Cross-Directional Processes: Modeling, Identification and Robust Control

by

Mohammed E. Ammar

B.Sc., Cairo University
M.Sc., Cairo University

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

Doctor of Philosophy

in

The Faculty of Graduate Studies
(Electrical and Computer Engineering)

The University Of British Columbia
(Vancouver)
July 2009
© Mohammed E. Ammar 2009
Abstract

The aim of this work is to improve the identification of cross-directional (CD) models as well as to simplify the robust control design problem. Providing computationally efficient techniques for identification and robust control will facilitate the implementation of an autonomous CD control system. These are challenging problems as CD processes are large scale distributed parameter systems. The conventional model for a CD process is a spatial interaction matrix cascaded by a low order transfer function in the temporal domain. This representation results in a large dimensional multi-variable model.

Model uncertainties in the process are inevitable and rise from several sources. As the CD process models are usually identified by input-output data from bump tests, there is a demand for better identification techniques that minimizes the uncertainties in CD mapping and response shape models. Mapping and misalignment detection are problems that are specific to CD processes due to the configuration of the paper machine and its unconventional scanning method. These problems are of great practical importance for industrial implementations.

This work proposes modeling the CD process as a two-dimensional (2D) system that is spatially noncausal. The spatial noncausal transfer functions facilitate input design and identification in the spatial domain. Both noncausal finite impulse response (FIR) models and rational transfer functions are used to model the CD response. The FIR model representation is convenient for input design and identification in the CD while the spatial noncausal rational transfer function is more suitable for robust control design. The 2D representation for the plant and the controller is convenient for implementation in an autonomous control scheme with iterative feedback tuning or adaptive control.

Robust stability criteria are developed to investigate the stability of the process under feedback. Employing the 2D representation results in criteria that are computationally efficient as the 2D stability conditions are replaced by a set of simple 1D problems. A robust stability criterion that is based on the $\nu$-gap stability criterion provides bounds for robust stability against perturbations in the plant or the controller. This feature permits design-
Abstract

ing a two-dimensional controller in an adaptive control scheme or a simple re-tuning of an existing controller through iterative feedback tuning. This property is convenient when switching between different grades of paper. Another stability criterion based on the concept of phase margins is developed to determine the closed-loop’s tolerance to misalignment uncertainties.

Finally, a simple loop-shaping technique using spatial transfer functions is proposed to shape the closed-loop’s two-dimensional frequency response. The noncausal spatial transfer function provides a convenient tool for improving the closed-loop’s performance at low temporal frequencies. An adaptive control scheme is implemented to retune the system after a grade change. The developed techniques for identification and robust control can be used as a part of a more sophisticated autonomous control system.
# Table of Contents

Abstract .................................................. ii

Table of Contents ........................................ iv

List of Tables ............................................ viii

List of Figures ........................................... ix

Acknowledgements ......................................... xii

Nomenclature ............................................. xiii

1 Introduction ............................................. 1
   1.1 Paper Machine Cross-Directional Processes ............ 1
   1.2 Cross-Directional Control ............................... 3
       1.2.1 Basis Weight ..................................... 3
       1.2.2 Moisture ........................................ 4
       1.2.3 Caliper ......................................... 4
   1.3 Challenges with Cross-Directional Control .............. 5
       1.3.1 Multi-Variable Process ............................ 5
       1.3.2 Ill-Conditioned Process ......................... 7
       1.3.3 Model Uncertainty .............................. 8
   1.4 Literature Review .................................... 9
       1.4.1 Identification and Input Design .................. 9
       1.4.2 Robust Stability ................................ 11
       1.4.3 Control Design ................................. 11
   1.5 Motivations and Contributions of this Work ............ 12
       1.5.1 Motivations ..................................... 12
       1.5.2 Contributions ................................... 13
   1.6 Thesis Overview ..................................... 14

iv
# Table of Contents

2 A Two-Dimensional Model for the CD Process ........................................ 16  
2.1 Conventional CD Model ................................................................. 16  
2.2 A Two-Dimensional ARMAX Model .................................................. 19  
2.3 A Two-Dimensional Spatially Noncausal Model ................................. 20  
  2.3.1 Non-Causality in Space .......................................................... 20  
  2.3.2 A Spatial Noncausal FIR ......................................................... 21  
  2.3.3 Region of Convergence (ROC) for a Two-Sided z-Transform .......... 22  
  2.3.4 A Spatial Noncausal Transfer Function .................................... 24  
2.4 Frequency Response of the Spatial Models ....................................... 26  
2.5 A 2D Closed-Loop Transfer Function for Non-Square Systems ............ 28  
  2.5.1 Frequency Domain Analysis for Non-Square Systems .................. 29  
  2.5.2 A 2D Closed-Loop Transfer Function ..................................... 30  
2.6 Summary ......................................................................................... 34  

3 Identification and Input Design in Open-Loop ..................................... 36  
3.1 Introduction ...................................................................................... 36  
3.2 CD Response Shape Model .............................................................. 36  
  3.2.1 Identification of Noncausal FIR Models .................................... 37  
  3.2.2 Identification of Noncausal ARX Models ................................. 41  
3.3 CD Mapping Model .......................................................................... 44  
3.4 Input Design for CD Model Identification ........................................ 46  
  3.4.1 White Gaussian Noise Input ....................................................... 48  
  3.4.2 Weighted White Gaussian Noise Input ....................................... 49  
  3.4.3 Input Design Minimizing the Variance ...................................... 49  
3.5 Paper Machine Simulator Results ..................................................... 52  
  3.5.1 Mapping Detection .................................................................... 52  
  3.5.2 Misalignment Detection ............................................................ 54  
  3.5.3 CD Response Shape Identification ............................................ 54  
3.6 Industrial Experiments ...................................................................... 63  
  3.6.1 Noncausal FIR Model ............................................................... 65  
  3.6.2 Noncausal ARX Model ............................................................. 67  
3.7 Summary ......................................................................................... 69  

4 Identification and Input Design in Closed-Loop .................................. 72  
4.1 Introduction ...................................................................................... 72  
4.2 Closed-Loop Identification from Uncorrelated Input-Output Data .......... 73  
  4.2.1 CD Mapping Model Identification in Closed-Loop .................... 75  
  4.2.2 Misalignment Detection ............................................................ 76
# Table of Contents

4.2.3 CD Response Shape Model Identification in Closed-Loop ........................................ 77

4.3 Closed-Loop Identification from Steady-State Data .................................................. 81
  4.3.1 Example 1 ................................................................................................................ 83
  4.3.2 Example 2 ................................................................................................................ 85

4.4 Summary .................................................................................................................... 86

5 Robust Stability Criteria for CD Processes ................................................................. 88
  5.1 Introduction ................................................................................................................. 88
  5.2 Stability Conditions for a 2D Spatially Noncausal System ........................................ 88
  5.3 The 2D Robust Stability Theorem in the Temporal Domain .................................... 90
    5.3.1 The ν-Gap Metric ................................................................................................. 90
    5.3.2 A 2D Robust Stability Criterion Using Temporal ν-Gaps ................................. 93
  5.4 The 2D Robust Stability Theorem in the Spatial Domain ......................................... 94
    5.4.1 Developing a Noncausal ν-Gap ......................................................................... 94
    5.4.2 A 2D Robust Stability Criterion Using Spatial Noncausal ν-Gaps .................. 96
  5.5 Illustrative Example .................................................................................................. 97
    5.5.1 Internal Stability of the Actual Plant .................................................................. 99
    5.5.2 Simulation Results ............................................................................................. 100
  5.6 A Robust Stability Criterion for Misalignment Uncertainties Using a 2D Phase Margin Concept ................................................................. 107
    5.6.1 Two-Dimensional Phase Margins ....................................................................... 109
    5.6.2 Modeling Misalignment in Non-Square Systems ............................................... 110
    5.6.3 Illustrative Example ............................................................................................ 111
  5.7 Summary .................................................................................................................. 113

6 CD Loop-Shaping Using Spatial Transfer Functions .................................................. 115
  6.1 Introduction ............................................................................................................... 115
  6.2 The CD Loop-Shaping Technique .......................................................................... 115
    6.2.1 Controller Structure ........................................................................................... 116
    6.2.2 CD Loop-Shaping Using Spatial Pole Placement ............................................. 119
    6.2.3 Simulation Results ............................................................................................. 122
  6.3 The Adaptive Control Technique ............................................................................. 130
    6.3.1 Simulation Results ............................................................................................. 130
  6.4 Summary .................................................................................................................. 137
# Table of Contents

7 Conclusions and Future Work ........................................... 139
   7.1 Conclusions ....................................................... 139
   7.2 Future Work ....................................................... 141

Bibliography ................................................................. 143
List of Tables

5.1 Maximum allowable misalignment at the critical points . . . 111
List of Figures

1.1 Wide view of the paper machine (Artwork courtesy of Honeywell) ........................................ 2
1.2 The scanning sensor’s zig-zag path ................................................................. 3
1.3 A bump response in 3D ........................................................................... 6
1.4 Singular value decomposition .................................................................... 7

2.1 ROC for a causal transfer function ......................................................... 23
2.2 ROC for an anti-causal transfer function ............................................... 23
2.3 ROC for a noncausal transfer function ..................................................... 24
2.4 Frequency response of the spatial transfer function vs FFT of the spatial matrix ............................................................. 28
2.5 SVD of the band diagonal matrix versus FFT of the approximate circulant matrix .................................................................................. 29
2.6 Matching the spatial frequency response of the square matrix in the high resolution $q_d^*\text{FFT}(G_{2sq})$, rectangular matrix ($G_{2rec}$) and the transfer function in the high resolution $q_d^*\text{Freqz}(G_{2HR})$ ........................................... 31
2.7 The multi-variable loop with mapping .................................................... 31
2.8 The two-dimensional equivalent loop in the high resolution ................ 32
2.9 Slow input fast output multirate system ................................................. 32
2.10 FFT of the spatial matrix vs transfer function frequency response where: LR is the low resolution matrix, LR-TF is the low resolution transfer function, HR is the high resolution matrix, HR-TF is the high resolution transfer function, .............................................. 34

3.1 Alignment in the CD process ............................................................... 45
3.2 The FFT of a standard bump test in the spatial domain ...................... 47
3.3 FFT of a bump test vs white gaussian noise ......................................... 49
3.4 Mapping detection test .......................................................................... 53
3.5 Mapping detection in open-loop: offsets in actuators responses .......... 53
3.6 Misalignment Bump test ......................................................................... 54
3.7 Misalignment detection in open-loop: three misalignment cases ......... 55
List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.8</td>
<td>Input signals in the CD</td>
<td>56</td>
</tr>
<tr>
<td>3.9</td>
<td>Bump test in open-loop</td>
<td>57</td>
</tr>
<tr>
<td>3.10</td>
<td>White Gaussian Noise (WGN) experiment in open-loop</td>
<td>59</td>
</tr>
<tr>
<td>3.11</td>
<td>The frequency response of the weighting functions</td>
<td>60</td>
</tr>
<tr>
<td>3.12</td>
<td>FFT of white gaussian noise vs weighted white noise</td>
<td>60</td>
</tr>
<tr>
<td>3.13</td>
<td>Weighted White Gaussian Noise (WGN 1) experiment in open-loop</td>
<td>61</td>
</tr>
<tr>
<td>3.14</td>
<td>Weighted White Gaussian Noise (WGN 2) experiment in open-loop</td>
<td>62</td>
</tr>
<tr>
<td>3.15</td>
<td>Input Design minimizing the variance</td>
<td>64</td>
</tr>
<tr>
<td>3.16</td>
<td>Industrial Bump Test</td>
<td>66</td>
</tr>
<tr>
<td>3.17</td>
<td>Identification experiment using a WGN input signal</td>
<td>67</td>
</tr>
<tr>
<td>3.18</td>
<td>Industrial identification (FIR model)</td>
<td>68</td>
</tr>
<tr>
<td>3.19</td>
<td>Identification experiment for an ARX model</td>
<td>69</td>
</tr>
<tr>
<td>3.20</td>
<td>Industrial identification (ARX model)</td>
<td>70</td>
</tr>
<tr>
<td>3.21</td>
<td>Residual analysis for the ARX model</td>
<td>71</td>
</tr>
<tr>
<td>4.1</td>
<td>Uncorrelated input-output data in closed-loop</td>
<td>74</td>
</tr>
<tr>
<td>4.2</td>
<td>Closed-loop configuration</td>
<td>74</td>
</tr>
<tr>
<td>4.3</td>
<td>Mapping detection in closed-loop: Offsets in actuators responses</td>
<td>76</td>
</tr>
<tr>
<td>4.4</td>
<td>Misalignment detection in closed-loop: three misalignment cases</td>
<td>76</td>
</tr>
<tr>
<td>4.5</td>
<td>White Gaussian Noise (WGN) experiment in closed-loop</td>
<td>78</td>
</tr>
<tr>
<td>4.6</td>
<td>Weighted White Gaussian Noise (WGN 1) experiment in closed-loop</td>
<td>79</td>
</tr>
<tr>
<td>4.7</td>
<td>Weighted White Gaussian Noise (WGN 2) experiment in closed-loop</td>
<td>80</td>
</tr>
<tr>
<td>4.8</td>
<td>Input Design minimizing the variance in closed-loop</td>
<td>82</td>
</tr>
<tr>
<td>4.9</td>
<td>Input output steady-state data</td>
<td>83</td>
</tr>
<tr>
<td>4.10</td>
<td>The sensitivity function</td>
<td>84</td>
</tr>
<tr>
<td>4.11</td>
<td>The input sensitivity function</td>
<td>84</td>
</tr>
<tr>
<td>4.12</td>
<td>CD response</td>
<td>85</td>
</tr>
<tr>
<td>4.13</td>
<td>Spatial frequency response</td>
<td>86</td>
</tr>
<tr>
<td>4.14</td>
<td>Input output steady-state data</td>
<td>87</td>
</tr>
<tr>
<td>4.15</td>
<td>CD response</td>
<td>87</td>
</tr>
<tr>
<td>5.1</td>
<td>Closed-loop configuration</td>
<td>92</td>
</tr>
<tr>
<td>5.2</td>
<td>The CD negative feedback loop</td>
<td>98</td>
</tr>
<tr>
<td>5.3</td>
<td>The rootmaps of the nominal loop</td>
<td>101</td>
</tr>
<tr>
<td>5.4</td>
<td>Spatial responses of the nominal and perturbed plants</td>
<td>102</td>
</tr>
</tbody>
</table>
List of Figures

5.5 Perturbed plant $G_1$ with the nominal controller ........... 104
5.6 Perturbed plant $G_1$ with controller $K_1$ ............... 105
5.7 Perturbed plant $G_1$ with controller $K_2$ ............... 106
5.8 Perturbed plant $G_1$ with controllers $K_1$ and $K_2$ ....... 108
5.9 Multi-rate loop in the low resolution ..................... 110
5.10 Rootmaps for the different misalignment cases .......... 112
5.11 Spatial steady-state profile for the different misalignment cases 114

6.1 The industrial feedback loop ........................... 116
6.2 Controllers’ pole-zero map ................................. 118
6.3 Closed-loop attenuation band .............................. 118
6.4 The closed-loop steady-state spatial transfer function .... 119
6.5 A flow chart for the design procedure .................... 121
6.6 The steady-state profile for the different controllers .... 125
6.7 The steady-state error spectrum for the different controllers 125
6.8 The steady-state spatial sensitivity function for the three loops 126
6.9 The 2D sensitivity function for the two-dimensional loop-shaped system ................. 127
6.10 The 2D sensitivity function for the spatial loop-shaped system 128
6.11 The difference in the magnitude between the 2D sensitivity functions .................. 128
6.12 The spatial sensitivity function at steady-state computed from the transfer function vs the spatial matrix 129
6.13 The 2D spatial loop-shaped controller at high spatial and temporal frequencies .......... 130
6.14 The spatial responses for the two grades .................. 131
6.15 Closed-loop identification of the new spatial response .... 132
6.16 The steady-state output profile for the two controllers .... 133
6.17 The steady-state error spectrum for the two controllers .... 134
6.18 The steady-state spatial sensitivity function for the two controllers ............. 134
6.19 The 2D sensitivity function for the 2D loop-shaped system after the grade change ........ 135
6.20 The 2D sensitivity function for the spatial loop-shaped system in the adaptive scheme .... 136
6.21 The 2D spatial loop-shaped adaptive controller at high spatial and temporal frequencies .......... 137
Acknowledgements

I would like to express my sincere gratitude to my supervisor Prof. Guy A. Dumont for his guidance and support throughout my postgraduate studies at the University of British Columbia. Many thanks go to my supervisory committee members: Prof. Michael Davies and Prof. Philip Loewen for their valuable suggestions and advice during the course of this project.

The efforts of my committee readers, Dr. Greg Stewart and Dr. Joseph Yan is gratefully appreciated. Special thanks are due to Dr. Greg Stewart for his invaluable input and technical comments.

I wish to thank Honeywell process solutions, Vancouver operations for technical support of this project. I gratefully acknowledge Grant Emmond and John Ball from Canfor mills, Prince George for their help in conducting the industrial experiments.

Over the past years, I have benefited from the technical discussions with my fellow members of the CD control group at the pulp and paper center, Dr. Bhushan Gopaluni, Dr. Jun Yan, Fazel Farahmand and Soroush Karimzadeh.

Finally, I would like to thank my parents Prof. Elsayed Ammar and Prof. Ahdab Elmorshedy for their love, support and encouragement throughout my life.
Nomenclature

\( \alpha \) The attenuation parameter
\( \bar{M} \) The average information matrix per data sample
\( \delta \) The divergence parameter
\( \gamma \) Normalization parameter
\( \lambda^{-1} \) The spatial left shift transform operator
\( \lambda^{-1}_N \) The spatial left shift transform operator in the high resolution
\( \mathbb{C} \) The field of complex numbers
\( \mathbb{R} \) The field of real numbers
\( \omega \) The temporal frequency (cycles/sec)
\( \Upsilon \) The condition number
\( \nu \) The spatial frequency (cycles/meter)
\( c_j \) The coordinate of the response center for actuator \( j \)
\( d_s \) The distance between the measuring points
\( M \) The Fisher information matrix
\( m_f \) The model order of the FIR model representing the causal spatial response in the measurement’s resolution
\( M_t \) The number of measurements in the MD
\( m_t \) A point’s coordinate in time
\( m_y \) The number of measurement positions
\( n_c \) The model order for the rational transfer function representing the causal spatial response
\( n_f \) The model order for the FIR representing the causal spatial response
\( n_k \) The model order for the causal part of the controller’s spatial transfer function
\( n_m \) Misalignment in the low resolution transfer function
Nomenclature

$N_t$ The number of measurements in the CD
$N_u$ The number of actuator arrays
$n_n$ The number of actuators
$n_x$ A point’s coordinate in space
$N_y$ The number of measured properties
$n'_m$ Misalignment in the high resolution transfer function
$q$ The right shift operator in space
$w$ The width of the actuator response
$z^{-1}$ The backward shift operator in temporal transfer functions
Chapter 1

Introduction

1.1 Paper Machine Cross-Directional Processes

Paper making is a web forming process where a slurry of water and cellulose fibres are transformed into a sheet of paper. The paper machine can be divided into four sections: The wet end section, the press section, the drying section and the post drying operations section. A typical Fourdrinier paper machine is shown in Figure 1.1.

In the wet end section, the mixture of water and pulp fibres (with only 0.5 % fibre concentration) is delivered onto a wire screen that is moving at speeds that could be in excess of 100 km/hr in the machine direction (MD). A significant amount of water is removed by gravity and suction devices, resulting in formation of a paper sheet with 20 % fibre concentration leaving the machine's wet end.

The sheet is heated by steam boxes before and during the press section where it is pressed and de-watered between counter-rotating rollers. Next, the sheet enters the dryer section with a consistency of about 40 %. In the dryer section, the sheet is further dewatered by a series of steam heated dryer cans and leaves the section with a 5 ~ 9 % water content. Afterwards, the sheet enters the post drying operations section where it passes through a series of rotating rollers known as the calendar stack to control sheet thickness.

The direction of sheet travel is known as the machine direction or MD while the direction perpendicular to the sheet travel is known as the cross direction or CD. The paper profile is monitored through a set of scanning sensors traversing back and forth across the sheet, taking between 15 ~ 45 seconds to scan the width of the sheet. As the sheet is moving in the MD, the sensors measure along a zig-zag path as shown in Figure 1.2. The scanned data is processed to extract the MD and CD information. The industrial custom is to use an exponential filter to separate the measurements into estimates of the CD and MD profiles. This filtering produces more accurate estimation at the expense of slowing down the observed dynamics of the system. The industrial approach to paper machine control considers the MD
and the CD problems separately. MD control is concerned with controlling the average value of each scan while the control of the dynamically varying zero-mean error profile is known as CD control.

The most important properties of the paper sheet are moisture, basis weight and caliper. The quality of the paper sheet is defined in terms of the variance of these properties. These properties are controlled through an array of identical actuators distributed across the width of the paper machine.

The actuator arrays act in the cross direction (CD) as the paper sheets move along the machine direction (MD). The actuator arrays include slice lip actuators, dilution actuators, steam boxes, rewet showers and induction heaters. Each actuator array can affect more than one property. Typically, an actuator array has 30 to 300 actuators while a measurement array has 200 to 1000 data points across the CD. The actuators in one array are assumed to be identical so the process can be considered spatially invariant if the irregularities at the edges are neglected. As for MD control, the overall paper weight is controlled by manipulating the flow rate of the pulp into the paper machine and the speed of the machine. The MD moisture content is controlled by manipulating the overall steam flow into the dryer section. This work focuses only on the CD control problem.
1.2 Cross-Directional Control

1.2.1 Basis Weight

Basis weight or grammage is the weight per unit area. It is expressed in grams per square meter (gsm) or pounds per ream (lbs/ream). CD control of the weight of a paper sheet is achieved by actuators at the headbox. The actuators distribute the pulp fibres evenly across the width of the wire mesh so that variability of the basis weight CD profile is minimized. The dynamics of the basis weight control include a significant dead time component as the headbox is located the furthest away from the scanning sensor. Basis weight profile control is important not only for paper strength reasons, but also due to the fact that a poor quality weight profile will propagate downstream and appear as disturbances in both the moisture and caliper profiles. The two most common weight control actuators are slice lip actuators and dilution actuators. As the dynamics of both actuators are fast compared to the scan time, the process dynamics are dominated by the sensor filtering and the transport delay.

Slice lip actuators have been the traditional way to control basis weight. The slice has an adjustable top lip and a fixed bottom lip. The upper lip can also be locally bent by the use of CD actuators. As the mixture of water and
pulp exits the headbox through the slice, increasing the actuator opening delivers more fibres resulting in a heavier sheet in the localized area around that actuator. The spacing \( x \) and the total number of actuators \( n \) in a slice lip array can vary significantly, it can be anywhere between \( x = 7 \sim 20 \text{cm} \) and \( n = 50 \sim 118 \) [74]. Slice lip actuators are subject to bending constraints to prevent damaging the slice lip through excessive flexing and bending.

Dilution actuators inject a stream of low consistency water into the pulp entering the headbox. Increasing the flow reduces the concentration of pulp fibres and results in a reduced basis weight. This method is known as consistency profiling which is quite new for basis weight control [87]. The response to a dilution actuators is very well localized without the side lobes that are observed in slice lip actuators’ response. There are different industrial implementations of dilution actuators. A typical configuration has a spacing \( x = 3.5 \text{cm} \) and \( n = 150 \sim 200 \) actuators while another configuration has a spacing \( x = 6 \sim 7 \text{ cm} \) and \( n = 60 \sim 80 \) actuators.

1.2.2 Moisture

The moisture profile highly affects the sheet strength as overdrying the paper sheet damages the fibres. Disturbances in the moisture profile propagate downstream and affect the caliper profiles. The main types of CD moisture control actuators are steam boxes and rewet showers [14].

Using steam boxes, the sheet temperature is increased by condensing steam in and on the sheet. The higher sheet temperature increases water fluidity so that it can be removed efficiently in the press section. An array of steam box actuators have from \( n = 55 \sim 171 \) and the actuator spacing is usually between \( x = 7.5 \sim 15 \text{ cm} \). Steam boxes are generally slow actuators with a time constant of approximately 200 \sim 250 \text{ seconds}.

Rewet shower actuators apply an atomized water spray to the dry spots in the paper sheet. Rewet showers have very fast dynamics compared to the scan time. An array of actuators can have from \( n = 50 \sim 120 \) actuators with a spacing of \( x = 7 \sim 15 \text{ cm} \).

1.2.3 Caliper

The paper caliper typically targets the range from \( 70 \sim 300 \mu m \). The paper sheet goes through a series of counter-rotating rollers located at the dry end of the paper machine. The caliper profile is controlled by changing the diameter of the rollers using induction heating actuators. The array of actuators is mounted 5mm from the roller. A high frequency alternating current is
used to produce eddy currents near the surface of the roller thus expanding it by heating. Each actuator in the array is controlled independently and heats the roller locally. An array of induction heating actuators typically has \( n = 100 \sim 150 \) actuators with a spacing of 7.5 cm. The dynamics of these actuators are very slow and have a small dead time as the stack is located close to the scanning sensor.

