Chirped-pulse laser amplifier and passive enhancement cavity for generation of extreme ultraviolet light

by

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Abstract

Extreme ultraviolet (EUV) light has many potential applications, including spectroscopy and scattering experiments in physical chemistry and atmospheric science. The dominant method for producing high-flux coherent radiation in this spectral range is synchrotron radiation produced from highly subscribed national-scale facilities such as the Canadian Light Source. An alternative to synchrotron radiation is high harmonic generation (HHG), a nonlinear optical process requiring high optical intensities. This thesis describes the development of an optical amplifier and passive enhancement cavity in order to realize a table-top EUV source. A chirped-pulse ytterbium-doped fiber amplifier system outputs 20 W average power from an initial mode-locked laser outputting pulses at 80 MHz and 160 mW average power. The pulses, of duration $\sim 250$ fs after the amplifier, are coupled to a high-finesse cavity which further increases the power by a factor of 500. The peak intensity achieved in the cavity is over $1 \times 10^{14}$ W/cm$^2$ and is an order of magnitude above the intensity required to drive HHG in xenon gas.
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List of Abbreviations

3-WEM  three-wavelength extinction measurements
AC  autocorrelation
AOM  acousto-optic modulator
AR  anti-reflective
ASE  amplified spontaneous emission
CEO  carrier-envelope offset
CPA  chirped-pulse amplification
EC  enhancement cavity
EDFA  erbium-doped fiber amplifier
EUV  extreme ultraviolet
FEL  free-electron laser
FEM  finite element method
FFC  femtosecond frequency comb
FFT  Fast Fourier Transform
FSR  free spectral range
FWHM  full-width at half-maximum
GVD  group-velocity dispersion
HHG  high harmonic generation
HR  high reflectivity
HWP  half-wave plate
IC   input coupler
NIR  near-infrared
NLSE nonlinear Schrödinger equation
PBS  polarizing beam-spitter
PCF  photonic crystal fiber
PZT  piezoelectric transducer
QWP  quarter-wave plate
SHG  second-harmonic generation
SMF  single-mode fiber
SPM  self-phase modulation
THG  third-harmonic generation
TOD  third-order dispersion
UV   ultraviolet
WDM  wavelength-division multiplexer
YDFA ytterbium-doped fiber amplifier
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Chapter 1

Introduction and Motivation

In fields such as physical chemistry and precision metrology, optical scattering experiments provide a wealth of information on the spatial and spectral characteristics of matter. Diffraction imposes a fundamental limit to the size resolution of any optical measurement; in general, the minimum length scale that can be observed is proportional to the wavelength of light used in the experiment. Second- and third-harmonic generation (SHG and THG respectively) are, by now, ubiquitous techniques which can extend coherent light from lasers in the visible and near-infrared (NIR) (visible: $400 \leq \lambda \leq 750$ nm, NIR: $750 \leq \lambda \leq 1450$ nm) into the ultraviolet (UV) ($\lambda \leq 400$ nm). However, the crystals used for second-harmonic generation (SHG) and third-harmonic generation (THG) begin to absorb light in the UV; this is particularly the case as wavelengths decrease into the region of the extreme ultraviolet (EUV) ($10 \leq \lambda \leq 121$ nm).

This thesis addresses the need for compact, high-flux light sources in the EUV region of the electromagnetic spectrum, to be applied to optical scattering experiments. Here our approach is to use high harmonic generation (HHG), a nonlinear
process that uses strong optical fields to ionize noble gases. Currently HHG has been employed to generate EUV radiation in ultrafast laser systems with kilohertz pulse repetition rates. Here we describe the development of a combined chirped-pulse amplification (CPA) laser system and passive enhancement cavity to build up the high optical power needed to drive the HHG process using the full repetition rate of an 80 MHz femtosecond frequency comb (FFC). This chapter provides an overview of EUV generation techniques as well as a review of HHG.

1.1 Current Approaches to EUV Generation

Currently, EUV generation techniques fall into two broad categories. The traditional method for generating EUV is via synchrotron radiation. This light is produced as a consequence of the acceleration of relativistic electrons in a magnetic field. Historically in the high-energy physics community, synchrotron radiation was an undesired parasitic effect of particle acceleration, as the radiated energy is bled off of the kinetic energy of the electron beam. However, this radiation mechanism is exploited in undulator-based or ’wiggler’ EUV sources. Modern synchrotron accelerators are built with straight sections in the electron beam path, so that experiments interacting with the electron beam can be inserted. In the case of an undulator, the insertion device is a series of magnets arranged with alternating polarity along the path of the beam. This causes a temporally-varying magnetic field (in the frame of the moving electron), and the electrons follow an oscillating path between the magnets. In the rest frame of the electron this is equivalent to antenna radiation, for which the wavelength appears shorter in the laboratory frame due to Lorenz contraction. The typical undulator has many pairs of alternating magnets, and the oscillatory motion of the electron in the magnetic field is in phase
with the photons that are emitted; this means that the electric fields associated with each oscillation add constructively and a highly coherent and bright EUV beam is produced. This mode of operation is referred to as a free-electron laser (FEL), in analogy to the highly coherent and monochromatic light emitted by a conventional laser – here, the gain medium is the free electrons being subjected to this spatial modulation.

While the use of synchrotron sources to generate EUV results in a high-quality, high-brightness beam, the large (national-scale) size and corresponding cost of such sources makes this approach an expensive one to pursue. Beam time at facilities such as the Canadian Light Source in Saskatchewan and the Stanford Synchrotron Radiation Lightsource in California is highly subscribed, and researchers must insert and remove their apparatus from the beam line with each run of experiments.

1.2 High Harmonic Generation

Another method to generate coherent EUV is HHG, which is the eventual goal of this project. Broadly, HHG is a process that exploits the nonlinear response of a noble gas subject to strong optical fields.

A semi-classical model of HHG (often termed the three-step model) was first proposed in [1]. The instantaneous electric field of the incident optical beam distorts the Coulomb potential seen by the valence electron of an atom in the nonlinear medium (usually a noble gas such as Xe or Ar). This yields a nonzero probability for the electron to escape the parent atom via tunneling (ie. the electron is ionized). Under the electric field of linearly polarized light, the electron is accelerated, first away, and then back toward, the parent ion. Upon recombination, the additional
Species | $I_s$ ($W/cm^2$)  
--- | ---  
Xe | $1.2 \times 10^{13}$  
Kr | $2.5 \times 10^{13}$  
Ar | $3.0 \times 10^{13}$  
Ne | $4.1 \times 10^{14}$  
He | $6.2 \times 10^{14}$

*Table 1.1: Saturation ionization intensities of some noble gases.*

kinetic energy given to the electron, given by [2]

$$E = 3.2U_p + E^0$$

is emitted as an EUV photon. Here, $U_p$ is the ponderomotive potential (the energy gained from the electron’s excursion in the applied field) and $E^0$ is the ionization potential of the atom.

There are further subtleties to efficient generation of high-flux EUV (eg. phase-matching), which will be addressed later in this thesis. However, the first technical challenge is to attain the high optical intensities necessary for ionizing the gas atoms to drive the HHG process. Table 1.1 lists the saturation ionization intensities $I_s$ of some noble gases, from [3]. The high required intensity necessitates the use of pulsed lasers, and indeed a leading method to generate short, intense pulses of light for HHG is the use of multipass regenerative titanium-sapphire (Ti:Sapph) amplifiers with $\sim 13$ W average power at kilohertz repetition rates (resulting in mJ pulse energies) [4]. This relatively low repetition rate results in poor spectral resolution [5], but is necessary to attain the high peak intensities required for HHG – given a certain average power, lowering the repetition rate, via eg. pulse-picking, divides the same energy between fewer pulses, thereby increasing the peak inten-
sity. In order to access the full repetition rate (on the order of tens of MHz) of a modelocked femtosecond frequency comb (FFC) \cite{6} while maintaining a high peak intensity, this thesis extends the work of \cite{7} where a FFC is coupled to a passive optical resonator. By coherent pulse addition (discussed in Section 2.4.2) the intracavity intensity is enhanced by several hundred times. The enhancement cavity also serves to stabilize the FFC against phase and amplitude noise, which may otherwise be written onto the generated EUV. A system similar to that discussed in this thesis has been reported to attain intensities on the order of $3 \times 10^{14} \text{ W/cm}^2$; this is easily enough power to drive the HHG process \cite{8}.
Chapter 2

Background

In order to attain the high peak intensities necessary to generate EUV via HHG, the output of a stable laser and amplifier system was coupled to a passive enhancement cavity in order to facilitate the coherent addition of many femtosecond optical pulses. This chapter firstly introduces ytterbium-doped fiber amplifiers (YDFAs) as a source of high-power laser pulses. Section 2.2 discusses the propagation of laser pulses in optical fiber. In particular, for high-power, ultrafast pulses, there are both linear and nonlinear effects that broaden the pulses and limit their peak power. Section 2.3 therefore introduces chirped-pulse amplification as a method to mitigate these detrimental processes. Having then described the generation of high-power, ultrafast pulses, the final section of this chapter describes the enhancement cavity and its stabilization to the laser in order to achieve coherent pulse addition and optical amplification.
2.1 Ytterbium-doped Fiber Amplifiers

Historically, rare-earth ion-doped fiber amplification was developed to address the problem of signal attenuation of glass fiber in long-haul optical fiber networks. In this application, fiber-based gain media offer superior properties to bulk amplifiers, such as ease of coupling and insensitivity to pump and signal polarization [9]. This culminated in the development of the erbium-doped fiber amplifier (EDFA) in 1987 [10] which has revolutionized the telecommunications industry. The Er$^{3+}$ emission at $\sim 1.55 \mu$m is well suited for optical fiber communications links because this is also the window where the losses in silica glass is lowest. The other common telecom band, around $\sim 1.3 \mu$m, is attractive because silica has roughly zero dispersion in this wavelength region, but rare-earth-doped fiber sources in this wavelength range (typically based on neodymium or praseodymium ions) suffer from poor performance due to undesired effects such as excited-state absorption and parasitic radiative transitions [9]. The use of ytterbium as a gain medium in optical fiber was first demonstrated in a Yb-doped fiber laser in 1988 [11], with increasing interest in the use of Yb-doped fiber in ytterbium-doped fiber amplifiers (YDFAS) through the subsequent decade [12]. This interest is driven by several attractive properties of the Yb gain medium, including high wall-plug efficiency and the availability of high-power laser pump diodes in the Yb gain region.

2.1.1 Spectroscopic Properties of Yb Gain Media

The energy diagram of a typical Yb$^{3+}$ system is shown below in Figure 2.1. There are two Stark-split energy manifolds (the $^2P_{7/2}$ and $^2P_{5/2}$ levels) relevant in optical processes. There are two absorption lines; one between the lowest Stark levels of the ground and excited states (at 976 nm), and between the lowest ground state and
Figure 2.1: Energy levels of Yb:glass system and pumping scheme.

the upper Stark manifold (at 940 nm). The upper Stark levels of the excited state are relatively short-lived and they decay via phonon-mediated processes to the lowest Stark level of the excited state. From here there are radiative transitions to the upper manifold of the ground state (at 1030 nm) and directly to the ground state (at 976 nm). Homogeneous and inhomogeneous broadening mechanisms render the upper Stark sublevels unresolvable [12], which results in the wide shoulders of the emission and absorption cross-sections as shown in Figure 2.2.

