Incentives for Retailers Competing on Price and Inventory

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Abstract

This dissertation studies three topics in supply chain management. Consider a decentralized supply chain, where a manufacturer distributes products through competing retailers. Incentive conflicts among players often occur in such supply chains since the players have different objectives. This research seeks to help the manufacturer understand downstream retailers' incentives and provide managerial guidelines such as coordinating mechanisms and optimal strategies under existing contractual agreements.

The first essay considers a manufacturer who distributes a product line (consisting of different product variants) through competing retailers. Due to the substitution between different product variants, as well as between different retailers, the incentive problems associated with distributing a product line are more complicated than that of distributing a single product. We characterize retailers' incentive distortions, and construct contracts that achieve channel coordination. Using numerical simulations, we study how the retailers' incentives and contracts are affected by underlying model parameters.

The second essay investigates firms' incentives for transshipment in a decentralized supply chain. Transshipment price and the control of transshipment parameters are key factors that affect the manufacturer's and retailers' incentives for transshipment. We identify conditions under which the manufacturer and retailers are better off and worse off under transshipment. We also compare the decentralized retailer supply chain with one where the retailers are under joint ownership (a "chain store"). We obtain two surprising results. First, the manufacturer may prefer dealing with the chain store rather than with decentralized retailers. Second, chain store retailers may earn lower profits than decentralized retailers.

The third essay examines the impact of a gray market on the firms in a decentralized supply chain. Under certain conditions, a gray market's positive effect, i.e., the demand generating effect, dominates the negative effect, i.e., the demand loss in authorized channels, and increases the manufacturer's profit. However, the manufacturer also can be hurt by a gray market. In some cases, the manufacturer can use the linear wholesale price to deter retailers from transshipping to a gray market. However, the deterrence may not always be successful, and the manufacturer needs to employ other approaches such as penalty terms to terminate a gray market.

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Co-authorship Statement

The three papers (Chapters 2, 3 and 4) in this dissertation were co-authored with my research supervisors Harish Krishnan and S. Thomas McCormick. The first paper (Chapter 2) has been submitted to POM (Production and Operations Management); and the second paper (Chapter 3) has been submitted to MSOM (Manufacturing and Service Operations Management). I was the first author of these papers and my contribution to the papers is as follows:

For each paper, I came up with initial ideas and identified the research questions in the discussions with my supervisors. I performed most of the analysis, derived most of the results and wrote the manuscript. My supervisors assisted me with some difficult parts in the analysis, and edited the manuscript into an appropriate style for submission.

Chapter 1

INTRODUCTION

In a decentralized supply chain, where firms have different objectives, incentive conflicts often occur, which can lead to inefficiency (Cachon, 2003). This dissertation considers a manufacturing firm which has an independent distribution network consisting of competing retailers. This dissertation has three separate but related essays which contribute to the operations literature in analyzing several important issues in supply chain management. The goal of this research is to help manufacturers understand the incentives of their downstream retailers, and devise coordinating mechanisms and other strategic responses when managing its distribution channel.

A retail company makes several decisions. The central interest of the traditional operations literature is the inventory decision of a retailer who faces demand uncertainty. (Of course, there are other operational decisions such as capacity and location choices of interests in operations literature.) On the other hand, marketing research is concerned with a different set of decisions: pricing, service, advertisement and promotion, etc. Yet researchers in both areas have recognized that both streams of decisions of a firm are inseparable. (For instance, the inventory level of a good may affect consumer choice and demand, and consequently impact the pricing strategy.) In particular, when competing with a rival retail company, the retailer's marketing and operations decisions affect the demand at both firms, and hence it is crucial not to isolate one from another in the analysis. This research, focusing on a decentralized distribution network, builds on a framework where retailers' competition results from price-based and inventory-based customer substitution. That is, the retailers compete on both price and inventory.

A retailer's decisions may deviate from the ones that lead to the centrally optimal outcome. Much of the existing work on this subject is set in either a vertical supply chain with a single upstream firm (such as a manufacturer) and a single downstream firm (such as a retailer), or a horizontal structure with competition (such as the competition between retailers). However, the models we consider possess both the horizontal and vertical components. Two types of externalities arise in this setting and influence the retailers' incentive distortions: the vertical externality between the upstream and downstream firms (often referred to as double marginalization), and the horizontal externality between retailers due to competition. The manufacturer may employ contracts to align the retailers' incentives in order to enhance efficiency. In particular, a coordinating contract (or complete contract) is one that aligns the retailers' incentives to the centrally optimal ones. Yet in many manufacturer-retailer relationships, a coordinating contract is not feasible or too complicated to implement, and the manufacturer is concerned about choosing the optimal parameters under a given (feasible) contract.

The dissertation considers three scenarios in the following three chapters. In the first scenario (Chapter 2), the retailers carry a product line with multiple product variants. In the second scenario (Chapter 3), the retailers may engage in transshipment, an activity that mitigates inventory risk. In the third scenario (Chapter 4), the retailers may sell the product in a gray market, which can increase demand and reduce inventory risk but also cause a demand loss. In each scenario, the incentive distortions are investigated.

More importantly, this research seeks to find the optimal strategies for the manufacturer. Each essay proposes coordinating contracts, under which the first-best profit is achieved and allocated between the firms. The last two essays also consider a linear wholesale price contract, which cannot lead to the first-best profit but is widely used due to its simplicity. The manufacturer's optimal strategies regarding retailers' transshipment and gray market activities under this contract are examined.

Chapter 2, titled " Distributing a Product Line in a Decentralized Supply Chain" investigates the retailers' incentives in setting price and inventory levels when they distribute a product line of a manufacturer. The distribution of a product line, consisting of multiple product variants, is a common problem for nearly every manufacturer. However, few papers have studied the incentive issues in a product line distribution problem, in contrast to the vast literature which examines those issues in the context of a single product (see Cachon (2003) for a review of single-product distribution problems). Due to the substitution between different product variants, as well as between different retailers, the incentive problems associated with distributing a product line are more complicated than that of distributing a single product.

Starting with Hotelling (1929), the economics and marketing literatures extensively study the product line design problems, focusing on the positioning or differentiation of a product line in terms of the products' quality, variety, and price. (See Manez and Waterson (2001) for a survey of this literature.) Firms' inventory decisions have typically been omitted from this research. There is also a significant body of literature in operations that deals with the joint optimization of inventory and assortment decisions. (See Mahajan and van Ryzin (1998) for a review.) These models typically assume that retail prices are determined exogenously. Recent studies consider both the price and inventory decisions of a *monopolist* designing a product line (see, e.g., Netessine and Taylor (2007) and Carlton and Dana (2006)).

Our contribution to the product line literature is that we look at *distribution* and *coordination* issues in a decentralized supply chain with downstream competition. Two papers are most relevant to this chapter: Villas-Boas (1998) uses a deterministic-demand model and shows that when dealing with independent retailers, the manufacturer should increase the differentiation between the products, as compared to the centralized supply chain. But channel coordination cannot be achieved in this case. Marvel and Peck (2008) incorporate demand uncertainty and show that the decentralized retailers' prices and inventories can be coordinated, when the consumers' disutility from switching between products is large. Unlike Villas-Boas (1998), this chapter considers demand uncertainty, and so the firms make both price and inventory decisions. Marvel and Peck (2008) have a very simple model of demand uncertainty, and their assumption of perfect retail competition weakens the retailers' role. In our model, we consider uncertain price-sensitive demand with a general distribution. We also assume a retail duopoly, so the retailers have some market power and are not price takers.

Three models are built on very general assumptions. In the first model, consumer demands are random, but the retail prices are fixed. The demand of each product variant is random. The retailers choose inventory levels. If a product variant at a retailer is stocked out, the customer searches for either the same variant at another retailer or another variant at the same retailer, which is called "between-store spillover" or "inside-store spillover". (An alternative form of inventory-based substitution is transshipment, which is considered in Chapter 3.)

In the second model, demands are deterministic and the retailers make retail price decisions. The demand of a product variant is decreasing in the price of the product and increasing in the other variants. In the final model, the retailers make both price and inventory decisions. For each model, we characterize retailers' incentive distortions, and construct contracts that achieve channel coordination. We also use simulations to determine the optimal value of contract parameters. The results of this chapter offer a guideline for manufacturers designing contracts when distributing a product line through competing retailers.

Chapter 3, titled " Incentives for Transshipment in Decentralized Supply Chains with Competing Retailers", examines the impact of transshipment in a decentralized supply chain. Transshipment occurs when a retailer who is stocked out of a product obtains the product from another retailer. In a centralized system (where the retailers are owned by the manufacturer), due to the greater ability to match supply and demand, transshipment reduces inventory risk and increases system profit. However, in a decentralized system, the firms who maximize their own profits may have different incentives for transshipment.

The first contribution of this chapter is that we examine, for the first time, transshipment incentives in a decentralized supply chain where the retailers are independent from the manufacturer and also from each other, i.e., in a vertically *and* horizontally decentralized supply chain. (As we discuss in the literature review, the existing literature considers either a horizontally decentralized supply chain or a vertically decentralized supply chain, but not both.) We show that transshipment price and the control of transshipment decisions determine whether the firms benefit from, or are hurt, by transshipment.

We also look at the impact of retail centralization on the manufacturer and retailers. We compare the decentralized supply chain with competing retailers and a model where the retailers are centralized (but still independent from the manufacturer). We show that, counter-intuitively, the the manufacturer may not always prefer dealing with decentralized retailers; and retailers are not necessarily better off being centralized.

Chapter 4, titled " Transshipping to the Gray Market: the Impact on a Decentralized Supply Chain" examines the firms' incentives for a gray market. A gray market refers to the sale of a product by a distributor not authorized by the manufacturer. A gray market can be harmful to a manufacturer, since it erodes authorized dealers' sales, hurts consumers' goodwill, and damages brand images (Cespedes et al., 1988; Lowe and McCrohan, 1988). In practice, however, some manufacturers tolerate or even knowingly use gray markets. A recent stream of qualitative research claims that a gray market may be beneficial to manufacturers. For instance, a gray market may increase total sales through reaching untapped consumer segments, offer market intelligence for the firms to learn about markets, and reduce firms' inventory risk (Antia and Bergen, 2004).

In this chapter, we examine the impact of a gray market on the firms in a decentralized supply chain. We characterize conditions under which a gray market benefits and hurts the different players. This turns out to be a result of the tradeoff between a gray market's negative and positive impact. While a gray market causes a demand loss of the authorized channel, it also reaches a new consumer segment that has not been tapped by the authorized channel. (In an extension to stochastic demand, we also recognize a gray market's "risk reducing effect"; that is, it can be used as a channel to salvage the unsold inventory from the authorized channel.) We also recommend strategies for manufacturers to cope with a gray market under different conditions.

This dissertation sheds light on several important issues in supply chain management. Some actions in distribution channels such as transshipment and selling into the gray markets may benefit a centralized system. However, due to the conflicting interests of a decentralized supply chain, these actions may not be beneficial for everyone; or even worse, they may result in everyone being worse off. This dissertation gives manufacturing companies a better understanding of incentives in their distribution channels. For managers who deal with such issues, we provide guidelines such as proposed coordinating contracts that achieve the system optimal outcome as well as optimal strategies under the current contractual agreements.

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Chapter 2

DISTRIBUTING A PRODUCT LINE IN A DECENTRALIZED SUPPLY CHAIN[1]

2.1 Introduction

The distribution of a product line, consisting of multiple product variants, is a central problem that every manufacturer faces. For example, the computer manufacturer Lenovo periodically releases a new line of personal computers. Before launching the product line, Lenovo needs to design contracts with its retailers. As the leader of the supply chain, when designing the contracts, Lenovo is concerned not only with its own profit, but also with the best method for coordinating retailers' incentives in order to achieve maximum efficiency of the supply chain.

It is well known that incentive conflicts arise in decentralized channels, which can lead to inefficiency. Several papers have investigated these incentive conflicts and their resolutions (see Cachon (2003) for a review). Nevertheless, few papers have studied the incentive issues in the context of a *product line* distribution problem. However, most products sold in the market are sold to retailers in assortments of different qualities ("vertical" differentiation) or in variants that differ in features such as size or colors

[1]A version of this chapter has been submitted for publication. Shao, J., Krishnan, H. and McCormick, S. T. Distributing a Product Line in a Decentralized Supply Chain.

("horizontal" differentiation). Due to the substitution between different product variants, as well as different retailers, the incentive issues involved in distributing a product line are more complicated than those for distributing a single product. Therefore, simply replicating the contract for a single product is, in general, not sufficient for channel coordination.

Consider the following illustration of incentive conflicts arising during the distribution of a product line. Starter Sportswear was licensed to manufacturer sports jackets bearing the trademarks of the teams in the major sports leagues, such as the National Basketball Association (NBA) (see the details of Starter case in Marvel and Peck (2001)). Starter required each retailer to carry its full product line, including the jackets of local and non-local teams. Starter wanted the retailers to maintain sufficient inventories to provide all customers with a high service level. Many retailers, however, preferred a lower service level for non-local team jackets than Starter did. Hence, Starter imposed minimum order requirements on retailers, which were tailored to the retailers' markets. This arrangement worked well until a large retailer, Trans Sport, started ordering products from Starter in large quantities, and then reselling to the other retailers in any amount the retailers requested. Despite the $7 per jacket service fee Trans Sport charged, retailers switched to ordering from Trans Sport instead of Starter, in order to avoid Starter's minimum order quantities. (Soon after, Starter refused to deal with Trans Sport, and Trans Sport sued Starter for intentionally eliminating it as a competitor. The case was ultimately decided in Starter's favor.)

As illustrated by the Starter case, retailers carrying a product line may make inventory and price decisions that are not optimal for the entire system. Our first objective in this chapter is to understand the retailers' incentive distortions in the decentralized supply chain. Our second objective in this chapter is to design contracts for the manufacturer that will fix retailers' incentive distortions and achieve channel coordination. We adopt the framework of Krishnan and Winter (2007) but extend their *single* product to incorporate the incentive issues inherent in the distribution of a *product line*.

Starting with Hotelling (1929), product line design problems have been extensively studied. The economics and marketing literatures typically study issues such as the positioning or differentiation of a product line in terms of the products' quality, variety, and price. (See Manez and Waterson (2001) for a survey of this literature.) Firms' inventory decisions have typically been omitted from this research.

There is also a significant body of literature in operations that deals with the joint optimization of inventory and assortment decisions of monopoly firms (Smith and Agrawal, 1999; van Ryzin and Mahajan, 1999; Gaur and Honhon, 2006), and competitive firms (Singh et al., 2006). (See Mahajan and van Ryzin (1998) for a review.) These models typically assume that retail prices are determined exogenously.

Recent studies consider both the price and inventory decisions of a monopolist designing a product line. Netessine and Taylor (2007) consider a model where a firm determines the prices and qualities of the product variants, and they incorporate the firm's inventory decision within an EOQ model. They show that an increase in production cost may cause the firm to reduce the number of product variants in the product line. Carlton and Dana (2006) suggest that demand uncertainty may be another factor that affects a firm's optimal choice of product variety.

Our contribution to the product line literature is that we look at *distribution* and *coordination* issues in a decentralized supply chain with downstream competition. Two papers are most relevant to our work. Villas-Boas (1998) investigates a monopoly manufacturer who distributes two vertically differentiated products. The manufacturer decides the quality and wholesale prices of the products, while the independent retailers choose retail prices under deterministic demand. Villas-Boas (1998) shows that in order to maximize its profit the manufacturer should increase the differentiation between the products (as compared to the centralized supply chain). But channel coordination cannot be achieved in this case. Marvel and Peck (2008) consider a monopoly manufacturer who sells a horizontally differentiated product line through competitive retailers. They incorporate demand uncertainty and show that the decentralized retailers' prices and inventories can be coordinated, when the consumers' disutility from switching between products is large.

Unlike Villas-Boas (1998), this chapter considers demand uncertainty, and so the firms make both price and inventory decisions, (although we do not study product line *design* issues as Villas-Boas (1998) does). Marvel and Peck (2008) have a very simple model of demand uncertainty, and their assumption of perfect retail competition weakens the retailers' role. In our model, we consider uncertain price-sensitive demand with a general distribution. We also assume a retail duopoly, so the retailers have some market power and are not price takers.

The remainder of this chapter is organized as follows. In Section 2.2 we set up the modeling framework. We study three models in this framework, one where retailers compete on inventory (Section 2.3), another on price (Section 2.4), and the final on *both* price and inventory (Section 2.5). In each model, we investigate the retailers' incentives in price and/or inventory, identify contracts that can coordinate the channel, and using numerical simulations study how the retailers' incentives and contracts are affected by underlying model parameters. In Section 2.6, we discuss the results of the chapter, and conclude with some suggestions for future research.

2.2 Modeling Framework

A monopoly manufacturer sells a product line, consisting of two product variants, through two identical retailers. The two product variants, A and B, are horizontally differentiated, (i.e. they differ in terms of features). They could be computers with different styles, cereals with different flavors, clothes of different colors, etc. (We can also apply the modeling framework to vertically differentiated product lines and reach similar results. See the discussion in Section 2.6.) The marginal cost for producing the two products is the same, and is denoted by c. We assume that the products are perishable, such as fashion or seasonal goods, and consider only a single time period.

We consider a centralized system as a benchmark, where the two retailers are owned by the manufacturer. Therefore, the manufacturer makes the central decisions (i.e., the retail price p_{ki} and quantity y_{ki} of product k at retailer i, $k = A, B$, $i = 1, 2$). We let $\mathbf{p} \equiv (p_{ki})$, and $\mathbf{y} \equiv (y_{ki})$. Throughout this chapter, the first superscript k indicates product, and the second superscript i indicates retailer; and boldface letters are vectors. The manufacturer's objective in the centralized system is to maximize the total profit of the system.

Our main focus is on the decentralized system, where the two retailers are independent from the manufacturer and compete with each other. The game has two stages. In the first stage, the manufacturer designs a take-it-or-leave-it contract and offers it to the retailers, specifying the wholesale prices of the two products, w_A and w_B, and other contract parameters, such as fixed fees. (The use of fixed fees enables the contract to arbitrarily allocate rents. Therefore it is always the interest of the manufacturer to pursue channel coordination (Cachon, 2003).) In the second stage, the retailers decide whether to accept the contract. If they accept it, the retailers simultaneously decide the retail prices and order quantities for the products. Finally, demand occurs, and the retailers sell the products to consumers at the retail prices. All firms are risk-neutral.

Under this framework, we investigate retailers' incentives using three separate models. In the first model, we assume that the retail prices are exogenous (for instance, dictated by the manufacturer), and that the retailers only make inventory decisions. In the second model, we assume that demands are deterministic, so retailers only decide retail prices. Finally, in the third model, we examine retailers' incentives for both price and inventory.

2.3 Model 1: Retailers Compete Only on Inventory

We start our analysis by analyzing retailers' inventory incentives, assuming that the retail prices are fixed. The initial demand for product variant k

at retailer i (also referred to as "product" ki in the following) is defined as the number of customers who search for product ki as their most preferred product, and is denoted by ξ_{ki}. The ξ_{ki}'s are random variables with arbitrary distributions, and are not perfectly correlated. We let $\xi \equiv (\xi_{ki})$ be the vector of the demands.

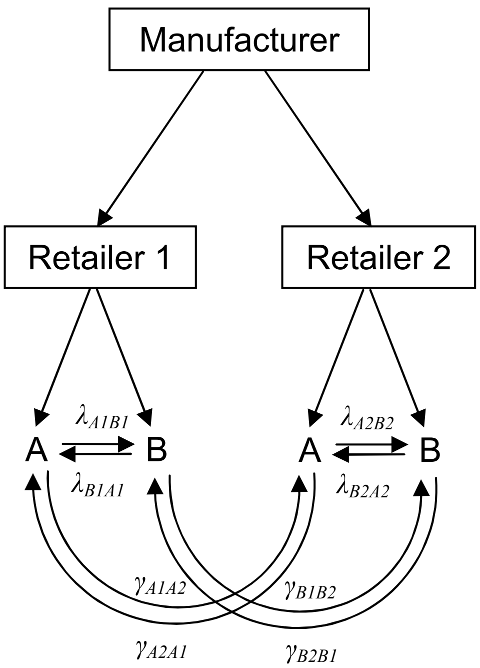

Figure 2.1: Product line and customer spillover

If the initial demand for a product exceeds the inventory level of the product, unsatisfied customers may search for a substitute. This process is called customer "spillover." There are two types of customer spillover: (1) *between*-store spillover, represented by γ, indicates those customers who go to the other retailer and in search of the same product, e.g., from $A1$ to $A2$; (2) *inside*-store spillover, represented by λ, indicates those customers who switch to the other product stocked by the same retailer, e.g., from $A1$ to $B1$. See Figure 2.1 for an illustration of both types of customer spillover.

We define the spillover *rate* as the proportion of unsatisfied customers who are willing to spill over; this can depend on the random demand realization ξ. In particular, $\gamma_{kikj}(\xi)$ is the between-store spillover rate of product k from store i to store j; and $\lambda_{kili}(\xi)$ is the inside-store spillover rate from product k to product l at retailer i.

Since each customer either spills over to the other product or the other store, $\lambda_{kili}(\xi)+\gamma_{kikj}(\xi) \leq 1$; and $1-\lambda_{kili}(\xi)-\gamma_{kikj}(\xi)$ of customers go home without searching. We assume that unsatisfied consumers do not switch both store and product. That is, customers who prefer $A1$ will not buy $B2$. The total demand for product k at retailer i, D_{ki} is equal to the sum of the initial demand plus the potential between-store and inside-store spillover demands, i.e., $D_{ki}(\mathbf{y};\xi) = \xi_{ki} + \gamma_{kjki}(\xi)(\xi_{kj} - y_{kj})^+ + \lambda_{liki}(\xi)(\xi_{li} - y_{li})^+$.

Let $S_{ki}(\mathbf{y};\xi)$ denote the actual sales of product k at retailer i, so that $S_{ki}(\mathbf{y};\xi) = \min(D_{ki}(\mathbf{y};\xi), y_{ki})$. The expected profits of retailer 1, $\pi_1(\mathbf{y};\xi)$, and the centralized firm, $\Pi(\mathbf{y};\xi)$, are given by

$$\pi_1(\mathbf{y};\xi) = p_{A1}ES_{A1}(\mathbf{y};\xi) + p_{B1}ES_{B1}(\mathbf{y};\xi) - w_A y_{A1} - w_B y_{B1} - F_1, \quad (2.1)$$

and

$$\Pi(\mathbf{y};\xi) = \sum_{k=A,B}\sum_{i=1,2} p_{ki}ES_{ki}(\mathbf{y};\xi) \; - \sum_{k=A,B}\sum_{i=1,2} c y_{ki}, \quad (2.2)$$

where F_1 is the fixed fee retailer 1 pays to the manufacturer for the right of carrying the products. (Hereafter, we only address the analysis for retailer 1, as retailer 2's behavior is symmetric.)

2.3.1 Comparing Decentralized and Centralized Inventories

In order to find how the retailers' inventory decisions deviate from the central optimum, we compare the first order conditions of the decentralized retailers' optimization problem with those of the centralized firm. (We assume that the second order conditions in the centralized and decentralized systems are satisfied. Relaxing this assumption will not affect the existence and direction of the distortions.)

We take the first derivatives of the centralized and decentralized profits, with respect to retailer 1's decision variables y_{A1} and y_{B1} (for convenience, we suppress the arguments in the profit and sales functions):

$$\frac{\partial \pi_1}{\partial y_{A1}} = \frac{\partial \Pi}{\partial y_{A1}} \quad \overbrace{-(w_A - c)}^{\text{vertical externality}} \quad \overbrace{-p_{A2}\frac{\partial ES_{A2}}{\partial y_{A1}}}^{\text{horizontal externality}} \tag{2.3}$$

$$\frac{\partial \pi_1}{\partial y_{B1}} = \frac{\partial \Pi}{\partial y_{B1}} \quad - (w_B - c) \quad - p_{B2}\frac{\partial ES_{B2}}{\partial y_{B1}}. \tag{2.4}$$

The terms labeled "vertical externality" and "horizontal externality" cause retailer 1 to choose inventory levels different than the centralized inventories. The first term captures the (vertical) externality imposed on the manufacturer: when retailer 1 increases the inventory y_{k1} from the centralized level, the manufacturer will collect its margin $w_k - c \geq 0$. The more inventory the retailer stocks, the more the manufacturer will benefit from it. Hence, the vertical externality gives retailer 1 the incentive to stock less than the centralized firm. The second term is the (horizontal) externality on profits at retailer 2. We notice that $\partial ES_{k2}/\partial y_{k1} = -\gamma_{k1k2}(\xi)Pr(\xi_{k1} > y_{k1}, D_{k2} < y_{k2}) \leq 0$. When retailer 1 increases y_{k1} from the centralized level, there will be fewer customers spilling over to retailer 2, and therefore, retailer 2 will make less sales and profits. So the horizontal externality gives retailer 1 the incentive to stock more inventory than the centralized firm. (Note that $\partial ES_{l2}/\partial y_{k1} = 0$; a change in inventory of product $k1$ does not affect the expected sales of product $l2$.)

The two externalities distort retailer 1's decisions in opposite directions. Whether the retailer will stock more or less than the centralized level depends on the contract the manufacturer chooses, and in particular, on the wholesale prices.

Note that when the manufacturer deals with a single retailer, due to the lack of horizontal competition, the manufacturer's markup makes the retailer order less than the centrally optimal quantity. Therefore, if the manufacturer sets the wholesale prices at its marginal cost c, it eliminates the vertical externality for the retailer, and fixes the retailer's inventory distortion. If the manufacturer can extract profits through a fixed fee, then a two-part pricing contract, referred to as the residual-claimancy contract in the following sections, easily coordinates the single retailer channel. (The residual-claimancy contract is so called because it is as if the manufacturer sells the whole project to the retailer upfront, and then collects rent through the fixed fee.)

However, due to the duopoly retail competition, under a residual-claimancy contract, the vertical externality is eliminated, but the horizontal externality still remains. Hence, the retailer has an incentive to stock more inventory than the centralized firm.

Proposition 2.1 *Under a residual-claimancy contract, where $w_A = w_B = c$, the decentralized retailer has incentive to increase the inventory of any product from the centralized level. (Proofs are deferred to the Appendix.)*

Note that the vertical externality only depends upon the manufacturer's markup, whereas the horizontal externality can be affected by multiple factors including, for instance, the spillover between retailers and the spillover within retailers. However, between the two types of spillover, the between-store spillover is the key factor that causes retailers' inventory distortions. If there is no between-store spillover ($\gamma_{kikj}(\xi) = 0$, $k = A, B$, $i, j = 1, 2$), the retailers are actually not competing with each other in terms of inventory, as the prices are fixed. When the manufacturer's markup is zero, as in the residual-claimancy contract, the decentralized retailers will just stock the same amount as the centralized firm.

The inside-store spillover is not the reason for the retailers' inventory distortions. In fact, the degree of spillover within retailer i has no effect on retailer i's inventory distortion. Note that the spillover within retailer i cancels out when we subtract the centralized and decentralized first order conditions. However, the inside-store spillover within retailer j does impact the inventory distortion of retailer i. For instance, the spillover from $B2$ to $A2$ is another source of $B2$'s demand, therefore, the spillover from $B2$ to $A2$ ($\gamma_{B2A2}(\xi)$) will affect the marginal effect of the spillover from $A1$ to $A2$. We summarize as follows:

Remark 2.1 *(1) If the between-store spillover for product k, $\gamma_{kikj}(\xi)$, is zero, then the decentralized retailers' inventory decision for product k is not distorted.*
(2) The inside-store spillover within retailer i does not distort retailer i's inventory decision; but the spillover within retailer j distorts retailer i's inventory decision.

2.3.2 Contracts that Fix Inventory Distortions

As noted above, the single-retailer channel is easily coordinated with the residual-claimancy contract, even when the retailer carries a product line. However, when retailers compete, the manufacturer must be aware of, and distinguish between, the impact of inside-store and between store spillover. If there is no customer spillover between retailers, setting the wholesale prices at the production cost will be sufficient for coordination. But when there is between-store spillover, the residual-claimancy contract fails to achieve coordination. Furthermore, when the products are not identical, in terms of their between-store spillover rates, etc., we cannot simply use a uniform contract to coordinate both products. We now consider several coordinating contracts.

Consider, first, a *quantity fixing* contract. Quantity fixing contracts impose restrictions on the retailers' order quantities. There are two types of contracts. *Quantity forcing* imposes a minimum quota on retailers' orders, while *quantity rationing* imposes a maximum quota on retailers' orders (Tirole, 1998). In our model, if the manufacturer sets wholesale prices equal to the production cost, the retailers tend to overstock. Hence, we need a quantity rationing contract, where the maximum quota is equal to the centralized optimal inventory, in order to keep the retailers from ordering too much.

