqs. (3.7) and (3.8) we can write,

$$P_i = \frac{-1}{1.6} \frac{(\sigma^2 + J_i)(2^{R_i} - 1)}{h_i^2} \ln(5BER_{spec}). \quad (3.9)$$

Further, we define $k(i) = \frac{\partial L_i}{\partial P_i}$ and then, substituting Eq. (3.2) in Eq. (3.5) we can write,

$$\min_{R_i \in \mathcal{R}} \sum_{i=1}^N k_i P_i. \quad (3.10)$$

By substituting Eq. (3.9) in Eq. (3.10) we can write,

$$\min_{R_i \in \mathcal{R}} \sum_{i=1}^N \frac{-k_i \ln(5BER_{spec})}{1.6} \frac{(\sigma^2 + J_i)(2^{R_i} - 1)}{h_i^2}. \quad (3.11)$$

Now using, Lagrange optimization in Eq. (3.6) and Eq. (3.11), we can write

$$L = \sum_{i=1}^N \frac{-k_i \ln(5BER_{spec})}{1.6} \frac{(\sigma^2 + J_i)(2^{R_i} - 1)}{h_i^2} - \lambda \left(\sum_{i=1}^N R_i - R_{spec} \right). \quad (3.12)$$

For the moment, we do not confine R_i to integers and now differentiating Eq. (3.12) with respect to R_i we can write,

$$\frac{\partial L}{\partial R_i} = \frac{-k_i \ln(5BER_{spec})}{1.6} \frac{(\sigma^2 + J_i)(2^{R_i} \ln(2))}{h_i^2} - \lambda = 0. \quad (3.13)$$

Now, from Eq. (3.13) we can write,

$$R_i = \log_2(\lambda) + \log_2\left(\frac{-1.6h_i^2}{k_i \ln(5BER_{spec})(\sigma^2 + J_i) \ln(2)}\right). \quad (3.14)$$

Substituting Eq. (3.14) into Eq. (3.6), we can write

$$\log_2(\lambda) = \frac{R_{spec}}{N} + \frac{1}{N} \sum_{i=1}^N \log_2\left(\frac{-k_i \ln(5BER_{spec})(\sigma^2 + J_i) \ln(2)}{1.6h_i^2}\right). \quad (3.15)$$

Now, from Eq. (3.14) and Eq. (3.15), we can write

$$R_i = \frac{R_{spec}}{N} + \frac{1}{N} \sum_{i=1}^N \log_2 \left(\frac{-k_i \ln(5BER_{spec})(\sigma^2 + J_i) \ln(2)}{h_i^2 1.6} \right) + \log_2 \left(\frac{-1.6h_i^2}{k_i \ln(5BER_{spec})(\sigma^2 + J_i) \ln(2)} \right). \quad (3.16)$$

If R_i in Eq. (3.16) comes out to be negative for some subcarriers, zero bit is assigned to those particular subcarriers and we reiterate the whole scheme for remaining subcarriers. Now, as rates R_i can only be integers, we round R_i to nearest integer R_{qi} and the round off error $\Delta R_i = R_i - R_{qi}$ is determined. The next part of the scheme is adopted from [24]. The sum $\sum_{i=1}^N R_{qi}$ is calculated and if it is larger (smaller) than the R_{spec} , then the rate of the channel with the smallest (largest) ΔR_i is decremented (incremented). The algorithm stops when $\sum_{i=1}^N R_{qi} = R_{spec}$.

3.4 Modifications in the Existing Schemes

3.4.1 Scheme B

The Hughes-Hartogs algorithm [23] incrementally allocates an integer number of bits at the cost of high computational complexity. The algorithm works in a manner such that bits are added successively to the subcarrier which will require least amount of power for the specified BER. But for the CR scenario, the goal is to minimize the interference introduced to the primary user's band. Hence we allocate the bits in a manner such that successively bits are added to the subcarrier which will introduce least amount of interference to the primary user band. The modified algorithm works as follows

1. The subcarrier by which the interference introduced to the primary user band will be least for the specified BER in assigning one more bit is searched.

2. The bit is assigned to the subcarrier which introduces minimum interference.
3. Keep on repeating till the given data rate is achieved.

It is obvious that the above algorithm requires extensive searching and hence it is slow. However, it is optimal as the bits are loaded in a manner such that the total interference will be minimized. Although the scheme B is optimal, it is very slow for practical applications as compared to scheme A which has a closed form expression.

3.4.2 Scheme C

The Chow et al. algorithm [24] omits intensive sorting as it allocates rate among the subcarriers according to the channel capacity approximation. The goal of the algorithm is to minimize the transmitted power or to maximize the noise margin given a data rate and a target BER, where noise margin (γ_{margin}) is defined as the amount of additional noise that can be tolerated, while still achieving the specified BER. The algorithm can be described as follows [24]

1. Initialize $\gamma_{margin} = 0$, IterateCount(number of iterations) = 0, UsedCar(total number of subcarriers) = N and $\varepsilon(i)$ (energy of a particular subcarrier) = 1.
2. For $\forall i$ calculate

$$R(i) = \log_2\left(1 + \frac{SNR(i)}{\tau + \gamma_{margin}}\right) \quad (3.17)$$

where τ is the SNR gap in the gap approximation [33].

$$\hat{R}(i) = \text{round}[R(i)] \quad (3.18)$$

where $\hat{R}(i)$ is an integer number of bits that are assigned to a particular subcarrier.

$$diff(i) = R(i) - \hat{R}(i). \quad (3.19)$$

$$\text{If } \hat{R}(i) = 0, \text{UsedCar} = \text{UsedCar} - 1. \quad (3.20)$$

Now, if $R_{total} = \sum_{i=1}^N \hat{R}(i) = 0$, the algorithm is stopped.

3. If $R_{total} \neq 0$, the new γ_{margin} is calculated according to:

$$\gamma_{margin} = \gamma_{margin} + 10 \log_{10} \left(2^{\frac{R_{total} - R_{spec}}{\text{UsedCar}}} \right) \quad (3.21)$$

where R_{spec} is the given data rate. Increment IterateCount by 1.

4. If $R_{total} \neq R_{spec}$ and IterateCount < MaxCount (maximum number of allowed iterations), let UsedCar = N and go to step 2, else go to step 5. It should be noted that if the algorithm does not converge after MaxCount iterations, convergence is forced using step 5.
5. If $R_{total} > R_{spec}$, then one bit is subtracted from the subcarrier which has the minimum diff(i) and it is repeated until R_{total} becomes equal to R_{spec} . On the other hand, if $R_{total} < R_{spec}$, then one bit is added to the subcarrier which has the maximum diff(i) and it is repeated until R_{total} becomes equal to R_{spec} .
6. Power of each subcarrier is adjusted such that the BER of each subcarrier (here, we exclude the subcarriers which have been assigned 0 bits) is equal to the specified BER for the given bit allocation $\hat{R}(i)$.

Basically, the algorithm first finds the optimal system performance margin (in steps 1 to 4), and then if the algorithm does not converges in MaxCount iterations, forced convergence is imposed (in step 5) and finally, it adjusts the input energy distribution.

Now, for the CR scenario we change the Eq. (3.17), so that the bits are allocated more to the sub-carriers which are far from the primary user band, as they

introduce less interference. We define $const(i)$ as follows

$$const(i) = (1/k(i)) / \sum_{i=1}^N (1/k(i)). \quad (3.22)$$

It should be noted that the subcarriers which are near to the primary user band introduce more interference and hence, has a higher value of $k(i)$ as compared to the subcarriers which are far from the primary user band. For the CR scenario, we modify Eq. (3.17) as follows

$$R(i) = \log_2(1 + \frac{const(i)SNR(i)}{\tau + \gamma_{margin}}). \quad (3.23)$$

Hence, by introducing $const(i)$, we are allocating less bits to subcarriers which are near to the primary user band as they produce more interference and more bits are given to the subcarriers which are far from the primary user band as they produce less interference. Rest of the algorithm remains same.

3.5 Numerical Results

In the numerical results presented in this section, we use the values of T_s to be 4μ seconds. We assume the values of Δf to be 0.3125 MHz, which is same as subcarrier frequency spacing in wireless local area network (LAN) standards [30], [31]. The value of B has been taken to be 0.3125 MHz. Noise variance of $10^{-6}W$ is assumed. The channel fading power gain h_i is assumed to be independent and identically distributed (i.i.d.) Rayleigh faded with an average channel power gain as 5dB. A specified BER of 10^{-7} is used and the value of τ is taken to be 9.8dB as in [24]. The value of amplitude P_{PU} is assumed to be 1mW. The value of $MaxCount$ is taken to be 10. Further, we assume that there are 10 subcarriers for CR users, 5 on each side of the PU band. ²

In Fig. 3.2, we present the interference introduced to the primary user band

²In practice the value of total number of CR subcarriers would be high, but for simplicity in the simulation analysis we have assumed the value to be 10.

versus given data rate plots for the proposed schemes A, B and C, and conventional Hughes-Hartogs scheme and Chow et al. scheme. The plotted data rates are average of 100,000 independent simulation runs. From Fig. 3.2, we observe that for a given data rate existing Hughes-Hartogs and Chow et al. schemes introduce more interference to the primary user band as compared to schemes A, B and C. Scheme B is optimal and introduces the minimum interference to the primary user band. Scheme A is sub-optimal and it introduces less interference as compared to sub-optimal scheme C and same as optimal scheme A. It should be noted that the sub-optimal scheme A has similar performance as the optimal scheme B because the quantization error does not produce any visual degradation in the performance. Further in Fig. 3.2, we have marked the power values for some selected data rates. We observe from these values that the conventional optimal Hughes-Hartogs scheme, which minimizes the power, requires the least power for a given data rate. Our proposed schemes A, B and C require higher transmit power than the Hughes-Hartogs scheme as they assign more power to the distant subcarriers compared to the nearest subcarriers in order to reduce the interference to the primary user band.

3.6 Effect of Subcarriers Nulling Mechanism

In [17], the authors have studied the effect of nulling the subcarriers and have shown that the interference introduced to the primary user band can be reduced by nulling the subcarriers which are adjacent to the primary user band. The reason behind it is that the adjacent subcarriers produce the maximum amount of interference. It also implies that for a given capacity more power will have to be allocated to the remaining subcarriers, so that the target capacity can be achieved. As a consequence one can expect that higher power needs to be allocated and which might lead to higher interference. Therefore, nulling creates a trade-off. Here we study the effect of nulling on the proposed scheme C, Hughes-Hartogs scheme and Chow et al. scheme. It should be noted that schemes A and B perform optimally and hence, nulling has no significance for these schemes.

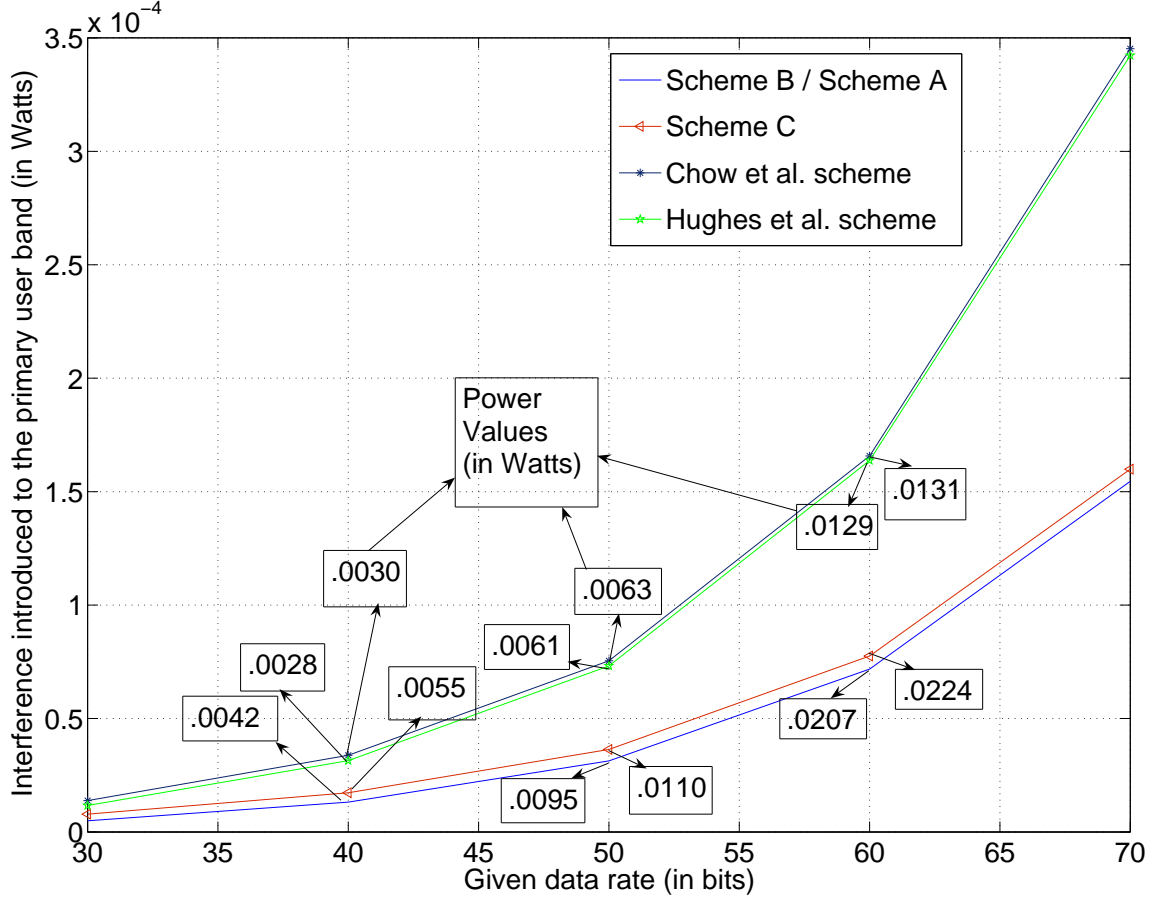


Figure 3.2: Interference introduced to the primary user band versus given data rate (BER = 10^{-7})

In Fig. 3.3, we plot the interference introduced to the primary user band vs. given data rate plots for the proposed scheme C under various nulling. Here, by one nulling we mean that we null one subcarrier from each sides of the primary user band that are nearest to it and similarly for higher values of nulling. Similar results have been plotted in Figs. 3.4 and 3.5, for Hughes-Hartogs scheme and Chow et al. scheme. In these plots we have also plotted the data rate of the optimal scheme for sake of comparison. From Figs. 3.3, 3.4 and 3.5, we can observe

that in several data points after nulling the performance of scheme C, Hughes-Hartogs scheme and Chow et al. scheme improves as compared to no nulling, but still the optimal scheme performs the best and transmits maximum data rate for a given interference threshold. We did not consider more number of nulling as the performance degrades and they have been checked via simulation. Also, from the plots we observe that for most of the data points two nulling performs the worst. It implies that after one nulling the more power that has been allocated to the remaining subcarriers dominates the reduction in interference due to the nulling of adjacent subcarriers. In Figs. 3.3, 3.4 and 3.5, we have also presented the power values used by the schemes under various nulling. The values presented can be used as an important design parameter in designing OFDM-based cognitive radio systems.

3.7 Conclusions

In this chapter we have studied the interference performance of the well-known discrete bit loading algorithms when they are used for OFDM-based CR systems. In order to minimize the interference to the primary user's band, a suboptimal algorithm has been proposed. We have also proposed two schemes based on modifications in the existing discrete bit loading schemes namely the Hughes-Hartogs [23] and Chow et al. [24] schemes. Presented numerical results demonstrate the strength of our proposed schemes.

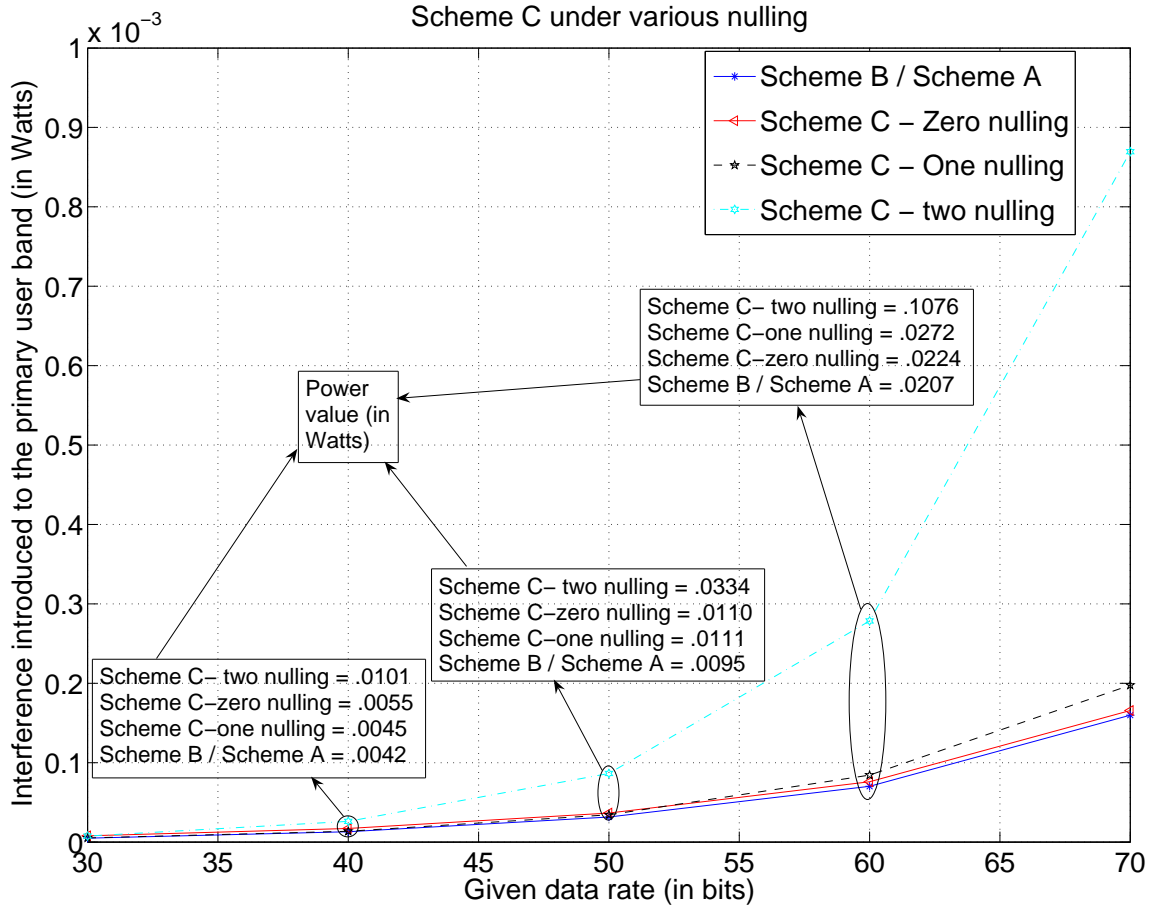


Figure 3.3: Interference introduced to the primary user band versus given data rate ($\text{BER} = 10^{-7}$) for Scheme C under various nulling.

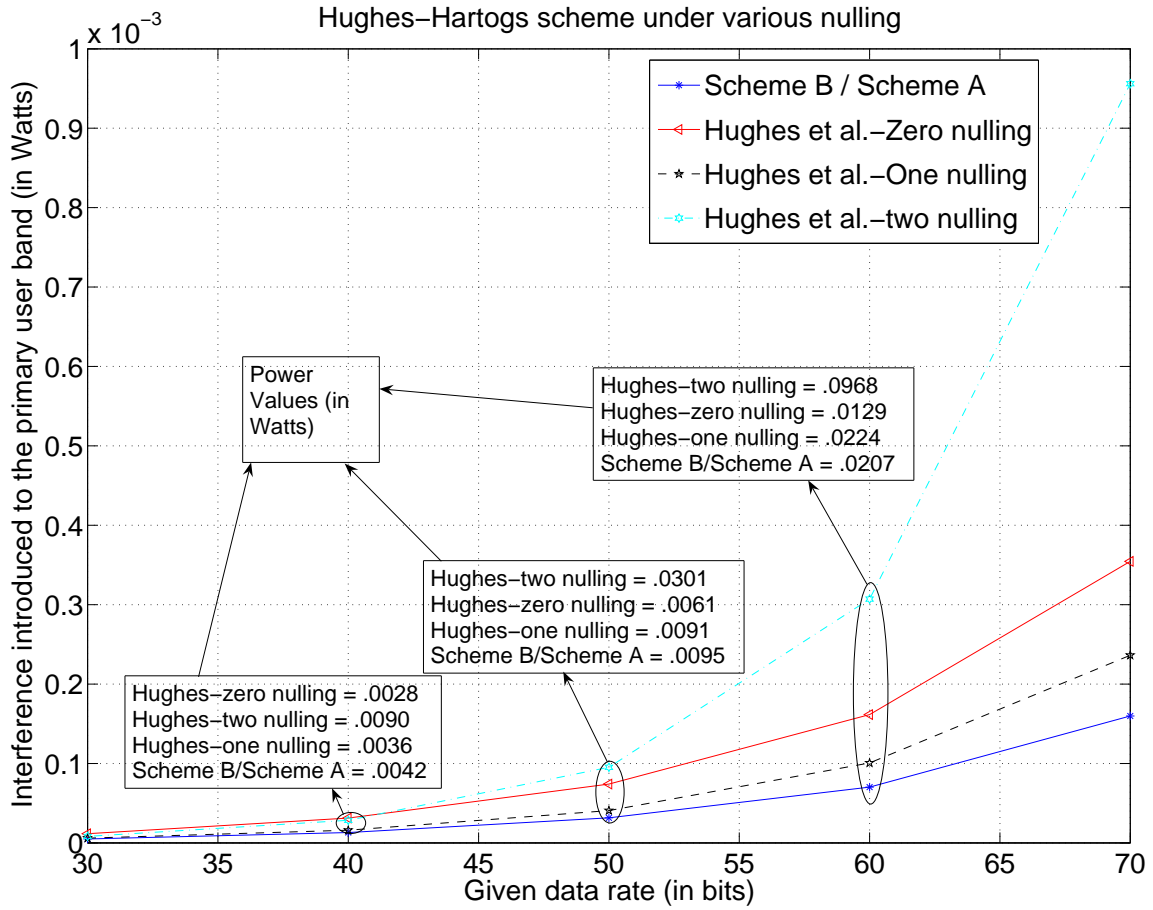


Figure 3.4: Interference introduced to the primary user band versus given data rate ($\text{BER} = 10^{-7}$) for Hughes-Hartogs scheme under various nulling.

