Designing Performance Based Contracts in Supply Chains

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Abstract

The three essays in this thesis address the design of performance-based contracts in decentralized supply chains when a supplier's effort is unobservable.

The first two essays explore various issues in the design of service level agreements (SLAs), a type of performance-based incentive scheme widely used for outsourcing manufacturing and services. We consider a supply chain in which a supplier manages the supply of a durable product for a buyer and the buyer contracts with the supplier on the supplier's inventory service level. The SLAs are discontinuous incentive schemes with a multi-period review strategy, and the supplier's performance measure is the ready rate (1 - stockout rate).

The first essay (Chapter 2) investigates the effectiveness of two common types of SLAs: a lump-sum penalty SLA and a linear-penalty SLA. The key finding is that when the supplier can observe the performance history and dynamically adjust the investment in inventory to affect her review period performance, to mitigate the supplier's incentive for strategic behavior, the penalty should be dependent on the degree of the supplier's performance deviation from the target.

The second essay (Chapter 3) focuses on the effectiveness of performance measures in SLAs. The problem is similar to that in the first essay, but the supplier can invest both in inventory and in inventory replenishment lead time. We consider two inventory performance measures: the immediate ready rate and the time-window ready rate, and find that there exists a unique positive time window such that a ready rate with window induces the first-best investments. Our findings demonstrate the importance of choosing the right performance measure to align a supplier's incentive.

The third essay (Chapter 4) investigates the design of performance-based volume incentive schemes in the form of allocating business between suppliers when a buyer maximizes his long-run discounted payoff from repeated dual sourcing. We consider both the case where a supplier's effort cost is proportional to her volume of business and the case where the cost is independent of her volume. We find that to induce and maintain suppliers' competition over time, the optimal scheme depends on each supplier's current share of business and is generally not a simple rank-order tournament; handicapping the definition of winner can do better than a simple first-past-the-post rule.

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Dedication

To my parents and my sister, my husband Wenhai, and my beloved daughter Yuyan, who always give me unconditional love and support in my life.

Chapter 1

Introduction

1.1 Motivation

Outsourcing manufacturing and services is a common practice in both the public and private sectors. As supply chains become more decentralized and the suppliers perform tasks on behalf of the buyer, effectively managing these suppliers becomes vital for a buyer's success in business. Mechanism design for aligning a supplier's incentive with the buyer's interest has consequently drawn attention from operations researchers. This has been driving a stream of literature at the interface of operations management and economics, which addresses the contract design for various operations management problems such as inventory management, production management, capacity investment, and quality control.

Traditional contracts often focus on how the work is performed and the payment is either a fixed price or cost/revenue sharing. For example, a contract may dictate the inventory level or production capacity in which a supplier should invest. However, implementing such a type of contract could be infeasible or expensive in many situations because a supplier's effort is often unobservable to the buyer and costly to monitor, which is known as the moral hazard problem in the principal-agent theory. Performance-based contracting has therefore gained a wider use in recent years due to its key feature of contracting on outcomes instead of dictating how the work is done. Service level agreements (SLAs) are a common type of performance-based contract for managing suppliers. In an SLA, the performance, or outcome of a task desired by the buyer is specified in terms of a service level target. According to a survey (Oblicore Inc. 2007), 91% of organizations use SLAs for managing suppliers, internal agreements, or external customer agreements. Performance-based incentives are also often used together with competition in the form of allocating business between multiple suppliers based on the suppliers' performance (volume incentive). Dyer and Ouchi (1993) report that Japanese firms usually employ a 'two-vendor policy' to motivate suppliers to innovate and improve performance. Sun Microsystems allocates demand among multiple suppliers using a scorecard system (Farlow et al. 1996). An empirical study by Bensaou (1999) also shows that Japanese buyers typically split their purchases among multiple suppliers and then demand that the suppliers make specialized investments to obtain and keep their business.

Despite the widespread use of performance-based contracts and volume incentives in practice, there is little theoretical research in the literature on their design. This thesis attempts to understand performance-based incentives as well as volume incentive schemes by studying issues in the design of SLAs for a single supplier and volume incentives under supplier competition. Some features of the performance-based incentive schemes distinct from traditional contracts present new issues which haven't been fully addressed in the literature. For example, performance-based contracts are typically associated with one or more performance measure, which creates the basis for compensation. Because SLAs are very context dependent, we study a particular application to inventory management, in this case for a durable product. To measure the inventory performance, off-the-shelf (immediate) fulfillment rates are often used in theory, but time-window fulfillment rates are more commonly used in practice (LaLonde and Zinszer 1976, LaLonde et al. 1988). We therefore also examine the effectiveness of both types of performance measures for incentive alignment.

1.2 Related Literature

The research in this thesis is at the interface of operations management and economics, and relates to four areas of literature: inventory management, supply chain contracting, non-cooperative games driven by demand allocation, and principal-agent theory. The following is an overview of the relevant research work. More detailed review of the literature can be found in each essay.

1.2.1 Inventory management

A number of papers in the inventory management literature investigate inventory performance measures. Examples are Schneider (1981), Choi et al. (2004), Boyaci and Gallego (2001), Wang et al. (2005), Thomas (2005), and Katok et al. (2008). Choi et al. (2004) investigate choosing supplier performance measures for vendor-managed-inventory when the supplier's capacity and inventory policy are private information. The performance measure in their study is mainly the ready rate. Off-the-shelf (immediate) fulfillment rates are often used and studied in theory to measure the inventory performance, but time-window fulfillment rates are more commonly employed in practice. Among the few papers that consider time-window performance measures, Boyaci and Gallego (2001) study the problem of minimizing average inventory costs subject to fill-rate and fill-rate-with-window service-level constraints in serial and assembly systems; and in an (s,S) inventory system with service level target represented by a time-window ready rate, Wang et al. (2005) find a significant tradeoff between the window length and the inventory costs, and suggest that a longer fulfillment window and lower price may be used for price-sensitive but time-insensitive customers.

The majority of the inventory management literature considers performance measures in the long run using expected performance (see Zipkin 2000, for example, for an introduction on the inventory theory in the literature). This is because the research questions are generally operations-oriented for an integrated supply chain. Those considering incentive issues tend to assume a contract can be on the long-run expected performance (see, e.g., Choi et al. 2004), which can have problems in implementation. When an SLA is used for a supplier with unobservable effort, the supplier's performance measure in a finite review period is a random variable, which often differs more or less from the long-run expectation. A contract based on the expected performance (service level) alone, either being unobservable or needing too long a review period gives no basis to the supplier for identifying and rectifying underperformance and thus cannot provide an adequate incentive.

For an SLA to work, it is critical to understand and employ in its design the probability distribution of its performance over the (typically short) review period in order for incentive to provide the right alignment. Thomas (2005) and Katok et al. (2008) consider the fill rate in a finite horizon. Thomas uses simulation to illustrate its distribution, and Katok et al. use an experimental method for analysis. Both papers examine how the length of a review period and the bonus/penalty affect the agent's choice of the base-stock level. They only investigate a supplier's stationary inventory policy but do not consider time-window fulfillment rates.

To address the above issues, for the problem of contracting for inventory management service, we are therefore interested in the design of an SLA on the basis of a random performance measure, as well as the difference between the incentive provided by an immediate fulfillment rate and that by a time-window one. These two issues are examined in the first and the second essays respectively.

1.2.2 Supply chain contracting

We introduce two types of contracts which are related to the SLAs studied in the first two essays.

Supply chain coordination contracts

The coordination of inventory investment (order quantity) with contracts has been extensively studied in the supply chain literature. A comprehensive review can be found in Cachon (2003), which includes the coordination of single-period single or multiple newsvendor problem as well as the coordination of infinite-horizon inventory investment on a durable product. In particular, for a single-location base-stock model with linear backorder cost, the optimal coordination contract is essentially a cost sharing one, with the retailer (agent) and the seller (principal) each bears a portion of the total supply chain cost (inventory holding and backorder costs). Employing such a contract means micromanaging the agent because the inventory level and the backorders at any time need to be recorded, which will definitely result in large administrative and transaction costs. Moreover, when the backorder cost is nonlinear, the coordination contract may have a complex form that is difficult to implement, which will not be an issue when using SLAs.

Performance-based contracts

The design of performance-based contracts has drawn operations researchers' attention in recent years. Examples can be seen in Plambeck and Zenios (2000), Plambeck and Zenios (2003), Kim et al. (2007), and Kim et al. (2009). Plambeck and Zenios (2000) consider a principal delegating operational control of a production system to an agent who can exert unobservable effort to maintain the system, and investigate the compensation scheme dependent on the observed system state transition. Plambeck and Zenios (2003) study a vendor-managed-inventory problem in which an agent chooses a privately known production rate to build inventory for the principal, and derive a contract based on the observed inventory level. Both papers consider a dynamic principal-agent problem with a risk-averse agent. Kim et al. (2007) study an after-sales service supply chain in which a customer operates assembled systems with each system consisting of distinct parts each provided by a different supplier. If any of the parts fails, the system is down, and that part has to be replaced by a spare part. When there is no spare part available, a backorder occurs. Each supplier determines her stock level of spare parts, which is unobservable to the customer. Facing a system uptime requirement, the customer offers contracts to the suppliers. The authors propose a contract linear in the backorders of each part, and show that it induces the first-best solutions when all parties are risk neutral. They only consider suppliers' stationary policies in a static formulation, and study a coordination problem with no supplier competition. Kim et al. (2009) focus on the choice of performance measure in a contract for restoration service of mission-critical systems. A risk-averse supplier makes a one-time service capacity investment unobservable to the customer of a system with infrequent but costly failure. The supplier's incentives under the performance measures of sample-average downtime and cumulative downtime are compared.

The SLAs on inventory management in our study are based on an aggregate performance measure – the demand fulfillment rate, rather than on the individual performance outcome such as the number of backorders. This type of contract differs from most of the aforementioned papers on inventory coordination contract and performance-based contracts. Thus it is interesting to find out its structure and understand how it generates the incentive for investment.

1.2.3 Non-cooperative games driven by demand allocation

In the operations management literature, a few papers investigate firms' competitive behavior under an exogenous demand allocation mechanism, often driven by the switching behavior of customers in the market, which is dependent on the firms' realized service levels. Lippman and McCardle (1997) consider a single-period multiple-competitive-newsvendor problem in which each newsvendor chooses an inventory level to meet a random demand and a rule specifies the allocation of initial market demand among the firms and the allocation of excess demand among those firms with remaining inventory. They derive the equilibrium inventory levels under specific allocation rules. Hall and Porteus (2000) and Liu et al. (2007) consider a multi-period competitive newsvendor problem where two firms make capacity (inventory) decision in each period, and the demand for each firm is dependent on the realized level of customer service (product availability) in the prior period. Both papers look for the firms' equilibrium behaviors in the dynamic game.

Some other papers consider a buyer specifying the allocation rule. Cachon and Lariviere (1999) investigate a special allocation rule commonly used in the automobile industry by considering a single supplier allocating capacity to multiple retailers based on their past sales. A two-period game is studied.

Both Benjaafar et al. (2007) and Cachon and Zhang (2007) consider a buyer outsourcing a fixed demand at a fixed unit price to multiple suppliers. Benjaafar et al. (2007) examine two competitive mechanisms used for outsourcing to a set of potential suppliers in a single-period setting. One mechanism is allocating the whole demand to one supplier with the probability of being selected increasing with her committed service level, the other allocating the demand to each supplier in proportion to her committed service level. Under both cases, it is assumed that the contractual promises of the suppliers regarding effort or service level are enforceable. The authors compare the service quality the buyer can achieve under both mechanisms. Cachon and Zhang (2007) study a queuing system where each supplier's service time is determined by her capacity investment, and the buyer allocates the demand among multiple suppliers based on their service times to minimize the average service time over an infinite horizon. Suppliers are homogeneous in terms of their capacity costs. Each supplier chooses a capacity level to maximize her own profit. Several commonly used allocation rules are evaluated and an optimal rule is proposed.

There exists a vast literature on sourcing policies for the problem of a buyer awarding a divisible business to one or more suppliers among multiple suppliers. The research questions are often related to the design of competitive mechanisms in the form of bidding and the suppliers' competitive behaviors under the bidding rule. Elmaghraby (2000) provides a survey on the research of this topic in the operations research and economics literature. Most of the models are for a one-time decision problem. The incentive from future business is largely not considered.

Since the suppliers' competitive behaviors that are influenced by future business and by the design of competitive mechanisms with no future consideration have been studied separately in the literature, the natural research question would be: How to design a competitive mechanism with the suppliers' incentive coming from the future business, i.e., a rule for allocating the buyer's future business among the suppliers based on their past performance? What is the impact of a supplier's current share of business on the form of the optimal allocation rule? How to design an allocation rule to maintain the suppliers' competition over time? All these questions are addressed in the third essay.

1.2.4 Principal-agent theory

Recent years have seen the application of principal-agent theory to operations management problems for incentive contract design. We also employ this theory as an analytical tool for all the research problems in this thesis. Bolton and Dewatripont (2005) provide a broad coverage of literature on principal-agent theory. All three essays in the thesis are on the repeated moral hazard problem, with the first two essays on a single agent and the third on multiple agents.

The SLAs studied in the first two essays are a blend of multiple-period review strategies and discontinuous incentive schemes. Radner (1985) investigates multiple-period review strategies for a repeated principal-agent game. The agent's performance in a period is a noisy signal of her effort level in that period. The agent's performance is reviewed every R periods. If the performance is within a margin of error of the expected output under the desired efficient effort level, the agent 'passes' the review. Otherwise they enter a penalty phase of M periods. The penalty in the SLAs under our study is monetary, not a subsequent phase of a non-cooperative game. With discontinuous incentive schemes, the payment from a principal to an agent changes only when some threshold of good or bad performance is reached (McMillan 1992). They are typically derived in single-period models. When this type of incentive scheme is used together with a multi-period review strategy, some new issues emerge.

Spear and Srivastava (1987) study a repeated moral hazard problem with discounting between a principal and an agent, and show that history dependence can be represented by using the agent's expected utility as a state, and thus the problem of characterizing the optimal contract of such a model can be reduced to a constrained static variational problem. Monetary compensation is used for an incentive. The third essay considers the problem between a principal and two competing agents, and the compensation is in the form of future demand. The optimal allocation rule is also derived from a constrained static variational formulation, but we are also concerned about the equilibrium of the agents' dynamic game.

Competitive compensation schemes studied in the economics literature can come in the form of rank-order tournament or relative performance evaluation (RPE), generally for a single-period problem. The relevant research can be found, for example, in Lazear and Rosen (1981), Green and Stokey (1983), Hart (1983), Holmstrom (1982), and Nalebuff and Stiglitz (1983). RPE compensates the agents based on their output levels, and the total compensation varies with the agents' realized output levels. In our case the total demand to be split is a constant. In tournaments, rewards are based on rank order of the individuals, not on their actual output levels. Lazear and Rosen (1981) show that for risk-neutral agents rank-order tournaments work as well as independent contracts; and for agents with known heterogeneous ability, handicapping will improve the efficiency of the tournaments. We want to find out if rank-order tournaments are still optimal when incentives come from future business, and how the demand allocation varies with the agents' outputs. Both questions are examined in the third essay.

1.3 Research Methodology and Findings

All three essays study the incentive mechanism design in the presence of suppliers' moral hazard problem. We therefore apply principal-agent theory to the analysis in each essay.

The first two essays deal with a single supplier's service level agreement design and examine specific forms of discontinuous incentive schemes, so the focus is mainly to find out if by choosing the contract parameters carefully, the buyer's optimization problem constrained by the supplier's incentive compatibility and individual rationality (participation) constraints can result in the same optimal solution as the first best (the optimal solution to the unconstrained problem). In the first essay, the analysis is complicated by the existence of the supplier's strategic behavior over time due to the multi-period review structure of SLAs, so we need to use a dynamic programming technique to solve for the supplier's optimal policies under the contract offered by the buyer.

The third essay studies a single-principal/two-agent/multi-period problem, in which the principal (buyer) designs an allocation rule to dictate how the two agents (suppliers) play a non-cooperative stochastic game. Therefore, in addition to principal-agent theory, we also apply stochastic game theory to the suppliers' problems and look for the suppliers' subgame perfect Nash equilibrium in a finite-horizon game as well as their stationary Nash equilibrium in an infinite-horizon game.

The contribution and findings of each essay are as follow.

1. The main contribution of the first essay is to argue for a methodology for studying the design of SLAs, specifically applying principal-agent theory. We have investigated two frequently observed forms of SLAs: the lump-sum penalty SLA and the linear-penalty one. We have identified the issue of potential supplier's strategic behavior under an SLA when the supplier can observe the performance history and dynamically adjust her effort level to affect her review period performance. We find that to mitigate the supplier's incentive for strategic behavior, the penalty should be dependent on the amount of supplier's performance deviation from the target. In particular, a simple linear-penalty SLA can do well over a lump-sum penalty one for this purpose. When using a linear-penalty SLA, the performance threshold should be kept close to the target, i.e., the allowable deviation of the performance from the target should be small.

2. The second essay investigates the effectiveness of performance measures in SLAs for a multi-task problem. Specifically, we examine two types of linear-penalty SLAs using either the immediate or time-window ready rate as a performance measure when the supplier can make privately observed investments in inventory and supply lead time. We find that there exists a unique positive time window such that a ready rate with window induces the first-best investment, and so an SLA using only the immediate ready rate generally cannot induce the first-best investment. The immediate ready rate can induce near optimal outcome when the buyer's cost for delayed demand fulfillment is linear in the length of delay, but the efficiency loss is higher when the cost is convex. Our findings demonstrate the importance of choosing the right performance measure to align a supplier's incentive, and provide some theoretical basis for the use of time-window fulfillment rate in practice for incentive purpose.

3. The main contribution of the third essay is to examine special features of performance-based volume incentives under supplier competition over time, and tackles the topic of volume incentives for operations management which has been little studied in the literature. For both the case where a supplier's cost of effort to perform is proportional to the volume of business and the case where it's independent of the volume, we have found: to induce and maintain suppliers' competition over time, the optimal volume incentive scheme is generally not a simple rank-order tournament;

handicapping the definition of winner can do well over a simple first-past-the-post rule even when the suppliers have an identical capability of doing the work, and the role of handicapping in volume incentive is either to enhance the suppliers' competition intensity or to provide a stronger incentive to the supplier with a larger share; and volume incentives often need to take into account each supplier's current share of business even when a supplier's effort cost is independent of the business volume. All these special features of volume incentives differ from those shown for the monetary incentives in the literature. We have also found that by limiting a supplier 's maximum share in each period, volume incentive schemes can always induce a unique stationary Nash equilibrium in the suppliers' stochastic game over an infinite horizon.

Performance-based incentive schemes for operations management have opened an area with many issues for research. For example, because SLAs are context dependent and we have only studied the application to inventory management, future research can investigate SLAs for other types of services such as those for health care, logistics, and maintenance. The second essay has demonstrated the importance of choosing the right performance measure for incentive alignment. Future research can be investigating performance measures for various applications. This thesis has focused on a risk neutral buyer and suppliers. In practice, firms especially the small ones often do not want to bear a lot of risk. A firm's risk attitude may have a big impact on the effectiveness of a contract and its associated performance measures. Future research can therefore be on the optimal structure of performance-based incentives under risk-averse buyer or suppliers. Adverse selection is another issue that often exists in reality but has not been fully examined in the literature. It refers to the situation where an agent's capability to do the work is privately known but unobservable to the principal. In the presence of adverse selection, the performance measure and particularly the performance target need to be carefully selected and the structure and the values of the contract parameters may be affected as well. Choi et al. (2004) study the choice of performance measures for vendor-managed-inventory when the supplier's capacity and inventory policy are unknown to the buyer. The third essay has looked into volume incentives under competition. How monetary incentive interacts with competitive mechanism hasn't been well investigated and understood in the literature. With competing suppliers, some conclusions can differ from those for a single agent. The design of performance-based monetary incentives under competition can be an avenue for future research.

1.4 Structure of the Thesis

The rest of this thesis consists of three chapters from two to four. Each chapter is a stand-alone paper with an introduction, a literature review, the main body and a conclusion. Following them is the bibliography for all three chapters. The mathematical proofs for each chapter are in the appendices at the end of the thesis.

Chapter 2

Designing Service Level Agreements For Inventory Management

2.1 Introduction

Service level agreements (SLAs) are a common type of performance-based contract for managing suppliers. A survey by Oblicore Inc. in 2007 found that 91% of organizations use SLAs for managing suppliers, internal agreements, or external customer agreements. In an SLA, the performance, or outcome of a task desired by the buyer is identified and a service level target specified. The buyer neither dictates nor needs to know how the work is done; the vendor can freely choose the most cost-efficient way. As described by the US Office of Federal Procurement Policy about Performance-Based Contracting:

The Performance-Based Acquisition (PBA) means an acquisition structured around the results to be achieved as opposed to the manner in which the work is to be performed (see, e.g., Acquisition Central website).

In return, the buyer pays a fixed price over a certain time period. As a fixed price alone is not enough to guarantee the required performance, incentives are needed. For example, a penalty might be imposed when the vendor underperforms over a period of time.

Despite widespread use of SLAs for outsourcing manufacturing and services, few papers address their design. This chapter examines some fundamental issues of SLA design by studying an application to inventory management from a principal-agent perspective. Specifically, consider a single supplier and a single buyer, where the supplier manages the supply of a single product for the buyer, and can invest in inventory to meet a service level target, but the investment level is unobservable to the buyer — a moral hazard problem in agency theory.

A problem with SLA design is that they are very context dependent. However, companies do need to answer five questions. First, what performance measure should be used. Since the performance is reviewed over a period of time, this measure should be an aggregate one. Second, what performance target is appropriate, as a higher performance means the buyer pays more. A performance target should consider both the buyer's valuation of performance and the vendor's cost of doing the task. The third question is how frequently performance will be reviewed. Any sufficiently complex task will result in some natural variation (noise) in performance, which will be affected by the review frequency. It is undesirable to penalize minor deviations from the target that are due purely to noise, so the fourth question is how much deviation is allowed from the target. Lastly, what penalty the vendor should pay when performance exceeds the allowable deviation. Here we mainly address the last two questions. Our objective is to study key issues in SLA design and the effectiveness of different forms of penalties.

We do not explicitly consider the buyer's backorder cost, but take the target service level as exogenous. In Section 2.8.1 we discuss the situations under which an SLA is preferred over a traditional coordination contract, e.g., when the backorder cost is nonlinear. We investigate two choices of a penalty that the buyer might employ to manage the supplier. The first is where the supplier incurs a lump-sum penalty if her review phase performance is below a performance threshold. The second is a linear-penalty SLA, where the supplier incurs a penalty linear in the amount of deviation from a performance threshold. For example, again from the Acquisition Central website (on Performance-Based Service Acquisition), we have:

Example 1. "The firm-fixed-price for this ... shall be reduced by 2% if the performance standard is not met." (A lump-sum penalty)

Example 2. "For each 5% degradation in ... performance observed ..., the firm-fixed-price for ... will be reduced by 1%." (A linear-based penalty)

Under an SLA, the supplier's inventory performance is reviewed every R periods, the review phase. Unlike most inventory models we must measure performance (a random variable) over the finite period R. A contract based on the expected performance (service level) alone, being either unobservable or needing too long a review phase gives no basis to the supplier for identifying and rectifying underperformance and thus cannot provide an adequate incentive. Therefore we need the distribution of review phase performance. The commonly used inventory performance measures in both the practice and the literature are fill rate and stockout rate. Fill rate is the long-run fraction of demands that are filled immediately. The steady state distribution of the fill rate is very hard to derive. The only study on its distribution is by Thomas (2005) using simulation for a static periodic-review base-stock model with zero leadtime. Our problem is more than obtaining the distribution of a performance measure because we need to find out the supplier's optimal response and the buyer's optimal choice of contract parameters given a performance measure. For simplicity in exposition, most of this chapter focuses on the ready rate — the long-run fraction of periods that demands are filled from stock, which is equal to 1 − stockout rate. But the major insights hold for the fill rate as well. The conventional fill rate and ready rate are performance measures of the immediate order fulfillment. In practice, time window fulfillment rates are more commonly used (LaLonde et al. 1988), so we also consider the ready rate with a window — the long-run fraction of periods that demands are filled within a pre-specified time window. We provide a theoretical approximation to the distributions of the immediate and time window ready rates.

In our problem, the optimal inventory policy for the integrated supply chain is a static one. We first address the SLA design problem assuming the supplier employs a static inventory policy. However, the supplier can observe her performance during the review phase and adjust her inventory level, inducing supplier's strategic behavior. We investigate the different incentives of the two penalty regimes in achieving the target performance and avoiding sub-optimal dynamic behavior. We find that although both the lump-sum penalty and linear-penalty SLAs can induce a nonstrategic supplier to choose the first-best (system optimal) base-stock level, they can result in very different inventory investments in the case of a strategic supplier. Specifically, under a lump-sum penalty SLA, a strategic supplier can achieve a significant cost saving from using a dynamic inventory policy; but such cost saving is minimal under a linear-penalty SLA.

The main contribution of this chapter is to argue for a methodology for studying the design of SLAs, specifically applying principal-agent theory. Our results have direct managerial implications to the design of SLAs. When the supplier can observe the performance history and dynamically adjust her effort level to affect her review phase performance, to mitigate the supplier's incentive for strategic behavior, the penalty should be dependent on the degree of supplier's performance deviation from the target. In particular, a simple linear-penalty SLA can do well over a lump-sum penalty one for this purpose. When using a linear-penalty SLA, the performance threshold should be kept close to the target, i.e., the allowable deviation of the performance from the target should be small; as a consequence, the penalty rate in a linear-penalty SLA is small so that the supplier has a low chance to pay an 'unaffordable' penalty. Another drawback with the lump-sum penalty SLA is that the penalty is generally large, which may not be feasible in practice. Although the application is to SLAs in inventory management, the lessons are applicable to SLA design in other situations.

The rest of this chapter is organized as follows. Section 2.2 reviews the literature. Section 2.3 describes the model, with the distribution of the ready rate derived in Section 2.4. Section 2.5 studies the buyer's problem and the supplier's long-run average cost under the two SLAs. Section 2.6 investigates the supplier's strategic behavior and Section 2.7 presents numerical results. The chapter concludes with a discussion in Section 2.8 and a summary in Section 2.9.

2.2 Literature Review

Most inventory management literature considers performance measures in the long run using expected performance. In a finite review phase, and the fact that the buyer cannot (and does not wish to) observe (micromanage) supplier effort, it is critical to use the distribution of random performance to try and unravel whether poor performance results from weak effort by the supplier or simply noise. Punishing suppliers for mere noise means suppliers overinvest, and charge more, an inefficient outcome. Thomas (2005) and Katok et al. (2008) consider fill rate in a finite horizon, using a static periodic-review base-stock model with zero lead time. The former considers a lump-sum penalty SLA; the latter a lump-sum bonus one, which gives the supplier a fixed bonus if the actual fill rate is above a threshold. Thomas uses simulation to investigate how the length of a review phase R and the penalty affect the optimal base-stock level. Katok et al. use an experimental method to examine how R and the bonus affect the human subjects' choice of the base-stock level. These two papers only consider the lump-sum penalty/bonus incentive, and do not examine the supplier's strategic (dynamic) behavior during a review phase.

Multiple-period review strategies are first studied by Radner (1985) for a repeated principal-agent game. The agent's performance in a period is a noisy signal of her effort level in that period, and the performance in each period is independently and identically distributed (i.i.d.). The agent's performance is reviewed every R periods. If performance is within a margin of error of the expected output under the desired efficient effort level, the agent 'passes' the review. Otherwise they enter a penalty phase of M periods.

In the supply chain context, Ren et al. (2008) apply a modified multiple-period review strategy to an information sharing game between a buyer and a supplier in a decentralized supply chain. In each period, the market demand is a function of a demand state and some normally distributed random variable, the buyer privately observes the realized demand state and sends a forecast to the supplier. A review strategy is used to evaluate whether the buyer truthfully shares the demand state information. The review strategy in their model differs from the one in Radner (1985) in two aspects. First, the review of the buyer's truthfulness of information sharing is started right at the beginning of each review phase and is conducted every period instead of only once at the end of a review phase. Second, the review phase is not fixed, R periods is the maximum length, but the phase can be terminated earlier once a review indicates that the buyer will have no incentive to share information truthfully during the rest of the review phase.

Both papers show that the two parties' payoffs can be arbitrarily close to the equilibrium efficient payoffs when R is sufficiently large. In our model, the SLA is also a multiple-period review strategy, but there are two major differences. Our review period length R is exogenous, and our penalty is monetary, not a subsequent phase of a noncooperative game. Also in our model, the performance in each period can be correlated. We and Radner (1985) study a moral hazard problem and Ren et al. (2008) a hidden information one.

Choi et al. (2004) investigate choosing supplier performance measures in a vendor-managed-inventory context. The supplier's capacity and inventory policy are private information, so the buyer sets performance measures for the supplier to meet an end-customer service level target. They show that in a capacitated supply chain, the supplier's service level is in general not sufficient to guarantee the target customer service level, and they propose a menu of contracts instead. Although supplier's actions are unobservable in their model, Choi et al. focus on the choice of performance measures rather than the noise in the observed measures and the penalty for failing to meet targets. The performance measure in their study is mainly the ready rate.

The economics literature studies discontinuous incentive schemes where the payment from a principal to an agent changes only when some threshold of good or bad performance is reached (McMillan 1992). These are typically single-period models. We study discontinuous incentive schemes under a multi-period review strategy, which bring about issues that do not exist in a single-period model.

As we employ a principal-agent framework, a reasonable question is whether our work is simply a special case. The answer is 'yes' at the most abstract level, but the implementation details differentiate them markedly. In the principal-agent literature, when both parties are risk neutral, a fixed-fee contract can be used to induce the first-best effort level, under which the buyer pays the supplier a fixed fee and the supplier bears all the system costs. Our study differs in two aspects. First, the SLAs in our study have a specific payment structure in that the supplier only pays a penalty for underperformance. So an SLA gives a specific form of risk-sharing rule for the two parties. Secondly, in the principal-agent literature, the distribution of the performance usually has a simple form such as an additive noise. As we focus on specific applications, the specific distribution of the performance is critical and typically quite complex.

The application of SLAs to call center outsourcing is studied by Milner and Olsen (2008) and Hasija et al. (2008). Both papers consider SLAs based on the expected performance and do not consider a multi-period review strategy.

2.3 Model Description

Consider a supply chain consisting of a single supplier and a single buyer, where the supplier manages the supply of a single product for the buyer near the buyer's site. Both parties are risk neutral. The demand is stochastic, and the demand in each period is i.i.d. with mean λ and standard deviation σ. Assume the distribution of single-period demand is unimodal and can be either discrete or continuous. The supplier owns the inventory and incurs a constant unit inventory holding cost h per time period. The supplier uses a periodic-review base-stock policy with a constant inventory replenishment lead time L and a base-stock level S. Her only choice is the base-stock level. At the beginning of period t, the supplier determines the base-stock level and places an order. The order placed at time t arrives by the end of period

$t + L$ and is used to fill demands occurred before the end of period $t + L$.

If a demand is not filled immediately, it is backlogged, and the *buyer* incurs the backorder (delay) cost, $C_D(y)$, which is increasing and convex in the length of delay y. The buyer pays a unit transfer price p for the product to the supplier. The supplier's unit ordering cost c is constant. Under our assumptions on the inventory holding cost and backorder cost, the optimal inventory policy for the integrated supply chain is a static base-stock (order-up-to-S) policy, which has been shown in, for example, Zipkin (2000).

In order to induce the supplier to invest in inventory, the buyer contracts with the supplier on the supplier's inventory service level. This SLA uses some aggregate level of performance, sets a target service level, and reviews the supplier's performance every R periods (a review phase). Assume $R > \max\{L, 1\}$.

2.4 Performance Measure

Commonly used measures of inventory performance are fill rate and ready rate. Fill rate is the long-run fraction of demands that are filled immediately. Ready rate is the long-run fraction of periods in which demands are filled immediately, which measures the inventory availability and is equal to 1 − stockout rate. The ready rate and fill rate in our model are the α-type and γ-type service levels as defined in Schneider (1981) and used by Choi et al. (2004). For simplicity in exposition, we mainly focus on the ready rate. We note that the major insights apply to fill rate as well. We consider two types of ready rates: the conventional immediate ready rate and the time window ready rate, which measures the performance of filling demands within a delivery time window.

2.4.1 Performance measure under a multi-period review strategy

The supplier's performance is evaluated every review phase of length R demand periods. Let $D(t)$ be the demand in t periods, $D(t) \geq 0$. Let $D[t, \tau)$ denote the demand in the interval $[t, \tau)$, i.e., from period t through period $\tau - 1$. Let W $(0 \leq W \leq L)$ be the delivery time window. $W = 0$ means immediate demand fulfillment, and $W > 0$ means demand fulfillment within a time window W. Let the performance indicator for period t $(1 \leq t \leq R)$ be X_t^W, $X_t^W = \mathbf{1}\{D[t-L, t+1-W) \leq S_{t-L}\} \in \{0,1\}$, where $X_t^W = 1$ and $X_t^W = 0$ represent the situations where there is no or some demand delayed longer than time W at the end of period t, and S_{t-L} is the supplier's base-stock level chosen in period $t-L$.

We define $\Pr\{X_t^W = 1\} = \Pr\{D[t-L, t+1-W) \leq S_{t-L}\} = \Pr\{D(L+1-W) \leq S_{t-L}\} = F_{L+1-W}(S_{t-L})$, where $F_n(\cdot)$ is the cumulative distribution function (cdf) of demands in n periods. Let $f_n(\cdot)$ denote the corresponding probability density function (pdf) for continuous demand, or the corresponding probability mass function for discrete demand. For continuous demand, assume $F_n(\cdot)$ and $f_n(\cdot)$ are continuous.

Let the supplier's cumulative performance during a review phase be $\eta_R^W = \sum_{t=1}^{R} X_t^W$. η_R^W is the number of periods without delay longer than W, and $0 \leq \eta_R^W \leq R$. So the ready rate in a review phase is $A_R^W = \eta_R^W / R$, and is random. A_R^0 is the review phase immediate ready rate, and A_R^W $(W > 0)$ the review phase time window ready rate. In each period of a review phase, the supplier receives a constant unit transfer price. Given a performance threshold α for the review phase ready rate A_R^W, if the observed $A_R^W \leq \alpha$, i.e., $\eta_R^W \leq R\alpha$, then the supplier incurs a penalty.

2.4.2 Distribution of ready rate under a static base-stock policy

Under a static base-stock policy, the supplier uses the same base-stock level S in every period. Because the demand in each period is i.i.d, X_t^W has an identical distribution for each t. $\Pr\{X_t^W = 1\} = F_{L+1-W}(S)$, and the ready rate (in the long run) is $A^W = \lim_{R\to\infty} A_R^W = F_{L+1-W}(S)$.

The case when $L = 0$:

We only need to consider the immediate ready rate, $W = 0$. The probability of no stockout in a period is $\Pr\{X_t^0 = 1\} = \Pr\{D(1) \leq S\} = F_1(S)$. Because the demand in each period is i.i.d, the performance in a period X_t is i.i.d. So the cumulative performance η_R^0 has a binomial distribution $B(R, F_1(S))$, and $\Pr\{\eta_R^0 = j\} = \binom{R}{j}(F_1(S))^j(1-F_1(S))^{R-j}$. Moreover, the distribution of the review phase ready rate A_R^0 is approximately normal $N(F_1(S), \sqrt{\frac{F_1(S)(1-F_1(S))}{R}})$.

The case when $L > 0$:

We can have either $W = 0$ or $W > 0$. The performance in each period can be correlated. The performance outcomes in any two periods, X_i^W and X_j^W, are independent for $|i-j| \geq L+1-W$ because $D[i-L, i+1-W)$ and $D[j-L, j+1-W)$ have no periods overlapped; and they are correlated for $|i-j| \leq L-W$ because $D[i-L, i+1-W)$ and $D[j-L, j+1-W)$ have some periods in common. Proposition 2.1 describes the theoretical distribution for η_R^W.

Proposition 2.1 *Under a static periodic-review base-stock policy with base-stock level* S, $\frac{\eta_R^W - E(\eta_R^W)}{\sigma_R^W}$ *converges in distribution to a standard normal random variable as* R *approaches* ∞, *where*

$$E(\eta_R^W) = \sum_{t=1}^{R} E(X_t^W) = RF_{L+1-W}(S),$$

and variance

$$\begin{aligned}(\sigma_R^W)^2 &= RF_{L+1-W}(S) - [(L-W)(2R-L+W-1)+R](F_{L+1-W}(S))^2 \\ &\quad + \sum_{n=1}^{L-W} 2(R-(L+1)+n+W)\int_0^S (F_{L+1-n-W}(S-x))^2 dF_n(x). \quad (2.1)\end{aligned}$$

Proposition 2.1 implies that for sufficiently large R, η_R^W is approximately normally distributed with mean $RF_{L+1-W}(S)$ and standard deviation σ_R^W. Therefore, in our numerical analysis, we will use normal approximation for the distribution of η_R^W. Because the supplier's review phase ready rate $A_R^W = \eta_R^W/R$, Corollary 2.1 follows.

Corollary 2.1 *Under a static base-stock policy with base-stock level* S, $\frac{A_R^W - E(A_R^W)}{\sigma_R^W/R}$ *converges in distribution to a standard normal random variable as* R *approaches* ∞, *where*

$$E(A_R^W) = F_{L+1-W}(S).$$

So for sufficiently large R, A_R^W is approximately normally distributed with mean $F_{L+1-W}(S)$ and standard deviation σ_R^W/R. Proposition 2.2 shows the effect of the length of a review phase, R, on the variability of performance measure A_R^W.

Proposition 2.2 $Var(A_R^W)$ *is decreasing in* R.

Proposition 2.2 implies that a long review phase reduces the variability of the supplier's performance outcome. The fill rate distribution obtained by Thomas (2005) using simulation shows a similar property.

2.5 Buyer's Problem

In this section we consider a nonstrategic supplier, assuming the supplier uses a static inventory policy. We derive the optimal contract parameters for the buyer's problem. In Section 2.6 we consider the dynamic inventory policy and study a strategic supplier's problem under an optimal contract.

Consider two types of SLAs. The first one has a lump-sum penalty under which a supplier pays the buyer a lump-sum penalty K if the supplier's ready rate with window W during a review phase, A_R^W, is not above the performance threshold α. The second type is a linear-penalty SLA, under which if A_R^W is no more than α, then the supplier will pay the buyer a penalty proportional to the difference between A_R^W and α. Specifically, for the realized number of periods without stockout i, $i \leq R\alpha$, the supplier will pay a penalty $K(R\alpha + 1 - i)$. Let $C_P(A_R^W, \alpha, K|S)$ denote the supplier's average penalty per period when the supplier chooses a static base-stock policy with base-stock level S and the realized review phase ready rate is A_R^W, given the ready rate threshold α and penalty parameter K.

The supplier's objective is to maximize her long-run average profit, which is her long-run average revenue minus cost, including the inventory holding cost, the expected penalty and the ordering cost. Because of full backlogging, all demands are filled. So the supplier's base-stock level in each period does not affect the average ordering cost. Without loss of generality we assume the unit ordering cost $c = 0$.

Given the lead time L and the base-stock level S, the buyer's average cost is

$$U_L(S) = p\lambda + \lambda EC_D(y|S) - EC_P(A_R^W, \alpha, K|S),$$

and the supplier's average profit is

$$\pi_L(S) = p\lambda - hE[S - D(L+1-W)]^+ - EC_P(A_R^W, \alpha, K|S),$$

where $D(L+1-W)$ is the realized demand in $L+1-W$ periods.

The buyer's problem is to choose contract parameters p, α and K such that

$$\begin{array}{ll} \min_{p,\alpha,K,S} & U_L(S) \\ \text{subject to} & (IR): \pi_L(S) \geq \overline{\pi} \\ & (IC): S \in \arg\max_{\widehat{S}} \pi_L(\widehat{S}) \end{array},$$

where $\overline{\pi}$ is the supplier's reservation profit per period, and the first constraint is called the individual rationality (IR) constraint, which guarantees the supplier to gain at least her reservation profit so that she will accept the contract; and the second constraint is called the incentive compatibility (IC) constraint, which reflects the supplier's optimal solution given the contract offered by the buyer. The above formulation with the (IR) and (IC) constraints is a standard formulation for incentive mechanism design in agency theory.

The buyer can choose p to make the supplier earn only her reservation profit and extract the rest of supply chain profit[1]. So the buyer's problem can be reformulated as

$$\begin{array}{rl} \min_{\alpha,K,S} & hE[S-D(L+1-W)]^{+}+\lambda EC_{D}(y|S) \\ \text{subject to} & S\in\arg\max_{\widehat{S}}\pi_{L}(\widehat{S}) \end{array} . \tag{2.2}$$

Let S^{*} be the optimal solution without constraint (2.2). Under our assumptions on $C_D(y|S)$, S^{*} minimizes the long-run average supply chain cost. So the target ready rate is $F_{L+1-W}(S^{*})$, and the buyer's problem is to choose α and K to induce the first-best base-stock level S^{*} and ready rate $F_{L+1-W}(S^{*})$. In practice, a reasonable performance threshold should not be above the performance target, so we only consider $\alpha \leq F_{L+1-W}(S^{*})$.

2.5.1 Choice of contract parameters

Let $V_L(S)$ denote the supplier's average cost under a static base-stock S policy. Under the buyer's contract, the supplier's average revenue $p\lambda$ does not affect S, so the supplier's profit-maximizing problem is equivalent to minimizing $V_L(S)$. Because the review phase ready rate $A_R^W = \eta_R^W/R$, in the supplier's average penalty $EC_P(A_R^W, \alpha, K|S)$, A_R^W can be replaced by η_R^W/R.

Under a lump-sum penalty SLA with the ready rate threshold α and lump-sum penalty K, the supplier's problem is

$$\min_{S} V_L(S) = hE[S-D(L+1-W)]^{+} + \frac{K}{R}\sum_{i=0}^{R\alpha}\Pr\{\eta_R^W = i|S\}. \tag{2.3}$$

Under a linear-penalty SLA with threshold α and penalty rate K, the supplier's

[1]The unit transfer price p can be regarded as a fixed fee. By choosing a higher p, the buyer can realize any desired profit division between the two parties.

problem is

$$\min_S V_L(S) = hE[S - D(L+1-W)]^+ + \frac{K}{R}\sum_{i=0}^{R\alpha}(R\alpha + 1 - i)\Pr\{\eta_R^W = i|S\}. \quad (2.4)$$

For $L = 0$ and $W = 0$,

$$\Pr\{\eta_R^0 = i|S\} = \binom{R}{i}(F_1(S))^i(1 - F_1(S))^{R-i}, \quad (2.5)$$

where

$$F_1(S) = \Pr\{D(1) \leq S\}.$$

For $L > 0$ and $0 \leq W \leq L$, using the result in Section 2.4.2, the distribution of η_R^W is approximately normal with mean $RF_{L+1-W}(S)$ and standard deviation σ_R^W, where σ_R^W is given by (2.1). Using a continuity correction,

$$\begin{aligned}
\Pr\{\eta_R^W &= 0|S\} = \Phi(z_0), \\
\Pr\{\eta_R^W &= i|S\} = \Phi(z_i) - \Phi(z_{i-1}) \quad 0 < i < R, \\
\Pr\{\eta_R^W &= R|S\} = 1 - \Phi(z_{R-1}), \quad (2.6)
\end{aligned}$$

where $\Phi(\cdot)$ is the cdf of the standard normal distribution and $z_i = \frac{i+0.5-RF_{L+1-W}(S)}{\sigma_R^W}$.

Now what values of parameters (α, K) ensure that the supplier chooses S^*, the first best integrated solution? We approach this in two steps; first, the necessary condition for the supplier's problem gives us a set of (α, K) candidates for each type of SLA, and then we look into the unimodality of the supplier's cost function under the (α, K) candidates.

Proposition 2.3 *The (α, K) values that ensure S^* satisfies the first order necessary conditions for optimality of either (2.3) and (2.4) are such that for continuous demand, there is a unique optimal $K^*(\alpha)$ for a given α; and for discrete demand, there is an interval of optimal $K^*(\alpha)$, $[\underline{K}^*(\alpha), \overline{K}^*(\alpha)]$, for a given α.*

Formulas for both can be found in Appendix A. Proposition 2.3 identifies multiple optimal candidates (α, K) for both discrete and continuous demands and both types of penalties. Because η_R^W only takes integer values, we pay particular attention to Θ, the set of $(R\alpha, K)$ values under which S^* is the supplier's local optimum; thus

$\Theta = \{(R\alpha, K)|0 < \alpha \leq F_{L+1-W}(S^*),\ R\alpha$ is integer, $K \in [\underline{K}^*(\alpha), \overline{K}^*(\alpha)]\}$. Note $\underline{K}^*(\alpha) = \overline{K}^*(\alpha) = K^*(\alpha)$ in the continuous case. We also note that Θ can be empty.

2.5.2 Unimodality of the supplier's objective function

The candidate parameters (α, K) above are derived from the necessary conditions for optimality, but may not be sufficient. In this section we examine the unimodality of the supplier's average cost under the (α, K) pairs derived above. If the supplier's cost function is unimodal then we can be assured that S^* is a global optimum. Otherwise the solution might be only a local optimum and care needs to be exercised. The supplier's cost function is very complex due to the complicated distribution function of the performance measure. Even with strong assumptions on the demand distribution, it is generally difficult to prove its unimodality.[2] We show that the supplier's cost function under a lump-sum penalty SLA is generally not unimodal. To show its unimodality under a linear-penalty SLA, in particular for $L > 0$, we have to rely on numerical results.

Lump-sum penalty SLA

For $(R\alpha, K) \in \Theta$, $V_L(S)$, the supplier's average cost under a static base-stock S policy, may not be unimodal and S^* not a global optimum. Consider the following example.

Consider a normal demand distribution $N(10, \sqrt{10})$, which can also be considered as an approximation for a Poisson demand with $\lambda = 10$; $h = 1$, $R = 30$, $L = W = 0$, $S^* = 14$, and hence the performance target $F_{L+1-W}(S^*) = F_1(S^*) = 90\%$. Figure 2.1 shows five supplier's cost functions $V_0(S)$ under different pairs of parameters

$(R\alpha, K) \in \{(23, 283), (24, 146), (25, 92), (26, 80), (27, 77)\} \subset \Theta$. Observe that in this continuous example the $K^*(\alpha)$ are unique, not intervals. For example $(27, 77)$ gives $\alpha = 27/30 = 90\%$ and $(24, 146)$ gives $\alpha = 80\%$ compared to the target of $F_1(S^*) = 90\%$. The plots have increasing $R\alpha$ corresponding to a decreasing intercept on the vertical cost axis. Each curve has $S^* = 14$ as a local optimum, however none of the curves is unimodal and S^* is the global optimum only for $R\alpha = 23$ and 24. For the remaining three pairs, $S = 0$ is the global optimum and S^* only local. Intuitively, under a lump-sum penalty SLA, reducing the base-stock level will only

[2] For the special case when $L = 0$ and $f_1'(S^*) < 0$, we can prove that S^* is a local optimum for the supplier's problem with either penalty regime. Interested readers can refer to the Appendix.

increase the supplier's probability of being penalized, not the penalty itself. For a large performance threshold, the penalty is small; hence the supplier may prefer not to stock at all because the holding cost saved may exceed the expected penalty increase. Thus $S = 0$ can be the global optimum for large α.

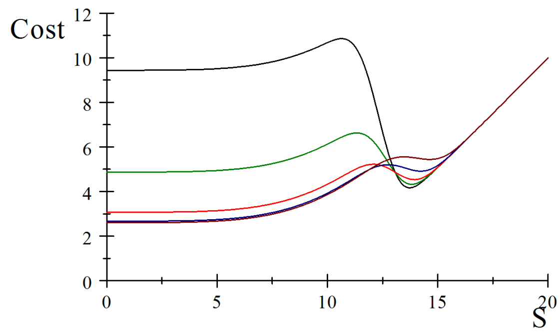

Figure 2.1: Supplier's cost function (lump-sum penalty, L=0)

Linear-penalty SLA

For the same example but with a linear penalty, Figure 2.2 plots the five supplier's cost functions for $(R\alpha, K) \in \{(23, 171), (24, 79), (25, 43), (26, 27), (27, 20)\} \subset \Theta$. Again, the curve with higher $R\alpha$ has lower intercept on the vertical cost axis, and the first-order condition gives $S^* = 14$ for each curve, but now Figure 2.2 shows that they are all unimodal and S^* is the global optimum.

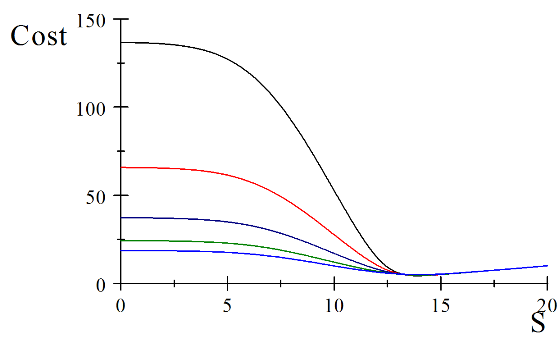

Figure 2.2: Supplier's cost function (linear penalty, L=0)

Comparing figures 2.1 and 2.2, observe the cost of choosing too small a value of S is much more with a linear than a lump-sum penalty. This is because once the target has been missed, with a lump-sum penalty any further deterioration is costless. With the linear penalty however, the expected cost keeps rising as performance deteriorates. For $L = 1$, similar patterns can be found.

Thus there are multiple choices of optimal (α, K) for either penalty regime. The linear-penalty SLA appears more likely unimodal than the lump-sum penalty case. This is because the supplier's penalty increases with the amount of performance deviation from the target. When reducing the base-stock level, the supplier will not only increase the probability of being penalized, but also the probability of paying a higher penalty, so the expected penalty increase will offset the holding cost saved. From Section 2.5.1 and this section, we can conclude that in the case of a lump-sum penalty SLA, $V_L(S^*)$ is likely to be the global minimum only under relatively small thresholds. The linear-penalty SLA is more likely to have S^* as the global optimum.

2.6 Supplier's Strategic Behavior

The optimal contract parameters are derived above under a static inventory policy. However the supplier, aware of her performance, may prefer to dynamically adjust inventory as the review period progresses. If her performance so far is good with few periods left in that review phase, her probability of exceeding the performance threshold α is large, then she has an incentive to decrease inventory to reduce cost. If the supplier's probability of exceeding α is small due to poor performance, she may have an incentive to increase inventory to repair the damage or alternatively to abandon any chance of improving and at least reduce cost by not keeping any further inventory.

In this section we formulate the dynamic program for computing a strategic supplier's optimal average cost under a dynamic inventory policy and the optimal contract obtained above. We first investigate the supplier's cost minimization problem in a single review phase under a contract offered, ignoring the impact of the inventory policy on the subsequent review phases. After we obtain the supplier's minimum cost from a single-review-phase problem, we then calculate the supplier's minimum long-run average cost.

In this chapter, we mainly study the cases of $L = 0$ and 1 with $W = 0$. But the major findings also hold for the situation of $L > 1$ and $0 \leq W \leq L$. For a positive lead time $L > 0$, the supplier's performance realization from her action in any period is delayed until L periods later. So when choosing the base-stock level in period t, the supplier has to anticipate possible performance outcomes in periods $t, t+1, ..., t+L-1$, which depend on the base-stock levels chosen in period $t-l$ ($l = 1, ..., L$).

2.6.1 Single-review-phase problem

Consider a single review phase with R periods. The decision epochs = $\{1, 2, ..., R, R+1\}$, and there is no decision made in period $R+1$. Let

d_t = demand in period t;

S_t = inventory order-up-to level chosen in period t; and

η_t = supplier's performance history — number of periods without a stockout up to the end of period t, $0 \le \eta_t \le t$ for $1 \le t \le R$.

Assume the supplier can always observe her performance history.

For the dynamic programming problem we consider discrete demand, but, without loss of generality, we scale demand so that 1 unit is 'small'. The purpose of this will be clear in Section 2.7 and is only to make the exposition clearer. The dynamic program for $L = 0$ is a straightforward derivation from $L = 1$, so below we will only give the $L = 1$ case.

By Proposition 2.1 and letting $L = 1$ and $W = 0$, the distribution of η_R is approximately normal with mean $RF_2(S)$ and standard deviation σ_R, where

$$\sigma_R^2 = RF_2(S) - (3R-2)(F_2(S))^2 + 2(R-1)\sum_{d=0}^{S}(F_1(S-d))^2 f_1(d).$$

Let $\widehat{I}_t$ = the supplier's inventory position (inventory plus the order made in the last period) at the beginning of period t, before an order is placed in period t; and

$I_t = \max\{-1, \widehat{I}_t\}$.

So the base-stock level chosen in period t, $S_t \ge I_t$.

Let $\pi_t^1(S_t|i, I_t)$ denote the supplier's cost to go from period t in a single review phase given performance history i, I_t and S_t; and $\pi_t^1(i, I_t) = \min_{S_t \ge I_t} \pi_t^1(S_t|i, I_t)$ denote the supplier's optimal cost to go from period t given i and I_t.

Because the supplier does not incur backorder cost, for any negative net inventory in a period the supplier has the same immediate cost (zero inventory holding cost) and performance outcome (stockout) in that period. Thus in the state space, we can use a single state -1 to represent all the states of negative inventory.