A *two-part pricing* contract can also achieve coordination. It is possible to set wholesale prices such that the vertical and horizontal externalities cancel each other out. Specifically, we define $\mathbf{y}^* \equiv (y^*_{ki}), k = A, B, i = 1, 2$ as the optimal inventories in the centralized system, then from equations

(2.3) and (2.4), we can obtain the coordinating wholesale prices:

$$w_A = c - p_{A2} \frac{\partial ES_{A2}}{\partial y_{A1}}\bigg|_{\mathbf{y}^*} \tag{2.5}$$

$$w_B = c - p_{B2} \frac{\partial ES_{B2}}{\partial y_{B1}}\bigg|_{\mathbf{y}^*}. \tag{2.6}$$

Since retailer 2 will lose sales as retailer 1 increases inventory, the partial derivatives on the right hand side are negative. Therefore, these wholesale prices are greater than the manufacturer's marginal cost c. Furthermore, if the two products have different between-store spillover rates, the partial derivatives in (2.5) and (2.6) will not be equal, and the two products end up with different wholesale prices. We will discuss this more in the example in Section 2.3.3.

If the manufacturer prefers charging equal wholesale prices across the product line, it can consider using a *buyback* contract. In a buyback contract, the manufacturer offers the retailers a per unit buyback price for each product, so that the retailers can return the leftover inventory to the manufacturer at the end of the selling season, and collect the buyback prices (see, for example, Pasternack (1985)). Letting b_A and b_B be the buyback prices of products A and B, retailer 1's profit under the buyback contract becomes

$$\begin{aligned} E\pi_1^b(\mathbf{y};\xi) =& p_{A1}ES_{A1}(\mathbf{y};\xi) + p_{B1}ES_{B1}(\mathbf{y};\xi)) - w(y_{A1}+y_{B1}) \\ &+ b_A(y_{A1} - ES_{A1}(\mathbf{y};\xi)) + b_B(y_{B1} - ES_{B1}(\mathbf{y};\xi)) - F_1. \end{aligned} \tag{2.7}$$

Comparing the centralized and decentralized first order conditions under the buyback prices:

$$\begin{aligned} \frac{\partial \pi_1^b}{\partial y_{A1}} =& \frac{\partial \Pi}{\partial y_{A1}} \underbrace{-(w_A - c)}_{\text{vertical externality}} \overbrace{+b_A(1 - \frac{\partial ES_{A1}}{\partial y_{A1}})}^{\text{vertical externality}} \overbrace{-p_{A2}\frac{\partial ES_{A2}}{\partial y_{A1}}}^{\text{horizontal externality}} \\ =& 0, \end{aligned} \tag{2.8}$$

$$\begin{aligned} \frac{\partial \pi_1^b}{\partial y_{B1}} =& \frac{\partial \Pi}{\partial y_{B1}} - (w_B - c) + b_B(1 - \frac{\partial ES_{B1}}{\partial y_{B1}}) - p_{B2}\frac{\partial ES_{B2}}{\partial y_{B1}} \\ =& 0. \end{aligned} \tag{2.9}$$

Notice that in equations (2.8) and (2.9) the buyback prices generate one more vertical externality. This externality gives the retailer incentive to stock more inventory than the centralized firm does. Under the buyback policy, the decentralized retailer gets compensation from the manufacturer when he has ordered too much. However, this is only an internal transfer for the centralized firm.

With the buyback prices, it is possible for the two products to have equal wholesale prices, even when they have different spillover rates. We let $w_A = w_B = w$, and obtain the coordinating buyback prices:

$$b_A = \left.\frac{w - c + p_{A2}\partial ES_{A2}/\partial y_{A1}}{1 - \partial ES_{A1}/\partial y_{A1}}\right|_{\mathbf{y}^*} \tag{2.10}$$

$$b_B = \left.\frac{w - c + p_{B2}\partial ES_{B2}/\partial y_{B1}}{1 - \partial ES_{B1}/\partial y_{B1}}\right|_{\mathbf{y}^*}. \tag{2.11}$$

The buyback prices are functions of the wholesale price w. In determining the buyback prices and wholesale price, the following inequalities must be satisfied:

$$0 \le b_k \le w \le p \tag{2.12}$$

Therefore, in (2.10) and (2.11), the wholesale price must be greater than or equal to the production cost c in order for the buyback prices to be nonnegative. In particular, when the between-store spillover rates are positive, a horizontal externality exists, so the wholesale price must be strictly greater than the production cost. This means the manufacturer collects rents from the retailers not only through the fixed fee but also through the positive markup. For instance, if the manufacturer decides to extract all the channel profit, the retailers will be left with zero profit:

$$\begin{aligned} E\pi_1^b(\mathbf{y}^*, \xi) &= p_{A1}ES_{A1}(\mathbf{y}^*, \xi) + p_{B1}ES_{B1}(\mathbf{y}^*, \xi) - w(y_{A1}^* + y_{B1}^*) \\ &+ b_A(y_{A1}^* - ES_{A1}(\mathbf{y}^*, \xi)) + b_B(y_{B1}^* - ES_{B1}(\mathbf{y}^*, \xi)) - F_1 = 0. \end{aligned} \tag{2.13}$$

Nevertheless, the manufacturer has the freedom to choose any combination of wholesale price and fixed fee, as long as equation (2.13) is satisfied. Once the wholesale price and fixed fee are chosen, buyback prices can be determined through equations (2.10) and (2.11).

We summarize the above contracts in the following proposition.

Proposition 2.2 *The following contracts can coordinate the retailers' inventories for the product line:*
(1) Identical linear wholesale prices ($w_A = w_B = c$) with quantity rationing, and a fixed fee;
(2) Differentiated linear wholesale prices, where the wholesale prices are given by (2.5) and (2.6), and a fixed fee;
(3) Identical linear wholesale prices, differentiated buyback prices, given by (2.10) and (2.11), and a fixed fee.

2.3.3 Spatial Model Example

In this section, we provide more insights on retailers' inventory incentives and contracts. We consider a special case of the general model: a spatial model of consumer choice.

Assume that the four products, $A1$, $B1$, $A2$, and $B2$ are located at the four vertices of a unit square. Customers are uniformly distributed along each of the four edges (see Figure 2.2). The location of a customer determines the customer's preference for each of the products.

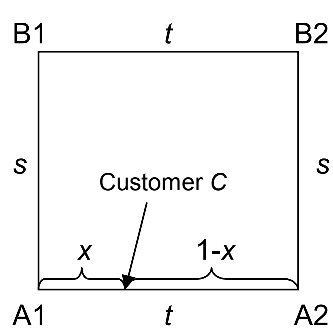

Figure 2.2: Spatial model

Each customer obtains a gross utility u from buying a product (A or B), and incurs a negative utility proportional to the "distance" from the product. On the edges $A1A2$ and $B1B2$, customers incur a "travel" cost; and on the edges $A1B1$ and $A2B2$ they incur a "switching" cost. The customer's net utility is the gross utility minus the price of the product and the necessary travel and switching costs. For example, customer C in Figure 2.2 will obtain utility $u - p_{A1} - tx$ from buying product $A1$, and utility $u - p_{B1} - t(1-x)$ from buying $A2$, where p_{A1} and p_{B1} are the prices and tx and $t(1-x)$ are the travel costs incurred. Note that t is the cost per unit distance travelled. Similarly, the location of customers on the edges $A1B1$ and $A2B2$ represents their willingness to switch between products. The term s represents the switching cost per unit distance "travelled" along these edges.

To incorporate demand uncertainty in this model, let θ_A, θ_B, θ_1, and θ_2 be random variables representing the random number of customers at each point on the edges $A1A2$, $B1B2$, $A1B1$, and $A2B2$.

Initial Demands and Spillover Demands

For simplicity, let the prices of the four products be equal, and denote it by p. The initial demand for each of the four products is given by the set of customers who obtain their highest (positive) utility from purchasing that product.

For any product, the initial demand can be determined by computing the utilities of the customers on the two edges adjacent to the product; this depends on the values of t and s. Consider the edge $A1A2$. For high values of t, the set of customers on this edge who obtain positive utility from both products $A1$ and $A2$ will be zero. For low values of t, this set will be non-empty.

In this case, customers will first attempt to obtain the product that gives them the higher utility. If their preferred product is stocked out, they spill over to the other product. If t (or s) is very small, it is possible that all customers on an edge obtain positive utility from both customers on the edge. If t (or s) is sufficiently high, it is possible that the set of customers who obtain positive utility from both products is empty. In general, the spillover rate is calculated as a proportion of the total initial demand for a product that will "spillover" to another product.

The travel cost t is an indicator of the degree of competition between the retailers. When t is low, all customers on an edge are willing to spill over and the between store spillover rate is at its upper bound. When t is high, there is no competition between retailers. When t takes intermediate values, the competition becomes weaker as t increases.

Incentive Distortions

To illustrate the incentive distortions, we first assume that the manufacturer uses the residual-claimancy contract, i.e., $w_A = w_B = c$. We set $u = 4, c = 1, w_A = w_B = 1$, and find the centrally optimal inventory and the decentralized equilibrium inventory for different values of t and s; assuming that the θ's are i.i.d. and uniformly distributed between 0 and 1. (Our results also hold for other common distributions, including normal and exponential distributions.) For the centralized firm, the four products are all symmetric, so when customers' switching cost or travel cost varies, it will have the same effect on the centralized inventory. Hence, we fix s at 0.5, and show how the centralized inventory varies in t, at various levels of retail price p.

When t is low, both the initial demand and spillover demand on the horizontal edges are at their upper bounds, and independent of t. Therefore, the inventory level is constant as t increases in this range. When t is high, there is no spillover demand, and the initial demand decreases in t. When t takes intermediate values, the spillover rate is (stochastically) decreasing in t. When t is in this range, the centralized inventory decreases if p is small, increases if p is large, and first decreasing then increasing if p takes intermediate values. This is demonstrated in Figure 2.3.

Compared with the no spillover case, the customer spillover between retailers has two opposite effects on the centralized firm's inventory decisions:

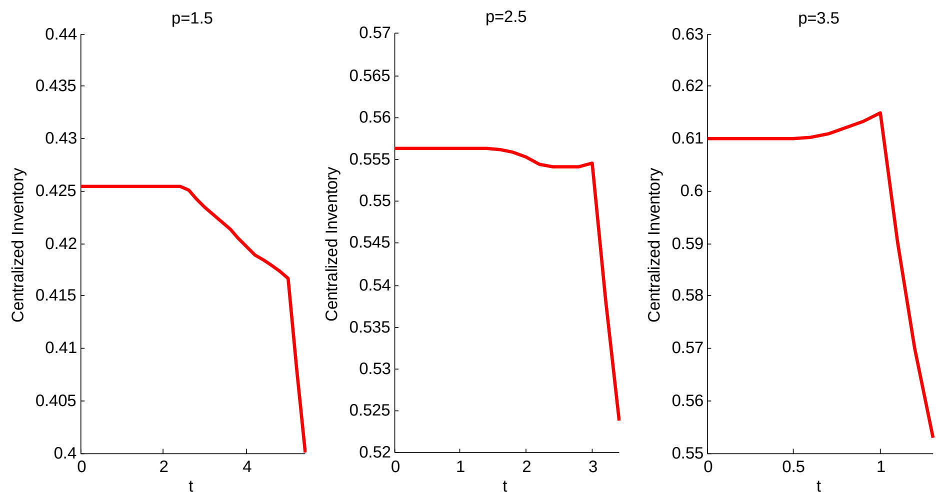

Figure 2.3: Centralized inventory $(u = 4, c = 1)$

(1) a "demand effect;" and (2) a "pooling effect." The demand effect is that each retailer faces more demand when customers spill over; this induces the centralized firm needs to raise inventory to satisfy the increased demands. The pooling effect is that, with customer spillover, there is a second chance for the centralized firm to satisfy demand with the stock at the other location; this induces the centralized firm to lower inventories.

For low values of t, the between-store spillover rates are at their upper bound. As t increases, the between-store spillover rates become stochastically smaller, and both the demand and pooling effects become weaker. The centralized firm tries to balance these two effects under different levels of the critical fractile, $(p - c)/p$. When the critical fractile is small, the safety stock is low and the centralized firm benefits more from the demand effect than the pooling effect. Therefore the centralized inventory decreases as t increases. When the critical fractile is high, the pooling effect dominates the demand effect; the centralized inventory can increase as t increases.

In the decentralized system, the pooling effect does not exist. Therefore, the decentralized inventory always decreases in t (see Figure 2.4).

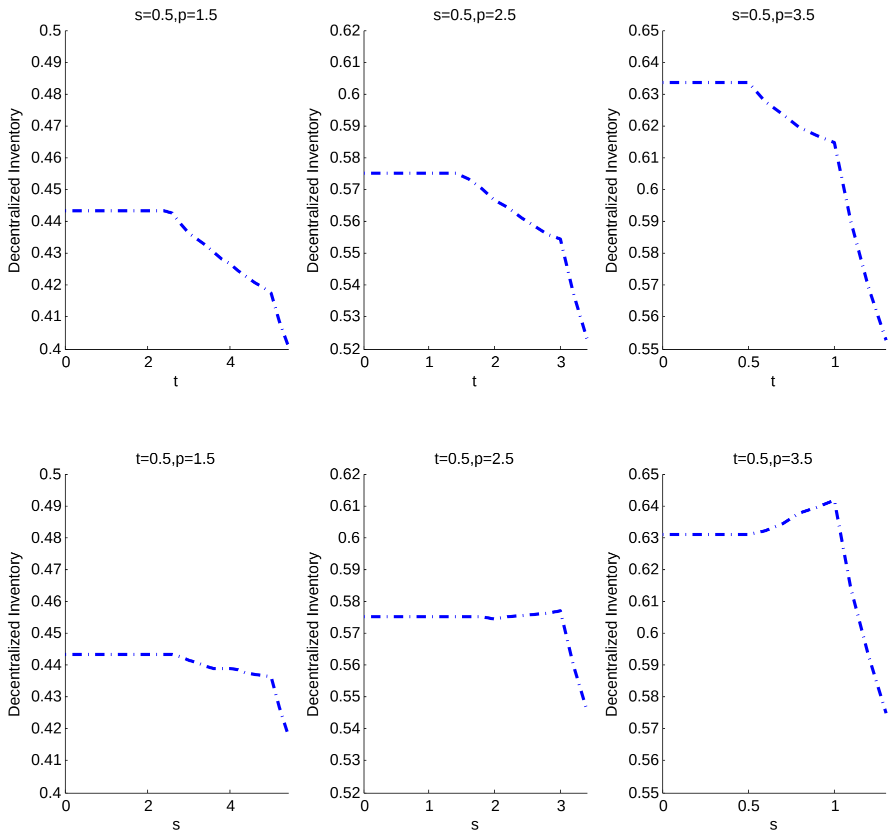

Figure 2.4: Decentralized inventory ($u = 4, w_A = w_B = 1$)

Observation 2.1 *(1) The centralized inventory may decrease, increase, or first decrease then increase in the travel cost t; (2) The decentralized inventory always decreases in the travel cost t.*

As the travel cost t becomes large, the decentralized inventory approaches the centralized inventory; the spillover rates go to zero. See Figure 2.5 for the difference between the decentralized and centralized inventories.

Observation 2.2 *When the retail price is fixed, the distortion between the decentralized and centralized inventories decreases as the travel cost increases.*

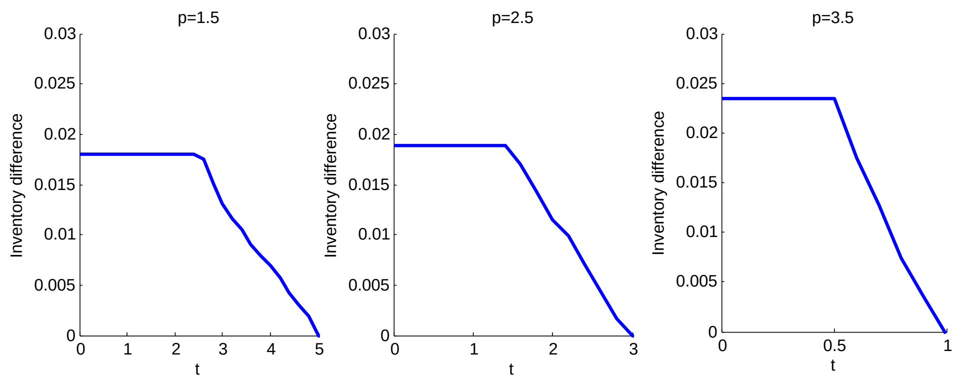

Figure 2.5: Difference between centralized and decentralized inventories ($u = 4, c = 1, w_A = w_B = 1$)

We also conducted experiments for the case where the customers have different travel costs for the two products. Without loss of generality, suppose product A has a lower travel cost than B, i.e., $t_A < t_B$. This measure indicates that product A has more loyal customers than does B, as the customers are more willing to travel in search of product A. Therefore, the retailers tend to compete more fiercely on product A than B. In terms of inventory decisions, the decentralized retailers will overstock product A more than product B, i.e. the distortion in product A's inventory decision is greater.

Coordinating Contracts

We next illustrate how the manufacturer should set contract parameters in order to fix the inventory distortions. Recall, from Proposition 2.2, that a two-part pricing contract can coordinate the channel. Clearly, the optimal value of the wholesale price will depend on t. For low values of t, the wholesale price is independent of t because the spillover rate is at its upper bound. As t increases, the inventory distortion decreases and w decreases till it reaches the marginal cost. See Figure 2.6.

Observation 2.3 *In the two-part pricing contract in Proposition 2.2, the wholesale price is non-increasing as the travel cost increases.*

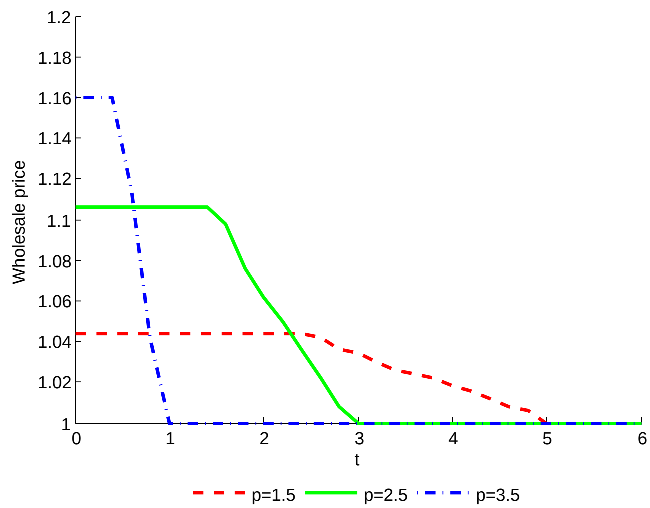

Figure 2.6: Inventory contract - two-part pricing ($u = 4, c = 1$)

Now consider the buyback contract from Proposition 2.2. In this case, both products have the same wholesale price but different buyback prices. We set the wholesale price equal to the coordinating wholesale price at $t = 0$ (see Figure 2.6). We then solve for the buyback prices that will induce the retailers to order the coordinating inventory level. When t is small, the wholesale price alone is able to achieve coordination, and the buyback price is equal to zero. As t increases, a positive buyback price is needed to encourage the retailers to order the right inventory level. The buyback price increases as t increases. For sufficiently high t, the inventory distortion disappears, and the buyback price remains constant for further increases in t. See Figure 2.7.

Observation 2.4 *In the buyback contract in Proposition 2.2, the buyback price is nondecreasing as the travel cost increases.*

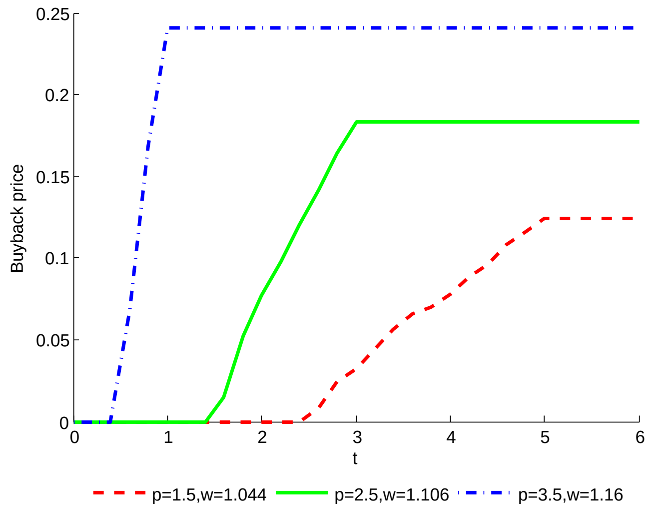

Figure 2.7: Inventory contract - buyback ($u = 4, c = 1$)

2.4 Model 2: Retailers Compete Only on Price

We now consider incentive distortions when demands are deterministic and retailers compete only on price. The demand for product k at retailer i, D_{ki}, is a continuous and differentiable function of the retail prices. The function, D_{ki} is decreasing in the price of product ki, and is increasing in the prices of the other products, i.e.,

$$\frac{\partial D_{ki}}{\partial p_{ki}} < 0, \quad \frac{\partial D_{kj}}{\partial p_{ki}} > 0, \quad \frac{\partial D_{li}}{\partial p_{ki}} > 0, \quad \frac{\partial D_{lj}}{\partial p_{ki}} > 0. \tag{2.14}$$

We also assume that

$$\left|\frac{\partial D_{ki}}{\partial p_{ki}}\right| > \frac{\partial D_{li}}{\partial p_{ki}} + \frac{\partial D_{kj}}{\partial p_{ki}} + \frac{\partial D_{lj}}{\partial p_{ki}} \quad k, l = A, B, i, j = 1, 2. \tag{2.15}$$

This implies that the impact of a change in the price of a product on its own demand is always greater than the overall impact on its substitutes. For instance, when p_{ki} increases, the decrease in D_{ki} will exceed the total increase in li, kj and ki, i.e., the demand in the whole system is decreased as the price of one product increases.

Retailer 1's profit in the decentralized system $\pi_1(\mathbf{p})$, and the centralized profit $\Pi(\mathbf{p})$, are as follows:

$$\pi_1(\mathbf{p}) = (p_{A1} - w_A)D_{A1}(\mathbf{p}) + (p_{B1} - w_B)D_{B1}(\mathbf{p}) - F_1 \tag{2.16}$$

$$\begin{aligned}\Pi(\mathbf{p}) = &(p_{A1} - c)D_{A1}(\mathbf{p}) + (p_{B1} - c)D_{B1}(\mathbf{p}) + (p_{A2} - c)D_{A2}(\mathbf{p}) \\ &+ (p_{B2} - c)D_{B2}(\mathbf{p}).\end{aligned} \tag{2.17}$$

2.4.1 Comparing Decentralized and Centralized Decisions on Price

Comparing the centralized and decentralized first order conditions in terms of retailer 1's two price variables, we get:

$$\begin{aligned}\frac{\partial \pi_1}{\partial p_{A1}} = \frac{\partial \Pi}{\partial p_{A1}} &- \overbrace{(w_A - c)\frac{\partial D_{A1}}{\partial p_{A1}}}^{\text{vertical externality}} - \overbrace{(w_B - c)\frac{\partial D_{B1}}{\partial p_{A1}}}^{\text{vertical externality}} - \overbrace{(p_{A2} - c)\frac{\partial D_{A2}}{\partial p_{A1}}}^{\text{horizontal externality}} \\ &- \overbrace{(p_{B2} - c)\frac{\partial D_{B2}}{\partial p_{A1}}}^{\text{horizontal externality}}\end{aligned} \tag{2.18}$$

$$\begin{aligned}\frac{\partial \pi_1}{\partial p_{B1}} = \frac{\partial \Pi}{\partial p_{B1}} &- \overbrace{(w_B - c)\frac{\partial D_{B1}}{\partial p_{B1}}}^{\text{vertical externality}} - \overbrace{(w_A - c)\frac{\partial D_{A1}}{\partial p_{B1}}}^{\text{vertical externality}} - \overbrace{(p_{B2} - c)\frac{\partial D_{B2}}{\partial p_{B1}}}^{\text{horizontal externality}} \\ &- \overbrace{(p_{A2} - c)\frac{\partial D_{A2}}{\partial p_{B1}}}^{\text{horizontal externality}}\end{aligned} \tag{2.19}$$

Unlike the inventory only model, here we have two vertical externalities and two horizontal externalities in each equation. When retailer 1 increases the price of any product, the manufacturer will lose its margin on the decreased demand of this product (the first vertical externality), but will collect the margin on the additional demand of the other product at the same retailer (the second vertical externality). (If the retailer carries a single product, only the first vertical externality exists, and it distorts retailer 1's price upwards. This is the traditional "double marginalization" or "double markup" effect known in the economics literature (e.g., Spengler (1950)).)

Similarly, when retailer 1 increases the price of one of his products, both of the products sold by retailer 2 will increase in demand (the two horizontal externalities). As an analogy to the spillover in the inventory model, the horizontal externalities are caused by the (price-based) substitution relationship *between* retailers.

To determine the net distortion in price, consider again the residual-claimancy contract, where $w_A = w_B = c$. In the single-retailer system, the residual-claimancy contract is able to align the retailer's price decision, for the same reason as in the inventory model: there is no horizontal competition, and it will be sufficient to coordinate the channel once the vertical externalities are eliminated. However, in the duopoly-retailer channel, the residual-claimancy contract cannot resolve the price distortions, because of the horizontal externalities.

2.4.2 Contracts that Fix Price Distortions

Under the residual-claimancy contract, the retailers tend to reduce prices from the centrally optimal levels. A simple way to fix that is to impose retail price floors, i.e., fixed lower bounds on the retail prices.

However, price floors have been subject to extensive anti-trust scrutiny and, at different point in time, have been illegal in many countries (Krishnan and Winter, 2007). An alternative approach is to choose the proper wholesale prices such that the externalities in (2.18) and (2.19) simply cancel out. Define $\mathbf{p}^* \equiv (p_{ki}), k = A, B, i = 1, 2$ as the optimal prices in the centralized system, then the coordinating wholesale prices for products A and B are as follows:

$$
\begin{aligned}
w_A = c + & \left. \frac{\frac{\partial D_{B1}}{\partial p_{A1}} [(p_{B2} - c)\frac{\partial D_{B2}}{\partial p_{B1}} + (p_{A2} - c)\frac{\partial D_{A2}}{\partial p_{B1}}]}{\frac{\partial D_{A1}}{\partial p_{A1}} \frac{\partial D_{B1}}{\partial p_{B1}} - \frac{\partial D_{B1}}{\partial p_{A1}} \frac{\partial D_{A1}}{\partial p_{B1}}} \right|_{\mathbf{p}^*} \\
- & \left. \frac{\frac{\partial D_{B1}}{\partial p_{B1}} [(p_{A2} - c)\frac{\partial D_{A2}}{\partial p_{A1}} + (p_{B2} - c)\frac{\partial D_{B2}}{\partial p_{A1}}]}{\frac{\partial D_{A1}}{\partial p_{A1}} \frac{\partial D_{B1}}{\partial p_{B1}} - \frac{\partial D_{B1}}{\partial p_{A1}} \frac{\partial D_{A1}}{\partial p_{B1}}} \right|_{\mathbf{p}^*}, \qquad (2.20) \\
w_B = c + & \left. \frac{\frac{\partial D_{A1}}{\partial p_{B1}} [(p_{A2} - c)\frac{\partial D_{A2}}{\partial p_{A1}} + (p_{B2} - c)\frac{\partial D_{B2}}{\partial p_{A1}}]}{\frac{\partial D_{A1}}{\partial p_{A1}} \frac{\partial D_{B1}}{\partial p_{B1}} - \frac{\partial D_{B1}}{\partial p_{A1}} \frac{\partial D_{A1}}{\partial p_{B1}}} \right|_{\mathbf{p}^*} \\
- & \left. \frac{\frac{\partial D_{A1}}{\partial p_{A1}} [(p_{B2} - c)\frac{\partial D_{B2}}{\partial p_{B1}} + (p_{A2} - c)\frac{\partial D_{A2}}{\partial p_{B1}}]}{\frac{\partial D_{A1}}{\partial p_{A1}} \frac{\partial D_{B1}}{\partial p_{B1}} - \frac{\partial D_{B1}}{\partial p_{A1}} \frac{\partial D_{A1}}{\partial p_{B1}}} \right|_{\mathbf{p}^*}. \qquad (2.21)
\end{aligned}
$$

By assumptions (2.14) and (2.15), the numerators of the fractions on the right hand sides are positive. Since the demand functions of the two products A and B are generally not symmetric, the wholesale prices for the two products will be different.

Proposition 2.3 *The following contracts coordinate the retailers' prices for the product line:*
(1) Identical linear wholesale prices ($w_A = w_B = c$), differentiated price floors, and a fixed fee;
(2) Differentiated linear wholesale prices, given by (2.20) and (2.21), and a fixed fee.

2.4.3 Spatial Model with Retailers' Price Decisions

To illustrate the retailers' price distortions and corresponding contracts, we consider the following model: in the spatial model of Section 2.3.3 let $\theta_A = \theta_B = \theta_1 = \theta_2 = 1$; i.e., demand is deterministic. Retailers choose prices, and not inventories. Note that there will be no spillover. Figure 2.8 plots the centralized and decentralized prices as t varies (assuming fixed s).