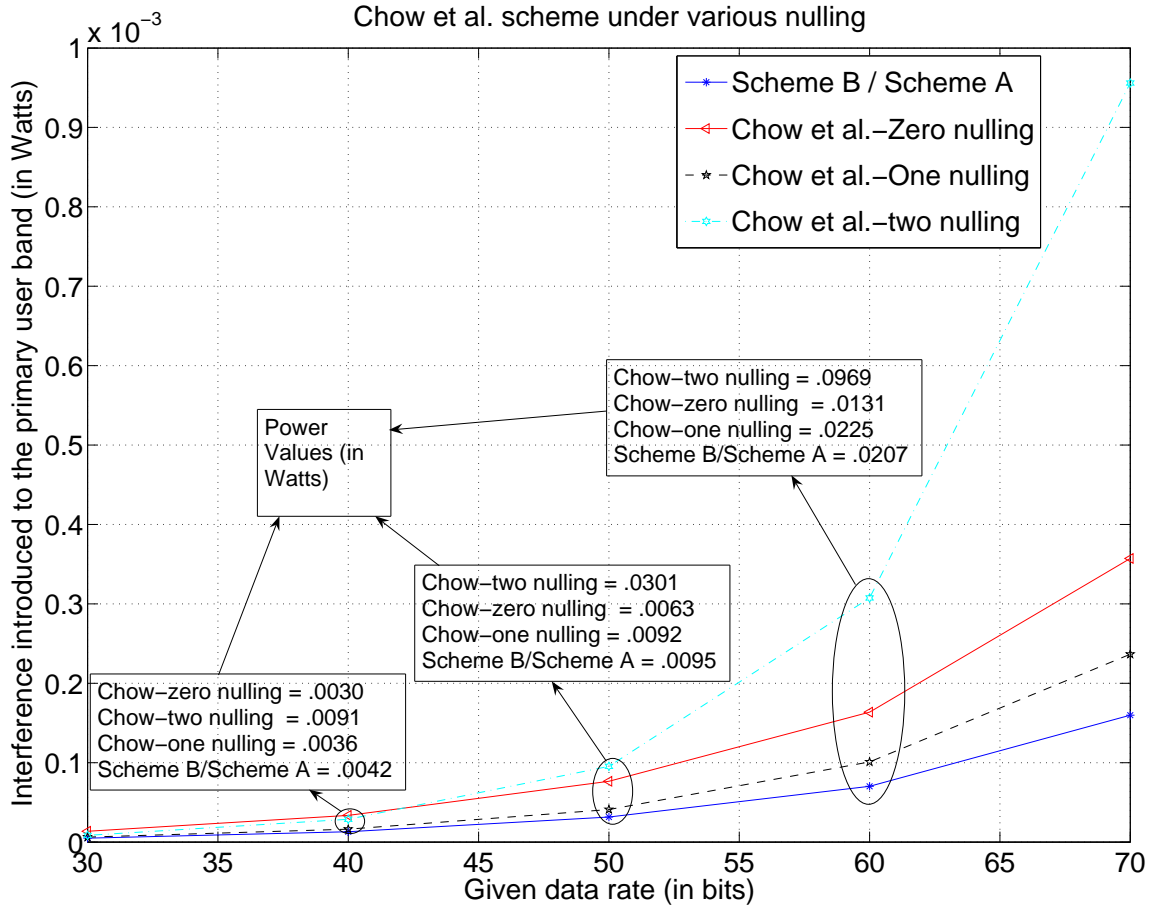


Figure 3.5: Interference introduced to the primary user band versus given data rate ($\text{BER} = 10^{-7}$) for Chow et al. scheme under various nulling.

Chapter 4

Adaptive Power Loading for OFDM-based Cognitive Radio Systems with Statistical Interference Constraint¹

4.1 Introduction

The utilization of valuable radio spectrum resource can be improved significantly by using the cognitive radio (CR) technology [6]. Orthogonal frequency division multiplexing (OFDM), because of its flexibility in allocating the spectrum, has been recognized as an air interface technology for CR systems [9]. For example, implementation details and some of the advantages of OFDM-based CR systems have been discussed in [22]. Because of the coexistence of CR and primary users in side-by-side bands, mutual interference between these users is the limit-

¹A paper based on the research work presented in this chapter has been published as: Gaurav Bansal, Md. Jahangir Hossain, and Vijay K. Bhargava, “Adaptive Power Loading for OFDM-based Cognitive Radio Systems with Statistical Interference Constraint”, accepted in *IEEE Transactions on Wireless Communications*, Feb., 2011.

ing factor in order to achieve a good performance for CR systems [17]. Use of the classical power allocation algorithms e.g., well known water-filling algorithm for CR systems may result in higher interference to the primary user (PU) bands. In [34], we proposed a power loading algorithm that maximize the downlink transmission capacity of the CR user while keeping the total interference introduced to different PU bands below a specified threshold. A distributed algorithm for optimal resource allocation in orthogonal frequency division multiplexing access (OFDMA)-based CR systems has been proposed in [35]. Several other resource allocation schemes for OFDM-based CR systems have been proposed in [36], [37], [38], [39].

Most of the above mentioned works assumed that the instantaneous channel qualities from the CR transmitter to both CR and PU receivers are known perfectly at the CR transmitter. In general, when perfect channel state information (CSI) is available at the CR transmitter, it can be exploited to increase the transmission rate of CR users [40]. Further, it is shown in [41] that if the channel fading gain between PU transmitter and PU receiver along with the channel gains between CR transmitter to CR and PU receivers is known at the CR transmitter, a superior power-control policy can be designed. However, it may not be a practical assumption that the perfect CSI is known at the CR transmitter. In particular although the fading gains among CR transmitter and its receiver can be known at the CR transmitter via feedback channel, it is difficult, if not impossible, to estimate the instantaneous channel fading gains between CR transmitter and PU receivers. For some scenarios, the CR transmitter can have information about the statistics of the channel fading gain among the CR transmitter and the PU receivers. For example, in [42], authors have argued that from the pilot signals transmitted by a PU, the mean value of the random channel fading gain between a PU receiver and the CR transmitter can be estimated.

In a recent work in [43] resource allocation algorithms have been proposed for multiple input multiple output (MIMO) OFDM-based CR systems where imperfect CSI is assumed at the CR transmitter. However, the authors have as-

sumed a bounded channel uncertainty model, where the channel estimation error is bounded by some threshold. For such a bounded uncertainty model, interference introduced to PU band has been guaranteed to be below a specified interference threshold. Similar, bounded uncertainty model is used in [44],[45]. Further, in a recent work in [46], a statistical channel uncertainty model is assumed where average interference introduced to the PU band is kept below the specified limit. In this chapter, we propose a probabilistic interference model for OFDM-based CR systems where channel fading gain can have any value (i.e., it is not bounded) and interference is guaranteed in a statistical manner.

Following our earlier work in [34], there are three key contributions in this chapter compared to the work in [34]. First, the proposed power allocation scheme requires the knowledge of channel fading statistics² instead of instantaneous channel fading gain among the CR transmitter and PU receivers. To this regard, we have proposed a probabilistic model of interference threshold where interference can be guaranteed only in a statistical manner. Second, the considered co-existence scenario in this chapter is more generalized than the one presented in [34]. In particular, in the system model presented in this chapter, different PU receivers may have different interference constraints. As such it provisions for different quality of transmission for different PUs. Third, the presented model also considers that the CR transmitter has a maximum transmit power limit. The power limitation shows two regions in achievable capacity of the CR user; one is power limited region and other is interference limited region.

As such the transmission capacity of CR user is maximized for different statistical interference constraints imposed by different PUs. We also propose a reduced complexity suboptimal power allocation scheme. Our simulation results demonstrate the strength of our proposed schemes compared to the classical schemes for the CR scenario. Specifically, we show that the optimal scheme can load more power into CR users' OFDM sub-carriers' in order to achieve higher transmis-

²We assume a non-line-of-sight (NLOS) propagation environment. So that the random amplitude fading gain can be modeled as Rayleigh distributed and by statistics we refer to both Rayleigh distribution and its mean value.

sion capacity while keeping the interference below a given threshold with certain probability. The optimal scheme can achieve higher transmission capacity than classical schemes. The suboptimal scheme which has same complexity as uniform-loading scheme can achieve higher transmission capacity than the uniform power-loading based scheme. Water-filling scheme, which is optimal power allocation for conventional OFDM-based systems and has higher complexity, outperforms the sub-optimal scheme.

The organization of this chapter is as follows. While Section 4.2 describes the system model, Section 4.3 presents the problem formulation and the optimal solution for power allocation over CR OFDM subcarriers. In Section 4.4, we propose a low-complexity suboptimal scheme and present the classical schemes. Selected numerical results and complexity analysis of proposed schemes is presented in Section 4.5. Finally, Section 4.6 concludes the chapter.

4.2 System Model

In the frequency domain, we consider that CR user and L PU users co-exist in side-by-side bands as depicted in Fig. 4.1 (similar co-existence model is used in [34]). We assume that the bandwidth available to the CR user is divided into N subcarriers with bandwidth Δf Hz. The L PUs are assumed to be occupying frequency bands of bandwidth B_1, B_2, \dots, B_L , respectively. In spatial domain, depicted in Fig. 4.2, there are two types of channel gains (i) h_i^{ss} is the channel gain between CR transmitter and the CR receiver in i th OFDM subcarrier (it is assumed that h_i^{ss} 's are independent and identically distributed (i.i.d.)), and (ii) h_l^{sp} is the channel gain between the CR transmitter and l th ($l = 1, 2, \dots, L$) PU receiver (assumed to be i.i.d.). We assume that the PU bands experience flat fading. We assume that the instantaneous channel gain h_i^{ss} is known at the CR transmitter. However, it is assumed that instead of instantaneous channel gain h_l^{sp} , the distribution type and the corresponding distribution parameters are known at CR transmitter.

For ideal modulation and coding scheme, the transmission rate of the CR user in i th subcarrier, C_i is connected via Shannon formula and can be expressed as

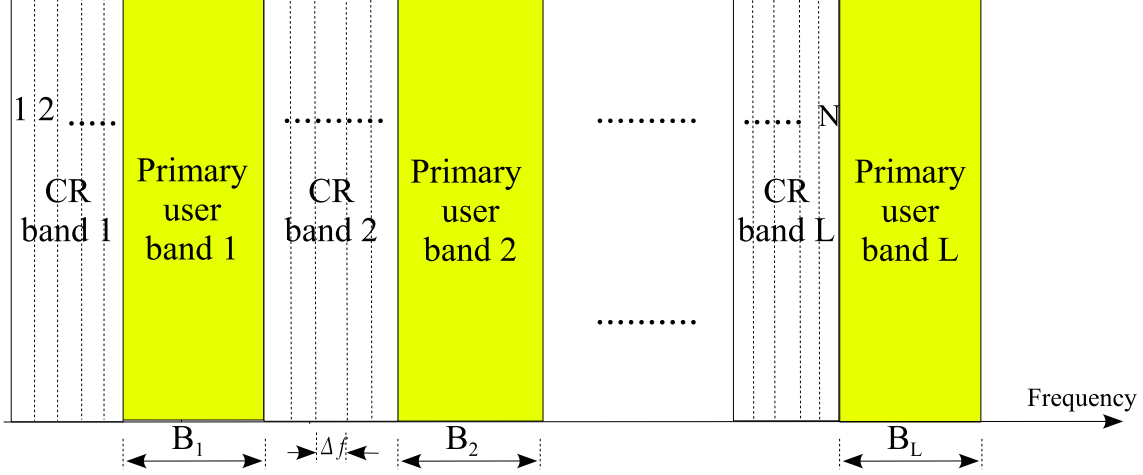


Figure 4.1: Co-existence of PU and CR users in the frequency domain.

[29]

$$C_i = \Delta f \log_2 \left(1 + \frac{|h_i^{ss}|^2 P_i}{\sigma^2 + \sum_{l=1}^L J_i^{(l)}} \right) \quad (4.1)$$

where P_i is the power of the CR user in the i th subcarrier, σ^2 is the additive white Gaussian noise (AWGN) variance and $J_i^{(l)}$ denotes the interference introduced by l th PU transmitter in the i th subcarrier of CR user. It is assumed that the interference $J_i^{(l)}$ is AWGN. We assume that the CR receiver is a smart receiver and the interference value $J_i^{(l)}$ can be measured at the CR receiver and hence, is known at the CR transmitter via a feedback channel. The interference introduced by i th CR subcarrier transmission in the l th PU band can be expressed as [17], [34],

$$I_i^{(l)} = P_i |h_l^{sp}|^2 T_s \int_{d_{il}-B_l/2}^{d_{il}+B_l/2} \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 df \quad (4.2)$$

where T_s is the symbol duration and d_{il} represents the spectral distance between the i th CR subcarrier and l th PU band.

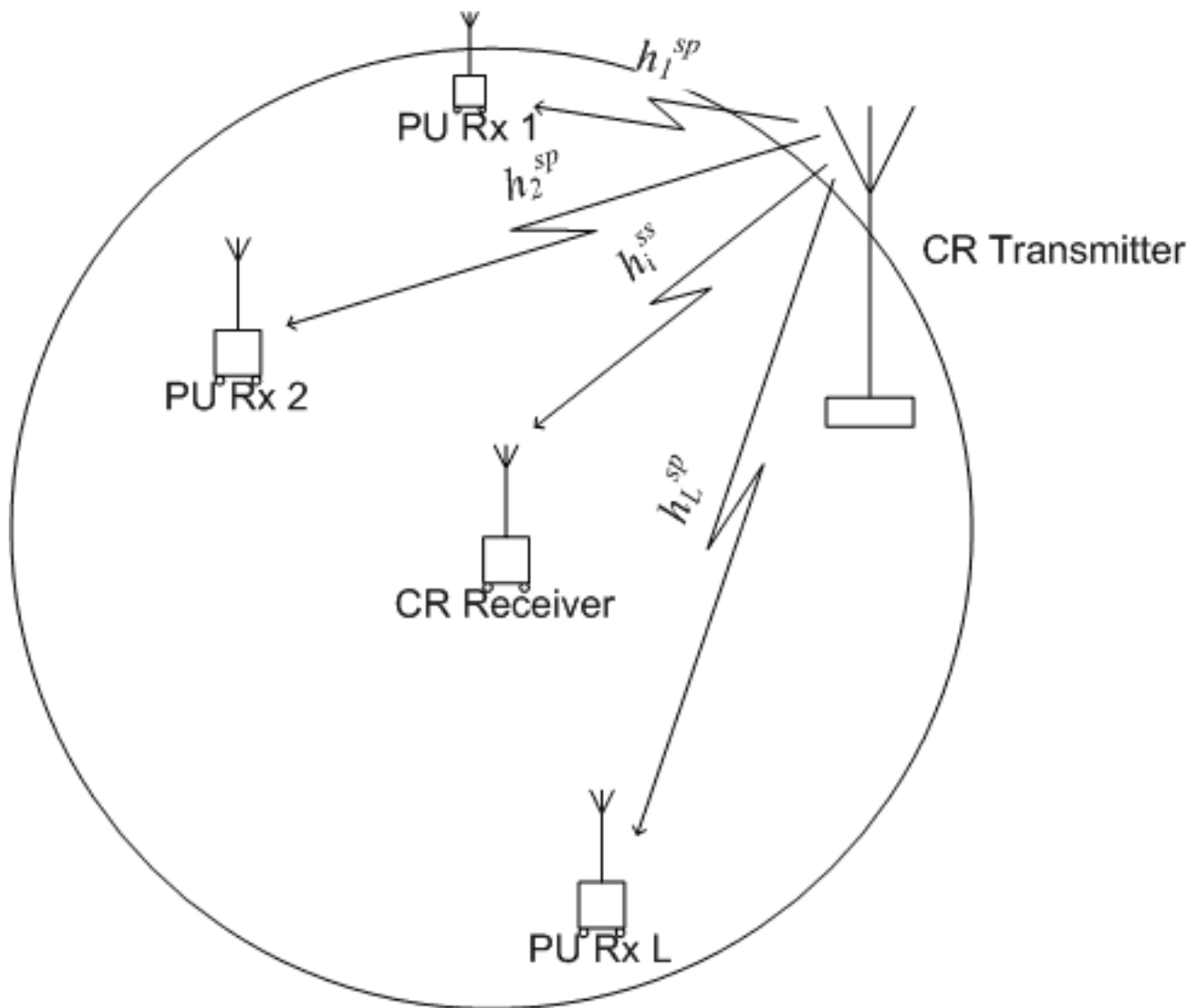


Figure 4.2: Co-existence of PU and CR users in the spatial domain.

4.3 Problem Formulation and Optimal Power Allocation

Our design goal is to find power values for each subcarrier, P_i ($i = 1, 2, \dots, N$) for given instantaneous fading gains h_i^{ss} and the total transmit power budget P_T . As such the total transmission capacity of CR user, C is maximized while the probability that the total interference to L PU bands is kept below the thresholds $I_{th}^{(l)}$ ($l = 1, 2, \dots, L$) with the probability value a or above. Mathematically, the problem in our hand can be formulated as a constrained optimization problem as follows,

$$C = \max_{P_i} \Delta f \sum_{i=1}^N \log_2 \left(1 + \frac{|h_i^{ss}|^2 P_i}{\sigma^2 + \sum_{l=1}^L J_i^{(l)}} \right) \quad (4.3)$$

subject to,

$$\Pr. \left(\sum_{i=1}^N I_i^{(l)}(d_{il}, P_i) \leq I_{th}^{(l)} \right) \geq a, \quad \forall l = 1, 2, \dots, L, \quad (4.4)$$

$$P_i \geq 0, \quad \forall i = 1, 2, \dots, N, \quad (4.5)$$

and,

$$\sum_{i=1}^N P_i \leq P_T. \quad (4.6)$$

Now the probabilistic interference constraint in Eq. (4.4) for a flat channel fading in l^{th} PU band can be written as,

$$\Pr. \left(|h_l^{sp}|^2 \sum_{i=1}^N K_i^{(l)} P_i \leq I_{th}^{(l)} \right) \geq a, \quad \forall l = 1, 2, \dots, L, \quad (4.7)$$

where $K_i^{(l)} = T_s \int_{d_{il}-B_l/2}^{d_{il}+B_l/2} \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 df$. Since h_l^{sp} is assumed to be Rayleigh distributed with a known parameter λ_l , the distribution of $|h_l^{sp}|^2$ corresponds to an exponential distribution with the parameter λ_l^2 . The constraint in Eq. (4.7) can be

evaluated in closed form for the Rayleigh fading case as follows,

$$1 - e^{-\frac{1}{2\lambda_l^2 \sum_{i=1}^N P_i K_i^{(l)}} I_{th}^{(l)}} \geq a, \quad \forall l = 1, 2, \dots, L. \quad (4.8)$$

After some mathematical manipulations, Eq. (4.8) can be written as

$$\sum_{i=1}^N P_i K_i^{(l)} \leq \frac{I_{th}^{(l)}}{2\lambda_l^2 (-\ln(1-a))} \quad \forall l = 1, 2, \dots, L. \quad (4.9)$$

Theorem 1: The power profile for which the total transmission capacity in Eq. (4.3) is maximized for the given constraints in Eqs. (4.5), (4.6), and (4.9), can be written as,

$$P_i^* = \left[w_i - \frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \right]^+ \quad \forall i = 1, 2, \dots, N, \quad (4.10)$$

where $w_i = \frac{1}{\beta + \sum_{l=1}^L \gamma_l K_i^{(l)}}$ and β , and γ_l are deterministic Lagrange parameters.

Proof: We use the fact that minimization of negative value of the concave function in Eq. (4.3) is equivalent to its maximization. Lagrange parameters γ_l , μ_i , and β are introduced for the inequality constraints in Eqs. (4.5), (4.6), and (4.9), respectively. The Karush-Kuhn-Tucker (KKT) conditions can be written as

follows [28],

$$P_i \geq 0, \quad \forall i = 1, 2, \dots, N, \quad (4.11)$$

$$\sum_{i=1}^N P_i - P_T \leq 0, \quad (4.12)$$

$$\sum_{i=1}^N P_i K_i^{(l)} \leq \frac{I_{th}^{(l)}}{2\lambda_l^2(-\ln(1-a))}, \quad \forall l = 1, 2, \dots, L \quad (4.13)$$

$$\mu_i \geq 0, \quad \forall i = 1, 2, \dots, N, \quad (4.14)$$

$$\mu_i P_i = 0, \quad \forall i = 1, 2, \dots, N, \quad (4.15)$$

$$\beta \geq 0, \quad (4.16)$$

$$\beta \left(\sum_{i=1}^N P_i - P_T \right) = 0, \quad (4.17)$$

$$\gamma_l \geq 0, \quad \forall l = 1, 2, \dots, L \quad (4.18)$$

$$\gamma_l \left(\sum_{i=1}^N P_i K_i^{(l)} - \frac{I_{th}^{(l)}}{2\lambda_l^2(-\ln(1-a))} \right) = 0, \quad \forall l = 1, 2, \dots, L \quad (4.19)$$

$$-\frac{1}{\left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} + P_i \right)} - \mu_i + \beta + \sum_{l=1}^L \gamma_l K_i^{(l)} = 0, \quad \forall i = 1, 2, \dots, N. \quad (4.20)$$

Now we can eliminate μ_i , and can write Eqs. (4.14), (4.15), and (4.20) as follows,

$$\frac{1}{\left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} + P_i \right)} \leq \beta + \sum_{l=1}^L \gamma_l K_i^{(l)}, \quad \forall i = 1, 2, \dots, N, \quad (4.21)$$

$$P_i \beta - \frac{P_i}{\left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} + P_i \right)} + P_i \sum_{l=1}^L \gamma_l K_i^{(l)} = 0, \quad \forall i = 1, 2, \dots, N, \quad (4.22)$$

If $\beta + \sum_{l=1}^L \gamma_l K_i^{(l)} < \frac{1}{\left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2}\right)}$, then Eq. (4.21) can only hold if $P_i^* > 0$,

which by solving Eq. (4.22) gives, $P_i^* = \frac{1}{\beta + \sum_{l=1}^L \gamma_l K_i^{(l)}} - \frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2}$. On the other hand, if $\beta + \sum_{l=1}^L \gamma_l K_i^{(l)} \geq \frac{1}{\left(\frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2}\right)}$ then $P_i^* > 0$ is impossible, because it would violate Eq. (4.22). Hence, the optimal power profile can be written as,

$$P_i^* = \left[w_i - \frac{\sigma^2 + \sum_{l=1}^L J_i^{(l)}}{|h_i^{ss}|^2} \right]^+ \quad \forall i = 1, 2, \dots, N, \quad (4.23)$$

where $w_i = \frac{1}{\beta + \sum_{l=1}^L \gamma_l K_i^{(l)}}$.

Solving for $L + 1$ Lagrange parameters (β , and $\gamma_l, l = 1, 2, \dots, L$) can be computationally complex. The Newton's method can be used to find the Lagrange parameters in a quadratic complexity [28]. The interior point method can also be used to maximize the total transmission capacity in Eq. (4.3), given the constraints in Eqs. (4.5), (4.6), and (4.9). The complexity using interior point method would be $O(N^3)$. As the complexity of the proposed algorithm can be quite high, in what follows, we propose a low-complexity sub-optimal scheme.

4.4 Suboptimal and Classical Power Loading Schemes

In this section we propose a low complexity sub-optimal scheme. We also describe classical schemes namely; uniform loading scheme and water-filling scheme which are used for conventional OFDM systems. It is important to mention that classical schemes are also suboptimal in context of CR systems as they do not take interference constraint into account.

4.4.1 Newly Proposed Suboptimal Scheme

The power has to be assigned to N CR subcarriers such that the capacity of CR users can be maximized while all $L + 1$ constraints (total power constraint from Eq. (4.6) and L interference constraints from Eq. (4.9)) are satisfied. The complexity of the optimal scheme comes from the fact that these $L + 1$ constraints have to be met simultaneously. In order to reduce such complexity, we follow a two step procedure as follows. First, we keep only one of the $L + 1$ constraints and find power allocation in each subcarrier. Let us denote the power allocation value for i th subcarrier when j th ($j = 1, 2, \dots, L + 1$) constraint is kept by P_i^j . It is important to mention that solving the optimization problem with only one constraint is less complex and obviously P_i^j is a suboptimal value. Since for a given subcarrier we will have $L + 1$ power allocation values corresponding to each constraint. In the second step, for a given subcarrier the minimum of $L + 1$ power values ($\{P_i^j\}_{j=1}^{L+1}$) is chosen. Therefore, all the constraints can be satisfied simultaneously in a suboptimal fashion.