The resulting pumping scheme is a four level system: Atoms are pumped either into the upper excited-state manifold where they quickly decay, or directly into the lowest excited state manifold. However, the close spacing between the lower level of the laser transition and the ground state means that Yb$^{3+}$ exhibits quasi-three level characteristics – that is, there is a significant population of atoms in the lower level of the laser transition in thermal equilibirum. Effectively, the gain spectrum of the medium depends strongly on the pumping level: An unpumped medium exhibits strong absorption which renders it opaque to light at resonant frequencies,
and the gain increases with increasing pump level until the gain becomes zero (ie. the material is transparent), and then positive (light is amplified). This behaviour implies that YDFAS must be pumped with a minimum amount of power which depends on the amount of absorption present in the gain fiber (which depends in turn on the doping level, glass type and fiber length) in order for the seed light to be amplified as opposed to simply absorbed by an unpumped section of fiber. The pump power and fiber length must therefore be chosen in order to ensure that the fiber has positive gain throughout the majority of the fiber and at least zero gain at the end farthest from the pump [12].

2.2 Ultrafast Laser Amplification in Optical Fiber

The evolution of a laser pulse as it propagates through an optical medium is described by the nonlinear Schrödinger equation (NLSE) with gain:

![Figure 2.2: Emission/Absorption cross-sections of Yb-doped glass.](image)
Here $E$ is the (slowly-varying) envelope of the electric field at time $t$ and $z$ is the distance along the fiber. In this equation the time $t$ is taken to be in a reference frame that moves along $z$ at the group velocity $v_g$ of the pulse; this is related to the time $T$ in the laboratory frame via $t = T - z/v_g$ [9]. Materially this results in the lowest-order derivative $\partial E/\partial t$ to be zero. The second term in brackets contains nonlinear effects, chief of which is self-phase modulation (SPM), and higher-order nonlinear interactions (such as pulse self-steepening and the delayed Raman gain). It can readily be seen that the linear gain/loss coefficient $\alpha$ causes the pulse to be attenuated if $\alpha > 0$ and amplified if $\alpha < 0$. Furthermore, having $\alpha$ constant implies flat gain across the spectrum of the pulse. A finite gain bandwidth involves integrating the susceptibility of the medium to obtain a time-domain picture of the gain bandwidth [13] and is beyond the scope of this thesis. Here, flat gain is assumed for the spectral bandwidths of interest due to the wide gain bandwidth of the Yb$^{3+}$ system compared to the signal spectrum of interest ($\sim 40$ nm vs. 8 nm full-width at half-maximum (FWHM), respectively).

The pulse evolution $E(z, t)$ as it travels through the medium can be calculated from solving Equation 2.1. As the NLSE is a nonlinear partial differential equation, it cannot be solved analytically except for some specific cases. The dominant general method to solve this equation numerically is the split-step Fourier method [9]. The split-step Fourier method is faster by up to two orders of magnitude from other approaches such as finite difference methods due to the speed of the Fast Fourier Transform (FFT) algorithm. In essence, the method works by performing a FFT
on the system – the differential operators in Equation 2.1 then become multiplicative terms (the usual factors of $i\omega$) and propagation of the pulse through a single space- and time-step amounts to an algebraic operation. In the following sections this method will be used to illustrate the effects of dispersion and nonlinearity on an optical pulse, and it will be used to model the performance of a CPA system in the vein of [14]. The split-step Fourier method implementation used here is due to [15].

### 2.2.1 Group Velocity Dispersion and Pulse Broadening

Dispersion is a phenomenon typically observed in light travelling through an optical medium. Fundamentally, it is encapsulated by the fact that light at different frequencies propagates with different wavevectors. It is described quantitatively by the terms in Equation 2.1 denoted by $\beta_2, \beta_3, \cdots$. These coefficients are obtained by decomposing the electric field of an optical pulse into its Fourier modes:

$$E(z,t) = \int a(\omega) e^{i(\omega t - \beta(\omega) z)} \, d\omega = \int \tilde{E}(\omega, z) e^{i\omega t}$$  \hspace{1cm} (2.2)

where $a(\omega)$ is the amplitude of the Fourier component at frequency $\omega$. The Fourier amplitude $\tilde{E}(\omega, z) = a(\omega) e^{-i\beta(\omega) z}$ includes the spatial dependence via the spectral phase $\phi = \beta(\omega) z$. Dispersion arises due to the frequency dependence of $\phi$, which changes the spectral phase imposed on each Fourier component $\tilde{E}(\omega, z)$. For instance, for propagation through an optical medium with index of refraction $n(\omega)$, $\beta$ depends on frequency via the speed of light in the material, which is related to
the speed of light in vacuum $c_0$:

$$\beta(\omega) = \omega \frac{n(\omega)}{c_0} \quad (2.3)$$

The frequency-dependent phase shift implies that each spectral component of the envelope experiences a different phase shift (and thus a different time delay) through the medium.

Taking $\beta(\omega)$ to be some general function of frequency, then, one can expand the propagation constant about $\omega_0$ to yield

$$\beta(\omega) = \beta(\omega_0) + (\omega - \omega_0) \frac{\partial \beta}{\partial \omega} + \frac{(\omega - \omega_0)^2}{2!} \frac{\partial^2 \beta}{\partial \omega^2} + \frac{(\omega - \omega_0)^3}{3!} \frac{\partial^3 \beta}{\partial \omega^3} + \cdots \quad (2.4)$$

And so the dispersive properties of the medium can be described by successive derivatives of $\beta$ with respect to frequency. The zeroth-order term is simply the propagation of the central frequency. The first derivative has units of the reciprocal of a velocity. Namely, this derivative describes the group velocity of the wave $v_g = \partial \omega / \partial \beta$, the speed with which the envelope of an optical pulse travels through the medium (a justification of this fact can be found in [16]). Multiplied by a length $z$, $\tau = z / v_g$ is termed the group delay: It is the amount of time that it takes for an optical pulse to propagate through a length $z$ of the medium. The existence of higher-order derivatives of $\beta$ means that the different frequency components that comprise the envelope of the optical pulse also propagate at different group velocities. Not only is the pulse delayed as it propagates through a dispersive medium, but shape of the pulse envelope is also affected. This can be seen by considering two identical signals of frequencies $\omega_0$ and $\omega_0 + \delta \omega$ and examining
their group delay after propagation through a distance \( z \) of some medium:

\[
\delta \tau = \frac{\partial \tau}{\partial \omega} \delta \omega = \frac{\partial}{\partial \omega} \left( \frac{z}{v_g} \right) \delta \omega = \frac{\partial}{\partial \omega} \frac{\partial \beta}{\partial \omega} z \delta \omega
\]  (2.5)

The differential group delay depends on the second derivative of \( \beta \) with respect to frequency – this is group velocity dispersion. By examining Equation 2.5, for \( \frac{\partial^2 \beta}{\partial \omega^2} > 0 \) lower frequency (ie. red) components of a pulse suffer a shorter group delay than higher frequency (ie. blue) components of the pulse. This is termed normal dispersion. The opposite case, \( \frac{\partial^2 \beta}{\partial \omega^2} < 0 \) is termed anomalous dispersion. Dispersion specified in this way has units of, for example, fs\(^2\)/mm.

There exists an alternate convention for the specification of dispersion typically used in the telecommunications community, where derivatives are taken of the refractive index with respect to wavelength. This is more straightforward in cases where dispersion is described by Sellmeier-type equations ([17]). Here, the group velocity is given by \( v_g = c_0/n(\lambda) \). Expanding the refractive index to first order yields the group index

\[
n(\lambda) = n_0 - \lambda \frac{\partial n}{\partial \lambda_0}
\]  (2.6)

Taking the second derivative and multiplying by a prefactor to work out the units yields the dispersion coefficient (from [16] and [18]):

\[
D_{\lambda} = -\frac{\lambda_0}{c_0} \frac{\partial^2 n}{\partial \lambda_0^2} = -\frac{\omega_0^2}{2 \pi c_0} \frac{\partial^2 \beta}{\partial \omega^2} \left[ \frac{ps}{nm \cdot km} \right]
\]  (2.7)

Note that the sign convention is reversed: \( D_{\lambda} < 0 \) for normal dispersion and \( D_{\lambda} > 0 \) for anomalous dispersion. These units for the dispersion are useful because
a quick estimate for the temporal broadening $\tau_p$ (in terms of FWHM pulse duration) of a light pulse of spectral FWHM bandwidth $\Delta\lambda$ after propagation through a length $z$ of fiber with this dispersion can be found via

$$\tau_p = \Delta\lambda z |D_\lambda|$$

(2.8)

Using the approximate expression $\lambda/d\lambda \approx \omega/d\omega$ one can formulate a similar expression based on the FWHM bandwidth in units of frequency (angular or periodic):

$$\tau_p = \Delta\omega z \frac{\partial^2 \beta}{\partial\omega^2} = \frac{\Delta f}{2\pi} \frac{z}{z} \frac{\partial^2 \beta}{\partial\omega^2}$$

(2.9)

Figure 2.3 illustrates both the sign conventions of dispersion as well as its ef-
fect on the pulse width. By separating the frequency components of the pulse in space (or equivalently, temporally), the envelope of the pulse is widened (according to Equations 2.8 and 2.9 above), and so by conservation of energy the peak power of the pulse is lowered. This time dependent change in frequency is termed chirp. In some applications, such as fiber optic communications, dispersion is an undesired quantity as the dispersion in optical fiber causes consecutive pulses to interfere with each other, placing an upper limit on the length of fiber communications links without needing to receive, decode, re-encode and retransmit the signal via an optical repeater. However, dispersion plays a central role in ultrafast science, where dispersion is used to manipulate the pulse width (and subsequently the peak power). This can affect the modelocking mechanism in the realization of femtosecond laser sources [19, 20]. The use of controlled dispersion to shape the pulse in this way is also the central concept behind CPA, which is covered in more detail in Sections 2.3 and 3.2.

There also exist higher order derivatives of the spectral phase, chief of which is responsible for third-order dispersion (TOD). In most optical systems the effect of dispersion is dominated by the second-order group-velocity dispersion (GVD), as locally the refractive index is essentially quadratic over the bandwidths considered here. For instance, for typical values of dispersion in a system such as single-mode optical fiber (GVD ≈ 21.9 ps²/km), there is no visible difference to the temporal pulse profile between including the fiber’s TOD of ≈ 21.9 ps²/km and not (the illustration is therefore not shown). The simulated effect of a large amount of TOD (in the absence of GVD is shown in Figure 2.4. This is equivalent to the TOD introduced by propagation through 10 km of optical fiber, the GVD of which would be enough to stretch a 140 fs optical pulse to over 1.5 ns. For the relatively modest
amounts of GVD introduced into the system for this project and for the bandwidths considered, TOD can therefore be largely neglected.

### 2.2.2 Self-phase Modulation and Nonlinear Phase Shift

The nonlinear effects on the propagating pulse are encapsulated in the last term of Equation 2.1 and the parameter $\gamma$ controls the size of these effects:

$$
\gamma = n_2 \frac{2\pi}{\lambda} \frac{1}{A_{\text{eff}}}
$$

(2.10)

$\lambda$ is the optical wavelength, $n_2$ is the Kerr coefficient of the fiber material, and $A_{\text{eff}}$ is the effective area of the transverse mode that propagates in the fiber. For parameters typical to single mode fiber at 1 $\mu$m ($A_{\text{eff}} \sim 30 \mu m^2$, $n_2 \sim 3 \times 10^{-20} m^2/W$) the nonlinear coefficient works out to be $\sim 0.01 W^{-1} m^{-1}$. There are higher
order terms which depend on higher powers of $|E|$ and derivatives of $E$ but at pulse
lengths on the order of at least $\approx 100$ fs these effects (such as self-steepening and
the Raman response) are not important to the pulse evolution [9].