Price competition induces decentralized retailers to always choose a lower price than the centralized firm. As t increases, the customers are less willing

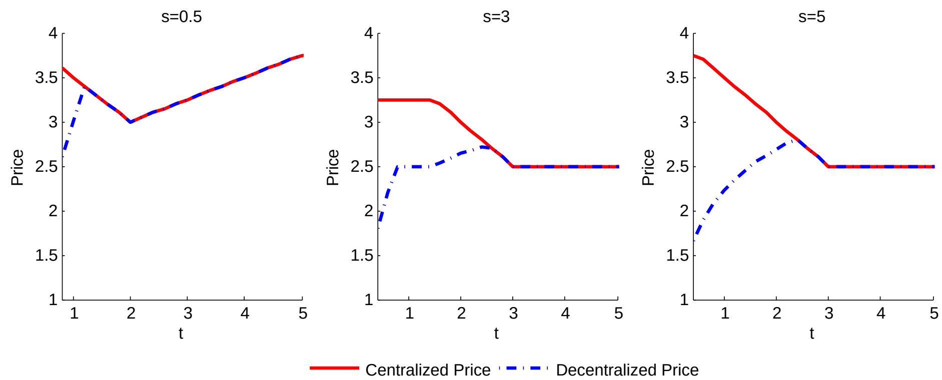

Figure 2.8: Centralized and decentralized prices ($u = 4, c = 1, w_A = w_B = 1$)

to travel between the retailers, which decreases price competition. Therefore, as travel cost t increases, the decentralized price approaches the centralized level. Although the centralized and decentralized prices are not monotonic in t, the distortion between them is decreasing in t. See Figure 2.9.

Observation 2.5 *The price distortion is decreasing as the travel cost increases.*

To fix the distortion, we consider the two-part pricing contract in Proposition 2.3. For each value of t, there exists at least one (and sometimes more than one) wholesale price that is able to coordinate the decentralized price. (Figure 2.10 shows the range of coordinating wholesale prices – the dark line shows the lowest coordinating value of wholesale price for each t.) When t is small, the wholesale prices are greater than the production cost. The purpose is to force the retailers to raise prices up to the centralized levels. At large t, where there is no price distortion, the residual-claimancy contract (the wholesale prices are equal to the production cost) coordinates the supply chain.

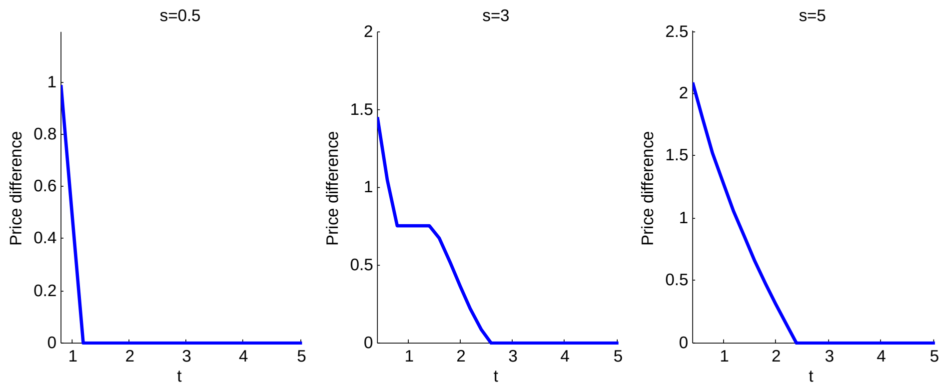

Figure 2.9: Difference between centralized and decentralized prices ($u = 4, c = 1, w_A = w_B = 1$)

Observation 2.6 *In the two-part pricing contract, the manufacturer should decrease the wholesale price as the travel cost increases.*

2.5 Model 3: Retailers Compete on Price and Inventory

Thus far we have analyzed the retailers' price and inventory incentives separately. Now we will consider the retailers' incentives when they make both price and inventory decisions. We now assume that the initial demand for product k at retailer i, $\xi_{ki}(\mathbf{p};\theta_{ki})$ depends upon the prices of the four products and a random variable θ_{ki}. Let $\boldsymbol{\theta} \equiv (\theta_{ki})$. We assume that θ_{ki}'s are not perfectly correlated. The spillover rates are not only sensitive to the retail prices, but also sensitive to the random variables. The overall demand for product k at retailer i is $D_{ki}(\mathbf{p},\mathbf{y};\boldsymbol{\theta}) = \xi_{ki}(\mathbf{p};\boldsymbol{\theta}) + \gamma_{kjki}(\mathbf{p};\boldsymbol{\theta})(\xi_{kj}(\mathbf{p};\boldsymbol{\theta}) - y_{kj})^+ + \lambda_{liki}(\mathbf{p};\boldsymbol{\theta})(\xi_{li}(\mathbf{p};\boldsymbol{\theta}) - y_{li})^+, k,l = A,B, i,j = 1,2$. The sales of product k at retailer i is $S_{ki}(\mathbf{p},\mathbf{y};\boldsymbol{\theta}) = \min\{D_{ki}(\mathbf{p},\mathbf{y};\boldsymbol{\theta}), y_{ki}\}$, and the expected

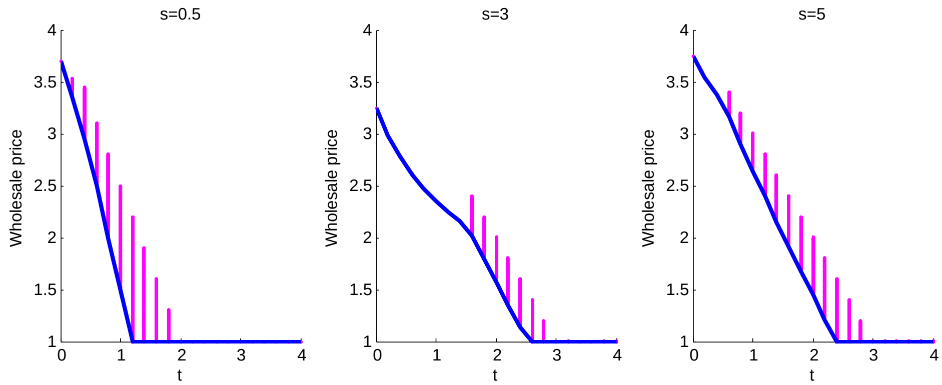

Figure 2.10: Price contract - two-part pricing ($u = 4, c = 1$)

profit functions are given by

$$\pi_1(\mathbf{p}, \mathbf{y}; \boldsymbol{\theta}) = p_{A1} ES_{A1}(\mathbf{p}, \mathbf{y}; \boldsymbol{\theta}) + p_{B1} ES_{B1}(\mathbf{p}, \mathbf{y}; \boldsymbol{\theta}) - w_A y_{A1} - w_B y_{B1} - F_1 \tag{2.22}$$

$$\Pi(\mathbf{p}, \mathbf{y}; \boldsymbol{\theta}) = \sum_{k=A,B} \sum_{i=1,2} (p_{ki} ES_{ki}(\mathbf{p}, \mathbf{y}; \boldsymbol{\theta}) - c y_{ki}). \tag{2.23}$$

2.5.1 Comparing Decentralized and Centralized Decisions on Price and Inventory

When we compare the decentralized and centralized first order conditions, there are four equations with respect to retailer 1's inventory and price variables:

$$
\begin{aligned}
\frac{\partial \pi_1}{\partial y_{A1}} &= \frac{\partial \Pi}{\partial y_{A1}} \quad \overbrace{-(w_A - c)}^{\text{vertical externality}} \quad \overbrace{-p_{A2}\frac{\partial ES_{A2}}{\partial y_{A1}}}^{\text{horizontal externality}} \quad \overbrace{\phantom{-p_{B2}\frac{\partial ES_{B2}}{\partial y_{A1}}}}^{\text{horizontal externality}} && (2.24) \\
\frac{\partial \pi_1}{\partial y_{B1}} &= \frac{\partial \Pi}{\partial y_{B1}} \quad -(w_B - c) \quad -p_{B2}\frac{\partial ES_{B2}}{\partial y_{B1}} && (2.25) \\
\frac{\partial \pi_1}{\partial p_{A1}} &= \frac{\partial \Pi}{\partial p_{A1}} \quad -p_{A2}\frac{\partial ES_{A2}}{\partial p_{A1}} \quad -p_{B2}\frac{\partial ES_{B2}}{\partial p_{A1}} && (2.26) \\
\frac{\partial \pi_1}{\partial p_{B1}} &= \frac{\partial \Pi}{\partial p_{B1}} \quad -p_{A2}\frac{\partial ES_{A2}}{\partial p_{B1}} \quad -p_{B2}\frac{\partial ES_{B2}}{\partial p_{B1}} && (2.27)
\end{aligned}
$$

The inventory equations (2.24) and (2.25) are the same as those in the inventory model. But in contrast to the price model, the vertical externalities are missing from the pricing equations (2.26) and (2.27); because they show up in the inventory equations. The signs of the horizontal externalities are subject to the ways in which retailer 2's sales are impacted by retailer 1's price decisions. Unlike the price model, when retailer 1 increases price, retailer 2 might not simply catch the additional demands. Because of the shrinkage of retailer 1's demand, retailer 2 will also lose some spillover demand from retailer 1. These are referred to as the "direct effect" and "fill-rate effect" by Krishnan and Winter (2007). However, there is evidence from simulations showing that in most of the cases the "direct effect" dominates the "fill-rate effect" (Krishnan and Winter, 2007). This implies that the horizontal externalities are usually negative, and distort the retailer prices downwards.

2.5.2 Contracts that Fix Price and Inventory Distortions

Given the distortions in retailers' price and inventory decisions, the manufacturer needs contractual provisions in order to fix both distortions. Since vertical externalities are missing from the pricing equations, the manufacturer can not use the wholesale prices to manipulate the retailers' prices. Nevertheless, the retail price floors are still a straightforward way to fix retailers' incentives for cutting prices. Once the price decisions are controlled, the retailers' inventory incentives can be aligned through the contracts we have found in the inventory model.

Proposition 2.4 *The following contracts can coordinate retailers' price and inventory decisions when they carry a product line:*
(1) Identical linear wholesale prices, quantity rationing, differentiated retail price floors, and a fixed fee.
(2) Differentiated linear wholesale prices, as given by (2.5) and (2.6), differentiated retail price floors, and a fixed fee.
(3) Differentiated buyback prices, as given by (2.11) and (2.10), identical linear wholesale prices, differentiated retail price floors, and a fixed fee. (If the fill-rate effect dominates the direct effect, retailers tend to increase prices from the centralized level, so the manufacturer should use price ceilings instead of price floors.)

The buyback contract works by generating new vertical externalities in both the inventory and price equations. Therefore, it is possible to use buyback prices, combined with the wholesale prices to fix both the inventory and price distortions, although, if the two products are asymmetric, they will have different wholesale prices as well as different buyback prices. This contract is particularly helpful where price floors are illegal.

Proposition 2.5 *The manufacturer can achieve channel coordination through the contract with differentiated linear wholesale prices, differentiated buyback* *prices, and a fixed fee, where the buyback prices are given by*

$$b_A = \left. \frac{\frac{\partial ES_{B1}}{\partial p_{A1}}(p_{A2}\frac{\partial ES_{A2}}{\partial p_{B1}} + p_{B2}\frac{\partial ES_{B2}}{\partial p_{B1}}) - \frac{\partial ES_{B1}}{\partial p_{B1}}(p_{A2}\frac{\partial ES_{A2}}{\partial p_{A1}} + p_{B2}\frac{\partial ES_{B2}}{\partial p_{A1}})}{\frac{\partial ES_{A1}}{\partial p_{A1}}\frac{\partial ES_{B1}}{\partial p_{B1}} - \frac{\partial ES_{B1}}{\partial p_{A1}}\frac{\partial ES_{A1}}{\partial p_{B1}}} \right|_{\mathbf{p}^*,\mathbf{y}^*} \tag{2.28}$$

$$b_B = \left. \frac{\frac{\partial ES_{A1}}{\partial p_{B1}}(p_{A2}\frac{\partial ES_{A2}}{\partial p_{A1}} + p_{B2}\frac{\partial ES_{B2}}{\partial p_{A1}}) - \frac{\partial ES_{A1}}{\partial p_{A1}}(p_{A2}\frac{\partial ES_{A2}}{\partial p_{B1}} + p_{B2}\frac{\partial ES_{B2}}{\partial p_{B1}})}{\frac{\partial ES_{A1}}{\partial p_{A1}}\frac{\partial ES_{B1}}{\partial p_{B1}} - \frac{\partial ES_{B1}}{\partial p_{A1}}\frac{\partial ES_{A1}}{\partial p_{B1}}} \right|_{\mathbf{p}^*,\mathbf{y}^*} \tag{2.29}$$

and the wholesale prices w_A and w_B are given by

$$w_A = c + [b_A(1 - \frac{\partial ES_{A1}}{\partial y_{A1}}) - p_{A2}\frac{\partial ES_{A2}}{\partial y_{A1}}]\Big|_{\mathbf{p}^*,\mathbf{y}^*} \tag{2.30}$$

$$w_B = c + [b_B(1 - \frac{\partial ES_{B1}}{\partial y_{B1}}) - p_{B2}\frac{\partial ES_{B2}}{\partial y_{B1}}]\Big|_{\mathbf{p}^*,\mathbf{y}^*}. \tag{2.31}$$

2.5.3 Spatial Model with Retailers' Price and Inventory Decisions

We now consider the retailers' price and inventory decisions in our spatial model. The initial demands are dependent on the retail prices that the retailers choose. The density of customers on each edge is random. Since the retailers' order quantities might not meet the initial demands, spillover can occurs. Since the spillover rates depend on the initial demands, they are sensitive to the randomness of customer densities as well as the retail prices. Of course, the spillover rates are also affected by the other parameters in the spatial model, such as travel cost and switching cost.

Through simulation, we find that the decentralized retailers tend to price lower, and stock more than the centralized firm (see Figure 2.11). However, as t increases, the retailers' decisions get closer to the centralized decisions. The increase in t decreases the degree of price-based as well as inventory-based substitution, therefore both the distortions in price and inventory are decreasing in t.

We also compute the wholesale prices and buyback prices that elicit the

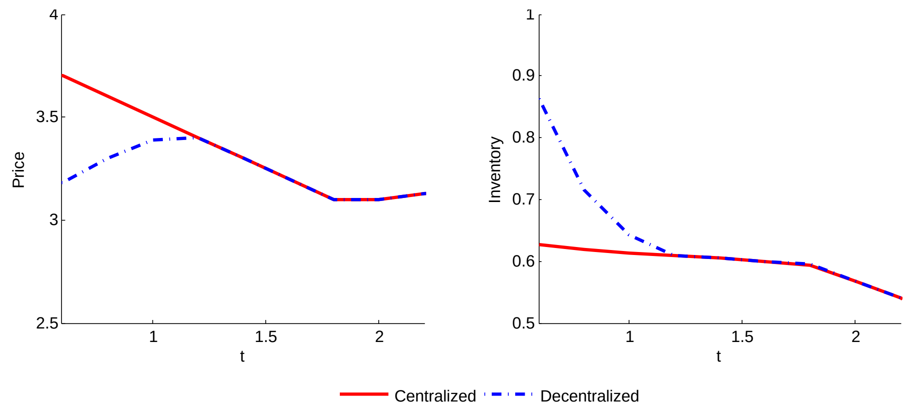

Figure 2.11: Price and inventory distortions ($u = 4, c = 1, w_A = w_B = 1, s = 0.5$)

centralized price and inventory (Proposition 2.5). Figure 2.12 shows that for each travel cost, there could be multiple pairs of wholesale price and buyback price that are able to fix the distortions. The manufacturer can choose any one of them for coordination. Note that when t is large (≥ 1.2), the decentralized and centralized decisions are overlapping, so the contract with wholesale price equal to 1, and buyback price equal to 0 (residual-claimancy contract) can achieve coordination.

Fixing the buyback price at certain levels, we can observe how the wholesale price varies in the travel cost. The three graphs in the first row of Figure 2.13 show that the wholesale price decreases in the travel cost for a given buyback price. Similarly, the three graphs in the second row of Figure 2.13 show that for a fixed wholesale price, the buyback price increases in the travel cost.

2.6 Concluding Remarks

The distribution of a product line, consisting of multiple product variants, is a common problem for nearly every manufacturer. The objective of this chapter is to understand the downstream retailers' incentives in terms of

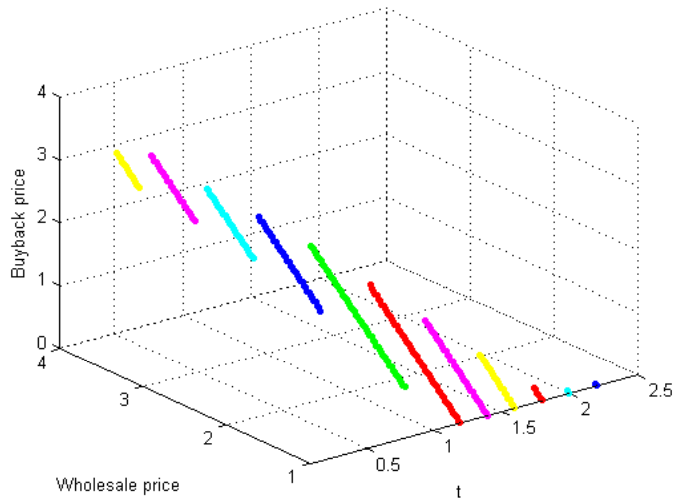

Figure 2.12: Price and inventory contract ($u = 4, c = 1, s = 0.5$)

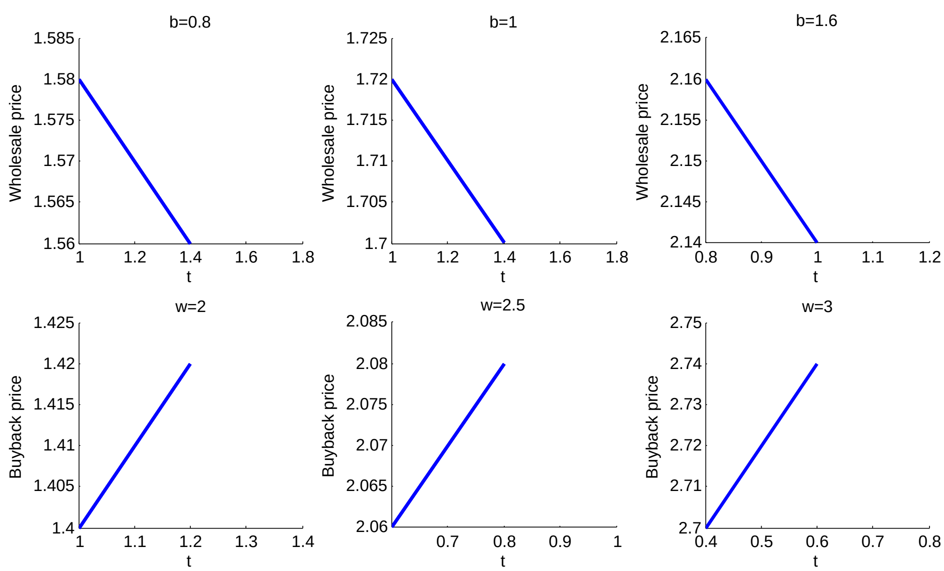

Figure 2.13: Price and inventory contract - planes ($u = 4, c = 1, s = 0.5$)

price and inventory in a multi-product multi-agent decentralized supply chain, and explore various contracts that achieve efficiency. We consider the case where a manufacturer produces and sells horizontally differentiated products via two downstream retailers. In the centralized system, the manufacturer makes decisions about selling prices in the consumer market and the quantities to produce, to maximize the system profit. In the decentralized system, where the retailers optimize their own objectives, incentive distortions emerge due to the vertical and horizontal externalities. We determine the structure of coordinating contracts in three cases: (1) retailers compete on inventory alone; (2) retailers compete on price alone; and (3) retailers compete on price and inventory. In each case, we use simulations to determine the optimal value of contract parameters.

Our results have managerial implications for firms distributing a product line through competing retailers. Consider the Starter case described in the introduction. Starter was using a quantity forcing contract with the retailers, i.e., imposing lower requirements on the retailers' orders. As we showed in Proposition 2.2(1), quantity forcing, as a supplement to uniform wholesale prices, can be used to coordinate inventory incentives. However, in the Starter case, retailers were able to circumvent starters quantity restrictions by buying product from Trans Sport, a large retailer which was "bootlegging" merchandise to smaller retailers. To prevent this, Starter sued Trans Sport. Alternatively, Starter could have relied on a different contract to circumvent this problem. Specifically, Starter could have offered to buy back unsold product from retailers who bought directly from Starter; and the buyback prices could have been tailored to induce the optimal inventory decisions (see Proposition 2.2(3)).

In our model, we focused on the analysis of horizontally differentiated product lines. Most product lines, however, have both horizontal and vertical differentiation. Nevertheless, our framework can also be applied to vertically differentiated product lines. The contracts that we found will still apply, although the different production costs of the vertically differentiated products will need to be accounted for to calculate the contract parameters.

Some of the assumptions in this chapter can be easily relaxed. For instance, the two product variants can be extended to n variants; duopoly retailers can be extended to oligopoly retailers, etc. Another simplification we made is to assume that the consumer searches for a substitute product when encountering an inventory shortage. In practice, the retailer may also offer to transship the product from another retailer, and then sell to the consumer. This is referred to as retail transshipment in supply chain literature (see Rudi et al. (2001) for a review). Future research may explore the decentralized retailers' incentives in the case of retailer transshipment (see, e.g., the third chapter of this dissertation). It will also be interesting to examine the social welfare of the contracts we proposed in the chapter. Another suggestion for future study is the channel coordination issues in inter-brand competition, i.e., how the manufacturer coordinates the downstream retailers, when they also carry the product lines of the other manufacturers.

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Chapter 3

INCENTIVES FOR TRANSSHIPMENT IN DECENTRALIZED SUPPLY CHAINS WITH COMPETING RETAILERS[1]

3.1 Introduction

Consider a customer who visits a car dealer (Dealer A) and finds that the model that she is looking for is out of stock. Suppose now that another dealer (Dealer B) has the same model in stock. Transshipment occurs when Dealer A, who has unsatisfied demand, obtains the product from Dealer B, who has unsold inventory.

Due to the greater ability to match supply and demand, transshipment can improve the performance of the supply chain. Perhaps as a result, many manufacturers, such as Honda, Caterpillar, and IBM facilitate transhipment by providing retailers with information systems that make inventory at all locations visible to supply chain members (Anupindi and Bassok, 1999; Zhao et al., 2005; Zarley, 17 February, 1992). But is transshipment always beneficial to all the members of the supply chain? Evidence shows that some

[1]A version of this chapter has been submitted for publication. Shao, J., Krishnan, H. and McCormick, S. T. Incentives for Transshipment in Decentralized Supply Chains with Competing Retailers.

manufacturers, such as BBI Enterprises, prohibit transshipment among their retailers. (See BBI Enterprises's agreement with its dealer, Cingular Wireless (Cingular Wireless, 5 December, 2005).)

The first contribution of our chapter is that we examine, for the first time, transshipment incentives in a decentralized supply chain where the retailers are independent from the manufacturer and also from each other, i.e., in a vertically *and* horizontally decentralized supply chain. (As we discuss in the literature review, the existing literature considers either a horizontally decentralized supply chain or a vertically decentralized supply chain, but not both.) We show that transshipment price and the control of transshipment decisions determine whether the firms benefit from, or are hurt, by transshipment.

We also compare the "completely" decentralized supply chain with one where downstream firms are under joint ownership. In the presence of transshipment, decentralized retailers may seek opportunities to join together in order to enhance their power in the supply chain. Through centralization, the retailers can avoid unnecessary competition and may thereby be able to reduce costs and increase profits. Prior work has studied the impact of transshipment when the manufacturer deals with a chain store (Dong and Rudi, 2004; Zhang, 2005). However, are decentralized retailers always better off when they centralize? Does the manufacturer prefer dealing with decentralized retailers or a chain store? Our analysis shows a counter-intuitive result, that is, the manufacturer may prefer dealing with a chain store; and the decentralized retailers may be better off than the chain store.

The remainder of this chapter is organized as follows. Section 3.2 reviews the related literature. Section 3.3 presents the modeling framework. In Section 3.4, we study the manufacturer's and decentralized retailers' incentives for transshipment under a linear wholesale price contract. In Section 3.5, we compare decentralized retailers and the case of a chain store. Section 3.6 discusses some model assumptions and suggests future research directions. Finally, we conclude this chapter in Section 3.7.

3.2 Literature Review

Traditional work on transshipment focuses on the optimal inventory and transshipment policies for a vertically integrated supply chain (see Krishnan and Rao (1965), Tagaras (1989), Robinson (1990), Wee and Dada (2005), Herer et al. (2006), etc.). There are two streams of recent research that study transshipment in decentralized supply chains.

One stream examines a *horizontally* decentralized supply chain; that is, transshipment occurs between locations that are not owned by one firm. The upstream supplier of the locations is not explicitly modelled in this stream of research. In particular, Rudi et al. (2001) compare the equilibrium inventory levels under transshipment and under no transshipment. Hu et al. (2007) extend the work of Rudi et al. (2001) to the uncertain capacities of the locations. Zhao et al. (2005) consider a dynamic game between decentralized retailers in multiple periods and propose a base-stock inventory and transshipment policy for each retailer. Zhao and Atkins (2009) compare the game between two retailers with transshipment and that with customer search. All the above papers take the non-cooperative game theoretical approach and assume duopoly retailers.[2]

The second stream studies a *vertically* decentralized supply chain with a single manufacturer and a chain store retailer. Assuming a normal demand distribution, Dong and Rudi (2004) show that, under mild assumptions, the manufacturer is better off from transshipment. Zhang (2005) generalizes the results of Dong and Rudi (2004) to an arbitrary demand distribution.

Our work differs from the existing literature as we examine transshipment in a completely decentralized supply chain. We consider both the downstream retail competition (in inventory) and the upstream manufacturer's decisions.

Furthermore, in most of the literature, the parameters of the transshipment decision are assumed to be exogenously determined. In this chapter,

[2]If there are more than two retailers, the analysis is complicated in that excess inventory needs to be allocated among multiple retailers facing a stock-out. In this setting, a cooperative game theory framework is more appropriate; see, for instance, Anupindi et al. (2001).

we allow the firms to determine whether, and at what transshipment price, transshipment will occur. Lee and Whang (2002) consider the transfer price in a secondary market, which is similar to transshipment, though in their model there are an infinite number of retailers and the retailers are price-takers. Rudi et al. (2001) analyze several cases where two retailers with asymmetric bargaining powers set the transshipment price. However, both papers assume that the manufacturer's wholesale price is fixed.

When manufacturers' decisions are fixed, it is well recognized that the (inventory) centralization can be beneficial to the retailers due to the pooling effect (see Hartman et al. (2000), Müllera et al. (2002), Chen and Zhang (2006), etc). Anupindi et al. (2001), Granot and Sošić (2003) and Sošić (2006) consider the scenario of retailers' "coopetition," that is, the retailers unilaterally determine the inventory they stock, but cooperatively determine how much inventory they want to share through transshipment.

Three papers take into account the vertical interaction between the manufacturer and retailers in the process of retail centralization. Under an endogenous wholesale price, Netessine and Zhang (2005) compare the inventory levels of the decentralized retailers and a chain store, in the cases where the retailers' products are complementary and substitutable. Anupindi and Bassok (1999) analyze an alternative to transshipment, i.e., customer search. They show that under a wholesale price only contract, the manufacturer may prefer retail decentralization or centralization, depending on the rate of customer search. While the two papers consider retail centralization in contexts other than transshipment, both papers do not discuss the impact of centralization on the *retailers*. Özen et al. (2008) show that if retailers reallocate inventories after observing demand signals, the retailers are better off but the manufacturer's profit may either increase or decrease.

3.3 Model

Consider a single period model where a monopolist produces a single product at production cost c per unit, and distributes it through two retailers.[3] The retailers are independent from the manufacturer and from each other. (In Section 3.5, we also consider the chain store case, i.e., the retailers are jointly owned but independent from the manufacturer.)

We assume that the retailers are identical, and denote by s the per unit "transshipment price" that the retailers pay to each other to obtain the transshipped goods. (In the chain store case, the transshipment price does not exist.) For simplicity, assume that the cost incurred during transshipment, e.g., the transportation cost, is zero. (Relaxing this assumption does not affect our analysis and results.) To avoid trivial outcomes, we assume that $s \in [0, p]$, where p is the fixed retail price.

The supply chain's decisions are a three stage process (see the timeline in Figure 3.1).[4] In stage 1, the firms decide whether the retailers should transship, and at what transshipment price. We consider three cases (Section 3.4.3) where the manufacturer has an increasing control of the above decisions.

In stage 2, with full knowledge of the decisions made in stage 1, the manufacturer offers a take-it-or-leave-it contract to the retailers, specifying the wholesale price w.[5] If the manufacturer has chosen any transshipment parameter in the first stage, it is also included in the contract. (The above contract, i.e. the wholesale price and potentially the transshipment parameters, will not in general be able to coordinate the supply chain; we discuss a coordinating contract in Appendix B.1.)

In stage 3, the retailers simultaneously decide order quantities y_1 and y_2

[3]We follow the assumption of duopoly retailers as does most of the transshipment literature that adopts the non-cooperative game theory methodology. See the discussion of key assumptions in Section 3.6.

[4]We discuss an alternative sequence of events in Section 3.6.