Specifically, to satisfy L interference constraints given in Eq. (4.9), power is allocated according to ladder profile as in [34]. It is based on the heuristics, that subcarriers closer to PU bands introduce more interference and hence, less power should be allocated to these subcarriers. Power is allocated in each CR subcarrier such that it is inversely proportional to the factor $K_i^{(l)}$. It should be noted that parameter $K_i^{(l)}$ depends on the spectral distance between CR subcarrier and the PU band. The closer CR subcarrier is to PU band, higher is the value of $K_i^{(l)}$ and hence, less power is allocated in that particular subcarrier. Power in the i th subcarrier because of the l th interference constraint is,

$$P_i^{(l)} = P/K_i^{(l)} \quad \forall l = 1, 2, \dots, L \quad (4.24)$$

where P can be calculated by assuming strict equality on the l th interference con-

straint ($I_{th}^{(l)}$) in Eq. (4.9) and can be written as,

$$P = \frac{I_{th}^{(l)}}{2N\lambda_l^2 \ln \frac{1}{(1-a)}}. \quad (4.25)$$

In order to satisfy the power constraint given in Eq. (4.6), we assume that equal power is allocated over all subcarriers, and corresponding power profile for total power constraint can be written as,

$$P_i^{(L+1)} = P_T/N. \quad (4.26)$$

For every subcarrier we choose the power which is minimum among all $L + 1$ power values and can be written as,

$$P_i^{subopt} = \min\{P_i^{(1)}, P_i^{(2)}, \dots, P_i^{(L+1)}\} \quad \forall i = 1, 2, \dots, N. \quad (4.27)$$

By selecting the minimum power value from $L + 1$ power values (corresponding to each of $L + 1$ constraints), all constraints are satisfied. However it may happen that none of these constraint is met strictly. Hence finally, in order to maximize the capacity we scale the power profile (P_i^{subopt}) until one of the constraints is met strictly.

4.4.2 Uniform Loading Scheme

In uniform loading scheme, equal power is allocated to all subcarriers such that all $L + 1$ constraints in Eqs. (4.6), and, (4.9) can be satisfied. By assuming equal power and solving Eq. (4.9) to satisfy the strict equality on l th interference constraint ($I_{th}^{(l)}$), the corresponding power for the i th subcarrier can be written as,

$$P_i^{(l)} = P = \frac{I_{th}^{(l)}}{2 \sum_{i=1}^N K_i^{(l)} \lambda_l^2 \ln \frac{1}{(1-a)}} \quad \forall l = 1, 2, \dots, L. \quad (4.28)$$

The power allocation for constraint in Eq. (4.6) remains same as in Eq. (4.26). The final power allocation to each subcarrier is done according to Eq. (4.27). It should be noted that at least one of these $L + 1$ constraints will be met strictly and hence, scaling of power value is not required.

4.4.3 Waterfilling Scheme

In water-filling scheme, we use the total power allocated by uniform loading scheme as the power constraint. The power value for i th subcarrier, denoted by $P_i^{(WF)}$, are obtained using the standard water-filling algorithm [29]. The power values will satisfy the total power constraint given in Eq. (4.6), however we check if the power values satisfy the interference constraints specified in Eq. (4.9). If a particular interference constraint is not satisfied, we reduce the power in each subcarrier $P_i^{(WF)}$ such that the all interference constraints are satisfied. Also, if none of these interference constraints is met strictly, the power value $P_i^{(WF)}$ is increased until one of these interference constraints is met strictly.

4.5 Numerical Results and Complexity of Algorithms

In this section we present a numerical example where we assume that there are three PU bands ($L=3$), and there are twenty eight OFDM subcarriers ($N = 28$) for CR user.³ The value of T_s has been taken to be 4μ seconds. Here we assume the values of Δf to be 0.3125 MHz, which is same as subcarrier frequency spacing in wireless local area network (LAN) standards [30], [31]. The values of B_1 , B_2 , and B_3 have been assigned to be 1 MHz, 2 MHz, and 5 MHz respectively. AWGN variance, (σ^2) is equal to 10^{-10} W and the channel gains are assumed to be Rayleigh distribution with average channel power gains of h_i^{ss} , h_1^{sp} , h_2^{sp} , and h_3^{sp} equal to -5dB, -3dB, -5dB, and -7dB respectively. The value of J_i^l is equal

³In practice the values of L and N would be high, but for simplicity in the simulation analysis we have assumed the values to be 3 and 28, respectively. It should be noted that the trends in the results presented in this chapter would still hold for other values of L and N .

to 1×10^{-8} W. The values of $I_{th}^{(1)}$, and $I_{th}^{(3)}$ have been assumed to be 1×10^{-6} W, and 5×10^{-6} W. Simulations are run for 100,000 independent iterations to obtain average transmitted data rate for different algorithms under consideration.

In Fig. 4.3, we plot the achievable maximum transmission rate for the CR user versus the total power budget for various schemes. The value of $I_{th}^{(2)}$ has been fixed to 2×10^{-6} W, and the value of a has been considered to be 0.95. From this figure, we observe that the optimal scheme is able to achieve the highest transmission rate for a given power budget. Further, water-filling scheme outperforms the sub-optimal scheme which performs better than the uniform power loading scheme. It should be noted that as we increase the power budget for CR user, the interference constraint becomes dominant and the transmission capacity of CR user does not increase as the power budget increases. This is expected as in this region the CR system operates in an interference limited scenario.

In Fig. 4.4, we plot the achievable transmitted data rate for the CR user versus interference threshold for second PU band, $(I_{th}^{(2)})$ for all the schemes under consideration. The value of total transmit power, P_T has been assumed to be 5×10^{-5} W. Again, we observe that the optimal scheme achieves higher capacity than that of other schemes. The water-filling scheme achieves higher capacity than the sub-optimal scheme. On the other hand, the suboptimal scheme achieves higher capacity than the uniform scheme. The capacity versus interference threshold curve saturates after a certain value of $I_{th}^{(2)}$. The reason is that although $I_{th}^{(2)}$ is relaxed by increasing its value, other constraints ($I_{th}^{(1)}$, $I_{th}^{(3)}$, and P_T) becomes dominant.

In Fig. 4.5, we plot achievable maximum transmitted data rate for the CR user versus probability a . The values of P_T , and $I_{th}^{(2)}$, is assumed to be 5×10^{-5} W, and 2×10^{-6} W, respectively. As expected, we observe that the optimal scheme performs better than the other schemes. Also, water-filling scheme outperforms the suboptimal scheme and the suboptimal scheme has an improved capacity performance compared to the uniform power loading scheme. It is observed from Fig. 4.5 that as expected as the value of a increases, the achievable capacity of CR user decreases for a given power budget and interference thresholds.

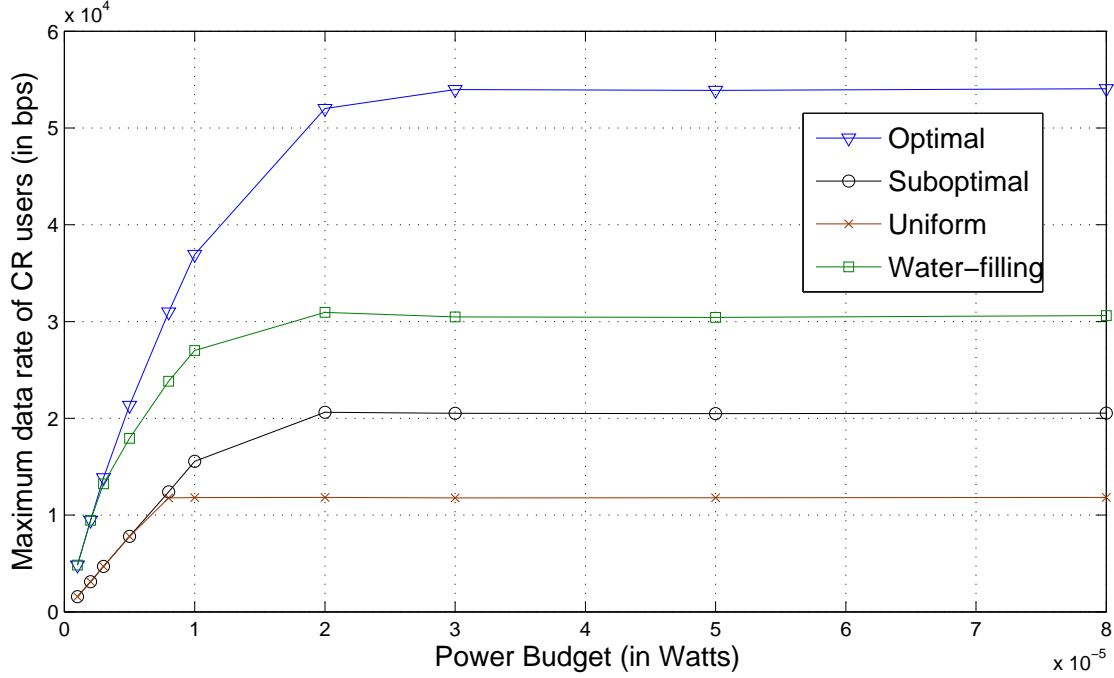


Figure 4.3: Maximum transmitted data rate versus power budget (P_T) for CR users.

Complexities of various algorithms under consideration are presented in Table I. It can be seen that the complexity of the proposed optimal scheme is higher compared to other schemes. Our proposed suboptimal scheme which performs better than the uniform loading scheme has a similar complexity. Water-filling scheme has a higher complexity than both the uniform loading scheme and the proposed suboptimal scheme.

4.6 Conclusions

In this chapter, we have developed an optimal power allocation algorithm for the orthogonal frequency division multiplexing (OFDM)-based CR system. Instead of instantaneous channel fading gain between the PU receiver and the CR transmitter, the developed optimal power allocation scheme requires the fading

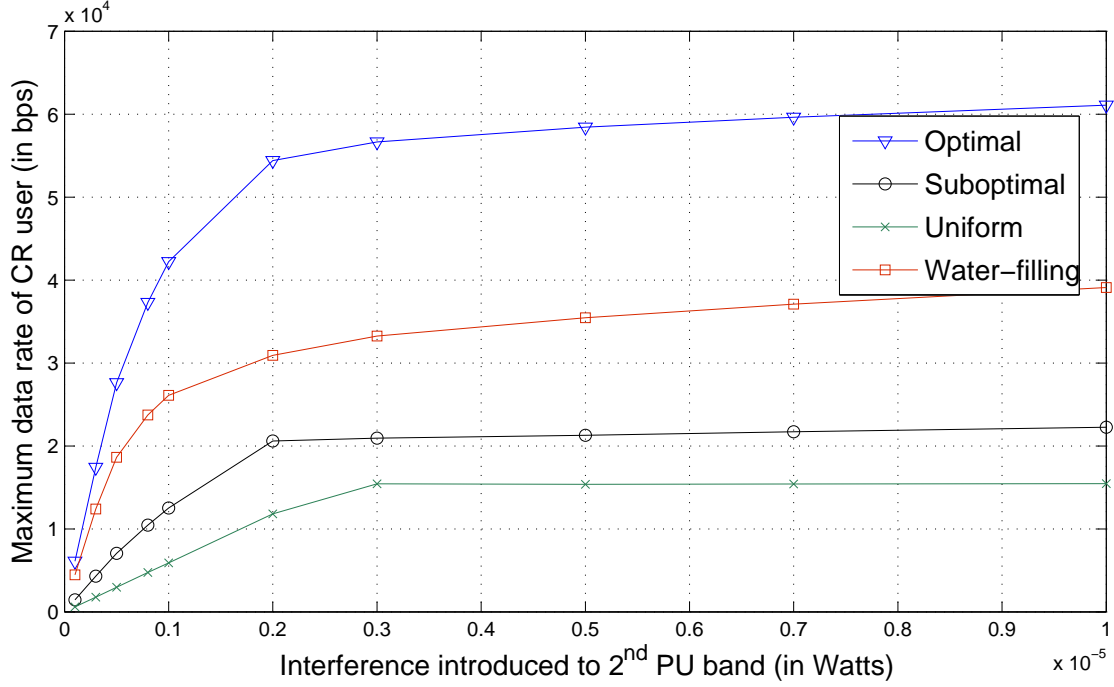


Figure 4.4: Maximum transmitted data rate versus interference threshold for 2^{nd} PU band ($I_{th}^{(2)}$) for CR users.

Table 4.1: Complexity of different schemes

Scheme	Complexity
Proposed optimal scheme	$O(N^3)$
Proposed suboptimal scheme	$O(L * N)$
Uniform Loading scheme	$O(L * N)$
Water-filling scheme	$O(L * N * \log(N))$

statistics and parameters to be known at the CR transmitter. As such the transmission rate of the CR user is maximized for a given power budget and different probabilistic interference constraints imposed by different PU systems. We also proposed and investigated performance of a low complexity suboptimal power allocation scheme. Presented selected numerical results showed that our proposed

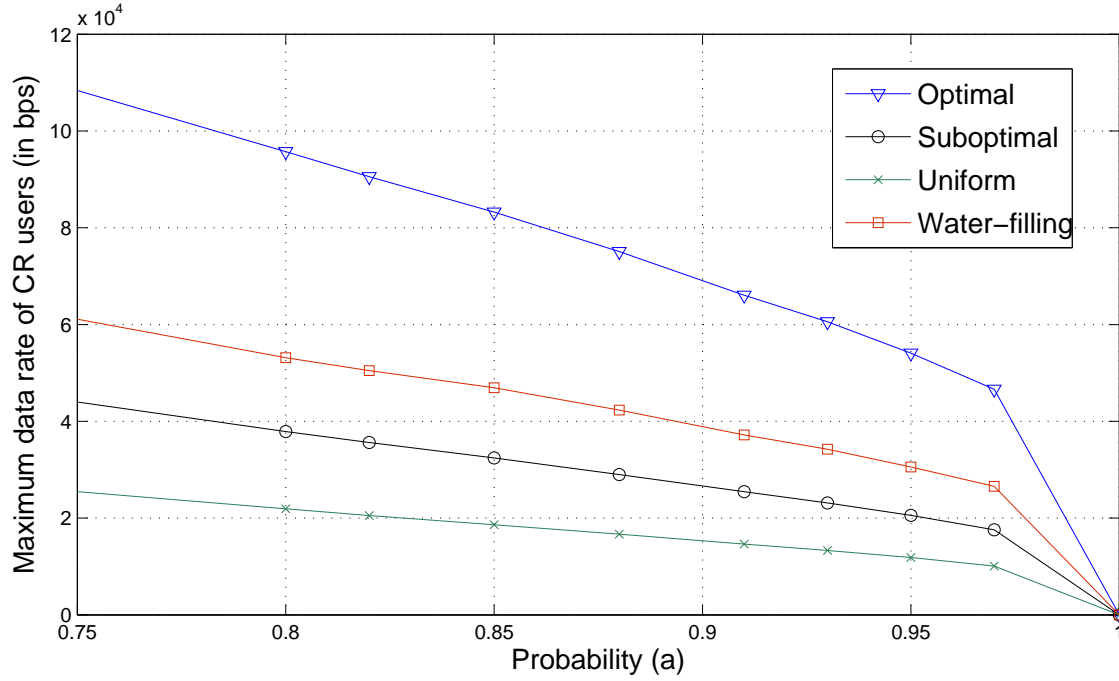


Figure 4.5: Maximum transmitted data rate versus probability (a) with which instantaneous interference introduced to the PU band remains below interference threshold (I_{th}) for CR users.

optimal power allocation achieves significantly higher transmission capacity for CR user compared to the classical power allocation schemes namely, uniform and water-filling power allocation schemes that is used for conventional OFDM-based system. The suboptimal scheme achieved better performance than the uniform loading scheme.

Chapter 5

Power allocation for multiuser OFDMA-based Cognitive Radio Systems with Joint Overlay and Underlay Architecture¹

5.1 Introduction

In recent years, cognitive radio (CR) has been proposed as a technology to improve the spectrum efficiency by giving an opportunistic access of the unused spectrum to unlicensed users[6]. Most CR systems in the literature implement orthogonal frequency division multiplexing (OFDM) as a modulation technology because of its flexibility in allocating spectrum resources[9]. Under interference temperature constraints, the total interference power at PU receivers limits the allowed transmitting power at the CR receiver level. A CR transmitter may adapt its power to the changing interference levels to PU receivers in order to maximize

¹A paper based on the research work presented in this chapter is to be submitted as Gaurav Bansal, Md. Jahangir Hossain, and Vijay K. Bhargava, “Power allocation for multiuser OFDMA-based Cognitive Radio Systems with Joint Overlay and Underlay Architecture”

its capacity in an overlay fashion [34], [36]. In such schemes, power is only allocated to frequency bands where primary user (PU) is not present and only mutual interference between PU and CR users is considered. In [37] resource allocation algorithms have been proposed where power is loaded in an underlay fashion (i.e. power is allocated to frequency bands where PU is already present and only co-channel interference between PU and CR is considered).

A hybrid overlay and underlay cognitive radio model is proposed in [47] to adapt coding and modulation to the interference that PU causes to CR but it does not include the interference caused by power leakage from CR in overlay bands to PU and arbitrarily fixes the underlay CR power, while it could be adapted to current CR interference levels. Including these elements to the model makes adaptive power allocation a critical factor in order to maximize the CR capacity. In [48], a joint overlay-underlay two-switch model is presented that either transmits in overlay or in underlay format.

Schemes which perform a joint power allocation employing both overlay and underlay waveforms do not exist. By considering either pure overlay or underlay only power transmissions, the entire degrees of freedom are not utilized. For example, while allocating power in an overlay fashion, entire power is allocated in overlay subcarriers (subcarriers where PU is not present). For a given interference threshold, it might seem that overlay power allocation would be able to maximize transmission capacity, as loading CR power in underlay subcarriers (subcarrier where PU is present) introduces more interference to the PU band. However, underlay subcarriers might have a very good channel quality for CR users. By allocating power in overlay only fashion, underlay subcarriers can not be exploited and hence, is not optimal.

In this chapter, we propose an optimal scheme for capacity maximization which allocates power to entire bandwidth (i.e., both in overlay and underlay fashion), while keeping the interference introduced to the PU bands within a prescribed threshold, and keeping the total transmission power within a budget. As the complexity of optimal scheme can be high, a suboptimal scheme has also been

proposed. It is based on the heuristic that as underlay subcarriers introduce more interference to the PU band, less power should be allocated to them. Also, we have used an equal power profile for underlay subcarriers and ladder-based power profile for overlay subcarriers. Further, we have compared our proposed schemes to schemes where power is allocated in either overlay or underlay only fashion (classical schemes). Presented numerical results demonstrate that significant improvement in achievable capacity is achieved by loading power in a joint overlay and underlay fashion, as compared to either overlay or underlay only. Further, we present the power profile of the proposed optimal scheme, which shows that indeed transmission capacity is maximized when power is allocated jointly to both overlay and underlay subcarriers.

The organization of the chapter is as follows. The system model is presented in Section 5.2 and the problem is formulated in Section 5.3. An optimal scheme is proposed in Section 5.4. In Section 5.5, we present a low-complexity sub-optimal scheme. Classical schemes are presented in Section 5.6 and numerical results are presented in Section 5.7. The chapter is concluded in Section 5.8.

5.2 System Model

We assume that using spectrum sensing, bands where spectrum holes are present have been identified, and spectrum holes and PU exist in side-by-side band. One possible co-existence scenario is depicted in Fig. 5.1, where spectrum holes are labeled as CR bands. It is assumed that the M available bands are divided into N spectrum holes (or CR subcarriers) and L PU subcarriers, each of bandwidth Δf Hz. As shown in Fig. 5.1, the CR transmitter may allocate a portion of its total power budget P_T in any number of overlay (CR) and underlay (PU) subcarriers. We assume a *downlink transmission* where base station is transmitting to K CR users.

We assume the CR power transmission to be an ideal Nyquist pulse, and the

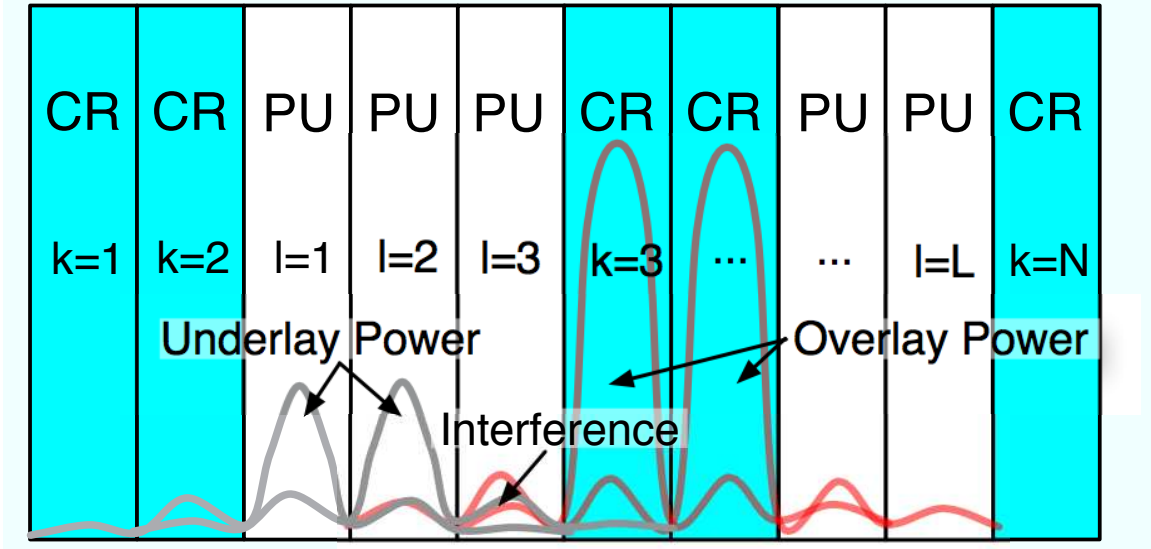


Figure 5.1: Underlay and overlay power allocation

power density spectrum in the k^{th} band can be written as,

$$\phi_k(f) = p(k)T_s \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2, \quad (5.1)$$

where T_s is the symbol duration, and $p(k)$ is the total power loaded in the k^{th} subcarrier.

If overlay power (p_O) is loaded in overlay subcarriers (where PU is not present), the overlay interference ($i_O(l)$) introduced to the l^{th} PU band can be expressed as [34], [17]

$$i_O(l) = \sum_{u=1}^K \sum_{k=1}^N p_O(u, k) |h_{SP}(l)|^2 T_s \int_{d_{kl}-\Delta f/2}^{d_{kl}+\Delta f/2} \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 df \quad (5.2)$$

where $h_{SP}(l)$ is the channel gain between CR base station and PU receiver in the l^{th} subcarrier, T_s is the symbol duration, d_{kl} represents the spectral distance between the k^{th} CR subcarrier and l^{th} PU band, $p_O(u, k)$ represents the overlay power loaded by the u^{th} user in the k^{th} overlay band. We assume that the PU

bands experience flat fading.

