Abstract

Electronic Speckle Pattern Interferometry (ESPI) provides a sensitive technique for measuring surface deformations. The technique involves comparison of the speckle phase angles within surface images measured before and after material deformation. This phase angle comparison requires that the speckle positions be consistent in all images. A lateral shift between images by just one pixel substantially degrades ESPI measurements, while a shift of two or more pixels typically causes complete speckle decorrelation and compromises the measurement entirely.

To prevent such lateral motions, the specimen and the optical system must be rigidly fixed. This requirement typically prevents use of the ESPI method in applications outside laboratories or where it is necessary to remove the specimen from the optical setup between ESPI measurements. Here, Digital Image Correlation (DIC) is used to track speckle motion caused by specimen displacement between ESPI measurements. The measured images can then be mathematically shifted to restore the original speckle locations, thereby recorrelating the ESPI measurements. Examples are presented where ESPI measurements are successfully made with specimen shifts in excess of 60 pixels. A novel ESPI measurement technique where the specimen is removed in between ESPI measurements is also developed and validated.
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Nomenclature

$A_m$  Complex amplitude of light wave
$a_m$  Amplitude of light wave
$\phi_m$  Phase of wave
$I$  Light intensity
$S$  Speckle size
$\lambda$  Source wavelength
$F$  Focal length of lens
$a$  Aperture diameter
$M$  Magnification
d_o  Distance from object to lens
d_i  Distance from lens to imaging plane
$f$  f-number of lens
$A_{m1}, A_{m2}$  Complex amplitudes of reference and illumination beams
$I_1, I_2$  Intensities of reference and illumination beams
$\phi$  Phase of interference
$\Delta \phi$  Phase change of interference pattern due to surface displacement
$I, I'$  Intensity distribution before and after surface displacement
$\phi'$  Phase of interference after surface displacement
$I_n$  Intensity distribution of nth phase stepped image
$A$  Mean of phase stepped set
$B$  Modulation (amplitude) of phase stepped set
$\beta_n$  Phase step of nth phase stepped image ($0^\circ, 90^\circ, 180^\circ, 270^\circ$)
$\overrightarrow{K_s}$  Sensitivity vector, bisector of illumination and reference beams
$\overrightarrow{d_s}$  Surface displacement at specimen
$\overrightarrow{k_1}, \overrightarrow{k_2}$  Propagation vectors of reference and illumination beams
$\rho$  Fringe pattern, expressed on range of $[0,1]$
c(s,t)  Correlation coefficient
$f(x,y)$  Potential match area from the target image
w(x,y)  Template being matched from the base image
$NCC(s,t)$  Normalized cross correlation coefficient
$z$  Normalized cross correlation coefficient distribution
\( \sigma_x, \sigma_y, \tau_{xy} \) In plane stress components

\( S_y \) Material yield strength

\( R^2 \) Pearson product moment correlation coefficient (equivalent to NCC)

\( d_p \) Physical pixel size

\( \delta \) Optical path length change

\( \theta_1, \theta_2 \) Angles of incidence of reference and illumination beams (beam 1 and 2)

\( \psi_1, \psi_2 \) Cone angles of reference and illumination beams

\( \delta_L, \delta_R \) Change in optical path length at right and left side of image

\( \phi_L, \phi_R \) Change in phase at right and left side of image

\( \Delta \phi \) Phase change across image due to specimen shift

\( D \) Image width at specimen

\( d \) Specimen shift

\( L \) Average beam path length (illumination and reference)

\( P, Q, R \) 2D linear equation coefficients describing phase gradient
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Dedication

To My Father
1 Introduction

1.1 Electronic Speckle Pattern Interferometry

Interferometry is a powerful optical tool that has many applications within engineering and physics. When two at least partially coherent light waves are superimposed, a spatially modulated intensity pattern is observed in the region of superposition. A common example of this phenomenon is the coloured fringes that occur at the surface of an oil slick.

Electronic speckle pattern interferometry (ESPI) is an interferometric technique that utilizes a highly coherent light source, generally a laser. The laser light is split and recombined at an object’s surface. The resulting interference, or speckle, pattern can then be used to infer information about the surface itself. Most commonly in-plane or out-of-plane displacements and or vibrations are measured. Figure 1.1 shows a typical ESPI setup. In this case displacements occurring at the object’s surface in the direction of the sensitivity vector can be measured.

Some typical applications of ESPI techniques include; full field measurement of surface deformations [1], material defect detection [2,3] and vibration mode shape measurement. [4] Although these are typical uses of ESPI, the range of applications is extensive. More unique applications of ESPI include the detection of buried landmines [5] and the condition monitoring of historical mosaics [6] for example. Within this thesis the particular ESPI application studied is the measurement of residual stress.
The speckle images recorded within ESPI measurements are inherently random in nature. The phase of one speckle relative to its neighbours’ is completely random. The speckle images are interference patterns created by laser light and modulated by the laser light and surface roughness of the object under investigation. An enlarged view of a typical speckle image is shown in Figure 1.2. Although the speckle pattern itself is random, small surface deformations cause each speckle to change phase consistently. This phase shift is then visible by subtraction, and the randomness of the images is effectively removed. Problems arise when rigid body
motions cause the object to shift significantly, changing the pixel location of the speckles from one set of images to the other. When this occurs, the phase change information is lost and the measurement is compromised.

1.2 Challenges of ESPI

One of the major challenges inherent to ESPI measurements is that they are very sensitive to environmental disturbances. In order to obtain accurate results from ESPI data, the specimen, and in particular area under investigation, must not shift during the measurement. Even small rigid-body motions can compromise the measurement entirely. If shifts do occur the measurement can easily be lost. This is due to the way surface information is recorded. The characteristic speckle pattern created by the surface features of the specimen under investigation is completely random. Information about the surface deformations is obtained by comparing surface speckle patterns before and after some mechanical change. These minute changes cause the speckle pattern to change phase accordingly with the surface, so that the surface information can be extracted in the form of a fringe pattern as shown in Figure 1.3. Each
light and dark fringe represents surface displacements of half the wavelength of the light source used.

Problems can arise however when the surface, and therefore the speckle pattern, is shifted significantly. If a speckle physically moves from one pixel location to another from image to image, the critical phase information is lost and the resulting fringe pattern is severely damaged. Often, due to the extremely small (micrometer) scale of ESPI measurements, environmental disturbances such as air currents or vibrations are enough to induce rigid body motions that result in speckle decorrelation and corrupt the ESPI measurement.

All of the preceding examples of ESPI applications are currently performed in a laboratory setting, where the environment can be precisely controlled. However, in an industrial or outdoor environment, there exists disturbances that will cause rigid body motions during measurements and this will corrupt the measurement data. This problem seriously limits the use of ESPI measurements in field applications.

1.3 Proposed Method: Hybrid ESPI - Digital Image Correlation

Digital Image Correlation
DIC is a well established, non contact optical technique. It involves storing digital images of an object in various states, and then mathematically comparing them to obtain information about object displacements or deformations. Frequently, DIC methods are used to compare many sub regions of images to obtain full field displacement or deformation data. [8] In order to compare and match images accurately, the surface of the object under investigation must
contain a pattern or optical features. DIC can be performed with a wide variety of surface patterns, including grids and dots, however it is most effective when using a completely random pattern. Recently, [9-11] work has been done involving utilizing painted and laser speckle patterns as the surface pattern for DIC measurements, leading to the emergence of digital speckle correlation (DSC) measurements.

The method used within this thesis is a novel hybrid ESPI-digital image correlation (EDIC) technique. An ESPI measurement is performed normally, and the specimen planar rigid body motions that occur during measurement are determined from the existing ESPI speckle images using DIC. Any rotations that may have occurred are not addressed using this procedure. Using this DIC data, the ESPI images can be mathematically shifted to compensate for any rigid body motions that may have occurred during measurement. From these new corrected images, accurate displacement data can be obtained.

1.4 **Residual Stresses and the Hole Drilling Method**

Residual stresses are self-equilibrating stresses that are locked into a material and may exist without any external loading or thermal gradients. The majority of manufacturing operations, including turning, grinding, heat-treating, surface hardening and welding, introduce residual stresses into the workpiece. Residual stresses can be quite large and can significantly influence the behaviour of a material, especially with respect to fatigue life and dimensional stability. Additionally residual stresses can impair material strength, and in extreme cases cause major structural failure. Conversely, residual stresses can have beneficial effects, for example, fatigue life improvement due to shot peening.
The hole drilling method for measuring residual stresses is a very popular, well-established method. One of its main advantages is that it inflicts minimal damage to the specimen. A small hole, typically a few mm in diameter, is drilled into the specimen to a depth approximately equal to its diameter. The surface deformations around the hole that result due to residual stress relaxation are then measured. Traditionally strain gauges are used to measure these surface displacements. From these surface displacements, the stress that existed within the removed material can then be determined via an inverse calculation.

With the advent of low cost, high resolution CCD cameras, ESPI has recently been applied to measure surface deformations within the hole drilling method. [12] Using an optical method such as ESPI presents many advantages when compared with traditional strain gauge methods. The main advantages include the relatively high speed at which measurements can be performed. Since all the surface information is contained within images captured via the camera, there is no need to attach anything to the surface of the specimen itself. The per-measurement cost using an ESPI setup is also very low. The initial cost is significantly higher when comparing ESPI to traditional strain gauge methods; however in an industrial setting in which many measurements need to be performed, the additional cost is easily mitigated.

The challenges of ESPI persist when performing hole drilling measurements; if the specimen moves even slightly before and after the hole is drilled, the entire measurement is compromised. This necessitates elaborate, extremely rigid fixtures to hold the specimen accurately, as well as careful control of the measurement environment. As well, since the specimen must be so rigidly fixed, the mechanism to drill the hole must be integrated into the
ESPI system. The action of drilling the hole may itself cause the specimen to shift, compromising the measurement. More practically, the chips from the drilling can also damage the specimen’s surface and interfere with the ESPI measurement.

If it were possible to relax the constraints on specimen shifting before and after hole drilling, it would allow ESPI to be performed in less isolated environments. It would also be possible to perform much more rapid ESPI measurements by eliminating the need for a very rigid fixture to hold the specimen. By removing the specimen to drill the hole and then replacing it, the drilling procedure could be separated from the ESPI system and performed in an area where the vibrations, cutting forces and chips from the process would not disturb the measurement.

Currently due to the inherent sensitivity of the ESPI method this is almost impossible. It therefore would be very valuable to develop a method to mitigate the negative effects of these rigid body motions so that accurate ESPI measurements can be performed even under non-ideal conditions, and so that specimen removal and replacement would be possible.

