Essays on Nominal Rigidities, Financial Constraints and Transfers

by

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Abstract

One principal research in macroeconomics is concerned with the importance of nominal rigidities. This dissertation applies nominal rigidities in closed- and open-economy models to study issues on firms' pricing decisions, optimal monetary policy in a financially constrained economy, and the choice of exchange rate regime in the presence of transfer problem. Chapter 1 presents empirical evidence for price and wage stickiness, and the development of models with nominal rigidities in macroeconomics and international finance.

Chapter 2 incorporates state-dependent pricing in a closed-economy model to explain the asymmetric responses of output and prices to monetary shocks. The model focuses on the effects of strategic complementarity and substitutability in firms' pricing decisions, as well as the mixed strategies used by an individual firm. The strategic interactions among firms' pricing decisions lead to asymmetric response of prices and output to monetary shocks. The model implies asymmetries in positive versus negative monetary shocks, and the asymmetric responses are affected by the degree of real rigidity in marginal cost, the magnitude of price-adjusting costs and the market power of an individual firm.

Chapter 3 studies the optimal monetary policy of a small open economy with nominal rigidities and exchange-rate sensitive collateral constraints. This model attempts to explain the observed monetary policy behaviour of emerging markets. The model implies pro-cyclical optimal monetary policy when the collateral constraint binds, and an economy with large external shocks that may favour a fixed exchange rate, which are consistent with the observed features of monetary policy used by emerging markets.

The last chapter studies the transfer problem using a two-country DSGE model with nominal rigidities. The model compares the effects of a transfer shock under flexible and sticky wages, as well as under fixed and floating exchange rate regimes. The results of this model are consistent with the conventional wisdom in international macroeconomics with nominal rigidities, which suggests that a flexible exchange rate can help reduce internal instability after some negative shocks via exchange rate adjustment. However, the welfare analysis of this model implies the donor country is better off to maintain the gold standard instead of going to a floating exchange rate, even with nominal rigidities.

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Chapter 1

Nominal Rigidities in Macroeconomics

...though the high price of commodities be a necessary consequence of the increase of gold and silver, yet it follows not immediately upon that increase... At first, no alternation is perceived; by degrees the price rises, first of one commodity, then of another...

David Hume (1752)[1]

A central research area in macroeconomics deals with the importance of nominal rigidities. Some two hundred and fifty years ago, British philosopher David Hume observed that prices would only adjust slowly after a monetary shock. General observations reveal that prices and wages take time to adjust in response to shocks in the economy. Along with increasing interests in the monetary transmission mechanism, studies on nominal rigidities have continued to develop since the mid-20th century.

1.1 Empirical evidence on nominal rigidities

Extensive empirical studies have used microeconomic data to evaluate the frequency on price and wage adjustment to show the importance of nominal rigidities.[2] Blinder (1994) and Blinder et al. (1998) survey firms' pricing policies, and find that price adjustments generally lag behind the change in money stock, which is consistent with conventional macroeconomics findings. Blinder (1994) finds that the mode frequency of price adjustment is one, that is, most of the interviewed firms (about 40 percent) adjust their prices annually, while 78 percent of the firms only adjust their prices quarterly

[1] Reproduced from Rotwein (1970).

[2] Taylor (1999) conducts a detailed survey on the empirical evidence for stickiness of prices and wages.

or less often. Bils and Klenow (2004) use monthly CPI data and find that the median duration of prices is about 4.3 months. In a recent study by Eichenbaum, Jaimovich and Rebelo (2008), the authors use weekly scanner data from a major US retailer to analyze price stickiness, and find that nominal rigidities do not take the form of sticky prices, but rather inertial reference prices and costs (that is, the most common prices and costs within a given quarter). The average duration of reference prices in their model is about one year, which brings nominal rigidities to the economy even when prices of individual goods change frequently.

Cecchetti (1984) uses data from major union contracts to examine wage rigidities in the US and finds that the length of wage contract is sensitive to the inflation rate. He finds that the average duration of wage contracts was 7 quarters in the 1950s and 1960s. Nevertheless, in high inflation periods, the average wage change was about one year in the 1970s. Kahn (1997) uses longitudinal microeconomic data to test the distribution of annual nominal wage and salary changes, and finds substantial stickiness of nominal wages for workers who were staying with the same employer over the year from the 1970s to the 1980s. Heckel, Bihan and Montornès (2008) provide evidence on wage stickiness in France. Using quarterly firm level survey data for the period from 1998 to 2005, they find that the quarterly wage change was about 35 percent in France. They also find evidence of downward wage rigidity and staggered wage changes across firms.

Taylor (1999) summarizes the general empirical findings on price-setting and wage-settings to indicate that the frequency of price and wage adjustment is about one year, and that it depends on the average rate of inflation. In addition, price-settings and wage-settings are heterogeneous and unsynchronized across industries and firms. These findings imply the need for nominal rigidities in macroeconomic models.

1.2 Development of models with nominal rigidities

Rigidities in price and wage are key assumptions of Keynesian models. Prices and wages are simply assumed to adjust slowly in response to changes in aggregate demand, and these nominal rigidities are the key frictions that lead to the non-neutral impacts of monetary policy. The introduction of rational expectations into macroeconomics in the early 1970s, and Lucas' (1976) critique of macroeconometric policy made researchers aware of the importance of expectations for price and wage dynamics. Since then, researchers have begun to incorporate nominal rigidities with rational expectations in economy-wide models to study the links between real and nominal variables.

In the first-generation New Keynesian models, Gray (1976), Fischer (1977), and Phelps and Taylor (1977) assume prices or wages are predetermined and that they remain fixed during the contract period. Since prices and wages become fully flexible after the contract period, these models imply long-run monetary neutrality and they cannot generate persistent effects of monetary shocks on real activities.

Staggered contract models were introduced in the late-1970s and early-1980s to explain the persistence of monetary policy. These models are based on observations that prices and wages are set at fixed values for fairly long periods, and the price-settings and wage-settings are staggered so that the contract periods overlap with one another. When firms set the pricing contracts, they must consider the future expectations and past decisions. Taylor (1979, 1980) examines the effects of staggered wages with fully flexible prices, with the assumption that wages are set fixed for n periods. In each period, $\frac{1}{n}th$ of the firms adjust their contracted wages. Taylor's model implies that staggered pricing leads to sluggish aggregate price adjustment and a "humped-shaped" response of output to wage shocks. Sticky price and sticky wage models like Taylor's can explain Gordon's (1982) findings where lags of nominal wages and prices are important in these macro models.

Calvo (1983) develops an alternative model of staggered pricing. An individual price-setter faces a constant probability of adjusting prices at any instant. Given a large number of independent price-setters in the economy, a constant proportion of prices is adjusted at any instant. Taylor (1999) refers to these types of models as "random duration models", as the length of contract is "stochastic and independent and identically distributed across contracts."(Calvo (1982)).

In the original staggered contract models, price-settings and wage-setting rules are only reduced form equations. The second-generation New Keynesian literature, originating from Rotemberg (1982), Mankiw (1985), Svensson (1986), and Blanchard and Kiyotaki (1987), provides the microfoundation for deriving the price-setting and wage-setting mechanism optimally through maximization problems. In these models, firms produce differentiated goods, and are monopolistically competitive in good markets so that individual firms have a certain degree of market power. According to Arrow (1959), the pricing decision is made meaningful by the firms' market power, however, monopolistic competition by itself cannot generate nominal stickiness. Small real costs of changing prices and wages are needed to account for the nominal rigidities. Svensson (1986) and Blanchard and Kiyotaki (1987) incorporate money in their static models with monopolistic competition, thus making the study of the effects of monetary shocks on aggregate demand feasible.

Later, the "New Neoclassical Synthesis" framework combines Keynesian elements of monopolistic competitions and nominal rigidities with the dynamic general equilibrium framework that is associated with the Real Business Cycle (RBC) literature.[3] In such a dynamic optimizing setting, researchers can study the persistence effects of monetary policy and the dynamics of real and nominal variables. This stream of literature mainly focuses on reproducing the observed behaviours of macroeconomic variables, such as output and inflation, in response to monetary policy shocks. King and Wolman (1999) and Chari, Kehoe and McGrattan (2000) incorporate staggered pricing and rational expectations into an otherwise RBC framework. King and Wolman (1999) focus on the optimal monetary policy when sticky prices are present in the economy. Chari, Kehoe and McGrattan (2000) find that staggered price-setting alone cannot generate realistic business cycle fluctuations, unless the value of the contract multiplier is unreasonably large.

These studies usually abstract from wage rigidities. Models with only wage rigidities have been criticized, as wage rigidities imply counter-cyclical movements of real wages, while empirical findings in the RBC literature and VAR analysis give mildly pro-cyclical movements.[4] Woodford (2003, p.235) argues that allowing for wage rigidities does not matter much if one only wants to study the comovement of inflation and output, since the effects of wage stickiness can be reproduced using different parameter values and shocks in the sticky price model. Nevertheless, Taylor (2007) criticizes the absence of wage rigidities in the recent research, as he believes sticky wages still provide some sort of inflation dynamics. Andersen (1998) points out that staggered wages are more likely to generate persistence than would sticky prices, since labour demand determines the quantities in the labour market when wages are staggered. Erceg et al. (2000) use a model with staggered wages and prices to study the optimal monetary policy. Christiano et al. (2005) and Levin et al. (2005) assume that prices and wages are both sticky in their models, and argue that wage rigidity is more crucial than price stickiness in producing more realistic effects of monetary policy.

[3]See Goodfriend and King (1997), Clarida, Galí and Gertler (1999), and Galí (2002) for further discussions.

[4]For example, Christiano et al. (1999, 2005).

Blanchard and Fischer (1989, p.388) point out that the frequency of price and wage adjustments and the real effects of money depend on the form of pricing rules used by the price-setters. Until now, the models being discussed assume that adjustments of prices and wages are some functions of time: prices and wages only adjust at fixed or randomly selected times. This stream of models is called *time-dependent pricing* models. In contrast, models with price and wage adjustments as a function of state are called *state-dependent pricing* models: price and wage decisions are endogenous, depending on the state of the world.

Caplin and Spulber (1987) develop a state-dependent pricing model in which price-setters face fixed costs of price adjustment and follow a one-sided (S, s) pricing policy.[5] They find that money can be neutral and price stickiness may disappear in aggregate, as all price-setters may adjust their prices when the shocks are large enough and prices are initially uniformly distributed. This model is not very satisfactory in analyzing the persistent effects of shocks, but it incorporates the microeconomics idea of state-dependent pricing into a macroeconomics model. Caplin and Leahy (1991) set up a dynamic model with state-dependent pricing where money has systematic effects on output as firms follow a two-sided (S, s) rule. They find that the effects of monetary shock on output depend on the existing output level in the economy: expansionary monetary policy is more effective when the output level is low, while contractionary policy is more effective when the output level is high.

In their seminal work, Dotsey, King and Wolman (1999) incorporate state-dependent pricing into a dynamic general equilibrium environment by modifying Calvo's (1983) staggered contract framework. The frequency and timing of price adjustment are endogenous in their model, in which the frequency of price adjustment varies with the average inflation rate. They also find that this type of state-dependent pricing model shares some characteristics with the time-dependent pricing model in that money has short-term effects on real variables but long-run neutrality continues to hold.

Dotsey, King and Wolman (1999) provide a tractable model of state-dependent pricing. Other authors subsequently extend this endogenous pricing framework to study the business cycle behaviour and the effects of nominal rigidities. Burstein (2006) and Devereux and Siu (2007) use state-dependent pricing models to study business

[5]The general idea about (S, s) pricing rule is that whenever the price deviates a certain level from its optimal value, firms should adjust their prices. For more discussion on (S, s) pricing, see Barro (1972) and Sheshinski and Weiss (1977, 1983).

cycle asymmetries. Gertler and Leahy (2006) analyze the Phillips curve in a state-dependent pricing model and find evidence that matches the microeconomic evidence on the magnitude and timing of price adjustment.

The normative and positive implications of time- and state-dependent pricing models can be quite diverse. Klenow and Kryvtsov (2008) empirically distinguish between the time- and state-dependent pricing models using microdata underlying the CPI, and argue that state-dependent pricing models of the type of Dotsey, King and Wolman (1999) are consistent with the data in the US. Woodford (2003, p.142), however, argues that the evidence is insufficient to conclude that the state-dependent pricing model should be regarded as being more realistic than the time-dependent pricing model. Thus, the study of nominal rigidities in macroeconomics continues to remain crucial.

1.3 Open economy framework

At the same time as the first-generation Keynesian models, Dornbusch (1976) extends the sticky price Mundell-Fleming model to study the volatility of nominal exchange rate under a floating exchange rate regime. His "overshooting" model implies that home currency depreciates by a larger amount than does the long-run depreciation after a positive money supply shock to have money market clearing via gradual appreciation when prices are sticky. Although the original work of Dornbusch (1976) does not have any microfoundation, it does "mark the birth of modern international macroeconomics."(Rogoff (2002)).[6]

Since the pioneering model of Obstfeld and Rogoff (1995), "New Open Economy Macroeconomics" has become the dominant research area in international finance. This stream of literature uses open-economy dynamic general equilibrium models with nominal rigidities and market imperfections to study issues like international business cycles, optimal international monetary policy and exchange rate regime, and international monetary policy interdependence.[7] The original *Redux* model of Obstfeld and Rogoff (1995) assumes prices are set one period in advance, and an unexpected increase in money supply leads to increases in output and consumption and a nominal depre-

[6]Obstfeld (2001) discusses the modern history of international macroeconomics.

[7]Obstfeld (2001), Lane (2002) and Bowman and Doyle (2003) provide comprehensive surveys on the research in New Open Economy Macroeconomics.

ciation. Unlike the Dornbusch overshooting model, long-run money neutrality does not hold in the *Redux* model, and exchange rate overshooting does not occur. Chari, Kehoe and McGrattan (2002) extend the analysis of Obstfeld and Rogoff and evaluate the effects of nominal rigidities on real exchange rate fluctuations. Their model can generate realistic real exchange rate persistence when firms are assumed to adjust their prices infrequently. Kollmann (2001) finds that prices and wages need to be sticky for a long period to get persistent real exchange rate fluctuations. Other models emphasize sticky wages; for instance, Corsetti and Pesenti (2002) and Obstfeld and Rogoff (2000), who argue that the results of their models are closer to reality.

In an open economy model, nominal prices can be set at domestic or foreign currency. In the *Redux* model, prices are set using *producer currency pricing* (PCP), which implies complete exchange rate pass-through. One important extension by Betts and Devereux (1996, 2000) allows *Pricing to Market* (PTM) in the modified *Redux* model. Betts and Devereux assume producers follow *local currency pricing* (LCP): firms set home prices in domestic currency and export prices in foreign currency. Since prices are sticky, movements in the nominal exchange rate lead to real exchange rate fluctuations. Market segmentation prevents international arbitrage and leads to zero exchange rate pass-through in the short-run. They also find that the results of PTM models are more inline with the evidence on international business cycles.[8]

Nominal rigidities also play an important role in open-economy optimal monetary policy and exchange rate regime evaluation. Devereux and Engel (1998) analyze the welfare under fixed and flexible exchange rate regimes and show that the choice of exchange rate arrangement depends on the nature of pricing. Clarida, Galí and Gertler (2001), Galí and Monacelli (2005), and Corsetti and Pesenti (2005) find that optimal monetary policy should target domestic variables as in the closed-economy models in the presence of complete exchange rate pass-through.[9] On the other hand, with incomplete pass-through, the monetary authority has an incentive to react to the exchange rate. (Devereux and Engel (2003), and Corsenti and Pesenti (2005)). Devereux and Engel (2003) use a welfare-based model to examine the optimal monetary policy and

[8]Other studies that incorporate the LCP assumption include Devereux and Engel (1998, 2003); Obstfeld and Rogoff (1998, 2000); Kollmann (2001); Bergin and Feenstra (2001); and Chari, Kehoe, and McGrattan (2002).

[9]This is because the effects of monetary policy on exchange rate will not affect foreign prices which is denominated in foreign currency.

find that under LCP, optimal monetary policy leads to a fixed exchange rate.[10]

1.4 From here ...

From the brief discussion on the evidence and development of price and wage stickiness in macroeconomics, and implications of nominal rigidities in an open economy framework, one cannot disregard the importance of nominal rigidities in studying macro and open-economy models. This dissertation applies nominal rigidities in closed- and open-economy models to study issues on firms' pricing decisions, optimal monetary policy in a financially constrained economy, and the choice of exchange rate regime in the presence of a transfer problem.

Chapter 2 applies the state-dependent pricing in a closed-economy model to explain the asymmetric responses of output and prices to monetary shocks. Extensive empirical evidence deals with business cycle asymmetries and, in this chapter, the model explains these asymmetries based on the strategic pricing decisions of firms. The model follows Devereux and Siu (2007), which incorporates state-dependent pricing into a dynamic general equilibrium model to examine the business cycle asymmetries. Their model combines the characteristics of time- and state-dependent pricing, which makes the comparison of the responses of aggregate prices and output to monetary policy shocks under the two pricing schemes feasible. They calibrate their model using US data, and find that the time-dependent and state-dependent pricing models give very different results on the responses to monetary policy shocks.

Instead of simulating and matching the model with the business cycle asymmetries in the US as in Devereux and Siu (2007), the model in Chapter 2 uses a one-period model, which focuses on the effects of strategic complementarity and that of substitutability in firms' pricing decisions. The main contribution of this one-period framework with a common fixed cost of price adjustment is to make the study of mixed strategies in firms' pricing decisions tractable, which add to the literature on state-dependent pricing models that focus on the pure-strategy equilibrium. In this model, firms are allowed to adjust their prices ex-post by paying a small fixed cost, and are allowed to follow mixed strategies in their pricing decisions. The strategic interactions among firms' pricing decisions lead to asymmetric responses of prices and output to

[10] Devereux, Lane and Xu (2006) study optimal monetary policy in emerging economies, and find that optimal monetary policy is affected by the degree of exchange rate pass-through.

monetary shocks. Thus, these interactions explain the observed asymmetries in the business cycles. The model implies asymmetries in positive versus negative monetary shocks, and that the asymmetric responses and the mixed strategies used by firms are affected by the degree of real rigidity in marginal cost, the magnitude of price-adjusting costs, and the market power of firms.

Chapter 3 studies the optimal monetary policy of a small open economy with nominal rigidities and exchange-rate sensitive collateral constraints. This model attempts to explain the observed monetary policy behaviour of emerging markets. Developed economies generally have counter-cyclical monetary policy; however, evidence shows that emerging markets tend to adopt pro-cyclical monetary policy in face of shocks. In addition, empirical evidence supports the view that financial vulnerability is an important constraint on macroeconomic policy in emerging markets. This model adds to the extensive literature on financial constraints in emerging market economies, for instance, Aghion, Bacchetta and Banerjee (2000, 2001, 2004), Choi and Cook (2004), and Christiano, Gust and Roldos (2004). This model differs from other papers in the sense that an optimal, state-contingent, welfare-maximizing monetary policy rule with commitment can be derived. This analysis allows the comparison of differences in monetary policy practice between the developed and emerging economies. The model in this chapter assumes that firms must finance their intermediate imported goods purchases with trade credit denominated in foreign currency, which is repaid by firms after they sell their final goods at the end of the period. The foreign-currency trade credit acts as a collateral constraint, which may limit firms' access to the intermediate imported goods when a devaluation takes place in the domestic currency. The optimal monetary policy rules of the unconstrained and constrained economies can be derived from this simple model. The model implies pro-cyclical optimal monetary policy when the collateral constraint binds, and that an economy with large external shocks may favor a fixed exchange rate, which can explain the observed features of monetary policy used by emerging markets.

Since the specie-flow mechanism discussed by Hume adds to the debate between Keynes and Ohlin on war reparations, the shifts in wealth across countries, which is called the transfer problem, has been a central topic in international macroeconomics. Extensive research on the transfer problem has been done in the past century; however, most of the existing models are generally static (for example, Keynes, 1929; Ohlin, 1929; Johnes, 1970; and Bezmen, 2006). Devereux and Smith (2007) use a real, DSGE model to study the quantitative, macroeconomic transfer effects of the Franco-Prussian War indemnity of 1871-1873, and their model explains the historical paths of French net exports and the terms of trade.

The last chapter of this dissertation also studies the transfer problem using a dynamic, two-country model, following Devereux and Smith (2007). However, money is incorporated into this model, and nominal rigidities may exist in the economy. These allow the theoretical study on the responses of the economy to a transfer shock under flexible and sticky wages, as well as under fixed and floating exchange rate regimes. The results of this model are consistent with the conventional wisdom in international macroeconomics with nominal rigidities, suggesting that a flexible exchange rate can help to reduce internal instability after some shocks via exchange rate adjustment. Nevertheless, the welfare analysis of this model implies that the donor country is better off maintaining the gold standard instead of going to a floating exchange rate, even with nominal rigidities.

Chapter 2

Strategic Pricing Decisions in a State-Dependent Pricing Model

Empirical evidence shows that output and prices respond asymmetrically to positive versus negative and big versus small shocks in the economy. This chapter uses a simple one-period, state-dependent pricing model to explain the asymmetric responses to monetary shocks. We find asymmetries in the effects of monetary shocks on output and prices, which can be explained by the strategic interactions in firms' pricing decisions. We also examine firms' pricing decisions when firms are allowed to follow mixed strategies. The magnitude of the shocks and the number of price-adjusting firms in the economy affect the mixed strategies that an individual firm follows. The model also shows that the degree of real rigidity in marginal cost, the costs of price adjustment and the firm's market power affect the asymmetric responses to monetary shocks, as well as the range of shocks that lead to mixed strategies in firms' pricing decisions.

2.1 Introduction

A large literature in macroeconomics explains economic fluctuations using models with nominal rigidities. The models in the literature can be categorized into two main ways. The first category is the *time-dependent* pricing models, for instance Taylor (1980) and Calvo (1983), which assumes that firms leave their prices unadjusted for some fixed amount of time.[11] The number of price-adjusting firms is pre-determined exogenously, and each firm is constrained to adjust its price at pre-specified times. Firms are not allowed to respond to shocks between price-adjusting periods, and can only choose the

[11]For a literature review on time-dependent pricing, see Taylor (1999), Goodfriend and King (1997) and Galí (2002).

prices to which they adjust, but not the timing of the price-adjustment.

As an alternative, the second category of models, the *state-dependent* pricing models, assumes that firms' pricing decisions depend on the state of the economy. Firms are assumed to face some small costs of price adjustment, and that they adjust only when the benefits outweigh the costs of price adjustment. State-dependent pricing is first introduced in microeconomic models, where a firm changes its price when price deviates from its optimal value (Barro, 1972). Dotsey, King and Wolman (1999) incorporate state-dependent pricing into a dynamic general equilibrium environment.[12]

Recent research has focused on the asymmetric effects of monetary shocks on output and prices. Empirical works provide mixed evidence on the asymmetries in the effects of monetary shocks. The seminal work by Cover (1992) studies the asymmetric effects of monetary policy on aggregate output in the US, and he finds that monetary contractions reduce output by a greater degree than does the amount of increase from equally-sized monetary expansions. De Long and Summers (1988), Karras (1996), and Florio (2005) also find evidence supporting the findings of Cover. On the other hand, Ravn and Sola (1996), Weise (1999), and Fischer, Sahay and Vegh (2002) find asymmetries in the effects of large versus small monetary shocks, but not in positive versus negative shocks.

Theoretical models that explain the asymmetric effects of monetary shocks are mainly built on the assumptions of costly price adjustment (menu cost models), positive trend inflation, and/or state-dependent pricing. Menu cost models suggest that firms face a small fixed cost for changing their prices, and firms adjust their prices only if the gains from adjusting prices exceed the costs of changing prices. Caballero and Engel (1992), Tsiddon (1993), and Ball and Mankiw (1994) use the sticky-price model with menu costs, and show that menu costs lead to asymmetric output and price responses to monetary shocks in the presence of positive trend inflation.[13] In the presence of a negative shock, firms have a smaller incentive to reduce their prices by paying the menu costs, as the positive trend inflation lowers firms' relative prices automatically between adjustments. However, a positive shock in the economy increases firms' desired relative prices while the trend inflation lowers them. Thus, firms are more willing to pay the costs to increase their prices. In contrast, Burstein (2006) and Devereux and Siu (2007)

[12] Other studies on state-dependent pricing include Caplin and Spulber (1987), Caplin and Leahy (1991, 1997), Ireland (1997), Burstein (2006) and Devereux and Siu (2007).

[13] Senda (2001) points out that the degree of asymmetry will lessen if the increase in the inflation rate is beyond some threshold level.

use state-dependent pricing models to study the business cycle asymmetries. Burstein (2006) finds that the economy responds asymmetrically to monetary expansions and contractions since firms are more averse to a lower than optimal relative price than to one that is too high in his model. On the other hand, Devereux and Siu (2007) find that the business cycle asymmetries are due to the strategic linkage among firms in their pricing decisions, and to the positive covariance of aggregate prices and marginal cost in the equilibrium.

The idea of strategic complementarity originally comes from the "coordination failure" literature, where an agent's optimal level of an economic activity depends positively on other agents' activity.[14] Ball and Romer (1991) link the "coordination failure" model with the "menu cost" approach to study firms' pricing decisions. They argue that nominal rigidity arises because of a failure in coordinating price adjustments among firms. They also find that firms' prices are strategic complements; that is, a greater flexibility of one firm's price increases the incentives for other firms to increase their price flexibility. In a recent research, John and Wolman (2008) use a state-dependent pricing model to extend Ball and Romer's analysis in a dynamic setting. The price complementarity in their model depends on the value of the discount factor, in particular, a low value of the discount factor leads to a high degree of complementarity. They also find that a region exists between flexible and sticky prices that contains no pure-strategy steady-state equilibrium, where firms randomize their choices of price flexibility.[15]

The model in this chapter follows the work of Devereux and Siu (2007), with the focus on the strategic interactions among firms in their pricing decisions. Devereux and Siu (2007) incorporate state-dependent pricing into a dynamic general equilibrium model to examine the business cycle asymmetries. Their model combines the characteristics of time- and state-dependent pricing, which makes the comparison of the responses of aggregate prices and output to monetary policy shocks under the two pricing schemes feasible. Instead of simulating and matching the model with the business cycle asymmetries in the US as in Devereux and Siu (2007), this chapter focuses on the effects of strategic complementarity and that of substitutability in firms' pricing decisions. We use a one-period state-dependent pricing model with fixed cost of price

[14]For example, see Diamond (1982), Bryant (1983), and Cooper and John (1988).

[15]John and Wolman (2008) argue that the nonexistence of a pure-strategy equilibrium in that region is due to the discontinuity in the steady-state best-response correspondence of the firms, where an adjusting firm is indifferent between the adjusted price and the pre-set price strategies.

adjustment to study the effects of monetary shocks on firms' pricing decisions and on the economy. We find the asymmetric responses of aggregate output and prices to positive and negative money shocks in this single-period model without positive trend inflation. Prices are downward sticky in face of negative money shocks and more flexible in response to positive shocks of the same magnitude. Firms' prices are *strategic complements* in the presence of positive monetary shocks: with a larger proportion of firms in the economy adjusting their prices, the greater is an individual firm's incentive to increase its price. For negative monetary shocks, firms' prices are *strategic substitutes*; that is, a firm has a smaller incentive to adjust its price when other firms adjust.

The main contribution of this one-period framework with a common fixed cost of price adjustment is to make the study of mixed strategies in firms' pricing decisions tractable, which adds to the literature on state-dependent pricing models that has focused on the pure-strategy equilibrium. Our model shows the existence of a region between flexible and sticky prices where a firm adjusts its price with some endogenously determined probability. We find that firms only follow mixed strategies in face of some negative range of monetary shocks. A firm has a greater probability of lowering its price after negative monetary shocks when it expects that more firms in the economy are adjusting. In addition, the greater the magnitude of the negative shock, the higher is the probability that a firm lowers its price. Therefore, the probability of price adjustment depends on the magnitude of shocks and the behaviour of others firms in the economy.

We also find that the effects of strategic complementarity and substitutability depend on the elasticity of real wage with respect to output and the firm's market power. When the marginal cost is less sensitive to output fluctuations; that is, with a higher degree of "real rigidity," as defined by Ball and Romer (1990), the effect of strategic substitutability is dampened by the increase in strategic complementarity. When marginal cost is insensitive to negative output fluctuations (and the negative money shocks), an individual firm finds the gain in profit from lowering its price reduces. Therefore, it has less incentive to adjust its price unless many other firms are also adjusting. The effect of strategic substitutability also lessens when the firm's market power increases. Thus, when the market becomes more competitive, a firm has a larger incentive to lower its price in response to negative monetary shocks even if many other firms are also adjusting.

The rest of this chapter is organized as follows. Section 2.2 sets up a one-period model. Section 2.3 examines the gains from adjusting prices and Section 2.4 studies the firm's pricing decisions. Section 2.5 concludes.

2.2 The model

Consider a one-period model of an economy with consumers and firms. Households consume differentiated goods and provide labour services to the final goods firms. They receive incomes from wages and from firms' profits. Firms are monopolistically competitive, and use labour hired in a competitive labour market for production.

2.2.1 Firm i

Assume there is a continuum of firms along the unit interval. Each firm i is a monopolist and produces a differentiated good using labour alone. The production function of firm i is:

$$Y_i = H_i \tag{2.1}$$

Since the labour market is competitive, the nominal marginal cost of production is given by the nominal wage, W.

Assume all firms must set their output prices before observing the state of the world. All firms in the economy choose the same price, $\bar{P}$, as all firms are ex-ante identical. However, a firm may choose to adjust its price ex-post by paying a fixed cost, κ. We can think of this cost as the physical ("menu") cost of price adjustment, or any cost incurred in changing prices. Assume that all firms face the same fixed price-adjusting cost.[16] Therefore, there are two types of firms in the economy ex-post; one with pre-set output price, and the other with adjusted price. Let s denotes the portion of firms in the economy that choose to adjust their prices ex-post.[17]

[16]As our interest is in the strategic interactions in pricing decisions and the mixed strategies faced by a firm, for simplicity, we assume identical fixed cost for each firm in this one-period model. We cannot generate results with infrequent heterogeneous price changes with uniform fixed menu costs. For models with stochastic fixed cost of price adjustment, see Dotsey et al. (1999), and Devereux and Siu (2007).

[17]Since the firms lie on the unit interval, we can also view s as firm i's probability of price adjustment. We use s as the probability faced by a firm and the portion of adjusting firms interchangeably in this chapter.

Prior to the realization of the state of the world, firm i's problem is to maximize expected profit by choosing its price, P_i, taking into account that it may choose to adjust its price ex-post with probability s:

$$\max_{P_i} \; E_i \left\{ \Phi \left[(1-s) X_i (P_i - W) + s \tilde{\Pi} \right] \right\} \tag{2.2}$$

where

$$X_i = \left(\frac{P_i}{P} \right)^{-\lambda} X \tag{2.3}$$

X_i is the Dixit-Stiglitz demand faced by firm i and X is the aggregate market demand.[18] $\lambda > 1$ is the firm's own price elasticity of demand, Φ is the firm's state contingent discount factor,[19] P is the price level in the economy, and $\tilde{\Pi}$ is the gross profit that firm i can earn if it adjusts its price ex-post.

Solving this firm's profit-maximizing problem, firm i's ex-ante optimal price is:

$$\bar{P}_i = \hat{\lambda} \frac{E_i \left\{ \Phi P^{\lambda} X W \right\}}{E_i \left\{ \Phi P^{\lambda} X \right\}} \tag{2.4}$$

where $\hat{\lambda} = \frac{\lambda}{\lambda - 1}$ is the markup factor of the monopolistic firm.

After observing the state of the world, firm i can choose to reset its price to $\tilde{P}$ (by paying a fixed cost, κ), which equals to a fixed markup over the marginal cost:

$$\tilde{P}_i = \hat{\lambda} W \tag{2.5}$$

If the state of the world is known ex-ante, (2.4) and (2.5) give the same price, and there is no (gross) gain in profit by adjusting the output price ex-post.

Firm i's profit depends on its pricing decision. Let $\Xi = \{W, X, \bar{P}, s\}$. If the firm chooses to maintain its pre-set price after observing the state of the world, its profit is:

$$\bar{\Pi}(\Xi) = (\bar{P}_i - W) X_i = (\bar{P}_i - W) \left(\frac{\bar{P}_i}{P} \right)^{-\lambda} X \tag{2.6}$$

[18] Demand function, X_i, is obtained from the general equilibrium model below.

[19] The firm's state contingent discount factor, Φ, is determined from the households' preference, as households are the owners of the firm.

If the firm adjusts its price, then its profit becomes:

$$\tilde{\Pi}(\Xi) = (\tilde{P}_i - W)X_i = \frac{\hat{\lambda}^{-\lambda}}{\lambda - 1} W^{1-\lambda} P^{\lambda} X \tag{2.7}$$

The firm chooses to adjust its price to $\tilde{P}$ whenever the gross gain in profit exceeds the fixed cost of price adjustment. That is, firm i adjusts its price whenever:

$$\Delta(\Xi) \equiv \tilde{\Pi}(\Xi) - \bar{\Pi}(\Xi) \geq \kappa \tag{2.8}$$

where $\Delta(\Xi)$ represents the gross gain from price adjustment.

2.2.2 Households

A representative household maximizes its utility by choosing consumption of each good and labour supply subject to her budget constraint. Household's preference is given as:

$$\mathcal{U} = \frac{C^{1-\phi}}{1-\phi} - \eta H \tag{2.9}$$

where $\phi > 0$, and C is a composite consumption, which is characterized by a CES aggregator over the unit measure of differentiated goods:

$$C = \left[\int_0^1 C_i^{\frac{\lambda-1}{\lambda}} di \right]^{\frac{\lambda}{\lambda-1}} \tag{2.10}$$

Household receives wage income, profits of firms, and a lump-sum transfer from the monetary authority, and uses these incomes on consumption and money holdings. Thus, the household's budget constraint is characterized by:

$$PC + M = WH + \int_0^1 \Pi_i di + M_0 + T \tag{2.11}$$

where P is the price index described by equation (2.16), M is the household's choice of money holdings, M_0 is the initial money holdings, T is the total transfer from the monetary authority, and $\int_0^1 \Pi_i di$ is the total profits of the final goods firms, given to household in form of dividends. Assume that household must hold money in order to consume. Thus, household's money holdings must satisfy the cash-in-advance constraint[20]:

$$PC \leq M \tag{2.12}$$

Household's demand for good i is derived as:

$$C_i = \left(\frac{P_i}{P}\right)^{-\lambda} C \tag{2.13}$$

The household problem gives the labour supply condition:

$$W = \eta P C^{\phi} \tag{2.14}$$

Combining this with the cash-in-advance constraint, we have:

$$W = \eta P^{1-\phi} M^{\phi} \tag{2.15}$$

From the household's optimization, we get $\Phi = \frac{1}{PC^{\phi}}$; that is, the stochastic discount factor of the firm's profit maximization is equal to the marginal utility of household. We can use this since all firms are owned by households.

2.2.3 Equilibrium

Since a portion s of firms in the economy will choose to adjust ex-post, we can derive the aggregate price level in the symmetric equilibrium as:

$$P = \left[\int_0^s \tilde{P}_i^{1-\lambda} di + \int_{s+1}^1 \bar{P}_i^{1-\lambda} di\right]^{\frac{1}{1-\lambda}} = \left[(1-s)\bar{P}^{1-\lambda} + s\tilde{P}^{1-\lambda}\right]^{\frac{1}{1-\lambda}} \tag{2.16}$$

Define a symmetric, imperfectly competitive equilibrium, given any monetary policy rule, as the set of allocations, $\Theta = \{C, H, M\}$ and the set of prices, $\wp = \{\bar{P}, \tilde{P}, W\}$ such that:

(1) Firms maximize their expected profits by choosing optimal prices;

(2) Households maximize their utility over consumption and labour supply subject to

[20]The CIA constraint binds in this one-period model. We introduce money to this state-dependent pricing model through the CIA constraint. Money-in-utility model can also serve the same purpose, but it complicates the algebra of this model while producing the same qualitative results.

budget constraints and CIA constraint;
(3) The money market clears:

$$M = M_0 + T \tag{2.17}$$

(4) Labour market clears:

$$H = Y \tag{2.18}$$

(5) The goods market clearing condition implies:

$$X = Y = C \tag{2.19}$$

In particular, the total output is:

$$Y = (1-s)\bar{Y} + s\tilde{Y} = X$$

where $\bar{Y}$ denotes the output of firms that do not adjust ex-post, while $\tilde{Y}$ represents the output of firms that adjust their prices ex-post:

$$\bar{Y} = \left(\frac{\bar{P}}{P}\right)^{-\lambda} C, \qquad \tilde{Y} = \left(\frac{\tilde{P}}{P}\right)^{-\lambda} C$$

2.3 Gains from price adjustment

In this section, we would like to look at the gains from ex-post price adjustment in order to understand the pricing decision of a firm when it observes the decisions of other firms. We look at two extreme cases: all firms adjust ex-post and none of the firms adjusts.