### 1.3 Challenges with Cross-Directional Control

#### 1.3.1 Multi-Variable Process

CD processes are large multi-variable spatially distributed systems that can have from 30-300 actuators and 200-2000 measurement points. It is standard in CD control to assume separability between the dynamic and spatial responses [20],[30],[59],[74] and [77]. The implication is that the actuator's spatial response shape is fixed while the magnitude of the response grows with time until it reaches steady-state.

**Single-Array System**

Considering a single property controlled by a single actuator array, the discrete time multi-variable model of the process is:

\[
y(z) = G(z)u(z) + d(z) \tag{1.1}
\]

Here \( y(z) \), \( u(z) \) are the \( z \)-transforms of the high resolution measurement profile \( y(t) \) and the actuator set-point profile \( u(t) \), respectively.

\( y(t) \in \mathbb{R}^{m_y} \), \( u(t) \in \mathbb{R}^{n_u} \) where \( \mathbb{R} \) denotes the field of real numbers.

Assuming that the time and spatial responses are separable, the process transfer matrix \( G(z) \in \mathbb{C}^{m_y \times n_u} \) is given by:

\[
G(z) = G_0 \cdot T(z) \tag{1.2}
\]

\[
T(z) = \frac{z^{-Td}}{1 - az^{-1}} \tag{1.3}
\]

- \( G_0 \) is a spatial interaction matrix
- \( T(z) \) is the discrete time model

Figure 1.3 illustrates a bump response in 3D. A bump test can be seen as a step input in the MD and an impulse in the CD.
Chapter 1. Introduction

Figure 1.3: A bump response in 3D

Multi-Array System

As discussed in section 1.2, each actuator array affects more than one property. A multiple array CD process model has been proposed in [10], [43], [25] and [26] to represent the interactions between the actuator arrays. The model is composed of blocks of single array systems as follows:

\[
Y(z) = G(z)U(z) \tag{1.4}
\]

\[
Y(z) = \begin{bmatrix}
y_1(z) \\
\vdots \\
y_N(z)
\end{bmatrix}
\]

\[
U(z) = \begin{bmatrix}
u_1(z) \\
\vdots \\
u_N(z)
\end{bmatrix}
\]

\[
G(z) = \begin{bmatrix}
G_{11}T_{11}(z) & \cdots & G_{1N_{u}}T_{1N_{u}}(z) \\
\vdots & \ddots & \vdots \\
G_{N_p1}T_{N_p1}(z) & \cdots & G_{N_pN_{u}}T_{N_pN_{u}}(z)
\end{bmatrix}
\]
Chapter 1. Introduction

Here $G_{ji} T_{ji}(z)$ are the spatial interaction matrix and the process dynamics between actuator array $i$ and the measurement profile $j$. $N_u$ and $N_p$ are the numbers of actuator arrays and sheet properties respectively. With all the CD actuators and controlled properties in one model, the dimension of the matrix $G(z)$ could be as large as $6000 \times 600$.

1.3.2 Ill-Conditioned Process

The CD process is known to be severely ill-conditioned as shown in [41],[55],[68] and [74]. The condition number ($T$), which is the ratio between the largest and the smallest singular value, could reach thousands.

$$T(G_0) = \frac{\sigma(G_0)}{\sigma(G_0)} \gg 1$$

Figure 1.4 shows the singular values of a typical CD basis weight process.

![Figure 1.4: Singular value decomposition](image)

Attempting to control the low gain directions of the process would result in a system that is not robust to model uncertainties. The controller needs to provide larger magnitudes of actuation in these directions when the sign of the gain is not known for certain.
Chapter 1. Introduction

One strategy to deal with this problem is to apply partial control at a subset of the singular values. Another approach is achieved by designing the controller to have low gains in the weakly controllable directions.

1.3.3 Model Uncertainty

In order to simplify the large dimensional multi-variable control problem, the actuator responses are assumed to be identical and the CD process is assumed to be spatially invariant. Another standard assumption is the separability of dynamical and spatial response. This is just an approximation for the real process resulting in a mismatch between the true process and the model. Model uncertainties arise from other sources due to the physical aspects of the process. The main sources for these uncertainties are:

1. Alignment: The sheet travels several hundred meters between the head-box and the scanning sensor where it may shrink. Moreover, paper machines have more measuring points than actuators. This results in a mismatch between the true actuator's center of response and the predicted one.

2. Sheet wandering: The sheet of paper may wander back and forth in the CD by several centimeters resulting in a change in the alignment.

3. MD-CD separation: Due to the zig-zag path of the scanning sensor, the sensor aliases certain MD variations into the CD profiles.

4. Paper machine speed: Sometimes the machine is sped up or slowed down by the operators. This alters the spatial response as well as the transport delay of the process.

5. Different grades: In many paper mills, the controller parameters are not retuned for different grades of paper. As the actuator's response depends on the grade, this results in a model uncertainty that the controller has to cope with.

6. Faulty actuators: One or more actuator may fail or partially fail. Such an actuator will respond differently than the rest of the actuators in the array.

7. Edge effects: The CD spatial response at the edges is quite different from that in the middle of the sheet.

No model can accurately represent the true process while keeping the problem tractable. Assuming separability and spatial invariance, the controllers are designed for the process and afterwards robustness to model uncertainties are investigated.
1.4 Literature Review

Identification of the cross-directional process is key to a satisfactory system performance. As modeling uncertainties resulting from input-output identification are inevitable, guaranteeing robust stability and performance is very crucial for CD processes. This work addresses three problems: identification and input design, robust stability and control design. The following sections present a literature review on each of these topics.

1.4.1 Identification and Input Design

Due to economic constraints, identification experiments are usually severely limited making the identification process challenging. Currently, industrial identification schemes do not explicitly attempt to minimize control-relevant uncertainty. The industrial identification technique uses bump test data. In a bump test, a few actuators are stepped while the CD controller is running in manual mode. The sheet response is observed and the input-output data are used to fit the process model assuming identical actuator responses. Bump tests do not provide enough information about the system’s response in the high spatial frequency range. This can lead to an uncertain gain sign resulting in non-robustly controllable frequencies. The presence of uncontrollable CD wavelengths can result in the accumulation of large controller action. In the temporal domain, pseudo-random binary signals have been applied to an industrial paper machine instead of step inputs [47].

A parametric model approach was implemented in industrial identification [35] and [36]. The algorithm is based on generalized least squares identification where an iterative procedure alternating between the temporal and spatial models is used. The technique is used in an industrial identification software package installed in many paper mills. Estimating the parameters of a two-dimensional autoregressive moving average with exogenous input (ARMAX) model using generalized least squares was presented in [45] and [46].

The identification of CD models is a twofold problem. The first problem is identifying the CD mapping model to determine the actual actuators’ center of responses. The second problem is to identify the CD response model [35] and [36]. The parametric model identification does not deliver an uncertainty region for the identified parameters nor provide a direct spatial frequency representation for control design purposes.
Chapter 1. Introduction

Identifying the mapping model is crucial for robust CD control. The mapping model aligns the center of response of each actuator with a measurement box. In paper machines, the alignment problem results from the fact that the paper sheet can shrink significantly by the time it reaches the scanning sensor. As there are more measurement points than actuators, the alignment problem becomes even more challenging. In [36], a technique is presented to identify a parametric mapping model through a fuzzy logic model of the shrinkage profile. The algorithm requires an estimate of the CD response model and minimizes the model fit error between the measured profile and the predicted profile [37].

As bump tests excite only a few spatial frequencies, the model uncertainty at the rest of the unexcited spatial frequencies might be large. The insufficiency of the information induced by bump tests raises the need for input design [30] and [84]. Experiment design was the subject of many earlier studies. Some of the design objectives were D-optimality, G-optimality, L-optimality and E-optimality each involving a cost function of the average per data sample information matrix $\bar{M}$. In [28] and [29], a spatial input signal was used to minimize the uncertainties in the steady-state process gains. As the optimization problem was non convex, a simulated annealing algorithm was used to solve it. The simulated annealing algorithm requires a significant computational effort to find a global optimum making it unsuitable for large scale processes. In [54], an input design technique minimizing the uncertainties in the singular value decomposition (SVD) of multi-variable models was discussed. This technique is suitable for multi-variable processes with few inputs and outputs. Recent work has used convex optimization to obtain numerical solutions for some experiment design problems.

Most of the previous work on identification of CD processes was performed in open-loop. A technique for estimating the impulse response of an actuator in closed-loop was presented in [20]. This method is not strictly non-invasive as a small perturbation must be introduced to the system to ensure identifiability. In [22], a method of extracting the open-loop responses of the actuators from an estimate of the closed-loop response is given. A perturbation signal that is persistently exciting is added to the actuators set-points. Because of the inherent time delay within the system, the open-loop impulse response will appear as terms in the closed-loop response right after the time delay. The estimation is restricted to a few actuators to limit the number of scans required to identify an accurate spatial response.

An indirect closed-loop identification technique was developed for an unconstrained model predictive controller for a paper machine in [69]. The technique uses the concept of Markov parameters to identify a step-response
model. Five actuator locations were selected and identified from 300 data samples where a high signal-to-noise (SNR) ratio of 20:1 was used in the simulations. The computational demand limits the on-line capabilities of this technique.

1.4.2 Robust Stability

Modeling uncertainties result either from inaccurate input-output identification or from incorrect mapping. Misalignment appears when some of the measurement positions are mapped incorrectly to actuators that are not aligned with these CD positions. Performing robust stability analysis using the large multi-variable model in the temporal domain is computationally inefficient. The information from the columns of the matrix is redundant as the actuators responses are assumed identical for the purpose of control design.

In [23], robust stability was investigated for a set of single input single output (SISO) systems resulting from a singular value decomposition of the interaction matrix. The structured singular value (\( \mu \) analysis) was proposed in [13] to provide a more general technique. In [78] and [77] an unstructured uncertainty analysis was presented where the authors used circulant matrix theory. The spatial matrix was first approximated by a circulant matrix which was then decomposed by Fourier matrices into a set of SISO systems. Structured and unstructured uncertainty analysis was investigated in [24] using the same technique. In [38],[39] and [40] structured singular value analysis was used to address the robustness of multidimensional systems against structured uncertainties. The multidimensional system was represented by a linear fractional transformation (LFT) model to perform the stability analysis.

The conventional additive unstructured uncertainty is inconvenient to investigate the effect of mis-mapping on the system’s stability. In [80], the shift property of the Fourier transform was used to study the effect of mis-mapping on stability. In that work, the mis-mapping shift was assumed to be constant across an array of identical actuators along the CD. The proposed method was applied to a simplified version of the CD controller.

1.4.3 Control Design

The CD controller is designed to attenuate the effect of the disturbances while providing robustness against modeling uncertainties. Minimum variance control [44] and two-dimensional control techniques [45],[46] were used.
for CD control. One approach to simplify the design problem is to reduce the dimensionality through basis function representations [55] and [67]. In [20],[21] and [23] singular value decomposition reduced the problem into a set of SISO transfer functions. In [75], [76] the spatial matrix was first approximated by a circulant matrix then decomposed by Fourier matrices into a set of SISO systems at different spatial frequencies. A temporal controller was designed for each spatial frequency and then the MIMO controller was obtained by the inverse Fourier matrices. This led to the two dimensional loop shaping technique for industrial CD controllers [74], [77] and [78]. However, this design technique can not be used in an adaptive control scheme as it results in a non-localized controller and a few iterations are performed until a trade-off between performance and localization is achieved. The fact that the rectangular circulant matrices can be decomposed in the same fashion as square circulant matrices facilitated designing an industrial model predictive control (MPC) of CD processes in [24]. A criterion was developed for evaluating the selection of tuning parameters of a MPC controller using the two-dimensional (temporal and spatial) frequency analysis. The design techniques based on two-dimensional frequency analysis are considered to be the state of the art in CD control design.

1.5 Motivations and Contributions of this Work

This work attempts to improve the identification of CD models and simplify the challenging multi-variable robust control design problem.

1.5.1 Motivations

In the pulp and paper industry, there is a continuous demand for better CD models as well as controllers that are robust to model uncertainties. In many paper mills, the same tuning parameters are used for the production of different grades of paper. As the actuator’s response changes with the grade, tuning the controller for each grade should be autonomous.

As discussed earlier, model uncertainties in the process are inevitable and rise from several sources. The CD process models are usually identified by input-output data from bump tests. There is a demand for better identification techniques that minimize the uncertainties in CD mapping and response shape models.

Mapping and misalignment detection are problems that are specific to CD processes due to the configuration of the paper machine and its particular scanning method. These problems are of great practical importance.
for industrial implementations. The available mapping model techniques require a prior estimate of the response shape model and are restricted to the open-loop mode.

As for CD response shape identification, input signals that excite high spatial modes are needed. The earlier work on input design for CD models neither invokes control relevant identification nor enables focusing on a range of spatial frequencies. This feature would be useful in minimizing the uncertainties at high spatial frequencies which is crucial for robust stability. A higher spatial bandwidth can be attained if the uncertainties in the mid-range spatial frequencies are quantified and minimized.

A few techniques were developed for the identification of CD models in closed-loop. The large dimension of the multi-variable model renders this problem very challenging. In [69], a high signal to noise (SNR) ratio of 20:1 was used in the simulation and the data from 300 scans were used. Moreover, the authors state that their technique is not computationally efficient. The identification method in [22] would not provide enough information to identify an accurate model in a low signal-to-noise ratio.

The existing methods for robust stability require decomposition of the system transfer matrix to cut down the computational effort when dealing with the large multi-variable process. In addition, some of these methods were restricted to handle one or a few uncertainty structures. A direct relation between the spatial frequency domain uncertainty and the stability bounds is not clear in the mathematical representation of uncertainties in the multi-variable model.

In order to keep the control design problems tractable, many techniques were proposed in the literature to reduce the dimensionality. The available control design techniques are inconvenient for implementation in an autonomous control system. The large dimensionality is not really necessary as it is standard to assume that all the actuators have identical responses for identification and control design purposes.

### 1.5.2 Contributions

The main contributions of this work are summarized as follows:

1. Replacing the large dimensional spatial interaction matrices with non-causal spatial models. These models better explain the spatial frequency response of the CD process and the controllers. In previous work, the non-causal model was proposed to investigate robust stability only.
2. Developing a two-dimensional closed-loop transfer function for CD pro-
Chapter 1. Introduction

cesses running under feedback in the simplified case of an equal number of measurement points and actuators (square systems) as well as the case with more measurement points than actuators (non-square systems).

3. Identification of noncausal spatial models in open-loop and closed-loop modes. The techniques are based on least squares identification. Input design experiments are conducted to minimize uncertainties in the spatial FIR models in both modes.

4. Presenting a technique to identify the CD mapping model. The method is suitable for both open-loop and closed-loop modes.

5. Developing robust stability criteria for the two dimensional process. These stability criteria are convenient for an autonomous control scheme as they are computationally efficient.

6. A CD loop-shaping technique using the spatial transfer function is proposed. The loop-shaping technique developed in this work is incorporated in an adaptive control scheme.

1.6 Thesis Overview

In chapter 2, the large dimensional interaction matrices of CD processes are replaced by noncausal spatial models. The spatial models provide an insight into the concept of spatial frequencies especially in systems with more measurement points than actuators. An equivalent two-dimensional closed-loop model for CD processes is developed in the simplified case of an equal number of measurement points and actuators as well as the case with more measurement points than actuators. This representation is convenient for predicting the spatial frequency response and spatial localization in closed-loop.

A novel method for identifying the CD mapping model and detecting misalignment using noncausal spatial FIR models is presented in Chapter 3. This method does not require a prior estimate of the response shape and works in closed-loop. A technique is developed for CD response shape identification where the response is modeled by noncausal FIR models as well as noncausal rational transfer function. These spatial models facilitate control relevant identification in the spatial domain. Input design techniques improving the spatial models are discussed. The designed input signals are tested on a paper machine simulator. Industrial identification experiments were conducted to identify FIR models and autoregressive with exogenous input (ARX) models.

In chapter 4, methods for closed-loop identification of CD mapping mod-
Chapter 1. Introduction

els and CD response models are developed. For systems with relatively short time constants, it is shown that the process inherent transport delay provides uncorrelated input-output data which can be used in identification of the spatial models. Open-loop input design techniques are applied in identification of the response models from correlation-free input-output data. CD response shape identification from steady-state data is proposed for systems with large time constants. CD mapping models are detected through identifying noncausal FIR models as proposed in chapter 3.

In chapter 5, two robust stability criteria for CD processes are presented through modeling the CD process by a two-dimensional transfer function and using results from the multidimensional digital stability theory. The first criterion is based on the \( \nu \)-gap stability criterion which provides a sufficient condition for stability. This test replaces the complex 2D robust stability problem by a set of simple 1D problems making it computationally very efficient.

Another robust stability criterion to determine the maximum allowable misalignment uncertainty in CD processes is proposed. Stability is investigated by extending the concept of a phase margin to the two-dimensional case.

A novel CD control design method through spatial loop shaping is proposed in chapter 6. The design is performed on the spatial model at temporal steady-state. The noncausal rational transfer function is used to model the spatial response permitting loop-shaping the spatial frequencies with the conventional techniques developed for the temporal domain. An adaptive control scheme is developed using the CD loop-shaping technique. The spatial response is identified in closed-loop and is followed by redesigning the controller.

Finally, conclusions and future work ideas are discussed in chapter 7.
Chapter 2

A Two-Dimensional Model for the CD Process

2.1 Conventional CD Model

The CD process is a spatially distributed process that describes the relation between an actuator array and a controlled property. The control action is introduced through an array of identical actuators across the sheet and the output profile is measured using identical sensors at different points across the CD direction. The process is modeled by linear dynamical systems distributed in the spatial dimension and discretized both dynamically and spatially. The CD process is usually modeled as a constant spatial interaction matrix cascaded with a linear low-order transfer function with dead time. This spatial interaction matrix has the actuator response as the entries in each of its columns. The response is shifted in space corresponding to the respective actuator position. The cross-directional web process is a very large multi-variable process that is generally ill conditioned.

The discrete time multi-variable process model is:

\[ y(z) = G(z)u(z) + d(z) \] (2.1)

where \( y(z) \), \( u(z) \) are the z-transforms of the high resolution measurement profile \( y(t) \) and the actuator set-point profile \( u(t) \) respectively. \( y(t) \in \mathbb{R}^{m_y} \), \( u(t) \in \mathbb{R}^{n_u} \) where \( \mathbb{R} \) denotes the field of real numbers.

Assuming that the time and spatial responses are separable, the process transfer matrix \( G(z) \in \mathbb{C}^{m_y \times n_u} \) is given by:

\[ G(z) = G_0 \cdot T(z) \] (2.2)

\[ T(z) = \frac{z^{-T_d}}{1 - az^{-1}} \] (2.3)
Chapter 2. A Two-Dimensional Model for the CD Process

$G_0$ is a spatial interaction matrix

$T(z)$ is the discrete time model

$T_d$ is the dead time

A parametric model approach was implemented in industrial identification [35] and [36]. This approach uses the separability between the spatial and temporal responses. An iterative procedure alternating between the temporal and spatial model is used. The algorithm is based on generalized least squares identification. The technique is used in an industrial identification software installed in many paper mills.

The entries of the spatial interaction matrix are:

$$G_i^j = g(d_s i - c_j)$$  \hspace{1cm} (2.4)

In this expression:

$d_s$ is the distance between the measuring points

$c_j$ is the coordinate of the response center for actuator $j$

$\gamma$ is a normalization parameter

$w$ defines the width of the actuator response

$\alpha$ is the attenuation parameter defining the size of the negative lobes

$\delta$ is the divergence parameter which defines the presence of two maximums and the distance between them

The actuator responses are assumed to be identical but they are truncated at the edges of the sheet due to the finite width of the sheet. For a system with the same number of actuators and measurement points, $G_0$ is a $n_u \times n_u$ band-diagonal symmetric Toeplitz matrix. In order to perform spatial frequency analysis on these matrices an assumption of infinite width must be invoked to avoid the need for boundary conditions at the edges.
Some web processes such as the blown extrusion of plastic films are modeled by circulant matrices as the web is extruded as a tube ([20] and [44]). These models can be decomposed using Fourier matrices and do not require the infinite width assumption. The corresponding circulant matrix is $G_0$.

Circulant matrices have some useful properties that simplify the large multi-variable control design and robustness problem. The properties of circulant matrices that are of interest are: Symmetric circulant matrices are diagonalized by pre- and post-multiplication by the real Fourier matrix. The singular values of a symmetric circulant matrix are equal to the magnitude of the eigenvalues [49]. This property allows the multi-variable controller to be designed in terms of a family of independent SISO processes. The singular vectors of symmetric circulant matrices are harmonic functions of the spatial variable.

It has been shown in [78] that a circulant symmetric controller is sufficient to control a symmetric circulant process. Similar properties exist for
rectangular circulant matrices [24]. In the CD process, as the difference between the narrow band diagonal symmetric Toeplitz matrix and the circulant matrix is a small perturbation, approximating the spatial interaction matrix by a circulant matrix was employed in recent work on CD control to simplify the control design and analysis [74],[77] and [78]. For a multi-variable system represented by a symmetric circulant matrix, the directionality is independent of the dynamical frequency as the transfer matrix has the same singular vectors at all frequencies. Although stability of a multi-variable system is defined in terms of its eigenvalues, the performance and stability robustness are investigated through the singular values of the system [19].

2.2 A Two-Dimensional ARMAX Model

In [45],[46] and [89] a 2D autoregressive moving average with exogenous input (ARMAX) model was used in identifying the CD process and the linkage between the multi-variable model and two dimensional systems was explored.

The process was represented by the following model:

\[ A(z, \lambda) y(m_t, n_x) = z^{-\nu} B(z, \lambda) u(m_t, n_x) + C(z, \lambda) e(m_t, n_x) \]

where \( y(m_t, n_x) \) is the output, \( u(m_t, n_x) \) is the input and \( e(m_t, n_x) \) is white noise with variance \( \sigma^2 \).

The two-variable polynomials \( A, B \) and \( C \) are given by:

\[
A(z, \lambda) = \sum_{i=0}^{N_x} \sum_{j=0}^{M_t} a_{i,j} z^{-i} \lambda^{-j} + \sum_{j=1}^{M_t} \sum_{i=1}^{N_x} a_{-i,j} z^{-j} \lambda^{i}
\]

\[
B(z, \lambda) = \sum_{i=0}^{N_x} \sum_{j=0}^{M_t} b_{i,j} z^{-i} \lambda^{-j} + \sum_{j=1}^{M_t} \sum_{i=1}^{N_x} b_{-i,j} z^{-j} \lambda^{i}
\]

\[
C(z, \lambda) = \sum_{i=0}^{N_x} \sum_{j=0}^{M_t} c_{i,j} z^{-j} \lambda^{-i} + \sum_{j=1}^{M_t} \sum_{i=1}^{N_x} c_{-i,j} z^{-j} \lambda^{i}
\]

The backwards shift operators \( z^{-1} \) and \( \lambda^{-1} \) operate in the horizontal (MD) and vertical (CD) directions respectively.

\[
\lambda^{-1} y(m_t, n_x) = y(m_t, n_x - 1)
\]

\[
z^{-1} y(m_t, n_x) = y(m_t - 1, n_x)
\]

The coordinates \((m_t, n_x)\) represent the position in the plane with respect to some arbitrary origin towards the bottom-left corner of the plane.
Chapter 2. A Two-Dimensional Model for the CD Process

A, B and C are all non-symmetric half plane (NSHP) causal. The term \( z^{-v} \) models the delay in the temporal domain. The local supports are assumed to be truncated at the edges to develop the appropriate prediction and control algorithms.

2.3 A Two-Dimensional Spatially Noncausal Model

Modeling the CD process as a two-dimensional system is appealing as the response propagates in time and space. The aforementioned two-dimensional model permits prediction and minimum variance control over the plane of finite width. However, this model did not use the standard assumption of separability between the temporal and spatial responses as it focused on recursive prediction at each point in the plane.

As the CD process is assumed to be spatially invariant except at the edges, the output profile at steady-state is the convolution of the input profile and a single actuator response. The process is noncausal in the spatial domain as bumping an actuator affects the profile on both sides.

This is a convolution of the input \( u \) with a noncausal response \( g(k) \) which decays to zero after \( k_0 \) samples on both sides.

\[
y(k) = \sum_{i=-k_0}^{k_0} g(k - i)u(i) \tag{2.5}
\]

In the following sections, spatial non-causality is discussed and used to simplify the previous work on two-dimensional modeling of the process.

2.3.1 Non-Causality in Space

In [32] and [63], problems that rise from dealing with the doubly-infinite time axis were pointed out. Spatial non-causality appears in applications such as CD processes, image processing and digital filtering [33]. In multidimensional digital filter theory, the stability of spatially noncausal filters was addressed [53] and [79].

In [15],[16] and [17] noncausal pseudo state-space models are proposed to represent noncausal multidimensional systems. Stability and closed-loop control design were investigated using the linear fractional transformations (LFT).
In the two dimensional ARMAX model presented in section 2.2, all the polynomials in the 2D ARMAX model were non-symmetric half plane (NSHP) causal where non-causality in space was accounted for by positive and negative powers of the transform operator in space. This representation resembles the standard form of modeling two-dimensional spatially noncausal filters. While this representation is convenient for recursive estimation, it does not really simplify robust control design nor provide an insight into the spatial frequency domain.