The state space is $\{(\eta_{t-1}, I_t) : 0 \le \eta_{t-1} \le t-1, -1 \le I_t \le \overline{S}\}$, where $1 \le t \le R+1$ and $\overline{S}$ is a large number such that $F_2(\overline{S}) \approx 1$.

Actions (inventory order-up-to level in a period): $S \in \{0, 1, .., \overline{S}\}$

State transition:

$$I_{t+1} = \max\{-1, S_t - d_t\}, \eta_t = \begin{cases} \eta_{t-1} + 1 & \text{if } I_t \ge d_t \\ \eta_{t-1} & \text{if } I_t < d_t \end{cases}.$$

Let $\eta_{t-1} = i$ for $1 \le t \le R+1$. Note that $\eta_0 = 0$. All the expectation calculations 'E' below are on d_t.

Rewards:

$$\begin{aligned} & r_t((i, I_t), S_t) = hE[I_t - d_t]^+ \qquad 1 \le t \le R, \\ & \text{lump-sum penalty: } r_{R+1}((i, I_{R+1})) = \begin{cases} K & \text{if } i \le R\alpha \\ 0 & \text{if } i > R\alpha \end{cases}, \\ & \text{linear penalty: } r_{R+1}((i, I_{R+1})) \text{ is similarly defined by} \\ & \qquad \text{replacing } K \text{ by } K(R\alpha + 1 - i). \end{aligned} \tag{2.7}$$

The supplier's base-stock level decision made in period R of a review phase cannot affect her performance in that review phase, but it determines the inventory holding cost and performance outcome in period 1 of the next review phase. So the optimal base-stock level S_R in period R is $S_R = \arg\min_{S \ge I_R} E\pi_1(0, \max\{-1, S - d_R\})$,

where $E\pi_1(0, \max\{-1, S-d_R\}) = \sum_{d=0}^{S} \Pr\{D(1) = d\}\pi_1(0, S-d) + \overline{F}_1(S)\pi_1(0, -1)$ is the supplier's expected cost to go from period 1 of the next review phase. We conjecture that if possible, the supplier will choose the first-best base-stock level S^* in period R. So $S_R = \max\{I_R, S^*\}$.

Transition probabilities:

$$p_t((j, I)|(i, u), S) = \begin{cases} \Pr\{D(1) = S - I\} & j = i+1, S \ge u, 0 \le S - I \le u, I \ge 0 \\ \Pr\{D(1) = S - I\} & j = i, S \ge u, S - I > u, I \ge 0 \\ \Pr\{D(1) > S\} & j = i, S \ge u, I = -1 \\ 0 & \text{otherwise} \end{cases}.$$

The dynamic program is

$$\pi^1_{R+1}(i, I_{R+1}) = r_{R+1}((i, I_{R+1})),$$
$$\text{for } 1 \leq t \leq R:$$
$$\pi^1_t(i, I_t) = hE[I_t - d_t]^+ + \min_{S_t \geq I_t}[E\{\pi^1_{t+1}(i+1, S_t - d_t)|d_t \leq I_t\}$$
$$+E\{\pi^1_{t+1}(i, \max\{-1, S_t - d_t\})|d_t > I_t\}].$$

From the dynamic program, we can obtain $\pi^1_1(0, I_1)$ for various opening inventory level I_1. The optimal base-stock policy obtained for this single review phase is not optimal in general for the infinite horizon problem because the terminal reward r_{R+1} ignores the inventory holding cost in the early periods of the next review phase resulting from the supplier's base-stock choice in period R of the current review phase.

2.6.2 Infinite-horizon problem

Instead of treating the infinite horizon as consisting of many time periods, we can think of each review phase as one time period in the infinite horizon, i.e., the infinite horizon consists of many review phases. So all of the supplier's base-stock policies in the n^{th} finite review phase can be denoted by a vector $\overrightarrow{\gamma}_n = (\gamma^n_1, \gamma^n_2, ..., \gamma^n_R) \in \Gamma$, where γ^n_t is the supplier's base-stock policy in period t of the n^{th} review phase, and Γ is the set of possible base-stock policies in a review phase. Although the supplier's base-stock policy within a review phase depends on her performance history in that review phase and so is history dependent, $\overrightarrow{\gamma}_n$ and $\pi^1_1(0, I_1)$ only depend on I_1, the opening inventory of a review phase. Let the opening inventory of a review phase be the system states. Thus the system states and Γ do not vary with time, and the state transitions (from the opening inventory of a review phase to that of the subsequent phase) as well as the supplier's rewards (expected total cost in a review phase) are Markovian. So the supplier's problem in an infinite horizon consisting of review phases is a Markov decision process. Assume the supplier only uses deterministic base-stock inventory policies. Due to demand uncertainty, this MDP is unichain.

To find out the supplier's optimal average cost in an infinite horizon, we use value iteration. Let $\overline{\pi}^1(I)$ denote the supplier's expected total cost in a single review phase with an opening inventory I as derived above. Let $\overline{\pi}^n(I)$ denote the supplier's expected total cost in n review phases with an opening inventory I for the first review phase. With n review phases, the review phases are indexed reversely from 1 to n, i.e., the last review phase is indexed by 1 and the first by n.

The algorithm is as follows.

1. Let $\overline{\pi}^1(I) = \pi_1^1(0, I)$, $\varepsilon = 0.01$, and set $n = 1$.

2. For each $I \in \{-1, 0, .., \overline{S}\}$, compute $\overline{\pi}^{n+1}(I)$ by applying the dynamic program as defined above for a single-review-phase problem but with the terminal reward $r_{R+1}((i, I_{R+1}))$ added by $\overline{\pi}^n(I_{R+1})$.

3. If $\max_I\{\overline{\pi}^{n+1}(I) - \overline{\pi}^n(I)\} - \min_I\{\overline{\pi}^{n+1}(I) - \overline{\pi}^n(I)\} < \varepsilon$, then go to step 4. Otherwise, increment n by 1 and return to step 2.

4. Let $V^D = \frac{1}{R}\max_I\{\overline{\pi}^{n+1}(I) - \overline{\pi}^n(I)\}$. Then V^D is an approximation to the supplier's optimal long-run average cost.

2.7 Numerical Analysis

We numerically investigate the supplier's incentive for strategic/dynamic behavior using either the lump-sum or linear penalty SLA. We compare the supplier's optimal average cost when using a static base-stock S^* policy with that under a dynamic policy. The numerical results demonstrate that the strategic supplier's gain under a lump-sum penalty SLA can be large, but that under an optimal linear-penalty SLA is minimal. It is also shown that a longer inventory replenishment lead time reduces such gain under both regimes.

Consider two distributions: a Poisson demand with arrival rate λ and a Normal(λ, σ) demand per period.[3] With a normal demand, we can study the impact of various parameters on the supplier's gain from a dynamic inventory policy while keeping the performance target fixed. For the dynamic policy we discretize the normal demand. $\Pr\{D(1) = d\} = \Phi(\frac{d-\lambda}{\sigma}) - \Phi(\frac{d-1-\lambda}{\sigma})$ for $d > 0$, and $\Pr\{D(1) = 0\} = \Phi(\frac{-\lambda}{\sigma})$. Now it can be seen why we scaled demand so that 1 unit was 'small'. Leadtime $L \in \{0, 1\}$. The performance threshold α is chosen such that $R\alpha$ is an integer. The range for the optimal penalty, $[\underline{K}^*(\alpha), \overline{K}^*(\alpha)]$, is obtained using the formulas in Section 2.5.1.

[3] The problem of negative demands with the normal distribution is not significant in our examples, as is ignored below. Only nonnegative demands are considered in the numerical analysis by using truncated normal distribution.

2.7.1 Lump-sum penalty SLA

Under a lump-sum penalty SLA, if the supplier's ready rate during a review phase does not exceed the performance threshold α, then the supplier will pay the buyer a lump-sum penalty K. We show that in this case the supplier can benefit significantly from a dynamic policy.

- Poisson demand

The following parameter values are used: $h = 1$, $R = 30$. With $\lambda = 8, 9, 10$, for $L = 0$, we have $S^* = 12, 13, 14$; and for $L = 1$, $S^* = 21, 24, 26$.

First consider the static base-stock policy.

Consider $L = 0$. For $\lambda = 10$ and $R\alpha = 24$ (i.e., $\alpha = 80\%$), $[\underline{K}^*(\alpha), \overline{K}^*(\alpha)] = [146, 853]$. It can be checked that $S^* = 14$ is the global optimum for the static inventory policy for any K in $[146, 853]$. For $K = 146$, 200 and 300, the supplier's cost savings from using a dynamic policy are 20.3%, 20.9% and 22.4%, respectively. So even for the same threshold α, the supplier's percentage cost saving from a dynamic policy varies with the penalty and is increasing in K.

Although by definition all $(R\alpha, K) \in \Theta$ give S^*, this might not always be a global optimum. For example, with $R\alpha = 25$ and using the smallest K in the interval $[92, 318]$, S^* is not the global optimum. Let $\underline{k}^*(\alpha) = \min\{K | K \in [\underline{K}^*(\alpha), \overline{K}^*(\alpha)]$ and S^* is the supplier's global optimum$\}$ if it exists, then any $K \in [\underline{k}^*(\alpha), \overline{K}^*(\alpha)]$ can induce the supplier to choose S^*. If the interval is empty we say that $K(\alpha) =$ 'Infeasible'. Define $\Theta^* = \{(R\alpha, K) | 0 < \alpha \leq F_{L+1-W}(S^*)$, $R\alpha$ is integer, $K \in [\underline{k}^*(\alpha), \overline{K}^*(\alpha)]$ s.t. the interval is not empty$\}$. So $\Theta^* \subseteq \Theta$ is the set of $(R\alpha, K)$ values under which S^* is the supplier's global optimum for a static policy.

Now consider the dynamic base-stock policy. Table 2.1 shows the supplier's percentage cost saving from using a dynamic policy under various $R\alpha$ and $\underline{k}^*(\alpha)$ for $L = 0$ and 1. Note that for $R\alpha = 27$ (i.e. $\alpha = 90\%$) the result is 'Infeasible'. The results show that the supplier can benefit significantly from a dynamic policy. The smaller the value α, the greater motivation for dynamic behavior, so the buyer should choose the performance threshold α as close to the performance target as possible. Regardless of such choices however, under a lump-sum penalty SLA, the supplier has a considerable incentive to adopt a dynamic inventory policy and increase costs for the buyer.

Table 2.1: Supplier's cost saving (lump-sum penalty,Poisson demand)

		λ		
	$R\alpha$	8	9	10
$L = 0$	24	26.14%	22.95%	20.28%
	25	18.41%	16.27%	15.21%
	26	13.62%	14.69%	Infeasible
$L = 1$	24	16.70%	20.51%	18.50%
	25	14.24%	15.21%	14.41%
	26	Infeasible	Infeasible	Infeasible

Table 2.2: Supplier's cost saving (lump-sum penalty,normal demand)

		σ			
	$R\alpha$	5	10	15	20
$L = 0$	24	20.09%	20.79%	21.09%	21.19%
	25	15.20%	15.21%	15.23%	15.25%
$L = 1$	24	16.68%	18.13%	18.50%	18.51%
	25	15.18%	15.83%	Infeasible	Infeasible

- Normal demand

We use normal demand to better study the impact of lead time on the supplier's cost saving from using a dynamic policy. The following parameter values are used: $h = 1$, $R = 30$, $\lambda = 50$. With $\sigma = 5, 10, 15, 20$, for $L = 0$, we have $S^* = 57, 64, 71, 78$; and for $L = 1$, $S^* = 110, 120, 130, 140$. In all scenarios the target ready rate is the same, about 92%. For $L = 0$ and 1, and $R\alpha = 26$ and 27, $S = 0$ is the global optimum and the first-best S^* is only a local optimum for all values of K in their intervals, that is $\underline{k}^*(\alpha)$ does not exist. For $L = 1$, $R\alpha = 24$ and 25, S^* is the global optimum in some of the cases, thus $\underline{k}^*(\alpha)$ is within the interval. Table 2.2 shows the supplier's percentage cost saving from using a dynamic inventory policy. They are all very large. Comparing the results for $L = 0$ and 1, we can see that given the same performance threshold, the supplier's percentage cost savings from using a dynamic policy decreases as the lead time increases from 0 to 1.

2.7.2 Linear-penalty SLA

We have shown that a lump-sum penalty SLA will greatly induce supplier's strategic behavior. Now we investigate the ability of a linear-penalty SLA to discourage

Table 2.3: Supplier's cost saving (linear penalty,Poisson demand)

$L=0$				$L=1$			
Target=91.7%		Target=95.1%		Target=92.2%		Target=94.8%	
Deviation	%saving	Deviation	%saving	Deviation	%saving	Deviation	%saving
1.7%	2.04%	1.8%	1.38%	2.2%	2.63%	1.4%	1.16%
5.0%	6.32%	5.1%	6.62%	5.5%	6.64%	4.8%	5.22%
8.3%	12.66%	8.5%	17.77%	8.9%	12.07%	8.1%	10.94%
11.7%	20.09%	11.8%	23.11%	12.2%	18.32%	11.4%	17.77%

supplier's strategic behavior. We also investigate the effects of lead time L, demand variability $\frac{\sigma}{\lambda}$, and the length of a review phase R, on the supplier's cost saving from using a dynamic policy under a linear-penalty SLA. For all the scenarios in the numerical examples below, the supplier's cost functions have been checked to be unimodal using plots under the optimal penalty K given α.

- Poisson demand

We use $h=1$ and $R=30$. We compare the supplier's optimal average cost under a static base-stock S^* policy with that under a dynamic base-stock policy, using a penalty $K=\underline{k}^*(\alpha)$. We note that for all the numerical examples here we have found that $\underline{k}^*(\alpha)=\underline{K}^*(\alpha)$.

To compare the supplier's cost saving under various performance thresholds α, we use a demand rate $\lambda=10$. For $L=0$, $S^*=14$ and 15 with the target service levels = 91.7% and 95.1%, respectively; and for $L=1$, $S^*=26$ and 27 with the target service levels = 92.2% and 94.8%, respectively. Table 2.3 show the results for $L=0$ and 1. The threshold α is represented by the allowable deviation from the target, which is equal to the target ready rate $-$ α. Similar conclusion can be drawn here as that under a lump-sum penalty SLA. The greater the allowable deviation from the target (thus the smaller α), the greater the supplier's percentage cost saving. Therefore, to reduce the supplier's cost saving from using a dynamic policy, the performance threshold should be chosen to be as close to the performance target as possible.

We also investigate the supplier's cost saving from using a dynamic policy for different demand rates, using the smallest possible allowable deviation from the target (the largest possible α). For $\lambda=7,8,9,10$, the corresponding parameter values for $L=0$ are $S^*=11,12,13,14$; and those for $L=1$ are $S^*=19,21,25,26$. Table 2.4 shows the results. α is again represented by the allowable deviation from the

Table 2.4: Supplier's cost saving (linear penalty,Poisson demand)

	$L=0$			$L=1$		
λ	Target	Deviation	%saving	Target	Deviation	%saving
7	94.7%	1.3%	1.10%	92.3%	2.3%	2.62%
8	93.6%	3.6%	3.87%	91.1%	1.1%	1.89%
9	92.6%	2.6%	2.80%	95.5%	2.2%	1.50%
10	91.7%	1.7%	2.04%	92.2%	2.2%	2.63%

Table 2.5: Supplier's cost saving (linear penalty,normal demand)

$R\alpha$	$L=0$	$L=1$
24	20.85%	18.39%
25	13.90%	12.58%
26	7.40%	6.57%
27	2.64%	2.63%

target. The supplier's percentage cost saving is small in all the cases. So under a linear-penalty SLA with a performance threshold close to the target, the supplier will have little incentive to adopt a dynamic inventory policy. A linear-penalty SLA can greatly mitigate the supplier's strategic behavior.

- Normal demand

The following parameter values are used: $h=1$, $R=30$, $\lambda=50$, $\sigma \in \{5,10,15,20\}$, $L \in \{0,1\}$. Note that we use the same demand process (single-period demand distribution) for both $L=0$ and 1, so we can investigate the supplier's cost saving from a dynamic inventory policy as the lead time increases from 0 to 1.

We first compare the supplier's cost saving under various α, using $\lambda=50$ and $\sigma=15$ for the single-period demand. For $L=0$, $S^*=71$ with the target service level = 91.9%; and for $L=1$, $S^*=130$ with the target service level = 92.1%. The results are shown in Table 2.5. The supplier's percentage cost saving decreases with α, indicating that a tight allowable performance deviation from the target is required to limit the supplier's gain from strategic behavior. The results also indicate that given the same performance threshold, a longer leadtime will reduce the supplier's cost saving from strategic behavior.

Next, we investigate the effects of demand variability and leadtime on the supplier's cost saving from using a dynamic policy. We fix $\lambda=50$ while changing σ, so the coefficient of variation of demand $\frac{\sigma}{\lambda} \in \{0.1,0.2,0.3,0.4\}$; we use the smallest

Table 2.6: Supplier's cost saving (linear penalty,normal demand)

	Target $\approx 92\%$		Target $\approx 96.5\%$	
σ/λ	$L=0$	$L=1$	$L=0$	$L=1$
0.1	2.41%	2.06%	2.72%	2.17%
0.2	2.51%	2.07%	2.81%	2.44%
0.3	2.64%	2.63%	2.90%	2.45%
0.4	2.68%	2.67%	2.95%	2.45%

possible deviation from the target (the largest $\alpha <$ target and $R\alpha$ is integer). For $\sigma \in \{5, 10, 15, 20\}$, S^* is chosen so that the target service level is equal for $L = 0, 1$ and different σ. We consider two target service levels: one is about 92% and the other about 96.5%. Table 2.6 lists the numerical results. Each column shows that the supplier's gain from strategic behavior increases with the demand variability. Intuitively, when demand is less variable, the optimal safety stock under a static inventory policy is small, and the optimal safety stock under a dynamic inventory policy will not deviate much from the static one, so the supplier's gain from a dynamic policy will be relatively small. Comparing the two columns for each target service level, we can see that the supplier's cost saving from a dynamic policy decreases as the leadtime increases from 0 to 1. So a longer leadtime will likely mitigate the supplier's incentive for strategic behavior. We will discuss the insights from this result in Section 2.8.

- Effect of length of a review phase R

We also evaluate the impact of the length of a review phase on the supplier's percentage cost saving from a dynamic policy. We use $\lambda = 50$ and $\sigma = 15$ for single-period demand, $R \in \{30, 60\}$, $L \in \{0, 1\}$, S^* is chosen so that the target service levels for $L = 0$ and 1 are both about 92%, and the same performance threshold $\alpha = 90\%$ (thus the same allowable deviation from the target). Table 2.7 compares the supplier's cost saving under different values of R with the other parameters fixed. It indicates that as the review phase becomes longer, if the performance threshold remains unchanged, then the supplier will benefit more from a dynamic inventory policy. This can be explained by Proposition 2.2, which says that the variance of the supplier's review phase ready rate decreases with R. So if α is fixed, then as R increases, the absolute allowable performance deviation from the target remains constant, but the relative allowable deviation increases, giving the supplier more flexibility to dynamically adjust the base-stock level. This implies that as R increases,

Table 2.7: Effect of R (linear penalty,normal demand)

	$L = 0$	$L = 1$
$R = 30$	2.64%	2.63%
$R = 60$	4.12%	4.08%

there should be less allowable performance deviation from the target (thus bigger α) in order to discourage the supplier from strategic behavior. We have assumed that R is exogenous because in practice R can be determined by other factors such as transaction costs and the accounting policy.

2.8 Discussion

The numerical results have shown that under a linear-penalty SLA with the performance threshold close to the target ready rate, the supplier's gain from using a dynamic inventory policy instead of a static one is small. In practice, implementing a dynamic inventory policy is more complicated than implementing a static one. To implement a dynamic inventory policy, the supplier has to determine the ordering quantity based on not only the inventory on hand and the performance history in the current review phase, but also every order placed in the past yet not arrived. So the implementation cost of a dynamic policy is higher than that of a static one. In this chapter, we do not explicitly model the supplier's costs of implementing an inventory policy. When the complexity and cost of implementing a policy is taken into account, a dynamic inventory policy will bring less benefit. So under a linear-penalty SLA with carefully chosen contract parameters, the supplier will have little incentive for strategic behavior.

2.8.1 SLA vs. traditional coordination contract

To provide an incentive to the supplier for inventory investment, the buyer can use either a traditional supply chain coordination contract or an SLA. Traditional contracts coordinate the supply chain via a holding cost and backorder cost transfer payment. Cachon (2003) provides an analysis on this type of contract in a single-location base-stock model with linear backorder cost. As a special case of this contract, the supplier pays a penalty on each individual delayed delivery. To micro-manage the supplier in this way, the buyer will have to make a detailed record of every single demand. When demands occur frequently, the buyer will incur large administrative and transaction costs. With an SLA, the buyer only needs to measure the supplier's aggregate performance over a period of time. So an SLA is preferable over a traditional contract for frequent demands.

In most inventory theory, the backorder cost is generally linear in the amount of delay. But in practice, the backorder cost is often nonlinear; a short delay may not matter much, but a long delay is very costly. In this case, if a traditional coordination contract is used, then the contract will likely be quite complicated in order to correctly align the supplier's incentive with the supply chain. However, an SLA has a simple form and is easy to implement even if the backorder cost has a complex form, as in our model, the backorder cost is convex in the length of delay.

Service levels for inventory performance are widely used in practice. The most frequently cited reason is that backorder costs are hard to measure because a stockout may affect not only a firm but also external parties (e.g., customers), so firms prefer to set a desired performance target. This makes traditional coordination contracts hard to design. But with an SLA, the performance target can be set exactly as desired.

2.8.2 Lump-sum penalty SLA vs. linear-penalty SLA

A lump-sum penalty SLA is a highly discontinuous incentive scheme used with a multi-period review strategy. For a single period problem with a risk-neutral buyer and supplier, any optimal combination of the performance threshold and penalty can induce the supplier to choose the first-best effort level, and thus a lump-sum penalty SLA can be optimal. However, the multi-period review strategy brings in additional issues for the design of discontinuous incentive schemes. In our model, the supplier can observe her performance history throughout the review phase and can adjust her effort level at any time to affect her performance outcome at the end of the review phase. In this case, as demonstrated by the numerical results in Section 2.7, a discontinuous incentive scheme with a lump-sum penalty will cause supplier's strategic behavior. If either condition is violated, i.e., the supplier cannot observe her performance history or cannot adjust her effort level dynamically, then a lump-sum penalty SLA will be less vulnerable to supplier's strategic behavior.

From the formulation of the supplier's dynamic program in Section 2.6.1, we can see that under a lump-sum penalty SLA, after observing the performance history in any period of a review phase, the supplier can take the following strategy. If her past performance has already exceeded the threshold (call it an 'early pass'), she will not put in any effort during the remainder of the review phase. If her past performance has been poor enough so that she will have no chance to attain the threshold in the current review phase (call it an 'early failure'), then she will also not put any effort into the remaining periods of the review phase. In other situations, she will choose effort levels depending on the past performance and the number of periods left in that review phase. In practice, a service level target such as the ready rate for inventory performance is generally very high (above 90%), so the chance for an early pass is very small. On the other hand, the chance for an early failure is large. Under a lump-sum penalty SLA, if an early failure occurs, the supplier's penalty is fixed no matter how poor her performance is. Thus a lump-sum penalty SLA will not mitigate the supplier's strategic behavior after an early failure. A linear-penalty SLA makes the supplier's penalty for poor performance linear in the amount of deviation from the threshold, and so can better mitigate the supplier's strategic behavior after an early failure.

In the case of a positive inventory replenishment lead time, orders placed in a period will arrive L periods later, and the performance as a result of any effort will be revealed after a time lag of L periods. Because of the information delay, the supplier cannot respond to the performance history as effectively as in the case of no delay. Moreover, when the number of periods left in a review phase is less than L, the supplier cannot adjust the base-stock level to affect her performance at the end of the current review phase. So we can anticipate that the supplier will benefit less from her strategic behavior in the case of a positive lead time. This is supported by the numerical results in Section 2.7 comparing the supplier's percentage cost savings from a dynamic policy for $L = 0$ and $L = 1$. In practice, the target ready rate or fill rate is usually high. Due to this and the positive lead time, unless a review phase is extremely long and/or α is small, there is little chance that the supplier will know she can meet the performance threshold with probability 1 before a review phase ends. At the low performance side, the linear-penalty scheme will provide the supplier with an incentive to prevent her performance from getting worse once it falls below α.

The linear-penalty SLA we study here does not provide incentives concerning performance beyond a threshold. So to induce a desired target service level, the

Table 2.8: Expected penalty vs. supplier's total cost

λ	σ	L	Lump-sum	Linear
10	3	0	8.9%	12.9%
10	4	0	9.3%	13.1%
10	3	1	5.1%	13.8%
10	4	1	5.1%	14.3%

Table 2.9: Actual penalty vs. supplier's total cost

L		Lump-sum	Linear					
0	Prob	9.03%	22.01%	13.05%	5.96%	2.18%	0.66%	0.17%
	ratio	99.3%	16.3%	32.5%	48.7%	65.0%	81.2%	97.4%
1	Prob	3.28%	21.79%	14.73%	7.06%	2.40%	0.58%	0.10%
	ratio	156.8%	16.1%	32.2%	48.4%	64.5%	80.6%	96.7%

allowable deviation for the supplier performance from the target should be small, i.e., α is not too far from the target. This has been shown by the numerical results on the supplier's cost savings under different values of α.

A linear-penalty SLA imposes a small penalty on the bad outcomes more likely due to random variability in the performance than to wrong effort levels. This can be seen by comparing the size of the supplier's penalty with her revenue under both types of penalty schemes in two ways. Note that the supplier's revenue is at least her total cost, including the inventory holding cost and expected penalty. So we use the supplier's total cost as a proxy for her revenue. The first way is to compare the supplier's expected penalty with her total cost, and the ratios are shown in Table 2.8 using normal demand distributions. The second way is to compare the supplier's actual penalty with her total cost, as shown in Table 2.9 using a Normal$(10, 3)$, in which for a lump-sum penalty, the probability of incurring a penalty and the penalty to total cost ratio are provided; and for a linear penalty, both the probability of incurring a penalty level and the actual penalty to total cost ratio are provided. The results show that although the expected penalty under a lump-sum penalty SLA is small, the actual penalty could exceed the supplier's revenue, making the supplier earn nothing. On the contrary, the supplier is unlikely to pay a large penalty under a linear-penalty SLA. Therefore, a linear-penalty SLA is a mild penalty scheme compared with a lump-sum penalty one and thus induces a more stable inventory investment.

2.8.3 Extensions

When studying supplier's strategic behavior under an SLA, we have mainly focused on the immediate ready rate as a performance measure for inventory and considered a periodic-review base-stock policy. The main findings also extend to time-window ready rate and fill rate as well as a continuous-review inventory policy. When a time-window ready rate is used with $W \leq L$, the performance indicator for period t , $X_t^W = \mathbf{1}\{D[t-L, t+1-W) \leq S_{t-L}\}$, and the supplier decides a base-stock level in each period to fill demands in the subsequent $L+1-W$ periods instead of $L+1$ periods. So a time window can reduce the supplier's inventory risk and inventory cost. Let $L' = L - W$. Because an order placed in a period will still arrive L periods later, the supplier's decision problem is like the one with lead time L' but the performance resulting from a base-stock level decision is realized after $L' + W$ periods, so the supplier cannot adjust the base-stock level under a dynamic policy as timely as in the case of lead time equal to L'. Therefore, the supplier's gain from using a dynamic inventory policy given lead time L under a time-window-W ready rate will not exceed that under an immediate ready rate given lead time $L-W$. When the fill rate is used as the performance measure, supplier's incentive for strategic behavior still exists, and the supplier's objective is to dynamically adjust the base-stock level to affect the proportion of demands filled on time in a review phase instead of the proportion of periods with all demands filled on time. This will only slightly change the calculation of the supplier's expected penalty, but the issues and insights from the ready rate as performance measure all hold here.

Now suppose the supplier uses a continuous-review base-stock policy ($L > 1$). Because the supplier's performance is revealed at the end of each period, assume the supplier chooses the base-stock level once every period. The only difference between the two inventory policies is that the supplier's inventory cost is relatively lower under a continuous-review policy. The supplier's ability to dynamically adjust the base-stock level and the effect of the information lag due to positive lead time are similar under both policies. So the insights from a periodic-review policy also hold for a continuous-review policy.

When the ready rate is used as inventory performance measure, the supplier may have other strategic behavior in addition to adopting a dynamic inventory policy. For example, if the supplier cannot fill all demands in a particular period so that the performance in that period will be bad no matter what proportion of the demands are filled, then the supplier may hold inventory instead of filling partial demands in order to save the transportation cost in that period. This will increase the supplier's inventory holding cost. In our model, the supplier manages inventory near the buyer's site, so the transportation cost is negligible. In other situations where the transportation cost is large relative to the holding cost, then the fill rate may be preferred because it measures the proportion of demands filled.

2.9 Conclusions

In this chapter, we have provided a methodology for studying service level agreements by applying the principal-agent theory to the design and choice of contract parameters of SLAs. Using a single-location uncapacitated inventory management problem, we have identified issues in the design of SLAs for inventory management, where the ready rate is the performance measure. In the case of a positive inventory replenishment lead time, the supplier's performance in each period can be correlated. The ready rate in a finite review phase is a random variable; we have shown that its distribution is approximately normal for a long review phase. We have studied two types of SLAs: a lump-sum penalty SLA and a linear-penalty one. Due to their multi-period review structure, SLAs with a target service level provide the supplier with an incentive for strategic dynamic behavior. Specifically, we have found that under a lump-sum penalty SLA, the supplier will have a significant incentive for strategic behavior. On the other hand, a simple linear-penalty SLA can greatly mitigate supplier's strategic behavior. This has implication for the design of SLAs in general: when the supplier can observe the performance history and dynamically adjust her effort level to affect her review phase performance, to mitigate the supplier's incentive for strategic behavior, the penalty should be dependent on the amount of supplier's performance deviation from the target. To effectively mitigate the supplier's incentive for strategic behavior using a linear-penalty SLA, the allowable deviation of the performance from the target service level should be small. For the application of SLAs to inventory management in particular, a positive inventory replenishment lead time and a high fulfillment rate target can further mitigate such strategic behavior.

Chapter 3

Managing Supplier's Delivery Performance With Service Level Agreements

3.1 Introduction

With the increased outsourcing of manufacturing and services to suppliers comes a need for better contractual agreements between suppliers and buyers. One of the most widely employed contractual instruments is a type of performance-based contracts called Service Level Agreements (SLAs). A survey by Oblicore Inc. in 2007 revealed that 91% of organizations use SLAs for managing suppliers, internal agreements, or external customer agreements. According to the Office of Federal Procurement Policy at the Office of Management and Budget, the US Federal Government expects agencies to make half their service contracts performance-based acquisitions in fiscal 2008, an increase from the goal of 45% for 2007. SLAs are typically employed when the buyer neither wants or is not able to micromanage the supplier and has no interest in how the product or service is delivered; but is interested only in the outcome.

SLAs are often used when the parties involved have a long-term relationship, where the transactions are not one time. Since a fixed price alone is not enough to guarantee the delivery of the required performance, positive and/or negative performance incentives are needed. For example, a penalty might be imposed when the supplier underperforms compared to some target service level. The penalty is not based on daily transactions but performance over a period of time; reducing the administrative costs of enforcement and freeing the buyer to concentrate on their core business.

Despite the widespread use of SLAs in practice, there is little theoretical research on their design. In Chapter 2 studying an application to inventory management, we identified five fundamental issues of SLA design that need to be answered: what performance measure should be used, what performance target is appropriate, how frequently the performance should be reviewed, how much deviation of the performance is allowed from the target, and what penalty the supplier should pay when the performance exceeds the allowable deviation. The performance measure should align the supplier's incentive with that of the buying firm. The allowable deviation and the penalty together determine how strong the incentive is for the supplier. We mainly addressed within the framework of a simple inventory management problem where the supplier's decision variable is the base-stock level alone. In this chapter, we concentrate on the first two questions and briefly address the other three, within a more complex problem. The natural framework for the study of SLAs is from a principal-agent perspective, and that is the perspective we take.

Specifically, we study a single-item inventory system with a continuous-review base-stock policy, stochastic and stationary demand, and full backlogging. We consider a supply chain consisting of a single supplier and a single buyer, where the supplier can invest both in inventory and in inventory replenishment lead time to meet a service level target, and both investments are unobservable to the buyer. The supplier owns the inventory and incurs a linear inventory holding cost. For each delayed delivery, the buyer incurs a cost which is a convex and increasing function of the amount of delay. An SLA uses a multi-period review strategy, under which the supplier's inventory performance is reviewed every R periods (called a review phase), and if it is below a pre-specified performance threshold, then the supplier will pay a penalty linear in the amount of performance deviation from the threshold.

Fill rate and stockout rate are commonly used in both the practice and the literature for measuring delivery and inventory performance. Most inventory management literature studies performance measures in the long run using expected performance. But the performance measure in a finite review phase is a random variable. When the supplier's actions are unobservable, it is important to know the distribution of the performance measure in order to provide an incentive to the supplier. It is very difficult to derive the distribution of fill rate. Interested readers can refer to Thomas (2005) for its distribution obtained using simulation in a static periodic-review base-stock model with zero lead time. For the same reason as in Chapter 2, we focus mainly on the ready rate, which is the long-run fraction of time that demands are filled immediately from the stock. It measures inventory availability, and is equal to $1-$ stockout rate. The conventional ready rate and fill rate are measures of the off-the-shelf or immediate order fulfillment performance. In practice, time-window fulfillment rates are more commonly used than off-the-shelf performance measures (LaLonde and Zinszer 1976, LaLonde et al. 1988). In Quick Response and other forms of time-based competition, the performance measure for customer service is often the ability to meet delivery promises, where the promised time window is usually small. For example, around 1995, Hewlett-Packard aimed at a 93% fulfillment rate within 3 days, and IBM PC and Compaq 95% within 5 days (Hausman et al. 1998). When the performance measure is based on on-time delivery by a supplier to a buyer, time-window fulfillment rates are also used. Therefore, we also study another form of ready rate, the ready rate with a window, which is the long-run fraction of time that demands are filled within a pre-specified time window. We study the ready rate for simplicity in exposition, but similar insights can be obtained when either the immediate or time-window fill rate is used as the performance measure.

Because the supplier is responsible for investments in both inventory and lead time and is evaluated by a single performance measure — the ready rate, our problem is a multi-task agency problem with a single output. Since both increasing the stock level and reducing the replenishment lead time can achieve a better performance, the supplier's two tasks are substitutes.

The objective of this chapter is twofold. First, we address the design of SLAs in supply management, including choosing the performance measure, determining the performance target, allowable deviation and penalties for underperformance. Specifically, we examine two types of SLAs using either the immediate or time-window ready rates as performance measures. Second, we compare these two forms of SLAs in terms of the average supply chain cost. We show that when the supplier employs a static inventory policy, can invest both in inventory level and in supply lead time, with the investments unobservable to the buyer, an SLA using the time-window ready rate can induce the supplier to make the investments compatible with overall supply chain optimization. An SLA using only the immediate ready rate generally cannot induce this first-best investment. We also discuss the issue of using a single performance measure for aligning the supplier's incentive when the supplier has multiple ways to achieve inventory performance. The time window in the performance measures plays three roles. It aligns the supplier's tradeoff between inventory and lead time investments with that of the supply chain, allocates inventory risk between the buyer and a supplier, and to some extent transfers the buyer's delay cost structure to the supplier.

The rest of this chapter is organized as follows. Section 3.2 reviews the literature. Section 3.3 describes the model and provides mathematical expressions for the waiting time distribution. Section 3.4 examines SLAs using the immediate or time-window ready rate as a performance measure and discusses the issue of incentive alignment using a single performance measure. Numerical analyses are in Section 3.5. We conclude in Section 3.6.

3.2 Literature Review

This chapter relates to three primary literatures: agency theory, inventory management, and supply chain contracting and coordination.

Multiple-period review strategies have been studied in the economics literature by Radner (1985) for a repeated principal-agent game. Ren et al. (2008) investigate a modified strategy for an information-sharing game between a buyer and a supplier in a supply chain context. Both papers use trigger strategies as punishments for noncooperation. Details of these approaches can be found in Chapter 2, which studies a multi-period review strategy in a service level agreement for inventory management. The review strategy differs from those in Radner (1985) and Ren et al. (2008) in two major aspects: the length of a review phase R is exogenous and the penalty is a monetary payment instead of a phase of noncooperative game. Moreover, the performance outcome of the agent (supplier) in each period can be correlated. The current chapter studies a multi-task moral hazard problem, whereas both Radner (1985) and Chapter 2 study single-task moral hazard problems and Ren et al. (2008) a hidden information one.

The immediate fulfillment rate is commonly employed in the inventory management literature, however in practice, time-window fulfillment rates are more common. Boyaci and Gallego (2001) study the problem of minimizing average inventory costs subject to fill-rate and fill-rate-with-window service-level constraints in serial and assembly systems. In an (s, S) inventory system with service level target represented by a time-window ready rate, Wang et al. (2005) find a significant tradeoff between the window length and the inventory costs, and suggests that a longer fulfillment window and lower price may be used for price-sensitive but time-insensitive customers. The above papers study inventory management from a single agent perspective, and do not deal with incentive issues in a decentralized system. Our concern is the use of either the immediate or time-window ready rate as a performance measure to induce investments by an independent supplier.

The inventory management literature generally considers performance measures in the long run using expected performance. In practice, a supplier's delivery performance will be evaluated over a finite period of time. At the end of a finite review phase R, the buyer, unable to observe the supplier's effort, observes a single noisy performance signal. How is the buyer to unscramble poor effort from 'bad' random effects? The buyer needs to know how the supplier's efforts affect the random distribution of performance. Thomas (2008) uses simulation to investigate the distribution of fill rate in a static periodic-review base-stock model with zero lead time and Erlang demand. Chapter 2 provides a theoretical approximation for the distribution of the review-phase immediate/time-window ready rate under a static periodic-review base-stock policy with general demand and discrete lead time. In this chapter, we employ a similar result under a continuous-review base-stock policy and continuous lead time.

Choi et al. (2004) investigate choosing supplier performance measures in a vendor-managed-inventory context. The production of the supplier and the manufacturer are capacitated. The supplier holds inventory, and her capacity and inventory policy are private information. So the buyer chooses performance measures for the supplier. Choi et al. study both the ready rate and the fill rate, and demonstrate that in a capacitated supply chain, the supplier's service level is in general not sufficient to guarantee the manufacturer's target customer service level. They propose a menu of contracts with different combinations of the ready rate and expected backorders. In our model, the buyer incurs a cost for each delayed delivery, and determines the service level target for the supplier. Moreover, we do not consider production by the buyer, and the supplier's supply is uncapacitated. Although the supplier's actions are unobservable in their model, Choi et al. only study immediate fulfillment rates in the long run, and focus on the choice of performance measures, ignoring the variability in the observed performance measures and the penalty for failing to meet a target. We study both the immediate and time-window ready rates, and compare the efficiency of each performance measure at aligning the supplier's incentive.

In all the aforementioned studies, the target fill rate or ready rate and the time window are assumed to be given. We allow both the time window and the target ready rate to be endogenous.

Our study is also related to incentive contracting on inventory management. In this stream of literature, principal-agent theory is applied to inventory management in decentralized systems. Bolton and Dewatripont (2005) provide a broad coverage of literature on incentive contracts. Literature on moral hazard problems in agency contracting can be found therein. Corbett (2001) studies the allocation of decision rights between a single buyer and a single supplier in an order-quantity/reorder-point (Q, r) inventory system with stochastic and stationary demand and backlogging. The delivery lead time between the two parties is constant. Consignment stock is studied, where the supplier holds inventory at the buyer's site and bears the holding cost until the goods are sold to the final customer. Corbett considers two situations: one in which the buyer is the principal and the supplier has private information about her setup cost; another in which the supplier is the principal and the buyer has private information about backorder costs. In our model, the supplier carries inventory and incurs the holding costs, and there is information asymmetry on the supplier's base-stock level and lead time.

Lutze and Özer (2008) examine promised lead time contracts offered by a supplier to a buyer, under which the buyer places orders in advance and the supplier guarantees the shipment of full order on time after a promised lead time. Both the supplier and the buyer hold inventory. They investigate how a promised lead time contract can be used to share inventory risk between a buyer and a supplier. Our model demonstrates that both the service level agreement structure and the window in the inventory performance measure allow the two parties to share inventory risk. Lutze and Özer study an adverse selection problem, where the buyer has private information about his shortage cost, but there is no uncertainty in the supplier's performance to meet the promised delivery lead time. We study a moral hazard problem, in which only the supplier holds inventory, the buyer offers the contract, and the supplier's performance is a random variable.

Kim et al. (2007) study performance-based contracting between a single buyer and multiple suppliers in after-sales service supply chains. The buyer is a customer of assembled systems, where each system consists of some distinct parts. Each type of spare part is stocked by a different supplier. If any of the parts fails, the system is down, and that part has to be replaced by a spare part. Failed parts are repaired and then returned to the spare part stock. When there is no spare part available, a backorder occurs. Each supplier determines her stock level of spare parts, which is unobservable to the customer. Facing a system uptime requirement, the customer offers contracts to the suppliers. Because the uptime requirement is equivalent to a system backorder target, the authors propose a contract linear in the backorders of each part, and show that it induces the first-best solutions when all parties are risk neutral. In our problem, the contract is based on the supplier's aggregate delivery performance — the ready rate, not on individual backorders.

3.3 Model and Preliminaries

Consider a supply chain consisting of a single supplier (she) and a single buyer (he) with the supplier producing a single product for the buyer. The supplier makes to stock and the buyer makes to order. Without loss of generality, we assume that the supplier's unit ordering and processing costs are zero. The supplier holds inventory at a unit cost of h per period of time, and replenishes her inventory from an unlimited supply source at a constant lead time L using an order-up-to-S inventory policy. Assume the supplier can invest in the replenishment lead time with cost $C_r(L)$ for lead time L. The lead time between the buyer and the supplier is taken as zero, representing a situation where the supplier holds inventory at a site near the buyer, such as a vendor-managed-inventory (VMI) program. Customer demands are stochastic and stationary. Demands in each period of time are independently and identically distributed (i.i.d.) with mean λ and standard deviation σ. The buyer incurs a cost of waiting if a demand for the product cannot be filled immediately. The buyer's processing time is negligible and is assumed to be zero.

Both the buyer and the supplier are risk neutral. Assume the distribution of the demand and the supplier's inventory holding and lead time costs are common information.

In order to induce the supplier to invest in inventory and lead time, the buyer contracts with the supplier on the supplier's inventory service level. The service level agreement uses a multi-period review strategy, under which the supplier's delivery performance is evaluated every R periods (a review phase). As the review period progresses the supplier has an incentive to dynamically (state-dependent) change her stock level S depending on her performance to date. In the earlier chapter, we investigated this issue and concluded that such strategic behavior was mitigated with the choice of penalties proportional to the deviation and that the supplier gains little from adopting them given a small allowable deviation from the target. Such dynamic inventory policies will also have higher implementation costs than a static one. Therefore, to allow us to focus on other aspects of SLA design we assume throughout this chapter that only such linear penalties are used and that the supplier uses a static inventory policy.

The following notation is used throughout this chapter.

Supplier's decision variables:

S : base-stock level

L : inventory replenishment lead time

Buyer's decision variables:

W : time window

A_W : supplier's expected ready rate (with window W)

p : unit transfer price

α : performance threshold for A

K : penalty rate — penalty paid by the supplier to the buyer per 1% below α

Other:

λ : demand rate

σ : standard deviation of demand per period

R : length of a review phase, assumed to be large compared with likely lead times L

A : supplier's realized ready rate (with window) in a review phase

$C_r(L)$: cost of attaining lead time L for each unit of demand

$C_D(y)$: buyer's cost of delay per unit demand if the demand is filled after y periods

$C(S, L)$: average supply chain cost if S and L are chosen

$C_B(S, L)$: buyer's average cost if the supplier chooses S and L

$\pi(S, L)$: supplier's average profit if she chooses S and L

$\overline{I}(S, L)$: average inventory level if S and L are chosen

$D(t)$: demand in t periods

$D(t, u]$: demand in the interval $(t, u]$.

Because working with discrete-valued demand and decision variables S and L makes our analysis much more complex, and our purpose is to gain insights in incentive contracting, we use continuous demand, S and L as an approximation in the analysis. We assume that the supplier uses a continuous-review inventory policy.

Let the cumulative distribution function (cdf) and probability density function (pdf) of $D(t)$ be denoted by $F(x|t)$ and $f(x|t)$, respectively. $F(x|t) = \Pr\{D(t) \leq x\}$. The average inventory level given base-stock level S and lead time L is

$$\overline{I}(S, L) = E([S - D(L)]^+) = \int_{x<S} (S - x) f(x|L) dx = S - \int_{x<S} \overline{F}(x|L) dx, \qquad (3.1)$$

where $\overline{F}(x|L) = 1 - F(x|L)$. Let $F_w(y|S, L)$ and $f_w(y|S, L)$ denote the cdf and pdf of the waiting time w given S and L, respectively. The distribution of the waiting time (see Appendix B for derivation) is

$$F_w(y|S, L) = \begin{cases} 0 & \text{for } y < 0 \\ \Pr\{D(L - y) \leq S\} & \text{for } y \in [0, L] \\ 1 & \text{for } y > L. \end{cases} \qquad (3.2)$$

It follows from (3.2) that

$$f_w(y|S, L) = \begin{cases} \Pr\{D(L) \leq S\} & \text{for } y = 0 \\ \frac{dF_w(y|S,L)}{dy} & \text{for } y \in (0, L) \\ 0 & \text{othewise.} \end{cases}$$

Assume the delay cost function $C_D(\cdot)$ and lead time cost function $C_r(\cdot)$ are continuous and differentiable; $C_D'(\cdot) > 0$, $C_D''(\cdot) \geq 0$, $C_D(0) = 0$; $C_r'(\cdot) < 0$, $C_r''(\cdot) > 0$, $\lim_{L\to 0} C_r(L) = \infty$, and $\lim_{L\to \widetilde{L}} C_r'(L) = 0$, where $0 < \widetilde{L} \leq \infty$. Without loss of generality, we assume that at time 0, the inventory is S, the base-stock level. We consider the situation where the supply chain optimal (first-best) base-stock level $S^* > 0$ and lead time $L^* < \widetilde{L}$, i.e., it is optimal for the supply chain to invest in both inventory and lead time.

3.4 Optimal Ready-Rate Contract

A typical SLA will be of the following form, somewhat abbreviated for simplicity.

The buyer will pay $25 for each part. The target is to ensure that parts are available 95% of the time. Every 90 days (one quarter), a review will determine the % of time that parts are available within 1 day, and if this figure falls beneath 93% a penalty of $200 per 1% below 93% will be deducted from the buyer's invoice.

The buyer makes a service level agreement with the supplier under a long-term relationship of the form $(R, W, A_W, p, \alpha, K)$. The supplier's inventory performance is reviewed every R periods (90 days), which constitute a review phase. The performance measure is the ready rate A with window W (1 day), $A \in [0, 1]$. The distribution of A will be discussed below. The target service level is A_W (95%). A transfer price p ($25) is paid for each unit of demand. If the supplier's performance falls below α (93%), then the supplier is charged with a linear penalty proportional to the difference between the actual performance and α (K ($200) per 1% below α). The buyer chooses the SLA to minimize his long-run average cost, including the payment to the supplier, the expected order delay cost, minus the penalty paid by the supplier. Given the SLA offered, the supplier chooses S and L to maximize her long-run average profit.

In practice, a reasonable performance threshold α should be below the performance target A_W. So to provide an incentive to the supplier, the candidate K and α must be such that $K > 0$ and $\alpha \in (0, A_W)$.

We assume R is exogenous because in practice R can be determined by other factors such as transaction costs and the accounting policy. Katok et al. (2008) use experimental methods to examine the effect of review periods in a finite-horizon periodic-review base-stock inventory model. The inventory replenishment lead time is zero. They find that longer review periods may be more effective than shorter ones at inducing service improvements. In our model, the supplier's performance is reviewed every R periods repeatedly. The supplier makes investment to maximize her long-run average profit over repeated review periods. The supplier chooses the lead time once, which is unchanged over time; there is no emergency expediting.

3.4.1 Performance measure

The immediate ready rate and time-window ready rate are two common measures for inventory performance in practice. The immediate ready rate is the fraction of time that demands are filled immediately; the time-window ready rate is the fraction of time that demands are filled within a time window. When demand arrivals see time averages (e.g., Poisson process), the ready rate (with window) is equal to the fill rate (with window), the fraction of demands that are filled immediately (within a time window). We study the ready rate for ease of exposition, but the major findings still hold if the fill rate is used as performance measure.

Let $A_W = A_W(S, L)$ denote the supplier's expected ready rate with window W if she chooses the base-stock level S and lead time L. Note that A_0 is the supplier's expected immediate ready rate. Using the waiting time distribution in (3.2), we obtain

$$A_W = \Pr\{w \leq W|S, L\} = \Pr\{D(L - W) \leq S\} = F_w(W|S, L) \quad W \in [0, L]. \quad (3.3)$$

We assume that the supplier's delivery performance is evaluated at the end of every period. Note that the review-phase ready rate with window W is the proportion of periods in the review phase that at the end of the period no demand is delayed longer than time W. Denote the distribution of the review-phase ready rate $A \in [0, 1]$ with the mean A_W by $\Psi(A|A_W)$. $\Psi(A|A_W) \in [0, 1]$, $\Psi(1|A_W) = 1$ and $\Psi'(A|A_W) > 0$. Let its pdf be denoted by $\psi(A|A_W)$.

Proposition 3.1 *Under a static continuous-review base-stock policy with base-stock level S and lead time L, $\frac{A-A_W}{\sigma_W}$ converges in distribution to a standard normal random variable as R approaches ∞, where*

$$\sigma_W^2 = \frac{1}{R^2}(RA_W - R^2 A_W^2 + 2\sum_{i<j} P_{ij}(S, L, W)), \quad (3.4)$$

and $P_{ij}(S, L, W) = \Pr\{D(i - L, i - W] \leq S, D(j - L, j - W] \leq S\}$ $(1 \leq i, j \leq R)$ is the probability that both performance outcomes in periods i and j are good.

Note that A is the supplier's realized ready rate with window W ($W \geq 0$) in a review phase. So when the review phase is sufficiently long, A is approximately normally distributed with mean A_W and standard deviation σ_W, and $\Psi(A|A_W) = \Phi(\frac{A-A_W}{\sigma_W})$ and $\psi(A|A_W) = \frac{1}{\sigma_W}\phi(\frac{A-A_W}{\sigma_W})$, where $\Phi(\cdot)$ and $\phi(\cdot)$ are the cdf and pdf of the standard normal distribution. It can be shown that σ_W is decreasing in R, meaning that the supplier's performance measure is more accurate with a longer review phase.

3.4.2 First-best solution

Using the result in Proposition 3.1, the supplier's expected average penalty under the SLA is

$$\frac{K}{R}\frac{\int_{A<\alpha}(\alpha-A)d\Psi(A|A_W)}{1\%} = \frac{100K}{R}\int_{A<\alpha}\Psi(A|A_W)dA.$$

The buyer's expected average cost and the supplier's expected average profit given the SLA are

$$EC_B(S,L) = p\lambda - \frac{100K}{R}\int_{A<\alpha}\Psi(A|A_W)dA + \lambda EC_D(y|S,L) \tag{3.5}$$

and

$$E\pi(S,L) = p\lambda - \frac{100K}{R}\int_{A<\alpha}\Psi(A|A_W)dA - h\overline{I}(S,L) - \lambda C_r(L), \tag{3.6}$$

respectively.

If the supplier's choice of S and L are observable and verifiable, then the optimal (first-best) contract is the solution to the following problem:

$$\begin{aligned} &\min_{W,p,\alpha,K,S,L} EC_B(S,L) \\ &\text{subject to } E\pi(S,L) \geq \overline{\pi} \end{aligned} \tag{3.7}$$

where $E\pi(S,L)$ is given by (3.6), A_W by (3.3), and $\overline{\pi}$ is the supplier's reservation profit per period. Constraint (3.7) is called the individual-rationality (IR) constraint in agency theory. It ensures that the supplier will expect to earn from this contract at least her reservation profit, and thus accepts the contract.

The expected total average cost of the supply chain is

$$EC(S,L) = h\overline{I}(S,L) + \lambda EC_D(y|S,L) + \lambda C_r(L). \tag{3.8}$$

Assume $EC(S,L)$ is jointly unimodal in S and L.[4] Proposition 3.2 characterizes the optimal solutions of the supply chain and the buyer.

[4]The property of $EC(S,L)$ depends on the demand distribution, delay cost $C_D(y|S,L)$ and lead time cost $C_r(L)$. So it is generally difficult to prove unimodality of $EC(S,L)$ in S and L. But for normal demand, linear delay cost and $C_r(L)$ with certain property, $EC(S,L)$ can be shown to be unimodal. See the Appendix for details.

Proposition 3.2 *(S^*, L^*) is the solution to*

$$hF(S|L) + \lambda \frac{\partial EC_D(y|S,L)}{\partial S} = 0 \tag{3.9}$$

$$-h\int_{x<S} \frac{\partial \overline{F}(x|L)}{\partial L} dx + \lambda \frac{\partial EC_D(y|S,L)}{\partial L} + \lambda C_r'(L) = 0, \tag{3.10}$$

and it is the buyer's optimal solution when S and L are observable and verifiable.