1.5 Literature Review

1.5.1 ESPI Robustness

The sensitivity to outside disturbances and delicate nature of ESPI equipment and measurements are well known. Currently the predominant method of circumventing these limitations is to perform measurements in a very controlled, isolated environment. Some work however has been done with the goal of increasing ESPI robustness to provide a more direct solution to the delicate nature of the measurement.
Ritter et al. [13] first investigated using a small Michelson interferometer in conjunction with a conventional out of plane ESPI system to remove the effect of out of plane vibrations between the ESPI system and the specimen. This was accomplished by using the information from the Michelson interferometer as an additional signal fed into the phase stepper of the larger system. By synchronizing the phase stepping mechanism with the environmental vibrations, detrimental effects of these vibrations could be reduced for environmental vibrations with frequencies up to 100 Hz.

Chau [14] proposed an alternate method of correcting for environmental vibrations within digital shearing speckle interferometry (DSSI), a measurement technique similar to ESPI. This process continually monitored a DIC subset relatively unaffected by the measured displacements within the images acquired. By continuously computing the correlation coefficient, an algorithm was developed that only acquired images at moments when the subsets happened to be well correlated. Using this result, DSSI fringes were able to be obtained for vibrations with an amplitude of up to 1 mm and frequencies of 25 Hz.

Findeis et al. [15] performed experiments using ESPI to measure thermal stresses in the presence of mechanical environmental vibrations induced by a compressor. By increasing the shutter speed of the camera significantly and utilizing a variety of mechanical and opto-electrical shutters, the effect of the environmental vibrations was mitigated significantly and reasonable measurements were possible for the environmental disturbances tested. Mechanical shutters were constructed in the form of discs with various slots to mechanically simulate pulsed laser illumination. By synchronizing the frequency of the rotating slots with the
exposure time of the camera, the effects of the environmental vibrations were able to be somewhat mitigated. For the best performance however this type of solution was found to be unwieldy and too large for more portable systems. Optical shutters were also investigated in the form of a liquid crystal modulator (LCM) and lithium niobate crystal. The LCM proved too slow to be effective, however the lithium niobate crystal was able to accommodate the much shorter exposure times required to diminish the effect of the environmental vibrations.

Reu and Hansche [16] recently suggested that speckle correlation may be recovered for ESPI measurements with displacements on the same order as the speckle size at the specimen. Similar to the method developed in this thesis, by utilizing DIC data in conjunction with ESPI images, they showed speckle correlation could be reasonably recovered. Unfortunately details of the results and methods were not discussed and the idea appears not to have been pursued any further.

1.5.2 Digital Image Correlation Using Laser Speckles

The use of laser speckles within DIC measurements has experienced rapid growth in recent years, mainly due to the increased popularity and growth of digital image correlation methods in general. Initial work was performed by Peters, Ranson, Chu and Sutton [17-19] who first examined using DIC methods in combination with laser speckles to measure full field displacements and rigid body motions for specimens with optically rough surfaces.

Takai and Asakura [20] examined the effectiveness of using laser speckle based DIC measurements to measure full field displacements of a beam in bending and a specimen undergoing thermal expansion. They determined that the upper limit of measurable
displacement was controlled by the DIC subset size used for matching, and the lower limit of
displacement was somewhat controlled by the speckle size and pixel resolution of the imaging
system.

Chen and Chiang [21] as well as Sjödahl and Benckert [22] examined the optimum laser speckle
size for displacement, and work by both authors suggested speckle size selection according to
the Nyquist, or Shannon sampling theorem produced reasonably accurate results. Chen and
Chiang examined in plane displacements of 200µm of an aluminum plate and performed
various measurements using a constant speckle size and varying the camera used in order to
vary the sampling frequency. They determined using a camera with a physical pixel size of
approximately half (0.528) the speckle size produced the best results, where the correlation
coefficient peak was the best defined. Another important conclusion from this work involved
identifying the spatial dependence of laser speckles; the pattern will not translate exactly with
the surface of a specimen. Taking a slightly different approach, Sjödahl and Benckert performed
a similar investigation. They examined the effectiveness of DIC measurements on computer
generated speckle images with speckles of various sizes and using various subset sizes. They
determined that using speckles of approximately twice the pixel dimensions of the camera
produced good results. Also, by keeping the ratio of subset to speckle size above 10, the DIC
measurement was successful about 80% of the time.

1.6 Research Goals

Currently ESPI is largely limited to laboratory use within very controlled environments. In many
applications however, measurements must be performed in non-ideal environments. These
environments can introduce rigid body motions during measurements that completely compromise the measurement itself. Although previous work has been done to minimize the effect of environmental displacements, specimen shifting still remains a significant problem for ESPI measurements. Therefore, the main goal of this research is to develop a method to counteract the effect of rigid body motions on ESPI measurements, with the ultimate goal of helping to move ESPI from a fragile laboratory technique to a more robust field measurement method.

In the case of ESPI applied to hole-drilling, this new method would provide better resistance to environment-induced rigid-body motions. By counteracting specimen shifting, specimen removal and replacement would also be possible in simple fixtures. This would allow drilling and its negative effects to be separated from the ESPI system as well as increasing measurement speed and accuracy.

1.7 Overview of Experiments

First, the performance and accuracy of DIC using ESPI speckle images (EDIC) will be evaluated using the experimental apparatus. The effectiveness of using DIC with both laser and painted speckles will also be investigated. The performance of the EDIC method for residual stress hole drilling measurements will then be examined by performing measurements involving various rigid body motions and disturbances. Finally the possibility of the removal and replacement of the specimen after drilling will be investigated.
2 Electronic Speckle Pattern Interferometry (ESPI)

2.1 The Nature of Light and the Laser Speckle Phenomenon

Light transmission can be modeled as the propagation of a harmonic plane wave. Its amplitude and phase can be mathematically represented by a complex phasor:

\[ A_m = a_m \exp(-i\phi_m) \]  \hspace{1cm} (2.1)

where: \( a_m \) = Amplitude of the wave \n\( \phi_m \) = Phase of wave

CCD cameras and similar sensors measure light intensity, which can be expressed as:

\[ I = |A_m|^2 \]  \hspace{1cm} (2.2)

A random ‘speckle’ pattern is created when illuminating a surface with a coherent source, i.e., a source where all parts of the light have fixed relative phase. The pattern forms due to the constructive and destructive interference of light waves reflecting from the surface features. A typical imaging setup with a detector array and lens is shown in Figure 2.1. If light waves interfere at the imaging plane constructively, then a bright speckle will be formed, if they interfere destructively then a dark speckle will be formed.
The coherence of the source is important because without it there is no fixed phase relationship between the rays of light shown in Figure 2.1, and thus no fixed positions of constructive or destructive interference. Figure 2.2 (a) shows a specimen illuminated with a coherent source and the speckle pattern is clearly visible. It is characterized by the random dark and light speckles caused by the coherent interference. Figure 2.2 (b) shows the same specimen illuminated using conventional white light. Since the source is not coherent, a speckle pattern is not visible, and the only features in the image are the physical markings on the surface itself.

Since the phase or amplitude of the light cannot be measured directly, the speckle pattern is recorded as values of intensity, which can be expressed as shown in equation (2.2). The speckle pattern itself can be considered as a set of surface markers virtually attached to the surface. As the surface moves, the speckles move with it. The speckles also change intensity with motion.
due to their spatial dependence; as the surface moves the incident light will interfere
differently causing the speckles to change phase from light to dark. This is the governing
principle behind several speckle pattern techniques including ESPI.

![Figure 2.2 – (a) Specimen illuminated with coherent light (b) Specimen illuminated with white light](image)

### 2.2 Speckle Size

The size of the speckles within an image can also be controlled by varying the optical setup
used. Considering the optical setup shown in Figure 2.3, the size of the speckles contained
within an image can be estimated as: [27]

\[
S = \lambda \frac{F(1 + M)}{a} = \lambda f (1 + M)
\]  

(2.3)

where:
- \( S \) = Speckle size
- \( \lambda \) = Source wavelength
- \( F \) = focal length of the lens
- \( a \) = aperture size
- \( M \) = magnification, defined as \( \frac{d_i}{d_o} \)
- \( f' \) = f number of lens defined as \( \frac{F}{a} \)
For an optical system with a set source and with components fixed at set distances, the speckle size of an image can be controlled by varying the f number of the imaging system. Depending on the type of measurement, controlling the size of the speckles within an image can be advantageous - the resolution of several types of speckle measurements is controlled by the speckle size. Figure 2.4 shows two images taken of the same specimen with only the f number

Figure 2.3 - Speckle image optical setup (Adapted from [27])

Figure 2.4 - Speckle image taken with (a) f = 8 (b) f = 32
of the lens and the camera exposure time varied to let in equivalent amounts of light.

Comparing the two images it is quite apparent how varying the speckle size can affect the resolution of the image – the hole clearly visible in Figure 2.4 (a) is nearly completely obscured in Figure 2.4 (b).

2.3 A Typical ESPI Optical Setup

Figure 2.5 shows a typical optical setup used in ESPI measurements. There are several main components: a coherent light source, a beam splitter, a phase stepper and a camera to record the speckle images. ESPI is typically used to evaluate full-field surface displacements or vibrations. The general procedure for an ESPI measurement is to record the speckle pattern created by splitting and recombining light from a coherent source. Speckle patterns are recorded before and after some mechanical displacement. These speckle pattern recorded second is modulated by surface displacements; by examining the phase change of the speckles before and after displacement occurs, information about the surface displacement can be inferred.
The intensity distribution measured on the CCD can be expressed as the superposition of the illumination and reference beams as:

\[ I = |A_{m1} + A_{m2}|^2 \]  

(2.4)

Applying equation (2.2),

\[ I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi \]  

(2.5)
where $\phi$ is the phase of the interference pattern created by the reference and illumination beams and $I_1$ and $I_2$ are the intensities of the illumination and reference beams respectively.

For a surface displacement $\bar{d}$, the intensity distribution becomes:

$$I' = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\phi + \Delta \phi)$$

(2.6)

where $\Delta \phi$ is the phase change caused by the motion of the surface. A challenge of ESPI measurements is how to determine this phase change $\Delta \phi$. Since the only measurable quantity is intensity, an additional technique is needed to evaluate the phase change of the speckles due to the movement of the measured surface.