Consider only the nonlinear part of the NLSE in Equation 2.1. That is, dis-

cipation can be neglected here by setting $\beta(\omega)$ to be a constant with respect to
frequency. Expressing the complex envelope of the pulse $E(z,t)$ in units of the
normalized amplitude $U(z,t)$ such that

$$E(z,t) = \sqrt{P_0} e^{-\alpha z/2} U(z,t) \quad (2.11)$$

we can write the relevant part of the NLSE (after [9]) as

$$\frac{\partial U}{\partial z} = i\gamma P_0 e^{-\alpha z} |U|^2 U \quad (2.12)$$

where $P_0$ is the incident (peak) power on the amplifier, which is given by

$$P_0 = A \frac{P_{\text{avg}}}{f_{\text{rep}} \tau} = A \frac{E_{\text{pulse}}}{\tau} \quad (2.13)$$

where $P_{\text{avg}}$ is the average power of the laser, $f_{\text{rep}}$ is the repetition rate, and
$\tau$ is the FWHM pulse duration. The prefactor $A$ normalizes $E_{\text{pulse}}$ to a pulse of
unit energy and unit FWHM pulse duration. For sech$^2$-shaped pulses, for example,
$A \approx 0.881$. Using Equation 2.13, an 80 MHz train of 150 fs pulses at 1 W average
power has a peak power of over 70 kW.

Equation 2.12 has the solution

$$U(z,t) = U(0,t) e^{i\phi z} \quad (2.14)$$
Figure 2.5: Calculated nonlinear spectral broadening of a transform-limited 140 fs pulse under various amounts of SPM, specified by the maximum phase shift $\phi_{\text{max}}$ undergone at the pulse peak.

The nonlinear phase shift $\phi_{\text{NL}}$ is related to the product $\gamma P_0$ and the length of the fiber, which is normalized to the gain or attenuation in the fiber (given by $\alpha$, which is defined as in Equation \(2.1\)):

$$\phi_{\text{NL}} = |U(0,t)|^2 z_e \gamma P_0, \quad z_e = \frac{1 - e^{-\alpha z}}{\alpha}$$  \hspace{1cm} (2.15)

It is evident that the nonlinear term in the NLSE gives rise to an intensity-dependent phase shift from Equations \(2.14\) and \(2.15\), and that $(\gamma P_0)^{-1}$ plays the role of a characteristic length. Specifically, the nonlinear phase shift attains a maximum at the center of the pulse, where $t = 0$ and so for unit intensity $|U(0,0)|^2 = 1$,

$$\phi_{\text{max}} = z_e \gamma P_0 = \frac{z_e}{L_{\text{NL}}}$$  \hspace{1cm} (2.16)

So the nonlinear length given by $L_{\text{NL}} = (\gamma P_0)^{-1}$ is the effective propagation distance for which $\phi_{\text{NL}}(z,t) \leq 1$. SPM leads to distortion of the pulse spectrum as
The nonlinear phase shift $\phi_{NL}$ varies with time due to the temporal envelope $U(z,t)$. A time-dependent phase shifts frequency components at different points in the pulse by different amounts. The oscillatory appearance of the spectrum is characteristic of SPM as the phase shifts cause different parts of the pulse to interfere with itself.

The addition of GVD in combination with SPM causes a nonlinear chirp [9]. The phase shift caused by SPM delays each frequency component of the pulse by a different amount of time, and GVD further acts to separate these components temporally. However, as SPM causes a nonlinear distortion of the spectrum, the chirp generated is not the simple linear chirp that is induced by GVD and thus cannot be completely compensated by the introduction of dispersion. Figure 2.5 illustrates the effect of SPM on an optical spectrum. Combined with GVD the time dependent spectral broadening also gives rise to a chirp, as shown in Figure 2.6. These simulations are generated via the split-step Fourier method discussed in Section 2.2 with pulse and fiber parameters close to what is used in this project, with the exception of the gain and pulse power which are tuned in order to generate illustrative figures of the pulse and spectral broadening. An additional complication from the spectral broadening which is not modelled here is that the flat gain approximation (where the pulse spectrum is much narrower than the gain bandwidth of the amplifier) may also break down. This causes a narrowing of the pulse spectrum, which necessarily increases the pulse duration by the time-bandwidth product.

Whether SPM-induced broadening is a problem in an amplifier system can be estimated by examining the total nonlinear phase accumulated in a length of fiber.
Figure 2.6: Simulated temporal (top) and spectral broadening (bottom) from GVD and SPM. Input (black) is a 140 fs transform limited pulse, which undergoes SPM specified by $\phi_{\text{max}}=10\pi$. The GVD introduced is equal to propagation through 3.5 m of silica glass.

This can be calculated by taking Equation 2.16 and taking the length $z$ to be small,

$$d\phi = \gamma P(z,0) dz$$

(2.17)

Note that the effective length $z_e$ is not used here as the gain is modelled explicitly via $P(z,0)$. The differential phase shift for a small propagation length can then be integrated to obtain a total phase shift

$$\phi_{\text{NL}} = \int_0^L \gamma P(z,0) dz$$

(2.18)

For an amplifier following Equation 2.1 the power of the pulse increases exponen-
ially through the amplifier, such that \( P(z,0) = P(0,0)e^{-\alpha z} \). The effect of varying \( \phi_{NL} \) on the properties of a pulse propagating through an amplifier is discussed in detail in [14]. For \( \phi_{NL} \ll 1 \) the amplifier is essentially linear and the input pulse shape can be maintained, resulting in maximum peak power.

### 2.3 Chirped-Pulse Amplification

Originally developed for radar signal transmission, CPA was first applied to optical amplifiers in 1985 by Strickland and Mourou [21]. The general aim of CPA is to lower the peak power of a pulsed signal source in order to reduce the effect of nonlinear distortion and to prevent damage of signal handling components.

In a CPA system, a large amount of GVD (here, on the order of \( 10^6 \text{ fs}^2 \)) is deliberately introduced (either via a length of optical fiber, or by bulk optics as discussed in Section 3.2), lengthening the pulse temporally. This serves to decrease the peak power according to Equation 2.13. This in turn reduces the nonlinear phase shift \( \phi_{NL} \) across the entire pulse, which mitigates the spectral distortion and nonlinear chirp due to SPM. The amplified pulse is then compressed to close to its original pulse duration by applying dispersion of opposite sign and roughly equal magnitude. The amount of GVD introduced by the stretcher and compressor typically differ in order to compensate for dispersion in the gain medium and other optics. Due to residual nonlinear effects and higher-order dispersion mismatch between the stretcher and compressor the compressed pulse is longer than the input pulse [14].

Figure 2.7 illustrates the effect of CPA on improving the peak power obtained by a fiber amplifier. Here the split-step Fourier method was used to compute the output pulse of a CPA system based on a 140 fs, 80 mW average power pulse.
Simulated CPA output as a function of stretching ratio, based on the parameters of the experimental setup. The input is a transform-limited 140 fs pulse of 80 mW average power (at 80 MHz repetition rate). This is stretched to the listed pulse durations and amplified to 30 W average power. The pulse is compressed using an equal magnitude of GVD of opposite sign. In the absence of SPM the maximum peak power is reached independently of the stretching ratio.

Train at 80 MHz that is amplified to 30 W average power. Differing amounts of dispersion are introduced, resulting in the stretched pulse widths $\tau$ listed in the figure. The gain fiber is modelled to be similar to the Yb-doped fiber used in the amplifier as constructed, in terms of dispersion and nonlinear parameter $\gamma$. After amplification the pulse is recompressed with an equivalent amount of dispersion, but with opposite sign. The fine structure at the edge of the pulse is due to the nonlinear chirp introduced by the combination of SPM and GVD, which cannot be recompressed by GVD (in the compressor) alone.

The amplifier discussed in this thesis operates with $\phi_{NL} \sim 1$, which according to [14] implies that optimally the amplified pulse should be capable of compression to within a factor of 1.5 of the input pulse duration.
2.4 Optical Enhancement Cavities

An optical resonator is a direct analogue to a resonant electric circuit: Energy is stored in the resonator, and the degree to which energy is stored depends on the natural frequency of the resonator and the frequency of the incoming radiation. This frequency selectivity is used for many applications where a narrow laser linewidth is necessary; in this case the cavity acts as a filter. However, the application here is to use the huge intensity stored in the cavity to generate EUV via HHG as described in Section 1.2. This section describes the mechanism by which energy is stored in the ring cavity when coupled with a FFC, and the Pound-Drever-Hall locking scheme which stabilizes the cavity length with respect to the laser repetition rate.

2.4.1 Power Enhancement in a Ring Cavity

A general ring resonator is shown in Figure 2.8. The achievable intensity in such a cavity depends on the quality of the resonator, or alternatively, how much energy leaks from the resonator via parasitic effects such as imperfect reflectors and scattering from air and other media. Let \( r = |r|e^{i\xi} \) and \( t = |t|e^{i\eta} \) equal the complex amplitude reflectivity and transmissivity of a mirror. The corresponding power re-
Reflectivity and transmissivity are given by $R = |r|^2$ and and $T = |t|^2$ ($R$ and $T$ are related via $T = 1 - R$). Then the initial electric field coupled into the cavity due to a monochromatic plane wave incident on the input coupler (IC) is:

$$E_0^{\text{cav}} = E_0 t_{\text{IC}}$$

where $E_0$ is some initial (complex) amplitude. The amplitude transmissivity of the IC is denoted by $t_{\text{IC}}$, and the square root is taken as the expression is given for an electric field. The superscript on $E_{\text{cav}}$ denotes the number of round trips completed around the cavity by the circulating field. At this point the circulating field is reflected off of $N$ mirrors with reflectivity coefficients $r_1, r_2, \cdots r_N$. Assume that the modulus of the complex reflectivity is the same between the mirrors, but with a different phase. Then the lumped reflectivity of the cavity mirrors besides the IC is $|r|^N e^{i\theta}$, where $\theta$ is the total phase from reflection off the mirrors. After one round-trip the field at the input coupler from this single plane wave is

$$E_1^{\text{cav}} = E_0 t_{\text{IC}} |r|^N |r_{\text{IC}}| e^{i(\phi + \Theta)}$$

Here, $\Theta = \theta + \zeta_{\text{IC}}$ includes the phase shift off of reflection of the IC. In addition to reflection off of the mirrors, the circulating field also picks up a phase $\phi = \beta d$ due to propagation through the optical cavity of total path length $d$. This circulating field is added to a new field transmitted through the IC (also described by Equation 2.19) and this sum field makes a round trip, where a new field is added on, and so on, leading to the series expression for the circulating field after $n$ round-trips in
Letting \( |r_{\text{cav}}| = |r|^N|r_{\text{IC}}| \) denote a lumped cavity reflectivity which takes into account all of the mirrors, the geometric series sums (in the limit of many round trips around the cavity) to

\[
E_{\text{cav}} = E_0t_{\text{IC}} \frac{1}{1 - |r_{\text{cav}}|e^{i(\phi + \Theta)}}
\]  
(2.22)

The resulting intracavity intensity obtained by squaring the field is

\[
I_{\text{cav}} = |E_{\text{cav}}|^2 = \frac{I_0T_{\text{IC}}}{1 + |r_{\text{cav}}|^2 - 2|r_{\text{cav}}|\cos(\phi + \Theta)}
\]  
(2.23)

where \( I_0 = |E_0|^2 \) is power of the light incident on the IC. Using the half-angle identity \( 2\sin^2\phi/2 = 1 - \cos \phi \) this can be rearranged to

\[
I_{\text{cav}} = \frac{I_0T_{\text{IC}}}{(1 - |r_{\text{cav}}|^2)^2} \frac{1}{1 + \frac{4|r_{\text{cav}}|}{(1 - |r_{\text{cav}}|^2)}\sin^2\frac{\phi + \Theta}{2}} = \frac{I_0T_{\text{IC}}}{(1 - |r_{\text{cav}}|^2)^2} \frac{1}{1 + \left(\frac{2\sin^2\phi/2}{\pi}\right)^2 \sin^2\frac{\phi + \Theta}{2}}
\]  
(2.24)