The corresponding optimal expected average supply chain cost is $EC(S^*, L^*)$.

Next we investigate the optimal SLA using either a time-window ready rate or an immediate ready rate as the performance measure when the supplier's choice of S and L are unobservable. We call the SLA using the time-window ready rate, a ready-rate-with-window contract and the SLA using the immediate ready rate, a ready-rate-without-window contract.

3.4.3 Ready-rate-with-window contract

When the supplier's choice of S and L are unobservable, the supplier's optimal choice is contingent on the contract parameters. The buyer's optimization problem is

$$\begin{aligned} &\min_{W,p,\alpha,K,S,L} EC_B(S,L) \\ &\text{subject to } (IR) : E\pi(S,L) \geq \overline{\pi} \\ &\qquad (IC) : (S,L) \in \arg\max_{\widehat{s},\widehat{L}} E\pi(\widehat{S},\widehat{L}) \end{aligned} \tag{3.11}$$

We make the following three assumptions throughout this chapter.

Assumption 3.1 $\frac{\partial F_w(y|S,L)}{\partial S} / \frac{\partial F_w(y|S,L)}{\partial L}$ *is strictly monotonic in $y \in [0, L]$.*

Many probability distributions such as those in the location-scale family and Poisson distribution satisfy this assumption.

Assumption 3.2 *For any $S, L > 0$ such that $A_0 > 0.5$,* $\frac{\partial(\int_{A<\alpha} \Psi(A|A_W)dA)/\partial S}{\partial(\int_{A<\alpha} \Psi(A|A_W)dA)/\partial L} = \frac{\partial A_W/\partial S}{\partial A_W/\partial L}$ *for any $W \in [0, L]$.*

Note that A_0 is the expected immediate ready rate, and is usually greater than 50% in practice. This means that when the performance measure is the ready rate with window W, the ratio of the marginal changes in the supplier's expected penalty with respect to S and L is the same as that of the performance target. For normal distribution, this assumption is approximately satisfied. Theoretical validation is not easy, but the validation can be done through numerical examples (see Appendix B).

Assumption 3.3 *The first-order conditions for the supplier's optimization problem given the contract offered by the buyer are sufficient.*

This allows us to replace the (IC) constraint (3.11) by the first-order conditions for the supplier's problem. Even for fixed L and no lead time cost it is difficult to prove the unimodality of the supplier's profit function $E\pi(S, L)$ (see Chapter 2). So we have to rely on numerical results to check unimodality. Now we present the main result of this chapter.

Theorem 3.1 *Assume that the optimal solution for the problem of the supply chain is interior, then there exists a unique optimal time window $W^* \in (0, L^*)$ that induces the first-best effort levels.*

Theorem 3.1 implies that as long as the relative change in the supplier's expected penalty with respect to S and L is the same as that in the expected performance, then the buyer can always find a time window W^* and use the ready rate with window W^* as the performance measure to coordinate the supply chain.

Let $A_W^* = F(S^*|L^* - W^*)$. Corollary 3.1 follows from the proof of Theorem 3.1.

Corollary 3.1 *W^* is such that*

$$\frac{\partial EC_D(y|S^*, L^*)/\partial S}{\partial EC_D(y|S^*, L^*)/\partial L} = \frac{\partial A_W^*/\partial S}{\partial A_W^*/\partial L}. \tag{3.12}$$

To understand the role of W^* in the optimal SLA, note that $\frac{\partial EC_D(y|S,L)/\partial S}{\partial EC_D(y|S,L)/\partial L}$ is the marginal rate of technical substitution (MRTS) for the expected delay cost in economics theory, $\frac{\partial A_W}{\partial S}$ and $\frac{\partial A_W}{\partial L}$ are the marginal change in the expected performance A_W with respect to the base-stock level S and lead time L, respectively. So in the service level agreement here, the role of the optimal window W^* is to set a right performance measure (and target) so that the optimal MRTS in the integrated system is transferred to the supplier, and the supplier's investments in inventory and lead time are perfectly balanced.

Because Theorem 3.1 implies that the ready rate with window W^* is the unique ready rate that induces the first-best effort levels and $W^* > 0$, we can conclude that the immediate ready rate is suboptimal. Corollary 3.2 follows.

Corollary 3.2 *The SLA using the immediate ready rate as performance measure cannot induce the first-best effort levels.*

So in general, a service level agreement using the immediate ready rate as the performance measure is suboptimal.

Proposition 3.3 *The first-best ready-rate contract is such that:*

1) W^* is characterized by (3.12), where (S^*, L^*) is the first-best solution determined by (3.9) and (3.10);

2) $A_W^* = F(S^*|L^* - W^*)$;

3) there are multiple choices of (K^*, α^*) such that $K^* > 0$, $\alpha^* \in (0, A_W^*)$ and satisfy

$$K^* = -\frac{RhF(S^*|L^*)}{100\int_{A<\alpha^*} \frac{\partial \Psi(A|A_W^*)}{\partial S} dA} \text{ and } \int_{A<\alpha^*} \frac{\partial \Psi(A|A_W^*)}{\partial S} dA < 0;$$

4) $p^* = \frac{\overline{\pi}}{\lambda} + \frac{100K^*}{\lambda R} \int_{A<\alpha^*} \Psi(A|A_W^*) dA + \frac{h}{\lambda}\overline{I}(S^*, L^*) + C_r(L^*).$

Proposition 3.3 implies that in practice, managers have many choices for the threshold performance level α and the penalty rate K. Note that the threshold performance level determines the allowable deviation of the supplier's performance from the target. As noted in Section 3.3, in Chapter 2 we had found it best that α should be close to the performance target to mitigate the supplier's strategic behavior. So although the choice of the optimal α is not unique, it should not be too far from the target.

Proposition 3.4 *If the demand distribution is normal, then for fixed $\frac{\sigma}{\lambda}$, L^*, W^* and A_W^* are independent of λ, and S^* is proportional to λ.*

Proposition 3.4 implies that for normal demand and the same coefficient of variation, the first-best inventory replenishment lead time, the optimal window and the performance target in the optimal ready-rate-with-window contract are identical, and the first-best base-stock level is proportional to the demand rate. This has implications for implementing an SLA. If the demand rate changes, as long as the coefficient of variation of demand remains the same, the optimal window W^* and the target ready rate A_W^* in the SLA need not be changed.

Next, we have a result for the first-best immediate ready rate A_0^* for general demand distributions and linear delay costs under the optimal ready-rate-with-window contract. A linear delay cost means $C_D(y) = \delta y$, where $\delta > 0$.

Proposition 3.5 *If the delay cost is linear, then* $A_0^* = \frac{\delta}{h+\delta}$.

Proposition 3.5 indicates that for a linear delay cost, even if the supplier can invest to attain a different lead time, it will not affect the first-best immediate ready rate A_0^*. A_0^* is only determined by the holding cost to delay cost ratio h/δ.

3.4.4 Ready-rate-without-window contract

A ready-rate-without-window contract has the same interpretation as a ready-rate-with-window contract except that the supplier's delivery performance is measured in terms of the immediate ready rate, i.e., $W = 0$. So the buyer's optimization problem is similar to that under a ready-rate-with-window contract with $W = 0$, and the optimal solution is provided in Proposition 3.6.

Proposition 3.6 *Under a ready-rate-without-window contract,*

1) the optimal (S^{**}, L^{**}) *that the buyer can induce is the solution to the constrained optimization problem:*

$$\min_{p,\alpha,K,S,L} EC(S,L) \tag{3.13}$$

$$\text{subject to } hF(S|L)\frac{\partial A_0/\partial L}{\partial A_0/\partial S} + h\int_{x<S}\frac{\partial \overline{F}(x|L)}{\partial L}dx - \lambda C_r'(L) = 0; \tag{3.14}$$

and

2) the optimal ready-rate-without-window contract is such that:

the optimal immediate ready rate $A_0^{**} = F(S^{**}|L^{**})$;

there are multiple choices of (K^{**}, α^{**}) *such that* $K^{**} > 0$, $\alpha^{**} \in (0, A_0^{**})$ *and satisfy*

$$K^{**} = -\frac{RhF(S^{**}|L^{**})}{100\int_{A<\alpha^{**}} \frac{\partial\Psi(A|A_0^{**})}{\partial S} dA} \text{ and } \int_{A<\alpha^{**}} \frac{\partial\Psi(A|A_0^{**})}{\partial S} dA < 0;$$

and

$$p^{**} = \frac{\pi}{\lambda} + \frac{100K^{**}}{\lambda R}\int_{A<\alpha^{**}} \Psi(A|A_0^{**})dA + \frac{h}{\lambda}\overline{I}(S^{**}, L^{**}) + C_r(L^{**}).$$

Proposition 3.7 is a counterpart of Proposition 3.4 under a ready-rate-without-window contract.

Proposition 3.7 *If the demand distribution is normal, then for fixed $\frac{\sigma}{\lambda}$, L^{**} and A_0^{**} are independent of λ, and S^{**} is proportional to λ.*

Similar to the result in Proposition 3.4, Proposition 3.7 implies that when the demand rate changes, as long as the coefficient of variation is not changed, the performance target in the optimal ready-rate-without-window contract is still optimal. Both Propositions 3.4 and 3.7 imply that the optimal performance target in a ready-rate contract is dependent on the coefficient of variation of demand. Corollary 3.3 follows from Propositions 3.4 and 3.7 and the fact that $\frac{EC(S,L)}{\lambda}$ is independent of λ for fixed $\frac{\sigma}{\lambda}$.

Corollary 3.3 *For normal demand, the system loss – the % increase in the average supply chain cost from not using a window in the contract is identical when $\frac{\sigma}{\lambda}$ is constant.*

The results in Propositions 3.4 & 3.7 and Corollary 3.3 will be useful for conducting numerical analysis in Section 3.5.

3.4.5 Incentive alignment using an inventory performance measure

In the supplier's expected average penalty, let $\Pi(S, L, W) = \int_{A<\alpha} \Psi(A|A_W)dA$, then $\frac{\partial\Pi(S,L,W)/\partial S}{\partial\Pi(S,L,W)/\partial L} = \theta\frac{\partial A_W/\partial S}{\partial A_W/\partial L}$, where

$$\theta = \frac{1 - \frac{\partial\sigma_W/\partial S}{\partial A_W/\partial S}\phi(\frac{\alpha-A_W}{\sigma_W})/\Phi(\frac{\alpha-A_W}{\sigma_W})}{1 - \frac{\partial\sigma_W/\partial L}{\partial A_W/\partial L}\phi(\frac{\alpha-A_W}{\sigma_W})/\Phi(\frac{\alpha-A_W}{\sigma_W})}. \tag{3.15}$$

The derivation of θ is from (B.4) in Appendix B. So Assumption 3.2 holds if and only if $\frac{\partial\sigma_W/\partial S}{\partial A_W/\partial S} = \frac{\partial\sigma_W/\partial L}{\partial A_W/\partial L}$ for any $S, L > 0$ with $A_0 > 0.5$. From Proposition 3.1, if the inventory replenishment lead time $L \leq 1$, then the performance outcomes in any two periods i and j are independent, and RA, the number of periods in a review phase that have good performance (no demand is delayed longer than the window W), has a binomial distribution with independent outcomes and $\widehat{\sigma}_W^2 = \frac{A_W(1-A_W)}{R}$, so $\frac{\partial\sigma_W/\partial S}{\partial A_W/\partial S} = \frac{\partial\sigma_W/\partial L}{\partial A_W/\partial L}$ holds. For $L > 1$, if any two periods i and j differ by less than L periods, then the performance outcomes in these two periods are positively correlated. When changing the base-stock level S and lead time L, the relative change in the performance variability σ_W due to S and L may be different from that in the performance target A_W, i.e., $\frac{\partial\sigma_W/\partial S}{\partial A_W/\partial S} = \frac{\partial\sigma_W/\partial L}{\partial A_W/\partial L}$ may not hold. Other order fulfillment rates for measuring inventory performance such as the fill rate have similar properties for positive lead times. From the proof for Theorem 3.1, for a demand distribution with $\frac{\partial\sigma_W/\partial S}{\partial A_W/\partial S} \neq \frac{\partial\sigma_W/\partial L}{\partial A_W/\partial L}$, there may not exist a $W \in [0, L]$ such that a ready rate with window W induces the supplier to make the first-best investment (S^*, L^*), or the optimal time window W may not be unique. This implies that for inventory management in a decentralized system, to effectively align a supplier's incentive with the buyer's when the supplier has multiple ways to perform, a single aggregate performance measure such as the ready rate may not be sufficient. This is due to the performance variability resulting from positive inventory replenishment lead time. If this is the case, then other performance measures are needed to complement the fulfillment rate. In Choi et al. (2004), a ready rate target for the supplier is not a sufficient guarantee due to the capacity constraints of both the buyer and the supplier, and they propose average backorders as a second performance measure. But in a finite-horizon, this measure is also a random variable, and its variability also needs consideration when designing an SLA.

3.5 Numerical Analysis

We use numerical examples to illustrate how the first-best S^*, L^* and optimal contract parameters are affected by the demand rate λ, the variability of demand σ, and related costs including the inventory holding cost, lead time cost and delay cost. Moreover, because both the immediate and time-window ready rates are seen in practice, we compare the system performance (the optimal average supply chain cost) under both types of SLAs.

Assume the demand per period has a normal distribution. The lead time cost function is $C_r(L) = \frac{r}{L}$ $(r > 0)$. To simplify the notation, let $z = \frac{S-\lambda L}{\sigma\sqrt{L}}$. Following the results in Propositions 3.4 & 3.7 and Corollary 3.3, under the SLA using either type of ready rate as performance measure, the optimal lead time is independent of $\frac{\sigma}{\lambda}$ and the optimal base-stock level is proportional to λ; and the system loss from using the immediate ready rate instead of the time-window one is also independent of $\frac{\sigma}{\lambda}$. So we only need to examine the results for demands with different coefficient of variation of demand by varying σ with λ fixed.

In the inventory management literature, the delay cost is known as the backorder cost and is usually assumed to be linear in the waiting time. In reality, however, it may be nonlinear. For example, if the buyer provides after-sales services, customers may not mind waiting for one or two days to have their laptops, televisions or cars etc. fixed, in which case the supplier provides spare parts to the buyer; but the loss of customers' goodwill goes up quickly when their waiting time is beyond their tolerance. In the automobile industry, if a customer's preferred vehicle model is not immediately available at a car dealer, the customer is often willing to wait for a few days before receiving it; but if the customer has to wait longer, then she may leave and go to another dealer. Therefore, we consider two types of delay costs: linear delay cost with $C_D(y) = \delta y$, and convex delay cost with $C_D(y) = \delta y^2$, where $\delta > 0$.

To be consistent with our continuous approximation of the underlying model, we report the continuous-valued optimal solutions in the numerical examples. We use the following parameter values: $h \in \{1, 2\}, \delta \in \{2, 10, 20\}, r \in \{2, 4\}, \lambda = 10$, and $\sigma \in \{1, 2, ..., 5\}$. All the formulas for the calculations can be found in Appendix B.

3.5.1 Linear delay cost

Linear delay cost is a common assumption in the theoretical analysis. This represents the situation where the marginal cost of a delayed delivery does not vary with the amount of delay. Under linear delay cost,the optimal window W^* can be computed from the simple formula below:

$$W_L^* = L^* - \frac{S^*}{\lambda - 2\lambda C_r'(L^*)/h}. \tag{3.16}$$

To examine the optimal ready-rate-with-window contract, we compute the first-best (S^*, L^*) and W_L^* for the parameters given above. The results are plotted in Figures 3.1-3.3.

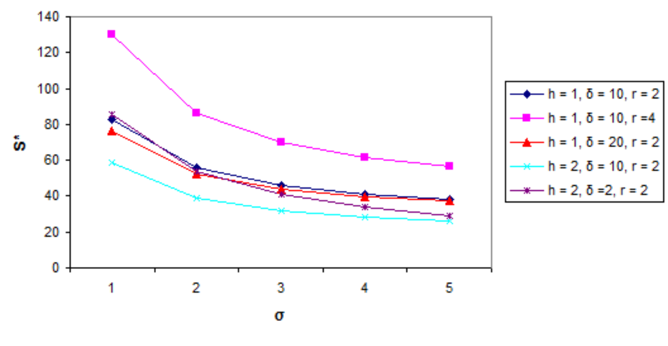

Figure 3.1: First-best base-stock level (linear delay cost)

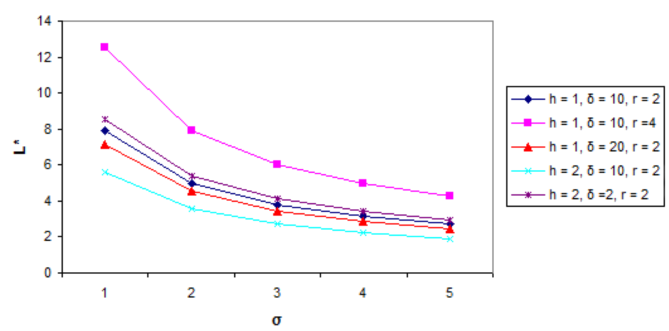

Figure 3.2: First-best lead time (linear delay cost)

Figures 3.1-3.3 indicate that the optimal window increases with the coefficient of variation. Because greater demand variability poses higher inventory risk on the supplier, we can interpret the optimal window as being used for sharing the inventory risk between the buyer and the supplier. For fixed demand rate, both the first-best base-stock level and lead time go down with the variability of demand (σ). Similar results are found for the optimal base-stock level and lead time under a ready-rate-without-window contract. Intuitively, with shorter lead time, the supply chain can respond more quickly to demand with large variability.

We also examine the performance of a ready-rate-without-window contract by comparing its optimal average supply chain cost C^{**} with the first-best one C^* and

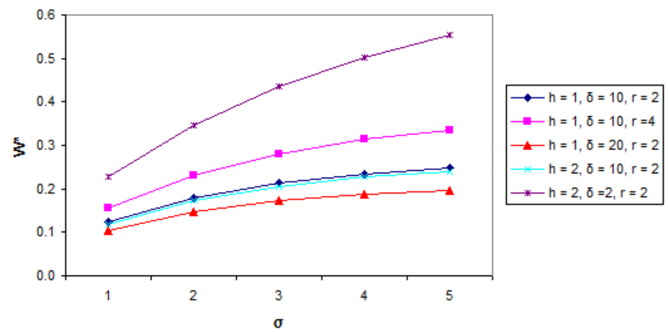

Figure 3.3: Optimal window (ready rate with window, linear delay)

Table 3.1: Cost increase from not using a window contract (linear delay cost)

Parameters	% Cost Incre
$h = 1, \delta = 10, r = 2$	0.69%
$h = 1, \delta = 10, r = 4$	0.69%
$h = 1, \delta = 20, r = 2$	0.44%
$h = 2, \delta = 10, r = 2$	1.12%
$h = 2,\ \delta = 2, r = 2$	3.65%

computing the percentage cost increase from not using a window contract. Table 3.1 demonstrates its performance for different demand variability and related costs.

It turns out that with other parameters fixed, the system loss — the percentage increase in the average supply chain cost, from using a ready-rate-without-window contract, is independent of the coefficient of variation of demand for the fixed demand rate. With other parameters fixed, the system loss is independent of the lead time cost, decreases with the delay cost, and increases with the holding cost. Proposition 3.5 has shown that for linear delay cost, the first-best expected immediate ready rate $A_0^* = \frac{\delta}{h+\delta} = \frac{1}{1+h/\delta}$. The numerical results indicate that for small h/δ ratio (high target service level), the system efficiency loss from using an immediate ready rate as performance measure is small. So when the holding cost h is small compared with the delay cost δ, an immediate ready rate will induce the system performance close to the optimal; if h is close to δ, then a time-window ready rate should be used.

3.5.2 Convex delay cost

In practice, the cost of delayed delivery to a buyer may not be linear in the amount of delay. The marginal cost of delay is increasing in the amount of delay, i.e., the delay cost is convex. Convex delay cost is often assumed for customers' value of service time in customer service models.

Similar to the analysis for linear delay cost, we examine the optimal ready-rate-with-window contract by computing the first-best solution (S^*, L^*) and the optimal window W_C^* for the parameters given above. The optimal window W^* can be computed from the formula

$$W_C^* = L^* - \frac{S^*\Phi(z^*)}{\lambda\Phi(z^*) - \frac{2\lambda}{h}C_r'(L^*) - \frac{\sigma}{\sqrt{L^*}}\phi(z^*)}, \tag{3.17}$$

where $z^* = \frac{S^* - \lambda L^*}{\sigma\sqrt{L^*}}$.

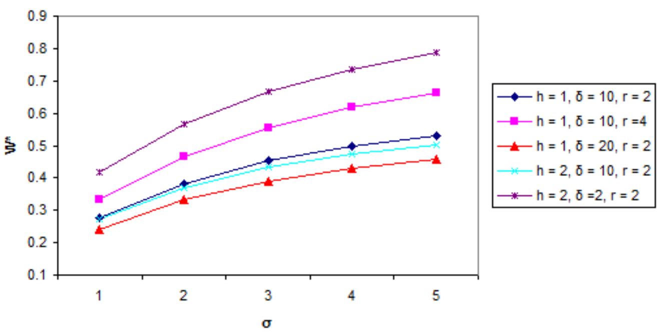

Figure 3.4: Optimal window (ready rate with window, convex delay)

Similar patterns are found for convex delay cost about the first-best base-stock level S^* and lead time L^*, the optimal window W_C^*, as well as the optimal base-stock level S^{**} and lead time L^{**} under a ready-rate-without-window contract as those in the case of linear delay cost, and similar conclusions can be made. The optimal window here can be regarded as being used for sharing the inventory risk between the buyer and the supplier. Compared with linear delay cost, convex delay cost is smaller for short delay and larger for long delay. So the optimal window also plays a role of partially transferring the buyer's delay cost structure to the supplier. This can be seen by comparing Figure 3.3 with Figure 3.4, where the optimal window under

Table 3.2: Cost increase from not using a window contract (convex delay cost)

	$h=1,\delta=10$	$h=1,\delta=10$	$h=1,\delta=20$	$h=2,\delta=10$	$h=2,\delta=2$
σ	$r=2$	$r=4$	$r=2$	$r=2$	$r=2$
1	3.78%	3.35%	2.75%	5.74%	10.46%
2	3.38%	2.95%	2.50%	5.17%	9.43%
3	3.35%	2.89%	2.52%	5.14%	9.16%
4	3.46%	2.94%	2.64%	5.30%	9.20%
5	3.63%	3.05%	2.79%	5.54%	9.39%

convex delay cost is generally larger than that under linear delay cost. For the system loss from not using a window contract, Table 3.2 shows that unlike that in the case of linear delay cost, it varies with the coefficient of variation of demand and is not monotonic. With other parameters fixed, the system loss decreases with the delay cost, but increases with the holding cost. Similar pattern has been found for linear delay cost. However, the effect of lead time cost on the system loss is different. Here the system loss decreases with the lead time cost. In all the scenarios, the system loss is much greater than that of linear delay cost.

From the above numerical results for linear and convex delay costs, we can see that a time-window fulfillment rate is preferred to an immediate one for measuring a supplier's delivery performance in two situations. One situation is when the buyer's cost of delay is not large compared with the inventory holding cost of the product. An example is perishable products such as expensive electronics, of which the inventory holding cost also includes the depreciation of the product. Another situation is when the marginal cost of a delay increases with the length of delay, that is, the delay cost is small for short delay but very large for long delay, often the case in reality. Linear delay cost is commonly used in theoretical analysis.

3.5.3 Effect of the length of review phase R

We first consider an SLA using the optimal time-window ready rate. We have assumed that both the buyer and the supplier optimize their long-run average payoff, so under a ready-rate-with-window contract, the optimal (S^*, L^*) and thus W^* (and A_W^*) are not affected by R. This can be seen from Propositions 3.2 and 3.3. Similarly, under an SLA using the optimal immediate ready rate, the optimal (S^{**}, L^{**}) and thus A_0^{**} are not affected by R, which follows from Proposition 3.6.

For both types of SLAs, let $m = \left|\frac{\alpha - A_W}{\sigma_W}\right|$, $W \geq 0$. m is the relative allowable deviation of the performance from the target. Because the variability of performance σ_W is decreasing in R and $\alpha < A_W$, for fixed m, the performance threshold α is increasing in R, i.e., the threshold should be closer to the target for large R.

3.6 Conclusions

In this chapter we examine the design of service level agreements in decentralized supply chains when a supplier has multiple ways to do things. Specifically, we study ready-rate contracts for managing a supplier's delivery performance from a principal-agent perspective, in which the supplier can invest both in inventory and in replenishment lead time, and the investments are unobservable to the buyer. Therefore, we study a multi-task moral hazard problem.

The SLA in our study is a linear-penalty scheme under a multi-period review strategy. We investigate two common measures of inventory performance observed in practice — the immediate ready rate and the time-window ready rate, and show that under a static continuous-review base-stock policy, the distribution of the supplier's review-phase ready rate is approximately normal. Because a single performance measure is used but the supplier has two choices — inventory and lead time — to affect the performance, we examine the effectiveness of immediate and time-window ready rates at aligning the supplier's incentive with that of the buyer. We also find that due to positive inventory replenishment lead time, the performance outcome in each period can be correlated. As a result, the relative change in the variance of the review-phase ready rate with respect to the base-stock level and lead time may be different from that of the performance target. In this case, a single performance measure such as the ready rate cannot align the supplier's incentive to make first-best investment in inventory and lead time, and additional performance measure is needed.

We propose a ready-rate-with-window contract, and show that under some mild conditions the SLA with a time-window ready rate as performance measure always induces the supplier to choose the first-best level of investment in inventory and lead time. For normal demand, the optimal window is identical for demands with the same coefficient of variation; and the greater the coefficient of variation in demand, the larger the optimal window.

We find that for linear cost of delayed delivery, a simpler form of ready rate contract, ready-rate-without-window contract, is near optimal. However, in the cases where the delay cost is convex, a ready-rate-without-window contract can result in high system loss — increase in the average supply chain cost; thus a ready-rate-with-window contract is preferred. The gain from using a ready-rate-with-window contract instead of a ready-rate-without-window one increases with the convexity of the delay cost. Therefore, a simple inclusion of window in performance measures can make a big difference.

The window used in the ready-rate contract plays three roles. First, it transfers the MRTS of the expected average delay cost to the supplier to balance the supplier's investment in inventory and lead time, making the supplier's tradeoff between inventory and lead-time investments same as that of the supply chain. Second, it facilitates the sharing of inventory risk (cost from excessive inventory and inventory shortage from stochastic demand) between a buyer and a supplier by allowing a small delay of delivery when evaluating the supplier's delivery performance. Moreover, the window to some extent transfers the buyer's delay cost structure to the supplier. This finding provides a theoretical support for the common use of time-window fulfillment rate in practice.

Chapter 4

Volume Incentive Through Performance-Based Allocation Of Demand

4.1 Introduction

Buyers of products or services commonly employ multiple suppliers rather than a single one. There are a large number of reasons why this might happen. For example in the case of some accident (e.g., supply disruption) the buyer does not want to lose his single source of product or service. Even without accidents the buyer does not want to be in a 'holdup' situation if switching to another supplier is not easy. In cases where the purchase amount is uncertain or the supply available is uncertain, then a backup or emergency source is needed. Finally of course there is the simple need often to keep supply price competition vigorous by maintaining a stable of suppliers. This chapter however assumes away all these reasons in order to concentrate on another widespread motivation. A great deal of innovation in the quality and performance of products and services is expected to be done by suppliers. The suppliers can make an effort (invest resources) to improve their performance, which benefits the buyer. Such investment is costly to the supplier and is often unobservable to the buyer. Through sourcing from multiple suppliers, the buyer can create competition between the suppliers and motivate them to invest for better performance. Dyer and Ouchi (1993) report that Japanese firms usually employ a 'two-vendor policy' to motivate suppliers to innovate and improve performance. An empirical study by Bensaou (1999) also shows that Japanese buyers typically split their purchases among multiple suppliers and then demand that the suppliers make specialized investments to obtain and keep their business.

In this chapter we assume away the risk of supply disruption by confining uncertainty only to the results of the investment, by all parties being risk neutral, by supply being fixed to one standardized unit (one million components or ten thousand payrolls serviced or 1,000 kilometers of yellow lines down the centre of city streets maintained), and by suppliers being essentially uncapacitated. In addition the price per unit is fixed. The only way for a supplier to gain a larger share of next period's (e.g., year's) contract for this unit is to improve quality or performance. To improve quality the buyer will announce in advance how next year's shares will depend on this year's observed performance or quality. This performance-based allocation of business provides an incentive in the form of business volume, which is commonly used in practice. For example, Toyota adjusts business volume between just two suppliers based on their performance to achieve effective competition between the suppliers (Dyer et al. 1998). Sun Microsystems allocates demand among multiple suppliers using a scorecard system (Farlow et al. 1996). When allocating business between two suppliers based on their performance, the buyer's objective is to keep suppliers competitive in terms of quality, delivery, or whatever supplier's performance characteristic the buyer deems important (Spekman 1988, Hahn, Kim & Kim 1986), and to motivate suppliers to improve by providing positive incentives (in the form of increased business volumes) or negative incentives (in the form of decreased business or competition). The buyer seeks for better supplier performance rather than the optimal supply chain efficiency. The actual measurement of quality or performance will be kept as simple as possible in this chapter but the key assumption is that the buyer can observe the supplier's performance or quality only and this is a noisy signal of the supplier's effort level.

Despite its widespread use, this volume incentive is seldom studied in the literature. In the absence of direct monetary incentives, volume incentives in the form of delayed rewards or penalties from gaining or losing future business may be a substitute. The objective of this research is to study special features in the performance-based volume incentive schemes and the effectiveness of different forms of volume incentives. We look for answers to the following questions: How can a buyer use the allocation of demand to induce competition between two suppliers to obtain better performance when the suppliers' investments are unobservable? What is the buyer's optimal volume incentive scheme that maintains the suppliers' competition over time?

Specifically, we consider a buyer repeatedly outsourcing a fixed amount of divisible service or product from two suppliers. The suppliers can make effort to improve their performance, and in order to isolate the key structure of what drives an optimal allocation we make the two suppliers equal in every way. The effort levels are unobservable to the buyer – a moral hazard problem in agency theory. The buyer allocates his business between the two suppliers based on their past performance in order to maximize his long-run discounted payoff from repeated dual sourcing. A key part of the modeling is the suppliers' cost function. We examine two cases; where a supplier's effort cost is proportional to her share of business (the proportional case) and the case where the cost is independent of her share (the independent case). An example of the latter might be where a new product is developed in a laboratory and then can be applied in their supplier's factory without further tooling; compared to the former case where the same product is made but the process improvement means that all the machines must be upgraded. An innovation of new software to payroll management that could then be rolled out to all accounts would be the latter, but one which needed the reformatting of accounts one by one would be the former. A new yellow paint formulation that dried quicker would be the latter but a drying process that cost per kilometer would be the former. An important example here is 'learning by doing'. A firm with a large share might in some circumstances have much more opportunity to innovate at a lower cost. The company with 999 of the 1,000 kilometers might have greater testing costs than the one with 1 kilometer, but the cost *per* kilometer is likely a lot less. Of course in practice this is going to be a lot more complex, but using these two cases and seeing how the results depend critically on them gives us insights into the importance of including this aspect.

A natural allocation rule by the buyer that comes readily to mind is to give all the business to the one with the better performance, often termed a 'winner-take-all' or WTA rule, essentially treating the competition as a rank-order tournament. Our finding addresses both the A, the 'all', of this acronym and the W, the 'winner'. In the proportional case the optimal is not 'all', but in the independent case it essentially is 'all'. However in neither case is the definition of 'winner' a simply 'first-past-the-post'. We find that winning must be relative to current shares, essentially an endowment with which a company enters this year's competition. For symmetric suppliers with an identical cost function, in both cases, the optimal rule of business allocation is what we might term a 'handicapped' one. A parallel might be drawn to competitive sailing where point handicaps reflect equipment endowments or past successes, or to the game of golf where success is handicapped. Both examples are mainly to ensure a vigorous competition, the handicapped player has to work harder to win by being given a handicap at the start. Thus the competition may also look like a 'fair' one. In some horse races, handicaps give extra weight into the saddles in order to make it a 'fair' race, although this may be more to do with making the punting more interesting with a more evenly balanced field. Thus the supplier with a better performance in the current period may not get a bigger share in the next period. So in the proportional case, although the optimal allocation rule is not a WTA one, numerical results show that a handicapped-winner-take-all (HWTA) rule can perform well compared with the optimal when the variability (noise) in the performance measure is small, but worse when the variability (noise) is large, while a common rank-order tournament type of allocation rule, simple WTA (SWTA) rule, always performs far worse compared to both the HWTA rule and the optimal one. In the 'independent' case, the optimal allocation rule for a finite horizon problem is a HWTA one. Both a share-dependent HWTA and a SWTA allocation rules are studied for an infinite horizon problem, and numerical results indicate that a HWTA rule can often perform much better than the SWTA one. Therefore, when the incentive comes from the allocation of business among competing suppliers, each supplier's current share of business plays an important role, and using a handicap can be very effective for incentive provision.

The main contribution of this chapter is to examine these special features of performance-based volume incentive schemes. Our results have direct managerial implications to the design of volume incentive contracts in practice. To induce competition among suppliers and maintain the competition over time, the optimal volume incentive scheme is generally not a simple rank-order tournament, which has been shown in literature to be effective under the monetary incentive scheme. Instead, handicapping the definition of winner can do well over a simple first-past-the-post rule and the optimal rule may not be to give all the demand to one company. Performance-based volume incentives often need to take into account each supplier's current share of business.

The rest of this chapter is organized as follows. Section 4.2 reviews the literature. Section 4.3 describes the model. Section 4.4 studies the buyer's problem under the two types of supplier's effort cost and presents numerical results. The chapter concludes with a summary and a discussion of future work in Section 4.5.

4.2 Literature Review

Our study is at the interface of operations management and economics, and thus is related to both streams of literature.

There exists a vast literature on dual (multiple) sourcing. Elmaghraby (2000) provides a survey on the research in operations research and economics literature on sourcing strategies for the problem of a buyer awarding a divisible business to one or more suppliers among multiple suppliers. The research questions are mostly one-time decision problems which are related to the design of competitive mechanisms in the form of bidding and the suppliers' competitive behavior under the bidding rule.

In the operations management literature, a number of papers investigate the effect of demand allocation on the behavior of competing firms.

Lippman and McCardle (1997) study a single-period competitive newsvendor problem in which each newsvendor chooses an inventory level to meet a random demand and a rule specifies the allocation of initial market demand among the firms as well as the allocation of excess demand among firms with remaining inventory. They investigate the relationship between four specific allocation rules and equilibrium inventory levels. Both Hall and Porteus (2000) and Liu et al. (2007) consider a multi-period competitive newsvendor problem where two firms make capacity (inventory) decision in each period, and the demand for each firm is dependent on the realized level of customer service (product availability) in the prior period. The firms' equilibrium behavior in the dynamic game is identified. In all three papers, firms' incentive for competition is governed by an exogenous demand allocation mechanism driven by the switching behavior of customers in the market, which is dependent on the firms' realized service levels in the current (first paper) or prior period (the other two papers), and there is no buyer dictating the supplier competition. In our model, a buyer designs the incentive mechanism – a demand allocation rule which is based on the firms' past performance levels. So the focus of our study is on the design of a demand allocation mechanism.

Our study is closely related to two papers. Both Cachon and Zhang (2007) and Benjaafar et al. (2007) consider a buyer outsourcing a fixed demand at a fixed unit price to multiple suppliers. Cachon and Zhang (2007) study a queuing system where each supplier's service time is determined by the capacity she invests, and the buyer allocates the demand among multiple suppliers based on their service times to minimize the average service time over an infinite horizon. Suppliers are homogeneous in terms of their capacity costs. Each supplier chooses a capacity level to maximize her own profit. The authors evaluate several allocation rules and show that performance-based allocation may not motivate suppliers to improve service times. In Benjaafar et al. (2007), a buyer outsources the demand to a set of potential suppliers. Competition between suppliers is created either by allocating the whole demand to one supplier with the probability of being selected increasing with her committed service level – market-seeking (MS) approach, or by allocating the demand to each supplier in proportion to her committed service level – market-augmenting (MA) approach. In the MA case, each supplier's service level is assumed to be independent of the demand allocated to her. Under both cases, it is assumed that the contractual promises of the suppliers regarding effort or service level are enforceable. The suppliers are heterogeneous in production and service costs. Each supplier chooses a committed service level to maximize her expected profit. The authors compare the service quality the buyer can achieve under the MA and MS mechanisms. Neither paper considers the hidden action problem. There is no noise in the suppliers' performance outcome, and the demand allocation is based on the suppliers' observable effort levels or expected performance in the first paper and on the suppliers' committed performance in the latter.

Cachon and Lariviere (1999) study a special allocation rule commonly used in the automobile industry by considering a single supplier allocating capacity to multiple retailers based on their past sales. They examine a two-period game under a given allocation rule, so their focus of study is not on the design of allocation rule.

In all the aforementioned papers, only Cachon and Zhang (2007) examine the optimal allocation rule. Our chapter differs from these papers by investigating the design of volume incentives which are on the basis of past performance and studying a multi-period multi-agent moral hazard problem.

Our research is also related to the economics literature. Spear and Srivastava (1987) study a repeated moral hazard problem with discounting between a principal and an agent, and show that history dependence can be represented by using the agent's expected utility as a state, and thus the problem of characterizing the optimal contract of such a model can be reduced to a constrained static variational problem. Monetary compensation is used for an incentive. We study a repeated moral hazard problem between a principal and two competing agents, the compensation is in the form of future demand, and the state is Supplier 1's current share of the business.

Lewis and Yildirim (2002) examine the design of competitive mechanisms for dual sourcing with supplier learning by doing. Each time only one supplier is selected through bidding. The buyer faces an adverse selection problem because the suppliers' production cost is private information. Supplier's investment in performance is not in consideration.

In the economics literature, incentive schemes are usually on the basis of monetary reward or penalty. Competitive compensation schemes can come in the form of rank-order tournament or relative performance evaluation. The relevant research can be found, for example, in Lazear and Rosen (1981), Green and Stokey (1983), Hart (1983), Holmstrom (1982), and Nalebuff and Stiglitz (1983). The problems are generally for a single period. Relative performance evaluation (RPE) compensates the agents based on their output levels, and is often used when there is a common shock to the agents' performance, which is not considered in our problem. The total compensation in RPE varies with the agents' realized output levels, but in our case the total demand to be split is a constant. In tournaments, rewards are based on the rank order of the individuals, not on their actual output levels. Lazear and Rosen (1981) show that for risk-neutral agents rank-order tournaments work as well as independent contracts; and for agents with known heterogeneous ability, handicapping will improve the efficiency of the tournaments. We find that when incentives are from future business, rank-order tournaments are generally not optimal, and handicapping significantly improves the efficiency even when the agents are homogeneous in ability.

4.3 Model Description

Consider a buyer outsourcing the supply of a fixed one unit of a divisible product or service from two suppliers repeatedly over an infinite horizon. Both the buyer and the two suppliers are risk neutral. For tractability we make a number of simplifying assumptions. Each supplier can make effort to improve her performance (e.g., delivery, quality, cost, etc.). For instance, when contracting for inventory management, a supplier's demand fulfillment performance can be measured by the fill rate. In each period t ($t = 1, 2, ...$), Supplier i's realized performance $x_t^i = e_t^i + \varepsilon_i$ ($i = 1, 2$), where e_t^i is Supplier i's effort level in period t, ε_1 and ε_2 are independently and identically distributed (i.i.d.) with the probability distribution $N(0, \sigma)$[5]. In this chapter, we use a supplier's effort level to refer to her target performance level. The suppliers have identical effort cost function, and the cost of effort takes the form $C(e, \beta) = g(\beta)\frac{be^2}{2}$, where $b > 0$, β is a supplier's share of demand in a period, $g(\beta) > 0$ and $g'(\beta) \geq 0$. It is important that marginal increases in performance are increasingly costly to achieve. The actual quadratic nature is a matter of convenience. Other strictly convex functions are possible but the analysis would be formidable. The effort cost is common information. Only two special cases of $g(\beta)$ are considered: $g(\beta) = \beta$ and $g(\beta) = 1$, the proportional and independent cases respectively. The unit transfer price p of the product or service between the buyer and each supplier is identical and constant in every period, which can reflect a dominant market price. The unit cost of supplying the product or service c is constant in every period. Consequently, the unit profit of supplying the product or service $m = p - c$ is also constant and identical for both suppliers. All parties have a common discount factor $\gamma \in (0, 1)$.

Let α_t denote Supplier 1's share of demand in period t. So Supplier 2's share in period t is $1 - \alpha_t$. The state in period t is α_t, Supplier 1's share in that period, $\alpha_t \in [0, 1]$. Each supplier's feasible action set is $A = [0, \widehat{e}]$, where $\widehat{e}$ is a sufficiently large number.

The buyer's allocation rule for the next period is restricted for simplicity to be based on the current share and performance. Rules such as based the average of previous year's shares or performance levels are not considered. In a repeated moral hazard problem between a principal and an agent, Spear and Srivastava (1987) have shown that history dependence in the compensation scheme can be represented by using the agent's expected utility as a state, thus the optimal compensation scheme is independent of the history of the agent's performance and compensation. In our problem, because a supplier's expected future payoff is directly linked to her next period share of business, by analogy the optimal allocation rule is likely to depend only on the suppliers' immediate past performance outcomes and shares. So the restriction does not necessarily limit our findings.

The sequence of events is as follows. At the beginning of the horizon, the buyer announces an allocation rule to be used for each period and gives each supplier an

[5]The case of correlated noise has been studied but essentially no added insights were available and the complexity was greatly increased. The normal assumption is just for tractability and appears reasonably benign.

initial share of the business, with the two suppliers' total share equal to one. In every period t (except for period 1), the buyer allocates his business between the two suppliers based on the suppliers' performance levels in the previous period and the allocation rule. The suppliers choose their effort levels simultaneously and incur the effort costs. Their performance levels are realized at the end of the period and observed by all parties.

We study a moral hazard problem, where the buyer can only observe both suppliers' realized performance but not their effort levels in each period. Therefore, the share of demand allocated to each supplier in a period can only be based on immediate past performance realizations and demand allocations.

Let v_t^i and v_t^B denote Supplier i's profit and the buyer's payoff from period t onwards. The buyer's payoff will be taken as the discounted weighted average quality (or performance) level in each period. So for $t \geq 1$, the buyer's payoff to go and the suppliers' profits to go from period t onwards are

$$\begin{aligned} v_t^B(\alpha_t) &= E(\sum_{\tau=t}^{\infty} \gamma^{\tau-1}[\alpha_\tau x_\tau^1 + (1-\alpha_\tau)x_\tau^2]) \\ &= \sum_{\tau=t}^{\infty} \gamma^{\tau-1}[\alpha_\tau e_\tau^1 + (1-\alpha_\tau)e_\tau^2], \end{aligned} \tag{4.1}$$

and

$$\begin{aligned} v_t^1(\alpha_t) &= \sum_{\tau=t}^{\infty} \gamma^{\tau-1}[m\alpha_\tau - \frac{bg(\alpha_\tau)(e_\tau^1)^2}{2}], \\ v_t^2(1-\alpha_t) &= \sum_{\tau=t}^{\infty} \gamma^{\tau-1}[m(1-\alpha_\tau) - \frac{bg(1-\alpha_\tau)(e_\tau^2)^2}{2}]. \end{aligned}$$

The buyer uses a stationary allocation rule $\beta_\alpha(x_1, x_2)$, which states that given Supplier 1's share in a period $= \alpha$ and the suppliers' performance outcomes (x_1, x_2) in that period, Supplier 1's share in the next period is $\beta_\alpha(x_1, x_2)$. Under the allocation rule $\beta_\alpha(x_1, x_2)$, the two suppliers play a stochastic game in an infinite horizon. If the two suppliers' equilibrium policies are stationary, then the buyer's problem can be represented as a static variational problem (omitting the time index in the notations), with the buyer's payoff at period 1 being

$$v_B(\alpha) = \alpha_1 e_1 + (1-\alpha_1)e_2 + \gamma \int\int v_B(\beta_\alpha(x_1,x_2)) f(x_1|e_1) f(x_2|e_2) dx_1 dx_2, \quad (4.2)$$

where $f(x|e) = \frac{1}{\sigma}\phi(\frac{x-e}{\sigma})$ and $\phi(\cdot)$ is the probability density function (pdf) of the normal distribution, and the payoffs of suppliers 1 and 2 at period 1 being

$$v_1(\alpha) = \alpha m - \frac{bg(\alpha)(e_1)^2}{2} + \gamma \int\int v_1(\beta_\alpha(x_1,x_2)) f(x_1|e_1) f(x_2|e_2) dx_1 dx_2, \quad (4.3)$$

$$\begin{aligned} v_2(1-\alpha) &= (1-\alpha)m - \frac{bg(1-\alpha)(e_2)^2}{2} \\ &+ \gamma \int\int v_2(1-\beta_\alpha(x_1,x_2)) f(x_1|e_1) f(x_2|e_2) dx_1 dx_2. \end{aligned} \quad (4.4)$$

In general we would like to keep the analysis simpler so that any allocation $\beta \in [0,1]$ was possible. Two circumstances prohibit this. First, given a buyer's allocation rule, the suppliers will play a stochastic game, so that we must ensure proper conditions for the Nash equilibrium to exist. Secondly, as the formulation of the buyer's problem is basically a repeated principal-agent formulation with the outcome of the suppliers' game as the agent, the participation of the suppliers needs to be ensured via the participation (individual rationality) constraints. Both of these considerations can place limits on the size of β that the buyer can employ. The main results can be best appreciated by thinking that β is between 0 and 1; however to do the modeling correctly we have to calculate the limits that β can feasibly take. By the symmetry of the two suppliers, the limits are identical for the two suppliers. Let $\overline{\beta}$ and $\underline{\beta}$ denote a supplier's maximum and minimum shares in a period. The actual values will be addressed later. Note that $\underline{\beta} = 1 - \overline{\beta}$.

We are interested in the form of the buyer's optimal stationary allocation rules. For this purpose, we first derive the optimal allocation rule $\beta^*_\alpha(x_1,x_2)$ from the static formulation of the buyer's infinite-horizon problem, under the assumption that the two suppliers use stationary policies to play the stochastic game; we then check that under this $\beta^*_\alpha(x_1,x_2)$, the suppliers' infinite-horizon stochastic game has a unique Nash equilibrium which is stationary and is the one derived from the static formulation. Let e^*_1 and e^*_2 denote the optimal stationary-policy effort levels of suppliers 1 and 2 under an optimal allocation rule $\beta^*_\alpha(x_1,x_2)$.

As discussed above we consider two special forms of $g(\beta)$: $g(\beta) = \beta$, representing a demand-dependent effort cost which is linear in the demand β; and $g(\beta) = 1$, a demand-independent effort cost.

4.4 Buyer's Problem

The buyer designs a demand allocation rule to maximize the long-run discounted aggregate performance of the suppliers over the horizon. Given the allocation rule, the suppliers choose their effort levels in each period to maximize their respective long-run discounted profit. So the suppliers play a stochastic game governed by the buyer's allocation rule.

To simplify our analysis, assume the suppliers' incentive compatibility constraints can be written as first-order conditions. The validity of this assumption will be checked later for each specific allocation rule[6]. To focus on the form of the optimal allocation rule and each party's equilibrium result, we mainly present the static formulation of the buyer's problem in the main body, leaving in the appendix the dynamic formulation of each party's problem and the verification of the existence of a unique stationary Nash equilibrium in the suppliers' stochastic game[7]. So the buyer's problem is to choose an allocation rule $\beta_\alpha(x_1, x_2)$ such that

$$\begin{aligned}
\max_{\beta_\alpha(x_1,x_2)} \quad & v_B(\alpha) \\
\text{subject to} \quad & -g(\alpha)be_1 + \gamma \int\int v_1(\beta_\alpha(x_1,x_2)) f^1(x_1|e_1) f(x_2|e_2) dx_1 dx_2 = 0 \\
& -g(1-\alpha)be_2 + \gamma \int\int v_2(1-\beta_\alpha(x_1,x_2)) f(x_1|e_1) f^2(x_2|e_2) dx_1 dx_2 = 0 \\
& v_1(\alpha) \geq 0 \\
& v_2(1-\alpha) \geq 0 \\
& 1-\overline{\beta} \leq \beta_\alpha(x_1,x_2) \leq \overline{\beta}.
\end{aligned} \tag{4.5}$$

The first two constraints are the incentive compatibility constraints for suppliers 1 and 2 respectively, where $f^i = \partial f / \partial e_i$. The third and fourth constraints are the

[6]Generally an agent's incentive compatibility constraint can be replaced by both a first-order condition and a second-order condition (i.e., concave objective function - a sufficient condition for the extreme point to be the global optimum). In the appendix we use the seond-order condition as the sufficient condition for the Nash equilibrium.

[7]We provide in the appendix the proof of the existence of a unique stationary Nash equilibrium in the suppliers' stochastic game under the optimal general allocation rule for the case of $g(\beta) = \beta$. The proof for other forms of allocation rules follows similar methodology and is thus omitted.

suppliers' individual rationality constraints which guarantee each supplier's long-run discounted payoff to be nonnegative.

4.4.1 Volume incentive under proportional effort cost

As discussed above, the proportional effort cost is taken to be the case of $g(\beta) = \beta$.

Allocation rules

We first study the buyer's optimal allocation rule which maximizes his long-run discounted payoff, and investigate some simple heuristics. We then obtain numerical results for each type of allocation rule to examine the efficiency of simple heuristics compared to the optimal rule.

- Optimal allocation rule

As the details of Theorem 4.1 make the main message a bit opaque we shall discuss the main message as-if $\overline{\beta} = 1$ and $\underline{\beta} = 0$. In Figure 4.1 the axes are the performance outcomes of the two suppliers. The point (e_2^*, e_1^*) is where the optimal efforts should be, and would be if the signal was not noisy. The dashed straight line is the 45° line. The optimal allocation for the buyer would be to allocate all demand to Supplier 1 or 2 in all quadrants based on the point (e_2^*, e_1^*) except for quadrant (+,+). Thus Supplier 1 gets all when $x_1 - e_1^* > x_2 - e_2^*$ and vice versa. However in the (+,+) quadrant emanating from (e_2^*, e_1^*), Supplier 1 should get a share β where $S(\beta) = \frac{x_1 - e_1^*}{x_2 - e_2^*}$ and we have

$$S(\beta) = \frac{m + H/(1-\beta)^2}{m + H/\beta^2}, \tag{4.6}$$

where $H > 0$ is defined in Theorem 4.1.

Because Theorem 4.1 indicates that e_1^* and e_2^* are functions of α, the optimal allocation rule for the infinite horizon problem is not a WTA one and is a function of each supplier's share in the current period. Since the two suppliers' optimal effort levels in a period differ when they have unequal shares, the optimal allocation rule is a handicapped rule in the sense that the suppliers are not compared by their actual performance but by the deviation from their respective target performance. Although the two suppliers are symmetric in terms of their effort cost function, for unequal starting shares in a period, their marginal costs for the same effort level differ

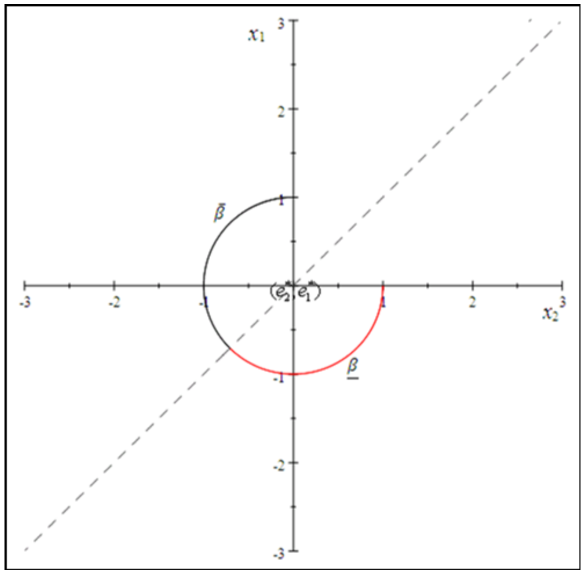

Figure 4.1: Optimal allocation rule with maximum share = 1

because of the demand-dependent effort cost structure. Therefore, the optimal rule uses a handicap to cope with unequal marginal effort cost. The supplier with a larger current share has a lower optimal effort level and a lower handicap under the optimal rule. This is because a larger share results in a higher marginal effort cost. It can also be seen from (4.7) that when both suppliers overperform, the optimal allocation rule dictates that for a range of $\frac{x_1-e_1^*}{x_2-e_2^*}$ values, Supplier 1's share in the subsequent period is dependent on the two suppliers' relative deviations from their respective target performance, regardless of how good their actual performance levels are. Moreover, the optimal allocation rule is symmetric in the suppliers' performance deviation from their respective target one.

Theorem 4.1 gives the result but with the correct limits to β.