If additional underlay power (p_U) is loaded in underlay subcarriers (where PU is present), the underlay interference ($i_U(l)$) introduced to the l^{th} PU band can be expressed as

$$i_U(l) = \sum_{u=1}^K \sum_{m=1}^L p_U(u, m) |h_{SP}(l)|^2 T_s \int_{d_{ml}-\Delta f/2}^{d_{ml}+\Delta f/2} \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 df \quad (5.3)$$

where $p_U(u, m)$ represents the underlay power loaded by the u^{th} user in the m^{th} underlay band.

The total interference introduced to the l^{th} PU band can be written as

$$i^{PU}(l) = i_O(l) + i_U(l), \quad (5.4)$$

which after reformulation can be written as,

$$i^{PU}(u, l) = |h_{SP}(l)|^2 \sum_{u=1}^K \sum_{k=1}^M \rho(u, k) p(u, k) f_{\text{dist}}(d_{k,l}) \quad (5.5)$$

where $\rho(u, k)$ indicates which user is occupying a particular subcarrier. It can have only value of either 0 or 1, as only one user can occupy a particular subcarrier. The spectral distance factor $0 \leq f_{\text{dist}}(k, l) \leq 1$ can be calculated as:

$$f_{\text{dist}}(k, l) = T_s \int_{(k-l-1/2)\Delta f}^{(k-l+1/2)\Delta f} \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 df \quad (5.6)$$

The capacities due to overlay transmission and underlay transmission have been denoted by C_U , and C_O and can be expressed as,

$$C_O = \Delta f \sum_{u=1}^K \sum_{k=1}^N \log \left(1 + \frac{|h_{SS}(u, k)|^2 p_O(u, k)}{\sigma^2 + j(k)} \right), \quad (5.7)$$

$$C_U = \Delta f \sum_{u=1}^K \sum_{l=1}^L \log \left(1 + \frac{|h_{SS}(u, l)|^2 p_U(u, l)}{\sigma^2 + j(l)} \right) \quad (5.8)$$

where $h_{SS}(u, k)$ are independent and identically distributed (i.i.d.) channel gains between CR base station and the u^{th} CR user in the k^{th} subcarrier, σ^2 denotes the additive white Gaussian noise (AWGN) variance, and $j(k)$ denotes the interference because of all PU bands into the CR transmission in k^{th} subcarrier. We assume interference $j(k)$ to be AWGN.

The total capacity of CR users can be written as

$$C = C_O + C_U \quad (5.9)$$

which can be reformulated as,

$$C = \Delta f \sum_{u=1}^K \sum_{k=1}^M \rho(u, k) \log \left(1 + \frac{|h_{SS}(u, k)|^2 p(u, k)}{\sigma^2 + j(k)} \right) \quad (5.10)$$

In this chapter, we assume that the channel gain $h_{SS}(u, k)$ between CR base station and CR receivers can be estimated by the feedback between CR base station and receivers, and is known perfectly at the CR base station. However, the channel gain $h_{SP}(l)$ between CR base station and PU receivers is not known perfectly at the CR base station. In [42], the authors have argued that from the pilot signals transmitted by PU user, mean and the distribution of the fading gains between CR base station and PU receiver can be estimated. Hence, in this chapter we assume that the CR base station knows the statistics (mean and the distribution) of the channel gains between itself and the PU receivers.

5.3 Problem Formulation

The optimization problem is to maximize the transmission capacity of K CR users while keeping the total interference introduced to the PU band below a threshold and total transmission power below a specified power constraint. Now, as we only know the statistics of channel gain $h_{SP}(l)$, the interference can be guaranteed only in a statistical manner, i.e., it can be guaranteed with any given probability (a or above) that the interference introduced to the PU band remains below a specified

interference threshold. The mathematical formulation of the optimization problem is:

$$\max_{p(u,k), \rho(u,k)} \sum_{u=1}^K \sum_{k=1}^M \rho(u,k) \log_2 \left(1 + \frac{|h_{ss}(u,k)|^2 p(u,k)}{\sigma^2 + j(k)} \right) \quad (5.11)$$

subject to,

$$Pr. \left(|h_{SP}(l)|^2 \sum_{u=1}^K \sum_{k=1}^M \rho(u,k) p(u,k) f_{\text{dist}}(d_{k,l}) \leq I_{th}^{(l)} \right) > a \quad \text{for } l = 1, 2, \dots, L, \quad (5.12)$$

$$p(u,k) \geq 0, \quad \text{for } u = 1, 2, \dots, K, \text{ and } k = 1, 2, \dots, M \quad (5.13)$$

$$\sum_{u=1}^K \sum_{k=1}^M \rho(u,k) p(u,k) \leq P_T, \quad (5.14)$$

$$\rho(u,k) = \{0, 1\} \quad \text{for } u = 1, 2, \dots, K, \text{ and } k = 1, 2, \dots, M \quad (5.15)$$

$$\sum_{u=1}^K \rho(u,k) = 1 \quad \text{for } k = 1, 2, \dots, M \quad (5.16)$$

where $Pr.$ denotes the probability and $\rho(u,k)$ indicates which user is occupying a particular subcarrier. It can only have value of either 1 or 0, as a particular subcarrier can be occupied only by one user.

Now, assuming $h_{SP}(l)$ to be Rayleigh distributed with a known parameter λ_l , the distribution of $|h_{SP}(l)|^2$ is exponential with the parameter λ_l^2 . Hence, the interference constraint in Eq. (5.12) can be written as,

$$1 - e^{-\frac{I_{th}^{(l)}}{2\lambda_l^2 \sum_{u=1}^K \sum_{k=1}^M \rho(u,k) p(u,k) f_{\text{dist}}(d_{k,l})}} \geq a, \quad \text{for } l = 1, 2, \dots, L \quad (5.17)$$

After mathematical manipulations, Eq. (5.17) can be rewritten as,

$$\sum_{u=1}^K \sum_{k=1}^M \rho(u,k) p(u,k) f_{\text{dist}}(d_{k,l}) \leq \frac{I_{th}^{(l)}}{2\lambda_l^2 (-\ln(1-a))}, \quad \text{for } l = 1, 2, \dots, L \quad (5.18)$$

It should be noted that the optimization problem for maximizing Eq. (5.11)

given constraints in Eqs. (5.13), (5.14), (5.15), (5.16), and (5.18), is generally a NP-hard problem, which is hard to solve optimally. However in this chapter. we have assumed a downlink scenario and an optimal solution has been proposed in the next section.

5.4 Optimal Scheme

In this section we propose optimal solution for the optimization problem given in Eqs. (5.11), (5.13), (5.14), (5.15), (5.16), and (5.18), by decoupling the subcarrier and power allocation. First, we propose a subcarrier allocation scheme and prove that even after decoupling, the proposed solution would be optimal.

5.4.1 Subcarrier Allocation

Since the goal in Eq. (5.11) is to maximize capacity, we allocate a particular subcarrier to a user with the highest signal to noise ratio (SNR) for that subcarrier. Thus $\rho(u, k) = 1$ for $u = u_k$, or 0 otherwise, where

$$u_k = \arg \max_u \frac{|h_{ss}(u, k)|^2}{\sigma^2 + j(k)}, \quad \text{for } k = 1, 2, \dots, M \quad (5.19)$$

Theorem1: The decoupling of power and subcarrier allocation, performed by allocating subcarrier according to Eq. (5.19) is optimal.

Proof: We will prove *Theorem1* by contradiction.

Lets assume that a optimal scheme exists where a user u_i with highest channel gain $h_{ss}(u_i, k)$ in the k^{th} subcarrier and the k^{th} subcarrier is not assigned to user u_i . Instead k^{th} subcarrier is assigned to user u_j with channel gain $h_{ss}(u_j, k)$. Lets assume that optimal power $p(u_j, k)$ is loaded. As it is a optimal scheme $p(u_j, k)$ will satisfy constraints in Eqs. (5.13), (5.14), and (5.18).

Now, if we assign the k^{th} subcarrier with same power $p(u_j, k)$ but to user u_i . The constraints in Eqs. (5.13), (5.14), and (5.18) will still be satisfied. But capacity in Eq. (5.11) would be higher as $h_{ss}(u_i, k) > h_{ss}(u_j, k)$. So, assigning k^{th}

subcarrier to user u_j is not optimal. Hence, proved by contradiction.

5.4.2 Power Allocation

Now after performing subcarrier allocation, the optimization problem can be written as:

$$\max_{p(u,k)} \sum_{u=1}^K \sum_{k \in \Omega_u} \rho(u,k) \log_2 \left(1 + \frac{|h_{ss}(u,k)|^2 p(u,k)}{\sigma^2 + j(k)} \right) \quad (5.20)$$

subject to,

$$\sum_{u=1}^K \sum_{k \in \Omega_u} p(u,k) f_{\text{dist}}(d_{k,l}) \leq \frac{I_{th}^{(l)}}{2\lambda_l^2(-\ln(1-a))}, \quad \text{for } l = 1, 2, \dots, L \quad (5.21)$$

$$p(u,k) \geq 0, \quad \text{for } u = 1, 2, \dots, K, \text{ and } k \in \Omega_u \quad (5.22)$$

$$\sum_{u=1}^K \sum_{k \in \Omega_u} p(u,k) \leq P_T, \quad (5.23)$$

where Ω_u is the set of subcarriers assigned to user u by performing subcarrier allocation according to Eq. (5.19).

The optimization problem specified in Eqs. (5.20), (5.21), (5.22), and (5.23), is convex in $p(u,k)$. We define a set S_1 such that, $p(u,k) \in S_1 = \{p(u,k) \geq 0; \forall u, k \in \Omega_u\}$. Introducing Lagrangian coefficients α_l , and γ , corresponding to constraints in Eqs. (5.21), and (5.23), respectively, the Lagrangian over set S_1 can be written as:

$$\begin{aligned} \ell = & \sum_{u=1}^K \sum_{k \in \Omega_u} \log_2 \left(1 + \frac{|h_{ss}(u,k)|^2 p(u,k)}{\sigma^2 + j(k)} \right) \\ & + \sum_{l=1}^L \alpha_l \left(\frac{I_{th}^{(l)}}{2\lambda_l^2(-\ln(1-a))} - \sum_{u=1}^K \sum_{k \in \Omega_u} p(u,k) f_{\text{dist}}(d_{k,l}) \right) \\ & + \gamma \left(P_T - \sum_{u=1}^K \sum_{k \in \Omega_u} p(u,k) \right) \end{aligned} \quad (5.24)$$

The Karush-Kuhn-Tucker (KKT) conditions [28] can be written as:

$$\frac{|h_{ss}(u, k)|^2}{\sigma^2 + j(k) + |h_{ss}(u, k)|^2 p(u, k)} - \alpha_l f_{\text{dist}}(d_{k,l}) - \gamma = 0 \quad (5.25)$$

$$\alpha_l \left(\frac{I_{th}^{(l)}}{2\lambda^2(l)(-\ln(1-a))} - \sum_{u=1}^K \sum_{k \in \Omega_u} p(u, k) f_{\text{dist}}(d_{k,l}) \right) = 0 \quad \text{for } l = 1, 2, \dots, L \quad (5.26)$$

$$\gamma \left(P_T - \sum_{u=1}^K \sum_{k \in \Omega_u} p(u, k) \right) = 0 \quad (5.27)$$

$$\alpha_l \geq 0 \quad \text{for } l = 1, 2, \dots, L \quad (5.28)$$

$$\gamma \geq 0 \quad (5.29)$$

From Eq. (5.25), the power profile can be written as:

$$p(u, k) = \left[\frac{1}{\alpha_l f_{\text{dist}}(d_{k,l}) + \gamma} - \frac{\sigma^2 + j(k)}{|h_{ss}(u, k)|^2} \right]^+ \quad (5.30)$$

As the optimization problem in hand is a convex problem, the Lagrange coefficients α_l and γ in Eq. (5.30) can be calculated using Newton's method or interior point method such that the KKT conditions in the Eqs. (5.26), (5.27), (5.28) and (5.29) are satisfied. As the Lagrange parameters need to be computed, the complexity of optimal scheme can be high. Hence, in the next scheme we propose a low-complexity suboptimal scheme.

5.5 Suboptimal Scheme

In the optimal scheme proposed in the previous section it has been shown that decoupling subcarrier and power allocation is optimal. However, the scheme can be computationally complex because of the proposed optimal power allocation technique employed in the optimal scheme. The complexity of subcarrier allocation scheme is $O(KM)$, but the complexity of power allocation technique would

be exponential.

In the suboptimal scheme proposed in this section, we first allocate subcarriers according to Eq. (5.19). Now, we propose a suboptimal power allocation such that the complexity is lower than exponential. Based on the heuristic that the underlay subcarriers introduce more total interference to the PU band compared to the overlay subcarriers, we allocate less power to underlay subcarriers compared to overlay subcarriers.

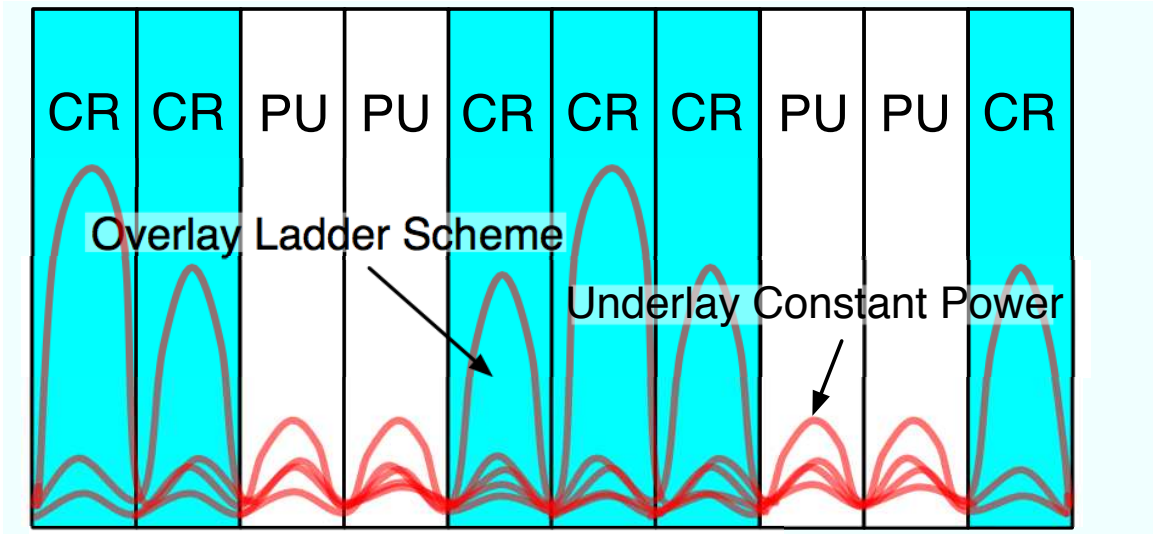


Figure 5.2: Power profile for suboptimal scheme

We allocate equal low power to all the underlay subcarriers. The power is loaded in the overlay subcarriers in a ladder profile, as in [34], which is based on the heuristic that the subcarriers which are closer to PU band introduce more interference to the PU band and hence are allocated less power. The power profile for the suboptimal scheme is depicted in Fig. 5.2.

The power profile for underlay subcarriers which are all allocated equal power can be expressed as,

$$p_U(l) = P_{underlay} \quad \forall l \in \{1, 2, \dots, L\} \quad (5.31)$$

The overlay subcarriers are allocated power in a ladder fashion where step size of the ladder is constant. Basically, we allocate $P_{overlay}$ power to the subcarriers adjacent to PU band, then we allocate $2P_{overlay}$ power to the subcarriers that are right next to them, and so on. Mathematically, the power profile for overlay subcarriers can be expressed as,

$$p_O(k) = P_{overlay} * i \quad \forall k \in \{1, 2, \dots, N\} \quad (5.32)$$

Now, we introduce a design factor x by which underlay power is less than the overlay subcarrier and can be expressed as,

$$P_{overlay} = x * P_{underlay} \quad \forall l \in \{1, 2, \dots, L\} \quad (5.33)$$

The factor x is empirically determined in the simulations. Now, the power profile can be determined if the value of $P_{underlay}$ is known. The values of $P_{underlay}$ is determined such that both total power budget in Eq. (5.23) and L interference threshold constraints in Eq. 5.21 can be met at the same time. Basically, $L + 1$ values of $P_{underlay}$, which satisfy the total power constraint and L interference thresholds are determined. The value which is minimum is chosen as it will satisfy all the constraints.

Without loss of generality, we can group overlay and underlay subcarriers such that the first L subcarriers are underlay subcarriers and remaining N subcarriers are overlay subcarriers. The power profile which will satisfy the total power constraint can be written as:

$$P_{underlay}^{(1)} * L + x * P_{underlay}^{(1)} * \sum_{i=1}^N i = P_T \quad (5.34)$$

Now, from Eq. 5.34, the value of $P_{underlay}^{(1)}$ can be calculated as:

$$P_{underlay}^{(1)} = P_T / (L + x * N * (N + 1) / 2) \quad (5.35)$$

The power profile (denoted by $P_{underlay}^{(l+1)}$) which will satisfy the l^{th} interference constraint in Eq. 5.21 can be written as:

$$P_{underlay}^{(l+1)} * \sum_{k=1}^L f_{\text{dist}}(d_{k,l}) + x * P_{underlay}^{(l+1)} * \left(\sum_{k=L+1}^{N+L} f_{\text{dist}}(d_{k,l}) * (k-L) \right) = \frac{I_{th}^{(l)}}{2\lambda_l^2(-\ln(1-a))} \quad (5.36)$$

Now, from Eq. 5.36, the value of $P_{underlay}^{(l)}$ can be calculated as:

$$P_{underlay}^{(l+1)} = \frac{I_{th}^{(l)}}{2\lambda_l^2(-\ln(1-a)) * (\sum_{k=1}^L f_{\text{dist}}(d_{k,l}) + x * \sum_{k=L+1}^{N+L} f_{\text{dist}}(d_{k,l}) * (k-L))} \quad (5.37)$$

Finally, the value of $P_{underlay}$ would be chosen as:

$$P_{underlay} = \min\{P_{underlay}^{(1)}, P_{underlay}^{(2)}, \dots, P_{underlay}^{(L+1)}\} \quad (5.38)$$

5.6 Classical Schemes

In this section, we propose overlay only and underlay only schemes for comparison to our proposed optimal and suboptimal schemes.

5.6.1 Overlay Only Scheme

In overlay only scheme, we allocate power in an optimal fashion, but only in overlay subcarriers. Basically, all the power is allocated to N overlay (CR) subcarriers, and L underlay (PU) subcarriers are nulled (assigned zero power). The power is allocated in overlay subcarriers such that the total transmission capacity is maximized, while maintaining total power budget constraint and keeping the total interference introduced to the PU band below a specified threshold. Specifically, the power profile for overlay only scheme (p_{OO}) is obtained by solving the following optimization problem:

$$\max_{p_{OO}} C_O \quad (5.39)$$

subject to,

$$Pr(i_O(l) \leq I_{th}^{(l)}) > a \quad \text{for } l = 1, 2, \dots, L \quad (5.40)$$

$$p_{OO}(u, i) \geq 0 \quad \text{for } u = 1, 2, \dots, K \text{ and } i = 1, 2, \dots, N \quad (5.41)$$

$$\sum_{u=1}^K \sum_{i=1}^N p_{OO}(u, i) \leq P_T \quad (5.42)$$

where $i_O(l)$ and C_O are defined in Eqs. (5.2) and (5.7) respectively.

Now the subcarriers are allocated to CR users according to Eq. (5.19) but only to N overlay subcarriers and can be written as,

$$u_i = \arg \max_u \frac{|h_{SS}(u, i)|^2}{\sigma^2 + j(i)}, \quad \text{for } i = 1, 2, \dots, N \quad (5.43)$$

The power allocation can be determined using optimal scheme and the corresponding power profile can be written as,

$$p_{OO}(u, i) = \left[\frac{1}{\alpha_l^{OO} f_{\text{dist}}(d_{i,l}) + \gamma^{OO}} - \frac{\sigma^2 + j(i)}{|h_{SS}(i)|^2} \right]^+ \quad (5.44)$$

where Lagrange coefficients α_l^{OO} and γ^{OO} are determined such that constraint in Eqs. (5.40), (5.41), and (5.42) are satisfied.

5.6.2 Underlay Only Scheme

Similarly, in underlay only scheme, we allocate power in an optimal fashion, but only in underlay subcarriers. All the power is allocated to L underlay (PU) subcarriers and N overlay (CR) subcarriers are nulled. The power profile for underlay only scheme (p_{UO}) is obtained by solving the following optimization problem:

$$\max_{p_{UO}} C_U \quad (5.45)$$

subject to,

$$Pr(i_U(l) \leq I_{th}^{(l)}) > a \quad \text{for } l = 1, 2, \dots, L \quad (5.46)$$

$$p_{UO}(u, i) \geq 0 \quad \text{for } u = 1, 2, \dots, K \quad \text{and } i = 1, 2, \dots, L \quad (5.47)$$

$$\sum_{u=1}^K \sum_{i=1}^L p_{UO}(u, i) \leq P_T \quad (5.48)$$

where $i_U(l)$ and C_U are defined in Eqs. (5.3) and (5.8) respectively.

The subcarriers are allocated to CR users according to Eq. (5.19) but only to L underlay subcarriers,

$$u_i = \arg \max_u \frac{|h_{SS}(u, i)|^2}{\sigma^2 + j(i)}, \quad \text{for } i = 1, 2, \dots, L \quad (5.49)$$

The power allocation can be determined using optimal scheme and the corresponding power profile,

$$p_{UO}(u, i) = \left[\frac{1}{\alpha_l^{UO} f_{\text{dist}}(d_{i,l}) + \gamma^{UO}} - \frac{\sigma^2 + j(i)}{|h_{SS}(i)|^2} \right]^+ \quad (5.50)$$

where Lagrange coefficients α_l^{UO} and γ^{UO} are determined such that constraint in Eqs. (5.46), (5.47), and (5.48) are satisfied.

5.7 Numerical Results

The values of T_s has been taken to be $4\mu\text{seconds}$. Here we assume the values of Δf to be 0.3125 MHz, which is same as subcarrier frequency spacing in wireless local area network (LAN) standards [30], [31]. The values of L and N have been assumed to be 8 each. The value of total number of users K have been assumed to be 4.² Noise variance (σ^2) has been taken to be 10^{-8}W . The value of interference $j(k)$, has been generated randomly with a mean of $1 \times 10^{-8}\text{W}$. The value of probability (a) with which the interference introduced to the PU band remains below I_{th} has been taken to be 0.95. The channel gain $h_{SS}(u, k)$ has been assumed

²In practice the values of L , N and K would be high, but for simplicity in the simulation analysis we have assumed the values to be 8, 8 and 4 respectively. Also, it should be noted that the trends in the results presented in this chapter would still hold for other values of L , N and K .

to be Rayleigh distributed with a mean of $-10dB$. The mean (λ_l) of the channel gains $h_{SP}(1)$, $h_{SP}(2)$, $h_{SP}(3)$, $h_{SP}(4)$, $h_{SP}(5)$, $h_{SP}(6)$, $h_{SP}(7)$, and $h_{SP}(8)$, have been assumed to be $-5dB$, $-7dB$, $-10dB$, $-5dB$, $-7dB$, $-10dB$, $-5dB$, and $-7dB$, respectively. For the suboptimal scheme, our simulation empirically showed 4 to be the best value for the factor x .