### 2.4 Phase Stepping

The observed intensity distribution corresponding to the speckle pattern created by splitting and recombining the coherent light source is described by equation (2.5). After surface displacements occur, the phase difference between the two beams becomes:

$$I' = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi'$$

(2.7)

where: $I_1, I_2$ = the intensities of the illumination and reference beam respectively

$\phi' = \phi + \Delta \phi = \text{phase of interference pattern after surface displacement}$

By combining equations (2.5) and (2.7), it can be seen that:

$$\Delta \phi = \phi' - \phi$$

(2.8)
Thus, the phase can be measured before \( (\phi) \) and after \( (\phi') \) the surface displacement then the phase change due to the displacements \( \Delta\phi \) can be determined. Simplifying, (2.5) can be expressed as:

\[
I = A + B \cos \phi
\]  \hspace{1cm} (2.9)

To solve for the phase difference, a system of equations in the form of (2.9) must be formed. This is done by phase stepping [29] schematically shown in Figure 2.6:

![Figure 2.6 - Phase stepping schematic (Adapted from [30])](image)

Sets of images, \( I \) and \( J \) are recorded before and after the surface displacement. Considering the images before displacement, the set of images can then be written as:

\[
I_n = A + B \cos(\phi + \beta_n)
\]  \hspace{1cm} (2.10)

With \( \beta_n \) being a known set of phase steps, usually in 90° increments. Equation (2.10) can also be rewritten using the additive cosine identity as:
\[ I_n = A + u \cos \beta_n + v \sin \beta_n \]  
\[ \text{(2.11)} \]

where: 
\[ u = B \cos(\phi) \]
\[ v = -B \sin(\phi) \]

The system of equations can be re-written as:

\[
\begin{bmatrix}
I_1 \\
\vdots \\
I_n
\end{bmatrix} = 
\begin{bmatrix}
1 & \cos \beta_1 & \sin \beta_1 \\
\vdots & \vdots & \vdots \\
1 & \cos \beta_n & \sin \beta_n
\end{bmatrix}
\begin{bmatrix}
A \\
u \\
v
\end{bmatrix}
\]

\[ \text{(2.12)} \]

\[ d = Gm \]

where:
\[ d = \begin{bmatrix} I_1 \\ \vdots \\
I_n \end{bmatrix}, G = \begin{bmatrix} 1 & \cos \beta_1 & \sin \beta_1 \\ \vdots & \vdots & \vdots \\ 1 & \cos \beta_n & \sin \beta_n \end{bmatrix}, m = \begin{bmatrix} A \\ u \\ v \end{bmatrix} \]

Forming a least squares solution by minimizing the error yields:

\[ \text{error} = d - Gm \]
\[ \min \|d - Gm\| \Rightarrow \frac{d}{dm}\|d - Gm\| = 0 \]
\[ \therefore G^T Gm = G^T d \]

\[ \text{(2.13)} \]

Simplifying equation (2.13) yields:

\[
\begin{bmatrix}
\sum_{n} \cos \beta_n \\
\sum_{n} \cos^2 \beta_n \\
\sum_{n} \sin \beta_n \cos \beta_n \\
\sum_{n} \sin \beta_n \\
\sum_{n} \sin \beta_n \cos \beta_n \\
\sum_{n} \sin^2 \beta_n
\end{bmatrix}
\begin{bmatrix}
A \\
u \\
v
\end{bmatrix} = 
\begin{bmatrix}
\sum_{n} I_n \\
\sum_{n} I_n \cos \beta_n \\
\sum_{n} I_n \sin \beta_n
\end{bmatrix}
\]

\[ \text{(2.14)} \]
Equation (2.14) is the general form of a least squares solution for a phase stepping scheme involving \( n \) steps. In this work a common 5 step scheme [29] is used with \( \beta_n = [0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ] \). Using these values, equation (2.14) can be solved to yield the desired phase difference between the illumination and reference beams as: [31]

\[
\tan \phi = \frac{7(I_4 - I_2)}{4I_1 - I_2 - 6I_3 - I_4 + 4I_5}
\]

(2.15)

By considering both phase stepped sets of images, \( \phi \) and \( \phi' \) can be determined, yielding the phase change due to the change in optical path length, \( \Delta \phi \). Finally, this change can be related to actual surface displacement by using the sensitivity vector \( \vec{K_s} \):

\[
\Delta \phi = \frac{2\pi}{\lambda} \vec{K_s} \cdot \vec{d_s}
\]

(2.16)

where:

\[
\vec{K_s} = \vec{k_2} - \vec{k_1}
\]

\[
|\vec{k_n}| = \frac{2\pi}{\lambda}
\]

\( \vec{k_2} \) and \( \vec{k_1} \) are defined as the propagation vectors of the illumination and reference beams respectively, as illustrated in Figure 2.7.
This phase difference can be plotted on the range of \([0,1]\) forming a fringe pattern as:

\[
\rho = \frac{1}{2}(1 + \cos \Delta \phi)
\]

Figure 2.8 (a) shows an example synthetic fringe pattern. The pattern can be visualized as a topographic map of the surface displacement in the direction of the sensitivity vector, where each pair of light and dark fringes represents a displacement of a wavelength of the light source.

### 2.5 Fringe Patterns and Phase Unwrapping

The phase difference evaluated in equation (2.15) lies on the range of \([-\pi, \pi]\). Angles outside this range are “wrapped” into the range by addition or subtraction of multiples of \(2\pi\). To determine the absolute phase difference due to change in optical path length \(\Delta \phi\) over the entire image this data must be unwrapped. Figure 2.8 (b) shows the phase difference along the
line shown in Figure 2.8 (a). The phase experiences several jumps as the unwrapped phase reaches the limits of \([-\pi, \pi]\). An “unwrapping” process is required to add \(\pm 2\pi\) where these jumps occur to restore the unwrapped “true” value of the phase difference that is not constrained on the range \([-\pi, \pi]\). This unwrapped phase is then used to determine the surface displacements. Here, a common phase unwrapping algorithm [32] is used.

Figure 2.8 - (a) Fringe pattern (b) Wrapped phase difference (c) Unwrapped phase difference
2.6 Speckle Decorrelation and Its Effects

ESPI involves measuring the phase change of speckles caused by surface movement. This phase change can be measured providing the speckles remain at the same pixel locations during all measurements. Speckle decorrelation occurs when the surface moves far enough so that the speckle moves to a different pixel location.

![Figure 2.9 - Speckle decorrelation](image)

Figure 2.9 (a) illustrates a typical speckle pattern. One speckle in particular is marked ‘X’ occupying the pixels marked 1-4. If the specimen displaces to the right, the speckle marked “X” could move from the upper left in Figure 2.9 (a) to the upper right in Figure 2.9 (b). In that case, the pixels at the upper left then measure a different speckle whose phase is random relative to the original speckle. Thus, the speckle phase becomes decorrelated and local surface displacement information is lost.

Speckle decorrelation is the reason that ESPI measurements are so sensitive to environmental disturbances. If the specimen shifts, even by very small amounts, the speckles become decorrelated, the phase information is lost and the fringe pattern is destroyed.
Figure 2.10 - The effect of speckle decorrelation

Figure 2.10 shows the effect of speckle decorrelation on an ESPI fringe pattern. The white circles and dimensions representing the hole drilled were added to the figure separately. Figure 2.10 (a) shows an undisturbed ESPI hole drilling fringe pattern where no rigid body motions have occurred during the measurement. Figure 2.10 (b) shows the same fringe pattern if a motion of one pixel (15 μm) to the right had occurred after drilling. The pixel decorrelation has introduced substantial noise and the quality of the fringe pattern is greatly decreased. Figure 2.10 (c) shows the fringe pattern where the surface has moved 2 pixels (30 μm) to the right after drilling. The thin line in Figure 2.10 (c) is placed for scale and is 2 pixels wide. In this case the fringe pattern is completely destroyed with only noise remaining. These figures clearly illustrate how severely speckle decorrelation can affect ESPI measurements. Even very small motions of the surface can completely destroy the measurements.

Speckle motion can be caused by a number of factors. These include air turbulence, mechanical vibrations, specimen motion within its fixture or, in the case of hole drilling ESPI measurements, the drilling process. Simply touching the workpiece can even cause sufficient motion to decorrelate the measurement. Speckle decorrelation is a serious problem for ESPI measurements.
measurements that is currently preventing more widespread use of the method. If specimen motion could be counteracted, then the robustness of the ESPI method could be significantly improved. Being able to recover accurate measurements even in the presence of rigid body motions would represent a solid step towards moving ESPI from a sensitive lab technique to more robust field use.
3 Digital Image Correlation & ESPI Fringe Correction

3.1 Digital Image Correlation

Digital Image Correlation (DIC) is a popular optical method used to determine relative motion between a pair of images. It is based on the principle of image matching; a template or subset from the first image is selected and a match is found within a second image after some displacement has occurred. Digital images are recorded as intensity distributions, as shown in Figure 3.1. Each pixel holds a value from 0 to a maximum value determined by the bit depth of the camera used. 0 represents black, and the maximum value represents white. Intermediate values represent shades of grey.

![Digital image storage of a painted speckle image](image)

**Figure 3.1 - Digital image storage of a painted speckle image**
A correlation coefficient which relates the template to its potential match can be defined as [33]:

\[
c(s,t) = \sum_{x} \sum_{y} f(x,y)w(x-s,y-t)
\]  

(3.1)

\[
\text{Figure 3.2 - The correlation coefficient (Adapted from [33])}
\]

where the terms are defined as shown in Figure 3.2 and the sums are computed over the pixel area currently covered by the template, \( w(x-s, y-t) \). \( f(x,y) \) represents the second image where the template is being searched for, with size \( M \times N \). \( w(x,y) \) is the template, or feature of interest, selected from the first image. By moving a template sized area, \( w(x-s, y-t) \), over
some range of $s$ and $t$ in $f(x, y)$, the distribution of the correlation coefficient can be built up. For each position $(s,t)$ of the template, the correlation coefficient, $c(s,t)$, can be computed. This coefficient will reach a maximum where the best match of $w(x, y)$ and $f(x, y)$ is found. This process is illustrated in Figure 3.3.

![Figure 3.3 – Template matching using the correlation coefficient](image)

(a) Base image $f(x, y)$ (b) Template $w(x, y)$ (c) Correlation coefficient distribution $c(s,t)$

(Adapted from [34])

Figure 3.3 (c) shows the correlation coefficient distribution, with higher values represented as brighter areas. The correlation coefficient is shown to reach a maximum at the location where the template is matched correctly; however areas where poorer matches were found, around the other characters, appear as well.

One of the difficulties with equation (4.1) is its dependence on the amplitudes of $f(x, y)$ and $w(x, y)$. If these functions change amplitudes the values of the coefficient will change accordingly. In order to remove this dependence, equation (4.1) can be normalized to form the normalized correlation coefficient (NCC) [33]:
Equation (3.2) removes the dependence on amplitude by removing the mean intensities of the template and area under review and normalizing to a unit magnitude. This new coefficient now falls on the range of [-1,1]. By finding the maximum of the NCC, the displacement of the template from one image to another can be determined, as shown in Figure 3.4.

In this case, the bounded area enclosing the letter ‘A’ is the template. By determining its motion using the NCC within image 2, the displacement \( \vec{d} \) of the feature from image 1 to 2 can be determined.

By repeating this process for many small templates, it is possible to build up a set of displacement data for the entire image. It should be noted however that due to the nature of equation (3.2), the matching process suffers near the edge of the images since the template will partially be placed over areas where there are no image data. In these cases the images are padded with blank values; however accuracy still suffers greatly.

Another important consideration when performing DIC measurements is the character of the features within the images. Although images with a variety of features such as grids, lines or...
dots can be used in DIC, the best results are obtained using random patterns with high information content. If a periodic pattern is used, it is possible for the coefficient to register several identical maxima as the template is periodically matched according to the pattern. This is known as the correspondence problem in image matching and illustrated in Figure 3.5.