Theorem 4.1 *For supplier's cost function with $g(\beta) = \beta$:*
1. The buyer's optimal allocation rule is characterized as below: given Supplier 1's share in the current period equal to α,
for $x_1 > e_1^*$ and $x_2 > e_2^*$:
if $x_1 - e_1^* > S(\overline{\beta})(x_2 - e_2^*)$, $\beta_\alpha^{1*}(x_1, x_2) = \overline{\beta}$; if $x_1 - e_1^* < S(\underline{\beta})(x_2 - e_2^*)$, $\beta_\alpha^{1*}(x_1, x_2) = \underline{\beta}$;

otherwise, $\beta_{\alpha}^{1*}(x_1, x_2)$ *is determined by*

$$S(\beta_{\alpha}^{1*}(x_1, x_2)) = \frac{x_1 - e_1^*}{x_2 - e_2^*}; \tag{4.7}$$

for $x_1 < e_1^*$ *or* $x_2 < e_2^*$*:* $\beta_{\alpha}^{1*}(x_1, x_2) = \overline{\beta}$ *if* $x_1 - e_1^* > x_2 - e_2^*$ *and* $\beta_{\alpha}^{1*}(x_1, x_2) = \underline{\beta}$ *otherwise;*
where $S(\beta)$ *is defined by* (4.6), $e_1^* = \frac{\sqrt{2H}}{\alpha\sqrt{b}}, e_2^* = \frac{\sqrt{2H}}{(1-\alpha)\sqrt{b}}$, *and* $H > 0$ *is the solution to*

$$\frac{\sigma}{\gamma}\sqrt{2bH} = \int y_1(\widehat{\beta}^*(y_1, y_2)m - \frac{H}{\widehat{\beta}^*(y_1, y_2)})\phi(y_1)\phi(y_2)dy_1dy_2, \tag{4.8}$$

$y_i \sim N(0, 1)$*, and* $\widehat{\beta}^*(y_1, y_2) = \beta_{\alpha}^{1*}(e_1^* + \sigma y_1, e_2^* + \sigma y_2)$ *and is independent of* α*.*
2. Under $\beta_{\alpha}^{1*}(x_1, x_2)$, (e_1^*, e_2^*) *is the unique stationary Nash equilibrium in the suppliers' stochastic game.*

The correct version of Figure 4.1 is shown below. The interpretation is the same.

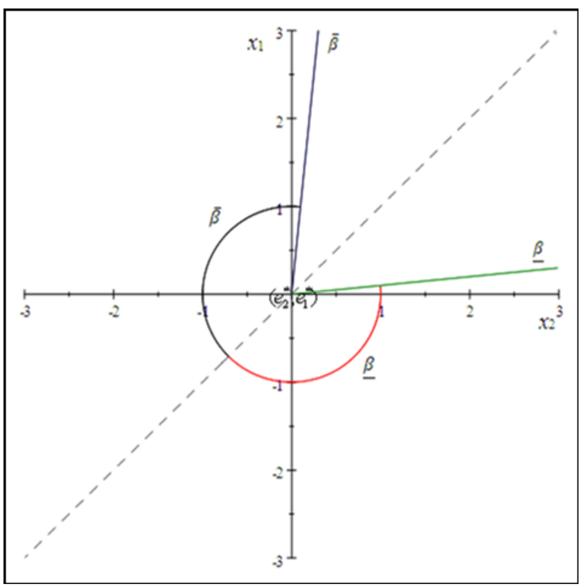

Figure 4.2: Optimal allocation rule with maximum share < 1

Corollary 4.1 provides the formulas for calculating the value functions of the buyer and the suppliers.

Corollary 4.1 *Under the optimal allocation rule $\beta_\alpha^{1*}(x_1, x_2)$, given Supplier 1's initial share α, the suppliers' value functions*

$$v_1^*(\alpha) = v_2^*(\alpha) = v^*(\alpha)$$

for any α due to symmetry,

$$\begin{aligned} v^*(\alpha) &= \alpha m - \frac{H}{\alpha} + \gamma V, \\ V &= \frac{1}{1-\gamma}\int\int (\widehat{\beta}^*(y_1, y_2)m - \frac{H}{\widehat{\beta}^*(y_1, y_2)})\phi(y_1)\phi(y_2)dy_1dy_2, \end{aligned}$$

and the buyer's value function is

$$v_B^* = \frac{2\sqrt{2H}}{(1-\gamma)\sqrt{b}}.$$

From the suppliers' value functions in Corollary 4.1, we can see that under the optimal allocation rule a supplier's expected future payoff V is independent of her current share. Therefore, the optimal allocation rule uses a handicap to provide the suppliers with different level of incentive, but it is also a 'fair' rule in that it gives the suppliers equal expected future payoff.

Also note that the nonlinear equation (4.8) may not have a unique positive solution of H. Because v_B^* is increasing in H, the buyer would prefer the largest H. However, the nonnegativity of $v^*(\alpha)$ and V may restrict such choices because they may become negative at the largest H. In fact, in the later section of numerical analysis, we have found that in some cases there are two solutions of H to (4.8), but both $v^*(\underline{\beta})$ and V are always negative at the bigger H, thus only the smaller H is used.

- Simple heuristics

Due to the complexity of the optimal allocation rule, we also investigate simpler rules. Consider the family of WTA allocation rules with the following form[8]

$$\beta_\alpha^{2*}(x_1, x_2) = \begin{cases} \widehat{\beta} & x_1 - x_2 > \theta_\alpha \\ 1-\widehat{\beta} & x_1 - x_2 < \theta_\alpha \end{cases}, \tag{4.9}$$

[8]We ignore the case of equality, i.e., $x_1 - x_2 = \theta_\alpha$, because the probability for this to occur is zero. Similarly for all the WTA rules we are investigating hereafter.

where θ_α and $\widehat{\beta} \in (\frac{1}{2}, 1)$ are parameters to be determined. Note that we use WTA to refer to both HWTA and SWTA allocation rules.

Theorem 4.2 *For supplier's cost function with $g(\beta) = \beta$, among the family of WTA allocation rules as defined in (4.9), the optimal rule has $\theta_\alpha^* \neq 0$ for $\alpha \neq \frac{1}{2}$.*

Let

$$\Delta v(\widehat{\beta}) = \frac{4\pi b\sigma^2 \widehat{\beta}(1-\widehat{\beta})}{\gamma^2(2\widehat{\beta}-1)}(1 - \sqrt{1 - \frac{m\gamma^2}{2\pi b\sigma^2}\frac{(2\widehat{\beta}-1)^2}{\widehat{\beta}(1-\widehat{\beta})}}),$$

then we obtain Corollary 4.2 following Theorem 4.2.

Corollary 4.2 *Given any $\widehat{\beta} \in (\frac{1}{2}, 1)$, the optimal*

$$\theta_\alpha^* = e_1^* - e_2^* = \frac{\gamma \Delta v(\widehat{\beta})}{2\sqrt{\pi} b\sigma}\frac{1-2\alpha}{\alpha(1-\alpha)};$$

the optimal β^* is a boundary solution, and is constrained by

$$\begin{aligned} v(1-\widehat{\beta}) = \frac{m}{1-\gamma} - \frac{1}{2}(\frac{1}{(1-\gamma)(2\widehat{\beta}-1)} + 1)\Delta v(\widehat{\beta}) \geq 0, \\ \frac{\gamma \Delta v(\widehat{\beta})}{2\sigma^2}\phi(1) \leq b(1-\widehat{\beta}); \end{aligned} \tag{4.10}$$

and the value functions of the buyer and the suppliers are

$$\begin{aligned} v_B^*(\alpha) &= \frac{\gamma}{1-\gamma}\frac{\Delta v(\beta^*)}{\sqrt{\pi} b\sigma}, \\ v^*(\alpha) &= \alpha m - \frac{(\gamma \Delta v(\beta^*))^2}{8\pi b\sigma^2 \alpha} + \frac{\gamma}{2}(v^*(\beta^*) + v^*(1-\beta^*)). \end{aligned}$$

It is obvious from Theorem 4.2 result that the optimal 'take all' allocation rule is also a handicapped one with the definition of θ_α^*. The reasoning is similar to that for the optimal general allocation rule. Again, the supplier with a larger current share has a lower optimal effort level and a lower handicap under the optimal rule, and the optimal 'take-all' allocation rule is a 'fair' rule because both suppliers have equal expected future payoff. We also note that $v_B^*(\alpha)$ is independent of α. This is due to the special structure of each supplier's effort cost which is proportional to the demand and quadratic in the effort level.

The simplest form of a WTA allocation rule is the one with $\theta_\alpha^* = 0$, i.e., the SWTA rule,

$$\beta_\alpha^{3*}(x_1, x_2) = \begin{cases} \widehat{\beta} & x_1 > x_2 \\ 1 - \widehat{\beta} & x_1 < x_2 \end{cases} . \tag{4.11}$$

A WTA scheme is usually used in a tournament. It has been extensively studied in the economics literature for a single-period monetary incentive problem, where the winner receives an extra reward in addition to a common compensation to each supplier. Such an incentive scheme has been shown to be optimal when the two suppliers are symmetric.

Let $\overline{e}$ be the solution to

$$\overline{e} = -\frac{\gamma}{\sqrt{2}b\sigma}\phi(\frac{\overline{e}}{\sqrt{2}\sigma})(\frac{(2\widehat{\beta}-1)^2}{\widehat{\beta}(1-\widehat{\beta})}m + \frac{b}{2}\overline{e}^2)[2\gamma\Phi(\frac{-\overline{e}}{\sqrt{2}\sigma}) + 1 - \gamma]^{-1}, \tag{4.12}$$

where $\Phi(\cdot)$ is the cumulative distribution function (cdf) of the normal distribution.

Proposition 4.1 *Under the SWTA rule defined in (4.11), the optimal $\beta^* < 1$ and is not necessarily a boundary solution; the optimal effort levels of a supplier with a current share $\widehat{\beta}$ and $1 - \widehat{\beta}$ are $e^*_{\widehat{\beta}} = \frac{(1-\widehat{\beta})\overline{e}}{1-2\widehat{\beta}}$ and $e^*_{1-\widehat{\beta}} = \frac{\widehat{\beta}\overline{e}}{1-2\widehat{\beta}}$ respectively.*

Corollary 4.3 follows from Theorem 4.1 and Theorem 4.2.

Corollary 4.3 *For supplier's cost function with $g(\beta) = \beta$, any WTA allocation rule is suboptimal, and a HWTA rule provides better incentive than a SWTA rule.*

Numerical analysis

The method for numerical calculation of the optimal H from (4.8) and the calculation of e_1^*, e_2^*, $v_B^*(\alpha)$ and $v^*(\alpha)$ can be found in Subsection 6 of Appendix C. Tables 4.1, 4.2 and 4.3 show the numerical analysis results under the optimal allocation rule, the HWTA and SWTA rules respectively[9]. Under all three rules, a supplier's maximum (minimum) share $\overline{\beta}$ ($\underline{\beta}$) goes up (down) with σ. The reason is that when a performance measure becomes noisier, the suppliers' Nash equilibrium can still be maintained at a larger gap in their shares. In return, the more extreme maximum/minimum share can provide a stronger incentive to the suppliers and counter the disincentive effect of the

[9]We use the minimum share $\underline{\beta}$ obtained for the HWTA rule as that for the optimal rule so that the comparison is on the structure of the rules only without the interference of the minimum share.

γ	σ	b	m	$\underline{\beta}$	v_B^*	V	$v^*(\overline{\beta})$	$v^*(\underline{\beta})$
0.9	0.5	1	1	25.9%	6.08	3.81	3.51	4.11
0.9	1	1	1	11.3%	5.48	3.25	2.70	3.77
0.9	2	1	1	3.5%	3.65	2.90	2.18	3.56
0.9	3	1	1	1.7%	2.66	2.75	1.96	3.45
0.9	0.5	1	2	33.6%	7.83	8.28	7.56	8.00
0.9	1	1	2	18.1%	8.56	7.00	5.97	7.00
0.9	2	1	2	6.5%	6.51	6.11	4.75	6.37
0.9	3	1	2	3.2%	4.94	5.76	4.25	6.12
0.9	0.5	2	1	18.1%	4.28	3.50	3.08	3.91
0.9	1	2	1	6.5%	3.25	3.05	2.41	3.65
0.9	2	2	1	1.8%	1.97	2.77	1.98	3.47
0.9	3	2	1	0.8%	1.40	2.64	1.80	3.36
0.5	0.5	1	1	17.9%	0.84	0.85	0.48	1.22
0.5	1	1	1	6.3%	0.59	0.83	0.31	1.34
0.5	2	1	1	1.7%	0.33	0.82	0.23	1.39
0.5	3	1	1	0.8%	0.23	0.82	0.21	1.40
0.5	0.5	2	1	11.0%	0.52	0.84	0.38	1.29
0.5	1	2	1	3.4%	0.32	0.83	0.26	1.37
0.5	2	2	1	0.9%	0.17	0.83	0.21	1.40
0.5	3	2	1	0.4%	0.12	0.83	0.21	1.41

Table 4.1: Optimal allocation rule (proportional case)

performance variability. On the other hand, as $\underline{\beta}$ becomes smaller, the supplier with a share $\underline{\beta}$ will put more effort and incur higher effort cost, which does not substantially benefit the buyer overall because this supplier only contributes to a small portion of the buyer's business, but does greatly reduce the supplier's profit. However, the buyer would prefer this since he is maximizing his own payoff and a larger $\overline{\beta}$ will strengthen the suppliers' incentive.

Table 4.4 compares the effectiveness of the three allocation rules by calculating the percentage increase in the buyer's long-run discounted payoff from using a HWTA rule instead of a SWTA rule, and from using the optimal allocation rule instead of a HWTA rule. In all the cases, a HWTA allocation rule is much more effective than a SWTA one. The buyer's long-run discounted payoff from using a HWTA rule can be more than doubled of that under a SWTA rule. The incentive can be further strengthened by using the optimal but more complex allocation rule, with the improvement generally small for less variable performance measure and large for

γ	σ	b	m	$\underline{\beta}$	v_B^*	V	$v^*(\overline{\beta})$	$v^*(\underline{\beta})$
0.9	0.5	1	1	25.9%	6.05	3.81	3.51	4.11
0.9	1	1	1	11.3%	5.29	3.26	2.74	3.78
0.9	2	1	1	3.5%	3.30	3.01	2.36	3.66
0.9	3	1	1	1.7%	2.31	2.95	2.27	3.63
0.9	0.5	1	2	33.6%	7.82	8.28	7.90	8.67
0.9	1	1	2	18.1%	8.43	7.00	6.17	7.83
0.9	2	1	2	6.5%	6.09	6.20	5.00	7.40
0.9	3	1	2	3.2%	4.44	5.99	4.68	7.30
0.9	0.5	2	1	18.1%	4.21	3.50	3.09	3.92
0.9	1	2	1	6.5%	3.05	3.10	2.50	3.70
0.9	2	2	1	1.8%	1.72	2.95	2.27	3.63
0.9	3	2	1	0.8%	1.18	2.92	2.23	3.62
0.5	0.5	1	1	17.9%	0.83	0.85	0.48	1.22
0.5	1	1	1	6.3%	0.58	0.82	0.30	1.34
0.5	2	1	1	1.7%	0.32	0.81	0.23	1.38
0.5	3	1	1	0.8%	0.22	0.80	0.22	1.39
0.5	0.5	2	1	11.0%	0.51	0.83	0.38	1.29
0.5	1	2	1	3.4%	0.31	0.81	0.26	1.37
0.5	2	2	1	0.9%	0.17	0.81	0.22	1.39
0.5	3	2	1	0.4%	0.11	0.80	0.21	1.40

Table 4.2: HWTA allocation rule (proportional case)

γ	σ	b	m	$\underline{\beta}$	v_B^*	$v^*(\overline{\beta})$	$v^*(\underline{\beta})$
0.9	0.5	1	1	25.0%	3.21	4.85	4.47
0.9	1	1	1	16.4%	2.52	4.98	4.44
0.9	2	1	1	9.6%	1.71	5.14	4.44
0.9	3	1	1	6.7%	1.29	5.22	4.45
0.9	0.5	1	2	29.5%	4.88	9.59	8.98
0.9	1	1	2	20.5%	4.09	9.82	8.90
0.9	2	1	2	12.7%	2.99	10.12	8.87
0.9	3	1	2	9.1%	2.33	10.31	8.88
0.9	0.5	2	1	20.5%	2.05	4.91	4.45
0.9	1	2	1	12.7%	1.49	5.06	4.44
0.9	2	2	1	7.1%	0.95	5.21	4.45
0.9	3	2	1	4.9%	0.70	5.29	4.46
0.5	0.5	1	1	20.8%	0.47	1.21	0.71
0.5	1	1	1	12.5%	0.35	1.30	0.63
0.5	2	1	1	6.7%	0.22	1.38	0.58
0.5	3	1	1	4.5%	0.16	1.41	0.55
0.5	0.5	2	1	16.4%	0.29	1.25	0.67
0.5	1	2	1	9.3%	0.20	1.34	0.60
0.5	2	2	1	4.8%	0.12	1.41	0.56
0.5	3	2	1	3.2%	0.08	1.43	0.54

Table 4.3: SWTA allocation rule (proportional case)

γ	σ	b	m	HWTA vs SWTA	Optimal vs HWTA
0.9	0.5	1	1	88.6%	0.5%
0.9	1	1	1	110.2%	3.7%
0.9	2	1	1	92.7%	10.6%
0.9	3	1	1	79.2%	15.3%
0.9	0.5	1	2	60.5%	0.1%
0.9	1	1	2	106.1%	1.6%
0.9	2	1	2	103.9%	6.8%
0.9	3	1	2	90.7%	11.2%
0.9	0.5	2	1	106.1%	1.6%
0.9	1	2	1	103.8%	6.8%
0.9	2	2	1	81.1%	14.6%
0.9	3	2	1	69.5%	19.1%
0.5	0.5	1	1	76.6%	0.4%
0.5	1	1	1	69.0%	1.1%
0.5	2	1	1	48.7%	2.9%
0.5	3	1	1	39.6%	3.7%
0.5	0.5	2	1	76.7%	0.8%
0.5	1	2	1	58.6%	1.6%
0.5	2	2	1	40.7%	3.6%
0.5	3	2	1	33.8%	4.2%

Table 4.4: Comparison between rules (% increase in buyer's payoff)

noisier performance measure. Overall, the improvement in the buyer's payoff from using the optimal rule over a HWTA one or from using a HWTA rule over a SWTA one is better for a larger discount factor. Apparently a handicap can be very effective for incentive provision.

4.4.2 Volume incentive under demand-independent effort cost

In Section 4.4.1 we have studied the optimal allocation rule and two simple WTA rules for the volume incentive design in the case of proportional effort cost. We have found that the optimal allocation rule is not WTA and is handicapped. Sometimes, a supplier's cost of investment is independent of her share of demand, such as a fixed investment in technology or innovation. Would the finding for the proportional effort cost still hold for the demand-independent effort cost? We will answer this question in this section by studying the buyer's problem in the case of $g(\beta) = 1$.

Allocation rules

We first study the buyer's optimal allocation rule for a finite-horizon problem, which maximizes his expected payoff over the horizon. We then investigate some simple heuristics for the infinite-horizon problem, and use numerical analysis to compare the buyer's expected payoff under different heuristics.

- Optimal allocation rule for finite-horizon problem

Before investigating the buyer's optimal allocation rule for an infinite-horizon problem, we first consider a finite-horizon problem with T periods and study the optimal rule for this problem, which is not necessarily a stationary rule. Let α_t denote Supplier 1's share in period t, e_t^{i*} and x_t^i denote Supplier i's target and realized performance in period t, and $\beta_{t+1} \in (\frac{1}{2}, 1]$ be the maximum share a supplier can have in period $t+1$. Also let $\Gamma_t = \sqrt{(\alpha_t)^2 + (1-\alpha_t)^2}$. Theorem 4.3 provides the optimal allocation rule for the finite-horizon problem in the case of demand-independent effort cost.

Theorem 4.3 *For supplier's cost function with $g(\beta) = 1$, the buyer's optimal allocation rule for a finite-horizon problem is a HWTA rule with the form:*

$$\beta_{t+1}^{\alpha}(x_t^1, x_t^2) = \begin{cases} \beta_{t+1} & \alpha_t(x_t^1 - e_t^{1*}) > (1-\alpha_t)(x_t^2 - e_t^{2*}) \\ 1-\beta_{t+1} & \alpha_t(x_t^1 - e_t^{1*}) < (1-\alpha_t)(x_t^2 - e_t^{2*}) \end{cases}, \tag{4.13}$$

where

$$\begin{aligned} e_t^{1*} &= \frac{\gamma \Delta v_{t+1}(\beta_{t+1})}{\sqrt{2\pi} b\sigma} \frac{\alpha_t}{\Gamma_t}, \\ e_t^{2*} &= \frac{\gamma \Delta v_{t+1}(\beta_{t+1})}{\sqrt{2\pi} b\sigma} \frac{1-\alpha_t}{\Gamma_t}, \end{aligned} \tag{4.14}$$

and

$$\Delta v_{t+1}(\beta_{t+1}) = v_{t+1}^1(\beta_{t+1}) - v_{t+1}^1(1-\beta_{t+1}),$$

with $v_{T+1}^1(\alpha_{T+1}) = \alpha_{T+1}m$ and $v_{T+1}^2(1-\alpha_{T+1}) = (1-\alpha_{T+1})m$. Under this rule, $\{(e_t^{1*}, e_t^{2*})\}_{t \in \{1,2,\ldots T\}}$ constitutes a subgame perfect Nash equilibrium.

- Simple heuristic for infinite-horizon problem

Since Theorem 4.3 has demonstrated that the optimal allocation rule for each period of a finite horizon takes the same form of a HWTA, we would expect the buyer's optimal stationary rule for an infinite horizon is also a HWTA one. Nonetheless, we shouldn't immediately jump at the conclusion that the optimal rule for an infinite horizon takes the same form as the optimal one for a finite horizon. Noting that with a bang-bang type of allocation rule, in the finite-horizon problem, the choice of the optimal maximum share for a period does not affect those optimal maximum shares and thus the suppliers' optimal payoffs in the subsequent periods; but in the infinite-horizon problem with a stationary allocation rule, the optimal maximum share for each period changes simultaneously. In the following analysis, we come back to the infinite-horizon problem. As the search for the optimal (WTA) rule is not straightforward, we shall demonstrate the key insights by examining WTA rules with the form

$$\beta_\alpha^{4*}(x_1, x_2) = \begin{cases} \widehat{\beta} & x_1 > k_\alpha x_2 + \theta_\alpha \\ 1 - \widehat{\beta} & x_1 < k_\alpha x_2 + \theta_\alpha \end{cases}, \tag{4.15}$$

where θ_α is to be determined, $k_\alpha \in (0, 1]$ for $\alpha \geq 0.5$ and $k_\alpha \geq 1$ for $\alpha < 0.5$, both θ_α and k_α are functions of α, and $\widehat{\beta} \in (\frac{1}{2}, 1]$. Let $\Upsilon_\alpha = \sqrt{1 + k_\alpha^2}$. Theorem 4.4 provides the optimal θ_α and the value functions of the buyer and the suppliers under the allocation rule (4.15).

Theorem 4.4 *For supplier's cost function with $g(\beta) = 1$, among the family of WTA allocation rules as defined in (4.15), the optimal rule has $\theta_\alpha^* = e_1^* - k_\alpha e_2^*$ for any α, where e_1^* and e_2^* are the optimal target performance levels of suppliers 1 and 2 under this optimal allocation rule,*

$$e_1^* = \frac{\gamma \Delta v^*(\widehat{\beta})}{\sqrt{2\pi} b\sigma} \frac{1}{\sqrt{k_\alpha^2 + 1}}, \tag{4.16}$$

$$e_2^* = \frac{\gamma \Delta v^*(\widehat{\beta})}{\sqrt{2\pi} b\sigma} \frac{k_\alpha}{\sqrt{k_\alpha^2 + 1}}, \tag{4.17}$$

with

$$\Delta v^*(\widehat{\beta}) = \begin{cases} \frac{2\pi b\sigma^2(1+k_{\widehat{\beta}}^2)}{\gamma^2(1-k_{\widehat{\beta}}^2)}\left(\sqrt{1 + \frac{\gamma^2 m(2\widehat{\beta}-1)}{\pi b\sigma^2}\frac{1-k_{\widehat{\beta}}^2}{1+k_{\widehat{\beta}}^2}} - 1\right) & k_{\widehat{\beta}} \neq 1 \\ m(2\widehat{\beta} - 1) & k_{\widehat{\beta}} = 1 \end{cases}. \tag{4.18}$$

$\theta^*_\alpha \neq 0$ for $k_\alpha \neq 1$, and $\theta^*_\alpha = 0$ for $k_\alpha = 1$. The suppliers' value functions

$$v^*_1(\alpha) = v^*_2(\alpha) = v^*(\alpha)$$

for any α due to symmetry,

$$v^*(\alpha) = m\alpha - \frac{(\gamma\Delta v^*(\widehat{\beta}))^2}{4\pi b\sigma^2(k^2_\alpha + 1)} + \frac{\gamma}{2}(v^*(\widehat{\beta}) + v^*(1-\widehat{\beta})), \tag{4.19}$$

and the buyer's value function is

$$v^*_B(\alpha) = \frac{\sqrt{2\pi}\sigma(1+k^2_{\widehat{\beta}})}{\gamma(1-\gamma)\Upsilon_\alpha(1-k^2_{\widehat{\beta}})}[\alpha + k_\alpha(1-\alpha)](\sqrt{1+\frac{\gamma^2 m(2\widehat{\beta}-1)}{\pi b\sigma^2}\frac{1-k^2_{\widehat{\beta}}}{1+k^2_{\widehat{\beta}}}} - 1).$$

Let $y_i = \frac{x_i - e^*_i}{\sigma}$ measure the standardized deviation of Supplier i's performance from the target. Then (4.15) with $\theta^*_\alpha = e^*_1 - k_\alpha e^*_2$ becomes

$$\widehat{\beta}^{4*}_\alpha(y_1, y_2) = \begin{cases} \widehat{\beta} & y_1 > k_\alpha y_2 \\ 1-\widehat{\beta} & y_1 < k_\alpha y_2 \end{cases}.$$

Under the allocation rule (4.15) with $\theta^*_\alpha = e^*_1 - k_\alpha e^*_2$, at the optimal effort levels, each supplier has equal chance to be the winner, but the whole (y_1, y_2) plane is split into two equal areas by the straight line $y_1 = k_\alpha y_2$ instead of the 45° line. From (4.16), (4.17) and (4.19), noting that the expected future payoff of the supplier with share α is $\frac{\gamma}{2}(v^*(\widehat{\beta}) + v^*(1-\widehat{\beta}))$, we can see that the allocation rule has an interesting property that it makes the two suppliers' expected future payoffs identical (the two suppliers have equal chance to be the winner) but provides the suppliers with different level of incentive if $k_\alpha \neq 1$. When $k_\alpha = 1$, due to the demand-independent effort cost, the suppliers' optimal effort levels are equal. The allocation rule (4.15) with $k_\alpha = 1$ is a SWTA one.

We are particularly interested in two special cases of $\beta^{4*}_\alpha(x_1, x_2)$, where $k_\alpha = \frac{1-\alpha}{\alpha}$ or 1. Corollaries 4.4 and 4.5 follow from Theorem 4.4 by letting $k_\alpha = \frac{1-\alpha}{\alpha}$ and 1 in (4.15), respectively. The effect of $k_\alpha \neq 1$ will be seen from the subsequent numerical analysis results.

Corollary 4.4 *Under the allocation rule $\beta^{4*}_\alpha(x_1, x_2)$ with $k_\alpha = \frac{1-\alpha}{\alpha}$, the optimal rule* *has*

$$\theta_\alpha^* = \frac{\gamma \Delta v^*(\widehat{\beta})}{\sqrt{2\pi} b\sigma} \frac{2\alpha - 1}{\alpha\sqrt{\widehat{\beta}^2 + (1-\widehat{\beta})^2}},$$

under this rule $e_\beta^* = \frac{\beta}{1-\beta} e_{1-\beta}^*$, *and the optimal* β^* *is a boundary solution.*

Corollary 4.5 *Under the allocation rule* $\beta_\alpha^{4*}(x_1, x_2)$ *with* $k_\alpha = 1$, *the optimal rule has* $\theta_\alpha^* = 0$ *for any* α, *and under this rule* $e_1^* = e_2^*$; *the optimal* β^* *is a boundary solution.*

Numerical analysis

We compare the effectiveness of two heuristics: a SWTA rule, and a HWTA one with $k_\alpha = \frac{1-\alpha}{\alpha}$. For all the cases but one in the numerical analysis here, the limit to a supplier's maximum share is in fact 1 (the buyer's total business) because both the Nash equilibrium condition and the suppliers' participation constraints set very loose boundaries to a supplier's share to be mathematically more than 1. In reality, there is often a minimum order quantity specified by a supplier. Therefore, in the analysis below, we impose a minimum share of 10% on each supplier. Table 4.5 compares the payoffs of the buyer and the suppliers under both rules. It is noted that for the case where $\gamma = 0.9, \sigma = 0.5, b = 1$ and $m = 2$, a supplier's minimum share is limited to 12% due to the Nash equilibrium condition, which causes the HWTA rule to work worse than the SWTA one. In all the other cases where the same minimum share applies to both rules, the HWTA rule provides better incentive in terms of the buyer's long-run discounted payoff. This is because the performance of the supplier with a larger share of business is more important to the buyer, and thus the buyer would often like to ensure that supplier to perform well in order to obtain a high aggregate performance of the two suppliers. For this purpose, the buyer can provide a stronger incentive to the supplier with a larger share by giving that supplier a higher award for performing above the expectation. Here a simple way to achieve this is to let $k_\alpha = \frac{1-\alpha}{\alpha}$, so that one unit of extra effort will be worth α units of overperformance for the supplier with a share of α, compared to $1-\alpha$ units for the other supplier.

However, this HWTA allocation rule may not always work better than a SWTA rule, as shown in Table 4.6. Here for the case where $\gamma = 0.9, \sigma = 0.5, b = 1$ and $m = 3$, under both rules, the Nash equilibrium condition sets a boundary to a supplier's minimum share, which is tighter under the HWTA rule. The first comparison of

				SWTA				HWTA				
γ	σ	b	m	v_B^*	V	$v^*(\overline{\beta})$	$v^*(\underline{\beta})$	v_B^*	V	$v^*(\overline{\beta})$	$v^*(\underline{\beta})$	%increase
0.9	0.5	1	1	4.06	4.17	4.57	3.77	4.44	4.40	4.74	4.06	9.3%
0.9	1	1	1	2.03	4.79	5.19	4.39	2.48	4.81	5.19	4.43	22.2%
0.9	2	1	1	1.02	4.95	5.35	4.55	1.28	4.95	5.34	4.55	26.5%
0.9	3	1	1	0.68	4.98	5.38	4.58	0.86	4.96	5.38	4.55	27.4%
0.9	0.5	1	2	8.12	6.70	7.50	5.90	7.45	8.23	8.82	7.65	−8.4%
0.9	1	1	2	4.06	9.17	9.97	8.37	4.76	9.31	10.04	8.58	17.3%
0.9	2	1	2	2.03	9.79	10.59	8.99	2.54	9.80	10.58	9.02	25.0%
0.9	3	1	2	1.35	9.91	10.71	9.11	1.72	9.91	10.70	9.12	26.7%
0.9	0.5	2	1	2.03	4.59	4.99	4.19	2.38	4.65	5.02	4.29	17.3%
0.9	1	2	1	1.02	4.90	5.30	4.50	1.27	4.90	5.29	4.51	25.0%
0.9	2	2	1	0.51	4.97	5.37	4.57	0.65	4.97	5.37	4.58	27.3%
0.9	3	2	1	0.34	4.99	5.39	4.59	0.43	4.99	5.39	4.59	27.7%
0.5	0.5	1	1	0.45	0.95	1.35	0.55	0.55	0.95	1.33	0.58	21.0%
0.5	1	1	1	0.23	0.99	1.39	0.59	0.28	0.99	1.38	0.59	26.1%
0.5	2	1	1	0.11	1.00	1.40	0.60	0.14	1.00	1.40	0.60	27.6%
0.5	3	1	1	0.08	1.00	1.40	0.60	0.10	1.00	1.40	0.60	27.4%
0.5	0.5	2	1	0.23	0.97	1.37	0.57	0.28	0.98	1.36	0.59	24.3%
0.5	1	2	1	0.11	0.99	1.39	0.59	0.14	0.99	1.39	0.60	27.1%
0.5	2	2	1	0.06	1.00	1.40	0.60	0.07	1.00	1.40	0.60	27.8%
0.5	3	2	1	0.04	1.00	1.40	0.60	0.05	1.00	1.40	0.60	28.0%

Table 4.5: Comparison of SWTA and HWTA (independent case)

	$\underline{\beta}$	v_B^*	V	$v^*(\overline{\beta})$	$v^*(\underline{\beta})$	HWTM vs SWTM
SWTA	11.7%	11.66	8.20	9.35	7.06	−38.1%
	23.5%	8.09	11.73	12.53	10.93	−10.7%
HWTA	23.5%	7.22	12.97	13.60	12.34	

Table 4.6: Comparison of SWTA and HWTA (independent case)

these two rules is by using the individual minimum share, 11.7% for the SWTA rule and 23.5% for the HWTA one, which shows a 38.1% lower of the buyer's long-run discounted payoff under the HWTA rule than that under the SWTA rule. The second comparison is by using the same minimum share, 23.5%, which still shows a 10.8% lower of the buyer's payoff under the HWTA rule. This is because the two suppliers' incentives also come from $\Delta v^*(\widehat{\beta})$, the gain in a supplier's payoff from a high starting share $\widehat{\beta}$ instead of a low one $1 - \widehat{\beta}$. The HWTA rule induces more effort from a supplier with a larger share, which results in higher effort cost and smaller $\Delta v^*(\widehat{\beta})$; while the SWTA rule always induces equal supplier effort and a $\Delta v^*(\widehat{\beta})$ independent of $\widehat{\beta}$. Therefore, in some situations a non-handicapped SWTA rule can perform better than the HWTA rule with $k_\alpha = \frac{1-\alpha}{\alpha}$.

4.5 Conclusions and Future Work

In this chapter, we have studied the design of performance-based volume incentive schemes, a type of incentive scheme widely used in practice but not well studied yet in the literature. We have considered a buyer repeatedly outsourcing a service or a product from two suppliers, under the assumptions of risk neutral parties, reliable and uncapacitated supply, no setup fee, zero switching cost, and unobservable supplier effort. We have focused on two types of supplier effort cost: the proportional case where the effort cost is proportional to the share of demand, and the demand-independent case where the effort cost is independent of the share. We have found that to maintain suppliers' competition over time, the optimal demand allocation rule is dependent on the suppliers' current shares, and is not a simple rank-order tournament. Even when the supplier's effort cost is demand independent, each supplier's current share of business still plays an important role in the allocation rule, and using a handicap can greatly improve the efficiency of volume incentive schemes. Handicapping plays two roles in incentive provision. It can level the field and enhance the suppliers' competition when the 'more important' supplier is at a disadvantage in the competition, as seen in the case of proportional effort cost. Alternatively, it gives the 'more important' supplier an advantage and makes that supplier work harder than the other supplier, as seen in the case of independent effort cost. Numerical results have indicated that for proportional effort cost and performance with small variability, a handicapped rank-order tournament induces an outcome which is near optimal.

Other factors can be considered in the study of volume incentive mechanism design. For example, changing demand may result in a supplier's switching cost such as the cost of adjusting the dedicated capacity to meet the demand and avoid low utilization; a buyer may be concerned about the variability in the suppliers' performance and thus has a mean-variance type of objective; or the buyer faces unreliable supply such as supply disruption or random yield. Since under our simple assumptions the problem and the analysis are already complex, the analysis with the above factors would be better conducted using simulation or experiments.

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Appendix A

Proof for Chapter 2

Proof of Proposition 2.1

We first compute the variance of η_R^W.

For $L > 0$ and $0 \leq W \leq L$,

$E(\eta_R^W) = \sum_{t=1}^{R} E(X_t^W) = RF_{L+1-W}(S)$ $E((\eta_R^W)^2) = E[(\sum_{i=1}^{R} X_i^W)(\sum_{j=1}^{R} X_j^W)]$

$= \sum_{i=1}^{R}\sum_{j=1}^{R} E(X_i^W X_j^W) = \sum_{i=1}^{R}\sum_{j=1}^{R} E(\mathbf{1}\{D[i-L, i+1-W) \leq S\} \times \mathbf{1}\{D[j-L, j+1-W) \leq S\})$

The number of (i,i) terms is R, and the sum of these terms is

$M_1 = RE(\mathbf{1}\{D(L+1-W) \leq S\}) = RF_{L+1-W}(S)$.

The number of (i,j) terms with X_i^W and X_j^W independent is

$2 \sum_{i=1}^{R-(L+1-W)} i = \sum_{i=1}^{R-(L+1-W)} i + \sum_{j=1}^{R-(L+1-W)} (R-L+W-j)$

$= \sum_{i=1}^{R-(L+1-W)} (i+R-L+W-i) = (R-L+W)(R-L+W-1)$.

This is the number of pairs of X_i^W and X_j^W which differ by at least $L+1-W$ periods.

The sum of these terms is

$M_2 = (R-L+W)(R-L+W-1)E(\mathbf{1}\{D(L+1-W) \leq S\})E(\mathbf{1}\{D(L+1-W) \leq S\})$ $= (R-L+W)(R-L+W-1)(F_{L+1-W}(S))^2$.

The number of (i,j) terms with n $(1 \leq n \leq L-W)$ periods of demands in common is $2(R-(L+1)+n+W)$. This is the number of pairs of X_i^W and X_j^W which differ by exactly $L+1-n-W$ periods. For $i < j$, $j = i+L+1-n-W$,

$E(X_i^W X_j^W) = \Pr\{D[i-L, i+1-W) \leq S, D[j-L, j+1-W) \leq S\}$

$= \Pr\{D[i-L, j-L) + D(n) \leq S, D(n) + D[i+1-W, j+1-W) \leq S\}$

$= \int_0^S (\Pr\{D(L+1-n-W) \le S-x\})^2 dF_n(x)$
$= \int_0^S (F_{L+1-n-W}(S-x))^2 dF_n(x)$

$Cov(D[i-L, i+1-W), D[j-L, j+1-W))$
$= Cov(D[i-L, j-L) + D[j-L, i+1-W), D[j-L, i+1-W) + D[i+1-W, j+1-W))$
$= Var(D(n)) > 0$ and increases with n
$\Rightarrow D[i-L, i+1-W)$ and $D[j-L, j+1-W)$ are positively correlated,
and $\int_0^S (F_{L+1-n-W}(S-x))^2 dF_n(x)$ increases with $n \Rightarrow$
$E(X_i^W X_j^W) = \Pr\{D[i-L, i+1-W) \le S\} \Pr\{D[j-L, j+1-W) \le S | D[i-L, i+1-W) \le S\} \ge F_{L+1-W}(S) \Pr\{D[j-L, j+1-W) \le S\} = (F_{L+1-W}(S))^2$
So

$$E(X_i^W X_j^W) \ge (F_{L+1-W}(S))^2. \tag{A.1}$$

The sum of the terms with each pair differing by exactly $L+1-n-W$ periods is
$M_3 = \sum_{n=1}^{L-W} 2(R-(L+1)+n+W) \int_0^S (F_{L+1-n-W}(S-x))^2 dF_n(x)$.

So $E((\eta_R^W)^2) = M_1 + M_2 + M_3$, and
$(\sigma_R^W)^2 = Var(\eta_R^W) = E((\eta_R^W)^2) - (E(\eta_R^W))^2$
$= RF_{L+1-W}(S) + (R-L+W)(R-L+W-1)(F_{L+1-W}(S))^2$
$+ \sum_{n=1}^{L-W} 2(R-(L+1)+n+W) \int_0^S (F_{L+1-n-W}(S-x))^2 dF_n(x) - R^2(F_{L+1-W}(S))^2$
$= RF_{L+1-W}(S) - [(L-W)(2R-L+W-1)+R](F_{L+1-W}(S))^2$
$+ \sum_{n=1}^{L-W} 2(R-(L+1)+n+W) \int_0^S (F_{L+1-n-W}(S-x))^2 dF_n(x)$.
By (A.1), $(\sigma_R^W)^2 \ge RF_{L+1-W}(S) - [(L-W)(2R-L+W-1)+R](F_{L+1-W}(S))^2 + \sum_{n=1}^{L-W} 2(R-(L+1)+n+W)(F_{L+1-W}(S))^2 = RF_{L+1-W}(S)(1-F_{L+1-W}(S))$.

$\lim_{R\to\infty} \frac{(\sigma_R^W)^2}{R^{2/3}} \ge \lim_{R\to\infty} R^{1/3} F_{L+1-W}(S)(1-F_{L+1-W}(S)) = \infty$.

The sequence $\{X_t\}$ is $(L+1-W)$-dependent because any subsequence $\{X_{t_j}, j \ge 1\} \subset \{X_t\}$, with $t_j + L + 1 - W < t_{j+1}$ for every $j \ge 1$, is a sequence of independent random variables. Moreover, $X_t \le 1$ for all t.

Applying Theorem 7.3.1 (Chung 1974 page 214), $\frac{\eta_R^W - E(\eta_R^W)}{\sigma_R^W}$ converges in distribution to a standard normal random variable Z, $Z \sim N(0,1)$ as R approaches ∞.

Proof of Proposition 2.2

$Var(A_R^W) = \frac{(\sigma_R^W)^2}{R^2}$,

$Var(A_{R+1}^W) - Var(A_R^W) = \frac{(\sigma_{R+1}^W)^2}{(R+1)^2} - \frac{(\sigma_R^W)^2}{R^2}$

$= -(\frac{1}{R} - \frac{1}{R+1})F_{L+1-W}(S) - [(L-W)(\frac{2}{R+1} - \frac{2}{R} + \frac{L+1-W}{R^2} - \frac{L+1-W}{(R+1)^2}) + \frac{1}{R+1} - \frac{1}{R}](F_{L+1-W}(S))^2 - 2\sum_{n=1}^{L-W}(\frac{1}{R} - \frac{1}{R+1} + \frac{L+1-n-W}{(R+1)^2} - \frac{L+1-n-W}{R^2})\int_0^S (F_{L+1-n-W}(S-x))^2 dF_n(x)$

$= -(\frac{1}{R} - \frac{1}{R+1})F_{L+1-W}(S)(1 - F_{L+1-W}(S))$

$-2\sum_{n=1}^{L-W}(\frac{1}{R} - \frac{1}{R+1} + \frac{L+1-n-W}{(R+1)^2} - \frac{L+1-n-W}{R^2})(\int_0^S (F_{L+1-n-W}(S-x))^2 dF_n(x) - (F_{L+1-W}(S))^2)$

$= -(\frac{1}{R} - \frac{1}{R+1})F_{L+1-W}(S)(1 - F_{L+1-W}(S)) - \Upsilon$

where $\Upsilon = 2\sum_{n=1}^{L-W} \frac{R(R+1)-(2R+1)(L+1-n-W)}{R^2(R+1)^2}(\int_0^S (F_{L+1-n-W}(S-x))^2 dF_n(x) - (F_{L+1-W}(S))^2)$.

Let $\Delta_n = R(R+1) - (2R+1)(L+1-n-W)$. Δ_n is increasing in n.

By the assumption $R > L$, $R \geq L+1$, so

$\Delta_1 = R(R+1) - (2R+1)(L-W)$ can be negative when R is small, for example, $R = L+1$;

$\Delta_L \geq R^2 - R - 1 > 0$ because $R > 1$.

Let $m_n = \int_0^S (F_{L+1-n-W}(S-x))^2 dF_n(x) - (F_{L+1-W}(S))^2$.

From the proof of Proposition 2.1, $m_n > 0$ and $\int_0^S (F_{L+1-n-W}(S-x))^2 dF_n(x)$ increases with n.

So if $\Delta_1 < 0$, let $\overline{n} = \max\{n|\Delta_n < 0\}$,

$\Upsilon \geq 2m_{\overline{n}} \sum_{n=1}^{L-W} \frac{R(R+1)-(2R+1)(L+1-n-W)}{R^2(R+1)^2} = m_{\overline{n}}(L-W)\frac{(2R+2)R-(2R+1)(L+1-W)}{R^2(R+1)^2} \geq 0$;

if $\Delta_1 \geq 0$, then $\Upsilon \geq 0$.

So $Var(A_{R+1}^W) - Var(A_R^W) < 0$, $Var(A_R^W)$ is decreasing in R.

Proof of Proposition 2.3

Lump-sum penalty SLA, $L = W = 0$: optimal K given α

$V_0(S) = hE[S - D(1)]^+ + \frac{K}{R}\sum_{i=0}^{R\alpha} \Pr\{\eta_R^0 = i|S\}$, where $\Pr\{\eta_R^0 = i|S\}$ is given by (2.5).

1. If the demand and S are continuous, then

$\frac{dV_0(S)}{dS} = hF_1(S) + \frac{K}{R}\sum_{i=0}^{R\alpha}\binom{R}{i}[i(F_1(S))^{i-1}(1-F_1(S))^{R-i} - (R-i)(F_1(S))^i(1-F_1(S))^{R-i-1}]f_1(S)$

$= hF_1(S) - K\binom{R-1}{R\alpha}(F_1(S))^{R\alpha}(1-F_1(S))^{R(1-\alpha)-1}f_1(S)$ gives

$$\frac{dV_0(S)}{dS} = F_1(S)[h - K\binom{R-1}{R\alpha}(F_1(S))^{R\alpha-1}(1-F_1(S))^{R(1-\alpha)-1}f_1(S)]. \tag{A.2}$$

Given any $\alpha < F_1(S^*)$, $\frac{dV_0(S^*)}{dS} = 0 \Rightarrow$the optimal

$$K^*(\alpha) = \frac{h}{\binom{R-1}{R\alpha}(F_1(S^*))^{R\alpha-1}(1-F_1(S^*))^{R(1-\alpha)-1}f_1(S^*)}, \tag{A.3}$$

where $f_1(\cdot)$ is the pdf of single-period demand.

2. If the demand and S are discrete, then given any α, to induce the supplier to choose S^*, $K^*(\alpha)$ should be chosen such that

$$V_0(S^*+1) - V_0(S^*) \geq 0 \text{ and } V_0(S^*-1) - V_0(S^*) \geq 0. \tag{A.4}$$

Because $E[S-d]^+ = \sum_{d=0}^{S}(S-d)f_1(d)$ and $f_1(d) = \Pr\{D(1) = d\}$, $(A.4) \Rightarrow$

$hF_1(S^*) + \frac{K}{R}\sum_{i=0}^{R\alpha}(\Pr\{\eta_R^0 = i|S^*+1\} - \Pr\{\eta_R^0 = i|S^*\}) \geq 0,$

and $-hF_1(S^*-1) + \frac{K}{R}\sum_{i=0}^{R\alpha}(\Pr\{\eta_R^0 = i|S^*-1\} - \Pr\{\eta_R^0 = i|S^*\}) \geq 0.$

It can be seen from $(A.2)$ that $\sum_{i=0}^{R\alpha}\Pr\{\eta_R^0 = i|S\}$ is decreasing in S, so

$\sum_{i=0}^{R\alpha}(\Pr\{\eta_R^0 = i|S^*\} - \Pr\{\eta_R^0 = i|S^*+1\}) > 0,$

and $\sum_{i=0}^{R\alpha}(\Pr\{\eta_R^0 = i|S^*-1\} - \Pr\{\eta_R^0 = i|S^*\}) > 0.$

So the interval of optimal $K^*(\alpha)$ is

$[\underline{K}^*(\alpha), \overline{K}^*(\alpha)] = [\frac{RhF_1(S^*-1)}{\sum_{i=0}^{R\alpha}(\Pr\{\eta_R^0=i|S^*-1\}-\Pr\{\eta_R^0=i|S^*\})}, \frac{RhF_1(S^*)}{\sum_{i=0}^{R\alpha}(\Pr\{\eta_R^0=i|S^*\}-\Pr\{\eta_R^0=i|S^*+1\})}].$

Lump-sum penalty SLA, $L > 0, \mathbf{0} \leq W \leq L$ *: optimal* K *given* α

Let $z_i(x) = \frac{i+0.5-RF_{L+1-W}(x)}{\sigma_R^W}$.

From (2.3) and (2.6),

$V_L(S) = hE[S - D(L+1-W)]^+ + \frac{K}{R}\Phi(z_{R\alpha}(S))$.

1. If the demand and S are continuous, then

$\frac{dV_L(S)}{dS} = hF_{L+1-W}(S) + \frac{K}{R}\phi(z_{R\alpha}(S))\frac{dz_{R\alpha}(S)}{dS}$.

Letting $\frac{dV_L(S^*)}{dS} = 0$ we can obtain $K^*(\alpha) = -\frac{RhF_{L+1-W}(S^*)}{\phi(z_{R\alpha}(S))\frac{dz_{R\alpha}(S)}{dS}}$.

2. If the demand and S are discrete, given any α, to induce the supplier to choose S^*, $K^*(\alpha)$ should be chosen such that

$V_L(S^*+1) - V_L(S^*) \geq 0$ and $V_L(S^*-1) - V_L(S^*) \geq 0 \Rightarrow$

$hF_{L+1-W}(S^*) + \frac{K}{R}[\Phi(z_{R\alpha}(S^*+1)) - \Phi(z_{R\alpha}(S^*))] \geq 0,$

and

$-hF_{L+1-W}(S^*-1) + \frac{K}{R}[\Phi(z_{R\alpha}(S^*-1)) - \Phi(z_{R\alpha}(S^*))] \geq 0.$

It follows that $[\underline{K}^*(\alpha), \overline{K}^*(\alpha)] = [\frac{RhF_{L+1-W}(S^*-1)}{\Phi(z_{R\alpha}(S^*-1))-\Phi(z_{R\alpha}(S^*))}, \frac{RhF_{L+1-W}(S^*)}{\Phi(z_{R\alpha}(S^*))-\Phi(z_{R\alpha}(S^*+1))}]$.

Linear penalty SLA, $L = W = 0$ *: optimal* K *given* α

$V_0(S) = hE[S - D(1)]^+ + \frac{K}{R}\sum_{i=0}^{R\alpha}(R\alpha+1-i)\Pr\{\eta_R^0 = i|S\},$

where $\Pr\{\eta_R^0 = i|S\}$ is given by (2.5).

1. If the demand and S are continuous, then

$\frac{dV_0(S)}{dS} = hF_1(S)+\frac{K}{R}\sum_{i=0}^{R\alpha}(R\alpha+1-i)\binom{R}{i}[i(F_1(S))^{i-1}(1-F_1(S))^{R-i}-(R-i)(F_1(S))^i(1-F_1(S))^{R-i-1}]f_1(S) \Rightarrow$

$$\frac{dV_0(S)}{dS} = hF_1(S) - K\sum_{i=0}^{R\alpha}\binom{R-1}{i}(F_1(S))^i(1-F_1(S))^{R-i-1}f_1(S). \tag{A.5}$$

Given $\alpha < F_1(S^*)$, $\frac{dV_0(S^*)}{dS} = 0 \Rightarrow$

$$K^*(\alpha) = \frac{hF_1(S^*)}{\sum_{i=0}^{R\alpha}\binom{R-1}{i}(F_1(S^*))^i(1-F_1(S^*))^{R-i-1}f_1(S^*)}. \tag{A.6}$$

2. If the demand and S are discrete, then given any α, to induce the supplier to choose S^*, $K^*(\alpha)$ should be chosen such that

$V_0(S^*+1)-V_0(S^*)\geq 0$ and $V_0(S^*-1)-V_0(S^*)\geq 0 \Rightarrow$

$$hF_1(S^*)+\frac{K}{R}\sum_{i=0}^{R\alpha}(R\alpha+1-i)(\Pr\{\eta_R^0=i|S^*+1\}-\Pr\{\eta_R^0=i|S^*\})\geq 0, \quad \text{(A.7)}$$

and

$$-hF_1(S^*-1)+\frac{K}{R}\sum_{i=0}^{R\alpha}(R\alpha+1-i)(\Pr\{\eta_R^0=i|S^*-1\}-\Pr\{\eta_R^0=i|S^*\})\geq 0. \quad \text{(A.8)}$$

It can be seen from $(A.5)$ that the second term in $V_0(S)$ is decreasing in S, so $\sum_{i=0}^{R\alpha}(R\alpha+1-i)(\Pr\{\eta_R^0=i|S^*\}-\Pr\{\eta_R^0=i|S^*+1\})>0$,

and $\sum_{i=0}^{R\alpha}(R\alpha+1-i)(\Pr\{\eta_R^0=i|S^*-1\}-\Pr\{\eta_R^0=i|S^*\})>0$.

Then it follows from $(A.7)$ and $(A.8)$ that $[\underline{K}^*(\alpha),\overline{K}^*(\alpha)]=$
$[\frac{RhF_1(S^*-1)}{\sum_{i=0}^{R\alpha}(R\alpha+1-i)(\Pr\{\eta_R^0=i|S^*-1\}-\Pr\{\eta_R^0=i|S^*\})},\frac{RhF_1(S^*)}{\sum_{i=0}^{R\alpha}(R\alpha+1-i)(\Pr\{\eta_R^0=i|S^*\}-\Pr\{\eta_R^0=i|S^*+1\})}]$.