First, we fix the value of $I_{th}^{(l)}$, for every l , to be 1×10^{-5} Watts and in Fig. 5.3, we plot the maximum achievable capacity versus total power budget for proposed optimal scheme, suboptimal scheme, overlay only scheme and underlay only scheme. From Fig. 5.3, it can be observed that the proposed optimal scheme achieves significantly higher transmission capacity compared to suboptimal, overlay and underlay only schemes. It can also be observed that even proposed low-complexity suboptimal scheme outperforms overlay and underlay only scheme. In Fig. 5.4, we plot the total transmitted power for various schemes. A very interesting point can be noted that when the power budget is greater than 1×10^{-3} Watts, overlay only scheme loads more power than optimal scheme in CR subcarriers. *Hence, the proposed optimal scheme not only loads more capacity but also achieves it while using less power.* At higher power budget (greater than 1×10^{-3} Watts), the interference constraint becomes the boundary constraint and there is more power available to load. But overlay only scheme is not able to load the power judiciously, as it does not use the entire degrees of freedom available by loading power in all the subcarriers (overlay as well as underlay subcarriers) and hence end up using more transmission power while achieving less capacity. It should also be noted that the proposed suboptimal scheme needs significantly lower power as compared to overlay only scheme while it achieves higher capacity.

Further in Fig. 5.5, we fix the total power budget to 1×10^{-3} Watts and plot the maximum achievable capacity versus interference threshold for the proposed optimal scheme, suboptimal scheme, overlay only scheme and underlay only scheme. Again it can be seen from Fig. 5.5, that proposed optimal and suboptimal schemes outperform both overlay and underlay only schemes. It should be noted that for

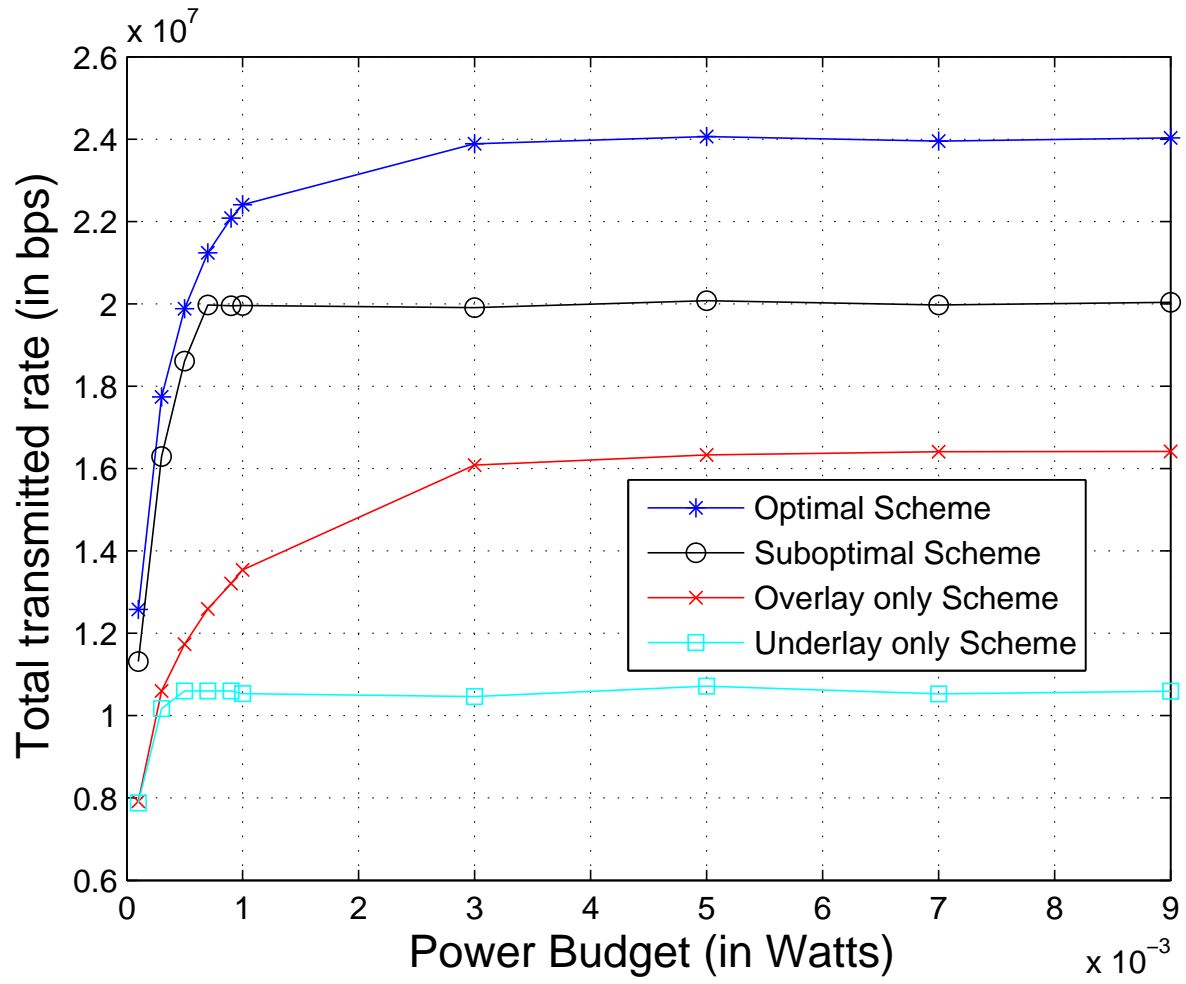


Figure 5.3: Total transmitted rate versus total power budget for various schemes

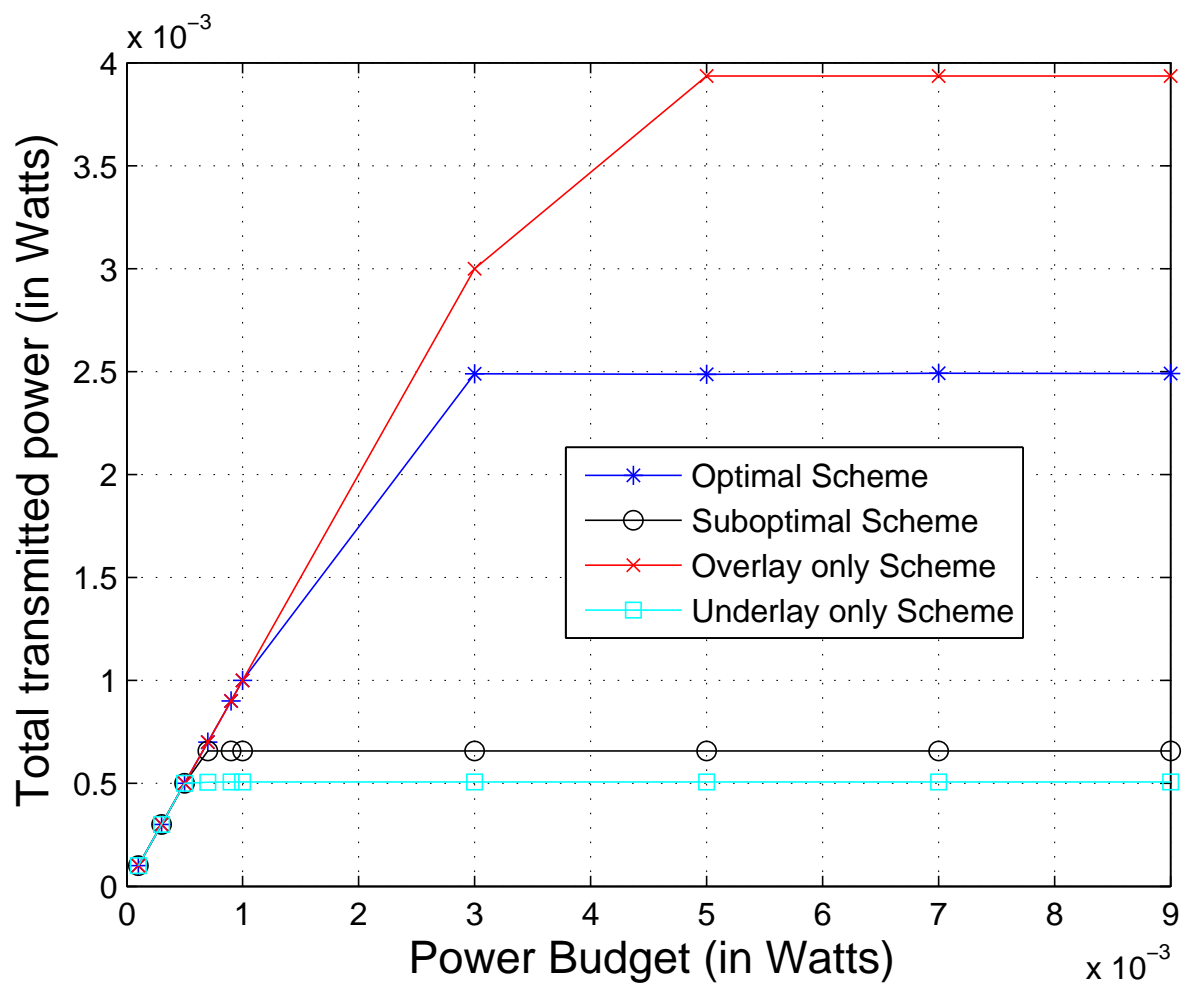


Figure 5.4: Total transmitted power versus total power budget for various schemes

higher interference threshold (greater than 4×10^{-5} Watts) overlay and underlay scheme achieves the same capacity. As for higher values of interference threshold, total power constraint (set to 1×10^{-3} Watts) becomes the limiting constraint. Both overlay and underlay only schemes has same degrees of freedom, as they can both load power in 8 ($N = L = 8$) subcarriers, they both perform the same. However, optimal and suboptimal scheme can load power into 16 ($N + L = 16$) subcarriers and hence, achieves higher capacity. In Fig. 5.6, we plot the total transmitted power for various schemes. It can be noted that for higher interference threshold total power constraint becomes the boundary constraint and all scheme loads same total power in subcarriers.

In Fig. 5.7, we have plotted the power profile for various proposed schemes depicting the power that has been loaded into various subcarriers. We have fixed the values of $I_{th}^{(l)}$ to be 1×10^{-6} Watts and total power budget to 10×10^{-3} Watts (these values are chosen such that total power budget is very high and interference constraint is acting as the boundary constraint). Further, for this simulation we have taken the mean (λ_l) of the channel gains $h_{SP}(l)$ for every l to be $-10dB$ (when we take different values of the mean(λ_l) for various PU users, bands with higher mean values are assigned lower power as they introduce more interference. Hence, power profile varies with it. Here, we want to study how various schemes load power irrespective of the differences of the channel gain and hence, we assumes the same value for all l . In Fig. 5.9, we will study the case when the mean value (λ_l) is different for different PU bands.) Now, from Fig. 5.7, it can be seen that overlay and underlay only schemes loads power only in overlay and underlay only subcarriers. The proposed optimal and suboptimal scheme loads power in every subcarrier. Also, as the interference constraint is the boundary constraint, less power is allocated to underlay subcarriers as they introduce higher interference to the PU band.

In Fig. 5.8, we have changed the values of $I_{th}^{(l)}$ to be 1×10^{-4} Watts and total power budget to 1×10^{-4} Watts (these values are chosen such that the interference threshold is high and total power constraint is acting as the boundary constraint).

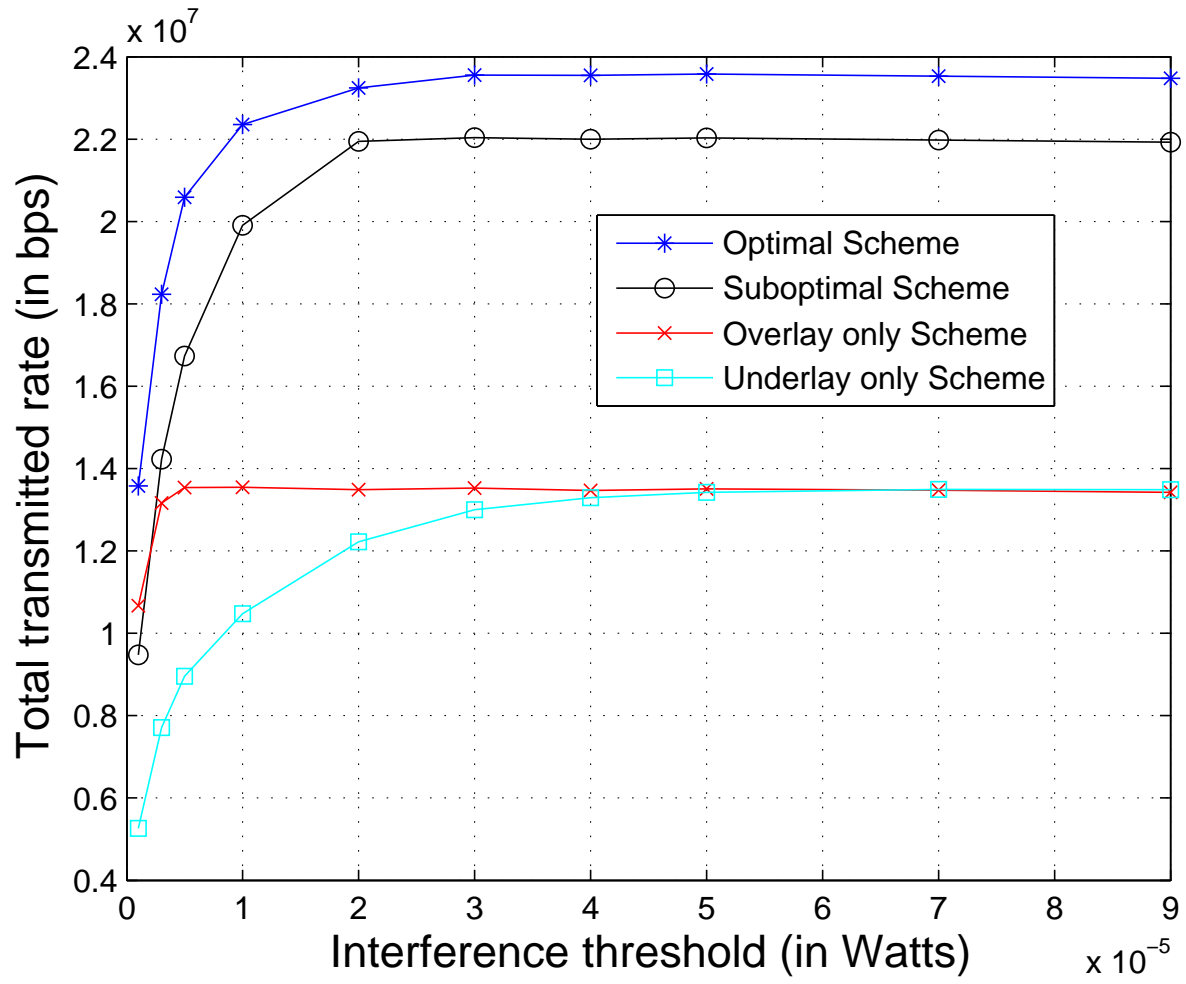


Figure 5.5: Total transmitted rate versus interference threshold for various schemes

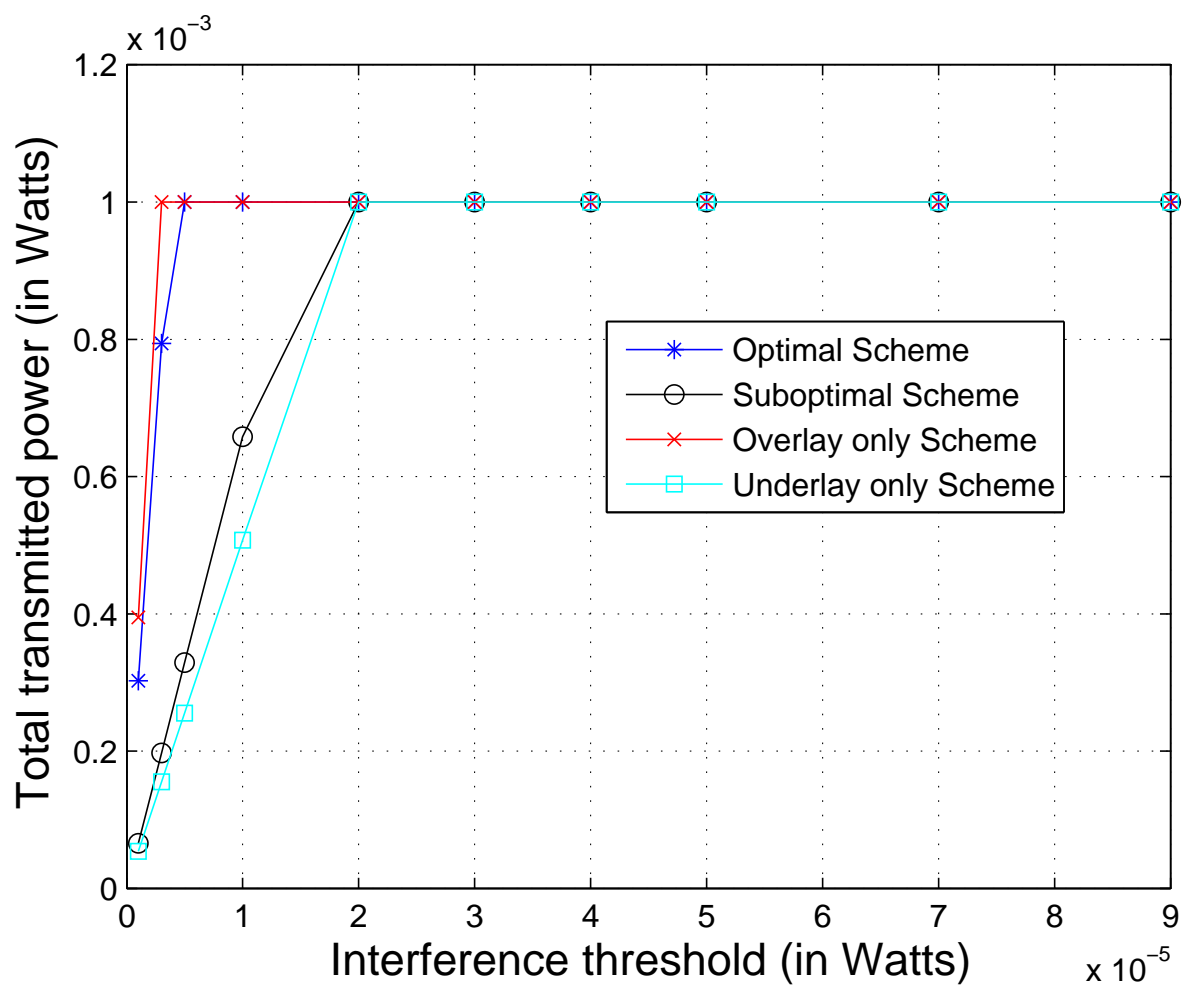


Figure 5.6: Total transmitted power versus interference threshold for various schemes

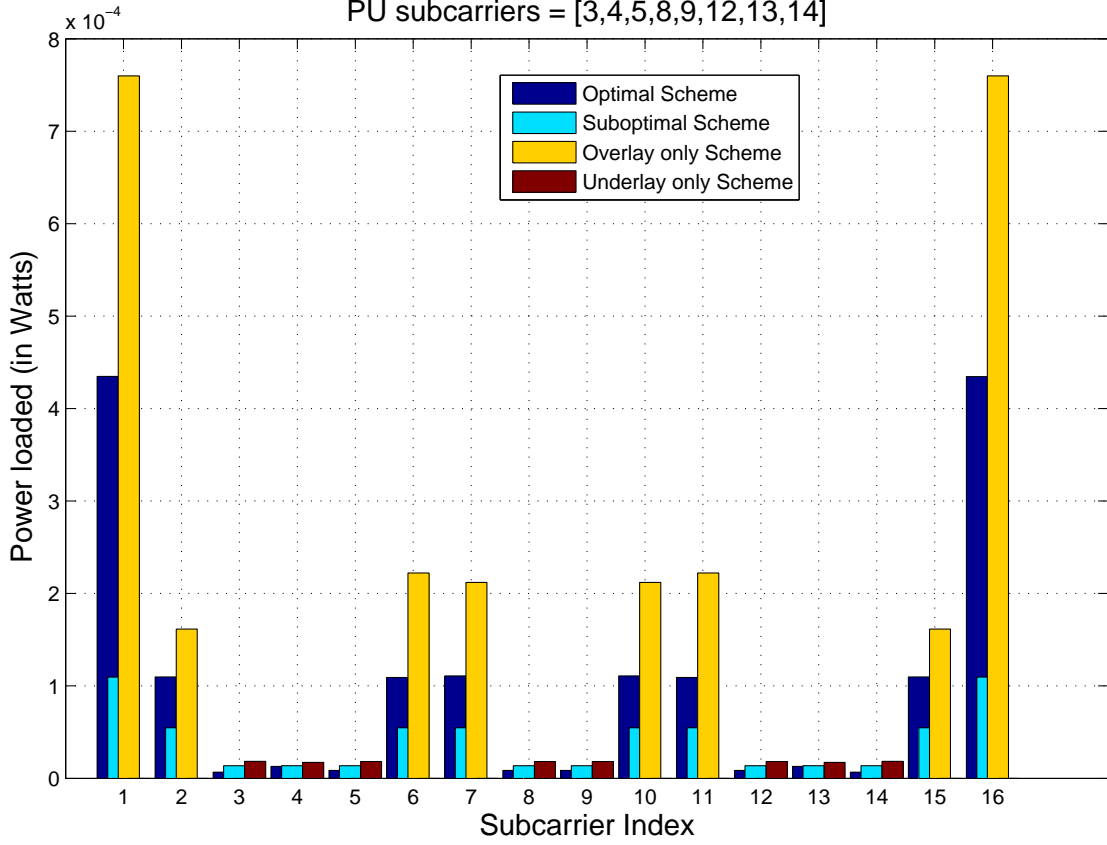


Figure 5.7: Power profile for various schemes for $I_{th}^{(l)} = 1 \times 10^{-6}$ Watts and total power budget = 10×10^{-3} Watts

Here, we observe that overlay only allocated the same power 1.25×10^{-5} Watts (1.25×10^{-5} Watts $\times 8 = 1 \times 10^{-4}$ Watts) in every overlay subcarrier. Underlay only scheme has also loaded 1.25×10^{-5} Watts in every underlay subcarrier. The reason is that as the interference threshold is very high, underlay and overlay subcarriers are similar. The proposed optimal scheme loads $.625 \times 10^{-5}$ Watts ($.625 \times 10^{-5}$ Watts $\times 16 = 1 \times 10^{-4}$ Watts) in every subcarrier. Because of the diversity gain obtained by loading power in 16 ($N + L$) subcarriers, optimal scheme achieves higher capacity for same power budget.