If the template, indicated by the dark box, is moved to any one of the positions indicated by the vectors, a perfect match will be found. Therefore, in this case, the motion of the template can only be determined as a scalar multiple of the pattern size.

In order to avoid this problem, high information random speckle patterns are often used to ensure that no false positives are reported and that the template displacement solution is unique.

3.1.1 Digital Image Correlation Using Painted Speckles

A popular choice for image correlation is a randomly sprayed painted speckle pattern applied to the surface under investigation. An example is given in Figure 3.1. Painted speckle patterns exhibit the desired qualities for DIC measurements – they are random, and have high information content so long as the size of the paint speckles is controlled. Often paint from a simple spray can is sufficient. The advantage of a painted pattern lies with the fact that the speckles are physical; they are durable and have no spatial dependence. Their positions also do not change significantly as the surface under investigation moves, unless very large
deformations occur. This durability and consistency aids in producing accurate DIC results.

Because of the desirable characteristics, recently the use of painted speckle DIC has been applied within various areas of experimental mechanics including tensile testing [9], strain field measurement [11] and residual stress measurement. [35,36]

3.1.2 Digital Image Correlation Using Laser Speckles

The characteristic speckle pattern created using a laser light source also contains many of the same characteristics as a painted speckle pattern – the pattern is highly random, unique and has dense information content. All of these qualities make laser speckle images an attractive choice as the basis for DIC measurements. Laser speckles also have the added advantage of not needing physical application; all that is required is a coherent monochromatic light source and the object to have a suitable surface finish with roughness on the scale of the source wavelength. The main disadvantage of using laser speckles is the impermanence of the pattern. As the surface moves, the speckles move along with the surface. However, the local illumination gradually changes from place to place, and so the pattern of the speckles correspondingly changes. In addition, the phase of the speckle pattern can change, causing light speckles to become dark, and vice-versa. These behaviours present obvious problems for DIC measurements as the template matching process suffers due to both the change in phase and speckle shape. Despite these challenges, laser speckle DIC measurements have been applied in similar areas to that of painted speckle DIC, with the majority of the applications aimed at strain and displacement measurements [10,24]. Due to the influence of the surface features on the laser speckle pattern, additional applications involving surface condition
monitoring [25] have also recently been explored. These measurements involve monitoring the correlation coefficient for a certain area as the surface condition changes.

### 3.2 ESPI Fringe Correction Procedure

Currently due to their inherent sensitivity to relative motions and environmental disturbances, ESPI measurements are limited largely to laboratory use in very controlled environments. Specimen rigid-body motions cause speckle decorrelation, which corrupts the critical phase information necessary to determine the surface displacement. The ESPI fringe correction procedure proposed here represents a method to correct rigid-body motions by re-correlating speckle images and therefore restoring the ESPI fringe map. A similar idea was first suggested by Reu and Hansche [16] but was never pursued.

The procedure makes use of DIC methods to first determine the rigid-body motions that have occurred during the ESPI measurement. This is done using the captured ESPI image sets as the basis of DIC displacement measurements. By using these laser speckle images, the rigid body motions that occurred between the image sets can be determined. Once these motions are known, the affected ESPI image set can be mathematically shifted in order to ensure the speckle phase information is re-correlated. Figure 3.6 shows an outline of the fringe correction procedure.
In order to determine the rigid body motions of the specimen, correlation subsets, or patches, are selected circumferentially around the centre of the measurement; a 60x60 pixel subset is used. The search range can be set accordingly, and the normalized cross correlation coefficient, in the equivalent form of the Pearson product-moment correlation coefficient, is formed over the search area. Figure 3.7 shows an example of how the subsets are selected - each box represents a separate subset. For each subset, a NCC distribution is built up over the selected search area by shifting the template pixel by pixel over the search area. From that distribution, the maximum value of the NCC is found. Figure 3.8 shows a NCC distribution and its maximum for one subset.
Figure 3.7 - DIC subset selection

Figure 3.8 - NCC distribution and maximum
Forming the NCC distribution using equation (3.2) allows matching to integer pixel values. To achieve sub-pixel resolution, a 2D quadratic function is fit to the maximum value and the surrounding 8 pixels. Starting with the general form of a 2D quadratic, there are 6 unknown constants, a-f:

\[ z = a + bx + cy + dx^2 + exy + fy^2 \]  \hspace{1cm} (3.3)

The NCC maximum and surrounding pixels are defined as shown in Figure 3.9, with 5 being the pixel where the maximum value of the NCC is located.

![Figure 3.9 - NCC maximum and surrounding pixels](image)

Since the pixels are unit spaced, a matrix equation can be formed by considering the coordinates of each of the 9 pixels as:
This system is overdetermined since there are 9 equations and only 6 coefficients. Here, it is chosen to fit the coefficients $a$, $b$, $c$, $d$, $f$ exactly to the central five points 2 4 5 6 8 and coefficient $e$ to the average of the corner points 1 3 7 9. This procedure focuses the fit on the central points.

The coefficients can then be determined by addition and subtraction as:

\[
\begin{align*}
a &= z_5 \\
b &= (z_6 - z_4) / 2 \\
c &= (z_8 - z_2) / 2 \\
d &= (z_4 + z_6) / 2 - z_5 \\
e &= (z_1 - z_3 - z_7 + z_9) / 4 \\
f &= (z_2 + z_8) / 2 - z_5
\end{align*}
\]  

(3.5)

Once the quadratic is formed, directional derivatives are calculated in order to find its maximum. Continuing from equation (3.3):

\[
\begin{align*}
\frac{\partial z}{\partial x} &= b + 2dx + ey = 0 \\
\frac{\partial z}{\partial y} &= c + ex + 2fy = 0
\end{align*}
\]  

(3.6)
Equations (3.6) can finally be solved to yield the co-ordinates of the maximum of the NCC relative to the maximum pixel as:

\[ x = \frac{-2bf + ce}{4df - e^2} \]
\[ y = \frac{be - 2cd}{4df - e^2} \]  

(3.7)

These results are added to the maximum integer pixel value to allow sub-pixel interpolation.

Once the relevant rigid body motions are determined, the affected set of ESPI images is shifted accordingly. Bi-linear interpolation is used to account for the sub-pixel image shifts.

Mathematical shifting of the image sets to account for the rigid-body motions re-establishes speckle correlation. The speckles in the second set of images now occupy the same pixels as they originally did within the first set. This pixel re-registration allows accurate phase data to be recovered and the surface displacement solution to be largely restored. At the image edge, there is no information to move into the shifted area, so image recorrelation is not possible there. Figure 3.9 shows an example of the loss of pixels near the image edge. This loss can become quite large for large image shifts.

To summarize, digital image correlation has been investigated and a basic algorithm developed. This algorithm will be used as the main component of the ESPI fringe correction process discussed. The aim of the process is to mitigate the negative effects of specimen rigid
body motions within ESPI measurements. If this fringe correction process can be proven effective, the robustness of ESPI measurements will be significantly improved, potentially allowing measurements to be performed in more adverse environments as well as other related benefits.
4 Experimental Validation

4.1 Hybrid ESPI – DIC Apparatus

An experimental apparatus capable of making hole drilling ESPI measurements was constructed to test the feasibility and effectiveness of the proposed DIC fringe correction procedure. Figure 4.1 illustrates the layout and major components of the experimental apparatus.

![Apparatus schematic](image)

**Figure 4.1 - Apparatus schematic**

Table 4.1 lists the major components of the apparatus. The ESPI portion of the apparatus – encompassing the beam splitter mirror and piezoelectric actuator, was custom designed and built. Chip suction was also implemented within the drilling apparatus to
minimize the spread of the metallic particles cut from the hole and to reduce their adverse impact on optical components and measured surfaces. The photos in Figure 4.2- Figure 4.4 illustrate the apparatus.

Figure 4.2 – ESPI beam splitter assembly

Figure 4.3 – Adjustable mirror

Figure 4.4 - Drill assembly
Table 4.1 - Apparatus components

<table>
<thead>
<tr>
<th>Component</th>
<th>Make/Model</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser</td>
<td>JDS Uniphase CDPS532M</td>
<td>532nm wavelength, 50mW</td>
</tr>
<tr>
<td>Camera</td>
<td>Prosilica EC 750</td>
<td>640 x 480 resolution, 8 bit grayscale color depth with telecentric lens</td>
</tr>
<tr>
<td>Piezoelectric Actuator</td>
<td>Piezomechanik PST 150/7x7/7</td>
<td>9µm maximum extension</td>
</tr>
<tr>
<td>3-Axis Stage</td>
<td>Daedal</td>
<td>Drill and specimen mounting possible</td>
</tr>
<tr>
<td>High Speed Drill</td>
<td>NSK EMS-3041</td>
<td>25k rpm max speed</td>
</tr>
<tr>
<td>Drill Stage</td>
<td>Newport CMA-25CCCL</td>
<td>Spring return, lead screw driven</td>
</tr>
</tbody>
</table>

The objective of the apparatus design was flexibility. Each beam can be adjusted independently, as can the vertical mirror placed near the specimen. This allows the area of the specimen under investigation to be uniformly illuminated by coherent beams oriented with the proper incidence angles. By repositioning and adjusting the components, a range of specimens can easily be accommodated. Attaching the selected specimen directly to the 3 axis table allows known specimen rigid-body motions to be accurately created. This allows the effectiveness of EDIC as well as the DIC fringe correction procedure to be explored systematically.

4.2 ESPI System Validation

Upon completion of the ESPI apparatus, it was necessary to first validate that it provided accurate measurements. Since residual stress hole drilling measurements are used as the test case for DIC fringe correction within this work, measurement of a specimen with a known residual stress field was examined.
The specimen used was a ring and plug specimen provided by Dr. M. Steinzig of the Los Alamos National Laboratory, New Mexico. This specimen had previously been calibrated at Los Alamos. By measuring the specimen using the newly constructed experimental apparatus and comparing the results to those obtained in Los Alamos the new apparatus could be verified. Figure 4.5 shows the ESPI fringe result from a hole drilling measurement performed on the calibrated specimen.

![Figure 4.5 - Fringe result using calibrated specimen](image)

Although frequently used to compare the accuracy of ESPI fringes within this research, the hole drilling residual stress solution will not be discussed in detail within this thesis. The focus of this work deals with ESPI measurements; residual stress hole drilling is only used as a test case to provide ESPI fringes. All residual stresses calculated are done so exactly in the manner described in [12].
The induced stress field within the specimen according to the measurements and calculations performed in Los Alamos is compared with the solution measured using the experimental apparatus in Table 4.2.