In [40], modeling the CD process using a separable two-dimensional system is proposed to study the effect of structured uncertainties on the system's stability. The process is modeled by a causal transfer function in the temporal domain cascaded by a noncausal transfer function in the spatial domain. The multidimensional system has to be well posed: i.e., its pulse response is absolutely summable along the spatial coordinates. The process is assumed to be both linear time invariant (LTI) and linear space invariant (LSI). The LSI assumption can be thought of as either an infinite sheet of paper or a tube where there are no edge effects and all the actuators have identical responses. This assumption is equivalent to the circulant matrix approximation for a band diagonal matrix.

This work adopts the separable two-dimensional representation from [40] to demonstrate its advantages in identification in the spatial domain, robust control and shaping the two-dimensional frequency domain. Two possible model structures are a noncausal FIR model and a noncausal rational transfer function. The FIR model is more convenient for identification and input design while the rational transfer function is more suitable for control design and robust stability analysis. The frequency responses of the proposed transfer functions are shown to be equivalent to the spatial frequency decomposition of the corresponding circulant spatial matrices. A noncausal rational transfer function has two sets of poles, causal and anti-causal poles that are responsible for the response in the positive and negative directions of space respectively.

2.3.2 A Spatial Noncausal FIR

The spatial impulse response can be represented by a two-sided $\lambda$-transform ($z$-transform in space) [38] and [39]:

$$G_{FIR}(\lambda, \lambda^{-1}) = g_{n_f} \lambda^{n_f} + \ldots + g_1 \lambda^1 + g_0 + g_1 \lambda^{-1} + \ldots + g_{n_f} \lambda^{-n_f} \quad (2.6)$$

where $\lambda^{-1}$ is the spatial left shift transform operator.
Chapter 2. A Two-Dimensional Model for the CD Process

Employing the separability assumption, the CD process is modeled by a noncausal spatial FIR cascaded by a temporal transfer function giving the following two-dimensional system.

\[ y(z, \lambda) = G_{FIR}(\lambda, \lambda^{-1})T(z)u(z, \lambda) + d(z, \lambda) \] (2.7)

\( G_{FIR}(\lambda, \lambda^{-1}) \) replaces the spatial interaction matrix.
\( T(z) \) is the temporal transfer function.

2.3.3 Region of Convergence (ROC) for a Two-Sided z-Transform

In this section, a brief review of the regions of convergence for the z-transform is provided as it is essential for the stability analysis of the rational noncausal transfer function.

For a causal transfer function \( X_c(z) \) computed as a right handed z-transform:

\[ X_c(z) = \sum_{i=0}^{\infty} a^i z^{-i} = 1 + a z^{-1} + a^2 z^{-2} + a^3 z^{-3} + ... \]

The series is absolutely summable for \( |az^{-1}| < 1 \) which can be rewritten as \( |z| > |a| \). The ROC for the causal transfer function is the shaded area in Figure 2.1. If \( X_c(z) \) has more than one pole, the ROC extends outward from the outermost pole.

For an anti-causal transfer function \( X_{ac}(z) \) computed as a left handed z-transform:

\[ X(z) = \sum_{i=\infty}^{0} b^i z^{-i} = 1 + b^{-1} z^1 + b^{-2} z^2 + b^{-3} z^3 + ... \]

The series is absolutely summable for \( |b^{-1}z| < 1 \) which can be rewritten as \( |z| < |b| \). The ROC for the causal transfer function is the shaded area in Figure 2.2. If \( X_{ac}(z) \) has more than one pole, the ROC extends inward from the innermost pole.

For a noncausal transfer function \( X_{nc}(z) \) where:

\[ X_{nc}(z) = X_c(z) + X_{ac}(z) \]
Chapter 2. A Two-Dimensional Model for the CD Process

Figure 2.1: ROC for a causal transfer function

Figure 2.2: ROC for an anti-causal transfer function
Chapter 2. A Two-Dimensional Model for the CD Process

The ROC for the noncausal transfer function is the shaded area in Figure 2.3. The expansion of the noncausal transfer function is absolutely summable as long as it is analytic in the convergence ring $|a| < |z| < |b|$ and contains the unit circle.

![Figure 2.3: ROC for a noncausal transfer function](image)

2.3.4 A Spatial Noncausal Transfer Function

The spatial response can be modeled by a two-sided $\lambda$-transform using a rational transfer function and taking non-causality into consideration. The response is composed of two parts, one propagating to the left and the other to the right [39].

\[ y_{nc}(\lambda, \lambda^{-1}) = y_c(\lambda^{-1}) + y_{ac}(\lambda) \]  

(2.8)

The causal part ($y_c$) of the actuator's impulse response is represented with the following model:

\[ y_c(k) = -\sum_{i=1}^{n_c} a_i y(k-i) + \sum_{i=0}^{n_r} b_i u(k-i) \]  

(2.9)
Chapter 2. A Two-Dimensional Model for the CD Process

The transfer function of this causal part is given by:

\[ G_c(\lambda^{-1}) = \frac{b_0 + b_1 \lambda^{-1} + \ldots + b_{nc} \lambda^{-nc}}{1 + a_1 \lambda^{-1} + \ldots + a_{nc} \lambda^{-nc}} \]  

(2.10)

The anti-causal part is symmetrical to the causal part, it can be modeled by the same transfer functions but with positive powers of \( \lambda \).

The noncausal transfer function is:

\[ G_{nc}(\lambda, \lambda^{-1}) = G_c(\lambda^{-1}) + G_{ac}(\lambda) \]  

(2.11)

\[ G_{nc}(\lambda, \lambda^{-1}) = \frac{c_{nc} \lambda^{nc} + \ldots + c_1 \lambda + c_0 + c_1 \lambda^{-1} + \ldots + c_{nc} \lambda^{-nc}}{d_{nc} \lambda^{nc} + \ldots + d_1 \lambda + d_0 + d_1 \lambda^{-1} + \ldots + d_{nc} \lambda^{-nc}} \]  

(2.12)

The Wiener-Hopf decomposition allows expressing this transfer function as:

\[ G_{nc}(\lambda, \lambda^{-1}) = G'_c(\lambda^{-1}) \cdot G'_{ac}(\lambda) \]  

(2.13)

The spatial process is open-loop stable as the response decays after \( k_0 \) samples on both sides. In the rational transfer function, this reflects in two sets of stable poles. The causal poles lie inside the unit circle and are responsible for the response on the right side of the bumped actuator while the anti-causal poles lie outside the unit circle and are responsible for the response on the left side. The parameters of the noncausal transfer function can be estimated using least squares identification from input-output data using \((n_c)\) forward and \((n_c)\) backward measurements at each point as shown in section 3.2.2. The identified model has \((n_c)\) causal and \((n_c)\) anti-causal poles. Adopting this noncausal spatial model reduces the large dimensionality introduced by the interaction matrix. This spatial noncausal transfer function is convenient for identification and control design in the spatial domain.

It is highly unlikely that non-minimum phase systems would be required to model the spatial causal response in CD process. Zeros outside the unit circle usually appear when a continuous time system, that has a pole-zero excess of more than one in the \( s \)-domain, with zero-order hold or a fraction time delay is transformed to the \( z \)-domain [66]. Even if non-minimum phase
zeros appear in the causal model, this should not be problematic as unstable pole-zero cancelations can be easily avoided in the spatial controller design. Given the spatial response of an actuator, the causal and anti-causal responses can be modeled with separate rational transfer functions leaving no room for mistaking a causal zero for an anti-causal zero.

Employing the separability assumption, the CD process is modeled by the noncausal spatial model cascaded by a temporal transfer function giving the following two-dimensional system.

\[
y(z, \lambda) = G_{nc}(\lambda, \lambda^{-1})T(z)u(z, \lambda) + d(z, \lambda) \quad (2.14)
\]

\(G_{nc}(\lambda, \lambda^{-1})\) replaces the spatial interaction matrix.

\(T(z)\) is the temporal transfer function.

As the sampling time in CD control systems is determined by the scanner, the control action is applied after a full scan when the whole CD profile is available. This fact makes it reasonable to identify a noncausal spatial model from the available CD profile.

The two-dimensional (2D) frequency domain can be shaped using a few parameters that represent the 2D process. This simple model permits extending loop-shaping techniques to multi-array systems.

### 2.4 Frequency Response of the Spatial Models

In order to perform spatial frequency analysis, an assumption of infinite width must be invoked to avoid the need for boundary conditions at the edges. Some web processes such as the blown extrusion of plastic films are modeled by circulant matrices as the web is extruded as a tube [20] and [44]. Circulant matrices are decomposed using Fourier matrices and do not require the infinite width assumption. As for CD processes where the web is produced as a sheet, the identical actuator responses are truncated at the edges resulting in spatial matrices that are band diagonal [84]. In spatial frequency analysis, boundary conditions are ignored [41] as the symmetric toeplitz band diagonal matrix is approximated by a circulant matrix. As the difference between the narrow band diagonal symmetric toeplitz matrix and the circulant matrix is a small perturbation, this is an acceptable approximation that has been employed in recent work on CD control [74],[77] and
In [42], it has been proven that the band diagonal matrix and its circulant approximation are asymptotically equivalent, i.e., both are bounded in the operator norm. Moreover, the eigen-values of both matrices are asymptotically equally distributed. In order to distinguish between spatial and temporal domain expressions, the symbol \( v \) is used to represent spatial frequencies while the standard \( \omega \) is reserved for the temporal frequencies. For illustration purposes in this work, whenever the actuator spacing is not specified, decomposing the spatial response by \( m \)-point discrete Fourier transform is interpreted as decomposing the signal into spatial harmonic functions with \( m \) equally spaced frequencies between \( v = 0 \) and \( v = 2\pi \) [65]. The frequency response is symmetrical around \( v = \pi \) so only the frequency range \([0, \pi]\) is shown in the plots.

The following example shows that the frequency response of a noncausal transfer function is the same as the spatial modes obtained through decomposing a circulant matrix by Fourier matrices. Let the circulant matrix \( G_{1sp} \), which has the actuator’s spatial responses as its columns be:

\[
G_{1sp} = \text{toeplitz}\{1.4352, 0.38, -0.3, -0.2201, 0.0151, 0.0718, 0.0203, -0.0147, 0, \ldots, 0, -0.0147, 0.0203, 0.0718, 0.0151, -0.2201, -0.3, 0.38\}
\]

where \text{toeplitz} is the MATLAB command which returns the symmetric or Hermitian Toeplitz matrix formed from the given vector.

The spatial response was identified as the following noncausal transfer function using the technique from [2].

\[
G_{1s}(\lambda, \lambda^{-1}) = \frac{1}{0.2497\lambda^2 - 0.3725\lambda^1 + \lambda^0 - 0.3725\lambda^{-1} + 0.2497\lambda^{-2}}
\]

Figure 2.4 shows that the frequency response (Freqz) of the transfer function \( G_{1s} \) is identical to the spatial modes of the circulant matrix \( G_{1sp} \) obtained by the decomposition by Fourier matrices.

The spatial interaction matrix for the basis weight process from [74] is used to compare the singular values to the spatial modes of the circulant matrix approximation. Figure 2.5 shows that the singular values of the symmetric Toeplitz band diagonal matrix \( G_{2t} \) are almost identical to those of its circulant matrix approximation \( G_{2c} \). In this example, the circulant matrix was chosen as one that has a monotonic spatial mode function to be easily compared to the singular values.

\[
G_{2t} = \text{toeplitz}\{-0.0814, -0.0455, -0.0047, 0.0017, 0.0003, 0, \ldots, 0\}
\]
Chapter 2. A Two-Dimensional Model for the CD Process

Figure 2.4: Frequency response of the spatial transfer function vs FFT of the spatial matrix

\[ G_2 = \text{toeplitz}\{-0.0814, -0.0455, -0.0047, 0.0017, 0.0003, 0, ..., 0, 0.0003, 0.0017, -0.0047, -0.0455\} \]

2.5 A 2D Closed-Loop Transfer Function for Non-Square Systems

In many paper machines there are more measurement points than there are actuators (in this work, these processes are referred to as non-square systems). The adopted noncausal spatial transfer functions provide models that are more convenient for frequency domain analysis. As there are two profile resolutions, a low resolution for the actuator profile and a high resolution for the measurement profile, multi-rate system theory could be used to study the system at the output’s high resolution. The low resolution control error profile is constructed from the high resolution error profile through filtering with an anti-aliasing window and/or weighted averaging across an actuator zone [41]. Sampling distances in the spatial transfer functions are analogous to sampling times in conventional temporal models.
In CD control design, usually the high-resolution measurement profile is mapped down to the actuators resolution resulting in a square spatial interaction matrix. In this case, the closed-loop system can be modeled easily by spatial transfer functions. Mapping the real process to a square system is appealing for control design but is inconvenient when studying the misalignment problem as the mis-map could be a fraction of the actuator spacing. To address this problem, rectangular matrices relating the whole measurement profile to the actuator array are required. Replacing these matrices by spatial noncausal transfer functions results in a multi-rate system as there are more outputs than inputs. This is analogous to sampling the output at a faster rate than that of the input in the temporal domain.

In this work, an equivalent 2D closed-loop transfer function in the faster sampling rate is developed and is shown to agree with the multi-rate signal theory [83] and with earlier work on spectral models for CD control [41].

2.5.1 Frequency Domain Analysis for Non-Square Systems
The number of measurement positions $m_y$ is assumed to be an integer multiple of the number of actuators $n_u$. An integer factor is not a must as in
Chapter 2. A Two-Dimensional Model for the CD Process

the case of a rational ratio, the controller and plant can be converted to a common multiple of the sampling distances.

The high resolution transfer function is given by:

\[ G(\lambda_N, \lambda_N^{-1}) = \frac{G_n(\lambda_N, \lambda_N^{-1})}{G_d(\lambda_N, \lambda_N^{-1})} \]

where \( \lambda_N^{-1} \) is the spatial left shift operator in the high resolution.

As the spatial noncausal transfer function at the fast sampling rate (high resolution) corresponds to an \((m_y \times m_u)\) spatial matrix, a scaling factor of \( q_d = 1/\sqrt{(m_y/n_u)} \) is required to match the frequency response of this transfer function with the spatial modes obtained from the Fourier transform of the \((m_y \times n_u)\) spatial matrix. In the following analysis, the low resolution square spatial interaction matrix \( G_2 \) from section 2.4 is replaced by a rectangular spatial matrix \( G_{2_{rec}} \) where \( m_y = 5n_u \) and a square matrix in the high resolution \( G_{2_{sq}} \). The spatial response in the high resolution is modeled by the transfer function \( G_{2_{HR}} \). Matching the resultant spatial frequency responses using the factor \( q_d \) is shown in Figure 2.6. The difference at low frequencies is due to restricting the order of the high resolution transfer function for computational purposes.

2.5.2 A 2D Closed-Loop Transfer Function

The closed-loop with the CD controller under investigation is shown in Figure 2.7. The controller in the CD process gets a mapped signal at the actuator’s low resolution and produces the output at the actuator’s positions. In order to find the equivalent loop in the high resolution a downsampler has to be included in the loop to account for the mapping. The downsampler is considered a linear but time-varying system [82].

In [70], it has been shown that when downsampling a signal \( y(t) \) with sampling time \( T_s \) by a factor of \( N \) to \( y'(t) \) with sampling time \( T'_s \), aliasing will be avoided if \( T_s \leq \frac{T'_s}{N} \) where \( \Omega \) is the Nyquist frequency of the original signal. If the inequality is satisfied, \( y'(z) = \frac{1}{N} y(z) \).

This is the case with CD control systems where an anti-aliasing window is used to filter the high resolution profile. The mapping results in a decimation factor appearing in the frequency domain. In the two-dimensional
Figure 2.6: Matching the spatial frequency response of the square matrix in the high resolution $q_d^*\text{FFT}(G_{2_{sq}})$, rectangular matrix ($G_{2_{rec}}$) and the transfer function in the high resolution $q_d^*\text{Freqz}(G_{2_{HR}})$

Figure 2.7: The multi-variable loop with mapping
closed-loop transfer function this is accounted for by a scaling factor of $(n_u/m_y)$. Taking into account the previous scaling factors arising in the high resolution spatial λ-transform, the two-dimensional closed-loop transfer function is shown in Figure 2.8:

\[
D(z, \lambda) = I(z, \lambda)
\]

It has been shown in [56] that the equivalent of an all-digital multi-rate system with a slow rate input and a fast rate output as in Figure 2.9 can be represented by the following transfer function:

\[
Y(z) = C(z)R(z)
\]

\[
C(z) = \sum_{k=0}^{\infty} c(kT_s/N)z^{-k/N}
\]
where

\[ C(z)_N = C(z)|_{z = z^{1/N}, T_s = T_s/N} \]

By letting \( z_N = z^{1/N}, z^{k/N} \) becomes \( z_N^k \).

The expansion of \( C(z)_N \) in powers of \( z^{-1/N} \) gives the values at the fast-rate sampling instants of \( t_s = kT_s \). For the digital controller, the output is non-zero only at every \( T_s \) seconds (the slow-rate sampling time). The same result is stated for discrete signals in [70].

Consider the spatial matrix \( C_{sp} \in \mathbb{R}^{n_x \times n_y} \) and the corresponding spatial noncausal FIR \( C_s(\lambda, \lambda^{-1}) \) for a system at the actuator’s low resolution \( n_u \). Assume that this spatial matrix is part of the feedback controller where a low resolution error profile is fed to the controller and a low resolution control profile is computed for the actuators.

Let the FIR model in the low resolution be:

\[ C_s(\lambda, \lambda^{-1}) = c_3\lambda^3 + c_2\lambda^2 + c_1\lambda^1 + c_0\lambda^0 + c_1\lambda^{-1} + c_2\lambda^{-2} + c_3\lambda^{-3} \]

Following the multi-rate theory, the equivalent spatial controller at 5 times the sampling rate is given by:

\[ C_s(\lambda_N, \lambda_N^{-1}) = c_3\lambda_N^{15} + c_2\lambda_N^{10} + c_1\lambda_N^{5} + c_0\lambda_N^0 + c_1\lambda_N^{-5} + c_2\lambda_N^{-10} + c_3\lambda_N^{-15} \]

The frequency domain equivalence is illustrated by the following example:

\[ C_{sp} = \text{toeplitz}\{-4.1226, -1.9487, 0.915, 0.9408, 0, \ldots, 0\} \]

The equivalent low resolution FIR model is:

\[ C_s(\lambda, \lambda^{-1}) = 0.9408\lambda^3 + 0.915\lambda^2 - 1.9487\lambda^1 - 4.1226\lambda^0 - 1.9487\lambda^{-1} + 0.915\lambda^{-2} + 0.9408\lambda^{-3} \]

The high resolution FIR model is:

\[ C_s(\lambda_N, \lambda_N^{-1}) = 0.9408\lambda_N^{15} + 0.915\lambda_N^{10} - 1.9487\lambda_N^{5} - 4.1226\lambda_N^0 - 1.9487\lambda_N^{-5} + 0.915\lambda_N^{-10} + 0.9408\lambda_N^{-15} \]
Chapter 2. A Two-Dimensional Model for the CD Process

This model equivalency is confirmed by matching the frequency response of both the low and high resolution spatial matrices with those of the transfer functions at the frequency range corresponding to the low resolution as shown in Figure 2.10.

![Figure 2.10: FFT of the spatial matrix vs transfer function frequency response](image)

In [41], it has been shown that controlling a high-resolution profile with a lower resolution actuator profile results in an aliasing effect. This fact is explained by the high resolution equivalent controller model. Frequency domain analysis at the measurement profile resolution is easily performed using the equivalent two-dimensional model.

### 2.6 Summary

In this chapter, the large dimension interaction matrix is replaced by non-causal transfer functions in space resulting in a two-dimensional (2D) spatially noncausal model. The spatial model can be either a FIR model or a rational transfer function depending on what is more convenient for the
analysis.

In processes with an equal number of actuators and measurement points (square systems), a two-dimensional closed-loop transfer function can be readily obtained by representing the controller by a 2D spatially noncausal model. The 2D closed-loop transfer function can help explain and predict the spatial bandwidth, spatial localization and streaks when running in a feedback loop. In order to address the industrial case where there are more measurement points than there are actuators (non-square systems), results from the multi-rate digital theory are used to develop an equivalent 2D closed-loop model to the process.
Chapter 3

Identification and Input Design in Open-Loop

3.1 Introduction

The identification of CD models is a twofold problem. The first problem is identifying the CD mapping model to determine the actual actuators' center of responses. The second problem is to identify the CD response model [35] and [36].

The spatial response is modeled by the noncausal transfer functions proposed in chapter 2 instead of the parametric function presented in [35] and [36]. The parametric model identification does not deliver an uncertainty region for the identified parameters and lacks a direct relation with control design tools. On the other hand, adopting noncausal spatial transfer functions has many advantages for identification and control design. These models provide simple means to detect the mapping model and misalignment as well as enabling control relevant identification and delivering parameter uncertainty regions. This work is presented in [6].

In this chapter, identification and input design for CD processes are investigated. In the next section least squares identification techniques are presented. A method for detecting the mapping model is proposed in section 3.3. Input design is addressed in section 3.4. The various input design signals are tested on an industrial paper machine simulator. Industrial identification results are given in section 3.6. The simulator and industrial identification experiments illustrate the advantage of using the designed inputs and the spatial model representation.

3.2 CD Response Shape Model

This work focuses on identifying the CD response shape as the dynamics are simple and modeled by a first-order process with time delay. Steady-state
Chapter 3. Identification and Input Design in Open-Loop

data is considered here, so the steady-state value of the temporal transfer function is included as a factor multiplied by the spatial transfer function. The spatial process can be modeled either by a FIR or a rational transfer function in space. The FIR coefficients are the elements in one column of the spatial interaction matrix with the powers of \( \lambda \) accounting for the shift introduced in position. The FIR model is identified directly from the input-output data with a least-squares algorithm. This model structure requires identifying quite a few parameters to model systems with wide spatial response. A more convenient model for control design is the spatial rational transfer function identified within an autoregressive with exogenous input (ARX) model structure. The rational transfer function needs fewer parameters to model systems. This transfer function can be used to shape the spatial sensitivity functions using design techniques from the temporal domain. Conventional control techniques can be modified to accommodate non-causality then applied to the spatial domain. Both models deliver an uncertainty set for the parameters, this is quite an advantage over standard CD models. Temporal domain experiment design tools can be applied to maximize the information matrices from the identification experiment with respect to a certain measure.

3.2.1 Identification of Noncausal FIR Models

The FIR coefficients are the elements in one column of the spatial interaction matrix with the powers of \( \lambda \) accounting for the shift introduced in position. In the conventional multi-variable model representation the response is truncated at the edges. In [30], the assumption of a symmetrical and identical actuator response was used to identify the coefficients of the CD response. The linear equations between the input profile and the output profile were rearranged to cast the problem as a function of the symmetrical response coefficients, thus avoiding the need to address the non-causality issue in the spatial domain. In this work, it is shown that the noncausal FIR model can be identified consistently from the input-output data with a least-squares algorithm. Adopting the noncausal FIR model provides insight into the spatial frequency response of the actuator arrays as well as delivering an uncertainty set for the parameters. This is quite an advantage over the parametric model in equation (2.4). Moreover, the proposed technique is useful in identifying the mapping model, as shown in section 3.3.

Introducing a spatial input signal \( u(x) \), the output profile at steady-state
Chapter 3. Identification and Input Design in Open-Loop

\( y(x) \) is:

\[
y(x) = \left[ g_n q^n + \ldots + g_1 q + g_0 + g_{-1} q^{-1} + \ldots + g_{-n_f} q^{-nf} \right] u(x) + e(x) \tag{3.1}
\]

where \( q \) is the right shift operator in space

\( e(x) \) is assumed to be white noise

The general case without assuming symmetry in the actuator's response is treated, the symmetrical model representation requires a simple change in the regressor. The non-symmetrical representation is used later on in identifying the mapping model.

No Symmetry Enforced in the Model

The observed data in a linear regression form is given by:

\[
y(x) = \varphi^T(x) \theta_0 + e(x) \tag{3.2}
\]

\[
\varphi(x) = [u(x + n_f), \ldots, u(x + 1), u(x), u(x - 1), \ldots, u(x - n_f)]^T
\]

\[
\theta_0 = [g_{n_f}, \ldots, g_1, g_0, g_{-1}, \ldots, g_{-n_f}]
\]

\( \varphi(x) \) is deterministic as the regressor contains samples of the input signal only

\( e(x) \) is a sequence of independent random variables with zero mean values and variance \( \sigma_0 \)

From [62], the prediction error is:

\[
\varepsilon(x, \theta) = y(x) - \varphi^T(x) \theta
\]

The least-squares criterion gives the vector \( \theta \) that minimizes:

\[
V_N(\theta, Z^N) = \frac{1}{N} \sum_{x=1}^{N} \frac{1}{2} [y(x) - \varphi^T(x) \theta]^2
\]

Let \( Z^N \) denote the set of collected data where \( N \) is the number of data points (in the CD).

\[
Z^N = [y(1), u(1), y(2), u(2), \ldots, y(N), u(N)]
\]
Chapter 3. Identification and Input Design in Open-Loop

\[ \hat{\theta}^{LS}_{N} = \arg \min_{\theta} V_{N}(\theta, Z^{N}) = \left[ \frac{1}{N} \sum_{x=1}^{N} \varphi(x)\varphi^{T}(x) \right]^{-1} \sum_{x=1}^{N} \frac{1}{N} \varphi^{T}(x)y(x) \]

The spatial non-causality does not cause any problems with least-squares identification in this model. The noise vector is a sequence of independent random variables with zero mean values and consequently the output data at different positions are not correlated allowing for a consistent least squares estimation of the parameters. The following results from [62] are cited here as they are used in identification, input design and computing the uncertainty regions.