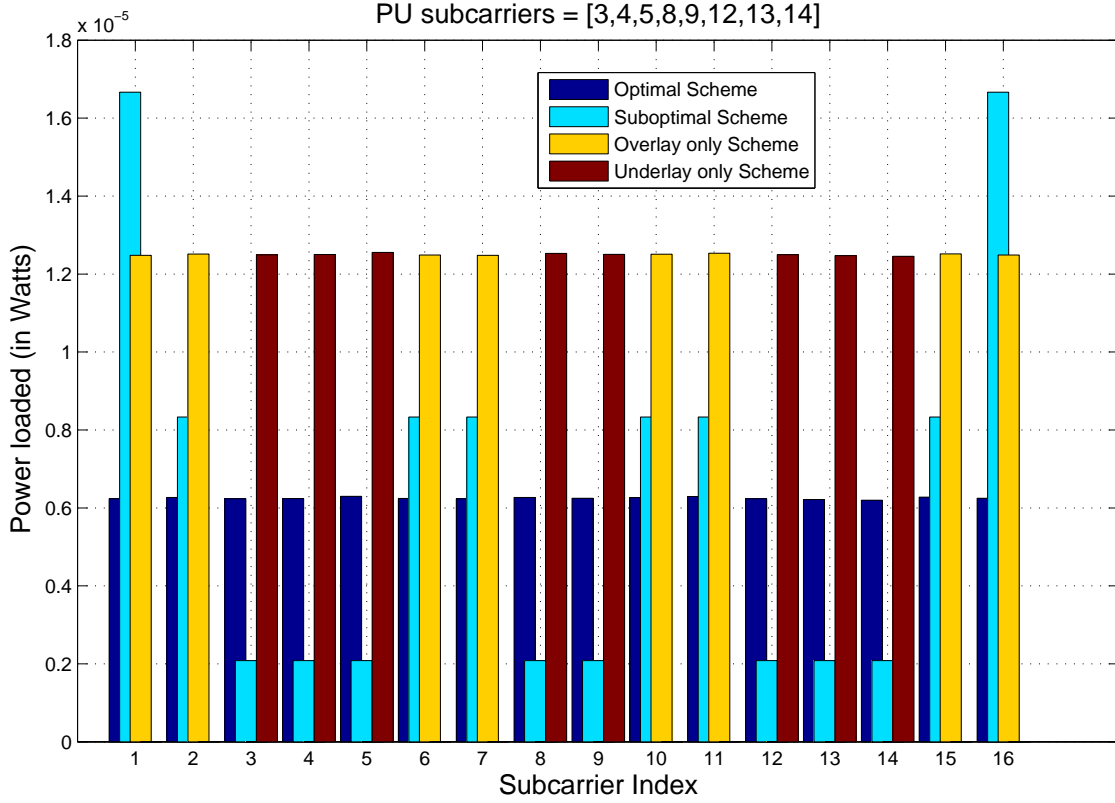


Figure 5.8: Power profile for various schemes for $I_{th}^{(l)} = 1 \times 10^{-4}$ Watts and total power budget = 1×10^{-4} Watts

In Fig. 5.9, we have fixed the values of $I_{th}^{(l)}$ to be 1×10^{-6} Watts and total power budget to 10×10^{-3} Watts. Now, the mean (λ_l) of the channel gains $h_{SP}(1)$, $h_{SP}(2)$, $h_{SP}(3)$, $h_{SP}(4)$, $h_{SP}(5)$, $h_{SP}(6)$, $h_{SP}(7)$, and $h_{SP}(8)$, have been assumed to be $-5dB$, $-7dB$, $-10dB$, $-5dB$, $-10dB$, $-10dB$, $-7dB$, and $-5dB$, respectively. We can observe from the Fig. 5.9, that more power is loaded into the bands with low values of mean (λ_l). Like in subcarrier 5 the mean is $-10dB$ and hence, more power in both underlay only and optimal scheme is loaded. However, suboptimal scheme loads same power in all underlay subcarriers and hence, same power does not increase with change in the values of the mean (λ_l).

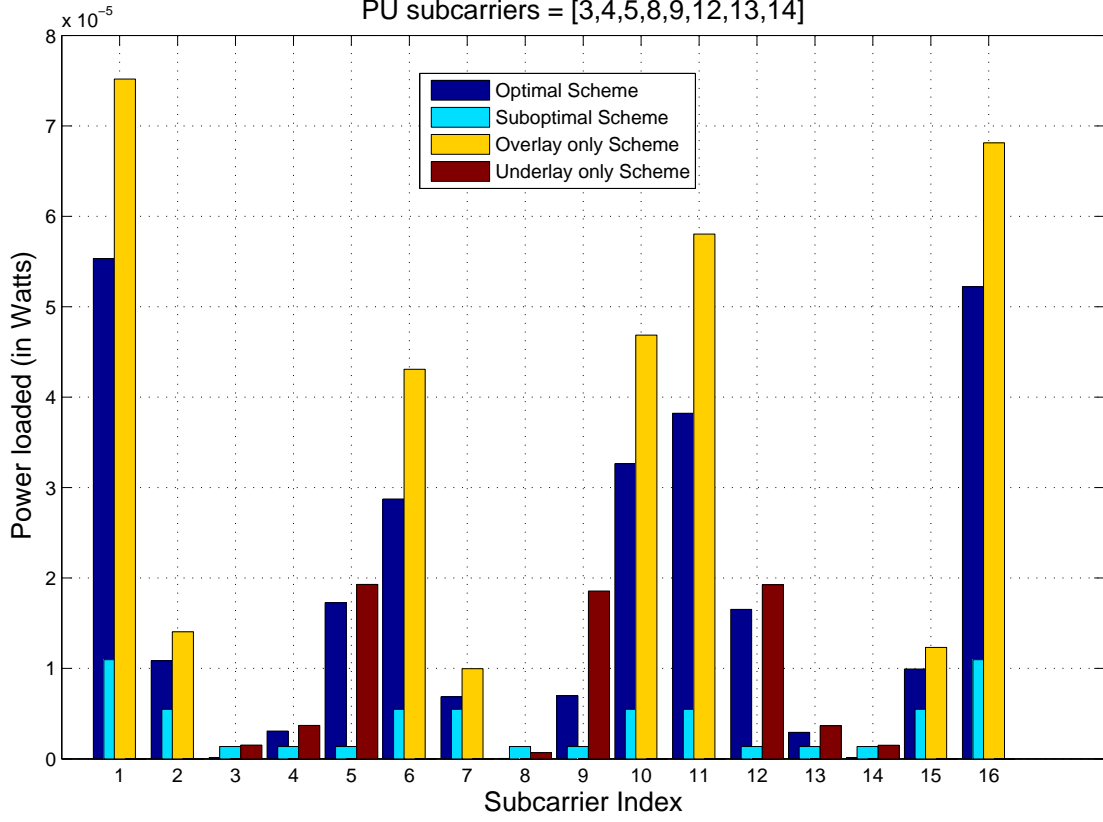


Figure 5.9: Power profile for various schemes for $I_{th}^{(l)} = 1 \times 10^{-6}$ Watts and total power budget = 10×10^{-3} Watts but for different values of λ_l

Further, in Fig. 5.10, we fix the values of $I_{th}^{(l)}$ to be 1×10^{-5} Watts and total power budget to 1×10^{-3} Watts. We change the number of CR users from 1 to 10. It can be seen from Fig. 5.10, that the capacity obtained by various schemes increase as the number of CR users increase. As base station assigns subcarrier to a user with highest channel gain, increase in number of users increases diversity and hence, capacity increases. In Fig. 5.11, we vary the values of probability (a) with which the interference introduced to the PU band has to remain below I_{th} . It can be seen that for $a = 1$, the capacity is 0, as it is not possible that with

probability 1 instantaneous interference remains below I_{th} . The capacity obtained increases as we decrease the value of a . It should be noted that for sufficiently high values of a , significant capacity can be obtained (like at $a = 0.95$, capacity of 1×10^7 bps can be achieved).

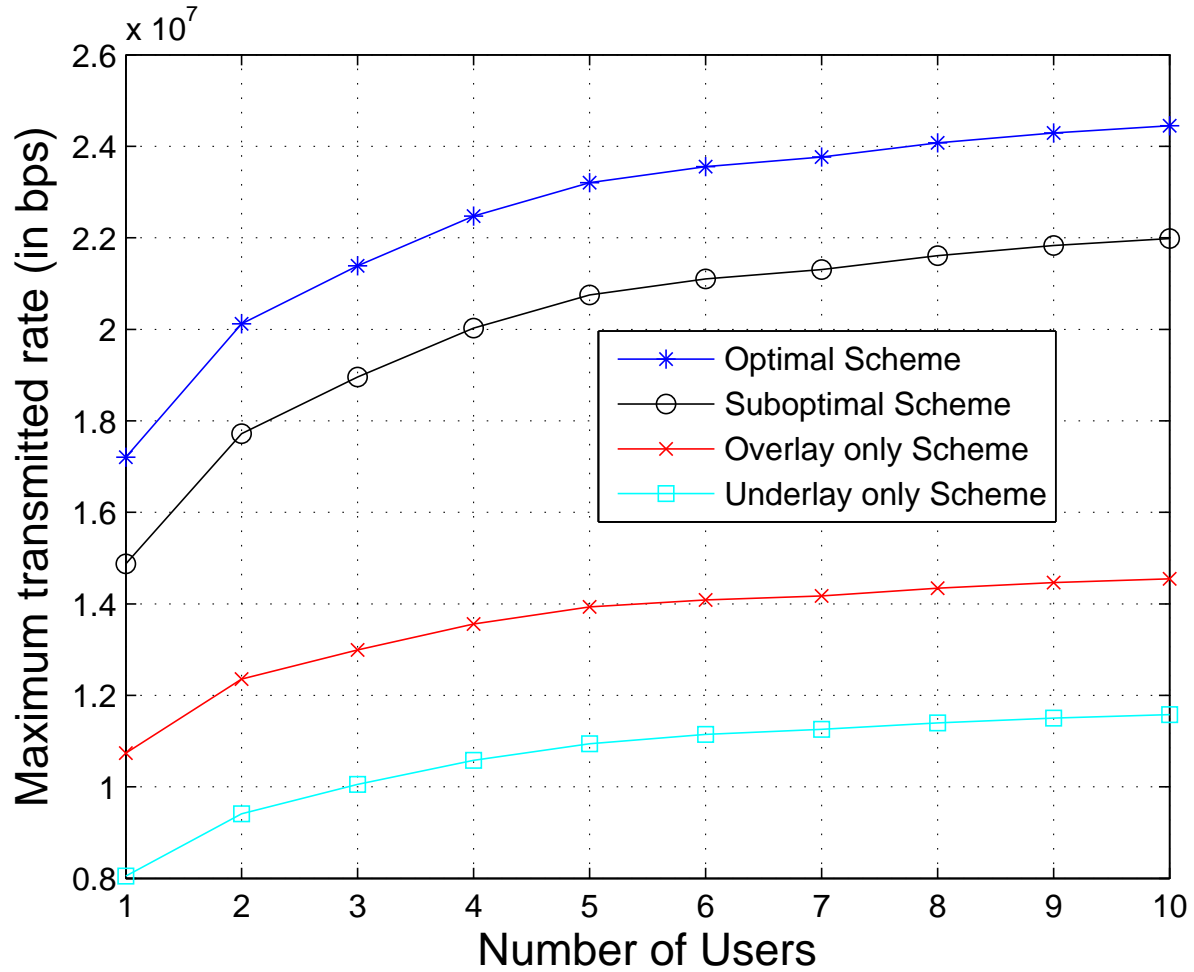


Figure 5.10: Total transmitted rate versus number of CR users for various schemes

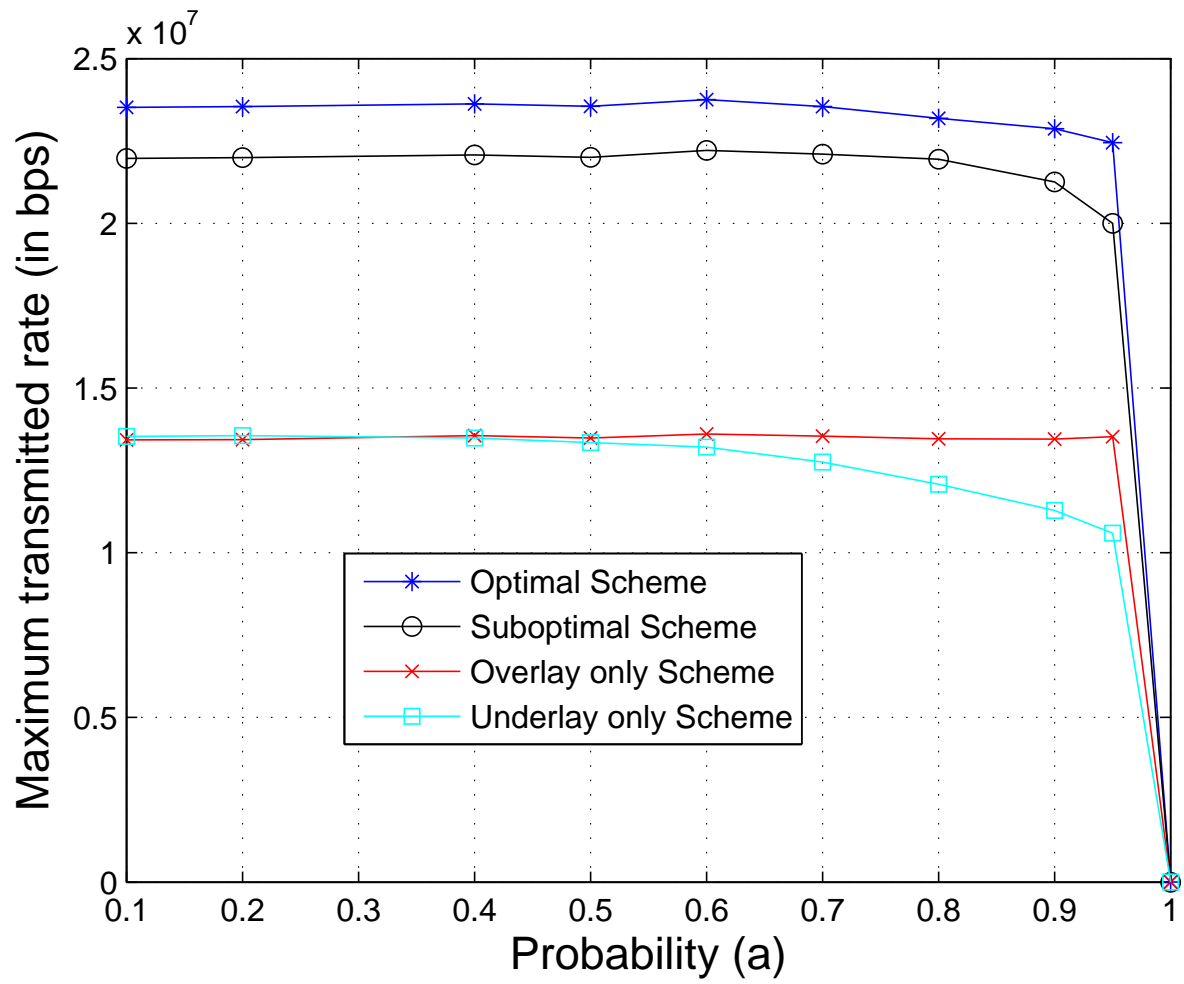


Figure 5.11: Total transmitted rate versus probability (a) with which instantaneous interference introduced to the PU band remains below interference threshold (I_{th}) for various schemes

5.8 Conclusions

In this chapter, we have proposed two schemes employing joint overlay and underlay power allocation for OFDM-based CR systems. The first scheme is an optimal scheme. It is based on Lagrange formulation that maximizes the down-link capacity of CR users, while maintaining a total power budget and keeping the interference introduced to the PU band below a threshold. The second scheme is a suboptimal scheme which provides a low complexity alternative to the first one, but nevertheless, performs comparably with the proposed optimal scheme. Both schemes significantly outperform classical schemes (overlay only or underlay only schemes), as we demonstrated numerically using simulations. Comparison of the complexity of various schemes shows that the proposed suboptimal scheme provides the aforementioned advantage in performance at a far lower cost.

Chapter 6

Relay and Power Allocation Schemes for OFDM-based Cognitive Radio Systems¹

6.1 Introduction

Cognitive radio (CR) has been proposed as a technology to improve the spectrum efficiency by opportunistically using unused licensed spectrum of a primary user (PU) [6]. In literature, orthogonal frequency division multiplexing (OFDM) has been recognized as a potential transmission technique for CR systems [6], [9]. However, in OFDM-based CR systems, even with near-perfect spectrum sensing, secondary users (SU) can produce significant interference to PUs operating in the adjacent bands and vice-a-versa. In [34], power allocation schemes have been proposed which maximize the capacity of CR user while keeping the interference introduced to the PU operating in these adjacent bands below a specified threshold.

Nevertheless, for the case when CR systems have a weak channel between

¹A paper based on the research work presented in this chapter has been submitted as: Dinesh Bharadia, Gaurav Bansal, Praveen Kaligineedi, and Vijay K. Bhargava, “Relay and Power Allocation Schemes for OFDM-based Cognitive Radio Systems” under minor revision in a journal.

source and destination, reliable communication might not be possible because the CR users can not transmit at high power as they might introduce unacceptable interference to PU in the adjacent bands. Therefore, the direct link of transmission between source and destination might not always be available [49], [50]. Instead, cooperative communications can be used to generate an alternative path between source and destination via relays and diversity can be achieved [51]. Also, using cooperative communications the data can be reliably transmitted by using lower power and hence, reducing the interference introduced to the PU band.

Cooperative transmission techniques have been proposed for direct sequence ultra wide-band (DS-UWB) based CR systems in [52] with the aim of avoiding interference to a PU present in the same band and maximizing the signal to noise ratio (SNR) at the destination. In this chapter, an OFDM based CR system is considered in which the channel between the source and destination is under a deep fade and hence, a direct link is not available. Therefore, the source transmits to the destination via CR based relays. Decode and forward (DF) relays which decode the transmitted signal and retransmit it to the destination are considered. For such a system, we investigate relay and power allocation schemes such that the Shannon capacity is maximized while taking into consideration the interference caused to the PUs in the adjacent bands as well as the total power consumed for transmission. We show that joint relay and power allocation is a mixed-integer optimization problem which is highly complex to solve. Hence, in this chapter, we solve the problem in two steps: We first allocate relays to sub-carriers assuming equal power allocation and then power is allocated to each relay. We explore both optimal and sub-optimal schemes for relay and power allocation. Further, numerical results are presented which shows the strength of proposed schemes.

The organization of the chapter is as follows. The system model is described in Section 6.2. The optimization problem and the proposed suboptimal schemes are presented in Section 6.3. In Section 6.4, the numerical results are discussed and finally the conclusions are drawn in Section 6.5.

6.2 System Model

We consider OFDM-based CR system with relays in the network as shown in Fig. 6.1. We assume that the channel between CR source (S) and CR destination (D) is weak and that the direct link does not exist between S and D, as shown in Fig. 6.1 with a dotted line. Here, we assume that spectrum sensing has been performed and the source and the relays have the knowledge of the bands available for transmission. Further, we assume that there are K relays and N CR subcarriers in the network. The relays $\{R_k\}_{k=1}^K$ are perfectly synchronized in time and frequency domain. Let h_{Sk}^i and h_{kD}^i denote the instantaneous channel gains of i^{th} subcarrier for CR user from S to relay R_k and from relay R_k to D, respectively. h_{SP}^l and h_{kP}^l denote the instantaneous channel gains between S and the l^{th} PU receiver, and between R_k and the l^{th} PU receiver, respectively.

We assume that a feedback channel exists between S and relays, and between relays and D. Hence, the knowledge of instantaneous channel gains h_{Sk}^i and h_{kD}^i through conventional techniques of estimation and feedback has been assumed. Further, we assume that CR network has a perfect knowledge of channel state information between PU and itself [34], [53], [39]. Therefore, channel gains h_{SP}^l and h_{kP}^l are assumed to be known at S (CR source) and CR relays respectively. These channel gains can be obtained by estimating the received signal power from primary terminal and assuming the channel reciprocity. However, it should be noted that the relay and power allocation schemes proposed in this work can be easily extended to a scenario where only the statistics of channel fading instead of perfect channel gains are known. For such a scenario, interference introduced to PU band can be guaranteed in a statistical manner, i.e., it can be guaranteed with any given probability that the interference introduced to the PU band always remains below interference threshold. Following the work in [54], it can be shown that when channel fading statistics follow a Rayleigh distribution, the interference constraint still remains linear and convex optimization techniques proposed in the next section will still be applicable.

In spectral domain, side-by-side access model has been considered as in [34].

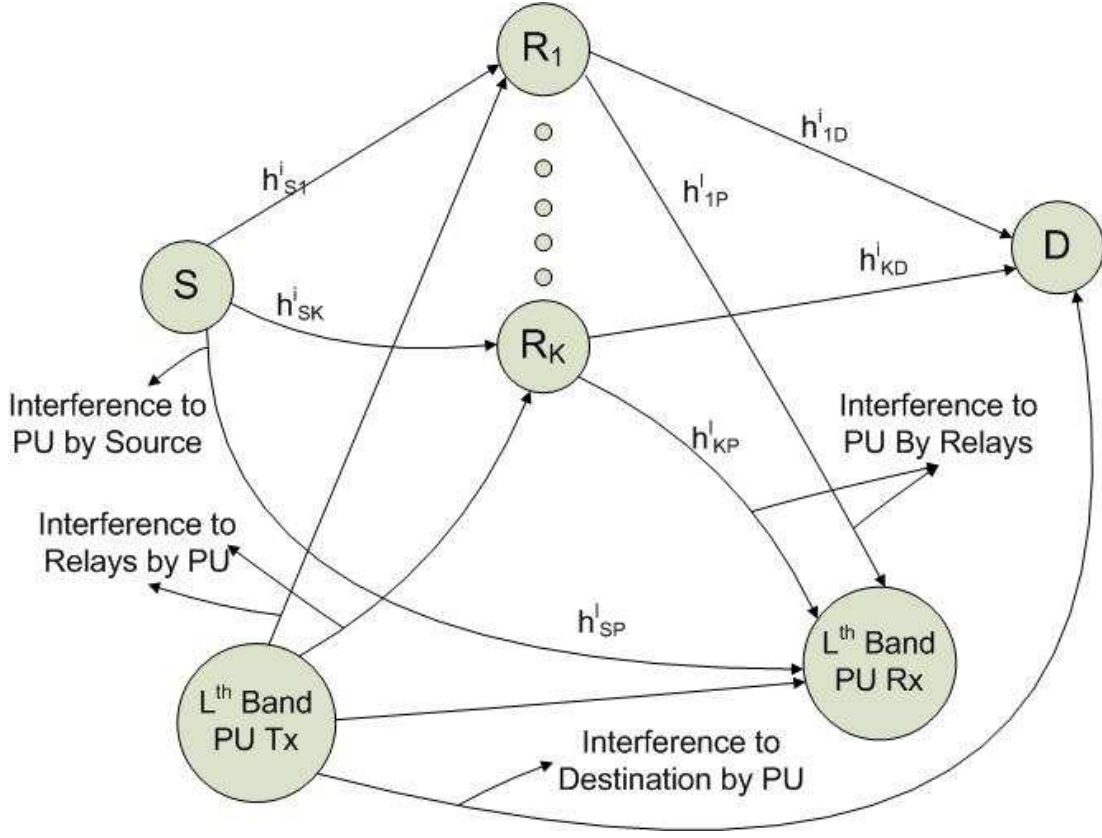


Figure 6.1: System model

It is assumed that there are L PU bands, with l^{th} PU having a bandwidth B_l . The remaining unused spectrum is divided among N subcarriers each with bandwidth Δf . In first time slot, S transmits information to R_k on i^{th} subcarrier with power P_{Sk}^i and in the second time slot, R_k transmits information to D on i^{th} subcarrier with power P_{kD}^i . Shannon capacity of i^{th} subcarrier is given by

$$C_{Sk}^i = \Delta f \log_2 \left(1 + \frac{|h_{Sk}^i|^2 P_{Sk}^i}{\sigma_k^2 + \sum_{l=1}^L J_{ik}^l} \right), \quad (6.1)$$

$$C_{kD}^i = \Delta f \log_2 \left(1 + \frac{|h_{kD}^i|^2 P_{kD}^i}{\sigma^2 + \sum_{l=1}^L J_i^l} \right) \quad (6.2)$$

where C_{Sk}^i and C_{kD}^i represent capacity of the channels between S to R_k and R_k to D, respectively. σ_k^2 and σ^2 are assumed to be the variance of additive white Gaussian noise (AWGN) at R_k and D, respectively. The interference introduced by the l^{th} PU to R_k and D on the i^{th} subcarrier are denoted by J_{ik}^l and J_i^l respectively. We assume PU interference J_{ik}^l and J_i^l to be AWGN and it is assumed that interferences can be measured by the CR network [34].