[5]Note that the transshipment price set in stage 1 does not depend on the knowledge of wholesale price. This is because at the time transshipment occurs, the wholesale price that the retailers have paid is sunk and it is always profitable for the retailers to transship as long as $0 \leq s \leq p$. Transshipment price can be greater or less than the wholesale price.

before demand is realized. (In the chain store case, the chain store determines the order quantities at both locations.)

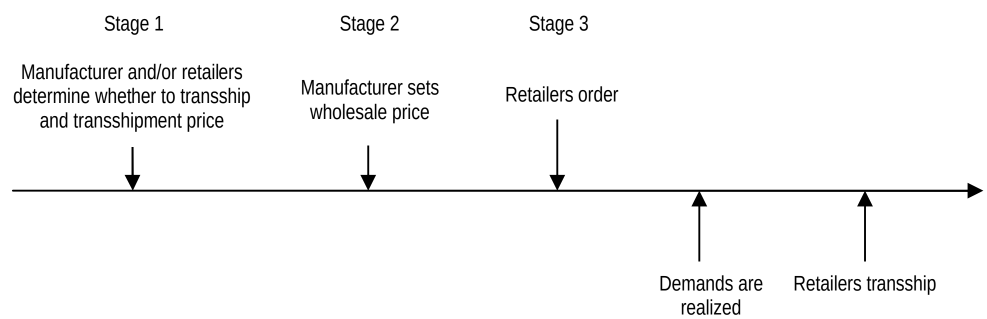

Figure 3.1: Timeline of the game

Retailer i faces random demand ξ_i, $i = 1, 2$, which has an arbitrary distribution. The demands at the retailers may be correlated. The distributions of ξ_1 and ξ_2 are identical, and let $F(\cdot)$ and $f(\cdot)$ denote their cumulative distribution function (CDF) and probability distribution function (PDF) respectively. Assume that the demand distribution functions are continuous and differentiable.

After demands are realized, an individual retailer may find that its inventory is either too small to satisfy its demand or higher than its demand. When out of stock, the retailer does not have a second chance to replenish from the manufacturer. However, it may transship from the other retailer if the latter has leftover inventory. The number of transshipped units from retailer i to j is given by $T_i = \min((y_i - \xi_i)^+, (\xi_j - y_j)^+)$, which is the minimum of i's excess inventory and j's excess demand. Note that after the realization of demand, it is in the interest of a stocked out retailer to use transshipment to satisfy as much demand as possible; it is also in the interest of an over-stocked retailer to transship as many units as requested.

3.4 Completely Decentralized Supply Chain

We first examine the firms' incentives for transshipment in the completely decentralized supply chain, where the retailers are independent from each other and from the manufacturer. We analyze the game by backward induction, starting with the retailers' inventory game in stage 3. In this stage, we look at the impact of transshipment by comparing two cases: (1) where the retailers transship (indicated by superscript "DT" which stands for "decentralized retailers transship") and; (2) do not transship (indicated by superscript "NT"). We then consider the second stage where the manufacturer optimally sets the wholesale price. Finally, we consider the first stage where the firms determine the parameters of the transshipment decision.

3.4.1 Stage 3: Retailers' Inventory Game

For given transshipment price and wholesale price, the expected profit of retailer i is given by

$$\pi_i = pE\min(\xi_i, y_i) + sET_i + (p-s)ET_j - wy_i. \tag{3.1}$$

We can show that a unique Nash equilibrium exists in the retailers' inventory game and the equilibrium is symmetric. (The details of the proof are omitted because they are almost identical to a similar proof in Rudi et al. (2001).) Denote by $y^{DT}(w,s)$ the equilibrium inventory of a retailer under transshipment, which is obtained as the solution to the best response functions:

$$p(1-F(y_i)) - w + s\frac{\partial ET_i}{\partial y_i} + (p-s)\frac{\partial ET_j}{\partial y_i} = 0,\ i,j = 1,2,\ i \neq j. \tag{3.2}$$

The retailer's equilibrium inventory under no transshipment, denoted by $y^{NT}(w)$, is the newsvendor quantity, given by $F(y^{NT}) = (p-w)/p$, because in this case each retailer's inventory is unaffected by the other's inventory decision.

Now compare y^{DT} with y^{NT} (we drop the arguments for expositional convenience). Under transshipment, there are two forces that cause retailer i's inventory choice to deviate from y^{NT}. First (for any inventory choice of retailer j), when retailer i *increases* inventory from y^{NT}, it will transship more *to* retailer j at the end of the period (in expectation). Therefore, retailer i gains profit by collecting the transshipment price s from retailer j on the extra units. Second, when retailer i *decreases* inventory from y^{NT}, it will transship more *from* retailer j at the end of the period (in expectation). It pays retailer j the transshipment price s on these units and then sells to consumers at p; that is, it collects a margin $(p - s)$ on these units.

The two forces pull retailer i's optimal inventory choice under transshipment in opposite directions. The net effect depends on the magnitude of the two margins, i.e., s and $p - s$, as well as the demand distribution; therefore, it is hard to determine in general. However, it becomes clear in the two extreme cases where $s = 0$ and $s = p$. In the first case, the second force becomes zero and it is optimal for the retailer to *decrease* its inventory from y^{NT}. In the second case, the first force becomes zero and an *increase* in inventory from y^{NT} is optimal. (The two cases are proved in Propositions 2 and 3 of Rudi et al. (2001).) Furthermore,

Lemma 3.1 *Each retailer's order quantity under transshipment, y^{DT}, is strictly monotonically increasing in the transshipment price s.*

(See the proof in Appendix B.2.) Lemma 3.1 is similar to Proposition 2 of Rudi et al. (2001) with asymmetric retailers; but here we provide a new approach to the proof. See Figure 3.2 for an illustration of Lemma 3.1.

As the wholesale price is fixed, the manufacturer's profits, $2(w - c)y^{NT}$ under no transshipment, and $2(w - c)y^{DT}$ under transshipment, are completely determined by the retailers' order quantities. Therefore, whether the manufacturer benefits from transshipment depends only on whether the retailers order more under transshipment. From the above discussion and Lemma 3.1, it follows that for any fixed wholesale price there exists a unique transshipment price $\tilde{s}(w)$ such that the manufacturer makes a higher profit under transshipment when $s \geq \tilde{s}(w)$; and it makes a lower profit under transshipment when $s < \tilde{s}(w)$.

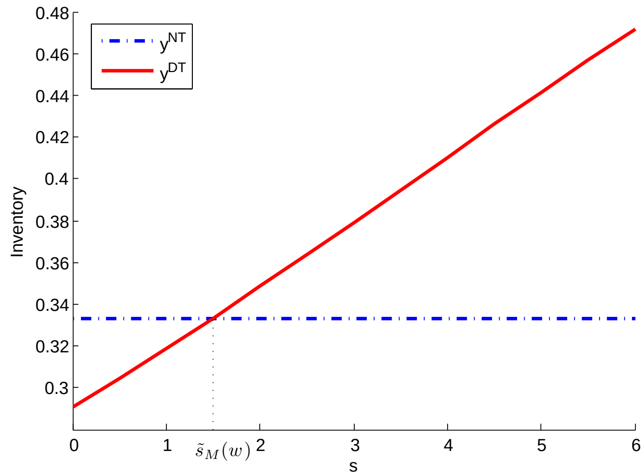

Figure 3.2: Retailer's inventory when wholesale price is fixed ($p = 6, w = 4, c = 1, \xi_i \sim$ Uniform[0, 1])

Next we look at the retailers' profits. When the retailers transship, each retailer's profit depends on the other retailer's inventory choice. However, no matter what inventory level the other retailer chooses, if a retailer orders the newsvendor inventory, it will obtain the expected profit under no transshipment plus an extra profit through transshipment. Therefore, when the retailer optimally chooses its inventory, it will obtain an even higher profit. As a result, if the wholesale price is fixed, the retailers always benefit from transshipment. See Figure 3.3 for an illustration.

3.4.2 Stage 2: Manufacturer Sets Wholesale Price

In stage 2, the manufacturer's problem under transshipment is

$$\max_{w} \ \pi_M = 2(w - c)y^{DT}(w, s), \tag{3.3}$$

where $y^{DT}(w, s)$ is the (symmetric) equilibrium inventory derived from stage 3 for given wholesale price and transshipment price. The following analysis can be applied whether or not the manufacturer's problem is unimodal.

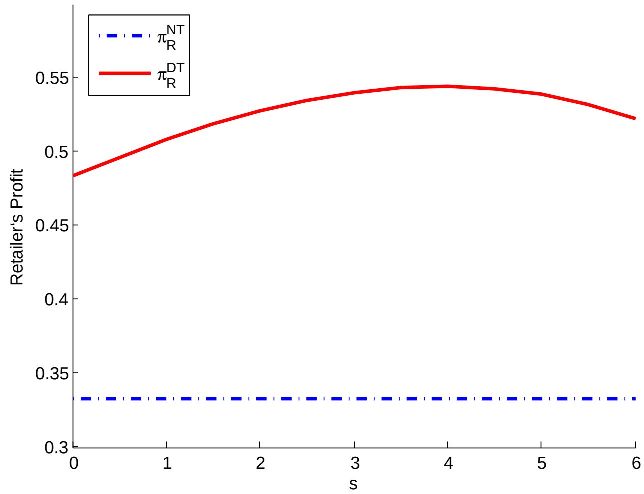

Figure 3.3: Retailer's profit when wholesale price is fixed (Here π_R^{NT} and π_R^{DT} represent one retailer's expected profits at equilibrium under no transshipment and under transshipment respectively. And $p = 6$,$w = 4, c = 1, \xi_i \sim$ Uniform$[0, 1]$.)

Manufacturer's profit

We first consider the manufacturer's profit in the two extreme cases where $s = 0$ and $s = p$. At $s = p$, for a fixed wholesale price, recall that the retailers always have incentives to stock more under transshipment, leading to higher manufacturer profits. This is true for all wholesale prices, including the optimal "no transshipment" wholesale price. Therefore, the manufacturer's optimal profit under transshipment will always be higher than the optimal profit under no transshipment. By a similar argument, the manufacturer always makes lower profits under transshipment at $s = 0$.

Lemma 3.2 *When the manufacturer optimally sets the wholesale price, it makes a higher profit under transshipment at $s = p$, and a lower profit under transshipment at $s = 0$.*

(See the proof in Appendix B.2.)

We next characterize the manufacturer's profit for all values of s in the range 0 to p.

Lemma 3.3 *When the manufacturer optimally sets the wholesale price, the manufacturer's profit is monotonically increasing in the transshipment price* s.

(See the proof in Appendix B.2.)

From Lemmas 3.2 and 3.3, we have:

Proposition 3.1 *When the manufacturer optimally sets the wholesale price, there exists a unique transshipment price* $\hat{s}_M$*, such that the manufacturer prefers transshipment when* $s \geq \hat{s}_M$*, and no transshipment when* $s < \hat{s}_M$*. The manufacturer obtains the highest profit under transshipment at* $s = p$*.*

(See Figure 3.4 for an illustration.) When the transshipment price is high, the retailers have incentives to stock more inventory under transshipment; and the manufacturer can increase the wholesale price to extract the retailers' benefits from transshipment. At a low transshipment price, however, the retailers stock less under transshipment. When increasing the wholesale price, the manufacturer will lose even more orders from the retailers. Thus the manufacturer's ability to extract retailers' profits is limited; and it turns out that the manufacturer will be worse off if the retailers transship.

Retailers' profits

Recall that when the wholesale price is fixed, the retailers always benefit from transshipment. However,

Observation 3.1 *When the manufacturer optimally sets the wholesale price, the retailers can be worse off under transshipment.*

For an example of Observation 3.1, assume that the demands at the two retailers have independent uniform distribution on the support $[0, b]$. When the retail price is relatively small ($p/c \leq 1.952$), the retailers are always better off as a result of transshipment at any transshipment price. However, when $p/c > 1.952$, the retailers are worse off under transshipment at large transshipment prices. (See Figure 3.5.)

The intuition is as follows. When the retail price is high, the upper limit of transshipment price increases. When the transshipment price is high, the

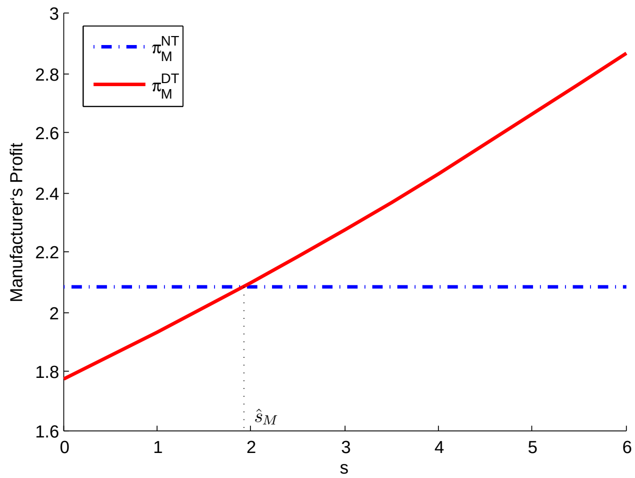

Figure 3.4: Manufacturer's profit when it sets wholesale price (Here π_M^{NT} and π_M^{DT} represent the manufacturer's optimal profits under no transshipment and under transshipment respectively. And $p = 6, c = 1, \xi_i \sim$ Uniform$[0, 1]$.)

retailers have stronger incentives to over-stock under transshipment. Therefore, the manufacturer can increase wholesale price to take advantage of retailers' inventory competition and extract more profits from the retailers. As a result, the retailers are worse off from transshipment.

3.4.3 Stage 1: Firms Determine Parameters of Transshipment Decision

Now consider the first stage of the game where the firms determine the parameters of the transshipment decision. In reality, both the manufacturer and the retailers may have different levels of control over these parameters, i.e., whether to transship and at what transshipment price. Here we provide three cases where the manufacturer has an increasing amount of control. This is certainly not an exhaustive list; we do not aim to enumerate all possibilities. Rather, we aim to provide a way to analyze how the transshipment decisions are being made and who benefits from transshipment in various

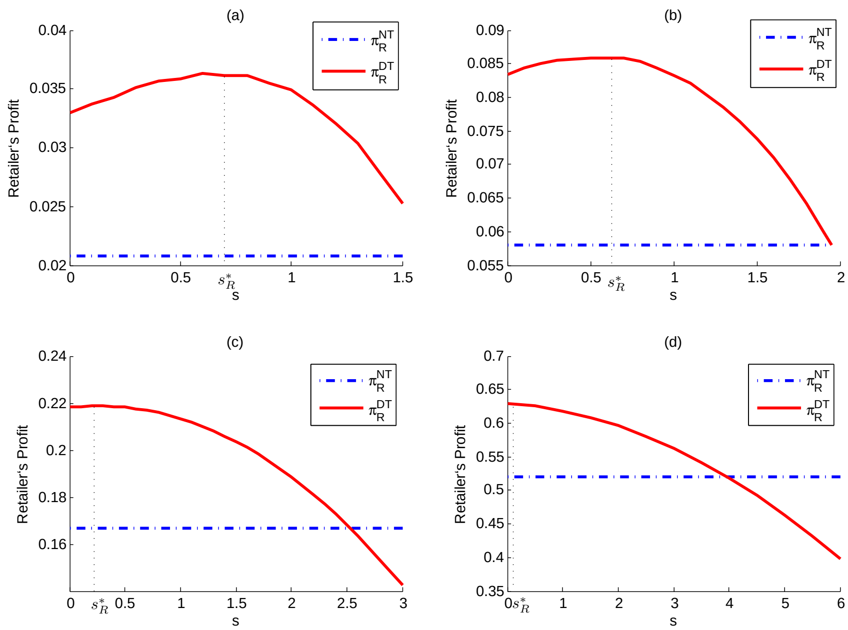

Figure 3.5: Retailer's profit when manufacturer sets wholesale price ($c = 1, \xi_i \sim$ Uniform$[0, 1]$ (a) $p = 1.5$; (b) $p = 1.952$; (c) $p = 3$; (d) $p = 6$)

situations.

Case 1: Manufacturer has no control.

This corresponds to the case of a weak manufacturer (whose only decision is to set the wholesale price) and powerful retailers, who jointly decide whether to transship and the transshipment price.

In Figure 3.5, although the retailers can be worse off under transshipment, there always exists a transshipment price at which the retailers benefit from transshipment. We can obtain analytically the retailers' profits under transshipment and under no transshipment for uniform distributions on $[0, b]$.[6] Denote by s_R^* the transshipment price at which the retailers ob-

[6] We observe a similar result for other probability distributions as well, including the exponential. However, for some demand distributions and parameter values, the retailers make less profit under transshipment at *all* transshipment prices. An example is the uniform distribution on $[a, b]$ ($a > 0$). The details and discussions are given in Appendix

tain the highest profits under transshipment. Combined with Proposition 3.1, this gives us the following result:

Proposition 3.2 *Suppose the retailers benefit from transshipment at* s_R^* *and the manufacturer has no control of the parameters of the transshipment decision. The manufacturer is better off from retailers' transshipment if* $s_R^* \geq \hat{s}_M$ *and worse off if* $s_R^* < \hat{s}_M$*.*[7]

For example, in Figure 3.6, the retailers will choose to transship and set the transshipment price at s_R^*; the manufacturer is better off in (a) but worse off in (b).

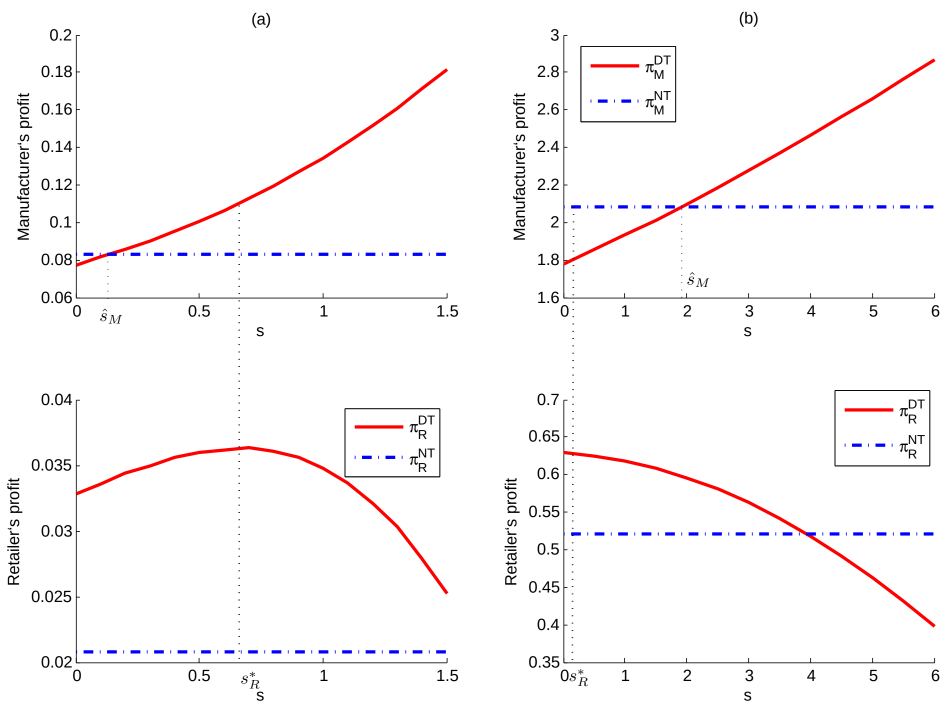

Figure 3.6: Firms' profits ($c = 1, \xi_i \sim$ Uniform$[0, 1]$ (a) $p = 1.5$; (b) $p = 6$)

B.3.

[7]Note that if the retailers are worse off at s_R^*, then transshipment will never occur.

The implication here is that if the manufacturer has little power compared with downstream retailers, the manufacturer may suffer from the negative impact of transshipment: if the transshipment price that the retailers optimally choose is low, the manufacturer does not get enough orders from the retailers as they "pool" their inventories.

Case 2: Manufacturer has partial control.

This case is similar to case 1 except that the manufacturer has the power to disallow retailers' transshipment. (But the manufacturer cannot control the transshipment price if the retailers do transship.) The manufacturer knows that if the retailers transship they will set the transshipment price at s_R^*. So the manufacturer needs only to determine whether it will be worse off by transshipment at s_R^*. Since the manufacturer benefits from transshipment only if $s_R^* \geq \hat{s}_M$, it allows transshipment when this condition is satisfied. For example, in Figure 3.6 (a), the manufacturer will allow the retailers to transship; but not in Figure 3.6 (b).

Thus, when the manufacturer has partial control of transshipment parameters, it can use its control to protect itself from being hurt by transshipment.[8]

Case 3: Manufacturer has complete control.

In this case the manufacturer is powerful. It can enforce or prohibit retailers' transshipment and also choose the transshipment price. From Proposition 3.1, we get

Proposition 3.3 *When the manufacturer has complete control, it always forces the retailers to transship and sets the transshipment price $s = p$. The manufacturer is always better off as a result of transshipment.*

Notice that while the manufacturer provides information systems and enforces transshipment, it also needs to dictate and monitor the retailers'

[8]As we show in Appendix B.3, there are cases where the retailers are always hurt by transshipment. In this case, the retailers would simply avoid transshipment even if the manufacturer allows it.

transshipment price. And while the manufacturer always benefits, the retailers' profits may decline. Figure 3.6 (a) is an example where the retailers also benefit by the transshipment decisions enforced by the manufacturer; and Figure 3.6 (b) is an example where the retailers are hurt by the manufacturer's decisions. This leads to the following observation:

Observation 3.2 *When the manufacturer has complete control of the parameters of the transshipment decision, the retailers may be worse off as a result of transshipment.*

Table 3.1 summarizes the results of this section.

Table 3.1: Manufacturer's and decentralized retailers' benefits under transshipment

Stage	w	Transshipment parameters	Manufacturer	Retailers
3	Fixed	Fixed	Worse off at low s; better off at high s	Better off
2	Manufacturer sets w	Fixed	Worse off at low s; better off at high s	Depending on s, demand distribution and critical fractile, $(p-c)/p$
1		Retailers have control	Depending on the transshipment price that retailers choose, s_R*	Can prevent transshipment to ensure being no worse off
		Both have control	Can prevent transshipment to ensure being no worse off	Can prevent transshipment to ensure being no worse off
		Manufacturer has control	Better off	Depending on demand distribution and critical fractile, $(p-c)/p$

3.5 Decentralized Retailers vs. a Chain Store Retailer

A chain store owns the two retail locations but it is still independent from the manufacturer. It makes inventory decisions to maximize the joint profits of the two locations. The existing literature shows that the manufacturer can benefit from or be hurt by transshipment between the locations of a chain store (Dong and Rudi, 2004; Zhang, 2005). The chain store's preference for transshipment has not been shown explicitly in the literature. (Under a normal demand distribution, Dong and Rudi (2004) show that the retailer's profit is generally decreasing as the effect of risk pooling is increasing, e.g., when the demand correlation between retailers increases. However, they do not show whether the retailer is better or worse off compared with the *no transshipment* case.) Through numerical experiments, we find that the chain store retailer can be worse off as a result of transshipment. (For example, for uniform demand on $[0, b]$, the chain store benefits from transshipment when $p/c < 2.113$ and it is worse off under transshipment when $p/c > 2.113$.)

Our focus here is on comparing the decentralized retailers with the chain store. Assume that the decentralized retailers and the chain store both transship, and consider (1) whether the manufacturer prefers the chain store or the decentralized retailers; and (2) whether the retailers make more profits by being centralized (i.e., as in the chain store) or decentralized. (Our approach is to compare a decentralized retailer's profit with the profit of a location of the chain store, which, by symmetry, equals one half of the total profit of the chain store.) We analyze these issues, again, in the reverse order starting from stage 3.

Recall that superscript DT indicates the case where decentralized retailers transship; now let superscript CT represent the case where the chain store transships between its locations.

3.5.1 Stage 3: Retailers' Inventory Decisions

Consider, first, for a fixed wholesale price whether the manufacturer makes more profits dealing with the chain store or decentralized retailers. We again only need to compare the inventory levels.

Lemma 3.4 *Under transshipment, for a given wholesale price, when the transshipment price $s = 0$ the decentralized retailers order less than the chain store; and when $s = p$ the decentralized retailers order more than the chain store.*

(See the proof in Appendix B.2.)[9]

The intuition behind Lemma 3.4 is that the decentralized retailers ignore the impact of their inventory decisions on the other retailers, while the chain store internalizes the impact. Denote by $y^{CT}(w)$ the optimal inventory of a location of the chain store. Note that y^{CT} is independent of s because the transshipment price is irrelevant for the chain store. In the decentralized case, if retailer i *increases* inventory from y^{CT}, it gains profit by transshipping *to* retailer j; if retailer i *decreases* inventory from y^{CT}, it gains profit by transshipping *from* retailer j. It is again not clear in general in which direction retailer i is going to move. However, at $s = 0$, the margin of transshipping *out* is zero; therefore, retailer i will decrease inventory from y^{CT}. (Retailer j hence will transship more to retailer i for free, and its profit declines; but retailer i ignores this externality.) Similarly, at $s = p$ the argument is reversed and retailer i stocks more than y^{CT}.

Now consider the impact on the manufacturer's profit. Combining Lemma 3.4 with the monotonicity of decentralized retailers' inventory (Lemma 3.1), we have the following result. For any fixed wholesale price w, there exists a unique transshipment price $\tilde{s}_M^{CT}(w)$ such that the manufacturer obtains a higher profit dealing with the chain store if $s \leq \tilde{s}_M^{CT}(w)$; and it obtains a higher profit dealing with the decentralized retailers if $s > \tilde{s}_M^{CT}(w)$. See Figure 3.7 for an illustration.

Now consider the impact on the retailers' profits. For a fixed wholesale price, the chain store makes the inventory decisions at the two locations. Its feasible set includes the decentralized retailers' equilibrium inventories. Therefore, for a given wholesale price, the chain store always makes more profits than the decentralized retailers. See Figure 3.8 for an illustration of retailers' profits.

Both the chain store and decentralized retailers benefit from transshipment due to the inventory pooling effect. That is, the risk of having unsold

[9] Propositions 3 and 4 of Rudi et al. (2001) are similar to Lemma 3.4 in the case of asymmetric retailers. However, Hu et al. (2007) provide a counterexample to Propositions 3 and 4 of Rudi et al. (2001). Here we use a different proof and show that they hold for symmetric retailers.

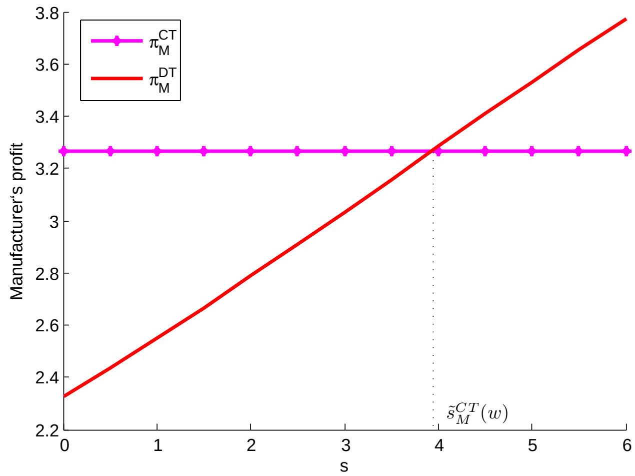

Figure 3.7: Manufacturer's profit when wholesale price is fixed (Here π_M^{CT} and π_M^{DT} represent the manufacturer's optimal profits when it deals with the chain store and decentralized retailers respectively. And $p = 6, w = 4, c = 1, \xi_i \sim \text{Uniform}[0, 1]$.)

inventory or unsatisfied demand is reduced by transshipment. However, decentralized retailers deviate from the chain store's inventory level and therefore, in general, can not achieve as much benefit from transshipment as the chain store does. Specifically,

Lemma 3.5 *For a given wholesale price w,*
(a) a decentralized retailer's profit is increasing in s when s is close to 0, and decreasing in s when s is close to p;
(b) at the unique transshipment price $\tilde{s}_M^{CT}(w) \in (0, p)$, a decentralized retailer makes the greatest profit, which is equal to the profit of a location of the chain store; at any other transshipment prices, it makes a lower profit than a location of the chain store.[10]

(See the proof in Appendix B.2.)

[10] Note that $\tilde{s}_M^{CT}(w)$ is referred to as the "coordinating transshipment price" in Hu et al. (2007); see discussions in Appendix B.1.

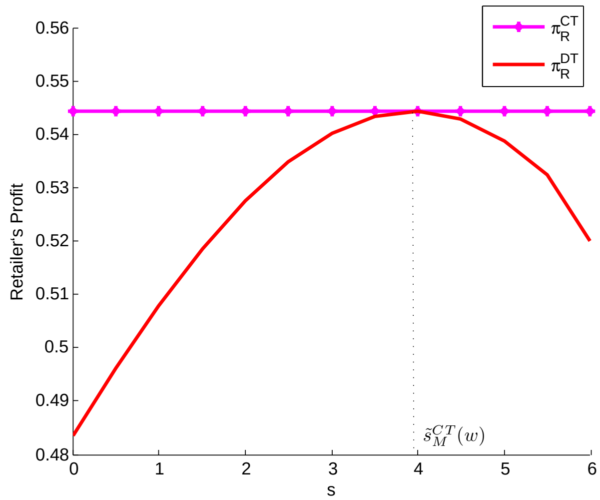

Figure 3.8: Retailers' profits when wholesale price is fixed (Here π_R^{CT} represents the profit of one of the chain store's locations and π_R^{DT} represents a decentralized retailer's profit. And $p = 6, w = 4, c = 1, \xi_i \sim \text{Uniform}[0, 1]$.)