- **Convergence and Consistency**

\[ \hat{\theta}^{LS}_{N} - \theta_{0} = \left[ \frac{1}{N} \sum_{x=1}^{N} \varphi(x)\varphi^{T}(x) \right]^{-1} \sum_{x=1}^{N} \frac{1}{N} \varphi^{T}(x)e(x) \]

Assume the first sum, which is deterministic, converges to an invertible matrix \( R_{\varphi}(0) \)

\[ \frac{1}{N} \sum_{x=1}^{N} \varphi(x)\varphi^{T}(x) \to R_{\varphi}(0) \text{ as } N \to \infty \]

and the second sum consists of random variables with zero mean values:

\[ E e(x) = 0 \]

Theorem (2.3) in [62] shows that:

\[ \frac{1}{N} \sum_{x=1}^{N} \varphi(x)e(x) \to 0 \text{ w.p. } 1 \text{ as } N \to \infty \]

\[ \hat{\theta}^{LS}_{N} \to \theta_{0} \text{ w.p. } 1 \text{ as } N \to \infty \]

\( \hat{\theta}^{LS}_{N} \) is a strongly consistent estimate of \( \theta_{0} \)

- **Bias and Variance**

The estimate is unbiased: \( E \hat{\theta}^{LS}_{N} = \theta_{0} \)

The covariance matrix of \( \hat{\theta}^{LS}_{N} \) is given by:

\[ P_{N} = Cov \hat{\theta}^{LS}_{N} = \lambda_{0} \left[ \sum_{x=1}^{N} \varphi(x)\varphi^{T}(x) \right]^{-1} \]

39
• Distribution of the Estimates

Assuming the vector of disturbance terms has a gaussian distribution:

\[ \hat{\theta}_N^{LS} \sim N(\theta_0, P_N) \]

• Confidence Intervals

\[ \hat{\theta}_N^{LS(i)} - \theta_0 \in N(0, P_N^{(i)}) \]

\[ (\hat{\theta}_N^{LS} - \theta_0)^T P_N^{-1} (\hat{\theta}_N^{LS} - \theta_0) \in \chi^2(d) \]

\[ d \text{ is the dimension of } \hat{\theta}_N^{LS} \]

The probability that:

\[ \left| \hat{\theta}_N^{LS} - \theta_0 \right|^2 \frac{1}{P_N^{(i)}} = (\hat{\theta}_N^{LS} - \theta_0)^T P_N^{-1} (\hat{\theta}_N^{LS} - \theta_0) \geq \alpha \]

is \( \chi^2_\alpha(d) \), the \( \alpha \) level of the \( \chi^2(d) \) distribution.

This expression defines an ellipsoid in \( \mathbb{R}^d \) whose shape is determined by \( P_N \).

In recent work [11], it has been proven that in order to construct confidence regions with probability \( \alpha \) by gluing together the frequency ellipses \( \chi^2(d) \) should be replaced with \( \chi_d \) where \( Pr(\chi^2(d) < \chi^2) = \alpha \).

• Expressions for the Asymptotic Covariance Matrix

\[ V_N(\theta, Z^N) = \frac{1}{N} \sum_{x=1}^{N} e(x, \theta)^2 \]

The covariance matrix of asymptotic distribution \( P_\theta \) is:

\[ P_\theta = \lambda_0 \left[ \bar{E} \psi(x, \theta_0) \psi^T(x, \theta_0) \right]^{-1} \]

\[ \psi(x, \theta) = \frac{d}{d\theta} \hat{y}(x|\theta) \]

\[ \psi(\lambda, \theta) = [\lambda^{n_1}, \lambda^{n_1-1}, \ldots, \lambda^0, \ldots, \lambda^{-n_f+1}, \lambda^{-n_f}] y(\lambda) \]

The transform operator in space is \( \lambda = e^{j\nu} \).

\[ \psi(e^{j\nu}, \theta) = [e^{j(n_f-1)\nu}, \ldots, 1, \ldots, e^{-j(n_f-1)\nu}, e^{-jn_f \nu}] y(e^{j\nu}) \] (3.3)
Chapter 3. Identification and Input Design in Open-Loop

\[ \Phi_\varphi = \left[ \begin{array}{c} \lambda^{n_f} \\ \lambda^{n_f-1} \\ \vdots \\ \lambda^{-n_f} \\ \lambda^{-n_f+1} \\ \vdots \\ \lambda^{-n_f} \end{array} \right] \Phi_u \left[ \begin{array}{cccc} \lambda^{-n_f} & \lambda^{-n_f+1} & \cdots & \lambda^{n_f-1} & \lambda^{n_f} \end{array} \right] \]

\[ \text{Cov} \; \hat{\theta}_N \approx \frac{1}{N} \left[ \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{\lambda_0} \psi(e^{j\nu}, \theta_0) \Phi_u \psi^T(e^{j\nu}, \theta_0) d\nu \right]^{-1} \]

\[ E(\theta_0 - \hat{\theta})(\theta_0 - \hat{\theta})^T \approx \frac{N}{2\pi} \int_{-\pi}^{\pi} \frac{\Phi_u}{\lambda_0} \left[ \begin{array}{cccc} \lambda^{n_f} & \lambda^{n_f-1} \\ \vdots & \vdots \\ \lambda^{-n_f+1} & \lambda^{-n_f} \end{array} \right] \left[ \begin{array}{cccc} \lambda^{n_f} & \lambda^{n_f-1} \\ \vdots & \vdots \\ \lambda^{-n_f+1} & \lambda^{-n_f} \end{array} \right]^* d\nu = M^{-1} \]

(3.4)

The Fisher information matrix \( M \) is the inverse of the covariance matrix. The average information matrix per data sample is \( \bar{M} = M/N \).

**Enforcing Symmetry in the Model**

As the CD process is assumed to be symmetrical in space, symmetry can be enforced in the model structure with the following regression vector:

\[ \varphi_{\text{sym}}(x) = [(u(x + n_f) + u(x - n_f)), \ldots, (u(x + 1) + u(x - 1)), u(x)]^T \]

In this case, the parameter vector is:

\[ \theta_{0, \text{sym}} = [g_{n_f}, \ldots, g_1, g_0] \]

**3.2.2 Identification of Noncausal ARX Models**

To the best of the authors' knowledge all the previous work on noncausal identification dealt with autoregressive (AR) or autoregressive moving average (ARMA) models where the systems were driven with unknown input
signals as in [72] and [81]. Some results are formulated in [9] for error-in-
variable processes with non-causality. In [90], it was observed that the least 
squares solution for a bilateral linear autoregression is biased. In the CD 
process the input signal is known exactly, this eliminates the identifiability 
problem mentioned in [60] that arises in the identification of noncausal sys-
tems driven solely by gaussian noise. However, non-causality in the rational 
transfer function structure results in a correlation between the outputs and 
the noise even if the noise vector itself is composed of independent random 
variables. This correlation results in a bias in least-squares identification 
but can be easily eliminated using the Instrumental Variables (IV) methods.

The ARX structure is:

\[ y(x) = \frac{B(q)}{A(q)} u(x) + \frac{1}{A(q)} e(x) \]  

(3.5)

\( u(x) \) is the input signal

\( y(x) \) is the output signal

\( e(x) \) is normally distributed noise with covariance \( \lambda_0 \)

\( q \) is the right shift operator

Replacing \( q \) with \( \lambda \), the transform in the \( \lambda \) domain is:

\[ y(\lambda) = \frac{b_n \lambda^n + \cdots + b_0 + \cdots + b_{-n_b} \lambda^{-n_b}}{a_n \lambda^n + \cdots + a_{-n_a} \lambda^{-n_a}} u(\lambda) + \frac{1}{a_n \lambda^n + \cdots + a_{-n_a} \lambda^{-n_a}} e(\lambda) \]  

(3.6)

The observed data in a linear regression form is given by:

\[ y(x) = \varphi^T(x) \theta_0 + e(x) \]  

(3.7)

\( \varphi(x) = [-y(x + n_a), \ldots, -y(x + 1), -y(x - 1), \ldots, -y(x - n_a), \]

\( u(x + n_b), \ldots, u(x + 1), u(x), u(x - 1), \ldots, u(x - n_b) \)

\( \theta_0 = [a_n, \ldots, a_1, a_{-1}, \ldots, a_{-n_a}, b_n, \ldots, b_0, \ldots, b_{-n_b}]^T \)
$\varphi(x)$ is the regressor that contains output and input data

$e(x)$ is a sequence of independent random variables with zero mean values and variance $\sigma_0$.

From [62], the prediction error is:

$$\varepsilon(x, \theta) = y(x) - \varphi^T(x)\theta$$

The least-squares criterion gives the vector $\theta$ that minimizes:

$$V_N(\theta, Z^N) = \frac{1}{N} \sum_{x=1}^{N} \frac{1}{2} [y(x) - \varphi^T(x)\theta]^2$$

Where $Z^N$ is the set of collected data, $N$ is the number of data points.

$$Z^N = [y(1), u(1), y(2), u(2), ..., y(N), u(N)]$$

$$\hat{\theta}_{LS}^N = \arg\min\ V_N(\theta, Z^N)$$

$$= \left[ \frac{1}{N} \sum_{x=1}^{N} \varphi(x)\varphi^T(x) \right]^{-1} \sum_{x=1}^{N} \frac{1}{N} \varphi^T(x)y(x)$$

The least squares estimate for this noncausal model is biased as the noise vector is correlated with the regressor. A simple solution to eliminate this correlation is the instrumental-variable methods. Following [62], a correlation vector $\zeta(x)$ (whose elements are the instrumental variables) is generated from an initial estimate of the system’s parameters such that:

$$\bar{E} \zeta(x)\varphi^T(x) \text{ is non singular}$$

$$\bar{E} \zeta(x)e_0(x) = 0$$

This gives:

$$\theta_{IV}^N = \text{sol} \left\{ \frac{1}{N} \sum_{x=1}^{N} \zeta(x)[y(x) - \varphi^T(x)\theta] = 0 \right\}$$

$$\hat{\theta}_{IV}^N = \left[ \frac{1}{N} \sum_{x=1}^{N} \zeta(x)\varphi^T(x) \right]^{-1} \sum_{x=1}^{N} \frac{1}{N} \zeta^T(x)y(x)$$
In the frequency domain, the prediction error expression for the ARX structure is:

\[
V_N(\theta, Z^N) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \hat{G}(e^{j\nu}) - G(e^{j\nu}, \theta) \right|^2 \frac{|U_N(\nu)|^2}{|H(e^{j\nu}, \theta)|^2} d\nu
\]

(3.8)

\(\nu\) represents the spatial frequency

For the ARX model:

\[
G(e^{j\nu}, \theta) = \frac{B(e^{j\nu}, \theta)}{A(e^{j\nu}, \theta)}
\]

\[
H(e^{j\nu}, \theta) = \frac{1}{A(e^{j\nu}, \theta)}
\]

\(\hat{G}(e^{j\nu})\) is the empirical transfer function estimate.

This expression shows that the prediction-error method fits the empirical transfer function estimate with a weighted norm corresponding to the model signal-to-noise ratio at each frequency [62]. In the CD process, the gain rolls off to zero in the high frequency range resulting in a small signal-noise-ratio. This fact might lead to incorrect model estimates that focus on fitting the low spatial frequency response. In order to avoid this, the assumption of a symmetrical response will be enforced in the identification. Symmetry can either be enforced in model structure or by running the identification process in one direction then back again in the other direction.

A frequency-domain expression for the covariance matrix of an IV estimate is given by (9.85) in [62]. This expression can be used for control relevant identification for the ARX model.

### 3.3 CD Mapping Model

In modern paper machines, the CD measurement profile has a higher resolution than that of the actuator profile as there are more measurement points than there are actuators. Identifying a higher order FIR model from the high resolution profile provides information about the mapping model. Regarding the CD response shape, identifying a low order FIR model from a low resolution measurement profile is sufficient for the purpose of feedback control as the actuator array can not remove disturbances at frequencies higher than the actuator nyquist frequency [20].
Identifying the mapping model is crucial for robust CD control. The mapping model aligns the center of response of each actuator with a measurement box as shown in Figure 3.1. In paper machines, the alignment problem results from the fact that the paper sheet can shrink significantly by the time it reaches the scanning sensor. Moreover, as there are more measurement points than actuators, the alignment problem becomes even more challenging. In [36], a technique is presented to identify a parametric mapping model through a fuzzy logic model of the shrinkage profile. The algorithm requires an estimate for the CD response model and minimizes the model fit error between the measured profile and the predicted profile [37].

![Figure 3.1: Alignment in the CD process](image)

In this work, a technique is proposed for detecting the actuators’ center of responses without the need for an initial estimate of the CD response shape. The CD response is identified by a noncausal spatial FIR model from input-output data without enforcing symmetry. Starting from some assumed alignment, the actual alignment will reflect in the shift in the axis of symmetry of the identified response. The initial alignment assumption is merely used as a reference in the identification algorithm and by no means affects the results.

In order to determine the alignment, the noncausal FIR model must be
in the measurement profile resolution. This is achieved through a high order
FIR model where the sampling distance is the spacing between the measure-
ment boxes. The left shift operator in space $\lambda_{HR}^{-1}$ accounts for one position
in the measurement profile.

The high resolution noncausal FIR without the symmetry assumption is:

$$G_{HR}(\lambda_{HR}, \lambda_{HR}^{-1}) = g'_{m_{1}} \lambda_{HR}^{m_{1}} + \ldots + g'_{1} \lambda_{HR} + g_{0} + g'_{-1} \lambda_{HR}^{-1} + \ldots + g'_{-m_{1}} \lambda_{HR}^{-m_{1}}$$  (3.9)

For example, assuming the ratio between the number of measurement points
to actuators in equation (3.1) is an integer $r$, the coefficients in the high res-
olution can be computed by interpolating $(r - 1)$ points between each $g_{i}$ and
$g_{i-1}$.

The mapping information can be identified within zones of few actuators
given the input-output data is generated from a rich spatial input signal.
Identification of the mapping model requires data from a single scan with
a SNR ratio smaller than the standard bump test. In the case of sheet
wandering, misalignment can be detected from a low SNR bump test given
an accurate mapping model. Mapping and misalignment detection through
the proposed technique was tested on an industrial paper machine simulator.
The simulation results are given in section 3.5.

### 3.4 Input Design for CD Model Identification

Figure 3.2 shows the FFT of a bump profile in the spatial domain when 10
actuators are bumped. As bump tests excite only a few spatial frequencies,
the model uncertainty at the rest of the unexcited spatial frequencies might
be large. The insufficiency of the information induced by bump tests raises
the need for input design [30] and [84].

Experiment design was the subject of many earlier studies. Some of
the design objectives were D-optimality, G-optimality, L-optimality and E-
optimality; each involving a cost function of the average per data sample
information matrix $\hat{M}$. A D-optimal design minimizes the determinant of
the generalized variance which is equivalent to minimizing the uncertainty
ellipsoids.

In [28],[29] and [30] a spatial input signal was used to minimize the un-
certainties in the steady-state process gains. As the optimization problem
was non convex, a simulated annealing algorithm was used to solve it. The
simulated annealing algorithm requires a significant computational effort to find a global optimum making it computationally inefficient for large scale processes. In [54], an input design technique minimizing the uncertainties in the SVD of multi-variable models was discussed. This technique is suitable for multi-variable processes with few inputs and outputs. Recent work has used convex optimization to obtain numerical solutions for some experiment design problems. In [61], an optimal FIR filtered white noise input was designed for identification experiments using convex optimization. In [48], an optimal input design technique minimizing the worst case $\nu$-gap over all the plants in the uncertainty region resulting from prediction error identification was computed using convex optimization.

Applying an input design technique to the large multi-variable system in the temporal domain is challenging and will not improve the CD models significantly. As the temporal process is simple and usually modeled by a first-order transfer function with time delay, this work will focus on input design for the spatial domain. Assuming that the actuators have identical responses, the spatial response is modeled by a noncausal FIR in space permitting the application of time domain SISO experiment design techniques to the spatial models. Input design techniques from [62] and [51] as well as white gaussian noise signals were adopted for the identification of FIR mod-
Chapter 3. Identification and Input Design in Open-Loop

els. The identification experiments deliver parameter uncertainty regions in open-loop and closed-loop.

3.4.1 White Gaussian Noise Input

The frequency domain expression for the asymptotic variance for the non-causal FIR from equation (3.4) is:

\[ E(\theta_0 - \hat{\theta})(\theta_0 - \hat{\theta})^T \approx \left[ \frac{N}{2\pi} \int_{-\pi}^{\pi} \frac{\Phi_u(v)}{\lambda_0} \begin{bmatrix} 1 & \lambda^1 & \lambda^2 & \cdots & \lambda^{2n_f} \\ \lambda^{-1} & 1 \\ \lambda^{-2} & 1 \\ \vdots & 1 \\ \lambda^{-2n_f} & 1 \end{bmatrix} dv \right]^{-1} \]

For a white noise input where \( \Phi_u(v) = 1 \), the information matrix \( M \) and its inverse are diagonal thus satisfying the condition for D-optimality as in part (3) of Theorem 8 [64]. The input signal maximizes \( |\tilde{M}| \) which is equivalent to minimizing the uncertainty ellipsoids.

Moreover, the white noise spatial input signal excites all the spatial frequencies thus providing models with smaller uncertainties at high spatial frequencies. As the gain rolls off to zero in CD processes at high spatial frequencies, accurate identification of the gain sign is necessary if the actuator array is required to remove disturbances at higher frequencies. The spectrum of a white gaussian noise signal is compared to that of a bump test in Figure 3.3.

An identification experiment with a white gaussian noise signal is used to identify the CD response model in the simulator discussed in section 3.5. The identification experiment delivers uncertainty bounds for the spatial frequency response. In addition, an industrial experiment was performed with a spatial input signal generated from white gaussian noise to identify FIR models.
3.4.2 Weighted White Gaussian Noise Input

In [62], it was proven that with a pure input power constraint and a stable system, it is always optimal to use open-loop identification. The input power should be spent at frequencies where the noise is high or an accurate model is important. These requirements are satisfied by the following formula:

\[ \Phi_U(v) = \mu \sqrt{\Phi_e(v)C(v)} \]

\( \Phi_U \) is the spectrum of the input signal
\( \Phi_e \) is the spectrum of the noise signal
\( C(v) \) is a frequency weighting function
\( \mu \) is a factor to meet the input power constraint

3.4.3 Input Design Minimizing the Variance

The work in [52] provides an input design technique for linear time variant (LTI) SISO systems in a prediction error framework. The problem is cast as a convex optimization problem and solved using linear matrix inequalities (LMI) [12]. A finite dimensional parametrization of the input spectrum is proposed. With this parametrization, the inverse covariance matrix \( P^{-1}(\theta_0) \)
Chapter 3. Identification and Input Design in Open-Loop

is expressed as an affine function of the parameters defining the input spectrum. Quality constraints that can be cast as being convex in $P^{-1}(\theta_0)$ are addressed.

An input design technique for linearly parametrized models from [51] is convenient to modify for noncausal FIR models. The input signal is designed to minimize the variance of the frequency function estimate keeping the variance less than a pre-specified bound $b(v)$. The technique restricts the maximum input amplitude indirectly through the root mean square value. Positive and negative powers of the shift operator are included in the FIR model equations to model non-causality and the finite sample variance expression from Lemma (4.1) in [51] is used.

$$\min_{\alpha_k, k=0, \ldots, N-1} \sum_{k=0}^{N-1} \alpha_k$$

subject to

$$\frac{1}{N} \Gamma^*(e^{j\nu}) \Gamma(r_{uu}(\tau))^{-1} \Gamma(e^{j\nu}) \leq \frac{b(v)}{|F(e^{j\nu})|^2}$$

$$\alpha_k = |U_k|^2$$

Where $N$ is the number of data points.

$$y(x) = G_0(q)u(x) + e_0(x)$$

Let $u(x) \in \mathbb{R}$ be defined for $x = -(n_d - 1), \ldots, -1$ where $n_d > 0$ and

$$u(x + N) = u(x), \quad x = -(n_d - 1), \ldots, -1$$

$e_0(x)$ is white gaussian noise

$F(q) = 1$ for FIR models

The input signal $u(x)$ can be expressed as:

$$u(x) = \sum_{k=0}^{N-1} U_k e^{jv_0 k x}, \quad x = -(n_d - 1), \ldots, N - 1$$

where $v_0 = \frac{2\pi}{N}$

This input design technique is modified to accommodate noncausal FIR models by introducing positive powers of the shift operator in $F(q)$.
Chapter 3. Identification and Input Design in Open-Loop

\[ G_0(q, q^{-1}) = \theta_0^T \Gamma(q, q^{-1}) \]

\[ \Gamma(q, q^{-1}) = [q^{n_d/2}, \ldots, 1, \ldots, q^{-n_d/2}]^T \]

\( q^{-1} \) is the unit shift operator.

\[ \Gamma(e^{j\nu}, e^{-j\nu}) = [e^{(n_d/2)j\nu}, \ldots, e^{j\nu}, e^{0j\nu}, e^{-j\nu}, \ldots, e^{-(n_d/2)j\nu}]^T \]

The covariance function for \( u(x) \) is given by:

\[ r_{uu}(\tau) = \frac{1}{N} \sum_{x=0}^{N-1} u(x)u(x - |\tau|), \quad |\tau| \leq n_d - 1 \]

When only \( m_d < N \) spectral lines of \( u(x) \) are non-zero:

\[ u(x) = \sum_{k=1}^{m_d} \hat{U}_ke^{j\nu_kx}, \quad x = -(n_d - 1), \ldots, N - 1 \]

where \( \hat{U}_k = U_l \) and \( v_k = v_0l \) for some \( 0 \leq l < N - 1 \)

Now, the covariance function can be written as:

\[ r_{uu}(\tau) = \sum_{k=1}^{m_d} |\hat{U}_k|^2 e^{j\nu_k\tau}, \quad |\tau| \leq n_d - 1 \]

Using Schur complements the constraint (3.10) can be written as:

\[
\min_{\sum_{k=0}^{n_d} |U_k|^2} \quad \text{subject to} \quad \text{Var} G(e^{j\nu}) \leq b(\nu)
\]

\[ \Lambda(\nu) \triangleq \begin{bmatrix} \frac{Nb(\nu)}{\lambda_0} & \Gamma^*(e^{j\nu}) \\ \Gamma(e^{j\nu}) & T(r_{uu}(\tau)) \end{bmatrix} \geq 0, \quad 0 \leq \nu \leq \pi \]

where \( T(t(\tau)) \) is an \( n_d \times n_d \) toeplitz matrix with \( t_{ij} = t(i - j) \)

The constraint becomes convex as the squared magnitude of the DFT-coefficients of the input signal appear linearly in the toeplitz matrix \( T(r_{uu}(\tau)) \).

An approximate solution to the semi-infinite problem is computed by sampling the positivity condition over the spatial frequency axis. The input signal is realized by a sum of sinusoids where the phases are chosen randomly.
Chapter 3. Identification and Input Design in Open-Loop

3.5 Paper Machine Simulator Results

The proposed techniques for identifying the CD mapping and response shape models were tested on a benchmark industrial paper machine simulator provided by Honeywell process solutions, Vancouver operations. This simulator is the tool used to validate identification and control techniques before conducting field experiments.

The CD process model is chosen as an array of slice lip actuators controlling the basis weight profile of a newsprint paper machine as given in [24]. The CD spatial response is given by the parametric function in equation (2.4) with $d_a = 0.07$, $\gamma = 0.84$, $w = 19.5$, $\alpha = 1.56$ and $\delta = 0.13$. The actuator spacing results in an array of actuators with a spatial nyquist frequency of $7.1428$ cycles/meter. The process has first order dynamics with $T_d = 2$ and $a = 0.3679$ in equation (2.3) and the sampling time is taken to be 30 seconds, which is typical for CD processes. The process has 64 actuators and 320 measurement points.

In this work, the CD models are identified from spatial input signals so the relevant signal-to-noise ratio (SNR) should be computed from the CD profile. However, as the SNR of a conventional bump test is computed in the MD, an MD SNR is computed in this work at the actuator with the maximum movement in order to compare the designed signals to the conventional bump test. In [69], a high SNR ratio of 20 was used in the identification experiments. The SNR in the control relevant identification experiment in [30] is computed as 25 from the given noise variance. In this work, the bump test in the identification experiment had a MD SNR of 14. The input signals’ magnitudes in the rest of the tests were chosen to restrict the maximum actuator movement to be less than that of a bump test.

3.5.1 Mapping Detection

A white gaussian noise input signal with an SNR of 8 was applied in open-loop to identify the CD mapping model. A linear mapping function is adopted in this simulation to illustrate the idea. The method can identify nonlinear mapping functions as in [35] using curve fitting techniques for the detected centers of responses. The input-output data is shown in Figure 3.4.