The interference introduced to PU band by CR transmission is as follows [17], [34]

$$I_{SPU} = \sum_{l=1}^L \sum_{i=1}^N \sum_{k=1}^K \rho_k^i |h_{SP}^l|^2 P_{Sk}^i T_s G_i^l, \quad (6.3)$$

$$I_{RPU} = \sum_{l=1}^L \sum_{i=1}^N \sum_{k=1}^K \rho_k^i |h_{kP}^l|^2 P_{kD}^i T_s G_i^l \quad (6.4)$$

where $G_i^l = \int_{d_{il}-B_l/2}^{d_{il}+B_l/2} \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 df$, d_{il} represents the spectral distance between the i^{th} CR subcarrier and the l^{th} PU band, T_s is the symbol duration, ρ_k^i is a binary decision variable which indicates i^{th} subcarrier is allocated to R_k , I_{SPU} and I_{RPU} represent total interference introduced to the PU band by transmission from S to all relays, and by transmission from all relays to D, respectively.

6.3 Problem Formulation and Proposed Schemes

The optimization problem can be expressed as follows,

$$\max_{\rho_k^i, P_{Sk}^i, P_{kD}^i} \sum_{i=1}^N \sum_{k=1}^K \rho_k^i \min \{C_{Sk}^i, C_{kD}^i\} \quad (6.5)$$

subject to,

$$\begin{aligned}
I_{SPU} &\leq I_{th}, \\
I_{RPU} &\leq I_{th}, \\
\sum_{k=1}^K \rho_k^i &= 1, \forall i \\
\rho_k^i &\in \{0, 1\}, \forall k, i. \\
P_{Sk}^i &\geq 0, P_{kD}^i \geq 0, \forall k, i, \\
\sum_{i=1}^N \sum_{k=1}^K \rho_k^i (P_{sk}^i + P_{kD}^i) &\leq P_T,
\end{aligned} \tag{6.6}$$

where P_T is the maximum total power that can be used for transmission, I_{th} is the maximum allowed interference to the PU band.

We use an approach similar to [55] to solve Eq. (6.5). It can be shown that the minimum of the capacities in the objective function in Eq. (6.5) is maximized if the SNR at both relay and D are equal for the given total power constraint [56]. Therefore,

$$\frac{|h_{Sk}^i|^2 P_{Sk}^i}{\sigma_k^2 + \sum_{l=1}^L J_{ik}^l} = \frac{|h_{kD}^i|^2 P_{kD}^i}{\sigma^2 + \sum_{l=1}^L J_i^l} \tag{6.7}$$

Using Eq. (6.7), the optimization problem in Eqs. (6.5), (6.6) can be reformulated as

$$\min_{\rho_k^i, P_k^i} \sum_{i=1}^N \sum_{k=1}^K -\rho_k^i \{ \Delta f \log_2(1 + \eta_k^i P_k^i) \} \tag{6.8}$$

subject to,

$$\begin{aligned}
\sum_{l=1}^L \sum_{i=1}^N \sum_{k=1}^K \rho_k^i B_{ik}^l P_k^i &\leq I_{th} \\
\sum_{l=1}^L \sum_{i=1}^N \sum_{k=1}^K \rho_k^i A_{ik}^l P_k^i &\leq I_{th} \\
\sum_{i=1}^N \sum_{k=1}^K \rho_k^i P_k^i &\leq P_T \\
\sum_{k=1}^K \rho_k^i &= 1 \quad \forall i \\
P_k^i &\geq 0, \quad \forall k, i, \\
\rho_k^i &\in \{0, 1\}, \quad \forall k, i.
\end{aligned} \tag{6.9}$$

where $P_k^i = P_{Sk}^i + P_{kD}^i$, $e_k^i = \frac{|h_{Sk}^i|^2}{\sigma_k^2 + \sum_{l=1}^L J_{ik}^l}$, $d_k^i = \frac{|h_{kD}^i|^2}{\sigma^2 + \sum_{l=1}^L J_i^l}$, $\eta_k^i = \frac{e_k^i d_k^i}{e_k^i + d_k^i}$, $B_{ik}^l = |h_{SP}^l|^2 \frac{d_k^i}{e_k^i + d_k^i} T_s G_i^l$, $A_{ik}^l = |h_{kP}^l|^2 \frac{e_k^i}{e_k^i + d_k^i} T_s G_i^l$

The optimization problem in Eq. (6.8), given the constraints in Eq. (6.9), is a mixed integer optimization problem and is *NP*-hard. Hence, in this chapter we propose sub-optimal schemes. The optimization problem is solved in two steps. First while assuming equal power allocation, we sub-optimally assign a relay to each subcarrier and then we optimally allocate the power to the relays for the assigned subcarriers.

6.3.1 Relay Allocation

In this subsection, we consider relay assignment techniques assuming equal power allocation to all subcarriers (i.e. $P_k^i = P_T/N$). Optimal relay assignment requires solving a discrete optimization problem which is *NP*-hard. Therefore, we propose three low-complexity suboptimal schemes to allocate relays to subcarriers as follows.

Scheme A

In this scheme, the integrality constraint on ρ_k^i is relaxed (ρ_k^i can have any real value between 0 and 1) which makes the problem in Eqs. (6.8), (6.9) a linear continuous optimization problem. Further while allocating equal power to all subcarriers ($P_k^i = P_T/N$), interference constraints ($I_{SPU} \leq I_{th}$, and $I_{RPU} \leq I_{th}$) might not satisfy. Hence, we relax $\sum_{k=1}^K \rho_k^i = 1$ constraint to $\sum_{k=1}^K \rho_k^i \leq 1$, so that a feasible solution is guaranteed. After solving this linear program (LP) for ρ_k^i , the relay R_k with highest ρ_k^i is assigned to the i^{th} subcarrier.

Hence, K_A which denotes the relay allocated to the i^{th} subcarrier using Scheme A can be written as,

$$K_A(i) = \arg \max_k \rho_k^i \quad \forall i \quad (6.10)$$

where ρ_k^i is calculated as described above. The complexity of this algorithm is $O(K^3N^3)$ as it involves solving a LP.

Scheme B

The previous scheme used linear programming to allocate the relay to a particular subcarrier. Here we reformulate the objective function in Eq. (6.8) as follows,

$$\min_{\rho_k^i, P_k^i} \sum_{i=1}^N \sum_{k=1}^K -\Delta f \log_2(1 + \rho_k^i \eta_k^i P_k^i) \quad (6.11)$$

The reformulated objective function in Eq. (6.11) is equivalent to the one in Eq. (6.8), for binary values of ρ_k^i . While allocating equal power to all subcarriers, we use the reformulated objective function to propose a sub-optimal closed form solution for ρ_k^i .

Relaxing the integrality constraint on ρ_k^i makes the reformulated problem convex. As in Scheme A, we relax $\sum_{k=1}^K \rho_k^i = 1$ constraint to $\sum_{k=1}^K \rho_k^i \leq 1$. Now, the

convex problem in Eq. (6.11) is as follows,

$$\min_{\rho_k^i} \sum_{i=1}^N \sum_{k=1}^K - \{ \Delta f \log_2(1 + \rho_k^i \xi_k^i) \} \quad (6.12)$$

subject to,

$$\begin{aligned} \sum_{l=1}^L \sum_{i=1}^N \sum_{k=1}^K \rho_k^i B_{ik}^l P_T &\leq N I_{th} \\ \sum_{l=1}^L \sum_{i=1}^N \sum_{k=1}^K \rho_k^i A_{ik}^l P_T &\leq N I_{th} \\ \sum_{k=1}^K \rho_k^i &\leq 1 \quad \forall i \\ \rho_k^i &\in (0, 1], \quad \forall k, i. \end{aligned} \quad (6.13)$$

where $\xi_k^i = \frac{\eta_k^i P_T}{N}$.

The above optimization problem described in Eqs. (6.12), and (6.13) is a convex optimization problem and applying Karush-Kuhn-Tucker (KKT) conditions [28], closed form solution is obtained as follows,

$$\rho_k^i = \max \left\{ 0, \frac{\Delta f}{\lambda_1 (\sum_{l=1}^L B_{ik}^l P_T) + \lambda_2 (\sum_{l=1}^L A_{ik}^l P_T) + v_i} - \frac{1}{\xi_k^i} \right\} \quad (6.14)$$

where $\lambda_1 \geq 0$, $\lambda_2 \geq 0$, and $v_i \geq 0 \quad \forall i$, are deterministic Lagrange parameters, which

are determined using following equations:

$$v_i \left(\sum_{k=1}^K \rho_k^i - 1 \right) = 0 \quad \forall i, \quad (6.15)$$

$$\lambda_1 \left\{ \sum_{l=1}^L \sum_{i=1}^N \sum_{k=1}^K \rho_k^i B_{ik}^l P_T - N I_{th} \right\} = 0 \quad (6.16)$$

$$\lambda_2 \left\{ \sum_{l=1}^L \sum_{i=1}^N \sum_{k=1}^K \rho_k^i A_{ik}^l P_T - N I_{th} \right\} = 0 \quad (6.17)$$

Now, K_B which denotes the relay allocated to the i^{th} subcarrier using Scheme B can be written as,

$$K_B(i) = \arg \max_k \rho_k^i \quad \forall i \quad (6.18)$$

where ρ_k^i is calculated using Scheme B. The complexity of the above proposed scheme is $O(K^3 N^3)$ as it involves solving linear system of equations, Eqs. (15)-(17). Further motivated by the closed form solution of ρ_k^i obtained in Eq. (6.14), we propose a low-complexity heuristic scheme in next section.

Scheme C

We propose a heuristic scheme for assignment of subcarriers for relays based on Eq. (6.14). ξ_k^i is the effective signal to interference-plus-noise ratio (SINR) when relay R_k is assigned to the i^{th} subcarrier, $\sum_{l=1}^L B_{ik}^l$ is proportional to the cumulative interference introduced by the CR source S to the PU bands, and $\sum_{l=1}^L A_{ik}^l$ is proportional to the cumulative interference introduced by relay R_k to the PU bands. Hence, we propose a heuristic scheme where ρ_k^i are assigned as follows,

$$\rho_k^i = \frac{\xi_k^i}{\sum_{l=1}^L B_{ik}^l + \sum_{l=1}^L A_{ik}^l}. \quad (6.19)$$

ρ_k^i in the above equation represents the ratio of the channel gain for the transmission between S and D, using relay k for subcarrier i to the interference caused to

the PU by this transmission. Intuitively, high ρ_k^i would imply that the relay k can give better throughput (higher channel gain) at lower interference to the PU band over the i^{th} subcarrier. Therefore, the relay R_k with maximum ρ_k^i is allocated to the i^{th} subcarrier. Hence,

$$K_C(i) = \arg \max_k \rho_k^i \quad \forall i \quad (6.20)$$

where ρ_k^i is calculated using Scheme C. The computational complexity of this heuristic scheme is $O(KN)$. After allocating relays to subcarriers, in the next section we propose a method for power allocation to the subcarriers.

6.3.2 Power Allocation

Now, for a given allocation of relay to the subcarriers, denoted by $K(i)$ (which could be $K_A(i)$, $K_B(i)$, or $K_C(i)$), the optimization problem in Eq. (6.8) given the constraints in Eq. (6.9) can be rewritten as,

$$\max_{P_{K(i)}^i} \sum_{i=1}^N \Delta f \log_2(1 + \eta_{K(i)}^i P_{K(i)}^i) \quad (6.21)$$

subject to,

$$\begin{aligned} \sum_{l=1}^L \sum_{i=1}^N B_{iK(i)}^l P_{K(i)}^i &\leq I_{th}, \\ \sum_{l=1}^L \sum_{i=1}^N A_{iK(i)}^l P_{K(i)}^i &\leq I_{th} \\ \sum_{i=1}^N P_{K(i)}^i &\leq P_T, \\ P_{K(i)}^i &\geq 0, \quad \forall i, \end{aligned} \quad (6.22)$$

This is a convex optimization problem with linear constraints and can be solved by using KKT conditions. The closed form solution is as follows,

$$P_{K(i)}^i = \max \left\{ 0, \frac{\Delta f}{\lambda_4 \{\sum_{l=1}^L B_{iK(i)}^l\} + \lambda_5 \{\sum_{l=1}^L A_{iK(i)}^l\} + \lambda_6} - \frac{1}{\eta_{K(i)}^i} \right\} \quad (6.23)$$

where $\lambda_4 \geq 0$, $\lambda_5 \geq 0$ and $\lambda_6 \geq 0$ are deterministic Lagrange parameters, which are determined using following equations:

$$\lambda_6 \left(\sum_{i=1}^N P_{K(i)}^i - P_T \right) = 0, \quad (6.24)$$

$$\lambda_4 \left\{ \sum_{l=1}^L \sum_{i=1}^N B_{iK(i)}^l P_{K(i)}^i - I_{th} \right\} = 0 \quad (6.25)$$

$$\lambda_5 \left\{ \sum_{l=1}^L \sum_{i=1}^N A_{iK(i)}^l P_{K(i)}^i - I_{th} \right\} = 0 \quad (6.26)$$

The computational complexity involved in assigning power to the subcarriers is $O(N^3)$.

6.3.3 Optimal Solution

For assessing the numerical strength of our proposed scheme, we propose an optimal solution which is obtained by exhaustive search and hence has a very high complexity. Specifically, all possible combinations of allocating relays to subcarriers are considered, and for a given relay allocation power is allocated in an optimal fashion i.e., the allocation which maximizes the capacity for given interference and total power constraint is chosen. The optimal solution requires an exhaustive search, and hence the complexity of optimal solution is $O(K^N \cdot N^3)$.

6.4 Numerical Results

The values of L and N are chosen to be 2 and 6, respectively.² The simulation model is shown in Fig. 6.2. Here we assume the values of Δf to be 0.3125 MHz, which is same as subcarrier frequency spacing in wireless local area network (LAN) standards [30], [31]. The values of B_1 , and B_2 are assumed to be 1 MHz, and 2 MHz, respectively. The values of σ^2 , σ_k^2 are assumed to be equal to 10^{-3} W. The values of interferences J_i^l , J_{ik}^l has been taken to be 10^{-6} W. The channel gains h_{kP}^l , h_{SP}^l , h_{Sk}^i , h_{kD}^i follow Rayleigh fading with a mean power gain of -10 dB. The maximum total power (P_T) is assumed to be 10^{-3} W. The simulations have been conducted for 10000 iterations, so that we can obtain average capacity values for various schemes under consideration.

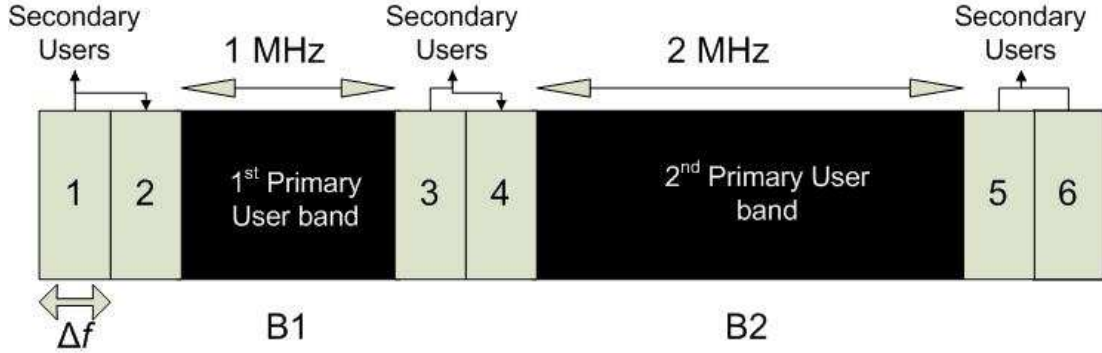


Figure 6.2: PU and CR user distribution in frequency domain

In Fig. 6.3, we plot capacity obtained by CR users under different schemes versus the interference threshold to PU (I_{th}), in a CR system with three relays. We have also plotted the results of the optimal solution. It can be observed in Fig. 6.3, that Scheme A and Scheme B perform similar and, are very close to optimal scheme. We can also observe that Scheme C performs a bit worse than the other schemes. However, note that Scheme C is based on heuristics and has a relatively

²In practice the values of L and N would be high, but for simplicity in the simulation analysis we have assumed the values to be 2 and 6, respectively. Also, it should be noted that the trends in the results presented in this chapter would still hold for other values of L and N .

low computational complexity.

In Fig. 6.4, we compare capacity obtained by CR users under different schemes with increasing number of relays but keeping interference threshold (I_{th}) constant to 10^{-6} W. We observe that as the number of relay increases, the performance of the proposed sub-optimal schemes decrease as compared to the optimal solution.

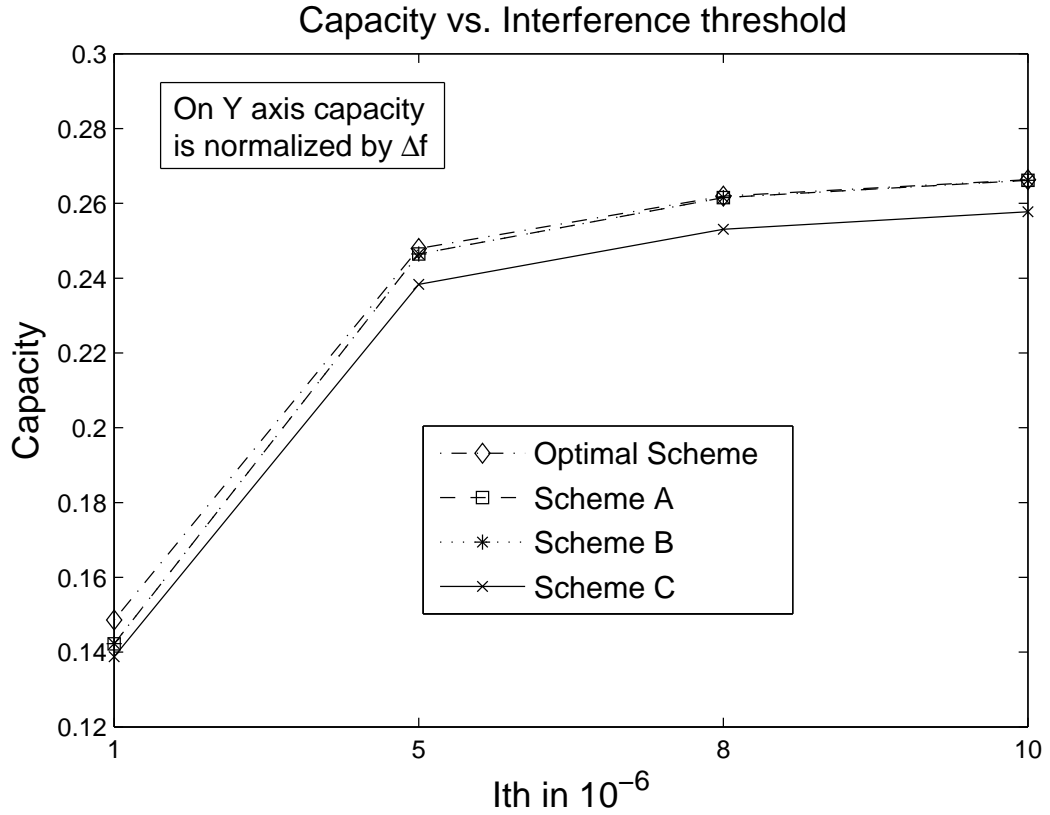


Figure 6.3: Capacity of CR users versus interference threshold (I_{th}) for a fixed number (3) of relays.

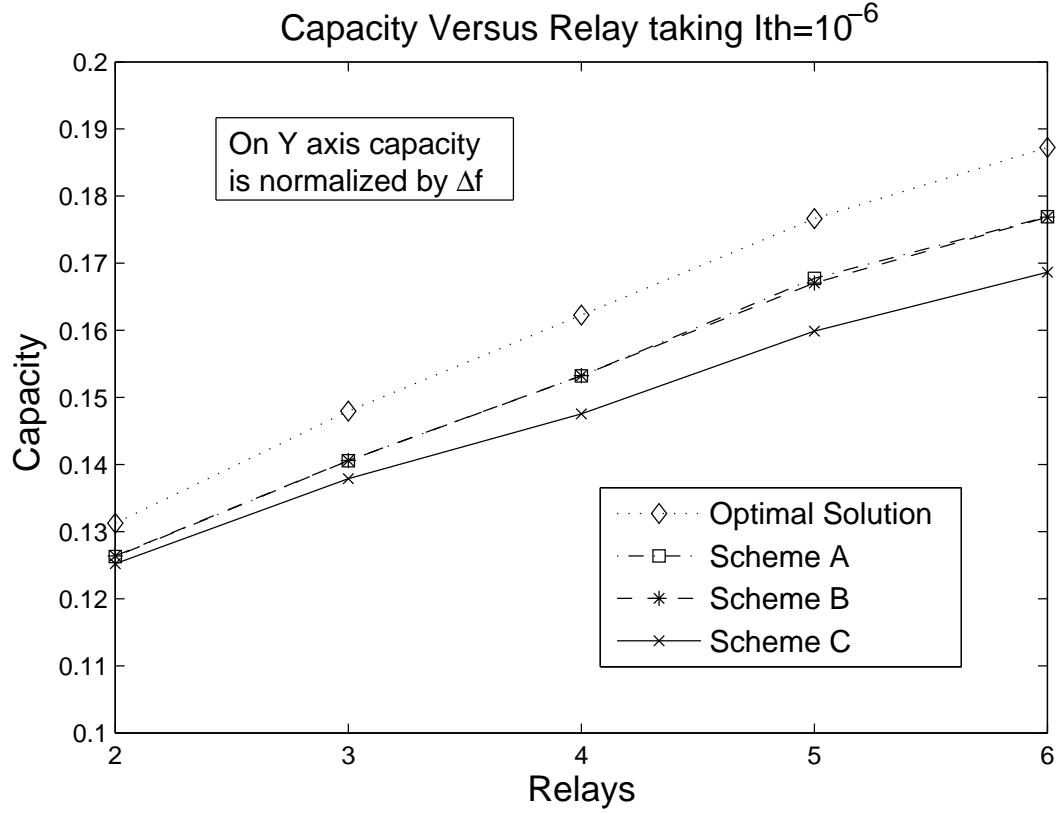


Figure 6.4: Capacity of CR users versus relays for a fixed value (10^{-6} Watts) of interference threshold (I_{th}).