3.5.2 Stage 2: Manufacturer Sets Wholesale Price

Manufacturer's profit

The manufacturer may prefer dealing with either the chain store or decentralized retailers, depending on the transshipment price between the decentralized retailers:

Proposition 3.4 *When the manufacturer optimally sets the wholesale price, there exists a unique transshipment price $\hat{s}_M^{CT}$ such that when the transshipment price is less than $\hat{s}_M^{CT}$ the manufacturer prefers dealing with the chain store, and when the transshipment price is greater than $\hat{s}_M^{CT}$ it prefers dealing with decentralized retailers.*

(The argument is similar to that of Proposition 3.1.) See Figure 3.9 for an illustration.

The intuition is as follows. The manufacturer can set the wholesale price to extract more profits from retailers who have stronger incentives to stock.

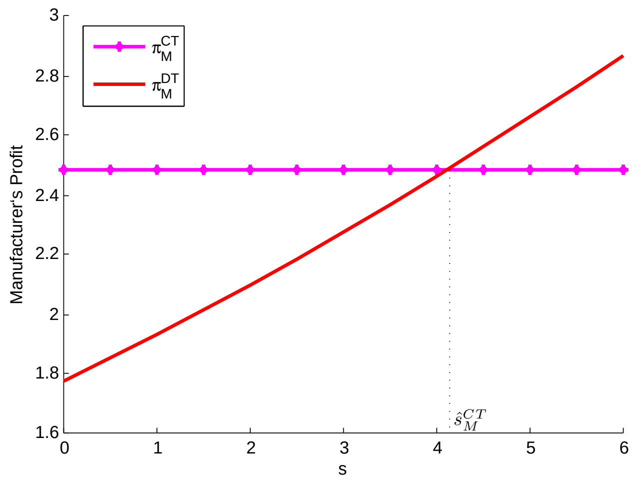

Figure 3.9: Manufacturer's profit when it sets wholesale price ($p = 6, c = 1, \xi_i \sim$ Uniform$[0, 1]$)

When the transshipment price is low, it is the chain store who orders more; when the transshipment price is high, it is the decentralized retailers who order more.

The threshold $\hat{s}_M^{CT}$ can be quite large relative to the retail price, which implies that there can be a large proportion of transshipment prices under which the manufacturer prefers the chain store. For example, for uniform distribution on $[0, b]$, if the critical fractile $(p - c)/p = 83.3\%$, then $\hat{s}_M^{CT}/p = 68.4\%$. In other words, as long as the transshipment price is no higher than 68.4% of the retail price, the manufacturer would rather dealing with the chain store than decentralized retailers. When the critical fractile is smaller than 83.3%, $\hat{s}_M^{CT}/p$ is even higher.

Retailers' profits

It is natural to think that the chain store makes more profits than the decentralized retailers. This is certainly true in the following example: assuming the demands have uniform distribution on $[0, b]$, we can show that the chain store makes more profits than the decentralized retailers when $s = p$. (See Figure 3.10.) However, from Figure 3.10 we also have the following observation:

Observation 3.3 *When the manufacturer optimally sets the wholesale price, the decentralized retailers can make more profits than the chain store under transshipment.*

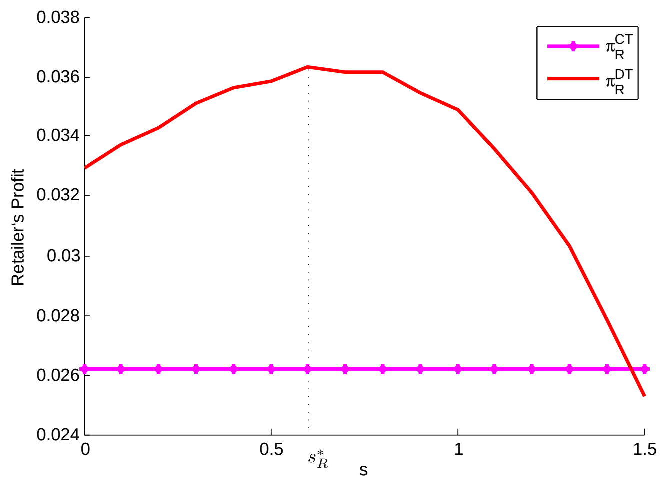

Figure 3.10: Retailers' profits when manufacturer sets wholesale price ($p = 1.5, c = 1, \xi_i \sim$ Uniform$[0, 1]$)

This seems counter-intuitive, since the chain store has stronger downstream power than the decentralized retailers. However, for a fixed wholesale price, the chain store effectively pools inventories under transshipment and benefits from a reduction in risk. The decentralized retailers do not enjoy as much benefit because of competition. This makes the manufacturer's demand from the chain store less elastic than that from the decentralized retailers. Therefore, when the manufacturer sets the wholesale price, it is able to extract more profits from the chain store. Only when the transshipment price is very large, can the manufacturer extract the decentralized retailers' profits more, due to their aggressive over-stocking behavior.

Furthermore, the proportion of transshipment prices at which the decentralized retailers are better off than the chain store can be very large. For example, if demand has uniform distribution on $[0, b]$, for $0.17 \leq (p-c)/p \leq$

0.98, as long as the transshipment price is no larger than 95% of the retail price, the retailers will prefer being decentralized. For some demand distributions, such as truncated normal, the decentralized retailers make more profits than the chain store for *all* transshipment prices.

3.5.3 Stage 1: Firms Determine Transshipment Price

Now consider the first stage where either the manufacturer or the decentralized retailers choose the transshipment price. Since we only consider the case where both the decentralized retailers and the chain store transship, the firms do not need to decide whether to transship. Note that when the manufacturer deals with the chain store, the transshipment price does not exist, and hence, the firms do not make any decisions in stage 1.

Case 1: Decentralized retailers control the transshipment price.

When the decentralized retailers control the transshipment price, they will choose to transship at s_R^*, where they obtain the greatest profits. Note that in Figure 3.11, at s_R^* the decentralized retailers will make more profits than the chain store. Therefore, under this scenario, the retailers always prefer being decentralized if they have control. This is analytically true for uniform distributions on the interval $[0, b]$. Unfortunately, we are unable to prove it for a general demand distribution. However, simulations show that it holds for other common distributions such as the truncated normal and exponential for a large range of parameter values.

From Proposition 3.4, the manufacturer prefers the decentralized retailers if $s_R^* \geq \hat{s}_M^{CT}$; it prefers the chain store otherwise. However, in simulations we only observe the case where the manufacturer prefers the chain store. Figure 3.11 shows an example.

Case 2: Manufacturer controls the transshipment price.

Recall that if the manufacturer can control transshipment price when it deals with the decentralized retailers, it will always set the transshipment price equal to p (Proposition 3.3). By Proposition 3.4, the manufacturer makes a higher profit when dealing with the decentralized retailers at $s = p$

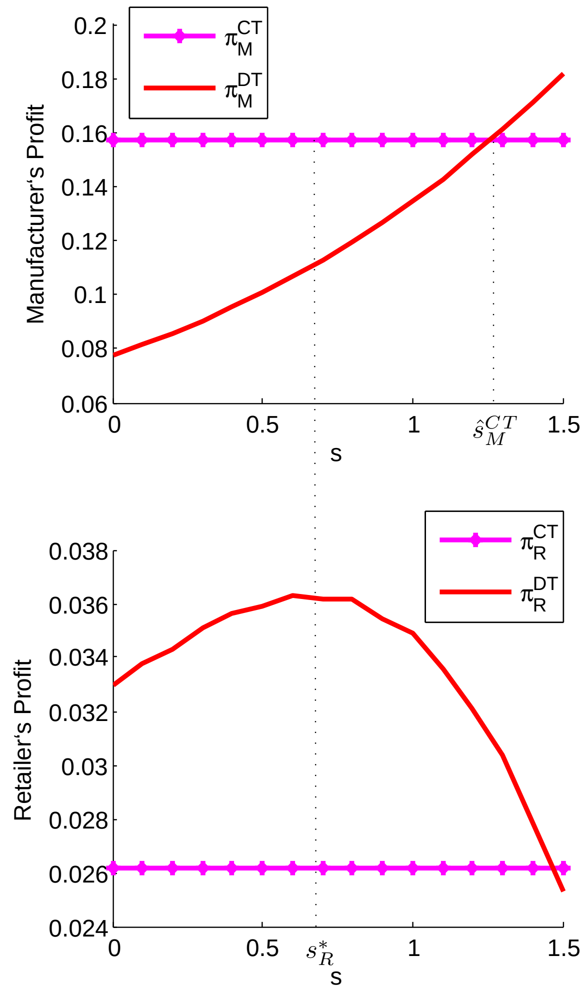

Figure 3.11: Firms' profits ($p = 1.5, c = 1, \xi_i \sim \text{Uniform}[0, 1]$)

than when dealing with the chain store. Therefore,

Proposition 3.5 *When the manufacturer controls the transshipment price, it always prefers dealing with decentralized retailers.*

For uniform distribution on $[0, b]$, at $s = p$ each decentralized retailer makes less profit than a location of the chain store. For some other distributions, such as truncated normal, the decentralized retailer may make a higher profit than a location of the chain store.

Table 3.2 summarizes the results of the section.

Table 3.2: Decentralized retailers vs. a chain store

Stage	*w*	*s*	Manufacturer (M)	Retailers
3	Fixed	Fixed	Prefers a chain store at low *s*; prefers competing retailers at high *s*	Better off by forming a chain store
2	M sets *w*	Fixed	Prefers a chain store at low *s*; prefers competing retailers at high *s*	Better off being decentralized for most transshipment prices (numerical result)
1		Competing Retailers set *s*	Prefers a chain store (numerical result)	Better off being decentralized (analytical result for Uniform [0,b])
		M sets *s*	Prefers competing retailers	Depending on demand distribution and critical fractile, $(p-c)/p$

3.6 Discussion

In this section, we discuss some assumptions that we make in the model and suggest some future research directions. First, in this chapter we assume that the retail price is fixed. However, in the real world, the retailers often make price decisions as well. We are aware of some cases where the manufacturer does not allow retailers to transship because of the possible negative impact of transshipment on retailers' pricing decisions. (This is obtained from the personal communication with Enver Yücesan during his visit at UBC in 2008.) Future research should incorporate retailers' pricing decisions and examine whether the players' incentives for transshipment will change. (Dong and Durbin (2005) implicitly incorporate the retail price of a product, which is determined by a market clear mechanism in a secondary market.)

Future research may also explore the case of a larger number of retailers. Since there can be multiple over-stocked retailers as well as multiple under-stocked retailers, the model needs to specify the allocation rule regarding how over-stocked retailers transship residual inventory to satisfy the under-stocked retailers' residual demand.

Another simplifying assumption of the model is that the retailers are symmetric. Suppose the retailers are asymmetric, then it is natural to expect that the transshipment prices charged by the two firms are not necessarily equal. There will be more cases in terms of who determines transshipment parameters. For instance, the retailers may still jointly determine both transshipment prices; or an individual retailer may set its own transshipment price. However, we conjecture that the insights from the current analysis will still hold.

Moreover, an alternative sequence of events is that the manufacturer sets the wholesale price before the firms determine the parameters of the transshipment decision. If the manufacturer has full control of the transshipment parameters, then the current results remain unchanged, since stage 1 and stage 2 are indeed one stage.

If the (decentralized) retailers determine transshipment parameters, from Lemma 3.5, in stage 2 it is always profitable for the (decentralized) retailers to transship. The (decentralized) retailers will set the transshipment price at $\tilde{s}_M^{CT}$ for a given wholesale price w, and thereby obtain the chain store's profit. Hence it is equivalent to analyzing the problem where a manufacturer deals with a chain store, which has been studied by Dong and Rudi (2004) and Zhang (2005). From Dong and Rudi (2004) and Zhang (2005) and our previous analysis, we can obtain similar results under the alternative sequence of events described above. That is, the firms' benefit from transshipment depends on their control of transshipment parameters. Under this alternative model, however, we will lose the comparative statics with respect to the transshipment price, which is crucial for us to understand the underlying cause for the firm's incentives. Hence we focus on the current model which provides us with valuable insights.

Finally, our single-period model fits industries with long production lead time and short life cycles such as fashion goods. An interesting extension is to utilize a multi-period model to examine the incentive issues in other cases.

3.7 Concluding Remarks

In this chapter, we investigate the incentives for transshipment in a supply chain where the manufacturer sells through decentralized retailers. The key take-away from this chapter is that both the manufacturer and retailers can be harmed by transshipment. The firms' control of the parameters of the transshipment decision determines whether they benefit from or are hurt by transshipment.

We also compare the supply chain with decentralized retailers and that with a chain store. When the manufacturer controls the transshipment price, it prefers dealing with decentralized retailers. When the decentralized retailers control the transshipment price, they make more profits than the chain store; and the manufacturer may prefer dealing with the chain store.

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Chapter 4

TRANSSHIPPING TO THE GRAY MARKET: THE IMPACT ON A DECENTRALIZED SUPPLY CHAIN[1]

4.1 Introduction

A gray market refers to the sale of a product by a distributor not authorized by the manufacturer. Unlike a black market which sells counterfeits, the goods in a gray market are "genuinely branded merchandise" (Bucklin, 1993). The goods in a gray market may be imported from another country (if the manufacturer has a globalized market), or diverted from domestic authorized channels. The former is also referred to as parallel importation.

The major cause of parallel importation is the manufacturer's price discrimination between countries, which creates an arbitrage opportunity. There can be several sources for a domestic gray market: authorized dealers who order in bulk to take advantage of the manufacturers' quantity discounts, manufacturers who rely on volume to drive down their production costs, authorized dealers who are eager to salvage unsold inventories by the

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end of a selling season.

Gray markets emerge in a broad variety of industries: personal computers, cell phones, automobiles, apparel and footwear, pharmaceuticals, cosmetics, watches, cameras and publishing, among many others. In the United States, there are 7 billion to 10 billion dollars worth of products sold in gray markets every year (Cespedes et al., 1988). In the European Union, gray market sales have grown to 7.4 billion euros. Gray market activities in developing countries are even more widespread and growing rapidly (Antia and Bergen, 2004). A gray market distributor can be a discount house, a mail-order retailer, a web-based seller, or even large supermarket chains such as K-Mart and Tesco (Berman, 2004; Lowe and McCrohan, 1988; Antia and Bergen, 2004).

A gray market can be harmful to the manufacturer and the authorized dealers in many ways. A gray market typically sells the goods in discount, which has a negative impact on the manufacturer's pricing strategy. It erodes the sales in the authorized channel. For example, in the 1980's, up to 21% of the Mercedes-Benz's cars old in the United States were purchased in gray markets (Lowe and McCrohan, 1989). A gray market may also cause the loss of consumers' goodwill and damage the brand image.[2] Many articles view gray markets as completely negative and suggest ways for manufacturers and authorized dealers to "cure" this "disease" (see, e.g., Berman (2004); Myers and Griffith (1999)).

Despite the obvious harm that can be done by a gray market, some industrial practitioners such as IBM tolerate it (Baerji, 1990). A survey of 460 manufacturers shows that the firms' tolerance of a gray market varies from 0 to 50% (0 and 100% indicate no and full tolerance respectively) (Bergen et al., 1998). Firms tolerate gray market activities not only because of the cost of terminating them. A recent stream of qualitative research claims that a gray market may be beneficial to manufacturers. For instance, a gray market may increase total sales through reaching untapped consumer

[2]The legal status of a gray market has not been certain, as it depends on the judicial system and the court's interpretation of the law (see Gallini and Hollis (1999); Duhan and Sheffet (1988)).

segments, offer market intelligence for the firms to learn about markets, and reduce firms' inventory risk (Antia and Bergen, 2004).

In this chapter, we examine the impact of a gray market on the firms in a decentralized supply chain. We characterize conditions under which a gray market benefits and hurts the different players. This turns out to be a result of the tradeoff between a gray market's negative and positive impact. While a gray market causes a demand loss of the authorized channel, it also reaches a new consumer segment that has not been tapped by the authorized channel. (In an extension to stochastic demand (Section 4.7), we also recognize a gray market's "risk reducing effect"; that is, it can be used as a channel to salvage the unsold inventory from the authorized channel.) We also recommend strategies for manufacturers to cope with a gray market under different conditions.

Ahmadi and Yang (2000) use an economics model to identify the gray markets' profitability to a manufacturer. The manufacturer, however, owns the authorized distribution channel. But in practice, gray markets are often observed in decentralized supply chains, where the downstream (authorized) distributors are independent from the upstream manufacturers. Conflicting incentives for gray markets may emerge due to firm's different objectives in a decentralized supply chain. For instance, manufactures such as Beanie Babies and Ty Inc., Caterpillar, Volvo and Chanel suffer from a profit loss as a result of their authorized retailers' transshipment to gray markets (Antia and Bergen, 2004). In some other cases, the manufacturers' profits are increased, but the authorized retailers' sales are eroded by gray markets (Lowe and McCrohan, 1988).

We start with a single-retailer model and examine the n-retailer case in an extension (Section 4.6). Since our n-retailer model assumes that the retailers are symmetric, there is no retail price difference among retailers. Our model, therefore, fits more closely the case of domestic gray markets (although we may argue that our model explains a part of the causes of parallel importation).

Although the gray market does benefit a manufacturer under certain conditions, it is not a distribution strategy which a manufacturer plans for.

Notice that an outlet store resembles a gray market. An outlet store typically sells the same products as a traditional store but offers a lower price and provides little service such as in-store demonstration and sales assistance. (An outlet store is often owned and directly operated by the manufacturer.) Then in the cases where a gray market is profitable, can the manufacturer simply extract gray market profits through outlet stores?

Notice that in many situations an outlet store is not feasible. For instance, a high-end brand being sold in an outlet store may cause premium customers' confusion and goodwill loss. Although the gray market could lead to similar problems, it is perceived as "further removed from the manufacturer" (Ahmadi and Yang, 2000).

In the rest of the chapter, we review the literature that is related in the next section. We then describe the model in Section 4.3. In Section 4.4, we analyze the impact of the gray market on a benchmark case, an integrated supply chain. Then in Section 4.5 we focus on examining the impact of the gray market on a decentralized supply chain. In Sections 4.6 and 4.7, we consider two extensions to the main model: the cases of n retailers and stochastic demand. Finally in Section 4.8, we provide concluding remarks.

4.2 Literature Review

4.2.1 Qualitative Research on Gray Markets

There is a substantial body of qualitative research on gray market, which describes the sources of a gray market, the harm and potential benefit of a gray market, the firms' reaction to a gray market, and provides remedies and strategies for firms to cope with a gray market (see, e.g., Cespedes et al. (1988), Lowe and McCrohan (1988), Myers and Griffith (1999), Berman (2004) and Antia and Bergen (2004)).

4.2.2 Quantitative Research on Gray Markets

In contrast to the huge body of qualitative papers (see above), the quantitative research on gray markets is extremely sparse. Several papers use economics models to examine a gray market. Ahmadi and Yang (2000) consider the case of parallel imports between two countries, which arise due to the manufacturer's differentiated pricing policy in the two countries. They assume that the manufacturer centrally decides the retail prices in the two countries. A single parallel importer (multiple importers are considered in an extension) arbitrages by importing the product from the lower-priced country to the higher-priced country. They conclude that in some cases the parallel imports increase the manufacturer's profit. The explanation is that parallel imports create another version of the product in the higher-priced country, which enables product differentiation and consequently price discrimination.

Fargeix and Perloff (1987) consider the competition between a manufacturer's product and the products imported from a foreign country. However, the foreign products are made by other companies. Therefore, these foreign products are not really gray market goods. Instead, they are just low-cost substitutes of the manufacturer's product (these foreign products might constitute black markets though). The competition of the foreign products lowers the retail price of the manufacturer's product and also lowers the services that the manufacturer's retailers provide due to free-riding. It turns out that if the manufacturer raises wholesale price, it may benefit from the foreign competition; and the social welfare may also increase as a result of the foreign competition.

Similar to Fargeix and Perloff (1987), Baerji (1990) considers a competitive fringe which produces a perfect substitute for the manufacturer's product. A single retailer carries the product of the competitive fringe and competes with the manufacturer's authorized retailers. Again, this competitive fringe does not constitute a gray market, since a gray market product is the "genuinely branded merchandise" of the manufacturer (Bucklin, 1993). These two models (Fargeix and Perloff (1987) and Baerji (1990)) essentially analyze interbrand competition rather than gray markets.

Empirical work on gray markets has also been limited. A major reason is that the data of gray markets are difficult to obtain, due to the confidential nature of firms' reactions to gray market activities. Antia et al. (2006)

conduct a survey of US-based manufacturers of personal care products. They find that the deterrence of gray market activities is only effective when the punishment of gray market activities is conducted severely, speedily and with certainty. Huang et al. (2004) use survey data to examine consumers' attitude toward gray market goods. They show that consumers' intention of purchasing gray market goods are affected by consumers' risk averseness and"high price, high quality; low price, low quality" inferences, etc.

4.2.3 Exclusive Territories

Gray markets often accompany exclusive territories. The manufacturer assigns retailers' exclusive territories in order to eliminate the competition between retailers. However, the retailers often violate the arrangement and sell in the other retailers' territories, which is referred to as "bootlegging". Bootlegging constitutes a gray market, since it is sales that are not permitted by the manufacturer. There are extensive economics papers on exclusive territories. Here we only list two papers that examine the *violation* of exclusive territories. Dutta et al. (1994) build an analytical model where the retailers may bootleg. The optimal policy of the manufacturer is to tolerate the retailers' bootlegging to a certain extent. Bergen et al. (1998) empirically verify that there exist manufacturers who tolerate bootlegging. And they find the factors that influence the degree of tolerance include the free-riding potential, the difficulty of detecting the violation, and the maturity of the product in its life cycle.

4.2.4 Management of Inventory Risk

In many manufacturing industries, it is important and necessary for retail stores to hold inventory in order to meet variable consumer demand. However, the unsold inventory at the end of a selling season often has limited value and the disposal of unsold inventory can be costly. The gray market can be a channel to salvage the excess inventory and hence reduce the retailers' inventory risks. Therefore, our research is also related to the operations literature on inventory management.

In particular, Lee and Whang (2002) is the quite relevant to this chapter. They study a secondary market where retailers who have excess inventory trade with retailers who have unsatisfied demand. However, a secondary market is typically legal and permitted by the manufacturer, which is different from a gray market. The major differences between their model and ours are that Lee and Whang (2002) do not consider the manufacturer's decision in the game, and that the retailers do not make price decisions.

4.3 Model

Consider the following scenario. A monopoly manufacturer produces a product at a per unit production cost c, and it distributes the product through a single retailer. (In Section 4.6, we extend the model to n retailers.) We start our analysis with an integrated supply chain as a benchmark, where the manufacturer controls the retailing function. We then consider the decentralized supply chain where the retailer is independent from the manufacturer. In the decentralized case, the manufacturer offers a linear wholesale price contract, w, to the retailer (see the diagram of the decentralized supply chain in Figure 4.1).

In the absence of the gray market, the retailer faces a linear demand, $D_N = 1 - \alpha p$ in the "primary market", where subscript "N" refers to "no gray market".[3] (In the main model, demand is deterministic; we discuss the case of stochastic demand in Extension 2 in Section 4.7.) When the product is also sold in the gray market, the retailer's primary market suffers a demand loss, l; so the retailer's demand in the primary market becomes $D_G = 1 - \alpha p - l$, where subscript "G" refers to "in the presence of a gray market". For the ease of analysis, we assume l is a constant. (There could be other ways to model the "demand loss"; see Section 4.8 for a discussions.)

The demand in the gray market is denoted by $\delta = 1 - \beta g$, where g is the price of the product in the gray market. Note that α and β represent the

[3]We assume linear demand function for analytical tractability. A linear demand function comes from the case where consumers have a quadratic untility function and large enough incomes (Vives, 1999).

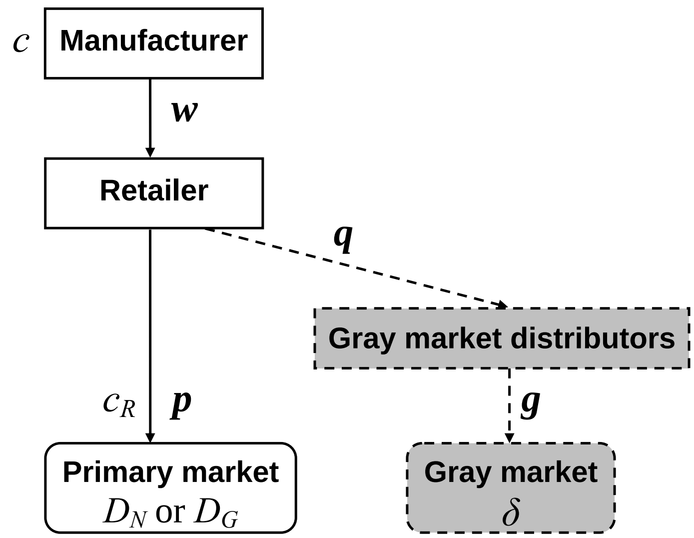

Figure 4.1: Decentralized supply chain diagram

price sensitivities in the two markets. Since the demand in the gray market is more price sensitive than that in the primary market, we assume $\beta > \alpha$. To avoid trivial solutions, we also make the following assumptions:

$$1 - l - \alpha c - \alpha c_R > 0; \tag{4.1}$$

otherwise it is always unprofitable to sell in the primary market in the presence of the gray market; and

$$1 - \beta c > 0; \tag{4.2}$$

otherwise it is always unprofitable to sell in the gray market.

Assume that there are infinite number of unauthorized distributors in the gray market and each makes zero profit under perfect competition. Hence these unauthorized distributors can be left out of the analysis of the game. The authorized retailer influences the gray market price through the quantity that it transships to the gray market, rather than setting the selling price in the gray market directly. So in addition to the retail price at the authorized store, p, the retailer also determines the transshipment quantity to gray market, q, which in turn determines a market clearing price in the gray market. We make the following assumption to ensure $p > g$, i.e., the price in the gray market is lower than the price in the primary market.

$$\beta(1 - l + \alpha c_R) - \alpha > 0 \tag{4.3}$$

From (4.1) and (4.3) we obtain the upper bound of l:

$$l < \min\{1 - \alpha c - \alpha c_R, (\beta + \alpha\beta c_R - \alpha)/\beta\} \equiv l_U. \tag{4.4}$$

That l exceeds the upper bound will lead to an unrealistic outcome: the retailer sells exclusively in the gray market; or it sells in both markets but the gray market price exceeds the primary market price.

The retailer incurs a per unit cost at his authorized store $c_R \geq 0$ for providing services. We normalize the corresponding cost for selling in the gray market to zero, which implies that the gray market sales provide no service.

4.4 Integrated Supply Chain

Consider, first, a vertically integrated supply chain, where the retailer is owned by the manufacturer. The manufacturer sets the retail price in the primary market. It also determines whether to transship the product to the gray market, and if it transships, the quantity to be transshipped to the gray market.

We first consider the case where the manufacturer never sells in the gray market (referred to as "NGM" hereafter). (For example, the manufacturer is unaware of the existence of the gray market, or it is unable to reach the gray market distributors, etc.) The supply chain profit is simply its profit in the primary market:

$$\Pi_N = (p - c - c_R)(1 - \alpha p). \tag{4.5}$$

We can easily solve (4.5) and obtain the optimal profit of the supply chain:

$$\Pi_N^* = \frac{(1 - \alpha c - \alpha c_R)^2}{4\alpha}. \tag{4.6}$$

Now consider the case where the manufacturer considers the gray market as an option (referred to as "GM" hereafter). The manufacturer may choose not to sell in the gray market; then it obtains the profit in (4.6). If the manufacturer chooses to sell in the gray market, the profit is the sum of the profits in the primary market and gray market:

$$\Pi_G = (p - c - c_R)(1 - \alpha p - l) + (g - c)q, \tag{4.7}$$

where the price in the gray market g is determined by a market clearing mechanism, i.e., supply equals demand, or

$$1 - \beta g = q \Rightarrow g = \frac{1 - q}{\beta}. \tag{4.8}$$

Maximizing (4.7) over p and q yields

$$\tilde{\Pi}_G = \frac{[1 - l - \alpha(c + c_R)]^2}{4\alpha} + \frac{(1 - \beta c)^2}{4\beta}. \tag{4.9}$$

Note that $\tilde{\Pi}_G$ is not necessarily the optimal profit under GM, so we use "$\sim$" instead of "$*$" to indicate it. Denote by Π_G^* the supply chain's optimal profit under GM, i.e., $\Pi_G^* = \tilde{\Pi}_G$ if $\tilde{\Pi}_G > \Pi_N^*$, and $\Pi_G^* = \Pi_N^*$ if $\tilde{\Pi}_G \leq \Pi_N^*$. The integrated supply chain makes a greater profit under GM than under NGM if and only if

$$\tilde{\Pi}_G - \Pi_N^* > 0 \tag{4.10}$$

$$\Leftrightarrow \beta l^2 - 2\beta(1 - \alpha c - \alpha c_R)l + \alpha(1 - \beta c_R)^2 > 0. \tag{4.11}$$

Condition (4.11) involves several parameters. In order to understand the intuition, notice that the left hand side of (4.11) is a parabola in l. Let it equal zero and we obtain two roots. Define l_{SC} as the smaller root ("*SC*" stands for supply chain), i.e., $l_{SC} \equiv 1 - \sqrt{(1-\alpha c-\alpha c_R)^2 - \alpha(1-\beta c)^2/\beta} - \alpha c - \alpha c_R$. (The larger root is greater than l_U and therefore ignored from Assumption (4.4).) So we have $\tilde{\Pi}_G - \Pi_N^* > 0 \Leftrightarrow l < l_{SC}$. Thus,

Proposition 4.1 *If $l < l_{SC}$, then the integrated supply chain is strictly better off from selling into the gray market. If $l \geq l_{SC}$, it chooses not to sell in the gray market and hence is no worse off.*

(See the proof in Appendix C.)