(1) When all other firms adjust, $s = 1$

When all other firms in the economy adjust their prices ex-post, from (2.16), the price level in the economy becomes: $P = \tilde{P} = \hat{\lambda} W$. There is only one type of firms in the economy, that is, the price-adjusting firms, so that the total profits received by the household is $\int \Pi = \tilde{\Pi}$.

Money demand becomes:

$$M = WH + \tilde{\Pi} = \hat{\lambda} W X \tag{2.20}$$

and wage is linear in money:

$$W = \varphi M \tag{2.21}$$

where $\varphi = \eta^{\frac{1}{\phi}} \lambda^{\frac{1-\phi}{\phi}}$.

Firm i's optimal pre-set price is:

$$\bar{P} = \hat{\lambda}\varphi \frac{E\left\{M^{\lambda}\right\}}{E\left\{M^{\lambda-1}\right\}} \tag{2.22}$$

Using (2.6), (2.7), (2.20) and (2.21), we can derive firm i's profits under the pre-set and adjusted prices as:

$$\bar{\Pi}(M|_{s=1}) = \hat{\lambda}^{\lambda-1}\varphi^{\lambda}\bar{P}^{-\lambda}M^{\lambda+1}\left(\frac{\bar{P}}{\varphi M} - 1\right) \tag{2.23}$$

$$\tilde{\Pi}(M|_{s=1}) = \frac{1}{\lambda-1}\frac{M}{\hat{\lambda}} \tag{2.24}$$

Then firm i's gross gain from price adjustment, given that all other firms are adjusting ex-post, is:

$$\Delta(M|_{s=1}) \equiv \tilde{\Pi}(M|_{s=1}) - \bar{\Pi}(M|_{s=1}) = \frac{M}{\hat{\lambda}}\left[\frac{1}{\lambda-1} - \bar{P}^{-\lambda}\hat{\lambda}^{\lambda}\varphi^{\lambda}M^{\lambda}\left(\frac{\bar{P}}{\varphi M} - 1\right)\right] \tag{2.25}$$

(2) When all other firms do not adjust, $s = 0$

When no firm in the economy chooses to adjust its price ex-post, the price level becomes $P = \bar{P}$, and $\bar{P}$ is given as:

$$\bar{P} = (\hat{\lambda}\eta)^{\frac{1}{\phi}}\left[\frac{E\left\{M\right\}}{E\left\{M^{1-\phi}\right\}}\right]^{\frac{1}{\phi}} \tag{2.26}$$

Money balances and wage become:

$$M = \bar{P}X \tag{2.27}$$
$$W = \eta X^{\phi-1}M \tag{2.28}$$

Firm i's profits under pre-set and adjusted prices are:

$$\bar{\Pi}(M|_{s=0}) = M\left(1-\eta\bar{P}^{-\phi}M^{\phi}\right) \tag{2.29}$$
$$\tilde{\Pi}(M|_{s=0}) = \frac{\hat{\lambda}^{-\lambda}}{\lambda-1}\eta^{1-\lambda}\bar{P}^{-\phi(1-\lambda)}M^{\phi(1-\lambda)+1} \tag{2.30}$$

and firm i's gross gain from price adjustment, given that no other firm chooses to deviate from the pre-set prices ex-post, is:

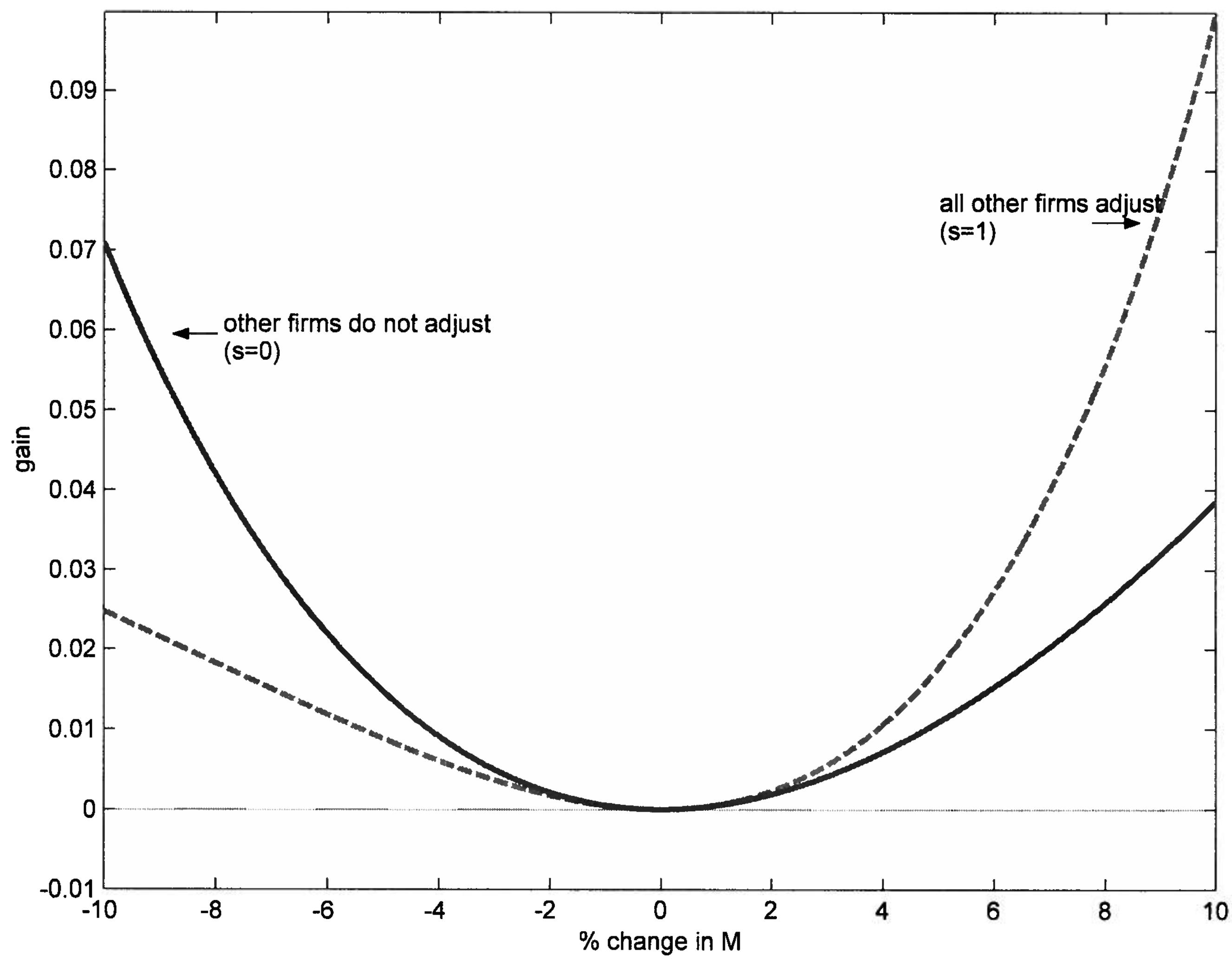
$$\Delta(M|_{s=0}) \equiv \tilde{\Pi}(M|_{s=0}) - \bar{\Pi}(M|_{s=0}) = M\left[\frac{\hat{\lambda}^{-\lambda}}{\lambda-1}\eta^{1-\lambda}\bar{P}^{-\phi(1-\lambda)}M^{\phi(1-\lambda)} - 1 + \eta\bar{P}^{-\phi}M^{\phi}\right] \tag{2.31}$$

Figure 2.1 plots the gross gains from price adjustment, equations (2.25) and (2.31), as functions of money shock. We use $\lambda = 11$, which is consistent to a 10 percent markup reported in Basu and Fernald (1997). We use $\phi = 1$ as the benchmark, where household's preference becomes logarithmic in consumption. The dashed line corresponds to firm i's gain from adjusting price when all other firms adjust their prices ex-post, while the solid line represents the case when all firms choose to maintain their pre-set prices. We can see that there are asymmetries in the gross gain functions. Since firm i's incentive to adjust ex-post depends on the gross gain and the fixed adjusting cost, Figure 2.1 also implies that firm i's pricing decision responds asymmetrically to exogenous monetary shocks. Note that the gains from price adjustment with $s \in (0, 1)$ lie between the dashed and solid lines.

When $s = 1$, that is, when all other firms in the economy adjust their prices ex-post, there is a larger incentive for firm i to adjust its price in response to positive money shocks than to negative shocks. Firm i faces a higher nominal wage (that is, higher nominal marginal cost) after a positive money shock, as other firms increase their prices which lead to a higher price level after the shock. Firm i's demand increases if it remains to charge $\bar{P}$. If the positive shock is large enough, however, firm i's marginal profit falls or even becomes negative, and hence, leads to a lower profit even if its demand

Figure 2.1: Individual Firm's Gross Gains From Price Adjustment

$(\phi = 1)$



increases. Therefore, firm i has a large incentive to follow other firms to increase its price when there are positive money shocks. This is the *strategic complementarity* in firm i's pricing decision: the greater the share of other firms adjusting their prices in face of a money shock, the greater is the incentive of firm i to adjust its own price.[21]

On the other hand, when there is a negative monetary shock in the economy, firm i has a smaller incentive to adjust its price when other firms adjust. When other firms lower their prices, the aggregate price level falls ex-post, which leads to a fall in firm i's demand and profit when it chooses to maintain $\bar{P}$. Firm i's profit, however, also drops even if it reduces its price ex-post, as the aggregate output in the economy and firm i's demand fall after a negative shock. The gross gain from lowering the price is smaller, therefore, firm i has a smaller incentive to adjust its price after a negative money shock when other firms in the economy choose to adjust. This is the *strategic substitutability* in firm i's pricing decision: the greater the share of other firms adjusting their prices in face of a money shock, the smaller is the incentive of firm i to adjust its own price.

When other firms maintain their prices ex-post (that is, $s = 0$), the asymmetry in firm i's pricing decision reverses. For positive money shocks, firm i has a smaller incentive to adjust when other firms do not adjust, while for negative money shocks, the firm has a larger incentive to adjust. When there is a negative money shock, firm i's demand increases if it lowers its price. Firm i's profit would fall by a smaller amount or even increase if the increase in demand is large enough. Therefore, firm i has an incentive to adjust when other firms do not adjust because the effect of strategic substitutability becomes larger. On the other hand, firm i is less willing to increase its price in face of positive money shocks, since its profit may fall due to a fall in demand.

Now consider the case with $\phi < 1$, that is, the elasticity of real wage with respect to output is less than unity. Recall from equation (2.15), we have:

$$W = \eta P^{1-\phi} M^{\phi}$$

In the benchmark case, we use $\phi = 1$. The nominal wages move linearly with the money stock, or equivalently, real wages only respond to fluctuations in aggregate

[21]Burstein and Hellwig (2007) call this strategic interaction between firms' pricing decisions as *aggregate pricing complementarity.* In their model, they also focus on *firm-level pricing complementarity,* which arises from the change in firm's marginal cost through the changes in price level and firm's relative price. In our model, since we adopt a constant return to scale production function, the change in firm's marginal cost is solely due to the exogenous money shock. Individual firms interact only because of the change in aggregate price level.

output (equation (2.14)). When ϕ approaches zero, nominal wages are less responsive to changes in money stock and output, and the changes in nominal wages are mainly induced by the changes in aggregate price level. Therefore, with $\phi < 1$, the presence of real rigidities further increases the interaction of firm i's price to other firms' prices, since the aggregate price level has a larger effect on firm i's marginal cost.

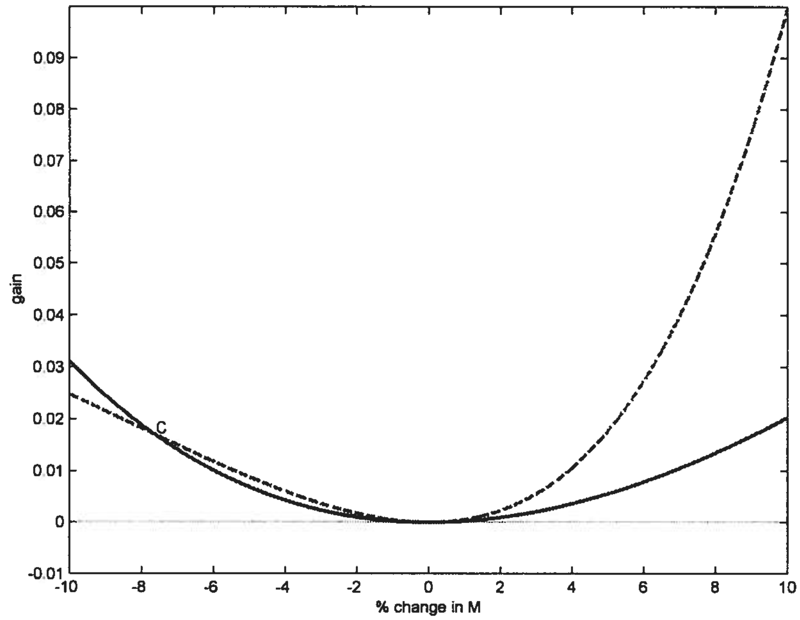
We plot firm i's gross gain functions with $\phi = 0.7$ and $\phi = 0.5$ in Figure 2.2. Panel (a) shows the gross gain functions when $\phi = 0.7$, and we find that there are two crossings of the gross gain functions. When comparing Figure 2.2 to the case with $\phi = 1$, the gross gain from price adjustment given $s = 0$ (the solid line) becomes flatter and less asymmetric within the $\pm 10\%$ money shock range, especially with respect to the range of negative money shocks. When $\phi < 1$, wages do not decrease one for one with the fall in money stock. If firm i lowers its price while other firms do not adjust, firm i's profit will be smaller than that under the case when $\phi = 1$. The increase in firm i's demand is now smaller, and hence, the gain from price adjustment is reduced. Therefore, firm i has less incentive to adjust. Firm i's pricing decision becomes more dependent on other firms' prices when there are real rigidities, and the effect of strategic substitutability in pricing decision decreases. When we further lower the value of ϕ (that is, increase the degree of real rigidities), panel (b) shows that the gross gain function given $s = 0$ becomes more flatter. The solid and dashed lines do not cross each other within the $\pm$ 10% money shock range, implying the effect of strategic substitutability is dominated by that of strategic complementarity in this range.[22] In general, with lower value of ϕ, that is, when wages are less responsive to changes in money stock (and hence, the aggregate output), the gain from price adjustment falls when other firms do not adjust, and the effect of strategic substitutability diminishes as ϕ approaches 0.

[22]The two lines do cross each other when the money shock is -15.78%. However, for low value of ϕ, the two functions are only tangent to each other at 0% money shock, and do not cross each other even for a $\pm$ 100% change in money stock.

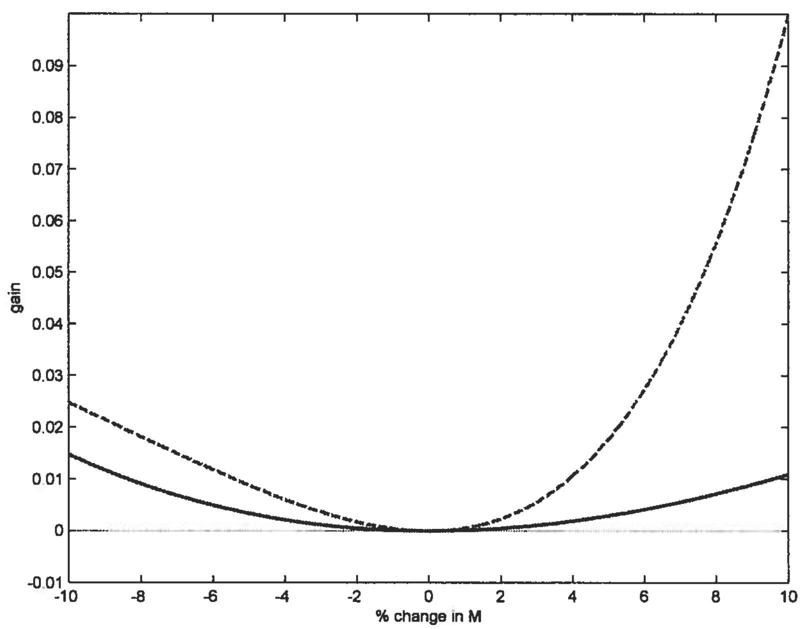
Figure 2.2: Effects of ϕ on Gross Gains From Price Adjustment

Dashed line: gain when $s = 1$; Solid line: gain when $s = 0$.

$(a) \quad \phi = 0.7$



$(b) \quad \phi = 0.5$



2.4 The pricing decision

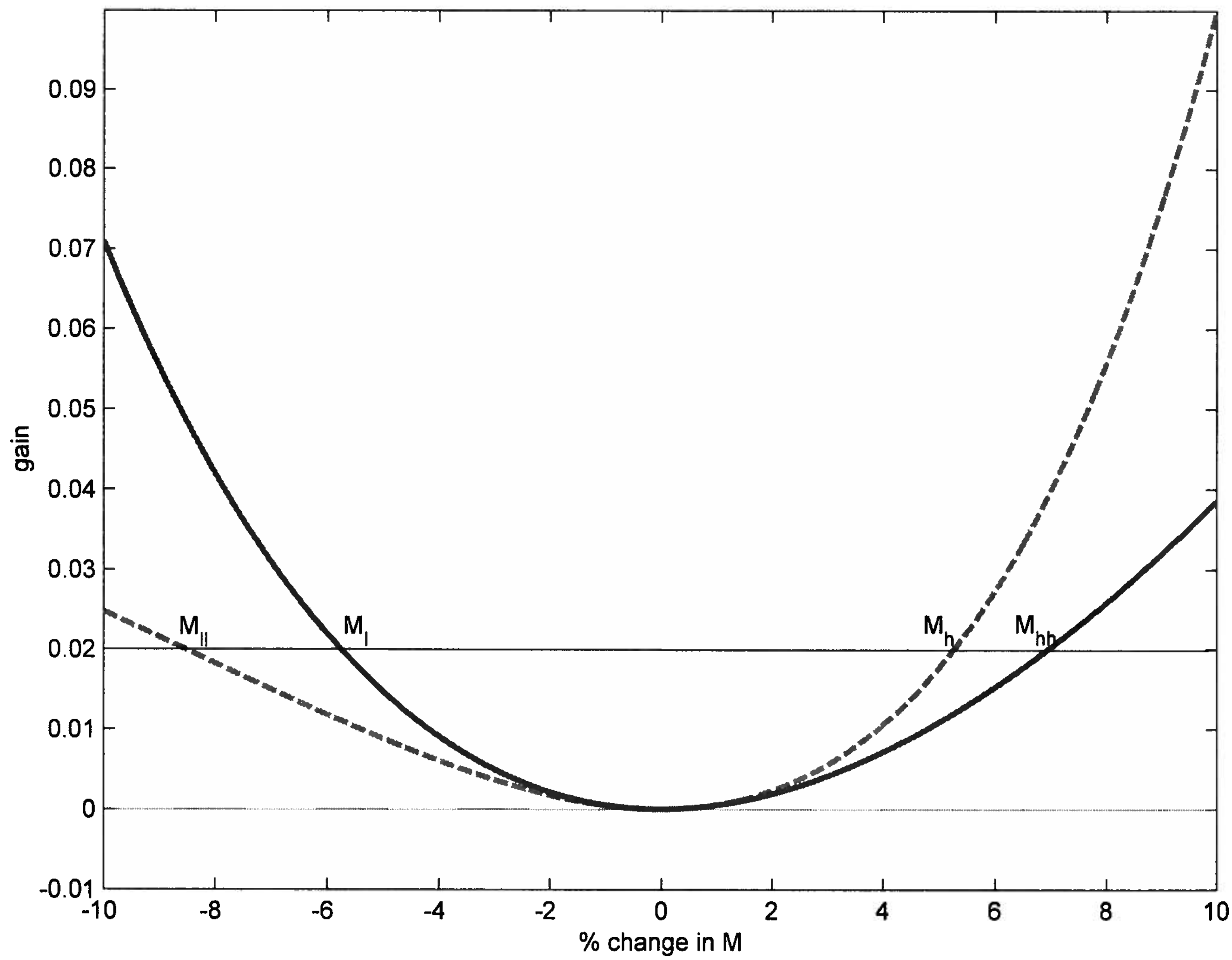
From equation (2.8), we know that an individual firm's pricing decision depends on the gross gain from and the fixed cost of price adjustment. We use the gross gain functions to determine firm i's pricing decision. We arbitrarily assume the fixed cost of price adjustment, κ, equals to 0.02, and it is represented by a horizontal line in Figure 2.3 (which is a modified version of Figure 2.1 for illustration purpose). With $\phi = 1$, the $\kappa = 0.02$ line intersects the gross gain curves at four points, M_{ll}, M_l, M_h and M_{hh}, respectively. For any negative money shock larger than M_{ll} (in absolute term), firm i certainly chooses to lower its price ex-post, as the gross gain from price adjustment is larger than the fixed cost κ, regardless of the number of price-adjusting firms in the economy. For such a large negative shock, all firms find that it is profitable to lower their prices and expect other firms will also adjust. All firms adjust ex-post, and thus, $s = 1$ is sustainable. Similarly, for any positive shocks that are greater than M_h, firm i also pays the fixed cost and adjusts its price ex-post.[23]

For relatively small negative and positive shocks that lie between M_l and M_h, firm i does not adjust its price ex-post, as the net gain from price adjustment is negative for all values of s. Firm i is better off without a price-adjustment. The region between M_{ll} and M_l is a region between the sticky-price and flexible-price pure strategies, where firm i randomizes its pricing decision. In this region, firm i is indifferent between the new flexible price and the pre-set sticky price in response to negative monetary shocks. Firm i uses mixed strategies in its pricing decision, in particular, it adjusts its price with probability $s \in (0, 1)$. The probability that firm i adjusts its price depends on the share of other adjusting firms in the economy. For negative shocks slightly greater than M_l, firm i adjusts only if s is close to 0. If firm i expects a lot of firms are adjusting, it will not adjust as the net gain is negative. Thus, firm i only has a small incentive to adjust when the shocks are relatively small in this range, and its incentive depends on the pricing decisions of other firms. While for larger negative shocks, firm i has a larger tendency to adjust (that is, when s is closer to 1) as net gain increases. Therefore, a typical firm i's pricing decision depends on the gross gain from and the fixed cost of price adjustment. Using the notations in Figure 2.3, we have:

[23]For positive monetary shocks, the effect of strategic complementarity always dominates the effect of strategic substitutability. That is, the $\Delta(M|_{s=1})$ curve always lies above the $\Delta(M|_{s=0})$ curve. Therefore, as long as the net gain of price adjustment, given $s = 1$, is positive, firm i adjusts its price ex-post.

Figure 2.3: Individual Firm's Gross Gain and Pricing Decisions

($\phi = 1$)



Proposition 1. *Given $\Delta(M|_{s=1})$ intersects κ at M_{ll} and M_h, while $\Delta(M|_{s=0})$ intersects κ at M_l and M_{hh}, where $M_{ll} < M_l < \bar{M} < M_h < M_{hh}$. Then the probability that firm i adjusts its price in response to monetary shocks are:*

$$\begin{array}{lllll} I: & \Delta(M,s) > \kappa & \Rightarrow & s = 1 & \text{for } M \geq M_h \\ II: & \Delta(M,s) < \kappa & \Rightarrow & s = 0 & \text{for } M \in [M_l, M_h) \\ III: & \Delta(M,s) = \kappa & \Rightarrow & s \in (0,1) & \text{for } M \in (M_{ll}, M_l) \\ IV: & \Delta(M,s) > \kappa & \Rightarrow & s = 1 & M \leq M_{ll} \end{array}$$

where $\bar{M}$ denotes the steady-state money stock.

Proof. See Appendix. □

When there are real rigidities (that is, $\phi < 1$), the range of negative shocks that leads to mixed strategies in firm i's pricing decision becomes smaller. For instance, we can see from panel (a) of Figure 2.2 that when $\phi = 0.7$ with $\kappa = 0.02$, the range of negative money shocks that leads to $s \in (0,1)$ is smaller (that is, the horizontal distance between the two functions is narrower). In addition, when we lower the fixed cost such that κ is below point C, firm i uses only pure strategies in its pricing decision: it adjusts whenever $\Delta(M|_{s=1})$ is greater than κ, and remains sticky when $\Delta(M|_{s=1})$ is less than κ. In panel (b) when the value of ϕ is low, since the $\Delta(M|_{s=0})$ curve is always flatter than the $\Delta(M|_{s=1})$ curve for any low value of κ, firm i's pricing decision only depends on the gross gain from price adjustment given $s = 1$. Thus, our model shows that lower the elasticity of real wage with respect to output, lower is the uncertainty in an individual firm's pricing decision, as the effect of strategic substitutability is dominated by that of strategic complementarity.

How can we determine firm i's probability of price adjustment when firm i faces a negative money shock that lies in Region III of proposition 1?

When firm i uses mixed strategies in its pricing decision (that is, $s \in (0,1)$), the money demand condition becomes:

$$M = WX + (1-s)\bar{\Pi} + s\tilde{\Pi} \tag{2.32}$$

where

$$\bar{\Pi} = (\bar{P} - W)\left(\frac{\bar{P}}{P}\right)^{-\lambda} X = \bar{P}^{1-\lambda}P^{\lambda-1}M - \eta\bar{P}^{-\lambda}P^{\lambda-\phi}M^{\phi+1} \tag{2.33}$$

$$\tilde{\Pi} = \frac{\hat{\lambda}^{-\lambda}}{\lambda - 1}W^{1-\lambda}P^{\lambda}X = \frac{\hat{\lambda}^{-\lambda}}{\lambda - 1}\eta^{1-\lambda}P^{-\phi(1-\lambda)}M^{\phi(1-\lambda)+1} \tag{2.34}$$

The price level in the economy is:

$$P = \left[(1-s)\bar{P}^{1-\lambda} + s\tilde{P}^{1-\lambda}\right]^{\frac{1}{1-\lambda}} \tag{2.35}$$

Firm i pre-sets its price at:

$$\bar{P} = \hat{\lambda}\frac{E\left\{\Phi P^{\lambda}XW\right\}}{E\left\{\Phi P^{\lambda}X\right\}} = \hat{\lambda}\eta\frac{E\left\{X^{1-\lambda}M^{\lambda}\right\}}{E\left\{X^{2-\phi-\lambda}M^{\lambda-1}\right\}} \tag{2.36}$$

and the wage equation can be written as:

$$W = \eta P^{1-\phi}M^{\phi} = \eta X^{\phi-1}M \tag{2.37}$$

From proposition 1, we know that firm i's gross gain from price adjustment must satisfy:

$$\Delta(M, \Xi) \equiv \tilde{\Pi}(M, \Xi) - \bar{\Pi}(M, \Xi) = \kappa \tag{2.38}$$

Substituting equation (2.37) and using the CIA constraint (2.12), equations (2.32), (2.36) and (2.38) can be used to solve for $\{X, \bar{P}, s\}$, for a given level of money stock, M.

Since we cannot solve this model analytically, we derive the probability of adjusting price numerically. Following the previous section, we continue to use $\lambda = 11$. Without loss of generality, we set the weight on labour supply in household's utility to one, that is $\eta = 1$. Again, we use $\phi = 1$ in the benchmark case, and use $\phi < 1$ for later experiments.

A key parameter to be characterized is the fixed cost of price adjustment, κ. As we have seen before, the value of κ plays an important role in firms' pricing decisions. In our benchmark calibration, we set the cost of adjusting price to 1 percent of the firm's steady-state revenue. We set this value by referring to some empirical studies on costs of price adjustment. For instance, Zbaracki et al.(2004) use the data of a large multi-product industrial manufacturer in the US to calculate the costs of adjusting prices. They categorize three types of price-adjusting costs, namely, managerial costs, customer costs and physical (menu) costs of changing prices, and the sum of these costs is about 1.23 percent of the firm's annual revenue. The managerial costs include costs in gathering information, making decisions, and communicating among departments within the firm, and these are estimated to be 0.28 percent of the revenue. 0.91 percent of the revenue are devoted as the customer costs, which are the costs of communicating and negotiating the price changes with customers. The physical costs, or menu costs of changing prices are about 0.04 percent of firm's annual revenue, and they are the actual costs of issuing the new prices.

Levy et al. (1997) measure the actual menu costs at four multi-store retail supermarket chains in the US. They find that the average annual menu costs of a store is 0.7 percent of revenues. Although these findings are consistent with the common sense that the costs of price adjustment are generally very small when they are compared to the overall costs and revenues of a firm's activities, Levy et al. argue that the menu costs they find are large enough to form a barrier to price changes, and have macroeconomic significance, especially when they are applied to other industries or markets. We believe that our choice of κ is reasonable, as it lies between the values found by the above studies.[24]

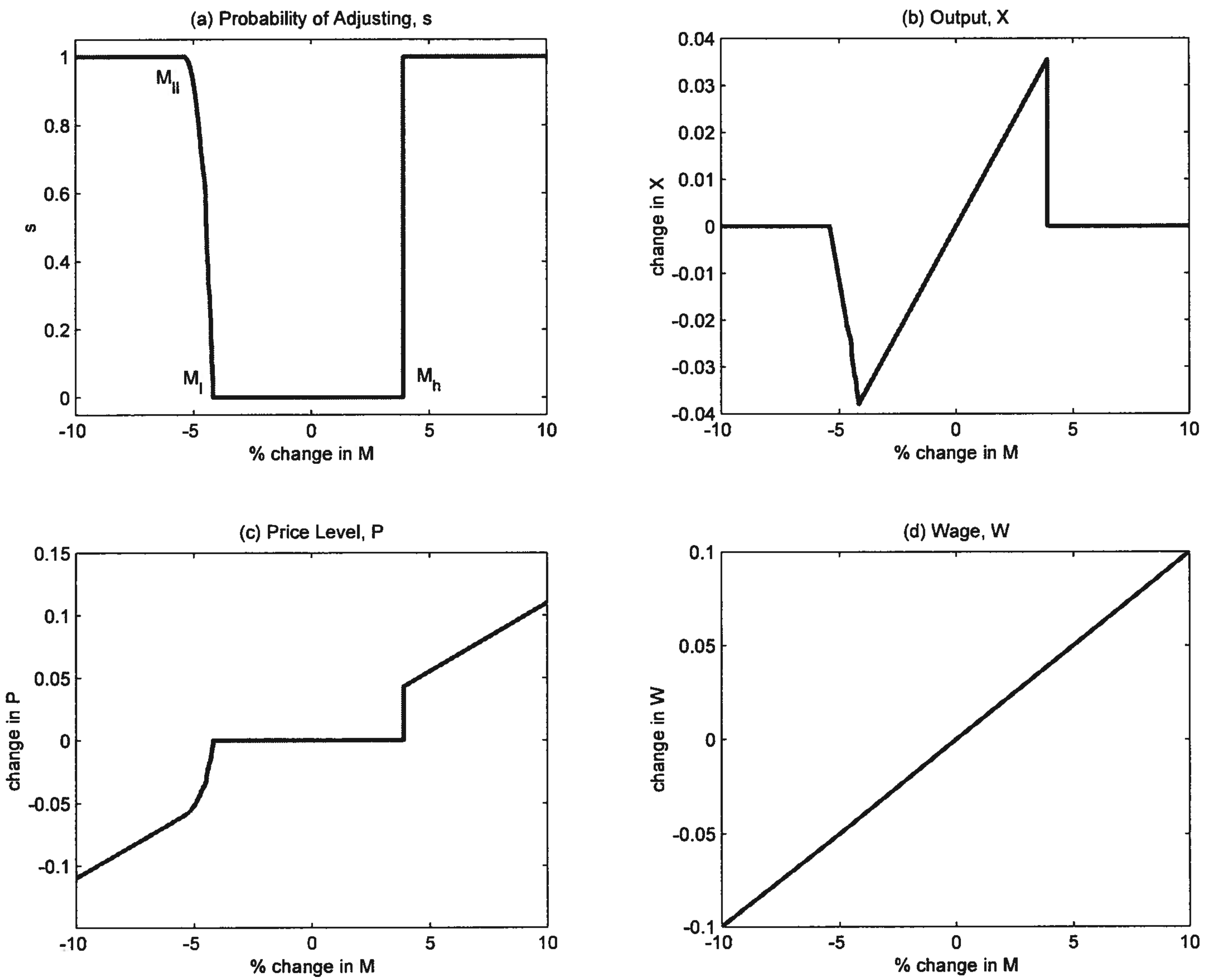
(1) Benchmark case

Figure 2.4 shows the benchmark calibration with fixed cost of price adjustment equals 1 percent of firm's steady-state revenue. We plot the responses of aggregate output, price level and wage to exogenous money shocks that lie within the [-10%, +10%] range. Panel (a) shows the probability of adjusting price that an individual firm i faces. This also measures the proportion of firms in the economy that adjust ex-post. Positive money shocks greater than +3.90% (that is, point M_h in the graph) induce all firms in the economy to adjust, and prices become flexible. On the other hand, all firms

[24]Slade (1998) uses weekly retail prices and sales of saltine crackers in a small US town to estimate the costs associated with price adjustment. She finds that the magnitude of fixed-adjustment costs is substantially higher than that of the variable-adjustment costs, approximately in the ratio of 15:1. Since we assume fixed cost of price adjustment in our model, if we apply this ratio to the total costs of price adjustment that are found in Zbarack et al. (2004), the fixed-adjustment costs account for about 1.1 percent of firm's annual revenue.

choose to adjust only when the negative shock is more negative than -5.41% (point M_{ll}). This is the asymmetry discussed in Section 2.3: other firms' prices are strategic complements to firm i's price when there are positive money shocks in the economy, while their prices are strategic substitutes to firm i's price if shocks are negative.

Figure 2.4: Responses to Monetary Shocks: Benchmark Model, $\phi = 1$, $\kappa = 1\%$ SS revenue



For negative shocks that lie between (-5.41%, -4.18%)(that is, between points M_{ll} and M_l), firm i's willingness to lower its price is increasing with the (absolute) magnitude of the shock and the share of adjusting firms in the economy. Larger the negative shock, greater is the loss of profits if an individual firm maintains its pre-set price due to the fall in aggregate output and firm's demand, and hence, greater is the number of firms deviate from their pre-set prices. Price level starts to fall when more firms reduce their prices, and it is more profitable for firm i to adjust. Therefore, firm i is more willing to lower its price to avoid further loss in profit.

Panel (b) of Figure 2.4 presents the output changes in response to exogenous money shocks. When no firm chooses to adjust its price, output increases when there is a positive money shock, and decreases when money stock falls. When all firms adjust ex-post and prices are flexible in the economy, money neutrality holds. When firm i follows mixed strategies in the range of (-5.41%, -4.18%), the magnitude of the fall in output becomes smaller when the shocks are getting more negative. More firms in the economy start to lower their prices when the shocks get more negative. Price level falls, which helps to lessen the fall in output. At the point where all firms have lowered their prices, money neutrality restores.[25]

Table 2.1a reports the distribution of output and price level under ±1%, ±5% and ±10% money shocks respectively. When the shocks are within ±1%, output and prices respond to shocks symmetrically. Firms do not adjust their prices so that price level is sticky in this range. For larger monetary shocks, firms' pricing decisions and the responses of the economy become asymmetric. When shocks lie between the [-5%, +5%] interval, the average increase in prices with respect to positive shocks is larger than the average price reduction in response to negative money shocks. Firms use mixed strategies in their pricing decisions in the range [-5%, -4.18%], in which the price level lies between the pre-set and adjusted prices. This lessens the average price deviation from the steady-state value, since for positive shocks, firms only follow pure strategies in their pricing decisions and prices jump from sticky to flexible prices. However, the average price adjustment deviates more from the steady-state value in response to negative shocks than that to positive shocks when shocks are in the ±10% range. Therefore, we find asymmetries in the responses of output and price level to positive versus negative shocks, as well as asymmetries in small versus big shocks.