6.5 Conclusions

In this chapter, we have presented relay and power allocation schemes for OFDM-based CR systems. The capacity of CR user is maximized in a relaying environment. The constraints on total transmission power and interference introduced to the PU band have been taken into account. The problem is a mixed-integer optimization problem and hence, an analytical optimal solution is hard to find. In this chapter, we have proposed three suboptimal relay and power allocation techniques. Complexity analysis and numerical results are presented.

Chapter 7

Conclusions and Future Research Directions

7.1 Conclusions

In this thesis, we have developed various dynamic resource allocation schemes for OFDM-based CR systems. We have made following six major contributions in this dissertation.

First, we have developed an optimal power loading algorithm that maximizes the downlink transmission data rate of the CR user while the interference introduced to the PU remains within a given limit. We have also proposed two sub-optimal power loading algorithms that have less complexity but can achieve a performance close to the optimal one. Presented numerical results show that the classical loading algorithms used for conventional wireless networks perform the worst for the CR scenario among the schemes considered. We have also studied the effect of nulling mechanisms on the performances of various schemes. Selected numerical results have demonstrated that the optimal scheme performs the best, and that the one-nulling case achieves better data rate performance than cases with a greater number of nullings, as well as zero-nulling cases. Finally, we have studied the case where the channel gain information is not perfectly known at the

CR transmitter and found that even in this case proposed schemes perform better than the classical schemes. The work has been published in [57], [34].

Second, we have studied the interference performance of the well-known discrete bit loading algorithms when they are used for OFDM-based CR systems. In order to minimize the interference to the primary user's band, a suboptimal algorithm has been proposed. We have also proposed two schemes based on modifications in the existing discrete bit loading schemes namely the Hughes-Hartogs [23] and Chow et al. [24] schemes. Presented numerical results demonstrate the strength of our proposed schemes. The work has been published in [58], [59].

Third, we have developed an optimal power allocation algorithm for the OFDM-based CR system. Instead of instantaneous channel fading gain between the PU receiver and the CR transmitter, the developed optimal power allocation scheme requires the fading statistics and parameters to be known at the CR transmitter. As such the transmission rate of the CR user is maximized for a given power budget and different probabilistic interference constraints imposed by different PU systems. We also proposed and investigated performance of a low complexity suboptimal power allocation scheme. Presented selected numerical results showed that our proposed optimal power allocation achieves significantly higher transmission capacity for CR user compared to the classical power allocation schemes namely, uniform and water-filling power allocation schemes that is used for conventional OFDM-based system. The suboptimal scheme achieved better performance than the uniform loading scheme. The work has been published as [60] and the journal version is to be submitted as [54].

Fourth, we have proposed two schemes employing joint overlay and underlay power allocation for OFDMA-based CR systems. The first scheme is an optimal scheme. It is based on Lagrange formulation that maximizes the downlink capacity of CR users, while maintaining a total power budget and keeping the interference introduced to the PU band below a threshold. The second scheme is a suboptimal scheme which provides a low complexity alternative to the first one, but nevertheless, performs comparably with the proposed optimal scheme.

Both schemes significantly outperform classical schemes (overlay only or underlay only schemes), as we demonstrated numerically using simulations. The work is under preparation to be submitted as [61].

Finally, we have presented relay and power allocation schemes for OFDM-based CR systems. The capacity of CR user is maximized in a relaying environment. The constraints on total transmission power and interference introduced to the PU band have been taken into account. The problem is a mixed-integer optimization problem and hence, an analytical optimal solution is hard to find. In this chapter, we have proposed three suboptimal relay and power allocation techniques. Complexity analysis and numerical results are presented. The work has been submitted for publication as [62].

7.2 Future Work and Work in Progress

The research conducted in this thesis have given rise to many challenging and interesting problems. Our current works are addressing various of these problems. A brief description of some of these issues is as follows.

7.2.1 Power Allocation Schemes for OFDM-based CR Systems with Imperfect Sensing

One of the important tasks of CR user is to reliably detect the available spectrum. In this thesis, we assumed perfect spectrum sensing. However, as CR employs sensing mechanism to determine the presence of PU, there might be an error in the detection of PU. The probability of missed detection would determine the amount of interference caused to the PU by the CR system. The probability of false alarm would determine the throughput of the CR system. Recently, some work has been done in [63], [64], where resource allocation schemes have been designed while incorporating the interference due to errors in spectrum sensing but the mutual interference considered in this thesis (which exist even CR and PU are in side-by-side bands) have not been considered. An interesting future research would be to

take not only the mutual interference but also the interference introduced due to errors in spectrum sensing into account. Dynamic resource allocation algorithms needs to be designed for such a system model.

7.2.2 Cross Layer Design of OFDM-based CR Systems

Cross-layer design can be a very useful tool for efficient utilization of resources available in wireless networks [65]. How efficiently can the physical layer functionalities (e.g. spectrum sensing, channel quality etc.) interact with the upper-layer MAC, network and transport-layer protocols like scheduling, routing and rate control, can play a very crucial part in achieving higher spectrum efficiency. The work conducted in this thesis (power control) for CR systems can be extended to optimization across various layers. Specifically, cross-layer schemes can be devised that will allow joint optimization of some or all of the following parameters: power allocation, (physical layer attributes), routing (MAC layer attribute), and rate (transport layer attribute) while making sure that the interference introduced to the PU bands remains below a specified threshold.

7.2.3 Resource Allocation Schemes for Cooperative Relaying for CR Networks

The work conducted in Chapter 6 of this thesis assumes dedicated relays employed for achieving the diversity gain. However, recently cooperative relaying has been proposed to improve spectrum diversity in CR networks [66], where CR users can act as a relay for each other. Large benefits in terms of improving efficiency and fairness of resource sharing can be gained by cooperation among CR nodes. Specifically, some CR users because of their low traffic demand do not need to use the entire available bandwidth (spectrum holes). However, they can improve spectrum efficiency by acting as relays for the CR user's who have high traffic demand but low available bandwidth. For such a network resource (relay, power etc.) allocation algorithms need to be designed. The work conducted in Chapter 6 of the thesis can be extended to design dynamic resource allocation algorithms

for cooperative relaying for CR networks.

Bibliography

- [1] R. W. Broderson, A. Wolisz, D. Cabric, S. M. Mishra, and D. Willkomm, "CORVUS: A cognitive radio approach for usage of virtual unlicensed spectrum," *white paper* submitted at the University of Berkeley, CA, Jul. 2004. → pages 1, 4
- [2] Federal Communications Commission, "Spectrum Policy Task Force," Rep. ET Docket no. 02-135, Nov. 2002. → pages 1
- [3] J. Mitola, "The software radio architecture," in *IEEE Commun. Mag.*, vol. 33, no. 5, pp. 26-38, May 1995. → pages 1
- [4] J. Mitola III, "Cognitive radio for flexible mobile multimedia communications," in *Proc. of 6th International Workshop on Mobile Multimedia Commun. (MoMuC'99)*, San Diego, CA, pp. 310, Nov. 1999. → pages 2, 4
- [5] J. Mitola III and G. Q. Maguire Jr., "Cognitive radios: Making software radios more personal," *IEEE Personal Commun. Mag.*, vol. 6, pp. 13-18, Aug. 1999. → pages 2
- [6] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE Journal on Select. Areas Commun.*, vol. 23, no. 2, pp. 201-220, Feb. 2005. → pages 2, 43, 60, 77, 104
- [7] D. Cabric, S. M. Mishra, and R. W. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," in *Proc. of Asilomar Conf. on Signals, Systems, and Computers*, pp. 772-776, Pacific Grove, CA, Nov. 2004. → pages 2

- [8] S.M. Mishra, A. Sahai, and R. Brodersen, "Cooperative sensing among cognitive radios," in *Proc. of IEEE Int. Conf. Commun. (ICC' 06)*, Istanbul, Turkey, June 2006. → pages 2
- [9] T. Weiss and F. K. Jondral, "Spectrum pooling: an innovative strategy for the enhancement of spectrum efficiency," *IEEE Commun. Mag.*, vol. 43, no. 3, pp. S8-S14, Mar. 2004. → pages 3, 4, 15, 44, 60, 77, 104
- [10] I. F. Akyildiz, W. Y. Lee, M. C. Vuran, and S. Mohanty, "NeXt generation/dynamic spectrum access/cognitive radio wireless networks: a survey," *Computer Networks Journal (Elsevier)*, vol. 50, pp. 2127–2159, Sept. 2006. → pages 4
- [11] Darpa XG Working group, "The xg architectural framework," rfc v1.0 2003. → pages 4
- [12] Darpa XG Working group, "The xg vision," rfc v1.0 2003. → pages 4
- [13] C. Cordeiro, K. Challapali, D. Birru, and S. Shankar, "IEEE 802.22: the first worldwide wireless standard based on cognitive radios," in *Proc. of IEEE Int. Symposium on Dynamic Spectrum Access Networks (DySPAN'05)*, pp. 328-337, Nov. 2005.
- [14] M. M. Buddhikot, P. Kolody, S. Miller, K. Ryan, and J. Evans, "DIMSUNNet: new directions in wireless networking using coordinated dynamic spectrum access," in *Proc. of IEEE Int. Symposium on a World of Wireless, Mobile and Multimedia Networks (WoWMoM'05)*, pp. 78-85, Jun. 2005. → pages 4
- [15] I. F. Akyildiz and Y. Li, "OCRA: OFDM-based cognitive radio networks," *Broadband and Wireless Networking Laboratory, Georgia Institute of Technology, Technical Report*, Mar. 2006. → pages 4
- [16] L. Xu, R. Tonjes, T. Paila, W. Hansmann, M. Frank, and M. Albrecht, "DRiVE-ing to the internet: dynamic radio for ip services in vehicular environments," in *Proc. of 25th Annual IEEE Conference on Local Computer Networks*, pp. 281-289, Nov. 2000. → pages 4
- [17] T. Weiss, J. Hillenbrand, A. Krohn, and F. K. Jondral, "Mutual interference in OFDM-based spectrum pooling systems," in *Proc. of IEEE Vehicular Technol. Conf. (VTC'04)*, vol. 4, pp. 1873-1877, May 2004. → pages 4

- [18] T. Keller, and L. Hanzo, "Multicarrier modulation: a convenient framework for time-frequency processing in wireless communications," in *Proc. of the IEE*, vol. 88, no. 5, pp. 611-640, May 2000.
- [19] A. T. Toyserkani, J. Ayan, S. Naik, Y. Made, and O. Al-Askary, "Sub-carrier based adaptive modulation in HIPERLAN/2 system," in *Proc. IEEE Int. Conf. Commun.*, Paris, France, June 2004 pp. 3460-3464. → pages 5, 7, 15, 16, 18, 20, 21, 31, 44, 45, 46, 47, 54, 61, 64, 80, 108
- [20] L. Goldfeld and V. Lyandres, "Capacity of the multicarrier channel with frequency-selective Nakagami fading," *IEICE Trans. Commun.*, vol. E83-B, no. 3, pp. 697-702, March 2000. → pages 8, 15
- [21] C. Y. Wong, R. S. Cheng, K. B. Letaief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, power, and power allocation," *IEEE J. Select. Areas Commun.*, vol. 17, no. 10, pp. 1747-1758, October 1999. → pages 8
→ pages 8
→ pages 8
- [22] A. M. Wyglinski, "Effects of bit allocation on non-contiguous multicarrier-based cognitive radio transceivers," in *Proc. 64th IEEE Veh. Technol. Conf. - Fall*, (Montreal, Canada), Sept. 2006. → pages 9, 16, 60
- [23] J. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Commun. Mag.*, vol. 28, no. 5, pp. 5-14, May 1990. → pages 9, 11, 44, 45, 50, 56, 120
- [24] P. Chow, J. Cioffi, and J. Bingham, "A practical discrete multitone transceiver loading algorithm for data transmission over spectrally shaped channels," *IEEE Trans. on Commun.*, vol. 43, no. 2/3/4, pp. 773-775, Feb./Mar./Apr. 1995. → pages 9, 11, 44, 45, 50, 51, 53, 56, 120
- [25] I. Budiarjo, H. Nikookar, and L. P. Ligthart, "Combined spectrum pooling and adaptive bit loading for cognitive radio OFDM based system", in *Proc. of Symposium on Communications and Vehicular Technology*, pp. 73-76, Nov. 2006, Liege.
- [26] H. A. Mahmoud, and H. Arslan, "Sidelobes suppression in OFDM-based spectrum sharing systems using adaptive symbol transition", *IEEE Commun. Letters*, Vol. 12, pp. 133-135, Feb. 2008.

- [27] Q. Lu, T. Peng, and W. Wang, "Efficient multiuser water-filling algorithm under interference temperature constraints in OFDMA-based cognitive radio networks", in *Proc. IEEE Int. Symp. Microwave, Antenna, Propagation, and EMC Technologies for Wireless Commun.*, pp. 174-177, Feb. 2007.
- [28] S. Boyd, and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004. → pages 16
- [29] T. M. Cover, and J. A. Thomas, *Elements of Information Theory*, John Wiley and Sons, 1991. → pages 16
- [30] IEEE802.11a, *Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: High-speed Physical Layer in the 5 GHz Band*, IEEE, Tech. Rep. 1999. → pages 16
- [31] ETSI-TS-101475, *Broadband Radio Access Networks (BRAN); HIPERLAN Type 2; Physical (PHY) Layer*, ETSI, Tech. Rep., 2001. → pages 23, 68, 69, 86, 112
 → pages 31, 64, 72
 → pages 33, 53, 72, 91, 116
 → pages 33, 53, 72, 91, 116
- [32] S. T. Chung, and A. J. Goldsmith, "Degrees of freedom in adaptive modulation: A unified view," *IEEE Trans. on Commun.*, vol. 49, no. 9, pp. 1561-1571, Sep. 2001. → pages 48
- [33] J. M. Cioffi, "A multicarrier primer," *ANSI T1E1.4 Committee Contribution*, pp. 91-157, Nov. 1991. → pages 51
- [34] G. Bansal, Md. J. Hossain, and V. K. Bhargava, "Optimal and suboptimal power allocation schemes for OFDM-based cognitive radio systems," *IEEE Trans. on Wireless Commun.*, vol. 7, No. 11, pp. 4710-4718, Nov. 2008. → pages 61, 62, 63, 64, 70, 78, 80, 87, 104, 106, 108, 120
- [35] P. Cheng, Z. Zhang, H. Huang, and P. Qiu, "A distributed algorithm for optimal resource allocation in cognitive OFDMA systems," in *Proc. of the IEEE International Conf. on Commun. (ICC'08)*, pp. 4718-4723, Beijing, China, May 2008. → pages 61
- [36] D. Ngo, C. Tellumbra, and H. H. Nguyen, "Resource allocation for OFDM-based cognitive radio multicast networks," in *Proc. of the IEEE*

- Wireless Commun. and Networking Conf. (WCNC'09)*, pp. 1-6, Budapest, Hungary, April 2009. → pages 61, 78
- [37] P. Wang, M. Zhao, L. Xiao, S. Zhou, and J. Wang, "Power allocation in OFDM-based cognitive radio systems," in *Proc. of the IEEE Global Telecommun. Conf. (GLOBECOM'07)*, pp. 4061-4065, Washington DC, USA, Nov. 2007. → pages 61, 78
- [38] Z. Hasan, G. Bansal, E. Hossain, and V. K. Bhargava, "Energy-efficient power allocation in OFDM-based cognitive radio systems: A risk return model," *IEEE Trans. on Wireless Commun.*, vol. 8, no. 12, pp. 6078-6088, Dec. 2009. → pages 61
- [39] X. Kang, H. Garg, Y.C. Liang, and R. Zhang, "Optimal power allocation for OFDM-based cognitive radio with new primary transmission protection criteria," *IEEE Trans. on Wireless Commun.*, vol. 9, no. 6, pp. 2066-2075, June 2010. → pages 61, 106
- [40] A. Soysal, S. Ulukus, and C. Clancy, "Channel estimation and adaptive M-QAM in cognitive radio links," in *Proc. of the IEEE International Conf. on Commun. (ICC'08)*, pp. 4043-4047, Beijing, China, May 2008. → pages 61
- [41] R. Zhang, "Optimal power control over fading cognitive radio channels by exploiting primary user CSI," in *Proc. of the IEEE Global Telecommun. Conf. (GLOBECOM'08)*, pp. 1-5, New Orleans, USA, Dec. 2008 → pages 61
- [42] D. I. Kim, L. B. Le, and E. Hossain, "Joint rate and power allocation for cognitive radios in dynamic spectrum access environment," *IEEE Trans. on Wireless Commun.*, vol. 7, no. 12, pp. 5517-5527, Dec. 2008. → pages 61, 82
- [43] T. Al-Khasib, M. Shenouda, and L. Lampe, "Single and multiple carrier designs for cognitive radio systems," in *Proc. of the IEEE International Conf. on Commun. (ICC'10)*, Cape Town, South Africa, May 2010. → pages 61
- [44] K. Phan, S. Vorobyov, N. Sidiropoulos, and C. Tellambura, "Spectrum sharing in wireless networks via QoS-aware secondary multicast beamforming," *IEEE Trans. on Signal Process.*, vol. 57, no. 6, pp. 2323-2335, June 2009. → pages 62

- [45] L. Zhang, Y.-C. Linag, and Y. Xin, "Robust cognitive beamforming with partial channel state information," in *Proc. of Conf. Inform. Sciences and Systems*, pp. 890-895, March 2008. → pages 62
- [46] T. Al-Khasib, M. Shenouda, and L. Lampe, "Dynamic spectrum management for multiple-antenna cognitive radio systems: design with imperfect CSI," submitted to *IEEE Trans. on Wireless Commun.*, 2010. → pages 62
- [47] V. Chakravarthy, L. Xue, Z. Ruolin, W. Zhiqiang and M. Temple, "A novel hybrid overlay/underlay Cognitive Radio waveform in frequency selective fading channels," in *Cognitive Radio Oriented Wireless Networks and Communications, 2009. CROWNCOM '09. 4th International Conference on*, pp. 1-6, Jun. 2009, Hanover, Germany. → pages 78
- [48] S. Srinivasa, S.A. Jafar, "The Throughput Potential of Cognitive Radio: A Theoretical Perspective," in *Signals, Systems and Computers, 2006. ACSSC '06. Fortieth Asilomar Conference on*, pp. 221-225, Nov. 2006, Pacific Grove, CA, USA. → pages 78
- [49] I. Hammerstrom, and A. Wittneben, "On the optimal power allocation for nonregenerative OFDM relay links," in *Proc. of IEEE International Conf. on Comm. (ICC' 06)*, pp. 4463-4468, June 2006. → pages 105
- [50] D. Zhang, Y. Wang, and J. Lu, "Qos aware relay selection and subcarrier allocation in cooperative OFDMA systems," *IEEE Comm. Letters*, vol. 14, no. 4, pp. 294-296, April 2010. → pages 105
- [51] K. J. Ray Liu, Ahmed K. Sadek, Weifeng Su, and Andres Kwasinski, "Cooperative communications and networking," Cambridge University Press, 2009. → pages 105
- [52] J. Mietzner, L. Lampe, and R. Schober, "Distributed transmit power allocation for relay-assisted cognitive-radio systems," in *Asilomar Conf'07*, pp. 792-796, Nov. 2007. → pages 105
- [53] S. M. Almalfouh, and G. L. Stuber, "Interference-aware power allocation in cognitive radio networks with imperfect spectrum sensing," in *Proc. of IEEE International Conf. on Comm. (ICC' 10)*, pp. 1-6, May 2010. → pages 106

- [54] G. Bansal, Md. J. Hossain, and V. K. Bhargava, "Adaptive power loading for OFDM-based cognitive radio systems with statistical interference constraint," accepted in IEEE Trans. on Wireless Comm., Feb. 2011. → pages 106, 120
- [55] Harin Jeong, and Jae Hong Lee, "Adaptive relay selection for regenerative OFDMA relay networks with fairness constraint," in IEEE Vehicular Tech. Conf. (VTC'08), Sep. 2008. → pages 109
- [56] W. Ying, Q. Xin-chun, W. Tong, and L. Bao-ling, "Power allocation and subcarrier paring algorithm for regenerative OFDM relay system," in IEEE Vehicular Tech. Conf. (VTC '07), pp. 2727-2731, Apr. 2007. → pages 109
- [57] Gaurav Bansal, Md. Jahangir Hossain, and Vijay K. Bhargava, "Adaptive Power Loading for OFDM-based Cognitive Radio Systems", in *Proceedings of IEEE International Conference on Communications (ICC'07)*, pp. 5137-5142, Glasgow, Scotland, June 2007. → pages 120
- [58] Gaurav Bansal, Md. Jahangir Hossain, and Vijay K. Bhargava, "Discrete bit loading for OFDM-based cognitive radio systems", in *Proceedings of National Conference on Communications (NCC'07)*, pp. 1104-1109, Jan. 2007, Kanpur, India. → pages 120
- [59] Gaurav Bansal, Md. Jahangir Hossain, and Vijay K. Bhargava, "Link Adaptation in OFDM-based Cognitive Radio Systems", in *Cognitive Radio Communications Networks, Springer-Verlag*, pp. 189-212, 2007. → pages 120
- [60] Gaurav Bansal, Olivier Duval, and Francois Gagnon, "Joint Overlay and Underlay Power Allocation Scheme for OFDM-based Cognitive Radio Systems", in *Proceedings of IEEE Vehicular Technology Conference (VTC'10)*, pp. 1-5, Taipei, Taiwan, May 2010. → pages 120
- [61] Gaurav Bansal, Md. Jahangir Hossain, and Vijay K. Bhargava, "Power allocation for multiuser OFDMA-based Cognitive Radio Systems with Joint Overlay and Underlay Architecture", to be submitted. → pages 121
- [62] Dinesh Bharadia, Gaurav Bansal, Praveen Kaligineedi, and Vijay K. Bhargava, "Relay and Power Allocation Schemes for OFDM-based Cognitive Radio Systems", under minor revision in a journal. → pages 121

- [63] Gaurav Bansal, Praveen Kaligineedi, and Vijay K. Bhargava, "Joint Sensing and Power Loading algorithms for OFDM-based Cognitive Radio systems", in *Proceedings of IEEE Wireless Communications and Networking Conference (WCNC'10)*, pp. 1-5, Sydney, Australia, April 2010. → pages 121
- [64] Sami M. Almalfouh, and Gordon L. Stuber, "Interference-Aware Power Allocation in Cognitive Radio Networks with Imperfect Spectrum Sensing", in *Proceedings of IEEE Internal Conference on Communications (ICC'10)*, Cape Town, South Africa, June 2010. → pages 121
- [65] H. Jiang, W. Zhuang, and X. Shen, "Cross-Layer Design for Resource Allocation in 3G Wireless Networks and Beyond," in *IEEE Communications Magazine*, vol. 43, no. 12, pp. 120-126, Dec. 2005. → pages 122
- [66] Q. Zhang, J. Jia, and J. Zhang, "Cooperative Relay to Improve Diversity in Cognitive Radio Networks", in *IEEE Communications Magazine*, vol. 47, no. 2, pp. 111-117, Feb. 2009. → pages 122