Since in an integrated supply chain the manufacturer can always choose not to sell in the gray market, it will never be worse off under GM. However, if it always ignores the gray market, in some cases it may lose the additional profit from the gray market.

We plot l_{SC} in Figure 4.2: (a) shows l_{SC} in an l–β space and (b) in an l–c_R space. (The rest of the parameters are fixed respectively.)

From Figures (a) and (b), the gray market is profitable if l is small, β is small, and c_R is large, which results from the tradeoff between the negative and positive effects of the gray market. In our model, the gray market's negative effect is the demand loss in the primary market, represented by parameter l. The positive effect of the gray market is the profit generated by the gray market, which depends on parameters c_R and β. We further decompose the positive effect into (1) the "cost advantage" and (2) the "volume effect".

(1) "Cost advantage". The gray market reaches the price-sensitive customers who do not care much about the service provided in the authorized store. This gives the gray market an advantage of saving the cost c_R. Therefore, although the gray market price is lower than the price in the primary market, the gray market margin may not be very low. (If c_R is sufficiently large, the gray market margin may exceed the primary market margin.) The larger c_R is, the stronger incentive the supply chain has for selling into the gray market. (2) "Volume effect". Parameter β indicates the price sensitivity of the gray market. The smaller β is, the more capable the gray market is of generating sales, and the more attractive it is to the supply chain.

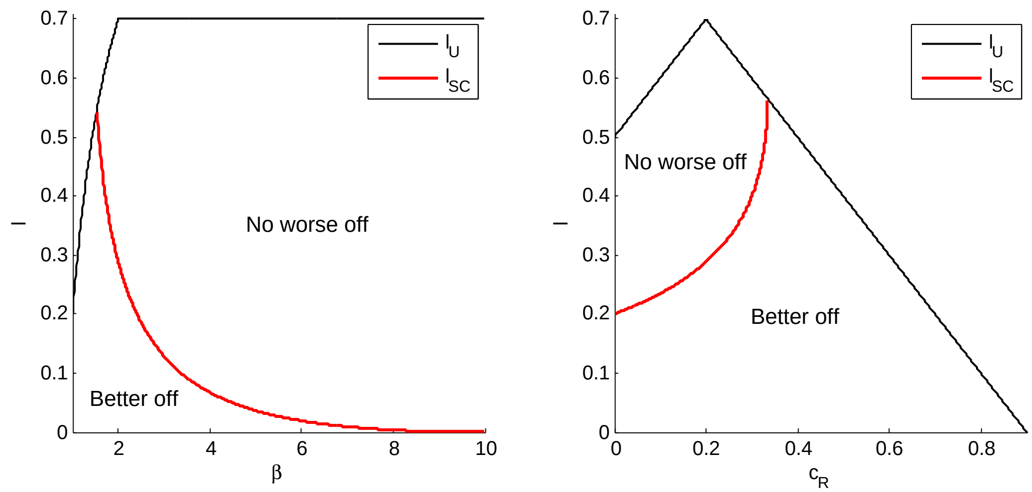

Figure 4.2: Integrated supply chain ($c = 0.1, \alpha = 1$; (a): $c_R = 0.2$; (b): $\beta = 2$)

If the cost advantage and the volume effect dominate the demand loss in the primary market, the gray market benefits the supply chain.

4.5 Decentralized Supply Chain

An integrated supply chain can exploit the gray market when it is beneficial, and avoid it if it is harmful. However, in a decentralized supply chain, since the members have different incentives, the impact of a gray market is not as straightforward. It is possible, e.g., that the retailer may benefit from the gray market to the detriment of the manufacturer. We start our analysis by formulating the firms' profit functions. Under NGM, the retailer sets the retail price in the primary market to maximize its own profit (the retailer's profit is denoted by π and the manufacturer's profit is denoted by Π):

$$\max_{p} \pi_N = (p - w - c_R)(1 - \alpha p). \tag{4.12}$$

As the leader of the Stackelberg game, the manufacturer solves

$$\max_{w} \Pi_N = (w - c)(1 - \alpha p). \tag{4.13}$$

The equilibrium under NGM is as follows:

$$\begin{aligned} w_N^* &= \frac{1 + \alpha c - \alpha c_R}{2\alpha},\ p_N^* = \frac{1 + \alpha w_N^* + \alpha c_R}{2\alpha}, \\ \Pi_N^* &= \frac{(1 - \alpha c - \alpha c_R)^2}{8\alpha},\ \pi_N^* = \frac{(1 - \alpha c - \alpha c_R)^2}{16\alpha}. \end{aligned} \tag{4.14}$$

If the retailer decides to transship to the gray market, the retailer also determines the quantity that it transship to the gray market q:

$$\max_{p,q} \pi_G = (p - w - c_R)(1 - \alpha p - l) + (\frac{1 - q}{\beta} - c)q. \tag{4.15}$$

The manufacturer solves, correspondingly,

$$\max_{w} \Pi_G = (w - c)(1 - l - \alpha p + q). \tag{4.16}$$

Solving (4.15) and (4.16) yields

$$\begin{aligned} &\tilde{w}_G = \frac{2 - l + \alpha c - \alpha c_R + \beta c}{2(\alpha + \beta)},\ \tilde{p}_G = \frac{1 - l + \alpha(\tilde{w}_G + c_R)}{2\alpha},\ \tilde{q} = \frac{1 - \beta \tilde{w}_G}{2}, \\ &\tilde{g} = \frac{1 + \beta \tilde{w}_G}{2\beta},\ \Pi_G(\tilde{w}_G) = \frac{(2 - l - \alpha c + \alpha c_R - \beta c)^2}{8(\alpha + \beta)}, \\ &\pi_G(\tilde{w}_G) = \frac{(1 - l - \alpha \tilde{w}_G + \alpha c_R)^2}{4\alpha} + \frac{(1 - \beta \tilde{w}_G)^2}{4\beta}. \end{aligned} \tag{4.17}$$

Again, (4.17) is not necessarily the equilibrium under GM, for the same reason as in the integrated case, and therefore is again indicated by "$\sim$" instead of "$*$". We will first examine the equilibrium under GM and compare it with the equilibrium under NGM in two special cases ($l = 0$ and $c_R = 0$); then we analyze the general case in Section 4.5.3.

4.5.1 Special Case 1: $l = 0$

We first examine a special case where $l = 0$. That is, the demand in the primary market is not affected by the existence of the gray market. Thus the gray market does not have any direct negative impact on the system. As a result, both the manufacturer and retailer will not be hurt by the gray market.

Proposition 4.2 *If $l = 0$, then both the manufacturer and retailer are no worse off from gray market.*

(See the proof in Appendix C.)

When $l = 0$, both the manufacturer and retailer can be strictly better off from gray market; but this is not always the case. We first look at the manufacturer's profit and it is easy to obtain

$$\begin{aligned} &\Pi_G(\tilde{w}_G) > \Pi_N(w_N^*) \\ \Leftrightarrow &\alpha c^2\beta^2 - (1 + 2\alpha c - 2\alpha c_R - \alpha^2 c^2 + \alpha^2 c_R^2)\beta + \alpha(3 - 2\alpha c - 2\alpha c_R) > 0. \end{aligned} \tag{4.18}$$

Condition (4.18) does not involve l as $l = 0$. So we transform it into

$$\begin{aligned} \beta < &\frac{1 + 2\alpha c - 2\alpha c_R - \alpha^2 c^2 + \alpha^2 c_R^2}{2\alpha c^2} \\ &- \frac{(1 - \alpha c - \alpha c_R)}{2\alpha c^2}\sqrt{(1 - \alpha c - \alpha c_R)^2 + 8\alpha c - 4\alpha^2 c_R^2 c} \equiv \beta_M. \end{aligned} \tag{4.19}$$

and have

Proposition 4.3 *Suppose $l = 0$.*

(a) If market condition $\beta < \beta_M$ is satisfied, then the manufacturer sets $w = \tilde{w}_G$ and the retailer transships to the gray market; the manufacturer and retailer are strictly better off from the gray market.

(b) If $\beta \geq \beta_M$, then the manufacturer sets $w = w_N^*$ and the retailer does not transship to the gray market; the manufacturer and retailer obtain the profits under no gray market.

(See the proof in Appendix C.)

In Figure 4.3, both firms are better off from the gray market in the region below β_M. This implies that when c_R is small (the cost advantage is weak), the gray market is profitable only if β is sufficiently low to generate enough volume. Similarly, when β is large, the gray market is profitable only if c_R is sufficiently large. In sum, the overall positive effect must be significant enough for the gray market to be profitable.

Note that in the region above the threshold β_M in Figure 4.3, the optimal wholesale price for the manufacturer is not $\tilde{w}_G$. This is because, if the manufacturer sets $w = \tilde{w}_G$, the retailer will actually transship zero units to the gray market. (In (4.17), $\tilde{q} < 0$ because β is large.) Therefore, the maximum profit that the manufacturer can get is Π_N^*. Hence, the manufacturer sets $w = w_N^*$ and obtains the profit under NGM.

4.5.2 Special Case 2: $c_R = 0$

In this special case, the retailer's marginal cost at the authorized store c_R is zero, which represents the industries that do not require services and sales effort at authorized stores. This implies that the gray market does not have the cost advantage. Whether the gray market is beneficial only depends on the tradeoff between the volume effect and the demand loss in primary market. Nevertheless,

Proposition 4.4 *If $c_R = 0$, then both the manufacturer and retailer are no worse off from the gray market.*

(See the proof in Appendix C.)

That $c_R = 0$ means the margin in the gray market can never exceed the margin in the primary market as the gray market price is always lower than the primary market price. For the retailer to have an incentive to transship, the gray market must be able to generate enough volume, which in turn calls

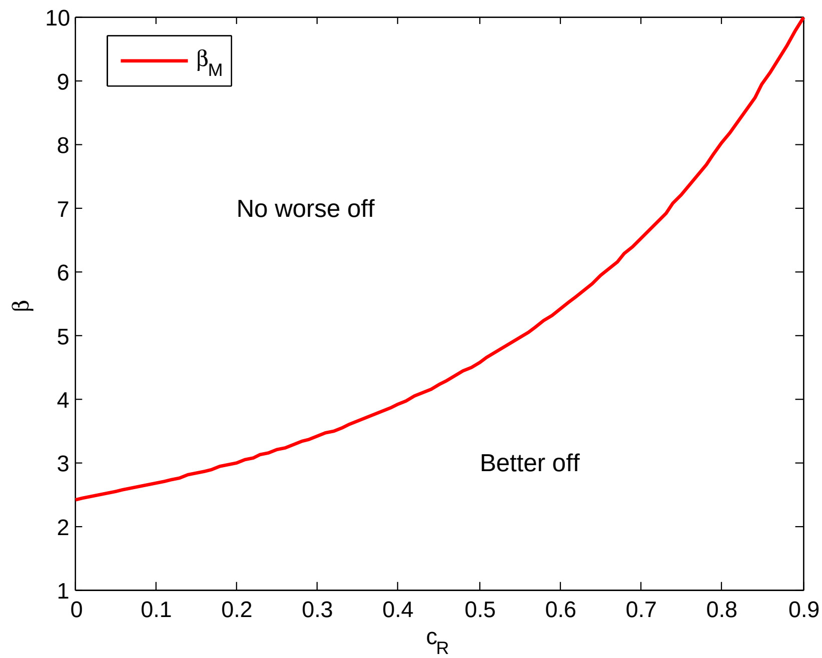

Figure 4.3: Manufacturer's threshold in special case 1 ($l = 0, c = 0.1, \alpha = 1$)

for a large retailer order. Thus the retailer's incentive becomes consistent with that of the manufacturer's. As a result, the firms either both prefer the gray market, or both do not prefer the gray market.

Then under what conditions do the firms prefer transshipping to the gray market? Examine the manufacturer's profit first. Using the same technique as before, define l_M as the smaller root of $\Pi_G(\tilde{w}_G) - \Pi_N(w_N^*) = 0$. (The other root is greater than l_U and therefore ignored.) And we have $\Pi_G(\tilde{w}_G) > \Pi_N(w_N^*)$ if and only if $l < l_M$,

Then we look at the retailer's profit. Let $\pi_G(\tilde{w}_G) - \pi_N(w_N^*) = 0$ and obtain two roots. Denote the one which is no greater than l_U by l_{RL}, then we have $\pi_G(\tilde{w}_G) > \pi_N(w_N^*)$ if $l < l_{RL}$.

We can show that threshold l_M is tangent to l_{RL} and $l_M < l_{RL}$ (see Figure 4.4). For $c_R = 0$, l_U intersects l_M and l_{RL} at their tangency point (see Figure 4.4). Therefore, for $l < l_M \leq l_{RL}$ both firms prefer selling in the gray market. For $l \geq l_{RL} \geq l_M$, both firms prefer NGM.

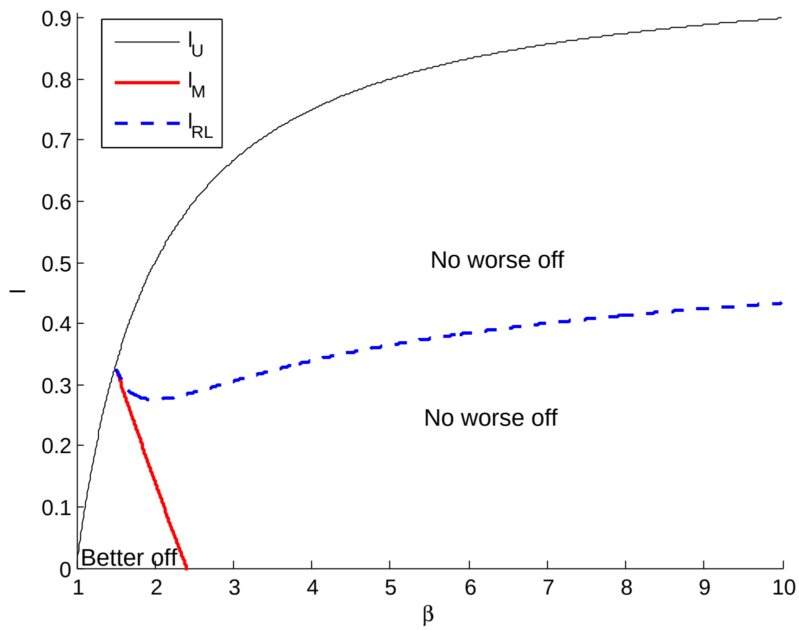

Figure 4.4: Manufacturer's threshold in special case 2 ($c = 0.1, c_R = 0, \alpha = 1$)

For $l \in (l_M, l_{RL})$, the retailer prefers the gray market since $\pi_G(\tilde{w}_G) > \pi_N(w_N^*)$. However, if the manufacturer sets $w = \tilde{w}_G$, the manufacturer obtains less profit than under NGM. Nevertheless, if the manufacturer sets $w = w_N^*$, the retailer will choose NGM (by Proposition 4.4). This means that for $l \in (l_M, l_{RL})$ both firms are no worse off as well. We summarize the analysis in the following proposition:

Proposition 4.5 *Suppose $c_R = 0$ and focus on the region where $l \leq l_U$.*

(a) If $l < l_M$, then the manufacturer sets $w = \tilde{w}_G$ and the retailer transships to the gray market. The manufacturer and retailers are strictly better off from the gray market.

(b) If $l \geq l_M$, then the manufacturer sets $w = w_N^*$ and the retailer does not transship to the gray market. The manufacturer and retailer obtain the

profits under no gray market.

Proposition 4.5 is shown in Figure 4.4. The intuition is straightforward: The gray market is beneficial only if the demand loss l is small and the gray market demand is not very price sensitive (small β).

In the above two special cases ($l = 0$ or $c_R = 0$), the manufacturer is either strictly better off from the gray market, or no worse off by setting the wholesale price to w_N^* to prevent the retailer from transshipping to the gray market. However, we will show next in the general case, the manufacturer may be hurt by the gray market.

4.5.3 General Case

In the general case, l and c_R are not necessarily zero. A similar result to the special cases is that the firms may benefit from the gray market. However, the manufacturer can also be hurt by the gray market. Define l_M in the same fashion as in Section 4.5.2. Then for $l < l_M$ the manufacturer prefers GM and for $l \geq l_M$ the manufacturer prefers NGM.

Next consider the retailer's profit. Now both roots of equation $\pi_G(\tilde{w}_G) - \pi_N(w_N^*) = 0$ (in terms of l) are no greater than l_U. Denote them as l_{RL} and l_{RH} ($l_{RH} \geq l_{RL}$). For $l_{RL} \leq l \leq l_{RH}$, we have $\pi_G(\tilde{w}_G) \leq \pi_N(w_N^*)$; i.e., the retailer prefers NGM. Otherwise, the retailer prefers GM.

Again, l_M is tangent to l_{RL} and $l_M \leq l_{RL}$. Therefore, for $l < l_M$, we must have $l < l_{RL}$, i.e., both firms prefer the gray market. If $l \geq l_M$, the manufacturer prefers NGM. However, if the manufacturer tries to deter the retailer from transshipping to the gray market by setting $w = w_N^*$, the deterrence may not be successful.

Define l_N as the larger root of $\pi_G(w_N^*) - \pi_N(w_N^*) = 0$ and we have

Proposition 4.6 *Focus on the region where $l \leq l_U$.*

(a) If $l < l_M$, then the manufacturer sets $w = \tilde{w}_G$ and retailer transships to the gray market. The manufacturer and retailer are strictly better off from the gray market.

(b) If $l_M \le l \le l_N$, then the manufacturer sets $w = w_N^*$ and the retailer does not transship to the gray market. Hence, the manufacturer and retailer obtain the profits under no gray market.

(c) If $l > \max(l_M, l_N)$, then the manufacturer will be hurt by the gray market.

(See the proof in Appendix C.)

Again, we plot the thresholds l_M and l_N against β and c_R in Figures 4.5 and 4.6. First note that both figures show that the case where the manufacturer is worse off from the gray market happens when l is sufficiently large, i.e., the negative effect is big enough.

What is more, Figure 4.5 shows that if β is sufficiently small, the gray market's volume effect benefits both firms. If β is very large, both firms prefer no gray market. However, for the intermediate values of β (and sufficiently large l), the manufacturer and retailer have different incentives: The retailer is attracted by the gray market and transships even under w_N^*. And the manufacturer's profit is reduced by the negative effect of gray market. (Numerically we find that the retailer is still no worse off in this case.)

Similarly, Figure 4.6 illustrates the firms' incentives in terms of c_R. If c_R is sufficiently large, both firms benefit from the cost advantage of gray market. If c_R is small, both firms find the gray market unattractive. If c_R is of intermediate values, the manufacturer suffers from the incentive misalignment.

In the case where the manufacturer is worse off, it can adopt mechanisms other than the wholesale price contract to deter or limit gray market. As indicated in Section 4.4, vertical integration is a way to eliminate the harm of the gray market. Alternatively, the manufacturer can employ coordinating mechanisms to align the retailer's incentives. (Under our single-retailer deterministic model, a two-part pricing contract is sufficient for supply chain coordination.) When the retailer is coordinated under such a contract, the manufacturer can achieve the profit of the integrated supply chain.

If the supply chain cannot be coordinated, in order to eliminate or re-

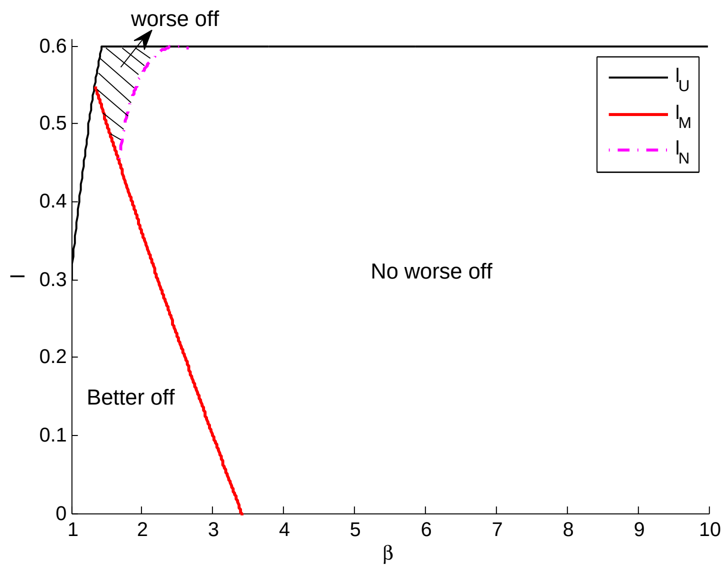

Figure 4.5: Manufacturer's threshold in the general case (a) ($c = 0.1, \alpha = 1, c_R = 0.3$)

duce the harm of the gray market, the manufacturer must strengthen the management and control of the distribution channel. It can apply penalty terms in the contract if any gray market activity is detected. For example, General Motors Corp. fines dealers who sell in the gray market to restrain such behaviors (Antia and Bergen, 2004). In practice, some manufacturers such as Kodak take even more severe actions when dealing with the gray market problem, e.g., terminating the dealership if a dealer is found to be transshipping to the gray market (Bezerra, 1998). Such terms, however, require that the manufacturer closely monitors dealers' activities and can be costly.

Table 4.1 summarizes the results for the case of a single retailer and deterministic demand.

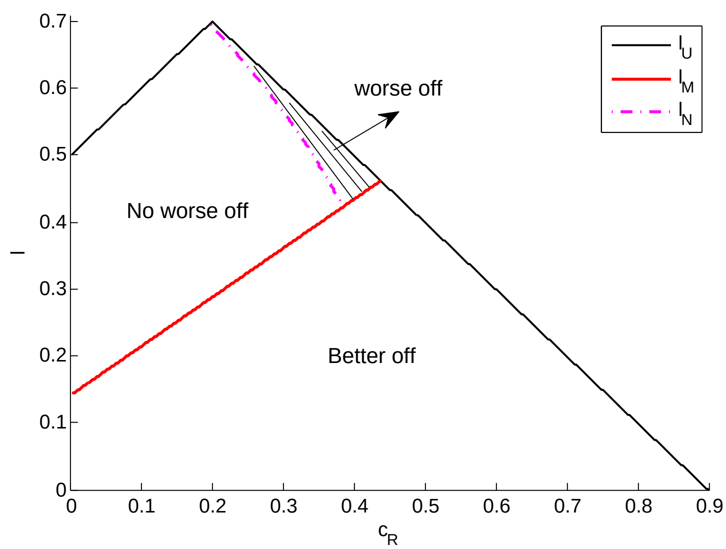

Figure 4.6: Manufacturer's threshold in the general case (b) ($c = 0.1, \alpha = 1, \beta = 2$)

Table 4.1: Impact of a gray market

Integrated supply chain		$l < l_{SC}$ Both firms (manufacturer and retailer) better off	$l \geq l_{SC}$ Both firms no worse off	
Decentralized supply chain	l=0	$\beta < \beta_M$ Both firms better off	$\beta \geq \beta_M$ Both firms no worse off	
	c_R=0	$l < l_M$ Both firms better off	$l \geq l_M$ Both firms no worse off	
	General case	$l < l_M$ Both firms better off	$l_M \leq l \leq l_N$ Both firms no worse off	$l > \max(l_M, l_N)$ Manufacturer worse off; retailer no worse off

4.6 Extension 1: n retailers

In many supply chains, the manufacturer has a network of downstream retailers. Consider the case where the manufacturer has n symmetric retailers and the manufacturer employs exclusive territories. That is, the retailers are only allowed to sell the product in their own territories but not others. Therefore the retailers do not compete head-to-head in their primary market. However, if the retailers transship the product to the gray market, competition may occur: the gray market distributors may buy from any retailer and resell in any retailer's territory.

For simplicity, assume that the total demand in a primary or a gray market is not increasing in the number of retailers.[4] In the absence of gray market, the single retailer's demand defined in Section 4.5 is evenly divided among n retailers, and each retailer faces demand $D_i = (1 - \alpha p_i)/n$ in its primary market. The gray market could be across the retailers' territories, since the gray market distributors are not restricted by the manufacturer's exclusive territories arrangement. Therefore, the overall gray market demand for *all* retailers is $\delta = 1 - \beta g$, where the gray market price g is again determined from the market clear mechanism. (In equilibrium, all retailers make the same decisions regarding whether to transship to the gray market due to the symmetry between them.) Once the gray market emerges, each retailer's demand in the primary market suffers a loss l/n, and the new demand is $D_i = (1 - l - \alpha p_i)/n$. Again, each retailer incurs a per unit cost c_R at its primary store; and we continue to assume (4.1), (4.2) and (4.3) to eliminate trivial outcomes.

In an integrated supply chain, all retailers are owned by the manufacturer and the manufacturer makes central decisions. Since the total demand is the same as in the single retailer case, the impact of the gray market is the same as before. That is, if $l < l_{SC}$ (where l_{SC} is defined in Section 4.5), the integrated supply chain is strictly better off from the gray market; otherwise, it does not sell in the gray market and is no worse off.

In the decentralized supply chain, under NGM, the retailers do not compete and each retailer shares $1/n$ of the single retailer's demand. Therefore, each retailer obtains $1/n$ of the single retailer's profit. The manufacturer's

[4] The total demand could increase in n due to the "retail network effect" (see Fargeix and Perloff (1987) for a model with such a consideration). In our model, we want to show that even without the total demand increasing effect, the manufacturer still enjoys a benefit from the gray market as a result of oligopoly retailers.

profit comes from the n retailers' orders and hence is the same as that in the single retailer case.

However, the emergence of the gray market introduces a (quantity) competition between the retailers and increases the retailers' order quantities. The manufacturer benefits from this competition. As the number of retailers n goes up, the manufacturer makes a higher profit under GM.

Proposition 4.7 *For given parameter values, there exist two thresholds* $1 \leq \hat{n}_1 \leq \hat{n}_2$ *such that*

(a) for $n < \hat{n}_1$ *the manufacturer is strictly worse off from the gray market; and*

(b) for $n > \hat{n}_2$ *the manufacturer is strictly better off from the gray market.*

(See the proof in Appendix C.)

This proposition implies that if the manufacturer is better off from the gray market dealing with n retailers, then it is also better off from the gray market dealing with more than n retailers. Figure 4.7 shows the manufacturer's thresholds l_M and l_N in the cases of one and three retailers respectively. As n increases, both l_M and l_N move upward. Thus the region where the manufacturer is strictly better off from the gray market (below l_M^n) is expanding, and the region where the manufacturer is worse off from the gray market (below l_N^n and above l_M^n) is shrinking.

Furthermore, as n approaches infinity, the manufacturer is never worse off from the gray market. This is because the retailers have perfect competition in gray market; the retailers make zero profit from the gray market, while the manufacturer extracts all the benefit from the gray market.

4.7 Extension 2: Stochastic Demand

In the main model we assumed that the retailer's demand is deterministic, which is a common assumption of the gray market literature in economics and marketing. The main model helps us understand the gray market's "cost advantage" and "volume effect" and the tradeoff with the "demand

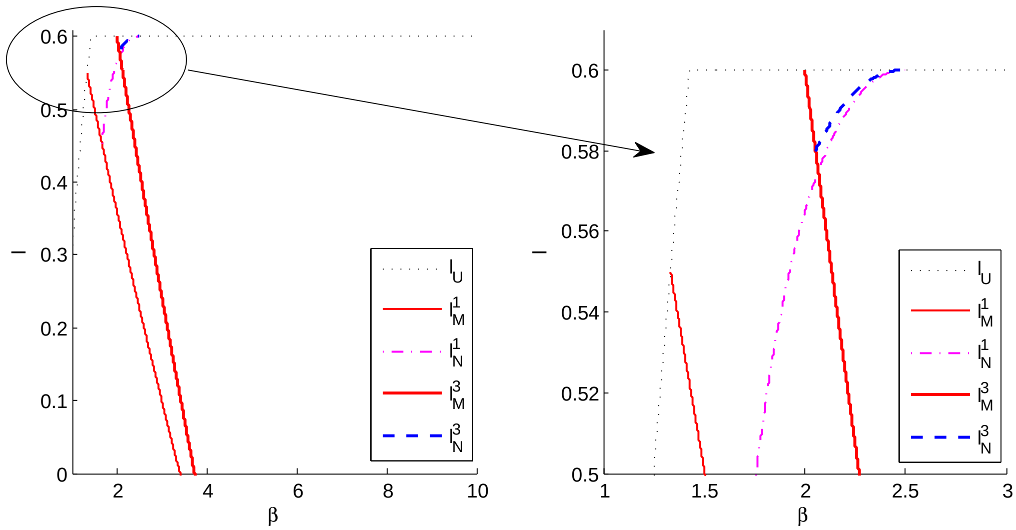

Figure 4.7: Manufacturer's thresholds with n retailers for $c = 0.1, \alpha = 1$, and $c_R = 0.3$ (From now on we only show thresholds such as l_M and l_N against β.)

loss" effect. However, in most industries, uncertainty in retail demand is commonplace and the management of retail inventory is well recognized. Some retailers use the gray market as a channel to salvage their leftover stocks. For example, near the end of a selling season, the retailers of brand apparel or footwear tend to dump their unsold inventories into a gray market, in order to save the inventory holding cost and empty the shelf space for the new season (Antia and Bergen, 2004). It turns out, therefore, that another benefit of the gray market is its ability to help retailers reduce inventory risk. The consideration of demand uncertainty in studying gray markets is an important extension of our main model.