An incorrect shift of two measurement boxes was introduced in the mapping function for all the actuators’ responses. Figures 3.5(a), 3.5(b) and 3.5(c) show the detected responses for actuators 23, 33 and 43. The correct alignment can be easily detected from the identified responses as they
all show a shift of two measurement boxes in the axis of symmetry of the identified FIR model. The rich input signal enables detection of the correct mapping function within zones of four actuators using data from a single scan. A prior estimate of the response shape is not necessary to detect the mapping function in this method.
3.5.2 Misalignment Detection

Estimating the shift in the center of responses permits detecting misalignment in the case of sheet wandering. Given an accurate CD mapping model, an FIR model is identified where the amount of misalignment reflects in a shift in the axis of symmetry of the identified model. Misalignments of 1, 2 and 3 measurement boxes were introduced in the simulator. A bump test with a SNR of 3.5 is performed, six actuators were bumped as shown in Figure 3.6.

![Bump Test](image)

Figure 3.6: Misalignment Bump test

The identified CD responses are shown in Figures 3.7(a), 3.7(b) and 3.7(c). The technique managed to detect the correct misalignment in the three cases using data generated from a low SNR bump test.

3.5.3 CD Response Shape Identification

Open-loop identification experiments were conducted on the industrial paper machine simulator. Four input signals are compared to conventional bump tests to illustrate the benefits of using the different spatial signals. The input signals are white gaussian noise, two frequency weighted white gaussian noise signals and an input signal designed to minimize the variance in the estimate as discussed in section 3.4.3. The maximum actuator
movement in the white gaussian noise signals is smaller than that of a bump test while that of the designed input is comparable to bump tests as seen in Figures 3.8(a) and 3.8(b). The data from each identification experiment was collected from 5 scans in open-loop to identify a FIR model. The measurement profile was down-sampled to the actuator's resolution.

For illustration purposes, the hard frequency-domain model error bounds are computed according to [88] and plotted for each experiment. A symmetry assumption is enforced in the model structure resulting in zero-phase systems.

- Bump Test

The identification data was collected from a bump test with an SNR of 14 giving the CD response shape shown in Figure 3.9(a). Figure 3.9(b) gives the frequency response of the identified model along with the uncertainty bounds generated from the least squares identification. The sign of the frequency response is known with confidence up to 6.5 cycles/meter. The identification experiment resulted in acceptable uncertainty bounds at the expense of a relatively high SNR.

- White Gaussian Noise (WGN)

A white gaussian noise signal was applied in the spatial domain resulting in an identification experiment with a SNR of 8. The CD response and the spatial frequency response along with the uncertainty bounds are shown in Figures 3.10(a) and 3.10(b). The input signal resulted in satisfactory uncertainty bounds throughout the whole frequency range. The identified
Chapter 3. Identification and Input Design in Open-Loop

Figure 3.8: Input signals in the CD

(a) Bump test versus input design

(b) WGN, WGN weighted(1) and WGN weighted (2)
Chapter 3. Identification and Input Design in Open-Loop

Figure 3.9: Bump test in open-loop
model is almost the same as the one obtained from the bump test with comparable uncertainty bounds. The maximum actuator movement in this experiment is 80% that of a bump test resulting in a smaller SNR. The sign of the frequency response is known with confidence up to 6 cycles/meter.

- Frequency Weighted White Gaussian Noise

Two spatial frequency weighting functions are shown in Figure 3.11. The first frequency weighting function focuses on the mid-range frequencies while the second increases the input’s power in the high frequency range. Two input signals were generated by filtering white gaussian noise through the weighting functions. The choice of the appropriate weighting function depends on the range of frequency where an accurate model is required. The spectrum of the weighted white noise signals versus that of white gaussian noise is plotted in Figure 3.12. It is clear how each frequency weighting function focuses more signal power at a specific frequency range.

The first identification experiment had a SNR of 3.58 where the input signal (WGN1) was generated by filtering white gaussian noise through the weighting function (1). The maximum actuator movement was about 50% of the bump test. The CD response and the spatial frequency response along with the uncertainty bounds for the are shown in Figures 3.13(a) and 3.13(b). From Figure 3.12, the input signal has most of its power in the range 2 ~ 6 cycles/m resulting in tighter uncertainty bounds in this frequency range. The sign of the frequency response is known with confidence up to 5.8 cycles/meter.

The maximum actuator movement in the second experiment was about 70% of that in a bump test resulting in a SNR of 7.22. The CD response and the spatial frequency response along with the uncertainty bounds are shown in Figures 3.14(a) and 3.14(b). The identification experiment minimizes the uncertainty bounds significantly at high frequencies (5 ~ 7 cycles/meter) as the input (WGN2) signal’s power is focused in this range as seen in Figure 3.12. The sign of the frequency response is known with confidence up to 7 cycles/meter.

- Input Minimizing the Variance

An input designed to minimize the variance in the transfer function estimate was generated according to section 3.4.3. The optimization problem is solved using the YALMIP toolbox in MATLAB resulting in an input signal realized by a sum of sinusoids generated at a set of discrete frequencies.
Chapter 3. Identification and Input Design in Open-Loop

Figure 3.10: White Gaussian Noise (WGN) experiment in open-loop

(a) CD response

(b) Spatial frequency response and uncertainty bounds

Figure 3.10: White Gaussian Noise (WGN) experiment in open-loop
Figure 3.11: The frequency response of the weighting functions

Figure 3.12: FFT of white gaussian noise vs weighted white noise
Figure 3.13: Weighted White Gaussian Noise (WGN 1) experiment in open-loop
Chapter 3. Identification and Input Design in Open-Loop

Figure 3.14: Weighted White Gaussian Noise (WGN 2) experiment in open-loop

(a) CD response

(b) Spatial frequency response and uncertainty bounds
The identification experiment had a SNR of 12.5. The CD response and the spatial frequency response along with the uncertainty bounds are shown in Figures 3.15(a) and 3.15(b). The designed input signal minimizes the uncertainty bounds significantly in the mid-frequency range and provides satisfactory bounds in the low frequency range. The sign of the frequency response is known with confidence up to 6 cycles/meter.

- Simulation Results Summary

The proposed noncausal spatial FIR model facilitates identification and input design in the spatial domain. The identification experiments deliver uncertainty bounds that can guarantee controller robustness throughout a spatial frequency range. The results show the superiority of the various input signals to bump tests as they result in better spatial models with tighter uncertainty bounds at lower SNR values.

3.6 Industrial Experiments

The standard industrial identification method is bump tests [84]. A few actuators across the machine are bumped in open-loop and the output profile is recorded. The actuators are chosen so that their responses do not overlap. In the paper mill where the identification experiment was performed, the spatial response is identified by the peak and the average of two readings on both sides. This simplistic approach results in identifying responses that were deemed incorrect based on the mill personnel’s experience. The control engineers have a rough idea about the nominal models resulting in the best performance for each grade. The bump tests are merely used as a tool for alignment and the identified responses that are too different from the nominal one are discarded. The identification experiments were conducted to identify the basis weight process for dilution actuators. The paper machine in the identification experiments had 78 actuators and 600 data-boxes.

The nominal model from the manufacturer’s manual is:

\[ G_d(\lambda, \lambda^{-1}) = -0.007\lambda^1 - 0.03\lambda^0 - 0.007\lambda^{-1} \]

Figure 3.16(a) shows the measurement profile after a bump response. The profile is corrupted with high measurement noise and some of the bump responses are not obvious in the profile.
Identified response from the Low resolution profile

(a) CD response

(b) Spatial frequency response and uncertainty bounds

Figure 3.15: Input Design minimizing the variance
The industrial identification tool estimated an incorrect spatial response
\( G_{\text{ind}}(\lambda, \lambda^{-1}) \):
\[
G_{\text{ind}}(\lambda, \lambda^{-1}) = -0.0016\lambda^2 - 0.0051\lambda^1 - 0.0203\lambda^3 - 0.0051\lambda^{-1} - 0.0016\lambda^{-2}
\]
The identified spatial response is shown in Figure 3.16(b).

The bump test resulted in an incorrect spatial model that has been discarded. This poor result shows the need for better inputs and identification techniques for CD spatial responses. Two identification experiments were performed in the spatial domain, one with a white gaussian input signal and the other with a sum of sinusoids. The alignment information has been used to down-sample the measurement profile to the actuator’s profile resolution. In order to avoid edge effects, the first four actuators on each side were not actuated in the tests. A total of 350 data points from 5 scans were used in the identification.

### 3.6.1 Noncausal FIR Model

The identification experiment with white gaussian noise gave the model:
\[
G_{\text{FIR}}(\lambda, \lambda^{-1}) = -0.0065\lambda^1 - 0.0288\lambda^0 - 0.0065\lambda^{-1}
\]
The input output data from the identification experiment is shown in Figure 3.17. The maximum actuator movement was limited to that of a standard bump test to perform the experiment at the typical SNR of 8. The identified spatial response is shown in Figure 3.18(a) and the spatial frequency response along with the uncertainty bounds is shown in 3.18(b). This identification experiment resulted in an accurate model with tight uncertainty bound. This is attributed to the large data set collected from performing the identification in the CD. An important issue to consider is choosing the most suitable FIR model order as the industrial noise level affected the higher order FIR models. The initial guess for the model order can be obtained from a rough idea about the typical response shape of the actuator. Statistical methods to measure the goodness of fit can be used to check the estimated model. The Akaike information criterion (AIC) is a convenient tool for model selection as it describes the trade off between bias and variance. It offers a relative measure of the information lost with a given model and ranks the competing models with the one having the lowest AIC being the best.
Chapter 3. Identification and Input Design in Open-Loop

(a) Input-output response

(b) Estimated Spatial response

Figure 3.16: Industrial Bump Test
3.6.2 Noncausal ARX Model

In a second identification experiment, the spatial input signal was generated from a sum of sinusoids. The input-output data is shown in Figure 3.19. The experiment was performed with the typical SNR of a bump test.

The spatial response is modeled by:

\[ G(\lambda, \lambda^{-1}) = \frac{-0.0280\lambda^0}{0.0263\lambda^2 - 0.1893\lambda^1 + 1\lambda^0 - 0.2142\lambda^{-1} + 0.0228\lambda^{-2}} \]

The spatial input signal resulted in an accurate estimate of the spatial response as seen in Figure 3.20(a). The spatial frequency response of this identified ARX model is a good estimate of the true model response as shown in Figure 3.20(b).

The identified ARX model from the experiment was tested using residual analysis. The 99% confidence regions are shown in Figure 3.21. The model identified from the identification experiment is a good fit as the autocorrelation and cross-correlation functions lie within the confidence region. This ARX structure is best suited for control design purposes as will be discussed in chapter 6.
Figure 3.18. Industrial identification (FIR model)
3.7 Summary

A novel method for identifying the CD mapping model and detecting misalignment using noncausal spatial FIR models is presented. Spatial input signals were designed to generate richer data and minimize the uncertainties in the frequency domain. The input signals are tested on a benchmark industrial paper machine simulator.

Industrial identification experiments were conducted to identify FIR and ARX models. The experiments were performed at the typical SNR of the bump tests and resulted in better CD models. The FIR model frequency response was identified with confidence throughout the whole frequency range. The ARX model accurately represents the nominal model's response at high spatial frequencies.

The simulator and industrial experiments showed that identification in the spatial domain using the noncausal models outperform the standard bump tests significantly with almost the same maximum actuator movements (same SNR).
Figure 3.20: Industrial identification (ARX model)
Chapter 3. Identification and Input Design in Open-Loop

Figure 3.21: Residual analysis for the ARX model
Chapter 4

Identification and Input Design in Closed-Loop

4.1 Introduction

The industrial custom is to identify CD models from bump tests in open-loop. This technique has to be initiated manually and requires the controller be turned off during the test. In this section, closed-loop identification techniques for CD processes are proposed to enable online identification in adaptive control schemes.

Most of the previous work on identification of CD processes was performed in open-loop. A technique for estimating the impulse response of an actuator in closed-loop was described in [20]. This method is not strictly non-invasive as a small perturbation must be introduced to the system to ensure identifiability. In [22], a method of extracting the open-loop responses of the actuators from an estimate of the closed-loop response is given. A perturbation signal that is persistently exciting is added to the actuators set-points. Because of the inherent time delay within the system, the open-loop impulse response will appear as terms in the closed-loop response right after the time delay. The test was restricted to a few actuators to limit the number of scans required to identify an accurate spatial response. In [69], an indirect closed-loop identification technique was developed for a CD process controlled by an unconstrained model predictive controller where the concept of Markov parameters is used to identify a step-response model. Five actuator locations were selected and identified from 300 data samples. A significantly large signal-to-noise ratio of 20:1 was used in the simulations. The computational demand limits the on-line capabilities of this technique.

In this chapter, a method for closed-loop identification of CD mapping models and CD response models is developed based on the technique proposed in [22].
4.2 Closed-Loop Identification from Uncorrelated Input-Output Data

The previous work on this subject struggled when dealing with the huge dimensionality. A large number of data samples were required when considering a few actuator locations across the sheet. Adopting the noncausal spatial transfer function and performing the identification in the spatial domain requires using only a few scans depending on the number of actuators and the signal to noise ratio. The inherent time delay will be utilized in a way similar to the work in [22] but with data from fewer scans. Limiting data collection to the few scans up till the time where the controller’s opposing action appears in the response provides uncorrelated input-output data. The system is virtually running in an open-loop for a few scans and consistent estimates of the process will be obtained from the collected data. Basically, this is an open-loop identification of the CD process due to the buffering provided by the time delay plus one sampling time. This method is appropriate for identifying the CD response shape in processes with short time constants. In these processes, the elapsed time till the instant where the measurements are feedback to the controller is enough to get a clear spatial response in the measurement profile with the standard signal-to-noise ratio. In addition, this method is proposed for identification of the CD mapping model in closed-loop for processes with both short and long time constants. The time provided by the transport delay plus one sampling time is enough for many CD processes to reach steady-state as the dynamics of the process are fast compared to the sampling time. Most of the time required to obtain the MD response is spent in MD/CD separation and data filtering. In some papers on CD control, the dynamics of the process is assumed to be a pure time delay resulting from the transport delay [41].

The idea behind the method is illustrated through the following example. A step input for the duration of the process transport delay plus one sampling time is applied to the actuator setpoint. The MD response to the step input is shown in Figure 4.1. In reality, the process responds as shown in the top plot while the controller is feedback the delayed response measured by the scanner in the bottom plot. Assuming the transport delay is 30 sec and the sampling time is 30 sec, the controller receives the measurement profile and applies the control action at $t = 60$ sec. While the effect of the controller’s opposing action can be seen in the top plot at $t = 60$ sec, it appears in the measurement profile lagging by the transport delay duration. The buffering introduced by the transport delay provides uncorrelated
input-output data up till $t = 90$ sec.

Figure 4.1: Uncorrelated input-output data in closed-loop

Consider the closed-loop in Figure 4.2:

The output profile is given by:

$$Y(z) = [I + G(z)K(z)]^{-1}G(z)U(z) + \eta(z)$$

(4.1)

where:

$$\eta(z) = [I + G(z)K(z)]^{-1}D(z)$$
Chapter 4. Identification and Input Design in Closed-Loop

The open-loop transfer function can be written as:

\[
G(z) = G_0 \cdot T(z) \\
T(z) = t_0 + t_1 z^{-1} + t_2 z^{-2} + t_3 z^{-3} + \ldots
\]  

(4.2)

As the process is stable, the higher \( t_i \) terms are decreasing in magnitude. Due to the inherent time delay of the process one or more of the first terms in the expansion will be zero. Following [22], assume that the time delay is two scans so that \( t_0 = 0 \) and \( t_1 = 0 \).

The sensitivity function can be expanded as:

\[
[I + G(z)K(z)]^{-1} = [I - G(z)K(z) + (G(z)K(z))^2 - (G(z)K(z))^3 + \ldots] \quad (4.3)
\]

\[
[I + G(z)K(z)]^{-1} = [I + S_1(z) + S_2(z) + S_3(z) + \ldots] \quad (4.4)
\]

Now, the output profile can be expressed as:

\[
Y(z) = [H_2(z^{-2}) + H_3(z^{-3}) + H_4(z^{-4}) + \ldots] U(z) + \eta(z) \quad (4.5)
\]

where \( H_2 = t_2 \cdot G_0 \)

Using this result, the open-loop spatial response of the process can be estimated in closed-loop due to the buffering provided by the transport delay. The closed-loop simulation was conducted with a basic control feedback law where \( K = [G^T G + \rho I]^{-1} G^T \).

4.2.1 CD Mapping Model Identification in Closed-Loop

For the simulated process, the collected data by the end of the third scan after the transport delay is uncorrelated with the previous control action. The mapping test was performed with a white gaussian noise signal resulting in a SNR of 8 as shown in Figure 3.4. The assumed CD mapping model has a shift of 2 measurement positions at each actuator zone. From a single scan, the correct mapping information was detected within zones of four actuators each. Figures 4.3(a), 4.3(b) and 4.3(c) show the detected responses for actuators 23, 33 and 43. The proposed technique managed to identify the correct mapping function for the three actuators.
4.2.2 Misalignment Detection

The uncorrelated input-output data provides means to estimate the misalignment produced by sheet wandering as in the open-loop case. Misalignments of 1, 2 and 3 measurement boxes were introduced in closed-loop. In each simulation, a bump test with a SNR of 3.5 was applied for the duration of three scans as shown in Figure 3.6. Figures 4.4(a), 4.4(b) and 4.4(c) show that the technique estimated the correct misalignment in all three cases from the low SNR bump test.

Figure 4.4: Misalignment detection in closed-loop: three misalignment cases
4.2.3 CD Response Shape Model Identification in Closed-Loop

Identification experiments were conducted using the industrial paper machine simulator in closed-loop. In all the simulations, the spatial input signal was fixed for the duration of three scans to generate a single scan of correlation-free data. The buffering provided by the time delay plus one sampling time (90 sec) allows the MD response to reach 90% of its steady-state value. Using a prior estimate of the dynamics, the CD response at steady-state is computed. The input signals that were applied in the open-loop case are investigated in closed-loop. In order to provide good estimates with acceptable uncertainty bounds, data was collected from 5 identification experiments for each input signal.

- White Gaussian Noise

An identification experiment with a white gaussian noise signal input resulting in a SNR of 5.35 provided the CD response shape and frequency response estimate shown in Figures 4.5(a) and 4.5(b).

The technique identifies the spatial response correctly and provides satisfactory uncertainty bounds from a low SNR identification experiment. The gain sign is known with confidence up to 6 cycles/meter.

- Frequency Weighted White Gaussian Noise

In the first identification experiment, white gaussian noise was filtered through the weighting filter (1) presented in section 3.5.3 resulting in an SNR of 10. The estimated CD response is shown in Figure 4.6(a) while the spatial frequency response and the uncertainty bounds are plotted in Figure 4.6(b). The identification experiment provided tight uncertainty bounds in the mid frequency range 2 – 5.5 as expected from the input (WGN1) signal's spectrum as seen in Figure 3.12. The sign of the frequency response is identified with confidence up to 5.8 cycles/meter.

In another experiment, white gaussian noise was filtered through the weighting filter (2). The estimated CD response is shown in Figure 4.7(a) while the spatial frequency response along with the uncertainty bounds are plotted in Figure 4.7(b). The identification experiment had a SNR of 11 and provided tight uncertainty bounds in the high frequency range. The sign of the frequency response is identified with confidence up to 7 cycles/meter.
Chapter 4. Identification and Input Design in Closed-Loop

Figure 4.5: White Gaussian Noise (WGN) experiment in closed-loop

(a) CD response

(b) Spatial frequency response and uncertainty bounds
Chapter 4. Identification and Input Design in Closed-Loop

(a) CD response

(b) Spatial frequency response and uncertainty bounds

Figure 4.6: Weighted White Gaussian Noise (WGN 1) experiment in closed-loop
Figure 4.7: Weighted White Gaussian Noise (WGN 2) experiment in closed-loop
Chapter 4. Identification and Input Design in Closed-Loop

- Input Design Minimizing the Variance

The input designed to minimize the variance in open-loop from section 3.4.3 is applied in closed-loop. The identification experiment had a SNR of 12.5. The estimated CD response and spatial frequency response are shown in Figures 4.8(a) and 4.8(b). The sign of the frequency response is identified with confidence up to 6 cycles/meter. The designed input resulted in an accurate CD model with tight uncertainty bounds from a reasonable SNR.

- Simulation Results Summary

The closed-loop identification technique from uncorrelated input-output data resulted in accurate CD models and satisfactory uncertainty bounds. In order to obtain good estimates, data had to be collected from five scans at typical SNR ratios for open-loop identification. In each experiment, the input signal is applied for the duration of three time samples. Although the test has to be repeated a few times to collect enough data, the SNR and the number of scans required are much less than previous work on closed-loop identification of CD processes [22] and [69].

4.3 Closed-Loop Identification from Steady-State Data

For some CD processes that have a large time constant, the buffering provided by the time delay is not enough for the MD response to reach steady-state. At typical SNR values, the measurement profile after the time delay would not provide informative data. A method for identifying the CD model in closed-loop from steady-state data is developed in this section. As the technique focuses on estimating the spatial model, an excitation spatial signal is introduced at the actuators setpoints. The signal is fixed in the temporal domain until the process reaches steady-state. Assuming identical actuator responses, two FIR models are identified between the reference signal and both the actuator profile and the measurement profile. This method is one of the variants of the joint input-output techniques closed-loop identification [62]. The technique is applied to the spatial domain by replacing the time axis with the spatial axis and permitting for non-causality. The plant's spatial response is estimated from the identified models. The technique is illustrated by the following equations:

\[ y(\lambda) = G_{ry}(\lambda, \lambda^{-1})r(\lambda) + v_1(\lambda) \]  (4.6)

81
Chapter 4. Identification and Input Design in Closed-Loop

Figure 4.8: Input Design minimizing the variance in closed-Loop
Chapter 4. Identification and Input Design in Closed-Loop

\[ u(\lambda) = G_{ru}(\lambda, \lambda^{-1})r(\lambda) + v_2(\lambda) \]  \hspace{1cm} (4.7)

where

\[ G_{ry}(\lambda, \lambda^{-1}) = G_s(\lambda, \lambda^{-1})[1 + G_s(\lambda, \lambda^{-1})K(\lambda, \lambda^{-1})]^{-1} \]

\[ G_{ru}(\lambda, \lambda^{-1}) = [1 + G_s(\lambda, \lambda^{-1})K(\lambda, \lambda^{-1})]^{-1} \]

\( v_1(\lambda), v_2(\lambda) \) are white noise signals

The spatial model is given by:

\[ G_s(\lambda, \lambda^{-1}) = G_{ry}(\lambda, \lambda^{-1})G_{ru}(\lambda, \lambda^{-1})^{-1} \]

### 4.3.1 Example 1

An identification experiment was conducted using the paper machine simulator with an SNR of 6.17. Data was collected from the fifth scan following the application of the input signal. The input-output data is shown in Figure 4.9.

![Input output steady-state data](image)

Figure 4.9: Input output steady-state data

Figures 4.10 and 4.11 show the estimated sensitivity functions.
Figure 4.10: The sensitivity function

Figure 4.11: The input sensitivity function
The estimated CD response and the respective spatial frequency response are shown in Figures 4.12 and 4.13. The technique provides a good estimate of the CD response from a short experiment in closed-loop. Further work is required to compute the uncertainty bounds resulting from this technique.

![Figure 4.12: CD response](image)

**4.3.2 Example 2**

A closed-loop identification experiment was run on Honeywell's simulator for the devronizer actuators controlling the moisture profile. The process was simulated with 26 actuators and 52 measurement points and the output profile was provided at both the low and high resolutions. No information was provided about the controller in the loop. The CD spatial response is given by the parametric function in equation (2.4) where \( d = 24.05, \gamma = 0.0045, \omega = 659.45, a = 7 \) and \( \delta = 0 \). The actuator's movements were limited to 10% of the maximum allowable and data were collected from 20 scans. The input-output data is shown in Figure 4.14 and the estimated CD response is shown in Figure 4.15.

The CD response is given by the following transfer function:

\[
G_{nc}(\lambda, \lambda^{-1}) = \frac{0.004428\lambda}{0.0004583\lambda^2 + 0.1143\lambda + 0.1143\lambda^{-1} + 0.0004583\lambda^{-2}}
\]
Chapter 4. Identification and Input Design in Closed-Loop

The technique managed to identify an accurate model from a few scans in low SNR values.

4.4 Summary

In this chapter, methods for closed-loop identification of CD mapping models and CD response models are developed. For systems with relatively short time constants, it is shown that the process's inherent transport delay permits collecting uncorrelated input-output data that can be used in the identification of the spatial models. Open-loop input design techniques are applied in identification of the response models from uncorrelated input-output data. The CD response of processes with long time constants is estimated from steady-state data using a variant of the joint input-output technique.
Figure 4.14: Input output steady-state data

Figure 4.15: CD response
Chapter 5

Robust Stability Criteria for CD Processes

5.1 Introduction

As the process is analogous to a two-dimensional (2D) spatially noncausal digital filter, robust stability of the CD process is investigated using results from the stability theorems for multidimensional digital filters.