[25]This is an "unrealistic" result of this one-period model. In practice, a large fall in money stock leads to fall in output.

Table 2.1: Distribution of Variables
($\lambda = 11$, $\kappa = 1\%$ of steady-state revenue)

(a) Benchmark ($\phi = 1$):

M shocks	-1%	$+1\%$	-5%	$+5\%$	-10%	$+10\%$
$E(X)$	0.9046	0.9136	0.8893	0.9229	0.8990	0.9160
σ_X	0.0026	0.0026	0.0106	0.0117	0.0123	0.0108
$E(P)$	1.1	1.1	1.0946	1.1107	1.0561	1.1466
σ_P	8.66×10^{-15}	8.66×10^{-15}	0.0134	0.0203	0.0413	0.0403

(b) $\phi = 0.7$:

M shocks	-1%	$+1\%$	-5%	$+5\%$	-10%	$+10\%$
$E(X)$	0.8683	0.8771	0.8509	0.8860	0.8600	0.8794
σ_X	0.0025	0.0025	0.01265	0.0112	0.0154	0.0103
$E(P)$	1.1459	1.1459	1.1459	1.1571	1.1053	1.1944
σ_P	4.44×10^{-16}	4.44×10^{-16}	5.00×10^{-14}	0.0212	0.04522	0.0419

(2) With some degree of real rigidity

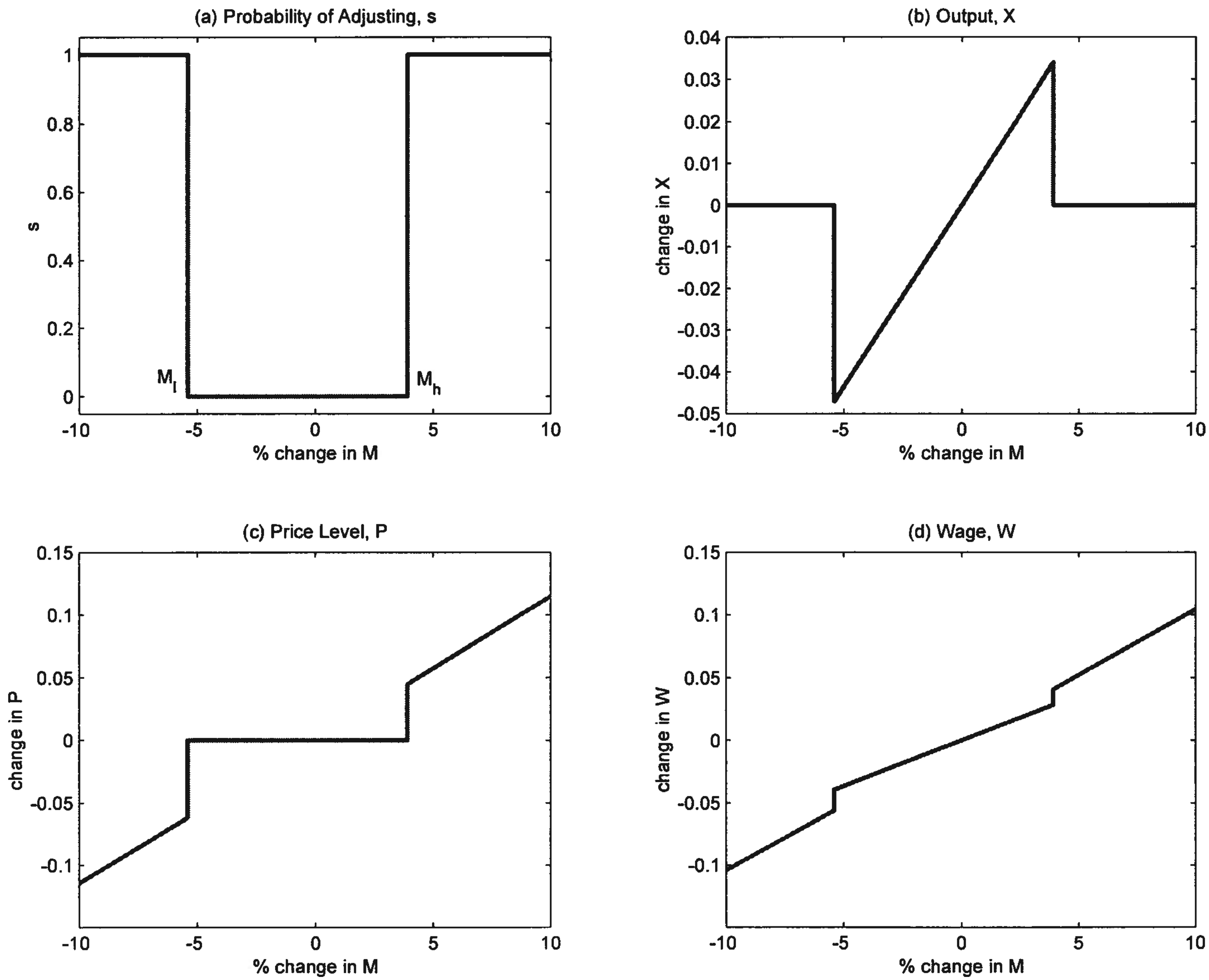
We also look at how the probability of adjusting, s, and output respond to exogenous money shocks when the elasticity of real wage with respect to output is less than unity. We use $\phi = 0.7$ and the results are shown in Figure 2.5.[26] When fixed costs are only 1 percent of the firm's steady-state revenue, firm i increases its price when the positive money shock is greater than +3.90%, and lowers its price when the negative shock is greater than -5.41%. These values are the same as those of the benchmark when $\phi = 1$; however, the difference is that when $\phi = 0.7$, firm i does not have any uncertainties in its price-adjusting decision.[27] Firm i either adjusts in response to large (positive and negative) shocks with probability $s = 1$, or does not adjust with $s = 1$. Firm i follows pure strategies in its pricing decision. As a result, either no firm adjusts so that prices are sticky in the economy when the magnitude of shocks is between (-5.41%, +3.90%), or all firms adjust when shocks lie outside this range.

When we compare Table 2.1b to 2.1a, we find that the presence of real rigidities increases the standard deviations of price level for ±5% and ±10% monetary shocks. With low fixed costs of adjustment, the economy moves from $\bar{P}$ to $\tilde{P}$ instantaneously when the monetary shocks exceed the threshold levels. The lack of mixed strategies induces larger deviations in output and price adjustments.

[26]Lower values of ϕ also produce the same qualitative results. For illustrative purpose, we choose $\phi = 0.7$.

[27]This is because in the specification of our model, change in ϕ does not affect the $\Delta(M|_{s=1})$ curve while the $\Delta(M|_{s=0})$ curve becomes flatter. With low fixed costs of price adjustment, $\Delta(M|_{s=1})$ always outweighs the $\Delta(M|_{s=0})$, and every firm expects other firms will adjust in face of a negative shock as long as the net gain is positive.

Figure 2.5: Responses to Monetary Shocks: Low ϕ; $\phi = 0.7$, $\kappa = 1\%$ SS revenue



(3) Fixed price-adjusting costs

To illustrate the effects of fixed cost of price adjustment on firm's pricing decision, we now increase the fixed cost of price adjustment to 5 percent of firm's steady-state revenue. Figure 2.6 shows the firm i's pricing decision and the response of aggregate output when the fixed costs are high. The upper panels, (a) and (b), show the case when $\phi = 1$. With high fixed cost of price adjustment, firm i does not adjust its price unless positive money shocks are greater than +7.66%. For negative shocks, it only adjusts if the shocks are more negative than -20.47%. The lower net gain from price adjustment due to the higher fixed adjustment cost increases the degree of price rigidity in the economy. In addition, with higher fixed adjustment cost, firms' responses to positive and negative shocks become more asymmetric: all firms only adjust when the (absolute) magnitude of negative shock is 12.81% higher than that of a positive shock (compare to 1.51% when κ is only 1 percent of the steady-state revenue).

Higher fixed adjustment cost also leads to a greater range of negative money shocks, (-20.47%, -8.62%), that gives $s \in (0, 1)$. With high cost of price adjustment, fewer firms have the incentive to adjust for a given magnitude of negative shock since the net gain from adjusting decreases. Price level falls by a smaller magnitude, and firm i gains less from price adjustment via the effect of strategic pricing interaction. As a result, the economy does not have full price flexibility for a larger range of negative shocks, and all firms lower their prices only for a greater magnitude of negative shocks. The distribution of price level in Table 2.2a illustrates this result numerically.

Figure 2.6: Responses to Monetary Shocks: High Fixed Cost; $\kappa = 5\%$ SS revenue

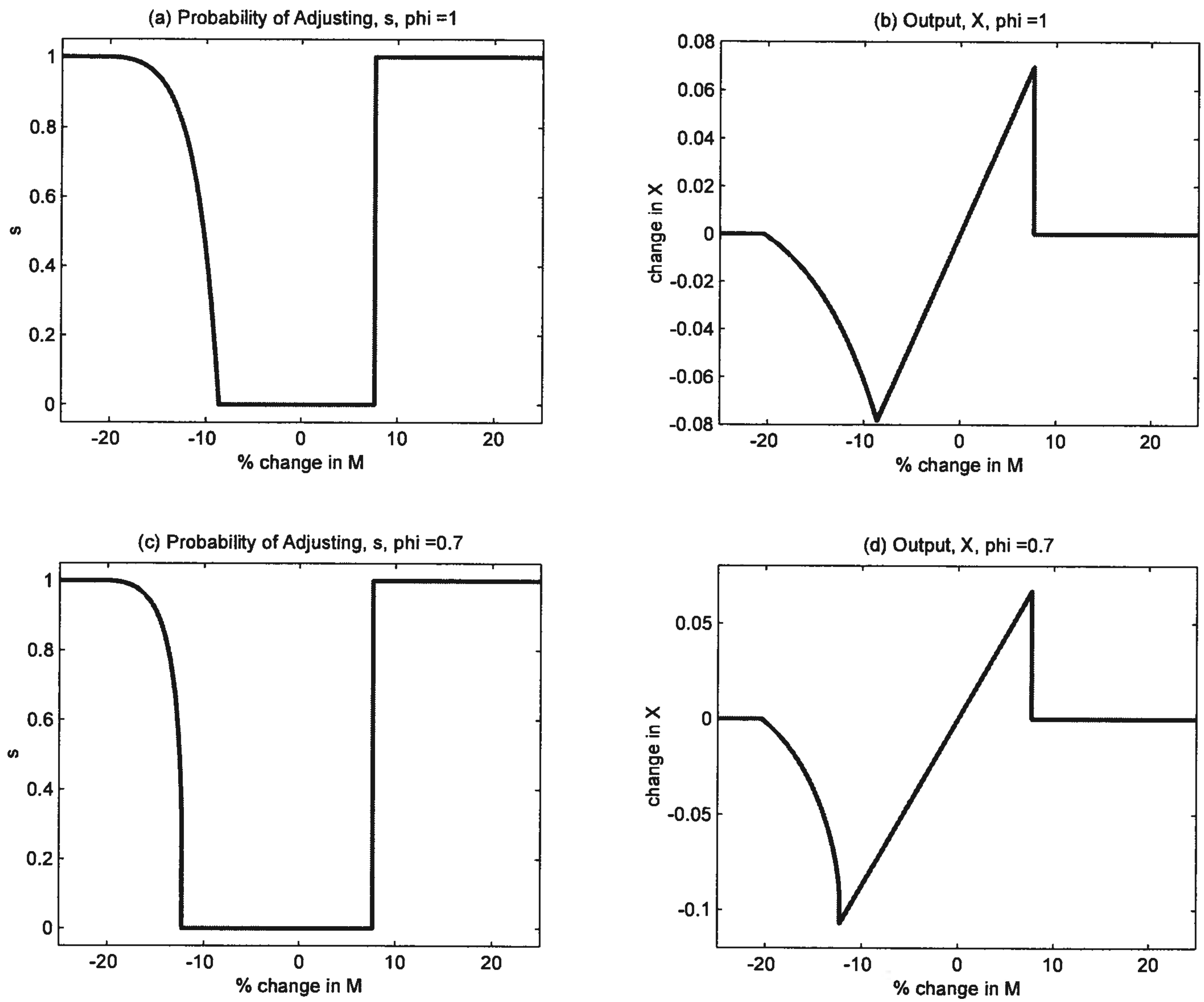


Table 2.2: High Cost of Price Adjustment, 5% of Steady-State Revenue ($\lambda = 11$)

(a) Benchmark ($\phi = 1$):

M shocks	-1%	$+1\%$	-5%	$+5\%$	-10%	$+10\%$
$E(X)$	0.9046	0.9136	0.8864	0.9318	0.8657	0.9358
σ_X	0.0026	0.0026	0.0131	0.0131	0.0236	0.0230
$E(P)$	1.1	1.1	1.1	1.1	1.0561	1.1466
σ_P	8.66×10^{-15}	8.66×10^{-15}	4.49×10^{-14}	4.49×10^{-14}	0.0142	0.0413

(b) $\phi = 0.7$:

M shocks	-1%	$+1\%$	-5%	$+5\%$	-10%	$+10\%$
$E(X)$	0.8683	0.8771	0.85099	0.8945	0.8291	0.8983
σ_X	0.0025	0.0025	0.0126	0.0126	0.0252	0.0220
$E(P)$	1.1459	1.1459	1.1459	1.1459	1.1459	1.1695
σ_P	4.44×10^{-16}	4.44×10^{-16}	5.00×10^{-14}	5.00×10^{-14}	2.03×10^{-13}	0.0430

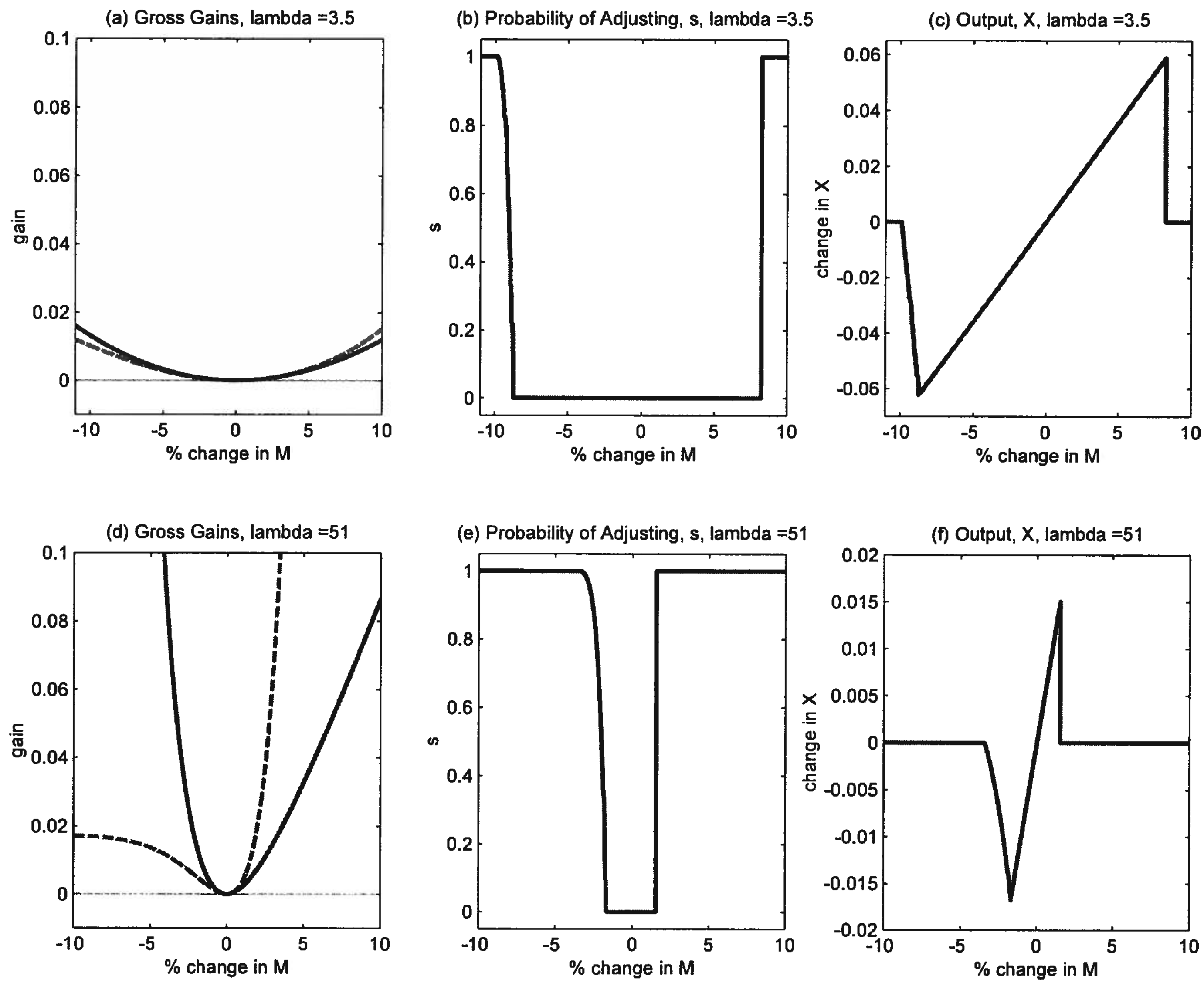
As we have mentioned before, when the elasticity of real wage with respect to output is less than unity, either all firms adjust or none adjusts when the fixed adjustment cost is low. However, with a higher fixed cost of price adjustment, a range of negative money shocks that yields less than full price flexibility in the economy exists. Firm i chooses mixed strategies when shocks lie within this range. We can see from panel (c) of Figure 2.6 that when shocks are in the range of (-20.47%, -12.29%), firm i's probability of adjusting its price is between 0 and 1, and the fraction of firms in the economy that adjust is less than unity. Output falls by a smaller amount in response to negative shocks when s is getting larger, and this is shown in panel (d). Firms only adjust their prices in response to larger shocks when the cost of price adjustment is high. Table 2.2b shows that firms do not adjust even for a -10% shock. Firms have less incentive to adjust as gains from adjusting decrease, and their pricing decisions more depend on other firms' behaviour. The effect of strategic substitutability becomes more significant when the fixed adjustment cost is high. Therefore, for some range of negative shocks, some firms adjust and some remain sticky ex-post.

(4) Firm's market power

We also examine the effect of firm's market power on its pricing decision. We find that with more market power, a firm tends to have smaller incentive to adjust its price in response to monetary shocks, while it has a larger incentive to adjust when the market competitiveness increases. Figure 2.7 and Table 2.3 illustrate this result with fixed cost equals to 1 percent of the steady-state revenue and $\phi = 1$. Panel (a) to (c) of Figure 2.7 show the case when firm i has a large market power, which is represented by a high firm's price markup;[28] in particular, we have price markup equals to 40 percent of the firm's marginal cost (that is, $\lambda = 3.5$). With a larger market power, firm's gross gain from price adjustment is smaller, and this is represented by the flatter gross gain functions in panel (a). When other firms in the economy do not lower their prices in face of negative monetary shocks, firm i also has a small incentive to reduce its price. Firm i's own price adjustment can lower the aggregate price level because of its market power, which lessens the gain from lowering prices. As a result, the effect of strategic substitutability in pricing decision becomes smaller. We can see from panel (b) that firm i only adjusts in response to larger positive and negative monetary shocks: +8.26% and -9.92%, and its price remains sticky for a larger range of monetary shocks (from -8.74% to +8.26%). Market power tends to increase the price stickiness in the economy, and this finding is consistent with the result of John and Wolman (2008).

[28] Managerial economics uses Lerner Index, $L = 1/\lambda$, to measure a firm's market power. When $L = 0$, the market is competitive and the firm has no market power, while $L \to 1$ implies the firm has larger market power.

Figure 2.7: Responses to Monetary Shocks: Effects of Market Competitiveness



Dashed Line: gain when s = 1; Solid line: gain when s = 0.

Table 2.3: Market Power
($\phi = 1$, $\kappa = 1\%$ of steady-state revenue)

(a) $\lambda = 3.5$ (40% price markup:

M shocks	-1%	$+1\%$	-5%	$+5\%$	-10%	$+10\%$
$E(X)$	0.7107	0.7179	0.6964	0.7321	0.6836	0.7386
σ_X	0.0021	0.0021	0.0103	0.0103	0.0181	0.0191
$E(P)$	1.4	1.4	1.4	1.4	1.3873	1.4223
σ_P	1.27×10^{-14}	1.27×10^{-14}	1.13×10^{-13}	1.13×10^{-13}	0.0359	0.0486

(b) $\lambda = 51$ (2% price markup):

M shocks	-1%	$+1\%$	-5%	$+5\%$	-10%	$+10\%$
$E(X)$	0.9755	0.9853	0.9751	0.9827	0.9778	0.9816
σ_X	0.0028	0.0028	0.0053	0.0042	0.0046	0.0032
$E(P)$	1.02	1.02	0.9987	1.0431	0.9711	1.0698
σ_P	1.24×10^{-14}	1.24×10^{-14}	0.0184	0.0176	0.0322	0.0312

In a more competitive environment, the effect of strategic substitutability becomes larger. Panels (d) to (f) show the case when firm's markup is only 2 percent of its marginal cost. With negative money shocks, firm i has a large incentive to adjust when other firms do not adjust (the steep solid line in panel (d)). When firm i lowers its price, its demand increases by a large amount as goods are more substitutable among each other, and hence, firm i can increase profits by lowering its price. With a smaller market power, firm i has a larger incentive to adjust in response to money shocks, and the range of sticky price is narrower (from -1.72% to +1.53%). The firm finds that it is profitable to increase its price when the money shock is only +1.53%, and to lower its price when the shock is -3.44%. Therefore, prices are more flexible in the economy when the market is more competitive.

2.5 Concluding remarks

Business cycle asymmetries have been widely discussed in recent economic studies. Evidence suggests that the economy responds to monetary shocks asymmetrically. This raises the concern about the effectiveness of monetary policies when the economy is at different phrases of the business cycle.

This chapter develops a simple one-period state-dependent pricing model to study the strategic interactions in firms' pricing decisions, and we show that strategic complementarity and substitutability in pricing decision can explain the asymmetric responses of the economy to monetary shocks. We find that the effects of monetary shocks are asymmetric: firms have a larger incentive to increase their prices in response to positive money shocks, and have a smaller incentive to lower their prices with negative shocks of the same magnitude.

This simple model also allows us to examine the mixed strategies that a firm chooses in response to some range of negative shocks. We find that an individual firm's mixed strategies in its pricing decision depend on the magnitude of the shocks, as well as on the proportion of price-adjusting firms in the economy. In addition, when a firm's marginal cost has some real rigidity, the firm only chooses to adjust or remain sticky with certainty if the fixed adjustment cost is low.

We may extend the model to a dynamic setting and use it to see how well it matches the data. The mixed strategies analysis may become more complicated as the firm's future decisions must be taken into account. To match the observation that prices are revised upward more frequently but in smaller magnitude than they are revised downward, we also have to take the heterogeneous menu costs into consideration.

Chapter 3

A Simple Model of Optimal Monetary Policy with Financial Constraints

Recent experience suggests that the operation of monetary policy in emerging market economies is severely limited by the presence of financial constraints. This is seen in the tendency to follow contractionary monetary policy during crises, and from the observation that these countries pursue much more stable exchange rates than do high-income advanced economies, despite having a more volatile external environment. This chapter analyzes the use of monetary policy in an open economy where exchange-rate sensitive collateral constraints may bind in some states of the world. The appeal of the model is that it allows for a complete analytical description of the effects of collateral constraints, and admits a full characterization of welfare-maximizing monetary policy rules. The model can explain the two empirical features of emerging market monetary policy described above - in particular, that optimal monetary policy may be pro-cyclical under binding collateral constraints, and an economy with large external shocks may favor a fixed exchange rate, even though flexible exchange rates are preferred when external shocks are smaller.[29]

[29] This chapter is based on the joint work with Michael Devereux.

3.1 Introduction

In developed economies, monetary policy is generally counter-cyclical. For instance, the widespread consensus is that policy should be eased in a recession. By contrast, from the recent experience of emerging market economies, monetary policy has often been pro-cyclical, raising interest rates during a crisis, usually in order to defend the exchange rate. For example, after the Asian-Russian crisis of 1997-98, interest rates fell in the US, Australia, Canada, and most other developed economies, while they rose in almost all emerging market economies (Edwards, 2003). Related evidence from Calvo and Reinhart (2002) indicates that many emerging economies place a high weight on exchange rate stability, even in face of large macroeconomic shocks which in principle would call for exchange rate adjustment.[30]

Why do we see such a contrast between the policy responses of developed economies and that of emerging markets? One explanation is market confidence. Many of these economies have a history of bad policy, so that in a crisis, raising interest rates to restore the confidence of international capital markets is more important than attempting to stabilize the domestic economy. Nevertheless, many economists (e.g. Krugman,1998; and Stiglitz, 2002) have questioned this, arguing that tight monetary policies exacerbate the crisis rather than generating confidence.

An alternative explanation for the differences in policy is that emerging market economies are more financially vulnerable, and in the presence of a mismatch between domestic assets and foreign liabilities in the balance sheets, an exchange rate depreciation may be more of a hindrance than a help.

Empirical evidence supports the view that financial vulnerability is an important constraint on macroeconomic policy in emerging markets (Goldstein, Kaminsky, and Reinhart, 2000). These countries can almost never issue external debt in their own currency, and weak domestic financial institutions mean that the balance sheet effects are an important limitation on domestic production (Eichengreen and Hausmann, 2005).

A growing literature has developed models where the balance sheet constraints impinge upon the workings of monetary policy and exchange rates.[31] Many of these papers treat financial vulnerability as a collateral constraint that limit firm's investment or production financing. Due to foreign currency denominated debt, these collateral

[30] In face of external demand shocks, a fixed exchange rate is a pro-cyclical monetary policy rule.
[31] See references below.

constraints are likely to be sensitive to movements in the exchange rate.

This chapter develops a very simple model of monetary policy making in an open economy, in the presence of sometimes binding collateral constraints which are related to trade credit financing. IMF (2003) notes that many recent emerging market crises were characterized by a very large decline in trade financing, and that this may have played a substantial role in exacerbating the crises. Figure 3.1 shows the trade credit flow of selected emerging markets. Sharp falls in trade finance were observed in Korea in 1997-1998, and in Brazil and Thailand in both 1997-1998 and in 1999-2000.

The main contribution of this chapter is to construct an optimal, welfare-maximizing monetary policy rule which takes collateral constraints into account. We find that an optimal rule calls for a conventional monetary policy in normal times, for small shocks. In this case, the exchange rate acts as a shock absorber, and helps to stabilize the real economy in face of external shocks.

Still, in face of large negative shocks that cause collateral constraints to bind, the optimal rule requires a counter-cyclical policy response. This is because when collateral constraints bind, exchange rate adjustment may be de-stabilizing. We can therefore rationalize why monetary policy should be pro-cyclical during a crisis, within the context of a welfare-maximizing optimal monetary policy problem.[32] The key reason to tighten monetary policy in face of a negative shock is that this policy relaxes the collateral constraints facing the economy. In general, however, we find that monetary policy should not be so pro-cyclical as to actually undo the collateral constraints entirely.

The model can help to explain why some countries might prefer exchange rate stability, even in face of large external shocks, which in the absence of financial constraints, would require substantial movement in exchange rates. Somewhat paradoxically, it is precisely when shocks are large and financial constraints may be binding that exchange rate stability may be desirable. With smaller shocks, which allow for adjustment without hitting collateral constraints, a flexible exchange rate is better.

An important consideration in the comparison of exchange rate regimes is the stock of outstanding foreign currency debt. When the debt outstanding is high, in-

[32]In this simple model, a financial crisis occurs when the collateral constraint becomes binding, when the economy is hit by a negative external shock. According to IMF's World Economic Outlook (1998), external conditions play an important role in causing financial crises, especially in emerging markets. In this model, a large fall in foreign demand in high debt-to-net worth ratio economy causes financial crises since collateral constraint limits the borrowing ability of firms. This causes an adverse effect on the real economy, as discussed in details in the rest of this chapter.

creasing the chances that the collateral constraint will bind, a fixed exchange rate is more likely to dominate a (non-optimal) floating exchange rate rule. This accords closely with the empirical evidence in Devereux and Lane (2003).

A substantial number of papers have explored different aspects of monetary policy in the presence of financial constraints.[33] Aghion, Bacchetta and Banerjee (2000, 2001) present models that can analyze monetary policy in situations where collateral constraints bind. In Aghion, Bacchetta and Banerjee (2001), they present a simple *IS-LM* type graph, close in spirit to our approach below, and discuss alternative monetary policies. Cook (2004) shows that the negative balance sheet effect of a devaluation can be enough to cause a fall in output when investment borrowing is limited by domestic firms' net worth. Choi and Cook (2004) extend this model to allow for the role of banks, and show that a fixed exchange rate can enhance welfare by stabilizing the banks' balance sheets. Christiano, Gust and Roldos (2004) introduce collateral constraints in financing trade credit, as we do, and show that monetary policy may be contractionary with binding collateral constraints. Braggion, Christiano and Roldos (2007) compute an optimal interest rate rule as a response to a financial crisis. Benigno et al. (2008) derive an optimal stabilization policy in terms of distortionary taxes when the country faces credit constraints.[34]

Our model differs from these papers in the sense that we are able (due to the simple set-up of the model) to derive the ex-ante welfare-maximizing monetary policy with commitment, that is, taking account of how wages are set, and allowing for the fact that collateral constraints are only sometimes binding. Thus, our monetary policy rule is state-contingent, where the state of the world is determined by an external demand shock, but also contingent on whether or not collateral constraints bind. The monetary policy problem uses exactly the same approach as used in the recent literature on optimal monetary policy in open economy models with nominal rigidities (for example, Obstfeld and Rogoff, 2000). Our model, in fact, nests a standard open economy sticky-wage environment, when collateral constraints are absent, or never binding. Thus, the

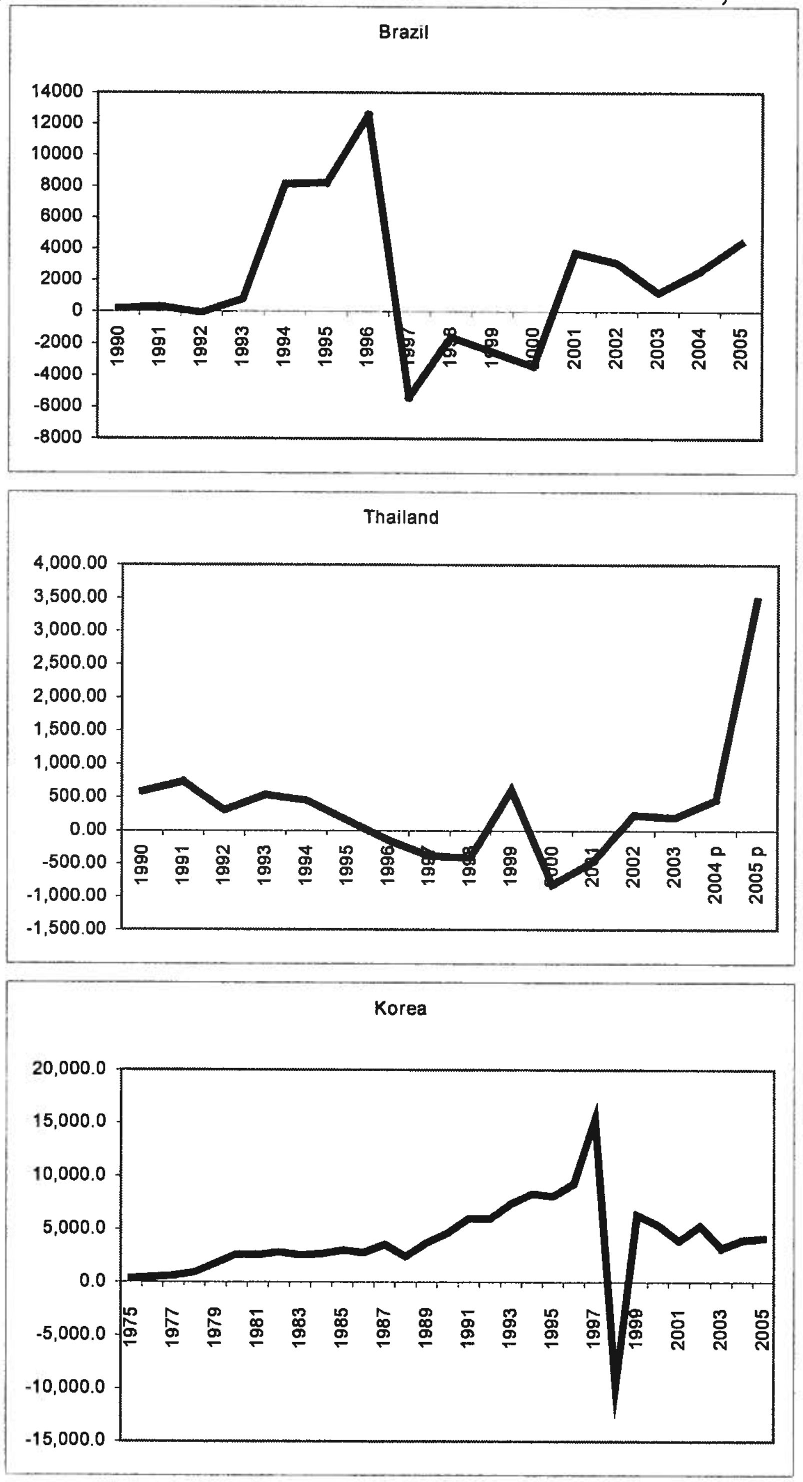
[33] See for instance, Aghion, Bacchetta and Banerjee (2000, 2001, 2004), Braggion, Christiano, and Roldos (2006), Burnside, Eichenbaum, and Rebelo (2001), Caballero and Krishnamurty (2001, 2005), Céspedes, Chang and Velasco (2003, 2004), Cook (2004), Choi and Cook (2004), Devereux and Lane (2003), Devereux, Lane and Xu (2006), Christiano, Gust and Roldos (2004), Mendoza (2002), and Mendoza and Smith (2002).

[34] Braggion, Christiano and Roldos (2007) and Benigno et al. (2008) study the optimal policy problem in a more complicated DSGE setting, while our model focuses on a simple non-dynamic policy issue.

analysis allows us to be precise about the nature and extent of differences in monetary policy stance between developed economies and financially vulnerable, emerging market countries.

This chapter is structured as follows. In Section 3.2, we present a simple open economy model with sometimes binding trade credit constraint. Section 3.3 provides a diagrammatic analysis on our model. We look at the optimal monetary policy under the case with and without credit constraint in Section 3.4, which also gives some numeral results. Section 3.5 concludes.

Figure 3.1: Trade Credits for Selected Countries, 1990-2005



Sources: Trade credits data are derived from countries' central banks' data.

Brazil: Banco Central Do Brasil; Series: Net short-term trade credits (in million US$)

Thailand: Bank of Thailand; Series: Net trade credit / Net flow of Private financial (in million US$)

Korea: Bank of Korea; Series: Net trade credits (in Business Sector Bill).

3.2 The model

Consider a one-period model of a small open economy with consumers and firms.[35] There is a continuum of households along the unit interval, consuming home and foreign produced goods, and providing heterogeneous labour services to final goods firms. Firms are competitive, and use both local labour and an intermediate imported good for production. Firms must finance their import purchases with trade credit extended by foreign exporters, which is repaid when they sell their output at the end of the period. However, as in Aghion et al. (2001), and Mendoza and Smith (2002), these importing firms may face collateral constraints related to their net worth. Their net worth comprises fixed domestic-currency denominated assets, less foreign currency denominated debt. When there is a large jump in the exchange rate, the collateral constraints may become binding, and firms then become rationed in their purchase of intermediates.

Households-workers set nominal wages in advance, before the realization of the state of the world.[36]

3.2.1 Firms

Final goods are produced using labour and an intermediate imported good. Labour is differentiated across households, so that households have market power in wage setting. We can define the aggregate labour composite as:

$$H = \left[\int_0^1 H(i)^{1-\frac{1}{\rho}} di \right]^{\frac{1}{1-\frac{1}{\rho}}} \tag{3.1}$$

[35] Since the constraint on external financing falls on imported raw materials, rather than investment, a one-period decision making structure is sufficient to map out the key elements of our model. A similar model is presented in Devereux and Lane (2003). In the case where investment finance is distorted by collateral constraints, as in Aghion et al.(2001), Mendoza and Smith (2002), or Céspedes et al. (2003), it would be necessary to construct an explicitly dynamic model. The virtue of the present analysis is that all features of the model be explicitly characterized in a simple analytical way, and we may also conduct a standard optimal monetary policy analysis. See IMF (2003) for evidence on the importance of trade financing disruptions in exacerbating the effects of crises in emerging markets.