The prefactor in Equation 2.24 is then an amplitude – it is the maximum intensity attained by light field, as the second factor is unitless and attains a maximum value of unity. This latter factor depends on the phase per round trip \( \phi \) (plus a constant phase \( \Theta \) per round trip from the mirror reflectivity), which is determined by the laser frequency \( \nu \) via

\[
\phi = kd = \frac{2\pi d}{\lambda} = \frac{2\pi \nu d}{c} = \frac{2\pi \nu}{\nu_F}
\]  
(2.25)
having defined $\nu_F = c/d$ as the free spectral range of the cavity – effectively, the natural frequency of the resonator. Clearly, Equation 2.24 attains a maximum when the argument to the sine is an integer multiple of $\pi$, that is,

$$\phi + \Theta = \frac{\pi \nu}{\nu_F} + \Theta = \pi$$

(2.26)

Successive cavity resonances are spaced by integer multiples of the free spectral range. This fact has important implications for the enhancement of an ultrafast train of pulses, which will be explored in the next section. The quantity $\mathcal{F}$ is termed the finesse and it determines the width of these cavity resonances in the frequency domain, as illustrated in Figure 2.9

$$\mathcal{F} = \frac{\pi \sqrt{|r_{cav}|}}{1 - |r_{cav}|}$$

(2.27)

An infinite finesse means that the cavity lines have infinitely fine width; as the finesse is reduced the cavity lines broaden, accepting frequencies in an increas-
ingly large neighborhood about $\nu_F$. Specifically, the FWHM linewidth of the cavity resonances is related to the free spectral range (FSR) via

$$\Delta \nu = \frac{\nu_F}{F} \quad \text{for} \quad F \gg 1 \quad (2.28)$$

The finesse is also related to the photon lifetime, which is the average time that a photon spends in the cavity before it is lost through leakage or absorption [22]:

$$\tau_p = \frac{\mathcal{F}}{2\pi \nu_F} \quad (2.29)$$

The buildup factor is then the power in the cavity normalized to the incident power and so is expressed by the prefactor in Equation 2.24 divided by the incident intensity $I_0$:

$$\frac{I_{\text{max}}}{I_0} = \frac{T_{IC}}{(1 - |r|N|r_{IC}|)^2} = \frac{1 - R_{IC}}{(1 - \sqrt{R^N R_{IC}})^2} \quad (2.30)$$

As shown in Figure 2.10, for a given $R^N$ (the power reflectivity of all of the mirrors in the cavity except for the input coupler) the buildup depends strongly on the power reflectivity of the input coupler $R_{IC}$. If the input coupler is too reflective, then the amount of light coupled into the cavity (which depends on the transmissivity of the input coupler $T_{IC} = 1 - R_{IC}$) is less than the cavity is capable of sustaining given its finesse. If the input coupler is not reflective enough, then an excessive amount of power leaks out of the cavity. The buildup factor peaks at a case which is said to be impedance matched, that is, $R_{IC} = R^N$. In this case, Equation 2.30 simplifies to

$$\frac{I_{\text{max}}}{I_0} = \frac{1}{1 - R_{IC} R^N} \approx \frac{\mathcal{F}}{\pi} \quad \text{(for } 1 - R_{IC}, 1 - R^N \ll 1) \quad (2.31)$$
2.4.2 Cavity Enhancement of a Mode-locked Laser

In the time domain, the output of an ultrafast laser can be thought of as a train of ultrashort optical pulses. These short pulses are generated when a fixed phase relationship is established between the longitudinal modes of the cavity, a phenomenon termed mode-locking [23]. Equivalently in the frequency domain, the spectrum of a mode-locked laser consists of regularly spaced narrow lines corresponding to the longitudinal modes of the lasing cavity. This characteristic spectrum is known as a frequency comb and these have found many applications in precision metrology and frequency synthesis [6, 24].

A frequency comb is characterized by two parameters: The repetition rate $f_{rep}$, which describes the spacing between adjacent frequency components, and the
carrier-envelope offset (CEO) frequency $f_o$, which together specify the frequency $\nu_n$ of the n-th component of the frequency comb by

$$\nu_n = nf_{\text{rep}} + f_o$$

Equivalently [25], in the time domain the same ultrafast pulse train is described by the period between pulses $T = 1/f_{\text{rep}}$ and the CEO phase $\Delta \phi_{ce}$. The effect of the carrier-envelope offset is most readily understood in this picture. In any dispersive material, the difference between group and phase velocities means that the phase of each successive pulse (as measured at any two consistent points; for example, the peak of the instantaneous electric field and the peak of the pulse envelope) changes by an amount equal to the CEO phase $\Delta \phi_{ce}$. To see how this translates to the CEO frequency, consider the spectrum of this pulse train obtained by the interference of many successive pulses. By the usual Fourier relationship relating constant shifts in time with a linear change in the spectral phase, evidently the comb lines are found at intervals spaced by $f_{\text{rep}} = 1/T$, where all of the spectral phases line up. By the dual Fourier property that a linear change in the temporal phase is equivalent to a constant frequency shift, adding a linear variation of the carrier-envelope phase means that the point (in frequency space) at which the spectral phases of successive pulses line up is shifted by a constant amount (from [26]),

$$f_o = \frac{f_{\text{rep}}}{2\pi} \Delta \phi_{ce}$$

Figure 2.11 illustrates these two visualizations of the frequency comb as well as the parameters described above. The spectrum of the frequency comb consists of
The problem of coupling an ultrafast train of pulses into an enhancement cavity (EC) can therefore be thought of in the frequency domain. In the original case of a monochromatic (CW) beam coupled into an enhancement cavity (EC), the task is simply to tune the laser frequency to one of the resonant lines of the EC (which may itself have an offset frequency due to the difference between phase and group velocities in the cavity). Coupling a frequency comb to an EC for a high buildup requires simultaneous synchronization of the repetition rates $f_{rep}$ and CEO frequencies $f_o$ of the laser oscillator and cavity resonator. Furthermore, the
dispersion of the cavity must be limited in order to ensure good overlap between the
types, since in the frequency domain, dispersion translates to an uneven spacing
between successive cavity transmission peaks [27]. A varying spacing between
comb lines limits the effective bandwidth of the EC, and furthermore the pulse is
stretched by an amount per pass [22]

\[ \frac{\Delta \tau}{\tau} = \frac{\partial^2 \beta}{\partial \omega^2} \frac{\pi^2 c^2 (\Delta \lambda)^2}{\ln(2) \lambda^4} \]  

(2.34)

where the ratio \( \Delta \tau / \tau \) is the fractional change per pass compared to a transform
limited pulse, and \( \partial^2 \beta / \partial \omega^2 \) is the net intracavity dispersion. Taking a transform
limited pulse at \( \lambda = 1037 \text{ nm} \) and a pulse width of \( \sim 130 \text{ fs} \), the maximum net
per-pass cavity dispersion required to limit the pulse broadening to 10% after 160
round-trips (roughly the photon lifetime of a 80 MHz cavity with \( F = 1000, \tau_p \approx
2 \mu s \)) is \( \approx 7 \text{ fs}^2 \). Evidently, as the finesse of the cavity is increased (or if the laser
pulse duration is shortened), the duration of the stored pulse in the cavity becomes
more sensitive to the effects of intracavity GVD.

The effect of frequency mismatch between the laser oscillator and enhancement
cavity is described in [28]. The relative intracavity power is scaled by a factor

\[ A(p) \propto \frac{1}{\sqrt{1 + p^2 \left( \frac{\Delta \varphi}{\lambda_0} \right)^2}} \]  

(2.35)

where \( p \) is the mismatch between the laser and cavity frequencies measured in units
of the repetition rate,

\[ p \propto \frac{f_{\text{rep}} - f_{\text{rep(EC)}}}{f_{\text{rep}}} + \frac{f_o - f_{o(EC)}}{f_{\text{rep}}} \]  

(2.36)
This imposes an envelope on the spectrum of the modes that the EC can support, since for \( p \neq 0 \) the enhancement of the cavity is reduced, or alternatively, an effective finesse is introduced:

\[
\frac{I_{\text{max}}}{I_0} = \frac{\left( \frac{2F_e}{\pi} \right)^2 \frac{I_0}{4}}{1 + \left( \frac{2F_e}{\pi} \right)^2 \sin^2 p\pi} \tag{2.37}
\]

where the effective finesse \( F_e \) is

\[
F_e = \frac{F}{A(p)} \tag{2.38}
\]

Note that the buildup factor as defined in Equation 2.37 is periodic in \( p \): For integers the intracavity power attains a local maximum whose value is modulated by the envelope function \( A(p) \). The expression of Equation 2.38 as an effective finesse also highlights the fact that cavity lines away from \( p = 0 \) also broaden as they decrease in height. As the cavity finesse or laser bandwidth increases, the envelope \( A(p) \) narrows: This is important both from the perspective of controlling the repetition rate of the laser as described in 2.4.3, and because the envelope limits the buildup in the cavity. Furthermore, an offset frequency difference \( f_o - f_{o(EC)} \) shifts the comb lines underneath the envelope, which again lowers the total cavity enhancement. However, note that if \( p \) drifts by a total of \( \pi \) due to offset frequency mismatch, one can simply reindex the modes of the cavity and see that cavity modes are at the same position under the envelope \( A(p) \) as in the case \( p = 0 \). This means that tunability of the offset frequency need only span half of a free spectral range.

Incidentally, in the absence of an octave-spanning spectrum to directly measure
the CEO frequency via the $f$-to-2$f$ technique [26], this latter phenomenon can be turned around to optimize the offset frequency difference between the laser and cavity. Direct observation of the central (highest) and two adjacent lines of the cavity transmission (or reflection) signal as the cavity length is varied can be used to roughly match $f_o$ of the laser and cavity. As shown in Figure 2.12, if the offset frequency difference is optimized, the maximum of $A(p)$ coincides with a cavity line, and so the two adjacent cavity lines will be equally displaced (at $p = \pm 1$) from the central line and so will have equal amplitude. This is particularly useful in the present work, as the offset frequency adjustment method used here does not affect the laser repetition rate (see Section 3.1.1), unlike Kerr-lens-modelocked Ti:Sapph systems which are currently ubiquitous in the field of nonlinear optics. Coupled with the exceptional stability of fiber laser and amplifier systems, this means that the offset frequency can be quickly optimized by observing the cavity transmission on a photodiode while sweeping the cavity length using a piezoelectric transducer (PZT) or other actuator (which maps the oscilloscope sweep time to frequency), and adjusting the offset frequency to maximize the height of the central peak and to make the height of the two adjacent peaks equal.

**2.4.3 Pound-Drever-Hall Locking Scheme**

The Pound-Drever-Hall locking scheme is a technique originally used to stabilize the frequency of a laser by locking it to a reference Fabry-Perot cavity, and most famously used in the Laser Interferometer Gravitational Wave Observatory (LIGO) [29][30]. In this project, the Pound-Drever-Hall technique is instead used to lock the cavity length to the laser repetition rate. In this case, the sensor and actuator in the control problem are both associated with the cavity. The control scheme is
Figure 2.12: Optimized (left) and un-optimized (right) offset frequency and its effect on the overlap between frequency comb lines and the cavity peaks.