Table 4.2 - Calibrated vs. measured stresses

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_x$</th>
<th>$\sigma_y$</th>
<th>$\tau_{xy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated Stress (MPa)</td>
<td>35.6</td>
<td>57.5</td>
<td>-0.3</td>
</tr>
<tr>
<td>Measured Stress (MPa)</td>
<td>31.4</td>
<td>53.2</td>
<td>-2.9</td>
</tr>
<tr>
<td>Difference (MPa)</td>
<td>4.2</td>
<td>4.3</td>
<td>2.6</td>
</tr>
<tr>
<td>% Difference w.r.t. $S_y$ (%)</td>
<td>1.5</td>
<td>1.5</td>
<td>0.9</td>
</tr>
</tbody>
</table>

The measured stresses agree with the calibrated values within about 4MPa. This is an encouraging result and gives credibility to the experimental system. The stress difference is modest compared with practical residual stresses, which can be in the range of some hundreds of MPa. Expressing the difference as a percentage of the yield strength of the material gives a more relative indication of the scale of the differences in the measurement. These errors are all less than 2%, and this again indicates the measurements are in good agreement.

Comparing the two results they are shown to be in good agreement, differing by only a few MPa when comparing each of the three in plane stress components. Since the capacity of the constructed system to measure residual stresses accurately has now been proven, the investigation of ESPI fringe correction using this apparatus can be explored with greater confidence.
4.3 DIC Measurements Using ESPI Images

Examining a typical ESPI measurement, a large amount of data is available for analysis. When using a 4-step phase stepping algorithm, 8 total images are therefore available for each ESPI measurement; 4 captured initially - the reference set, and 4 captured after some mechanical change - the fringe set. Each of the images within each set is phase stepped according to the scheme described in section 2.4. Due to the phase dependent nature of laser speckles, this causes the images within each set to vary significantly. Figure 4.6 shows sections from a set of 4 phase-stepped images. These sections are taken from the same location of each image. The phase-stepping among these images causes them to appear very different, even though they share the same underlying speckle pattern. Speckles are seen to change phase from light to dark as well as to change shape from image to image.

![Figure 4.6 - Sections of a phase stepped set of images](image)

This image variation becomes the first challenge when attempting to utilize ESPI images within DIC measurements. When examining an arbitrary pair of images, one from the reference set and one from the fringe set, it is possible that the speckles could be in phase, out of phase, or somewhere in between. For successful DIC measurements it is imperative that the pattern being matched remains consistent from image to image. Because phase variation has a direct effect on intensity, using these phase-stepped images directly is not suitable for DIC image
analysis. Therefore, it is essential to develop some means of mitigating this pattern fluctuation due to phase variation.

Returning to equation (2.10), the intensity distribution created by the illumination and reference beams within a phase stepped image can be expressed as:

\[
I_n = A + B \cos(\phi + \beta_n) \quad (4.1)
\]

In order to construct images suitable for DIC analysis, the quantities \( A \) and \( B \) which are seen to be phase independent, are of interest. By solving equation (2.18) in the case of a 4 step algorithm, these quantities can be expressed in terms of the four phase-stepped image intensities at a given pixel as:

\[
A = \frac{I_1 + I_2 + I_3 + I_4}{4} \quad B = \sqrt{\frac{(I_4 - I_2)^2 + (I_3 - I_1)^2}{2}} \quad (4.2)
\]

Therefore, \( A \) and \( B \) ‘images’ can be constructed from the 4 images within a phase stepped set. To test the feasibility and effectiveness of utilizing these new phase independent images within DIC measurements, statistical tests of DIC capabilities were conducted on an example specimen mechanically displaced by eighteen 0.1mm increments up to 1.8mm using the three-axis table. In order to minimize the backlash errors within the table, the specimen was moved backwards and forwards each time before recording images to take up slack in the table drive system. Sets of four phase-stepped ESPI images were measured after each displacement. For the
magnification used, a 0.1mm displacement corresponded to 6.1 pixels. DIC evaluations of the image displacement were done at one hundred 50x50 pixel patches in a 10x10 grid spanning the images. The standard deviations of these DIC evaluations and the average correlation coefficients $R^2$ within the patches gave indications of the statistical quality of the results. Figure 4.7 shows the standard deviation of the DIC displacement estimates vs. amount of image shift. A lower standard deviation indicates less statistical uncertainty and hence greater expected computational precision. In general, the standard deviation increases with image shift, indicating that the speckle pattern does not simply translate, as would happen with images of displaced physical surface features. ESPI speckle patterns from displaced surfaces slowly change due to differences in the illumination at different points in space.

![Figure 4.7 - Standard deviation of DIC displacement estimates vs. amount of image shift](image-url)
The lowest of the four lines in Figure 4.7 shows the DIC results using the constant “A” from equation (4.2). The adjacent bold line shows the results from “B” image, which are comparable to the “A” results but deteriorate at higher image shifts. The “Min. Image” and “Max. Image” lines respectively show the minimum and maximum standard deviations observed among the 16 possible combinations of four initial stepped images with four final stepped images. The “Min. Image” results derive from the ESPI images that happen to be in phase, and give results similar to those from the “A” images. However, it is not known in advance which particular ESPI images are in phase. Thus, it would be necessary to test all image combinations to find them.

![Figure 4.8 - Average correlation coefficient $R^2$ vs. amount of image shift](image-url)
which is not a very practical procedure. The “Max. Image” curve represents case when the chosen images are out of phase, and thus are poorly correlated. The large “Max. Image” standard deviations at small image shifts occur because the speckle patterns remain mostly intact in this range, and so adverse relative phase has a large effect. At larger image shifts, the speckle patterns become distorted, and so the additional distortion caused by adverse phase is no longer so influential.

Figure 4.8 shows the average correlation coefficient $R^2$ for the same correlation patches as in Figure 4.7. Higher $R^2$ is desirable, and thus the trends in Figure 4.8 support the conclusions from Figure 4.7. The correlations decrease with amount of image shift, showing the changes in speckle patterns with surface displacement. The $R^2$ values are much lower than the range 0.95 to 0.99 typically achieved with images of physical surface features [8]. This lower correlation occurs because speckle patterns respond to local changes in illumination as well as to surface displacements. Image “A” in Figure 4.8 achieve the best (highest) correlation coefficients; image “B” and the raw ESPI images are consistently lower. Thus, based on both Figure 4.7 and Figure 4.8, the general use of “A” images for DIC analysis is indicated. Based on this result whenever an ESPI based DIC (EDIC) calculation is performed, an “A” image will first be formed for each phase stepped set of images and then the DIC method will be applied.
4.4 Measuring Surface Motions Using EDIC & the Effect of Speckle Size

Having now established an effective method of using phase stepped ESPI images within DIC measurements, the fringe correction process is one step closer to reality. The next logical step is to investigate the range and accuracy of these measurements. Of particular interest is how these qualities compare to those of physical feature based DIC. In addition, previous work involving conventional painted [37] and laser [22] speckle based DIC measurements suggests that the size of the speckles used affects measurement accuracy. Bearing this in mind, the effect of laser speckle size on EDIC measurements will also be investigated.

To this end, a series of experiments was conducted using a flat specimen mounted on the 3-axis motorized table. Considering a co-ordinate system oriented on the specimen as shown in Figure 4.9, the specimen was moved from an initial position over a range of 3mm in both the x and y directions in 0.1mm increments. Again, the table was run backwards and forwards for each set in order to minimize the slack errors inherent to the table’s drive system. For each phase stepped set, 18 60x60 pixel subsets were selected as before on a circular grid as shown in Figure 3.6. The central area of the image was used to avoid measurements losing correlation due to the search area being constrained by the edges of the image. Using these subsets 18 individual DIC measurements were performed and an average recorded. The DIC result was then compared with the applied displacement, converted into pixel units using a calibration done by recording images of a precision scale.
placed on the surface of the specimen. The entire process was then repeated for a range of laser speckle sizes. Recalling equation (2.3), the variation in speckle size was achieved by changing the f# of the lens of the CCD camera. In order to compensate for the change in light incident to the CCD due to this change, the exposure time of the camera was adjusted accordingly to achieve a consistent average intensity over the entire image. The camera parameters for the range of speckle sizes examined are listed in Table 4.3. The speckle size listed represents an estimate calculated using equation (2.3).

**Table 4.3 - Camera parameters and speckle sizes**

<table>
<thead>
<tr>
<th>$f$</th>
<th>5.6</th>
<th>8</th>
<th>11</th>
<th>16</th>
<th>22</th>
<th>27</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exposure time (ms)</td>
<td>1</td>
<td>3</td>
<td>7</td>
<td>18</td>
<td>40</td>
<td>70</td>
<td>125</td>
</tr>
<tr>
<td>Speckle Size, $S$, (px/s)</td>
<td>0.61</td>
<td>0.87</td>
<td>1.19</td>
<td>1.73</td>
<td>2.38</td>
<td>2.92</td>
<td>3.46</td>
</tr>
<tr>
<td>Mean Intensity, $\bar{I}$</td>
<td>101</td>
<td>108</td>
<td>104.7</td>
<td>101.3</td>
<td>100.1</td>
<td>101.4</td>
<td>101.2</td>
</tr>
</tbody>
</table>

A higher f# represents a smaller aperture and therefore larger speckles. By closing the aperture down smaller speckles are unable to form and all that remains are the larger ones [38]. In addition to laser speckles, a painted speckle pattern was also examined. This pattern was created directly on the surface of the specimen using a can of conventional black spray paint. When recording images of the painted pattern, only a single image was required as no phase stepping occurred. Specimen illumination was accomplished using diffuse white light from a fluorescent lamp. The experimental results for specimen shifts in the x direction are shown in Figure 4.10. The ideal results obtained from the table itself are shown as a line with unity slope. The accuracy of the various EDIC measurements can then be measured by how closely the
corresponding data matches the table data. Examining Figure 4.10, it becomes very evident that speckle size has a significant effect on the effectiveness of the EDIC measurements. Both

![Figure 4.10 - X Direction EDIC Measurements](image)

the largest and smallest speckles, 3.46px/s and 0.61px/s respectively, perform poorly. The large speckles maintain a reasonably accurate result until a displacement of 82.6 px, or 1.4mm. Beyond that point the solution deteriorates rapidly and deviates from the true displacement more than any speckle size tested. On the other end of the size spectrum, the smallest speckles lose accuracy at even smaller displacements, deviating at displacements of 70.9 px or 1.1mm. When compared to the largest speckles, the smallest speckles tested did provide a closer
agreement with the applied displacements however. Although neither the smallest or largest speckles performed well, the results improved within the middle of the speckle size range. Speckles 0.87px/s in size, the next smallest speckles, began deviating at displacements of 94.5px followed by the second largest speckles of 2.92px/s at 129px displacement.

It is evident examining the data that the trend is towards an optimum speckle size that maximizes the EDIC effectiveness. Consistent with this trend, the datasets representing the midsize speckles, 1.19px/s and 1.73px/s are shown to perform the best, maintaining good agreement with the table displacement right up to the maximum displacement measured of 3mm or 177.2px. Over this range the calculation error remains sub pixel except in a few outlying cases.