[36] Sticky wage model predicts counter-cyclical real wages in response to a negative shock (that is, real wage increases and output falls, while leaving the nominal wages unchanged). There is evidence for pro-cyclical real wages from panel-data based estimation of real business cycle models (for instance, Bils (1985) using US data). However, recent studies show that the movement of real wage reacts differently to different shocks in the economy. Fleischman (1999) shows that real wage is counter-cyclical to labour supply shock and aggregate demand shocks in the US economy.

where $H(i)$ is employment of household i, and $\rho > 1$ is the elasticity of substitution between labour varieties.

The production function for final goods is given by:

$$Y = AH^{\omega}I^{1-\omega} \tag{3.2}$$

where A is a constant productivity term, and I represents the imported intermediate input.

Firms' profits are defined as:

$$\Pi = P_h Y - WH - Sq^* I \tag{3.3}$$

Firms maximize profits taking the nominal wage, W, and the foreign currency price of intermediate imports, q^*, as given. S is the nominal exchange rate.

In addition, firms are assumed to face a collateral constraint, related to net worth. The constraint is represented as

$$Sq^* I \leq N - SD^* \tag{3.4}$$

where N is the domestic currency denominated assets and D^* is the pre-existing foreign currency liabilities of the importers.[37] This collateral constraint may limit access to intermediate imported goods when the economy faces a large devaluation. In this sense, it captures the importance of *currency mismatch* between assets and liabilities, a phenomenon that has been emphasized by many commentators on emerging market crises (e.g. Eichengreen and Hausmann, 2005). The constraint is *not always binding.* We might think that in normal times, the collateral constraint holds with strict inequality, and firms can freely import intermediates at the world price. On the other hand, following a large devaluation, this constraint may be binding, and firms will be

[37]As in Mendoza and Smith (2002), we can motivate this collateral constraint by the difficulties of enforcing international contracts. It can be argued that the model would be more realistic when there are a separate group of intermediate importing firms who purchased intermediates from abroad, subject to trade-credit related collateral constraints, and sell intermediates to final goods firms. But in fact, the aggregate results in that case would be identical to those in this model, so to avoid excess notation, we simply assume that final goods firms are subject to these constraints. Finally, for all the analysis, we assume that parameters and initial conditions are such that $N - SD^* > 0$. Without this, GDP would always be zero.

constrained.[38]

Constraints not binding

When firms are unconstrained, assuming free entry, profit maximization problem gives the price of home-produced goods:

$$P_h = \kappa \frac{W^{\omega}(Sq^*)^{1-\omega}}{A} \tag{3.5}$$

where $\kappa = \left(\frac{1}{1-\omega}\right)^{1-\omega}\left(\frac{1}{\omega}\right)^{\omega}$.

Binding constraints

When the collateral constraint is binding, $Sq^*I = N - SD^*$, then we have $I = \frac{N-SD^*}{Sq^*}$. Firms choose employment to maximize profits, and we get the implicit labour demand function:

$$W(i) = \frac{\omega A H^{\omega} I^{1-\omega}}{H}\left(\frac{H(i)}{H}\right)^{-\frac{1}{\rho}} P_h. \tag{3.6}$$

In a symmetric equilibrium as described below, $H(i) = H$ and $W(i) = W$, where $W(i)$ is the nominal wage set by household i. We therefore get the optimal employment condition:

$$P_h \omega \frac{Y}{H} = W \tag{3.7}$$

and output:

$$Y = AH^{\omega}\left(\frac{N - SD^*}{Sq^*}\right)^{1-\omega} \tag{3.8}$$

3.2.2 Households

Household i, $i \in [0, 1]$, has preference given by:

$$\ln C(i) + \chi \ln\left(\frac{M(i)}{P}\right) - \eta \frac{H(i)^{1+\psi}}{1+\psi} \tag{3.9}$$

[38] We note that a fundamental simplification of our model is to take N and D^* as exogenous (although as discussed below, we assume that N is proportional to the expected money stock, so as to rule out the possibility that its real value can be altered by systematic monetary policy). In a dynamic model, we would need to track these stocks over time. But in this one period framework, we can take the collateral as given at the beginning of the period.

where $C(i)$ is a composite of the consumption of home and foreign goods, given by:

$$C(i) = C_h(i)^{\alpha} C_f(i)^{1-\alpha} \tag{3.10}$$

and P is the price index, given by $P = \left(\frac{P_h}{\alpha}\right)^{\alpha} \left(\frac{SP_f^*}{1-\alpha}\right)^{1-\alpha}$, where P_f^* is the foreign currency price of foreign goods. α represents the relative preference for home goods. $M(i)$ is the quantity of domestic money held, and ψ is the elasticity of labour supply. Households face the budget constraint:

$$PC(i) + M(i) = W(i)H(i) + M_0(i) + T(i) + \Pi \tag{3.11}$$

where $M_0(i)$ is initial money holdings, T is total transfer from the monetary authority, and Π is total profits of the final good firms.[39]

Households choose money balances and consumption of each good to maximize utility, subject to their budget constraint. We get the demand for each good, $C_h(i)$ and $C_f(i)$, and that of money balances:

$$C_h(i) = \frac{\alpha PC(i)}{P_h} \tag{3.12}$$

$$C_f(i) = \frac{(1-\alpha)PC(i)}{P_f} \tag{3.13}$$

$$M(i) = \chi PC(i) \tag{3.14}$$

We assume that nominal wages are pre-set ex-ante, and cannot adjust to shocks within the period. Each household i faces a downward-sloping labour demand curve with elasticity ρ, given in equation (3.6). The expected utility-maximizing wage is:

$$W(i) = \eta \frac{\rho}{\rho - 1} \frac{E\{H(i)^{1+\psi}\}}{E\left\{\frac{H(i)}{PC(i)}\right\}} \tag{3.15}$$

[39]The final good firms provide profits in forms of dividends to the households. We assume the firms use the net worth as collateral when purchasing intermediate goods from abroad,and we also assume that the firms do not return the net worth to the households. We may think of this as the firms set aside some assets only for the purchase of foreign intermediate goods. Thus, the net worth does not enter the households' budget constraint.

3.2.3 Equilibrium

We assume that foreign demand for the home good is unit elastic, and is given by:

$$X^d = \tilde{X}\frac{S}{P_h}, \tag{3.16}$$

where $\tilde{X}$ is an exogenous stochastic foreign demand shift term.

We focus on symmetric equilibria in the sense that: $C(i) = C$, $H(i) = H$, $W(i) = W$, $M(i) = M$, $M_0(i) = M_0$ and $T(i) = T$, $\forall i \in [0,1]$. Define a symmetric, imperfectly competitive equilibrium, given any monetary policy rule, as the set of allocations, $\Theta = \{C, H, M\}$ and the set of prices, $\wp = \{W^s, S, P_h\}$ given P_f^*, q^* such that:

1. Firms maximize profits;
2. The wage is set by households to maximize expected utility;
3. Households maximize their utility over consumption and real money balances subject to ex-post budget constraint;
4. The money market clears:
$$M = M_0 + T \tag{3.17}$$
5. The home goods market clears:
$$Y = \alpha\frac{PC}{P_h} + \tilde{X}\frac{S}{P_h}. \tag{3.18}$$

The equilibrium conditions must be characterized separately under the two regimes, depending upon whether or not the collateral constraint binds.

Equilibrium conditions without collateral constraints

When the collateral constraint is not binding, the equilibrium conditions are characterized as follows. Money market clearing and profit maximization imply:

$$PC = WH = P_hY - Sq^*I = P_hY - (1-\omega)P_hY = \omega P_hY \tag{3.19}$$

which implies:

$$M = \chi PC = \chi\omega P_h Y \tag{3.20}$$

The market clearing condition can be written as:

$$Y = \alpha\omega Y + \tilde{X}\frac{S}{P_h}. \tag{3.21}$$

Along with the optimal pricing equation,

$$P_h = \kappa\frac{W^{\omega}(Sq^*)^{1-\omega}}{A}, \tag{3.22}$$

equations (3.20) - (3.22) can be used to solve for $\{P_h, S, Y\}$, conditional on $\tilde{X}$ and the pre-set wage W. Equation (3.15) then determines the wage, given the distribution of employment, prices and consumption.

Equilibrium conditions with collateral constraints

When the collateral constraint is binding, the household's budget constraint becomes:

$$PC = P_h Y - N + SD^* \tag{3.23}$$

Then the money market equilibrium is given by:

$$M = \chi PC = \chi\left(P_h Y - N + SD^*\right). \tag{3.24}$$

The goods market clearing condition can then be written as:

$$Y = \alpha\left(Y - \frac{N - SD^*}{P_h}\right) + \tilde{X}\frac{S}{P_h}. \tag{3.25}$$

Together with the profit maximization condition and the production function:

$$P_h\omega\frac{Y}{H} = W \tag{3.26}$$

$$Y = AH^{\omega}\left(\frac{N - SD^*}{Sq^*}\right)^{1-\omega}, \tag{3.27}$$

equations (3.24) - (3.27) solve for the variables $\{P_h, H, S, Y\}$, conditional on $\tilde{X}$ and the nominal wage. Again, equation (3.15) determines the nominal wage.

3.2.4 The nature of the collateral constraint

What determines whether the collateral constraint binds? From the properties of the economy in the *unconstrained region*, we have:

$$Sq^*I = (1-\omega)P_hY = \frac{1-\omega}{\omega}\frac{M}{\chi} \tag{3.28}$$

Hence, total spending on intermediate imports in the unconstrained economy depends only on the domestic money supply. We can therefore write the collateral constraint as:

$$\frac{1-\omega}{\omega}\frac{M}{\chi} \leq N - SD^* \tag{3.29}$$

We can then define the cut-off exchange rate $\bar{S}$, at which the collateral constraint will just bind, as:

$$\bar{S} = \frac{1}{D^*}\left[N - \frac{1-\omega}{\omega}\frac{M}{\chi}\right] \tag{3.30}$$

When the nominal exchange rate is below $\bar{S}$ ($S < \bar{S}$), the constraint doesn't bind. When $S \geq \bar{S}$, however, firms are restricted by the collateral constraint.

In the analysis below, we will make the regularity assumption $\frac{1-\omega}{\omega}\frac{M}{\chi} < N$. This implies that the collateral constraint will not bind in an economy without foreign currency debt. Note that, since $N - SD^* > 0$, there always exists a monetary policy rule (a small enough M) for which the collateral constraint does not bind in any state of the world.

This highlights a particular property of the model. By following a contractionary monetary policy, the collateral constraint becomes less binding on two counts. First, M falls so that nominal demand for intermediate goods falls. But also, the fall in M will reduce S, so that $N - SD^*$ rises. Hence, the left hand side of (3.29) falls, and the right hand side rises, easy the collateral constraint on two counts. Equivalently, we could say that a contractionary monetary policy reduces S and raises the cut-off exchange rate $\bar{S}$.

3.3 A diagrammatic analysis

Given a fixed nominal wage, the behavior of the model under each regime can be illustrated in a very simple fashion.

3.3.1 Unconstrained regime

In the unconstrained regime, the economy behaves as a simple Mundell-Fleming type model. This is described by equations (3.20)-(3.22). Substituting from (3.22) into (3.20) and (3.21) gives the two equations:

$$M = \chi\omega\kappa\frac{W^{\omega}(Sq^{*})^{1-\omega}}{A}Y \tag{3.31}$$

$$Y = \frac{1}{1-\alpha\omega}A\tilde{X}\frac{S^{\omega}}{\kappa W^{\omega}(q^{*})^{1-\omega}}. \tag{3.32}$$

Equation (3.31) gives the money market clearing condition, while (3.32) gives the goods market clearing condition. The first equation describes a downward sloping schedule in S, Y space, while the second describes an upward sloping schedule. These are described as the LM and IS curves in Figure 3.2a, respectively. A fall in foreign demand, $\tilde{X}$, will shift the IS schedule back to the left, while a rise in the money supply shifts the LM schedule to the right.

3.3.2 Constrained regime

When the collateral constraint binds, the conditions analogous to (3.31) and (3.32) are different. From (3.24)-(3.27), we can combine profit maximizing of firms with the money market clearing condition to get:

$$M = \chi\left(W\omega A^{-\frac{1}{\omega}}Y^{\frac{1}{\omega}}\left[\frac{N-SD^{*}}{q^{*}S}\right]^{\frac{\omega-1}{\omega}} - (N-SD^{*})\right) \tag{3.33}$$

The equivalent condition for the goods market equilibrium condition can be derived as:

$$Y^{\frac{1}{\omega}} = \frac{1}{1-\alpha}\frac{[\tilde{X}S - \alpha(N - SD^*)]}{W\omega A^{-\frac{1}{\omega}}}\left[\frac{N - SD^*}{Sq^*}\right]^{\frac{1-\omega}{\omega}} \tag{3.34}$$

Equation (3.33) represents the money market clearing condition when the economy is in the collateral constrained region. As before, it is represented by a downward sloping relationship in S, Y space, illustrated as the constrained LM schedule in Figure 3.2b. An exchange rate depreciation first of all reduces nominal purchases of intermediate imports, and *ceteris paribus*, raises nominal income and the demand for money. But there is a secondary effect of a depreciation, coming from a rise in the home good price, which also raises the demand for money. In both cases, for a given money stock, output must fall to allow the money market to clear. In the region where the collateral constraint is just binding (point C in Figure 3.2b), the constrained LM schedule is always flatter than that in the unconstrained region.[40]

Equation (3.34) describes the goods market clearing relationship between output and the exchange rate. Again, there are two effects to take into account. First, an exchange rate depreciation directly increases demand for the home good, because it raises foreign demand, and increases home nominal income by a reduction in payments on intermediate imports (since $N - SD^*$ must fall). But the depreciation also raises the home goods price P_h, which reduces demand. The extent of the rise in the price of the home good depends on two features of the model; a), the ratio of foreign currency debt to net worth, $\frac{SD^*}{N-SD^*}$, which we define as the *leverage ratio*, and b) the share of intermediate imports in production, $1 - \omega$. The higher is the leverage ratio, and the smaller is ω, the more likely it is that an exchange rate depreciation has a negative impact on total demand, through equation (3.34). Unlike the unconstrained economy, it is possible that (3.34) implies a *negative* relationship between output and the exchange rate. The constrained IS schedule is illustrated in Figure 3.2b and 3.2c. If the constrained IS schedule is negatively sloped, it is always steeper than that of the constrained LM schedule, under a reasonable range of parameter values.[41] This

[40]We can show this by taking a log-linear approximation of (3.31) and (3.33) around the point where the collateral constraint is just binding. The slope of the unconstrained LM schedule at this point is $-\frac{1}{1-\omega}$, while the slope of the constrained LM schedule is $\frac{-\omega^2}{(1-\omega)(1+l(1+\omega))}$, where $l = \frac{SD^*}{N-SD^*}$ is the leverage ratio (see below).

[41]The condition can be derived by taking a log-linear approximation of equations (3.33) and (3.34). The slope of the negatively sloped IS is $\frac{1-\alpha}{(1-\alpha\omega)\omega+\alpha(1-\omega)\omega l-(1-\alpha)(1-\omega)(1+l)} < 0$, while the slope of LM

ensures that equilibria are locally unique, even in the presence of sometimes binding collateral constraints. Figure 3.2d shows the case when there are multiple equilibria locally.

We can use (3.31)-(3.32) and (3.33)-(3.34) and Figure 3.3 to discuss the implications of the model for the response to world demand shocks and the conduct of monetary policy.

Cushioning world demand shocks

The response of the economy to fluctuations in world demand $\tilde{X}$ depends critically upon whether the collateral constraint binds or not. The threshold between the unconstrained and the constrained regimes in Figure 3.2 is given by the cut-off exchange rate $\bar{S}$. When there is a very low value of foreign currency debt, D^*, $\bar{S}$ will be very high, and the constrained regime is less likely to be operative.

Take the case where the collateral constraint never binds (for instance, when there is a very low value of D^*, and thus a low leverage ratio, so $\bar{S}$ is very high), and equilibrium is characterized by the lower part of Figure 3.3a. Furthermore, assume that monetary policy targets nominal income, which is equivalent in this case to a fixed money supply. Then, after a fall in $\tilde{X}$, the exchange rate will depreciate, hence mitigating the fall in GDP.

On the other hand, if fluctuations in $\tilde{X}$ occur in the constrained region, so that equilibrium is characterized by the upper part of Figure 3.3a and 3.3b, then the characteristics of adjustment are substantially different. Assume that the leverage ratio is high enough so that the IS schedule is negatively sloped (Figure 3.3b). Then a fall in $\tilde{X}$ shifts the IS curve back to the left (from IS' to IS''). At a given exchange rate, output will fall (point $D_{s_{fixed}}$). Again, a monetary rule which keeps M constant ensures the LM curve does not shift, so the exchange rate will depreciate. In contrast to the unconstrained case, now the depreciation is *destabilizing* (point D''). GDP falls by more after the depreciation, and the fall in output would be mitigated by preventing the depreciation.[42]

This comparison illustrates a critical difference between the workings of flexible

is $-\frac{1}{(1-\omega)(1+l(1+\omega))}$. From these, we can show that IS is steeper than LM if $l(1-\omega)(2\alpha-1) > \alpha\omega - 1$.

[42] Fixed exchange rate is stabilizing in constrained region only if the IS is negatively sloped. When the leverage ratio is not high enough, less output fall under the flexible exchange rate than under the fixed exchange rate in the constrained region (see Figure 3.3a)

exchange rates in economies with and without financial frictions. Without frictions, the exchange rate acts as a shock absorber in the standard way, and in response to external shocks, a flexible exchange rate is stabilizing. But with severe balance-sheet related financial frictions, arising from large foreign-currency debt positions, the exchange rate no longer acts as a macro-economic shock absorber, and endogenous movements in the exchange rate may de-stabilize the economy.

Large versus small shocks

It is clear from Figure 3.3 that shocks can push the equilibrium from one region to another. A large negative shock to world demand can shift the IS curve back so that the economy moves from the unconstrained region to the constrained region. Since the cut-off exchange rate $\bar{S}$ is independent of $\tilde{X}$, this happens if the required nominal depreciation would entail an exchange rate greater than $\bar{S}$.

This has two implications. First, the economy may behave quite differently for large versus small shocks. In response to moderate macro-economic shocks, the fluctuation in the real economy may be quite modest and movements in exchange rates are stabilizing. But when shocks are large, exchange rate adjustment is de-stabilizing, and can produce much larger movements in real income (in the case of negative shocks).

Secondly, the model predicts a distinct non-linearity. Large positive world demand shocks have a smaller positive effect on output than the negative effect of large negative shocks. Again, this carries implications for monetary policy.

Monetary policy and fear of floating

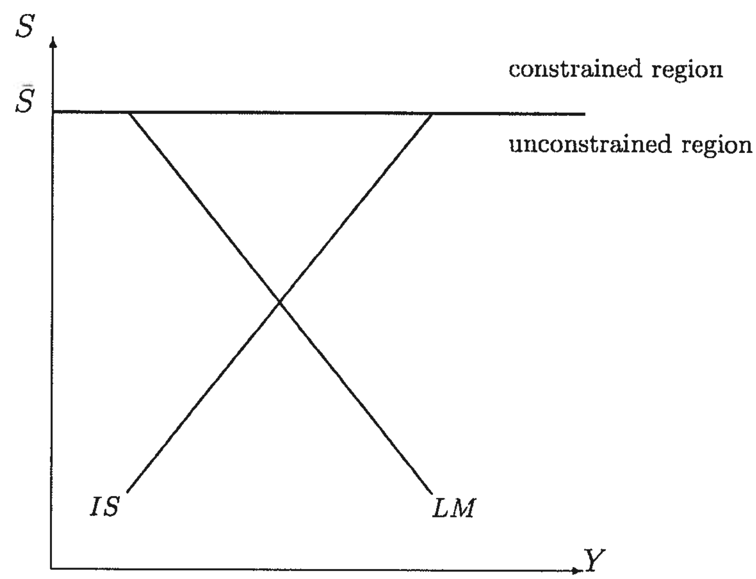
The discussion until now assumes that the country follows a constant money supply rule. But if the economy is in the constrained region, it is doubtful whether such a rule is desirable. In the next section, we compute the exact utility-maximizing monetary policy rule. Before doing this, however, we can make some immediate observations about the model's implications for monetary policy.

Calvo and Reinhart (2002) uncover the puzzling fact that some developing and emerging market economies seem to prevent their exchange rate from adjusting to shocks, even though they experience bigger shocks than the high income countries. This "fear of floating" seems clearly at variance with the standard Mundell-Fleming intuition on the value of exchange rate adjustment. But we can see that such an aversion to exchange rate adjustment with large shocks may be precisely what a model with sometimes-binding collateral constraints implies. If a monetary authority had to choose either a commitment to a stable exchange rate, or a fully flexible exchange rate, then in the presence of large shocks, and with a high foreign currency debt position, the fixed exchange rate may dominate a flexible exchange rate. Paradoxically, this is *less* likely to be the case when the country is subject to a lower volatility of external shocks. Thus, with financial frictions of the type described here, the relationship between the volatility of macro shocks and the benefits of flexible exchange rate becomes complicated. It is likely to be different for countries with large shocks and high foreign debt positions than for countries without these attributes.

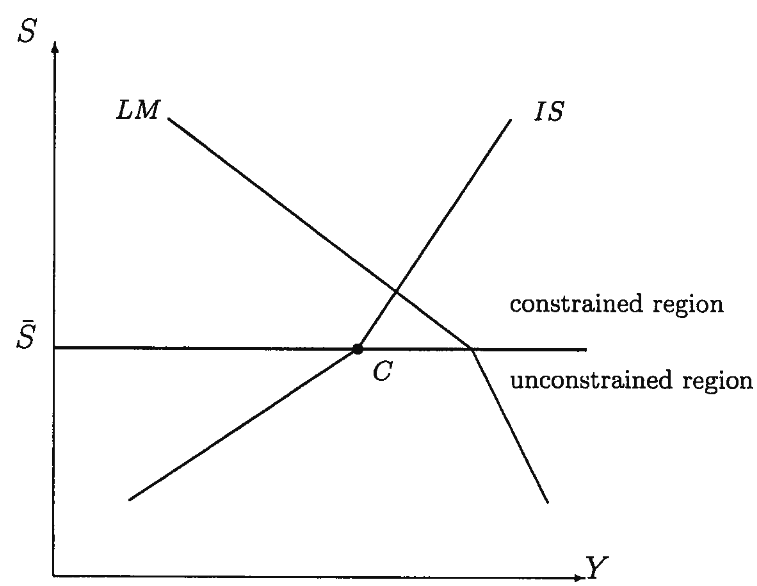
If a country wishes to use monetary policy to prevent exchange rate adjustment in the collateral constrained region, it can do so by shifting the LM curve *back* to the left. With a high leverage ratio, and when the economy is in the constrained region, monetary policy is *contractionary*, and GDP can be stabilized by a monetary policy tightening after a negative shock. Fixing the exchange rate at $\bar{S}$ prevents the economy from entering the constrained region, and leads to a lower output loss after a negative shock than allowing the exchange rate to float with a constant money supply.

But in order to undo the collateral constraint, the monetary authority does not need to keep the exchange rate fixed at $\bar{S}$. This is because the cut-off exchange rate itself depends upon the monetary policy rule (equation (3.30)). Take Figure 3.4, where the economy is at point D in the constrained region after a negative world demand shock. By reducing the money supply, the central bank can shift the LM curve to the left and raise GDP. At the same time, the reduction in the money supply raises $\bar{S}$, and reduces the area over which the collateral constraint binds. In this sense, a pro-cyclical monetary policy has two benefits when a negative shock pushes the economy into a financially constrained equilibrium - it both reduces the exchange rate and raises the threshold exchange rate at which the economy becomes financially vulnerable.

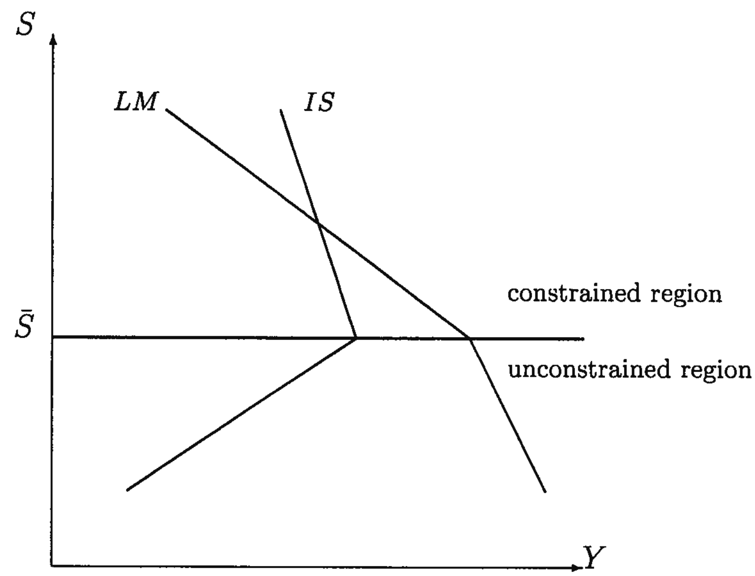
Figure 3.2: *IS* - *LM* Diagrams



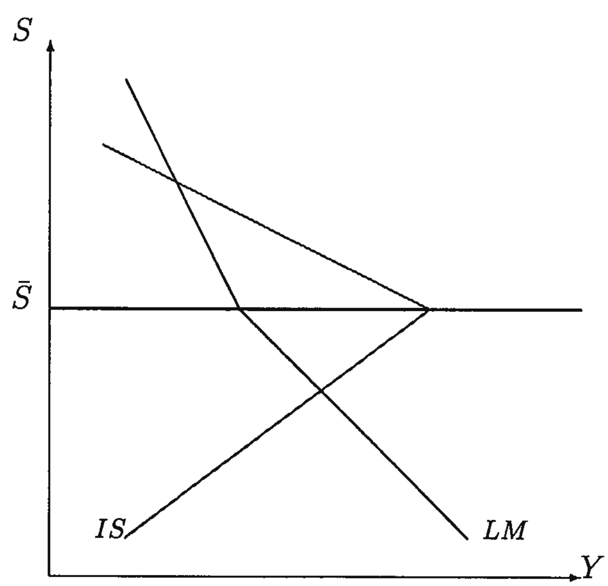
(a): Unconstrained Region



(b): Constrained Region – Positive *IS*

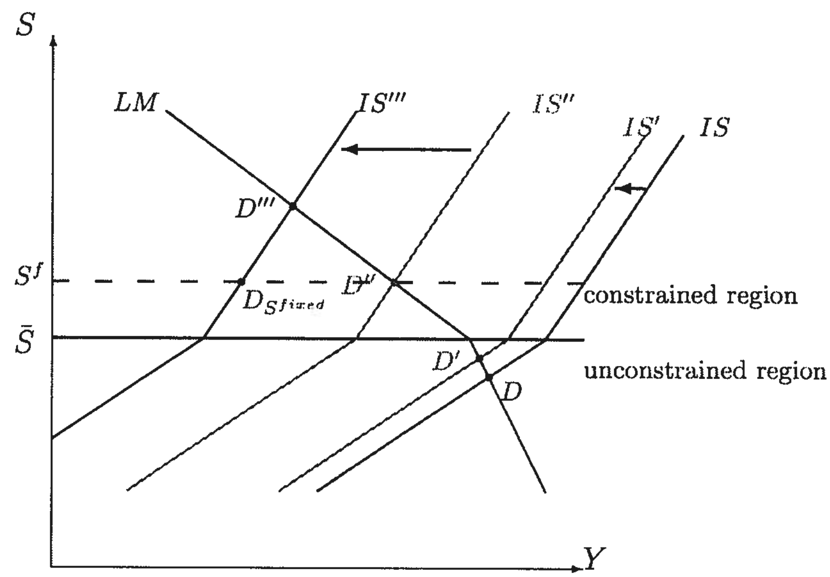


(c): Constrained Region – Negative *IS*

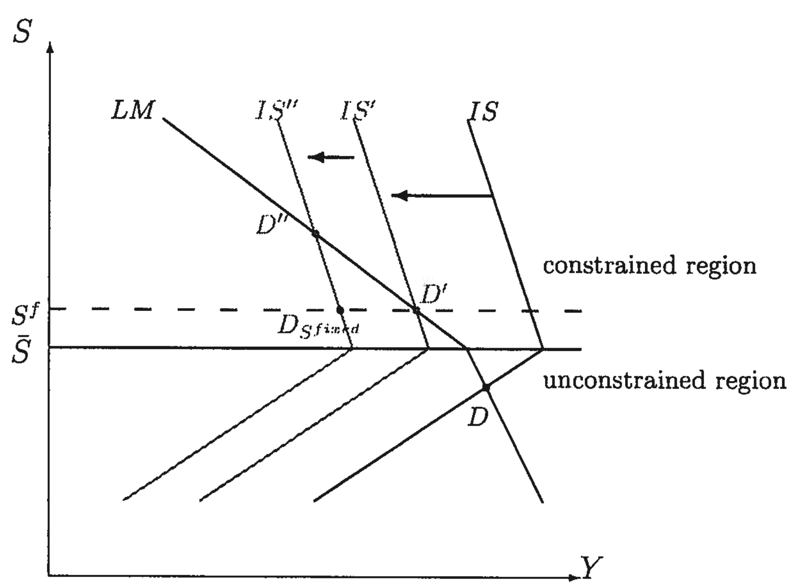


(d): Negative *IS* with Multiple Equilibria

Figure 3.3: A Fall in Export

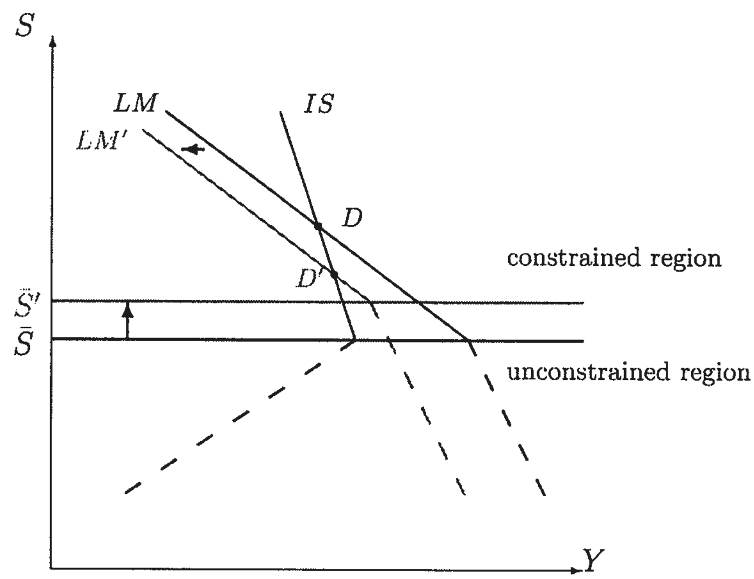


(a): **Positive** $\boldsymbol{IS}$; $\downarrow \tilde{X} \Rightarrow \downarrow Y, \uparrow S$.



(b): **Negative** $\boldsymbol{IS}$; $\downarrow \tilde{X} \Rightarrow \downarrow Y, \uparrow S$.

Figure 3.4: A Contractionary Monetary Policy



Constrained Region – Negative *IS*; $\downarrow M \Rightarrow \uparrow Y, \downarrow S$

3.4 Optimal monetary policy

What is the optimal monetary policy to follow in this economy? Here we define an optimal monetary policy with commitment as that which maximizes expected utility of the representative home agent, taking into account the way in which the wage is set (c.f. equation (3.15)). In the absence of collateral constraints, the optimal monetary policy follows as a simple application of the results of recent literature (for example Obstfeld and Rogoff, 2000), and is very easy to describe.

3.4.1 Optimal monetary policy without collateral constraints

When the collateral constraint does not bind, optimal monetary policy (with commitment) is the one that replicates the *flexible* wage equilibrium allocation.

Proposition 2 (Optimal Monetary Policy in the Unconstrained Economy).

When the collateral constraint does not bind, the optimal monetary policy is a fixed *level of the money stock, $M = \overline{M}$.*

Proof. See Appendix. $\square$

The intuition behind this proposition is in two parts. First, the model has the property that in the unconstrained region, the optimal pre-set wage is independent of the distribution of $\tilde{X}$. To see this note from the profit-maximizing employment condition, $WH = \omega P_h Y$, and also that $\omega P_h Y = \frac{M}{\chi}$. Then from (3.15), it follows that

$$W = \left[\eta \frac{\rho}{\rho - 1}\right]^{\frac{1}{1+\psi}} \frac{1}{\chi} \left[E\left\{M^{1+\psi}\right\}\right]^{\frac{1}{1+\psi}} \tag{3.35}$$

If the money stock is constant, then the pre-set nominal wage is the same as the wage that would obtain in the flexible wage economy. Hence a fixed money stock supports the flexible wage allocation.

The second part of the reasoning behind proposition 2 relates to the optimality of the flexible wage allocation. The flexible wage allocation is inefficient, due to a) monopoly wage-setting, and b) the economy has international market power over the sale of its good, as evidenced by the foreign demand schedule (3.16). A social planner would wish to exploit this market power. Nevertheless, in the economy without collateral constraints, these other inefficiencies dichotomize from the inefficiency due to nominal wage-setting. Monetary policy under commitment cannot systematically influence the equilibrium real wage, or the equilibrium terms of trade faced by the economy. Hence, the best that the monetary authority can do, under commitment, and without collateral constraints, is to achieve the flexible wage allocation. It does this by following a constant money supply (or equivalently, constant nominal income) rule.[43]

[43] Of course, it would follow the same rule even if an optimal package of taxes and subsidies sustained the fully efficient real wage and terms of trade.

3.4.2 Optimal monetary policy with sometimes binding constraints

When the collateral constraints may bind, the nature of monetary policy is substantially different. To explore this, we must frame the problem in an appropriate way.

There are two features of the collateral constrained economy that alter the monetary policy problem in an artificial sense. The first is that we have defined financial assets N in nominal terms. This introduces a nominal non-neutrality into the economy which would allow the monetary authority to systematically alter real magnitudes, even when monetary policy is chosen with commitment. To avoid this, we assume that financial assets are set proportional to the expected money supply. Hence, we assume that $N = \bar{N}E(M)$, where $\bar{N}$ is constant. This means that the monetary authority cannot systematically alter the real value of N.[44]

The second feature of the model that must be addressed is the presence of the multiple distortions discussed in the previous sub-section. The distortions due to monopoly wage-setting and optimal tariff considerations dichotomize from the monetary policy problem in the economy without binding collateral constraints. But this is not true in the economy where the constraints may occasionally bind. The reason is that average employment and output is in general influenced by the fraction of the total state space over which the constraint will bind, and this fraction is itself affected by the monetary policy rule. In order to avoid the convolution of the optimal monetary stabilization rule with these monopoly distortions, we assume that a set of optimal taxes and subsidies is chosen so that in the absence of nominal wage-setting, the equilibrium allocation in the unconstrained economy is socially optimal (first-best). This ensures that the monetary authority has no incentive to push average output down by forcing the economy to operate for more time in the constrained region, thereby raising the average terms of trade. In the appendix, the following result is shown:

Proposition 3. *The social planning optimal allocation of the unconstrained economy is supported by a) a tax on intermediate imports in the amount* $\tau_I = -1 + \frac{(1-\omega)}{(1-\alpha\omega)\phi_I}$,

[44] We could think N as being determined in an ex-ante contract between imported intermediate suppliers and final goods firms, where N can adjust to expected changes in the money supply, but is not state-contingent. More generally, in a dynamic model, N would be related to previous period earnings of firms.

and b) a tax on employment for firms in the amount $\tau_H = -1 + \frac{(\rho-1)}{\rho}\frac{1}{\alpha(1-\frac{1-\omega}{1+\tau_I})}$, *where*

$$\phi_I = \left[\frac{1}{2}\frac{\omega\,\alpha^2 - \alpha - 1 + \alpha\,\omega + \sqrt{\omega^2\,\alpha^4 + 2\,\omega\,\alpha^3 - 2\,\omega^2\,\alpha^3 - 3\,\alpha^2 + 2\,\alpha + 1 - 2\,\alpha\,\omega + \alpha^2\,\omega^2}}{\alpha\,(-1+\alpha\,\omega)}\right]$$

represents the socially optimal value of intermediate imports, as a fraction of the normalized foreign demand $\frac{X}{q^*}$.