Figure 2.13: Simplified diagram of the Pound-Drever-Hall locking scheme.

depicted schematically in Figure 2.13

The error signal is derived from the cavity reflection, which decreases sharply as the cavity is brought into resonance with the laser (due to the fact that the cavity absorbs and builds up the laser light that is resonant with itself). However, the
cavity reflection alone is not a useful error signal as it is neither linear nor centered about zero. Both of these problems are ameliorated by instead taking the derivative of the cavity reflection. This is accomplished by dithering the actuator: The PZT that controls the cavity length is modulated sinusoidally at a relatively high frequency (~ 1 MHz). Since the dither signal is far outside the control bandwidth of the servo, the PZT can driven by the sum of the control and dither signal and fulfil the role of both actuator and sensor. This distinguishes the two sides of the resonance curve due to the relative phase of the modulation and the error signal. If the cavity is below resonance with respect to the laser, then increasing the frequency (shortening the cavity) decreases the cavity reflection; conversely if the cavity is above resonance, then increasing the frequency increases the cavity reflection. Mixing the error signal with the dither signal recovers the phase of the two signals (via the DC part of the mixer output). A simulated resonance curve and its derivative (the Pound-Drever-Hall error signal) is shown in Figure 2.14. Near resonance, the signal is essentially linear. Figure 2.15 illustrates the effect of the cavity finesse on the slope of the linear region of the error signal near resonance. More quantitively, consider the intensity of a plane wave that is reflected off of the cavity input coupler. This is of the same form as the field that is coupled into the cavity, as in Equation 2.19, and consists of the coherent sum of the promptly reflected beam and the leakage from the stored beam in the cavity (Equation 2.22):

\[
E_{ref} = E_0 \left( r_{ic} - \frac{T_{lc}|r_{cav}|e^{i\theta}}{1 - |r_{cav}|e^{i(\phi+\Theta)}} \right) = E_0 F(\omega) \quad (2.39)
\]

The reflection coefficient \( F \) is a function of frequency via the phase \( \phi \) which is the same as defined in Equation 2.25. As before, the cavity is on resonance when
the phase is an integer multiple of $2\pi$ — in other words, the reflection vanishes ($F = 0$) the cavity length is an integer multiple of wavelengths (alternatively, the laser frequency is an integer multiple of cavity free spectral ranges).

Suppose that the frequency of the cavity (or the repetition rate of the laser) is modulated (dithered) by changing the cavity length at a frequency $\Omega$. This results in sidebands appearing on the resonance signal about the central frequency $\omega$ separated by the dither frequency. In terms of power, the power of the reflected signal is split amongst the carrier frequency and sidebands:

$$P_{ref} = P_c + 2P_s + 2P_{2s} + \cdots$$  \hspace{1cm} (2.40)

In the case of a small modulation depth, the power is contained almost entirely in the first set of sidebands (at $\omega \pm \Omega$) and so $P_{ref} \approx P_c + 2P_s$, but this approximation is made here only for convenience; the analysis does not change materially with

**Figure 2.14:** Simulated cavity transmission and Pound-Drever-Hall signal.
the addition of higher-order sidebands. The reflected field can then be written as:

\[ E_{ref} = E_c F(\omega) e^{i\omega t} + E_s F(\omega + \Omega) e^{i(\omega + \Omega)t} - E_s F(\omega - \Omega) e^{i(\omega - \Omega)t} \]  

(2.41)

Squaring the field to obtain the power yields products of the carrier and sidebands:

\[
P_{ref} = |F(\omega)|^2 + |F(\omega + \Omega)|^2 + |F(\omega - \Omega)|^2 \\
+ 2\sqrt{P_c P_s} \text{Re} [F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega)] \cos \Omega t \\
+ 2\sqrt{P_c P_s} \text{Im} [F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega)] \sin \Omega t \\
+ (\text{higher-order terms in } \Omega) \]  

(2.42)

Mixing this with the original dither signal (at frequency \( \Omega \)) yields sum and difference frequencies, the latter of which is the portion of \( P_{ref} \) that is proportional to \( \sin \Omega t \). Through the mixer this signal is shifted back to base-band, and forms the Pound-Drever-Hall error signal:

\[ \varepsilon = 2\sqrt{P_c P_s} \text{Im} [F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega)] \]  

(2.43)

The usefulness of this error signal is apparent near resonance. Here, assuming that the dither frequency is high enough that the carrier is completely transmitted while the sidebands are completely reflected (ie. \( F(\omega) \ll 1 \) and \( F(\omega \pm \Omega) = -1 \)), the reflected power simplifies to

\[ P_{ref} \approx 2P_s - 4\sqrt{P_c P_s} \text{Im} [F(\omega)] \sin \Omega t \]  

(2.44)
The reflection coefficient $F(\omega)$ is rewritten as a function of a small perturbation of the cavity length $\delta d$ about $d = 0$, via an expansion of the optical phase about $\phi = 0$:

$$F(\omega) = \left( r_{ic} - \frac{T_{ic}r_{cav}e^{i\phi}}{1 - r_{cav}e^{i\phi}} \right) \approx i \frac{r_{cav}T_{ic}}{1 - r_{cav}r_{ic}} \sin \phi \approx i \frac{r_{cav}T_{ic}}{1 - r_{cav}r_{ic}} \frac{2\pi \delta d}{\lambda} \quad (2.45)$$

The approximation in (2.45) removes terms of order $\sin^2 \phi$ which otherwise would have appeared in the expression. From this we determine that

$$\epsilon = -4\sqrt{P_cP_s}\text{Im}[F(\omega)] \propto \mathcal{F} \delta d \quad (2.46)$$

Near resonance, the Pound-Drever-Hall error signal is linear and the slope of the error signal is proportional to the cavity finesse. Figure 2.15 illustrates the effect of the cavity finesse on the slope of the linear region of the error signal near resonance. As the cavity finesse increases, the slope of the error signal increases. Furthermore, the size of the linear region of the error signal (which is the controllable range of the loop) decreases. This reduces the magnitude of excursions of the laser repetition rate away from resonance with the cavity. However, the sensitivity of the cavity to transient impulses (such as acoustic noise or bumping the optical table) is increased, as the effective range of the control loop is reduced and it is easier to knock the control system out-of-loop.
Figure 2.15: Calculated effect of cavity finesse (from low (blue) to high (red) finesse) on the slope of the error signal and length of linear region.
Chapter 3

Results and Discussion

This chapter describes the design and construction of a CPA system and passive enhancement cavity capable of producing the high peak powers necessary for HHG. A schematic of the system is shown in Figure 3.1 and each section is described in the subsequent sections of this chapter. A commercial mode-locked laser module provides a femtosecond frequency comb whose offset frequency is adjusted by a double-pass acousto-optic modulator (AOM) module. The resulting signal is chirped via a diffraction grating stretcher. After the stretcher, the combined efficiency of the AOM module and stretcher yields an average power of approximately 15 mW. This signal is then amplified, first by a Yb-doped single-clad fiber amplifier (the preamplifier) to approximately 80 mW, and then by a Yb-doped double-clad photonic crystal fiber amplifier (the power amplifier) to approximately 30 W. The preamplifier ensures that sufficient power is incident on the power amplifier in order to drive signal gain, and to decrease the offset frequency dependence of the signal power due to the AOM module. After compression to a pulse width of approximately 250 fs, the pulses (now at 20 W average power) are coupled into
Figure 3.1: Block diagram of complete CPA system and enhancement cavity. Pulses from a mode-locked laser oscillator (OSC) goes through a double-pass AOM setup to adjust the offset frequency. The pulses are chirped by a grating stretcher. After two amplification stages, the pulses are dechirped by a grating compressor and coupled into the enhancement cavity.

the passive enhancement cavity which simultaneously builds the power up by a factor of approximately 500, and focuses the beam at a point in the cavity. This setup yields an optical intensity at the intracavity focus of over $1 \times 10^{14} \text{ W/cm}^2$, sufficient to realize EUV generation through HHG as discussed previously.

3.1 Laser Oscillator

The laser oscillator used in this project is a passively mode-locked commercial module (Origami 10, Onefive GmbH, Switzerland) providing approximately 140 mW of average output power at 1038 nm with a hyperbolic secant-shaped pulse width of 140 fs. The FWHM bandwidth is 8.31 nm which implies a near transform-limited pulse ($\Delta \tau \Delta \nu \approx 0.32$). The 80 MHz repetition rate of the laser puts the peak
Voltage (mV) | Power (mW) | Center $\lambda$ (nm) | $\Delta \lambda$ (FWHM,nm) | $\Delta \tau_p$ (fs) 
---|---|---|---|---
1000  | 93  | 1035  | 6.32  | 181  
1100  | 110 | 1036  | 7.01  | 163  
1200  | 126 | 1037.1| 7.69  | 149  
1300  | 143 | 1038  | 8.31  | 138  
1400  | 160 | 1039.3| 8.92  | 129  
1500  | 173 | 1040.4| 9.51  | 121  
1550  | 181 | 1040.8| 9.78  | 118  

Table 3.1: Operating parameters of Onefive oscillator.

The instantaneous power of the pulse at $\sim 1.1$ kW. The laser module includes a PZT acuator capable of adjusting the repetition rate (by the cavity length) by 100 Hz. Additionally the pump current is adjustable by a control voltage between 1000 and 1550 mV. Changes in the pump current change the operating parameters of the laser, including the output power, center wavelength, spectral bandwidth, and pulse duration, as shown in Table 3.1. The variation in laser parameters as a function of pump power arises from the interplay between self-phase modulation (SPM) and group-velocity dispersion (GVD) in the soliton mode-locking process [31]. The data is taken from the manufacturer specification sheet.

3.1.1 Acousto-optic Modulator Frequency Shifter

One method to adjust the offset frequency $f_{ceo}$ of a mode-locked laser is to adjust the pump power of the oscillator. This method is problematic for passively mode-locked fiber lasers, as the interplay between the pump power (and therefore the intracavity intensity) and the mode-locking mechanism (such as SPM-induced spectral broadening or nonlinear polarization rotation) means that tuning of the pump power also changes other operating parameters of the laser, such as output power, spectral width, and pulse duration. The Onefive laser used in this system...
has a pump current tuning feature, but as expected the laser operating parameters vary across a relatively wide range, as shown in Table 3.1.

Another method to adjust the offset frequency is to launch the pulses through an AOM. An AOM uses a PZT to set up a pressure wave pattern that travels through a crystal (e.g., glass or, in this case, PbMoO₄). The changes in density result in a periodic refractive index modulation which acts as a diffraction grating. As in a diffraction grating illuminated at normal incidence, the beam emerges from the AOM at an angle $\theta$:

$$\sin \theta = \frac{m\lambda}{2\Lambda}$$  \hspace{1cm} (3.1)

where $m$ is the diffracted order (usually one is interested in the first order, $m = 1$), $\lambda$ is the optical wavelength, and $\Lambda$ is the wavelength of the acoustic wave. One difference of diffraction due to an AOM from diffraction due to gratings is that the optical wave is Doppler-shifted by the travelling acoustic wave, that is, $f_o = f_i + mF$ where $f_i$ is the input wave, $f_o$ is the output wave, and $F$ is the frequency of the acoustic wave. In the spectral picture, this directly shifts the frequency of the laser, and therefore AOMs are often used to precisely tune the frequency of lasers in precision cooling and trapping experiments ([32]). For a femtosecond frequency comb, the AOM shifts all of the lines by the acoustic frequency, while keeping the comb spacing constant. One problem with sending the light through only an AOM is that by Equation [3.1] the diffracted angle varies as the acoustic frequency is swept; the diffraction efficiency of the AOM also exhibits an acoustic frequency dependence. The latter is immaterial to the current application, as the signal is then sent through an amplifier chain whose output power depends only weakly on the
initial power.