The painted speckles are shown to give accurate results, matching the applied displacements very closely. This increased accuracy is attributed to the fact that painted speckles are physical features that move with the specimen surface. They are subject to a different limitation caused by the possible presence of geometrical distortions due to aberrations in the imaging optics. However, this is a minor effect with good quality optics. This quality is in stark contrast with laser speckles, which as previously discussed in section 2, do have a spatial dependence relating to the phase of the interference. This causes the speckle pattern to change as the specimen moves, even though the phase dependence has been mitigated by using the technique described in section 4.3. Since DIC measurements rely on the consistency of the pattern being matched this difference indicates that painted speckles will always yield more accurate results than laser speckles. This is confirmed in the experimental results.
When examining EDIC measurements in the y direction, much the same trend is observed. These experimental results are shown in Figure 4.11. Again, the largest and smallest speckles lose accuracy the fastest but in this case 2.92px/s is the next speckle size to fail as opposed to 0.87px/s as in the x direction. The general trend however remains the same, with 1.19px/s and 1.73px/s again performing the best of those speckle sizes tested. It should be noted that the last data points shown in Figure 4.11 are especially inaccurate because the search area of several DIC subsets was curtailed by the bounds of the image causing the average measurement to be severely affected. This is due to the 4:3 aspect ratio of the image itself; the increased data available in the x direction allows displacements to be measured at greater distances than in the y direction. This trend is clearly visible when examining the EDIC results. Although painted DIC remains accurate to 2.8mm this is most likely due to the greater strength of correlation when using painted physical features when compared to the spatially dependent laser speckles. Even with the subset being only partially matched, the correlation using painted speckles is strong enough to maintain a reasonably accurate result.
Figure 4.12 shows the measurement range of each speckle size in both the x and y directions.

The trend towards an optimal speckle size around 1.19px/s and 1.73px/s, where the measurement range is greatest, is clearly shown for both directions.
To examine more clearly the effectiveness of the most accurate speckle sizes F11 and F16 as Examining the data for 1.19px/s, 1.73px/s as well as the painted speckles, the absolute average error and total error were calculated as shown in Table 4.4, omitting the last measurements in the y direction because of the aspect ratio issue.

Table 4.4 - EDIC measurement standard deviation for painted and laser speckles

<table>
<thead>
<tr>
<th></th>
<th>1.19px/s</th>
<th>1.73px/s</th>
<th>Painted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Error (px)</td>
<td>0.60</td>
<td>0.58</td>
<td>0.46</td>
</tr>
<tr>
<td>Total Error (px)</td>
<td>32.01</td>
<td>30.56</td>
<td>25.10</td>
</tr>
</tbody>
</table>

Figure 4.12 - Measurement range vs. Speckle Size
These data yield much clearer results showing that the laser speckle size of 1.73px/s, where \( f = 16 \) yields speckles that are the most accurate when used within EDIC measurements. Using this size the average error of 0.58px is slightly lower when compared with the average error of 0.60px when using speckles 1.19px in size. This trend is also echoed when considering the total error across all measurements. At 1.73px/s, the average speckle has a size that is larger than the conventional size of 1px. Also, at 1.2px/s the EDIC error is very similar. This suggests speckles 1-2px in size are the most effective for EDIC measurements. Painted speckles again, as expected, are shown to be more effective than laser speckles, with both an average error and total error lower than either laser speckle size. The average error of 0.46px for painted speckles is relatively high when compared with typical error associated with the DIC method, however errors in the range of 0.001 to 0.5px have been reported within previous work [8,23], which is consistent with this result.

The deterioration in DIC correlation accuracy at the extremes of speckle size conforms to theoretical expectations. For a given patch size, DIC correlation accuracy varies approximately inversely with the average linear size of the features within the analyzed images. Thus, the larger speckle sizes associated with larger \( f \) give lower correlation accuracy. The DIC accuracy increases as the speckle size reduces with smaller \( f \). However, with the smallest values of \( f \), the average speckle size reduces below the CCD pixel size and cannot be adequately resolved by the camera, thereby also reducing correlation accuracy.

This concept was investigated for single beam DSC measurements by Feiel and Wilksch [24] who, based on the work of Sjödahl and Benckert [22] proposed an optimum f number be
chosen according to the following criteria, in order to minimize aliasing errors while maintaining the highest resolution possible:

\[ f = 2 \frac{d_p}{(1 + M) \lambda} \]  \hspace{1cm} (4.3)

Where:
- \( d_p \) = Physical pixel size
- \( M \) = Magnification
- \( \lambda \) = Source wavelength

Equation (4.3) is based on the Nyquist criterion, which states that in order to accurately measure a signal containing a maximum frequency of \( Y \), the signal must be sampled at a frequency of \( 2Y \). Physically, the speckle pattern contains speckles at a frequency of the inverse of the speckle size as defined in equation (2.3). The sampling frequency in this case is defined as the inverse of the pixel size. By this reasoning, equation (4.3) was developed to yield the corresponding f number in line with the Nyquist criterion and speckle size as defined by equation (2.3). Using equation (4.3) for the case of the experimental apparatus, the optimum \( f = 18.49 \) shows good agreement with the experimental result of \( f = 16 \). This indicates that EDIC measurements share similarities to single beam DSC measurements.

The particular DIC algorithm used here has been applied in recent work [39] with good results, so the errors in displacement measurement are most certainly due to physical imperfections within the measurement setup. Determining the field of view using a scale introduces certain inaccuracies inherent to the scale itself. Due to physical constraints the camera itself had to be placed at a slight angle to the specimen, decreasing the CCD surface sensitivity to planar displacements. Finally, the 3-axis table itself is not perfectly accurate and the lead screw drive
system contains inherent backlash errors. DIC accuracy could be improved by using a higher resolution camera with a smaller physical pixel size and greater bit depth with a higher magnification lens [8]. In the fringe correction application discussed here, DIC is used only to evaluate bulk specimen motions but not to measure specimen surface displacements directly. Therefore, for this application, the lower accuracy achieved using ESPI images is acceptable. Although more complex DIC algorithms exist [23], this particular one is attractive for its ease of implementation as well as speed. A typical DIC measurement encompassing 18 individual subsets was completed in a few seconds on a 2.14 Ghz processor once the images were recorded.

Through these experiments three key results are obtained. First, using the constructed experimental apparatus, surface displacements can be measured accurately with sub pixel resolution using EDIC in excess of 3mm or 177.2px in the x direction and up to 2.1mm or 137.0px in the y direction. This difference in measurement range can be attributed to the aspect ratio of the images used as well as the optical setup. Second, the size of the laser speckles used has a very significant affect on the accuracy and range of the EDIC measurement, with the best results being obtained for this setup using an f number of 16 which corresponds to an average speckle size of approximately 1.73px. Third, using painted speckles consistently produces more accurate DIC results than the proposed EDIC method. By comparison, using this experimental apparatus, both painted DIC and EDIC measurements produce errors on the same scale. Considering the accuracy when using painted speckles, it is on the lower end of the reported range. This is mainly due to various inaccuracies inherent to the apparatus and method itself. The slightly lower accuracy of EDIC should not be a significant concern within the
ESPI fringe correction method since the hole drilling surface displacements are measured using ESPI and DIC is used only to re-correlate the ESPI images.

4.5 **Null Fringe Correction**

Previous experiments have shown that EDIC measurements are capable of measuring surface displacements of significant distances with sub-pixel accuracy. This is an encouraging result and bodes well for the fringe correction process. A simplified fringe correction is now investigated – one in which no measurement is actually performed and the data is blank. Ideally if the fringe correction is successful, the recorrelation process will yield a completely null fringe. These experiments will provide a good starting point for examining the viability of the fringe correction process.

The procedure used for these tests is similar to that used in section 4.4. Using a nominally flat specimen, a set of phase-stepped images is first recorded to serve as the reference set. The specimen is then displaced a known amount using the table and a second set of fringe images is recorded. The table displacement is calculated using an EDIC solution Based on 18 circumferentially spaced 60x60 pixel patches. Finally the fringe set of images are shifted to reverse the table motion and re-correlate the laser speckles. Figure 4.13 shows an example set of measurements.
Figure 4.13 - (a) Baseline zero fringe (b) Corrupted fringe after 0.5mm displacement (c) Corrected fringe after image shifting

Figure 4.13(a) shows the baseline zero fringe obtained by recording consecutive sets of phase stepped images without moving the table. In the ideal case this fringe would be completely blank; however slight texture created by measurement noise is visible. This noise is caused by the superposition of a random speckle pattern on a regular CCD pixel grid. Figure 4.13 (b) shows the fringe obtained after the specimen has been shifted 0.5mm in the positive x direction. As expected, the speckle correlation is completely lost and the resulting fringe pattern is destroyed. Finally, after performing the EDIC measurement, the fringe images are shifted accordingly to correct for the 0.5mm displacement. The resulting fringe is shown in Figure 4.13 (c), which shows that the speckle correlation has been restored, but with a large phase gradient superimposed across the image. This initial result shows promise – speckle correlation can indeed be recovered, however the cause and correction of the resulting phase gradient needs to be determined.
4.5.1 Phase Gradients – Cause and Correction

Upon investigation, the cause of the phase gradient visible in Figure 4.13 (c) was found to be a combination of several factors. Within the experimental apparatus, a beam expander is placed directly in front of the laser source, which makes both the illumination and reference beams conical in shape. This conical shape is one of the factors that causes phase gradients within corrected ESPI fringes. Consider the ideal case of a perfectly flat specimen imaged asymmetrically by two conical beams at angles \( \theta_1 \) and \( \theta_2 \) as shown in Figure 4.14. If the specimen is then shifted right by a distance \( d \), the path length of beam 1 increases by \( d \sin \theta_1 \) as shown in Figure 4.15 and similarly the path length of beam 2 decreases by \( d \sin \theta_2 \). The relative path length therefore changes by:

\[
\delta = d(\sin \theta_1 + \sin \theta_2)
\]  

Equation (4.4)

The beams can be idealized as originating from point sources at a distance \( L \) from the surface, being conical in shape due to the beam expander and illuminating an image \( D \) wide as shown in Figure 4.16. The cone angles of the beams can be described as
Because of the conical shape of the beam, the beam angles, or angles of incidence, vary across the image. Considering only a one-dimensional image, the beam angles on the left side will be \( \theta_1 - \frac{\psi_1}{2} \), \( \theta_2 + \frac{\psi_2}{2} \) and on the right they will be \( \theta_1 + \frac{\psi_1}{2} \), \( \theta_2 - \frac{\psi_2}{2} \) respectively. Considering the change in path length at the left and right side of the image yields:

\[
\delta_L = d \left( \sin \left( \theta_1 - \frac{\psi_1}{2} \right) + \left( \theta_2 + \frac{\psi_2}{2} \right) \right)
\]
\[
\delta_R = d \left( \sin \left( \theta_1 + \frac{\psi_1}{2} \right) + \left( \theta_2 - \frac{\psi_2}{2} \right) \right)
\]

Finally calculating the total path length based on equation (4.6) and simplifying using trigonometric identities yields:

\[
\delta_L - \delta_R = 2d \left( \cos \theta_2 \sin \frac{\psi_2}{2} - \cos \theta_1 \sin \frac{\psi_1}{2} \right)
\]