Two-dimensional robust stability criteria based on the $\nu$-gap stability theorem are proposed for both the temporal and spatial domains. This will provide a general framework for investigating stability in closed-loop configurations when dealing with model uncertainties. Moreover, this stability criterion provides stability conditions for the closed-loop with a controller other than the nominal controller. This feature permits designing a two-dimensional controller in an adaptive control scheme or a simple re-tuning of an existing controller through iterative feedback tuning.

A two-dimensional robust stability test is developed to determine tolerance to mis-mapping errors in systems with more measurement points than actuators. Stability is investigated through a two-dimensional phase margin concept.

5.2 Stability Conditions for a 2D Spatially Noncausal System

Ensuring robust stability in closed-loop for CD systems is crucial in control design. Instability can be caused by the uncertainties in either the temporal model or the spatial one. In [21], it is shown how uncertainties in the actuator response, in particular at high spatial frequencies, can render the system unstable. An example of this is actuator picketing which occurs when a controller chases non-robustly controllable components of the error [47].

Representing the CD process as a two-dimensional transfer function models
Chapter 5. Robust Stability Criteria for CD Processes

it as a multidimensional digital filter. The CD process is analogous to a 2D spatially noncausal filter where a steady-state output $y(m_t, n_x)$ depends on all input values $x(m_t - i, n_x - j)$ with $i \geq 0$ and $\infty > j > -\infty$. Indices $m_t$ and $n_x$ represent points in time and space respectively. Consider a digital filter of this type

$$F(z_1, z_2) = \frac{B_F(z_1, z_2)}{A_F(z_1, z_2)} \quad (5.1)$$

As shown in [53], the stability criterion is

$$A_F(z_1, z_2) \neq 0, \text{ when } |z_1| \leq 1, |z_2| = 1 \quad (5.2)$$

In [79], the stability criterion was reformulated as the following logically equivalent but simpler pair of conditions

$$\text{for some } b, |b| = 1, A_F(z_1, b) \neq 0, \text{ when } |z_1| \leq 1 \quad (5.3)$$

$$A_F(z_1, z_2) \neq 0, \text{ when } |z_1| = 1, |z_2| = 1 \quad (5.4)$$

Note that these conditions deal with $z = e^{-sk}$ where $s$ is a complex number and $k$ is a constant [50]. To be consistent with the conventional notation, let $z_1 = z$ where $z = e^{j\omega}$ and change condition (5.3) to:

$$\text{for some } b, |b| = 1, A_F(z, b) \neq 0, \text{ when } |z| \geq 1 \quad (5.5)$$

Consider a CD process $G(t, x)$ controlled by a 2D controller $-K(t, x)$ in a negative feedback loop. Let their 2D z-transforms be $G(z, \lambda)$ and $-K(z, \lambda)$. Assuming that the 2D controller is stable and the CD plants are open-loop stable, checking the poles of $[1 - G(z, \lambda)K(z, \lambda)]^{-1}$ is sufficient to guarantee stability.

The characteristic polynomial of the nominal model is:

$$A_{F0}(z, \lambda) = [1 - G_0(z, \lambda)K(z, \lambda)] \quad (5.6)$$

while that of the perturbed plant $G_1(z, \lambda)$ is:

$$A_{F1}(z, \lambda) = [1 - G_1(z, \lambda)K(z, \lambda)] \quad (5.7)$$

A stability test based on conditions (5.4 and 5.5) can be performed in a Nyquist-like test for a set of 1D systems in one of the discrete transform variables at discrete frequencies of the other variable spanning the range from $[0, 2\pi]$ [18].
In sections (5.3) and (5.4), two 2D robust stability criteria based on the $\nu$-gap theorem are developed to check these stability conditions. The first criterion is investigated using a set of temporal causal transfer functions at a set of discrete spatial frequencies while the second criterion investigates a set of noncausal spatial transfer functions at a set of discrete temporal frequencies. While the perturbation in (5.7) is assumed to be in the plant model, a loop with a perturbed version of the controller can be investigated in a similar manner. Satisfying the sufficient condition for stability in each criterion, closed-loop stability is guaranteed against perturbations in either the plant or the controller. As shown in [71] and [73] condition (5.4) can be checked graphically from the rootmaps of the temporal transfer functions $[1 - G_1(z, e^{j\omega})K(z, e^{j\omega})]^{-1}$ or from the spatial transfer functions $[1 - G_1(e^{j\omega}, \lambda)K(e^{j\omega}, \lambda)]^{-1}$. The rootmap plots the path of closed-loop poles of a set of 1D transfer functions at discrete frequencies of the other variable.

5.3 The 2D Robust Stability Theorem in the Temporal Domain

5.3.1 The $\nu$-Gap Metric

Let $\mathbb{R}$ and $\mathbb{C}$ denote the fields of real and complex numbers respectively. Let $Z := \{z \in \mathbb{C} : |z| > 1\}$ and $\delta Z$ the boundary of this set. $L_\infty$ is a Banach space of matrix or scalar valued functions bounded on $\delta Z$. $H_\infty$ is a subspace of $L_\infty$ with functions that are analytic in $Z$. $\mathbb{R}$ represents the space of all real transfer functions. $\mathbb{RH}_\infty$ is the space of real rational transfer matrices in $L_\infty$ (no poles on $\delta Z$). $\mathbb{RH}_\infty$ is the space of stable rational transfer functions (functions with no poles outside the unit circle for discrete systems).

Let $G_{i} = \begin{bmatrix} N_i \\ M_i \end{bmatrix}$ represent the normalized right graph symbol of $G_i$. 

$\tilde{G}_{Ni} = [-M_i \ N_i]$ is the normalized left graph symbol of $G_i$. 

$G_i = N_i M_i^{-1} = \tilde{M}_i^{-1} \tilde{N}_i$ 

$\{N_i, M_i\}$ and $\{\tilde{N}_i, \tilde{M}_i\}$ are the right and left coprime factorization of $G_i$ respectively.

Let $G_K$ and $\tilde{G}_K$ denote the right and left normalized graph symbols of a controller $K$. 

$G_K = \begin{bmatrix} M_K \\ N_K \end{bmatrix}$, $\tilde{G}_K := [-\tilde{N}_K \ \tilde{M}_K]$, 

90
Chapter 5. Robust Stability Criteria for CD Processes

Where $N_i, M_i, \tilde{N}_i, \tilde{M}_i \in \Re H_\infty$

$\text{wno}(G)$ denotes the winding number evaluated on the standard Nyquist contour indented around any imaginary poles of $G$ and $(\ast)$ denotes the conjugate transpose.

The $\nu$-gap metric between two plants $G_0$ and $G_1$ ($\delta_\nu(G_0, G_1)$) is the gap between their $L_2$ graph spaces when the winding condition is satisfied, this only depends on their frequency responses [85]. For SISO systems, the $\nu$-gap is equal to the chordal distance between the projections of their Nyquist diagrams onto the Riemann sphere.

$$
\delta_\nu(G_0, G_1) := \left\{ \begin{array}{ll}
\| \tilde{G}_{G_1} G_{G_0} \|_\infty, & \text{if } \det(G_{G_1}^* G_{G_0})(e^{j\omega}) \neq 0 \forall \omega \\
1, & \text{otherwise}
\end{array} \right.
$$ (5.8)

Given that $\delta_\nu(G_0, G_1) < 1$:

$$
\delta_\nu(G_0, G_1) = \sup_\omega \kappa(G_1, G_0)(e^{j\omega}) \text{ if } \text{wno}(G_{G_1}^* G_{G_0}) = 0
$$ (5.9)

where $\kappa(G_0, G_1)(e^{j\omega})$ is the point-wise chordal distance

$$
\kappa(G_0, G_1) = \sigma[ (I + G_1 G_1^*)^{-1/2}(G_1 - G_0)(I + G_0^* G_0)^{-1/2} ]
$$ (5.10)

This measure evaluates how close the performance of two plants will be when running in a closed-loop.

Consider the following closed-loop configuration:

A generalized stability margin for the transfer function from $[U_1 \ U_2]$ to $[Y_1 \ Y_2]$ for a plant $G$ and controller $K$ in a feedback loop can be defined as:

$$
b_{G,K} := \left\{ \begin{array}{ll}
\| \left[ \begin{array}{c} G \\ 0 \end{array} \right] (I-KG)^{-1} \left[ \begin{array}{cc} -K & I \end{array} \right] \|_\infty^{-1} & \text{if } |G,K| \text{ is stable} \\
0 & \text{otherwise}
\end{array} \right.
$$ (5.11)

$$
b_{G,K} = \inf_\omega \rho(G, K)(e^{j\omega})
$$ (5.12)

Where:

$$
\rho(G_0, K) = \sigma^{-1}[ (I + G_0^* G_0)^{1/2}(I - KG_0)^{-1}(I + KK^*)^{1/2} ]
$$ (5.13)
In the scalar case:

\[ \rho(G_0, K)(e^{j\omega}) = \kappa(G_0, 1/K)(e^{j\omega}) \]  

(5.14)

It has been proven in [85] that any controller stabilizing a nominal plant \( G_0 \) with a generalized stability margin \( b_{G_0, K} \) will stabilize all plants \( G_1 \) satisfying \( \delta_\nu(G_0, G_1) \leq \beta \) if, and only if, \( \beta < b_{G_0, K} \).

The closed-loop transfer matrix

\[
\begin{bmatrix}
    G_0 \\
    I
\end{bmatrix}
(I - KG_0)^{-1}
\begin{bmatrix}
    -K \\
    I
\end{bmatrix} = G_{GO}(\tilde{G}_K G_{GO})^{-1}\tilde{G}_K
\]  

(5.15)

The closed-loop stability in terms of the coprime factorizations can be checked from the following equivalent conditions:

1) \((\tilde{G}_K G_{GO})^{-1} \in H_\infty\)

2) \(\det(\tilde{G}_K G_{GO})(e^{j\omega}) \neq 0 \forall \omega \) and \(\text{wnc det}(\tilde{G}_K G_{GO}) = 0\).

Robust stability is guaranteed if the temporal \( \nu \)-gap \( \delta_\nu[G_0(z), G_1(z)] \) is less than the generalized stability margin \( b_{G_0, K}(z) \). However, a frequency dependent version of the \( \nu \)-gap stability criterion given in [86] and [91] is less conservative when determining robust stability. The inequality \( k(G_1, G_0)(e^{j\omega}) < b_{G_0, K}(e^{j\omega}) \) is checked frequency by frequency given that
δ_v(G_0(z), G_1(z)) < 1. Lemma 3.6 in [86] states that for a stable loop [G_0, K] and \( \kappa(G_0(e^{j\omega}), G_1(e^{j\omega})) < \rho(G_0(e^{j\omega}), K(e^{j\omega})) \forall \omega \), then [G_1, K] is stable if, and only if, \( \text{uno det}(G_1^* G_0) = 0 \).

### 5.3.2 A 2D Robust Stability Criterion Using Temporal \( \nu \)-Gaps

This criterion provides a general framework for stability in closed-loop configurations for two-dimensional systems. The 2D Robust stability can be investigated through a set of SISO temporal transfer functions at a set of discrete spatial frequencies as discussed in [18].

The perturbed 2D loop is stable if the nominal loop is stable and the temporal \( \nu \)-gaps \( \delta_v(G_0(z, e^{j\nu}), G_1(z, e^{j\nu})) \) are less than their corresponding generalized stability margins \( b_{G_0,K}(z, e^{j\nu}) \) evaluated at a set of discrete spatial frequencies \( \nu \in [0, 2\pi] \). It is sufficient to choose \( n_u \) discrete spatial frequencies to span the range from \( [0, 2\pi] \) as these are the spatial modes of the circulant matrix.

\[
b_{G_0,K}(z, e^{j\nu}) := \left\| \begin{bmatrix} G_0(z, e^{j\nu}) & 1 \\ 1 - G_0(z, e^{j\nu}) K(z, e^{j\nu}) & -K(z, e^{j\nu}) \end{bmatrix} \right\|^{-1}_{\infty}
\]

(5.16)

This test turns to be very close to the one employed in [77] and [78] with the difference that here the set of spatial frequencies can be chosen arbitrarily. Employing the frequency-by-frequency version of the \( \nu \)-gap stability criterion, stability conditions (5.4 and 5.5) will not be violated if for a set of spatial frequencies \( \nu \) spanning \( [0, 2\pi] \), the inequality \( \kappa(G_0, G_1)(e^{j\omega}, e^{j\nu}) < \rho_{G_0,K}(e^{j\omega}, e^{j\nu}) \forall \nu \) is satisfied given that the temporal \( \nu \)-gaps \( \delta_v(G_0(z, e^{j\nu}), G_1(z, e^{j\nu})) < 1 \). The robust stability test requires computing two surfaces, one for the pointwise chordal distances (denote it \( \nu \)-surface) and the other for the stability margins (denote it \( b_{G_0,K} \) surface) over a dense grid of spatial and temporal frequencies. As long as the \( \nu \) surface is below the \( b_{G_0,K} \) surface everywhere, robust stability is guaranteed.
5.4 The 2D Robust Stability Theorem in the Spatial Domain

As the dynamics are quite simple and are usually modeled by a low-order transfer function with a time delay, this section will focus on the uncertainties arising from the spatial part. Spatial uncertainties that rise from the CD response shape identification or the mapping function are easily modeled in the noncausal spatial transfer function. Misalignment is modeled with powers of the shift operator in space. The spatial transfer function relates uncertainties in the spatial frequency domain directly to the 2D system’s stability. Moreover, this model delivers performance measures especially at temporal steady-state and provides more insight into the expected spatial frequency response in closed-loop.

As noncausal time domain systems are not physically realizable, stability of noncausal systems has not drawn much attention from researchers in the field. A new notion for stability has to be used to investigate stability of noncausal systems taking into consideration the direction of the response as discussed in [40]. Stability is attained if the impulse response $g(k)$ decays exponentially on both sides.

$$g(i) = \sum_{i=-k_0}^{-1} g(i)z^{-i} + \sum_{i=0}^{k_0} g(i)z^{-i} \quad (5.17)$$

As a decaying response in the negative axis can be computed in terms of poles with magnitude greater than one, a discrete noncausal system is stable if the causal and anti-causal poles lie inside and outside the unit circle respectively. Robust stability is guaranteed as long as none of the closed-loop poles crosses the unit circle to the other side (this would cause a growing term in the series).

In CD processes where the spatial response is open-loop stable and symmetric, tracking the two sets of poles with a perturbed system in the loop is an easy task. There is no room for confusion between an unstable causal pole and a stable anti-causal pole. The $\nu$-gap metric from [85] is very convenient to ensure stability for a noncausal system as it measures how far a perturbed plant is from singularity (poles on the unit circle). A minor modification is required to accommodate the stable anti-causal poles.

5.4.1 Developing a Noncausal $\nu$-Gap

For a stable loop $[G_0, K]$, if $\kappa(G_0, 1/K)(e^{j\omega}) > \kappa(G_0, G_1)(e^{j\omega})$ then $G_1(e^{j\omega}) \neq 1/K(e^{j\omega})$ and $|1 - G_1(e^{j\omega})K(e^{j\omega})| \neq 0$ for any frequency $\omega$ meaning that...
the closed-loop $[G_1, K]$ can have no poles on the unit circle. This is true for a continuous path of plants from $G_0$ to $G_1$ satisfying the inequality \cite{86}. In the aforementioned reference, there is a brief discussion on how the onset of instability in systems, whether causal or noncausal, is associated with a pole crossing the unit circle to the other region as the closed-loop system will fail to lie in either $H_\infty$ or $L_\infty$ (\cite{86}, section 1.5, pages 55-57). Using this idea, a noncausal $\nu$-gap metric is developed through modifying the conventional metric to accommodate noncausal systems. Causal systems are stable if they lie in the $H_\infty$ space. Stability for noncausal symmetrical systems can be defined in the $L_\infty$ space by ensuring that the causal and anti-causal poles lie inside and outside the unit circle respectively. The stability criterion for the noncausal system is no longer $(\hat{G}_K G_{G0})^{-1} \in H_\infty$, it must be modified to take into consideration the stable anti-causal poles of the process. As this work deals with symmetric models, the new stability condition is $(\hat{G}_K G_{G0})^{-1} \in L_\infty$ with $\text{wno}$ of $n/2$ where $n$ is the number of closed-loop poles in these symmetric systems. Starting from a stable system, a generalized stability margin $b_{G_0,K}$ can be defined by equation (5.11) whenever the winding number condition is satisfied. This measure indicates how far the transfer matrix is from singularity. The stability of the nominal loop must be guaranteed beforehand to distinguish between any unstable causal poles and stable anti-causal poles. In order to permit asymmetrical parameter uncertainties, plants that are symmetrical in model structure, i.e. have the same number of causal and anti-causal poles, are considered symmetrical when it comes to 2D robust analysis.

**Lemma 1:**

Let $G_0, G_1$ and $K$ be three spatially noncausal symmetrical, transfer functions such that $[G_0, K]$ is stable and $\kappa(G_0, G_1)(e^{j\nu}) < \rho(G_0, K)(e^{j\nu}) \forall \nu$, then $[G_1, K]$ is stable if, and only if, $\text{wno} \det(\hat{G}_K G_{G0}) = 0$.

**Proof:**

To ensure stability for the perturbed noncausal symmetrical system, the following must be met:

i) $\det(\hat{G}_K G_{G1})(e^{j\nu}) \neq 0 \forall \nu$

ii) $\text{wno} \det(\hat{G}_K G_{G1}) = n/2$

The result is a consequence of lemma 3.6 in \cite{86} taking into account that $[G_0, K]$ has stable anti-causal poles giving a $\text{wno} \det(\hat{G}_K G_{G0}) = n/2$. 

95
5.4.2 A 2D Robust Stability Criterion Using Spatial Noncausal $\nu$-Gaps

Stability is guaranteed if the set of spatial noncausal transfer functions $[1 - G_0(e^{j\omega}, \lambda)K(e^{j\omega}, \lambda)]^{-1}$ for $\omega \in [0, 2\pi]$ have no poles lying on or crossing the unit circle and condition (5.5) is satisfied at a spatial frequency.

The same idea that has been used in section 5.3 is applied in the spatial domain. The 2D robust stability is investigated through a set of SISO spatial transfer functions at a set of discrete temporal frequencies. The frequency-dependent version of the $\nu$-gap stability criterion from [86] and [91] is used. The generalized stability margins are defined by the following formula when $\text{wno det}(\hat{G}_K G_0(e^{j\omega}, \lambda)) = n/2$

$$b_{G_0,K}(e^{j\omega}, \lambda) := \left[ \left| G_0(e^{j\omega}, \lambda) \right| - \left| G_0(i, \lambda)K(c^3, \lambda) \right| - \left| K(c^3, \lambda) \right| \right]^{-1}$$

(5.18)

**Theorem 1:**

For a stable closed-loop CD process with a nominal plant $G_0$ and a controller $K$, robust stability is guaranteed for all perturbed plants $G_1$ satisfying $\delta_{\nu}[G_0(e^{j\omega}, \lambda), G_1(e^{j\omega}, \lambda)] < 1$ & $\delta_{\nu}[G_0(z, e^{jv}), G_1(z, e^{jv})] < 1 \forall \omega_i, v_j \in [0, 2\pi]$ if:

$$\kappa(G_1, G_0)(e^{j\omega}, e^{jv}) < \rho_{G_0,K}(e^{j\omega}, e^{jv}) \forall \omega, v \in [0, 2\pi]$$

(5.19)

**Proof:**

1. $\delta_{\nu}[G_0(e^{j\omega}, \lambda), G_1(e^{j\omega}, \lambda)] < 1 \forall \omega_i \in [0, 2\pi]$

2. $\text{wno det}(\hat{G}_1 G_0)(e^{j\omega}, \lambda) = 0 \forall \omega_i \in [0, 2\pi]$

3. $\text{wno det}(\hat{G}_K G_0)(e^{j\omega}, \lambda) = n/2 \forall \omega_i \in [0, 2\pi]$

At a temporal frequency $\omega_i$, if $\forall v$:

$$\kappa(G_1, G_0)(e^{j\omega_i}, e^{jv}) < \rho_{G_0,K}(e^{j\omega_i}, e^{jv})$$

(5.20)

From Lemma 1, $[G_1(e^{j\omega_i}, \lambda), K(e^{j\omega_i}, \lambda)]$ is stable.

If $\forall \omega$ condition (5.20) is satisfied:

$A_1(z, \lambda) \neq 0, \text{when } |z| = 1, |\lambda| = 1$ (condition 5.4)
2) At some spatial frequency \( \upsilon, \lambda = e^{j\upsilon_\lambda} \):

\[
\therefore \kappa(G_0, G_1)(e^{j\omega}, e^{j\upsilon_\lambda}) < \rho G_0, K(e^{j\omega}, e^{j\upsilon_\lambda}) \forall \omega
\]  

(5.21)

From lemma 3.6 in [86]:

\[
\therefore A_1(z, \lambda) \neq 0, \text{when } |z| \geq 1 \text{ (condition 5.5) (Q.E.D)}
\]

Condition (5.4) is satisfied given (5.19) as:

\[
\therefore \rho G_0, K(e^{j\omega}, e^{j\upsilon_\lambda}) = \kappa(G_0, 1/K)(e^{j\omega}, e^{j\upsilon_\lambda})
\]

\[
\therefore G_1(e^{j\omega}, e^{j\upsilon_\lambda}) \neq 1/K(e^{j\omega}, e^{j\upsilon_\lambda})
\]

\[
\therefore |1 - G_1(e^{j\omega}, e^{j\upsilon_\lambda}) K(e^{j\omega}, e^{j\upsilon_\lambda})| \neq 0 \forall \omega, \upsilon
\]

\[
\therefore A_1(z, \lambda) \neq 0, \text{when } |z| = 1, |\lambda| = 1
\]

A surface for the pointwise chordal distances (\( \nu \)-surface) and another one for the stability margins (\( b_{G_0, K} \) surface) are computed over a dense grid of spatial and temporal frequencies. As long as the \( \nu \)-surface is below the \( b_{G_0, K} \) surface everywhere, robust stability is guaranteed. As these two surfaces are computed from the 2D frequency response of the plants and the controller, computing the surfaces from the temporal or spatial transfer functions gives the same results.

### 5.5 Illustrative Example

In this section, the two-dimensional robust stability criterion is illustrated by an example. Consider the industrial feedback loop for the weight process from [74] as it provides a common controller structure for paper machines. All the perturbed plants satisfy \( \delta_\nu[G_0(e^{j\omega_i}, \lambda), G_1(e^{j\omega_j}, \lambda)] < 1 \) and \( \delta_\nu[G_0(z, e^{j\upsilon_j}), G_1(z, e^{j\upsilon_j})] < 1 \forall \omega_i, \upsilon_j \in [0, 2\pi] \). The process has 226 actuators and 226 measurement points and runs in the closed-loop configuration shown in Figure 5.2.

The process model is:

\[
y(z) = G(z)u(z) + d(z)
\]  

(5.22)

\[
G(z) = (I + Az^{-1})^{-1}(Bz^{-T_4})
\]
Figure 5.2: The CD negative feedback loop

\[ D(z) = (I + H_z^{-1})^{-1}(I + E_z^{-1}) \]
\[ d(z) = D(z)dn(z) \]
\[ B = \text{toeplitz}\{-0.0827, -0.0463, -0.0030, 0.0164, 0.0141, 0, \ldots, 0\} \]
\[ A = \text{toeplitz}\{-0.8221, 0, 0, \ldots, 0\} \]
\[ E = \text{toeplitz}\{-0.8221, 0, 0, \ldots, 0\} \]
\[ H = \text{toeplitz}\{-0.999, 0, 0, \ldots, 0\} \]
\[ A, B, H, E \in \mathbb{R}^{226 \times 226} \]

\( T_d = 3 \) is the dead time.
\( dn(t) \) is Gaussian white noise.
Toeplitz represents a toeplitz matrix with the given vector as its first column.
The sampling time is 25 sec and the spacing between the actuators is 0.035 meter.

The process transfer matrix can be rewritten as:

\[ G(z) = G_0 \frac{z^{-T_d}}{1 - 0.8221z^{-1}} \text{ where } G_0 = B \]

The multi-variable controller is given by:

\[ K(z) = (I - S \cdot z^{-1})^{-1}c(z)C \tag{5.23} \]
Chapter 5. Robust Stability Criteria for CD Processes

\[ c(z) = \frac{(1 - \alpha_c)(1 - \alpha_c z^{-1})}{1 + (1 - \alpha_c) \sum_{i=1}^{d_c-1} z^{-i}} \]

\[ c(z) \] is the z-transform of a Dahlin controller.

The spatial plant response was identified using the noncausal identification technique developed in section 3.2.2:

\[ G_{0S}(\lambda, \lambda^{-1}) = \frac{-0.0266\lambda^0}{0.227\lambda^2 - 0.62\lambda^1 + \lambda^0 - 0.62\lambda^{-1} + 0.227\lambda^{-2}} \]

The two-dimensional nominal plant is:

\[ G_{02D}(z, \lambda) = G_{0S}(\lambda, \lambda^{-1}) \cdot T_0(z) \quad (5.24) \]

\[ T_0(z) = \frac{z^{-T_d}}{1 - 0.8221z^{-1}} \]

5.5.1 Internal Stability of the Actual Plant

The two-dimensional representation is equivalent to approximating the process with circulant matrices where the boundary conditions at the edges are ignored. Modeling the finite width CD process by an infinite spatial transfer function significantly simplifies the control design and stability problem of the large multi-variable system. The circulant matrix approximation has been used in recent work on CD control [77], [78], [24] and [27]. The result of neglecting the edge effects on the system’s internal stability is investigated using the small gain theorem as in [74]. Considering the feedback loop in Figure 5.2, the true system involves band diagonal matrices \( A_t(z), B_t(z), C_t(z) \) and \( S_t(z) \). Their symmetric circulant approximations are \( A(z), B(z), C(z) \) and \( S(z) \).