In this section, we go back to the single retailer's setting and focus on the comparison between the results in stochastic and deterministic demand models. Assume that the "size" of the retailer's primary market is represented by a random variable ξ. That is, the demand in the primary market is $\xi(1-\alpha p)$ with no gray market and $\xi(1-l-\alpha p)$ with a gray market. (In order to incorporate uncertainty in the demand function, we use a multiplicative form. An alternative is an additive demand form, which, however, creates technical problems such as a negative demand. See details in Krishnan (2009).) Suppose ξ has mean 1 and a continuous density. The retailer orders from the manufacturer before ξ is realized; so with no gray market, the retailer is a price sensitive newsvendor. For simplicity, assume that the gray market demand $\delta = 1 - \beta g$ is still deterministic through the market clearing mechanism. Let the retailer's order quantity be y (y is for both markets). At the end of the period, any leftover inventory, which has not been sold in either market, has zero salvage value.

Under GM, the sequence of events is as follows (see Figure 4.8). The manufacturer first offers the wholesale price w. And then the retailer determines p and y. The market size ξ is realized, which gives the retailer a signal on the strength of the market. After observing ξ, the retailer decides whether to transship to the gray market, and if so, how much to transship to the gray market. Let v denote the realized value of ξ. If the retailer does not transship to the gray market, the demand in the primary market is $v(1-\alpha p) \equiv D_N$. But if the retailer transships to the gray market, the primary market loses some customers and the demand is $v(1-\alpha p-l) \equiv D_G$.

If the retailer does not transship, it sells $\min(y, D_N)$ in the primary market. If the retailer transships, assume that the retailer fills the primary market demand with priority, i.e., it sells $\min(y, D_G)$ in the primary market, and there are $(y-D_G)^+$ units left[5]. Due to the uncertainty of ξ, the leftover inventory $(y-D_G)^+$ could be large (up to y). It may not be optimal to transship all leftover inventory to the gray market. Since the gray market demand is deterministic, we can actually calculate the optimal quantity to transship into the gray market. Recall that in Section 4.5 we calculated $\tilde{q}$ (4.17) as the optimal quantity for gray market. However, in the stochastic model, at the time when the retailer determines q, the purchasing cost (w) is sunk; so the retailer needs to choose a quantity q to maximize the *revenue* in the gray market, $(1-q)/\beta \cdot q$. It is easy to obtain that the optimal quantity

[5] Notation x^+ represents the positive part of x, i.e., $\max(x, 0)$.

is 1/2. The inventory left may be greater than or less than 1/2. So the retailer transships $\min((y - D_G)^+, 1/2)$ to the gray market.

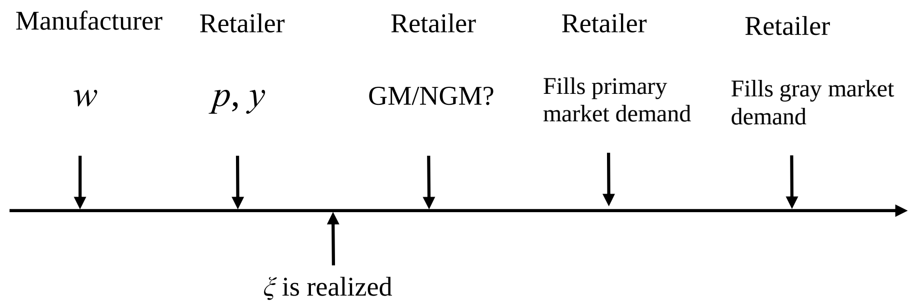

Figure 4.8: Sequence of Events under Stochastic Demand

We summarize the timeline in the decentralized supply chain as follows (depicted in Figure 4.8):

1. The manufacturer offers the wholesale price w;
2. The retailer orders y units from the manufacturer and sets retail price p in the primary market;
3. The retailer observes ξ;
4. The retailer decides whether to transship to the gray market;
5. The retailer fills demand in the primary market;
6. The retailer fills demand in the gray market (if applicable).

4.7.1 Integrated Supply Chain

For the integrated supply chain, step 1 does not exist; and in step 2 the supply chain determines p and y. Consider, first, the case where the integrated supply chain ignores the gray market, i.e., it never sells in gray market. Under NGM, the integrated supply chain is a price sensitive newsvendor, and it solves

$$\max_{p,y} \Pi_N = (p - c_R)E \min(D, y) - cy. \tag{4.20}$$

where $E(\cdot)$ stands for expectation.

If the integrated supply chain considers selling into the gray market, for each realized value v of ξ, it chooses between "not sell in gray market" (indicated by superscript "NS" in the following) and "sell in gray market" (indicated by superscript "S" in the following). The corresponding profits are

$$\Pi_G^{NS} = (p - c_R) \min(v(1 - \alpha p), y) - cy, \tag{4.21}$$

and

$$\begin{aligned} \Pi_G^S =&(p - c_R) \min(v(1 - l - \alpha p), y)+ \\ &\frac{1 - \min((y - D_G)^+, 1/2)}{\beta} \cdot \min((y - D_G)^+, 1/2) - cy. \end{aligned} \tag{4.22}$$

Ex ante, the supply chain chooses p and y to maximize the expected profit.

If ξ belongs to the type of distributions with increasing failure rate (IFR), the objective function under NGM (4.20) is unimodal and has a unique maximum. (The proof is similar to Petruzzi and Dada (1999).) We use the first order conditions to determine the optimal p and y. However, the problem under GM in general is not well-behaved. So we use simulation and global search to numerically find the optimal profit.

In the numerical examples, we let ξ be normally distributed with mean 1 and standard deviation $\sigma \in \mathbb{R}^+$. Recall that in the deterministic model, l_{SC} is defined such that for $l < l_{SC}$, the supply chain is strictly better off from the gray market; for $l \geq l_{SC}$, it chooses not to sell into the gray market. With stochastic demand we find a similar threshold l_{SC} and have

Observation 4.1 *As σ, the standard deviation of ξ increases, the threshold l_{SC} moves upwards, i.e., the region where the supply chain is better off from* *the gray market expands.*

See l_{SC} for $\sigma = 0$ (ξ is deterministic), $\sigma = 0.1$ and $\sigma = 0.2$ in Figure 4.9. As the demand becomes more variable (σ increases), with no gray market, the profit of the supply chain declines. The gray market, however, turns out to be a channel for inventory salvaging and therefore reduces the risk of overstocking. As demand variability increases, the gray market's risk reducing effect becomes stronger.

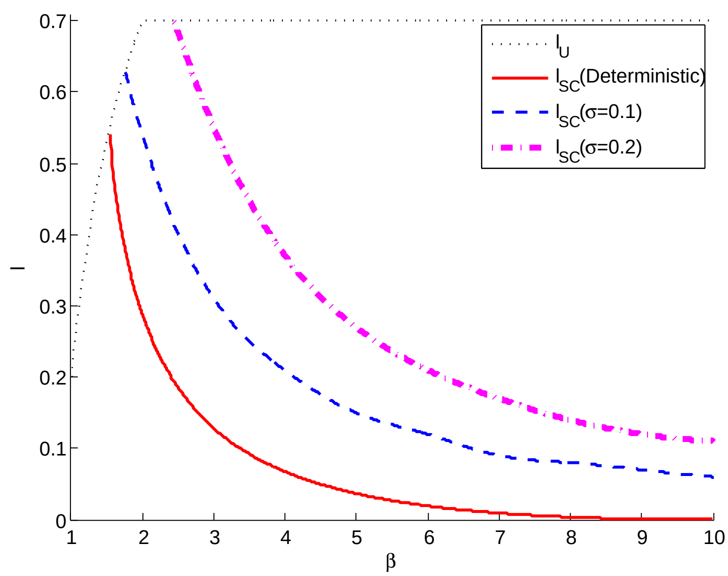

Figure 4.9: Integrated supply chain - stochastic demand ($c = 0.1, \alpha = 1, c_R = 0.3$)

Notice that for $\beta \geq 1/c$ ($1/c = 10$ in the above example), if the demand is deterministic, then the supply chain always chooses not to sell in the gray market since the margin in the gray market is negative. However, when the demand is stochastic, even for $\beta \geq 1/c$, it is still profitable to use the gray market. This is because, ex post, the production cost is sunk, and it is profitable to salvage some unsold inventory in the gray market. (The optimal level of inventory to be salvaged in the gray market is $\min((y - D_G)^+, 1/2)$ as we analyzed previously.) For instance, if $l = 0$, it is always profitable to utilize the gray market for all values of β.

4.7.2 Decentralized Supply Chain

In a decentralized supply chain, the retailer determines the price in the primary market and order quantity for both markets ex ante (before ξ is realized); and it decides whether to transship to the gray market ex post (after ξ is realized). The manufacturer, taking the retailer's decisions into account, selects a wholesale price that maximizes its profit. The analysis and simulation method is similar to the case of integrated supply chain.

Recall that under deterministic demand, the manufacturer is strictly better off from the gray market if $l < l_M$ and worse off if $l > \max(l_M, l_N)$. For $l_M \leq l \leq l_N$, the retailer does not sell in the gray market under w_N and the manufacturer obtains the profit under NGM. Under stochastic demand, we again focus on searching for thresholds l_M and l_N and exploring the additional benefit of a gray market under demand uncertainty.

However, as the demand uncertainty (σ) increases, the change in these thresholds are not monotonic. (See Figures 4.10 and 4.11.) For instance, we expect that l_M moves upward as σ goes up, which indicates that the parameter set where the manufacturer is strictly better off from the gray market expands. But we observe that l_M moves downward as σ increases from 0 to 0.1 for intermediate values of β.

Similarly, as demand uncertainty increases, l_N does not move in a single direction either. (Figure 4.11) shows that l_N first moves upward as σ increases from 0 to 0.1, and then moves downward as σ increases from 0.1 to 0.2). Therefore, the region where the manufacturer is worse off from the gray market is not monotonically shrinking as σ increases. This might be a result of the fact that both the manufacturer and retailer are eager to extract the extra benefit of the gray market.

Observation 4.2 *In the decentralized case, we observe the extra benefit of a gray market due to the risk reducing effect under demand uncertainty. However, this benefit is not monotonically increasing as the demand uncertainty goes up (i.e., σ increases).*

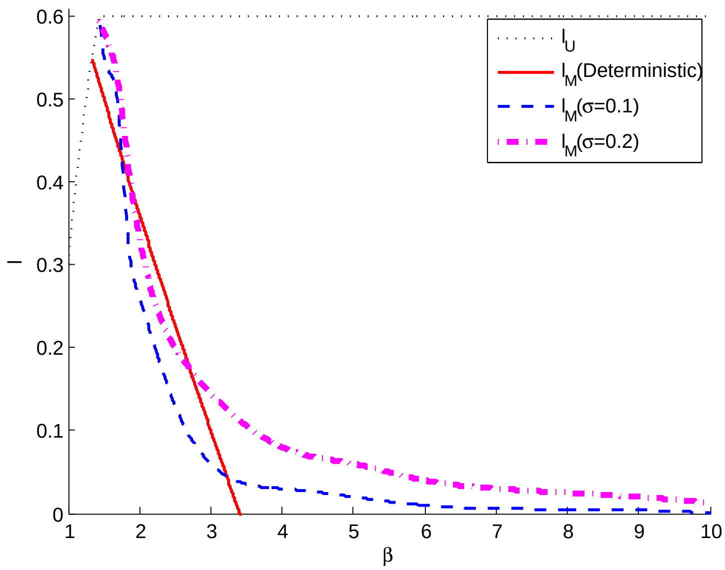

Figure 4.10: Manufacturer's threshold l_M under stochastic demand ($c = 0.1, \alpha = 1, c_R = 0.3$)

4.8 Concluding Remarks

Gray markets are an important issue in many industries. Despite the harm that a gray market can do, manufacturers may also benefit from a gray market in some cases. However, the existing analytical research on gray markets has been limited. This chapter is the first attempt to investigate the impact of gray markets on the firms in a vertically decentralized supply chain.

In the main model we consider a monopoly manufacturer who deals with a single authorized retailer. We first look at a benchmark case where the retailer is integrated with the manufacturer. The integrated firm can be strictly better off from the gray market and it will never be hurt since it controls the only source for a gray market. We identify two positive effects of the gray market: the (service) cost advantage and the volume effect, which trade off with the gray market's negative impact, i.e., the demand loss in

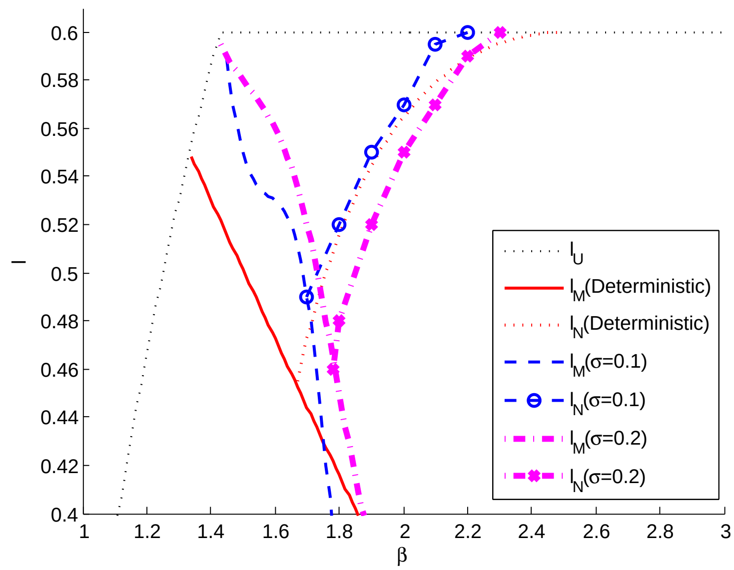

Figure 4.11: Manufacturer's thresholds l_M and l_N under stochastic demand $(c = 0.1, \alpha = 1, c_R = 0.3)$

the authorized channel.

In a decentralized supply chain, we show that in some special cases the manufacturer is never worse off from the gray markets and can be better off under certain parameter values. One special case is that the existence of a gray market does not reduce the demand in authorized channel. Of course, in reality the gray market often hurts the demand in the authorized channel. But if this impact is at a relatively low level, selling into the gray market does increase the profits for both the manufacturer and retailer. In another special case the marginal cost at the authorized retail store is zero, therefore the gray market loses the cost advantage. However, because the retailer's incentive is consistent with that of the manufacturer's, the manufacturer is still no worse off in this case.

In the general case, the manufacturer may benefit from or be hurt by the gray market. Under some conditions, the manufacturer can use the wholesale price to deter the retailer from transshipping to the gray market.

However, under some other conditions where the demand loss in the authorized channel is large while the gray market's cost advantage and volume effect are both small, the deterrence is unsuccessful. Therefore, the manufacturer has to use other approaches such as penalty terms to terminate or limit the gray market.

In one extension we examine the case of multiple retailers. Similar to the single retailer case, the manufacturer is possibly worse off from the gray market. However, the downstream competition benefits the manufacturer. Therefore, the gray market is more beneficial to the manufacturer as the number of retailers increases.

In another extension we consider the case where demand is stochastic. Since the retailer can salvage the unsold inventory in the gray market, the gray market reduces the inventory risk. The risk reducing effect is increasing in demand variability. The gray market is more profitable as the demand becomes more uncertain. However, in the decentralized supply chain, this benefit of the gray market is not monotonically increasing as the demand uncertainty goes up.

Due to some assumptions that we make for analytical tractability, this chapter has some limitations. For instance, we assume that the demand loss in the primary market, l is a constant. The underlying consumer utility is as follows. In the absence of the gray market, suppose a representative consumer's utility function is $U(y) = ay - (b/2)y^2$, where y is the consumption of the product and $a, b \geq 0$. The net utility of the consumer is $U(y) - py$. Following from the first order condition of the consumer's utility maximization problem, the demand function is $y = a/b - (1/b)p$. Let $a = b = 1/\alpha$ and $y = D_N$, we then get the demand function D_N. And the consumer's utility is $U(y) = (1/\alpha)y - (1/2\alpha)y^2$.

With a gray market, let $a = (1-l)/\alpha$ and $b = 1/\alpha$ in the utility function and the new demand function in primary market D_G follows. Now rewrite the utility function as $U(y) = (1/\alpha)y - (1/2\alpha)y^2 - (l/\alpha)y$. Compare it with the consumer's utility with no gray market: With a gray market, for each unit consumed, the consumer suffers a utility loss which equals l/α.

In reality, however, l might be dependent on factors such as the price difference between the primary and gray markets. We believe that this model will give us similar results regarding firms' incentives for the gray market. However, this alternative model is less tractable and the thresholds such as l_M and l_N in Proposition 4.6 will not be in closed form. So we use the current model to show the basic insights.

Another simplifying assumption is that there are an infinite number of gray market distributors. If the distributors are oligopolists and have certain market power, we expect that the manufacturer and authorized retailer may not benefit as much from the gray market. What is more, in the n-retailer case we assume that the retailers are symmetric. However, this model does not capture the situation where a gray market is the result of price differentiation/discrimination between retail territories (a special case of which is parallel imports between countries). Future research may start with an analysis of two (asymmetric) retailers as Ahmadi and Yang (2000) do in their model. (However, Ahmadi and Yang (2000) study a vertically integrated system.)

We also assume that the manufacturer and retailer sign a wholesale price contract. Other types of supply chain contracts may be explored (such as the two-part pricing contract mentioned in the chapter). Furthermore, the relationship between the upstream and downstream firms in practice can be more complicated. For example, in the stochastic model, we assume that the retailer determines whether to transship to the gray market after demand realization. It is possible that the retailer signals the manufacturer its action ex ante. The signal may be beneficial or harmful to the firms, which can be studied in future research. Finally, it would be interesting to collect data and conduct empirical research to verify the results of the theoretical model.

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Chapter 5

CONCLUSIONS

This dissertation focuses on a manufacturer who distributes a single product (Chapters 3 and 4) or a product line (Chapter 2) through competing retailers. We study the retailers' incentives in three scenarios in supply chain management. Chapter 2 studies a product line distribution problem. The retailers compete in price *and* inventory and optimize their own objectives. The retailers' price and inventory decisions deviate from the centrally optimal decisions due to the vertical and horizontal externalities. We characterize the retailers' incentive distortions in three cases: (1) retailers compete on inventory alone; (2) retailers compete on price alone; and (3) retailers compete on price and inventory. In each case, we determine the structure of coordinating contracts which achieve the first-best outcomes. We then use simulations to determine the optimal value of contract parameters, and study how the retailers' incentives and contracts are affected by underlying model parameters.

Some of the assumptions in this chapter can be easily relaxed. For instance, the two product variants can be extended to n variants; duopoly retailers can be extended to oligopoly retailers. However, the model also has limitations. We assume that when encountering a stock-out, the consumer searches for a substitute product. In practice, the retailer may transship the product from another retailer and then sell to the consumer (referred to as "transshipment"). The scenario of transshipment is considered in Chapter 3. However, consumer search and transshipment may both occur; and an interesting extension is to explore the incentive issues in such a case. (Zhao and Atkins (2009) compare the two cases where consumer search and transshipment occur exclusively.) Another extension is to study the channel coordination issues in *inter-brand* competition, i.e., the downstream retailers also carry the product lines of the other manufacturers. Future research can also examine the social welfare of the contracts we propose in this chapter.

In Chapter 3, we investigate the firms' incentives for transshipment in a decentralized supply chain. In the first part of this chapter, we consider a completely decentralized supply chain; that is, the downstream retailers are independent from the manufacturer, and the retailers compete with each other. The existing literature has shown the incentives for transshipment in a *vertically* decentralized supply chain, where the manufacturer deals with a centralized retailer which has multiple locations, i.e., a chain store (Dong and Rudi, 2004; Zhang, 2005). However, this chapter is the first to examine this issue in a vertically *and* horizontally decentralized supply chain.

We show that for fixed wholesale price and transshipment price, the retailers always prefer transshipment for any transshipment price. However, the manufacturer can be worse off at low transshipment prices. When the manufacturer optimally sets the wholesale price, the manufacturer can still be worse off under low transshipment prices; and retailers can also be harmed by transshipment. If the manufacturer controls the parameters of the transshipment decisions, it can ensure that it is never hurt by transshipment, while the retailers may be worse off. Similarly, if the retailers control the parameters of the transshipment decisions, they will never be worse off, but the manufacturer may be hurt.

In the second part of this chapter, we then compare the supply chain with competing retailers and one with a chain store. When the manufacturer controls the transshipment price, it prefers dealing with competing retailers, while the competing retailers may make more profits than the chain store. When the competing retailers control the transshipment price, they always make more profits than the chain store (numerical result); and the manufacturer may prefer dealing with the chain store.

A few simplifying assumptions in the model may be relaxed in future research. First, we assume that the retail price is fixed. However, in the real world, the retailers often make price decisions as well. Future research should incorporate retailers' pricing decisions and examine whether the players' incentives for transshipment will be changed. Second, an alternative to the current sequence of events is that the manufacturer sets the wholesale price before the manufacturer and/or retailers determine the parameters in transshipment decisions. Based on some preliminary work we did in the current model and existing literature, we would obtain results similar to that under the current model.

Furthermore, future research may also explore the case of a larger number of retailers and the case where the retailers are asymmetric. Finally, our single-period model fits industries with long production lead time and short life cycles such as fashion goods. An interesting extension is to utilize a multi-period model to examine the incentive issues in other cases.

Chapter 4 considers the scenario where an unauthorized distribution channel emerges, referred to as “gray market”. Despite the harm of the gray market, the existing literature shows that the gray market can be beneficial to a monopolist who has an integrated distribution channel (Ahmadi and Yang, 2000). However, this chapter is the first quantitative work which studies the impact of gray markets on the firms in a *decentralized* supply chain.

In the basic model we assume that the manufacturer has a single retailer in the authorized channel. In the benchmark case where the retailer is integrated by the manufacturer, the manufacturer will never be hurt since it controls the only source for gray market. Furthermore, the manufacturer can be strictly better off from gray market when the positive effect of the gray market, i.e., the profit increasing effect dominates its negative effect, i.e., the demand loss in the authorized channel.

In the decentralized case, where the retailer is independent from the manufacturer, the manufacturer may benefit from or be hurt by the gray market. In some cases, the manufacturer can use the wholesale price to deter the retailer from transshipping to the gray market. However, the deterrence with the wholesale price alone is not always successful, and hence the manufacturer needs to use other approaches to terminate gray market, such as a penalty on the retailer if any gray market activity is detected.

In one extension of the basic model we examine the case of multiple retailers. The results in the single retailer model still hold, i.e., the gray market may increase or decrease the manufacturer's profit in different cases. What is more, the gray market is more beneficial to the manufacturer as the number of retailers increases. This is because the gray market incurs a (quantity) competition between the retailers, which increases the retailers' order quantities. In another extension we consider the case where demand is stochastic. Since the retailer can salvage the unsold inventory in the gray market, the gray market reduces the inventory risk. The risk reducing effect is increasing in demand variability. As a result, the gray market is more profitable as the demand becomes more uncertain. However, in the decentralized supply chain, this benefit of gray market is not monotonically increasing as the demand uncertainty goes up.

Due to some assumptions that we make for analytical tractability, this chapter has limitations. For instance, we assume that there is an infinite number of distributors in the gray market. Future research may consider the case where the gray market distributors have certain market power. Future research may also extend the model to asymmetric retailers and consider other types of supply chain contracts. Finally, it is very important to collect data and conduct empirical research to verify the results of the theoretical model.

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Appendix A

Proofs for Chapter 2

Proof of Proposition 2.1: Consider product $A1$. Under the residual-claimancy contract, in equations (2.3) the vertical externalities become zero. The horizontal externality $p_{A2}\partial ES_{A2}/\partial y_{A1} = -p_{A2}\gamma_{A1A2}Pr(\xi_{A1} > y_{A1}, D_{A2} < y_{A2}) \leq 0$. Evaluating equation (2.3) at the centrally optimal inventories, $\mathbf{y}^*$, yields $\partial\pi_1/\partial y_{A1}|_{\mathbf{y}^*} \geq 0$. This implies that when we fix the other decision variables y_{A2}, y_{B2} and y_{B1} at the centrally optimal levels, retailer 1 will not choose the centrally optimal inventory of product $A1$. If we assume the second order conditions hold (i.e. π_1 is quasi-concave in y_{A1}), it follows that retailer 1 will increase the inventory of $A1$ from the centrally optimal inventory level. $\square$

Proof of Proposition 2.2: 1) When the wholesale prices are equal to the production cost, the retailer has incentive to overstock in the decentralized system. Therefore, the manufacturer should use a quantity rationing contract, i.e., to impose upper bounds on the retailers' order quantities. And the upper bounds should be equal to the centrally optimal inventory levels, $\mathbf{y}^*$.

2) Consider product $A1$. The vertical externality $w_A - c \geq 0$. Evaluating equation (2.3) at $\mathbf{y}^*$, the first term on the right hand side $\partial\Pi/\partial y_{A1}|_{\mathbf{y}^*} = 0$. In order for the decentralized retailer to choose the centrally optimal inventories $\mathbf{y}^*$, we must have $\partial\pi_1/\partial y_{A1}|_{\mathbf{y}^*} = 0$. Therefore, $w_A = c - p_{A2}\partial ES_{A2}/\partial y_{A1}|_{\mathbf{y}^*}$.

3) Consider product $A1$. Under the buyback contract, evaluate equation (2.8) at $\mathbf{y}^*$. For fixed w_A, we can find the buyback price b_A that sets $\partial\pi_1/\partial y_{A1}|_{\mathbf{y}^*} = 0$. This yields the buyback price in (2.10). $\square$

Proof of Proposition 2.5: With the buyback prices b_A and b_B, retailer 1's profit is

$$E\pi_1^b = p_{A1}ES_{A1}+p_{B1}ES_{B1}-w(y_{A1}+y_{B1})+b_AE(y_{A1}-S_{A1})+b_BE(y_{B1}-S_{B1}). \tag{A.1}$$

Therefore, we obtain retailer 1's equations with respect to his prices and inventories as follows:

$$\frac{\partial \pi_1^b}{\partial p_{A1}} = \frac{\partial \Pi}{\partial p_{A1}} - (b_A\frac{\partial ES_{A1}}{\partial p_{A1}} + b_B\frac{\partial ES_{B1}}{\partial p_{A1}}) - (p_{A2}\frac{\partial ES_{A2}}{\partial p_{A1}} + p_{B2}\frac{\partial ES_{B2}}{\partial p_{A1}}) \tag{A.2}$$

$$\frac{\partial \pi_1^b}{\partial p_{B1}} = \frac{\partial \Pi}{\partial p_{B1}} - (b_A\frac{\partial ES_{A1}}{\partial p_{B1}} + b_B\frac{\partial ES_{B1}}{\partial p_{B1}}) - (p_{A2}\frac{\partial ES_{A2}}{\partial p_{B1}} + p_{B2}\frac{\partial ES_{B2}}{\partial p_{B1}}) \tag{A.3}$$

$$\frac{\partial \pi_1^b}{\partial y_{A1}} = \frac{\partial \Pi}{\partial y_{A1}} - (w_A - c) + b_A(1 - \frac{\partial ES_{A1}}{\partial y_{A1}}) - p\frac{\partial ES_{A2}}{\partial y_{A1}} \tag{A.4}$$

$$\frac{\partial \pi_1^b}{\partial y_{B1}} = \frac{\partial \Pi}{\partial y_{B1}} - (w_B - c) + b_B(1 - \frac{\partial ES_{B1}}{\partial y_{B1}}) - p\frac{\partial ES_{B2}}{\partial y_{B1}}. \tag{A.5}$$

In order for retailer 1 to choose the centrally optimal decisions $(\mathbf{p}^*, \mathbf{y}^*)$, we must have

$$p_{A2}\frac{\partial ES_{A2}}{\partial p_{A1}}\bigg|_{\mathbf{p}^*,\mathbf{y}^*} + p_{B2}\frac{\partial ES_{B2}}{\partial p_{A1}}\bigg|_{\mathbf{p}^*,\mathbf{y}^*} + b_A\frac{\partial ES_{A1}}{\partial p_{A1}}\bigg|_{\mathbf{p}^*,\mathbf{y}^*} + b_B\frac{\partial ES_{B1}}{\partial p_{A1}}\bigg|_{\mathbf{p}^*,\mathbf{y}^*} = 0 \tag{A.6}$$

$$p_{A2}\frac{\partial ES_{A2}}{\partial p_{B1}}\bigg|_{\mathbf{p}^*,\mathbf{y}^*} + p_{B2}\frac{\partial ES_{B2}}{\partial p_{B1}}\bigg|_{\mathbf{p}^*,\mathbf{y}^*} + b_A\frac{\partial ES_{A1}}{\partial p_{B1}}\bigg|_{\mathbf{p}^*,\mathbf{y}^*} + b_B\frac{\partial ES_{B1}}{\partial p_{B1}}\bigg|_{\mathbf{p}^*,\mathbf{y}^*} = 0 \tag{A.7}$$

$$-(w_A - c) + b_A\,(1 - \frac{\partial ES_{A1}}{\partial y_{A1}}\bigg|_{\mathbf{p}^*,\mathbf{y}^*}) - p_{A2}\frac{\partial ES_{A2}}{\partial y_{A1}}\bigg|_{\mathbf{p}^*,\mathbf{y}^*} = 0 \tag{A.8}$$

$$-(w_B - c) + b_B(1 - \frac{\partial ES_{B1}}{\partial y_{B1}}\bigg|_{\mathbf{p}^*,\mathbf{y}^*}) - p_{B2}\frac{\partial ES_{B2}}{\partial y_{B1}}\bigg|_{\mathbf{p}^*,\mathbf{y}^*} = 0. \tag{A.9}$$

Solving the simultaneous equations yields the buyback prices and wholesale prices in Proposition 2.5. □

Appendix B

Additional Materials for Chapter 3

B.1 Supply Chain Coordination under Transshipment

Despite the fact that the linear wholesale price contract is broadly used in practice (Cachon, 2003), it fails to coordinate the supply chain in our model as it does in many other cases (e.g., a simple single-manufacturer-single-retailer setting). The manufacturer should desire a coordinating contract which achieves a higher supply chain profit, if the contract can arbitrarily allocate the supply chain profit between the manufacturer and retailers (Cachon, 2003). In this section, we discuss the supply chain coordination under transshipment and the incentives for transshipment if the supply chain can be coordinated.