Linear penalty SLA, $L>0, \mathbf{0}\leq W\leq L$ *: optimal* K *given* α

$V_L(S)=hE[S-D(L+1-W)]^+ +\frac{K}{R}\sum_{i=0}^{R\alpha}(R\alpha+1-i)\Pr\{\eta_R^W=i|S\}$

1. If the demand and S are continuous, then using the results in Proposition 2.1, the approximation for $V_L(S)$ is

$V_L(S)=hE[S-D(L+1-W)]^+ +\frac{K}{R}\int_{x\leq R\alpha}(R\alpha-x)d\Phi(\frac{x-RF_{L+1-W}(S)}{\sigma_R^W})$

$=hE[S-D(L+1-W)]^+ +\frac{K}{R}\int_{x\leq R\alpha}\Phi(\frac{x-RF_{L+1-W}(S)}{\sigma_R^W})dx$

$\Rightarrow$

$\frac{dV_L(S)}{dS}=hF_{L+1-W}(S)+\frac{K}{R}\int_{x\leq R\alpha}\frac{d}{dS}\Phi(\frac{x-RF_{L+1-W}(S)}{\sigma_R^W})dx$

$=hF_{L+1-W}(S)+\frac{K}{R}[-Rf_{L+1-W}(S)\Phi(\frac{R\alpha-RF_{L+1-W}(S)}{\sigma_R^W})+\frac{d\sigma_R^W}{dS}\phi(\frac{R\alpha-RF_{L+1-W}(S)}{\sigma_R^W})]$.

Letting $\frac{dV_L(S^*)}{dS}=0$ we can obtain

$K^*(\alpha)=\frac{RhF_{L+1-W}(S^*)}{Rf_{L+1-W}(S)\Phi(\frac{R\alpha-RF_{L+1-W}(S^*)}{\sigma_R^W})-\frac{d\sigma_R^W}{dS}\phi(\frac{R\alpha-RF_{L+1-W}(S^*)}{\sigma_R^W})}$, where σ_R^W is given by (2.1).

2. If the demand and S are discrete, given any α, to induce the supplier to choose

S^*, $K^*(\alpha)$ should be chosen such that

$V_L(S^*+1) - V_L(S^*) \geq 0$ and $V_L(S^*-1) - V_L(S^*) \geq 0$

$\Rightarrow hF_{L+1-W}(S^*) + \frac{K}{R}\sum_{i=0}^{R\alpha}(R\alpha+1-i)(\Pr\{\eta_R^W = i|S^*+1\} - \Pr\{\eta_R^W = i|S^*\}) \geq 0,$

and

$-hF_{L+1-W}(S^*-1) + \frac{K}{R}\sum_{i=0}^{R\alpha}(R\alpha+1-i)(\Pr\{\eta_R^W = i|S^*-1\} - \Pr\{\eta_R^W = i|S^*\}) \geq 0.$

So $[\underline{K}^*(\alpha), \overline{K}^*(\alpha)]$

$= [\frac{RhF_{L+1-W}(S^*-1)}{\sum_{i=0}^{R\alpha}(R\alpha+1-i)(\Pr\{\eta_R^W=i|S^*-1\}-\Pr\{\eta_R^W=i|S^*\})}, \frac{RhF_{L+1-W}(S^*)}{\sum_{i=0}^{R\alpha}(R\alpha+1-i)(\Pr\{\eta_R^W=i|S^*\}-\Pr\{\eta_R^W=i|S^*+1\})}],$

where $\Pr\{\eta_R^W = i|S^*\}$ is given by (2.6).

Proposition 2.3 follows from all the above derivations for (α, K) candidates.

Derivation for Section 2.5.2 Unimodality of supplier's objective function

The derivation below is based on continuous-valued demand and base-stock level S.

Lump-sum penalty SLA, $L = 0$:

Substituting $(A.3)$ into $(A.2) \Rightarrow$

at α and $K^*(\alpha)$, $\frac{dV_0(S)}{dS} = hF_1(S)(1 - \frac{(F_1(S))^{R\alpha-2}(1-F_1(S))^{R(1-\alpha)}f_1(S)}{(F_1(S^*))^{R\alpha-2}(1-F_1(S^*))^{R(1-\alpha)}f_1(S^*)})$

$\frac{d[(F_1(S))^{R\alpha-2}(1-F_1(S))^{R(1-\alpha)}f_1(S)]}{dS} = (F_1(S))^{R\alpha-3}(1-F_1(S))^{R(1-\alpha)-1}[(f_1(S))^2(R\alpha-2-(R-2)F_1(S)) + f_1'(S)F_1(S)(1-F_1(S))].$

$\alpha < F_1(S^*), F_1'(S) > 0, 0 \leq F_1(S) \leq 1$

$\Rightarrow R\alpha - 2 - (R-2)F_1(S) = R(\alpha - F_1(S)) - 2(1-F_1(S)) < 0$ for $S \geq S^*$;

moreover, $\frac{R\alpha-2}{R-2} < \alpha < F_1(S^*), F_1'(S) > 0 \Rightarrow F_1^{-1}(\frac{R\alpha-2}{R-2}) < S^*$

$\Rightarrow R\alpha - 2 - (R-2)F_1(S) < 0$ for $F_1^{-1}(\frac{R\alpha-2}{R-2}) < S < S^*$.

Because it is assumed that the distribution of single-period demand is unimodal, if $f_1'(S^*) < 0$, then $f_1'(S) < 0$ for $S \geq S^*$. This is true for distributions in the location-scale family and Poisson distribution with $S^* > \lambda$. Because $f_1(S)$ is continuous,

for $S > S^*$, $\frac{d[(F_1(S))^{R\alpha-2}(1-F_1(S))^{R(1-\alpha)}f_1(S)]}{dS} < 0$ and $\frac{dV_0(S)}{dS} > 0$; $\frac{dV_0(S^*)}{dS} = 0$, and

for $S < S^*$ and S close to S^*, $\frac{d[(F_1(S))^{R\alpha-2}(1-F_1(S))^{R(1-\alpha)}f_1(S)]}{dS} < 0$, and so $\frac{dV_0(S)}{dS} < 0$.

Thus S^* is a local optimum.

But $V_0(S)$ may be not unimodal. For small S and $S < S^*$, $\frac{d[(F_1(S))^{R\alpha-2}(1-F_1(S))^{R(1-\alpha)}f_1(S)]}{dS} > 0$. Depending on the value of S^* and $R\alpha$, it is possible that for $S < S^*$, $\frac{dV_0(S)}{dS} > 0$. This can be seen from Figure 2.1.

Linear penalty SLA, $L = 0$:

Substituting $(A.6)$ into $(A.5) \Rightarrow$ at α and $K^*(\alpha)$,

$$\frac{dV_0(S)}{dS} = h[F_1(S) - F_1(S^*)\frac{\sum_{i=0}^{R\alpha-1}\binom{R-1}{i}(F_1(S))^i(1-F_1(S))^{R-i-1}f_1(S)}{\sum_{i=0}^{R\alpha-1}\binom{R-1}{i}(F_1(S^*))^i(1-F_1(S^*))^{R-i-1}f_1(S^*)}].$$

Let $\Lambda_1 = \sum_{i=0}^{R\alpha-1}\binom{R-1}{i}(F_1(S))^i(1-F_1(S))^{R-i-1}$.

$$\frac{d\Lambda_1}{dS} = -(R-1)\binom{R-2}{R\alpha-1}(F_1(S))^{R\alpha-1}(1-F_1(S))^{R(1-\alpha)-1}f_1(S) < 0. \qquad \text{(A.9)}$$

Using the same assumption for the case of lump-sum penalty SLA and $L = 0$, $f_1'(S^*) < 0$,

1. for $S > S^*$, $F_1(S) > F_1(S^*)$ and $f_1(S) < f_1(S^*)$, then together with $(A.9)$, $\frac{dV_0(S)}{dS} > 0$ for $S > S^*$;

2. for $S < S^*$, $F_1(S) < F_1(S^*)$, and by $(A.9)$,

$$\sum_{i=0}^{R\alpha-1}\binom{R-1}{i}(F_1(S^*))^i(1-F_1(S^*))^{R-i-1} < \sum_{i=0}^{R\alpha-1}\binom{R-1}{i}(F_1(S))^i(1-F_1(S))^{R-i-1}, \qquad \text{(A.10)}$$

for S close to S^*, because $f_1'(S)$ is continuous, $f_1'(S) < 0$ and $f_1(S) > f_1(S^*)$, so $\frac{dV_0(S)}{dS} < 0$ for $S < S^*$ and close to S^*;

for $S < S^*$ and not close to S^*, $\frac{dV_0(S)}{dS} < 0$ if S satisfies

$$\frac{f_1(S^*)}{F_1(S^*)}\sum_{i=0}^{R\alpha-1}\binom{R-1}{i}(F_1(S^*))^i(1-F_1(S^*))^{R-i-1}$$
$$< \frac{f_1(S)}{F_1(S)}\sum_{i=0}^{R\alpha-1}\binom{R-1}{i}(F_1(S))^i(1-F_1(S))^{R-i-1}. \qquad \text{(A.11)}$$

The above inequality depends on the value of S^* and $R\alpha$. Because $\frac{1}{F_1(S^*)} < \frac{1}{F_1(S)}$ and $(A.10)$, if S^* is sufficiently large so that $\frac{f_1(S)}{f_1(S^*)}$ is not too small, then $(A.11)$ will hold.

Therefore, S^* is a local optimum. If $(A.11)$ holds for all $S < S^*$, then $V_0(S)$ is unimodal and S^* is a global optimum.

Dynamic program for $L = 0$ (Section 2.6.1)

Compared to the case of $L = 1$, the definition of I_t is modified as

$I_t =$ the supplier's inventory on hand at the beginning of period t, before an order is placed in period t.

Because the supplier does not incur the backorder cost, for any nonpositive net inventory in a period, the supplier has the same immediate cost (zero inventory holding cost). Due to a zero lead time, the inventory performance in any period is determined by the base-stock level rather than the net inventory in that period. Note that a zero base-stock level dominates any negative base-stock levels because the inventory costs in both cases are zero and the supplier's performance under the former choice may be better than that under the latter when the demand in a period is zero. Thus in the state space, we can use 0 to represent all the states of nonpositive inventory.

The state space is $\{(\eta_{t-1}, I_t) : 0 \le \eta_{t-1} \le t-1, 0 \le I_t \le \overline{S}\}$ where $1 \le t \le R+1$ and $\overline{S}$ is a large number such that $F_1(\overline{S}) \approx 1$.

Actions (inventory order-up-to level in a period): $S \in \{0, 1, 2, ..., \overline{S}\}$

State transition:

$$I_{t+1} = [S_t - d_t]^+, \eta_t = \begin{cases} \eta_{t-1} + 1 & \text{if } S_t \ge d_t \\ \eta_{t-1} & \text{if } S_t < d_t \end{cases}.$$

Rewards:

$$r_t((i, I_t), S_t) = hE[S_t - d_t]^+ \qquad 1 \le t \le R,$$

and the definition of $r_{R+1}((i, I_{R+1}))$ is the same as that in (2.7). Notice that in this single-review-phase model there is no consequence of having closing inventory I_{R+1}.

Transition probabilities:

$$p_t((j, I)|(i, u), S) = \begin{cases} \Pr\{D(1) = S - I\} & \text{if } j = i+1, S \ge u, I \ge 0 \\ \Pr\{D(1) > S\} & \text{if } j = i, S \ge u, I = 0 \\ 0 & \text{otherwise} \end{cases}.$$

In period t, for the order-up-to level $S_t \ge I_t$,

$$\pi_t^1(S_t|i, I_t) = hE[S_t - d_t]^+ + E\{\pi_{t+1}^1(i+1, S_t - d_t)|S_t \ge d_t\} + \overline{F}_1(S_t)\pi_{t+1}^1(i, 0).$$

The optimal cost to go from period t is $\pi_t^1(i, I_t) = \min_{S_t \ge I_t} \pi_t^1(S_t|i, I_t)$.

The dynamic program is

$\pi^1_{R+1}(i, I_{R+1}) = r_{R+1}((i, I_{R+1}))$;

for $1 \leq t \leq R$: $\pi^1_t(i, I_t) = \min_{S_t \geq I_t} \pi^1_t(S_t | i, I_t)$.

Appendix B

Proof for Chapter 3

Derivation of formula (3.2)

The waiting time distribution given S and L is derived as follows.

Let w_t denote the waiting time of a demand at time t. For the demand at time t to be filled by time $t + y$, the inventory position at time $t + y - L$, denoted by $IP(t + y - L)$, should satisfy the demands occuring in the interval $(t + y - L, t]$. Therefore, the distribution of w_t given S and L is

$\Pr\{w_t \leq y|S, L\} = \Pr\{D(t + y - L, t] \leq IP(t + y - L)\}$,

where $D(t+y-L, t]$ is the demand during $(t+y-L, t]$, and $IP(t+y-L)$ is the constant S. Note that $D(t + y - L, t] = D(L - y)$. So for $0 \leq y < L$,

$\Pr\{w_t \leq y|S, L\} = \Pr\{D(L - y) \leq S\}$, which is independent of t. Therefore,

$F_w(y|S, L) = \Pr\{w \leq y|S, L\} = \lim_{t\to\infty} \Pr\{w_t \leq y|S, L\} = \Pr\{D(L - y) \leq S\}$.

We also define $F_w(y|S, L) = 0$ for $y < 0$ and $F_w(y|S, L) = 1$ for $y \geq L$.

Proof of Proposition 3.1

Referring to the proof of Proposition 2.1 in the previous chapter, the proof for the distribution of A is similar. Note that a periodic-review base-stock policy is considered therein and the timing of demand is defined differently. So in the proof there, L is generally replaced by $L+1$. In this chapter, a continuous-review base-stock policy is studied, and L is not replaced by $L + 1$. Moreover, both L and W are continuous.

The definition of X_t^W, the performance indicator for period t $(1 \leq t \leq R)$, is modified as:

$\Pr\{X_t^W = 1\} = \Pr\{D(t - L, t - W] \leq S\} = \Pr\{D(L - W) \leq S\} = A_W$.

In the proof of Proposition 2.1, the derivation of the variance of the realized ready rate with window W is modified as follows:

$$\sigma_W^2 = \frac{1}{R^2}[\sum_{i=1}^{R} Var(X_i^W) + 2\sum_{i<j} cov(X_i^W, X_j^W)]$$

$$= \frac{1}{R^2}[\sum_{i=1}^{R} Var(X_i^W) + 2\sum_{i<j}(E(X_i^W X_j^W) - E(X_i^W)E(X_j^W))]$$

$= \frac{1}{R^2}[RA_W(1-A_W) + 2\sum_{i<j} E(X_i^W X_j^W) - R(R-1)A_W^2]$.

So (3.4) follows.

Alternatively, σ_W^2 can be written in a similar way as in the previous chapter:

$$\sigma_W^2 = \frac{1}{R^2}[RF(S|L-W) - ((L_C-1)(2R-L_C)+R)(F(S|L-W))^2 + 2\sum_{n=1}^{L_C-1}(R-L+n)P_n(S,L)], \quad \text{(B.1)}$$

where $L_C = \lceil L-W \rceil$, where $\lceil x \rceil$ gives the ceiling of x; and

$$P_n(S,L) = \int_{-\infty}^{\infty} (F(S-x|n))^2 dF(x|L-W-n) \quad \text{(B.2)}$$

is the probability that the performance outcomes in any two periods are both good, where the two periods differ by n periods, $1 \leq n \leq L_C - 1$. Note that the definition of the third term is different from that in the proof in the previous chapter, but they are equivalent.

Unimodality of Total Supply Chain Cost $EC(S,L)$

Linear delay cost:

Consider normal demand, linear delay cost $C_D(y|S,L) = \delta y$ ($\delta > 0$), and $C_r(L) = rL^{-b}$ ($b > 0$).

Using the relationship between the average backorders and average waiting time (see page 192 of Zipkin (2000)),

$\overline{B}(S,L) = \lambda E(y|S,L)$, where $\overline{B}(S,L)$ is the average backorders.

So $\delta\overline{B}(S,L) = \lambda EC_D(y|S,L)$.

$\overline{B}(S,L) = E([D(L)-S]^+) = \int_{x>S}(x-S)f(x|L)dx$

$= \int_x (x-S)f(x|L)dx + \int_{x<S}(S-x)f(x|L)dx = \overline{I}(S,L) - (S-\lambda L)$

$\Rightarrow EC(S,L) = (h+\delta)\overline{I}(S,L) - \delta(S-\lambda L) + \lambda C_r(L)$.

Because $\lim_{L\to 0} C_r(L) = \infty$, we only need to consider $L > 0$.

1) Unimodality of $EC(S,L)$ in S for given L

Given $L > 0$,

$C_1(S,L) = \frac{\partial EC(S,L)}{\partial S} = (h+\delta)F(S|L) - \delta$,

$C_{11}(S,L) = \frac{\partial^2 EC(S,L)}{\partial S^2} = (h+\delta)f(S|L) > 0$

so $EC(S,L)$ is convex in S for given L.

Let $\widehat{S}(L)$ be the solution to $C_1(S,L)=0$, $F(\widehat{S}(L)|L)=\frac{\delta}{h+\delta}$, $\frac{\partial F(\widehat{S}(L)|L)}{\partial L}=0$ for any $L>0$.

$\widehat{S}(L)$ is the minimizer of $EC(S,L)$ given L.

2) Unimodality of $EC(\widehat{S}(L),L)$ in L

For normal demand, let $z=\frac{S-\lambda L}{\sigma\sqrt{L}}$,

$F(S|L)=\Phi(z)$, $f(S|L)=\phi(z)$ and $\overline{I}(S,L)=(S-\lambda L)\Phi(z)+\sigma\sqrt{L}\phi(z)$.

So $C_2(S,L)=\frac{\partial EC(S,L)}{\partial L}=(h+\delta)[-\lambda\Phi(z)+\frac{\sigma}{2\sqrt{L}}\phi(z)]+\delta\lambda+\lambda C_r'(L)$.

By the Envelope Theorem, and noting that $\Phi(\frac{\widehat{S}(L)-\lambda L}{\sigma\sqrt{L}})=\frac{\delta}{h+\delta}$,

$\frac{dEC(\widehat{S}(L),L)}{dL}=C_2(\widehat{S}(L),L)=(h+\delta)\frac{\sigma}{2\sqrt{L}}\phi(z^*)-\frac{\lambda b}{L^{b+1}}$,where $z^*=\Phi^{-1}(\frac{\delta}{h+\delta})$.

Let $\widetilde{L}$ be such that $\widetilde{L}^{b+0.5}=\frac{2\lambda b}{(h+\delta)\sigma\phi(z^*)}$.

Then $\frac{dEC(\widehat{S}(L),L)}{dL}<0$ for $L<\widetilde{L}$, $\frac{dEC(\widehat{S}(\widetilde{L}),\widetilde{L})}{dL}=0$ and $\frac{dEC(\widehat{S}(L),L)}{dL}>0$ for $L>\widetilde{L}$.

So $EC(\widehat{S}(L),L)$ is unimodal in L, and $EC(S,L)$ is unimodal in (S,L).

Convex delay cost:

For convex delay cost $C_D(y|S,L)=\delta y^2$, it is not easy to prove that $EC(S,L)$ is unimodal. Instead, we have to use the plot of $EC(S,L)$ for each specific case to check its unimodality. For example, for the case of $h=2,\delta=10,r=2,\lambda=4$ and $\sigma=2$, $EC(S,L)$ is plotted in the Figure B.1, which indicates that it is unimodal.

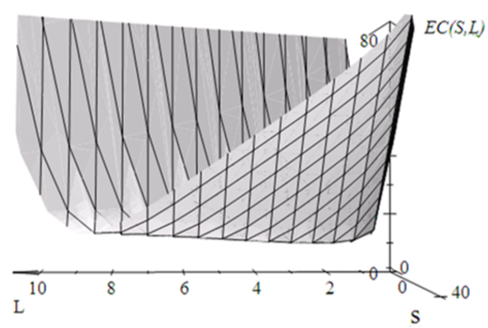

Figure B.1: EC(S,L) under convex delay cost

Proof of Proposition 3.2

At the optimum, constraint (3.7) is binding because otherwise the buyer's objective function value can be reduced by decreasing p while (3.7) remains to hold. Solving for p from (3.7) with equality and subsituting it into the objective function, we obtain the buyer's unconstrained optimization problem:

$\min_{S,L} EC_B(S,L) = h\overline{I}(S,L) + \lambda EC_D(y|S,L) + \lambda C_r(L) + \overline{\pi}$.

So $EC_B(S,L) = EC(S,L) + \overline{\pi}$.

The first-order derivatives of the objective function with respect to S and L are

$\frac{\partial EC_B(S,L)}{\partial S} = \frac{\partial EC(S,L)}{\partial S} = hF(S|L) + \lambda \frac{\partial EC_D(y|S,L)}{\partial S}$

and

$\frac{\partial EC_B(S,L)}{\partial L} = \frac{\partial EC(S,L)}{\partial L} = -h\int_{x<S} \frac{\partial \overline{F}(x|L)}{\partial L} dx + \lambda \frac{\partial EC_D(y|S,L)}{\partial L} + \lambda C_r'(L)$.

Then by the unimodality of $EC(S,L)$, the first-best solution (S^*, L^*) is the solution to (3.9) and (3.10).

Checking if normal distribution satisfies Assumption 3.2

Let

$$\Pi(S,L,W) = \int_{A<\alpha} \Psi(A|A_W) dA = \int_{A<\alpha} \Phi(\frac{A-A_W}{\sigma_W}) dA. \quad \text{(B.3)}$$

$\frac{\partial \Pi(S,L,W)}{\partial S} = \int_{A<\alpha} \phi(\frac{A-A_W}{\sigma_W}) \frac{\partial}{\partial S}(\frac{A-A_W}{\sigma_W}) dA = -\int_{A<\alpha} \phi(\frac{A-A_W}{\sigma_W})(\frac{1}{\sigma_W}\frac{\partial A_W}{\partial S} + \frac{A-A_W}{(\sigma_W)^2}\frac{\partial \sigma_W}{\partial S}) dA$
$= -\Phi(\frac{\alpha-A_W}{\sigma_W})\frac{\partial A_W}{\partial S} + \phi(\frac{\alpha-A_W}{\sigma_W})\frac{\partial \sigma_W}{\partial S}$.

Similarly, $\frac{\partial \Pi(S,L,W)}{\partial L} = -\Phi(\frac{\alpha-A_W}{\sigma_W})\frac{\partial A_W}{\partial L} + \phi(\frac{\alpha-A_W}{\sigma_W})\frac{\partial \sigma_W}{\partial L}$.

Let $\frac{\frac{\partial \Pi(S,L,W)}{\partial S}}{\frac{\partial \Pi(S,L,W)}{\partial L}} = \theta \frac{\partial A_W/\partial S}{\partial A_W/\partial L}$, where

$$\theta = \frac{1 - \frac{\partial \sigma_W/\partial S}{\partial A_W/\partial S}\phi(\frac{\alpha-A_W}{\sigma_W})/\Phi(\frac{\alpha-A_W}{\sigma_W})}{1 - \frac{\partial \sigma_W/\partial L}{\partial A_W/\partial L}\phi(\frac{\alpha-A_W}{\sigma_W})/\Phi(\frac{\alpha-A_W}{\sigma_W})}. \quad \text{(B.4)}$$

It follows from (B.1) and (B.2) that

$\frac{\partial \sigma_W}{\partial S} = \frac{1}{2\sigma_W}\frac{\partial \sigma_W^2}{\partial S} = \frac{1}{2R^2\sigma_W}\frac{\partial A_W}{\partial S}[R - 2((L_C-1)(2R-L_C)+R)F_w(W|S,L) + 2\sum_{n=1}^{L_C-1}(R - L + n)\frac{\partial P_n(S,L)/\partial S}{\partial A_W/\partial S}]$,

$\frac{\partial \sigma_W}{\partial L} = \frac{1}{2R^2\sigma_W}\frac{\partial A_W}{\partial L}[R - 2((L_C-1)(2R-L_C)+R)F_w(W|S,L) + 2\sum_{n=1}^{L_C-1}(R - L + n)\frac{\partial P_n(S,L)/\partial L}{\partial A_W/\partial L}]$.

Table B.1: Value of θ for normal demand

$L=4.2$			$L=4.8$		
S	$R=100$	$R=300$	S	$R=100$	$R=300$
42	0.99	0.99	48	0.99	0.99
44	0.99	0.99	50	0.99	0.99
46	0.99	0.99	52	0.99	0.99
48	0.99	0.99	54	0.99	0.99
50	0.99	0.99	56	0.99	0.99
52	0.99	0.99	58	0.99	0.99
54	0.99	0.99	60	0.99	0.99
56	0.99	0.99	62	0.99	0.99
58	0.99	0.99	64	0.99	0.99
60	0.99	0.99	66	0.99	0.99
62	0.99	0.99	68	0.99	0.99

We use numerical examples to check if $\theta = 1$ for normal distribution. Table B.1 presents the value of θ for normal distribution with $\lambda = 10$ and $\sigma = 3$, $L \in \{4.2, 4.8\}$, $R \in \{100, 300\}$, $S \geq \lambda L$, and $\frac{\alpha - A_W}{\sigma_W} = -0.02$ (the relative allowable deviation of the performance from the target is 2%). It shows that $\theta \approx 1$ in all the scenarios. So for normal demand distribution, Assumption 3.2 approximately holds.

Proof of Theorem 3.1

From the assumption that $\lim_{L\to 0} C_r(L) = \infty$, $L^* > 0$.

Using (B.3), when the supplier's choices of S and L are unobservable, the first-order conditions for (3.6) are

$$\frac{\partial E\pi(S,L)}{\partial S} = -\frac{100K}{R}\frac{\partial \Pi(S,L,W)}{\partial S} - hF(S|L) = 0, \tag{B.5}$$

$$\frac{\partial E\pi(S,L)}{\partial L} = -\frac{100K}{R}\frac{\partial \Pi(S,L,W)}{\partial L} + h\int_{x<S}\frac{\partial \overline{F}(x|L)}{\partial L}dx - \lambda C_r'(L) = 0. \tag{B.6}$$

Given Assumption 3.3, constraint (3.11) can be replaced by (B.5) and (B.6). Because at the optimum, constraint (3.7) is binding, so p can be solved from (3.7) by taking equality in (3.7) and then substituted into the objective function. The buyer's problem becomes

$$\min_{S,L} EC_B(S,L) = h\overline{I}(S,L) + \lambda EC_D(y|S,L) + \lambda C_r(L) + \overline{\pi}$$
$$\text{subject to} \qquad \text{(B.5) and (B.6).}$$

Comparing (B.5) with (3.9), and (B.6) with (3.10), if the first-best solution (S^*, L^*) is also the solution to (B.5) and (B.6), then at (S^*, L^*),

$$\lambda \frac{\partial EC_D(y|S^*, L^*)}{\partial S} = \frac{100K}{R} \frac{\partial \Pi(S^*, L^*, W)}{\partial S}, \tag{B.7}$$
$$\lambda \frac{\partial EC_D(y|S^*, L^*)}{\partial L} = \frac{100K}{R} \frac{\partial \Pi(S^*, L^*, W)}{\partial L}. \tag{B.8}$$

$EC_D(y|S^*, L^*) = \int_0^{L^*} C_D(y) dF_w(y|S^*, L^*) = C_D(L^*) - \int_0^{L^*} F_w(y|S^*, L^*) C_D'(y) dy.$

By Assumption 3.2, $\theta = 1$. So (B.8) $\div$ (B.7)$\Rightarrow$
$\frac{\partial EC_D(y|S^*,L^*)/\partial L}{\partial EC_D(y|S^*,L^*)/\partial S} = \frac{\frac{\partial \Pi(S^*,L^*,W)}{\partial L}}{\frac{\partial \Pi(S^*,L^*,W)}{\partial S}} = \frac{\frac{\partial A_W^*}{\partial L}}{\frac{\partial A_W^*}{\partial S}} \Rightarrow \frac{\int_0^{L^*} \frac{\partial F_w(y|S^*,L^*)}{\partial L} C_D'(y) dy}{\int_0^{L^*} \frac{\partial F_w(y|S^*,L^*)}{\partial S} C_D'(y) dy} = \frac{\frac{\partial F_w(W|S^*,L^*)}{\partial L}}{\frac{\partial F_w(W|S^*,L^*)}{\partial S}}$, or equivalently,

$$\int_0^{L^*} \left(\frac{\frac{\partial F_w(y|S^*,L^*)}{\partial S}}{\frac{\partial F_w(y|S^*,L^*)}{\partial L}} - \frac{\frac{\partial F_w(W|S^*,L^*)}{\partial S}}{\frac{\partial F_w(W|S^*,L^*)}{\partial L}} \right) \frac{\partial F_w(y|S^*, L^*)}{\partial L} C_D'(y) dy = 0. \tag{B.9}$$

If $S^* > 0$, for (S^*, L^*) to be the solution to the supplier's first-order conditions (B.5) and (B.6), it is only required that there exists $W \in [0, L^*]$ such that (B.9) holds.

Let

$$g(W) = \int_0^{L^*} \left(\frac{\frac{\partial F_w(y|S^*,L^*)}{\partial S}}{\frac{\partial F_w(y|S^*,L^*)}{\partial L}} - \frac{\frac{\partial F_w(W|S^*,L^*)}{\partial S}}{\frac{\partial F_w(W|S^*,L^*)}{\partial L}} \right) \frac{\partial F_w(y|S^*, L^*)}{\partial L} C_D'(y) dy, \tag{B.10}$$

and $g_0(y|W) = \left(\frac{\frac{\partial F_w(y|S^*,L^*)}{\partial S}}{\frac{\partial F_w(y|S^*,L^*)}{\partial L}} - \frac{\frac{\partial F_w(W|S^*,L^*)}{\partial S}}{\frac{\partial F_w(W|S^*,L^*)}{\partial L}} \right) \frac{\partial F_w(y|S^*,L^*)}{\partial L} C_D'(y)$.

$\frac{\partial F_w(y|S^*,L^*)}{\partial L} = \frac{\partial \Pr(D(L^*-y) \le S^*)}{\partial L} < 0.$

It has been assumed that $C_D'(y) > 0$, and $\frac{\partial F_w(y|S^*,L^*)}{\partial S} / \frac{\partial F_w(y|S^*,L^*)}{\partial L}$ is strictly monotonic in y. Consider the case where $\frac{\partial F_w(y|S^*,L^*)}{\partial S} / \frac{\partial F_w(y|S^*,L^*)}{\partial L}$ is increasing in y.

For $W = 0$, $g_0(0|W) = 0$, and $g_0(y|W) < 0$ for $y \in (0, L^*]$. So $g(0) < 0$.

For $W = L^*$, $g_0(L^*|W) = 0$, and $g_0(y|W) > 0$ for $y \in [0, L^*)$. So $g(L^*) > 0$.

Similarly when $\frac{\partial F_w(y|S^*,L^*)}{\partial S} / \frac{\partial F_w(y|S^*,L^*)}{\partial L}$ is decreasing in y, we can show that $g(0) >$

0 and $g(L^*) < 0$.

Because $g(W)$ is continuous on $[0, L^*]$, by the Intermediate Value Theorem, there exists $W \in [0, L^*]$ such that $g(W) = 0$, and (B.9) holds. Because $g(W)$ is strictly monotonic in W, this W is unique; and $g(0) \neq 0$, $g(L^*) \neq 0$, so the optimal $W^* \in (0, L^*)$.

Proof of Proposition 3.3

The first-best K and α are determined by the supplier's first-order conditions at the first-best solution (S^*, L^*) and the optimal time window W^*:

$$\frac{100K}{R}\int_{A<\alpha}\frac{\partial\Psi(A|A^*_W)}{\partial S}dA + hF(S^*|L^*) = 0 \tag{B.11}$$

$$\frac{100K}{R}\int_{A<\alpha}\frac{\partial\Psi(A|A^*_W)}{\partial L}dA - h\int_{x<S^*}\frac{\partial\overline{F}(x|L^*)}{\partial L}dx + \lambda C'_r(L^*) = 0, \tag{B.12}$$

where $A^*_W = F(S^*|L^* - W^*)$. Following Assumption 3.2, either of the equations is redundant for solving K and α. Because $F(S^*|L^*) > 0$, the optimal α should be such that $\int_{A<\alpha}\frac{\partial\Psi(A|A^*_W)}{\partial S}dA < 0$. So any $K > 0$ and $\alpha \in (0, A^*_W)$ satisfying $\int_{A<\alpha}\frac{\partial\Psi(A|A^*_W)}{\partial S}dA < 0$ and either equation, e.g. (B.11), are the optimal contract parameters for a ready-rate-with-window contract.

Proof of Proposition 3.4

Let $(\cdot)$ and $\varphi(\cdot)$ denote the cdf and pdf of the standardized demand, respectively. Also let $s = \frac{S}{\lambda}$, then $S = s\lambda$.

Then the first-order conditions (3.9) and (3.10) can be represented as functions of s and L:

$h(\frac{s-L}{(\sigma/\lambda)\sqrt{L}}) - \int_0^L C'_D(y)\frac{1}{(\sigma/\lambda)\sqrt{L-y}}\varphi(\frac{s-(L-y)}{(\sigma/\lambda)\sqrt{L-y}})dy = 0$

and

$h[-(\frac{s-L}{(\sigma/\lambda)\sqrt{L}}) + \frac{\sigma}{\lambda}\frac{1}{2\sqrt{L}}\varphi(\frac{s-L}{(\sigma/\lambda)\sqrt{L}})] + \int_0^L C'_D(y)\frac{s+(L-y)}{2(\sigma/\lambda)(L-y)^{3/2}}\varphi(\frac{s-(L-y)}{(\sigma/\lambda)\sqrt{L-y}})dy + C'_r(L) = 0.$

Let (s^*, L^*) denote the solution to the above two equations. For fixed $\frac{\sigma}{\lambda}$, (s^*, L^*) is independent of λ, and $A^*_W = (\frac{s^*-(L^*-W^*)}{(\sigma/\lambda)\sqrt{L^*-W^*}})$ is independent of λ;

$EC_D(y|S^*, L^*) = \int_0^{L^*} C_D(y)d(\frac{s^*-(L^*-y)}{(\sigma/\lambda)\sqrt{L^*-y}})$ is independent of λ, $\frac{\partial A_{W^*}}{\partial L}$ and $\frac{\partial EC_D(y|S^*,L^*)}{\partial L}$ are independent of λ. Because $S^* = \lambda s^*$, S^* is proportional to λ, $\frac{\partial A_{W^*}}{\partial S} = \frac{1}{\lambda}\frac{\partial A_{W^*}}{\partial s}$ and $\frac{\partial EC_D(y|S^*,L^*)}{\partial S} = \frac{1}{\lambda}\frac{\partial EC_D(y|S^*,L^*)}{\partial s}$. So for fixed $\frac{\sigma}{\lambda}$, (3.12) is independent of λ, and W^*, the solution to (3.12), is independent of λ.

Proof of Proposition 3.5

If $C_D(y) = \delta y$, then $\frac{1}{\delta}EC_D(y|S,L)$ is the average waiting time and $\frac{\lambda}{\delta}EC_D(y|S,L)$ the average backorders, and $EC_D(y|S,L) = \frac{\delta}{\lambda}(\overline{I}(S,L) - (S-\lambda L))$.

The first-order condition for the expected average supply chain cost, (3.9), becomes

$$(h+\delta)F(S|L) - \delta = 0.$$

Using the result in Theorem 3.1, under the optimal ready-rate-with-window contract, the supplier's optimal base-stock level and lead time are the first-best S^* and L^*, and thus satisfy the above first-order condition. So $A_0^* = F(S^*|L^*) = \frac{\delta}{h+\delta}$.

Proof of Proposition 3.6

The results in Proposition 3.6 are similar to those in Proposition 3.3 with $W = 0$. (3.14) follows from (B.5), (B.6) and Assumption 3.2.

Proof of Proposition 3.7

Following the definition of $(\cdot)$, $\varphi(\cdot)$ and s in the proof of Proposition 3.4, (3.1) becomes

$$\overline{I}(S,L) = S - \int_{x<S}(\tfrac{x/\lambda - L}{(\sigma/\lambda)\sqrt{L}})dx = \lambda(s - \int_{y<s}(\tfrac{y-L}{(\sigma/\lambda)\sqrt{L}})dy).$$

Using the result in Proposition 3.6, the optimization problem for solving the optimal S^{**} and L^{**} under the optimal ready-rate-without-window contract can be reformulated in terms of s and L as follows.

$$\min_{p,\alpha,K,s,L} EC(s,L) = \lambda[h(s - \int_{y<s}(\tfrac{y-L}{(\sigma/\lambda)\sqrt{L}})dy) + C_r(L) + \int_0^L C_D(y)d(\tfrac{s-(L-y)}{(\sigma/\lambda)\sqrt{L-y}})]$$

subject to

$$h\lambda\tfrac{s+L}{2L}(\tfrac{s-L}{(\sigma/\lambda)\sqrt{L}}) + h\lambda\int_{y<s}\tfrac{\partial}{\partial L}((\tfrac{y-L}{(\sigma/\lambda)\sqrt{L}}))dy - \lambda C_r'(L) = 0$$

It can be simplified as

$$\min_{p,\alpha,K,s,L} h(s - \int_{y<s}(\tfrac{y-L}{(\sigma/\lambda)\sqrt{L}})dy) + C_r(L) + \int_0^L C_D(y)d(\tfrac{s-(L-y)}{(\sigma/\lambda)\sqrt{L-y}})$$

subject to

$$h[\tfrac{s+L}{2L}(\tfrac{s-L}{(\sigma/\lambda)\sqrt{L}}) + \int_{y<s}\tfrac{\partial}{\partial L}((\tfrac{y-L}{(\sigma/\lambda)\sqrt{L}}))dy] - C_r'(L) = 0$$

The formulation is independent of λ for fixed $\frac{\sigma}{\lambda}$. So for fixed $\frac{\sigma}{\lambda}$, the optimal (s^{**}, L^{**}) is independent of λ, $A_0^{**} = (\frac{s^{**}-L^{**}}{(\sigma/\lambda)\sqrt{L^{**}}})$ is independent of λ, and $S^{**} = \lambda s^{**}$ is proportional to λ.

For all the following derivation of formulas, we define

$z = \frac{S-\lambda L}{\sigma\sqrt{L}}$, $z^* = \frac{S^*-\lambda L^*}{\sigma\sqrt{L^*}}$, $z_y = \frac{S-\lambda(L-y)}{\sigma\sqrt{L-y}}$, $z_T = \frac{S-\lambda(L-T)}{\sigma\sqrt{L-T}}$ and $z_T^+ = \frac{S+\lambda(L-T)}{\sigma\sqrt{L-T}}$.

Section 3.5.1. Linear delay cost: formulas

It is assumed that the demand distribution is normal, $C_D(y) = \delta y$ and $C_r(L) = \frac{r}{L}$. The average supply chain cost is

$$EC_L(S, L) = (h + \delta)\sigma\sqrt{L}(z\Phi(z) + \phi(z)) - \delta(S - \lambda L) + \frac{\lambda r}{L} \tag{B.13}$$

(3.9) and (3.10), the first-order conditions for the first-best solution (S^*, L^*), become

$$\begin{aligned} (h + \delta)\Phi(z) &= \delta & \text{(B.14)} \\ (h + \delta)(\Phi(z) - \frac{\sigma}{2\lambda\sqrt{L}}\phi(z)) - C_r'(L) &= \delta. & \text{(B.15)} \end{aligned}$$

The subscript L in $C_L(S, L)$ and W_L^* denotes linear delay cost, and $C_r'(L) = -\frac{r}{L^2}$.

Constraint (3.14) in the optimization problem under the ready-rate-without-window contract is

$$h(\frac{S - \lambda L}{2\lambda L}\Phi(z) + \frac{\sigma}{2\lambda\sqrt{L}}\phi(z)) + C_r'(L) = 0. \tag{B.16}$$

1. Derivation of (B.13)

Applying following formulas on page 206 of Zipkin (2000) to the objective function in (3.8),

$$\overline{I}(S, L) = \sigma\sqrt{L}[z\Phi(z) + \phi(z)] \tag{B.17}$$

and

$$EC_D(y|S, L) = \frac{\delta}{\lambda}[\overline{I}(S, L) - (S - \lambda L)] = \frac{\delta}{\lambda}\sigma\sqrt{L}[-z\overline{\Phi}(z) + \phi(z)], \tag{B.18}$$

where $\overline{\Phi}(\cdot) = 1 - \Phi(\cdot)$.

2. Derivation of (B.14) and (B.15)

They are obtained by differentiating (B.13) w.r.t. S and L, respectively.

3. Derivation of (3.16)

$A_W^*(S^*, L^*) = \Phi(\frac{S^* - \lambda(L^* - W^*)}{\sigma\sqrt{L^* - W^*}}) \Rightarrow \frac{\partial A_W^*(S^*, L^*)/\partial S}{\partial A_W^*(S^*, L^*)/\partial L} = -\frac{2(L^* - W^*)}{S^* + \lambda(L^* - W^*)}$.

(B.18) $\Rightarrow \frac{\partial EC_D(y|S^*, L^*)/\partial S}{\partial EC_D(y|S^*, L^*)/\partial L} = \frac{-\overline{\Phi}(z^*)}{\lambda\overline{\Phi}(z^*) + \frac{\sigma}{2\sqrt{L^*}}\phi(z^*)}$. Substituting the above results into (3.12),

$$-\frac{2(L^* - W^*)}{S^* + \lambda(L^* - W^*)} = \frac{-\overline{\Phi}(z^*)}{\lambda\overline{\Phi}(z^*) + \frac{\sigma}{2\sqrt{L^*}}\phi(z^*)}. \tag{B.19}$$

(B.14) and (B.15) $\Rightarrow \overline{\Phi}(z^*) = \frac{h}{h+\delta}$ and $\frac{\sigma}{2\sqrt{L^*}}\phi(z^*) = -\frac{\lambda}{h+\delta}C_r'(L^*)$. Substituting into (B.19) gives (3.16).

Section 3.5.2. Convex delay cost: formulas

It is assumed that the demand distribution is normal, $C_D(y) = \delta y^2$ and $C_r(L) = \frac{r}{L}$. The average supply chain cost is

$$\begin{aligned} EC_C(S,L) \;&=\; [h(S-\lambda L) - \frac{\delta}{\lambda}((S-\lambda L)^2 + \frac{\sigma^2}{\lambda}(2S - \lambda L + \frac{3\sigma^2}{2\lambda}))]\Phi(z) \\ &+\delta\frac{\sigma^2}{\lambda^2}(S+\lambda L - \frac{3\sigma^2}{2\lambda})e^{2S\lambda/\sigma^2}\overline{\Phi}(z^+) + (h + \frac{\delta}{\lambda}(\lambda L - S - \frac{3\sigma^2}{\lambda}))\sigma\sqrt{L}\phi(z) \\ &+\lambda C_r(L) + \frac{\delta}{\lambda}((S-\lambda L)^2 + \frac{\sigma^2}{\lambda}(2S - \lambda L + \frac{3\sigma^2}{2\lambda})), \end{aligned} \quad \text{(B.20)}$$

where $z^+ = \frac{S+\lambda L}{\sigma\sqrt{L}}$. (3.9) and (3.10), the first-order conditions for the first-best solution (S^*, L^*) become

$$h\Phi(z) - \frac{4\delta\sigma\sqrt{L}}{\lambda}\phi(z) - 2\delta(L - \frac{S}{\lambda} - \frac{\sigma^2}{\lambda^2})\overline{\Phi}(z) + 2\delta(L + \frac{S}{\lambda} - \frac{\sigma^2}{\lambda^2})e^{2S\lambda/\sigma^2}\overline{\Phi}(z^+) = 0 \quad \text{(B.21)}$$

$$h\Phi(z) - (\frac{h}{2L} + 2\delta)\frac{\sigma\sqrt{L}}{\lambda}\phi(z) - 2\delta(L - \frac{S}{\lambda} - \frac{\sigma^2}{2\lambda^2})\overline{\Phi}(z) - \frac{\delta\sigma^2}{\lambda^2}e^{2S\lambda/\sigma^2}\overline{\Phi}(z^+) - C_r'(L) = 0. \quad \text{(B.22)}$$

The subscript C in $C_C(S,L)$ and W_C^* denotes convex delay cost. Note that the constraint in the optimization problem under the ready-rate-without-window contract for convex delay cost is the same as that for linear delay cost, given by (B.16).

1. Derivation of (B.20)

$$C_C(S,L) = h\sigma\sqrt{L}(z\Phi(z) + \phi(z)) + \lambda\delta\int_0^L y^2\phi(z_y)\frac{S+\lambda(L-y)}{2\sigma(L-y)^{3/2}}dy + \lambda C_r(L) \quad \text{(B.23)}$$

$$\begin{aligned} &\int_T^L y^2\phi(z_y)\tfrac{S+\lambda(L-y)}{2\sigma(L-y)^{3/2}}dy = -\int_T^L y^2 d\overline{\Phi}(z_y) = T^2\overline{\Phi}(z_T) + 2\int_T^L y\overline{\Phi}(z_y)dy \\ &= T^2\overline{\Phi}(z_T) - \tfrac{2\sigma^2}{\lambda^2}\int_{z_T}^{+\infty}(L - \tfrac{\sigma^2}{\lambda^2}(\tfrac{S\lambda}{\sigma^2} + \tfrac{z_y^2}{2} - z_y\sqrt{\tfrac{S\lambda}{\sigma^2} + \tfrac{z_y^2}{4}}))(z_y - \sqrt{\tfrac{S\lambda}{\sigma^2} + \tfrac{z_y^2}{4}} - \tfrac{z_y^2}{4\sqrt{\frac{S\lambda}{\sigma^2} + \frac{z_y^2}{4}}})\overline{\Phi}(z_y)dz_y \\ &= T^2\overline{\Phi}(z_T) + \tfrac{2\sigma^2 L}{\lambda^2}[(\overline{\Phi}(z_y)z_y\sqrt{\tfrac{S\lambda}{\sigma^2} + \tfrac{z_y^2}{4}} + z_y\overline{\Phi}(z_y))|_{z_T}^{+\infty} + \int_{z_T}^{+\infty} z_y\sqrt{\tfrac{S\lambda}{\sigma^2} + \tfrac{z_y^2}{4}}\phi(z_y)dz_y - \\ &\int_{z_T}^{+\infty}\overline{\Phi}(z_y)dz_y] \end{aligned}$$

$-\frac{2\sigma^4}{\lambda^4}[(\overline{\Phi}(z_y)z_y(\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z_y^2}{4}})^3+\overline{\Phi}(z_y)\frac{z_y^3}{4}\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z_y^2}{4}}-\overline{\Phi}(z_y)z_y^2(\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z_y^2}{4}})^2)|_{z_T}^{+\infty}$
$+\int_{z_T}^{+\infty}z_y(\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z_y^2}{4}})^3\phi(z_y)dz_y+\int_{z_T}^{+\infty}\frac{z_y^3}{4}\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z_y^2}{4}}\phi(z_y)dz_y-\int_{z_T}^{+\infty}z_y^2(\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z_y^2}{4}})^2\phi(z_y)dz_y]$

Let $x=2\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z_y^2}{4}}, x_T=2\sqrt{\frac{S\lambda}{\sigma^2}+\frac{1}{4}(\frac{S-\lambda(L-T)}{\sigma\sqrt{L-T}})^2}=z_T^+$.
$\int_{z_T}^{+\infty}z_y(\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z_y^2}{4}})^3\phi(z_y)dz_y=\int_{z_T}^{+\infty}\frac{1}{4}(2\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z_y^2}{4}})^3\phi(z_y)d(\frac{S\lambda}{\sigma^2}+\frac{z_y^2}{4})$
$=e^{2S\lambda/\sigma^2}[-\frac{1}{8}x^3\phi(x)|_{x_T}^{+\infty}-\frac{3}{8}x\phi(x)|_{x_T}^{+\infty}+\int_{x_T}^{+\infty}\frac{3}{8}\phi(x)dx]\Rightarrow$
$\int_{z_T}^{+\infty}z_y(\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z_y^2}{4}})^3\phi(z_y)dz_y=\frac{1}{8}(z_T^+)^3\phi(z_T)+\frac{3}{8}z_T^+\phi(z_T)+\frac{3}{8}e^{2S\lambda/\sigma^2}\overline{\Phi}(z_T^+)$

$\int_{z_T}^{+\infty}\frac{z_y^3}{4}\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z_y^2}{4}}\phi(z_y)dz_y=2\int_{z_T}^{+\infty}\frac{z_y^2}{4}\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z_y^2}{4}}\phi(z_y)d(\frac{S\lambda}{\sigma^2}+\frac{z_y^2}{4})$
$=e^{2S\lambda/\sigma^2}\int_{x_T}^{+\infty}\frac{x}{2}(\frac{x^2}{4}-\frac{S\lambda}{\sigma^2})x\phi(x)dx=e^{2S\lambda/\sigma^2}(\int_{x_T}^{+\infty}\frac{x^4}{8}\phi(x)dx-\int_{x_T}^{+\infty}\frac{S\lambda}{2\sigma^2}x^2\phi(x)dx)$
using the result above,
$\int_{z_T}^{+\infty}\frac{z_y^3}{4}\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z_y^2}{4}}\phi(z_y)dz_y$
$=(\frac{1}{8}(z_T^+)^3+\frac{3}{8}z_T^+-\frac{S\lambda}{2\sigma^2}z_T^+)\phi(z_T)+(\frac{3}{8}-\frac{S\lambda}{2\sigma^2})e^{2S\lambda/\sigma^2}\overline{\Phi}(z_T^+)$

$\int_{z_T}^{+\infty}z_y^2(\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z_y^2}{4}})^2\phi(z_y)dz_y=\int_{z_T}^{+\infty}\frac{S\lambda}{\sigma^2}z_y^2\phi(z_y)dz_y+\int_{z_T}^{+\infty}\frac{z_y^4}{4}\phi(z_y)dz_y$
$=-\int_{z_T}^{+\infty}\frac{S\lambda}{\sigma^2}z_yd\phi(z_y)-\int_{z_T}^{+\infty}\frac{z_y^3}{4}d\phi(z_y)$
$=\frac{S\lambda}{\sigma^2}z_T\phi(z_T)+\frac{1}{4}(z_T)^3\phi(z_T)+\frac{3}{4}z_T\phi(z_T)+\frac{S\lambda}{\sigma^2}\overline{\Phi}(z_T)+\frac{3}{4}\overline{\Phi}(z_T)$
$\Rightarrow$
$\int_T^L y^2\phi(z_y)\frac{S+\lambda(L-y)}{2\sigma(L-y)^{3/2}}dy$
$=T^2\overline{\Phi}(z_T)+\frac{2\sigma^2L}{\lambda^2}[-\frac{1}{2}(2\lambda\frac{S-\lambda(L-T)}{\sigma^2}+1)\overline{\Phi}(z_T)+\frac{\lambda\sqrt{L-T}}{\sigma}\phi(z_T)+\frac{1}{2}e^{2S\lambda/\sigma^2}\overline{\Phi}(z_T^+)]$
$-\frac{2\sigma^4}{\lambda^4}[(-z_T(\frac{1}{2}z_T^+)^3-\frac{1}{8}(z_T)^3z_T^++(z_T)^2(\frac{1}{2}z_T^+)^2)\overline{\Phi}(z_T)-\frac{S\lambda}{\sigma^2}\overline{\Phi}(z_T)-\frac{3}{4}\overline{\Phi}(z_T)$
$+(\frac{3}{4}-\frac{S\lambda}{2\sigma^2})e^{2S\lambda/\sigma^2}\overline{\Phi}(z_T^+)+\frac{1}{4}(z_T^+)^3\phi(z_T)-\frac{1}{4}(z_T)^3\phi(z_T)$
$+(\frac{3}{4}z_T^+-\frac{3}{4}z_T-\frac{S\lambda}{2\sigma^2}z_T^+-\frac{S\lambda}{\sigma^2}z_T)\phi(z_T)]$
$=T^2\overline{\Phi}(z_T)+\frac{2\sigma^2L}{\lambda^2}[-(\lambda\frac{S-\lambda(L-T)}{\sigma^2}+\frac{1}{2})\overline{\Phi}(z_T)+\frac{\lambda\sqrt{L-T}}{\sigma}\phi(z_T)+\frac{1}{2}e^{2S\lambda/\sigma^2}\overline{\Phi}(z_T^+)]$
$+\frac{S^2-\lambda^2(L-T)^2}{\lambda^2}\overline{\Phi}(z_T)+\frac{2S\sigma^2}{\lambda^3}\overline{\Phi}(z_T)+\frac{3\sigma^4}{2\lambda^4}\overline{\Phi}(z_T)+\frac{\sigma^4}{\lambda^4}(\frac{S\lambda}{\sigma^2}-\frac{3}{2})e^{2S\lambda/\sigma^2}\overline{\Phi}(z_T^+)$
$-\frac{\sigma}{\lambda}\sqrt{L-T}((L-T)+\frac{3\sigma^2}{\lambda^2}+\frac{S}{\lambda})\phi(z_T)\Rightarrow$

$$\int_0^L y^2\phi(z_y)\frac{S+\lambda(L-y)}{2\sigma(L-y)^{3/2}}dy=((\frac{S}{\lambda}-L)^2+\frac{\sigma^2}{\lambda^2}(\frac{2S}{\lambda}+\frac{3\sigma^2}{2\lambda^2}-L))\overline{\Phi}(z)$$
$$+\frac{\sigma^2}{\lambda^2}(\frac{S}{\lambda}+L-\frac{3}{2}\frac{\sigma^2}{\lambda^2})e^{2S\lambda/\sigma^2}\overline{\Phi}(\frac{S+\lambda L}{\sigma\sqrt{L}})+\frac{\sigma}{\lambda}\sqrt{L}(L-\frac{S}{\lambda}-\frac{3\sigma^2}{\lambda^2})\phi(z) \quad (B.24)$$

Substituting (B.24) into (B.23) gives (B.20).