To address the pointing change as the acoustic frequency is swept, a double-pass AOM setup was used in order to adjust the offset frequency of the laser. The design is based off the double-pass AOM setup of Donley, originally used to tune the frequency of a laser for precision experiments ([32]), and is shown in Figure 3.2. The input pulse first passes through a polarizing beam-spitter (PBS), and the polarization of the light is adjusted by a half-wave plate (HWP) upstream of the setup in order to pass the maximum amount of light through the PBS. The beam is resized using a Galilean telescope constructed from one converging lens and a diverging lens whose focal length is half that of the converging lens. The selection of the beam size is determined by two competing factors. The diffraction efficiency through the AOM favours a large beam size while the time response of the AOM is faster with a smaller beam size. The telescope was realized with a +100 mm and -50 mm focal length lens spaced approximately 7 mm apart, giving a collimated beam of diameter $\sim 320 \mu m$ which is passed through the AOM (1205C-1-869, Isomet Corporation, USA). A 150 mm focal length lens is placed after the AOM such that the diffracted beams propagate displaced but parallel to the zeroth-order, and an iris placed after the lens to block all beams except for the first order. The polarization of the light is made circular by an appropriately-oriented quarter-wave plate (QWP) and retro-reflected by a mirror placed at the focal length away from the 150 mm lens, such that the first order beam retraces the original path. This reflector scheme ensures that the light emerging from the AOM propagates in a direction that is independent of the acoustic frequency. Due to the QWP and the flip in handedness of polarization (defined with respect to the original frame of reference), the modulated light emerging colinearly with the input light is of an
orthogonal polarization and so is coupled out by the PBS. A photograph of the completed AOM module is shown in Figure 3.3. The alignment of the AOM module is done in parts: Firstly the telescope is aligned by observing the pointing of the beam downrange, and the spot size is measured using a beam profiler (BeamView, Coherent Incorporated, USA) in order to check the collimation of the beam. The AOM is inserted and aligned for maximum transmission into the first order. The position of the final lens is set by checking that the diffracted beam remains parallel to the optical axis as the AOM frequency is tuned. Lastly, the reflector is inserted and aligned so that the returning beam is coincident with the input beam on all of the elements on reflection.

To verify the operation of the AOM module the transmission of the optical cavity was monitored as the AOM frequency was tuned. As in Figure 2.12 the relative intensity of the cavity lines changed as the frequency was swept, with the maximum amplitude of the central line occurring when the two adjacent cavity lines were of equal height. As for the power efficiency of the AOM module, Figure 3.4 plots the output of the double-pass AOM module. However, as mentioned earlier the reason for using the double-pass configuration is to mitigate the pointing change of the output beam as the frequency is tuned. To that end, a more important metric
is the output power from the preamplifier (the details of which are given in Section 3.3), and that is also plotted in Figure 3.4. It is observed that the FWHM bandwidth of the composite AOM-preamplifier system is greater than the tunable range of the AOM driver. Additionally, the output power of this system is such that for a preamplifier pump current within the normal operating range of this system (I_p ≤ 600 mA) the output power of the AOM module throughout its frequency tuning range is sufficient to drive signal gain in the preamplifier, as discussed in Section 3.3. This is evidenced by the shape of the preamplifier output spectrum remaining similar throughout the frequency range of the AOM, as shown in Figure 3.5.

3.2 Dispersion Management via Diffraction Gratings

The pulse is manipulated temporally by a pulse compressor whose layout was first used by Treacy to compress pulses from a Nd:glass laser [33] and is depicted schematically in Figure 3.6. Here the input pulse is incident on the first of a pair of diffraction gratings. The spectrally dispersed light is recollimated by a second grating placed parallel to the first and is then normally incident on a retroreflector.
Figure 3.4: Relative power output of double-pass AOM module as a function of frequency. The FWHM bandwidth of the AOM module is nearly the entire tuning range of the AOM driver. The preamplifier further decreases the frequency dependence of the power incident on the power amplifier, ensuring that the latter is sufficiently seeded for power gain.

Figure 3.5: Preamplifier output spectra with varying AOM drive frequency. The similarity of these spectra shows that the AOM module output power is sufficient to seed the preamplifier throughout the frequency tuning range.
(either a roof prism or a mirror tilted slightly upward). The returning light retraces the path of the input light (except displaced above or below the input). By virtue of the different path lengths traversed in the compressor by different frequencies of light, the pulse is chirped – equivalently, GVD is introduced.

### 3.2.1 Grating Compressor

From [23] the GVD introduced by the compressor is given by

\[
\beta_2 \equiv \frac{\partial^2 \phi}{\partial \omega^2} = -\frac{\lambda}{2\pi c^2} \left(\frac{\lambda}{d}\right)^2 \frac{b}{\cos^2(\theta_d)}
\]  

(3.2)

and the TOD of the compressor is given by [34]

\[
\beta_3 \equiv \frac{\partial^3 \phi}{\partial \omega^2} = -\beta_2 \frac{3\lambda}{2\pi c} \left(1 + \frac{\lambda}{d \cos^2 \theta}\right)
\]  

(3.3)
where $d$ is the grating period, $b$ is the separation between the gratings (as measured along the diffracted beam as opposed to the grating normals), and $\theta_d$ is the angle of the diffracted beam with respect to the grating normal. Note also that the sign of the TOD is always opposite to that of the GVD.

The physical layout of the compressor in this system is depicted in Figure 3.7. Fused-silica transmission gratings with a pitch of 1250 l/mm (Ibsen Photonics A/S, Denmark) were selected for the compressor in order to handle the high power (30 W at 80 MHz repetition rate, pulse duration 20 ps) and high efficiency requirement. The beam is coupled into the compressor by reflection off of a D-shaped mirror, and the retroreflecting mirror is tilted upward such that output pulse passes over the D-shaped mirror and exits the compressor. The alignment of the compressor gratings can be checked by ensuring that all of the angular dispersion is removed after the first strike on the second transmission grating. The retroreflector is aligned by first overlapping the input and returning beams on all of the elements of the compressor to align the horizontal displacement, then finally tilting the returning beam up to clear the D-shaped mirror and exit the compressor. A photograph of the completed pulse compressor is shown in Figure 3.8.

### 3.2.2 Grating Stretcher

A pulse compressor based on the design of [33] is only capable of introducing GVD of a single sign, due to the fact that for a fixed diffraction angle, all the other quantities in equation 3.2 are of a fixed (positive) sign. In the case of the Treacy compressor, the dispersion introduced is anomalous (ie. $\beta_2 < 0$). One method for introducing the opposite sign of GVD (normal GVD, ie. $\beta_2 > 0$) is to use a length of single-mode fiber (SMF), which has normal GVD at wavelengths close to 1 µm.
Figure 3.7: Top view of compressor layout. The dashed box indicates the D-shaped mirror (D) that separates the input and output beams. The gratings (G) diffract the beam which is retroreflected by a mirror tilted upwards (RR). The quantities $b$ and $\theta_d$ are defined as in Equation 3.2.

Inset: Side view of compressor layout. The tilt of RR causes the reflected beam to clear the mirror D and exit the compressor.

However, this suffers from a lack of adjustability (short of repeatedly cutting and splicing the fiber to size), as well as an incomplete compensation of dispersion, as the TOD of SMF as well as the Treacy compressor are of the same sign.

A modification of this design due to Martinez [35] involves inserting a unity-magnification telescope between the diffraction gratings, as shown in Figure 3.9. This effectively shifts the point $b = 0$ in Equation 3.2 from zero spacing between the gratings to placement of the gratings at the two focal planes of the telescope. When the gratings are moved farther away from the focal plane, the distance between the gratings $b > 0$ and the system introduces anomalous GVD as described above. Conversely, when the gratings are moved closer to the telescope, $b < 0$ and normal GVD is introduced. The magnitude of the dispersion is the same regardless.
Figure 3.8: Photograph of transmission grating compressor. The pulse enters the compressor via reflection off of the D-shaped mirror (D), is diffracted by the gratings (G), and the output beam is tilted upwards by the retro-reflecting mirror (RR) such that it passes over the same mirror.

of the sign of $b$ [34]. It can be shown via a ray-optics analysis [16] that the beam shape is not distorted as the distance between the gratings and telescope is changed. As noted in [35], there exists a plane of symmetry about which the stretcher design can be folded via a flat mirror (i.e. the internal focal plane of the telescope), in order to save space and optical components. An additional benefit is that since the same grating is used to disperse and reassemble the beam, any misalignment with respect to the parallel orientation of two separate gratings is avoided.

The layout of the stretcher is depicted in Figure 3.10, which is based on the design of a commercially-produced femtosecond regenerative CPA (Spitfire Pro, Spectra-Physics, USA). A photograph of the completed stretcher is shown in Figure 3.11. Reflective optics are used instead of lenses for ease of alignment and to
avoid aberrations. The input beam strikes a 1200 l/mm ruled reflection grating (G) and the dispersed beam is incident above the center of a $f = 45$ cm concave mirror (CM). The curved mirror is tilted up so that the reflection passes over the grating and is incident on the folding mirror (FM) placed in the focal plane of the curved mirror. The face of the flat mirror is parallel to the focal plane so the reflected beam is strikes the curved mirror slightly below-center. This reflection is level and strikes the grating just below the initial spot. The resulting beam emerges parallel to the input beam but is displaced slightly below and so strikes the retro-reflector (RR) which is constructed from two mirrors in order to allow the input beam into the stretcher. The retro-reflector displaces the beam up such that it strikes the grating above the input spot. The reflection from the grating follows a similar path to the
Figure 3.10: Top view schematic of pulse stretcher layout. The dashed box indicates a D-shaped mirror over which the input beam passes to strike the grating (G); the vertically-displaced output beam is reflected off of this mirror out of the stretcher. The concave mirror (CM) along with the folding mirror (FM) forms a telescope to invert the dispersion of the stretcher. Inset: Side view of retroreflector (RR), which reflects and displaces the beam inside the stretcher (solid) while still allowing the input beam (dotted) into the stretcher.

first pass, but is displaced lower on the return path (since the beam was displaced higher than on the first pass), which allows extraction of the final reflection off the grating via a D-shaped mirror. The alignment of the stretcher is easily checked by observing the four spots on the reflection grating. The two mirrors on the retroreflector are adjusted in order to remove any horizontal displacement between the spots, and the curved mirror and folding mirror are responsible for setting the relative spacing between the spots. The stretcher is properly aligned when these spots on the grating are evenly spaced and centered about a single vertical axis.

3.2.3 Discussion

The power efficiency of the stretcher, defined as the final reflection off of the grating divided by the power initially incident on the grating, was measured to be
Figure 3.11: Photograph of grating pulse stretcher. The dashed line represents the output beam reflected off of the D-shaped mirror (D).

15.1%. This is consistent with the nominal diffraction efficiency of ~ 65% for this reflection grating. The low efficiency of the grating stretcher is acceptable in this system because of the use of a preamplifier which ensures that sufficient light is delivered to the power amplifier for signal gain. Furthermore, gain saturation of the power amplifier ([9]) ensures that for a fiber amplifier pumped with sufficient power, the output power becomes only weakly dependent on the input signal power; it will be shown later that this is indeed the case in this amplifier system.