Let $\mathbf{y}^*$ be the vector of the integrated firm's optimal inventory level; then we have

Proposition 2.8 *In a supply chain where retailers transship, coordination can be achieved through a contract with a linear wholesale price w^*, a linear transshipment price s^*, and a fixed fee, where*

$$s^* = \left. \frac{w - c + p\partial ET_i/\partial y_i}{\partial ET_i/\partial y_i - \partial ET_j/\partial y_i} \right|_{\mathbf{y}^*} \tag{B.1}$$

and

$$w^* \leq c - p\frac{\partial ET_j}{\partial y_i}|_{\mathbf{y}^*}. \tag{B.2}$$

Proof of Proposition 2.8:

Denote Π as the expected profit of the integrated firm, and π_i as the expected profit of retailer i in the decentralized supply chain (retailers are symmetric), then

$$\Pi = \sum_{i=1,2}(pE\min(\xi_i, y_i) + pET_i - cy_i), \tag{B.3}$$

$$\pi_i = pE\min(\xi_i, y_i) + sET_i + (p-s)ET_j - wy_i. \tag{B.4}$$

Compare the first order conditions of the integrated firm and retailer i with respect to retailer i's decision variable y_i and have

$$\frac{\partial \pi_i}{\partial y_i} = \frac{\partial \Pi}{\partial y_i} - (w-c) - (p-s)\frac{\partial ET_i}{\partial y_i} - s\frac{\partial ET_j}{\partial y_i}. \tag{B.5}$$

In order to have retailer i make the same inventory decision as the integrated firm, we must have

$$-(w-c) - (p-s)\frac{\partial ET_i}{\partial y_i} - s\frac{\partial ET_j}{\partial y_i} = 0. \tag{B.6}$$

Rearranging the terms in (B.6) yields (B.1), the coordinating transshipment price for a given wholesale price. Furthermore, this transshipment price must be less than or equal to p, which gives us condition (B.2). $\square$

Note that the coordinating contract is not unique; there are infinite pairs of wholesale price and transshipment price that achieve coordination.

Under a fixed wholesale price, Rudi et al. (2001) and Hu et al. (2007) discuss the existence of "coordinating transshipment price". (Hu et al. (2007) show that "coordinating transshipment price" only exists for symmetric retailers.) With no fixed fee, however, the "coordinating transshipment price" and the wholesale price alone cannot achieve coordination.

Under the coordinating contract, transshipment is beneficial to the system due to its risk pooling effect. Therefore,

Corollary 2.1 *If the supply chain can be coordinated, the manufacturer and retailers will prefer transshipment.*

B.2 Proofs for Chapter 3

For some of the proofs, we need to use the following lemma:

Lemma B.1 *At equilibrium, $\partial ET_i/\partial y_i > 0$, and $\partial ET_i/\partial y_j < 0, i, j = 1, 2$.*

Proof of Lemma B.1: Since $T_i = \min((y_i - \xi_i)^+, (\xi_j - y_j)^+)$, by the assumption of continuity in demand distribution functions, we have

$$\frac{\partial ET_i}{\partial y_i} = Pr(\xi_i < y_i, \xi_i + \xi_j > y_i + y_j) \geq 0, \tag{B.7}$$

$$\frac{\partial ET_i}{\partial y_j} = -Pr(\xi_j > y_j, \xi_i + \xi_j < y_i + y_j) \leq 0 \tag{B.8}$$

At equilibrium the above inequalities are strict. □

Proof of Lemma 3.1: By Implicit Function Theorem,

$$\frac{\partial y^{DT}}{\partial s} = \frac{-\frac{\partial^2 \pi_i}{\partial y_i \partial s}\frac{\partial^2 \pi_j}{\partial y_j^2} + \frac{\partial^2 \pi_i}{\partial y_i \partial y_j}\frac{\partial^2 \pi_j}{\partial y_j \partial s}}{|H|} = \frac{\frac{\partial^2 \pi_i}{\partial y_i \partial s}\left(\frac{\partial^2 \pi_i}{\partial y_i \partial y_j} - \frac{\partial^2 \pi_i}{\partial y_j^2}\right)}{|H|}, \tag{B.9}$$

where $|H|$ is the positive determinant of the Hessian matrix, and the second equality follows from the symmetry between the retailers. Now look at the numerator. First, by Lemma B.1 we have $\partial^2 \pi_i/\partial y_i \partial s = \partial ET_i/\partial y_i - \partial ET_j/\partial y_i > 0$.

Define the following marginal probabilities (following the notation of Rudi et al. (2001)):

$$b^1_{ij} = Pr(\xi_i < y_i) f_{D|\xi_i<y_i}(y_i + y_j), \tag{B.10}$$
$$b^2_{ij} = Pr(D > y_i + y_j) f_{\xi_i|D>y_i+y_j}(y_i), \tag{B.11}$$
$$g^1_{ij} = Pr(\xi_i > y_i) f_{D|\xi_i>y_i}(y_i + y_j), \tag{B.12}$$
$$g^2_{ij} = Pr(D < y_i + y_j) f_{\xi_i|D<y_i+y_j}(y_i), \tag{B.13}$$
$$\text{and} \qquad a_i = f(y_i), \tag{B.14}$$

where $D = \xi_1 + \xi_2$; then

$$\frac{\partial^2 \pi_i}{\partial y_i \partial y_j} = -[s b^1_{ij} + (p-s) g^1_{ij}], \tag{B.15}$$
$$\frac{\partial^2 \pi_i}{\partial y_i^2} = -[s(b^1_{ij} - b^2_{ij}) + (p-s)(g^1_{ij} - g^2_{ij}) + p a_i]. \tag{B.16}$$

So $\partial^2\pi_i/\partial y_i \partial y_j - \partial^2\pi_i/\partial y_i^2 = pa_i - sb^2_{ij} - (p-s)g^2_{ij}$. Since $a_i > b^2_{ij}$ and $a_i > g^2_{ij}$, we have $\partial^2\pi_i/\partial y_i \partial y_j - \partial^2\pi_i/\partial y_i^2 > 0$. It follows that $\partial y^{DT}/\partial s > 0$ at equilibrium. □

Proof of Lemma 3.2: For any given wholesale price w, a retailer's optimal inventory under no transshipment is given by $F(y^{NT}(w)) = (p-w)/p$. Under transshipment the retailer's equilibrium inventory when $s = p$ is given by $F(y^{DT}(w, s = p)) = (p-w)/p + \partial ET_i/\partial y_i$ from (3.2).

From Lemma B.1 and the monotonicity of function $F(\cdot)$, it follows that for any w, $y^{DT}(w, s = p) > y^{NT}(w)$. It certainly holds for the manufacturer's optimal wholesale price under no transshipment, denoted by w^{NT}. Under transshipment, if the manufacturer sets $w = w^{NT}$, the manufacturer's profit will be $2(w^{NT} - c)y^{DT}(w = w^{NT}, s = p)$, which is greater than $2(w^{NT} - c)y^{NT}(w = w^{NT})$, i.e., the manufacturer's *optimal* profit under *no* transshipment. When the manufacturer sets the optimal wholesale price under transshipment, it will make even more profits.

The argument is similar for $s = 0$. □

Proof of Lemma 3.3: In stage 3, for given transshipment price s and wholesale price w, retailer i's equilibrium inventory is given by (3.2). Rearrange (3.2) and have

$$w = p[1 - F(y_i)] + s\frac{\partial ET_i}{\partial y_i} + (p - s)\frac{\partial ET_j}{\partial y_i}. \tag{B.17}$$

Substitute (B.17) into the manufacturer's expected profit function (3.3) and have:

$$\pi_M^{DT} = 2\left(p[1 - F(y_i)] + s\frac{\partial ET_i}{\partial y_i} + (p - s)\frac{\partial ET_j}{\partial y_i} - c\right) y^{DT}. \tag{B.18}$$

(This transformation facilitates the computation of the following derivative.)

Denote the manufacturer's optimal expected profit by π_M^{DT*} and take its first derivative with respect to s:

$$\frac{d\pi_M^{DT*}}{ds} = \frac{\partial \pi_M^{DT*}}{\partial s} = 2\left(\frac{\partial ET_i}{\partial y_i} - \frac{\partial ET_j}{\partial y_i}\right) y^{DT}, \tag{B.19}$$

where the first equality follows from the Envelope Theorem. From Lemma B.1 it follows that $d\pi_M^{DT*}/ds > 0$. $\square$

Proof of Lemma 3.4: We first derive the optimal inventory of the chain store for a given wholesale price. Denote by Π^{CT} the expected profit of the chain store (both locations) under transshipment, and recall that $D = \xi_1 + \xi_2$; then

$$\Pi^{CT} = \sum_{i=1,2} [pE\min(\xi_i, y_i) + pET_i - wy_i] = pE\min(D, y_i + y_j) - w(y_i + y_j). \tag{B.20}$$

Denote by $F_D(\cdot)$ the cumulative distribution function of D. The first order condition of (B.20) with respect to y_i, is

$$pPr(D > y_i + y_j) - w = 0. \tag{B.21}$$

Let y^{CT} be the optimal inventory of a location of the chain store (the locations are symmetric), which, from (B.21) is given by

$$F_D(2y^{CT}) = \frac{p-w}{p}. \tag{B.22}$$

The equilibrium inventory of a decentralized retailer can be derived similarly and is given by

$$F_D(2y^{DT}) = \frac{p-w}{p} - \frac{\partial ET_i}{\partial y_i}, \text{ at } s=0; \tag{B.23}$$

$$F_D(2y^{DT}) = \frac{p-w}{p} - \frac{\partial ET_j}{\partial y_i}, \text{ at } s=p. \tag{B.24}$$

Now compare (B.22) with (B.23) and (B.24). From Lemma B.1 and the monotonicity of $F_D(\cdot)$, we have $y^{DT} < y^{CT}$ at $s=0$, and $y^{DT} > y^{CT}$ at $s=p$. $\square$

Proof of Lemma 3.5: (a) Take the first derivative of retailer i's profit at equilibrium with respect to s: (For the purpose of demonstration, we suppress the arguments in the profit functions.)

$$\frac{d\pi_i^{DT}}{ds} = \frac{\partial \pi_i^{DT}}{\partial s} + \frac{\partial \pi_i^{DT}}{\partial y_i}\frac{\partial y_i}{\partial s} + \frac{\partial \pi_i^{DT}}{\partial y_j}\frac{\partial y_j}{\partial s}. \tag{B.25}$$

The first term on the right hand side $\partial \pi_i^{DT}/\partial s = -ET_i + ET_j$, which is equal to zero at equilibrium. Since $\partial \pi_i^{DT}/\partial y_i = 0$ at equilibrium, the second term also equals zero. In the last term, $\partial y_j/\partial s > 0$ by Lemma 3.1. In addition,

$$\frac{\partial \pi_i^{DT}}{\partial y_j} = s\frac{\partial ET_i}{\partial y_j} + (p-s)\frac{\partial ET_j}{\partial y_j}, \tag{B.26}$$

which is greater than zero when $s=0$, and less than zero when $s=p$ from Lemma B.1. The desired result follows.

(b) For any given wholesale price, the manufacturer's profit is the same when dealing with the decentralized retailers and the chain store at $\tilde{s}_M^{CT}$. This implies that the decentralized retailers' order quantity is the same as that of the chain store at $\tilde{s}_M^{CT}$. Therefore, at $\tilde{s}_M^{CT}$ the decentralized retailers obtain the chain store's profit. From the result in (a), $0 < \tilde{s}_M^{CT} < p$. $\square$

B.3 Examples where Retailers are Worse off under Transshipment at All Transshipment Price

In this supplement we show that decentralized retailers can be worse off under transshipment at *all* transshipment price. We present two examples where demand follows the uniform distribution on $[a, b]$ ($a > 0$) and the truncated normal distribution (negative values are truncated off). (See Figure B.1 for an illustration of a retailer's profit for Uniform[1,4].)

For the above distributions, this special case may occur when the coefficient of variation (CV) of demands (i.e., the ratio of standard deviation to mean) is moderate, and the retail price is sufficiently large. Figure B.2 shows the parameter sets where this special case occurs for the above two distributions. (The special case does not occur if the demand has the uniform distribution on $[0, b]$. This is because for Uniform$[0, b]$, CV=0.577, which is the upper bound of all uniform distributions.)

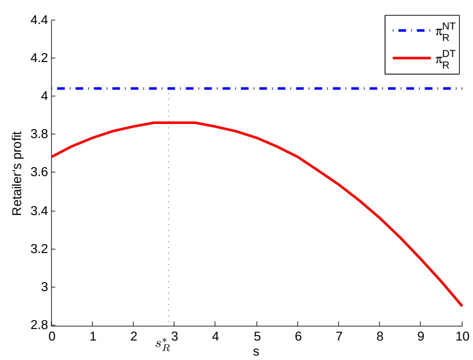

Figure B.1: Retailers are worse off under transshipment at all transshipment prices ($p = 10, c = 1, \xi_i \sim$ Uniform$[1, 4]$)

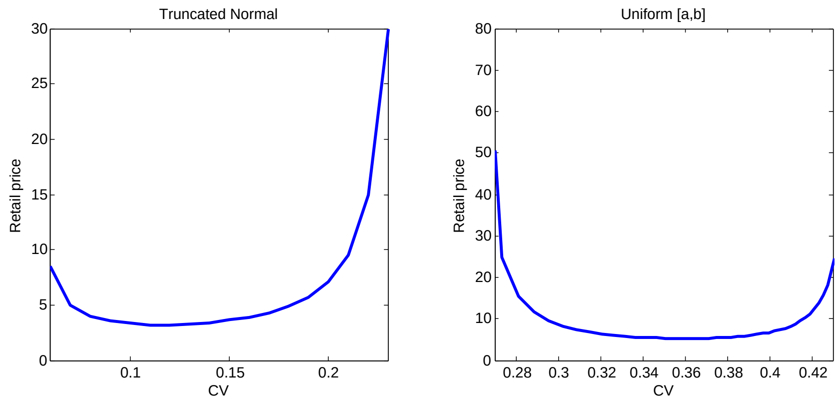

Figure B.2: Parameter values under which the special case occurs

We observe that this special case is less likely to occur if CV is sufficiently large or small. When CV is large, although under transshipment the retailers' profits are lower at large transshipment prices (e.g., $s = p$; see Figure B.3.), their profits are higher at small transshipment prices (e.g., $s = 0$; see Figure B.3). This is because for a large CV, the risk pooling effect of transshipment is strong. Recall that in such a case, at a large transshipment price, the retailers have strong incentives to over-stock under transshipment, which enables the manufacturer to set a high wholesale price and extract profits from the retailers. At a small transshipment price, however, the retailers' incentives to under-stock are also strong. This limits the manufacturer's ability to raise the wholesale price and extract retailers profits. As a result, the retailers end up being better off.

By a similar argument, when CV is *small*, the retailers benefit from transshipment at *high* transshipment prices (see Figure B.3). Thus, when CV is sufficiently large or small, the retailers are better off from transshipment at *some* transshipment prices. When CV is of moderate values, however, the retailers are worse off at all transshipment prices.

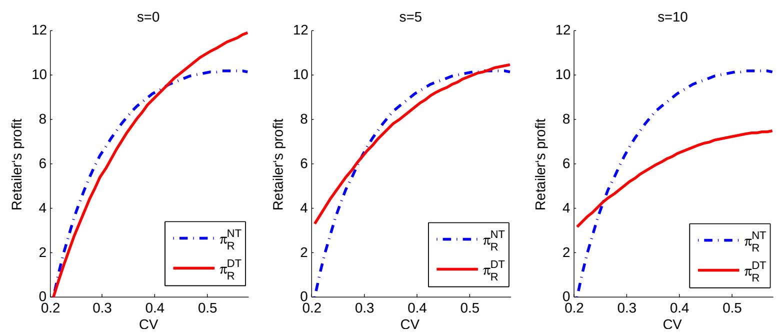

Figure B.3: Retailer's profit under fixed transshipment prices ($p = 10, c = 1, \xi_i \sim \text{Uniform}[1, 4]$)

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Appendix C

Proofs for Chapter 4

Proof of Proposition 4.1: If $l < l_{SC}$, then $\tilde{\Pi}_G - \Pi_N^* > 0$ and the supply chain prefers the outcome under GM. If $l \geq l_{SC}$, then $\tilde{\Pi}_G > \Pi_N^* > 0$, i.e., the supply chain would be worse off from selling into gray market. Therefore, it does not transship to gray market and enjoy the outcome under NGM. $\square$

Proof of Proposition 4.2: (a) "The manufacturer is no worse off": The proof is by contradiction. Denote the manufacturer's optimal wholesale price and profit under GM by w_G^* and Π_G^*. Suppose that $\Pi_G^* < \Pi_N^*$. Let the manufacturer set $w = w_N^*$ under GM. The retailer' profit function in the primary market is the same as that under NGM; therefore, the retailer's order quantity in the primary market is the same as NGM. In addition, the retailer's order quantities in the gray market $q_i \geq 0$. Therefore, under GM, the overall order quantity in both markets is no less than that under NGM. This implies that the manufacturer obtains a profit that is no less than Π_N^*. When the manufacturer sets the optimal wholesale price under GM, it will obtain even more profit (not in the strict sense), which leads to a contradiction.

(b) "The retailer is no worse off": We only need to examine the retailer's profit in the cases where the manufacturer sets $w = w_N^*$ and $w = \tilde{w}_G$. When the manufacturer sets $w = w_N^*$, the retailer gets at least the profit under NGM, π_N^*. When the manufacturer sets $w = \tilde{w}_G$, the retailer's profit $\pi_G(\tilde{w}_G)$ is given by (4.17). Compare it with $\pi_N(w_N^*)$:

$$\pi_G(\tilde{w}_G) - \pi_N(w_N^*) = [Ac_R^2 + Bc_R + C]/[16\alpha\beta(\alpha + \beta)], \tag{C.1}$$

where $A = 3\alpha^2\beta^3$, $B = -6\alpha\beta(\beta - \alpha)$, and $C = -5\alpha\beta + \alpha\beta^3c^2 - 2\beta\alpha^2c -$

$2\alpha\beta^2c + 4\alpha^2 + \alpha^2\beta^2c^2 + 3\beta^2$.

The numerator is a parabola in c_R, whose smallest value is

$$(4AC - B^2)/(4A) = 12\alpha^3\beta^2(\alpha + \beta)(1 - \beta c)^2/4A > 0. \tag{C.2}$$

Therefore, under $\tilde{w}_G$, the retailer is better off from gray market. $\square$

Proof of Proposition 4.3: For $\beta < \beta_M$, the manufacturer will set $w = \tilde{w}_G$. From (C.1) and (C.2), we know that under $\tilde{w}_G$ the retailer is also strictly better off from GM.

For $\beta \leq \beta_M$, since the manufacturer would be worse off under GM, so it sets $w = w_N^*$. Under w_N^* it is not profitable for the retailer to transship to gray market. This is because the margin on gray market $g(w_N^*) - w_N^* = (1 - \beta w_N^*)/2\beta < 0$ for $\beta \leq \beta_M$ and $l = 0$. In other words, under w_N^*, the retailer will not transship to the gray market. $\square$

Proof of Proposition 4.4: We first prove that "the manufacturer is no worse off". The proof is again by contradiction. Suppose the manufacturer's optimal profit under GM, $\Pi_G^*(w_G^*) < \Pi_N^*(w_N^*)$. Let the manufacturer set w_N^* under GM. Then the retailer's profit at w_N^* is

$$\pi_G(w_N^*) = (p_G(w_N^*) - w_N^*)(1 - \alpha p_G(w_N^*) - l) + (g(w_N^*) - w_N^*)q(w_N^*) \tag{C.3}$$

$$\leq (p_G(w_N^*) - w_N^*)(1 - \alpha p_G(w_N^*) - l + q(w_N^*)) \tag{C.4}$$

$$< (p_{iG}(w_N^*) - w_N^*)(1 - \alpha p_N(w_N^*)) \tag{C.5}$$

$$\leq (p_N(w_N^*) - w_N^*)(1 - \alpha p_N(w_N^*)) = \Pi_N(w_N^*). \tag{C.6}$$

The first inequality is due to the fact that $p^G(w_N^*)) \geq g(w_N^*))$. Note that in (C.4), $(1 - \alpha p_G(w_N^*) - l + q(w_N^*))$ is the retailer's overall order quantity under w_N^*). The second inequality comes from the assumption that $\Pi_G(\tilde{w}_G) < \Pi_N^*(w_N^*)$ and $\Pi_G(w_N^*) \leq \Pi_G(\tilde{w}_G)$. To see the last inequality, we need to compare the retail prices under GM and NGM. Under GM, the retail price in the primary market is $p_G(w_N^*) = (1 + \alpha w_N^* - l)/(2\alpha)$; and under NGM the retail price is $p_N(w_N^*) = (1 + \alpha w_N^*)/(2\alpha)$. So $p_G(w_N^*) \leq p_N(w_N^*)$

and the last inequality holds.

Therefore, under GM, if the manufacturer sets $w = w_N^*$, the retailer compares his profits under GM and NGM, and he will choose NGM and order the quantity under NGM; so the manufacturer obtains $\Pi_N^*(w_N^*)$. When the manufacturer sets the optimal wholesale price under GM, it will obtain even more profit; that is, $\Pi_G(\tilde{w}_G) \geq \Pi_N^*(w_N^*)$. This contradicts the assumption $\Pi_G(\tilde{w}_G) < \Pi_N(w_N^*)$.

We then prove that "the retailer is no worse off". When the manufacturer sets $w = w_N^*$, the retailer can always choose not to transship in the gray market and gets the profit under NGM, π_N^*. When the manufacturer sets $w = \tilde{w}_G$, note that $\tilde{w}_G < w_N^*$ for $c_R = 0$. So if the retailer chooses not to transship to the gray market, then it gets $\pi_N(\tilde{w}_G) > \pi_N^*(w_N^*)$. □

Proof of Proposition 4.6: The proof is very similar to Propositions 4.5 and 4.1. For $l \leq l_N$, $\pi_G(w_N^*) \leq \pi_N(w_N^*)$, i.e., the retailer prefers NGM under w_N^*. Hence, the manufacturer and retailer obtain their profits under NGM. For $l > l_N$, however, $\pi_G(w_N^*) > \pi_N(w_N^*)$, i.e., under w_N^* the retailer still prefers transshipping to the gray market. So the manufacturer cannot achieve $\Pi_N(w_N^*)$ and therefore is worse off. □

For the proof of Proposition 4.7, we need to use the following two lemmas:

Lemma C.1 *Let $k2 > k1 \geq 1$. For given parameter values,*

(1) if the manufacturer is better off from gray market with $k1$ retailers, then it is also better off with $k2$ retailers;

(2) if the manufacturer is worse off from gray market with $k2$ retailers, then it is also worse off with $k1$ retailers.

Proof of Lemma C.1: (1): Notice that under NGM, the manufacturer's optimal wholesale price and profit are the same for any number of retailers. Under GM, suppose the manufacturer's optimal wholesale price is w_G^* with $k1$ retailers. If the manufacturer has $k2, k2 > k1$ retailers, let the manufacturer set $w = w_G^*$. The retail price in each retailer's primary market is the same as before, which equals $(1 - l + \alpha w_G^*)/(2\alpha)$. (This is because the retailers still do not compete in their primary markets.) And the total quantity that the $k2$ retailers order for their primary markets is the same as that of $k1$ retailers.

However, the quantity each retailer orders to sell into the gray market is $(1 - \beta w_G^*)/(n + 1)$ for n retailers, which depends on n. The total quantity sold in the gray market is $n(1-\beta w_G^*)/(n+1)$. Since $k2(1-\beta w_G^*)/(k2+1) > k1(1-\beta w_G^*)/(k1+1)$, the overall order quantity that the manufacturer gets from $k2$ retailers is greater than that from $k1$ retailers. This implies that setting $w = w_G^*$, the manufacturer makes a bigger profit with $k2$ retailers than with $k1$ retailers. When the manufacturer sets the optimal wholesale price with $k2$ retailers, it will make even more profit (not in the strict sense).

(2): Suppose the man is worse off from gray market for $k2$ retailers. In other words, under GM when the manufacturer sets $w = w_N^*$, the retailers still transship to the gray market, and thus the manufacturer cannot achieve its profit under NGM. For $k1$ retailers ($k1 < k2$), under $w = w_N^*$, if the retailers also choose to transship to the gray market, then the result in (b) follows. (Notice that the optimal wholesale price under NGM, w_N^* is the same for any n.) A retailer's profit in the primary market is the same for any n. Its profit in gray market under w_N^* is $n(1 - \beta w_N^*)/(n + 1)^2$ for n, which is decreasing in n. Therefore, the retailer's total profit under the same wholesale price w_N^* is higher for $k1$ retailers than for $k2$ retailers. Since the retailers choose to transship to gray market for $k2$ retailers, this implies that the retailers also choose to transship to gray market for $k2$ retailers. Thus (2) follows. $\square$

Lemma C.2 *For all parameter values, as n approaches infinity, the manufacturer will never be worse off from gray market.*

Proof of Lemma C.2: Recall that l_N is the root of $\pi_G(w_N^*) - \pi_N(w_N^*) = 0$; for $l > l_N$, if the manufacturer sets $w = w_N^*$, then $\pi_G(w_N^*) > \pi_N(w_N^*)$, i.e., the retailer still chooses to transship to the gray market. As n approaches infinity, $l_N - > l_U$; that is, for all l, if the manufacturer sets $w = w_N^*$, the retailer will choose NGM. $\square$

Proof of Proposition 4.7: First notice that from Lemma C.2, $\hat{n}_1$ is finite. By Proposition 4.6, when $n = 1$, for given parameter values there are three cases: the manufacturer is strictly better off, no worse off, and strictly worse off from the gray market. In case 1, i.e., for $n = 1$ the manufacturer is strictly better off from the gray market, we have $\hat{n}_1 = \hat{n}_2 = 1$ from Lemma C.1, that is, the manufacturer is strictly better off from the gray market for any number of retailers.

In case 2, i.e., for $n = 1$ the manufacturer is no worse off from the gray market, there are two sub-cases: the manufacturer is either no worse off or strictly better off for $n = \infty$. From Lemmas C.1, we have the following: If it is no worse off for $n = \infty$, then $\hat{n}_1 = 1$ (the manufacturer is no worse off for all n) and $\hat{n}_2 = \infty$ (the manufacturer is *not* strictly better off for any n); If it is strictly better off for $n = \infty$, then $\hat{n}_1 = 1$ (the manufacturer is no worse off for all n) and $\hat{n}_1 \leq \hat{n}_2 \leq \infty$.

Similarly, we apply Lemmas C.1 to case 3, where the manufacturer is strictly worse off from the gray market for $n = 1$. There are again two sub-cases: If the manufacturer is no worse off for $n = \infty$, then $1 < \hat{n}_1 < \infty$ (the second inequality from Lemma C.2) and $\hat{n}_2 = \infty$. If the manufacturer is strictly better off from the gray market for $n = \infty$, then $1 < \hat{n}_1 < \infty$ (the second inequality from Lemma C.2) and $\hat{n}_1 \leq \hat{n}_2 \leq \infty$. $\square$