2. Derivation of (B.21) and (B.22)

The first-order conditions (3.9) and (3.10) become

$$h\Phi(z) - \frac{2\lambda\delta}{\sigma}\int_0^L \frac{y}{\sqrt{L-y}}\phi(z_y)dy = 0 \tag{B.25}$$

and

$$h(\Phi(z) - \frac{\sigma}{2\lambda\sqrt{L}}\phi(z)) - \frac{\delta}{\sigma}(S\int_0^L \frac{y}{(L-y)^{3/2}}\phi(z_y)dy + \lambda\int_0^L \frac{y}{\sqrt{L-y}}\phi(z_y)dy) - C_r'(L) = 0. \tag{B.26}$$

The integrals are derived as below.

2.1 $\int_T^L \frac{y}{\sqrt{L-y}}\phi(z_y)dy$ $(0 \le T < L)$

$\int_T^L \frac{y}{\sqrt{L-y}}\phi(z_y)dy = L\int_T^L \frac{1}{\sqrt{L-y}}\phi(z_y)dy - \int_T^L \sqrt{L-y}\phi(z_y)dy$

$\int_T^L \sqrt{L-y}\phi(z_y)dy = \frac{\sigma^3}{\lambda^3}\int_{z_T}^{+\infty}(\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z^2}{4}} - \frac{z}{2})(\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z^2}{4}} - \frac{z}{2} - \frac{z}{2}\frac{\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z^2}{4}} - \frac{z}{2}}{\sqrt{\frac{S\lambda}{\sigma^2}+\frac{z^2}{4}}})\phi(z)dz$

$= \overline{\Phi}(z_T)(\frac{S\sigma}{\lambda^2} + \frac{\sigma^3}{\lambda^3}) - \frac{2\sigma^2}{\lambda^2}\sqrt{L-T}\phi(z_T) - \frac{\sigma^3}{\lambda^3}e^{2S\lambda/\sigma^2}\overline{\Phi}(z_T^+) + \frac{S\sigma}{\lambda^2}e^{2S\lambda/\sigma^2}\overline{\Phi}(z_T^+)$

$\Rightarrow$

$\int_T^L \frac{y}{\sqrt{L-y}}\phi(z_y)dy = L\frac{\sigma}{\lambda}[\overline{\Phi}(z_T) - e^{2S\lambda/\sigma^2}\overline{\Phi}(z_T^+)] - \overline{\Phi}(z_T)(\frac{S\sigma}{\lambda^2} + \frac{\sigma^3}{\lambda^3})$
$+\frac{2\sigma^2}{\lambda^2}\sqrt{L-T}\phi(z_T) + \frac{\sigma^3}{\lambda^3}e^{2S\lambda/\sigma^2}\overline{\Phi}(z_T^+) - \frac{S\sigma}{\lambda^2}e^{2S\lambda/\sigma^2}\overline{\Phi}(z_T^+)$
$= \frac{\sigma}{\lambda}[(L - (\frac{S}{\lambda} + \frac{\sigma^2}{\lambda^2}))\overline{\Phi}(z_T) - (L + \frac{S}{\lambda} - \frac{\sigma^2}{\lambda^2})e^{2S\lambda/\sigma^2}\overline{\Phi}(z_T^+)] + \frac{2\sigma^2}{\lambda^2}\sqrt{L-T}\phi(z_T) \Rightarrow$

$$\int_0^L \frac{y}{\sqrt{L-y}}\phi(z_y)dy = \frac{\sigma}{\lambda}[(L-(\frac{S}{\lambda}+\frac{\sigma^2}{\lambda^2}))\overline{\Phi}(z) - (L+\frac{S}{\lambda}-\frac{\sigma^2}{\lambda^2})e^{2S\lambda/\sigma^2}\overline{\Phi}(\frac{S+\lambda L}{\sigma\sqrt{L}})] + \frac{2\sigma^2}{\lambda^2}\sqrt{L}\phi(z) \tag{B.27}$$

2.2 $\int_0^L \frac{y}{(L-y)^{3/2}}\phi(z_y)dy$

$\int_0^L \frac{y}{(L-y)^{3/2}}\phi(z_y)dy = -\int_0^L \frac{L-y}{(L-y)^{3/2}}\phi(z_y)dy + \int_0^L \frac{L}{(L-y)^{3/2}}\phi(z_y)dy$
$= L\int_0^L \frac{1}{(L-y)^{3/2}}\phi(z_y)dy - \int_0^L \frac{1}{\sqrt{L-y}}\phi(z_y)dy$
$= L\frac{\sigma}{S}(\overline{\Phi}(z) + e^{2S\lambda/\sigma^2}\overline{\Phi}(\frac{S+\lambda L}{\sigma\sqrt{L}})) - \frac{\sigma}{\lambda}(\overline{\Phi}(z) - e^{2S\lambda/\sigma^2}\overline{\Phi}(\frac{S+\lambda L}{\sigma\sqrt{L}})) \Rightarrow$

$$\int_0^L \frac{y}{(L-y)^{3/2}}\phi(z_y)dy = \frac{\sigma}{\lambda}[(L\frac{\lambda}{S} - 1)\overline{\Phi}(z) + (L\frac{\lambda}{S} + 1)e^{2S\lambda/\sigma^2}\overline{\Phi}(\frac{S+\lambda L}{\sigma\sqrt{L}})]. \tag{B.28}$$

(B.21) and (B.22) follow by substituting (B.27) and (B.28) into (B.25) and (B.26).

3. Derivation of (3.17)

Let $z_y^* = \frac{S^*-\lambda(L^*-y)}{\sigma\sqrt{L^*-y}}$ and $z^{+*} = \frac{S^*+\lambda L^*}{\sigma\sqrt{L^*}}$.

(3.12) $\Rightarrow W_C^* = L^* - \frac{\int_0^{L^*} \frac{y}{\sqrt{L^*-y}} \phi(z_y^*) dy}{\int_0^{L^*} \frac{y}{L^*-y} \frac{1}{\sqrt{L^*-y}} \phi(z_y^*) dy}.$

Using (B.27) and (B.28),

$W_C^* = L^* - \frac{\frac{\sigma}{\lambda}[(L^*-(\frac{S^*}{\lambda}+\frac{\sigma^2}{\lambda^2}))\overline{\Phi}(z^*)-(L^*+\frac{S^*}{\lambda}-\frac{\sigma^2}{\lambda^2})e^{2S^*\lambda/\sigma^2}\overline{\Phi}(z^{+*})]+\frac{2\sigma}{\lambda}\sqrt{L^*}\phi(z^*)]}{\frac{\sigma}{\lambda}[(L^*\frac{\lambda}{S^*}-1)\overline{\Phi}(z^*)+(L^*\frac{\lambda}{S^*}+1)e^{2S^*\lambda/\sigma^2}\overline{\Phi}(z^{+*})]}$

$= L^* - \frac{S^*}{\lambda} \frac{(L^*-\frac{S^*}{\lambda}-\frac{\sigma^2}{\lambda^2})\overline{\Phi}(z^*)-(L+\frac{S^*}{\lambda}-\frac{\sigma^2}{\lambda^2})e^{2S^*\lambda/\sigma^2}\overline{\Phi}(z^{+*})+\frac{2\sigma}{\lambda}\sqrt{L^*}\phi(z^*)}{(L^*-\frac{S^*}{\lambda})\overline{\Phi}(z^*)+(L^*+\frac{S^*}{\lambda})e^{2S^*\lambda/\sigma^2}\overline{\Phi}(z^{+*})}$

First-order condition (B.21) $\Rightarrow$ at the optimal (S^*, L^*),

$(L^* - \frac{S^*}{\lambda} - \frac{\sigma^2}{\lambda^2})\overline{\Phi}(z^*) - (L + \frac{S^*}{\lambda} - \frac{\sigma^2}{\lambda^2})e^{2S^*\lambda/\sigma^2}\overline{\Phi}(z^{+*}) + \frac{2\sigma}{\lambda}\sqrt{L^*}\phi(z^*) = \frac{h}{2\delta}\Phi(z^*),$

substituting it into the above formula for W_C^* gives

$W_C^* = L^* - \frac{S^*}{\lambda} \frac{\frac{h}{2\delta}\Phi(z^*)}{(L^*-\frac{S^*}{\lambda})\overline{\Phi}(z^*)+(L^*+\frac{S^*}{\lambda})e^{2S^*\lambda/\sigma^2}\overline{\Phi}(z^{+*})}.$

(B.21) $-2\times$(B.22) $\Rightarrow$

$(L^* + \frac{S^*}{\lambda})e^{2S^*\lambda/\sigma^2}\overline{\Phi}(z^{+*}) = h\Phi(z^*) - \frac{h}{L^*}\frac{\sigma}{\lambda}\sqrt{L^*}\phi(z^*) - 2\delta(L^* - \frac{S^*}{\lambda})\overline{\Phi}(z^*) + 2C_r'(L^*)$

substituting it into the above formula for $W_C^* \Rightarrow$ (3.17).

Appendix C

Proof for Chapter 4

1. Definitions and preliminaries

Let $\Psi(\beta|H) = \beta m - \frac{H}{\beta}$ and $S(\beta|H) = \frac{m+H/(1-\beta)^2}{m+H/\beta^2}$, where $H \geq 0$. $\Psi'(\beta|H) = m + \frac{H}{\beta^2} > 0$.

We use $\Psi(\beta)$ and $S(\beta)$ instead of $\Psi(\beta|H)$ and $S(\beta|H)$ whenever there is no ambiguity.

The following results will be used in the proofs.

$$\int_a^b (y^2-1)\phi(y)dy = a\phi(a) - b\phi(b), \tag{C.1}$$

$$\int_a^b y^2\phi(y)dy = a\phi(a) - b\phi(b) + \Phi(b) - \Phi(a). \tag{C.2}$$

2. Formulation of period t problem

Let Supplier 1's share in period t be α_t. Under the allocation rule $\beta^{\alpha}_{t+1}(x^1_t, x^2_t)$ for period t, the buyer's payoff to go is

$$v^B_t(\alpha_t) = \alpha_t e^1_t + (1-\alpha_t)e^2_t + \gamma\int_{x^1_t}\int_{x^2_t} v^B_{t+1}(\beta^{\alpha}_{t+1}(x^1_t, x^2_t)) f(x^1_t|e^1_t) f(x^2_t|e^2_t) dx^1_t dx^2_t, \tag{C.3}$$

the profits to go of supplier 1 and 2 are

$$v^1_t(\alpha_t) = m\alpha_t - \frac{bg(\alpha_t)(e^1_t)^2}{2} + \gamma\int_{x^1_t}\int_{x^2_t} v^1_{t+1}(\beta^{\alpha}_{t+1}(x^1_t, x^2_t)) f(x^1_t|e^1_t) f(x^2_t|e^2_t) dx^1_t dx^2_t, \tag{C.4}$$

and

$$\begin{aligned} v^2_t(1-\alpha_t) &= m(1-\alpha_t) - \frac{bg(1-\alpha_t)(e^1_t)^2}{2} \\ &+\gamma\int_{x^1_t}\int_{x^2_t} v^2_{t+1}(1-\beta^{\alpha}_{t+1}(x^1_t, x^2_t)) f(x^1_t|e^1_t) f(x^2_t|e^2_t) dx^1_t dx^2_t. \end{aligned} \tag{C.5}$$

First-order conditions $\Rightarrow$
$\frac{\partial v_t^1(\alpha_t)}{\partial e_t^1} = -bg(\alpha_t)e_t^1 + \gamma \int_{x_t^1}\int_{x_t^2} v_{t+1}^1(\beta_{t+1}^{\alpha}(x_t^1, x_t^2))f^1(x_t^1|e_t^1)f(x_t^2|e_t^2)dx_t^1 dx_t^2 = 0,$
$\frac{\partial v_t^2(1-\alpha_t)}{\partial e_t^2} = -bg(1-\alpha_t)e_t^2 + \gamma \int_{x_t^1}\int_{x_t^2} v_{t+1}^2(1-\beta_{t+1}^{\alpha}(x_t^1, x_t^2))f(x_t^1|e_t^1)f^2(x_t^2|e_t^2)dx_t^1 dx_t^2$
$= 0 \Rightarrow$

$$e_t^1 = \frac{\gamma}{bg(\alpha_t)}\int_{x_t^1}\int_{x_t^2} v_{t+1}^1(\beta_{t+1}^{\alpha}(x_t^1, x_t^2))f^1(x_t^1|e_t^1)f(x_t^2|e_t^2)dx_t^1 dx_t^2, \quad \text{(C.6)}$$

$$e_t^2 = \frac{\gamma}{bg(1-\alpha_t)}\int_{x_t^1}\int_{x_t^2} v_{t+1}^2(1-\beta_{t+1}^{\alpha}(x_t^1, x_t^2))f(x_t^1|e_t^1)f^2(x_t^2|e_t^2)dx_t^1 dx_t^2. \quad \text{(C.7)}$$

$(C.6)$ and $(C.7)$ are necessary conditions for (e_t^1, e_t^2) to be a Nash equilibrium in period t.

3. Proof of Theorem 4.1

Due to the complexity of the problem and our focus on the suppliers' incentive issues, we will ignore the suppliers' individual rationality constraints in the following analysis and restrict a supplier's share to a range by letting $\overline{\beta}$ and $\underline{\beta}$ be the upper and lower bounds for a supplier's share in a period, where $\underline{\beta}$ can be exogeneously defined by some constraints such as the minimum order quantity and also ensures the (IR) constraint is satisfied.

The proof of this theorem consists of two parts. In the first part, we derive the optimal allocation rule from the static formulation of the buyer's infinite-horizon problem, under the assumption that the two suppliers use stationary policies to play the stochastic game; in the second part, we check that under the derived optimal allocation rule, the suppliers' infinite-horizon stochastic game has a unique Nash equilibrium which is stationary and is the one derived from the static formulation.

- Part 1. Derive the optimal allocation rule

We first consider the buyer's problem with the incentive compatibility constraints only. Other constraints will define the upper and lower bounds for Supplier 1's share in the next period. Let $y_i = \frac{x_i - e_i^*}{\sigma}$, then $y_i \sim N(0,1)$ and from the first-order conditions in (4.5), the suppliers' best responses to an allocation rule $\beta_\alpha(x_1, x_2)$ are

$$e_1^* = \frac{\gamma}{\alpha b\sigma}\int\int v(\widehat{\beta}_\alpha(y_1, y_2))y_1\phi(y_1)\phi(y_2)dy_1dy_2, \quad (C.8)$$
$$e_2^* = \frac{\gamma}{(1-\alpha)b\sigma}\int\int v(1-\widehat{\beta}_\alpha(y_1, y_2))y_2\phi(y_1)\phi(y_2)dy_1dy_2, \quad (C.9)$$

where $\widehat{\beta}_\alpha(y_1, y_2) = \beta_\alpha(e_1^* + \sigma y_1, e_2^* + \sigma y_2)$, and $v(\beta) = v_1(\beta) = v_2(\beta)$ for any β due to symmetry.

Substituting the suppliers' best response functions into the buyer's objective function, we obtain the buyer's unconstrained optimization problem, in which the buyer only needs to choose an allocation rule in terms of the suppliers' standardized performance:

$v_B(\alpha) = \max_{\widehat{\beta}_\alpha}\{\gamma\int\int[\frac{1}{b\sigma}y_1v(\widehat{\beta}_\alpha(y_1, y_2)) + \frac{1}{b\sigma}y_2v(1-\widehat{\beta}_\alpha(y_1, y_2))$
$+v_B(\widehat{\beta}_\alpha(y_1, y_2))]\phi(y_1)\phi(y_2)dy_1dy_2\}$.

Let $G(\widehat{\beta}_\alpha(y_1, y_2)) = \frac{1}{b\sigma}y_1v(\widehat{\beta}_\alpha(y_1, y_2)) + \frac{1}{b\sigma}y_2v(1-\widehat{\beta}_\alpha(y_1, y_2)) + v_B(\widehat{\beta}_\alpha(y_1, y_2))$.
Pointwise optimization of $v_B(\alpha)$ w.r.t. $\widehat{\beta}_\alpha(y_1, y_2) \Rightarrow$
$G'(\widehat{\beta}_\alpha(y_1, y_2)) = \frac{1}{b\sigma}y_1v'(\widehat{\beta}_\alpha(y_1, y_2)) - \frac{1}{b\sigma}y_2v'(1-\widehat{\beta}_\alpha(y_1, y_2)) + v_B'(\widehat{\beta}_\alpha(y_1, y_2))$.
So the optimal $\widehat{\beta}^*_\alpha(y_1, y_2)$ is independent of α, and we omit the subscript α in $\widehat{\beta}_\alpha(y_1, y_2)$ and $\widehat{\beta}^*_\alpha(y_1, y_2)$ from now on. It follows that

$$\begin{aligned} v_B(\alpha) = \gamma\int\int[&\frac{1}{b\sigma}y_1v(\widehat{\beta}^*(y_1, y_2)) + \frac{1}{b\sigma}y_2v(1-\widehat{\beta}^*(y_1, y_2)) \\ &+v_B(\widehat{\beta}^*(y_1, y_2))]\phi(y_1)\phi(y_2)dy_1dy_2 = v_B \end{aligned} \quad (C.10)$$

is independent of α.

Let $H = \frac{\gamma^2}{2b\sigma^2}(\int\int y_1v(\widehat{\beta}^*(y_1, y_2))\phi(y_1)\phi(y_2)dy_1dy_2)^2$. Substituting it into $(C.8)$ and $(C.9)$, we get $e_1^* = \frac{\sqrt{2H}}{\alpha\sqrt{b}}, e_2^* = \frac{\sqrt{2H}}{(1-\alpha)\sqrt{b}}$, then due to symmetry of the two suppliers, $v_1(\alpha) = v_2(\alpha) = v(\alpha)$, where

$$v(\alpha) = \alpha m - \frac{H}{\alpha} + \gamma\int\int v(\widehat{\beta}^*(y_1, y_2))\phi(y_1)\phi(y_2)dy_1dy_2. \quad (C.11)$$

Note that both H and the last term in the above formula are independent of α and $H > 0$, so

$v'(\alpha) = m + \frac{H}{\alpha^2} > 0$ and $v''(\alpha) = -\frac{2H}{\alpha^3} < 0$,
$G'(\widehat{\beta}(y_1, y_2)) = \frac{1}{b\sigma}y_1(m + \frac{H}{(\widehat{\beta}(y_1,y_2))^2}) - \frac{1}{b\sigma}y_2(m + \frac{H}{(1-\widehat{\beta}(y_1,y_2))^2})$,
$G''(\widehat{\beta}(y_1, y_2)) = -\frac{1}{b\sigma}y_1\frac{2H}{(\widehat{\beta}(y_1,y_2))^3} - \frac{1}{b\sigma}y_2\frac{2H}{(1-\widehat{\beta}(y_1,y_2))^3}$.

So we have

(1) for $y_1 > 0$ and $y_2 > 0$: $G(\widehat{\beta}(y_1, y_2))$ is concave, the optimal $\widehat{\beta}^*(y_1, y_2)$ is either determined by $G'(\widehat{\beta}(y_1, y_2)) = 0$,

or at the boundary with $\widehat{\beta}^*(y_1, y_2) = \overline{\beta}$ if $G'(\overline{\beta}) > 0$ and $\widehat{\beta}^*(y_1, y_2) = \underline{\beta}$ if $G'(\underline{\beta}) < 0$;

(2) for $y_1 < 0$ and $y_2 < 0$: $G(\widehat{\beta}(y_1, y_2))$ is convex, $\widehat{\beta}^*(y_1, y_2) = \overline{\beta}$ if $G(\overline{\beta}) > G(1-\overline{\beta})$ and $\widehat{\beta}^*(y_1, y_2) = \underline{\beta}$ otherwise;

(3) for $y_1 > 0$ and $y_2 < 0$: $G'(\widehat{\beta}(y_1, y_2)) > 0$, $\widehat{\beta}^*(y_1, y_2) = \overline{\beta}$;

(4) for $y_1 < 0$ and $y_2 > 0$: $G'(\widehat{\beta}(y_1, y_2)) < 0$, $\widehat{\beta}^*(y_1, y_2) = \underline{\beta}$.

In the above analysis,
$G'(\widehat{\beta}(y_1, y_2)) > 0$ is equivalent to $y_1 > S(\widehat{\beta}(y_1, y_2))y_2$;
$G'(\widehat{\beta}(y_1, y_2)) < 0$ is equivalent to $y_1 < S(\widehat{\beta}(y_1, y_2))y_2$;
$G(\overline{\beta}) > G(1 - \overline{\beta})$ is equivalent to
$\frac{1}{b\sigma}y_1v(\overline{\beta}) + \frac{1}{b\sigma}y_2v(1 - \overline{\beta}) > \frac{1}{b\sigma}y_1v(1 - \overline{\beta}) + \frac{1}{b\sigma}y_2v(\overline{\beta})$,
$\Leftrightarrow y_1(v(\overline{\beta}) - v(1 - \overline{\beta})) > y_2(v(\overline{\beta}) - v(1 - \overline{\beta})) \Leftrightarrow y_1 > y_2$.

Because $y_i = \frac{x_i - e_i^*}{\sigma}$, the above analysis on $\widehat{\beta}^*(y_1, y_2)$ immediately translates to $\beta_\alpha^{1*}(x_1, x_2)$ based on x_1 and x_2. So the optimal rule in Theorem 4.1 follows. Let

$$C = \int\int y_1v(\widehat{\beta}^*(y_1, y_2))\phi(y_1)\phi(y_2)dy_1dy_2, \tag{C.12}$$

so

$$H = \frac{\gamma^2C^2}{2b\sigma^2}. \tag{C.13}$$

In $(C.11)$, let $V = \int\int v(\widehat{\beta}^*(y_1, y_2))\phi(y_1)\phi(y_2)dy_1dy_2$, $v(\alpha) = \alpha m - \frac{H}{\alpha} + \gamma V$, take $\alpha = \widehat{\beta}^*(y_1, y_2)$, multiply both sides by y_1, and integrate both sides over y_1 and y_2,
$\int\int y_1v(\widehat{\beta}^*(y_1, y_2))\phi(y_1)\phi(y_2)dy_1dy_2$
$= \int\int y_1(\widehat{\beta}^*(y_1, y_2)m - \frac{H}{\widehat{\beta}^*(y_1,y_2)})\phi(y_1)\phi(y_2)dy_1dy_2 + \gamma V\int\int y_1\phi(y_1)\phi(y_2)dy_1dy_2$
$\Rightarrow C = \int y_1(\widehat{\beta}^*(y_1, y_2)m - \frac{H}{\widehat{\beta}^*(y_1,y_2)})\phi(y_1)\phi(y_2)dy_1dy_2$,
together with $(C.13) \Rightarrow$
$\frac{\sigma}{\gamma}\sqrt{2bH} = \int\int y_1(\widehat{\beta}^*(y_1, y_2)m - \frac{H}{\widehat{\beta}^*(y_1,y_2)})\phi(y_1)\phi(y_2)dy_1dy_2$.

Check the suppliers' second-order conditions (Nash equilibrium):

Under the optimal rule $\beta_\alpha^{1*}(x_1, x_2)$,

$\frac{\partial^2 v_1(\alpha)}{(\partial e_1)^2} = -\alpha b + \gamma \int\int v_1(\beta_\alpha^{1*}(x_1, x_2)) f^{11}(x_1|e_1) f(x_2|e_2) dx_1 dx_2$

$= -\alpha b + \gamma \int\int v_1(\beta_\alpha^{1*}(x_1, x_2)) \frac{1}{\sigma^4}[(\frac{x_1-e_1}{\sigma})^2 - 1]\phi(\frac{x_1-e_1}{\sigma})\phi(\frac{x_2-e_2}{\sigma}) dx_1 dx_2.$

At $(e_1, e_2) = (e_1^*, e_2^*)$,

$\frac{\partial^2 v_1(\alpha)}{(\partial e_1)^2}|_{(e_1^*, e_2^*)} = -\alpha b + \gamma \int\int (\widehat{\beta}^*(y_1, y_2)m - \frac{H}{\widehat{\beta}^*(y_1,y_2)} + \gamma V) \frac{1}{\sigma^2}[(y_1)^2 - 1]\phi(y_1)\phi(y_2) dy_1 dy_2$

$= -\alpha b + \frac{\gamma}{\sigma^2} \int\int [(y_1)^2 - 1]\Psi(\widehat{\beta}^*(y_1, y_2))\phi(y_1)\phi(y_2) dy_1 dy_2.$

Using polar coordinates

$$y_1 = r\sin\theta, y_2 = r\cos\theta, dy_1 dy_2 = r dr d\theta, \theta \sim U[0, 2\pi], \tan\theta = \frac{y_1}{y_2} = S(\widehat{\beta}^*). \quad \text{(C.14)}$$

Let $\theta_{\min}$ and $\theta_{\max}$ correspond to $\underline{\beta}$ and $\overline{\beta}$.

$\int_{y_1}\int_{y_2} y_1^2 \Psi(\widehat{\beta}^*(y_1, y_2))\phi(y_1)\phi(y_2) dy_1 dy_2$

$= \Lambda_1 + \Psi(\overline{\beta}) \int_{y_2<0}\int_{y_2}^0 y_1^2 \phi(y_1)\phi(y_2) dy_1 dy_2 + \Psi(\underline{\beta}) \int_{y_2<0}\int_{y_1<y_2} y_1^2 \phi(y_1)\phi(y_2) dy_1 dy_2$

$+\Psi(\overline{\beta}) \int_{y_1>0, y_2<0} y_1^2 \phi(y_1)\phi(y_2) dy_1 dy_2 + \Psi(\underline{\beta}) \int_{y_1<0, y_2>0} y_1^2 \phi(y_1)\phi(y_2) dy_1 dy_2,$

where

$\Lambda_1 = \int_{r=0}^{\infty}\int_{\theta=0}^{\pi/2} r^2 (\sin\theta)^2 \Psi(\widehat{\beta}^*(y_1, y_2)) \frac{1}{2\pi} e^{-r^2/2} r dr d\theta$

$= \int_{\theta=0}^{\pi/2} (\sin\theta)^2 \Psi(\widehat{\beta}^*(y_1, y_2)) \frac{1}{\pi}[\int_{r=0}^{\infty} \frac{r^2}{2} e^{-r^2/2} d(\frac{r^2}{2})] d\theta$

$= \frac{1}{\pi} \int_{\theta=0}^{\pi/2} (\sin\theta)^2 \Psi(\widehat{\beta}^*(y_1, y_2)) d\theta$

$= \frac{1}{\pi} \int_{\theta_{\min}}^{\theta_{\max}} (\sin\theta)^2 \Psi(\widehat{\beta}^*(y_1, y_2)) d\theta + \frac{1}{\pi}\Psi(\overline{\beta})(\frac{\pi}{4} - \frac{\theta_{\max}}{2} + \frac{1}{4}\sin(2\theta_{\max})) + \frac{1}{\pi}\Psi(\underline{\beta})(\frac{\theta_{\min}}{2} - \frac{1}{4}\sin(2\theta_{\min})).$

By $(C.2)$,

$\int_{y_2<0}\int_{y_2}^0 y_1^2 \phi(y_1)\phi(y_2) dy_1 dy_2 = \int_{y_2<0} [y_2\phi(y_2) + \frac{1}{2} - \Phi(y_2)]\phi(y_2) dy_2$

$= \int_{y_2<0} y_2 \phi(y_2)\phi(y_2) dy_2 + \frac{1}{2}\int_{y_2<0} \phi(y_2) dy_2 - \int_{y_2<0} \Phi(y_2)\phi(y_2) dy_2$

$= -\frac{1}{4\pi} + \frac{1}{4} - \int_{y_2<0} \Phi(y_2)\phi(y_2) dy_2,$

$$\int_{y_2<0}\Phi(y_2)\phi(y_2)dy_2 = (\Phi(y_2))^2|_{-\infty}^{0} - \int_{y_2<0}\Phi(y_2)\phi(y_2)dy_2$$
$$\Rightarrow \int_{y_2<0}\Phi(y_2)\phi(y_2)dy_2 = \tfrac{1}{8}$$
$$\Rightarrow \int_{y_2<0}\int_{y_2}^{0} y_1^2\phi(y_1)\phi(y_2)dy_1dy_2 = \tfrac{1}{8} - \tfrac{1}{4\pi}.$$

$$\int_{y_2<0}\int_{y_1<y_2} y_1^2\phi(y_1)\phi(y_2)dy_1dy_2 = \int_{y_2<0}[-y_2\phi(y_2) + \Phi(y_2)]\phi(y_2)dy_2$$
$$= -\int_{y_2<0} y_2\phi(y_2)\phi(y_2)dy_2 + \int_{y_2<0}\Phi(y_2)\phi(y_2)dy_2 = \tfrac{1}{4\pi} + \tfrac{1}{8},$$
$$\int_{y_1>0,y_2<0} y_1^2\phi(y_1)\phi(y_2)dy_1dy_2 = \tfrac{1}{2}\int_{y_2<0}\phi(y_2)dy_2 = \tfrac{1}{4},$$
$$\int_{y_1<0,y_2>0} y_1^2\phi(y_1)\phi(y_2)dy_1dy_2 = \tfrac{1}{2}\int_{y_2>0}\phi(y_2)dy_2 = \tfrac{1}{4}.$$

So $\int_{y_1}\int_{y_2} y_1^2\Psi(\widehat{\beta}^*(y_1,y_2))\phi(y_1)\phi(y_2)dy_1dy_2$
$= \Lambda_1 + \Psi(\overline{\beta})(\tfrac{3}{8} - \tfrac{1}{4\pi}) + \Psi(\underline{\beta})(\tfrac{3}{8} + \tfrac{1}{4\pi}).$

$$\int_{y_1}\int_{y_2}\Psi(\widehat{\beta}^*(y_1,y_2))\phi(y_1)\phi(y_2)dy_1dy_2$$
$$= \Lambda_2 + \Psi(\overline{\beta})\int_{y_2<0}\int_{y_2}^{0}\phi(y_1)\phi(y_2)dy_1dy_2 + \Psi(\underline{\beta})\int_{y_2<0}\int_{y_1<y_2}\phi(y_1)\phi(y_2)dy_1dy_2$$
$$+\Psi(\overline{\beta})\int_{y_1>0,y_2<0}\phi(y_1)\phi(y_2)dy_1dy_2 + \Psi(\underline{\beta})\int_{y_1<0,y_2>0}\phi(y_1)\phi(y_2)dy_1dy_2$$
$$= \Lambda_2 + \tfrac{1}{8}\Psi(\overline{\beta}) + \tfrac{1}{8}\Psi(\underline{\beta}) + \tfrac{1}{4}(\Psi(\overline{\beta}) + \Psi(\underline{\beta})) \Rightarrow$$

$$\int_{y_1}\int_{y_2}\Psi(\widehat{\beta}^*(y_1,y_2))\phi(y_1)\phi(y_2)dy_1dy_2 = \Lambda_2 + \frac{3}{8}(\Psi(\overline{\beta}) + \Psi(\underline{\beta})), \tag{C.15}$$

where $\Lambda_2 = \int_{y_1>0,y_2>0}\Psi(\widehat{\beta}^*(y_1,y_2))\phi(y_1)\phi(y_2)dy_1dy_2$
$= \int_{r=0}^{\infty}\int_{\theta=0}^{\pi/2}\Psi(\widehat{\beta}^*)\tfrac{1}{2\pi}e^{-r^2/2}rdrd\theta = \int_{\theta=0}^{\pi/2}\Psi(\widehat{\beta}^*)\tfrac{1}{2\pi}d\theta \Rightarrow$

$$\Lambda_2 = \frac{1}{2\pi}\int_{\theta_{\min}}^{\theta_{\max}}\Psi(\widehat{\beta}^*)d\theta + \frac{1}{2\pi}\Psi(\underline{\beta})\theta_{\min} + \frac{1}{2\pi}\Psi(\overline{\beta})(\pi/2 - \theta_{\max}). \tag{C.16}$$

$$\tfrac{\partial^2 v_1(\alpha)}{(\partial e_1)^2}|_{(e_1^*,e_2^*)} = -\alpha b + \tfrac{\gamma}{\sigma^2}[\Lambda_1 + \Psi(\overline{\beta})(\tfrac{3}{8} - \tfrac{1}{4\pi}) + \Psi(\underline{\beta})(\tfrac{3}{8} + \tfrac{1}{4\pi}) - (\Lambda_2 + \tfrac{3}{8}(\Psi(\overline{\beta}) + \Psi(\underline{\beta})))]$$

$= -\alpha b + \frac{\gamma}{\pi\sigma^2}[\int_{\theta_{\min}}^{\theta_{\max}} ((\sin\theta)^2 - \frac{1}{2})\Psi(\widehat{\beta}^*(y_1, y_2))d\theta + \frac{1}{4}(\sin(2\theta_{\max}) - 1)(\Psi(\overline{\beta}) - \Psi(\underline{\beta}))]$,

note that $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$,

$\frac{\partial^2 v_1(\alpha)}{(\partial e_1)^2}|_{(e_1^*, e_2^*)} \leq -\alpha b + \frac{\gamma}{\pi\sigma^2}[\Psi(\overline{\beta})\int_{\pi/4}^{\theta_{\max}} ((\sin\theta)^2 - \frac{1}{2})d\theta + \Psi(\underline{\beta})\int_{\theta_{\min}}^{\pi/4} ((\sin\theta)^2 - \frac{1}{2})d\theta$
$+\frac{1}{4}(\sin(2\theta_{\max}) - 1)(\Psi(\overline{\beta}) - \Psi(\underline{\beta}))]$,

because $(\sin\theta)^2 = \frac{1-\cos 2\theta}{2}$,

$\frac{\partial^2 v_1(\alpha)}{(\partial e_1)^2}|_{(e_1^*, e_2^*)} \leq -\alpha b + \frac{\gamma}{\pi\sigma^2}[-\frac{1}{2}\Psi(\overline{\beta})\int_{\pi/4}^{\theta_{\max}} \cos 2\theta d\theta - \frac{1}{2}\Psi(\underline{\beta})\int_{\theta_{\min}}^{\pi/4} \cos 2\theta d\theta$
$+\frac{1}{4}(\sin(2\theta_{\max}) - 1)(\Psi(\overline{\beta}) - \Psi(\underline{\beta}))]$
$= -\alpha b + \frac{\gamma}{\pi\sigma^2}[-\frac{1}{4}\Psi(\overline{\beta})(\sin(2\theta_{\max}) - 1) - \frac{1}{4}\Psi(\underline{\beta})(1 - \sin(2\theta_{\min})) + \frac{1}{4}(\sin(2\theta_{\max}) - 1)(\Psi(\overline{\beta}) - \Psi(\underline{\beta}))]$
$= -\alpha b + \frac{\gamma}{4\pi\sigma^2}\Psi(\underline{\beta})(\sin(2\theta_{\min}) - \sin(2\theta_{\max}))$
$= -\alpha b + \frac{\gamma}{4\pi\sigma^2}\Psi(\underline{\beta})(\sin(2\theta_{\min}) - \sin(2(\frac{\pi}{2} - \theta_{\min}))) = -\alpha b$,

similarly $\frac{\partial^2 v_2(1-\alpha)}{(\partial e_2)^2}|_{(e_1^*, e_2^*)} \leq -(1-\alpha)b$, so (e_1^*, e_2^*) is a Nash equilibrium when the suppliers only use stationary policies.

Uniqueness of Nash equilibrium:

Let $a_i = \frac{e_i - e_i^*}{\sigma}$, $z_i = y_i - a_i$. By $(C.1)$,

$\frac{\partial^2 v_1(\alpha)}{(\partial e_1)^2} = -\alpha b + \gamma \int\int \Psi(\beta_\alpha^{1*}(x_1, x_2))\frac{1}{\sigma^4}[(\frac{x_1 - e_1}{\sigma})^2 - 1]\phi(\frac{x_1 - e_1}{\sigma})\phi(\frac{x_2 - e_2}{\sigma})dx_1 dx_2$
$+\gamma V \int\int \frac{1}{\sigma^4}[(\frac{x_1 - e_1}{\sigma})^2 - 1]\phi(\frac{x_1 - e_1}{\sigma})\phi(\frac{x_2 - e_2}{\sigma})dx_1 dx_2$
$= -\alpha b + \frac{\gamma}{\sigma^2}\int\int \Psi(\widehat{\beta}^*(y_1, y_2))[(y_1 - a_1)^2 - 1]\phi(y_1 - a_1)\phi(y_2 - a_2)dy_1 dy_2$
$= -\alpha b + \frac{\gamma}{\sigma^2}[\Psi(\overline{\beta})\int_{-\infty}^{0}(\int_{y_2}^{\infty} [(y_1 - a_1)^2 - 1]\phi(y_1 - a_1)dy_1)\phi(y_2 - a_2)dy_2$
$+\Psi(\underline{\beta})\int_{-\infty}^{0}(\int_{-\infty}^{y_2} [(y_1 - a_1)^2 - 1]\phi(y_1 - a_1)dy_1)\phi(y_2 - a_2)dy_2$
$+\Psi(\underline{\beta})\int_{0}^{\infty}(\int_{-\infty}^{0} [(y_1 - a_1)^2 - 1]\phi(y_1 - a_1)dy_1)\phi(y_2 - a_2)dy_2$
$+\int_{0}^{\infty}(\int_{0}^{\infty} \Psi(\widehat{\beta}^*(y_1, y_2))[(y_1 - a_1)^2 - 1]\phi(y_1 - a_1)dy_1)\phi(y_2 - a_2)dy_2]$
$= -\alpha b + \frac{\gamma}{\sigma^2}[\Psi(\overline{\beta})\int_{-\infty}^{0}(\int_{y_2 - a_1}^{\infty} (z_1^2 - 1)\phi(z_1)dz_1)\phi(y_2 - a_2)dy_2$
$+\Psi(\underline{\beta})\int_{-\infty}^{0}(\int_{-\infty}^{y_2 - a_1} (z_1^2 - 1)\phi(z_1)dz_1)\phi(y_2 - a_2)dy_2$
$+\Psi(\underline{\beta})\int_{0}^{\infty}(\int_{-\infty}^{-a_1} (z_1^2 - 1)\phi(z_1)dz_1)\phi(y_2 - a_2)dy_2$

$+\int_{-a_1}^{\infty}(\int_{-a_2}^{\infty}\Psi(\widetilde{\beta}^*_\alpha(z_1,z_2))(z_1^2-1)\phi(z_1)dz_1)\phi(z_2)dz_2$
$= -\alpha b + \frac{\gamma}{\sigma^2}[(\Psi(\overline{\beta})-\Psi(\underline{\beta}))\int_{-\infty}^{0}(y_2-a_1)\phi(y_2-a_1)\phi(y_2-a_2)dy_2$
$+\Psi(\underline{\beta})a_1\phi(a_1)\Phi(a_2)+\int_{-a_1}^{\infty}(\int_{-a_2}^{\infty}\Psi(\widetilde{\beta}^*_\alpha(z_1,z_2))(z_1^2-1)\phi(z_1)dz_1)\phi(z_2)dz_2.$

$\int_{-\infty}^{0}(y-a_1)\phi(y-a_1)\phi(y-a_2)dy$
$=\frac{1}{2\pi}\int_{-\infty}^{0}(y-a_1)\exp[-\frac{1}{2}((y-a_1)^2+(y-a_2)^2)]dy$
$=\frac{1}{2\pi}\exp[-(\frac{a_1-a_2}{2})^2]\int_{-\infty}^{0}(y-a_1)\exp[-(y-\frac{a_1+a_2}{2})^2]dy$
$=\frac{1}{2\pi}\exp[-(\frac{a_1-a_2}{2})^2][\int_{-\infty}^{0}(y-\frac{a_1+a_2}{2})\exp[-(y-\frac{a_1+a_2}{2})^2]dy$
$-\frac{a_1-a_2}{2}\int_{-\infty}^{0}\exp[-(y-\frac{a_1+a_2}{2})^2]dy]$
$=\frac{1}{\sqrt{2\pi}}\exp[-(\frac{a_1-a_2}{2})^2][-\frac{1}{2\sqrt{2\pi}}\exp[-(\frac{a_1+a_2}{2})^2]-\frac{a_1-a_2}{2\sqrt{2}}\int_{-\infty}^{0}\frac{\sqrt{2}}{\sqrt{2\pi}}\exp[-\frac{1}{2}(\sqrt{2}(y-\frac{a_1+a_2}{2}))^2]dy]$
$=\frac{L}{2\sqrt{2\pi}},$
where $L=-\frac{1}{\sqrt{2\pi}}\exp[-\frac{1}{2}(a_1^2+a_2^2)]-\frac{a_1-a_2}{\sqrt{2}}\exp[-(\frac{a_1-a_2}{2})^2]\Phi(-\frac{a_1+a_2}{\sqrt{2}})$.

Let $\overline{y}$ and $\underline{y}$ be the solutions to $(y_1-a_1)^2-1=0$, $\overline{y}-a_1=1$, $\underline{y}-a_1=-1$, $(y_1-a_1)^2-1\geq 0$ for $y_1\geq\overline{y}$ and $y_1\leq\underline{y}$. Because $\underline{\beta}\leq\widetilde{\beta}^*_\alpha(z_1,z_2)\leq\overline{\beta}$,
$\frac{\partial^2 v_1(\alpha)}{(\partial e_1)^2}\leq -\alpha b+\frac{\gamma}{\sigma^2}[(\Psi(\overline{\beta})-\Psi(\underline{\beta}))\frac{L}{2\sqrt{2\pi}}+\Psi(\underline{\beta})a_1\phi(a_1)\Phi(a_2)$
$+\int_0^{\infty}\{\Psi(\overline{\beta})\int_{\overline{y}}^{\infty}[(y_1-a_1)^2-1]\phi(y_1-a_1)dy_1+\Psi(\overline{\beta})\int_0^{\underline{y}}[(y_1-a_1)^2-1]\phi(y_1-a_1)dy_1$
$+\Psi(\underline{\beta})\int_{\underline{y}}^{\overline{y}}[(y_1-a_1)^2-1]\phi(y_1-a_1)dy_1\}\phi(y_2-a_2)dy_2]$
$=-\alpha b+\frac{\gamma}{\sigma^2}[(\Psi(\overline{\beta})-\Psi(\underline{\beta}))\frac{L}{2\sqrt{2\pi}}+\Psi(\underline{\beta})a_1\phi(a_1)\Phi(a_2)$
$+\int_0^{\infty}\{\Psi(\overline{\beta})\int_{\overline{y}-a_1}^{\infty}(z_1^2-1)\phi(z_1)dz_1+\Psi(\overline{\beta})\int_{-a_1}^{\underline{y}-a_1}(z_1^2-1)\phi(z_1)dz_1$
$+\Psi(\underline{\beta})\int_{\underline{y}-a_1}^{\overline{y}-a_1}(z_1^2-1)\phi(z_1)dz_1\}\phi(y_2-a_2)dy_2]$
$=-\alpha b+\frac{\gamma}{\sigma^2}[(\Psi(\overline{\beta})-\Psi(\underline{\beta}))\frac{L}{2\sqrt{2\pi}}-(\Psi(\overline{\beta})-\Psi(\underline{\beta}))a_1\phi(a_1)\Phi(a_2)$
$+(\Psi(\overline{\beta})-\Psi(\underline{\beta}))((\overline{y}-a_1)\phi(\overline{y}-a_1)-(\underline{y}-a_1)\phi(\underline{y}-a_1))\Phi(a_2)]$
$=-\alpha b+\frac{\gamma}{\sigma^2}(\Psi(\overline{\beta})-\Psi(\underline{\beta}))[-\frac{1}{2\sqrt{2\pi}}\phi(\sqrt{a_1^2+a_2^2})-\frac{a_1-a_2}{2}\phi(\frac{a_1-a_2}{\sqrt{2}})\Phi(-\frac{a_1+a_2}{\sqrt{2}})-a_1\phi(a_1)\Phi(a_2)+2\phi(1)\Phi(a_2)]$

$= -\alpha b + \frac{\gamma}{\sigma^2}(\Psi(\overline{\beta}) - \Psi(\underline{\beta}))[-\frac{1}{2}\phi(a_1)\phi(a_2) + \frac{a_2 - a_1}{2}\phi(\frac{a_1 - a_2}{\sqrt{2}})(1 - \Phi(\frac{a_1 + a_2}{\sqrt{2}})) - a_1\phi(a_1)\Phi(a_2) + 2\phi(1)\Phi(a_2)]$

because $x\phi(x)$ is maximized at $x = 1$ and $-x\phi(x)$ is maximized at $x = -1$, relaxing each term in the [ ] individually, we obtain

$\frac{\partial^2 v_1(\alpha)}{(\partial e_1)^2} \leq -\alpha b + \frac{\gamma}{\sigma^2}(\Psi(\overline{\beta}) - \Psi(\underline{\beta}))[-\frac{1}{2}\phi(a_1)\phi(a_2) + \frac{a_2 - a_1}{2}\phi(\frac{a_1 - a_2}{\sqrt{2}}) - a_1\phi(a_1)\Phi(a_2)$
$+2\phi(1)\Phi(a_2)]$
$\leq -\alpha b + \frac{\gamma}{\sigma^2}(\Psi(\overline{\beta}) - \Psi(\underline{\beta}))[\frac{1}{\sqrt{2}}\phi(1) + 3\phi(1)\Phi(a_2)]$
$\leq -\alpha b + \frac{\gamma(2\overline{\beta} - 1)}{\sigma^2}(m + \frac{H}{\overline{\beta}(1 - \overline{\beta})})(\frac{1}{\sqrt{2}} + 3)\phi(1).$

Because $1 - \overline{\beta} \leq \alpha \leq \overline{\beta}$, the sufficient condition for the existence of a unique Nash equilibrium when the suppliers only use stationary policies is

$$\frac{\gamma(2\overline{\beta} - 1)}{\sigma^2}(m + \frac{H}{\overline{\beta}(1 - \overline{\beta})})(\frac{1}{\sqrt{2}} + 3)\phi(1) < b(1 - \overline{\beta}). \qquad \text{(C.17)}$$

Because $(C.17)$ holds at $\overline{\beta} = \frac{1}{2}$ with strict inequality, the set of the values of $\overline{\beta}$ such that $\overline{\beta} > \frac{1}{2}$ and $(C.17)$ holds is nonempty.

- Part 2. Prove the existence of a unique stationary Nash equilibrium in the suppliers' infinite-horizon stochastic game under $\beta_\alpha^{1*}(x_1, x_2)$

The proof consists of two steps: the first step is on a finite-horizon problem and uses backward recursion to show the existence of a unique subgame perfect Nash equilibrium in the two suppliers' finite-horizon stochastic game; and the second step shows when the horizon goes to infinity, the subgame perfect equilibrium becomes a stationary equilibrium.

Step 1. Finite-horizon problem:

First consider a T-period model, $T < \infty$. Assume given Supplier 1's share in Period $T+1$, α_{T+1}, the terminal values of each party take the following form:

$v_{T+1}^B(\alpha_{T+1}) = V_{T+1}^B = \frac{2\sqrt{2H_{T+1}}}{\sqrt{b}}, v_{T+1}^1(\alpha_{T+1}) = \alpha_{T+1}m - \frac{H_{T+1}}{\alpha_{T+1}} + \gamma V_{T+1},$
$v_{T+1}^2(1 - \alpha_{T+1}) = (1 - \alpha_{T+1})m - \frac{H_{T+1}}{1 - \alpha_{T+1}} + \gamma V_{T+1},$

where $\alpha_{T+1} \in [\underline{\beta}, \overline{\beta}]$, and $H_{T+1} \geq 0$ and V_{T+1} are independent of α_{T+1}. Because there is little restriction on the value of H_{T+1} and V_{T+1}, the above form has included the case of no future payoff for the buyer beyond period T when $H_{T+1} = 0$ and the case of no future payoff for the suppliers beyond period T when $H_{T+1} = V_{T+1} = 0$.

Motivated by Part 1 of the proof, we consider an allocation rule of the following form for the finite horizon problem.

Given Supplier 1's share in period t, α_t, the optimal allocation rule for period $t+1$ ($t \leq T$), $\beta^{\alpha*}_{t+1}(x^1_t, x^2_t)$, takes the form

1. for $x^1_t > e^{1*}_t$ and $x^2_t > e^{2*}_t$:

if $x^1_t - e^{1*}_t > S(\overline{\beta}|H_t)(x^2_t - e^{2*}_t)$, $\beta^{\alpha*}_{t+1}(x^1_t, x^2_t) = \overline{\beta}$; if $x^1_t - e^{1*}_t < S(\underline{\beta}|H_t)(x^2_t - e^{2*}_t)$, $\beta^{\alpha*}_{t+1}(x^1_t, x^2_t) = \underline{\beta}$; otherwise, $\beta^{\alpha*}_{t+1}(x^1_t, x^2_t)$ is determined by $S(\beta^{\alpha*}_{t+1}(x^1_t, x^2_t)|H_t) = \frac{x^1_t - e^{1*}_t}{x^2_t - e^{2*}_t}$;

2. for $x^1_t < e^{1*}_t$ or $x^2_t < e^{2*}_t$: $\beta^{\alpha*}_{t+1}(x^1_t, x^2_t) = \overline{\beta}$ if $x^1_t - e^{1*}_t > x^2_t - e^{2*}_t$ and $\beta^{\alpha*}_{t+1}(x^1_t, x^2_t) = \underline{\beta}$ otherwise, where

$$e^{1*}_t = \frac{\sqrt{2H_t}}{\alpha_t\sqrt{b}}, e^{2*}_t = \frac{\sqrt{2H_t}}{(1-\alpha_t)\sqrt{b}}, \tag{C.18}$$

and H_t ($t = T, T-1, ...1$) is calculated as

$$H_t = \frac{\gamma^2}{2b\sigma^2}\left(\int y_1(\widehat{\beta}^*_{t+1}(y_1, y_2)m - \frac{H_{t+1}}{\widehat{\beta}^*_{t+1}(y_1, y_2)})\phi(y_1)\phi(y_2)dy_1dy_2\right)^2 > 0, \tag{C.19}$$

where $\widehat{\beta}^*_{t+1}(y_1, y_2) = \beta^{\alpha*}_{t+1}(e^{1*}_t + \sigma y_1, e^{2*}_t + \sigma y_2)$.

In period T, let $y_i = \frac{x^i_T - e^{i*}_T}{\sigma}$ ($i = 1, 2$), from (C.6) and (C.7),

$e^1_T = \frac{\gamma}{b\alpha_T}\int_{x^1_T}\int_{x^2_T}(\beta^{\alpha*}_{T+1}(x^1_T, x^2_T)m - \frac{H_{T+1}}{\beta^{\alpha*}_{T+1}(x^1_T, x^2_T)} + \gamma V_{T+1})\frac{x^1_T - e^1_T}{\sigma^2}f(x^1_T|e^1_T)f(x^2_T|e^2_T)dx^1_T dx^2_T$

$= \frac{\gamma}{b\alpha_T\sigma}\int_{y_1}\int_{y_2}(\widehat{\beta}^*_{T+1}(y_1, y_2)m - \frac{H_{T+1}}{\widehat{\beta}^*_{T+1}(y_1, y_2)})(y_1 - \frac{e^1_T - e^{1*}_T}{\sigma})\phi(y_1 - \frac{e^1_T - e^{1*}_T}{\sigma})\phi(y_2 - \frac{e^2_T - e^{2*}_T}{\sigma})dy_1dy_2$,

and

$e^2_T = \frac{\gamma}{b(1-\alpha_t)\sigma}\int_{y_1}\int_{y_2}((1 - \widehat{\beta}^*_{T+1}(y_1, y_2))m - \frac{H_{T+1}}{1 - \widehat{\beta}^*_{T+1}(y_1, y_2)})(y_2 - \frac{e^2_T - e^{2*}_T}{\sigma})\phi(y_1 - \frac{e^1_T - e^{1*}_T}{\sigma})\phi(y_2 - \frac{e^2_T - e^{2*}_T}{\sigma})dy_1dy_2$.

It is obvious that $\widehat{\beta}^*_{T+1}(y_1, y_2)$ is symmetrical in y_1 and y_2, i.e., $\widehat{\beta}^*_{T+1}(y_1, y_2) = 1 - \widehat{\beta}^*_{T+1}(y_2, y_1)$ for any (y_1, y_2). So

$\int y_1(\widehat{\beta}^*_{T+1}(y_1, y_2)m - \frac{H_{T+1}}{\widehat{\beta}^*_{T+1}(y_1, y_2)})\phi(y_1)\phi(y_2)dy_1dy_2$

$= \int y_2((1 - \widehat{\beta}^*_{T+1}(y_1, y_2))m - \frac{H_{T+1}}{1 - \widehat{\beta}^*_{T+1}(y_1, y_2)})\phi(y_1)\phi(y_2)dy_1dy_2$.

Because letting $e^i_T = e^{i*}_T$ in the two equations above for e^1_T and e^2_T will make both equations hold simultaneously, obviously $e^{1*}_T = \frac{\sqrt{2H_T}}{\alpha_T\sqrt{b}}, e^{2*}_T = \frac{\sqrt{2H_T}}{(1-\alpha_T)\sqrt{b}}$ is a solution to (C.6) and (C.7) for period T problem.

To show (e_T^{1*}, e_T^{2*}) is a unique Nash equilibrium in period T, consider the suppliers' second-order conditions. Following similar analysis as above for the second-order conditions in the static formuation, but with H and V replaced by H_{T+1} and V_{T+1}, we can show that

at $(e_T^1, e_T^2) = (e_T^{1*}, e_T^{2*})$, $\frac{\partial^2 v_T^1(\alpha_T)}{(\partial e_T^1)^2}|_{(e_T^{1*}, e_T^{2*})} \leq -\alpha_T b$ and $\frac{\partial^2 v_T^2(1-\alpha_T)}{(\partial e_T^2)^2}|_{(e_T^{1*}, e_T^{2*})} \leq -(1-\alpha_T)b$, so (e_T^{1*}, e_T^{2*}) is a Nash equilibrium;

moreover, the sufficient condition for the existence of a unique Nash equilibrium in period T is $\frac{2\gamma(2\overline{\beta}-1)}{\sigma^2}(m + \frac{H_{T+1}}{\overline{\beta}(1-\overline{\beta})})\phi(1) < b(1-\overline{\beta})$.