The efficiency of the grating compressor is measured to be 68.4%. This is consistent with the grating diffraction efficiency that is quoted by the manufacturer (≥ 90%), since the beam is diffracted two times through each grating in this setup; this results in the delivery of over 20 W to the enhancement cavity.

An intensity autocorrelation (AC) was performed in order to verify the operation of the stretcher and compressor system. The autocorrelator used (FR-103MN, Femtochrome Research Inc., USA) is not capable of measuring pulse durations be-
Table 3.2: Dispersion properties of some optical components. + indicates normal dispersion, - indicates anomalous dispersion.

<table>
<thead>
<tr>
<th>Component</th>
<th>GVD</th>
<th>TOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMF (1030 nm)</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Silica glass</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Stretcher</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Compressor</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 3.2: Dispersion properties of some optical components. + indicates normal dispersion, - indicates anomalous dispersion.
yond ∼ 15 ps, so an autocorrelation of the stretched pulse was not obtained. The AC trace out of the oscillator (through an optical isolator and a PBS) measured 40 µs, with a calibration factor of 6.225 ps/ms, which implies a pulse duration of 162 fs assuming a sech² pulse shape. After amplification to ∼ 20 W (the lower power is due to the use of a different pump diode that was subsequently replaced) pulse duration out of the compressor was observed on the autocorrelator. By tuning the grating spacing on the compressor the shortest AC trace observed measured 51.5 µs, with a calibration factor of 7.30 ps/ms implying a pulse duration of 243 fs assuming a sech² pulse shape. The discrepancy in pulse duration between the input and output is due primarily to residual nonlinearities in the amplifier (as discussed earlier, here φ_{NL} ≈ 1 where to avoid nonlinear distortion one must have φ_{NL} ≪ 1). In addition, there may be uncompensated TOD or other higher-order dispersion in the amplifier system.

In principle, since the stretcher and compressor both have two degrees of freedom, it is possible to completely compensate for arbitrary dispersion in the amplifier material itself – as shown in Figure 3.12, which plots the a calculation of the dispersion introduced by a stretcher (or compressor) as a function of the grating separation and the incident angle. As the grating separation is tuned, the GVD and TOD vary linearly. The incident angle controls the ratio of TOD to GVD. These
two controls allow the stretcher to span the dispersion space, and by intentionally mismatching the dispersion of the compressor with respect to the stretcher, the material dispersion in the amplifier system (the dispersion of some components is given in Table 3.2 as an example) can be cancelled out to achieve maximum peak power. This is illustrated in Figure 3.13. Here, the split-step Fourier method was used to model the peak power of a CPA system with parameters similar to the real system. Here the stretcher parameters were held constant ($b = 25$ cm, $\theta_i = 37.5$ degrees) and the compressor was tuned in a neighborhood about this value. It is
Figure 3.13: Simulated peak power of CPA system given a fixed stretcher geometry and variable compressor geometry. Setting one degree of freedom and optimizing the peak power based on the other degree of freedom results in the optimal peak power irrespective of the specific operating point of the compressor.

observed that for each angle, there corresponds a grating spacing that maximizes the peak power, and that the value of the peak power does not significantly depend on the particular choice of the angle/spacing pair.

In practice, this calculation shows that for a fixed stretching ratio (via fixing the geometry of the stretcher), one can obtain the optimal peak intensity by fixing one degree of freedom (eg. the diffraction angle) of the compressor, then tuning the second degree of freedom (eg. the grating separation) to maximize the power. Assuming that SPM limits the compressed peak power, the only way to further optimize the system at this point is to increase the stretching ratio by modifying the stretcher.
3.3 Amplifier Chain

The amplifier chain consists of a fiber preamplifier whose output seeds a high-power fiber amplifier that provides the majority of the power gain in the system. The preamplifier is needed in order to have sufficient power incident on the power amplifier so that the input signal saturates the gain of the Yb-doped fiber. This ensures that the power amplifier output consists of amplified signal light as opposed to amplified spontaneous emission (ASE). This requires approximately 75 mW of seed power, as measured on a similar Yb-fiber amplifier setup. As the preamplifier gain medium consists of a shorter length of fiber that is pumped at a lower power, it requires less incident power for ASE suppression.

3.3.1 Preamplifier

A schematic of the preamplifier is shown in Figure 3.14. The gain medium of the preamplifier is a ~45 cm length of Yb-doped phosphosilicate glass fiber (YB-19-
Figure 3.15: Output power of preamplifier as a function of input power. The AOM module drive frequency provides a convenient way to vary the power incident on the preamplifier.

The Yb-doped fiber is pumped in a co-propagating configuration by a 500 mW fiber-coupled laser diode (JDS Uniphase, Canada) spliced to the gain fiber via a 980/1030 nm wavelength-division multiplexer (WDM) (Lightel Corporation, USA). An additional WDM serves as an isolator to prevent any 1030 nm back-reflection from entering the pump diode.

The output power of the preamplifier pumped at 600 mA (corresponding to 350 mW of optical power, from the manufacturer datasheet) is shown in Figure 3.15 as a function of the input power. The input power was varied by tuning the frequency of the AOM module upstream of the preamplifier (discussed previously in Section 3.1.1), as this is the most common mode of large-scale power fluctuations at the input of the preamplifier and thus represents the most realistic measure of performance in this regard. The scatter in the data is due to the fact that the AOM is scanned through its entire range, and so there is some walk-off of the beam alignment due to imperfections of the AOM module alignment. Nevertheless, throughout nearly the entire range of the amplifier the optical spectrum appears as in Figure 59.
The FWHM bandwidth was found to be 8.7 nm, and there is no apparent ASE signal. The similarity of the output spectrum across the tuning range of the AOM to the input spectrum (from Figure 3.5) suggests both that the gain of the preamplifier is saturated and that the amplifier is sufficiently seeded for ASE suppression. Even then, what ASE is generated by the amplifier is rejected on reflection off the first mirror in the power amplifier, as described below.

### 3.3.2 Power Amplifier

The power amplifier is pictured in Figure 3.17. The gain medium consists of a 2.75 m length of Yb-doped photonic crystal fiber (PCF) (DC-170-40-Yb-2, NKT Photonics A/S, Denmark) pumped in a counter-propagating configuration by a fiber-coupled 70 W diode laser emitting at 976 nm (nLight Corporation, USA). The ~100 mW of seed light from the preamplifier is focused onto the input end of
Figure 3.17: Schematic of the power amplifier. The signal is coupled into and out of the Yb-doped photonic crystal fiber (Yb-PCF) via 980/1030 nm dichroic mirrors (DM), which transmit pump light from the laser diode (LD).

The signal light is coupled into and out of the amplifier system by a pair of dichroic mirrors (DM) (Layertec GmbH, Germany) which are anti-reflective (AR)-coated at 980 nm and high reflectivity (HR)-coated at 1030 nm in order to pass the pump light and reflect the signal light. The dichroic mirror at the input serves both to dump any unabosred pump light, and to reject ASE generated by the preamplifier. A photograph of the pump-coupling and output optics.
is shown in Figure 3.18.

Light is guided in the PCF by microstructuring of the fiber cross-section, as opposed to a step in refractive index which is typical for conventional SMF. In order to model pulse propagation through the PCF via the split-step Fourier method as described in Section 2.2, the dispersion of this microstructured fiber was simulated using a finite element method (FEM) mode solver (MODE Solutions, Lumerical Inc., Canada). The fiber core is surrounded by four rings of air holes of diameter \( \sim 2.3 \, \mu m \) spaced 10.8 \( \mu m \) apart, as specified by the manufacturer and similar to the
The dispersion is found to be very close to the material dispersion of the glass; that is, the geometric dispersion due to the waveguide nature of the fiber is only responsible for $\sim 1\%$ of the total dispersion.

The gain saturation of the power amplifier was checked by again varying the AOM frequency through its entire range. As established earlier, this corresponds to a relatively small deviation on the input signal to the power amplifier (due to the gain saturation of the preamplifier), but this is a realistic test for the operating conditions of this amplifier. A test involving insufficient light to suppress the ASE in the power amplifier was not attempted as this could result in irreparable damage to the gain fiber due to giant pulse generation. At full pump power (corresponding to 71 W incident on the gain fiber) the power amplifier output varied by less than 1\% through the entire AOM tuning range. As an additional check, the spectrum out of the power amplifier is measured while varying the pump current to the preampli-
fier in order to reduce its output power. By reducing the preamplifier pump current to 400 mA (from 600), the light incident on the power amplifier is nearly cut in half (to 65 mW). As shown in Figure 3.19, the spectra are nearly indistinguishable, which suggests that the gain of the power amplifier is indeed saturated.

Figure 3.20 is a spectrum of the power amplifier output at full power (31 W output). The FWHM bandwidth is approximately 8.8 nm. However, the shape of the spectrum (compared with, eg. Figure 3.16) has been distorted in a fashion characteristic of parabolic pulses resulting from amplification under high $\phi_{NL}$ [14]. This may be mitigated in future by increasing the stretching ratio, but as demonstrated earlier, the pulse can still be compressed to $\sim 250$ fs duration which allows for a peak intensity of over $1 \times 10^{14}$ W/cm$^2$ in the enhancement cavity. Additionally the central wavelength of the spectrum is shifted, likely due to the slope of the Yb fiber gain spectrum pumped at this power level [12].

### 3.4 Passive Enhancement Cavity

The enhancement cavity (EC) in this project is a ring resonator consisting of seven mirrors. The layout of the cavity is shown in Figure 3.21, and Table 3.3 summarizes some important properties of each cavity mirror. A relatively large number of mirrors is used in order to attain the cavity length corresponding to a repetition rate of 80 MHz ($L \approx 2.75$ m) while also minimizing the angle of incidence of the high reflectivity (HR) mirrors. An additional constraint on the cavity layout is due to mirror 5, which is designed for a 70° angle of incidence. The angle on this mirror is specified in order to allow for output coupling of the EUV from the cavity [5] via a 200 nm pitch grating etched on the surface of the mirror (by Ibsen Photonics, Denmark). With the exception of the input coupler (IC), all mirrors are specified
Figure 3.20: Normalized optical spectrum out of power amplifier (purple) with sech$^2$ fit (red) that measures 8.8 nm FWHM. The oscillator spectrum (grey) is plotted for reference.

to $\geq 99.9\%$ reflectivity at $1030 \pm 20$ nm. The IC is chosen based on a tradeoff between impedance-matching and ability to lock the cavity in the Pound-Drever-Hall scheme. Impedance-matching the cavity in order to optimize the buildup factor favours a high reflectivity, but a very high reflectivity (and therefore a high finesse) results in a very narrow cavity linewidth function $A(p)$ as discussed in Section 2.4.3, which reduces the control loop bandwidth and results in a less stable lock. In practice it is easiest to let the cavity finesse be determined almost entirely by the input coupler ($1 - R_N \gg 1 - R_{IC}$), such that typical input couplers used are of reflectivity $\sim 99.7\%$ (compared with $R_N = (99.99\%)^6 = 99.97\%$). For instance, this particular choice of input coupler results in a cavity finesse $\approx 1700$ and a theoretical buildup factor of $\approx 700$.

The selection of 7.5 and 15 cm radius-of-curvature mirrors results in a intra-
Figure 3.21: Scale drawing of enhancement cavity. Location of the intracavity focus is indicated by the red arrow. The properties of each numbered mirror are described in the text.