Proof. See Appendix. □

The combination of a tax on employment and tax on purchases on intermediate imports ensures that the monopoly distortion in wage-setting is eliminated, and the optimal-tariff level of the terms of trade is attained. Note that with respect to employment, the tax may be positive or negative, depending on the strength of the monopoly distortion in wage-setting (which tends to reduce employment below the optimal level), and the optimal-tariff level of employment (which tends to reduce employment below the price-taking competitive level, in order to improve the terms of trade). The tax on intermediate imports is always positive however. Note that the results of proposition 2 hold whether or not these taxes-subsidies are in place.

Calibration

Let $\tilde{X}$ takes on three values, $\tilde{X}_1, \tilde{X}_2, \tilde{X}_3$. We think of $\tilde{X}_1$ and $\tilde{X}_2$ as high and low values of foreign demand, but giving variation in a range which does not lead to a binding collateral constraint. On the other hand, $\tilde{X}_3$ is a 'crash' state, in the sense that it represents a sharp fall in foreign demand. This will generally lead the collateral constraint to bind. Thus, a situation in which all shock variance is concentrated over $\tilde{X}_1$ and $\tilde{X}_2$ would be typical of a high income advanced economy, whereas realizations of $\tilde{X}_3$ would be characteristic of an emerging market economy.

Our model has only a small number of parameters that need to be calibrated. First, we set $\omega = 0.6$ so that the share of intermediate goods in production is 40 percent. This is consistent with the estimates given for intermediate imports as a fraction of GDP in Braggion et al.(2003) for Thailand. In other countries, the share of intermediate imports is higher, and would imply a lower value of ω. This would strengthen the case for a pro-cyclical monetary policy. Hence we see $\omega = 0.6$ as a conservative estimate. We set $\alpha = 0.5$, on the calculation that for Asian economies, imports are up to half of the GDP (e.g. Thailand), and half of these imports go to consumption goods. Since about half of the GDP is in the non-traded sector, which is absent from our model, it is appropriate to make consumption of imports equal to 50 percent of traded sector GDP. We assume that state 1 and state 2 occur with equal probability, equal to 0.475, and a 'crash' occurs with probability 0.05.[45] The values of the foreign demand $\tilde{X}$ are set so that, separately, $\tilde{X}_1$ and $\tilde{X}_2$ would generate a 2 percent standard deviation in GDP, for a fixed money stock. The 'crash' state is determined so that the GDP in this state falls by 10 percent, for a given money stock. This is roughly the fall in GDP seen in emerging market crises.

An empirical counterpart for the leverage ratio at the aggregate level can be seen as the ratio of short-term debt to usable foreign exchange rate reserves. Radelet and Sachs (1998) report estimates for this for emerging market countries prior to the Asian crisis. The estimates vary considerably across countries (see also Chang and Velasco, 2000). Many countries have leverage ratios exceeding unity. In the experiment below, we vary the leverage ratio between 0.25 and 3.

Optimal monetary rules

When the economy may move between the constrained and unconstrained regions, depending on external shocks and the monetary rules followed, the optimal monetary policy rule must be derived numerically. Assume a discrete distribution of $\tilde{X}$. Then, let monetary policy be represented as a state-contingent response vector $M_i = M(\tilde{X}_i)$, where we solve for the values of M_i that maximize the expected utility of the home representative individual. Since the response of the economy to any systematic monetary rule is neutral, we normalize so that $M(\tilde{X}_1) = 1$, where $\tilde{X}_1$ is the highest value of the foreign demand. In an economy without binding collateral constraints, Proposition 2 would then ensure that $M_i = 1$ obtains for all i. The influence of collateral constraints is then seen to the extent that $M_i \neq 1$ for some i.

Table 3.1 reports the distribution of output, consumption, employment, and the exchange rate for the model, under a fixed money rule, a fixed exchange rate rule, and the optimal monetary policy rule, for a range of values for the leverage ratio. In

[45] Typically, the empirical models of crisis probabilities in emerging markets have predicted very low crisis probabilities, in the range 5-10 percent, even as the economies get very close to crises. See Berg and Patillo (1999).

addition, the table reports the values of expected utility and the optimal monetary rule in each case. The appendix describes the solution for the optimal rule in more detail.

For a low leverage ratio, i.e. $l = 0.25$, the collateral constraint does not bind in the 'crash' state under the fixed money rule (which is equivalent to a nominal income target). In this case, this is the optimal rule. Most of the time, output will fluctuate between state 1 and 2, with a low variance. In the rare state 3, output will fall substantially. But since the collateral constraint never binds, the exchange rate acts as a 'shock absorber' for all states. The fixed money rule clearly welfare-dominates a fixed exchange rule.

As the leverage ratio rises however, the optimal rule prescribes a monetary contraction in the low state of the world. Now the exchange rate adjusts freely only when output fluctuates between states 1 and 2, but in the crash state, the monetary rule limits the exchange rate depreciation. For a leverage ratio of 0.5, the optimal rule requires a 1 percent money contraction. This contraction slightly reduces output, since, in terms of Figure 3.2, the IS curve is still upward sloping for a leverage ratio of 0.5. A monetary contraction is desirable nonetheless, because by limiting the exchange rate depreciation, it relaxes the collateral constraint. This allows for an increase in the quantity of intermediate imports in production. In this way, the monetary rule is designed to undo the effects of the trade credit financing constraints that hit the collateral constrained economy in a crisis.[46]

As the leverage ratio rises even more, there is a larger fall in output in the 'crash' state. Then the optimal monetary rule calls for an even greater monetary contraction in this state. Hence, we find that the greater is the output loss in the crash, the *more pro-cyclical should be the monetary response.* In this case, the monetary contraction is actually expansionary. When the leverage ratio is above 0.6 (for this calibration), the IS curve is negatively sloped in the constrained region, and the monetary contraction actually raises output in the crash state. Although the contraction still reduces employment, the rise in intermediate imports more than offsets this, so that GDP rises.

The optimal monetary policy in this model can therefore rationalize the observation that emerging economies tend to follow pro-cyclical monetary policies. In our model, they do so not to shore up credibility - we are focusing solely on optimal mone-

[46]In utility terms, the monetary contraction has conflicting effects. It directly reduces consumption, but also reduces employment. The gain in utility comes about because, due to the rise in intermediate imports, the utility cost of the fall in consumption is offset by the benefit of a fall in employment.

tary policy with commitment, so that full credibility is assumed. Rather, the monetary contraction in a crisis is an optimal response to the financing constraint - by limiting the size of the nominal exchange rate depreciation, it relaxes the constraint. If the collateral constraint is severe enough, the monetary contraction will in fact raise output, but this is not a pre-requisite for a pro-cyclical monetary rule. With a less severe collateral constraint, a monetary contraction is still optimal, even though it reduces output.

The optimal monetary policy in the crash state trades off the benefits of relaxing the financing constraint against the costs of the monetary contraction - in all cases the contraction reduces consumption. It is always feasible for the monetary contraction to be great enough so as to eliminate this constraint, however, we find that this is not optimal. For our calibration (and all other experiments we conducted) an optimal rule will not be so great as to entirely undo the collateral constraint.

Hard peg versus flexible exchange rates

A general property of the optimal monetary rule is that as long as the collateral constraint binds, there is less exchange rate depreciation than would occur under a fixed money stock rule. But note that the optimal monetary rule does not fix the exchange rate. In the unconstrained region, it allows the exchange rate to adjust freely to shocks, and narrows the range of exchange rate movements in the bad state.

What if this optimal monetary response is infeasible? Is it possible that the monetary policy maker would prefer to operate under a fully fixed exchange rate(for instance, a currency board, or dollarized system), rather than a nominal income targeting rule? This question underlies the debate between the argument for emerging economies to move towards "inflation targeting" regimes, allowing the exchange rate to float freely,[47] or alternatively adopt a 'hard peg'. In our context, there is a trade-off between the benefits of exchange rate adjustment in the unconstrained economy, and the costs of exchange rate depreciation in the constrained economy. While the optimal monetary rule exploits this trade-off in the best way possible, in light of this recent debate, a relevant comparison is between the nominal income target and a fixed exchange

[47] A constant nominal income rule followed in the unconstrained model above is the exact theoretical counterpart of the argument for inflation targeting in dynamic sticky-price models such as those of Woodford (2003). In both cases (absent cost-push shocks), the rule sustains the flexible price/wage allocation.

rate.

In Figure 3.5 we illustrate expected utility under each regime, as a function of the leverage ratio. Note that under our calibration, a crash state occurs only with 5 percent probability. For relatively low leverage ratios, this ensures that a nominal income target will dominate a fully fixed exchange rate. But as the leverage ratio rises above 2.2, the expected utility cost of exchange rate depreciation in a crash (even though it occurs with low probability) offsets the benefits of exchange rate adjustment in the high states, and a fixed exchange rate actually dominates.

The empirical evidence in Devereux and Lane (2003) closely accords with this theoretical finding. They find that countries with a larger stock of portfolio liabilities against a given trading partner tend to minimize bilateral exchange rate volatility against that partner. Thus, for emerging market countries with weak financial constraints, the net debt position must be taken into account, as well as the standard optimal currency area factors, in the assessment of optimal exchange rate policy.

This gives a precise sense in which an emerging market economy may display a "fear of floating", in the Calvo and Reinhart (2002) terminology. An observer may feel that a fixed exchange rate is inefficient, by preventing the economy from adjusting to small shocks, which occur frequently. But in fact there is a "peso problem" in making judgements based on small samples. In reality the authorities may be choosing a fixed exchange rate optimally (if the choice is only between fixed and floating), with knowledge of the consequences of a very costly, but low probability crash event.

An alternative perspective on the fear of floating may be seen in the different magnitude of shocks. Table 3.2 shows the comparison of fixed and flexible exchange rates between two economies with equal leverage ratios (l=1.5). In the first comparison, the crash state is associated with a 12 percent fall in output for a fixed money stock rule, while in the second, the fall in output is 30 percent. For this example, the second economy displays a larger "fear of floating", in the sense that it would choose a fixed exchange rate over a free float, despite the fact that it is subjected to a more volatile external environment. The key driving force is that the consequences of exchange rate depreciation in the crash state is more damaging for the more volatile economy.

Flexible wage equilibrium

Up until now, all the analyses are based on the assumption that wages are preset by households and are sticky within the period. When wages are flexible, the prediction of this model will be different. When wages are sticky, the IS shifts to the left and LM remains unaffected when the economy is hit by a negative foreign demand shock. However, when wages are flexible, wages will fall in response to a negative external shock to clear the goods and the money market. This fall in wage shifts the IS back to the right, and shifts the LM to the right as well. Output will fall less than the case under sticky wages, and exchange rate depreciates less as well. Flexible nominal wages help to stabilize the economy from an external shock, and hence, the economy is less likely to move to the constrained region after a shock.

3.5 Concluding remarks

We have developed a very simple macroeconomic model of optimal monetary policy in an emerging market economy, where the economy is subject to occasionally binding collateral constraints. The model can rationalize why it makes sense for monetary policy to be tight during a financial crisis, because the channels of monetary policy are very different when collateral constraints bind in a crisis than in normal times, without financial constraints. The model can also help to explain a predominance of the 'fear of floating' in emerging market economies, and the fact that empirically, this phenomenon seems to be greater among more heavily indebted countries, and also that paradoxically, this tendency is greater among countries that are experiencing relatively larger external shocks.

In developing a very simple framework which is analogous to the monetary policy analysis carried out in the recent literature on general equilibrium sticky price models, we see our model as a complement rather than as a substitute for the extensive literature on financial constraints in emerging market economies. An obvious drawback of our model is that it says nothing about the implications of capital flows for monetary policy. Nevertheless, it has an advantage in being specific about the design of optimal monetary policy that takes account of financial vulnerabilities. We leave for future work an extension of our approach to deal with capital flows.

Table 3.1: Distribution of Variables (at optimal M)

(a) $l = 0.25$

	Fixed M	Fixed ER	Optimal M
Max EU	-0.8318	-0.8321	-0.8318
Optimal M	–	–	[1, 1, 1]
Y	[0.2573, 0.2521, 0.2353]	[0.2627, 0.2495, 0.2101]	[0.2573, 0.2521, 0.2353]
$E(Y)$	0.2537	0.2538	0.2537
σ_Y	0.0133	0.0317	0.0133
C	[0.6024, 0.5811, 0.5152]	[0.6086, 0.5782, 0.4869]	[0.6024, 0.5811, 0.5152]
$E(C)$	0.5879	0.5881	0.5879
σ_C	0.0526	0.0733	0.0526
H	[0.3000, 0.3000, 0.3000]	[0.3105, 0.2950, 0.2484]	[0.3000, 0.3000, 0.3000]
$E(H)$	0.3000	0.3000	0.3000
σ_H	0	0.0374	0
S	[0.6285, 0.6615, 0.7856]	[0.6000, 0.6000, 0.6000]	[0.6285, 0.6615, 0.7856]
$E(S)$	0.6520	0.6000	0.6520
σ_S	0.0961	0	0.0961

($\tilde{X} = [1, 0.95, 0.8]$, $\pi = [0.475, 0.475, 0.05]$)

(b) $l = 0.5$

	Fixed M	Fixed ER	Optimal M
Max EU	-0.8318	-0.8321	-0.8318
Optimal M	–	–	[1, 1, 0.99]
Y	[0.2573, 0.2521, 0.2327]	[0.2627, 0.2495, 0.2101]	[0.2574, 0.2522, 0.2327]
$E(Y)$	0.2536	0.2538	0.2537
σ_Y	0.0150	0.0317	0.0151
C	[0.6024, 0.5811, 0.5147]	[0.6086, 0.5782, 0.4869]	[0.6024, 0.5812, 0.5135]
$E(C)$	0.5879	0.5881	0.5879
σ_C	0.0530	0.0733	0.0538
H	[0.3000, 0.3000, 0.2990]	[0.3105, 0.2950, 0.2484]	[0.3001, 0.3001, 0.2967]
$E(H)$	0.3000	0.3000	0.3000
σ_H	0.0007	0.0374	0.0024
S	[0.6285, 0.6615, 0.7810]	[0.6000, 0.6000, 0.6000]	[0.6285, 0.6615, 0.7754]
$E(S)$	0.6518	0.6000	0.6515
σ_S	0.0931	0	0.0894

($\tilde{X} = [1, 0.95, 0.8]$, $\pi = [0.475, 0.475, 0.05]$)

(c) $l = 1$

	Fixed M	Fixed ER	Optimal M
Max EU	-0.8318	-0.8321	-0.8318
Optimal M	–	–	[1, 1, 0.96]
Y	[0.2573, 0.2521, 0.2246]	[0.2627, 0.2495, 0.2101]	[0.2576, 0.2524, 0.2257]
$E(Y)$	0.2532	0.2538	0.2535
σ_Y	0.0205	0.0317	0.0199
C	[0.6024, 0.5811, 0.5126]	[0.6086, 0.5782, 0.4869]	[0.6027, 0.5815, 0.5084]
$E(C)$	0.5878	0.5881	0.5879
σ_C	0.0543	0.0733	0.0574
H	[0.3000, 0.3000, 0.2961]	[0.3105, 0.2950, 0.2484]	[0.3006, 0.3006, 0.2870]
$E(H)$	0.2998	0.3000	0.2999
σ_H	0.0026	0.0374	0.0091
S	[0.6285, 0.6615, 0.7673]	[0.6000, 0.6000, 0.6000]	[0.6285, 0.6615, 0.7470]
$E(S)$	0.6511	0.6000	0.6501
σ_S	0.0840	0	0.0707

($\tilde{X} = [1, 0.95, 0.8]$, $\pi = [0.475, 0.475, 0.05]$)

(d) $l = 1.5$

	Fixed M	Fixed ER	Optimal M
Max EU	-0.8319	-0.8321	-0.8318
Optimal M	–	–	[1, 1, 0.93]
Y	[0.2573, 0.2521, 0.2176]	[0.2627, 0.2495, 0.2101]	[0.2579, 0.2526, 0.2218]
$E(Y)$	0.2528	0.2538	0.2536
σ_Y	0.0251	0.0317	0.0227
C	[0.6024, 0.5811, 0.5103]	[0.6086, 0.5782, 0.4869]	[0.6030, 0.5817, 0.5038]
$E(C)$	0.5877	0.5881	0.5879
σ_C	0.0558	0.0733	0.0606
H	[0.3000, 0.3000, 0.2937]	[0.3105, 0.2950, 0.2484]	[0.3010, 0.3010, 0.2785]
$E(H)$	0.2997	0.3000	0.2999
σ_H	0.0042	0.0374	0.0152
S	[0.6285, 0.6615, 0.7561]	[0.6000, 0.6000, 0.6000]	[0.6285, 0.6615, 0.7238]
$E(S)$	0.6506	0.6000	0.6489
σ_S	0.0767	0	0.0556

($\tilde{X} = [1, 0.95, 0.8]$, $\pi = [0.475, 0.475, 0.05]$)

(e) $l = 3$

	Fixed M	Fixed ER	Optimal M
Max EU	-0.8324	-0.8321	-0.8319
Optimal M	–	–	[1, 1, 0.89]
Y	[0.2573, 0.2521, 0.2015]	[0.2627, 0.2495, 0.2101]	[0.2582, 0.2529, 0.2160]
$E(Y)$	0.2520	0.2538	0.2536
σ_Y	0.0359	0.0317	0.0267
C	[0.6024, 0.5811, 0.5033]	[0.6086, 0.5782, 0.4869]	[0.6033, 0.5821, 0.4975]
$E(C)$	0.5873	0.5881	0.5879
σ_C	0.0605	0.0733	0.0650
H	[0.3000, 0.3000, 0.2887]	[0.3105, 0.2950, 0.2484]	[0.3017, 0.3017, 0.2670]
$E(H)$	0.2994	0.3000	0.2999
σ_H	0.0076	0.0374	0.0233
S	[0.6285, 0.6615, 0.7326]	[0.6000, 0.6000, 0.6000]	[0.6285, 0.6615, 0.6923]
$E(S)$	0.6494	0.6000	0.6474
σ_S	0.0613	0	0.0359

($\tilde{X} = [1, 0.95, 0.8]$, $\pi = [0.475, 0.475, 0.05]$)

Figure 3.5: Maximum Expected Utility, Different Policy Rules

$(\tilde{X} = [1, 0.95, 0.8];\ \pi = [0.475, 0.475, 0.05])$

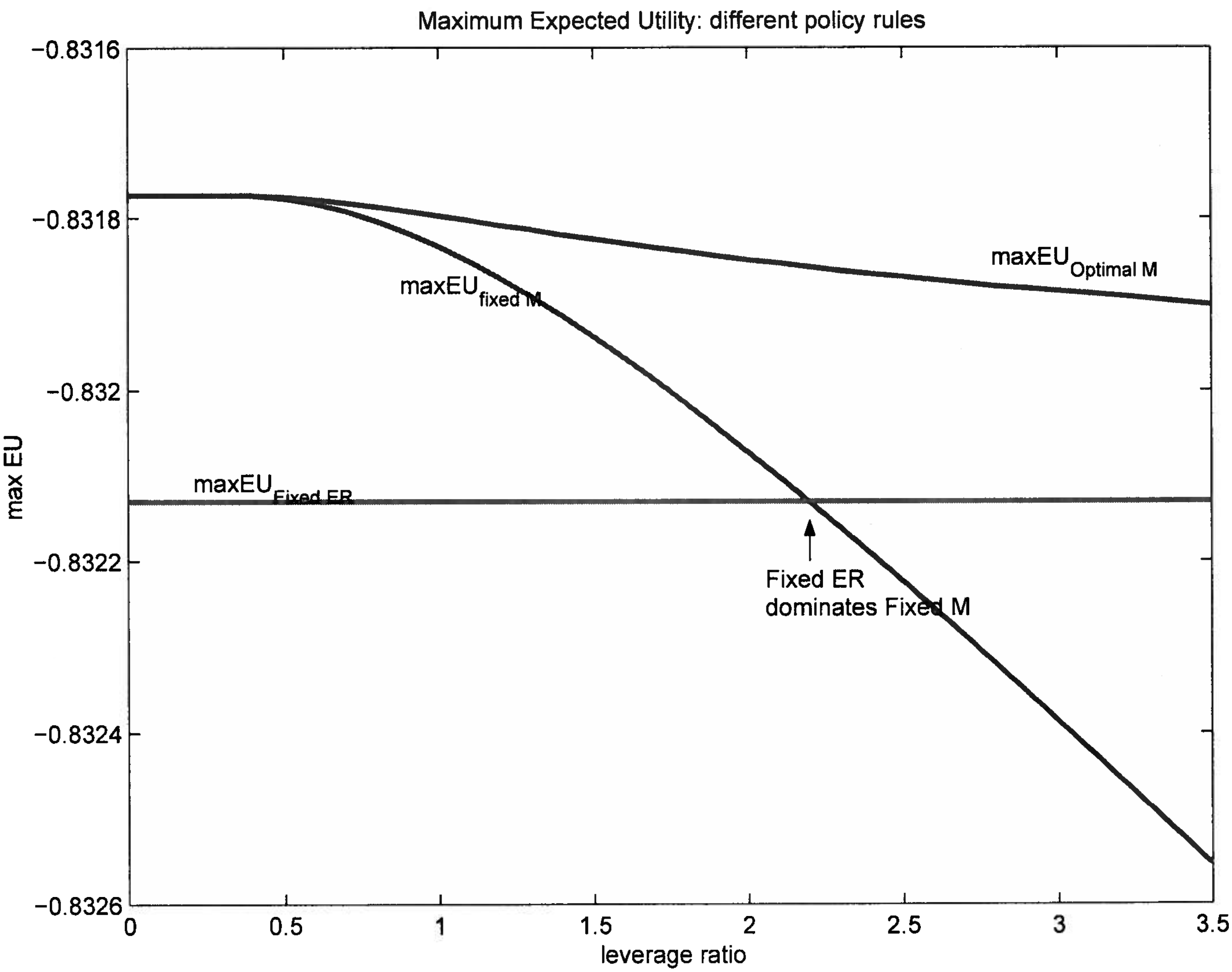


Table 3.2: Different Magnitudes of Shocks, $l = 1.5$

$X = [1, 0.95, 0.82]$			
	Fixed M	Fixed ER	Optimal M
Max EU	-0.831	-0.8312	-0.8309
Optimal M	–	–	[1, 1, 0.94]
Y	[0.2573, 0.2521, 0.2230]	[0.2625, 0.2494, 0.2153]	[0.2578, 0.2525, 0.2264]
$E(Y)$	0.2531	0.2539	0.2537
σ_Y	0.0215	0.0282	0.0196
C	[0.6024, 0.5811, 0.5204]	[0.6084, 0.5780, 0.4989]	[0.6029, 0.5816, 0.5144]
$E(C)$	0.5882	0.5885	0.5884
σ_C	0.0492	0.0653	0.0535
H	[0.3000, 0.3000, 0.2948]	[0.3102, 0.2946, 0.2543]	[0.3009, 0.3009, 0.2818]
$E(H)$	0.2997	0.3000	0.2999
σ_H	0.0035	0.0333	0.0129
S	[0.6285, 0.6615, 0.7427]	[0.6000, 0.6000, 0.6000]	[0.6285, 0.6615, 0.7154]
$E(S)$	0.6499	0.6000	0.6485
σ_S	0.0678	0	0.0502

$X = [1, 0.95, 0.67]$			
	Fixed M	Fixed ER	Optimal M
Max EU	-0.8391	-0.8390	-0.8382
Optimal M	–	–	[1, 1, 0.84]
Y	[0.2573, 0.2521, 0.1773]	[0.2637, 0.2506, 0.1767]	[0.2585, 0.2533, 0.1913]
$E(Y)$	0.2508	0.2531	0.2527
σ_Y	0.0522	0.0546	0.0436
C	[0.6024, 0.5811, 0.4388]	[0.6098, 0.5793, 0.4086]	[0.6038, 0.5825, 0.4313]
$E(C)$	0.5841	0.5853	0.5851
σ_C	0.1036	0.1262	0.1095
H	[0.3000, 0.3000, 0.2856]	[0.3126, 0.2970, 0.2094]	[0.3024, 0.3024, 0.2514]
$E(H)$	0.2993	0.3000	0.2998
σ_H	0.0097	0.0647	0.0343
S	[0.6285, 0.6615, 0.8573]	[0.6000, 0.6000, 0.6000]	[0.6285, 0.6615, 0.7735]
$E(S)$	0.6556	0.6000	0.6514
σ_S	0.1440	0	0.0881

Chapter 4

Transfer Problem and Exchange Rate Regimes

Different forms of shifts in wealth between countries have brought the transfer problem back into focus in international macroeconomics in recent years. In this chapter, we study the choice of exchange rate regimes when a transfer takes place from home to foreign country using a two-country DSGE model. Our results support the "orthodox" view of Keynes (1929) that a transfer leads to terms of trade deterioration in the donor country, regardless of the choice of exchange rate regimes. Historical evidence shows that donor countries usually chose to abandon the specie-standard after some negative shocks, so that the countries had more flexible monetary policies to alleviate the transfer effects. Our model also implies flexible exchange rate can help to reduce the negative effects on consumption in the donor country. In contrast, however, the welfare analysis suggests that abandoning the gold standard and going to a floating exchange rate cannot make the donor country better off. This result holds even if nominal rigidities are present.[48]

4.1 Introduction

The economic effects of an international payment between countries, which is referred to as the "transfer problem" in international economics, have been studied extensively since the debate between Keynes (1929) and Ohlin (1929) in the 1920s. Transfer is a shift in wealth, which is a shift in purchasing power, between countries within a reasonably short period of time, and it exists in different forms. In economic history, the transfer between countries usually occurred in terms of a reparation payment after

[48]This chapter is based on the joint work with Michael Devereux.

a war from the defeated party to the defeater. For instance: the Franco-Prussian war payment of 1871 - 1873; the reparation payment by Germany to the Allies after WWII; and the Gulf War reparation in the 1990s. Economic aid given to countries in need, in the event of a humanitarian crisis, or aid given to assist the development of a less developed country is also viewed as a transfer. In recent years, the rising prices of resources in the global economy also give rise to a concern about transfers of income and wealth from the oil-importing countries to the oil-producing countries. In addition, global trade imbalances in which incomes are transferred from the trade-deficit countries (such as the US) to the trade-surplus countries (for instance, China) have led to studies on the current account adjustments.[49] All of these issues have brought the transfer problem back into focus in international macroeconomics.

A vast amount of literature has been written about the transfer problem in the past century. The transfer problem is originally studied in real models, which focused on the effects on terms of trade. Keynes (1929) and Ohlin (1929) study the German reparations after WWI, and focus on the impacts of transfers on terms of trade and real exchange rates. Keynes (1929) argues that a transfer leads to terms of trade and real exchange rate deterioration in the donor country, which leaves the donor with an extra burden. Ohlin (1929), however, criticizes Keynes' analysis, and argues that this "orthodox" view may not hold, since income effects can make the terms of trade adjustments redundant. The controversy surrounding the transfer problem is later discussed based on the marginal propensities to import and save out of the transfer payments and receipts (see Johnson, 1955 and Samuelson, 1952).[50] Other studies focus on taste differences (Jones, 1970), and the existence of trade impediments and market distortions (Bhagwati et al., 1983 and Bezmen, 2006).

All of these previous models on the transfer problem are generally static.[51] Relatively few works have been done using a dynamic, general-equilibrium macroeconomics model to address the transfer problem. In this paper, the model is similar to that

[49] For example, see Mckinnon (2007), Corsetti, Martin and Pesenti (2008).

[50] In general, based on the full multiplier analysis in a world without trade impediments, three general outcomes are possible for the terms of trade in response to a transfer: (1) the terms of trade will not be affected by a negative transfer if the relevant sum of marginal propensities sum to unity; (2) if the sum of marginal propensities is less than one, terms of trade will deteriorate and the donor country will suffer a secondary burden; (3) when the sum of relevant marginal propensities to import and save exceeds unity, terms of trade will improve at the transfer paying countries and it may enjoy gains from the transfer (Morrison, 1992).

[51] See Chipman (1974), and Brakman and van Marrewijk (1998) for literature reviews on the transfer problem.

of Devereux and Smith (2007), in which they use a real, DSGE model to study the quantitative, macroeconomic transfer effects of the Franco-Prussian War indemnity of 1871-1873, and their model explains the historical paths of French net exports and the terms of trade. Our model differs from theirs, however, in that the transfer effect is studied in a two-country world with money and nominal rigidities. We focus on the effects of a transfer on consumption, output, terms of trade, and welfare, under different exchange rate regimes. We look at the transfer effects under both the fixed exchange rate and the flexible exchange rate regimes. Specifically, the fixed exchange rate is modeled as a gold standard as well as a hard peg maintained by the home country. We then compare the transfer effects under different regimes and ask how the paying country's economy reacts to the negative transfer when the choice of exchange rate regime is different.

The optimal choice of exchange rate regime has also been studied extensively in the literature.[52] Countries' choice of exchange rate system ranges from the fully fixed exchange rate and specie-standards to the floating exchange rate regime. Fixed exchange rate regime provides exchange rate stability, but a floating exchange rate has the advantage of monetary independence that allows the governments to use monetary policies in response to country-specific shocks in the presence of nominal rigidities (Friedman, 1953). Mundell (1963) incorporates capital mobility to Friedman's (1953) analysis and finds that the choice between fixed and flexible exchange rates depends on the degree of capital mobility and the source of shocks (that is, real or nominal shocks).[53]

In past centuries, the international monetary system has alternated between a commodity-based fixed rate system and a fiat money floating rate system. The classical gold standard was once the dominant international monetary system of the developed countries in the late-19^{th} and early-20^{th} centuries, where the currencies of these countries were fixed to gold at some rates set by their governments. The gold standard was good for maintaining the external balance through the price-specie flow mechanism, but not for maintaining the internal balance. This specie-standard emerged as a system that made international investment and trade more efficient in its time.[54]

[52]Bordo (2003) looks at the choice of exchange rate regime in a historical perspective.

[53]With international capital mobility, the floating exchange rate provides insulation against real shocks, while fixed exchange rate serves as a nominal anchor in response to nominal shocks.

[54]For instance, see Bordo and Rockoff (1996), Estevadeordal, Frantz and Taylor (2003), and Obstfeld and Taylor (2003).

After the WWI, however, the reconstituted gold standard failed, and all developed countries abandoned the gold standard system in the early-1930s. Countries followed a floating exchange rate in the interwar period. The creation of the Bretton Woods adjustable peg system in 1944 was aimed "to combine the advantages of the gold standard (sound money) with those of floating (flexibility and independence)." (Bordo, 2003) After the breakdown of the Bretton Woods system in the 1970s, due to difficulties in controlling the capital mobility, most developed countries either went to floating exchange rates or joined the European Monetary Union (EMU).

A trade-off exists between the ability of national governments to affect the national monetary conditions and the commitment to gold standard or to a fixed exchange rate. Therefore, the common practice of governments in the gold standard system was to go off the gold during major crises, such as wars or economic downturns (Frieden, 1997).[55] For instance, the British government suspended gold convertibility in the midst of Napoleonic Wars in 1797. Moreover, during the Great Depression, Great Britain, Germany, most of Central and Eastern Europe, Canada and Japan went off the gold standard in 1931.[56] Eichengreen (1985) even states that "the historical record provides little support for the theorist's vision of the gold standard as an ideal monetary regime." Yeager (1984) notes that proponents of the gold standard suggest a gold standard during peacetime with temporary departures during wartime, with the idea that it is more effective to finance government spending during wartime by paper money. Chernyshoff, Jacks and Taylor (2005) use a static model and argue that the failure of the gold standard after the pre-war period was due to the increase in nominal rigidities so that the gold standard could not help to absorb macroeconomic volatility. Following these arguments, it appears that if a country needs to pay a transfer to another country, especially in the presence of nominal rigidities, the national government should abandon the gold standard, at least temporarily, to allow the domestic monetary policies to alleviate some of the negative effects of the transfer on the domestic economy.

Given these arguments, we set up a model to answer two questions: what are the economic effects of a transfer of wealth between countries? Is the donor country better off if it goes off the gold standard during the payment of transfer? From these

[55] Frieden (1997) points out that the failure of the gold standard after the interwar period was due to the lack of an international focal point and to the weak domestic political support for an international monetary cooperation.

[56] The United States suspended the gold convertibility in 1933.

questions, we try to shed light on aspects of the optimal exchange rate regime in the presence of an international transfer.

Our model shows that a transfer from home to foreign country leads to terms of trade deterioration in the donor country under all exchange rate regimes studied, regardless of whether or not nominal rigidities exist. Consumption falls in the donor countries in all regimes, except in those under the flexible exchange rate with nominal rigidities. Welfare of the donor country also drops as the transfer shifts the wealth of the donor country to the recipient country. We also find that the flexible exchange rate fails to alleviate some of the negative effects of the transfer, in comparison to the case under the gold standard, and welfare is lower under the flexible exchange rate regime compared to that under the gold standard and the fixed exchange rate. This result does not agree with the standard argument in the literature about going off the gold standard in face of a transfer shock.

The rest of this chapter is organized as follows. Section 4.2 develops a two-country model of the effects of transfer. Section 4.3 examines the effects of transfer under different exchange rate regimes. Section 4.4 studies the welfare effects of transfer and section 4.5 concludes.

4.2 The model

Consider an infinite horizon two-country model without capital. There is a continuum of households along the unit interval, consuming home- and foreign-produced goods, and providing heterogeneous labour services to final goods firms. Firms are competitive, using local labour for production. Nominal wages are set in advance by households-workers, before the production takes place.

4.2.1 Households

There are two countries, Home and Foreign, in this model. Home households' preferences are given by:

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\chi}{1-\varepsilon} \left(\frac{M_t}{P_t} \right)^{1-\varepsilon} - \frac{\eta}{1+\psi} H_t^{1+\psi} \right) \tag{4.1}$$

where M_t is the quantity of domestic money held by households, $\frac{1}{\psi}$ is the elasticity of labour supply, and C_t is the composite of consumption of home- (X) and foreign-produced (M) goods, given by:

$$C_t = \left(\gamma^{\frac{1}{\theta}} C_{Xt}^{1-\frac{1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{Mt}^{1-\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\theta}}} \tag{4.2}$$

γ represents the relative preference for home-produced goods and θ is the elasticity of substitution between good X and good M. We assume $\gamma > \frac{1}{2}$, indicating there is some home bias in households' preferences. The price index is given by:

$$P_t = \left(\gamma P_{Xt}^{1-\theta} + (1-\gamma) P_{Mt}^{1-\theta} \right)^{\frac{1}{1-\theta}} \tag{4.3}$$

Households receive their wage income, inter-country transfers, initial money balances, and investment income.[57] With those incomes, they purchase consumer goods, hold money and accumulate bonds:

$$P_t C_t + B_{t+1} + M_t = W_t H_t + M_{t-1} + T_t + (1+i_t) B_t \tag{4.4}$$

[57] We assume the international transfers are distributed (withdrawn) directly to (from) the households by the government.

Households choose money balances and consumption of home and foreign goods to maximize utility, subject to their budget constraint. The demand for each good, C_{Xt} and C_{Mt}, the demand for money balances and the corresponding Euler equation are:

$$C_{Xt} = \gamma \left(\frac{P_{Xt}}{P_t}\right)^{-\theta} C_t \tag{4.5}$$

$$C_{Mt} = (1-\gamma) \left(\frac{P_{Mt}}{P_t}\right)^{-\theta} C_t \tag{4.6}$$

$$\frac{M_t}{P_t} = \frac{\chi^{\frac{1}{\varepsilon}} C_t^{\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_{t+1}}\right)^{\frac{1}{\varepsilon}}} \tag{4.7}$$

$$\frac{1}{1+i_{t+1}} = \beta \frac{P_t C_t^{\sigma}}{P_{t+1} C_{t+1}^{\sigma}} \tag{4.8}$$

Foreign economy conditions are analogous to the home conditions, and foreign variables are denoted by an asterisk.