For small cone angles, and substituting equation (4.5) this yields:

\[
\delta_L - \delta_R = \frac{dD}{L} \left( \cos^2 \theta_2 - \cos^2 \theta_1 \right)
\]

Utilizing trigonometric identities, equation (4.8) simplifies to:
\[
\delta_L - \delta_R = \frac{dD}{L} \sin\left(\theta_1 + \theta_2\right) \sin\left(\theta_1 - \theta_2\right)
\]  \tag{4.9}

Phase difference can then be related to the change in path length using the wavelength of the source as:

\[
\phi_L - \phi_R = (\delta_L - \delta_R) \cdot \left(\frac{2\pi}{\lambda}\right)
\]  \tag{4.10}

Simplifying equation (4.10) yields:

\[
\Delta \phi = \frac{2\pi dD}{\lambda L} \sin\left(\theta_1 + \theta_2\right) \sin\left(\theta_1 - \theta_2\right)
\]  \tag{4.11}

Where:
- \(d\) = Specimen shift distance
- \(D\) = Image width at specimen
- \(\lambda\) = Source wavelength
- \(L\) = Beam path length
- \(\theta_1\) = Beam 1 angle of incidence
- \(\theta_2\) = Beam 2 angle of incidence

The phase gradient due to the conical illumination beams and angles of incidence can be calculated using equation (4.11). To validate this equation, an experiment was performed where the flat mirror that reflects the illumination beam was mounted on a precision turntable. By rotating the mirror set angular increments, the angle of incidence of the reference beam was varied. Sets of images were recorded before and after a constant specimen shift of 0.25mm in the x direction. The fringe set of images were then shifted back into alignment according to the EDIC result and the number fringes measured numerically using a least squares solution in the same manner described later in this section. Figure 4.17 shows the results of the mirror turning experiment.
As shown in the figure, the experimental data matches the results from equation (4.11) quite well. The difference between the two results may partly be due to the difficulty of accurately identifying the illumination distance “L” because the position of the source focal point could not be exactly identified.

The remaining phase gradients are caused by the change in the relative path length of the illumination and reference beams from one side of the image to the other. These changes in path length can be attributed to a combination of factors. Any specimen curvature in the z direction will cause phase gradients as the specimen is moved since the relative path lengths of
the illumination and reference beams will change according to the curvature. Specimen motion that is not completely parallel to the x-y plane of the specimen will also change the relative path lengths across an image and introduce a phase gradient, as will specimen rotation.

To summarize; if the angles of incidence of the two beams is not constant across the specimen surface and specimen motion occurs, the path lengths of the two beams will not change equally and a phase gradient will occur. Specimen rotation will also cause phase gradients for the same reason. For nominally flat specimens, adding lenses to create parallel illuminating beams and adjusting the optical equipment to create constant angles of incidence will allow planar motions to produce very little relative phase change. After adding the lenses and adjusting the vertical mirror near the specimen to create nominally equal angles of incidence, the phase gradients of various planar motions in the x and y direction after fringe correction are shown in Figure 4.18.

**Figure 4.18 - Phase gradients for various specimen shifts**

The top row of figures shows specimen motions in the x direction, and the bottom row shifts in the y direction. The relative size of the shifts can be seen by observing the band of information
loss visible at the edge of the fringe patterns. The noise content clearly increases with specimen shift due to the speckle pattern gradually changing shape. As the speckles change shape, some of the speckle correlation is lost upon shifting the images back and the noise is increased. Therefore it is expected that fringe correction will only be feasible up to a certain range which is to be investigated in future experiments.

Comparison of Figure 4.13 with Figure 4.18 shows that the residual phase gradient has been largely corrected. A slight phase gradient is still visible with larger specimen motions however, even with the addition of parallel lenses and beam adjustment. It should also be noted that using physical methods to correct phase gradients requires the equipment to be adjusted independently for each specimen measured. The fringes in Figure 4.18 were obtained after careful adjustment to illustrate that this type of gradient correction is possible. In a more practical application however it is very likely that phase gradients will still occur; even for the case shown after careful adjustment slight phase gradients exist. Therefore an additional method to systematically correct this artifact would be very beneficial.

To this end, phase gradient correction by mathematical methods was investigated. A least-squares solution, similar to that used to solve for pixel phase (equations (2.17)-(2.20)) is used to estimate these phase gradients. The gradient is considered as a bi-linear function in the form of:

\[ \phi = P + Qx + Ry \]  

(4.12)

Considering an area within the image bounded by two concentric circles containing \( n \) pixels:
where \( x_n, y_n \) are the co-ordinates of the pixel with the origin equal to that of the circles bounding the area and \( \phi_n \) being the phase of the pixel.

This equation is over determined and can be solved using the least-square method. The polynomial coefficients can be determined as:

\[
P = \sum_{i=1}^{n} \phi_i \quad Q = \sum_{i=1}^{n} \phi x_i^2 \quad R = \sum_{i=1}^{n} \phi y_i^2
\]

Once the phase gradient is calculated using equation (4.14) it can be subtracted from the fringe pattern, yielding a more accurate result of the true measurement fringe. Figure 4.19 (a) shows a corrected zero fringe with a specimen motion of 0.5mm; the phase gradient is clearly visible. Figure 4.19 (b) shows the same fringe after the phase gradient is subtracted using the least squares solution.

\[
\begin{bmatrix}
1 & x_1 & y_1 \\
1 & x_2 & y_2 \\
\vdots & \vdots & \vdots \\
1 & x_n & y_n
\end{bmatrix} \begin{bmatrix}
P \\
Q \\
R
\end{bmatrix} = \begin{bmatrix}
\phi_1 \\
\phi_2 \\
\vdots \\
\phi_n
\end{bmatrix}
\] (4.13)
The fringe after phase subtraction is clearly much cleaner; the gradient is removed leaving a fringe pattern much closer to the ideal case of a completely white image.

Correcting for the phase gradients in the case of residual stress hole drilling measurements serves mainly to help avoid unwrapping errors. The stress calculation itself has provisions for removing the effect of the gradients; however unwrapping errors can still damage the stress solution. By removing the phase gradients, the possibility of extraneous unwrapping errors is significantly reduced. For ESPI measurements of other kinds however, mitigating phase gradients may prove to be more significant.

To summarize, although phase gradients are created using the proposed ESPI fringe correction process, they can be corrected using a combination of physical and mathematical methods. These experiments show that after such treatment, a restored zero fringe is shown to have good agreement with the ideal case of a completely null image. Noise is clearly visible and will increase with larger specimen shifts as the speckles change shape, but within a certain range, the ESPI fringe correction process is shown to be feasible for the case of blank fringes.

4.6 Validation of ESPI Fringe Correction Using Hole Drilling

The fringe correction process has been shown to be feasible for correcting a null measurement fringe which is an important result. Ideally however this process will be used to correct rigid body motions affecting actual ESPI measurements. Therefore, in order to test the effectiveness of the process in an actual application, the process was applied to actual ESPI hole drilling measurements. These series of experiments involved comparing a baseline measurement performed under controlled conditions with a corrupted measurement containing some
specimen rigid-body motion. The fringe correction procedure was performed on the corrupted measurement, and the recovered fringe pattern was used as the basis of the residual stress solution. This recovered solution was then compared with the solution obtained using the baseline measurement to establish the effectiveness of the correction process. By repeating this experiment for various specimen motions, a range over which fringe correction is possible was determined.

The specimen used for these tests was the same calibrated specimen used in section 4.2. Since this specimen contained a well-known residual stress distribution, a measurement which matches this stress was known to be accurate and serves as the baseline. So long as the recovered measurement matches the baseline, its accuracy can be safely assumed.

The initial tests performed examined the maximum range of specimen motion that could be corrected. The results for the various specimen directions are shown in Figures 4.20-4.23. The dark black lines represent the baseline measurement performed in section 4.2. If the residual stress solution remains close to the line, then the process is successful. Both positive and negative Z direction specimen shifts were considered since it was thought the depth of field of the lens may have an effect on the range of correction. Examining the results, it’s quite apparent that the procedure is effective over a specific range, after which the solution deteriorates rapidly. If a deviation of ± 3 MPa is considered, the correction is effective over the ranges shown in Table 4.5. The standard deviation is also tabulated for each stress component within this table.
Figure 4.20 - Recovered stress solution vs. X direction specimen shift

Figure 4.21 - Recovered stress solution vs. Y direction specimen shift
Figure 4.22 – Recovered stress solution vs. +Z direction specimen shift

Figure 4.23 - Recovered stress solution vs. -Z direction specimen shift
Table 4.5 - Range and deviation of fringe correction process

<table>
<thead>
<tr>
<th>Specimen Shift Direction</th>
<th>Range (mm)</th>
<th>Standard Deviation (Mpa)</th>
<th>$\sigma_x$</th>
<th>$\sigma_y$</th>
<th>$\tau_{xy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>1.4mm</td>
<td></td>
<td>0.283</td>
<td>0.642</td>
<td>0.243</td>
</tr>
<tr>
<td>Y</td>
<td>1.2mm</td>
<td></td>
<td>1.086</td>
<td>0.928</td>
<td>0.356</td>
</tr>
<tr>
<td>+Z</td>
<td>0.8mm</td>
<td></td>
<td>0.516</td>
<td>1.065</td>
<td>0.186</td>
</tr>
<tr>
<td>-Z</td>
<td>0.9mm</td>
<td></td>
<td>0.445</td>
<td>1.230</td>
<td>0.339</td>
</tr>
</tbody>
</table>

The data in the table show that specimen motions in the out-of-plane (Z) direction have the most detrimental effect on the measurement recovery, as the range of correction in these directions were both significantly lower than the in plane (X,Y) directions. This follows from the fact that specimen motions in the z direction change the path length of both ESPI beams much more than in-plane motions. As the pattern changes the re-correlation process suffers and the measurement is lost. Another trend is observed examining the standard deviations of the various in plane stress components. The y component of stress exhibits the highest deviation across nearly all specimen motions. This may be attributed to the fact that the optical setup of the ESPI system is aligned to measure displacements in the x direction only. For y-stresses, such displacements occur only through the action of Poisson’s ratio, and are only about one third the size of the corresponding displacements for x-stresses. Thus, measurement of x-displacements gives sensitivity to y-stresses of about one third the sensitivity to x-stresses. This trend is evident when considering the graphs. The y component of stress consistently deviates from the true solution first while the x component of stress, which is more directly measured, maintains agreement some distance longer. If the system was adapted to measure all three components of stress evenly, by changing the sensitivity vector such that x and y displacements
are measured equally, the range of correction for this application could most likely be extended.