At the Nash equilibrium (e_T^{1*}, e_T^{2*}), from $(C.3)$ and noting that V_{T+1}^B is independent of α_{T+1},

$v_T^B(\alpha_T) = V_T^B = \frac{2\sqrt{2H_T}}{\sqrt{b}} + \gamma V_{T+1}^B$,

$v_T^1(\alpha_T) = \alpha_T m - \frac{H_T}{\alpha_T} + \gamma V_T$, $v_T^2(1-\alpha_T) = (1-\alpha_T)m - \frac{H_T}{1-\alpha_T} + \gamma V_T$,

where

$$V_T = \int v_{T+1}^1(\widehat{\beta}_{T+1}^*(y_1, y_2))\phi(y_1)\phi(y_2)dy_1dy_2$$
$$= \int (\widehat{\beta}_{T+1}^*(y_1, y_2)m - \frac{H_{T+1}}{\widehat{\beta}_{T+1}^*(y_1,y_2)})\phi(y_1)\phi(y_2)dy_1dy_2 + V_{T+1}.$$

Suppose the above results hold for period $t+1$ ($t \leq T-1$), i.e., under the condition $\frac{2\gamma(2\overline{\beta}-1)}{\sigma^2}(m + \frac{H_{t+2}}{\overline{\beta}(1-\overline{\beta})})\phi(1) < b(1-\overline{\beta})$, $(e_{t+1}^{1*}, e_{t+1}^{2*})$ defined by $(C.18)$ and $(C.19)$ with the subscript t replaced by $t+1$ is the unique Nash equilibrium of the suppliers in period $t+1$, and at this equilibrium,

$v_{t+1}^B(\alpha_{t+1}) = V_{t+1}^B = \frac{2\sqrt{2H_{t+1}}}{\sqrt{b}} + \gamma V_{t+2}^B$,

$v_{t+1}^1(\alpha_{t+1}) = \alpha_{t+1} m - \frac{H_{t+1}}{\alpha_{t+1}} + \gamma V_{t+1}$, $v_{t+1}^2(1-\alpha_{t+1}) = (1-\alpha_{t+1})m - \frac{H_{t+1}}{1-\alpha_{t+1}} + \gamma V_{t+1}$,

where

$$V_{t+1} = \int (\widehat{\beta}_{t+2}^*(y_1, y_2)m - \frac{H_{t+2}}{\widehat{\beta}_{t+2}^*(y_1,y_2)})\phi(y_1)\phi(y_2)dy_1dy_2 + V_{t+2}.$$

Then using the backward recursive argument and similar analysis as that for period T problem, we can show that in period t, under the condition

$$\frac{2\gamma(2\overline{\beta}-1)}{\sigma^2}(m + \frac{H_{t+1}}{\overline{\beta}(1-\overline{\beta})})\phi(1) < b(1-\overline{\beta}), \tag{C.20}$$

(e_t^{1*}, e_t^{2*}) defined by $(C.18)$ and $(C.19)$ is the unique Nash equilibrium of the suppliers in period t, and at this equilibrium,

$v_t^B(\alpha_t) = V_t^B$, $v_t^1(\alpha_t) = \alpha_t m - \frac{H_t}{\alpha_t} + \gamma V_t$, $v_t^2(1-\alpha_t) = (1-\alpha_t)m - \frac{H_t}{1-\alpha_t} + \gamma V_t$, where

$$V_t = \int (\widehat{\beta}^*_{t+1}(y_1, y_2)m - \frac{H_{t+1}}{\widehat{\beta}^*_{t+1}(y_1, y_2)})\phi(y_1)\phi(y_2)dy_1dy_2 + V_{t+1}, \tag{C.21}$$

and

$$V_t^B = \frac{2\sqrt{2H_t}}{\sqrt{b}} + \gamma V_{t+1}^B. \tag{C.22}$$

Because $(C.20)$ holds at $\overline{\beta} = \frac{1}{2}$, the set of the values of $\overline{\beta}$ such that $\overline{\beta} > \frac{1}{2}$ and $(C.20)$ holds is nonempty. So under condition $(C.20)$, $\{(e_t^{1*}, e_t^{2*})\}_{t=1,2,\ldots,T}$ constitutes a unique subgame perfect Nash equilibrium.

Step 2. Infinite-horizon problem:

When $T \to \infty$, the Nash equilibrium is stationary if there exists a solution to

$$F(H_\infty) = H_\infty - \frac{\gamma^2}{2b\sigma^2}(\int y_1(\widehat{\beta}^*(y_1, y_2)m - \frac{H_\infty}{\widehat{\beta}^*(y_1, y_2)})\phi(y_1)\phi(y_2)dy_1dy_2)^2 = 0. \tag{C.23}$$

Then in period t, given α_t, $e_t^{1*} = e_1^* = \frac{\sqrt{2H_\infty}}{\alpha_t\sqrt{b}}, e_t^{2*} = e_2^* = \frac{\sqrt{2H_\infty}}{(1-\alpha_t)\sqrt{b}}$,

the allocation rule is in fact the optimal allocation rule derived in the Part 1 proof; and from $(C.21)$ and $(C.22)$,

$V_\infty = \frac{1}{1-\gamma}\int(\widehat{\beta}^*(y_1, y_2)m - \frac{H_\infty}{\widehat{\beta}^*(y_1,y_2)})\phi(y_1)\phi(y_2)dy_1dy_2$, $V_\infty^B = \frac{2\sqrt{2H_\infty}}{(1-\gamma)\sqrt{b}}$.

It is obvious that $F(H_\infty)$ is continuous in H_∞. To prove the existence of a solution to $(C.23)$, we can find a bound to $F(H_\infty)$ using the fact that $\underline{\beta} \leq \widehat{\beta}^*(y_1, y_2) \leq \overline{\beta}$. For simplicity of notation, we omit H_∞ in $\Psi(\widehat{\beta}^*(y_1, y_2)|H_\infty)$ in the analysis.

Using polar coordinates defined in $(C.14)$, in $F(H_\infty)$, let

$B = \int_{y_1}\int_{y_2} y_1\Psi(\widehat{\beta}^*(y_1, y_2))\phi(y_1)\phi(y_2)dy_1dy_2$

$= \Lambda_3 + \Psi(\overline{\beta})\int_{y_2<0}\int_{y_2}^0 y_1\phi(y_1)\phi(y_2)dy_1dy_2 + \Psi(\underline{\beta})\int_{y_2<0}\int_{y_1<y_2} y_1\phi(y_1)\phi(y_2)dy_1dy_2$

$+\Psi(\overline{\beta})\int_{y_1>0,y_2<0} y_1\phi(y_1)\phi(y_2)dy_1dy_2 + \Psi(\underline{\beta})\int_{y_1<0,y_2>0} y_1\phi(y_1)\phi(y_2)dy_1dy_2]$

$= \Lambda_3 + \Psi(\overline{\beta})\int_{y_2<0}\phi(y_2)(\phi(y_2) - \phi(0))dy_2 - \Psi(\underline{\beta})\int_{y_2<0}(\phi(y_2))^2dy_2 + \frac{1}{2}(\Psi(\overline{\beta}) - \Psi(\underline{\beta}))\phi(0)$

$= \Lambda_3 + \Psi(\overline{\beta})(\frac{1}{4\sqrt{\pi}} - \frac{\phi(0)}{2}) - \Psi(\underline{\beta})\frac{1}{4\sqrt{\pi}} + \frac{1}{2}(\Psi(\overline{\beta}) - \Psi(\underline{\beta}))\phi(0) \Rightarrow$

$$B = \Lambda_3 + \Psi(\overline{\beta})\frac{1}{4\sqrt{\pi}} - \Psi(\underline{\beta})\frac{1}{4\sqrt{\pi}}(\sqrt{2} + 1), \tag{C.24}$$

where $\Lambda_3 = \int_{y_1>0,y_2>0} y_1 \Psi(\widehat{\beta}^*(y_1,y_2))\phi(y_1)\phi(y_2)dy_1dy_2$
$= \int_{r=0}^{\infty}\int_{\theta=0}^{\pi/2} \Psi(\widehat{\beta}^*) r\sin\theta \frac{1}{2\pi} e^{-r^2/2} r dr d\theta = \sqrt{\frac{\pi}{2}} \int_{\theta=0}^{\pi/2} \Psi(\widehat{\beta}^*)\sin\theta \frac{1}{2\pi} d\theta \Rightarrow$

$$\Lambda_3 = \frac{1}{2\sqrt{2\pi}}\left[\int_{\theta_{\min}}^{\theta_{\max}} \Psi(\widehat{\beta}^*)\sin\theta d\theta + \Psi(\underline{\beta})(1-\cos\theta_{\min}) + \Psi(\overline{\beta})\cos\theta_{\max}\right]. \tag{C.25}$$

Because $\Psi'(\beta|H_\infty) > 0$ and $H_\infty \geq 0$,
$\Lambda_3 < \frac{1}{2\sqrt{2\pi}}\Psi(\overline{\beta})\int_{\theta_{\min}}^{\theta_{\max}} \sin\theta d\theta + \frac{1}{2\sqrt{2\pi}}\Psi(\underline{\beta})(1-\cos\theta_{\min}) + \frac{1}{2\sqrt{2\pi}}\Psi(\overline{\beta})\cos\theta_{\max}$
$= \frac{1}{2\sqrt{2\pi}}[\Psi(\underline{\beta}) + (\Psi(\overline{\beta}) - \Psi(\underline{\beta}))\cos\theta_{\min}] \Rightarrow$
$B < \frac{1}{2\sqrt{2\pi}}[\frac{1}{\sqrt{2}} + \cos\theta_{\min}](\Psi(\overline{\beta}) - \Psi(\underline{\beta}))$;
$\Lambda_3 > \frac{1}{2\sqrt{2\pi}}\Psi(\underline{\beta})\int_{\theta_{\min}}^{\theta_{\max}} \sin\theta d\theta + \frac{1}{2\sqrt{2\pi}}\Psi(\underline{\beta})(1-\cos\theta_{\min}) + \frac{1}{2\sqrt{2\pi}}\Psi(\overline{\beta})\cos\theta_{\max}$
$= \frac{1}{2\sqrt{2\pi}}[\Psi(\underline{\beta}) + (\Psi(\overline{\beta}) - \Psi(\underline{\beta}))\cos\theta_{\max}] \Rightarrow$
$B > \frac{1}{2\sqrt{2\pi}}[\frac{1}{\sqrt{2}} + \cos\theta_{\max}](\Psi(\overline{\beta}) - \Psi(\underline{\beta})) > 0$.

Let $F_1(x) = x - \frac{\gamma^2}{16\pi b\sigma^2}(\frac{1}{\sqrt{2}} + \cos\theta_{\min})^2(\Psi(\overline{\beta}) - \Psi(\underline{\beta}))^2$
$= x - J(\theta_{\min})(m + \frac{x}{\overline{\beta}\underline{\beta}})^2$,
and $F_2(x) = x - J(\theta_{\max})(m + \frac{x}{\overline{\beta}\underline{\beta}})^2$,
where $J(\theta) = \frac{\gamma^2}{16\pi b\sigma^2}(2\overline{\beta} - 1)^2(\frac{1}{\sqrt{2}} + \cos\theta)^2$. So for any $x \geq 0$,

$$F_1(x) < F(x) < F_2(x). \tag{C.26}$$

$F_1'(x) = 1 - 2J(\theta_{\min})(m + \frac{x}{\overline{\beta}\underline{\beta}})\frac{1}{\overline{\beta}\underline{\beta}}, F_1''(x) = -2J(\theta_{\min})\frac{1}{(\overline{\beta}\underline{\beta})^2} < 0$;
$F_2'(x) = 1 - 2J(\theta_{\max})(m + \frac{x}{\overline{\beta}\underline{\beta}})\frac{1}{\overline{\beta}\underline{\beta}}, F_2''(x) = -2J(\theta_{\max})\frac{1}{(\overline{\beta}\underline{\beta})^2} < 0$.

Let x^* be the solution to $F_1'(x) = 0$, so $F_1(x)$ takes the maximum at $x^* = \overline{\beta}\underline{\beta}(\frac{\overline{\beta}\underline{\beta}}{2J(\theta_{\min})} - m)$,

and $F_1(x^*) = \overline{\beta}\underline{\beta}(\frac{\overline{\beta}\underline{\beta}}{4J(\theta_{\min})} - m)$.

Because $F_1(0) < 0$ and $F_2(0) < 0$, the sufficient condition for the existence of a solution to $(C.23)$ is $F_1(x^*) \geq 0$, which is equivalent to

$$\frac{(2\overline{\beta} - 1)^2}{\overline{\beta}(1 - \overline{\beta})} \leq \frac{4\pi b\sigma^2}{\gamma^2 m(\frac{1}{\sqrt{2}} + \cos\theta_{\min})^2}. \tag{C.27}$$

Because $(C.27)$ holds at $\overline{\beta} = \frac{1}{2}$ with strict inequality, the set of the values of $\overline{\beta}$

such that $\overline{\beta} > \frac{1}{2}$ and $(C.27)$ holds is nonempty. $(C.27)$ sets a lower bound to $\underline{\beta}$ with $\underline{\beta} > 0$, and guarantees at least one solution to $F_1(x) = 0$.

$(C.27) \Rightarrow F_2'(0) = 1 - 2J(\theta_{\max})\frac{m}{\overline{\beta}\underline{\beta}} > 1 - 2J(\theta_{\min})\frac{m}{\overline{\beta}\underline{\beta}} \geq 1 - \frac{1}{2} = \frac{1}{2}$;

for sufficiently large x, $F_2'(x) < 0$, so $(C.27)$ guarantees two solutions to $F_2(x) = 0$. Let the bigger solution to $F_2(x) = 0$ be $\overline{x}$, and the smaller one be $\underline{x}$. Obviously $x^* \in [\underline{x}, \overline{x}]$. It follows from $(C.26)$ that $F(\underline{x}) < 0$, $F(\overline{x}) < 0$ and $F(x^*) > F_1(x^*) \geq 0$. By the Intermediate Value Theorem, there exists at least one $\widetilde{x} \in [\underline{x}, \overline{x}]$ such that $F(\widetilde{x}) = 0$. Because $F(x) < F_2(x) < 0$ for $x > 0$ and $x \notin [\underline{x}, \overline{x}]$, there is no fixed point outside $[\underline{x}, \overline{x}]$. So under $(C.27)$, (e_1^*, e_2^*) constitutes a unique stationary Nash equilibrium. It is noted that $(C.27)$ is a sufficient condition which can be relaxed because $F_1(x)$ only gives a lower bound to $F(x)$, even if $F_1(x^*) < 0$, there may still exist a solution to $F(x) = 0$ when the maximum value of $F_2(x)$ is positive.

It is also noted that there could be multiple fixed points in $[\underline{x}, \overline{x}]$. As will be shown in Corollary 4.1, the buyer's long-run discounted payoff $v_B^* = \frac{2\sqrt{2H}}{(1-\gamma)\sqrt{b}}$. So when there are multiple fixed points in $[\underline{x}, \overline{x}]$, the largest fixed point while making the (IR) constraint and Nash constraint hold is optimal.

4. Proof of Corollary 4.1

From the proof of Theorem 4.1, $G'(\widehat{\beta}^*(y_1, y_2)) = 0 \Rightarrow$

$$S(\widehat{\beta}^*(y_1, y_2)) = \frac{y_1}{y_2} = R. \tag{C.28}$$

Because

$\int\int y_1 v(\widehat{\beta}^*(y_1, y_2))\phi(y_1)\phi(y_2)dy_1dy_2 = \int\int y_2 v(1 - \widehat{\beta}^*(y_1, y_2))\phi(y_1)\phi(y_2)dy_1dy_2$,

$(C.10) \Rightarrow$

$v_B = \gamma\frac{2}{b\sigma}\int\int y_1 v(\widehat{\beta}^*(y_1, y_2))\phi(y_1)\phi(y_2)dy_1dy_2 + \gamma v_B\int\int \phi(y_1)\phi(y_2)dy_1dy_2$

$\Rightarrow v_B = \frac{2\sqrt{2H}}{(1-\gamma)\sqrt{b}}$.

In $(C.11)$, using the definition of V in the proof of Theorem 4.1, take $\alpha = \widehat{\beta}^*(y_1, y_2)$ and integrate both sides over y_1 and y_2,

$V = \int\int (\widehat{\beta}^*(y_1, y_2)m - \frac{H}{\widehat{\beta}^*(y_1,y_2)})\phi(y_1)\phi(y_2)dy_1dy_2 + \gamma V\int\int \phi(y_1)\phi(y_2)dy_1dy_2$

$\Rightarrow V = \frac{1}{1-\gamma}\int\int (\widehat{\beta}^*(y_1, y_2)m - \frac{H}{\widehat{\beta}^*(y_1,y_2)})\phi(y_1)\phi(y_2)dy_1dy_2$.

Substituting this into $(C.11)$ gives the formula for $v^*(\alpha)$.

In the following subsections 5 to 7, for simplicity of notation, we use β for $\widehat{\beta}$.

5. Proof of Theorem 4.2

For simplicity of notation, we use θ for θ_α. Under the allocation rule defined by (4.9), Supplier 1's long-run discounted payoff is

$$v_1(\alpha) = \alpha m - \frac{\alpha b(e_1)^2}{2} + \gamma[v_1(\beta)\overline{\Phi}(\frac{\theta-(e_1-e_2)}{\sqrt{2}\sigma}) + v_1(1-\beta)\Phi(\frac{\theta-(e_1-e_2)}{\sqrt{2}\sigma})]. \quad (C.29)$$

FOC $\Rightarrow$

$$-\alpha b e_1 + \gamma(v_1(\beta) - v_1(1-\beta))\frac{1}{\sqrt{2}\sigma}\phi(\frac{\theta-(e_1-e_2)}{\sqrt{2}\sigma}) = 0. \quad (C.30)$$

Supplier 2's long-run discounted payoff is

$$\begin{aligned} v_2(1-\alpha) &= (1-\alpha)m - \frac{(1-\alpha)b(e_2)^2}{2} \\ &\quad + \gamma[v_2(1-\beta)\overline{\Phi}(\frac{\theta-(e_1-e_2)}{\sqrt{2}\sigma}) + v_2(\beta)\Phi(\frac{\theta-(e_1-e_2)}{\sqrt{2}\sigma})]. \end{aligned} \quad (C.31)$$

FOC $\Rightarrow$

$$-(1-\alpha)b e_2 + \gamma(v_2(\beta) - v_2(1-\beta))\frac{1}{\sqrt{2}\sigma}\phi(\frac{\theta-(e_1-e_2)}{\sqrt{2}\sigma}) = 0. \quad (C.32)$$

Because $v_1(\alpha) = v_2(\alpha)$ for any α, denote it by $v(\alpha)$ and let $\Delta v(\alpha) = v(\alpha) - v(1-\alpha)$. For an allocation rule to provide incentive to suppliers, we need $\Delta v(\beta) > 0$.

The buyer's problem is

$$v_B(\alpha) = \max_{\beta,\theta}\{\alpha e_1 + (1-\alpha)e_2 + \gamma[v_B(\beta)\overline{\Phi}(\frac{\theta-(e_1-e_2)}{\sqrt{2}\sigma}) + v_B(1-\beta)\Phi(\frac{\theta-(e_1-e_2)}{\sqrt{2}\sigma})]\} \quad (C.33)$$

subject to $(C.30)$ and $(C.32)$.

Note that $v_B(\beta) = v_B(1-\beta)$. Let η_1 and η_2 be the Lagrangian multipliers of $(C.30)$ and $(C.32)$.

$\frac{\partial \mathcal{L}}{\partial \theta} = -(\eta_1+\eta_2)\gamma\Delta v(\beta)\frac{\theta-(e_1^*-e_2^*)}{2\sigma^2}\frac{1}{\sqrt{2}\sigma}\phi(\frac{\theta-(e_1^*-e_2^*)}{\sqrt{2}\sigma}) = 0 \Rightarrow$ the optimal

$$\theta^* = e_1^* - e_2^*. \quad (C.34)$$

It follows from $(C.30)$, $(C.32)$ and $(C.34)$ that

$$e_1^* = \frac{\gamma\Delta v(\beta)}{\alpha\sqrt{2}b\sigma}\phi\left(\frac{\theta^* - (e_1^* - e_2^*)}{\sqrt{2}\sigma}\right) = \frac{\gamma\Delta v(\beta)}{2\alpha\sqrt{\pi}b\sigma}, \quad (C.35)$$

$$e_2^* = \frac{\gamma\Delta v(\beta)}{2(1-\alpha)\sqrt{\pi}b\sigma}. \quad (C.36)$$

For $\alpha \neq \frac{1}{2}$, $e_1^* \neq e_2^*$. So $\theta^* \neq 0$.

6. Proof of Corollary 4.2

The formula for θ_α^* follows from $(C.35)$ and $(C.36)$. Substituting this formula, $(C.35)$ and $(C.36)$ into the buyer's objective function, $(C.29)$ and $(C.31)$, we obtain

$v_B(\alpha) = \frac{\gamma}{1-\gamma}\frac{\Delta v(\beta)}{\sqrt{\pi}b\sigma}$,

$v(\alpha) = \alpha m - \frac{(\gamma\Delta v(\beta))^2}{8\pi\alpha b\sigma^2} + \frac{\gamma}{2}(v(\beta) + v(1-\beta)) \Rightarrow$

$$v(\beta) = \beta m - \frac{(\gamma\Delta v(\beta))^2}{8\pi\beta b\sigma^2} + \frac{\gamma}{2}(v(\beta) + v(1-\beta)), \quad (C.37)$$

$$v(1-\beta) = (1-\beta)m - \frac{(\gamma\Delta v(\beta))^2}{8\pi(1-\beta)b\sigma^2} + \frac{\gamma}{2}(v(\beta) + v(1-\beta)). \quad (C.38)$$

$\Rightarrow \Delta v(\beta) = (2\beta-1)m + \frac{(\gamma\Delta v(\beta))^2(2\beta-1)}{8\pi b\sigma^2\beta(1-\beta)} \Rightarrow \Delta v(\beta)$ is the solution to

$$\frac{\gamma^2(2\beta-1)}{8\pi b\sigma^2\beta(1-\beta)}(\Delta v(\beta))^2 - \Delta v(\beta) + (2\beta-1)m = 0. \quad (C.39)$$

Let $W = \frac{\gamma^2}{8\pi b\sigma^2}\frac{1-2\beta}{\beta(1-\beta)}$. Then $\Delta v(\beta) = \frac{1}{2W}(-1 \pm \sqrt{1 + 4W(2\beta-1)m})$.

For $\beta > \frac{1}{2}$, $W < 0$ and $\sqrt{1 + 4W(2\beta-1)m} < 1$; and for there existing a solution to $(C.39)$, we need

$$1 + 4W(2\beta-1)m \geq 0. \quad (C.40)$$

This could constrain the value of the optimal β. $\Delta v(\beta) = 0$ at $\beta = \frac{1}{2}$, so

$$\Delta v(\beta) = \frac{1}{-2W}(1 - \sqrt{1 + 4W(2\beta-1)m}). \quad (C.41)$$

Let $\Sigma v(\beta) = v(\beta) + v(1-\beta)$. $(C.37) + (C.38) \Rightarrow$

$$(1-\gamma)\Sigma v(\beta) = m - \frac{(\gamma\Delta v(\beta))^2}{8\pi b\sigma^2\beta(1-\beta)}. \quad (C.42)$$

It follows that $v(\beta) = \frac{1}{2}(\Sigma v(\beta) + \Delta v(\beta)), v(1-\beta) = \frac{1}{2}(\Sigma v(\beta) - \Delta v(\beta))$.

Let $K = \frac{\gamma^2}{8\pi b\sigma^2}$. For $\beta > \frac{1}{2}$,
$\frac{\partial W}{\partial \beta} = -K(\frac{1}{\beta^2} + \frac{1}{(1-\beta)^2}) < 0, \frac{\partial}{\partial \beta}(\frac{1}{-2W}) = \frac{1}{2W^2}\frac{\partial W}{\partial \beta} < 0$,
$\frac{\partial}{\partial \beta}(\sqrt{1+4W(2\beta-1)m}) = \frac{1}{2}(1+4W(2\beta-1)m)^{-\frac{1}{2}}[-4(2\beta-1)mK(\frac{1}{\beta^2} + \frac{1}{(1-\beta)^2}) + 8Wm]$
$= -2(1+4W(2\beta-1)m)^{-\frac{1}{2}}m[(2\beta-1)K(\frac{1}{\beta^2} + \frac{1}{(1-\beta)^2}) + \frac{2K(2\beta-1)}{\beta(1-\beta)}]$
$= -2(1+4W(2\beta-1)m)^{-\frac{1}{2}}\frac{K(2\beta-1)m}{\beta^2(1-\beta)^2} = 2(1+4W(2\beta-1)m)^{-\frac{1}{2}}\frac{mW}{\beta(1-\beta)}$.

By $(C.41)$,
$\frac{\partial \Delta v(\beta)}{\partial \beta} = \frac{1}{2W^2}\frac{\partial W}{\partial \beta}(1-\sqrt{1+4W(2\beta-1)m}) + (1+4W(2\beta-1)m)^{-\frac{1}{2}}\frac{m}{\beta(1-\beta)}$
$= -\frac{1}{2K(2\beta-1)^2}(\beta^2+(1-\beta)^2)(1-\sqrt{1+4W(2\beta-1)m}) + (1+4W(2\beta-1)m)^{-\frac{1}{2}}\frac{m}{\beta(1-\beta)}$
$= (1-\frac{4K(2\beta-1)^2m}{\beta(1-\beta)})^{-\frac{1}{2}}[\frac{1}{2K(2\beta-1)^2}(1-\sqrt{1-\frac{4K(2\beta-1)^2m}{\beta(1-\beta)}})(\beta^2+(1-\beta)^2) - \frac{m(2\beta-1)^2}{\beta(1-\beta)}]$.

At $\beta = \frac{1}{2}$, $(1-\frac{4K(2\beta-1)^2m}{\beta(1-\beta)})^{-\frac{1}{2}} = 1, \beta^2 + (1-\beta)^2 = \frac{1}{2}, (2\beta-1)^2\frac{m}{\beta(1-\beta)} = 0$,
$\lim_{\beta\to\frac{1}{2}} \frac{1}{2K(2\beta-1)^2}(1-\sqrt{1-\frac{4K(2\beta-1)^2m}{\beta(1-\beta)}}) = -\lim_{\beta\to\frac{1}{2}} \frac{1}{8K(2\beta-1)\beta}2(1+4W(2\beta-1)m)^{-\frac{1}{2}}\frac{mW}{\beta(1-\beta)}$
$= \lim_{\beta\to\frac{1}{2}} \frac{m}{4\beta^3(1-\beta)^2}(1+4W(2\beta-1)m)^{-\frac{1}{2}} = 8m$,
so

$$\lim_{\beta\to\frac{1}{2}} \frac{\partial \Delta v(\beta)}{\partial \beta} > 0. \tag{C.43}$$

$\frac{\partial^2 \Delta v(\beta)}{\partial \beta^2} = \frac{1}{2K(1+4W(2\beta-1)m)^{\frac{3}{2}}(2\beta-1)^3}[\left(1-\sqrt{1+4W(2\beta-1)m}\right)^2\left(2\sqrt{1+4W(2\beta-1)m}+1\right) + \frac{4m^2K^2(2\beta-1)^6}{\beta^3(1-\beta)^3}]$
$\Rightarrow \frac{\partial^2 \Delta v(\beta)}{\partial \beta^2} > 0$ for $\beta > \frac{1}{2}$ and $\frac{\partial^2 \Delta v(\beta)}{\partial \beta^2} < 0$ for $\beta < \frac{1}{2}$;

so $\Delta v(\beta)$ is convex in $\beta > \frac{1}{2}$ and concave in $\beta < \frac{1}{2}$. By $(C.43)$, $\frac{\partial \Delta v(\beta)}{\partial \beta} > 0$ for β satisfying $(C.40)$; because $v_B(\beta) = \frac{\gamma}{1-\gamma}\frac{\Delta v(\beta)}{\sqrt{\pi}b\sigma}$, the optimal β^* should be as large as possible and is constrained by $(C.40)$.

- Supplier's individual rationality constraint

$(C.39)$ and $(C.42) \Rightarrow \Sigma v(\beta) = \frac{1}{1-\gamma}[2m - \frac{1}{2\beta-1}\Delta v(\beta)]$,
so $v(1-\beta) = \frac{m}{1-\gamma} - \frac{1}{2}(1+\frac{1}{(1-\gamma)(2\beta-1)})\Delta v(\beta) \Rightarrow$

$$v(1-\beta) = \frac{m}{1-\gamma} + \frac{1}{4W}(1+\frac{1}{(1-\gamma)(2\beta-1)})(1-\sqrt{1+4W(2\beta-1)m}) \geq 0. \tag{C.44}$$

- Nash equilibrium condition (suppliers' second-order conditions)

We need to check if at the optimal β^*, the Nash equilibrium between the two suppliers exists and is unique.

SOC for Supplier 1's and Supplier 2's problems:
$\frac{\partial^2 v_1(\alpha)}{(\partial e_1)^2} = -\alpha b + \gamma \Delta v(\beta) \frac{1}{\sqrt{2}\sigma} \frac{\theta-(e_1-e_2)}{2\sigma^2} \phi(\frac{\theta-(e_1-e_2)}{\sqrt{2}\sigma})$
and $\frac{\partial^2 v_2(1-\alpha)}{(\partial e_2)^2} = -(1-\alpha)b - \gamma \Delta v(\beta) \frac{1}{\sqrt{2}\sigma} \frac{\theta-(e_1-e_2)}{2\sigma^2} \phi(\frac{\theta-(e_1-e_2)}{\sqrt{2}\sigma})$.

Using the fact that $x\phi(x)$ is maximized at $x = 1$, the sufficient condition for the existence of a unique Nash equilibrium is

$$\frac{\gamma \Delta v(\beta)}{2\sigma^2} \phi(1) \leq b \min\{\alpha, 1-\alpha\} = b(1-\beta). \tag{C.45}$$

This condition also guarantees that using the suppliers' FOCs for the incentive compatibility constraints is sufficient.

So the optimal β^* is constrained by $(C.40)$, $(C.44)$ and $(C.45)$.

$\frac{\partial v(1-\beta)}{\partial \beta} = \frac{1}{(1-\gamma)(2\beta-1)^2} \frac{1}{-2W} (1 - \sqrt{1+4W(2\beta-1)m})$
$-\frac{1}{2}(1 + \frac{1}{(1-\gamma)(2\beta-1)})[-\frac{1}{2K(2\beta-1)^2}(\beta^2 + (1-\beta)^2)(1 - \sqrt{1+4W(2\beta-1)m}) + (1 + 4W(2\beta-1)m)^{-\frac{1}{2}} \frac{m}{\beta(1-\beta)}]$
$= (1 - \sqrt{1+4W(2\beta-1)m}) \frac{1}{4K(2\beta-1)^2} (\beta^2 + (1-\beta)^2 + \frac{1}{(1-\gamma)(2\beta-1)})$
$-(1+4W(2\beta-1)m)^{-\frac{1}{2}} \frac{m}{2\beta(1-\beta)} - \frac{1}{2} \frac{1}{(1-\gamma)(2\beta-1)} (1+4W(2\beta-1)m)^{-\frac{1}{2}} \frac{m}{\beta(1-\beta)}$
$= (1+4W(2\beta-1)m)^{-\frac{1}{2}} [(1+4W(2\beta-1)m)^{\frac{1}{2}} \frac{1}{4K(2\beta-1)^2} (\beta^2 + (1-\beta)^2 + \frac{1}{(1-\gamma)(2\beta-1)})$
$-\frac{1}{4K(2\beta-1)^2}(\beta^2+(1-\beta)^2+\frac{1}{(1-\gamma)(2\beta-1)}) + \frac{m}{\beta(1-\beta)}(\beta^2+(1-\beta)^2+\frac{1}{(1-\gamma)(2\beta-1)}) - \frac{m}{2\beta(1-\beta)} - \frac{1}{2}\frac{1}{(1-\gamma)(2\beta-1)} \frac{m}{\beta(1-\beta)}]$
$= (1+4W(2\beta-1)m)^{-\frac{1}{2}} \{-[(1-(1+4W(2\beta-1)m)^{\frac{1}{2}}) \frac{1}{4K(2\beta-1)^2} - \frac{m}{\beta(1-\beta)}](\beta^2 + (1-\beta)^2 + \frac{1}{(1-\gamma)(2\beta-1)}) - \frac{m}{2\beta(1-\beta)} - \frac{1}{(1-\gamma)(2\beta-1)} \frac{m}{2\beta(1-\beta)}\}$
$= -(1+4W(2\beta-1)m)^{-\frac{1}{2}} [\frac{1}{4K(2\beta-1)^2}(1-(1+4W(2\beta-1)m)^{\frac{1}{2}})(1+4W(2\beta-1)(\beta^2 + (1-\beta)^2 + \frac{1}{(1-\gamma)(2\beta-1)}) + \frac{m}{2\beta(1-\beta)}(1 + \frac{1}{(1-\gamma)(2\beta-1)})]$, so

$$\frac{\partial v(1-\beta)}{\partial \beta} < 0 \text{ for } \beta > \frac{1}{2}. \tag{C.46}$$

Let $\widetilde{\beta} > \frac{1}{2}$ be the solution to $1 + 4W(2\beta-1)m = 0$. At $\widetilde{\beta}$,
$v(1-\widetilde{\beta}) = \frac{m}{1-\gamma} + \frac{1}{4W}(\frac{1}{(1-\gamma)(2\widetilde{\beta}-1)} + 1)$
$= \frac{1}{4W(1-\gamma)(2\widetilde{\beta}-1)}[4W(2\widetilde{\beta}-1)m + 1 + (1-\gamma)(2\widetilde{\beta}-1)] = \frac{1}{4W} < 0$,
by $(C.46)$, the solution to $v(1-\beta) = 0$ is smaller than $\widetilde{\beta}$;
for $\beta > \frac{1}{2}$, $\frac{\partial}{\partial \beta}(1 + 4W(2\beta-1)m) = \frac{4mW}{\beta(1-\beta)} < 0$. So $(C.40)$ holds at any $\beta \leq \widetilde{\beta}$,

$(C.44)$ implies $(C.40)$. The optimal β^* is a boundary solution constrained by $(C.44)$ and $(C.45)$.

7. Proof of Proposition 4.1

Note that we only consider $\beta > \frac{1}{2}$. Let $\overline{e} = e^*_\beta - e^*_{1-\beta}$. From $(C.30)$ and $(C.32)$, $\overline{e}$ solves

$$\overline{e} = \frac{\gamma \Delta v(\beta)}{\sqrt{2}b\sigma}\phi(\frac{\overline{e}}{\sqrt{2}\sigma})\frac{1-2\beta}{\beta(1-\beta)}. \tag{C.47}$$

So $e^*_\beta = \frac{\gamma\Delta v(\beta)}{\sqrt{2}\beta b\sigma}\phi(\frac{\overline{e}}{\sqrt{2}\sigma}) = \frac{(1-\beta)\overline{e}}{1-2\beta}, e^*_{1-\beta} = \frac{\beta\overline{e}}{1-2\beta}$.

$(C.29)$ and $(C.31)$ become
$v_1(\beta) = \beta m - \beta\frac{b(e_\beta)^2}{2} + \gamma[v_1(\beta)\overline{\Phi}(\frac{-\overline{e}}{\sqrt{2}\sigma}) + v_1(1-\beta)\Phi(\frac{-\overline{e}}{\sqrt{2}\sigma})]$,
$v_2(1-\beta) = (1-\beta)m - (1-\beta)\frac{b(e_{1-\beta})^2}{2} + \gamma[v_2(1-\beta)\overline{\Phi}(\frac{-\overline{e}}{\sqrt{2}\sigma}) + v_2(\beta)\Phi(\frac{-\overline{e}}{\sqrt{2}\sigma})]$
$\Rightarrow \Sigma v(\beta) = \frac{1}{1-\gamma}(m - \frac{b\beta(1-\beta)}{2(2\beta-1)^2}(\overline{e})^2)$;
$\Delta v(\beta) = (2\beta-1)m + \frac{b}{2}\frac{\beta(1-\beta)}{2\beta-1}\overline{e}^2 - \gamma\Delta v(\beta)(2\Phi(\frac{-\overline{e}}{\sqrt{2}\sigma}) - 1)$
$\Rightarrow \Delta v(\beta) = [1+\gamma(2\Phi(\frac{-\overline{e}}{\sqrt{2}\sigma}) - 1)]^{-1}[(2\beta-1)m + \frac{b}{2}\frac{\beta(1-\beta)}{2\beta-1}\overline{e}^2]$,
substituting into $(C.47)$,

$$\overline{e} = -\frac{\gamma}{\sqrt{2}b\sigma}\phi(\frac{\overline{e}}{\sqrt{2}\sigma})(\frac{(2\beta-1)^2}{\beta(1-\beta)}m + \frac{b}{2}\overline{e}^2)[2\gamma\Phi(\frac{-\overline{e}}{\sqrt{2}\sigma}) + 1 - \gamma]^{-1}. \tag{C.48}$$

So given any β, the optimal $\overline{e}$ can be solved from $(C.48)$.

$\frac{\partial\overline{e}}{\partial\beta} = \frac{\gamma}{\sqrt{2}b\sigma}\frac{\overline{e}}{2\sigma^2}\phi(\frac{\overline{e}}{\sqrt{2}\sigma})(\frac{(2\beta-1)^2}{\beta(1-\beta)}m + \frac{b}{2}\overline{e}^2)[2\gamma\Phi(\frac{-\overline{e}}{\sqrt{2}\sigma}) + 1 - \gamma]^{-1}\frac{\partial\overline{e}}{\partial\beta}$
$-\frac{\gamma}{\sqrt{2}b\sigma}\phi(\frac{\overline{e}}{\sqrt{2}\sigma})(\frac{(2\beta-1)m}{\beta^2(1-\beta)^2} + b\overline{e}\frac{\partial\overline{e}}{\partial\beta})[2\gamma\Phi(\frac{-\overline{e}}{\sqrt{2}\sigma}) + 1 - \gamma]^{-1}$
$-\frac{\gamma^2}{\sigma^2}(\phi(\frac{\overline{e}}{\sqrt{2}\sigma}))^2(\frac{(2\beta-1)^2}{\beta(1-\beta)}m + \frac{b}{2}\overline{e}^2)[2\gamma\Phi(\frac{-\overline{e}}{\sqrt{2}\sigma}) + 1 - \gamma]^{-2}\frac{\partial\overline{e}}{\partial\beta}$
$\Rightarrow \frac{\partial\overline{e}}{\partial\beta} = [(\frac{1}{\overline{e}} + \frac{\overline{e}}{2\sigma^2})(\frac{(2\beta-1)^2}{\beta(1-\beta)}m + \frac{b}{2}\overline{e}^2) + b\overline{e}]^{-1}\frac{(2\beta-1)m}{\beta^2(1-\beta)^2}$
$\Rightarrow \frac{\partial\overline{e}}{\partial\beta} < 0$ for $\beta > \frac{1}{2}$.

Because $v_B(\beta) = v_B(1-\beta)$, we only consider $v_B(\beta)$ for $\beta > \frac{1}{2}$.
From $(C.33)$, $v_B(\beta) = -\frac{2\beta(1-\beta)\overline{e}}{(1-\gamma)(2\beta-1)}$.
$\frac{\partial v_B(\beta)}{\partial\beta} = \frac{2(2\beta^2-2\beta+1)}{(1-\gamma)(2\beta-1)^2}\overline{e} - \frac{2\beta(1-\beta)}{(1-\gamma)(2\beta-1)}\frac{\partial\overline{e}}{\partial\beta}$
$= \frac{2(2\beta^2-2\beta+1)}{(1-\gamma)(2\beta-1)^2}\overline{e} - \frac{2m}{(1-\gamma)\beta(1-\beta)}[(\frac{1}{\overline{e}} + \frac{\overline{e}}{2\sigma^2})(\frac{(2\beta-1)^2}{\beta(1-\beta)}m + \frac{b}{2}\overline{e}^2) + b\overline{e}]^{-1}$
$= \frac{2}{1-\gamma}[\frac{2\beta^2-2\beta+1}{(2\beta-1)^2}\overline{e} - \frac{m}{\beta(1-\beta)}((\frac{1}{\overline{e}} + \frac{\overline{e}}{2\sigma^2})(\frac{(2\beta-1)^2}{\beta(1-\beta)}m + \frac{b}{2}\overline{e}^2) + b\overline{e})^{-1}]$
$\Rightarrow$
$\frac{\partial^2 v_B(\beta)}{\partial\beta^2} = \frac{2}{(1-\gamma)}\{-\frac{2}{(2\beta-1)^3}\overline{e} + \frac{(2\beta^2-2\beta+1)}{(2\beta-1)^2}\frac{\partial\overline{e}}{\partial\beta} - \frac{(2\beta-1)m}{\beta^2(\beta-1)^2}[(\frac{1}{\overline{e}} + \frac{\overline{e}}{2\sigma^2})(\frac{(2\beta-1)^2}{\beta(1-\beta)}m + \frac{b}{2}\overline{e}^2) + b\overline{e}]^{-1}$

$$+\frac{m}{\beta(1-\beta)}[(\frac{1}{\overline{e}}+\frac{\overline{e}}{2\sigma^2})(\frac{(2\beta-1)^2}{\beta(1-\beta)}m+\frac{b}{2}\overline{e}^2)+b\overline{e}]^{-2}[(-\frac{1}{\overline{e}^2}+\frac{1}{2\sigma^2})(\frac{(2\beta-1)^2}{\beta(1-\beta)}m+\frac{b}{2}\overline{e}^2)\frac{\partial\overline{e}}{\partial\beta}$$
$$+(\frac{1}{\overline{e}}+\frac{\overline{e}}{2\sigma^2})(\frac{(2\beta-1)m}{\beta^2(\beta-1)^2}+b\overline{e}\frac{\partial\overline{e}}{\partial\beta})+b\frac{\partial\overline{e}}{\partial\beta}]\}$$
$$=\frac{2}{(1-\gamma)}\{-\frac{2}{(2\beta-1)^3}\overline{e}+\frac{\left(2\beta^2-2\beta+1\right)}{(2\beta-1)^2}\frac{\partial\overline{e}}{\partial\beta}-\frac{\partial\overline{e}}{\partial\beta}$$
$$+\frac{\partial\overline{e}}{\partial\beta}\frac{\beta(1-\beta)}{(2\beta-1)}[(\frac{1}{\overline{e}}+\frac{\overline{e}}{2\sigma^2})(\frac{(2\beta-1)^2}{\beta(1-\beta)}m+\frac{b}{2}\overline{e}^2)+b\overline{e}]^{-1}[(-\frac{1}{\overline{e}^2}+\frac{1}{2\sigma^2})(\frac{(2\beta-1)^2}{\beta(1-\beta)}m+\frac{b}{2}\overline{e}^2)\frac{\partial\overline{e}}{\partial\beta}$$
$$+(\frac{1}{\overline{e}}+\frac{\overline{e}}{2\sigma^2})(\frac{(2\beta-1)m}{\beta^2(\beta-1)^2}+b\overline{e}\frac{\partial\overline{e}}{\partial\beta})+b\frac{\partial\overline{e}}{\partial\beta}]\}$$
$$=\frac{2}{(1-\gamma)}\{-\frac{2}{(2\beta-1)^3}\overline{e}+\frac{2\beta(1-\beta)}{(2\beta-1)^2}\frac{\partial\overline{e}}{\partial\beta}+\frac{\partial\overline{e}}{\partial\beta}\frac{\beta(1-\beta)}{(2\beta-1)}[(\frac{1}{\overline{e}}+\frac{\overline{e}}{2\sigma^2})(\frac{(2\beta-1)^2}{\beta(1-\beta)}m+\frac{b}{2}\overline{e}^2)+b\overline{e}]^{-1}[\frac{\partial\overline{e}}{\partial\beta}[(-\frac{1}{\overline{e}^2}+$$
$$\frac{1}{2\sigma^2})\frac{(2\beta-1)^2}{\beta(1-\beta)}m+\frac{3b}{2}(1+\frac{\overline{e}^2}{2\sigma^2})]+(\frac{1}{\overline{e}}+\frac{\overline{e}}{2\sigma^2})\frac{(2\beta-1)m}{\beta^2(\beta-1)^2}]\}$$
$$=\frac{2}{(1-\gamma)}\{-\frac{1}{4\sigma^2}\frac{\overline{e}^2}{\beta(1-\beta)(2\beta-1)^2}(\beta\,(1-\beta)\,(\overline{e}^2+\frac{3}{2\sigma^2})+2m\,(2\beta-1)^2)$$
$$+\frac{\partial\overline{e}}{\partial\beta}\frac{\beta(1-\beta)}{(2\beta-1)}[(\frac{1}{\overline{e}}+\frac{\overline{e}}{2\sigma^2})(\frac{(2\beta-1)^2}{\beta(1-\beta)}m+\frac{b}{2}\overline{e}^2)+b\overline{e}]^{-2}\frac{(2\beta-1)m}{\beta^2(1-\beta)^2}[\frac{3}{2\sigma^2}\frac{(2\beta-1)^2}{\beta(1-\beta)}m+3b+\frac{7b\overline{e}^2}{4\sigma^2}+(\frac{\overline{e}}{2\sigma^2})^2(\frac{(2\beta-1)^2}{\beta(1-\beta)}m+$$
$$\frac{b\overline{e}^2}{2})]\}.$$

For $\beta > \frac{1}{2}$, because $\frac{\partial\overline{e}}{\partial\beta} < 0$, $\frac{\partial^2 v_B(\beta)}{\partial\beta^2} < 0$, thus $v_B(\beta)$ is concave in β.

It follows from $(C.48)$ that

$[2\gamma\Phi(\frac{-\overline{e}}{\sqrt{2}\sigma})+1-\gamma]\overline{e}/\phi(\frac{\overline{e}}{\sqrt{2}\sigma}) = -\frac{\gamma}{\sqrt{2}b\sigma}(\frac{(2\beta-1)^2}{\beta(1-\beta)}m+\frac{b}{2}\overline{e}^2)$, so $\overline{e}\to-\infty$ as $\beta\to 1$.

$\frac{\partial v_B(\beta)}{\partial\beta} = \frac{2}{1-\gamma}[\frac{2\beta^2-2\beta+1}{(2\beta-1)^2}\overline{e}-\frac{m}{(\frac{1}{\overline{e}}+\frac{\overline{e}}{2\sigma^2})((2\beta-1)^2m+\frac{b}{2}\beta(1-\beta)\overline{e}^2)+b\beta(1-\beta)\overline{e}}]$

$\frac{m}{(\frac{1}{\overline{e}}+\frac{\overline{e}}{2\sigma^2})((2\beta-1)^2m+\frac{b}{2}\beta(1-\beta)\overline{e}^2)+b\beta(1-\beta)\overline{e}} \to 0$ as $\beta\to 1$, so $\frac{\partial v_B(\beta)}{\partial\beta}\to-\infty$ as $\beta\to 1$. Because $v_B(\beta)$ is concave in β, the solution of β to $\frac{\partial v_B(\beta)}{\partial\beta}=0$ is smaller than 1, so the optimal $\beta^* < 1$.

It follows from the formulas for $\Sigma v(\beta)$ and $\Delta v(\beta)$ that

$v(1-\beta) = \frac{1}{2}[\frac{1}{1-\gamma}(m-\frac{b\beta(1-\beta)}{2(2\beta-1)^2}(\overline{e})^2)-(1+\gamma(2\Phi(\frac{-\overline{e}}{\sqrt{2}\sigma})-1))^{-1}((2\beta-1)m+\frac{b\beta(1-\beta)}{2(2\beta-1)}\overline{e}^2)]$.

Similar to the case under the HWTA rule, the sufficient condition for the existence of a unique Nash equilibrium between the two suppliers is $(C.44)$ together with $(C.48)$. The optimal β^* obtained from $\frac{\partial v_B(\beta)}{\partial\beta}=0$ needs to be checked with the Nash equilibrium condition and the supplier's individual rationality constraint ($v(1-\beta)\geq 0$ together with $(C.48)$). If either constraint is binding, then the optimal β^* is a boundary solution.

8. Method for numerical calculation for the optimal allocation rule $\beta_\alpha^{1*}(x_1,x_2)$ and $g(\beta)=\beta$

- Computing the right hand side of (4.8)

Using polar coordinates defined in $(C.14)$, $\tan\theta = \frac{y_1}{y_2} = \frac{m+H/(1-\beta)^2}{m+H/\beta^2}$. By $(C.24)$ and $(C.25)$, the right hand side of (4.8) is

$B = \frac{1}{2\sqrt{2\pi}}[\int_{\theta_{\min}}^{\theta_{\max}} \Psi(\beta)\sin\theta d\theta + \Psi(\overline{\beta})(\frac{1}{\sqrt{2}} + \cos\theta_{\max}) - \Psi(\underline{\beta})(\frac{1}{\sqrt{2}} + \cos\theta_{\min})]$.
$(C.28) \Rightarrow$

$$(R-1)m\beta^4 - 2(R-1)m\beta^3 + (R-1)(m+H)\beta^2 - 2RH\beta + RH = 0, \qquad \text{(C.49)}$$

where $R = \tan\theta$. So the value of β corresponding to θ is determined by $(C.49)$, and $\int_{\theta_{\min}}^{\theta_{\max}} \Psi(\beta)\sin\theta d\theta$ can be computed numerically by adding $\Psi(\beta)\sin\theta$ over $\theta \in [\theta_{\min}, \theta_{\max}]$.

- Steps used for numerical calculation of v_B, $v(\alpha)$ and e_1^*, e_2^*

Step 1. Compute H. A search method is used. For any H in a range of values, using $(C.49)$ to compute β corresponding to θ and calculate $\int_{\theta_{\min}}^{\theta_{\max}} \Psi(\beta)\sin\theta d\theta$, then using the above numerical method to obtain the RHS of (4.8). If (4.8) holds at a value of H, then H is optimal under the optimal allocation rule $\widehat{\beta}^*(y_1, y_2)$.

Step 2. Using the result in Corollary 4.1 and Theorem 4.1,
$v_B = \frac{2\sqrt{2H}}{(1-\gamma)\sqrt{b}}, e_1^* = \frac{\sqrt{2H}}{\alpha\sqrt{b}}, e_2^* = \frac{\sqrt{2H}}{(1-\alpha)\sqrt{b}}$.
Step 3. Compute $v(\alpha)$.
Using the result in Corollary 4.1,

$$V = \frac{1}{1-\gamma}\int\int(\widehat{\beta}^*(y_1, y_2)m - \frac{H}{\widehat{\beta}^*(y_1, y_2)})\phi(y_1)\phi(y_2)dy_1dy_2. \qquad \text{(C.50)}$$

By $(C.15)$ and $(C.16)$,
$\int_{y_1}\int_{y_2} \Psi(\widehat{\beta}^*(y_1, y_2))\phi(y_1)\phi(y_2)dy_1dy_2 = \frac{1}{2\pi}\int_{\theta_{\min}}^{\theta_{\max}} \Psi(\widehat{\beta}^*(y_1, y_2))d\theta + \Psi(\underline{\beta})(\frac{3}{8} + \frac{\theta_{\min}}{2\pi}) + \Psi(\overline{\beta})(\frac{5}{8} - \frac{\theta_{\max}}{2\pi})$.

Similar to the calculation for the RHS of (4.8) as above, $\int_{\theta_{\min}}^{\theta_{\max}} \Psi(\widehat{\beta}^*(y_1, y_2))d\theta$ can be computed numerically by using the optimal H in Step 1, and adding $\Psi(\beta)$ over $\theta \in [\theta_{\min}, \theta_{\max}]$. This gives the calculation of V, then for any α, $v(\alpha) = \alpha m - \frac{H}{\alpha} + \gamma V$.

9. Proof of Theorem 4.3

We use backward induction to solve this problem. Let $y_t^i = \frac{x_t^i - e_t^{i*}}{\sigma}$, where $i = 1, 2$, $t = 1, 2, ...$, e_t^{i*} is Supplier i's optimal effort level in period t.

Because there is no future business from the buyer beyond period $T+1$, in period $T+1$, both suppliers have no incentive for investment in performance, so both suppliers' optimal effort levels are 0, $v^B_{T+1}(\alpha_{T+1}) = 0, v^1_{T+1}(\alpha_{T+1}) = \alpha_{T+1}m, v^2_{T+1}(1-\alpha_{T+1}) = (1-\alpha_{T+1})m$.