4.2.2 Firms

Final goods X and M are produced competitively using differentiated domestic labour inputs by home and foreign respectively. An individual producer in the home country has the production function:

$$X_t = \left(\int_0^1 h_t(i)^{1-\frac{1}{\lambda}} di\right)^{\frac{1}{1-\frac{1}{\lambda}}} \tag{4.9}$$

where $\lambda > 1$ is the elasticity of substitution between labour varieties, and $h_t(i)$ is the employment from individual i. This gives the labour demand for type i labour as:

$$h_t(i) = \left(\frac{W_t(i)}{P_{Xt}}\right)^{-\lambda} X_t \tag{4.10}$$

Firms are competitive, and the price of the home goods in a symmetric equilibrium is determined as:

$$P_{Xt} = \left(\int_0^1 W_t(i)^{1-\lambda} di \right)^{\frac{1}{1-\lambda}} = W_t \tag{4.11}$$

The home goods market clearing condition is:

$$H_t = \gamma \left(\frac{P_{Xt}}{P_t} \right)^{-\theta} C_t + (1-\gamma) \left(\frac{P_{Xt}}{P_t^*} \right)^{-\theta} C_t^* \tag{4.12}$$

4.2.3 Wage-setting

We assume each individual i is a monopolistic supplier of her own type of labour. Labour supply is chosen to maximize utility, and the utility-maximizing wage is derived as:

$$W_t(i) = \eta \frac{\lambda}{\lambda - 1} P_t C_t^{\sigma} h_t(i)^{\psi} \tag{4.13}$$

That is, wage is set as a markup over the "marginal disutility" of labour.

4.3 Effects of a transfer

In this section, we would like to investigate the effects of a transfer under different exchange rate regimes. First, we study the transfer effects under the fixed exchange rate regime, which is modeled as a gold standard setting. We also look at the case when the fixed exchange rate regime is maintained by the home country, which adjusts its domestic money supply endogenously to maintain the fixed rate. Then, the effects of transfer under the flexible exchange rate regime will be examined.

We also assume there are incomplete markets in the economy. The effects of a transfer will be washed out if we have a perfectly pooled equilibrium. Thus, we assume a transfer takes place unexpectedly at the initial period (period 0). Finally, we assume that nominal wages may be fixed in advance. We simply assume that the wages are fixed for only one period, and they can fully adjust after the first period. We will study the impacts of the transfer in the economy with flexible wages, and then compare the results with those of the fixed wage economy.

4.3.1 Gold standard economy

Assume there is a fixed world stock of gold, $\bar{M}$, and home and foreign countries can increase or reduce their share of world gold holding via balance of payments surpluses or deficits. The nominal money stock in the model is then thought of as species, and therefore, all prices are in terms of gold. Gold serves as the medium of exchange in this model. International bonds are also gold denominated.[58]

Flexible wage (gold standard)

Using the setup in Section 4.2, we can write the full flexible wage symmetric equilibrium system under the gold standard as:

$$\frac{M_t}{P_t} = \frac{\chi^{\frac{1}{\varepsilon}} C_t^{\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_{t+1}}\right)^{\frac{1}{\varepsilon}}} \tag{4.14}$$

$$\frac{M_t^*}{P_t^*} = \frac{\chi^{\frac{1}{\varepsilon}} C_t^{*\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_{t+1}}\right)^{\frac{1}{\varepsilon}}} \tag{4.15}$$

$$W_t = \eta \frac{\lambda}{\lambda - 1} P_t C_t^{\sigma} H_t^{\psi} \tag{4.16}$$

$$W_t^* = \eta \frac{\lambda}{\lambda - 1} P_t^* C_t^{*\sigma} H_t^{*\psi} \tag{4.17}$$

$$H_t = \gamma \left(\frac{P_{Xt}}{P_t}\right)^{-\theta} C_t + (1-\gamma)\left(\frac{P_{Xt}}{P_t^*}\right)^{-\theta} C_t^* \tag{4.18}$$

$$H_t^* = (1-\gamma)\left(\frac{P_{Mt}}{P_t}\right)^{-\theta} C_t + \gamma \left(\frac{P_{Mt}}{P_t^*}\right)^{-\theta} C_t^* \tag{4.19}$$

$$P_t C_t + B_{t+1} + M_t = P_{Xt} H_t + M_{t-1} + T_t + (1+i_t) B_t \tag{4.20}$$

[58] We may also model the gold standard regime by assuming the home and foreign governments fix the prices of their domestic currency in terms of a specified amount of gold, with free convertibility. This setup will add extra equations to the model, linking the conversion rate between gold and currencies, however, the results will be the same as this commodity model. For simplicity, we use this commodity model for analysis.

$$\begin{aligned}
\frac{1}{1+i_{t+1}} &= \beta\frac{P_t C_t^{\sigma}}{P_{t+1}C_{t+1}^{\sigma}} && (4.21)\\
\frac{1}{1+i_{t+1}} &= \beta\frac{P_t^* C_t^{*\sigma}}{P_{t+1}^* C_{t+1}^{*\sigma}} && (4.22)
\end{aligned}$$

$$\begin{aligned}
P_t &= \left[\gamma P_{Xt}^{1-\theta} + (1-\gamma)(P_{Mt})^{1-\theta}\right]^{\frac{1}{1-\theta}} && (4.23)\\
P_t^* &= \left[(1-\gamma)P_{Xt}^{1-\theta} + \gamma P_{Mt}^{1-\theta}\right]^{\frac{1}{1-\theta}} && (4.24)
\end{aligned}$$

$$\begin{aligned}
P_{Xt} &= W_t && (4.25)\\
P_{Mt} &= W_t^* && (4.26)
\end{aligned}$$

$$\bar{M} = M_t + M_t^* \qquad (4.27)$$

(4.14) and (4.15) are the home and foreign money demand conditions, (4.16) and (4.17) are the equilibrium wage equations. Home and foreign goods market clearing conditions are given as (4.18) and (4.19). The money (gold) market clearing condition is (4.27).

In this economy, terms of trade is denoted as:

$$TOT_t = \frac{P_{Xt}}{P_{Mt}} \qquad (4.28)$$

We log-linearize the model around the initial steady state to derive some useful analytical properties of the equilibrium. In the steady state, we have $C = C^* = \bar{C}$, $H = H^* = \bar{H}$, $M = M^* = \bar{M}$, $\frac{1}{1+i} = \beta$, $\frac{P_X}{P_M} = 1$, and $B = 0$. Let $\hat{z} = \ln\left(Z_t/\bar{Z}\right)$, that is, the hat variables are log differences from the steady state.[59]

The log-linearized version of (4.18) and (4.19) can be interpreted as the world demand schedule of home and foreign goods:

$$\begin{aligned}
\hat{h}_t &= -\theta\hat{p}_{Xt} + \gamma\hat{c}_t + (1-\gamma)\hat{c}_t^* + \gamma\theta\hat{p}_t + (1-\gamma)\theta\hat{p}_t^* && (4.29)\\
\hat{h}_t^* &= -\theta\hat{p}_{Mt} + (1-\gamma)\hat{c}_t + \gamma\hat{c}_t^* + (1-\gamma)\theta\hat{p}_t + \gamma\theta\hat{p}_t^* && (4.30)
\end{aligned}$$

[59]See Appendix C.1 for the full log-linearized system.

With a fixed stock of gold in the world economy, log-linearized gold market clearing condition becomes:

$$0 = \hat{m}_t + \hat{m}_t^* \tag{4.31}$$

The two money market conditions imply that the relative money demand depends on the consumption difference and the terms of trade ($\hat{\tau}_t$):

$$\hat{m}_t - \hat{m}_t^* = (2\gamma - 1)\hat{\tau}_t + \frac{\sigma}{\varepsilon}(\hat{c}_t - \hat{c}_t^*) \tag{4.32}$$

The output difference also depends on the consumption difference and the terms of trade, and it can be derived from the two goods market clearing conditions, (4.29) and (4.30):

$$\hat{h}_t - \hat{h}_t^* = -4\theta\gamma(1-\gamma)\hat{\tau}_t + (2\gamma - 1)(\hat{c}_t - \hat{c}_t^*) \tag{4.33}$$

Using the labour market conditions and goods market clearing conditions, we get an expression for the terms of trade as:

$$\hat{\tau}_t = \left[\frac{\sigma + \psi(2\gamma - 1)}{2(1-\gamma) + 4\psi\theta\gamma(1-\gamma)}\right](\hat{c}_t - \hat{c}_t^*) \equiv \Phi(\hat{c}_t - \hat{c}_t^*) \tag{4.34}$$

That is, the change in terms of trade also depends on the consumption difference.

Proposition 4 (Real interest rates, world consumption and output). *Under the gold standard when wages are flexible, the real interest rates are equalized across countries, and the world consumption and output changes are zero.*

Proof. The difference of home and foreign Euler equations is:

$$\sigma\left[(\hat{c}_{t+1} - \hat{c}_{t+1}^*) - (\hat{c}_t - \hat{c}_t^*)\right] = -(2\gamma - 1)(\hat{\tau}_{t+1} - \hat{\tau}_t)$$

Combining this equation with equation (4.34), we can show:

$$\hat{c}_{t+1} - \hat{c}_t = \hat{c}_{t+1}^* - \hat{c}_t^* \tag{4.35}$$

which implies that real interest rates are equalized across countries.

By adding the home and foreign market clearing conditions and the pricing equations, we get $\hat{c}_t + \hat{c}_t^* = \hat{h}_t + \hat{h}_t^*$. Using this condition together with the labour market equilibrium conditions, we get $\hat{c}_t + \hat{c}_t^* = 0$, and hence:

$$\hat{c}_t + \hat{c}_t^* = \hat{h}_t + \hat{h}_t^* = 0 \tag{4.36}$$

which also implies:

$$\hat{c}_t - \hat{c}_t^* = 2\hat{c}_t \tag{4.37}$$

□

Using the fact that real interest rates are equalized across countries and the movement in the world output is zero, as well as the condition that $\hat{m}_t = -\hat{m}_t^*$ from the world gold market clearing condition, we can write the money balances, labour and terms of trade expressions ((4.32),(4.33),(4.34)) as:

$$\hat{m}_t = \frac{2\gamma - 1}{2}\hat{\tau}_t + \frac{\sigma}{\varepsilon}\hat{c}_t \tag{4.38}$$

$$\hat{h}_t = -2\theta\gamma(1-\gamma)\hat{\tau}_t + (2\gamma - 1)\hat{c}_t = [-4\theta\gamma\Phi(1-\gamma) + (2\gamma - 1)]\hat{c}_t \tag{4.39}$$

$$\hat{\tau}_t = \left[\frac{\sigma + \psi(2\gamma - 1)}{2(1-\gamma) + 4\psi\theta\gamma(1-\gamma)}\right] 2\hat{c}_t \equiv 2\Phi\hat{c}_t \tag{4.40}$$

We use the home country budget constraint to study the impact of a transfer on consumption, output and terms of trade. The log-linearized home budget constraint is given as:

$$\hat{c}_t + \frac{dB_{t+1}}{\bar{P}\bar{C}} + \omega\hat{m}_t = (1-\gamma)\hat{\tau}_t + \hat{h}_t + \omega\hat{m}_{t-1} + \frac{dT_t}{\bar{P}\bar{C}} + \frac{1}{\beta}\frac{dB_t}{\bar{P}\bar{C}} \tag{4.41}$$

where $\omega = \frac{\bar{M}}{\bar{P}\bar{C}}$ and $dZ_t = Z_t - \bar{Z}$.

Substitute in (4.38), (4.39) and (4.40), we can get:

$$\begin{aligned} & \hat{c}_t + \frac{dB_{t+1}}{\bar{P}\bar{C}} + \omega\left((2\gamma - 1)\Phi\hat{c}_t + \frac{\sigma}{\varepsilon}\hat{c}_t\right) \\ = \; & (1-\gamma)2\Phi\hat{c}_t + (2\gamma-1)\hat{c}_t - 2\theta\gamma(1-\gamma)2\Phi\hat{c}_t + \omega\left((2\gamma-1)\Phi\hat{c}_{t-1} + \frac{\sigma}{\varepsilon}\hat{c}_{t-1}\right) + \frac{1}{\beta}\frac{dB_t}{\bar{P}\bar{C}} + \frac{dT_t}{\bar{P}\bar{C}} \end{aligned} \tag{4.42}$$

We can now use (4.42) to work out the impact of a one-time, unanticipated transfer away from the home country in period 0 (that is, $dT_0 < 0$).

Period 0:
In the period of the transfer taking place, with $\hat{c}_{-1} = 0$ and $dB_0 = 0$, then (4.42) becomes:

$$\hat{c}_0 + \frac{dB_1}{\bar{P}\bar{C}} + \omega\left((2\gamma-1)\Phi\hat{c}_0 + \frac{\sigma}{\varepsilon}\hat{c}_0\right) = (1-\gamma)2\Phi\hat{c}_0 + (2\gamma-1)\hat{c}_0 - 2\theta\gamma(1-\gamma)2\Phi\hat{c}_0 + \frac{dT_0}{\bar{P}\bar{C}}$$

Period 1 and onwards:
Since we already know that $\hat{c}_{t+1} = \hat{c}_t$ for $t \geq 0$, to keep consumption the same in every future period after 1, we must have $dB_{t+1} = dB_t$ for $t \geq 1$. Then the period budget constraint becomes:

$$\begin{aligned} & \hat{c}_0 + \frac{dB_1}{\bar{P}\bar{C}} + \omega\left((2\gamma-1)\Phi\hat{c}_0 + \frac{\sigma}{\varepsilon}\hat{c}_0\right) \\ = \; & (1-\gamma)2\Phi\hat{c}_0 + (2\gamma-1)\hat{c}_0 - 2\theta\gamma(1-\gamma)2\Phi\hat{c}_0 + \omega\left((2\gamma-1)\Phi\hat{c}_0 + \frac{\sigma}{\varepsilon}\hat{c}_0\right) + \frac{1}{\beta}\frac{dB_1}{\bar{P}\bar{C}} \end{aligned}$$

Take the difference of period 0 and period 1 budget constraints, we can get an expression for the capital account:

$$\frac{dB_1}{\bar{P}\bar{C}} = \beta\left(\frac{dT_o}{\bar{P}\bar{C}} - \omega\left((2\gamma-1)\Phi\hat{c}_0 + \frac{\sigma}{\varepsilon}\hat{c}_0\right)\right) \tag{4.43}$$

Substitute this back to the period zero condition, rearrange, and we can solve for $\hat{c}_0$:

$$\hat{c}_0 = \frac{1-\beta}{\Delta}\frac{dT_0}{\bar{P}\bar{C}} \tag{4.44}$$

where $\Delta = 2(1-\gamma)(1-\Phi(1-2\theta\gamma)) + (1-\beta)\omega\left((2\gamma-1)\Phi + \frac{\sigma}{\varepsilon}\right)$. The sufficient condition for the denominator, Δ, to be positive is that $\theta \geq \frac{1}{2\gamma}$.[60]

Thus, given a large enough elasticity of substitution between home and foreign

[60] Because $1 - \Phi(1-2\theta\gamma) > 0$ when $\theta \geq \frac{1}{2\gamma}$.

goods, the impact of a transfer away from home on consumption is negative:

$$\hat{c}_0 = \frac{1-\beta}{\Delta}\frac{dT_0}{\bar{P}\bar{C}} < 0$$

The negative transfer also induces an immediate balance of payments deficit at home country, and the home terms of trade deteriorates:

$$\hat{m}_0 = \left((2\gamma - 1)\Phi + \frac{\sigma}{\varepsilon}\right)\hat{c}_0 < 0$$

$$\hat{\tau}_0 = 2\Phi\hat{c}_0 < 0$$

The transfer effect on home output depends on the elasticity of labour supply.[61] If the elasticity of labour supply is zero, that is, $\psi \to \infty$, the transfer has no effect on output, and we have:

$$\hat{h}_0 = 0$$

With infinite elasticity of labour supply, $\psi = 0$, we have:

$$\hat{h}_0 = -\left[1 - 2\gamma(1-\sigma\theta)\right]\hat{c}_0 > 0$$

Home output increases in response to a negative transfer if $\sigma\theta > 1$. The home capital account becomes negative after a transfer takes place:

$$\frac{dB_1}{\bar{P}\bar{C}} = \beta\frac{dT_0}{\bar{P}\bar{C}}\left(1 - \frac{(1-\beta)\omega\left((2\gamma-1)\Phi + \frac{\sigma}{\varepsilon}\right)}{\Delta}\right) < 0$$

Therefore, the home country borrows abroad in period 0 to partly cushion the impacts of the transfer on consumption and gold holdings.

Sticky Wage (Gold Standard)

Now we look at the implication of the transfer with sticky nominal wages in the first period (that is, at the period of the transfer that takes place, so that $P_{X0} = W_0 = \bar{W}$ and $P^*_{M0} = W^*_0 = \bar{W}^*$). After realizing the negative transfer shock, wages can fully

[61] In later sections, we calibrate the transfer effect using unit elasticity of labour supply. We find that with $\psi = 1$, output increases after a negative transfer under flexible wage.

adjust from the second period onwards. When wages are fixed in period 0, then prices are also fixed.

The system of equilibrium conditions describing the first period is presented in Appendix C.1.[62] World demand schedules of home and foreign goods are:

$$\hat{h}_0 = \gamma\hat{c}_0 + (1-\gamma)\hat{c}_0^* \tag{4.45}$$

$$\hat{h}_0^* = (1-\gamma)\hat{c}_0 + \gamma\hat{c}_0^* \tag{4.46}$$

Period 0 gold market clearing condition and budget constraint are:

$$0 = \hat{m}_0 + \hat{m}_0^* \tag{4.47}$$

and

$$\hat{c}_0 + \frac{dB_1}{\bar{P}\bar{C}} + \omega\hat{m}_0 = \hat{h}_0 + \frac{dT_0}{\bar{P}\bar{C}} \tag{4.48}$$

The home and foreign money demand equations and the gold market clearing condition (4.47) give:

$$\hat{m}_0 - \hat{m}_0^* = 2\hat{m}_0 = \frac{\sigma}{\varepsilon}(\hat{c}_0 - \hat{c}_0^*) \tag{4.49}$$

The difference of the Euler equations yield:

$$\hat{c}_1 = \frac{\sigma(\hat{c}_0 - \hat{c}_0^*)}{2\left[\sigma + (2\gamma - 1)\Phi\right]} \tag{4.50}$$

Then, the goods market clearing conditions imply:

$$\hat{h}_0 - \hat{h}_0^* = (2\gamma - 1)(\hat{c}_0 - \hat{c}_0^*) \tag{4.51}$$

We use the difference of the home and foreign country period 0 budget constraints,

[62] In this wage stickiness case, we omit the wage-setting equations, the zero-profit conditions, and the definitions of the consumer price index when we solve the model. To solve for the impact of transfer in the first period when wages are sticky, we must use the solution from the flexible wage system, because the sticky wage system contains variables dated period 1 (in which wages are flexible). Thus, we need to tie both solutions together.

together with equations (4.49), (4.50) and (4.51) to get:

$$(\hat{c}_0 - \hat{c}_0^*)\left(2(1-\gamma) + \frac{\omega\sigma}{\varepsilon}\right) + \frac{2dB_1}{\bar{P}\bar{C}} = \frac{2dT_0}{\bar{P}\bar{C}} \tag{4.52}$$

Since equations (4.50) and (4.52) give us two conditions in three variables: $\{\hat{c}_1, (\hat{c}_0 - \hat{c}_0^*), \frac{dB_1}{\bar{P}\bar{C}}\}$, we need to use the period 1 and onwards conditions to solve for all three variables. Period 1 and period 2 budget constraints give an expression for $\hat{c}_1$:

$$\hat{c}_1 = \frac{\frac{\omega\sigma}{2\varepsilon}(\hat{c}_0 - \hat{c}_0^*)(1-\beta) + \frac{1-\beta}{\beta}\frac{dB_1}{\bar{P}\bar{C}}}{\Delta} \tag{4.53}$$

We derive this equation using the fact that, from period 1 onwards, consumption is the same for all period ($\hat{c}_{t+1} = \hat{c}_t$ for $t \geq 1$), and from period 2 onwards, the stock of bond holdings will remain the same (that is, $dB_{t+1} = dB_t$ for $t \geq 2$).

Using this equation and equation (4.52), we get an equation of consumption difference at period 0:

$$\hat{c}_0 - \hat{c}_0^* = \frac{2}{\left(2(1-\gamma) + \frac{\omega\sigma}{\varepsilon}(1-\beta)\right) + \left(\frac{\Delta\sigma}{\sigma+(2\gamma-1)\Phi}\frac{\beta}{1-\beta}\right)}\frac{dT_0}{\bar{P}\bar{C}} \tag{4.54}$$

The denominator is positive as long as $\Delta > 0$, or $\theta \geq \frac{1}{2\gamma}$. Thus, a transfer from the home country reduces home relative consumption in period 0.

From (4.54) and the goods market clearing conditions in the home country, we can state the following results:

Proposition 5 (Gold standard, sticky wages). *Under the gold standard and when wages are sticky in period 0, a negative transfer away from home leads to a fall in consumption, a fall in output, and a balance of payments deficit in period 0; however, terms of trade is not affected.*

Proof. In order to find the impacts of the negative transfer on consumption and output, we use the two money market equilibrium conditions in period 0 to get:

$$\hat{m}_0 + \hat{m}_0^* = \frac{\sigma}{\varepsilon}(\hat{c}_0 + \hat{c}_0^*) + \varphi\left(\sigma(\hat{c}_0 + \hat{c}_0^*) - \sigma(\hat{c}_1 + \hat{c}_1^*) + (\hat{p}_1 + \hat{p}_1^*)\right)$$

where $\varphi = \frac{\beta}{\varepsilon}(1-\beta)^{\frac{1}{\varepsilon}-1}$.

We know that $\hat{m}_0+\hat{m}_0^* = 0$ and $\hat{c}_1+\hat{c}_1^* = 0$ from the gold market clearing condition and the zero movement in world output. Moreover, from the period 1 money market conditions onwards, we have $\hat{p}_1 + \hat{p}_1^* = 0$. These conditions imply $\hat{c}_0 + \hat{c}_0^* = 0$. Thus, even with sticky wages, the transfer does not affect world consumption and output in the period that the transfer takes place, that is, $\hat{h}_0 + \hat{h}_0^* = 0$. Using these facts and equations (4.54), (4.51) and (4.49), we can get:

$$\begin{aligned}
\hat{c}_0 &= \frac{1}{\left(2(1-\gamma)+\frac{\omega\sigma}{\varepsilon}(1-\beta)\right)+\left(\frac{\Delta\sigma}{\sigma+(2\gamma-1)\Phi}\frac{\beta}{1-\beta}\right)}\frac{dT_0}{\bar{P}\bar{C}} < 0 \\
\hat{h}_0 &= \gamma\hat{c}_0 + (1-\gamma)\hat{c}_0^* = (2\gamma-1)\hat{c}_0 < 0 \\
\hat{m}_0 &= \frac{\sigma}{\varepsilon}\frac{1}{\left(2(1-\gamma)+\frac{\omega\sigma}{\varepsilon}(1-\beta)\right)+\left(\frac{\Delta\sigma}{\sigma+(2\gamma-1)\Phi}\frac{\beta}{1-\beta}\right)}\frac{dT_0}{\bar{P}\bar{C}} < 0
\end{aligned}$$

Since wages are sticky, prices are fixed as well. Thus, $\hat{\tau}_0 = \hat{p}_{Xt} - \hat{p}_{Mt} = 0$. □

Therefore, a transfer away from home to foreign country leads to a fall in period 0 home consumption when wages are sticky. There is a fall in home output rather than an increase when there are nominal rigidities.[63] The transfer does not affect the terms of trade when wages are sticky. Therefore, the negative transfer reduces home country output by lowering the demand for home product, and increases the foreign country output by raising the demand for foreign's product.

[63]This result holds for any value of the elasticity of labour supply.

4.3.2 Fixed exchange rate regime

Now assume that both home and foreign leave the gold standard, but instead, the home country maintains a fixed exchange rate with the foreign country by adjusting its domestic money supply endogenously.[64] We may think of this situation as one that the home country wants to have a hard peg arrangement, such as a currency board, to maintain exchange rate stability within the country. Home and foreign now have their own fiat currency, and the home money supply expands or contracts endogenously with the balance of payments and the state of the economy. What will be the effects of a transfer on home country with such an exchange rate arrangement in period 0?

Assume that the exchange rate is set at $\bar{S}$ so that $S_t = \bar{S}$ for $t \geq 0$. Once the two countries have suspended the gold standard, the condition $\bar{M} = M_t + M_t^*$ no longer holds. Prices are no longer in terms of species, but rather, in fiat money. Home and foreign monetary policies follow: $M_t = M_{t-1} + \mu_t$ and $M_t^* = M_{t-1}^* + \mu_t^*$, respectively, in which the foreign policy is exogenously determined by the foreign government, but the money supply at home is endogenously determined to maintain the fixed exchange rate.[65]

Flexible wage (fixed exchange rate)

The log-linearized terms of trade under this fixed exchange rate regime is:

$$\hat{\tau}_t = \hat{p}_{Xt} - \hat{p}_{Mt}^* \tag{4.55}$$

as $\hat{s}_t = 0$. Since the foreign monetary policy is exogenous, we have $\hat{m}_t^* = 0$. Change in home money stock, $\hat{m}_t$, on the other hand, is endogenously determined by the money balances conditions and the pricing rules:

$$\hat{m}_t = \left((2\gamma - 1)\Phi + \frac{\sigma}{\varepsilon}\right)(\hat{c}_t - \hat{c}_t^*) \tag{4.56}$$

[64]On the other hand, foreign country's monetary policy is assumed to be exogenous.

[65]The full system of equilibrium conditions and log-linearized system can be found in Appendix C.2.

From the market clearing conditions and pricing equations, we have:

$$\hat{h}_t - \hat{h}_t^* = -4\theta\gamma(1-\gamma)\hat{\tau}_t + (2\gamma - 1)(\hat{c}_t - \hat{c}_t^*) \tag{4.57}$$

which gives the same equation as (4.33) in the gold standard system.

Besides, the Euler equations and pricing equations yield:

$$\hat{\tau}_t = \Phi(\hat{c}_t - \hat{c}_t^*) \tag{4.58}$$

which is the same terms of trade expression as (4.34). Similar to the gold standard case, we can show equalization of real interest rates and zero world output movements:

$$\hat{c}_{t+1} - \hat{c}_t = \hat{c}_{t+1}^* - \hat{c}_t^*$$

$$\hat{c}_t + \hat{c}_t^* = \hat{h}_t + \hat{h}_t^* = 0$$

These conditions are equivalent to those of proposition 5 under the gold standard regime.

From the home log-linearized budget constraint, labour equations and pricing equations, we get an expression for studying the impact of an unanticipated transfer in period t:

$$(1-\gamma)\left(1 - \Phi(1-2\gamma\theta)\right) 2\hat{c}_t = \frac{dT_t}{\bar{P}\bar{C}} + \frac{1}{\beta}\frac{dB_t}{\bar{P}\bar{C}} - \frac{dB_{t+1}}{\bar{P}\bar{C}} \tag{4.59}$$

We perform the same exercise as those under the gold standard regime to derive the period 0 and period 1 budget constraints using (4.59), and use the fact that $\hat{c}_1 = \hat{c}_0$ and $dB_1 = dB_2$ to get:

$$\frac{dB_1}{\bar{P}\bar{C}} = \beta\frac{dT_0}{\bar{P}\bar{C}}$$

and,

$$\hat{c}_0 = \frac{1-\beta}{2(1-\gamma)\left(1-\Phi(1-2\gamma\theta)\right)}\frac{dT_0}{\bar{P}\bar{C}} < 0 \tag{4.60}$$

When we compare (4.60) to the corresponding expression under the gold standard, we find that the denominator of the gold standard expression, Δ, is smaller than the one in (4.60). This implies that the negative transfer has a larger negative effect on period 0 home consumption under the fixed exchange rate regime than that under the gold standard. That is, home consumption falls by a larger amount after a negative transfer under fixed exchange rate regime.

We can also show a fall in money balances and a terms of trade deterioration in the home economy:

$$\hat{m}_0 = \left((2\gamma - 1)\Phi + \frac{\sigma}{\varepsilon}\right) 2\hat{c}_0 < 0$$

$$\hat{\tau}_0 = 2\Phi\hat{c}_0 < 0$$

Since the fall in period 0 home consumption is larger under the fixed exchange rate regime, we can deduce that the negative transfer induces a larger balance of payments deficit and a larger terms of trade deterioration under the fixed exchange rate regime than under the gold standard.[66]

For home output, if the elasticity of labour supply is zero, then there is no change in home output (that is, $\hat{h}_0 = 0$). If the elasticity of labour supply is infinite, then:

$$\hat{h}_0 = -\left[1 - 2\gamma(1 - \sigma\theta)\right]\hat{c}_0 > 0$$

That is, output increases, but by a larger amount than that under the gold standard.

Sticky wage (fixed exchange rate)

We now turn to the sticky wage environment. Using the same assumption as before, wages are sticky for one period only, and they can be adjusted fully from the second period onwards. Then we have: $P_{X0} = W_0 = \bar{W}$; $P^*_{M0} = W^*_0 = \bar{W}^*$; and $S_t = \bar{S}$ for $t \geq 0$.

Since $\hat{m}^*_0 = 0$, home money demand equation become:

$$\hat{m}_0 = \frac{\sigma}{\varepsilon}(\hat{c}_0 - \hat{c}^*_0) \tag{4.61}$$

[66] Under the gold standard, we have $\hat{m}_0^{GS} = \left((2\gamma - 1)\Phi + \frac{\sigma}{\varepsilon}\right)\hat{c}_0 = \left((2\gamma - 1)\Phi + \frac{\sigma}{\varepsilon}\right)\frac{1-\beta}{\Delta}\frac{dT_0}{PC}$. Equation (4.3.2) implies: $\hat{m}_0^{fixed} = \left((2\gamma - 1)\Phi + \frac{\sigma}{\varepsilon}\right)2\hat{c}_0 = \left((2\gamma - 1)\Phi + \frac{\sigma}{\varepsilon}\right)\frac{1-\beta}{(1-\gamma)(1-\Phi(1-2\gamma\theta))}\frac{dT_0}{PC}$. Since $\Delta > (1-\gamma)(1-\Phi(1-2\gamma\theta))$, we must have $0 > \hat{m}_0^{GS} > \hat{m}_0^{fixed}$.

Home and foreign labour market conditions imply:

$$\hat{h}_0 - \hat{h}_0^* = (2\gamma - 1)(\hat{c}_0 - \hat{c}_0^*) \tag{4.62}$$

Euler equations and the period 1 pricing equations yield:

$$\hat{c}_1 = \frac{\sigma(\hat{c}_0 - \hat{c}_0^*)}{2\left[\sigma + \Phi(2\gamma - 1)\right]} \tag{4.63}$$

Using equation (4.59), $\hat{c}_1 = \hat{c}_2$, and $dB_2 = dB_3$, to derive the period 1 and period 2 home budget constraints and get:

$$\hat{c}_1 = \frac{1-\beta}{\beta} \frac{1}{2(1-\gamma)\left(1 - \Phi(1 - 2\gamma\theta)\right)} \frac{dB_1}{\bar{P}\bar{C}} \tag{4.64}$$

Using (4.64) and (4.63), we can solve for $\frac{dB_1}{\bar{P}\bar{C}}$:

$$\frac{dB_1}{\bar{P}\bar{C}} = \frac{\beta}{1-\beta} \left[\frac{(1-\gamma)\left(1 - \Phi(1 - 2\gamma\theta)\right)\sigma}{\sigma + \Phi(2\gamma - 1)}\right] (\hat{c}_0 - \hat{c}_0^*) \tag{4.65}$$

The difference of the home and foreign period 0 budget constraints and equation (4.65) imply:

$$\hat{c}_0 - \hat{c}_0^* = \frac{1}{(1-\gamma)\left[1 + \frac{\beta}{1-\beta} \frac{(1-\Phi(1-2\gamma\theta))\sigma}{\sigma+\Phi(2\gamma-1)}\right]} \frac{dT_0}{\bar{P}\bar{C}} < 0 \tag{4.66}$$

if $\theta \geq \frac{1}{2\gamma}$.

The effects of a transfer under the fixed exchange rate regime with sticky wages are stated as follows:

Proposition 6 (Fixed exchange rate, sticky wages). *Under the fixed exchange rate regime and when wages are sticky in period 0, a negative transfer away from home reduces period 0 consumption, output level and money balances in the home country. Terms of trade is not affected by the negative transfer.*

Proof. To solve for home period 0 consumption, we use home and foreign money market conditions and the Euler equations to get:

$$\hat{c}_0 = \left[\frac{2(\sigma + \Phi(2\gamma - 1))}{\beta\sigma(\frac{1}{\varepsilon} - 1)} + 1\right] \hat{c}_0^* \tag{4.67}$$

Use this equation together with (4.66) and (4.61), we can derive the $\hat{c}_0$ and $\hat{m}_0$ expressions respectively:

$$\begin{aligned}
\hat{c}_0 &= \frac{2\left(\sigma + \Phi(2\gamma - 1)\right) + \beta\sigma\left(\frac{1}{\varepsilon} - 1\right)}{2(1-\gamma)\left[\left(\sigma + \Phi(2\gamma - 1)\right) + \frac{\beta}{1-\beta}\sigma\left(1 - \Phi(1 - 2\gamma\theta)\right)\right]} \frac{dT_0}{\bar{P}\bar{C}} < 0 \\
\hat{m}_0 &= \frac{\sigma}{\varepsilon} \frac{1}{(1-\gamma)\left[1 + \frac{\beta}{1-\beta}\frac{(1-\Phi(1-2\gamma\theta))\sigma}{\sigma + \Phi(2\gamma - 1)}\right]} \frac{dT_0}{\bar{P}\bar{C}} < 0
\end{aligned}$$

To solve for the impact of a negative transfer on home output, we use the $\hat{c}_0$ expression and the home market clearing condition to get:

$$\hat{h}_0 = \frac{2\gamma\left(\sigma + \Phi(2\gamma - 1)\right) + \beta\sigma(\frac{1}{\varepsilon} - 1)}{2\left(\sigma + \Phi(2\gamma - 1)\right) + \beta\sigma(\frac{1}{\varepsilon} - 1)} \hat{c}_0 < 0$$

for $\psi \geq 0$. Since wages are sticky, which imply prices are fixed as well, thus, $\hat{\tau}_0 = \hat{p}_{Xt} - \hat{p}^*_{Mt} = 0$. □

4.3.3 Flexible exchange rate regime

We assume both home and foreign follow a flexible exchange rate regime, and their money supplies are exogenously determined by each country's government.[67] With flexible exchange rate, the uncovered interest rate parity must hold in a world of perfect foresight:

$$(1 + i^*_{t+1}) = (1 + i_{t+1})\frac{S_t}{S_{t+1}} \tag{4.68}$$

However, purchasing power parity does not hold in this economy even if the law of one price holds for individual tradable goods, because of the presence of home bias in domestically produced goods in both countries.

The log-linearized terms of trade under the flexible exchange rate regime is:

$$\hat{\tau}_t = \hat{p}_{Xt} - \hat{p}^*_{Mt} - \hat{s}_t \tag{4.69}$$

The home and foreign labour market conditions imply:

$$\hat{h}_t - \hat{h}^*_t = -4\gamma\theta(1-\gamma)\hat{\tau}_t + (2\gamma - 1)(\hat{c}_t - \hat{c}^*_t) \tag{4.70}$$

(4.70) and the home and foreign wage equations yield:

$$\hat{\tau}_t = \Phi(\hat{c}_t - \hat{c}^*_t) \tag{4.71}$$

Note that this condition is the same as the one under the gold standard and the fixed exchange rate regime. That is, when wages are flexible, terms of trade is linearly proportional to the home and foreign consumption difference.

We can show using the Euler equations, pricing equations, goods market and labour market conditions, together with the terms of trade expression (4.71) that:

$$\hat{c}_{t+1} - \hat{c}_t = \hat{c}^*_{t+1} - \hat{c}^*_t \tag{4.72}$$

[67] The full system of equilibrium conditions under the flexible exchange rate regime is in Appendix C.3.

$$\hat{c}_t + \hat{c}_t^* = \hat{h}_t + \hat{h}_t^* = 0 \tag{4.73}$$

These conditions are equivalent to those under both the gold standard and fixed exchange rate regime. Therefore, real interest rates are equalized across countries, and the world consumption and output have zero movements under all three exchange rate regimes when wages are flexible.

Using the period 0 and period 1 budget constraints to solve for $\hat{c}_0$:

$$\hat{c}_0 = \frac{1-\beta}{2(1-\gamma)[1-\Phi(1-2\theta\gamma)]} \frac{dT_0}{\bar{P}\bar{C}} < 0 \tag{4.74}$$

The sufficient condition for positive coefficient is $\theta \geq \frac{1}{2\gamma}$. A large elasticity of substitution between home and foreign goods leads to a negative transfer impact on period 0 home consumption under flexible exchange rate regime.