### 4.7 Specimen Removal and Replacement

As well as increasing the resilience of ESPI measurements to specimen rigid body motions, the ESPI fringe correction process developed here has now made possible a novel measurement method. Throughout previous experiments the drilling process was integrated within the apparatus – the drill was moved into position after the reference set of images was recorded. This can cause problems however as chips created during drilling can contaminate the optics and the cutting forces themselves can disturb the optical equipment. If drilling could be performed at a separate location these problems could be avoided, and the apparatus itself could be simplified. This provided the motivation for the following experiments.

The specimen used for these tests was developed for previous work [40] for hole drilling residual stress measurements. To establish a baseline residual stress result, a series of 5 measurements was performed with the specimen fixed rigidly to the optical table. The results are shown in Figure 4.24, with the average for each in plane stress component plotted as a straight line.
Examining these results, they are quite closely grouped around the averages indicating reasonable consistency. This average stress solution therefore represents a reasonable baseline residual stress solution for this particular specimen. If a measurement produced using specimen removal and replacement can produce a similar solution it can be successfully validated.
To accomplish specimen removal and replacement, an additional fixture was constructed to hold the specimen in the path of the ESPI beams. This fixture is shown in Figure 4.25. The process of these experiments consisted of aligning the specimen in the fixture within the existing ESPI system. The fixture was then bolted to the optical table. The hole location was next marked on the specimen and the set of reference images was recorded. The specimen was then removed from the fixture and bolted to a separate fixture at a different location and then drilled with the same drilling system used in previous experiments. After drilling, the specimen was replaced in the fixture and the fringe set of images was recorded. In order to test the repeatability of the replacement procedure the specimen was then removed and replaced an additional 10 times, with a new fringe set of images being recorded each time. The results of the experiment are shown in Figure 4.26, with the average of all 10 tests again shown as a straight line. Considering the results, there is definite variation; however the measurement does show promise. The removal and replacement process is shown to have good repeatability with all the measurements spaced closely around the average. A numerical comparison between the baseline tests where the specimen was fixed, and the removal and replacement experiments is shown in Table 4.6.
Comparing both tests numerically shows that they are in good agreement, with the stress component in the y direction varying more than the other two. Again this can be attributed to the low sensitivity to displacements in the y direction. This difference may also be attributed due to the need to reposition the specimen and fixture independently of the previous tests.
Although care was taken to try and return the specimen to the same position used in the fixed tests, small deviations could cause a slightly different measurement due to the beam angles of the illumination and reference beam changing. Since the removal and replacement results are consistently lower than those of the fixed tests, this could certainly be the case. A graphical comparison of the fringes obtained during both the baseline and removal and replacement tests are shown in Figure 4.27

![Figure 4.27 - (a) Baseline fringe (b) Removal and replacement fringe before correction (c) Removal and replacement fringe after correction](image)

This figure clearly shows the effectiveness of the fringe correction process as well as good agreement between the baseline fixed specimen measurements and the removal and replacement measurements. The effectiveness of the correction process is clearly evident when considering Figure 4.27 (b) and (c). (b) shows a completely corrupted fringe pattern, while after correction (c) shows a fringe pattern with good contrast and well defined fringes. Comparing Figure 4.27 (a) and (c) the similarity is evident; the fringe patterns show very close agreement, indicating that a comparable solution can indeed be obtained using specimen removal and replacement, even using the simple fixture that was constructed. The noise content is slightly higher in the corrected fringe however this is typical of the correction process as previously discussed and does not affect the measurement significantly.
The experimental results presented in this section work to investigate and validate the various aspects of the ESPI fringe correction process. In order generate accurate ESPI data, an adjustable ESPI hole drilling apparatus was constructed and validated using a calibrated specimen. In order to make use of phase stepped ESPI images for DIC measurements (EDIC), a phase independent method was successfully developed. The size effect of the speckles within these ESPI images was investigated and an optimum speckle size of 1.73px, which yielded the largest EDIC measurement range, was determined for this optical setup. Null fringe correction, representing the most basic application of the fringe correction process, was investigated and shown to be successful; however a phase gradient artifact was encountered. The cause of these phase gradients was determined analytically, validated experimentally and corrected using a combination of both physical and mathematical methods. Once the fringe correction process was established, it was applied to ESPI hole drilling measurements. Using the 3 axis table, specimen motions were systematically created within hole drilling measurements. By comparing recovered solutions after fringe correction with a baseline measurement performed in controlled conditions, the accuracy of the process could be assessed. This allowed the effective range of the correction process to be determined. Finally, a new ESPI measurement method involving removal and replacement of the specimen was investigated. This type of measurement was made possible due to the flexibility afforded by the fringe correction process. It was shown that this type of new measurement is accurate and repeatable when compared with a baseline measurement performed in the conventional manner.
5 Conclusions and Future Work

5.1 Conclusions

A novel adaptable ESPI system was designed and constructed

The ESPI apparatus developed here was designed and constructed for use when measuring residual stresses using the hole drilling method. The apparatus was validated using a specimen with a known residual stress field; results were obtained and shown to be in good agreement with the stresses contained in the specimen.

Phase independent DIC using ESPI phase stepped image sets (EDIC) was developed

By constructing a phase independent image from the available images within a stepped set, accurate DIC results could be obtained using the existing images within an ESPI measurement. Statistical tests determined that using an average (“A”) image yielded the best results of any possible combination of phase-stepped images or other phase independent quantities.

Using this new EDIC method, the effect of laser speckle size was investigated systematically by varying the optical configuration of the apparatus. It was determined that speckle size did have a significant effect on EDIC effectiveness, and an optimum lens aperture F16 was determined to yield the most accurate results for this particular apparatus. This agrees well with previous work done on single beam DSC measurements. When compared with conventional painted speckles, EDIC was found to be less accurate, but the errors within both methods were comparable. The difference in accuracy can be attributed to the spatial dependency of laser speckles. Physical surface features do not suffer from this dependency and therefore generally
yield more accurate DIC results. Using EDIC, displacements in excess of 150 pixels or 2.5mm were measured successfully with sub pixel error.

**An ESPI fringe correction process was developed to correct for specimen motion**

Utilizing EDIC measurements, an ESPI fringe correction process was developed and successfully applied to residual stress hole drilling measurements. Significant specimen rigid body motions occurring between the recordings of phase stepped sets were able to be corrected using this method.

Phase gradients resulting from this fringe correction were investigated and were found to be mostly caused by the use of conical illumination beams. This mechanism was shown analytically and experimentally. The phase gradients were largely mitigated by the addition of lenses to convert the conical illumination into parallel illumination. In addition, the optical equipment was carefully adjusted to ensure equal angles of incidence for both the reference and illumination beams. Even with careful optical adjustments, some gradients remained due to specimen rotations and curvature. To address these remaining gradients, mathematical correction by a least squares solution and subtraction method was developed.

Using a precision 3-axis table, specimen motions were inserted into actual ESPI residual stress measurements after the hole drilling had occurred. Using the fringe correction process, measurements could be recovered accurately for specimen motions of up to 1.4mm in the x direction, 1.1mm in the y direction and 0.8mm and 0.9mm in the positive and negative z directions respectively. These results were validated by matching them to conventional ESPI measurements performed on the same specimen. Without the fringe correction process,
motions of approximately 0.030mm in any direction would compromise the measurement entirely. The fringe correction process is therefore shown to improve the ability of ESPI to measure stress accurately, independent of these motions, by 2 orders of magnitude.

This result is encouraging with respect to helping ESPI move to more practical use in the field, especially in terms of a portable ESPI system. Within such an application, it would be very possible for the specimen to shift after drilling, due to either environmental disturbances or the drilling itself. Using the fringe correction process developed, no additional equipment or data are required, and the ability of the system to handle specimen motions is significantly improved.

**Specimen removal and replacement was investigated**

A novel residual stress hole drilling technique was also made possible by the fringe correction process. Previously the drilling process had been integrated within the experimental apparatus; however measurements where the specimen was removed for drilling and replaced within the system were investigated. A simple fixture was developed for this purpose, and a new specimen was fabricated. This specimen was then measured 5 times using the conventional measurement method with the specimen fixed, and a baseline residual stress solution was established by taking the average of these tests. The specimen and fixture were then aligned within the system and the new process was tested. In this case the specimen was removed after the reference set of images was recorded, drilled at a separate location with the same drilling system used previously, and replaced within the fixture. The fringe set of images was then recorded and the fringe correction procedure was applied. Using this method, results
were obtained that were comparable to those done in the baseline tests with the specimen fixed. In order to test the repeatability of the method, the specimen was removed and replaced an additional 9 times and a new set of fringe images was recorded. The results for all 10 tests were shown to be in good agreement indicating that the process possesses good repeatability.

This is an exciting result as it creates several new possibilities. The drilling system can now easily be separated from the ESPI apparatus. By doing this, the optical system itself can be simplified, and components can be placed in better alignment. In addition, the chips created during the cutting process can be kept separate from the delicate optical equipment. Even with the chip suction system implemented, specimen chips were still a problem within the apparatus and they would often obstruct the vertical mirror in particular. This problem can now be avoided. In addition, the data corruption caused by the cutting forces induced during the drilling process can also be avoided using specimen removal and replacement. Specimen removal and replacement also opens the possibility of measuring a single specimen at multiple attitudes within a single system. For instance, consider a hole drilling measurement performed on a thin plate. Two measurements can now be performed simultaneously on the front and back faces of the plate by simply taking a reference set of images with the specimen in the fixture, flipping the specimen and recording reference images of the back face. By repeating this process after the hole is drilled, displacement data for the front and back face can be obtained simultaneously. Previously it would be necessary to devise an extremely complicated ESPI system to achieve this type of measurement however through this work it is now possible using the conventional ESPI system constructed.
5.2 Future Work

Investigate correction of individual stepped images for active correction

This work has been shown to correct specimen motions occurring in between recording sets of phase-stepped images. Vibrations may cause the specimen to shift during the acquisition of a phase-stepped set, causing the steps to deviate from the desired 90-degree increments. Some work was done examining the feasibility of correcting individual images within a set, however the tests done were mainly inconclusive because the induced vibrations were too fast to be accurately captured by the camera; significant blurring occurred which destroyed the speckle correlation. The use of stroboscopic laser illumination in combination with a higher speed camera could reduce this blurring. If this is possible, then individual images within a set could possibly be corrected in the same manner as full sets within this work, so long as the imaging frequency was high enough to minimize the displacement between each image in a set. This could allow another significant increase in ESPI system robustness and could mitigate the effect of vibrations and other disturbances common to field environments.

Extend fringe correction to in plane rotations

Within this work, a fringe correction process is developed that can account for planar motions and out of plane rotations. It is possible however that in plane (xy) specimen rotations could occur as well. Therefore, it follows logically that adapting the process to account for these rotations as well as investigating the performance of rotation correction should be investigated in the future.
Develop a portable ESPI system

The ESPI fringe correction process represents a solid step towards constructing a viable ESPI system for portable measurements. Using this method as well as the adjustable design developed for the experimental apparatus, it become practical to develop a feasible system that could be used for making mobile residual stress measurements. If work was done to develop single image correction for more active specimen motion correction the robustness of a portable system could be even further increased.
References


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