<table>
<thead>
<tr>
<th>Mirror</th>
<th>Radius of Curvature</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>∞</td>
<td>Input Coupler</td>
</tr>
<tr>
<td>2</td>
<td>∞</td>
<td>Scanning PZT</td>
</tr>
<tr>
<td>3</td>
<td>∞</td>
<td>Dither PZT</td>
</tr>
<tr>
<td>4</td>
<td>7.5 cm</td>
<td>Left Curved Mirror</td>
</tr>
<tr>
<td>5</td>
<td>∞</td>
<td>EUV Output Coupler</td>
</tr>
<tr>
<td>6</td>
<td>15 cm</td>
<td>Right Curved Mirror</td>
</tr>
<tr>
<td>7</td>
<td>∞</td>
<td>Cavity Transmission</td>
</tr>
</tbody>
</table>

Table 3.3: Properties of cavity mirrors.

cavity beam waist radius $w_0 \approx 16 \mu m$ located between mirror 4 and mirror 5. The lowest-order eigenmode of the cavity is calculated using a Gaussian ray-transfer-matrix [16] program (Winlase, Future Laser Technologies, USA). The resulting depth of focus is $2z_0 = 1.49$ mm. A calculated map of the temporal peak intensity as a function of the distance $r$ away from the beam axis and the position $z$ about the intracavity focus is shown in Figure 3.22. With beam parameters that are comparable to what has been achieved with this system (500 times buildup factor, 20 W incident power, 250 fs pulse duration) there is a focal region of volume $\sim 1$ mm$^3$ where the optical intensity is high enough to effect the ionization of, for example, xenon ($I \sim 3 \times 10^{13}$ W/cm$^2$). This relatively large focal volume compared to similar systems [22] allows for a longer interaction region for HHG, which will increase the EUV photon flux.
3.4.1 Mode-matching

In order to ensure that incident pulses on the cavity add coherently to the pulse train already stored in the resonator at each round-trip, the spatial properties of the frequency comb must be adjusted in addition to the spectral properties. Since the normal modes of a ring resonator are Gaussian, the spatial properties of the beam at any point along the optical axis $z$ can be described completely by the complex parameter $q(z)$. The mode-matching task is then to find some configuration of lenses that causes the $q$-parameter of the input beam to be equal to that of the lowest-order eigenmode (the Gaussian TEM$_{00}$ mode) of the cavity when measured at the input coupler (IC). In reality, there is an additional complication in that both

Figure 3.22: Calculated optical intensity of the Gaussian mode in the cavity based on the present mirror selection, assuming 500 times buildup of 20 W incident power, 250 fs pulses at 80 MHz.
the cavity eigenmode and laser spot can be astigmatic; that is, the $q$-parameter may differ in the tangential and sagittal planes. In principle this can be addressed by using cylindrical lenses and mode-matching each plane separately, but in practice the use of spherical lenses results in satisfactory mode-matching to realize a high enhancement factor.

The cavity $q$-parameter at the IC is found from modelling the cavity using ray-transfer matrices as above. The laser mode at the IC was determined by measuring the spot size at multiple points out of the power amplifier using a beam profiler (Beamview USB, Coherent Inc., USA). The beam properties can then be found by measuring the distance from the beam profile measurements to the output of the power amplifier. Assuming that the lens at the output of the power amplifier collimates the beam emerging from the PCF, the beam size $w_o$ and Rayleigh range $z_R$ of the laser can be calculated. This information is equivalent to the $q$ parameter.
by the relation \( \text{Im}(q) = z_R \). The beam radius as measured \( \sim 1 \) m away from the power amplifier output lens is plotted in Figure 3.23. This was fit to the usual Gaussian beam waist expression

\[
    w(z) = w_0 \sqrt{1 + \left( \frac{z - z_0}{z_R} \right)^2}
\]

(3.4)

where \( w_0 \) denotes the minimum waist, \( z_0 \) is the location of the minimum waist along the optical axis, and \( z_R \) is the Rayleigh range, which is fixed to the diffraction-limited value

\[
    z_R = \frac{\pi w_0^2}{\lambda}
\]

(3.5)

where \( \lambda \) is the wavelength. This is done in order to ensure a self-consistent fit – a direct measurement of the Rayleigh range (eg. by beam profile measurements closer to the focus) cannot be made in this case because anything inserted into the beam path close to the power amplifier output would necessarily block the pump light into the amplifier. The beam waist is calculated to be 0.68 mm at the focus, which is estimated to be 99.4 cm away from the point at which the beam profile measurements were taken. This is consistent with measurements of the distance between the power amplifier output and the beam profiling location as made by a ruler.

Using this information the beam can be propagated through the distance between the amplifier and input coupler by ray transfer matrices. At the input coupler the beam, as estimated from the measurements made above, has a complex parameter \( q = 700 + 679i \) mm. This is contrasted with the lowest-order eigenmode of the cavity which has \( q = -1260 + 957i \) mm. A telescope consisting of two lenses

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is inserted to resize the beam (ie. change the $q$ parameter of the laser) in order to match the mode to the eigenmode of the cavity. Using this method it was found that +75 mm and +50 mm focal length lenses spaced $\sim 128$ mm apart matched the $q$ parameters of the cavity and the laser to within 10% ($q = -1372 + 958i$ mm). The spacing between the lenses and the relative distance between the telescope and the IC were adjusted in order to further tune the mode-matching.

### 3.4.2 Pound-Drever-Hall Lock

A Pound-Drever-Hall cavity locking scheme was implemented as described in Section 2.4.3. The system was built as depicted earlier in Figure 2.13 with some minor differences. An attenuator is inserted between the dither frequency generator and the summing junction in order to control the ratio of dither to controller output that is sent to the actuator (the dither PZT from Table 3.3). An additional PZT actuator (the scanning PZT) is added to the cavity outside of the control loop. This PZT is used to fine-tune the cavity length without the vibration associated with motorized actuators. A map of the error signal similar to Figure 2.14 is observed on the cavity transmission by ramping the position of the scanning PZT through several free spectral ranges. This allows a user to adjust the cavity length close enough to resonance so that the error signal is in the linear region – far away from resonance, the error signal drops to zero so the controller cannot compensate for the length mismatch. It is also possible to stabilize the cavity to a mode that is mismatched by an integer number of free spectral ranges, since the error signal is again linear in this region. By observing the relative peak heights of the cavity transmission as the scanning PZT is swept through several of these transmission peaks, the bias point of the scanning PZT can be adjusted in order to select the central cavity resonance,
which results in the highest transmitted (and therefore intra-cavity) intensity when locked.

3.4.3 Finesse and Power Enhancement

The finesse of the EC is measured using a cavity ring-down measurement [37]. Here, the resonant frequency of the EC is scanned by ramping the PZT that controls the cavity length. As the cavity is brought onto resonance, it accepts light from the laser and power is built up in the cavity. As the cavity moves off resonance, any incident light on it is reflected. At this point, any transmission through the cavity is due entirely to the light that was stored while the cavity was on resonance. The transient response of the cavity transmission, which follows an exponential decay, is therefore a direct measurement of the cavity photon lifetime $\tau_p$ from Equation 2.29, from which the finesse can be calculated. The EC resonant frequency is scanned by driving the fast PZT actuator with a $\sim 1$ kHz triangle wave and the drive signal is made small enough so that only the central cavity resonance is within the range of the scan. The cavity response is monitored by observing the output voltage of a photodiode on an oscilloscope. Figure 3.24 shows a plot of the cavity transmission signal in such a measurement. Here, the finesse measured is approximately 1700. The oscillations in the signal are due to interference between the stored light in the cavity and the incident field on the IC – even off-resonance, there is still a small amount of leakage through the IC which beats with the stored light in the cavity, which is Doppler-shifted due to the PZT-modulated mirror. Careful cleaning and alignment of the cavity mirrors yielded a finesse $\mathcal{F} \approx 2100$ using a 99.99% reflective IC, which implies a maximum finesse of 2500 as calculated by Equation 2.27. This discrepancy is likely due to residual misalignment of the cavity, and the
Figure 3.24: Cavity ring-down measurement used to determine finesse. Here \( \tau_p \approx 3.4 \mu s \), which corresponds to a finesse of 1700.

finite size of the cavity mirrors which acts as an aperture whose far-field diffraction pattern siphons power from the resonant mode of the cavity.

The buildup factor \( I_{\text{max}}/I_0 \) in the cavity was measured by monitoring the leakage through a cavity mirror. First, the IC is removed and the beam (of known input power) is circulated through the cavity; the leakage power is measured in order to calibrate the ratio of leakage power to intracavity power. The IC is then replaced and the cavity is realigned and locked. The leakage power in this case can be compared to the incident power in order to determine the buildup factor of the cavity. It was found that the buildup power for the cavity was approximately 500 (when the finesse was 2000). The observed buildup factor is less than the ideal value of \( \mathcal{F}/\pi \) because of imperfect mode-matching (which results in the loss of a fraction of the incident power) and the absence of impedance-matching, which is restricted by the availability of input couplers of varying reflectivity. Nevertheless, a buildup factor of 500 is sufficient to achieve intracavity intensities of over \( 1 \times 10^{14} \) W/cm\(^2\).
Chapter 4

Conclusions and Future Work

This thesis has demonstrated a laser setup capable of optical intensities sufficient to generate extreme ultraviolet (EUV) radiation via high harmonic generation (HHG). A Yb-doped fiber-based chirped-pulse amplification (CPA) system was designed to amplify a 80 MHz femtosecond frequency comb (FFC) of 140 fs pulse duration to over 20 W average power with a final pulse duration of 250 fs (220 nJ pulse energy). The output of the CPA is coupled to a passive enhancement cavity (EC). Active stabilization of the cavity free spectral range to the frequency comb spacing, as well as adjustment of the carrier-envelope offset frequency via a double-pass acousto-optic modulator (AOM) yields a further enhancement factor of 500. This results in an estimated intracavity peak intensity of over $1 \times 10^{14}$ W/cm$^2$.

The immediate next step in realizing EUV generation in this setup, which is inserting a gas jet into the enhancement cavity, has been demonstrated elsewhere [7]. While the estimated peak intensity of this system is certainly sufficient to achieve ionization of noble gases of interest, there are some issues that preclude efficient generation of EUV. In analogy to SHG and THG processes, phase-matching between
the driving optical field and the resulting EUV field is a vital factor in efficient EUV generation [38]. In the case of single-pass, low repetition-rate setups there exist schemes to improve the phase-matching by the use of hollow-fiber interaction regions [39], but the high finesse required in the EC precludes such an approach. It is expected that phase-matching in this case will involve optimization of the gas flow profile in the interaction region and adjustment of the focal volume size and location within the cavity.

Nevertheless, it is expected that this system will lead to a realization of a EUV source suitable for size metrology of nanoparticles via three-wavelength extinction measurements (3-WEM) [40]. This method involves measuring the attenuation (via Mie scattering) of three separate wavelengths of light through a sample. From these three measurements the extinction coefficients (which are functions of the wavelengths, particle size distribution, and complex refractive index) at each wavelength can be computed from Mie theory and the size distribution can be inferred from this. This method has been applied using laser sources in the visible region [41], but this limits the size resolution of such a measurement to $\geq \sim 200$ nm. Application of EUV radiation to this measurement will improve the size resolution by over an order of magnitude. The EUV source discussed in this thesis is particularly well-suited to 3-WEM experiments, due to the high flux generated by a high repetition rate frequency comb. Furthermore it is expected that due to the coherence of the HHG process, the ultrastable amplitude and phase structure of the optical FFC will be transferred to the EUV signal, resulting in a low-noise source suitable for measurements whose accuracy depends directly on the amplitude stability of the laser. Nanoparticle size measurements made using the EUV source developed in this thesis can be expected to find application in such fields as
atmospheric chemistry, reaction kinetics, and drug delivery.
Bibliography