The transfer away from home will cause the home's terms of trade to deteriorate, but it does not have any impact on the exogenous money supply at period 0:

$$\hat{\tau}_0 = 2\Phi\hat{c}_0 < 0$$

$$\hat{m}_0 = 0$$

The effect of transfer on period 0 home output also depends on the elasticity of labour supply. In particular, the transfer does not affect the output level if the elasticity of labour supply is zero, while the transfer increases output if the elasticity of labour supply is infinite:

$$\hat{h}_0 = -[1-2\gamma(1-\theta\sigma)]\hat{c}_0 > 0$$

Capital account becomes negative when the home country gives a transfer to foreign country, that is, again, home country needs to borrow to finance the transfer:

$$\frac{dB_1}{\bar{P}\bar{C}} = \beta\frac{dT_0}{\bar{P}\bar{C}} < 0$$

When prices and wages are flexible, we show that the effects of transfer on the home economy are the same under both fixed and flexible exchange rate regimes. With nominal flexibility, prices can freely adjust in response to shocks to the economy, and hence, the choice of exchange rate regime does not generate any real effect on the economy.[68]

Sticky wage (flexible exchange rate)

We use the same assumption as in the previous two sub-sections that wages are fixed for the first period only, and they can fully adjust from the second period onwards. From period 1 onwards, the equilibrium conditions will be the same as those under the flexible wage case.

Period 0 Euler equations and period 1 pricing equations give:

$$\hat{c}_1 = \frac{\sigma(\hat{c}_0 - \hat{c}_0^*) + (1 - 2\gamma)\hat{s}_0}{2\left[\sigma + (2\gamma - 1)\Phi\right]} \tag{4.75}$$

When we compare this equation to the analogous equations under the gold standard and the fixed exchange rate (equations (4.50) and (4.63)), we can see that exchange-rate pass-through affects period 1 consumption under the flexible exchange rate.

Period 1 and 2 budget constraints imply:

$$\hat{c}_1 = \frac{1-\beta}{\beta}\frac{1}{2(1-\gamma)\left[1 - \Phi(1 - 2\gamma\theta)\right]}\frac{dB_1}{\bar{P}\bar{C}} \tag{4.76}$$

We can show from the money market conditions, good market conditions, and the Euler equations that:

$$c_0 + c_0^* = h_0 + h_0^* = 0$$

as home and foreign money supplies are exogenous ($m_0 = m_0^* = 0$). Then the exchange rate is derived as:

$$\hat{s}_0 = -\frac{\sigma}{(1-\gamma)\left(\beta + \varepsilon(1-\beta)\right)}\hat{c}_0 \tag{4.77}$$

Using this expression together with (4.75) and (4.76), we can derive the effect of

[68]However, in the specie-standard economy, the economy is also constrained by the stock of species even if there is no nominal rigidity. Therefore, the effects of transfer on the donor country are different under the gold standard.

transfer on period 0 consumption as:

$$\hat{c}_0 = \frac{1}{2(1-\gamma) + (2\gamma\theta - 1)\frac{\sigma}{\varepsilon} + \frac{\beta}{1-\beta}\Psi} \frac{dT_0}{\bar{P}\bar{C}} \tag{4.78}$$

where $\Psi = \left[\frac{(1-\gamma)(1-\Phi(1-2\gamma\theta))}{\sigma+(2\gamma-1)\Phi}\right]\left[2\sigma - \frac{\sigma(1-2\gamma)}{(1-\gamma)(\beta+\varepsilon(1-\beta))}\right]$.

The denominator of this equation is positive if $\theta \geq \frac{1}{2\gamma}$. This indicates that a negative transfer reduces the home consumption in period 0.

We state the effects of transfer at period 0 under the sticky-wage flexible exchange rate environment as follows:

Proposition 7 (Flexible exchange rate, sticky wages). *Under the flexible exchange rate regime and when wages are sticky in period 0, a negative transfer away from home reduces period 0 home consumption. Home output increases in response to the negative transfer, and the terms of trade deteriorates in period 0.*

Proof. The transfer effect on consumption can be seen directly from equation (4.78); that is:

$$\hat{c}_0 = \frac{1}{2(1-\gamma) + (2\gamma\theta - 1)\frac{\sigma}{\varepsilon} + \frac{\beta}{1-\beta}\Psi} \frac{dT_0}{\bar{P}\bar{C}} < 0$$

From home labour market condition, we find the period 0 home output is:

$$\hat{h}_0 = \left[2\gamma\left(1 - \frac{\sigma}{\varepsilon}\theta\right) - 1\right]\hat{c}_0$$

The coefficient is negative for large θ. Thus, home output increases after a negative transfer: $\hat{h}_0 > 0$.

Therefore, under the flexible exchange rate regime with sticky wages, home consumption is lowered by the transfer from home to foreign, and output increases when the value of θ is reasonably large. Terms of trade falls as the nominal exchange rate depreciates ($\hat{s}_0 > 0$) after the transfer takes place. Home money balances and terms of trade are:

$$\hat{m}_0 = 0$$

$$\hat{\tau}_0 = \frac{\sigma}{(1-\gamma)\left(\beta + \varepsilon(1-\beta)\right)}\hat{c}_0 < 0$$

$\square$

4.3.4 Comparing the transfer effects under different policy regimes

We look at the effects of a negative transfer under the gold standard, the fixed exchange rate and the flexible exchange rate regimes, with both flexible and sticky wages in the previous sub-sections. Table 4.1 summarizes the results when wages are flexible.

Table 4.1: Effects of Transfer on Home Country, Flexible Wages

Regime	$\hat{c}_0$	Regime	$\hat{m}_0$
Gold Std	$\hat{c}_0^{GS} = \frac{1-\beta}{\Delta}\frac{dT_0}{PC} < 0$	Gold Std	$\hat{m}_0^{GS} = ((2\gamma-1)\Phi + \frac{\sigma}{\varepsilon})\hat{c}_0^{GS} < 0$
Fixed ER	$\hat{c}_0^{fixed} = \frac{1-\beta}{2(1-\gamma)[1-\Phi(1-2\gamma\theta)]}\frac{dT_0}{PC} < 0$	Fixed ER	$\hat{m}_0^{fixed} = ((2\gamma-1)\Phi + \frac{\sigma}{\varepsilon})2\hat{c}_0^{fixed} < 0$
Flexible ER	$\hat{c}_0^{flex} = \frac{1-\beta}{2(1-\gamma)[1-\Phi(1-2\gamma\theta)]}\frac{dT_0}{PC} < 0$	Flexible ER	$\hat{m}_0^{flex} = 0$
Regime	$\hat{h}_0$ $(\psi \to \infty)$	Regime	$\hat{\tau}_0$
Gold Std	$\hat{h}_0^{GS} = 0$	Gold Std	$\hat{\tau}_0^{GS} = 2\Phi\hat{c}_0^{GS} < 0$
Fixed ER	$\hat{h}_0^{fixed} = 0$	Fixed ER	$\hat{\tau}_0^{fixed} = 2\Phi\hat{c}_0^{fixed} < 0$
Flexible ER	$\hat{h}_0^{flex} = 0$	Flexible ER	$\hat{\tau}_0^{flex} = 2\Phi\hat{c}_0^{flex} < 0$
Regime	$\hat{h}_0$ $(\psi = 0)$	Regime	$\frac{dB_1}{PC}$
Gold Std	$\hat{h}_0^{GS} = -[1-2\gamma(1-\sigma\theta)]\hat{c}_0^{GS} > 0$	Gold Std	$\frac{dB_1^{GS}}{PC} = \beta\frac{dT_0}{PC}[1 - \frac{(1-\beta)\omega((2\gamma-1)\Phi+\frac{\sigma}{\varepsilon})}{\Delta}] < 0$
Fixed ER	$\hat{h}_0^{fixed} = -[1-2\gamma(1-\sigma\theta)]\hat{c}_0^{fixed} > 0$	Fixed ER	$\frac{dB_1^{fixed}}{PC} = \beta\frac{dT_0}{PC} < 0$
Flexible ER	$\hat{h}_0^{flex} = -[1-2\gamma(1-\sigma\theta)]\hat{c}_0^{flex} > 0$	Flexible ER	$\frac{dB_1^{flex}}{PC} = \beta\frac{dT_0}{PC} < 0$

where
$\Delta = 2(1-\gamma)(1-\Phi(1-2\theta\gamma)) + (1-\beta)\omega((2\gamma-1)\Phi + \frac{\sigma}{\varepsilon})$ and $\Phi = \frac{\sigma+\psi(2\gamma-1)}{2(1-\gamma)+4\psi\theta\gamma(1-\gamma)}$.

Several results can be drawn from Table 4.1. When wages are flexible, nominal exchange rate flexibility becomes immaterial in the fiat money world as prices and wages can fully adjust in response to the negative transfer. The change in consumption, output, terms of trade, and capital account are the same under the flexible and fixed exchange rates. The changes in money balances are different under these two regimes since the home government adjusts its money supply endogenously to maintain a pre-set exchange rate under the fixed exchange rate regime, while the money supply is exogenously determined under the flexible exchange rate.

When comparing the results to those of the gold standard, the negative transfer leads to a larger fall in consumption under the flexible and fixed exchange rate regimes. Terms of trade also deteriorates by a smaller amount under the gold standard than the other two fiat money regimes. When home pays the transfer to foreign country, there is an outflow of gold under the gold standard. Home country has a balance of payments deficit while the foreign has a surplus. Prices then fall in the home country and increase in foreign, which can be explained by the price-specie flow model of Hume (1752). The fall in home good demand due to the fall in home wealth will be offset by the increase in demand caused by the fall in price. Since the home and foreign governments cannot print money in this gold standard environment, when the foreign country imports more home produced goods, there's an export of gold from foreign back to the home country. Gold standard is then served as a shock absorber when a transfer takes place.

When the elasticity of labour supply is infinite, home output increases by a larger amount under the flexible and fixed exchange rate regimes. Since the negative transfer leads to a more severe terms of trade deterioration under the flexible and fixed exchange rate, home goods become relatively cheaper than foreign goods. This terms of trade effect induces a larger foreign demand for home goods. In addition, with home bias, home households also demand more home goods than foreign goods. Therefore, home output increases by a larger amount to meet the increase in demand due to the direct income effect of transfer, and the indirect effect through the terms of trade.

To summarize the degree of transfer effects when wages are flexible, we have:

Proposition 8 (Effects of transfer under flexible wages). *When wages are flexible, the transfer effects on home consumption, output, terms of trade and capital account are the same under both the fixed and flexible exchange rate regimes. Under the gold standard, however, home consumption falls by a smaller amount and output increases by less. Terms of trade also deteriorates by a smaller magnitude, which leads to a* *smaller fall in capital account. That is:*

$$
\begin{aligned}
0 &> \hat{c}_0^{GS} > \hat{c}_0^{flex} = \hat{c}_0^{fixed} \\
\hat{h}_0^{flex} &= \hat{h}_0^{fixed} > \hat{h}_0^{GS} > 0 \qquad (when\ \psi = 0) \\
0 &= \hat{m}_0^{flex} > \hat{m}_0^{GS} > \hat{m}_0^{fixed} \\
0 &> \hat{\tau}_0^{GS} > \hat{\tau}_0^{flex} = \hat{\tau}_0^{fixed} \\
0 &> \frac{dB_1^{GS}}{\bar{P}\bar{C}} > \frac{dB_1^{flex}}{\bar{P}\bar{C}} = \frac{dB_1^{fixed}}{\bar{P}\bar{C}}
\end{aligned}
$$

Proof. The proof follows the results of Table 4.1. □

Therefore, suspending the gold standard does not necessarily lessen the negative effects of a transfer on the home economy when prices and wages are flexible. Instead, gold standard can help to stabilize the donor country's economy in response to a negative transfer. What happens when there are nominal rigidities?

Table 4.2 summarizes the effects of transfer on the home economy when wages are sticky.

When wages are sticky, the transfer effects on consumption, output, terms of trade, and balance of payments under different exchange rate regimes cannot be compared analytically . We calibrate the transfer effects for comparison, and the parameter values are described in Table 4.3.

The coefficient of relative risk aversion, σ, usually takes values in the interval [1,6] in the literature, and we set $\sigma = 2$. We assume the discount factor, β, equals to 0.96, so that the steady-state real interest rate is about 4 percent. The elasticity of labour supply is set to unity (that is, $\psi = 1$), following Christiano, Eichenbaum, and Evans (1997). The elasticity of substitution between home and foreign goods is estimated to be between [1,2] in the literature (Chari et al. (1998)), and we set $\theta = 1.5$, following Backus et al. (1994). We assume $\gamma = 0.75$, that is, households have home bias towards domestically produced goods in both countries. The steady-state money balances to consumption ratio, ω, is set to 1, which matches the average value of M1 to nominal consumption ratio in the data.[69] Chari, Kehoe and McGrattan (2000) get an estimate of the elasticity of money demand of 0.39, and Hoffman, Rasche and Tieslau (1995)

[69] Collard and Dellas (2005) find that $\frac{M1}{PC} = 1.245$ in the period from 1960 to 2000 in the US, and find after 1980, the average value is about 0.75.

Table 4.2: Effects of Transfer on Home Country, Sticky Wages

Regime	$\hat{c}_0$
Gold Standard	$\hat{c}_0^{GS} = \frac{1}{[2(1-\gamma)+\frac{\omega\sigma}{\varepsilon}(1-\beta)]+[\frac{\beta}{1-\beta}\frac{\Delta\sigma}{\sigma+(2\gamma-1)\Phi}]}\frac{dT_0}{PC} < 0$
Fixed ER	$\hat{c}_0^{fixed} = \frac{2[\sigma+\Phi(2\gamma-1)]+\beta\sigma(\frac{1}{\varepsilon}-1)}{2(1-\gamma)[(\sigma+\Phi(2\gamma-1))+\frac{\beta}{1-\beta}\sigma(1-\Phi(1-2\gamma\theta))]}\frac{dT_0}{PC} < 0$
Flexible ER	$\hat{c}_0^{flex} = \frac{1}{2(1-\gamma)+(2\gamma\theta-1)\frac{\sigma}{\varepsilon}+\frac{\beta}{1-\beta}\Psi}\frac{dT_0}{PC} < 0$
Regime	$\hat{h}_0\ (\psi \geq 0)$
Gold Standard	$\hat{h}_0^{GS} = (2\gamma-1)\hat{c}_0^{GS} < 0$
Fixed ER	$\hat{h}_0^{fixed} = \frac{2\gamma(\sigma+\Phi(2\gamma-1))+\beta\sigma\left(\frac{1}{\varepsilon}-1\right)}{2(\sigma+\Phi(2\gamma-1))+\beta\sigma\left(\frac{1}{\varepsilon}-1\right)}\hat{c}_0^{fixed} < 0$
Flexible ER	$\hat{h}_0^{flex} = [2\gamma(1-\frac{\sigma}{\varepsilon}\theta)-1]\hat{c}_0^{flex} > 0$
Regime	$\hat{m}_0$
Gold Standard	$\hat{m}_0^{GS} = \frac{\sigma}{\varepsilon}\frac{1}{(2(1-\gamma)+\omega\frac{\sigma}{\varepsilon}(1-\beta))+(\frac{\Delta\sigma}{\sigma+(2\gamma-1)\Phi}\frac{\beta}{1-\beta})}\frac{dT_0}{PC} < 0$
Fixed ER	$\hat{m}_0^{fixed} = \frac{\sigma}{\varepsilon}\frac{1}{(1-\gamma)[1+\frac{\beta}{1-\beta}\frac{(1-\Phi(1-2\gamma\theta))\sigma}{\sigma+\Phi(2\gamma-1)}]}\frac{dT_0}{PC} < 0$
Flexible ER	$\hat{m}_0^{flex} = 0$
Regime	$\hat{\tau}_0$
Gold Standard	$\hat{\tau}_0^{GS} = 0$
Fixed ER	$\hat{\tau}_0^{fixed} = 0$
Flexible ER	$\hat{\tau}_0^{flex} = \frac{\sigma}{(1-\gamma)(\beta+\varepsilon(1-\beta))}\hat{c}_0^{flex} < 0$
Regime	$\frac{dB_1}{PC}$
Gold Standard	$\frac{dB_1}{PC}^{GS} = \frac{\beta}{1-\beta}\frac{\frac{\Delta\sigma}{\sigma+(2\gamma-1)\Phi}-\omega\frac{\sigma}{\varepsilon}(1-\beta)}{2(1-\gamma)+\omega\frac{\sigma}{\varepsilon}(1-\beta)+\frac{\Delta\sigma}{\sigma+(2\gamma-1)\Phi}\frac{\beta}{1-\beta}}\frac{dT_0}{PC} < 0$
Fixed ER	$\frac{dB_1}{PC}^{fixed} = \frac{\beta}{1-\beta}\frac{(1-\Phi(1-2\gamma\theta))\sigma}{(\sigma+\Phi(2\gamma-1))[1+\frac{\beta}{1-\beta}\frac{(1-\Phi(1-2\gamma\theta))\sigma}{\sigma+\Phi(2\gamma-1)}]}\frac{dT_0}{PC} < 0$
Flexible ER	$\frac{dB_1}{PC}^{flex} = \frac{\beta}{1-\beta}\frac{\Psi}{2(1-\gamma)+(2\gamma\theta-1)\frac{\sigma}{\varepsilon}+\frac{\beta}{1-\beta}\Psi}\frac{dT_0}{PC} < 0$

Table 4.3: Benchmark Calibration: Parameter Values

Parameter	Value	Description
σ	2	Coefficient of relative risk aversion
η	1	Weight on labour supply in period utility
ψ	1	Inverse of elasticity of labour supply
β	0.96	Discount factor (annual real interest rate = $\frac{1-\beta}{\beta}$)
ε	2	Inverse of elasticity of money
θ	1.5	Elasticity of substitution between imports and exports
γ	0.75	Relative preference for domestically produced goods
ω	1	Steady-state money balances to consumption ratio, $\frac{\bar{M}}{\bar{P}\bar{C}}$
$\eta\frac{\lambda}{\lambda-1}$	1.1	Gross steady-state markup

find the elasticity of money demand is about 0.5 in the US and Canada; thus, we set $\varepsilon = 2$. In the steady state, λ determines the markup of price over marginal costs in the wage-setting equation. We assume a 10 percent markup so that we get $\lambda = 11$.

Using these parameter values, we calibrate the effects of a transfer from home to foreign on different home variables, and the results are presented in Table 4.4.[70]

Table 4.4: Effects of Transfer on Home Country under Sticky Wages, Benchmark Calibration

Change in	Gold Standard	Fixed ER	Flexible ER
$\hat{c}_0$	-0.0369	-0.0640	-0.0194
$\hat{h}_0$	-0.0185	-0.0446	0.0340
$\hat{m}_0$	-0.0369	-0.0774	0
$\hat{\tau}_0$	0	0	-0.1495
$\frac{dB_1}{\bar{P}\bar{C}}$	-0.9447	-0.9807	-0.9660

[70]The calibration is performed by setting $\frac{dT_0}{\bar{P}\bar{C}} = -1$, that is, home transfers one unit of $\bar{P}\bar{C}$ to the foreign country.

Comparing these calibrated results, we can summarize the effects of transfer under sticky wages as follows:

Proposition 9 (Effects of transfer under sticky wages). *When wages are sticky, the responses of home economy to a negative transfer are different under the gold standard, the fixed exchange rate and the flexible exchange rate regimes. Consumption falls by the least amount when exchange rate is flexible, under the benchmark calibration. Output increases under the flexible exchange rate, but falls under the gold standard and fixed exchange rate. To summarize, we have:*

$$\begin{aligned}
0 &> \hat{c}_0^{flex} > \hat{c}_0^{GS} > \hat{c}_0^{fixed} \\
\hat{h}_0^{flex} &> 0 > \hat{h}_0^{GS} > \hat{h}_0^{fixed} \\
0 &= \hat{m}_0^{flex} > \hat{m}_0^{GS} > \hat{m}_0^{fixed} \\
0 &= \hat{\tau}_0^{GS} = \hat{\tau}_0^{fixed} > \hat{\tau}_0^{flex} \\
0 &> \frac{dB_1^{GS}}{\bar{P}\bar{C}} > \frac{dB_1^{flex}}{\bar{P}\bar{C}} > \frac{dB_1^{fixed}}{\bar{P}\bar{C}}
\end{aligned}$$

Proof. The results follow from Table 4.4. □

When there are nominal rigidities, the exchange rate regime matters. The effects of transfer on the home economy are different under the flexible and fixed exchange rate.

Under the benchmark calibration with sticky wages and flexible exchange rate, the negative transfer also leads to the smallest fall in consumption, just like the case when wages are flexible. Under the fixed exchange rate regime, consumption falls by the largest amount. Prices and wages are not able to adjust in response to the transfer; in addition, the home government needs to further adjust its money supply to maintain the fixed exchange rate, which exacerbates the negative effect of the transfer on consumption.

One major difference in the results under the flexible and sticky wage models is the transfer effects on home output (compare Table 4.1, 4.2 and 4.4). When wages are flexible, home output increases under all three exchange rate regimes after the transfer takes place.[71] The increase in output is due to the increase in foreign demand, since foreign wealth increases and home goods are relatively cheaper than foreign goods through the terms of trade adjustment.

When wages are sticky, home output falls under the gold standard and the fixed exchange rate. In the presence of home bias in domestically produced goods, the increase in foreign demand is not large enough to offset the fall in home demand due to the wealth effect. Besides, when wages are sticky, the terms of trade is irresponsive to the negative transfer under the gold standard and the fixed exchange rate. There is no indirect effect that increases home goods demand via the terms of trade deterioration, and hence, there is a net decrease in home output after a transfer when there are nominal rigidities.

However, when exchange rate is flexible and wages are sticky, output change remains positive after the transfer takes place, which is the same as in the flexible wage environment. When wages are sticky, the change in exchange rate affects the general price level at home. The exchange rate depreciation (and hence, terms of trade deterioration) at home increases foreign demand, which increases the home output. Together with the increase in foreign demand for home good that arises from the increase in foreign wealth, the net change in demand for home goods is positive. Therefore, a transfer from home leads to a rise in home output under the sticky-wage, flexible exchange rate environment.

Table 4.5 shows the transfer effects when we lower θ to 0.5, and ω to 0.5 respectively. The results in proposition 9 continue to hold when the elasticity of substitution between imports and exports, and the money to consumption ratio are low.[72] Since flexible exchange rate can reduce the fall in consumption and lead to an increase in output after a negative transfer, going off the gold helps stabilizing the home consumption fluctuations when there are nominal rigidities.

[71] This is true for $\psi \geq 0$.

[72] We find that the results in proposition 9 does not hold when there is no home bias ($\gamma = 0.5$), or when the value of ω is unreasonably large. In these cases, gold standard leads to a smaller drop in consumption than the flexible exchange rate after the transfer.

Table 4.5: Effects of Transfer on Home Country under Sticky Wages, Different Parameter Values

(a) $\theta = 0.5$

Change in	Gold Standard	Fixed ER	Flexible ER
$\hat{c}_0$	-0.2564	-0.6880	-0.2396
$\hat{h}_0$	-0.1282	-0.4880	0.05991
$\hat{m}_0$	-0.2564	-0.800	0
$\hat{\tau}_0$	0	0	-1.8433
$\frac{dB_1}{PC}$	-0.6154	-0.800	-0.9401

(b) $\omega = 0.5$

Change in	Gold Standard	Fixed ER	Flexible ER
$\hat{c}_0$	-0.0378	-0.0640	-0.0194 9
$\hat{h}_0$	-0.0189	-0.0446	0.0340
$\hat{m}_0$	-0.0378	-0.0774	0
$\hat{\tau}_0$	0	0	-0.1495
$\frac{dB_1}{PC}$	-0.9622	-0.9807	-0.9660

4.4 Welfare effect

In this section, we look at the effects of transfer on home welfare. We measure welfare by the change in consumer's utility. Recall that the home household's utility is given as:

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\chi}{1-\varepsilon} \left(\frac{M_t}{P_t} \right)^{1-\varepsilon} - \frac{\eta}{1+\psi} H_t^{1+\psi} \right) \tag{4.79}$$

Following Obstfeld and Rogoff (1995), we can separate the welfare changes into two parts by writing the utility as $\mathcal{U} = \mathcal{U}^R + \mathcal{U}^M$, where $\mathcal{U}^R$ represents the change in utility due to consumption and output changes, while $\mathcal{U}^M$ indicates the welfare changes that are induced by changes in real money balances. Since the economy reaches the steady state from period 1 onwards, therefore, we can write $\mathcal{U}^R$ and $\mathcal{U}^M$ as:

$$\mathcal{U}^R = \left(\frac{C_0^{1-\sigma}}{1-\sigma} - \frac{\eta}{1+\psi} H_0^{1+\psi} \right) + \frac{\beta}{1-\beta} \left(\frac{C_1^{1-\sigma}}{1-\sigma} - \frac{\eta}{1+\psi} H_1^{1+\psi} \right) \tag{4.80}$$

and

$$\mathcal{U}^M = \frac{\chi}{1-\varepsilon}\left(\frac{M_0}{P_0}\right)^{1-\varepsilon} + \frac{\beta}{1-\beta}\frac{\chi}{1-\varepsilon}\left(\frac{M_1}{P_1}\right)^{1-\varepsilon} \tag{4.81}$$

We approximate the welfare effects by total differentiating the household's utility around the steady state, and the changes in welfare become:

$$d\mathcal{U}^R = \bar{C}^{1-\sigma}\left[\hat{c}_0 - \Gamma\hat{h}_0 + \frac{\beta}{1-\beta}\left(\hat{c}_1 - \Gamma\hat{h}_1\right)\right] \tag{4.82}$$

and

$$d\mathcal{U}^M = \bar{C}^{1-\sigma}\Lambda\left[(\hat{m}_0 - \hat{p}_0) + \frac{\beta}{1-\beta}(\hat{m}_1 - \hat{p}_1)\right] \tag{4.83}$$

where $\Gamma = \frac{\lambda-1}{\lambda}\gamma^{\frac{1}{\theta-1}}\left[\gamma^{\frac{1}{1-\theta}} + (1-\gamma)^{\frac{1}{1-\theta}}\right]$ and $\Lambda = \chi^{\frac{1}{\varepsilon}}(1-\beta)^{\frac{\varepsilon-1}{\varepsilon}}\bar{C}^{\frac{\sigma}{\varepsilon}-1}$.

Obstfeld and Rogoff (1995) analyze the effects of consumption and output changes on welfare, but not the real-balance effect. When they study the effects of a monetary shock on $d\mathcal{U}^R$, they find that a monetary expansion has first order positive effect on welfare through the increase in world consumption. As marginal utility of money is positive in their model, positive monetary shock is welfare-improving, which would not reverse the positive welfare effect. Thus, they do not have to calculate $d\mathcal{U}^M$ and the results are still unambiguous.

However, this may not be true in our model, in which we focus on the effects of a transfer, especially when the economy is under the gold standard. When home makes a transfer to foreign, foreign country will increase its demand for home goods, as well as the demand for gold. This would lessen the fall in home consumption and lead to a smaller welfare loss under the gold standard through the changes in consumption and output. On the other hand, because of the fixed supply of gold in the world economy, the increase in demand for gold by the foreign country will make home to lose some of its gold in addition to the transfer. This additional transfer of gold from home to foreign may have negative welfare effects through the real balances. Therefore, it is important to include $d\mathcal{U}^M$ in our welfare analysis, as the fall in home gold holding may generate welfare loss which can overturn our results.

We calibrate the effects of negative transfer on home's welfare using (4.82) and (4.83), and the calibrated results of the benchmark model are presented in Table 4.6.

When wages are flexible, a transfer from home to foreign country lowers the welfare of home country through the fall in consumption and disutility of labour supply

Table 4.6: Effects of Transfer on Welfare, Benchmark Calibration

Flexible Wages Regime	$d\mathcal{U}^R$	+	$d\mathcal{U}^M$	=	$d\mathcal{U}$
Gold Std	-25.9616		-0.085198		-26.0468
Fixed ER	-27.2187		-0.089323		-27.3080
Flex ER	-27.2187		-0.089323		-27.3080
Sticky Wages Regime	$d\mathcal{U}^R$	+	$d\mathcal{U}^M$	=	$d\mathcal{U}$
Gold Std	-24.5389		-0.052188		-24.5911
Fixed ER	-25.0603		-0.097702		-25.1580
Flex ER	-27.2161		-0.091165		-27.3072

($d\mathcal{U}_R$). The welfare drops by a larger amount under the flexible (and fixed) exchange rate than that under the gold standard. The fall in real money balances also worsens the home welfare ($d\mathcal{U}_M$), but by a much smaller magnitude. Although home loses some of its gold holding in addition to the transfer, its impact is not large enough to wash out the "gain" in welfare by staying in the gold standard. Money balances also lower the welfare, under both the flexible and fixed exchange rates. This suggests that going off the gold standard will worsen the welfare of the donor country when wages are fully flexible.

Similar results hold when wages are sticky. From Table 4.6, a negative transfer leads to a smaller welfare drop under the gold standard than under the other exchange rate regimes. The welfare falls by the largest amount under the flexible exchange rate, since the increase in home output due to direct wealth effect and indirect terms of trade effect lead to a large disutility of labour, which makes households worse off. Thus, the donor country would be better off if it stayed in the gold standard after a transfer takes place, even if there are nominal rigidities.

We find that the welfare ranking is insensitive to the value of θ and γ, but is sensitive to the value of ω. Table 4.7 shows the welfare effect when $\omega = 0.5$. We find that the gold standard gives the highest welfare when wages are flexible, and the fixed exchange rate regime gives the highest when there are nominal rigidities. Flexible exchange rate regime, however, yields the lowest welfare under both the flexible wages

Table 4.7: Effects of Transfer on Welfare, Low ω ($\omega = 0.5$)

Flexible Wages Regime	$d\mathcal{U}^R$	+	$d\mathcal{U}^M$	=	$d\mathcal{U}$
Gold Std	-26.5753		-0.043606		-26.6189
Fixed ER	-27.2187		-0.044662		-27.2633
Flex ER	-27.2187		-0.044662		-27.2633
Sticky Wages Regime	$d\mathcal{U}^R$	+	$d\mathcal{U}^M$	=	$d\mathcal{U}$
Gold Std	-25.1194		-0.026718		-25.1461
Fixed ER	-25.0603		-0.048851		-25.1091
Flex ER	-27.2161		-0.045583		-27.2617

and sticky wages, and this result holds for any reasonable value of ω.[73]

4.5 Concluding remarks

We develop a two-country model to study the transfer problem under different exchange rate regimes. Keynes' "orthodox" view is also justified as the terms of trade deteriorates regardless of the choice of exchange rate systems. The conventional wisdom suggests it is optimal for countries to abandon the gold standard in face of economic crises such as war or economic downturn. In terms of stabilizing the domestic consumption, our model suggests that the donor country should follow the conventional wisdom and go floating. However, our welfare analysis implies the transfer paying country is better off to stay within the gold standard system since the gold standard regime can absorb some of the negative effects from the transfer, even with nominal rigidities.

The gold standard or the fixed exchange rate, however, is difficult to be maintained by the developed countries in today's highly integrated world. The costs of losing domestic policy autonomy are larger than the benefits from exchange rate stability to the developed countries. Developing countries, on the other hand, tend to go for exchange rate stability because of the "fear of floating". The existence of this bipolarity cannot be explained in this simple model. Furthermore, capital and investment

[73]For some unreasonably large values of ω, we find that instead of the flexible exchange rate, but the fixed exchange rate gives the lowest welfare under the sticky wages.

are absent from this model. A promising future extension to this study would be to include capital in the model to see if the same surprising results remind.

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Appendix A

Appendix for Chapter 2

A.1 Proof of Proposition 1

Proof.

For region I, $\Delta(M|_{s=1}) > \kappa$, firm i's net gain from price adjustment is positive when $s = 1$. In this region, $\Delta(M|_{s=0}) < \kappa$, but $s = 0$ is not sustainable as every firm expects other firms to adjust after the shocks. To avoid loss in possible gain from price adjustment, a marginal firm i will choose to adjust and we have $s = 1$.

For region II, since $\forall s$, $\Delta(M, s) < \kappa$ when $M \in [M_l, M_h)$. Firm i's net gain from price adjustment is negative, and hence, it will not adjust its price ex-post. Therefore, $s = 0$.

For region IV, the results follow from equation (2.8) that $\forall s$, $\Delta(M, s) > \kappa$, firm i's net gain from price adjustment is positive. Then it decreases its prices in response to negative shocks with probability 1.

For region III, suppose M' lies between M_{ll} and M_l such that: $M_{ll} < M' < M_l < \bar{M}$. We already have $\Delta(M \leq M_{ll}, s) > \kappa$, $\Delta(M \in (M_l, \bar{M}), s) < \kappa$, and given $\Delta(M)$ is continuous and concave up (convex) with $\Delta(\bar{M}) = 0$.[74] Then $\exists\ M' \in (M_{ll}, M_l)$ such that $\Delta(M', s) = \kappa$.

Since M_{ll} is the intersect of $\Delta(M|_{s=1})$ and κ, then $\Delta(M_{ll}|_{s=1}) = \kappa$. Similarly, $\Delta(M|_{s=0})$ and κ intersect at M_l, then $\Delta(M_l|_{s=0}) = \kappa$. The convexity and continuity of Δ imply:

$$\Delta(M'|_{s=1}) < \kappa \quad and \quad \Delta(M'|_{s=0}) > \kappa$$

[74] $\Delta(\bar{M}) = 0$ can be shown using equations (2.6) and (2.7).

Therefore, for a given M', $\exists\ s \in (0,1)$ such that $\Delta(M'|_{s\in(0,1)}) = \kappa$. $\square$

Appendix B

Appendix for Chapter 3

B.1 Solving the model

B.1.1 Solutions for the unconstrained economy

The unconstrained economy is described by the following equations:

$$
\begin{aligned}
P_h &= \kappa \frac{W^{\omega}(Sq^*)^{1-\omega}}{A} && \text{(B.1)}\\
PC &= WH = \omega P_h Y && \text{(B.2)}\\
M &= \chi PC && \text{(B.3)}\\
W &= \eta \frac{\rho}{\rho - 1} \frac{E\{H^{1+\psi}\}}{E\left\{\frac{H}{PC}\right\}} && \text{(B.4)}\\
Y &= \alpha\omega Y + \tilde{X}\frac{S}{P_h} && \text{(B.5)}
\end{aligned}
$$

These five equations are used to solve for the five variables: $\{W, H(\sigma), P_h(\sigma), S(\sigma), C(\sigma)\}$, where σ is the ex-post state of the world, depending on the foreign demand, $\tilde{X}$.

Equations (B.2) and (B.4) are used to derive the consumption expression. We combine equations (B.2) and (B.3) to get the employment solution. The resulting solutions are:

$$
\begin{aligned}
C^u &= \frac{\omega}{1-\alpha\omega}\frac{\tilde{X}S^u}{P^u} && \text{(B.6)}\\
H^u &= \frac{M}{\chi W^u} && \text{(B.7)}
\end{aligned}
$$

We substitute this employment solution into equation (B.4) to solve for the wage, given by:

$$W^u = \left[\eta\frac{\rho}{\rho-1}\right]^{\frac{1}{1+\psi}} \frac{1}{\chi}\left[E\left\{M^{1+\psi}\right\}\right]^{\frac{1}{1+\psi}} \tag{B.8}$$

(B.1) gives the price of home produced goods, and the nominal exchange rate can be solved by using equations (B.2), (B.3) and (B.4).

Using these, we may prove Proposition 2.

B.1.2 Proof of Proposition 2

Proof. From (B.6), (B.7), and (B.8), we may write out expected utility in the unconstrained economy as:

$$EU = \Gamma + E\omega \ln\left(\frac{S}{W}\right) \tag{B.9}$$

where Γ is a constant function of parameters. Without loss of generality, assume that $\tilde{X}$ takes on a discrete distribution $\tilde{X} \in \{X(1)..X(N)\}$, with probabilities $\{\pi_1..\pi_N\}$. Let the monetary policy be defined as $M_i = M(X_i)$, $i = 1..N$. Since there are N states, there are only $N-1$ degrees of freedom with respect to monetary policy. Hence we normalize so that $M_1 = 1$. Then (B.9) may be re-written as:

$$EU = \Gamma' + E\omega \ln\left(\frac{M}{E\left\{M^{1+\psi}\right\})^{\frac{1}{1+\psi}}}\right) \tag{B.10}$$

where Γ' is again a constant function of parameters. The first order condition for an expected utility maximizing choice of monetary policy is

$$\frac{\pi_i}{M_i} = \pi_i \frac{M_i^{\psi}}{E\left\{M^{1+\psi}\right\}^{\frac{1}{1+\psi}}} \tag{B.11}$$

From (B.11), it is clear that $M_i = M = 1$, for all i. This establishes the proof.