The prototype for the pairwise relationships here is:

\[ H_t(z) = H(z) - \delta H(z) \]

\[ \delta H(z) = \text{toeplitz}\{0, 0, \ldots, 0, h_m, h_{m-1}, \ldots, h_1\} \quad (5.25) \]

The true system plant and controller are given by:

\[ C_t(z) = [I + A_t(z)]^{-1}B_t(z) \]

\[ K_t(z) = [I + S_t(z)]^{-1}C_t(z) \quad (5.26) \]
Chapter 5. Robust Stability Criteria for CD Processes

The circulant plant and controller are:

\[ G(z) = [I + A(z)]^{-1}B(z) \]
\[ K(z) = [I + S(z)]^{-1}C(z) \]  (5.27)

Internal stability of the circulant closed-loop is equivalent to the condition that the \((2n_x \times 2n_u)\) transfer matrix:

\[ L(z) = \begin{bmatrix} I + S(z) & C(z) \\ B(z) & I + A(z) \end{bmatrix} \]  (5.28)

is invertible in \(\Re H_\infty\) for all \(|z| \geq 1\).

The true system internal stability can be investigated by expressing \(L_\Delta(z)\) in terms of \(L(z)\) and the perturbation.

\[ L_\Delta(z) = \begin{bmatrix} I + S_\Delta(z) & C_\Delta(z) \\ B_\Delta(z) & I + A_\Delta(z) \end{bmatrix} \]  (5.29)

The small gain theorem shows that if \(L(z)\) is stable, the closed-loop \(L_\Delta(z)\) is internally stable if:

\[ \left\| \begin{bmatrix} \delta S(z) & \delta C(z) \\ \delta B(z) & \delta A(z) \end{bmatrix} \cdot \begin{bmatrix} I + S(z) & C(z) \\ B(z) & I + A(z) \end{bmatrix}^{-1} \right\|_\infty < 1 \]  (5.30)

Simulation results showed that this norm does not depend on the systems dimension so it can be computed from a low dimension multi-variable system.

5.5.2 Simulation Results

The feedback loop was simulated with the two-dimensional loop-shaped multi-variable controller designed in [74].

\[ G_0, S, C \in \mathbb{R}^{226 \times 226} \]

where:

\[ S = toeplitz\{0.993, 0.0023, 0.001, 0.0002, 0, \ldots, 0\} \]
\[ C = toeplitz\{-4.1226, -1.9487, 0.915, 0.9408, 0, \ldots, 0\} \]
\[ d_c = 3, \alpha_c = 0.8221, \alpha_c = 0.8221 \]
The spatial matrices are replaced by the following spatial FIR transfer functions:

\[
S(\lambda, \lambda^{-1}) = 0.0002\lambda^3 + 0.001\lambda^2 + 0.0023\lambda + 0.993\lambda^0 + 0.0023\lambda^{-1} + 0.001\lambda^{-2} + 0.0002\lambda^{-3}
\]

\[
C(\lambda, \lambda^{-1}) = 0.9408\lambda^3 + 0.915\lambda^2 - 1.9487\lambda + 4.1226\lambda^0 - 1.9487\lambda^{-1} + 0.915\lambda^{-2} + 0.9408\lambda^{-3}
\]

The feedback controller in a 2D transfer function form is:

\[
K_{02D}(z, \lambda) = \left[1 - S(\lambda, \lambda^{-1})z^{-1}\right]^{-1}c(z)C(\lambda, \lambda^{-1})
\]  

(5.31)

The nominal loop is stable as can be seen from the rootmap plots. All the poles of the temporal transfer functions lie inside the unit circle in Figure 5.3(a). There is an equal number of causal and anti-causal spatial poles lying inside and outside the unit circle respectively as shown in Figure 5.3(b). The norm of condition (5.30) regarding the edge effects is satisfied for the plant \( G_{02D} \) with a value of 0.9291, thus ensuring stability of the true plant.

In this example, a perturbed version of the spatial plant is studied. A temporal perturbation was introduced by assuming a pole value of \( \alpha_p = 0.8 \).
The perturbed spatial transfer function is given by:

\[ G_{1S}(\lambda, \lambda^{-1}) = \frac{-0.0092\lambda^0}{0.2300\lambda^2 - 0.6944\lambda^1 + \lambda^0 - 0.6944\lambda^{-1} + 0.2300\lambda^{-2}} \]

The spatial response of the perturbed plant is shown in Figure 5.4.

![Spatial response of the perturbed plant](image)

Figure 5.4: Spatial responses of the nominal and perturbed plants

The temporal transfer function is given by:

\[ T_1(z) = \frac{z^{-T_d}}{1 - a_p z^{-1}} \]  

(5.32)

The two-dimensional perturbed plant is:

\[ G_{12D}(z, \lambda) = G_{1S}(\lambda, \lambda^{-1}) \cdot T_1(z) \]  

(5.33)

In order to investigate the effect of retuning the controller on the closed-loop, three simulations were conducted. The first simulation was run with the multi-variable controller from [74], the other two simulations were run with re-tuned versions of the controller. The new controllers are simply \( K_{12D} = 1.2 \times K0_{2D} \) and \( K_{22D} = 1.5 \times K0_{2D} \). The analysis was performed in both the temporal and spatial domains on a grid of 62 temporal frequencies.
between $[0, 2\pi]$ and 226 spatial frequencies between $[0, 2\pi]$. The stability margin surface and the $\nu$-gap surface are plotted up to the temporal and spatial nyquist frequencies. Investigating the closed-loop with the nominal controller $K_{02D}$ and the perturbed plant $G_{12D}$, the robust stability criterion guarantees stability for the perturbed loop. The stability margin surfaces for $[G_{02D}, K_{02D}]$ and the $\nu$-gap surface for $(G_{02D}, G_{12D})$ calculated from the temporal and spatial transfer functions are shown in 5.5(a) and 5.5(b) respectively. As the $\nu$ surface is below the stability margin surface everywhere, the sufficient condition for stability is satisfied. This is validated by plotting the rootmaps of the corresponding set of 1D transfer functions as shown in 5.5(c) and 5.5(d) as the rootmaps do not intersect with the unit circle; condition (5.4). The rootmap of the temporal transfer functions in 5.5(c) has all its poles inside the unit circle. On the other hand, the rootmap of the noncausal spatial transfer functions in 5.5(d) has two sets with an equal number of spatial poles, a set of causal poles and a set of anti-causal poles lying inside and outside the unit circle respectively.

The analysis shows that the closed-loop with controller $K_{12D}$ and the perturbed plant $G_{12D}$ is stable. The stability margin surface for $[G_{02D}, K_{12D}]$ does not intersect with the $\nu$-gap surface $(G_{02D}, G_{12D})$. The rootmaps of the corresponding set of 1D transfer functions are shown in 5.6(c) and 5.6(d). All the poles of the temporal transfer functions lie inside the unit circle 5.6(c). The rootmap of the noncausal spatial transfer functions in 5.6(d) shows that the causal and anti-causal poles are located closer to the unit circle than in the previous case.

The closed-loop with controller $K_{22D}$ and the perturbed plant $G_{12D}$ turns out to be unstable. Figures 5.7(a) and 5.7(b) do not guarantee stability as the stability margin surface for $[G_{02D}, K_{22D}]$ intersects with the $\nu$-gap surface $(G_{02D}, G_{12D})$ violating condition (5.19). The rootmaps of the corresponding set of 1D transfer functions in Figures 5.7(c) and 5.7(d) intersect with the unit circle. The rootmap of the temporal transfer functions has poles outside the unit circle and the rootmap of the spatial transfer functions has poles lying on the unit circle. The 2D $\nu$-gap robust stability analysis guarantees stability for the perturbed loop with controllers $K_{02D}$ and $K_{12D}$ but not for $K_{22D}$.

Given the stability of $[G_{12D}, K_{02D}]$, the same results can be obtained if the analysis was performed by fixing the plant $G_{12D}$ in the stability mar-
Chapter 5. Robust Stability Criteria for CD Processes

(a) Stability margin surface ($G_0, K_0$) vs $\nu$-gap surface ($G_0, G_1$) (Temporal)

(b) Stability margin surface ($G_0, K_0$) vs $\nu$-gap surface ($G_0, G_1$) (Spatial)

(c) Rootmap of the temporal transfer functions

(d) Rootmap of the spatial transfer functions

Figure 5.5: Perturbed plant $G_1$ with the nominal controller
Chapter 5. Robust Stability Criteria for CD Processes

(a) Stability margin surface $(G_0,K_1)$ vs $\nu$-gap surface $(G_0,G_1)$ (Temporal)

(b) Stability margin surface $(G_0,K_1)$ vs $\nu$-gap surface $(G_0,G_1)$ (Spatial)

(c) Rootmap of the temporal transfer

(d) Rootmap of the spatial transfer

Figure 5.6: Perturbed plant $G_1$ with controller $K_1$
Chapter 5. Robust Stability Criteria for CD Processes

Figure 5.7: Perturbed plant G1 with controller K2
gin analysis and calculating the \( \nu \)-gap surfaces between \((K_{02D}, K_{12D})\) and \((K_{02D}, K_{22D})\). Figures 5.8(a) and 5.8(b) show that the stability margin surface for \([G_1, K_{02D}]\) is above the \( \nu \)-gap surface for \((K_{02D}, K_{12D})\) everywhere guaranteeing the stability of the loop \([G_1, K_{12D}]\). On the other hand, stability of the loop \([G_1, K_{22D}]\) is not guaranteed as the stability margin surface for \([G_1, K_{02D}]\) and the \( \nu \)-gap surface for \((K_{02D}, K_{22D})\) intersect as shown in Figures 5.8(c) and 5.8(d). This property permits robust control design online either through iterative feedback tuning or adaptive control schemes. An existing controller can be re-tuned online with guaranteed stability as long as the 2D robust stability criterion is satisfied. The investigation is computationally very efficient. Moreover, if the tuning is to be restricted to the controller’s spatial matrices only, the stability criterion simplifies to the \( \nu \)-gap metric between the spatial transfer functions of the two controllers.

5.6 A Robust Stability Criterion for Misalignment Uncertainties Using a 2D Phase Margin Concept

In [80], the shift property of the Fourier transform was used to study the effect of mis-mapping on stability. The proposed method was applied to a simplified version of the CD controller. In this section, robust stability of the CD process against misalignment uncertainties is investigated using phase margins. Two-dimensional models are adopted for the plant and the controller. In order to handle systems which have more measurement points than actuators (non-square systems), the equivalent model in the high resolution that is developed in section 2.5 is used. This stability criterion was presented in [5].

As only misalignment uncertainty is investigated, a simple criterion can be used given the stability of the nominal loop \([K_{2D}, G_{02D}]\). Misalignment is represented by powers of the shift operator \( \lambda \) in the spatial transfer function. Marginal stability of the two-dimensional system occurs when spatial poles of the characteristic polynomial lie on the unit circle. For this analysis, a controller \( K(t, z) \) in a negative feedback loop is investigated changing equation (5.7) to \([1 + G_{12D}(z, \lambda)K_{2D}(z, \lambda)]\). As misalignment only contributes to the phase of \( G_{12D}(z, \lambda)K_{2D}(z, \lambda) \) and does not change the magnitude, determining the phase margins at the frequencies where \( |G_{02D}(e^{j\omega}, e^{j\nu})K_{2D}(e^{j\omega}, e^{j\nu})| = 1 \) is used to check robustness to misalignment uncertainties. The maximum allowable misalignment is computed from the spatial frequency and the associated phase margin. If the
Chapter 5. Robust Stability Criteria for CD Processes

(a) Stability margin surface (G1,K0) vs $\nu$-gap surface (K0,K1) (Temporal) vs $\nu$-gap surface (K0,K1) (Spatial)

(b) Stability margin surface for (G1,K0) vs $\nu$-gap surface (K0,K2) (Temporal) vs $\nu$-gap surface (K0,K2) (Spatial)

Figure 5.8: Perturbed plant G1 with controllers K1 and K2
added phase shift at any of the determined frequencies does not make $G_{12D}(e^{j\omega}, e^{j\nu})K_{2D}(e^{j\omega}, e^{j\nu}) = -1$ then the system remains stable as:

$$
\therefore |1 + G_{12D}(e^{j\omega}, e^{j\nu})K_{2D}(e^{j\omega}, e^{j\nu})| \neq 0 \quad \forall \omega, \nu \in [0, 2\pi]
$$

The developed criterion is studied in processes with more measurement points than actuators (non-square systems) as in square systems a mis-map of only one actuator zone will be too close to instability if not unstable. Nonetheless, applying the criterion to square systems is straightforward.

### 5.6.1 Two-Dimensional Phase Margins

This section discusses the robust stability criterion based on the allowable phase shift contributed by misalignment uncertainties only. The formulas are given for systems with equal number of actuators and measurements, i.e. for a single rate system.

For a loop with misalignment uncertainty $\lambda_{nm}$:

$$
G_{12D}(z, \lambda)K(z, \lambda) = \lambda_{nm}G_{02D}(z, \lambda)K(z, \lambda)
$$

(5.34)

Given the stability of the nominal loop, the perturbed loop is stable if:

$$
A_1(e^{j\omega}, \lambda) \neq 0 \quad \text{when} \quad |\lambda| = 1 \quad \forall \omega \in [0, 2\pi]
$$

$$
G_{12D}(e^{j\omega}, e^{j\nu})K(e^{j\omega}, e^{j\nu}) \neq -1
$$

Misalignment uncertainty contributes a phase shift only:

$$
|G_{02D}(e^{j\omega}, e^{j\nu})K(e^{j\omega}, e^{j\nu})| = |G_{12D}(e^{j\omega}, e^{j\nu})K(e^{j\omega}, e^{j\nu})|
$$

$$
\text{phase}[G_{12D}(e^{j\omega}, e^{j\nu})K(e^{j\omega}, e^{j\nu})] = \text{phase}[G_{02D}(e^{j\omega}, e^{j\nu})K(e^{j\omega}, e^{j\nu})] + \text{phase}[e^{j\nu}]_{\text{nm}}
$$

Determine the phase margins $\phi_{(\omega_i,\nu_i)}$ at the set of points $(\omega_i, \nu_i)$ where:

$$
|G_{02D}(e^{j\omega_i}, e^{j\nu_i})K(e^{j\omega_i}, e^{j\nu_i})| = 1
$$

Maximum tolerable misalignment $(n_m)$:

$$
n_m = \min_{i} (\phi_{(\omega_i,\nu_i)}/\nu_i)
$$

(5.35)
5.6.2 Modeling Misalignment in Non-Square Systems

To investigate the allowable mismap in non-square systems using the developed criterion, two approaches are proposed. The first approach is to consider the square system model in the low resolution while permitting fractional powers of the shift operator in space to account for shifts in the measurement resolution. The second approach is to adopt the high resolution equivalent transfer function developed in section 2.5. Sampling distances in the adopted spatial transfer functions are analogous to sampling times in conventional temporal models.

Modeling Misalignment in the Low Resolution Transfer Function

This method computes the two-dimensional phase margins for the slow rate system. The power of the shift operator \( n_m \) in equation (5.34) is replaced by \( n_m \) multiples of the fraction \( (n_u/m_y) \). As the spatial noncausal transfer function at the low sampling rate (low resolution) corresponds to an \((n_u \times n_u)\) spatial matrix, a scaling factor of \( q_d = \sqrt{(m_y/n_u)} \) is required to match the frequency response of this transfer function with the spatial modes obtained from the Fourier transform of the \((m_y \times n_u)\) spatial matrix. Decimation of the high resolution profile with an anti-aliasing window results in the scaling factor \( n_u/m_y \) as discussed in section 2.5. A low resolution equivalent to the non-square system is shown in Figure 5.9.

![Figure 5.9: Multi-rate loop in the low resolution](image-url)
Chapter 5. Robust Stability Criteria for CD Processes

Modeling Misalignment in the High Resolution Transfer Function

The high resolution equivalent model developed in section 2.5 is used for this analysis. The power of the shift operator $n_m$ in equation (5.34) is replaced by $n_m^5$.

5.6.3 Illustrative Example

In this section, the two-dimensional robust stability criterion for misalignment uncertainties is illustrated by the industrial model for the weight process from [74] and [77] that has been studied in section 5.5. The ratio of measurement boxes to actuators is 5.

Low Resolution Transfer Function Analysis

Table 5.1 presents the phase margins at the set of critical points $(\omega_i, v_i)$ on the 2D frequency grid that had the given spatial frequencies:

<table>
<thead>
<tr>
<th>$v_i$</th>
<th>0.6283</th>
<th>0.7414</th>
<th>0.7477</th>
<th>0.9865</th>
<th>0.9927</th>
<th>1.087</th>
<th>1.7153</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{v_i}$</td>
<td>1.4290</td>
<td>1.3968</td>
<td>1.3973</td>
<td>1.4165</td>
<td>1.4170</td>
<td>1.4724</td>
<td>3.1416</td>
</tr>
</tbody>
</table>

Table 5.1: Maximum allowable misalignment at the critical points

The maximum allowable misalignment as a fraction of the actuators spacing is : $6/5$ which corresponds to 6 measurement positions.

High Resolution Transfer Function Analysis

Using the proposed robust stability criterion, the maximum allowable misalignment is found to be 6 measurement positions, slightly higher than one actuator spacing. This result is confirmed by plotting the rootmaps of the loops with misalignments. The rootmap plots the poles of the spatial transfer functions at different temporal frequencies. Stability is attained if there are not any intersections with the unit circle. Figures 5.10(a), 5.10(b), 5.10(c), 5.10(d) and 5.10(e) show the rootmaps of the nominal loop and loops with misalignments of 5, 6, 7 and 8 measurement positions respectively. It is clear that the loop is stable up to a misalignment of 6 measurement positions as the rootmaps for the loops with misalignment of 7 and 8 intersect the unit circle because of anti-causal poles crossing the unit circle causing instability.
Figure 5.10: Rootmaps for the different misalignment cases
This result is validated by simulation results, at a misalignment of 7 measurement positions oscillations start to appear in the output profile as the closed-loop spatial poles lie on the unit circle causing marginal stability. Figures 5.11(a), 5.11(b) and 5.11(c) show the steady-state response for loops with misalignments of 5, 6 and 7 respectively assuming the oscillations does not result in a sheet break.

As seen from the given example, this method provides a simple criterion to determine the maximum tolerable misalignment in the CD feedback loop. Moreover, this analysis shows that oscillations appearing in the CD profile can be attributed to mis-mapping errors. A similar observation was addressed in [31]. The noncausal spatial model representation relates these streaks (oscillations in the output profile) to spatial poles that are too close to the unit circle resulting in marginal stability.

5.7 Summary

A novel robust stability criterion for CD processes is presented through modeling the process by a 2D transfer function and using results from multidimensional digital stability theory. This test replaces the large dimension multi-variable robust stability problem by a set of simple 1D problems making it computationally very efficient. The analysis can be performed in the temporal or the spatial domain depending on which is more convenient to investigate the controller's robustness and performance. The criterion is based on the \( \nu \)-gap robust stability theorem which provides a sufficient condition for stability when dealing with perturbations in either the plant or the controller. In the temporal domain, the conventional \( \nu \)-gap robust stability analysis is performed for a set of causal temporal transfer functions at a set of discrete spatial frequencies. A noncausal \( \nu \)-gap is proposed to perform the analysis in the spatial domain. The stability conditions are investigated through noncausal spatial transfer functions at a set of discrete temporal frequencies.

Another robust stability criterion to determine the maximum allowable misalignment uncertainty in CD processes is proposed. Stability is investigated by extending the concept of a phase margin to the two-dimensional case.
Figure 5.11: Spatial steady-state profile for the different misalignment cases
Chapter 6

CD Loop-Shaping Using Spatial Transfer Functions

6.1 Introduction

This chapter presents a control design technique for cross-directional (CD) controllers using spatial loop-shaping. The controller is designed to satisfy the performance and robustness conditions at temporal steady state. The performance and robustness of the designed controller are then investigated on a grid of temporal and spatial frequencies. As the CD process is a regulation problem attenuating spatial disturbances, mainly at low dynamical frequencies, performing control design at steady-state is sufficient. This approach simplifies the complicated design problem thus permitting the use of adaptive control. The CD process is modeled by the spatial noncausal transfer function and the closed-loop spatial frequency response is shaped using pole placement. This technique shapes the singular values of the MIMO system at steady-state and consequently the $H_{\infty}$ norm.

The CD loop-shaping technique is developed in section 6.2 and illustrated with a simulation example. In section 6.3, the control design technique is used in an adaptive control scheme. The spatial transfer function is identified in closed-loop and used in the CD loop-shaping technique. In section 6.3.1, simulation results are provided to show the advantages of the adaptive technique with the introduction of a grade change.

6.2 The CD Loop-Shaping Technique

CD Control design using the large multi-variable model in the temporal domain is a challenging problem. In [74],[77] and [78] the spatial circulant matrix approximation was used to decompose the large multi-variable system into a set of SISO systems at different spatial frequencies. The performance requirements were enforced at low spatial and dynamical frequencies where most of the disturbances are. The low dynamical frequency range was up to
Chapter 6. CD Loop-Shaping Using Spatial Transfer Functions

\(10^{-4}\) Hz). The robustness requirements must be observed at high spatial and dynamical frequencies where the process gain is small. The controller was designed to satisfy the robustness requirements for high spatial frequencies at temporal steady state as the gain of the Dahlin controller rolls off quickly for high dynamical frequencies. The two-dimensional loop-shaping technique has been successfully implemented in many paper mills. However, this design technique can not be used in an adaptive control as it results in a non-localized controller and a few iterations are performed until a trade-off between performance and localization is achieved.

In this work, a technique for designing CD controllers is proposed. The design problem is simplified by fixing the dynamics of the controller and focusing on shaping the spatial frequency response at temporal steady-state. Designing the controller for steady-state performance and robustness was proposed in [34]. Basically, this is a spatial frequency loop-shaping which in turn shapes the maximum and minimum singular values of the MIMO system. Square systems with the same number \(n_a\) of measurement points and actuators are addressed. This is not unusual as mapping the measurement error profile to the actuator’s resolution is standard in CD processes. The performance of the designed controller is compared to those of an industrially tuned controller and a controller designed using the two-dimensional frequency loop-shaping from [75] and [76].

### 6.2.1 Controller Structure

The industrial controller structure that was presented in section 5.5 is adopted here. The feedback loop is shown in Figure 6.1 where the sensitivity function is given by \([I + G(z)K(z)]^{-1}\).

![Figure 6.1: The industrial feedback loop](image)

The spatial matrices are replaced by the corresponding spatial transfer functions:
Chapter 6. CD Loop-Shaping Using Spatial Transfer Functions

\[ G_0 \to G(\lambda, \lambda^{-1}) \]
\[ C \to C(\lambda, \lambda^{-1}) \]
\[ S \to S(\lambda, \lambda^{-1}) \]

\[ \lambda = e^{j\nu} \text{ where } \nu \text{ is the spatial frequency.} \]

The 2D feedback controller in transfer function form is:

\[ K(z, \lambda) = \left[1 - S(\lambda, \lambda^{-1})z^{-1}\right]^{-1}c(z)C(\lambda, \lambda^{-1}) \quad (6.1) \]

\[ c(z) = \frac{(1 - \alpha_c)[1 - \alpha_c z^{-1}]}{1 + (1 - \alpha_c) \sum_{i=1}^{d-1} z^{-i}} \]

At temporal steady-state, \( z = 1 \), the spatial transfer function becomes:

\[ K(\lambda, \lambda^{-1}) = \frac{c(1) \cdot C(\lambda, \lambda^{-1})}{1 - S(\lambda, \lambda^{-1})} \]
\[ c(1) = \frac{(1 - \alpha_c)(1 - \alpha_c)}{1 + (1 - \alpha_c)(1 + 1)} \]

The controller is then expressed in terms of rational polynomials. Let's assume that \( c(1) = 1 \) for now and drop it from the equations.

\[ K(\lambda, \lambda^{-1}) = \frac{C(\lambda, \lambda^{-1})}{D(\lambda, \lambda^{-1})} \]
\[ C(\lambda, \lambda^{-1}) = c_{n_k} \lambda^{n_k} + \ldots + c_1 \lambda + c_0 + c_1 \lambda^{-1} + \ldots + c_{n_k} \lambda^{-n_k} \]
\[ D(\lambda, \lambda^{-1}) = 1\lambda^0 - S(\lambda, \lambda^{-1}) \]
\[ D(\lambda, \lambda^{-1}) = -s_{n_k} \lambda^{n_k} - \ldots - s_1 \lambda + (1 - s_0) \lambda^0 - s_1 \lambda^{-1} - \ldots - s_{n_k} \lambda^{-n_k} \]

The spatial bandwidth of the actuator's array in closed-loop can be controlled easily through the noncausal transfer function representation. The following example shows how the spatial bandwidth of the process \( G_0 \) in closed-loop is determined by the location of the spatial poles and zeros of the spatial controller. Figure 6.2 plots the pole-zero map for two spatial controllers \( K_1 \) and \( K_2 \). In closed-loop, controller \( K_2 \) results in a larger attenuation band as seen in the sensitivity function in Figure 6.3.
Figure 6.2: Controllers' pole-zero map

Figure 6.3: Closed-loop attenuation band
6.2.2 CD Loop-Shaping Using Spatial Pole Placement

In [58], a sensitivity function shaping method using pole placement is given. The output sensitivity function is a good indicator of performance and robust stability of the closed-loop as the modulus margin is the inverse of its infinity norm. The method is adaptable to the spatial domain with minor modifications. As the process is symmetric noncausal, adding a causal pole or zero must be accompanied by its corresponding anti-causal pole or zero (its reciprocal). This eliminates any restrictions on any unstable poles or zeros as here these are stable as long as they correspond to the anti-causal response. The 2D stability check developed in sections 5.2 was used in investigating the system’s stability and robustness. As the CD process is a regulation problem, spatial open-loop poles are chosen as closed-loop ones by default unless different response localization is desired [57]. The pre-filter in the place pole technique was dropped in this design because in most of the cases the reference is set to zero. When a set-point \( R(t) \) other than zero is desired two poles at \( (\lambda = 1) \) are introduced in the controller to work as a spatial integrator to bring the sensitivity function to zero at \( (\nu = 0) \).