- Period T problem:

The buyer's profit to go is

$$v^B_T(\alpha_T) = \alpha_T e^1_T + (1-\alpha_T)e^2_T. \tag{C.51}$$

Supplier 1's expected profit to go is
$v^1_T(\alpha_T) = m\alpha_T - \frac{b(e^1_T)^2}{2} + \gamma m \int_{x^1_T}\int_{x^2_T} \beta^\alpha_{T+1}(x^1_T, x^2_T) f(x^1_T|e^1_T) f(x^2_T|e^2_T) dx^1_T dx^2_T$
FOC $\Rightarrow$
$\frac{\partial v^1_T(\alpha_T)}{\partial e^1_T} = -be^1_T + \gamma m \int_{x^1_T}\int_{x^2_T} \beta^\alpha_{T+1}(x^1_T, x^2_T) f^1(x^1_T|e^1_T) f(x^2_T|e^2_T) dx^1_T dx^2_T = 0 \Rightarrow$

$$e^1_T = \frac{\gamma m}{b} \int_{x^1_T}\int_{x^2_T} \beta^\alpha_{T+1}(x^1_T, x^2_T) f^1(x^1_T|e^1_T) f(x^2_T|e^2_T) dx^1_T dx^2_T. \tag{C.52}$$

Similarly, the FOC for Supplier 2's expected profit to go is
$\frac{\partial v^2_T(1-\alpha_T)}{\partial e^2_T} = -be^2_T + \gamma m \int_{x^1_T}\int_{x^2_T} (1-\beta^\alpha_{T+1}(x^1_T, x^2_T)) f(x^1_T|e^1_T) f^2(x^2_T|e^2_T) dx^1_T dx^2_T = 0 \Rightarrow$

$$e^2_T = -\frac{\gamma m}{b} \int_{x^1_T}\int_{x^2_T} \beta^\alpha_{T+1}(x^1_T, x^2_T) f(x^1_T|e^1_T) f^2(x^2_T|e^2_T) dx^1_T dx^2_T. \tag{C.53}$$

$(C.52)$ and $(C.53)$ are necessary conditions for (e^1_T, e^2_T) to be a Nash equilibrium.

Substituting $(C.52)$ and $(C.53)$ into $(C.51)$, the buyer's problem becomes
$v^B_T(\alpha_T) = \int_{y^1_T}\int_{y^2_T} \widehat{\beta}^\alpha_{T+1}(y^1_T, y^2_T) \frac{\gamma m}{b\sigma}[\alpha_T y^1_T - (1-\alpha_T)y^2_T]\phi(y^1_T)\phi(y^2_T) dy^1_T dy^2_T.$
Pointwise optimization w.r.t. $\widehat{\beta}^{T+1}_\alpha$ $\Rightarrow$
the optimal value is determined by the sign of $\frac{\gamma m}{b\sigma}[\alpha_T y^1_T - (1-\alpha_T)y^2_T]\phi(y^1_T)\phi(y^2_T)$.
So to induce (e^1_T, e^2_T) defined by $(C.52)$ and $(C.53)$, the optimal allocation rule is a HWTA one such that

$$\widehat{\beta}^\alpha_{T+1}(y^1_T, y^2_T) = \begin{cases} \beta_{T+1} & \alpha_T y^1_T > (1-\alpha_T)y^2_T \\ 1-\beta_{T+1} & \alpha_T y^1_T < (1-\alpha_T)y^2_T \end{cases}, \tag{C.54}$$

and

$$\beta^{\alpha}_{T+1}(x^1_T, x^2_T) = \begin{cases} \beta_{T+1} & \alpha_T(x^1_T - e^{1*}_T) > (1-\alpha_T)(x^2_T - e^{2*}_T) \\ 1-\beta_{T+1} & \alpha_T(x^1_T - e^{1*}_T) < (1-\alpha_T)(x^2_T - e^{2*}_T) \end{cases}.$$

Using the fact that

$$\begin{aligned} \int_y \phi(y)\phi(\frac{1-\alpha}{\alpha}y)dy &= \frac{\alpha}{\sqrt{2\pi}\Gamma}\int_y \frac{\Gamma}{\alpha}\phi(\frac{\Gamma}{\alpha}y)dy = \frac{\alpha}{\sqrt{2\pi}\Gamma}, \\ \int_y \phi(y)\phi(\frac{\alpha}{1-\alpha}y)dy &= \frac{1-\alpha}{\sqrt{2\pi}\Gamma}\int_y \frac{\Gamma}{1-\alpha}\phi(\frac{\Gamma}{1-\alpha}y)dy = \frac{1-\alpha}{\sqrt{2\pi}\Gamma}, \end{aligned} \tag{C.55}$$

where $\Gamma = \sqrt{\alpha^2 + (1-\alpha)^2}$,

$e^1_T = \frac{\gamma m}{b\sigma}\int_{y^2_T}[\beta_{T+1}\int^{\infty}_{y^2_T\frac{1-\alpha_2}{\alpha_2}} y^1_T\phi(y^1_T)dy^1_T + (1-\beta_{T+1})\int^{y^2_T\frac{1-\alpha_2}{\alpha_2}}_{-\infty} y^1_T\phi(y^1_T)dy^1_T]\phi(y^2_T)dy^2_T$

$= \frac{\gamma m}{b\sigma}\int_{y^2_T}(2\beta_{T+1}-1)\phi(y^2_T\frac{1-\alpha_2}{\alpha_2})\phi(y^2_T)dy^2_T = \frac{\gamma m(2\beta_{T+1}-1)}{\sqrt{2\pi}b\sigma}\frac{\alpha_T}{\Gamma_T}.$

So $(C.52)$, $(C.53)$ and $(C.54)$ $\Rightarrow e^{1*}_T = \frac{\gamma m(2\beta_{T+1}-1)\alpha_T}{\sqrt{2\pi}b\sigma\Gamma_T}, e^{2*}_T = \frac{\gamma m(2\beta_{T+1}-1)(1-\alpha_T)}{\sqrt{2\pi}b\sigma\Gamma_T}$,
at (e^{1*}_T, e^{2*}_T),
$v^1_T(\alpha_T) = m(\alpha_T + \frac{\gamma}{2}) - \frac{b}{2}(e^{1*}_T)^2 = m(\alpha_T + \frac{\gamma}{2}) - \frac{b}{4\pi}(\frac{\gamma m(2\beta_{T+1}-1)\alpha_T}{b\sigma\Gamma_T})^2$,
$v^2_T(1-\alpha_T) = m(1-\alpha_T + \frac{\gamma}{2}) - \frac{b}{2}(e^{2*}_T)^2 = m(1-\alpha_T + \frac{\gamma}{2}) - \frac{b}{4\pi}(\frac{\gamma m(2\beta_{T+1}-1)(1-\alpha_T)}{b\sigma\Gamma_T})^2$,
$\Delta v_T(\alpha_T) = v^1_T(\alpha_T) - v^1_T(1-\alpha_T) = (2\alpha_T - 1)[m - \frac{1}{4\pi b}(\frac{\gamma m(2\beta_{T+1}-1)}{\sigma\Gamma_T})^2]$
$v^B_T(\alpha_T) = \alpha_T e^{1*}_T + (1-\alpha_T)e^{2*}_T = \frac{\gamma m(2\beta_{T+1}-1)\Gamma_T}{\sqrt{2\pi}b\sigma}$.
Because $v^B_T(\alpha_T)$ is increasing in β_{T+1}, the optimal β_{T+1} is a boundary solution.

$\frac{\partial\Gamma_T}{\partial\alpha_T} = \frac{1}{2\Gamma_T}\frac{\partial\Gamma^2_T}{\partial\alpha_T} = \frac{2\alpha_T-1}{\Gamma_T}, \frac{\partial^2\Gamma_T}{\partial\alpha^2_T} = \frac{1}{\Gamma^2_T}[2\Gamma_T - \frac{(2\alpha_T-1)^2}{\Gamma_T}] = \frac{1}{\Gamma^3_T} \Rightarrow$

$$\frac{\partial v^B_T(\alpha_T)}{\partial\alpha_T} = \frac{\gamma m(2\beta_{T+1}-1)}{\sqrt{2\pi}b\sigma}\frac{2\alpha_T-1}{\Gamma_T}, \tag{C.56}$$

$\frac{\partial v^B_T(\alpha_T)}{\partial\alpha_T} = 0$ at $\alpha_T = \frac{1}{2}$.

$$\frac{\partial^2 v^B_T(\alpha_T)}{(\partial\alpha_T)^2} = \frac{\gamma m(2\beta_{T+1}-1)}{\sqrt{2\pi}b\sigma}\frac{1}{\Gamma^3_T} > 0, \tag{C.57}$$

so $v^B_T(\alpha_T)$ is convex and is minimized at $\alpha_T = \frac{1}{2}$, at $\alpha_T = \frac{1}{2}$, $\Gamma_T^2 = \frac{1}{2}$, and $\frac{\partial^2 v_T^B(\alpha_T)}{(\partial\alpha_T)^2} = \frac{2\gamma m(2\beta_{T+1}-1)}{\sqrt{\pi}b\sigma}$.

$$\frac{\partial v_T^1(\alpha_T)}{\partial\alpha_T} = m - \frac{\gamma^2 m^2(2\beta_{T+1}-1)^2\alpha_T(1-\alpha_T)}{2\pi b\sigma^2\Gamma_T^4}, \tag{C.58}$$

so for $\frac{\partial v_T^1(\alpha_T)}{\partial\alpha_T} > 0$, because $\frac{\gamma^2 m(2\beta_{T+1}-1)^2\alpha_T(1-\alpha_T)}{2\pi b\sigma^2\Gamma_T^4}$ is maximized at $\alpha_T = \frac{1}{2}$, it is required that $\frac{\gamma^2 m(2\beta_{T+1}-1)^2}{2\pi b\sigma^2} < 1$.

$$\frac{\partial v_T^2(1-\alpha_T)}{\partial\alpha_T} = -m(1 - \frac{\gamma^2 m(2\beta_{T+1}-1)^2\alpha_T(1-\alpha_T)}{2\pi b\sigma^2\Gamma_T^4}) = -\frac{\partial v_T^1(\alpha_T)}{\partial\alpha_T}, \tag{C.59}$$

$\frac{\partial\Delta v_T(\alpha_T)}{\partial\alpha_T} = \frac{\partial v_T^1(\alpha_T)}{\partial\alpha_T} - \frac{\partial v_T^2(1-\alpha_T)}{\partial\alpha_T} = 2\frac{\partial v_T^1(\alpha_T)}{\partial\alpha_T} > 0,$

$$\begin{aligned}\frac{\partial^2 v_T^1(\alpha_T)}{(\partial\alpha_T)^2} &= \frac{\gamma^2 m^2(2\beta_{T+1}-1)^2(2\alpha_T-1)(1+2\alpha_T-2(\alpha_T)^2)}{2\pi b\sigma^2\Gamma_T^6},\\ \frac{\partial^2 v_T^2(1-\alpha_T)}{(\partial\alpha_T)^2} &= -\frac{\partial^2 v_T^1(\alpha_T)}{(\partial\alpha_T)^2}.\end{aligned} \tag{C.60}$$

- Period $T-1$ problem:

The buyer's payoff to go and the suppliers' profits to go are defined by $(C.3)$ to $(C.5)$ with $t = T-1$ and with $g(\alpha_t) = g(1-\alpha_t) = 1$. $(C.6)$ and $(C.7)$ become

$e_{T-1}^1 = \frac{\gamma}{b\sigma}\int_{y_{T-1}^1}\int_{y_{T-1}^2} v_T^1(\widehat{\beta}_T^\alpha(y_{T-1}^1, y_{T-1}^2))y_{T-1}^1\phi(y_{T-1}^1)\phi(y_{T-1}^2)dy_{T-1}^1 dy_{T-1}^2,$

$e_{T-1}^2 = \frac{\gamma}{b\sigma}\int_{y_{T-1}^1}\int_{y_{T-1}^2} v_T^2(1-\widehat{\beta}_T^\alpha(y_{T-1}^1, y_{T-1}^2))y_{T-1}^2\phi(y_{T-1}^1)\phi(y_{T-1}^2)dy_{T-1}^1 dy_{T-1}^2.$

Substituted into the buyer's objective function,

$v_{T-1}^B(\alpha_{T-1}) = \gamma\int_{y_{T-1}^1}\int_{y_{T-1}^2}[\frac{1}{b\sigma}(\alpha_{T-1}v_T^1(\widehat{\beta}_T^\alpha)y_{T-1}^1 + (1-\alpha_{T-1})v_T^2(1-\widehat{\beta}_T^\alpha)y_{T-1}^2)$
$+v_T^B(\widehat{\beta}_T^\alpha)]\phi(y_{T-1}^1)\phi(y_{T-1}^2)dy_{T-1}^1 dy_{T-1}^2.$

Let $G(\widehat{\beta}_T^\alpha) = \frac{\alpha_{T-1}}{b\sigma}y_{T-1}^1 v_T^1(\widehat{\beta}_T^\alpha) + \frac{(1-\alpha_{T-1})}{b\sigma}y_{T-1}^2 v_T^2(1-\widehat{\beta}_T^\alpha) + v_T^B(\widehat{\beta}_T^\alpha)$.

Pointwise optimization w.r.t. $\widehat{\beta}_T^\alpha$ and by $(C.56)$, $(C.58)$ and $(C.59) \Rightarrow$

$G'(\widehat{\beta}_T^\alpha) = \frac{1}{b\sigma}\frac{\partial v_T^1(\widehat{\beta}_T^\alpha)}{\partial\widehat{\beta}_T^\alpha}(\alpha_{T-1}y_{T-1}^1 - (1-\alpha_{T-1})y_{T-1}^2) + \frac{\partial v_T^B(\widehat{\beta}_T^\alpha)}{\partial\widehat{\beta}_T^\alpha},$

by $(C.57)$ and $(C.60)$,

$G''(\widehat{\beta}_T^\alpha) = \frac{1}{b\sigma}\frac{\partial^2 v_T^1(\widehat{\beta}_T^\alpha)}{(\partial\widehat{\beta}_T^\alpha)^2}(\alpha_{T-1}y_{T-1}^1 - (1-\alpha_{T-1})y_{T-1}^2) + \frac{\partial^2 v_T^B(\widehat{\beta}_T^\alpha)}{(\partial\widehat{\beta}_T^\alpha)^2}.$

Because the distribution of a supplier's performance given her effort level $f(x|e)$ is normal and satisfies the MLRP, the allocation rule should be such that $\beta_T^\alpha(x_{T-1}^1, x_{T-1}^2)$

is increasing in x^1_{T-1} and decreasing in x^2_{T-1}, and $\widehat{\beta}^{\alpha}_T(y^1_{T-1}, y^2_{T-1})$ is increasing in y^1_{T-1} and decreasing in y^2_{T-1}, so

for $\alpha_{T-1}y^1_{T-1} > (1-\alpha_{T-1})y^2_{T-1}$, $G'(\widehat{\beta}^{\alpha}_T) > 0$ and $G''(\widehat{\beta}^{\alpha}_T) > 0$ for $\widehat{\beta}^{\alpha}_T > \frac{1}{2}$, the optimal $\widehat{\beta}^{\alpha*}_T(y^1_{T-1}, y^2_{T-1}) = \beta_T$,

for $\alpha_{T-1}y^1_{T-1} < (1-\alpha_{T-1})y^2_{T-1}$, $G'(\widehat{\beta}^{\alpha}_T) < 0$ and $G''(\widehat{\beta}^{\alpha}_T) > 0$ for $\widehat{\beta}^{\alpha}_T < \frac{1}{2}$, the optimal $\widehat{\beta}^{\alpha*}_T(y^1_{T-1}, y^2_{T-1}) = 1-\beta_T$.

Thus the optimal allocation rule is a HWTA one,

$$\widehat{\beta}^{\alpha}_T(y^1_{T-1}, y^2_{T-1}) = \begin{cases} \beta_T & \alpha_{T-1}y^1_{T-1} > (1-\alpha_{T-1})y^2_{T-1} \\ 1-\beta_T & \alpha_{T-1}y^1_{T-1} < (1-\alpha_{T-1})y^2_{T-1} \end{cases}. \tag{C.61}$$

Using $(C.55)$,

$v^B_{T-1}(\alpha_{T-1}) = \frac{\gamma\Gamma_{T-1}}{\sqrt{2\pi}b\sigma}\Delta v_T(\beta_T) + \frac{\gamma}{2}[v^B_T(\beta_T) + v^B_T(1-\beta_T)] \Rightarrow$

$\frac{\partial v^B_{T-1}(\alpha_{T-1})}{\partial\alpha_{T-1}} = \gamma\frac{2\alpha_{T-1}-1}{\sqrt{2\pi}b\sigma\Gamma_{T-1}}\Delta v_T(\beta_T)$,

$\frac{\partial^2 v^B_{T-1}(\alpha_{T-1})}{\partial\alpha^2_{T-1}} = \frac{2\gamma}{\sqrt{2\pi}b\sigma\Gamma_{T-1}}\Delta v_T(\beta_T) - \gamma\frac{2\alpha_{T-1}-1}{\sqrt{2\pi}b\sigma}\Delta v_T(\beta_T)\frac{2\alpha_{T-1}-1}{\Gamma^3_{T-1}}$

$= \frac{\gamma}{\sqrt{2\pi}b\sigma}\Delta v_T(\beta_T)\frac{1}{\Gamma^3_{T-1}} > 0$.

$e^1_{T-1} = \frac{\gamma}{b\sigma}\Delta v_T(\beta_T)\frac{\alpha_{T-1}}{\sqrt{2\pi}\Gamma_{T-1}}, e^2_{T-1} = \frac{\gamma}{b\sigma}\Delta v_T(\beta_T)\frac{1-\alpha_{T-1}}{\sqrt{2\pi}\Gamma_{T-1}}, \frac{\partial e^1_{T-1}}{\partial\alpha_{T-1}} = \frac{\gamma}{\sqrt{2\pi}b\sigma}\Delta v_T(\beta_T)\frac{1-\alpha_{T-1}}{\Gamma^3_{T-1}}$.

$v^1_{T-1}(\alpha_{T-1}) = m\alpha_{T-1} - \frac{b(e^1_{T-1})^2}{2} + \frac{\gamma}{2}[v^1_T(\alpha_T) + v^2_T(1-\alpha_T)]$,

$\frac{\partial v^1_{T-1}(\alpha_{T-1})}{\partial\alpha_{T-1}} = m - \frac{\gamma^2}{2\pi b\sigma^2}(\Delta v_T(\beta_T))^2\frac{\alpha_{T-1}(1-\alpha_{T-1})}{\Gamma^4_{T-1}}$,

$\frac{\partial^2 v^1_{T-1}(\alpha_{T-1})}{(\partial\alpha_{T-1})^2} = \frac{\gamma^2}{2\pi b\sigma^2}(\Delta v_T(\beta_T))^2\frac{(2\alpha_{T-1}-1)(1+2\alpha_{T-1}-2\alpha^2_{T-1})}{\Gamma^6_{T-1}}$.

- Period t problem:

Based on the results for periods T and $T-1$, suppose in period $t+1$,

$e^1_{t+1} = \frac{\gamma\Delta v_{t+2}(\beta_{t+2})}{\sqrt{2\pi}b\sigma}\frac{\alpha_{t+1}}{\Gamma_{t+1}}, e^2_{t+1} = \frac{\gamma\Delta v_{t+2}(\beta_{t+2})}{\sqrt{2\pi}b\sigma}\frac{1-\alpha_{t+1}}{\Gamma_{t+1}}$;

$\frac{\partial v^B_{t+1}(\alpha_{t+1})}{\partial\alpha_{t+1}} = \gamma\frac{2\alpha_{t+1}-1}{\sqrt{2\pi}b\sigma\Gamma_{t+1}}\Delta v_{t+2}(\beta_{t+2}) \Rightarrow$

$\frac{\partial v^B_{t+1}(\alpha_{t+1})}{\partial\alpha_{t+1}} > 0$ for $\alpha_{t+1} > \frac{1}{2}$, $\frac{\partial v^B_{t+1}(\alpha_{t+1})}{\partial\alpha_{t+1}} < 0$ for $\alpha_{t+1} < \frac{1}{2}$;

$\frac{\partial^2 v^B_{t+1}(\alpha_{t+1})}{\partial\alpha^2_{t+1}} = \frac{\gamma}{\sqrt{2\pi}b\sigma}\Delta v_{t+2}(\beta_{t+2})\frac{1}{\Gamma^3_{t+1}} > 0$,

$\frac{\partial v^1_{t+1}(\alpha_{t+1})}{\partial\alpha_{t+1}} = m - \frac{\gamma^2}{2\pi b\sigma^2}(\Delta v_{t+2}(\beta_{t+2}))^2\frac{\alpha_{t+1}(1-\alpha_{t+1})}{\Gamma^4_{t+1}} > 0$,

$\frac{\partial^2 v^1_{t+1}(\alpha_{t+1})}{(\partial\alpha_{t+1})^2} = \frac{\gamma^2(2\alpha_{t+1}-1)(1+2\alpha_{t+1}-2(\alpha_{t+1})^2)}{2\pi b\sigma^2\Gamma^6_{t+1}}(\Delta v_{t+2}(\beta_{t+2}))^2$

$\Rightarrow \frac{\partial^2 v^1_{t+1}(\alpha_{t+1})}{(\partial\alpha_{t+1})^2} > 0$ for $\alpha_{t+1} > \frac{1}{2}$, $\frac{\partial^2 v^1_{t+1}(\alpha_{t+1})}{(\partial\alpha_{t+1})^2} < 0$ for $\alpha_{t+1} < \frac{1}{2}$;

$$\frac{\partial v^2_{t+1}(1-\alpha_{t+1})}{\partial\alpha_{t+1}} = -\frac{\partial v^1_{t+1}(\alpha_{t+1})}{\partial\alpha_{t+1}}, \frac{\partial^2 v^2_{t+1}(1-\alpha_{t+1})}{(\partial\alpha_{t+1})^2} = -\frac{\partial^2 v^1_{t+1}(\alpha_{t+1})}{(\partial\alpha_{t+1})^2}. \qquad \text{(C.62)}$$

Then in period t, referring to the formulation defined in Subsection 2 of Appendix C with $g(\alpha_t) = g(1-\alpha_t) = 1$,

$e^1_t = \frac{\gamma}{b\sigma}\int_{y^1_t}\int_{y^2_t} v^1_{t+1}(\widehat{\beta}^{\alpha}_{t+1}(y^1_t, y^2_t))y^1_t\phi(y^1_t)\phi(y^2_t)dy^1_t dy^2_t$,

$e^2_t = \frac{\gamma}{b\sigma}\int_{y^1_t}\int_{y^2_t} v^2_{t+1}(1-\widehat{\beta}^{\alpha}_{t+1}(y^1_t, y^2_t))y^2_t\phi(y^1_t)\phi(y^2_t)dy^1_t dy^2_t$.

substituted into the buyer's objective function,

$v^B_t(\alpha_t) = \gamma\int_{y^1_t}\int_{y^2_t}[\frac{1}{b\sigma}(\alpha_t v^1_{t+1}(\widehat{\beta}^{\alpha}_{t+1})y^1_t + (1-\alpha_t)v^2_{t+1}(1-\widehat{\beta}^{\alpha}_{t+1})y^2_t)$

$+v^B_{t+1}(\widehat{\beta}^{\alpha}_{t+1})]\phi(y^1_t)\phi(y^2_t)dy^1_t dy^2_t$.

Let $G(\widehat{\beta}^{\alpha}_{t+1}) = \frac{\alpha_t}{b\sigma}y^1_t v^1_{t+1}(\widehat{\beta}^{\alpha}_{t+1}) + \frac{(1-\alpha_t)}{b\sigma}y^2_t v^2_{t+1}(1-\widehat{\beta}^{\alpha}_{t+1}) + v^B_{t+1}(\widehat{\beta}^{\alpha}_{t+1})$.

Pointwise optimization w.r.t. $\widehat{\alpha}_{t+1}$ and by $(C.62) \Rightarrow$

$G'(\widehat{\beta}^{\alpha}_{t+1}) = \frac{1}{b\sigma}\frac{\partial v^1_{t+1}(\widehat{\beta}^{\alpha}_{t+1})}{\partial\widehat{\beta}^{\alpha}_{t+1}}(\alpha_t y^1_t - (1-\alpha_t)y^2_t) + \frac{\partial v^B_{t+1}(\widehat{\beta}^{\alpha}_{t+1})}{\partial\widehat{\beta}^{\alpha}_{t+1}}$,

by $(C.62)$,

$G''(\widehat{\beta}^{\alpha}_{t+1}) = \frac{1}{b\sigma}\frac{\partial^2 v^1_{t+1}(\widehat{\beta}^{\alpha}_{t+1})}{(\partial\widehat{\beta}^{\alpha}_{t+1})^2}(\alpha_t y^1_t - (1-\alpha_t)y^2_t) + \frac{\partial^2 v^B_{t+1}(\widehat{\beta}^{\alpha}_{t+1})}{(\partial\widehat{\beta}^{\alpha}_{t+1})^2}$.

Because the distribution of a supplier's performance given her effort level $f(x|e)$ is normal and satisfies the MLRP, the allocation rule should be such that $\beta^{\alpha}_{t+1}(x^1_t, x^2_t)$ is increasing in x^1_t and decreasing in x^2_t, and $\widehat{\beta}^{\alpha}_{t+1}(y^1_t, y^2_t)$ is increasing in y^1_t and decreasing in y^2_t, so

for $\alpha_t y^1_t > (1-\alpha_t)y^2_t$, $G'(\widehat{\beta}^{\alpha}_{t+1}) > 0$ and $G''(\widehat{\beta}^{\alpha}_{t+1}) > 0$ for $\widehat{\alpha}_{t+1} > \frac{1}{2}$, the optimal $\widehat{\beta}^{\alpha}_{t+1}(y^1_t, y^2_t) = \beta_{t+1}$,

for $\alpha_t y^1_t < (1-\alpha_t)y^2_t$, $G'(\widehat{\beta}^{\alpha}_{t+1}) < 0$ and $G''(\widehat{\beta}^{\alpha}_{t+1}) > 0$ for $\widehat{\alpha}_{t+1} < \frac{1}{2}$, the optimal $\widehat{\beta}^{\alpha}_{t+1}(y^1_t, y^2_t) = 1-\beta_{t+1}$.

Thus the optimal allocation rule for period $t+1$ is again a bang-bang one,

$$\widehat{\beta}^{\alpha}_{t+1}(y^1_t, y^2_t) = \begin{cases} \beta_{t+1} & \alpha_t y^1_t > (1-\alpha_t)y^2_t \\ 1-\beta_{t+1} & \alpha_t y^1_t < (1-\alpha_t)y^2_t \end{cases}.$$

Using $(C.55)$,

$v^B_t(\alpha_t) = \frac{\gamma\Gamma_t}{\sqrt{2\pi}b\sigma}\Delta v_{t+1}(\beta_{t+1}) + \frac{\gamma}{2}[v^B_{t+1}(\beta_{t+1}) + v^B_{t+1}(1-\beta_{t+1})]$,

$\frac{\partial v^B_t(\alpha_t)}{\partial\alpha_t} = \gamma\frac{2\alpha_t - 1}{\sqrt{2\pi}b\sigma\Gamma_t}\Delta v_{t+1}(\beta_{t+1})$, $\frac{\partial^2 v^B_t(\alpha_t)}{\partial\alpha^2_t} = \frac{\gamma}{\sqrt{2\pi}b\sigma}\Delta v_{t+1}(\beta_{t+1})\frac{1}{\Gamma^3_t}$.

$e_t^{1*} = \frac{\gamma}{b\sigma}\Delta v_{t+1}(\beta_{t+1})\frac{\alpha_t}{\sqrt{2\pi}\Gamma_t}, e_t^{2*} = \frac{\gamma}{b\sigma}\Delta v_{t+1}(\beta_{t+1})\frac{1-\alpha_t}{\sqrt{2\pi}\Gamma_t}, \frac{\partial e_t^1}{\partial \alpha_t} = \frac{\gamma}{\sqrt{2\pi}b\sigma}\Delta v_{t+1}(\beta_{t+1})\frac{1-\alpha_t}{\Gamma_t^3}.$

$v_t^1(\alpha_t) = m\alpha_t - \frac{b(e_t^1)^2}{2} + \frac{\gamma}{2}[v_{t+1}^1(\alpha_{t+1}) + v_{t+1}^2(1-\alpha_{t+1})],$
$\frac{\partial v_t^1(\alpha_t)}{\partial \alpha_t} = m - \frac{\gamma^2}{2\pi b\sigma^2}(\Delta v_{t+1}(\beta_{t+1}))^2\frac{\alpha_t(1-\alpha_t)}{\Gamma_t^4},$
$\frac{\partial^2 v_t^1(\alpha_t)}{(\partial\alpha_t)^2} = \frac{\gamma^2(2\alpha_t-1)(1+2\alpha_t-2(\alpha_t)^2)}{2\pi b\sigma^2\Gamma_t^6}(\Delta v_{t+1}(\beta_{t+1}))^2.$

So all the value functions in period t have the same properties as those in period $t+1$. By recursion, the optimal allocation rule for every period takes the same form as in $(C.61)$.

Nash equilibrium condition:

Note that $\int_a^b ((y)^2-1)\phi(y)dy = a\phi(a) - b\phi(b)$.

Second-order conditions for the suppliers' period t problems:

at (e_t^{1*}, e_t^{2*}),

$\frac{\partial^2 v_t^1(\alpha_t)}{(\partial e_t^1)^2} = -b + \frac{\gamma}{\sigma^2}\int_{x_t^1}\int_{x_t^2} v_{t+1}^1(\beta_{t+1}^{\alpha}(x_t^1,x_t^2))((\frac{x_t^1-e_t^{1*}}{\sigma})^2-1)f(x_t^1|e_t^{1*})f(x_t^2|e_t^{2*})dx_t^1dx_t^2$
$= -b + \frac{\gamma}{\sigma^2}\int_{y_t^1}\int_{y_t^2} v_{t+1}^1(\widehat{\beta}_{t+1}^{\alpha}(y_t^1,y_t^2))((y_t^1)^2-1)\phi(y_t^1)\phi(y_t^2)dy_t^1dy_t^2$
$= -b + \frac{\gamma}{\sigma^2}\int_{y_t^2}[v_{t+1}^1(\overline{\beta})\int_{y_t^2\frac{1-\alpha_t}{\alpha_t}}^{\infty}((y_t^1)^2-1)\phi(y_t^1)dy_t^1 + v_{t+1}^1(1-\overline{\beta})\int_{-\infty}^{y_t^2\frac{1-\alpha_t}{\alpha_t}}((y_t^1)^2-1)\phi(y_t^1)dy_t^1]\phi(y_t^2)dy_t^2$
$= -b + \frac{\gamma}{\sigma^2}[v_{t+1}^1(\overline{\beta})\int_{y_t^2} y_t^2\frac{1-\alpha_t}{\alpha_t}\phi(y_t^2\frac{1-\alpha_t}{\alpha_t})\phi(y_t^2)dy_t^2 - v_{t+1}^1(1-\overline{\beta})\int_{y_t^2} y_t^2\frac{1-\alpha_t}{\alpha_t}\phi(y_t^2\frac{1-\alpha_t}{\alpha_t})\phi(y_t^2)dy_t^2]$
$= -b < 0,$

similarly $\frac{\partial^2 v_t^2(1-\alpha_t)}{(\partial e_t^2)^2} = -b$, so (e_t^{1*}, e_t^{2*}) is a Nash equilibrium, and $\{(e_t^{1*}, e_t^{2*})\}_{t=1,2...,T}$ constitutes a subgame perfect Nash equilibrium.

Let $\varepsilon_i = \frac{e_t^i - e_t^{i*}}{\sigma}$.

$\frac{\partial^2 v_t^1(\alpha_t)}{(\partial e_t^1)^2} = -b + \frac{\gamma}{\sigma^2}\int_{x_t^1}\int_{x_t^2} v_{t+1}^1(\beta_{t+1}^{\alpha}(x_t^1,x_t^2))((\frac{x_t^1-e_t^1}{\sigma})^2-1)f(x_t^1|e_t^1)f(x_t^2|e_t^2)dx_t^1dx_t^2$
$= -b + \frac{\gamma}{\sigma^2}[\int_{y_t^2}\int_{y_t^2\frac{1-\alpha_t}{\alpha_t}}^{\infty} v_{t+1}^1(\beta_{t+1})((y_t^1-\varepsilon_1)^2-1)\phi(y_t^1-\varepsilon_1)\phi(y_t^2-\varepsilon_2)dy_t^1dy_t^2$
$+\int_{y_t^2}\int_{-\infty}^{y_t^2\frac{1-\alpha_t}{\alpha_t}} v_{t+1}^1(1-\beta_{t+1})((y_t^1-\varepsilon_1)^2-1)\phi(y_t^1-\varepsilon_1)\phi(y_t^2-\varepsilon_2)dy_t^1dy_t^2]$
$= -b + \frac{\gamma}{\sigma^2}[\int_{z_t^2}\int_{(z_t^2+\varepsilon_2)\frac{1-\alpha_t}{\alpha_t}}^{\infty} v_{t+1}^1(\beta_{t+1})((z_t^1)^2-1)\phi(z_t^1)\phi(z_t^2)dz_t^1dz_t^2$
$+\int_{z_t^2}\int_{-\infty}^{(z_t^2+\varepsilon_2)\frac{1-\alpha_t}{\alpha_t}} v_{t+1}^1(1-\beta_{t+1})((z_t^1)^2-1)\phi(z_t^1)\phi(z_t^2)dz_t^1dz_t^2]$

$= -b + \frac{\gamma}{\sigma^2}\Delta v_{t+1}(\beta_{t+1})\int_{z_t^2}(z_t^2+\varepsilon_2)\frac{1-\alpha_t}{\alpha_t}\phi((z_t^2+\varepsilon_2)\frac{1-\alpha_t}{\alpha_t})\phi(z_t^2)dz_t^2$

$= -b + \frac{\gamma}{\sigma^2}\frac{1-\alpha_t}{\alpha_t}\Delta v_{t+1}(\beta_{t+1})\int_{z_t^2}(z_t^2+\varepsilon_2\frac{(1-\alpha_t)^2}{\Gamma_t^2}+\varepsilon_2\frac{\alpha_t^2}{\Gamma_t^2})\phi(\frac{\Gamma_t}{\alpha_t}(z_t^2+\varepsilon_2\frac{(1-\alpha_t)^2}{\Gamma_t^2}))\phi(\varepsilon_2\frac{(1-\alpha_t)}{\Gamma_t})dz_t^2$

$= -b + \frac{\gamma}{\sigma^2}\frac{\alpha_t^2(1-\alpha_t)}{\Gamma_t^3}\Delta v_{t+1}(\beta_{t+1})\varepsilon_2\phi(\varepsilon_2\frac{1-\alpha_t}{\Gamma_t})$

$\leq -b + \frac{\gamma}{\sigma^2}\frac{\alpha_t^2}{\Gamma_t^2}\Delta v_{t+1}(\beta_{t+1})\phi(1)$,

similarly $\frac{\partial^2 v_t^2(\alpha_t)}{(\partial e_t^2)^2} \leq -b + \frac{\gamma}{\sigma^2}\frac{(1-\alpha_t)^2}{\Gamma_t^2}\Delta v_{t+1}(\beta_{t+1})\phi(1)$. So the sufficient condition for the existence of a unique subgame perfect Nash equilibrium in the T period problem is

$\frac{\gamma}{\sigma^2}\frac{\beta_t^2}{\Gamma_t^2}\Delta v_{t+1}(\beta_{t+1})\phi(1) \leq b$, which sets a boundary to the value of β_{t+1}.

10. Proof of Theorem 4.4

To simplify the notation, we omit the subscripts of k_α, θ_α and Υ_α when there is no ambiguity, and use β for $\widehat{\beta}$.

The suppliers' optimal effort levels

$e_1^* = \frac{\gamma}{b}\int_{x_1}\int_{x_2} v_1(\beta_\alpha^{4*}(x_1,x_2))f^1(x_1|e_1^*)f(x_2|e_2^*)dx_1dx_2$

$= \frac{\gamma}{b\sigma}\int_{y_2}[v_1(\beta)\int_{ky_2+\frac{1}{\sigma}(\theta-(e_1^*-ke_2^*))}^{\infty} y_1\phi(y_1)dy_1$

$+v_1(1-\beta)\int_{-\infty}^{ky_2+\frac{1}{\sigma}(\theta-(e_1^*-ke_2^*))} y_1\phi(y_1)dy_1]\phi(y_2)dy_2$

$= \frac{\gamma}{b\sigma}\int_{y_2}(v_1(\beta)-v_1(1-\beta))\phi(ky_2+\frac{1}{\sigma}(\theta-(e_1^*-ke_2^*)))\phi(y_2)dy_2$

$\Rightarrow e_1^* = \frac{\gamma\Delta v^*(\beta)}{b\sigma}\frac{1}{\sqrt{2\pi}\Upsilon}\exp[-\frac{1}{2\Upsilon^2\sigma^2}(\theta-(e_1^*-ke_2^*))^2]$,

and similarly

$e_2^* = \frac{\gamma\Delta v^*(\beta)}{b\sigma}\frac{k}{\sqrt{2\pi}\Upsilon}\exp[-\frac{1}{2\Upsilon^2\sigma^2}(\theta-(e_1^*-ke_2^*))^2] \Rightarrow \theta^* = e_1^* - ke_2^*$, and (4.16) and (4.17) follow.

For $e_1^*, e_2^* > 0$, we need $k > 0$ and $\Delta v^*(\beta) > 0$.

$v_B^*(\alpha) = \alpha\frac{\gamma\Delta v^*(\beta)}{b\sigma\sqrt{2\pi}\Upsilon}+k(1-\alpha)\frac{\gamma\Delta v^*(\beta)}{b\sigma\sqrt{2\pi}\Upsilon}+\gamma\int_{y_2}[v_B^*(\beta)\int_{ky_2}^{\infty}\phi(y_1)dy_1+v_B^*(1-\beta)\int_{-\infty}^{ky_2}\phi(y_1)dy_1]\phi(y_2)dy_2$

$\Rightarrow$

$$v_B^*(\alpha) = \frac{\gamma\Delta v^*(\beta)}{\sqrt{2\pi}b\sigma\Upsilon}[\alpha+k(1-\alpha)] + \frac{\gamma}{2}[v_B^*(\beta)+v_B^*(1-\beta)], \tag{C.63}$$

$v_B^*(\beta) = \frac{\gamma\Delta v^*(\beta)}{\sqrt{2\pi}b\sigma\Upsilon_\beta}[\beta+k_\beta(1-\beta)] + \frac{\gamma}{2}[v_B^*(\beta)+v_B^*(1-\beta)]$,

$v_B^*(1-\beta) = \frac{\gamma\Delta v^*(\beta)}{\sqrt{2\pi}b\sigma\Upsilon_{1-\beta}}[1-\beta+k_{1-\beta}\beta] + \frac{\gamma}{2}[v_B^*(\beta)+v_B^*(1-\beta)]$.

Due to symmetry, $k_{1-\beta} = 1/k_\beta$, it follows that $v_B^*(\beta) = v_B^*(1-\beta)$,

$v_1^*(\alpha) = v_2^*(\alpha) = v^*(\alpha)$,
$v^*(\alpha) = m\alpha - \frac{1}{4\pi b}(\frac{\gamma\Delta v^*(\beta)}{\sigma}\frac{1}{\Upsilon})^2 + \gamma\int_{y_2}[v^*(\beta)\int_{ky_2}^{\infty}\phi(y_1)dy_1$
$+v^*(1-\beta)\int_{-\infty}^{ky_2}\phi(y_1)dy_1]\phi(y_2)dy_2$
$= m\alpha - \frac{1}{4\pi b}(\frac{\gamma\Delta v^*(\beta)}{\sigma}\frac{1}{\Upsilon})^2 + \frac{\gamma}{2}(v^*(\beta) + v^*(1-\beta))$.

FOC of the supplier's problem:
$\frac{\partial v_1(\alpha)}{\partial e_1} = -be_1 + \gamma\int_{x_1}\int_{x_2} v_1(\beta_\alpha^4(x_1,x_2))f^1(x_1|e_1)f(x_2|e_2)dx_1dx_2 \Rightarrow$

$$\frac{\partial v_1(\alpha)}{\partial e_1} = -be_1 + \frac{\gamma\Delta v(\beta)}{\sqrt{2\pi}\Upsilon\sigma}\exp[-\frac{1}{2\Upsilon^2\sigma^2}(e_1^* - ke_2^* - (e_1 - ke_2))^2], \tag{C.64}$$

$\frac{\partial v_2(1-\alpha)}{\partial e_2} = -be_2 + \frac{\gamma\Delta v(\beta)}{\sigma}\frac{k}{\sqrt{2\pi}\Upsilon}\exp[-\frac{1}{2\Upsilon^2\sigma^2}(e_1^* - ke_2^* - (e_1 - ke_2))^2]$,
$\Rightarrow$ at any equilibrium (e_1, e_2), $e_2 = ke_1$, so at an equilibrium (e_1, e_2),
$\frac{\partial v_1(\alpha)}{\partial e_1} = -be_1 + \frac{\gamma\Delta v(\beta)}{\sigma}\frac{1}{\sqrt{2\pi}\Upsilon}\exp[-\frac{1-k^2}{2\sigma^2(1+k^2)}(e_1^* - e_1)^2]$,
$\frac{\partial v_2(1-\alpha)}{\partial e_2} = -be_2 + \frac{\gamma\Delta v(\beta)}{\sigma}\frac{k}{\sqrt{2\pi}\Upsilon}\exp[-\frac{1-k^2}{2\sigma^2(1+k^2)}(e_1^* - e_1)^2]$.
$\frac{\partial v_1(\alpha)}{\partial e_1} = 0 \Rightarrow$

$$e_1 = \frac{\gamma\Delta v(\beta)}{\sqrt{2\pi}\Upsilon b\sigma}\exp[-\frac{1-k^2}{2\sigma^2(1+k^2)}(e_1^* - e_1)^2], \tag{C.65}$$

substituting $(C.65)$ and $e_2 = ke_1$ into the supplier's value function, we obtain
$\Delta v(\beta) = m(2\beta - 1) - \frac{1}{4\pi b}(\frac{\gamma\Delta v(\beta)}{\sigma}\exp[-\frac{1-k_\beta^2}{2\sigma^2(1+k_\beta^2)}(e_1^* - e_1)^2])^2\frac{1-k_\beta^2}{1+k_\beta^2} \Rightarrow$
for $k_\beta \neq 1$,
$\Delta v(\beta) = \frac{2\pi b\sigma^2(1+k_\beta^2)}{\gamma^2(1-k_\beta^2)}(\sqrt{1 + \frac{\gamma^2\exp[-\frac{1-k_\beta^2}{\sigma^2(1+k_\beta^2)}(e_1^*-e_1)^2]m(2\beta-1)}{\pi b\sigma^2}\frac{1-k_\beta^2}{1+k_\beta^2}} - 1)$,
$\Delta v^*(\beta) = \frac{2\pi b\sigma^2(1+k_\beta^2)}{\gamma^2(1-k_\beta^2)}(\sqrt{1 + \frac{\gamma^2 m(2\beta-1)}{\pi b\sigma^2}\frac{1-k_\beta^2}{1+k_\beta^2}} - 1)$;
and for $k_\beta = 1$, $\Delta v(\beta) = m(2\beta - 1)$,
then from $(C.63)$, $v_B^*(\alpha) = \frac{\sqrt{2\pi}\sigma(1+k_\beta^2)}{\gamma(1-\gamma)\Upsilon(1-k_\beta^2)}[\alpha + k(1-\alpha)](\sqrt{1 + \frac{\gamma^2 m(2\beta-1)}{\pi b\sigma^2}\frac{1-k_\beta^2}{1+k_\beta^2}} - 1)$.

From $(C.64)$,
$\frac{\partial^2 v_1(\alpha)}{(\partial e_1)^2} = -b + \frac{\gamma\Delta v(\beta)}{\sigma\sqrt{2\pi}\Upsilon^3\sigma^2}\exp[-\frac{1}{2\sigma^2(1+k^2)}(e_1^* - ke_2^* - (e_1 - ke_2))^2](e_1^* - ke_2^* - (e_1 - ke_2))$
$\leq -b + \frac{\gamma\Delta v(\beta)}{(1+k^2)\sigma^2}\phi(1)$,
similarly $\frac{\partial^2 v_2(\alpha)}{(\partial e_2)^2} \leq -b + \frac{\gamma\Delta v(\beta)}{(1+k^2)\sigma^2}\phi(1)$.

For $k_\beta \leq 1$, $\Delta v(\beta) \leq \Delta v^*(\beta)$. So the sufficient condition for the existence of a unique Nash equilibrium is

$$\frac{\gamma \Delta v^*(\beta)}{(1+k^2)\sigma^2}\phi(1) \leq b. \tag{C.66}$$

11. Proof of Corollary 4.4

Letting $k_\alpha = \frac{1-\alpha}{\alpha}$ in Theorem 4.4, we obtain the formula for θ^*_α, e^*_β and $e^*_{1-\beta}$, then $e^*_\beta = \frac{\beta}{1-\beta}e^*_{1-\beta}$. Also

$v^*_B(\alpha) = \frac{\sqrt{2\pi}\sigma\Gamma^3}{\gamma(1-\gamma)(2\beta-1)}(\sqrt{1+\frac{\gamma^2 m(2\beta-1)^2}{\pi b\sigma^2\Gamma^2}}-1)$ is independent of α.

$$\frac{\partial v^*_B(\beta)}{\partial\beta} = \frac{\sqrt{2\pi}\sigma}{\gamma(1-\gamma)}[\frac{3\Gamma(2\beta-1)^2-2\Gamma^3}{(2\beta-1)^2}(\sqrt{1+\frac{\gamma^2 m(2\beta-1)^2}{\pi b\sigma^2\Gamma^2}}-1)$$
$$+\frac{\Gamma^3}{2(2\beta-1)\sqrt{1+\frac{\gamma^2 m(2\beta-1)^2}{\pi b\sigma^2\Gamma^2}}}\frac{\gamma^2 m}{\pi b\sigma^2}\frac{4(2\beta-1)\Gamma^2-2(2\beta-1)^3}{\Gamma^4}]$$
$$=\frac{\sqrt{2\pi}\sigma}{\gamma(1-\gamma)\sqrt{1+\frac{\gamma^2 m(2\beta-1)^2}{\pi b\sigma^2\Gamma^2}}}[\Gamma\frac{8\beta^2-8\beta+1}{(2\beta-1)^2}(1+\frac{\gamma^2 m(2\beta-1)^2}{\pi b\sigma^2\Gamma^2}-\sqrt{1+\frac{\gamma^2 m(2\beta-1)^2}{\pi b\sigma^2\Gamma^2}})+\frac{\gamma^2 m}{\pi b\Gamma\sigma^2}]$$
$$=\frac{\sqrt{2\pi}\sigma}{\gamma(1-\gamma)\Gamma(2\beta-1)^2\sqrt{1+\frac{\gamma^2 m(2\beta-1)^2}{\pi b\sigma^2\Gamma^2}}}[\frac{2m}{\pi b}(\frac{\gamma}{\sigma})^2(2\beta-1)^4+\Gamma^2(8\beta^2-8\beta+1)$$
$$-\Gamma^2(8\beta^2-8\beta+1)\sqrt{1+\frac{\gamma^2 m(2\beta-1)^2}{\pi b\sigma^2\Gamma^2}}]$$
$$=\frac{\sqrt{2\pi}\sigma\Gamma}{\gamma(1-\gamma)(2\beta-1)^2\sqrt{1+\frac{\gamma^2 m(2\beta-1)^2}{\pi b\sigma^2\Gamma^2}}}[\frac{2m}{\pi b}(\frac{\gamma}{\sigma})^2\frac{(2\beta-1)^4}{\Gamma^2}+(2(2\beta-1)^2-1)(1-\sqrt{1+\frac{\gamma^2 m(2\beta-1)^2}{\pi b\sigma^2\Gamma^2}})].$$

For $2(2\beta-1)^2 \leq 1$, i.e., $\beta \leq \frac{1}{2}+\frac{\sqrt{2}}{4}$, $\frac{\partial v_B(\beta)}{\partial\beta} > 0$;

for $\beta > \frac{1}{2}+\frac{\sqrt{2}}{4}$, $2(2\beta-1)^2-1>0$, and noting that $1-\sqrt{1+a} > -a$ for any $a>0$,

$$\frac{\partial v^*_B(\beta)}{\partial\beta} > \frac{\sqrt{2\pi}\sigma\Gamma}{\gamma(1-\gamma)(2\beta-1)^2\sqrt{1+\frac{\gamma^2 m(2\beta-1)^2}{\pi b\sigma^2\Gamma^2}}}\frac{m(2\beta-1)^2}{\pi b\Gamma^2}(\frac{\gamma}{\sigma})^2[2(2\beta-1)^2-(2(2\beta-1)^2-1)]$$
$$=\frac{\sqrt{2}\gamma m}{(1-\gamma)b\sigma\Gamma\sqrt{\pi}\sqrt{1+\frac{\gamma^2 m(2\beta-1)^2}{\pi b\sigma^2\Gamma^2}}} > 0,$$

so $\frac{\partial v^*_B(\beta)}{\partial\beta} > 0$ for any $\beta > \frac{1}{2}$, therefore, the optimal β^* is a boundary solution and is constrained by the (IR) constraint or Nash equilibrium constraint.

Letting $k_\beta = \frac{1-\beta}{\beta}$ in (4.18),

$\Delta v^*(\beta) = \frac{2\pi b\sigma^2(\beta^2+(1-\beta)^2)}{\gamma^2(2\beta-1)}(\sqrt{1+\frac{\gamma^2 m(2\beta-1)^2}{\pi b\sigma^2(\beta^2+(1-\beta)^2)}}-1)$.

By $(C.66)$, because the $\frac{\gamma\Delta v^*(\beta)}{(1+k^2)\sigma^2}\phi(1)$ is decreasing in k, the Nash equilibrium condition becomes

$\frac{\sqrt{2\pi}b\beta^2}{\gamma(2\beta-1)\sqrt{e}}(\sqrt{1+\frac{\gamma^2 m(2\beta-1)^2}{\pi b\sigma^2(\beta^2+(1-\beta)^2)}}-1) \leq b.$

(4.19) with $k_\alpha = \frac{1-\alpha}{\alpha} \Rightarrow$
$\Sigma v^*(\beta) = v^*(\beta) + v^*(1-\beta) = \frac{1}{1-\gamma}[m - \frac{(\gamma \Delta v^*(\beta))^2}{4\pi b\sigma^2}]$. So
$v^*(\beta) = \frac{1}{2}(\Sigma v^*(\beta) + \Delta v^*(\beta)) = \frac{1}{2}(\frac{1}{1-\gamma}[m - \frac{(\gamma \Delta v^*(\beta))^2}{4\pi b\sigma^2}] + \Delta v^*(\beta))$,
$v^*(1-\beta) = \frac{1}{2}(\Sigma v^*(\beta) - \Delta v^*(\beta)) = \frac{1}{2}(\frac{1}{1-\gamma}[m - \frac{(\gamma \Delta v^*(\beta))^2}{4\pi b\sigma^2}] - \Delta v^*(\beta))$.
The (IR) constraint is $v(1-\beta) \geq 0$.

12. Proof of Corollary 4.5

From the results in Theorem 4.4, letting $k_\alpha = 1$,
$\Delta v(\beta) = (2\beta - 1)m$, $\Sigma v(\beta) = \frac{1}{1-\gamma}[m - \frac{(\gamma(2\beta-1)m)^2}{4\pi b\sigma^2}]$,
$e_1^* = e_2^* = \frac{\gamma(2\beta-1)m}{2\sqrt{\pi}b\sigma}, \theta^* = 0$.

It follows that
$v(\beta) = \frac{1}{2}(\frac{1}{1-\gamma}[m - \frac{(\gamma(2\beta-1)m)^2}{4\pi b\sigma^2}] + (2\beta-1)m)$,
$v(1-\beta) = \frac{1}{2}(\frac{1}{1-\gamma}[m - \frac{(\gamma(2\beta-1)m)^2}{4\pi b\sigma^2}] - (2\beta-1)m)$,
$v_B(\beta) = \frac{\gamma}{1-\gamma}\frac{\Delta v(\beta)}{2\sqrt{\pi}b\sigma} = \frac{\gamma}{1-\gamma}\frac{(2\beta-1)m}{2\sqrt{\pi}b\sigma}$.

For $v(1-\beta) \geq 0$, $\frac{1}{2}(\frac{1}{1-\gamma}[m - \frac{(\gamma(2\beta-1)m)^2}{4\pi b\sigma^2}] - (2\beta-1)m) \geq 0 \Rightarrow$
$\frac{\gamma^2 m(2\beta-1)^2}{4\pi b\sigma^2} + (1-\gamma)(2\beta-1) \leq 1 \Rightarrow$

$$\beta \leq \frac{1}{2} + \frac{1}{m\gamma^2}(\sqrt{(\pi b\sigma^2(1-\gamma))^2 + \pi b\sigma^2 m\gamma^2} - \pi b\sigma^2(1-\gamma)). \tag{C.67}$$

Second-order condition for Supplier 1's problem
$\frac{\partial^2 v_1(\alpha)}{(\partial e_1)^2} = -b + \gamma(2\beta-1)\frac{m}{2\sigma^2}\frac{-(e_1-e_2)}{\sqrt{2}\sigma}\phi(\frac{-(e_1-e_2)}{\sqrt{2}\sigma}) \leq -b + \frac{\gamma m(2\beta-1)}{2\sigma^2\sqrt{2\pi e}}$, so $\frac{\partial^2 v_1(\alpha)}{(\partial e_1)^2} \leq 0$ for

$$\beta \leq \frac{1}{2} + \frac{b\sigma^2\sqrt{2\pi e}}{\gamma m}, \tag{C.68}$$

which can be obtained similarly for Supplier 2's problem.

Because $v_B(\beta)$ is increasing in β for any β, the optimal β is the boundary solution constrained by $(C.67)$ and $(C.68)$.