□

B.1.3 Solutions for the constrained economy

The corresponding representation for the *constrained* economy is as follows:

$$P_h \omega \frac{Y}{H} = W \tag{B.12}$$

$$PC = P_h Y - Sq^* I = P_h Y - N + SD^* \tag{B.13}$$

$$M = \chi PC = \chi \left[P_h Y - N + SD^*\right] \tag{B.14}$$

$$W = \eta \frac{\rho}{\rho - 1} \frac{E\{H^{1+\psi}\}}{E\left\{\frac{H}{PC}\right\}} \tag{B.15}$$

$$Y = \alpha \left(Y - \frac{N - SD^*}{P_h} \right) + \tilde{X} \frac{S}{P_h} \tag{B.16}$$

$$Y = AH^{\omega} \left(\frac{N - SD^*}{Sq^*} \right)^{1-\omega} \tag{B.17}$$

These six equations can be solved for the six variables: $\{W, H(\sigma), P_h(\sigma), S(\sigma), C(\sigma), Y(\sigma)\}$, where σ is the ex-post state of the world.

Using (B.13), (B.14), and (B.16), we can solve for the nominal exchange rate:

$$S^c = \frac{1-\alpha}{\chi} \frac{M}{\tilde{X} + D^*} + \frac{N}{\tilde{X} + D^*} \tag{B.18}$$

From (A12), (A13) and (A14), we can obtain the solution for consumption and employment as:

$$C^c = \frac{\omega}{1-\alpha} \left[\frac{\tilde{X} S^c}{P^c} - \alpha \left(\frac{N}{P^c} - \frac{S^c D^*}{P^c} \right) \right] \tag{B.19}$$

$$H^c = \frac{\omega \left[\frac{M}{\chi} + N - S^c D^* \right]}{W^c} \tag{B.20}$$

B.1.4 Proof of Proposition 3

Proof. Take a social planner who wishes to maximize utility of the home agent, facing the foreign demand function (3.16). The constraints faced by the planner are:

$$P_h C_h + C_m = P_h A H^{\omega} I^{1-\omega} - I \tag{B.21}$$

$$P_h A H^{\omega} I^{1-\omega} = C_h + \tilde{X} \tag{B.22}$$

The planner will choose the consumption allocation so that $C_m = \frac{1-\alpha}{\alpha} P_h C_h$. Using this in (B.21) and (B.22), we can express the problem of the planner as the choice of I and H so as to maximize:

$$EU = \ln(\tilde{X} - I) - \alpha \ln(\tilde{X} - \alpha I) + \alpha\omega ln(H) + \alpha(1-\omega)\ln(I) - \eta \frac{H^{1+\psi}}{1+\psi} \tag{B.23}$$

The planner's optimal choice of H and I are represented as

$$H = \left(\frac{\alpha\omega}{\eta}\right)^{\frac{1}{1+\psi}}$$

$$I = \tilde{X}\left[\frac{1}{2}\,\frac{\omega\,\alpha^2 - \alpha - 1 + \alpha\,\omega + \sqrt{\omega^2\,\alpha^4 + 2\,\omega\,\alpha^3 - 2\,\omega^2\,\alpha^3 - 3\,\alpha^2 + 2\,\alpha + 1 - 2\,\alpha\,\omega + \alpha^2\,\omega^2}}{\alpha\,(-1 + \alpha\,\omega)}\right]$$

Using the taxes described in the Proposition, we may ensure that employment and intermediate imports satisfies these two equations. □

B.2 Numerical solution

We look for the optimal monetary policy in the constrained region numerically. We assume the monetary policy rule is some function of the state of the world, $\tilde{X}$. Denote this policy rule as $\mathbf{M}(\tilde{X})$. In particular, assume $\tilde{X}$ takes on three realizations: $\tilde{X} = \{\tilde{X}_1, \tilde{X}_2, \tilde{X}_3\}$ with probabilities $\{\pi_1, \pi_2, \pi_3\}$. We choose $\tilde{X}$ such that, for a given leverage ratio, collateral constraint becomes binding when the foreign demand for home good is $\tilde{X}_3$.

The optimal monetary policy rule in the constrained region is the vector of state-contingent money response, $M_i = M(\tilde{X}_i)$, $i = \{1,2,3\}$, that maximizes the expected utility of home households. The optimal policy is solved in the following steps.

1. Set the initial state-contingent money response as:

$$\mathbf{M^0} = [M_1^0, M_2^0, M_3^0] = [1,1,1] \tag{B.24}$$

2. Solve for the preset optimal nominal wage, given $\mathbf{M^0}$:

$$W(\mathbf{M^0}) = \Lambda \frac{E\{H(\mathbf{M^0})^{1+\psi}\}}{E\left\{\frac{H(\mathbf{M^0})}{P(\mathbf{M^0})C(\mathbf{M^0})}\right\}} = \Lambda \frac{\sum_{i=1}^{3}\{H(M_i^0)^{1+\psi}\}}{\sum_{i=1}^{3}\left\{\frac{H(M_i^0)}{P(M_i^0)C(M_i^0)}\right\}} \tag{B.25}$$

   where Λ is a function of constant parameters. Given this wage, we can solve for the other variables.

3. Use $W(\mathbf{M^0})$ to solve for other variables and compute the expected utility. Denote the resulting expected utility as EU_0.

4. Define a vector $\delta_{\mathbf{M}}{}^j = [\delta_{M1}^j, \delta_{M2}^j, \delta_{M3}^j]$, $j \in \mathcal{J}$, where $\mathcal{J}$ is the policy space. This vector represents the exogenous money change of policy j, carried out by the policymaker. Denote the new state-contingent money response as $\mathbf{M^1}$, given by:

$$\mathbf{M^1} = \mathbf{M^0} - \delta_{\mathbf{M}}{}^j \tag{B.26}$$

5. Given $\mathbf{M^1}$, solve for the preset optimal nominal wage $W(\mathbf{M^1})$ as in Step 2. Then compute the expected utility as in Step 3, denoted it by EU_1.

6. Repeat Steps 4 and 5 n times. The state-contingent money response at the k^{th} iteration is:

$$\mathbf{M^k} = \mathbf{M^0} - k\delta_{\mathbf{M}}{}^j, \tag{B.27}$$

and the corresponding expected utility is EU_k.

7. The optimal money supply under policy j is the $\mathbf{M^k}$ that gives the highest expected utility. Call the highest expected utility under policy j as EU^j_{max}

8. Repeat Steps 4 to 7 for different money policies (characterized by $\delta_{\mathbf{M}}{}^j$, $j \in \mathcal{J}$).

9. Compare EU^j_{max}, $\forall j \in \mathcal{J}$. The optimal policy rule is characterized by the $\delta_{\mathbf{M}}{}^j$ that gives the highest EU^j_{max}.

We choose the following parameters values to illustrate our numerical results:

Table B.1: Parameter Values for the Unconstrained Model

ψ	0	Elasticity of labour supply
χ	1	Coefficient on money balance in utility function
ω	0.6	Share of labour in Cobb-Douglas production function
α	0.5	Consumption share in home goods
Λ	1	Coefficient on the optimal nominal wage
A	1	Productivity in Cobb-Douglas function
q^*	1	Foreign price of imported intermediate goods
P_f^*	1	Foreign price of foreign produced final goods

Appendix C

Appendix for Chapter 4

C.1 Equilibrium conditions under the gold standard

(a) Flexible wage

The log-linearized version of conditions (4.14) to (4.26) are described as follows:

$$\hat{m}_t - \hat{p}_t = \frac{\sigma}{\varepsilon}\hat{c}_t - \frac{1}{\varepsilon}\left(\frac{1}{1+\bar{i}}\right)\hat{i}_{t+1} \tag{C.1}$$

$$\hat{m}_t^* - \hat{p}_t^* = \frac{\sigma}{\varepsilon}\hat{c}_t^* - \frac{1}{\varepsilon}\left(\frac{1}{1+\bar{i}}\right)\hat{i}_{t+1} \tag{C.2}$$

$$\hat{w}_t = \hat{p}_t + \sigma\hat{c}_t + \psi\hat{h}_t \tag{C.3}$$

$$\hat{w}_t^* = \hat{p}_t^* + \sigma\hat{c}_t^* + \psi\hat{h}_t^* \tag{C.4}$$

$$\hat{h}_t = -\theta\hat{p}_{Xt} + \gamma\hat{c}_t + (1-\gamma)\hat{c}_t^* + \gamma\theta\hat{p}_t + (1-\gamma)\theta\hat{p}_t^* \tag{C.5}$$

$$\hat{h}_t^* = -\theta\hat{p}_{Mt} + (1-\gamma)\hat{c}_t + \gamma\hat{c}_t^* + (1-\gamma)\theta\hat{p}_t + \gamma\theta\hat{p}_t^* \tag{C.6}$$

$$\hat{c}_t + \frac{dB_{t+1}}{\bar{P}\bar{C}} + \omega\hat{m}_t = (\hat{p}_{Xt} - \hat{p}_t) + \hat{h}_t + \omega\hat{m}_{t-1} + \frac{dT_t}{\bar{P}\bar{C}} + (1+\bar{i})\frac{dB_t}{\bar{P}\bar{C}} \tag{C.7}$$

$$
\begin{aligned}
-\frac{\bar{i}}{1+\bar{i}}\hat{i}_{t+1} &= \hat{p}_t + \sigma\hat{c}_t - \hat{p}_{t+1} - \sigma\hat{c}_{t+1} && \text{(C.8)} \\
-\frac{\bar{i}}{1+\bar{i}}\hat{i}_{t+1} &= \hat{p}_t^* + \sigma\hat{c}_t^* - \hat{p}_{t+1}^* - \sigma\hat{c}_{t+1}^* && \text{(C.9)}
\end{aligned}
$$

$$
\begin{aligned}
\hat{p}_t &= \gamma\hat{p}_{Xt} + (1-\gamma)\hat{p}_{Mt}^* && \text{(C.10)} \\
\hat{p}_t^* &= (1-\gamma)\hat{p}_{Xt} + \gamma\hat{p}_{Mt}^* && \text{(C.11)}
\end{aligned}
$$

$$
\begin{aligned}
\hat{p}_{Xt} &= \hat{w}_t && \text{(C.12)} \\
\hat{p}_{Mt}^* &= \hat{w}_t^* && \text{(C.13)}
\end{aligned}
$$

where:

$\omega = \frac{\bar{M}}{\bar{P}\bar{C}} = \left(\frac{\chi}{1-\beta}\right)^{1-\varepsilon}\left[\gamma^{\frac{1}{\theta-1}}\frac{1}{\eta}\frac{\lambda-1}{\lambda}\left(\gamma^{\frac{1}{1-\theta}} + (1-\gamma)^{\frac{1}{1-\theta}}\right)^{-\psi}\right]^{\frac{\frac{\sigma}{\varepsilon}-1}{\sigma+\psi}}$,

$dZ_t = Z_t - \bar{Z}$. Also, at equilibrium, $\frac{1}{1+\bar{i}} = \beta$.

(b) Sticky wage

The equations describing the first period (period 0) can be described as:

$$
\begin{aligned}
\frac{M_0}{P_0} &= \frac{\chi^{\frac{1}{\varepsilon}} C_0^{\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_1}\right)^{\frac{1}{\varepsilon}}} && \text{(C.14)} \\
\frac{M_0^*}{P_0^*} &= \frac{\chi^{\frac{1}{\varepsilon}} C_0^{*\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_1}\right)^{\frac{1}{\varepsilon}}} && \text{(C.15)}
\end{aligned}
$$

$$
\begin{aligned}
H_0 &= \gamma\left(\frac{P_{X0}}{P_0}\right)^{-\theta} C_0 + (1-\gamma)\left(\frac{P_{X0}}{P_0^*}\right)^{-\theta} C_0^* && \text{(C.16)} \\
H_0^* &= (1-\gamma)\left(\frac{P_{M0}}{P_0}\right)^{-\theta} C_0 + \gamma\left(\frac{P_{M0}}{P_0^*}\right)^{-\theta} C_0^* && \text{(C.17)}
\end{aligned}
$$

$$P_0C_0 + B_1 + M_0 = P_{X0}H_0 + M_{-1} + T_0 \tag{C.18}$$

$$\frac{1}{1+i_1} = \beta\frac{P_0C_0^{\sigma}}{P_1C_1^{\sigma}} \tag{C.19}$$

$$\frac{1}{1+i_1} = \beta\frac{P_0^*C_0^{*\sigma}}{P_1^*C_1^{*\sigma}} \tag{C.20}$$

$$\bar{M} = M_0 + M_0^* \tag{C.21}$$

This system gives 8 equations in 8 variables: $\{C_0, C_0^*, M_0, M_0^*, H_0, H_0^*, B_1, i_1\}$.

We log-linearized these conditions and get the following:

$$\hat{m}_0 = \frac{\sigma}{\varepsilon}\hat{c}_0 - \frac{1}{\varepsilon}\left(\frac{1}{1+\bar{i}}\right)\hat{i}_1 \tag{C.22}$$

$$\hat{m}_0^* = \frac{\sigma}{\varepsilon}\hat{c}_0^* - \frac{1}{\varepsilon}\left(\frac{1}{1+\bar{i}}\right)\hat{i}_1 \tag{C.23}$$

$$\hat{h}_0 = \gamma\hat{c}_0 + (1-\gamma)\hat{c}_0^* \tag{C.24}$$

$$\hat{h}_0^* = (1-\gamma)\hat{c}_0 + \gamma\hat{c}_0^* \tag{C.25}$$

$$\hat{c}_0 + \frac{dB_1}{\bar{P}\bar{C}} + \omega\hat{m}_0 = \hat{h}_0 + \frac{dT_0}{\bar{P}\bar{C}} \tag{C.26}$$

$$-\frac{\bar{i}}{1+\bar{i}}\hat{i}_1 = \sigma\hat{c}_0 - \hat{p}_1 - \sigma\hat{c}_1 \tag{C.27}$$

$$-\frac{\bar{i}}{1+\bar{i}}\hat{i}_1 = \sigma\hat{c}_0^* - \hat{p}_1^* - \sigma\hat{c}_1^* \tag{C.28}$$

$$0 = \hat{m}_0 + \hat{m}_0^* \tag{C.29}$$

C.2 Equilibrium conditions under fixed exchange rate regime

(a) Flexible wage

The full equilibrium conditions under fixed exchange rate regime with flexible wages are as follows:

$$\frac{M_t}{P_t} = \frac{\chi^{\frac{1}{\varepsilon}} C_t^{\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_{t+1}}\right)^{\frac{1}{\varepsilon}}} \tag{C.30}$$

$$\frac{M_t^*}{P_t^*} = \frac{\chi^{\frac{1}{\varepsilon}} C_t^{*\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_{t+1}^*}\right)^{\frac{1}{\varepsilon}}} = \frac{\chi^{\frac{1}{\varepsilon}} C_t^{*\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_{t+1}}\frac{\bar{S}}{\bar{S}}\right)^{\frac{1}{\varepsilon}}} = \frac{\chi^{\frac{1}{\varepsilon}} C_t^{*\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_{t+1}}\right)^{\frac{1}{\varepsilon}}} \tag{C.31}$$

$$W_t = \eta \frac{\lambda}{\lambda - 1} P_t C_t^{\sigma} H_t^{\psi} \tag{C.32}$$

$$W_t^* = \eta \frac{\lambda}{\lambda - 1} P_t^* C_t^{*\sigma} H_t^{*\psi} \tag{C.33}$$

$$H_t = \gamma \left(\frac{P_{Xt}}{P_t}\right)^{-\theta} C_t + (1-\gamma) \left(\frac{P_{Xt}}{\bar{S} P_t^*}\right)^{-\theta} C_t^* \tag{C.34}$$

$$H_t^* = (1-\gamma) \left(\frac{\bar{S} P_{Mt}^*}{P_t}\right)^{-\theta} C_t + \gamma \left(\frac{P_{Mt}^*}{P_t^*}\right) C_t^* \tag{C.35}$$

$$P_t C_t + B_{t+1} + M_t = W_t H_t + M_{t-1} + \mu_t + T_t + (1+i_t) B_t \tag{C.36}$$

$$\frac{1}{1+i_{t+1}} = \beta \frac{P_t C_t^{\sigma}}{P_{t+1} C_{t+1}^{\sigma}} \tag{C.37}$$

$$\frac{1}{1+i_{t+1}} = \beta \frac{P_t^* C_t^{*\sigma}}{P_{t+1}^* C_{t+1}^{*\sigma}} \frac{\bar{S}}{\bar{S}} = \beta \frac{P_t^* C_t^{*\sigma}}{P_{t+1}^* C_{t+1}^{*\sigma}} \tag{C.38}$$

$$P_t = \left[\gamma P_{Xt}^{1-\theta} + (1-\gamma)(\bar{S}P_{Mt}^{*})^{1-\theta}\right]^{\frac{1}{1-\theta}} \quad (C.39)$$

$$P_t^* = \left[(1-\gamma)\left(\frac{P_{Xt}}{\bar{S}}\right)^{1-\theta} + \gamma P_{Mt}^{*1-\theta}\right]^{\frac{1}{1-\theta}} \quad (C.40)$$

$$P_{Xt} = W_t \quad (C.41)$$

$$P_{Mt}^* = W_t^* \quad (C.42)$$

$$M_t = M_{t-1} + \mu_t \quad (C.43)$$

$$M_t^* = M_{t-1}^* + \mu_t^* \quad (C.44)$$

Now, the terms of trade is:

$$TOT = \frac{P_{Xt}}{P_{Mt}} = \frac{P_{Xt}}{\bar{S}P_{Mt}^*} \quad (C.45)$$

The log-linearized version of conditions (C.30) to (C.42) are described as follows:

$$\hat{m}_t - \hat{p}_t = \frac{\sigma}{\varepsilon}\hat{c}_t - \frac{1}{\varepsilon}\left(\frac{1}{1+\bar{i}}\right)\hat{i}_{t+1} \quad (C.46)$$

$$\hat{m}_t^* - \hat{p}_t^* = \frac{\sigma}{\varepsilon}\hat{c}_t^* - \frac{1}{\varepsilon}\left(\frac{1}{1+\bar{i}}\right)\hat{i}_{t+1} \quad (C.47)$$

$$\hat{w}_t = \hat{p}_t + \sigma\hat{c}_t + \psi\hat{h}_t \quad (C.48)$$

$$\hat{w}_t^* = \hat{p}_t^* + \sigma\hat{c}_t^* + \psi\hat{h}_t^* \quad (C.49)$$

$$\hat{h}_t = -\theta\hat{p}_{Xt} + \gamma\hat{c}_t + (1-\gamma)\hat{c}_t^* + \gamma\theta\hat{p}_t + (1-\gamma)\theta\hat{p}_t^* \quad (C.50)$$

$$\hat{h}_t^* = -\theta\hat{p}_{Mt}^* + (1-\gamma)\hat{c}_t + \gamma\hat{c}_t^* + (1-\gamma)\theta\hat{p}_t + \gamma\theta\hat{p}_t^* \quad (C.51)$$

$$\hat{c}_t + \frac{dB_{t+1}}{\bar{P}\bar{C}} + \omega\hat{m}_t = (\hat{p}_{Xt} - \hat{p}_t) + \hat{h}_t + \omega\hat{m}_{t-1} + \frac{d\mu_t}{\bar{P}\bar{C}} + \frac{dT_t}{\bar{P}\bar{C}} + (1+\bar{i})\frac{dB_t}{\bar{P}\bar{C}} \quad (C.52)$$

$$-\frac{\bar{i}}{1+\bar{i}}\hat{i}_{t+1} = \hat{p}_t + \sigma\hat{c}_t - \hat{p}_{t+1} - \sigma\hat{c}_{t+1} \quad \text{(C.53)}$$

$$-\frac{\bar{i}}{1+\bar{i}}\hat{i}_{t+1} = \hat{p}_t^* + \sigma\hat{c}_t^* - \hat{p}_{t+1}^* - \sigma\hat{c}_{t+1}^* \quad \text{(C.54)}$$

$$\hat{p}_t = \gamma\hat{p}_{Xt} + (1-\gamma)\hat{p}_{Mt}^* \quad \text{(C.55)}$$

$$\hat{p}_t^* = (1-\gamma)\hat{p}_{Xt} + \gamma\hat{p}_{Mt}^* \quad \text{(C.56)}$$

$$\hat{p}_{Xt} = \hat{w}_t \quad \text{(C.57)}$$

$$\hat{p}_{Mt}^* = \hat{w}_t^* \quad \text{(C.58)}$$

$$\hat{m}_t = \hat{m}_{t-1} + \frac{d\mu_t}{\bar{M}} \quad \text{(C.59)}$$

$$\hat{m}_t^* = \hat{m}_{t-1}^* + \frac{d\mu_t^*}{\bar{M}} \quad \text{(C.60)}$$

(b) Sticky wage

The period 0 equilibrium conditions now become:

$$\frac{M_0}{P_0} = \frac{\chi^{\frac{1}{\varepsilon}} C_0^{\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_1}\right)^{\frac{1}{\varepsilon}}} \quad \text{(C.61)}$$

$$\frac{M_0^*}{P_0^*} = \frac{\chi^{\frac{1}{\varepsilon}} C_0^{*\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_1^*}\right)^{\frac{1}{\varepsilon}}} = \frac{\chi^{\frac{1}{\varepsilon}} C_0^{*\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_1}\frac{S}{\bar{S}}\right)^{\frac{1}{\varepsilon}}} = \frac{\chi^{\frac{1}{\varepsilon}} C_0^{*\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_1}\right)^{\frac{1}{\varepsilon}}} \quad \text{(C.62)}$$

$$H_0 = \gamma \left(\frac{P_{X0}}{P_0}\right)^{-\theta} C_0 + (1-\gamma) \left(\frac{P_{X0}}{\bar{S} P_0^*}\right)^{-\theta} C_0^* \tag{C.63}$$

$$H_0^* = (1-\gamma) \left(\frac{\bar{S} P_{M0}^*}{P_0}\right)^{-\theta} C_0 + \gamma \left(\frac{P_{M0}^*}{P_0^*}\right)^{-\theta} C_0^* \tag{C.64}$$

$$P_0 C_0 + B_1 + M_0 = W_0 H_0 + M_{-1} + \mu_0 + T_0 + (1+i_0) B_0 \tag{C.65}$$

$$\frac{1}{1+i_1} = \beta \frac{P_0 C_0^\sigma}{P_1 C_1^\sigma} \tag{C.66}$$

$$\frac{1}{1+i_1} = \beta \frac{P_0^* C_0^{*\sigma}}{P_1^* C_1^{*\sigma}} \frac{\bar{S}}{\bar{S}} = \beta \frac{P_0^* C_0^{*\sigma}}{P_1^* C_1^{*\sigma}} \tag{C.67}$$

$$M_0 = M_{-1} + \mu_0 \tag{C.68}$$

$$M_0^* = M_{-1}^* + \mu_0^* \tag{C.69}$$

We log-linearized these conditions and get the following:

$$\hat{m}_0 = \frac{\sigma}{\varepsilon} \hat{c}_0 - \frac{\beta}{\varepsilon} \hat{i}_1 \tag{C.70}$$

$$\hat{m}_0^* = \frac{\sigma}{\varepsilon} \hat{c}_0^* - \frac{\beta}{\varepsilon} \hat{i}_1 \tag{C.71}$$

$$\hat{h}_0 = \gamma \hat{c}_0 + (1-\gamma) \hat{c}_0^* \tag{C.72}$$

$$\hat{h}_0^* = (1-\gamma) \hat{c}_0 + \gamma \hat{c}_0^* \tag{C.73}$$

$$\hat{c}_0 + \frac{dB_1}{\bar{P}\bar{C}} + \omega \hat{m}_0 = \hat{h}_0 + \omega \hat{m}_{-1} + \frac{d\mu_0}{\bar{P}\bar{C}} + \frac{dT_0}{\bar{P}\bar{C}} + \frac{1}{\beta} \frac{dB_0}{\bar{P}\bar{C}} \tag{C.74}$$

$$-\frac{\bar{i}}{1+\bar{i}} \hat{i}_1 = \sigma \hat{c}_0 - \hat{p}_1 - \sigma \hat{c}_1 \tag{C.75}$$

$$-\frac{\bar{i}}{1+\bar{i}} \hat{i}_1 = \sigma \hat{c}_0^* - \hat{p}_1^* - \sigma \hat{c}_1^* \tag{C.76}$$

$$\hat{m}_0 = \hat{m}_{-1} + \frac{d\mu_0}{\bar{M}} \tag{C.77}$$

$$\hat{m}_0^* = \hat{m}_{-1}^* + \frac{d\mu_0^*}{\bar{M}} \tag{C.78}$$

We get these expressions using the fact that $\hat{w}_0 = \hat{w}_0^* = \hat{p}_0 = \hat{p}_0^* = \hat{p}_{X0} = \hat{p}_{M0}^* = \hat{s}_t = 0$ because wages are fixed for 1 period, and exchange rates are fixed over time.

C.3 Equilibrium conditions under flexible exchange rate regime

(a) Flexible wage

Then, the equilibrium conditions under the flexible exchange rate regime are as follows:

$$\frac{M_t}{P_t} = \frac{\chi^{\frac{1}{\varepsilon}} C_t^{\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_{t+1}}\right)^{\frac{1}{\varepsilon}}} \tag{C.79}$$

$$\frac{M_t^*}{P_t^*} = \frac{\chi^{\frac{1}{\varepsilon}} C_t^{*\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_{t+1}^*}\right)^{\frac{1}{\varepsilon}}} = \frac{\chi^{\frac{1}{\varepsilon}} C_t^{*\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_{t+1}} \frac{S_{t+1}}{S_t}\right)^{\frac{1}{\varepsilon}}} \tag{C.80}$$

$$W_t = \eta \frac{\lambda}{\lambda - 1} P_t C_t^{\sigma} H_t^{\psi} \tag{C.81}$$

$$W_t^* = \eta \frac{\lambda}{\lambda - 1} P_t^* C_t^{*\sigma} H_t^{*\psi} \tag{C.82}$$

$$H_t = \gamma \left(\frac{P_{Xt}}{P_t}\right)^{-\theta} C_t + (1-\gamma)\left(\frac{P_{Xt}}{S_t P_t^*}\right)^{-\theta} C_t^* \tag{C.83}$$

$$H_t^* = (1-\gamma)\left(\frac{S_t P_{Mt}^*}{P_t}\right)^{-\theta} C_t + \gamma \left(\frac{P_{Mt}^*}{P_t^*}\right) C_t^* \tag{C.84}$$

$$P_t C_t + B_{t+1} + M_t = W_t H_t + M_{t-1} + \mu_t + T_t + (1+i_t) B_t \tag{C.85}$$

$$\frac{1}{1+i_{t+1}} = \beta \frac{P_t C_t^{\sigma}}{P_{t+1} C_{t+1}^{\sigma}} \tag{C.86}$$

$$\frac{1}{1+i_{t+1}} = \beta \frac{P_t^* C_t^{*\sigma}}{P_{t+1}^* C_{t+1}^{*\sigma}} \frac{S_t}{S_{t+1}} \tag{C.87}$$

$$P_t = \left[\gamma P_{Xt}^{1-\theta} + (1-\gamma)(S_t P_{Mt}^*)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{C.88}$$

$$P_t^* = \left[(1-\gamma)\left(\frac{P_{Xt}}{S_t}\right)^{1-\theta} + \gamma P_{Mt}^{*1-\theta}\right]^{\frac{1}{1-\theta}} \tag{C.89}$$

$$P_{Xt} = W_t \tag{C.90}$$

$$P_{Mt}^* = W_t^* \tag{C.91}$$

$$M_t = M_{t-1} + \mu_t \tag{C.92}$$

$$M_t^* = M_{t-1}^* + \mu_t^* \tag{C.93}$$

(C.92) and (C.93) are the exogenous money supply rules in the home and foreign countries respectively, where μ_t and μ_t^* are the exogenous monetary transfers within the country. This system gives 15 equations to solve for 15 variables:
$\{C_t, C_t^*, M_t, M_t^*, P_t, P_t^*, i_{t+1}, W_t, W_t^*, H_t, H_t^*, P_{Xt}, P_{Mt}^*, S_t, B_{t+1}\}$.

The terms of trade under this regime is denoted as:

$$TOT = \frac{P_{Xt}}{S_t P_{Mt}^*}$$

We log-linearized the above system and get the following equations:

$$\hat{m}_t - \hat{p}_t = \frac{\sigma}{\varepsilon}\hat{c}_t - \frac{\beta}{\varepsilon}\hat{i}_{t+1} \tag{C.94}$$

$$\hat{m}_t^* - \hat{p}_t^* = \frac{\sigma}{\varepsilon}\hat{c}_t^* - \frac{\beta}{\varepsilon}\hat{i}_{t+1} - \frac{1}{\varepsilon}\frac{\beta}{1-\beta}(\hat{s}_t - \hat{s}_{t+1}) \tag{C.95}$$

$$\hat{w}_t = \hat{p}_t + \sigma\hat{c}_t + \psi\hat{h}_t \tag{C.96}$$

$$\hat{w}_t^* = \hat{p}_t^* + \sigma\hat{c}_t^* + \psi\hat{h}_t^* \tag{C.97}$$

$$\hat{h}_t = -\theta\hat{p}_{Xt} + \gamma\hat{c}_t + (1-\gamma)\hat{c}_t^* + \gamma\theta\hat{p}_t + (1-\gamma)\theta\hat{p}_t^* + (1-\gamma)\theta\hat{s}_t \qquad (C.98)$$

$$\hat{h}_t^* = -\theta\hat{p}_{Mt}^* + (1-\gamma)\hat{c}_t + \gamma\hat{c}_t^* + (1-\gamma)\theta\hat{p}_t + \gamma\theta\hat{p}_t^* - (1-\gamma)\theta\hat{s}_t \qquad (C.99)$$

$$\hat{c}_t + \frac{dB_{t+1}}{\bar{P}\bar{C}} + \omega\hat{m}_t = (\hat{p}_{Xt} - \hat{p}_t) + \hat{h}_t + \omega\hat{m}_{t-1} + \frac{d\mu_t}{\bar{P}\bar{C}} + \frac{dT_t}{\bar{P}\bar{C}} + (1+\bar{i})\frac{dB_t}{\bar{P}\bar{C}} \qquad (C.100)$$

$$-\frac{\bar{i}}{1+\bar{i}}\hat{i}_{t+1} = \hat{p}_t + \sigma\hat{c}_t - \hat{p}_{t+1} - \sigma\hat{c}_{t+1} \qquad (C.101)$$

$$-\frac{\bar{i}}{1+\bar{i}}\hat{i}_{t+1} = \hat{p}_t^* + \sigma\hat{c}_t^* - \hat{p}_{t+1}^* - \sigma\hat{c}_{t+1}^* + \hat{s}_t - \hat{s}_{t+1} \qquad (C.102)$$

$$\hat{p}_t = \gamma\hat{p}_{Xt} + (1-\gamma)(\hat{s}_t + \hat{p}_{Mt}^*) \qquad (C.103)$$

$$\hat{p}_t^* = (1-\gamma)(\hat{p}_{Xt} - \hat{s}_t) + \gamma\hat{p}_{Mt}^* \qquad (C.104)$$

$$\hat{p}_{Xt} = \hat{w}_t \qquad (C.105)$$

$$\hat{p}_{Mt}^* = \hat{w}_t^* \qquad (C.106)$$

$$\hat{m}_t = \hat{m}_{t-1} + \frac{d\mu_t}{\bar{M}} \qquad (C.107)$$

$$\hat{m}_t^* = \hat{m}_{t-1}^* + \frac{d\mu_t^*}{\bar{M}} \qquad (C.108)$$

(b) Sticky wage

The conditions describing the flexible exchange rate equilibrium at period 0 with sticky wages are:

$$\frac{M_0}{P_0} = \frac{\chi^{\frac{1}{\varepsilon}} C_0^{\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_1}\right)^{\frac{1}{\varepsilon}}} \tag{C.109}$$

$$\frac{M_0^*}{P_0^*} = \frac{\chi^{\frac{1}{\varepsilon}} C_0^{*\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_1^*}\right)^{\frac{1}{\varepsilon}}} = \frac{\chi^{\frac{1}{\varepsilon}} C_0^{*\frac{\sigma}{\varepsilon}}}{\left(1 - \frac{1}{1+i_1}\frac{S_1}{S_0}\right)^{\frac{1}{\varepsilon}}} \tag{C.110}$$

$$H_0 = \gamma \left(\frac{P_{X0}}{P_0}\right)^{-\theta} C_0 + (1-\gamma)\left(\frac{P_{X0}}{S_0 P_0^*}\right)^{-\theta} C_0^* \tag{C.111}$$

$$H_0^* = (1-\gamma)\left(\frac{S_0 P_{M0}^*}{P_0}\right)^{-\theta} C_0 + \gamma \left(\frac{P_{M0}^*}{P_0^*}\right)^{-\theta} C_0^* \tag{C.112}$$

$$P_0 C_0 + B_1 + M_0 = W_0 H_0 + M_{-1} + \mu_0 + T_0 + (1+i_0) B_0 \tag{C.113}$$

$$\frac{1}{1+i_1} = \beta \frac{P_0 C_0^\sigma}{P_1 C_1^\sigma} \tag{C.114}$$

$$\frac{1}{1+i_1} = \beta \frac{P_0^* C_0^{*\sigma}}{P_1^* C_1^{*\sigma}} \frac{S_0}{S_1} \tag{C.115}$$

$$M_0 = M_{-1} + \mu_0 \tag{C.116}$$

$$M_0^* = M_{-1}^* + \mu_0^* \tag{C.117}$$

The log-linearized conditions are:

$$\hat{m}_0 - (1-\gamma)\hat{s}_0 = \frac{\sigma}{\varepsilon}\hat{c}_0 - \frac{\beta}{\varepsilon}\hat{i}_1 \tag{C.118}$$

$$\hat{m}_0^* + (1-\gamma)\hat{s}_0 = \frac{\sigma}{\varepsilon}\hat{c}_0^* - \frac{\beta}{\varepsilon}\hat{i}_1 - \frac{1}{\varepsilon}\frac{\beta}{1-\beta}(\hat{s}_0 - \hat{s}_1) \tag{C.119}$$

$$\hat{h}_0 = \gamma\hat{c}_0 + (1-\gamma)\hat{c}_0^* + 2\gamma\theta(1-\gamma)\hat{s}_0 \tag{C.120}$$

$$\hat{h}_0^* = (1-\gamma)\hat{c}_0 + \gamma\hat{c}_0^* - 2\gamma\theta(1-\gamma)\hat{s}_0 \tag{C.121}$$

$$\hat{c}_0 + \frac{dB_1}{\bar{P}\bar{C}} + \omega\hat{m}_0 = -(1-\gamma)\hat{s}_0 + \hat{h}_0 + \omega\hat{m}_{-1} + \frac{d\mu_0}{\bar{P}\bar{C}} + \frac{dT_0}{\bar{P}\bar{C}} + \frac{1}{\beta}\frac{dB_0}{\bar{P}\bar{C}} \tag{C.122}$$

$$-\frac{\bar{i}}{1+\bar{i}}\hat{i}_1 = \sigma\hat{c}_0 - \hat{p}_1 - \sigma\hat{c}_1 + (1-\gamma)\hat{s}_0 \tag{C.123}$$

$$-\frac{\bar{i}}{1+\bar{i}}\hat{i}_1 = \sigma\hat{c}_0^* - \hat{p}_1^* - \sigma\hat{c}_1^* + \hat{s}_0 - \hat{s}_1 - (1-\gamma)\hat{s}_0 \tag{C.124}$$

$$\hat{m}_0 = \hat{m}_{-1} + \frac{d\mu_0}{\bar{M}} \tag{C.125}$$

$$\hat{m}_0^* = \hat{m}_{-1}^* + \frac{d\mu_0^*}{\bar{M}} \tag{C.126}$$