Some rules for pole placement in the CD process were developed as follows: Symmetry is enforced in the controller transfer functions, controller zeros are placed at high frequencies to reduce the controller gain in the high spatial frequency range and the controller poles are not to be placed on the unit circle. The proposed technique was presented in [3].

![Figure 6.4: The closed-loop steady-state spatial transfer function](image)

The plant to be controlled is:

\[
G(\lambda, \lambda^{-1}) = \frac{B(\lambda, \lambda^{-1})}{A(\lambda, \lambda^{-1})}
\]
The controller in pole zero form

\[ K(\lambda, \lambda^{-1}) = \frac{C(\lambda, \lambda^{-1})}{D(\lambda, \lambda^{-1})} = \frac{(1 - z_1 \lambda^{-1})(1 - z_1 \lambda^1)(1 - z_n \lambda^{-n_k})(1 - z_n \lambda^{n_k})}{(1 - p_1 \lambda^{-1})(1 - p_1 \lambda^1)(1 - p_n \lambda^{-n_k})(1 - p_n \lambda^{n_k})} \]

where

\[ C(\lambda, \lambda^{-1}) = C'(\lambda, \lambda^{-1}) H_C(\lambda, \lambda^{-1}) \]
\[ D(\lambda, \lambda^{-1}) = D'(\lambda, \lambda^{-1}) H_D(\lambda, \lambda^{-1}) \]

\( H_C(\lambda, \lambda^{-1}), H_D(\lambda, \lambda^{-1}) \) are pre-specified fixed parts in \( C(\lambda, \lambda^{-1}) \) and \( D(\lambda, \lambda^{-1}) \) respectively.

The output sensitivity function \( S_{yd} \) is:

\[ S_{yd}(\lambda, \lambda^{-1}) = \frac{A(\lambda, \lambda^{-1}) D(\lambda, \lambda^{-1})}{A(\lambda, \lambda^{-1}) D(\lambda, \lambda^{-1}) + B(\lambda, \lambda^{-1}) C(\lambda, \lambda^{-1})} \]

The closed-loop characteristic equation:

\[ P_{cl}(\lambda, \lambda^{-1}) = A(\lambda, \lambda^{-1}) D(\lambda, \lambda^{-1}) + B(\lambda, \lambda^{-1}) C(\lambda, \lambda^{-1}) \]
\[ = P_{dom}(\lambda, \lambda^{-1}) \cdot P_{aux}(\lambda, \lambda^{-1}) \quad (6.2) \]

\( P_{dom}, P_{aux} \) correspond to the dominant and auxiliary closed-loop poles.

The design procedure is given by the following steps:

1. Specify the sensitivity function template, the plant model, the desired closed-loop properties (spatial bandwidth, localization) and the fixed parts of the controller \( H_C(\lambda, \lambda^{-1}) \) and \( H_D(\lambda, \lambda^{-1}) \).
2. Solve the Diophantine equation (6.2) to get \( C'(\lambda, \lambda^{-1}) \) and \( D'(\lambda, \lambda^{-1}) \).
3. Check if the resultant sensitivity function and controller frequency response are satisfactory.
4. Check the 2D stability and robustness.
5. If the design is not acceptable change any of the fixed parts \( (H_C, H_D \text{ or } P_{aux}) \) to improve the design and go back to step 2 (The changes are performed according to the rules given in [58],[57]).

The flow chart given in Figure 6.5 illustrates the design procedure.
Figure 6.5: A flow chart for the design procedure
6.2.3 Simulation Results

In this section, the designed controller performance is compared to that of the two-dimensional loop shaping technique. The industrial model for the weight process from [74] is simulated with three controllers: an industrial controller, the two dimensional loop-shaped controller and the spatial loop-shaped controller. This particular paper machine has 226 consistency profiling actuators. The controllers had the structure shown in Figure 6.1 and the reference input was set to 40.

The process model is:

\[ y(z) = G(z)u(z) + d(z) \]  \hspace{1cm} (6.3)

\[ G(z) = (I - Az^{-1})^{-1}(Bz^{-T_d}) \]

\[ D_t(z) = (I - Hz^{-1})^{-1}(I - Ez^{-1}) \]

\[ d(z) = D_t(z)d_n(z) \]

\[ B = \text{toeplitz}\{-0.0814, -0.0455, -0.0047, 0.0017, 0.0003, 0, ..., 0\} \]

\[ A = \text{toeplitz}\{0.8221, 0, 0, 0, ..., 0\} \]

\[ E = \text{toeplitz}\{0.8221, 0, 0, 0, ..., 0\} \]

\[ H = \text{toeplitz}\{0.999, 0, 0, 0, ..., 0\} \]

\[ A, B, H, E \in \mathbb{R}^{226 \times 226} \]

\( T_d = 3 \) is the dead time.

\( d_n(t) \) is Gaussian white noise.

The spacing between actuators is 0.035 meter.

Toeplitz represents a toeplitz matrix with the given vector as its first column.

The process transfer matrix can be rewritten as:

\[ G(z) = G_0 z^{-T_d} \text{toeplitz}\{-0.8221, 0, 0, ..., 0\} \]

where \( G_0 = B \)

The plant’s spatial response was identified using the noncausal identification technique developed in section 3.2.2:

\[ G_s(\lambda, \lambda^{-1}) = \frac{-0.0056 \lambda^2 - 0.0422 \lambda^1 - 0.0763 - 0.0422 \lambda^{-1} - 0.0056 \lambda^{-2}}{0.0433\lambda^2 - 0.0605\lambda^1 + 1 - 0.0605\lambda^{-1} + 0.0433\lambda^{-2}} \]
The spatial response of the transfer function is identical to the response in the interaction matrix columns. The norm of the transfer function is almost equal to that of the interaction matrix.

\[ \|G_s\|_\infty = 0.178 \quad , \quad \|G_0\|_\infty = 0.1778 \]

The spacing between the actuators results in an actuator array with a Nyquist frequency of 14.28 cycles/meter.

The industrial controller designed with the traditional empirical tuning rules is:

\[ S_{\text{ind}} = \text{toeplitz}\{0.9595, 0.0202, 0, \ldots, 0\} \]
\[ C_{\text{ind}} = -6.5494 \cdot S_{\text{ind}} \]
\[ d_{\text{ind}} = 3, \alpha_{\text{ind}} = 0.7316, \gamma_{\text{ind}} = 0.8365 \]

The two-dimensional loop-shaped controller is:

\[ S_c = \text{toeplitz}\{0.993, 0.0023, 0.001, 0.0002, 0, \ldots, 0\} \]
\[ C_c = \text{toeplitz}\{-4.1226, -1.9487, 0.915, 0.9408, 0.0027, 0.0011, 0.0002, 0, \ldots, 0\} \]
\[ d_c = 3, \alpha_c = 0.8221, \gamma_c = 0.8221 \]

The frequency domain specifications with the feedback in the two-dimensional loop-shaping technique were derived as bounds on the shape of \( |K(e^{j\omega}, e^{j\nu})| \). The performance and robustness constraints were enforced at the sets \( \Omega_l \) and \( \Omega_h \) respectively.

\[ \Omega_l = \{ (\omega, \nu) : |\nu| \leq 5.5m^{-1}, |\omega| \leq 10^{-4}Hz, |K(e^{j\omega}, e^{j\nu})| \geq 22 \} \]  
\[ \Omega_h = \{ (\omega, \nu) : |G(e^{j\omega}, e^{j\nu})| \leq 0.1, |K(e^{j\omega}, e^{j\nu})| \leq 5 \} \]

These constraints were included as a condition to be satisfied in the spatial loop shaping design resulting in the following controller:

\[ K(\lambda, \lambda^{-1}) = \frac{-0.363\lambda^3 - 0.34\lambda^2 + 1.367\lambda^1 + 2.727\lambda^0 + 1.367\lambda^{-1} - 0.34\lambda^{-2} - 0.363\lambda^{-3}}{10^2(2.43\lambda^3 + 2.55\lambda^2 + 13.63\lambda^1 - 26.91\lambda^0 + 13.63\lambda^1 + 2.55\lambda^{-2} + 2.4\lambda^{-3})} \]

The Dahlin controller was tuned with the same parameters as in the two-dimensional loop-shaping case giving \( c(z)|_{z=1} = 0.023 \) (Dahlin controller at
steady-state).

\[ C(\lambda, \lambda^{-1}) = 0.36\lambda^2 + 0.34\lambda - 1.367\lambda^4 - 2.7268 - 1.367\lambda^{-1} + 0.34\lambda^{-2} + 0.36\lambda^{-3} \]

\[ S(\lambda, \lambda^{-1}) = .0006\lambda^3 - .0006\lambda^2 + .0032\lambda^1 + .9937 + .0032\lambda^{-1} - .0006\lambda^{-2} + .0006\lambda^{-3} \]

The controller was realized by the following spatial matrices:

\[ C_{sp} = \text{toeplitz}\{-2.7268, -1.367, 0.34, 0.36, 0, \ldots, 0\} \]

\[ S_{sp} = \text{toeplitz}\{0.9937, 0.0032, -0.0006, 0.0006, 0, \ldots, 0\} \]

**Performance**

The estimated standard deviation of the paper properties is used as a measure for paper quality [1]. The conventional quality index \( \sigma \) for a measurement profile \( y(k) \in R_{y}^m \) with discrete-time index \( k = 1, \ldots, N \) is given by:

\[
\sigma = 2\sqrt{\sum_{k=1}^{N} \frac{(y(k)-\bar{y})^T(y(k)-\bar{y})}{m_y N-1}}
\]

(6.6)

\[ \bar{y} = [y_{avg}, y_{avg}, \ldots, y_{avg}] \quad , \quad y_{avg} = \frac{1}{N} \sum_{k=1}^{N} \sum_{j=1}^{m_y} y_{ij}(k) \]

where \( y_{avg} \) is the overall mean of the two-dimensional array of measurements.

The steady-state profile with the three controllers is shown in Figure 6.6. The error spectrum in Figure 6.7 shows that the spatial loop-shaping technique outperformed the empirically tuned industrial controller significantly and performed comparably to the two-dimensional loop-shaped controller. The 2-\( \sigma \) in the output profile of the two-dimensional loop shaped controller is 0.6479 while that of the spatially loop-shaped controller is 0.664. These results are consistent with the sensitivity function plots shown in Figure 6.8. The spatially loop-shaped controller has almost the same disturbance rejection band as the two-dimensional loop-shaped controller as seen in Figure 6.8. The two-dimensional plots of the sensitivity functions for the two-dimensional loop-shaping and the spatial loop-shaping techniques are shown in Figures 6.9 and 6.10. The difference in magnitude between the latter and the former is given in Figure 6.11. The difference shows that the spatial loop-shaped

124
Chapter 6. CD Loop-Shaping Using Spatial Transfer Functions

Figure 6.6: The steady-state profile for the different controllers

Figure 6.7: The steady-state error spectrum for the different controllers
controller has almost the same disturbance rejection band at steady-state. At low dynamical frequencies and mid-range spatial frequencies, the spatial loop-shaped controller's sensitivity is slightly higher than the two dimensional loop-shaped one while the opposite is true at mid-range dynamical frequencies and mid-range spatial frequencies. This is advantageous as it minimizes the magnitude of the sensitivity function in this frequency range bringing it closer to unity. If the 2D sensitivity function turns to be unsatisfactory, the dynamical part of the controller can be re-tuned given the computed spatial matrices from the spatial loop-shaping technique.

Figure 6.12 shows that the loop sensitivity function computed from spatial transfer functions is almost identical to the actual multi-variable sensitivity function at steady-state obtained from spatial matrices. The slight difference is due to neglecting the edge effects by assuming identical actuator responses everywhere.

Robustness

A quick stability check was included in the design algorithm using the method developed for the two-dimensional transfer function [7] and [8].
Figure 6.9: The 2D sensitivity function for the two-dimensional loop-shaped system
Chapter 6. CD Loop-Shaping Using Spatial Transfer Functions

Figure 6.10: The 2D sensitivity function for the spatial loop-shaped system

Figure 6.11: The difference in the magnitude between the 2D sensitivity functions
Figure 6.12: The spatial sensitivity function at steady-state computed from the transfer function vs the spatial matrix
condition $|K(e^{j\omega}, e^{j\nu})| < 5$ is satisfied for all high spatial and dynamical frequencies. The gain of the controller rolls off quickly in the temporal domain, this is shown in Figure 6.13.

![Controller 2D frequency response](image)

Figure 6.13: The 2D spatial loop-shaped controller at high spatial and temporal frequencies

### 6.3 The Adaptive Control Technique

The adaptive control scheme is achieved through a closed-loop identification experiment followed by loop-shaping the steady-state sensitivity function using spatial pole placement as discussed in section 6.2.2. The method for closed-loop identification discussed in section 4.3 is adopted in the adaptive control scheme. This technique was presented in [4].

#### 6.3.1 Simulation Results

In this section, the adaptive controller’s performance is investigated. The same industrial model with the two dimensional loop-shaping controller from section (6.2.3) is studied. As a small grade change is introduced in closed-loop after 50 scans, the performance deteriorates with the existing controller.
Prior to the grade change, the quality index $2-\sigma$ had a value of 0.64. The adaptive control scheme is triggered when the quality index exceeds a threshold value of 0.75. A closed-loop identification experiment is performed where a reference signal is introduced at the actuators set-points in the 100th scan and the spatial response is identified from steady-state input-output data. As the magnitude of the reference signal is restricted to be small, data is collected from 10 scans to identify the steady-state spatial response. The new spatial response is shown in Figure 6.14. The estimated response is shown in Figure 6.15.

The controller is redesigned using spatial loop shaping based on the estimated model. The constraints in equations (6.4,6.5) were included as conditions to be satisfied in the adaptive spatial loop-shaping design.

![Spatial responses for the two grades](image)

Figure 6.14: The spatial responses for the two grades

The new response is modeled by:

$$G_{s1}(\lambda, \lambda^{-1}) = \frac{-0.01\lambda^2 - 0.02\lambda^1 - 0.048\lambda^0 - 0.02\lambda^{-1} - 0.01\lambda^{-2}}{0.136\lambda^2 - 0.21\lambda^1 + 1\lambda^0 - 0.21\lambda^{-1} + 0.136\lambda^{-2}}$$

The designed controller is:

$$K(\lambda, \lambda^{-1}) = \frac{-0.2648\lambda^3 + 0.03185\lambda^2 + 1.98\lambda^1 + 3.392\lambda^0 + 1.98\lambda^{-1} + 0.03185\lambda^{-2} - 0.2648\lambda^{-3}}{10^{-2}(3.527\lambda^3 - 3.619\lambda^2 + 9.373\lambda^1 - 18.56\lambda^0 + 9.373\lambda^{-1} - 3.619\lambda^{-2} + 3.527\lambda^{-3})}$$
Chapter 6. CD Loop-Shaping Using Spatial Transfer Functions

The spatial response estimated in closed-loop

Figure 6.15: Closed-loop identification of the new spatial response

The Dahlin controller was tuned with the same parameters as in the two-dimensional loop-shaping case giving $c(z)|_{z=1} = 0.023$ (Dahlin controller at steady-state).

This gave:

$$S(\lambda, \lambda^{-1}) = 0.0008\lambda^3 - 0.0008\lambda^2 + 0.0022\lambda^1 + 0.9957$$

$$+ 0.0022\lambda^{-1} - 0.0008\lambda^{-2} + 0.0008\lambda^{-3}$$

$$C(\lambda, \lambda^{-1}) = 0.265\lambda^3 - 0.032\lambda^2 - 1.978\lambda^1 - 3.3917$$

$$- 1.978\lambda^{-1} - 0.032\lambda^{-2} + 0.265\lambda^{-3}$$

The controller was realized by the following spatial matrices:

$$C_{sp} = toeplitz\{-3.3917, -1.978, -0.032, 0.265, 0, \ldots, 0\}$$

$$S_{sp} = toeplitz\{0.9957, 0.0022, -0.0008, 0.0008, 0, \ldots, 0\}$$
Performance

The steady-state output profile with the two controllers is shown in Figure 6.16. Plotting the error spectrums in Figure 6.17 shows that the adaptive controller designed by the spatial loop-shaping technique performed better than the two-dimensional loop-shaped controller with the new spatial response. The adaptive controller managed to remove more low and mid-range frequency errors resulting in a 2-σ value of 0.68. Figure 6.18 shows that the new loop has a larger attenuation band for disturbance rejection at steady-state.

![Response with the 2D loop-shaping controller](image1)

![Response with the spatial loop-shaping controller](image2)

Figure 6.16: The steady-state output profile for the two controllers

The two-dimensional plots of the sensitivity function for the two-dimensional loop-shaping and the adaptive spatial loop-shaping techniques (Figures 6.19 and 6.20) show that the loop with the new controller has better disturbance rejection at low spatial and dynamical frequencies.

Robustness

A quick stability check was included in the design algorithm using the method developed for the two-dimensional transfer function in section 5.2. The constraint $|K(e^{j\omega}, e^{j\nu})| < 5$ from equation (6.4) is satisfied for all high
Figure 6.17: The steady-state error spectrum for the two controllers

Figure 6.18: The steady-state spatial sensitivity function for the two controllers
Figure 6.19: The 2D sensitivity function for the 2D loop-shaped system after the grade change
Chapter 6.  

*CD Loop-Shaping Using Spatial Transfer Functions*

Figure 6.20: The 2D sensitivity function for the spatial loop-shaped system in the adaptive scheme
Chapter 6. CD Loop-Shaping Using Spatial Transfer Functions

spatial and dynamical frequencies. The gain of the controller rolls off quickly in the temporal domain as shown in Figure 6.21.

![Controller 2D frequency response](image)

Figure 6.21: The 2D spatial loop-shaped adaptive controller at high spatial and temporal frequencies

6.4 Summary

The 2D representation simplifies the challenging problem of designing a robust controller by using 2D frequency loop-shaping. A novel CD control design method through spatial loop shaping is presented. The design is performed on the spatial model at temporal steady state. The noncausal spatial transfer function permits loop-shaping the spatial frequencies with the conventional techniques developed for the temporal frequencies. This approach reduces the challenging problem of designing a large dimensional multi-variable controller to a simple design one which can be done in few seconds and results in an improved controller performance. An adaptive control scheme is developed using the CD loop-shaping technique. The spatial response is identified in closed-loop and is followed by redesigning the controller. The proposed adaptive scheme improved controller performance
significantly when a grade change was introduced in closed-loop.

Loop-shaping the CD controller is performed at steady-state then the two-dimensional frequency response is investigated. It is straightforward to perform the loop-shaping technique on the whole two-dimensional frequency grid given that a few parameters are sufficient to model the process in the 2D representation. The robust control design technique can be readily extended to multi-array systems.
Chapter 7

Conclusions and Future Work

7.1 Conclusions

This work has focused on improving the identification of CD models as well as simplifying the robust control design problem. The CD process is a large scale multi-variable process that evolves in two-dimensions and is generally ill-conditioned. The large dimensionality makes these goals challenging and even intractable unless the problem is simplified through basis functions representations or decomposition by Fourier matrices. As it is standard to assume that the actuators have identical responses, the information from the columns of the spatial interaction matrices is redundant. The large dimension is unnecessary and the only merit of the interaction matrix is to account for the shift in space using downward shifts in the columns. Moreover, the available CD response shape functions are inconvenient for control relevant identification and control design. In order to eliminate the unnecessary complications, the large dimensional interaction matrices of CD processes are replaced by noncausal spatial models. This representation is equivalent to the circulant matrix approximation that has been used in recent work on CD control.

Employing the separability condition between the spatial and the temporal domain, a two-dimensional (2D) spatially noncausal representation is proposed to model the CD process. This representation requires a few parameters to model the process and a simple 2D closed-loop transfer function can be easily obtained upon representing the CD controller as a 2D model. Using results from the multirate digital theory, a 2D closed-loop equivalent to processes with more measurement points than actuators is developed. This transfer function is very useful in predicting and explaining spatial phenomena in these systems.

A novel method for identifying the CD mapping model and detecting misalignment using noncausal spatial FIR models is presented in chapter 3.
As the response is assumed to be symmetrical, the offset in the axis of symmetry of the identified model is the distance between the actuator geometric position and its actual center of response. The method is tested on a benchmark industrial paper machine simulator and is shown to provide accurate estimates. Nancausal spatial transfer functions simplify the identification technique and deliver parameter uncertainty regions instead of assuming arbitrary uncertainties. The spatial domain identification enables focusing the signal's power in a specific range of spatial frequencies that might be more important for the sake of control or robust stability. Moreover, these models facilitate control relevant identification using experiment design techniques from the temporal domain. A technique that minimizes the variance in the identified model was modified for the spatial domain and used in designing an identification experiment. The results from the industrial paper machine simulator and industrial identification experiments show the improvements in the CD models provided by identification in the spatial domain as well as the superiority of the designed inputs over the standard bump test.

In chapter 4, methods for closed-loop identification of CD mapping models and CD response models are developed. For systems with relatively short time constants, it is shown that the process's inherent transport delay provides uncorrelated input-output data which can be used in identification of the spatial models as if the system is running in open-loop. Performing the identification in the spatial domain provides enough data to identify accurate models from a few scans even in low signal-to-noise ratios. As in chapter 3, CD mapping models are detected through identifying noncausal FIR models. Open-loop input design techniques are applied in identification of the response models from uncorrelated input-output data. CD response shape identification from steady-state data is proposed for systems with large time constants.

A novel robust stability test for CD processes is developed through adopting the 2D transfer function and using results from multidimensional digital stability theory. The test is based on the $\nu$-gap stability criterion which provides a sufficient condition for stability. Moreover, the $\nu$-gap stability criterion provides bounds for robust stability when the controller is replaced by another one in the loop. This feature permits designing a two-dimensional controller in an adaptive control scheme or a simple online re-tuning of an existing controller through iterative feedback tuning. This property is convenient when switching between different grades of paper. This test can be investigated in either the temporal or the spatial domain. The spatial domain analysis is more useful when focusing on uncertainties in the CD model or for the sake of robust performance at temporal steady-
Another robust stability criterion to determine the maximum allowable misalignment uncertainty in CD processes is proposed. Stability is investigated by extending the concept of a phase margin to the two-dimensional case. Both criteria replace the large multi-variable robust stability problem by a set of simple one dimensional problems making it computationally very efficient.

A new CD control design method through spatial loop shaping is presented in chapter 6. A noncausal transfer function is used to model the spatial response allowing loop-shaping the spatial frequencies with the conventional techniques developed for the temporal frequencies. The design is performed on the spatial model at temporal steady state. This approach reduces the challenging problem of designing a large dimensional multi-variable controller to a simple design one which can be done in few seconds and results in an improved controller performance.

An adaptive control scheme is developed using the CD loop-shaping technique. The spatial response is identified in closed-loop and is followed by redesigning the controller. The proposed adaptive scheme improved controller performance significantly when a grade change was introduced in closed-loop.

### 7.2 Future Work

Input design techniques were applied in the identification of the spatial non-causal FIR models. Control relevant identification for the ARX structure should be explored as this model is more convenient for control design and robust stability analysis.

The multi-rate equivalent spatial transfer functions can be explored to improve controlling the high resolution measurement profile with the low resolution actuator profile.

In this work, spatial loop shaping was performed using the pole placement technique in an adaptive control scheme. Another possible approach is re-tuning the controller online using iterative feedback tuning. The wind-surfing approach to iterative control design can be readily applied to the spatial domain. Following an identification experiment that minimizes the uncertainty in a specific frequency range, the controller should be re-tuned to be more aggressive in that frequency range.

Loop-shaping the CD controller is performed at steady-state then the
two-dimensional frequency response is investigated. It is possible to perform the loop-shaping technique on the whole two-dimensional frequency grid given that a few parameters are sufficient to model the process in the 2D representation. The robust control design technique can be extended to multi-array systems.

An autonomous CD control system that uses the 2D model can be implemented upon integrating the identification and robust control techniques developed in this work.
Bibliography


150


